Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/1.1.2.4-e-x-^m-a+b-x^2-^p-c+d-x^2-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [884]. This is test number [8].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (884)	0.00 (0)
Mathematica	100.00 (884)	0.00 (0)
Fricas	96.83 (856)	3.17 (28)
Maple	96.83 (856)	3.17 (28)
Giac	92.99 (822)	7.01 (62)
Mupad	82.58 (730)	17.42 (154)
Maxima	77.15 (682)	22.85 (202)
IntegrateAlgebraic	62.22 (550)	37.78 (334)
Sympy	58.37 (516)	% 41.63 (368)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

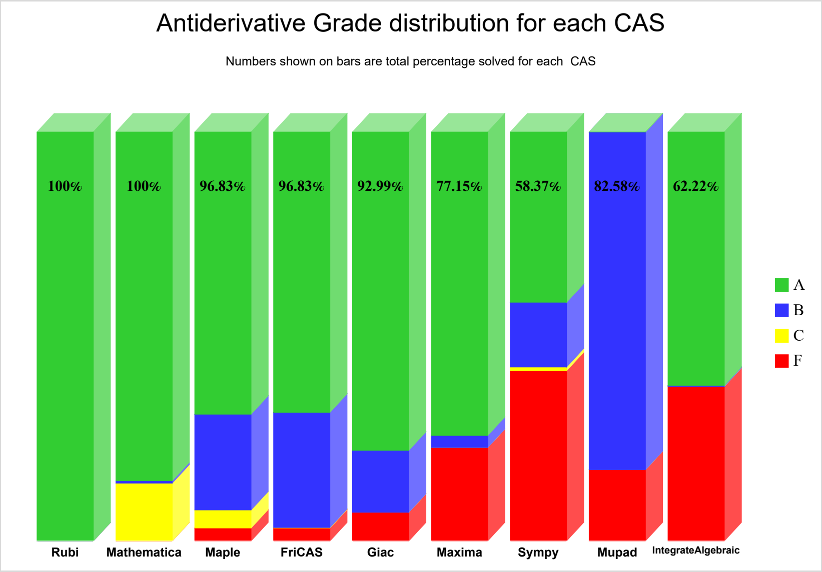
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

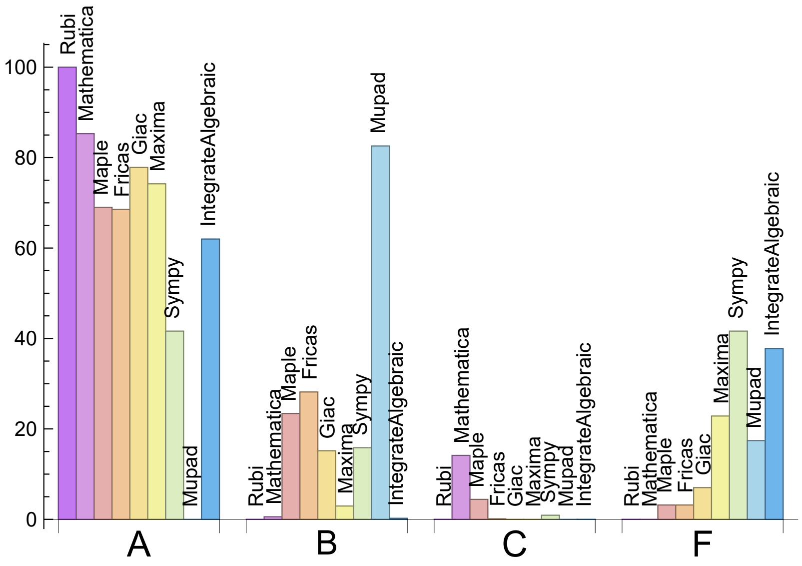
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	85.29	0.57	14.14	0.00
Giac	77.83	15.16	0.00	7.01
Maxima	74.21	2.94	0.00	22.85
Maple	69.00	23.42	4.41	3.17
Fricas	68.55	28.17	0.11	3.17
IntegrateAlgebraic	61.99	0.23	0.00	37.78
Sympy	41.63	15.84	0.90	41.63
Mupad	N/A	82.58	0.00	17.42

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	28	100.00 %	0.00 %	0.00 %
Fricas	28	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	334	100.00 %	0.00 %	0.00 %
Giac	62	72.58 %	0.00 %	27.42 %
Maxima	202	69.80 %	0.00 %	30.20 %
Sympy	368	51.90 %	48.10 %	0.00 %
Mupad	154	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

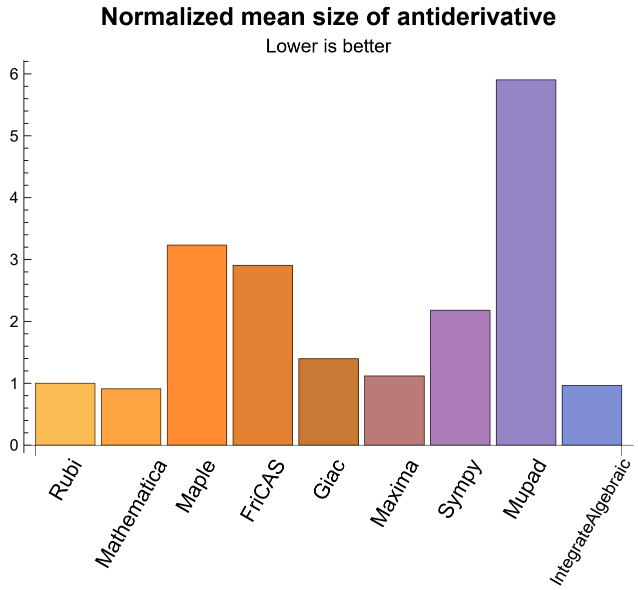
1.3 Performance

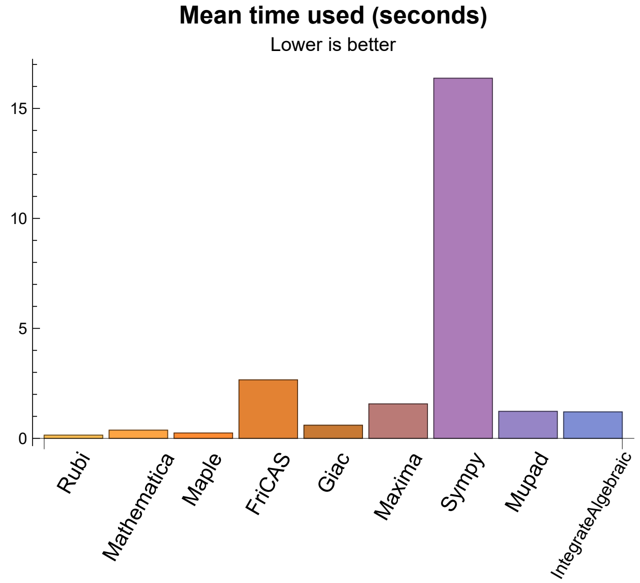
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.15	148.77	1.00	112.50	1.00
Mathematica	0.37	130.38	0.91	95.00	0.93
Maple	0.25	477.69	3.23	147.00	1.27
Maxima	1.57	168.72	1.12	119.50	1.04
Fricas	2.66	495.38	2.91	221.00	2.11
Sympy	16.37	223.26	2.18	133.00	1.40
Giac	0.60	214.05	1.40	135.00	1.14
Mupad	1.23	2633.48	5.90	119.00	1.12
IntegrateAlgebraic	1.20	155.47	0.97	135.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {434, 435, 436, 437, 438, 439, 440, 441, 442, 693, 699, 706, 708, 744, 751, 758, 760, 811, 825, 832, 833, 834, 835, 836, 837, 838, 839, 840, 847, 848, 849, 850, 851, 852, 853, 854, 855, 862}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

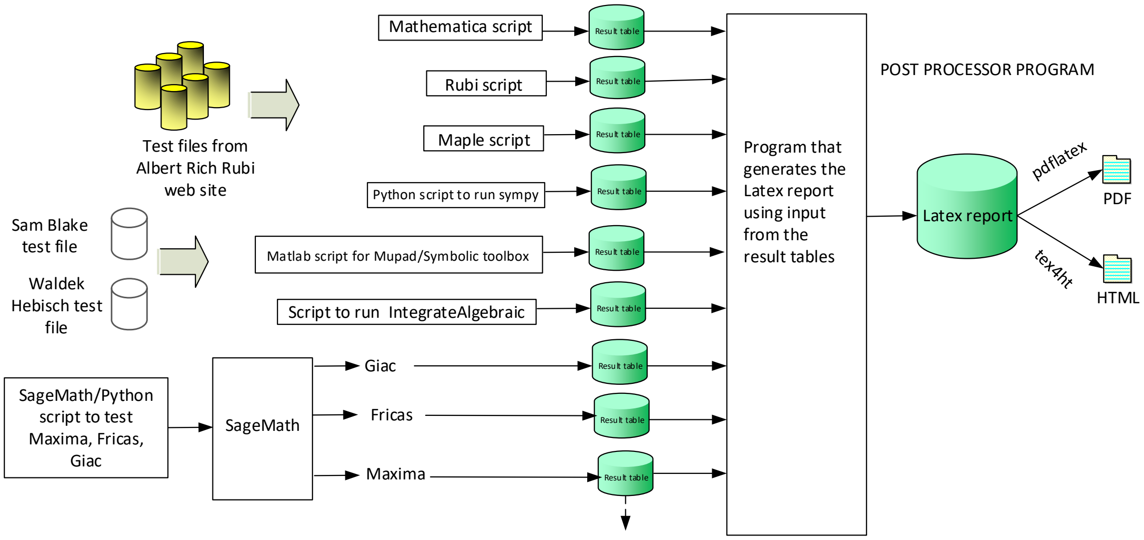
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form “{n,n,..}” which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,213,214,215,216,217,218,219,220,221,222,223,224,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,257,258,259,260,261,262,263,264,265,266,267,268,269,270,271,272,273,274,275,276,277,278,279,280,281,282,283,284,285,286,287,288,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,315,316,317,318,319,320,321,322,323,324,325,326,327,328,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,353,354,355,357,359,361,363,365,367,369,371,373,374,375,376,377,378,379,380,381,382,383,384,385,386,387,388,389,390,391,392,393,394,395,396,397,398,399,400,401,402,403,404,405,407,408,409,410,411,412,413,414,415,416,417,418,419,420,421,422,443,444,445,446,447,448,449,450,451,452,453,454,455,456,457,458,459,460,461,462,463,464,465,466,467,468,469,470,471,472,473,474,475,476,477,478,479,480,481,482,483,484,485,486,487,488,489,490,491,492,493,494,495,496,497,499,501,503,504,505,506,507,508,509,510,511,515,516,518,520,521,522,523,524,525,526,527,528,534,535,537,538,539,540,541,542,543,544,545,546,547,548,550,551,552,553,554,555,556,557,558,559,561,563,565,566,567,568,569,570,571,572,573,575,577,579,580,581,582,583,584,585,586,587,588,589,590,591,592,593,594,595,596,597,598,599,600,601,602,603,604,605,607,608,609,610,611,612,613,614,615,616,618,619,620,621,622,623,624,625,626,627,628,629,630,631,632,633,634,636,637,638,640,642,643,644,645,646,648,650,652,653,654,655,656,657,658,659,660,661,662,663,665,666,667,668,669,670,671,672,673,674,676,677,678,679,680,681,682,683,684,685,686,687,688,689,690,691,692,694,695,696,697,701,703,704,710,712,713,714,715,716,717,718,719,720,721,722,723,724,726,727,729,730,731,732,733,735,736,737,738,740,741,742,743,745,746,747,748,753,755,757,762,764,766,767,768,769,770,771,772,773,774,775,776,777,778,779,780,781,782,783,784,785,786,787,788,789,790,791,792,793,794,795,796,797,798,799,800,801,802,805,806,807,808,809,810,812,813,814,816,817,818,826,827,828,829,841,842,843,844,845,846,856,857,858,859,860,861,863,864,865,866,867,868,871,872,873,876,877,878,881,882,883,884 }

B grade: { 31,47,108,803,804 }

C grade: { 356,358,360,362,364,366,368,370,372,406,423,424,425,426,427,428,429,430,431,432,433,434,435,436,437,438,439,440,441,442,498,500,502,512,513,514,517,519,529, 530, 531, 532, 533, 536, 549, 560, 562, 564, 574, 576, 578, 606, 617, 635, 639, 641, 647, 649, 651, 664, 675, 693, 698, 699, 700, 702, 705, 706, 707, 708, 709, 711, 725, 728, 734, 739, 744, 749, 750, 751, 752, 754, 756, 758, 759, 760, 761, 763, 765, 811, 815, 819, 820, 821, 822, 823, 824, 825, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 847, 848, 849, 850, 851, 852, 853, 854, 855, 862, 869, 870, 874, 875, 879, 880 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 220, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 313, 314, 316, 317, 318, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 407, 408, 409, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 785, 786, 795, 797, 798, 799, 800, 801, 802, 866, 867, 868, 871, 872, 873, 876, 877, 878, 881, 882, 883, 884 }

B grade: { 29, 31, 47, 108, 118, 218, 219, 221, 227, 251, 252, 254, 256, 257, 280, 281, 283, 284, 310, 311, 312, 315, 319, 320, 321, 322, 323, 324, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 411, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 496, 515, 517, 519, 532, 534, 536, 586, 606, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 792, 793, 794, 796, 803, 804 }

C grade: { 805, 806, 807, 809, 811, 812, 813, 814, 815, 819, 820, 821, 822, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 841, 842, 843, 844, 845, 846, 847, 848, 849, 856, 857, 858, 859, 860, 861, 862 }

F grade: { 808, 810, 816, 817, 818, 823, 824, 835, 836, 837, 838, 839, 840, 850, 851, 852, 853, 854, 855, 863, 864, 865, 869, 870, 874, 875, 879, 880 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 254, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 307, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 785, 786, 805, 806, 807, 812, 813, 814, 815, 819, 820, 821, 822, 826, 827, 828, 829, 841, 842, 843, 844, 856, 857, 858, 859 }

B grade: { 31, 47, 108, 251, 253, 255, 256, 259, 302, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 515, 534, 536, 644 }

C grade: { }

F grade: { 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 808, 809, 810, 811, 816, 817, 818, 823, 824, 825, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 188, 189, 190, 192, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 282, 286, 288, 289, 290, 291, 292, 293, 295, 297, 299, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 599, 600, 601, 602, 603, 604, 605, 606, 608, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 688, 689, 690, 691, 697, 698, 713, 714, 720, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 745, 747, 748, 767, 768, 769, 771, 772, 773, 774, 775, 776, 778, 779, 780, 781, 782, 783, 784, 785, 787, 788, 791, 792, 795, 797, 799, 805, 806, 807, 808, 809, 810, 812, 813, 814, 815, 816, 817, 818, 826, 827, 828, 829, 830, 831, 837, 838, 841, 842, 843, 856, 857, 858, 859, 860, 861, 866, 867, 868, 871, 872, 873, 876, 877, 878, 881, 882, 883, 884 }

B grade: { 31, 47, 95, 96, 108, 162, 184, 187, 191, 193, 194, 196, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 278, 281, 283, 284, 285, 287, 294, 296, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 470, 471, 472, 524, 535, 598, 607, 609, 685, 686, 687, 692, 693, 694, 695, 696, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 715, 716, 717, 718, 719, 721, 741, 742, 743, 744, 746, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 770, 777, 786, 789, 790, 793, 794, 796, 798, 800, 801, 802, 803, 804, 811, 819, 820, 821, 822, 823, 824, 825, 832, 833, 834, 835, 836, 839, 840, 844, 845, 846, 847, 848, 850, 851, 852, 853, 854, 855, 862, 869, 870, 879, 880 }

C grade: { 849 }

F grade: { 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 863, 864, 865, 874, 875 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 53, 57, 59, 60, 61, 63, 65, 67, 69, 71, 72, 73, 75, 76, 77, 78, 79, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 194, 195, 196, 197, 198, 200, 201, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 261, 262, 263, 264, 265, 267, 268, 269, 272, 274, 276, 278, 281, 283, 285, 287, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 359, 360, 361, 368, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 399, 400, 401, 402, 403, 404, 410, 411, 425, 426, 427, 428, 437, 486, 487, 488, 490, 491, 492, 493, 494, 495, 496, 503, 505, 507, 509, 511, 512, 520, 521, 522, 524, 526, 528, 530, 537, 539, 540, 541, 542, 543, 544, 545, 546, 547, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 564, 566, 568, 570, 572, 574, 580, 581, 582, 583, 584, 589, 590, 596, 598, 600, 602, 607, 609, 611, 613, 619, 621, 622, 623, 624, 625, 626, 631, 633, 635, 643, 645, 647, 659, 661, 663, 668, 670, 672, 677, 679, 681, 687, 688, 698, 700, 707, 709, 786, 826, 827, 828, 829, 859, 871, 876 }

B grade: { 29, 31, 47, 56, 58, 62, 64, 66, 68, 70, 74, 80, 84, 86, 101, 104, 105, 108, 116, 130, 132, 135, 137, 162, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 193, 199, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 228, 229, 230, 231, 232, 233, 235, 237, 243, 245, 251, 253, 255, 266, 270, 271, 273, 275, 277, 279, 280, 282, 284, 286, 288, 290, 292, 302, 304, 311, 313, 489, 497, 498, 499, 501, 504, 506, 508, 510, 513, 514, 516, 518, 523, 525, 527, 529, 531, 533, 535, 538, 548, 549, 550, 560, 561, 563, 565, 567, 569, 571, 573, 575, 576, 577, 578, 579, 587, 588, 591, 592, 593, 594, 595, 597, 599, 601, 603, 605, 608, 610, 612, 614, 616, 618, 620, 628, 866, 881 }

C grade: { 107, 863, 864, 865, 869, 870, 874, 875 }

F grade: { 52, 54, 55, 234, 236, 238, 239, 240, 241, 242, 244, 246, 247, 248, 249, 250, 252, 254, 256, 257, 258, 259, 260, 289, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 305, 306, 307, 308, 309, 310, 312, 314, 315, 316, 317, 318, 357, 358, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 397, 398, 405, 406, 407, 408, 409, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 429, 430, 431, 432, 433, 434, 435, 436, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 500, 502, 515, 517, 519, 532, 534, 536, 585, 586, 604, 606, 615, 617, 627, 629, 630, 632, 634, 636, 637, 638, 639, 640, 641, 642, 644, 646, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 662, 664, 665, 666, 667, 669, 671, 673, 674, 675, 676, 678, 680, 682, 683, 684, 685, 686, 689, 690, 691, 692, 693, 694, 695, 696, 697, 699, 701, 702, 703, 704, 705, 706, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 862, 867, 868, 872, 873, 877, 878, 879, 880, 882, 883, 884 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 309, 310, 312, 313, 314, 316, 318, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 475, 476, 477, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 498, 500, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 513, 515, 517, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 532, 534, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 549, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 562, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 627, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 653, 654, 655, 656, 657, 659, 661, 663, 665, 668, 670, 672, 674, 677, 679, 681, 683, 685, 686, 687, 688, 689, 692, 693, 696, 697, 698, 699, 700, 701, 702, 703, 705, 707, 709, 711, 714, 716, 718, 720, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 757, 759, 761, 763, 765, 767, 768, 769, 773, 774, 775, 779, 780, 781, 785, 786, 787, 788, 789, 793, 803, 804, 805, 806, 807, 808, 809, 810, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 826, 827, 828, 829, 830, 831, 841, 842, 843, 844, 845, 846, 856, 857, 858, 859, 860, 861 }

B grade: { 31, 47, 108, 129, 162, 177, 179, 249, 251, 257, 259, 281, 283, 296, 306, 308, 311, 315, 317, 319, 320, 321, 322, 323, 324, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 463, 471, 472, 473, 474, 478, 479, 480, 481, 482, 483, 495, 497, 499, 501, 512, 514, 516, 518, 531, 533, 535, 546, 548, 550, 561, 563, 565, 577, 579, 590, 591, 592, 593, 594, 605, 616, 626, 628, 638, 640, 650, 652, 664, 666, 675, 694, 704, 706, 708, 710, 712, 713, 715, 717, 719, 721, 722, 724, 726, 728, 730, 731, 733, 735, 739, 740, 742, 744, 746, 748, 749, 751, 753, 755, 758, 760, 762, 764, 766, 771, 772, 777, 778, 783, 784, 790, 791, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802 }

C grade: { }

F grade: { 658, 660, 662, 667, 669, 671, 673, 676, 678, 680, 682, 684, 690, 691, 695, 737, 770, 776, 782, 811, 825, 832, 833, 834, 835, 836, 837, 838, 839, 840, 847, 848, 849, 850, 851, 852, 853, 854, 855, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 490, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 507, 509, 510, 511, 513, 515, 516, 517, 518, 519, 520, 522, 524, 526, 527, 528, 530, 532, 534, 535, 536, 537, 539, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 552, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 570, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 592, 593, 594, 596, 598, 600, 602, 604, 606, 607, 609, 611, 613, 615, 617, 619, 621, 623, 624, 625, 627, 628, 629, 631, 633, 635, 637, 638, 639, 640, 641, 643, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 659, 661, 663, 665, 668, 670, 672, 674, 677, 679, 681, 683, 685, 686, 687, 688, 689, 696, 698, 700, 702, 705, 707, 709, 711, 714, 716, 718, 720, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 759, 761, 763, 765, 767, 768, 769, 770, 771, 772, 785, 786, 787, 788, 789, 790, 791, 792, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 826, 827, 828, 829, 830, 831, 841, 842, 843, 844, 845, 846, 856, 857, 858, 859, 860, 861, 866, 867, 868, 871, 872, 873, 876, 877, 878, 881, 882, 883, 884 }

C grade: { }

F grade: { 487, 489, 491, 504, 506, 508, 512, 514, 521, 523, 525, 529, 531, 533, 538, 540, 551, 553, 555, 567, 569, 571, 587, 588, 589, 590, 591, 595, 597, 599, 601, 603, 605, 608, 610, 612, 614, 616, 618, 620, 622, 626, 630, 632, 634, 636, 642, 644, 646, 658, 660, 662, 664, 666, 667, 669, 671, 673, 675, 676, 678, 680, 682, 684, 690, 691, 692, 693, 694, 695, 697, 699, 701, 703, 704, 706, 708, 710, 712, 713, 715, 717, 719, 721, 722, 724, 726, 728, 730, 731, 733, 735, 737, 739, 740, 742, 744, 746, 748, 749, 751, 753, 755, 757, 758, 760, 762, 764, 766, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 793, 794, 796, 811, 825, 832, 833, 834, 835, 836, 837, 838, 839, 840, 847, 848, 849, 850, 851, 852, 853, 854, 855, 862, 863, 864, 865, 869, 870, 874, 875, 879, 880 }

2.1.9 IntegrateAlgebraic

A grade: { 120, 122, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 768, 769, 770, 771, 774, 775, 776, 777, 781, 782, 783, 785, 786, 788, 789, 790, 791, 793, 794, 795, 796, 797, 798, 800, 801, 803, 804, 805, 806, 807, 808, 809, 810, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884 }

B grade: { 535, 692 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 767, 772, 773, 778, 779, 780, 784, 787, 792, 799, 802, 811 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.020	0.013	0.022	1.342	0.384	0.103	0.301	0.180	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	27	29	29	29	28	0
N.S.	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85	0.00
time (sec)	N/A	0.032	0.022	0.001	1.350	0.371	0.065	0.384	0.038	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	26	26	26	25	0
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89	0.00
time (sec)	N/A	0.013	0.005	0.002	1.336	0.399	0.063	0.410	0.040	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	28	28	25	27	30	26	0
N.S.	1	1.00	1.00	0.97	0.97	0.86	0.93	1.03	0.90	0.00
time (sec)	N/A	0.021	0.018	0.019	1.311	0.474	0.113	0.307	0.063	0.001

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	24	24	28	20	23	24	0
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92	0.00
time (sec)	N/A	0.016	0.008	0.048	1.388	0.403	0.105	0.280	0.077	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	28	30	26	42	25	0
N.S.	1	1.00	1.00	0.90	0.97	1.03	0.90	1.45	0.86	0.00
time (sec)	N/A	0.020	0.010	0.010	1.350	0.483	0.184	0.412	0.059	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	27	25	26	29	27	28	26	0
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00	0.00
time (sec)	N/A	0.015	0.031	0.005	1.375	0.415	0.205	0.318	0.033	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	31	28	30	31	29	39	29	0
N.S.	1	1.00	1.07	0.97	1.03	1.07	1.00	1.34	1.00	0.00
time (sec)	N/A	0.020	0.016	0.005	1.385	0.479	0.359	0.320	0.071	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	33	28	29	29	32	31	29	0
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94	0.00
time (sec)	N/A	0.015	0.010	0.011	1.395	0.455	0.366	0.291	0.036	0.001

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	35	28	29	29	32	31	30	0
N.S.	1	1.00	1.06	0.85	0.88	0.88	0.97	0.94	0.91	0.00
time (sec)	N/A	0.022	0.017	0.006	1.375	0.426	0.490	0.335	0.036	0.001

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	53	56	53	51	0
N.S.	1	1.00	1.00	0.95	0.93	0.96	1.02	0.96	0.93	0.00
time (sec)	N/A	0.036	0.007	0.001	1.379	0.402	0.075	0.312	0.076	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	51	52	51	53	53	53	51	0
N.S.	1	1.00	1.21	1.24	1.21	1.26	1.26	1.26	1.21	0.00
time (sec)	N/A	0.064	0.010	0.003	1.370	0.382	0.076	0.285	0.044	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	53	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96	0.00
time (sec)	N/A	0.022	0.006	0.005	1.329	0.381	0.074	0.282	0.043	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	51	51	52	49	49	53	48	0
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12	0.00
time (sec)	N/A	0.032	0.018	0.007	1.302	0.479	0.144	0.308	0.038	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	49	48	53	48	48	48	0
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00	0.00
time (sec)	N/A	0.027	0.034	0.007	1.308	0.474	0.138	0.373	0.046	0.001

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	49	50	52	54	48	70	48	0
N.S.	1	1.00	0.96	0.98	1.02	1.06	0.94	1.37	0.94	0.00
time (sec)	N/A	0.046	0.033	0.013	1.361	0.445	0.236	0.330	0.042	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	50	46	50	52	51	50	50	0
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04	0.00
time (sec)	N/A	0.030	0.018	0.013	1.375	0.427	0.253	0.353	0.066	0.001

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	50	51	54	55	51	72	51	0
N.S.	1	1.00	0.98	1.00	1.06	1.08	1.00	1.41	1.00	0.00
time (sec)	N/A	0.038	0.022	0.022	1.389	0.413	0.522	0.342	0.081	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	45	51	53	54	53	50	0
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04	0.00
time (sec)	N/A	0.028	0.018	0.015	1.353	0.420	0.603	0.410	0.044	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	54	52	55	55	56	66	51	0
N.S.	1	1.00	1.06	1.02	1.08	1.08	1.10	1.29	1.00	0.00
time (sec)	N/A	0.037	0.027	0.008	1.010	0.408	0.997	0.321	0.085	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	56	48	53	53	58	55	52	0
N.S.	1	1.00	1.06	0.91	1.00	1.00	1.09	1.04	0.98	0.00
time (sec)	N/A	0.027	0.029	0.005	0.981	0.430	1.061	0.373	0.034	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	55	48	53	53	58	55	53	0
N.S.	1	1.00	1.15	1.00	1.10	1.10	1.21	1.15	1.10	0.00
time (sec)	N/A	0.031	0.032	0.006	1.046	0.418	1.480	0.375	0.055	0.001

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	136	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91	0.00
time (sec)	N/A	0.153	0.020	0.005	0.997	0.363	0.095	0.365	0.201	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	138	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.18	1.07	0.91	0.00
time (sec)	N/A	0.092	0.034	0.000	0.977	0.366	0.092	0.331	0.041	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	117	124	119	125	136	125	107	0
N.S.	1	1.00	0.96	1.02	0.98	1.02	1.11	1.02	0.88	0.00
time (sec)	N/A	0.280	0.015	0.000	0.997	0.389	0.093	0.337	0.039	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	136	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91	0.00
time (sec)	N/A	0.070	0.031	0.000	1.062	0.383	0.093	0.369	0.040	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	107	124	119	124	133	124	107	0
N.S.	1	1.00	1.13	1.31	1.25	1.31	1.40	1.31	1.13	0.00
time (sec)	N/A	0.213	0.041	0.001	0.978	0.354	0.092	0.306	0.039	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	125	136	125	107	0
N.S.	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91	0.00
time (sec)	N/A	0.070	0.014	0.002	1.105	0.378	0.094	0.311	0.039	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	114	124	118	123	131	123	106	0
N.S.	1	1.00	1.70	1.85	1.76	1.84	1.96	1.84	1.58	0.00
time (sec)	N/A	0.143	0.033	0.001	1.011	0.379	0.093	0.437	0.040	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	124	119	124	134	124	106	0
N.S.	1	1.00	1.00	1.06	1.02	1.06	1.15	1.06	0.91	0.00
time (sec)	N/A	0.066	0.014	0.003	0.978	0.371	0.094	0.361	0.040	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	107	124	119	124	133	124	107	0
N.S.	1	1.00	2.55	2.95	2.83	2.95	3.17	2.95	2.55	0.00
time (sec)	N/A	0.066	0.039	0.002	1.015	0.375	0.092	0.345	0.039	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	109	121	115	121	129	121	103	0
N.S.	1	1.00	1.00	1.11	1.06	1.11	1.18	1.11	0.94	0.00
time (sec)	N/A	0.060	0.013	0.001	1.010	0.367	0.095	0.330	0.040	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	113	124	120	117	134	126	105	0
N.S.	1	1.00	1.28	1.41	1.36	1.33	1.52	1.43	1.19	0.00
time (sec)	N/A	0.065	0.044	0.004	1.034	0.411	0.241	0.323	0.043	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	121	116	121	126	120	104	0
N.S.	1	1.00	1.00	1.12	1.07	1.12	1.17	1.11	0.96	0.00
time (sec)	N/A	0.062	0.027	0.004	1.129	0.427	0.239	0.307	0.042	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	115	123	120	123	131	145	105	0
N.S.	1	1.00	1.02	1.09	1.06	1.09	1.16	1.28	0.93	0.00
time (sec)	N/A	0.113	0.051	0.008	1.097	0.443	0.345	0.384	0.076	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	110	118	118	121	128	122	106	0
N.S.	1	1.00	1.02	1.09	1.09	1.12	1.19	1.13	0.98	0.00
time (sec)	N/A	0.061	0.044	0.005	1.030	0.403	0.356	0.372	0.043	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	124	122	123	128	149	113	0
N.S.	1	1.00	1.00	1.11	1.09	1.10	1.14	1.33	1.01	0.00
time (sec)	N/A	0.104	0.035	0.009	1.037	0.417	0.697	0.346	0.048	0.001

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	111	113	120	121	129	123	111	0
N.S.	1	1.00	1.00	1.02	1.08	1.09	1.16	1.11	1.00	0.00
time (sec)	N/A	0.062	0.034	0.006	0.948	0.421	0.771	0.366	0.072	0.001

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	116	124	123	123	128	151	118	0
N.S.	1	1.00	1.02	1.09	1.08	1.08	1.12	1.32	1.04	0.00
time (sec)	N/A	0.101	0.036	0.009	1.063	0.469	1.414	0.340	0.080	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	111	108	120	121	131	124	116	0
N.S.	1	1.00	1.00	0.97	1.08	1.09	1.18	1.12	1.05	0.00
time (sec)	N/A	0.062	0.033	0.007	1.016	0.417	1.616	0.317	0.101	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	116	124	123	123	129	150	122	0
N.S.	1	1.00	1.04	1.11	1.10	1.10	1.15	1.34	1.09	0.00
time (sec)	N/A	0.097	0.054	0.008	0.958	0.427	2.731	0.366	0.062	0.001

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	115	102	119	121	129	123	119	0
N.S.	1	1.00	1.06	0.94	1.10	1.12	1.19	1.14	1.10	0.00
time (sec)	N/A	0.066	0.029	0.007	1.078	0.731	3.090	0.368	0.073	0.001

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	116	123	123	123	129	147	121	0
N.S.	1	1.00	1.03	1.09	1.09	1.09	1.14	1.30	1.07	0.00
time (sec)	N/A	0.089	0.053	0.010	1.023	0.431	5.004	0.430	0.109	0.001

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	122	101	119	121	131	125	119	0
N.S.	1	1.00	1.13	0.94	1.10	1.12	1.21	1.16	1.10	0.00
time (sec)	N/A	0.062	0.039	0.006	1.023	0.457	6.685	0.363	0.075	0.001

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	118	124	123	123	133	138	121	0
N.S.	1	1.00	1.30	1.36	1.35	1.35	1.46	1.52	1.33	0.00
time (sec)	N/A	0.056	0.053	0.008	1.094	0.438	8.684	0.403	0.120	0.001

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	119	104	121	121	134	127	120	0
N.S.	1	1.00	1.05	0.92	1.07	1.07	1.19	1.12	1.06	0.00
time (sec)	N/A	0.062	0.029	0.007	1.070	0.409	14.040	0.270	0.092	0.001

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	118	104	121	121	134	127	121	0
N.S.	1	1.00	2.46	2.17	2.52	2.52	2.79	2.65	2.52	0.00
time (sec)	N/A	0.030	0.028	0.007	1.059	0.428	15.801	0.405	0.062	0.001

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	121	104	121	121	134	127	121	0
N.S.	1	1.00	1.03	0.89	1.03	1.03	1.15	1.09	1.03	0.00
time (sec)	N/A	0.059	0.029	0.007	0.968	0.404	36.444	0.302	0.065	0.001

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	121	104	121	121	134	127	120	0
N.S.	1	1.00	1.59	1.37	1.59	1.59	1.76	1.67	1.58	0.00
time (sec)	N/A	0.051	0.028	0.007	1.087	0.451	32.115	0.317	0.092	0.001

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	104	121	121	134	127	122	0
N.S.	1	1.00	1.00	0.89	1.03	1.03	1.15	1.09	1.04	0.00
time (sec)	N/A	0.059	0.040	0.006	1.137	0.396	94.112	0.305	0.094	0.001

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	121	104	121	121	134	127	121	0
N.S.	1	1.00	1.03	0.89	1.03	1.03	1.15	1.09	1.03	0.00
time (sec)	N/A	0.084	0.027	0.007	1.116	0.406	65.260	0.336	0.063	0.001

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	104	121	121	0	127	122	0
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04	0.00
time (sec)	N/A	0.059	0.039	0.007	1.038	0.417	0.000	0.422	0.094	0.001

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	121	104	121	121	134	127	122	0
N.S.	1	1.00	1.03	0.89	1.03	1.03	1.15	1.09	1.04	0.00
time (sec)	N/A	0.083	0.029	0.007	1.001	0.421	127.672	0.344	0.091	0.001

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	104	121	121	0	127	122	0
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04	0.00
time (sec)	N/A	0.062	0.039	0.006	1.033	0.454	0.000	0.285	0.094	0.001

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	121	104	121	121	0	127	122	0
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.04	0.00
time (sec)	N/A	0.085	0.030	0.006	1.008	0.440	0.000	0.452	0.064	0.001

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	116	100	228	180	108	118	0
N.S.	1	1.00	1.00	1.18	1.02	2.33	1.84	1.10	1.20	0.00
time (sec)	N/A	0.061	0.080	0.010	2.415	0.460	0.414	0.333	0.049	0.001

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	71	86	74	75	70	77	76	0
N.S.	1	1.00	0.95	1.15	0.99	1.00	0.93	1.03	1.01	0.00
time (sec)	N/A	0.087	0.031	0.005	1.026	0.448	0.323	0.308	0.057	0.001

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	92	78	178	153	85	96	0
N.S.	1	1.00	1.00	1.19	1.01	2.31	1.99	1.10	1.25	0.00
time (sec)	N/A	0.050	0.051	0.004	2.463	0.477	0.375	0.325	0.089	0.001

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	47	62	50	51	46	52	52	0
N.S.	1	1.00	0.87	1.15	0.93	0.94	0.85	0.96	0.96	0.00
time (sec)	N/A	0.056	0.020	0.003	1.028	0.432	0.288	0.267	0.098	0.001

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	68	53	129	90	57	70	0
N.S.	1	1.00	0.98	1.17	0.91	2.22	1.55	0.98	1.21	0.00
time (sec)	N/A	0.036	0.037	0.004	2.228	0.461	0.332	0.393	0.100	0.001

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	31	40	31	30	27	32	31	0
N.S.	1	1.00	0.89	1.14	0.89	0.86	0.77	0.91	0.89	0.00
time (sec)	N/A	0.030	0.011	0.005	1.094	0.385	0.246	0.345	0.053	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	40	45	34	99	82	34	31	0
N.S.	1	1.00	1.03	1.15	0.87	2.54	2.10	0.87	0.79	0.00
time (sec)	N/A	0.017	0.024	0.004	2.499	0.456	0.282	0.278	0.052	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	37	35	32	26	36	32	0
N.S.	1	1.00	1.00	1.09	1.03	0.94	0.76	1.06	0.94	0.00
time (sec)	N/A	0.034	0.011	0.006	1.017	0.466	0.695	0.290	0.135	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	42	48	36	105	82	36	35	0
N.S.	1	1.00	0.98	1.12	0.84	2.44	1.91	0.84	0.81	0.00
time (sec)	N/A	0.021	0.025	0.015	2.358	0.454	0.341	0.300	0.057	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	49	56	48	47	41	71	46	0
N.S.	1	1.00	0.98	1.12	0.96	0.94	0.82	1.42	0.92	0.00
time (sec)	N/A	0.048	0.021	0.010	1.039	0.456	0.712	0.325	0.132	0.001

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	60	72	56	135	129	57	53	0
N.S.	1	1.00	1.02	1.22	0.95	2.29	2.19	0.97	0.90	0.00
time (sec)	N/A	0.039	0.049	0.009	2.394	0.446	0.417	0.323	0.104	0.001

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	70	81	70	73	61	100	70	0
N.S.	1	1.00	1.01	1.17	1.01	1.06	0.88	1.45	1.01	0.00
time (sec)	N/A	0.060	0.027	0.010	1.116	0.473	0.827	0.362	0.134	0.001

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	78	96	79	184	163	81	70	0
N.S.	1	1.00	0.98	1.20	0.99	2.30	2.04	1.01	0.88	0.00
time (sec)	N/A	0.050	0.050	0.008	2.464	0.482	0.487	0.302	0.097	0.001

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	96	107	96	98	88	126	92	0
N.S.	1	1.00	1.03	1.15	1.03	1.05	0.95	1.35	0.99	0.00
time (sec)	N/A	0.082	0.032	0.010	0.966	0.449	0.896	0.276	0.136	0.001

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	101	120	103	234	187	106	89	0
N.S.	1	1.00	1.02	1.21	1.04	2.36	1.89	1.07	0.90	0.00
time (sec)	N/A	0.068	0.066	0.010	2.479	0.450	0.548	0.372	0.121	0.001

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	113	146	131	172	131	159	181	0
N.S.	1	1.00	0.90	1.16	1.04	1.37	1.04	1.26	1.44	0.00
time (sec)	N/A	0.169	0.071	0.024	1.088	0.395	0.788	0.349	0.101	0.001

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	134	155	136	350	238	139	203	0
N.S.	1	1.00	1.02	1.18	1.04	2.67	1.82	1.06	1.55	0.00
time (sec)	N/A	0.150	0.099	0.021	2.390	0.472	0.790	0.419	0.053	0.001

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	93	122	107	148	104	135	121	0
N.S.	1	1.00	0.89	1.17	1.03	1.42	1.00	1.30	1.16	0.00
time (sec)	N/A	0.126	0.059	0.015	0.989	0.419	0.723	0.320	0.094	0.001

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	111	132	112	298	211	115	141	0
N.S.	1	1.00	1.01	1.20	1.02	2.71	1.92	1.05	1.28	0.00
time (sec)	N/A	0.114	0.081	0.010	2.230	0.473	0.718	0.265	0.048	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	72	98	82	121	78	106	86	0
N.S.	1	1.00	0.88	1.20	1.00	1.48	0.95	1.29	1.05	0.00
time (sec)	N/A	0.087	0.067	0.016	1.040	0.405	0.661	0.277	0.066	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	89	105	85	240	129	88	104	0
N.S.	1	1.00	1.02	1.21	0.98	2.76	1.48	1.01	1.20	0.00
time (sec)	N/A	0.070	0.077	0.010	2.421	0.427	0.641	0.301	0.069	0.001

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	50	74	60	81	56	91	62	0
N.S.	1	1.00	0.83	1.23	1.00	1.35	0.93	1.52	1.03	0.00
time (sec)	N/A	0.058	0.033	0.010	1.031	0.412	0.574	0.339	0.070	0.001

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	68	82	61	208	114	59	59	0
N.S.	1	1.00	1.01	1.22	0.91	3.10	1.70	0.88	0.88	0.00
time (sec)	N/A	0.050	0.066	0.011	2.358	0.460	0.533	0.342	0.121	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	47	40	44	36	65	37	0
N.S.	1	1.00	1.00	1.15	0.98	1.07	0.88	1.59	0.90	0.00
time (sec)	N/A	0.036	0.012	0.012	1.092	0.409	0.366	0.246	0.085	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	68	57	182	112	57	51	0
N.S.	1	1.00	1.00	1.08	0.90	2.89	1.78	0.90	0.81	0.00
time (sec)	N/A	0.022	0.044	0.009	2.411	0.465	0.398	0.294	0.116	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	53	51	70	46	63	47	0
N.S.	1	1.00	0.90	1.04	1.00	1.37	0.90	1.24	0.92	0.00
time (sec)	N/A	0.045	0.028	0.015	1.116	0.426	0.415	0.326	0.154	0.001

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	70	85	63	210	114	62	63	0
N.S.	1	1.00	0.99	1.20	0.89	2.96	1.61	0.87	0.89	0.00
time (sec)	N/A	0.051	0.030	0.013	2.148	0.482	0.492	0.405	0.125	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	64	86	76	117	70	82	78	0
N.S.	1	1.00	0.84	1.13	1.00	1.54	0.92	1.08	1.03	0.00
time (sec)	N/A	0.075	0.045	0.019	1.063	0.415	0.884	0.295	0.139	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	90	110	93	250	184	85	83	0
N.S.	1	1.00	1.00	1.22	1.03	2.78	2.04	0.94	0.92	0.00
time (sec)	N/A	0.106	0.076	0.021	2.452	0.459	0.593	0.432	0.139	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	85	114	106	154	100	150	100	0
N.S.	1	1.00	0.88	1.18	1.09	1.59	1.03	1.55	1.03	0.00
time (sec)	N/A	0.095	0.091	0.020	1.126	0.434	1.021	0.392	0.137	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	112	136	119	308	218	112	104	0
N.S.	1	1.00	0.99	1.20	1.05	2.73	1.93	0.99	0.92	0.00
time (sec)	N/A	0.186	0.073	0.016	2.306	0.459	0.676	0.304	0.152	0.001

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	110	143	136	184	129	178	126	0
N.S.	1	1.00	0.89	1.15	1.10	1.48	1.04	1.44	1.02	0.00
time (sec)	N/A	0.130	0.092	0.020	1.102	0.465	1.098	0.386	0.161	0.001

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	136	182	165	231	170	183	225	0
N.S.	1	1.00	0.91	1.21	1.10	1.54	1.13	1.22	1.50	0.00
time (sec)	N/A	0.230	0.080	0.023	1.125	0.440	1.605	0.400	0.120	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	116	158	141	205	143	159	155	0
N.S.	1	1.00	0.91	1.23	1.10	1.60	1.12	1.24	1.21	0.00
time (sec)	N/A	0.166	0.068	0.017	1.063	0.431	1.511	0.328	0.073	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	94	134	116	179	119	132	118	0
N.S.	1	1.00	0.86	1.23	1.06	1.64	1.09	1.21	1.08	0.00
time (sec)	N/A	0.121	0.061	0.015	1.090	0.447	1.414	0.363	0.078	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	92	109	94	142	94	93	95	0
N.S.	1	1.00	1.05	1.24	1.07	1.61	1.07	1.06	1.08	0.00
time (sec)	N/A	0.090	0.036	0.013	1.081	0.430	1.250	0.362	0.129	0.001

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	64	80	72	89	70	61	70	0
N.S.	1	1.00	0.97	1.21	1.09	1.35	1.06	0.92	1.06	0.00
time (sec)	N/A	0.066	0.023	0.011	1.009	0.425	0.907	0.411	0.108	0.001

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	30	39	42	42	42	28	44	0
N.S.	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38	0.00
time (sec)	N/A	0.020	0.012	0.009	1.056	0.438	0.521	0.362	0.070	0.001

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	59	68	77	119	75	76	71	0
N.S.	1	1.00	0.87	1.00	1.13	1.75	1.10	1.12	1.04	0.00
time (sec)	N/A	0.060	0.045	0.014	1.120	0.451	0.580	0.382	0.178	0.001

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	87	118	109	197	107	138	107	0
N.S.	1	1.00	0.86	1.17	1.08	1.95	1.06	1.37	1.06	0.00
time (sec)	N/A	0.105	0.061	0.017	0.999	0.554	1.052	0.359	0.105	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	108	150	137	229	136	133	131	0
N.S.	1	1.00	0.87	1.21	1.10	1.85	1.10	1.07	1.06	0.00
time (sec)	N/A	0.130	0.075	0.018	1.061	0.432	1.217	0.352	0.156	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	135	180	170	267	165	201	155	0
N.S.	1	1.00	0.91	1.21	1.14	1.79	1.11	1.35	1.04	0.00
time (sec)	N/A	0.166	0.115	0.020	1.029	0.436	1.264	0.395	0.181	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	158	198	171	468	280	162	246	0
N.S.	1	1.00	1.00	1.25	1.08	2.96	1.77	1.03	1.56	0.00
time (sec)	N/A	0.283	0.089	0.015	2.347	0.471	1.441	0.483	0.115	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	133	174	147	416	252	138	177	0
N.S.	1	1.00	0.96	1.26	1.07	3.01	1.83	1.00	1.28	0.00
time (sec)	N/A	0.218	0.096	0.039	2.392	0.460	1.364	0.360	0.101	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	113	147	120	358	214	111	138	0
N.S.	1	1.00	0.97	1.27	1.03	3.09	1.84	0.96	1.19	0.00
time (sec)	N/A	0.152	0.082	0.012	2.342	0.448	1.252	0.382	0.111	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	91	122	94	328	194	80	92	0
N.S.	1	1.00	0.97	1.30	1.00	3.49	2.06	0.85	0.98	0.00
time (sec)	N/A	0.086	0.070	0.013	2.293	0.499	1.065	0.412	0.253	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	83	89	92	301	155	78	82	0
N.S.	1	1.00	0.93	1.00	1.03	3.38	1.74	0.88	0.92	0.00
time (sec)	N/A	0.062	0.081	0.010	2.373	0.460	0.733	0.336	0.240	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	84	90	92	300	150	78	82	0
N.S.	1	1.00	0.91	0.98	1.00	3.26	1.63	0.85	0.89	0.00
time (sec)	N/A	0.032	0.061	0.008	2.516	0.439	0.545	0.341	0.191	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	96	125	96	324	194	82	113	0
N.S.	1	1.00	0.99	1.29	0.99	3.34	2.00	0.85	1.16	0.00
time (sec)	N/A	0.100	0.058	0.013	2.428	0.463	0.673	0.298	0.191	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	116	152	128	368	226	108	114	0
N.S.	1	1.00	0.99	1.30	1.09	3.15	1.93	0.92	0.97	0.00
time (sec)	N/A	0.165	0.089	0.017	2.285	0.480	0.774	0.325	0.197	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	139	177	154	426	260	135	135	0
N.S.	1	1.00	0.98	1.25	1.08	3.00	1.83	0.95	0.95	0.00
time (sec)	N/A	0.330	0.097	0.027	2.361	0.451	0.868	0.356	0.195	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	14	12	12	26	12	12	0
N.S.	1	1.00	1.00	1.17	1.00	1.00	2.17	1.00	1.00	0.00
time (sec)	N/A	0.006	0.006	0.003	2.287	0.402	0.157	0.348	0.075	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	28	34	23	23	22	25	11	0
N.S.	1	1.00	2.55	3.09	2.09	2.09	2.00	2.27	1.00	0.00
time (sec)	N/A	0.007	0.010	0.005	0.993	0.466	0.166	0.364	0.111	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	10	16	10	10	7	11	10	0
N.S.	1	1.00	0.91	1.45	0.91	0.91	0.64	1.00	0.91	0.00
time (sec)	N/A	0.003	0.004	0.005	1.034	0.411	0.086	0.427	0.077	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	10	9	9	5	7	9	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00	0.00
time (sec)	N/A	0.003	0.004	0.007	1.021	0.427	0.087	0.349	0.082	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	20	14	15	16	0
N.S.	1	1.00	1.00	0.84	0.79	1.05	0.74	0.79	0.84	0.00
time (sec)	N/A	0.004	0.061	0.007	2.300	0.413	0.108	0.292	0.072	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	21	15	15	17	0
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.79	0.79	0.89	0.00
time (sec)	N/A	0.004	0.007	0.005	2.344	0.409	0.105	0.312	0.070	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	19	10	14	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.36	0.71	1.00	1.00	0.00
time (sec)	N/A	0.004	0.006	0.006	2.292	0.411	0.102	0.343	0.024	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	14	15	14	14	8	14	12	0
N.S.	1	1.00	1.17	1.25	1.17	1.17	0.67	1.17	1.00	0.00
time (sec)	N/A	0.004	0.008	0.006	1.001	0.413	0.184	0.322	0.044	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	14	15	14	14	8	14	12	0
N.S.	1	1.00	1.17	1.25	1.17	1.17	0.67	1.17	1.00	0.00
time (sec)	N/A	0.004	0.005	0.006	1.052	0.605	0.181	0.349	0.002	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	37	49	98	75	36	31	0
N.S.	1	1.00	1.00	0.95	1.26	2.51	1.92	0.92	0.79	0.00
time (sec)	N/A	0.018	0.021	0.004	2.422	0.454	0.291	0.293	0.140	0.001

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	25	30	39	27	25	30	0
N.S.	1	1.00	1.00	0.71	0.86	1.11	0.77	0.71	0.86	0.00
time (sec)	N/A	0.007	0.009	0.008	2.469	0.482	0.133	0.409	0.080	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	30	16	16	17	11	20	0
N.S.	1	1.00	1.00	2.14	1.14	1.14	1.21	0.79	1.43	0.00
time (sec)	N/A	0.008	0.006	0.012	1.029	0.761	0.128	0.269	0.036	0.001

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1	1	1	2	1	1	0	1	1	0
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	1.048	0.698	0.060	0.303	0.002	0.236

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	6	6	8
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75	1.00
time (sec)	N/A	0.002	0.000	0.002	1.063	0.604	0.073	0.325	0.023	0.008

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	6	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75	0.00
time (sec)	N/A	0.001	0.000	0.001	0.968	0.666	0.071	0.336	0.011	0.081

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	5	6	6	8
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75	1.00
time (sec)	N/A	0.002	0.000	0.000	0.981	0.599	0.071	0.316	0.013	0.007

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	2	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	0.00
time (sec)	N/A	0.001	0.000	0.001	1.051	0.604	0.066	0.294	0.006	0.133

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	4	5	7	4	3	5	4	0
N.S.	1	1.00	1.00	1.25	1.75	1.00	0.75	1.25	1.00	0.00
time (sec)	N/A	0.001	0.000	0.001	1.021	0.623	0.073	0.268	0.014	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	6	7	6	6	3	6	6	0
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	1.00	1.00	0.00
time (sec)	N/A	0.001	0.000	0.001	1.088	0.618	0.072	0.322	0.013	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	6	0
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	0.00
time (sec)	N/A	0.001	0.000	0.001	1.059	0.634	0.075	0.372	0.017	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	26	25	24	22	47	23	0
N.S.	1	1.00	1.00	0.90	0.86	0.83	0.76	1.62	0.79	0.00
time (sec)	N/A	0.021	0.005	0.002	1.082	0.650	0.150	0.381	0.040	0.001

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	29	28	86	58	28	25	0
N.S.	1	1.00	1.00	0.88	0.85	2.61	1.76	0.85	0.76	0.00
time (sec)	N/A	0.015	0.008	0.003	2.575	0.602	0.162	0.272	0.040	0.001

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	14	12	63	14	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	3.94	0.88	0.00
time (sec)	N/A	0.005	0.002	0.002	1.150	0.610	0.124	0.300	0.031	0.001

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	17	16	69	54	16	17	0
N.S.	1	1.00	1.00	0.68	0.64	2.76	2.16	0.64	0.68	0.00
time (sec)	N/A	0.008	0.005	0.003	2.227	0.769	0.148	0.329	0.046	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	25	21	17	26	19	0
N.S.	1	1.00	1.00	0.96	1.04	0.88	0.71	1.08	0.79	0.00
time (sec)	N/A	0.013	0.005	0.004	1.000	0.706	0.217	0.269	0.100	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	32	31	86	66	31	28	0
N.S.	1	1.00	1.00	0.89	0.86	2.39	1.83	0.86	0.78	0.00
time (sec)	N/A	0.015	0.015	0.004	2.346	0.563	0.193	0.351	0.100	0.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	37	35	36	36	32	47	34	0
N.S.	1	1.00	0.97	0.92	0.95	0.95	0.84	1.24	0.89	0.00
time (sec)	N/A	0.024	0.007	0.008	1.038	0.614	0.280	0.410	0.109	0.001

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	28	32	34	40	31	32	31	0
N.S.	1	1.00	0.80	0.91	0.97	1.14	0.89	0.91	0.89	0.00
time (sec)	N/A	0.027	0.009	0.009	1.027	0.604	0.205	0.350	0.094	0.001

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	38	38	128	80	37	35	0
N.S.	1	1.00	1.00	0.81	0.81	2.72	1.70	0.79	0.74	0.00
time (sec)	N/A	0.015	0.022	0.007	2.388	0.658	0.222	0.350	0.044	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	16	16	15	15	15	0
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.88	0.88	0.88	0.00
time (sec)	N/A	0.004	0.002	0.001	1.003	0.565	0.175	0.433	0.027	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	38	37	128	80	37	35	0
N.S.	1	1.00	1.00	0.81	0.79	2.72	1.70	0.79	0.74	0.00
time (sec)	N/A	0.013	0.023	0.004	2.401	0.578	0.228	0.384	0.044	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	34	38	40	54	36	51	37	0
N.S.	1	1.00	0.83	0.93	0.98	1.32	0.88	1.24	0.90	0.00
time (sec)	N/A	0.030	0.014	0.012	1.008	0.641	0.306	0.373	0.097	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	56	49	52	144	94	50	48	0
N.S.	1	1.00	0.93	0.82	0.87	2.40	1.57	0.83	0.80	0.00
time (sec)	N/A	0.022	0.036	0.010	2.400	0.584	0.310	0.331	0.069	0.001

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	42	50	57	80	53	56	55	0
N.S.	1	1.00	0.79	0.94	1.08	1.51	1.00	1.06	1.04	0.00
time (sec)	N/A	0.041	0.035	0.015	1.050	0.529	0.378	0.300	0.104	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	53	56	53	51	0
N.S.	1	1.00	1.00	0.95	0.93	0.96	1.02	0.96	0.93	0.00
time (sec)	N/A	0.035	0.009	0.001	1.040	0.499	0.074	0.296	0.094	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	53	53	53	51	0
N.S.	1	1.00	1.00	0.95	0.93	0.96	0.96	0.96	0.93	0.00
time (sec)	N/A	0.061	0.009	0.001	0.989	0.537	0.075	0.315	0.045	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	52	51	53	56	53	51	0
N.S.	1	1.00	1.00	0.95	0.93	0.96	1.02	0.96	0.93	0.00
time (sec)	N/A	0.030	0.008	0.002	0.956	0.535	0.073	0.434	0.093	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	51	52	51	53	53	53	51	0
N.S.	1	1.00	1.21	1.24	1.21	1.26	1.26	1.26	1.21	0.00
time (sec)	N/A	0.059	0.013	0.002	1.069	0.436	0.073	0.426	0.044	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	53	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96	0.00
time (sec)	N/A	0.025	0.008	0.000	1.053	0.529	0.073	0.242	0.042	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	51	51	52	49	49	53	48	0
N.S.	1	1.00	1.19	1.19	1.21	1.14	1.14	1.23	1.12	0.00
time (sec)	N/A	0.031	0.016	0.002	1.050	0.567	0.142	0.353	0.038	0.001

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	49	48	53	48	48	48	0
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00	0.00
time (sec)	N/A	0.026	0.016	0.005	1.069	0.600	0.137	0.334	0.050	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	49	50	52	54	48	70	48	0
N.S.	1	1.00	0.96	0.98	1.02	1.06	0.94	1.37	0.94	0.00
time (sec)	N/A	0.042	0.024	0.006	1.072	0.571	0.233	0.399	0.090	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	50	46	50	52	51	50	50	0
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04	0.00
time (sec)	N/A	0.028	0.019	0.004	1.045	0.491	0.245	0.419	0.046	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	87	90	85	94	100	94	78	0
N.S.	1	1.00	1.00	1.03	0.98	1.08	1.15	1.08	0.90	0.00
time (sec)	N/A	0.062	0.017	0.000	1.073	0.536	0.085	0.431	0.050	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	81	90	85	94	92	94	78	0
N.S.	1	1.00	0.93	1.03	0.98	1.08	1.06	1.08	0.90	0.00
time (sec)	N/A	0.106	0.025	0.001	1.154	0.474	0.087	0.410	0.028	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	87	90	85	94	100	94	78	0
N.S.	1	1.00	1.00	1.03	0.98	1.08	1.15	1.08	0.90	0.00
time (sec)	N/A	0.053	0.018	0.001	1.072	0.503	0.085	0.262	0.029	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	81	90	85	94	94	94	78	0
N.S.	1	1.00	1.14	1.27	1.20	1.32	1.32	1.32	1.10	0.00
time (sec)	N/A	0.105	0.022	0.000	1.022	0.626	0.084	0.313	0.028	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	87	82	91	97	91	75	0
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.18	1.11	0.91	0.00
time (sec)	N/A	0.039	0.016	0.001	1.063	0.744	0.083	0.426	0.029	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	90	85	82	85	92	74	0
N.S.	1	1.00	1.00	1.12	1.06	1.02	1.06	1.15	0.92	0.00
time (sec)	N/A	0.075	0.026	0.003	1.066	0.671	0.184	0.446	0.033	0.001

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	91	83	87	92	90	76	0
N.S.	1	1.00	1.00	1.12	1.02	1.07	1.14	1.11	0.94	0.00
time (sec)	N/A	0.043	0.040	0.005	1.097	0.702	0.178	0.273	0.031	0.001

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	83	93	85	88	87	114	82	0
N.S.	1	1.00	0.99	1.11	1.01	1.05	1.04	1.36	0.98	0.00
time (sec)	N/A	0.078	0.041	0.007	1.049	0.570	0.279	0.291	0.038	0.001

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	81	84	87	92	88	82	0
N.S.	1	1.00	1.00	1.01	1.05	1.09	1.15	1.10	1.02	0.00
time (sec)	N/A	0.053	0.041	0.007	1.174	0.695	0.295	0.323	0.053	0.001

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	127	128	127	135	143	135	119	0
N.S.	1	1.00	1.00	1.01	1.00	1.06	1.13	1.06	0.94	0.00
time (sec)	N/A	0.088	0.027	0.001	1.021	0.559	0.093	0.353	0.096	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	119	128	127	135	138	135	119	0
N.S.	1	1.00	1.12	1.21	1.20	1.27	1.30	1.27	1.12	0.00
time (sec)	N/A	0.225	0.033	0.000	1.024	0.444	0.092	0.383	0.037	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	127	128	127	135	143	135	119	0
N.S.	1	1.00	1.00	1.01	1.00	1.06	1.13	1.06	0.94	0.00
time (sec)	N/A	0.072	0.022	0.001	1.038	0.670	0.093	0.371	0.037	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	119	128	127	134	136	134	118	0
N.S.	1	1.00	1.68	1.80	1.79	1.89	1.92	1.89	1.66	0.00
time (sec)	N/A	0.118	0.034	0.001	1.064	0.486	0.091	0.322	0.037	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	122	125	124	131	136	131	116	0
N.S.	1	1.00	1.00	1.02	1.02	1.07	1.11	1.07	0.95	0.00
time (sec)	N/A	0.054	0.021	0.001	1.012	0.589	0.090	0.341	0.039	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	123	132	128	125	133	134	116	0
N.S.	1	1.00	1.00	1.07	1.04	1.02	1.08	1.09	0.94	0.00
time (sec)	N/A	0.104	0.032	0.005	1.103	0.560	0.232	0.417	0.043	0.001

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	120	131	124	129	131	130	115	0
N.S.	1	1.00	1.00	1.09	1.03	1.08	1.09	1.08	0.96	0.00
time (sec)	N/A	0.056	0.038	0.005	1.004	0.565	0.224	0.384	0.087	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	120	134	128	131	133	160	121	0
N.S.	1	1.00	0.98	1.09	1.04	1.07	1.08	1.30	0.98	0.00
time (sec)	N/A	0.101	0.047	0.006	1.060	0.661	0.325	0.317	0.090	0.001

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	120	124	126	129	131	129	121	0
N.S.	1	1.00	1.00	1.03	1.05	1.08	1.09	1.08	1.01	0.00
time (sec)	N/A	0.060	0.044	0.006	1.093	0.711	0.346	0.276	0.041	0.001

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	104	176	139	302	246	153	169	0
N.S.	1	1.00	1.00	1.69	1.34	2.90	2.37	1.47	1.62	0.00
time (sec)	N/A	0.074	0.096	0.007	2.415	0.631	0.528	0.310	0.057	0.001

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	82	124	100	101	83	107	106	0
N.S.	1	1.00	1.04	1.57	1.27	1.28	1.05	1.35	1.34	0.00
time (sec)	N/A	0.084	0.042	0.004	1.049	0.460	0.416	0.337	0.062	0.001

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	83	135	104	228	194	113	128	0
N.S.	1	1.00	1.00	1.63	1.25	2.75	2.34	1.36	1.54	0.00
time (sec)	N/A	0.064	0.071	0.005	2.418	0.573	0.485	0.259	0.062	0.001

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	49	85	65	66	49	67	68	0
N.S.	1	1.00	0.80	1.39	1.07	1.08	0.80	1.10	1.11	0.00
time (sec)	N/A	0.049	0.024	0.003	1.033	0.901	0.363	0.324	0.116	0.001

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	95	68	179	172	72	90	0
N.S.	1	1.00	0.94	1.51	1.08	2.84	2.73	1.14	1.43	0.00
time (sec)	N/A	0.039	0.051	0.004	2.337	0.837	0.418	0.277	0.074	0.001

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	50	69	61	59	41	62	58	0
N.S.	1	1.00	0.98	1.35	1.20	1.16	0.80	1.22	1.14	0.00
time (sec)	N/A	0.046	0.028	0.005	1.099	0.848	1.215	0.325	0.159	0.001

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	85	63	164	165	63	80	0
N.S.	1	1.00	1.00	1.55	1.15	2.98	3.00	1.15	1.45	0.00
time (sec)	N/A	0.049	0.047	0.007	2.389	0.879	0.543	0.340	0.124	0.001

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	60	81	70	74	49	91	67	0
N.S.	1	1.00	1.03	1.40	1.21	1.28	0.84	1.57	1.16	0.00
time (sec)	N/A	0.058	0.031	0.007	1.044	0.872	1.397	0.399	0.173	0.001

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	64	98	71	192	172	71	90	0
N.S.	1	1.00	0.97	1.48	1.08	2.91	2.61	1.08	1.36	0.00
time (sec)	N/A	0.055	0.056	0.008	2.331	0.578	0.645	0.325	0.131	0.001

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	72	116	96	98	66	139	93	0
N.S.	1	1.00	0.96	1.55	1.28	1.31	0.88	1.85	1.24	0.00
time (sec)	N/A	0.067	0.045	0.008	0.978	0.892	1.324	0.327	0.160	0.001

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	86	143	107	236	207	112	129	0
N.S.	1	1.00	0.99	1.64	1.23	2.71	2.38	1.29	1.48	0.00
time (sec)	N/A	0.065	0.073	0.006	2.440	1.118	0.763	0.318	0.089	0.001

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	108	160	134	136	105	184	129	0
N.S.	1	1.00	1.10	1.63	1.37	1.39	1.07	1.88	1.32	0.00
time (sec)	N/A	0.083	0.064	0.009	0.967	0.623	1.507	0.323	0.125	0.001

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	138	196	149	400	286	156	200	0
N.S.	1	1.00	0.95	1.35	1.03	2.76	1.97	1.08	1.38	0.00
time (sec)	N/A	0.135	0.095	0.013	2.379	0.796	0.991	0.374	0.137	0.001

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	87	142	107	161	99	163	112	0
N.S.	1	1.00	0.97	1.58	1.19	1.79	1.10	1.81	1.24	0.00
time (sec)	N/A	0.098	0.063	0.011	1.047	0.597	0.943	0.345	0.122	0.001

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	105	156	109	342	246	114	146	0
N.S.	1	1.00	0.89	1.32	0.92	2.90	2.08	0.97	1.24	0.00
time (sec)	N/A	0.110	0.072	0.010	2.348	0.804	0.872	0.300	0.079	0.001

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	56	97	74	101	68	110	77	0
N.S.	1	1.00	0.90	1.56	1.19	1.63	1.10	1.77	1.24	0.00
time (sec)	N/A	0.058	0.045	0.010	1.048	0.795	0.773	0.327	0.083	0.001

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	89	129	96	302	236	95	124	0
N.S.	1	1.00	1.09	1.57	1.17	3.68	2.88	1.16	1.51	0.00
time (sec)	N/A	0.107	0.060	0.010	2.385	0.915	0.728	0.356	0.158	0.001

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	70	94	86	116	80	99	80	0
N.S.	1	1.00	1.04	1.40	1.28	1.73	1.19	1.48	1.19	0.00
time (sec)	N/A	0.064	0.044	0.014	1.069	0.655	1.251	0.317	0.124	0.001

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	91	131	100	305	238	102	128	0
N.S.	1	1.00	0.86	1.24	0.94	2.88	2.25	0.96	1.21	0.00
time (sec)	N/A	0.076	0.060	0.028	2.494	0.798	0.869	0.323	0.190	0.001

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	72	114	100	159	92	109	100	0
N.S.	1	1.00	0.89	1.41	1.23	1.96	1.14	1.35	1.23	0.00
time (sec)	N/A	0.079	0.095	0.017	1.069	0.891	1.362	0.302	0.092	0.001

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	107	161	118	356	248	111	147	0
N.S.	1	1.00	0.85	1.28	0.94	2.83	1.97	0.88	1.17	0.00
time (sec)	N/A	0.135	0.065	0.016	2.490	0.789	0.993	0.387	0.192	0.001

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	148	223	159	522	240	154	159	0
N.S.	1	1.00	0.91	1.37	0.98	3.20	1.47	0.94	0.98	0.00
time (sec)	N/A	0.158	0.093	0.015	2.373	0.713	1.782	0.346	0.160	0.001

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	114	155	120	178	122	107	123	0
N.S.	1	1.00	1.15	1.57	1.21	1.80	1.23	1.08	1.24	0.00
time (sec)	N/A	0.100	0.050	0.015	1.124	0.988	2.107	0.338	0.147	0.001

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	130	196	143	475	223	133	135	0
N.S.	1	1.00	1.02	1.54	1.13	3.74	1.76	1.05	1.06	0.00
time (sec)	N/A	0.125	0.100	0.013	2.399	0.738	1.498	0.395	0.174	0.001

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	75	105	87	108	87	76	83	0
N.S.	1	1.00	1.12	1.57	1.30	1.61	1.30	1.13	1.24	0.00
time (sec)	N/A	0.068	0.026	0.011	1.022	0.605	1.274	0.439	0.071	0.001

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	121	147	138	449	223	126	130	0
N.S.	1	1.00	1.04	1.27	1.19	3.87	1.92	1.09	1.12	0.00
time (sec)	N/A	0.077	0.097	0.011	2.325	0.776	0.996	0.309	0.199	0.001

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	103	112	109	163	107	110	106	0
N.S.	1	1.00	1.20	1.30	1.27	1.90	1.24	1.28	1.23	0.00
time (sec)	N/A	0.083	0.047	0.015	1.005	1.178	1.177	0.413	0.126	0.001

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	151	133	199	146	475	224	135	135	0
N.S.	1	0.99	0.88	1.31	0.96	3.12	1.47	0.89	0.89	0.00
time (sec)	N/A	0.109	0.090	0.015	2.411	0.854	1.199	0.308	0.182	0.001

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	99	149	142	256	139	177	132	0
N.S.	1	1.00	0.93	1.41	1.34	2.42	1.31	1.67	1.25	0.00
time (sec)	N/A	0.114	0.091	0.019	1.017	0.681	1.819	0.385	0.113	0.001

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	148	227	167	536	240	151	156	0
N.S.	1	1.00	0.92	1.41	1.04	3.33	1.49	0.94	0.97	0.00
time (sec)	N/A	0.191	0.074	0.014	2.301	0.928	1.340	0.374	0.195	0.001

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	71	86	74	75	70	77	76	0
N.S.	1	1.00	0.95	1.15	0.99	1.00	0.93	1.03	1.01	0.00
time (sec)	N/A	0.085	0.030	0.003	1.020	0.782	0.316	0.320	0.116	0.001

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	77	92	77	178	153	84	96	0
N.S.	1	1.00	1.00	1.19	1.00	2.31	1.99	1.09	1.25	0.00
time (sec)	N/A	0.052	0.050	0.004	2.534	0.939	0.364	0.394	0.117	0.001

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	47	62	50	51	46	52	52	0
N.S.	1	1.00	0.87	1.15	0.93	0.94	0.85	0.96	0.96	0.00
time (sec)	N/A	0.056	0.019	0.003	1.051	0.748	0.279	0.332	0.064	0.001

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	68	54	129	90	58	70	0
N.S.	1	1.00	0.98	1.17	0.93	2.22	1.55	1.00	1.21	0.00
time (sec)	N/A	0.034	0.039	0.010	2.162	0.847	0.327	0.437	0.070	0.001

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	31	40	31	29	27	32	31	0
N.S.	1	1.00	0.89	1.14	0.89	0.83	0.77	0.91	0.89	0.00
time (sec)	N/A	0.031	0.012	0.004	0.983	0.762	0.242	0.274	0.105	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	40	45	33	98	82	33	32	0
N.S.	1	1.00	1.03	1.15	0.85	2.51	2.10	0.85	0.82	0.00
time (sec)	N/A	0.015	0.025	0.003	2.274	0.762	0.277	0.344	0.051	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	37	35	33	26	36	32	0
N.S.	1	1.00	1.00	1.09	1.03	0.97	0.76	1.06	0.94	0.00
time (sec)	N/A	0.031	0.012	0.004	1.109	0.739	0.673	0.303	0.076	0.001

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	42	48	37	105	82	37	34	0
N.S.	1	1.00	0.98	1.12	0.86	2.44	1.91	0.86	0.79	0.00
time (sec)	N/A	0.020	0.025	0.006	2.463	0.658	0.329	0.268	0.112	0.001

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	49	56	48	48	41	72	45	0
N.S.	1	1.00	0.98	1.12	0.96	0.96	0.82	1.44	0.90	0.00
time (sec)	N/A	0.045	0.022	0.007	1.045	0.785	0.692	0.332	0.145	0.001

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	60	72	56	136	129	57	53	0
N.S.	1	1.00	1.02	1.22	0.95	2.31	2.19	0.97	0.90	0.00
time (sec)	N/A	0.036	0.054	0.009	2.327	1.163	0.408	0.305	0.120	0.001

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	116	165	137	138	122	148	146	0
N.S.	1	1.00	1.13	1.60	1.33	1.34	1.18	1.44	1.42	0.00
time (sec)	N/A	0.122	0.053	0.005	1.107	0.785	0.460	0.318	0.110	0.001

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	105	176	140	304	246	153	169	0
N.S.	1	1.00	1.00	1.68	1.33	2.90	2.34	1.46	1.61	0.00
time (sec)	N/A	0.069	0.097	0.005	2.402	0.766	0.540	0.411	0.096	0.001

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	82	124	101	102	83	107	106	0
N.S.	1	1.00	1.02	1.55	1.26	1.28	1.04	1.34	1.32	0.00
time (sec)	N/A	0.086	0.039	0.003	1.034	0.614	0.420	0.352	0.113	0.001

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	135	105	230	194	113	128	0
N.S.	1	1.00	1.00	1.61	1.25	2.74	2.31	1.35	1.52	0.00
time (sec)	N/A	0.064	0.074	0.004	2.457	0.926	0.489	0.302	0.120	0.001

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	49	85	66	67	49	67	68	0
N.S.	1	1.00	0.80	1.39	1.08	1.10	0.80	1.10	1.11	0.00
time (sec)	N/A	0.049	0.023	0.004	1.121	1.050	0.368	0.377	0.065	0.001

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	59	95	69	181	172	72	90	0
N.S.	1	1.00	0.94	1.51	1.10	2.87	2.73	1.14	1.43	0.00
time (sec)	N/A	0.042	0.051	0.004	2.404	0.779	0.437	0.323	0.074	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	50	69	61	59	41	62	58	0
N.S.	1	1.00	0.98	1.35	1.20	1.16	0.80	1.22	1.14	0.00
time (sec)	N/A	0.048	0.027	0.005	1.110	0.854	1.227	0.325	0.166	0.001

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	55	85	63	164	165	63	80	0
N.S.	1	1.00	1.00	1.55	1.15	2.98	3.00	1.15	1.45	0.00
time (sec)	N/A	0.050	0.054	0.007	2.431	0.861	0.555	0.295	0.074	0.001

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	60	81	69	73	49	90	67	0
N.S.	1	1.00	1.03	1.40	1.19	1.26	0.84	1.55	1.16	0.00
time (sec)	N/A	0.058	0.028	0.007	1.128	0.824	1.391	0.341	0.183	0.001

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	66	98	70	190	172	72	90	0
N.S.	1	1.00	1.03	1.53	1.09	2.97	2.69	1.12	1.41	0.00
time (sec)	N/A	0.055	0.063	0.007	2.456	0.889	0.673	0.329	0.134	0.001

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	128	263	219	220	201	238	236	0
N.S.	1	1.00	0.93	1.91	1.59	1.59	1.46	1.72	1.71	0.00
time (sec)	N/A	0.178	0.071	0.007	1.201	0.839	0.603	0.322	0.120	0.001

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	140	276	222	468	343	241	260	0
N.S.	1	1.00	1.00	1.97	1.59	3.34	2.45	1.72	1.86	0.00
time (sec)	N/A	0.097	0.047	0.006	2.584	0.753	0.721	0.297	0.115	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	125	205	168	169	144	180	178	0
N.S.	1	1.00	1.09	1.78	1.46	1.47	1.25	1.57	1.55	0.00
time (sec)	N/A	0.125	0.057	0.005	1.039	1.040	0.548	0.393	0.054	0.001

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	118	218	172	364	274	184	199	0
N.S.	1	1.00	0.99	1.83	1.45	3.06	2.30	1.55	1.67	0.00
time (sec)	N/A	0.085	0.042	0.005	2.350	0.786	0.666	0.451	0.102	0.001

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	82	149	119	120	94	124	123	0
N.S.	1	1.00	0.94	1.71	1.37	1.38	1.08	1.43	1.41	0.00
time (sec)	N/A	0.081	0.032	0.003	1.058	0.655	0.494	0.288	0.060	0.001

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	92	161	122	292	238	129	146	0
N.S.	1	1.00	0.94	1.64	1.24	2.98	2.43	1.32	1.49	0.00
time (sec)	N/A	0.056	0.067	0.005	2.432	1.047	0.603	0.358	0.124	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	65	116	98	101	65	99	97	0
N.S.	1	1.00	0.89	1.59	1.34	1.38	0.89	1.36	1.33	0.00
time (sec)	N/A	0.077	0.032	0.008	1.156	0.514	1.774	0.356	0.149	0.001

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	76	135	101	253	221	104	118	0
N.S.	1	1.00	0.99	1.75	1.31	3.29	2.87	1.35	1.53	0.00
time (sec)	N/A	0.064	0.033	0.007	2.369	0.751	0.807	0.360	0.075	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	75	114	97	105	63	120	95	0
N.S.	1	1.00	1.03	1.56	1.33	1.44	0.86	1.64	1.30	0.00
time (sec)	N/A	0.076	0.038	0.009	1.077	0.925	2.214	0.308	0.168	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	135	98	256	221	100	122	0
N.S.	1	1.00	1.00	1.82	1.32	3.46	2.99	1.35	1.65	0.00
time (sec)	N/A	0.066	0.042	0.010	2.441	0.587	1.130	0.368	0.145	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	65	68	72	201	70	68	0
N.S.	1	1.00	0.94	0.93	0.97	1.03	2.87	1.00	0.97	0.00
time (sec)	N/A	0.067	0.032	0.009	1.002	0.693	179.213	0.367	0.323	0.001

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	74	73	72	391	921	72	343	0
N.S.	1	1.00	0.95	0.94	0.92	5.01	11.81	0.92	4.40	0.00
time (sec)	N/A	0.083	0.090	0.009	2.353	0.949	12.395	0.379	0.612	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	43	50	49	42	144	51	51	0
N.S.	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96	0.00
time (sec)	N/A	0.049	0.026	0.009	1.032	0.782	2.341	0.300	0.271	0.001

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	61	55	54	309	570	54	133	0
N.S.	1	1.00	0.87	0.79	0.77	4.41	8.14	0.77	1.90	0.00
time (sec)	N/A	0.034	0.053	0.009	2.310	0.839	2.183	0.339	0.344	0.001

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	31	42	41	31	138	51	148	0
N.S.	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	3.29	0.00
time (sec)	N/A	0.026	0.017	0.007	1.075	0.963	0.982	0.371	0.198	0.001

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	61	55	54	292	712	54	135	0
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93	0.00
time (sec)	N/A	0.027	0.043	0.009	2.315	0.885	2.825	0.389	0.343	0.001

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	54	59	61	54	0	73	58	0
N.S.	1	1.00	0.87	0.95	0.98	0.87	0.00	1.18	0.94	0.00
time (sec)	N/A	0.061	0.028	0.010	1.035	1.002	0.000	0.306	0.303	0.001

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	76	76	75	384	1093	75	338	0
N.S.	1	1.00	0.94	0.94	0.93	4.74	13.49	0.93	4.17	0.00
time (sec)	N/A	0.086	0.086	0.010	2.395	1.201	12.233	0.354	0.628	0.001

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	88	87	87	99	0	112	87	0
N.S.	1	1.00	1.01	1.00	1.00	1.14	0.00	1.29	1.00	0.00
time (sec)	N/A	0.091	0.044	0.014	1.019	1.236	0.000	0.356	0.348	0.001

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	101	98	96	560	1353	98	367	0
N.S.	1	1.00	1.01	0.98	0.96	5.60	13.53	0.98	3.67	0.00
time (sec)	N/A	0.177	0.131	0.013	2.412	1.157	40.892	0.352	0.615	0.001

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	124	117	127	0	167	118	0
N.S.	1	1.00	1.00	1.04	0.98	1.07	0.00	1.40	0.99	0.00
time (sec)	N/A	0.127	0.055	0.016	1.113	3.086	0.000	0.311	0.431	0.001

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	135	141	131	669	0	139	397	0
N.S.	1	1.00	1.01	1.05	0.98	4.99	0.00	1.04	2.96	0.00
time (sec)	N/A	0.227	0.123	0.014	2.449	1.017	0.000	0.330	0.625	0.001

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	147	184	165	155	0	239	165	0
N.S.	1	1.00	0.95	1.19	1.06	1.00	0.00	1.54	1.06	0.00
time (sec)	N/A	0.169	0.065	0.017	1.117	4.458	0.000	0.309	0.486	0.001

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	91	136	130	162	0	152	169	0
N.S.	1	1.00	0.98	1.46	1.40	1.74	0.00	1.63	1.82	0.00
time (sec)	N/A	0.087	0.049	0.013	1.064	1.070	0.000	0.279	0.454	0.001

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	108	144	132	718	0	121	3572	0
N.S.	1	1.00	1.00	1.33	1.22	6.65	0.00	1.12	33.07	0.00
time (sec)	N/A	0.085	0.138	0.013	2.488	1.024	0.000	0.345	0.944	0.001

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	95	105	117	253	91	173	0
N.S.	1	1.00	1.00	1.28	1.42	1.58	3.42	1.23	2.34	0.00
time (sec)	N/A	0.064	0.040	0.012	1.110	0.857	1.919	0.374	0.267	0.001

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	90	134	119	705	0	110	3154	0
N.S.	1	1.00	0.87	1.29	1.14	6.78	0.00	1.06	30.33	0.00
time (sec)	N/A	0.063	0.127	0.012	2.446	1.048	0.000	0.321	0.785	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	90	99	103	248	85	160	0
N.S.	1	1.00	0.94	1.29	1.41	1.47	3.54	1.21	2.29	0.00
time (sec)	N/A	0.054	0.031	0.013	1.030	0.795	1.890	0.340	0.271	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	95	144	133	711	0	122	3637	0
N.S.	1	1.00	0.87	1.32	1.22	6.52	0.00	1.12	33.37	0.00
time (sec)	N/A	0.079	0.173	0.010	2.351	1.308	0.000	0.358	0.978	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	98	139	138	219	0	185	127	0
N.S.	1	1.00	0.98	1.39	1.38	2.19	0.00	1.85	1.27	0.00
time (sec)	N/A	0.101	0.095	0.017	0.986	2.648	0.000	0.387	0.708	0.001

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	123	169	178	1005	0	164	432	0
N.S.	1	1.00	0.85	1.17	1.24	6.98	0.00	1.14	3.00	0.00
time (sec)	N/A	0.198	0.261	0.016	2.304	1.522	0.000	0.384	0.829	0.001

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	117	170	188	302	0	257	171	0
N.S.	1	1.00	0.93	1.35	1.49	2.40	0.00	2.04	1.36	0.00
time (sec)	N/A	0.144	0.238	0.020	1.133	4.994	0.000	0.349	0.928	0.001

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	142	191	236	1281	0	165	469	0
N.S.	1	1.00	0.75	1.01	1.25	6.78	0.00	0.87	2.48	0.00
time (sec)	N/A	0.272	0.387	0.017	2.374	3.284	0.000	0.342	0.938	0.001

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	99	218	236	290	418	232	370	0
N.S.	1	1.00	0.85	1.88	2.03	2.50	3.60	2.00	3.19	0.00
time (sec)	N/A	0.113	0.098	0.013	1.227	0.899	3.263	0.334	0.524	0.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	154	299	264	1573	0	204	5754	0
N.S.	1	1.00	0.98	1.90	1.68	10.02	0.00	1.30	36.65	0.00
time (sec)	N/A	0.172	0.273	0.014	2.519	1.659	0.000	0.328	1.984	0.001

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	77	177	217	256	411	174	343	0
N.S.	1	1.00	0.77	1.77	2.17	2.56	4.11	1.74	3.43	0.00
time (sec)	N/A	0.092	0.112	0.013	1.144	0.788	2.974	0.434	0.467	0.001

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	151	298	266	1587	0	206	5898	0
N.S.	1	1.00	0.97	1.92	1.72	10.24	0.00	1.33	38.05	0.00
time (sec)	N/A	0.140	0.218	0.011	2.300	1.865	0.000	0.373	1.985	0.001

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	98	176	211	254	391	174	340	0
N.S.	1	1.00	1.00	1.80	2.15	2.59	3.99	1.78	3.47	0.00
time (sec)	N/A	0.075	0.052	0.013	1.058	0.562	2.867	0.383	0.349	0.001

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	158	310	277	1585	0	217	6033	0
N.S.	1	1.00	0.99	1.94	1.73	9.91	0.00	1.36	37.71	0.00
time (sec)	N/A	0.189	0.238	0.013	2.334	2.643	0.000	0.367	2.368	0.001

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	141	286	278	520	0	315	246	0
N.S.	1	1.00	0.95	1.92	1.87	3.49	0.00	2.11	1.65	0.00
time (sec)	N/A	0.152	0.267	0.023	1.253	8.892	0.000	0.427	1.401	0.001

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	172	335	352	1991	0	236	738	0
N.S.	1	1.00	0.82	1.59	1.67	9.44	0.00	1.12	3.50	0.00
time (sec)	N/A	0.309	0.376	0.017	2.538	4.426	0.000	0.332	1.305	0.001

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	171	322	364	640	0	357	314	0
N.S.	1	1.00	0.96	1.81	2.04	3.60	0.00	2.01	1.76	0.00
time (sec)	N/A	0.213	0.419	0.021	1.281	18.491	0.000	0.338	1.504	0.001

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	196	362	440	2397	0	256	785	0
N.S.	1	1.00	0.73	1.34	1.63	8.88	0.00	0.95	2.91	0.00
time (sec)	N/A	0.434	0.431	0.020	2.578	12.310	0.000	0.314	1.422	0.001

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	17	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81	0.00
time (sec)	N/A	0.013	0.004	0.007	0.953	0.965	0.108	0.282	0.148	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	89	105	84	240	129	88	104	0
N.S.	1	1.00	1.02	1.21	0.97	2.76	1.48	1.01	1.20	0.00
time (sec)	N/A	0.071	0.073	0.011	2.244	0.946	0.629	0.392	0.084	0.001

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	50	74	59	78	56	90	63	0
N.S.	1	1.00	0.83	1.23	0.98	1.30	0.93	1.50	1.05	0.00
time (sec)	N/A	0.060	0.035	0.011	0.993	1.009	0.563	0.310	0.076	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	68	82	60	202	114	58	59	0
N.S.	1	1.00	1.01	1.22	0.90	3.01	1.70	0.87	0.88	0.00
time (sec)	N/A	0.051	0.067	0.010	2.325	0.918	0.527	0.298	0.181	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	47	40	45	36	65	37	0
N.S.	1	1.00	1.00	1.15	0.98	1.10	0.88	1.59	0.90	0.00
time (sec)	N/A	0.036	0.012	0.010	1.068	0.846	0.361	0.337	0.148	0.001

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	68	57	181	112	57	51	0
N.S.	1	1.00	1.00	1.08	0.90	2.87	1.78	0.90	0.81	0.00
time (sec)	N/A	0.022	0.042	0.009	2.394	0.803	0.395	0.399	0.181	0.001

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	46	53	51	71	46	63	47	0
N.S.	1	1.00	0.90	1.04	1.00	1.39	0.90	1.24	0.92	0.00
time (sec)	N/A	0.044	0.028	0.012	1.074	0.840	0.415	0.317	0.056	0.001

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	70	85	65	214	114	64	61	0
N.S.	1	1.00	0.99	1.20	0.92	3.01	1.61	0.90	0.86	0.00
time (sec)	N/A	0.053	0.032	0.010	2.231	0.951	0.488	0.355	0.188	0.001

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	64	86	78	122	70	84	74	0
N.S.	1	1.00	0.84	1.13	1.03	1.61	0.92	1.11	0.97	0.00
time (sec)	N/A	0.074	0.045	0.015	1.001	0.845	0.867	0.285	0.110	0.001

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	90	110	93	250	184	86	84	0
N.S.	1	1.00	1.00	1.22	1.03	2.78	2.04	0.96	0.93	0.00
time (sec)	N/A	0.106	0.068	0.013	2.363	1.016	0.590	0.349	0.199	0.001

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	138	196	149	400	286	156	200	0
N.S.	1	1.00	0.95	1.35	1.03	2.76	1.97	1.08	1.38	0.00
time (sec)	N/A	0.132	0.092	0.013	2.417	0.956	0.982	0.449	0.172	0.001

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	87	142	107	160	99	163	112	0
N.S.	1	1.00	0.99	1.61	1.22	1.82	1.12	1.85	1.27	0.00
time (sec)	N/A	0.106	0.062	0.012	1.002	1.001	0.926	0.283	0.072	0.001

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	105	156	109	342	246	114	148	0
N.S.	1	1.00	0.91	1.34	0.94	2.95	2.12	0.98	1.28	0.00
time (sec)	N/A	0.111	0.071	0.011	2.412	0.783	0.858	0.365	0.176	0.001

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	56	97	73	101	68	111	77	0
N.S.	1	1.00	0.92	1.59	1.20	1.66	1.11	1.82	1.26	0.00
time (sec)	N/A	0.058	0.045	0.013	1.042	0.549	0.757	0.341	0.257	0.001

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	88	129	95	297	236	94	124	0
N.S.	1	1.00	1.07	1.57	1.16	3.62	2.88	1.15	1.51	0.00
time (sec)	N/A	0.099	0.059	0.010	2.469	0.931	0.713	0.288	0.283	0.001

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	70	94	86	117	80	99	80	0
N.S.	1	1.00	1.04	1.40	1.28	1.75	1.19	1.48	1.19	0.00
time (sec)	N/A	0.065	0.041	0.011	1.070	0.886	1.240	0.340	0.308	0.001

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	91	131	101	308	238	103	128	0
N.S.	1	1.00	0.88	1.27	0.98	2.99	2.31	1.00	1.24	0.00
time (sec)	N/A	0.078	0.069	0.013	2.286	0.975	0.873	0.325	0.139	0.001

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	72	114	100	159	92	109	100	0
N.S.	1	1.00	0.90	1.42	1.25	1.99	1.15	1.36	1.25	0.00
time (sec)	N/A	0.081	0.103	0.014	0.999	0.902	1.365	0.337	0.200	0.001

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	125	107	161	118	356	248	112	146	0
N.S.	1	0.98	0.84	1.27	0.93	2.80	1.95	0.88	1.15	0.00
time (sec)	N/A	0.143	0.073	0.013	2.439	0.906	0.996	0.411	0.254	0.001

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	151	302	228	580	389	241	328	0
N.S.	1	1.00	0.89	1.79	1.35	3.43	2.30	1.43	1.94	0.00
time (sec)	N/A	0.156	0.085	0.011	2.626	0.954	1.362	0.391	0.199	0.001

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	106	229	174	254	163	249	194	0
N.S.	1	1.00	0.91	1.96	1.49	2.17	1.39	2.13	1.66	0.00
time (sec)	N/A	0.148	0.089	0.013	1.028	0.829	1.343	0.372	0.174	0.001

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	125	247	176	508	338	184	232	0
N.S.	1	1.00	0.85	1.68	1.20	3.46	2.30	1.25	1.58	0.00
time (sec)	N/A	0.190	0.074	0.011	2.290	0.957	1.222	0.358	0.065	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	127	168	124	181	112	183	130	0
N.S.	1	1.00	1.44	1.91	1.41	2.06	1.27	2.08	1.48	0.00
time (sec)	N/A	0.093	0.046	0.012	1.001	0.995	1.156	0.474	0.180	0.001

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	106	205	147	442	314	152	182	0
N.S.	1	1.00	1.00	1.93	1.39	4.17	2.96	1.43	1.72	0.00
time (sec)	N/A	0.091	0.063	0.009	2.345	0.967	1.062	0.358	0.192	0.001

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	111	146	122	178	110	150	122	0
N.S.	1	1.00	1.26	1.66	1.39	2.02	1.25	1.70	1.39	0.00
time (sec)	N/A	0.084	0.101	0.024	0.992	0.849	2.329	0.414	0.117	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	94	189	140	412	309	143	173	0
N.S.	1	1.00	0.72	1.44	1.07	3.15	2.36	1.09	1.32	0.00
time (sec)	N/A	0.133	0.064	0.015	2.324	0.580	1.538	0.344	0.224	0.001

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	87	156	141	209	128	157	135	0
N.S.	1	1.00	0.89	1.59	1.44	2.13	1.31	1.60	1.38	0.00
time (sec)	N/A	0.106	0.096	0.015	1.099	0.886	3.180	0.293	0.270	0.001

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	109	209	159	458	321	150	183	0
N.S.	1	1.00	0.74	1.42	1.08	3.12	2.18	1.02	1.24	0.00
time (sec)	N/A	0.141	0.068	0.017	2.396	0.680	1.893	0.299	0.259	0.001

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	95	144	133	726	0	122	3558	0
N.S.	1	1.00	0.87	1.32	1.22	6.66	0.00	1.12	32.64	0.00
time (sec)	N/A	0.085	0.148	0.013	2.430	1.100	0.000	0.300	1.059	0.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	95	105	117	253	92	172	0
N.S.	1	1.00	1.00	1.28	1.42	1.58	3.42	1.24	2.32	0.00
time (sec)	N/A	0.065	0.035	0.013	1.075	0.818	1.906	0.379	0.313	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	104	134	119	704	0	110	3153	0
N.S.	1	1.00	1.00	1.29	1.14	6.77	0.00	1.06	30.32	0.00
time (sec)	N/A	0.065	0.136	0.011	2.404	1.017	0.000	0.426	0.892	0.001

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	66	90	99	103	248	85	161	0
N.S.	1	1.00	0.94	1.29	1.41	1.47	3.54	1.21	2.30	0.00
time (sec)	N/A	0.052	0.028	0.013	1.090	0.572	1.807	0.313	0.320	0.001

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	109	144	132	699	0	121	3649	0
N.S.	1	1.00	1.01	1.33	1.22	6.47	0.00	1.12	33.79	0.00
time (sec)	N/A	0.080	0.140	0.010	2.423	0.915	0.000	0.379	1.015	0.001

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	97	139	137	218	0	183	127	0
N.S.	1	1.00	0.98	1.40	1.38	2.20	0.00	1.85	1.28	0.00
time (sec)	N/A	0.105	0.116	0.017	1.116	2.616	0.000	0.302	0.734	0.001

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	123	169	178	1003	0	164	2400	0
N.S.	1	1.00	0.85	1.17	1.24	6.97	0.00	1.14	16.67	0.00
time (sec)	N/A	0.206	0.180	0.016	2.255	1.626	0.000	0.368	1.001	0.001

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	119	170	189	303	0	257	171	0
N.S.	1	1.00	0.94	1.35	1.50	2.40	0.00	2.04	1.36	0.00
time (sec)	N/A	0.146	0.156	0.020	1.126	5.584	0.000	0.347	1.024	0.001

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	142	191	236	1281	0	165	4654	0
N.S.	1	1.00	0.75	1.01	1.25	6.78	0.00	0.87	24.62	0.00
time (sec)	N/A	0.277	0.286	0.017	2.462	3.478	0.000	0.378	1.232	0.001

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	155	209	258	356	0	281	217	0
N.S.	1	1.00	0.97	1.31	1.61	2.22	0.00	1.76	1.36	0.00
time (sec)	N/A	0.194	0.200	0.024	1.123	12.769	0.000	0.388	1.107	0.001

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	179	234	303	1489	0	207	2737	0
N.S.	1	1.00	0.72	0.94	1.21	5.96	0.00	0.83	10.95	0.00
time (sec)	N/A	0.409	0.312	0.017	2.408	6.929	0.000	0.393	1.285	0.001

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	202	268	339	410	0	354	278	0
N.S.	1	1.00	0.96	1.28	1.61	1.95	0.00	1.69	1.32	0.00
time (sec)	N/A	0.249	0.277	0.030	1.173	19.484	0.000	0.396	1.226	0.001

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	133	222	249	1407	0	198	5395	0
N.S.	1	1.00	0.82	1.37	1.54	8.69	0.00	1.22	33.30	0.00
time (sec)	N/A	0.164	0.214	0.016	2.540	1.558	0.000	0.338	1.843	0.001

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	86	188	228	296	507	178	522	0
N.S.	1	1.00	0.80	1.76	2.13	2.77	4.74	1.66	4.88	0.00
time (sec)	N/A	0.108	0.096	0.017	1.270	1.401	4.031	0.394	0.530	0.001

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	137	222	249	1387	0	196	5236	0
N.S.	1	1.00	0.93	1.51	1.69	9.44	0.00	1.33	35.62	0.00
time (sec)	N/A	0.134	0.174	0.019	2.423	1.950	0.000	0.360	1.669	0.001

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	77	143	215	253	410	163	378	0
N.S.	1	1.00	0.84	1.55	2.34	2.75	4.46	1.77	4.11	0.00
time (sec)	N/A	0.079	0.079	0.017	1.063	1.046	2.760	0.368	0.287	0.001

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	136	238	294	1681	0	232	6183	0
N.S.	1	1.00	0.81	1.43	1.76	10.07	0.00	1.39	37.02	0.00
time (sec)	N/A	0.198	0.333	0.015	2.552	3.057	0.000	0.445	2.066	0.001

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	133	225	295	540	0	321	193	0
N.S.	1	1.00	0.94	1.60	2.09	3.83	0.00	2.28	1.37	0.00
time (sec)	N/A	0.169	0.228	0.032	1.204	11.968	0.000	0.391	1.437	0.001

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	158	261	378	2113	0	321	3747	0
N.S.	1	1.00	0.72	1.20	1.73	9.69	0.00	1.47	17.19	0.00
time (sec)	N/A	0.311	0.295	0.019	2.496	6.149	0.000	0.415	1.691	0.001

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	157	254	381	667	0	333	313	0
N.S.	1	1.00	1.01	1.63	2.44	4.28	0.00	2.13	2.01	0.00
time (sec)	N/A	0.209	0.219	0.029	1.211	25.064	0.000	0.390	1.635	0.001

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	178	285	460	2457	0	275	3978	0
N.S.	1	1.00	0.66	1.05	1.70	9.07	0.00	1.01	14.68	0.00
time (sec)	N/A	0.448	0.395	0.023	2.681	13.745	0.000	0.352	1.885	0.001

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	166	388	443	2859	0	301	7515	0
N.S.	1	1.00	0.80	1.87	2.14	13.81	0.00	1.45	36.30	0.00
time (sec)	N/A	0.277	0.355	0.017	2.538	2.898	0.000	0.407	2.926	0.001

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	121	283	415	598	784	267	926	0
N.S.	1	1.00	0.85	1.99	2.92	4.21	5.52	1.88	6.52	0.00
time (sec)	N/A	0.153	0.106	0.019	1.231	1.177	7.955	0.485	0.704	0.001

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	171	391	473	2891	0	317	7929	0
N.S.	1	1.00	0.86	1.96	2.36	14.46	0.00	1.58	39.64	0.00
time (sec)	N/A	0.252	0.409	0.017	2.632	4.600	0.000	0.360	2.767	0.001

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	107	234	394	507	643	229	707	0
N.S.	1	1.00	0.85	1.86	3.13	4.02	5.10	1.82	5.61	0.00
time (sec)	N/A	0.109	0.134	0.020	1.263	0.934	10.063	0.433	0.586	0.001

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	197	403	529	3239	0	332	8649	0
N.S.	1	1.00	0.86	1.75	2.30	14.08	0.00	1.44	37.60	0.00
time (sec)	N/A	0.301	0.411	0.017	2.682	8.744	0.000	0.445	3.001	0.001

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	187	374	527	1058	0	470	472	0
N.S.	1	1.00	0.97	1.95	2.74	5.51	0.00	2.45	2.46	0.00
time (sec)	N/A	0.241	0.273	0.026	1.342	30.217	0.000	0.361	2.070	0.001

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	210	428	639	3753	0	430	5060	0
N.S.	1	1.00	0.71	1.44	2.15	12.64	0.00	1.45	17.04	0.00
time (sec)	N/A	0.497	0.424	0.023	2.748	19.648	0.000	0.503	2.804	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	208	405	651	1227	0	638	549	0
N.S.	1	1.00	0.97	1.88	3.03	5.71	0.00	2.97	2.55	0.00
time (sec)	N/A	0.294	0.321	0.031	1.481	72.333	0.000	0.452	2.420	0.001

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	230	455	738	4225	0	367	1161	0
N.S.	1	1.00	0.61	1.21	1.96	11.21	0.00	0.97	3.08	0.00
time (sec)	N/A	0.686	0.456	0.026	2.781	47.778	0.000	0.411	2.290	0.001

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	89	474	129	379	2069	593	289	0
N.S.	1	1.00	0.93	4.94	1.34	3.95	21.55	6.18	3.01	0.00
time (sec)	N/A	0.065	0.115	0.025	1.072	1.259	3.077	0.441	0.495	0.072

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	262	91	215	1044	332	177	0
N.S.	1	1.00	0.93	3.69	1.28	3.03	14.70	4.68	2.49	0.00
time (sec)	N/A	0.042	0.049	0.007	1.014	0.931	1.792	0.445	0.343	0.039

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	110	53	92	410	143	95	0
N.S.	1	1.00	0.93	2.44	1.18	2.04	9.11	3.18	2.11	0.00
time (sec)	N/A	0.021	0.033	0.004	1.062	0.766	0.943	0.364	0.300	0.028

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	141	976	215	773	4345	1192	443	0
N.S.	1	1.00	0.93	6.46	1.42	5.12	28.77	7.89	2.93	0.00
time (sec)	N/A	0.089	0.100	0.011	1.105	1.205	5.320	0.587	0.658	0.142

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	101	569	153	442	2363	703	302	0
N.S.	1	1.00	0.93	5.22	1.40	4.06	21.68	6.45	2.77	0.00
time (sec)	N/A	0.062	0.089	0.009	1.020	1.043	3.185	0.383	0.469	0.066

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	262	91	215	1044	332	177	0
N.S.	1	1.00	0.93	3.69	1.28	3.03	14.70	4.68	2.49	0.00
time (sec)	N/A	0.037	0.060	0.007	0.945	1.026	1.758	0.426	0.355	0.040

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.016	0.021	0.006	1.051	1.019	10.206	0.355	0.199	0.024

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.016	0.014	0.006	1.061	0.709	5.303	0.317	0.193	0.023

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	32	46	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79	1.05
time (sec)	N/A	0.016	0.014	0.004	1.088	0.986	2.412	0.344	0.043	0.021

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	32	27	30	37	29	31	41
N.S.	1	1.00	0.85	0.82	0.69	0.77	0.95	0.74	0.79	1.05
time (sec)	N/A	0.015	0.014	0.005	0.951	1.023	1.990	0.296	0.041	0.021

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	32	27	29	44	29	31	41
N.S.	1	1.00	0.89	0.86	0.73	0.78	1.19	0.78	0.84	1.11
time (sec)	N/A	0.015	0.014	0.005	1.060	0.833	0.778	0.307	0.191	0.021

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	35	32	27	29	44	29	31	35
N.S.	1	1.00	0.95	0.86	0.73	0.78	1.19	0.78	0.84	0.95
time (sec)	N/A	0.015	0.010	0.007	1.104	1.006	0.994	0.348	0.192	0.024

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	36	32	27	29	42	29	31	35
N.S.	1	1.00	0.97	0.86	0.73	0.78	1.14	0.78	0.84	0.95
time (sec)	N/A	0.016	0.013	0.004	1.032	1.037	1.194	0.333	0.040	0.026

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	36	32	29	29	42	31	31	35
N.S.	1	1.00	0.97	0.86	0.78	0.78	1.14	0.84	0.84	0.95
time (sec)	N/A	0.016	0.013	0.004	1.037	0.847	1.761	0.371	0.038	0.031

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.030	0.031	0.008	1.157	1.070	19.432	0.275	0.215	0.039

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	51	56	80	53	51	69
N.S.	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.030	0.028	0.006	1.103	1.070	10.849	0.294	0.052	0.035

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.030	0.029	0.008	1.080	1.327	5.711	0.315	0.053	0.032

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	54	66	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.86	1.05	0.84	0.81	1.10
time (sec)	N/A	0.029	0.028	0.005	1.055	1.167	2.531	0.310	0.052	0.037

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	53	56	51	53	78	53	51	69
N.S.	1	1.00	0.87	0.92	0.84	0.87	1.28	0.87	0.84	1.13
time (sec)	N/A	0.032	0.027	0.006	0.955	1.340	2.108	0.264	0.048	0.032

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	60	56	51	53	78	53	51	59
N.S.	1	1.00	0.98	0.92	0.84	0.87	1.28	0.87	0.84	0.97
time (sec)	N/A	0.031	0.020	0.007	1.032	1.172	2.418	0.364	0.051	0.041

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	57	56	51	53	76	53	51	59
N.S.	1	1.00	0.93	0.92	0.84	0.87	1.25	0.87	0.84	0.97
time (sec)	N/A	0.029	0.018	0.006	0.987	1.079	2.956	0.411	0.052	0.043

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	57	56	53	53	76	55	55	59
N.S.	1	1.00	0.93	0.92	0.87	0.87	1.25	0.90	0.90	0.97
time (sec)	N/A	0.029	0.017	0.007	0.986	1.045	4.014	0.396	0.248	0.042

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.042	0.042	0.008	1.135	1.234	34.248	0.373	0.043	0.048

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.041	0.043	0.006	1.042	0.793	20.628	0.358	0.034	0.042

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	80	73	78	114	77	69	97
N.S.	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81	1.14
time (sec)	N/A	0.040	0.039	0.007	1.087	1.280	11.404	0.378	0.049	0.040

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	71	80	73	76	95	77	69	97
N.S.	1	1.00	0.84	0.94	0.86	0.89	1.12	0.91	0.81	1.14
time (sec)	N/A	0.041	0.038	0.007	1.165	0.893	3.338	0.397	0.030	0.038

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	71	80	73	75	112	77	69	97
N.S.	1	1.00	0.86	0.96	0.88	0.90	1.35	0.93	0.83	1.17
time (sec)	N/A	0.040	0.039	0.008	1.064	0.761	4.825	0.247	0.034	0.041

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	81	80	73	75	110	77	69	83
N.S.	1	1.00	0.98	0.96	0.88	0.90	1.33	0.93	0.83	1.00
time (sec)	N/A	0.044	0.022	0.006	0.959	1.031	5.326	0.298	0.033	0.054

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	78	80	73	75	110	77	69	83
N.S.	1	1.00	0.94	0.96	0.88	0.90	1.33	0.93	0.83	1.00
time (sec)	N/A	0.043	0.024	0.016	1.147	1.017	6.057	0.409	0.033	0.050

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	78	80	75	75	107	79	72	83
N.S.	1	1.00	0.96	0.99	0.93	0.93	1.32	0.98	0.89	1.02
time (sec)	N/A	0.043	0.023	0.006	1.013	1.179	8.684	0.285	0.059	0.056

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	227	330	259	714	434	298	788	181
N.S.	1	1.00	0.82	1.20	0.94	2.59	1.57	1.08	2.86	0.66
time (sec)	N/A	0.259	0.289	0.022	2.488	0.990	127.297	0.371	0.378	0.289

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	110	308	214	899	502	264	92	160
N.S.	1	1.00	0.43	1.20	0.83	3.50	1.95	1.03	0.36	0.62
time (sec)	N/A	0.205	0.133	0.010	2.414	0.933	54.693	0.325	0.179	0.199

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	208	299	235	660	393	263	789	158
N.S.	1	1.00	0.82	1.17	0.92	2.59	1.54	1.03	3.09	0.62
time (sec)	N/A	0.202	0.202	0.009	2.241	1.292	16.605	0.408	0.380	0.185

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	95	280	194	834	459	251	71	135
N.S.	1	1.00	0.40	1.18	0.82	3.52	1.94	1.06	0.30	0.57
time (sec)	N/A	0.178	0.073	0.010	2.329	1.131	6.758	0.417	0.154	0.189

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	166	277	218	645	355	251	739	134
N.S.	1	1.00	0.71	1.18	0.93	2.74	1.51	1.07	3.14	0.57
time (sec)	N/A	0.176	0.130	0.007	2.359	0.822	6.499	0.417	0.239	0.189

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	235	74	277	194	843	206	251	71	135
N.S.	1	1.00	0.31	1.18	0.83	3.59	0.88	1.07	0.30	0.57
time (sec)	N/A	0.181	0.084	0.012	2.420	1.127	19.842	0.370	0.157	0.188

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	168	280	218	653	364	251	811	136
N.S.	1	1.00	0.71	1.18	0.92	2.76	1.54	1.06	3.42	0.57
time (sec)	N/A	0.177	0.139	0.010	2.373	1.406	31.263	0.364	0.384	0.186

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	46	299	213	883	366	268	90	160
N.S.	1	1.00	0.18	1.17	0.84	3.46	1.44	1.05	0.35	0.63
time (sec)	N/A	0.206	0.015	0.015	2.378	1.037	124.887	0.348	0.308	0.207

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	385	339	271	748	0	298	823	208
N.S.	1	1.00	1.24	1.09	0.87	2.41	0.00	0.96	2.65	0.67
time (sec)	N/A	0.243	0.428	0.017	2.459	1.606	0.000	0.510	0.365	0.618

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	136	317	223	925	0	283	106	167
N.S.	1	1.00	0.47	1.10	0.77	3.20	0.00	0.98	0.37	0.58
time (sec)	N/A	0.219	0.196	0.017	2.393	1.451	0.000	0.419	0.197	0.665

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	284	284	353	323	250	725	984	283	744	167
N.S.	1	1.00	1.24	1.14	0.88	2.55	3.46	1.00	2.62	0.59
time (sec)	N/A	0.209	0.370	0.017	2.489	1.009	84.943	0.434	0.372	0.647

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	95	305	217	912	162	273	91	160
N.S.	1	1.00	0.36	1.17	0.83	3.49	0.62	1.05	0.35	0.61
time (sec)	N/A	0.182	0.121	0.017	2.280	1.208	32.132	0.365	0.313	0.574

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	203	305	241	717	959	273	750	160
N.S.	1	1.00	0.78	1.17	0.92	2.75	3.67	1.05	2.87	0.61
time (sec)	N/A	0.181	0.234	0.014	2.465	1.152	78.536	0.378	0.429	0.579

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	117	323	222	920	0	278	104	167
N.S.	1	1.00	0.40	1.12	0.77	3.18	0.00	0.96	0.36	0.58
time (sec)	N/A	0.212	0.216	0.019	2.428	1.054	0.000	0.464	0.320	0.620

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	289	289	355	317	251	741	0	283	859	170
N.S.	1	1.00	1.23	1.10	0.87	2.56	0.00	0.98	2.97	0.59
time (sec)	N/A	0.214	0.391	0.019	2.706	1.603	0.000	0.497	0.469	0.614

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	151	339	250	974	0	303	121	200
N.S.	1	1.00	0.49	1.09	0.81	3.14	0.00	0.98	0.39	0.65
time (sec)	N/A	0.237	0.434	0.020	2.310	1.437	0.000	0.537	0.176	0.592

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	316	316	402	363	283	793	0	304	760	189
N.S.	1	1.00	1.27	1.15	0.90	2.51	0.00	0.96	2.41	0.60
time (sec)	N/A	0.240	0.435	0.021	2.466	1.171	0.000	0.406	0.398	0.716

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	137	325	251	990	0	293	122	180
N.S.	1	1.00	0.47	1.11	0.86	3.38	0.00	1.00	0.42	0.61
time (sec)	N/A	0.216	0.207	0.020	2.369	1.307	0.000	0.467	0.185	0.731

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	389	334	280	806	0	298	799	191
N.S.	1	1.00	1.31	1.12	0.94	2.70	0.00	1.00	2.68	0.64
time (sec)	N/A	0.212	0.432	0.017	2.377	1.203	0.000	0.519	0.471	0.738

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	298	298	62	335	253	1005	299	298	124	182
N.S.	1	1.00	0.21	1.12	0.85	3.37	1.00	1.00	0.42	0.61
time (sec)	N/A	0.216	0.050	0.020	2.425	0.873	152.840	0.522	0.322	0.723

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	293	293	230	325	276	793	0	293	780	181
N.S.	1	1.00	0.78	1.11	0.94	2.71	0.00	1.00	2.66	0.62
time (sec)	N/A	0.212	0.280	0.019	2.329	0.607	0.000	0.439	0.479	0.700

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	147	363	255	988	0	300	133	190
N.S.	1	1.00	0.46	1.13	0.79	3.07	0.00	0.93	0.41	0.59
time (sec)	N/A	0.234	0.181	0.023	2.593	0.823	0.000	0.409	0.187	0.732

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	400	357	285	809	0	304	888	192
N.S.	1	1.00	1.24	1.11	0.89	2.51	0.00	0.94	2.76	0.60
time (sec)	N/A	0.237	0.415	0.020	2.433	1.144	0.000	0.445	0.534	0.621

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	343	343	189	381	285	1043	0	326	152	224
N.S.	1	1.00	0.55	1.11	0.83	3.04	0.00	0.95	0.44	0.65
time (sec)	N/A	0.262	0.467	0.026	2.455	1.468	0.000	0.541	0.353	0.636

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.033	0.031	0.007	0.999	1.048	19.900	0.399	0.061	0.039

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	56	51	56	80	53	51	69
N.S.	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.029	0.029	0.006	1.026	0.847	11.155	0.363	0.048	0.034

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	56	80	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81	1.10
time (sec)	N/A	0.030	0.028	0.008	0.961	1.156	5.888	0.318	0.048	0.034

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	56	51	54	66	53	51	69
N.S.	1	1.00	0.84	0.89	0.81	0.86	1.05	0.84	0.81	1.10
time (sec)	N/A	0.029	0.029	0.007	1.005	0.762	2.570	0.331	0.052	0.040

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	53	56	51	53	78	53	51	69
N.S.	1	1.00	0.87	0.92	0.84	0.87	1.28	0.87	0.84	1.13
time (sec)	N/A	0.029	0.029	0.006	0.995	1.270	2.117	0.282	0.046	0.034

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	60	56	51	53	78	53	51	59
N.S.	1	1.00	0.98	0.92	0.84	0.87	1.28	0.87	0.84	0.97
time (sec)	N/A	0.031	0.021	0.009	1.039	1.079	2.314	0.271	0.051	0.041

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	57	56	51	53	76	53	51	59
N.S.	1	1.00	0.93	0.92	0.84	0.87	1.25	0.87	0.84	0.97
time (sec)	N/A	0.029	0.019	0.006	1.100	0.820	2.762	0.329	0.052	0.043

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	57	56	53	53	76	55	55	59
N.S.	1	1.00	0.93	0.92	0.87	0.87	1.25	0.90	0.90	0.97
time (sec)	N/A	0.029	0.017	0.007	1.083	0.783	3.831	0.345	0.053	0.042

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	97	85	90	136	94	78	116
N.S.	1	1.00	1.00	1.00	0.88	0.93	1.40	0.97	0.80	1.20
time (sec)	N/A	0.049	0.035	0.007	1.022	1.039	31.909	0.342	0.051	0.053

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	97	85	90	136	94	78	116
N.S.	1	1.00	1.00	1.00	0.88	0.93	1.40	0.97	0.80	1.20
time (sec)	N/A	0.046	0.034	0.007	1.170	0.891	20.276	0.361	0.033	0.052

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	97	85	90	136	94	78	116
N.S.	1	1.00	1.00	1.00	0.88	0.93	1.40	0.97	0.80	1.20
time (sec)	N/A	0.050	0.033	0.007	1.092	1.272	10.866	0.440	0.032	0.057

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	83	97	85	88	110	94	78	116
N.S.	1	1.00	0.86	1.00	0.88	0.91	1.13	0.97	0.80	1.20
time (sec)	N/A	0.047	0.035	0.008	1.061	0.693	3.380	0.418	0.032	0.050

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	97	85	87	134	94	78	116
N.S.	1	1.00	1.00	1.02	0.89	0.92	1.41	0.99	0.82	1.22
time (sec)	N/A	0.046	0.031	0.008	0.950	0.963	4.776	0.308	0.032	0.057

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	83	97	85	87	134	94	78	100
N.S.	1	1.00	0.87	1.02	0.89	0.92	1.41	0.99	0.82	1.05
time (sec)	N/A	0.048	0.033	0.007	1.035	1.048	5.255	0.392	0.034	0.047

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	83	97	85	87	133	94	78	100
N.S.	1	1.00	0.87	1.02	0.89	0.92	1.40	0.99	0.82	1.05
time (sec)	N/A	0.047	0.035	0.009	1.008	0.871	6.068	0.368	0.033	0.047

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	83	97	87	87	133	96	86	100
N.S.	1	1.00	0.87	1.02	0.92	0.92	1.40	1.01	0.91	1.05
time (sec)	N/A	0.047	0.039	0.007	1.193	1.087	8.044	0.432	0.061	0.070

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	139	138	127	132	192	135	119	163
N.S.	1	1.00	1.00	0.99	0.91	0.95	1.38	0.97	0.86	1.17
time (sec)	N/A	0.065	0.042	0.008	1.064	0.874	53.082	0.348	0.214	0.075

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	139	138	127	132	192	135	119	163
N.S.	1	1.00	1.00	0.99	0.91	0.95	1.38	0.97	0.86	1.17
time (sec)	N/A	0.065	0.041	0.007	1.060	0.769	33.452	0.443	0.039	0.072

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	139	138	127	132	192	135	119	163
N.S.	1	1.00	1.00	0.99	0.91	0.95	1.38	0.97	0.86	1.17
time (sec)	N/A	0.063	0.038	0.010	1.087	0.753	19.975	0.307	0.040	0.069

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	139	138	127	130	155	135	119	163
N.S.	1	1.00	1.00	0.99	0.91	0.94	1.12	0.97	0.86	1.17
time (sec)	N/A	0.064	0.038	0.009	1.034	0.846	4.364	0.385	0.042	0.070

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	137	138	127	129	190	135	119	163
N.S.	1	1.00	1.00	1.01	0.93	0.94	1.39	0.99	0.87	1.19
time (sec)	N/A	0.065	0.041	0.009	1.078	0.818	9.682	0.289	0.039	0.069

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	137	138	127	129	189	135	119	141
N.S.	1	1.00	1.00	1.01	0.93	0.94	1.38	0.99	0.87	1.03
time (sec)	N/A	0.064	0.055	0.007	1.113	1.071	10.561	0.307	0.044	0.064

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	137	138	127	129	189	135	119	141
N.S.	1	1.00	1.00	1.01	0.93	0.94	1.38	0.99	0.87	1.03
time (sec)	N/A	0.064	0.054	0.010	1.122	1.285	12.050	0.445	0.042	0.086

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	121	138	129	129	185	137	125	141
N.S.	1	1.00	0.88	1.01	0.94	0.94	1.35	1.00	0.91	1.03
time (sec)	N/A	0.063	0.047	0.009	1.077	1.025	15.274	0.337	0.043	0.067

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	311	311	299	545	360	1334	0	436	1202	238
N.S.	1	1.00	0.96	1.75	1.16	4.29	0.00	1.40	3.86	0.77
time (sec)	N/A	0.313	0.140	0.020	2.439	1.409	0.000	0.459	0.382	0.260

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	290	276	504	263	1701	0	385	435	198
N.S.	1	1.00	0.95	1.74	0.91	5.87	0.00	1.33	1.50	0.68
time (sec)	N/A	0.254	0.111	0.012	2.447	1.209	0.000	0.458	0.311	0.237

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	276	495	324	1268	656	385	1175	199
N.S.	1	1.00	0.96	1.72	1.12	4.40	2.28	1.34	4.08	0.69
time (sec)	N/A	0.240	0.109	0.013	2.433	1.374	53.529	0.460	0.351	0.225

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	249	461	229	1629	87	361	390	163
N.S.	1	1.00	0.93	1.72	0.85	6.08	0.32	1.35	1.46	0.61
time (sec)	N/A	0.233	0.119	0.011	2.505	1.103	11.401	0.540	0.176	0.205

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	249	452	291	1245	597	360	1107	161
N.S.	1	1.00	0.94	1.70	1.09	4.68	2.24	1.35	4.16	0.61
time (sec)	N/A	0.212	0.116	0.011	2.408	1.284	16.758	0.399	0.371	0.200

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	261	439	223	1636	394	344	416	162
N.S.	1	1.00	1.00	1.69	0.86	6.29	1.52	1.32	1.60	0.62
time (sec)	N/A	0.266	0.132	0.015	2.383	0.961	36.722	0.480	0.343	0.218

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	261	439	286	1253	576	344	1201	164
N.S.	1	1.00	1.00	1.69	1.10	4.82	2.22	1.32	4.62	0.63
time (sec)	N/A	0.265	0.127	0.014	2.409	0.733	31.822	0.405	0.381	0.212

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	254	452	229	1648	406	353	417	165
N.S.	1	1.00	0.95	1.69	0.86	6.17	1.52	1.32	1.56	0.62
time (sec)	N/A	0.280	0.121	0.017	2.469	1.218	125.079	0.472	0.335	0.209

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	254	461	293	1252	0	354	1209	165
N.S.	1	1.00	0.94	1.71	1.09	4.65	0.00	1.32	4.49	0.61
time (sec)	N/A	0.276	0.127	0.015	2.357	1.343	0.000	0.478	0.441	0.204

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	101	495	263	1686	0	390	451	204
N.S.	1	1.00	0.35	1.72	0.91	5.85	0.00	1.35	1.57	0.71
time (sec)	N/A	0.316	0.210	0.016	2.596	1.472	0.000	0.474	0.351	0.241

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	372	563	377	1373	0	440	1367	284
N.S.	1	1.00	0.99	1.50	1.01	3.66	0.00	1.17	3.65	0.76
time (sec)	N/A	0.437	0.356	0.021	2.467	0.771	0.000	0.394	0.444	0.778

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	337	523	272	1733	0	413	160	233
N.S.	1	1.00	0.97	1.51	0.79	5.01	0.00	1.19	0.46	0.67
time (sec)	N/A	0.317	0.188	0.020	2.416	0.837	0.000	0.480	0.216	0.743

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	333	523	336	1334	0	408	1238	231
N.S.	1	1.00	0.96	1.51	0.97	3.86	0.00	1.18	3.58	0.67
time (sec)	N/A	0.329	0.179	0.020	2.449	1.160	0.000	0.469	0.263	0.710

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	310	310	319	499	258	1723	173	388	137	215
N.S.	1	1.00	1.03	1.61	0.83	5.56	0.56	1.25	0.44	0.69
time (sec)	N/A	0.282	0.181	0.019	2.514	1.460	42.058	0.460	0.401	0.790

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	312	312	318	496	327	1341	1574	388	1267	214
N.S.	1	1.00	1.02	1.59	1.05	4.30	5.04	1.24	4.06	0.69
time (sec)	N/A	0.342	0.180	0.018	2.582	1.146	79.526	0.495	0.458	0.775

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	333	333	317	495	260	1739	0	389	138	221
N.S.	1	1.00	0.95	1.49	0.78	5.22	0.00	1.17	0.41	0.66
time (sec)	N/A	0.328	0.195	0.020	2.376	1.441	0.000	0.534	0.394	0.757

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	332	332	315	498	326	1340	0	384	1340	219
N.S.	1	1.00	0.95	1.50	0.98	4.04	0.00	1.16	4.04	0.66
time (sec)	N/A	0.343	0.190	0.021	2.434	1.378	0.000	0.444	0.561	0.737

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	363	333	524	275	1737	0	401	152	237
N.S.	1	1.00	0.92	1.44	0.76	4.79	0.00	1.10	0.42	0.65
time (sec)	N/A	0.377	0.197	0.025	2.381	1.558	0.000	0.500	0.233	0.761

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	440	440	383	590	389	1427	0	451	1426	272
N.S.	1	1.00	0.87	1.34	0.88	3.24	0.00	1.02	3.24	0.62
time (sec)	N/A	0.381	0.473	0.025	2.438	1.441	0.000	0.444	0.480	0.908

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	401	401	363	562	306	1813	0	427	197	254
N.S.	1	1.00	0.91	1.40	0.76	4.52	0.00	1.06	0.49	0.63
time (sec)	N/A	0.343	0.241	0.026	2.541	1.242	0.000	0.473	0.419	1.021

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	402	402	361	568	374	1420	0	426	1236	253
N.S.	1	1.00	0.90	1.41	0.93	3.53	0.00	1.06	3.07	0.63
time (sec)	N/A	0.330	0.386	0.023	2.440	1.293	0.000	0.426	0.441	1.096

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	364	364	339	514	297	1811	0	416	184	242
N.S.	1	1.00	0.93	1.41	0.82	4.98	0.00	1.14	0.51	0.66
time (sec)	N/A	0.293	0.207	0.023	2.545	1.582	0.000	0.521	0.395	0.869

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	364	364	339	514	366	1416	0	416	1419	242
N.S.	1	1.00	0.93	1.41	1.01	3.89	0.00	1.14	3.90	0.66
time (sec)	N/A	0.297	0.204	0.021	2.431	1.557	0.000	0.530	0.606	0.853

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	399	398	364	568	307	1819	0	427	192	260
N.S.	1	1.00	0.91	1.42	0.77	4.56	0.00	1.07	0.48	0.65
time (sec)	N/A	0.395	0.412	0.025	2.521	1.842	0.000	0.536	0.399	0.999

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	402	401	365	562	377	1433	0	426	1508	260
N.S.	1	1.00	0.91	1.40	0.94	3.56	0.00	1.06	3.75	0.65
time (sec)	N/A	0.406	0.261	0.023	2.439	1.582	0.000	0.493	0.635	0.967

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	439	438	382	590	324	1832	0	444	208	278
N.S.	1	1.00	0.87	1.34	0.74	4.17	0.00	1.01	0.47	0.63
time (sec)	N/A	0.450	0.387	0.028	2.411	2.007	0.000	0.527	0.237	0.981

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	328	328	132	721	331	2528	0	531	634	255
N.S.	1	1.00	0.40	2.20	1.01	7.71	0.00	1.62	1.93	0.78
time (sec)	N/A	0.293	0.393	0.014	2.466	1.657	0.000	0.523	0.371	0.275

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	326	326	133	712	437	1898	0	531	1564	254
N.S.	1	1.00	0.41	2.18	1.34	5.82	0.00	1.63	4.80	0.78
time (sec)	N/A	0.277	0.401	0.013	2.418	1.487	0.000	0.446	0.391	0.267

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	306	306	97	659	282	2441	874	490	574	199
N.S.	1	1.00	0.32	2.15	0.92	7.98	2.86	1.60	1.88	0.65
time (sec)	N/A	0.245	0.376	0.013	2.419	1.603	93.472	0.486	0.157	0.250

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	304	304	96	650	390	1862	874	490	1460	200
N.S.	1	1.00	0.32	2.14	1.28	6.12	2.88	1.61	4.80	0.66
time (sec)	N/A	0.249	0.363	0.012	2.439	1.662	71.753	0.426	0.377	0.253

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	284	284	89	622	261	2442	598	462	580	190
N.S.	1	1.00	0.31	2.19	0.92	8.60	2.11	1.63	2.04	0.67
time (sec)	N/A	0.290	0.352	0.015	2.397	1.555	161.473	0.455	0.173	0.258

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	284	284	89	616	368	1866	828	461	1561	189
N.S.	1	1.00	0.31	2.17	1.30	6.57	2.92	1.62	5.50	0.67
time (sec)	N/A	0.267	0.380	0.016	2.508	1.362	125.947	0.450	0.223	0.251

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	283	283	88	616	259	2451	0	455	583	188
N.S.	1	1.00	0.31	2.18	0.92	8.66	0.00	1.61	2.06	0.66
time (sec)	N/A	0.275	0.371	0.017	2.420	1.469	0.000	0.435	0.367	0.244

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	283	283	88	622	368	1861	0	455	1564	189
N.S.	1	1.00	0.31	2.20	1.30	6.58	0.00	1.61	5.53	0.67
time (sec)	N/A	0.258	0.380	0.019	2.587	1.452	0.000	0.445	0.254	0.243

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	303	303	101	650	281	2459	0	483	591	207
N.S.	1	1.00	0.33	2.15	0.93	8.12	0.00	1.59	1.95	0.68
time (sec)	N/A	0.291	0.384	0.016	2.526	1.761	0.000	0.471	0.365	0.255

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	305	102	659	389	1866	0	483	1580	206
N.S.	1	1.00	0.33	2.16	1.28	6.12	0.00	1.58	5.18	0.68
time (sec)	N/A	0.268	0.379	0.016	2.373	1.488	0.000	0.505	0.507	0.254

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	325	325	148	712	330	2512	0	536	639	262
N.S.	1	1.00	0.46	2.19	1.02	7.73	0.00	1.65	1.97	0.81
time (sec)	N/A	0.306	0.425	0.018	2.420	1.807	0.000	0.447	0.386	0.277

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	409	409	419	804	499	2014	0	600	1850	344
N.S.	1	1.00	1.02	1.97	1.22	4.92	0.00	1.47	4.52	0.84
time (sec)	N/A	0.467	2.695	0.021	2.463	1.413	0.000	0.494	0.304	0.582

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	374	374	377	748	337	2542	0	552	681	280
N.S.	1	1.00	1.01	2.00	0.90	6.80	0.00	1.48	1.82	0.75
time (sec)	N/A	0.415	2.217	0.023	2.361	1.729	0.000	0.599	0.454	0.559

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	386	386	377	748	445	1961	0	552	1691	280
N.S.	1	1.00	0.98	1.94	1.15	5.08	0.00	1.43	4.38	0.73
time (sec)	N/A	0.529	2.693	0.020	2.346	1.063	0.000	0.502	0.446	0.540

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	376	376	355	706	309	2531	173	516	616	246
N.S.	1	1.00	0.94	1.88	0.82	6.73	0.46	1.37	1.64	0.65
time (sec)	N/A	0.426	2.635	0.020	2.514	1.386	136.463	0.498	0.231	0.519

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	340	340	358	697	412	1944	0	511	1636	246
N.S.	1	1.00	1.05	2.05	1.21	5.72	0.00	1.50	4.81	0.72
time (sec)	N/A	0.388	2.635	0.022	2.580	1.365	0.000	0.427	0.251	0.511

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	368	368	355	682	304	2547	0	504	657	244
N.S.	1	1.00	0.96	1.85	0.83	6.92	0.00	1.37	1.79	0.66
time (sec)	N/A	0.431	2.387	0.024	2.480	1.273	0.000	0.503	0.412	0.568

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	367	367	350	682	415	1967	0	501	1759	244
N.S.	1	1.00	0.95	1.86	1.13	5.36	0.00	1.37	4.79	0.66
time (sec)	N/A	0.411	2.672	0.023	2.467	1.420	0.000	0.484	0.473	0.551

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	376	376	353	697	315	2549	0	505	656	255
N.S.	1	1.00	0.94	1.85	0.84	6.78	0.00	1.34	1.74	0.68
time (sec)	N/A	0.429	2.188	0.025	2.540	1.195	0.000	0.573	0.251	0.656

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F(-1)	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	376	376	353	706	424	1955	0	509	1746	255
N.S.	1	1.00	0.94	1.88	1.13	5.20	0.00	1.35	4.64	0.68
time (sec)	N/A	0.416	2.400	0.022	2.550	0.940	0.000	0.462	0.606	0.635

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	478	478	411	351	390	1422	0	476	7892	282
N.S.	1	1.00	0.86	0.73	0.82	2.97	0.00	1.00	16.51	0.59
time (sec)	N/A	0.557	0.233	0.015	2.613	7.232	0.000	0.763	1.934	0.603

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	476	476	409	339	384	1388	0	476	6428	278
N.S.	1	1.00	0.86	0.71	0.81	2.92	0.00	1.00	13.50	0.58
time (sec)	N/A	0.489	0.189	0.017	2.591	2.121	0.000	0.757	1.602	0.566

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	463	463	142	328	369	1385	0	457	2609	263
N.S.	1	1.00	0.31	0.71	0.80	2.99	0.00	0.99	5.63	0.57
time (sec)	N/A	0.359	0.113	0.013	2.603	1.664	0.000	0.687	1.268	0.491

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	463	463	364	304	367	1249	0	441	5963	265
N.S.	1	1.00	0.79	0.66	0.79	2.70	0.00	0.95	12.88	0.57
time (sec)	N/A	0.363	0.110	0.013	2.105	1.152	0.000	0.645	1.419	0.460

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	463	463	364	304	369	1285	0	481	6701	263
N.S.	1	1.00	0.79	0.66	0.80	2.78	0.00	1.04	14.47	0.57
time (sec)	N/A	0.358	0.139	0.013	2.573	1.156	0.000	0.767	1.154	0.499

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	463	463	364	328	371	1365	0	441	8785	265
N.S.	1	1.00	0.79	0.71	0.80	2.95	0.00	0.95	18.97	0.57
time (sec)	N/A	0.347	0.122	0.013	2.502	1.799	0.000	0.650	1.675	0.525

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	476	476	409	339	390	1421	0	492	6038	280
N.S.	1	1.00	0.86	0.71	0.82	2.99	0.00	1.03	12.68	0.59
time (sec)	N/A	0.535	0.233	0.016	2.545	2.488	0.000	0.858	1.643	0.623

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	478	478	411	351	396	1431	0	476	7540	280
N.S.	1	1.00	0.86	0.73	0.83	2.99	0.00	1.00	15.77	0.59
time (sec)	N/A	0.481	0.239	0.016	2.384	15.950	0.000	0.705	2.395	0.640

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	498	498	437	375	411	1484	0	487	4643	296
N.S.	1	1.00	0.88	0.75	0.83	2.98	0.00	0.98	9.32	0.59
time (sec)	N/A	0.687	0.278	0.020	2.547	21.817	0.000	0.740	2.732	0.692

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	570	570	563	582	494	3393	0	718	22978	353
N.S.	1	1.00	0.99	1.02	0.87	5.95	0.00	1.26	40.31	0.62
time (sec)	N/A	0.846	0.357	0.020	2.025	111.542	0.000	1.108	3.254	1.201

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	536	536	527	566	450	3524	0	681	19871	323
N.S.	1	1.00	0.98	1.06	0.84	6.57	0.00	1.27	37.07	0.60
time (sec)	N/A	0.592	0.312	0.019	2.532	86.411	0.000	1.245	2.527	1.053

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	532	532	523	533	468	3224	0	669	21485	320
N.S.	1	1.00	0.98	1.00	0.88	6.06	0.00	1.26	40.39	0.60
time (sec)	N/A	0.529	0.300	0.018	2.465	17.800	0.000	0.977	2.484	0.986

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	528	528	522	528	436	3393	0	683	18673	307
N.S.	1	1.00	0.99	1.00	0.83	6.43	0.00	1.29	35.37	0.58
time (sec)	N/A	0.591	0.291	0.018	2.450	32.468	0.000	0.934	2.480	0.965

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	528	528	522	528	461	3171	0	655	20689	308
N.S.	1	1.00	0.99	1.00	0.87	6.01	0.00	1.24	39.18	0.58
time (sec)	N/A	0.475	0.297	0.019	2.662	13.986	0.000	1.128	2.359	0.919

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	536	536	523	533	450	3457	0	701	19453	323
N.S.	1	1.00	0.98	0.99	0.84	6.45	0.00	1.31	36.29	0.60
time (sec)	N/A	0.598	0.416	0.018	2.528	47.517	0.000	1.284	2.386	1.142

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	536	536	526	566	489	3310	0	673	21987	322
N.S.	1	1.00	0.98	1.06	0.91	6.18	0.00	1.26	41.02	0.60
time (sec)	N/A	0.533	0.422	0.018	2.418	55.341	0.000	0.970	2.734	1.126

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	570	570	540	582	494	3630	0	725	21370	352
N.S.	1	1.00	0.95	1.02	0.87	6.37	0.00	1.27	37.49	0.62
time (sec)	N/A	0.750	2.174	0.023	2.578	88.692	0.000	1.169	4.091	1.325

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	570	570	602	588	539	0	0	718	27743	353
N.S.	1	1.00	1.06	1.03	0.95	0.00	0.00	1.26	48.67	0.62
time (sec)	N/A	0.823	6.146	0.021	2.488	0.000	0.000	1.151	6.584	1.286

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	618	618	621	612	551	0	0	715	17850	410
N.S.	1	1.00	1.00	0.99	0.89	0.00	0.00	1.16	28.88	0.66
time (sec)	N/A	0.962	6.108	0.025	2.496	0.000	0.000	1.231	5.835	1.459

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	631	631	640	839	653	0	0	944	35251	378
N.S.	1	1.00	1.01	1.33	1.03	0.00	0.00	1.50	55.87	0.60
time (sec)	N/A	0.814	0.539	0.021	2.635	0.000	0.000	1.567	4.219	1.986

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	628	628	544	839	583	0	0	963	31866	369
N.S.	1	1.00	0.87	1.34	0.93	0.00	0.00	1.53	50.74	0.59
time (sec)	N/A	0.761	0.748	0.023	2.610	0.000	0.000	1.648	4.521	1.435

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	627	627	543	848	654	0	0	946	36160	369
N.S.	1	1.00	0.87	1.35	1.04	0.00	0.00	1.51	57.67	0.59
time (sec)	N/A	0.710	0.730	0.021	2.610	0.000	0.000	1.401	4.255	1.387

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	633	633	620	855	594	0	0	968	32735	382
N.S.	1	1.00	0.98	1.35	0.94	0.00	0.00	1.53	51.71	0.60
time (sec)	N/A	0.817	0.860	0.022	2.518	0.000	0.000	1.608	4.354	1.635

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	633	633	620	882	675	0	0	960	36997	383
N.S.	1	1.00	0.98	1.39	1.07	0.00	0.00	1.52	58.45	0.61
time (sec)	N/A	0.832	0.855	0.020	2.613	0.000	0.000	1.456	4.415	1.606

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	681	681	699	900	668	0	0	987	33717	458
N.S.	1	1.00	1.03	1.32	0.98	0.00	0.00	1.45	49.51	0.67
time (sec)	N/A	1.006	6.166	0.027	2.842	0.000	0.000	1.441	10.855	1.474

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	681	681	701	906	755	0	0	995	44524	457
N.S.	1	1.00	1.03	1.33	1.11	0.00	0.00	1.46	65.38	0.67
time (sec)	N/A	0.928	6.178	0.028	2.619	0.000	0.000	1.715	10.808	1.456

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	743	743	720	933	756	0	0	1000	36917	546
N.S.	1	1.00	0.97	1.26	1.02	0.00	0.00	1.35	49.69	0.73
time (sec)	N/A	1.233	6.178	0.028	2.757	0.000	0.000	1.860	12.546	1.340

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	624	624	585	740	617	5375	0	912	34921	369
N.S.	1	1.00	0.94	1.19	0.99	8.61	0.00	1.46	55.96	0.59
time (sec)	N/A	0.757	0.924	0.023	2.520	106.841	0.000	1.298	3.985	1.432

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	609	609	583	740	567	5814	0	952	30956	366
N.S.	1	1.00	0.96	1.22	0.93	9.55	0.00	1.56	50.83	0.60
time (sec)	N/A	0.796	0.952	0.023	2.531	158.694	0.000	1.757	3.965	2.050

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	601	601	575	770	620	5474	0	904	34586	362
N.S.	1	1.00	0.96	1.28	1.03	9.11	0.00	1.50	57.55	0.60
time (sec)	N/A	0.691	1.033	0.021	2.576	130.914	0.000	1.731	3.788	2.003

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	624	624	589	778	610	0	0	973	32506	391
N.S.	1	1.00	0.94	1.25	0.98	0.00	0.00	1.56	52.09	0.63
time (sec)	N/A	0.858	2.093	0.024	2.483	0.000	0.000	1.642	4.560	1.666

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	628	628	661	808	678	0	0	977	37332	391
N.S.	1	1.00	1.05	1.29	1.08	0.00	0.00	1.56	59.45	0.62
time (sec)	N/A	0.862	6.211	0.025	2.589	0.000	0.000	1.286	5.070	1.669

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	676	676	674	825	694	0	0	1035	33548	472
N.S.	1	1.00	1.00	1.22	1.03	0.00	0.00	1.53	49.63	0.70
time (sec)	N/A	1.056	6.158	0.027	2.935	0.000	0.000	1.560	12.376	1.403

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	676	676	676	825	761	0	0	1012	44436	472
N.S.	1	1.00	1.00	1.22	1.13	0.00	0.00	1.50	65.73	0.70
time (sec)	N/A	0.967	6.155	0.027	2.689	0.000	0.000	2.032	10.589	1.425

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	731	731	696	849	774	0	0	1015	36571	564
N.S.	1	1.00	0.95	1.16	1.06	0.00	0.00	1.39	50.03	0.77
time (sec)	N/A	1.276	6.175	0.031	2.751	0.000	0.000	1.961	14.273	1.410

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	718	718	604	1066	855	0	0	1193	48950	456
N.S.	1	1.00	0.84	1.48	1.19	0.00	0.00	1.66	68.18	0.64
time (sec)	N/A	1.042	1.409	0.026	2.650	0.000	0.000	2.213	7.732	2.432

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	703	703	604	1067	791	0	0	1238	44169	467
N.S.	1	1.00	0.86	1.52	1.13	0.00	0.00	1.76	62.83	0.66
time (sec)	N/A	1.020	2.007	0.026	2.778	0.000	0.000	2.573	7.631	3.365

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	703	703	603	1094	889	0	0	1217	50125	465
N.S.	1	1.00	0.86	1.56	1.26	0.00	0.00	1.73	71.30	0.66
time (sec)	N/A	1.005	2.394	0.027	2.816	0.000	0.000	2.322	7.006	3.527

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	739	739	760	1100	845	0	0	1233	45858	510
N.S.	1	1.00	1.03	1.49	1.14	0.00	0.00	1.67	62.05	0.69
time (sec)	N/A	1.159	6.281	0.029	2.883	0.000	0.000	2.588	9.581	2.587

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	739	739	760	1124	951	0	0	1253	150312	510
N.S.	1	1.00	1.03	1.52	1.29	0.00	0.00	1.70	203.40	0.69
time (sec)	N/A	0.978	6.329	0.026	2.753	0.000	0.000	2.535	16.833	2.466

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	805	805	773	1143	955	0	0	1333	127276	627
N.S.	1	1.00	0.96	1.42	1.19	0.00	0.00	1.66	158.11	0.78
time (sec)	N/A	1.400	6.252	0.036	3.065	0.000	0.000	3.047	24.918	2.544

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	805	805	775	1143	1064	0	0	1278	180372	627
N.S.	1	1.00	0.96	1.42	1.32	0.00	0.00	1.59	224.06	0.78
time (sec)	N/A	1.322	6.264	0.033	2.796	0.000	0.000	2.535	19.058	2.593

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	881	881	797	1170	1066	0	0	1289	143600	753
N.S.	1	1.00	0.90	1.33	1.21	0.00	0.00	1.46	163.00	0.85
time (sec)	N/A	1.691	6.268	0.040	2.883	0.000	0.000	2.729	22.141	2.286

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	75	77	132	99	212	104	96	80
N.S.	1	1.00	0.73	0.75	1.28	0.96	2.06	1.01	0.93	0.78
time (sec)	N/A	0.087	0.056	0.008	0.974	1.371	2.217	0.312	0.644	0.049

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	130	181	166	257	286	132	-1	127
N.S.	1	1.00	0.84	1.17	1.07	1.66	1.85	0.85	-0.01	0.82
time (sec)	N/A	0.072	0.283	0.016	1.003	1.107	17.645	0.387	0.000	0.194

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	57	53	90	75	162	73	76	56
N.S.	1	1.00	0.78	0.73	1.23	1.03	2.22	1.00	1.04	0.77
time (sec)	N/A	0.062	0.039	0.007	1.093	1.289	0.949	0.297	0.556	0.040

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	108	139	124	206	226	100	-1	103
N.S.	1	1.00	0.89	1.14	1.02	1.69	1.85	0.82	-0.01	0.84
time (sec)	N/A	0.059	0.216	0.009	1.012	0.809	12.085	0.398	0.000	0.128

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	34	31	50	50	110	44	53	34
N.S.	1	1.00	0.74	0.67	1.09	1.09	2.39	0.96	1.15	0.74
time (sec)	N/A	0.036	0.022	0.005	0.982	0.962	0.371	0.311	0.537	0.027

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	96	81	155	144	69	-1	77
N.S.	1	1.00	0.98	1.10	0.93	1.78	1.66	0.79	-0.01	0.89
time (sec)	N/A	0.028	0.157	0.006	1.116	0.929	6.304	0.494	0.000	0.103

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	59	57	45	123	76	60	47	59
N.S.	1	1.00	1.00	0.97	0.76	2.08	1.29	1.02	0.80	1.00
time (sec)	N/A	0.044	0.034	0.009	0.951	0.892	25.641	0.407	0.881	0.051

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	71	93	59	134	107	84	94	67
N.S.	1	1.00	0.85	1.11	0.70	1.60	1.27	1.00	1.12	0.80
time (sec)	N/A	0.033	0.155	0.010	1.037	0.952	4.415	0.421	1.262	0.124

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	63	106	83	141	107	68	68	65
N.S.	1	1.00	0.75	1.26	0.99	1.68	1.27	0.81	0.81	0.77
time (sec)	N/A	0.063	0.040	0.010	1.099	0.855	43.703	0.361	1.347	0.106

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	81	75	48	137	107	151	76	70
N.S.	1	1.00	1.23	1.14	0.73	2.08	1.62	2.29	1.15	1.06
time (sec)	N/A	0.025	0.173	0.011	1.082	0.881	3.354	0.521	1.467	0.119

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	93	153	130	170	144	120	93	79
N.S.	1	1.00	1.06	1.74	1.48	1.93	1.64	1.36	1.06	0.90
time (sec)	N/A	0.069	0.070	0.013	1.045	0.821	143.855	0.349	1.697	0.112

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	40	37	56	55	119	232	97	62
N.S.	1	1.00	0.75	0.70	1.06	1.04	2.25	4.38	1.83	1.17
time (sec)	N/A	0.021	0.014	0.006	1.076	0.912	3.226	0.425	0.896	0.134

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	61	197	174	221	226	140	134	104
N.S.	1	1.00	0.51	1.64	1.45	1.84	1.88	1.17	1.12	0.87
time (sec)	N/A	0.094	0.022	0.013	1.130	0.911	144.349	0.312	2.200	0.167

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	63	59	96	81	442	288	132	86
N.S.	1	1.00	0.75	0.70	1.14	0.96	5.26	3.43	1.57	1.02
time (sec)	N/A	0.034	0.029	0.005	1.098	0.884	3.888	0.465	1.094	0.154

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	62	239	216	269	0	194	173	128
N.S.	1	1.00	0.40	1.53	1.38	1.72	0.00	1.24	1.11	0.82
time (sec)	N/A	0.120	0.022	0.021	1.167	0.999	0.000	0.452	2.680	0.190

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	81	83	138	105	957	344	174	110
N.S.	1	1.00	0.69	0.71	1.18	0.90	8.18	2.94	1.49	0.94
time (sec)	N/A	0.055	0.036	0.009	0.987	0.908	5.240	0.421	1.463	0.185

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	62	281	258	317	0	230	209	152
N.S.	1	1.00	0.33	1.49	1.37	1.68	0.00	1.22	1.11	0.80
time (sec)	N/A	0.147	0.029	0.034	1.167	1.095	0.000	0.335	3.497	0.256

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	78	77	132	124	260	104	117	80
N.S.	1	1.00	0.76	0.75	1.28	1.20	2.52	1.01	1.14	0.78
time (sec)	N/A	0.077	0.058	0.008	1.053	0.850	8.362	0.426	0.742	0.055

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	150	219	204	299	345	159	-1	150
N.S.	1	1.00	0.80	1.16	1.09	1.59	1.84	0.85	-0.01	0.80
time (sec)	N/A	0.096	0.313	0.014	1.011	1.029	51.181	0.415	0.000	0.206

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	57	53	90	99	209	73	96	56
N.S.	1	1.00	0.78	0.73	1.23	1.36	2.86	1.00	1.32	0.77
time (sec)	N/A	0.059	0.038	0.007	0.996	0.838	2.724	0.281	0.627	0.042

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	130	177	162	260	287	133	-1	127
N.S.	1	1.00	0.84	1.14	1.05	1.68	1.85	0.86	-0.01	0.82
time (sec)	N/A	0.067	0.256	0.010	1.049	1.249	26.192	0.376	0.000	0.186

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	34	31	50	73	158	44	76	34
N.S.	1	1.00	0.74	0.67	1.09	1.59	3.43	0.96	1.65	0.74
time (sec)	N/A	0.032	0.023	0.004	1.011	0.959	1.537	0.426	0.563	0.029

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	109	131	116	207	253	102	-1	102
N.S.	1	1.00	0.92	1.11	0.98	1.75	2.14	0.86	-0.01	0.86
time (sec)	N/A	0.041	0.206	0.007	0.963	0.903	16.545	0.416	0.000	0.164

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	70	58	170	71	79	60	83
N.S.	1	1.00	1.00	0.92	0.76	2.24	0.93	1.04	0.79	1.09
time (sec)	N/A	0.053	0.065	0.011	1.035	0.868	61.943	0.354	0.972	0.062

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	87	125	91	182	216	114	80	86
N.S.	1	1.00	0.80	1.15	0.83	1.67	1.98	1.05	0.73	0.79
time (sec)	N/A	0.041	0.191	0.010	1.140	0.710	12.197	0.436	1.504	0.205

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	80	132	109	167	184	103	94	85
N.S.	1	1.00	0.73	1.20	0.99	1.52	1.67	0.94	0.85	0.77
time (sec)	N/A	0.080	0.054	0.010	1.030	0.909	71.665	0.406	1.491	0.135

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	83	168	115	166	202	207	-1	88
N.S.	1	1.00	0.70	1.41	0.97	1.39	1.70	1.74	-0.01	0.74
time (sec)	N/A	0.047	0.083	0.012	1.009	0.979	7.754	0.522	0.000	0.232

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	59	184	161	189	216	131	104	83
N.S.	1	1.00	0.51	1.60	1.40	1.64	1.88	1.14	0.90	0.72
time (sec)	N/A	0.084	0.029	0.012	1.050	0.935	154.891	0.471	1.903	0.146

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	76	115	88	184	184	236	-1	92
N.S.	1	1.00	0.88	1.34	1.02	2.14	2.14	2.74	-0.01	1.07
time (sec)	N/A	0.035	0.037	0.010	1.109	0.818	6.741	0.443	0.000	0.212

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	119	233	210	222	0	159	130	103
N.S.	1	1.00	0.99	1.94	1.75	1.85	0.00	1.32	1.08	0.86
time (sec)	N/A	0.096	0.082	0.014	1.089	0.982	0.000	0.335	2.626	0.187

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	40	37	56	78	518	344	128	86
N.S.	1	1.00	0.75	0.70	1.06	1.47	9.77	6.49	2.42	1.62
time (sec)	N/A	0.022	0.016	0.006	1.050	0.960	6.486	0.466	1.518	0.221

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	62	275	252	271	0	194	169	128
N.S.	1	1.00	0.40	1.76	1.62	1.74	0.00	1.24	1.08	0.82
time (sec)	N/A	0.123	0.028	0.021	1.119	0.709	0.000	0.414	3.571	0.228

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	63	59	96	105	1408	400	170	110
N.S.	1	1.00	0.75	0.70	1.14	1.25	16.76	4.76	2.02	1.31
time (sec)	N/A	0.036	0.044	0.007	1.221	0.997	7.149	0.495	2.134	0.253

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	61	317	294	317	0	212	205	152
N.S.	1	1.00	0.33	1.72	1.60	1.72	0.00	1.15	1.11	0.83
time (sec)	N/A	0.146	0.043	0.036	1.202	1.078	0.000	0.480	4.889	0.258

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	78	77	132	147	313	104	136	80
N.S.	1	1.00	0.76	0.75	1.28	1.43	3.04	1.01	1.32	0.78
time (sec)	N/A	0.076	0.069	0.009	1.118	0.679	10.417	0.342	0.815	0.055

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	172	257	242	355	405	195	-1	175
N.S.	1	1.00	0.78	1.16	1.10	1.61	1.83	0.88	-0.00	0.79
time (sec)	N/A	0.103	0.451	0.018	1.118	1.057	83.278	0.352	0.000	0.286

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	57	53	90	122	260	73	115	56
N.S.	1	1.00	0.78	0.73	1.23	1.67	3.56	1.00	1.58	0.77
time (sec)	N/A	0.056	0.044	0.006	1.061	0.724	8.852	0.407	0.689	0.044

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	151	215	200	308	348	165	-1	151
N.S.	1	1.00	0.80	1.14	1.06	1.64	1.85	0.88	-0.01	0.80
time (sec)	N/A	0.088	0.311	0.007	1.056	1.137	59.841	0.406	0.000	0.277

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	34	31	50	97	209	44	44	34
N.S.	1	1.00	0.74	0.67	1.09	2.11	4.54	0.96	0.96	0.74
time (sec)	N/A	0.035	0.027	0.003	1.027	0.913	5.658	0.461	0.675	0.029

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	130	166	151	257	316	134	-1	126
N.S.	1	1.00	0.87	1.11	1.01	1.72	2.12	0.90	-0.01	0.85
time (sec)	N/A	0.054	0.225	0.006	1.130	1.091	32.842	0.508	0.000	0.233

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	88	85	73	220	88	97	78	107
N.S.	1	1.00	0.93	0.89	0.77	2.32	0.93	1.02	0.82	1.13
time (sec)	N/A	0.064	0.099	0.008	1.058	1.105	82.468	0.368	1.035	0.082

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	125	158	124	236	306	146	80	112
N.S.	1	1.00	0.92	1.16	0.91	1.74	2.25	1.07	0.59	0.82
time (sec)	N/A	0.053	0.358	0.010	0.986	1.128	24.288	0.542	1.916	0.254

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	105	161	138	221	296	139	132	109
N.S.	1	1.00	0.78	1.19	1.02	1.64	2.19	1.03	0.98	0.81
time (sec)	N/A	0.100	0.072	0.010	1.060	1.197	68.238	0.369	1.854	0.188

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	84	204	151	220	299	238	-1	115
N.S.	1	1.00	0.58	1.40	1.03	1.51	2.05	1.63	-0.01	0.79
time (sec)	N/A	0.060	0.037	0.012	1.054	0.883	15.708	0.471	0.000	0.381

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	60	213	190	221	279	171	144	112
N.S.	1	1.00	0.42	1.49	1.33	1.55	1.95	1.20	1.01	0.78
time (sec)	N/A	0.101	0.031	0.011	1.058	0.941	165.194	0.352	2.586	0.175

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	84	251	198	220	292	321	-1	112
N.S.	1	1.00	0.55	1.65	1.30	1.45	1.92	2.11	-0.01	0.74
time (sec)	N/A	0.062	0.043	0.013	1.115	1.118	11.539	0.572	0.000	0.287

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	61	266	243	241	0	167	150	109
N.S.	1	1.00	0.41	1.79	1.63	1.62	0.00	1.12	1.01	0.73
time (sec)	N/A	0.110	0.032	0.015	1.196	1.032	0.000	0.363	3.369	0.181

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	78	155	128	234	592	320	-1	116
N.S.	1	1.00	0.72	1.44	1.19	2.17	5.48	2.96	-0.01	1.07
time (sec)	N/A	0.046	0.092	0.017	1.089	0.854	14.265	0.576	0.000	0.300

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	140	311	288	272	0	195	169	128
N.S.	1	1.00	0.92	2.05	1.89	1.79	0.00	1.28	1.11	0.84
time (sec)	N/A	0.118	0.142	0.022	1.167	0.939	0.000	0.444	4.581	0.221

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	40	37	56	102	1489	456	170	110
N.S.	1	1.00	0.75	0.70	1.06	1.92	28.09	8.60	3.21	2.08
time (sec)	N/A	0.022	0.017	0.006	1.052	1.114	13.950	0.473	2.718	0.324

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	62	353	330	319	0	230	205	152
N.S.	1	1.00	0.33	1.87	1.75	1.69	0.00	1.22	1.08	0.80
time (sec)	N/A	0.148	0.028	0.043	1.270	0.977	0.000	0.348	6.060	0.297

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	78	77	132	76	172	101	80	80
N.S.	1	1.00	0.78	0.77	1.32	0.76	1.72	1.01	0.80	0.80
time (sec)	N/A	0.076	0.053	0.007	0.959	1.021	2.502	0.305	0.666	0.055

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	100	143	128	211	235	107	-1	103
N.S.	1	1.00	0.82	1.17	1.05	1.73	1.93	0.88	-0.01	0.84
time (sec)	N/A	0.053	0.097	0.010	1.062	0.989	12.430	0.453	0.000	0.182

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	56	53	90	52	121	69	57	56
N.S.	1	1.00	0.79	0.75	1.27	0.73	1.70	0.97	0.80	0.79
time (sec)	N/A	0.055	0.036	0.007	1.024	0.845	1.217	0.334	0.597	0.038

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	74	101	86	162	150	75	-1	79
N.S.	1	1.00	0.83	1.13	0.97	1.82	1.69	0.84	-0.01	0.89
time (sec)	N/A	0.036	0.063	0.008	1.083	0.707	7.779	0.359	0.000	0.137

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	33	30	49	29	70	38	34	33
N.S.	1	1.00	0.77	0.70	1.14	0.67	1.63	0.88	0.79	0.77
time (sec)	N/A	0.034	0.025	0.006	0.995	0.830	0.647	0.305	0.568	0.026

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	57	62	47	110	126	48	86	59
N.S.	1	1.00	0.98	1.07	0.81	1.90	2.17	0.83	1.48	1.02
time (sec)	N/A	0.017	0.018	0.005	1.158	1.120	3.306	0.398	1.071	0.059

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	45	33	102	61	38	35	43
N.S.	1	1.00	1.00	1.05	0.77	2.37	1.42	0.88	0.81	1.00
time (sec)	N/A	0.033	0.022	0.008	1.055	0.966	13.200	0.366	1.018	0.038

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	41	33	109	99	58	40	50
N.S.	1	1.00	1.00	0.87	0.70	2.32	2.11	1.23	0.85	1.06
time (sec)	N/A	0.018	0.018	0.010	1.095	0.672	2.645	0.414	0.737	0.079

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	60	79	56	124	66	62	60	58
N.S.	1	1.00	1.03	1.36	0.97	2.14	1.14	1.07	1.03	1.00
time (sec)	N/A	0.046	0.032	0.010	1.106	0.888	39.416	0.373	1.342	0.073

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	39	36	56	34	70	120	35	40
N.S.	1	1.00	0.74	0.68	1.06	0.64	1.32	2.26	0.66	0.75
time (sec)	N/A	0.020	0.014	0.006	1.121	0.990	3.869	0.433	0.578	0.094

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	83	119	96	171	150	121	99	80
N.S.	1	1.00	0.92	1.32	1.07	1.90	1.67	1.34	1.10	0.89
time (sec)	N/A	0.068	0.171	0.010	1.064	1.079	88.380	0.341	1.533	0.116

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	62	59	96	58	355	176	58	62
N.S.	1	1.00	0.74	0.70	1.14	0.69	4.23	2.10	0.69	0.74
time (sec)	N/A	0.035	0.020	0.006	1.021	1.164	3.174	0.420	0.682	0.121

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	61	161	138	223	235	158	140	104
N.S.	1	1.00	0.50	1.31	1.12	1.81	1.91	1.28	1.14	0.85
time (sec)	N/A	0.094	0.020	0.015	0.996	0.740	133.683	0.378	1.679	0.208

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	84	83	138	82	819	232	105	86
N.S.	1	1.00	0.72	0.71	1.18	0.70	7.00	1.98	0.90	0.74
time (sec)	N/A	0.047	0.030	0.006	1.098	1.062	3.679	0.416	0.733	0.149

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	131	185	170	325	233	136	-1	127
N.S.	1	1.00	0.86	1.22	1.12	2.14	1.53	0.89	-0.01	0.84
time (sec)	N/A	0.068	0.164	0.019	1.060	1.152	35.608	0.370	0.000	0.238

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	77	77	132	88	172	113	89	80
N.S.	1	1.00	0.78	0.78	1.33	0.89	1.74	1.14	0.90	0.81
time (sec)	N/A	0.076	0.045	0.006	1.089	0.715	3.638	0.344	0.819	0.051

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	108	141	126	274	177	104	-1	101
N.S.	1	1.00	0.91	1.18	1.06	2.30	1.49	0.87	-0.01	0.85
time (sec)	N/A	0.051	0.117	0.011	1.146	1.079	16.631	0.373	0.000	0.184

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	55	52	89	63	117	77	59	55
N.S.	1	1.00	0.82	0.78	1.33	0.94	1.75	1.15	0.88	0.82
time (sec)	N/A	0.056	0.031	0.005	1.043	0.872	2.022	0.378	0.679	0.042

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	86	97	82	213	114	70	-1	75
N.S.	1	1.00	1.04	1.17	0.99	2.57	1.37	0.84	-0.01	0.90
time (sec)	N/A	0.057	0.100	0.010	1.007	0.713	14.258	0.467	0.000	0.151

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	30	30	49	40	66	36	30	30
N.S.	1	1.00	0.73	0.73	1.20	0.98	1.61	0.88	0.73	0.73
time (sec)	N/A	0.032	0.020	0.004	1.040	1.063	0.677	0.432	0.595	0.031

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	70	54	46	168	60	51	53	58
N.S.	1	1.00	1.30	1.00	0.85	3.11	1.11	0.94	0.98	1.07
time (sec)	N/A	0.018	0.053	0.007	1.006	1.038	10.196	0.439	0.772	0.094

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	53	60	48	167	48	52	50	53
N.S.	1	1.00	1.00	1.13	0.91	3.15	0.91	0.98	0.94	1.00
time (sec)	N/A	0.039	0.027	0.010	1.057	0.948	23.905	0.289	1.058	0.071

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	36	36	51	43	68	57	46	36
N.S.	1	1.00	0.77	0.77	1.09	0.91	1.45	1.21	0.98	0.77
time (sec)	N/A	0.019	0.012	0.005	1.045	1.091	13.666	0.506	0.538	0.078

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	57	109	86	232	262	99	90	77
N.S.	1	1.00	0.66	1.27	1.00	2.70	3.05	1.15	1.05	0.90
time (sec)	N/A	0.067	0.019	0.010	1.079	1.217	41.260	0.354	1.440	0.141

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	61	58	92	68	284	181	57	62
N.S.	1	1.00	0.74	0.71	1.12	0.83	3.46	2.21	0.70	0.76
time (sec)	N/A	0.031	0.018	0.007	1.008	0.947	9.987	0.572	0.665	0.129

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	120	60	153	130	287	180	137	134	102
N.S.	1	1.02	0.51	1.30	1.10	2.43	1.53	1.16	1.14	0.86
time (sec)	N/A	0.088	0.073	0.012	1.044	1.048	84.399	0.371	1.907	0.164

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	60	83	134	94	593	294	82	86
N.S.	1	1.00	0.52	0.72	1.17	0.82	5.16	2.56	0.71	0.75
time (sec)	N/A	0.045	0.024	0.007	1.052	1.038	17.525	0.474	0.826	0.163

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	62	197	174	341	236	180	178	128
N.S.	1	1.00	0.41	1.29	1.14	2.23	1.54	1.18	1.16	0.84
time (sec)	N/A	0.117	0.022	0.014	1.049	0.745	141.330	0.380	2.246	0.235

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	71	107	176	117	1030	407	148	110
N.S.	1	1.00	0.48	0.72	1.19	0.79	6.96	2.75	1.00	0.74
time (sec)	N/A	0.061	0.028	0.007	1.063	1.160	19.080	0.541	1.074	0.201

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	98	101	174	123	437	141	122	104
N.S.	1	1.00	0.77	0.79	1.36	0.96	3.41	1.10	0.95	0.81
time (sec)	N/A	0.102	0.069	0.007	1.212	1.200	4.207	0.364	1.005	0.063

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	139	181	210	392	804	148	-1	125
N.S.	1	1.00	0.93	1.21	1.41	2.63	5.40	0.99	-0.01	0.84
time (sec)	N/A	0.062	0.235	0.020	1.094	1.294	38.816	0.578	0.000	0.276

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	73	76	131	98	337	104	89	79
N.S.	1	1.00	0.75	0.78	1.35	1.01	3.47	1.07	0.92	0.81
time (sec)	N/A	0.076	0.049	0.007	1.161	0.859	1.987	0.446	0.862	0.053

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	116	134	160	333	675	112	-1	98
N.S.	1	1.00	1.02	1.18	1.40	2.92	5.92	0.98	-0.01	0.86
time (sec)	N/A	0.090	0.260	0.010	1.126	0.703	20.567	0.501	0.000	0.209

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	54	53	89	75	240	62	59	56
N.S.	1	1.00	0.79	0.78	1.31	1.10	3.53	0.91	0.87	0.82
time (sec)	N/A	0.054	0.034	0.006	1.074	0.865	1.972	0.337	0.668	0.042

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	96	92	103	245	352	69	-1	75
N.S.	1	1.00	1.25	1.19	1.34	3.18	4.57	0.90	-0.01	0.97
time (sec)	N/A	0.030	0.162	0.010	1.017	0.946	14.517	0.383	0.000	0.143

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	34	30	50	52	143	32	32	34
N.S.	1	1.00	0.77	0.68	1.14	1.18	3.25	0.73	0.73	0.77
time (sec)	N/A	0.033	0.022	0.004	1.017	1.030	1.475	0.340	0.542	0.033

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	37	34	68	54	144	40	33	37
N.S.	1	1.00	0.79	0.72	1.45	1.15	3.06	0.85	0.70	0.79
time (sec)	N/A	0.010	0.014	0.005	1.008	1.039	11.433	0.402	0.552	0.098

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	61	75	63	241	66	66	65	69
N.S.	1	1.00	0.85	1.04	0.88	3.35	0.92	0.92	0.90	0.96
time (sec)	N/A	0.049	0.022	0.010	1.038	1.129	42.168	0.310	1.114	0.079

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	60	59	85	77	265	101	68	62
N.S.	1	1.00	0.78	0.77	1.10	1.00	3.44	1.31	0.88	0.81
time (sec)	N/A	0.027	0.037	0.005	1.126	0.951	21.736	0.421	0.619	0.129

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	57	140	117	349	1608	101	126	99
N.S.	1	1.00	0.50	1.24	1.04	3.09	14.23	0.89	1.12	0.88
time (sec)	N/A	0.087	0.039	0.018	0.953	1.079	68.639	0.358	1.546	0.144

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	79	82	128	101	524	224	123	86
N.S.	1	1.00	0.73	0.76	1.19	0.94	4.85	2.07	1.14	0.80
time (sec)	N/A	0.049	0.025	0.007	1.126	0.723	30.025	0.468	0.762	0.148

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	150	60	187	164	407	1323	165	176	126
N.S.	1	1.03	0.41	1.28	1.12	2.79	9.06	1.13	1.21	0.86
time (sec)	N/A	0.110	0.022	0.013	1.100	1.061	139.239	0.411	2.017	0.194

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	72	107	172	129	944	336	231	110
N.S.	1	1.00	0.49	0.73	1.18	0.88	6.47	2.30	1.58	0.75
time (sec)	N/A	0.057	0.043	0.007	1.181	1.230	60.742	0.578	0.999	0.211

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	132	149	249	179	389	204	171	152
N.S.	1	1.00	0.84	0.95	1.59	1.14	2.48	1.30	1.09	0.97
time (sec)	N/A	0.129	0.132	0.008	1.079	1.534	5.958	0.287	0.756	0.067

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	99	108	181	140	308	150	137	111
N.S.	1	1.00	0.87	0.95	1.59	1.23	2.70	1.32	1.20	0.97
time (sec)	N/A	0.096	0.112	0.008	1.034	1.138	3.241	0.353	0.654	0.057

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	67	69	115	103	226	98	101	72
N.S.	1	1.00	0.87	0.90	1.49	1.34	2.94	1.27	1.31	0.94
time (sec)	N/A	0.060	0.066	0.007	0.919	0.959	1.603	0.292	0.626	0.039

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	93	100	88	207	90	101	135	100
N.S.	1	1.00	1.01	1.09	0.96	2.25	0.98	1.10	1.47	1.09
time (sec)	N/A	0.084	0.137	0.013	1.025	0.864	72.465	0.316	0.695	0.093

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	87	132	109	211	148	89	103	94
N.S.	1	1.00	0.80	1.21	1.00	1.94	1.36	0.82	0.94	0.86
time (sec)	N/A	0.087	0.114	0.014	1.064	1.602	73.979	0.380	1.059	0.135

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	140	104	207	173	225	0	153	137	104
N.S.	1	0.98	0.73	1.45	1.21	1.57	0.00	1.07	0.96	0.73
time (sec)	N/A	0.160	0.121	0.016	1.072	1.268	0.000	0.355	1.328	0.221

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	142	281	247	276	0	222	193	135
N.S.	1	1.00	0.95	1.89	1.66	1.85	0.00	1.49	1.30	0.91
time (sec)	N/A	0.149	0.174	0.017	1.023	1.812	0.000	0.485	1.838	0.229

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	188	157	259	237	341	411	174	-1	173
N.S.	1	0.98	0.82	1.36	1.24	1.79	2.15	0.91	-0.01	0.91
time (sec)	N/A	0.186	0.162	0.015	1.110	1.661	21.912	0.492	0.000	0.217

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	122	190	168	262	291	128	-1	132
N.S.	1	1.00	0.82	1.28	1.13	1.76	1.95	0.86	-0.01	0.89
time (sec)	N/A	0.095	0.132	0.009	1.004	1.769	13.722	0.385	0.000	0.178

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	130	99	163	120	215	219	126	-1	106
N.S.	1	0.98	0.74	1.23	0.90	1.62	1.65	0.95	-0.01	0.80
time (sec)	N/A	0.086	0.176	0.013	1.078	1.425	9.049	0.471	0.000	0.190

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	91	122	86	210	170	188	-1	97
N.S.	1	1.00	0.82	1.10	0.77	1.89	1.53	1.69	-0.01	0.87
time (sec)	N/A	0.065	0.174	0.013	1.076	1.638	5.465	0.457	0.000	0.213

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	104	123	94	221	199	403	-1	113
N.S.	1	1.00	1.01	1.19	0.91	2.15	1.93	3.91	-0.01	1.10
time (sec)	N/A	0.058	0.147	0.015	1.014	1.519	5.500	0.466	0.000	0.213

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	100	76	78	124	107	510	490	181	120
N.S.	1	1.01	0.77	0.79	1.25	1.08	5.15	4.95	1.83	1.21
time (sec)	N/A	0.075	0.052	0.008	1.099	1.430	4.514	0.444	1.632	0.206

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	144	108	117	190	147	1061	579	249	161
N.S.	1	1.01	0.76	0.82	1.33	1.03	7.42	4.05	1.74	1.13
time (sec)	N/A	0.132	0.148	0.008	1.162	2.038	5.359	0.438	2.291	0.242

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	190	141	158	258	185	1856	668	317	202
N.S.	1	1.01	0.75	0.84	1.37	0.98	9.82	3.53	1.68	1.07
time (sec)	N/A	0.171	0.138	0.007	1.121	2.329	8.284	0.421	2.991	0.288

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	278	225	389	367	494	598	263	-1	255
N.S.	1	0.99	0.80	1.38	1.31	1.76	2.13	0.94	-0.00	0.91
time (sec)	N/A	0.266	0.212	0.024	1.136	2.506	83.439	0.494	0.000	0.369

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	100	108	181	179	384	150	170	111
N.S.	1	1.00	0.88	0.95	1.59	1.57	3.37	1.32	1.49	0.97
time (sec)	N/A	0.090	0.166	0.010	1.113	1.196	8.399	0.374	0.823	0.069

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	235	232	193	321	299	419	505	219	-1	214
N.S.	1	0.99	0.82	1.37	1.27	1.78	2.15	0.93	-0.00	0.91
time (sec)	N/A	0.218	0.178	0.013	1.040	1.874	53.694	0.431	0.000	0.311

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	67	69	115	141	303	98	136	72
N.S.	1	1.00	0.87	0.90	1.49	1.83	3.94	1.27	1.77	0.94
time (sec)	N/A	0.060	0.047	0.009	1.001	1.402	3.678	0.319	0.726	0.048

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	196	196	159	249	227	344	440	175	-1	173
N.S.	1	1.00	0.81	1.27	1.16	1.76	2.24	0.89	-0.01	0.88
time (sec)	N/A	0.124	0.095	0.009	1.145	1.308	30.552	0.462	0.000	0.267

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	110	115	103	282	109	121	191	140
N.S.	1	1.00	0.99	1.04	0.93	2.54	0.98	1.09	1.72	1.26
time (sec)	N/A	0.094	0.077	0.011	1.112	1.407	103.741	0.382	0.703	0.111

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	172	135	221	178	293	367	173	-1	147
N.S.	1	0.98	0.77	1.26	1.02	1.67	2.10	0.99	-0.01	0.84
time (sec)	N/A	0.119	0.149	0.013	1.144	1.362	22.413	0.496	0.000	0.297

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	108	161	138	267	303	126	201	135
N.S.	1	1.00	0.79	1.18	1.01	1.96	2.23	0.93	1.48	0.99
time (sec)	N/A	0.109	0.094	0.013	1.145	1.627	88.857	0.365	1.167	0.169

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	181	118	241	177	266	352	262	-1	122
N.S.	1	0.98	0.64	1.31	0.96	1.45	1.91	1.42	-0.01	0.66
time (sec)	N/A	0.130	0.104	0.015	1.129	1.576	14.127	0.463	0.000	0.323

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	178	116	256	222	267	0	182	208	119
N.S.	1	0.98	0.64	1.41	1.23	1.48	0.00	1.01	1.15	0.66
time (sec)	N/A	0.207	0.112	0.014	1.129	1.474	0.000	0.346	1.645	0.191

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	113	203	147	266	304	407	-1	138
N.S.	1	1.00	0.77	1.38	1.00	1.81	2.07	2.77	-0.01	0.94
time (sec)	N/A	0.090	0.111	0.016	1.066	1.560	8.786	0.532	0.000	0.343

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	184	92	335	301	301	0	259	215	144
N.S.	1	0.98	0.49	1.79	1.61	1.61	0.00	1.39	1.15	0.77
time (sec)	N/A	0.222	0.048	0.016	0.953	1.691	0.000	0.322	2.345	0.243

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	99	108	181	216	468	150	207	111
N.S.	1	1.00	0.87	0.95	1.59	1.89	4.11	1.32	1.82	0.97
time (sec)	N/A	0.090	0.109	0.009	0.965	1.521	19.940	0.272	0.918	0.066

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	278	226	383	361	495	602	265	-1	255
N.S.	1	0.99	0.80	1.36	1.28	1.76	2.14	0.94	-0.00	0.91
time (sec)	N/A	0.258	0.165	0.020	0.966	2.124	96.020	0.486	0.000	0.406

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	67	69	115	178	384	98	98	72
N.S.	1	1.00	0.87	0.90	1.49	2.31	4.99	1.27	1.27	0.94
time (sec)	N/A	0.059	0.050	0.007	0.894	1.457	14.545	0.323	0.797	0.049

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	192	308	286	420	537	221	-1	214
N.S.	1	1.00	0.80	1.28	1.19	1.75	2.24	0.92	-0.00	0.89
time (sec)	N/A	0.152	0.138	0.019	0.976	1.884	96.678	0.507	0.000	0.367

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	123	132	120	360	128	141	249	181
N.S.	1	1.00	0.93	1.00	0.91	2.73	0.97	1.07	1.89	1.37
time (sec)	N/A	0.111	0.119	0.013	0.892	1.643	128.707	0.443	0.775	0.119

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	214	174	278	235	375	496	219	-1	190
N.S.	1	0.99	0.80	1.28	1.08	1.73	2.29	1.01	-0.00	0.88
time (sec)	N/A	0.141	0.130	0.012	0.966	1.692	44.324	0.564	0.000	0.397

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	122	193	170	349	518	165	274	176
N.S.	1	1.00	0.75	1.19	1.05	2.15	3.20	1.02	1.69	1.09
time (sec)	N/A	0.132	0.244	0.013	0.948	1.575	91.815	0.503	1.451	0.192

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	219	155	298	234	346	490	307	-1	165
N.S.	1	0.98	0.70	1.34	1.05	1.55	2.20	1.38	-0.00	0.74
time (sec)	N/A	0.175	0.128	0.016	0.991	1.584	24.420	0.600	0.000	0.382

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	219	153	305	271	319	0	242	262	166
N.S.	1	0.99	0.69	1.37	1.22	1.44	0.00	1.09	1.18	0.75
time (sec)	N/A	0.253	0.113	0.016	0.942	1.497	0.000	0.526	1.885	0.204

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	225	158	369	285	318	474	510	-1	169
N.S.	1	0.99	0.69	1.62	1.25	1.39	2.08	2.24	-0.00	0.74
time (sec)	N/A	0.160	0.172	0.016	0.956	1.051	20.408	0.544	0.000	0.484

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	219	92	387	353	347	0	286	301	162
N.S.	1	0.99	0.41	1.74	1.59	1.56	0.00	1.29	1.36	0.73
time (sec)	N/A	0.252	0.061	0.018	0.960	1.696	0.000	0.469	2.723	0.262

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	159	265	243	344	422	178	-1	173
N.S.	1	1.00	0.82	1.37	1.25	1.77	2.18	0.92	-0.01	0.89
time (sec)	N/A	0.155	0.158	0.018	0.940	1.174	28.183	0.414	0.000	0.312

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	99	108	181	103	240	137	105	111
N.S.	1	1.00	0.88	0.96	1.62	0.92	2.14	1.22	0.94	0.99
time (sec)	N/A	0.089	0.101	0.010	0.906	1.404	1.810	0.358	0.664	0.057

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	125	197	175	267	301	135	-1	132
N.S.	1	1.00	0.86	1.35	1.20	1.83	2.06	0.92	-0.01	0.90
time (sec)	N/A	0.140	0.129	0.011	0.916	0.918	16.346	0.372	0.000	0.223

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	66	69	114	68	158	84	68	72
N.S.	1	1.00	0.89	0.93	1.54	0.92	2.14	1.14	0.92	0.97
time (sec)	N/A	0.054	0.052	0.007	0.892	1.476	1.885	0.333	0.655	0.042

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	91	131	109	194	238	91	-1	95
N.S.	1	1.00	0.85	1.22	1.02	1.81	2.22	0.85	-0.01	0.89
time (sec)	N/A	0.059	0.059	0.009	0.886	1.429	14.272	0.493	0.000	0.110

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	63	87	75	157	76	82	77	67
N.S.	1	1.00	0.84	1.16	1.00	2.09	1.01	1.09	1.03	0.89
time (sec)	N/A	0.071	0.078	0.014	0.868	1.662	53.563	0.295	0.718	0.067

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	76	88	73	165	155	93	125	81
N.S.	1	1.00	0.93	1.07	0.89	2.01	1.89	1.13	1.52	0.99
time (sec)	N/A	0.045	0.079	0.012	0.883	0.970	8.038	0.463	1.464	0.145

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	77	100	77	175	99	81	65	79
N.S.	1	1.00	0.96	1.25	0.96	2.19	1.24	1.01	0.81	0.99
time (sec)	N/A	0.068	0.065	0.018	0.914	1.540	130.447	0.492	0.936	0.127

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	72	85	77	173	158	156	-1	77
N.S.	1	1.00	0.86	1.01	0.92	2.06	1.88	1.86	-0.01	0.92
time (sec)	N/A	0.050	0.109	0.013	0.883	1.539	5.416	0.459	0.000	0.144

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	92	157	123	204	0	140	129	96
N.S.	1	1.00	0.87	1.48	1.16	1.92	0.00	1.32	1.22	0.91
time (sec)	N/A	0.106	0.113	0.012	0.892	0.820	0.000	0.385	1.063	0.182

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	100	74	78	124	73	391	312	77	81
N.S.	1	1.01	0.75	0.79	1.25	0.74	3.95	3.15	0.78	0.82
time (sec)	N/A	0.072	0.022	0.007	0.907	1.308	7.484	0.458	0.727	0.151

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	135	224	190	279	0	241	207	135
N.S.	1	1.00	0.89	1.48	1.26	1.85	0.00	1.60	1.37	0.89
time (sec)	N/A	0.161	0.249	0.014	0.918	1.428	0.000	0.341	1.148	0.334

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	158	263	241	431	0	175	-1	171
N.S.	1	1.00	0.80	1.34	1.22	2.19	0.00	0.89	-0.01	0.87
time (sec)	N/A	0.149	0.158	0.022	0.959	1.441	0.000	0.476	0.000	0.308

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	97	108	180	115	236	149	107	111
N.S.	1	1.00	0.90	1.00	1.67	1.06	2.19	1.38	0.99	1.03
time (sec)	N/A	0.086	0.059	0.007	0.924	0.751	1.991	0.407	0.885	0.059

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	124	192	170	350	0	131	-1	129
N.S.	1	1.00	0.82	1.26	1.12	2.30	0.00	0.86	-0.01	0.85
time (sec)	N/A	0.117	0.118	0.010	0.866	1.716	0.000	0.426	0.000	0.218

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	65	69	115	79	155	92	75	71
N.S.	1	1.00	0.89	0.95	1.58	1.08	2.12	1.26	1.03	0.97
time (sec)	N/A	0.056	0.037	0.006	0.845	1.562	1.291	0.448	0.674	0.045

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	93	123	108	275	0	92	-1	99
N.S.	1	1.00	0.88	1.16	1.02	2.59	0.00	0.87	-0.01	0.93
time (sec)	N/A	0.054	0.110	0.009	0.915	1.380	0.000	0.458	0.000	0.164

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	62	102	90	232	70	82	76	78
N.S.	1	1.00	0.83	1.36	1.20	3.09	0.93	1.09	1.01	1.04
time (sec)	N/A	0.075	0.042	0.011	0.910	1.536	36.046	0.327	0.863	0.077

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	87	81	99	91	239	0	104	-1	92
N.S.	1	0.96	0.89	1.09	1.00	2.63	0.00	1.14	-0.01	1.01
time (sec)	N/A	0.063	0.104	0.011	0.902	1.549	0.000	0.467	0.000	0.149

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	96	135	112	292	0	140	119	103
N.S.	1	1.00	0.93	1.31	1.09	2.83	0.00	1.36	1.16	1.00
time (sec)	N/A	0.098	0.085	0.012	0.856	1.191	0.000	0.389	1.080	0.216

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	98	74	77	117	85	0	199	76	81
N.S.	1	1.01	0.76	0.79	1.21	0.88	0.00	2.05	0.78	0.84
time (sec)	N/A	0.072	0.024	0.006	0.884	1.362	0.000	0.446	0.760	0.156

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	89	211	177	364	0	163	179	132
N.S.	1	1.00	0.61	1.46	1.22	2.51	0.00	1.12	1.23	0.91
time (sec)	N/A	0.167	0.037	0.013	0.851	0.998	0.000	0.350	1.064	0.275

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	105	117	184	121	0	452	116	120
N.S.	1	1.00	0.74	0.83	1.30	0.86	0.00	3.21	0.82	0.85
time (sec)	N/A	0.114	0.086	0.009	0.833	1.671	0.000	0.503	0.847	0.200

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	193	92	281	247	447	0	267	246	174
N.S.	1	1.02	0.48	1.48	1.30	2.35	0.00	1.41	1.29	0.92
time (sec)	N/A	0.218	0.038	0.016	0.946	1.642	0.000	0.403	1.340	0.277

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	156	255	296	522	0	190	-1	168
N.S.	1	1.00	0.77	1.26	1.47	2.58	0.00	0.94	-0.00	0.83
time (sec)	N/A	0.155	0.180	0.022	0.886	1.356	0.000	0.507	0.000	0.351

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	98	108	181	124	454	140	107	110
N.S.	1	1.00	0.89	0.98	1.65	1.13	4.13	1.27	0.97	1.00
time (sec)	N/A	0.090	0.058	0.008	0.910	1.515	2.946	0.393	0.780	0.061

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	118	185	211	409	0	130	-1	129
N.S.	1	1.00	0.98	1.53	1.74	3.38	0.00	1.07	-0.01	1.07
time (sec)	N/A	0.108	0.115	0.012	0.865	1.228	0.000	0.469	0.000	0.250

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	67	68	114	91	303	79	76	72
N.S.	1	1.00	0.93	0.94	1.58	1.26	4.21	1.10	1.06	1.00
time (sec)	N/A	0.058	0.039	0.006	0.923	1.116	1.377	0.456	0.629	0.052

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	101	136	147	321	0	105	-1	107
N.S.	1	1.00	0.96	1.30	1.40	3.06	0.00	1.00	-0.01	1.02
time (sec)	N/A	0.050	0.141	0.009	0.950	1.392	0.000	0.521	0.000	0.187

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	67	120	108	316	87	102	90	99
N.S.	1	1.00	0.76	1.36	1.23	3.59	0.99	1.16	1.02	1.12
time (sec)	N/A	0.092	0.044	0.013	0.930	1.168	44.361	0.437	0.803	0.108

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	76	78	132	92	0	117	77	80
N.S.	1	1.00	0.84	0.87	1.47	1.02	0.00	1.30	0.86	0.89
time (sec)	N/A	0.046	0.027	0.008	0.928	0.988	0.000	0.431	0.647	0.171

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	105	169	146	426	0	128	147	130
N.S.	1	1.00	0.80	1.29	1.11	3.25	0.00	0.98	1.12	0.99
time (sec)	N/A	0.117	0.046	0.017	0.964	1.100	0.000	0.372	0.897	0.217

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	130	107	116	175	130	0	258	187	120
N.S.	1	0.99	0.82	0.89	1.34	0.99	0.00	1.97	1.43	0.92
time (sec)	N/A	0.125	0.073	0.007	0.931	1.340	0.000	0.445	0.733	0.176

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	90	265	231	537	0	210	216	171
N.S.	1	1.00	0.49	1.43	1.25	2.90	0.00	1.14	1.17	0.92
time (sec)	N/A	0.216	0.037	0.017	0.938	1.332	0.000	0.403	1.110	0.227

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	142	158	244	171	0	509	298	161
N.S.	1	1.00	0.78	0.86	1.33	0.93	0.00	2.78	1.63	0.88
time (sec)	N/A	0.166	0.182	0.009	0.964	1.277	0.000	0.540	0.979	0.266

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	56	53	67	147	0	70	51	71
N.S.	1	1.00	0.78	0.74	0.93	2.04	0.00	0.97	0.71	0.99
time (sec)	N/A	0.026	0.070	0.031	1.826	1.260	0.000	0.409	0.648	0.068

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	44	38	49	126	0	46	37	60
N.S.	1	1.00	0.85	0.73	0.94	2.42	0.00	0.88	0.71	1.15
time (sec)	N/A	0.016	0.017	0.006	1.793	1.235	0.000	0.430	0.618	0.049

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	24	23	94	0	23	23	42
N.S.	1	1.00	1.00	0.71	0.68	2.76	0.00	0.68	0.68	1.24
time (sec)	N/A	0.008	0.008	0.006	1.970	1.246	0.000	0.358	0.609	0.025

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	46	36	35	132	0	41	38	64
N.S.	1	1.00	0.92	0.72	0.70	2.64	0.00	0.82	0.76	1.28
time (sec)	N/A	0.017	0.018	0.010	1.989	0.791	0.000	0.283	0.620	0.051

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	58	58	52	157	0	60	53	75
N.S.	1	1.00	0.85	0.85	0.76	2.31	0.00	0.88	0.78	1.10
time (sec)	N/A	0.024	0.024	0.013	2.010	1.370	0.000	0.309	0.633	0.065

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	148	1088	0	857	0	0	-1	200
N.S.	1	1.00	0.94	6.93	0.00	5.46	0.00	0.00	-0.01	1.27
time (sec)	N/A	0.231	0.230	0.073	0.000	1.880	0.000	0.000	0.000	0.403

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	85	963	0	295	87	96	86	96
N.S.	1	1.00	0.97	10.94	0.00	3.35	0.99	1.09	0.98	1.09
time (sec)	N/A	0.081	0.068	0.013	0.000	1.429	7.230	0.317	0.592	0.108

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	108	1010	0	690	0	0	-1	167
N.S.	1	1.00	0.96	9.02	0.00	6.16	0.00	0.00	-0.01	1.49
time (sec)	N/A	0.096	0.130	0.013	0.000	1.181	0.000	0.000	0.000	0.187

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	936	0	255	61	64	53	74
N.S.	1	1.00	1.00	14.40	0.00	3.92	0.94	0.98	0.82	1.14
time (sec)	N/A	0.054	0.027	0.011	0.000	1.370	4.887	0.301	0.586	0.084

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	84	948	0	596	0	0	-1	138
N.S.	1	1.00	1.04	11.70	0.00	7.36	0.00	0.00	-0.01	1.70
time (sec)	N/A	0.044	0.035	0.009	0.000	0.964	0.000	0.000	0.000	0.203

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	78	984	0	578	78	78	103	91
N.S.	1	1.00	0.98	12.30	0.00	7.22	0.98	0.98	1.29	1.14
time (sec)	N/A	0.073	0.034	0.013	0.000	1.530	9.586	0.388	0.725	0.076

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	51	1017	0	273	0	117	-1	122
N.S.	1	1.00	0.73	14.53	0.00	3.90	0.00	1.67	-0.01	1.74
time (sec)	N/A	0.051	0.019	0.020	0.000	1.209	0.000	3.646	0.000	0.199

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	107	1054	0	708	0	106	268	122
N.S.	1	1.00	0.95	9.33	0.00	6.27	0.00	0.94	2.37	1.08
time (sec)	N/A	0.120	0.174	0.014	0.000	1.638	0.000	0.467	0.992	0.269

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	93	1059	0	325	0	215	-1	148
N.S.	1	1.00	0.89	10.09	0.00	3.10	0.00	2.05	-0.01	1.41
time (sec)	N/A	0.120	5.160	0.013	0.000	1.431	0.000	5.430	0.000	0.263

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	196	2081	0	1119	0	0	-1	234
N.S.	1	1.00	0.93	9.91	0.00	5.33	0.00	0.00	-0.00	1.11
time (sec)	N/A	0.403	0.310	0.022	0.000	7.624	0.000	0.000	0.000	0.416

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	108	1897	0	397	104	151	179	135
N.S.	1	1.00	0.94	16.50	0.00	3.45	0.90	1.31	1.56	1.17
time (sec)	N/A	0.106	0.124	0.015	0.000	1.379	46.420	0.368	0.655	0.181

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	139	1973	0	894	0	0	-1	178
N.S.	1	1.00	0.88	12.49	0.00	5.66	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.252	0.235	0.012	0.000	2.993	0.000	0.000	0.000	0.335

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	83	1856	0	303	80	112	98	93
N.S.	1	1.00	0.91	20.40	0.00	3.33	0.88	1.23	1.08	1.02
time (sec)	N/A	0.076	0.093	0.013	0.000	1.619	33.909	0.376	0.623	0.131

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	110	1875	0	721	0	0	-1	173
N.S.	1	1.00	0.97	16.59	0.00	6.38	0.00	0.00	-0.01	1.53
time (sec)	N/A	0.096	0.236	0.013	0.000	1.557	0.000	0.000	0.000	0.293

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	102	1919	0	682	92	110	711	106
N.S.	1	1.00	1.06	19.99	0.00	7.10	0.96	1.15	7.41	1.10
time (sec)	N/A	0.107	0.072	0.011	0.000	2.425	29.872	0.375	0.806	0.163

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	105	1956	0	718	0	0	-1	157
N.S.	1	1.00	1.03	19.18	0.00	7.04	0.00	0.00	-0.01	1.54
time (sec)	N/A	0.091	0.206	0.015	0.000	1.949	0.000	0.000	0.000	0.269

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	108	2003	0	732	0	120	560	128
N.S.	1	1.00	0.95	17.57	0.00	6.42	0.00	1.05	4.91	1.12
time (sec)	N/A	0.141	0.166	0.013	0.000	3.187	0.000	0.412	1.159	0.246

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	53	2089	0	331	0	256	-1	144
N.S.	1	1.00	0.52	20.48	0.00	3.25	0.00	2.51	-0.01	1.41
time (sec)	N/A	0.126	0.018	0.013	0.000	2.056	0.000	5.350	0.000	0.313

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	247	3373	0	1443	0	0	-1	320
N.S.	1	1.00	0.85	11.59	0.00	4.96	0.00	0.00	-0.00	1.10
time (sec)	N/A	0.575	0.373	0.023	0.000	21.592	0.000	0.000	0.000	0.774

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	136	3127	0	527	144	228	251	191
N.S.	1	1.00	0.94	21.72	0.00	3.66	1.00	1.58	1.74	1.33
time (sec)	N/A	0.150	0.308	0.013	0.000	1.165	84.634	0.467	0.603	0.193

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	217	187	3235	0	1161	0	0	-1	246
N.S.	1	1.00	0.86	14.91	0.00	5.35	0.00	0.00	-0.00	1.13
time (sec)	N/A	0.392	0.158	0.015	0.000	7.142	0.000	0.000	0.000	0.468

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	113	3078	0	405	117	184	137	131
N.S.	1	1.00	0.95	25.87	0.00	3.40	0.98	1.55	1.15	1.10
time (sec)	N/A	0.107	0.160	0.012	0.000	0.934	50.764	0.435	0.605	0.134

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	140	3101	0	931	0	0	-1	199
N.S.	1	1.00	0.90	19.88	0.00	5.97	0.00	0.00	-0.01	1.28
time (sec)	N/A	0.180	0.115	0.013	0.000	3.574	0.000	0.000	0.000	0.436

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	114	3148	0	837	119	163	2094	129
N.S.	1	1.00	0.92	25.39	0.00	6.75	0.96	1.31	16.89	1.04
time (sec)	N/A	0.191	0.118	0.016	0.000	3.470	67.140	0.383	1.000	0.175

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	132	3191	0	887	0	0	-1	180
N.S.	1	1.00	0.91	22.01	0.00	6.12	0.00	0.00	-0.01	1.24
time (sec)	N/A	0.197	0.092	0.015	0.000	2.502	0.000	0.000	0.000	0.398

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	125	3247	0	891	0	158	1428	145
N.S.	1	1.00	0.87	22.55	0.00	6.19	0.00	1.10	9.92	1.01
time (sec)	N/A	0.244	0.142	0.015	0.000	4.393	0.000	0.386	1.368	0.256

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	125	3346	0	901	0	0	-1	172
N.S.	1	1.00	0.96	25.74	0.00	6.93	0.00	0.00	-0.01	1.32
time (sec)	N/A	0.178	0.102	0.017	0.000	2.052	0.000	0.000	0.000	0.489

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	89	362	0	390	0	105	100	99
N.S.	1	1.00	0.89	3.62	0.00	3.90	0.00	1.05	1.00	0.99
time (sec)	N/A	0.106	0.131	0.022	0.000	1.272	0.000	0.380	0.689	0.153

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	318	0	306	0	64	57	78
N.S.	1	1.00	1.00	4.68	0.00	4.50	0.00	0.94	0.84	1.15
time (sec)	N/A	0.064	0.061	0.014	0.000	1.095	0.000	0.345	0.637	0.081

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	300	0	231	36	39	39	59
N.S.	1	1.00	1.00	6.12	0.00	4.71	0.73	0.80	0.80	1.20
time (sec)	N/A	0.043	0.015	0.010	0.000	1.202	6.895	0.325	0.648	0.048

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	78	331	0	603	63	70	651	90
N.S.	1	1.00	0.98	4.14	0.00	7.54	0.79	0.88	8.14	1.12
time (sec)	N/A	0.074	0.085	0.013	0.000	1.227	14.308	0.299	0.881	0.109

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	109	385	0	734	0	103	396	125
N.S.	1	1.00	0.95	3.35	0.00	6.38	0.00	0.90	3.44	1.09
time (sec)	N/A	0.115	0.380	0.015	0.000	1.387	0.000	0.351	1.112	0.206

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	112	386	0	717	0	0	-1	170
N.S.	1	1.00	0.98	3.39	0.00	6.29	0.00	0.00	-0.01	1.49
time (sec)	N/A	0.094	0.179	0.014	0.000	1.098	0.000	0.000	0.000	0.327

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	85	337	0	616	0	0	-1	137
N.S.	1	1.00	1.04	4.11	0.00	7.51	0.00	0.00	-0.01	1.67
time (sec)	N/A	0.050	0.068	0.013	0.000	1.258	0.000	0.000	0.000	0.198

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	306	0	241	0	70	-1	103
N.S.	1	1.00	1.00	6.24	0.00	4.92	0.00	1.43	-0.02	2.10
time (sec)	N/A	0.019	0.015	0.013	0.000	1.018	0.000	0.327	0.000	0.001

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	177	334	0	324	0	111	-1	137
N.S.	1	1.00	2.39	4.51	0.00	4.38	0.00	1.50	-0.01	1.85
time (sec)	N/A	0.053	3.943	0.015	0.000	1.307	0.000	0.320	0.000	0.211

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	96	379	0	414	0	195	-1	159
N.S.	1	1.00	0.87	3.45	0.00	3.76	0.00	1.77	-0.01	1.45
time (sec)	N/A	0.123	5.123	0.015	0.000	1.450	0.000	5.362	0.000	0.308

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	111	720	0	977	0	0	-1	165
N.S.	1	1.00	1.02	6.61	0.00	8.96	0.00	0.00	-0.01	1.51
time (sec)	N/A	0.101	0.176	0.020	0.000	1.737	0.000	0.000	0.000	0.341

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	86	653	0	428	0	78	64	87
N.S.	1	1.00	1.12	8.48	0.00	5.56	0.00	1.01	0.83	1.13
time (sec)	N/A	0.072	0.075	0.013	0.000	1.138	0.000	0.466	0.755	0.116

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	111	653	0	334	0	103	-1	126
N.S.	1	1.00	1.50	8.82	0.00	4.51	0.00	1.39	-0.01	1.70
time (sec)	N/A	0.049	0.381	0.017	0.000	1.220	0.000	0.464	0.000	0.239

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	50	618	0	323	61	71	61	81
N.S.	1	1.00	0.69	8.58	0.00	4.49	0.85	0.99	0.85	1.12
time (sec)	N/A	0.056	0.018	0.013	0.000	1.124	19.962	0.366	0.735	0.089

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	79	79	236	628	0	442	0	107	-1	133
N.S.	1	1.00	2.99	7.95	0.00	5.59	0.00	1.35	-0.01	1.68
time (sec)	N/A	0.037	2.961	0.010	0.000	1.171	0.000	0.306	0.000	0.001

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	87	681	0	959	94	110	2296	118
N.S.	1	1.00	0.81	6.36	0.00	8.96	0.88	1.03	21.46	1.10
time (sec)	N/A	0.112	0.037	0.015	0.000	2.021	21.003	0.364	1.415	0.201

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	102	695	0	560	0	152	-1	164
N.S.	1	1.00	0.82	5.60	0.00	4.52	0.00	1.23	-0.01	1.32
time (sec)	N/A	0.117	5.207	0.017	0.000	1.440	0.000	3.563	0.000	0.447

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	117	763	0	1291	0	172	3025	162
N.S.	1	1.00	0.75	4.89	0.00	8.28	0.00	1.10	19.39	1.04
time (sec)	N/A	0.216	0.044	0.018	0.000	2.656	0.000	0.450	1.979	0.306

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	124	762	0	706	0	275	-1	225
N.S.	1	1.00	0.70	4.33	0.00	4.01	0.00	1.56	-0.01	1.28
time (sec)	N/A	0.218	5.248	0.018	0.000	1.751	0.000	3.615	0.000	0.547

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	160	1207	0	524	0	304	-1	148
N.S.	1	1.00	1.37	10.32	0.00	4.48	0.00	2.60	-0.01	1.26
time (sec)	N/A	0.113	0.295	0.029	0.000	1.943	0.000	0.451	0.000	0.436

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	77	1123	0	535	0	127	110	109
N.S.	1	1.00	0.75	10.90	0.00	5.19	0.00	1.23	1.07	1.06
time (sec)	N/A	0.092	0.029	0.018	0.000	1.088	0.000	0.317	0.927	0.175

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	115	115	257	1134	0	550	0	291	-1	156
N.S.	1	1.00	2.23	9.86	0.00	4.78	0.00	2.53	-0.01	1.36
time (sec)	N/A	0.088	3.081	0.015	0.000	1.792	0.000	0.454	0.000	0.412

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	52	1086	0	511	85	118	103	101
N.S.	1	1.00	0.53	11.08	0.00	5.21	0.87	1.20	1.05	1.03
time (sec)	N/A	0.085	0.022	0.013	0.000	1.146	25.053	0.346	0.850	0.150

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	122	122	1385	1086	0	764	0	321	-1	171
N.S.	1	1.00	11.35	8.90	0.00	6.26	0.00	2.63	-0.01	1.40
time (sec)	N/A	0.101	6.261	0.013	0.000	1.609	0.000	0.513	0.000	0.439

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	90	1186	0	1711	133	176	4558	152
N.S.	1	1.00	0.62	8.18	0.00	11.80	0.92	1.21	31.43	1.05
time (sec)	N/A	0.185	0.032	0.016	0.000	3.201	26.340	0.343	2.481	0.334

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	143	1192	0	934	0	366	-1	241
N.S.	1	1.00	0.80	6.70	0.00	5.25	0.00	2.06	-0.01	1.35
time (sec)	N/A	0.236	5.268	0.017	0.000	2.170	0.000	3.802	0.000	0.611

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	118	1289	0	2219	0	211	5409	237
N.S.	1	1.00	0.56	6.11	0.00	10.52	0.00	1.00	25.64	1.12
time (sec)	N/A	0.322	0.053	0.016	0.000	6.035	0.000	0.362	3.420	0.747

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	160	1285	0	1128	0	490	-1	319
N.S.	1	1.00	0.65	5.24	0.00	4.60	0.00	2.00	-0.00	1.30
time (sec)	N/A	0.361	5.362	0.020	0.000	3.410	0.000	4.333	0.000	0.894

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	134	2615	0	1002	0	288	-1	169
N.S.	1	1.00	0.89	17.43	0.00	6.68	0.00	1.92	-0.01	1.13
time (sec)	N/A	0.169	0.180	0.026	0.000	1.947	0.000	0.603	0.000	0.901

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	117	2543	0	436	0	101	102	108
N.S.	1	1.00	0.86	18.70	0.00	3.21	0.00	0.74	0.75	0.79
time (sec)	N/A	0.111	0.145	0.014	0.000	0.966	0.000	0.319	0.902	0.277

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	118	2547	0	1069	0	251	-1	145
N.S.	1	1.00	0.98	21.22	0.00	8.91	0.00	2.09	-0.01	1.21
time (sec)	N/A	0.083	0.083	0.013	0.000	1.110	0.000	0.490	0.000	0.600

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	80	1617	0	356	0	79	70	90
N.S.	1	1.00	1.00	20.21	0.00	4.45	0.00	0.99	0.88	1.12
time (sec)	N/A	0.062	0.084	0.013	0.000	1.107	0.000	0.396	0.738	0.170

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	112	2559	0	369	0	218	-1	145
N.S.	1	1.00	1.37	31.21	0.00	4.50	0.00	2.66	-0.01	1.77
time (sec)	N/A	0.035	0.293	0.012	0.000	1.115	0.000	3.789	0.000	0.531

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	112	2585	0	1054	0	113	996	129
N.S.	1	1.00	0.94	21.72	0.00	8.86	0.00	0.95	8.37	1.08
time (sec)	N/A	0.112	0.219	0.014	0.000	1.506	0.000	0.356	1.157	0.528

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	101	2618	0	458	0	329	-1	133
N.S.	1	1.00	0.89	23.17	0.00	4.05	0.00	2.91	-0.01	1.18
time (sec)	N/A	0.110	5.073	0.020	0.000	1.128	0.000	4.066	0.000	0.530

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	190	2669	0	1043	0	183	1193	157
N.S.	1	1.00	1.19	16.79	0.00	6.56	0.00	1.15	7.50	0.99
time (sec)	N/A	0.209	0.233	0.022	0.000	1.639	0.000	0.360	1.689	0.561

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	120	2667	0	602	0	361	-1	170
N.S.	1	1.00	0.82	18.14	0.00	4.10	0.00	2.46	-0.01	1.16
time (sec)	N/A	0.203	5.144	0.019	0.000	1.316	0.000	4.516	0.000	0.916

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	192	4795	0	1249	0	394	-1	211
N.S.	1	1.00	0.97	24.34	0.00	6.34	0.00	2.00	-0.01	1.07
time (sec)	N/A	0.338	0.168	0.026	0.000	1.836	0.000	0.541	0.000	0.782

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	125	4673	0	413	0	173	183	150
N.S.	1	1.00	0.77	28.67	0.00	2.53	0.00	1.06	1.12	0.92
time (sec)	N/A	0.136	0.109	0.017	0.000	0.939	0.000	0.356	0.986	0.322

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	155	4685	0	996	0	336	-1	175
N.S.	1	1.00	1.04	31.44	0.00	6.68	0.00	2.26	-0.01	1.17
time (sec)	N/A	0.164	0.177	0.015	0.000	1.503	0.000	0.513	0.000	0.664

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	54	2821	0	333	0	122	117	106
N.S.	1	1.00	0.55	28.49	0.00	3.36	0.00	1.23	1.18	1.07
time (sec)	N/A	0.077	0.021	0.013	0.000	1.305	0.000	0.429	0.905	0.277

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	141	4689	0	903	0	315	-1	155
N.S.	1	1.00	1.08	35.79	0.00	6.89	0.00	2.40	-0.01	1.18
time (sec)	N/A	0.089	0.155	0.015	0.000	1.682	0.000	0.488	0.000	0.639

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	122	4718	0	883	0	154	488	154
N.S.	1	1.00	0.95	36.57	0.00	6.84	0.00	1.19	3.78	1.19
time (sec)	N/A	0.142	0.162	0.016	0.000	1.640	0.000	0.287	1.101	0.330

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	52	4764	0	351	0	412	-1	155
N.S.	1	1.00	0.41	37.22	0.00	2.74	0.00	3.22	-0.01	1.21
time (sec)	N/A	0.124	0.014	0.019	0.000	0.987	0.000	3.489	0.000	0.602

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	142	4820	0	1034	0	216	441	179
N.S.	1	1.00	0.84	28.35	0.00	6.08	0.00	1.27	2.59	1.05
time (sec)	N/A	0.255	0.324	0.016	0.000	1.555	0.000	0.439	1.737	0.672

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	131	4908	0	443	0	442	-1	154
N.S.	1	1.00	0.79	29.57	0.00	2.67	0.00	2.66	-0.01	0.93
time (sec)	N/A	0.240	5.160	0.017	0.000	1.154	0.000	4.270	0.000	0.634

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	219	7611	0	1697	0	521	-1	299
N.S.	1	1.00	0.85	29.50	0.00	6.58	0.00	2.02	-0.00	1.16
time (sec)	N/A	0.448	0.292	0.029	0.000	6.212	0.000	0.602	0.000	1.191

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	164	7443	0	573	0	264	276	205
N.S.	1	1.00	0.83	37.59	0.00	2.89	0.00	1.33	1.39	1.04
time (sec)	N/A	0.183	0.432	0.016	0.000	0.877	0.000	0.398	1.172	0.428

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	173	7459	0	1379	0	446	-1	245
N.S.	1	1.00	0.89	38.25	0.00	7.07	0.00	2.29	-0.01	1.26
time (sec)	N/A	0.244	0.230	0.019	0.000	3.139	0.000	0.572	0.000	1.022

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	54	4363	0	453	0	197	172	182
N.S.	1	1.00	0.43	34.63	0.00	3.60	0.00	1.56	1.37	1.44
time (sec)	N/A	0.103	0.021	0.014	0.000	0.947	0.000	0.397	1.036	0.355

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	143	7451	0	1228	0	407	-1	209
N.S.	1	1.00	0.82	42.82	0.00	7.06	0.00	2.34	-0.01	1.20
time (sec)	N/A	0.219	0.208	0.016	0.000	2.375	0.000	0.610	0.000	0.799

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	158	7477	0	1132	0	206	1321	164
N.S.	1	1.00	0.99	46.73	0.00	7.08	0.00	1.29	8.26	1.02
time (sec)	N/A	0.218	0.210	0.018	0.000	3.481	0.000	0.345	1.310	0.352

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	168	150	7529	0	1184	0	0	-1	203
N.S.	1	1.00	0.89	44.82	0.00	7.05	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.189	0.149	0.018	0.000	1.846	0.000	0.000	0.000	0.748

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	175	7590	0	1266	0	283	1152	231
N.S.	1	1.00	0.97	42.17	0.00	7.03	0.00	1.57	6.40	1.28
time (sec)	N/A	0.270	0.453	0.020	0.000	3.825	0.000	0.348	1.990	0.522

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	54	7705	0	483	0	496	-1	175
N.S.	1	1.00	0.31	43.78	0.00	2.74	0.00	2.82	-0.01	0.99
time (sec)	N/A	0.247	0.018	0.020	0.000	1.175	0.000	4.942	0.000	0.928

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	129	846	0	1053	0	284	-1	160
N.S.	1	1.00	0.98	6.41	0.00	7.98	0.00	2.15	-0.01	1.21
time (sec)	N/A	0.112	0.194	0.021	0.000	1.651	0.000	0.575	0.000	0.947

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	98	807	0	450	0	116	93	109
N.S.	1	1.00	0.99	8.15	0.00	4.55	0.00	1.17	0.94	1.10
time (sec)	N/A	0.085	0.085	0.015	0.000	0.915	0.000	0.334	0.892	0.172

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	124	817	0	418	0	231	-1	142
N.S.	1	1.00	1.39	9.18	0.00	4.70	0.00	2.60	-0.01	1.60
time (sec)	N/A	0.053	0.443	0.017	0.000	1.017	0.000	3.835	0.000	0.653

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	85	513	0	404	0	93	82	97
N.S.	1	1.00	0.98	5.90	0.00	4.64	0.00	1.07	0.94	1.11
time (sec)	N/A	0.068	0.088	0.012	0.000	0.847	0.000	0.281	0.785	0.189

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	100	100	405	823	0	459	0	225	-1	122
N.S.	1	1.00	4.05	8.23	0.00	4.59	0.00	2.25	-0.01	1.22
time (sec)	N/A	0.053	4.087	0.013	0.000	0.946	0.000	0.545	0.000	0.001

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	123	838	0	1037	0	138	3023	144
N.S.	1	1.00	0.95	6.45	0.00	7.98	0.00	1.06	23.25	1.11
time (sec)	N/A	0.136	0.242	0.016	0.000	1.865	0.000	0.312	1.937	0.313

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	116	841	0	600	0	396	-1	158
N.S.	1	1.00	0.79	5.72	0.00	4.08	0.00	2.69	-0.01	1.07
time (sec)	N/A	0.136	5.161	0.014	0.000	1.111	0.000	3.941	0.000	0.948

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	163	899	0	1407	0	257	3837	187
N.S.	1	1.00	0.88	4.86	0.00	7.61	0.00	1.39	20.74	1.01
time (sec)	N/A	0.243	0.570	0.016	0.000	2.769	0.000	0.336	2.853	0.738

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	136	893	0	758	0	375	-1	216
N.S.	1	1.00	0.66	4.33	0.00	3.68	0.00	1.82	-0.00	1.05
time (sec)	N/A	0.252	5.249	0.020	0.000	1.638	0.000	4.748	0.000	1.209

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	54	1498	0	552	0	298	-1	160
N.S.	1	1.00	0.42	11.52	0.00	4.25	0.00	2.29	-0.01	1.23
time (sec)	N/A	0.102	0.019	0.027	0.000	1.572	0.000	4.327	0.000	0.800

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	91	1456	0	732	0	181	142	123
N.S.	1	1.00	0.68	10.87	0.00	5.46	0.00	1.35	1.06	0.92
time (sec)	N/A	0.119	0.033	0.015	0.000	1.179	0.000	0.391	1.101	0.311

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	123	123	133	1453	0	744	0	299	-1	135
N.S.	1	1.00	1.08	11.81	0.00	6.05	0.00	2.43	-0.01	1.10
time (sec)	N/A	0.091	1.176	0.017	0.000	1.726	0.000	4.450	0.000	0.726

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	52	989	0	537	0	153	130	113
N.S.	1	1.00	0.46	8.75	0.00	4.75	0.00	1.35	1.15	1.00
time (sec)	N/A	0.085	0.016	0.012	0.000	0.997	0.000	0.392	0.986	0.290

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	120	1461	0	854	0	318	-1	163
N.S.	1	1.00	0.85	10.29	0.00	6.01	0.00	2.24	-0.01	1.15
time (sec)	N/A	0.102	5.245	0.014	0.000	1.748	0.000	4.532	0.000	0.759

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	123	1672	0	1992	0	225	5227	181
N.S.	1	1.00	0.72	9.84	0.00	11.72	0.00	1.32	30.75	1.06
time (sec)	N/A	0.244	0.132	0.016	0.000	5.882	0.000	0.348	3.405	0.709

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	145	1524	0	1018	0	554	-1	244
N.S.	1	1.00	0.71	7.43	0.00	4.97	0.00	2.70	-0.00	1.19
time (sec)	N/A	0.268	5.368	0.021	0.000	1.827	0.000	4.766	0.000	1.192

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	189	1778	0	2554	0	367	4286	272
N.S.	1	1.00	0.78	7.38	0.00	10.60	0.00	1.52	17.78	1.13
time (sec)	N/A	0.336	0.131	0.018	0.000	11.546	0.000	0.363	4.901	0.990

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	167	1608	0	1252	0	486	-1	324
N.S.	1	1.00	0.60	5.81	0.00	4.52	0.00	1.75	-0.00	1.17
time (sec)	N/A	0.396	5.487	0.020	0.000	3.020	0.000	5.810	0.000	1.929

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	174	174	133	2463	0	1008	0	594	-1	189
N.S.	1	1.00	0.76	14.16	0.00	5.79	0.00	3.41	-0.01	1.09
time (sec)	N/A	0.211	1.166	0.027	0.000	2.590	0.000	4.489	0.000	1.429

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	93	2400	0	993	0	260	193	185
N.S.	1	1.00	0.55	14.12	0.00	5.84	0.00	1.53	1.14	1.09
time (sec)	N/A	0.165	0.034	0.020	0.000	1.537	0.000	0.364	1.431	0.477

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	163	163	211	2369	0	1292	0	595	-1	198
N.S.	1	1.00	1.29	14.53	0.00	7.93	0.00	3.65	-0.01	1.21
time (sec)	N/A	0.160	2.428	0.019	0.000	2.928	0.000	4.518	0.000	1.275

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	54	1639	0	895	0	226	171	151
N.S.	1	1.00	0.39	11.71	0.00	6.39	0.00	1.61	1.22	1.08
time (sec)	N/A	0.103	0.019	0.016	0.000	1.138	0.000	0.565	1.243	0.357

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	170	2405	0	1434	0	619	-1	251
N.S.	1	1.00	0.85	11.97	0.00	7.13	0.00	3.08	-0.00	1.25
time (sec)	N/A	0.222	5.509	0.017	0.000	3.219	0.000	4.907	0.000	1.466

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	114	2837	0	3403	0	298	8467	276
N.S.	1	1.00	0.51	12.61	0.00	15.12	0.00	1.32	37.63	1.23
time (sec)	N/A	0.328	0.118	0.019	0.000	15.720	0.000	0.422	5.137	0.653

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	188	2513	0	1662	0	938	-1	368
N.S.	1	1.00	0.67	9.01	0.00	5.96	0.00	3.36	-0.00	1.32
time (sec)	N/A	0.443	5.590	0.023	0.000	3.963	0.000	5.335	0.000	1.574

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	304	304	190	2980	0	4115	0	505	5800	397
N.S.	1	1.00	0.62	9.80	0.00	13.54	0.00	1.66	19.08	1.31
time (sec)	N/A	0.484	0.138	0.020	0.000	29.743	0.000	0.425	6.582	1.331

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	362	362	210	2623	0	1890	0	789	-1	480
N.S.	1	1.00	0.58	7.25	0.00	5.22	0.00	2.18	-0.00	1.33
time (sec)	N/A	0.596	5.868	0.024	0.000	5.631	0.000	6.410	0.000	2.079

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	187	532	0	442	0	226	993	0
N.S.	1	1.00	0.89	2.55	0.00	2.11	0.00	1.08	4.75	0.00
time (sec)	N/A	0.249	0.452	0.084	0.000	1.960	0.000	0.480	54.785	1.480

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	138	339	0	334	0	159	639	227
N.S.	1	1.00	1.01	2.47	0.00	2.44	0.00	1.16	4.66	1.66
time (sec)	N/A	0.127	0.309	0.017	0.000	1.419	0.000	0.564	25.181	1.086

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	116	198	0	259	0	106	280	117
N.S.	1	1.00	1.35	2.30	0.00	3.01	0.00	1.23	3.26	1.36
time (sec)	N/A	0.075	0.286	0.014	0.000	1.624	0.000	0.550	4.998	0.501

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	129	177	0	777	0	0	4638	92
N.S.	1	1.00	1.40	1.92	0.00	8.45	0.00	0.00	50.41	1.00
time (sec)	N/A	0.095	0.142	0.033	0.000	1.879	0.000	0.000	19.831	0.645

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	89	207	0	280	0	434	477	117
N.S.	1	1.00	1.00	2.33	0.00	3.15	0.00	4.88	5.36	1.31
time (sec)	N/A	0.072	0.065	0.034	0.000	1.640	0.000	2.102	6.405	0.909

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	125	355	0	358	0	1107	955	0
N.S.	1	1.00	0.87	2.48	0.00	2.50	0.00	7.74	6.68	0.00
time (sec)	N/A	0.116	0.078	0.030	0.000	1.855	0.000	2.274	21.518	1.417

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	231	770	0	574	0	305	-1	0
N.S.	1	1.00	0.84	2.79	0.00	2.08	0.00	1.11	-0.00	0.00
time (sec)	N/A	0.325	0.600	0.038	0.000	1.035	0.000	0.654	0.000	2.840

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	173	532	0	440	0	225	-1	242
N.S.	1	1.00	0.93	2.84	0.00	2.35	0.00	1.20	-0.01	1.29
time (sec)	N/A	0.172	0.444	0.022	0.000	1.533	0.000	0.497	0.000	2.078

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	131	337	0	334	0	149	-1	150
N.S.	1	1.00	1.05	2.70	0.00	2.67	0.00	1.19	-0.01	1.20
time (sec)	N/A	0.104	0.384	0.016	0.000	0.983	0.000	0.515	0.000	1.208

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	195	287	0	918	0	0	-1	169
N.S.	1	1.00	1.47	2.16	0.00	6.90	0.00	0.00	-0.01	1.27
time (sec)	N/A	0.140	0.768	0.019	0.000	3.094	0.000	0.000	0.000	1.432

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	172	298	0	958	0	498	-1	167
N.S.	1	1.00	1.26	2.19	0.00	7.04	0.00	3.66	-0.01	1.23
time (sec)	N/A	0.140	1.014	0.021	0.000	3.083	0.000	0.755	0.000	1.664

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	110	352	0	360	0	1101	-1	0
N.S.	1	1.00	0.84	2.69	0.00	2.75	0.00	8.40	-0.01	0.00
time (sec)	N/A	0.106	0.072	0.020	0.000	2.131	0.000	2.288	0.000	2.569

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	271	1054	0	734	0	398	-1	0
N.S.	1	1.00	0.80	3.10	0.00	2.16	0.00	1.17	-0.00	0.00
time (sec)	N/A	0.418	1.201	0.044	0.000	1.048	0.000	0.684	0.000	3.335

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	214	770	0	574	0	304	-1	0
N.S.	1	1.00	0.90	3.25	0.00	2.42	0.00	1.28	-0.00	0.00
time (sec)	N/A	0.230	0.585	0.023	0.000	1.349	0.000	0.571	0.000	3.096

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	164	529	0	440	0	210	-1	180
N.S.	1	1.00	1.00	3.23	0.00	2.68	0.00	1.28	-0.01	1.10
time (sec)	N/A	0.141	0.560	0.019	0.000	1.491	0.000	0.497	0.000	1.863

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	213	446	0	1075	0	0	-1	276
N.S.	1	1.00	1.14	2.39	0.00	5.75	0.00	0.00	-0.01	1.48
time (sec)	N/A	0.226	0.666	0.021	0.000	10.419	0.000	0.000	0.000	2.272

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	196	423	0	1097	0	558	-1	269
N.S.	1	1.00	1.05	2.26	0.00	5.87	0.00	2.98	-0.01	1.44
time (sec)	N/A	0.230	1.060	0.021	0.000	8.105	0.000	0.867	0.000	2.415

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	206	464	0	1123	0	1175	-1	0
N.S.	1	1.00	1.07	2.42	0.00	5.85	0.00	6.12	-0.01	0.00
time (sec)	N/A	0.201	1.248	0.022	0.000	6.706	0.000	1.179	0.000	3.040

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	44	81	46	72	0	40	414	96
N.S.	1	1.00	0.68	1.25	0.71	1.11	0.00	0.62	6.37	1.48
time (sec)	N/A	0.049	0.026	0.029	2.013	0.709	0.000	0.381	12.038	0.530

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	37	60	27	65	66	33	206	59
N.S.	1	1.00	0.95	1.54	0.69	1.67	1.69	0.85	5.28	1.51
time (sec)	N/A	0.031	0.025	0.013	2.001	1.254	6.109	0.452	2.741	0.163

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	154	340	0	336	0	160	550	0
N.S.	1	1.00	1.09	2.41	0.00	2.38	0.00	1.13	3.90	0.00
time (sec)	N/A	0.159	0.275	0.035	0.000	1.540	0.000	0.547	22.538	1.674

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	123	200	0	256	0	104	279	122
N.S.	1	1.00	1.40	2.27	0.00	2.91	0.00	1.18	3.17	1.39
time (sec)	N/A	0.093	0.179	0.021	0.000	1.199	0.000	0.396	5.146	1.073

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	82	103	0	194	0	54	49	45
N.S.	1	1.00	1.82	2.29	0.00	4.31	0.00	1.20	1.09	1.00
time (sec)	N/A	0.055	0.066	0.019	0.000	1.241	0.000	0.416	1.224	0.533

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	103	0	204	0	89	136	46
N.S.	1	1.00	1.00	2.24	0.00	4.43	0.00	1.93	2.96	1.00
time (sec)	N/A	0.047	0.013	0.025	0.000	1.219	0.000	0.429	3.451	0.774

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	91	209	0	278	0	413	481	120
N.S.	1	1.00	1.00	2.30	0.00	3.05	0.00	4.54	5.29	1.32
time (sec)	N/A	0.079	0.053	0.024	0.000	1.382	0.000	0.483	6.696	1.138

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	126	355	0	360	0	1015	962	0
N.S.	1	1.00	0.85	2.38	0.00	2.42	0.00	6.81	6.46	0.00
time (sec)	N/A	0.137	0.089	0.029	0.000	2.036	0.000	1.971	21.575	1.681

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	185	553	0	498	0	192	-1	169
N.S.	1	1.00	1.43	4.29	0.00	3.86	0.00	1.49	-0.01	1.31
time (sec)	N/A	0.164	0.354	0.049	0.000	1.448	0.000	0.540	0.000	2.328

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	118	320	0	367	0	135	-1	83
N.S.	1	1.00	1.42	3.86	0.00	4.42	0.00	1.63	-0.01	1.00
time (sec)	N/A	0.089	0.518	0.026	0.000	1.436	0.000	0.480	0.000	1.751

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	33	30	0	48	0	70	45	34
N.S.	1	1.00	0.97	0.88	0.00	1.41	0.00	2.06	1.32	1.00
time (sec)	N/A	0.027	0.011	0.005	0.000	1.179	0.000	0.486	1.212	0.102

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	214	651	0	706	0	333	-1	114
N.S.	1	1.00	1.56	4.75	0.00	5.15	0.00	2.43	-0.01	0.83
time (sec)	N/A	0.141	0.483	0.046	0.000	1.706	0.000	0.861	0.000	2.744

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	54	63	0	128	0	214	139	64
N.S.	1	1.00	0.61	0.71	0.00	1.44	0.00	2.40	1.56	0.72
time (sec)	N/A	0.066	0.027	0.008	0.000	1.550	0.000	0.633	1.406	2.282

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	52	60	0	126	0	129	137	65
N.S.	1	1.00	0.70	0.81	0.00	1.70	0.00	1.74	1.85	0.88
time (sec)	N/A	0.044	0.016	0.007	0.000	1.434	0.000	0.447	1.337	1.810

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	91	119	0	262	0	597	220	0
N.S.	1	1.00	0.59	0.77	0.00	1.70	0.00	3.88	1.43	0.00
time (sec)	N/A	0.180	0.057	0.010	0.000	1.578	0.000	0.874	1.599	3.708

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	91	125	0	269	0	472	227	119
N.S.	1	1.00	0.66	0.91	0.00	1.95	0.00	3.42	1.64	0.86
time (sec)	N/A	0.101	0.042	0.012	0.000	2.093	0.000	0.915	1.544	2.563

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	83	113	0	259	0	243	216	95
N.S.	1	1.00	0.73	1.00	0.00	2.29	0.00	2.15	1.91	0.84
time (sec)	N/A	0.064	0.029	0.012	0.000	2.085	0.000	0.480	1.470	2.147

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	217	151	213	0	451	0	1036	336	0
N.S.	1	1.00	0.70	0.98	0.00	2.08	0.00	4.77	1.55	0.00
time (sec)	N/A	0.269	0.080	0.013	0.000	3.460	0.000	1.240	1.861	3.837

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	108	108	0	201	0	57	48	46
N.S.	1	1.00	2.30	2.30	0.00	4.28	0.00	1.21	1.02	0.98
time (sec)	N/A	0.057	0.092	0.046	0.000	1.387	0.000	0.426	1.271	0.563

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	109	111	0	203	0	57	51	48
N.S.	1	1.00	2.27	2.31	0.00	4.23	0.00	1.19	1.06	1.00
time (sec)	N/A	0.058	0.092	0.054	0.000	1.457	0.000	0.382	1.249	0.575

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	106	655	108	102	0	108	128	140
N.S.	1	1.00	0.97	6.01	0.99	0.94	0.00	0.99	1.17	1.28
time (sec)	N/A	0.088	0.133	7.176	1.938	0.920	0.000	0.423	0.889	0.156

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	90	754	97	122	0	97	117	135
N.S.	1	1.00	0.96	8.02	1.03	1.30	0.00	1.03	1.24	1.44
time (sec)	N/A	0.065	0.033	7.982	1.999	1.318	0.000	0.511	0.862	0.125

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	67	460	86	86	0	86	106	120
N.S.	1	1.00	0.85	5.82	1.09	1.09	0.00	1.09	1.34	1.52
time (sec)	N/A	0.052	0.018	6.315	2.022	1.405	0.000	0.465	1.051	0.098

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	127	0	0	177	0	149	256	201
N.S.	1	1.00	0.93	0.00	0.00	1.30	0.00	1.10	1.88	1.48
time (sec)	N/A	0.099	0.040	1.504	0.000	1.373	0.000	0.411	0.954	0.195

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	93	754	0	115	0	100	120	138
N.S.	1	1.00	0.96	7.77	0.00	1.19	0.00	1.03	1.24	1.42
time (sec)	N/A	0.074	0.046	6.053	0.000	1.133	0.000	0.370	0.886	0.167

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	178	0	0	217	0	177	397	226
N.S.	1	1.00	1.03	0.00	0.00	1.26	0.00	1.03	2.31	1.31
time (sec)	N/A	0.130	0.077	1.638	0.000	1.174	0.000	0.452	0.982	0.331

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	113	113	118	938	0	1943	0	0	-1	0
N.S.	1	1.00	1.04	8.30	0.00	17.19	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.013	0.036	42.308	0.000	4.820	0.000	0.000	0.000	0.001

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	120	770	126	133	0	126	148	154
N.S.	1	1.00	0.90	5.79	0.95	1.00	0.00	0.95	1.11	1.16
time (sec)	N/A	0.086	0.191	5.976	1.981	1.031	0.000	0.481	0.924	0.212

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	110	490	115	153	0	115	137	149
N.S.	1	1.00	0.95	4.22	0.99	1.32	0.00	0.99	1.18	1.28
time (sec)	N/A	0.077	0.169	6.766	1.958	1.148	0.000	0.425	0.888	0.162

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	97	758	104	121	0	104	126	142
N.S.	1	1.00	0.96	7.50	1.03	1.20	0.00	1.03	1.25	1.41
time (sec)	N/A	0.066	0.108	5.052	1.947	1.036	0.000	0.411	0.862	0.160

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	34	657	104	125	0	104	126	142
N.S.	1	1.00	0.34	6.50	1.03	1.24	0.00	1.03	1.25	1.41
time (sec)	N/A	0.063	0.008	4.947	2.030	0.960	0.000	0.388	0.860	0.133

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	148	0	0	227	0	167	375	223
N.S.	1	1.00	0.94	0.00	0.00	1.44	0.00	1.06	2.37	1.41
time (sec)	N/A	0.111	0.238	1.632	0.000	1.292	0.000	0.546	0.933	0.261

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	171	0	0	238	0	190	409	233
N.S.	1	1.00	0.93	0.00	0.00	1.30	0.00	1.04	2.23	1.27
time (sec)	N/A	0.130	0.180	1.669	0.000	1.170	0.000	0.420	0.988	0.343

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	194	0	0	265	0	181	416	236
N.S.	1	1.00	0.93	0.00	0.00	1.27	0.00	0.87	2.00	1.13
time (sec)	N/A	0.147	0.179	1.687	0.000	0.703	0.000	0.451	1.013	0.368

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	46	217	151	253	0	160	82	124
N.S.	1	1.00	0.34	1.60	1.11	1.86	0.00	1.18	0.60	0.91
time (sec)	N/A	0.088	0.030	2.848	1.965	1.284	0.000	0.447	0.909	0.183

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	42	211	140	246	0	140	71	117
N.S.	1	1.00	0.35	1.74	1.16	2.03	0.00	1.16	0.59	0.97
time (sec)	N/A	0.065	0.023	2.513	2.067	1.166	0.000	0.520	0.170	0.150

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	36	206	129	241	0	129	60	112
N.S.	1	1.00	0.34	1.94	1.22	2.27	0.00	1.22	0.57	1.06
time (sec)	N/A	0.046	0.012	1.725	2.043	1.138	0.000	0.432	0.153	0.125

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	34	189	118	189	0	118	49	97
N.S.	1	1.00	0.37	2.08	1.30	2.08	0.00	1.30	0.54	1.07
time (sec)	N/A	0.013	0.008	3.965	1.902	1.395	0.000	0.367	0.147	0.118

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	73	0	0	270	0	216	91	151
N.S.	1	1.00	0.50	0.00	0.00	1.86	0.00	1.49	0.63	1.04
time (sec)	N/A	0.083	0.095	11.909	0.000	1.468	0.000	0.421	0.971	0.190

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	102	0	0	307	0	192	109	169
N.S.	1	1.00	0.63	0.00	0.00	1.88	0.00	1.18	0.67	1.04
time (sec)	N/A	0.133	0.054	9.181	0.000	1.272	0.000	0.586	1.009	0.323

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	135	186	0	553	0	0	-1	137
N.S.	1	1.00	1.12	1.55	0.00	4.61	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.015	0.033	2.055	0.000	6.600	0.000	0.000	0.000	0.001

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	57	148	74	64	88	75	62	60
N.S.	1	1.00	0.73	1.90	0.95	0.82	1.13	0.96	0.79	0.77
time (sec)	N/A	0.053	0.043	1.080	1.862	0.901	24.082	0.307	0.126	0.042

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	51	141	63	57	75	64	51	53
N.S.	1	1.00	0.81	2.24	1.00	0.90	1.19	1.02	0.81	0.84
time (sec)	N/A	0.048	0.041	0.965	1.929	0.914	18.983	0.495	0.851	0.038

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	44	136	52	52	58	53	36	48
N.S.	1	1.00	0.92	2.83	1.08	1.08	1.21	1.10	0.75	1.00
time (sec)	N/A	0.033	0.011	1.013	1.986	0.953	14.208	0.481	0.865	0.035

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	125	41	41	42	42	25	33
N.S.	1	1.00	1.00	3.79	1.24	1.24	1.27	1.27	0.76	1.00
time (sec)	N/A	0.022	0.006	0.928	2.019	0.905	8.986	0.359	0.164	0.030

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	63	301	0	215	0	155	77	122
N.S.	1	1.00	0.36	1.74	0.00	1.24	0.00	0.90	0.45	0.71
time (sec)	N/A	0.132	0.017	5.782	0.000	0.775	0.000	0.357	0.193	0.132

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	77	315	0	252	0	169	82	140
N.S.	1	1.00	0.40	1.65	0.00	1.32	0.00	0.88	0.43	0.73
time (sec)	N/A	0.147	0.033	5.086	0.000	1.033	0.000	0.503	0.226	0.225

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	61	127	137	0	104	0	0	-1	63
N.S.	1	1.00	2.08	2.25	0.00	1.70	0.00	0.00	-0.02	1.03
time (sec)	N/A	0.009	0.022	1.292	0.000	6.596	0.000	0.000	0.000	0.001

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	129	129	37	186	0	282	0	0	-1	136
N.S.	1	1.00	0.29	1.44	0.00	2.19	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.023	0.042	7.444	0.000	1.083	0.000	0.000	0.000	2.176

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	37	186	0	282	0	0	-1	136
N.S.	1	1.00	0.31	1.55	0.00	2.35	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.023	0.041	7.247	0.000	1.083	0.000	0.000	0.000	2.238

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	124	124	39	0	0	393	0	0	-1	136
N.S.	1	1.00	0.31	0.00	0.00	3.17	0.00	0.00	-0.01	1.10
time (sec)	N/A	0.031	0.048	0.283	0.000	1.104	0.000	0.000	0.000	2.285

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	119	39	0	0	403	0	0	-1	140
N.S.	1	1.00	0.33	0.00	0.00	3.39	0.00	0.00	-0.01	1.18
time (sec)	N/A	0.032	0.055	0.088	0.000	0.945	0.000	0.000	0.000	2.307

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	65	0	0	171	0	0	-1	136
N.S.	1	1.00	0.54	0.00	0.00	1.42	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.028	0.063	0.095	0.000	0.922	0.000	0.000	0.000	2.257

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	65	0	0	171	0	0	-1	136
N.S.	1	1.00	0.54	0.00	0.00	1.42	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.026	0.058	0.171	0.000	0.821	0.000	0.000	0.000	2.235

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	115	115	67	0	0	207	0	0	-1	136
N.S.	1	1.00	0.58	0.00	0.00	1.80	0.00	0.00	-0.01	1.18
time (sec)	N/A	0.035	0.062	0.321	0.000	0.908	0.000	0.000	0.000	2.378

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	119	119	68	0	0	211	0	0	-1	140
N.S.	1	1.00	0.57	0.00	0.00	1.77	0.00	0.00	-0.01	1.18
time (sec)	N/A	0.037	0.072	0.149	0.000	0.964	0.000	0.000	0.000	2.363

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	179	541	151	198	0	160	82	124
N.S.	1	1.00	0.95	2.88	0.80	1.05	0.00	0.85	0.44	0.66
time (sec)	N/A	0.211	0.110	2.974	2.008	0.829	0.000	0.347	0.883	0.172

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	163	536	140	193	0	140	71	119
N.S.	1	1.00	0.94	3.10	0.81	1.12	0.00	0.81	0.41	0.69
time (sec)	N/A	0.180	0.066	2.165	2.079	1.049	0.000	0.416	0.217	0.160

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	146	528	129	186	0	129	60	112
N.S.	1	1.00	0.92	3.34	0.82	1.18	0.00	0.82	0.38	0.71
time (sec)	N/A	0.152	0.040	2.393	2.009	0.974	0.000	0.456	0.166	0.133

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	117	189	118	230	0	118	49	97
N.S.	1	1.00	0.82	1.32	0.83	1.61	0.00	0.83	0.34	0.68
time (sec)	N/A	0.118	0.027	1.801	2.019	1.052	0.000	0.379	0.955	0.116

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	156	560	0	315	0	210	91	151
N.S.	1	1.00	0.79	2.84	0.00	1.60	0.00	1.07	0.46	0.77
time (sec)	N/A	0.189	0.060	15.084	0.000	1.012	0.000	0.505	0.241	0.192

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	210	1540	0	352	0	192	107	169
N.S.	1	1.00	0.98	7.16	0.00	1.64	0.00	0.89	0.50	0.79
time (sec)	N/A	0.241	0.089	23.050	0.000	0.961	0.000	0.575	1.007	0.314

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	120	120	37	186	0	282	0	0	-1	136
N.S.	1	1.00	0.31	1.55	0.00	2.35	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.023	0.032	0.000	0.000	1.141	0.000	0.000	0.000	0.001

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	61	52	137	0	104	0	0	-1	61
N.S.	1	1.00	0.85	2.25	0.00	1.70	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.016	0.041	1.283	0.000	0.866	0.000	0.000	0.000	2.053

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	61	52	138	0	115	0	0	-1	79
N.S.	1	1.00	0.85	2.26	0.00	1.89	0.00	0.00	-0.02	1.30
time (sec)	N/A	0.017	0.097	1.324	0.000	0.928	0.000	0.000	0.000	2.110

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	72	72	54	0	0	275	0	0	-1	72
N.S.	1	1.00	0.75	0.00	0.00	3.82	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.024	0.053	0.089	0.000	0.952	0.000	0.000	0.000	2.184

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	74	74	55	0	0	274	0	0	-1	92
N.S.	1	1.00	0.74	0.00	0.00	3.70	0.00	0.00	-0.01	1.24
time (sec)	N/A	0.026	0.061	0.086	0.000	0.808	0.000	0.000	0.000	2.197

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	85	85	66	0	0	145	0	0	-1	87
N.S.	1	1.00	0.78	0.00	0.00	1.71	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.026	0.106	0.084	0.000	0.838	0.000	0.000	0.000	2.213

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	85	85	67	0	0	145	0	0	-1	87
N.S.	1	1.00	0.79	0.00	0.00	1.71	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.024	0.059	0.084	0.000	0.845	0.000	0.000	0.000	2.207

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	96	96	68	0	0	207	0	0	-1	98
N.S.	1	1.00	0.71	0.00	0.00	2.16	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.033	0.067	0.091	0.000	0.735	0.000	0.000	0.000	2.324

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	98	98	70	0	0	211	0	0	-1	100
N.S.	1	1.00	0.71	0.00	0.00	2.15	0.00	0.00	-0.01	1.02
time (sec)	N/A	0.033	0.064	0.082	0.000	0.961	0.000	0.000	0.000	2.318

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	58	424	74	64	0	75	62	60
N.S.	1	1.00	0.74	5.44	0.95	0.82	0.00	0.96	0.79	0.77
time (sec)	N/A	0.051	0.037	0.864	2.023	0.797	0.000	0.283	0.105	0.046

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	53	420	63	59	0	64	51	55
N.S.	1	1.00	0.84	6.67	1.00	0.94	0.00	1.02	0.81	0.87
time (sec)	N/A	0.046	0.028	0.828	1.951	0.942	0.000	0.443	0.123	0.040

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	44	412	52	52	0	53	36	48
N.S.	1	1.00	0.92	8.58	1.08	1.08	0.00	1.10	0.75	1.00
time (sec)	N/A	0.032	0.011	0.757	1.809	0.893	0.000	0.406	0.116	0.037

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	125	41	41	42	42	25	33
N.S.	1	1.00	1.00	3.79	1.24	1.24	1.27	1.27	0.76	1.00
time (sec)	N/A	0.021	0.006	0.530	1.846	0.741	9.731	0.424	0.868	0.030

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	150	299	0	215	0	155	77	122
N.S.	1	1.00	0.87	1.73	0.00	1.24	0.00	0.90	0.45	0.71
time (sec)	N/A	0.126	0.051	3.500	0.000	0.819	0.000	0.409	0.912	0.132

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	181	916	0	252	0	169	81	140
N.S.	1	1.00	0.95	4.80	0.00	1.32	0.00	0.88	0.42	0.73
time (sec)	N/A	0.146	0.054	5.956	0.000	0.843	0.000	0.499	0.214	0.238

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	61	61	52	137	0	104	0	0	-1	61
N.S.	1	1.00	0.85	2.25	0.00	1.70	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.016	0.031	0.000	0.000	0.649	0.000	0.000	0.000	0.001

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	131	0	0	0	94	0	-1	206
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.54	0.00	-0.01	1.19
time (sec)	N/A	0.126	0.175	0.115	0.000	0.000	41.582	0.000	0.000	2.197

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	112	0	0	0	92	0	-1	168
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.68	0.00	-0.01	1.24
time (sec)	N/A	0.094	0.100	0.060	0.000	0.000	5.831	0.000	0.000	1.202

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	100	0	0	0	85	0	-1	145
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.75	0.00	-0.01	1.28
time (sec)	N/A	0.080	0.032	0.064	0.000	0.000	10.423	0.000	0.000	0.951

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	44	39	0	43	121	0	49	55
N.S.	1	1.00	0.66	0.58	0.00	0.64	1.81	0.00	0.73	0.82
time (sec)	N/A	0.037	0.019	0.007	0.000	1.159	59.813	0.000	1.167	1.661

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	72	62	0	66	0	0	75	84
N.S.	1	1.00	0.69	0.60	0.00	0.63	0.00	0.00	0.72	0.81
time (sec)	N/A	0.051	0.041	0.006	0.000	1.553	0.000	0.000	1.212	1.341

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	94	86	0	90	0	0	100	114
N.S.	1	1.00	0.67	0.61	0.00	0.64	0.00	0.00	0.71	0.81
time (sec)	N/A	0.070	0.052	0.008	0.000	0.702	0.000	0.000	1.259	2.226

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	77	0	0	916	94	0	-1	213
N.S.	1	1.00	0.45	0.00	0.00	5.36	0.55	0.00	-0.01	1.25
time (sec)	N/A	0.108	0.122	0.121	0.000	1.102	32.825	0.000	0.000	3.911

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	68	0	0	384	83	0	-1	168
N.S.	1	1.00	0.56	0.00	0.00	3.15	0.68	0.00	-0.01	1.38
time (sec)	N/A	0.072	0.063	0.060	0.000	1.060	16.057	0.000	0.000	2.362

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	45	39	0	57	117	0	70	72
N.S.	1	1.00	0.67	0.58	0.00	0.85	1.75	0.00	1.04	1.07
time (sec)	N/A	0.031	0.022	0.007	0.000	0.822	64.285	0.000	1.201	1.834

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	71	62	0	80	0	0	101	100
N.S.	1	1.00	0.68	0.60	0.00	0.77	0.00	0.00	0.97	0.96
time (sec)	N/A	0.047	0.030	0.007	0.000	1.198	0.000	0.000	1.232	2.528

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	68	86	0	105	0	0	125	130
N.S.	1	1.00	0.48	0.61	0.00	0.74	0.00	0.00	0.89	0.92
time (sec)	N/A	0.065	0.042	0.008	0.000	1.090	0.000	0.000	1.271	3.974

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	184	184	77	0	0	0	94	0	-1	220
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.51	0.00	-0.01	1.20
time (sec)	N/A	0.119	0.126	0.111	0.000	0.000	166.392	0.000	0.000	23.696

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	69	0	0	0	87	0	-1	173
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.70	0.00	-0.01	1.38
time (sec)	N/A	0.076	0.060	0.054	0.000	0.000	21.768	0.000	0.000	22.949

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	44	40	0	56	119	0	69	71
N.S.	1	1.00	0.68	0.62	0.00	0.86	1.83	0.00	1.06	1.09
time (sec)	N/A	0.031	0.020	0.006	0.000	1.744	79.594	0.000	1.177	24.377

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	66	62	0	81	0	0	101	100
N.S.	1	1.00	0.63	0.60	0.00	0.78	0.00	0.00	0.97	0.96
time (sec)	N/A	0.052	0.029	0.008	0.000	2.417	0.000	0.000	1.209	27.332

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	88	86	0	105	0	0	125	130
N.S.	1	1.00	0.62	0.61	0.00	0.74	0.00	0.00	0.89	0.92
time (sec)	N/A	0.066	0.042	0.007	0.000	1.156	0.000	0.000	1.262	39.694

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	91	0	0	997	0	0	-1	260
N.S.	1	1.00	0.41	0.00	0.00	4.51	0.00	0.00	-0.00	1.18
time (sec)	N/A	0.135	0.124	0.112	0.000	1.750	0.000	0.000	0.000	30.800

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	77	0	0	448	0	0	-1	206
N.S.	1	1.00	0.52	0.00	0.00	3.01	0.00	0.00	-0.01	1.38
time (sec)	N/A	0.087	0.081	0.058	0.000	1.531	0.000	0.000	0.000	28.608

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	44	39	0	62	230	0	79	89
N.S.	1	1.00	0.56	0.49	0.00	0.78	2.91	0.00	1.00	1.13
time (sec)	N/A	0.033	0.046	0.006	0.000	1.326	165.872	0.000	1.215	64.347

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	65	62	0	95	0	0	115	100
N.S.	1	1.00	0.62	0.60	0.00	0.91	0.00	0.00	1.11	0.96
time (sec)	N/A	0.048	0.043	0.007	0.000	1.155	0.000	0.000	1.310	27.469

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	94	86	0	119	0	0	144	130
N.S.	1	1.00	0.67	0.61	0.00	0.84	0.00	0.00	1.02	0.92
time (sec)	N/A	0.069	0.051	0.007	0.000	1.258	0.000	0.000	1.338	38.391

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	79	110	0	143	0	0	156	160
N.S.	1	1.00	0.44	0.62	0.00	0.80	0.00	0.00	0.88	0.90
time (sec)	N/A	0.090	0.058	0.008	0.000	1.083	0.000	0.000	1.406	45.915

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [846] had the largest ratio of [.5833]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	18	0.056
2	A	3	2	1.00	16	0.125
3	A	2	1	1.00	15	0.067
4	A	3	2	1.00	18	0.111
5	A	2	1	1.00	18	0.056
6	A	3	2	1.00	18	0.111
7	A	2	1	1.00	18	0.056
8	A	3	2	1.00	18	0.111
9	A	2	1	1.00	18	0.056
10	A	3	2	1.00	18	0.111
11	A	2	1	1.00	20	0.050
12	A	3	2	1.00	18	0.111
13	A	2	1	1.00	17	0.059
14	A	4	3	1.00	20	0.150
15	A	2	1	1.00	20	0.050
16	A	3	2	1.00	20	0.100
17	A	2	1	1.00	20	0.050
18	A	3	2	1.00	20	0.100
19	A	2	1	1.00	20	0.050
20	A	3	2	1.00	20	0.100
21	A	2	1	1.00	20	0.050
22	A	3	3	1.00	20	0.150
23	A	3	2	1.00	20	0.100
24	A	2	1	1.00	20	0.050
25	A	3	2	1.00	20	0.100
26	A	2	1	1.00	20	0.050

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	3	2	1.00	20	0.100
28	A	2	1	1.00	20	0.050
29	A	3	2	1.00	20	0.100
30	A	2	1	1.00	20	0.050
31	A	3	2	1.00	18	0.111
32	A	2	1	1.00	17	0.059
33	A	4	3	1.00	20	0.150
34	A	2	1	1.00	20	0.050
35	A	3	2	1.00	20	0.100
36	A	2	1	1.00	20	0.050
37	A	3	2	1.00	20	0.100
38	A	2	1	1.00	20	0.050
39	A	3	2	1.00	20	0.100
40	A	2	1	1.00	20	0.050
41	A	3	2	1.00	20	0.100
42	A	2	1	1.00	20	0.050
43	A	3	2	1.00	20	0.100
44	A	2	1	1.00	20	0.050
45	A	4	3	1.00	20	0.150
46	A	2	1	1.00	20	0.050
47	A	3	3	1.00	20	0.150
48	A	2	1	1.00	20	0.050
49	A	4	4	1.00	20	0.200
50	A	2	1	1.00	20	0.050
51	A	3	2	1.00	20	0.100
52	A	2	1	1.00	20	0.050
53	A	3	2	1.00	20	0.100
54	A	2	1	1.00	20	0.050
55	A	3	2	1.00	20	0.100
56	A	4	3	1.00	20	0.150
57	A	3	2	1.00	20	0.100
58	A	4	3	1.00	20	0.150
59	A	3	2	1.00	20	0.100
60	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	18	0.111
62	A	2	2	1.00	17	0.118
63	A	3	2	1.00	20	0.100
64	A	2	2	1.00	20	0.100
65	A	3	2	1.00	20	0.100
66	A	3	3	1.00	20	0.150
67	A	3	2	1.00	20	0.100
68	A	4	3	1.00	20	0.150
69	A	3	2	1.00	20	0.100
70	A	5	3	1.00	20	0.150
71	A	3	2	1.00	20	0.100
72	A	4	3	1.00	20	0.150
73	A	3	2	1.00	20	0.100
74	A	4	3	1.00	20	0.150
75	A	3	2	1.00	20	0.100
76	A	4	3	1.00	20	0.150
77	A	3	2	1.00	20	0.100
78	A	3	3	1.00	20	0.150
79	A	3	2	1.00	18	0.111
80	A	2	2	1.00	17	0.118
81	A	3	2	1.00	20	0.100
82	A	3	3	1.00	20	0.150
83	A	3	2	1.00	20	0.100
84	A	4	3	1.00	20	0.150
85	A	3	2	1.00	20	0.100
86	A	4	3	1.00	20	0.150
87	A	3	2	1.00	20	0.100
88	A	3	2	1.00	20	0.100
89	A	3	2	1.00	20	0.100
90	A	3	2	1.00	20	0.100
91	A	3	2	1.00	20	0.100
92	A	3	2	1.00	20	0.100
93	A	2	2	1.00	18	0.111
94	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	3	2	1.00	20	0.100
96	A	3	2	1.00	20	0.100
97	A	3	2	1.00	20	0.100
98	A	5	4	1.00	20	0.200
99	A	5	4	1.00	20	0.200
100	A	5	4	1.00	20	0.200
101	A	4	4	1.00	20	0.200
102	A	3	3	1.00	20	0.150
103	A	3	3	1.00	17	0.176
104	A	4	3	1.00	20	0.150
105	A	5	4	1.00	20	0.200
106	A	5	4	1.00	20	0.200
107	A	2	2	1.00	15	0.133
108	A	2	2	1.00	17	0.118
109	A	1	1	1.00	13	0.077
110	A	1	1	1.00	15	0.067
111	A	2	2	1.00	15	0.133
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	13	0.154
114	A	1	1	1.00	19	0.053
115	A	1	1	1.00	18	0.056
116	A	2	2	1.00	18	0.111
117	A	3	3	1.00	13	0.231
118	A	2	2	1.00	18	0.111
119	A	2	2	1.00	17	0.118
120	A	2	2	1.00	23	0.087
121	A	2	2	1.00	23	0.087
122	A	2	2	1.00	21	0.095
123	A	2	2	1.00	20	0.100
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	23	0.087
126	A	2	2	1.00	23	0.087
127	A	4	3	1.00	23	0.130
128	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	2	2	1.00	21	0.095
130	A	2	2	1.00	20	0.100
131	A	5	5	1.00	23	0.217
132	A	3	3	1.00	23	0.130
133	A	4	3	1.00	23	0.130
134	A	4	3	1.00	23	0.130
135	A	3	3	1.00	23	0.130
136	A	2	2	1.00	21	0.095
137	A	3	3	1.00	20	0.150
138	A	4	3	1.00	23	0.130
139	A	4	4	1.00	23	0.174
140	A	4	3	1.00	23	0.130
141	A	2	1	1.00	20	0.050
142	A	3	2	1.00	20	0.100
143	A	2	1	1.00	20	0.050
144	A	3	2	1.00	18	0.111
145	A	2	1	1.00	17	0.059
146	A	4	3	1.00	20	0.150
147	A	2	1	1.00	20	0.050
148	A	3	2	1.00	20	0.100
149	A	2	1	1.00	20	0.050
150	A	2	1	1.00	22	0.045
151	A	3	2	1.00	22	0.091
152	A	2	1	1.00	22	0.045
153	A	3	2	1.00	20	0.100
154	A	2	1	1.00	19	0.053
155	A	3	2	1.00	22	0.091
156	A	2	1	1.00	22	0.045
157	A	3	2	1.00	22	0.091
158	A	2	1	1.00	22	0.045
159	A	2	1	1.00	22	0.045
160	A	3	2	1.00	22	0.091
161	A	2	1	1.00	22	0.045
162	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	2	1	1.00	19	0.053
164	A	3	2	1.00	22	0.091
165	A	2	1	1.00	22	0.045
166	A	3	2	1.00	22	0.091
167	A	2	1	1.00	22	0.045
168	A	3	2	1.00	22	0.091
169	A	3	2	1.00	22	0.091
170	A	3	2	1.00	22	0.091
171	A	3	2	1.00	20	0.100
172	A	3	2	1.00	19	0.105
173	A	3	2	1.00	22	0.091
174	A	3	2	1.00	22	0.091
175	A	3	2	1.00	22	0.091
176	A	3	2	1.00	22	0.091
177	A	3	2	1.00	22	0.091
178	A	3	2	1.00	22	0.091
179	A	3	2	1.00	22	0.091
180	A	5	4	1.00	22	0.182
181	A	3	2	1.00	22	0.091
182	A	4	4	1.00	22	0.182
183	A	3	2	1.00	20	0.100
184	A	4	3	1.00	19	0.158
185	A	3	2	1.00	22	0.091
186	A	3	3	1.00	22	0.136
187	A	3	2	1.00	22	0.091
188	A	4	4	1.00	22	0.182
189	A	5	4	1.00	22	0.182
190	A	3	2	1.00	22	0.091
191	A	4	4	1.00	22	0.182
192	A	3	2	1.00	20	0.100
193	A	3	3	1.00	19	0.158
194	A	3	2	1.00	22	0.091
195	A	4	4	0.99	22	0.182
196	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	5	4	1.00	22	0.182
198	A	3	2	1.00	20	0.100
199	A	4	3	1.00	20	0.150
200	A	3	2	1.00	20	0.100
201	A	3	3	1.00	20	0.150
202	A	3	2	1.00	18	0.111
203	A	2	2	1.00	17	0.118
204	A	3	2	1.00	20	0.100
205	A	2	2	1.00	20	0.100
206	A	3	2	1.00	20	0.100
207	A	3	3	1.00	20	0.150
208	A	3	2	1.00	22	0.091
209	A	3	2	1.00	22	0.091
210	A	3	2	1.00	22	0.091
211	A	3	2	1.00	22	0.091
212	A	3	2	1.00	20	0.100
213	A	3	2	1.00	19	0.105
214	A	3	2	1.00	22	0.091
215	A	3	2	1.00	22	0.091
216	A	3	2	1.00	22	0.091
217	A	3	2	1.00	22	0.091
218	A	3	2	1.00	22	0.091
219	A	3	2	1.00	22	0.091
220	A	3	2	1.00	22	0.091
221	A	3	2	1.00	22	0.091
222	A	3	2	1.00	20	0.100
223	A	3	2	1.00	19	0.105
224	A	3	2	1.00	22	0.091
225	A	3	2	1.00	22	0.091
226	A	3	2	1.00	22	0.091
227	A	3	2	1.00	22	0.091
228	A	3	2	1.00	22	0.091
229	A	4	3	1.00	22	0.136
230	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	3	2	1.00	22	0.091
232	A	4	3	1.00	20	0.150
233	A	3	2	1.00	19	0.105
234	A	3	2	1.00	22	0.091
235	A	4	3	1.00	22	0.136
236	A	3	2	1.00	22	0.091
237	A	5	4	1.00	22	0.182
238	A	3	2	1.00	22	0.091
239	A	6	4	1.00	22	0.182
240	A	3	2	1.00	22	0.091
241	A	3	2	1.00	22	0.091
242	A	4	3	1.00	22	0.136
243	A	3	2	1.00	22	0.091
244	A	4	3	1.00	22	0.136
245	A	3	2	1.00	20	0.100
246	A	4	3	1.00	19	0.158
247	A	3	2	1.00	22	0.091
248	A	5	4	1.00	22	0.182
249	A	3	2	1.00	22	0.091
250	A	6	4	1.00	22	0.182
251	A	3	2	1.00	22	0.091
252	A	5	4	1.00	22	0.182
253	A	3	2	1.00	22	0.091
254	A	5	4	1.00	22	0.182
255	A	3	2	1.00	20	0.100
256	A	5	4	1.00	19	0.210
257	A	3	2	1.00	22	0.091
258	A	6	5	1.00	22	0.227
259	A	3	2	1.00	22	0.091
260	A	7	5	1.00	22	0.227
261	A	4	3	1.00	16	0.188
262	A	4	3	1.00	20	0.150
263	A	3	2	1.00	20	0.100
264	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	3	2	1.00	18	0.111
266	A	2	2	1.00	17	0.118
267	A	3	2	1.00	20	0.100
268	A	3	3	1.00	20	0.150
269	A	3	2	1.00	20	0.100
270	A	4	3	1.00	20	0.150
271	A	5	4	1.00	22	0.182
272	A	3	2	1.00	22	0.091
273	A	4	4	1.00	22	0.182
274	A	3	2	1.00	20	0.100
275	A	4	3	1.00	19	0.158
276	A	3	2	1.00	22	0.091
277	A	3	3	1.00	22	0.136
278	A	3	2	1.00	22	0.091
279	A	4	4	0.98	22	0.182
280	A	4	3	1.00	22	0.136
281	A	3	2	1.00	22	0.091
282	A	5	4	1.00	22	0.182
283	A	3	2	1.00	20	0.100
284	A	4	3	1.00	19	0.158
285	A	3	2	1.00	22	0.091
286	A	4	3	1.00	22	0.136
287	A	3	2	1.00	22	0.091
288	A	4	3	1.00	22	0.136
289	A	4	3	1.00	22	0.136
290	A	3	2	1.00	22	0.091
291	A	4	3	1.00	22	0.136
292	A	3	2	1.00	20	0.100
293	A	4	3	1.00	19	0.158
294	A	3	2	1.00	22	0.091
295	A	5	4	1.00	22	0.182
296	A	3	2	1.00	22	0.091
297	A	6	4	1.00	22	0.182
298	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	7	4	1.00	22	0.182
300	A	3	2	1.00	22	0.091
301	A	5	4	1.00	22	0.182
302	A	3	2	1.00	22	0.091
303	A	5	4	1.00	22	0.182
304	A	3	2	1.00	20	0.100
305	A	5	4	1.00	19	0.210
306	A	3	2	1.00	22	0.091
307	A	6	5	1.00	22	0.227
308	A	3	2	1.00	22	0.091
309	A	7	5	1.00	22	0.227
310	A	6	4	1.00	22	0.182
311	A	3	2	1.00	22	0.091
312	A	6	4	1.00	22	0.182
313	A	3	2	1.00	20	0.100
314	A	6	4	1.00	19	0.210
315	A	3	2	1.00	22	0.091
316	A	7	5	1.00	22	0.227
317	A	3	2	1.00	22	0.091
318	A	8	5	1.00	22	0.227
319	A	2	1	1.00	20	0.050
320	A	2	1	1.00	20	0.050
321	A	2	1	1.00	18	0.056
322	A	2	1	1.00	22	0.045
323	A	2	1	1.00	22	0.045
324	A	2	1	1.00	20	0.050
325	A	2	1	1.00	20	0.050
326	A	2	1	1.00	20	0.050
327	A	2	1	1.00	20	0.050
328	A	2	1	1.00	20	0.050
329	A	2	1	1.00	20	0.050
330	A	2	1	1.00	20	0.050
331	A	2	1	1.00	20	0.050
332	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	2	1	1.00	22	0.045
334	A	2	1	1.00	22	0.045
335	A	2	1	1.00	22	0.045
336	A	2	1	1.00	22	0.045
337	A	2	1	1.00	22	0.045
338	A	2	1	1.00	22	0.045
339	A	2	1	1.00	22	0.045
340	A	2	1	1.00	22	0.045
341	A	2	1	1.00	22	0.045
342	A	2	1	1.00	22	0.045
343	A	2	1	1.00	22	0.045
344	A	2	1	1.00	22	0.045
345	A	2	1	1.00	22	0.045
346	A	2	1	1.00	22	0.045
347	A	2	1	1.00	22	0.045
348	A	2	1	1.00	22	0.045
349	A	13	9	1.00	22	0.409
350	A	12	9	1.00	22	0.409
351	A	12	9	1.00	22	0.409
352	A	11	8	1.00	22	0.364
353	A	11	8	1.00	22	0.364
354	A	11	8	1.00	22	0.364
355	A	11	8	1.00	22	0.364
356	A	12	9	1.00	22	0.409
357	A	13	9	1.00	22	0.409
358	A	12	9	1.00	22	0.409
359	A	12	9	1.00	22	0.409
360	A	11	8	1.00	22	0.364
361	A	11	8	1.00	22	0.364
362	A	12	9	1.00	22	0.409
363	A	12	9	1.00	22	0.409
364	A	13	9	1.00	22	0.409
365	A	13	10	1.00	22	0.454
366	A	12	9	1.00	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	12	9	1.00	22	0.409
368	A	12	9	1.00	22	0.409
369	A	12	9	1.00	22	0.409
370	A	13	10	1.00	22	0.454
371	A	13	10	1.00	22	0.454
372	A	14	10	1.00	22	0.454
373	A	2	1	1.00	22	0.045
374	A	2	1	1.00	22	0.045
375	A	2	1	1.00	22	0.045
376	A	2	1	1.00	22	0.045
377	A	2	1	1.00	22	0.045
378	A	2	1	1.00	22	0.045
379	A	2	1	1.00	22	0.045
380	A	2	1	1.00	22	0.045
381	A	2	1	1.00	24	0.042
382	A	2	1	1.00	24	0.042
383	A	2	1	1.00	24	0.042
384	A	2	1	1.00	24	0.042
385	A	2	1	1.00	24	0.042
386	A	2	1	1.00	24	0.042
387	A	2	1	1.00	24	0.042
388	A	2	1	1.00	24	0.042
389	A	2	1	1.00	24	0.042
390	A	2	1	1.00	24	0.042
391	A	2	1	1.00	24	0.042
392	A	2	1	1.00	24	0.042
393	A	2	1	1.00	24	0.042
394	A	2	1	1.00	24	0.042
395	A	2	1	1.00	24	0.042
396	A	2	1	1.00	24	0.042
397	A	14	9	1.00	24	0.375
398	A	13	9	1.00	24	0.375
399	A	13	9	1.00	24	0.375
400	A	12	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	12	8	1.00	24	0.333
402	A	12	9	1.00	24	0.375
403	A	12	9	1.00	24	0.375
404	A	12	9	1.00	24	0.375
405	A	12	9	1.00	24	0.375
406	A	13	10	1.00	24	0.417
407	A	14	10	1.00	24	0.417
408	A	13	10	1.00	24	0.417
409	A	13	10	1.00	24	0.417
410	A	12	9	1.00	24	0.375
411	A	12	9	1.00	24	0.375
412	A	12	9	1.00	24	0.375
413	A	12	9	1.00	24	0.375
414	A	13	10	1.00	24	0.417
415	A	14	10	1.00	24	0.417
416	A	13	10	1.00	24	0.417
417	A	13	10	1.00	24	0.417
418	A	12	9	1.00	24	0.375
419	A	12	9	1.00	24	0.375
420	A	13	10	1.00	24	0.417
421	A	13	10	1.00	24	0.417
422	A	14	11	1.00	24	0.458
423	A	13	9	1.00	24	0.375
424	A	13	9	1.00	24	0.375
425	A	12	8	1.00	24	0.333
426	A	12	8	1.00	24	0.333
427	A	12	8	1.00	24	0.333
428	A	12	8	1.00	24	0.333
429	A	12	8	1.00	24	0.333
430	A	12	8	1.00	24	0.333
431	A	12	8	1.00	24	0.333
432	A	12	8	1.00	24	0.333
433	A	12	8	1.00	24	0.333
434	A	13	9	1.00	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	A	13	9	1.00	24	0.375
436	A	14	10	1.00	24	0.417
437	A	13	9	1.00	24	0.375
438	A	13	9	1.00	24	0.375
439	A	13	9	1.00	24	0.375
440	A	13	9	1.00	24	0.375
441	A	13	9	1.00	24	0.375
442	A	13	9	1.00	24	0.375
443	A	22	9	1.00	24	0.375
444	A	21	9	1.00	24	0.375
445	A	20	8	1.00	24	0.333
446	A	20	8	1.00	24	0.333
447	A	20	8	1.00	24	0.333
448	A	20	8	1.00	24	0.333
449	A	22	9	1.00	24	0.375
450	A	21	9	1.00	24	0.375
451	A	23	10	1.00	24	0.417
452	A	22	10	1.00	24	0.417
453	A	22	9	1.00	24	0.375
454	A	21	9	1.00	24	0.375
455	A	22	9	1.00	24	0.375
456	A	21	9	1.00	24	0.375
457	A	22	9	1.00	24	0.375
458	A	21	9	1.00	24	0.375
459	A	23	10	1.00	24	0.417
460	A	22	10	1.00	24	0.417
461	A	24	10	1.00	24	0.417
462	A	22	10	1.00	24	0.417
463	A	23	10	1.00	24	0.417
464	A	22	10	1.00	24	0.417
465	A	23	10	1.00	24	0.417
466	A	22	10	1.00	24	0.417
467	A	24	11	1.00	24	0.458
468	A	23	11	1.00	24	0.458

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	25	11	1.00	24	0.458
470	A	22	10	1.00	24	0.417
471	A	23	10	1.00	24	0.417
472	A	22	10	1.00	24	0.417
473	A	23	10	1.00	24	0.417
474	A	22	10	1.00	24	0.417
475	A	24	11	1.00	24	0.458
476	A	23	11	1.00	24	0.458
477	A	25	11	1.00	24	0.458
478	A	23	10	1.00	24	0.417
479	A	24	10	1.00	24	0.417
480	A	23	10	1.00	24	0.417
481	A	24	10	1.00	24	0.417
482	A	23	10	1.00	24	0.417
483	A	25	11	1.00	24	0.458
484	A	24	11	1.00	24	0.458
485	A	26	11	1.00	24	0.458
486	A	3	2	1.00	22	0.091
487	A	6	5	1.00	22	0.227
488	A	3	2	1.00	22	0.091
489	A	5	5	1.00	22	0.227
490	A	3	2	1.00	20	0.100
491	A	4	4	1.00	19	0.210
492	A	5	5	1.00	22	0.227
493	A	4	4	1.00	22	0.182
494	A	5	5	1.00	22	0.227
495	A	4	4	1.00	22	0.182
496	A	5	5	1.00	22	0.227
497	A	2	2	1.00	22	0.091
498	A	6	6	1.00	22	0.273
499	A	3	3	1.00	22	0.136
500	A	7	6	1.00	22	0.273
501	A	4	3	1.00	22	0.136
502	A	8	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
503	A	3	2	1.00	22	0.091
504	A	7	5	1.00	22	0.227
505	A	3	2	1.00	22	0.091
506	A	6	5	1.00	22	0.227
507	A	3	2	1.00	20	0.100
508	A	5	4	1.00	19	0.210
509	A	6	5	1.00	22	0.227
510	A	5	4	1.00	22	0.182
511	A	6	5	1.00	22	0.227
512	A	5	5	1.00	22	0.227
513	A	6	6	1.00	22	0.273
514	A	5	4	1.00	22	0.182
515	A	6	5	1.00	22	0.227
516	A	2	2	1.00	22	0.091
517	A	7	6	1.00	22	0.273
518	A	3	3	1.00	22	0.136
519	A	8	6	1.00	22	0.273
520	A	3	2	1.00	22	0.091
521	A	8	5	1.00	22	0.227
522	A	3	2	1.00	22	0.091
523	A	7	5	1.00	22	0.227
524	A	3	2	1.00	20	0.100
525	A	6	4	1.00	19	0.210
526	A	7	5	1.00	22	0.227
527	A	6	4	1.00	22	0.182
528	A	7	5	1.00	22	0.227
529	A	6	5	1.00	22	0.227
530	A	7	6	1.00	22	0.273
531	A	6	5	1.00	22	0.227
532	A	7	6	1.00	22	0.273
533	A	6	4	1.00	22	0.182
534	A	7	5	1.00	22	0.227
535	A	2	2	1.00	22	0.091
536	A	8	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
537	A	3	2	1.00	22	0.091
538	A	5	4	1.00	22	0.182
539	A	3	2	1.00	22	0.091
540	A	4	4	1.00	22	0.182
541	A	3	2	1.00	20	0.100
542	A	3	3	1.00	19	0.158
543	A	4	4	1.00	22	0.182
544	A	3	3	1.00	22	0.136
545	A	4	4	1.00	22	0.182
546	A	2	2	1.00	22	0.091
547	A	5	5	1.00	22	0.227
548	A	3	3	1.00	22	0.136
549	A	6	5	1.00	22	0.227
550	A	4	3	1.00	22	0.136
551	A	6	5	1.00	22	0.227
552	A	3	2	1.00	22	0.091
553	A	5	5	1.00	22	0.227
554	A	3	2	1.00	22	0.091
555	A	4	4	1.00	22	0.182
556	A	3	2	1.00	20	0.100
557	A	3	3	1.00	19	0.158
558	A	4	4	1.00	22	0.182
559	A	2	2	1.00	22	0.091
560	A	5	5	1.00	22	0.227
561	A	3	3	1.00	22	0.136
562	A	6	5	1.02	22	0.227
563	A	4	3	1.00	22	0.136
564	A	7	5	1.00	22	0.227
565	A	5	3	1.00	22	0.136
566	A	3	2	1.00	22	0.091
567	A	6	5	1.00	22	0.227
568	A	3	2	1.00	22	0.091
569	A	5	5	1.00	22	0.227
570	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
571	A	4	4	1.00	22	0.182
572	A	3	2	1.00	20	0.100
573	A	2	2	1.00	19	0.105
574	A	5	5	1.00	22	0.227
575	A	3	3	1.00	22	0.136
576	A	6	5	1.00	22	0.227
577	A	4	4	1.00	22	0.182
578	A	7	5	1.03	22	0.227
579	A	5	4	1.00	22	0.182
580	A	3	2	1.00	24	0.083
581	A	3	2	1.00	24	0.083
582	A	3	2	1.00	22	0.091
583	A	6	5	1.00	24	0.208
584	A	6	6	1.00	24	0.250
585	A	6	6	0.98	24	0.250
586	A	6	6	1.00	24	0.250
587	A	6	6	0.98	24	0.250
588	A	5	5	1.00	21	0.238
589	A	5	5	0.98	24	0.208
590	A	5	5	1.00	24	0.208
591	A	5	5	1.00	24	0.208
592	A	3	3	1.01	24	0.125
593	A	4	4	1.01	24	0.167
594	A	5	4	1.01	24	0.167
595	A	8	6	0.99	24	0.250
596	A	3	2	1.00	24	0.083
597	A	7	6	0.99	24	0.250
598	A	3	2	1.00	22	0.091
599	A	6	5	1.00	21	0.238
600	A	7	5	1.00	24	0.208
601	A	6	5	0.98	24	0.208
602	A	7	6	1.00	24	0.250
603	A	6	5	0.98	24	0.208
604	A	7	6	0.98	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
605	A	6	6	1.00	24	0.250
606	A	7	7	0.98	24	0.292
607	A	3	2	1.00	24	0.083
608	A	8	6	0.99	24	0.250
609	A	3	2	1.00	22	0.091
610	A	7	5	1.00	21	0.238
611	A	8	5	1.00	24	0.208
612	A	7	5	0.99	24	0.208
613	A	8	6	1.00	24	0.250
614	A	7	5	0.98	24	0.208
615	A	8	6	0.99	24	0.250
616	A	7	6	0.99	24	0.250
617	A	8	7	0.99	24	0.292
618	A	6	5	1.00	24	0.208
619	A	3	2	1.00	24	0.083
620	A	5	5	1.00	24	0.208
621	A	3	2	1.00	22	0.091
622	A	4	4	1.00	21	0.190
623	A	5	4	1.00	24	0.167
624	A	4	4	1.00	24	0.167
625	A	5	5	1.00	24	0.208
626	A	4	4	1.00	24	0.167
627	A	5	5	1.00	24	0.208
628	A	3	3	1.01	24	0.125
629	A	6	6	1.00	24	0.250
630	A	6	5	1.00	24	0.208
631	A	3	2	1.00	24	0.083
632	A	5	5	1.00	24	0.208
633	A	3	2	1.00	22	0.091
634	A	4	4	1.00	21	0.190
635	A	5	4	1.00	24	0.167
636	A	4	4	0.96	24	0.167
637	A	5	5	1.00	24	0.208
638	A	3	3	1.01	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
639	A	6	6	1.00	24	0.250
640	A	4	4	1.00	24	0.167
641	A	7	6	1.02	24	0.250
642	A	6	6	1.00	24	0.250
643	A	3	2	1.00	24	0.083
644	A	5	5	1.00	24	0.208
645	A	3	2	1.00	22	0.091
646	A	4	4	1.00	21	0.190
647	A	5	4	1.00	24	0.167
648	A	3	3	1.00	24	0.125
649	A	6	6	1.00	24	0.250
650	A	4	4	0.99	24	0.167
651	A	7	6	1.00	24	0.250
652	A	5	5	1.00	24	0.208
653	A	4	3	1.00	22	0.136
654	A	3	3	1.00	22	0.136
655	A	2	2	1.00	20	0.100
656	A	3	3	1.00	22	0.136
657	A	4	3	1.00	22	0.136
658	A	7	7	1.00	24	0.292
659	A	5	5	1.00	24	0.208
660	A	6	6	1.00	24	0.250
661	A	4	4	1.00	22	0.182
662	A	5	5	1.00	21	0.238
663	A	6	4	1.00	24	0.167
664	A	4	4	1.00	24	0.167
665	A	7	5	1.00	24	0.208
666	A	5	5	1.00	24	0.208
667	A	8	7	1.00	24	0.292
668	A	6	5	1.00	24	0.208
669	A	7	7	1.00	24	0.292
670	A	5	4	1.00	22	0.182
671	A	6	6	1.00	21	0.286
672	A	7	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
673	A	6	6	1.00	24	0.250
674	A	7	5	1.00	24	0.208
675	A	5	5	1.00	24	0.208
676	A	9	8	1.00	24	0.333
677	A	7	5	1.00	24	0.208
678	A	8	8	1.00	24	0.333
679	A	6	4	1.00	22	0.182
680	A	7	7	1.00	21	0.333
681	A	8	6	1.00	24	0.250
682	A	7	7	1.00	24	0.292
683	A	8	6	1.00	24	0.250
684	A	7	7	1.00	24	0.292
685	A	5	4	1.00	24	0.167
686	A	4	4	1.00	24	0.167
687	A	3	3	1.00	22	0.136
688	A	6	4	1.00	24	0.167
689	A	7	5	1.00	24	0.208
690	A	6	6	1.00	24	0.250
691	A	5	5	1.00	24	0.208
692	A	2	2	1.00	21	0.095
693	A	4	4	1.00	24	0.167
694	A	5	5	1.00	24	0.208
695	A	6	6	1.00	24	0.250
696	A	4	4	1.00	24	0.167
697	A	4	4	1.00	24	0.167
698	A	4	4	1.00	22	0.182
699	A	3	3	1.00	21	0.143
700	A	7	5	1.00	24	0.208
701	A	5	5	1.00	24	0.208
702	A	8	6	1.00	24	0.250
703	A	6	5	1.00	24	0.208
704	A	5	5	1.00	24	0.208
705	A	5	5	1.00	24	0.208
706	A	5	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
707	A	5	4	1.00	22	0.182
708	A	5	5	1.00	21	0.238
709	A	8	6	1.00	24	0.250
710	A	6	6	1.00	24	0.250
711	A	9	6	1.00	24	0.250
712	A	7	6	1.00	24	0.250
713	A	7	7	1.00	24	0.292
714	A	5	5	1.00	24	0.208
715	A	6	6	1.00	24	0.250
716	A	4	4	1.00	22	0.182
717	A	3	3	1.00	21	0.143
718	A	7	5	1.00	24	0.208
719	A	5	5	1.00	24	0.208
720	A	8	6	1.00	24	0.250
721	A	6	5	1.00	24	0.208
722	A	8	8	1.00	24	0.333
723	A	6	5	1.00	24	0.208
724	A	7	7	1.00	24	0.292
725	A	5	5	1.00	22	0.227
726	A	6	6	1.00	21	0.286
727	A	7	5	1.00	24	0.208
728	A	5	5	1.00	24	0.208
729	A	8	6	1.00	24	0.250
730	A	6	5	1.00	24	0.208
731	A	9	8	1.00	24	0.333
732	A	7	5	1.00	24	0.208
733	A	8	7	1.00	24	0.292
734	A	6	5	1.00	22	0.227
735	A	7	7	1.00	21	0.333
736	A	8	6	1.00	24	0.250
737	A	7	7	1.00	24	0.292
738	A	8	6	1.00	24	0.250
739	A	6	6	1.00	24	0.250
740	A	6	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
741	A	4	4	1.00	24	0.167
742	A	4	4	1.00	24	0.167
743	A	4	4	1.00	22	0.182
744	A	3	3	1.00	21	0.143
745	A	7	5	1.00	24	0.208
746	A	5	5	1.00	24	0.208
747	A	8	6	1.00	24	0.250
748	A	6	5	1.00	24	0.208
749	A	5	5	1.00	24	0.208
750	A	5	5	1.00	24	0.208
751	A	5	5	1.00	24	0.208
752	A	5	4	1.00	22	0.182
753	A	5	5	1.00	21	0.238
754	A	8	6	1.00	24	0.250
755	A	6	6	1.00	24	0.250
756	A	9	7	1.00	24	0.292
757	A	7	6	1.00	24	0.250
758	A	6	5	1.00	24	0.208
759	A	6	5	1.00	24	0.208
760	A	6	5	1.00	24	0.208
761	A	6	4	1.00	22	0.182
762	A	6	5	1.00	21	0.238
763	A	9	6	1.00	24	0.250
764	A	7	6	1.00	24	0.250
765	A	10	7	1.00	24	0.292
766	A	8	6	1.00	24	0.250
767	A	7	7	1.00	26	0.269
768	A	6	6	1.00	26	0.231
769	A	5	5	1.00	24	0.208
770	A	7	7	1.00	26	0.269
771	A	4	4	1.00	26	0.154
772	A	5	5	1.00	26	0.192
773	A	8	7	1.00	26	0.269
774	A	7	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
775	A	6	5	1.00	24	0.208
776	A	8	8	1.00	26	0.308
777	A	8	8	1.00	26	0.308
778	A	5	4	1.00	26	0.154
779	A	9	7	1.00	26	0.269
780	A	8	6	1.00	26	0.231
781	A	7	5	1.00	24	0.208
782	A	9	9	1.00	26	0.346
783	A	9	9	1.00	26	0.346
784	A	9	9	1.00	26	0.346
785	A	6	6	1.00	26	0.231
786	A	5	5	1.00	24	0.208
787	A	6	6	1.00	26	0.231
788	A	5	5	1.00	26	0.192
789	A	4	4	1.00	24	0.167
790	A	3	3	1.00	26	0.115
791	A	4	4	1.00	26	0.154
792	A	6	6	1.00	26	0.231
793	A	6	6	1.00	26	0.231
794	A	5	5	1.00	26	0.192
795	A	2	2	1.00	24	0.083
796	A	6	6	1.00	26	0.231
797	A	3	3	1.00	26	0.115
798	A	3	3	1.00	24	0.125
799	A	4	4	1.00	26	0.154
800	A	4	4	1.00	26	0.154
801	A	4	3	1.00	24	0.125
802	A	5	5	1.00	26	0.192
803	A	4	4	1.00	25	0.160
804	A	4	4	1.00	26	0.154
805	A	7	6	1.00	22	0.273
806	A	6	6	1.00	22	0.273
807	A	5	5	1.00	20	0.250
808	A	10	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
809	A	7	7	1.00	22	0.318
810	A	12	9	1.00	22	0.409
811	A	1	1	1.00	19	0.053
812	A	7	7	1.00	22	0.318
813	A	7	7	1.00	22	0.318
814	A	6	6	1.00	22	0.273
815	A	6	6	1.00	20	0.300
816	A	11	8	1.00	22	0.364
817	A	12	9	1.00	22	0.409
818	A	13	9	1.00	22	0.409
819	A	10	5	1.00	24	0.208
820	A	7	5	1.00	24	0.208
821	A	4	3	1.00	24	0.125
822	A	1	1	1.00	22	0.045
823	A	8	7	1.00	24	0.292
824	A	14	8	1.00	24	0.333
825	A	1	1	1.00	21	0.048
826	A	7	6	1.00	24	0.250
827	A	7	6	1.00	24	0.250
828	A	6	6	1.00	24	0.250
829	A	5	5	1.00	22	0.227
830	A	16	12	1.00	24	0.500
831	A	17	13	1.00	24	0.542
832	A	1	1	1.00	21	0.048
833	A	1	1	1.00	24	0.042
834	A	1	1	1.00	24	0.042
835	A	1	1	1.00	24	0.042
836	A	1	1	1.00	26	0.038
837	A	1	1	1.00	26	0.038
838	A	1	1	1.00	26	0.038
839	A	1	1	1.00	26	0.038
840	A	1	1	1.00	28	0.036
841	A	20	12	1.00	24	0.500
842	A	17	12	1.00	24	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
843	A	14	10	1.00	24	0.417
844	A	11	8	1.00	22	0.364
845	A	18	13	1.00	24	0.542
846	A	24	14	1.00	24	0.583
847	A	1	1	1.00	24	0.042
848	A	1	1	1.00	24	0.042
849	A	1	1	1.00	24	0.042
850	A	1	1	1.00	24	0.042
851	A	1	1	1.00	26	0.038
852	A	1	1	1.00	28	0.036
853	A	1	1	1.00	28	0.036
854	A	1	1	1.00	28	0.036
855	A	1	1	1.00	30	0.033
856	A	7	6	1.00	24	0.250
857	A	7	6	1.00	24	0.250
858	A	6	6	1.00	24	0.250
859	A	5	5	1.00	22	0.227
860	A	16	12	1.00	24	0.500
861	A	17	13	1.00	24	0.542
862	A	1	1	1.00	24	0.042
863	A	7	7	1.00	26	0.269
864	A	6	6	1.00	26	0.231
865	A	6	6	1.00	26	0.231
866	A	2	2	1.00	26	0.077
867	A	3	3	1.00	26	0.115
868	A	4	3	1.00	26	0.115
869	A	7	7	1.00	26	0.269
870	A	6	6	1.00	26	0.231
871	A	2	2	1.00	26	0.077
872	A	3	3	1.00	26	0.115
873	A	4	3	1.00	26	0.115
874	A	7	7	1.00	26	0.269
875	A	6	6	1.00	26	0.231
876	A	2	2	1.00	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
877	A	3	3	1.00	26	0.115
878	A	4	3	1.00	26	0.115
879	A	8	8	1.00	26	0.308
880	A	7	7	1.00	26	0.269
881	A	2	2	1.00	26	0.077
882	A	3	3	1.00	26	0.115
883	A	4	3	1.00	26	0.115
884	A	5	3	1.00	26	0.115

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2 \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^2\right) \left(A+Bx^2\right) \, dx &= \int \left(aAx^2+(Ab+aB)x^4+bBx^6\right) \, dx\\ &= \frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + bx^2\right) \left(A + Bx^2\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)*(A + B*x^2), x]
```

fricas [A] time = 0.38, size = 29, normalized size = 0.88

$$\frac{1}{7}x^7bB + \frac{1}{5}x^5aB + \frac{1}{5}x^5bA + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/3*x^3*a*A
```

giac [A] time = 0.30, size = 29, normalized size = 0.88

$$\frac{1}{7} \, Bbx^7 + \frac{1}{5} \, Bax^5 + \frac{1}{5} \, Abx^5 + \frac{1}{3} \, Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b*x^7 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/3*A*a*x^3
```

maple [A] time = 0.02, size = 28, normalized size = 0.85

$$\frac{Bb\,x^7}{7} + \frac{Aa\,x^3}{3} + \frac{(Ab + Ba)\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

maxima [A] time = 1.34, size = 27, normalized size = 0.82

$$\frac{1}{7} \, Bbx^7 + \frac{1}{5} \, (Ba + Ab)x^5 + \frac{1}{3} \, Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

mupad [B] time = 0.18, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^7}{7}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^3)/3 + (B*b*x^7)/7
```

sympy [A] time = 0.10, size = 29, normalized size = 0.88

$$\frac{Aax^{3}}{3}+\frac{Bbx^{7}}{7}+x^{5}\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**3/3 + B*b*x**7/7 + x**5*(A*b/5 + B*a/5)
```

3.2 $\int x\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 43}

$$\frac{1}{4}x^4(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(A+Bx^2\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(aA+(Ab+aB)x+bBx^2\right)\,dx,x,x^2\right)\\&=\frac{1}{2}aAx^2+\frac{1}{4}(Ab+aB)x^4+\frac{1}{6}bBx^6\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{1}{4}x^4(aB + Ab) + \frac{1}{2}aAx^2 + \frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + bx^2\right)\left(A + Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)*(A + B*x^2), x]
```

fricas [A] time = 0.37, size = 29, normalized size = 0.88

$$\frac{1}{6}x^6bB + \frac{1}{4}x^4aB + \frac{1}{4}x^4bA + \frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + 1/2*x^2*a*A
```

giac [A] time = 0.38, size = 29, normalized size = 0.88

$$\frac{1}{6}\,Bbx^6 + \frac{1}{4}\,Bax^4 + \frac{1}{4}\,Abx^4 + \frac{1}{2}\,Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/6*B*b*x^6 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/2*A*a*x^2
```

maple [A] time = 0.00, size = 28, normalized size = 0.85

$$\frac{Bb\,x^6}{6} + \frac{Aa\,x^2}{2} + \frac{(Ab + Ba)\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

maxima [A] time = 1.35, size = 27, normalized size = 0.82

$$\frac{1}{6}\,Bbx^{6}+\frac{1}{4}\,(Ba+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

mupad [B] time = 0.04, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^4+\frac{A\,a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^4*((A*b)/4 + (B*a)/4) + (A*a*x^2)/2 + (B*b*x^6)/6
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Aax^{2}}{2}+\frac{Bbx^{6}}{6}+x^{4}\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**6/6 + x**4*(A*b/4 + B*a/4)
```

3.3 $\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.067, Rules used = {373}

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aA+(Ab+aB)x^2+bBx^4\right)\,dx\\ &= aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)*(A + B*x^2), x]
```

fricas [A] time = 0.40, size = 26, normalized size = 0.93

$$\frac{1}{5}x^5bB+\frac{1}{3}x^3aB+\frac{1}{3}x^3bA+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b*B + 1/3*x^3*a*B + 1/3*x^3*b*A + x*a*A
```

giac [A] time = 0.41, size = 26, normalized size = 0.93

$$\frac{1}{5}\,Bbx^5+\frac{1}{3}\,Bax^3+\frac{1}{3}\,Abx^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$\frac{Bb\,x^5}{5}+Aax+\frac{(Ab+Ba)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A),x)
```

```
[Out] a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

maxima [A] time = 1.34, size = 24, normalized size = 0.86

$$\frac{1}{5}\,Bbx^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^3+A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^3*((A*b)/3 + (B*a)/3) + A*a*x + (B*b*x^5)/5
```

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$Aax+\frac{Bbx^5}{5}+x^3\left(\frac{Ab}{3}+\frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A),x)
```

```
[Out] A*a*x + B*b*x**5/5 + x**3*(A*b/3 + B*a/3)
```

3.4 $\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}x^2(aB+Ab)+aA\log(x)+\frac{1}{4}bBx^4$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\frac{1}{2}x^2(aB+Ab)+aA\log(x)+\frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)dx,x,x^2\right)\\ &= \frac{1}{2}(Ab+aB)x^2+\frac{1}{4}bBx^4+aA\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{1}{2}x^2(aB + Ab) + aA\log(x) + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] ((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(A + Bx^2)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x, x]
```

fricas [A] time = 0.47, size = 25, normalized size = 0.86

$$\frac{1}{4}\,Bbx^4 + \frac{1}{2}\,(Ba + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + A*a*log(x)
```

giac [A] time = 0.31, size = 30, normalized size = 1.03

$$\frac{1}{4}\,Bbx^4 + \frac{1}{2}\,Bax^2 + \frac{1}{2}\,Abx^2 + \frac{1}{2}\,Aa\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + 1/2*A*a*log(x^2)
```

maple [A] time = 0.02, size = 28, normalized size = 0.97

$$\frac{Bb\,x^4}{4} + \frac{Ab\,x^2}{2} + \frac{Ba\,x^2}{2} + Aa\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x,x)
```

```
[Out] 1/4*b*B*x^4+1/2*A*x^2*b+1/2*B*x^2*a+a*A*ln(x)
```

maxima [**A**] time = 1.31, size = 28, normalized size = 0.97

$$\frac{1}{4}\,Bbx^{4} + \frac{1}{2}\,(Ba + Ab)x^{2} + \frac{1}{2}\,Aa\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + 1/2*A*a*log(x^2)
```

mupad [**B**] time = 0.06, size = 26, normalized size = 0.90

$$x^{2}\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right) + \frac{B\,b\,x^{4}}{4} + A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x,x)
```

```
[Out] x^2*((A*b)/2 + (B*a)/2) + (B*b*x^4)/4 + A*a*log(x)
```

sympy [**A**] time = 0.11, size = 27, normalized size = 0.93

$$Aa\log{\left(x \right)} + \frac{Bbx^{4}}{4} + x^{2}\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x,x)
```

```
[Out] A*a*log(x) + B*b*x**4/4 + x**2*(A*b/2 + B*a/2)
```

3.5 $\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=26

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^2}+bBx^2\right)dx\\ &= -\frac{aA}{x}+(Ab+aB)x+\frac{1}{3}bBx^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^2, x]
```

fricas [A] time = 0.40, size = 28, normalized size = 1.08

$$\frac{Bbx^4 + 3\,(Ba + Ab)x^2 - 3\,Aa}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^4 + 3*(B*a + A*b)*x^2 - 3*A*a)/x
```

giac [A] time = 0.28, size = 23, normalized size = 0.88

$$\frac{1}{3}\,Bbx^3 + Bax + Abx - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*B*b*x^3 + B*a*x + A*b*x - A*a/x
```

maple [A] time = 0.05, size = 24, normalized size = 0.92

$$\frac{Bb\,x^3}{3} + Abx + Bax - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/3*b*B*x^3+A*b*x+B*a*x-a*A/x
```

maxima [A] time = 1.39, size = 24, normalized size = 0.92

$$\frac{1}{3}\,Bbx^3 + (Ba + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*b*x^3 + (B*a + A*b)*x - A*a/x
```

mupad [B] time = 0.08, size = 24, normalized size = 0.92

$$x\,\left(A\,b+B\,a\right)-\frac{A\,a}{x}+\frac{B\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^2,x)
```

```
[Out] x*(A*b + B*a) - (A*a)/x + (B*b*x^3)/3
```

sympy [A] time = 0.11, size = 20, normalized size = 0.77

$$-\frac{Aa}{x}+\frac{Bbx^3}{3}+x\left(Ab+Ba\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a/x + B*b*x**3/3 + x*(A*b + B*a)
```

3.6 $\int \frac{(a+bx^2)(A+Bx^2)}{x^3} \, dx$

Optimal. Leaf size=29

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\log(x)(aB+Ab)-\frac{aA}{2x^2}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)(A+Bx^2)}{x^3} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right) dx, x, x^2\right) \\
&= -\frac{aA}{2x^2}+\frac{1}{2}bBx^2+(Ab+aB)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\log(x)(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)(A + Bx^2)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^3, x]
```

fricas [A] time = 0.48, size = 30, normalized size = 1.03

$$\frac{Bbx^4 + 2\,(Ba + Ab)x^2\log(x) - Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b*x^4 + 2*(B*a + A*b)*x^2*log(x) - A*a)/x^2
```

giac [A] time = 0.41, size = 42, normalized size = 1.45

$$\frac{1}{2}\,Bbx^2 + \frac{1}{2}\,(Ba + Ab)\log\left(x^2\right) - \frac{Bax^2 + Abx^2 + Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*(B*a*x^2 + A*b*x^2 + A*a)/x^2
```

maple [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{Bb\,x^2}{2} + Ab\ln(x) + Ba\ln(x) - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^3,x)
```

```
[Out] 1/2*b*B*x^2-1/2*a*A/x^2+A*ln(x)*b+B*ln(x)*a
```

maxima [A] time = 1.35, size = 28, normalized size = 0.97

$$\frac{1}{2}\,Bbx^2+\frac{1}{2}\,(Ba+Ab)\log\left(x^2\right)-\frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*A*a/x^2
```

mupad [B] time = 0.06, size = 25, normalized size = 0.86

$$\ln(x)\,\left(A\,b+B\,a\right)-\frac{A\,a}{2\,x^2}+\frac{B\,b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^3,x)
```

```
[Out] log(x)*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^2)/2
```

sympy [A] time = 0.18, size = 26, normalized size = 0.90

$$-\frac{Aa}{2x^2}+\frac{Bbx^2}{2}+(Ab+Ba)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a/(2*x**2) + B*b*x**2/2 + (A*b + B*a)*log(x)
```

3.7 $\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=26

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (A*b + a*B)/x + b*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx &= \int \left(bB+\frac{aA}{x^4}+\frac{Ab+aB}{x^2}\right)dx\\ &= -\frac{aA}{3x^3}-\frac{Ab+aB}{x}+bBx\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.04

$$\frac{-aB-Ab}{x}-\frac{aA}{3x^3}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (-(A*b) - a*B)/x + b*B*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^4, x]
```

fricas [A] time = 0.41, size = 29, normalized size = 1.12

$$\frac{3\,Bbx^4-3\,(Ba+Ab)x^2-Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b*x^4 - 3*(B*a + A*b)*x^2 - A*a)/x^3
```

giac [A] time = 0.32, size = 28, normalized size = 1.08

$$Bbx-\frac{3\,Bax^2+3\,Abx^2+Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] B*b*x - 1/3*(3*B*a*x^2 + 3*A*b*x^2 + A*a)/x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.96

$$Bbx-\frac{Aa}{3x^3}-\frac{Ab+Ba}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^4,x)
```

```
[Out] b*B*x-(A*b+B*a)/x-1/3*a*A/x^3
```

maxima [A] time = 1.38, size = 26, normalized size = 1.00

$$Bbx-\frac{3\,(Ba+Ab)x^2+Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] B*b*x - 1/3*(3*(B*a + A*b)*x^2 + A*a)/x^3
```

mupad [B] time = 0.03, size = 26, normalized size = 1.00

$$Bbx-\frac{(Ab+Ba)\,x^2+\frac{Aa}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^4,x)
```

```
[Out] B*b*x - ((A*a)/3 + x^2*(A*b + B*a))/x^3
```

sympy [A] time = 0.21, size = 27, normalized size = 1.04

$$Bbx+\frac{-Aa+x^2\,(-3Ab-3Ba)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**4,x)
```

```
[Out] B*b*x + (-A*a + x**2*(-3*A*b - 3*B*a))/(3*x**3)
```

3.8 $\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=29

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/(2*x^2) + b*B*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right)dx, x, x^2\right)\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{2x^2}+bB\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.07

$$\frac{-aB-Ab}{2x^2}-\frac{aA}{4x^4}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 + (-(A*b) - a*B)/(2*x^2) + b*B*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^5, x]
```

fricas [A] time = 0.48, size = 31, normalized size = 1.07

$$\frac{4\,Bbx^4\log(x)-2\,(Ba+Ab)x^2-Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*B*b*x^4*log(x) - 2*(B*a + A*b)*x^2 - A*a)/x^4
```

giac [A] time = 0.32, size = 39, normalized size = 1.34

$$\frac{1}{2}\,Bb\log\left(x^2\right)-\frac{3\,Bbx^4+2\,Bax^2+2\,Abx^2+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b*log(x^2) - 1/4*(3*B*b*x^4 + 2*B*a*x^2 + 2*A*b*x^2 + A*a)/x^4
```

maple [A] time = 0.00, size = 28, normalized size = 0.97

$$Bb\ln(x)-\frac{Ab}{2x^2}-\frac{Ba}{2x^2}-\frac{Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/4*a*A/x^4-1/2/x^2*A*b-1/2/x^2*B*a+b*B*ln(x)
```

maxima [**A**] time = 1.39, size = 30, normalized size = 1.03

$$\frac{1}{2}\,Bb\log\left(x^{2}\right)-\frac{2\,(Ba+Ab)x^{2}+Aa}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b*log(x^2) - 1/4*(2*(B*a + A*b)*x^2 + A*a)/x^4
```

mupad [**B**] time = 0.07, size = 29, normalized size = 1.00

$$B\,b\,\ln(x)-\frac{\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,x^{2}+\frac{A\,a}{4}}{x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^5,x)
```

```
[Out] B*b*log(x) - ((A*a)/4 + x^2*((A*b)/2 + (B*a)/2))/x^4
```

sympy [**A**] time = 0.36, size = 29, normalized size = 1.00

$$Bb\log\left(x\right)+\frac{-Aa+x^{2}\left(-2Ab-2Ba\right)}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**5,x)
```

```
[Out] B*b*log(x) + (-A*a + x**2*(-2*A*b - 2*B*a))/(4*x**4)
```

3.9 $\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=31

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Rubi [**A**] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(3*x^3) - (b*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx &= \int \left(\frac{aA}{x^6}+\frac{Ab+aB}{x^4}+\frac{bB}{x^2}\right)dx\\ &= -\frac{aA}{5x^5}-\frac{Ab+aB}{3x^3}-\frac{bB}{x}\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 33, normalized size = 1.06

$$\frac{-aB-Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 + (-(A*b) - a*B)/(3*x^3) - (b*B)/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^6, x]
```

fricas [A] time = 0.46, size = 29, normalized size = 0.94

$$-\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

giac [A] time = 0.29, size = 31, normalized size = 1.00

$$-\frac{15\,Bbx^4+5\,Bax^2+5\,Abx^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*B*a*x^2 + 5*A*b*x^2 + 3*A*a)/x^5
```

maple [A] time = 0.01, size = 28, normalized size = 0.90

$$-\frac{Bb}{x}-\frac{Aa}{5x^5}-\frac{Ab+Ba}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^6,x)
```

```
[Out] -b*B/x-1/3*(A*b+B*a)/x^3-1/5*a*A/x^5
```

maxima [A] time = 1.40, size = 29, normalized size = 0.94

$$-\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

mupad [B] time = 0.04, size = 29, normalized size = 0.94

$$-\frac{B\,b\,x^4+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^2+\frac{A\,a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^6,x)
```

```
[Out] -((A*a)/5 + x^2*((A*b)/3 + (B*a)/3) + B*b*x^4)/x^5
```

sympy [A] time = 0.37, size = 32, normalized size = 1.03

$$\frac{-3Aa-15Bbx^4+x^2\left(-5Ab-5Ba\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**6,x)
```

```
[Out] (-3*A*a - 15*B*b*x**4 + x**2*(-5*A*b - 5*B*a))/(15*x**5)
```

3.10 $\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx$

Optimal. Leaf size=33

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(4*x^4) - (b*B)/(2*x^2)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{aA}{x^4}+\frac{Ab+aB}{x^3}+\frac{bB}{x^2}\right)dx,x,x^2\right)\\ &= -\frac{aA}{6x^6}-\frac{Ab+aB}{4x^4}-\frac{bB}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.06

$$\frac{-aB-Ab}{4x^4}-\frac{aA}{6x^6}-\frac{bB}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(4*x^4) - (b*B)/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^7, x]
```

fricas [A] time = 0.43, size = 29, normalized size = 0.88

$$-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

giac [A] time = 0.33, size = 31, normalized size = 0.94

$$-\frac{6\,Bbx^4+3\,Bax^2+3\,Abx^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*B*b*x^4 + 3*B*a*x^2 + 3*A*b*x^2 + 2*A*a)/x^6
```

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-\frac{Bb}{2x^2}-\frac{Aa}{6x^6}-\frac{Ab+Ba}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/4*(A*b+B*a)/x^4-1/6*a*A/x^6-1/2*b*B/x^2
```

maxima [**A**] time = 1.37, size = 29, normalized size = 0.88

$$-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

mupad [**B**] time = 0.04, size = 30, normalized size = 0.91

$$-\frac{\frac{B\,b\,x^4}{2}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^2+\frac{A\,a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^7,x)
```

```
[Out] -((A*a)/6 + x^2*((A*b)/4 + (B*a)/4) + (B*b*x^4)/2)/x^6
```

sympy [**A**] time = 0.49, size = 32, normalized size = 0.97

$$\frac{-2Aa-6Bbx^4+x^2\,(-3Ab-3Ba)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**7,x)
```

```
[Out] (-2*A*a - 6*B*b*x**4 + x**2*(-3*A*b - 3*B*a))/(12*x**6)
```

3.11 $\int x^2 \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx &= \int \left(a^2Ax^2+a(2Ab+aB)x^4+b(Ab+2aB)x^6+b^2Bx^8\right)\,dx\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{9}b^2Bx^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(A + B*x^2), x]
```

fricas [A] time = 0.40, size = 53, normalized size = 0.96

$$\frac{1}{9}x^9b^2B + \frac{2}{7}x^7baB + \frac{1}{7}x^7b^2A + \frac{1}{5}x^5a^2B + \frac{2}{5}x^5baA + \frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/5*x^5*a^2*B + 2/5*x^5*b*a
*A + 1/3*x^3*a^2*A
```

giac [A] time = 0.31, size = 53, normalized size = 0.96

$$\frac{1}{9}\,Bb^2x^9 + \frac{2}{7}\,Babx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{5}\,Ba^2x^5 + \frac{2}{5}\,Aabx^5 + \frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/5*B*a^2*x^5 + 2/5*A*a*b*x
^5 + 1/3*A*a^2*x^3
```

maple [A] time = 0.00, size = 52, normalized size = 0.95

$$\frac{B\,b^2x^9}{9} + \frac{\left(b^2A + 2abB\right)x^7}{7} + \frac{A\,a^2x^3}{3} + \frac{\left(2abA + a^2B\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/9*b^2*B*x^9+1/7*(A*b^2+2*B*a*b)*x^7+1/5*(2*A*a*b+B*a^2)*x^5+1/3*a^2*A*x^3
```

maxima [A] time = 1.38, size = 51, normalized size = 0.93

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{7}\left(2\,Bab+Ab^2\right)x^7+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 + 2*
A*a*b)*x^5
```

mupad [B] time = 0.08, size = 51, normalized size = 0.93

$$x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (A*a^2*x^3)
/3 + (B*b^2*x^9)/9
```

sympy [A] time = 0.07, size = 56, normalized size = 1.02

$$\frac{Aa^2x^3}{3}+\frac{Bb^2x^9}{9}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^5\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**9/9 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

3.12 $\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^3(Ab-aB)}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}$$

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^3(Ab-aB)}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^3)/(6*b^2) + (B*(a + b*x^2)^4)/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right)dx,x,x^2\right)\\&=\frac{(Ab-aB)\left(a+bx^2\right)^3}{6b^2}+\frac{B\left(a+bx^2\right)^4}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2A+4bx^4(2aB+Ab)+6ax^2(aB+2Ab)+3b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (x^2*(12*a^2*A + 6*a*(2*A*b + a*B)*x^2 + 4*b*(A*b + 2*a*B)*x^4 + 3*b^2*B*x^
6))/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^2*(A + B*x^2), x]
```

fricas [A] time = 0.38, size = 53, normalized size = 1.26

$$\frac{1}{8}x^8b^2B+\frac{1}{3}x^6baB+\frac{1}{6}x^6b^2A+\frac{1}{4}x^4a^2B+\frac{1}{2}x^4baA+\frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2*B + 1/3*x^6*b*a*B + 1/6*x^6*b^2*A + 1/4*x^4*a^2*B + 1/2*x^4*b*a
*A + 1/2*x^2*a^2*A
```

giac [A] time = 0.28, size = 53, normalized size = 1.26

$$\frac{1}{8}\,Bb^2x^8+\frac{1}{3}\,Babx^6+\frac{1}{6}\,Ab^2x^6+\frac{1}{4}\,Ba^2x^4+\frac{1}{2}\,Aabx^4+\frac{1}{2}\,Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/4*B*a^2*x^4 + 1/2*A*a*b*x
^4 + 1/2*A*a^2*x^2
```

maple [A] time = 0.00, size = 52, normalized size = 1.24

$$\frac{B\,b^2x^8}{8}+\frac{\left(b^2A+2abB\right)x^6}{6}+\frac{A\,a^2x^2}{2}+\frac{\left(2abA+a^2B\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/8*b^2*B*x^8+1/6*(A*b^2+2*B*a*b)*x^6+1/4*(2*A*a*b+B*a^2)*x^4+1/2*a^2*A*x^2
```

maxima [A] time = 1.37, size = 51, normalized size = 1.21

$$\frac{1}{8}Bb^2x^8 + \frac{1}{6}\left(2\,Bab + Ab^2\right)x^6 + \frac{1}{2}Aa^2x^2 + \frac{1}{4}\left(Ba^2 + 2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(B*a^2 + 2*
A*a*b)*x^4
```

mupad [B] time = 0.04, size = 51, normalized size = 1.21

$$x^4\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^6*((A*b^2)/6 + (B*a*b)/3) + (A*a^2*x^2)/2 +
 (B*b^2*x^8)/8
```

sympy [A] time = 0.08, size = 53, normalized size = 1.26

$$\frac{Aa^2x^2}{2}+\frac{Bb^2x^8}{8}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**8/8 + x**6*(A*b**2/6 + B*a*b/3) + x**4*(A*a*b/2 +
 B*a**2/4)
```

3.13 $\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=50

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx &= \int \left(a^2A+a(2Ab+aB)x^2+b(Ab+2aB)x^4+b^2Bx^6\right) \, dx\\ &= a^2Ax+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{5}b(Ab+2aB)x^5+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(A + B*x^2), x]
```

fricas [A] time = 0.38, size = 50, normalized size = 1.00

$$\frac{1}{7}x^7b^2B+\frac{2}{5}x^5baB+\frac{1}{5}x^5b^2A+\frac{1}{3}x^3a^2B+\frac{2}{3}x^3baA+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2*B + 2/5*x^5*b*a*B + 1/5*x^5*b^2*A + 1/3*x^3*a^2*B + 2/3*x^3*b*a
*A + x*a^2*A
```

giac [A] time = 0.28, size = 50, normalized size = 1.00

$$\frac{1}{7}\,Bb^2x^7+\frac{2}{5}\,Babx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*x
```

maple [A] time = 0.00, size = 49, normalized size = 0.98

$$\frac{B\,b^2x^7}{7}+\frac{\left(b^2A+2abB\right)x^5}{5}+A\,a^2x+\frac{\left(2abA+a^2B\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 1/7*b^2*B*x^7+1/5*(A*b^2+2*B*a*b)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

maxima [A] time = 1.33, size = 48, normalized size = 0.96

$$\frac{1}{7}\,Bb^2x^7+\frac{1}{5}\left(2\,Bab+Ab^2\right)x^5+Aa^2x+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b)
*x^3
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^5\,\left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+\frac{B\,b^2\,x^7}{7}+A\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^5*((A*b^2)/5 + (2*B*a*b)/5) + (B*b^2*x^7)
/7 + A*a^2*x
```

sympy [A] time = 0.07, size = 53, normalized size = 1.06

$$Aa^2x+\frac{Bb^2x^7}{7}+x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}\right)+x^3\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**7/7 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**3*(2*A*a*b/3 +
B*a**2/3)
```

3.14 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x}\,dx$

Optimal. Leaf size=43

$$a^2A\log(x)+aAbx^2+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 80, 43}

$$a^2A\log(x)+aAbx^2+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] a*A*b*x^2 + (A*b^2*x^4)/4 + (B*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{2}A\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{2}A\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx,x,x^2\right)\\
&= aAbx^2+\frac{1}{4}Ab^2x^4+\frac{B\left(a+bx^2\right)^3}{6b}+a^2A\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.19

$$a^2A\log(x)+\frac{1}{4}bx^4(2aB+Ab)+\frac{1}{2}ax^2(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^6)/6 + a^2*A*L
og[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x, x]
```

fricas [A] time = 0.48, size = 49, normalized size = 1.14

$$\frac{1}{6}\,Bb^2x^6+\frac{1}{4}\left(2\,Bab+Ab^2\right)x^4+Aa^2\log(x)+\frac{1}{2}\left(Ba^2+2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + A*a^2*log(x) + 1/2*(B*a^2 + 2*A
*a*b)*x^2
```

giac [A] time = 0.31, size = 53, normalized size = 1.23

$$\frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Ba^2x^2 + Aabx^2 + \frac{1}{2}Aa^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*B*a^2*x^2 + A*a*b*x^2 +
 1/2*A*a^2*log(x^2)
```

maple [A] time = 0.01, size = 51, normalized size = 1.19

$$\frac{B\,b^2x^6}{6} + \frac{A\,b^2x^4}{4} + \frac{Bab\,x^4}{2} + Aab\,x^2 + \frac{B\,a^2x^2}{2} + A\,a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x,x)
```

```
[Out] 1/6*B*b^2*x^6+1/4*A*b^2*x^4+1/2*B*x^4*a*b+a*A*b*x^2+1/2*B*x^2*a^2+a^2*A*ln(
x)
```

maxima [A] time = 1.30, size = 52, normalized size = 1.21

$$\frac{1}{6}Bb^2x^6 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + \frac{1}{2}Aa^2\log\left(x^2\right) + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/2*A*a^2*log(x^2) + 1/2*(B*a^2
 + 2*A*a*b)*x^2
```

mupad [B] time = 0.04, size = 48, normalized size = 1.12

$$x^2\left(\frac{B\,a^2}{2} + A\,b\,a\right) + x^4\left(\frac{A\,b^2}{4} + \frac{B\,a\,b}{2}\right) + \frac{B\,b^2\,x^6}{6} + A\,a^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x,x)
```

```
[Out] x^2*((B*a^2)/2 + A*a*b) + x^4*((A*b^2)/4 + (B*a*b)/2) + (B*b^2*x^6)/6 + A*a
^2*log(x)
```

sympy [A] time = 0.14, size = 49, normalized size = 1.14

$$Aa^2\log(x) + \frac{Bb^2x^6}{6} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**6/6 + x**4*(A*b**2/4 + B*a*b/2) + x**2*(A*a*b + B
*a**2/2)
```

3.15 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

Rubi [**A**] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^2}\,dx&=\int \left(a(2Ab+aB)+\frac{a^2A}{x^2}+b(Ab+2aB)x^2+b^2Bx^4\right)dx\\&=-\frac{a^2A}{x}+a(2Ab+aB)x+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{5}b^2Bx^5\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 48, normalized size = 1.00

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^2, x]
```

fricas [A] time = 0.47, size = 53, normalized size = 1.10

$$\frac{3\,Bb^2x^6+5\left(2\,Bab+Ab^2\right)x^4-15\,Aa^2+15\left(Ba^2+2\,Aab\right)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^6 + 5*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 15*(B*a^2 + 2*A*a*
b)*x^2)/x
```

giac [A] time = 0.37, size = 48, normalized size = 1.00

$$\frac{1}{5}\,Bb^2x^5+\frac{2}{3}\,Babx^3+\frac{1}{3}\,Ab^2x^3+Ba^2x+2\,Aabx-\frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + B*a^2*x + 2*A*a*b*x - A*a^2
/x
```

maple [A] time = 0.01, size = 49, normalized size = 1.02

$$\frac{B\,b^2x^5}{5}+\frac{A\,b^2x^3}{3}+\frac{2Bab\,x^3}{3}+2Aabx+B\,a^2x-\frac{A\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^2,x)
```

```
[Out] 1/5*b^2*B*x^5+1/3*A*x^3*b^2+2/3*B*x^3*a*b+2*a*b*A*x+a^2*B*x-a^2*A/x
```

maxima [**A**] time = 1.31, size = 48, normalized size = 1.00

$$\frac{1}{5}Bb^2x^5 + \frac{1}{3}\left(2\,Bab + Ab^2\right)x^3 - \frac{Aa^2}{x} + \left(Ba^2 + 2\,Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/3*(2*B*a*b + A*b^2)*x^3 - A*a^2/x + (B*a^2 + 2*A*a*b)*x
```

mupad [**B**] time = 0.05, size = 48, normalized size = 1.00

$$x^3\left(\frac{A\,b^2}{3} + \frac{2\,B\,a\,b}{3}\right) + x\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{x} + \frac{B\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^2,x)
```

```
[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/x + (B*b^2*x^
5)/5
```

sympy [**A**] time = 0.14, size = 48, normalized size = 1.00

$$-\frac{Aa^2}{x} + \frac{Bb^2x^5}{5} + x^3\left(\frac{Ab^2}{3} + \frac{2Bab}{3}\right) + x\left(2Aab + Ba^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**5/5 + x**3*(A*b**2/3 + 2*B*a*b/3) + x*(2*A*a*b + B*a*
*2)
```

3.16 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{2x^2}+\frac{1}{2}bx^2(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x^4)/4 + a*(2*A*b + a*B
)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right)dx,x,x^2\right)\\ &= -\frac{a^2A}{2x^2}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{4}b^2Bx^4+a(2Ab+aB)\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2A}{x^2}+2bx^2(2aB+Ab)+4a\log(x)(aB+2Ab)+b^2Bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*A)/x^2 + 2*b*(A*b + 2*a*B)*x^2 + b^2*B*x^4 + 4*a*(2*A*b + a*B)*Log
[x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^3, x]
```

fricas [A] time = 0.45, size = 54, normalized size = 1.06

$$\frac{Bb^2x^6+2\left(2\,Bab+Ab^2\right)x^4+4\left(Ba^2+2\,Aab\right)x^2\log(x)-2\,Aa^2}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^6 + 2*(2*B*a*b + A*b^2)*x^4 + 4*(B*a^2 + 2*A*a*b)*x^2*log(x) -
 2*A*a^2)/x^2
```

giac [A] time = 0.33, size = 70, normalized size = 1.37

$$\frac{1}{4}Bb^2x^4 + Babx^2 + \frac{1}{2}Ab^2x^2 + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)\log\left(x^2\right) - \frac{Ba^2x^2 + 2\,Aabx^2 + Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + B*a*b*x^2 + 1/2*A*b^2*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
- 1/2*(B*a^2*x^2 + 2*A*a*b*x^2 + A*a^2)/x^2
```

maple [A] time = 0.01, size = 50, normalized size = 0.98

$$\frac{B\,b^2x^4}{4} + \frac{A\,b^2x^2}{2} + Bab\,x^2 + 2Aab\ln(x) + B\,a^2\ln(x) - \frac{A\,a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^3,x)
```

```
[Out] 1/4*b^2*B*x^4+1/2*A*x^2*b^2+B*x^2*a*b-1/2*a^2*A/x^2+2*A*ln(x)*a*b+B*ln(x)*a
^2
```

maxima [A] time = 1.36, size = 52, normalized size = 1.02

$$\frac{1}{4}Bb^2x^4 + \frac{1}{2}\left(2\,Bab + Ab^2\right)x^2 + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)\log\left(x^2\right) - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + 1/2*(2*B*a*b + A*b^2)*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
- 1/2*A*a^2/x^2
```

mupad [B] time = 0.04, size = 48, normalized size = 0.94

$$x^2\left(\frac{A\,b^2}{2} + B\,a\,b\right) + \ln(x)\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{2\,x^2} + \frac{B\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^3,x)
```

```
[Out] x^2*((A*b^2)/2 + B*a*b) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b
^2*x^4)/4
```

sympy [A] time = 0.24, size = 48, normalized size = 0.94

$$-\frac{Aa^2}{2x^2}+\frac{Bb^2x^4}{4}+a\left(2Ab+Ba\right)\log\left(x\right)+x^2\left(\frac{Ab^2}{2}+Bab\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**4/4 + a*(2*A*b + B*a)*log(x) + x**2*(A*b**2/2
+ B*a*b)
```

3.17 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^4}\,dx &= \int \left(b(Ab+2aB)+\frac{a^2A}{x^4}+\frac{a(2Ab+aB)}{x^2}+b^2Bx^2\right)dx\\ &= -\frac{a^2A}{3x^3}-\frac{a(2Ab+aB)}{x}+b(Ab+2aB)x+\frac{1}{3}b^2Bx^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.04

$$\frac{a^2(-B)-2aAb}{x}-\frac{a^2A}{3x^3}+bx(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 + (-2*a*A*b - a^2*B)/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^4, x]
```

fricas [A] time = 0.43, size = 52, normalized size = 1.08

$$\frac{Bb^2x^6+3\left(2\,Bab+Ab^2\right)x^4-Aa^2-3\left(Ba^2+2\,Aab\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + 3*(2*B*a*b + A*b^2)*x^4 - A*a^2 - 3*(B*a^2 + 2*A*a*b)*x^2)
/x^3
```

giac [A] time = 0.35, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bb^2x^3+2\,Babx+Ab^2x-\frac{3\,Ba^2x^2+6\,Aabx^2+Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + 2*B*a*b*x + A*b^2*x - 1/3*(3*B*a^2*x^2 + 6*A*a*b*x^2 + A*a^
2)/x^3
```

maple [A] time = 0.01, size = 46, normalized size = 0.96

$$\frac{B\,b^2x^3}{3}+A\,b^2x+2Babx-\frac{A\,a^2}{3x^3}-\frac{(2Ab+Ba)\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^4,x)
```

```
[Out] 1/3*b^2*B*x^3+b^2*A*x+2*a*b*B*x-a*(2*A*b+B*a)/x-1/3*a^2*A/x^3
```

maxima [A] time = 1.37, size = 50, normalized size = 1.04

$$\frac{1}{3}\,Bb^2x^3 + \left(2\,Bab + Ab^2\right)x - \frac{Aa^2 + 3\left(Ba^2 + 2\,Aab\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*x - 1/3*(A*a^2 + 3*(B*a^2 + 2*A*a*b)*x^2)
/x^3
```

mupad [B] time = 0.07, size = 50, normalized size = 1.04

$$x\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^2\,\left(B\,a^2+2\,A\,b\,a\right)+\frac{A\,a^2}{3}}{x^3}+\frac{B\,b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^4,x)
```

```
[Out] x*(A*b^2 + 2*B*a*b) - (x^2*(B*a^2 + 2*A*a*b) + (A*a^2)/3)/x^3 + (B*b^2*x^3)
/3
```

sympy [A] time = 0.25, size = 51, normalized size = 1.06

$$\frac{Bb^2x^3}{3} + x\left(Ab^2 + 2Bab\right) + \frac{-Aa^2 + x^2\left(-6Aab - 3Ba^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**2*x**3/3 + x*(A*b**2 + 2*B*a*b) + (-A*a**2 + x**2*(-6*A*a*b - 3*B*a**2
))/(3*x**3)
```

3.18 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^5}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{4x^4}-\frac{a(aB+2Ab)}{2x^2}+b\log(x)(2aB+Ab)+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/(2*x^2) + (b^2*B*x^2)/2 + b*(A*b + 2*a
*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(b^2B+\frac{a^2A}{x^3}+\frac{a(2Ab+aB)}{x^2}+\frac{b(Ab+2aB)}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{2x^2}+\frac{1}{2}b^2Bx^2+b(Ab+2aB)\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.98

$$b\log(x)(2aB+Ab)-\frac{a^2\left(A+2Bx^2\right)+4aAbx^2-2b^2Bx^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(4*a*A*b*x^2 - 2*b^2*B*x^6 + a^2*(A + 2*B*x^2))/x^4 + b*(A*b + 2*a*B)*
Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^5, x]
```

fricas [A] time = 0.41, size = 55, normalized size = 1.08

$$\frac{2\,Bb^2x^6+4\left(2\,Bab+Ab^2\right)x^4\log(x)-Aa^2-2\left(Ba^2+2\,Aab\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^2*x^6 + 4*(2*B*a*b + A*b^2)*x^4*log(x) - A*a^2 - 2*(B*a^2 + 2*A*
a*b)*x^2)/x^4
```

giac [**A**] time = 0.34, size = 72, normalized size = 1.41

$$\frac{1}{2}\,Bb^2x^2+\frac{1}{2}\left(2\,Bab+Ab^2\right)\log\left(x^2\right)-\frac{6\,Babx^4+3\,Ab^2x^4+2\,Ba^2x^2+4\,Aabx^2+Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(6*B*a*b*x^4 + 3*A*b^2
*x^4 + 2*B*a^2*x^2 + 4*A*a*b*x^2 + A*a^2)/x^4
```

maple [**A**] time = 0.02, size = 51, normalized size = 1.00

$$\frac{B\,b^2x^2}{2}+A\,b^2\ln(x)+2Bab\ln(x)-\frac{Aab}{x^2}-\frac{B\,a^2}{2x^2}-\frac{A\,a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^5,x)
```

```
[Out] 1/2*b^2*B*x^2-1/4*a^2*A/x^4-a/x^2*A*b-1/2*a^2/x^2*B+A*ln(x)*b^2+2*B*ln(x)*a
*b
```

maxima [**A**] time = 1.39, size = 54, normalized size = 1.06

$$\frac{1}{2}\,Bb^2x^2+\frac{1}{2}\left(2\,Bab+Ab^2\right)\log\left(x^2\right)-\frac{Aa^2+2\left(Ba^2+2\,Aab\right)x^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(A*a^2 + 2*(B*a^2 + 2*
A*a*b)*x^2)/x^4
```

mupad [**B**] time = 0.08, size = 51, normalized size = 1.00

$$\ln(x)\,\left(A\,b^2+2\,B\,a\,b\right)-\frac{x^2\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{4}}{x^4}+\frac{B\,b^2\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^5,x)
```

```
[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^2*((B*a^2)/2 + A*a*b) + (A*a^2)/4)/x^4 + (B*b
^2*x^2)/2
```

sympy [A] time = 0.52, size = 51, normalized size = 1.00

$$\frac{Bb^2x^2}{2} + b\,(Ab + 2Ba)\log{(x)} + \frac{-Aa^2 + x^2\left(-4Aab - 2Ba^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**2*x**2/2 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**2*(-4*A*a*b - 2*B*a*
*2))/(4*x**4)
```

3.19 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^6}\,dx &= \int \left(b^2B+\frac{a^2A}{x^6}+\frac{a(2Ab+aB)}{x^4}+\frac{b(Ab+2aB)}{x^2}\right)dx\\ &= -\frac{a^2A}{5x^5}-\frac{a(2Ab+aB)}{3x^3}-\frac{b(Ab+2aB)}{x}+b^2Bx\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{a^2A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B*
x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^6, x]
```

fricas [A] time = 0.42, size = 53, normalized size = 1.10

$$\frac{15\,Bb^2x^6-15\left(2\,Bab+Ab^2\right)x^4-3\,Aa^2-5\left(Ba^2+2\,Aab\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*B*b^2*x^6 - 15*(2*B*a*b + A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 + 2*A*a*
b)*x^2)/x^5
```

giac [A] time = 0.41, size = 53, normalized size = 1.10

$$Bb^2x-\frac{30\,Babx^4+15\,Ab^2x^4+5\,Ba^2x^2+10\,Aabx^2+3\,Aa^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/15*(30*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 + 10*A*a*b*x^2 +
3*A*a^2)/x^5
```

maple [A] time = 0.02, size = 45, normalized size = 0.94

$$B\,b^2x-\frac{(Ab+2Ba)\,b}{x}-\frac{A\,a^2}{5x^5}-\frac{(2Ab+Ba)\,a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

maxima [A] time = 1.35, size = 51, normalized size = 1.06

$$Bb^2x - \frac{15\left(2\,Bab + Ab^2\right)x^4 + 3\,Aa^2 + 5\left(Ba^2 + 2\,Aab\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/15*(15*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 + 2*A*a*b)*x^
2)/x^5
```

mupad [B] time = 0.04, size = 50, normalized size = 1.04

$$B\,b^2\,x - \frac{x^2\,\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^4\,\left(A\,b^2 + 2\,B\,a\,b\right) + \frac{A\,a^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^6,x)
```

```
[Out] B*b^2*x - (x^2*((B*a^2)/3 + (2*A*a*b)/3) + x^4*(A*b^2 + 2*B*a*b) + (A*a^2)/
5)/x^5
```

sympy [A] time = 0.60, size = 54, normalized size = 1.12

$$Bb^2x + \frac{-3Aa^2 + x^4\left(-15Ab^2 - 30Bab\right) + x^2\left(-10Aab - 5Ba^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**2*x + (-3*A*a**2 + x**4*(-15*A*b**2 - 30*B*a*b) + x**2*(-10*A*a*b - 5*
B*a**2))/(15*x**5)
```

3.20 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{2x^2}+b^2B\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{2x^2}+b^2B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^2A}{x^4}+\frac{a(2Ab+aB)}{x^3}+\frac{b(Ab+2aB)}{x^2}+\frac{b^2B}{x}\right)dx,x,x^2\right)\\ &= -\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{2x^2}+b^2B\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.06

$$b^2B\log(x)-\frac{a^2\left(2A+3Bx^2\right)+6abx^2\left(A+2Bx^2\right)+6Ab^2x^4}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] -1/12*(6*A*b^2*x^4 + 6*a*b*x^2*(A + 2*B*x^2) + a^2*(2*A + 3*B*x^2))/x^6 + b
^2*B*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^7, x]
```

fricas [A] time = 0.41, size = 55, normalized size = 1.08

$$\frac{12\,Bb^2x^6\log(x)-6\left(2\,Bab+Ab^2\right)x^4-2\,Aa^2-3\left(Ba^2+2\,Aab\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*B*b^2*x^6*log(x) - 6*(2*B*a*b + A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 +
2*A*a*b)*x^2)/x^6
```

giac [A] time = 0.32, size = 66, normalized size = 1.29

$$\frac{1}{2}\,Bb^2\log\left(x^2\right) - \frac{11\,Bb^2x^6 + 12\,Babx^4 + 6\,Ab^2x^4 + 3\,Ba^2x^2 + 6\,Aabx^2 + 2\,Aa^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(11*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 3*B*
a^2*x^2 + 6*A*a*b*x^2 + 2*A*a^2)/x^6
```

maple [A] time = 0.01, size = 52, normalized size = 1.02

$$B\,b^2\ln(x) - \frac{A\,b^2}{2x^2} - \frac{Bab}{x^2} - \frac{Aab}{2x^4} - \frac{B\,a^2}{4x^4} - \frac{A\,a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^7,x)
```

```
[Out] -1/2*a/x^4*A*b-1/4*a^2/x^4*B-1/2*b^2/x^2*A-b/x^2*B*a-1/6*a^2*A/x^6+b^2*B*ln
(x)
```

maxima [A] time = 1.01, size = 55, normalized size = 1.08

$$\frac{1}{2}\,Bb^2\log\left(x^2\right) - \frac{6\left(2\,Bab + Ab^2\right)x^4 + 2\,Aa^2 + 3\left(Ba^2 + 2\,Aab\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*log(x^2) - 1/12*(6*(2*B*a*b + A*b^2)*x^4 + 2*A*a^2 + 3*(B*a^2 + 2
*A*a*b)*x^2)/x^6
```

mupad [B] time = 0.09, size = 51, normalized size = 1.00

$$B\,b^2\,\ln(x) - \frac{x^2\,\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^4\,\left(\frac{A\,b^2}{2} + B\,a\,b\right) + \frac{A\,a^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^7,x)
```

```
[Out] B*b^2*log(x) - (x^2*((B*a^2)/4 + (A*a*b)/2) + x^4*((A*b^2)/2 + B*a*b) + (A*
a^2)/6)/x^6
```

sympy [A] time = 1.00, size = 56, normalized size = 1.10

$$Bb^2\log(x)+\frac{-2Aa^2+x^4\left(-6Ab^2-12Bab\right)+x^2\left(-6Aab-3Ba^2\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**2*log(x) + (-2*A*a**2 + x**4*(-6*A*b**2 - 12*B*a*b) + x**2*(-6*A*a*b -
 3*B*a**2))/(12*x**6)
```

3.21 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{x}$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(3*x^3) -
(b^2*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^8}\,dx &= \int \left(\frac{a^2A}{x^8}+\frac{a(2Ab+aB)}{x^6}+\frac{b(Ab+2aB)}{x^4}+\frac{b^2B}{x^2}\right)dx\\ &= -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{3x^3}-\frac{b^2B}{x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.06

$$-\frac{3a^2\left(5A+7Bx^2\right)+14abx^2\left(3A+5Bx^2\right)+35b^2x^4\left(A+3Bx^2\right)}{105x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] -1/105*(35*b^2*x^4*(A + 3*B*x^2) + 14*a*b*x^2*(3*A + 5*B*x^2) + 3*a^2*(5*A
+ 7*B*x^2))/x^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^8, x]
```

fricas [A] time = 0.43, size = 53, normalized size = 1.00

$$-\frac{105\,Bb^2x^6+35\left(2\,Bab+Ab^2\right)x^4+15\,Aa^2+21\left(Ba^2+2\,Aab\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 + 2
*A*a*b)*x^2)/x^7
```

giac [A] time = 0.37, size = 55, normalized size = 1.04

$$-\frac{105\,Bb^2x^6+70\,Babx^4+35\,Ab^2x^4+21\,Ba^2x^2+42\,Aabx^2+15\,Aa^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 70*B*a*b*x^4 + 35*A*b^2*x^4 + 21*B*a^2*x^2 + 42*A*a
*b*x^2 + 15*A*a^2)/x^7
```

maple [A] time = 0.00, size = 48, normalized size = 0.91

$$-\frac{B\,b^2}{x}-\frac{(Ab+2Ba)\,b}{3x^3}-\frac{A\,a^2}{7x^7}-\frac{(2Ab+Ba)\,a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^8,x)
```

```
[Out] -1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/x
```

maxima [A] time = 0.98, size = 53, normalized size = 1.00

$$-\frac{105\,Bb^2x^6+35\left(2\,Bab+Ab^2\right)x^4+15\,Aa^2+21\left(Ba^2+2\,Aab\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 + 2
*A*a*b)*x^2)/x^7
```

mupad [B] time = 0.03, size = 52, normalized size = 0.98

$$-\frac{x^2\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^4\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)+\frac{A\,a^2}{7}+B\,b^2\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^8,x)
```

```
[Out] -(x^2*((B*a^2)/5 + (2*A*a*b)/5) + x^4*((A*b^2)/3 + (2*B*a*b)/3) + (A*a^2)/7
 + B*b^2*x^6)/x^7
```

sympy [A] time = 1.06, size = 58, normalized size = 1.09

$$\frac{-15Aa^2-105Bb^2x^6+x^4\left(-35Ab^2-70Bab\right)+x^2\left(-42Aab-21Ba^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**8,x)
```

```
[Out] (-15*A*a**2 - 105*B*b**2*x**6 + x**4*(-35*A*b**2 - 70*B*a*b) + x**2*(-42*A*
a*b - 21*B*a**2))/(105*x**7)
```

3.22 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^9}\,dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^2\right)^3(Ab-4aB)}{24a^2x^6}-\frac{A\left(a+bx^2\right)^3}{8ax^8}$$

Rubi [**A**] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 37}

$$\frac{\left(a+bx^2\right)^3(Ab-4aB)}{24a^2x^6}-\frac{A\left(a+bx^2\right)^3}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -(A*(a + b*x^2)^3)/(8*a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(24*a^2*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^9}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^5}\,dx,x,x^2\right)$$
$$=-\frac{A\left(a+bx^2\right)^3}{8ax^8}+\frac{(-Ab+4aB)\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^4}\,dx,x,x^2\right)}{8a}$$
$$=-\frac{A\left(a+bx^2\right)^3}{8ax^8}+\frac{(Ab-4aB)\left(a+bx^2\right)^3}{24a^2x^6}$$

Mathematica **[A]** time = 0.03, size = 55, normalized size = 1.15

$$-\frac{a^2\left(3A+4Bx^2\right)+4abx^2\left(2A+3Bx^2\right)+6b^2x^4\left(A+2Bx^2\right)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] -1/24*(6*b^2*x^4*(A + 2*B*x^2) + 4*a*b*x^2*(2*A + 3*B*x^2) + a^2*(3*A + 4*B
*x^2))/x^8
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^9, x]
```

fricas **[A]** time = 0.42, size = 53, normalized size = 1.10

$$-\frac{12\,Bb^2x^6+6\left(2\,Bab+Ab^2\right)x^4+3\,Aa^2+4\left(Ba^2+2\,Aab\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a*
b)*x^2)/x^8
```

giac [A] time = 0.38, size = 55, normalized size = 1.15

$$-\frac{12 \, B b^{2} x^{6} + 12 \, B a b x^{4} + 6 \, A b^{2} x^{4} + 4 \, B a^{2} x^{2} + 8 \, A a b x^{2} + 3 \, A a^{2}}{24 \, x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 4*B*a^2*x^2 + 8*A*a*b*x^
2 + 3*A*a^2)/x^8
```

maple [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{B\,b^{2}}{2x^{2}}-\frac{(Ab+2Ba)\,b}{4x^{4}}-\frac{A\,a^{2}}{8x^{8}}-\frac{(2Ab+Ba)\,a}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^9,x)
```

```
[Out] -1/8*A*a^2/x^8-1/4*b*(A*b+2*B*a)/x^4-1/2*B*b^2/x^2-1/6*a*(2*A*b+B*a)/x^6
```

maxima [A] time = 1.05, size = 53, normalized size = 1.10

$$-\frac{12 \, B b^{2} x^{6} + 6 \, {\left(2 \, B a b + A b^{2}\right)} x^{4} + 3 \, A a^{2} + 4 \, {\left(B a^{2} + 2 \, A a b\right)} x^{2}}{24 \, x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] -1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a*
b)*x^2)/x^8
```

mupad [B] time = 0.05, size = 53, normalized size = 1.10

$$-\frac{x^2\,\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^4\,\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2}{8}+\frac{B\,b^2\,x^6}{2}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^9,x)
```

```
[Out] -(x^2*((B*a^2)/6 + (A*a*b)/3) + x^4*((A*b^2)/4 + (B*a*b)/2) + (A*a^2)/8 + (
B*b^2*x^6)/2)/x^8
```

sympy [A] time = 1.48, size = 58, normalized size = 1.21

$$\frac{-3Aa^2 - 12Bb^2x^6 + x^4\left(-6Ab^2 - 12Bab\right) + x^2\left(-8Aab - 4Ba^2\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**9,x)
```

```
[Out] (-3*A*a**2 - 12*B*b**2*x**6 + x**4*(-6*A*b**2 - 12*B*a*b) + x**2*(-8*A*a*b
- 4*B*a**2))/(24*x**8)
```

3.23 $\int x^9 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+A$$

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x^4(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(a^5Ax^4+a^4(5Ab+aB)x^5+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)\right.\right.\\ &= \frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4(5Ab+aB)x^{12}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4x^{12}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{18}ab^3x^{18}(2aB+Ab)+\frac{1}{22}b^5Bx^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^14
)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 + (
b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^9*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.36, size = 125, normalized size = 1.07

$$\frac{1}{22}x^{22}b^5B+\frac{1}{4}x^{20}b^4aB+\frac{1}{20}x^{20}b^5A+\frac{5}{9}x^{18}b^3a^2B+\frac{5}{18}x^{18}b^4aA+\frac{5}{8}x^{16}b^2a^3B+\frac{5}{8}x^{16}b^3a^2A+\frac{5}{14}x^{14}ba^4B+\frac{5}{7}x^{14}b^2a^3A+\frac{1}{12}x^{12}a^5B+\frac{5}{12}x^{12}ba^4A+\frac{1}{10}x^{10}a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5*B + 1/4*x^20*b^4*a*B + 1/20*x^20*b^5*A + 5/9*x^18*b^3*a^2*B +
 5/18*x^18*b^4*a*A + 5/8*x^16*b^2*a^3*B + 5/8*x^16*b^3*a^2*A + 5/14*x^14*b*
a^4*B + 5/7*x^14*b^2*a^3*A + 1/12*x^12*a^5*B + 5/12*x^12*b*a^4*A + 1/10*x^1
0*a^5*A
```

giac [A] time = 0.37, size = 125, normalized size = 1.07

$$\frac{1}{22}Bb^5x^{22}+\frac{1}{4}Bab^4x^{20}+\frac{1}{20}Ab^5x^{20}+\frac{5}{9}Ba^2b^3x^{18}+\frac{5}{18}Aab^4x^{18}+\frac{5}{8}Ba^3b^2x^{16}+\frac{5}{8}Aa^2b^3x^{16}+\frac{5}{14}Ba^4bx^{14}+\frac{5}{7}Aa^3b^2x^{14}+\frac{1}{12}Ba^5x^{12}+\frac{5}{12}Aa^4bx^{12}+\frac{1}{10}Aa^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 5/9*B*a^2*b^3*x^18 +
 5/18*A*a*b^4*x^18 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/14*B*a^4*b
*x^14 + 5/7*A*a^3*b^2*x^14 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/10*A*a
^5*x^10
```

maple [A] time = 0.00, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{22}}{22}+\frac{\left(b^5A+5a\,b^4B\right)x^{20}}{20}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{18}}{18}+\frac{A\,a^5x^{10}}{10}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{16}}{16}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{14}}{14}+\frac{\left(5a^4bA+a^5B\right)x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/22*b^5*B*x^22+1/20*(A*b^5+5*B*a*b^4)*x^20+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/12*(5*A*a^4*b+B*a^5)*x^12+1/10*a^5*A*x^10
```

maxima [A] time = 1.00, size = 119, normalized size = 1.02

$$\frac{1}{22}\,Bb^5x^{22}+\frac{1}{20}\left(5\,Bab^4+Ab^5\right)x^{20}+\frac{5}{18}\left(2\,Ba^2b^3+Aab^4\right)x^{18}+\frac{5}{8}\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{1}{10}\,Aa^5x^{10}+\frac{5}{14}\left(Ba^4b+2\,Aa^3b^2\right)x^{14}+\frac{1}{12}\left(Ba^5+5\,Aa^4b\right)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^5*x^10 + 5/14*(B*a^4
*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

mupad [B] time = 0.20, size = 107, normalized size = 0.91

$$x^{12}\,\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{20}\,\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right)+\frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{22}}{22}+\frac{5\,a^2\,b^2\,x^{16}\,\left(A\,b+B\,a\right)}{8}+\frac{5\,a^3\,b\,x^{14}\,\left(2\,A\,b+B\,a\right)}{14}+\frac{5\,a\,b^3\,x^{18}\,\left(A\,b+2\,B\,a\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a
^5*x^10)/10 + (B*b^5*x^22)/22 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x
^14*(2*A*b + B*a))/14 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

sympy [A] time = 0.09, size = 136, normalized size = 1.16

$$\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{22}}{22}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{18}\left(\frac{5Aab^4}{18}+\frac{5Ba^2b^3}{9}\right)+x^{16}\left(\frac{5Aa^2b^3}{8}+\frac{5Ba^3b^2}{8}\right)+x^{14}\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+x^{12}\left(\frac{5Aa^4b}{12}+\frac{Ba^5}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**22/22 + x**20*(A*b**5/20 + B*a*b**4/4) + x**18*
(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b**2/
8) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

3.24 $\int x^8 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)$$

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{9}a^5Ax^9+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^2\right)^5\left(A+Bx^2\right)dx &= \int \left(a^5Ax^8+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x^{14}+5\right.\\ &= \frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{2}{3}a^2b^2(Ab+aB)x^{15}+\end{aligned}$$

Mathematica [A] time = 0.03, size = 117, normalized size = 1.00

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^
4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.37, size = 125, normalized size = 1.07

$$\frac{1}{21}x^{21}b^5B+\frac{5}{19}x^{19}b^4aB+\frac{1}{19}x^{19}b^5A+\frac{10}{17}x^{17}b^3a^2B+\frac{5}{17}x^{17}b^4aA+\frac{2}{3}x^{15}b^2a^3B+\frac{2}{3}x^{15}b^3a^2A+\frac{5}{13}x^{13}ba^4B+\frac{10}{13}x^{13}b^2a^3A+\frac{1}{11}x^{11}a^5B+\frac{5}{11}x^{11}ba^4A+\frac{1}{9}x^9a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5*B + 5/19*x^19*b^4*a*B + 1/19*x^19*b^5*A + 10/17*x^17*b^3*a^2*
B + 5/17*x^17*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b^3*a^2*A + 5/13*x^13
*b*a^4*B + 10/13*x^13*b^2*a^3*A + 1/11*x^11*a^5*B + 5/11*x^11*b*a^4*A + 1/9
*x^9*a^5*A
```

giac [A] time = 0.33, size = 125, normalized size = 1.07

$$\frac{1}{21}Bb^5x^{21}+\frac{5}{19}Bab^4x^{19}+\frac{1}{19}Ab^5x^{19}+\frac{10}{17}Ba^2b^3x^{17}+\frac{5}{17}Aab^4x^{17}+\frac{2}{3}Ba^3b^2x^{15}+\frac{2}{3}Aa^2b^3x^{15}+\frac{5}{13}Ba^4bx^{13}+\frac{10}{13}Aa^3b^2x^{13}+\frac{1}{11}Ba^5x^{11}+\frac{5}{11}Aa^4bx^{11}+\frac{1}{9}Aa^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 10/17*B*a^2*b^3*x^1
7 + 5/17*A*a*b^4*x^17 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/9
*A*a^5*x^9
```

maple [A] time = 0.00, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{21}}{21}+\frac{\left(b^5A+5a\,b^4B\right)x^{19}}{19}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{17}}{17}+\frac{A\,a^5x^9}{9}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{15}}{15}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{13}}{13}+\frac{\left(5a^4bA+a^5B\right)x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/21*b^5*B*x^21+1/19*(A*b^5+5*B*a*b^4)*x^19+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/11*(5*A*a^4*b+B*a^5)*x^11+1/9*a^5*A*x^9
```

maxima [A] time = 0.98, size = 119, normalized size = 1.02

$$\frac{1}{21}Bb^5x^{21}+\frac{1}{19}\left(5\,Bab^4+Ab^5\right)x^{19}+\frac{5}{17}\left(2\,Ba^2b^3+Aab^4\right)x^{17}+\frac{2}{3}\left(Ba^3b^2+Aa^2b^3\right)x^{15}+\frac{1}{9}Aa^5x^9+\frac{5}{13}\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{11}\left(Ba^5+5\,Aa^4b\right)x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5*x^9 + 5/13*(B*a^4*b
 + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right)+\frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{21}}{21}+\frac{2\,a^2\,b^2\,x^{15}\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^{13}\,\left(2\,A\,b+B\,a\right)}{13}+\frac{5\,a\,b^3\,x^{17}\,\left(A\,b+2\,B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (
A*a^5*x^9)/9 + (B*b^5*x^21)/21 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*
x^13*(2*A*b + B*a))/13 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

sympy [A] time = 0.09, size = 138, normalized size = 1.18

$$\frac{Aa^5x^9}{9}+\frac{Bb^5x^{21}}{21}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{17}\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{15}\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{13}\left(\frac{10Aa^3b^2}{13}+\frac{5Ba^4b}{13}\right)+x^{11}\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**21/21 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**17
*(5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b*
*2/3) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**11*(5*A*a**4*b/11 +
B*a**5/11)
```

3.25 $\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)dx$

Optimal. Leaf size=122

$$-\frac{a^3\left(a+bx^2\right)^6(Ab-aB)}{12b^5}+\frac{a^2\left(a+bx^2\right)^7(3Ab-4aB)}{14b^5}+\frac{\left(a+bx^2\right)^9(Ab-4aB)}{18b^5}-\frac{3a\left(a+bx^2\right)^8(Ab-2aB)}{16b^5}+\frac{B\left(a+\right.}{20}$$

Rubi [A] time = 0.28, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{a^2\left(a+bx^2\right)^7(3Ab-4aB)}{14b^5}-\frac{a^3\left(a+bx^2\right)^6(Ab-aB)}{12b^5}+\frac{\left(a+bx^2\right)^9(Ab-4aB)}{18b^5}-\frac{3a\left(a+bx^2\right)^8(Ab-2aB)}{16b^5}+\frac{B\left(a+bx^2\right)^{10}}{20b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -(a^3*(A*b - a*B)*(a + b*x^2)^6)/(12*b^5) + (a^2*(3*A*b - 4*a*B)*(a + b*x^2
)^7)/(14*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(16*b^5) + ((A*b - 4*a*B)
*(a + b*x^2)^9)/(18*b^5) + (B*(a + b*x^2)^10)/(20*b^5)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int x^3(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{a^3(-Ab+aB)(a+bx)^5}{b^4}-\frac{a^2(-3Ab+4aB)(a+bx)^6}{b^4}+\frac{3a(-Ab+\right.\right.\\ &= -\frac{a^3(Ab-aB)\left(a+bx^2\right)^6}{12b^5}+\frac{a^2(3Ab-4aB)\left(a+bx^2\right)^7}{14b^5}-\frac{3a(Ab-2aB)\left(a+bx\right.}{16b^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 0.96

$$\frac{1}{8}a^5Ax^8+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{12}a^3bx^{12}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{18}b^4x^{18}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{20}b^5Bx^{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^12)/
12 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^
4*(A*b + 5*a*B)*x^18)/18 + (b^5*B*x^20)/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.39, size = 125, normalized size = 1.02

$$\frac{1}{20}x^{20}b^5B+\frac{5}{18}x^{18}b^4aB+\frac{1}{18}x^{18}b^5A+\frac{5}{8}x^{16}b^3a^2B+\frac{5}{16}x^{16}b^4aA+\frac{5}{7}x^{14}b^2a^3B+\frac{5}{7}x^{14}b^3a^2A+\frac{5}{12}x^{12}ba^4B+\frac{5}{6}x^{12}b^2a^3A+\frac{1}{10}x^{10}a^5B+\frac{1}{2}x^{10}ba^4A+\frac{1}{8}x^8a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 5/8*x^16*b^3*a^2*B
+ 5/16*x^16*b^4*a*A + 5/7*x^14*b^2*a^3*B + 5/7*x^14*b^3*a^2*A + 5/12*x^12*b
*a^4*B + 5/6*x^12*b^2*a^3*A + 1/10*x^10*a^5*B + 1/2*x^10*b*a^4*A + 1/8*x^8*
a^5*A
```

giac [A] time = 0.34, size = 125, normalized size = 1.02

$$\frac{1}{20}Bb^5x^{20} + \frac{5}{18}Bab^4x^{18} + \frac{1}{18}Ab^5x^{18} + \frac{5}{8}Ba^2b^3x^{16} + \frac{5}{16}Aab^4x^{16} + \frac{5}{7}Ba^3b^2x^{14} + \frac{5}{7}Aa^2b^3x^{14} + \frac{5}{12}Ba^4bx^{12} + \frac{5}{6}Aa^3b^2x^{12} + \frac{1}{10}Ba^5x^{10} + \frac{1}{2}Aa^4bx^{10} + \frac{1}{8}Aa^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/8*A*a^
5*x^8
```

maple [A] time = 0.00, size = 124, normalized size = 1.02

$$\frac{B\,b^5x^{20}}{20} + \frac{\left(b^5A + 5a\,b^4B\right)x^{18}}{18} + \frac{\left(5a\,b^4A + 10a^2b^3B\right)x^{16}}{16} + \frac{A\,a^5x^8}{8} + \frac{\left(10a^2b^3A + 10a^3b^2B\right)x^{14}}{14} + \frac{\left(10a^3b^2A + 5a^4bB\right)x^{12}}{12} + \frac{\left(5a^4bA + a^5B\right)x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/20*b^5*B*x^20+1/18*(A*b^5+5*B*a*b^4)*x^18+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/10*(5*A*a^4*b+B*a^5)*x^10+1/8*a^5*A*x^8
```

maxima [A] time = 1.00, size = 119, normalized size = 0.98

$$\frac{1}{20}Bb^5x^{20} + \frac{1}{18}\left(5\,Bab^4 + Ab^5\right)x^{18} + \frac{5}{16}\left(2\,Ba^2b^3 + Aab^4\right)x^{16} + \frac{5}{7}\left(Ba^3b^2 + Aa^2b^3\right)x^{14} + \frac{1}{8}Aa^5x^8 + \frac{5}{12}\left(Ba^4b + 2\,Aa^3b^2\right)x^{12} + \frac{1}{10}\left(Ba^5 + 5\,Aa^4b\right)x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5*x^8 + 5/12*(B*a^4*b
 + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

mupad [B] time = 0.04, size = 107, normalized size = 0.88

$$x^{10}\left(\frac{B\,a^5}{10} + \frac{A\,b\,a^4}{2}\right) + x^{18}\left(\frac{A\,b^5}{18} + \frac{5\,B\,a\,b^4}{18}\right) + \frac{A\,a^5\,x^8}{8} + \frac{B\,b^5\,x^{20}}{20} + \frac{5\,a^2\,b^2\,x^{14}\,\left(A\,b + B\,a\right)}{7} + \frac{5\,a^3\,b\,x^{12}\,\left(2\,A\,b + B\,a\right)}{12} + \frac{5\,a\,b^3\,x^{16}\,\left(A\,b + 2\,B\,a\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a
^5*x^8)/8 + (B*b^5*x^20)/20 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^1
2*(2*A*b + B*a))/12 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

sympy [A] time = 0.09, size = 136, normalized size = 1.11

$$\frac{Aa^{5}x^{8}}{8}+\frac{Bb^{5}x^{20}}{20}+x^{18}\left(\frac{Ab^{5}}{18}+\frac{5Bab^{4}}{18}\right)+x^{16}\left(\frac{5Aab^{4}}{16}+\frac{5Ba^{2}b^{3}}{8}\right)+x^{14}\left(\frac{5Aa^{2}b^{3}}{7}+\frac{5Ba^{3}b^{2}}{7}\right)+x^{12}\left(\frac{5Aa^{3}b^{2}}{6}+\frac{5Ba^{4}b}{12}\right)+x^{10}\left(\frac{Aa^{4}b}{2}+\frac{Ba^{5}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**20/20 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2
/7) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**10*(A*a**4*b/2 + B*a**5/
10)
```

3.26 $\int x^6 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=117

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \int \left(a^5Ax^6+a^4(5Ab+aB)x^8+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^{12}+5ab\right.\\ &= \frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{1}{3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 117, normalized size = 1.00

$$\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4x^9(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{1}{3}ab^3x^{15}(2aB+Ab)+\frac{1}{19}b^5Bx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4*(
A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.38, size = 125, normalized size = 1.07

$$\frac{1}{19}x^{19}b^5B+\frac{5}{17}x^{17}b^4aB+\frac{1}{17}x^{17}b^5A+\frac{2}{3}x^{15}b^3a^2B+\frac{1}{3}x^{15}b^4aA+\frac{10}{13}x^{13}b^2a^3B+\frac{10}{13}x^{13}b^3a^2A+\frac{5}{11}x^{11}ba^4B+\frac{10}{11}x^{11}b^2a^3A+\frac{1}{9}x^9a^5B+\frac{5}{9}x^9ba^4A+\frac{1}{7}x^7a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5*B + 5/17*x^17*b^4*a*B + 1/17*x^17*b^5*A + 2/3*x^15*b^3*a^2*B
+ 1/3*x^15*b^4*a*A + 10/13*x^13*b^2*a^3*B + 10/13*x^13*b^3*a^2*A + 5/11*x^1
1*b*a^4*B + 10/11*x^11*b^2*a^3*A + 1/9*x^9*a^5*B + 5/9*x^9*b*a^4*A + 1/7*x^
7*a^5*A
```

giac [A] time = 0.37, size = 125, normalized size = 1.07

$$\frac{1}{19}Bb^5x^{19}+\frac{5}{17}Bab^4x^{17}+\frac{1}{17}Ab^5x^{17}+\frac{2}{3}Ba^2b^3x^{15}+\frac{1}{3}Aab^4x^{15}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{5}{11}Ba^4bx^{11}+\frac{10}{11}Aa^3b^2x^{11}+\frac{1}{9}Ba^5x^9+\frac{5}{9}Aa^4bx^9+\frac{1}{7}Aa^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 5/11*B*a
^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/7*A*
a^5*x^7
```

maple [A] time = 0.00, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{19}}{19}+\frac{\left(b^5A+5a\,b^4B\right)x^{17}}{17}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{15}}{15}+\frac{A\,a^5x^7}{7}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{13}}{13}+\frac{\left(10a^3b^2A+5a^4bB\right)x^{11}}{11}+\frac{\left(5a^4bA+a^5B\right)x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/19*b^5*B*x^19+1/17*(A*b^5+5*B*a*b^4)*x^17+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/9*(5*A*a^4*b+B*a^5)*x^9+1/7*a^5*A*x^7
```

maxima [A] time = 1.06, size = 119, normalized size = 1.02

$$\frac{1}{19}\,Bb^{5}x^{19}+\frac{1}{17}\left(5\,Bab^{4}+Ab^{5}\right)x^{17}+\frac{1}{3}\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{15}+\frac{10}{13}\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{13}+\frac{1}{7}\,Aa^{5}x^{7}+\frac{5}{11}\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{11}+\frac{1}{9}\left(Ba^{5}+5\,Aa^{4}b\right)x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^5*x^7 + 5/11*(B*a^4*
b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^{9}\left(\frac{B\,a^{5}}{9}+\frac{5\,A\,b\,a^{4}}{9}\right)+x^{17}\left(\frac{A\,b^{5}}{17}+\frac{5\,B\,a\,b^{4}}{17}\right)+\frac{A\,a^{5}\,x^{7}}{7}+\frac{B\,b^{5}\,x^{19}}{19}+\frac{10\,a^{2}\,b^{2}\,x^{13}\,\left(A\,b+B\,a\right)}{13}+\frac{5\,a^{3}\,b\,x^{11}\,\left(2\,A\,b+B\,a\right)}{11}+\frac{a\,b^{3}\,x^{15}\,\left(A\,b+2\,B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a
^5*x^7)/7 + (B*b^5*x^19)/19 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (5*a^3*b*x
^11*(2*A*b + B*a))/11 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

sympy [A] time = 0.09, size = 136, normalized size = 1.16

$$\frac{Aa^{5}x^{7}}{7}+\frac{Bb^{5}x^{19}}{19}+x^{17}\left(\frac{Ab^{5}}{17}+\frac{5Bab^{4}}{17}\right)+x^{15}\left(\frac{Aab^{4}}{3}+\frac{2Ba^{2}b^{3}}{3}\right)+x^{13}\left(\frac{10Aa^{2}b^{3}}{13}+\frac{10Ba^{3}b^{2}}{13}\right)+x^{11}\left(\frac{10Aa^{3}b^{2}}{11}+\frac{5Ba^{4}b}{11}\right)+x^{9}\left(\frac{5Aa^{4}b}{9}+\frac{Ba^{5}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**19/19 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b**2
/13) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**9*(5*A*a**4*b/9 + B*a
**5/9)
```

3.27 $\int x^5 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=95

$$\frac{a^2 \left(a+bx^2\right)^6 (Ab-aB)}{12b^4}+\frac{\left(a+bx^2\right)^8 (Ab-3aB)}{16b^4}-\frac{a \left(a+bx^2\right)^7 (2Ab-3aB)}{14b^4}+\frac{B \left(a+bx^2\right)^9}{18b^4}$$

Rubi [A] time = 0.21, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{a^2 \left(a+bx^2\right)^6 (Ab-aB)}{12b^4}+\frac{\left(a+bx^2\right)^8 (Ab-3aB)}{16b^4}-\frac{a \left(a+bx^2\right)^7 (2Ab-3aB)}{14b^4}+\frac{B \left(a+bx^2\right)^9}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^6)/(12*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2)^7
)/(14*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(16*b^4) + (B*(a + b*x^2)^9)/(18
*b^4)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^6}{b^3}+\frac{(Ab-3aB}{b}\right.\right.\\&=\frac{a^2(Ab-aB)\left(a+bx^2\right)^6}{12b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^7}{14b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^8}{16b^4}+\end{aligned}$$

Mathematica [A] time = 0.04, size = 107, normalized size = 1.13

$$\frac{x^6\left(168a^5A+126a^4x^2(aB+5Ab)+504a^3bx^4(aB+2Ab)+840a^2b^2x^6(aB+Ab)+63b^4x^{10}(5aB+Ab)+360ab^3x^8(2aB+Ab)+56b^5Bx^{12}\right)}{1008}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^6*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^2 + 504*a^3*b*(2*A*b + a*B)*x^4 +
 840*a^2*b^2*(A*b + a*B)*x^6 + 360*a*b^3*(A*b + 2*a*B)*x^8 + 63*b^4*(A*b +
5*a*B)*x^10 + 56*b^5*B*x^12))/1008
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.35, size = 124, normalized size = 1.31

$$\frac{1}{18}x^{18}b^5B+\frac{5}{16}x^{16}b^4aB+\frac{1}{16}x^{16}b^5A+\frac{5}{7}x^{14}b^3a^2B+\frac{5}{14}x^{14}b^4aA+\frac{5}{6}x^{12}b^2a^3B+\frac{5}{6}x^{12}b^3a^2A+\frac{1}{2}x^{10}ba^4B+x^{10}b^2a^3A+\frac{1}{8}x^8a^5B+\frac{5}{8}x^8ba^4A+\frac{1}{6}x^6a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^5*B + 5/16*x^16*b^4*a*B + 1/16*x^16*b^5*A + 5/7*x^14*b^3*a^2*B
+ 5/14*x^14*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3*a^2*A + 1/2*x^10*b*
a^4*B + x^10*b^2*a^3*A + 1/8*x^8*a^5*B + 5/8*x^8*b*a^4*A + 1/6*x^6*a^5*A
```

giac [A] time = 0.31, size = 124, normalized size = 1.31

$$\frac{1}{18} Bb^5x^{18} + \frac{5}{16} Bab^4x^{16} + \frac{1}{16} Ab^5x^{16} + \frac{5}{7} Ba^2b^3x^{14} + \frac{5}{14} Aab^4x^{14} + \frac{5}{6} Ba^3b^2x^{12} + \frac{5}{6} Aa^2b^3x^{12} + \frac{1}{2} Ba^4bx^{10} + Aa^3b^2x^{10} + \frac{1}{8} Ba^5x^8 + \frac{5}{8} Aa^4bx^8 + \frac{1}{6} Aa^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 1/2*B*a^4*b
*x^10 + A*a^3*b^2*x^10 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/6*A*a^5*x^6
```

maple [A] time = 0.00, size = 124, normalized size = 1.31

$$\frac{B\,b^5x^{18}}{18} + \frac{\left(b^5A + 5a\,b^4B\right)x^{16}}{16} + \frac{\left(5a\,b^4A + 10a^2b^3B\right)x^{14}}{14} + \frac{A\,a^5x^6}{6} + \frac{\left(10a^2b^3A + 10a^3b^2B\right)x^{12}}{12} + \frac{\left(10a^3b^2A + 5a^4bB\right)x^{10}}{10} + \frac{\left(5a^4bA + a^5B\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/18*b^5*B*x^18+1/16*(A*b^5+5*B*a*b^4)*x^16+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/8*(5*A*a^4*b+B*a^5)*x^8+1/6*a^5*A*x^6
```

maxima [A] time = 0.98, size = 119, normalized size = 1.25

$$\frac{1}{18} Bb^5x^{18} + \frac{1}{16}\left(5\,Bab^4 + Ab^5\right)x^{16} + \frac{5}{14}\left(2\,Ba^2b^3 + Aab^4\right)x^{14} + \frac{5}{6}\left(Ba^3b^2 + Aa^2b^3\right)x^{12} + \frac{1}{6} Aa^5x^6 + \frac{1}{2}\left(Ba^4b + 2\,Aa^3b^2\right)x^{10} + \frac{1}{8}\left(Ba^5 + 5\,Aa^4b\right)x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5*x^6 + 1/2*(B*a^4*b
+ 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

mupad [B] time = 0.04, size = 107, normalized size = 1.13

$$x^8\left(\frac{B\,a^5}{8} + \frac{5\,A\,b\,a^4}{8}\right) + x^{16}\left(\frac{A\,b^5}{16} + \frac{5\,B\,a\,b^4}{16}\right) + \frac{A\,a^5\,x^6}{6} + \frac{B\,b^5\,x^{18}}{18} + \frac{5\,a^2\,b^2\,x^{12}\,\left(A\,b + B\,a\right)}{6} + \frac{a^3\,b\,x^{10}\,\left(2\,A\,b + B\,a\right)}{2} + \frac{5\,a\,b^3\,x^{14}\,\left(A\,b + 2\,B\,a\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (A*a
^5*x^6)/6 + (B*b^5*x^18)/18 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (a^3*b*x^10*
(2*A*b + B*a))/2 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

sympy [A] time = 0.09, size = 133, normalized size = 1.40

$$\frac{Aa^5x^6}{6} + \frac{Bb^5x^{18}}{18} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{14}\left(\frac{5Aab^4}{14} + \frac{5Ba^2b^3}{7}\right) + x^{12}\left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^{10}\left(Aa^3b^2 + \frac{Ba^4b}{2}\right) + x^8\left(\frac{5Aa^4b}{8} + \frac{Ba^5}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**18/18 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2
/6) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**8*(5*A*a**4*b/8 + B*a**5/8)
```

3.28 $\int x^4 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=117

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*
(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx &= \int \left(a^5Ax^4+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^8+10a^2b^2(Ab+aB)x^{10}+5a\right.\\ &= \frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{5}{9}a^3bx^9(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*
(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.38, size = 125, normalized size = 1.07

$$\frac{1}{17}x^{17}b^5B + \frac{1}{3}x^{15}b^4aB + \frac{1}{15}x^{15}b^5A + \frac{10}{13}x^{13}b^3a^2B + \frac{5}{13}x^{13}b^4aA + \frac{10}{11}x^{11}b^2a^3B + \frac{10}{11}x^{11}b^3a^2A + \frac{5}{9}x^9ba^4B + \frac{10}{9}x^9b^2a^3A + \frac{1}{7}x^7a^5B + \frac{5}{7}x^7ba^4A + \frac{1}{5}x^5a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^5*B + 1/3*x^15*b^4*a*B + 1/15*x^15*b^5*A + 10/13*x^13*b^3*a^2*B
 + 5/13*x^13*b^4*a*A + 10/11*x^11*b^2*a^3*B + 10/11*x^11*b^3*a^2*A + 5/9*x^
9*b*a^4*B + 10/9*x^9*b^2*a^3*A + 1/7*x^7*a^5*B + 5/7*x^7*b*a^4*A + 1/5*x^5*
a^5*A
```

giac [A] time = 0.31, size = 125, normalized size = 1.07

$$\frac{1}{17}Bb^5x^{17} + \frac{1}{3}Bab^4x^{15} + \frac{1}{15}Ab^5x^{15} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{7}Ba^5x^7 + \frac{5}{7}Aa^4bx^7 + \frac{1}{5}Aa^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 10/13*B*a^2*b^3*x^13
 + 5/13*A*a*b^4*x^13 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/9*B*
a^4*b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/5*A*a^
5*x^5
```

maple [A] time = 0.00, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{17}}{17} + \frac{\left(b^5A + 5a\,b^4B\right)x^{15}}{15} + \frac{\left(5a\,b^4A + 10a^2b^3B\right)x^{13}}{13} + \frac{A\,a^5x^5}{5} + \frac{\left(10a^2b^3A + 10a^3b^2B\right)x^{11}}{11} + \frac{\left(10a^3b^2A + 5a^4bB\right)x^9}{9} + \frac{\left(5a^4bA + a^5B\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/17*b^5*B*x^17+1/15*(A*b^5+5*B*a*b^4)*x^15+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x
^13+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+
1/7*(5*A*a^4*b+B*a^5)*x^7+1/5*a^5*A*x^5
```

maxima [A] time = 1.11, size = 119, normalized size = 1.02

$$\frac{1}{17} B b^5 x^{17} + \frac{1}{15} \left(5 B a b^4 + A b^5\right) x^{15} + \frac{5}{13} \left(2 B a^2 b^3 + A a b^4\right) x^{13} + \frac{10}{11} \left(B a^3 b^2 + A a^2 b^3\right) x^{11} + \frac{1}{5} A a^5 x^5 + \frac{5}{9} \left(B a^4 b + 2 A a^3 b^2\right) x^9 + \frac{1}{7} \left(B a^5 + 5 A a^4 b\right) x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a^5*x^5 + 5/9*(B*a^4*
b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^7 \left(\frac{B a^5}{7} + \frac{5 A b a^4}{7}\right) + x^{15} \left(\frac{A b^5}{15} + \frac{B a b^4}{3}\right) + \frac{A a^5 x^5}{5} + \frac{B b^5 x^{17}}{17} + \frac{10 a^2 b^2 x^{11} (A b + B a)}{11} + \frac{5 a^3 b x^9 (2 A b + B a)}{9} + \frac{5 a b^3 x^{13} (A b + 2 B a)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (A*a^5*
x^5)/5 + (B*b^5*x^17)/17 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^9*
(2*A*b + B*a))/9 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

sympy [A] time = 0.09, size = 136, normalized size = 1.16

$$\frac{A a^5 x^5}{5} + \frac{B b^5 x^{17}}{17} + x^{15} \left(\frac{A b^5}{15} + \frac{B a b^4}{3}\right) + x^{13} \left(\frac{5 A a b^4}{13} + \frac{10 B a^2 b^3}{13}\right) + x^{11} \left(\frac{10 A a^2 b^3}{11} + \frac{10 B a^3 b^2}{11}\right) + x^9 \left(\frac{10 A a^3 b^2}{9} + \frac{5 B a^4 b}{9}\right) + x^7 \left(\frac{5 A a^4 b}{7} + \frac{B a^5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**17/17 + x**15*(A*b**5/15 + B*a*b**4/3) + x**13*(5
*A*a*b**4/13 + 10*B*a**2*b**3/13) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b*
*2/11) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**7*(5*A*a**4*b/7 + B*a*
*5/7)
```

3.29 $\int x^3 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$

Optimal. Leaf size=67

$$\frac{\left(a+bx^2\right)^7 (Ab-2aB)}{14b^3}-\frac{a\left(a+bx^2\right)^6 (Ab-aB)}{12b^3}+\frac{B\left(a+bx^2\right)^8}{16b^3}$$

Rubi **[A]** time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{\left(a+bx^2\right)^7 (Ab-2aB)}{14b^3}-\frac{a\left(a+bx^2\right)^6 (Ab-aB)}{12b^3}+\frac{B\left(a+bx^2\right)^8}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^6)/(12*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^7)/(14
*b^3) + (B*(a + b*x^2)^8)/(16*b^3)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^5}{b^2} + \frac{(Ab-2aB)(a+bx)^6}{b^2} + \frac{B(a+bx)^7}{b^2}\right) dx\right.\\ &= -\frac{a(Ab-aB)\left(a+bx^2\right)^6}{12b^3} + \frac{(Ab-2aB)\left(a+bx^2\right)^7}{14b^3} + \frac{B\left(a+bx^2\right)^8}{16b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 114, normalized size = 1.70

$$\frac{1}{4}a^5Ax^4 + \frac{1}{6}a^4x^6(aB+5Ab) + \frac{5}{8}a^3bx^8(aB+2Ab) + a^2b^2x^{10}(aB+Ab) + \frac{1}{14}b^4x^{14}(5aB+Ab) + \frac{5}{12}ab^3x^{12}(2aB+Ab) + \frac{1}{16}b^5Bx^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5
*a*B)*x^14)/14 + (b^5*B*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.38, size = 123, normalized size = 1.84

$$\frac{1}{16}x^{16}b^5B + \frac{5}{14}x^{14}b^4aB + \frac{1}{14}x^{14}b^5A + \frac{5}{6}x^{12}b^3a^2B + \frac{5}{12}x^{12}b^4aA + x^{10}b^2a^3B + x^{10}b^3a^2A + \frac{5}{8}x^8ba^4B + \frac{5}{4}x^8b^2a^3A + \frac{1}{6}x^6a^5B + \frac{5}{6}x^6ba^4A + \frac{1}{4}x^4a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^5*B + 5/14*x^14*b^4*a*B + 1/14*x^14*b^5*A + 5/6*x^12*b^3*a^2*B
+ 5/12*x^12*b^4*a*A + x^10*b^2*a^3*B + x^10*b^3*a^2*A + 5/8*x^8*b*a^4*B + 5
/4*x^8*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*a^4*A + 1/4*x^4*a^5*A
```

giac [A] time = 0.44, size = 123, normalized size = 1.84

$$\frac{1}{16}Bb^5x^{16}+\frac{5}{14}Bab^4x^{14}+\frac{1}{14}Ab^5x^{14}+\frac{5}{6}Ba^2b^3x^{12}+\frac{5}{12}Aab^4x^{12}+Ba^3b^2x^{10}+Aa^2b^3x^{10}+\frac{5}{8}Ba^4bx^8+\frac{5}{4}Aa^3b^2x^8+\frac{1}{6}Ba^5x^6+\frac{5}{6}Aa^4bx^6+\frac{1}{4}Aa^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 5/6*B*a^2*b^3*x^12
+ 5/12*A*a*b^4*x^12 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/8*B*a^4*b*x^8 + 5
/4*A*a^3*b^2*x^8 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/4*A*a^5*x^4
```

maple [B] time = 0.00, size = 124, normalized size = 1.85

$$\frac{B\,b^5x^{16}}{16}+\frac{\left(b^5A+5a\,b^4B\right)x^{14}}{14}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{12}}{12}+\frac{A\,a^5x^4}{4}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{10}}{10}+\frac{\left(10a^3b^2A+5a^4bB\right)x^8}{8}+\frac{\left(5a^4bA+a^5B\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/16*b^5*B*x^16+1/14*(A*b^5+5*B*a*b^4)*x^14+1/12*(5*A*a*b^4+10*B*a^2*b^3)*x
^12+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/4*a^5*A*x^4
```

maxima [A] time = 1.01, size = 118, normalized size = 1.76

$$\frac{1}{16}Bb^5x^{16}+\frac{1}{14}\left(5Bab^4+Ab^5\right)x^{14}+\frac{5}{12}\left(2Ba^2b^3+Aab^4\right)x^{12}+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{1}{4}Aa^5x^4+\frac{5}{8}\left(Ba^4b+2Aa^3b^2\right)x^8+\frac{1}{6}\left(Ba^5+5Aa^4b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4 + 5/8*(B*a^4*b + 2*
A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

mupad [B] time = 0.04, size = 106, normalized size = 1.58

$$x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{14}\left(\frac{A\,b^5}{14}+\frac{5\,B\,a\,b^4}{14}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{16}}{16}+a^2\,b^2\,x^{10}\,\left(A\,b+B\,a\right)+\frac{5\,a^3\,b\,x^8\,\left(2\,A\,b+B\,a\right)}{8}+\frac{5\,a\,b^3\,x^{12}\,\left(A\,b+2\,B\,a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) + (A*a
^5*x^4)/4 + (B*b^5*x^16)/16 + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^8*(2*A*
b + B*a))/8 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

sympy [B] time = 0.09, size = 131, normalized size = 1.96

$$\frac{Aa^5x^4}{4}+\frac{Bb^5x^{16}}{16}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{12}\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^{10}\left(Aa^2b^3+Ba^3b^2\right)+x^8\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right)+x^6\left(\frac{5Aa^4b}{6}+\frac{Ba^5}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**16/16 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**12
*(5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x*
*8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

3.30 $\int x^2\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=117

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{3}a^5Ax^3+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx &= \int \left(a^5Ax^2+a^4(5Ab+aB)x^4+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^8+5ab^3(\right.\\ &= \frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{11}a\end{aligned}$$

Mathematica [A] time = 0.01, size = 117, normalized size = 1.00

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 +
 (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A
*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.37, size = 124, normalized size = 1.06

$$\frac{1}{15}x^{15}b^5B+\frac{5}{13}x^{13}b^4aB+\frac{1}{13}x^{13}b^5A+\frac{10}{11}x^{11}b^3a^2B+\frac{5}{11}x^{11}b^4aA+\frac{10}{9}x^9b^2a^3B+\frac{10}{9}x^9b^3a^2A+\frac{5}{7}x^7ba^4B+\frac{10}{7}x^7b^2a^3A+\frac{1}{5}x^5a^5B+x^5ba^4A+\frac{1}{3}x^3a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^5*B + 5/13*x^13*b^4*a*B + 1/13*x^13*b^5*A + 10/11*x^11*b^3*a^2*
B + 5/11*x^11*b^4*a*A + 10/9*x^9*b^2*a^3*B + 10/9*x^9*b^3*a^2*A + 5/7*x^7*b
*a^4*B + 10/7*x^7*b^2*a^3*A + 1/5*x^5*a^5*B + x^5*b*a^4*A + 1/3*x^3*a^5*A
```

giac [A] time = 0.36, size = 124, normalized size = 1.06

$$\frac{1}{15}Bb^5x^{15}+\frac{5}{13}Bab^4x^{13}+\frac{1}{13}Ab^5x^{13}+\frac{10}{11}Ba^2b^3x^{11}+\frac{5}{11}Aab^4x^{11}+\frac{10}{9}Ba^3b^2x^9+\frac{10}{9}Aa^2b^3x^9+\frac{5}{7}Ba^4bx^7+\frac{10}{7}Aa^3b^2x^7+\frac{1}{5}Ba^5x^5+Aa^4bx^5+\frac{1}{3}Aa^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/7*B*a^4
*b*x^7 + 10/7*A*a^3*b^2*x^7 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/3*A*a^5*x^3
```

maple [A] time = 0.00, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{15}}{15}+\frac{\left(b^5A+5a\,b^4B\right)x^{13}}{13}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{11}}{11}+\frac{A\,a^5x^3}{3}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^9}{9}+\frac{\left(10a^3b^2A+5a^4bB\right)x^7}{7}+\frac{\left(5a^4bA+a^5B\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/15*b^5*B*x^15+1/13*(A*b^5+5*B*a*b^4)*x^13+1/11*(5*A*a*b^4+10*B*a^2*b^3)*x
^11+1/9*(10*A*a^2*b^3+10*B*a^3*b^2)*x^9+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+1/
5*(5*A*a^4*b+B*a^5)*x^5+1/3*a^5*A*x^3
```

maxima [A] time = 0.98, size = 119, normalized size = 1.02

$$\frac{1}{15}\,Bb^5x^{15} + \frac{1}{13}\left(5\,Bab^4 + Ab^5\right)x^{13} + \frac{5}{11}\left(2\,Ba^2b^3 + Aab^4\right)x^{11} + \frac{10}{9}\left(Ba^3b^2 + Aa^2b^3\right)x^9 + \frac{1}{3}\,Aa^5x^3 + \frac{5}{7}\left(Ba^4b + 2\,Aa^3b^2\right)x^7 + \frac{1}{5}\left(Ba^5 + 5\,Aa^4b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5*x^3 + 5/7*(B*a^4*b
+ 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

mupad [B] time = 0.04, size = 106, normalized size = 0.91

$$x^5\left(\frac{B\,a^5}{5} + A\,b\,a^4\right) + x^{13}\left(\frac{A\,b^5}{13} + \frac{5\,B\,a\,b^4}{13}\right) + \frac{A\,a^5\,x^3}{3} + \frac{B\,b^5\,x^{15}}{15} + \frac{10\,a^2\,b^2\,x^9\,\left(A\,b + B\,a\right)}{9} + \frac{5\,a^3\,b\,x^7\,\left(2\,A\,b + B\,a\right)}{7} + \frac{5\,a\,b^3\,x^{11}\,\left(A\,b + 2\,B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^5*((B*a^5)/5 + A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) + (A*a^5*x^3
)/3 + (B*b^5*x^15)/15 + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^7*(2*A*
b + B*a))/7 + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

sympy [A] time = 0.09, size = 134, normalized size = 1.15

$$\frac{Aa^5x^3}{3} + \frac{Bb^5x^{15}}{15} + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) + x^{11}\left(\frac{5Aab^4}{11} + \frac{10Ba^2b^3}{11}\right) + x^9\left(\frac{10Aa^2b^3}{9} + \frac{10Ba^3b^2}{9}\right) + x^7\left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^5\left(Aa^4b + \frac{Ba^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**15/15 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x**11
*(5*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b
**2/9) + x**7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**5*(A*a**4*b + B*a**5/5
)
```

3.31 $\int x\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^6(Ab-aB)}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}$$

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^6(Ab-aB)}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^6)/(12*b^2) + (B*(a + b*x^2)^7)/(14*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^5(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)dx,x,x^2\right)\\&=\frac{(Ab-aB)\left(a+bx^2\right)^6}{12b^2}+\frac{B\left(a+bx^2\right)^7}{14b^2}\end{aligned}$$

Mathematica [B] time = 0.04, size = 107, normalized size = 2.55

$$\frac{1}{84}x^2\left(42a^5A+21a^4x^2(aB+5Ab)+70a^3bx^4(aB+2Ab)+105a^2b^2x^6(aB+Ab)+7b^4x^{10}(5aB+Ab)+42ab^3x^8(2aB+Ab)+6b^5Bx^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] (x^2*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^2 + 70*a^3*b*(2*A*b + a*B)*x^4 + 10
5*a^2*b^2*(A*b + a*B)*x^6 + 42*a*b^3*(A*b + 2*a*B)*x^8 + 7*b^4*(A*b + 5*a*B
)*x^10 + 6*b^5*B*x^12))/84
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^5\left(A+Bx^2\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [B] time = 0.37, size = 124, normalized size = 2.95

$$\frac{1}{14}x^{14}b^5B+\frac{5}{12}x^{12}b^4aB+\frac{1}{12}x^{12}b^5A+x^{10}b^3a^2B+\frac{1}{2}x^{10}b^4aA+\frac{5}{4}x^8b^2a^3B+\frac{5}{4}x^8b^3a^2A+\frac{5}{6}x^6ba^4B+\frac{5}{3}x^6b^2a^3A+\frac{1}{4}x^4a^5B+\frac{5}{4}x^4ba^4A+\frac{1}{2}x^2a^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^5*B + 5/12*x^12*b^4*a*B + 1/12*x^12*b^5*A + x^10*b^3*a^2*B + 1/
2*x^10*b^4*a*A + 5/4*x^8*b^2*a^3*B + 5/4*x^8*b^3*a^2*A + 5/6*x^6*b*a^4*B +
5/3*x^6*b^2*a^3*A + 1/4*x^4*a^5*B + 5/4*x^4*b*a^4*A + 1/2*x^2*a^5*A
```

giac [B] time = 0.34, size = 124, normalized size = 2.95

$$\frac{1}{14}Bb^5x^{14}+\frac{5}{12}Bab^4x^{12}+\frac{1}{12}Ab^5x^{12}+Ba^2b^3x^{10}+\frac{1}{2}Aab^4x^{10}+\frac{5}{4}Ba^3b^2x^8+\frac{5}{4}Aa^2b^3x^8+\frac{5}{6}Ba^4bx^6+\frac{5}{3}Aa^3b^2x^6+\frac{1}{4}Ba^5x^4+\frac{5}{4}Aa^4bx^4+\frac{1}{2}Aa^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + 5/6*B*a^4*b*x^6 +
5/3*A*a^3*b^2*x^6 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + 1/2*A*a^5*x^2
```

maple [B] time = 0.00, size = 124, normalized size = 2.95

$$\frac{B\,b^5x^{14}}{14}+\frac{\left(b^5A+5a\,b^4B\right)x^{12}}{12}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{10}}{10}+\frac{A\,a^5x^2}{2}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^8}{8}+\frac{\left(10a^3b^2A+5a^4bB\right)x^6}{6}+\frac{\left(5a^4bA+a^5B\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/14*b^5*B*x^14+1/12*(A*b^5+5*B*a*b^4)*x^12+1/10*(5*A*a*b^4+10*B*a^2*b^3)*x
^10+1/8*(10*A*a^2*b^3+10*B*a^3*b^2)*x^8+1/6*(10*A*a^3*b^2+5*B*a^4*b)*x^6+1/
4*(5*A*a^4*b+B*a^5)*x^4+1/2*a^5*A*x^2
```

maxima [B] time = 1.01, size = 119, normalized size = 2.83

$$\frac{1}{14}\,Bb^5x^{14}+\frac{1}{12}\left(5\,Bab^4+Ab^5\right)x^{12}+\frac{1}{2}\left(2\,Ba^2b^3+Aab^4\right)x^{10}+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\frac{1}{2}\,Aa^5x^2+\frac{5}{6}\left(Ba^4b+2\,Aa^3b^2\right)x^6+\frac{1}{4}\left(Ba^5+5\,Aa^4b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x^2 + 5/6*(B*a^4*b +
2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

mupad [B] time = 0.04, size = 107, normalized size = 2.55

$$x^4\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)+x^{12}\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+\frac{A\,a^5\,x^2}{2}+\frac{B\,b^5\,x^{14}}{14}+\frac{5\,a^2\,b^2\,x^8\,\left(A\,b+B\,a\right)}{4}+\frac{5\,a^3\,b\,x^6\,\left(2\,A\,b+B\,a\right)}{6}+\frac{a\,b^3\,x^{10}\,\left(A\,b+2\,B\,a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + (A*a
^5*x^2)/2 + (B*b^5*x^14)/14 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + (5*a^3*b*x^6*
(2*A*b + B*a))/6 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

sympy [B] time = 0.09, size = 133, normalized size = 3.17

$$\frac{Aa^5x^2}{2}+\frac{Bb^5x^{14}}{14}+x^{12}\left(\frac{Ab^5}{12}+\frac{5Bab^4}{12}\right)+x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^8\left(\frac{5Aa^2b^3}{4}+\frac{5Ba^3b^2}{4}\right)+x^6\left(\frac{5Aa^3b^2}{3}+\frac{5Ba^4b}{6}\right)+x^4\left(\frac{5Aa^4b}{4}+\frac{Ba^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**14/14 + x**12*(A*b**5/12 + 5*B*a*b**4/12) + x**10
*(A*a*b**4/2 + B*a**2*b**3) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4) + x*
*6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

3.32 $\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=109

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{3}a^4x^3(aB+5Ab)+a^5Ax+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx = \int \left(a^5A+a^4(5Ab+aB)x^2+5a^3b(2Ab+aB)x^4+10a^2b^2(Ab+aB)x^6+5ab^3(Ab+\right.$$

$$=a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3(Ab+$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.00

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^2
*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^
11)/11 + (b^5*B*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^5*(A + B*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^5*(A + B*x^2), x]
```

fricas [A] time = 0.37, size = 121, normalized size = 1.11

$$\frac{1}{13}x^{13}b^5B+\frac{5}{11}x^{11}b^4aB+\frac{1}{11}x^{11}b^5A+\frac{10}{9}x^9b^3a^2B+\frac{5}{9}x^9b^4aA+\frac{10}{7}x^7b^2a^3B+\frac{10}{7}x^7b^3a^2A+x^5ba^4B+2x^5b^2a^3A+\frac{1}{3}x^3a^5B+\frac{5}{3}x^3ba^4A+xa^5A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^5*B + 5/11*x^11*b^4*a*B + 1/11*x^11*b^5*A + 10/9*x^9*b^3*a^2*B
+ 5/9*x^9*b^4*a*A + 10/7*x^7*b^2*a^3*B + 10/7*x^7*b^3*a^2*A + x^5*b*a^4*B +
 2*x^5*b^2*a^3*A + 1/3*x^3*a^5*B + 5/3*x^3*b*a^4*A + x*a^5*A
```

giac [A] time = 0.33, size = 121, normalized size = 1.11

$$\frac{1}{13}Bb^5x^{13}+\frac{5}{11}Bab^4x^{11}+\frac{1}{11}Ab^5x^{11}+\frac{10}{9}Ba^2b^3x^9+\frac{5}{9}Aab^4x^9+\frac{10}{7}Ba^3b^2x^7+\frac{10}{7}Aa^2b^3x^7+Ba^4bx^5+2Aa^3b^2x^5+\frac{1}{3}Ba^5x^3+\frac{5}{3}Aa^4bx^3+Aa^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + B*a^4*b*x^5 +
 2*A*a^3*b^2*x^5 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*x
```

maple [A] time = 0.00, size = 121, normalized size = 1.11

$$\frac{B\,b^5x^{13}}{13}+\frac{\left(b^5A+5a\,b^4B\right)x^{11}}{11}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^9}{9}+A\,a^5x+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^7}{7}+\frac{\left(10a^3b^2A+5a^4bB\right)x^5}{5}+\frac{\left(5a^4bA+a^5B\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A),x)
```

```
[Out] 1/13*b^5*B*x^13+1/11*(A*b^5+5*B*a*b^4)*x^11+1/9*(5*A*a*b^4+10*B*a^2*b^3)*x^
9+1/7*(10*A*a^2*b^3+10*B*a^3*b^2)*x^7+1/5*(10*A*a^3*b^2+5*B*a^4*b)*x^5+1/3*
(5*A*a^4*b+B*a^5)*x^3+a^5*A*x
```

maxima [A] time = 1.01, size = 115, normalized size = 1.06

$$\frac{1}{13}\,Bb^{5}x^{13}+\frac{1}{11}\left(5\,Bab^{4}+Ab^{5}\right)x^{11}+\frac{5}{9}\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{9}+\frac{10}{7}\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{7}+Aa^{5}x+\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{5}+\frac{1}{3}\left(Ba^{5}+5\,Aa^{4}b\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (B*a^4*b + 2*A*a^3*b^
2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

mupad [B] time = 0.04, size = 103, normalized size = 0.94

$$x^{3}\left(\frac{B\,a^{5}}{3}+\frac{5\,A\,b\,a^{4}}{3}\right)+x^{11}\left(\frac{A\,b^{5}}{11}+\frac{5\,B\,a\,b^{4}}{11}\right)+\frac{B\,b^{5}\,x^{13}}{13}+A\,a^{5}\,x+\frac{10\,a^{2}\,b^{2}\,x^{7}\,\left(A\,b+B\,a\right)}{7}+a^{3}\,b\,x^{5}\,\left(2\,A\,b+B\,a\right)+\frac{5\,a\,b^{3}\,x^{9}\,\left(A\,b+2\,B\,a\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^5,x)
```

```
[Out] x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^11*((A*b^5)/11 + (5*B*a*b^4)/11) + (B*b
^5*x^13)/13 + A*a^5*x + (10*a^2*b^2*x^7*(A*b + B*a))/7 + a^3*b*x^5*(2*A*b +
 B*a) + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

sympy [A] time = 0.09, size = 129, normalized size = 1.18

$$Aa^{5}x+\frac{Bb^{5}x^{13}}{13}+x^{11}\left(\frac{Ab^{5}}{11}+\frac{5Bab^{4}}{11}\right)+x^{9}\left(\frac{5Aab^{4}}{9}+\frac{10Ba^{2}b^{3}}{9}\right)+x^{7}\left(\frac{10Aa^{2}b^{3}}{7}+\frac{10Ba^{3}b^{2}}{7}\right)+x^{5}\left(2Aa^{3}b^{2}+Ba^{4}b\right)+x^{3}\left(\frac{5Aa^{4}b}{3}+\frac{Ba^{5}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**13/13 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**9*(5*A*
a*b**4/9 + 10*B*a**2*b**3/9) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7) +
 x**5*(2*A*a**3*b**2 + B*a**4*b) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

3.33 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=88

$$a^5A\log(x)+\frac{5}{2}a^4Abx^2+\frac{5}{2}a^3Ab^2x^4+\frac{5}{3}a^2Ab^3x^6+\frac{5}{8}aAb^4x^8+\frac{B\left(a+bx^2\right)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

Rubi **[A]** time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 80, 43}

$$\frac{5}{3}a^2Ab^3x^6+\frac{5}{2}a^3Ab^2x^4+\frac{5}{2}a^4Abx^2+a^5A\log(x)+\frac{5}{8}aAb^4x^8+\frac{B\left(a+bx^2\right)^6}{12b}+\frac{1}{10}Ab^5x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (5*a^4*A*b*x^2)/2 + (5*a^3*A*b^2*x^4)/2 + (5*a^2*A*b^3*x^6)/3 + (5*a*A*b^4*
x^8)/8 + (A*b^5*x^10)/10 + (B*(a + b*x^2)^6)/(12*b) + a^5*A*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(a+bx^2\right)^6}{12b}+\frac{1}{2}A\,\text{Subst}\left(\int \frac{(a+bx)^5}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(a+bx^2\right)^6}{12b}+\frac{1}{2}A\,\text{Subst}\left(\int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)\right.\\
&= \frac{5}{2}a^4Abx^2+\frac{5}{2}a^3Ab^2x^4+\frac{5}{3}a^2Ab^3x^6+\frac{5}{8}aAb^4x^8+\frac{1}{10}Ab^5x^{10}+\frac{B\left(a+bx^2\right)^6}{12b}+a^5A
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 113, normalized size = 1.28

$$a^5A\log(x)+\frac{1}{2}a^4x^2(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{12}b^5Bx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^2)/2 + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (5*a^2*b^2*(A*b
 + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a*B)*x^10)/1
0 + (b^5*B*x^12)/12 + a^5*A*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x, x]
```

fricas [A] time = 0.41, size = 117, normalized size = 1.33

$$\frac{1}{12}\,Bb^5x^{12}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{8}\left(2\,Ba^2b^3+Aab^4\right)x^8+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+Aa^5\log(x)+\frac{5}{4}\left(Ba^4b+2\,Aa^3b^2\right)x^4+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + A*a^5*log(x) + 5/4*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

giac [A] time = 0.32, size = 126, normalized size = 1.43

$$\frac{1}{12}\,Bb^{5}x^{12}+\frac{1}{2}\,Bab^{4}x^{10}+\frac{1}{10}\,Ab^{5}x^{10}+\frac{5}{4}\,Ba^{2}b^{3}x^{8}+\frac{5}{8}\,Aab^{4}x^{8}+\frac{5}{3}\,Ba^{3}b^{2}x^{6}+\frac{5}{3}\,Aa^{2}b^{3}x^{6}+\frac{5}{4}\,Ba^{4}bx^{4}+\frac{5}{2}\,Aa^{3}b^{2}x^{4}+\frac{1}{2}\,Ba^{5}x^{2}+\frac{5}{2}\,Aa^{4}bx^{2}+\frac{1}{2}\,Aa^{5}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 5/4*B*a^2*b^3*x^8 +
5/8*A*a*b^4*x^8 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/4*B*a^4*b*x^4 +
 5/2*A*a^3*b^2*x^4 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 + 1/2*A*a^5*log(x^2)
```

maple [A] time = 0.00, size = 124, normalized size = 1.41

$$\frac{B\,b^{5}x^{12}}{12}+\frac{A\,b^{5}x^{10}}{10}+\frac{Ba\,b^{4}x^{10}}{2}+\frac{5Aa\,b^{4}x^{8}}{8}+\frac{5B\,a^{2}b^{3}x^{8}}{4}+\frac{5A\,a^{2}b^{3}x^{6}}{3}+\frac{5B\,a^{3}b^{2}x^{6}}{3}+\frac{5A\,a^{3}b^{2}x^{4}}{2}+\frac{5B\,a^{4}b\,x^{4}}{4}+\frac{5A\,a^{4}b\,x^{2}}{2}+\frac{B\,a^{5}x^{2}}{2}+A\,a^{5}\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x,x)
```

```
[Out] 1/12*B*b^5*x^12+1/10*A*b^5*x^10+1/2*B*x^10*a*b^4+5/8*a*A*b^4*x^8+5/4*B*x^8*
a^2*b^3+5/3*a^2*A*b^3*x^6+5/3*B*x^6*a^3*b^2+5/2*a^3*A*b^2*x^4+5/4*B*x^4*a^4
*b+5/2*a^4*A*b*x^2+1/2*B*x^2*a^5+a^5*A*ln(x)
```

maxima [A] time = 1.03, size = 120, normalized size = 1.36

$$\frac{1}{12}\,Bb^{5}x^{12}+\frac{1}{10}\,\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+\frac{5}{8}\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+\frac{5}{3}\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+\frac{1}{2}\,Aa^{5}\log\left(x^{2}\right)+\frac{5}{4}\,\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+\frac{1}{2}\,\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1/2*A*a^5*log(x^2) + 5/4*(B*a^4*
b + 2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

mupad [B] time = 0.04, size = 105, normalized size = 1.19

$$x^{2}\,\left(\frac{B\,a^{5}}{2}+\frac{5\,A\,b\,a^{4}}{2}\right)+x^{10}\,\left(\frac{A\,b^{5}}{10}+\frac{B\,a\,b^{4}}{2}\right)+\frac{B\,b^{5}\,x^{12}}{12}+A\,a^{5}\,\ln(x)+\frac{5\,a^{2}\,b^{2}\,x^{6}\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^{3}\,b\,x^{4}\,\left(2\,A\,b+B\,a\right)}{4}+\frac{5\,a\,b^{3}\,x^{8}\,\left(A\,b+2\,B\,a\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x,x)
```

```
[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^10*((A*b^5)/10 + (B*a*b^4)/2) + (B*b^5*
x^12)/12 + A*a^5*log(x) + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^4*(2*A
*b + B*a))/4 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

sympy [A] time = 0.24, size = 134, normalized size = 1.52

$$Aa^5\log(x)+\frac{Bb^5x^{12}}{12}+x^{10}\left(\frac{Ab^5}{10}+\frac{Bab^4}{2}\right)+x^8\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^6\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right)+x^4\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x^2\left(\frac{5Aa^4b}{2}+\frac{Ba^5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**12/12 + x**10*(A*b**5/10 + B*a*b**4/2) + x**8*(5*
A*a*b**4/8 + 5*B*a**2*b**3/4) + x**6*(5*A*a**2*b**3/3 + 5*B*a**3*b**2/3) +
x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

3.34 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$2a^2b^2x^5(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5A}{x}+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
9)/9 + (b^5*B*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2}\,dx = \int \Bigg(a^4(5Ab+aB)+\frac{a^5A}{x^2}+5a^3b(2Ab+aB)x^2+10a^2b^2(Ab+aB)x^4+5ab^3(Ab+$$

$$= -\frac{a^5A}{x}+a^4(5Ab+aB)x+\frac{5}{3}a^3b(2Ab+aB)x^3+2a^2b^2(Ab+aB)x^5+\frac{5}{7}ab^3(Ab+$$

Mathematica [A] time = 0.03, size = 108, normalized size = 1.00

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] -((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2*
b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*x^
9)/9 + (b^5*B*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^2, x]
```

fricas [A] time = 0.43, size = 121, normalized size = 1.12

$$\frac{63\,Bb^5x^{12}+77\left(5\,Bab^4+Ab^5\right)x^{10}+495\left(2\,Ba^2b^3+Aab^4\right)x^8+1386\left(Ba^3b^2+Aa^2b^3\right)x^6-693\,Aa^5+1155\left(Ba^4b+2\,Aa^3b^2\right)x^4+693\left(Ba^5+5\,Aa^4b\right)x^2}{693\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^12 + 77*(5*B*a*b^4 + A*b^5)*x^10 + 495*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 693*A*a^5 + 1155*(B*a^4*b +
2*A*a^3*b^2)*x^4 + 693*(B*a^5 + 5*A*a^4*b)*x^2)/x
```

giac [A] time = 0.31, size = 120, normalized size = 1.11

$$\frac{1}{11}\,Bb^5x^{11}+\frac{5}{9}\,Bab^4x^9+\frac{1}{9}\,Ab^5x^9+\frac{10}{7}\,Ba^2b^3x^7+\frac{5}{7}\,Aab^4x^7+2\,Ba^3b^2x^5+2\,Aa^2b^3x^5+\frac{5}{3}\,Ba^4bx^3+\frac{10}{3}\,Aa^3b^2x^3+Ba^5x+5\,Aa^4bx-\frac{Aa^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 10/7*B*a^2*b^3*x^7 + 5/
7*A*a*b^4*x^7 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/3*B*a^4*b*x^3 + 10/3*
A*a^3*b^2*x^3 + B*a^5*x + 5*A*a^4*b*x - A*a^5/x
```

maple [A] time = 0.00, size = 121, normalized size = 1.12

$$\frac{B\,b^5x^{11}}{11}+\frac{A\,b^5x^9}{9}+\frac{5Ba\,b^4x^9}{9}+\frac{5Aa\,b^4x^7}{7}+\frac{10B\,a^2b^3x^7}{7}+2A\,a^2b^3x^5+2B\,a^3b^2x^5+\frac{10A\,a^3b^2x^3}{3}+\frac{5B\,a^4b\,x^3}{3}+5A\,a^4bx+B\,a^5x-\frac{A\,a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^2,x)
```

```
[Out] 1/11*b^5*B*x^11+1/9*A*x^9*b^5+5/9*B*x^9*a*b^4+5/7*A*x^7*a*b^4+10/7*B*x^7*a^
2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+10/3*A*x^3*a^3*b^2+5/3*B*x^3*a^4*b+5*
a^4*b*A*x+a^5*B*x-a^5*A/x
```

maxima [A] time = 1.13, size = 116, normalized size = 1.07

$$\frac{1}{11}\,Bb^5x^{11} + \frac{1}{9}\left(5\,Bab^4 + Ab^5\right)x^9 + \frac{5}{7}\left(2\,Ba^2b^3 + Aab^4\right)x^7 + 2\left(Ba^3b^2 + Aa^2b^3\right)x^5 - \frac{Aa^5}{x} + \frac{5}{3}\left(Ba^4b + 2\,Aa^3b^2\right)x^3 + \left(Ba^5 + 5\,Aa^4b\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/7*(2*B*a^2*b^3 + A*a*b^4)
*x^7 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 - A*a^5/x + 5/3*(B*a^4*b + 2*A*a^3*b^2
)*x^3 + (B*a^5 + 5*A*a^4*b)*x
```

mupad [B] time = 0.04, size = 104, normalized size = 0.96

$$x\left(B\,a^5 + 5\,A\,b\,a^4\right) + x^9\left(\frac{A\,b^5}{9} + \frac{5\,B\,a\,b^4}{9}\right) - \frac{A\,a^5}{x} + \frac{B\,b^5\,x^{11}}{11} + 2\,a^2\,b^2\,x^5\,\left(A\,b + B\,a\right) + \frac{5\,a^3\,b\,x^3\,\left(2\,A\,b + B\,a\right)}{3} + \frac{5\,a\,b^3\,x^7\,\left(A\,b + 2\,B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^2,x)
```

```
[Out] x*(B*a^5 + 5*A*a^4*b) + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) - (A*a^5)/x + (B*b^
5*x^11)/11 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^3*(2*A*b + B*a))/3 + (5
*a*b^3*x^7*(A*b + 2*B*a))/7
```

sympy [A] time = 0.24, size = 126, normalized size = 1.17

$$-\frac{Aa^5}{x} + \frac{Bb^5x^{11}}{11} + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^7\left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^5\left(2Aa^2b^3 + 2Ba^3b^2\right) + x^3\left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right) + x\left(5Aa^4b + Ba^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**11/11 + x**9*(A*b**5/9 + 5*B*a*b**4/9) + x**7*(5*A*a*
b**4/7 + 10*B*a**2*b**3/7) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**3*(1
0*A*a**3*b**2/3 + 5*B*a**4*b/3) + x*(5*A*a**4*b + B*a**5)
```

3.35 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^3}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{2x^2}+a^4\log(x)(aB+5Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{10}b^5Bx$$

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{2x^2}+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^2}+\frac{a^4(5Ab+aB)}{x}+10a^2b^2(Ab+aB)x+5a\right.\right.\\ &= -\frac{a^5A}{2x^2}+\frac{5}{2}a^3b(2Ab+aB)x^2+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{8}b^4(A\end{aligned}$$

Mathematica [A] time = 0.05, size = 115, normalized size = 1.02

$$-\frac{a^5A}{2x^2}+\frac{5}{2}a^3bx^2(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\log(x)\left(a^5B+5a^4Ab\right)+\frac{1}{8}b^4x^8(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*x
^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*B*
x^10)/10 + (5*a^4*A*b + a^5*B)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^3, x]
```

fricas [A] time = 0.44, size = 123, normalized size = 1.09

$$\frac{12\,Bb^5x^{12}+15\left(5\,Bab^4+Ab^5\right)x^{10}+100\left(2\,Ba^2b^3+Aab^4\right)x^8+300\left(Ba^3b^2+Aa^2b^3\right)x^6-60\,Aa^5+300\left(Ba^4b+2\,Aa^3b^2\right)x^4+120\left(Ba^5+5\,Aa^4b\right)x^2\log(x)}{120\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(12*B*b^5*x^12 + 15*(5*B*a*b^4 + A*b^5)*x^10 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 60*A*a^5 + 300*(B*a^4*b + 2*A
*a^3*b^2)*x^4 + 120*(B*a^5 + 5*A*a^4*b)*x^2*log(x))/x^2
```

giac [A] time = 0.38, size = 145, normalized size = 1.28

$$\frac{1}{10}\,Bb^5x^{10}+\frac{5}{8}\,Bab^4x^8+\frac{1}{8}\,Ab^5x^8+\frac{5}{3}\,Ba^2b^3x^6+\frac{5}{6}\,Aab^4x^6+\frac{5}{2}\,Ba^3b^2x^4+\frac{5}{2}\,Aa^2b^3x^4+\frac{5}{2}\,Ba^4bx^2+5\,Aa^3b^2x^2+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)\log\left(x^2\right)-\frac{Ba^5x^2+5\,Aa^4bx^2+Aa^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2) - 1/2*(B*a^5*x^2 + 5*A*a^4
*b*x^2 + A*a^5)/x^2
```

maple [A] time = 0.01, size = 123, normalized size = 1.09

$$\frac{B\,b^5x^{10}}{10}+\frac{A\,b^5x^8}{8}+\frac{5Ba\,b^4x^8}{8}+\frac{5Aa\,b^4x^6}{6}+\frac{5B\,a^2b^3x^6}{3}+\frac{5A\,a^2b^3x^4}{2}+\frac{5B\,a^3b^2x^4}{2}+5A\,a^3b^2x^2+\frac{5B\,a^4b\,x^2}{2}+5A\,a^4b\ln(x)+B\,a^5\ln(x)-\frac{A\,a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^3,x)
```

```
[Out] 1/10*b^5*B*x^10+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+5/6*A*x^6*a*b^4+5/3*B*x^6*a^2
*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+5*A*x^2*a^3*b^2+5/2*B*x^2*a^4*b-1/
2*a^5*A/x^2+5*A*ln(x)*a^4*b+B*ln(x)*a^5
```

maxima [A] time = 1.10, size = 120, normalized size = 1.06

$$\frac{1}{10}\,Bb^5x^{10}+\frac{1}{8}\left(5\,Bab^4+Ab^5\right)x^8+\frac{5}{6}\left(2\,Ba^2b^3+Aab^4\right)x^6+\frac{5}{2}\left(Ba^3b^2+Aa^2b^3\right)x^4-\frac{Aa^5}{2\,x^2}+\frac{5}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^2+\frac{1}{2}\left(Ba^5+5\,Aa^4b\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 - 1/2*A*a^5/x^2 + 5/2*(B*a^4*b + 2*A
*a^3*b^2)*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2)
```

mupad [B] time = 0.08, size = 105, normalized size = 0.93

$$x^8\left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right)+\ln(x)\,\left(B\,a^5+5\,A\,b\,a^4\right)-\frac{A\,a^5}{2\,x^2}+\frac{B\,b^5\,x^{10}}{10}+\frac{5\,a^2\,b^2\,x^4\,\left(A\,b+B\,a\right)}{2}+\frac{5\,a^3\,b\,x^2\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^3\,x^6\,\left(A\,b+2\,B\,a\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^3,x)
```

```
[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(2*x
^2) + (B*b^5*x^10)/10 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + (5*a^3*b*x^2*(2*A*b
 + B*a))/2 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

sympy [A] time = 0.35, size = 131, normalized size = 1.16

$$-\frac{Aa^5}{2x^2} + \frac{Bb^5x^{10}}{10} + a^4(5Ab + Ba)\log(x) + x^8\left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^6\left(\frac{5Aab^4}{6} + \frac{5Ba^2b^3}{3}\right) + x^4\left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x^2\left(5Aa^3b^2 + \frac{5Ba^4b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**10/10 + a**4*(5*A*b + B*a)*log(x) + x**8*(A*b*
*5/8 + 5*B*a*b**4/8) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**4*(5*A*a*
*2*b**3/2 + 5*B*a**3*b**2/2) + x**2*(5*A*a**3*b**2 + 5*B*a**4*b/2)
```

3.36 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^4}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{3x^3}-\frac{a^4(aB+5Ab)}{x}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{3}a^2b^2x^3(aB+Ab)+5a^3bx(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{3x^3}+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a^
2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^7
)/7 + (b^5*B*x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^4}\,dx &= \int \Bigg(5a^3b(2Ab+aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x^2+5ab^3(Ab+2a\\ &= -\frac{a^5A}{3x^3}-\frac{a^4(5Ab+aB)}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab+2aB\end{aligned}$$

Mathematica [A] time = 0.04, size = 110, normalized size = 1.02

$$-\frac{a^5A}{3x^3}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{a^5(-B)-5a^4Ab}{x}+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (-5*a^4*A*b - a^5*B)/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a
^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x^
7)/7 + (b^5*B*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^4, x]
```

fricas [A] time = 0.40, size = 121, normalized size = 1.12

$$\frac{7\,Bb^5x^{12}+9\left(5\,Bab^4+Ab^5\right)x^{10}+63\left(2\,Ba^2b^3+Aab^4\right)x^8+210\left(Ba^3b^2+Aa^2b^3\right)x^6-21\,Aa^5+315\left(Ba^4b+2\,Aa^3b^2\right)x^4-63\left(Ba^5+5\,Aa^4b\right)x^2}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^5*x^12 + 9*(5*B*a*b^4 + A*b^5)*x^10 + 63*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 + 315*(B*a^4*b + 2*A*a^3
*b^2)*x^4 - 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

giac [A] time = 0.37, size = 122, normalized size = 1.13

$$\frac{1}{9}\,Bb^5x^9+\frac{5}{7}\,Bab^4x^7+\frac{1}{7}\,Ab^5x^7+2\,Ba^2b^3x^5+Aab^4x^5+\frac{10}{3}\,Ba^3b^2x^3+\frac{10}{3}\,Aa^2b^3x^3+5\,Ba^4bx+10\,Aa^3b^2x-\frac{3\,Ba^5x^2+15\,Aa^4bx^2+Aa^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 2*B*a^2*b^3*x^5 + A*a*b^4
*x^5 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*B*a^4*b*x + 10*A*a^3*b^2
*x - 1/3*(3*B*a^5*x^2 + 15*A*a^4*b*x^2 + A*a^5)/x^3
```

maple [A] time = 0.00, size = 118, normalized size = 1.09

$$\frac{B\,b^5x^9}{9}+\frac{A\,b^5x^7}{7}+\frac{5Ba\,b^4x^7}{7}+Aa\,b^4x^5+2B\,a^2b^3x^5+\frac{10A\,a^2b^3x^3}{3}+\frac{10B\,a^3b^2x^3}{3}+10A\,a^3b^2x+5B\,a^4bx-\frac{A\,a^5}{3x^3}-\frac{(5Ab+Ba)\,a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^4,x)
```

```
[Out] 1/9*b^5*B*x^9+1/7*A*x^7*b^5+5/7*B*x^7*a*b^4+A*x^5*a*b^4+2*B*x^5*a^2*b^3+10/
3*A*x^3*a^2*b^3+10/3*B*x^3*a^3*b^2+10*a^3*b^2*A*x+5*a^4*b*B*x-a^4*(5*A*b+B*
a)/x-1/3*a^5*A/x^3
```

maxima [A] time = 1.03, size = 118, normalized size = 1.09

$$\frac{1}{9}\,Bb^5x^9+\frac{1}{7}\left(5\,Bab^4+Ab^5\right)x^7+\left(2\,Ba^2b^3+Aab^4\right)x^5+\frac{10}{3}\left(Ba^3b^2+Aa^2b^3\right)x^3+5\left(Ba^4b+2\,Aa^3b^2\right)x-\frac{Aa^5+3\left(Ba^5+5\,Aa^4b\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + (2*B*a^2*b^3 + A*a*b^4)*x^5 +
 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/3*(A*a^
5 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

mupad [B] time = 0.04, size = 106, normalized size = 0.98

$$x^7\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)-\frac{\frac{A\,a^5}{3}+x^2\,\left(B\,a^5+5\,A\,b\,a^4\right)}{x^3}+\frac{B\,b^5\,x^9}{9}+\frac{10\,a^2\,b^2\,x^3\,\left(A\,b+B\,a\right)}{3}+5\,a^3\,b\,x\,\left(2\,A\,b+B\,a\right)+a\,b^3\,x^5\,\left(A\,b+2\,B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^4,x)
```

```
[Out] x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/3 + x^2*(B*a^5 + 5*A*a^4*b))/x^3
 + (B*b^5*x^9)/9 + (10*a^2*b^2*x^3*(A*b + B*a))/3 + 5*a^3*b*x*(2*A*b + B*a)
 + a*b^3*x^5*(A*b + 2*B*a)
```

sympy [A] time = 0.36, size = 128, normalized size = 1.19

$$\frac{Bb^5x^9}{9}+x^7\left(\frac{Ab^5}{7}+\frac{5Bab^4}{7}\right)+x^5\left(Aab^4+2Ba^2b^3\right)+x^3\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right)+x\left(10Aa^3b^2+5Ba^4b\right)+\frac{-Aa^5+x^2\left(-15Aa^4b-3Ba^5\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**4,x)
```

```
[Out] B*b**5*x**9/9 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**5*(A*a*b**4 + 2*B*a**2*
b**3) + x**3*(10*A*a**2*b**3/3 + 10*B*a**3*b**2/3) + x*(10*A*a**3*b**2 + 5*
B*a**4*b) + (-A*a**5 + x**2*(-15*A*a**4*b - 3*B*a**5))/(3*x**3)
```

3.37 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)-\frac{a^5A}{4x^4}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^5}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\Bigg(\int \Bigg(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}+5ab^3(\\
&= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{2x^2}+5a^2b^2(Ab+aB)x^2+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{6}b^4(Ab+5aB
\end{aligned}
$$

Mathematica [**A**]   time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3b\log(x)(aB+2Ab)+5a^2b^2x^2(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
+ (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)/8
 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

IntegrateAlgebraic [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^5, x]
```

fricas [**A**]   time = 0.42, size = 123, normalized size = 1.10

$$\frac{3\,Bb^5x^{12}+4\left(5\,Bab^4+Ab^5\right)x^{10}+30\left(2\,Ba^2b^3+Aab^4\right)x^8+120\left(Ba^3b^2+Aa^2b^3\right)x^6-6\,Aa^5+120\left(Ba^4b+2\,Aa^3b^2\right)x^4\log(x)-12\left(Ba^5+5\,Aa^4b\right)x^2}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^12 + 4*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 6*A*a^5 + 120*(B*a^4*b + 2*A*a^3*
b^2)*x^4*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

giac [A] time = 0.35, size = 149, normalized size = 1.33

$$\frac{1}{8}\,Bb^{5}x^{8}+\frac{5}{6}\,Bab^{4}x^{6}+\frac{1}{6}\,Ab^{5}x^{6}+\frac{5}{2}\,Ba^{2}b^{3}x^{4}+\frac{5}{4}\,Aab^{4}x^{4}+5\,Ba^{3}b^{2}x^{2}+5\,Aa^{2}b^{3}x^{2}+\frac{5}{2}\,\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)\log\left(x^{2}\right)-\frac{15\,Ba^{4}bx^{4}+30\,Aa^{3}b^{2}x^{4}+2\,Ba^{5}x^{2}+10\,Aa^{4}bx^{2}+Aa^{5}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 5/2*B*a^2*b^3*x^4 + 5/4*A
*a*b^4*x^4 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2
)*log(x^2) - 1/4*(15*B*a^4*b*x^4 + 30*A*a^3*b^2*x^4 + 2*B*a^5*x^2 + 10*A*a^
4*b*x^2 + A*a^5)/x^4
```

maple [A] time = 0.01, size = 124, normalized size = 1.11

$$\frac{B\,b^{5}x^{8}}{8}+\frac{A\,b^{5}x^{6}}{6}+\frac{5Ba\,b^{4}x^{6}}{6}+\frac{5Aa\,b^{4}x^{4}}{4}+\frac{5B\,a^{2}b^{3}x^{4}}{2}+5A\,a^{2}b^{3}x^{2}+5B\,a^{3}b^{2}x^{2}+10A\,a^{3}b^{2}\ln(x)+5B\,a^{4}b\ln(x)-\frac{5A\,a^{4}b}{2x^{2}}-\frac{B\,a^{5}}{2x^{2}}-\frac{A\,a^{5}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^5,x)
```

```
[Out] 1/8*b^5*B*x^8+1/6*A*x^6*b^5+5/6*B*x^6*a*b^4+5/4*A*x^4*a*b^4+5/2*B*x^4*a^2*b
^3+5*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2-1/4*a^5*A/x^4-5/2*a^4/x^2*A*b-1/2*a^5/x^
2*B+10*A*ln(x)*a^3*b^2+5*B*ln(x)*a^4*b
```

maxima [A] time = 1.04, size = 122, normalized size = 1.09

$$\frac{1}{8}\,Bb^{5}x^{8}+\frac{1}{6}\,\left(5\,Bab^{4}+Ab^{5}\right)x^{6}+\frac{5}{4}\,\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{4}+5\,\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{2}+\frac{5}{2}\,\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)\log\left(x^{2}\right)-\frac{Aa^{5}+2\,\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/4*(2*B*a^2*b^3 + A*a*b^4)*x
^4 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*log(x^2) -
 1/4*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

mupad [B] time = 0.05, size = 113, normalized size = 1.01

$$\ln(x)\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{\frac{A\,a^5}{4}+x^2\,\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)}{x^4}+x^6\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+\frac{B\,b^5\,x^8}{8}+5\,a^2\,b^2\,x^2\,\left(A\,b+B\,a\right)+\frac{5\,a\,b^3\,x^4\,\left(A\,b+2\,B\,a\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^5,x)
```

```
[Out] log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/4 + x^2*((B*a^5)/2 + (5*A*a^4*
b)/2))/x^4 + x^6*((A*b^5)/6 + (5*B*a*b^4)/6) + (B*b^5*x^8)/8 + 5*a^2*b^2*x^
2*(A*b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

sympy [A] time = 0.70, size = 128, normalized size = 1.14

$$\frac{Bb^5x^8}{8} + 5a^3b\,(2Ab + Ba)\log{(x)} + x^6\left(\frac{Ab^5}{6} + \frac{5Bab^4}{6}\right) + x^4\left(\frac{5Aab^4}{4} + \frac{5Ba^2b^3}{2}\right) + x^2\left(5Aa^2b^3 + 5Ba^3b^2\right) + \frac{-Aa^5 + x^2\left(-10Aa^4b - 2Ba^5\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**5,x)
```

```
[Out] B*b**5*x**8/8 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b**4
/6) + x**4*(5*A*a*b**4/4 + 5*B*a**2*b**3/2) + x**2*(5*A*a**2*b**3 + 5*B*a**
3*b**2) + (-A*a**5 + x**2*(-10*A*a**4*b - 2*B*a**5))/(4*x**4)
```

3.38 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=111

$$-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2x(aB+Ab)+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$10a^2b^2x(aB+Ab)-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5A}{5x^5}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^6}\,dx=\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^2}+5ab^3(Ab+2a$$

$$=-\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab+2a$$

Mathematica [A] time = 0.03, size = 111, normalized size = 1.00

$$-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2x(aB+Ab)+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
+ 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*
a*B)*x^5)/5 + (b^5*B*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5 \left(A+Bx^2\right)}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^6, x]
```

fricas [A] time = 0.42, size = 121, normalized size = 1.09

$$\frac{15\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+175\left(2\,Ba^2b^3+Aab^4\right)x^8+1050\left(Ba^3b^2+Aa^2b^3\right)x^6-21\,Aa^5-525\left(Ba^4b+2\,Aa^3b^2\right)x^4-35\left(Ba^5+5\,Aa^4b\right)x^2}{105\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 175*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 - 525*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 35*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

giac [A] time = 0.37, size = 123, normalized size = 1.11

$$\frac{1}{7}\,Bb^5x^7+Bab^4x^5+\frac{1}{5}\,Ab^5x^5+\frac{10}{3}\,Ba^2b^3x^3+\frac{5}{3}\,Aab^4x^3+10\,Ba^3b^2x+10\,Aa^2b^3x-\frac{75\,Ba^4bx^4+150\,Aa^3b^2x^4+5\,Ba^5x^2+25\,Aa^4bx^2+3\,Aa^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 10/3*B*a^2*b^3*x^3 + 5/3*A*a*
b^4*x^3 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/15*(75*B*a^4*b*x^4 + 150*A*a^
3*b^2*x^4 + 5*B*a^5*x^2 + 25*A*a^4*b*x^2 + 3*A*a^5)/x^5
```

maple [A] time = 0.01, size = 113, normalized size = 1.02

$$\frac{B\,b^5x^7}{7}+\frac{A\,b^5x^5}{5}+Ba\,b^4x^5+\frac{5Aa\,b^4x^3}{3}+\frac{10B\,a^2b^3x^3}{3}+10A\,a^2b^3x+10B\,a^3b^2x-\frac{5\,(2Ab+Ba)\,a^3b}{x}-\frac{A\,a^5}{5x^5}-\frac{(5Ab+Ba)\,a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^6,x)
```

```
[Out] 1/7*b^5*B*x^7+1/5*A*x^5*b^5+B*x^5*a*b^4+5/3*A*x^3*a*b^4+10/3*B*x^3*a^2*b^3+
10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^5*A/x^5-5*a^3*b*(2*A*b+B*a)/x-1/3*a^4*(
5*A*b+B*a)/x^3
```

maxima [**A**]   time = 0.95, size = 120, normalized size = 1.08

$$\frac{1}{7}\,Bb^5x^7 + \frac{1}{5}\left(5\,Bab^4 + Ab^5\right)x^5 + \frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^3 + 10\left(Ba^3b^2 + Aa^2b^3\right)x - \frac{3\,Aa^5 + 75\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 5\left(Ba^5 + 5\,Aa^4b\right)x^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/15*(3*A*a^5 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^4 + 5*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

mupad [**B**]   time = 0.07, size = 111, normalized size = 1.00

$$x^5\left(\frac{A\,b^5}{5} + B\,a\,b^4\right) - \frac{\frac{A\,a^5}{5} + x^4\,\left(5\,B\,a^4\,b + 10\,A\,a^3\,b^2\right) + x^2\,\left(\frac{B\,a^5}{3} + \frac{5\,A\,b\,a^4}{3}\right)}{x^5} + \frac{B\,b^5\,x^7}{7} + 10\,a^2\,b^2\,x\,\left(A\,b + B\,a\right) + \frac{5\,a\,b^3\,x^3\,\left(A\,b + 2\,B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^6,x)
```

```
[Out] x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/5 + x^4*(10*A*a^3*b^2 + 5*B*a^4*b) + x
^2*((B*a^5)/3 + (5*A*a^4*b)/3))/x^5 + (B*b^5*x^7)/7 + 10*a^2*b^2*x*(A*b + B
*a) + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

sympy [**A**]   time = 0.77, size = 129, normalized size = 1.16

$$\frac{Bb^5x^7}{7} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^3\left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) + x\left(10Aa^2b^3 + 10Ba^3b^2\right) + \frac{-3Aa^5 + x^4\left(-150Aa^3b^2 - 75Ba^4b\right) + x^2\left(-25Aa^4b - 5Ba^5\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**6,x)
```

```
[Out] B*b**5*x**7/7 + x**5*(A*b**5/5 + B*a*b**4) + x**3*(5*A*a*b**4/3 + 10*B*a**2
*b**3/3) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-3*A*a**5 + x**4*(-150*A*
a**3*b**2 - 75*B*a**4*b) + x**2*(-25*A*a**4*b - 5*B*a**5))/(15*x**5)
```

3.39 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^7} \, dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{a^5A}{6x^6}+\frac{1}{4}b^4x^4(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{6}b^5Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4*(A*b + 5*a*B)*x^4)/4 + (b^5*B*
x^6)/6 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\Bigg(\int \Bigg(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}+10a^2\\ &= -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{2x^2}+\frac{5}{2}ab^3(Ab+2aB)x^2+\frac{1}{4}b^4(Ab+5aB\end{aligned}$$

Mathematica [A] time = 0.04, size = 116, normalized size = 1.02

$$\frac{1}{12}\left(-\frac{a^5\left(2A+3Bx^2\right)}{x^6}-\frac{15a^4b\left(A+2Bx^2\right)}{x^4}-\frac{60a^3Ab^2}{x^2}+120a^2b^2\log(x)(aB+Ab)+60a^2b^3Bx^2+15ab^4x^2\left(2A+Bx^2\right)+b^5x^4\left(3A+2Bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] ((-60*a^3*A*b^2)/x^2 + 60*a^2*b^3*B*x^2 + 15*a*b^4*x^2*(2*A + B*x^2) - (15*
a^4*b*(A + 2*B*x^2))/x^4 + b^5*x^4*(3*A + 2*B*x^2) - (a^5*(2*A + 3*B*x^2))/
x^6 + 120*a^2*b^2*(A*b + a*B)*Log[x])/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^7, x]
```

fricas [A] time = 0.47, size = 123, normalized size = 1.08

$$\frac{2\,Bb^5x^{12}+3\left(5\,Bab^4+Ab^5\right)x^{10}+30\left(2\,Ba^2b^3+Aab^4\right)x^8+120\left(Ba^3b^2+Aa^2b^3\right)x^6\log(x)-2\,Aa^5-30\left(Ba^4b+2\,Aa^3b^2\right)x^4-3\left(Ba^5+5\,Aa^4b\right)x^2}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^12 + 3*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6*log(x) - 2*A*a^5 - 30*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6
```

giac [A] time = 0.34, size = 151, normalized size = 1.32

$$\frac{1}{6}\,Bb^{5}x^{6} + \frac{5}{4}\,Bab^{4}x^{4} + \frac{1}{4}\,Ab^{5}x^{4} + 5\,Ba^{2}b^{3}x^{2} + \frac{5}{2}\,Aab^{4}x^{2} + 5\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)\log\left(x^{2}\right) - \frac{110\,Ba^{3}b^{2}x^{6} + 110\,Aa^{2}b^{3}x^{6} + 30\,Ba^{4}bx^{4} + 60\,Aa^{3}b^{2}x^{4} + 3\,Ba^{5}x^{2} + 15\,Aa^{4}bx^{2} + 2\,Aa^{5}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 5*B*a^2*b^3*x^2 + 5/2*A*a
*b^4*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(110*B*a^3*b^2*x^6 + 1
10*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 3*B*a^5*x^2 + 15*A*a
^4*b*x^2 + 2*A*a^5)/x^6
```

maple [A] time = 0.01, size = 124, normalized size = 1.09

$$\frac{B\,b^{5}x^{6}}{6} + \frac{A\,b^{5}x^{4}}{4} + \frac{5Ba\,b^{4}x^{4}}{4} + \frac{5Aa\,b^{4}x^{2}}{2} + 5B\,a^{2}b^{3}x^{2} + 10A\,a^{2}b^{3}\ln(x) + 10B\,a^{3}b^{2}\ln(x) - \frac{5A\,a^{3}b^{2}}{x^{2}} - \frac{5B\,a^{4}b}{2x^{2}} - \frac{5A\,a^{4}b}{4x^{4}} - \frac{B\,a^{5}}{4x^{4}} - \frac{A\,a^{5}}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^7,x)
```

```
[Out] 1/6*b^5*B*x^6+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5/2*A*x^2*a*b^4+5*B*x^2*a^2*b^3
-5/4*a^4/x^4*A*b-1/4*a^5/x^4*B-1/6*a^5*A/x^6-5*a^3*b^2/x^2*A-5/2*a^4*b/x^2*
B+10*A*ln(x)*a^2*b^3+10*B*ln(x)*a^3*b^2
```

maxima [A] time = 1.06, size = 123, normalized size = 1.08

$$\frac{1}{6}\,Bb^{5}x^{6} + \frac{1}{4}\left(5\,Bab^{4} + Ab^{5}\right)x^{4} + \frac{5}{2}\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{2} + 5\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)\log\left(x^{2}\right) - \frac{2\,Aa^{5} + 30\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{4} + 3\left(Ba^{5} + 5\,Aa^{4}b\right)x^{2}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(2*A*a^5 + 30*(B*a^4*b + 2*A
*a^3*b^2)*x^4 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6
```

mupad [B] time = 0.08, size = 118, normalized size = 1.04

$$x^{4}\left(\frac{A\,b^{5}}{4} + \frac{5\,B\,a\,b^{4}}{4}\right) - \frac{\frac{A\,a^{5}}{6} + x^{4}\left(\frac{5\,B\,a^{4}\,b}{2} + 5\,A\,a^{3}\,b^{2}\right) + x^{2}\left(\frac{B\,a^{5}}{4} + \frac{5\,A\,b\,a^{4}}{4}\right)}{x^{6}} + \ln(x)\left(10\,B\,a^{3}\,b^{2} + 10\,A\,a^{2}\,b^{3}\right) + \frac{B\,b^{5}\,x^{6}}{6} + \frac{5\,a\,b^{3}\,x^{2}\,\left(A\,b + 2\,B\,a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^7,x)
```

```
[Out] x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/6 + x^4*(5*A*a^3*b^2 + (5*B*a^4*
b)/2) + x^2*((B*a^5)/4 + (5*A*a^4*b)/4))/x^6 + log(x)*(10*A*a^2*b^3 + 10*B*
a^3*b^2) + (B*b^5*x^6)/6 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

sympy [A] time = 1.41, size = 128, normalized size = 1.12

$$\frac{Bb^5x^6}{6} + 10a^2b^2\left(Ab + Ba\right)\log\left(x\right) + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x^2\left(\frac{5Aab^4}{2} + 5Ba^2b^3\right) + \frac{-2Aa^5 + x^4\left(-60Aa^3b^2 - 30Ba^4b\right) + x^2\left(-15Aa^4b - 3Ba^5\right)}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**7,x)
```

```
[Out] B*b**5*x**6/6 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**4*(A*b**5/4 + 5*B*a*b*
*4/4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*b**3) + (-2*A*a**5 + x**4*(-60*A*a**3
*b**2 - 30*B*a**4*b) + x**2*(-15*A*a**4*b - 3*B*a**5))/(12*x**6)
```

3.40 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^8}\,dx$

Optimal. Leaf size=111

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^5A}{7x^7}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^8}\,dx &= \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^8}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+a}{x^2}\right.\\ &= -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+2aB)x\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 1.00

$$-\frac{a^5A}{7x^7}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b +
5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^8, x]
```

fricas [A] time = 0.42, size = 121, normalized size = 1.09

$$\frac{21\,Bb^5x^{12}+35\,\left(5\,Bab^4+Ab^5\right)x^{10}+525\,\left(2\,Ba^2b^3+Aab^4\right)x^8-1050\,\left(Ba^3b^2+Aa^2b^3\right)x^6-15\,Aa^5-175\,\left(Ba^4b+2\,Aa^3b^2\right)x^4-21\,\left(Ba^5+5\,Aa^4b\right)x^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(21*B*b^5*x^12 + 35*(5*B*a*b^4 + A*b^5)*x^10 + 525*(2*B*a^2*b^3 + A*a
*b^4)*x^8 - 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 15*A*a^5 - 175*(B*a^4*b + 2*
A*a^3*b^2)*x^4 - 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

giac [A] time = 0.32, size = 124, normalized size = 1.12

$$\frac{1}{5}\,Bb^5x^5+\frac{5}{3}\,Bab^4x^3+\frac{1}{3}\,Ab^5x^3+10\,Ba^2b^3x+5\,Aab^4x-\frac{1050\,Ba^3b^2x^6+1050\,Aa^2b^3x^6+175\,Ba^4bx^4+350\,Aa^3b^2x^4+21\,Ba^5x^2+105\,Aa^4bx^2+15\,Aa^5}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/105*(1050*B*a^3*b^2*x^6 + 1050*A*a^2*b^3*x^6 + 175*B*a^4*b*x^4 + 35
0*A*a^3*b^2*x^4 + 21*B*a^5*x^2 + 105*A*a^4*b*x^2 + 15*A*a^5)/x^7
```

maple [A] time = 0.01, size = 108, normalized size = 0.97

$$\frac{B\,b^5x^5}{5}+\frac{A\,b^5x^3}{3}+\frac{5Ba\,b^4x^3}{3}+5Aa\,b^4x+10B\,a^2b^3x-\frac{10\,(Ab+Ba)\,a^2b^2}{x}-\frac{5\,(2Ab+Ba)\,a^3b}{3x^3}-\frac{A\,a^5}{7x^7}-\frac{(5Ab+Ba)\,a^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^8,x)
```

```
[Out] 1/5*b^5*B*x^5+1/3*A*x^3*b^5+5/3*B*x^3*a*b^4+5*a*b^4*A*x+10*a^2*b^3*B*x-1/5*
a^4*(5*A*b+B*a)/x^5-10*a^2*b^2*(A*b+B*a)/x-1/7*a^5*A/x^7-5/3*a^3*b*(2*A*b+B
*a)/x^3
```

maxima [**A**] time = 1.02, size = 120, normalized size = 1.08

$$\frac{1}{5}\,Bb^{5}x^{5}+\frac{1}{3}\left(5\,Bab^{4}+Ab^{5}\right)x^{3}+5\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x-\frac{1050\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+15\,Aa^{5}+175\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+21\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{105\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/105*(1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 15*A*a^5 + 175*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

mupad [**B**] time = 0.10, size = 116, normalized size = 1.05

$$x^{3}\left(\frac{A\,b^{5}}{3}+\frac{5\,B\,a\,b^{4}}{3}\right)-\frac{\frac{A\,a^{5}}{7}+x^{4}\left(\frac{5\,B\,a^{4}\,b}{3}+\frac{10\,A\,a^{3}\,b^{2}}{3}\right)+x^{2}\left(\frac{B\,a^{5}}{5}+A\,b\,a^{4}\right)+x^{6}\left(10\,B\,a^{3}\,b^{2}+10\,A\,a^{2}\,b^{3}\right)}{x^{7}}+\frac{B\,b^{5}\,x^{5}}{5}+5\,a\,b^{3}\,x\,\left(A\,b+2\,B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^8,x)
```

```
[Out] x^3*((A*b^5)/3 + (5*B*a*b^4)/3) - ((A*a^5)/7 + x^4*((10*A*a^3*b^2)/3 + (5*B
*a^4*b)/3) + x^2*((B*a^5)/5 + A*a^4*b) + x^6*(10*A*a^2*b^3 + 10*B*a^3*b^2))
/x^7 + (B*b^5*x^5)/5 + 5*a*b^3*x*(A*b + 2*B*a)
```

sympy [**A**] time = 1.62, size = 131, normalized size = 1.18

$$\frac{Bb^{5}x^{5}}{5}+x^{3}\left(\frac{Ab^{5}}{3}+\frac{5Bab^{4}}{3}\right)+x\left(5Aab^{4}+10Ba^{2}b^{3}\right)+\frac{-15Aa^{5}+x^{6}\left(-1050Aa^{2}b^{3}-1050Ba^{3}b^{2}\right)+x^{4}\left(-350Aa^{3}b^{2}-175Ba^{4}b\right)+x^{2}\left(-105Aa^{4}b-21Ba^{5}\right)}{105x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**8,x)
```

```
[Out] B*b**5*x**5/5 + x**3*(A*b**5/3 + 5*B*a*b**4/3) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) + (-15*A*a**5 + x**6*(-1050*A*a**2*b**3 - 1050*B*a**3*b**2) + x**4*(-
350*A*a**3*b**2 - 175*B*a**4*b) + x**2*(-105*A*a**4*b - 21*B*a**5))/(105*x*
*7)
```

3.41 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^9}\,dx$

Optimal. Leaf size=112

$$-\frac{a^5A}{8x^8}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^5A}{8x^8}+\frac{1}{2}b^4x^2(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(4
*x^4) - (5*a^2*b^2*(A*b + a*B))/x^2 + (b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^
4)/4 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^9}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(b^4(Ab+5aB)+\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{10a^2b^2(}{x}\right.\right.\\
&= -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{5a^2b^2(Ab+aB)}{x^2}+\frac{1}{2}b^4(Ab+5aB)x^2
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 116, normalized size = 1.04

$$5ab^3\log(x)(2aB+Ab)-\frac{a^5\left(3A+4Bx^2\right)+10a^4bx^2\left(2A+3Bx^2\right)+60a^3b^2x^4\left(A+2Bx^2\right)+120a^2Ab^3x^6-60ab^4Bx^{10}-6b^5x^{10}\left(2A+Bx^2\right)}{24x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] -1/24*(120*a^2*A*b^3*x^6 - 60*a*b^4*B*x^10 - 6*b^5*x^10*(2*A + B*x^2) + 60*
a^3*b^2*x^4*(A + 2*B*x^2) + 10*a^4*b*x^2*(2*A + 3*B*x^2) + a^5*(3*A + 4*B*x
^2))/x^8 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^9, x]
```

fricas [A] time = 0.43, size = 123, normalized size = 1.10

$$\frac{6\,Bb^5x^{12}+12\left(5\,Bab^4+Ab^5\right)x^{10}+120\left(2\,Ba^2b^3+Aab^4\right)x^8\log(x)-120\left(Ba^3b^2+Aa^2b^3\right)x^6-3\,Aa^5-30\left(Ba^4b+2\,Aa^3b^2\right)x^4-4\left(Ba^5+5\,Aa^4b\right)x^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(6*B*b^5*x^12 + 12*(5*B*a*b^4 + A*b^5)*x^10 + 120*(2*B*a^2*b^3 + A*a*b
^4)*x^8*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 3*A*a^5 - 30*(B*a^4*b +
2*A*a^3*b^2)*x^4 - 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

giac [A] time = 0.37, size = 150, normalized size = 1.34

$$\frac{1}{4} \, Bb^{5}x^{4} + \frac{5}{2} \, Bab^{4}x^{2} + \frac{1}{2} \, Ab^{5}x^{2} + \frac{5}{2} \, {\left(2 \, Ba^{2}b^{3} + Aab^{4}\right)} \log\left(x^{2}\right) - \frac{250 \, Ba^{2}b^{3}x^{8} + 125 \, Aab^{4}x^{8} + 120 \, Ba^{3}b^{2}x^{6} + 120 \, Aa^{2}b^{3}x^{6} + 30 \, Ba^{4}bx^{4} + 60 \, Aa^{3}b^{2}x^{4} + 4 \, Ba^{5}x^{2} + 20 \, Aa^{4}bx^{2} + 3 \, Aa^{5}}{24 \, x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^
4)*log(x^2) - 1/24*(250*B*a^2*b^3*x^8 + 125*A*a*b^4*x^8 + 120*B*a^3*b^2*x^6
 + 120*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 4*B*a^5*x^2 + 20
*A*a^4*b*x^2 + 3*A*a^5)/x^8
```

maple [A] time = 0.01, size = 124, normalized size = 1.11

$$\frac{B\,b^{5}x^{4}}{4}+\frac{A\,b^{5}x^{2}}{2}+\frac{5Ba\,b^{4}x^{2}}{2}+5Aa\,b^{4}\ln(x)+10B\,a^{2}b^{3}\ln(x)-\frac{5A\,a^{2}b^{3}}{x^{2}}-\frac{5B\,a^{3}b^{2}}{x^{2}}-\frac{5A\,a^{3}b^{2}}{2x^{4}}-\frac{5B\,a^{4}b}{4x^{4}}-\frac{5A\,a^{4}b}{6x^{6}}-\frac{B\,a^{5}}{6x^{6}}-\frac{A\,a^{5}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^9,x)
```

```
[Out] 1/4*b^5*B*x^4+1/2*A*x^2*b^5+5/2*B*x^2*a*b^4-5/2*a^3*b^2/x^4*A-5/4*a^4*b/x^4
*B-5/6*a^4/x^6*A*b-1/6*a^5/x^6*B-1/8*a^5*A/x^8-5*a^2*b^3/x^2*A-5*a^3*b^2/x^
2*B+5*A*ln(x)*a*b^4+10*B*ln(x)*a^2*b^3
```

maxima [A] time = 0.96, size = 123, normalized size = 1.10

$$\frac{1}{4} \, Bb^{5}x^{4} + \frac{1}{2} \, {\left(5 \, Bab^{4} + Ab^{5}\right)} x^{2} + \frac{5}{2} \, {\left(2 \, Ba^{2}b^{3} + Aab^{4}\right)} \log\left(x^{2}\right) - \frac{120 \, {\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)} x^{6} + 3 \, Aa^{5} + 30 \, {\left(Ba^{4}b + 2 \, Aa^{3}b^{2}\right)} x^{4} + 4 \, {\left(Ba^{5} + 5 \, Aa^{4}b\right)} x^{2}}{24 \, x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^2) - 1/24*(120*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3*A*a^5 + 30*(B*a^4*b + 2
*A*a^3*b^2)*x^4 + 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

mupad [B] time = 0.06, size = 122, normalized size = 1.09

$$\ln\left(x\right)\,\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)-\frac{\frac{A\,a^5}{8}+x^4\,\left(\frac{5\,B\,a^4\,b}{4}+\frac{5\,A\,a^3\,b^2}{2}\right)+x^2\,\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^6\,\left(5\,B\,a^3\,b^2+5\,A\,a^2\,b^3\right)}{x^8}+x^2\,\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+\frac{B\,b^5\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^9,x)
```

```
[Out] log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/8 + x^4*((5*A*a^3*b^2)/2 + (5*
B*a^4*b)/4) + x^2*((B*a^5)/6 + (5*A*a^4*b)/6) + x^6*(5*A*a^2*b^3 + 5*B*a^3*
b^2))/x^8 + x^2*((A*b^5)/2 + (5*B*a*b^4)/2) + (B*b^5*x^4)/4
```

sympy [A] time = 2.73, size = 129, normalized size = 1.15

$$\frac{Bb^5x^4}{4} + 5ab^3\left(Ab + 2Ba\right)\log\left(x\right) + x^2\left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right) + \frac{-3Aa^5 + x^6\left(-120Aa^2b^3 - 120Ba^3b^2\right) + x^4\left(-60Aa^3b^2 - 30Ba^4b\right) + x^2\left(-20Aa^4b - 4Ba^5\right)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**9,x)
```

```
[Out] B*b**5*x**4/4 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**2*(A*b**5/2 + 5*B*a*b**4
/2) + (-3*A*a**5 + x**6*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**4*(-60*A*
a**3*b**2 - 30*B*a**4*b) + x**2*(-20*A*a**4*b - 4*B*a**5))/(24*x**8)
```

3.42 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{9x^9}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{10a^2b^2(aB+Ab)}{3x^3}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{3}b^5Bx^3$$

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^5A}{9x^9}-\frac{5ab^3(2aB+Ab)}{x}+b^4x(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b + a*B))/x^5
- (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*a*B))/x + b^4*(A*b +
 5*a*B)*x + (b^5*B*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{10}}\,dx &= \int \Bigg(b^4(Ab+5aB)+\frac{a^5A}{x^{10}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(Ab+aB)}{x^4}\\ &= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{10a^2b^2(Ab+aB)}{3x^3}-\frac{5ab^3(Ab+2aB)}{x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 115, normalized size = 1.06

$$-\frac{a^5\left(7A+9Bx^2\right)+9a^4bx^2\left(5A+7Bx^2\right)+42a^3b^2x^4\left(3A+5Bx^2\right)+210a^2b^3x^6\left(A+3Bx^2\right)+315ab^4x^8\left(A-Bx^2\right)-21b^5x^{10}\left(3A+Bx^2\right)}{63x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] -1/63*(315*a*b^4*x^8*(A - B*x^2) - 21*b^5*x^10*(3*A + B*x^2) + 210*a^2*b^3*
x^6*(A + 3*B*x^2) + 42*a^3*b^2*x^4*(3*A + 5*B*x^2) + 9*a^4*b*x^2*(5*A + 7*B
*x^2) + a^5*(7*A + 9*B*x^2))/x^9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^10, x]
```

fricas [A] time = 0.73, size = 121, normalized size = 1.12

$$\frac{21\,Bb^5x^{12} + 63\left(5\,Bab^4 + Ab^5\right)x^{10} - 315\left(2\,Ba^2b^3 + Aab^4\right)x^8 - 210\left(Ba^3b^2 + Aa^2b^3\right)x^6 - 7\,Aa^5 - 63\left(Ba^4b + 2\,Aa^3b^2\right)x^4 - 9\left(Ba^5 + 5\,Aa^4b\right)x^2}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*B*b^5*x^12 + 63*(5*B*a*b^4 + A*b^5)*x^10 - 315*(2*B*a^2*b^3 + A*a*
b^4)*x^8 - 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 7*A*a^5 - 63*(B*a^4*b + 2*A*a^
3*b^2)*x^4 - 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

giac [A] time = 0.37, size = 123, normalized size = 1.14

$$\frac{1}{3}\,Bb^5x^3 + 5\,Bab^4x + Ab^5x - \frac{630\,Ba^2b^3x^8 + 315\,Aab^4x^8 + 210\,Ba^3b^2x^6 + 210\,Aa^2b^3x^6 + 63\,Ba^4bx^4 + 126\,Aa^3b^2x^4 + 9\,Ba^5x^2 + 45\,Aa^4bx^2 + 7\,Aa^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + 5*B*a*b^4*x + A*b^5*x - 1/63*(630*B*a^2*b^3*x^8 + 315*A*a*b
^4*x^8 + 210*B*a^3*b^2*x^6 + 210*A*a^2*b^3*x^6 + 63*B*a^4*b*x^4 + 126*A*a^3
*b^2*x^4 + 9*B*a^5*x^2 + 45*A*a^4*b*x^2 + 7*A*a^5)/x^9
```

maple [A] time = 0.01, size = 102, normalized size = 0.94

$$\frac{B\,b^5x^3}{3} + A\,b^5x + 5Ba\,b^4x - \frac{5\,(Ab+2Ba)\,a\,b^3}{x} - \frac{10\,(Ab+Ba)\,a^2b^2}{3x^3} - \frac{(2Ab+Ba)\,a^3b}{x^5} - \frac{A\,a^5}{9x^9} - \frac{(5Ab+Ba)\,a^4}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^10,x)
```

```
[Out] 1/3*b^5*B*x^3+b^5*A*x+5*a*b^4*B*x-a^3*b*(2*A*b+B*a)/x^5-5*a*b^3*(A*b+2*B*a)
/x-1/7*a^4*(5*A*b+B*a)/x^7-1/9*a^5*A/x^9-10/3*a^2*b^2*(A*b+B*a)/x^3
```

maxima [A] time = 1.08, size = 119, normalized size = 1.10

$$\frac{1}{3}\,Bb^{5}x^{3}+\left(5\,Bab^{4}+Ab^{5}\right)x-\frac{315\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+210\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+7\,Aa^{5}+63\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+9\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{63\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*x - 1/63*(315*(2*B*a^2*b^3 + A*a*b^4)*x
^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 7*A*a^5 + 63*(B*a^4*b + 2*A*a^3*b^2)
*x^4 + 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

mupad [B] time = 0.07, size = 119, normalized size = 1.10

$$x\,\left(A\,b^5+5\,B\,a\,b^4\right)-\frac{\frac{A\,a^5}{9}+x^4\,\left(B\,a^4\,b+2\,A\,a^3\,b^2\right)+x^8\,\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)+x^2\,\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^6\,\left(\frac{10\,B\,a^3\,b^2}{3}+\frac{10\,A\,a^2\,b^3}{3}\right)}{x^9}+\frac{B\,b^5\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^10,x)
```

```
[Out] x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/9 + x^4*(2*A*a^3*b^2 + B*a^4*b) + x^8*(10*
B*a^2*b^3 + 5*A*a*b^4) + x^2*((B*a^5)/7 + (5*A*a^4*b)/7) + x^6*((10*A*a^2*b
^3)/3 + (10*B*a^3*b^2)/3))/x^9 + (B*b^5*x^3)/3
```

sympy [A] time = 3.09, size = 129, normalized size = 1.19

$$\frac{Bb^{5}x^{3}}{3}+x\left(Ab^{5}+5Bab^{4}\right)+\frac{-7Aa^{5}+x^{8}\left(-315Aab^{4}-630Ba^{2}b^{3}\right)+x^{6}\left(-210Aa^{2}b^{3}-210Ba^{3}b^{2}\right)+x^{4}\left(-126Aa^{3}b^{2}-63Ba^{4}b\right)+x^{2}\left(-45Aa^{4}b-9Ba^{5}\right)}{63x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**10,x)
```

```
[Out] B*b**5*x**3/3 + x*(A*b**5 + 5*B*a*b**4) + (-7*A*a**5 + x**8*(-315*A*a*b**4
- 630*B*a**2*b**3) + x**6*(-210*A*a**2*b**3 - 210*B*a**3*b**2) + x**4*(-126
*A*a**3*b**2 - 63*B*a**4*b) + x**2*(-45*A*a**4*b - 9*B*a**5))/(63*x**9)
```

3.43 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{11}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{5a^2b^2(aB+Ab)}{2x^4}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{2}b^5Bx^2$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^5A}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4\log(x)(5aB+Ab)+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(8*x^8) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/(2*x^2)
 + (b^5*B*x^2)/2 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\Bigg(\int \Bigg(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^3}\\ &= -\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 1.03

$$b^4\log(x)(5aB+Ab)-\frac{3a^5\left(4A+5Bx^2\right)+25a^4bx^2\left(3A+4Bx^2\right)+100a^3b^2x^4\left(2A+3Bx^2\right)+300a^2b^3x^6\left(A+2Bx^2\right)+300aAb^4x^8-60b^5Bx^{12}}{120x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] -1/120*(300*a*A*b^4*x^8 - 60*b^5*B*x^12 + 300*a^2*b^3*x^6*(A + 2*B*x^2) + 1
00*a^3*b^2*x^4*(2*A + 3*B*x^2) + 25*a^4*b*x^2*(3*A + 4*B*x^2) + 3*a^5*(4*A
+ 5*B*x^2))/x^10 + b^4*(A*b + 5*a*B)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^11, x]
```

fricas [A] time = 0.43, size = 123, normalized size = 1.09

$$\frac{60\,Bb^5x^{12}+120\left(5\,Bab^4+Ab^5\right)x^{10}\log(x)-300\left(2\,Ba^2b^3+Aab^4\right)x^8-300\left(Ba^3b^2+Aa^2b^3\right)x^6-12\,Aa^5-100\left(Ba^4b+2\,Aa^3b^2\right)x^4-15\left(Ba^5+5\,Aa^4b\right)x^2}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(60*B*b^5*x^12 + 120*(5*B*a*b^4 + A*b^5)*x^10*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^8 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 12*A*a^5 - 100*(B*a^4
*b + 2*A*a^3*b^2)*x^4 - 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

giac [A] time = 0.43, size = 147, normalized size = 1.30

$$\frac{1}{2} Bb^5x^2 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)\log\left(x^2\right) - \frac{685\,Bab^4x^{10} + 137\,Ab^5x^{10} + 600\,Ba^2b^3x^8 + 300\,Aab^4x^8 + 300\,Ba^3b^2x^6 + 300\,Aa^2b^3x^6 + 100\,Ba^4bx^4 + 200\,Aa^3b^2x^4 + 15\,Ba^5x^2 + 75\,Aa^4bx^2 + 12\,Aa^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(685*B*a*b^4*x^10
+ 137*A*b^5*x^10 + 600*B*a^2*b^3*x^8 + 300*A*a*b^4*x^8 + 300*B*a^3*b^2*x^6
+ 300*A*a^2*b^3*x^6 + 100*B*a^4*b*x^4 + 200*A*a^3*b^2*x^4 + 15*B*a^5*x^2 +
75*A*a^4*b*x^2 + 12*A*a^5)/x^10
```

maple [A] time = 0.01, size = 123, normalized size = 1.09

$$\frac{B\,b^5x^2}{2} + A\,b^5\ln(x) + 5Ba\,b^4\ln(x) - \frac{5Aa\,b^4}{2x^2} - \frac{5B\,a^2b^3}{x^2} - \frac{5A\,a^2b^3}{2x^4} - \frac{5B\,a^3b^2}{2x^4} - \frac{5A\,a^3b^2}{3x^6} - \frac{5B\,a^4b}{6x^6} - \frac{5A\,a^4b}{8x^8} - \frac{B\,a^5}{8x^8} - \frac{A\,a^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^11,x)
```

```
[Out] 1/2*b^5*B*x^2-5/2*a^2*b^3/x^4*A-5/2*a^3*b^2/x^4*B-1/10*a^5*A/x^10-5/3*a^3*b
^2/x^6*A-5/6*a^4*b/x^6*B-5/8*a^4/x^8*A*b-1/8*a^5/x^8*B-5/2*a*b^4/x^2*A-5*a^
2*b^3/x^2*B+A*ln(x)*b^5+5*B*ln(x)*a*b^4
```

maxima [A] time = 1.02, size = 123, normalized size = 1.09

$$\frac{1}{2} Bb^5x^2 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)\log\left(x^2\right) - \frac{300\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 300\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 12\,Aa^5 + 100\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 15\left(Ba^5 + 5\,Aa^4b\right)x^2}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 12*A*a^5 + 100*(B*a^4*b
+ 2*A*a^3*b^2)*x^4 + 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

mupad [B] time = 0.11, size = 121, normalized size = 1.07

$$\ln(x)\,\left(A\,b^5 + 5\,B\,a\,b^4\right) - \frac{\frac{A\,a^5}{10} + x^8\,\left(5\,B\,a^2\,b^3 + \frac{5\,A\,a\,b^4}{2}\right) + x^4\,\left(\frac{5\,B\,a^4\,b}{6} + \frac{5\,A\,a^3\,b^2}{3}\right) + x^2\,\left(\frac{B\,a^5}{8} + \frac{5\,A\,b\,a^4}{8}\right) + x^6\,\left(\frac{5\,B\,a^3\,b^2}{2} + \frac{5\,A\,a^2\,b^3}{2}\right)}{x^{10}} + \frac{B\,b^5\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^11,x)
```

```
[Out] log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/10 + x^8*(5*B*a^2*b^3 + (5*A*a*b^4)/2
) + x^4*((5*A*a^3*b^2)/3 + (5*B*a^4*b)/6) + x^2*((B*a^5)/8 + (5*A*a^4*b)/8)
 + x^6*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^10 + (B*b^5*x^2)/2
```

sympy [A] time = 5.00, size = 129, normalized size = 1.14

$$\frac{Bb^5x^2}{2}+b^4\left(Ab+5Ba\right)\log\left(x\right)+\frac{-12Aa^5+x^8\left(-300Aab^4-600Ba^2b^3\right)+x^6\left(-300Aa^2b^3-300Ba^3b^2\right)+x^4\left(-200Aa^3b^2-100Ba^4b\right)+x^2\left(-75Aa^4b-15Ba^5\right)}{120x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**11,x)
```

```
[Out] B*b**5*x**2/2 + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**8*(-300*A*a*b*
*4 - 600*B*a**2*b**3) + x**6*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**4*(-
200*A*a**3*b**2 - 100*B*a**4*b) + x**2*(-75*A*a**4*b - 15*B*a**5))/(120*x**
10)
```

3.44 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx$

Optimal. Leaf size=108

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^5Bx$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5A}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{3x^3}-\frac{b^4(5aB+Ab)}{x}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(3*x^3) - (
b^4*(A*b + 5*a*B))/x + b^5*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeBQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{12}}\,dx=\int \left(b^5B+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^{10}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^6}+\right.$$

$$=-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{3x^3}$$

Mathematica [A] time = 0.04, size = 122, normalized size = 1.13

$$-\frac{a^5\left(9A+11Bx^2\right)}{99x^{11}}-\frac{5a^4b\left(7A+9Bx^2\right)}{63x^9}-\frac{2a^3b^2\left(5A+7Bx^2\right)}{7x^7}-\frac{2a^2b^3\left(3A+5Bx^2\right)}{3x^5}-\frac{5ab^4\left(A+3Bx^2\right)}{3x^3}-\frac{Ab^5}{x}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] -((A*b^5)/x) + b^5*B*x - (5*a*b^4*(A + 3*B*x^2))/(3*x^3) - (2*a^2*b^3*(3*A
+ 5*B*x^2))/(3*x^5) - (2*a^3*b^2*(5*A + 7*B*x^2))/(7*x^7) - (5*a^4*b*(7*A +
 9*B*x^2))/(63*x^9) - (a^5*(9*A + 11*B*x^2))/(99*x^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^12, x]
```

fricas [A] time = 0.46, size = 121, normalized size = 1.12

$$\frac{693\,Bb^5x^{12}-693\left(5\,Bab^4+Ab^5\right)x^{10}-1155\left(2\,Ba^2b^3+Aab^4\right)x^8-1386\left(Ba^3b^2+Aa^2b^3\right)x^6-63\,Aa^5-495\left(Ba^4b+2\,Aa^3b^2\right)x^4-77\left(Ba^5+5\,Aa^4b\right)x^2}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/693*(693*B*b^5*x^12 - 693*(5*B*a*b^4 + A*b^5)*x^10 - 1155*(2*B*a^2*b^3 +
A*a*b^4)*x^8 - 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 63*A*a^5 - 495*(B*a^4*b +
 2*A*a^3*b^2)*x^4 - 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

giac [A] time = 0.36, size = 125, normalized size = 1.16

$$Bb^5x-\frac{3465\,Bab^4x^{10}+693\,Ab^5x^{10}+2310\,Ba^2b^3x^8+1155\,Aab^4x^8+1386\,Ba^3b^2x^6+1386\,Aa^2b^3x^6+495\,Ba^4bx^4+990\,Aa^3b^2x^4+77\,Ba^5x^2+385\,Aa^4bx^2+63\,Aa^5}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/693*(3465*B*a*b^4*x^10 + 693*A*b^5*x^10 + 2310*B*a^2*b^3*x^8 +
1155*A*a*b^4*x^8 + 1386*B*a^3*b^2*x^6 + 1386*A*a^2*b^3*x^6 + 495*B*a^4*b*x^
4 + 990*A*a^3*b^2*x^4 + 77*B*a^5*x^2 + 385*A*a^4*b*x^2 + 63*A*a^5)/x^11
```

maple [A] time = 0.01, size = 101, normalized size = 0.94

$$B\,b^5x-\frac{(Ab+5Ba)\,b^4}{x}-\frac{5\,(Ab+2Ba)\,a\,b^3}{3x^3}-\frac{2\,(Ab+Ba)\,a^2b^2}{x^5}-\frac{5\,(2Ab+Ba)\,a^3b}{7x^7}-\frac{A\,a^5}{11x^{11}}-\frac{(5Ab+Ba)\,a^4}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^12,x)
```

```
[Out] -1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^7-2*a^2*b^
2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*a)/x+b^5*B*x
```

maxima [A] time = 1.02, size = 119, normalized size = 1.10

$$Bb^5x - \frac{693\left(5\,Bab^4 + Ab^5\right)x^{10} + 1155\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 1386\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 63\,Aa^5 + 495\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 77\left(Ba^5 + 5\,Aa^4b\right)x^2}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/693*(693*(5*B*a*b^4 + A*b^5)*x^10 + 1155*(2*B*a^2*b^3 + A*a*b^4
)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 63*A*a^5 + 495*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

mupad [B] time = 0.07, size = 119, normalized size = 1.10

$$B\,b^5\,x - \frac{\frac{A\,a^5}{11} + x^8\,\left(\frac{10\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{3}\right) + x^4\,\left(\frac{5\,B\,a^4\,b}{7} + \frac{10\,A\,a^3\,b^2}{7}\right) + x^2\,\left(\frac{B\,a^5}{9} + \frac{5\,A\,b\,a^4}{9}\right) + x^{10}\,\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^6\,\left(2\,B\,a^3\,b^2 + 2\,A\,a^2\,b^3\right)}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^12,x)
```

```
[Out] B*b^5*x - ((A*a^5)/11 + x^8*((10*B*a^2*b^3)/3 + (5*A*a*b^4)/3) + x^4*((10*A
*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^2*((B*a^5)/9 + (5*A*a^4*b)/9) + x^10*(A*b^
5 + 5*B*a*b^4) + x^6*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^11
```

sympy [A] time = 6.68, size = 131, normalized size = 1.21

$$Bb^5x + \frac{-63Aa^5 + x^{10}\left(-693Ab^5 - 3465Bab^4\right) + x^8\left(-1155Aab^4 - 2310Ba^2b^3\right) + x^6\left(-1386Aa^2b^3 - 1386Ba^3b^2\right) + x^4\left(-990Aa^3b^2 - 495Ba^4b\right) + x^2\left(-385Aa^4b - 77Ba^5\right)}{693x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**12,x)
```

```
[Out] B*b**5*x + (-63*A*a**5 + x**10*(-693*A*b**5 - 3465*B*a*b**4) + x**8*(-1155*
A*a*b**4 - 2310*B*a**2*b**3) + x**6*(-1386*A*a**2*b**3 - 1386*B*a**3*b**2)
+ x**4*(-990*A*a**3*b**2 - 495*B*a**4*b) + x**2*(-385*A*a**4*b - 77*B*a**5)
)/(693*x**11)
```

3.45 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}} dx$

Optimal. Leaf size=91

$$-\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{A(a+bx^2)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

Rubi **[A]** time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 43}

$$-\frac{5a^2b^3B}{2x^4}-\frac{5a^3b^2B}{3x^6}-\frac{5a^4bB}{8x^8}-\frac{a^5B}{10x^{10}}-\frac{A(a+bx^2)^6}{12ax^{12}}-\frac{5ab^4B}{2x^2}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -(a^5*B)/(10*x^10) - (5*a^4*b*B)/(8*x^8) - (5*a^3*b^2*B)/(3*x^6) - (5*a^2*b
^3*B)/(2*x^4) - (5*a*b^4*B)/(2*x^2) - (A*(a + b*x^2)^6)/(12*a*x^12) + b^5*B
*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{13}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^6}{12ax^{12}}+\frac{1}{2}B\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^6}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^6}{12ax^{12}}+\frac{1}{2}B\operatorname{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx,\right.\\
&= -\frac{a^5B}{10x^{10}}-\frac{5a^4bB}{8x^8}-\frac{5a^3b^2B}{3x^6}-\frac{5a^2b^3B}{2x^4}-\frac{5ab^4B}{2x^2}-\frac{A\left(a+bx^2\right)^6}{12ax^{12}}+b^5B\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 118, normalized size = 1.30

$$b^5B\log(x)-\frac{2a^5\left(5A+6Bx^2\right)+15a^4bx^2\left(4A+5Bx^2\right)+50a^3b^2x^4\left(3A+4Bx^2\right)+100a^2b^3x^6\left(2A+3Bx^2\right)+150ab^4x^8\left(A+2Bx^2\right)+60Ab^5x^{10}}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] -1/120*(60*A*b^5*x^10 + 150*a*b^4*x^8*(A + 2*B*x^2) + 100*a^2*b^3*x^6*(2*A
+ 3*B*x^2) + 50*a^3*b^2*x^4*(3*A + 4*B*x^2) + 15*a^4*b*x^2*(4*A + 5*B*x^2)
+ 2*a^5*(5*A + 6*B*x^2))/x^12 + b^5*B*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^13, x]
```

fricas [A] time = 0.44, size = 123, normalized size = 1.35

$$\frac{120\,Bb^5x^{12}\log(x)-60\left(5\,Bab^4+Ab^5\right)x^{10}-150\left(2\,Ba^2b^3+Aab^4\right)x^8-200\left(Ba^3b^2+Aa^2b^3\right)x^6-10\,Aa^5-75\left(Ba^4b+2\,Aa^3b^2\right)x^4-12\left(Ba^5+5\,Aa^4b\right)x^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*B*b^5*x^12*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^10 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^8 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 10*A*a^5 - 75*(B*a^4*
b + 2*A*a^3*b^2)*x^4 - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

giac [A]   time = 0.40, size = 138, normalized size = 1.52

$$\frac{1}{2}\,Bb^5\log\left(x^2\right) - \frac{147\,Bb^5x^{12} + 300\,Bab^4x^{10} + 60\,Ab^5x^{10} + 300\,Ba^2b^3x^8 + 150\,Aab^4x^8 + 200\,Ba^3b^2x^6 + 200\,Aa^2b^3x^6 + 75\,Ba^4bx^4 + 150\,Aa^3b^2x^4 + 12\,Ba^5x^2 + 60\,Aa^4bx^2 + 10\,Aa^5}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(147*B*b^5*x^12 + 300*B*a*b^4*x^10 + 60*A*b^5*x^
10 + 300*B*a^2*b^3*x^8 + 150*A*a*b^4*x^8 + 200*B*a^3*b^2*x^6 + 200*A*a^2*b^
3*x^6 + 75*B*a^4*b*x^4 + 150*A*a^3*b^2*x^4 + 12*B*a^5*x^2 + 60*A*a^4*b*x^2
+ 10*A*a^5)/x^12
```

maple [A]   time = 0.01, size = 124, normalized size = 1.36

$$B\,b^5\ln(x) - \frac{A\,b^5}{2x^2} - \frac{5Ba\,b^4}{2x^2} - \frac{5Aa\,b^4}{4x^4} - \frac{5B\,a^2b^3}{2x^4} - \frac{5A\,a^2b^3}{3x^6} - \frac{5B\,a^3b^2}{3x^6} - \frac{5A\,a^3b^2}{4x^8} - \frac{5B\,a^4b}{8x^8} - \frac{A\,a^4b}{2x^{10}} - \frac{B\,a^5}{10x^{10}} - \frac{A\,a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^13,x)
```

```
[Out] -5/4*a*b^4/x^4*A-5/2*a^2*b^3*B/x^4-1/2*a^4/x^10*A*b-1/10*a^5*B/x^10-5/3*a^2
*b^3/x^6*A-5/3*a^3*b^2*B/x^6-5/4*a^3*b^2/x^8*A-5/8*a^4*b*B/x^8-1/2*b^5/x^2*
A-5/2*a*b^4*B/x^2-1/12*A*a^5/x^12+b^5*B*ln(x)
```

maxima [A]   time = 1.09, size = 123, normalized size = 1.35

$$\frac{1}{2}\,Bb^5\log\left(x^2\right) - \frac{60\left(5\,Bab^4 + Ab^5\right)x^{10} + 150\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 200\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 10\,Aa^5 + 75\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 12\left(Ba^5 + 5\,Aa^4b\right)x^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*log(x^2) - 1/120*(60*(5*B*a*b^4 + A*b^5)*x^10 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 10*A*a^5 + 75*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

mupad [B]   time = 0.12, size = 121, normalized size = 1.33

$$B\,b^5\,\ln(x) - \frac{\frac{A\,a^5}{12} + x^8\left(\frac{5\,B\,a^2\,b^3}{2} + \frac{5\,A\,a\,b^4}{4}\right) + x^4\left(\frac{5\,B\,a^4\,b}{8} + \frac{5\,A\,a^3\,b^2}{4}\right) + x^2\left(\frac{B\,a^5}{10} + \frac{A\,b\,a^4}{2}\right) + x^{10}\left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) + x^6\left(\frac{5\,B\,a^3\,b^2}{3} + \frac{5\,A\,a^2\,b^3}{3}\right)}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^13,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/12 + x^8*((5*B*a^2*b^3)/2 + (5*A*a*b^4)/4) + x^4*((
5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^2*((B*a^5)/10 + (A*a^4*b)/2) + x^10*((A
*b^5)/2 + (5*B*a*b^4)/2) + x^6*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^12
```

sympy [**A**] time = 8.68, size = 133, normalized size = 1.46

$$Bb^5\log(x) + \frac{-10Aa^5 + x^{10}\left(-60Ab^5 - 300Bab^4\right) + x^8\left(-150Aab^4 - 300Ba^2b^3\right) + x^6\left(-200Aa^2b^3 - 200Ba^3b^2\right) + x^4\left(-150Aa^3b^2 - 75Ba^4b\right) + x^2\left(-60Aa^4b - 12Ba^5\right)}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**13,x)
```

```
[Out] B*b**5*log(x) + (-10*A*a**5 + x**10*(-60*A*b**5 - 300*B*a*b**4) + x**8*(-15
0*A*a*b**4 - 300*B*a**2*b**3) + x**6*(-200*A*a**2*b**3 - 200*B*a**3*b**2) +
 x**4*(-150*A*a**3*b**2 - 75*B*a**4*b) + x**2*(-60*A*a**4*b - 12*B*a**5))/(
120*x**12)
```

3.46 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{x}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5A}{13x^{13}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(A*b + 2*a*B))/x^5 -
(b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx = \int \left(\frac{a^5A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^{10}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(Ab+2}{x^6}\right.$$

$$=-\frac{a^5A}{13x^{13}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{ab^3(Ab+2aB}{x^5}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 1.05

$$-\frac{63a^5\left(11A+13Bx^2\right)+455a^4bx^2\left(9A+11Bx^2\right)+1430a^3b^2x^4\left(7A+9Bx^2\right)+2574a^2b^3x^6\left(5A+7Bx^2\right)+3003ab^4x^8\left(3A+5Bx^2\right)+3003b^5x^{10}\left(A+3Bx^2\right)}{9009x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] -1/9009*(3003*b^5*x^10*(A + 3*B*x^2) + 3003*a*b^4*x^8*(3*A + 5*B*x^2) + 257
4*a^2*b^3*x^6*(5*A + 7*B*x^2) + 1430*a^3*b^2*x^4*(7*A + 9*B*x^2) + 455*a^4*
b*x^2*(9*A + 11*B*x^2) + 63*a^5*(11*A + 13*B*x^2))/x^13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^14, x]
```

fricas [A] time = 0.41, size = 121, normalized size = 1.07

$$-\frac{9009\,Bb^5x^{12}+3003\left(5\,Bab^4+Ab^5\right)x^{10}+9009\left(2\,Ba^2b^3+Aab^4\right)x^8+12870\left(Ba^3b^2+Aa^2b^3\right)x^6+693\,Aa^5+5005\left(Ba^4b+2\,Aa^3b^2\right)x^4+819\left(Ba^5+5\,Aa^4b\right)x^2}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

giac [A] time = 0.27, size = 127, normalized size = 1.12

$$-\frac{9009\,Bb^5x^{12}+15015\,Bab^4x^{10}+3003\,Ab^5x^{10}+18018\,Ba^2b^3x^8+9009\,Aab^4x^8+12870\,Ba^3b^2x^6+12870\,Aa^2b^3x^6+5005\,Ba^4bx^4+10010\,Aa^3b^2x^4+819\,Ba^5x^2+4095\,Aa^4bx^2+693\,Aa^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 15015*B*a*b^4*x^10 + 3003*A*b^5*x^10 + 18018*B*a
^2*b^3*x^8 + 9009*A*a*b^4*x^8 + 12870*B*a^3*b^2*x^6 + 12870*A*a^2*b^3*x^6 +
 5005*B*a^4*b*x^4 + 10010*A*a^3*b^2*x^4 + 819*B*a^5*x^2 + 4095*A*a^4*b*x^2
+ 693*A*a^5)/x^13
```

maple [A] time = 0.01, size = 104, normalized size = 0.92

$$-\frac{B\,b^5}{x}-\frac{(Ab+5Ba)\,b^4}{3x^3}-\frac{(Ab+2Ba)\,a\,b^3}{x^5}-\frac{10\,(Ab+Ba)\,a^2b^2}{7x^7}-\frac{5\,(2Ab+Ba)\,a^3b}{9x^9}-\frac{A\,a^5}{13x^{13}}-\frac{(5Ab+Ba)\,a^4}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^14,x)
```

```
[Out] -1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/x^9-10/7*a
^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b+5*B*a)/x^3-b^5*B/x
```

maxima [A] time = 1.07, size = 121, normalized size = 1.07

$$-\frac{9009\,Bb^{5}x^{12} + 3003\,\left(5\,Bab^{4} + Ab^{5}\right)x^{10} + 9009\,\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{8} + 12870\,\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{6} + 693\,Aa^{5} + 5005\,\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{4} + 819\,\left(Ba^{5} + 5\,Aa^{4}b\right)x^{2}}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b^
3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(B*
a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

mupad [B] time = 0.09, size = 120, normalized size = 1.06

$$-\frac{\frac{A\,a^{5}}{13} + x^{8}\,\left(2\,B\,a^{2}\,b^{3} + A\,a\,b^{4}\right) + x^{4}\,\left(\frac{5\,B\,a^{4}\,b}{9} + \frac{10\,A\,a^{3}\,b^{2}}{9}\right) + x^{2}\,\left(\frac{B\,a^{5}}{11} + \frac{5\,A\,b\,a^{4}}{11}\right) + x^{10}\,\left(\frac{A\,b^{5}}{3} + \frac{5\,B\,a\,b^{4}}{3}\right) + x^{6}\,\left(\frac{10\,B\,a^{3}\,b^{2}}{7} + \frac{10\,A\,a^{2}\,b^{3}}{7}\right) + B\,b^{5}\,x^{12}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^14,x)
```

```
[Out] -((A*a^5)/13 + x^8*(2*B*a^2*b^3 + A*a*b^4) + x^4*((10*A*a^3*b^2)/9 + (5*B*a
^4*b)/9) + x^2*((B*a^5)/11 + (5*A*a^4*b)/11) + x^10*((A*b^5)/3 + (5*B*a*b^4
)/3) + x^6*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7) + B*b^5*x^12)/x^13
```

sympy [A] time = 14.04, size = 134, normalized size = 1.19

$$\frac{-693Aa^{5} - 9009Bb^{5}x^{12} + x^{10}\left(-3003Ab^{5} - 15015Bab^{4}\right) + x^{8}\left(-9009Aab^{4} - 18018Ba^{2}b^{3}\right) + x^{6}\left(-12870Aa^{2}b^{3} - 12870Ba^{3}b^{2}\right) + x^{4}\left(-10010Aa^{3}b^{2} - 5005Ba^{4}b\right) + x^{2}\left(-4095Aa^{4}b - 819Ba^{5}\right)}{9009x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

```
[Out] (-693*A*a**5 - 9009*B*b**5*x**12 + x**10*(-3003*A*b**5 - 15015*B*a*b**4) +
x**8*(-9009*A*a*b**4 - 18018*B*a**2*b**3) + x**6*(-12870*A*a**2*b**3 - 1287
0*B*a**3*b**2) + x**4*(-10010*A*a**3*b**2 - 5005*B*a**4*b) + x**2*(-4095*A*
a**4*b - 819*B*a**5))/(9009*x**13)
```

3.47 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}} \, dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^2\right)^6 (Ab-7aB)}{84a^2x^{12}}-\frac{A\left(a+bx^2\right)^6}{14ax^{14}}$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 37}

$$\frac{\left(a+bx^2\right)^6 (Ab-7aB)}{84a^2x^{12}}-\frac{A\left(a+bx^2\right)^6}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -(A*(a + b*x^2)^6)/(14*a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(84*a^2*x^12
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{15}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^6}{14ax^{14}}+\frac{(-Ab+7aB)\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^2\right)}{14a}\\
&= -\frac{A\left(a+bx^2\right)^6}{14ax^{14}}+\frac{(Ab-7aB)\left(a+bx^2\right)^6}{84a^2x^{12}}
\end{aligned}
$$

Mathematica [B] time = 0.03, size = 118, normalized size = 2.46

$$
-\frac{a^5\left(6A+7Bx^2\right)+7a^4bx^2\left(5A+6Bx^2\right)+21a^3b^2x^4\left(4A+5Bx^2\right)+35a^2b^3x^6\left(3A+4Bx^2\right)+35ab^4x^8\left(2A+3Bx^2\right)+21b^5x^{10}\left(A+2Bx^2\right)}{84x^{14}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] -1/84*(21*b^5*x^10*(A + 2*B*x^2) + 35*a*b^4*x^8*(2*A + 3*B*x^2) + 35*a^2*b^
3*x^6*(3*A + 4*B*x^2) + 21*a^3*b^2*x^4*(4*A + 5*B*x^2) + 7*a^4*b*x^2*(5*A +
 6*B*x^2) + a^5*(6*A + 7*B*x^2))/x^14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{15}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^15, x]
```

fricas [B] time = 0.43, size = 121, normalized size = 2.52

$$
-\frac{42\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+70\left(2\,Ba^2b^3+Aab^4\right)x^8+105\left(Ba^3b^2+Aa^2b^3\right)x^6+6\,Aa^5+42\left(Ba^4b+2\,Aa^3b^2\right)x^4+7\left(Ba^5+5\,Aa^4b\right)x^2}{84\,x^{14}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a*
b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

giac [B] time = 0.40, size = 127, normalized size = 2.65

$$-\frac{42\,Bb^5x^{12}+105\,Bab^4x^{10}+21\,Ab^5x^{10}+140\,Ba^2b^3x^8+70\,Aab^4x^8+105\,Ba^3b^2x^6+105\,Aa^2b^3x^6+42\,Ba^4bx^4+84\,Aa^3b^2x^4+7\,Ba^5x^2+35\,Aa^4bx^2+6\,Aa^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 105*B*a*b^4*x^10 + 21*A*b^5*x^10 + 140*B*a^2*b^3*x^8
 + 70*A*a*b^4*x^8 + 105*B*a^3*b^2*x^6 + 105*A*a^2*b^3*x^6 + 42*B*a^4*b*x^4
+ 84*A*a^3*b^2*x^4 + 7*B*a^5*x^2 + 35*A*a^4*b*x^2 + 6*A*a^5)/x^14
```

maple [B] time = 0.01, size = 104, normalized size = 2.17

$$-\frac{B\,b^5}{2x^2}-\frac{(Ab+5Ba)\,b^4}{4x^4}-\frac{5\,(Ab+2Ba)\,a\,b^3}{6x^6}-\frac{5\,(Ab+Ba)\,a^2b^2}{4x^8}-\frac{(2Ab+Ba)\,a^3b}{2x^{10}}-\frac{A\,a^5}{14x^{14}}-\frac{(5Ab+Ba)\,a^4}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^15,x)
```

```
[Out] -1/2*a^3*b*(2*A*b+B*a)/x^10-5/6*a*b^3*(A*b+2*B*a)/x^6-5/4*a^2*b^2*(A*b+B*a)
/x^8-1/2*B*b^5/x^2-1/14*A*a^5/x^14-1/4*b^4*(A*b+5*B*a)/x^4-1/12*a^4*(5*A*b+
B*a)/x^12
```

maxima [B] time = 1.06, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{12}+21\left(5\,Bab^4+Ab^5\right)x^{10}+70\left(2\,Ba^2b^3+Aab^4\right)x^8+105\left(Ba^3b^2+Aa^2b^3\right)x^6+6\,Aa^5+42\left(Ba^4b+2\,Aa^3b^2\right)x^4+7\left(Ba^5+5\,Aa^4b\right)x^2}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="maxima")
```

```
[Out] -1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a*
b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*a^
3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

mupad [B] time = 0.06, size = 121, normalized size = 2.52

$$-\frac{\frac{A\,a^5}{14}+x^4\,\left(\frac{B\,a^4\,b}{2}+A\,a^3\,b^2\right)+x^8\,\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^2\,\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{10}\,\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{4}+\frac{5\,A\,a^2\,b^3}{4}\right)+\frac{B\,b^5\,x^{12}}{2}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^15,x)
```

```
[Out] -((A*a^5)/14 + x^4*(A*a^3*b^2 + (B*a^4*b)/2) + x^8*((5*B*a^2*b^3)/3 + (5*A*
a*b^4)/6) + x^2*((B*a^5)/12 + (5*A*a^4*b)/12) + x^10*((A*b^5)/4 + (5*B*a*b^
4)/4) + x^6*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4) + (B*b^5*x^12)/2)/x^14
```

sympy [B] time = 15.80, size = 134, normalized size = 2.79

$$\frac{-6Aa^5-42Bb^5x^{12}+x^{10}\left(-21Ab^5-105Bab^4\right)+x^8\left(-70Aab^4-140Ba^2b^3\right)+x^6\left(-105Aa^2b^3-105Ba^3b^2\right)+x^4\left(-84Aa^3b^2-42Ba^4b\right)+x^2\left(-35Aa^4b-7Ba^5\right)}{84x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**15,x)
```

```
[Out] (-6*A*a**5 - 42*B*b**5*x**12 + x**10*(-21*A*b**5 - 105*B*a*b**4) + x**8*(-7
0*A*a*b**4 - 140*B*a**2*b**3) + x**6*(-105*A*a**2*b**3 - 105*B*a**3*b**2) +
 x**4*(-84*A*a**3*b**2 - 42*B*a**4*b) + x**2*(-35*A*a**4*b - 7*B*a**5))/(84
*x**14)
```

3.48 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{16}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{3x^3}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5B}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b^3*(A*b + 2*a*B))/(7
*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(3*x^3)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{16}}\,dx &= \int \left(\frac{a^5A}{x^{16}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^{10}}+\frac{5ab^3(Ab+2a}{x^8}\right.\\ &= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{10a^2b^2(Ab+aB)}{9x^9}-\frac{5ab^3(Ab+2aB)}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 1.03

$$-\frac{231a^5\left(13A+15Bx^2\right)+1575a^4bx^2\left(11A+13Bx^2\right)+4550a^3b^2x^4\left(9A+11Bx^2\right)+7150a^2b^3x^6\left(7A+9Bx^2\right)+6435ab^4x^8\left(5A+7Bx^2\right)+3003b^5x^{10}\left(3A+5Bx^2\right)}{45045x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] -1/45045*(3003*b^5*x^10*(3*A + 5*B*x^2) + 6435*a*b^4*x^8*(5*A + 7*B*x^2) +
7150*a^2*b^3*x^6*(7*A + 9*B*x^2) + 4550*a^3*b^2*x^4*(9*A + 11*B*x^2) + 1575
*a^4*b*x^2*(11*A + 13*B*x^2) + 231*a^5*(13*A + 15*B*x^2))/x^15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^16, x]
```

fricas [A] time = 0.40, size = 121, normalized size = 1.03

$$-\frac{15015\,Bb^5x^{12}+9009\left(5\,Bab^4+Ab^5\right)x^{10}+32175\left(2\,Ba^2b^3+Aab^4\right)x^8+50050\left(Ba^3b^2+Aa^2b^3\right)x^6+3003\,Aa^5+20475\left(Ba^4b+2\,Aa^3b^2\right)x^4+3465\left(Ba^5+5\,Aa^4b\right)x^2}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 2047
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

giac [A] time = 0.30, size = 127, normalized size = 1.09

$$-\frac{15015\,Bb^5x^{12}+45045\,Bab^4x^{10}+9009\,Ab^5x^{10}+64350\,Ba^2b^3x^8+32175\,Aab^4x^8+50050\,Ba^3b^2x^6+50050\,Aa^2b^3x^6+20475\,Ba^4bx^4+40950\,Aa^3b^2x^4+3465\,Ba^5x^2+17325\,Aa^4bx^2+3003\,Aa^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 45045*B*a*b^4*x^10 + 9009*A*b^5*x^10 + 64350*B
*a^2*b^3*x^8 + 32175*A*a*b^4*x^8 + 50050*B*a^3*b^2*x^6 + 50050*A*a^2*b^3*x^
6 + 20475*B*a^4*b*x^4 + 40950*A*a^3*b^2*x^4 + 3465*B*a^5*x^2 + 17325*A*a^4*
b*x^2 + 3003*A*a^5)/x^15
```

maple [A] time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{3x^3}-\frac{(Ab+5Ba)\,b^4}{5x^5}-\frac{5\,(Ab+2Ba)\,a\,b^3}{7x^7}-\frac{10\,(Ab+Ba)\,a^2b^2}{9x^9}-\frac{5\,(2Ab+Ba)\,a^3b}{11x^{11}}-\frac{A\,a^5}{15x^{15}}-\frac{(5Ab+Ba)\,a^4}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^16,x)
```

```
[Out] -1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)/x^11-10/9
*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^4*(A*b+5*B*a)/x^5-1/
3*b^5*B/x^3
```

maxima **[A]** time = 0.97, size = 121, normalized size = 1.03

$$-\frac{15015\,Bb^{5}x^{12}+9009\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+32175\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+50050\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+3003\,Aa^{5}+20475\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+3465\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^2
*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 2047
5*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

mupad **[B]** time = 0.06, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^{5}}{15}+x^{8}\,\left(\frac{10\,B\,a^{2}\,b^{3}}{7}+\frac{5\,A\,a\,b^{4}}{7}\right)+x^{4}\,\left(\frac{5\,B\,a^{4}\,b}{11}+\frac{10\,A\,a^{3}\,b^{2}}{11}\right)+x^{2}\,\left(\frac{B\,a^{5}}{13}+\frac{5\,A\,b\,a^{4}}{13}\right)+x^{10}\,\left(\frac{A\,b^{5}}{5}+B\,a\,b^{4}\right)+x^{6}\,\left(\frac{10\,B\,a^{3}\,b^{2}}{9}+\frac{10\,A\,a^{2}\,b^{3}}{9}\right)+\frac{B\,b^{5}\,x^{12}}{3}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^16,x)
```

```
[Out] -((A*a^5)/15 + x^8*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^4*((10*A*a^3*b^2)
/11 + (5*B*a^4*b)/11) + x^2*((B*a^5)/13 + (5*A*a^4*b)/13) + x^10*((A*b^5)/5
 + B*a*b^4) + x^6*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9) + (B*b^5*x^12)/3)/x
^15
```

sympy **[A]** time = 36.44, size = 134, normalized size = 1.15

$$\frac{-3003Aa^{5}-15015Bb^{5}x^{12}+x^{10}\left(-9009Ab^{5}-45045Bab^{4}\right)+x^{8}\left(-32175Aab^{4}-64350Ba^{2}b^{3}\right)+x^{6}\left(-50050Aa^{2}b^{3}-50050Ba^{3}b^{2}\right)+x^{4}\left(-40950Aa^{3}b^{2}-20475Ba^{4}b\right)+x^{2}\left(-17325Aa^{4}b-3465Ba^{5}\right)}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

```
[Out] (-3003*A*a**5 - 15015*B*b**5*x**12 + x**10*(-9009*A*b**5 - 45045*B*a*b**4)
+ x**8*(-32175*A*a*b**4 - 64350*B*a**2*b**3) + x**6*(-50050*A*a**2*b**3 - 5
0050*B*a**3*b**2) + x**4*(-40950*A*a**3*b**2 - 20475*B*a**4*b) + x**2*(-173
25*A*a**4*b - 3465*B*a**5))/(45045*x**15)
```

3.49 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx$

Optimal. Leaf size=76

$$-\frac{b(a+bx^2)^6(Ab-4aB)}{336a^3x^{12}}+\frac{(a+bx^2)^6(Ab-4aB)}{56a^2x^{14}}-\frac{A(a+bx^2)^6}{16ax^{16}}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 78, 45, 37}

$$-\frac{b(a+bx^2)^6(Ab-4aB)}{336a^3x^{12}}+\frac{(a+bx^2)^6(Ab-4aB)}{56a^2x^{14}}-\frac{A(a+bx^2)^6}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -(A*(a + b*x^2)^6)/(16*a*x^16) + ((A*b - 4*a*B)*(a + b*x^2)^6)/(56*a^2*x^14
) - (b*(A*b - 4*a*B)*(a + b*x^2)^6)/(336*a^3*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{17}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^9}\,dx, x, x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^6}{16ax^{16}}+\frac{(-2Ab+8aB)\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^8}\,dx, x, x^2\right)}{16a}\\
&= -\frac{A\left(a+bx^2\right)^6}{16ax^{16}}+\frac{(Ab-4aB)\left(a+bx^2\right)^6}{56a^2x^{14}}+\frac{(b(Ab-4aB))\,\text{Subst}\Big(\int \frac{(a+bx)^5}{x^7}\,dx, x, x^2}{56a^2}\\
&= -\frac{A\left(a+bx^2\right)^6}{16ax^{16}}+\frac{(Ab-4aB)\left(a+bx^2\right)^6}{56a^2x^{14}}-\frac{b(Ab-4aB)\left(a+bx^2\right)^6}{336a^3x^{12}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 121, normalized size = 1.59

$$
-\frac{3a^5\left(7A+8Bx^2\right)+20a^4bx^2\left(6A+7Bx^2\right)+56a^3b^2x^4\left(5A+6Bx^2\right)+84a^2b^3x^6\left(4A+5Bx^2\right)+70ab^4x^8\left(3A+4Bx^2\right)+28b^5x^{10}\left(2A+3Bx^2\right)}{336x^{16}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] -1/336*(28*b^5*x^10*(2*A + 3*B*x^2) + 70*a*b^4*x^8*(3*A + 4*B*x^2) + 84*a^2
*b^3*x^6*(4*A + 5*B*x^2) + 56*a^3*b^2*x^4*(5*A + 6*B*x^2) + 20*a^4*b*x^2*(6
*A + 7*B*x^2) + 3*a^5*(7*A + 8*B*x^2))/x^16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{17}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^17, x]
```

fricas **[A]** time = 0.45, size = 121, normalized size = 1.59

$$-\frac{84\,Bb^{5}x^{12} + 56\,\left(5\,Bab^{4} + Ab^{5}\right)x^{10} + 210\,\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{8} + 336\,\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{6} + 21\,Aa^{5} + 140\,\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{4} + 24\,\left(Ba^{5} + 5\,Aa^{4}b\right)x^{2}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

giac **[A]** time = 0.32, size = 127, normalized size = 1.67

$$-\frac{84\,Bb^{5}x^{12} + 280\,Bab^{4}x^{10} + 56\,Ab^{5}x^{10} + 420\,Ba^{2}b^{3}x^{8} + 210\,Aab^{4}x^{8} + 336\,Ba^{3}b^{2}x^{6} + 336\,Aa^{2}b^{3}x^{6} + 140\,Ba^{4}bx^{4} + 280\,Aa^{3}b^{2}x^{4} + 24\,Ba^{5}x^{2} + 120\,Aa^{4}bx^{2} + 21\,Aa^{5}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 280*B*a*b^4*x^10 + 56*A*b^5*x^10 + 420*B*a^2*b^3*x^
8 + 210*A*a*b^4*x^8 + 336*B*a^3*b^2*x^6 + 336*A*a^2*b^3*x^6 + 140*B*a^4*b*x
^4 + 280*A*a^3*b^2*x^4 + 24*B*a^5*x^2 + 120*A*a^4*b*x^2 + 21*A*a^5)/x^16
```

maple **[A]** time = 0.01, size = 104, normalized size = 1.37

$$-\frac{B\,b^{5}}{4x^{4}}-\frac{(Ab+5Ba)\,b^{4}}{6x^{6}}-\frac{5\,(Ab+2Ba)\,a\,b^{3}}{8x^{8}}-\frac{(Ab+Ba)\,a^{2}b^{2}}{x^{10}}-\frac{5\,(2Ab+Ba)\,a^{3}b}{12x^{12}}-\frac{A\,a^{5}}{16x^{16}}-\frac{(5Ab+Ba)\,a^{4}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^17,x)
```

```
[Out] -1/16*A*a^5/x^16-a^2*b^2*(A*b+B*a)/x^10-1/6*b^4*(A*b+5*B*a)/x^6-5/8*a*b^3*(
A*b+2*B*a)/x^8-1/4*B*b^5/x^4-1/14*a^4*(5*A*b+B*a)/x^14-5/12*a^3*b*(2*A*b+B*
a)/x^12
```

maxima **[A]** time = 1.09, size = 121, normalized size = 1.59

$$-\frac{84\,Bb^{5}x^{12} + 56\,\left(5\,Bab^{4} + Ab^{5}\right)x^{10} + 210\,\left(2\,Ba^{2}b^{3} + Aab^{4}\right)x^{8} + 336\,\left(Ba^{3}b^{2} + Aa^{2}b^{3}\right)x^{6} + 21\,Aa^{5} + 140\,\left(Ba^{4}b + 2\,Aa^{3}b^{2}\right)x^{4} + 24\,\left(Ba^{5} + 5\,Aa^{4}b\right)x^{2}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="maxima")
```

```
[Out] -1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b + 2*
A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

mupad [B] time = 0.09, size = 120, normalized size = 1.58

$$-\frac{\frac{A\,a^5}{16}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{12}+\frac{5\,A\,a^3\,b^2}{6}\right)+x^2\,\left(\frac{B\,a^5}{14}+\frac{5\,A\,b\,a^4}{14}\right)+x^{10}\,\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^6\,\left(B\,a^3\,b^2+A\,a^2\,b^3\right)+\frac{B\,b^5\,x^{12}}{4}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^17,x)
```

```
[Out] -((A*a^5)/16 + x^8*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^4*((5*A*a^3*b^2)/6
 + (5*B*a^4*b)/12) + x^2*((B*a^5)/14 + (5*A*a^4*b)/14) + x^10*((A*b^5)/6 +
(5*B*a*b^4)/6) + x^6*(A*a^2*b^3 + B*a^3*b^2) + (B*b^5*x^12)/4)/x^16
```

sympy [A] time = 32.11, size = 134, normalized size = 1.76

$$\frac{-21Aa^5-84Bb^5x^{12}+x^{10}\left(-56Ab^5-280Bab^4\right)+x^8\left(-210Aab^4-420Ba^2b^3\right)+x^6\left(-336Aa^2b^3-336Ba^3b^2\right)+x^4\left(-280Aa^3b^2-140Ba^4b\right)+x^2\left(-120Aa^4b-24Ba^5\right)}{336x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**17,x)
```

```
[Out] (-21*A*a**5 - 84*B*b**5*x**12 + x**10*(-56*A*b**5 - 280*B*a*b**4) + x**8*(-
210*A*a*b**4 - 420*B*a**2*b**3) + x**6*(-336*A*a**2*b**3 - 336*B*a**3*b**2)
 + x**4*(-280*A*a**3*b**2 - 140*B*a**4*b) + x**2*(-120*A*a**4*b - 24*B*a**5
))/(336*x**16)
```

3.50 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{17x^{17}}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx &= \int \left(\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{16}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(Ab+2}{x^{10}}\right.\\ &= -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2a}{9x^9}\end{aligned}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 1.00

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5 \left(A+Bx^2\right)}{x^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^18, x]
```

fricas [A] time = 0.40, size = 121, normalized size = 1.03

$$-\frac{153153\,Bb^5x^{12} + 109395\,\left(5\,Bab^4 + Ab^5\right)x^{10} + 425425\,\left(2\,Ba^2b^3 + Aab^4\right)x^8 + 696150\,\left(Ba^3b^2 + Aa^2b^3\right)x^6 + 45045\,Aa^5 + 294525\,\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + 51051\,\left(Ba^5 + 5\,Aa^4b\right)x^2}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="fricas")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a^5
 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

giac [A] time = 0.30, size = 127, normalized size = 1.09

$$-\frac{153153\,Bb^5x^{12} + 546975\,Bab^4x^{10} + 109395\,Ab^5x^{10} + 850850\,Ba^2b^3x^8 + 425425\,Aab^4x^8 + 696150\,Ba^3b^2x^6 + 696150\,Aa^2b^3x^6 + 294525\,Ba^4bx^4 + 589050\,Aa^3b^2x^4 + 51051\,Ba^5x^2 + 255255\,Aa^4bx^2 + 45045\,Aa^5}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="giac")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 546975*B*a*b^4*x^10 + 109395*A*b^5*x^10 + 85
0850*B*a^2*b^3*x^8 + 425425*A*a*b^4*x^8 + 696150*B*a^3*b^2*x^6 + 696150*A*a
^2*b^3*x^6 + 294525*B*a^4*b*x^4 + 589050*A*a^3*b^2*x^4 + 51051*B*a^5*x^2 +
255255*A*a^4*b*x^2 + 45045*A*a^5)/x^17
```

maple [A] time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{5x^5} - \frac{(Ab+5Ba)\,b^4}{7x^7} - \frac{5\,(Ab+2Ba)\,a\,b^3}{9x^9} - \frac{10\,(Ab+Ba)\,a^2b^2}{11x^{11}} - \frac{5\,(2Ab+Ba)\,a^3b}{13x^{13}} - \frac{A\,a^5}{17x^{17}} - \frac{(5Ab+Ba)\,a^4}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^18,x)
```

```
[Out] -1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)/x^13-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*b^4*(A*b+5*B*a)/x^7-
1/5*b^5*B/x^5
```

maxima [A] time = 1.14, size = 121, normalized size = 1.03

$$-\frac{153153\,Bb^{5}x^{12}+109395\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+425425\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+696150\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+45045\,Aa^{5}+294525\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+51051\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{765765\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="maxima")
```

```
[Out] -1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2*
B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a^5
 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x^17
```

mupad [B] time = 0.09, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^{5}}{17}+x^{8}\left(\frac{10\,B\,a^{2}\,b^{3}}{9}+\frac{5\,A\,a\,b^{4}}{9}\right)+x^{4}\left(\frac{5\,B\,a^{4}\,b}{13}+\frac{10\,A\,a^{3}\,b^{2}}{13}\right)+x^{2}\left(\frac{B\,a^{5}}{15}+\frac{A\,b\,a^{4}}{3}\right)+x^{10}\left(\frac{A\,b^{5}}{7}+\frac{5\,B\,a\,b^{4}}{7}\right)+x^{6}\left(\frac{10\,B\,a^{3}\,b^{2}}{11}+\frac{10\,A\,a^{2}\,b^{3}}{11}\right)+\frac{B\,b^{5}\,x^{12}}{5}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^18,x)
```

```
[Out] -((A*a^5)/17 + x^8*((10*B*a^2*b^3)/9 + (5*A*a*b^4)/9) + x^4*((10*A*a^3*b^2)
/13 + (5*B*a^4*b)/13) + x^2*((B*a^5)/15 + (A*a^4*b)/3) + x^10*((A*b^5)/7 +
(5*B*a*b^4)/7) + x^6*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^12)
/5)/x^17
```

sympy [A] time = 94.11, size = 134, normalized size = 1.15

$$\frac{-45045Aa^{5}-153153Bb^{5}x^{12}+x^{10}\left(-109395Ab^{5}-546975Bab^{4}\right)+x^{8}\left(-425425Aab^{4}-850850Ba^{2}b^{3}\right)+x^{6}\left(-696150Aa^{2}b^{3}-696150Ba^{3}b^{2}\right)+x^{4}\left(-589050Aa^{3}b^{2}-294525Ba^{4}b\right)+x^{2}\left(-255255Aa^{4}b-51051Ba^{5}\right)}{765765x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

```
[Out] (-45045*A*a**5 - 153153*B*b**5*x**12 + x**10*(-109395*A*b**5 - 546975*B*a*b
**4) + x**8*(-425425*A*a*b**4 - 850850*B*a**2*b**3) + x**6*(-696150*A*a**2*
b**3 - 696150*B*a**3*b**2) + x**4*(-589050*A*a**3*b**2 - 294525*B*a**4*b) +
 x**2*(-255255*A*a**4*b - 51051*B*a**5))/(765765*x**17)
```

3.51 $\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{18x^{18}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{6x^6}$$

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{6x^{12}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5A}{18x^{18}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{8x^8}-\frac{b^5B}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -(a^5*A)/(18*x^18) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (5*a^2*b^2*(A*b + a*B))/(6*x^12) - (a*b^3*(A*b + 2*a*B))/(2*x
^10) - (b^4*(A*b + 5*a*B))/(8*x^8) - (b^5*B)/(6*x^6)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{19}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{10}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a^5A}{x^{10}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^7}+\right.\right.\\ &= -\frac{a^5A}{18x^{18}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{5a^2b^2(Ab+aB)}{6x^{12}}-\frac{ab^3(Ab+2aB)}{2x^{10}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 1.03

$$-\frac{7a^5\left(8A+9Bx^2\right)+45a^4bx^2\left(7A+8Bx^2\right)+120a^3b^2x^4\left(6A+7Bx^2\right)+168a^2b^3x^6\left(5A+6Bx^2\right)+126ab^4x^8\left(4A+5Bx^2\right)+42b^5x^{10}\left(3A+4Bx^2\right)}{1008x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] -1/1008*(42*b^5*x^10*(3*A + 4*B*x^2) + 126*a*b^4*x^8*(4*A + 5*B*x^2) + 168*
a^2*b^3*x^6*(5*A + 6*B*x^2) + 120*a^3*b^2*x^4*(6*A + 7*B*x^2) + 45*a^4*b*x^
2*(7*A + 8*B*x^2) + 7*a^5*(8*A + 9*B*x^2))/x^18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^19, x]
```

fricas [A] time = 0.41, size = 121, normalized size = 1.03

$$-\frac{168\,Bb^5x^{12}+126\left(5\,Bab^4+Ab^5\right)x^{10}+504\left(2\,Ba^2b^3+Aab^4\right)x^8+840\left(Ba^3b^2+Aa^2b^3\right)x^6+56\,Aa^5+360\left(Ba^4b+2\,Aa^3b^2\right)x^4+63\left(Ba^5+5\,Aa^4b\right)x^2}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

giac [A] time = 0.34, size = 127, normalized size = 1.09

$$-\frac{168\,Bb^{5}x^{12}+630\,Bab^{4}x^{10}+126\,Ab^{5}x^{10}+1008\,Ba^{2}b^{3}x^{8}+504\,Aab^{4}x^{8}+840\,Ba^{3}b^{2}x^{6}+840\,Aa^{2}b^{3}x^{6}+360\,Ba^{4}bx^{4}+720\,Aa^{3}b^{2}x^{4}+63\,Ba^{5}x^{2}+315\,Aa^{4}bx^{2}+56\,Aa^{5}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 630*B*a*b^4*x^10 + 126*A*b^5*x^10 + 1008*B*a^2*b^
3*x^8 + 504*A*a*b^4*x^8 + 840*B*a^3*b^2*x^6 + 840*A*a^2*b^3*x^6 + 360*B*a^4
*b*x^4 + 720*A*a^3*b^2*x^4 + 63*B*a^5*x^2 + 315*A*a^4*b*x^2 + 56*A*a^5)/x^1
8
```

maple [A] time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^{5}}{6x^{6}}-\frac{\left(Ab+5Ba\right)b^{4}}{8x^{8}}-\frac{\left(Ab+2Ba\right)a\,b^{3}}{2x^{10}}-\frac{5\left(Ab+Ba\right)a^{2}b^{2}}{6x^{12}}-\frac{5\left(2Ab+Ba\right)a^{3}b}{14x^{14}}-\frac{A\,a^{5}}{18x^{18}}-\frac{\left(5Ab+Ba\right)a^{4}}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^19,x)
```

```
[Out] -1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)/x^14-5/6*
a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b^4*(A*b+5*B*a)/x^8-1
/6*b^5*B/x^6
```

maxima [A] time = 1.12, size = 121, normalized size = 1.03

$$-\frac{168\,Bb^{5}x^{12}+126\left(5\,Bab^{4}+Ab^{5}\right)x^{10}+504\left(2\,Ba^{2}b^{3}+Aab^{4}\right)x^{8}+840\left(Ba^{3}b^{2}+Aa^{2}b^{3}\right)x^{6}+56\,Aa^{5}+360\left(Ba^{4}b+2\,Aa^{3}b^{2}\right)x^{4}+63\left(Ba^{5}+5\,Aa^{4}b\right)x^{2}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="maxima")
```

```
[Out] -1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

mupad [B] time = 0.06, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{18}+x^8\,\left(B\,a^2\,b^3+\frac{A\,a\,b^4}{2}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^2\,\left(\frac{B\,a^5}{16}+\frac{5\,A\,b\,a^4}{16}\right)+x^{10}\,\left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{6}+\frac{5\,A\,a^2\,b^3}{6}\right)+\frac{B\,b^5\,x^{12}}{6}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^19,x)
```

```
[Out] -((A*a^5)/18 + x^8*(B*a^2*b^3 + (A*a*b^4)/2) + x^4*((5*A*a^3*b^2)/7 + (5*B*
a^4*b)/14) + x^2*((B*a^5)/16 + (5*A*a^4*b)/16) + x^10*((A*b^5)/8 + (5*B*a*b
^4)/8) + x^6*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^12)/6)/x^18
```

sympy [A] time = 65.26, size = 134, normalized size = 1.15

$$\frac{-56Aa^5 - 168Bb^5x^{12} + x^{10}\left(-126Ab^5 - 630Bab^4\right) + x^8\left(-504Aab^4 - 1008Ba^2b^3\right) + x^6\left(-840Aa^2b^3 - 840Ba^3b^2\right) + x^4\left(-720Aa^3b^2 - 360Ba^4b\right) + x^2\left(-315Aa^4b - 63Ba^5\right)}{1008x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

```
[Out] (-56*A*a**5 - 168*B*b**5*x**12 + x**10*(-126*A*b**5 - 630*B*a*b**4) + x**8*
(-504*A*a*b**4 - 1008*B*a**2*b**3) + x**6*(-840*A*a**2*b**3 - 840*B*a**3*b*
*2) + x**4*(-720*A*a**3*b**2 - 360*B*a**4*b) + x**2*(-315*A*a**4*b - 63*B*a
**5))/(1008*x**18)
```

3.52 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{20}}\,dx &= \int \bigg(\frac{a^5A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{16}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(Ab+2a}{x^{12}}\\ &= -\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{a^3b(2Ab+aB)}{3x^{15}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{5ab^3(Ab+2aB)}{11x^{11}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 1.00

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] -1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))/
(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(11
*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^20, x]
```

fricas [A] time = 0.42, size = 121, normalized size = 1.03

$$-\frac{415701\,Bb^5x^{12}+323323\left(5\,Bab^4+Ab^5\right)x^{10}+1322685\left(2\,Ba^2b^3+Aab^4\right)x^8+2238390\left(Ba^3b^2+Aa^2b^3\right)x^6+153153\,Aa^5+969969\left(Ba^4b+2\,Aa^3b^2\right)x^4+171171\left(Ba^5+5\,Aa^4b\right)x^2}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153*A
*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x^2)
/x^19
```

giac [A] time = 0.42, size = 127, normalized size = 1.09

$$-\frac{415701\,Bb^5x^{12}+1616615\,Bab^4x^{10}+323323\,Ab^5x^{10}+2645370\,Ba^2b^3x^8+1322685\,Aab^4x^8+2238390\,Ba^3b^2x^6+2238390\,Aa^2b^3x^6+969969\,Ba^4bx^4+1939938\,Aa^3b^2x^4+171171\,Ba^5x^2+855855\,Aa^4bx^2+153153\,Aa^5}{2909907\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 1616615*B*a*b^4*x^10 + 323323*A*b^5*x^10 +
2645370*B*a^2*b^3*x^8 + 1322685*A*a*b^4*x^8 + 2238390*B*a^3*b^2*x^6 + 22383
90*A*a^2*b^3*x^6 + 969969*B*a^4*b*x^4 + 1939938*A*a^3*b^2*x^4 + 171171*B*a^
5*x^2 + 855855*A*a^4*b*x^2 + 153153*A*a^5)/x^19
```

maple [A] time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{7x^7}-\frac{(Ab+5Ba)\,b^4}{9x^9}-\frac{5\,(Ab+2Ba)\,a\,b^3}{11x^{11}}-\frac{10\,(Ab+Ba)\,a^2b^2}{13x^{13}}-\frac{(2Ab+Ba)\,a^3b}{3x^{15}}-\frac{A\,a^5}{19x^{19}}-\frac{(5Ab+Ba)\,a^4}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^20,x)
```

```
[Out] -1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/x^15-10/13
*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9*b^4*(A*b+5*B*a)/x^9
-1/7*b^5*B/x^7
```

maxima [A] time = 1.04, size = 121, normalized size = 1.03

$$-\frac{415701 \, B b^{5} x^{12} + 323323 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{10} + 1322685 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{8} + 2238390 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{6} + 153153 \, A a^{5} + 969969 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{4} + 171171 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{2}}{2909907 \, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153*A
*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x^2)
/x^19
```

mupad [B] time = 0.09, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{19}+x^4\,\left(\frac{B\,a^4\,b}{3}+\frac{2\,A\,a^3\,b^2}{3}\right)+x^8\,\left(\frac{10\,B\,a^2\,b^3}{11}+\frac{5\,A\,a\,b^4}{11}\right)+x^2\,\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{10}\,\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+x^6\,\left(\frac{10\,B\,a^3\,b^2}{13}+\frac{10\,A\,a^2\,b^3}{13}\right)+\frac{B\,b^5\,x^{12}}{7}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^20,x)
```

```
[Out] -((A*a^5)/19 + x^4*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^8*((10*B*a^2*b^3)/11
 + (5*A*a*b^4)/11) + x^2*((B*a^5)/17 + (5*A*a^4*b)/17) + x^10*((A*b^5)/9 +
(5*B*a*b^4)/9) + x^6*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^12)
/7)/x^19
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**20,x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{21}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{20x^{20}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^5B}{8x^8}$$

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{7x^{14}}-\frac{a^4(aB+5Ab)}{18x^{18}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{12x^{12}}-\frac{b^4(5aB+Ab)}{10x^{10}}-\frac{b^5B}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(18*x^18) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (5*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b^3*(A*b + 2*a*B))/(1
2*x^12) - (b^4*(A*b + 5*a*B))/(10*x^10) - (b^5*B)/(8*x^8)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{21}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{11}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a^5A}{x^{11}}+\frac{a^4(5Ab+aB)}{x^{10}}+\frac{5a^3b(2Ab+aB)}{x^9}+\frac{10a^2b^2(Ab+aB)}{x^8}+5ab^3\right.\right.\\ &= -\frac{a^5A}{20x^{20}}-\frac{a^4(5Ab+aB)}{18x^{18}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{5a^2b^2(Ab+aB)}{7x^{14}}-\frac{5ab^3(Ab+2aB)}{12x^{12}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 121, normalized size = 1.03

$$-\frac{28a^5\left(9A+10Bx^2\right)+175a^4bx^2\left(8A+9Bx^2\right)+450a^3b^2x^4\left(7A+8Bx^2\right)+600a^2b^3x^6\left(6A+7Bx^2\right)+420ab^4x^8\left(5A+6Bx^2\right)+126b^5x^{10}\left(4A+5Bx^2\right)}{5040x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] -1/5040*(126*b^5*x^10*(4*A + 5*B*x^2) + 420*a*b^4*x^8*(5*A + 6*B*x^2) + 600
*a^2*b^3*x^6*(6*A + 7*B*x^2) + 450*a^3*b^2*x^4*(7*A + 8*B*x^2) + 175*a^4*b*
x^2*(8*A + 9*B*x^2) + 28*a^5*(9*A + 10*B*x^2))/x^20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{21}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^21, x]
```

fricas [A] time = 0.42, size = 121, normalized size = 1.03

$$-\frac{630\,Bb^5x^{12}+504\left(5\,Bab^4+Ab^5\right)x^{10}+2100\left(2\,Ba^2b^3+Aab^4\right)x^8+3600\left(Ba^3b^2+Aa^2b^3\right)x^6+252\,Aa^5+1575\left(Ba^4b+2\,Aa^3b^2\right)x^4+280\left(Ba^5+5\,Aa^4b\right)x^2}{5040\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

giac [A]   time = 0.34, size = 127, normalized size = 1.09

$$-\frac{630 \, B b^{5} x^{12} + 2520 \, B a b^{4} x^{10} + 504 \, A b^{5} x^{10} + 4200 \, B a^{2} b^{3} x^{8} + 2100 \, A a b^{4} x^{8} + 3600 \, B a^{3} b^{2} x^{6} + 3600 \, A a^{2} b^{3} x^{6} + 1575 \, B a^{4} b x^{4} + 3150 \, A a^{3} b^{2} x^{4} + 280 \, B a^{5} x^{2} + 1400 \, A a^{4} b x^{2} + 252 \, A a^{5}}{5040 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 2520*B*a*b^4*x^10 + 504*A*b^5*x^10 + 4200*B*a^2*b
^3*x^8 + 2100*A*a*b^4*x^8 + 3600*B*a^3*b^2*x^6 + 3600*A*a^2*b^3*x^6 + 1575*
B*a^4*b*x^4 + 3150*A*a^3*b^2*x^4 + 280*B*a^5*x^2 + 1400*A*a^4*b*x^2 + 252*A
*a^5)/x^20
```

maple [A]   time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^{5}}{8x^{8}}-\frac{\left(Ab+5Ba\right)b^{4}}{10x^{10}}-\frac{5\left(Ab+2Ba\right)a\,b^{3}}{12x^{12}}-\frac{5\left(Ab+Ba\right)a^{2}b^{2}}{7x^{14}}-\frac{5\left(2Ab+Ba\right)a^{3}b}{16x^{16}}-\frac{A\,a^{5}}{20x^{20}}-\frac{\left(5Ab+Ba\right)a^{4}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^21,x)
```

```
[Out] -1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)/x^16-5/7*
a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10*b^4*(A*b+5*B*a)/x^1
0-1/8*b^5*B/x^8
```

maxima [A]   time = 1.00, size = 121, normalized size = 1.03

$$-\frac{630 \, B b^{5} x^{12} + 504 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{10} + 2100 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{8} + 3600 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{6} + 252 \, A a^{5} + 1575 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{4} + 280 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{2}}{5040 \, x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a^4
*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

mupad [B]   time = 0.09, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{20}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{6}+\frac{5\,A\,a\,b^4}{12}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{16}+\frac{5\,A\,a^3\,b^2}{8}\right)+x^2\,\left(\frac{B\,a^5}{18}+\frac{5\,A\,b\,a^4}{18}\right)+x^{10}\,\left(\frac{A\,b^5}{10}+\frac{B\,a\,b^4}{2}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{7}+\frac{5\,A\,a^2\,b^3}{7}\right)+\frac{B\,b^5\,x^{12}}{8}}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^21,x)
```

```
[Out] -((A*a^5)/20 + x^8*((5*B*a^2*b^3)/6 + (5*A*a*b^4)/12) + x^4*((5*A*a^3*b^2)/
8 + (5*B*a^4*b)/16) + x^2*((B*a^5)/18 + (5*A*a^4*b)/18) + x^10*((A*b^5)/10
+ (B*a*b^4)/2) + x^6*((5*A*a^2*b^3)/7 + (5*B*a^3*b^2)/7) + (B*b^5*x^12)/8)/
x^20
```

sympy [A] time = 127.67, size = 134, normalized size = 1.15

$$\frac{-252Aa^5 - 630Bb^5x^{12} + x^{10}\left(-504Ab^5 - 2520Bab^4\right) + x^8\left(-2100Aab^4 - 4200Ba^2b^3\right) + x^6\left(-3600Aa^2b^3 - 3600Ba^3b^2\right) + x^4\left(-3150Aa^3b^2 - 1575Ba^4b\right) + x^2\left(-1400Aa^4b - 280Ba^5\right)}{5040x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

```
[Out] (-252*A*a**5 - 630*B*b**5*x**12 + x**10*(-504*A*b**5 - 2520*B*a*b**4) + x**
8*(-2100*A*a*b**4 - 4200*B*a**2*b**3) + x**6*(-3600*A*a**2*b**3 - 3600*B*a*
*3*b**2) + x**4*(-3150*A*a**3*b**2 - 1575*B*a**4*b) + x**2*(-1400*A*a**4*b
- 280*B*a**5))/(5040*x**20)
```

3.54 $\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{22}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{a^5A}{21x^{21}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -(a^5*A)/(21*x^21) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{22}} \, dx &= \int \left(\frac{a^5A}{x^{22}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{18}}+\frac{10a^2b^2(Ab+aB)}{x^{16}}+\frac{5ab^3(Ab+2}{x^{14}}\right. \\ &= -\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}}-\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab+2aB}{13x^{13}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 117, normalized size = 1.00

$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] -1/21*(a^5*A)/x^21 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/(1
3*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{22}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^22, x]
```

fricas [**A**] time = 0.45, size = 121, normalized size = 1.03

$$-\frac{323323\,Bb^5x^{12}+264537\left(5\,Bab^4+Ab^5\right)x^{10}+1119195\left(2\,Ba^2b^3+Aab^4\right)x^8+1939938\left(Ba^3b^2+Aa^2b^3\right)x^6+138567\,Aa^5+855855\left(Ba^4b+2\,Aa^3b^2\right)x^4+153153\left(Ba^5+5\,Aa^4b\right)x^2}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567*A
*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x^2)
/x^21
```

giac [**A**] time = 0.28, size = 127, normalized size = 1.09

$$-\frac{323323\,Bb^5x^{12}+1322685\,Bab^4x^{10}+264537\,Ab^5x^{10}+2238390\,Ba^2b^3x^8+1119195\,Aab^4x^8+1939938\,Ba^3b^2x^6+1939938\,Aa^2b^3x^6+855855\,Ba^4bx^4+1711710\,Aa^3b^2x^4+153153\,Ba^5x^2+765765\,Aa^4bx^2+138567\,Aa^5}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 1322685*B*a*b^4*x^10 + 264537*A*b^5*x^10 +
2238390*B*a^2*b^3*x^8 + 1119195*A*a*b^4*x^8 + 1939938*B*a^3*b^2*x^6 + 19399
38*A*a^2*b^3*x^6 + 855855*B*a^4*b*x^4 + 1711710*A*a^3*b^2*x^4 + 153153*B*a^
5*x^2 + 765765*A*a^4*b*x^2 + 138567*A*a^5)/x^21
```

maple [**A**] time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{9x^9}-\frac{(Ab+5Ba)\,b^4}{11x^{11}}-\frac{5\,(Ab+2Ba)\,a\,b^3}{13x^{13}}-\frac{2\,(Ab+Ba)\,a^2b^2}{3x^{15}}-\frac{5\,(2Ab+Ba)\,a^3b}{17x^{17}}-\frac{A\,a^5}{21x^{21}}-\frac{(5Ab+Ba)\,a^4}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^22,x)
```

```
[Out] -1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)/x^17-2/3*
a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11*b^4*(A*b+5*B*a)/x^1
1-1/9*b^5*B/x^9
```

maxima [A] time = 1.03, size = 121, normalized size = 1.03

$$-\frac{323323\,Bb^5x^{12}+264537\,\left(5\,Bab^4+Ab^5\right)x^{10}+1119195\,\left(2\,Ba^2b^3+Aab^4\right)x^8+1939938\,\left(Ba^3b^2+Aa^2b^3\right)x^6+138567\,Aa^5+855855\,\left(Ba^4b+2\,Aa^3b^2\right)x^4+153153\,\left(Ba^5+5\,Aa^4b\right)x^2}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*(
2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567*A
*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x^2)
/x^21
```

mupad [B] time = 0.09, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{21}+x^8\,\left(\frac{10\,B\,a^2\,b^3}{13}+\frac{5\,A\,a\,b^4}{13}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{17}+\frac{10\,A\,a^3\,b^2}{17}\right)+x^2\,\left(\frac{B\,a^5}{19}+\frac{5\,A\,b\,a^4}{19}\right)+x^{10}\,\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)+x^6\,\left(\frac{2\,B\,a^3\,b^2}{3}+\frac{2\,A\,a^2\,b^3}{3}\right)+\frac{B\,b^5\,x^{12}}{9}}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^22,x)
```

```
[Out] -((A*a^5)/21 + x^8*((10*B*a^2*b^3)/13 + (5*A*a*b^4)/13) + x^4*((10*A*a^3*b^
2)/17 + (5*B*a^4*b)/17) + x^2*((B*a^5)/19 + (5*A*a^4*b)/19) + x^10*((A*b^5)
/11 + (5*B*a*b^4)/11) + x^6*((2*A*a^2*b^3)/3 + (2*B*a^3*b^2)/3) + (B*b^5*x^
12)/9)/x^21
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**22,x)
```

```
[Out] Timed out
```

3.55 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{23}}\,dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^5B}{10x^{10}}$$

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{8x^{16}}-\frac{a^4(aB+5Ab)}{20x^{20}}-\frac{5a^3b(aB+2Ab)}{18x^{18}}-\frac{a^5A}{22x^{22}}-\frac{5ab^3(2aB+Ab)}{14x^{14}}-\frac{b^4(5aB+Ab)}{12x^{12}}-\frac{b^5B}{10x^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(20*x^20) - (5*a^3*b*(2*A*b + a*B)
)/(18*x^18) - (5*a^2*b^2*(A*b + a*B))/(8*x^16) - (5*a*b^3*(A*b + 2*a*B))/(1
4*x^14) - (b^4*(A*b + 5*a*B))/(12*x^12) - (b^5*B)/(10*x^10)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{23}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^{12}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^{11}}+\frac{5a^3b(2Ab+aB)}{x^{10}}+\frac{10a^2b^2(Ab+aB)}{x^9}+ 5a\right.\right.\\ &= -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{20x^{20}}-\frac{5a^3b(2Ab+aB)}{18x^{18}}-\frac{5a^2b^2(Ab+aB)}{8x^{16}}-\frac{5ab^3(Ab+2aB}{14x^{14}}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 121, normalized size = 1.03

$$-\frac{126a^5\left(10A+11Bx^2\right)+770a^4bx^2\left(9A+10Bx^2\right)+1925a^3b^2x^4\left(8A+9Bx^2\right)+2475a^2b^3x^6\left(7A+8Bx^2\right)+1650ab^4x^8\left(6A+7Bx^2\right)+462b^5x^{10}\left(5A+6Bx^2\right)}{27720x^{22}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] -1/27720*(462*b^5*x^10*(5*A + 6*B*x^2) + 1650*a*b^4*x^8*(6*A + 7*B*x^2) + 2
475*a^2*b^3*x^6*(7*A + 8*B*x^2) + 1925*a^3*b^2*x^4*(8*A + 9*B*x^2) + 770*a^
4*b*x^2*(9*A + 10*B*x^2) + 126*a^5*(10*A + 11*B*x^2))/x^22
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{23}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^5*(A + B*x^2))/x^23, x]
```

fricas [**A**] time = 0.44, size = 121, normalized size = 1.03

$$-\frac{2772\,Bb^5x^{12}+2310\left(5\,Bab^4+Ab^5\right)x^{10}+9900\left(2\,Ba^2b^3+Aab^4\right)x^8+17325\left(Ba^3b^2+Aa^2b^3\right)x^6+1260\,Aa^5+7700\left(Ba^4b+2\,Aa^3b^2\right)x^4+1386\left(Ba^5+5\,Aa^4b\right)x^2}{27720\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

giac [A] time = 0.45, size = 127, normalized size = 1.09

$$-\frac{2772 \, B b^{5} x^{12} + 11550 \, B a b^{4} x^{10} + 2310 \, A b^{5} x^{10} + 19800 \, B a^{2} b^{3} x^{8} + 9900 \, A a b^{4} x^{8} + 17325 \, B a^{3} b^{2} x^{6} + 17325 \, A a^{2} b^{3} x^{6} + 7700 \, B a^{4} b x^{4} + 15400 \, A a^{3} b^{2} x^{4} + 1386 \, B a^{5} x^{2} + 6930 \, A a^{4} b x^{2} + 1260 \, A a^{5}}{27720 \, x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 11550*B*a*b^4*x^10 + 2310*A*b^5*x^10 + 19800*B*
a^2*b^3*x^8 + 9900*A*a*b^4*x^8 + 17325*B*a^3*b^2*x^6 + 17325*A*a^2*b^3*x^6
+ 7700*B*a^4*b*x^4 + 15400*A*a^3*b^2*x^4 + 1386*B*a^5*x^2 + 6930*A*a^4*b*x^
2 + 1260*A*a^5)/x^22
```

maple [A] time = 0.01, size = 104, normalized size = 0.89

$$-\frac{B\,b^{5}}{10x^{10}}-\frac{\left(Ab+5Ba\right)b^{4}}{12x^{12}}-\frac{5\left(Ab+2Ba\right)a\,b^{3}}{14x^{14}}-\frac{5\left(Ab+Ba\right)a^{2}b^{2}}{8x^{16}}-\frac{5\left(2Ab+Ba\right)a^{3}b}{18x^{18}}-\frac{A\,a^{5}}{22x^{22}}-\frac{\left(5Ab+Ba\right)a^{4}}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^5*(B*x^2+A)/x^23,x)
```

```
[Out] -1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)/x^18-5/8*
a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12*b^4*(A*b+5*B*a)/x^1
2-1/10*b^5*B/x^10
```

maxima [A] time = 1.01, size = 121, normalized size = 1.03

$$-\frac{2772 \, B b^{5} x^{12} + 2310 \, {\left(5 \, B a b^{4} + A b^{5}\right)} x^{10} + 9900 \, {\left(2 \, B a^{2} b^{3} + A a b^{4}\right)} x^{8} + 17325 \, {\left(B a^{3} b^{2} + A a^{2} b^{3}\right)} x^{6} + 1260 \, A a^{5} + 7700 \, {\left(B a^{4} b + 2 \, A a^{3} b^{2}\right)} x^{4} + 1386 \, {\left(B a^{5} + 5 \, A a^{4} b\right)} x^{2}}{27720 \, x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

mupad [B] time = 0.06, size = 122, normalized size = 1.04

$$-\frac{\frac{A\,a^5}{22}+x^8\,\left(\frac{5\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{14}\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{18}+\frac{5\,A\,a^3\,b^2}{9}\right)+x^2\,\left(\frac{B\,a^5}{20}+\frac{A\,b\,a^4}{4}\right)+x^{10}\,\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+x^6\,\left(\frac{5\,B\,a^3\,b^2}{8}+\frac{5\,A\,a^2\,b^3}{8}\right)+\frac{B\,b^5\,x^{12}}{10}}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^5)/x^23,x)
```

```
[Out] -((A*a^5)/22 + x^8*((5*B*a^2*b^3)/7 + (5*A*a*b^4)/14) + x^4*((5*A*a^3*b^2)/
9 + (5*B*a^4*b)/18) + x^2*((B*a^5)/20 + (A*a^4*b)/4) + x^10*((A*b^5)/12 + (
5*B*a*b^4)/12) + x^6*((5*A*a^2*b^3)/8 + (5*B*a^3*b^2)/8) + (B*b^5*x^12)/10)
/x^22
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**5*(B*x**2+A)/x**23,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{x^6\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=98

$$-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{a^2x(Ab-aB)}{b^4}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{x^5(Ab-aB)}{5b^2}+\frac{Bx^7}{7b}$$

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {459, 302, 205}

$$\frac{a^2x(Ab-aB)}{b^4}-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x^5(Ab-aB)}{5b^2}-\frac{ax^3(Ab-aB)}{3b^3}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*x)/b^4 - (a*(A*b - a*B)*x^3)/(3*b^3) + ((A*b - a*B)*x^5)/(
5*b^2) + (B*x^7)/(7*b) - (a^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{Bx^7}{7b}-\frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^2}\,dx}{7b}\\
&= \frac{Bx^7}{7b}-\frac{(-7Ab+7aB)\int \left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx}{7b}\\
&= \frac{a^2(Ab-aB)x}{b^4}-\frac{a(Ab-aB)x^3}{3b^3}+\frac{(Ab-aB)x^5}{5b^2}+\frac{Bx^7}{7b}-\frac{\left(a^3(Ab-aB)\right)\int \frac{1}{a+bx^2}\,dx}{b^4}\\
&= \frac{a^2(Ab-aB)x}{b^4}-\frac{a(Ab-aB)x^3}{3b^3}+\frac{(Ab-aB)x^5}{5b^2}+\frac{Bx^7}{7b}-\frac{a^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}
\end{aligned}$$

Mathematica [A]   time = 0.08, size = 98, normalized size = 1.00

$$\frac{a^{5/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^2x(aB-Ab)}{b^4}+\frac{ax^3(aB-Ab)}{3b^3}+\frac{x^5(Ab-aB)}{5b^2}+\frac{Bx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(A*b) + a*B)*x)/b^4) + (a*(-(A*b) + a*B)*x^3)/(3*b^3) + ((A*b - a*
B)*x^5)/(5*b^2) + (B*x^7)/(7*b) + (a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/b^(9/2)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(A+Bx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2), x]
```

fricas [A]   time = 0.46, size = 228, normalized size = 2.33

$$\left[\frac{30\,Bb^3x^7-42\left(Bab^2-Ab^3\right)x^5+70\left(Ba^2b-Aab^2\right)x^3-105\left(Ba^3-Aa^2b\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\left(Ba^3-Aa^2b\right)x}{210\,b^4},\frac{15\,Bb^3x^7-21\left(Bab^2-Ab^3\right)x^5+35\left(Ba^2b-Aab^2\right)x^3+105\left(Ba^3-Aa^2b\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-105\left(Ba^3-Aa^2b\right)x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*B*b^3*x^7 - 42*(B*a*b^2 - A*b^3)*x^5 + 70*(B*a^2*b - A*a*b^2)*x^
3 - 105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*
x^2 + a)) - 210*(B*a^3 - A*a^2*b)*x)/b^4, 1/105*(15*B*b^3*x^7 - 21*(B*a*b^2
 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 + 105*(B*a^3 - A*a^2*b)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 105*(B*a^3 - A*a^2*b)*x)/b^4]
```

giac [A] time = 0.33, size = 108, normalized size = 1.10

$$\frac{\left(Ba^4 - Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,Bb^6x^7 - 21\,Bab^5x^5 + 21\,Ab^6x^5 + 35\,Ba^2b^4x^3 - 35\,Aab^5x^3 - 105\,Ba^3b^3x + 105\,Aa^2b^4x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^6*x
^7 - 21*B*a*b^5*x^5 + 21*A*b^6*x^5 + 35*B*a^2*b^4*x^3 - 35*A*a*b^5*x^3 - 10
5*B*a^3*b^3*x + 105*A*a^2*b^4*x)/b^7
```

maple [A] time = 0.01, size = 116, normalized size = 1.18

$$\frac{B\,x^7}{7b} + \frac{A\,x^5}{5b} - \frac{Ba\,x^5}{5b^2} - \frac{Aa\,x^3}{3b^2} + \frac{B\,a^2x^3}{3b^3} - \frac{A\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} + \frac{B\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^4} + \frac{A\,a^2x}{b^3} - \frac{B\,a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/7*B*x^7/b+1/5/b*A*x^5-1/5/b^2*B*x^5*a-1/3/b^2*A*x^3*a+1/3/b^3*B*x^3*a^2+1
/b^3*A*a^2*x-1/b^4*B*a^3*x-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*A+a^
4/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*B
```

maxima [A] time = 2.41, size = 100, normalized size = 1.02

$$\frac{\left(Ba^4 - Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,Bb^3x^7 - 21\,\left(Bab^2 - Ab^3\right)x^5 + 35\,\left(Ba^2b - Aab^2\right)x^3 - 105\,\left(Ba^3 - Aa^2b\right)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^3*x
^7 - 21*(B*a*b^2 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 - 105*(B*a^3 - A
*a^2*b)*x)/b^4
```

mupad [B] time = 0.05, size = 118, normalized size = 1.20

$$x^5\left(\frac{A}{5\,b}-\frac{B\,a}{5\,b^2}\right)+\frac{B\,x^7}{7\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(A\,b-B\,a)}{B\,a^4-A\,a^3\,b}\right)\,(A\,b-B\,a)}{b^{9/2}}-\frac{a\,x^3\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{3\,b}+\frac{a^2\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^5*(A/(5*b) - (B*a)/(5*b^2)) + (B*x^7)/(7*b) + (a^(5/2)*atan((a^(5/2)*b^(1
/2)*x*(A*b - B*a))/(B*a^4 - A*a^3*b))*(A*b - B*a))/b^(9/2) - (a*x^3*(A/b -
(B*a)/b^2))/(3*b) + (a^2*x*(A/b - (B*a)/b^2))/b^2
```

sympy [B] time = 0.41, size = 180, normalized size = 1.84

$$\frac{Bx^7}{7b}+x^5\left(\frac{A}{5b}-\frac{Ba}{5b^2}\right)+x^3\left(-\frac{Aa}{3b^2}+\frac{Ba^2}{3b^3}\right)+x\left(\frac{Aa^2}{b^3}-\frac{Ba^3}{b^4}\right)-\frac{\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)}{-Aa^2b+Ba^3}+x\right)}{2}+\frac{\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)\log\left(\frac{b^4\sqrt{-\frac{a^5}{b^9}}\,(-Ab+Ba)}{-Aa^2b+Ba^3}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**7/(7*b) + x**5*(A/(5*b) - B*a/(5*b**2)) + x**3*(-A*a/(3*b**2) + B*a**2
/(3*b**3)) + x*(A*a**2/b**3 - B*a**3/b**4) - sqrt(-a**5/b**9)*(-A*b + B*a)*
log(-b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b + B*a**3) + x)/2 + sqrt(
-a**5/b**9)*(-A*b + B*a)*log(b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b
+ B*a**3) + x)/2
```

3.57 $\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{x^4(Ab-aB)}{4b^2}+\frac{Bx^6}{6b}$$

Rubi [**A**] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}+\frac{x^4(Ab-aB)}{4b^2}-\frac{ax^2(Ab-aB)}{2b^3}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*x^2)/(2*b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^6)/(6*b) +
(a^2*(A*b - a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^3}+\frac{(Ab-aB)x}{b^2}+\frac{Bx^2}{b}-\frac{a^2(-Ab+aB)}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{a(Ab-aB)x^2}{2b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^6}{6b}+\frac{a^2(Ab-aB)\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica **[A]** time = 0.03, size = 71, normalized size = 0.95

$$\frac{bx^2\left(6a^2B-3ab\left(2A+Bx^2\right)+b^2x^2\left(3A+2Bx^2\right)\right)+6a^2(Ab-aB)\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*B - 3*a*b*(2*A + B*x^2) + b^2*x^2*(3*A + 2*B*x^2)) + 6*a^2*(A
*b - a*B)*Log[a + b*x^2])/(12*b^4)
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2), x]
```

fricas **[A]** time = 0.45, size = 75, normalized size = 1.00

$$\frac{2\,Bb^3x^6-3\left(Bab^2-Ab^3\right)x^4+6\left(Ba^2b-Aab^2\right)x^2-6\left(Ba^3-Aa^2b\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^3*x^6 - 3*(B*a*b^2 - A*b^3)*x^4 + 6*(B*a^2*b - A*a*b^2)*x^2 - 6
*(B*a^3 - A*a^2*b)*log(b*x^2 + a))/b^4
```

giac [A] time = 0.31, size = 77, normalized size = 1.03

$$\frac{2\,Bb^2x^6-3\,Babx^4+3\,Ab^2x^4+6\,Ba^2x^2-6\,Aabx^2}{12\,b^3}-\frac{\left(Ba^3-Aa^2b\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*B*a*b*x^4 + 3*A*b^2*x^4 + 6*B*a^2*x^2 - 6*A*a*b*x^2)/
b^3 - 1/2*(B*a^3 - A*a^2*b)*log(abs(b*x^2 + a))/b^4
```

maple [A] time = 0.00, size = 86, normalized size = 1.15

$$\frac{B\,x^6}{6b}+\frac{A\,x^4}{4b}-\frac{Ba\,x^4}{4b^2}-\frac{Aa\,x^2}{2b^2}+\frac{B\,a^2x^2}{2b^3}+\frac{A\,a^2\ln\left(b\,x^2+a\right)}{2b^3}-\frac{B\,a^3\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/6*B*x^6/b+1/4/b*A*x^4-1/4/b^2*B*x^4*a-1/2/b^2*A*x^2*a+1/2/b^3*B*x^2*a^2+1
/2*a^2/b^3*ln(b*x^2+a)*A-1/2*a^3/b^4*ln(b*x^2+a)*B
```

maxima [A] time = 1.03, size = 74, normalized size = 0.99

$$\frac{2\,Bb^2x^6-3\left(Bab-Ab^2\right)x^4+6\left(Ba^2-Aab\right)x^2}{12\,b^3}-\frac{\left(Ba^3-Aa^2b\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*B*b^2*x^6 - 3*(B*a*b - A*b^2)*x^4 + 6*(B*a^2 - A*a*b)*x^2)/b^3 - 1/
2*(B*a^3 - A*a^2*b)*log(b*x^2 + a)/b^4
```

mupad [B] time = 0.06, size = 76, normalized size = 1.01

$$x^4\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)+\frac{B\,x^6}{6\,b}-\frac{\ln\left(b\,x^2+a\right)\,\left(B\,a^3-A\,a^2\,b\right)}{2\,b^4}-\frac{a\,x^2\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^6)/(6*b) - (log(a + b*x^2)*(B*a^3 - A*
a^2*b))/(2*b^4) - (a*x^2*(A/b - (B*a)/b^2))/(2*b)
```

sympy [A] time = 0.32, size = 70, normalized size = 0.93

$$\frac{Bx^6}{6b} - \frac{a^2\left(-Ab+Ba\right)\log\left(a+bx^2\right)}{2b^4} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x^2\left(-\frac{Aa}{2b^2} + \frac{Ba^2}{2b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**6/(6*b) - a**2*(-A*b + B*a)*log(a + b*x**2)/(2*b**4) + x**4*(A/(4*b) -
 B*a/(4*b**2)) + x**2*(-A*a/(2*b**2) + B*a**2/(2*b**3))
```

3.58 $\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(Ab-aB)}{b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^5}{5b}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {459, 302, 205}

$$\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^3(Ab-aB)}{3b^2}-\frac{ax(Ab-aB)}{b^3}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) + (a^(
3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{Bx^5}{5b}-\frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^2}\,dx}{5b}\\ &= \frac{Bx^5}{5b}-\frac{(-5Ab+5aB)\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{5b}\\ &= -\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^5}{5b}+\frac{\left(a^2(Ab-aB)\right)\int \frac{1}{a+bx^2}\,dx}{b^3}\\ &= -\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^5}{5b}+\frac{a^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.00

$$-\frac{a^{3/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{ax(aB-Ab)}{b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(A*b) + a*B)*x)/b^3 + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) - (a^(
3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.48, size = 178, normalized size = 2.31

$$\left[\frac{6\,Bb^2x^5-10\left(Bab-Ab^2\right)x^3-15\left(Ba^2-Aab\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\left(Ba^2-Aab\right)x}{30\,b^3},\ \frac{3\,Bb^2x^5-5\left(Bab-Ab^2\right)x^3-15\left(Ba^2-Aab\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\left(Ba^2-Aab\right)x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*B*b^2*x^5 - 10*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(-a/b)
*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(B*a^2 - A*a*b)*x)/b^
3, 1/15*(3*B*b^2*x^5 - 5*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(a/b)
*arctan(b*x*sqrt(a/b)/a) + 15*(B*a^2 - A*a*b)*x)/b^3]
```

giac [A] time = 0.33, size = 85, normalized size = 1.10

$$-\frac{\left(Ba^3-Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,Bb^4x^5-5\,Bab^3x^3+5\,Ab^4x^3+15\,Ba^2b^2x-15\,Aab^3x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^4*x^
5 - 5*B*a*b^3*x^3 + 5*A*b^4*x^3 + 15*B*a^2*b^2*x - 15*A*a*b^3*x)/b^5
```

maple [A] time = 0.00, size = 92, normalized size = 1.19

$$\frac{B\,x^5}{5b}+\frac{A\,x^3}{3b}-\frac{Ba\,x^3}{3b^2}+\frac{A\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{B\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}-\frac{Aax}{b^2}+\frac{B\,a^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/5*B*x^5/b+1/3/b*A*x^3-1/3/b^2*B*x^3*a-1/b^2*a*A*x+1/b^3*a^2*B*x+a^2/b^2/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)*B
```

maxima [A] time = 2.46, size = 78, normalized size = 1.01

$$-\frac{\left(Ba^3-Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,Bb^2x^5-5\,\left(Bab-Ab^2\right)x^3+15\,\left(Ba^2-Aab\right)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^2*x^
5 - 5*(B*a*b - A*b^2)*x^3 + 15*(B*a^2 - A*a*b)*x)/b^3
```

mupad [B] time = 0.09, size = 96, normalized size = 1.25

$$x^3\,\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^2}\right)+\frac{B\,x^5}{5\,b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(A\,b-B\,a\right)}{B\,a^3-A\,a^2\,b}\right)\,\left(A\,b-B\,a\right)}{b^{7/2}}-\frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (B*x^5)/(5*b) - (a^(3/2)*atan((a^(3/2)*b^(1
/2)*x*(A*b - B*a))/(B*a^3 - A*a^2*b))*(A*b - B*a))/b^(7/2) - (a*x*(A/b - (B
*a)/b^2))/b
```

sympy [B] time = 0.37, size = 153, normalized size = 1.99

$$\frac{Bx^5}{5b}+x^3\left(\frac{A}{3b}-\frac{Ba}{3b^2}\right)+x\left(-\frac{Aa}{b^2}+\frac{Ba^2}{b^3}\right)+\frac{\sqrt{-\frac{a^3}{b^7}}\,(-Ab+Ba)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(-Ab+Ba)}{-Aab+Ba^2}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^7}}\,(-Ab+Ba)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(-Ab+Ba)}{-Aab+Ba^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**5/(5*b) + x**3*(A/(3*b) - B*a/(3*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
+ sqrt(-a**3/b**7)*(-A*b + B*a)*log(-b**3*sqrt(-a**3/b**7)*(-A*b + B*a)/(-A
*a*b + B*a**2) + x)/2 - sqrt(-a**3/b**7)*(-A*b + B*a)*log(b**3*sqrt(-a**3/b
**7)*(-A*b + B*a)/(-A*a*b + B*a**2) + x)/2
```

3.59 $\int \frac{x^3\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=54

$$-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aB)}{2b^2}+\frac{Bx^4}{4b}$$

Rubi **[A]** time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{x^2(Ab-aB)}{2b^2}-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}+\frac{Bx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^4)/(4*b) - (a*(A*b - a*B)*Log[a + b*x^2])/
(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^4}{4b}-\frac{a(Ab-aB)\log\left(a+bx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.87

$$\frac{bx^2\left(-2aB+2Ab+bBx^2\right)+2a(aB-Ab)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*A*b - 2*a*B + b*B*x^2) + 2*a*(-(A*b) + a*B)*Log[a + b*x^2])/(4*b^
3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.43, size = 51, normalized size = 0.94

$$\frac{Bb^2x^4-2\left(Bab-Ab^2\right)x^2+2\left(Ba^2-Aab\right)\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^2*x^4 - 2*(B*a*b - A*b^2)*x^2 + 2*(B*a^2 - A*a*b)*log(b*x^2 + a))/
b^3
```

giac [A] time = 0.27, size = 52, normalized size = 0.96

$$\frac{Bbx^4-2\,Bax^2+2\,Abx^2}{4\,b^2}+\frac{\left(Ba^2-Aab\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*b*x^4 - 2*B*a*x^2 + 2*A*b*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(abs(b*x
^2 + a))/b^3
```

maple [A] time = 0.00, size = 62, normalized size = 1.15

$$\frac{B\,x^4}{4b}+\frac{A\,x^2}{2b}-\frac{Ba\,x^2}{2b^2}-\frac{Aa\ln\left(b\,x^2+a\right)}{2b^2}+\frac{B\,a^2\ln\left(b\,x^2+a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a-1/2*a/b^2*ln(b*x^2+a)*A+1/2*a^2/b^3
*ln(b*x^2+a)*B
```

maxima [A] time = 1.03, size = 50, normalized size = 0.93

$$\frac{Bbx^4-2\,(Ba-Ab)x^2}{4\,b^2}+\frac{\left(Ba^2-Aab\right)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 2*(B*a - A*b)*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(b*x^2 + a)/
b^3
```

mupad [B] time = 0.10, size = 52, normalized size = 0.96

$$x^2\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(B\,a^2-A\,a\,b\right)}{2\,b^3}+\frac{B\,x^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^2*(A/(2*b) - (B*a)/(2*b^2)) + (log(a + b*x^2)*(B*a^2 - A*a*b))/(2*b^3) +
(B*x^4)/(4*b)
```

sympy [A] time = 0.29, size = 46, normalized size = 0.85

$$\frac{Bx^4}{4b} + \frac{a(-Ab + Ba)\log\left(a + bx^2\right)}{2b^3} + x^2\left(\frac{A}{2b} - \frac{Ba}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**4/(4*b) + a*(-A*b + B*a)*log(a + b*x**2)/(2*b**3) + x**2*(A/(2*b) - B*
a/(2*b**2))
```

3.60 $\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^3}{3b}$$

Rubi [**A**] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {459, 321, 205}

$$\frac{x(Ab-aB)}{b^2}-\frac{\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) - (Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{Bx^3}{3b}-\frac{(-3Ab+3aB)\int \frac{x^2}{a+bx^2}\,dx}{3b}\\ &= \frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{(a(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{b^2}\\ &= \frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.98

$$\frac{\sqrt{a}\,(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aB)}{b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) + (Sqrt[a]*(-(A*b) + a*B)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.46, size = 129, normalized size = 2.22

$$\left[\frac{2\,Bbx^3-3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Ba-Ab)x}{6\,b^2},\ \frac{Bbx^3+3\,(Ba-Ab)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,(Ba-Ab)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*b*x^3 - 3*(B*a - A*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) - 6*(B*a - A*b)*x)/b^2, 1/3*(B*b*x^3 + 3*(B*a - A*b)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) - 3*(B*a - A*b)*x)/b^2]
```

giac [A] time = 0.39, size = 57, normalized size = 0.98

$$\frac{\left(Ba^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{Bb^2x^3 - 3\,Babx + 3\,Ab^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b^2*x^3 - 3*
B*a*b*x + 3*A*b^2*x)/b^3
```

maple [A] time = 0.00, size = 68, normalized size = 1.17

$$\frac{B\,x^3}{3b} - \frac{Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{B\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} + \frac{Ax}{b} - \frac{Bax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/b+1/b*A*x-1/b^2*B*a*x-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A
+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.23, size = 53, normalized size = 0.91

$$\frac{\left(Ba^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{Bbx^3 - 3\,(Ba - Ab)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b*x^3 - 3*(B
*a - A*b)*x)/b^2
```

mupad [B] time = 0.10, size = 70, normalized size = 1.21

$$x\left(\frac{A}{b} - \frac{B\,a}{b^2}\right) + \frac{B\,x^3}{3\,b} + \frac{\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(A\,b - B\,a)}{B\,a^2 - A\,a\,b}\right)(A\,b - B\,a)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x*(A/b - (B*a)/b^2) + (B*x^3)/(3*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(A*b
 - B*a))/(B*a^2 - A*a*b))*(A*b - B*a))/b^(5/2)
```

sympy [A] time = 0.33, size = 90, normalized size = 1.55

$$\frac{Bx^3}{3b}+x\left(\frac{A}{b}-\frac{Ba}{b^2}\right)-\frac{\sqrt{-\frac{a}{b^5}}\,(-Ab+Ba)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^5}}\,(-Ab+Ba)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**3/(3*b) + x*(A/b - B*a/b**2) - sqrt(-a/b**5)*(-A*b + B*a)*log(-b**2*sq
rt(-a/b**5) + x)/2 + sqrt(-a/b**5)*(-A*b + B*a)*log(b**2*sqrt(-a/b**5) + x)
/2
```

3.61 $\int \frac{x\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=35

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2b^2}+\frac{Bx^2}{2b}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2b^2}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (B*x^2)/(2*b) + ((A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{Bx^2}{2b}+\frac{(Ab-aB)\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.89

$$\frac{(Ab-aB)\log\left(a+bx^2\right)+bBx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (b*B*x^2 + (A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.39, size = 30, normalized size = 0.86

$$\frac{Bbx^2-(Ba-Ab)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(B*b*x^2 - (B*a - A*b)*log(b*x^2 + a))/b^2
```

giac [A] time = 0.34, size = 32, normalized size = 0.91

$$\frac{Bx^2}{2\,b}-\frac{(Ba-Ab)\log\left(\left|bx^2+a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/b^2
```

maple [A] time = 0.00, size = 40, normalized size = 1.14

$$\frac{B\,x^2}{2b}+\frac{A\ln\left(b\,x^2+a\right)}{2b}-\frac{Ba\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 1/2*B*x^2/b+1/2/b*ln(b*x^2+a)*A-1/2/b^2*ln(b*x^2+a)*B*a
```

maxima [A] time = 1.09, size = 31, normalized size = 0.89

$$\frac{Bx^2}{2\,b} - \frac{(Ba - Ab)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b - 1/2*(B*a - A*b)*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.05, size = 31, normalized size = 0.89

$$\frac{B\,x^2}{2\,b} + \frac{\ln\left(b\,x^2 + a\right)\,(A\,b - B\,a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] (B*x^2)/(2*b) + (log(a + b*x^2)*(A*b - B*a))/(2*b^2)
```

sympy [A] time = 0.25, size = 27, normalized size = 0.77

$$\frac{Bx^2}{2b} - \frac{(-Ab + Ba)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x**2/(2*b) - (-A*b + B*a)*log(a + b*x**2)/(2*b**2)
```

3.62 $\int \frac{A+Bx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Bx}{b}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a+bx^2}\,dx &= \frac{Bx}{b}-\frac{(-Ab+aB)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{Bx}{b}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 40, normalized size = 1.03

$$\frac{Bx}{b}-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] (B*x)/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2), x]
```

fricas [**A**] time = 0.46, size = 99, normalized size = 2.54

$$\left[\frac{2\,Babx+(Ba-Ab)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{Babx-(Ba-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x + (B*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (B*a*b*x - (B*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

giac [**A**] time = 0.28, size = 34, normalized size = 0.87

$$\frac{Bx}{b}-\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.00, size = 45, normalized size = 1.15

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b}+\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a),x)
```

```
[Out] B*x/b+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A-1/b/(a*b)^(1/2)*arctan(1/(a
*b)^(1/2)*b*x)*B*a
```

maxima [A] time = 2.50, size = 34, normalized size = 0.87

$$\frac{Bx}{b}-\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.05, size = 31, normalized size = 0.79

$$\frac{B\,x}{b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2),x)
```

```
[Out] (B*x)/b + (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(1/2)*b^(3/2))
```

sympy [B] time = 0.28, size = 82, normalized size = 2.10

$$\frac{Bx}{b}+\frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab+Ba)\log\left(-ab\sqrt{-\frac{1}{ab^3}}\,+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(-Ab+Ba)\log\left(ab\sqrt{-\frac{1}{ab^3}}\,+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a),x)
```

```
[Out] B*x/b + sqrt(-1/(a*b**3))*(-A*b + B*a)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
sqrt(-1/(a*b**3))*(-A*b + B*a)*log(a*b*sqrt(-1/(a*b**3)) + x)/2
```

3.63 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 72}

$$\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^2])/(2*a*b)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx&=\frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB}{a(a+bx)}\right)dx,x,x^2\right)\\&=\frac{A\log(x)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{2ab}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{(aB-Ab)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^2])/(2*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)), x]
```

fricas [A] time = 0.47, size = 32, normalized size = 0.94

$$\frac{2\,Ab\log(x)+(Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*b*log(x) + (B*a - A*b)*log(b*x^2 + a))/(a*b)
```

giac [A] time = 0.29, size = 36, normalized size = 1.06

$$\frac{A\log\left(x^2\right)}{2\,a}+\frac{(Ba-Ab)\log\left(\left|bx^2+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/(a*b)
```

maple [A] time = 0.01, size = 37, normalized size = 1.09

$$\frac{A\ln(x)}{a}-\frac{A\ln\left(b\,x^2+a\right)}{2a}+\frac{B\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a),x)
```

```
[Out] -1/2/a*ln(b*x^2+a)*A+1/2/b*ln(b*x^2+a)*B+A*ln(x)/a
```

maxima [A] time = 1.02, size = 35, normalized size = 1.03

$$\frac{A\log\left(x^2\right)}{2\,a}+\frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(b*x^2 + a)/(a*b)
```

mupad [B] time = 0.13, size = 32, normalized size = 0.94

$$\frac{A\,\ln(x)}{a}-\frac{\ln\left(b\,x^2+a\right)\,(A\,b-B\,a)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)),x)
```

```
[Out] (A*log(x))/a - (log(a + b*x^2)*(A*b - B*a))/(2*a*b)
```

sympy [A] time = 0.70, size = 26, normalized size = 0.76

$$\frac{A\log(x)}{a}+\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a),x)
```

```
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**2)/(2*a*b)
```

3.64 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Rubi **[A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 205}

$$-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx &= -\frac{A}{ax}-\frac{(Ab-aB)\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{A}{ax}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.98

$$\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 0.45, size = 105, normalized size = 2.44

$$\left[\frac{(Ba-Ab)\sqrt{-ab}\,x\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,Aab}{2\,a^2bx},\ \frac{(Ba-Ab)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-Aab}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*((B*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*A*a*b)/(a^2*b*x), ((B*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) - A*
a*b)/(a^2*b*x)]
```

giac [A] time = 0.30, size = 36, normalized size = 0.84

$$\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

maple [A] time = 0.02, size = 48, normalized size = 1.12

$$-\frac{Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a),x)
```

```
[Out] -1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A*b+1/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)*B-A/a/x
```

maxima [A] time = 2.36, size = 36, normalized size = 0.84

$$\frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

mupad [B] time = 0.06, size = 35, normalized size = 0.81

$$-\frac{A}{a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b-B\,a)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)),x)
```

```
[Out] - A/(a*x) - (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(3/2)*b^(1/2))
```

sympy [B] time = 0.34, size = 82, normalized size = 1.91

$$-\frac{A}{ax}-\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(-Ab+Ba)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a),x)
```

```
[Out] -A/(a*x) - sqrt(-1/(a**3*b))*(-A*b + B*a)*log(-a**2*sqrt(-1/(a**3*b)) + x)/
2 + sqrt(-1/(a**3*b))*(-A*b + B*a)*log(a**2*sqrt(-1/(a**3*b)) + x)/2
```

3.65 $\int \frac{A+Bx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=50

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -A/(2*a*x^2) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])/(2*a
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)dx, x, x^2\right)\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.98

$$\frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)(aB-Ab)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*A/(a*x^2) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2])
/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 0.46, size = 47, normalized size = 0.94

$$-\frac{(Ba-Ab)x^2\log\left(bx^2+a\right)-2\,(Ba-Ab)x^2\log(x)+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*((B*a - A*b)*x^2*log(b*x^2 + a) - 2*(B*a - A*b)*x^2*log(x) + A*a)/(a^2
*x^2)
```

giac [A] time = 0.32, size = 71, normalized size = 1.42

$$\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{\left(Bab-Ab^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}-\frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*(B*a*b - A*b^2)*log(abs(b*x^2 + a))/(a^2
*b) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

maple [A] time = 0.01, size = 56, normalized size = 1.12

$$-\frac{Ab\ln(x)}{a^2}+\frac{Ab\ln\left(b\,x^2+a\right)}{2a^2}+\frac{B\ln(x)}{a}-\frac{B\ln\left(b\,x^2+a\right)}{2a}-\frac{A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a),x)
```

```
[Out] 1/2/a^2*ln(b*x^2+a)*A*b-1/2/a*ln(b*x^2+a)*B-1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a
*ln(x)*B
```

maxima [A] time = 1.04, size = 48, normalized size = 0.96

$$-\frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,a^2}+\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*log(b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*A/
(a*x^2)
```

mupad [B] time = 0.13, size = 46, normalized size = 0.92

$$\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-B\,a\right)}{2\,a^2}-\frac{A}{2\,a\,x^2}-\frac{\ln(x)\,\left(A\,b-B\,a\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(A*b - B*a))/(2*a^2) - A/(2*a*x^2) - (log(x)*(A*b - B*a))/a
^2
```

sympy [A] time = 0.71, size = 41, normalized size = 0.82

$$-\frac{A}{2ax^2} + \frac{(-Ab + Ba)\log(x)}{a^2} - \frac{(-Ab + Ba)\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**2)/(2*
a**2)
```

3.66 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {453, 325, 205}

$$\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) + (Sqrt[b]*(A*b - a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx &= -\frac{A}{3ax^3}-\frac{(3Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{3a}\\ &= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{(b(Ab-aB))\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\sqrt{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.02

$$-\frac{\sqrt{b}\,(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[b]*(-(A*b) + a*B)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)), x]
```

fricas [A] time = 0.45, size = 135, normalized size = 2.29

$$\left[-\frac{3\,(Ba-Ab)x^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,(Ba-Ab)x^2+2\,Aa}{6\,a^2x^3},\,-\frac{3\,(Ba-Ab)x^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(B*a - A*b)*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 6*(B*a - A*b)*x^2 + 2*A*a)/(a^2*x^3), -1/3*(3*(B*a - A*b)*x^3*sq
rt(b/a)*arctan(x*sqrt(b/a)) + 3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)]
```

giac [A] time = 0.32, size = 57, normalized size = 0.97

$$-\frac{\left(Bab - Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} - \frac{3\,Bax^2 - 3\,Abx^2 + Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*B*a*x^2 - 3
*A*b*x^2 + A*a)/(a^2*x^3)
```

maple [A] time = 0.01, size = 72, normalized size = 1.22

$$\frac{A\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2} - \frac{Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} + \frac{Ab}{a^2x} - \frac{B}{ax} - \frac{A}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a),x)
```

```
[Out] b^2/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A-b/a/(a*b)^(1/2)*arctan(1/(a
*b)^(1/2)*b*x)*B-1/3*A/a/x^3+1/a^2/x*A*b-1/a/x*B
```

maxima [A] time = 2.39, size = 56, normalized size = 0.95

$$-\frac{\left(Bab - Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} - \frac{3\,(Ba - Ab)x^2 + Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*(B*a - A*b)
*x^2 + A*a)/(a^2*x^3)
```

mupad [B] time = 0.10, size = 53, normalized size = 0.90

$$\frac{\sqrt{b}\;\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b - B\,a\right)}{a^{5/2}} - \frac{\frac{A}{3\,a} - \frac{x^2\,\left(A\,b-B\,a\right)}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(5/2) - (A/(3*a) - (x^2*(
A*b - B*a))/a^2)/x^3
```

sympy [B] time = 0.42, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}\,(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2}+\frac{-Aa+x^2\,(3Ab-3Ba)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(-A*b + B*a)*log(-a**3*sqrt(-b/a**5)*(-A*b + B*a)/(-A*b**2 +
B*a*b) + x)/2 - sqrt(-b/a**5)*(-A*b + B*a)*log(a**3*sqrt(-b/a**5)*(-A*b + B
*a)/(-A*b**2 + B*a*b) + x)/2 + (-A*a + x**2*(3*A*b - 3*B*a))/(3*a**2*x**3)
```

3.67 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=69

$$-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{2a^2x^2}-\frac{A}{4ax^4}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{Ab-aB}{2a^2x^2}-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(2*a^2*x^2) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{A}{4ax^4}+\frac{Ab-aB}{2a^2x^2}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 1.01

$$\frac{4bx^4\log(x)(Ab-aB)-a\left(aA+2aBx^2-2Abx^2\right)+2bx^4(aB-Ab)\log\left(a+bx^2\right)}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^2 + 2*a*B*x^2)) + 4*b*(A*b - a*B)*x^4*Log[x] + 2*b*(-(A
*b) + a*B)*x^4*Log[a + b*x^2])/(4*a^3*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)), x]
```

fricas [A] time = 0.47, size = 73, normalized size = 1.06

$$\frac{2\left(Bab-Ab^2\right)x^4\log\left(bx^2+a\right)-4\left(Bab-Ab^2\right)x^4\log(x)-Aa^2-2\left(Ba^2-Aab\right)x^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a*b - A*b^2)*x^4*log(b*x^2 + a) - 4*(B*a*b - A*b^2)*x^4*log(x) -
A*a^2 - 2*(B*a^2 - A*a*b)*x^2)/(a^3*x^4)
```

giac [A] time = 0.36, size = 100, normalized size = 1.45

$$-\frac{\left(Bab - Ab^2\right)\log\left(x^2\right)}{2\,a^3} + \frac{\left(Bab^2 - Ab^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^3b} + \frac{3\,Babx^4 - 3\,Ab^2x^4 - 2\,Ba^2x^2 + 2\,Aabx^2 - Aa^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b - A*b^2)*log(x^2)/a^3 + 1/2*(B*a*b^2 - A*b^3)*log(abs(b*x^2 + a
))/(a^3*b) + 1/4*(3*B*a*b*x^4 - 3*A*b^2*x^4 - 2*B*a^2*x^2 + 2*A*a*b*x^2 - A
*a^2)/(a^3*x^4)
```

maple [A] time = 0.01, size = 81, normalized size = 1.17

$$\frac{A\,b^2\ln(x)}{a^3} - \frac{A\,b^2\ln\left(b\,x^2 + a\right)}{2a^3} - \frac{Bb\ln(x)}{a^2} + \frac{Bb\ln\left(b\,x^2 + a\right)}{2a^2} + \frac{Ab}{2a^2x^2} - \frac{B}{2a\,x^2} - \frac{A}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a),x)
```

```
[Out] -1/2*b^2/a^3*ln(b*x^2+a)*A+1/2*b/a^2*ln(b*x^2+a)*B-1/4*A/a/x^4+1/2/a^2/x^2*
A*b-1/2/a/x^2*B+1/a^3*b^2*ln(x)*A-1/a^2*b*ln(x)*B
```

maxima [A] time = 1.12, size = 70, normalized size = 1.01

$$\frac{\left(Bab - Ab^2\right)\log\left(bx^2 + a\right)}{2\,a^3} - \frac{\left(Bab - Ab^2\right)\log\left(x^2\right)}{2\,a^3} - \frac{2\,(Ba - Ab)x^2 + Aa}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b - A*b^2)*log(b*x^2 + a)/a^3 - 1/2*(B*a*b - A*b^2)*log(x^2)/a^3 -
 1/4*(2*(B*a - A*b)*x^2 + A*a)/(a^2*x^4)
```

mupad [B] time = 0.13, size = 70, normalized size = 1.01

$$\frac{\ln(x)\,\left(A\,b^2 - B\,a\,b\right)}{a^3} - \frac{\ln\left(b\,x^2 + a\right)\,\left(A\,b^2 - B\,a\,b\right)}{2\,a^3} - \frac{\frac{A}{4\,a} - \frac{x^2\,(A\,b - B\,a)}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)),x)
```

```
[Out] (log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^2)*(A*b^2 - B*a*b))/(2*a^3) - (
A/(4*a) - (x^2*(A*b - B*a))/(2*a^2))/x^4
```

sympy [A] time = 0.83, size = 61, normalized size = 0.88

$$\frac{-Aa + x^2 (2Ab - 2Ba)}{4a^2x^4} - \frac{b(-Ab + Ba)\log(x)}{a^3} + \frac{b(-Ab + Ba)\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a),x)
```

```
[Out] (-A*a + x**2*(2*A*b - 2*B*a))/(4*a**2*x**4) - b*(-A*b + B*a)*log(x)/a**3 +
b*(-A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

3.68 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=80

$$-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}-\frac{b(Ab-aB)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{5ax^5}$$

Rubi [**A**] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {453, 325, 205}

$$-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(3*a^2*x^3) - (b*(A*b - a*B))/(a^3*x) - (b^(3/2)
*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx &= -\frac{A}{5ax^5}-\frac{(5Ab-5aB)\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{5a}\\
&= -\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}+\frac{(b(Ab-aB))\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{\left(b^2(Ab-aB)\right)\int \frac{1}{a+bx^2}\,dx}{a^3}\\
&= -\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{b^{3/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica **[A]** time = 0.05, size = 78, normalized size = 0.98

$$\frac{b^{3/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}+\frac{b(aB-Ab)}{a^3x}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*A/(a*x^5) + (A*b - a*B)/(3*a^2*x^3) + (b*(-(A*b) + a*B))/(a^3*x) + (b^
(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)), x]
```

fricas **[A]** time = 0.48, size = 184, normalized size = 2.30

$$\left[-\frac{15\left(Bab-Ab^2\right)x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-30\left(Bab-Ab^2\right)x^4+6Aa^2+10\left(Ba^2-Aab\right)x^2}{30a^3x^5},\ \frac{15\left(Bab-Ab^2\right)x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+15\left(Bab-Ab^2\right)x^4-3Aa^2-5\left(Ba^2-Aab\right)x^2}{15a^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(B*a*b - A*b^2)*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) - 30*(B*a*b - A*b^2)*x^4 + 6*A*a^2 + 10*(B*a^2 - A*a*b)*x^2)
/(a^3*x^5), 1/15*(15*(B*a*b - A*b^2)*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15
*(B*a*b - A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)]
```

giac [A] time = 0.30, size = 81, normalized size = 1.01

$$\frac{\left(Bab^2 - Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{15\,Babx^4 - 15\,Ab^2x^4 - 5\,Ba^2x^2 + 5\,Aabx^2 - 3\,Aa^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*B*a*b*x^
4 - 15*A*b^2*x^4 - 5*B*a^2*x^2 + 5*A*a*b*x^2 - 3*A*a^2)/(a^3*x^5)
```

maple [A] time = 0.01, size = 96, normalized size = 1.20

$$-\frac{A\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3} + \frac{B\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2} - \frac{A\,b^2}{a^3x} + \frac{Bb}{a^2x} + \frac{Ab}{3a^2x^3} - \frac{B}{3a\,x^3} - \frac{A}{5a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a),x)
```

```
[Out] -b^3/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+b^2/a^2/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x)*B-1/5*A/a/x^5+1/3/a^2/x^3*A*b-1/3/a/x^3*B-1/a^3*b^2/x*A
+1/a^2*b/x*B
```

maxima [A] time = 2.46, size = 79, normalized size = 0.99

$$\frac{\left(Bab^2 - Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{15\left(Bab - Ab^2\right)x^4 - 3\,Aa^2 - 5\left(Ba^2 - Aab\right)x^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*(B*a*b -
 A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)
```

mupad [**B**] time = 0.10, size = 70, normalized size = 0.88

$$-\frac{\frac{A}{5\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{3\,a^2}+\frac{b\,x^4\,\left(A\,b-B\,a\right)}{a^3}}{x^5}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)),x)
```

```
[Out] - (A/(5*a) - (x^2*(A*b - B*a))/(3*a^2) + (b*x^4*(A*b - B*a))/a^3)/x^5 - (b^
(3/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(7/2)
```

sympy [**B**] time = 0.49, size = 163, normalized size = 2.04

$$-\frac{\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)}{-Ab^3+Bab^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}\,(-Ab+Ba)}{-Ab^3+Bab^2}+x\right)}{2}+\frac{-3Aa^2+x^4\left(-15Ab^2+15Bab\right)+x^2\left(5Aab-5Ba^2\right)}{15a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**7)*(-A*b + B*a)*log(-a**4*sqrt(-b**3/a**7)*(-A*b + B*a)/(-A*
b**3 + B*a*b**2) + x)/2 + sqrt(-b**3/a**7)*(-A*b + B*a)*log(a**4*sqrt(-b**3
/a**7)*(-A*b + B*a)/(-A*b**3 + B*a*b**2) + x)/2 + (-3*A*a**2 + x**4*(-15*A*
b**2 + 15*B*a*b) + x**2*(5*A*a*b - 5*B*a**2))/(15*a**3*x**5)
```

3.69 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=93

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Rubi **[A]** time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{b^2\log(x)(Ab-aB)}{a^4}-\frac{b(Ab-aB)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(2*a^3*x^2) - (b^2
*(A*b - a*B)*Log[x])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x^2])/(2*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{ax^4}+\frac{-Ab+aB}{a^2x^3}-\frac{b(-Ab+aB)}{a^3x^2}+\frac{b^2(-Ab+aB)}{a^4x}-\frac{b^3(-Ab+aB)}{a^4(a+bx)}\right)dx,x,x\right.\\ &= -\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{2a^3x^2}-\frac{b^2(Ab-aB)\log(x)}{a^4}+\frac{b^2(Ab-aB)\log\left(a+bx^2\right)}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 96, normalized size = 1.03

$$\frac{\left(Ab^3-ab^2B\right)\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)\left(ab^2B-Ab^3\right)}{a^4}+\frac{b(aB-Ab)}{2a^3x^2}+\frac{Ab-aB}{4a^2x^4}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] -1/6*A/(a*x^6) + (A*b - a*B)/(4*a^2*x^4) + (b*(-(A*b) + a*B))/(2*a^3*x^2) +
 ((-(A*b^3) + a*b^2*B)*Log[x])/a^4 + ((A*b^3 - a*b^2*B)*Log[a + b*x^2])/(2*
a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)), x]
```

fricas [A] time = 0.45, size = 98, normalized size = 1.05

$$-\frac{6\left(Bab^2-Ab^3\right)x^6\log\left(bx^2+a\right)-12\left(Bab^2-Ab^3\right)x^6\log(x)-6\left(Ba^2b-Aab^2\right)x^4+2\,Aa^3+3\left(Ba^3-Aa^2b\right)x^2}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*(B*a*b^2 - A*b^3)*x^6*log(b*x^2 + a) - 12*(B*a*b^2 - A*b^3)*x^6*lo
g(x) - 6*(B*a^2*b - A*a*b^2)*x^4 + 2*A*a^3 + 3*(B*a^3 - A*a^2*b)*x^2)/(a^4*
x^6)
```

giac [**A**] time = 0.28, size = 126, normalized size = 1.35

$$\frac{\left(Bab^2 - Ab^3\right)\log\left(x^2\right)}{2\,a^4} - \frac{\left(Bab^3 - Ab^4\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^4b} - \frac{11\,Bab^2x^6 - 11\,Ab^3x^6 - 6\,Ba^2bx^4 + 6\,Aab^2x^4 + 3\,Ba^3x^2 - 3\,Aa^2bx^2 + 2\,Aa^3}{12\,a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b^2 - A*b^3)*log(x^2)/a^4 - 1/2*(B*a*b^3 - A*b^4)*log(abs(b*x^2 +
a))/(a^4*b) - 1/12*(11*B*a*b^2*x^6 - 11*A*b^3*x^6 - 6*B*a^2*b*x^4 + 6*A*a*b
^2*x^4 + 3*B*a^3*x^2 - 3*A*a^2*b*x^2 + 2*A*a^3)/(a^4*x^6)
```

maple [**A**] time = 0.01, size = 107, normalized size = 1.15

$$-\frac{A\,b^3\ln(x)}{a^4} + \frac{A\,b^3\ln\left(b\,x^2 + a\right)}{2a^4} + \frac{B\,b^2\ln(x)}{a^3} - \frac{B\,b^2\ln\left(b\,x^2 + a\right)}{2a^3} - \frac{A\,b^2}{2a^3x^2} + \frac{Bb}{2a^2x^2} + \frac{Ab}{4a^2x^4} - \frac{B}{4a\,x^4} - \frac{A}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a),x)
```

```
[Out] 1/2*b^3/a^4*ln(b*x^2+a)*A-1/2*b^2/a^3*ln(b*x^2+a)*B-1/6*A/a/x^6+1/4/a^2/x^4
*A*b-1/4/a/x^4*B-1/2/a^3*b^2/x^2*A+1/2/a^2*b/x^2*B-1/a^4*b^3*ln(x)*A+1/a^3*
b^2*ln(x)*B
```

maxima [**A**] time = 0.97, size = 96, normalized size = 1.03

$$-\frac{\left(Bab^2 - Ab^3\right)\log\left(bx^2 + a\right)}{2\,a^4} + \frac{\left(Bab^2 - Ab^3\right)\log\left(x^2\right)}{2\,a^4} + \frac{6\left(Bab - Ab^2\right)x^4 - 2\,Aa^2 - 3\left(Ba^2 - Aab\right)x^2}{12\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*b^2 - A*b^3)*log(b*x^2 + a)/a^4 + 1/2*(B*a*b^2 - A*b^3)*log(x^2)/
a^4 + 1/12*(6*(B*a*b - A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 - A*a*b)*x^2)/(a^3*x
^6)
```

mupad [**B**] time = 0.14, size = 92, normalized size = 0.99

$$\frac{\ln\left(b\,x^2 + a\right)\,\left(A\,b^3 - B\,a\,b^2\right)}{2\,a^4} - \frac{\frac{A}{6\,a} - \frac{x^2\,\left(A\,b - B\,a\right)}{4\,a^2} + \frac{b\,x^4\,\left(A\,b - B\,a\right)}{2\,a^3}}{x^6} - \frac{\ln(x)\,\left(A\,b^3 - B\,a\,b^2\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(A*b^3 - B*a*b^2))/(2*a^4) - (A/(6*a) - (x^2*(A*b - B*a))/(
4*a^2) + (b*x^4*(A*b - B*a))/(2*a^3))/x^6 - (log(x)*(A*b^3 - B*a*b^2))/a^4
```

sympy [A] time = 0.90, size = 88, normalized size = 0.95

$$\frac{-2Aa^2 + x^4\left(-6Ab^2 + 6Bab\right) + x^2\left(3Aab - 3Ba^2\right)}{12a^3x^6} + \frac{b^2\left(-Ab + Ba\right)\log\left(x\right)}{a^4} - \frac{b^2\left(-Ab + Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a),x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 6*B*a*b) + x**2*(3*A*a*b - 3*B*a**2))/(12*a*
*3*x**6) + b**2*(-A*b + B*a)*log(x)/a**4 - b**2*(-A*b + B*a)*log(a/b + x**2
)/(2*a**4)
```

3.70 $\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=99

$$\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}+\frac{b^2(Ab-aB)}{a^4x}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {453, 325, 205}

$$\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(5*a^2*x^5) - (b*(A*b - a*B))/(3*a^3*x^3) + (b^2
*(A*b - a*B))/(a^4*x) + (b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a
^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx &= -\frac{A}{7ax^7}-\frac{(7Ab-7aB)\int \frac{1}{x^6\left(a+bx^2\right)}\,dx}{7a}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}+\frac{(b(Ab-aB))\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{a^2}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}-\frac{\left(b^2(Ab-aB)\right)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a^3}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{b^2(Ab-aB)}{a^4x}+\frac{\left(b^3(Ab-aB)\right)\int \frac{1}{a+bx^2}\,dx}{a^4}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 101, normalized size = 1.02

$$
-\frac{b^{5/2}(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}-\frac{b^2(aB-Ab)}{a^4x}+\frac{b(aB-Ab)}{3a^3x^3}+\frac{Ab-aB}{5a^2x^5}-\frac{A}{7ax^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*A/(a*x^7) + (A*b - a*B)/(5*a^2*x^5) + (b*(-(A*b) + a*B))/(3*a^3*x^3) -
 (b^2*(-(A*b) + a*B))/(a^4*x) - (b^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^8*(a + b*x^2)), x]
```

fricas [A] time = 0.45, size = 234, normalized size = 2.36

$$\left[-\frac{105\left(Bab^{2}-Ab^{3}\right)x^{7}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+210\left(Bab^{2}-Ab^{3}\right)x^{6}-70\left(Ba^{2}b-Aab^{2}\right)x^{4}+30\,Aa^{3}+42\left(Ba^{3}-Aa^{2}b\right)x^{2}}{210\,a^{4}x^{7}},-\frac{105\left(Bab^{2}-Ab^{3}\right)x^{7}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+105\left(Bab^{2}-Ab^{3}\right)x^{6}-35\left(Ba^{2}b-Aab^{2}\right)x^{4}+15\,Aa^{3}+21\left(Ba^{3}-Aa^{2}b\right)x^{2}}{105\,a^{4}x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/210*(105*(B*a*b^2 - A*b^3)*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) + 210*(B*a*b^2 - A*b^3)*x^6 - 70*(B*a^2*b - A*a*b^2)*x^4
 + 30*A*a^3 + 42*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7), -1/105*(105*(B*a*b^2 - A
*b^3)*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(B*a*b^2 - A*b^3)*x^6 - 35*(B
*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7)]
```

giac [A] time = 0.37, size = 106, normalized size = 1.07

$$-\frac{\left(Bab^{3}-Ab^{4}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{4}}-\frac{105\,Bab^{2}x^{6}-105\,Ab^{3}x^{6}-35\,Ba^{2}bx^{4}+35\,Aab^{2}x^{4}+21\,Ba^{3}x^{2}-21\,Aa^{2}bx^{2}+15\,Aa^{3}}{105\,a^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*B*a*b
^2*x^6 - 105*A*b^3*x^6 - 35*B*a^2*b*x^4 + 35*A*a*b^2*x^4 + 21*B*a^3*x^2 - 2
1*A*a^2*b*x^2 + 15*A*a^3)/(a^4*x^7)
```

maple [A] time = 0.01, size = 120, normalized size = 1.21

$$\frac{A\,b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^{4}}-\frac{B\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^{3}}+\frac{A\,b^{3}}{a^{4}x}-\frac{B\,b^{2}}{a^{3}x}-\frac{A\,b^{2}}{3a^{3}x^{3}}+\frac{Bb}{3a^{2}x^{3}}+\frac{Ab}{5a^{2}x^{5}}-\frac{B}{5a\,x^{5}}-\frac{A}{7a\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a),x)
```

```
[Out] b^4/a^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A-b^3/a^3/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*B-1/7*A/a/x^7+1/5/a^2/x^5*A*b-1/5/a/x^5*B-1/3/a^3*b^2/x^
3*A+1/3/a^2*b/x^3*B+1/a^4*b^3/x*A-1/a^3*b^2/x*B
```

maxima [A] time = 2.48, size = 103, normalized size = 1.04

$$-\frac{\left(Bab^{3}-Ab^{4}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{4}}-\frac{105\left(Bab^{2}-Ab^{3}\right)x^{6}-35\left(Ba^{2}b-Aab^{2}\right)x^{4}+15\,Aa^{3}+21\left(Ba^{3}-Aa^{2}b\right)x^{2}}{105\,a^{4}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*(B*a*
b^2 - A*b^3)*x^6 - 35*(B*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^
2*b)*x^2)/(a^4*x^7)
```

mupad [B] time = 0.12, size = 89, normalized size = 0.90

$$\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{9/2}}-\frac{\frac{A}{7\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{5\,a^2}-\frac{b^2\,x^6\,\left(A\,b-B\,a\right)}{a^4}+\frac{b\,x^4\,\left(A\,b-B\,a\right)}{3\,a^3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(9/2) - (A/(7*a) - (x^2*(
A*b - B*a))/(5*a^2) - (b^2*x^6*(A*b - B*a))/a^4 + (b*x^4*(A*b - B*a))/(3*a^
3))/x^7
```

sympy [B] time = 0.55, size = 187, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^9}}\,(-Ab+Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}\,(-Ab+Ba)}{-Ab^4+Bab^3}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^9}}\,(-Ab+Ba)\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}\,(-Ab+Ba)}{-Ab^4+Bab^3}+x\right)}{2}+\frac{-15Aa^3+x^6\left(105Ab^3-105Bab^2\right)+x^4\left(-35Aab^2+35Ba^2b\right)+x^2\left(21Aa^2b-21Ba^3\right)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**9)*(-A*b + B*a)*log(-a**5*sqrt(-b**5/a**9)*(-A*b + B*a)/(-A*b
**4 + B*a*b**3) + x)/2 - sqrt(-b**5/a**9)*(-A*b + B*a)*log(a**5*sqrt(-b**5/
a**9)*(-A*b + B*a)/(-A*b**4 + B*a*b**3) + x)/2 + (-15*A*a**3 + x**6*(105*A*
b**3 - 105*B*a*b**2) + x**4*(-35*A*a*b**2 + 35*B*a**2*b) + x**2*(21*A*a**2*
b - 21*B*a**3))/(105*a**4*x**7)
```

3.71 $\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{a^4(Ab-aB)}{2b^6\left(a+bx^2\right)}-\frac{a^3(4Ab-5aB)\log\left(a+bx^2\right)}{2b^6}+\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{x^6(Ab-2aB)}{6b^3}+\frac{Bx^8}{8b^2}$$

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a^2x^2(3Ab-4aB)}{2b^5}-\frac{a^4(Ab-aB)}{2b^6\left(a+bx^2\right)}-\frac{a^3(4Ab-5aB)\log\left(a+bx^2\right)}{2b^6}+\frac{x^6(Ab-2aB)}{6b^3}-\frac{ax^4(2Ab-3aB)}{4b^4}+\frac{Bx^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x^2)/(2*b^5) - (a*(2*A*b - 3*a*B)*x^4)/(4*b^4) + ((A*b
 - 2*a*B)*x^6)/(6*b^3) + (B*x^8)/(8*b^2) - (a^4*(A*b - a*B))/(2*b^6*(a + b*
x^2)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/(2*b^6)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9 \left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^2(-3Ab+4aB)}{b^5} + \frac{a(-2Ab+3aB)x}{b^4} + \frac{(Ab-2aB)x^2}{b^3} + \frac{Bx^3}{b^2} - \frac{a^4(-Ab+}{b^5(a+b}\right.\right.$$

$$= \frac{a^2(3Ab-4aB)x^2}{2b^5} - \frac{a(2Ab-3aB)x^4}{4b^4} + \frac{(Ab-2aB)x^6}{6b^3} + \frac{Bx^8}{8b^2} - \frac{a^4(Ab-aB)}{2b^6\left(a+bx^2\right)} - \frac{a^3(4Ab-}{}$$

Mathematica [A] time = 0.07, size = 113, normalized size = 0.90

$$\frac{\frac{12a^4(aB-Ab)}{a+bx^2} + 12a^3(5aB-4Ab)\log\left(a+bx^2\right) - 12a^2bx^2(4aB-3Ab) + 4b^3x^6(Ab-2aB) + 6ab^2x^4(3aB-2Ab) + 3b^4Bx^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-12*a^2*b*(-3*A*b + 4*a*B)*x^2 + 6*a*b^2*(-2*A*b + 3*a*B)*x^4 + 4*b^3*(A*b
 - 2*a*B)*x^6 + 3*b^4*B*x^8 + (12*a^4*(-(A*b) + a*B))/(a + b*x^2) + 12*a^3*
(-4*A*b + 5*a*B)*Log[a + b*x^2])/(24*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9 \left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.39, size = 172, normalized size = 1.37

$$\frac{3\,Bb^5x^{10} - \left(5\,Bab^4 - 4\,Ab^5\right)x^8 + 2\left(5\,Ba^2b^3 - 4\,Aab^4\right)x^6 + 12\,Ba^5 - 12\,Aa^4b - 6\left(5\,Ba^3b^2 - 4\,Aa^2b^3\right)x^4 - 12\left(4\,Ba^4b - 3\,Aa^3b^2\right)x^2 + 12\left(5\,Ba^5 - 4\,Aa^4b + \left(5\,Ba^4b - 4\,Aa^3b^2\right)x^2\right)\log\left(bx^2+a\right)}{24\left(b^7x^2 + ab^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^5*x^10 - (5*B*a*b^4 - 4*A*b^5)*x^8 + 2*(5*B*a^2*b^3 - 4*A*a*b^4
)*x^6 + 12*B*a^5 - 12*A*a^4*b - 6*(5*B*a^3*b^2 - 4*A*a^2*b^3)*x^4 - 12*(4*B
```

```
*a^4*b - 3*A*a^3*b^2)*x^2 + 12*(5*B*a^5 - 4*A*a^4*b + (5*B*a^4*b - 4*A*a^3*
b^2)*x^2)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

giac [A] time = 0.35, size = 159, normalized size = 1.26

$$\frac{\left(5\,Ba^{4}-4\,Aa^{3}b\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,b^{6}}-\frac{5\,Ba^{4}bx^{2}-4\,Aa^{3}b^{2}x^{2}+4\,Ba^{5}-3\,Aa^{4}b}{2\,\left(bx^{2}+a\right)b^{6}}+\frac{3\,Bb^{6}x^{8}-8\,Bab^{5}x^{6}+4\,Ab^{6}x^{6}+18\,Ba^{2}b^{4}x^{4}-12\,Aab^{5}x^{4}-48\,Ba^{3}b^{3}x^{2}+36\,Aa^{2}b^{4}x^{2}}{24\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^4 - 4*A*a^3*b)*log(abs(b*x^2 + a))/b^6 - 1/2*(5*B*a^4*b*x^2 - 4*
A*a^3*b^2*x^2 + 4*B*a^5 - 3*A*a^4*b)/((b*x^2 + a)*b^6) + 1/24*(3*B*b^6*x^8
- 8*B*a*b^5*x^6 + 4*A*b^6*x^6 + 18*B*a^2*b^4*x^4 - 12*A*a*b^5*x^4 - 48*B*a^
3*b^3*x^2 + 36*A*a^2*b^4*x^2)/b^8
```

maple [A] time = 0.02, size = 146, normalized size = 1.16

$$\frac{B\,x^{8}}{8b^{2}}+\frac{A\,x^{6}}{6b^{2}}-\frac{Ba\,x^{6}}{3b^{3}}-\frac{Aa\,x^{4}}{2b^{3}}+\frac{3B\,a^{2}x^{4}}{4b^{4}}+\frac{3A\,a^{2}x^{2}}{2b^{4}}-\frac{2B\,a^{3}x^{2}}{b^{5}}-\frac{A\,a^{4}}{2\left(b\,x^{2}+a\right)b^{5}}-\frac{2A\,a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}+\frac{B\,a^{5}}{2\left(b\,x^{2}+a\right)b^{6}}+\frac{5B\,a^{4}\ln\left(b\,x^{2}+a\right)}{2b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/8*B*x^8/b^2+1/6/b^2*x^6*A-1/3/b^3*x^6*B*a-1/2/b^3*x^4*A*a+3/4/b^4*x^4*B*a
^2+3/2/b^4*x^2*A*a^2-2/b^5*x^2*B*a^3-1/2*a^4/b^5/(b*x^2+a)*A+1/2*a^5/b^6/(b
*x^2+a)*B-2*a^3/b^5*ln(b*x^2+a)*A+5/2*a^4/b^6*ln(b*x^2+a)*B
```

maxima [A] time = 1.09, size = 131, normalized size = 1.04

$$\frac{Ba^{5}-Aa^{4}b}{2\,\left(b^{7}x^{2}+ab^{6}\right)}+\frac{3\,Bb^{3}x^{8}-4\,\left(2\,Bab^{2}-Ab^{3}\right)x^{6}+6\,\left(3\,Ba^{2}b-2\,Aab^{2}\right)x^{4}-12\,\left(4\,Ba^{3}-3\,Aa^{2}b\right)x^{2}}{24\,b^{5}}+\frac{\left(5\,Ba^{4}-4\,Aa^{3}b\right)\log\left(bx^{2}+a\right)}{2\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^5 - A*a^4*b)/(b^7*x^2 + a*b^6) + 1/24*(3*B*b^3*x^8 - 4*(2*B*a*b^2
- A*b^3)*x^6 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^4 - 12*(4*B*a^3 - 3*A*a^2*b)*x^2
)/b^5 + 1/2*(5*B*a^4 - 4*A*a^3*b)*log(b*x^2 + a)/b^6
```

mupad [B] time = 0.10, size = 181, normalized size = 1.44

$$x^{2}\left(\frac{a\left(\frac{2\,a\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{b}+\frac{B\,a^{2}}{b^{4}}\right)}{b}-\frac{a^{2}\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{2\,b^{2}}\right)+x^{6}\left(\frac{A}{6\,b^{2}}-\frac{B\,a}{3\,b^{3}}\right)-x^{4}\left(\frac{a\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{2\,b}+\frac{B\,a^{2}}{4\,b^{4}}\right)+\frac{B\,x^{8}}{8\,b^{2}}+\frac{\ln\left(b\,x^{2}+a\right)\left(5\,B\,a^{4}-4\,A\,a^{3}\,b\right)}{2\,b^{6}}+\frac{B\,a^{5}-A\,a^{4}\,b}{2\,b\left(b^{6}\,x^{2}+a\,b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^2*((a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (2
*B*a)/b^3))/(2*b^2)) + x^6*(A/(6*b^2) - (B*a)/(3*b^3)) - x^4*((a*(A/b^2 - (
2*B*a)/b^3))/(2*b) + (B*a^2)/(4*b^4)) + (B*x^8)/(8*b^2) + (log(a + b*x^2)*(
5*B*a^4 - 4*A*a^3*b))/(2*b^6) + (B*a^5 - A*a^4*b)/(2*b*(a*b^5 + b^6*x^2))
```

sympy [A] time = 0.79, size = 131, normalized size = 1.04

$$\frac{Bx^8}{8b^2} + \frac{a^3 \left(-4Ab + 5Ba\right) \log\left(a + bx^2\right)}{2b^6} + x^6 \left(\frac{A}{6b^2} - \frac{Ba}{3b^3}\right) + x^4 \left(-\frac{Aa}{2b^3} + \frac{3Ba^2}{4b^4}\right) + x^2 \left(\frac{3Aa^2}{2b^4} - \frac{2Ba^3}{b^5}\right) + \frac{-Aa^4b + Ba^5}{2ab^6 + 2b^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**8/(8*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x**2)/(2*b**6) + x**6*(A/
(6*b**2) - B*a/(3*b**3)) + x**4*(-A*a/(2*b**3) + 3*B*a**2/(4*b**4)) + x**2*
(3*A*a**2/(2*b**4) - 2*B*a**3/b**5) + (-A*a**4*b + B*a**5)/(2*a*b**6 + 2*b*
*7*x**2)
```

3.72 $\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{a^3x(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{x^5(Ab-2aB)}{5b^3}+\frac{Bx^7}{7b^2}$$

Rubi [A] time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 1810, 205}

$$\frac{a^3x(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2x(3Ab-4aB)}{b^5}-\frac{a^{5/2}(7Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x^5(Ab-2aB)}{5b^3}-\frac{ax^3(2Ab-3aB)}{3b^4}+\frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a^2*(3*A*b - 4*a*B)*x)/b^5 - (a*(2*A*b - 3*a*B)*x^3)/(3*b^4) + ((A*b - 2*a
*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + (a^3*(A*b - a*B)*x)/(2*b^5*(a + b*x^2)
) - (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^8 \left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{a^3(Ab-aB)x}{2b^5\left(a+bx^2\right)} - \frac{\int \frac{a^3(Ab-aB)-2a^2b(Ab-aB)x^2+2ab^2(Ab-aB)x^4-2b^3(Ab-aB)x^6-2b^4Bx^8}{a+bx^2}\,dx}{2b^5}$$

$$= \frac{a^3(Ab-aB)x}{2b^5\left(a+bx^2\right)} - \frac{\int \Big(-2a^2(3Ab-4aB)+2ab(2Ab-3aB)x^2-2b^2(Ab-2aB)x^4-2b^3Bx^6}{2b^5}$$

$$= \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{a^3(Ab-aB)x}{2b^5\left(a+bx^2\right)} - \frac{\Big(a^3(7Ab-}{}$$

$$= \frac{a^2(3Ab-4aB)x}{b^5} - \frac{a(2Ab-3aB)x^3}{3b^4} + \frac{(Ab-2aB)x^5}{5b^3} + \frac{Bx^7}{7b^2} + \frac{a^3(Ab-aB)x}{2b^5\left(a+bx^2\right)} - \frac{a^{5/2}(7Ab-}{}$$

Mathematica [A] time = 0.10, size = 134, normalized size = 1.02

$$\frac{a^{5/2}(9aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}} - \frac{a^2x(4aB-3Ab)}{b^5} + \frac{x\left(a^3Ab-a^4B\right)}{2b^5\left(a+bx^2\right)} + \frac{ax^3(3aB-2Ab)}{3b^4} + \frac{x^5(Ab-2aB)}{5b^3} + \frac{Bx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a^2*(-3*A*b + 4*a*B)*x)/b^5) + (a*(-2*A*b + 3*a*B)*x^3)/(3*b^4) + ((A*b
- 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + ((a^3*A*b - a^4*B)*x)/(2*b^5*(a +
 b*x^2)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/
2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.47, size = 350, normalized size = 2.67

$$\left[\frac{60\,Bb^4x^9 - 12\left(9\,Bab^3 - 7\,Ab^4\right)x^7 + 28\left(9\,Ba^2b^2 - 7\,Aab^3\right)x^5 - 140\left(9\,Ba^3b - 7\,Aa^2b^2\right)x^3 - 105\left(9\,Ba^4 - 7\,Aa^3b + \left(9\,Ba^3b - 7\,Aa^2b^2\right)x^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 210\left(9\,Ba^4 - 7\,Aa^3b\right)x}{420\left(b^6x^2 + ab^5\right)}, \frac{30\,Bb^4x^9 - 6\left(9\,Bab^3 - 7\,Ab^4\right)x^7 + 14\left(9\,Ba^2b^2 - 7\,Aab^3\right)x^5 - 70\left(9\,Ba^3b - 7\,Aa^2b^2\right)x^3 + 105\left(9\,Ba^4 - 7\,Aa^3b + \left(9\,Ba^3b - 7\,Aa^2b^2\right)x^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 105\left(9\,Ba^4 - 7\,Aa^3b\right)x}{210\left(b^6x^2 + ab^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*B*b^4*x^9 - 12*(9*B*a*b^3 - 7*A*b^4)*x^7 + 28*(9*B*a^2*b^2 - 7*A
*a*b^3)*x^5 - 140*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 7*A*a^3*b
+ (9*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) - 210*(9*B*a^4 - 7*A*a^3*b)*x)/(b^6*x^2 + a*b^5), 1/210*(3
0*B*b^4*x^9 - 6*(9*B*a*b^3 - 7*A*b^4)*x^7 + 14*(9*B*a^2*b^2 - 7*A*a*b^3)*x^
5 - 70*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 + 105*(9*B*a^4 - 7*A*a^3*b + (9*B*a^3*
b - 7*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(9*B*a^4 - 7*
A*a^3*b)*x)/(b^6*x^2 + a*b^5)]
```

giac [A] time = 0.42, size = 139, normalized size = 1.06

$$\frac{\left(9\,Ba^4 - 7\,Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5} - \frac{Ba^4x - Aa^3bx}{2\left(bx^2 + a\right)b^5} + \frac{15\,Bb^{12}x^7 - 42\,Bab^{11}x^5 + 21\,Ab^{12}x^5 + 105\,Ba^2b^{10}x^3 - 70\,Aab^{11}x^3 - 420\,Ba^3b^9x + 315\,Aa^2b^{10}x}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*(B*a^
4*x - A*a^3*b*x)/((b*x^2 + a)*b^5) + 1/105*(15*B*b^12*x^7 - 42*B*a*b^11*x^5
 + 21*A*b^12*x^5 + 105*B*a^2*b^10*x^3 - 70*A*a*b^11*x^3 - 420*B*a^3*b^9*x +
 315*A*a^2*b^10*x)/b^14
```

maple [A] time = 0.02, size = 155, normalized size = 1.18

$$\frac{B\,x^7}{7b^2} + \frac{A\,x^5}{5b^2} - \frac{2Ba\,x^5}{5b^3} - \frac{2Aa\,x^3}{3b^3} + \frac{B\,a^2x^3}{b^4} + \frac{A\,a^3x}{2\left(b\,x^2 + a\right)b^4} - \frac{7A\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^4} - \frac{B\,a^4x}{2\left(b\,x^2 + a\right)b^5} + \frac{9B\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^5} + \frac{3A\,a^2x}{b^4} - \frac{4B\,a^3x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/7*B*x^7/b^2+1/5/b^2*A*x^5-2/5/b^3*B*x^5*a-2/3/b^3*A*x^3*a+1/b^4*B*x^3*a^2
+3/b^4*A*a^2*x-4/b^5*B*a^3*x+1/2*a^3/b^4*x/(b*x^2+a)*A-1/2*a^4/b^5*x/(b*x^2
+a)*B-7/2*a^3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+9/2*a^4/b^5/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.39, size = 136, normalized size = 1.04

$$-\frac{\left(Ba^{4}-Aa^{3}b\right)x}{2\left(b^{6}x^{2}+ab^{5}\right)}+\frac{\left(9\,Ba^{4}-7\,Aa^{3}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{5}}+\frac{15\,Bb^{3}x^{7}-21\left(2\,Bab^{2}-Ab^{3}\right)x^{5}+35\left(3\,Ba^{2}b-2\,Aab^{2}\right)x^{3}-105\left(4\,Ba^{3}-3\,Aa^{2}b\right)x}{105\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^4 - A*a^3*b)*x/(b^6*x^2 + a*b^5) + 1/2*(9*B*a^4 - 7*A*a^3*b)*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/105*(15*B*b^3*x^7 - 21*(2*B*a*b^2 - A
*b^3)*x^5 + 35*(3*B*a^2*b - 2*A*a*b^2)*x^3 - 105*(4*B*a^3 - 3*A*a^2*b)*x)/b
^5
```

mupad [B] time = 0.05, size = 203, normalized size = 1.55

$$x\left(\frac{2\,a\left(\frac{2\,a\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{b}+\frac{B\,a^{2}}{b^{4}}\right)}{b}-\frac{a^{2}\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{b^{2}}\right)+x^{5}\left(\frac{A}{5\,b^{2}}-\frac{2\,B\,a}{5\,b^{3}}\right)-x^{3}\left(\frac{2\,a\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{3\,b}+\frac{B\,a^{2}}{3\,b^{4}}\right)+\frac{B\,x^{7}}{7\,b^{2}}-\frac{x\left(\frac{B\,a^{4}}{2}-\frac{A\,a^{3}\,b}{2}\right)}{b^{6}\,x^{2}+a\,b^{5}}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(7\,A\,b-9\,B\,a)}{9\,B\,a^{4}-7\,A\,a^{3}\,b}\right)(7\,A\,b-9\,B\,a)}{2\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x*((2*a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (2
*B*a)/b^3))/b^2) + x^5*(A/(5*b^2) - (2*B*a)/(5*b^3)) - x^3*((2*a*(A/b^2 - (
2*B*a)/b^3))/(3*b) + (B*a^2)/(3*b^4)) + (B*x^7)/(7*b^2) - (x*((B*a^4)/2 - (
A*a^3*b)/2))/(a*b^5 + b^6*x^2) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(7*A*b -
9*B*a))/(9*B*a^4 - 7*A*a^3*b))*(7*A*b - 9*B*a))/(2*b^(11/2))
```

sympy [A] time = 0.79, size = 238, normalized size = 1.82

$$\frac{Bx^{7}}{7b^{2}}+x^{5}\left(\frac{A}{5b^{2}}-\frac{2Ba}{5b^{3}}\right)+x^{3}\left(-\frac{2Aa}{3b^{3}}+\frac{Ba^{2}}{b^{4}}\right)+x\left(\frac{3Aa^{2}}{b^{4}}-\frac{4Ba^{3}}{b^{5}}\right)+\frac{x\left(Aa^{3}b-Ba^{4}\right)}{2ab^{5}+2b^{6}x^{2}}-\frac{\sqrt{-\frac{a^{5}}{b^{11}}}\,(-7Ab+9Ba)\log\left(-\frac{b^{5}\sqrt{-\frac{a^{5}}{b^{11}}}\,(-7Ab+9Ba)}{-7Aa^{2}b+9Ba^{3}}+x\right)}{4}+\frac{\sqrt{-\frac{a^{5}}{b^{11}}}\,(-7Ab+9Ba)\log\left(\frac{b^{5}\sqrt{-\frac{a^{5}}{b^{11}}}\,(-7Ab+9Ba)}{-7Aa^{2}b+9Ba^{3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**5*(A/(5*b**2) - 2*B*a/(5*b**3)) + x**3*(-2*A*a/(3*b**3
) + B*a**2/b**4) + x*(3*A*a**2/b**4 - 4*B*a**3/b**5) + x*(A*a**3*b - B*a**4
)/(2*a*b**5 + 2*b**6*x**2) - sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)*log(-b**5*s
qrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-7*A*a**2*b + 9*B*a**3) + x)/4 + sqrt(-a
**5/b**11)*(-7*A*b + 9*B*a)*log(b**5*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-7
*A*a**2*b + 9*B*a**3) + x)/4
```

3.73 $\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=104

$$\frac{a^3(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2(3Ab-4aB)\log\left(a+bx^2\right)}{2b^5}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{x^4(Ab-2aB)}{4b^3}+\frac{Bx^6}{6b^2}$$

Rubi [**A**] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a^3(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2(3Ab-4aB)\log\left(a+bx^2\right)}{2b^5}+\frac{x^4(Ab-2aB)}{4b^3}-\frac{ax^2(2Ab-3aB)}{2b^4}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(a*(2*A*b - 3*a*B)*x^2)/(2*b^4) + ((A*b - 2*a*B)*x^4)/(4*b^3) + (B*x^6)/(6
*b^2) + (a^3*(A*b - a*B))/(2*b^5*(a + b*x^2)) + (a^2*(3*A*b - 4*a*B)*Log[a
+ b*x^2])/(2*b^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\Bigg(\int \Bigg(\frac{a(-2Ab+3aB)}{b^4}+\frac{(Ab-2aB)x}{b^3}+\frac{Bx^2}{b^2}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^2}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)}\\ &= -\frac{a(2Ab-3aB)x^2}{2b^4}+\frac{(Ab-2aB)x^4}{4b^3}+\frac{Bx^6}{6b^2}+\frac{a^3(Ab-aB)}{2b^5\left(a+bx^2\right)}+\frac{a^2(3Ab-4aB)\log\big(a+bx^2}{2b^5}\end{aligned}$$

Mathematica [**A**] time = 0.06, size = 93, normalized size = 0.89

$$\frac{\frac{6a^3(Ab-aB)}{a+bx^2}+6a^2(3Ab-4aB)\log\left(a+bx^2\right)+3b^2x^4(Ab-2aB)+6abx^2(3aB-2Ab)+2b^3Bx^6}{12b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (6*a*b*(-2*A*b + 3*a*B)*x^2 + 3*b^2*(A*b - 2*a*B)*x^4 + 2*b^3*B*x^6 + (6*a^
3*(A*b - a*B))/(a + b*x^2) + 6*a^2*(3*A*b - 4*a*B)*Log[a + b*x^2])/(12*b^5)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [**A**] time = 0.42, size = 148, normalized size = 1.42

$$\frac{2\,Bb^4x^8-\left(4\,Bab^3-3\,Ab^4\right)x^6-6\,Ba^4+6\,Aa^3b+3\left(4\,Ba^2b^2-3\,Aab^3\right)x^4+6\left(3\,Ba^3b-2\,Aa^2b^2\right)x^2-6\left(4\,Ba^4-3\,Aa^3b+\left(4\,Ba^3b-3\,Aa^2b^2\right)x^2\right)\log\left(bx^2+a\right)}{12\left(b^6x^2+ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^4*x^8 - (4*B*a*b^3 - 3*A*b^4)*x^6 - 6*B*a^4 + 6*A*a^3*b + 3*(4*
B*a^2*b^2 - 3*A*a*b^3)*x^4 + 6*(3*B*a^3*b - 2*A*a^2*b^2)*x^2 - 6*(4*B*a^4 -
```

```
 3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^2 + a*b^
5)
```

giac [A] time = 0.32, size = 135, normalized size = 1.30

$$-\frac{\left(4\,Ba^{3}-3\,Aa^{2}b\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,b^{5}}+\frac{2\,Bb^{4}x^{6}-6\,Bab^{3}x^{4}+3\,Ab^{4}x^{4}+18\,Ba^{2}b^{2}x^{2}-12\,Aab^{3}x^{2}}{12\,b^{6}}+\frac{4\,Ba^{3}bx^{2}-3\,Aa^{2}b^{2}x^{2}+3\,Ba^{4}-2\,Aa^{3}b}{2\left(bx^{2}+a\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^5 + 1/12*(2*B*b^4*x^6 - 6*
B*a*b^3*x^4 + 3*A*b^4*x^4 + 18*B*a^2*b^2*x^2 - 12*A*a*b^3*x^2)/b^6 + 1/2*(4
*B*a^3*b*x^2 - 3*A*a^2*b^2*x^2 + 3*B*a^4 - 2*A*a^3*b)/((b*x^2 + a)*b^5)
```

maple [A] time = 0.02, size = 122, normalized size = 1.17

$$\frac{B\,x^{6}}{6b^{2}}+\frac{A\,x^{4}}{4b^{2}}-\frac{Ba\,x^{4}}{2b^{3}}-\frac{Aa\,x^{2}}{b^{3}}+\frac{3B\,a^{2}x^{2}}{2b^{4}}+\frac{A\,a^{3}}{2\left(b\,x^{2}+a\right)b^{4}}+\frac{3A\,a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}-\frac{B\,a^{4}}{2\left(b\,x^{2}+a\right)b^{5}}-\frac{2B\,a^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/6*B*x^6/b^2+1/4/b^2*A*x^4-1/2/b^3*B*x^4*a-1/b^3*A*x^2*a+3/2/b^4*B*x^2*a^2
+1/2*a^3/b^4/(b*x^2+a)*A-1/2*a^4/b^5/(b*x^2+a)*B+3/2*a^2/b^4*ln(b*x^2+a)*A-
2*a^3/b^5*ln(b*x^2+a)*B
```

maxima [A] time = 0.99, size = 107, normalized size = 1.03

$$-\frac{Ba^{4}-Aa^{3}b}{2\left(b^{6}x^{2}+ab^{5}\right)}+\frac{2\,Bb^{2}x^{6}-3\left(2\,Bab-Ab^{2}\right)x^{4}+6\left(3\,Ba^{2}-2\,Aab\right)x^{2}}{12\,b^{4}}-\frac{\left(4\,Ba^{3}-3\,Aa^{2}b\right)\log\left(bx^{2}+a\right)}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^4 - A*a^3*b)/(b^6*x^2 + a*b^5) + 1/12*(2*B*b^2*x^6 - 3*(2*B*a*b -
 A*b^2)*x^4 + 6*(3*B*a^2 - 2*A*a*b)*x^2)/b^4 - 1/2*(4*B*a^3 - 3*A*a^2*b)*lo
g(b*x^2 + a)/b^5
```

mupad [B] time = 0.09, size = 121, normalized size = 1.16

$$x^{4}\left(\frac{A}{4\,b^{2}}-\frac{B\,a}{2\,b^{3}}\right)-x^{2}\left(\frac{a\left(\frac{A}{b^{2}}-\frac{2\,B\,a}{b^{3}}\right)}{b}+\frac{B\,a^{2}}{2\,b^{4}}\right)+\frac{B\,x^{6}}{6\,b^{2}}-\frac{\ln\left(b\,x^{2}+a\right)\left(4\,B\,a^{3}-3\,A\,a^{2}\,b\right)}{2\,b^{5}}-\frac{B\,a^{4}-A\,a^{3}\,b}{2\,b\,\left(b^{5}\,x^{2}+a\,b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x^2*((a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2
)/(2*b^4)) + (B*x^6)/(6*b^2) - (log(a + b*x^2)*(4*B*a^3 - 3*A*a^2*b))/(2*b^
5) - (B*a^4 - A*a^3*b)/(2*b*(a*b^4 + b^5*x^2))
```

sympy [A] time = 0.72, size = 104, normalized size = 1.00

$$\frac{Bx^6}{6b^2} - \frac{a^2\left(-3Ab + 4Ba\right)\log\left(a + bx^2\right)}{2b^5} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x^2\left(-\frac{Aa}{b^3} + \frac{3Ba^2}{2b^4}\right) + \frac{Aa^3b - Ba^4}{2ab^5 + 2b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x**2)/(2*b**5) + x**4*(A/
(4*b**2) - B*a/(2*b**3)) + x**2*(-A*a/b**3 + 3*B*a**2/(2*b**4)) + (A*a**3*b
 - B*a**4)/(2*a*b**5 + 2*b**6*x**2)
```

3.74 $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{a^2x(Ab-aB)}{2b^4\left(a+bx^2\right)}-\frac{ax(2Ab-3aB)}{b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^5}{5b^2}$$

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 1810, 205}

$$-\frac{a^2x(Ab-aB)}{2b^4\left(a+bx^2\right)}+\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{ax(2Ab-3aB)}{b^4}+\frac{Bx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((a*(2*A*b - 3*a*B)*x)/b^4) + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2
) - (a^2*(A*b - a*B)*x)/(2*b^4*(a + b*x^2)) + (a^(3/2)*(5*A*b - 7*a*B)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)}-\frac{\int \frac{-a^2(Ab-aB)+2ab(Ab-aB)x^2-2b^2(Ab-aB)x^4-2b^3Bx^6}{a+bx^2}\,dx}{2b^4}\\
&= -\frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)}-\frac{\int \left(2a(2Ab-3aB)-2b(Ab-2aB)x^2-2b^2Bx^4+\frac{-5a^2Ab+7a^3B}{a+bx^2}\right)dx}{2b^4}\\
&= -\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^5}{5b^2}-\frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)}+\frac{\left(a^2(5Ab-7aB)\right)\int \frac{1}{a+bx^2}\,dx}{2b^4}\\
&= -\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^5}{5b^2}-\frac{a^2(Ab-aB)x}{2b^4\left(a+bx^2\right)}+\frac{a^{3/2}(5Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}
\end{aligned}
$$

Mathematica [A]   time = 0.08, size = 111, normalized size = 1.01

$$
-\frac{a^{3/2}(7aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}-\frac{x\left(a^2Ab-a^3B\right)}{2b^4\left(a+bx^2\right)}+\frac{ax(3aB-2Ab)}{b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^5}{5b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*A*b + 3*a*B)*x)/b^4 + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2)
- ((a^2*A*b - a^3*B)*x)/(2*b^4*(a + b*x^2)) - (a^(3/2)*(-5*A*b + 7*a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [A]   time = 0.47, size = 298, normalized size = 2.71

$$
\left[\frac{12\,Bb^3x^7-4\left(7\,Bab^2-5\,Ab^3\right)x^5+20\left(7\,Ba^2b-5\,Aab^2\right)x^3-15\left(7\,Ba^3-5\,Aa^2b+\left(7\,Ba^2b-5\,Aab^2\right)x^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\left(7\,Ba^3-5\,Aa^2b\right)x}{60\left(b^5x^2+ab^4\right)},\ \frac{6\,Bb^3x^7-2\left(7\,Bab^2-5\,Ab^3\right)x^5+10\left(7\,Ba^2b-5\,Aab^2\right)x^3-15\left(7\,Ba^3-5\,Aa^2b+\left(7\,Ba^2b-5\,Aab^2\right)x^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\left(7\,Ba^3-5\,Aa^2b\right)x}{30\left(b^5x^2+ab^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*B*b^3*x^7 - 4*(7*B*a*b^2 - 5*A*b^3)*x^5 + 20*(7*B*a^2*b - 5*A*a*b
^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(-a/b)
*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(7*B*a^3 - 5*A*a^2*b)
*x)/(b^5*x^2 + a*b^4), 1/30*(6*B*b^3*x^7 - 2*(7*B*a*b^2 - 5*A*b^3)*x^5 + 10
*(7*B*a^2*b - 5*A*a*b^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a
*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(7*B*a^3 - 5*A*a^2*b)*x)/
(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.27, size = 115, normalized size = 1.05

$$-\frac{\left(7\,Ba^{3}-5\,Aa^{2}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}+\frac{Ba^{3}x-Aa^{2}bx}{2\left(bx^{2}+a\right)b^{4}}+\frac{3\,Bb^{8}x^{5}-10\,Bab^{7}x^{3}+5\,Ab^{8}x^{3}+45\,Ba^{2}b^{6}x-30\,Aab^{7}x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*(B*a
^3*x - A*a^2*b*x)/((b*x^2 + a)*b^4) + 1/15*(3*B*b^8*x^5 - 10*B*a*b^7*x^3 +
5*A*b^8*x^3 + 45*B*a^2*b^6*x - 30*A*a*b^7*x)/b^10
```

maple [A] time = 0.01, size = 132, normalized size = 1.20

$$\frac{B\,x^{5}}{5b^{2}}+\frac{A\,x^{3}}{3b^{2}}-\frac{2Ba\,x^{3}}{3b^{3}}-\frac{A\,a^{2}x}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{5A\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}+\frac{B\,a^{3}x}{2\left(b\,x^{2}+a\right)b^{4}}-\frac{7B\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{4}}-\frac{2Aax}{b^{3}}+\frac{3B\,a^{2}x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/5*B*x^5/b^2+1/3/b^2*A*x^3-2/3/b^3*B*x^3*a-2/b^3*a*A*x+3/b^4*a^2*B*x-1/2*a
^2/b^3*x/(b*x^2+a)*A+1/2*a^3/b^4*x/(b*x^2+a)*B+5/2*a^2/b^3/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)*A-7/2*a^3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.23, size = 112, normalized size = 1.02

$$\frac{\left(Ba^{3}-Aa^{2}b\right)x}{2\left(b^{5}x^{2}+ab^{4}\right)}-\frac{\left(7\,Ba^{3}-5\,Aa^{2}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}+\frac{3\,Bb^{2}x^{5}-5\left(2\,Bab-Ab^{2}\right)x^{3}+15\left(3\,Ba^{2}-2\,Aab\right)x}{15\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^3 - A*a^2*b)*x/(b^5*x^2 + a*b^4) - 1/2*(7*B*a^3 - 5*A*a^2*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/15*(3*B*b^2*x^5 - 5*(2*B*a*b - A*b^2)*
x^3 + 15*(3*B*a^2 - 2*A*a*b)*x)/b^4
```

mupad [B] time = 0.05, size = 141, normalized size = 1.28

$$x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)-x\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^5}{5\,b^2}+\frac{x\left(\frac{B\,a^3}{2}-\frac{A\,a^2\,b}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(5\,A\,b-7\,B\,a)}{7\,B\,a^3-5\,A\,a^2\,b}\right)(5\,A\,b-7\,B\,a)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a
^2)/b^4) + (B*x^5)/(5*b^2) + (x*((B*a^3)/2 - (A*a^2*b)/2))/(a*b^4 + b^5*x^2
) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(5*A*b - 7*B*a))/(7*B*a^3 - 5*A*a^2*b)
)*(5*A*b - 7*B*a))/(2*b^(9/2))
```

sympy [B] time = 0.72, size = 211, normalized size = 1.92

$$\frac{Bx^5}{5b^2}+x^3\left(\frac{A}{3b^2}-\frac{2Ba}{3b^3}\right)+x\left(-\frac{2Aa}{b^3}+\frac{3Ba^2}{b^4}\right)+\frac{x\left(-Aa^2b+Ba^3\right)}{2ab^4+2b^5x^2}+\frac{\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)}{-5Aab+7Ba^2}+x\right)}{4}-\frac{\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(-5Ab+7Ba)}{-5Aab+7Ba^2}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x**3*(A/(3*b**2) - 2*B*a/(3*b**3)) + x*(-2*A*a/b**3 + 3*B
*a**2/b**4) + x*(-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-a**3/
b**9)*(-5*A*b + 7*B*a)*log(-b**4*sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*A*a*
b + 7*B*a**2) + x)/4 - sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)*log(b**4*sqrt(-a**
3/b**9)*(-5*A*b + 7*B*a)/(-5*A*a*b + 7*B*a**2) + x)/4
```

3.75 $\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2(Ab-aB)}{2b^4\left(a+bx^2\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(Ab-2aB)}{2b^3}+\frac{Bx^4}{4b^2}$$

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{2b^4\left(a+bx^2\right)}+\frac{x^2(Ab-2aB)}{2b^3}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^2)/(2*b^3) + (B*x^4)/(4*b^2) - (a^2*(A*b - a*B))/(2*b^4*(a
 + b*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5 \left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^4}{4b^2}-\frac{a^2(Ab-aB)}{2b^4\left(a+bx^2\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [**A**] time = 0.07, size = 72, normalized size = 0.88

$$
\frac{\frac{2a^2(aB-Ab)}{a+bx^2}+2bx^2(Ab-2aB)+2a(3aB-2Ab)\log\left(a+bx^2\right)+b^2Bx^4}{4b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^2 + b^2*B*x^4 + (2*a^2*(-(A*b) + a*B))/(a + b*x^2) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(4*b^4)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [**A**] time = 0.41, size = 121, normalized size = 1.48

$$
\frac{Bb^3x^6-\left(3\,Bab^2-2\,Ab^3\right)x^4+2\,Ba^3-2\,Aa^2b-2\left(2\,Ba^2b-Aab^2\right)x^2+2\left(3\,Ba^3-2\,Aa^2b+\left(3\,Ba^2b-2\,Aab^2\right)x^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^3*x^6 - (3*B*a*b^2 - 2*A*b^3)*x^4 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^2 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^2)
*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.28, size = 106, normalized size = 1.29

$$\frac{\left(3\,Ba^2 - 2\,Aab\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{Bb^2x^4 - 4\,Babx^2 + 2\,Ab^2x^2}{4\,b^4} - \frac{3\,Ba^2bx^2 - 2\,Aab^2x^2 + 2\,Ba^3 - Aa^2b}{2\,\left(bx^2 + a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(B*b^2*x^4 - 4*B*a*b*
x^2 + 2*A*b^2*x^2)/b^4 - 1/2*(3*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 2*B*a^3 - A*a
^2*b)/((b*x^2 + a)*b^4)
```

maple [A] time = 0.02, size = 98, normalized size = 1.20

$$\frac{B\,x^4}{4b^2} + \frac{A\,x^2}{2b^2} - \frac{Ba\,x^2}{b^3} - \frac{A\,a^2}{2\left(b\,x^2 + a\right)b^3} - \frac{Aa\ln\left(b\,x^2 + a\right)}{b^3} + \frac{B\,a^3}{2\left(b\,x^2 + a\right)b^4} + \frac{3B\,a^2\ln\left(b\,x^2 + a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*B*x^4/b^2+1/2/b^2*A*x^2-1/b^3*B*x^2*a-1/2*a^2/b^3/(b*x^2+a)*A+1/2*a^3/b
^4/(b*x^2+a)*B-a/b^3*ln(b*x^2+a)*A+3/2*a^2/b^4*ln(b*x^2+a)*B
```

maxima [A] time = 1.04, size = 82, normalized size = 1.00

$$\frac{Ba^3 - Aa^2b}{2\left(b^5x^2 + ab^4\right)} + \frac{Bbx^4 - 2\,(2\,Ba - Ab)x^2}{4\,b^3} + \frac{\left(3\,Ba^2 - 2\,Aab\right)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a^3 - A*a^2*b)/(b^5*x^2 + a*b^4) + 1/4*(B*b*x^4 - 2*(2*B*a - A*b)*x^
2)/b^3 + 1/2*(3*B*a^2 - 2*A*a*b)*log(b*x^2 + a)/b^4
```

mupad [B] time = 0.07, size = 86, normalized size = 1.05

$$x^2\left(\frac{A}{2\,b^2} - \frac{B\,a}{b^3}\right) + \frac{\ln\left(b\,x^2 + a\right)\,\left(3\,B\,a^2 - 2\,A\,a\,b\right)}{2\,b^4} + \frac{B\,x^4}{4\,b^2} + \frac{B\,a^3 - A\,a^2\,b}{2\,b\,\left(b^4\,x^2 + a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^2*(A/(2*b^2) - (B*a)/b^3) + (log(a + b*x^2)*(3*B*a^2 - 2*A*a*b))/(2*b^4)
+ (B*x^4)/(4*b^2) + (B*a^3 - A*a^2*b)/(2*b*(a*b^3 + b^4*x^2))
```

sympy [A] time = 0.66, size = 78, normalized size = 0.95

$$\frac{Bx^4}{4b^2} + \frac{a\left(-2Ab + 3Ba\right)\log\left(a + bx^2\right)}{2b^4} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{-Aa^2b + Ba^3}{2ab^4 + 2b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**4) + x**2*(A/(2*
b**2) - B*a/b**3) + (-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2)
```

3.76 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}\,(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(Ab-aB)}{2b^3\left(a+bx^2\right)}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^3}{3b^2}$$

Rubi [**A**] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 1153, 205}

$$\frac{ax(Ab-aB)}{2b^3\left(a+bx^2\right)}+\frac{x(Ab-2aB)}{b^3}-\frac{\sqrt{a}\,(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + (a*(A*b - a*B)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)}-\frac{\int \frac{a(Ab-aB)-2b(Ab-aB)x^2-2b^2Bx^4}{a+bx^2}\,dx}{2b^3}\\
&= \frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)}-\frac{\int \left(-2(Ab-2aB)-2bBx^2+\frac{3aAb-5a^2B}{a+bx^2}\right)dx}{2b^3}\\
&= \frac{(Ab-2aB)x}{b^3}+\frac{Bx^3}{3b^2}+\frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)}-\frac{(a(3Ab-5aB))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= \frac{(Ab-2aB)x}{b^3}+\frac{Bx^3}{3b^2}+\frac{a(Ab-aB)x}{2b^3\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}
\end{aligned}
$$

Mathematica [**A**] time = 0.08, size = 89, normalized size = 1.02

$$\frac{x\left(aAb-a^2B\right)}{2b^3\left(a+bx^2\right)}+\frac{\sqrt{a}\,(5aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(Ab-2aB)}{b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
[Out] ((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + ((a*A*b - a^2*B)*x)/(2*b^3*(a + b
*x^2)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.43, size = 240, normalized size = 2.76

$$\left[\frac{4\,Bb^{2}x^{5}-4\left(5\,Bab-3\,Ab^{2}\right)x^{3}-3\left(5\,Ba^{2}-3\,Aab+\left(5\,Bab-3\,Ab^{2}\right)x^{2}\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\left(5\,Ba^{2}-3\,Aab\right)x}{12\left(b^{4}x^{2}+ab^{3}\right)},\frac{2\,Bb^{2}x^{5}-2\left(5\,Bab-3\,Ab^{2}\right)x^{3}+3\left(5\,Ba^{2}-3\,Aab+\left(5\,Bab-3\,Ab^{2}\right)x^{2}\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\left(5\,Ba^{2}-3\,Aab\right)x}{6\left(b^{4}x^{2}+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*B*b^2*x^5 - 4*(5*B*a*b - 3*A*b^2)*x^3 - 3*(5*B*a^2 - 3*A*a*b + (5*
B*a*b - 3*A*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) - 6*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3), 1/6*(2*B*b^2*x^5 - 2*(5
*B*a*b - 3*A*b^2)*x^3 + 3*(5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x^2)*sqr
t(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3)
]
```

giac [A] time = 0.30, size = 88, normalized size = 1.01

$$\frac{\left(5\,Ba^{2}-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}-\frac{Ba^{2}x-Aabx}{2\left(bx^{2}+a\right)b^{3}}+\frac{Bb^{4}x^{3}-6\,Bab^{3}x+3\,Ab^{4}x}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(B*a^2*
x - A*a*b*x)/((b*x^2 + a)*b^3) + 1/3*(B*b^4*x^3 - 6*B*a*b^3*x + 3*A*b^4*x)/
b^6
```

maple [A] time = 0.01, size = 105, normalized size = 1.21

$$\frac{B\,x^{3}}{3b^{2}}+\frac{Aax}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{2}}-\frac{B\,a^{2}x}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{5B\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}+\frac{Ax}{b^{2}}-\frac{2Bax}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*B*x^3/b^2+1/b^2*A*x-2/b^3*B*a*x+1/2*a/b^2*x/(b*x^2+a)*A-1/2*a^2/b^3*x/(
b*x^2+a)*B-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+5/2*a^2/b^3/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.42, size = 85, normalized size = 0.98

$$-\frac{\left(Ba^{2}-Aab\right)x}{2\left(b^{4}x^{2}+ab^{3}\right)}+\frac{\left(5\,Ba^{2}-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}+\frac{Bbx^{3}-3\,(2\,Ba-Ab)x}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^2 - A*a*b)*x/(b^4*x^2 + a*b^3) + 1/2*(5*B*a^2 - 3*A*a*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(B*b*x^3 - 3*(2*B*a - A*b)*x)/b^3
```

mupad [B] time = 0.07, size = 104, normalized size = 1.20

$$x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{2}-\frac{A\,a\,b}{2}\right)}{b^4\,x^2+a\,b^3}+\frac{B\,x^3}{3\,b^2}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(3\,A\,b-5\,B\,a)}{5\,B\,a^2-3\,A\,a\,b}\right)(3\,A\,b-5\,B\,a)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/2 - (A*a*b)/2))/(a*b^3 + b^4*x^2) + (
B*x^3)/(3*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*A*b - 5*B*a))/(5*B*a^2
 - 3*A*a*b))*(3*A*b - 5*B*a))/(2*b^(7/2))
```

sympy [A] time = 0.64, size = 129, normalized size = 1.48

$$\frac{Bx^3}{3b^2}+x\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)+\frac{x\left(Aab-Ba^2\right)}{2ab^3+2b^4x^2}-\frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab+5Ba)\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{\sqrt{-\frac{a}{b^7}}\,(-3Ab+5Ba)\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(2*a*b**3 +
2*b**4*x**2) - sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(-b**3*sqrt(-a/b**7) + x)/
4 + sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(b**3*sqrt(-a/b**7) + x)/4
```

3.77 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=60

$$\frac{a(Ab-aB)}{2b^3(a+bx^2)}+\frac{(Ab-2aB)\log(a+bx^2)}{2b^3}+\frac{Bx^2}{2b^2}$$

Rubi **[A]** time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a(Ab-aB)}{2b^3(a+bx^2)}+\frac{(Ab-2aB)\log(a+bx^2)}{2b^3}+\frac{Bx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x^2)/(2*b^2) + (a*(A*b - a*B))/(2*b^3*(a + b*x^2)) + ((A*b - 2*a*B)*Log[
a + b*x^2])/(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{B}{b^2}+\frac{a(-Ab+aB)}{b^2(a+bx)^2}+\frac{Ab-2aB}{b^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{Bx^2}{2b^2}+\frac{a(Ab-aB)}{2b^3\left(a+bx^2\right)}+\frac{(Ab-2aB)\log\left(a+bx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab-aB)}{a+bx^2}+(Ab-2aB)\log\left(a+bx^2\right)+bBx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*B*x^2 + (a*(A*b - a*B))/(a + b*x^2) + (A*b - 2*a*B)*Log[a + b*x^2])/(2*b
^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.41, size = 81, normalized size = 1.35

$$\frac{Bb^2x^4+Babx^2-Ba^2+Aab-\left(2\,Ba^2-Aab+\left(2\,Bab-Ab^2\right)x^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(B*b^2*x^4 + B*a*b*x^2 - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (2*B*a*b -
A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

giac [A]   time = 0.34, size = 91, normalized size = 1.52

$$\frac{\frac{\left(bx^2+a\right)B}{b^2}+\frac{(2\,Ba-Ab)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^2}-\frac{\frac{Ba^2b}{bx^2+a}-\frac{Aab^2}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^2 + a) - A*a*b^2/(b*x^2 + a))/b^3)/b
```

maple [A]   time = 0.01, size = 74, normalized size = 1.23

$$\frac{B\,x^2}{2b^2}+\frac{Aa}{2\left(b\,x^2+a\right)b^2}+\frac{A\ln\left(b\,x^2+a\right)}{2b^2}-\frac{B\,a^2}{2\left(b\,x^2+a\right)b^3}-\frac{Ba\ln\left(b\,x^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/b^2+1/2/b^2*a/(b*x^2+a)*A-1/2/b^3*a^2/(b*x^2+a)*B+1/2/b^2*ln(b*x^
2+a)*A-1/b^3*ln(b*x^2+a)*B*a
```

maxima [A]   time = 1.03, size = 60, normalized size = 1.00

$$\frac{Bx^2}{2\,b^2}-\frac{Ba^2-Aab}{2\left(b^4x^2+ab^3\right)}-\frac{(2\,Ba-Ab)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b^2 - 1/2*(B*a^2 - A*a*b)/(b^4*x^2 + a*b^3) - 1/2*(2*B*a - A*b)*l
og(b*x^2 + a)/b^3
```

mupad [B]   time = 0.07, size = 62, normalized size = 1.03

$$\frac{B\,x^2}{2\,b^2}+\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-2\,B\,a\right)}{2\,b^3}-\frac{B\,a^2-A\,a\,b}{2\,b\,\left(b^3\,x^2+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (B*x^2)/(2*b^2) + (log(a + b*x^2)*(A*b - 2*B*a))/(2*b^3) - (B*a^2 - A*a*b)/
(2*b*(a*b^2 + b^3*x^2))
```

sympy [A] time = 0.57, size = 56, normalized size = 0.93

$$\frac{Bx^2}{2b^2}+\frac{Aab-Ba^2}{2ab^3+2b^4x^2}-\frac{(-Ab+2Ba)\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + (A*a*b - B*a**2)/(2*a*b**3 + 2*b**4*x**2) - (-A*b + 2*B*a
)*log(a + b*x**2)/(2*b**3)
```

3.78 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=67

$$\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{Bx}{b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 388, 205}

$$-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) + ((A*b - 3*a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{(Ab-aB)x}{2b^2\left(a+bx^2\right)}-\frac{\int \frac{-Ab+aB-2bBx^2}{a+bx^2}\,dx}{2b^2}\\ &= \frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2\left(a+bx^2\right)}+\frac{(Ab-3aB)\int \frac{1}{a+bx^2}\,dx}{2b^2}\\ &= \frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2\left(a+bx^2\right)}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 1.01

$$-\frac{(3aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(Ab-aB)}{2b^2\left(a+bx^2\right)}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) - ((-(A*b) + 3*a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.46, size = 208, normalized size = 3.10

$$\left[\frac{4\,Bab^2x^3+\left(3\,Ba^2-Aab+\left(3\,Bab-Ab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(3\,Ba^2b-Aab^2\right)x}{4\left(ab^4x^2+a^2b^3\right)},\ \frac{2\,Bab^2x^3-\left(3\,Ba^2-Aab+\left(3\,Bab-Ab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Ba^2b-Aab^2\right)x}{2\left(ab^4x^2+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*B*a*b^2*x^3 + (3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x^2)*sqrt(-a*b)*
log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^2*b - A*a*b^2)*x)/
(a*b^4*x^2 + a^2*b^3), 1/2*(2*B*a*b^2*x^3 - (3*B*a^2 - A*a*b + (3*B*a*b - A
*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*B*a^2*b - A*a*b^2)*x)/(a*b^
4*x^2 + a^2*b^3)]
```

giac [A] time = 0.34, size = 59, normalized size = 0.88

$$\frac{Bx}{b^2} - \frac{(3\,Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2} + \frac{Bax - Abx}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(B*
a*x - A*b*x)/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 82, normalized size = 1.22

$$-\frac{Ax}{2\left(b\,x^2 + a\right)b} + \frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b} + \frac{Bax}{2\left(b\,x^2 + a\right)b^2} - \frac{3Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2} + \frac{Bx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] B*x/b^2-1/2/b*x/(b*x^2+a)*A+1/2/b^2*x/(b*x^2+a)*B*a+1/2/b/(a*b)^(1/2)*arcta
n(1/(a*b)^(1/2)*b*x)*A-3/2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B*a
```

maxima [A] time = 2.36, size = 61, normalized size = 0.91

$$\frac{(Ba - Ab)x}{2\left(b^3x^2 + ab^2\right)} + \frac{Bx}{b^2} - \frac{(3\,Ba - Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)*x/(b^3*x^2 + a*b^2) + B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 0.12, size = 59, normalized size = 0.88

$$\frac{B\,x}{b^2}-\frac{x\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-3\,B\,a\right)}{2\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (B*x)/b^2 - (x*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (atan((b^(1/2)*x)/a
^(1/2))*(A*b - 3*B*a))/(2*a^(1/2)*b^(5/2))
```

sympy [A] time = 0.53, size = 114, normalized size = 1.70

$$\frac{Bx}{b^2}+\frac{x\left(-Ab+Ba\right)}{2ab^2+2b^3x^2}+\frac{\sqrt{-\frac{1}{ab^5}}\left(-Ab+3Ba\right)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^5}}\left(-Ab+3Ba\right)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(-A*
b + 3*B*a)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(-A*b +
 3*B*a)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/4
```

3.79 $\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=41

$$\frac{B\log\left(a+bx^2\right)}{2b^2}-\frac{Ab-aB}{2b^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{B\log\left(a+bx^2\right)}{2b^2}-\frac{Ab-aB}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(A*b - a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{Ab-aB}{2b^2\left(a+bx^2\right)}+\frac{B\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica **[A]** time = 0.01, size = 41, normalized size = 1.00

$$\frac{aB-Ab}{2b^2\left(a+bx^2\right)}+\frac{B\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^2,x]
[Out] (-(A*b) + a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2)^2,x]
[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2)^2, x]
```

fricas **[A]** time = 0.41, size = 44, normalized size = 1.07

$$\frac{Ba-Ab+\left(Bbx^2+Ba\right)\log\left(bx^2+a\right)}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/2*(B*a - A*b + (B*b*x^2 + B*a)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.25, size = 65, normalized size = 1.59

$$-\frac{B\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2\,b}-\frac{A}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
 - 1/2*A/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 47, normalized size = 1.15

$$-\frac{A}{2\left(b\,x^2+a\right)b}+\frac{Ba}{2\left(b\,x^2+a\right)b^2}+\frac{B\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2+a)*A+1/2/b^2/(b*x^2+a)*B*a+1/2*B*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.09, size = 40, normalized size = 0.98

$$\frac{Ba-Ab}{2\left(b^3x^2+ab^2\right)}+\frac{B\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)/(b^3*x^2 + a*b^2) + 1/2*B*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.08, size = 37, normalized size = 0.90

$$\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2}-\frac{A\,b-B\,a}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (B*log(a + b*x^2))/(2*b^2) - (A*b - B*a)/(2*b^2*(a + b*x^2))
```

sympy [A]   time = 0.37, size = 36, normalized size = 0.88

$$\frac{B\log\left(a+bx^2\right)}{2b^2}+\frac{-Ab+Ba}{2ab^2+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**2) + (-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2)
```

3.80 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\left(a+bx^2\right)^2}\,dx &= \frac{(Ab-aB)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aB)\int \frac{1}{a+bx^2}\,dx}{2ab}\\ &= \frac{(Ab-aB)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.00

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(aB-Ab)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(A*b) + a*B)*x)/(a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.46, size = 182, normalized size = 2.89

$$\left[-\frac{\left(Ba^2+Aab+\left(Bab+Ab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(Ba^2b-Aab^2\right)x}{4\left(a^2b^3x^2+a^3b^2\right)},\ \frac{\left(Ba^2+Aab+\left(Bab+Ab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-\left(Ba^2b-Aab^2\right)x}{2\left(a^2b^3x^2+a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*(B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^
```

```
2), 1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) - (B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.29, size = 57, normalized size = 0.90

$$\frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}-\frac{Bax-Abx}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(B*a*x - A*b*x)
/((b*x^2 + a)*a*b)
```

maple [A] time = 0.01, size = 68, normalized size = 1.08

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}+\frac{(Ab-Ba)\,x}{2\left(b\,x^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A
+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.41, size = 57, normalized size = 0.90

$$-\frac{(Ba-Ab)x}{2\left(ab^2x^2+a^2b\right)}+\frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*x/(a*b^2*x^2 + a^2*b) + 1/2*(B*a + A*b)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.12, size = 51, normalized size = 0.81

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b+B\,a\right)}{2\,a^{3/2}\,b^{3/2}}+\frac{x\,\left(A\,b-B\,a\right)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(2*a^(3/2)*b^(3/2)) + (x*(A*b - B*a
))/(2*a*b*(a + b*x^2))
```

sympy [B] time = 0.40, size = 112, normalized size = 1.78

$$\frac{x\left(Ab-Ba\right)}{2a^{2}b+2ab^{2}x^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\left(Ab+Ba\right)\log\left(-a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b^{3}}}\left(Ab+Ba\right)\log\left(a^{2}b\sqrt{-\frac{1}{a^{3}b^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(A*b + B*a)
*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(A*b + B*a)
*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

3.81 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*B)/(2*a*b*(a + b*x^2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/(2*a^
2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{Ab-aB}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{a(Ab-aB)}{b\left(a+bx^2\right)}-A\log\left(a+bx^2\right)+2A\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^2)) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)^2), x]
```

fricas [A] time = 0.43, size = 70, normalized size = 1.37

$$-\frac{Ba^2-Aab+\left(Ab^2x^2+Aab\right)\log\left(bx^2+a\right)-2\left(Ab^2x^2+Aab\right)\log(x)}{2\left(a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(B*a^2 - A*a*b + (A*b^2*x^2 + A*a*b)*log(b*x^2 + a) - 2*(A*b^2*x^2 + A
*a*b)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

giac [**A**] time = 0.33, size = 63, normalized size = 1.24

$$\frac{A\log\left(x^{2}\right)}{2\,a^{2}}-\frac{A\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{2}}+\frac{Ab^{2}x^{2}-Ba^{2}+2\,Aab}{2\left(bx^{2}+a\right)a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a^2 - 1/2*A*log(abs(b*x^2 + a))/a^2 + 1/2*(A*b^2*x^2 - B*a^2
 + 2*A*a*b)/((b*x^2 + a)*a^2*b)
```

maple [**A**] time = 0.02, size = 53, normalized size = 1.04

$$\frac{A}{2\left(b\,x^{2}+a\right)a}+\frac{A\ln(x)}{a^{2}}-\frac{A\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{B}{2\left(b\,x^{2}+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)*A-1/2/b/(b*x^2+a)*B-1/2*A*ln(b*x^2+a)/a^2+A*ln(x)/a^2
```

maxima [**A**] time = 1.12, size = 51, normalized size = 1.00

$$-\frac{Ba-Ab}{2\left(ab^{2}x^{2}+a^{2}b\right)}-\frac{A\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{A\log\left(x^{2}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)/a^2 + 1/2*A*log
(x^2)/a^2
```

mupad [**B**] time = 0.15, size = 47, normalized size = 0.92

$$\frac{A\,\ln(x)}{a^{2}}-\frac{A\,\ln\left(b\,x^{2}+a\right)}{2\,a^{2}}+\frac{A\,b-B\,a}{2\,a\,b\,\left(b\,x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (A*log(x))/a^2 - (A*log(a + b*x^2))/(2*a^2) + (A*b - B*a)/(2*a*b*(a + b*x^2
))
```

sympy [**A**]   time = 0.42, size = 46, normalized size = 0.90

$$\frac{A\log(x)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^2\right)}{2a^2} + \frac{Ab - Ba}{2a^2b + 2ab^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**2)/(2*a**2) + (A*b - B*a)/(2*a**2*b + 2*a*b*
*2*x**2)
```

3.82 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(Ab-aB)}{2a^2\left(a+bx^2\right)}-\frac{A}{a^2x}$$

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {456, 453, 205}

$$-\frac{x(Ab-aB)}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) - ((A*b - a*B)*x)/(2*a^2*(a + b*x^2)) - ((3*A*b - a*B)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)}-\frac{1}{2}\int \frac{-\frac{2A}{a}+\frac{(Ab-aB)x^2}{a^2}}{x^2\left(a+bx^2\right)}\,dx\\
&= -\frac{A}{a^2x}-\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{A}{a^2x}-\frac{(Ab-aB)x}{2a^2\left(a+bx^2\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [**A**] time = 0.03, size = 70, normalized size = 0.99

$$
\frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{x(aB-Ab)}{2a^2\left(a+bx^2\right)}-\frac{A}{a^2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + ((-(A*b) + a*B)*x)/(2*a^2*(a + b*x^2)) + ((-3*A*b + a*B)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)^2), x]
```

fricas [**A**] time = 0.48, size = 210, normalized size = 2.96

$$
\left[-\frac{4\,Aa^2b-2\left(Ba^2b-3\,Aab^2\right)x^2-\left(\left(Bab-3\,Ab^2\right)x^3+\left(Ba^2-3\,Aab\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^3b^2x^3+a^4bx\right)},-\frac{2\,Aa^2b-\left(Ba^2b-3\,Aab^2\right)x^2-\left(\left(Bab-3\,Ab^2\right)x^3+\left(Ba^2-3\,Aab\right)x\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^3b^2x^3+a^4bx\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - ((B*a*b - 3*A*b^2)*x^3 + (
B*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2)*x^2 - (
(B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a^3*b^2*x^3 + a^4*b*x)]
```

giac [A] time = 0.41, size = 62, normalized size = 0.87

$$\frac{(Ba - 3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2} + \frac{Bax^2 - 3\,Abx^2 - 2\,Aa}{2\left(bx^3 + ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(B*a*x^2 - 3*
A*b*x^2 - 2*A*a)/((b*x^3 + a*x)*a^2)
```

maple [A] time = 0.01, size = 85, normalized size = 1.20

$$-\frac{Abx}{2\left(b\,x^2 + a\right)a^2} - \frac{3Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\ a^2} + \frac{Bx}{2\left(b\,x^2 + a\right)a} + \frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\ a} - \frac{A}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2*x/(b*x^2+a)*A*b+1/2/a*x/(b*x^2+a)*B-3/2/a^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*A*b+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B-A/a^2/x
```

maxima [A] time = 2.15, size = 63, normalized size = 0.89

$$\frac{(Ba - 3\,Ab)x^2 - 2\,Aa}{2\left(a^2bx^3 + a^3x\right)} + \frac{(Ba - 3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 3*A*b)*x^2 - 2*A*a)/(a^2*b*x^3 + a^3*x) + 1/2*(B*a - 3*A*b)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 0.12, size = 63, normalized size = 0.89

$$-\frac{\frac{A}{a}+\frac{x^2\,(3\,A\,b-B\,a)}{2\,a^2}}{b\,x^3+a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,A\,b-B\,a)}{2\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - (A/a + (x^2*(3*A*b - B*a))/(2*a^2))/(a*x + b*x^3) - (atan((b^(1/2)*x)/a^(
1/2))*(3*A*b - B*a))/(2*a^(5/2)*b^(1/2))
```

sympy [A] time = 0.49, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(-3Ab+Ba)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(-3Ab+Ba)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2Aa+x^2\,(-3Ab+Ba)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(-3*A*b + B*a)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt
(-1/(a**5*b))*(-3*A*b + B*a)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*A*a +
x**2*(-3*A*b + B*a))/(2*a**3*x + 2*a**2*b*x**3)
```

3.83 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=76

$$\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{A}{2a^2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -A/(2*a^2*x^2) - (A*b - a*B)/(2*a^2*(a + b*x^2)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{a^2x^2}+\frac{-2Ab+aB}{a^3x}-\frac{b(-Ab+aB)}{a^2(a+bx)^2}-\frac{b(-2Ab+aB)}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{A}{2a^2x^2}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB-Ab)}{a+bx^2}+(2Ab-aB)\log\left(a+bx^2\right)+2\log(x)(aB-2Ab)-\frac{aA}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) + (a*(-(A*b) + a*B))/(a + b*x^2) + 2*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)^2), x]
```

fricas [A] time = 0.41, size = 117, normalized size = 1.54

$$-\frac{Aa^2-\left(Ba^2-2\,Aab\right)x^2+\left(\left(Bab-2\,Ab^2\right)x^4+\left(Ba^2-2\,Aab\right)x^2\right)\log\left(bx^2+a\right)-2\left(\left(Bab-2\,Ab^2\right)x^4+\left(Ba^2-2\,Aab\right)x^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(A*a^2 - (B*a^2 - 2*A*a*b)*x^2 + ((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A
*a*b)*x^2)*log(b*x^2 + a) - 2*((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A*a*b)*x^
2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.30, size = 82, normalized size = 1.08

$$\frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3}+\frac{Bax^2-2\,Abx^2-Aa}{2\left(bx^4+ax^2\right)a^2}-\frac{\left(Bab-2\,Ab^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 2*A*b)*log(x^2)/a^3 + 1/2*(B*a*x^2 - 2*A*b*x^2 - A*a)/((b*x^4 +
a*x^2)*a^2) - 1/2*(B*a*b - 2*A*b^2)*log(abs(b*x^2 + a))/(a^3*b)
```

maple [A] time = 0.02, size = 86, normalized size = 1.13

$$-\frac{Ab}{2\left(b\,x^2+a\right)a^2}-\frac{2Ab\ln(x)}{a^3}+\frac{Ab\ln\left(b\,x^2+a\right)}{a^3}+\frac{B}{2\left(b\,x^2+a\right)a}+\frac{B\ln(x)}{a^2}-\frac{B\ln\left(b\,x^2+a\right)}{2a^2}-\frac{A}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2*b/(b*x^2+a)*A+1/2/a/(b*x^2+a)*B+1/a^3*b*ln(b*x^2+a)*A-1/2/a^2*ln(b
*x^2+a)*B-1/2*A/a^2/x^2-2/a^3*ln(x)*A*b+1/a^2*ln(x)*B
```

maxima [A] time = 1.06, size = 76, normalized size = 1.00

$$\frac{(Ba-2\,Ab)x^2-Aa}{2\left(a^2bx^4+a^3x^2\right)}-\frac{(Ba-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^3*x^2) - 1/2*(B*a - 2*A*b)*log
(b*x^2 + a)/a^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

mupad [B] time = 0.14, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^2+a\right)\,\left(2\,A\,b-B\,a\right)}{2\,a^3}-\frac{\frac{A}{2\,a}+\frac{x^2\,\left(2\,A\,b-B\,a\right)}{2\,a^2}}{b\,x^4+a\,x^2}-\frac{\ln(x)\,\left(2\,A\,b-B\,a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(2*A*b - B*a))/(2*a^3) - (A/(2*a) + (x^2*(2*A*b - B*a))/(2*
a^2))/(a*x^2 + b*x^4) - (log(x)*(2*A*b - B*a))/a^3
```

sympy [A] time = 0.88, size = 70, normalized size = 0.92

$$\frac{-Aa+x^{2}\left(-2Ab+Ba\right)}{2a^{3}x^{2}+2a^{2}bx^{4}}+\frac{\left(-2Ab+Ba\right)\log\left(x\right)}{a^{3}}-\frac{\left(-2Ab+Ba\right)\log\left(\frac{a}{b}+x^{2}\right)}{2a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-A*a + x**2*(-2*A*b + B*a))/(2*a**3*x**2 + 2*a**2*b*x**4) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

3.84 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{b}\,(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}-\frac{A}{3a^2x^3}$$

Rubi [**A**] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {456, 1261, 205}

$$\frac{bx(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}+\frac{\sqrt{b}\,(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -A/(3*a^2*x^3) + (2*A*b - a*B)/(a^3*x) + (b*(A*b - a*B)*x)/(2*a^3*(a + b*x^
2)) + (Sqrt[b]*(5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx &= \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \frac{-\frac{2A}{ab}+\frac{2(Ab-aB)x^2}{a^2b}-\frac{(Ab-aB)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx\\ &= \frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \left(-\frac{2A}{a^2bx^4}-\frac{2(-2Ab+aB)}{a^3bx^2}+\frac{-5Ab+3aB}{a^3\left(a+bx^2\right)}\right)dx\\ &= -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}+\frac{(b(5Ab-3aB))\int \frac{1}{a+bx^2}\,dx}{2a^3}\\ &= -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x}{2a^3\left(a+bx^2\right)}+\frac{\sqrt{b}\,(5Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 1.00

$$-\frac{\sqrt{b}\,(3aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{bx(aB-Ab)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{a^3x}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*A/(a^2*x^3) + (2*A*b - a*B)/(a^3*x) - (b*(-(A*b) + a*B)*x)/(2*a^3*(a +
 b*x^2)) - (Sqrt[b]*(-5*A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)^2), x]
```

fricas [**A**] time = 0.46, size = 250, normalized size = 2.78

$$\left[-\frac{6\left(3\,Bab-5\,Ab^{2}\right)x^{4}+4\,Aa^{2}+4\left(3\,Ba^{2}-5\,Aab\right)x^{2}+3\left(\left(3\,Bab-5\,Ab^{2}\right)x^{5}+\left(3\,Ba^{2}-5\,Aab\right)x^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{12\left(a^{3}bx^{5}+a^{4}x^{3}\right)},-\frac{3\left(3\,Bab-5\,Ab^{2}\right)x^{4}+2\,Aa^{2}+2\left(3\,Ba^{2}-5\,Aab\right)x^{2}+3\left(\left(3\,Bab-5\,Ab^{2}\right)x^{5}+\left(3\,Ba^{2}-5\,Aab\right)x^{3}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{6\left(a^{3}bx^{5}+a^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(3*B*a*b - 5*A*b^2)*x^4 + 4*A*a^2 + 4*(3*B*a^2 - 5*A*a*b)*x^2 + 3
*((3*B*a*b - 5*A*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(-b/a)*log((b*x^2
+ 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), -1/6*(3*(3*B*a
*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2 + 3*((3*B*a*b - 5*A
*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3*b*
x^5 + a^4*x^3)]
```

giac [**A**] time = 0.43, size = 85, normalized size = 0.94

$$-\frac{\left(3\,Bab-5\,Ab^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}-\frac{Babx-Ab^{2}x}{2\left(bx^{2}+a\right)a^{3}}-\frac{3\,Bax^{2}-6\,Abx^{2}+Aa}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(B*a*b
*x - A*b^2*x)/((b*x^2 + a)*a^3) - 1/3*(3*B*a*x^2 - 6*A*b*x^2 + A*a)/(a^3*x^
3)
```

maple [**A**] time = 0.02, size = 110, normalized size = 1.22

$$\frac{A\,b^{2}x}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5A\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{Bbx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{2Ab}{a^{3}x}-\frac{B}{a^{2}x}-\frac{A}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a^3*b^2*x/(b*x^2+a)*A-1/2/a^2*b*x/(b*x^2+a)*B+5/2/a^3*b^2/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x)*A-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*
B-1/3*A/a^2/x^3+2/a^3/x*A*b-1/a^2/x*B
```

maxima [**A**] time = 2.45, size = 93, normalized size = 1.03

$$-\frac{3\left(3\,Bab-5\,Ab^{2}\right)x^{4}+2\,Aa^{2}+2\left(3\,Ba^{2}-5\,Aab\right)x^{2}}{6\left(a^{3}bx^{5}+a^{4}x^{3}\right)}-\frac{\left(3\,Bab-5\,Ab^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2)/(a^3
*b*x^5 + a^4*x^3) - 1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3)
```

mupad [B] time = 0.14, size = 83, normalized size = 0.92

$$\frac{\frac{x^2\,\left(5\,A\,b-3\,B\,a\right)}{3\,a^2}-\frac{A}{3\,a}+\frac{b\,x^4\,\left(5\,A\,b-3\,B\,a\right)}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(5\,A\,b-3\,B\,a\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(5*A*b - 3*B*a))/(3*a^2) - A/(3*a) + (b*x^4*(5*A*b - 3*B*a))/(2*a^3))
/(a*x^3 + b*x^5) + (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(5*A*b - 3*B*a))/(2*a
^(7/2))
```

sympy [B] time = 0.59, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\,(-5Ab+3Ba)\log{\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}\,(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}}{4}-\frac{\sqrt{-\frac{b}{a^7}}\,(-5Ab+3Ba)\log{\left(\frac{a^4\sqrt{-\frac{b}{a^7}}\,(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}}{4}+\frac{-2Aa^2+x^4\left(15Ab^2-9Bab\right)+x^2\left(10Aab-6Ba^2\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(-a**4*sqrt(-b/a**7)*(-5*A*b + 3*B*a)/(-5
*A*b**2 + 3*B*a*b) + x)/4 - sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(a**4*sqrt(-b
/a**7)*(-5*A*b + 3*B*a)/(-5*A*b**2 + 3*B*a*b) + x)/4 + (-2*A*a**2 + x**4*(1
5*A*b**2 - 9*B*a*b) + x**2*(10*A*a*b - 6*B*a**2))/(6*a**4*x**3 + 6*a**3*b*x
**5)
```

3.85 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{A}{4a^2x^4}$$

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{2Ab-aB}{2a^3x^2}-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -A/(4*a^2*x^4) + (2*A*b - a*B)/(2*a^3*x^2) + (b*(A*b - a*B))/(2*a^3*(a + b*
x^2)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^2])
/(2*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2aB)}{a^4(a+bx)}\right.\right.\\ &= -\frac{A}{4a^2x^4}+\frac{2Ab-aB}{2a^3x^2}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx\right.}{2a^4}\end{aligned}$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^4}+\frac{2ab(aB-Ab)}{a+bx^2}+\frac{2a(aB-2Ab)}{x^2}+2b(3Ab-2aB)\log\left(a+bx^2\right)-4b\log(x)(3Ab-2aB)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] -1/4*((a^2*A)/x^4 + (2*a*(-2*A*b + a*B))/x^2 + (2*a*b*(-(A*b) + a*B))/(a +
b*x^2) - 4*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^2])/a
^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)^2), x]
```

fricas [A] time = 0.43, size = 154, normalized size = 1.59

$$-\frac{2\left(2\,Ba^2b-3\,Aab^2\right)x^4+Aa^3+\left(2\,Ba^3-3\,Aa^2b\right)x^2-2\left(\left(2\,Bab^2-3\,Ab^3\right)x^6+\left(2\,Ba^2b-3\,Aab^2\right)x^4\right)\log\left(bx^2+a\right)+4\left(\left(2\,Bab^2-3\,Ab^3\right)x^6+\left(2\,Ba^2b-3\,Aab^2\right)x^4\right)\log(x)}{4\left(a^4bx^6+a^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a^2*b - 3*A*a*b^2)*x^4 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^2 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(b*x^2 + a) +
```

```
 4*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(x))/(a^4*b
*x^6 + a^5*x^4)
```

giac [A] time = 0.39, size = 150, normalized size = 1.55

$$-\frac{\left(2\,Bab-3\,Ab^{2}\right)\log\left(x^{2}\right)}{2\,a^{4}}+\frac{\left(2\,Bab^{2}-3\,Ab^{3}\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{4}b}-\frac{2\,Bab^{2}x^{2}-3\,Ab^{3}x^{2}+3\,Ba^{2}b-4\,Aab^{2}}{2\left(bx^{2}+a\right)a^{4}}+\frac{6\,Babx^{4}-9\,Ab^{2}x^{4}-2\,Ba^{2}x^{2}+4\,Aabx^{2}-Aa^{2}}{4\,a^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*b - 3*A*b^2)*log(x^2)/a^4 + 1/2*(2*B*a*b^2 - 3*A*b^3)*log(abs(b
*x^2 + a))/(a^4*b) - 1/2*(2*B*a*b^2*x^2 - 3*A*b^3*x^2 + 3*B*a^2*b - 4*A*a*b
^2)/((b*x^2 + a)*a^4) + 1/4*(6*B*a*b*x^4 - 9*A*b^2*x^4 - 2*B*a^2*x^2 + 4*A*
a*b*x^2 - A*a^2)/(a^4*x^4)
```

maple [A] time = 0.02, size = 114, normalized size = 1.18

$$\frac{A\,b^{2}}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{3A\,b^{2}\ln(x)}{a^{4}}-\frac{3A\,b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{4}}-\frac{Bb}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{2Bb\ln(x)}{a^{3}}+\frac{Bb\ln\left(b\,x^{2}+a\right)}{a^{3}}+\frac{Ab}{a^{3}x^{2}}-\frac{B}{2a^{2}x^{2}}-\frac{A}{4a^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a^3*b^2/(b*x^2+a)*A-1/2/a^2*b/(b*x^2+a)*B-3/2/a^4*b^2*ln(b*x^2+a)*A+1/a
^3*b*ln(b*x^2+a)*B-1/4*A/a^2/x^4+1/a^3/x^2*A*b-1/2/a^2/x^2*B+3*b^2/a^4*ln(x
)*A-2*b/a^3*ln(x)*B
```

maxima [A] time = 1.13, size = 106, normalized size = 1.09

$$-\frac{2\left(2\,Bab-3\,Ab^{2}\right)x^{4}+Aa^{2}+\left(2\,Ba^{2}-3\,Aab\right)x^{2}}{4\left(a^{3}bx^{6}+a^{4}x^{4}\right)}+\frac{\left(2\,Bab-3\,Ab^{2}\right)\log\left(bx^{2}+a\right)}{2\,a^{4}}-\frac{\left(2\,Bab-3\,Ab^{2}\right)\log\left(x^{2}\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(2*B*a*b - 3*A*b^2)*x^4 + A*a^2 + (2*B*a^2 - 3*A*a*b)*x^2)/(a^3*b*x
^6 + a^4*x^4) + 1/2*(2*B*a*b - 3*A*b^2)*log(b*x^2 + a)/a^4 - 1/2*(2*B*a*b -
 3*A*b^2)*log(x^2)/a^4
```

mupad [B] time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^{2}\,(3\,A\,b-2\,B\,a)}{4\,a^{2}}-\frac{A}{4\,a}+\frac{b\,x^{4}\,(3\,A\,b-2\,B\,a)}{2\,a^{3}}}{b\,x^{6}+a\,x^{4}}-\frac{\ln\left(b\,x^{2}+a\right)\,\left(3\,A\,b^{2}-2\,B\,a\,b\right)}{2\,a^{4}}+\frac{\ln(x)\,\left(3\,A\,b^{2}-2\,B\,a\,b\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(3*A*b - 2*B*a))/(4*a^2) - A/(4*a) + (b*x^4*(3*A*b - 2*B*a))/(2*a^3))
/(a*x^4 + b*x^6) - (log(a + b*x^2)*(3*A*b^2 - 2*B*a*b))/(2*a^4) + (log(x)*(
3*A*b^2 - 2*B*a*b))/a^4
```

sympy [A] time = 1.02, size = 100, normalized size = 1.03

$$\frac{-Aa^2 + x^4\left(6Ab^2 - 4Bab\right) + x^2\left(3Aab - 2Ba^2\right)}{4a^4x^4 + 4a^3bx^6} - \frac{b\left(-3Ab + 2Ba\right)\log\left(x\right)}{a^4} + \frac{b\left(-3Ab + 2Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**2,x)
```

```
[Out] (-A*a**2 + x**4*(6*A*b**2 - 4*B*a*b) + x**2*(3*A*a*b - 2*B*a**2))/(4*a**4*x
**4 + 4*a**3*b*x**6) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a)*
log(a/b + x**2)/(2*a**4)
```

3.86 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b(3Ab-2aB)}{a^4x}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{5a^2x^5}$$

Rubi [A] time = 0.19, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {456, 1802, 205}

$$-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -A/(5*a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) - (b*(3*A*b - 2*a*B))/(a^4*x) -
(b^2*(A*b - a*B)*x)/(2*a^4*(a + b*x^2)) - (b^(3/2)*(7*A*b - 5*a*B)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx &= -\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{1}{2}b^2\int \frac{-\frac{2A}{ab^2}+\frac{2(Ab-aB)x^2}{a^2b^2}-\frac{2(Ab-aB)x^4}{a^3b}+\frac{(Ab-aB)x^6}{a^4}}{x^6\left(a+bx^2\right)}\,dx\\
&= -\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{1}{2}b^2\int \left(-\frac{2A}{a^2b^2x^6}-\frac{2(-2Ab+aB)}{a^3b^2x^4}+\frac{2(-3Ab+2aB)}{a^4bx^2}+\frac{7Ab-5aB}{a^4\left(a+bx^2\right)}\right)\\
&= -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{\left(b^2(7Ab-5aB)\right)\int \frac{1}{a+bx^2}\,dx}{2a^4}\\
&= -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x}-\frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.99

$$\frac{b^{3/2}(5aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}}+\frac{b^2x(aB-Ab)}{2a^4\left(a+bx^2\right)}+\frac{b(2aB-3Ab)}{a^4x}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*A/(a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(-3*A*b + 2*a*B))/(a^4*x)
 + (b^2*(-(A*b) + a*B)*x)/(2*a^4*(a + b*x^2)) + (b^(3/2)*(-7*A*b + 5*a*B)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)^2), x]
```

fricas [A] time = 0.46, size = 308, normalized size = 2.73

$$\left[\frac{30\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{6}+20\left(5\,Ba^{2}b-7\,Aab^{2}\right)x^{4}-12\,Aa^{3}-4\left(5\,Ba^{3}-7\,Aa^{2}b\right)x^{2}-15\left(\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{7}+\left(5\,Ba^{2}b-7\,Aab^{2}\right)x^{5}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{60\left(a^{4}bx^{7}+a^{5}x^{5}\right)},\ \frac{15\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{6}+10\left(5\,Ba^{2}b-7\,Aab^{2}\right)x^{4}-6\,Aa^{3}-2\left(5\,Ba^{3}-7\,Aa^{2}b\right)x^{2}+15\left(\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{7}+\left(5\,Ba^{2}b-7\,Aab^{2}\right)x^{5}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\left(a^{4}bx^{7}+a^{5}x^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(30*(5*B*a*b^2 - 7*A*b^3)*x^6 + 20*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 12*A
*a^3 - 4*(5*B*a^3 - 7*A*a^2*b)*x^2 - 15*((5*B*a*b^2 - 7*A*b^3)*x^7 + (5*B*a
^2*b - 7*A*a*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^4*b*x^7 + a^5*x^5), 1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B
*a^2*b - 7*A*a*b^2)*x^4 - 6*A*a^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2 + 15*((5*B*
a*b^2 - 7*A*b^3)*x^7 + (5*B*a^2*b - 7*A*a*b^2)*x^5)*sqrt(b/a)*arctan(x*sqrt
(b/a)))/(a^4*b*x^7 + a^5*x^5)]
```

giac [A] time = 0.30, size = 112, normalized size = 0.99

$$\frac{\left(5\,Bab^{2}-7\,Ab^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{4}}+\frac{Bab^{2}x-Ab^{3}x}{2\left(bx^{2}+a\right)a^{4}}+\frac{30\,Babx^{4}-45\,Ab^{2}x^{4}-5\,Ba^{2}x^{2}+10\,Aabx^{2}-3\,Aa^{2}}{15\,a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*B*a*b^2 - 7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/2*(B*a*
b^2*x - A*b^3*x)/((b*x^2 + a)*a^4) + 1/15*(30*B*a*b*x^4 - 45*A*b^2*x^4 - 5*
B*a^2*x^2 + 10*A*a*b*x^2 - 3*A*a^2)/(a^4*x^5)
```

maple [A] time = 0.02, size = 136, normalized size = 1.20

$$-\frac{A\,b^{3}x}{2\left(b\,x^{2}+a\right)a^{4}}-\frac{7A\,b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{4}}+\frac{B\,b^{2}x}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5B\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{3A\,b^{2}}{a^{4}x}+\frac{2Bb}{a^{3}x}+\frac{2Ab}{3a^{3}x^{3}}-\frac{B}{3a^{2}x^{3}}-\frac{A}{5a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^4*b^3*x/(b*x^2+a)*A+1/2/a^3*b^2*x/(b*x^2+a)*B-7/2/a^4*b^3/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)*A+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)*B-1/5*A/a^2/x^5+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B-3*b^2/a^4/x*A+2*b/a^3/x*
B
```

maxima [A] time = 2.31, size = 119, normalized size = 1.05

$$\frac{15\left(5\,Bab^{2}-7\,Ab^{3}\right)x^{6}+10\left(5\,Ba^{2}b-7\,Aab^{2}\right)x^{4}-6\,Aa^{3}-2\left(5\,Ba^{3}-7\,Aa^{2}b\right)x^{2}}{30\left(a^{4}bx^{7}+a^{5}x^{5}\right)}+\frac{\left(5\,Bab^{2}-7\,Ab^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 6*A*a
^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2)/(a^4*b*x^7 + a^5*x^5) + 1/2*(5*B*a*b^2 -
7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [B] time = 0.15, size = 104, normalized size = 0.92

$$-\frac{\frac{A}{5\,a}-\frac{x^2\,\left(7\,A\,b-5\,B\,a\right)}{15\,a^2}+\frac{b^2\,x^6\,\left(7\,A\,b-5\,B\,a\right)}{2\,a^4}+\frac{b\,x^4\,\left(7\,A\,b-5\,B\,a\right)}{3\,a^3}}{b\,x^7+a\,x^5}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(7\,A\,b-5\,B\,a\right)}{2\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (A/(5*a) - (x^2*(7*A*b - 5*B*a))/(15*a^2) + (b^2*x^6*(7*A*b - 5*B*a))/(2*
a^4) + (b*x^4*(7*A*b - 5*B*a))/(3*a^3))/(a*x^5 + b*x^7) - (b^(3/2)*atan((b^
(1/2)*x)/a^(1/2))*(7*A*b - 5*B*a))/(2*a^(9/2))
```

sympy [B] time = 0.68, size = 218, normalized size = 1.93

$$-\frac{\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}\,(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{-6Aa^3+x^6\left(-105Ab^3+75Bab^2\right)+x^4\left(-70Aab^2+50Ba^2b\right)+x^2\left(14Aa^2b-10Ba^3\right)}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log(-a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B
*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log
(a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + (
-6*A*a**3 + x**6*(-105*A*b**3 + 75*B*a*b**2) + x**4*(-70*A*a*b**2 + 50*B*a*
*2*b) + x**2*(14*A*a**2*b - 10*B*a**3))/(30*a**5*x**5 + 30*a**4*b*x**7)
```

3.87 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{b^2(4Ab-3aB)\log\left(a+bx^2\right)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)}+\frac{b^2(4Ab-3aB)\log\left(a+bx^2\right)}{2a^5}-\frac{b^2\log(x)(4Ab-3aB)}{a^5}-\frac{b(3Ab-2aB)}{2a^4x^2}+\frac{2Ab-aB}{4a^3x^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(4*a^3*x^4) - (b*(3*A*b - 2*a*B))/(2*a^4*x^2
) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)) - (b^2*(4*A*b - 3*a*B)*Log[x])/a^
5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(2*a^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{a^2x^4}+\frac{-2Ab+aB}{a^3x^3}-\frac{b(-3Ab+2aB)}{a^4x^2}+\frac{b^2(-4Ab+3aB)}{a^5x}-\frac{b^3(-Ab+aB)}{a^4(a+bx)^2}\right.\right.\\ &= -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{4a^3x^4}-\frac{b(3Ab-2aB)}{2a^4x^2}-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)}-\frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(4Ab}{}\end{aligned}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 0.89

$$\frac{-\frac{2a^3A}{x^6}-\frac{3a^2(aB-2Ab)}{x^4}+\frac{6ab^2(aB-Ab)}{a+bx^2}+6b^2(4Ab-3aB)\log\left(a+bx^2\right)+12b^2\log(x)(3aB-4Ab)+\frac{6ab(2aB-3Ab)}{x^2}}{12a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-2*A*b + a*B))/x^4 + (6*a*b*(-3*A*b + 2*a*B))/x^2
 + (6*a*b^2*(-(A*b) + a*B))/(a + b*x^2) + 12*b^2*(-4*A*b + 3*a*B)*Log[x] +
6*b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(12*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)^2), x]
```

fricas [A] time = 0.47, size = 184, normalized size = 1.48

$$\frac{6\left(3\,Ba^2b^2-4\,Aab^3\right)x^6-2\,Aa^4+3\left(3\,Ba^3b-4\,Aa^2b^2\right)x^4-\left(3\,Ba^4-4\,Aa^3b\right)x^2-6\left(\left(3\,Bab^3-4\,Ab^4\right)x^8+\left(3\,Ba^2b^2-4\,Aab^3\right)x^6\right)\log\left(bx^2+a\right)+12\left(\left(3\,Bab^3-4\,Ab^4\right)x^8+\left(3\,Ba^2b^2-4\,Aab^3\right)x^6\right)\log(x)}{12\left(a^5bx^8+a^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 2*A*a^4 + 3*(3*B*a^3*b - 4*A*a^2*b^
2)*x^4 - (3*B*a^4 - 4*A*a^3*b)*x^2 - 6*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3*B*a^
```

```
2*b^2 - 4*A*a*b^3)*x^6)*log(b*x^2 + a) + 12*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3
*B*a^2*b^2 - 4*A*a*b^3)*x^6)*log(x))/(a^5*b*x^8 + a^6*x^6)
```

giac [A] time = 0.39, size = 178, normalized size = 1.44

$$\frac{\left(3\,Bab^{2}-4\,Ab^{3}\right)\log\left(x^{2}\right)}{2\,a^{5}}-\frac{\left(3\,Bab^{3}-4\,Ab^{4}\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{5}b}+\frac{3\,Bab^{3}x^{2}-4\,Ab^{4}x^{2}+4\,Ba^{2}b^{2}-5\,Aab^{3}}{2\left(bx^{2}+a\right)a^{5}}-\frac{33\,Bab^{2}x^{6}-44\,Ab^{3}x^{6}-12\,Ba^{2}bx^{4}+18\,Aab^{2}x^{4}+3\,Ba^{3}x^{2}-6\,Aa^{2}bx^{2}+2\,Aa^{3}}{12\,a^{5}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5 - 1/2*(3*B*a*b^3 - 4*A*b^4)*log(abs(
b*x^2 + a))/(a^5*b) + 1/2*(3*B*a*b^3*x^2 - 4*A*b^4*x^2 + 4*B*a^2*b^2 - 5*A*
a*b^3)/((b*x^2 + a)*a^5) - 1/12*(33*B*a*b^2*x^6 - 44*A*b^3*x^6 - 12*B*a^2*b
*x^4 + 18*A*a*b^2*x^4 + 3*B*a^3*x^2 - 6*A*a^2*b*x^2 + 2*A*a^3)/(a^5*x^6)
```

maple [A] time = 0.02, size = 143, normalized size = 1.15

$$-\frac{A\,b^{3}}{2\left(b\,x^{2}+a\right)a^{4}}-\frac{4A\,b^{3}\ln(x)}{a^{5}}+\frac{2A\,b^{3}\ln\left(b\,x^{2}+a\right)}{a^{5}}+\frac{B\,b^{2}}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{3B\,b^{2}\ln(x)}{a^{4}}-\frac{3B\,b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{4}}-\frac{3A\,b^{2}}{2a^{4}x^{2}}+\frac{Bb}{a^{3}x^{2}}+\frac{Ab}{2a^{3}x^{4}}-\frac{B}{4a^{2}x^{4}}-\frac{A}{6a^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^4*b^3/(b*x^2+a)*A+1/2/a^3*b^2/(b*x^2+a)*B+2/a^5*b^3*ln(b*x^2+a)*A-3/
2/a^4*b^2*ln(b*x^2+a)*B-1/6*A/a^2/x^6+1/2/a^3/x^4*A*b-1/4/a^2/x^4*B-3/2*b^2
/a^4/x^2*A+b/a^3/x^2*B-4*b^3/a^5*ln(x)*A+3*b^2/a^4*ln(x)*B
```

maxima [A] time = 1.10, size = 136, normalized size = 1.10

$$\frac{6\left(3\,Bab^{2}-4\,Ab^{3}\right)x^{6}+3\left(3\,Ba^{2}b-4\,Aab^{2}\right)x^{4}-2\,Aa^{3}-\left(3\,Ba^{3}-4\,Aa^{2}b\right)x^{2}}{12\left(a^{4}bx^{8}+a^{5}x^{6}\right)}-\frac{\left(3\,Bab^{2}-4\,Ab^{3}\right)\log\left(bx^{2}+a\right)}{2\,a^{5}}+\frac{\left(3\,Bab^{2}-4\,Ab^{3}\right)\log\left(x^{2}\right)}{2\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(3*B*a*b^2 - 4*A*b^3)*x^6 + 3*(3*B*a^2*b - 4*A*a*b^2)*x^4 - 2*A*a^3
 - (3*B*a^3 - 4*A*a^2*b)*x^2)/(a^4*b*x^8 + a^5*x^6) - 1/2*(3*B*a*b^2 - 4*A*
b^3)*log(b*x^2 + a)/a^5 + 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5
```

mupad [B] time = 0.16, size = 126, normalized size = 1.02

$$\frac{\ln\left(b\,x^{2}+a\right)\,\left(4\,A\,b^{3}-3\,B\,a\,b^{2}\right)}{2\,a^{5}}-\frac{\frac{A}{6\,a}-\frac{x^{2}\,\left(4\,A\,b-3\,B\,a\right)}{12\,a^{2}}+\frac{b^{2}\,x^{6}\,\left(4\,A\,b-3\,B\,a\right)}{2\,a^{4}}+\frac{b\,x^{4}\,\left(4\,A\,b-3\,B\,a\right)}{4\,a^{3}}}{b\,x^{8}+a\,x^{6}}-\frac{\ln(x)\,\left(4\,A\,b^{3}-3\,B\,a\,b^{2}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(4*A*b^3 - 3*B*a*b^2))/(2*a^5) - (A/(6*a) - (x^2*(4*A*b - 3
*B*a))/(12*a^2) + (b^2*x^6*(4*A*b - 3*B*a))/(2*a^4) + (b*x^4*(4*A*b - 3*B*a
))/(4*a^3))/(a*x^6 + b*x^8) - (log(x)*(4*A*b^3 - 3*B*a*b^2))/a^5
```

sympy [A] time = 1.10, size = 129, normalized size = 1.04

$$\frac{-2Aa^3 + x^6\left(-24Ab^3 + 18Bab^2\right) + x^4\left(-12Aab^2 + 9Ba^2b\right) + x^2\left(4Aa^2b - 3Ba^3\right)}{12a^5x^6 + 12a^4bx^8} + \frac{b^2\left(-4Ab + 3Ba\right)\log\left(x\right)}{a^5} - \frac{b^2\left(-4Ab + 3Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**2,x)
```

```
[Out] (-2*A*a**3 + x**6*(-24*A*b**3 + 18*B*a*b**2) + x**4*(-12*A*a*b**2 + 9*B*a**
2*b) + x**2*(4*A*a**2*b - 3*B*a**3))/(12*a**5*x**6 + 12*a**4*b*x**8) + b**2
*(-4*A*b + 3*B*a)*log(x)/a**5 - b**2*(-4*A*b + 3*B*a)*log(a/b + x**2)/(2*a*
*5)
```

3.88 $\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=150

$$\frac{a^5(Ab-aB)}{4b^7\left(a+bx^2\right)^2}-\frac{a^4(5Ab-6aB)}{2b^7\left(a+bx^2\right)}-\frac{5a^3(2Ab-3aB)\log\left(a+bx^2\right)}{2b^7}+\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{3ax^4(Ab-2aB)}{4b^5}+\frac{x^6(Ab-3a}{6b^4}$$

Rubi [A] time = 0.23, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a^2x^2(3Ab-5aB)}{b^6}-\frac{a^4(5Ab-6aB)}{2b^7\left(a+bx^2\right)}+\frac{a^5(Ab-aB)}{4b^7\left(a+bx^2\right)^2}-\frac{5a^3(2Ab-3aB)\log\left(a+bx^2\right)}{2b^7}+\frac{x^6(Ab-3aB)}{6b^4}-\frac{3ax^4(Ab-2aB)}{4b^5}+\frac{Bx^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(4*b^5) + ((A*b - 3
*a*B)*x^6)/(6*b^4) + (B*x^8)/(8*b^3) + (a^5*(A*b - a*B))/(4*b^7*(a + b*x^2)
^2) - (a^4*(5*A*b - 6*a*B))/(2*b^7*(a + b*x^2)) - (5*a^3*(2*A*b - 3*a*B)*Lo
g[a + b*x^2])/(2*b^7)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^5(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{2a^2(-3Ab+5aB)}{b^6}+\frac{3a(-Ab+2aB)x}{b^5}+\frac{(Ab-3aB)x^2}{b^4}+\frac{Bx^3}{b^3}+\frac{a^5(-Ab}{b^6(a+}\right.\right.\\ &= \frac{a^2(3Ab-5aB)x^2}{b^6}-\frac{3a(Ab-2aB)x^4}{4b^5}+\frac{(Ab-3aB)x^6}{6b^4}+\frac{Bx^8}{8b^3}+\frac{a^5(Ab-aB)}{4b^7\left(a+bx^2\right)^2}-\frac{a^4(5Ab}{2b^7\left(a\right.}\end{aligned}$$

Mathematica [A] time = 0.08, size = 136, normalized size = 0.91

$$\frac{\frac{6a^5(Ab-aB)}{\left(a+bx^2\right)^2}+\frac{12a^4(6aB-5Ab)}{a+bx^2}+60a^3(3aB-2Ab)\log\left(a+bx^2\right)-24a^2bx^2(5aB-3Ab)+4b^3x^6(Ab-3aB)+18ab^2x^4(2aB-Ab)+3b^4Bx^8}{24b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-24*a^2*b*(-3*A*b + 5*a*B)*x^2 + 18*a*b^2*(-(A*b) + 2*a*B)*x^4 + 4*b^3*(A*
b - 3*a*B)*x^6 + 3*b^4*B*x^8 + (6*a^5*(A*b - a*B))/(a + b*x^2)^2 + (12*a^4*
(-5*A*b + 6*a*B))/(a + b*x^2) + 60*a^3*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(24
*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^11*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.44, size = 231, normalized size = 1.54

$$\frac{3Bb^6x^{12}-2\left(3Bab^5-2Ab^6\right)x^{10}+5\left(3Ba^2b^4-2Aab^5\right)x^8+66Ba^6-54Aa^5b-20\left(3Ba^3b^3-2Aa^2b^4\right)x^6-6\left(34Ba^4b^2-21Aa^3b^3\right)x^4-12\left(4Ba^5b-Aa^4b^2\right)x^2+60\left(3Ba^6-2Aa^5b+\left(3Ba^4b^2-2Aa^3b^3\right)x^4+2\left(3Ba^5b-2Aa^4b^2\right)x^2\right)\log\left(bx^2+a\right)}{24\left(b^9x^4+2ab^8x^2+a^2b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*B*b^6*x^12 - 2*(3*B*a*b^5 - 2*A*b^6)*x^10 + 5*(3*B*a^2*b^4 - 2*A*a*
b^5)*x^8 + 66*B*a^6 - 54*A*a^5*b - 20*(3*B*a^3*b^3 - 2*A*a^2*b^4)*x^6 - 6*(
34*B*a^4*b^2 - 21*A*a^3*b^3)*x^4 - 12*(4*B*a^5*b - A*a^4*b^2)*x^2 + 60*(3*B
*a^6 - 2*A*a^5*b + (3*B*a^4*b^2 - 2*A*a^3*b^3)*x^4 + 2*(3*B*a^5*b - 2*A*a^4
*b^2)*x^2)*log(b*x^2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

giac [A] time = 0.40, size = 183, normalized size = 1.22

$$\frac{5\left(3\,Ba^4-2\,Aa^3b\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^7}-\frac{45\,Ba^4b^2x^4-30\,Aa^3b^3x^4+78\,Ba^5bx^2-50\,Aa^4b^2x^2+34\,Ba^6-21\,Aa^5b}{4\left(bx^2+a\right)^2b^7}+\frac{3\,Bb^9x^8-12\,Bab^8x^6+4\,Ab^9x^6+36\,Ba^2b^7x^4-18\,Aab^8x^4-120\,Ba^3b^6x^2+72\,Aa^2b^7x^2}{24\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/2*(3*B*a^4 - 2*A*a^3*b)*log(abs(b*x^2 + a))/b^7 - 1/4*(45*B*a^4*b^2*x^4 -
 30*A*a^3*b^3*x^4 + 78*B*a^5*b*x^2 - 50*A*a^4*b^2*x^2 + 34*B*a^6 - 21*A*a^5
*b)/((b*x^2 + a)^2*b^7) + 1/24*(3*B*b^9*x^8 - 12*B*a*b^8*x^6 + 4*A*b^9*x^6
+ 36*B*a^2*b^7*x^4 - 18*A*a*b^8*x^4 - 120*B*a^3*b^6*x^2 + 72*A*a^2*b^7*x^2)
/b^12
```

maple [A] time = 0.02, size = 182, normalized size = 1.21

$$\frac{B\,x^8}{8b^3}+\frac{A\,x^6}{6b^3}-\frac{Ba\,x^6}{2b^4}-\frac{3Aa\,x^4}{4b^4}+\frac{3B\,a^2x^4}{2b^5}+\frac{A\,a^5}{4\left(b\,x^2+a\right)^2b^6}+\frac{3A\,a^2x^2}{b^5}-\frac{B\,a^6}{4\left(b\,x^2+a\right)^2b^7}-\frac{5B\,a^3x^2}{b^6}-\frac{5A\,a^4}{2\left(b\,x^2+a\right)b^6}-\frac{5A\,a^3\ln\left(b\,x^2+a\right)}{b^6}+\frac{3B\,a^5}{\left(b\,x^2+a\right)b^7}+\frac{15B\,a^4\ln\left(b\,x^2+a\right)}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/8*B*x^8/b^3+1/6/b^3*x^6*A-1/2/b^4*x^6*B*a-3/4/b^4*x^4*a*A+3/2/b^5*x^4*a^2
*B+3/b^5*A*x^2*a^2-5/b^6*B*x^2*a^3-5/2*a^4/b^6/(b*x^2+a)*A+3*a^5/b^7/(b*x^2
+a)*B+1/4*a^5/b^6/(b*x^2+a)^2*A-1/4*a^6/b^7/(b*x^2+a)^2*B-5*a^3/b^6*ln(b*x^
2+a)*A+15/2*a^4/b^7*ln(b*x^2+a)*B
```

maxima [A] time = 1.12, size = 165, normalized size = 1.10

$$\frac{11\,Ba^6-9\,Aa^5b+2\left(6\,Ba^5b-5\,Aa^4b^2\right)x^2}{4\left(b^9x^4+2\,ab^8x^2+a^2b^7\right)}+\frac{3\,Bb^3x^8-4\left(3\,Bab^2-Ab^3\right)x^6+18\left(2\,Ba^2b-Aab^2\right)x^4-24\left(5\,Ba^3-3\,Aa^2b\right)x^2}{24\,b^6}+\frac{5\left(3\,Ba^4-2\,Aa^3b\right)\log\left(bx^2+a\right)}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(11*B*a^6 - 9*A*a^5*b + 2*(6*B*a^5*b - 5*A*a^4*b^2)*x^2)/(b^9*x^4 + 2*a
*b^8*x^2 + a^2*b^7) + 1/24*(3*B*b^3*x^8 - 4*(3*B*a*b^2 - A*b^3)*x^6 + 18*(2
*B*a^2*b - A*a*b^2)*x^4 - 24*(5*B*a^3 - 3*A*a^2*b)*x^2)/b^6 + 5/2*(3*B*a^4
- 2*A*a^3*b)*log(b*x^2 + a)/b^7
```

mupad [B] time = 0.12, size = 225, normalized size = 1.50

$$\frac{\frac{11\,B\,a^6-9\,A\,a^5\,b}{4\,b}+x^2\,\left(3\,B\,a^5-\frac{5\,A\,a^4\,b}{2}\right)}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}-x^2\,\left(\frac{B\,a^3}{2\,b^6}-\frac{3\,a\,\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)}{2\,b}+\frac{3\,a^2\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{2\,b^2}\right)+x^6\,\left(\frac{A}{6\,b^3}-\frac{B\,a}{2\,b^4}\right)-x^4\,\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{4\,b}+\frac{3\,B\,a^2}{4\,b^5}\right)+\frac{B\,x^8}{8\,b^3}+\frac{\ln\left(b\,x^2+a\right)\,\left(15\,B\,a^4-10\,A\,a^3\,b\right)}{2\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((11*B*a^6 - 9*A*a^5*b)/(4*b) + x^2*(3*B*a^5 - (5*A*a^4*b)/2))/(a^2*b^6 + b
^8*x^4 + 2*a*b^7*x^2) - x^2*((B*a^3)/(2*b^6) - (3*a*((3*a*(A/b^3 - (3*B*a)/
b^4))/b + (3*B*a^2)/b^5))/(2*b) + (3*a^2*(A/b^3 - (3*B*a)/b^4))/(2*b^2)) +
x^6*(A/(6*b^3) - (B*a)/(2*b^4)) - x^4*((3*a*(A/b^3 - (3*B*a)/b^4))/(4*b) +
(3*B*a^2)/(4*b^5)) + (B*x^8)/(8*b^3) + (log(a + b*x^2)*(15*B*a^4 - 10*A*a^3
*b))/(2*b^7)
```

sympy [A] time = 1.61, size = 170, normalized size = 1.13

$$\frac{Bx^8}{8b^3}+\frac{5a^3\left(-2Ab+3Ba\right)\log\left(a+bx^2\right)}{2b^7}+x^6\left(\frac{A}{6b^3}-\frac{Ba}{2b^4}\right)+x^4\left(-\frac{3Aa}{4b^4}+\frac{3Ba^2}{2b^5}\right)+x^2\left(\frac{3Aa^2}{b^5}-\frac{5Ba^3}{b^6}\right)+\frac{-9Aa^5b+11Ba^6+x^2\left(-10Aa^4b^2+12Ba^5b\right)}{4a^2b^7+8ab^8x^2+4b^9x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**8/(8*b**3) + 5*a**3*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**7) + x**6*(
A/(6*b**3) - B*a/(2*b**4)) + x**4*(-3*A*a/(4*b**4) + 3*B*a**2/(2*b**5)) + x
**2*(3*A*a**2/b**5 - 5*B*a**3/b**6) + (-9*A*a**5*b + 11*B*a**6 + x**2*(-10*
A*a**4*b**2 + 12*B*a**5*b))/(4*a**2*b**7 + 8*a*b**8*x**2 + 4*b**9*x**4)
```

3.89 $\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=128

$$-\frac{a^4(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^3(4Ab-5aB)}{2b^6\left(a+bx^2\right)}+\frac{a^2(3Ab-5aB)\log\left(a+bx^2\right)}{b^6}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{x^4(Ab-3aB)}{4b^4}+\frac{Bx^6}{6b^3}$$

Rubi [A] time = 0.17, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a^3(4Ab-5aB)}{2b^6\left(a+bx^2\right)}-\frac{a^4(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^2(3Ab-5aB)\log\left(a+bx^2\right)}{b^6}+\frac{x^4(Ab-3aB)}{4b^4}-\frac{3ax^2(Ab-2aB)}{2b^5}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x^2)/(2*b^5) + ((A*b - 3*a*B)*x^4)/(4*b^4) + (B*x^6)/(6
*b^3) - (a^4*(A*b - a*B))/(4*b^6*(a + b*x^2)^2) + (a^3*(4*A*b - 5*a*B))/(2*
b^6*(a + b*x^2)) + (a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/b^6
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{3a(-Ab+2aB)}{b^5} + \frac{(Ab-3aB)x}{b^4} + \frac{Bx^2}{b^3} - \frac{a^4(-Ab+aB)}{b^5(a+bx)^3} + \frac{a^3(-4Ab+5aB)}{b^5(a+bx)^2}\right.\right.$$

$$= -\frac{3a(Ab-2aB)x^2}{2b^5} + \frac{(Ab-3aB)x^4}{4b^4} + \frac{Bx^6}{6b^3} - \frac{a^4(Ab-aB)}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(4Ab-5aB)}{2b^6\left(a+bx^2\right)} + a^2(3Ab -$$

Mathematica [A] time = 0.07, size = 116, normalized size = 0.91

$$\frac{\frac{3a^4(aB-Ab)}{\left(a+bx^2\right)^2} + \frac{6a^3(4Ab-5aB)}{a+bx^2} + 12a^2(3Ab-5aB)\log\left(a+bx^2\right) + 3b^2x^4(Ab-3aB) + 18abx^2(2aB-Ab) + 2b^3Bx^6}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (18*a*b*(-(A*b) + 2*a*B)*x^2 + 3*b^2*(A*b - 3*a*B)*x^4 + 2*b^3*B*x^6 + (3*a
^4*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a^3*(4*A*b - 5*a*B))/(a + b*x^2) + 12
*a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/(12*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^9*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.43, size = 205, normalized size = 1.60

$$\frac{2\,Bb^5x^{10} - \left(5\,Bab^4 - 3\,Ab^5\right)x^8 + 4\left(5\,Ba^2b^3 - 3\,Aab^4\right)x^6 - 27\,Ba^5 + 21\,Aa^4b + 3\left(21\,Ba^3b^2 - 11\,Aa^2b^3\right)x^4 + 6\left(Ba^4b + Aa^3b^2\right)x^2 - 12\left(5\,Ba^5 - 3\,Aa^4b + \left(5\,Ba^3b^2 - 3\,Aa^2b^3\right)x^4 + 2\left(5\,Ba^4b - 3\,Aa^3b^2\right)x^2\right)\log\left(bx^2+a\right)}{12\left(b^8x^4 + 2\,ab^7x^2 + a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*B*b^5*x^10 - (5*B*a*b^4 - 3*A*b^5)*x^8 + 4*(5*B*a^2*b^3 - 3*A*a*b^4
)*x^6 - 27*B*a^5 + 21*A*a^4*b + 3*(21*B*a^3*b^2 - 11*A*a^2*b^3)*x^4 + 6*(B*
a^4*b + A*a^3*b^2)*x^2 - 12*(5*B*a^5 - 3*A*a^4*b + (5*B*a^3*b^2 - 3*A*a^2*b
^3)*x^4 + 2*(5*B*a^4*b - 3*A*a^3*b^2)*x^2)*log(b*x^2 + a))/(b^8*x^4 + 2*a*b
^7*x^2 + a^2*b^6)
```

giac [A] time = 0.33, size = 159, normalized size = 1.24

$$-\frac{\left(5\,Ba^{3}-3\,Aa^{2}b\right)\log\left(\left|bx^{2}+a\right|\right)}{b^{6}}+\frac{30\,Ba^{3}b^{2}x^{4}-18\,Aa^{2}b^{3}x^{4}+50\,Ba^{4}bx^{2}-28\,Aa^{3}b^{2}x^{2}+21\,Ba^{5}-11\,Aa^{4}b}{4\left(bx^{2}+a\right)^{2}b^{6}}+\frac{2\,Bb^{6}x^{6}-9\,Bab^{5}x^{4}+3\,Ab^{6}x^{4}+36\,Ba^{2}b^{4}x^{2}-18\,Aab^{5}x^{2}}{12\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(5*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^6 + 1/4*(30*B*a^3*b^2*x^4 - 18
*A*a^2*b^3*x^4 + 50*B*a^4*b*x^2 - 28*A*a^3*b^2*x^2 + 21*B*a^5 - 11*A*a^4*b)
/((b*x^2 + a)^2*b^6) + 1/12*(2*B*b^6*x^6 - 9*B*a*b^5*x^4 + 3*A*b^6*x^4 + 36
*B*a^2*b^4*x^2 - 18*A*a*b^5*x^2)/b^9
```

maple [A] time = 0.02, size = 158, normalized size = 1.23

$$\frac{B\,x^{6}}{6b^{3}}+\frac{A\,x^{4}}{4b^{3}}-\frac{3Ba\,x^{4}}{4b^{4}}-\frac{A\,a^{4}}{4\left(b\,x^{2}+a\right)^{2}b^{5}}-\frac{3Aa\,x^{2}}{2b^{4}}+\frac{B\,a^{5}}{4\left(b\,x^{2}+a\right)^{2}b^{6}}+\frac{3B\,a^{2}x^{2}}{b^{5}}+\frac{2A\,a^{3}}{\left(b\,x^{2}+a\right)b^{5}}+\frac{3A\,a^{2}\ln\left(b\,x^{2}+a\right)}{b^{5}}-\frac{5B\,a^{4}}{2\left(b\,x^{2}+a\right)b^{6}}-\frac{5B\,a^{3}\ln\left(b\,x^{2}+a\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/6*B*x^6/b^3+1/4/b^3*A*x^4-3/4/b^4*B*x^4*a-3/2/b^4*A*x^2*a+3/b^5*B*x^2*a^2
+2*a^3/b^5/(b*x^2+a)*A-5/2*a^4/b^6/(b*x^2+a)*B-1/4*a^4/b^5/(b*x^2+a)^2*A+1/
4*a^5/b^6/(b*x^2+a)^2*B+3*a^2/b^5*ln(b*x^2+a)*A-5*a^3/b^6*ln(b*x^2+a)*B
```

maxima [A] time = 1.06, size = 141, normalized size = 1.10

$$-\frac{9\,Ba^{5}-7\,Aa^{4}b+2\left(5\,Ba^{4}b-4\,Aa^{3}b^{2}\right)x^{2}}{4\left(b^{8}x^{4}+2\,ab^{7}x^{2}+a^{2}b^{6}\right)}+\frac{2\,Bb^{2}x^{6}-3\left(3\,Bab-Ab^{2}\right)x^{4}+18\left(2\,Ba^{2}-Aab\right)x^{2}}{12\,b^{5}}-\frac{\left(5\,Ba^{3}-3\,Aa^{2}b\right)\log\left(bx^{2}+a\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*B*a^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x^2)/(b^8*x^4 + 2*a
*b^7*x^2 + a^2*b^6) + 1/12*(2*B*b^2*x^6 - 3*(3*B*a*b - A*b^2)*x^4 + 18*(2*B
*a^2 - A*a*b)*x^2)/b^5 - (5*B*a^3 - 3*A*a^2*b)*log(b*x^2 + a)/b^6
```

mupad [B] time = 0.07, size = 155, normalized size = 1.21

$$x^{4}\,\left(\frac{A}{4\,b^{3}}-\frac{3\,B\,a}{4\,b^{4}}\right)-\frac{\frac{9\,B\,a^{5}-7\,A\,a^{4}\,b}{4\,b}+x^{2}\,\left(\frac{5\,B\,a^{4}}{2}-2\,A\,a^{3}\,b\right)}{a^{2}\,b^{5}+2\,a\,b^{6}\,x^{2}+b^{7}\,x^{4}}-x^{2}\,\left(\frac{3\,a\,\left(\frac{A}{b^{3}}-\frac{3\,B\,a}{b^{4}}\right)}{2\,b}+\frac{3\,B\,a^{2}}{2\,b^{5}}\right)+\frac{B\,x^{6}}{6\,b^{3}}-\frac{\ln\left(b\,x^{2}+a\right)\,\left(5\,B\,a^{3}-3\,A\,a^{2}\,b\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] x^4*(A/(4*b^3) - (3*B*a)/(4*b^4)) - ((9*B*a^5 - 7*A*a^4*b)/(4*b) + x^2*((5*
B*a^4)/2 - 2*A*a^3*b))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^2*((3*a*(A/b^3
 - (3*B*a)/b^4))/(2*b) + (3*B*a^2)/(2*b^5)) + (B*x^6)/(6*b^3) - (log(a + b*
x^2)*(5*B*a^3 - 3*A*a^2*b))/b^6
```

sympy [A] time = 1.51, size = 143, normalized size = 1.12

$$\frac{Bx^6}{6b^3}-\frac{a^2\,(-3Ab+5Ba)\log\left(a+bx^2\right)}{b^6}+x^4\left(\frac{A}{4b^3}-\frac{3Ba}{4b^4}\right)+x^2\left(-\frac{3Aa}{2b^4}+\frac{3Ba^2}{b^5}\right)+\frac{7Aa^4b-9Ba^5+x^2\left(8Aa^3b^2-10Ba^4b\right)}{4a^2b^6+8ab^7x^2+4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) - a**2*(-3*A*b + 5*B*a)*log(a + b*x**2)/b**6 + x**4*(A/(4*b
**3) - 3*B*a/(4*b**4)) + x**2*(-3*A*a/(2*b**4) + 3*B*a**2/b**5) + (7*A*a**4
*b - 9*B*a**5 + x**2*(8*A*a**3*b**2 - 10*B*a**4*b))/(4*a**2*b**6 + 8*a*b**7
*x**2 + 4*b**8*x**4)
```

3.90 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=109

$$\frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}+\frac{x^2(Ab-3aB)}{2b^4}+\frac{Bx^4}{4b^3}$$

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = $\{446, 77\}$

$$-\frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)}+\frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2}+\frac{x^2(Ab-3aB)}{2b^4}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}+\frac{Bx^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^2)/(2*b^4) + (B*x^4)/(4*b^3) + (a^3*(A*b - a*B))/(4*b^5*(a
 + b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(2*b^5*(a + b*x^2)) - (3*a*(A*b - 2*a*
B)*Log[a + b*x^2])/(2*b^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^3}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2}+\frac{3a(-Ab+2aB)}{b^4(a+bx)}\right) d\right. \\
&= \frac{(Ab-3aB)x^2}{2b^4}+\frac{Bx^4}{4b^3}+\frac{a^3(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{a^2(3Ab-4aB)}{2b^5\left(a+bx^2\right)}-\frac{3a(Ab-2aB)\log\left(a+bx^2\right)}{2b^5}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 0.86

$$\frac{\frac{a^3(Ab-aB)}{\left(a+bx^2\right)^2}+\frac{2a^2(4aB-3Ab)}{a+bx^2}+2bx^2(Ab-3aB)+6a(2aB-Ab)\log\left(a+bx^2\right)+b^2Bx^4}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^2 + b^2*B*x^4 + (a^3*(A*b - a*B))/(a + b*x^2)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^2) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^2])/(4*
b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.45, size = 179, normalized size = 1.64

$$\frac{Bb^4x^8-2\left(2\,Bab^3-Ab^4\right)x^6+7\,Ba^4-5\,Aa^3b-\left(11\,Ba^2b^2-4\,Aab^3\right)x^4+2\left(Ba^3b-2\,Aa^2b^2\right)x^2+6\left(2\,Ba^4-Aa^3b+\left(2\,Ba^2b^2-Aab^3\right)x^4+2\left(2\,Ba^3b-Aa^2b^2\right)x^2\right)\log\left(bx^2+a\right)}{4\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b^4*x^8 - 2*(2*B*a*b^3 - A*b^4)*x^6 + 7*B*a^4 - 5*A*a^3*b - (11*B*a^
2*b^2 - 4*A*a*b^3)*x^4 + 2*(B*a^3*b - 2*A*a^2*b^2)*x^2 + 6*(2*B*a^4 - A*a^3
*b + (2*B*a^2*b^2 - A*a*b^3)*x^4 + 2*(2*B*a^3*b - A*a^2*b^2)*x^2)*log(b*x^2
 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

giac [A] time = 0.36, size = 132, normalized size = 1.21

$$\frac{3\left(2\,Ba^2-Aab\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^5}+\frac{Bb^3x^4-6\,Bab^2x^2+2\,Ab^3x^2}{4\,b^6}-\frac{18\,Ba^2b^2x^4-9\,Aab^3x^4+28\,Ba^3bx^2-12\,Aa^2b^2x^2+11\,Ba^4-4\,Aa^3b}{4\left(bx^2+a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/2*(2*B*a^2 - A*a*b)*log(abs(b*x^2 + a))/b^5 + 1/4*(B*b^3*x^4 - 6*B*a*b^2*
x^2 + 2*A*b^3*x^2)/b^6 - 1/4*(18*B*a^2*b^2*x^4 - 9*A*a*b^3*x^4 + 28*B*a^3*b
*x^2 - 12*A*a^2*b^2*x^2 + 11*B*a^4 - 4*A*a^3*b)/((b*x^2 + a)^2*b^5)
```

maple [A] time = 0.02, size = 134, normalized size = 1.23

$$\frac{B\,x^4}{4b^3}+\frac{A\,a^3}{4\left(b\,x^2+a\right)^2b^4}+\frac{A\,x^2}{2b^3}-\frac{B\,a^4}{4\left(b\,x^2+a\right)^2b^5}-\frac{3Ba\,x^2}{2b^4}-\frac{3A\,a^2}{2\left(b\,x^2+a\right)b^4}-\frac{3Aa\ln\left(b\,x^2+a\right)}{2b^4}+\frac{2B\,a^3}{\left(b\,x^2+a\right)b^5}+\frac{3B\,a^2\ln\left(b\,x^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/4*B*x^4/b^3-3/2/b^4*B*x^2*a+1/2/b^3*A*x^2-3/2*a^2/b^4/(b*x^2+a)*A+2*a^3/b
^5/(b*x^2+a)*B+1/4*a^3/b^4/(b*x^2+a)^2*A-1/4*a^4/b^5/(b*x^2+a)^2*B-3/2*a/b^
4*ln(b*x^2+a)*A+3*a^2/b^5*ln(b*x^2+a)*B
```

maxima [A] time = 1.09, size = 116, normalized size = 1.06

$$\frac{7\,Ba^4-5\,Aa^3b+2\left(4\,Ba^3b-3\,Aa^2b^2\right)x^2}{4\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}+\frac{Bbx^4-2\,(3\,Ba-Ab)x^2}{4\,b^4}+\frac{3\left(2\,Ba^2-Aab\right)\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^2)/(b^7*x^4 + 2*a*
b^6*x^2 + a^2*b^5) + 1/4*(B*b*x^4 - 2*(3*B*a - A*b)*x^2)/b^4 + 3/2*(2*B*a^2
 - A*a*b)*log(b*x^2 + a)/b^5
```

mupad [B] time = 0.08, size = 118, normalized size = 1.08

$$\frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{4\,b}+x^2\,\left(2\,B\,a^3-\frac{3\,A\,a^2\,b}{2}\right)}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+x^2\,\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(6\,B\,a^2-3\,A\,a\,b\right)}{2\,b^5}+\frac{B\,x^4}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((7*B*a^4 - 5*A*a^3*b)/(4*b) + x^2*(2*B*a^3 - (3*A*a^2*b)/2))/(a^2*b^4 + b^
6*x^4 + 2*a*b^5*x^2) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) + (log(a + b*x^2)*
(6*B*a^2 - 3*A*a*b))/(2*b^5) + (B*x^4)/(4*b^3)
```

sympy [A] time = 1.41, size = 119, normalized size = 1.09

$$\frac{Bx^4}{4b^3}+\frac{3a\left(-Ab+2Ba\right)\log\left(a+bx^2\right)}{2b^5}+x^2\left(\frac{A}{2b^3}-\frac{3Ba}{2b^4}\right)+\frac{-5Aa^3b+7Ba^4+x^2\left(-6Aa^2b^2+8Ba^3b\right)}{4a^2b^5+8ab^6x^2+4b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + 3*a*(-A*b + 2*B*a)*log(a + b*x**2)/(2*b**5) + x**2*(A/(2*
b**3) - 3*B*a/(2*b**4)) + (-5*A*a**3*b + 7*B*a**4 + x**2*(-6*A*a**2*b**2 +
8*B*a**3*b))/(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4)
```

3.91 $\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=88

$$-\frac{a^2(Ab-aB)}{4b^4\left(a+bx^2\right)^2}+\frac{a(2Ab-3aB)}{2b^4\left(a+bx^2\right)}+\frac{(Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^2}{2b^3}$$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{4b^4\left(a+bx^2\right)^2}+\frac{a(2Ab-3aB)}{2b^4\left(a+bx^2\right)}+\frac{(Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) - (a^2*(A*b - a*B))/(4*b^4*(a + b*x^2)^2) + (a*(2*A*b - 3*a
*B))/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{B}{b^3}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^3}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{Bx^2}{2b^3}-\frac{a^2(Ab-aB)}{4b^4\left(a+bx^2\right)^2}+\frac{a(2Ab-3aB)}{2b^4\left(a+bx^2\right)}+\frac{(Ab-3aB)\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A]   time = 0.04, size = 92, normalized size = 1.05

$$
\frac{2aAb-3a^2B}{2b^4\left(a+bx^2\right)}+\frac{a^3B-a^2Ab}{4b^4\left(a+bx^2\right)^2}+\frac{(Ab-3aB)\log\left(a+bx^2\right)}{2b^4}+\frac{Bx^2}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x^2)/(2*b^3) + (-(a^2*A*b) + a^3*B)/(4*b^4*(a + b*x^2)^2) + (2*a*A*b - 3
*a^2*B)/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A]   time = 0.43, size = 142, normalized size = 1.61

$$
\frac{2\,Bb^3x^6+4\,Bab^2x^4-5\,Ba^3+3\,Aa^2b-4\left(Ba^2b-Aab^2\right)x^2-2\left(\left(3\,Bab^2-Ab^3\right)x^4+3\,Ba^3-Aa^2b+2\left(3\,Ba^2b-Aab^2\right)x^2\right)\log\left(bx^2+a\right)}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b^3*x^6 + 4*B*a*b^2*x^4 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^2 - 2*((3*B*a*b^2 - A*b^3)*x^4 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
*a*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [A] time = 0.36, size = 93, normalized size = 1.06

$$\frac{Bx^2}{2\,b^3}-\frac{(3\,Ba-Ab)\log\left(\left|bx^2+a\right|\right)}{2\,b^4}+\frac{9\,Bab^2x^4-3\,Ab^3x^4+12\,Ba^2bx^2-2\,Aab^2x^2+4\,Ba^3}{4\left(bx^2+a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(9*B*a*b^2*
x^4 - 3*A*b^3*x^4 + 12*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 4*B*a^3)/((b*x^2 + a)^
2*b^4)
```

maple [A] time = 0.01, size = 109, normalized size = 1.24

$$-\frac{A\,a^2}{4\left(b\,x^2+a\right)^2b^3}+\frac{B\,a^3}{4\left(b\,x^2+a\right)^2b^4}+\frac{B\,x^2}{2b^3}+\frac{Aa}{\left(b\,x^2+a\right)b^3}+\frac{A\ln\left(b\,x^2+a\right)}{2b^3}-\frac{3B\,a^2}{2\left(b\,x^2+a\right)b^4}-\frac{3Ba\ln\left(b\,x^2+a\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*B*x^2/b^3+1/b^3*a/(b*x^2+a)*A-3/2/b^4*a^2/(b*x^2+a)*B-1/4/b^3*a^2/(b*x^
2+a)^2*A+1/4/b^4*a^3/(b*x^2+a)^2*B+1/2/b^3*ln(b*x^2+a)*A-3/2/b^4*ln(b*x^2+a
)*B*a
```

maxima [A] time = 1.08, size = 94, normalized size = 1.07

$$-\frac{5\,Ba^3-3\,Aa^2b+2\left(3\,Ba^2b-2\,Aab^2\right)x^2}{4\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}+\frac{Bx^2}{2\,b^3}-\frac{(3\,Ba-Ab)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^2)/(b^6*x^4 + 2*a*b
^5*x^2 + a^2*b^4) + 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(b*x^2 + a)/b^4
```

mupad [B] time = 0.13, size = 95, normalized size = 1.08

$$\frac{B\,x^2}{2\,b^3}-\frac{x^2\,\left(\frac{3\,B\,a^2}{2}-A\,a\,b\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{4\,b}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-3\,B\,a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (B*x^2)/(2*b^3) - (x^2*((3*B*a^2)/2 - A*a*b) + (5*B*a^3 - 3*A*a^2*b)/(4*b))
/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (log(a + b*x^2)*(A*b - 3*B*a))/(2*b^4)
```

sympy [A] time = 1.25, size = 94, normalized size = 1.07

$$\frac{Bx^2}{2b^3} + \frac{3Aa^2b - 5Ba^3 + x^2\left(4Aab^2 - 6Ba^2b\right)}{4a^2b^4 + 8ab^5x^2 + 4b^6x^4} - \frac{(-Ab + 3Ba)\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (3*A*a**2*b - 5*B*a**3 + x**2*(4*A*a*b**2 - 6*B*a**2*b))/
(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - (-A*b + 3*B*a)*log(a + b*x**2
)/(2*b**4)
```

3.92 $\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=66

$$-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{B\log\left(a+bx^2\right)}{2b^3}$$

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{B\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*(A*b - a*B))/(4*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(2*b^3*(a + b*x^2)) +
 (B*Log[a + b*x^2])/(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3}+\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{B}{b^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{a(Ab-aB)}{4b^3\left(a+bx^2\right)^2}-\frac{Ab-2aB}{2b^3\left(a+bx^2\right)}+\frac{B\log\left(a+bx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.97

$$\frac{3a^2B-ab\left(A-4Bx^2\right)+2B\left(a+bx^2\right)^2\log\left(a+bx^2\right)-2Ab^2x^2}{4b^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^2 - a*b*(A - 4*B*x^2) + 2*B*(a + b*x^2)^2*Log[a + b*x^
2])/(4*b^3*(a + b*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.42, size = 89, normalized size = 1.35

$$\frac{3\,Ba^2-Aab+2\left(2\,Bab-Ab^2\right)x^2+2\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2 + 2*(B*b^2*x^4 + 2*B*a*b*x^2
 + B*a^2)*log(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.41, size = 61, normalized size = 0.92

$$\frac{B\log\left(\left|bx^2+a\right|\right)}{2\,b^3}+\frac{2\,(2\,Ba-Ab)x^2+\frac{3\,Ba^2-Aab}{b}}{4\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*log(abs(b*x^2 + a))/b^3 + 1/4*(2*(2*B*a - A*b)*x^2 + (3*B*a^2 - A*a*b
)/b)/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 80, normalized size = 1.21

$$\frac{Aa}{4\left(b\,x^2+a\right)^2b^2}-\frac{B\,a^2}{4\left(b\,x^2+a\right)^2b^3}-\frac{A}{2\left(b\,x^2+a\right)b^2}+\frac{Ba}{\left(b\,x^2+a\right)b^3}+\frac{B\ln\left(b\,x^2+a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] -1/2/b^2/(b*x^2+a)*A+1/b^3/(b*x^2+a)*B*a+1/4*a/b^2/(b*x^2+a)^2*A-1/4*a^2/b^
3/(b*x^2+a)^2*B+1/2*B*ln(b*x^2+a)/b^3
```

maxima [A] time = 1.01, size = 72, normalized size = 1.09

$$\frac{3\,Ba^2-Aab+2\left(2\,Bab-Ab^2\right)x^2}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{B\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^
2*b^3) + 1/2*B*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.11, size = 70, normalized size = 1.06

$$\frac{\frac{3\,B\,a^2-A\,a\,b}{4\,b^3}-\frac{x^2\,(A\,b-2\,B\,a)}{2\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((3*B*a^2 - A*a*b)/(4*b^3) - (x^2*(A*b - 2*B*a))/(2*b^2))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (B*log(a + b*x^2))/(2*b^3)
```

sympy [A] time = 0.91, size = 70, normalized size = 1.06

$$\frac{B\log\left(a+bx^2\right)}{2b^3}+\frac{-Aab+3Ba^2+x^2\left(-2Ab^2+4Bab\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*log(a + b*x**2)/(2*b**3) + (-A*a*b + 3*B*a**2 + x**2*(-2*A*b**2 + 4*B*a*b
))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
```

3.93 $\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{(A+Bx^2)^2}{4(a+bx^2)^2(Ab-aB)}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.111, Rules used = {444, 37}

$$-\frac{(A+Bx^2)^2}{4(a+bx^2)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -(A + B*x^2)^2/(4*(A*b - a*B)*(a + b*x^2)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^3}\,dx, x, x^2\right)$$
$$= -\frac{\left(A+Bx^2\right)^2}{4(Ab-aB)\left(a+bx^2\right)^2}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.94

$$-\frac{B\left(a+2bx^2\right)+Ab}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(A*b + B*(a + 2*b*x^2))/(b^2*(a + b*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.44, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^2+Ba+Ab}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

giac [A] time = 0.36, size = 28, normalized size = 0.88

$$-\frac{2\,Bbx^2+Ba+Ab}{4\left(bx^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/((b*x^2 + a)^2*b^2)
```

maple [A] time = 0.01, size = 39, normalized size = 1.22

$$-\frac{B}{2\left(b\,x^2+a\right)b^2}-\frac{Ab-Ba}{4\left(b\,x^2+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] -1/2*B/b^2/(b*x^2+a)-1/4*(A*b-B*a)/b^2/(b*x^2+a)^2
```

maxima [A] time = 1.06, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^2+Ba+Ab}{4\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

mupad [B] time = 0.07, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,b+B\,a}{4\,b^2}+\frac{B\,x^2}{2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] -((A*b + B*a)/(4*b^2) + (B*x^2)/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [A] time = 0.52, size = 42, normalized size = 1.31

$$\frac{-Ab-Ba-2Bbx^2}{4a^2b^2+8ab^3x^2+4b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] (-A*b - B*a - 2*B*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

3.94 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{A}{2a^2\left(a+bx^2\right)}+\frac{Ab-aB}{4ab\left(a+bx^2\right)^2}$$

Rubi [**A**] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{A}{2a^2\left(a+bx^2\right)}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*B)/(4*a*b*(a + b*x^2)^2) + A/(2*a^2*(a + b*x^2)) + (A*Log[x])/a^3
- (A*Log[a + b*x^2])/(2*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{Ab-aB}{4ab\left(a+bx^2\right)^2}+\frac{A}{2a^2\left(a+bx^2\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.87

$$
\frac{\frac{a\left(a^2(-B)+3aAb+2Ab^2x^2\right)}{b\left(a+bx^2\right)^2}-2A\log\left(a+bx^2\right)+4A\log(x)}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^2))/(b*(a + b*x^2)^2) + 4*A*Log[x] - 2*A*L
og[a + b*x^2])/(4*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)^3), x]
```

fricas [A] time = 0.45, size = 119, normalized size = 1.75

$$
\frac{2\,Aab^2x^2-Ba^3+3\,Aa^2b-2\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\log\left(bx^2+a\right)+4\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\log(x)}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*A*a*b^2*x^2 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a
^2*b)*log(b*x^2 + a) + 4*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

giac [A] time = 0.38, size = 76, normalized size = 1.12

$$\frac{A\log\left(x^2\right)}{2\,a^3}-\frac{A\log\left(\left|bx^2+a\right|\right)}{2\,a^3}+\frac{3\,Ab^3x^4+8\,Aab^2x^2-Ba^3+6\,Aa^2b}{4\left(bx^2+a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(x^2)/a^3 - 1/2*A*log(abs(b*x^2 + a))/a^3 + 1/4*(3*A*b^3*x^4 + 8*A
*a*b^2*x^2 - B*a^3 + 6*A*a^2*b)/((b*x^2 + a)^2*a^3*b)
```

maple [A] time = 0.01, size = 68, normalized size = 1.00

$$\frac{A}{4\left(b\,x^2+a\right)^2a}-\frac{B}{4\left(b\,x^2+a\right)^2b}+\frac{A}{2\left(b\,x^2+a\right)a^2}+\frac{A\ln(x)}{a^3}-\frac{A\ln\left(b\,x^2+a\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^3,x)
```

```
[Out] 1/2*A/a^2/(b*x^2+a)+1/4/a/(b*x^2+a)^2*A-1/4/b/(b*x^2+a)^2*B-1/2*A*ln(b*x^2+
a)/a^3+A*ln(x)/a^3
```

maxima [A] time = 1.12, size = 77, normalized size = 1.13

$$\frac{2\,Ab^2x^2-Ba^2+3\,Aab}{4\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*A*b^2*x^2 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) -
 1/2*A*log(b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3
```

mupad [B] time = 0.18, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,b-B\,a}{4\,a\,b}+\frac{A\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{A\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{A\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^3),x)
```

```
[Out] ((3*A*b - B*a)/(4*a*b) + (A*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (
A*log(a + b*x^2))/(2*a^3) + (A*log(x))/a^3
```

sympy **[A]** time = 0.58, size = 75, normalized size = 1.10

$$\frac{A\log{(x)}}{a^3}-\frac{A\log{\left(\frac{a}{b}+x^2\right)}}{2a^3}+\frac{3Aab+2Ab^2x^2-Ba^2}{4a^4b+8a^3b^2x^2+4a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**2)/(2*a**3) + (3*A*a*b + 2*A*b**2*x**2 - B*a
**2)/(4*a**4*b + 8*a**3*b**2*x**2 + 4*a**2*b**3*x**4)
```

3.95 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=101

$$\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -A/(2*a^3*x^2) - (A*b - a*B)/(4*a^2*(a + b*x^2)^2) - (2*A*b - a*B)/(2*a^3*(
a + b*x^2)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2])/(
2*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{A}{a^3x^2}+\frac{-3Ab+aB}{a^4x}-\frac{b(-Ab+aB)}{a^2(a+bx)^3}-\frac{b(-2Ab+aB)}{a^3(a+bx)^2}-\frac{b(-3Ab+aB)}{a^4(a+bx)}\right)dx,\right.\\ &= -\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx\right.}{2a^4}\end{aligned}$$

Mathematica [**A**] time = 0.06, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{\left(a+bx^2\right)^2}+\frac{2a(aB-2Ab)}{a+bx^2}+2(3Ab-aB)\log\left(a+bx^2\right)+4\log(x)(aB-3Ab)-\frac{2aA}{x^2}}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a*A)/x^2 + (a^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^2) + 4*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^2])/(4*a
^4)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)^3), x]
```

fricas [**B**] time = 0.55, size = 197, normalized size = 1.95

$$\frac{2\left(Ba^2b-3\,Aab^2\right)x^4-2\,Aa^3+3\left(Ba^3-3\,Aa^2b\right)x^2-2\left(\left(Bab^2-3\,Ab^3\right)x^6+2\left(Ba^2b-3\,Aab^2\right)x^4+\left(Ba^3-3\,Aa^2b\right)x^2\right)\log\left(bx^2+a\right)+4\left(\left(Bab^2-3\,Ab^3\right)x^6+2\left(Ba^2b-3\,Aab^2\right)x^4+\left(Ba^3-3\,Aa^2b\right)x^2\right)\log\left(x\right)}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*a^2*b - 3*A*a*b^2)*x^4 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^2 - 2*
((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3*A*a^2*b
)*x^2)*log(b*x^2 + a) + 4*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2
)*x^4 + (B*a^3 - 3*A*a^2*b)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x
^2)
```

giac [A] time = 0.36, size = 138, normalized size = 1.37

$$\frac{(Ba-3\,Ab)\log\left(x^2\right)}{2\,a^4}-\frac{\left(Bab-3\,Ab^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^4b}+\frac{3\,Bab^2x^4-9\,Ab^3x^4+8\,Ba^2bx^2-22\,Aab^2x^2+6\,Ba^3-14\,Aa^2b}{4\left(bx^2+a\right)^2a^4}-\frac{Bax^2-3\,Abx^2+Aa}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*b - 3*A*b^2)*log(abs(b*x^2 + a))/
(a^4*b) + 1/4*(3*B*a*b^2*x^4 - 9*A*b^3*x^4 + 8*B*a^2*b*x^2 - 22*A*a*b^2*x^2
 + 6*B*a^3 - 14*A*a^2*b)/((b*x^2 + a)^2*a^4) - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

maple [A] time = 0.02, size = 118, normalized size = 1.17

$$-\frac{Ab}{4\left(b\,x^2+a\right)^2a^2}+\frac{B}{4\left(b\,x^2+a\right)^2a}-\frac{Ab}{\left(b\,x^2+a\right)a^3}-\frac{3Ab\ln(x)}{a^4}+\frac{3Ab\ln\left(b\,x^2+a\right)}{2a^4}+\frac{B}{2\left(b\,x^2+a\right)a^2}+\frac{B\ln(x)}{a^3}-\frac{B\ln\left(b\,x^2+a\right)}{2a^3}-\frac{A}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/a^3*b/(b*x^2+a)*A+1/2/a^2/(b*x^2+a)*B-1/4/a^2*b/(b*x^2+a)^2*A+1/4/a/(b*x
^2+a)^2*B+3/2/a^4*b*ln(b*x^2+a)*A-1/2/a^3*ln(b*x^2+a)*B-1/2*A/a^3/x^2-3/a^4
*ln(x)*A*b+1/a^3*ln(x)*B
```

maxima [A] time = 1.00, size = 109, normalized size = 1.08

$$\frac{2\left(Bab-3\,Ab^2\right)x^4-2\,Aa^2+3\left(Ba^2-3\,Aab\right)x^2}{4\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}-\frac{(Ba-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ba-3\,Ab)\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(B*a*b - 3*A*b^2)*x^4 - 2*A*a^2 + 3*(B*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*
x^6 + 2*a^4*b*x^4 + a^5*x^2) - 1/2*(B*a - 3*A*b)*log(b*x^2 + a)/a^4 + 1/2*(
B*a - 3*A*b)*log(x^2)/a^4
```

mupad [B]   time = 0.10, size = 107, normalized size = 1.06

$$\frac{\ln\left(b\,x^2+a\right)\,\left(3\,A\,b-B\,a\right)}{2\,a^4}-\frac{\frac{A}{2\,a}+\frac{3\,x^2\,\left(3\,A\,b-B\,a\right)}{4\,a^2}+\frac{b\,x^4\,\left(3\,A\,b-B\,a\right)}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6}-\frac{\ln(x)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (log(a + b*x^2)*(3*A*b - B*a))/(2*a^4) - (A/(2*a) + (3*x^2*(3*A*b - B*a))/(
4*a^2) + (b*x^4*(3*A*b - B*a))/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) - (
log(x)*(3*A*b - B*a))/a^4
```

sympy [A]   time = 1.05, size = 107, normalized size = 1.06

$$\frac{-2Aa^2+x^4\left(-6Ab^2+2Bab\right)+x^2\left(-9Aab+3Ba^2\right)}{4a^5x^2+8a^4bx^4+4a^3b^2x^6}+\frac{(-3Ab+Ba)\log(x)}{a^4}-\frac{(-3Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**4*(-6*A*b**2 + 2*B*a*b) + x**2*(-9*A*a*b + 3*B*a**2))/(4*a*
*5*x**2 + 8*a**4*b*x**4 + 4*a**3*b**2*x**6) + (-3*A*b + B*a)*log(x)/a**4 -
(-3*A*b + B*a)*log(a/b + x**2)/(2*a**4)
```

3.96 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=124

$$-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{4a^3x^4}$$

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] -A/(4*a^3*x^4) + (3*A*b - a*B)/(2*a^4*x^2) + (b*(A*b - a*B))/(4*a^3*(a + b*
x^2)^2) + (b*(3*A*b - 2*a*B))/(2*a^4*(a + b*x^2)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/(2*a^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{a^3x^3} + \frac{-3Ab+aB}{a^4x^2} - \frac{3b(-2Ab+aB)}{a^5x} + \frac{b^2(-Ab+aB)}{a^3(a+bx)^3} + \frac{b^2(-3Ab+2aB)}{a^4(a+bx)^2}\right.\right.$$

$$= -\frac{A}{4a^3x^4} + \frac{3Ab-aB}{2a^4x^2} + \frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2} + \frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)} + \frac{3b(2Ab-aB)\log(x)}{a^5} - 3b(2Ab$$

Mathematica [A] time = 0.07, size = 108, normalized size = 0.87

$$\frac{\frac{a^2b(Ab-aB)}{\left(a+bx^2\right)^2} - \frac{a^2A}{x^4} + \frac{2ab(3Ab-2aB)}{a+bx^2} - \frac{2a(aB-3Ab)}{x^2} + 6b(aB-2Ab)\log\left(a+bx^2\right) + 12b\log(x)(2Ab-aB)}{4a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] (-((a^2*A)/x^4) - (2*a*(-3*A*b + a*B))/x^2 + (a^2*b*(A*b - a*B))/(a + b*x^2
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^2) + 12*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^2])/(4*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)^3), x]
```

fricas [B] time = 0.43, size = 229, normalized size = 1.85

$$-\frac{6\left(Ba^2b^2 - 2Aab^3\right)x^6 + Aa^4 + 9\left(Ba^3b - 2Aa^2b^2\right)x^4 + 2\left(Ba^4 - 2Aa^3b\right)x^2 - 6\left(\left(Bab^3 - 2Ab^4\right)x^8 + 2\left(Ba^2b^2 - 2Aab^3\right)x^6 + \left(Ba^3b - 2Aa^2b^2\right)x^4\right)\log\left(bx^2+a\right) + 12\left(\left(Bab^3 - 2Ab^4\right)x^8 + 2\left(Ba^2b^2 - 2Aab^3\right)x^6 + \left(Ba^3b - 2Aa^2b^2\right)x^4\right)\log(x)}{4\left(a^5b^2x^8 + 2a^6bx^6 + a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + A*a^4 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^2 - 6*((B*a*b^3 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 -
```

```
2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x^4)*log(b*x^2 + a) + 12*((B*a*b^3
 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x
^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

giac [A] time = 0.35, size = 133, normalized size = 1.07

$$-\frac{3\left(Bab-2\,Ab^{2}\right)\log\left(x^{2}\right)}{2\,a^{5}}+\frac{3\left(Bab^{2}-2\,Ab^{3}\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{5}b}-\frac{6\,Bab^{2}x^{6}-12\,Ab^{3}x^{6}+9\,Ba^{2}bx^{4}-18\,Aab^{2}x^{4}+2\,Ba^{3}x^{2}-4\,Aa^{2}bx^{2}+Aa^{3}}{4\left(bx^{4}+ax^{2}\right)^{2}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5 + 3/2*(B*a*b^2 - 2*A*b^3)*log(abs(b*x^2
 + a))/(a^5*b) - 1/4*(6*B*a*b^2*x^6 - 12*A*b^3*x^6 + 9*B*a^2*b*x^4 - 18*A*a
*b^2*x^4 + 2*B*a^3*x^2 - 4*A*a^2*b*x^2 + A*a^3)/((b*x^4 + a*x^2)^2*a^4)
```

maple [A] time = 0.02, size = 150, normalized size = 1.21

$$\frac{A\,b^{2}}{4\left(b\,x^{2}+a\right)^{2}a^{3}}-\frac{Bb}{4\left(b\,x^{2}+a\right)^{2}a^{2}}+\frac{3A\,b^{2}}{2\left(b\,x^{2}+a\right)a^{4}}+\frac{6A\,b^{2}\ln(x)}{a^{5}}-\frac{3A\,b^{2}\ln\left(b\,x^{2}+a\right)}{a^{5}}-\frac{Bb}{\left(b\,x^{2}+a\right)a^{3}}-\frac{3Bb\ln(x)}{a^{4}}+\frac{3Bb\ln\left(b\,x^{2}+a\right)}{2a^{4}}+\frac{3Ab}{2a^{4}x^{2}}-\frac{B}{2a^{3}x^{2}}-\frac{A}{4a^{3}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^3,x)
```

```
[Out] 3/2/a^4*b^2/(b*x^2+a)*A-1/a^3*b/(b*x^2+a)*B+1/4/a^3*b^2/(b*x^2+a)^2*A-1/4/a
^2*b/(b*x^2+a)^2*B-3/a^5*b^2*ln(b*x^2+a)*A+3/2/a^4*b*ln(b*x^2+a)*B-1/4*A/a^
3/x^4+3/2/a^4/x^2*A*b-1/2/a^3/x^2*B+6*b^2/a^5*ln(x)*A-3*b/a^4*ln(x)*B
```

maxima [A] time = 1.06, size = 137, normalized size = 1.10

$$-\frac{6\left(Bab^{2}-2\,Ab^{3}\right)x^{6}+9\left(Ba^{2}b-2\,Aab^{2}\right)x^{4}+Aa^{3}+2\left(Ba^{3}-2\,Aa^{2}b\right)x^{2}}{4\left(a^{4}b^{2}x^{8}+2\,a^{5}bx^{6}+a^{6}x^{4}\right)}+\frac{3\left(Bab-2\,Ab^{2}\right)\log\left(bx^{2}+a\right)}{2\,a^{5}}-\frac{3\left(Bab-2\,Ab^{2}\right)\log\left(x^{2}\right)}{2\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(B*a*b^2 - 2*A*b^3)*x^6 + 9*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^2)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4) + 3/2*(B*a*b
- 2*A*b^2)*log(b*x^2 + a)/a^5 - 3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5
```

mupad [B] time = 0.16, size = 131, normalized size = 1.06

$$\frac{\frac{x^{2}\,(2\,A\,b-B\,a)}{2\,a^{2}}-\frac{A}{4\,a}+\frac{3\,b^{2}\,x^{6}\,(2\,A\,b-B\,a)}{2\,a^{4}}+\frac{9\,b\,x^{4}\,(2\,A\,b-B\,a)}{4\,a^{3}}}{a^{2}\,x^{4}+2\,a\,b\,x^{6}+b^{2}\,x^{8}}-\frac{\ln\left(b\,x^{2}+a\right)\,\left(6\,A\,b^{2}-3\,B\,a\,b\right)}{2\,a^{5}}+\frac{\ln(x)\,\left(6\,A\,b^{2}-3\,B\,a\,b\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^3),x)
```

```
[Out] ((x^2*(2*A*b - B*a))/(2*a^2) - A/(4*a) + (3*b^2*x^6*(2*A*b - B*a))/(2*a^4)
+ (9*b*x^4*(2*A*b - B*a))/(4*a^3))/(a^2*x^4 + b^2*x^8 + 2*a*b*x^6) - (log(a
 + b*x^2)*(6*A*b^2 - 3*B*a*b))/(2*a^5) + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

sympy [A] time = 1.22, size = 136, normalized size = 1.10

$$\frac{-Aa^3 + x^6\left(12Ab^3 - 6Bab^2\right) + x^4\left(18Aab^2 - 9Ba^2b\right) + x^2\left(4Aa^2b - 2Ba^3\right)}{4a^6x^4 + 8a^5bx^6 + 4a^4b^2x^8} - \frac{3b\left(-2Ab + Ba\right)\log\left(x\right)}{a^5} + \frac{3b\left(-2Ab + Ba\right)\log\left(\frac{a}{b} + x^2\right)}{2a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**3,x)
```

```
[Out] (-A*a**3 + x**6*(12*A*b**3 - 6*B*a*b**2) + x**4*(18*A*a*b**2 - 9*B*a**2*b)
+ x**2*(4*A*a**2*b - 2*B*a**3))/(4*a**6*x**4 + 8*a**5*b*x**6 + 4*a**4*b**2*
x**8) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + 3*b*(-2*A*b + B*a)*log(a/b + x**2)
/(2*a**5)
```

3.97 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=149

$$\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{2a^5x^2}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-a}{4a^4x^4}$$

Rubi [A] time = 0.17, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)}-\frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{2b^2\log(x)(5Ab-3aB)}{a^6}-\frac{3b(2Ab-aB)}{2a^5x^2}+\frac{3Ab-aB}{4a^4x^4}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(4*a^4*x^4) - (3*b*(2*A*b - a*B))/(2*a^5*x^2
) - (b^2*(A*b - a*B))/(4*a^4*(a + b*x^2)^2) - (b^2*(4*A*b - 3*a*B))/(2*a^5*
(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x])/a^6 + (b^2*(5*A*b - 3*a*B)*Lo
g[a + b*x^2])/a^6
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\Bigg(\int\Bigg(\frac{A}{a^3x^4} + \frac{-3Ab+aB}{a^4x^3} - \frac{3b(-2Ab+aB)}{a^5x^2} + \frac{2b^2(-5Ab+3aB)}{a^6x} - \frac{b^3(-Ab+aB}{a^4(a+bx)^3}$$

$$= -\frac{A}{6a^3x^6} + \frac{3Ab-aB}{4a^4x^4} - \frac{3b(2Ab-aB)}{2a^5x^2} - \frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2} - \frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)} - \frac{2b^2(5Ab-3a}{a^6}$$

Mathematica [A] time = 0.11, size = 135, normalized size = 0.91

$$\frac{-\frac{2a^3A}{x^6} + \frac{3a^2b^2(aB-Ab)}{\left(a+bx^2\right)^2} - \frac{3a^2(aB-3Ab)}{x^4} + \frac{6ab^2(3aB-4Ab)}{a+bx^2} + 12b^2(5Ab-3aB)\log\left(a+bx^2\right) + 24b^2\log(x)(3aB-5Ab) + \frac{18ab(aB-2Ab)}{x^2}}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a^3*A)/x^6 - (3*a^2*(-3*A*b + a*B))/x^4 + (18*a*b*(-2*A*b + a*B))/x^2
+ (3*a^2*b^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a*b^2*(-4*A*b + 3*a*B))/(a
+ b*x^2) + 24*b^2*(-5*A*b + 3*a*B)*Log[x] + 12*b^2*(5*A*b - 3*a*B)*Log[a +
b*x^2])/(12*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)^3), x]
```

fricas [A] time = 0.44, size = 267, normalized size = 1.79

$$\frac{12\left(3Ba^2b^3-5Aab^4\right)x^8+18\left(3Ba^3b^2-5Aa^2b^3\right)x^6-2Aa^5+4\left(3Ba^4b-5Aa^3b^2\right)x^4-\left(3Ba^5-5Aa^4b\right)x^2-12\left(\left(3Bab^4-5Ab^5\right)x^{10}+2\left(3Ba^2b^3-5Aab^4\right)x^8+\left(3Ba^3b^2-5Aa^2b^3\right)x^6\right)\log\left(bx^2+a\right)+24\left(\left(3Bab^4-5Ab^5\right)x^{10}+2\left(3Ba^2b^3-5Aab^4\right)x^8+\left(3Ba^3b^2-5Aa^2b^3\right)x^6\right)\log(x)}{12\left(a^6b^2x^{10}+2a^7bx^8+a^8x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + 18*(3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6
 - 2*A*a^5 + 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^4 - (3*B*a^5 - 5*A*a^4*b)*x^2 -
12*((3*B*a*b^4 - 5*A*b^5)*x^10 + 2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3
*b^2 - 5*A*a^2*b^3)*x^6)*log(b*x^2 + a) + 24*((3*B*a*b^4 - 5*A*b^5)*x^10 +
2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6)*log(x))/
(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6)
```

giac [A] time = 0.40, size = 201, normalized size = 1.35

$$\frac{\left(3\,Bab^{2}-5\,Ab^{3}\right)\log\left(x^{2}\right)}{a^{6}}-\frac{\left(3\,Bab^{3}-5\,Ab^{4}\right)\log\left(\left|bx^{2}+a\right|\right)}{a^{6}b}+\frac{18\,Bab^{4}x^{4}-30\,Ab^{5}x^{4}+42\,Ba^{2}b^{3}x^{2}-68\,Aab^{4}x^{2}+25\,Ba^{3}b^{2}-39\,Aa^{2}b^{3}}{4\,\left(bx^{2}+a\right)^{2}a^{6}}-\frac{66\,Bab^{2}x^{6}-110\,Ab^{3}x^{6}-18\,Ba^{2}bx^{4}+36\,Aab^{2}x^{4}+3\,Ba^{3}x^{2}-9\,Aa^{2}bx^{2}+2\,Aa^{3}}{12\,a^{6}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6 - (3*B*a*b^3 - 5*A*b^4)*log(abs(b*x^2 +
a))/(a^6*b) + 1/4*(18*B*a*b^4*x^4 - 30*A*b^5*x^4 + 42*B*a^2*b^3*x^2 - 68*A*
a*b^4*x^2 + 25*B*a^3*b^2 - 39*A*a^2*b^3)/((b*x^2 + a)^2*a^6) - 1/12*(66*B*a
*b^2*x^6 - 110*A*b^3*x^6 - 18*B*a^2*b*x^4 + 36*A*a*b^2*x^4 + 3*B*a^3*x^2 -
9*A*a^2*b*x^2 + 2*A*a^3)/(a^6*x^6)
```

maple [A] time = 0.02, size = 180, normalized size = 1.21

$$-\frac{A\,b^{3}}{4\left(b\,x^{2}+a\right)^{2}a^{4}}+\frac{B\,b^{2}}{4\left(b\,x^{2}+a\right)^{2}a^{3}}-\frac{2A\,b^{3}}{\left(b\,x^{2}+a\right)a^{5}}-\frac{10A\,b^{3}\ln(x)}{a^{6}}+\frac{5A\,b^{3}\ln\left(b\,x^{2}+a\right)}{a^{6}}+\frac{3B\,b^{2}}{2\left(b\,x^{2}+a\right)a^{4}}+\frac{6B\,b^{2}\ln(x)}{a^{5}}-\frac{3B\,b^{2}\ln\left(b\,x^{2}+a\right)}{a^{5}}-\frac{3A\,b^{2}}{a^{5}x^{2}}+\frac{3Bb}{2a^{4}x^{2}}+\frac{3Ab}{4a^{4}x^{4}}-\frac{B}{4a^{3}x^{4}}-\frac{A}{6a^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^3,x)
```

```
[Out] -2/a^5*b^3/(b*x^2+a)*A+3/2/a^4*b^2/(b*x^2+a)*B-1/4/a^4*b^3/(b*x^2+a)^2*A+1/
4/a^3*b^2/(b*x^2+a)^2*B+5/a^6*b^3*ln(b*x^2+a)*A-3/a^5*b^2*ln(b*x^2+a)*B-1/6
*A/a^3/x^6+3/4/a^4/x^4*A*b-1/4/a^3/x^4*B-3*b^2/a^5/x^2*A+3/2*b/a^4/x^2*B-10
*b^3/a^6*ln(x)*A+6*b^2/a^5*ln(x)*B
```

maxima [A] time = 1.03, size = 170, normalized size = 1.14

$$\frac{12\left(3\,Bab^{3}-5\,Ab^{4}\right)x^{8}+18\left(3\,Ba^{2}b^{2}-5\,Aab^{3}\right)x^{6}-2\,Aa^{4}+4\left(3\,Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{4}-\left(3\,Ba^{4}-5\,Aa^{3}b\right)x^{2}}{12\left(a^{5}b^{2}x^{10}+2\,a^{6}bx^{8}+a^{7}x^{6}\right)}-\frac{\left(3\,Bab^{2}-5\,Ab^{3}\right)\log\left(bx^{2}+a\right)}{a^{6}}+\frac{\left(3\,Bab^{2}-5\,Ab^{3}\right)\log\left(x^{2}\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(3*B*a*b^3 - 5*A*b^4)*x^8 + 18*(3*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 2*A
*a^4 + 4*(3*B*a^3*b - 5*A*a^2*b^2)*x^4 - (3*B*a^4 - 5*A*a^3*b)*x^2)/(a^5*b^
2*x^10 + 2*a^6*b*x^8 + a^7*x^6) - (3*B*a*b^2 - 5*A*b^3)*log(b*x^2 + a)/a^6
+ (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6
```

mupad [B] time = 0.18, size = 155, normalized size = 1.04

$$\frac{\ln\left(b\,x^2+a\right)\,\left(5\,A\,b^3-3\,B\,a\,b^2\right)}{a^6}-\frac{\frac{A}{6\,a}-\frac{x^2\,(5\,A\,b-3\,B\,a)}{12\,a^2}+\frac{3\,b^2\,x^6\,(5\,A\,b-3\,B\,a)}{2\,a^4}+\frac{b^3\,x^8\,(5\,A\,b-3\,B\,a)}{a^5}+\frac{b\,x^4\,(5\,A\,b-3\,B\,a)}{3\,a^3}}{a^2\,x^6+2\,a\,b\,x^8+b^2\,x^{10}}-\frac{\ln(x)\,\left(10\,A\,b^3-6\,B\,a\,b^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^3),x)
```

```
[Out] (log(a + b*x^2)*(5*A*b^3 - 3*B*a*b^2))/a^6 - (A/(6*a) - (x^2*(5*A*b - 3*B*a
))/(12*a^2) + (3*b^2*x^6*(5*A*b - 3*B*a))/(2*a^4) + (b^3*x^8*(5*A*b - 3*B*a
))/a^5 + (b*x^4*(5*A*b - 3*B*a))/(3*a^3))/(a^2*x^6 + b^2*x^10 + 2*a*b*x^8)
- (log(x)*(10*A*b^3 - 6*B*a*b^2))/a^6
```

sympy [A] time = 1.26, size = 165, normalized size = 1.11

$$\frac{-2Aa^4+x^8\left(-60Ab^4+36Bab^3\right)+x^6\left(-90Aab^3+54Ba^2b^2\right)+x^4\left(-20Aa^2b^2+12Ba^3b\right)+x^2\left(5Aa^3b-3Ba^4\right)}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}}+\frac{2b^2\,(-5Ab+3Ba)\log{(x)}}{a^6}-\frac{b^2\,(-5Ab+3Ba)\log{\left(\frac{a}{b}+x^2\right)}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**3,x)
```

```
[Out] (-2*A*a**4 + x**8*(-60*A*b**4 + 36*B*a*b**3) + x**6*(-90*A*a*b**3 + 54*B*a*
*2*b**2) + x**4*(-20*A*a**2*b**2 + 12*B*a**3*b) + x**2*(5*A*a**3*b - 3*B*a*
*4))/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) + 2*b**2*(-5*A*b
+ 3*B*a)*log(x)/a**6 - b**2*(-5*A*b + 3*B*a)*log(a/b + x**2)/a**6
```

3.98 $\int \frac{x^{10}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=158

$$-\frac{9a^{5/2}(7Ab-11aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{ax^3(Ab-2aB)}{b^5}+x^5(A$$

Rubi [A] time = 0.28, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {455, 1814, 1810, 205}

$$\frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}+\frac{2a^2x(3Ab-5aB)}{b^6}-\frac{9a^{5/2}(7Ab-11aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}+\frac{x^5(Ab-3aB)}{5b^4}-\frac{ax^3(Ab-2aB)}{b^5}+\frac{Bx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*a^2*(3*A*b - 5*a*B)*x)/b^6 - (a*(A*b - 2*a*B)*x^3)/b^5 + ((A*b - 3*a*B)*
x^5)/(5*b^4) + (B*x^7)/(7*b^3) - (a^4*(A*b - a*B)*x)/(4*b^6*(a + b*x^2)^2)
+ (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) - (9*a^(5/2)*(7*A*b - 11*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} - \frac{\int \frac{-a^4(Ab-aB)+4a^3b(Ab-aB)x^2-4a^2b^2(Ab-aB)x^4+4ab^3(Ab-aB)x^6-4b^4(Ab-aB)x^8-4b^5Bx}{\left(a+bx^2\right)^2}}{4b^6}$$

$$= -\frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab-21aB)x}{8b^6\left(a+bx^2\right)} + \frac{\int \frac{-a^4(15Ab-19aB)+8a^3b(3Ab-4aB)x^2-8a^2b^2(2Ab-3aB)x^4+8}{a+bx^2}}{8ab^6}$$

$$= -\frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab-21aB)x}{8b^6\left(a+bx^2\right)} + \frac{\int \left(16a^3(3Ab-5aB)-24a^2b(Ab-2aB)x^2+8a\right.}{8}$$

$$= \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab}{8b^6(a}$$

$$= \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6\left(a+bx^2\right)^2} + \frac{a^3(17Ab}{8b^6(a}$$

Mathematica [A] time = 0.09, size = 158, normalized size = 1.00

$$\frac{9a^{5/2}(11aB-7Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}} + \frac{a^4x(aB-Ab)}{4b^6\left(a+bx^2\right)^2} + \frac{a^3x(17Ab-21aB)}{8b^6\left(a+bx^2\right)} - \frac{2a^2x(5aB-3Ab)}{b^6} + \frac{ax^3(2aB-Ab)}{b^5} + \frac{x^5(Ab-3aB)}{5b^4} + \frac{Bx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-2*a^2*(-3*A*b + 5*a*B)*x)/b^6 + (a*(-(A*b) + 2*a*B)*x^3)/b^5 + ((A*b - 3*
a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) + (a^4*(-(A*b) + a*B)*x)/(4*b^6*(a + b*
```

```
x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) + (9*a^(5/2)*(-7*A*
b + 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^10*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.47, size = 468, normalized size = 2.96

$$\left[\frac{80\,Bb^5x^{11}-16\left(11\,Bab^4-7\,Ab^5\right)x^9+48\left(11\,Ba^2b^3-7\,Aab^4\right)x^7-336\left(11\,Ba^3b^2-7\,Aa^2b^3\right)x^5-1050\left(11\,Ba^4b-7\,Aa^3b^2\right)x^3-315\left(11\,Ba^5-7\,Aa^4b+\left(11\,Ba^3b^2-7\,Aa^2b^3\right)x^4+2\left(11\,Ba^4b-7\,Aa^3b^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-630\left(11\,Ba^5-7\,Aa^4b\right)x}{560\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)},\ \frac{40\,Bb^5x^{11}-8\left(11\,Bab^4-7\,Ab^5\right)x^9+24\left(11\,Ba^2b^3-7\,Aab^4\right)x^7-168\left(11\,Ba^3b^2-7\,Aa^2b^3\right)x^5-525\left(11\,Ba^4b-7\,Aa^3b^2\right)x^3+315\left(11\,Ba^5-7\,Aa^4b+\left(11\,Ba^3b^2-7\,Aa^2b^3\right)x^4+2\left(11\,Ba^4b-7\,Aa^3b^2\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\left(11\,Ba^5-7\,Aa^4b\right)x}{280\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/560*(80*B*b^5*x^11 - 16*(11*B*a*b^4 - 7*A*b^5)*x^9 + 48*(11*B*a^2*b^3 -
7*A*a*b^4)*x^7 - 336*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 1050*(11*B*a^4*b -
7*A*a^3*b^2)*x^3 - 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2 - 7*A*a^2*b^3)
*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
(-a/b) - a)/(b*x^2 + a)) - 630*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7
*x^2 + a^2*b^6), 1/280*(40*B*b^5*x^11 - 8*(11*B*a*b^4 - 7*A*b^5)*x^9 + 24*(
11*B*a^2*b^3 - 7*A*a*b^4)*x^7 - 168*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 525*
(11*B*a^4*b - 7*A*a^3*b^2)*x^3 + 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2
- 7*A*a^2*b^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) - 315*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*
b^6)]
```

giac [A] time = 0.48, size = 162, normalized size = 1.03

$$\frac{9\left(11\,Ba^4-7\,Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}-\frac{21\,Ba^4bx^3-17\,Aa^3b^2x^3+19\,Ba^5x-15\,Aa^4bx}{8\left(bx^2+a\right)^2b^6}+\frac{5\,Bb^{18}x^7-21\,Bab^{17}x^5+7\,Ab^{18}x^5+70\,Ba^2b^{16}x^3-35\,Aab^{17}x^3-350\,Ba^3b^{15}x+210\,Aa^2b^{16}x}{35\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*
B*a^4*b*x^3 - 17*A*a^3*b^2*x^3 + 19*B*a^5*x - 15*A*a^4*b*x)/((b*x^2 + a)^2*
b^6) + 1/35*(5*B*b^18*x^7 - 21*B*a*b^17*x^5 + 7*A*b^18*x^5 + 70*B*a^2*b^16*
x^3 - 35*A*a*b^17*x^3 - 350*B*a^3*b^15*x + 210*A*a^2*b^16*x)/b^21
```

maple [A] time = 0.02, size = 198, normalized size = 1.25

$$\frac{B\,x^7}{7b^3}+\frac{17A\,a^3x^3}{8\left(b\,x^2+a\right)^2b^4}+\frac{A\,x^5}{5b^3}-\frac{21B\,a^4x^3}{8\left(b\,x^2+a\right)^2b^5}-\frac{3Ba\,x^5}{5b^4}+\frac{15A\,a^4x}{8\left(b\,x^2+a\right)^2b^5}-\frac{Aa\,x^3}{b^4}-\frac{19B\,a^5x}{8\left(b\,x^2+a\right)^2b^6}+\frac{2B\,a^2x^3}{b^5}-\frac{63A\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^5}+\frac{99B\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^6}+\frac{6A\,a^2x}{b^5}-\frac{10B\,a^3x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/7*B*x^7/b^3+1/5/b^3*A*x^5-3/5/b^4*B*x^5*a-1/b^4*A*x^3*a+2/b^5*B*x^3*a^2+6
/b^5*A*a^2*x-10/b^6*B*a^3*x+17/8*a^3/b^4/(b*x^2+a)^2*A*x^3-21/8*a^4/b^5/(b*
x^2+a)^2*B*x^3+15/8*a^4/b^5/(b*x^2+a)^2*A*x-19/8*a^5/b^6/(b*x^2+a)^2*B*x-63
/8*a^3/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+99/8*a^4/b^6/(a*b)^(1/2)
*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.35, size = 171, normalized size = 1.08

$$-\frac{\left(21\,Ba^4b-17\,Aa^3b^2\right)x^3+\left(19\,Ba^5-15\,Aa^4b\right)x}{8\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}+\frac{9\left(11\,Ba^4-7\,Aa^3b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{5\,Bb^3x^7-7\left(3\,Bab^2-Ab^3\right)x^5+35\left(2\,Ba^2b-Aab^2\right)x^3-70\left(5\,Ba^3-3\,Aa^2b\right)x}{35\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((21*B*a^4*b - 17*A*a^3*b^2)*x^3 + (19*B*a^5 - 15*A*a^4*b)*x)/(b^8*x^4
 + 2*a*b^7*x^2 + a^2*b^6) + 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) + 1/35*(5*B*b^3*x^7 - 7*(3*B*a*b^2 - A*b^3)*x^5 + 35*(2*B
*a^2*b - A*a*b^2)*x^3 - 70*(5*B*a^3 - 3*A*a^2*b)*x)/b^6
```

mupad [B] time = 0.11, size = 246, normalized size = 1.56

$$x^5\left(\frac{A}{5\,b^3}-\frac{3\,B\,a}{5\,b^4}\right)-\frac{x\left(\frac{19\,B\,a^5}{8}-\frac{15\,A\,a^4\,b}{8}\right)-x^3\left(\frac{17\,A\,a^3\,b^2}{8}-\frac{21\,B\,a^4\,b}{8}\right)}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}-x^3\left(\frac{a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{B\,a^2}{b^5}\right)-x\left(\frac{B\,a^3}{b^6}-\frac{3\,a\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)}{b}+\frac{3\,a^2\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b^2}\right)+\frac{B\,x^7}{7\,b^3}+\frac{9\,a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(7\,A\,b-11\,B\,a)}{11\,B\,a^4-7\,A\,a^3\,b}\right)(7\,A\,b-11\,B\,a)}{8\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] x^5*(A/(5*b^3) - (3*B*a)/(5*b^4)) - (x*((19*B*a^5)/8 - (15*A*a^4*b)/8) - x^
3*((17*A*a^3*b^2)/8 - (21*B*a^4*b)/8))/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) -
x^3*((a*(A/b^3 - (3*B*a)/b^4))/b + (B*a^2)/b^5) - x*((B*a^3)/b^6 - (3*a*((3
*a*(A/b^3 - (3*B*a)/b^4))/b + (3*B*a^2)/b^5))/b + (3*a^2*(A/b^3 - (3*B*a)/b
^4))/b^2) + (B*x^7)/(7*b^3) + (9*a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(7*A*b - 1
1*B*a))/(11*B*a^4 - 7*A*a^3*b))*(7*A*b - 11*B*a))/(8*b^(13/2))
```

sympy [A] time = 1.44, size = 280, normalized size = 1.77

$$\frac{Bx^7}{7b^3}+x^5\left(\frac{A}{5b^3}-\frac{3Ba}{5b^4}\right)+x^3\left(-\frac{Aa}{b^4}+\frac{2Ba^2}{b^5}\right)+x\left(\frac{6Aa^2}{b^5}-\frac{10Ba^3}{b^6}\right)-\frac{9\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)\log\left(-\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16}+\frac{9\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)\log\left(\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}\,(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16}+\frac{x^3\left(17Aa^3b^2-21Ba^4b\right)+x\left(15Aa^4b-19Ba^5\right)}{8a^2b^6+16ab^7x^2+8b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**7/(7*b**3) + x**5*(A/(5*b**3) - 3*B*a/(5*b**4)) + x**3*(-A*a/b**4 + 2*
B*a**2/b**5) + x*(6*A*a**2/b**5 - 10*B*a**3/b**6) - 9*sqrt(-a**5/b**13)*(-7
*A*b + 11*B*a)*log(-9*b**6*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*
b + 99*B*a**3) + x)/16 + 9*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)*log(9*b**6*s
qrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*b + 99*B*a**3) + x)/16 + (x*
*3*(17*A*a**3*b**2 - 21*B*a**4*b) + x*(15*A*a**4*b - 19*B*a**5))/(8*a**2*b*
*6 + 16*a*b**7*x**2 + 8*b**8*x**4)
```

3.99 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=138

$$\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{a^3x(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{a^2x(13Ab-17aB)}{8b^5\left(a+bx^2\right)}-\frac{3ax(Ab-2aB)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^5}{5b^3}$$

Rubi [A] time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {455, 1814, 1810, 205}

$$-\frac{a^2x(13Ab-17aB)}{8b^5\left(a+bx^2\right)}+\frac{a^3x(Ab-aB)}{4b^5\left(a+bx^2\right)^2}+\frac{7a^{3/2}(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{11/2}}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{3ax(Ab-2aB)}{b^5}+\frac{Bx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*(A*b - 2*a*B)*x)/b^5 + ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^5)/(5*b^3)
+ (a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - (a^2*(13*A*b - 17*a*B)*x)/(8*
b^5*(a + b*x^2)) + (7*a^(3/2)*(5*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*b^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2} - \frac{\int \frac{a^3(Ab-aB)-4a^2b(Ab-aB)x^2+4ab^2(Ab-aB)x^4-4b^3(Ab-aB)x^6-4b^4Bx^8}{\left(a+bx^2\right)^2}\,dx}{4b^5}$$

$$= \frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2} - \frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)} + \frac{\int \frac{a^3(11Ab-15aB)-8a^2b(2Ab-3aB)x^2+8ab^2(Ab-2aB)x^4+8ab^3Bx^6}{a+bx^2}}{8ab^5}$$

$$= \frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2} - \frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)} + \frac{\int \Big(-24a^2(Ab-2aB)+8ab(Ab-3aB)x^2+8ab^2Bx}{8ab^5}$$

$$= -\frac{3a(Ab-2aB)x}{b^5} + \frac{(Ab-3aB)x^3}{3b^4} + \frac{Bx^5}{5b^3} + \frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2} - \frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)} + \frac{\big(7a^2(5}{}$$

$$= -\frac{3a(Ab-2aB)x}{b^5} + \frac{(Ab-3aB)x^3}{3b^4} + \frac{Bx^5}{5b^3} + \frac{a^3(Ab-aB)x}{4b^5\left(a+bx^2\right)^2} - \frac{a^2(13Ab-17aB)x}{8b^5\left(a+bx^2\right)} + \frac{7a^{3/2}(5}{}$$

Mathematica [A] time = 0.10, size = 133, normalized size = 0.96

$$\frac{x\left(945a^4B-525a^3b\left(A-3Bx^2\right)+7a^2b^2x^2\left(72Bx^2-125A\right)-8ab^3x^4\left(35A+9Bx^2\right)+8b^4x^6\left(5A+3Bx^2\right)\right)}{120b^5\left(a+bx^2\right)^2} - \frac{7a^{3/2}(9aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*B - 525*a^3*b*(A - 3*B*x^2) + 8*b^4*x^6*(5*A + 3*B*x^2) - 8*a*b
^3*x^4*(35*A + 9*B*x^2) + 7*a^2*b^2*x^2*(-125*A + 72*B*x^2)))/(120*b^5*(a +
```

```
 b*x^2)^2) - (7*a^(3/2)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \left(A + Bx^2\right)}{\left(a + bx^2\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^8*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.46, size = 416, normalized size = 3.01

$$\left[\frac{48 \, B b^{4} x^{9} - 16 \, {\left(9 \, B a b^{3} - 5 \, A b^{4}\right)} x^{7} + 112 \, {\left(9 \, B a^{2} b^{2} - 5 \, A a b^{3}\right)} x^{5} + 350 \, {\left(9 \, B a^{3} b - 5 \, A a^{2} b^{2}\right)} x^{3} - 105 \, {\left(9 \, B a^{4} - 5 \, A a^{3} b + {\left(9 \, B a^{2} b^{2} - 5 \, A a b^{3}\right)} x^{4} + 2 \, {\left(9 \, B a^{3} b - 5 \, A a^{2} b^{2}\right)} x^{2}\right)} \sqrt{-\frac{a}{b}} \log\left(\frac{b x^{2} + 2 \, b x \sqrt{-\frac{a}{b}} - a}{b x^{2} + a}\right) + 210 \, {\left(9 \, B a^{4} - 5 \, A a^{3} b\right)} x}{240 \, {\left(b^{7} x^{4} + 2 \, a b^{6} x^{2} + a^{2} b^{5}\right)}}, \frac{24 \, B b^{4} x^{9} - 8 \, {\left(9 \, B a b^{3} - 5 \, A b^{4}\right)} x^{7} + 56 \, {\left(9 \, B a^{2} b^{2} - 5 \, A a b^{3}\right)} x^{5} + 175 \, {\left(9 \, B a^{3} b - 5 \, A a^{2} b^{2}\right)} x^{3} - 105 \, {\left(9 \, B a^{4} - 5 \, A a^{3} b + {\left(9 \, B a^{2} b^{2} - 5 \, A a b^{3}\right)} x^{4} + 2 \, {\left(9 \, B a^{3} b - 5 \, A a^{2} b^{2}\right)} x^{2}\right)} \sqrt{\frac{a}{b}} \arctan\left(\frac{b x \sqrt{\frac{a}{b}}}{a}\right) + 105 \, {\left(9 \, B a^{4} - 5 \, A a^{3} b\right)} x}{120 \, {\left(b^{7} x^{4} + 2 \, a b^{6} x^{2} + a^{2} b^{5}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*B*b^4*x^9 - 16*(9*B*a*b^3 - 5*A*b^4)*x^7 + 112*(9*B*a^2*b^2 - 5*
A*a*b^3)*x^5 + 350*(9*B*a^3*b - 5*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b
 + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(9*B*a^4 - 5*A*a^
3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/120*(24*B*b^4*x^9 - 8*(9*B*a*b
^3 - 5*A*b^4)*x^7 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^5 + 175*(9*B*a^3*b - 5*A
*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 +
2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(9
*B*a^4 - 5*A*a^3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.36, size = 138, normalized size = 1.00

$$-\frac{7 \, {\left(9 \, B a^{3} - 5 \, A a^{2} b\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{8 \, \sqrt{a b} b^{5}} + \frac{17 \, B a^{3} b x^{3} - 13 \, A a^{2} b^{2} x^{3} + 15 \, B a^{4} x - 11 \, A a^{3} b x}{8 \, {\left(b x^{2} + a\right)}^{2} b^{5}} + \frac{3 \, B b^{12} x^{5} - 15 \, B a b^{11} x^{3} + 5 \, A b^{12} x^{3} + 90 \, B a^{2} b^{10} x - 45 \, A a b^{11} x}{15 \, b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*
B*a^3*b*x^3 - 13*A*a^2*b^2*x^3 + 15*B*a^4*x - 11*A*a^3*b*x)/((b*x^2 + a)^2*
b^5) + 1/15*(3*B*b^12*x^5 - 15*B*a*b^11*x^3 + 5*A*b^12*x^3 + 90*B*a^2*b^10*
x - 45*A*a*b^11*x)/b^15
```

maple [A] time = 0.04, size = 174, normalized size = 1.26

$$-\frac{13Aa^2x^3}{8\left(bx^2+a\right)^2b^3}+\frac{17Ba^3x^3}{8\left(bx^2+a\right)^2b^4}+\frac{Bx^5}{5b^3}-\frac{11Aa^3x}{8\left(bx^2+a\right)^2b^4}+\frac{Ax^3}{3b^3}+\frac{15Ba^4x}{8\left(bx^2+a\right)^2b^5}-\frac{Bax^3}{b^4}+\frac{35Aa^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^4}-\frac{63Ba^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^5}-\frac{3Aax}{b^4}+\frac{6Ba^2x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/5*B*x^5/b^3+1/3/b^3*A*x^3-1/b^4*B*x^3*a-3/b^4*a*A*x+6/b^5*a^2*B*x-13/8*a^
2/b^3/(b*x^2+a)^2*A*x^3+17/8*a^3/b^4/(b*x^2+a)^2*B*x^3-11/8*a^3/b^4/(b*x^2+
a)^2*A*x+15/8*a^4/b^5/(b*x^2+a)^2*B*x+35/8*a^2/b^4/(a*b)^(1/2)*arctan(1/(a*
b)^(1/2)*b*x)*A-63/8*a^3/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.39, size = 147, normalized size = 1.07

$$\frac{\left(17\,Ba^3b-13\,Aa^2b^2\right)x^3+\left(15\,Ba^4-11\,Aa^3b\right)x}{8\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}-\frac{7\left(9\,Ba^3-5\,Aa^2b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{3\,Bb^2x^5-5\left(3\,Bab-Ab^2\right)x^3+45\left(2\,Ba^2-Aab\right)x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((17*B*a^3*b - 13*A*a^2*b^2)*x^3 + (15*B*a^4 - 11*A*a^3*b)*x)/(b^7*x^4
+ 2*a*b^6*x^2 + a^2*b^5) - 7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/15*(3*B*b^2*x^5 - 5*(3*B*a*b - A*b^2)*x^3 + 45*(2*B*a^2
 - A*a*b)*x)/b^5
```

mupad [B] time = 0.10, size = 177, normalized size = 1.28

$$\frac{x\left(\frac{15\,B\,a^4}{8}-\frac{11\,A\,a^3\,b}{8}\right)-x^3\left(\frac{13\,A\,a^2\,b^2}{8}-\frac{17\,B\,a^3\,b}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}-x\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)+x^3\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{B\,x^5}{5\,b^3}-\frac{7\,a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(5\,A\,b-9\,B\,a)}{9\,B\,a^3-5\,A\,a^2\,b}\right)(5\,A\,b-9\,B\,a)}{8\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (x*((15*B*a^4)/8 - (11*A*a^3*b)/8) - x^3*((13*A*a^2*b^2)/8 - (17*B*a^3*b)/8
))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x*((3*a*(A/b^3 - (3*B*a)/b^4))/b + (
3*B*a^2)/b^5) + x^3*(A/(3*b^3) - (B*a)/b^4) + (B*x^5)/(5*b^3) - (7*a^(3/2)*
atan((a^(3/2)*b^(1/2)*x*(5*A*b - 9*B*a))/(9*B*a^3 - 5*A*a^2*b))*(5*A*b - 9*
B*a))/(8*b^(11/2))
```

sympy [A] time = 1.36, size = 252, normalized size = 1.83

$$\frac{Bx^5}{5b^3}+x^3\left(\frac{A}{3b^3}-\frac{Ba}{b^4}\right)+x\left(-\frac{3Aa}{b^4}+\frac{6Ba^2}{b^5}\right)+\frac{7\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)\log\left(-\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16}-\frac{7\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)\log\left(\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}\,(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16}+\frac{x^3\left(-13Aa^2b^2+17Ba^3b\right)+x\left(-11Aa^3b+15Ba^4\right)}{8a^2b^5+16ab^6x^2+8b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + x**3*(A/(3*b**3) - B*a/b**4) + x*(-3*A*a/b**4 + 6*B*a**2/
b**5) + 7*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)*log(-7*b**5*sqrt(-a**3/b**11)*
(-5*A*b + 9*B*a)/(-35*A*a*b + 63*B*a**2) + x)/16 - 7*sqrt(-a**3/b**11)*(-5*
A*b + 9*B*a)*log(7*b**5*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)/(-35*A*a*b + 63*
B*a**2) + x)/16 + (x**3*(-13*A*a**2*b**2 + 17*B*a**3*b) + x*(-11*A*a**3*b +
 15*B*a**4))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4)
```

3.100 $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=116

$$-\frac{a^2x(Ab-aB)}{4b^4\left(a+bx^2\right)^2}-\frac{5\sqrt{a}\,(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{ax(9Ab-13aB)}{8b^4\left(a+bx^2\right)}+\frac{x(Ab-3aB)}{b^4}+\frac{Bx^3}{3b^3}$$

Rubi **[A]** time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {455, 1814, 1153, 205}

$$-\frac{a^2x(Ab-aB)}{4b^4\left(a+bx^2\right)^2}+\frac{ax(9Ab-13aB)}{8b^4\left(a+bx^2\right)}+\frac{x(Ab-3aB)}{b^4}-\frac{5\sqrt{a}\,(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 + (B*x^3)/(3*b^3) - (a^2*(A*b - a*B)*x)/(4*b^4*(a + b
*x^2)^2) + (a*(9*A*b - 13*a*B)*x)/(8*b^4*(a + b*x^2)) - (5*Sqrt[a]*(3*A*b -
 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= -\frac{a^2(Ab-aB)x}{4b^4\left(a+bx^2\right)^2}-\frac{\int \frac{-a^2(Ab-aB)+4ab(Ab-aB)x^2-4b^2(Ab-aB)x^4-4b^3Bx^6}{\left(a+bx^2\right)^2}\,dx}{4b^4}\\
&= -\frac{a^2(Ab-aB)x}{4b^4\left(a+bx^2\right)^2}+\frac{a(9Ab-13aB)x}{8b^4\left(a+bx^2\right)}+\frac{\int \frac{-a^2(7Ab-11aB)+8ab(Ab-2aB)x^2+8ab^2Bx^4}{a+bx^2}\,dx}{8ab^4}\\
&= -\frac{a^2(Ab-aB)x}{4b^4\left(a+bx^2\right)^2}+\frac{a(9Ab-13aB)x}{8b^4\left(a+bx^2\right)}+\frac{\int \left(8a(Ab-3aB)+8abBx^2+\frac{5\left(-3a^2Ab+7a^3B\right)}{a+bx^2}\right)dx}{8ab^4}\\
&= \frac{(Ab-3aB)x}{b^4}+\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)x}{4b^4\left(a+bx^2\right)^2}+\frac{a(9Ab-13aB)x}{8b^4\left(a+bx^2\right)}-\frac{(5a(3Ab-7aB))\int \frac{1}{a+bx^2}\,dx}{8b^4}\\
&= \frac{(Ab-3aB)x}{b^4}+\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)x}{4b^4\left(a+bx^2\right)^2}+\frac{a(9Ab-13aB)x}{8b^4\left(a+bx^2\right)}-\frac{5\sqrt{a}\,(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{8b^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.97

$$\frac{-105a^3Bx+5a^2bx\left(9A-35Bx^2\right)+ab^2x^3\left(75A-56Bx^2\right)+8b^3x^5\left(3A+Bx^2\right)}{24b^4\left(a+bx^2\right)^2}+\frac{5\sqrt{a}\,(7aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-105*a^3*B*x + a*b^2*x^3*(75*A - 56*B*x^2) + 5*a^2*b*x*(9*A - 35*B*x^2) +
8*b^3*x^5*(3*A + B*x^2))/(24*b^4*(a + b*x^2)^2) + (5*Sqrt[a]*(-3*A*b + 7*a*
B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.45, size = 358, normalized size = 3.09

$$\left[\frac{16\,Bb^3x^7-16\left(7\,Bab^2-3\,Ab^3\right)x^5-50\left(7\,Ba^2b-3\,Aab^2\right)x^3-15\left(\left(7\,Bab^2-3\,Ab^3\right)x^4+7\,Ba^3-3\,Aa^2b+2\left(7\,Ba^2b-3\,Aab^2\right)x^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\left(7\,Ba^3-3\,Aa^2b\right)x}{48\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)},\ \frac{8\,Bb^3x^7-8\left(7\,Bab^2-3\,Ab^3\right)x^5-25\left(7\,Ba^2b-3\,Aab^2\right)x^3+15\left(\left(7\,Bab^2-3\,Ab^3\right)x^4+7\,Ba^3-3\,Aa^2b+2\left(7\,Ba^2b-3\,Aab^2\right)x^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-15\left(7\,Ba^3-3\,Aa^2b\right)x}{24\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*B*b^3*x^7 - 16*(7*B*a*b^2 - 3*A*b^3)*x^5 - 50*(7*B*a^2*b - 3*A*a*
b^2)*x^3 - 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2
*b - 3*A*a*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) - 30*(7*B*a^3 - 3*A*a^2*b)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*
(8*B*b^3*x^7 - 8*(7*B*a*b^2 - 3*A*b^3)*x^5 - 25*(7*B*a^2*b - 3*A*a*b^2)*x^3
 + 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2*b - 3*A
*a*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 15*(7*B*a^3 - 3*A*a^2*b)*x
)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 0.38, size = 111, normalized size = 0.96

$$\frac{5\left(7\,Ba^2-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}-\frac{13\,Ba^2bx^3-9\,Aab^2x^3+11\,Ba^3x-7\,Aa^2bx}{8\left(bx^2+a\right)^2b^4}+\frac{Bb^6x^3-9\,Bab^5x+3\,Ab^6x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*B*a
^2*b*x^3 - 9*A*a*b^2*x^3 + 11*B*a^3*x - 7*A*a^2*b*x)/((b*x^2 + a)^2*b^4) +
1/3*(B*b^6*x^3 - 9*B*a*b^5*x + 3*A*b^6*x)/b^9
```

maple [A] time = 0.01, size = 147, normalized size = 1.27

$$\frac{9Aa\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{2}}-\frac{13B\,a^{2}x^{3}}{8\left(b\,x^{2}+a\right)^{2}b^{3}}+\frac{7A\,a^{2}x}{8\left(b\,x^{2}+a\right)^{2}b^{3}}-\frac{11B\,a^{3}x}{8\left(b\,x^{2}+a\right)^{2}b^{4}}+\frac{B\,x^{3}}{3b^{3}}-\frac{15Aa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{3}}+\frac{35B\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{4}}+\frac{Ax}{b^{3}}-\frac{3Bax}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 1/3*B*x^3/b^3+1/b^3*A*x-3/b^4*B*a*x+9/8*a/b^2/(b*x^2+a)^2*A*x^3-13/8*a^2/b^
3/(b*x^2+a)^2*B*x^3+7/8*a^2/b^3/(b*x^2+a)^2*A*x-11/8*a^3/b^4/(b*x^2+a)^2*B*
x-15/8*a/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+35/8*a^2/b^4/(a*b)^(1/
2)*arctan(1/(a*b)^(1/2)*b*x)*B
```

maxima [A] time = 2.34, size = 120, normalized size = 1.03

$$-\frac{\left(13\,Ba^{2}b-9\,Aab^{2}\right)x^{3}+\left(11\,Ba^{3}-7\,Aa^{2}b\right)x}{8\,\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)}+\frac{5\left(7\,Ba^{2}-3\,Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{4}}+\frac{Bbx^{3}-3\,(3\,Ba-Ab)x}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*B*a^2*b - 9*A*a*b^2)*x^3 + (11*B*a^3 - 7*A*a^2*b)*x)/(b^6*x^4 + 2
*a*b^5*x^2 + a^2*b^4) + 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^4) + 1/3*(B*b*x^3 - 3*(3*B*a - A*b)*x)/b^4
```

mupad [B] time = 0.11, size = 138, normalized size = 1.19

$$\frac{x^{3}\left(\frac{9\,A\,a\,b^{2}}{8}-\frac{13\,B\,a^{2}\,b}{8}\right)-x\left(\frac{11\,B\,a^{3}}{8}-\frac{7\,A\,a^{2}\,b}{8}\right)}{a^{2}\,b^{4}+2\,a\,b^{5}\,x^{2}+b^{6}\,x^{4}}+x\left(\frac{A}{b^{3}}-\frac{3\,B\,a}{b^{4}}\right)+\frac{B\,x^{3}}{3\,b^{3}}+\frac{5\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(3\,A\,b-7\,B\,a)}{7\,B\,a^{2}-3\,A\,a\,b}\right)(3\,A\,b-7\,B\,a)}{8\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (x^3*((9*A*a*b^2)/8 - (13*B*a^2*b)/8) - x*((11*B*a^3)/8 - (7*A*a^2*b)/8))/(
a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^3)/(3*b^3
) + (5*a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*A*b - 7*B*a))/(7*B*a^2 - 3*A*a*b)
)*(3*A*b - 7*B*a))/(8*b^(9/2))
```

sympy [A] time = 1.25, size = 214, normalized size = 1.84

$$\frac{Bx^{3}}{3b^{3}}+x\left(\frac{A}{b^{3}}-\frac{3Ba}{b^{4}}\right)-\frac{5\sqrt{-\frac{a}{b^{9}}}\,(-3Ab+7Ba)\log\left(-\frac{5b^{4}\sqrt{-\frac{a}{b^{9}}}\,(-3Ab+7Ba)}{-15Ab+35Ba}+x\right)}{16}+\frac{5\sqrt{-\frac{a}{b^{9}}}\,(-3Ab+7Ba)\log\left(\frac{5b^{4}\sqrt{-\frac{a}{b^{9}}}\,(-3Ab+7Ba)}{-15Ab+35Ba}+x\right)}{16}+\frac{x^{3}\left(9Aab^{2}-13Ba^{2}b\right)+x\left(7Aa^{2}b-11Ba^{3}\right)}{8a^{2}b^{4}+16ab^{5}x^{2}+8b^{6}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) + x*(A/b**3 - 3*B*a/b**4) - 5*sqrt(-a/b**9)*(-3*A*b + 7*B*a
)*log(-5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a)/(-15*A*b + 35*B*a) + x)/16 + 5
*sqrt(-a/b**9)*(-3*A*b + 7*B*a)*log(5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a)/(
-15*A*b + 35*B*a) + x)/16 + (x**3*(9*A*a*b**2 - 13*B*a**2*b) + x*(7*A*a**2*
b - 11*B*a**3))/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4)
```

3.101 $\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=94

$$\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{x(5Ab-9aB)}{8b^3\left(a+bx^2\right)}+\frac{ax(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{Bx}{b^3}$$

Rubi **[A]** time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {455, 1157, 388, 205}

$$-\frac{x(5Ab-9aB)}{8b^3\left(a+bx^2\right)}+\frac{ax(Ab-aB)}{4b^3\left(a+bx^2\right)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{Bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (B*x)/b^3 + (a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - ((5*A*b - 9*a*B)*x)/(
8*b^3*(a + b*x^2)) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[
a]*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{a(Ab-aB)x}{4b^3\left(a+bx^2\right)^2}-\frac{\int \frac{a(Ab-aB)-4b(Ab-aB)x^2-4b^2Bx^4}{\left(a+bx^2\right)^2}\,dx}{4b^3}\\
&= \frac{a(Ab-aB)x}{4b^3\left(a+bx^2\right)^2}-\frac{(5Ab-9aB)x}{8b^3\left(a+bx^2\right)}+\frac{\int \frac{a(3Ab-7aB)+8abBx^2}{a+bx^2}\,dx}{8ab^3}\\
&= \frac{Bx}{b^3}+\frac{a(Ab-aB)x}{4b^3\left(a+bx^2\right)^2}-\frac{(5Ab-9aB)x}{8b^3\left(a+bx^2\right)}+\frac{(3(Ab-5aB))\int \frac{1}{a+bx^2}\,dx}{8b^3}\\
&= \frac{Bx}{b^3}+\frac{a(Ab-aB)x}{4b^3\left(a+bx^2\right)^2}-\frac{(5Ab-9aB)x}{8b^3\left(a+bx^2\right)}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 91, normalized size = 0.97

$$
\frac{x\left(15a^2B+a\left(25bBx^2-3Ab\right)+b^2x^2\left(8Bx^2-5A\right)\right)}{8b^3\left(a+bx^2\right)^2}+\frac{3(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^2*B + b^2*x^2*(-5*A + 8*B*x^2) + a*(-3*A*b + 25*b*B*x^2)))/(8*b^3*
(a + b*x^2)^2) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b
^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.50, size = 328, normalized size = 3.49

$$\left[\frac{16\,Bab^3x^5+10\left(5\,Ba^2b^2-Aab^3\right)x^3+3\left(\left(5\,Bab^2-Ab^3\right)x^4+5\,Ba^3-Aa^2b+2\left(5\,Ba^2b-Aab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(5\,Ba^3b-Aa^2b^2\right)x}{16\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)},\ \frac{8\,Bab^3x^5+5\left(5\,Ba^2b^2-Aab^3\right)x^3-3\left(\left(5\,Bab^2-Ab^3\right)x^4+5\,Ba^3-Aa^2b+2\left(5\,Ba^2b-Aab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(5\,Ba^3b-Aa^2b^2\right)x}{8\left(ab^6x^4+2\,a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*B*a*b^3*x^5 + 10*(5*B*a^2*b^2 - A*a*b^3)*x^3 + 3*((5*B*a*b^2 - A*
b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*b - A*a*b^2)*x^2)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*B*a^3*b - A*a^2*b^2)*x)/(a
*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4), 1/8*(8*B*a*b^3*x^5 + 5*(5*B*a^2*b^2 -
A*a*b^3)*x^3 - 3*((5*B*a*b^2 - A*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*
b - A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*B*a^3*b - A*a^2*b^
2)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.41, size = 80, normalized size = 0.85

$$\frac{Bx}{b^3}-\frac{3\,(5\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{9\,Babx^3-5\,Ab^2x^3+7\,Ba^2x-3\,Aabx}{8\left(bx^2+a\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/8*(9*
B*a*b*x^3 - 5*A*b^2*x^3 + 7*B*a^2*x - 3*A*a*b*x)/((b*x^2 + a)^2*b^3)
```

maple [A] time = 0.01, size = 122, normalized size = 1.30

$$-\frac{5A\,x^3}{8\left(b\,x^2+a\right)^2b}+\frac{9Ba\,x^3}{8\left(b\,x^2+a\right)^2b^2}-\frac{3Aax}{8\left(b\,x^2+a\right)^2b^2}+\frac{7B\,a^2x}{8\left(b\,x^2+a\right)^2b^3}+\frac{3A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^2}-\frac{15Ba\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,b^3}+\frac{Bx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] B*x/b^3-5/8/b/(b*x^2+a)^2*A*x^3+9/8/b^2/(b*x^2+a)^2*B*x^3*a-3/8/b^2/(b*x^2+
a)^2*a*A*x+7/8/b^3/(b*x^2+a)^2*a^2*B*x+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)*A-15/8/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B*a
```

maxima [A] time = 2.29, size = 94, normalized size = 1.00

$$\frac{\left(9\,Bab-5\,Ab^2\right)x^3+\left(7\,Ba^2-3\,Aab\right)x}{8\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{Bx}{b^3}-\frac{3\,(5\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*B*a*b - 5*A*b^2)*x^3 + (7*B*a^2 - 3*A*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^
2 + a^2*b^3) + B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^3)
```

mupad [B] time = 0.25, size = 92, normalized size = 0.98

$$\frac{B\,x}{b^3}-\frac{x^3\,\left(\frac{5\,A\,b^2}{8}-\frac{9\,B\,a\,b}{8}\right)-x\,\left(\frac{7\,B\,a^2}{8}-\frac{3\,A\,a\,b}{8}\right)}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-5\,B\,a\right)}{8\,\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (B*x)/b^3 - (x^3*((5*A*b^2)/8 - (9*B*a*b)/8) - x*((7*B*a^2)/8 - (3*A*a*b)/8
))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (3*atan((b^(1/2)*x)/a^(1/2))*(A*b -
5*B*a))/(8*a^(1/2)*b^(7/2))
```

sympy [B] time = 1.06, size = 194, normalized size = 2.06

$$\frac{Bx}{b^3}+\frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(-\frac{3ab^3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16}-\frac{3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)\log\left(\frac{3ab^3\sqrt{-\frac{1}{ab^7}}\,(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16}+\frac{x^3\left(-5Ab^2+9Bab\right)+x\left(-3Aab+7Ba^2\right)}{8a^2b^3+16ab^4x^2+8b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] B*x/b**3 + 3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)*log(-3*a*b**3*sqrt(-1/(a*b**7
))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/16 - 3*sqrt(-1/(a*b**7))*(-A*b + 5
```

```
*B*a)*log(3*a*b**3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/
16 + (x**3*(-5*A*b**2 + 9*B*a*b) + x*(-3*A*a*b + 7*B*a**2))/(8*a**2*b**3 +
16*a*b**4*x**2 + 8*b**5*x**4)
```

3.102 $\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=89

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4b^2\left(a+bx^2\right)^2}$$

Rubi [**A**] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 385, 205}

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{x(Ab-5aB)}{8ab^2\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -((A*b - a*B)*x)/(4*b^2*(a + b*x^2)^2) + ((A*b - 5*a*B)*x)/(8*a*b^2*(a + b*
x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = -\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2}-\frac{\int \frac{-Ab+aB-4bBx^2}{\left(a+bx^2\right)^2}\,dx}{4b^2}$$

$$= -\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2}+\frac{(Ab-5aB)x}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3aB)\int \frac{1}{a+bx^2}\,dx}{8ab^2}$$

$$= -\frac{(Ab-aB)x}{4b^2\left(a+bx^2\right)^2}+\frac{(Ab-5aB)x}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.08, size = 83, normalized size = 0.93

$$\frac{\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{b}\,x\left(-3a^2B-ab\left(A+5Bx^2\right)+Ab^2x^2\right)}{a\left(a+bx^2\right)^2}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
[Out] ((Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - a*b*(A + 5*B*x^2)))/(a*(a + b*x^2)^2) +
 ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))/(8*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.46, size = 301, normalized size = 3.38

$$\left[-\frac{2\left(5\,Ba^{2}b^{2}-Aab^{3}\right)x^{3}+\left(\left(3\,Bab^{2}+Ab^{3}\right)x^{4}+3\,Ba^{3}+Aa^{2}b+2\left(3\,Ba^{2}b+Aab^{2}\right)x^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\left(3\,Ba^{3}b+Aa^{2}b^{2}\right)x}{16\left(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)},-\frac{\left(5\,Ba^{2}b^{2}-Aab^{3}\right)x^{3}-\left(\left(3\,Bab^{2}+Ab^{3}\right)x^{4}+3\,Ba^{3}+Aa^{2}b+2\left(3\,Ba^{2}b+Aab^{2}\right)x^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Ba^{3}b+Aa^{2}b^{2}\right)x}{8\left(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*B*a^2*b^2 - A*a*b^3)*x^3 + ((3*B*a*b^2 + A*b^3)*x^4 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*
b^4*x^2 + a^4*b^3), -1/8*((5*B*a^2*b^2 - A*a*b^3)*x^3 - ((3*B*a*b^2 + A*b^3
)*x^4 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) + (3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]
```

giac [A] time = 0.34, size = 78, normalized size = 0.88

$$\frac{(3\,Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^{2}}-\frac{5\,Babx^{3}-Ab^{2}x^{3}+3\,Ba^{2}x+Aabx}{8\left(bx^{2}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*B*a*b*x^
3 - A*b^2*x^3 + 3*B*a^2*x + A*a*b*x)/((b*x^2 + a)^2*a*b^2)
```

maple [A] time = 0.01, size = 89, normalized size = 1.00

$$\frac{A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab}+\frac{3B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;b^{2}}+\frac{\frac{(Ab-5Ba)x^{3}}{8ab}-\frac{(Ab+3Ba)x}{8b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(A*b-5*B*a)/a/b*x^3-1/8*(A*b+3*B*a)/b^2*x)/(b*x^2+a)^2+1/8/b/a/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+3/8/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*B
```

maxima [A] time = 2.37, size = 92, normalized size = 1.03

$$-\frac{\left(5\,Bab-Ab^{2}\right)x^{3}+\left(3\,Ba^{2}+Aab\right)x}{8\left(ab^{4}x^{4}+2\,a^{2}b^{3}x^{2}+a^{3}b^{2}\right)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*a*b - A*b^2)*x^3 + (3*B*a^2 + A*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x
^2 + a^3*b^2) + 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [B] time = 0.24, size = 82, normalized size = 0.92

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+3\,B\,a)}{8\,a^{3/2}\,b^{5/2}}-\frac{\frac{x\,(A\,b+3\,B\,a)}{8\,b^2}-\frac{x^3\,(A\,b-5\,B\,a)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(A*b + 3*B*a))/(8*a^(3/2)*b^(5/2)) - ((x*(A*b +
3*B*a))/(8*b^2) - (x^3*(A*b - 5*B*a))/(8*a*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [A] time = 0.73, size = 155, normalized size = 1.74

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(Ab+3Ba)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16}+\frac{x^3\left(Ab^2-5Bab\right)+x\left(-Aab-3Ba^2\right)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 + (x**3*(A*b**2 - 5*B*a*b) + x*(-A*a*b - 3*B*a**2))/(8*a**3*b**2
+ 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

3.103 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=92

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\left(a+bx^2\right)}+\frac{x(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {385, 199, 205}

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(aB+3Ab)}{8a^2b\left(a+bx^2\right)}+\frac{x(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{\left(a+bx^2\right)^3}\,dx &= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2}+\frac{(3Ab+aB)\int \frac{1}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2}+\frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)}+\frac{(3Ab+aB)\int \frac{1}{a+bx^2}\,dx}{8a^2b}\\
&= \frac{(Ab-aB)x}{4ab\left(a+bx^2\right)^2}+\frac{(3Ab+aB)x}{8a^2b\left(a+bx^2\right)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 84, normalized size = 0.91

$$
\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x\left(a^2(-B)+ab\left(5A+Bx^2\right)+3Ab^2x^2\right)}{8a^2b\left(a+bx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (x*(-(a^2*B) + 3*A*b^2*x^2 + a*b*(5*A + B*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{A+Bx^2}{\left(a+bx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2)^3, x]
```

fricas [A] time = 0.44, size = 300, normalized size = 3.26

$$
\left[\frac{2\left(Ba^2b^2+3\,Aab^3\right)x^3-\left(\left(Bab^2+3\,Ab^3\right)x^4+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\left(Ba^3b-5\,Aa^2b^2\right)x}{16\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)},\frac{\left(Ba^2b^2+3\,Aab^3\right)x^3+\left(\left(Bab^2+3\,Ab^3\right)x^4+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-\left(Ba^3b-5\,Aa^2b^2\right)x}{8\left(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(B*a^2*b^2 + 3*A*a*b^3)*x^3 - ((B*a*b^2 + 3*A*b^3)*x^4 + B*a^3 + 3
*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((B*a^2*b^2 + 3*A*a*b^3)*x^3 + ((B*a*b^2 + 3*A*b^3)*
x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

giac [A]   time = 0.34, size = 78, normalized size = 0.85

$$\frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{Babx^3+3\,Ab^2x^3-Ba^2x+5\,Aabx}{8\,\left(bx^2+a\right)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(B*a*b*x^3
+ 3*A*b^2*x^3 - B*a^2*x + 5*A*a*b*x)/((b*x^2 + a)^2*a^2*b)
```

maple [A]   time = 0.01, size = 90, normalized size = 0.98

$$\frac{3A\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^2}+\frac{B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;ab}+\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*b+B*a)/a^2*x^3+1/8*(5*A*b-B*a)/a/b*x)/(b*x^2+a)^2+3/8/a^2/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x)*A+1/8/a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*B
```

maxima [A]   time = 2.52, size = 92, normalized size = 1.00

$$\frac{\left(Bab+3\,Ab^2\right)x^3-\left(Ba^2-5\,Aab\right)x}{8\,\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*a*b + 3*A*b^2)*x^3 - (B*a^2 - 5*A*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*
x^2 + a^4*b) + 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

mupad [B] time = 0.19, size = 82, normalized size = 0.89

$$\frac{\frac{x^3\,\left(3\,A\,b+B\,a\right)}{8\,a^2}+\frac{x\,\left(5\,A\,b-B\,a\right)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,A\,b+B\,a\right)}{8\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^3,x)
```

```
[Out] ((x^3*(3*A*b + B*a))/(8*a^2) + (x*(5*A*b - B*a))/(8*a*b))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(3*A*b + B*a))/(8*a^(5/2)*b^(3/2))
```

sympy [A] time = 0.55, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ba)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(3Ab+Ba)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{x^3\left(3Ab^2+Bab\right)+x\left(5Aab-Ba^2\right)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(3*A*b**2 + B*a*b) + x*(5*A*a*b - B*a**2))/(8*a**4*b + 16*a**3*
b**2*x**2 + 8*a**2*b**3*x**4)
```

3.104 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{x(7Ab-3aB)}{8a^3\left(a+bx^2\right)}-\frac{A}{a^3x}-\frac{x(Ab-aB)}{4a^2\left(a+bx^2\right)^2}$$

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {456, 453, 205}

$$-\frac{x(7Ab-3aB)}{8a^3\left(a+bx^2\right)}-\frac{x(Ab-aB)}{4a^2\left(a+bx^2\right)^2}-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) - ((A*b - a*B)*x)/(4*a^2*(a + b*x^2)^2) - ((7*A*b - 3*a*B)*x)/
(8*a^3*(a + b*x^2)) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7
/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
```

```
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx &= -\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{1}{4}\int \frac{-\frac{4A}{a}+\frac{3(Ab-aB)x^2}{a^2}}{x^2\left(a+bx^2\right)^2}\,dx\\ &= -\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)}+\frac{1}{8}\int \frac{\frac{8A}{a^2}-\frac{(7Ab-3aB)x^2}{a^3}}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{A}{a^3x}-\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)}-\frac{(3(5Ab-aB))\int \frac{1}{a+bx^2}\,dx}{8a^3}\\ &= -\frac{A}{a^3x}-\frac{(Ab-aB)x}{4a^2\left(a+bx^2\right)^2}-\frac{(7Ab-3aB)x}{8a^3\left(a+bx^2\right)}-\frac{3(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.99

$$\frac{3(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{x(3aB-7Ab)}{8a^3\left(a+bx^2\right)}-\frac{A}{a^3x}+\frac{x(aB-Ab)}{4a^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + ((-(A*b) + a*B)*x)/(4*a^2*(a + b*x^2)^2) + ((-7*A*b + 3*a*B)
*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)^3), x]
```

fricas [A] time = 0.46, size = 324, normalized size = 3.34

$$\left[-\frac{16\,Aa^{3}b-6\left(Ba^{2}b^{2}-5\,Aab^{3}\right)x^{4}-10\left(Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{2}-3\left(\left(Bab^{2}-5\,Ab^{3}\right)x^{5}+2\left(Ba^{2}b-5\,Aab^{2}\right)x^{3}+\left(Ba^{3}-5\,Aa^{2}b\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{16\left(a^{4}b^{3}x^{5}+2\,a^{5}b^{2}x^{3}+a^{6}bx\right)},-\frac{8\,Aa^{3}b-3\left(Ba^{2}b^{2}-5\,Aab^{3}\right)x^{4}-5\left(Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{2}-3\left(\left(Bab^{2}-5\,Ab^{3}\right)x^{5}+2\left(Ba^{2}b-5\,Aab^{2}\right)x^{3}+\left(Ba^{3}-5\,Aa^{2}b\right)x\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{8\left(a^{4}b^{3}x^{5}+2\,a^{5}b^{2}x^{3}+a^{6}bx\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*A*a^3*b - 6*(B*a^2*b^2 - 5*A*a*b^3)*x^4 - 10*(B*a^3*b - 5*A*a^2*
b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^
3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*A*a^3*b - 3*(B*a^2*b^2 -
5*A*a*b^3)*x^4 - 5*(B*a^3*b - 5*A*a^2*b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5
 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^3 - 5*A*a^2*b)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

giac [A] time = 0.30, size = 82, normalized size = 0.85

$$\frac{3\,(Ba-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}-\frac{A}{a^{3}x}+\frac{3\,Babx^{3}-7\,Ab^{2}x^{3}+5\,Ba^{2}x-9\,Aabx}{8\left(bx^{2}+a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - A/(a^3*x) + 1/8*(
3*B*a*b*x^3 - 7*A*b^2*x^3 + 5*B*a^2*x - 9*A*a*b*x)/((b*x^2 + a)^2*a^3)
```

maple [A] time = 0.01, size = 125, normalized size = 1.29

$$-\frac{7A\,b^{2}x^{3}}{8\left(b\,x^{2}+a\right)^{2}a^{3}}+\frac{3Bb\,x^{3}}{8\left(b\,x^{2}+a\right)^{2}a^{2}}-\frac{9Abx}{8\left(b\,x^{2}+a\right)^{2}a^{2}}+\frac{5Bx}{8\left(b\,x^{2}+a\right)^{2}a}-\frac{15Ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^{3}}+\frac{3B\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^{2}}-\frac{A}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -7/8/a^3/(b*x^2+a)^2*A*x^3*b^2+3/8/a^2/(b*x^2+a)^2*B*x^3*b-9/8/a^2/(b*x^2+a
)^2*b*A*x+5/8/a/(b*x^2+a)^2*B*x-15/8/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x)*A*b+3/8/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B-A/a^3/x
```

maxima [A] time = 2.43, size = 96, normalized size = 0.99

$$\frac{3\left(Bab-5\,Ab^{2}\right)x^{4}-8\,Aa^{2}+5\left(Ba^{2}-5\,Aab\right)x^{2}}{8\left(a^{3}b^{2}x^{5}+2\,a^{4}bx^{3}+a^{5}x\right)}+\frac{3\left(Ba-5\,Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*a*b - 5*A*b^2)*x^4 - 8*A*a^2 + 5*(B*a^2 - 5*A*a*b)*x^2)/(a^3*b^2*
x^5 + 2*a^4*b*x^3 + a^5*x) + 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^3)
```

mupad [B] time = 0.19, size = 113, normalized size = 1.16

$$-\frac{\frac{A}{a}+\frac{5\,x^{2}\,(5\,A\,b-B\,a)}{8\,a^{2}}+\frac{3\,b\,x^{4}\,(5\,A\,b-B\,a)}{8\,a^{3}}}{a^{2}\,x+2\,a\,b\,x^{3}+b^{2}\,x^{5}}-\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{b}\,x\,(5\,A\,b-B\,a)}{\sqrt{a}\,(15\,A\,b-3\,B\,a)}\right)\,(5\,A\,b-B\,a)}{8\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (A/a + (5*x^2*(5*A*b - B*a))/(8*a^2) + (3*b*x^4*(5*A*b - B*a))/(8*a^3))/(
a^2*x + b^2*x^5 + 2*a*b*x^3) - (3*atan((3*b^(1/2)*x*(5*A*b - B*a))/(a^(1/2)
*(15*A*b - 3*B*a)))*(5*A*b - B*a))/(8*a^(7/2)*b^(1/2))
```

sympy [B] time = 0.67, size = 194, normalized size = 2.00

$$-\frac{3\sqrt{-\frac{1}{a^{7}b}}\,(-5Ab+Ba)\log\left(-\frac{3a^{4}\sqrt{-\frac{1}{a^{7}b}}\,(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^{7}b}}\,(-5Ab+Ba)\log\left(\frac{3a^{4}\sqrt{-\frac{1}{a^{7}b}}\,(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16}+\frac{-8Aa^{2}+x^{4}\left(-15Ab^{2}+3Bab\right)+x^{2}\left(-25Aab+5Ba^{2}\right)}{8a^{5}x+16a^{4}bx^{3}+8a^{3}b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(-3*a**4*sqrt(-1/(a**7*b))*(-5*A*b +
 B*a)/(-15*A*b + 3*B*a) + x)/16 + 3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(3*
a**4*sqrt(-1/(a**7*b))*(-5*A*b + B*a)/(-15*A*b + 3*B*a) + x)/16 + (-8*A*a**
2 + x**4*(-15*A*b**2 + 3*B*a*b) + x**2*(-25*A*a*b + 5*B*a**2))/(8*a**5*x +
16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

3.105 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=117

$$\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{bx(11Ab-7aB)}{8a^4\left(a+bx^2\right)}+\frac{3Ab-aB}{a^4x}+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{A}{3a^3x^3}$$

Rubi [A] time = 0.17, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {456, 1259, 1261, 205}

$$\frac{bx(11Ab-7aB)}{8a^4\left(a+bx^2\right)}+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}+\frac{3Ab-aB}{a^4x}+\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -A/(3*a^3*x^3) + (3*A*b - a*B)/(a^4*x) + (b*(A*b - a*B)*x)/(4*a^3*(a + b*x^
2)^2) + (b*(11*A*b - 7*a*B)*x)/(8*a^4*(a + b*x^2)) + (5*Sqrt[b]*(7*A*b - 3*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
```

```
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx &= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4}b\int \frac{-\frac{4A}{ab}+\frac{4(Ab-aB)x^2}{a^2b}-\frac{3(Ab-aB)x^4}{a^3}}{x^4\left(a+bx^2\right)^2}\,dx\\
&= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}-\frac{\int \frac{-8aAb+8b(2Ab-aB)x^2-\frac{b^2(11Ab-7aB)x^4}{a}}{x^4\left(a+bx^2\right)}\,dx}{8a^3b}\\
&= \frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}-\frac{\int \left(-\frac{8Ab}{x^4}-\frac{8b(-3Ab+aB)}{ax^2}+\frac{5b^2(-7Ab+3aB)}{a\left(a+bx^2\right)}\right)dx}{8a^3b}\\
&= -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{(5b(7Ab-3aB))\int \frac{1}{a+bx^2}\,dx}{8a^4}\\
&= -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{8a^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 116, normalized size = 0.99

$$\frac{5\sqrt{b}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{-8a^3\left(A+3Bx^2\right)+a^2bx^2\left(56A-75Bx^2\right)+5ab^2x^4\left(35A-9Bx^2\right)+105Ab^3x^6}{24a^4x^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (105*A*b^3*x^6 + a^2*b*x^2*(56*A - 75*B*x^2) + 5*a*b^2*x^4*(35*A - 9*B*x^2)
 - 8*a^3*(A + 3*B*x^2))/(24*a^4*x^3*(a + b*x^2)^2) + (5*Sqrt[b]*(7*A*b - 3*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)^3), x]
```

fricas [A]   time = 0.48, size = 368, normalized size = 3.15

$$\left[-\frac{30\left(3\,Bab^2-7\,Ab^3\right)x^6+50\left(3\,Ba^2b-7\,Aab^2\right)x^4+16\,Aa^3+16\left(3\,Ba^3-7\,Aa^2b\right)x^2+15\left(\left(3\,Bab^2-7\,Ab^3\right)x^7+2\left(3\,Ba^2b-7\,Aab^2\right)x^5+\left(3\,Ba^3-7\,Aa^2b\right)x^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)},\ -\frac{15\left(3\,Bab^2-7\,Ab^3\right)x^6+25\left(3\,Ba^2b-7\,Aab^2\right)x^4+8\,Aa^3+8\left(3\,Ba^3-7\,Aa^2b\right)x^2+15\left(\left(3\,Bab^2-7\,Ab^3\right)x^7+2\left(3\,Ba^2b-7\,Aab^2\right)x^5+\left(3\,Ba^3-7\,Aa^2b\right)x^3\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{24\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(30*(3*B*a*b^2 - 7*A*b^3)*x^6 + 50*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 16*
A*a^3 + 16*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(3
*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(-b/a)*log((b*x^
2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^
3), -1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 +
8*A*a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*(
3*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(b/a)*arctan(x*
sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

giac [A]   time = 0.32, size = 108, normalized size = 0.92

$$-\frac{5\left(3\,Bab-7\,Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}-\frac{7\,Bab^2x^3-11\,Ab^3x^3+9\,Ba^2bx-13\,Aab^2x}{8\left(bx^2+a\right)^2a^4}-\frac{3\,Bax^2-9\,Abx^2+Aa}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/8*(7*B*a
*b^2*x^3 - 11*A*b^3*x^3 + 9*B*a^2*b*x - 13*A*a*b^2*x)/((b*x^2 + a)^2*a^4) -
 1/3*(3*B*a*x^2 - 9*A*b*x^2 + A*a)/(a^4*x^3)
```

maple [**A**] time = 0.02, size = 152, normalized size = 1.30

$$\frac{11A\,b^3x^3}{8\left(b\,x^2+a\right)^2a^4}-\frac{7B\,b^2x^3}{8\left(b\,x^2+a\right)^2a^3}+\frac{13A\,b^2x}{8\left(b\,x^2+a\right)^2a^3}-\frac{9Bbx}{8\left(b\,x^2+a\right)^2a^2}+\frac{35A\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^4}-\frac{15Bb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\;a^3}+\frac{3Ab}{a^4x}-\frac{B}{a^3x}-\frac{A}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^3,x)
```

```
[Out] 11/8/a^4*b^3/(b*x^2+a)^2*A*x^3-7/8/a^3*b^2/(b*x^2+a)^2*B*x^3+13/8/a^3*b^2/(
b*x^2+a)^2*A*x-9/8/a^2*b/(b*x^2+a)^2*B*x+35/8/a^4*b^2/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)*A-15/8/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*B-1/3*A
/a^3/x^3+3/a^4/x*A*b-1/a^3/x*B
```

maxima [**A**] time = 2.29, size = 128, normalized size = 1.09

$$-\frac{15\left(3\,Bab^2-7\,Ab^3\right)x^6+25\left(3\,Ba^2b-7\,Aab^2\right)x^4+8\,Aa^3+8\left(3\,Ba^3-7\,Aa^2b\right)x^2}{24\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}-\frac{5\left(3\,Bab-7\,Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 8*A*
a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3) -
5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

mupad [**B**] time = 0.20, size = 114, normalized size = 0.97

$$\frac{\frac{x^2\,\left(7\,A\,b-3\,B\,a\right)}{3\,a^2}-\frac{A}{3\,a}+\frac{5\,b^2\,x^6\,\left(7\,A\,b-3\,B\,a\right)}{8\,a^4}+\frac{25\,b\,x^4\,\left(7\,A\,b-3\,B\,a\right)}{24\,a^3}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}+\frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(7\,A\,b-3\,B\,a\right)}{8\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^3),x)
```

```
[Out] ((x^2*(7*A*b - 3*B*a))/(3*a^2) - A/(3*a) + (5*b^2*x^6*(7*A*b - 3*B*a))/(8*a
^4) + (25*b*x^4*(7*A*b - 3*B*a))/(24*a^3))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
+ (5*b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(7*A*b - 3*B*a))/(8*a^(9/2))
```

sympy [**B**] time = 0.77, size = 226, normalized size = 1.93

$$\frac{5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)\log\left(-\frac{5a^5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}-\frac{5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)\log\left(\frac{5a^5\sqrt{-\frac{b}{a^9}}\,(-7Ab+3Ba)}{-35Ab^2+15Bab}+x\right)}{16}+\frac{-8Aa^3+x^6\left(105Ab^3-45Bab^2\right)+x^4\left(175Aab^2-75Ba^2b\right)+x^2\left(56Aa^2b-24Ba^3\right)}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

```
[Out] 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(-5*a**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)
/(-35*A*b**2 + 15*B*a*b) + x)/16 - 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(5*a
**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)/(-35*A*b**2 + 15*B*a*b) + x)/16 + (-8*A*
a**3 + x**6*(105*A*b**3 - 45*B*a*b**2) + x**4*(175*A*a*b**2 - 75*B*a**2*b)
+ x**2*(56*A*a**2*b - 24*B*a**3))/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a**4*
b**2*x**7)
```

3.106 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}-\frac{b^2x(15Ab-11aB)}{8a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2x(Ab-aB)}{4a^4\left(a+bx^2\right)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{A}{5a^3x^5}$$

Rubi [A] time = 0.33, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {456, 1805, 1802, 205}

$$-\frac{b^2x(15Ab-11aB)}{8a^5\left(a+bx^2\right)}-\frac{b^2x(Ab-aB)}{4a^4\left(a+bx^2\right)^2}-\frac{7b^{3/2}(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{A}{5a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -A/(5*a^3*x^5) + (3*A*b - a*B)/(3*a^4*x^3) - (3*b*(2*A*b - a*B))/(a^5*x) -
(b^2*(A*b - a*B)*x)/(4*a^4*(a + b*x^2)^2) - (b^2*(15*A*b - 11*a*B)*x)/(8*a^
5*(a + b*x^2)) - (7*b^(3/2)*(9*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(11/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4}b^2\int \frac{-\frac{4A}{ab^2}+\frac{4(Ab-aB)x^2}{a^2b^2}-\frac{4(Ab-aB)x^4}{a^3b}+\frac{3(Ab-aB)x^6}{a^4}}{x^6\left(a+bx^2\right)^2}\,dx$$

$$= -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}+\frac{b^2\int \frac{\frac{8A}{ab^2}-\frac{8(2Ab-aB)x^2}{a^2b^2}+\frac{8(3Ab-2aB)x^4}{a^3b}-\frac{(15Ab-11aB)x^6}{a^4}}{x^6\left(a+bx^2\right)}\,dx}{8a}$$

$$= -\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}+\frac{b^2\int \Big(\frac{8A}{a^2b^2x^6}+\frac{8(-3Ab+aB)}{a^3b^2x^4}-\frac{24(-2Ab+aB)}{a^4bx^2}+\frac{7(-9Ab+5a}{a^4\left(a+bx^2\right)}}{8a}$$

$$= -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\Big(7b^2(9Ab-$$

$$= -\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-7b^{3/2}(9Ab-$$

Mathematica [A] time = 0.10, size = 139, normalized size = 0.98

$$\frac{7b^{3/2}(5aB-9Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}}+\frac{-8a^4\left(3A+5Bx^2\right)+8a^3bx^2\left(9A+35Bx^2\right)+7a^2b^2x^4\left(125Bx^2-72A\right)+525ab^3x^6\left(Bx^2-3A\right)-945Ab^4x^8}{120a^5x^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] (-945*A*b^4*x^8 + 525*a*b^3*x^6*(-3*A + B*x^2) - 8*a^4*(3*A + 5*B*x^2) + 8*
a^3*b*x^2*(9*A + 35*B*x^2) + 7*a^2*b^2*x^4*(-72*A + 125*B*x^2))/(120*a^5*x^
5*(a + b*x^2)^2) + (7*b^(3/2)*(-9*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)^3), x]
```

fricas [A] time = 0.45, size = 426, normalized size = 3.00

$$\left[\frac{210\left(5Bab^3-9Ab^4\right)x^8+350\left(5Ba^2b^2-9Aab^3\right)x^6-48Aa^4+112\left(5Ba^3b-9Aa^2b^2\right)x^4-16\left(5Ba^4-9Aa^3b\right)x^2-105\left(\left(5Bab^3-9Ab^4\right)x^9+2\left(5Ba^2b^2-9Aab^3\right)x^7+\left(5Ba^3b-9Aa^2b^2\right)x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{240\left(a^5b^2x^9+2a^6bx^7+a^7x^5\right)},\ \frac{105\left(5Bab^3-9Ab^4\right)x^8+175\left(5Ba^2b^2-9Aab^3\right)x^6-24Aa^4+56\left(5Ba^3b-9Aa^2b^2\right)x^4-8\left(5Ba^4-9Aa^3b\right)x^2+105\left(\left(5Bab^3-9Ab^4\right)x^9+2\left(5Ba^2b^2-9Aab^3\right)x^7+\left(5Ba^3b-9Aa^2b^2\right)x^5\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{120\left(a^5b^2x^9+2a^6bx^7+a^7x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(210*(5*B*a*b^3 - 9*A*b^4)*x^8 + 350*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 -
 48*A*a^4 + 112*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 16*(5*B*a^4 - 9*A*a^3*b)*x^
2 - 105*((5*B*a*b^3 - 9*A*b^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B
*a^3*b - 9*A*a^2*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), 1/120*(105*(5*B*a*b^3 -
9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 - 24*A*a^4 + 56*(5*B*a^3*b
 - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2 + 105*((5*B*a*b^3 - 9*A*b
^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B*a^3*b - 9*A*a^2*b^2)*x^5)*
sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

giac [A] time = 0.36, size = 135, normalized size = 0.95

$$\frac{7\left(5Bab^2-9Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^5}+\frac{11Bab^3x^3-15Ab^4x^3+13Ba^2b^2x-17Aab^3x}{8\left(bx^2+a\right)^2a^5}+\frac{45Babx^4-90Ab^2x^4-5Ba^2x^2+15Aabx^2-3Aa^2}{15a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/8*(11*B
*a*b^3*x^3 - 15*A*b^4*x^3 + 13*B*a^2*b^2*x - 17*A*a*b^3*x)/((b*x^2 + a)^2*a
```

```
^5) + 1/15*(45*B*a*b*x^4 - 90*A*b^2*x^4 - 5*B*a^2*x^2 + 15*A*a*b*x^2 - 3*A*
a^2)/(a^5*x^5)
```

maple [A] time = 0.03, size = 177, normalized size = 1.25

$$-\frac{15Ab^4x^3}{8\left(bx^2+a\right)^2a^5}+\frac{11Bb^3x^3}{8\left(bx^2+a\right)^2a^4}-\frac{17Ab^3x}{8\left(bx^2+a\right)^2a^4}+\frac{13Bb^2x}{8\left(bx^2+a\right)^2a^3}-\frac{63Ab^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^5}+\frac{35Bb^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,a^4}-\frac{6Ab^2}{a^5x}+\frac{3Bb}{a^4x}+\frac{Ab}{a^4x^3}-\frac{B}{3a^3x^3}-\frac{A}{5a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^3,x)
```

```
[Out] -15/8/a^5*b^4/(b*x^2+a)^2*A*x^3+11/8/a^4*b^3/(b*x^2+a)^2*B*x^3-17/8/a^4*b^3
/(b*x^2+a)^2*A*x+13/8/a^3*b^2/(b*x^2+a)^2*B*x-63/8/a^5*b^3/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)*A+35/8/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*
B-1/5*A/a^3/x^5+1/a^4/x^3*A*b-1/3/a^3/x^3*B-6*b^2/a^5/x*A+3*b/a^4/x*B
```

maxima [A] time = 2.36, size = 154, normalized size = 1.08

$$\frac{105\left(5\,Bab^3-9\,Ab^4\right)x^8+175\left(5\,Ba^2b^2-9\,Aab^3\right)x^6-24\,Aa^4+56\left(5\,Ba^3b-9\,Aa^2b^2\right)x^4-8\left(5\,Ba^4-9\,Aa^3b\right)x^2}{120\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}+\frac{7\left(5\,Bab^2-9\,Ab^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(105*(5*B*a*b^3 - 9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 -
24*A*a^4 + 56*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2)/
(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) + 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

mupad [B] time = 0.20, size = 135, normalized size = 0.95

$$-\frac{\frac{A}{5\,a}-\frac{x^2\,\left(9\,A\,b-5\,B\,a\right)}{15\,a^2}+\frac{35\,b^2\,x^6\,\left(9\,A\,b-5\,B\,a\right)}{24\,a^4}+\frac{7\,b^3\,x^8\,\left(9\,A\,b-5\,B\,a\right)}{8\,a^5}+\frac{7\,b\,x^4\,\left(9\,A\,b-5\,B\,a\right)}{15\,a^3}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(9\,A\,b-5\,B\,a\right)}{8\,a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (A/(5*a) - (x^2*(9*A*b - 5*B*a))/(15*a^2) + (35*b^2*x^6*(9*A*b - 5*B*a))/
(24*a^4) + (7*b^3*x^8*(9*A*b - 5*B*a))/(8*a^5) + (7*b*x^4*(9*A*b - 5*B*a))/
(15*a^3))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (7*b^(3/2)*atan((b^(1/2)*x)/a^(
1/2))*(9*A*b - 5*B*a))/(8*a^(11/2))
```

sympy [A] time = 0.87, size = 260, normalized size = 1.83

$$-\frac{7\sqrt{-\frac{b^3}{a^{11}}}\left(-9Ab+5Ba\right)\log\left(-\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}\left(-9Ab+5Ba\right)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{7\sqrt{-\frac{b^3}{a^{11}}}\left(-9Ab+5Ba\right)\log\left(\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}\left(-9Ab+5Ba\right)}{-63Ab^3+35Bab^2}+x\right)}{16}+\frac{-24Aa^4+x^8\left(-945Ab^4+525Bab^3\right)+x^6\left(-1575Aab^3+875Ba^2b^2\right)+x^4\left(-504Aa^2b^2+280Ba^3b\right)+x^2\left(72Aa^3b-40Ba^4\right)}{120a^7x^5+240a^6bx^7+120a^5b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

```
[Out] -7*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)*log(-7*a**6*sqrt(-b**3/a**11)*(-9*A*b
 + 5*B*a)/(-63*A*b**3 + 35*B*a*b**2) + x)/16 + 7*sqrt(-b**3/a**11)*(-9*A*b
+ 5*B*a)*log(7*a**6*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)/(-63*A*b**3 + 35*B*a
*b**2) + x)/16 + (-24*A*a**4 + x**8*(-945*A*b**4 + 525*B*a*b**3) + x**6*(-1
575*A*a*b**3 + 875*B*a**2*b**2) + x**4*(-504*A*a**2*b**2 + 280*B*a**3*b) +
x**2*(72*A*a**3*b - 40*B*a**4))/(120*a**7*x**5 + 240*a**6*b*x**7 + 120*a**5
*b**2*x**9)
```

3.107 $\int \frac{a+bx^2}{1+x^2}\,dx$

Optimal. Leaf size=12

$$(a-b)\tan^{-1}(x)+bx$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 203}

$$(a-b)\tan^{-1}(x)+bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2),x]
```

```
[Out] b*x + (a - b)*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2}\,dx &= bx-(-a+b)\int \frac{1}{1+x^2}\,dx\\ &= bx+(a-b)\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$(a-b)\tan^{-1}(x)+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 + x^2),x]
```

```
[Out] b*x + (a - b)*ArcTan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(1 + x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(1 + x^2), x]
```

fricas [A] time = 0.40, size = 12, normalized size = 1.00

$$bx+(a-b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] b*x + (a - b)*arctan(x)
```

giac [A] time = 0.35, size = 12, normalized size = 1.00

$$bx+(a-b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] b*x + (a - b)*arctan(x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$a\arctan(x)+bx-b\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^2+1),x)
```

```
[Out] b*x+arctan(x)*a-arctan(x)*b
```

maxima [A] time = 2.29, size = 12, normalized size = 1.00

$$bx+(a-b)\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] b*x + (a - b)*arctan(x)
```

mupad [B] time = 0.08, size = 12, normalized size = 1.00

$$b\,x+\mathrm{atan}(x)\,\left(a-b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + 1),x)
```

```
[Out] b*x + atan(x)*(a - b)
```

sympy [C] time = 0.16, size = 26, normalized size = 2.17

$$bx - \frac{i\left(a - b\right)\log{\left(x - i \right)}}{2} + \frac{i\left(a - b\right)\log{\left(x + i \right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**2+1),x)
```

```
[Out] b*x - I*(a - b)*log(x - I)/2 + I*(a - b)*log(x + I)/2
```

3.108 $\int \frac{a+bx^2}{1-x^2} \, dx$

Optimal. Leaf size=11

$$(a+b)\tanh^{-1}(x)-bx$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 206}

$$(a+b)\tanh^{-1}(x)-bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] -(b*x) + (a + b)*ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1-x^2}\,dx &= -bx-(-a-b)\int \frac{1}{1-x^2}\,dx\\ &= -bx+(a+b)\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.01, size = 28, normalized size = 2.55

$$\frac{1}{2}(-(a+b)\log(1-x)+(a+b)\log(x+1)-2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] (-2*b*x - (a + b)*Log[1 - x] + (a + b)*Log[1 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{1-x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(1 - x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(1 - x^2), x]
```

fricas [B] time = 0.47, size = 23, normalized size = 2.09

$$-bx+\frac{1}{2}(a+b)\log(x+1)-\frac{1}{2}(a+b)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

giac [B] time = 0.36, size = 25, normalized size = 2.27

$$-bx+\frac{1}{2}(a+b)\log(|x+1|)-\frac{1}{2}(a+b)\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="giac")
```

```
[Out] -b*x + 1/2*(a + b)*log(abs(x + 1)) - 1/2*(a + b)*log(abs(x - 1))
```

maple [B] time = 0.00, size = 34, normalized size = 3.09

$$-\frac{a\ln(x-1)}{2}+\frac{a\ln(x+1)}{2}-bx-\frac{b\ln(x-1)}{2}+\frac{b\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(-x^2+1),x)
```

```
[Out] -b*x-1/2*ln(x-1)*a-1/2*ln(x-1)*b+1/2*ln(x+1)*a+1/2*ln(x+1)*b
```

maxima [B] time = 0.99, size = 23, normalized size = 2.09

$$-bx + \frac{1}{2}(a+b)\log(x+1) - \frac{1}{2}(a+b)\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

mupad [B] time = 0.11, size = 11, normalized size = 1.00

$$\operatorname{atanh}(x)\ (a+b) - b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a + b*x^2)/(x^2 - 1),x)
```

```
[Out] atanh(x)*(a + b) - b*x
```

sympy [B] time = 0.17, size = 22, normalized size = 2.00

$$-bx - \frac{(a+b)\log(x-1)}{2} + \frac{(a+b)\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(-x**2+1),x)
```

```
[Out] -b*x - (a + b)*log(x - 1)/2 + (a + b)*log(x + 1)/2
```

3.109 $\int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {383}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-1 + x^2)^2,x]
```

```
[Out] x/(1 - x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(-1+x^2\right)^2} \, dx = \frac{x}{1-x^2}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-1 + x^2)^2,x]
```

```
[Out] -(x/(-1 + x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(-1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(-1 + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(-1 + x^2)^2, x]
```

fricas [A] time = 0.41, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^2 - 1)
```

giac [A] time = 0.43, size = 11, normalized size = 1.00

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/(x - 1/x)
```

maple [A] time = 0.00, size = 16, normalized size = 1.45

$$-\frac{1}{2\,(x+1)}-\frac{1}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-1)^2,x)
```

```
[Out] -1/2/(x+1)-1/2/(x-1)
```

maxima [A] time = 1.03, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^2 - 1)
```

mupad [B] time = 0.08, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 - 1)^2,x)
```

```
[Out] -x/(x^2 - 1)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-1)**2,x)
```

```
[Out] -x/(x**2 - 1)
```

3.110 $\int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=9

$$\frac{x}{x^2+1}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {383}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx = \frac{x}{1+x^2}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^2)/(1 + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(1 - x^2)/(1 + x^2)^2, x]
```

fricas [A] time = 0.43, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

giac [A] time = 0.35, size = 7, normalized size = 0.78

$$\frac{1}{x+\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

maple [A] time = 0.01, size = 10, normalized size = 1.11

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)^2,x)
```

```
[Out] x/(x^2+1)
```

maxima [A] time = 1.02, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

mupad [B] time = 0.08, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^2 + 1)^2,x)
```

```
[Out] x/(x^2 + 1)
```

sympy [A] time = 0.09, size = 5, normalized size = 0.56

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1)
```

3.111 $\int \frac{3+2x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {385, 203}

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x^2}{\left(1+x^2\right)^2} \, dx &= \frac{x}{2\left(1+x^2\right)}+\frac{5}{2}\int \frac{1}{1+x^2} \, dx\\ &= \frac{x}{2\left(1+x^2\right)}+\frac{5}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.06, size = 19, normalized size = 1.00

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+2x^2}{\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 2*x^2)/(1 + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 + 2*x^2)/(1 + x^2)^2, x]
```

fricas [A] time = 0.41, size = 20, normalized size = 1.05

$$\frac{5\left(x^2+1\right)\arctan(x)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

giac [A] time = 0.29, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{x}{2x^2+2}+\frac{5\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)*x+5/2*arctan(x)
```

maxima [A] time = 2.30, size = 15, normalized size = 0.79

$$\frac{x}{2\left(x^2+1\right)}+\frac{5}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

mupad [B] time = 0.07, size = 16, normalized size = 0.84

$$\frac{5\,\mathrm{atan}(x)}{2}+\frac{x}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x^2 + 1)^2,x)
```

```
[Out] (5*atan(x))/2 + x/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{x}{2x^2+2}+\frac{5\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + 5*atan(x)/2
```

3.112 $\int \frac{-2+x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=19

$$-\frac{3x}{2\left(x^2+1\right)}-\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {385, 203}

$$-\frac{3x}{2\left(x^2+1\right)}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x^2}{\left(1+x^2\right)^2} \, dx &= -\frac{3x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1}{1+x^2} \, dx\\ &= -\frac{3x}{2\left(1+x^2\right)}-\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{3x}{2\left(x^2+1\right)}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] (-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2+x^2}{\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + x^2)/(1 + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(-2 + x^2)/(1 + x^2)^2, x]
```

fricas [A] time = 0.41, size = 21, normalized size = 1.11

$$-\frac{\left(x^2+1\right)\arctan(x)+3\,x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x) + 3*x)/(x^2 + 1)
```

giac [A] time = 0.31, size = 15, normalized size = 0.79

$$-\frac{3\,x}{2\left(x^2+1\right)}-\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{3x}{2\left(x^2+1\right)}-\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/(x^2+1)^2,x)
```

```
[Out] -3/2/(x^2+1)*x-1/2*arctan(x)
```

maxima [A] time = 2.34, size = 15, normalized size = 0.79

$$-\frac{3\,x}{2\left(x^2+1\right)}-\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.89

$$-\frac{\mathrm{atan}(x)}{2}-\frac{3\,x}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2)/(x^2 + 1)^2,x)
```

```
[Out] - atan(x)/2 - (3*x)/(2*(x^2 + 1))
```

sympy [A] time = 0.11, size = 15, normalized size = 0.79

$$-\frac{3x}{2x^2+2}-\frac{\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/(x**2+1)**2,x)
```

```
[Out] -3*x/(2*x**2 + 2) - atan(x)/2
```

3.113 $\int \frac{3+x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=14

$$\frac{x}{x^2+1}+2\tan^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {385, 203}

$$\frac{x}{x^2+1}+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2}{\left(1+x^2\right)^2} \, dx &= \frac{x}{1+x^2}+2\int \frac{1}{1+x^2} \, dx\\ &= \frac{x}{1+x^2}+2\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{x}{x^2+1}+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(1 + x^2)^2,x]
```

```
[Out] x/(1 + x^2) + 2*ArcTan[x]
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+x^2}{\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + x^2)/(1 + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 + x^2)/(1 + x^2)^2, x]
```

fricas **[A]** time = 0.41, size = 19, normalized size = 1.36

$$\frac{2\left(x^2+1\right)\arctan(x)+x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (2*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

giac **[A]** time = 0.34, size = 14, normalized size = 1.00

$$\frac{x}{x^2+1}+2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

maple **[A]** time = 0.01, size = 15, normalized size = 1.07

$$\frac{x}{x^2+1}+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2+1)^2,x)
```

```
[Out] 1/(x^2+1)*x+2*arctan(x)
```

maxima [A] time = 2.29, size = 14, normalized size = 1.00

$$\frac{x}{x^2+1}+2\ \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + 2*arctan(x)
```

mupad [B] time = 0.02, size = 14, normalized size = 1.00

$$2\,\mathrm{atan}(x)+\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/(x^2 + 1)^2,x)
```

```
[Out] 2*atan(x) + x/(x^2 + 1)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.71

$$\frac{x}{x^2+1}+2\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2+1)**2,x)
```

```
[Out] x/(x**2 + 1) + 2*atan(x)
```

3.114 $\int \frac{a+bx^2}{\left(-a+bx^2\right)^2} \, dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {383}

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(-a + b*x^2)^2,x]
```

```
[Out] x/(a - b*x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(-a+bx^2\right)^2} \, dx = \frac{x}{a-bx^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(-a + b*x^2)^2,x]
```

```
[Out] -(x/(-a + b*x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\left(-a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(-a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)/(-a + b*x^2)^2, x]
```

fricas [A] time = 0.41, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="fricas")
```

```
[Out] -x/(b*x^2 - a)
```

giac [A] time = 0.32, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="giac")
```

```
[Out] -x/(b*x^2 - a)
```

maple [A] time = 0.01, size = 15, normalized size = 1.25

$$-\frac{x}{b\,x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(b*x^2-a)^2,x)
```

```
[Out] -x/(b*x^2-a)
```

maxima [A] time = 1.00, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="maxima")
```

```
[Out] -x/(b*x^2 - a)
```

mupad [B] time = 0.04, size = 12, normalized size = 1.00

$$\frac{x}{a-b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(a - b*x^2)^2,x)
```

```
[Out] x/(a - b*x^2)
```

sympy [A] time = 0.18, size = 8, normalized size = 0.67

$$-\frac{x}{-a+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(b*x**2-a)**2,x)
```

```
[Out] -x/(-a + b*x**2)
```

3.115 $\int \frac{a+bx^2}{\left(a-bx^2\right)^2} \, dx$

Optimal. Leaf size=12

$$\frac{x}{a-bx^2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {383}

$$\frac{x}{a-bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(a - b*x^2)^2,x]
```

```
[Out] x/(a - b*x^2)
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\left(a-bx^2\right)^2} \, dx = \frac{x}{a-bx^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(a - b*x^2)^2,x]
```

```
[Out] -(x/(-a + b*x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^2}{\left(a-bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(a - b*x^2)^2,x]
[Out] IntegrateAlgebraic[(a + b*x^2)/(a - b*x^2)^2, x]
```

fricas [A] time = 0.61, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="fricas")
[Out] -x/(b*x^2 - a)
```

giac [A] time = 0.35, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="giac")
[Out] -x/(b*x^2 - a)
```

maple [A] time = 0.01, size = 15, normalized size = 1.25

$$-\frac{x}{b\,x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(-b*x^2+a)^2,x)
[Out] -1/(b*x^2-a)*x
```

maxima [A] time = 1.05, size = 14, normalized size = 1.17

$$-\frac{x}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -x/(b*x^2 - a)
```

mupad [B] time = 0.00, size = 12, normalized size = 1.00

$$\frac{x}{a-b\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(a - b*x^2)^2,x)
```

```
[Out] x/(a - b*x^2)
```

sympy [A] time = 0.18, size = 8, normalized size = 0.67

$$-\frac{x}{-a+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(-b*x**2+a)**2,x)
```

```
[Out] -x/(-a + b*x**2)
```

3.116 $\int \frac{A+Bx^2}{a-bx^2} \, dx$

Optimal. Leaf size=39

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{Bx}{b}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {388, 208}

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{a-bx^2}\,dx &= -\frac{Bx}{b}+\frac{(Ab+aB)\int \frac{1}{a-bx^2}\,dx}{b}\\ &= -\frac{Bx}{b}+\frac{(Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{(aB+Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] -((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+Bx^2}{a-bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a - b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x^2)/(a - b*x^2), x]
```

fricas [A] time = 0.45, size = 98, normalized size = 2.51

$$\left[-\frac{2\,Babx-(Ba+Ab)\sqrt{ab}\,\log\left(\frac{bx^2+2\,\sqrt{ab}\,x+a}{bx^2-a}\right)}{2\,ab^2},-\frac{Babx+(Ba+Ab)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*a*b*x - (B*a + A*b)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b
*x^2 - a)))/(a*b^2), -(B*a*b*x + (B*a + A*b)*sqrt(-a*b)*arctan(sqrt(-a*b)*x
/a))/(a*b^2)]
```

giac [A] time = 0.29, size = 36, normalized size = 0.92

$$-\frac{Bx}{b}-\frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="giac")
```

```
[Out] -B*x/b - (B*a + A*b)*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b)
```

maple [A] time = 0.00, size = 37, normalized size = 0.95

$$-\frac{Bx}{b}-\frac{(-Ab-Ba)\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(-b*x^2+a),x)
```

```
[Out] -B/b*x-(-A*b-B*a)/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.42, size = 49, normalized size = 1.26

$$-\frac{Bx}{b}-\frac{(Ba+Ab)\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="maxima")
```

```
[Out] -B*x/b - 1/2*(B*a + A*b)*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b
)*b)
```

mupad [B] time = 0.14, size = 31, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(A\,b+B\,a)}{\sqrt{a}\,b^{3/2}}-\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a - b*x^2),x)
```

```
[Out] (atanh((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(a^(1/2)*b^(3/2)) - (B*x)/b
```

sympy [B] time = 0.29, size = 75, normalized size = 1.92

$$-\frac{Bx}{b}-\frac{\sqrt{\frac{1}{ab^3}}\,(Ab+Ba)\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{2}+\frac{\sqrt{\frac{1}{ab^3}}\,(Ab+Ba)\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(-b*x**2+a),x)
```

```
[Out] -B*x/b - sqrt(1/(a*b**3))*(A*b + B*a)*log(-a*b*sqrt(1/(a*b**3)) + x)/2 + sq
rt(1/(a*b**3))*(A*b + B*a)*log(a*b*sqrt(1/(a*b**3)) + x)/2
```

3.117 $\int \frac{1+x^2}{\left(16+x^2\right)^3} \, dx$

Optimal. Leaf size=35

$$\frac{19x}{2048\left(x^2+16\right)}-\frac{15x}{64\left(x^2+16\right)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {385, 199, 203}

$$\frac{19x}{2048\left(x^2+16\right)}-\frac{15x}{64\left(x^2+16\right)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2}{\left(16+x^2\right)^3}\,dx &= -\frac{15x}{64\left(16+x^2\right)^2}+\frac{19}{64}\int \frac{1}{\left(16+x^2\right)^2}\,dx\\
&= -\frac{15x}{64\left(16+x^2\right)^2}+\frac{19x}{2048\left(16+x^2\right)}+\frac{19\int \frac{1}{16+x^2}\,dx}{2048}\\
&= -\frac{15x}{64\left(16+x^2\right)^2}+\frac{19x}{2048\left(16+x^2\right)}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{19x}{2048\left(x^2+16\right)}-\frac{15x}{64\left(x^2+16\right)^2}+\frac{19\tan^{-1}\left(\frac{x}{4}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] (-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/819
2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(16+x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(16 + x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(16 + x^2)^3, x]
```

fricas [A] time = 0.48, size = 39, normalized size = 1.11

$$\frac{76\,x^3+19\left(x^4+32\,x^2+256\right)\arctan\left(\frac{1}{4}\,x\right)-704\,x}{8192\left(x^4+32\,x^2+256\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(76*x^3 + 19*(x^4 + 32*x^2 + 256)*arctan(1/4*x) - 704*x)/(x^4 + 32*x
^2 + 256)
```

giac [A] time = 0.41, size = 25, normalized size = 0.71

$$\frac{19\,x^3-176\,x}{2048\,\left(x^2+16\right)^2}+\frac{19}{8192}\,\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^2 + 16)^2 + 19/8192*arctan(1/4*x)
```

maple [A] time = 0.01, size = 25, normalized size = 0.71

$$\frac{19\arctan\left(\frac{x}{4}\right)}{8192}+\frac{\frac{19}{2048}x^3-\frac{11}{128}x}{\left(x^2+16\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2+16)^3,x)
```

```
[Out] (19/2048*x^3-11/128*x)/(x^2+16)^2+19/8192*arctan(1/4*x)
```

maxima [A] time = 2.47, size = 30, normalized size = 0.86

$$\frac{19\,x^3-176\,x}{2048\,\left(x^4+32\,x^2+256\right)}+\frac{19}{8192}\,\arctan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+16)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(19*x^3 - 176*x)/(x^4 + 32*x^2 + 256) + 19/8192*arctan(1/4*x)
```

mupad [B] time = 0.08, size = 30, normalized size = 0.86

$$\frac{19\,\mathrm{atan}\left(\frac{x}{4}\right)}{8192}-\frac{\frac{11\,x}{128}-\frac{19\,x^3}{2048}}{x^4+32\,x^2+256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + 16)^3,x)
```

```
[Out] (19*atan(x/4))/8192 - ((11*x)/128 - (19*x^3)/2048)/(32*x^2 + x^4 + 256)
```

sympy [**A**] time = 0.13, size = 27, normalized size = 0.77

$$\frac{19x^3 - 176x}{2048x^4 + 65536x^2 + 524288} + \frac{19\,\mathrm{atan}\left(\frac{x}{4}\right)}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+16)**3,x)
```

```
[Out] (19*x**3 - 176*x)/(2048*x**4 + 65536*x**2 + 524288) + 19*atan(x/4)/8192
```

3.118 $\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 74}

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] -1/(4*x^4*(1 + x^2)^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{x^3(1+x)^3}\,dx, x, x^2\right)\\ &= -\frac{1}{4x^4\left(1+x^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + 2*x^2)/(x^5*(1 + x^2)^3), x]
```

fricas [A] time = 0.76, size = 16, normalized size = 1.14

$$-\frac{1}{4\left(x^8+2\,x^6+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

giac [A] time = 0.27, size = 11, normalized size = 0.79

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + x^2)^2
```

maple [B] time = 0.01, size = 30, normalized size = 2.14

$$\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{2\left(x^2+1\right)}-\frac{1}{4\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/x^5/(x^2+1)^3,x)
```

```
[Out] -1/2/(x^2+1)-1/4/(x^2+1)^2-1/4/x^4+1/2/x^2
```

maxima [A] time = 1.03, size = 16, normalized size = 1.14

$$-\frac{1}{4\left(x^8+2\,x^6+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

mupad [B] time = 0.04, size = 20, normalized size = 1.43

$$-\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(x^5*(x^2 + 1)^3),x)
```

```
[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)
```

sympy [A] time = 0.13, size = 17, normalized size = 1.21

$$-\frac{1}{4x^8+8x^6+4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/x**5/(x**2+1)**3,x)
```

```
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

3.119 $\int \frac{\left(1-x^2\right)^2}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=1

$$x$$

Rubi [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {21, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^2/(-1 + x^2)^2,x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{\left(1-x^2\right)^2}{\left(-1+x^2\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^2/(-1 + x^2)^2,x]
```

```
[Out] x
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(1-x^2\right)^2}{\left(-1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^2)^2/(-1 + x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(1 - x^2)^2/(-1 + x^2)^2, x]
```

fricas [A] time = 0.70, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 0.30, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2/(x^2-1)^2,x)
```

```
[Out] x
```

maxima [A] time = 1.05, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

sympy [A] time = 0.06, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2/(x**2-1)**2,x)
```

```
[Out] x
```

3.120 $\int \frac{x^3(ac+bcx^2)}{a+bx^2} dx$

Optimal. Leaf size=8

$$\frac{cx^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^4)/4
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3\left(ac+bcx^2\right)}{a+bx^2} dx &= c\int x^3\, dx \\ &= \frac{cx^4}{4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^4)/4
```

IntegrateAlgebraic [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^4)/4
```

fricas [A] time = 0.60, size = 6, normalized size = 0.75

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4
```

giac [A] time = 0.33, size = 6, normalized size = 0.75

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*c*x^4
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] 1/4*c*x^4
```

maxima [**A**] time = 1.06, size = 6, normalized size = 0.75

$$\frac{1}{4}\,cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4
```

mupad [**B**] time = 0.02, size = 6, normalized size = 0.75

$$\frac{c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*c + b*c*x^2))/(a + b*x^2),x)
```

```
[Out] (c*x^4)/4
```

sympy [**A**] time = 0.07, size = 5, normalized size = 0.62

$$\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**4/4
```

3.121 $\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=8

$$\frac{cx^3}{3}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^3)/3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(ac+bcx^2\right)}{a+bx^2}\,dx &= c\int x^2\,dx\\ &= \frac{cx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^3)/3
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(ac+bcx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^2*(a*c + b*c*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.67, size = 6, normalized size = 0.75

$$\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3
```

giac [A] time = 0.34, size = 6, normalized size = 0.75

$$\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] 1/3*c*x^3
```

maxima [A] time = 0.97, size = 6, normalized size = 0.75

$$\frac{1}{3}\,cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3
```

mupad [B] time = 0.01, size = 6, normalized size = 0.75

$$\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*c + b*c*x^2))/(a + b*x^2),x)
```

```
[Out] (c*x^3)/3
```

sympy [A] time = 0.07, size = 5, normalized size = 0.62

$$\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**3/3
```

3.122 $\int \frac{x(ac+bcx^2)}{a+bx^2} \, dx$

Optimal. Leaf size=8

$$\frac{cx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.095, Rules used = {21, 30}

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^2)/2
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x\left(ac+bcx^2\right)}{a+bx^2} \, dx &= c\int x \, dx \\ &= \frac{cx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^2)/2
```

IntegrateAlgebraic [A] time = 0.01, size = 8, normalized size = 1.00

$$\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

```
[Out] (c*x^2)/2
```

fricas [A] time = 0.60, size = 6, normalized size = 0.75

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*c*x^2
```

giac [A] time = 0.32, size = 6, normalized size = 0.75

$$\frac{1}{2}\,cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c*x^2
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\frac{c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] 1/2*c*x^2
```

maxima [A] time = 0.98, size = 6, normalized size = 0.75

$$\frac{1}{2}cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2
```

mupad [B] time = 0.01, size = 6, normalized size = 0.75

$$\frac{c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*c + b*c*x^2))/(a + b*x^2),x)
```

```
[Out] (c*x^2)/2
```

sympy [A] time = 0.07, size = 5, normalized size = 0.62

$$\frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x**2/2
```

3.123 $\int \frac{ac+bcx^2}{a+bx^2}\,dx$

Optimal. Leaf size=3

$$cx$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$cx$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2),x]
```

```
[Out] c*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{a+bx^2}\,dx &= c\int 1\,dx\\ &= cx\end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2),x]
```

```
[Out] c*x
```

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{ac + bcx^2}{a + bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(a + b*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a*c + b*c*x^2)/(a + b*x^2), x]
```

fricas [A] time = 0.60, size = 3, normalized size = 1.00

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] c*x
```

giac [A] time = 0.29, size = 3, normalized size = 1.00

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c*x
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a),x)
```

```
[Out] c*x
```

maxima [A] time = 1.05, size = 3, normalized size = 1.00

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*x
```

mupad [B] time = 0.01, size = 3, normalized size = 1.00

$$c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(a + b*x^2),x)
```

```
[Out] c*x
```

sympy [A] time = 0.07, size = 2, normalized size = 0.67

$$cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a),x)
```

```
[Out] c*x
```

3.124 $\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=4

$$c\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 29}

$$c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] c*Log[x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}\,dx &= c\int \frac{1}{x}\,dx\\ &= c\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] c*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x*(a + b*x^2)), x]
```

fricas [A] time = 0.62, size = 4, normalized size = 1.00

$$c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] c*log(x)
```

giac [A] time = 0.27, size = 5, normalized size = 1.25

$$c\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c*log(abs(x))
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a),x)
```

```
[Out] c*ln(x)
```

maxima [A] time = 1.02, size = 7, normalized size = 1.75

$$\frac{1}{2}\,c\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)
```

mupad [B] time = 0.01, size = 4, normalized size = 1.00

$$c\ \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x*(a + b*x^2)),x)
```

```
[Out] c*log(x)
```

sympy [A] time = 0.07, size = 3, normalized size = 0.75

$$c \log{\left (x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)
```

3.125 $\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=6

$$-\frac{c}{x}$$

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.087, Rules used = {21, 30}

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/x)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)}\,dx &= c\int \frac{1}{x^2}\,dx\\ &= -\frac{c}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac + bcx^2}{x^2 \left(a + bx^2\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 0.62, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -c/x
```

giac [A] time = 0.32, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/x
```

maple [A] time = 0.00, size = 7, normalized size = 1.17

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a),x)
```

```
[Out] -c/x
```

maxima [**A**] time = 1.09, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -c/x
```

mupad [**B**] time = 0.01, size = 6, normalized size = 1.00

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^2*(a + b*x^2)),x)
```

```
[Out] -c/x
```

sympy [**A**] time = 0.07, size = 3, normalized size = 0.50

$$-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a),x)
```

```
[Out] -c/x
```

3.126 $\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=8

$$-\frac{c}{2x^2}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {21, 30}

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -c/(2*x^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)}\,dx &= c\int \frac{1}{x^3}\,dx\\ &= -\frac{c}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]
[Out] -1/2*c/x^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 0.63, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="fricas")
[Out] -1/2*c/x^2
```

giac [A] time = 0.37, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="giac")
[Out] -1/2*c/x^2
```

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a),x)
[Out] -1/2*c/x^2
```

maxima [A] time = 1.06, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*c/x^2
```

mupad [B] time = 0.02, size = 6, normalized size = 0.75

$$-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^3*(a + b*x^2)),x)
```

```
[Out] -c/(2*x^2)
```

sympy [A] time = 0.07, size = 7, normalized size = 0.88

$$-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*x**2)
```

3.127 $\int \frac{x^3\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=29

$$\frac{cx^2}{2b}-\frac{ac\log\left(a+bx^2\right)}{2b^2}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 266, 43}

$$\frac{cx^2}{2b}-\frac{ac\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*x^2)/(2*b) - (a*c*Log[a + b*x^2])/(2*b^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx &= c\int \frac{x^3}{a+bx^2}\,dx\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{cx^2}{2b}-\frac{ac\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.00, size = 29, normalized size = 1.00

$$
c\left(\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] c*(x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2, x]
```

fricas [**A**] time = 0.65, size = 24, normalized size = 0.83

$$
\frac{bcx^2-ac\log\left(bx^2+a\right)}{2\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2 - a*c*log(b*x^2 + a))/b^2
```

giac [**A**] time = 0.38, size = 47, normalized size = 1.62

$$\frac{\frac{ac\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2\left|b\right|}\right)}{b}+\frac{\left(bx^2+a\right)c}{b}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b + (b*x^2 + a)*c/b)/b
```

maple [**A**] time = 0.00, size = 26, normalized size = 0.90

$$\frac{c\,x^2}{2b}-\frac{ac\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*c*x^2/b-1/2*a*c*ln(b*x^2+a)/b^2
```

maxima [**A**] time = 1.08, size = 25, normalized size = 0.86

$$\frac{cx^2}{2\,b}-\frac{ac\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2/b - 1/2*a*c*log(b*x^2 + a)/b^2
```

mupad [**B**] time = 0.04, size = 23, normalized size = 0.79

$$-\frac{c\,\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

```
[Out] -(c*(a*log(a + b*x^2) - b*x^2))/(2*b^2)
```

sympy [A] time = 0.15, size = 22, normalized size = 0.76

$$c\left(-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b))
```

3.128 $\int \frac{x^2\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{cx}{b}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 321, 205}

$$\frac{cx}{b}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*x)/b - (Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx &= c\int \frac{x^2}{a+bx^2}\,dx\\
&= \frac{cx}{b}-\frac{(ac)\int \frac{1}{a+bx^2}\,dx}{b}\\
&= \frac{cx}{b}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$
c\left(\frac{x}{b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] c*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.60, size = 86, normalized size = 2.61

$$
\left[\frac{c\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,cx}{2\,b},\,-\frac{c\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-cx}{b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*c*x)
/b, -(c*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - c*x)/b]
```

giac [**A**] time = 0.27, size = 28, normalized size = 0.85

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

maple [**A**] time = 0.00, size = 29, normalized size = 0.88

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] c*x/b-c*a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [**A**] time = 2.58, size = 28, normalized size = 0.85

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

mupad [**B**] time = 0.04, size = 25, normalized size = 0.76

$$\frac{c\,x}{b}-\frac{\sqrt{a}\;c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (c*x)/b - (a^(1/2)*c*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

sympy [A] time = 0.16, size = 58, normalized size = 1.76

$$c\left(\frac{\sqrt{-\frac{a}{b^{3}}}\log\left(-b\sqrt{-\frac{a}{b^{3}}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^{3}}}\log\left(b\sqrt{-\frac{a}{b^{3}}}+x\right)}{2}+\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/
b**3) + x)/2 + x/b)
```

3.129 $\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=16

$$\frac{c\log\left(a+bx^2\right)}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {21, 260}

$$\frac{c\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx &= c\int \frac{x}{a+bx^2}\,dx\\ &= \frac{c\log\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{c\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (c*Log[a + b*x^2])/(2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(a*c + b*c*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.61, size = 14, normalized size = 0.88

$$\frac{c\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

giac [B] time = 0.30, size = 63, normalized size = 3.94

$$-\frac{1}{2}\,c\left(\frac{\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b}-\frac{a}{\left(bx^2+a\right)b}\right)-\frac{ac}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b)) -
 1/2*a*c/((b*x^2 + a)*b)
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{c\ln\left(b\,x^{2}+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*c*ln(b*x^2+a)/b
```

maxima [A] time = 1.15, size = 14, normalized size = 0.88

$$\frac{c\log\left(bx^{2}+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/b
```

mupad [B] time = 0.03, size = 14, normalized size = 0.88

$$\frac{c\,\ln\left(b\,x^{2}+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (c*log(a + b*x^2))/(2*b)
```

sympy [A] time = 0.12, size = 12, normalized size = 0.75

$$\frac{c\log\left(a+bx^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*log(a + b*x**2)/(2*b)
```

3.130 $\int \frac{ac+bcx^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=25

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 205}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{a+bx^2}\,dx\\ &= \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.77, size = 69, normalized size = 2.76

$$\left[-\frac{\sqrt{-ab}\,c\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,c\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqr
t(a*b)*c*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.33, size = 16, normalized size = 0.64

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A]   time = 0.00, size = 17, normalized size = 0.68

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a)^2,x)
```

```
[Out] c/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A]   time = 2.23, size = 16, normalized size = 0.64

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B]   time = 0.05, size = 17, normalized size = 0.68

$$\frac{c \operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

sympy [B]   time = 0.15, size = 54, normalized size = 2.16

$$c \left(- \frac{\sqrt{- \frac{1}{ab}} \log\left(- a \sqrt{- \frac{1}{ab}} + x\right)}{2} + \frac{\sqrt{- \frac{1}{ab}} \log\left(a \sqrt{- \frac{1}{ab}} + x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

```
[Out] c*(-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt
(-1/(a*b)) + x)/2)
```

3.131 $\int \frac{ac+bcx^2}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=24

$$\frac{c\log(x)}{a}-\frac{c\log\left(a+bx^2\right)}{2a}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {21, 266, 36, 29, 31}

$$\frac{c\log(x)}{a}-\frac{c\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (c*Log[x])/a - (c*Log[a + b*x^2])/(2*a)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{x\left(a+bx^2\right)}\,dx\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{c\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}-\frac{(bc)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2a}\\ &= \frac{c\log(x)}{a}-\frac{c\log\left(a+bx^2\right)}{2a}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$c\left(\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] c*(Log[x]/a - Log[a + b*x^2]/(2*a))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x*(a + b*x^2)^2), x]
```

fricas [A] time = 0.71, size = 21, normalized size = 0.88

$$-\frac{c\log\left(bx^2+a\right)-2\,c\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(c*log(b*x^2 + a) - 2*c*log(x))/a
```

giac [A] time = 0.27, size = 26, normalized size = 1.08

$$\frac{c\log\left(x^2\right)}{2\,a}-\frac{c\log\left(\left|bx^2+a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*c*log(abs(b*x^2 + a))/a
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$\frac{c\ln(x)}{a}-\frac{c\ln\left(b\,x^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a)^2,x)
```

```
[Out] c*ln(x)/a-1/2*c*ln(b*x^2+a)/a
```

maxima [A] time = 1.00, size = 25, normalized size = 1.04

$$-\frac{c\log\left(bx^2+a\right)}{2\,a}+\frac{c\log\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*c*log(b*x^2 + a)/a + 1/2*c*log(x^2)/a
```

mupad [B] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{c\,\left(\ln\left(b\,x^2+a\right)-2\,\ln(x)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x*(a + b*x^2)^2),x)
```

```
[Out] -(c*(log(a + b*x^2) - 2*log(x)))/(2*a)
```

sympy [A] time = 0.22, size = 17, normalized size = 0.71

$$c\left(\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^{2}\right)}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a)**2,x)
```

```
[Out] c*(log(x)/a - log(a/b + x**2)/(2*a))
```

3.132 $\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 325, 205}

$$-\frac{\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx = c\int \frac{1}{x^2\left(a+bx^2\right)}\,dx$$
$$= -\frac{c}{ax} - \frac{(bc)\int \frac{1}{a+bx^2}\,dx}{a}$$
$$= -\frac{c}{ax} - \frac{\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$c\left(-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
[Out] c*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 0.56, size = 86, normalized size = 2.39

$$\left[\frac{cx\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,c}{2\,ax}, -\frac{cx\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+c}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(c*x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*c)
/(a*x), -(c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) + c)/(a*x)]
```

giac [A] time = 0.35, size = 31, normalized size = 0.86

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.89

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -c*b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-c/a/x
```

maxima [A] time = 2.35, size = 31, normalized size = 0.86

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

mupad [B] time = 0.10, size = 28, normalized size = 0.78

$$-\frac{c}{a\,x}-\frac{\sqrt{b}\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] - c/(a*x) - (b^(1/2)*c*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 0.19, size = 66, normalized size = 1.83

$$c\left(\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] c*(sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*
sqrt(-b/a**3)/b + x)/2 - 1/(a*x))
```

3.133 $\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{bc\log\left(a+bx^2\right)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 266, 44}

$$\frac{bc\log\left(a+bx^2\right)}{2a^2}-\frac{bc\log(x)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c/(2*a*x^2) - (b*c*Log[x])/a^2 + (b*c*Log[a + b*x^2])/(2*a^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx &= c\int \frac{1}{x^3\left(a+bx^2\right)}\,dx\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{c}{2ax^2}-\frac{bc\log(x)}{a^2}+\frac{bc\log\left(a+bx^2\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.97

$$
c\left(\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] c*(-1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2), x]
```

fricas [A] time = 0.61, size = 36, normalized size = 0.95

$$
\frac{bcx^2\log\left(bx^2+a\right)-2\,bcx^2\log(x)-ac}{2\,a^2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x^2*log(b*x^2 + a) - 2*b*c*x^2*log(x) - a*c)/(a^2*x^2)
```

giac [A] time = 0.41, size = 47, normalized size = 1.24

$$-\frac{bc\log\left(x^2\right)}{2\,a^2} + \frac{bc\log\left(\left|bx^2+a\right|\right)}{2\,a^2} + \frac{bcx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*b*c*log(x^2)/a^2 + 1/2*b*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 - a*
c)/(a^2*x^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$-\frac{bc\ln(x)}{a^2} + \frac{bc\ln\left(b\,x^2+a\right)}{2a^2} - \frac{c}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2*c/a/x^2-b*c*ln(x)/a^2+1/2*b*c*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.04, size = 36, normalized size = 0.95

$$\frac{bc\log\left(bx^2+a\right)}{2\,a^2} - \frac{bc\log\left(x^2\right)}{2\,a^2} - \frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*log(b*x^2 + a)/a^2 - 1/2*b*c*log(x^2)/a^2 - 1/2*c/(a*x^2)
```

mupad [B] time = 0.11, size = 34, normalized size = 0.89

$$\frac{b\,c\,\ln\left(b\,x^2+a\right)}{2\,a^2} - \frac{c}{2\,a\,x^2} - \frac{b\,c\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (b*c*log(a + b*x^2))/(2*a^2) - c/(2*a*x^2) - (b*c*log(x))/a^2
```

sympy [A] time = 0.28, size = 32, normalized size = 0.84

$$c\left(-\frac{1}{2ax^2} - \frac{b\log\left(x\right)}{a^2} + \frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] c*(-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2))
```

3.134 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=35

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 266, 43}

$$\frac{ac}{2b^2(a+bx^2)}+\frac{c\log(a+bx^2)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (a*c)/(2*b^2*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*b^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx &= c\int \frac{x^3}{\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}c\,\mathrm{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx,x,x^2\right) \\
&= \frac{ac}{2b^2\left(a+bx^2\right)}+\frac{c\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 28, normalized size = 0.80

$$\frac{c\left(\frac{a}{a+bx^2}+\log\left(a+bx^2\right)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (c*(a/(a + b*x^2) + Log[a + b*x^2]))/(2*b^2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3, x]
```

fricas [**A**] time = 0.60, size = 40, normalized size = 1.14

$$\frac{ac+\left(bcx^2+ac\right)\log\left(bx^2+a\right)}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c + (b*c*x^2 + a*c)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.35, size = 32, normalized size = 0.91

$$\frac{c\log\left(\left|bx^2+a\right|\right)}{2\,b^2}+\frac{ac}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
[Out] 1/2*c*log(abs(b*x^2 + a))/b^2 + 1/2*a*c/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{ac}{2\left(b\,x^2+a\right)b^2}+\frac{c\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
[Out] 1/2*a*c/b^2/(b*x^2+a)+1/2*c*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.03, size = 34, normalized size = 0.97

$$\frac{ac}{2\left(b^3x^2+ab^2\right)}+\frac{c\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/2*a*c/(b^3*x^2 + a*b^2) + 1/2*c*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.09, size = 31, normalized size = 0.89

$$\frac{c\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a\,c}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
[Out] (c*log(a + b*x^2))/(2*b^2) + (a*c)/(2*b^2*(a + b*x^2))
```

sympy [A] time = 0.21, size = 31, normalized size = 0.89

$$c\left(\frac{a}{2ab^2+2b^3x^2}+\frac{\log\left(a+bx^2\right)}{2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2))
```

3.135 $\int \frac{x^2\left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=47

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{cx}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 288, 205}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{cx}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -(c*x)/(2*b*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(3/
2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx &= c\int \frac{x^2}{\left(a+bx^2\right)^2}\,dx\\ &= -\frac{cx}{2b\left(a+bx^2\right)}+\frac{c\int \frac{1}{a+bx^2}\,dx}{2b}\\ &= -\frac{cx}{2b\left(a+bx^2\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$c\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] c*(-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3, x]
```

fricas [A] time = 0.66, size = 128, normalized size = 2.72

$$\left[-\frac{2\,abcx+\left(bcx^2+ac\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(ab^3x^2+a^2b^2\right)},-\frac{abcx-\left(bcx^2+ac\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*c*x + (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*c*x - (b*c*x^2 + a*c)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

giac [**A**] time = 0.35, size = 37, normalized size = 0.79

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}-\frac{cx}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*c*x/((b*x^2 + a)*b)
```

maple [**A**] time = 0.01, size = 38, normalized size = 0.81

$$-\frac{cx}{2\left(b\,x^2+a\right)b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c*x/b/(b*x^2+a)+1/2*c/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [**A**] time = 2.39, size = 38, normalized size = 0.81

$$-\frac{cx}{2\left(b^2x^2+ab\right)}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*c*x/(b^2*x^2 + a*b) + 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [**B**] time = 0.04, size = 35, normalized size = 0.74

$$\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{c\,x}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (c*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2)) - (c*x)/(2*b*(a + b*x^2))
```

sympy [B] time = 0.22, size = 80, normalized size = 1.70

$$c\left(-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3))
+ x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4)
```

3.136 $\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{c}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.095, Rules used = {21, 261}

$$-\frac{c}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -c/(2*b*(a + b*x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx &= c\int \frac{x}{\left(a+bx^2\right)^2}\,dx\\ &= -\frac{c}{2b\left(a+bx^2\right)}\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{c}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/2*c/(b*(a + b*x^2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(ac+bcx^2\right)}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(a*c + b*c*x^2))/(a + b*x^2)^3, x]
```

fricas [**A**] time = 0.56, size = 16, normalized size = 0.94

$$-\frac{c}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*c/(b^2*x^2 + a*b)
```

giac [**A**] time = 0.43, size = 15, normalized size = 0.88

$$-\frac{c}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*c/((b*x^2 + a)*b)
```

maple [**A**] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{c}{2\left(b\,x^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c/b/(b*x^2+a)
```

maxima [A] time = 1.00, size = 16, normalized size = 0.94

$$-\frac{c}{2\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*c/(b^2*x^2 + a*b)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$-\frac{c}{2\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

```
[Out] -c/(2*b*(a + b*x^2))
```

sympy [A] time = 0.17, size = 15, normalized size = 0.88

$$-\frac{c}{2ab+2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] -c/(2*a*b + 2*b**2*x**2)
```

3.137 $\int \frac{ac+bcx^2}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=47

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{cx}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {21, 199, 205}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{cx}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b
])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{\left(a+bx^2\right)^2}\,dx\\ &= \frac{cx}{2a\left(a+bx^2\right)}+\frac{c\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{cx}{2a\left(a+bx^2\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$c\left(\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
[Out] c*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(a + b*x^2)^3, x]
```

fricas [A] time = 0.58, size = 128, normalized size = 2.72

$$\left[\frac{2\,abcx-\left(bcx^2+ac\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)},\ \frac{abcx+\left(bcx^2+ac\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*c*x - (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
a)/(b*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*c*x + (b*c*x^2 + a*c)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.38, size = 37, normalized size = 0.79

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}+\frac{cx}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*c*x/((b*x^2 + a)*a)
```

maple [A] time = 0.00, size = 38, normalized size = 0.81

$$\frac{cx}{2\left(b\,x^2+a\right)a}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*c*x/a/(b*x^2+a)+1/2*c/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.40, size = 37, normalized size = 0.79

$$\frac{cx}{2\left(abx^2+a^2\right)}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x/(a*b*x^2 + a^2) + 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$\frac{c\,x}{2\,a\,\left(b\,x^2+a\right)}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(a + b*x^2)^3,x)
```

```
[Out] (c*x)/(2*a*(a + b*x^2)) + (c*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.23, size = 80, normalized size = 1.70

$$c\left(\frac{x}{2a^2+2abx^2}-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

```
[Out] c*(x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b))
+ x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4)
```

3.138 $\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=41

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 266, 44}

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{c}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] c/(2*a*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x\left(a+bx^2\right)^2}\,dx\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}c\,\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{c}{2a\left(a+bx^2\right)}+\frac{c\log(x)}{a^2}-\frac{c\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.83

$$\frac{c\left(\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] (c*(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2]))/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x*(a + b*x^2)^3), x]
```

fricas [A] time = 0.64, size = 54, normalized size = 1.32

$$\frac{ac-\left(bcx^2+ac\right)\log\left(bx^2+a\right)+2\left(bcx^2+ac\right)\log(x)}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a*c - (b*c*x^2 + a*c)*log(b*x^2 + a) + 2*(b*c*x^2 + a*c)*log(x))/(a^2*
b*x^2 + a^3)
```

giac [A] time = 0.37, size = 51, normalized size = 1.24

$$\frac{c\log\left(x^2\right)}{2\,a^2}-\frac{c\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bcx^2+2\,ac}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 + 2*a*c)/
((b*x^2 + a)*a^2)
```

maple [A] time = 0.01, size = 38, normalized size = 0.93

$$\frac{c}{2\left(b\,x^2+a\right)a}+\frac{c\ln(x)}{a^2}-\frac{c\ln\left(b\,x^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x/(b*x^2+a)^3,x)
```

```
[Out] 1/2*c/a/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.01, size = 40, normalized size = 0.98

$$\frac{c}{2\left(abx^2+a^2\right)}-\frac{c\log\left(bx^2+a\right)}{2\,a^2}+\frac{c\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*c/(a*b*x^2 + a^2) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(x^2)/a^2
```

mupad [B] time = 0.10, size = 37, normalized size = 0.90

$$\frac{c}{2\,a\,\left(b\,x^2+a\right)}-\frac{c\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{c\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x*(a + b*x^2)^3),x)
```

```
[Out] c/(2*a*(a + b*x^2)) - (c*log(a + b*x^2))/(2*a^2) + (c*log(x))/a^2
```

sympy **[A]** time = 0.31, size = 36, normalized size = 0.88

$$c\left(\frac{1}{2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x/(b*x**2+a)**3,x)
```

```
[Out] c*(1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2))
```

3.139 $\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=60

$$-\frac{3\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {21, 290, 325, 205}

$$-\frac{3\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (-3*c)/(2*a^2*x) + c/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*c*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(5/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx\\
&= \frac{c}{2ax\left(a+bx^2\right)}+\frac{(3c)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{(3bc)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{3c}{2a^2x}+\frac{c}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.93

$$
c\left(-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{1}{a^2x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] c*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(5/2)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3), x]
```

fricas [A] time = 0.58, size = 144, normalized size = 2.40

$$\left[-\frac{6\,bcx^{2}-3\left(bcx^{3}+acx\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+4\,ac}{4\left(a^{2}bx^{3}+a^{3}x\right)},\,-\frac{3\,bcx^{2}+3\left(bcx^{3}+acx\right)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,ac}{2\left(a^{2}bx^{3}+a^{3}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*c*x^2 - 3*(b*c*x^3 + a*c*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-
b/a) - a)/(b*x^2 + a)) + 4*a*c)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*c*x^2 + 3*(b
*c*x^3 + a*c*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a*c)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.33, size = 50, normalized size = 0.83

$$-\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}-\frac{3\,bcx^{2}+2\,ac}{2\left(bx^{3}+ax\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/((
b*x^3 + a*x)*a^2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.82

$$-\frac{bcx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}-\frac{c}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c/a^2*b*x/(b*x^2+a)-3/2*c/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-
c/a^2/x
```

maxima [A] time = 2.40, size = 52, normalized size = 0.87

$$-\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}-\frac{3\,bcx^{2}+2\,ac}{2\left(a^{2}bx^{3}+a^{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/(a
^2*b*x^3 + a^3*x)
```

mupad [**B**] time = 0.07, size = 48, normalized size = 0.80

$$-\frac{\frac{c}{a}+\frac{3\,b\,c\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] - (c/a + (3*b*c*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*c*atan((b^(1/2)*x)
/a^(1/2)))/(2*a^(5/2))
```

sympy [**A**] time = 0.31, size = 94, normalized size = 1.57

$$c\left(\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] c*(3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a
**3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3))
```

3.140 $\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=53

$$\frac{bc\log\left(a+bx^2\right)}{a^3}-\frac{2bc\log(x)}{a^3}-\frac{bc}{2a^2\left(a+bx^2\right)}-\frac{c}{2a^2x^2}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {21, 266, 44}

$$-\frac{bc}{2a^2\left(a+bx^2\right)}+\frac{bc\log\left(a+bx^2\right)}{a^3}-\frac{2bc\log(x)}{a^3}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -c/(2*a^2*x^2) - (b*c)/(2*a^2*(a + b*x^2)) - (2*b*c*Log[x])/a^3 + (b*c*Log[
a + b*x^2])/a^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx &= c\int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx\\
&= \frac{1}{2}c\,\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}c\,\mathrm{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{c}{2a^2x^2}-\frac{bc}{2a^2\left(a+bx^2\right)}-\frac{2bc\log(x)}{a^3}+\frac{bc\log\left(a+bx^2\right)}{a^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.79

$$
-\frac{c\left(a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)\right)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*(c*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2]))/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3), x]
```

fricas [A] time = 0.53, size = 80, normalized size = 1.51

$$
-\frac{2\,abcx^2+a^2c-2\left(b^2cx^4+abcx^2\right)\log\left(bx^2+a\right)+4\left(b^2cx^4+abcx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*c*x^2 + a^2*c - 2*(b^2*c*x^4 + a*b*c*x^2)*log(b*x^2 + a) + 4*(b
^2*c*x^4 + a*b*c*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.30, size = 56, normalized size = 1.06

$$-\frac{bc\log\left(x^2\right)}{a^3}+\frac{bc\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bcx^2+ac}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -b*c*log(x^2)/a^3 + b*c*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*c*x^2 + a*c)/((b
*x^4 + a*x^2)*a^2)
```

maple [A] time = 0.02, size = 50, normalized size = 0.94

$$-\frac{bc}{2\left(b\,x^2+a\right)a^2}-\frac{2bc\ln(x)}{a^3}+\frac{bc\ln\left(b\,x^2+a\right)}{a^3}-\frac{c}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x)
```

```
[Out] -1/2*c/a^2/x^2-1/2*b*c/a^2/(b*x^2+a)-2*b*c*ln(x)/a^3+b*c*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.05, size = 57, normalized size = 1.08

$$-\frac{2\,bcx^2+ac}{2\left(a^2bx^4+a^3x^2\right)}+\frac{bc\log\left(bx^2+a\right)}{a^3}-\frac{bc\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*c*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + b*c*log(b*x^2 + a)/a^3 - b*c
*log(x^2)/a^3
```

mupad [B] time = 0.10, size = 55, normalized size = 1.04

$$\frac{b\,c\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{c}{2\,a}+\frac{b\,c\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,c\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] (b*c*log(a + b*x^2))/a^3 - (c/(2*a) + (b*c*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b
*c*log(x))/a^3
```

sympy [A] time = 0.38, size = 53, normalized size = 1.00

$$c\left(\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log{(x)}}{a^3}+\frac{b\log{\left(\frac{a}{b}+x^2\right)}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] c*((-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(
a/b + x**2)/a**3)
```

3.141 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^4+a(2bc+ad)x^6+b(bc+2ad)x^8+b^2dx^{10}\right)\,dx\\&=\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*
x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [A] time = 0.50, size = 53, normalized size = 0.96

$$\frac{1}{11}x^{11}db^2+\frac{1}{9}x^9cb^2+\frac{2}{9}x^9dba+\frac{2}{7}x^7cba+\frac{1}{7}x^7da^2+\frac{1}{5}x^5ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/7*x^7*c*b*a + 1/7*x^7*d
*a^2 + 1/5*x^5*c*a^2
```

giac [A] time = 0.30, size = 53, normalized size = 0.96

$$\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\,b^2cx^9+\frac{2}{9}\,abdx^9+\frac{2}{7}\,abcx^7+\frac{1}{7}\,a^2dx^7+\frac{1}{5}\,a^2cx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/7*a*b*c*x^7 + 1/7*a^2*d
*x^7 + 1/5*a^2*c*x^5
```

maple [A] time = 0.00, size = 52, normalized size = 0.95

$$\frac{b^2d\,x^{11}}{11}+\frac{\left(2abd+b^2c\right)x^9}{9}+\frac{a^2c\,x^5}{5}+\frac{\left(a^2d+2abc\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/11*b^2*d*x^11+1/9*(2*a*b*d+b^2*c)*x^9+1/7*(a^2*d+2*a*b*c)*x^7+1/5*a^2*c*x
^5
```

maxima [A] time = 1.04, size = 51, normalized size = 0.93

$$\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\left(b^2c+2\,abd\right)x^9+\frac{1}{5}\,a^2cx^5+\frac{1}{7}\left(2\,abc+a^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7*(2*a*b*c
+ a^2*d)*x^7
```

mupad [B] time = 0.09, size = 51, normalized size = 0.93

$$x^7\left(\frac{d\,a^2}{7}+\frac{2\,b\,c\,a}{7}\right)+x^9\left(\frac{c\,b^2}{9}+\frac{2\,a\,d\,b}{9}\right)+\frac{a^2\,c\,x^5}{5}+\frac{b^2\,d\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^7*((a^2*d)/7 + (2*a*b*c)/7) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (a^2*c*x^5)
/5 + (b^2*d*x^11)/11
```

sympy [A] time = 0.07, size = 56, normalized size = 1.02

$$\frac{a^2cx^5}{5}+\frac{b^2dx^{11}}{11}+x^9\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^7\left(\frac{a^2d}{7}+\frac{2abc}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**5/5 + b**2*d*x**11/11 + x**9*(2*a*b*d/9 + b**2*c/9) + x**7*(a**2*
d/7 + 2*a*b*c/7)
```

3.142 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2(c+dx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2cx+a(2bc+ad)x^2+b(bc+2ad)x^3+b^2dx^4\right)\,dx,x,x^2\right)\\ &= \frac{1}{4}a^2cx^4+\frac{1}{6}a(2bc+ad)x^6+\frac{1}{8}b(bc+2ad)x^8+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{4}a^2cx^4+\frac{1}{8}bx^8(2ad+bc)+\frac{1}{6}ax^6(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d*
x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [A] time = 0.54, size = 53, normalized size = 0.96

$$\frac{1}{10}x^{10}db^2+\frac{1}{8}x^8cb^2+\frac{1}{4}x^8dba+\frac{1}{3}x^6cba+\frac{1}{6}x^6da^2+\frac{1}{4}x^4ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/8*x^8*c*b^2 + 1/4*x^8*d*b*a + 1/3*x^6*c*b*a + 1/6*x^6*d
*a^2 + 1/4*x^4*c*a^2
```

giac [A] time = 0.32, size = 53, normalized size = 0.96

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{8}\,b^2cx^8+\frac{1}{4}\,abdx^8+\frac{1}{3}\,abcx^6+\frac{1}{6}\,a^2dx^6+\frac{1}{4}\,a^2cx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*b^2*c*x^8 + 1/4*a*b*d*x^8 + 1/3*a*b*c*x^6 + 1/6*a^2*d
*x^6 + 1/4*a^2*c*x^4
```

maple [A] time = 0.00, size = 52, normalized size = 0.95

$$\frac{b^2d\,x^{10}}{10}+\frac{\left(2abd+b^2c\right)x^8}{8}+\frac{a^2c\,x^4}{4}+\frac{\left(a^2d+2abc\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/8*(2*a*b*d+b^2*c)*x^8+1/6*(a^2*d+2*a*b*c)*x^6+1/4*a^2*c*x
^4
```

maxima [**A**] time = 0.99, size = 51, normalized size = 0.93

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{8}\left(b^2c+2\,abd\right)x^8+\frac{1}{4}\,a^2cx^4+\frac{1}{6}\left(2\,abc+a^2d\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6*(2*a*b*c
+ a^2*d)*x^6
```

mupad [**B**] time = 0.05, size = 51, normalized size = 0.93

$$x^6\left(\frac{d\,a^2}{6}+\frac{b\,c\,a}{3}\right)+x^8\left(\frac{c\,b^2}{8}+\frac{a\,d\,b}{4}\right)+\frac{a^2\,c\,x^4}{4}+\frac{b^2\,d\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^6*((a^2*d)/6 + (a*b*c)/3) + x^8*((b^2*c)/8 + (a*b*d)/4) + (a^2*c*x^4)/4 +
 (b^2*d*x^10)/10
```

sympy [**A**] time = 0.07, size = 53, normalized size = 0.96

$$\frac{a^2cx^4}{4}+\frac{b^2dx^{10}}{10}+x^8\left(\frac{abd}{4}+\frac{b^2c}{8}\right)+x^6\left(\frac{a^2d}{6}+\frac{abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**4/4 + b**2*d*x**10/10 + x**8*(a*b*d/4 + b**2*c/8) + x**6*(a**2*d/
6 + a*b*c/3)
```

3.143 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2cx^2+a(2bc+ad)x^4+b(bc+2ad)x^6+b^2dx^8\right)\,dx\\ &= \frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*
x^9)/9
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [A]   time = 0.54, size = 53, normalized size = 0.96

$$\frac{1}{9}x^9db^2+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7dba+\frac{2}{5}x^5cba+\frac{1}{5}x^5da^2+\frac{1}{3}x^3ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 2/5*x^5*c*b*a + 1/5*x^5*d*a
^2 + 1/3*x^3*c*a^2
```

giac [A]   time = 0.43, size = 53, normalized size = 0.96

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{2}{5}\,abcx^5+\frac{1}{5}\,a^2dx^5+\frac{1}{3}\,a^2cx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 2/5*a*b*c*x^5 + 1/5*a^2*d*x
^5 + 1/3*a^2*c*x^3
```

maple [A]   time = 0.00, size = 52, normalized size = 0.95

$$\frac{b^2d\,x^9}{9}+\frac{\left(2abd+b^2c\right)x^7}{7}+\frac{a^2c\,x^3}{3}+\frac{\left(a^2d+2abc\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/9*b^2*d*x^9+1/7*(2*a*b*d+b^2*c)*x^7+1/5*(a^2*d+2*a*b*c)*x^5+1/3*a^2*c*x^3
```

maxima [A] time = 0.96, size = 51, normalized size = 0.93

$$\frac{1}{9}\,b^2dx^9+\frac{1}{7}\left(b^2c+2\,abd\right)x^7+\frac{1}{3}\,a^2cx^3+\frac{1}{5}\left(2\,abc+a^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(2*a*b*c +
a^2*d)*x^5
```

mupad [B] time = 0.09, size = 51, normalized size = 0.93

$$x^5\left(\frac{d\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{a^2\,c\,x^3}{3}+\frac{b^2\,d\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (a^2*c*x^3)
/3 + (b^2*d*x^9)/9
```

sympy [A] time = 0.07, size = 56, normalized size = 1.02

$$\frac{a^2cx^3}{3}+\frac{b^2dx^9}{9}+x^7\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**3/3 + b**2*d*x**9/9 + x**7*(2*a*b*d/7 + b**2*c/7) + x**5*(a**2*d/
5 + 2*a*b*c/5)
```

3.144 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^2\right)^3(bc-ad)}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}$$

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.111, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^3(bc-ad)}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)*(a + b*x^2)^3)/(6*b^2) + (d*(a + b*x^2)^4)/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(c+dx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx,x,x^2\right)\\&=\frac{(bc-ad)\left(a+bx^2\right)^3}{6b^2}+\frac{d\left(a+bx^2\right)^4}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{1}{24}x^2\left(12a^2c+4bx^4(2ad+bc)+6ax^2(ad+2bc)+3b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (x^2*(12*a^2*c + 6*a*(2*b*c + a*d)*x^2 + 4*b*(b*c + 2*a*d)*x^4 + 3*b^2*d*x^
6))/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [A] time = 0.44, size = 53, normalized size = 1.26

$$\frac{1}{8}x^8db^2+\frac{1}{6}x^6cb^2+\frac{1}{3}x^6dba+\frac{1}{2}x^4cba+\frac{1}{4}x^4da^2+\frac{1}{2}x^2ca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d*b^2 + 1/6*x^6*c*b^2 + 1/3*x^6*d*b*a + 1/2*x^4*c*b*a + 1/4*x^4*d*a
^2 + 1/2*x^2*c*a^2
```

giac [A] time = 0.43, size = 53, normalized size = 1.26

$$\frac{1}{8}b^2dx^8+\frac{1}{6}b^2cx^6+\frac{1}{3}abdx^6+\frac{1}{2}abcx^4+\frac{1}{4}a^2dx^4+\frac{1}{2}a^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*b^2*c*x^6 + 1/3*a*b*d*x^6 + 1/2*a*b*c*x^4 + 1/4*a^2*d*x
^4 + 1/2*a^2*c*x^2
```

maple [A] time = 0.00, size = 52, normalized size = 1.24

$$\frac{b^2d\,x^8}{8}+\frac{\left(2abd+b^2c\right)x^6}{6}+\frac{a^2c\,x^2}{2}+\frac{\left(a^2d+2abc\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/8*b^2*d*x^8+1/6*(2*a*b*d+b^2*c)*x^6+1/4*(a^2*d+2*a*b*c)*x^4+1/2*a^2*c*x^2
```

maxima [A] time = 1.07, size = 51, normalized size = 1.21

$$\frac{1}{8}\,b^2dx^8+\frac{1}{6}\left(b^2c+2\,abd\right)x^6+\frac{1}{2}\,a^2cx^2+\frac{1}{4}\left(2\,abc+a^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(2*a*b*c +
a^2*d)*x^4
```

mupad [B] time = 0.04, size = 51, normalized size = 1.21

$$x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^6\left(\frac{c\,b^2}{6}+\frac{a\,d\,b}{3}\right)+\frac{a^2\,c\,x^2}{2}+\frac{b^2\,d\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^4*((a^2*d)/4 + (a*b*c)/2) + x^6*((b^2*c)/6 + (a*b*d)/3) + (a^2*c*x^2)/2 +
 (b^2*d*x^8)/8
```

sympy [A] time = 0.07, size = 53, normalized size = 1.26

$$\frac{a^2cx^2}{2}+\frac{b^2dx^8}{8}+x^6\left(\frac{abd}{3}+\frac{b^2c}{6}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x**2/2 + b**2*d*x**8/8 + x**6*(a*b*d/3 + b**2*c/6) + x**4*(a**2*d/4
+ a*b*c/2)
```

3.145 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx &= \int \left(a^2c+a(2bc+ad)x^2+b(bc+2ad)x^4+b^2dx^6\right) \, dx\\ &= a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [A] time = 0.53, size = 50, normalized size = 1.00

$$\frac{1}{7}x^7db^2 + \frac{1}{5}x^5cb^2 + \frac{2}{5}x^5dba + \frac{2}{3}x^3cba + \frac{1}{3}x^3da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b^2 + 1/5*x^5*c*b^2 + 2/5*x^5*d*b*a + 2/3*x^3*c*b*a + 1/3*x^3*d*a
^2 + x*c*a^2
```

giac [A] time = 0.24, size = 50, normalized size = 1.00

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abdx^5 + \frac{2}{3}\,abcx^3 + \frac{1}{3}\,a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

maple [A] time = 0.00, size = 49, normalized size = 0.98

$$\frac{b^2d\,x^7}{7} + \frac{\left(2abd+b^2c\right)x^5}{5} + a^2cx + \frac{\left(a^2d+2abc\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

maxima [A] time = 1.05, size = 48, normalized size = 0.96

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\left(b^2c + 2\,abd\right)x^5 + a^2cx + \frac{1}{3}\left(2\,abc + a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^3\left(\frac{d\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^5\left(\frac{c\,b^2}{5}+\frac{2\,a\,d\,b}{5}\right)+\frac{b^2\,d\,x^7}{7}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7)
/7 + a^2*c*x
```

sympy [A] time = 0.07, size = 53, normalized size = 1.06

$$a^2cx+\frac{b^2dx^7}{7}+x^5\left(\frac{2abd}{5}+\frac{b^2c}{5}\right)+x^3\left(\frac{a^2d}{3}+\frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

3.146 $\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx$

Optimal. Leaf size=43

$$a^2c\log(x)+abcx^2+\frac{d\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2cx^4$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 80, 43}

$$a^2c\log(x)+abcx^2+\frac{d\left(a+bx^2\right)^3}{6b}+\frac{1}{4}b^2cx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] a*b*c*x^2 + (b^2*c*x^4)/4 + (d*(a + b*x^2)^3)/(6*b) + a^2*c*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)}{x}\,dx,x,x^2\right)\\ &= \frac{d\left(a+bx^2\right)^3}{6b}+\frac{1}{2}c\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx,x,x^2\right)\\ &= \frac{d\left(a+bx^2\right)^3}{6b}+\frac{1}{2}c\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx,x,x^2\right)\\ &= abcx^2+\frac{1}{4}b^2cx^4+\frac{d\left(a+bx^2\right)^3}{6b}+a^2c\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.19

$$a^2c\log(x)+\frac{1}{4}bx^4(2ad+bc)+\frac{1}{2}ax^2(ad+2bc)+\frac{1}{6}b^2dx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] (a*(2*b*c + a*d)*x^2)/2 + (b*(b*c + 2*a*d)*x^4)/4 + (b^2*d*x^6)/6 + a^2*c*L
og[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x, x]
```

fricas [A] time = 0.57, size = 49, normalized size = 1.14

$$\frac{1}{6}\,b^2dx^6+\frac{1}{4}\left(b^2c+2\,abd\right)x^4+a^2c\log(x)+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + a^2*c*log(x) + 1/2*(2*a*b*c + a
^2*d)*x^2
```

giac [A] time = 0.35, size = 53, normalized size = 1.23

$$\frac{1}{6}\,b^2dx^6+\frac{1}{4}\,b^2cx^4+\frac{1}{2}\,abdx^4+abcx^2+\frac{1}{2}\,a^2dx^2+\frac{1}{2}\,a^2c\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*b^2*c*x^4 + 1/2*a*b*d*x^4 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 1/2*a^2*c*log(x^2)
```

maple [A] time = 0.00, size = 51, normalized size = 1.19

$$\frac{b^2d\,x^6}{6}+\frac{abd\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2d\,x^2}{2}+abc\,x^2+a^2c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x,x)
```

```
[Out] 1/6*b^2*d*x^6+1/2*x^4*a*b*d+1/4*b^2*c*x^4+1/2*x^2*a^2*d+a*b*c*x^2+a^2*c*ln(
x)
```

maxima [A] time = 1.05, size = 52, normalized size = 1.21

$$\frac{1}{6}\,b^2dx^6+\frac{1}{4}\left(b^2c+2\,abd\right)x^4+\frac{1}{2}\,a^2c\log\left(x^2\right)+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + 1/2*a^2*c*log(x^2) + 1/2*(2*a*b
*c + a^2*d)*x^2
```

mupad [B] time = 0.04, size = 48, normalized size = 1.12

$$x^2\left(\frac{d\,a^2}{2}+b\,c\,a\right)+x^4\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{2}\right)+\frac{b^2\,d\,x^6}{6}+a^2\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x,x)
```

```
[Out] x^2*((a^2*d)/2 + a*b*c) + x^4*((b^2*c)/4 + (a*b*d)/2) + (b^2*d*x^6)/6 + a^2
*c*log(x)
```

sympy [A] time = 0.14, size = 49, normalized size = 1.14

$$a^2c\log{(x)} + \frac{b^2dx^6}{6} + x^4\left(\frac{abd}{2} + \frac{b^2c}{4}\right) + x^2\left(\frac{a^2d}{2} + abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x,x)
```

```
[Out] a**2*c*log(x) + b**2*d*x**6/6 + x**4*(a*b*d/2 + b**2*c/4) + x**2*(a**2*d/2
+ a*b*c)
```

3.147 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^2}\,dx &= \int \left(a(2bc+ad)+\frac{a^2c}{x^2}+b(bc+2ad)x^2+b^2dx^4\right)dx\\ &= -\frac{a^2c}{x}+a(2bc+ad)x+\frac{1}{3}b(bc+2ad)x^3+\frac{1}{5}b^2dx^5\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] -((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^2, x]
```

fricas [A] time = 0.60, size = 53, normalized size = 1.10

$$\frac{3\,b^2dx^6+5\left(b^2c+2\,abd\right)x^4-15\,a^2c+15\left(2\,abc+a^2d\right)x^2}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d*x^6 + 5*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 15*(2*a*b*c + a^2*
d)*x^2)/x
```

giac [A] time = 0.33, size = 48, normalized size = 1.00

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abdx^3+2\,abcx+a^2dx-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + 2*a*b*c*x + a^2*d*x - a^2*c
/x
```

maple [A] time = 0.00, size = 49, normalized size = 1.02

$$\frac{b^2d\,x^5}{5}+\frac{2abd\,x^3}{3}+\frac{b^2c\,x^3}{3}+a^2dx+2abcx-\frac{a^2c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^2,x)
```

```
[Out] 1/5*b^2*d*x^5+2/3*x^3*a*b*d+1/3*x^3*b^2*c+a^2*d*x+2*a*b*c*x-a^2*c/x
```

maxima [A] time = 1.07, size = 48, normalized size = 1.00

$$\frac{1}{5}\,b^2dx^5+\frac{1}{3}\left(b^2c+2\,abd\right)x^3-\frac{a^2c}{x}+\left(2\,abc+a^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d*x^5 + 1/3*(b^2*c + 2*a*b*d)*x^3 - a^2*c/x + (2*a*b*c + a^2*d)*x
```

mupad [B] time = 0.05, size = 48, normalized size = 1.00

$$x\,\left(d\,a^2+2\,b\,c\,a\right)+x^3\,\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)-\frac{a^2\,c}{x}+\frac{b^2\,d\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^2,x)
```

```
[Out] x*(a^2*d + 2*a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) - (a^2*c)/x + (b^2*d*x^
5)/5
```

sympy [A] time = 0.14, size = 48, normalized size = 1.00

$$-\frac{a^2c}{x}+\frac{b^2dx^5}{5}+x^3\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x\left(a^2d+2abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**2,x)
```

```
[Out] -a**2*c/x + b**2*d*x**5/5 + x**3*(2*a*b*d/3 + b**2*c/3) + x*(a**2*d + 2*a*b
*c)
```

3.148 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^3}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2c}{2x^2}+\frac{1}{2}bx^2(2ad+bc)+a\log(x)(ad+2bc)+\frac{1}{4}b^2dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] -(a^2*c)/(2*x^2) + (b*(b*c + 2*a*d)*x^2)/2 + (b^2*d*x^4)/4 + a*(2*b*c + a*d
)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(c+dx)}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(b(bc+2ad)+\frac{a^2c}{x^2}+\frac{a(2bc+ad)}{x}+b^2dx\right)dx,x,x^2\right)\\ &= -\frac{a^2c}{2x^2}+\frac{1}{2}b(bc+2ad)x^2+\frac{1}{4}b^2dx^4+a(2bc+ad)\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2a^2c}{x^2}+2bx^2(2ad+bc)+4a\log(x)(ad+2bc)+b^2dx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] ((-2*a^2*c)/x^2 + 2*b*(b*c + 2*a*d)*x^2 + b^2*d*x^4 + 4*a*(2*b*c + a*d)*Log
[x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^3, x]
```

fricas [A] time = 0.57, size = 54, normalized size = 1.06

$$\frac{b^2dx^6+2\left(b^2c+2\,abd\right)x^4+4\left(2\,abc+a^2d\right)x^2\log(x)-2\,a^2c}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^6 + 2*(b^2*c + 2*a*b*d)*x^4 + 4*(2*a*b*c + a^2*d)*x^2*log(x) -
 2*a^2*c)/x^2
```

giac **[A]** time = 0.40, size = 70, normalized size = 1.37

$$\frac{1}{4}\,b^2dx^4 + \frac{1}{2}\,b^2cx^2 + abdx^2 + \frac{1}{2}\left(2\,abc + a^2d\right)\log\left(x^2\right) - \frac{2\,abcx^2 + a^2dx^2 + a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*b^2*c*x^2 + a*b*d*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
- 1/2*(2*a*b*c*x^2 + a^2*d*x^2 + a^2*c)/x^2
```

maple **[A]** time = 0.01, size = 50, normalized size = 0.98

$$\frac{b^2d\,x^4}{4} + abd\,x^2 + \frac{b^2c\,x^2}{2} + a^2d\ln(x) + 2abc\ln(x) - \frac{a^2c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^3,x)
```

```
[Out] 1/4*b^2*d*x^4+x^2*a*b*d+1/2*b^2*c*x^2-1/2*a^2*c/x^2+ln(x)*a^2*d+2*ln(x)*a*b
*c
```

maxima **[A]** time = 1.07, size = 52, normalized size = 1.02

$$\frac{1}{4}\,b^2dx^4 + \frac{1}{2}\left(b^2c + 2\,abd\right)x^2 + \frac{1}{2}\left(2\,abc + a^2d\right)\log\left(x^2\right) - \frac{a^2c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + 1/2*(b^2*c + 2*a*b*d)*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
- 1/2*a^2*c/x^2
```

mupad **[B]** time = 0.09, size = 48, normalized size = 0.94

$$x^2\left(\frac{c\,b^2}{2} + a\,d\,b\right) + \ln(x)\,\left(d\,a^2 + 2\,b\,c\,a\right) - \frac{a^2\,c}{2\,x^2} + \frac{b^2\,d\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^3,x)
```

```
[Out] x^2*((b^2*c)/2 + a*b*d) + log(x)*(a^2*d + 2*a*b*c) - (a^2*c)/(2*x^2) + (b^2
*d*x^4)/4
```

sympy [A] time = 0.23, size = 48, normalized size = 0.94

$$-\frac{a^2c}{2x^2} + a\left(ad + 2bc\right)\log\left(x\right) + \frac{b^2dx^4}{4} + x^2\left(abd + \frac{b^2c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**3,x)
```

```
[Out] -a**2*c/(2*x**2) + a*(a*d + 2*b*c)*log(x) + b**2*d*x**4/4 + x**2*(a*b*d + b
**2*c/2)
```

3.149 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^4}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -(a^2*c)/(3*x^3) - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^4}\,dx &= \int \left(b(bc+2ad)+\frac{a^2c}{x^4}+\frac{a(2bc+ad)}{x^2}+b^2dx^2\right)dx\\ &= -\frac{a^2c}{3x^3}-\frac{a(2bc+ad)}{x}+b(bc+2ad)x+\frac{1}{3}b^2dx^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.04

$$\frac{a^2(-d)-2abc}{x}-\frac{a^2c}{3x^3}+bx(2ad+bc)+\frac{1}{3}b^2dx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] -1/3*(a^2*c)/x^3 + (-2*a*b*c - a^2*d)/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^4, x]
```

fricas [A] time = 0.49, size = 52, normalized size = 1.08

$$\frac{b^2dx^6+3\left(b^2c+2\,abd\right)x^4-a^2c-3\left(2\,abc+a^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d*x^6 + 3*(b^2*c + 2*a*b*d)*x^4 - a^2*c - 3*(2*a*b*c + a^2*d)*x^2)
/x^3
```

giac [A] time = 0.42, size = 50, normalized size = 1.04

$$\frac{1}{3}\,b^2dx^3+b^2cx+2\,abdx-\frac{6\,abcx^2+3\,a^2dx^2+a^2c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^2*d*x^3 + b^2*c*x + 2*a*b*d*x - 1/3*(6*a*b*c*x^2 + 3*a^2*d*x^2 + a^2*
c)/x^3
```

maple [A] time = 0.00, size = 46, normalized size = 0.96

$$\frac{b^2d\,x^3}{3}+2abdx+b^2cx-\frac{a^2c}{3x^3}-\frac{(ad+2bc)\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^4,x)
```

```
[Out] 1/3*b^2*d*x^3+2*a*b*d*x+b^2*c*x-a*(a*d+2*b*c)/x-1/3*a^2*c/x^3
```

maxima [A] time = 1.04, size = 50, normalized size = 1.04

$$\frac{1}{3}\,b^2dx^3 + \left(b^2c + 2\,abd\right)x - \frac{a^2c + 3\left(2\,abc + a^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*d*x^3 + (b^2*c + 2*a*b*d)*x - 1/3*(a^2*c + 3*(2*a*b*c + a^2*d)*x^2)
/x^3
```

mupad [B] time = 0.05, size = 50, normalized size = 1.04

$$x\,\left(c\,b^2 + 2\,a\,d\,b\right) - \frac{\frac{a^2\,c}{3} + x^2\,\left(d\,a^2 + 2\,b\,c\,a\right)}{x^3} + \frac{b^2\,d\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^4,x)
```

```
[Out] x*(b^2*c + 2*a*b*d) - ((a^2*c)/3 + x^2*(a^2*d + 2*a*b*c))/x^3 + (b^2*d*x^3)
/3
```

sympy [A] time = 0.25, size = 51, normalized size = 1.06

$$\frac{b^2dx^3}{3} + x\left(2abd + b^2c\right) + \frac{-a^2c + x^2\left(-3a^2d - 6abc\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**4,x)
```

```
[Out] b**2*d*x**3/3 + x*(2*a*b*d + b**2*c) + (-a**2*c + x**2*(-3*a**2*d - 6*a*b*c
))/(3*x**3)
```

3.150 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^4+2ac(bc+ad)x^6+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{10}+b\right.\\ &= \frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{11}bd(bc+ad)x^{11}+\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{5}a^2c^2x^5+\frac{2}{11}bdx^{11}(ad+bc)+\frac{2}{7}acx^7(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [A] time = 0.54, size = 94, normalized size = 1.08

$$\frac{1}{13}x^{13}d^2b^2 + \frac{2}{11}x^{11}dcb^2 + \frac{2}{11}x^{11}d^2ba + \frac{1}{9}x^9c^2b^2 + \frac{4}{9}x^9dcba + \frac{1}{9}x^9d^2a^2 + \frac{2}{7}x^7c^2ba + \frac{2}{7}x^7dca^2 + \frac{1}{5}x^5c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 2/11*x^11*d*c*b^2 + 2/11*x^11*d^2*b*a + 1/9*x^9*c^2*b^2
 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/7*x^7*c^2*b*a + 2/7*x^7*d*c*a^2 +
1/5*x^5*c^2*a^2
```

giac [A] time = 0.43, size = 94, normalized size = 1.08

$$\frac{1}{13}\,b^2d^2x^{13} + \frac{2}{11}\,b^2cdx^{11} + \frac{2}{11}\,abd^2x^{11} + \frac{1}{9}\,b^2c^2x^9 + \frac{4}{9}\,abcdx^9 + \frac{1}{9}\,a^2d^2x^9 + \frac{2}{7}\,abc^2x^7 + \frac{2}{7}\,a^2cdx^7 + \frac{1}{5}\,a^2c^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*b^2*c*d*x^11 + 2/11*a*b*d^2*x^11 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/7*a*b*c^2*x^7 + 2/7*a^2*c*d*x^7 +
1/5*a^2*c^2*x^5
```

maple [A] time = 0.00, size = 90, normalized size = 1.03

$$\frac{b^2d^2x^{13}}{13} + \frac{\left(2ab\,d^2 + 2b^2cd\right)x^{11}}{11} + \frac{a^2c^2x^5}{5} + \frac{\left(a^2d^2 + 4abcd + b^2c^2\right)x^9}{9} + \frac{\left(2a^2cd + 2ab\,c^2\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/11*(2*a*b*d^2+2*b^2*c*d)*x^11+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/7*(2*a^2*c*d+2*a*b*c^2)*x^7+1/5*a^2*c^2*x^5
```

maxima [A] time = 1.07, size = 85, normalized size = 0.98

$$\frac{1}{13}\,b^{2}d^{2}x^{13}+\frac{2}{11}\left(b^{2}cd+abd^{2}\right)x^{11}+\frac{1}{9}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{9}+\frac{1}{5}\,a^{2}c^{2}x^{5}+\frac{2}{7}\left(abc^{2}+a^{2}cd\right)x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2*c*d)*x^7
```

mupad [B] time = 0.05, size = 78, normalized size = 0.90

$$x^{9}\left(\frac{a^{2}\,d^{2}}{9}+\frac{4\,a\,b\,c\,d}{9}+\frac{b^{2}\,c^{2}}{9}\right)+\frac{a^{2}\,c^{2}\,x^{5}}{5}+\frac{b^{2}\,d^{2}\,x^{13}}{13}+\frac{2\,a\,c\,x^{7}\,\left(a\,d+b\,c\right)}{7}+\frac{2\,b\,d\,x^{11}\,\left(a\,d+b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + (a^2*c^2*x^5)/5 + (b^2*d^
2*x^13)/13 + (2*a*c*x^7*(a*d + b*c))/7 + (2*b*d*x^11*(a*d + b*c))/11
```

sympy [A] time = 0.09, size = 100, normalized size = 1.15

$$\frac{a^{2}c^{2}x^{5}}{5}+\frac{b^{2}d^{2}x^{13}}{13}+x^{11}\left(\frac{2abd^{2}}{11}+\frac{2b^{2}cd}{11}\right)+x^{9}\left(\frac{a^{2}d^{2}}{9}+\frac{4abcd}{9}+\frac{b^{2}c^{2}}{9}\right)+x^{7}\left(\frac{2a^{2}cd}{7}+\frac{2abc^{2}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**5/5 + b**2*d**2*x**13/13 + x**11*(2*a*b*d**2/11 + 2*b**2*c*d/1
1) + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**7*(2*a**2*c*d/7 +
2*a*b*c**2/7)
```

3.151 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

Rubi [**A**] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{1}{8}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{4}a^2c^2x^4+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{3}acx^6(ad+bc)+\frac{1}{12}b^2d^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^4)/4 + (a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x^8)/8 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/12
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^2(c+dx)^2\,dx,x,x^2\right)$$

$$=\frac{1}{2}\operatorname{Subst}\Big(\int\Big(a^2c^2x+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^3+2bd(bc+ad)$$

$$=\frac{1}{4}a^2c^2x^4+\frac{1}{3}ac(bc+ad)x^6+\frac{1}{8}\left(b^2c^2+4abcd+a^2d^2\right)x^8+\frac{1}{5}bd(bc+ad)x^{10}+$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.93

$$\frac{1}{120}x^4\left(15x^4\left(a^2d^2+4abcd+b^2c^2\right)+30a^2c^2+24bdx^6(ad+bc)+40acx^2(ad+bc)+10b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^4*(30*a^2*c^2 + 40*a*c*(b*c + a*d)*x^2 + 15*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^4 + 24*b*d*(b*c + a*d)*x^6 + 10*b^2*d^2*x^8))/120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [A] time = 0.47, size = 94, normalized size = 1.08

$$\frac{1}{12}x^{12}d^2b^2+\frac{1}{5}x^{10}dcb^2+\frac{1}{5}x^{10}d^2ba+\frac{1}{8}x^8c^2b^2+\frac{1}{2}x^8dcba+\frac{1}{8}x^8d^2a^2+\frac{1}{3}x^6c^2ba+\frac{1}{3}x^6dca^2+\frac{1}{4}x^4c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/8*x^8*c^2*b^2 +
 1/2*x^8*d*c*b*a + 1/8*x^8*d^2*a^2 + 1/3*x^6*c^2*b*a + 1/3*x^6*d*c*a^2 + 1/
4*x^4*c^2*a^2
```

giac [A] time = 0.41, size = 94, normalized size = 1.08

$$\frac{1}{12}\,b^2d^2x^{12}+\frac{1}{5}\,b^2cdx^{10}+\frac{1}{5}\,abd^2x^{10}+\frac{1}{8}\,b^2c^2x^8+\frac{1}{2}\,abcdx^8+\frac{1}{8}\,a^2d^2x^8+\frac{1}{3}\,abc^2x^6+\frac{1}{3}\,a^2cdx^6+\frac{1}{4}\,a^2c^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/8*b^2*c^2*x^8 +
 1/2*a*b*c*d*x^8 + 1/8*a^2*d^2*x^8 + 1/3*a*b*c^2*x^6 + 1/3*a^2*c*d*x^6 + 1/
4*a^2*c^2*x^4
```

maple [A] time = 0.00, size = 90, normalized size = 1.03

$$\frac{b^{2}d^{2}x^{12}}{12}+\frac{\left(2ab\,d^{2}+2b^{2}cd\right)x^{10}}{10}+\frac{a^{2}c^{2}x^{4}}{4}+\frac{\left(a^{2}d^{2}+4abcd+b^{2}c^{2}\right)x^{8}}{8}+\frac{\left(2a^{2}cd+2ab\,c^{2}\right)x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/12*b^2*d^2*x^12+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/8*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^8+1/6*(2*a^2*c*d+2*a*b*c^2)*x^6+1/4*a^2*c^2*x^4
```

maxima [A] time = 1.15, size = 85, normalized size = 0.98

$$\frac{1}{12}\,b^{2}d^{2}x^{12}+\frac{1}{5}\left(b^{2}cd+abd^{2}\right)x^{10}+\frac{1}{8}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{8}+\frac{1}{4}\,a^{2}c^{2}x^{4}+\frac{1}{3}\left(abc^{2}+a^{2}cd\right)x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*c*d)*x^6
```

mupad [B] time = 0.03, size = 78, normalized size = 0.90

$$x^{8}\left(\frac{a^{2}\,d^{2}}{8}+\frac{a\,b\,c\,d}{2}+\frac{b^{2}\,c^{2}}{8}\right)+\frac{a^{2}\,c^{2}\,x^{4}}{4}+\frac{b^{2}\,d^{2}\,x^{12}}{12}+\frac{a\,c\,x^{6}\,\left(a\,d+b\,c\right)}{3}+\frac{b\,d\,x^{10}\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^8*((a^2*d^2)/8 + (b^2*c^2)/8 + (a*b*c*d)/2) + (a^2*c^2*x^4)/4 + (b^2*d^2*
x^12)/12 + (a*c*x^6*(a*d + b*c))/3 + (b*d*x^10*(a*d + b*c))/5
```

sympy [A] time = 0.09, size = 92, normalized size = 1.06

$$\frac{a^{2}c^{2}x^{4}}{4}+\frac{b^{2}d^{2}x^{12}}{12}+x^{10}\left(\frac{abd^{2}}{5}+\frac{b^{2}cd}{5}\right)+x^{8}\left(\frac{a^{2}d^{2}}{8}+\frac{abcd}{2}+\frac{b^{2}c^{2}}{8}\right)+x^{6}\left(\frac{a^{2}cd}{3}+\frac{abc^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**4/4 + b**2*d**2*x**12/12 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x
**8*(a**2*d**2/8 + a*b*c*d/2 + b**2*c**2/8) + x**6*(a**2*c*d/3 + a*b*c**2/3
)
```

3.152 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=87

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^8+b^2\right.\\ &= \frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\end{aligned}$$

Mathematica [A] time = 0.02, size = 87, normalized size = 1.00

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [A] time = 0.50, size = 94, normalized size = 1.08

$$\frac{1}{11}x^{11}d^2b^2+\frac{2}{9}x^9dcb^2+\frac{2}{9}x^9d^2ba+\frac{1}{7}x^7c^2b^2+\frac{4}{7}x^7dcba+\frac{1}{7}x^7d^2a^2+\frac{2}{5}x^5c^2ba+\frac{2}{5}x^5dca^2+\frac{1}{3}x^3c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^2*b^2 + 2/9*x^9*d*c*b^2 + 2/9*x^9*d^2*b*a + 1/7*x^7*c^2*b^2 + 4
/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/5*x^5*d*c*a^2 + 1/3*
x^3*c^2*a^2
```

giac [A] time = 0.26, size = 94, normalized size = 1.08

$$\frac{1}{11}\,b^2d^2x^{11}+\frac{2}{9}\,b^2cdx^9+\frac{2}{9}\,abd^2x^9+\frac{1}{7}\,b^2c^2x^7+\frac{4}{7}\,abcdx^7+\frac{1}{7}\,a^2d^2x^7+\frac{2}{5}\,abc^2x^5+\frac{2}{5}\,a^2cdx^5+\frac{1}{3}\,a^2c^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*b^2*c*d*x^9 + 2/9*a*b*d^2*x^9 + 1/7*b^2*c^2*x^7 + 4
/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 + 1/3*
a^2*c^2*x^3
```

maple [A] time = 0.00, size = 90, normalized size = 1.03

$$\frac{b^2d^2x^{11}}{11}+\frac{\left(2ab\,d^2+2b^2cd\right)x^9}{9}+\frac{a^2c^2x^3}{3}+\frac{\left(a^2d^2+4abcd+b^2c^2\right)x^7}{7}+\frac{\left(2a^2cd+2ab\,c^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/11*b^2*d^2*x^11+1/9*(2*a*b*d^2+2*b^2*c*d)*x^9+1/7*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^7+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+1/3*a^2*c^2*x^3
```

maxima [A] time = 1.07, size = 85, normalized size = 0.98

$$\frac{1}{11}\,b^2d^2x^{11}+\frac{2}{9}\left(b^2cd+abd^2\right)x^9+\frac{1}{7}\left(b^2c^2+4\,abcd+a^2d^2\right)x^7+\frac{1}{3}\,a^2c^2x^3+\frac{2}{5}\left(abc^2+a^2cd\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c*d)*x^5
```

mupad [B] time = 0.03, size = 78, normalized size = 0.90

$$x^7\left(\frac{a^2\,d^2}{7}+\frac{4\,a\,b\,c\,d}{7}+\frac{b^2\,c^2}{7}\right)+\frac{a^2\,c^2\,x^3}{3}+\frac{b^2\,d^2\,x^{11}}{11}+\frac{2\,a\,c\,x^5\,\left(a\,d+b\,c\right)}{5}+\frac{2\,b\,d\,x^9\,\left(a\,d+b\,c\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + (a^2*c^2*x^3)/3 + (b^2*d^
2*x^11)/11 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^9*(a*d + b*c))/9
```

sympy [A] time = 0.08, size = 100, normalized size = 1.15

$$\frac{a^2c^2x^3}{3}+\frac{b^2d^2x^{11}}{11}+x^9\left(\frac{2abd^2}{9}+\frac{2b^2cd}{9}\right)+x^7\left(\frac{a^2d^2}{7}+\frac{4abcd}{7}+\frac{b^2c^2}{7}\right)+x^5\left(\frac{2a^2cd}{5}+\frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**3/3 + b**2*d**2*x**11/11 + x**9*(2*a*b*d**2/9 + 2*b**2*c*d/9)
+ x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**5*(2*a**2*c*d/5 + 2*a
*b*c**2/5)
```

3.153 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=71

$$\frac{d\left(a+bx^2\right)^4(bc-ad)}{4b^3}+\frac{\left(a+bx^2\right)^3(bc-ad)^2}{6b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}$$

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{d\left(a+bx^2\right)^4(bc-ad)}{4b^3}+\frac{\left(a+bx^2\right)^3(bc-ad)^2}{6b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x^2)^3)/(6*b^3) + (d*(b*c - a*d)*(a + b*x^2)^4)/(4*b^
3) + (d^2*(a + b*x^2)^5)/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(c+dx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx,x,x\right.\\ &= \frac{(bc-ad)^2\left(a+bx^2\right)^3}{6b^3}+\frac{d(bc-ad)\left(a+bx^2\right)^4}{4b^3}+\frac{d^2\left(a+bx^2\right)^5}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 1.14

$$\frac{1}{60}x^2\left(10x^4\left(a^2d^2+4abcd+b^2c^2\right)+30a^2c^2+15bdx^6(ad+bc)+30acx^2(ad+bc)+6b^2d^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (x^2*(30*a^2*c^2 + 30*a*c*(b*c + a*d)*x^2 + 10*(b^2*c^2 + 4*a*b*c*d + a^2*d
^2)*x^4 + 15*b*d*(b*c + a*d)*x^6 + 6*b^2*d^2*x^8))/60
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [A] time = 0.63, size = 94, normalized size = 1.32

$$\frac{1}{10}x^{10}d^2b^2+\frac{1}{4}x^8dcb^2+\frac{1}{4}x^8d^2ba+\frac{1}{6}x^6c^2b^2+\frac{2}{3}x^6dcba+\frac{1}{6}x^6d^2a^2+\frac{1}{2}x^4c^2ba+\frac{1}{2}x^4dca^2+\frac{1}{2}x^2c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b^2 + 1/4*x^8*d*c*b^2 + 1/4*x^8*d^2*b*a + 1/6*x^6*c^2*b^2 + 2
/3*x^6*d*c*b*a + 1/6*x^6*d^2*a^2 + 1/2*x^4*c^2*b*a + 1/2*x^4*d*c*a^2 + 1/2*
x^2*c^2*a^2
```

giac [A] time = 0.31, size = 94, normalized size = 1.32

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,b^2cdx^8+\frac{1}{4}\,abd^2x^8+\frac{1}{6}\,b^2c^2x^6+\frac{2}{3}\,abcdx^6+\frac{1}{6}\,a^2d^2x^6+\frac{1}{2}\,abc^2x^4+\frac{1}{2}\,a^2cdx^4+\frac{1}{2}\,a^2c^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*b^2*c*d*x^8 + 1/4*a*b*d^2*x^8 + 1/6*b^2*c^2*x^6 + 2
/3*a*b*c*d*x^6 + 1/6*a^2*d^2*x^6 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + 1/2*
a^2*c^2*x^2
```

maple [A] time = 0.00, size = 90, normalized size = 1.27

$$\frac{b^2d^2x^{10}}{10}+\frac{\left(2ab\,d^2+2b^2cd\right)x^8}{8}+\frac{a^2c^2x^2}{2}+\frac{\left(a^2d^2+4abcd+b^2c^2\right)x^6}{6}+\frac{\left(2a^2cd+2ab\,c^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/10*b^2*d^2*x^10+1/8*(2*a*b*d^2+2*b^2*c*d)*x^8+1/6*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^6+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+1/2*a^2*c^2*x^2
```

maxima [A] time = 1.02, size = 85, normalized size = 1.20

$$\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\left(b^2cd+abd^2\right)x^8+\frac{1}{6}\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\left(abc^2+a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

mupad [B] time = 0.03, size = 78, normalized size = 1.10

$$x^6\left(\frac{a^2\,d^2}{6}+\frac{2\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{6}\right)+\frac{a^2\,c^2\,x^2}{2}+\frac{b^2\,d^2\,x^{10}}{10}+\frac{a\,c\,x^4\,\left(a\,d+b\,c\right)}{2}+\frac{b\,d\,x^8\,\left(a\,d+b\,c\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^6*((a^2*d^2)/6 + (b^2*c^2)/6 + (2*a*b*c*d)/3) + (a^2*c^2*x^2)/2 + (b^2*d^
2*x^10)/10 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^8*(a*d + b*c))/4
```

sympy [A] time = 0.08, size = 94, normalized size = 1.32

$$\frac{a^2c^2x^2}{2}+\frac{b^2d^2x^{10}}{10}+x^8\left(\frac{abd^2}{4}+\frac{b^2cd}{4}\right)+x^6\left(\frac{a^2d^2}{6}+\frac{2abcd}{3}+\frac{b^2c^2}{6}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x**2/2 + b**2*d**2*x**10/10 + x**8*(a*b*d**2/4 + b**2*c*d/4) + x*
*6*(a**2*d**2/6 + 2*a*b*c*d/3 + b**2*c**2/6) + x**4*(a**2*c*d/2 + a*b*c**2/
2)
```

3.154 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2x^8\right) \\ &= a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.00

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^2\right)^2 \left(c + dx^2\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [A] time = 0.74, size = 91, normalized size = 1.11

$$\frac{1}{9}x^9d^2b^2 + \frac{2}{7}x^7dcb^2 + \frac{2}{7}x^7d^2ba + \frac{1}{5}x^5c^2b^2 + \frac{4}{5}x^5dcba + \frac{1}{5}x^5d^2a^2 + \frac{2}{3}x^3c^2ba + \frac{2}{3}x^3dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^2*b^2 + 2/7*x^7*d*c*b^2 + 2/7*x^7*d^2*b*a + 1/5*x^5*c^2*b^2 + 4/5
*x^5*d*c*b*a + 1/5*x^5*d^2*a^2 + 2/3*x^3*c^2*b*a + 2/3*x^3*d*c*a^2 + x*c^2*
a^2
```

giac [A] time = 0.43, size = 91, normalized size = 1.11

$$\frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\,b^2cdx^7 + \frac{2}{7}\,abd^2x^7 + \frac{1}{5}\,b^2c^2x^5 + \frac{4}{5}\,abcdx^5 + \frac{1}{5}\,a^2d^2x^5 + \frac{2}{3}\,abc^2x^3 + \frac{2}{3}\,a^2cdx^3 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

maple [A] time = 0.00, size = 87, normalized size = 1.06

$$\frac{b^2d^2x^9}{9} + \frac{\left(2ab\,d^2 + 2b^2cd\right)x^7}{7} + a^2c^2x + \frac{\left(a^2d^2 + 4abcd + b^2c^2\right)x^5}{5} + \frac{\left(2a^2cd + 2ab\,c^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

maxima [A] time = 1.06, size = 82, normalized size = 1.00

$$\frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\left(b^2cd+abd^2\right)x^7+\frac{1}{5}\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+a^2c^2x+\frac{2}{3}\left(abc^2+a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

mupad [B] time = 0.03, size = 75, normalized size = 0.91

$$x^5\,\left(\frac{a^2\,d^2}{5}+\frac{4\,a\,b\,c\,d}{5}+\frac{b^2\,c^2}{5}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^9}{9}+\frac{2\,a\,c\,x^3\,\left(a\,d+b\,c\right)}{3}+\frac{2\,b\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9)
/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

sympy [A] time = 0.08, size = 97, normalized size = 1.18

$$a^2c^2x+\frac{b^2d^2x^9}{9}+x^7\left(\frac{2abd^2}{7}+\frac{2b^2cd}{7}\right)+x^5\left(\frac{a^2d^2}{5}+\frac{4abcd}{5}+\frac{b^2c^2}{5}\right)+x^3\left(\frac{2a^2cd}{3}+\frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

3.155 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx$

Optimal. Leaf size=80

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(c+dx)^2}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(2ac(bc+ad)+\frac{a^2c^2}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+2bd(bc+ad)x^2+b\right.\right.\\ &= ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4+\frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2c^2\,\mathrm{lo}\end{aligned}$$

Mathematica [A] time = 0.03, size = 80, normalized size = 1.00

$$\frac{1}{4}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log(x)+\frac{1}{3}bdx^6(ad+bc)+acx^2(ad+bc)+\frac{1}{8}b^2d^2x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c +
 a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x, x]
```

fricas [A] time = 0.67, size = 82, normalized size = 1.02

$$\frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+a^2c^2\log(x)+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^4 + a^2*c^2*log(x) + (a*b*c^2 + a^2*c*d)*x^2
```

giac [A] time = 0.45, size = 92, normalized size = 1.15

$$\frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\,b^2cdx^6+\frac{1}{3}\,abd^2x^6+\frac{1}{4}\,b^2c^2x^4+abcdx^4+\frac{1}{4}\,a^2d^2x^4+abc^2x^2+a^2cdx^2+\frac{1}{2}\,a^2c^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*b^2*c*d*x^6 + 1/3*a*b*d^2*x^6 + 1/4*b^2*c^2*x^4 + a*b
*c*d*x^4 + 1/4*a^2*d^2*x^4 + a*b*c^2*x^2 + a^2*c*d*x^2 + 1/2*a^2*c^2*log(x^
2)
```

maple [A] time = 0.00, size = 90, normalized size = 1.12

$$\frac{b^2d^2x^8}{8}+\frac{ab\,d^2x^6}{3}+\frac{b^2cd\,x^6}{3}+\frac{a^2d^2x^4}{4}+abcd\,x^4+\frac{b^2c^2x^4}{4}+a^2cd\,x^2+ab\,c^2x^2+a^2c^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x,x)
```

```
[Out] 1/8*b^2*d^2*x^8+1/3*x^6*a*b*d^2+1/3*x^6*b^2*c*d+1/4*x^4*a^2*d^2+x^4*a*b*c*d
+1/4*x^4*b^2*c^2+x^2*a^2*c*d+x^2*a*b*c^2+a^2*c^2*ln(x)
```

maxima [A] time = 1.07, size = 85, normalized size = 1.06

$$\frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+\frac{1}{2}\,a^2c^2\log\left(x^2\right)+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^4 + 1/2*a^2*c^2*log(x^2) + (a*b*c^2 + a^2*c*d)*x^2
```

mupad [B] time = 0.03, size = 74, normalized size = 0.92

$$x^4\left(\frac{a^2\,d^2}{4}+a\,b\,c\,d+\frac{b^2\,c^2}{4}\right)+\frac{b^2\,d^2\,x^8}{8}+a^2\,c^2\,\ln(x)+a\,c\,x^2\,\left(a\,d+b\,c\right)+\frac{b\,d\,x^6\,\left(a\,d+b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x,x)
```

```
[Out] x^4*((a^2*d^2)/4 + (b^2*c^2)/4 + a*b*c*d) + (b^2*d^2*x^8)/8 + a^2*c^2*log(x
) + a*c*x^2*(a*d + b*c) + (b*d*x^6*(a*d + b*c))/3
```

sympy [A] time = 0.18, size = 85, normalized size = 1.06

$$a^2c^2\log\left(x\right)+\frac{b^2d^2x^8}{8}+x^6\left(\frac{abd^2}{3}+\frac{b^2cd}{3}\right)+x^4\left(\frac{a^2d^2}{4}+abcd+\frac{b^2c^2}{4}\right)+x^2\left(a^2cd+abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x,x)
```

```
[Out] a**2*c**2*log(x) + b**2*d**2*x**8/8 + x**6*(a*b*d**2/3 + b**2*c*d/3) + x**4
*(a**2*d**2/4 + a*b*c*d + b**2*c**2/4) + x**2*(a**2*c*d + a*b*c**2)
```

3.156 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx$

Optimal. Leaf size=81

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^2}\,dx &= \int \left(2ac(bc+ad)+\frac{a^2c^2}{x^2}+\left(b^2c^2+4abcd+a^2d^2\right)x^2+2bd(bc+ad)x^4+b^2d^2x^6\right)\\ &= -\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d^2x\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.00

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] -((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3
)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^2, x]
```

fricas [A] time = 0.70, size = 87, normalized size = 1.07

$$\frac{15\,b^2d^2x^8+42\left(b^2cd+abd^2\right)x^6+35\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-105\,a^2c^2+210\left(abc^2+a^2cd\right)x^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^2*x^8 + 42*(b^2*c*d + a*b*d^2)*x^6 + 35*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^4 - 105*a^2*c^2 + 210*(a*b*c^2 + a^2*c*d)*x^2)/x
```

giac [A] time = 0.27, size = 90, normalized size = 1.11

$$\frac{1}{7}\,b^2d^2x^7+\frac{2}{5}\,b^2cdx^5+\frac{2}{5}\,abd^2x^5+\frac{1}{3}\,b^2c^2x^3+\frac{4}{3}\,abcdx^3+\frac{1}{3}\,a^2d^2x^3+2\,abc^2x+2\,a^2cdx-\frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*b^2*c*d*x^5 + 2/5*a*b*d^2*x^5 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + 2*a*b*c^2*x + 2*a^2*c*d*x - a^2*c^2/x
```

maple [A] time = 0.00, size = 91, normalized size = 1.12

$$\frac{b^2d^2x^7}{7}+\frac{2ab\,d^2x^5}{5}+\frac{2b^2cd\,x^5}{5}+\frac{a^2d^2x^3}{3}+\frac{4abcd\,x^3}{3}+\frac{b^2c^2x^3}{3}+2a^2cdx+2ab\,c^2x-\frac{a^2c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^2,x)
```

```
[Out] 1/7*b^2*d^2*x^7+2/5*x^5*a*b*d^2+2/5*x^5*b^2*c*d+1/3*x^3*a^2*d^2+4/3*x^3*a*b
*c*d+1/3*x^3*b^2*c^2+2*a^2*c*d*x+2*a*b*c^2*x-a^2*c^2/x
```

maxima [A] time = 1.10, size = 83, normalized size = 1.02

$$\frac{1}{7}b^2d^2x^7+\frac{2}{5}\left(b^2cd+abd^2\right)x^5+\frac{1}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^3-\frac{a^2c^2}{x}+2\left(abc^2+a^2cd\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*x^7 + 2/5*(b^2*c*d + a*b*d^2)*x^5 + 1/3*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^3 - a^2*c^2/x + 2*(a*b*c^2 + a^2*c*d)*x
```

mupad [B] time = 0.03, size = 76, normalized size = 0.94

$$x^3\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)-\frac{a^2\,c^2}{x}+\frac{b^2\,d^2\,x^7}{7}+2\,a\,c\,x\,\left(a\,d+b\,c\right)+\frac{2\,b\,d\,x^5\,\left(a\,d+b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x)
```

```
[Out] x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) - (a^2*c^2)/x + (b^2*d^2*x^
7)/7 + 2*a*c*x*(a*d + b*c) + (2*b*d*x^5*(a*d + b*c))/5
```

sympy [A] time = 0.18, size = 92, normalized size = 1.14

$$-\frac{a^2c^2}{x}+\frac{b^2d^2x^7}{7}+x^5\left(\frac{2abd^2}{5}+\frac{2b^2cd}{5}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x\left(2a^2cd+2abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**2,x)
```

```
[Out] -a**2*c**2/x + b**2*d**2*x**7/7 + x**5*(2*a*b*d**2/5 + 2*b**2*c*d/5) + x**3
*(a**2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x*(2*a**2*c*d + 2*a*b*c**2)
```

3.157 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

Rubi [**A**] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{2}x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{2x^2}+\frac{1}{2}bdx^4(ad+bc)+2ac\log(x)(ad+bc)+\frac{1}{6}b^2d^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] -(a^2*c^2)/(2*x^2) + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)/2 + (b*d*(b*c +
a*d)*x^4)/2 + (b^2*d^2*x^6)/6 + 2*a*c*(b*c + a*d)*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^2}{x^2}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\Bigg(\int \Bigg(b^2c^2\left(1+\frac{ad(4bc+ad)}{b^2c^2}\right)+\frac{a^2c^2}{x^2}+\frac{2ac(bc+ad)}{x}+2bd(bc+ad)x+b^2$$

$$=-\frac{a^2c^2}{2x^2}+\frac{1}{2}\left(b^2c^2+4abcd+a^2d^2\right)x^2+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{6}b^2d^2x^6+2ac(bc+ad)\,\mathrm{lo}$$

Mathematica [A]   time = 0.04, size = 83, normalized size = 0.99

$$\frac{1}{6}\left(\frac{3a^2\left(d^2x^4-c^2\right)}{x^2}+3abdx^2\left(4c+dx^2\right)+12ac\log(x)(ad+bc)+b^2x^2\left(3c^2+3cdx^2+d^2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] (3*a*b*d*x^2*(4*c + d*x^2) + (3*a^2*(-c^2 + d^2*x^4))/x^2 + b^2*x^2*(3*c^2
+ 3*c*d*x^2 + d^2*x^4) + 12*a*c*(b*c + a*d)*Log[x])/6
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^3, x]
```

fricas [A]   time = 0.57, size = 88, normalized size = 1.05

$$\frac{b^2d^2x^8+3\left(b^2cd+abd^2\right)x^6+3\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-3\,a^2c^2+12\left(abc^2+a^2cd\right)x^2\log(x)}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^2*d^2*x^8 + 3*(b^2*c*d + a*b*d^2)*x^6 + 3*(b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^4 - 3*a^2*c^2 + 12*(a*b*c^2 + a^2*c*d)*x^2*log(x))/x^2
```

giac [A]   time = 0.29, size = 114, normalized size = 1.36

$$\frac{1}{6}\,b^2d^2x^6+\frac{1}{2}\,b^2cdx^4+\frac{1}{2}\,abd^2x^4+\frac{1}{2}\,b^2c^2x^2+2\,abcdx^2+\frac{1}{2}\,a^2d^2x^2+\left(abc^2+a^2cd\right)\log\left(x^2\right)-\frac{2\,abc^2x^2+2\,a^2cdx^2+a^2c^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/2*b^2*c^2*x^2 + 2*a
*b*c*d*x^2 + 1/2*a^2*d^2*x^2 + (a*b*c^2 + a^2*c*d)*log(x^2) - 1/2*(2*a*b*c^
2*x^2 + 2*a^2*c*d*x^2 + a^2*c^2)/x^2
```

maple [A] time = 0.01, size = 93, normalized size = 1.11

$$\frac{b^{2}d^{2}x^{6}}{6}+\frac{ab\,d^{2}x^{4}}{2}+\frac{b^{2}cd\,x^{4}}{2}+\frac{a^{2}d^{2}x^{2}}{2}+2abcd\,x^{2}+\frac{b^{2}c^{2}x^{2}}{2}+2a^{2}cd\ln(x)+2ab\,c^{2}\ln(x)-\frac{a^{2}c^{2}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^3,x)
```

```
[Out] 1/6*b^2*d^2*x^6+1/2*x^4*a*b*d^2+1/2*x^4*b^2*c*d+1/2*x^2*a^2*d^2+2*x^2*a*b*c
*d+1/2*x^2*b^2*c^2-1/2*a^2*c^2/x^2+2*ln(x)*a^2*c*d+2*ln(x)*a*b*c^2
```

maxima [A] time = 1.05, size = 85, normalized size = 1.01

$$\frac{1}{6}\,b^{2}d^{2}x^{6}+\frac{1}{2}\left(b^{2}cd+abd^{2}\right)x^{4}+\frac{1}{2}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{2}-\frac{a^{2}c^{2}}{2\,x^{2}}+\left(abc^{2}+a^{2}cd\right)\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^2*x^6 + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/2*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^2 - 1/2*a^2*c^2/x^2 + (a*b*c^2 + a^2*c*d)*log(x^2)
```

mupad [B] time = 0.04, size = 82, normalized size = 0.98

$$x^{2}\left(\frac{a^{2}\,d^{2}}{2}+2\,a\,b\,c\,d+\frac{b^{2}\,c^{2}}{2}\right)+\ln(x)\,\left(2\,d\,a^{2}\,c+2\,b\,a\,c^{2}\right)-\frac{a^{2}\,c^{2}}{2\,x^{2}}+\frac{b^{2}\,d^{2}\,x^{6}}{6}+\frac{b\,d\,x^{4}\,\left(a\,d+b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x)
```

```
[Out] x^2*((a^2*d^2)/2 + (b^2*c^2)/2 + 2*a*b*c*d) + log(x)*(2*a*b*c^2 + 2*a^2*c*d
) - (a^2*c^2)/(2*x^2) + (b^2*d^2*x^6)/6 + (b*d*x^4*(a*d + b*c))/2
```

sympy [A] time = 0.28, size = 87, normalized size = 1.04

$$-\frac{a^{2}c^{2}}{2x^{2}}+2ac\left(ad+bc\right)\log\left(x\right)+\frac{b^{2}d^{2}x^{6}}{6}+x^{4}\left(\frac{abd^{2}}{2}+\frac{b^{2}cd}{2}\right)+x^{2}\left(\frac{a^{2}d^{2}}{2}+2abcd+\frac{b^{2}c^{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**3,x)
```

```
[Out] -a**2*c**2/(2*x**2) + 2*a*c*(a*d + b*c)*log(x) + b**2*d**2*x**6/6 + x**4*(a
*b*d**2/2 + b**2*c*d/2) + x**2*(a**2*d**2/2 + 2*a*b*c*d + b**2*c**2/2)
```

3.158 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^4}\,dx$

Optimal. Leaf size=80

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -(a^2*c^2)/(3*x^3) - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^4}\,dx &= \int \left(b^2c^2\left(1+\frac{ad(4bc+ad)}{b^2c^2}\right)+\frac{a^2c^2}{x^4}+\frac{2ac(bc+ad)}{x^2}+2bd(bc+ad)x^2+b^2d^2x^4\right) \\ &= -\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 1.00

$$x\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*c^2)/x^3 - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^4, x]
```

fricas [A] time = 0.70, size = 87, normalized size = 1.09

$$\frac{3\,b^2d^2x^8+10\left(b^2cd+abd^2\right)x^6+15\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-5\,a^2c^2-30\left(abc^2+a^2cd\right)x^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^8 + 10*(b^2*c*d + a*b*d^2)*x^6 + 15*(b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^4 - 5*a^2*c^2 - 30*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

giac [A] time = 0.32, size = 88, normalized size = 1.10

$$\frac{1}{5}\,b^2d^2x^5+\frac{2}{3}\,b^2cdx^3+\frac{2}{3}\,abd^2x^3+b^2c^2x+4\,abcdx+a^2d^2x-\frac{6\,abc^2x^2+6\,a^2cdx^2+a^2c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*b^2*c*d*x^3 + 2/3*a*b*d^2*x^3 + b^2*c^2*x + 4*a*b*c*d
*x + a^2*d^2*x - 1/3*(6*a*b*c^2*x^2 + 6*a^2*c*d*x^2 + a^2*c^2)/x^3
```

maple [A] time = 0.01, size = 81, normalized size = 1.01

$$\frac{b^2d^2x^5}{5}+\frac{2ab\,d^2x^3}{3}+\frac{2b^2cd\,x^3}{3}+a^2d^2x+4abcdx+b^2c^2x-\frac{a^2c^2}{3x^3}-\frac{2\left(ad+bc\right)ac}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^4,x)
```

```
[Out] 1/5*b^2*d^2*x^5+2/3*x^3*a*b*d^2+2/3*x^3*b^2*c*d+a^2*d^2*x+4*a*b*c*d*x+b^2*c
^2*x-2*a*c*(a*d+b*c)/x-1/3*a^2*c^2/x^3
```

maxima [A] time = 1.17, size = 84, normalized size = 1.05

$$\frac{1}{5}\,b^2d^2x^5+\frac{2}{3}\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+4\,abcd+a^2d^2\right)x-\frac{a^2c^2+6\left(abc^2+a^2cd\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + 2/3*(b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x - 1/3*(a^2*c^2 + 6*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

mupad [B] time = 0.05, size = 82, normalized size = 1.02

$$x\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)-\frac{x^2\,\left(2\,d\,a^2\,c+2\,b\,a\,c^2\right)+\frac{a^2\,c^2}{3}}{x^3}+\frac{b^2\,d^2\,x^5}{5}+\frac{2\,b\,d\,x^3\,\left(a\,d+b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x)
```

```
[Out] x*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) - (x^2*(2*a*b*c^2 + 2*a^2*c*d) + (a^2*c^2
)/3)/x^3 + (b^2*d^2*x^5)/5 + (2*b*d*x^3*(a*d + b*c))/3
```

sympy [A] time = 0.30, size = 92, normalized size = 1.15

$$\frac{b^2d^2x^5}{5}+x^3\left(\frac{2abd^2}{3}+\frac{2b^2cd}{3}\right)+x\left(a^2d^2+4abcd+b^2c^2\right)+\frac{-a^2c^2+x^2\left(-6a^2cd-6abc^2\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**4,x)
```

```
[Out] b**2*d**2*x**5/5 + x**3*(2*a*b*d**2/3 + 2*b**2*c*d/3) + x*(a**2*d**2 + 4*a*
b*c*d + b**2*c**2) + (-a**2*c**2 + x**2*(-6*a**2*c*d - 6*a*b*c**2))/(3*x**3
)
```

3.159 $\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+$$

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int\left(a^2c^3x^4+ac^2(2bc+3ad)x^6+c\left(b^2c^2+6abcd+3a^2d^2\right)x^8+d\left(3b^2c^2+6abcd\right.\right.\\ &= \frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{11}d\left(3b^2c^2+6a\right.\end{aligned}$$

Mathematica [A] time = 0.03, size = 127, normalized size = 1.00

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d^2
*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [A] time = 0.56, size = 135, normalized size = 1.06

$$\frac{1}{15}x^{15}d^3b^2 + \frac{3}{13}x^{13}d^2cb^2 + \frac{2}{13}x^{13}d^3ba + \frac{3}{11}x^{11}dc^2b^2 + \frac{6}{11}x^{11}d^2cba + \frac{1}{11}x^{11}d^3a^2 + \frac{1}{9}x^9c^3b^2 + \frac{2}{3}x^9dc^2ba + \frac{1}{3}x^9d^2ca^2 + \frac{2}{7}x^7c^3ba + \frac{3}{7}x^7dc^2a^2 + \frac{1}{5}x^5c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/11*x^11*d*c
^2*b^2 + 6/11*x^11*d^2*c*b*a + 1/11*x^11*d^3*a^2 + 1/9*x^9*c^3*b^2 + 2/3*x^
9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/7*x^7*c^3*b*a + 3/7*x^7*d*c^2*a^2 + 1/5
*x^5*c^3*a^2
```

giac [A] time = 0.35, size = 135, normalized size = 1.06

$$\frac{1}{15}\,b^2d^3x^{15} + \frac{3}{13}\,b^2cd^2x^{13} + \frac{2}{13}\,abd^3x^{13} + \frac{3}{11}\,b^2c^2dx^{11} + \frac{6}{11}\,abcd^2x^{11} + \frac{1}{11}\,a^2d^3x^{11} + \frac{1}{9}\,b^2c^3x^9 + \frac{2}{3}\,abc^2dx^9 + \frac{1}{3}\,a^2cd^2x^9 + \frac{2}{7}\,abc^3x^7 + \frac{3}{7}\,a^2c^2dx^7 + \frac{1}{5}\,a^2c^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15*b^2*d^3*x^15 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/11*b^2*c^2*
d*x^11 + 6/11*a*b*c*d^2*x^11 + 1/11*a^2*d^3*x^11 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/7*a*b*c^3*x^7 + 3/7*a^2*c^2*d*x^7 + 1/5
*a^2*c^3*x^5
```

maple [A] time = 0.00, size = 128, normalized size = 1.01

$$\frac{b^2d^3x^{15}}{15} + \frac{\left(2ab\,d^3 + 3b^2c\,d^2\right)x^{13}}{13} + \frac{\left(a^2d^3 + 6abc\,d^2 + 3b^2c^2d\right)x^{11}}{11} + \frac{a^2c^3x^5}{5} + \frac{\left(3a^2c\,d^2 + 6ab\,c^2d + b^2c^3\right)x^9}{9} + \frac{\left(3a^2c^2d + 2ab\,c^3\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/15*b^2*d^3*x^15+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/11*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^11+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/7*(3*a^2*c
^2*d+2*a*b*c^3)*x^7+1/5*a^2*c^3*x^5
```

maxima [A] time = 1.02, size = 127, normalized size = 1.00

$$\frac{1}{15}\,b^2d^3x^{15}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{11}+\frac{1}{5}\,a^2c^3x^5+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+\frac{1}{7}\left(2\,abc^3+3\,a^2c^2d\right)x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^7
```

mupad [B] time = 0.10, size = 119, normalized size = 0.94

$$x^9\,\left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right)+x^{11}\,\left(\frac{a^2\,d^3}{11}+\frac{6\,a\,b\,c\,d^2}{11}+\frac{3\,b^2\,c^2\,d}{11}\right)+\frac{a^2\,c^3\,x^5}{5}+\frac{b^2\,d^3\,x^{15}}{15}+\frac{a\,c^2\,x^7\,\left(3\,a\,d+2\,b\,c\right)}{7}+\frac{b\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^11*((a^2*d^3)/11 +
(3*b^2*c^2*d)/11 + (6*a*b*c*d^2)/11) + (a^2*c^3*x^5)/5 + (b^2*d^3*x^15)/15
+ (a*c^2*x^7*(3*a*d + 2*b*c))/7 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

sympy [A] time = 0.09, size = 143, normalized size = 1.13

$$\frac{a^2c^3x^5}{5}+\frac{b^2d^3x^{15}}{15}+x^{13}\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{11}\left(\frac{a^2d^3}{11}+\frac{6abcd^2}{11}+\frac{3b^2c^2d}{11}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^7\left(\frac{3a^2c^2d}{7}+\frac{2abc^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**5/5 + b**2*d**3*x**15/15 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**
2/13) + x**11*(a**2*d**3/11 + 6*a*b*c*d**2/11 + 3*b**2*c**2*d/11) + x**9*(a
**2*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**7*(3*a**2*c**2*d/7 + 2*a*
b*c**3/7)
```

3.160 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=106

$$-\frac{b\left(c+dx^2\right)^6 (3bc-2ad)}{12d^4}+\frac{\left(c+dx^2\right)^5 (bc-ad)(3bc-ad)}{10d^4}-\frac{c\left(c+dx^2\right)^4 (bc-ad)^2}{8d^4}+\frac{b^2\left(c+dx^2\right)^7}{14d^4}$$

Rubi [A] time = 0.22, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^6 (3bc-2ad)}{12d^4}+\frac{\left(c+dx^2\right)^5 (bc-ad)(3bc-ad)}{10d^4}-\frac{c\left(c+dx^2\right)^4 (bc-ad)^2}{8d^4}+\frac{b^2\left(c+dx^2\right)^7}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^4)/(8*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^5)/(10*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/(12*d^4) + (b^2*(c +
 d*x^2)^7)/(14*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^2(c+dx)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\Bigg(\int\Bigg(-\frac{c(bc-ad)^2(c+dx)^3}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^4}{d^3}-b(3bc-2a\\ &= -\frac{c(bc-ad)^2\left(c+dx^2\right)^4}{8d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^5}{10d^4}-\frac{b(3bc-2ad)\big(c+dx}{12d^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 1.12

$$\frac{1}{840}x^4\left(84dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+105cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+210a^2c^3+140ac^2x^2(3ad+2bc)+70bd^2x^8(2ad+3bc)+60b^2d^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^4*(210*a^2*c^3 + 140*a*c^2*(2*b*c + 3*a*d)*x^2 + 105*c*(b^2*c^2 + 6*a*b*
c*d + 3*a^2*d^2)*x^4 + 84*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 70*b*d^
2*(3*b*c + 2*a*d)*x^8 + 60*b^2*d^3*x^10))/840
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [A] time = 0.44, size = 135, normalized size = 1.27

$$\frac{1}{14}x^{14}d^3b^2+\frac{1}{4}x^{12}d^2cb^2+\frac{1}{6}x^{12}d^3ba+\frac{3}{10}x^{10}dc^2b^2+\frac{3}{5}x^{10}d^2cba+\frac{1}{10}x^{10}d^3a^2+\frac{1}{8}x^8c^3b^2+\frac{3}{4}x^8dc^2ba+\frac{3}{8}x^8d^2ca^2+\frac{1}{3}x^6c^3ba+\frac{1}{2}x^6dc^2a^2+\frac{1}{4}x^4c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^3*b^2 + 1/4*x^12*d^2*c*b^2 + 1/6*x^12*d^3*b*a + 3/10*x^10*d*c^2
*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/8*x^8*c^3*b^2 + 3/4*x^8*d
*c^2*b*a + 3/8*x^8*d^2*c*a^2 + 1/3*x^6*c^3*b*a + 1/2*x^6*d*c^2*a^2 + 1/4*x^
4*c^3*a^2
```

giac [A] time = 0.38, size = 135, normalized size = 1.27

$$\frac{1}{14}\,b^{2}d^{3}x^{14}+\frac{1}{4}\,b^{2}cd^{2}x^{12}+\frac{1}{6}\,abd^{3}x^{12}+\frac{3}{10}\,b^{2}c^{2}dx^{10}+\frac{3}{5}\,abcd^{2}x^{10}+\frac{1}{10}\,a^{2}d^{3}x^{10}+\frac{1}{8}\,b^{2}c^{3}x^{8}+\frac{3}{4}\,abc^{2}dx^{8}+\frac{3}{8}\,a^{2}cd^{2}x^{8}+\frac{1}{3}\,abc^{3}x^{6}+\frac{1}{2}\,a^{2}c^{2}dx^{6}+\frac{1}{4}\,a^{2}c^{3}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/4*b^2*c*d^2*x^12 + 1/6*a*b*d^3*x^12 + 3/10*b^2*c^2*d*
x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/8*b^2*c^3*x^8 + 3/4*a*b*c
^2*d*x^8 + 3/8*a^2*c*d^2*x^8 + 1/3*a*b*c^3*x^6 + 1/2*a^2*c^2*d*x^6 + 1/4*a^
2*c^3*x^4
```

maple [A] time = 0.00, size = 128, normalized size = 1.21

$$\frac{b^{2}d^{3}x^{14}}{14}+\frac{\left(2ab\,d^{3}+3b^{2}c\,d^{2}\right)x^{12}}{12}+\frac{\left(a^{2}d^{3}+6abc\,d^{2}+3b^{2}c^{2}d\right)x^{10}}{10}+\frac{a^{2}c^{3}x^{4}}{4}+\frac{\left(3a^{2}c\,d^{2}+6ab\,c^{2}d+b^{2}c^{3}\right)x^{8}}{8}+\frac{\left(3a^{2}c^{2}d+2ab\,c^{3}\right)x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/14*b^2*d^3*x^14+1/12*(2*a*b*d^3+3*b^2*c*d^2)*x^12+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/8*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^8+1/6*(3*a^2*c
^2*d+2*a*b*c^3)*x^6+1/4*a^2*c^3*x^4
```

maxima [A] time = 1.02, size = 127, normalized size = 1.20

$$\frac{1}{14}\,b^{2}d^{3}x^{14}+\frac{1}{12}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{12}+\frac{1}{10}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{10}+\frac{1}{4}\,a^{2}c^{3}x^{4}+\frac{1}{8}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{8}+\frac{1}{6}\left(2\,abc^{3}+3\,a^{2}c^{2}d\right)x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6
```

mupad [B] time = 0.04, size = 119, normalized size = 1.12

$$x^{8}\left(\frac{3\,a^{2}\,c\,d^{2}}{8}+\frac{3\,a\,b\,c^{2}\,d}{4}+\frac{b^{2}\,c^{3}}{8}\right)+x^{10}\left(\frac{a^{2}\,d^{3}}{10}+\frac{3\,a\,b\,c\,d^{2}}{5}+\frac{3\,b^{2}\,c^{2}\,d}{10}\right)+\frac{a^{2}\,c^{3}\,x^{4}}{4}+\frac{b^{2}\,d^{3}\,x^{14}}{14}+\frac{a\,c^{2}\,x^{6}\,\left(3\,a\,d+2\,b\,c\right)}{6}+\frac{b\,d^{2}\,x^{12}\,\left(2\,a\,d+3\,b\,c\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^8*((b^2*c^3)/8 + (3*a^2*c*d^2)/8 + (3*a*b*c^2*d)/4) + x^10*((a^2*d^3)/10
+ (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + (a^2*c^3*x^4)/4 + (b^2*d^3*x^14)/14
 + (a*c^2*x^6*(3*a*d + 2*b*c))/6 + (b*d^2*x^12*(2*a*d + 3*b*c))/12
```

sympy [A] time = 0.09, size = 138, normalized size = 1.30

$$\frac{a^2c^3x^4}{4}+\frac{b^2d^3x^{14}}{14}+x^{12}\left(\frac{abd^3}{6}+\frac{b^2cd^2}{4}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^8\left(\frac{3a^2cd^2}{8}+\frac{3abc^2d}{4}+\frac{b^2c^3}{8}\right)+x^6\left(\frac{a^2c^2d}{2}+\frac{abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**4/4 + b**2*d**3*x**14/14 + x**12*(a*b*d**3/6 + b**2*c*d**2/4)
+ x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**8*(3*a**2*c
*d**2/8 + 3*a*b*c**2*d/4 + b**2*c**3/8) + x**6*(a**2*c**2*d/2 + a*b*c**3/3)
```

3.161 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=127

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+$$

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int\left(a^2c^3x^2+ac^2(2bc+3ad)x^4+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6abc\right.\right.\\ &= \frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+6\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 127, normalized size = 1.00

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d +
 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2*(
3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [A] time = 0.67, size = 135, normalized size = 1.06

$$\frac{1}{13}x^{13}d^3b^2+\frac{3}{11}x^{11}d^2cb^2+\frac{2}{11}x^{11}d^3ba+\frac{1}{3}x^9dc^2b^2+\frac{2}{3}x^9d^2cba+\frac{1}{9}x^9d^3a^2+\frac{1}{7}x^7c^3b^2+\frac{6}{7}x^7dc^2ba+\frac{3}{7}x^7d^2ca^2+\frac{2}{5}x^5c^3ba+\frac{3}{5}x^5dc^2a^2+\frac{1}{3}x^3c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b^2 + 3/11*x^11*d^2*c*b^2 + 2/11*x^11*d^3*b*a + 1/3*x^9*d*c^2
*b^2 + 2/3*x^9*d^2*c*b*a + 1/9*x^9*d^3*a^2 + 1/7*x^7*c^3*b^2 + 6/7*x^7*d*c^
2*b*a + 3/7*x^7*d^2*c*a^2 + 2/5*x^5*c^3*b*a + 3/5*x^5*d*c^2*a^2 + 1/3*x^3*c
^3*a^2
```

giac [A] time = 0.37, size = 135, normalized size = 1.06

$$\frac{1}{13}b^2d^3x^{13}+\frac{3}{11}b^2cd^2x^{11}+\frac{2}{11}abd^3x^{11}+\frac{1}{3}b^2c^2dx^9+\frac{2}{3}abcd^2x^9+\frac{1}{9}a^2d^3x^9+\frac{1}{7}b^2c^3x^7+\frac{6}{7}abc^2dx^7+\frac{3}{7}a^2cd^2x^7+\frac{2}{5}abc^3x^5+\frac{3}{5}a^2c^2dx^5+\frac{1}{3}a^2c^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^3*x^13 + 3/11*b^2*c*d^2*x^11 + 2/11*a*b*d^3*x^11 + 1/3*b^2*c^2*d
*x^9 + 2/3*a*b*c*d^2*x^9 + 1/9*a^2*d^3*x^9 + 1/7*b^2*c^3*x^7 + 6/7*a*b*c^2*
d*x^7 + 3/7*a^2*c*d^2*x^7 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + 1/3*a^2*c
^3*x^3
```

maple [A] time = 0.00, size = 128, normalized size = 1.01

$$\frac{b^2d^3x^{13}}{13}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{11}}{11}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^9}{9}+\frac{a^2c^3x^3}{3}+\frac{\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^7}{7}+\frac{\left(3a^2c^2d+2ab\,c^3\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/13*b^2*d^3*x^13+1/11*(2*a*b*d^3+3*b^2*c*d^2)*x^11+1/9*(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*x^9+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/5*(3*a^2*c^2
*d+2*a*b*c^3)*x^5+1/3*a^2*c^3*x^3
```

maxima [A]   time = 1.04, size = 127, normalized size = 1.00

$$\frac{1}{13}\,b^{2}d^{3}x^{13}+\frac{1}{11}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{11}+\frac{1}{9}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{9}+\frac{1}{3}\,a^{2}c^{3}x^{3}+\frac{1}{7}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{7}+\frac{1}{5}\left(2\,abc^{3}+3\,a^{2}c^{2}d\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

mupad [B]   time = 0.04, size = 119, normalized size = 0.94

$$x^{7}\,\left(\frac{3\,a^{2}\,c\,d^{2}}{7}+\frac{6\,a\,b\,c^{2}\,d}{7}+\frac{b^{2}\,c^{3}}{7}\right)+x^{9}\,\left(\frac{a^{2}\,d^{3}}{9}+\frac{2\,a\,b\,c\,d^{2}}{3}+\frac{b^{2}\,c^{2}\,d}{3}\right)+\frac{a^{2}\,c^{3}\,x^{3}}{3}+\frac{b^{2}\,d^{3}\,x^{13}}{13}+\frac{a\,c^{2}\,x^{5}\,\left(3\,a\,d+2\,b\,c\right)}{5}+\frac{b\,d^{2}\,x^{11}\,\left(2\,a\,d+3\,b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^9*((a^2*d^3)/9 +
(b^2*c^2*d)/3 + (2*a*b*c*d^2)/3) + (a^2*c^3*x^3)/3 + (b^2*d^3*x^13)/13 + (a
*c^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^11*(2*a*d + 3*b*c))/11
```

sympy [A]   time = 0.09, size = 143, normalized size = 1.13

$$\frac{a^{2}c^{3}x^{3}}{3}+\frac{b^{2}d^{3}x^{13}}{13}+x^{11}\left(\frac{2abd^{3}}{11}+\frac{3b^{2}cd^{2}}{11}\right)+x^{9}\left(\frac{a^{2}d^{3}}{9}+\frac{2abcd^{2}}{3}+\frac{b^{2}c^{2}d}{3}\right)+x^{7}\left(\frac{3a^{2}cd^{2}}{7}+\frac{6abc^{2}d}{7}+\frac{b^{2}c^{3}}{7}\right)+x^{5}\left(\frac{3a^{2}c^{2}d}{5}+\frac{2abc^{3}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**3/3 + b**2*d**3*x**13/13 + x**11*(2*a*b*d**3/11 + 3*b**2*c*d**
2/11) + x**9*(a**2*d**3/9 + 2*a*b*c*d**2/3 + b**2*c**2*d/3) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c*
*3/5)
```

3.162 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=71

$$-\frac{b\left(c+dx^2\right)^5(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^4(bc-ad)^2}{8d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}$$

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$-\frac{b\left(c+dx^2\right)^5(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^4(bc-ad)^2}{8d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^4)/(8*d^3) - (b*(b*c - a*d)*(c + d*x^2)^5)/(5*d^
3) + (b^2*(c + d*x^2)^6)/(12*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(c+dx)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx,x,\right.\\&=\frac{(bc-ad)^2\left(c+dx^2\right)^4}{8d^3}-\frac{b(bc-ad)\left(c+dx^2\right)^5}{5d^3}+\frac{b^2\left(c+dx^2\right)^6}{12d^3}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 119, normalized size = 1.68

$$\frac{1}{120}x^2\left(15dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+20cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+60a^2c^3+30ac^2x^2(3ad+2bc)+12bd^2x^8(2ad+3bc)+10b^2d^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (x^2*(60*a^2*c^3 + 30*a*c^2*(2*b*c + 3*a*d)*x^2 + 20*c*(b^2*c^2 + 6*a*b*c*d
 + 3*a^2*d^2)*x^4 + 15*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 12*b*d^2*(
3*b*c + 2*a*d)*x^8 + 10*b^2*d^3*x^10))/120
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [**B**] time = 0.49, size = 134, normalized size = 1.89

$$\frac{1}{12}x^{12}d^3b^2+\frac{3}{10}x^{10}d^2cb^2+\frac{1}{5}x^{10}d^3ba+\frac{3}{8}x^8dc^2b^2+\frac{3}{4}x^8d^2cba+\frac{1}{8}x^8d^3a^2+\frac{1}{6}x^6c^3b^2+x^6dc^2ba+\frac{1}{2}x^6d^2ca^2+\frac{1}{2}x^4c^3ba+\frac{3}{4}x^4dc^2a^2+\frac{1}{2}x^2c^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^3*b^2 + 3/10*x^10*d^2*c*b^2 + 1/5*x^10*d^3*b*a + 3/8*x^8*d*c^2*
b^2 + 3/4*x^8*d^2*c*b*a + 1/8*x^8*d^3*a^2 + 1/6*x^6*c^3*b^2 + x^6*d*c^2*b*a
 + 1/2*x^6*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*x^4*d*c^2*a^2 + 1/2*x^2*c^3*a^
2
```

giac [**B**] time = 0.32, size = 134, normalized size = 1.89

$$\frac{1}{12}b^2d^3x^{12}+\frac{3}{10}b^2cd^2x^{10}+\frac{1}{5}abd^3x^{10}+\frac{3}{8}b^2c^2dx^8+\frac{3}{4}abcd^2x^8+\frac{1}{8}a^2d^3x^8+\frac{1}{6}b^2c^3x^6+abc^2dx^6+\frac{1}{2}a^2cd^2x^6+\frac{1}{2}abc^3x^4+\frac{3}{4}a^2c^2dx^4+\frac{1}{2}a^2c^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/12*b^2*d^3*x^12 + 3/10*b^2*c*d^2*x^10 + 1/5*a*b*d^3*x^10 + 3/8*b^2*c^2*d*
x^8 + 3/4*a*b*c*d^2*x^8 + 1/8*a^2*d^3*x^8 + 1/6*b^2*c^3*x^6 + a*b*c^2*d*x^6
 + 1/2*a^2*c*d^2*x^6 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + 1/2*a^2*c^3*x^
2
```

maple [A] time = 0.00, size = 128, normalized size = 1.80

$$\frac{b^2d^3x^{12}}{12}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{10}}{10}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^8}{8}+\frac{a^2c^3x^2}{2}+\frac{\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^6}{6}+\frac{\left(3a^2c^2d+2ab\,c^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/12*b^2*d^3*x^12+1/10*(2*a*b*d^3+3*b^2*c*d^2)*x^10+1/8*(a^2*d^3+6*a*b*c*d^
2+3*b^2*c^2*d)*x^8+1/6*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^6+1/4*(3*a^2*c^2
*d+2*a*b*c^3)*x^4+1/2*a^2*c^3*x^2
```

maxima [A] time = 1.06, size = 127, normalized size = 1.79

$$\frac{1}{12}\,b^2d^3x^{12}+\frac{1}{10}\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+\frac{1}{8}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+\frac{1}{2}\,a^2c^3x^2+\frac{1}{6}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

mupad [B] time = 0.04, size = 118, normalized size = 1.66

$$x^6\left(\frac{a^2\,c\,d^2}{2}+a\,b\,c^2\,d+\frac{b^2\,c^3}{6}\right)+x^8\left(\frac{a^2\,d^3}{8}+\frac{3\,a\,b\,c\,d^2}{4}+\frac{3\,b^2\,c^2\,d}{8}\right)+\frac{a^2\,c^3\,x^2}{2}+\frac{b^2\,d^3\,x^{12}}{12}+\frac{a\,c^2\,x^4\,\left(3\,a\,d+2\,b\,c\right)}{4}+\frac{b\,d^2\,x^{10}\,\left(2\,a\,d+3\,b\,c\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^6*((b^2*c^3)/6 + (a^2*c*d^2)/2 + a*b*c^2*d) + x^8*((a^2*d^3)/8 + (3*b^2*c
^2*d)/8 + (3*a*b*c*d^2)/4) + (a^2*c^3*x^2)/2 + (b^2*d^3*x^12)/12 + (a*c^2*x
^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^10*(2*a*d + 3*b*c))/10
```

sympy [B] time = 0.09, size = 136, normalized size = 1.92

$$\frac{a^2c^3x^2}{2}+\frac{b^2d^3x^{12}}{12}+x^{10}\left(\frac{abd^3}{5}+\frac{3b^2cd^2}{10}\right)+x^8\left(\frac{a^2d^3}{8}+\frac{3abcd^2}{4}+\frac{3b^2c^2d}{8}\right)+x^6\left(\frac{a^2cd^2}{2}+abc^2d+\frac{b^2c^3}{6}\right)+x^4\left(\frac{3a^2c^2d}{4}+\frac{abc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x**2/2 + b**2*d**3*x**12/12 + x**10*(a*b*d**3/5 + 3*b**2*c*d**2/1
0) + x**8*(a**2*d**3/8 + 3*a*b*c*d**2/4 + 3*b**2*c**2*d/8) + x**6*(a**2*c*d
**2/2 + a*b*c**2*d + b**2*c**3/6) + x**4*(3*a**2*c**2*d/4 + a*b*c**3/2)
```

3.163 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b$$

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+d\left(3b^2c^2+6abcd+\right.\right.\\ &= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5+\frac{1}{7}d\left(3b^2c^2+6abcd\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [A] time = 0.59, size = 131, normalized size = 1.07

$$\frac{1}{11}x^{11}d^3b^2 + \frac{1}{3}x^9d^2cb^2 + \frac{2}{9}x^9d^3ba + \frac{3}{7}x^7dc^2b^2 + \frac{6}{7}x^7d^2cba + \frac{1}{7}x^7d^3a^2 + \frac{1}{5}x^5c^3b^2 + \frac{6}{5}x^5dc^2ba + \frac{3}{5}x^5d^2ca^2 + \frac{2}{3}x^3c^3ba + x^3dc^2a^2 + xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^3*b^2 + 1/3*x^9*d^2*c*b^2 + 2/9*x^9*d^3*b*a + 3/7*x^7*d*c^2*b^2
 + 6/7*x^7*d^2*c*b*a + 1/7*x^7*d^3*a^2 + 1/5*x^5*c^3*b^2 + 6/5*x^5*d*c^2*b*
a + 3/5*x^5*d^2*c*a^2 + 2/3*x^3*c^3*b*a + x^3*d*c^2*a^2 + x*c^3*a^2
```

giac [A] time = 0.34, size = 131, normalized size = 1.07

$$\frac{1}{11}b^2d^3x^{11} + \frac{1}{3}b^2cd^2x^9 + \frac{2}{9}abd^3x^9 + \frac{3}{7}b^2c^2dx^7 + \frac{6}{7}abcd^2x^7 + \frac{1}{7}a^2d^3x^7 + \frac{1}{5}b^2c^3x^5 + \frac{6}{5}abc^2dx^5 + \frac{3}{5}a^2cd^2x^5 + \frac{2}{3}abc^3x^3 + a^2c^2dx^3 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

maple [A] time = 0.00, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{11}}{11} + \frac{\left(2abd^3+3b^2cd^2\right)x^9}{9} + \frac{\left(a^2d^3+6abcd^2+3b^2c^2d\right)x^7}{7} + a^2c^3x + \frac{\left(3a^2cd^2+6abc^2d+b^2c^3\right)x^5}{5} + \frac{\left(3a^2c^2d+2abc^3\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*a*b*c*d^2+
3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^5+1/3*(3*a^2*c^2*d
+2*a*b*c^3)*x^3+a^2*c^3*x
```

maxima [A] time = 1.01, size = 124, normalized size = 1.02

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\left(3\,b^2cd^2+2\,abd^3\right)x^9+\frac{1}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+\frac{1}{3}\left(2\,abc^3+3\,a^2c^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

mupad [B] time = 0.04, size = 116, normalized size = 0.95

$$x^5\left(\frac{3\,a^2\,c\,d^2}{5}+\frac{6\,a\,b\,c^2\,d}{5}+\frac{b^2\,c^3}{5}\right)+x^7\left(\frac{a^2\,d^3}{7}+\frac{6\,a\,b\,c\,d^2}{7}+\frac{3\,b^2\,c^2\,d}{7}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{11}}{11}+\frac{a\,c^2\,x^3\,\left(3\,a\,d+2\,b\,c\right)}{3}+\frac{b\,d^2\,x^9\,\left(2\,a\,d+3\,b\,c\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
(3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c^2
*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

sympy [A] time = 0.09, size = 136, normalized size = 1.11

$$a^2c^3x+\frac{b^2d^3x^{11}}{11}+x^9\left(\frac{2abd^3}{9}+\frac{b^2cd^2}{3}\right)+x^7\left(\frac{a^2d^3}{7}+\frac{6abcd^2}{7}+\frac{3b^2c^2d}{7}\right)+x^5\left(\frac{3a^2cd^2}{5}+\frac{6abc^2d}{5}+\frac{b^2c^3}{5}\right)+x^3\left(a^2c^2d+\frac{2abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

3.164 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x}\,dx$

Optimal. Leaf size=123

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+$$

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+\frac{1}{10}b^2d^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^3}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ac^2(2bc+3ad)+\frac{a^2c^3}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+d\left(3b^2c^2+6ab\right.\right.\right.\\ &= \frac{1}{2}ac^2(2bc+3ad)x^2+\frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+\frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)x^6\end{aligned}$$

Mathematica [A] time = 0.03, size = 123, normalized size = 1.00

$$\frac{1}{6}dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{4}cx^4\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3\log(x)+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{8}bd^2x^8(2ad+3bc)+\frac{1}{10}b^2d^3x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4
 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x^8
)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x, x]
```

fricas [A] time = 0.56, size = 125, normalized size = 1.02

$$\frac{1}{10}b^2d^3x^{10}+\frac{1}{8}\left(3b^2cd^2+2abd^3\right)x^8+\frac{1}{6}\left(3b^2c^2d+6abcd^2+a^2d^3\right)x^6+a^2c^3\log(x)+\frac{1}{4}\left(b^2c^3+6abc^2d+3a^2cd^2\right)x^4+\frac{1}{2}\left(2abc^3+3a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3*log(x) + 1/4*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

giac [A] time = 0.42, size = 134, normalized size = 1.09

$$\frac{1}{10}b^2d^3x^{10}+\frac{3}{8}b^2cd^2x^8+\frac{1}{4}abd^3x^8+\frac{1}{2}b^2c^2dx^6+abcd^2x^6+\frac{1}{6}a^2d^3x^6+\frac{1}{4}b^2c^3x^4+\frac{3}{2}abc^2dx^4+\frac{3}{4}a^2cd^2x^4+abc^3x^2+\frac{3}{2}a^2c^2dx^2+\frac{1}{2}a^2c^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^3*x^10 + 3/8*b^2*c*d^2*x^8 + 1/4*a*b*d^3*x^8 + 1/2*b^2*c^2*d*x^6
 + a*b*c*d^2*x^6 + 1/6*a^2*d^3*x^6 + 1/4*b^2*c^3*x^4 + 3/2*a*b*c^2*d*x^4 +
3/4*a^2*c*d^2*x^4 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + 1/2*a^2*c^3*log(x^2)
```

maple [A] time = 0.00, size = 132, normalized size = 1.07

$$\frac{b^2d^3x^{10}}{10}+\frac{ab\,d^3x^8}{4}+\frac{3b^2c\,d^2x^8}{8}+\frac{a^2d^3x^6}{6}+abc\,d^2x^6+\frac{b^2c^2d\,x^6}{2}+\frac{3a^2c\,d^2x^4}{4}+\frac{3ab\,c^2d\,x^4}{2}+\frac{b^2c^3x^4}{4}+\frac{3a^2c^2d\,x^2}{2}+ab\,c^3x^2+a^2c^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x,x)
```

```
[Out] 1/10*b^2*d^3*x^10+1/4*x^8*a*b*d^3+3/8*x^8*b^2*c*d^2+1/6*x^6*a^2*d^3+x^6*a*b
*c*d^2+1/2*x^6*b^2*c^2*d+3/4*x^4*a^2*c*d^2+3/2*x^4*a*b*c^2*d+1/4*x^4*b^2*c^
3+3/2*x^2*a^2*c^2*d+x^2*a*b*c^3+a^2*c^3*ln(x)
```

maxima [A] time = 1.10, size = 128, normalized size = 1.04

$$\frac{1}{10}\,b^2d^3x^{10}+\frac{1}{8}\left(3\,b^2cd^2+2\,abd^3\right)x^8+\frac{1}{6}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+\frac{1}{2}\,a^2c^3\log\left(x^2\right)+\frac{1}{4}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+\frac{1}{2}\left(2\,abc^3+3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 + 1/2*a^2*c^3*log(x^2) + 1/4*(b^2*c^3 + 6*a*b*c^
2*d + 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

mupad [B] time = 0.04, size = 116, normalized size = 0.94

$$x^4\left(\frac{3\,a^2\,c\,d^2}{4}+\frac{3\,a\,b\,c^2\,d}{2}+\frac{b^2\,c^3}{4}\right)+x^6\left(\frac{a^2\,d^3}{6}+a\,b\,c\,d^2+\frac{b^2\,c^2\,d}{2}\right)+\frac{b^2\,d^3\,x^{10}}{10}+a^2\,c^3\,\ln(x)+\frac{a\,c^2\,x^2\,\left(3\,a\,d+2\,b\,c\right)}{2}+\frac{b\,d^2\,x^8\,\left(2\,a\,d+3\,b\,c\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x,x)
```

```
[Out] x^4*((b^2*c^3)/4 + (3*a^2*c*d^2)/4 + (3*a*b*c^2*d)/2) + x^6*((a^2*d^3)/6 +
(b^2*c^2*d)/2 + a*b*c*d^2) + (b^2*d^3*x^10)/10 + a^2*c^3*log(x) + (a*c^2*x^
2*(3*a*d + 2*b*c))/2 + (b*d^2*x^8*(2*a*d + 3*b*c))/8
```

sympy [A] time = 0.23, size = 133, normalized size = 1.08

$$a^2c^3\log\left(x\right)+\frac{b^2d^3x^{10}}{10}+x^8\left(\frac{abd^3}{4}+\frac{3b^2cd^2}{8}\right)+x^6\left(\frac{a^2d^3}{6}+abcd^2+\frac{b^2c^2d}{2}\right)+x^4\left(\frac{3a^2cd^2}{4}+\frac{3abc^2d}{2}+\frac{b^2c^3}{4}\right)+x^2\left(\frac{3a^2c^2d}{2}+abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x,x)
```

```
[Out] a**2*c**3*log(x) + b**2*d**3*x**10/10 + x**8*(a*b*d**3/4 + 3*b**2*c*d**2/8)
 + x**6*(a**2*d**3/6 + a*b*c*d**2 + b**2*c**2*d/2) + x**4*(3*a**2*c*d**2/4
+ 3*a*b*c**2*d/2 + b**2*c**3/4) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

3.165 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^2}\,dx$

Optimal. Leaf size=120

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^2}\,dx &= \int \Bigg(ac^2(2bc+3ad)+\frac{a^2c^3}{x^2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^2+d\left(3b^2c^2+6abcd+a^2\right.\\ &= -\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{5}d\left(3b^2c^2+6abcd+\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 120, normalized size = 1.00

$$\frac{1}{5}dx^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] -((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*
d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c +
 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2 \left(c+dx^2\right)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^2, x]
```

fricas [A] time = 0.57, size = 129, normalized size = 1.08

$$\frac{35\,b^2d^3x^{10} + 45\,\left(3\,b^2cd^2 + 2\,abd^3\right)x^8 + 63\,\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^6 - 315\,a^2c^3 + 105\,\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^4 + 315\,\left(2\,abc^3 + 3\,a^2c^2d\right)x^2}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^3*x^10 + 45*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 63*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 315*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d +
3*a^2*c*d^2)*x^4 + 315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x
```

giac [A] time = 0.38, size = 130, normalized size = 1.08

$$\frac{1}{9}\,b^2d^3x^9 + \frac{3}{7}\,b^2cd^2x^7 + \frac{2}{7}\,abd^3x^7 + \frac{3}{5}\,b^2c^2dx^5 + \frac{6}{5}\,abcd^2x^5 + \frac{1}{5}\,a^2d^3x^5 + \frac{1}{3}\,b^2c^3x^3 + 2\,abc^2dx^3 + a^2cd^2x^3 + 2\,abc^3x + 3\,a^2c^2dx - \frac{a^2c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^3*x^9 + 3/7*b^2*c*d^2*x^7 + 2/7*a*b*d^3*x^7 + 3/5*b^2*c^2*d*x^5 +
 6/5*a*b*c*d^2*x^5 + 1/5*a^2*d^3*x^5 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + 2*a*b*c^3*x + 3*a^2*c^2*d*x - a^2*c^3/x
```

maple [A] time = 0.00, size = 131, normalized size = 1.09

$$\frac{b^2d^3x^9}{9} + \frac{2ab\,d^3x^7}{7} + \frac{3b^2c\,d^2x^7}{7} + \frac{a^2d^3x^5}{5} + \frac{6abc\,d^2x^5}{5} + \frac{3b^2c^2d\,x^5}{5} + a^2c\,d^2x^3 + 2ab\,c^2d\,x^3 + \frac{b^2c^3x^3}{3} + 3a^2c^2dx + 2ab\,c^3x - \frac{a^2c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^2,x)
```

```
[Out] 1/9*b^2*d^3*x^9+2/7*x^7*a*b*d^3+3/7*x^7*b^2*c*d^2+1/5*x^5*a^2*d^3+6/5*x^5*a
*b*c*d^2+3/5*x^5*b^2*c^2*d+x^3*a^2*c*d^2+2*x^3*a*b*c^2*d+1/3*x^3*b^2*c^3+3*
a^2*c^2*d*x+2*a*b*c^3*x-a^2*c^3/x
```

maxima [A] time = 1.00, size = 124, normalized size = 1.03

$$\frac{1}{9}b^2d^3x^9+\frac{1}{7}\left(3b^2cd^2+2abd^3\right)x^7+\frac{1}{5}\left(3b^2c^2d+6abcd^2+a^2d^3\right)x^5-\frac{a^2c^3}{x}+\frac{1}{3}\left(b^2c^3+6abc^2d+3a^2cd^2\right)x^3+\left(2abc^3+3a^2c^2d\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^3*x^9 + 1/7*(3*b^2*c*d^2 + 2*a*b*d^3)*x^7 + 1/5*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^5 - a^2*c^3/x + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*x
```

mupad [B] time = 0.09, size = 115, normalized size = 0.96

$$x^3\left(a^2\,c\,d^2+2\,a\,b\,c^2\,d+\frac{b^2\,c^3}{3}\right)+x^5\left(\frac{a^2\,d^3}{5}+\frac{6\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{5}\right)-\frac{a^2\,c^3}{x}+\frac{b^2\,d^3\,x^9}{9}+\frac{b\,d^2\,x^7\,\left(2\,a\,d+3\,b\,c\right)}{7}+a\,c^2\,x\,\left(3\,a\,d+2\,b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x)
```

```
[Out] x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^5*((a^2*d^3)/5 + (3*b^2*c^2
*d)/5 + (6*a*b*c*d^2)/5) - (a^2*c^3)/x + (b^2*d^3*x^9)/9 + (b*d^2*x^7*(2*a*
d + 3*b*c))/7 + a*c^2*x*(3*a*d + 2*b*c)
```

sympy [A] time = 0.22, size = 131, normalized size = 1.09

$$-\frac{a^2c^3}{x}+\frac{b^2d^3x^9}{9}+x^7\left(\frac{2abd^3}{7}+\frac{3b^2cd^2}{7}\right)+x^5\left(\frac{a^2d^3}{5}+\frac{6abcd^2}{5}+\frac{3b^2c^2d}{5}\right)+x^3\left(a^2cd^2+2abc^2d+\frac{b^2c^3}{3}\right)+x\left(3a^2c^2d+2abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**2,x)
```

```
[Out] -a**2*c**3/x + b**2*d**3*x**9/9 + x**7*(2*a*b*d**3/7 + 3*b**2*c*d**2/7) + x
**5*(a**2*d**3/5 + 6*a*b*c*d**2/5 + 3*b**2*c**2*d/5) + x**3*(a**2*c*d**2 +
2*a*b*c**2*d + b**2*c**3/3) + x*(3*a**2*c**2*d + 2*a*b*c**3)
```

3.166 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}b^2$$

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}cx^2\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{2x^2}+ac^2\log(x)(3ad+2bc)+\frac{1}{6}bd^2x^6(2ad+3bc)+\frac{1}{8}b^2d^3x^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] -(a^2*c^3)/(2*x^2) + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2)/2 + (d*(3*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c + 2*a*d)*x^6)/6 + (b^2*
d^3*x^8)/8 + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2(c+dx)^3}{x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\Bigg(\int \bigg(c\left(b^2c^2+6abcd+3a^2d^2\right)+\frac{a^2c^3}{x^2}+\frac{ac^2(2bc+3ad)}{x}+d\big(3b^2c^2+6a\\ &= -\frac{a^2c^3}{2x^2}+\frac{1}{2}c\left(b^2c^2+6abcd+3a^2d^2\right)x^2+\frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4+\frac{1}{6}bd^2(3\end{aligned}$$

Mathematica [A] time = 0.05, size = 120, normalized size = 0.98

$$\frac{6a^2\left(-2c^3+6cd^2x^4+d^3x^6\right)+4abdx^4\left(18c^2+9cdx^2+2d^2x^4\right)+3b^2x^4\left(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6\right)}{24x^2}+ac^2\log(x)(3ad+2bc)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] (4*a*b*d*x^4*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 3*b^2*x^4*(4*c^3 + 6*c^2*d*
x^2 + 4*c*d^2*x^4 + d^3*x^6) + 6*a^2*(-2*c^3 + 6*c*d^2*x^4 + d^3*x^6))/(24*
x^2) + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^3, x]
```

fricas [A] time = 0.66, size = 131, normalized size = 1.07

$$\frac{3\,b^2d^3x^{10}+4\left(3\,b^2cd^2+2\,abd^3\right)x^8+6\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-12\,a^2c^3+12\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+24\left(2\,abc^3+3\,a^2c^2d\right)x^2\log(x)}{24\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*d^3*x^10 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 6*(3*b^2*c^2*d + 6
*a*b*c*d^2 + a^2*d^3)*x^6 - 12*a^2*c^3 + 12*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*
c*d^2)*x^4 + 24*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2*log(x))/x^2
```

giac [A] time = 0.32, size = 160, normalized size = 1.30

$$\frac{1}{8}\,b^2d^3x^8+\frac{1}{2}\,b^2cd^2x^6+\frac{1}{3}\,abd^3x^6+\frac{3}{4}\,b^2c^2dx^4+\frac{3}{2}\,abcd^2x^4+\frac{1}{4}\,a^2d^3x^4+\frac{1}{2}\,b^2c^3x^2+3\,abc^2dx^2+\frac{3}{2}\,a^2cd^2x^2+\frac{1}{2}\,\left(2\,abc^3+3\,a^2c^2d\right)\log\left(x^2\right)-\frac{2\,abc^3x^2+3\,a^2c^2dx^2+a^2c^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/2*b^2*c*d^2*x^6 + 1/3*a*b*d^3*x^6 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/2*b^2*c^3*x^2 + 3*a*b*c^2*d*x^2 +
3/2*a^2*c*d^2*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2) - 1/2*(2*a*b*c^3
*x^2 + 3*a^2*c^2*d*x^2 + a^2*c^3)/x^2
```

maple [A] time = 0.01, size = 134, normalized size = 1.09

$$\frac{b^2d^3x^8}{8}+\frac{ab\,d^3x^6}{3}+\frac{b^2c\,d^2x^6}{2}+\frac{a^2d^3x^4}{4}+\frac{3abc\,d^2x^4}{2}+\frac{3b^2c^2d\,x^4}{4}+\frac{3a^2c\,d^2x^2}{2}+3ab\,c^2d\,x^2+\frac{b^2c^3x^2}{2}+3a^2c^2d\ln(x)+2ab\,c^3\ln(x)-\frac{a^2c^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^3,x)
```

```
[Out] 1/8*b^2*d^3*x^8+1/3*x^6*a*b*d^3+1/2*x^6*b^2*c*d^2+1/4*x^4*a^2*d^3+3/2*x^4*a
*b*c*d^2+3/4*x^4*b^2*c^2*d+3/2*x^2*a^2*c*d^2+3*x^2*a*b*c^2*d+1/2*x^2*b^2*c^
3-1/2*a^2*c^3/x^2+3*ln(x)*a^2*c^2*d+2*ln(x)*a*b*c^3
```

maxima [A] time = 1.06, size = 128, normalized size = 1.04

$$\frac{1}{8}\,b^2d^3x^8+\frac{1}{6}\,\left(3\,b^2cd^2+2\,abd^3\right)x^6+\frac{1}{4}\,\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^4-\frac{a^2c^3}{2\,x^2}+\frac{1}{2}\,\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^2+\frac{1}{2}\,\left(2\,abc^3+3\,a^2c^2d\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*d^3*x^8 + 1/6*(3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 1/4*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^4 - 1/2*a^2*c^3/x^2 + 1/2*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2)
```

mupad [B] time = 0.09, size = 121, normalized size = 0.98

$$x^2\,\left(\frac{3\,a^2\,c\,d^2}{2}+3\,a\,b\,c^2\,d+\frac{b^2\,c^3}{2}\right)+x^4\,\left(\frac{a^2\,d^3}{4}+\frac{3\,a\,b\,c\,d^2}{2}+\frac{3\,b^2\,c^2\,d}{4}\right)+\ln(x)\,\left(3\,d\,a^2\,c^2+2\,b\,a\,c^3\right)-\frac{a^2\,c^3}{2\,x^2}+\frac{b^2\,d^3\,x^8}{8}+\frac{b\,d^2\,x^6\,\left(2\,a\,d+3\,b\,c\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x)
```

```
[Out] x^2*((b^2*c^3)/2 + (3*a^2*c*d^2)/2 + 3*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b
^2*c^2*d)/4 + (3*a*b*c*d^2)/2) + log(x)*(3*a^2*c^2*d + 2*a*b*c^3) - (a^2*c^
3)/(2*x^2) + (b^2*d^3*x^8)/8 + (b*d^2*x^6*(2*a*d + 3*b*c))/6
```

sympy [A] time = 0.33, size = 133, normalized size = 1.08

$$-\frac{a^2c^3}{2x^2}+ac^2\left(3ad+2bc\right)\log\left(x\right)+\frac{b^2d^3x^8}{8}+x^6\left(\frac{abd^3}{3}+\frac{b^2cd^2}{2}\right)+x^4\left(\frac{a^2d^3}{4}+\frac{3abcd^2}{2}+\frac{3b^2c^2d}{4}\right)+x^2\left(\frac{3a^2cd^2}{2}+3abc^2d+\frac{b^2c^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**3,x)
```

```
[Out] -a**2*c**3/(2*x**2) + a*c**2*(3*a*d + 2*b*c)*log(x) + b**2*d**3*x**8/8 + x*
*6*(a*b*d**3/3 + b**2*c*d**2/2) + x**4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b*
*2*c**2*d/4) + x**2*(3*a**2*c*d**2/2 + 3*a*b*c**2*d + b**2*c**3/2)
```

3.167 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^4}\,dx$

Optimal. Leaf size=120

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -(a^2*c^3)/(3*x^3) - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^4}\,dx &= \int \left(c\left(b^2c^2+6abcd+3a^2d^2\right)+\frac{a^2c^3}{x^4}+\frac{ac^2(2bc+3ad)}{x^2}+d\left(3b^2c^2+6abcd+a^2d^2\right)\right.\\ &= -\frac{a^2c^3}{3x^3}-\frac{ac^2(2bc+3ad)}{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x+\frac{1}{3}d\left(3b^2c^2+6abcd+a^2d^2\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 120, normalized size = 1.00

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] -1/3*(a^2*c^3)/x^3 - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d + 3
*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c +
 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

```
[Out] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^4, x]
```

fricas [A] time = 0.71, size = 129, normalized size = 1.08

$$\frac{15\,b^{2}d^{3}x^{10}+21\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{8}+35\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{6}-35\,a^{2}c^{3}+105\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{4}-105\left(2\,abc^{3}+3\,a^{2}c^{2}d\right)x^{2}}{105\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^10 + 21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 35*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 35*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^4 - 105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

giac [A] time = 0.28, size = 129, normalized size = 1.08

$$\frac{1}{7}\,b^{2}d^{3}x^{7}+\frac{3}{5}\,b^{2}cd^{2}x^{5}+\frac{2}{5}\,abd^{3}x^{5}+b^{2}c^{2}dx^{3}+2\,abcd^{2}x^{3}+\frac{1}{3}\,a^{2}d^{3}x^{3}+b^{2}c^{3}x+6\,abc^{2}dx+3\,a^{2}cd^{2}x-\frac{6\,abc^{3}x^{2}+9\,a^{2}c^{2}dx^{2}+a^{2}c^{3}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^3*x^7 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + b^2*c^2*d*x^3 + 2*a
*b*c*d^2*x^3 + 1/3*a^2*d^3*x^3 + b^2*c^3*x + 6*a*b*c^2*d*x + 3*a^2*c*d^2*x
- 1/3*(6*a*b*c^3*x^2 + 9*a^2*c^2*d*x^2 + a^2*c^3)/x^3
```

maple [A] time = 0.01, size = 124, normalized size = 1.03

$$\frac{b^{2}d^{3}x^{7}}{7}+\frac{2ab\,d^{3}x^{5}}{5}+\frac{3b^{2}c\,d^{2}x^{5}}{5}+\frac{a^{2}d^{3}x^{3}}{3}+2abc\,d^{2}x^{3}+b^{2}c^{2}d\,x^{3}+3a^{2}c\,d^{2}x+6ab\,c^{2}dx+b^{2}c^{3}x-\frac{a^{2}c^{3}}{3x^{3}}-\frac{\left(3ad+2bc\right)a\,c^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^4,x)
```

```
[Out] 1/7*b^2*d^3*x^7+2/5*x^5*a*b*d^3+3/5*x^5*b^2*c*d^2+1/3*x^3*a^2*d^3+2*x^3*a*b
*c*d^2+x^3*b^2*c^2*d+3*a^2*c*d^2*x+6*a*b*c^2*d*x+b^2*c^3*x-a*c^2*(3*a*d+2*b
*c)/x-1/3*a^2*c^3/x^3
```

maxima [A] time = 1.09, size = 126, normalized size = 1.05

$$\frac{1}{7}\,b^2d^3x^7+\frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5+\frac{1}{3}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3+\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x-\frac{a^2c^3+3\left(2\,abc^3+3\,a^2c^2d\right)x^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^3*x^7 + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/3*(3*b^2*c^2*d + 6*
a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x - 1/3*(a
^2*c^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

mupad [B] time = 0.04, size = 121, normalized size = 1.01

$$x^3\left(\frac{a^2\,d^3}{3}+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)-\frac{x^2\,\left(3\,d\,a^2\,c^2+2\,b\,a\,c^3\right)+\frac{a^2\,c^3}{3}}{x^3}+x\,\left(3\,a^2\,c\,d^2+6\,a\,b\,c^2\,d+b^2\,c^3\right)+\frac{b^2\,d^3\,x^7}{7}+\frac{b\,d^2\,x^5\,\left(2\,a\,d+3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x)
```

```
[Out] x^3*((a^2*d^3)/3 + b^2*c^2*d + 2*a*b*c*d^2) - (x^2*(3*a^2*c^2*d + 2*a*b*c^3
) + (a^2*c^3)/3)/x^3 + x*(b^2*c^3 + 3*a^2*c*d^2 + 6*a*b*c^2*d) + (b^2*d^3*x
^7)/7 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

sympy [A] time = 0.35, size = 131, normalized size = 1.09

$$\frac{b^2d^3x^7}{7}+x^5\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}\right)+x^3\left(\frac{a^2d^3}{3}+2abcd^2+b^2c^2d\right)+x\left(3a^2cd^2+6abc^2d+b^2c^3\right)+\frac{-a^2c^3+x^2\left(-9a^2c^2d-6abc^3\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**4,x)
```

```
[Out] b**2*d**3*x**7/7 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x**3*(a**2*d**3/
3 + 2*a*b*c*d**2 + b**2*c**2*d) + x*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c*
*3) + (-a**2*c**3 + x**2*(-9*a**2*c**2*d - 6*a*b*c**3))/(3*x**3)
```

3.168 $\int \frac{x^4\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=104

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{cx(bc-ad)^2}{d^4}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a*d)
*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/d^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(bc-ad)^2x^2}{d^3}-\frac{b(bc-2ad)x^4}{d^2}+\frac{b^2x^6}{d}+\frac{b^2c^4-2abc^3d+a^2c^2d^2}{d^4\left(c+dx^2\right)}\right)dx\\
&= -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{\left(c^2(bc-ad)^2\right)\int \frac{1}{c+dx^2}\,dx}{d^4}\\
&= -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 104, normalized size = 1.00

$$\frac{c^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(ad-bc)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -((c*(b*c - a*d)^2*x)/d^4) + ((-(b*c) + a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a
*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/d^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^2\right)^2}{c+dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2), x]
```

fricas [A] time = 0.63, size = 302, normalized size = 2.90

$$\left[\frac{30\,b^2d^3x^7-42\left(b^2cd^2-2\,abd^3\right)x^5+70\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^3+105\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{-\frac{c}{d}}\,\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-210\left(b^2c^3-2\,abc^2d+a^2cd^2\right)x}{210\,d^4},\ \frac{15\,b^2d^3x^7-21\left(b^2cd^2-2\,abd^3\right)x^5+35\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^3+105\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{\frac{c}{d}}\,\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-105\left(b^2c^3-2\,abc^2d+a^2cd^2\right)x}{105\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^2*d^3*x^7 - 42*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 70*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(-c
/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 210*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*x)/d^4, 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^
3)*x^5 + 35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) - 105*(b^2*c^3 - 2*a*b
*c^2*d + a^2*c*d^2)*x)/d^4]
```

giac [A] time = 0.31, size = 153, normalized size = 1.47

$$\frac{\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^4}+\frac{15\,b^2d^6x^7-21\,b^2cd^5x^5+42\,abd^6x^5+35\,b^2c^2d^4x^3-70\,abcd^5x^3+35\,a^2d^6x^3-105\,b^2c^3d^3x+210\,abc^2d^4x-105\,a^2cd^5x}{105\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4)
 + 1/105*(15*b^2*d^6*x^7 - 21*b^2*c*d^5*x^5 + 42*a*b*d^6*x^5 + 35*b^2*c^2*d
^4*x^3 - 70*a*b*c*d^5*x^3 + 35*a^2*d^6*x^3 - 105*b^2*c^3*d^3*x + 210*a*b*c^
2*d^4*x - 105*a^2*c*d^5*x)/d^7
```

maple [A] time = 0.01, size = 176, normalized size = 1.69

$$\frac{b^2x^7}{7d}+\frac{2ab\,x^5}{5d}-\frac{b^2c\,x^5}{5d^2}+\frac{a^2x^3}{3d}-\frac{2abc\,x^3}{3d^2}+\frac{b^2c^2x^3}{3d^3}+\frac{a^2c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^2}-\frac{2ab\,c^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^3}+\frac{b^2c^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^4}-\frac{a^2cx}{d^2}+\frac{2ab\,c^2x}{d^3}-\frac{b^2c^3x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/7*b^2*x^7/d+2/5/d*x^5*a*b-1/5/d^2*x^5*b^2*c+1/3/d*x^3*a^2-2/3/d^2*x^3*a*b
*c+1/3/d^3*x^3*b^2*c^2-1/d^2*a^2*c*x+2/d^3*a*b*c^2*x-1/d^4*b^2*c^3*x+c^2/d^
2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))*a^2-2*c^3/d^3/(c*d)^(1/2)*arctan(d*x/
(c*d)^(1/2))*a*b+c^4/d^4/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))*b^2
```

maxima [A] time = 2.41, size = 139, normalized size = 1.34

$$\frac{\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^4}+\frac{15\,b^2d^3x^7-21\,\left(b^2cd^2-2\,abd^3\right)x^5+35\,\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^3-105\,\left(b^2c^3-2\,abc^2d+a^2cd^2\right)x}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4)
 + 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 35*(b^2*c^2*d -
```

```
 2*a*b*c*d^2 + a^2*d^3)*x^3 - 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x)/d^
4
```

mupad [B] time = 0.06, size = 169, normalized size = 1.62

$$x^3\left(\frac{a^2}{3\,d}+\frac{c\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{3\,d}\right)-x^5\,\left(\frac{b^2\,c}{5\,d^2}-\frac{2\,a\,b}{5\,d}\right)+\frac{b^2\,x^7}{7\,d}+\frac{c^{3/2}\,\mathrm{atan}\left(\frac{c^{3/2}\,\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4}\right)\,{\left(a\,d-b\,c\right)}^2}{d^{9/2}}-\frac{c\,x\,\left(\frac{a^2}{d}+\frac{c\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^3*(a^2/(3*d) + (c*((b^2*c)/d^2 - (2*a*b)/d))/(3*d)) - x^5*((b^2*c)/(5*d^2
) - (2*a*b)/(5*d)) + (b^2*x^7)/(7*d) + (c^(3/2)*atan((c^(3/2)*d^(1/2)*x*(a*
d - b*c)^2)/(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))*(a*d - b*c)^2)/d^(9/2) -
 (c*x*(a^2/d + (c*((b^2*c)/d^2 - (2*a*b)/d))/d))/d
```

sympy [B] time = 0.53, size = 246, normalized size = 2.37

$$\frac{b^{2} x^{7}}{7 d} + x^{5} \left(\frac{2 a b}{5 d} - \frac{b^{2} c}{5 d^{2}}\right) + x^{3} \left(\frac{a^{2}}{3 d} - \frac{2 a b c}{3 d^{2}} + \frac{b^{2} c^{2}}{3 d^{3}}\right) + x \left(- \frac{a^{2} c}{d^{2}} + \frac{2 a b c^{2}}{d^{3}} - \frac{b^{2} c^{3}}{d^{4}}\right) - \frac{\sqrt{- \frac{c^{3}}{d^{9}}} \left(a d - b c\right)^{2} \log{\left(- \frac{d^{4} \sqrt{- \frac{c^{3}}{d^{9}}} \left(a d - b c\right)^{2}}{a^{2} c d^{2} - 2 a b c^{2} d + b^{2} c^{3}} + x \right)}}{2} + \frac{\sqrt{- \frac{c^{3}}{d^{9}}} \left(a d - b c\right)^{2} \log{\left(\frac{d^{4} \sqrt{- \frac{c^{3}}{d^{9}}} \left(a d - b c\right)^{2}}{a^{2} c d^{2} - 2 a b c^{2} d + b^{2} c^{3}} + x \right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**7/(7*d) + x**5*(2*a*b/(5*d) - b**2*c/(5*d**2)) + x**3*(a**2/(3*d) -
 2*a*b*c/(3*d**2) + b**2*c**2/(3*d**3)) + x*(-a**2*c/d**2 + 2*a*b*c**2/d**3
 - b**2*c**3/d**4) - sqrt(-c**3/d**9)*(a*d - b*c)**2*log(-d**4*sqrt(-c**3/d
**9)*(a*d - b*c)**2/(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2 + sqrt(
-c**3/d**9)*(a*d - b*c)**2*log(d**4*sqrt(-c**3/d**9)*(a*d - b*c)**2/(a**2*c
*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2
```

3.169 $\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx$

Optimal. Leaf size=79

$$-\frac{c(bc-ad)^2\log\left(c+dx^2\right)}{2d^4}+\frac{x^2(bc-ad)^2}{2d^3}-\frac{bx^4(bc-2ad)}{4d^2}+\frac{b^2x^6}{6d}$$

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{bx^4(bc-2ad)}{4d^2}+\frac{x^2(bc-ad)^2}{2d^3}-\frac{c(bc-ad)^2\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^6}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*d^3) - (b*(b*c - 2*a*d)*x^4)/(4*d^2) + (b^2*x^6)/(6*
d) - (c*(b*c - a*d)^2*Log[c + d*x^2])/(2*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3}-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^2}{d}-\frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2x^2}{2d^3}-\frac{b(bc-2ad)x^4}{4d^2}+\frac{b^2x^6}{6d}-\frac{c(bc-ad)^2\log\left(c+dx^2\right)}{2d^4}\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 82, normalized size = 1.04

$$\frac{dx^2\left(6a^2d^2+6abd\left(dx^2-2c\right)+b^2\left(6c^2-3cdx^2+2d^2x^4\right)\right)-6c(bc-ad)^2\log\left(c+dx^2\right)}{12d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (d*x^2*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^2) + b^2*(6*c^2 - 3*c*d*x^2 + 2*d^2
*x^4)) - 6*c*(b*c - a*d)^2*Log[c + d*x^2])/(12*d^4)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+bx^2\right)^2}{c+dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2), x]
```

fricas [**A**] time = 0.46, size = 101, normalized size = 1.28

$$\frac{2\,b^2d^3x^6-3\left(b^2cd^2-2\,abd^3\right)x^4+6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2-6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(dx^2+c\right)}{12\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^4 + 6*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*x^2 - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 + c)
)/d^4
```

giac [A] time = 0.34, size = 107, normalized size = 1.35

$$\frac{2\,b^2d^2x^6-3\,b^2cdx^4+6\,abd^2x^4+6\,b^2c^2x^2-12\,abcdx^2+6\,a^2d^2x^2}{12\,d^3}-\frac{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(\left|dx^2+c\right|\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*b^2*c*d*x^4 + 6*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a*
b*c*d*x^2 + 6*a^2*d^2*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*lo
g(abs(d*x^2 + c))/d^4
```

maple [A] time = 0.00, size = 124, normalized size = 1.57

$$\frac{b^2x^6}{6d}+\frac{ab\,x^4}{2d}-\frac{b^2c\,x^4}{4d^2}+\frac{a^2x^2}{2d}-\frac{abc\,x^2}{d^2}+\frac{b^2c^2x^2}{2d^3}-\frac{a^2c\ln\left(d\,x^2+c\right)}{2d^2}+\frac{ab\,c^2\ln\left(d\,x^2+c\right)}{d^3}-\frac{b^2c^3\ln\left(d\,x^2+c\right)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/6*b^2*x^6/d+1/2/d*x^4*a*b-1/4/d^2*x^4*b^2*c+1/2/d*x^2*a^2-1/d^2*x^2*a*b*c
+1/2/d^3*x^2*b^2*c^2-1/2*c/d^2*ln(d*x^2+c)*a^2+c^2/d^3*ln(d*x^2+c)*a*b-1/2*
c^3/d^4*ln(d*x^2+c)*b^2
```

maxima [A] time = 1.05, size = 100, normalized size = 1.27

$$\frac{2\,b^2d^2x^6-3\left(b^2cd-2\,abd^2\right)x^4+6\left(b^2c^2-2\,abcd+a^2d^2\right)x^2}{12\,d^3}-\frac{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(dx^2+c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 - 3*(b^2*c*d - 2*a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 + c
)/d^4
```

mupad [B] time = 0.06, size = 106, normalized size = 1.34

$$x^2\left(\frac{a^2}{2\,d}+\frac{c\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{2\,d}\right)-x^4\,\left(\frac{b^2\,c}{4\,d^2}-\frac{a\,b}{2\,d}\right)+\frac{b^2\,x^6}{6\,d}-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^2*(a^2/(2*d) + (c*((b^2*c)/d^2 - (2*a*b)/d))/(2*d)) - x^4*((b^2*c)/(4*d^2
) - (a*b)/(2*d)) + (b^2*x^6)/(6*d) - (log(c + d*x^2)*(b^2*c^3 + a^2*c*d^2 -
 2*a*b*c^2*d))/(2*d^4)
```

sympy [A] time = 0.42, size = 83, normalized size = 1.05

$$\frac{b^2x^6}{6d} - \frac{c\left(ad - bc\right)^2 \log\left(c + dx^2\right)}{2d^4} + x^4\left(\frac{ab}{2d} - \frac{b^2c}{4d^2}\right) + x^2\left(\frac{a^2}{2d} - \frac{abc}{d^2} + \frac{b^2c^2}{2d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**6/(6*d) - c*(a*d - b*c)**2*log(c + d*x**2)/(2*d**4) + x**4*(a*b/(2*
d) - b**2*c/(4*d**2)) + x**2*(a**2/(2*d) - a*b*c/d**2 + b**2*c**2/(2*d**3))
```

3.170 $\int \frac{x^2\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(bc-ad)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{bx^3(bc-2ad)}{3d^2}+\frac{x(bc-ad)^2}{d^3}-\frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d) - (
Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(\frac{(bc-ad)^2}{d^3}-\frac{b(bc-2ad)x^2}{d^2}+\frac{b^2x^4}{d}+\frac{-b^2c^3+2abc^2d-a^2cd^2}{d^3\left(c+dx^2\right)}\right)dx\\
&= \frac{(bc-ad)^2x}{d^3}-\frac{b(bc-2ad)x^3}{3d^2}+\frac{b^2x^5}{5d}-\frac{\left(c(bc-ad)^2\right)\int \frac{1}{c+dx^2}\,dx}{d^3}\\
&= \frac{(bc-ad)^2x}{d^3}-\frac{b(bc-2ad)x^3}{3d^2}+\frac{b^2x^5}{5d}-\frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 83, normalized size = 1.00

$$
-\frac{\sqrt{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(ad-bc)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] ((-(b*c) + a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d)
- (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\left(a+bx^2\right)^2}{c+dx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2), x]
```

fricas [A] time = 0.57, size = 228, normalized size = 2.75

$$
\left[\frac{6\,b^2d^2x^5-10\left(b^2cd-2\,abd^2\right)x^3+15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{c}{d}}\,\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+30\left(b^2c^2-2\,abcd+a^2d^2\right)x}{30\,d^3},\ \frac{3\,b^2d^2x^5-5\left(b^2cd-2\,abd^2\right)x^3-15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{\frac{c}{d}}\,\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+15\left(b^2c^2-2\,abcd+a^2d^2\right)x}{15\,d^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 - 10*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) +
```

```
30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3, 1/15*(3*b^2*d^2*x^5 - 5*(b^2*c*d
 - 2*a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c/d)*arctan(d*x
*sqrt(c/d)/c) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3]
```

giac [A] time = 0.26, size = 113, normalized size = 1.36

$$-\frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3} + \frac{3\,b^2d^4x^5 - 5\,b^2cd^3x^3 + 10\,abd^4x^3 + 15\,b^2c^2d^2x - 30\,abcd^3x + 15\,a^2d^4x}{15\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
+ 1/15*(3*b^2*d^4*x^5 - 5*b^2*c*d^3*x^3 + 10*a*b*d^4*x^3 + 15*b^2*c^2*d^2*x
 - 30*a*b*c*d^3*x + 15*a^2*d^4*x)/d^5
```

maple [A] time = 0.00, size = 135, normalized size = 1.63

$$\frac{b^2x^5}{5d} + \frac{2ab\,x^3}{3d} - \frac{b^2c\,x^3}{3d^2} - \frac{a^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d} + \frac{2ab\,c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^2} - \frac{b^2c^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^3} + \frac{a^2x}{d} - \frac{2abcx}{d^2} + \frac{b^2c^2x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2/3/d*x^3*a*b-1/3/d^2*x^3*b^2*c+1/d*a^2*x-2/d^2*a*b*c*x+1/d^3
*b^2*c^2*x-c/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+2*c^2/d^2/(c*d)^(1
/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b-c^3/d^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*
d*x)*b^2
```

maxima [A] time = 2.42, size = 104, normalized size = 1.25

$$-\frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3} + \frac{3\,b^2d^2x^5 - 5\left(b^2cd - 2\,abd^2\right)x^3 + 15\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
+ 1/15*(3*b^2*d^2*x^5 - 5*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x)/d^3
```

mupad [B] time = 0.06, size = 128, normalized size = 1.54

$$x\left(\frac{a^2}{d}+\frac{c\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{d}\right)-x^3\left(\frac{b^2\,c}{3\,d^2}-\frac{2\,a\,b}{3\,d}\right)+\frac{b^2\,x^5}{5\,d}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,x\,(a\,d-b\,c)^2}{a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3}\right)(a\,d-b\,c)^2}{d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x*(a^2/d + (c*((b^2*c)/d^2 - (2*a*b)/d))/d) - x^3*((b^2*c)/(3*d^2) - (2*a*b
)/(3*d)) + (b^2*x^5)/(5*d) - (c^(1/2)*atan((c^(1/2)*d^(1/2)*x*(a*d - b*c)^2
)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))*(a*d - b*c)^2)/d^(7/2)
```

sympy [B] time = 0.49, size = 194, normalized size = 2.34

$$\frac{b^2x^5}{5d}+x^3\left(\frac{2ab}{3d}-\frac{b^2c}{3d^2}\right)+x\left(\frac{a^2}{d}-\frac{2abc}{d^2}+\frac{b^2c^2}{d^3}\right)+\frac{\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2\log\left(-\frac{d^3\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2\log\left(\frac{d^3\sqrt{-\frac{c}{d^7}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**5/(5*d) + x**3*(2*a*b/(3*d) - b**2*c/(3*d**2)) + x*(a**2/d - 2*a*b*
c/d**2 + b**2*c**2/d**3) + sqrt(-c/d**7)*(a*d - b*c)**2*log(-d**3*sqrt(-c/d
**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-c/d*
*7)*(a*d - b*c)**2*log(d**3*sqrt(-c/d**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b
*c*d + b**2*c**2) + x)/2
```

3.171 $\int \frac{x\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=61

$$\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}-\frac{bx^2(bc-ad)}{2d^2}+\frac{\left(a+bx^2\right)^2}{4d}$$

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$-\frac{bx^2(bc-ad)}{2d^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}+\frac{\left(a+bx^2\right)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] -(b*(b*c - a*d)*x^2)/(2*d^2) + (a + b*x^2)^2/(4*d) + ((b*c - a*d)^2*Log[c +
 d*x^2])/(2*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx,x,x^2\right)\\
&= -\frac{b(bc-ad)x^2}{2d^2}+\frac{\left(a+bx^2\right)^2}{4d}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2d^3}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.80

$$\frac{bdx^2\left(4ad-2bc+bdx^2\right)+2(bc-ad)^2\log\left(c+dx^2\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (b*d*x^2*(-2*b*c + 4*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[c + d*x^2])/(4*d^
3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^2\right)^2}{c+dx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2), x]
```

fricas [A] time = 0.90, size = 66, normalized size = 1.08

$$\frac{b^2d^2x^4-2\left(b^2cd-2\,abd^2\right)x^2+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 - 2*(b^2*c*d - 2*a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(d*x^2 + c))/d^3
```

giac [A] time = 0.32, size = 67, normalized size = 1.10

$$\frac{b^2dx^4 - 2\,b^2cx^2 + 4\,abdx^2}{4\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|dx^2 + c\right|\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*b^2*c*x^2 + 4*a*b*d*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(d*x^2 + c))/d^3
```

maple [A] time = 0.00, size = 85, normalized size = 1.39

$$\frac{b^2x^4}{4d} + \frac{ab\,x^2}{d} - \frac{b^2c\,x^2}{2d^2} + \frac{a^2\ln\left(d\,x^2 + c\right)}{2d} - \frac{abc\ln\left(d\,x^2 + c\right)}{d^2} + \frac{b^2c^2\ln\left(d\,x^2 + c\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/4*b^2/d*x^4+b/d*a*x^2-1/2*b^2/d^2*c*x^2+1/2/d*ln(d*x^2+c)*a^2-1/d^2*ln(d*
x^2+c)*a*b*c+1/2/d^3*ln(d*x^2+c)*b^2*c^2
```

maxima [A] time = 1.03, size = 65, normalized size = 1.07

$$\frac{b^2dx^4 - 2\left(b^2c - 2\,abd\right)x^2}{4\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx^2 + c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*d*x^4 - 2*(b^2*c - 2*a*b*d)*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(d*x^2 + c)/d^3
```

mupad [B] time = 0.12, size = 68, normalized size = 1.11

$$\frac{b^2\,x^4}{4\,d} - x^2\left(\frac{b^2\,c}{2\,d^2} - \frac{a\,b}{d}\right) + \frac{\ln\left(d\,x^2 + c\right)\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] (b^2*x^4)/(4*d) - x^2*((b^2*c)/(2*d^2) - (a*b)/d) + (log(c + d*x^2)*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d))/(2*d^3)
```

sympy [A] time = 0.36, size = 49, normalized size = 0.80

$$\frac{b^2x^4}{4d}+x^2\left(\frac{ab}{d}-\frac{b^2c}{2d^2}\right)+\frac{(ad-bc)^2\log\left(c+dx^2\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**4/(4*d) + x**2*(a*b/d - b**2*c/(2*d**2)) + (a*d - b*c)**2*log(c + d
*x**2)/(2*d**3)
```

3.172 $\int \frac{\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$-\frac{bx(bc-2ad)}{d^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}+\frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^2}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^2\right)}\right)dx \\
&= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^2}\,dx}{d^2} \\
&= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.94

$$
\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{5/2}}+\frac{bx\left(6ad-3bc+bdx^2\right)}{3d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^2}{c+dx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2), x]
```

fricas [A] time = 0.84, size = 179, normalized size = 2.84

$$
\left[\frac{2\,b^2cd^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)-6\left(b^2c^2d-2\,abcd^2\right)x}{6\,cd^3},\frac{b^2cd^2x^3+3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-3\left(b^2c^2d-2\,abcd^2\right)x}{3\,cd^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

giac [A] time = 0.28, size = 72, normalized size = 1.14

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{b^2d^2x^3-3\,b^2cdx+6\,abd^2x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

maple [A] time = 0.00, size = 95, normalized size = 1.51

$$\frac{b^2x^3}{3d}+\frac{a^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{2abc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d}+\frac{b^2c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d^2}+\frac{2abx}{d}-\frac{b^2cx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/3*b^2*x^3/d+2*b/d*a*x-b^2/d^2*c*x+1/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)
*a^2-2/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*c+1/d^2/(c*d)^(1/2)*arct
an(1/(c*d)^(1/2)*d*x)*b^2*c^2
```

maxima [A] time = 2.34, size = 68, normalized size = 1.08

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}+\frac{b^2dx^3-3\left(b^2c-2\,abd\right)x}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

mupad [B] time = 0.07, size = 90, normalized size = 1.43

$$\frac{b^2\,x^3}{3\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2),x)
```

```
[Out] (b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*c
)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d^(
5/2))
```

sympy [B] time = 0.42, size = 172, normalized size = 2.73

$$\frac{b^2x^3}{3d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) - \frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c)
**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b
**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

3.173 $\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=51

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}+\frac{b^2x^2}{2d}$$

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}+\frac{b^2x^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*x^2)/(2*d) + (a^2*Log[x])/c - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^2}{d}+\frac{a^2}{cx}-\frac{(bc-ad)^2}{cd(c+dx)}\right)\,dx,x,x^2\right)\\
&= \frac{b^2x^2}{2d}+\frac{a^2\log(x)}{c}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2cd^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.98

$$\frac{2a^2d^2\log(x)-(bc-ad)^2\log\left(c+dx^2\right)+b^2cdx^2}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] (b^2*c*d*x^2 + 2*a^2*d^2*Log[x] - (b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)), x]
```

fricas [A] time = 0.85, size = 59, normalized size = 1.16

$$\frac{b^2cdx^2+2\,a^2d^2\log(x)-\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*d*x^2 + 2*a^2*d^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d
*x^2 + c))/(c*d^2)
```

giac [A] time = 0.32, size = 62, normalized size = 1.22

$$\frac{b^2x^2}{2\,d}+\frac{a^2\log\left(x^2\right)}{2\,c}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(d*x^2 + c))/(c*d^2)
```

maple [A] time = 0.00, size = 69, normalized size = 1.35

$$\frac{b^2x^2}{2d}+\frac{a^2\ln(x)}{c}-\frac{a^2\ln\left(d\,x^2+c\right)}{2c}+\frac{ab\ln\left(d\,x^2+c\right)}{d}-\frac{b^2c\ln\left(d\,x^2+c\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c),x)
```

```
[Out] 1/2*b^2*x^2/d-1/2/c*ln(d*x^2+c)*a^2+1/d*ln(d*x^2+c)*a*b-1/2*c/d^2*ln(d*x^2+
c)*b^2+a^2*ln(x)/c
```

maxima [A] time = 1.10, size = 61, normalized size = 1.20

$$\frac{b^2x^2}{2\,d}+\frac{a^2\log\left(x^2\right)}{2\,c}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(d*x^2 + c)/(c*d^2)
```

mupad [B] time = 0.16, size = 58, normalized size = 1.14

$$\frac{b^2\,x^2}{2\,d}+\frac{a^2\,\ln(x)}{c}-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)),x)
```

```
[Out] (b^2*x^2)/(2*d) + (a^2*log(x))/c - (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/(2*c*d^2)
```

sympy [A] time = 1.21, size = 41, normalized size = 0.80

$$\frac{a^2\log\left(x\right)}{c}+\frac{b^2x^2}{2d}-\frac{\left(ad-bc\right)^2\log\left(\frac{c}{d}+x^2\right)}{2cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c),x)
```

```
[Out] a**2*log(x)/c + b**2*x**2/(2*d) - (a*d - b*c)**2*log(c/d + x**2)/(2*c*d**2)
```

3.174 $\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=55

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Rubi [**A**] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)}\,dx &= \int \left(\frac{b^2}{d}+\frac{a^2}{cx^2}-\frac{(bc-ad)^2}{cd\left(c+dx^2\right)}\right)dx\\ &= -\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\int \frac{1}{c+dx^2}\,dx}{cd}\\ &= -\frac{a^2}{cx}+\frac{b^2x}{d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.00

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] -(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
3/2)*d^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)), x]
```

fricas [A] time = 0.88, size = 164, normalized size = 2.98

$$\left[\frac{2\,b^2c^2dx^2-2\,a^2cd^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}\,x\log\left(\frac{dx^2+2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{2\,c^2d^2x},\ \frac{b^2c^2dx^2-a^2cd^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}\,x\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{c^2d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b^2*c^2*d*x^2 - 2*a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
-c*d)*x*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^2*d^2*x), (b^2*c^
2*d*x^2 - a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x*arctan(sq
rt(c*d)*x/c))/(c^2*d^2*x)]
```

giac [A] time = 0.34, size = 63, normalized size = 1.15

$$\frac{b^2x}{d}-\frac{a^2}{cx}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c*d)
```

maple [A] time = 0.01, size = 85, normalized size = 1.55

$$-\frac{a^2d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c}+\frac{2ab\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{b^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;d}+\frac{b^2x}{d}-\frac{a^2}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c),x)
```

```
[Out] b^2*x/d-1/c*d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+2/(c*d)^(1/2)*arcta
n(1/(c*d)^(1/2)*d*x)*a*b-c/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-a^2/
c/x
```

maxima [A] time = 2.39, size = 63, normalized size = 1.15

$$\frac{b^2x}{d}-\frac{a^2}{cx}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c*d)
```

mupad [B] time = 0.12, size = 80, normalized size = 1.45

$$\frac{b^2\,x}{d}-\frac{a^2}{c\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{c^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)),x)
```

```
[Out] (b^2*x)/d - a^2/(c*x) - (atan((d^(1/2)*x*(a*d - b*c)^2)/(c^(1/2)*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(3/2)*d^(3/2))
```

sympy [B] time = 0.54, size = 165, normalized size = 3.00

$$-\frac{a^2}{cx}+\frac{b^2x}{d}+\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2\log\left(-\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2\log\left(\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c),x)
```

```
[Out] -a**2/(c*x) + b**2*x/d + sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(-c**2*d*sq
rt(-1/(c**3*d**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/
2 - sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(c**2*d*sqrt(-1/(c**3*d**3))*(a*
d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

3.175 $\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2cx^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}+\frac{a\log(x)(2bc-ad)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] -a^2/(2*c*x^2) + (a*(2*b*c - a*d)*Log[x])/c^2 + ((b*c - a*d)^2*Log[c + d*x^
2])/(2*c^2*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^2}{cx^2}-\frac{a(-2bc+ad)}{c^2x}+\frac{(bc-ad)^2}{c^2(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{2cx^2}+\frac{a(2bc-ad)\log(x)}{c^2}+\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.03

$$\frac{a^2(-c)d-2adx^2\log(x)(ad-2bc)+x^2(bc-ad)^2\log\left(c+dx^2\right)}{2c^2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] (-(a^2*c*d) - 2*a*d*(-2*b*c + a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[c + d
*x^2])/(2*c^2*d*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)), x]
```

fricas [A] time = 0.87, size = 74, normalized size = 1.28

$$-\frac{a^2cd-\left(b^2c^2-2\,abcd+a^2d^2\right)x^2\log\left(dx^2+c\right)-2\left(2\,abcd-a^2d^2\right)x^2\log(x)}{2\,c^2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c*d - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(d*x^2 + c) - 2*(2*a
*b*c*d - a^2*d^2)*x^2*log(x))/(c^2*d*x^2)
```

giac [A] time = 0.40, size = 91, normalized size = 1.57

$$\frac{\left(2\,abc-a^2d\right)\log\left(x^2\right)}{2\,c^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{2\,c^2d}-\frac{2\,abcx^2-a^2dx^2+a^2c}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(d*x^2 + c))/(c^2*d) - 1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/(c^2*x^2)
```

maple [A] time = 0.01, size = 81, normalized size = 1.40

$$-\frac{a^2d\ln(x)}{c^2}+\frac{a^2d\ln\left(d\,x^2+c\right)}{2c^2}+\frac{2ab\ln(x)}{c}-\frac{ab\ln\left(d\,x^2+c\right)}{c}+\frac{b^2\ln\left(d\,x^2+c\right)}{2d}-\frac{a^2}{2c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c),x)
```

```
[Out] 1/2/c^2*d*ln(d*x^2+c)*a^2-1/c*ln(d*x^2+c)*a*b+1/2/d*ln(d*x^2+c)*b^2-1/2*a^2
/c/x^2-a^2/c^2*ln(x)*d+2*a/c*ln(x)*b
```

maxima [A] time = 1.04, size = 70, normalized size = 1.21

$$\frac{\left(2\,abc-a^2d\right)\log\left(x^2\right)}{2\,c^2}-\frac{a^2}{2\,cx^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 - 1/2*a^2/(c*x^2) + 1/2*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*log(d*x^2 + c)/(c^2*d)
```

mupad [B] time = 0.17, size = 67, normalized size = 1.16

$$\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d}-\frac{a^2}{2\,c\,x^2}-\frac{\ln(x)\,\left(a^2\,d-2\,a\,b\,c\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)),x)
```

```
[Out] (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^2*d) - a^2/(2*c*x^2)
- (log(x)*(a^2*d - 2*a*b*c))/c^2
```

sympy [A] time = 1.40, size = 49, normalized size = 0.84

$$-\frac{a^2}{2cx^2} - \frac{a(ad - 2bc)\log(x)}{c^2} + \frac{(ad - bc)^2 \log\left(\frac{c}{d} + x^2\right)}{2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c),x)
```

```
[Out] -a**2/(2*c*x**2) - a*(a*d - 2*b*c)*log(x)/c**2 + (a*d - b*c)**2*log(c/d + x
**2)/(2*c**2*d)
```

3.176 $\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}-\frac{a(2bc-ad)}{c^2x}$$

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -a^2/(3*c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)}\,dx &= \int \left(\frac{a^2}{cx^4}-\frac{a(-2bc+ad)}{c^2x^2}+\frac{(bc-ad)^2}{c^2\left(c+dx^2\right)}\right)dx\\ &= -\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\int \frac{1}{c+dx^2}\,dx}{c^2}\\ &= -\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.97

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}+\frac{a(ad-2bc)}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] -1/3*a^2/(c*x^3) + (a*(-2*b*c + a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)), x]
```

fricas [A] time = 0.58, size = 192, normalized size = 2.91

$$\left[-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-cd}\,x^3\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\,a^2c^2d+6\left(2\,abc^2d-a^2cd^2\right)x^2}{6\,c^3dx^3},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}\,x^3\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-a^2c^2d-3\left(2\,abc^2d-a^2cd^2\right)x^2}{3\,c^3dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*x^3*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*a^2*c^2*d + 6*(2*a*b*c^2*d - a^2*c*d^2)*x^2)
/(c^3*d*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x^3*arctan(s
qrt(c*d)*x/c) - a^2*c^2*d - 3*(2*a*b*c^2*d - a^2*c*d^2)*x^2)/(c^3*d*x^3)]
```

giac [A] time = 0.33, size = 71, normalized size = 1.08

$$\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{2}}-\frac{6\,abcx^{2}-3\,a^{2}dx^{2}+a^{2}c}{3\,c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/3
*(6*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^2*x^3)
```

maple [A] time = 0.01, size = 98, normalized size = 1.48

$$\frac{a^{2}d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c^{2}}-\frac{2abd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c}+\frac{b^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{a^{2}d}{c^{2}x}-\frac{2ab}{cx}-\frac{a^{2}}{3c\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c),x)
```

```
[Out] 1/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2*d^2-2/c/(c*d)^(1/2)*arctan(
1/(c*d)^(1/2)*d*x)*a*b*d+1/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1/3*a^
2/c/x^3+a^2/c^2/x*d-2*a/c/x*b
```

maxima [A] time = 2.33, size = 71, normalized size = 1.08

$$\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{2}}-\frac{a^{2}c+3\left(2\,abc-a^{2}d\right)x^{2}}{3\,c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/3
*(a^2*c + 3*(2*a*b*c - a^2*d)*x^2)/(c^2*x^3)
```

mupad [B] time = 0.13, size = 90, normalized size = 1.36

$$\frac{a^{2}\,d}{c^{2}\,x}-\frac{a^{2}}{3\,c\,x^{3}}+\frac{a^{2}\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}+\frac{b^{2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}}-\frac{2\,a\,b}{c\,x}-\frac{2\,a\,b\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)),x)
```

```
[Out] (a^2*d)/(c^2*x) - a^2/(3*c*x^3) + (a^2*d^(3/2)*atan((d^(1/2)*x)/c^(1/2)))/c
^(5/2) + (b^2*atan((d^(1/2)*x)/c^(1/2)))/(c^(1/2)*d^(1/2)) - (2*a*b)/(c*x)
- (2*a*b*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/c^(3/2)
```

sympy [B] time = 0.65, size = 172, normalized size = 2.61

$$-\frac{\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2\log\left(-\frac{c^3\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2\log\left(\frac{c^3\sqrt{-\frac{1}{c^5d}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{-a^2c+x^2\left(3a^2d-6abc\right)}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c),x)
```

```
[Out] -sqrt(-1/(c**5*d))*(a*d - b*c)**2*log(-c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(c**5*d))*(a*d - b*c
)**2*log(c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**
2*c**2) + x)/2 + (-a**2*c + x**2*(3*a**2*d - 6*a*b*c))/(3*c**2*x**3)
```

3.177 $\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2}{4cx^4}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}-\frac{a(2bc-ad)}{2c^2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}+\frac{\log(x)(bc-ad)^2}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -a^2/(4*c*x^4) - (a*(2*b*c - a*d))/(2*c^2*x^2) + ((b*c - a*d)^2*Log[x])/c^3
 - ((b*c - a*d)^2*Log[c + d*x^2])/(2*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{cx^3}-\frac{a(-2bc+ad)}{c^2x^2}+\frac{(bc-ad)^2}{c^3x}-\frac{d(bc-ad)^2}{c^3(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{4cx^4}-\frac{a(2bc-ad)}{2c^2x^2}+\frac{(bc-ad)^2\log(x)}{c^3}-\frac{(bc-ad)^2\log\left(c+dx^2\right)}{2c^3}\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 72, normalized size = 0.96

$$-\frac{-4x^4\log(x)(bc-ad)^2+ac\left(ac-2adx^2+4bcx^2\right)+2x^4(bc-ad)^2\log\left(c+dx^2\right)}{4c^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] -1/4*(a*c*(a*c + 4*b*c*x^2 - 2*a*d*x^2) - 4*(b*c - a*d)^2*x^4*Log[x] + 2*(b
*c - a*d)^2*x^4*Log[c + d*x^2])/(c^3*x^4)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^5*(c + d*x^2)), x]
```

fricas [**A**] time = 0.89, size = 98, normalized size = 1.31

$$-\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)x^4\log\left(dx^2+c\right)-4\left(b^2c^2-2\,abcd+a^2d^2\right)x^4\log(x)+a^2c^2+2\left(2\,abc^2-a^2cd\right)x^2}{4\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(d*x^2 + c) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x^4*log(x) + a^2*c^2 + 2*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^
3*x^4)
```

giac [B] time = 0.33, size = 139, normalized size = 1.85

$$\frac{\left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \log \left(x^{2}\right)}{2 \, c^{3}}-\frac{\left(b^{2} c^{2} d-2 \, abcd^{2}+a^{2} d^{3}\right) \log \left(\left|dx^{2}+c\right|\right)}{2 \, c^{3} d}-\frac{3 \, b^{2} c^{2} x^{4}-6 \, abcdx^{4}+3 \, a^{2} d^{2} x^{4}+4 \, abc^{2} x^{2}-2 \, a^{2} cdx^{2}+a^{2} c^{2}}{4 \, c^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/2*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*log(abs(d*x^2 + c))/(c^3*d) - 1/4*(3*b^2*c^2*x^4 - 6*a*b*c*
d*x^4 + 3*a^2*d^2*x^4 + 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(c^3*x^4)
```

maple [A] time = 0.01, size = 116, normalized size = 1.55

$$\frac{a^{2} d^{2} \ln(x)}{c^{3}}-\frac{a^{2} d^{2} \ln \left(d \, x^{2}+c\right)}{2 c^{3}}-\frac{2 a b d \ln(x)}{c^{2}}+\frac{a b d \ln \left(d \, x^{2}+c\right)}{c^{2}}+\frac{b^{2} \ln(x)}{c}-\frac{b^{2} \ln \left(d \, x^{2}+c\right)}{2 c}+\frac{a^{2} d}{2 c^{2} x^{2}}-\frac{a b}{c \, x^{2}}-\frac{a^{2}}{4 c \, x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c),x)
```

```
[Out] -1/2/c^3*ln(d*x^2+c)*a^2*d^2+1/c^2*ln(d*x^2+c)*a*b*d-1/2/c*ln(d*x^2+c)*b^2-
1/4*a^2/c/x^4+1/c^3*ln(x)*a^2*d^2-2/c^2*ln(x)*a*b*d+1/c*ln(x)*b^2+1/2*a^2/c
^2/x^2*d-a/c/x^2*b
```

maxima [A] time = 0.98, size = 96, normalized size = 1.28

$$-\frac{\left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \log \left(dx^{2}+c\right)}{2 \, c^{3}}+\frac{\left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \log \left(x^{2}\right)}{2 \, c^{3}}-\frac{a^{2} c+2 \, \left(2 \, abc-a^{2} d\right) x^{2}}{4 \, c^{2} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d*x^2 + c)/c^3 + 1/2*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/4*(a^2*c + 2*(2*a*b*c - a^2*d)*x^2)/(c^
2*x^4)
```

mupad [B] time = 0.16, size = 93, normalized size = 1.24

$$\frac{\ln(x) \, \left(a^{2} \, d^{2}-2 \, a \, b \, c \, d+b^{2} \, c^{2}\right)}{c^{3}}-\frac{\frac{a^{2}}{4 \, c}-\frac{a \, x^{2} \, (a \, d-2 \, b \, c)}{2 \, c^{2}}}{x^{4}}-\frac{\ln \left(d \, x^{2}+c\right) \, \left(a^{2} \, d^{2}-2 \, a \, b \, c \, d+b^{2} \, c^{2}\right)}{2 \, c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)),x)
```

```
[Out] (log(x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/c^3 - (a^2/(4*c) - (a*x^2*(a*d - 2
*b*c))/(2*c^2))/x^4 - (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c
^3)
```

sympy [A] time = 1.32, size = 66, normalized size = 0.88

$$\frac{-a^2c + x^2\left(2a^2d - 4abc\right)}{4c^2x^4} + \frac{(ad - bc)^2\log(x)}{c^3} - \frac{(ad - bc)^2\log\left(\frac{c}{d} + x^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c),x)
```

```
[Out] (-a**2*c + x**2*(2*a**2*d - 4*a*b*c))/(4*c**2*x**4) + (a*d - b*c)**2*log(x)
/c**3 - (a*d - b*c)**2*log(c/d + x**2)/(2*c**3)
```

3.178 $\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=87

$$-\frac{a^2}{5cx^5}-\frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}-\frac{a(2bc-ad)}{3c^2x^3}$$

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -a^2/(5*c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) - (S
qrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)}\,dx &= \int\left(\frac{a^2}{cx^6}-\frac{a(-2bc+ad)}{c^2x^4}+\frac{(bc-ad)^2}{c^3x^2}-\frac{d(bc-ad)^2}{c^3\left(c+dx^2\right)}\right)dx\\
&= -\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\left(d(bc-ad)^2\right)\int\frac{1}{c+dx^2}\,dx}{c^3}\\
&= -\frac{a^2}{5cx^5}-\frac{a(2bc-ad)}{3c^2x^3}-\frac{(bc-ad)^2}{c^3x}-\frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.99

$$
-\frac{a^2}{5cx^5}-\frac{\sqrt{d}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}+\frac{a(ad-2bc)}{3c^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] -1/5*a^2/(c*x^5) + (a*(-2*b*c + a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) -
 (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^6*(c + d*x^2)), x]
```

fricas [A] time = 1.12, size = 236, normalized size = 2.71

$$
\left[\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)x^5\sqrt{-\frac{d}{c}}\,\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-30\left(b^2c^2-2\,abcd+a^2d^2\right)x^4-6\,a^2c^2-10\left(2\,abc^2-a^2cd\right)x^2}{30\,c^3x^5},\ -\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)x^5\sqrt{\frac{d}{c}}\,\arctan\left(x\sqrt{\frac{d}{c}}\right)+15\left(b^2c^2-2\,abcd+a^2d^2\right)x^4+3\,a^2c^2+5\left(2\,abc^2-a^2cd\right)x^2}{15\,c^3x^5}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x
*sqrt(-d/c) - c)/(d*x^2 + c)) - 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 - 6*
a^2*c^2 - 10*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5), -1/15*(15*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5)]
```

giac [A] time = 0.32, size = 112, normalized size = 1.29

$$-\frac{\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{3}}-\frac{15\,b^{2}c^{2}x^{4}-30\,abcdx^{4}+15\,a^{2}d^{2}x^{4}+10\,abc^{2}x^{2}-5\,a^{2}cdx^{2}+3\,a^{2}c^{2}}{15\,c^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
- 1/15*(15*b^2*c^2*x^4 - 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 + 10*a*b*c^2*x^2 -
 5*a^2*c*d*x^2 + 3*a^2*c^2)/(c^3*x^5)
```

maple [A] time = 0.01, size = 143, normalized size = 1.64

$$-\frac{a^{2}d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c^{3}}+\frac{2ab\,d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c^{2}}-\frac{b^{2}d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c}-\frac{a^{2}d^{2}}{c^{3}x}+\frac{2abd}{c^{2}x}-\frac{b^{2}}{cx}+\frac{a^{2}d}{3c^{2}x^{3}}-\frac{2ab}{3c\,x^{3}}-\frac{a^{2}}{5c\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c),x)
```

```
[Out] -d^3/c^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+2*d^2/c^2/(c*d)^(1/2)*ar
ctan(1/(c*d)^(1/2)*d*x)*a*b-d/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1
/5*a^2/c/x^5-1/c^3/x*a^2*d^2+2/c^2/x*a*b*d-1/c/x*b^2+1/3*a^2/c^2/x^3*d-2/3*
a/c/x^3*b
```

maxima [A] time = 2.44, size = 107, normalized size = 1.23

$$-\frac{\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c^{3}}-\frac{15\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{4}+3\,a^{2}c^{2}+5\left(2\,abc^{2}-a^{2}cd\right)x^{2}}{15\,c^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
- 1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2 -
 a^2*c*d)*x^2)/(c^3*x^5)
```

mupad [B] time = 0.09, size = 129, normalized size = 1.48

$$\frac{a^2\,d}{3\,c^2\,x^3}-\frac{b^2}{c\,x}-\frac{a^2}{5\,c\,x^5}-\frac{a^2\,d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{b^2\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a^2\,d^2}{c^3\,x}-\frac{2\,a\,b}{3\,c\,x^3}+\frac{2\,a\,b\,d}{c^2\,x}+\frac{2\,a\,b\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)),x)
```

```
[Out] (a^2*d)/(3*c^2*x^3) - b^2/(c*x) - a^2/(5*c*x^5) - (a^2*d^(5/2)*atan((d^(1/2
)*x)/c^(1/2)))/c^(7/2) - (b^2*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/c^(3/2) -
(a^2*d^2)/(c^3*x) - (2*a*b)/(3*c*x^3) + (2*a*b*d)/(c^2*x) + (2*a*b*d^(3/2)*
atan((d^(1/2)*x)/c^(1/2)))/c^(5/2)
```

sympy [B] time = 0.76, size = 207, normalized size = 2.38

$$\frac{\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2\log\left(-\frac{c^4\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2}-\frac{\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2\log\left(\frac{c^4\sqrt{-\frac{d}{c^7}}\,(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2}+\frac{-3a^2c^2+x^4\left(-15a^2d^2+30abcd-15b^2c^2\right)+x^2\left(5a^2cd-10abc^2\right)}{15c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c),x)
```

```
[Out] sqrt(-d/c**7)*(a*d - b*c)**2*log(-c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d
**3 - 2*a*b*c*d**2 + b**2*c**2*d) + x)/2 - sqrt(-d/c**7)*(a*d - b*c)**2*log
(c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d**3 - 2*a*b*c*d**2 + b**2*c**2*d)
 + x)/2 + (-3*a**2*c**2 + x**4*(-15*a**2*d**2 + 30*a*b*c*d - 15*b**2*c**2)
+ x**2*(5*a**2*c*d - 10*a*b*c**2))/(15*c**3*x**5)
```

3.179 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2}{6cx^6}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{a(2bc-ad)}{4c^2x^4}$$

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}-\frac{d\log(x)(bc-ad)^2}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -a^2/(6*c*x^6) - (a*(2*b*c - a*d))/(4*c^2*x^4) - (b*c - a*d)^2/(2*c^3*x^2)
- (d*(b*c - a*d)^2*Log[x])/c^4 + (d*(b*c - a*d)^2*Log[c + d*x^2])/(2*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^4(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{cx^4}-\frac{a(-2bc+ad)}{c^2x^3}+\frac{(bc-ad)^2}{c^3x^2}-\frac{d(bc-ad)^2}{c^4x}+\frac{d^2(bc-ad)^2}{c^4(c+dx)}\right)dx, x, x^2\right)\\
&= -\frac{a^2}{6cx^6}-\frac{a(2bc-ad)}{4c^2x^4}-\frac{(bc-ad)^2}{2c^3x^2}-\frac{d(bc-ad)^2\log(x)}{c^4}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{2c^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 108, normalized size = 1.10

$$
-\frac{c\left(a^2\left(2c^2-3cdx^2+6d^2x^4\right)+6abcx^2\left(c-2dx^2\right)+6b^2c^2x^4\right)+12dx^6\log(x)(bc-ad)^2-6dx^6(bc-ad)^2\log\left(c+dx^2\right)}{12c^4x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] -1/12*(c*(6*b^2*c^2*x^4 + 6*a*b*c*x^2*(c - 2*d*x^2) + a^2*(2*c^2 - 3*c*d*x^
2 + 6*d^2*x^4)) + 12*d*(b*c - a*d)^2*x^6*Log[x] - 6*d*(b*c - a*d)^2*x^6*Log
[c + d*x^2])/(c^4*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^7*(c + d*x^2)), x]
```

fricas [A] time = 0.62, size = 136, normalized size = 1.39

$$
\frac{6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^6\log\left(dx^2+c\right)-12\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^6\log(x)-2\,a^2c^3-6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)x^4-3\left(2\,abc^3-a^2c^2d\right)x^2}{12\,c^4x^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(d*x^2 + c) - 12*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(x) - 2*a^2*c^3 - 6*(b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*x^4 - 3*(2*a*b*c^3 - a^2*c^2*d)*x^2)/(c^4*x^6)
```

giac [B] time = 0.32, size = 184, normalized size = 1.88

$$-\frac{\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\log\left(x^{2}\right)}{2\,c^{4}}+\frac{\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)\log\left(\left|dx^{2}+c\right|\right)}{2\,c^{4}d}+\frac{11\,b^{2}c^{2}dx^{6}-22\,abcd^{2}x^{6}+11\,a^{2}d^{3}x^{6}-6\,b^{2}c^{3}x^{4}+12\,abc^{2}dx^{4}-6\,a^{2}cd^{2}x^{4}-6\,abc^{3}x^{2}+3\,a^{2}c^{2}dx^{2}-2\,a^{2}c^{3}}{12\,c^{4}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 + 1/2*(b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 + c))/(c^4*d) + 1/12*(11*b^2*c^2*d*x^6
 - 22*a*b*c*d^2*x^6 + 11*a^2*d^3*x^6 - 6*b^2*c^3*x^4 + 12*a*b*c^2*d*x^4 - 6
*a^2*c*d^2*x^4 - 6*a*b*c^3*x^2 + 3*a^2*c^2*d*x^2 - 2*a^2*c^3)/(c^4*x^6)
```

maple [A] time = 0.01, size = 160, normalized size = 1.63

$$-\frac{a^{2}d^{3}\ln(x)}{c^{4}}+\frac{a^{2}d^{3}\ln\left(d\,x^{2}+c\right)}{2c^{4}}+\frac{2ab\,d^{2}\ln(x)}{c^{3}}-\frac{ab\,d^{2}\ln\left(d\,x^{2}+c\right)}{c^{3}}-\frac{b^{2}d\ln(x)}{c^{2}}+\frac{b^{2}d\ln\left(d\,x^{2}+c\right)}{2c^{2}}-\frac{a^{2}d^{2}}{2c^{3}x^{2}}+\frac{abd}{c^{2}x^{2}}-\frac{b^{2}}{2c\,x^{2}}+\frac{a^{2}d}{4c^{2}x^{4}}-\frac{ab}{2c\,x^{4}}-\frac{a^{2}}{6c\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c),x)
```

```
[Out] 1/2*d^3/c^4*ln(d*x^2+c)*a^2-d^2/c^3*ln(d*x^2+c)*a*b+1/2*d/c^2*ln(d*x^2+c)*b
^2-1/6*a^2/c/x^6-1/2/c^3/x^2*a^2*d^2+1/c^2/x^2*a*b*d-1/2/c/x^2*b^2+1/4*a^2/
c^2/x^4*d-1/2*a/c/x^4*b-1/c^4*d^3*ln(x)*a^2+2/c^3*d^2*ln(x)*a*b-1/c^2*d*ln(
x)*b^2
```

maxima [A] time = 0.97, size = 134, normalized size = 1.37

$$\frac{\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\log\left(dx^{2}+c\right)}{2\,c^{4}}-\frac{\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\log\left(x^{2}\right)}{2\,c^{4}}-\frac{6\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{4}+2\,a^{2}c^{2}+3\left(2\,abc^{2}-a^{2}cd\right)x^{2}}{12\,c^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/c^4 - 1/2*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 - 1/12*(6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*x^4 + 2*a^2*c^2 + 3*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^6)
```

mupad [B] time = 0.12, size = 129, normalized size = 1.32

$$\frac{\ln\left(d\,x^{2}+c\right)\,\left(a^{2}\,d^{3}-2\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{2\,c^{4}}-\frac{\frac{a^{2}}{6\,c}+\frac{x^{4}\,\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}{2\,c^{3}}-\frac{a\,x^{2}\,\left(a\,d-2\,b\,c\right)}{4\,c^{2}}}{x^{6}}-\frac{\ln(x)\,\left(a^{2}\,d^{3}-2\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^7*(c + d*x^2)),x)
```

```
[Out] (log(c + d*x^2)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(2*c^4) - (a^2/(6*c) +
 (x^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^3) - (a*x^2*(a*d - 2*b*c))/(4*c
^2))/x^6 - (log(x)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/c^4
```

sympy [A] time = 1.51, size = 105, normalized size = 1.07

$$\frac{-2a^2c^2 + x^4\left(-6a^2d^2 + 12abcd - 6b^2c^2\right) + x^2\left(3a^2cd - 6abc^2\right)}{12c^3x^6} - \frac{d\left(ad - bc\right)^2\log\left(x\right)}{c^4} + \frac{d\left(ad - bc\right)^2\log\left(\frac{c}{d} + x^2\right)}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c),x)
```

```
[Out] (-2*a**2*c**2 + x**4*(-6*a**2*d**2 + 12*a*b*c*d - 6*b**2*c**2) + x**2*(3*a*
*2*c*d - 6*a*b*c**2))/(12*c**3*x**6) - d*(a*d - b*c)**2*log(x)/c**4 + d*(a*
d - b*c)**2*log(c/d + x**2)/(2*c**4)
```

3.180 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{c}\,(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x^5(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x^5}{5d^2}$$

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 302, 205}

$$\frac{x^5(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^3(7bc-3ad)(bc-ad)}{6cd^3}+\frac{x(7bc-3ad)(bc-ad)}{2d^4}-\frac{\sqrt{c}\,(7bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{9/2}}+\frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((7*b*c - 3*a*d)*(b*c - a*d)*x)/(2*d^4) - ((7*b*c - 3*a*d)*(b*c - a*d)*x^3)
/(6*c*d^3) + (b^2*x^5)/(5*d^2) + ((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2))
- (Sqrt[c]*(7*b*c - 3*a*d)*(b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9
/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^4\left(-2a^2d^2+5(bc-ad)^2-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{((7bc-3ad)(bc-ad))\int \frac{x^4}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \frac{((7bc-3ad)(bc-ad))\int \left(-\frac{c}{d^2}+\frac{x^2}{d}+\frac{c^2}{d^2\left(c+dx^2\right)}\right)dx}{2cd^2}$$

$$= \frac{(7bc-3ad)(bc-ad)x}{2d^4} - \frac{(7bc-3ad)(bc-ad)x^3}{6cd^3} + \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - (c(7bc-3ad)$$

$$= \frac{(7bc-3ad)(bc-ad)x}{2d^4} - \frac{(7bc-3ad)(bc-ad)x^3}{6cd^3} + \frac{b^2x^5}{5d^2} + \frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)} - \sqrt{c}\,(7bc-3a$$

Mathematica [A] time = 0.10, size = 138, normalized size = 0.95

$$-\frac{\sqrt{c}\left(3a^2d^2-10abcd+7b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{9/2}} + \frac{x\left(a^2d^2-4abcd+3b^2c^2\right)}{d^4} + \frac{cx(bc-ad)^2}{2d^4\left(c+dx^2\right)} - \frac{2bx^3(bc-ad)}{3d^3} + \frac{b^2x^5}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x)/d^4 - (2*b*(b*c - a*d)*x^3)/(3*d^3) +
 (b^2*x^5)/(5*d^2) + (c*(b*c - a*d)^2*x)/(2*d^4*(c + d*x^2)) - (Sqrt[c]*(7*
b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2, x]
```

fricas [A] time = 0.80, size = 400, normalized size = 2.76

$$\left[\frac{12\,b^2d^3x^7-4\left(7\,b^2cd^2-10\,abd^3\right)x^5+20\left(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3\right)x^3+15\left(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2+\left(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3\right)x^2\right)\sqrt{-\frac{c}{d}}\,\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+30\left(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2\right)x}{60\left(d^5x^2+cd^4\right)},\ \frac{6\,b^2d^3x^7-2\left(7\,b^2cd^2-10\,abd^3\right)x^5+10\left(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3\right)x^3-15\left(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2+\left(7\,b^2c^2d-10\,abcd^2+3\,a^2d^3\right)x^2\right)\sqrt{\frac{c}{d}}\,\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+15\left(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2\right)x}{30\left(d^5x^2+cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^2*d^3*x^7 - 4*(7*b^2*c*d^2 - 10*a*b*d^3)*x^5 + 20*(7*b^2*c^2*d
- 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^
2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*sqrt(-c/d)*log((d*x^2 - 2
*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 30*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2)*x)/(d^5*x^2 + c*d^4), 1/30*(6*b^2*d^3*x^7 - 2*(7*b^2*c*d^2 - 10*a*b*d^
3)*x^5 + 10*(7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 15*(7*b^2*c^3 -
10*a*b*c^2*d + 3*a^2*c*d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*
sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2)*x)/(d^5*x^2 + c*d^4)]
```

giac [A] time = 0.37, size = 156, normalized size = 1.08

$$-\frac{\left(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,d^4}+\frac{b^2c^3x-2\,abc^2dx+a^2cd^2x}{2\left(dx^2+c\right)d^4}+\frac{3\,b^2d^8x^5-10\,b^2cd^7x^3+10\,abd^8x^3+45\,b^2c^2d^6x-60\,abcd^7x+15\,a^2d^8x}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c
*d)*d^4) + 1/2*(b^2*c^3*x - 2*a*b*c^2*d*x + a^2*c*d^2*x)/((d*x^2 + c)*d^4)
+ 1/15*(3*b^2*d^8*x^5 - 10*b^2*c*d^7*x^3 + 10*a*b*d^8*x^3 + 45*b^2*c^2*d^6*
x - 60*a*b*c*d^7*x + 15*a^2*d^8*x)/d^10
```

maple [A] time = 0.01, size = 196, normalized size = 1.35

$$\frac{b^2x^5}{5d^2}+\frac{2ab\,x^3}{3d^2}-\frac{2b^2c\,x^3}{3d^3}+\frac{a^2cx}{2\left(d\,x^2+c\right)d^2}-\frac{3a^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d^2}-\frac{ab\,c^2x}{\left(d\,x^2+c\right)d^3}+\frac{5ab\,c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^3}+\frac{b^2c^3x}{2\left(d\,x^2+c\right)d^4}-\frac{7b^2c^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d^4}+\frac{a^2x}{d^2}-\frac{4abcx}{d^3}+\frac{3b^2c^2x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/5*b^2*x^5/d^2+2/3/d^2*x^3*a*b-2/3/d^3*x^3*b^2*c+1/d^2*a^2*x-4/d^3*a*b*c*x
+3/d^4*b^2*c^2*x+1/2*c/d^2*x/(d*x^2+c)*a^2-c^2/d^3*x/(d*x^2+c)*a*b+1/2*c^3/
d^4*x/(d*x^2+c)*b^2-3/2*c/d^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+5*c
^2/d^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b-7/2*c^3/d^4/(c*d)^(1/2)*ar
ctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 2.38, size = 149, normalized size = 1.03

$$\frac{\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)x}{2\left(d^{5}x^{2}+cd^{4}\right)}-\frac{\left(7\,b^{2}c^{3}-10\,abc^{2}d+3\,a^{2}cd^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,d^{4}}+\frac{3\,b^{2}d^{2}x^{5}-10\left(b^{2}cd-abd^{2}\right)x^{3}+15\left(3\,b^{2}c^{2}-4\,abcd+a^{2}d^{2}\right)x}{15\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x/(d^5*x^2 + c*d^4) - 1/2*(7*b^2*c^
3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4) + 1/1
5*(3*b^2*d^2*x^5 - 10*(b^2*c*d - a*b*d^2)*x^3 + 15*(3*b^2*c^2 - 4*a*b*c*d +
 a^2*d^2)*x)/d^4
```

mupad [B] time = 0.14, size = 200, normalized size = 1.38

$$x\left(\frac{a^{2}}{d^{2}}+\frac{2\,c\left(\frac{2\,b^{2}\,c}{d^{3}}-\frac{2\,a\,b}{d^{2}}\right)}{d}-\frac{b^{2}\,c^{2}}{d^{4}}\right)-x^{3}\left(\frac{2\,b^{2}\,c}{3\,d^{3}}-\frac{2\,a\,b}{3\,d^{2}}\right)+\frac{b^{2}\,x^{5}}{5\,d^{2}}+\frac{x\left(\frac{a^{2}\,c\,d^{2}}{2}-a\,b\,c^{2}\,d+\frac{b^{2}\,c^{3}}{2}\right)}{d^{5}\,x^{2}+c\,d^{4}}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,x\,(a\,d-b\,c)\,(3\,a\,d-7\,b\,c)}{3\,a^{2}\,c\,d^{2}-10\,a\,b\,c^{2}\,d+7\,b^{2}\,c^{3}}\right)\,(a\,d-b\,c)\,(3\,a\,d-7\,b\,c)}{2\,d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] x*(a^2/d^2 + (2*c*((2*b^2*c)/d^3 - (2*a*b)/d^2))/d - (b^2*c^2)/d^4) - x^3*(
(2*b^2*c)/(3*d^3) - (2*a*b)/(3*d^2)) + (b^2*x^5)/(5*d^2) + (x*((b^2*c^3)/2
+ (a^2*c*d^2)/2 - a*b*c^2*d))/(c*d^4 + d^5*x^2) - (c^(1/2)*atan((c^(1/2)*d^
(1/2)*x*(a*d - b*c)*(3*a*d - 7*b*c))/(7*b^2*c^3 + 3*a^2*c*d^2 - 10*a*b*c^2*
d))*(a*d - b*c)*(3*a*d - 7*b*c))/(2*d^(9/2))
```

sympy [B] time = 0.99, size = 286, normalized size = 1.97

$$\frac{b^{2}x^{5}}{5d^{2}}+x^{3}\left(\frac{2ab}{3d^{2}}-\frac{2b^{2}c}{3d^{3}}\right)+x\left(\frac{a^{2}}{d^{2}}-\frac{4abc}{d^{3}}+\frac{3b^{2}c^{2}}{d^{4}}\right)+\frac{x\left(a^{2}cd^{2}-2abc^{2}d+b^{2}c^{3}\right)}{2cd^{4}+2d^{5}x^{2}}+\frac{\sqrt{-\frac{c}{d^{9}}}\,(ad-bc)\,(3ad-7bc)\log\left(-\frac{d^{4}\sqrt{-\frac{c}{d^{9}}}\,(ad-bc)(3ad-7bc)}{3a^{2}d^{2}-10abcd+7b^{2}c^{2}}+x\right)}{4}-\frac{\sqrt{-\frac{c}{d^{9}}}\,(ad-bc)\,(3ad-7bc)\log\left(\frac{d^{4}\sqrt{-\frac{c}{d^{9}}}\,(ad-bc)(3ad-7bc)}{3a^{2}d^{2}-10abcd+7b^{2}c^{2}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**5/(5*d**2) + x**3*(2*a*b/(3*d**2) - 2*b**2*c/(3*d**3)) + x*(a**2/d*
*2 - 4*a*b*c/d**3 + 3*b**2*c**2/d**4) + x*(a**2*c*d**2 - 2*a*b*c**2*d + b**
2*c**3)/(2*c*d**4 + 2*d**5*x**2) + sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c
)*log(-d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b
*c*d + 7*b**2*c**2) + x)/4 - sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)*log(
d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b*c*d +
7*b**2*c**2) + x)/4
```

3.181 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=90

$$\frac{c(bc-ad)^2}{2d^4\left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log\left(c+dx^2\right)}{2d^4}-\frac{bx^2(bc-ad)}{d^3}+\frac{b^2x^4}{4d^2}$$

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{bx^2(bc-ad)}{d^3}+\frac{c(bc-ad)^2}{2d^4\left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^4}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*(b*c - a*d)*x^2)/d^3) + (b^2*x^4)/(4*d^2) + (c*(b*c - a*d)^2)/(2*d^4*(
c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2])/(2*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{2b(bc-ad)}{d^3}+\frac{b^2x}{d^2}-\frac{c(bc-ad)^2}{d^3(c+dx)^2}+\frac{(bc-ad)(3bc-ad)}{d^3(c+dx)}\right)dx, x, x^2\right)\\
&= -\frac{b(bc-ad)x^2}{d^3}+\frac{b^2x^4}{4d^2}+\frac{c(bc-ad)^2}{2d^4\left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log\left(c+dx^2\right)}{2d^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.97

$$
\frac{2\left(a^2d^2-4abcd+3b^2c^2\right)\log\left(c+dx^2\right)+4bdx^2(ad-bc)+\frac{2c(bc-ad)^2}{c+dx^2}+b^2d^2x^4}{4d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (4*b*d*(-(b*c) + a*d)*x^2 + b^2*d^2*x^4 + (2*c*(b*c - a*d)^2)/(c + d*x^2) +
 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(4*d^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2, x]
```

fricas [A] time = 0.60, size = 161, normalized size = 1.79

$$
\frac{b^2d^3x^6+2\,b^2c^3-4\,abc^2d+2\,a^2cd^2-\left(3\,b^2cd^2-4\,abd^3\right)x^4-4\left(b^2c^2d-abcd^2\right)x^2+2\left(3\,b^2c^3-4\,abc^2d+a^2cd^2+\left(3\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^2\right)\log\left(dx^2+c\right)}{4\left(d^5x^2+cd^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^3*x^6 + 2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 - (3*b^2*c*d^2 - 4
*a*b*d^3)*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*x^2 + 2*(3*b^2*c^3 - 4*a*b*c^2*d
```

```
+ a^2*c*d^2 + (3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2)*log(d*x^2 + c))/(d
^5*x^2 + c*d^4)
```

giac [A] time = 0.35, size = 163, normalized size = 1.81

$$\frac{\frac{\left(dx^2+c\right)^2\left(b^2-\frac{2\left(3\,b^2cd-2\,abd^2\right)}{\left(dx^2+c\right)d}\right)}{d^3}-\frac{2\left(3\,b^2c^2-4\,abcd+a^2d^2\right)\log\left(\frac{\left|dx^2+c\right|}{\left(dx^2+c\right)^2\left|d\right|}\right)}{d^3}+\frac{2\left(\frac{b^2c^3d^2}{dx^2+c}-\frac{2\,abc^2d^3}{dx^2+c}+\frac{a^2cd^4}{dx^2+c}\right)}{d^5}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((d*x^2 + c)^2*(b^2 - 2*(3*b^2*c*d - 2*a*b*d^2)/((d*x^2 + c)*d))/d^3 -
2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(abs(d*x^2 + c)/((d*x^2 + c)^2*abs(d
)))/d^3 + 2*(b^2*c^3*d^2/(d*x^2 + c) - 2*a*b*c^2*d^3/(d*x^2 + c) + a^2*c*d^
4/(d*x^2 + c))/d^5)/d
```

maple [A] time = 0.01, size = 142, normalized size = 1.58

$$\frac{b^2x^4}{4d^2}+\frac{ab\,x^2}{d^2}-\frac{b^2c\,x^2}{d^3}+\frac{a^2c}{2\left(d\,x^2+c\right)d^2}+\frac{a^2\ln\left(d\,x^2+c\right)}{2d^2}-\frac{ab\,c^2}{\left(d\,x^2+c\right)d^3}-\frac{2abc\ln\left(d\,x^2+c\right)}{d^3}+\frac{b^2c^3}{2\left(d\,x^2+c\right)d^4}+\frac{3b^2c^2\ln\left(d\,x^2+c\right)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/4*b^2*x^4/d^2+b/d^2*a*x^2-b^2/d^3*c*x^2+1/2/d^2*ln(d*x^2+c)*a^2-2/d^3*ln(
d*x^2+c)*a*b*c+3/2/d^4*ln(d*x^2+c)*b^2*c^2+1/2/d^2*c/(d*x^2+c)*a^2-1/d^3*c^
2/(d*x^2+c)*a*b+1/2/d^4*c^3/(d*x^2+c)*b^2
```

maxima [A] time = 1.05, size = 107, normalized size = 1.19

$$\frac{b^2c^3-2\,abc^2d+a^2cd^2}{2\left(d^5x^2+cd^4\right)}+\frac{b^2dx^4-4\left(b^2c-abd\right)x^2}{4\,d^3}+\frac{\left(3\,b^2c^2-4\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d^5*x^2 + c*d^4) + 1/4*(b^2*d*x^4
- 4*(b^2*c - a*b*d)*x^2)/d^3 + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(d*
x^2 + c)/d^4
```

mupad [B] time = 0.12, size = 112, normalized size = 1.24

$$\frac{a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3}{2\,d\,\left(d^4\,x^2+c\,d^3\right)}-x^2\,\left(\frac{b^2\,c}{d^3}-\frac{a\,b}{d^2}\right)+\frac{b^2\,x^4}{4\,d^2}+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)/(2*d*(c*d^3 + d^4*x^2)) - x^2*((b^2*c)/
d^3 - (a*b)/d^2) + (b^2*x^4)/(4*d^2) + (log(c + d*x^2)*(a^2*d^2 + 3*b^2*c^2
 - 4*a*b*c*d))/(2*d^4)
```

sympy [A] time = 0.94, size = 99, normalized size = 1.10

$$\frac{b^2x^4}{4d^2} + x^2\left(\frac{ab}{d^2} - \frac{b^2c}{d^3}\right) + \frac{a^2cd^2 - 2abc^2d + b^2c^3}{2cd^4 + 2d^5x^2} + \frac{(ad - 3bc)\,(ad - bc)\log\left(c + dx^2\right)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**4/(4*d**2) + x**2*(a*b/d**2 - b**2*c/d**3) + (a**2*c*d**2 - 2*a*b*c
**2*d + b**2*c**3)/(2*c*d**4 + 2*d**5*x**2) + (a*d - 3*b*c)*(a*d - b*c)*log
(c + d*x**2)/(2*d**4)
```

3.182 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=118

$$\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{x^3(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x^3}{3d^2}$$

Rubi [**A**] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 321, 205}

$$\frac{x^3(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x(bc-ad)(5bc-ad)}{2cd^3}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}+\frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*(5*b*c - a*d)*x)/(2*c*d^3) + (b^2*x^3)/(3*d^2) + ((b*c - a*d)
^2*x^3)/(2*c*d^2*(c + d*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(2*Sqrt[c]*d^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \frac{(bc-ad)^2x^3}{2cd^2\left(c+dx^2\right)}-\frac{\int \frac{x^2\left(3b^2c^2-6abcd+a^2d^2-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}\\ &= \frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2\left(c+dx^2\right)}-\frac{((bc-ad)(5bc-ad))\int \frac{x^2}{c+dx^2}\,dx}{2cd^2}\\ &= -\frac{(bc-ad)(5bc-ad)x}{2cd^3}+\frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2\left(c+dx^2\right)}+\frac{((bc-ad)(5bc-ad))\int \frac{1}{c+dx^2}\,dx}{2d^3}\\ &= -\frac{(bc-ad)(5bc-ad)x}{2cd^3}+\frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2\left(c+dx^2\right)}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.89

$$\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,d^{7/2}}-\frac{x(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{2bx(bc-ad)}{d^3}+\frac{b^2x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-2*b*(b*c - a*d)*x)/d^3 + (b^2*x^3)/(3*d^2) - ((b*c - a*d)^2*x)/(2*d^3*(c
+ d*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(2*Sqrt[c]*d^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2, x]
```

fricas [A] time = 0.80, size = 342, normalized size = 2.90

$$\left[\frac{4\,b^2cd^3x^5-4\left(5\,b^2c^2d^2-6\,abcd^3\right)x^3-3\left(5\,b^2c^3-6\,abc^2d+a^2cd^2+\left(5\,b^2c^2d-6\,abcd^2+a^2d^3\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)-6\left(5\,b^2c^3d-6\,abc^2d^2+a^2cd^3\right)x}{12\left(cd^5x^2+c^2d^4\right)},\ \frac{2\,b^2cd^3x^5-2\left(5\,b^2c^2d^2-6\,abcd^3\right)x^3+3\left(5\,b^2c^3-6\,abc^2d+a^2cd^2+\left(5\,b^2c^2d-6\,abcd^2+a^2d^3\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-3\left(5\,b^2c^3d-6\,abc^2d^2+a^2cd^3\right)x}{6\left(cd^5x^2+c^2d^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*c*d^3*x^5 - 4*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 - 3*(5*b^2*c^3
 - 6*a*b*c^2*d + a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqr
t(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^2*c^3*d - 6*
a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4), 1/6*(2*b^2*c*d^3*x^5 - 2
*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2
 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) - 3*(5*b^2*c^3*d - 6*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4)]
```

giac [A] time = 0.30, size = 114, normalized size = 0.97

$$\frac{\left(5\,b^2c^2-6\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,d^3}-\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(dx^2+c\right)d^3}+\frac{b^2d^4x^3-6\,b^2cd^3x+6\,abd^4x}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
 - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*d^3) + 1/3*(b^2*d
^4*x^3 - 6*b^2*c*d^3*x + 6*a*b*d^4*x)/d^6
```

maple [A] time = 0.01, size = 156, normalized size = 1.32

$$\frac{b^2x^3}{3d^2}-\frac{a^2x}{2\left(d\,x^2+c\right)d}+\frac{a^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d}+\frac{abcx}{\left(d\,x^2+c\right)d^2}-\frac{3abc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d^2}-\frac{b^2c^2x}{2\left(d\,x^2+c\right)d^3}+\frac{5b^2c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d^3}+\frac{2abx}{d^2}-\frac{2b^2cx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/3*b^2*x^3/d^2+2*b/d^2*a*x-2*b^2/d^3*c*x-1/2/d*x/(d*x^2+c)*a^2+1/d^2*x/(d*
x^2+c)*a*b*c-1/2/d^3*x/(d*x^2+c)*b^2*c^2+1/2/d/(c*d)^(1/2)*arctan(1/(c*d)^(
1/2)*d*x)*a^2-3/d^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*c+5/2/d^3/(c*
d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2*c^2
```

maxima [A] time = 2.35, size = 109, normalized size = 0.92

$$-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x}{2\left(d^{4}x^{2}+cd^{3}\right)}+\frac{\left(5\,b^{2}c^{2}-6\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,d^{3}}+\frac{b^{2}dx^{3}-6\left(b^{2}c-abd\right)x}{3\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(d^4*x^2 + c*d^3) + 1/2*(5*b^2*c^2 -
 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3) + 1/3*(b^2*d*x^
3 - 6*(b^2*c - a*b*d)*x)/d^3
```

mupad [B] time = 0.08, size = 146, normalized size = 1.24

$$\frac{b^{2}\,x^{3}}{3\,d^{2}}-\frac{x\left(\frac{a^{2}\,d^{2}}{2}-a\,b\,c\,d+\frac{b^{2}\,c^{2}}{2}\right)}{d^{4}\,x^{2}+c\,d^{3}}-x\left(\frac{2\,b^{2}\,c}{d^{3}}-\frac{2\,a\,b}{d^{2}}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\left(a\,d-5\,b\,c\right)}{\sqrt{c}\,\left(a^{2}\,d^{2}-6\,a\,b\,c\,d+5\,b^{2}\,c^{2}\right)}\right)\left(a\,d-b\,c\right)\,\left(a\,d-5\,b\,c\right)}{2\,\sqrt{c}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x^3)/(3*d^2) - (x*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(c*d^3 + d^4*
x^2) - x*((2*b^2*c)/d^3 - (2*a*b)/d^2) + (atan((d^(1/2)*x*(a*d - b*c)*(a*d
- 5*b*c))/(c^(1/2)*(a^2*d^2 + 5*b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(a*d - 5
*b*c))/(2*c^(1/2)*d^(7/2))
```

sympy [B] time = 0.87, size = 246, normalized size = 2.08

$$\frac{b^{2}x^{3}}{3d^{2}}+x\left(\frac{2ab}{d^{2}}-\frac{2b^{2}c}{d^{3}}\right)+\frac{x\left(-a^{2}d^{2}+2abcd-b^{2}c^{2}\right)}{2cd^{3}+2d^{4}x^{2}}-\frac{\sqrt{-\frac{1}{cd^{7}}}\,(ad-5bc)\,(ad-bc)\log\left(-\frac{cd^{3}\sqrt{-\frac{1}{cd^{7}}}\,(ad-5bc)(ad-bc)}{a^{2}d^{2}-6abcd+5b^{2}c^{2}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{cd^{7}}}\,(ad-5bc)\,(ad-bc)\log\left(\frac{cd^{3}\sqrt{-\frac{1}{cd^{7}}}\,(ad-5bc)(ad-bc)}{a^{2}d^{2}-6abcd+5b^{2}c^{2}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**3/(3*d**2) + x*(2*a*b/d**2 - 2*b**2*c/d**3) + x*(-a**2*d**2 + 2*a*b
*c*d - b**2*c**2)/(2*c*d**3 + 2*d**4*x**2) - sqrt(-1/(c*d**7))*(a*d - 5*b*c
)*(a*d - b*c)*log(-c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2
*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(
a*d - b*c)*log(c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**2*d**
2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4
```

3.183 $\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}+\frac{b^2x^2}{2d^2}$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}+\frac{b^2x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x^2)/(2*d^2) - (b*c - a*d)^2/(2*d^3*(c + d*x^2)) - (b*(b*c - a*d)*Log[
c + d*x^2])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)dx, x, x^2\right)\\
&= \frac{b^2x^2}{2d^2}-\frac{(bc-ad)^2}{2d^3\left(c+dx^2\right)}-\frac{b(bc-ad)\log\left(c+dx^2\right)}{d^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.90

$$
\frac{-\frac{(bc-ad)^2}{c+dx^2}+2b(ad-bc)\log\left(c+dx^2\right)+b^2dx^2}{2d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (b^2*d*x^2 - (b*c - a*d)^2/(c + d*x^2) + 2*b*(-(b*c) + a*d)*Log[c + d*x^2])
/(2*d^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2)^2, x]
```

fricas [A] time = 0.80, size = 101, normalized size = 1.63

$$
\frac{b^2d^2x^4+b^2cdx^2-b^2c^2+2\,abcd-a^2d^2-2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)\log\left(dx^2+c\right)}{2\left(d^4x^2+cd^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + b^2*c*d*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2
 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)*log(d*x^2 + c))/(d^4*x^2 + c*d^3)
```

giac [A] time = 0.33, size = 110, normalized size = 1.77

$$\frac{\left(dx^2+c\right)b^2}{2\,d^3}+\frac{\left(b^2c-abd\right)\log\left(\frac{\left|dx^2+c\right|}{\left(dx^2+c\right)^2\left|d\right|}\right)}{d^3}-\frac{\frac{b^2c^2d}{dx^2+c}-\frac{2\,abcd^2}{dx^2+c}+\frac{a^2d^3}{dx^2+c}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*x^2 + c)*b^2/d^3 + (b^2*c - a*b*d)*log(abs(d*x^2 + c)/((d*x^2 + c)^2
*abs(d)))/d^3 - 1/2*(b^2*c^2*d/(d*x^2 + c) - 2*a*b*c*d^2/(d*x^2 + c) + a^2*
d^3/(d*x^2 + c))/d^4
```

maple [A] time = 0.01, size = 97, normalized size = 1.56

$$\frac{b^2x^2}{2d^2}-\frac{a^2}{2\left(d\,x^2+c\right)d}+\frac{abc}{\left(d\,x^2+c\right)d^2}+\frac{ab\ln\left(d\,x^2+c\right)}{d^2}-\frac{b^2c^2}{2\left(d\,x^2+c\right)d^3}-\frac{b^2c\ln\left(d\,x^2+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^2*x^2/d^2+1/d^2*b*ln(d*x^2+c)*a-1/d^3*b^2*ln(d*x^2+c)*c-1/2/d/(d*x^2+
c)*a^2+1/d^2/(d*x^2+c)*a*b*c-1/2/d^3/(d*x^2+c)*b^2*c^2
```

maxima [A] time = 1.05, size = 74, normalized size = 1.19

$$\frac{b^2x^2}{2\,d^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(d^4x^2+cd^3\right)}-\frac{\left(b^2c-abd\right)\log\left(dx^2+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x^2 + c*d^3) - (
b^2*c - a*b*d)*log(d*x^2 + c)/d^3
```

mupad [B] time = 0.08, size = 77, normalized size = 1.24

$$\frac{b^2\,x^2}{2\,d^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{2\,d\,\left(d^3\,x^2+c\,d^2\right)}-\frac{\ln\left(d\,x^2+c\right)\,\left(b^2\,c-a\,b\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x^2)/(2*d^2) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*d*(c*d^2 + d^3*x^2))
 - (log(c + d*x^2)*(b^2*c - a*b*d))/d^3
```

sympy **[A]** time = 0.77, size = 68, normalized size = 1.10

$$\frac{b^2x^2}{2d^2}+\frac{b(ad-bc)\log\left(c+dx^2\right)}{d^3}+\frac{-a^2d^2+2abcd-b^2c^2}{2cd^3+2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x**2/(2*d**2) + b*(a*d - b*c)*log(c + d*x**2)/d**3 + (-a**2*d**2 + 2*a
*b*c*d - b**2*c**2)/(2*c*d**3 + 2*d**4*x**2)
```

3.184 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx &= \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{d^2\left(c+dx^2\right)^2}\right)dx\\
&= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(c+dx^2\right)^2}\,dx}{d^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)}-\frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2cd^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.09

$$
-\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^2, x]
```

fricas [B] time = 0.92, size = 302, normalized size = 3.68

$$
\left[\frac{4\,b^2c^2d^2x^3+\left(3\,b^2c^3-2\,abc^2d-a^2cd^2+\left(3\,b^2c^2d-2\,abcd^2-a^2d^3\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\left(3\,b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x}{4\left(c^2d^4x^2+c^3d^3\right)},\ \frac{2\,b^2c^2d^2x^3-\left(3\,b^2c^3-2\,abc^2d-a^2cd^2+\left(3\,b^2c^2d-2\,abcd^2-a^2d^3\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+\left(3\,b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x}{2\left(c^2d^4x^2+c^3d^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

giac [A] time = 0.36, size = 95, normalized size = 1.16

$$\frac{b^2x}{d^2} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(dx^2 + c\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

maple [A] time = 0.01, size = 129, normalized size = 1.57

$$\frac{a^2x}{2\left(d\,x^2+c\right)c} + \frac{a^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,c} - \frac{abx}{\left(d\,x^2+c\right)d} + \frac{ab\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\,d} + \frac{b^2cx}{2\left(d\,x^2+c\right)d^2} - \frac{3b^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\,d^2} + \frac{b^2x}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] b^2*x/d^2+1/2/c*x/(d*x^2+c)*a^2-1/d*x/(d*x^2+c)*a*b+1/2/d^2*c*x/(d*x^2+c)*b
^2+1/2/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+1/d/(c*d)^(1/2)*arctan(1
/(c*d)^(1/2)*d*x)*a*b-3/2/d^2*c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 2.38, size = 96, normalized size = 1.17

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{2\left(cd^3x^2 + c^2d^2\right)} + \frac{b^2x}{d^2} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 - 1
/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^2
)
```

mupad [**B**] time = 0.16, size = 124, normalized size = 1.51

$$\frac{b^2\,x}{d^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,\left(d^3\,x^2+c\,d^2\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\,\left(a\,d+3\,b\,c\right)}{\sqrt{c}\,\left(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+3\,b\,c\right)}{2\,c^{3/2}\,d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2)) +
 (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^2
+ 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

sympy [**B**] time = 0.73, size = 236, normalized size = 2.88

$$\frac{b^{2}x}{d^{2}}+\frac{x\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2c^{2}d^{2}+2cd^{3}x^{2}}-\frac{\sqrt{-\frac{1}{c^{3}d^{5}}}\left(ad-bc\right)\left(ad+3bc\right)\log\left(-\frac{c^{2}d^{2}\sqrt{-\frac{1}{c^{3}d^{5}}}\left(ad-bc\right)\left(ad+3bc\right)}{a^{2}d^{2}+2abcd-3b^{2}c^{2}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^{3}d^{5}}}\left(ad-bc\right)\left(ad+3bc\right)\log\left(\frac{c^{2}d^{2}\sqrt{-\frac{1}{c^{3}d^{5}}}\left(ad-bc\right)\left(ad+3bc\right)}{a^{2}d^{2}+2abcd-3b^{2}c^{2}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

3.185 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log(x)}{c^2}+\frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log(x)}{c^2}+\frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*c*d^2*(c + d*x^2)) + (a^2*Log[x])/c^2 - ((a^2/c^2 - b^2/d^
2)*Log[c + d*x^2])/2
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{c^2x}-\frac{(bc-ad)^2}{cd(c+dx)^2}+\frac{b^2c^2-a^2d^2}{c^2d(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{a^2\log(x)}{c^2}-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.04

$$\frac{2a^2\log(x)+\frac{(bc-ad)\left(\left(c+dx^2\right)(ad+bc)\log\left(c+dx^2\right)+c(bc-ad)\right)}{d^2\left(c+dx^2\right)}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] (2*a^2*Log[x] + ((b*c - a*d)*(c*(b*c - a*d) + (b*c + a*d)*(c + d*x^2)*Log[c
 + d*x^2]))/(d^2*(c + d*x^2)))/(2*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)^2), x]
```

fricas [A] time = 0.65, size = 116, normalized size = 1.73

$$\frac{b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^3-a^2cd^2+\left(b^2c^2d-a^2d^3\right)x^2\right)\log\left(dx^2+c\right)+2\left(a^2d^3x^2+a^2cd^2\right)\log(x)}{2\left(c^2d^3x^2+c^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^3 - a^2*c*d^2 + (b^2*c^2*d
- a^2*d^3)*x^2)*log(d*x^2 + c) + 2*(a^2*d^3*x^2 + a^2*c*d^2)*log(x))/(c^2*d
^3*x^2 + c^3*d^2)
```

giac [A] time = 0.32, size = 99, normalized size = 1.48

$$\frac{a^2\log\left(x^2\right)}{2\,c^2}+\frac{\left(b^2c^2-a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{2\,c^2d^2}-\frac{b^2c^2x^2-a^2d^2x^2+2\,abc^2-2\,a^2cd}{2\left(dx^2+c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - a^2*d^2)*log(abs(d*x^2 + c))/(c^2*d^2
) - 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((d*x^2 + c)*c^
2*d)
```

maple [A] time = 0.01, size = 94, normalized size = 1.40

$$\frac{a^2}{2\left(d\,x^2+c\right)c}+\frac{a^2\ln(x)}{c^2}-\frac{a^2\ln\left(d\,x^2+c\right)}{2c^2}-\frac{ab}{\left(d\,x^2+c\right)d}+\frac{b^2c}{2\left(d\,x^2+c\right)d^2}+\frac{b^2\ln\left(d\,x^2+c\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*ln(d*x^2+c)*a^2+1/2/d^2*ln(d*x^2+c)*b^2+1/2/c/(d*x^2+c)*a^2-1/d/(d
*x^2+c)*a*b+1/2*c/d^2/(d*x^2+c)*b^2+a^2*ln(x)/c^2
```

maxima [A] time = 1.07, size = 86, normalized size = 1.28

$$\frac{a^2\log\left(x^2\right)}{2\,c^2}+\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(cd^3x^2+c^2d^2\right)}+\frac{\left(b^2c^2-a^2d^2\right)\log\left(dx^2+c\right)}{2\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(c*d^3*x^2 + c^2
*d^2) + 1/2*(b^2*c^2 - a^2*d^2)*log(d*x^2 + c)/(c^2*d^2)
```

mupad [B] time = 0.12, size = 80, normalized size = 1.19

$$\frac{a^2\,\ln(x)}{c^2}+\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{2\,c\,d^2\,\left(d\,x^2+c\right)}-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-b^2\,c^2\right)}{2\,c^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^2),x)
```

```
[Out] (a^2*log(x))/c^2 + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*c*d^2*(c + d*x^2)) -
(log(c + d*x^2)*(a^2*d^2 - b^2*c^2))/(2*c^2*d^2)
```

sympy [A] time = 1.25, size = 80, normalized size = 1.19

$$\frac{a^2\log(x)}{c^2}+\frac{a^2d^2-2abcd+b^2c^2}{2c^2d^2+2cd^3x^2}-\frac{(ad-bc)\,(ad+bc)\log\left(\frac{c}{d}+x^2\right)}{2c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**2,x)
```

```
[Out] a**2*log(x)/c**2 + (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d
**3*x**2) - (a*d - b*c)*(a*d + b*c)*log(c/d + x**2)/(2*c**2*d**2)
```

3.186 $\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{x\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{a^2}{cx\left(c+dx^2\right)}+\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {462, 385, 205}

$$-\frac{x\left(3a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{a^2}{cx\left(c+dx^2\right)}+\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2))) - ((b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x)/(2*c^2*d*(
c + d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^
(5/2)*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
```

```
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^2}\,dx &= -\frac{a^2}{cx\left(c+dx^2\right)}+\frac{\int \frac{a(2bc-3ad)+b^2cx^2}{\left(c+dx^2\right)^2}\,dx}{c}\\ &= -\frac{a^2}{cx\left(c+dx^2\right)}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{3a^2d}{c}\right)x}{2c\left(c+dx^2\right)}+\frac{\left((bc-ad)(bc+3ad)\right)\int \frac{1}{c+dx^2}\,dx}{2c^2d}\\ &= -\frac{a^2}{cx\left(c+dx^2\right)}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{3a^2d}{c}\right)x}{2c\left(c+dx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.86

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}-\frac{a^2}{c^2x}-\frac{x(bc-ad)^2}{2c^2d\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] -(a^2/(c^2*x)) - ((b*c - a*d)^2*x)/(2*c^2*d*(c + d*x^2)) + ((b^2*c^2 + 2*a*
b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*d^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)^2), x]
```

fricas [A] time = 0.80, size = 305, normalized size = 2.88

$$\left[-\frac{4\,a^{2}c^{2}d^{2}+2\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+3\,a^{2}cd^{3}\right)x^{2}-\left(\left(b^{2}c^{2}d+2\,abcd^{2}-3\,a^{2}d^{3}\right)x^{3}+\left(b^{2}c^{3}+2\,abc^{2}d-3\,a^{2}cd^{2}\right)x\right)\sqrt{-cd}\,\log\left(\frac{dx^{2}+2\,\sqrt{-cd}\,x-c}{dx^{2}+c}\right)}{4\left(c^{3}d^{3}x^{3}+c^{4}d^{2}x\right)},-\frac{2\,a^{2}c^{2}d^{2}+\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+3\,a^{2}cd^{3}\right)x^{2}-\left(\left(b^{2}c^{2}d+2\,abcd^{2}-3\,a^{2}d^{3}\right)x^{3}+\left(b^{2}c^{3}+2\,abc^{2}d-3\,a^{2}cd^{2}\right)x\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{2\left(c^{3}d^{3}x^{3}+c^{4}d^{2}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*c^2*d^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2 - ((
b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c
*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^3*d^3
*x^3 + c^4*d^2*x), -1/2*(2*a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2
*c*d^3)*x^2 - ((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b
*c^2*d - 3*a^2*c*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^3*d^3*x^3 + c^
4*d^2*x)]
```

giac [A] time = 0.32, size = 102, normalized size = 0.96

$$\frac{\left(b^{2}c^{2}+2\,abcd-3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^{2}d}-\frac{b^{2}c^{2}x^{2}-2\,abcdx^{2}+3\,a^{2}d^{2}x^{2}+2\,a^{2}cd}{2\left(dx^{3}+cx\right)c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*
d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 3*a^2*d^2*x^2 + 2*a^2*c*d)/((d*x^3
+ c*x)*c^2*d)
```

maple [A] time = 0.03, size = 131, normalized size = 1.24

$$-\frac{a^{2}dx}{2\left(d\,x^{2}+c\right)c^{2}}-\frac{3a^{2}d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;c^{2}}+\frac{abx}{\left(d\,x^{2}+c\right)c}+\frac{ab\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c}-\frac{b^{2}x}{2\left(d\,x^{2}+c\right)d}+\frac{b^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;d}-\frac{a^{2}}{c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x/(d*x^2+c)*a^2+1/c*x/(d*x^2+c)*a*b-1/2/d*x/(d*x^2+c)*b^2-3/2/c^
2*d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+1/c/(c*d)^(1/2)*arctan(1/(c*d
)^(1/2)*d*x)*a*b+1/2/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-a^2/c^2/x
```

maxima [A] time = 2.49, size = 100, normalized size = 0.94

$$-\frac{2\,a^{2}cd+\left(b^{2}c^{2}-2\,abcd+3\,a^{2}d^{2}\right)x^{2}}{2\left(c^{2}d^{2}x^{3}+c^{3}dx\right)}+\frac{\left(b^{2}c^{2}+2\,abcd-3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x^2)/(c^2*d^2*x^3 + c^3
*d*x) + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c
*d)*c^2*d)
```

mupad [**B**] time = 0.19, size = 128, normalized size = 1.21

$$\frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{c}\,\left(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{2\,c^{5/2}\,d^{3/2}}-\frac{\frac{a^2}{c}+\frac{x^2\,\left(3\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d}}{d\,x^3+c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^2),x)
```

```
[Out] (atan((d^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(c^(1/2)*(b^2*c^2 - 3*a^2*d^2 +
 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*c^(5/2)*d^(3/2)) - (a^2/c + (x^
2*(3*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^2*d))/(c*x + d*x^3)
```

sympy [**B**] time = 0.87, size = 238, normalized size = 2.25

$$\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}\,(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4}+\frac{-2a^2cd+x^2\left(-3a^2d^2+2abcd-b^2c^2\right)}{2c^3dx+2c^2d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**2,x)
```

```
[Out] sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d*
*3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4
 - sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*
d**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)
/4 + (-2*a**2*c*d + x**2*(-3*a**2*d**2 + 2*a*b*c*d - b**2*c**2))/(2*c**3*d*
x + 2*c**2*d**2*x**3)
```

3.187 $\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{a^2}{2c^2x^2}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}+\frac{2a\log(x)(bc-ad)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -a^2/(2*c^2*x^2) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)) + (2*a*(b*c - a*d)*L
og[x])/c^3 - (a*(b*c - a*d)*Log[c + d*x^2])/c^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{c^2x^2}-\frac{2a(-bc+ad)}{c^3x}+\frac{(bc-ad)^2}{c^2(c+dx)^2}+\frac{2ad(-bc+ad)}{c^3(c+dx)}\right)dx,x,x^2\right)\\ &= -\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d\left(c+dx^2\right)}+\frac{2a(bc-ad)\log(x)}{c^3}-\frac{a(bc-ad)\log\left(c+dx^2\right)}{c^3}\end{aligned}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.89

$$-\frac{\frac{a^2c}{x^2}+\frac{c(bc-ad)^2}{d\left(c+dx^2\right)}-2a(ad-bc)\log\left(c+dx^2\right)+4a\log(x)(ad-bc)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] -1/2*((a^2*c)/x^2 + (c*(b*c - a*d)^2)/(d*(c + d*x^2)) + 4*a*(-(b*c) + a*d)*
Log[x] - 2*a*(-(b*c) + a*d)*Log[c + d*x^2])/c^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)^2), x]
```

fricas [B] time = 0.89, size = 159, normalized size = 1.96

$$-\frac{a^2c^2d+\left(b^2c^3-2\,abc^2d+2\,a^2cd^2\right)x^2+2\left(\left(abcd^2-a^2d^3\right)x^4+\left(abc^2d-a^2cd^2\right)x^2\right)\log\left(dx^2+c\right)-4\left(\left(abcd^2-a^2d^3\right)x^4+\left(abc^2d-a^2cd^2\right)x^2\right)\log(x)}{2\left(c^3d^2x^4+c^4dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c^2*d + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x^2 + 2*((a*b*c*d^2
 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(d*x^2 + c) - 4*((a*b*c*d
^2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(x))/(c^3*d^2*x^4 + c^4
*d*x^2)
```

giac [A] time = 0.30, size = 109, normalized size = 1.35

$$\frac{\left(abc-a^2d\right)\log\left(x^2\right)}{c^3}-\frac{\left(abcd-a^2d^2\right)\log\left(\left|dx^2+c\right|\right)}{c^3d}-\frac{b^2c^2x^2-2\,abcdx^2+2\,a^2d^2x^2+a^2cd}{2\left(dx^4+cx^2\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b*c - a^2*d)*log(x^2)/c^3 - (a*b*c*d - a^2*d^2)*log(abs(d*x^2 + c))/(c^3
*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 2*a^2*d^2*x^2 + a^2*c*d)/((d*x^4 +
 c*x^2)*c^2*d)
```

maple [A] time = 0.02, size = 114, normalized size = 1.41

$$-\frac{a^2d}{2\left(d\,x^2+c\right)c^2}-\frac{2a^2d\ln(x)}{c^3}+\frac{a^2d\ln\left(d\,x^2+c\right)}{c^3}+\frac{ab}{\left(d\,x^2+c\right)c}+\frac{2ab\ln(x)}{c^2}-\frac{ab\ln\left(d\,x^2+c\right)}{c^2}-\frac{b^2}{2\left(d\,x^2+c\right)d}-\frac{a^2}{2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^2,x)
```

```
[Out] 1/c^3*a^2*ln(d*x^2+c)*d-1/c^2*a*ln(d*x^2+c)*b-1/2/c^2/(d*x^2+c)*a^2*d+1/c/(
d*x^2+c)*a*b-1/2/d/(d*x^2+c)*b^2-1/2*a^2/c^2/x^2-2*a^2/c^3*ln(x)*d+2*a/c^2*
ln(x)*b
```

maxima [A] time = 1.07, size = 100, normalized size = 1.23

$$-\frac{a^2cd+\left(b^2c^2-2\,abcd+2\,a^2d^2\right)x^2}{2\left(c^2d^2x^4+c^3dx^2\right)}-\frac{\left(abc-a^2d\right)\log\left(dx^2+c\right)}{c^3}+\frac{\left(abc-a^2d\right)\log\left(x^2\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)/(c^2*d^2*x^4 + c^3*d
*x^2) - (a*b*c - a^2*d)*log(d*x^2 + c)/c^3 + (a*b*c - a^2*d)*log(x^2)/c^3
```

mupad [B] time = 0.09, size = 100, normalized size = 1.23

$$\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d-a\,b\,c\right)}{c^3}-\frac{\frac{a^2}{2\,c}+\frac{x^2\,\left(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d}}{d\,x^4+c\,x^2}-\frac{\ln(x)\,\left(2\,a^2\,d-2\,a\,b\,c\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^2),x)
```

```
[Out] (log(c + d*x^2)*(a^2*d - a*b*c))/c^3 - (a^2/(2*c) + (x^2*(2*a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))/(2*c^2*d))/(c*x^2 + d*x^4) - (log(x)*(2*a^2*d - 2*a*b*c))/
c^3
```

sympy **[A]** time = 1.36, size = 92, normalized size = 1.14

$$-\frac{2a\left(ad-bc\right)\log\left(x\right)}{c^3}+\frac{a\left(ad-bc\right)\log\left(\frac{c}{d}+x^2\right)}{c^3}+\frac{-a^2cd+x^2\left(-2a^2d^2+2abcd-b^2c^2\right)}{2c^3dx^2+2c^2d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**2,x)
```

```
[Out] -2*a*(a*d - b*c)*log(x)/c**3 + a*(a*d - b*c)*log(c/d + x**2)/c**3 + (-a**2*
c*d + x**2*(-2*a**2*d**2 + 2*a*b*c*d - b**2*c**2))/(2*c**3*d*x**2 + 2*c**2*
d**2*x**4)
```

3.188 $\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=126

$$\frac{x\left(5a^2d^2-6abcd+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}-\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a(6bc-5ad)}{3c^3x}$$

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 456, 453, 205}

$$\frac{x\left(5a^2d^2-6abcd+3b^2c^2\right)}{6c^3\left(c+dx^2\right)}-\frac{a^2}{3cx^3\left(c+dx^2\right)}-\frac{a(6bc-5ad)}{3c^3x}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -(a*(6*b*c - 5*a*d))/(3*c^3*x) - a^2/(3*c*x^3*(c + d*x^2)) + ((3*b^2*c^2 -
6*a*b*c*d + 5*a^2*d^2)*x)/(6*c^3*(c + d*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
```

```
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^2}\,dx = -\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{\int \frac{a(6bc-5ad)+3b^2cx^2}{x^2\left(c+dx^2\right)^2}\,dx}{3c}$$

$$= -\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{\left(3b^2c^2-6abcd+5a^2d^2\right)x}{6c^3\left(c+dx^2\right)}-\frac{\int \frac{-\frac{2a(6bc-5ad)}{c}-\left(3b^2-\frac{6abd}{c}+\frac{5a^2d^2}{c^2}\right)x^2}{x^2\left(c+dx^2\right)}\,dx}{6c}$$

$$= -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{\left(3b^2c^2-6abcd+5a^2d^2\right)x}{6c^3\left(c+dx^2\right)}+\frac{((bc-5ad)(bc-ad))\int}{2c^3}$$

$$= -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3\left(c+dx^2\right)}+\frac{\left(3b^2c^2-6abcd+5a^2d^2\right)x}{6c^3\left(c+dx^2\right)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}}{2c^{7/2}\sqrt{d}}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.85

$$\frac{\left(5a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}-\frac{a^2}{3c^2x^3}+\frac{x(bc-ad)^2}{2c^3\left(c+dx^2\right)}+\frac{2a(ad-bc)}{c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] -1/3*a^2/(c^2*x^3) + (2*a*(-(b*c) + a*d))/(c^3*x) + ((b*c - a*d)^2*x)/(2*c^
3*(c + d*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(2*c^(7/2)*Sqrt[d])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)^2), x]
```

fricas [A] time = 0.79, size = 356, normalized size = 2.83

$$\left[-\frac{4\,a^2c^3d-6\left(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3\right)x^4+4\left(6\,abc^3d-5\,a^2c^2d^2\right)x^2+3\left(\left(b^2c^2d-6\,abcd^2+5\,a^2d^3\right)x^5+\left(b^2c^3-6\,abc^2d+5\,a^2cd^2\right)x^3\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{12\left(c^4d^2x^5+c^5dx^3\right)},\ -\frac{2\,a^2c^3d-3\left(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3\right)x^4+2\left(6\,abc^3d-5\,a^2c^2d^2\right)x^2-3\left(\left(b^2c^2d-6\,abcd^2+5\,a^2d^3\right)x^5+\left(b^2c^3-6\,abc^2d+5\,a^2cd^2\right)x^3\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{6\left(c^4d^2x^5+c^5dx^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*c^3*d - 6*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 4*(
6*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2 + 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)
*x^5 + (b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^2*x^5 + c^5*d*x^3), -1/6*(2*a^2*c^3
*d - 3*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 2*(6*a*b*c^3*d - 5*a
^2*c^2*d^2)*x^2 - 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^5 + (b^2*c^3 -
 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^4*d^2*
x^5 + c^5*d*x^3)]
```

giac [A] time = 0.39, size = 111, normalized size = 0.88

$$\frac{\left(b^2c^2-6\,abcd+5\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^3}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(dx^2+c\right)c^3}-\frac{6\,abcx^2-6\,a^2dx^2+a^2c}{3\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
 + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c^3) - 1/3*(6*a*b
*c*x^2 - 6*a^2*d*x^2 + a^2*c)/(c^3*x^3)
```

maple [A] time = 0.02, size = 161, normalized size = 1.28

$$\frac{a^2d^2x}{2\left(d\,x^2+c\right)c^3}+\frac{5a^2d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;c^3}-\frac{abdx}{\left(d\,x^2+c\right)c^2}-\frac{3abd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c^2}+\frac{b^2x}{2\left(d\,x^2+c\right)c}+\frac{b^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}\;c}+\frac{2a^2d}{c^3x}-\frac{2ab}{c^2x}-\frac{a^2}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^2,x)
```

```
[Out] 1/2/c^3*x/(d*x^2+c)*a^2*d^2-1/c^2*x/(d*x^2+c)*a*b*d+1/2/c*x/(d*x^2+c)*b^2+5
/2/c^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2*d^2-3/c^2/(c*d)^(1/2)*arct
an(1/(c*d)^(1/2)*d*x)*a*b*d+1/2/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2
-1/3*a^2/c^2/x^3+2*a^2/c^3/x*d-2*a/c^2/x*b
```

maxima [A] time = 2.49, size = 118, normalized size = 0.94

$$\frac{3\left(b^2c^2-6\,abcd+5\,a^2d^2\right)x^4-2\,a^2c^2-2\left(6\,abc^2-5\,a^2cd\right)x^2}{6\left(c^3dx^5+c^4x^3\right)}+\frac{\left(b^2c^2-6\,abcd+5\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4 - 2*a^2*c^2 - 2*(6*a*b*c^2 - 5
*a^2*c*d)*x^2)/(c^3*d*x^5 + c^4*x^3) + 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2
)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
```

mupad [B] time = 0.19, size = 147, normalized size = 1.17

$$\frac{\frac{x^4\left(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^3}-\frac{a^2}{3\,c}+\frac{a\,x^2\,(5\,a\,d-6\,b\,c)}{3\,c^2}}{d\,x^5+c\,x^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(5\,a\,d-b\,c)}{\sqrt{c}\,\left(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}\right)(a\,d-b\,c)\,(5\,a\,d-b\,c)}{2\,c^{7/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^2),x)
```

```
[Out] ((x^4*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(2*c^3) - a^2/(3*c) + (a*x^2*(5*a*
d - 6*b*c))/(3*c^2))/(c*x^3 + d*x^5) + (atan((d^(1/2)*x*(a*d - b*c)*(5*a*d
- b*c))/(c^(1/2)*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(5*a*d - b
*c))/(2*c^(7/2)*d^(1/2))
```

sympy [B] time = 0.99, size = 248, normalized size = 1.97

$$-\frac{\sqrt{-\frac{1}{c^7d}}\,(ad-bc)\,(5ad-bc)\log\left(-\frac{c^4\sqrt{-\frac{1}{c^7d}}\,(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^7d}}\,(ad-bc)\,(5ad-bc)\log\left(\frac{c^4\sqrt{-\frac{1}{c^7d}}\,(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{-2a^2c^2+x^4\left(15a^2d^2-18abcd+3b^2c^2\right)+x^2\left(10a^2cd-12abc^2\right)}{6c^4x^3+6c^3dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**2,x)
```

```
[Out] -sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(-c**4*sqrt(-1/(c**7*d))*(a
*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + sqrt
(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(c**4*sqrt(-1/(c**7*d))*(a*d - b
*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + (-2*a**2*c
**2 + x**4*(15*a**2*d**2 - 18*a*b*c*d + 3*b**2*c**2) + x**2*(10*a**2*c*d -
12*a*b*c**2))/(6*c**4*x**3 + 6*c**3*d*x**5)
```

3.189 $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{9/2}}-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}-\frac{x(bc-ad)(9bc-ad)}{8d^4\left(c+dx^2\right)}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+$$

Rubi [A] time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 455, 1153, 205}

$$-\frac{x\left(a^2d^2-10abcd+13b^2c^2\right)}{4cd^4}+\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{9/2}}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{x(bc-ad)(9bc-ad)}{8d^4\left(c+dx^2\right)}+\frac{b^2x^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x)/(4*c*d^4) + (b^2*x^3)/(3*d^3) + ((
b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c - a*d)*x)/(
8*d^4*(c + d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*Sqrt[c]*d^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
```

```
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^4\left(-4a^2d^2+5(bc-ad)^2-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \frac{\int \frac{cd(bc-ad)(9bc-ad)-2d^2(bc-ad)(9bc-ad)x^2+8b^2cd^3x^4}{c+dx^2}\,dx}{8cd^5}$$

$$= \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \frac{\int \left(-2d\left(13b^2c^2-10abcd+a^2d^2\right)+8b^2cd^2x^2+\right.}{8cd^5}$$

$$= -\frac{\left(13b^2c^2-10abcd+a^2d^2\right)x}{4cd^4} + \frac{b^2x^3}{3d^3} + \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \left(35b^2c^2\right.$$

$$= -\frac{\left(13b^2c^2-10abcd+a^2d^2\right)x}{4cd^4} + \frac{b^2x^3}{3d^3} + \frac{(bc-ad)^2x^5}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\left(c+dx^2\right)} + \left(35b^2c^2\right.$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.91

$$\frac{\left(3a^2d^2-30abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{9/2}} - \frac{x\left(5a^2d^2-18abcd+13b^2c^2\right)}{8d^4\left(c+dx^2\right)} + \frac{cx(bc-ad)^2}{4d^4\left(c+dx^2\right)^2} - \frac{bx(3bc-2ad)}{d^4} + \frac{b^2x^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*(3*b*c - 2*a*d)*x)/d^4) + (b^2*x^3)/(3*d^3) + (c*(b*c - a*d)^2*x)/(4*d
^4*(c + d*x^2)^2) - ((13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*x)/(8*d^4*(c + d
*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*d^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3, x]
```

fricas [A] time = 0.71, size = 522, normalized size = 3.20

$$\left[\frac{16\,b^2cd^4x^7 - 16\left(7\,b^2c^2d^3 - 6\,abcd^4\right)x^5 - 10\left(35\,b^2c^3d^2 - 30\,abc^2d^3 + 3\,a^2cd^4\right)x^3 - 3\left(35\,b^2c^4 - 30\,abc^3d + 3\,a^2c^2d^2 + \left(35\,b^2c^2d^2 - 30\,abcd^3 + 3\,a^2d^4\right)x^4 + 2\left(35\,b^2c^3d - 30\,abc^2d^2 + 3\,a^2cd^3\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2 - 2\,\sqrt{-cd}\,x - c}{dx^2 + c}\right) - 6\left(35\,b^2c^4d - 30\,abc^3d^2 + 3\,a^2c^2d^3\right)x}{48\left(cd^7x^4 + 2\,c^2d^6x^2 + c^3d^5\right)}, \frac{8\,b^2cd^4x^7 - 8\left(7\,b^2c^2d^3 - 6\,abcd^4\right)x^5 - 5\left(35\,b^2c^3d^2 - 30\,abc^2d^3 + 3\,a^2cd^4\right)x^3 + 3\left(35\,b^2c^4 - 30\,abc^3d + 3\,a^2c^2d^2 + \left(35\,b^2c^2d^2 - 30\,abcd^3 + 3\,a^2d^4\right)x^4 + 2\left(35\,b^2c^3d - 30\,abc^2d^2 + 3\,a^2cd^3\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right) - 3\left(35\,b^2c^4d - 30\,abc^3d^2 + 3\,a^2c^2d^3\right)x}{24\left(cd^7x^4 + 2\,c^2d^6x^2 + c^3d^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b^2*c*d^4*x^7 - 16*(7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 10*(35*b^2
*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a^2*c*d^4)*x^3 - 3*(35*b^2*c^4 - 30*a*b*c^3*d
 + 3*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*
b^2*c^3*d - 30*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sq
rt(-c*d)*x - c)/(d*x^2 + c)) - 6*(35*b^2*c^4*d - 30*a*b*c^3*d^2 + 3*a^2*c^2
*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5), 1/24*(8*b^2*c*d^4*x^7 - 8*(
7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 5*(35*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a
^2*c*d^4)*x^3 + 3*(35*b^2*c^4 - 30*a*b*c^3*d + 3*a^2*c^2*d^2 + (35*b^2*c^2*
d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 30*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(35*b^2*c^4*d - 30*a*b*
c^3*d^2 + 3*a^2*c^2*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5)]
```

giac [A] time = 0.35, size = 154, normalized size = 0.94

$$\frac{\left(35\,b^2c^2 - 30\,abcd + 3\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,d^4} - \frac{13\,b^2c^2dx^3 - 18\,abcd^2x^3 + 5\,a^2d^3x^3 + 11\,b^2c^3x - 14\,abc^2dx + 3\,a^2cd^2x}{8\left(dx^2 + c\right)^2d^4} + \frac{b^2d^6x^3 - 9\,b^2cd^5x + 6\,abd^6x}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
d^4) - 1/8*(13*b^2*c^2*d*x^3 - 18*a*b*c*d^2*x^3 + 5*a^2*d^3*x^3 + 11*b^2*c^
```

```
3*x - 14*a*b*c^2*d*x + 3*a^2*c*d^2*x)/((d*x^2 + c)^2*d^4) + 1/3*(b^2*d^6*x^
3 - 9*b^2*c*d^5*x + 6*a*b*d^6*x)/d^9
```

maple [A] time = 0.02, size = 223, normalized size = 1.37

$$-\frac{5a^{2}x^{3}}{8\left(d\,x^{2}+c\right)^{2}d}+\frac{9abc\,x^{3}}{4\left(d\,x^{2}+c\right)^{2}d^{2}}-\frac{13b^{2}c^{2}x^{3}}{8\left(d\,x^{2}+c\right)^{2}d^{3}}-\frac{3a^{2}cx}{8\left(d\,x^{2}+c\right)^{2}d^{2}}+\frac{7ab\,c^{2}x}{4\left(d\,x^{2}+c\right)^{2}d^{3}}-\frac{11b^{2}c^{3}x}{8\left(d\,x^{2}+c\right)^{2}d^{4}}+\frac{b^{2}x^{3}}{3d^{3}}+\frac{3a^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,d^{2}}-\frac{15abc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\sqrt{cd}\,d^{3}}+\frac{35b^{2}c^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,d^{4}}+\frac{2abx}{d^{3}}-\frac{3b^{2}cx}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/3*b^2*x^3/d^3+2*b/d^3*a*x-3*b^2/d^4*c*x-5/8/d/(d*x^2+c)^2*x^3*a^2+9/4/d^2
/(d*x^2+c)^2*x^3*a*b*c-13/8/d^3/(d*x^2+c)^2*x^3*b^2*c^2-3/8/d^2/(d*x^2+c)^2
*a^2*c*x+7/4/d^3/(d*x^2+c)^2*a*b*c^2*x-11/8/d^4/(d*x^2+c)^2*b^2*c^3*x+3/8/d
^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2-15/4/d^3/(c*d)^(1/2)*arctan(1/
(c*d)^(1/2)*d*x)*a*b*c+35/8/d^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2*c
^2
```

maxima [A] time = 2.37, size = 159, normalized size = 0.98

$$-\frac{\left(13\,b^{2}c^{2}d-18\,abcd^{2}+5\,a^{2}d^{3}\right)x^{3}+\left(11\,b^{2}c^{3}-14\,abc^{2}d+3\,a^{2}cd^{2}\right)x}{8\left(d^{6}x^{4}+2\,cd^{5}x^{2}+c^{2}d^{4}\right)}+\frac{\left(35\,b^{2}c^{2}-30\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,d^{4}}+\frac{b^{2}dx^{3}-3\left(3\,b^{2}c-2\,abd\right)x}{3\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*x^3 + (11*b^2*c^3 - 14*a*b*
c^2*d + 3*a^2*c*d^2)*x)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) + 1/8*(35*b^2*c^2
 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4) + 1/3*(b^2
*d*x^3 - 3*(3*b^2*c - 2*a*b*d)*x)/d^4
```

mupad [B] time = 0.16, size = 159, normalized size = 0.98

$$\frac{b^2\,x^3}{3\,d^3}-\frac{\left(\frac{5\,a^2\,d^3}{8}-\frac{9\,a\,b\,c\,d^2}{4}+\frac{13\,b^2\,c^2\,d}{8}\right)\,x^3+\left(\frac{3\,a^2\,c\,d^2}{8}-\frac{7\,a\,b\,c^2\,d}{4}+\frac{11\,b^2\,c^3}{8}\right)\,x}{c^2\,d^4+2\,c\,d^5\,x^2+d^6\,x^4}-x\,\left(\frac{3\,b^2\,c}{d^4}-\frac{2\,a\,b}{d^3}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2-30\,a\,b\,c\,d+35\,b^2\,c^2\right)}{8\,\sqrt{c}\,d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*x^3)/(3*d^3) - (x^3*((5*a^2*d^3)/8 + (13*b^2*c^2*d)/8 - (9*a*b*c*d^2)/
4) + x*((11*b^2*c^3)/8 + (3*a^2*c*d^2)/8 - (7*a*b*c^2*d)/4))/(c^2*d^4 + d^6
*x^4 + 2*c*d^5*x^2) - x*((3*b^2*c)/d^4 - (2*a*b)/d^3) + (atan((d^(1/2)*x)/c
^(1/2))*(3*a^2*d^2 + 35*b^2*c^2 - 30*a*b*c*d))/(8*c^(1/2)*d^(9/2))
```

sympy [A] time = 1.78, size = 240, normalized size = 1.47

$$\frac{b^2x^3}{3d^3}+x\left(\frac{2ab}{d^3}-\frac{3b^2c}{d^4}\right)-\frac{\sqrt{-\frac{1}{cd^9}}\left(3a^2d^2-30abcd+35b^2c^2\right)\log\left(-cd^4\sqrt{-\frac{1}{cd^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{cd^9}}\left(3a^2d^2-30abcd+35b^2c^2\right)\log\left(cd^4\sqrt{-\frac{1}{cd^9}}+x\right)}{16}+\frac{x^3\left(-5a^2d^3+18abcd^2-13b^2c^2d\right)+x\left(-3a^2cd^2+14abc^2d-11b^2c^3\right)}{8c^2d^4+16cd^5x^2+8d^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**3/(3*d**3) + x*(2*a*b/d**3 - 3*b**2*c/d**4) - sqrt(-1/(c*d**9))*(3*
a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(-c*d**4*sqrt(-1/(c*d**9)) + x)/1
6 + sqrt(-1/(c*d**9))*(3*a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(c*d**4*
sqrt(-1/(c*d**9)) + x)/16 + (x**3*(-5*a**2*d**3 + 18*a*b*c*d**2 - 13*b**2*c
**2*d) + x*(-3*a**2*c*d**2 + 14*a*b*c**2*d - 11*b**2*c**3))/(8*c**2*d**4 +
16*c*d**5*x**2 + 8*d**6*x**4)
```

3.190 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=99

$$\frac{c(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4\left(c+dx^2\right)}-\frac{b(3bc-2ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^2}{2d^3}$$

Rubi [**A**] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{(3bc-ad)(bc-ad)}{2d^4\left(c+dx^2\right)}-\frac{b(3bc-2ad)\log\left(c+dx^2\right)}{2d^4}+\frac{b^2x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x^2)/(2*d^3) + (c*(b*c - a*d)^2)/(4*d^4*(c + d*x^2)^2) - ((b*c - a*d)*
(3*b*c - a*d))/(2*d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)*Log[c + d*x^2])/(2*
d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{b^2}{d^3}-\frac{c(bc-ad)^2}{d^3(c+dx)^3}+\frac{(bc-ad)(3bc-ad)}{d^3(c+dx)^2}-\frac{b(3bc-2ad)}{d^3(c+dx)}\right)dx, x, x^2\right)\\
&= \frac{b^2x^2}{2d^3}+\frac{c(bc-ad)^2}{4d^4\left(c+dx^2\right)^2}-\frac{(bc-ad)(3bc-ad)}{2d^4\left(c+dx^2\right)}-\frac{b(3bc-2ad)\log\left(c+dx^2\right)}{2d^4}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 114, normalized size = 1.15

$$
\frac{-a^2d^2\left(c+2dx^2\right)+2abcd\left(3c+4dx^2\right)-2b\left(c+dx^2\right)^2(3bc-2ad)\log\left(c+dx^2\right)+b^2\left(-5c^3-4c^2dx^2+4cd^2x^4+2d^3x^6\right)}{4d^4\left(c+dx^2\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2*(c + 2*d*x^2)) + 2*a*b*c*d*(3*c + 4*d*x^2) + b^2*(-5*c^3 - 4*c^2
*d*x^2 + 4*c*d^2*x^4 + 2*d^3*x^6) - 2*b*(3*b*c - 2*a*d)*(c + d*x^2)^2*Log[c
 + d*x^2])/(4*d^4*(c + d*x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3, x]
```

fricas [A] time = 0.99, size = 178, normalized size = 1.80

$$
\frac{2\,b^2d^3x^6+4\,b^2cd^2x^4-5\,b^2c^3+6\,abc^2d-a^2cd^2-2\left(2\,b^2c^2d-4\,abcd^2+a^2d^3\right)x^2-2\left(3\,b^2c^3-2\,abc^2d+\left(3\,b^2cd^2-2\,abd^3\right)x^4+2\left(3\,b^2c^2d-2\,abcd^2\right)x^2\right)\log\left(dx^2+c\right)}{4\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^3*x^6 + 4*b^2*c*d^2*x^4 - 5*b^2*c^3 + 6*a*b*c^2*d - a^2*c*d^2
- 2*(2*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2 - 2*(3*b^2*c^3 - 2*a*b*c^2*d
+ (3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 2*(3*b^2*c^2*d - 2*a*b*c*d^2)*x^2)*log(d*
x^2 + c))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

giac [A]   time = 0.34, size = 107, normalized size = 1.08

$$\frac{b^{2}x^{2}}{2\,d^{3}}-\frac{\left(3\,b^{2}c-2\,abd\right)\log\left(\left|dx^{2}+c\right|\right)}{2\,d^{4}}-\frac{5\,b^{2}c^{3}-6\,abc^{2}d+a^{2}cd^{2}+2\left(3\,b^{2}c^{2}d-4\,abcd^{2}+a^{2}d^{3}\right)x^{2}}{4\left(dx^{2}+c\right)^{2}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/2*(3*b^2*c - 2*a*b*d)*log(abs(d*x^2 + c))/d^4 - 1/4*(5*
b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)
*x^2)/((d*x^2 + c)^2*d^4)
```

maple [A]   time = 0.02, size = 155, normalized size = 1.57

$$\frac{a^{2}c}{4\left(d\,x^{2}+c\right)^{2}d^{2}}-\frac{ab\,c^{2}}{2\left(d\,x^{2}+c\right)^{2}d^{3}}+\frac{b^{2}c^{3}}{4\left(d\,x^{2}+c\right)^{2}d^{4}}+\frac{b^{2}x^{2}}{2d^{3}}-\frac{a^{2}}{2\left(d\,x^{2}+c\right)d^{2}}+\frac{2abc}{\left(d\,x^{2}+c\right)d^{3}}+\frac{ab\ln\left(d\,x^{2}+c\right)}{d^{3}}-\frac{3b^{2}c^{2}}{2\left(d\,x^{2}+c\right)d^{4}}-\frac{3b^{2}c\ln\left(d\,x^{2}+c\right)}{2d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2*x^2/d^3+1/4/d^2*c/(d*x^2+c)^2*a^2-1/2/d^3*c^2/(d*x^2+c)^2*a*b+1/4/d
^4*c^3/(d*x^2+c)^2*b^2+1/d^3*b*ln(d*x^2+c)*a-3/2/d^4*b^2*ln(d*x^2+c)*c-1/2/
d^2/(d*x^2+c)*a^2+2/d^3/(d*x^2+c)*a*b*c-3/2/d^4/(d*x^2+c)*b^2*c^2
```

maxima [A]   time = 1.12, size = 120, normalized size = 1.21

$$\frac{b^{2}x^{2}}{2\,d^{3}}-\frac{5\,b^{2}c^{3}-6\,abc^{2}d+a^{2}cd^{2}+2\left(3\,b^{2}c^{2}d-4\,abcd^{2}+a^{2}d^{3}\right)x^{2}}{4\left(d^{6}x^{4}+2\,cd^{5}x^{2}+c^{2}d^{4}\right)}-\frac{\left(3\,b^{2}c-2\,abd\right)\log\left(dx^{2}+c\right)}{2\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2/d^3 - 1/4*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d
 - 4*a*b*c*d^2 + a^2*d^3)*x^2)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) - 1/2*(3*b
^2*c - 2*a*b*d)*log(d*x^2 + c)/d^4
```

mupad [B]   time = 0.15, size = 123, normalized size = 1.24

$$\frac{b^{2}\,x^{2}}{2\,d^{3}}-\frac{\ln\left(d\,x^{2}+c\right)\,\left(3\,b^{2}\,c-2\,a\,b\,d\right)}{2\,d^{4}}-\frac{x^{2}\,\left(\frac{a^{2}\,d^{2}}{2}-2\,a\,b\,c\,d+\frac{3\,b^{2}\,c^{2}}{2}\right)+\frac{a^{2}\,c\,d^{2}-6\,a\,b\,c^{2}\,d+5\,b^{2}\,c^{3}}{4\,d}}{c^{2}\,d^{3}+2\,c\,d^{4}\,x^{2}+d^{5}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*x^2)/(2*d^3) - (log(c + d*x^2)*(3*b^2*c - 2*a*b*d))/(2*d^4) - (x^2*((a
^2*d^2)/2 + (3*b^2*c^2)/2 - 2*a*b*c*d) + (5*b^2*c^3 + a^2*c*d^2 - 6*a*b*c^2
*d)/(4*d))/(c^2*d^3 + d^5*x^4 + 2*c*d^4*x^2)
```

sympy [A] time = 2.11, size = 122, normalized size = 1.23

$$\frac{b^2x^2}{2d^3}+\frac{b(2ad-3bc)\log\left(c+dx^2\right)}{2d^4}+\frac{-a^2cd^2+6abc^2d-5b^2c^3+x^2\left(-2a^2d^3+8abcd^2-6b^2c^2d\right)}{4c^2d^4+8cd^5x^2+4d^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x**2/(2*d**3) + b*(2*a*d - 3*b*c)*log(c + d*x**2)/(2*d**4) + (-a**2*c*
d**2 + 6*a*b*c**2*d - 5*b**2*c**3 + x**2*(-2*a**2*d**3 + 8*a*b*c*d**2 - 6*b
**2*c**2*d))/(4*c**2*d**4 + 8*c*d**5*x**2 + 4*d**6*x**4)
```

3.191 $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x(bc-ad)(ad+7bc)}{8cd^3\left(c+dx^2\right)}+\frac{x^3(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{b^2x}{d^3}$$

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 455, 388, 205}

$$-\frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}+\frac{x^3(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{x(bc-ad)(ad+7bc)}{8cd^3\left(c+dx^2\right)}+\frac{b^2x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (b^2*x)/d^3 + ((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) + ((b*c - a*d)*(7
*b*c + a*d)*x)/(8*c*d^3*(c + d*x^2)) - ((15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
```

```
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= \frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2}-\frac{\int \frac{x^2\left(-4a^2d^2+3(bc-ad)^2-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}\\
&= \frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2}+\frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)}+\frac{\int \frac{-d(bc-ad)(7bc+ad)+8b^2cd^2x^2}{c+dx^2}\,dx}{8cd^4}\\
&= \frac{b^2x}{d^3}+\frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2}+\frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)}-\frac{\left(15b^2c^2-6abcd-a^2d^2\right)\int \frac{1}{c+dx^2}\,dx}{8cd^3}\\
&= \frac{b^2x}{d^3}+\frac{(bc-ad)^2x^3}{4cd^2\left(c+dx^2\right)^2}+\frac{(bc-ad)(7bc+ad)x}{8cd^3\left(c+dx^2\right)}-\frac{\left(15b^2c^2-6abcd-a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 130, normalized size = 1.02

$$
\frac{x\left(a^2d^2\left(dx^2-c\right)-2abcd\left(3c+5dx^2\right)+b^2c\left(15c^2+25cdx^2+8d^2x^4\right)\right)}{8cd^3\left(c+dx^2\right)^2}-\frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(3*c + 5*d*x^2) + b^2*c*(15*c^2 + 25*c
*d*x^2 + 8*d^2*x^4)))/(8*c*d^3*(c + d*x^2)^2) - ((15*b^2*c^2 - 6*a*b*c*d -
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3, x]
```

fricas [B] time = 0.74, size = 475, normalized size = 3.74

$$\left[\frac{16\,b^2c^2d^3x^5+2\left(25\,b^2c^3d^2-10\,abc^2d^3+a^2cd^4\right)x^3+\left(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+\left(15\,b^2c^2d^2-6\,abcd^3-a^2d^4\right)x^4+2\left(15\,b^2c^3d-6\,abc^2d^2-a^2cd^3\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\left(15\,b^2c^4d-6\,abc^3d^2-a^2c^2d^3\right)x}{16\left(c^2d^6x^4+2\,c^3d^5x^2+c^4d^4\right)},\ \frac{8\,b^2c^2d^3x^5+\left(25\,b^2c^3d^2-10\,abc^2d^3+a^2cd^4\right)x^3-\left(15\,b^2c^4-6\,abc^3d-a^2c^2d^2+\left(15\,b^2c^2d^2-6\,abcd^3-a^2d^4\right)x^4+2\left(15\,b^2c^3d-6\,abc^2d^2-a^2cd^3\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+\left(15\,b^2c^4d-6\,abc^3d^2-a^2c^2d^3\right)x}{8\left(c^2d^6x^4+2\,c^3d^5x^2+c^4d^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^2*c^2*d^3*x^5 + 2*(25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)
*x^3 + (15*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*
d^3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^2*c^4*d - 6*
a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4), 1/8*
(8*b^2*c^2*d^3*x^5 + (25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)*x^3 - (1
5*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*d^3 - a^2
*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) + (15*b^2*c^4*d - 6*a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d
^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4)]
```

giac [A] time = 0.39, size = 133, normalized size = 1.05

$$\frac{b^2x}{d^3}-\frac{\left(15\,b^2c^2-6\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,cd^3}+\frac{9\,b^2c^2dx^3-10\,abcd^2x^3+a^2d^3x^3+7\,b^2c^3x-6\,abc^2dx-a^2cd^2x}{8\left(dx^2+c\right)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^2*x/d^3 - 1/8*(15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(s
qrt(c*d)*c*d^3) + 1/8*(9*b^2*c^2*d*x^3 - 10*a*b*c*d^2*x^3 + a^2*d^3*x^3 + 7
*b^2*c^3*x - 6*a*b*c^2*d*x - a^2*c*d^2*x)/((d*x^2 + c)^2*c*d^3)
```

maple [A] time = 0.01, size = 196, normalized size = 1.54

$$\frac{a^2x^3}{8\left(dx^2+c\right)^2c}-\frac{5ab\,x^3}{4\left(dx^2+c\right)^2d}+\frac{9b^2c\,x^3}{8\left(dx^2+c\right)^2d^2}-\frac{a^2x}{8\left(dx^2+c\right)^2d}-\frac{3abcx}{4\left(dx^2+c\right)^2d^2}+\frac{7b^2c^2x}{8\left(dx^2+c\right)^2d^3}+\frac{a^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,cd}+\frac{3ab\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\sqrt{cd}\,d^2}-\frac{15b^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,d^3}+\frac{b^2x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] b^2*x/d^3+1/8/(d*x^2+c)^2/c*x^3*a^2-5/4/d/(d*x^2+c)^2*x^3*a*b+9/8/d^2/(d*x^
2+c)^2*x^3*b^2*c-1/8/d/(d*x^2+c)^2*a^2*x-3/4/d^2/(d*x^2+c)^2*a*b*c*x+7/8/d^
3/(d*x^2+c)^2*b^2*c^2*x+1/8/d/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+3
/4/d^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b-15/8/d^3*c/(c*d)^(1/2)*arc
tan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 2.40, size = 143, normalized size = 1.13

$$\frac{\left(9\,b^{2}c^{2}d-10\,abcd^{2}+a^{2}d^{3}\right)x^{3}+\left(7\,b^{2}c^{3}-6\,abc^{2}d-a^{2}cd^{2}\right)x}{8\left(cd^{5}x^{4}+2\,c^{2}d^{4}x^{2}+c^{3}d^{3}\right)}+\frac{b^{2}x}{d^{3}}-\frac{\left(15\,b^{2}c^{2}-6\,abcd-a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,cd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*b^2*c^2*d - 10*a*b*c*d^2 + a^2*d^3)*x^3 + (7*b^2*c^3 - 6*a*b*c^2*d
- a^2*c*d^2)*x)/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3) + b^2*x/d^3 - 1/8*(15
*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^3)
```

mupad [B] time = 0.17, size = 135, normalized size = 1.06

$$\frac{b^2\,x}{d^3}-\frac{x\,\left(\frac{a^2\,d^2}{8}+\frac{3\,a\,b\,c\,d}{4}-\frac{7\,b^2\,c^2}{8}\right)-\frac{x^3\,\left(a^2\,d^3-10\,a\,b\,c\,d^2+9\,b^2\,c^2\,d\right)}{8\,c}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(a^2\,d^2+6\,a\,b\,c\,d-15\,b^2\,c^2\right)}{8\,c^{3/2}\,d^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*x)/d^3 - (x*((a^2*d^2)/8 - (7*b^2*c^2)/8 + (3*a*b*c*d)/4) - (x^3*(a^2*
d^3 + 9*b^2*c^2*d - 10*a*b*c*d^2))/(8*c))/(c^2*d^3 + d^5*x^4 + 2*c*d^4*x^2)
 + (atan((d^(1/2)*x)/c^(1/2))*(a^2*d^2 - 15*b^2*c^2 + 6*a*b*c*d))/(8*c^(3/2
)*d^(7/2))
```

sympy [A] time = 1.50, size = 223, normalized size = 1.76

$$\frac{b^{2}x}{d^{3}}-\frac{\sqrt{-\frac{1}{c^{3}d^{7}}}\left(a^{2}d^{2}+6abcd-15b^{2}c^{2}\right)\log\left(-c^{2}d^{3}\sqrt{-\frac{1}{c^{3}d^{7}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^{3}d^{7}}}\left(a^{2}d^{2}+6abcd-15b^{2}c^{2}\right)\log\left(c^{2}d^{3}\sqrt{-\frac{1}{c^{3}d^{7}}}+x\right)}{16}+\frac{x^{3}\left(a^{2}d^{3}-10abcd^{2}+9b^{2}c^{2}d\right)+x\left(-a^{2}cd^{2}-6abc^{2}d+7b^{2}c^{3}\right)}{8c^{3}d^{3}+16c^{2}d^{4}x^{2}+8cd^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*x/d**3 - sqrt(-1/(c**3*d**7))*(a**2*d**2 + 6*a*b*c*d - 15*b**2*c**2)*l
og(-c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 + sqrt(-1/(c**3*d**7))*(a**2*d**
2 + 6*a*b*c*d - 15*b**2*c**2)*log(c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 +
(x**3*(a**2*d**3 - 10*a*b*c*d**2 + 9*b**2*c**2*d) + x*(-a**2*c*d**2 - 6*a*b
*c**2*d + 7*b**2*c**3))/(8*c**3*d**3 + 16*c**2*d**4*x**2 + 8*c*d**5*x**4)
```

3.192 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=67

$$\frac{b(bc-ad)}{d^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4d^3\left(c+dx^2\right)^2}+\frac{b^2\log\left(c+dx^2\right)}{2d^3}$$

Rubi **[A]** time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{b(bc-ad)}{d^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4d^3\left(c+dx^2\right)^2}+\frac{b^2\log\left(c+dx^2\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -(b*c - a*d)^2/(4*d^3*(c + d*x^2)^2) + (b*(b*c - a*d))/(d^3*(c + d*x^2)) +
(b^2*Log[c + d*x^2])/(2*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3}-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{b^2}{d^2(c+dx)}\right)dx, x, x^2\right)\\ &= -\frac{(bc-ad)^2}{4d^3\left(c+dx^2\right)^2}+\frac{b(bc-ad)}{d^3\left(c+dx^2\right)}+\frac{b^2\log\left(c+dx^2\right)}{2d^3}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 75, normalized size = 1.12

$$\frac{-a^2d^2-2abd\left(c+2dx^2\right)+b^2c\left(3c+4dx^2\right)+2b^2\left(c+dx^2\right)^2\log\left(c+dx^2\right)}{4d^3\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(c + 2*d*x^2) + b^2*c*(3*c + 4*d*x^2) + 2*b^2*(c + d*
x^2)^2*Log[c + d*x^2])/(4*d^3*(c + d*x^2)^2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2)^3, x]
```

fricas [**A**] time = 0.61, size = 108, normalized size = 1.61

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+2\,b^2cdx^2+b^2c^2\right)\log\left(dx^2+c\right)}{4\left(d^5x^4+2\,cd^4x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d
^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(d^5*x^4 + 2*c*d^4*x^2 +
c^2*d^3)
```

giac [A] time = 0.44, size = 76, normalized size = 1.13

$$\frac{b^2\log\left(\left|dx^2+c\right|\right)}{2\,d^3}+\frac{4\left(b^2c-abd\right)x^2+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{4\left(dx^2+c\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(abs(d*x^2 + c))/d^3 + 1/4*(4*(b^2*c - a*b*d)*x^2 + (3*b^2*c^2 -
 2*a*b*c*d - a^2*d^2)/d)/((d*x^2 + c)^2*d^2)
```

maple [A] time = 0.01, size = 105, normalized size = 1.57

$$-\frac{a^2}{4\left(d\,x^2+c\right)^2 d}+\frac{abc}{2\left(d\,x^2+c\right)^2 d^2}-\frac{b^2c^2}{4\left(d\,x^2+c\right)^2 d^3}-\frac{ab}{\left(d\,x^2+c\right)d^2}+\frac{b^2c}{\left(d\,x^2+c\right)d^3}+\frac{b^2\ln\left(d\,x^2+c\right)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/4/d/(d*x^2+c)^2*a^2+1/2/d^2/(d*x^2+c)^2*a*b*c-1/4/d^3/(d*x^2+c)^2*b^2*c^
2+1/2*b^2*ln(d*x^2+c)/d^3-b/d^2/(d*x^2+c)*a+b^2/d^3/(d*x^2+c)*c
```

maxima [A] time = 1.02, size = 87, normalized size = 1.30

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\left(b^2cd-abd^2\right)x^2}{4\left(d^5x^4+2\,cd^4x^2+c^2d^3\right)}+\frac{b^2\log\left(dx^2+c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2)/(d^5*x^4
+ 2*c*d^4*x^2 + c^2*d^3) + 1/2*b^2*log(d*x^2 + c)/d^3
```

mupad [B] time = 0.07, size = 83, normalized size = 1.24

$$\frac{b^2\,\ln\left(d\,x^2+c\right)}{2\,d^3}-\frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{4\,d^3}+\frac{b\,x^2\,\left(a\,d-b\,c\right)}{d^2}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (b^2*log(c + d*x^2))/(2*d^3) - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(4*d^3) +
 (b*x^2*(a*d - b*c))/d^2)/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

sympy [A] time = 1.27, size = 87, normalized size = 1.30

$$\frac{b^2\log\left(c+dx^2\right)}{2d^3}+\frac{-a^2d^2-2abcd+3b^2c^2+x^2\left(-4abd^2+4b^2cd\right)}{4c^2d^3+8cd^4x^2+4d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(c + d*x**2)/(2*d**3) + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x**
2*(-4*a*b*d**2 + 4*b**2*c*d))/(4*c**2*d**3 + 8*c*d**4*x**2 + 4*d**5*x**4)
```

3.193 $\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 205}

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^2)
*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{\left(c+dx^2\right)^2}\,dx}{4cd}\\ &= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\int \frac{1}{c+dx^2}\,dx}{8c^2d^2}\\ &= -\frac{(bc-ad)x\left(a+bx^2\right)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 121, normalized size = 1.04

$$\frac{x\left(a^2d^2\left(5c+3dx^2\right)-2abcd\left(c-dx^2\right)-b^2c^2\left(3c+5dx^2\right)\right)}{8c^2d^2\left(c+dx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3*c + 5*d*x
^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^3, x]
```

fricas [B] time = 0.78, size = 449, normalized size = 3.87

$$\left[-\frac{2 \left(5 b^{2} c^{3} d^{2}-2 a b c^{2} d^{3}-3 a^{2} c d^{4}\right) x^{3}+\left(3 b^{2} c^{4}+2 a b c^{3} d+3 a^{2} c^{2} d^{2}+\left(3 b^{2} c^{2} d^{2}+2 a b c d^{3}+3 a^{2} d^{4}\right) x^{4}+2 \left(3 b^{2} c^{3} d+2 a b c^{2} d^{2}+3 a^{2} c d^{3}\right) x^{2}\right) \sqrt{-c d} \log \left(\frac{d x^{2}-2 \sqrt{-c d}\, x-c}{d x^{2}+c}\right)+2 \left(3 b^{2} c^{4} d+2 a b c^{3} d^{2}-5 a^{2} c^{2} d^{3}\right) x}{16 \left(c^{3} d^{5} x^{4}+2 c^{4} d^{4} x^{2}+c^{5} d^{3}\right)}, -\frac{\left(5 b^{2} c^{3} d^{2}-2 a b c^{2} d^{3}-3 a^{2} c d^{4}\right) x^{3}-\left(3 b^{2} c^{4}+2 a b c^{3} d+3 a^{2} c^{2} d^{2}+\left(3 b^{2} c^{2} d^{2}+2 a b c d^{3}+3 a^{2} d^{4}\right) x^{4}+2 \left(3 b^{2} c^{3} d+2 a b c^{2} d^{2}+3 a^{2} c d^{3}\right) x^{2}\right) \sqrt{c d} \arctan \left(\frac{\sqrt{c d}\, x}{c}\right)+\left(3 b^{2} c^{4} d+2 a b c^{3} d^{2}-5 a^{2} c^{2} d^{3}\right) x}{8 \left(c^{3} d^{5} x^{4}+2 c^{4} d^{4} x^{2}+c^{5} d^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

giac [A] time = 0.31, size = 126, normalized size = 1.09

$$\frac{\left(3 b^{2} c^{2}+2 a b c d+3 a^{2} d^{2}\right) \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{8 \sqrt{c d}\, c^{2} d^{2}}-\frac{5 b^{2} c^{2} d x^{3}-2 a b c d^{2} x^{3}-3 a^{2} d^{3} x^{3}+3 b^{2} c^{3} x+2 a b c^{2} d x-5 a^{2} c d^{2} x}{8 \left(d x^{2}+c\right)^{2} c^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

maple [A] time = 0.01, size = 147, normalized size = 1.27

$$\frac{3 a^{2} \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{8 \sqrt{c d}\, c^{2}}+\frac{a b \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{4 \sqrt{c d}\, c d}+\frac{3 b^{2} \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{8 \sqrt{c d}\, d^{2}}+\frac{\frac{\left(3 a^{2} d^{2}+2 a b c d-5 b^{2} c^{2}\right) x^{3}}{8 c^{2} d}+\frac{\left(5 a^{2} d^{2}-2 a b c d-3 b^{2} c^{2}\right) x}{8 c\, d^{2}}}{\left(d\, x^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/d^2/c*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*
a^2+1/4/c/d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b+3/8/d^2/(c*d)^(1/2)*a
rctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 2.32, size = 138, normalized size = 1.19

$$-\frac{\left(5\,b^{2}c^{2}d-2\,abcd^{2}-3\,a^{2}d^{3}\right)x^{3}+\left(3\,b^{2}c^{3}+2\,abc^{2}d-5\,a^{2}cd^{2}\right)x}{8\,\left(c^{2}d^{4}x^{4}+2\,c^{3}d^{3}x^{2}+c^{4}d^{2}\right)}+\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2*
d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

mupad [B] time = 0.20, size = 130, normalized size = 1.12

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{5/2}}-\frac{\frac{x\,\left(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c\,d^2}-\frac{x^3\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2\right)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)*
d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a^2
*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

sympy [B] time = 1.00, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^{5}d^{5}}}\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\log\left(-c^{3}d^{2}\sqrt{-\frac{1}{c^{5}d^{5}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^{5}d^{5}}}\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\log\left(c^{3}d^{2}\sqrt{-\frac{1}{c^{5}d^{5}}}+x\right)}{16}+\frac{x^{3}\left(3a^{2}d^{3}+2abcd^{2}-5b^{2}c^{2}d\right)+x\left(5a^{2}cd^{2}-2abc^{2}d-3b^{2}c^{3}\right)}{8c^{4}d^{2}+16c^{3}d^{3}x^{2}+8c^{2}d^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

3.194 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx$

Optimal. Leaf size=86

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2\left(c+dx^2\right)}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2\left(c+dx^2\right)}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}+\frac{a^2\log(x)}{c^3}+\frac{(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b*c - a*d)^2/(4*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(2*(c + d*x^2))
 + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*c^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{c^3x}-\frac{(bc-ad)^2}{cd(c+dx)^3}+\frac{b^2c^2-a^2d^2}{c^2d(c+dx)^2}-\frac{a^2d}{c^3(c+dx)}\right)dx, x, x^2\right)\\ &= \frac{(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2\left(c+dx^2\right)}+\frac{a^2\log(x)}{c^3}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}\end{aligned}$$

Mathematica [**A**] time = 0.05, size = 103, normalized size = 1.20

$$\frac{a^2d^2-b^2c^2}{2c^2d^2\left(c+dx^2\right)}+\frac{a^2d^2-2abcd+b^2c^2}{4cd^2\left(c+dx^2\right)^2}-\frac{a^2\log\left(c+dx^2\right)}{2c^3}+\frac{a^2\log(x)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(4*c*d^2*(c + d*x^2)^2) + (-(b^2*c^2) + a^2
*d^2)/(2*c^2*d^2*(c + d*x^2)) + (a^2*Log[x])/c^3 - (a^2*Log[c + d*x^2])/(2*
c^3)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)^3), x]
```

fricas [**B**] time = 1.18, size = 163, normalized size = 1.90

$$-\frac{b^2c^4+2\,abc^3d-3\,a^2c^2d^2+2\left(b^2c^3d-a^2cd^3\right)x^2+2\left(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2\right)\log\left(dx^2+c\right)-4\left(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2\right)\log(x)}{4\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^4 + 2*a*b*c^3*d - 3*a^2*c^2*d^2 + 2*(b^2*c^3*d - a^2*c*d^3)*x^2
 + 2*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(d*x^2 + c) - 4*(a^2*
d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(x))/(c^3*d^4*x^4 + 2*c^4*d^3*x
^2 + c^5*d^2)
```

giac [A] time = 0.41, size = 110, normalized size = 1.28

$$\frac{a^2\log\left(x^2\right)}{2\,c^3}-\frac{a^2\log\left(\left|dx^2+c\right|\right)}{2\,c^3}+\frac{3\,a^2d^4x^4-2\,b^2c^3dx^2+8\,a^2cd^3x^2-b^2c^4-2\,abc^3d+6\,a^2c^2d^2}{4\left(dx^2+c\right)^2c^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(x^2)/c^3 - 1/2*a^2*log(abs(d*x^2 + c))/c^3 + 1/4*(3*a^2*d^4*x^4
 - 2*b^2*c^3*d*x^2 + 8*a^2*c*d^3*x^2 - b^2*c^4 - 2*a*b*c^3*d + 6*a^2*c^2*d^
2)/((d*x^2 + c)^2*c^3*d^2)
```

maple [A] time = 0.02, size = 112, normalized size = 1.30

$$\frac{a^2}{4\left(d\,x^2+c\right)^2c}-\frac{ab}{2\left(d\,x^2+c\right)^2d}+\frac{b^2c}{4\left(d\,x^2+c\right)^2d^2}+\frac{a^2}{2\left(d\,x^2+c\right)c^2}+\frac{a^2\ln(x)}{c^3}-\frac{a^2\ln\left(d\,x^2+c\right)}{2c^3}-\frac{b^2}{2\left(d\,x^2+c\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^3,x)
```

```
[Out] 1/4/c/(d*x^2+c)^2*a^2-1/2/d/(d*x^2+c)^2*a*b+1/4*c/d^2/(d*x^2+c)^2*b^2-1/2*a
^2*ln(d*x^2+c)/c^3+1/2/c^2/(d*x^2+c)*a^2-1/2/d^2/(d*x^2+c)*b^2+a^2*ln(x)/c^
3
```

maxima [A] time = 1.01, size = 109, normalized size = 1.27

$$-\frac{b^2c^3+2\,abc^2d-3\,a^2cd^2+2\left(b^2c^2d-a^2d^3\right)x^2}{4\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)}-\frac{a^2\log\left(dx^2+c\right)}{2\,c^3}+\frac{a^2\log\left(x^2\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2 + 2*(b^2*c^2*d - a^2*d^3)*x^2)/(c
^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) - 1/2*a^2*log(d*x^2 + c)/c^3 + 1/2*a^
2*log(x^2)/c^3
```

mupad **[B]** time = 0.13, size = 106, normalized size = 1.23

$$\frac{a^2\,\ln(x)}{c^3}-\frac{a^2\,\ln\left(d\,x^2+c\right)}{2\,c^3}-\frac{\frac{-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2}{4\,c\,d^2}-\frac{x^2\,\left(a^2\,d^2-b^2\,c^2\right)}{2\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^3),x)
```

```
[Out] (a^2*log(x))/c^3 - (a^2*log(c + d*x^2))/(2*c^3) - ((b^2*c^2 - 3*a^2*d^2 + 2
*a*b*c*d)/(4*c*d^2) - (x^2*(a^2*d^2 - b^2*c^2))/(2*c^2*d))/(c^2 + d^2*x^4 +
 2*c*d*x^2)
```

sympy **[A]** time = 1.18, size = 107, normalized size = 1.24

$$\frac{a^2\log\left(x\right)}{c^3}-\frac{a^2\log\left(\frac{c}{d}+x^2\right)}{2c^3}+\frac{3a^2cd^2-2abc^2d-b^2c^3+x^2\left(2a^2d^3-2b^2c^2d\right)}{4c^4d^2+8c^3d^3x^2+4c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**3,x)
```

```
[Out] a**2*log(x)/c**3 - a**2*log(c/d + x**2)/(2*c**3) + (3*a**2*c*d**2 - 2*a*b*c
**2*d - b**2*c**3 + x**2*(2*a**2*d**3 - 2*b**2*c**2*d))/(4*c**4*d**2 + 8*c*
*3*d**3*x**2 + 4*c**2*d**4*x**4)
```

3.195 $\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{x\left(5a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(3ad(2bc-5ad)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(3ad(2bc-5ad)+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 385, 199, 205}

$$-\frac{x\left(5a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(3ad(2bc-5ad)+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(\frac{3a(2bc-5ad)}{c^2}+\frac{b^2}{d}\right)}{8c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(4*c^2*d
*(c + d*x^2)^2) + ((b^2/d + (3*a*(2*b*c - 5*a*d))/c^2)*x)/(8*c*(c + d*x^2))
 + ((b^2*c^2 + 3*a*d*(2*b*c - 5*a*d))*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/
2)*d^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^3}\,dx &= -\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\int \frac{a(2bc-5ad)+b^2cx^2}{\left(c+dx^2\right)^3}\,dx}{c}\\
&= -\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x}{4c\left(c+dx^2\right)^2}+\frac{1}{4}\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)\int \frac{1}{\left(c+dx^2\right)^2}\,dx\\
&= -\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)x}{8c\left(c+dx^2\right)}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)\int \frac{1}{c+dx^2}\,dx}{8c}\\
&= -\frac{a^2}{cx\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{b^2}{d}+\frac{3a(2bc-5ad)}{c^2}\right)x}{8c\left(c+dx^2\right)}+\frac{\left(b^2c^2+6abcd-15a^2d^2\right)\mathrm{tan}}{8c^{7/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 133, normalized size = 0.88

$$\frac{\left(-15a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}+\frac{x\left(-7a^2d^2+6abcd+b^2c^2\right)}{8c^3d\left(c+dx^2\right)}-\frac{a^2}{c^3x}-\frac{x(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] -(a^2/(c^3*x)) - ((b*c - a*d)^2*x)/(4*c^2*d*(c + d*x^2)^2) + ((b^2*c^2 + 6*
a*b*c*d - 7*a^2*d^2)*x)/(8*c^3*d*(c + d*x^2)) + ((b^2*c^2 + 6*a*b*c*d - 15*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)^3), x]
```

fricas [A] time = 0.85, size = 475, normalized size = 3.12

$$\left[-\frac{16\,a^2c^3d^2-2\left(b^2c^3d^2+6\,abc^2d^3-15\,a^2cd^4\right)x^4+2\left(b^2c^4d-10\,abc^3d^2+25\,a^2c^2d^3\right)x^2-\left(\left(b^2c^2d^2+6\,abcd^3-15\,a^2d^4\right)x^5+2\left(b^2c^3d+6\,abc^2d^2-15\,a^2cd^3\right)x^3+\left(b^2c^4+6\,abc^3d-15\,a^2c^2d^2\right)x\right)\sqrt{-cd}\,\log\left(\frac{dx^2+2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{16\left(c^4d^4x^5+2\,c^5d^3x^3+c^6d^2x\right)},\ -\frac{8\,a^2c^3d^2-\left(b^2c^3d^2+6\,abc^2d^3-15\,a^2cd^4\right)x^4+\left(b^2c^4d-10\,abc^3d^2+25\,a^2c^2d^3\right)x^2-\left(\left(b^2c^2d^2+6\,abcd^3-15\,a^2d^4\right)x^5+2\left(b^2c^3d+6\,abc^2d^2-15\,a^2cd^3\right)x^3+\left(b^2c^4+6\,abc^3d-15\,a^2c^2d^2\right)x\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{8\left(c^4d^4x^5+2\,c^5d^3x^3+c^6d^2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^2*c^3*d^2 - 2*(b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^4
 + 2*(b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6*
a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*
x^3 + (b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x),
 -1/8*(8*a^2*c^3*d^2 - (b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^4 + (
b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6*a*b*c*
d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3)*x^3 +
(b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c)
)/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x)]
```

giac [A] time = 0.31, size = 135, normalized size = 0.89

$$-\frac{a^2}{c^3x}+\frac{\left(b^2c^2+6\,abcd-15\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^3d}+\frac{b^2c^2dx^3+6\,abcd^2x^3-7\,a^2d^3x^3-b^2c^3x+10\,abc^2dx-9\,a^2cd^2x}{8\left(dx^2+c\right)^2c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -a^2/(c^3*x) + 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d))
/(sqrt(c*d)*c^3*d) + 1/8*(b^2*c^2*d*x^3 + 6*a*b*c*d^2*x^3 - 7*a^2*d^3*x^3 -
 b^2*c^3*x + 10*a*b*c^2*d*x - 9*a^2*c*d^2*x)/((d*x^2 + c)^2*c^3*d)
```

maple [A] time = 0.02, size = 199, normalized size = 1.31

$$-\frac{7a^2d^2x^3}{8\left(d\,x^2+c\right)^2c^3}+\frac{3abd\,x^3}{4\left(d\,x^2+c\right)^2c^2}+\frac{b^2x^3}{8\left(d\,x^2+c\right)^2c}-\frac{9a^2dx}{8\left(d\,x^2+c\right)^2c^2}+\frac{5abx}{4\left(d\,x^2+c\right)^2c}-\frac{b^2x}{8\left(d\,x^2+c\right)^2d}-\frac{15a^2d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^3}+\frac{3ab\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\sqrt{cd}\,c^2}+\frac{b^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,cd}-\frac{a^2}{c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^3,x)
```

```
[Out] -7/8/c^3/(d*x^2+c)^2*x^3*a^2*d^2+3/4/c^2/(d*x^2+c)^2*x^3*a*b*d+1/8/c/(d*x^2
+c)^2*x^3*b^2-9/8/c^2/(d*x^2+c)^2*a^2*d*x+5/4/c/(d*x^2+c)^2*a*b*x-1/8/(d*x^
2+c)^2/d*x*b^2-15/8/c^3*d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+3/4/c^2
/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b+1/8/c/d/(c*d)^(1/2)*arctan(1/(c*
d)^(1/2)*d*x)*b^2-a^2/c^3/x
```

maxima [A] time = 2.41, size = 146, normalized size = 0.96

$$-\frac{8\,a^{2}c^{2}d-\left(b^{2}c^{2}d+6\,abcd^{2}-15\,a^{2}d^{3}\right)x^{4}+\left(b^{2}c^{3}-10\,abc^{2}d+25\,a^{2}cd^{2}\right)x^{2}}{8\left(c^{3}d^{3}x^{5}+2\,c^{4}d^{2}x^{3}+c^{5}dx\right)}+\frac{\left(b^{2}c^{2}+6\,abcd-15\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*c^2*d - (b^2*c^2*d + 6*a*b*c*d^2 - 15*a^2*d^3)*x^4 + (b^2*c^3 -
 10*a*b*c^2*d + 25*a^2*c*d^2)*x^2)/(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)
+ 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
^3*d)
```

mupad [B] time = 0.18, size = 135, normalized size = 0.89

$$\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(-15\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^{7/2}\,d^{3/2}}-\frac{\frac{a^2}{c}-\frac{x^4\,\left(-15\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^3}+\frac{x^2\,\left(25\,a^2\,d^2-10\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^2\,d}}{c^2\,x+2\,c\,d\,x^3+d^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^3),x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(b^2*c^2 - 15*a^2*d^2 + 6*a*b*c*d))/(8*c^(7/2)*d
^(3/2)) - (a^2/c - (x^4*(b^2*c^2 - 15*a^2*d^2 + 6*a*b*c*d))/(8*c^3) + (x^2*
(25*a^2*d^2 + b^2*c^2 - 10*a*b*c*d))/(8*c^2*d))/(c^2*x + d^2*x^5 + 2*c*d*x^
3)
```

sympy [A] time = 1.20, size = 224, normalized size = 1.47

$$\frac{\sqrt{-\frac{1}{c^{7}d^{3}}}\left(15a^{2}d^{2}-6abcd-b^{2}c^{2}\right)\log\left(-c^{4}d\sqrt{-\frac{1}{c^{7}d^{3}}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{c^{7}d^{3}}}\left(15a^{2}d^{2}-6abcd-b^{2}c^{2}\right)\log\left(c^{4}d\sqrt{-\frac{1}{c^{7}d^{3}}}+x\right)}{16}+\frac{-8a^{2}c^{2}d+x^{4}\left(-15a^{2}d^{3}+6abcd^{2}+b^{2}c^{2}d\right)+x^{2}\left(-25a^{2}cd^{2}+10abc^{2}d-b^{2}c^{3}\right)}{8c^{5}dx+16c^{4}d^{2}x^{3}+8c^{3}d^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**3,x)
```

```
[Out] sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d - b**2*c**2)*log(-c**4*d*sqr
t(-1/(c**7*d**3)) + x)/16 - sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d
- b**2*c**2)*log(c**4*d*sqrt(-1/(c**7*d**3)) + x)/16 + (-8*a**2*c**2*d + x*
*4*(-15*a**2*d**3 + 6*a*b*c*d**2 + b**2*c**2*d) + x**2*(-25*a**2*c*d**2 + 1
0*a*b*c**2*d - b**2*c**3))/(8*c**5*d*x + 16*c**4*d**2*x**3 + 8*c**3*d**3*x*
*5)
```

3.196 $\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=106

$$-\frac{a^2}{2c^3x^2}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}+\frac{a\log(x)(2bc-3ad)}{c^4}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}$$

Rubi **[A]** time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2c^3x^2}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}-\frac{a(2bc-3ad)\log\left(c+dx^2\right)}{2c^4}+\frac{a\log(x)(2bc-3ad)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] -a^2/(2*c^3*x^2) - (b*c - a*d)^2/(4*c^2*d*(c + d*x^2)^2) + (a*(b*c - a*d))/
(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x])/c^4 - (a*(2*b*c - 3*a*d)*Log
[c + d*x^2])/(2*c^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{c^3x^2}-\frac{a(-2bc+3ad)}{c^4x}+\frac{(bc-ad)^2}{c^2(c+dx)^3}+\frac{2ad(-bc+ad)}{c^3(c+dx)^2}+\frac{ad(-2bc+3ad)}{c^4(c+dx)}\right) a\right.\\
&= -\frac{a^2}{2c^3x^2}-\frac{(bc-ad)^2}{4c^2d\left(c+dx^2\right)^2}+\frac{a(bc-ad)}{c^3\left(c+dx^2\right)}+\frac{a(2bc-3ad)\log(x)}{c^4}-\frac{a(2bc-3ad)\log\left(c+d\right.}{2c^4}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 99, normalized size = 0.93

$$
\frac{-\frac{2a^2c}{x^2}-\frac{c^2(bc-ad)^2}{d\left(c+dx^2\right)^2}+\frac{4ac(bc-ad)}{c+dx^2}+2a(3ad-2bc)\log\left(c+dx^2\right)+4a\log(x)(2bc-3ad)}{4c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] ((-2*a^2*c)/x^2 - (c^2*(b*c - a*d)^2)/(d*(c + d*x^2)^2) + (4*a*c*(b*c - a*d
))/(c + d*x^2) + 4*a*(2*b*c - 3*a*d)*Log[x] + 2*a*(-2*b*c + 3*a*d)*Log[c +
d*x^2])/(4*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)^3), x]
```

fricas [B] time = 0.68, size = 256, normalized size = 2.42

$$
-\frac{2\,a^2c^3d-2\left(2\,abc^2d^2-3\,a^2cd^3\right)x^4+\left(b^2c^4-6\,abc^3d+9\,a^2c^2d^2\right)x^2+2\left(\left(2\,abcd^3-3\,a^2d^4\right)x^6+2\left(2\,abc^2d^2-3\,a^2cd^3\right)x^4+\left(2\,abc^3d-3\,a^2c^2d^2\right)x^2\right)\log\left(dx^2+c\right)-4\left(\left(2\,abcd^3-3\,a^2d^4\right)x^6+2\left(2\,abc^2d^2-3\,a^2cd^3\right)x^4+\left(2\,abc^3d-3\,a^2c^2d^2\right)x^2\right)\log(x)}{4\left(c^4d^3x^6+2\,c^5d^2x^4+c^6dx^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*c^3*d - 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (b^2*c^4 - 6*a*b*
c^3*d + 9*a^2*c^2*d^2)*x^2 + 2*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^
2*d^2 - 3*a^2*c*d^3)*x^4 + (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(d*x^2 + c
) - 4*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4
+ (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(x))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 +
 c^6*d*x^2)
```

giac [A] time = 0.39, size = 177, normalized size = 1.67

$$\frac{\left(2\,abc-3\,a^{2}d\right)\log\left(x^{2}\right)}{2\,c^{4}}-\frac{\left(2\,abcd-3\,a^{2}d^{2}\right)\log\left(\left|dx^{2}+c\right|\right)}{2\,c^{4}d}-\frac{2\,abcx^{2}-3\,a^{2}dx^{2}+a^{2}c}{2\,c^{4}x^{2}}+\frac{6\,abcd^{3}x^{4}-9\,a^{2}d^{4}x^{4}+16\,abc^{2}d^{2}x^{2}-22\,a^{2}cd^{3}x^{2}-b^{2}c^{4}+12\,abc^{3}d-14\,a^{2}c^{2}d^{2}}{4\left(dx^{2}+c\right)^{2}c^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4 - 1/2*(2*a*b*c*d - 3*a^2*d^2)*log(abs(
d*x^2 + c))/(c^4*d) - 1/2*(2*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^4*x^2) + 1
/4*(6*a*b*c*d^3*x^4 - 9*a^2*d^4*x^4 + 16*a*b*c^2*d^2*x^2 - 22*a^2*c*d^3*x^2
 - b^2*c^4 + 12*a*b*c^3*d - 14*a^2*c^2*d^2)/((d*x^2 + c)^2*c^4*d)
```

maple [A] time = 0.02, size = 149, normalized size = 1.41

$$-\frac{a^{2}d}{4\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{ab}{2\left(d\,x^{2}+c\right)^{2}c}-\frac{b^{2}}{4\left(d\,x^{2}+c\right)^{2}d}-\frac{a^{2}d}{\left(d\,x^{2}+c\right)c^{3}}-\frac{3a^{2}d\ln(x)}{c^{4}}+\frac{3a^{2}d\ln\left(d\,x^{2}+c\right)}{2c^{4}}+\frac{ab}{\left(d\,x^{2}+c\right)c^{2}}+\frac{2ab\ln(x)}{c^{3}}-\frac{ab\ln\left(d\,x^{2}+c\right)}{c^{3}}-\frac{a^{2}}{2c^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^3,x)
```

```
[Out] -1/4/c^2*d/(d*x^2+c)^2*a^2+1/2/c/(d*x^2+c)^2*a*b-1/4/d/(d*x^2+c)^2*b^2+3/2/
c^4*ln(d*x^2+c)*a^2*d-1/c^3*ln(d*x^2+c)*a*b-1/c^3/(d*x^2+c)*a^2*d+1/c^2/(d*
x^2+c)*a*b-1/2*a^2/c^3/x^2-3*a^2/c^4*ln(x)*d+2*a/c^3*ln(x)*b
```

maxima [A] time = 1.02, size = 142, normalized size = 1.34

$$-\frac{2\,a^{2}c^{2}d-2\left(2\,abcd^{2}-3\,a^{2}d^{3}\right)x^{4}+\left(b^{2}c^{3}-6\,abc^{2}d+9\,a^{2}cd^{2}\right)x^{2}}{4\left(c^{3}d^{3}x^{6}+2\,c^{4}d^{2}x^{4}+c^{5}dx^{2}\right)}-\frac{\left(2\,abc-3\,a^{2}d\right)\log\left(dx^{2}+c\right)}{2\,c^{4}}+\frac{\left(2\,abc-3\,a^{2}d\right)\log\left(x^{2}\right)}{2\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*c^2*d - 2*(2*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (b^2*c^3 - 6*a*b*c^2*
d + 9*a^2*c*d^2)*x^2)/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2) - 1/2*(2*a*
b*c - 3*a^2*d)*log(d*x^2 + c)/c^4 + 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4
```

mupad [B] time = 0.11, size = 132, normalized size = 1.25

$$\frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d-2\,a\,b\,c\right)}{2\,c^4}-\frac{\frac{a^2}{2\,c}+\frac{x^2\,\left(9\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{4\,c^2\,d}+\frac{a\,d\,x^4\,\left(3\,a\,d-2\,b\,c\right)}{2\,c^3}}{c^2\,x^2+2\,c\,d\,x^4+d^2\,x^6}-\frac{\ln(x)\,\left(3\,a^2\,d-2\,a\,b\,c\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^3),x)
```

```
[Out] (log(c + d*x^2)*(3*a^2*d - 2*a*b*c))/(2*c^4) - (a^2/(2*c) + (x^2*(9*a^2*d^2
 + b^2*c^2 - 6*a*b*c*d))/(4*c^2*d) + (a*d*x^4*(3*a*d - 2*b*c))/(2*c^3))/(c^
2*x^2 + d^2*x^6 + 2*c*d*x^4) - (log(x)*(3*a^2*d - 2*a*b*c))/c^4
```

sympy [A] time = 1.82, size = 139, normalized size = 1.31

$$-\frac{a\left(3ad-2bc\right)\log\left(x\right)}{c^4}+\frac{a\left(3ad-2bc\right)\log\left(\frac{c}{d}+x^2\right)}{2c^4}+\frac{-2a^2c^2d+x^4\left(-6a^2d^3+4abcd^2\right)+x^2\left(-9a^2cd^2+6abc^2d-b^2c^3\right)}{4c^5dx^2+8c^4d^2x^4+4c^3d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**3,x)
```

```
[Out] -a*(3*a*d - 2*b*c)*log(x)/c**4 + a*(3*a*d - 2*b*c)*log(c/d + x**2)/(2*c**4)
 + (-2*a**2*c**2*d + x**4*(-6*a**2*d**3 + 4*a*b*c*d**2) + x**2*(-9*a**2*c*d
**2 + 6*a*b*c**2*d - b**2*c**3))/(4*c**5*d*x**2 + 8*c**4*d**2*x**4 + 4*c**3
*d**3*x**6)
```

3.197 $\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=161

$$\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}+\frac{x\left(7a^2d^2-6abcd+3b^2c^2\right)}{12c^3\left(c+dx^2\right)^2}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{x(3bc-7ad)^2}{24c^4\left(c+dx^2\right)}-\frac{a(6bc-7}{3c^4x}$$

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 456, 453, 205}

$$\frac{x\left(7a^2d^2-6abcd+3b^2c^2\right)}{12c^3\left(c+dx^2\right)^2}+\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{x(3bc-7ad)^2}{24c^4\left(c+dx^2\right)}-\frac{a(6bc-7ad)}{3c^4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -(a*(6*b*c - 7*a*d))/(3*c^4*x) - a^2/(3*c*x^3*(c + d*x^2)^2) + ((3*b^2*c^2
- 6*a*b*c*d + 7*a^2*d^2)*x)/(12*c^3*(c + d*x^2)^2) + ((3*b*c - 7*a*d)^2*x)/
(24*c^4*(c + d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*c^(9/2)*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
```

```
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^3}\,dx = -\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\int \frac{a(6bc-7ad)+3b^2cx^2}{x^2\left(c+dx^2\right)^3}\,dx}{3c}$$

$$= -\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}-\frac{\int \frac{-\frac{4a(6bc-7ad)}{c}-3\left(3b^2-\frac{6abd}{c}+\frac{7a^2d^2}{c^2}\right)x^2}{x^2\left(c+dx^2\right)^2}\,dx}{12c}$$

$$= -\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\frac{\int \frac{\frac{8a(6bc-7ad)}{c^2}+\frac{(3bc-7ad)^2x^2}{c^3}}{x^2\left(c+dx^2\right)}}{24c}$$

$$= -\frac{a(6bc-7ad)}{3c^4x}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\left(3b^2c^2\right.$$

$$= -\frac{a(6bc-7ad)}{3c^4x}-\frac{a^2}{3cx^3\left(c+dx^2\right)^2}+\frac{\left(3b^2c^2-6abcd+7a^2d^2\right)x}{12c^3\left(c+dx^2\right)^2}+\frac{(3bc-7ad)^2x}{24c^4\left(c+dx^2\right)}+\left(3b^2c^2\right.$$

Mathematica [A] time = 0.07, size = 148, normalized size = 0.92

$$\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}+\frac{x\left(11a^2d^2-14abcd+3b^2c^2\right)}{8c^4\left(c+dx^2\right)}-\frac{a^2}{3c^3x^3}+\frac{a(3ad-2bc)}{c^4x}+\frac{x(bc-ad)^2}{4c^3\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] -1/3*a^2/(c^3*x^3) + (a*(-2*b*c + 3*a*d))/(c^4*x) + ((b*c - a*d)^2*x)/(4*c^
3*(c + d*x^2)^2) + ((3*b^2*c^2 - 14*a*b*c*d + 11*a^2*d^2)*x)/(8*c^4*(c + d*
x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])
/(8*c^(9/2)*Sqrt[d])
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)^3), x]
```

fricas **[A]** time = 0.93, size = 536, normalized size = 3.33

$$\left[-\frac{16\,a^2c^4d-6\left(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4\right)x^6-10\left(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3\right)x^4+16\left(6\,abc^4d-7\,a^2c^3d^2\right)x^2+3\left(\left(3\,b^2c^2d^2-30\,abcd^3+35\,a^2d^4\right)x^7+2\left(3\,b^2c^3d-30\,abc^2d^2+35\,a^2cd^3\right)x^5+\left(3\,b^2c^4-30\,abc^3d+35\,a^2c^2d^2\right)x^3\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\,\sqrt{-cd}\,x-c}{dx^2+c}\right)}{48\left(c^5d^3x^7+2\,c^6d^2x^5+c^7dx^3\right)},\,-\frac{8\,a^2c^4d-3\left(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4\right)x^6-5\left(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3\right)x^4+8\left(6\,abc^4d-7\,a^2c^3d^2\right)x^2-3\left(\left(3\,b^2c^2d^2-30\,abcd^3+35\,a^2d^4\right)x^7+2\left(3\,b^2c^3d-30\,abc^2d^2+35\,a^2cd^3\right)x^5+\left(3\,b^2c^4-30\,abc^3d+35\,a^2c^2d^2\right)x^3\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)}{24\left(c^5d^3x^7+2\,c^6d^2x^5+c^7dx^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^2*c^4*d - 6*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^
6 - 10*(3*b^2*c^4*d - 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 16*(6*a*b*c^4*
d - 7*a^2*c^3*d^2)*x^2 + 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7
 + 2*(3*b^2*c^3*d - 30*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*
b*c^3*d + 35*a^2*c^2*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/
(d*x^2 + c)))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^7*d*x^3), -1/24*(8*a^2*c^4*d
 - 3*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^6 - 5*(3*b^2*c^4*d -
 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 8*(6*a*b*c^4*d - 7*a^2*c^3*d^2)*x^2
 - 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7 + 2*(3*b^2*c^3*d - 30
*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*b*c^3*d + 35*a^2*c^2*d
^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^
7*d*x^3)]
```

giac **[A]** time = 0.37, size = 151, normalized size = 0.94

$$\frac{\left(3\,b^2c^2-30\,abcd+35\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^4}+\frac{3\,b^2c^2dx^3-14\,abcd^2x^3+11\,a^2d^3x^3+5\,b^2c^3x-18\,abc^2dx+13\,a^2cd^2x}{8\left(dx^2+c\right)^2c^4}-\frac{6\,abcx^2-9\,a^2dx^2+a^2c}{3\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
c^4) + 1/8*(3*b^2*c^2*d*x^3 - 14*a*b*c*d^2*x^3 + 11*a^2*d^3*x^3 + 5*b^2*c^3
*x - 18*a*b*c^2*d*x + 13*a^2*c*d^2*x)/((d*x^2 + c)^2*c^4) - 1/3*(6*a*b*c*x^
2 - 9*a^2*d*x^2 + a^2*c)/(c^4*x^3)
```

maple [A] time = 0.01, size = 227, normalized size = 1.41

$$\frac{11a^2d^3x^3}{8\left(dx^2+c\right)^2c^4}-\frac{7ab\,d^2x^3}{4\left(dx^2+c\right)^2c^3}+\frac{3b^2d\,x^3}{8\left(dx^2+c\right)^2c^2}+\frac{13a^2d^2x}{8\left(dx^2+c\right)^2c^3}-\frac{9abdx}{4\left(dx^2+c\right)^2c^2}+\frac{5b^2x}{8\left(dx^2+c\right)^2c}+\frac{35a^2d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^4}-\frac{15abd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\sqrt{cd}\,c^3}+\frac{3b^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}\,c^2}+\frac{3a^2d}{c^4x}-\frac{2ab}{c^3x}-\frac{a^2}{3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^3,x)
```

```
[Out] 11/8/c^4/(d*x^2+c)^2*x^3*a^2*d^3-7/4/c^3/(d*x^2+c)^2*x^3*a*b*d^2+3/8/c^2/(d
*x^2+c)^2*x^3*b^2*d+13/8/c^3/(d*x^2+c)^2*a^2*d^2*x-9/4/c^2/(d*x^2+c)^2*a*b*
d*x+5/8/c/(d*x^2+c)^2*b^2*x+35/8/c^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*
a^2*d^2-15/4/c^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*d+3/8/c^2/(c*d)^
(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1/3*a^2/c^3/x^3+3*a^2/c^4/x*d-2*a/c^3/x
*b
```

maxima [A] time = 2.30, size = 167, normalized size = 1.04

$$\frac{3\left(3\,b^2c^2d-30\,abcd^2+35\,a^2d^3\right)x^6-8\,a^2c^3+5\left(3\,b^2c^3-30\,abc^2d+35\,a^2cd^2\right)x^4-8\left(6\,abc^3-7\,a^2c^2d\right)x^2}{24\left(c^4d^2x^7+2\,c^5dx^5+c^6x^3\right)}+\frac{\left(3\,b^2c^2-30\,abcd+35\,a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(3*b^2*c^2*d - 30*a*b*c*d^2 + 35*a^2*d^3)*x^6 - 8*a^2*c^3 + 5*(3*b^
2*c^3 - 30*a*b*c^2*d + 35*a^2*c*d^2)*x^4 - 8*(6*a*b*c^3 - 7*a^2*c^2*d)*x^2)
/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3) + 1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a
^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^4)
```

mupad [B] time = 0.19, size = 156, normalized size = 0.97

$$\frac{\frac{5\,x^4\,\left(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2\right)}{24\,c^3}-\frac{a^2}{3\,c}+\frac{a\,x^2\,\left(7\,a\,d-6\,b\,c\right)}{3\,c^2}+\frac{d\,x^6\,\left(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^4}}{c^2\,x^3+2\,c\,d\,x^5+d^2\,x^7}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,c^{9/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^3),x)
```

```
[Out] ((5*x^4*(35*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/(24*c^3) - a^2/(3*c) + (a*x^
2*(7*a*d - 6*b*c))/(3*c^2) + (d*x^6*(35*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/
(8*c^4))/(c^2*x^3 + d^2*x^7 + 2*c*d*x^5) + (atan((d^(1/2)*x)/c^(1/2))*(35*a
^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/(8*c^(9/2)*d^(1/2))
```

sympy [A] time = 1.34, size = 240, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{c^9d}}\left(35a^2d^2-30abcd+3b^2c^2\right)\log\left(-c^5\sqrt{-\frac{1}{c^9d}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^9d}}\left(35a^2d^2-30abcd+3b^2c^2\right)\log\left(c^5\sqrt{-\frac{1}{c^9d}}+x\right)}{16}+\frac{-8a^2c^3+x^6\left(105a^2d^3-90abcd^2+9b^2c^2d\right)+x^4\left(175a^2cd^2-150abc^2d+15b^2c^3\right)+x^2\left(56a^2c^2d-48abc^3\right)}{24c^6x^3+48c^5dx^5+24c^4d^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b**2*c**2)*log(-c**5*sqrt
(-1/(c**9*d)) + x)/16 + sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b*
*2*c**2)*log(c**5*sqrt(-1/(c**9*d)) + x)/16 + (-8*a**2*c**3 + x**6*(105*a**
2*d**3 - 90*a*b*c*d**2 + 9*b**2*c**2*d) + x**4*(175*a**2*c*d**2 - 150*a*b*c
**2*d + 15*b**2*c**3) + x**2*(56*a**2*c**2*d - 48*a*b*c**3))/(24*c**6*x**3
+ 48*c**5*d*x**5 + 24*c**4*d**2*x**7)
```

3.198 $\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}-\frac{ax^2(bc-ad)}{2b^3}+\frac{x^4(bc-ad)}{4b^2}+\frac{dx^6}{6b}$$

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{x^4(bc-ad)}{4b^2}-\frac{ax^2(bc-ad)}{2b^3}+\frac{dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)*x^2)/(2*b^3) + ((b*c - a*d)*x^4)/(4*b^2) + (d*x^6)/(6*b) +
(a^2*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2(c+dx)}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a(-bc+ad)}{b^3}+\frac{(bc-ad)x}{b^2}+\frac{dx^2}{b}-\frac{a^2(-bc+ad)}{b^3(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{a(bc-ad)x^2}{2b^3}+\frac{(bc-ad)x^4}{4b^2}+\frac{dx^6}{6b}+\frac{a^2(bc-ad)\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 71, normalized size = 0.95

$$\frac{bx^2\left(6a^2d-3ab\left(2c+dx^2\right)+b^2x^2\left(3c+2dx^2\right)\right)+6a^2(bc-ad)\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d - 3*a*b*(2*c + d*x^2) + b^2*x^2*(3*c + 2*d*x^2)) + 6*a^2*(b
*c - a*d)*Log[a + b*x^2])/(12*b^4)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(c+dx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(c + d*x^2))/(a + b*x^2), x]
```

fricas [**A**] time = 0.78, size = 75, normalized size = 1.00

$$\frac{2\,b^3dx^6+3\left(b^3c-ab^2d\right)x^4-6\left(ab^2c-a^2bd\right)x^2+6\left(a^2bc-a^3d\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d*x^6 + 3*(b^3*c - a*b^2*d)*x^4 - 6*(a*b^2*c - a^2*b*d)*x^2 + 6
*(a^2*b*c - a^3*d)*log(b*x^2 + a))/b^4
```

giac [A] time = 0.32, size = 77, normalized size = 1.03

$$\frac{2\,b^2dx^6+3\,b^2cx^4-3\,abdx^4-6\,abcx^2+6\,a^2dx^2}{12\,b^3}+\frac{\left(a^2bc-a^3d\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*b^2*c*x^4 - 3*a*b*d*x^4 - 6*a*b*c*x^2 + 6*a^2*d*x^2)/
b^3 + 1/2*(a^2*b*c - a^3*d)*log(abs(b*x^2 + a))/b^4
```

maple [A] time = 0.00, size = 86, normalized size = 1.15

$$\frac{d\,x^6}{6b}-\frac{ad\,x^4}{4b^2}+\frac{c\,x^4}{4b}+\frac{a^2d\,x^2}{2b^3}-\frac{ac\,x^2}{2b^2}-\frac{a^3d\ln\left(b\,x^2+a\right)}{2b^4}+\frac{a^2c\ln\left(b\,x^2+a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/6*d*x^6/b-1/4/b^2*x^4*a*d+1/4/b*x^4*c+1/2/b^3*x^2*a^2*d-1/2/b^2*x^2*a*c-1
/2*a^3/b^4*ln(b*x^2+a)*d+1/2*a^2/b^3*ln(b*x^2+a)*c
```

maxima [A] time = 1.02, size = 74, normalized size = 0.99

$$\frac{2\,b^2dx^6+3\left(b^2c-abd\right)x^4-6\left(abc-a^2d\right)x^2}{12\,b^3}+\frac{\left(a^2bc-a^3d\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d*x^6 + 3*(b^2*c - a*b*d)*x^4 - 6*(a*b*c - a^2*d)*x^2)/b^3 + 1/
2*(a^2*b*c - a^3*d)*log(b*x^2 + a)/b^4
```

mupad [B] time = 0.12, size = 76, normalized size = 1.01

$$x^4\left(\frac{c}{4\,b}-\frac{a\,d}{4\,b^2}\right)+\frac{d\,x^6}{6\,b}-\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d-a^2\,b\,c\right)}{2\,b^4}-\frac{a\,x^2\,\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x^4*(c/(4*b) - (a*d)/(4*b^2)) + (d*x^6)/(6*b) - (log(a + b*x^2)*(a^3*d - a^
2*b*c))/(2*b^4) - (a*x^2*(c/b - (a*d)/b^2))/(2*b)
```

sympy [A] time = 0.32, size = 70, normalized size = 0.93

$$-\frac{a^2\,(ad-bc)\log\left(a+bx^2\right)}{2b^4}+x^4\left(-\frac{ad}{4b^2}+\frac{c}{4b}\right)+x^2\left(\frac{a^2d}{2b^3}-\frac{ac}{2b^2}\right)+\frac{dx^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)*log(a + b*x**2)/(2*b**4) + x**4*(-a*d/(4*b**2) + c/(4*b))
 + x**2*(a**2*d/(2*b**3) - a*c/(2*b**2)) + d*x**6/(6*b)
```

3.199 $\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{ax(bc-ad)}{b^3}+\frac{x^3(bc-ad)}{3b^2}+\frac{dx^5}{5b}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {459, 302, 205}

$$\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x^3(bc-ad)}{3b^2}-\frac{ax(bc-ad)}{b^3}+\frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*x)/b^3) + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) + (a^(
3/2)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{dx^5}{5b}-\frac{(-5bc+5ad)\int \frac{x^4}{a+bx^2}\,dx}{5b}\\
&= \frac{dx^5}{5b}-\frac{(-5bc+5ad)\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{5b}\\
&= -\frac{a(bc-ad)x}{b^3}+\frac{(bc-ad)x^3}{3b^2}+\frac{dx^5}{5b}+\frac{\left(a^2(bc-ad)\right)\int \frac{1}{a+bx^2}\,dx}{b^3}\\
&= -\frac{a(bc-ad)x}{b^3}+\frac{(bc-ad)x^3}{3b^2}+\frac{dx^5}{5b}+\frac{a^{3/2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.00

$$-\frac{a^{3/2}(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{ax(ad-bc)}{b^3}+\frac{x^3(bc-ad)}{3b^2}+\frac{dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)*x)/b^3 + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) - (a^(
3/2)*(-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.94, size = 178, normalized size = 2.31

$$\left[\frac{6\,b^2dx^5+10\left(b^2c-abd\right)x^3-15\left(abc-a^2d\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\left(abc-a^2d\right)x}{30\,b^3},\ \frac{3\,b^2dx^5+5\left(b^2c-abd\right)x^3+15\left(abc-a^2d\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-15\left(abc-a^2d\right)x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d*x^5 + 10*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*sqrt(-a/b)
*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 30*(a*b*c - a^2*d)*x)/b^
3, 1/15*(3*b^2*d*x^5 + 5*(b^2*c - a*b*d)*x^3 + 15*(a*b*c - a^2*d)*sqrt(a/b)
*arctan(b*x*sqrt(a/b)/a) - 15*(a*b*c - a^2*d)*x)/b^3]
```

giac [A] time = 0.39, size = 84, normalized size = 1.09

$$\frac{\left(a^2bc - a^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^4dx^5 + 5\,b^4cx^3 - 5\,ab^3dx^3 - 15\,ab^3cx + 15\,a^2b^2dx}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*d*x^5
 + 5*b^4*c*x^3 - 5*a*b^3*d*x^3 - 15*a*b^3*c*x + 15*a^2*b^2*d*x)/b^5
```

maple [A] time = 0.00, size = 92, normalized size = 1.19

$$\frac{d\,x^5}{5b} - \frac{ad\,x^3}{3b^2} + \frac{c\,x^3}{3b} - \frac{a^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b^3} + \frac{a^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ b^2} + \frac{a^2dx}{b^3} - \frac{acx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/5*d*x^5/b-1/3/b^2*x^3*a*d+1/3/b*x^3*c+1/b^3*a^2*d*x-1/b^2*a*c*x-a^3/b^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)*c
```

maxima [A] time = 2.53, size = 77, normalized size = 1.00

$$\frac{\left(a^2bc - a^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^2dx^5 + 5\left(b^2c - abd\right)x^3 - 15\left(abc - a^2d\right)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*d*x^5
 + 5*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*x)/b^3
```

mupad [B] time = 0.12, size = 96, normalized size = 1.25

$$x^3\,\left(\frac{c}{3\,b}-\frac{a\,d}{3\,b^2}\right)+\frac{d\,x^5}{5\,b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(a\,d-b\,c\right)}{a^3\,d-a^2\,b\,c}\right)\,\left(a\,d-b\,c\right)}{b^{7/2}}-\frac{a\,x\,\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x^3*(c/(3*b) - (a*d)/(3*b^2)) + (d*x^5)/(5*b) - (a^(3/2)*atan((a^(3/2)*b^(1
/2)*x*(a*d - b*c))/(a^3*d - a^2*b*c))*(a*d - b*c))/b^(7/2) - (a*x*(c/b - (a
*d)/b^2))/b
```

sympy [B] time = 0.36, size = 153, normalized size = 1.99

$$x^3\left(-\frac{ad}{3b^2}+\frac{c}{3b}\right)+x\left(\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)+\frac{\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)}{a^2d-abc}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}\,(ad-bc)}{a^2d-abc}+x\right)}{2}+\frac{dx^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d/(3*b**2) + c/(3*b)) + x*(a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b*
*7)*(a*d - b*c)*log(-b**3*sqrt(-a**3/b**7)*(a*d - b*c)/(a**2*d - a*b*c) + x
)/2 - sqrt(-a**3/b**7)*(a*d - b*c)*log(b**3*sqrt(-a**3/b**7)*(a*d - b*c)/(a
**2*d - a*b*c) + x)/2 + d*x**5/(5*b)
```

3.200 $\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=54

$$-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bc-ad)}{2b^2}+\frac{dx^4}{4b}$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{x^2(bc-ad)}{2b^2}-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}+\frac{dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x^2)/(2*b^2) + (d*x^4)/(4*b) - (a*(b*c - a*d)*Log[a + b*x^2])/
(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)}{a+bx}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{bc-ad}{b^2}+\frac{dx}{b}+\frac{a(-bc+ad)}{b^2(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{(bc-ad)x^2}{2b^2}+\frac{dx^4}{4b}-\frac{a(bc-ad)\log\left(a+bx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.87

$$\frac{bx^2\left(-2ad+2bc+bdx^2\right)+2a(ad-bc)\log\left(a+bx^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*x^2*(2*b*c - 2*a*d + b*d*x^2) + 2*a*(-(b*c) + a*d)*Log[a + b*x^2])/(4*b^
3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.75, size = 51, normalized size = 0.94

$$\frac{b^2dx^4+2\left(b^2c-abd\right)x^2-2\left(abc-a^2d\right)\log\left(bx^2+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d*x^4 + 2*(b^2*c - a*b*d)*x^2 - 2*(a*b*c - a^2*d)*log(b*x^2 + a))/
b^3
```

giac [A] time = 0.33, size = 52, normalized size = 0.96

$$\frac{bdx^4+2\,bcx^2-2\,adx^2}{4\,b^2}-\frac{\left(abc-a^2d\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x^4 + 2*b*c*x^2 - 2*a*d*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(abs(b*x
^2 + a))/b^3
```

maple [A] time = 0.00, size = 62, normalized size = 1.15

$$\frac{d\,x^4}{4b}-\frac{ad\,x^2}{2b^2}+\frac{c\,x^2}{2b}+\frac{a^2d\ln\left(b\,x^2+a\right)}{2b^3}-\frac{ac\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/4*d*x^4/b-1/2/b^2*a*d*x^2+1/2/b*c*x^2+1/2*a^2/b^3*ln(b*x^2+a)*d-1/2*a/b^2
*c*ln(b*x^2+a)
```

maxima [A] time = 1.05, size = 50, normalized size = 0.93

$$\frac{bdx^4+2\,(bc-ad)x^2}{4\,b^2}-\frac{\left(abc-a^2d\right)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d*x^4 + 2*(b*c - a*d)*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(b*x^2 + a)/
b^3
```

mupad [B] time = 0.06, size = 52, normalized size = 0.96

$$x^2\left(\frac{c}{2\,b}-\frac{a\,d}{2\,b^2}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(a^2\,d-a\,b\,c\right)}{2\,b^3}+\frac{d\,x^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x^2*(c/(2*b) - (a*d)/(2*b^2)) + (log(a + b*x^2)*(a^2*d - a*b*c))/(2*b^3) +
(d*x^4)/(4*b)
```

sympy [A] time = 0.28, size = 46, normalized size = 0.85

$$\frac{a(ad-bc)\log\left(a+bx^2\right)}{2b^3}+x^2\left(-\frac{ad}{2b^2}+\frac{c}{2b}\right)+\frac{dx^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)*log(a + b*x**2)/(2*b**3) + x**2*(-a*d/(2*b**2) + c/(2*b)) + d
*x**4/(4*b)
```

3.201 $\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{a}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bc-ad)}{b^2}+\frac{dx^3}{3b}$$

Rubi **[A]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {459, 321, 205}

$$\frac{x(bc-ad)}{b^2}-\frac{\sqrt{a}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) - (Sqrt[a]*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{dx^3}{3b}-\frac{(-3bc+3ad)\int \frac{x^2}{a+bx^2}\,dx}{3b}\\ &= \frac{(bc-ad)x}{b^2}+\frac{dx^3}{3b}-\frac{(a(bc-ad))\int \frac{1}{a+bx^2}\,dx}{b^2}\\ &= \frac{(bc-ad)x}{b^2}+\frac{dx^3}{3b}-\frac{\sqrt{a}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 57, normalized size = 0.98

$$\frac{\sqrt{a}\,(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(bc-ad)}{b^2}+\frac{dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) + (Sqrt[a]*(-(b*c) + a*d)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2))/(a + b*x^2), x]
```

fricas [**A**] time = 0.85, size = 129, normalized size = 2.22

$$\left[\frac{2\,bdx^3-3\,(bc-ad)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(bc-ad)x}{6\,b^2},\frac{bdx^3-3\,(bc-ad)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,(bc-ad)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*d*x^3 - 3*(b*c - a*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*c - a*d)*x)/b^2, 1/3*(b*d*x^3 - 3*(b*c - a*d)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*c - a*d)*x)/b^2]
```

giac [A] time = 0.44, size = 58, normalized size = 1.00

$$-\frac{\left(abc - a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{b^2dx^3 + 3\,b^2cx - 3\,abdx}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*d*x^3 + 3
*b^2*c*x - 3*a*b*d*x)/b^3
```

maple [A] time = 0.01, size = 68, normalized size = 1.17

$$\frac{d\,x^3}{3b} + \frac{a^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{adx}{b^2} + \frac{cx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/3*d*x^3/b-1/b^2*a*d*x+1/b*c*x+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*d-1/(a*b)^(1/2)*a/b*c*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.16, size = 54, normalized size = 0.93

$$-\frac{\left(abc - a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{bdx^3 + 3\,(bc - ad)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*d*x^3 + 3*(
b*c - a*d)*x)/b^2
```

mupad [B] time = 0.07, size = 70, normalized size = 1.21

$$x\left(\frac{c}{b} - \frac{a\,d}{b^2}\right) + \frac{d\,x^3}{3\,b} + \frac{\sqrt{a}\;\mathrm{atan}\left(\frac{\sqrt{a}\;\sqrt{b}\;x\,(a\,d-b\,c)}{a^2\,d-a\,b\,c}\right)\,(a\,d - b\,c)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] x*(c/b - (a*d)/b^2) + (d*x^3)/(3*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d
 - b*c))/(a^2*d - a*b*c))*(a*d - b*c))/b^(5/2)
```

sympy **[A]** time = 0.33, size = 90, normalized size = 1.55

$$x\left(-\frac{ad}{b^2}+\frac{c}{b}\right)-\frac{\sqrt{-\frac{a}{b^5}}\,(ad-bc)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^5}}\,(ad-bc)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{dx^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x*(-a*d/b**2 + c/b) - sqrt(-a/b**5)*(a*d - b*c)*log(-b**2*sqrt(-a/b**5) + x
)/2 + sqrt(-a/b**5)*(a*d - b*c)*log(b**2*sqrt(-a/b**5) + x)/2 + d*x**3/(3*b
)
```

3.202 $\int \frac{x(c+dx^2)}{a+bx^2} \, dx$

Optimal. Leaf size=35

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2b^2}+\frac{dx^2}{2b}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2b^2}+\frac{dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (d*x^2)/(2*b) + ((b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^2\right)}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{dx^2}{2b}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.89

$$\frac{(bc-ad)\log\left(a+bx^2\right)+bdx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] (b*d*x^2 + (b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^2\right)}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2))/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^2))/(a + b*x^2), x]
```

fricas [A] time = 0.76, size = 29, normalized size = 0.83

$$\frac{bdx^2+(bc-ad)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x^2 + (b*c - a*d)*log(b*x^2 + a))/b^2
```

giac [A] time = 0.27, size = 32, normalized size = 0.91

$$\frac{dx^2}{2\,b}+\frac{(bc-ad)\log\left(\left|bx^2+a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/b^2
```

maple [A] time = 0.00, size = 40, normalized size = 1.14

$$\frac{d\,x^2}{2b}-\frac{ad\ln\left(b\,x^2+a\right)}{2b^2}+\frac{c\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)/(b*x^2+a),x)
```

```
[Out] 1/2*d*x^2/b-1/2/b^2*ln(b*x^2+a)*a*d+1/2/b*c*ln(b*x^2+a)
```

maxima [A] time = 0.98, size = 31, normalized size = 0.89

$$\frac{dx^2}{2\,b} + \frac{(bc - ad)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b + 1/2*(b*c - a*d)*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.11, size = 31, normalized size = 0.89

$$\frac{d\,x^2}{2\,b} - \frac{\ln\left(b\,x^2 + a\right)\,(a\,d - b\,c)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2))/(a + b*x^2),x)
```

```
[Out] (d*x^2)/(2*b) - (log(a + b*x^2)*(a*d - b*c))/(2*b^2)
```

sympy [A] time = 0.24, size = 27, normalized size = 0.77

$$\frac{dx^2}{2b} - \frac{(ad - bc)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)/(b*x**2+a),x)
```

```
[Out] d*x**2/(2*b) - (a*d - b*c)*log(a + b*x**2)/(2*b**2)
```

3.203 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{dx}{b}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{a+bx^2}\,dx&=\frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}\\&=\frac{dx}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{c+dx^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2), x]
```

fricas [A] time = 0.76, size = 98, normalized size = 2.51

$$\left[\frac{2\,abdx+\sqrt{-ab}\,(bc-ad)\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{abdx+\sqrt{ab}\,(bc-ad)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

giac [A] time = 0.34, size = 33, normalized size = 0.85

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.00, size = 45, normalized size = 1.15

$$-\frac{ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a),x)
```

```
[Out] d*x/b-1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d+1/(a*b)^(1/2)*c*arctan(
1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.27, size = 33, normalized size = 0.85

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.05, size = 32, normalized size = 0.82

$$\frac{d\,x}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2),x)
```

```
[Out] (d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

sympy [B] time = 0.28, size = 82, normalized size = 2.10

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

3.204 $\int \frac{c+dx^2}{x\left(a+bx^2\right)} \, dx$

Optimal. Leaf size=34

$$\frac{c\log(x)}{a}-\frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 72}

$$\frac{c\log(x)}{a}-\frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (c*Log[x])/a - ((b*c - a*d)*Log[a + b*x^2])/(2*a*b)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{x\left(a+bx^2\right)} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx}{x(a+bx)} \, dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c}{ax}+\frac{-bc+ad}{a(a+bx)}\right) dx, x, x^2\right) \\
&= \frac{c\log(x)}{a}-\frac{(bc-ad)\log\left(a+bx^2\right)}{2ab}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{(ad-bc)\log\left(a+bx^2\right)}{2ab}+\frac{c\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] (c*Log[x])/a + ((-(b*c) + a*d)*Log[a + b*x^2])/(2*a*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{c+dx^2}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x*(a + b*x^2)), x]
```

fricas [A] time = 0.74, size = 33, normalized size = 0.97

$$\frac{2\,bc\log(x)-(bc-ad)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*log(x) - (b*c - a*d)*log(b*x^2 + a))/(a*b)
```

giac [A] time = 0.30, size = 36, normalized size = 1.06

$$\frac{c\log\left(x^2\right)}{2\,a}-\frac{(bc-ad)\log\left(\left|bx^2+a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/(a*b)
```

maple [A] time = 0.00, size = 37, normalized size = 1.09

$$\frac{c\ln(x)}{a}-\frac{c\ln\left(b\,x^2+a\right)}{2a}+\frac{d\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x/(b*x^2+a),x)
```

```
[Out] 1/2/b*ln(b*x^2+a)*d-1/2/a*c*ln(b*x^2+a)+1/a*c*ln(x)
```

maxima [A] time = 1.11, size = 35, normalized size = 1.03

$$\frac{c\log\left(x^2\right)}{2\,a}-\frac{(bc-ad)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(b*x^2 + a)/(a*b)
```

mupad [B] time = 0.08, size = 32, normalized size = 0.94

$$\frac{c\,\ln(x)}{a}+\frac{\ln\left(b\,x^2+a\right)\,(a\,d-b\,c)}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x*(a + b*x^2)),x)
```

```
[Out] (c*log(x))/a + (log(a + b*x^2)*(a*d - b*c))/(2*a*b)
```

sympy [A] time = 0.67, size = 26, normalized size = 0.76

$$\frac{c\log{(x)}}{a}+\frac{(ad-bc)\log{\left(\frac{a}{b}+x^2\right)}}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x/(b*x**2+a),x)
```

```
[Out] c*log(x)/a + (a*d - b*c)*log(a/b + x**2)/(2*a*b)
```

3.205 $\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=43

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Rubi [**A**] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {453, 205}

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}\,dx &= -\frac{c}{ax}-\frac{(bc-ad)\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{c}{ax}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.98

$$\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 0.66, size = 105, normalized size = 2.44

$$\left[\frac{\sqrt{-ab}\,(bc-ad)x\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,abc}{2\,a^2bx},-\frac{\sqrt{ab}\,(bc-ad)x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+abc}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*c - a*d)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*c)/(a^2*b*x), -(sqrt(a*b)*(b*c - a*d)*x*arctan(sqrt(a*b)*x/a) + a
*b*c)/(a^2*b*x)]
```

giac [A] time = 0.27, size = 37, normalized size = 0.86

$$-\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

maple [A] time = 0.01, size = 48, normalized size = 1.12

$$-\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^2/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-1/(a*b)^(1/2)/a*b*c*arctan(1/(a*b
)^(1/2)*b*x)-1/a*c/x
```

maxima [A] time = 2.46, size = 37, normalized size = 0.86

$$-\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

mupad [B] time = 0.11, size = 34, normalized size = 0.79

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d-b\,c\right)}{a^{3/2}\,\sqrt{b}}-\frac{c}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^2*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(3/2)*b^(1/2)) - c/(a*x)
```

sympy [B] time = 0.33, size = 82, normalized size = 1.91

$$-\frac{\sqrt{-\frac{1}{a^{3}b}}\,(ad-bc)\log\left(-a^{2}\sqrt{-\frac{1}{a^{3}b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^{3}b}}\,(ad-bc)\log\left(a^{2}\sqrt{-\frac{1}{a^{3}b}}+x\right)}{2}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*d - b*c)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*d - b*c)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - c/(a*x)
```

3.206 $\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=50

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{2ax^2}$$

Rubi **[A]** time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(bc-ad)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -c/(2*a*x^2) - ((b*c - a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])/(2*a
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{c+dx}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{c}{ax^2}+\frac{-bc+ad}{a^2x}-\frac{b(-bc+ad)}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c}{2ax^2}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.98

$$\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)(ad-bc)}{a^2}-\frac{c}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*c/(a*x^2) + ((-(b*c) + a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2])
/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 0.79, size = 48, normalized size = 0.96

$$\frac{(bc-ad)x^2\log\left(bx^2+a\right)-2\,(bc-ad)x^2\log(x)-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*((b*c - a*d)*x^2*log(b*x^2 + a) - 2*(b*c - a*d)*x^2*log(x) - a*c)/(a^2*
x^2)
```

giac [A] time = 0.33, size = 72, normalized size = 1.44

$$-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2c-abd\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bcx^2-adx^2-ac}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c - a*d)*log(x^2)/a^2 + 1/2*(b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^
2*b) + 1/2*(b*c*x^2 - a*d*x^2 - a*c)/(a^2*x^2)
```

maple [A] time = 0.01, size = 56, normalized size = 1.12

$$\frac{d\ln(x)}{a}-\frac{d\ln\left(b\,x^2+a\right)}{2a}-\frac{bc\ln(x)}{a^2}+\frac{bc\ln\left(b\,x^2+a\right)}{2a^2}-\frac{c}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^3/(b*x^2+a),x)
```

```
[Out] -1/2/a*ln(b*x^2+a)*d+1/2/a^2*b*c*ln(b*x^2+a)-1/2/a*c/x^2+1/a*ln(x)*d-1/a^2*
b*c*ln(x)
```

maxima [A] time = 1.04, size = 48, normalized size = 0.96

$$\frac{(bc-ad)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bc-ad)\log\left(x^2\right)}{2\,a^2}-\frac{c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*log(b*x^2 + a)/a^2 - 1/2*(b*c - a*d)*log(x^2)/a^2 - 1/2*c/(
a*x^2)
```

mupad [B] time = 0.15, size = 45, normalized size = 0.90

$$\frac{\ln(x)\,\left(a\,d-b\,c\right)}{a^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-b\,c\right)}{2\,a^2}-\frac{c}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (log(x)*(a*d - b*c))/a^2 - (log(a + b*x^2)*(a*d - b*c))/(2*a^2) - c/(2*a*x^
2)
```

sympy [A] time = 0.69, size = 41, normalized size = 0.82

$$-\frac{c}{2ax^2}+\frac{(ad-bc)\log(x)}{a^2}-\frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**3/(b*x**2+a),x)
```

```
[Out] -c/(2*a*x**2) + (a*d - b*c)*log(x)/a**2 - (a*d - b*c)*log(a/b + x**2)/(2*a*
*2)
```

3.207 $\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {453, 325, 205}

$$\frac{bc-ad}{a^2x}+\frac{\sqrt{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{c}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -c/(3*a*x^3) + (b*c - a*d)/(a^2*x) + (Sqrt[b]*(b*c - a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx &= -\frac{c}{3ax^3}-\frac{(3bc-3ad)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{3a}\\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{(b(bc-ad))\int \frac{1}{a+bx^2}\,dx}{a^2}\\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{\sqrt{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 60, normalized size = 1.02

$$
-\frac{\sqrt{b}\,(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{bc-ad}{a^2x}-\frac{c}{3ax^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) - (Sqrt[b]*(-(b*c) + a*d)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(5/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x^4*(a + b*x^2)), x]
```

fricas [A] time = 1.16, size = 136, normalized size = 2.31

$$
\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6\,(bc-ad)x^2+2\,ac}{6\,a^2x^3},\ \frac{3\,(bc-ad)x^3\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,(bc-ad)x^2-ac}{3\,a^2x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b*c - a*d)*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) - 6*(b*c - a*d)*x^2 + 2*a*c)/(a^2*x^3), 1/3*(3*(b*c - a*d)*x^3*sqr
t(b/a)*arctan(x*sqrt(b/a)) + 3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)]
```

giac [A] time = 0.31, size = 57, normalized size = 0.97

$$\frac{\left(b^2c-abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*c*x^2 - 3*
a*d*x^2 - a*c)/(a^2*x^3)
```

maple [A] time = 0.01, size = 72, normalized size = 1.22

$$-\frac{bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{b^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{d}{ax}+\frac{bc}{a^2x}-\frac{c}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^4/(b*x^2+a),x)
```

```
[Out] -b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+b^2/a^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*c-1/3*c/a/x^3-1/a/x*d+1/a^2/x*b*c
```

maxima [A] time = 2.33, size = 56, normalized size = 0.95

$$\frac{\left(b^2c-abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*(b*c - a*d)*
x^2 - a*c)/(a^2*x^3)
```

mupad [B] time = 0.12, size = 53, normalized size = 0.90

$$-\frac{\frac{c}{3\,a}+\frac{x^2\,(a\,d-b\,c)}{a^2}}{x^3}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^4*(a + b*x^2)),x)
```

```
[Out] - (c/(3*a) + (x^2*(a*d - b*c))/a^2)/x^3 - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)
)*(a*d - b*c))/a^(5/2)
```

sympy [B] time = 0.41, size = 129, normalized size = 2.19

$$\frac{\sqrt{-\frac{b}{a^5}}\,(ad-bc)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}\,(ad-bc)}{abd-b^2c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^5}}\,(ad-bc)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}\,(ad-bc)}{abd-b^2c}+x\right)}{2}+\frac{-ac+x^2\,(-3ad+3bc)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**5)*(a*d - b*c)*log(-a**3*sqrt(-b/a**5)*(a*d - b*c)/(a*b*d - b**2
*c) + x)/2 - sqrt(-b/a**5)*(a*d - b*c)*log(a**3*sqrt(-b/a**5)*(a*d - b*c)/(
a*b*d - b**2*c) + x)/2 + (-a*c + x**2*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

3.208 $\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=103

$$\frac{a^2(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{x^4(bc-ad)^2}{4b^3}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{d^2x^8}{8b}$$

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{a^2(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{x^4(bc-ad)^2}{4b^3}-\frac{ax^2(bc-ad)^2}{2b^4}+\frac{d^2x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)^2*x^2)/(2*b^4) + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c -
a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + (a^2*(b*c - a*d)^2*Log[a + b*x^2])/(2
*b^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(c+dx)^2}{a+bx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a(-bc+ad)^2}{b^4}+\frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^2}{b^2}+\frac{d^2x^3}{b}+\frac{a^2(-bc+ad)^2}{b^4(a+bx)}\right) dx,
\right. \\
&= -\frac{a(bc-ad)^2x^2}{2b^4}+\frac{(bc-ad)^2x^4}{4b^3}+\frac{d(2bc-ad)x^6}{6b^2}+\frac{d^2x^8}{8b}+\frac{a^2(bc-ad)^2\log\left(a+bx^2\right)}{2b^5}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 116, normalized size = 1.13

$$
\frac{\left(a^4d^2-2a^3bcd+a^2b^2c^2\right)\log\left(a+bx^2\right)}{2b^5}-\frac{ax^2(ad-bc)^2}{2b^4}+\frac{x^4(bc-ad)^2}{4b^3}+\frac{dx^6(2bc-ad)}{6b^2}+\frac{d^2x^8}{8b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -1/2*(a*(-(b*c) + a*d)^2*x^2)/b^4 + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*c
 - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + ((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2)*Log[a + b*x^2])/(2*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^5\left(c+dx^2\right)^2}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(c + d*x^2)^2)/(a + b*x^2), x]
```

fricas [A] time = 0.79, size = 138, normalized size = 1.34

$$
\frac{3\,b^4d^2x^8+4\left(2\,b^4cd-ab^3d^2\right)x^6+6\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4-12\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2+12\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\log\left(bx^2+a\right)}{24\,b^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^2*x^8 + 4*(2*b^4*c*d - a*b^3*d^2)*x^6 + 6*(b^4*c^2 - 2*a*b^3*
c*d + a^2*b^2*d^2)*x^4 - 12*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 + 1
2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a))/b^5
```

giac [A] time = 0.32, size = 148, normalized size = 1.44

$$\frac{3\,b^3 d^2 x^8 + 8\,b^3 c d x^6 - 4\,a b^2 d^2 x^6 + 6\,b^3 c^2 x^4 - 12\,a b^2 c d x^4 + 6\,a^2 b d^2 x^4 - 12\,a b^2 c^2 x^2 + 24\,a^2 b c d x^2 - 12\,a^3 d^2 x^2}{24\,b^4} + \frac{\left(a^2 b^2 c^2 - 2\,a^3 b c d + a^4 d^2\right) \log\left(\left|b x^2 + a\right|\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 8*b^3*c*d*x^6 - 4*a*b^2*d^2*x^6 + 6*b^3*c^2*x^4 - 12*
a*b^2*c*d*x^4 + 6*a^2*b*d^2*x^4 - 12*a*b^2*c^2*x^2 + 24*a^2*b*c*d*x^2 - 12*
a^3*d^2*x^2)/b^4 + 1/2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(abs(b*x^2
+ a))/b^5
```

maple [A] time = 0.00, size = 165, normalized size = 1.60

$$\frac{d^2 x^8}{8b} - \frac{a\,d^2 x^6}{6b^2} + \frac{c d\,x^6}{3b} + \frac{a^2 d^2 x^4}{4b^3} - \frac{a c d\,x^4}{2b^2} + \frac{c^2 x^4}{4b} - \frac{a^3 d^2 x^2}{2b^4} + \frac{a^2 c d\,x^2}{b^3} - \frac{a\,c^2 x^2}{2b^2} + \frac{a^4 d^2 \ln\left(b\,x^2 + a\right)}{2b^5} - \frac{a^3 c d \ln\left(b\,x^2 + a\right)}{b^4} + \frac{a^2 c^2 \ln\left(b\,x^2 + a\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/8*d^2*x^8/b-1/6/b^2*x^6*a*d^2+1/3/b*x^6*c*d+1/4/b^3*x^4*a^2*d^2-1/2/b^2*x
^4*a*c*d+1/4/b*x^4*c^2-1/2/b^4*x^2*a^3*d^2+1/b^3*x^2*a^2*c*d-1/2/b^2*x^2*a*
c^2+1/2*a^4/b^5*ln(b*x^2+a)*d^2-a^3/b^4*ln(b*x^2+a)*c*d+1/2*a^2/b^3*ln(b*x^
2+a)*c^2
```

maxima [A] time = 1.11, size = 137, normalized size = 1.33

$$\frac{3\,b^3 d^2 x^8 + 4\left(2\,b^3 c d - a b^2 d^2\right)x^6 + 6\left(b^3 c^2 - 2\,a b^2 c d + a^2 b d^2\right)x^4 - 12\left(a b^2 c^2 - 2\,a^2 b c d + a^3 d^2\right)x^2}{24\,b^4} + \frac{\left(a^2 b^2 c^2 - 2\,a^3 b c d + a^4 d^2\right)\log\left(b x^2 + a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^2*x^8 + 4*(2*b^3*c*d - a*b^2*d^2)*x^6 + 6*(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*x^4 - 12*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)/b^4 + 1/
2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a)/b^5
```

mupad [B] time = 0.11, size = 146, normalized size = 1.42

$$x^4 \left(\frac{c^2}{4\,b} + \frac{a\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right)}{4\,b}\right) - x^6 \left(\frac{a\,d^2}{6\,b^2} - \frac{c\,d}{3\,b}\right) + \frac{\ln\left(b\,x^2 + a\right)\left(a^4\,d^2 - 2\,a^3\,b\,c\,d + a^2\,b^2\,c^2\right)}{2\,b^5} + \frac{d^2\,x^8}{8\,b} - \frac{a\,x^2\left(\frac{c^2}{b} + \frac{a\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right)}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x^4*(c^2/(4*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(4*b)) - x^6*((a*d^2)/(6*b^2
) - (c*d)/(3*b)) + (log(a + b*x^2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))/(
2*b^5) + (d^2*x^8)/(8*b) - (a*x^2*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b)
)/(2*b)
```

sympy [A] time = 0.46, size = 122, normalized size = 1.18

$$\frac{a^2 (ad-bc)^2 \log \left(a+bx^2\right)}{2b^5}+x^6 \left(-\frac{ad^2}{6b^2}+\frac{cd}{3b}\right)+x^4 \left(\frac{a^2d^2}{4b^3}-\frac{acd}{2b^2}+\frac{c^2}{4b}\right)+x^2 \left(-\frac{a^3d^2}{2b^4}+\frac{a^2cd}{b^3}-\frac{ac^2}{2b^2}\right)+\frac{d^2x^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] a**2*(a*d - b*c)**2*log(a + b*x**2)/(2*b**5) + x**6*(-a*d**2/(6*b**2) + c*d
/(3*b)) + x**4*(a**2*d**2/(4*b**3) - a*c*d/(2*b**2) + c**2/(4*b)) + x**2*(-
a**3*d**2/(2*b**4) + a**2*c*d/b**3 - a*c**2/(2*b**2)) + d**2*x**8/(8*b)
```

3.209 $\int \frac{x^4\left(c+dx^2\right)^2}{a+bx^2}\,dx$

Optimal. Leaf size=105

$$\frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax(bc-ad)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

Rubi **[A]** time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{x^3(bc-ad)^2}{3b^3}-\frac{ax(bc-ad)^2}{b^4}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a*d)
*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \int \left(-\frac{a(bc-ad)^2}{b^4}+\frac{(bc-ad)^2x^2}{b^3}+\frac{d(2bc-ad)x^4}{b^2}+\frac{d^2x^6}{b}+\frac{a^2b^2c^2-2a^3bcd+a^4d^2}{b^4\left(a+bx^2\right)}\right)dx\\ &= -\frac{a(bc-ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{\left(a^2(bc-ad)^2\right)\int \frac{1}{a+bx^2}\,dx}{b^4}\\ &= -\frac{a(bc-ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{a^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 1.00

$$\frac{a^{3/2}(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{ax(ad-bc)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] -((a*(-(b*c) + a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a
*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(-(b*c) + a*d)^2*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2)^2)/(a + b*x^2), x]
```

fricas [A] time = 0.77, size = 304, normalized size = 2.90

$$\left[\frac{30\,b^3d^2x^7+42\left(2\,b^3cd-ab^2d^2\right)x^5+70\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^3+105\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)x}{210\,b^4},\ \frac{15\,b^3d^2x^7+21\left(2\,b^3cd-ab^2d^2\right)x^5+35\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^3+105\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-105\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)x}{105\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^2*x^7 + 42*(2*b^3*c*d - a*b^2*d^2)*x^5 + 70*(b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2)*x)/b^4, 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^
2)*x^5 + 35*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^
2*b*c*d + a^3*d^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(a*b^2*c^2 - 2*a
^2*b*c*d + a^3*d^2)*x)/b^4]
```

giac [A] time = 0.41, size = 153, normalized size = 1.46

$$\frac{\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^6d^2x^7 + 42\,b^6cdx^5 - 21\,ab^5d^2x^5 + 35\,b^6c^2x^3 - 70\,ab^5cdx^3 + 35\,a^2b^4d^2x^3 - 105\,ab^5c^2x + 210\,a^2b^4cdx - 105\,a^3b^3d^2x}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/105*(15*b^6*d^2*x^7 + 42*b^6*c*d*x^5 - 21*a*b^5*d^2*x^5 + 35*b^6*c^2*x
^3 - 70*a*b^5*c*d*x^3 + 35*a^2*b^4*d^2*x^3 - 105*a*b^5*c^2*x + 210*a^2*b^4*
c*d*x - 105*a^3*b^3*d^2*x)/b^7
```

maple [A] time = 0.00, size = 176, normalized size = 1.68

$$\frac{d^2x^7}{7b} - \frac{a\,d^2x^5}{5b^2} + \frac{2cd\,x^5}{5b} + \frac{a^2d^2x^3}{3b^3} - \frac{2acd\,x^3}{3b^2} + \frac{c^2x^3}{3b} + \frac{a^4d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^4} - \frac{2a^3cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3} + \frac{a^2c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{a^3d^2x}{b^4} + \frac{2a^2cdx}{b^3} - \frac{a\,c^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/7*d^2*x^7/b-1/5/b^2*x^5*a*d^2+2/5/b*x^5*c*d+1/3/b^3*x^3*a^2*d^2-2/3/b^2*x
^3*a*c*d+1/3/b*x^3*c^2-1/b^4*a^3*d^2*x+2/b^3*a^2*c*d*x-1/b^2*a*c^2*x+a^4/b^
4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2-2*a^3/b^3/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)*c*d+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.40, size = 140, normalized size = 1.33

$$\frac{\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^3d^2x^7 + 21\,\left(2\,b^3cd - ab^2d^2\right)x^5 + 35\,\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)x^3 - 105\,\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4)
 + 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^2)*x^5 + 35*(b^3*c^2 - 2
```

```
*a*b^2*c*d + a^2*b*d^2)*x^3 - 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x)/b^
4
```

mupad [B] time = 0.10, size = 169, normalized size = 1.61

$$x^3\left(\frac{c^2}{3\,b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{3\,b}\right)-x^5\,\left(\frac{a\,d^2}{5\,b^2}-\frac{2\,c\,d}{5\,b}\right)+\frac{d^2\,x^7}{7\,b}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(a\,d-b\,c)^2}{a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2}\right)\,(a\,d-b\,c)^2}{b^{9/2}}-\frac{a\,x\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x^3*(c^2/(3*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(3*b)) - x^5*((a*d^2)/(5*b^2
) - (2*c*d)/(5*b)) + (d^2*x^7)/(7*b) + (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(a*
d - b*c)^2)/(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))*(a*d - b*c)^2)/b^(9/2) -
 (a*x*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b))/b
```

sympy [B] time = 0.54, size = 246, normalized size = 2.34

$$x^5\left(-\frac{ad^2}{5b^2}+\frac{2cd}{5b}\right)+x^3\left(\frac{a^2d^2}{3b^3}-\frac{2acd}{3b^2}+\frac{c^2}{3b}\right)+x\left(-\frac{a^3d^2}{b^4}+\frac{2a^2cd}{b^3}-\frac{ac^2}{b^2}\right)-\frac{\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}\,(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2}+\frac{d^2x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**2/(5*b**2) + 2*c*d/(5*b)) + x**3*(a**2*d**2/(3*b**3) - 2*a*c*d/
(3*b**2) + c**2/(3*b)) + x*(-a**3*d**2/b**4 + 2*a**2*c*d/b**3 - a*c**2/b**2
) - sqrt(-a**3/b**9)*(a*d - b*c)**2*log(-b**4*sqrt(-a**3/b**9)*(a*d - b*c)*
*2/(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2) + x)/2 + sqrt(-a**3/b**9)*(a*d
- b*c)**2*log(b**4*sqrt(-a**3/b**9)*(a*d - b*c)**2/(a**3*d**2 - 2*a**2*b*c*
d + a*b**2*c**2) + x)/2 + d**2*x**7/(7*b)
```

3.210 $\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(bc-ad)^2}{2b^3}+\frac{dx^4(2bc-ad)}{4b^2}+\frac{d^2x^6}{6b}$$

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{dx^4(2bc-ad)}{4b^2}+\frac{x^2(bc-ad)^2}{2b^3}-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x^2)/(2*b^3) + (d*(2*b*c - a*d)*x^4)/(4*b^2) + (d^2*x^6)/(6*
b) - (a*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)^2}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(bc-ad)^2}{b^3}+\frac{d(2bc-ad)x}{b^2}+\frac{d^2x^2}{b}-\frac{a(-bc+ad)^2}{b^3(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2x^2}{2b^3}+\frac{d(2bc-ad)x^4}{4b^2}+\frac{d^2x^6}{6b}-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 82, normalized size = 1.02

$$\frac{bx^2\left(6a^2d^2-3abd\left(4c+dx^2\right)+2b^2\left(3c^2+3cdx^2+d^2x^4\right)\right)-6a(bc-ad)^2\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(6*a^2*d^2 - 3*a*b*d*(4*c + d*x^2) + 2*b^2*(3*c^2 + 3*c*d*x^2 + d^2*
x^4)) - 6*a*(b*c - a*d)^2*Log[a + b*x^2])/(12*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx^2\right)^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^2)^2)/(a + b*x^2), x]
```

fricas [A] time = 0.61, size = 102, normalized size = 1.28

$$\frac{2\,b^3d^2x^6+3\left(2\,b^3cd-ab^2d^2\right)x^4+6\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^2-6\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\log\left(bx^2+a\right)}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^6 + 3*(2*b^3*c*d - a*b^2*d^2)*x^4 + 6*(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2)*x^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 + a)
)/b^4
```

giac [A] time = 0.35, size = 107, normalized size = 1.34

$$\frac{2\,b^2d^2x^6+6\,b^2cdx^4-3\,abd^2x^4+6\,b^2c^2x^2-12\,abcdx^2+6\,a^2d^2x^2}{12\,b^3}-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 6*b^2*c*d*x^4 - 3*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a*
b*c*d*x^2 + 6*a^2*d^2*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*lo
g(abs(b*x^2 + a))/b^4
```

maple [A] time = 0.00, size = 124, normalized size = 1.55

$$\frac{d^2x^6}{6b}-\frac{a\,d^2x^4}{4b^2}+\frac{cd\,x^4}{2b}+\frac{a^2d^2x^2}{2b^3}-\frac{acd\,x^2}{b^2}+\frac{c^2x^2}{2b}-\frac{a^3d^2\ln\left(b\,x^2+a\right)}{2b^4}+\frac{a^2cd\ln\left(b\,x^2+a\right)}{b^3}-\frac{a\,c^2\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/6*d^2*x^6/b-1/4/b^2*x^4*a*d^2+1/2/b*x^4*c*d+1/2/b^3*x^2*a^2*d^2-1/b^2*x^2
*a*c*d+1/2/b*x^2*c^2-1/2*a^3/b^4*ln(b*x^2+a)*d^2+a^2/b^3*ln(b*x^2+a)*c*d-1/
2*a/b^2*ln(b*x^2+a)*c^2
```

maxima [A] time = 1.03, size = 101, normalized size = 1.26

$$\frac{2\,b^2d^2x^6+3\left(2\,b^2cd-abd^2\right)x^4+6\left(b^2c^2-2\,abcd+a^2d^2\right)x^2}{12\,b^3}-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^2*x^6 + 3*(2*b^2*c*d - a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 + a
)/b^4
```

mupad [B] time = 0.11, size = 106, normalized size = 1.32

$$x^2\left(\frac{c^2}{2\,b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{2\,b}\right)-x^4\,\left(\frac{a\,d^2}{4\,b^2}-\frac{c\,d}{2\,b}\right)-\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2\right)}{2\,b^4}+\frac{d^2\,x^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x^2*(c^2/(2*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(2*b)) - x^4*((a*d^2)/(4*b^2
) - (c*d)/(2*b)) - (log(a + b*x^2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))/(2*
b^4) + (d^2*x^6)/(6*b)
```

sympy [A] time = 0.42, size = 83, normalized size = 1.04

$$-\frac{a\left(ad-bc\right)^{2}\log\left(a+bx^{2}\right)}{2b^{4}}+x^{4}\left(-\frac{ad^{2}}{4b^{2}}+\frac{cd}{2b}\right)+x^{2}\left(\frac{a^{2}d^{2}}{2b^{3}}-\frac{acd}{b^{2}}+\frac{c^{2}}{2b}\right)+\frac{d^{2}x^{6}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -a*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4) + x**4*(-a*d**2/(4*b**2) + c*d/(
2*b)) + x**2*(a**2*d**2/(2*b**3) - a*c*d/b**2 + c**2/(2*b)) + d**2*x**6/(6*
b)
```

3.211 $\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{a}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{dx^3(2bc-ad)}{3b^2}+\frac{x(bc-ad)^2}{b^3}-\frac{\sqrt{a}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \int \left(\frac{(bc-ad)^2}{b^3}+\frac{d(2bc-ad)x^2}{b^2}+\frac{d^2x^4}{b}+\frac{-ab^2c^2+2a^2bcd-a^3d^2}{b^3\left(a+bx^2\right)}\right)dx\\ &= \frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\left(a(bc-ad)^2\right)\int \frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\sqrt{a}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 84, normalized size = 1.00

$$-\frac{\sqrt{a}\,(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) - (
Sqrt[a]*(-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx^2\right)^2}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2)^2)/(a + b*x^2), x]
```

fricas [A] time = 0.93, size = 230, normalized size = 2.74

$$\left[\frac{6\,b^2d^2x^5+10\left(2\,b^2cd-abd^2\right)x^3+15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\left(b^2c^2-2\,abcd+a^2d^2\right)x}{30\,b^3},\ \frac{3\,b^2d^2x^5+5\left(2\,b^2cd-abd^2\right)x^3-15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\left(b^2c^2-2\,abcd+a^2d^2\right)x}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^2*d^2*x^5 + 10*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) +
```

```
30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3, 1/15*(3*b^2*d^2*x^5 + 5*(2*b^2*c
*d - a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3]
```

giac [A] time = 0.30, size = 113, normalized size = 1.35

$$-\frac{\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{4}d^{2}x^{5}+10\,b^{4}cdx^{3}-5\,ab^{3}d^{2}x^{3}+15\,b^{4}c^{2}x-30\,ab^{3}cdx+15\,a^{2}b^{2}d^{2}x}{15\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^4*d^2*x^5 + 10*b^4*c*d*x^3 - 5*a*b^3*d^2*x^3 + 15*b^4*c^2*x - 3
0*a*b^3*c*d*x + 15*a^2*b^2*d^2*x)/b^5
```

maple [A] time = 0.00, size = 135, normalized size = 1.61

$$\frac{d^{2}x^{5}}{5b}-\frac{a\,d^{2}x^{3}}{3b^{2}}+\frac{2cd\,x^{3}}{3b}-\frac{a^{3}d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{3}}+\frac{2a^{2}cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{2}}-\frac{a\,c^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{a^{2}d^{2}x}{b^{3}}-\frac{2acdx}{b^{2}}+\frac{c^{2}x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/5*d^2*x^5/b-1/3/b^2*x^3*a*d^2+2/3/b*x^3*c*d+1/b^3*a^2*d^2*x-2/b^2*a*c*d*x
+1/b*c^2*x-a^3/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+2*a^2/b^2/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)*c^2
```

maxima [A] time = 2.46, size = 105, normalized size = 1.25

$$-\frac{\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{3\,b^{2}d^{2}x^{5}+5\left(2\,b^{2}cd-abd^{2}\right)x^{3}+15\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^2*d^2*x^5 + 5*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x)/b^3
```

mupad [B] time = 0.12, size = 128, normalized size = 1.52

$$x\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)-x^3\,\left(\frac{a\,d^2}{3\,b^2}-\frac{2\,c\,d}{3\,b}\right)+\frac{d^2\,x^5}{5\,b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^2}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] x*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b) - x^3*((a*d^2)/(3*b^2) - (2*c*d
)/(3*b)) + (d^2*x^5)/(5*b) - (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d - b*c)^2
)/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))*(a*d - b*c)^2)/b^(7/2)
```

sympy [B] time = 0.49, size = 194, normalized size = 2.31

$$x^3\left(-\frac{ad^2}{3b^2}+\frac{2cd}{3b}\right)+x\left(\frac{a^2d^2}{b^3}-\frac{2acd}{b^2}+\frac{c^2}{b}\right)+\frac{\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2\log\left(-\frac{b^3\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2\log\left(\frac{b^3\sqrt{-\frac{a}{b^7}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**2/(3*b**2) + 2*c*d/(3*b)) + x*(a**2*d**2/b**3 - 2*a*c*d/b**2 +
c**2/b) + sqrt(-a/b**7)*(a*d - b*c)**2*log(-b**3*sqrt(-a/b**7)*(a*d - b*c)*
*2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-a/b**7)*(a*d - b*c)**
2*log(b**3*sqrt(-a/b**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2)
 + x)/2 + d**2*x**5/(5*b)
```

3.212 $\int \frac{x\left(c+dx^2\right)^2}{a+bx^2}\,dx$

Optimal. Leaf size=61

$$\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2b^3}+\frac{dx^2(bc-ad)}{2b^2}+\frac{\left(c+dx^2\right)^2}{4b}$$

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{dx^2(bc-ad)}{2b^2}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2b^3}+\frac{\left(c+dx^2\right)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)*x^2)/(2*b^2) + (c + d*x^2)^2/(4*b) + ((b*c - a*d)^2*Log[a +
b*x^2])/(2*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^2}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx,x,x^2\right)\\
&= \frac{d(bc-ad)x^2}{2b^2}+\frac{\left(c+dx^2\right)^2}{4b}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.02, size = 49, normalized size = 0.80

$$
\frac{bdx^2\left(-2ad+4bc+bdx^2\right)+2(bc-ad)^2\log\left(a+bx^2\right)}{4b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(4*b*c - 2*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[a + b*x^2])/(4*b^3
)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(c+dx^2\right)^2}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^2)^2)/(a + b*x^2), x]
```

fricas [**A**] time = 1.05, size = 67, normalized size = 1.10

$$
\frac{b^2d^2x^4+2\left(2\,b^2cd-abd^2\right)x^2+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx^2+a\right)}{4\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^4 + 2*(2*b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(b*x^2 + a))/b^3
```

giac [A] time = 0.38, size = 67, normalized size = 1.10

$$\frac{bd^2x^4 + 4\,bcdx^2 - 2\,ad^2x^2}{4\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*b*c*d*x^2 - 2*a*d^2*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*log(abs(b*x^2 + a))/b^3
```

maple [A] time = 0.00, size = 85, normalized size = 1.39

$$\frac{d^2x^4}{4b} - \frac{a\,d^2x^2}{2b^2} + \frac{cd\,x^2}{b} + \frac{a^2d^2\ln\left(b\,x^2 + a\right)}{2b^3} - \frac{acd\ln\left(b\,x^2 + a\right)}{b^2} + \frac{c^2\ln\left(b\,x^2 + a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/4*d^2/b*x^4-1/2*d^2/b^2*a*x^2+d/b*c*x^2+1/2/b^3*ln(b*x^2+a)*a^2*d^2-1/b^2
*ln(b*x^2+a)*a*c*d+1/2/b*ln(b*x^2+a)*c^2
```

maxima [A] time = 1.12, size = 66, normalized size = 1.08

$$\frac{bd^2x^4 + 2\left(2\,bcd - ad^2\right)x^2}{4\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 2*(2*b*c*d - a*d^2)*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.06, size = 68, normalized size = 1.11

$$\frac{d^2\,x^4}{4\,b} - x^2\,\left(\frac{a\,d^2}{2\,b^2} - \frac{c\,d}{b}\right) + \frac{\ln\left(b\,x^2 + a\right)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^2)/(a + b*x^2),x)
```

```
[Out] (d^2*x^4)/(4*b) - x^2*((a*d^2)/(2*b^2) - (c*d)/b) + (log(a + b*x^2)*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d))/(2*b^3)
```

sympy [A] time = 0.37, size = 49, normalized size = 0.80

$$x^2\left(-\frac{ad^2}{2b^2}+\frac{cd}{b}\right)+\frac{d^2x^4}{4b}+\frac{(ad-bc)^2\log\left(a+bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x**2*(-a*d**2/(2*b**2) + c*d/b) + d**2*x**4/(4*b) + (a*d - b*c)**2*log(a +
b*x**2)/(2*b**3)
```

3.213 $\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.105, Rules used = {390, 205}

$$\frac{dx(2bc-ad)}{b^2}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{a+bx^2}\,dx &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^2\right)}\right)dx \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^2}\,dx}{b^2} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^3}{3b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}
\end{aligned}
$$

Mathematica [**A**] time = 0.05, size = 59, normalized size = 0.94

$$
\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{dx\left(-3ad+6bc+bdx^2\right)}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(c+dx^2\right)^2}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2), x]
```

fricas [**A**] time = 0.78, size = 181, normalized size = 2.87

$$
\left[\frac{2\,ab^2d^2x^3-3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(2\,ab^2cd-a^2bd^2\right)x}{6\,ab^3},\ \frac{ab^2d^2x^3+3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(2\,ab^2cd-a^2bd^2\right)x}{3\,ab^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

giac [A] time = 0.32, size = 72, normalized size = 1.14

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{b^2d^2x^3 + 6\,b^2cdx - 3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

maple [A] time = 0.00, size = 95, normalized size = 1.51

$$\frac{d^2x^3}{3b} + \frac{a^2d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2} - \frac{2acd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{a\,d^2x}{b^2} + \frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^2*x^3/b-d^2/b^2*a*x+2*d/b*c*x+1/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)*a^2*d^2-2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*c*d+1/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.40, size = 69, normalized size = 1.10

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{bd^2x^3 + 3\left(2\,bcd - ad^2\right)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

mupad [B] time = 0.07, size = 90, normalized size = 1.43

$$\frac{d^2\,x^3}{3\,b} - x\,\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2),x)
```

```
[Out] (d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*c
)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b^(
5/2))
```

sympy [B] time = 0.44, size = 172, normalized size = 2.73

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)-\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*s
qrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
+ sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

3.214 $\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2ab^2}+\frac{c^2\log(x)}{a}+\frac{d^2x^2}{2b}$$

Rubi [**A**] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2ab^2}+\frac{c^2\log(x)}{a}+\frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*x^2)/(2*b) + (c^2*Log[x])/a - ((b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(c+dx)^2}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{b}+\frac{c^2}{ax}-\frac{(-bc+ad)^2}{ab(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{d^2x^2}{2b}+\frac{c^2\log(x)}{a}-\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2ab^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.98

$$\frac{-(bc-ad)^2\log\left(a+bx^2\right)+abd^2x^2+2b^2c^2\log(x)}{2ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2 + 2*b^2*c^2*Log[x] - (b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x*(a + b*x^2)), x]
```

fricas [A] time = 0.85, size = 59, normalized size = 1.16

$$\frac{abd^2x^2+2\,b^2c^2\log(x)-\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx^2+a\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*d^2*x^2 + 2*b^2*c^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(b
*x^2 + a))/(a*b^2)
```

giac [A] time = 0.33, size = 62, normalized size = 1.22

$$\frac{d^2x^2}{2\,b}+\frac{c^2\log\left(x^2\right)}{2\,a}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(abs(b*x^2 + a))/(a*b^2)
```

maple [A] time = 0.00, size = 69, normalized size = 1.35

$$\frac{d^{2}x^{2}}{2b}-\frac{a\,d^{2}\ln\left(b\,x^{2}+a\right)}{2b^{2}}+\frac{c^{2}\ln(x)}{a}-\frac{c^{2}\ln\left(b\,x^{2}+a\right)}{2a}+\frac{cd\ln\left(b\,x^{2}+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x/(b*x^2+a),x)
```

```
[Out] 1/2*d^2*x^2/b-1/2*a/b^2*ln(b*x^2+a)*d^2+1/b*ln(b*x^2+a)*c*d-1/2/a*ln(b*x^2+
a)*c^2+c^2*ln(x)/a
```

maxima [A] time = 1.11, size = 61, normalized size = 1.20

$$\frac{d^{2}x^{2}}{2\,b}+\frac{c^{2}\log\left(x^{2}\right)}{2\,a}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(bx^{2}+a\right)}{2\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(b*x^2 + a)/(a*b^2)
```

mupad [B] time = 0.17, size = 58, normalized size = 1.14

$$\frac{d^{2}\,x^{2}}{2\,b}+\frac{c^{2}\,\ln(x)}{a}-\frac{\ln\left(b\,x^{2}+a\right)\,\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}{2\,a\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x*(a + b*x^2)),x)
```

```
[Out] (d^2*x^2)/(2*b) + (c^2*log(x))/a - (log(a + b*x^2)*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/(2*a*b^2)
```

sympy [A] time = 1.23, size = 41, normalized size = 0.80

$$\frac{d^{2}x^{2}}{2b}+\frac{c^{2}\log\left(x\right)}{a}-\frac{\left(ad-bc\right)^{2}\log\left(\frac{a}{b}+x^{2}\right)}{2ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x/(b*x**2+a),x)
```

```
[Out] d**2*x**2/(2*b) + c**2*log(x)/a - (a*d - b*c)**2*log(a/b + x**2)/(2*a*b**2)
```

3.215 $\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=55

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Rubi [**A**] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(
3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{d^2}{b}+\frac{c^2}{ax^2}-\frac{(-bc+ad)^2}{ab\left(a+bx^2\right)}\right)dx \\
&= -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\int \frac{1}{a+bx^2}\,dx}{ab} \\
&= -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.00

$$
-\frac{(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^2/(a*x)) + (d^2*x)/b - ((-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(3/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 0.86, size = 164, normalized size = 2.98

$$
\left[\frac{2\,a^2bd^2x^2-2\,ab^2c^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}\,x\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,a^2b^2x}, \frac{a^2bd^2x^2-ab^2c^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{a^2b^2x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*b*d^2*x^2 - 2*a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(
-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^2*x), (a^2*b*
d^2*x^2 - a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x*arctan(sq
rt(a*b)*x/a))/(a^2*b^2*x)]
```

giac [A] time = 0.30, size = 63, normalized size = 1.15

$$\frac{d^2x}{b} - \frac{c^2}{ax} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a*b)
```

maple [A] time = 0.01, size = 85, normalized size = 1.55

$$-\frac{a\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{b\,c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} + \frac{2cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d^2x}{b} - \frac{c^2}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^2/(b*x^2+a),x)
```

```
[Out] d^2*x/b-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*c*d-1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2-c^2/
a/x
```

maxima [A] time = 2.43, size = 63, normalized size = 1.15

$$\frac{d^2x}{b} - \frac{c^2}{ax} - \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.07, size = 80, normalized size = 1.45

$$\frac{d^2\,x}{b} - \frac{c^2}{a\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^2*(a + b*x^2)),x)
```

```
[Out] (d^2*x)/b - c^2/(a*x) - (atan((b^(1/2)*x*(a*d - b*c)^2)/(a^(1/2)*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(3/2)*b^(3/2))
```

sympy [B] time = 0.56, size = 165, normalized size = 3.00

$$\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2\log\left(-\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2\log\left(\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x}{b}-\frac{c^2}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**2/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(-a**2*b*sqrt(-1/(a**3*b**3))*(a*d -
 b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-1/(a**3*b**3))*
(a*d - b*c)**2*log(a**2*b*sqrt(-1/(a**3*b**3))*(a*d - b*c)**2/(a**2*d**2 -
2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x/b - c**2/(a*x)
```

3.216 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}-\frac{c\log(x)(bc-2ad)}{a^2}-\frac{c^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] -c^2/(2*a*x^2) - (c*(b*c - 2*a*d)*Log[x])/a^2 + ((b*c - a*d)^2*Log[a + b*x^
2])/(2*a^2*b)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c^2}{ax^2}+\frac{c(-bc+2ad)}{a^2x}+\frac{(-bc+ad)^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c^2}{2ax^2}-\frac{c(bc-2ad)\log(x)}{a^2}+\frac{(bc-ad)^2\log\left(a+bx^2\right)}{2a^2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.03

$$\frac{-abc^2-2bcx^2\log(x)(bc-2ad)+x^2(bc-ad)^2\log\left(a+bx^2\right)}{2a^2bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] (-(a*b*c^2) - 2*b*c*(b*c - 2*a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[a + b*
x^2])/(2*a^2*b*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 0.82, size = 73, normalized size = 1.26

$$-\frac{abc^2-\left(b^2c^2-2\,abcd+a^2d^2\right)x^2\log\left(bx^2+a\right)+2\left(b^2c^2-2\,abcd\right)x^2\log(x)}{2\,a^2bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(b*x^2 + a) + 2*(b^2
*c^2 - 2*a*b*c*d)*x^2*log(x))/(a^2*b*x^2)
```

giac [A] time = 0.34, size = 90, normalized size = 1.55

$$-\frac{\left(bc^2-2\,acd\right)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bc^2x^2-2\,acdx^2-ac^2}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*c^2*x^2 - 2*a*c*d*x^2 - a*c^2)/(a^2*x^2
)
```

maple [A] time = 0.01, size = 81, normalized size = 1.40

$$\frac{2cd\ln(x)}{a}-\frac{cd\ln\left(b\,x^2+a\right)}{a}-\frac{b\,c^2\ln(x)}{a^2}+\frac{b\,c^2\ln\left(b\,x^2+a\right)}{2a^2}+\frac{d^2\ln\left(b\,x^2+a\right)}{2b}-\frac{c^2}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^3/(b*x^2+a),x)
```

```
[Out] 1/2/b*ln(b*x^2+a)*d^2-1/a*ln(b*x^2+a)*c*d+1/2/a^2*b*ln(b*x^2+a)*c^2-1/2*c^2
/a/x^2+2*c/a*ln(x)*d-c^2/a^2*ln(x)*b
```

maxima [A] time = 1.13, size = 69, normalized size = 1.19

$$-\frac{\left(bc^2-2\,acd\right)\log\left(x^2\right)}{2\,a^2}-\frac{c^2}{2\,ax^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(bx^2+a\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 - 1/2*c^2/(a*x^2) + 1/2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*log(b*x^2 + a)/(a^2*b)
```

mupad [B] time = 0.18, size = 67, normalized size = 1.16

$$\frac{\ln\left(b\,x^2+a\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a^2\,b}-\frac{c^2}{2\,a\,x^2}-\frac{\ln(x)\,\left(b\,c^2-2\,a\,c\,d\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^3*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a^2*b) - c^2/(2*a*x^2)
- (log(x)*(b*c^2 - 2*a*c*d))/a^2
```

sympy [**A**] time = 1.39, size = 49, normalized size = 0.84

$$-\frac{c^2}{2ax^2} + \frac{c\,(2ad - bc)\log{(x)}}{a^2} + \frac{(ad - bc)^2\log{\left(\frac{a}{b} + x^2\right)}}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**3/(b*x**2+a),x)
```

```
[Out] -c**2/(2*a*x**2) + c*(2*a*d - b*c)*log(x)/a**2 + (a*d - b*c)**2*log(a/b + x
**2)/(2*a**2*b)
```

3.217 $\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=64

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}+\frac{c(bc-2ad)}{a^2x}-\frac{c^2}{3ax^3}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^2/(3*a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)}\,dx &= \int \left(\frac{c^2}{ax^4}+\frac{c(-bc+2ad)}{a^2x^2}+\frac{(-bc+ad)^2}{a^2\left(a+bx^2\right)}\right)dx\\ &= -\frac{c^2}{3ax^3}+\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{c^2}{3ax^3}+\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.03

$$\frac{(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}-\frac{c(2ad-bc)}{a^2x}-\frac{c^2}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c^2/(a*x^3) - (c*(-(b*c) + 2*a*d))/(a^2*x) + ((-(b*c) + a*d)^2*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x^4*(a + b*x^2)), x]
```

fricas [A] time = 0.89, size = 190, normalized size = 2.97

$$\left[-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ab}\,x^3\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,a^2bc^2-6\left(ab^2c^2-2\,a^2bcd\right)x^2}{6\,a^3bx^3},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}\,x^3\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-a^2bc^2+3\left(ab^2c^2-2\,a^2bcd\right)x^2}{3\,a^3bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*x^3*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*a^2*b*c^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)
/(a^3*b*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x^3*arctan(s
qrt(a*b)*x/a) - a^2*b*c^2 + 3*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)/(a^3*b*x^3)]
```

giac [A] time = 0.33, size = 72, normalized size = 1.12

$$\frac{\left(b^{2} c^{2}-2 \, a b c d+a^{2} d^{2}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} a^{2}}+\frac{3 \, b c^{2} x^{2}-6 \, a c d x^{2}-a c^{2}}{3 \, a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3
*(3*b*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^2*x^3)
```

maple [A] time = 0.01, size = 98, normalized size = 1.53

$$-\frac{2 b c d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a}+\frac{b^{2} c^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, a^{2}}+\frac{d^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}}-\frac{2 c d}{a x}+\frac{b \,c^{2}}{a^{2} x}-\frac{c^{2}}{3 a \,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^4/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2-2/a/(a*b)^(1/2)*arctan(1/(a*b)^
(1/2)*b*x)*b*c*d+1/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^2*c^2-1/3*c^
2/a/x^3-2*c/a/x*d+c^2/a^2/x*b
```

maxima [A] time = 2.46, size = 70, normalized size = 1.09

$$\frac{\left(b^{2} c^{2}-2 \, a b c d+a^{2} d^{2}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} a^{2}}-\frac{a c^{2}-3 \, \left(b c^{2}-2 \, a c d\right) x^{2}}{3 \, a^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3
*(a*c^2 - 3*(b*c^2 - 2*a*c*d)*x^2)/(a^2*x^3)
```

mupad [B] time = 0.13, size = 90, normalized size = 1.41

$$\frac{b\,c^2}{a^2\,x}-\frac{c^2}{3\,a\,x^3}+\frac{b^{3/2}\,c^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\,c\,d}{a\,x}-\frac{2\,\sqrt{b}\,c\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^4*(a + b*x^2)),x)
```

```
[Out] (b*c^2)/(a^2*x) - c^2/(3*a*x^3) + (b^(3/2)*c^2*atan((b^(1/2)*x)/a^(1/2)))/a
^(5/2) + (d^2*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (2*c*d)/(a*x)
- (2*b^(1/2)*c*d*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [**B**] time = 0.67, size = 172, normalized size = 2.69

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2\log\left(-\frac{a^3\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2\log\left(\frac{a^3\sqrt{-\frac{1}{a^5b}}\,(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{-ac^2+x^2\left(-6acd+3bc^2\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - b*c)**2*log(-a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**
2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(a**5*b))*(a*d - b*c
)**2*log(a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**
2*c**2) + x)/2 + (-a*c**2 + x**2*(-6*a*c*d + 3*b*c**2))/(3*a**2*x**3)
```

3.218 $\int \frac{x^5\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=138

$$\frac{a^2(bc-ad)^3\log\left(a+bx^2\right)}{2b^6}+\frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{6b^3}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{x^4(bc-ad)^3}{4b^4}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{d^3x^{10}}{10b}$$

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{6b^3}+\frac{a^2(bc-ad)^3\log\left(a+bx^2\right)}{2b^6}+\frac{d^2x^8(3bc-ad)}{8b^2}+\frac{x^4(bc-ad)^3}{4b^4}-\frac{ax^2(bc-ad)^3}{2b^5}+\frac{d^3x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -(a*(b*c - a*d)^3*x^2)/(2*b^5) + ((b*c - a*d)^3*x^4)/(4*b^4) + (d*(3*b^2*c^
2 - 3*a*b*c*d + a^2*d^2)*x^6)/(6*b^3) + (d^2*(3*b*c - a*d)*x^8)/(8*b^2) + (
d^3*x^10)/(10*b) + (a^2*(b*c - a*d)^3*Log[a + b*x^2])/(2*b^6)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(c+dx)^3}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a(-bc+ad)^3}{b^5}+\frac{(bc-ad)^3x}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^2}{b^3}+\frac{d^2(3bc-ad)x}{b^2}\right.\right.\\ &= -\frac{a(bc-ad)^3x^2}{2b^5}+\frac{(bc-ad)^3x^4}{4b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^6}{6b^3}+\frac{d^2(3bc-ad)x^8}{8b^2}+\frac{d^3x^{10}}{10b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 128, normalized size = 0.93

$$\frac{20b^3dx^6\left(a^2d^2-3abcd+3b^2c^2\right)+60a^2(bc-ad)^3\log\left(a+bx^2\right)+15b^4d^2x^8(3bc-ad)+30b^2x^4(bc-ad)^3+60abx^2(ad-bc)^3+12b^5d^3x^{10}}{120b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (60*a*b*(-(b*c) + a*d)^3*x^2 + 30*b^2*(b*c - a*d)^3*x^4 + 20*b^3*d*(3*b^2*c
^2 - 3*a*b*c*d + a^2*d^2)*x^6 + 15*b^4*d^2*(3*b*c - a*d)*x^8 + 12*b^5*d^3*x
^10 + 60*a^2*(b*c - a*d)^3*Log[a + b*x^2])/(120*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(c+dx^2\right)^3}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^5*(c + d*x^2)^3)/(a + b*x^2), x]
```

fricas [A] time = 0.84, size = 220, normalized size = 1.59

$$\frac{12\,b^5d^3x^{10}+15\left(3\,b^5cd^2-ab^4d^3\right)x^8+20\left(3\,b^5c^2d-3\,ab^4cd^2+a^2b^3d^3\right)x^6+30\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)x^4-60\left(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3\right)x^2+60\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\log\left(bx^2+a\right)}{120\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/120*(12*b^5*d^3*x^10 + 15*(3*b^5*c*d^2 - a*b^4*d^3)*x^8 + 20*(3*b^5*c^2*d
 - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^6 + 30*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b
^3*c*d^2 - a^3*b^2*d^3)*x^4 - 60*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c
```

```
*d^2 - a^4*b*d^3)*x^2 + 60*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*log(b*x^2 + a))/b^6
```

giac [A] time = 0.32, size = 238, normalized size = 1.72

$$\frac{12\,b^{4}d^{3}x^{10} + 45\,b^{4}cd^{2}x^{8} - 15\,ab^{3}d^{3}x^{8} + 60\,b^{4}c^{2}dx^{6} - 60\,ab^{3}cd^{2}x^{6} + 20\,a^{2}b^{2}d^{3}x^{6} + 30\,b^{4}c^{3}x^{4} - 90\,ab^{3}c^{2}dx^{4} + 90\,a^{2}b^{2}cd^{2}x^{4} - 30\,a^{3}bd^{3}x^{4} - 60\,ab^{3}c^{3}x^{2} + 180\,a^{2}b^{2}c^{2}dx^{2} - 180\,a^{3}bcd^{2}x^{2} + 60\,a^{4}d^{3}x^{2}}{120\,b^{5}} + \frac{\left(a^{2}b^{3}c^{3} - 3\,a^{3}b^{2}c^{2}d + 3\,a^{4}bcd^{2} - a^{5}d^{3}\right)\log\left(\left|bx^{2} + a\right|\right)}{2\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 45*b^4*c*d^2*x^8 - 15*a*b^3*d^3*x^8 + 60*b^4*c^2*d
*x^6 - 60*a*b^3*c*d^2*x^6 + 20*a^2*b^2*d^3*x^6 + 30*b^4*c^3*x^4 - 90*a*b^3*
c^2*d*x^4 + 90*a^2*b^2*c*d^2*x^4 - 30*a^3*b*d^3*x^4 - 60*a*b^3*c^3*x^2 + 18
0*a^2*b^2*c^2*d*x^2 - 180*a^3*b*c*d^2*x^2 + 60*a^4*d^3*x^2)/b^5 + 1/2*(a^2*
b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*log(abs(b*x^2 + a))/b^
6
```

maple [B] time = 0.01, size = 263, normalized size = 1.91

$$\frac{d^{3}x^{10}}{10b} - \frac{a\,d^{3}x^{8}}{8b^{2}} + \frac{3c\,d^{2}x^{8}}{8b} + \frac{a^{2}d^{3}x^{6}}{6b^{3}} - \frac{ac\,d^{2}x^{6}}{2b^{2}} + \frac{c^{2}d\,x^{6}}{2b} - \frac{a^{3}d^{3}x^{4}}{4b^{4}} + \frac{3a^{2}c\,d^{2}x^{4}}{4b^{3}} - \frac{3a\,c^{2}d\,x^{4}}{4b^{2}} + \frac{c^{3}x^{4}}{4b} + \frac{a^{4}d^{3}x^{2}}{2b^{5}} - \frac{3a^{3}c\,d^{2}x^{2}}{2b^{4}} + \frac{3a^{2}c^{2}d\,x^{2}}{2b^{3}} - \frac{a\,c^{3}x^{2}}{2b^{2}} - \frac{a^{5}d^{3}\ln\left(b\,x^{2} + a\right)}{2b^{6}} + \frac{3a^{4}c\,d^{2}\ln\left(b\,x^{2} + a\right)}{2b^{5}} - \frac{3a^{3}c^{2}d\ln\left(b\,x^{2} + a\right)}{2b^{4}} + \frac{a^{2}c^{3}\ln\left(b\,x^{2} + a\right)}{2b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/10*d^3*x^10/b-1/8/b^2*x^8*a*d^3+3/8/b*x^8*c*d^2+1/6/b^3*x^6*a^2*d^3-1/2/b
^2*x^6*a*c*d^2+1/2/b*x^6*c^2*d-1/4/b^4*x^4*a^3*d^3+3/4/b^3*x^4*a^2*c*d^2-3/
4/b^2*x^4*a*c^2*d+1/4/b*x^4*c^3+1/2/b^5*x^2*a^4*d^3-3/2/b^4*x^2*a^3*c*d^2+3
/2/b^3*x^2*a^2*c^2*d-1/2/b^2*x^2*a*c^3-1/2*a^5/b^6*ln(b*x^2+a)*d^3+3/2*a^4/
b^5*ln(b*x^2+a)*c*d^2-3/2*a^3/b^4*ln(b*x^2+a)*c^2*d+1/2*a^2/b^3*ln(b*x^2+a)
*c^3
```

maxima [A] time = 1.20, size = 219, normalized size = 1.59

$$\frac{12\,b^{4}d^{3}x^{10} + 15\left(3\,b^{4}cd^{2} - ab^{3}d^{3}\right)x^{8} + 20\left(3\,b^{4}c^{2}d - 3\,ab^{3}cd^{2} + a^{2}b^{2}d^{3}\right)x^{6} + 30\left(b^{4}c^{3} - 3\,ab^{3}c^{2}d + 3\,a^{2}b^{2}cd^{2} - a^{3}bd^{3}\right)x^{4} - 60\left(ab^{3}c^{3} - 3\,a^{2}b^{2}c^{2}d + 3\,a^{3}bcd^{2} - a^{4}d^{3}\right)x^{2}}{120\,b^{5}} + \frac{\left(a^{2}b^{3}c^{3} - 3\,a^{3}b^{2}c^{2}d + 3\,a^{4}bcd^{2} - a^{5}d^{3}\right)\log\left(bx^{2} + a\right)}{2\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/120*(12*b^4*d^3*x^10 + 15*(3*b^4*c*d^2 - a*b^3*d^3)*x^8 + 20*(3*b^4*c^2*d
 - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^6 + 30*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b
^2*c*d^2 - a^3*b*d^3)*x^4 - 60*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
 - a^4*d^3)*x^2)/b^5 + 1/2*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*log(b*x^2 + a)/b^6
```

mupad [**B**] time = 0.12, size = 236, normalized size = 1.71

$$x^4\left(\frac{c^3}{4\,b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{4\,b}\right)-x^8\left(\frac{a\,d^3}{8\,b^2}-\frac{3\,c\,d^2}{8\,b}\right)+x^6\left(\frac{c^2\,d}{2\,b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{6\,b}\right)-\frac{\ln\left(b\,x^2+a\right)\left(a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3\right)}{2\,b^6}+\frac{d^3\,x^{10}}{10\,b}-\frac{a\,x^2\left(\frac{c^3}{b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^4*(c^3/(4*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(4*b
)) - x^8*((a*d^3)/(8*b^2) - (3*c*d^2)/(8*b)) + x^6*((c^2*d)/(2*b) + (a*((a*
d^3)/b^2 - (3*c*d^2)/b))/(6*b)) - (log(a + b*x^2)*(a^5*d^3 - a^2*b^3*c^3 +
3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/(2*b^6) + (d^3*x^10)/(10*b) - (a*x^2*(c^3
/b - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/b))/(2*b)
```

sympy [**A**] time = 0.60, size = 201, normalized size = 1.46

$$-\frac{a^2\left(ad-bc\right)^3\log\left(a+bx^2\right)}{2b^6}+x^8\left(-\frac{ad^3}{8b^2}+\frac{3cd^2}{8b}\right)+x^6\left(\frac{a^2d^3}{6b^3}-\frac{acd^2}{2b^2}+\frac{c^2d}{2b}\right)+x^4\left(-\frac{a^3d^3}{4b^4}+\frac{3a^2cd^2}{4b^3}-\frac{3ac^2d}{4b^2}+\frac{c^3}{4b}\right)+x^2\left(\frac{a^4d^3}{2b^5}-\frac{3a^3cd^2}{2b^4}+\frac{3a^2c^2d}{2b^3}-\frac{ac^3}{2b^2}\right)+\frac{d^3x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] -a**2*(a*d - b*c)**3*log(a + b*x**2)/(2*b**6) + x**8*(-a*d**3/(8*b**2) + 3*
c*d**2/(8*b)) + x**6*(a**2*d**3/(6*b**3) - a*c*d**2/(2*b**2) + c**2*d/(2*b)
) + x**4*(-a**3*d**3/(4*b**4) + 3*a**2*c*d**2/(4*b**3) - 3*a*c**2*d/(4*b**2
) + c**3/(4*b)) + x**2*(a**4*d**3/(2*b**5) - 3*a**3*c*d**2/(2*b**4) + 3*a**
2*c**2*d/(2*b**3) - a*c**3/(2*b**2)) + d**3*x**10/(10*b)
```

3.219 $\int \frac{x^4\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=140

$$\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}-\frac{ax(bc-ad)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{a^{3/2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{x^3(bc-ad)^3}{3b^4}-\frac{ax(bc-ad)^3}{b^5}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^3}{a+bx^2}\,dx = \int \Bigg(-\frac{a(bc-ad)^3}{b^5}+\frac{(bc-ad)^3x^2}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^4}{b^3}+\frac{d^2(3bc-ad)x^6}{b^2}+\frac{d^3x^8}{b}$$

$$= -\frac{a(bc-ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}+$$

$$= -\frac{a(bc-ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}+$$

Mathematica [A] time = 0.05, size = 140, normalized size = 1.00

$$-\frac{a^{3/2}(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{dx^5\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{ax(ad-bc)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (a*(-(b*c) + a*d)^3*x)/b^5 + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^3*
x^9)/(9*b) - (a^(3/2)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)^3}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2)^3)/(a + b*x^2), x]
```

fricas [A] time = 0.75, size = 468, normalized size = 3.34

$$\left[\frac{70\,b^4d^3x^9+90\left(3\,b^4cd^2-ab^3d^3\right)x^7+126\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^5+210\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^3-315\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-630\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)x}{630\,b^5},\ \frac{35\,b^4d^3x^9+45\left(3\,b^4cd^2-ab^3d^3\right)x^7+63\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^5+105\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^3+315\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)x}{315\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*d^3*x^9 + 90*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 126*(3*b^4*c^2*
d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 210*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2 - a^4*d^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) -
 630*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5, 1/315*
(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 63*(3*b^4*c^2*d - 3*a*
b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d
^2 - a^3*b*d^3)*x^3 + 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5]
```

giac [A] time = 0.30, size = 241, normalized size = 1.72

$$\frac{\left(a^{2} b^{3} c^{3} - 3 \, a^{3} b^{2} c^{2} d + 3 \, a^{4} b c d^{2} - a^{5} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{5}} + \frac{35 \, b^{8} d^{3} x^{9} + 135 \, b^{8} c d^{2} x^{7} - 45 \, a b^{7} d^{3} x^{7} + 189 \, b^{8} c^{2} d x^{5} - 189 \, a b^{7} c d^{2} x^{5} + 63 \, a^{2} b^{6} d^{3} x^{5} + 105 \, b^{8} c^{3} x^{3} - 315 \, a b^{7} c^{2} d x^{3} + 315 \, a^{2} b^{6} c d^{2} x^{3} - 105 \, a^{3} b^{5} d^{3} x^{3} - 315 \, a b^{7} c^{3} x + 945 \, a^{2} b^{6} c^{2} d x - 945 \, a^{3} b^{5} c d^{2} x + 315 \, a^{4} b^{4} d^{3} x}{315 \, b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*d^3*x^9 + 135*b^8*c*d^2*x^7 - 45*a*b^7
*d^3*x^7 + 189*b^8*c^2*d*x^5 - 189*a*b^7*c*d^2*x^5 + 63*a^2*b^6*d^3*x^5 + 1
05*b^8*c^3*x^3 - 315*a*b^7*c^2*d*x^3 + 315*a^2*b^6*c*d^2*x^3 - 105*a^3*b^5*
d^3*x^3 - 315*a*b^7*c^3*x + 945*a^2*b^6*c^2*d*x - 945*a^3*b^5*c*d^2*x + 315
*a^4*b^4*d^3*x)/b^9
```

maple [B] time = 0.01, size = 276, normalized size = 1.97

$$\frac{d^{3} x^{9}}{9 b}-\frac{a \,d^{3} x^{7}}{7 b^{2}}+\frac{3 c \,d^{2} x^{7}}{7 b}+\frac{a^{2} d^{3} x^{5}}{5 b^{3}}-\frac{3 a c \,d^{2} x^{5}}{5 b^{2}}+\frac{3 c^{2} d \,x^{5}}{5 b}-\frac{a^{3} d^{3} x^{3}}{3 b^{4}}+\frac{a^{2} c \,d^{2} x^{3}}{b^{3}}-\frac{a \,c^{2} d \,x^{3}}{b^{2}}+\frac{c^{3} x^{3}}{3 b}-\frac{a^{5} d^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{5}}+\frac{3 a^{4} c \,d^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{4}}-\frac{3 a^{3} c^{2} d \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{3}}+\frac{a^{2} c^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{2}}+\frac{a^{4} d^{3} x}{b^{5}}-\frac{3 a^{3} c \,d^{2} x}{b^{4}}+\frac{3 a^{2} c^{2} d x}{b^{3}}-\frac{a \,c^{3} x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/7/b^2*x^7*a*d^3+3/7/b*x^7*c*d^2+1/5/b^3*x^5*a^2*d^3-3/5/b^2
*x^5*a*c*d^2+3/5/b*x^5*c^2*d-1/3/b^4*x^3*a^3*d^3+1/b^3*x^3*a^2*c*d^2-1/b^2*
x^3*a*c^2*d+1/3/b*x^3*c^3+1/b^5*a^4*d^3*x-3/b^4*a^3*c*d^2*x+3/b^3*a^2*c^2*d
*x-1/b^2*a*c^3*x-a^5/b^5/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3+3*a^4/b^
4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^2-3*a^3/b^3/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*c^2*d+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.58, size = 222, normalized size = 1.59

$$\frac{{\left(a^{2} b^{3} c^{3} - 3 \, a^{3} b^{2} c^{2} d + 3 \, a^{4} b c d^{2} - a^{5} d^{3}\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{5}} + \frac{35 \, b^{4} d^{3} x^{9} + 45 \, {\left(3 \, b^{4} c d^{2} - a b^{3} d^{3}\right)} x^{7} + 63 \, {\left(3 \, b^{4} c^{2} d - 3 \, a b^{3} c d^{2} + a^{2} b^{2} d^{3}\right)} x^{5} + 105 \, {\left(b^{4} c^{3} - 3 \, a b^{3} c^{2} d + 3 \, a^{2} b^{2} c d^{2} - a^{3} b d^{3}\right)} x^{3} - 315 \, {\left(a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + 3 \, a^{3} b c d^{2} - a^{4} d^{3}\right)} x}{315 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^3)
*x^7 + 63*(3*b^4*c^2*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 -
3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5
```

mupad [**B**] time = 0.11, size = 260, normalized size = 1.86

$$x^3\left(\frac{c^3}{3\,b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{3\,b}\right)-x^7\left(\frac{a\,d^3}{7\,b^2}-\frac{3\,c\,d^2}{7\,b}\right)+x^5\left(\frac{3\,c^2\,d}{5\,b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{5\,b}\right)+\frac{d^3\,x^9}{9\,b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(a\,d-b\,c)^3}{a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3}\right)(a\,d-b\,c)^3}{b^{11/2}}-\frac{a\,x\left(\frac{c^3}{b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^3*(c^3/(3*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(3*b
)) - x^7*((a*d^3)/(7*b^2) - (3*c*d^2)/(7*b)) + x^5*((3*c^2*d)/(5*b) + (a*((
a*d^3)/b^2 - (3*c*d^2)/b))/(5*b)) + (d^3*x^9)/(9*b) - (a^(3/2)*atan((a^(3/2
)*b^(1/2)*x*(a*d - b*c)^3)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4
*b*c*d^2))*(a*d - b*c)^3)/b^(11/2) - (a*x*(c^3/b - (a*((3*c^2*d)/b + (a*((a
*d^3)/b^2 - (3*c*d^2)/b))/b))/b))/b
```

sympy [**B**] time = 0.72, size = 343, normalized size = 2.45

$$x^7\left(-\frac{ad^3}{7b^2}+\frac{3cd^2}{7b}\right)+x^5\left(\frac{a^2d^3}{5b^3}-\frac{3acd^2}{5b^2}+\frac{3c^2d}{5b}\right)+x^3\left(-\frac{a^3d^3}{3b^4}+\frac{a^2cd^2}{b^3}-\frac{ac^2d}{b^2}+\frac{c^3}{3b}\right)+x\left(\frac{a^4d^3}{b^5}-\frac{3a^3cd^2}{b^4}+\frac{3a^2c^2d}{b^3}-\frac{ac^3}{b^2}\right)+\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\,(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}+\frac{d^3x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**7*(-a*d**3/(7*b**2) + 3*c*d**2/(7*b)) + x**5*(a**2*d**3/(5*b**3) - 3*a*c
*d**2/(5*b**2) + 3*c**2*d/(5*b)) + x**3*(-a**3*d**3/(3*b**4) + a**2*c*d**2/
b**3 - a*c**2*d/b**2 + c**3/(3*b)) + x*(a**4*d**3/b**5 - 3*a**3*c*d**2/b**4
 + 3*a**2*c**2*d/b**3 - a*c**3/b**2) + sqrt(-a**3/b**11)*(a*d - b*c)**3*log
(-b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a*
*2*b**2*c**2*d - a*b**3*c**3) + x)/2 - sqrt(-a**3/b**11)*(a*d - b*c)**3*log
(b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 + 3*a**
2*b**2*c**2*d - a*b**3*c**3) + x)/2 + d**3*x**9/(9*b)
```

3.220 $\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx$

Optimal. Leaf size=115

$$\frac{dx^4\left(a^2d^2-3abcd+3b^2c^2\right)}{4b^3}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}+\frac{x^2(bc-ad)^3}{2b^4}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{d^3x^8}{8b}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{dx^4\left(a^2d^2-3abcd+3b^2c^2\right)}{4b^3}+\frac{d^2x^6(3bc-ad)}{6b^2}+\frac{x^2(bc-ad)^3}{2b^4}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}+\frac{d^3x^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x^2)/(2*b^4) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^4)/(4*
b^3) + (d^2*(3*b*c - a*d)*x^6)/(6*b^2) + (d^3*x^8)/(8*b) - (a*(b*c - a*d)^3
*Log[a + b*x^2])/(2*b^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+dx)^3}{a+bx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\Bigg(\int\Bigg(\frac{(bc-ad)^3}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^3}{b}+\frac{a(-bc}{b^4(a} \\
&= \frac{(bc-ad)^3x^2}{2b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^2(3bc-ad)x^6}{6b^2}+\frac{d^3x^8}{8b}-\frac{a(bc-ad)^3\log(}{2b^5}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 125, normalized size = 1.09

$$
\frac{bx^2\left(-12a^3d^3+6a^2bd^2\left(6c+dx^2\right)-2ab^2d\left(18c^2+9cdx^2+2d^2x^4\right)+3b^3\left(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6\right)\right)+12a(ad-bc)^3\log\left(a+bx^2\right)}{24b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*x^2*(-12*a^3*d^3 + 6*a^2*b*d^2*(6*c + d*x^2) - 2*a*b^2*d*(18*c^2 + 9*c*d
*x^2 + 2*d^2*x^4) + 3*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d^2*x^4 + d^3*x^6)) +
12*a*(-(b*c) + a*d)^3*Log[a + b*x^2])/(24*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(c+dx^2\right)^3}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^2)^3)/(a + b*x^2), x]
```

fricas [A] time = 1.04, size = 169, normalized size = 1.47

$$
\frac{3\,b^4d^3x^8+4\left(3\,b^4cd^2-ab^3d^3\right)x^6+6\left(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3\right)x^4+12\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)x^2-12\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\log\left(bx^2+a\right)}{24\,b^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*d^3*x^8 + 4*(3*b^4*c*d^2 - a*b^3*d^3)*x^6 + 6*(3*b^4*c^2*d - 3*
a*b^3*c*d^2 + a^2*b^2*d^3)*x^4 + 12*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
```

```
d^2 - a^3*b*d^3)*x^2 - 12*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*log(b*x^2 + a))/b^5
```

giac [A] time = 0.39, size = 180, normalized size = 1.57

$$\frac{3\,b^3d^3x^8 + 12\,b^3cd^2x^6 - 4\,ab^2d^3x^6 + 18\,b^3c^2dx^4 - 18\,ab^2cd^2x^4 + 6\,a^2bd^3x^4 + 12\,b^3c^3x^2 - 36\,ab^2c^2dx^2 + 36\,a^2bcd^2x^2 - 12\,a^3d^3x^2}{24\,b^4} - \frac{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 12*b^3*c*d^2*x^6 - 4*a*b^2*d^3*x^6 + 18*b^3*c^2*d*x^4
 - 18*a*b^2*c*d^2*x^4 + 6*a^2*b*d^3*x^4 + 12*b^3*c^3*x^2 - 36*a*b^2*c^2*d*x
^2 + 36*a^2*b*c*d^2*x^2 - 12*a^3*d^3*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*
c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*log(abs(b*x^2 + a))/b^5
```

maple [A] time = 0.00, size = 205, normalized size = 1.78

$$\frac{d^3x^8}{8b} - \frac{a\,d^3x^6}{6b^2} + \frac{c\,d^2x^6}{2b} + \frac{a^2d^3x^4}{4b^3} - \frac{3ac\,d^2x^4}{4b^2} + \frac{3c^2d\,x^4}{4b} - \frac{a^3d^3x^2}{2b^4} + \frac{3a^2c\,d^2x^2}{2b^3} - \frac{3a\,c^2d\,x^2}{2b^2} + \frac{c^3x^2}{2b} + \frac{a^4d^3\ln\left(b\,x^2 + a\right)}{2b^5} - \frac{3a^3c\,d^2\ln\left(b\,x^2 + a\right)}{2b^4} + \frac{3a^2c^2d\ln\left(b\,x^2 + a\right)}{2b^3} - \frac{a\,c^3\ln\left(b\,x^2 + a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/8*d^3*x^8/b-1/6/b^2*x^6*a*d^3+1/2/b*x^6*c*d^2+1/4/b^3*x^4*a^2*d^3-3/4/b^2
*x^4*a*c*d^2+3/4/b*x^4*c^2*d-1/2/b^4*x^2*a^3*d^3+3/2/b^3*x^2*a^2*c*d^2-3/2/
b^2*x^2*a*c^2*d+1/2/b*x^2*c^3+1/2*a^4/b^5*ln(b*x^2+a)*d^3-3/2*a^3/b^4*ln(b*
x^2+a)*c*d^2+3/2*a^2/b^3*ln(b*x^2+a)*c^2*d-1/2*a/b^2*ln(b*x^2+a)*c^3
```

maxima [A] time = 1.04, size = 168, normalized size = 1.46

$$\frac{3\,b^3d^3x^8 + 4\,\left(3\,b^3cd^2 - ab^2d^3\right)x^6 + 6\,\left(3\,b^3c^2d - 3\,ab^2cd^2 + a^2bd^3\right)x^4 + 12\,\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x^2}{24\,b^4} - \frac{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\log\left(bx^2 + a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^3*d^3*x^8 + 4*(3*b^3*c*d^2 - a*b^2*d^3)*x^6 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^4 + 12*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^
4*d^3)*log(b*x^2 + a)/b^5
```

mupad [B] time = 0.05, size = 178, normalized size = 1.55

$$x^2\left(\frac{c^3}{2\,b} - \frac{a\left(\frac{3\,c^2\,d}{b} + \frac{a\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right)}{b}\right)}{2\,b}\right) - x^6\left(\frac{a\,d^3}{6\,b^2} - \frac{c\,d^2}{2\,b}\right) + x^4\left(\frac{3\,c^2\,d}{4\,b} + \frac{a\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right)}{4\,b}\right) + \frac{\ln\left(b\,x^2 + a\right)\left(a^4\,d^3 - 3\,a^3\,b\,c\,d^2 + 3\,a^2\,b^2\,c^2\,d - a\,b^3\,c^3\right)}{2\,b^5} + \frac{d^3\,x^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^2*(c^3/(2*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(2*b
)) - x^6*((a*d^3)/(6*b^2) - (c*d^2)/(2*b)) + x^4*((3*c^2*d)/(4*b) + (a*((a*
d^3)/b^2 - (3*c*d^2)/b))/(4*b)) + (log(a + b*x^2)*(a^4*d^3 - a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 3*a^3*b*c*d^2))/(2*b^5) + (d^3*x^8)/(8*b)
```

sympy [A] time = 0.55, size = 144, normalized size = 1.25

$$\frac{a\left(ad-bc\right)^{3}\log\left(a+bx^{2}\right)}{2b^{5}}+x^{6}\left(-\frac{ad^{3}}{6b^{2}}+\frac{cd^{2}}{2b}\right)+x^{4}\left(\frac{a^{2}d^{3}}{4b^{3}}-\frac{3acd^{2}}{4b^{2}}+\frac{3c^{2}d}{4b}\right)+x^{2}\left(-\frac{a^{3}d^{3}}{2b^{4}}+\frac{3a^{2}cd^{2}}{2b^{3}}-\frac{3ac^{2}d}{2b^{2}}+\frac{c^{3}}{2b}\right)+\frac{d^{3}x^{8}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] a*(a*d - b*c)**3*log(a + b*x**2)/(2*b**5) + x**6*(-a*d**3/(6*b**2) + c*d**2
/(2*b)) + x**4*(a**2*d**3/(4*b**3) - 3*a*c*d**2/(4*b**2) + 3*c**2*d/(4*b))
+ x**2*(-a**3*d**3/(2*b**4) + 3*a**2*c*d**2/(2*b**3) - 3*a*c**2*d/(2*b**2)
+ c**3/(2*b)) + d**3*x**8/(8*b)
```

3.221 $\int \frac{x^2\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=119

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}-\frac{\sqrt{a}\,(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}$$

Rubi [A] time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{x(bc-ad)^3}{b^4}-\frac{\sqrt{a}\,(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (Sqrt[a]*(b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \int \left(\frac{(bc-ad)^3}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^2}{b^3}+\frac{d^2(3bc-ad)x^4}{b^2}+\frac{d^3x^6}{b}+\frac{-ab^3c^3+3a^2b^2}{b^4\,(}\right.\\
&= \frac{(bc-ad)^3x}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^7}{7b}-\frac{\left(a(bc-ad)^3\right)\int\frac{}{a+}}{b^4}\\
&= \frac{(bc-ad)^3x}{b^4}+\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^3}{3b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^7}{7b}-\frac{\sqrt{a}\,(bc-ad)^3\tan^{}}{b^{9/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 118, normalized size = 0.99

$$
\frac{dx^3\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{\sqrt{a}\,(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) +
 (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) + (Sqrt[a]*(-(b*c) + a*d
)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\left(c+dx^2\right)^3}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2)^3)/(a + b*x^2), x]
```

fricas [A] time = 0.79, size = 364, normalized size = 3.06

$$
\left[\frac{30\,b^3d^3x^7+42\left(3\,b^3cd^2-ab^2d^3\right)x^5+70\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^3-105\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x}{210\,b^4},\ \frac{15\,b^3d^3x^7+21\left(3\,b^3cd^2-ab^2d^3\right)x^5+35\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^3-105\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x}{105\,b^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*d^3*x^7 + 42*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 70*(3*b^3*c^2*d
 - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
```

```
c*d^2 - a^3*d^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
 + 210*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4, 1/105*(1
5*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 35*(3*b^3*c^2*d - 3*a*b^
2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4]
```

giac [A] time = 0.45, size = 184, normalized size = 1.55

$$-\frac{\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{4}}+\frac{15\,b^{6}d^{3}x^{7}+63\,b^{6}cd^{2}x^{5}-21\,ab^{5}d^{3}x^{5}+105\,b^{6}c^{2}dx^{3}-105\,ab^{5}cd^{2}x^{3}+35\,a^{2}b^{4}d^{3}x^{3}+105\,b^{6}c^{3}x-315\,ab^{5}c^{2}dx+315\,a^{2}b^{4}cd^{2}x-105\,a^{3}b^{3}d^{3}x}{105\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^3*x^7 + 63*b^6*c*d^2*x^5 - 21*a*b^5*d
^3*x^5 + 105*b^6*c^2*d*x^3 - 105*a*b^5*c*d^2*x^3 + 35*a^2*b^4*d^3*x^3 + 105
*b^6*c^3*x - 315*a*b^5*c^2*d*x + 315*a^2*b^4*c*d^2*x - 105*a^3*b^3*d^3*x)/b
^7
```

maple [B] time = 0.00, size = 218, normalized size = 1.83

$$\frac{d^{3}x^{7}}{7b}-\frac{a\,d^{3}x^{5}}{5b^{2}}+\frac{3c\,d^{2}x^{5}}{5b}+\frac{a^{2}d^{3}x^{3}}{3b^{3}}-\frac{ac\,d^{2}x^{3}}{b^{2}}+\frac{c^{2}d\,x^{3}}{b}+\frac{a^{4}d^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{4}}-\frac{3a^{3}c\,d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{3}}+\frac{3a^{2}c^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{2}}-\frac{a\,c^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{a^{3}d^{3}x}{b^{4}}+\frac{3a^{2}c\,d^{2}x}{b^{3}}-\frac{3a\,c^{2}dx}{b^{2}}+\frac{c^{3}x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/5/b^2*x^5*a*d^3+3/5/b*x^5*c*d^2+1/3/b^3*x^3*a^2*d^3-1/b^2*x
^3*a*c*d^2+1/b*x^3*c^2*d-1/b^4*a^3*d^3*x+3/b^3*a^2*c*d^2*x-3/b^2*a*c^2*d*x+
1/b*c^3*x+a^4/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3-3*a^3/b^3/(a*b)
^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^2+3*a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)*c^2*d-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.35, size = 172, normalized size = 1.45

$$-\frac{\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{4}}+\frac{15\,b^{3}d^{3}x^{7}+21\,\left(3\,b^{3}cd^{2}-ab^{2}d^{3}\right)x^{5}+35\,\left(3\,b^{3}c^{2}d-3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{3}+105\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)x}{105\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3)*
```

```
x^5 + 35*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 105*(b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4
```

mupad [B] time = 0.10, size = 199, normalized size = 1.67

$$x^3\left(\frac{c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{3\,b}\right)-x^5\,\left(\frac{a\,d^3}{5\,b^2}-\frac{3\,c\,d^2}{5\,b}\right)+x\,\left(\frac{c^3}{b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)+\frac{d^3\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^3}{b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^3*((c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/(3*b)) - x^5*((a*d^3)/(5*b
^2) - (3*c*d^2)/(5*b)) + x*(c^3/b - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*
c*d^2)/b))/b))/b) + (d^3*x^7)/(7*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d
 - b*c)^3)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))*(a*d -
b*c)^3)/b^(9/2)
```

sympy [B] time = 0.67, size = 274, normalized size = 2.30

$$x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x^3\left(\frac{a^2d^3}{3b^3}-\frac{acd^2}{b^2}+\frac{c^2d}{b}\right)+x\left(-\frac{a^3d^3}{b^4}+\frac{3a^2cd^2}{b^3}-\frac{3ac^2d}{b^2}+\frac{c^3}{b}\right)-\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^7}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x**3*(a**2*d**3/(3*b**3) - a*c*d
**2/b**2 + c**2*d/b) + x*(-a**3*d**3/b**4 + 3*a**2*c*d**2/b**3 - 3*a*c**2*d
/b**2 + c**3/b) - sqrt(-a/b**9)*(a*d - b*c)**3*log(-b**4*sqrt(-a/b**9)*(a*d
 - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)
/2 + sqrt(-a/b**9)*(a*d - b*c)**3*log(b**4*sqrt(-a/b**9)*(a*d - b*c)**3/(a*
*3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**7
/(7*b)
```

3.222 $\int \frac{x\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=87

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}+\frac{dx^2(bc-ad)^2}{2b^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4b^2}+\frac{\left(c+dx^2\right)^3}{6b}$$

Rubi [**A**] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.100, Rules used = {444, 43}

$$\frac{dx^2(bc-ad)^2}{2b^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4b^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}+\frac{\left(c+dx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (d*(b*c - a*d)^2*x^2)/(2*b^3) + ((b*c - a*d)*(c + d*x^2)^2)/(4*b^2) + (c +
d*x^2)^3/(6*b) + ((b*c - a*d)^3*Log[a + b*x^2])/(2*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^3}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx, x, x^2\right)\\
&= \frac{d(bc-ad)^2x^2}{2b^3}+\frac{(bc-ad)\left(c+dx^2\right)^2}{4b^2}+\frac{\left(c+dx^2\right)^3}{6b}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 82, normalized size = 0.94

$$
\frac{bdx^2\left(6a^2d^2-3abd\left(6c+dx^2\right)+b^2\left(18c^2+9cdx^2+2d^2x^4\right)\right)+6(bc-ad)^3\log\left(a+bx^2\right)}{12b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (b*d*x^2*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x^2) + b^2*(18*c^2 + 9*c*d*x^2 + 2*d
^2*x^4)) + 6*(b*c - a*d)^3*Log[a + b*x^2])/(12*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(c+dx^2\right)^3}{a+bx^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^2)^3)/(a + b*x^2), x]
```

fricas [A] time = 0.66, size = 120, normalized size = 1.38

$$
\frac{2\,b^3d^3x^6+3\left(3\,b^3cd^2-ab^2d^3\right)x^4+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x^2+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{12\,b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^3*x^6 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 6*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(b*x^2 + a))/b^4
```

giac [A] time = 0.29, size = 124, normalized size = 1.43

$$\frac{2\,b^{2}d^{3}x^{6} + 9\,b^{2}cd^{2}x^{4} - 3\,abd^{3}x^{4} + 18\,b^{2}c^{2}dx^{2} - 18\,abcd^{2}x^{2} + 6\,a^{2}d^{3}x^{2}}{12\,b^{3}} + \frac{\left(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3}\right)\log\left(\left|bx^{2} + a\right|\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 9*b^2*c*d^2*x^4 - 3*a*b*d^3*x^4 + 18*b^2*c^2*d*x^2 -
18*a*b*c*d^2*x^2 + 6*a^2*d^3*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/b^4
```

maple [A] time = 0.00, size = 149, normalized size = 1.71

$$\frac{d^{3}x^{6}}{6b} - \frac{a\,d^{3}x^{4}}{4b^{2}} + \frac{3c\,d^{2}x^{4}}{4b} + \frac{a^{2}d^{3}x^{2}}{2b^{3}} - \frac{3ac\,d^{2}x^{2}}{2b^{2}} + \frac{3c^{2}d\,x^{2}}{2b} - \frac{a^{3}d^{3}\ln\left(b\,x^{2} + a\right)}{2b^{4}} + \frac{3a^{2}c\,d^{2}\ln\left(b\,x^{2} + a\right)}{2b^{3}} - \frac{3a\,c^{2}d\ln\left(b\,x^{2} + a\right)}{2b^{2}} + \frac{c^{3}\ln\left(b\,x^{2} + a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/6*d^3/b*x^6-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+1/2*d^3/b^3*x^2*a^2-3/2*d^2
/b^2*x^2*a*c+3/2*d/b*x^2*c^2-1/2/b^4*ln(b*x^2+a)*a^3*d^3+3/2/b^3*ln(b*x^2+a
)*a^2*c*d^2-3/2/b^2*ln(b*x^2+a)*a*c^2*d+1/2/b*ln(b*x^2+a)*c^3
```

maxima [A] time = 1.06, size = 119, normalized size = 1.37

$$\frac{2\,b^{2}d^{3}x^{6} + 3\left(3\,b^{2}cd^{2} - abd^{3}\right)x^{4} + 6\left(3\,b^{2}c^{2}d - 3\,abcd^{2} + a^{2}d^{3}\right)x^{2}}{12\,b^{3}} + \frac{\left(b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3}\right)\log\left(bx^{2} + a\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 6*(3*b^2*c^2*d - 3*a*
b*c*d^2 + a^2*d^3)*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(b*x^2 + a)/b^4
```

mupad [B] time = 0.06, size = 123, normalized size = 1.41

$$x^{2}\left(\frac{3\,c^{2}\,d}{2\,b} + \frac{a\left(\frac{a\,d^{3}}{b^{2}} - \frac{3\,c\,d^{2}}{b}\right)}{2\,b}\right) - x^{4}\left(\frac{a\,d^{3}}{4\,b^{2}} - \frac{3\,c\,d^{2}}{4\,b}\right) - \frac{\ln\left(b\,x^{2} + a\right)\left(a^{3}\,d^{3} - 3\,a^{2}\,b\,c\,d^{2} + 3\,a\,b^{2}\,c^{2}\,d - b^{3}\,c^{3}\right)}{2\,b^{4}} + \frac{d^{3}\,x^{6}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^2*((3*c^2*d)/(2*b) + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/(2*b)) - x^4*((a*d^3
)/(4*b^2) - (3*c*d^2)/(4*b)) - (log(a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))/(2*b^4) + (d^3*x^6)/(6*b)
```

sympy [A] time = 0.49, size = 94, normalized size = 1.08

$$x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x^2\left(\frac{a^2d^3}{2b^3}-\frac{3acd^2}{2b^2}+\frac{3c^2d}{2b}\right)+\frac{d^3x^6}{6b}-\frac{(ad-bc)^3\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x**2*(a**2*d**3/(2*b**3) - 3*a*c
*d**2/(2*b**2) + 3*c**2*d/(2*b)) + d**3*x**6/(6*b) - (a*d - b*c)**3*log(a +
 b*x**2)/(2*b**4)
```

3.223 $\int \frac{\left(c+dx^2\right)^3}{a+bx^2} \, dx$

Optimal. Leaf size=98

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {390, 205}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^4}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^2\right)}\right)\\ &= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{b^3}\\ &= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 0.94

$$\frac{dx\left(15a^2d^2-5abd\left(9c+dx^2\right)+3b^2\left(15c^2+5cdx^2+d^2x^4\right)\right)}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2), x]
```

fricas [A] time = 1.05, size = 292, normalized size = 2.98

$$\left[\frac{6\,ab^3d^3x^5+10\left(3\,ab^3cd^2-a^2b^2d^3\right)x^3+15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+30\left(3\,ab^3c^2d-3\,a^2b^2cd^2+a^3bd^3\right)x}{30\,ab^4},\ \frac{3\,ab^3d^3x^5+5\left(3\,ab^3cd^2-a^2b^2d^3\right)x^3+15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+15\left(3\,ab^3c^2d-3\,a^2b^2cd^2+a^3bd^3\right)x}{15\,ab^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

giac [A] time = 0.36, size = 129, normalized size = 1.32

$$\frac{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{3}} + \frac{3 \, b^{4} d^{3} x^{5} + 15 \, b^{4} c d^{2} x^{3} - 5 \, a b^{3} d^{3} x^{3} + 45 \, b^{4} c^{2} d x - 45 \, a b^{3} c d^{2} x + 15 \, a^{2} b^{2} d^{3} x}{15 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

maple [A] time = 0.00, size = 161, normalized size = 1.64

$$\frac{d^{3} x^{5}}{5 b}-\frac{a \,d^{3} x^{3}}{3 b^{2}}+\frac{c \,d^{2} x^{3}}{b}-\frac{a^{3} d^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{3}}+\frac{3 a^{2} c \,d^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{2}}-\frac{3 a \,c^{2} d \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b}+\frac{c^{3} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}}+\frac{a^{2} d^{3} x}{b^{3}}-\frac{3 a c \,d^{2} x}{b^{2}}+\frac{3 c^{2} d x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/5*d^3*x^5/b-1/3*d^3/b^2*x^3*a+d^2/b*x^3*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c*x+3
*d/b*c^2*x-1/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^3*d^3+3/b^2/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a^2*c*d^2-3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x)*a*c^2*d+1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.43, size = 122, normalized size = 1.24

$$\frac{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{3}} + \frac{3 \, b^{2} d^{3} x^{5} + 5 \, {\left(3 \, b^{2} c d^{2} - a b d^{3}\right)} x^{3} + 15 \, {\left(3 \, b^{2} c^{2} d - 3 \, a b c d^{2} + a^{2} d^{3}\right)} x}{15 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(
3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

mupad [**B**] time = 0.12, size = 146, normalized size = 1.49

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^3\,\left(\frac{a\,d^3}{3\,b^2}-\frac{c\,d^2}{b}\right)+\frac{d^3\,x^5}{5\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^3}{\sqrt{a}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
- (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(1/2
)*b^(7/2))
```

sympy [**B**] time = 0.60, size = 238, normalized size = 2.43

$$x^3\left(-\frac{ad^3}{3b^2}+\frac{cd^2}{b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3
) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
x)/2 + d**3*x**5/(5*b)
```

3.224 $\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=73

$$-\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2ab^3}+\frac{d^2x^2(3bc-ad)}{2b^2}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{d^2x^2(3bc-ad)}{2b^2}-\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2ab^3}+\frac{c^3\log(x)}{a}+\frac{d^3x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (d^2*(3*b*c - a*d)*x^2)/(2*b^2) + (d^3*x^4)/(4*b) + (c^3*Log[x])/a - ((b*c
- a*d)^3*Log[a + b*x^2])/(2*a*b^3)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^3}{x(a+bx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax}+\frac{d^3x}{b}+\frac{(-bc+ad)^3}{ab^2(a+bx)}\right) dx, x, x^2\right)\\ &= \frac{d^2(3bc-ad)x^2}{2b^2}+\frac{d^3x^4}{4b}+\frac{c^3\log(x)}{a}-\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2ab^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.89

$$\frac{abd^2x^2\left(-2ad+6bc+bdx^2\right)-2(bc-ad)^3\log\left(a+bx^2\right)+4b^3c^3\log(x)}{4ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] (a*b*d^2*x^2*(6*b*c - 2*a*d + b*d*x^2) + 4*b^3*c^3*Log[x] - 2*(b*c - a*d)^3
*Log[a + b*x^2])/(4*a*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x*(a + b*x^2)), x]
```

fricas [A] time = 0.51, size = 101, normalized size = 1.38

$$\frac{ab^2d^3x^4+4\,b^3c^3\log(x)+2\left(3\,ab^2cd^2-a^2bd^3\right)x^2-2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{4\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(a*b^2*d^3*x^4 + 4*b^3*c^3*log(x) + 2*(3*a*b^2*c*d^2 - a^2*b*d^3)*x^2 -
 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a))/(a*b
^3)
```

giac [A] time = 0.36, size = 99, normalized size = 1.36

$$\frac{c^3\log\left(x^2\right)}{2\,a}+\frac{bd^3x^4+6\,bcd^2x^2-2\,ad^3x^2}{4\,b^2}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\left|bx^2+a\right|\right)}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 6*b*c*d^2*x^2 - 2*a*d^3*x^2)/b^2 - 1/
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(
a*b^3)
```

maple [A] time = 0.01, size = 116, normalized size = 1.59

$$\frac{d^3x^4}{4b}-\frac{a\,d^3x^2}{2b^2}+\frac{3c\,d^2x^2}{2b}+\frac{a^2d^3\ln\left(b\,x^2+a\right)}{2b^3}-\frac{3ac\,d^2\ln\left(b\,x^2+a\right)}{2b^2}+\frac{c^3\ln(x)}{a}-\frac{c^3\ln\left(b\,x^2+a\right)}{2a}+\frac{3c^2d\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x/(b*x^2+a),x)
```

```
[Out] 1/4*d^3*x^4/b-1/2*d^3/b^2*a*x^2+3/2*d^2/b*c*x^2+1/2*a^2/b^3*ln(b*x^2+a)*d^3
-3/2*a/b^2*ln(b*x^2+a)*c*d^2+3/2/b*ln(b*x^2+a)*c^2*d-1/2/a*ln(b*x^2+a)*c^3+
c^3*ln(x)/a
```

maxima [A] time = 1.16, size = 98, normalized size = 1.34

$$\frac{c^3\log\left(x^2\right)}{2\,a}+\frac{bd^3x^4+2\left(3\,bcd^2-ad^3\right)x^2}{4\,b^2}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{2\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - a*d^3)*x^2)/b^2 - 1/2*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a*b^3)
```

mupad [B] time = 0.15, size = 97, normalized size = 1.33

$$\frac{d^3\,x^4}{4\,b}-x^2\,\left(\frac{a\,d^3}{2\,b^2}-\frac{3\,c\,d^2}{2\,b}\right)+\frac{c^3\,\ln(x)}{a}+\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x*(a + b*x^2)),x)
```

```
[Out] (d^3*x^4)/(4*b) - x^2*((a*d^3)/(2*b^2) - (3*c*d^2)/(2*b)) + (c^3*log(x))/a
+ (log(a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a
*b^3)
```

sympy [A] time = 1.77, size = 65, normalized size = 0.89

$$x^{2}\left(-\frac{ad^{3}}{2b^{2}}+\frac{3cd^{2}}{2b}\right)+\frac{d^{3}x^{4}}{4b}+\frac{c^{3}\log\left(x\right)}{a}+\frac{\left(ad-bc\right)^{3}\log\left(\frac{a}{b}+x^{2}\right)}{2ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x/(b*x**2+a),x)
```

```
[Out] x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + d**3*x**4/(4*b) + c**3*log(x)/a
+ (a*d - b*c)**3*log(a/b + x**2)/(2*a*b**3)
```

3.225 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=77

$$-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax^2}+\frac{d^3x^2}{b}+\frac{(-bc+ad)^3}{ab^2\left(a+bx^2\right)}\right)dx\\ &= -\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}-\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{ab^2}\\ &= -\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.99

$$\frac{(ad-bc)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) + ((-(b*c) + a*d
)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x^2*(a + b*x^2)), x]
```

fricas [A] time = 0.75, size = 253, normalized size = 3.29

$$\left[\frac{2\,a^2b^2d^3x^4-6\,ab^3c^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-ab}\,x\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(3\,a^2b^2cd^2-a^3bd^3\right)x^2}{6\,a^2b^3x},\ \frac{a^2b^2d^3x^4-3\,ab^3c^3-3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(3\,a^2b^2cd^2-a^3bd^3\right)x^2}{3\,a^2b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*d^3*x^4 - 6*a*b^3*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
```

```
)) + 6*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*d^3*x^4
 - 3*a*b^3*c^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b
^3*x)]
```

giac [A] time = 0.36, size = 104, normalized size = 1.35

$$-\frac{c^3}{ax}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}+\frac{b^2d^3x^3+9\,b^2cd^2x-3\,abd^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c^3/(a*x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/3*(b^2*d^3*x^3 + 9*b^2*c*d^2*x - 3*a*b*d^
3*x)/b^3
```

maple [A] time = 0.01, size = 135, normalized size = 1.75

$$\frac{d^3x^3}{3b}+\frac{a^2d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{3ac\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{b\,c^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{3c^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{a\,d^3x}{b^2}+\frac{3c\,d^2x}{b}-\frac{c^3}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^3*x^3/b-d^3/b^2*a*x+3*d^2/b*c*x+a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x)*d^3-3*a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^2+3/(a*b)^(1/2
)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d-1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*c^3-c^3/a/x
```

maxima [A] time = 2.37, size = 101, normalized size = 1.31

$$-\frac{c^3}{ax}+\frac{bd^3x^3+3\left(3\,bcd^2-ad^3\right)x}{3\,b^2}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -c^3/(a*x) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - a*d^3)*x)/b^2 - (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^
2)
```

mupad [B] time = 0.07, size = 118, normalized size = 1.53

$$\frac{d^3\,x^3}{3\,b} - \frac{c^3}{a\,x} - x\,\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,(a\,d-b\,c)^3}{a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^2*(a + b*x^2)),x)
```

```
[Out] (d^3*x^3)/(3*b) - c^3/(a*x) - x*((a*d^3)/b^2 - (3*c*d^2)/b) + (atan((b^(1/2
)*x*(a*d - b*c)^3)/(a^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2)))*(a*d - b*c)^3)/(a^(3/2)*b^(5/2))
```

sympy [B] time = 0.81, size = 221, normalized size = 2.87

$$x\left(-\frac{ad^3}{b^2}+\frac{3cd^2}{b}\right) - \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{d^3x^3}{3b} - \frac{c^3}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**3/b**2 + 3*c*d**2/b) - sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*log(-a*
*2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 +
3*a*b**2*c**2*d - b**3*c**3) + x)/2 + sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*l
og(a**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d*
*2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**3/(3*b) - c**3/(a*x)
```

3.226 $\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=73

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}-\frac{c^2\log(x)(bc-3ad)}{a^2}-\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] -c^3/(2*a*x^2) + (d^3*x^2)/(2*b) - (c^2*(b*c - 3*a*d)*Log[x])/a^2 + ((b*c -
 a*d)^3*Log[a + b*x^2])/(2*a^2*b^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^3}{b}+\frac{c^3}{ax^2}+\frac{c^2(-bc+3ad)}{a^2x}-\frac{(-bc+ad)^3}{a^2b(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}-\frac{c^2(bc-3ad)\log(x)}{a^2}+\frac{(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 75, normalized size = 1.03

$$\frac{-2b^2c^2x^2\log(x)(bc-3ad)+ab\left(ad^3x^4-bc^3\right)+x^2(bc-ad)^3\log\left(a+bx^2\right)}{2a^2b^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] (a*b*(-(b*c^3) + a*d^3*x^4) - 2*b^2*c^2*(b*c - 3*a*d)*x^2*Log[x] + (b*c - a
*d)^3*x^2*Log[a + b*x^2])/(2*a^2*b^2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x^3*(a + b*x^2)), x]
```

fricas [A] time = 0.92, size = 105, normalized size = 1.44

$$\frac{a^2bd^3x^4-ab^2c^3+\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^2\log\left(bx^2+a\right)-2\left(b^3c^3-3\,ab^2c^2d\right)x^2\log(x)}{2\,a^2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*d^3*x^4 - a*b^2*c^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*x^2*log(b*x^2 + a) - 2*(b^3*c^3 - 3*a*b^2*c^2*d)*x^2*log(x))/(a^2
*b^2*x^2)
```

giac [A] time = 0.31, size = 120, normalized size = 1.64

$$\frac{d^3x^2}{2\,b} - \frac{\left(bc^3 - 3\,ac^2d\right)\log\left(x^2\right)}{2\,a^2} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(\left|bx^2 + a\right|\right)}{2\,a^2b^2} + \frac{bc^3x^2 - 3\,ac^2dx^2 - ac^3}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^2) + 1/2*(b*
c^3*x^2 - 3*a*c^2*d*x^2 - a*c^3)/(a^2*x^2)
```

maple [A] time = 0.01, size = 114, normalized size = 1.56

$$\frac{d^3x^2}{2b} - \frac{a\,d^3\ln\left(b\,x^2 + a\right)}{2b^2} + \frac{3c^2d\ln(x)}{a} - \frac{3c^2d\ln\left(b\,x^2 + a\right)}{2a} - \frac{b\,c^3\ln(x)}{a^2} + \frac{b\,c^3\ln\left(b\,x^2 + a\right)}{2a^2} + \frac{3c\,d^2\ln\left(b\,x^2 + a\right)}{2b} - \frac{c^3}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^3/(b*x^2+a),x)
```

```
[Out] 1/2*d^3*x^2/b-1/2*a/b^2*ln(b*x^2+a)*d^3+3/2/b*ln(b*x^2+a)*c*d^2-3/2/a*ln(b*
x^2+a)*c^2*d+1/2/a^2*b*ln(b*x^2+a)*c^3-1/2*c^3/a/x^2+3*c^2/a*ln(x)*d-c^3/a^
2*ln(x)*b
```

maxima [A] time = 1.08, size = 97, normalized size = 1.33

$$\frac{d^3x^2}{2\,b} - \frac{c^3}{2\,ax^2} - \frac{\left(bc^3 - 3\,ac^2d\right)\log\left(x^2\right)}{2\,a^2} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\log\left(bx^2 + a\right)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b - 1/2*c^3/(a*x^2) - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/
2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a^2*b
^2)
```

mupad [B] time = 0.17, size = 95, normalized size = 1.30

$$\frac{d^3\,x^2}{2\,b} - \frac{c^3}{2\,a\,x^2} - \frac{\ln(x)\,\left(b\,c^3 - 3\,a\,c^2\,d\right)}{a^2} - \frac{\ln\left(b\,x^2 + a\right)\,\left(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3\right)}{2\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^3*(a + b*x^2)),x)
```

```
[Out] (d^3*x^2)/(2*b) - c^3/(2*a*x^2) - (log(x)*(b*c^3 - 3*a*c^2*d))/a^2 - (log(a
 + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2*b^2)
```

sympy [A] time = 2.21, size = 63, normalized size = 0.86

$$\frac{d^3x^2}{2b} - \frac{c^3}{2ax^2} + \frac{c^2 (3ad - bc) \log(x)}{a^2} - \frac{(ad - bc)^3 \log\left(\frac{a}{b} + x^2\right)}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**3/(b*x**2+a),x)
```

```
[Out] d**3*x**2/(2*b) - c**3/(2*a*x**2) + c**2*(3*a*d - b*c)*log(x)/a**2 - (a*d -
 b*c)**3*log(a/b + x**2)/(2*a**2*b**2)
```

3.227 $\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=74

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {461, 205}

$$\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -c^3/(3*a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)}\,dx &= \int \left(\frac{d^3}{b}+\frac{c^3}{ax^4}+\frac{c^2(-bc+3ad)}{a^2x^2}-\frac{(-bc+ad)^3}{a^2b\left(a+bx^2\right)}\right)dx\\
&= -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\int \frac{1}{a+bx^2}\,dx}{a^2b}\\
&= -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.00

$$
\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c^3/(a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x^4*(a + b*x^2)), x]
```

fricas [A] time = 0.59, size = 256, normalized size = 3.46

$$
\left[\frac{6\,a^3bd^3x^4-2\,a^2b^2c^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-ab}\,x^3\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(ab^3c^3-3\,a^2b^2c^2d\right)x^2}{6\,a^3b^2x^3},\ \frac{3\,a^3bd^3x^4-a^2b^2c^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}\,x^3\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(ab^3c^3-3\,a^2b^2c^2d\right)x^2}{3\,a^3b^2x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*d^3*x^4 - 2*a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x^3*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
```

```
 a)) + 6*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*d^3
*x^4 - a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/
(a^3*b^2*x^3)]
```

giac [A] time = 0.37, size = 100, normalized size = 1.35

$$\frac{d^3x}{b} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b} + \frac{3\,bc^3x^2 - 9\,ac^2dx^2 - ac^3}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2*b) + 1/3*(3*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^3)/(a^2
*x^3)
```

maple [B] time = 0.01, size = 135, normalized size = 1.82

$$-\frac{a\,d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} - \frac{3b\,c^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a} + \frac{b^2c^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2} + \frac{3c\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d^3x}{b} - \frac{3c^2d}{ax} + \frac{b\,c^3}{a^2x} - \frac{c^3}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^4/(b*x^2+a),x)
```

```
[Out] d^3*x/b-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3+3/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x)*c*d^2-3/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d+
1/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3-1/3*c^3/a/x^3-3*c^2/a/x
*d+c^3/a^2/x*b
```

maxima [A] time = 2.44, size = 98, normalized size = 1.32

$$\frac{d^3x}{b} + \frac{\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b} - \frac{ac^3 - 3\left(bc^3 - 3\,ac^2d\right)x^2}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^2*b) - 1/3*(a*c^3 - 3*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x
^3)
```

mupad [B] time = 0.14, size = 122, normalized size = 1.65

$$\frac{d^3\,x}{b}-\frac{\frac{b\,c^3}{3\,a}+\frac{b\,c^2\,x^2\,(3\,a\,d-b\,c)}{a^2}}{b\,x^3}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,(a\,d-b\,c)^3}{a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^4*(a + b*x^2)),x)
```

```
[Out] (d^3*x)/b - ((b*c^3)/(3*a) + (b*c^2*x^2*(3*a*d - b*c))/a^2)/(b*x^3) - (atan
((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(5/2)*b^(3/2))
```

sympy [B] time = 1.13, size = 221, normalized size = 2.99

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x}{b}+\frac{-ac^3+x^2\left(-9ac^2d+3bc^3\right)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(-a**3*b*sqrt(-1/(a**5*b**3))*(a*d -
 b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2
 - sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(a**3*b*sqrt(-1/(a**5*b**3))*(a*d
 - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)
/2 + d**3*x/b + (-a*c**3 + x**2*(-9*a*c**2*d + 3*b*c**3))/(3*a**2*x**3)
```

3.228 $\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2\log\left(a+bx^2\right)}{2b^2(bc-ad)}-\frac{c^2\log\left(c+dx^2\right)}{2d^2(bc-ad)}+\frac{x^2}{2bd}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2\log\left(a+bx^2\right)}{2b^2(bc-ad)}-\frac{c^2\log\left(c+dx^2\right)}{2d^2(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x^2/(2*b*d) + (a^2*Log[a + b*x^2])/(2*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^2
])/(2*d^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx&=\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^2\right)\\&=\frac{x^2}{2bd}+\frac{a^2\log\left(a+bx^2\right)}{2b^2(bc-ad)}-\frac{c^2\log\left(c+dx^2\right)}{2d^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^2\right)-b\left(dx^2(ad-bc)+bc^2\log\left(c+dx^2\right)\right)}{2b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^2] - b*(d*(-(b*c) + a*d)*x^2 + b*c^2*Log[c + d*x^2]))/
(2*b^2*d^2*(b*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.69, size = 72, normalized size = 1.03

$$\frac{a^2d^2\log\left(bx^2+a\right)-b^2c^2\log\left(dx^2+c\right)+\left(b^2cd-abd^2\right)x^2}{2\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*log(b*x^2 + a) - b^2*c^2*log(d*x^2 + c) + (b^2*c*d - a*b*d^2)*
x^2)/(b^3*c*d^2 - a*b^2*d^3)
```

giac [A] time = 0.37, size = 70, normalized size = 1.00

$$\frac{a^2\log\left(\left|bx^2+a\right|\right)}{2\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(\left|dx^2+c\right|\right)}{2\left(bcd^2-ad^3\right)}+\frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*log(abs(b*x^2 + a))/(b^3*c - a*b^2*d) - 1/2*c^2*log(abs(d*x^2 + c))
/(b*c*d^2 - a*d^3) + 1/2*x^2/(b*d)
```

maple [A] time = 0.01, size = 65, normalized size = 0.93

$$-\frac{a^2 \ln\left(b x^2 + a\right)}{2 (ad - bc) b^2} + \frac{c^2 \ln\left(d x^2 + c\right)}{2 (ad - bc) d^2} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*x^2/b/d-1/2*a^2/b^2/(a*d-b*c)*ln(b*x^2+a)+1/2*c^2/d^2/(a*d-b*c)*ln(d*x^
2+c)
```

maxima [A] time = 1.00, size = 68, normalized size = 0.97

$$\frac{a^2 \log\left(bx^2 + a\right)}{2 \left(b^3 c - ab^2 d\right)} - \frac{c^2 \log\left(dx^2 + c\right)}{2 \left(bcd^2 - ad^3\right)} + \frac{x^2}{2\, bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*log(b*x^2 + a)/(b^3*c - a*b^2*d) - 1/2*c^2*log(d*x^2 + c)/(b*c*d^2
- a*d^3) + 1/2*x^2/(b*d)
```

mupad [B] time = 0.32, size = 68, normalized size = 0.97

$$\frac{a^2 \ln\left(b\, x^2 + a\right)}{2\, b^3\, c - 2\, a\, b^2\, d} + \frac{c^2 \ln\left(d\, x^2 + c\right)}{2\, a\, d^3 - 2\, b\, c\, d^2} + \frac{x^2}{2\, b\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (a^2*log(a + b*x^2))/(2*b^3*c - 2*a*b^2*d) + (c^2*log(c + d*x^2))/(2*a*d^3
- 2*b*c*d^2) + x^2/(2*b*d)
```

sympy [B] time = 179.21, size = 201, normalized size = 2.87

$$-\frac{a^2 \log\left(x^2 + \frac{\frac{a^4 d^3}{b(ad-bc)} - \frac{2a^3 cd^2}{ad-bc} + \frac{a^2 bc^2 d}{ad-bc} + a^2 cd + abc^2}{a^2 d^2 + b^2 c^2}\right)}{2b^2 (ad - bc)} + \frac{c^2 \log\left(x^2 + \frac{-\frac{a^2 bc^2 d}{ad-bc} + a^2 cd + \frac{2ab^2 c^3}{ad-bc} + abc^2 - \frac{b^3 c^4}{d(ad-bc)}}{a^2 d^2 + b^2 c^2}\right)}{2d^2 (ad - bc)} + \frac{x^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -a**2*log(x**2 + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a
**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(2
*b**2*(a*d - b*c)) + c**2*log(x**2 + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d
 + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*
d**2 + b**2*c**2))/(2*d**2*(a*d - b*c)) + x**2/(2*b*d)
```

3.229 $\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=78

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {479, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] x/(b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*(b*c - a*d)) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(d^(3/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{x}{bd}-\frac{\int \frac{ac+(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{bd}\\
&= \frac{x}{bd}+\frac{a^2\int \frac{1}{a+bx^2}\,dx}{b(bc-ad)}-\frac{c^2\int \frac{1}{c+dx^2}\,dx}{d(bc-ad)}\\
&= \frac{x}{bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 74, normalized size = 0.95

$$\frac{\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{ax}{b}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{3/2}}+\frac{cx}{d}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((a*x)/b) + (c*x)/d + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(3/2))/(b*c - a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.95, size = 391, normalized size = 5.01

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\,(bc-ad)x}{2\left(b^2cd-abd^2\right)},\ \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+2\,(bc-ad)x}{2\left(b^2cd-abd^2\right)},\ -\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,(bc-ad)x}{2\left(b^2cd-abd^2\right)},\ \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+(bc-ad)x}{b^2cd-abd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + b*c
*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c - a*d)
*x)/(b^2*c*d - a*b*d^2), 1/2*(2*a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b*c
*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 2*(b*c - a*d)
*x)/(b^2*c*d - a*b*d^2), -1/2*(2*b*c*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + a*
d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b*c - a*d
)*x)/(b^2*c*d - a*b*d^2), (a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b*c*sqrt
(c/d)*arctan(d*x*sqrt(c/d)/c) + (b*c - a*d)*x)/(b^2*c*d - a*b*d^2)]
```

giac [A] time = 0.38, size = 72, normalized size = 0.92

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c-abd\right)\sqrt{ab}}-\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bcd-ad^2\right)\sqrt{cd}}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - c^2*arctan(d*x/sqrt
(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + x/(b*d)
```

maple [A] time = 0.01, size = 73, normalized size = 0.94

$$-\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}\,b}+\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}\,d}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] x/b/d-1/b*a^2/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+1/d*c^2/(a*d-
b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)
```

maxima [A] time = 2.35, size = 72, normalized size = 0.92

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c-abd\right)\sqrt{ab}}-\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bcd-ad^2\right)\sqrt{cd}}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - c^2*arctan(d*x/sqrt
(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + x/(b*d)
```

mupad [B] time = 0.61, size = 343, normalized size = 4.40

$$\frac{\ln\left(a^5\,b^4\,d^3-a^2\,b^7\,c^3+d^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+b^6\,c^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,b^4\,c-2\,a\,b^3\,d}-\frac{\ln\left(a^2\,b^7\,c^3-a^5\,b^4\,d^3+d^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+b^6\,c^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,\left(b^4\,c-a\,b^3\,d\right)}+\frac{x}{b\,d}-\frac{\ln\left(a^3\,c^2\,d^7-b^3\,c^5\,d^4+b^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+a^3\,d^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,\left(a\,d^4-b\,c\,d^3\right)}+\frac{\ln\left(b^3\,c^5\,d^4-a^3\,c^2\,d^7+b^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+a^3\,d^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,a\,d^4-2\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^5*b^4*d^3 - a^2*b^7*c^3 + d^3*x*(-a^3*b^3)^(3/2) + b^6*c^3*x*(-a^3*b
^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*b^4*c - 2*a*b^3*d) - (log(a^2*b^7*c^3 - a^5
*b^4*d^3 + d^3*x*(-a^3*b^3)^(3/2) + b^6*c^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^
(1/2))/(2*(b^4*c - a*b^3*d)) + x/(b*d) - (log(a^3*c^2*d^7 - b^3*c^5*d^4 + b
^3*x*(-c^3*d^3)^(3/2) + a^3*d^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(2*(a
*d^4 - b*c*d^3)) + (log(b^3*c^5*d^4 - a^3*c^2*d^7 + b^3*x*(-c^3*d^3)^(3/2)
+ a^3*d^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(2*a*d^4 - 2*b*c*d^3)
```

sympy [B] time = 12.39, size = 921, normalized size = 11.81

$$-\frac{\sqrt{-\frac{a^{3}}{b^{3}}}\log{\left(x+\frac{-\frac{a^{4}d^{4}\sqrt{-\frac{a^{3}}{b^{3}}}}{ad-bc}-\frac{a^{3}b^{3}d^{6}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}b^{4}cd^{5}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{ab^{5}c^{2}d^{4}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{6}c^{3}d^{3}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{4}c^{4}\sqrt{-\frac{a^{3}}{b^{3}}}}{ad-bc}}{a^{3}cd^{2}+a^{2}bc^{2}d+ab^{2}c^{3}}\right)}}{2\left(ad-bc\right)}+\frac{\sqrt{-\frac{a^{3}}{b^{3}}}\log{\left(x+\frac{\frac{a^{4}d^{4}\sqrt{-\frac{a^{3}}{b^{3}}}}{ad-bc}+\frac{a^{3}b^{3}d^{6}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{2}b^{4}cd^{5}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{ab^{5}c^{2}d^{4}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{6}c^{3}d^{3}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{4}c^{4}\sqrt{-\frac{a^{3}}{b^{3}}}}{ad-bc}}{a^{3}cd^{2}+a^{2}bc^{2}d+ab^{2}c^{3}}\right)}}{2\left(ad-bc\right)}-\frac{\sqrt{-\frac{c^{3}}{d^{3}}}\log{\left(x+\frac{-\frac{a^{4}d^{4}\sqrt{-\frac{c^{3}}{d^{3}}}}{ad-bc}-\frac{a^{3}b^{3}d^{6}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}b^{4}cd^{5}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{ab^{5}c^{2}d^{4}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{6}c^{3}d^{3}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{4}c^{4}\sqrt{-\frac{c^{3}}{d^{3}}}}{ad-bc}}{a^{3}cd^{2}+a^{2}bc^{2}d+ab^{2}c^{3}}\right)}}{2\left(ad-bc\right)}+\frac{\sqrt{-\frac{c^{3}}{d^{3}}}\log{\left(x+\frac{\frac{a^{4}d^{4}\sqrt{-\frac{c^{3}}{d^{3}}}}{ad-bc}+\frac{a^{3}b^{3}d^{6}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{2}b^{4}cd^{5}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{ab^{5}c^{2}d^{4}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{6}c^{3}d^{3}\left(-\frac{c^{3}}{d^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{4}c^{4}\sqrt{-\frac{c^{3}}{d^{3}}}}{ad-bc}}{a^{3}cd^{2}+a^{2}bc^{2}d+ab^{2}c^{3}}\right)}}{2\left(ad-bc\right)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**3*b
**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b**3)
**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c)**
3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-a**
3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2*(a*d -
 b*c)) + sqrt(-a**3/b**3)*log(x + (a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) +
 a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5*(-a**
3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d -
 b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c**4*sq
rt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2
*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x + (-a**4*d**4*sqrt(-c**3/d**3)/(a*d
- b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d*
*5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d**3)**(3/2
)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - b**4
*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c
**3))/(2*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x + (a**4*d**4*sqrt(-c**3/d**3
)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a**2*b*
*4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-c**3/d**3
)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)**3
 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a
*b**2*c**3))/(2*(a*d - b*c)) + x/(b*d)
```

3.230 $\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=53

$$\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(a*Log[a + b*x^2])/(2*b*(b*c - a*d)) + (c*Log[c + d*x^2])/(2*d*(b*c - a*d)
)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^2\right)\\
&= -\frac{a\log\left(a+bx^2\right)}{2b(bc-ad)}+\frac{c\log\left(c+dx^2\right)}{2d(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^2\right)-bc\log\left(c+dx^2\right)}{2b^2cd-2abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((a*d*Log[a + b*x^2] - b*c*Log[c + d*x^2])/(2*b^2*c*d - 2*a*b*d^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.78, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^2+a\right)-bc\log\left(dx^2+c\right)}{2\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d*log(b*x^2 + a) - b*c*log(d*x^2 + c))/(b^2*c*d - a*b*d^2)
```

giac [A] time = 0.30, size = 51, normalized size = 0.96

$$-\frac{a\log\left(\left|bx^2+a\right|\right)}{2\left(b^2c-abd\right)}+\frac{c\log\left(\left|dx^2+c\right|\right)}{2\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*a*log(abs(b*x^2 + a))/(b^2*c - a*b*d) + 1/2*c*log(abs(d*x^2 + c))/(b*c
*d - a*d^2)
```

maple [A] time = 0.01, size = 50, normalized size = 0.94

$$\frac{a \ln \left(b \,x^{2}+a \right)}{2 \left(a d -b c \right) b}-\frac{c \ln \left(d \,x^{2}+c \right)}{2 \left(a d -b c \right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*a/(a*d-b*c)/b*ln(b*x^2+a)-1/2*c/(a*d-b*c)/d*ln(d*x^2+c)
```

maxima [A] time = 1.03, size = 49, normalized size = 0.92

$$-\frac{a \log \left(b x^{2}+a\right)}{2 \left(b^{2} c-a b d\right)}+\frac{c \log \left(d x^{2}+c\right)}{2 \left(b c d-a d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c - a*b*d) + 1/2*c*log(d*x^2 + c)/(b*c*d - a*d^2
)
```

mupad [B] time = 0.27, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^2+a\right)}{2\,b^2\,c-2\,a\,b\,d}-\frac{c\,\ln\left(d\,x^2+c\right)}{2\,a\,d^2-2\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - (a*log(a + b*x^2))/(2*b^2*c - 2*a*b*d) - (c*log(c + d*x^2))/(2*a*d^2 - 2*
b*c*d)
```

sympy [B] time = 2.34, size = 144, normalized size = 2.72

$$\frac{a \log{\left(x^{2} + \frac{\frac{a^{3} d^{2}}{b \left(a d - b c\right)} - \frac{2 a^{2} c d}{a d - b c} + \frac{a b c^{2}}{a d - b c} + 2 a c}{a d + b c} \right)}}{2 b \left(a d - b c\right)} - \frac{c \log{\left(x^{2} + \frac{- \frac{a^{2} c d}{a d - b c} + \frac{2 a b c^{2}}{a d - b c} + 2 a c - \frac{b^{2} c^{3}}{d \left(a d - b c\right)}}{a d + b c} \right)}}{2 d \left(a d - b c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] a*log(x**2 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(2*b*(a*d - b*c)) - c*log(x**2 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(2*d*(a*d - b*c))
```

3.231 $\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {481, 205}

$$\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[c]*A
rcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{a\int \frac{1}{a+bx^2}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)}+\frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/Sqrt[d])/(b*c - a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.84, size = 309, normalized size = 4.41

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2(bc-ad)},-\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2(bc-ad)},\frac{2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{2(bc-ad)},-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + sqrt(-c
/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(a/b)*arctan(b*x*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d)
 - c)/(d*x^2 + c)))/(b*c - a*d), 1/2*(2*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) -
 sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b*c - a*d), -
(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x*sqrt(c/d)/c))/(b*
c - a*d)]
```

giac [A] time = 0.34, size = 54, normalized size = 0.77

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + c*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

maple [A] time = 0.01, size = 55, normalized size = 0.79

$$\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}-\frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] a/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-c/(a*d-b*c)/(c*d)^(1/2)*a
rctan(1/(c*d)^(1/2)*d*x)
```

maxima [A] time = 2.31, size = 54, normalized size = 0.77

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + c*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

mupad [B] time = 0.34, size = 133, normalized size = 1.90

$$\frac{\ln\left(a+x\,\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,b^2\,c-2\,a\,b\,d}-\frac{\ln\left(a-x\,\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(b^2\,c-a\,b\,d\right)}-\frac{\ln\left(c-x\,\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(a\,d^2-b\,c\,d\right)}+\frac{\ln\left(c+x\,\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,a\,d^2-2\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a + x*(-a*b)^(1/2))*(-a*b)^(1/2))/(2*b^2*c - 2*a*b*d) - (log(a - x*(-a
*b)^(1/2))*(-a*b)^(1/2))/(2*(b^2*c - a*b*d)) - (log(c - x*(-c*d)^(1/2))*(-c
*d)^(1/2))/(2*(a*d^2 - b*c*d)) + (log(c + x*(-c*d)^(1/2))*(-c*d)^(1/2))/(2*
a*d^2 - 2*b*c*d)
```

sympy [B] time = 2.18, size = 570, normalized size = 8.14

$$\frac{\sqrt{-\frac{a}{b}}\log\left(-\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2\left(ad-bc\right)}-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2\left(ad-bc\right)}+\frac{\sqrt{-\frac{c}{d}}\log\left(-\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2\left(ad-bc\right)}-\frac{\sqrt{-\frac{c}{d}}\log\left(\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-a/b)*log(-2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 + 4*a*b**2*c*d**
2*(-a/b)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-a/b)/(a*d - b*c) - 2*b**3*c**2*d
*(-a/b)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a*d - b
*c)) - sqrt(-a/b)*log(2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 - 4*a*b**2
*c*d**2*(-a/b)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-a/b)/(a*d - b*c) + 2*b**3*
c**2*d*(-a/b)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a
*d - b*c)) + sqrt(-c/d)*log(-2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)**3 + 4
*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-c/d)/(a*d - b*c) -
2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-c/d)/(a*d - b*c) + x
)/(2*(a*d - b*c)) - sqrt(-c/d)*log(2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)*
*3 - 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-c/d)/(a*d - b
*c) + 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-c/d)/(a*d - b*
c) + x)/(2*(a*d - b*c))
```

3.232 $\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=45

$$\frac{\log\left(a+bx^2\right)}{2(bc-ad)}-\frac{\log\left(c+dx^2\right)}{2(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {444, 36, 31}

$$\frac{\log\left(a+bx^2\right)}{2(bc-ad)}-\frac{\log\left(c+dx^2\right)}{2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] Log[a + b*x^2]/(2*(b*c - a*d)) - Log[c + d*x^2]/(2*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx, x, x^2\right)\\ &= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+dx}\,dx, x, x^2\right)}{2(bc-ad)}\\ &= \frac{\log\left(a+bx^2\right)}{2(bc-ad)} - \frac{\log\left(c+dx^2\right)}{2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log\left(a+bx^2\right)-\log\left(c+dx^2\right)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)),x]
[Out] (Log[a + b*x^2] - Log[c + d*x^2])/(2*b*c - 2*a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)),x]
[Out] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.96, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^2+a\right)-\log\left(dx^2+c\right)}{2\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
[Out] 1/2*(log(b*x^2 + a) - log(d*x^2 + c))/(b*c - a*d)
```

giac [A] time = 0.37, size = 51, normalized size = 1.13

$$\frac{b\log\left(\left|bx^2+a\right|\right)}{2\left(b^2c-abd\right)}-\frac{d\log\left(\left|dx^2+c\right|\right)}{2\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*log(abs(b*x^2 + a))/(b^2*c - a*b*d) - 1/2*d*log(abs(d*x^2 + c))/(b*c*
d - a*d^2)
```

maple [A] time = 0.01, size = 42, normalized size = 0.93

$$-\frac{\ln\left(b\,x^2+a\right)}{2\,(ad-bc)}+\frac{\ln\left(d\,x^2+c\right)}{2ad-2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2/(a*d-b*c)*ln(b*x^2+a)+1/2/(a*d-b*c)*ln(d*x^2+c)
```

maxima [A] time = 1.07, size = 41, normalized size = 0.91

$$\frac{\log\left(bx^2+a\right)}{2\,(bc-ad)}-\frac{\log\left(dx^2+c\right)}{2\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/(b*c - a*d) - 1/2*log(d*x^2 + c)/(b*c - a*d)
```

mupad [B] time = 0.20, size = 148, normalized size = 3.29

$$\frac{2\,\mathrm{atanh}\left(\frac{8\,b^2\,d^2\,x^2}{(2\,a\,d-2\,b\,c)\left(\frac{32\,a\,b^2\,c\,d^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}+\frac{16\,a\,b^2\,d^3\,x^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}+\frac{16\,b^3\,c\,d^2\,x^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}\right)}\right)}{2\,a\,d-2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (2*atanh((8*b^2*d^2*x^2)/((2*a*d - 2*b*c)*((32*a*b^2*c*d^2)/(4*a^2*d^2 + 4*
b^2*c^2 - 8*a*b*c*d) + (16*a*b^2*d^3*x^2)/(4*a^2*d^2 + 4*b^2*c^2 - 8*a*b*c*
```

```
d) + (16*b^3*c*d^2*x^2)/(4*a^2*d^2 + 4*b^2*c^2 - 8*a*b*c*d)))))/(2*a*d - 2*
b*c)
```

sympy [B] time = 0.98, size = 138, normalized size = 3.07

$$\frac{\log\left(x^2 + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{2\left(ad - bc\right)} - \frac{\log\left(x^2 + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{2\left(ad - bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] log(x**2 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(2*(a*d - b*c)) - log(x**2 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(2*(a*d - b*c))
```

3.233 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx &= \frac{b\int \frac{1}{a+bx^2}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 0.89, size = 292, normalized size = 4.17

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\ -\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)},\ \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\ \frac{\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{bc-ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

giac [A] time = 0.39, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

maple [A] time = 0.01, size = 55, normalized size = 0.79

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -b/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+d/(a*d-b*c)/(c*d)^(1/2)*
arctan(1/(c*d)^(1/2)*d*x)
```

maxima [A] time = 2.31, size = 54, normalized size = 0.77

$$\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

mupad [B] time = 0.34, size = 135, normalized size = 1.93

$$\frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c}-\frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)}-\frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)}+\frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-c
*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(-a
*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/(2*
b*c^2 - 2*a*c*d)
```

sympy [B] time = 2.82, size = 712, normalized size = 10.17

$$\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{-\frac{a^{4}cd^{3}\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{3}bc^{2}d^{2}\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}b^{2}c^{3}d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{2}d^{2}\sqrt{-\frac{b}{a}}}{ad-bc}-\frac{ab^{3}c^{4}\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{2}c^{2}\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{\frac{a^{4}cd^{3}\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{3}bc^{2}d^{2}\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{2}b^{2}c^{3}d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}d^{2}\sqrt{-\frac{b}{a}}}{ad-bc}+\frac{ab^{3}c^{4}\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{2}c^{2}\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}+\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{-\frac{a^{4}cd^{3}\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{3}bc^{2}d^{2}\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}b^{2}c^{3}d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{2}d^{2}\sqrt{-\frac{d}{c}}}{ad-bc}-\frac{ab^{3}c^{4}\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{2}c^{2}\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}-\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{\frac{a^{4}cd^{3}\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{3}bc^{2}d^{2}\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{a^{2}b^{2}c^{3}d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}d^{2}\sqrt{-\frac{d}{c}}}{ad-bc}+\frac{ab^{3}c^{4}\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{b^{2}c^{2}\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2
*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d -
b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d
 - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqr
t(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**
2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)
**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b
*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d
/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(
-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3
 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)
**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)
*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a
**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3
+ b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

3.234 $\int \frac{1}{x(a+bx^2)(c+dx^2)} \, dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^2])/(2*a*(b*c - a*d)) + (d*Log[c + d*x^2])/(2
*c*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^2\right)\\ &= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^2\right)}{2a(bc-ad)}+\frac{d\log\left(c+dx^2\right)}{2c(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^2\right)+ad\log\left(c+dx^2\right)-2ad\log(x)+2bc\log(x)}{2abc^2-2a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*b*c*Log[x] - 2*a*d*Log[x] - b*c*Log[a + b*x^2] + a*d*Log[c + d*x^2])/(2*
a*b*c^2 - 2*a^2*c*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 1.00, size = 54, normalized size = 0.87

$$-\frac{bc\log\left(bx^2+a\right)-ad\log\left(dx^2+c\right)-2\,(bc-ad)\log(x)}{2\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c*log(b*x^2 + a) - a*d*log(d*x^2 + c) - 2*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

giac [A] time = 0.31, size = 73, normalized size = 1.18

$$-\frac{b^2\log\left(\left|bx^2+a\right|\right)}{2\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^2+c\right|\right)}{2\left(bc^2d-acd^2\right)}+\frac{\log\left(x^2\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log(abs(b*x^2 + a))/(a*b^2*c - a^2*b*d) + 1/2*d^2*log(abs(d*x^2 +
c))/(b*c^2*d - a*c*d^2) + 1/2*log(x^2)/(a*c)
```

maple [**A**] time = 0.01, size = 59, normalized size = 0.95

$$\frac{b\ln\left(b\,x^{2}+a\right)}{2\,(ad-bc)\,a}-\frac{d\ln\left(d\,x^{2}+c\right)}{2\,(ad-bc)\,c}+\frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*b/a/(a*d-b*c)*ln(b*x^2+a)-1/2*d/c/(a*d-b*c)*ln(d*x^2+c)+ln(x)/a/c
```

maxima [**A**] time = 1.03, size = 61, normalized size = 0.98

$$-\frac{b\log\left(bx^{2}+a\right)}{2\left(abc-a^{2}d\right)}+\frac{d\log\left(dx^{2}+c\right)}{2\left(bc^{2}-acd\right)}+\frac{\log\left(x^{2}\right)}{2\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*log(b*x^2 + a)/(a*b*c - a^2*d) + 1/2*d*log(d*x^2 + c)/(b*c^2 - a*c*d
) + 1/2*log(x^2)/(a*c)
```

mupad [**B**] time = 0.30, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^{2}+a\right)}{2\,a^{2}\,d-2\,a\,b\,c}+\frac{d\,\ln\left(d\,x^{2}+c\right)}{2\,b\,c^{2}-2\,a\,c\,d}+\frac{\ln(x)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2*d - 2*a*b*c) + (d*log(c + d*x^2))/(2*b*c^2 - 2*a*
c*d) + log(x)/(a*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.235 $\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=81

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {480, 522, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}-\frac{1}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -(1/(a*c*x)) - (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d))
+ (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{acx}+\frac{\int \frac{-bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{ac}\\
&= -\frac{1}{acx}-\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^2}\,dx}{c(bc-ad)}\\
&= -\frac{1}{acx}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 76, normalized size = 0.94

$$
\frac{-\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{b}{a}+\frac{d^{3/2}x\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{d}{c}}{bcx-adx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-(b/a) + d/c - (b^(3/2)*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + (d^(3/2)*
x*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c*x - a*d*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 1.20, size = 384, normalized size = 4.74

$$
\left[-\frac{bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\left(abc^2-a^2cd\right)x}, \frac{2\,adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,bc+2\,ad}{2\left(abc^2-a^2cd\right)x}, -\frac{2\,bcx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\left(abc^2-a^2cd\right)x}, -\frac{bcx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+bc-ad}{\left(abc^2-a^2cd\right)x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + a
*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*c - 2
*a*d)/((a*b*c^2 - a^2*c*d)*x), 1/2*(2*a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) -
 b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*b*c +
 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -1/2*(2*b*c*x*sqrt(b/a)*arctan(x*sqrt(b/a)
) + a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*
c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -(b*c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) + b*c - a*d)/((a*b*c^2 - a^2*c*d)*x)]
```

giac [**A**] time = 0.35, size = 75, normalized size = 0.93

$$-\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(abc - a^2 d\right)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bc^2 - acd\right)\sqrt{cd}} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -b^2*arctan(b*x/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + d^2*arctan(d*x/sqr
t(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/(a*c*x)
```

maple [**A**] time = 0.01, size = 76, normalized size = 0.94

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad - bc)\sqrt{ab}\, a} - \frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad - bc)\sqrt{cd}\, c} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/a*b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/c*d^2/(a*d-b*c)/(
c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/a/c/x
```

maxima [**A**] time = 2.39, size = 75, normalized size = 0.93

$$-\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(abc - a^2 d\right)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bc^2 - acd\right)\sqrt{cd}} - \frac{1}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -b^2*arctan(b*x/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + d^2*arctan(d*x/sqr
t(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/(a*c*x)
```

mupad [B] time = 0.63, size = 338, normalized size = 4.17

$$\frac{\ln\left(a^3\,c^5\,d^4-b^3\,c^8\,d+a^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,b\,c^4-2\,a\,c^3\,d}-\frac{\ln\left(b^3\,c^8\,d-a^3\,c^5\,d^4+a^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,\left(b\,c^4-a\,c^3\,d\right)}-\frac{1}{a\,c\,x}-\frac{\ln\left(a^8\,b\,d^3-a^5\,b^4\,c^3+c^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+a^6\,d^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,\left(a^4\,d-a^3\,b\,c\right)}+\frac{\ln\left(a^5\,b^4\,c^3-a^8\,b\,d^3+c^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+a^6\,d^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,a^4\,d-2\,a^3\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^3*c^5*d^4 - b^3*c^8*d + a^3*x*(-c^3*d^3)^(3/2) + b^3*c^6*x*(-c^3*d^3
)^(1/2))*(-c^3*d^3)^(1/2))/(2*b*c^4 - 2*a*c^3*d) - (log(b^3*c^8*d - a^3*c^5
*d^4 + a^3*x*(-c^3*d^3)^(3/2) + b^3*c^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2
))/(2*(b*c^4 - a*c^3*d)) - 1/(a*c*x) - (log(a^8*b*d^3 - a^5*b^4*c^3 + c^3*x
*(-a^3*b^3)^(3/2) + a^6*d^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*(a^4*d
 - a^3*b*c)) + (log(a^5*b^4*c^3 - a^8*b*d^3 + c^3*x*(-a^3*b^3)^(3/2) + a^6*
d^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*a^4*d - 2*a^3*b*c)
```

sympy [B] time = 12.23, size = 1093, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)*
*3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2*c*
*5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/(a*
d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*b**
4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(a*d
 - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c))
 + sqrt(-b**3/a**3)*log(x + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)
**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*b**2*c
**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a**3)/(a
*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**3*b*
*4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**3)/(a*
d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*c)
) - sqrt(-d**3/c**3)*log(x + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2
*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d**3/c**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**3*
b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d**3/c**3)/(
a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b*
c)) + sqrt(-d**3/c**3)*log(x + (a**7*c**3*d**4*(-d**3/c**3)**(3/2)/(a*d - b
*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*b**
2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-d**3/c**3)
/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**3
*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-d**3/c**3)/
```

```
(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d - b
*c)) - 1/(a*c*x)
```

3.236 $\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(2*a*c*x^2) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^2])/(2*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right) d\right.\\ &= -\frac{1}{2acx^2}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^2\right)}{2a^2(bc-ad)}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^2\right)}{2c^2(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/2*1/(a*c*x^2) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^2])
/(2*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 1.24, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^2\log\left(bx^2+a\right)-a^2d^2x^2\log\left(dx^2+c\right)-abc^2+a^2cd-2\left(b^2c^2-a^2d^2\right)x^2\log(x)}{2\left(a^2bc^3-a^3c^2d\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log(b*x^2 + a) - a^2*d^2*x^2*log(d*x^2 + c) - a*b*c^2 + a^
2*c*d - 2*(b^2*c^2 - a^2*d^2)*x^2*log(x))/((a^2*b*c^3 - a^3*c^2*d)*x^2)
```

giac [A] time = 0.36, size = 112, normalized size = 1.29

$$\frac{b^3\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^2c-a^3bd\right)}-\frac{d^3\log\left(\left|dx^2+c\right|\right)}{2\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log\left(x^2\right)}{2\,a^2c^2}+\frac{bcx^2+adx^2-ac}{2\,a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(abs(b*x^2 + a))/(a^2*b^2*c - a^3*b*d) - 1/2*d^3*log(abs(d*x^2 +
 c))/(b*c^3*d - a*c^2*d^2) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) + 1/2*(b*c*
x^2 + a*d*x^2 - a*c)/(a^2*c^2*x^2)
```

maple [**A**] time = 0.01, size = 87, normalized size = 1.00

$$-\frac{b^2\ln\left(b\,x^2+a\right)}{2\,(ad-bc)\,a^2}+\frac{d^2\ln\left(d\,x^2+c\right)}{2\,(ad-bc)\,c^2}-\frac{d\ln(x)}{a\,c^2}-\frac{b\ln(x)}{a^2c}-\frac{1}{2ac\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2*b^2/a^2/(a*d-b*c)*ln(b*x^2+a)+1/2*d^2/c^2/(a*d-b*c)*ln(d*x^2+c)-1/2/a/
c/x^2-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

maxima [**A**] time = 1.02, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^2+a\right)}{2\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^2+c\right)}{2\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^2\right)}{2\,a^2c^2}-\frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(a^2*b*c - a^3*d) - 1/2*d^2*log(d*x^2 + c)/(b*c^3 -
a*c^2*d) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) - 1/2/(a*c*x^2)
```

mupad [**B**] time = 0.35, size = 87, normalized size = 1.00

$$-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^2+c\right)}{2\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{2\,a\,c\,x^2}-\frac{\ln(x)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - (b^2*log(a + b*x^2))/(2*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^2))/(2*(b*c
^3 - a*c^2*d)) - 1/(2*a*c*x^2) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.237 $\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=100

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Rubi [A] time = 0.18, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {480, 583, 522, 205}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2
)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{3ac}\\
&= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{3a^2c^2}\\
&= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{1}{c+dx^2}\,dx}{c^2(bc-ad)}\\
&= -\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 101, normalized size = 1.01

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(a^(5/2)*(-(b*c) + a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c
^(5/2)*(b*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 1.16, size = 560, normalized size = 5.60

$$\left[-\frac{3\,b^{2}c^{2}x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+3\,a^{2}d^{2}x^{3}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)+2\,abc^{2}-2\,a^{2}cd-6\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}}{6\left(a^{2}bc^{3}-a^{3}c^{2}d\right)x^{3}}, -\frac{6\,a^{2}d^{2}x^{3}\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+3\,b^{2}c^{2}x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+2\,abc^{2}-2\,a^{2}cd-6\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}}{6\left(a^{2}bc^{3}-a^{3}c^{2}d\right)x^{3}}, \frac{6\,b^{2}c^{2}x^{3}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-3\,a^{2}d^{2}x^{3}\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2}+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^{2}+c}\right)-2\,abc^{2}+2\,a^{2}cd+6\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}}{6\left(a^{2}bc^{3}-a^{3}c^{2}d\right)x^{3}}, \frac{3\,b^{2}c^{2}x^{3}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-3\,a^{2}d^{2}x^{3}\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-abc^{2}+a^{2}cd+3\left(b^{2}c^{2}-a^{2}d^{2}\right)x^{2}}{3\left(a^{2}bc^{3}-a^{3}c^{2}d\right)x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*b^2*c^2*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) + 3*a^2*d^2*x^3*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
 c)) + 2*a*b*c^2 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3
*c^2*d)*x^3), -1/6*(6*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*b^2*c^2
*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*a*b*c^2
 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/
6*(6*b^2*c^2*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(-d/c)*l
og((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*a*b*c^2 + 2*a^2*c*d + 6*
(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/3*(3*b^2*c^2*x^3*
sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c))
 - a*b*c^2 + a^2*c*d + 3*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*
x^3)]
```

giac [A] time = 0.35, size = 98, normalized size = 0.98

$$\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^{2}bc-a^{3}d\right)\sqrt{ab}}-\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bc^{3}-ac^{2}d\right)\sqrt{cd}}+\frac{3\,bcx^{2}+3\,adx^{2}-ac}{3\,a^{2}c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - d^3*arctan(d*x/sq
rt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/3*(3*b*c*x^2 + 3*a*d*x^2 - a*c)/
(a^2*c^2*x^3)
```

maple [A] time = 0.01, size = 98, normalized size = 0.98

$$-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)\sqrt{ab}\,a^{2}}+\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad-bc\right)\sqrt{cd}\,c^{2}}+\frac{d}{a\,c^{2}x}+\frac{b}{a^{2}cx}-\frac{1}{3ac\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/a^2*b^3/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+1/c^2*d^3/(a*d-b
*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/3/a/c/x^3+1/a/c^2/x*d+1/a^2/c/x
*b
```

maxima [A] time = 2.41, size = 96, normalized size = 0.96

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^2bc - a^3d\right)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bc^3 - ac^2d\right)\sqrt{cd}} + \frac{3\,(bc + ad)x^2 - ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - d^3*arctan(d*x/sq
rt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/3*(3*(b*c + a*d)*x^2 - a*c)/(a^2
*c^2*x^3)
```

mupad [B] time = 0.61, size = 367, normalized size = 3.67

$$\frac{\ln\left(a^{13}\,b^2\,d^5 - a^8\,b^7\,c^5 + c^5\,x\,{\left(-a^5\,b^5\right)}^{3/2} + a^{10}\,d^5\,x\,\sqrt{-a^5\,b^5}\right)\,\sqrt{-a^5\,b^5}}{2\,a^6\,d - 2\,a^5\,b\,c} - \frac{\ln\left(a^8\,b^7\,c^5 - a^{13}\,b^2\,d^5 + c^5\,x\,{\left(-a^5\,b^5\right)}^{3/2} + a^{10}\,d^5\,x\,\sqrt{-a^5\,b^5}\right)\,\sqrt{-a^5\,b^5}}{2\,\left(a^6\,d - a^5\,b\,c\right)} - \frac{\frac{1}{3\,a\,c} - \frac{x^2\,\left(a\,d+b\,c\right)}{a^2\,c^2}}{x^3} - \frac{\ln\left(a^5\,c^8\,d^7 - b^5\,c^{13}\,d^2 + a^5\,x\,{\left(-c^5\,d^5\right)}^{3/2} + b^5\,c^{10}\,x\,\sqrt{-c^5\,d^5}\right)\,\sqrt{-c^5\,d^5}}{2\,\left(b\,c^6 - a\,c^5\,d\right)} + \frac{\ln\left(b^5\,c^{13}\,d^2 - a^5\,c^8\,d^7 + a^5\,x\,{\left(-c^5\,d^5\right)}^{3/2} + b^5\,c^{10}\,x\,\sqrt{-c^5\,d^5}\right)\,\sqrt{-c^5\,d^5}}{2\,b\,c^6 - 2\,a\,c^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^13*b^2*d^5 - a^8*b^7*c^5 + c^5*x*(-a^5*b^5)^(3/2) + a^10*d^5*x*(-a^5
*b^5)^(1/2))*(-a^5*b^5)^(1/2))/(2*a^6*d - 2*a^5*b*c) - (log(a^8*b^7*c^5 - a
^13*b^2*d^5 + c^5*x*(-a^5*b^5)^(3/2) + a^10*d^5*x*(-a^5*b^5)^(1/2))*(-a^5*b
^5)^(1/2))/(2*(a^6*d - a^5*b*c)) - (1/(3*a*c) - (x^2*(a*d + b*c))/(a^2*c^2)
)/x^3 - (log(a^5*c^8*d^7 - b^5*c^13*d^2 + a^5*x*(-c^5*d^5)^(3/2) + b^5*c^10
*x*(-c^5*d^5)^(1/2))*(-c^5*d^5)^(1/2))/(2*(b*c^6 - a*c^5*d)) + (log(b^5*c^1
3*d^2 - a^5*c^8*d^7 + a^5*x*(-c^5*d^5)^(3/2) + b^5*c^10*x*(-c^5*d^5)^(1/2))
*(-c^5*d^5)^(1/2))/(2*b*c^6 - 2*a*c^5*d)
```

sympy [B] time = 40.89, size = 1353, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b**5/a**5)*log(x + (-a**10*c**5*d**5*(-b**5/a**5)**(3/2)/(a*d - b*c)*
*3 + 2*a**9*b*c**6*d**4*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a**8*b**2*c**7
```

```
*d**3*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a**8*d**8*sqrt(-b**5/a**5)/(a*d
- b*c) - a**7*b**3*c**8*d**2*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + 2*a**6*b*
*4*c**9*d*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 - a**5*b**5*c**10*(-b**5/a**5)
**(3/2)/(a*d - b*c)**3 - b**8*c**8*sqrt(-b**5/a**5)/(a*d - b*c))/(a**4*b**3
*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 + b**7*c*
*4*d**3))/(2*(a*d - b*c)) - sqrt(-b**5/a**5)*log(x + (a**10*c**5*d**5*(-b**
5/a**5)**(3/2)/(a*d - b*c)**3 - 2*a**9*b*c**6*d**4*(-b**5/a**5)**(3/2)/(a*d
 - b*c)**3 + a**8*b**2*c**7*d**3*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + a**8*
d**8*sqrt(-b**5/a**5)/(a*d - b*c) + a**7*b**3*c**8*d**2*(-b**5/a**5)**(3/2)
/(a*d - b*c)**3 - 2*a**6*b**4*c**9*d*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + a
**5*b**5*c**10*(-b**5/a**5)**(3/2)/(a*d - b*c)**3 + b**8*c**8*sqrt(-b**5/a*
*5)/(a*d - b*c))/(a**4*b**3*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 +
 a*b**6*c**3*d**4 + b**7*c**4*d**3))/(2*(a*d - b*c)) + sqrt(-d**5/c**5)*log
(x + (-a**10*c**5*d**5*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + 2*a**9*b*c**6*d
**4*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - a**8*b**2*c**7*d**3*(-d**5/c**5)**
(3/2)/(a*d - b*c)**3 - a**8*d**8*sqrt(-d**5/c**5)/(a*d - b*c) - a**7*b**3*c
**8*d**2*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + 2*a**6*b**4*c**9*d*(-d**5/c**
5)**(3/2)/(a*d - b*c)**3 - a**5*b**5*c**10*(-d**5/c**5)**(3/2)/(a*d - b*c)*
*3 - b**8*c**8*sqrt(-d**5/c**5)/(a*d - b*c))/(a**4*b**3*d**7 + a**3*b**4*c*
d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 + b**7*c**4*d**3))/(2*(a*d -
b*c)) - sqrt(-d**5/c**5)*log(x + (a**10*c**5*d**5*(-d**5/c**5)**(3/2)/(a*d
- b*c)**3 - 2*a**9*b*c**6*d**4*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**8*b*
*2*c**7*d**3*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**8*d**8*sqrt(-d**5/c**5
)/(a*d - b*c) + a**7*b**3*c**8*d**2*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 - 2*
a**6*b**4*c**9*d*(-d**5/c**5)**(3/2)/(a*d - b*c)**3 + a**5*b**5*c**10*(-d**
5/c**5)**(3/2)/(a*d - b*c)**3 + b**8*c**8*sqrt(-d**5/c**5)/(a*d - b*c))/(a*
*4*b**3*d**7 + a**3*b**4*c*d**6 + a**2*b**5*c**2*d**5 + a*b**6*c**3*d**4 +
b**7*c**4*d**3))/(2*(a*d - b*c)) + (-a*c + x**2*(3*a*d + 3*b*c))/(3*a**2*c*
*2*x**3)
```

3.238 $\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=119

$$-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(2*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^2])/(2*c^3*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+bx)}+\frac{}{c^3}\right.\right.\\ &= -\frac{1}{4acx^4}+\frac{bc+ad}{2a^2c^2x^2}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log(x)}{a^3c^3}-\frac{b^3\log\left(a+bx^2\right)}{2a^3(bc-ad)}+\frac{d^3\log\left(c+\right.}{2c^3(bc-}\end{aligned}$$

Mathematica [A] time = 0.05, size = 119, normalized size = 1.00

$$\frac{b^3\log\left(a+bx^2\right)}{2a^3(ad-bc)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^2\right)}{2c^3(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + (b*c + a*d)/(2*a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^2])/(2*a^3*(-(b*c) + a*d)) + (d^3
*Log[c + d*x^2])/(2*c^3*(b*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 3.09, size = 127, normalized size = 1.07

$$-\frac{2\,b^3c^3x^4\log\left(bx^2+a\right)-2\,a^3d^3x^4\log\left(dx^2+c\right)+a^2bc^3-a^3c^2d-4\left(b^3c^3-a^3d^3\right)x^4\log(x)-2\left(ab^2c^3-a^3cd^2\right)x^2}{4\left(a^3bc^4-a^4c^3d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3*x^4*log(b*x^2 + a) - 2*a^3*d^3*x^4*log(d*x^2 + c) + a^2*b*c
^3 - a^3*c^2*d - 4*(b^3*c^3 - a^3*d^3)*x^4*log(x) - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^4)
```

giac [A] time = 0.31, size = 167, normalized size = 1.40

$$-\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\left(a^3b^2c-a^4bd\right)}+\frac{d^4\log\left(\left|dx^2+c\right|\right)}{2\left(bc^4d-ac^3d^2\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^3c^3}-\frac{3\,b^2c^2x^4+3\,abcdx^4+3\,a^2d^2x^4-2\,abc^2x^2-2\,a^2cdx^2+a^2c^2}{4\,a^3c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*b^4*log(abs(b*x^2 + a))/(a^3*b^2*c - a^4*b*d) + 1/2*d^4*log(abs(d*x^2
+ c))/(b*c^4*d - a*c^3*d^2) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/(a
^3*c^3) - 1/4*(3*b^2*c^2*x^4 + 3*a*b*c*d*x^4 + 3*a^2*d^2*x^4 - 2*a*b*c^2*x^
2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^3*c^3*x^4)
```

maple [A] time = 0.02, size = 124, normalized size = 1.04

$$\frac{b^3\ln\left(b\,x^2+a\right)}{2\,(ad-bc)\,a^3}-\frac{d^3\ln\left(d\,x^2+c\right)}{2\,(ad-bc)\,c^3}+\frac{d^2\ln(x)}{a\,c^3}+\frac{bd\ln(x)}{a^2c^2}+\frac{b^2\ln(x)}{a^3c}+\frac{d}{2a\,c^2x^2}+\frac{b}{2a^2c\,x^2}-\frac{1}{4ac\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/2*b^3/a^3/(a*d-b*c)*ln(b*x^2+a)-1/2*d^3/c^3/(a*d-b*c)*ln(d*x^2+c)-1/4/a/c
/x^4+1/2/a/c^2/x^2*d+1/2/a^2/c/x^2*b+1/a/c^3*ln(x)*d^2+1/a^2/c^2*ln(x)*b*d+
1/a^3/c*ln(x)*b^2
```

maxima [A] time = 1.11, size = 117, normalized size = 0.98

$$-\frac{b^3\log\left(bx^2+a\right)}{2\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^2+c\right)}{2\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^3c^3}+\frac{2\,(bc+ad)x^2-ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a^3*b*c - a^4*d) + 1/2*d^3*log(d*x^2 + c)/(b*c^4 -
 a*c^3*d) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/(a^3*c^3) + 1/4*(2*(
b*c + a*d)*x^2 - a*c)/(a^2*c^2*x^4)
```

mupad [B] time = 0.43, size = 118, normalized size = 0.99

$$\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4\,d-2\,a^3\,b\,c}-\frac{\frac{1}{4\,a\,c}-\frac{x^2\,(a\,d+b\,c)}{2\,a^2\,c^2}}{x^4}+\frac{d^3\,\ln\left(d\,x^2+c\right)}{2\,b\,c^4-2\,a\,c^3\,d}+\frac{\ln(x)\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*a^4*d - 2*a^3*b*c) - (1/(4*a*c) - (x^2*(a*d + b*c))
/(2*a^2*c^2))/x^4 + (d^3*log(c + d*x^2))/(2*b*c^4 - 2*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=134

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Rubi [A] time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {480, 583, 522, 205}

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d))
 + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= -\frac{1}{5acx^5}+\frac{\int \frac{-5(bc+ad)-5bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{5ac}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{\int \frac{-15\left(b^2c^2+abcd+a^2d^2\right)-15bd(bc+ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{15a^2c^2}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{-15(bc+ad)\left(b^2c^2+a^2d^2\right)-15bd(b^2c^2+abcd+a^2d^2}{\left(a+bx^2\right)\left(c+dx^2\right)}}{15a^3c^3}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int \frac{1}{a+bx^2}\,dx}{a^3(bc-ad)}+\frac{d^4\int \frac{1}{c+dx^2}\,dx}{c^3(bc-ad)}\\
&= -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 135, normalized size = 1.01

$$
\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(ad-bc)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{-a^2d^2-abcd-b^2c^2}{a^3c^3x}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}-\frac{1}{5acx^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/5*1/(a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) + (-(b^2*c^2) - a*b*c*d - a^
2*d^2)/(a^3*c^3*x) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c)
 + a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(a+bx^2\right) \left(c+dx^2\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 1.02, size = 669, normalized size = 4.99

$$\left[-\frac{15\,b^3c^3x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+15\,a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\,a^2bc^3-6\,a^3c^2d+30\left(b^3c^3-a^3d^3\right)x^4-10\left(ab^2c^3-a^3cd^2\right)x^2}{30\left(a^3bc^4-a^4c^3d\right)x^5}, \frac{30\,a^3d^3x^5\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-15\,b^3c^3x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6\,a^2bc^3+6\,a^3c^2d-30\left(b^3c^3-a^3d^3\right)x^4+10\left(ab^2c^3-a^3cd^2\right)x^2}{30\left(a^3bc^4-a^4c^3d\right)x^5}, -\frac{30\,b^3c^3x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+15\,a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\,a^2bc^3-6\,a^3c^2d+30\left(b^3c^3-a^3d^3\right)x^4-10\left(ab^2c^3-a^3cd^2\right)x^2}{30\left(a^3bc^4-a^4c^3d\right)x^5}, -\frac{15\,b^3c^3x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-15\,a^3d^3x^5\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+3\,a^2bc^3-3\,a^3c^2d+15\left(b^3c^3-a^3d^3\right)x^4-5\left(ab^2c^3-a^3cd^2\right)x^2}{15\left(a^3bc^4-a^4c^3d\right)x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + 15*a^3*d^3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)) + 6*a^2*b*c^3 - 6*a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^
2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5), 1/30*(30*a^3*d^3*x^5
*sqrt(d/c)*arctan(x*sqrt(d/c)) - 15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6*a^2*b*c^3 + 6*a^3*c^2*d - 30*(b^3*c^3 -
 a^3*d^3)*x^4 + 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^
5), -1/30*(30*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15*a^3*d^3*x^5*sq
rt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 6*a^2*b*c^3 - 6*
a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((
a^3*b*c^4 - a^4*c^3*d)*x^5), -1/15*(15*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 15*a^3*d^3*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*a^2*b*c^3 - 3*a^3*
c^2*d + 15*(b^3*c^3 - a^3*d^3)*x^4 - 5*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b
*c^4 - a^4*c^3*d)*x^5)]
```

giac [A] time = 0.33, size = 139, normalized size = 1.04

$$-\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^3bc - a^4d\right)\sqrt{ab}} + \frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(bc^4 - ac^3d\right)\sqrt{cd}} - \frac{15\,b^2c^2x^4 + 15\,abcdx^4 + 15\,a^2d^2x^4 - 5\,abc^2x^2 - 5\,a^2cdx^2 + 3\,a^2c^2}{15\,a^3c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a^3*b*c - a^4*d)*sqrt(a*b)) + d^4*arctan(d*x/s
qrt(c*d))/((b*c^4 - a*c^3*d)*sqrt(c*d)) - 1/15*(15*b^2*c^2*x^4 + 15*a*b*c*d
```

```
*x^4 + 15*a^2*d^2*x^4 - 5*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(a^3*c^3
*x^5)
```

maple [A] time = 0.01, size = 141, normalized size = 1.05

$$\frac{b^{4} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d - b c\right) \sqrt{a b}\, a^{3}}-\frac{d^{4} \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{\left(a d - b c\right) \sqrt{c d}\, c^{3}}-\frac{d^{2}}{a\, c^{3} x}-\frac{b d}{a^{2} c^{2} x}-\frac{b^{2}}{a^{3} c x}+\frac{d}{3 a\, c^{2} x^{3}}+\frac{b}{3 a^{2} c\, x^{3}}-\frac{1}{5 a c\, x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/a^3*b^4/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/c^3*d^4/(a*d-b*
c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/5/a/c/x^5+1/3/a/c^2/x^3*d+1/3/a^
2/c/x^3*b-1/a/c^3/x*d^2-1/a^2/c^2/x*b*d-1/a^3/c/x*b^2
```

maxima [A] time = 2.45, size = 131, normalized size = 0.98

$$-\frac{b^{4} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{\left(a^{3} b c - a^{4} d\right) \sqrt{a b}}+\frac{d^{4} \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{\left(b c^{4} - a c^{3} d\right) \sqrt{c d}}-\frac{15 \left(b^{2} c^{2} + a b c d + a^{2} d^{2}\right) x^{4} + 3 \, a^{2} c^{2} - 5 \left(a b c^{2} + a^{2} c d\right) x^{2}}{15 \, a^{3} c^{3} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a^3*b*c - a^4*d)*sqrt(a*b)) + d^4*arctan(d*x/s
qrt(c*d))/((b*c^4 - a*c^3*d)*sqrt(c*d)) - 1/15*(15*(b^2*c^2 + a*b*c*d + a^2
*d^2)*x^4 + 3*a^2*c^2 - 5*(a*b*c^2 + a^2*c*d)*x^2)/(a^3*c^3*x^5)
```

mupad [B] time = 0.63, size = 397, normalized size = 2.96

$$\frac{\ln\left(a^{11}\,b^{10}\,c^7-a^{18}\,b^3\,d^7+c^7\,x\,{\left(-a^7\,b^7\right)}^{3/2}+a^{14}\,d^7\,x\,\sqrt{-a^7\,b^7}\right)\,\sqrt{-a^7\,b^7}}{2\,a^8\,d-2\,a^7\,b\,c}-\frac{\ln\left(a^{18}\,b^3\,d^7-a^{11}\,b^{10}\,c^7+c^7\,x\,{\left(-a^7\,b^7\right)}^{3/2}+a^{14}\,d^7\,x\,\sqrt{-a^7\,b^7}\right)\,\sqrt{-a^7\,b^7}}{2\,\left(a^8\,d-a^7\,b\,c\right)}-\frac{\frac{1}{5\,a\,c}-\frac{x^2\,\left(a\,d+b\,c\right)}{3\,a^2\,c^2}+\frac{x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}}{x^5}-\frac{\ln\left(b^7\,c^{18}\,d^3-a^7\,c^{11}\,d^{10}+a^7\,x\,{\left(-c^7\,d^7\right)}^{3/2}+b^7\,c^{14}\,x\,\sqrt{-c^7\,d^7}\right)\,\sqrt{-c^7\,d^7}}{2\,\left(b\,c^8-a\,c^7\,d\right)}+\frac{\ln\left(a^7\,c^{11}\,d^{10}-b^7\,c^{18}\,d^3+a^7\,x\,{\left(-c^7\,d^7\right)}^{3/2}+b^7\,c^{14}\,x\,\sqrt{-c^7\,d^7}\right)\,\sqrt{-c^7\,d^7}}{2\,b\,c^8-2\,a\,c^7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(a^11*b^10*c^7 - a^18*b^3*d^7 + c^7*x*(-a^7*b^7)^(3/2) + a^14*d^7*x*(-a
^7*b^7)^(1/2))*(-a^7*b^7)^(1/2))/(2*a^8*d - 2*a^7*b*c) - (log(a^18*b^3*d^7
- a^11*b^10*c^7 + c^7*x*(-a^7*b^7)^(3/2) + a^14*d^7*x*(-a^7*b^7)^(1/2))*(-a
^7*b^7)^(1/2))/(2*(a^8*d - a^7*b*c)) - (1/(5*a*c) - (x^2*(a*d + b*c))/(3*a^
2*c^2) + (x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^5 - (log(b^7*c^18
*d^3 - a^7*c^11*d^10 + a^7*x*(-c^7*d^7)^(3/2) + b^7*c^14*x*(-c^7*d^7)^(1/2)
)*(-c^7*d^7)^(1/2))/(2*(b*c^8 - a*c^7*d)) + (log(a^7*c^11*d^10 - b^7*c^18*d
```

```
^3 + a^7*x*(-c^7*d^7)^(3/2) + b^7*c^14*x*(-c^7*d^7)^(1/2))*(-c^7*d^7)^(1/2)
)/(2*b*c^8 - 2*a*c^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{1}{x^7\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=155

$$\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

Rubi [A] time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{a^2d^2+abcd+b^2c^2}{2a^3c^3x^2}-\frac{\log(x)(ad+bc)\left(a^2d^2+b^2c^2\right)}{a^4c^4}+\frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(2*a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Log[x])/(a^4*c^4) + (b
^4*Log[a + b*x^2])/(2*a^4*(b*c - a*d)) - (d^4*Log[c + d*x^2])/(2*c^4*(b*c -
 a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4(a+bx)(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{acx^4}+\frac{-bc-ad}{a^2c^2x^3}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x^2}-\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)}{a^4c^4x} -\right.\right.\\ &= -\frac{1}{6acx^6}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{2a^3c^3x^2}-\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\log(x)}{a^4c^4}+\frac{b^4}{2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 147, normalized size = 0.95

$$\frac{12x^6\log(x)\left(b^4c^4-a^4d^4\right)+a\left(a^3cd\left(-2c^2+3cdx^2-6d^2x^4\right)+6a^3d^4x^6\log\left(c+dx^2\right)+2a^2bc^4-3ab^2c^4x^2+6b^3c^4x^4\right)-6b^4c^4x^6\log\left(a+bx^2\right)}{12a^4c^4x^6(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (12*(b^4*c^4 - a^4*d^4)*x^6*Log[x] - 6*b^4*c^4*x^6*Log[a + b*x^2] + a*(2*a^
2*b*c^4 - 3*a*b^2*c^4*x^2 + 6*b^3*c^4*x^4 + a^3*c*d*(-2*c^2 + 3*c*d*x^2 - 6
*d^2*x^4) + 6*a^3*d^4*x^6*Log[c + d*x^2]))/(12*a^4*c^4*(-(b*c) + a*d)*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^2)*(c + d*x^2)), x]
```

fricas [A] time = 4.46, size = 155, normalized size = 1.00

$$\frac{6\,b^4c^4x^6\log\left(bx^2+a\right)-6\,a^4d^4x^6\log\left(dx^2+c\right)-2\,a^3bc^4+2\,a^4c^3d-12\left(b^4c^4-a^4d^4\right)x^6\log(x)-6\left(ab^3c^4-a^4cd^3\right)x^4+3\left(a^2b^2c^4-a^4c^2d^2\right)x^2}{12\left(a^4bc^5-a^5c^4d\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b^4*c^4*x^6*log(b*x^2 + a) - 6*a^4*d^4*x^6*log(d*x^2 + c) - 2*a^3*b
*c^4 + 2*a^4*c^3*d - 12*(b^4*c^4 - a^4*d^4)*x^6*log(x) - 6*(a*b^3*c^4 - a^4
```

```
*c*d^3)*x^4 + 3*(a^2*b^2*c^4 - a^4*c^2*d^2)*x^2)/((a^4*b*c^5 - a^5*c^4*d)*x
^6)
```

giac [A] time = 0.31, size = 239, normalized size = 1.54

$$\frac{b^5\log\left(|bx^2+a|\right)}{2\left(a^4b^2c-a^5bd\right)}-\frac{d^5\log\left(|dx^2+c|\right)}{2\left(bc^5d-ac^4d^2\right)}-\frac{\left(b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2\,a^4c^4}+\frac{11\,b^3c^3x^6+11\,ab^2c^2dx^6+11\,a^2bcd^2x^6+11\,a^3d^3x^6-6\,ab^2c^3x^4-6\,a^2bc^2dx^4-6\,a^3cd^2x^4+3\,a^2bc^3x^2+3\,a^3c^2dx^2-2\,a^3c^3}{12\,a^4c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(abs(b*x^2 + a))/(a^4*b^2*c - a^5*b*d) - 1/2*d^5*log(abs(d*x^2 +
 c))/(b*c^5*d - a*c^4*d^2) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3
*d^3)*log(x^2)/(a^4*c^4) + 1/12*(11*b^3*c^3*x^6 + 11*a*b^2*c^2*d*x^6 + 11*a
^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 6*a*b^2*c^3*x^4 - 6*a^2*b*c^2*d*x^4 - 6*a
^3*c*d^2*x^4 + 3*a^2*b*c^3*x^2 + 3*a^3*c^2*d*x^2 - 2*a^3*c^3)/(a^4*c^4*x^6)
```

maple [A] time = 0.02, size = 184, normalized size = 1.19

$$-\frac{b^4\ln\left(b\,x^2+a\right)}{2\,(ad-bc)\,a^4}+\frac{d^4\ln\left(d\,x^2+c\right)}{2\,(ad-bc)\,c^4}-\frac{d^3\ln(x)}{a\,c^4}-\frac{b\,d^2\ln(x)}{a^2c^3}-\frac{b^2d\ln(x)}{a^3c^2}-\frac{b^3\ln(x)}{a^4c}-\frac{d^2}{2a\,c^3x^2}-\frac{bd}{2a^2c^2x^2}-\frac{b^2}{2a^3c\,x^2}+\frac{d}{4a\,c^2x^4}+\frac{b}{4a^2c\,x^4}-\frac{1}{6ac\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/2*b^4/a^4/(a*d-b*c)*ln(b*x^2+a)+1/2*d^4/c^4/(a*d-b*c)*ln(d*x^2+c)-1/6/a/
c/x^6+1/4/a/c^2/x^4*d+1/4/a^2/c/x^4*b-1/2/a/c^3/x^2*d^2-1/2/a^2/c^2/x^2*b*d
-1/2/a^3/c/x^2*b^2-1/a/c^4*ln(x)*d^3-1/a^2/c^3*ln(x)*b*d^2-1/a^3/c^2*ln(x)*
b^2*d-1/a^4/c*ln(x)*b^3
```

maxima [A] time = 1.12, size = 165, normalized size = 1.06

$$\frac{b^4\log\left(bx^2+a\right)}{2\left(a^4bc-a^5d\right)}-\frac{d^4\log\left(dx^2+c\right)}{2\left(bc^5-ac^4d\right)}-\frac{\left(b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2\,a^4c^4}-\frac{6\left(b^2c^2+abcd+a^2d^2\right)x^4+2\,a^2c^2-3\left(abc^2+a^2cd\right)x^2}{12\,a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^4*b*c - a^5*d) - 1/2*d^4*log(d*x^2 + c)/(b*c^5 -
a*c^4*d) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^
4*c^4) - 1/12*(6*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 - 3*(a*b*c^2
 + a^2*c*d)*x^2)/(a^3*c^3*x^6)
```

mupad [B] time = 0.49, size = 165, normalized size = 1.06

$$-\frac{\frac{1}{6\,a\,c}-\frac{x^2\,(a\,d+b\,c)}{4\,a^2\,c^2}+\frac{x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{2\,a^3\,c^3}}{x^6}-\frac{b^4\,\ln\left(b\,x^2+a\right)}{2\,\left(a^5\,d-a^4\,b\,c\right)}-\frac{d^4\,\ln\left(d\,x^2+c\right)}{2\,\left(b\,c^5-a\,c^4\,d\right)}-\frac{\ln(x)\,\left(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}{a^4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - (1/(6*a*c) - (x^2*(a*d + b*c))/(4*a^2*c^2) + (x^4*(a^2*d^2 + b^2*c^2 + a*
b*c*d))/(2*a^3*c^3))/x^6 - (b^4*log(a + b*x^2))/(2*(a^5*d - a^4*b*c)) - (d^
4*log(c + d*x^2))/(2*(b*c^5 - a*c^4*d)) - (log(x)*(a^3*d^3 + b^3*c^3 + a*b^
2*c^2*d + a^2*b*c*d^2))/(a^4*c^4)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{x^5}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{2b^2\left(a+bx^2\right)(bc-ad)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -a^2/(2*b^2*(b*c - a*d)*(a + b*x^2)) - (a*(2*b*c - a*d)*Log[a + b*x^2])/(2*
b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(2*d*(b*c - a*d)^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)}+\frac{c^2}{(bc-ad)^2(c+dx)}\right)dx,\right.\\ &= -\frac{a^2}{2b^2(bc-ad)\left(a+bx^2\right)}-\frac{a(2bc-ad)\log\left(a+bx^2\right)}{2b^2(bc-ad)^2}+\frac{c^2\log\left(c+dx^2\right)}{2d(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.98

$$\frac{a^2d(ad-bc)+b^2c^2\left(a+bx^2\right)\log\left(c+dx^2\right)+ad\left(a+bx^2\right)(ad-2bc)\log\left(a+bx^2\right)}{2b^2d\left(a+bx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a^2*d*(-(b*c) + a*d) + a*d*(-2*b*c + a*d)*(a + b*x^2)*Log[a + b*x^2] + b^2
*c^2*(a + b*x^2)*Log[c + d*x^2])/(2*b^2*d*(b*c - a*d)^2*(a + b*x^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 1.07, size = 162, normalized size = 1.74

$$-\frac{a^2bcd-a^3d^2+\left(2\,a^2bcd-a^3d^2+\left(2\,ab^2cd-a^2bd^2\right)x^2\right)\log\left(bx^2+a\right)-\left(b^3c^2x^2+ab^2c^2\right)\log\left(dx^2+c\right)}{2\left(ab^4c^2d-2\,a^2b^3cd^2+a^3b^2d^3+\left(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d
^2)*x^2)*log(b*x^2 + a) - (b^3*c^2*x^2 + a*b^2*c^2)*log(d*x^2 + c))/(a*b^4*
```

```
c^2*d - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3 + (b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^
3*d^3)*x^2)
```

giac [A] time = 0.28, size = 152, normalized size = 1.63

$$\frac{c^{2} \log \left(\left|dx^{2}+c\right|\right)}{2 \left(b^{2} c^{2} d-2 \, abcd^{2}+a^{2} d^{3}\right)}-\frac{\left(2 \, abc-a^{2} d\right) \log \left(\left|bx^{2}+a\right|\right)}{2 \left(b^{4} c^{2}-2 \, ab^{3} cd+a^{2} b^{2} d^{2}\right)}+\frac{2 \, abcx^{2}-a^{2} dx^{2}+a^{2} c}{2 \left(b^{3} c^{2}-2 \, ab^{2} cd+a^{2} bd^{2}\right) \left(bx^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(abs(d*x^2 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*(2*a*
b*c - a^2*d)*log(abs(b*x^2 + a))/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2) + 1/
2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(b
*x^2 + a))
```

maple [A] time = 0.01, size = 136, normalized size = 1.46

$$\frac{a^{3} d}{2 \left(ad-bc\right)^{2} \left(b \,x^{2}+a\right) b^{2}}-\frac{a^{2} c}{2 \left(ad-bc\right)^{2} \left(b \,x^{2}+a\right) b}+\frac{a^{2} d \ln \left(b \,x^{2}+a\right)}{2 \left(ad-bc\right)^{2} b^{2}}-\frac{ac \ln \left(b \,x^{2}+a\right)}{\left(ad-bc\right)^{2} b}+\frac{c^{2} \ln \left(d \,x^{2}+c\right)}{2 \left(ad-bc\right)^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*a^3/(a*d-b*c)^2/b^2/(b*x^2+a)*d-1/2*a^2/(a*d-b*c)^2/b/(b*x^2+a)*c+1/2*a
^2/(a*d-b*c)^2/b^2*ln(b*x^2+a)*d-a/(a*d-b*c)^2/b*ln(b*x^2+a)*c+1/2*c^2/(a*d
-b*c)^2/d*ln(d*x^2+c)
```

maxima [A] time = 1.06, size = 130, normalized size = 1.40

$$\frac{c^{2} \log \left(dx^{2}+c\right)}{2 \left(b^{2} c^{2} d-2 \, abcd^{2}+a^{2} d^{3}\right)}-\frac{a^{2}}{2 \left(ab^{3} c-a^{2} b^{2} d+\left(b^{4} c-ab^{3} d\right) x^{2}\right)}-\frac{\left(2 \, abc-a^{2} d\right) \log \left(bx^{2}+a\right)}{2 \left(b^{4} c^{2}-2 \, ab^{3} cd+a^{2} b^{2} d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(d*x^2 + c)/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*a^2/(a*b^3
*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2) - 1/2*(2*a*b*c - a^2*d)*log(b*x^2 +
 a)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

mupad [B] time = 0.45, size = 169, normalized size = 1.82

$$\frac{a^2}{2\,\left(d\,a^2\,b^2+d\,a\,b^3\,x^2-c\,a\,b^3-c\,b^4\,x^2\right)}+\frac{c^2\,\ln\left(d\,x^2+c\right)}{2\,a^2\,d^3-4\,a\,b\,c\,d^2+2\,b^2\,c^2\,d}+\frac{a^2\,d\,\ln\left(b\,x^2+a\right)}{2\,a^2\,b^2\,d^2-4\,a\,b^3\,c\,d+2\,b^4\,c^2}-\frac{2\,a\,b\,c\,\ln\left(b\,x^2+a\right)}{2\,a^2\,b^2\,d^2-4\,a\,b^3\,c\,d+2\,b^4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] a^2/(2*(a^2*b^2*d - b^4*c*x^2 - a*b^3*c + a*b^3*d*x^2)) + (c^2*log(c + d*x^
2))/(2*a^2*d^3 + 2*b^2*c^2*d - 4*a*b*c*d^2) + (a^2*d*log(a + b*x^2))/(2*b^4
*c^2 + 2*a^2*b^2*d^2 - 4*a*b^3*c*d) - (2*a*b*c*log(a + b*x^2))/(2*b^4*c^2 +
 2*a^2*b^2*d^2 - 4*a*b^3*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.242 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=108

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {470, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}-\frac{cx}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*x)/(2*d*(b*c - a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[b]*(b*c - a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= -\frac{cx}{2d(bc-ad)\left(c+dx^2\right)}+\frac{\int \frac{ac+(bc-2ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2d(bc-ad)}\\ &= -\frac{cx}{2d(bc-ad)\left(c+dx^2\right)}+\frac{a^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}+\frac{(c(bc-3ad))\int \frac{1}{c+dx^2}\,dx}{2d(bc-ad)^2}\\ &= -\frac{cx}{2d(bc-ad)\left(c+dx^2\right)}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(bc-ad)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 108, normalized size = 1.00

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(ad-bc)^2}+\frac{\sqrt{c}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}+\frac{cx}{2d\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (c*x)/(2*d*(-(b*c) + a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[b]*(-(b*c) + a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [A] time = 1.02, size = 718, normalized size = 6.65

$$\left[\frac{2\left(ad^2x^2+acd\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-\left(bc^2-3acd+\left(bcd-3ad^2\right)x^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\left(bc^2-acd\right)x}{4\left(b^2c^3d-2abc^2d^2+a^2cd^3+\left(b^2c^2d^2-2abcd^3+a^2d^4\right)x^2\right)}, \frac{4\left(ad^2x^2+acd\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-\left(bc^2-3acd+\left(bcd-3ad^2\right)x^2\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\left(bc^2-acd\right)x}{4\left(b^2c^3d-2abc^2d^2+a^2cd^3+\left(b^2c^2d^2-2abcd^3+a^2d^4\right)x^2\right)}, \frac{\left(bc^2-3acd+\left(bcd-3ad^2\right)x^2\right)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+\left(ad^2x^2+acd\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-\left(bc^2-acd\right)x}{2\left(b^2c^3d-2abc^2d^2+a^2cd^3+\left(b^2c^2d^2-2abcd^3+a^2d^4\right)x^2\right)}, \frac{2\left(ad^2x^2+acd\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+\left(bc^2-3acd+\left(bcd-3ad^2\right)x^2\right)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-\left(bc^2-acd\right)x}{2\left(b^2c^3d-2abc^2d^2+a^2cd^3+\left(b^2c^2d^2-2abcd^3+a^2d^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*d^2*x^2 + a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) - (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x
^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d -
 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/
4*(4*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - (b*c^2 - 3*a*c
*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(
d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 +
 (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/2*((b*c^2 - 3*a*c*d + (b*c*d
 - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + (a*d^2*x^2 + a*c*d)*sq
rt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - (b*c^2 - a*c*d)*
x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*x^2), 1/2*(2*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c
) - (b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^2)]
```

giac [A] time = 0.34, size = 121, normalized size = 1.12

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{ab}} + \frac{\left(bc^2 - 3\,acd\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)\sqrt{cd}} - \frac{cx}{2\left(bcd - ad^2\right)\left(dx^2 + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/2
*(b*c^2 - 3*a*c*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*sqrt(c*d)) - 1/2*c*x/((b*c*d - a*d^2)*(d*x^2 + c))
```

maple [A] time = 0.01, size = 144, normalized size = 1.33

$$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}} + \frac{acx}{2\,(ad-bc)^2\left(d\,x^2+c\right)} - \frac{3ac\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(ad-bc)^2\sqrt{cd}} - \frac{b\,c^2x}{2\,(ad-bc)^2\left(d\,x^2+c\right)d} + \frac{b\,c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(ad-bc)^2\sqrt{cd}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+1/2*c/(a*d-b*c)^2*x/(
d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x/(d*x^2+c)*b-3/2*c/(a*d-b*c)^2/(c*d)^(1/2
```

```
)*arctan(1/(c*d)^(1/2)*d*x)*a+1/2*c^2/(a*d-b*c)^2/d/(c*d)^(1/2)*arctan(1/(c
*d)^(1/2)*d*x)*b
```

maxima [A] time = 2.49, size = 132, normalized size = 1.22

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{cx}{2\left(bc^2d-acd^2+\left(bcd^2-ad^3\right)x^2\right)}+\frac{\left(bc^2-3\,acd\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*c*x/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2) + 1/2*(b*c^2 - 3*a*c*d)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(c*d))
```

mupad [B] time = 0.94, size = 3572, normalized size = 33.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (c*x)/(2*d*(c + d*x^2)*(a*d - b*c)) - (atan(((((x*(b^5*c^4 + 4*a^4*b*d^4 +
9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))
 - ((-c*d^3)^(1/2)*(3*a*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^
2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*
d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*
a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2
*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4
)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*
c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c)*1i)/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b
*c*d^4)) + (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d)
)/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + ((-c*d^3)^(1/2)*(3*a*d - b*c)*(
(2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4
 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^
3) + (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*
a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6)
)/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d
^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b
*c)*1i)/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))/(((a^2*b^3*c^3)/2 - (5*a
^3*b^2*c^2*d)/2 + 3*a^4*b*c*d^2)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3
*a^2*b*c*d^3) - (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c
^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) - ((-c*d^3)^(1/2)*(3*a*d - b
*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^
```

```
3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b
*c*d^3) - (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3
- 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2
*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*
b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*
d - b*c))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)) + (((x*(b^5*c^4 + 4*a^4
*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*
b*c*d^2)) + ((-c*d^3)^(1/2)*(3*a*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d
^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 -
 b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x*(-c*d^3)^(1/2)*(3*a*d -
b*c)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7
 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*
a*b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3
 - 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c))/(4*(a^2*d^5 + b^2*c^2*d^3 -
 2*a*b*c*d^4))))*(-c*d^3)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^2*d^5 + b^2*c^2*d^3
 - 2*a*b*c*d^4)) - (atan(-(((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*
c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(a^3
*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) - (x*(-a^3*b)^(1/2)*(1
6*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a
^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d
)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*
c*d)) - (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(4*
(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/2)*1i)/(b^3*c^2 + a^2*b*d
^2 - 2*a*b^2*c*d) - ((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 -
 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(a^3*d^4 -
b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + (x*(-a^3*b)^(1/2)*(16*a^5*b
^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*
c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*
d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) +
 (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(4*(a^2*d^
3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/2)*1i)/(b^3*c^2 + a^2*b*d^2 - 2*
a*b^2*c*d))/(((a^2*b^3*c^3)/2 - (5*a^3*b^2*c^2*d)/2 + 3*a^4*b*c*d^2)/(a^3*d
^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((((-a^3*b)^(1/2)*((2*a
*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8
*a^4*b^3*c^2*d^5)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3
)) - (x*(-a^3*b)^(1/2)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4
- 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2
+ a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c
^2 + a^2*b*d^2 - 2*a*b^2*c*d)) - (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*
d^2 - 6*a*b^4*c^3*d))/(4*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/
2))/(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d) + ((((-a^3*b)^(1/2)*((2*a*b^6*c^5*d
^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c
^2*d^5)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + (x*(-
a^3*b)^(1/2)*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b
^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(b^3*c^2 + a^2*b*d^
```

```
2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(b^3*c^2 + a^2*b
*d^2 - 2*a*b^2*c*d)) + (x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*
b^4*c^3*d))/(4*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))*(-a^3*b)^(1/2))/(b^3*c
^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(-a^3*b)^(1/2)*1i)/(b^3*c^2 + a^2*b*d^2 - 2
*a*b^2*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.243 $\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{c}{2d\left(c+dx^2\right)(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Rubi [**A**] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{c}{2d\left(c+dx^2\right)(bc-ad)}-\frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -c/(2*d*(b*c - a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (
a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{ab}{(bc-ad)^2(a+bx)}+\frac{c}{(bc-ad)(c+dx)^2}+\frac{ad}{(-bc+ad)^2(c+dx)}\right) dx$$

$$= -\frac{c}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2} + \frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.00

$$\frac{c}{2d\left(c+dx^2\right)(ad-bc)} - \frac{a\log\left(a+bx^2\right)}{2(bc-ad)^2} + \frac{a\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] c/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2) +
 (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [A] time = 0.86, size = 117, normalized size = 1.58

$$-\frac{bc^2 - acd + \left(ad^2x^2 + acd\right)\log\left(bx^2+a\right) - \left(ad^2x^2 + acd\right)\log\left(dx^2+c\right)}{2\left(b^2c^3d - 2\,abc^2d^2 + a^2cd^3 + \left(b^2c^2d^2 - 2\,abcd^3 + a^2d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c^2 - a*c*d + (a*d^2*x^2 + a*c*d)*log(b*x^2 + a) - (a*d^2*x^2 + a*c
*d)*log(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*x^2)
```

giac [A] time = 0.37, size = 91, normalized size = 1.23

$$-\frac{\frac{ad^2\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{cd}{\left(bcd-ad^2\right)\left(dx^2+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3) + c*d/((b*c*d - a*d^2)*(d*x^2 + c)))/d
```

maple [A] time = 0.01, size = 95, normalized size = 1.28

$$\frac{ac}{2\left(ad-bc\right)^2\left(d\,x^2+c\right)}-\frac{a\ln\left(b\,x^2+a\right)}{2\left(ad-bc\right)^2}+\frac{a\ln\left(d\,x^2+c\right)}{2\left(ad-bc\right)^2}-\frac{b\,c^2}{2\left(ad-bc\right)^2\left(d\,x^2+c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*a/(a*d-b*c)^2*ln(b*x^2+a)+1/2/(a*d-b*c)^2*a*ln(d*x^2+c)+1/2/(a*d-b*c)^
2*c/(d*x^2+c)*a-1/2/(a*d-b*c)^2*c^2/d/(d*x^2+c)*b
```

maxima [A] time = 1.11, size = 105, normalized size = 1.42

$$-\frac{a\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{a\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{c}{2\left(bc^2d-acd^2+\left(bcd^2-ad^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a
*d^3)*x^2)
```

mupad [B] time = 0.27, size = 173, normalized size = 2.34

$$-\frac{b\,c^2-c\,\left(a\,d-a\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}\right)+a\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}}{2\,a^2\,c\,d^3+2\,a^2\,d^4\,x^2-4\,a\,b\,c^2\,d^2-4\,a\,b\,c\,d^3\,x^2+2\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] -(b*c^2 - c*(a*d - a*d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*
c*x^2))*2i) + a*d^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b
*c*x^2))*2i)/(2*a^2*c*d^3 + 2*b^2*c^3*d + 2*a^2*d^4*x^2 + 2*b^2*c^2*d^2*x^2
 - 4*a*b*c^2*d^2 - 4*a*b*c*d^3*x^2)
```

sympy [B] time = 1.92, size = 253, normalized size = 3.42

$$\frac{a\log\left(x^2+\frac{-\frac{a^4d^3}{(ad-bc)^2}+\frac{3a^3bcd^2}{(ad-bc)^2}-\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d+\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2\left(ad-bc\right)^2}-\frac{a\log\left(x^2+\frac{\frac{a^4d^3}{(ad-bc)^2}-\frac{3a^3bcd^2}{(ad-bc)^2}+\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d-\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2\left(ad-bc\right)^2}+\frac{c}{2acd^2-2bc^2d+x^2\left(2ad^3-2bcd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] a*log(x**2 + (-a**4*d**3/(a*d - b*c)**2 + 3*a**3*b*c*d**2/(a*d - b*c)**2 -
3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a**2*d + a*b**3*c**3/(a*d - b*c)**2 + a
*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) - a*log(x**2 + (a**4*d**3/(a*d - b*c)**
2 - 3*a**3*b*c*d**2/(a*d - b*c)**2 + 3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a*
*2*d - a*b**3*c**3/(a*d - b*c)**2 + a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) +
c/(2*a*c*d**2 - 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2))
```

3.244 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=104

$$\frac{x}{2\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}\,(bc-ad)^2}$$

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {471, 522, 205}

$$\frac{x}{2\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x/(2*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(b*c - a*d)^2 + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqr
t[d]*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= \frac{x}{2(bc-ad)\left(c+dx^2\right)}-\frac{\int \frac{a-bx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2(bc-ad)}\\
&= \frac{x}{2(bc-ad)\left(c+dx^2\right)}-\frac{(ab)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}+\frac{(bc+ad)\int \frac{1}{c+dx^2}\,dx}{2(bc-ad)^2}\\
&= \frac{x}{2(bc-ad)\left(c+dx^2\right)}-\frac{\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^2}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,\sqrt{d}\,(bc-ad)^2}
\end{aligned}$$

Mathematica [**A**] time = 0.13, size = 90, normalized size = 0.87

$$\frac{\frac{x(bc-ad)}{c+dx^2}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}}-2\sqrt{a}\,\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (((b*c - a*d)*x)/(c + d*x^2) - 2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]
] + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*(b*c -
a*d)^2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [**A**] time = 1.05, size = 705, normalized size = 6.78

$$\left[\frac{2\left(cd^2x^2+c^2d\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\left(bc^2d-acd^2\right)x}{4\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}, \frac{\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+\left(cd^2x^2+c^2d\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+\left(bc^2d-acd^2\right)x}{2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}, -\frac{4\left(cd^2x^2+c^2d\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)-2\left(bc^2d-acd^2\right)x}{4\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}, -\frac{2\left(cd^2x^2+c^2d\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-\left(bc^2d-acd^2\right)x}{2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2
*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a
*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2),
1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c)
+ (c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)) + (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^
2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2), -1/4*(4*(c*d^2*x^2 + c^2*d)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) + (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(
-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b*c^2*d - a*c*d^2)
*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2), -1/2*(2*(c*d^2*x^2 + c^2*d)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
) - (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*
c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

giac [A] time = 0.32, size = 110, normalized size = 1.06

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}+\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{x}{2\left(dx^2+c\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/
2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*x/((d*x^2 + c)*(b*c - a*d))
```

maple [A] time = 0.01, size = 134, normalized size = 1.29

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}}-\frac{adx}{2\,(ad-bc)^2\left(d\,x^2+c\right)}+\frac{ad\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(ad-bc)^2\sqrt{cd}}+\frac{bcx}{2\,(ad-bc)^2\left(d\,x^2+c\right)}+\frac{bc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(ad-bc)^2\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -b/(a*d-b*c)^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/2/(a*d-b*c)^2*x/(d
*x^2+c)*a*d+1/2/(a*d-b*c)^2*x/(d*x^2+c)*b*c+1/2/(a*d-b*c)^2/(c*d)^(1/2)*arc
```

```
tan(1/(c*d)^(1/2)*d*x)*a*d+1/2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)
*d*x)*b*c
```

maxima [**A**] time = 2.45, size = 119, normalized size = 1.14

$$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}+\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{x}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/
2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*x/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

mupad [**B**] time = 0.78, size = 3154, normalized size = 30.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (atan(-(((-a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*
a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16
*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 3
2*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)) - (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - ((-a
*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^
5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2
 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^
2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)) + (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(((a^2*b^3*d^2)/2
 + (a*b^4*c*d)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((-
a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d
^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^
2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c
^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d)) - (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + ((-a*b)^(1/2)*(((-
```

```
a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*
c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2)) + (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^
4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*
(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(-a*b)^(1/2)*1i)/(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d) - x/(2*(c + d*x^2)*(a*d - b*c)) + (atan((((-c*d)^(1/2)*((x*
(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)) - ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^5 -
 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2) - (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^
5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b
^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d^3 + b^2*c^3*d - 2*
a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a
*d + b*c)*1i)/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)) + ((-c*d)^(1/2)*(
(x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)) + ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*b^3*c*d^
5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2) + (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7
*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^
3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d^3 + b^2*c^3*d -
 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))
*(a*d + b*c)*1i)/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))/(((a^2*b^3*d^
2)/2 + (a*b^4*c*d)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) -
 ((-c*d)^(1/2)*((x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)) - ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2
- 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*
b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5
*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d
^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d -
2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2)) +
 ((-c*d)^(1/2)*((x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(2*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)) + ((-c*d)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2
- 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (x*(-c*d)^(1/2)*(a*d + b*c)*(16*a^5*
b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5
*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*c*d
^3 + b^2*c^3*d - 2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d -
2*a*b*c^2*d^2)))*(a*d + b*c))/(4*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2))))
*(-c*d)^(1/2)*(a*d + b*c)*1i)/(2*(a^2*c*d^3 + b^2*c^3*d - 2*a*b*c^2*d^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.245 $\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=70

$$\frac{1}{2(c+dx^2)(bc-ad)}+\frac{b\log(a+bx^2)}{2(bc-ad)^2}-\frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

Rubi [**A**] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 44}

$$\frac{1}{2(c+dx^2)(bc-ad)}+\frac{b\log(a+bx^2)}{2(bc-ad)^2}-\frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (b*L
og[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right)dx, x,\right.\\ &= \frac{1}{2(bc-ad)\left(c+dx^2\right)}+\frac{b\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{b\log\left(c+dx^2\right)}{2(bc-ad)^2}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 66, normalized size = 0.94

$$\frac{b\left(c+dx^2\right)\log\left(a+bx^2\right)-ad-b\left(c+dx^2\right)\log\left(c+dx^2\right)+bc}{2\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x^2)*Log[a + b*x^2] - b*(c + d*x^2)*Log[c + d*x^2])/(
2*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [**A**] time = 0.79, size = 103, normalized size = 1.47

$$\frac{bc-ad+\left(bdx^2+bc\right)\log\left(bx^2+a\right)-\left(bdx^2+bc\right)\log\left(dx^2+c\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c - a*d + (b*d*x^2 + b*c)*log(b*x^2 + a) - (b*d*x^2 + b*c)*log(d*x^2
 + c))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*x^2)
```

giac [A]   time = 0.34, size = 85, normalized size = 1.21

$$\frac{bd\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}+\frac{d}{2\left(bcd-ad^2\right)\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*d*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) + 1/2*d/((b*c*d - a*d^2)*(d*x^2 + c))
```

maple [A]   time = 0.01, size = 90, normalized size = 1.29

$$-\frac{ad}{2\left(ad-bc\right)^2\left(d\,x^2+c\right)}+\frac{bc}{2\left(ad-bc\right)^2\left(d\,x^2+c\right)}+\frac{b\ln\left(b\,x^2+a\right)}{2\left(ad-bc\right)^2}-\frac{b\ln\left(d\,x^2+c\right)}{2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b/(a*d-b*c)^2*ln(b*x^2+a)-1/2/(a*d-b*c)^2*b*ln(d*x^2+c)-1/2*d/(a*d-b*c)
^2/(d*x^2+c)*a+1/2/(a*d-b*c)^2/(d*x^2+c)*b*c
```

maxima [A]   time = 1.03, size = 99, normalized size = 1.41

$$\frac{b\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{b\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{1}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*b*log(d*x^2 + c)
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2
)
```

mupad [B]   time = 0.27, size = 160, normalized size = 2.29

$$\frac{-a\,d+c\,\left(b+b\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}\right)+b\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}}{2\,a^2\,c\,d^2+2\,a^2\,d^3\,x^2-4\,a\,b\,c^2\,d-4\,a\,b\,c\,d^2\,x^2+2\,b^2\,c^3+2\,b^2\,c^2\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (c*(b + b*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i) -
 a*d + b*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*
2i)/(2*b^2*c^3 + 2*a^2*c*d^2 + 2*a^2*d^3*x^2 + 2*b^2*c^2*d*x^2 - 4*a*b*c^2*
d - 4*a*b*c*d^2*x^2)
```

sympy [B] time = 1.89, size = 248, normalized size = 3.54

$$-\frac{b\log\left(x^2+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2\,(ad-bc)^2}+\frac{b\log\left(x^2+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2\,(ad-bc)^2}-\frac{1}{2acd-2bc^2+x^2\left(2ad^2-2bcd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] -b*log(x**2 + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)
**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b
**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2) + b*log(x**2 + (a**3*b*d**3/(a*d - b*
c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2
+ a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2)
 - 1/(2*a*c*d - 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d))
```

3.246 $\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2} \, dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= -\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2c(bc-ad)}\\ &= -\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^2}-\frac{(d(3bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}\\ &= -\frac{dx}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^2}-\frac{\sqrt{d}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.87

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}\,(ad-3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{dx(ad-bc)}{c\left(c+dx^2\right)}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [A] time = 1.31, size = 711, normalized size = 6.52

$$\left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)}, -\frac{\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\left(bcd-ad^2\right)x}{2\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)}, \frac{4\left(bcdx^2+bc^2\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)}, \frac{2\left(bcdx^2+bc^2\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\left(3bc^2-acd+\left(3bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(bcd-ad^2\right)x}{2\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x
^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1
/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
 - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*
log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

giac [A] time = 0.36, size = 122, normalized size = 1.12

$$\frac{b^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{2}c^{2} - 2\,abcd + a^{2}d^{2}\right)\sqrt{ab}} - \frac{\left(3\,bcd - ad^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^{2}c^{3} - 2\,abc^{2}d + a^{2}cd^{2}\right)\sqrt{cd}} - \frac{dx}{2\left(bc^{2} - acd\right)\left(dx^{2} + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

maple [A] time = 0.01, size = 144, normalized size = 1.32

$$\frac{a\,d^{2}x}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)c} + \frac{a\,d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{2}\sqrt{cd}\;c} + \frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^{2}\sqrt{ab}} - \frac{bdx}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)} - \frac{3bd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{2}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+1/2*d^2/(a*d-b*c)^2/c
*x/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x/(d*x^2+c)*b+1/2*d^2/(a*d-b*c)^2/c/(c*d)^
```

```
(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a-3/2*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(1/(c
*d)^(1/2)*d*x)*b
```

maxima [A] time = 2.35, size = 133, normalized size = 1.22

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{dx}{2\left(bc^3-ac^2d+\left(bc^2d-acd^2\right)x^2\right)}-\frac{\left(3\,bcd-ad^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

mupad [B] time = 0.98, size = 3637, normalized size = 33.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)*
((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7
+ 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 -
a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3
*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a
*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a
^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5
 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4
*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*
a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^
7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4
*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*
(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5 - a^3*c^2*d^3 + 3*a
^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*
d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*
```

```
c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 +
 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7
*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*
a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/
(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)) - ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^
3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) + (((4*b^7*c^
6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^
4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 -
3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6
*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 1
6*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2
*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c
^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))*(-c^3*
d)^(1/2)*(a*d - 3*b*c)*1i)/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) - (ata
n((((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 +
 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5
- a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (x*(-a*b^3)^(1/2)*(16*b
^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 4
8*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*
c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 +
a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^
4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*b^2*c^2 - 2
*a^2*b*c*d) - ((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*
b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(
2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^
(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*
(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*
b^2*c^2 - 2*a^2*b*c*d))/(((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d
^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3
*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (
x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 +
32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4
+ a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)
^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*
c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))))/(a^3*
d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - ((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5
 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-a*b^3)^(1/2)*((((4*b
^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a
^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^
3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*
```

```
d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5
*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^
2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
+ (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*
d^2 - 2*a*b*c^3*d))))/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2)*
1i)/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.247 $\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}-\frac{d}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\log(x)}{ac^2}$$

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}-\frac{d}{2c\left(c+dx^2\right)(bc-ad)}+\frac{\log(x)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -d/(2*c*(b*c - a*d)*(c + d*x^2)) + Log[x]/(a*c^2) - (b^2*Log[a + b*x^2])/(2
*a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\Bigg(\int \frac{1}{x(a+bx)(c+dx)^2}\,dx,x,x^2\Bigg)\\
&= \frac{1}{2}\,\mathrm{Subst}\Bigg(\int \Bigg(\frac{1}{ac^2x}-\frac{b^3}{a(-bc+ad)^2(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)^2}+\frac{d^2(2bc-}{c^2(bc-ad)^2(}\\
&= -\frac{d}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\log(x)}{ac^2}-\frac{b^2\log\left(a+bx^2\right)}{2a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.98

$$\frac{1}{2}\left(-\frac{b^2\log\left(a+bx^2\right)}{a(bc-ad)^2}+\frac{d(2bc-ad)\log\left(c+dx^2\right)}{c^2(bc-ad)^2}-\frac{d}{c\left(c+dx^2\right)(bc-ad)}+\frac{2\log(x)}{ac^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-(d/(c*(b*c - a*d)*(c + d*x^2))) + (2*Log[x])/(a*c^2) - (b^2*Log[a + b*x^2
])/(a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^2)
)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [B] time = 2.65, size = 219, normalized size = 2.19

$$-\frac{abc^2d-a^2cd^2+\left(b^2c^2dx^2+b^2c^3\right)\log\left(bx^2+a\right)-\left(2\,abc^2d-a^2cd^2+\left(2\,abcd^2-a^2d^3\right)x^2\right)\log\left(dx^2+c\right)-2\left(b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^2\right)\log(x)}{2\left(ab^2c^5-2\,a^2bc^4d+a^3c^3d^2+\left(ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c^2*d - a^2*c*d^2 + (b^2*c^2*d*x^2 + b^2*c^3)*log(b*x^2 + a) - (2
*a*b*c^2*d - a^2*c*d^2 + (2*a*b*c*d^2 - a^2*d^3)*x^2)*log(d*x^2 + c) - 2*(b
```

```
^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)
*log(x))/(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2 + (a*b^2*c^4*d - 2*a^2*b*
c^3*d^2 + a^3*c^2*d^3)*x^2)
```

giac [A] time = 0.39, size = 185, normalized size = 1.85

$$-\frac{b^3\log\left(\left|bx^2+a\right|\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}+\frac{\left(2\,bcd^2-ad^3\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)}-\frac{2\,bcd^2x^2-ad^3x^2+3\,bc^2d-2\,acd^2}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\left(dx^2+c\right)}+\frac{\log\left(x^2\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(abs(b*x^2 + a))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2) + 1/2*
(2*b*c*d^2 - a*d^3)*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3) - 1/2*(2*b*c*d^2*x^2 - a*d^3*x^2 + 3*b*c^2*d - 2*a*c*d^2)/((b^2*c^4
- 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)) + 1/2*log(x^2)/(a*c^2)
```

maple [A] time = 0.02, size = 139, normalized size = 1.39

$$\frac{a\,d^2}{2\,(ad-bc)^2\left(d\,x^2+c\right)c}-\frac{a\,d^2\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^2\,c^2}-\frac{b^2\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^2\,a}+\frac{bd\ln\left(d\,x^2+c\right)}{(ad-bc)^2\,c}-\frac{bd}{2\,(ad-bc)^2\left(d\,x^2+c\right)}+\frac{\ln(x)}{a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b^2/a/(a*d-b*c)^2*ln(b*x^2+a)-1/2*d^2/c^2/(a*d-b*c)^2*ln(d*x^2+c)*a+d/
c/(a*d-b*c)^2*ln(d*x^2+c)*b+1/2*d^2/c/(a*d-b*c)^2/(d*x^2+c)*a-1/2*d/(a*d-b*
c)^2/(d*x^2+c)*b+ln(x)/a/c^2
```

maxima [A] time = 0.99, size = 138, normalized size = 1.38

$$-\frac{b^2\log\left(bx^2+a\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)}+\frac{\left(2\,bcd-ad^2\right)\log\left(dx^2+c\right)}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)}-\frac{d}{2\left(bc^3-ac^2d+\left(bc^2d-acd^2\right)x^2\right)}+\frac{\log\left(x^2\right)}{2\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/2*(2*b*c*d
- a*d^2)*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/2*d/(b*c^
3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/2*log(x^2)/(a*c^2)
```

mupad [B] time = 0.71, size = 127, normalized size = 1.27

$$\frac{\ln(x)}{a\,c^2}-\frac{\ln\left(d\,x^2+c\right)\,\left(a\,d^2-2\,b\,c\,d\right)}{2\,a^2\,c^2\,d^2-4\,a\,b\,c^3\,d+2\,b^2\,c^4}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3\,d^2-4\,a^2\,b\,c\,d+2\,a\,b^2\,c^2}+\frac{d}{2\,c\,\left(d\,x^2+c\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] log(x)/(a*c^2) - (log(c + d*x^2)*(a*d^2 - 2*b*c*d))/(2*b^2*c^4 + 2*a^2*c^2*
d^2 - 4*a*b*c^3*d) - (b^2*log(a + b*x^2))/(2*a^3*d^2 + 2*a*b^2*c^2 - 4*a^2*
b*c*d) + d/(2*c*(c + d*x^2)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.248 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2} \, dx$

Optimal. Leaf size=144

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}-\frac{2bc-3ad}{2ac^2x(bc-ad)}-\frac{d}{2cx\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(2*b*c - 3*a*d)/(2*a*c^2*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x*(c + d*x^2)
) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^2) + (d^(3/2
)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = -\frac{d}{2c(bc-ad)x\left(c+dx^2\right)}+\frac{\int \frac{2bc-3ad-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$= -\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x\left(c+dx^2\right)}-\frac{\int \frac{2b^2c^2+2abcd-3a^2d^2+bd(2bc-3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2ac^2(bc-ad)}$$

$$= -\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x\left(c+dx^2\right)}-\frac{b^3\int \frac{1}{a+bx^2}\,dx}{a(bc-ad)^2}+\frac{\left(d^2(5bc-3ad)\right)\int}{2c^2(bc-ad)}$$

$$= -\frac{2bc-3ad}{2ac^2(bc-ad)x}-\frac{d}{2c(bc-ad)x\left(c+dx^2\right)}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2}+\frac{d^{3/2}(5bc-3ad)}{2c^{5/2}(bc}$$

Mathematica [A] time = 0.26, size = 123, normalized size = 0.85

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^2}+\frac{d^{3/2}(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}+\frac{d^2x}{2c^2\left(c+dx^2\right)(bc-ad)}-\frac{1}{ac^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(1/(a*c^2*x)) + (d^2*x)/(2*c^2*(b*c - a*d)*(c + d*x^2)) - (b^(5/2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*(5*b*c - 3*a*d)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a+bx^2\right) \left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [A] time = 1.52, size = 1005, normalized size = 6.98

$$\left[-\frac{4\,b^{2}c^{3} - 8\,abc^{2}d + 4\,a^{2}cd^{2} + 2\left(2\,b^{2}c^{2}d - 5\,abcd^{2} + 3\,a^{2}d^{3}\right)x^{2} - 2\left(b^{2}c^{2}dx^{3} + b^{2}c^{3}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2} - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^{2} + a}\right) + \left(\left(5\,abcd^{2} - 3\,a^{2}d^{3}\right)x^{3} + \left(5\,abc^{2}d - 3\,a^{2}cd^{2}\right)x\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2} - 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^{2} + c}\right)}{4\left(\left(ab^{2}c^{4}d - 2\,a^{2}bc^{3}d^{2} + a^{3}c^{2}d^{3}\right)x^{3} + \left(ab^{2}c^{5} - 2\,a^{2}bc^{4}d + a^{3}c^{3}d^{2}\right)x\right)}, -\frac{2\,b^{2}c^{3} - 4\,abc^{2}d + 2\,a^{2}cd^{2} + \left(2\,b^{2}c^{2}d - 5\,abcd^{2} + 3\,a^{2}d^{3}\right)x^{2} - \left(\left(5\,abcd^{2} - 3\,a^{2}d^{3}\right)x^{3} + \left(5\,abc^{2}d - 3\,a^{2}cd^{2}\right)x\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) - \left(b^{2}c^{2}dx^{3} + b^{2}c^{3}x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2} - 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^{2} + a}\right)}{2\left(\left(ab^{2}c^{4}d - 2\,a^{2}bc^{3}d^{2} + a^{3}c^{2}d^{3}\right)x^{3} + \left(ab^{2}c^{5} - 2\,a^{2}bc^{4}d + a^{3}c^{3}d^{2}\right)x\right)}, -\frac{4\,b^{2}c^{3} - 8\,abc^{2}d + 4\,a^{2}cd^{2} + 2\left(2\,b^{2}c^{2}d - 5\,abcd^{2} + 3\,a^{2}d^{3}\right)x^{2} + 4\left(b^{2}c^{2}dx^{3} + b^{2}c^{3}x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + \left(\left(5\,abcd^{2} - 3\,a^{2}d^{3}\right)x^{3} + \left(5\,abc^{2}d - 3\,a^{2}cd^{2}\right)x\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^{2} - 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^{2} + c}\right)}{4\left(\left(ab^{2}c^{4}d - 2\,a^{2}bc^{3}d^{2} + a^{3}c^{2}d^{3}\right)x^{3} + \left(ab^{2}c^{5} - 2\,a^{2}bc^{4}d + a^{3}c^{3}d^{2}\right)x\right)}, -\frac{2\,b^{2}c^{3} - 4\,abc^{2}d + 2\,a^{2}cd^{2} + \left(2\,b^{2}c^{2}d - 5\,abcd^{2} + 3\,a^{2}d^{3}\right)x^{2} + 2\left(b^{2}c^{2}dx^{3} + b^{2}c^{3}x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - \left(\left(5\,abcd^{2} - 3\,a^{2}d^{3}\right)x^{3} + \left(5\,abc^{2}d - 3\,a^{2}cd^{2}\right)x\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{2\left(\left(ab^{2}c^{4}d - 2\,a^{2}bc^{3}d^{2} + a^{3}c^{2}d^{3}\right)x^{3} + \left(ab^{2}c^{5} - 2\,a^{2}bc^{4}d + a^{3}c^{3}d^{2}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2
 + 3*a^2*d^3)*x^2 - 2*(b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5*a*b
*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x
^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -
 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d
^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 - ((5*a*b*c*d^2 - 3*a^2*d^
3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b^
2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -
 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d
^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 4*(b^2*c^2*d*x^3 + b^2
*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5
*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c
^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2
*c*d^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 2*(b^2*c^2*d*x^3 + b
^2*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 +
(5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a*b^2*c^4*d
 - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^
3*d^2)*x)]
```

giac [A] time = 0.38, size = 164, normalized size = 1.14

$$-\frac{b^{3} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^{2}c^{2} - 2\,a^{2}bcd + a^{3}d^{2}\right)\sqrt{ab}} + \frac{\left(5\,bcd^{2} - 3\,ad^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^{2}c^{4} - 2\,abc^{3}d + a^{2}c^{2}d^{2}\right)\sqrt{cd}} - \frac{2\,bcdx^{2} - 3\,ad^{2}x^{2} + 2\,bc^{2} - 2\,acd}{2\left(abc^{3} - a^{2}c^{2}d\right)\left(dx^{3} + cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
+ 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c*d*x^2 - 3*a*d^2*x^2 + 2*b*c^2 - 2*a*c
*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^3 + c*x))
```

maple [A] time = 0.02, size = 169, normalized size = 1.17

$$-\frac{a\,d^{3}x}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)c^{2}}-\frac{3a\,d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{2}\sqrt{cd}\;c^{2}}-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^{2}\sqrt{ab}\;a}+\frac{b\,d^{2}x}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)c}+\frac{5b\,d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{2}\sqrt{cd}\;c}-\frac{1}{a\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/a*b^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/2*d^3/c^2/(a*d
-b*c)^2*x/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2*x/(d*x^2+c)*b-3/2*d^3/c^2/(a*d-
b*c)^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a+5/2*d^2/c/(a*d-b*c)^2/(c*d)^
(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b-1/a/c^2/x
```

maxima [A] time = 2.30, size = 178, normalized size = 1.24

$$-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\sqrt{ab}}+\frac{\left(5\,bcd^{2}-3\,ad^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\sqrt{cd}}-\frac{2\,bc^{2}-2\,acd+\left(2\,bcd-3\,ad^{2}\right)x^{2}}{2\left(\left(abc^{3}d-a^{2}c^{2}d^{2}\right)x^{3}+\left(abc^{4}-a^{2}c^{3}d\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
+ 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c^2 - 2*a*c*d + (2*b*c*d - 3*a*d^2)*x^2
)/((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)
```

mupad [B] time = 0.83, size = 432, normalized size = 3.00

$$-\frac{\frac{1}{a\,c}+\frac{x^{2}\left(3\,a\,d^{2}-2\,b\,c\,d\right)}{2\,a\,c^{2}\left(a\,d-b\,c\right)}}{d\,x^{3}+c\,x}+\frac{\mathrm{atan}\left(\frac{b\,c^{5}\,x\left(-a^{3}\,b^{5}\right)^{3/2}\,4\mathrm{i}+a^{8}\,b\,d^{5}\,x\,\sqrt{-a^{3}\,b^{5}}\,9\mathrm{i}+a^{6}\,b^{3}\,c^{2}\,d^{3}\,x\,\sqrt{-a^{3}\,b^{5}}\,25\mathrm{i}-a^{7}\,b^{2}\,c\,d^{4}\,x\,\sqrt{-a^{3}\,b^{5}}\,30\mathrm{i}}{-9\,a^{10}\,b^{3}\,d^{5}+30\,a^{9}\,b^{4}\,c\,d^{4}-25\,a^{8}\,b^{5}\,c^{2}\,d^{3}+4\,a^{5}\,b^{8}\,c^{5}}\right)\sqrt{-a^{3}\,b^{5}}\,1\mathrm{i}}{a^{5}\,d^{2}-2\,a^{4}\,b\,c\,d+a^{3}\,b^{2}\,c^{2}}+\frac{\mathrm{atan}\left(\frac{a^{5}\,d^{3}\,x\left(-c^{5}\,d^{3}\right)^{3/2}\,9\mathrm{i}+b^{5}\,c^{10}\,d\,x\,\sqrt{-c^{5}\,d^{3}}\,4\mathrm{i}-a^{4}\,b\,c\,d^{2}\,x\left(-c^{5}\,d^{3}\right)^{3/2}\,30\mathrm{i}+a^{3}\,b^{2}\,c^{2}\,d\,x\left(-c^{5}\,d^{3}\right)^{3/2}\,25\mathrm{i}}{9\,a^{5}\,c^{8}\,d^{7}-30\,a^{4}\,b\,c^{9}\,d^{6}+25\,a^{3}\,b^{2}\,c^{10}\,d^{5}-4\,b^{5}\,c^{13}\,d^{2}}\right)\left(3\,a\,d-5\,b\,c\right)\sqrt{-c^{5}\,d^{3}}\,1\mathrm{i}}{2\left(a^{2}\,c^{5}\,d^{2}-2\,a\,b\,c^{6}\,d+b^{2}\,c^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (atan((b*c^5*x*(-a^3*b^5)^(3/2)*4i + a^8*b*d^5*x*(-a^3*b^5)^(1/2)*9i + a^6*
b^3*c^2*d^3*x*(-a^3*b^5)^(1/2)*25i - a^7*b^2*c*d^4*x*(-a^3*b^5)^(1/2)*30i)/
(4*a^5*b^8*c^5 - 9*a^10*b^3*d^5 + 30*a^9*b^4*c*d^4 - 25*a^8*b^5*c^2*d^3))*(
-a^3*b^5)^(1/2)*1i)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (1/(a*c) + (x^2
*(3*a*d^2 - 2*b*c*d))/(2*a*c^2*(a*d - b*c)))/(c*x + d*x^3) + (atan((a^5*d^3
*x*(-c^5*d^3)^(3/2)*9i + b^5*c^10*d*x*(-c^5*d^3)^(1/2)*4i - a^4*b*c*d^2*x*(
-c^5*d^3)^(3/2)*30i + a^3*b^2*c^2*d*x*(-c^5*d^3)^(3/2)*25i)/(9*a^5*c^8*d^7
- 4*b^5*c^13*d^2 - 30*a^4*b*c^9*d^6 + 25*a^3*b^2*c^10*d^5))*(3*a*d - 5*b*c)
*(-c^5*d^3)^(1/2)*1i)/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.249 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=126

$$\frac{b^3\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}-\frac{d^2(3bc-2ad)\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}+\frac{d^2}{2c^2\left(c+dx^2\right)(bc-ad)}-\frac{1}{2ac^2x^2}$$

Rubi [A] time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{b^3\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{\log(x)(2ad+bc)}{a^2c^3}+\frac{d^2}{2c^2\left(c+dx^2\right)(bc-ad)}-\frac{d^2(3bc-2ad)\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}-\frac{1}{2ac^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/(2*a*c^2*x^2) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c + 2*a*d)*Log
[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*(3*b*c -
 2*a*d)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ac^2x^2} + \frac{-bc-2ad}{a^2c^3x} + \frac{b^4}{a^2(-bc+ad)^2(a+bx)} - \frac{d^3}{c^2(bc-ad)(c+dx)^2}\right.\right.$$

$$= -\frac{1}{2ac^2x^2} + \frac{d^2}{2c^2(bc-ad)\left(c+dx^2\right)} - \frac{(bc+2ad)\log(x)}{a^2c^3} + \frac{b^3\log\left(a+bx^2\right)}{2a^2(bc-ad)^2} -$$

Mathematica [A] time = 0.24, size = 117, normalized size = 0.93

$$\frac{1}{2}\left(\frac{b^3\log\left(a+bx^2\right)}{a^2(bc-ad)^2} - \frac{2\log(x)(2ad+bc)}{a^2c^3} + \frac{\frac{cd^2}{\left(c+dx^2\right)(bc-ad)} + \frac{d^2(2ad-3bc)\log\left(c+dx^2\right)}{(bc-ad)^2} - \frac{c}{ax^2}}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((-2*(b*c + 2*a*d)*Log[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(a^2*(b*c - a*d
)^2) + (-(c/(a*x^2)) + (c*d^2)/((b*c - a*d)*(c + d*x^2)) + (d^2*(-3*b*c + 2
*a*d)*Log[c + d*x^2])/(b*c - a*d)^2)/c^3)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [B] time = 4.99, size = 302, normalized size = 2.40

$$-\frac{ab^2c^4 - 2\,a^2bc^3d + a^3c^2d^2 + \left(ab^2c^3d - 3\,a^2bc^2d^2 + 2\,a^3cd^3\right)x^2 - \left(b^3c^3dx^4 + b^3c^4x^2\right)\log\left(bx^2+a\right) + \left(\left(3\,a^2bcd^3 - 2\,a^3d^4\right)x^4 + \left(3\,a^2bc^2d^2 - 2\,a^3cd^3\right)x^2\right)\log\left(dx^2+c\right) + 2\left(\left(b^3c^3d - 3\,a^2bcd^3 + 2\,a^3d^4\right)x^4 + \left(b^3c^4 - 3\,a^2bc^2d^2 + 2\,a^3cd^3\right)x^2\right)\log(x)}{2\left(\left(a^2b^2c^5d - 2\,a^3bc^4d^2 + a^4c^3d^3\right)x^4 + \left(a^2b^2c^6 - 2\,a^3bc^5d + a^4c^4d^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 3*a^2*b*c^2*
d^2 + 2*a^3*c*d^3)*x^2 - (b^3*c^3*d*x^4 + b^3*c^4*x^2)*log(b*x^2 + a) + ((3
*a^2*b*c*d^3 - 2*a^3*d^4)*x^4 + (3*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*log(d*
x^2 + c) + 2*((b^3*c^3*d - 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^4 + (b^3*c^4 - 3*a^
2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*log(x))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 +
 a^4*c^3*d^3)*x^4 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^2)
```

giac [B] time = 0.35, size = 257, normalized size = 2.04

$$\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}-\frac{\left(3\,bcd^3-2\,ad^4\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)}+\frac{b^3c^2dx^4+b^3c^3x^2-2\,ab^2c^2dx^2+6\,a^2bcd^2x^2-4\,a^3d^3x^2-2\,ab^2c^3+4\,a^2bc^2d-2\,a^3cd^2}{4\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(dx^4+cx^2\right)}-\frac{(bc+2\,ad)\log\left(x^2\right)}{2\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^4*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2) - 1/2
*(3*b*c*d^3 - 2*a*d^4)*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2
*c^3*d^3) + 1/4*(b^3*c^2*d*x^4 + b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 + 6*a^2*b*
c*d^2*x^2 - 4*a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(d*x^4 + c*x^2)) - 1/2*(b*c + 2*a*
d)*log(x^2)/(a^2*c^3)
```

maple [A] time = 0.02, size = 170, normalized size = 1.35

$$-\frac{a\,d^3}{2\,(ad-bc)^2\left(d\,x^2+c\right)c^2}+\frac{a\,d^3\ln\left(d\,x^2+c\right)}{(ad-bc)^2\,c^3}+\frac{b^3\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^2\,a^2}+\frac{b\,d^2}{2\,(ad-bc)^2\left(d\,x^2+c\right)c}-\frac{3b\,d^2\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^2\,c^2}-\frac{2d\ln(x)}{a\,c^3}-\frac{b\ln(x)}{a^2c^2}-\frac{1}{2a\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^3/a^2/(a*d-b*c)^2*ln(b*x^2+a)+d^3/c^3/(a*d-b*c)^2*ln(d*x^2+c)*a-3/2*d
^2/c^2/(a*d-b*c)^2*ln(d*x^2+c)*b-1/2*d^3/c^2/(a*d-b*c)^2/(d*x^2+c)*a+1/2*d^
2/c/(a*d-b*c)^2/(d*x^2+c)*b-1/2/a/c^2/x^2-2/a/c^3*ln(x)*d-1/a^2/c^2*ln(x)*b
```

maxima [A] time = 1.13, size = 188, normalized size = 1.49

$$\frac{b^3\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}-\frac{\left(3\,bcd^2-2\,ad^3\right)\log\left(dx^2+c\right)}{2\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)}-\frac{bc^2-acd+\left(bcd-2\,ad^2\right)x^2}{2\left(\left(abc^3d-a^2c^2d^2\right)x^4+\left(abc^4-a^2c^3d\right)x^2\right)}-\frac{(bc+2\,ad)\log\left(x^2\right)}{2\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/2*(3*b*c*d
^2 - 2*a*d^3)*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(b
```

```
*c^2 - a*c*d + (b*c*d - 2*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b
*c^4 - a^2*c^3*d)*x^2) - 1/2*(b*c + 2*a*d)*log(x^2)/(a^2*c^3)
```

mupad [B] time = 0.93, size = 171, normalized size = 1.36

$$\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,\left(a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2\right)}-\frac{\frac{1}{2\,a\,c}+\frac{x^2\,\left(2\,a\,d^2-b\,c\,d\right)}{2\,a\,c^2\,\left(a\,d-b\,c\right)}}{d\,x^4+c\,x^2}+\frac{\ln\left(d\,x^2+c\right)\,\left(2\,a\,d^3-3\,b\,c\,d^2\right)}{2\,a^2\,c^3\,d^2-4\,a\,b\,c^4\,d+2\,b^2\,c^5}-\frac{\ln(x)\,\left(2\,a\,d+b\,c\right)}{a^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (b^3*log(a + b*x^2))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (1/(2*a*c)
 + (x^2*(2*a*d^2 - b*c*d))/(2*a*c^2*(a*d - b*c)))/(c*x^2 + d*x^4) + (log(c
+ d*x^2)*(2*a*d^3 - 3*b*c*d^2))/(2*b^2*c^5 + 2*a^2*c^3*d^2 - 4*a*b*c^4*d) -
 (log(x)*(2*a*d + b*c))/(a^2*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.250 $\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}+\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-}$$

Rubi [**A**] time = 0.27, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$\frac{-5a^2d^2+2abcd+2b^2c^2}{2a^2c^3x(bc-ad)}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{2bc-5ad}{6ac^2x^3(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(2*b*c - 5*a*d)/(6*a*c^2*(b*c - a*d)*x^3) + (2*b^2*c^2 + 2*a*b*c*d - 5*a^2
*d^2)/(2*a^2*c^3*(b*c - a*d)*x) - d/(2*c*(b*c - a*d)*x^3*(c + d*x^2)) + (b^
(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^2) - (d^(5/2)*(7*b*
c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx=-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{\int \frac{2bc-5ad-5bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$=-\frac{2bc-5ad}{6ac^2(bc-ad)x^3}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}-\frac{\int \frac{3\left(2b^2c^2+2abcd-5a^2d^2\right)+3bd(2bc-5ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,d}{6ac^2(bc-ad)}$$

$$=-\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{\int \frac{3\left(2b^3c^3\right.}{}}{}$$

$$=-\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{b^4\int\frac{1}{a+b}}{a^2(bc-}$$

$$=-\frac{2bc-5ad}{6ac^2(bc-ad)x^3}+\frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x}-\frac{d}{2c(bc-ad)x^3\left(c+dx^2\right)}+\frac{b^{7/2}\tan}{a^{5/2}(bc}$$

Mathematica [A] time = 0.39, size = 142, normalized size = 0.75

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^2}+\frac{2ad+bc}{a^2c^3x}-\frac{d^{5/2}(7bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}-\frac{d^3x}{2c^3\left(c+dx^2\right)(bc-ad)}-\frac{1}{3ac^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/3*1/(a*c^2*x^3) + (b*c + 2*a*d)/(a^2*c^3*x) - (d^3*x)/(2*c^3*(b*c - a*d)
*(c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*
d)^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)*(b
*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)^2), x]
```

fricas [A] time = 3.28, size = 1281, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a^
2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 +
 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 +
(7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5
+ (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a
^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^
4 - 2*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 + 3*((7
*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d
/c)*arctan(x*sqrt(d/c)) - 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b
*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^
2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1
/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a^2*b
*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*
a^3*c*d^3)*x^2 - 12*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(b/a)*arctan(x*sqrt(b
/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)
*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^2
*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d
+ a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*
(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 2*(3*b^3*c^4 - a*b^2*c^3*d
- 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt
(b/a)*arctan(x*sqrt(b/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c
```

```
^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^2*c^5*d -
 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^
4*d^2)*x^3)]
```

giac [A]   time = 0.34, size = 165, normalized size = 0.87

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{ab}}-\frac{d^3x}{2\left(bc^4-ac^3d\right)\left(dx^2+c\right)}-\frac{\left(7\,bcd^3-5\,ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\sqrt{cd}}+\frac{3\,bcx^2+6\,adx^2-ac}{3\,a^2c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b))
 - 1/2*d^3*x/((b*c^4 - a*c^3*d)*(d*x^2 + c)) - 1/2*(7*b*c*d^3 - 5*a*d^4)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)) + 1/3
*(3*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^2*c^3*x^3)
```

maple [A]   time = 0.02, size = 191, normalized size = 1.01

$$\frac{a\,d^4x}{2\left(ad-bc\right)^2\left(d\,x^2+c\right)c^3}+\frac{5a\,d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^2\sqrt{cd}\;c^3}+\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^2\sqrt{ab}\;a^2}-\frac{b\,d^3x}{2\left(ad-bc\right)^2\left(d\,x^2+c\right)c^2}-\frac{7b\,d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^2\sqrt{cd}\;c^2}+\frac{2d}{a\,c^3x}+\frac{b}{a^2c^2x}-\frac{1}{3a\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/a^2*b^4/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+1/2*d^4/c^3/(a*
d-b*c)^2*x/(d*x^2+c)*a-1/2*d^3/c^2/(a*d-b*c)^2*x/(d*x^2+c)*b+5/2*d^4/c^3/(a
*d-b*c)^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a-7/2*d^3/c^2/(a*d-b*c)^2/(
c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b-1/3/a/c^2/x^3+2/a/c^3/x*d+1/a^2/c^2/
x*b
```

maxima [A]   time = 2.37, size = 236, normalized size = 1.25

$$\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{ab}}-\frac{\left(7\,bcd^3-5\,ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\sqrt{cd}}-\frac{2\,abc^3-2\,a^2c^2d-3\left(2\,b^2c^2d+2\,abcd^2-5\,a^2d^3\right)x^4-2\left(3\,b^2c^3+2\,abc^2d-5\,a^2cd^2\right)x^2}{6\left(\left(a^2bc^4d-a^3c^3d^2\right)x^5+\left(a^2bc^5-a^3c^4d\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b))
 - 1/2*(7*b*c*d^3 - 5*a*d^4)*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d
+ a^2*c^3*d^2)*sqrt(c*d)) - 1/6*(2*a*b*c^3 - 2*a^2*c^2*d - 3*(2*b^2*c^2*d +
 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 - 2*(3*b^2*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x
^2)/((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)
```

mupad [B] time = 0.94, size = 469, normalized size = 2.48

$$-\frac{\frac{1}{3\,a\,c}-\frac{x^2\,\left(5\,a\,d+3\,b\,c\right)}{3\,a^2\,c^2}+\frac{x^4\,\left(-5\,a^2\,d^3+2\,a\,b\,c\,d^2+2\,b^2\,c^2\,d\right)}{2\,a^2\,c^3\,\left(a\,d-b\,c\right)}}{d\,x^5+c\,x^3}-\frac{\mathrm{atan}\left(\frac{b\,c^7\,x\,{\left(-a^5\,b^7\right)}^{3/2}\,4\mathrm{i}+a^{12}\,b\,d^7\,x\,\sqrt{-a^5\,b^7}\,25\mathrm{i}+a^{10}\,b^3\,c^2\,d^5\,x\,\sqrt{-a^5\,b^7}\,49\mathrm{i}-a^{11}\,b^2\,c\,d^6\,x\,\sqrt{-a^5\,b^7}\,70\mathrm{i}}{-25\,a^{15}\,b^4\,d^7+70\,a^{14}\,b^5\,c\,d^6-49\,a^{13}\,b^6\,c^2\,d^5+4\,a^8\,b^{11}\,c^7}\right)\,\sqrt{-a^5\,b^7}\,1\mathrm{i}}{a^7\,d^2-2\,a^6\,b\,c\,d+a^5\,b^2\,c^2}-\frac{\mathrm{atan}\left(\frac{a^7\,d^3\,x\,{\left(-c^7\,d^5\right)}^{3/2}\,25\mathrm{i}+b^7\,c^{14}\,d\,x\,\sqrt{-c^7\,d^5}\,4\mathrm{i}-a^6\,b\,c\,d^2\,x\,{\left(-c^7\,d^5\right)}^{3/2}\,70\mathrm{i}+a^5\,b^2\,c^2\,d\,x\,{\left(-c^7\,d^5\right)}^{3/2}\,49\mathrm{i}}{25\,a^7\,c^{11}\,d^{10}-70\,a^6\,b\,c^{12}\,d^9+49\,a^5\,b^2\,c^{13}\,d^8-4\,b^7\,c^{18}\,d^3}\right)\,\left(5\,a\,d-7\,b\,c\right)\,\sqrt{-c^7\,d^5}\,1\mathrm{i}}{2\,\left(a^2\,c^7\,d^2-2\,a\,b\,c^8\,d+b^2\,c^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] - (1/(3*a*c) - (x^2*(5*a*d + 3*b*c))/(3*a^2*c^2) + (x^4*(2*b^2*c^2*d - 5*a^
2*d^3 + 2*a*b*c*d^2))/(2*a^2*c^3*(a*d - b*c)))/(c*x^3 + d*x^5) - (atan((b*c
^7*x*(-a^5*b^7)^(3/2)*4i + a^12*b*d^7*x*(-a^5*b^7)^(1/2)*25i + a^10*b^3*c^2
*d^5*x*(-a^5*b^7)^(1/2)*49i - a^11*b^2*c*d^6*x*(-a^5*b^7)^(1/2)*70i)/(4*a^8
*b^11*c^7 - 25*a^15*b^4*d^7 + 70*a^14*b^5*c*d^6 - 49*a^13*b^6*c^2*d^5))*(-a
^5*b^7)^(1/2)*1i)/(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d) - (atan((a^7*d^3*x*
(-c^7*d^5)^(3/2)*25i + b^7*c^14*d*x*(-c^7*d^5)^(1/2)*4i - a^6*b*c*d^2*x*(-c
^7*d^5)^(3/2)*70i + a^5*b^2*c^2*d*x*(-c^7*d^5)^(3/2)*49i)/(25*a^7*c^11*d^10
 - 4*b^7*c^18*d^3 - 70*a^6*b*c^12*d^9 + 49*a^5*b^2*c^13*d^8))*(5*a*d - 7*b*
c)*(-c^7*d^5)^(1/2)*1i)/(2*(b^2*c^9 + a^2*c^7*d^2 - 2*a*b*c^8*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.251 $\int \frac{x^5}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=116

$$-\frac{a^2}{4b^2\left(a+bx^2\right)^2(bc-ad)}+\frac{a(2bc-ad)}{2b^2\left(a+bx^2\right)(bc-ad)^2}+\frac{c^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{c^2\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Rubi [A] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2}{4b^2\left(a+bx^2\right)^2(bc-ad)}+\frac{a(2bc-ad)}{2b^2\left(a+bx^2\right)(bc-ad)^2}+\frac{c^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{c^2\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] -a^2/(4*b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(2*b^2*(b*c - a*
d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (c^2*Log[c + d
*x^2])/(2*(b*c - a*d)^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2}+\frac{bc^2}{(bc-ad)^3(a+bx)}-\right.\right.\\ &= -\frac{a^2}{4b^2(bc-ad)\left(a+bx^2\right)^2}+\frac{a(2bc-ad)}{2b^2(bc-ad)^2\left(a+bx^2\right)}+\frac{c^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{c^2\log\left(c\right.}{2(bc-}\end{aligned}$$

Mathematica [A] time = 0.10, size = 99, normalized size = 0.85

$$\frac{-\frac{a^2(bc-ad)^2}{b^2\left(a+bx^2\right)^2}+\frac{2a(ad-2bc)(ad-bc)}{b^2\left(a+bx^2\right)}+2c^2\log\left(a+bx^2\right)-2c^2\log\left(c+dx^2\right)}{4(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (-((a^2*(b*c - a*d)^2)/(b^2*(a + b*x^2)^2)) + (2*a*(-2*b*c + a*d)*(-(b*c) +
 a*d))/(b^2*(a + b*x^2)) + 2*c^2*Log[a + b*x^2] - 2*c^2*Log[c + d*x^2])/(4*
(b*c - a*d)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^5/((a + b*x^2)^3*(c + d*x^2)), x]
```

fricas [B] time = 0.90, size = 290, normalized size = 2.50

$$\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\left(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2\right)x^2+2\left(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2\right)\log\left(bx^2+a\right)-2\left(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2\right)\log\left(dx^2+c\right)}{4\left(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3+\left(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3\right)x^4+2\left(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^2 + 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(b*x
^2 + a) - 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(d*x^2 + c))/(
a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3 + (b^7*c^3 -
3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 2*(a*b^6*c^3 - 3*a^2*b
^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2)
```

giac [B] time = 0.33, size = 232, normalized size = 2.00

$$\frac{bc^{2}\log\left(\left|bx^{2}+a\right|\right)}{2\left(b^{4}c^{3}-3\,ab^{3}c^{2}d+3\,a^{2}b^{2}cd^{2}-a^{3}bd^{3}\right)}-\frac{c^{2}d\log\left(\left|dx^{2}+c\right|\right)}{2\left(b^{3}c^{3}d-3\,ab^{2}c^{2}d^{2}+3\,a^{2}bcd^{3}-a^{3}d^{4}\right)}-\frac{3\,b^{4}c^{2}x^{4}+2\,ab^{3}c^{2}x^{2}+6\,a^{2}b^{2}cdx^{2}-2\,a^{3}bd^{2}x^{2}+4\,a^{3}bcd-a^{4}d^{2}}{4\left(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3}\right)\left(bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*c^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
a^3*b*d^3) - 1/2*c^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4) - 1/4*(3*b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + 6*a^2*b^2*
c*d*x^2 - 2*a^3*b*d^2*x^2 + 4*a^3*b*c*d - a^4*d^2)/((b^5*c^3 - 3*a*b^4*c^2*
d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(b*x^2 + a)^2)
```

maple [B] time = 0.01, size = 218, normalized size = 1.88

$$\frac{a^{4}d^{2}}{4\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)^{2}b^{2}}-\frac{a^{3}cd}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)^{2}b}+\frac{a^{2}c^{2}}{4\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)^{2}}-\frac{a^{3}d^{2}}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)b^{2}}+\frac{3a^{2}cd}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)b}-\frac{a\,c^{2}}{\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)}-\frac{c^{2}\ln\left(b\,x^{2}+a\right)}{2\left(ad-bc\right)^{3}}+\frac{c^{2}\ln\left(d\,x^{2}+c\right)}{2\left(ad-bc\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] -1/2/(a*d-b*c)^3*a^3/b^2/(b*x^2+a)*d^2+3/2/(a*d-b*c)^3*a^2/b/(b*x^2+a)*c*d-
1/(a*d-b*c)^3*a/(b*x^2+a)*c^2+1/4/(a*d-b*c)^3*a^4/b^2/(b*x^2+a)^2*d^2-1/2/(
a*d-b*c)^3*a^3/b/(b*x^2+a)^2*c*d+1/4/(a*d-b*c)^3*a^2/(b*x^2+a)^2*c^2-1/2/(a
*d-b*c)^3*c^2*ln(b*x^2+a)+1/2*c^2/(a*d-b*c)^3*ln(d*x^2+c)
```

maxima [B] time = 1.23, size = 236, normalized size = 2.03

$$\frac{c^{2}\log\left(bx^{2}+a\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{c^{2}\log\left(dx^{2}+c\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{3\,a^{2}bc-a^{3}d+2\left(2\,ab^{2}c-a^{2}bd\right)x^{2}}{4\left(a^{2}b^{4}c^{2}-2\,a^{3}b^{3}cd+a^{4}b^{2}d^{2}+\left(b^{6}c^{2}-2\,ab^{5}cd+a^{2}b^{4}d^{2}\right)x^{4}+2\left(ab^{5}c^{2}-2\,a^{2}b^{4}cd+a^{3}b^{3}d^{2}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*c^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/4*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^2)/(a^2*b^4*c^2 - 2*
```

```
a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 2*(
a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2)
```

mupad [B] time = 0.52, size = 370, normalized size = 3.19

$$\frac{b^3\left(4\,a\,c^2\,x^2+a\,c^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}\right)+b\,\left(2\,a^3\,d^2\,x^2-4\,a^3\,c\,d\right)+a^4\,d^2+b^2\,\left(3\,a^2\,c^2-6\,a^2\,c\,d\,x^2+a^2\,c^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}\right)+b^4\,c^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}}{-4\,a^5\,b^2\,d^3+12\,a^4\,b^3\,c\,d^2-8\,a^4\,b^3\,d^3\,x^2-12\,a^3\,b^4\,c^2\,d+24\,a^3\,b^4\,c\,d^2\,x^2-4\,a^3\,b^4\,d^3\,x^4+4\,a^2\,b^5\,c^3-24\,a^2\,b^5\,c^2\,d\,x^2+12\,a^2\,b^5\,c\,d^2\,x^4+8\,a\,b^6\,c^3\,x^2-12\,a\,b^6\,c^2\,d\,x^4+4\,b^7\,c^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^3*(c + d*x^2)),x)
```

```
[Out] (b^3*(4*a*c^2*x^2 + a*c^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x
^2 + b*c*x^2))*8i) + b*(2*a^3*d^2*x^2 - 4*a^3*c*d) + a^4*d^2 + b^2*(3*a^2*c
^2 + a^2*c^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i
 - 6*a^2*c*d*x^2) + b^4*c^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d
*x^2 + b*c*x^2))*4i)/(4*a^2*b^5*c^3 - 4*a^5*b^2*d^3 + 4*b^7*c^3*x^4 - 12*a^
3*b^4*c^2*d + 12*a^4*b^3*c*d^2 + 8*a*b^6*c^3*x^2 - 8*a^4*b^3*d^3*x^2 - 4*a^
3*b^4*d^3*x^4 - 12*a*b^6*c^2*d*x^4 - 24*a^2*b^5*c^2*d*x^2 + 24*a^3*b^4*c*d^
2*x^2 + 12*a^2*b^5*c*d^2*x^4)
```

sympy [B] time = 3.26, size = 418, normalized size = 3.60

$$\frac{c^2\log\left(x^2+\frac{-\frac{a^4c^2d^4}{(ad-bc)^3}+\frac{4a^3bc^3d^3}{(ad-bc)^3}-\frac{6a^2b^2c^4d^2}{(ad-bc)^3}+\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d-\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2\left(ad-bc\right)^3}-\frac{c^2\log\left(x^2+\frac{\frac{a^4c^2d^4}{(ad-bc)^3}-\frac{4a^3bc^3d^3}{(ad-bc)^3}+\frac{6a^2b^2c^4d^2}{(ad-bc)^3}-\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d+\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2\left(ad-bc\right)^3}+\frac{-a^3d+3a^2bc+x^2\left(-2a^2bd+4ab^2c\right)}{4a^4b^2d^2-8a^3b^3cd+4a^2b^4c^2+x^4\left(4a^2b^4d^2-8ab^5cd+4b^6c^2\right)+x^2\left(8a^3b^3d^2-16a^2b^4cd+8ab^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] c**2*log(x**2 + (-a**4*c**2*d**4/(a*d - b*c)**3 + 4*a**3*b*c**3*d**3/(a*d -
 b*c)**3 - 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 + 4*a*b**3*c**5*d/(a*d - b*
c)**3 + a*c**2*d - b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**2*d))/(2*(a*d
 - b*c)**3) - c**2*log(x**2 + (a**4*c**2*d**4/(a*d - b*c)**3 - 4*a**3*b*c**
3*d**3/(a*d - b*c)**3 + 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 - 4*a*b**3*c**
5*d/(a*d - b*c)**3 + a*c**2*d + b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**
2*d))/(2*(a*d - b*c)**3) + (-a**3*d + 3*a**2*b*c + x**2*(-2*a**2*b*d + 4*a*
b**2*c))/(4*a**4*b**2*d**2 - 8*a**3*b**3*c*d + 4*a**2*b**4*c**2 + x**4*(4*a
**2*b**4*d**2 - 8*a*b**5*c*d + 4*b**6*c**2) + x**2*(8*a**3*b**3*d**2 - 16*a
**2*b**4*c*d + 8*a*b**5*c**2))
```

3.252 $\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=157

$$\frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{3/2}(bc-ad)^3}+\frac{x(bc-5ad)}{8d\left(c+dx^2\right)(bc-ad)^2}-\frac{cx}{4d\left(c+dx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 527, 522, 205}

$$\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,d^{3/2}(bc-ad)^3}+\frac{a^{3/2}\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(bc-5ad)}{8d\left(c+dx^2\right)(bc-ad)^2}-\frac{cx}{4d\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(c*x)/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(8*d*(b*c - a*d)
^2*(c + d*x^2)) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)
^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*Sqr
t[c]*d^(3/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{ac+(bc-4ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4d(bc-ad)}\\
&= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{ac(bc+3ad)+bc(bc-5ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8cd(bc-ad)^2}\\
&= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2\left(c+dx^2\right)}+\frac{\left(a^2b\right)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}+\frac{(b^2c^2-6a}{}\\
&= -\frac{cx}{4d(bc-ad)\left(c+dx^2\right)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2\left(c+dx^2\right)}+\frac{a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(b^2c^2}{}
\end{aligned}
$$

Mathematica [A] time = 0.27, size = 154, normalized size = 0.98

$$
\frac{1}{8}\left(\frac{8a^{3/2}\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\,d^{3/2}(bc-ad)^3}+\frac{x(bc-5ad)}{d\left(c+dx^2\right)(bc-ad)^2}+\frac{2cx}{d\left(c+dx^2\right)^2(ad-bc)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*c*x)/(d*(-(b*c) + a*d)*(c + d*x^2)^2) + ((b*c - 5*a*d)*x)/(d*(b*c - a*d
)^2*(c + d*x^2)) + (8*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a
```

```
*d)^3 + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sq
rt[c]*d^(3/2)*(b*c - a*d)^3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 1.66, size = 1573, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 - 8*(a*c*d^4*x^4 +
 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)) - (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b
*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*d + 2
*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c
^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a
^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^
2*d^6)*x^2), 1/8*((b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + (b^2*c^
4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt
(c*d)*x/c) - 4*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((
b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - (b^2*c^4*d + 2*a*b*c^3*d^2 - 3*a
^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 2*
(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^2), 1/16*
(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 16*(a*c*d^4*x^4 + 2*a*
c^2*d^3*x^2 + a*c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (b^2*c^4 - 6*a*b
*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b
^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*d + 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*
x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^
4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3
 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^2), 1/8*((b^2*c^3*d^2
 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 8*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*
```

```
c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c
^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b
*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - (b^2*c^4*d +
 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b
*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 -
 a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*
c^2*d^6)*x^2)]
```

giac [A] time = 0.33, size = 204, normalized size = 1.30

$$\frac{a^{2}b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}+\frac{\left(b^{2}c^{2}-6\,abcd-3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{3}c^{3}d-3\,ab^{2}c^{2}d^{2}+3\,a^{2}bcd^{3}-a^{3}d^{4}\right)\sqrt{cd}}+\frac{bcdx^{3}-5\,ad^{2}x^{3}-bc^{2}x-3\,acdx}{8\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d)) + 1
/8*(b*c*d*x^3 - 5*a*d^2*x^3 - b*c^2*x - 3*a*c*d*x)/((b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(d*x^2 + c)^2)
```

maple [B] time = 0.01, size = 299, normalized size = 1.90

$$-\frac{5a^{2}d^{2}x^{3}}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{3abcd\,x^{3}}{4\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}-\frac{b^{2}c^{2}x^{3}}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}-\frac{3a^{2}cdx}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{ab\,c^{2}x}{4\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{b^{2}c^{3}x}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}d}-\frac{a^{2}b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^{3}\sqrt{ab}}+\frac{3a^{2}d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{3}\sqrt{cd}}+\frac{3abc\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^{3}\sqrt{cd}}-\frac{b^{2}c^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{3}\sqrt{cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -a^2*b/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-5/8/(a*d-b*c)^3/(d
*x^2+c)^2*x^3*a^2*d^2+3/4/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*b*c*d-1/8/(a*d-b*c)
^3/(d*x^2+c)^2*x^3*b^2*c^2-3/8/(a*d-b*c)^3/(d*x^2+c)^2*a^2*c*d*x+1/4/(a*d-b
*c)^3/(d*x^2+c)^2*a*b*c^2*x+1/8/(a*d-b*c)^3/(d*x^2+c)^2*c^3/d*x*b^2+3/8/(a*
d-b*c)^3*d/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+3/4/(a*d-b*c)^3/(c*d)^
(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*c-1/8/(a*d-b*c)^3/d/(c*d)^(1/2)*arctan(
1/(c*d)^(1/2)*d*x)*b^2*c^2
```

maxima [A] time = 2.52, size = 264, normalized size = 1.68

$$\frac{a^{2}b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}+\frac{\left(b^{2}c^{2}-6\,abcd-3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{3}c^{3}d-3\,ab^{2}c^{2}d^{2}+3\,a^{2}bcd^{3}-a^{3}d^{4}\right)\sqrt{cd}}+\frac{\left(bcd-5\,ad^{2}\right)x^{3}-\left(bc^{2}+3\,acd\right)x}{8\left(b^{2}c^{4}d-2\,abc^{3}d^{2}+a^{2}c^{2}d^{3}+\left(b^{2}c^{2}d^{3}-2\,abcd^{4}+a^{2}d^{5}\right)x^{4}+2\left(b^{2}c^{3}d^{2}-2\,abc^{2}d^{3}+a^{2}cd^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d)) + 1
/8*((b*c*d - 5*a*d^2)*x^3 - (b*c^2 + 3*a*c*d)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2
 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2
 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)
```

mupad **[B]** time = 1.98, size = 5754, normalized size = 36.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] (atan((((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 3
0*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) - (((96*a^8*b^2*d^9 + 32*a*b^9*c
^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*
a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7
 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 +
15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c
^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 -
1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4
*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b
^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*
b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))
*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b
^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) + ((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*
d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5
 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) + (((9
6*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 -
 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6
*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d
^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) + (x*(-c*d^3
)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d
^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*
a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c
*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(
3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^
3*d^4 - 3*a^2*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(16*(a^3*c
*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))/((a^2*b^6*c^3 + 1
5*a^5*b^3*d^3 - 11*a^3*b^5*c^2*d + 27*a^4*b^4*c*d^2)/(32*(a^6*d^7 + b^6*c^6
*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2
```

```
*c^2*d^5 - 6*a^5*b*c*d^6)) - ((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*d^4
+ 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b
^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) - (((96*a^
8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*
a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4
*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-c*d^3)^(1
/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 -
 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*
b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6
 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d -
4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^
2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^
4 - 3*a^2*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 -
b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) + ((-c*d^3)^(1/2)*((x*(b^
7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c
^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a
^3*b*c*d^4)) + (((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 9
6*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5
*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2
 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c
*d^6)) + (x*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d
^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*
b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^
2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*
(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)
))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c
^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*
c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5))))*
(-c*d^3)^(1/2)*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(8*(a^3*c*d^6 - b^3*c^
4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) - (atan((((-a^3*b)^(1/2)*(((-a^
3*b)^(1/2)*((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2
*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*
d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15
*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)
) - (x*(-a^3*b)^(1/2)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*
d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 12
80*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2
*c^2*d^3 - 4*a^3*b*c*d^4))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2)) - (x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^
2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2
*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*1i)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2)) - ((-a^3*b)^(1/2)*(((-a^3*b)^(1/2)*((96*a^8*b^2*d^9 + 32*a
*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4
```

```
+ 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a
^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) + (x*(-a^3*b)^(1/2)*(256*a^7*b^2
*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^
2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*
c^2*d^8))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*d^5
+ b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4))))/(2*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(b^7*c^4 + 73*a^4*b
^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*
d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*1i
)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((a^2*b^6*c^3 +
15*a^5*b^3*d^3 - 11*a^3*b^5*c^2*d + 27*a^4*b^4*c*d^2)/(32*(a^6*d^7 + b^6*c^
6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^
2*c^2*d^5 - 6*a^5*b*c*d^6)) + ((-a^3*b)^(1/2)*(((-a^3*b)^(1/2)*((96*a^8*b^2
*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b
^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*
d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a
^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-a^3*b)^(1/2)*(
256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^
9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 23
04*a^5*b^4*c^2*d^8))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^
4))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(b^7*c^4
 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d)
)/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*
c*d^4))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + ((-a^3*
b)^(1/2)*(((-a^3*b)^(1/2)*((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3
*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 14
40*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b
^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 -
 6*a^5*b*c*d^6)) + (x*(-a^3*b)^(1/2)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 -
1280*a*b^8*c^6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b
^6*c^4*d^6 - 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(64*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*
d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3
+ 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4))))/(2*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))))*(-a^3*b)^(1/2)*1i)/(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2) - ((x^3*(5*a*d - b*c))/(8*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)) + (c*x*(3*a*d + b*c))/(8*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)))/(
c^2 + d^2*x^4 + 2*c*d*x^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.253 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{a}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{c}{4d\left(c+dx^2\right)^2(bc-ad)}-\frac{ab\log\left(a+bx^2\right)}{2(bc-ad)^3}+\frac{ab\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{a}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{c}{4d\left(c+dx^2\right)^2(bc-ad)}-\frac{ab\log\left(a+bx^2\right)}{2(bc-ad)^3}+\frac{ab\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -c/(4*d*(b*c - a*d)*(c + d*x^2)^2) - a/(2*(b*c - a*d)^2*(c + d*x^2)) - (a*b
*Log[a + b*x^2])/(2*(b*c - a*d)^3) + (a*b*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{ab^2}{(bc-ad)^3(a+bx)}+\frac{c}{(bc-ad)(c+dx)^3}+\frac{ad}{(-bc+ad)^2(c+dx)^2}-\right.\right.\\
&= -\frac{c}{4d(bc-ad)\left(c+dx^2\right)^2}-\frac{a}{2(bc-ad)^2\left(c+dx^2\right)}-\frac{ab\log\left(a+bx^2\right)}{2(bc-ad)^3}+\frac{ab\log\left(c+\right.}{2(bc-ad}
\end{aligned}
$$

Mathematica [A]   time = 0.11, size = 77, normalized size = 0.77

$$
\frac{\frac{(ad-bc)\left(ad\left(c+2dx^2\right)+bc^2\right)}{d\left(c+dx^2\right)^2}+2ab\log\left(c+dx^2\right)-2ab\log\left(a+bx^2\right)}{4(bc-ad)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (((-(b*c) + a*d)*(b*c^2 + a*d*(c + 2*d*x^2)))/(d*(c + d*x^2)^2) - 2*a*b*Log
[a + b*x^2] + 2*a*b*Log[c + d*x^2])/(4*(b*c - a*d)^3)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B]   time = 0.79, size = 256, normalized size = 2.56

$$
-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2+2\left(abd^3x^4+2\,abcd^2x^2+abc^2d\right)\log\left(bx^2+a\right)-2\left(abd^3x^4+2\,abcd^2x^2+abc^2d\right)\log\left(dx^2+c\right)}{4\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^4+2\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5\right)x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^3)*x^2 + 2*(a*b*d^3*x^4 +
2*a*b*c*d^2*x^2 + a*b*c^2*d)*log(b*x^2 + a) - 2*(a*b*d^3*x^4 + 2*a*b*c*d^2*
x^2 + a*b*c^2*d)*log(d*x^2 + c))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3
*d^3 - a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d
^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*x
^2)
```

giac [A] time = 0.43, size = 174, normalized size = 1.74

$$-\frac{ab^2\log\left(\left|bx^2+a\right|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)}+\frac{abd\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)}-\frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2}{4\left(dx^2+c\right)^2(bc-ad)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*a*b^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3) + 1/2*a*b*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^
3)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3*d)
```

maple [A] time = 0.01, size = 177, normalized size = 1.77

$$\frac{a^2cd}{4\,(ad-bc)^3\left(d\,x^2+c\right)^2}-\frac{ab\,c^2}{2\,(ad-bc)^3\left(d\,x^2+c\right)^2}+\frac{b^2c^3}{4\,(ad-bc)^3\left(d\,x^2+c\right)^2d}-\frac{a^2d}{2\,(ad-bc)^3\left(d\,x^2+c\right)}+\frac{abc}{2\,(ad-bc)^3\left(d\,x^2+c\right)}+\frac{ab\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^3}-\frac{ab\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/2*a*b/(a*d-b*c)^3*ln(b*x^2+a)+1/4/(a*d-b*c)^3*c*d/(d*x^2+c)^2*a^2-1/2/(a*
d-b*c)^3*c^2/(d*x^2+c)^2*a*b+1/4/(a*d-b*c)^3*c^3/d/(d*x^2+c)^2*b^2-1/2/(a*d
-b*c)^3*a*b*ln(d*x^2+c)-1/2/(a*d-b*c)^3*a^2/(d*x^2+c)*d+1/2/(a*d-b*c)^3*a/(
d*x^2+c)*b*c
```

maxima [B] time = 1.14, size = 217, normalized size = 2.17

$$-\frac{ab\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{ab\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{2\,ad^2x^2+bc^2+acd}{4\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^4+2\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*b*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
 + 1/2*a*b*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3) - 1/4*(2*a*d^2*x^2 + b*c^2 + a*c*d)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2
```

```
*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c
^2*d^3 + a^2*c*d^4)*x^2)
```

mupad [B] time = 0.47, size = 343, normalized size = 3.43

$$-\frac{b^2\,c^3-a^2\,c\,d^2-2\,a^2\,d^3\,x^2+2\,a\,b\,c\,d^2\,x^2+a\,b\,c^2\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,d^3\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,c\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}}{-4\,a^3\,c^2\,d^4-8\,a^3\,c\,d^5\,x^2-4\,a^3\,d^6\,x^4+12\,a^2\,b\,c^3\,d^3+24\,a^2\,b\,c^2\,d^4\,x^2+12\,a^2\,b\,c\,d^5\,x^4-12\,a\,b^2\,c^4\,d^2-24\,a\,b^2\,c^3\,d^3\,x^2-12\,a\,b^2\,c^2\,d^4\,x^4+4\,b^3\,c^5\,d+8\,b^3\,c^4\,d^2\,x^2+4\,b^3\,c^3\,d^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] -(b^2*c^3 - a^2*c*d^2 - 2*a^2*d^3*x^2 + 2*a*b*c*d^2*x^2 + a*b*c^2*d*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*d^3*x^4*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*c*d^2*x^2*a
tan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i)/(4*b^3*c^5*d
 - 4*a^3*c^2*d^4 - 4*a^3*d^6*x^4 - 12*a*b^2*c^4*d^2 + 12*a^2*b*c^3*d^3 - 8*
a^3*c*d^5*x^2 + 8*b^3*c^4*d^2*x^2 + 4*b^3*c^3*d^3*x^4 + 12*a^2*b*c*d^5*x^4
- 24*a*b^2*c^3*d^3*x^2 + 24*a^2*b*c^2*d^4*x^2 - 12*a*b^2*c^2*d^4*x^4)
```

sympy [B] time = 2.97, size = 411, normalized size = 4.11

$$-\frac{ab\log\left(x^2+\frac{-\frac{a^5bd^4}{(ad-bc)^3}+\frac{4a^4b^2cd^3}{(ad-bc)^3}-\frac{6a^3b^3c^2d^2}{(ad-bc)^3}+\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd-\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2\left(ad-bc\right)^3}+\frac{ab\log\left(x^2+\frac{\frac{a^5bd^4}{(ad-bc)^3}-\frac{4a^4b^2cd^3}{(ad-bc)^3}+\frac{6a^3b^3c^2d^2}{(ad-bc)^3}-\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd+\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2\left(ad-bc\right)^3}+\frac{-acd-2ad^2x^2-bc^2}{4a^2c^2d^3-8abc^3d^2+4b^2c^4d+x^4\left(4a^2d^5-8abcd^4+4b^2c^2d^3\right)+x^2\left(8a^2cd^4-16abc^2d^3+8b^2c^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -a*b*log(x**2 + (-a**5*b*d**4/(a*d - b*c)**3 + 4*a**4*b**2*c*d**3/(a*d - b*
c)**3 - 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 + 4*a**2*b**4*c**3*d/(a*d - b*
c)**3 + a**2*b*d - a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(2*a*b**2*d))/(2*
(a*d - b*c)**3) + a*b*log(x**2 + (a**5*b*d**4/(a*d - b*c)**3 - 4*a**4*b**2*
c*d**3/(a*d - b*c)**3 + 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 - 4*a**2*b**4*
c**3*d/(a*d - b*c)**3 + a**2*b*d + a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(
2*a*b**2*d))/(2*(a*d - b*c)**3) + (-a*c*d - 2*a*d**2*x**2 - b*c**2)/(4*a**2
*c**2*d**3 - 8*a*b*c**3*d**2 + 4*b**2*c**4*d + x**4*(4*a**2*d**5 - 8*a*b*c*
d**4 + 4*b**2*c**2*d**3) + x**2*(8*a**2*c*d**4 - 16*a*b*c**2*d**3 + 8*b**2*
c**3*d**2))
```

3.254 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=155

$$\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}\,(bc-ad)^3}-\frac{\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(bc-ad)^2}+\frac{x}{4\left(c+dx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {471, 527, 522, 205}

$$\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}\,(bc-ad)^3}-\frac{\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{x(ad+3bc)}{8c\left(c+dx^2\right)(bc-ad)^2}+\frac{x}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x/(4*(b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(8*c*(b*c - a*d)^2*(c +
 d*x^2)) - (Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*d)^3 + ((
3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*Sq
rt[d]*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} - \frac{\int \frac{a-3bx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4(bc-ad)}$$

$$= \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(3bc+ad)x}{8c(bc-ad)^2\left(c+dx^2\right)} - \frac{\int \frac{a(5bc-ad)-b(3bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8c(bc-ad)^2}$$

$$= \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(3bc+ad)x}{8c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(ab^2\right)\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3} + \frac{\left(3b^2c^2+6abc\right.}{8c}$$

$$= \frac{x}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(3bc+ad)x}{8c(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(bc-ad)^3} + \frac{\left(3b^2c^2+\right.}{8}$$

Mathematica [A] time = 0.22, size = 151, normalized size = 0.97

$$\frac{1}{8}\left(\frac{\left(-a^2d^2+6abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}\sqrt{d}\,(bc-ad)^3} + \frac{8\sqrt{a}\,b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{(ad-bc)^3} + \frac{x(ad+3bc)}{c\left(c+dx^2\right)(bc-ad)^2} + \frac{2x}{\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((2*x)/((b*c - a*d)*(c + d*x^2)^2) + ((3*b*c + a*d)*x)/(c*(b*c - a*d)^2*(c
+ d*x^2)) + (8*Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(-(b*c) + a*d)^
```

```
3 + ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2
)*Sqrt[d]*(b*c - a*d)^3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 1.86, size = 1587, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^4
 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)) - (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a
*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*
sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^4*d -
 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5
*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3
*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^
3*d^5)*x^2), 1/8*((3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 + (3*b^2*
c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x
^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt
(c*d)*x/c) - 4*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((
b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + (5*b^2*c^4*d - 6*a*b*c^3*d^2 + a
^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4
 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^4 + 2*
(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2), 1/16*
(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 16*(b*c^2*d^3*x^4 + 2*
b*c^3*d^2*x^2 + b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (3*b^2*c^4 + 6*a
*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3
*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^4*d - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*
x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2
 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2), 1/8*((3*b^2*c^3*d
^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 +
```

```
b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c
^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a
*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (5*b^2*c^4*d
 - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c
^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a
^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*
c^3*d^5)*x^2)]
```

giac [A] time = 0.37, size = 206, normalized size = 1.33

$$-\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}}+\frac{\left(3\,b^2c^2+6\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\sqrt{cd}}+\frac{3\,bcdx^3+ad^2x^3+5\,bc^2x-acdx}{8\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\left(dx^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c
*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d)) +
1/8*(3*b*c*d*x^3 + a*d^2*x^3 + 5*b*c^2*x - a*c*d*x)/((b^2*c^3 - 2*a*b*c^2*d
 + a^2*c*d^2)*(d*x^2 + c)^2)
```

maple [B] time = 0.01, size = 298, normalized size = 1.92

$$\frac{a^2d^3x^3}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2c}+\frac{ab\,d^2x^3}{4\left(ad-bc\right)^3\left(d\,x^2+c\right)^2}-\frac{3b^2cd\,x^3}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2}-\frac{a^2d^2x}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2}+\frac{3abcdx}{4\left(ad-bc\right)^3\left(d\,x^2+c\right)^2}-\frac{5b^2c^2x}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2}+\frac{a^2d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^3\sqrt{cd}\,c}+\frac{a\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^3\sqrt{ab}}-\frac{3abd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^3\sqrt{cd}}-\frac{3b^2c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^3\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] a*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+1/8/(a*d-b*c)^3/(d*
x^2+c)^2*d^3/c*x^3*a^2+1/4/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*b*d^2-3/8/(a*d-b*c
)^3/(d*x^2+c)^2*x^3*b^2*c*d+3/4/(a*d-b*c)^3/(d*x^2+c)^2*a*b*c*d*x-5/8/(a*d-
b*c)^3/(d*x^2+c)^2*b^2*c^2*x-1/8/(a*d-b*c)^3/(d*x^2+c)^2*a^2*d^2*x+1/8/(a*d
-b*c)^3/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2*d^2-3/4/(a*d-b*c)^3/(c*
d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*d-3/8/(a*d-b*c)^3*c/(c*d)^(1/2)*arct
an(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [A] time = 2.30, size = 266, normalized size = 1.72

$$-\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{ab}}+\frac{\left(3\,b^2c^2+6\,abcd-a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\sqrt{cd}}+\frac{\left(3\,bcd+ad^2\right)x^3+\left(5\,bc^2-acd\right)x}{8\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^4+2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c
*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d)) +
1/8*((3*b*c*d + a*d^2)*x^3 + (5*b*c^2 - a*c*d)*x)/(b^2*c^5 - 2*a*b*c^4*d +
a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d
- 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

mupad [B] time = 1.98, size = 5898, normalized size = 38.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] (atan((((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2*b
^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c^4
*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^2*d
^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2
*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2 - 1280*a*b^
8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*
d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(
64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6 + a^4*c^2*d
^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*b^3)^(1/2))
/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(9*b^7*c^4*d
+ a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/
(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*
c^5*d)))*1i)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - ((-a
*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3
+ 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440
*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*
b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 -
6*a*b^5*c^7*d)) + (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 +
 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*
a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(64*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*
b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^
3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(9*b^7*c^4*d + a^4*b^3*d
^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^
6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*1i
)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((3*a^3*b^5*c*d^
3 - a^4*b^4*d^4 + 21*a^2*b^6*c^2*d^2 + 9*a*b^7*c^3*d)/(32*(b^6*c^8 + a^6*c^
2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*
b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + ((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*
a^8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c
^5*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)
```

```
/(64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3
*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-a*b^3)^(1/2)*(25
6*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^
6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8
+ 256*a^7*b^2*c^2*d^9))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3
*c^5*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - (x*(9*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^
4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a
^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + ((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9
- 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*
a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8
+ a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 +
 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2
- 1280*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a
^4*b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*
c^2*d^9))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6
+ a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*
b^3)^(1/2))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(9
*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5
*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2
 - 4*a*b^3*c^5*d))))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
)))*(-a*b^3)^(1/2)*1i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
- ((x*(a*d - 5*b*c))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x^3*(a*d^2 + 3*
b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 + 2*c*d*x^2)
- (atan((((-c^3*d)^(1/2)*((x*(9*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2
- 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^
3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)) - (((160*a*b^9*c^8*d^2 -
32*a^8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^
6*c^5*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d
^8)/(64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*
a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-c^3*d)^(1/2)*
(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d)*(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2
304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^
5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(512*(b^3*c^6*
d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3)*(b^4*c^6 + a^4*c^2*d^4
 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-c^3*d)^(1/2)*(3
*b^2*c^2 - a^2*d^2 + 6*a*b*c*d))/(16*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5
*d^2 + 3*a^2*b*c^4*d^3)))*(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d)*1i)/(16*(b^3*c^
6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3)) + ((-c^3*d)^(1/2)*(
(x*(9*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^
2*b^5*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^
4*d^2 - 4*a*b^3*c^5*d)) + (((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2
*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c
```

```
^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^2
*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b
^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x*(-c^3*d)^(1/2)*(3*b^2*c^2 - a^2*d^2 + 6*a
*b*c*d)*(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280
*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b
^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(512*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*
c^5*d^2 + 3*a^2*b*c^4*d^3)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2
*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-c^3*d)^(1/2)*(3*b^2*c^2 - a^2*d^2 + 6*a*b
*c*d))/(16*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3)))*
(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d)*1i)/(16*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^
2*c^5*d^2 + 3*a^2*b*c^4*d^3)))/((3*a^3*b^5*c*d^3 - a^4*b^4*d^4 + 21*a^2*b^6
*c^2*d^2 + 9*a*b^7*c^3*d)/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15
*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)
) - ((-c^3*d)^(1/2)*((x*(9*b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*
a^3*b^4*c*d^4 + 94*a^2*b^5*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c
^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)) - (((160*a*b^9*c^8*d^2 - 32*a^
8*b^2*c*d^9 - 992*a^2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5
*d^5 + 3040*a^5*b^5*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(
64*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b
^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-c^3*d)^(1/2)*(3*b^
2*c^2 - a^2*d^2 + 6*a*b*c*d)*(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2304*a
^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^5*b^4
*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*d^9))/(512*(b^3*c^6*d - a
^3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3)*(b^4*c^6 + a^4*c^2*d^4 - 4*
a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-c^3*d)^(1/2)*(3*b^2*
c^2 - a^2*d^2 + 6*a*b*c*d))/(16*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2
+ 3*a^2*b*c^4*d^3)))*(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d))/(16*(b^3*c^6*d - a^
3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3)) + ((-c^3*d)^(1/2)*((x*(9*b^
7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5*c^
2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 -
4*a*b^3*c^5*d)) + (((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2*b^8*c^7
*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c^4*d^6 -
 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^2*d^6 - 6
*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d
^4 - 6*a*b^5*c^7*d)) + (x*(-c^3*d)^(1/2)*(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d)*
(256*b^9*c^9*d^2 - 1280*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6
*c^6*d^5 - 1280*a^4*b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d
^8 + 256*a^7*b^2*c^2*d^9))/(512*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2
+ 3*a^2*b*c^4*d^3)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4
*d^2 - 4*a*b^3*c^5*d)))*(-c^3*d)^(1/2)*(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d))/(
16*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3)))*(3*b^2*c
^2 - a^2*d^2 + 6*a*b*c*d))/(16*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2 +
 3*a^2*b*c^4*d^3))))*(-c^3*d)^(1/2)*(3*b^2*c^2 - a^2*d^2 + 6*a*b*c*d)*1i)/(
8*(b^3*c^6*d - a^3*c^3*d^4 - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3))
```

sympy [**F(-1)**]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.255 $\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=98

$$\frac{b^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2\log\left(c+dx^2\right)}{2(bc-ad)^3}+\frac{b}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{1}{4\left(c+dx^2\right)^2(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 44}

$$\frac{b^2\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{b^2\log\left(c+dx^2\right)}{2(bc-ad)^3}+\frac{b}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{1}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] 1/(4*(b*c - a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (b^2*Lo
g[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^3}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^3}{(bc-ad)^3(a+bx)} - \frac{d}{(bc-ad)(c+dx)^3} - \frac{bd}{(bc-ad)^2(c+dx)^2} - \frac{}{(bc} \right.\right. \\
&= \frac{1}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{b}{2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^2\log\left(a+bx^2\right)}{2(bc-ad)^3} - \frac{b^2\log\left(c+dx^2\right.}{2(bc-ad)^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.05, size = 98, normalized size = 1.00

$$
\frac{b^2\log\left(a+bx^2\right)}{2(bc-ad)^3} - \frac{b^2\log\left(c+dx^2\right)}{2(bc-ad)^3} + \frac{b}{2\left(c+dx^2\right)(bc-ad)^2} - \frac{1}{4\left(c+dx^2\right)^2(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*1/((-(b*c) + a*d)*(c + d*x^2)^2) + b/(2*(b*c - a*d)^2*(c + d*x^2)) + (
b^2*Log[a + b*x^2])/(2*(b*c - a*d)^3) - (b^2*Log[c + d*x^2])/(2*(b*c - a*d)
^3)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [**B**] time = 0.56, size = 254, normalized size = 2.59

$$
\frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\left(b^2cd - abd^2\right)x^2 + 2\left(b^2d^2x^4 + 2\,b^2cdx^2 + b^2c^2\right)\log\left(bx^2+a\right) - 2\left(b^2d^2x^4 + 2\,b^2cdx^2 + b^2c^2\right)\log\left(dx^2+c\right)}{4\left(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3 + \left(b^3c^3d^2 - 3\,ab^2c^2d^3 + 3\,a^2bcd^4 - a^3d^5\right)x^4 + 2\left(b^3c^4d - 3\,ab^2c^3d^2 + 3\,a^2bc^2d^3 - a^3cd^4\right)x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d
^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + 2*b^2*c
*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^
2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)
*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)
```

giac [A] time = 0.38, size = 174, normalized size = 1.78

$$\frac{b^{3}\log\left(\left|bx^{2}+a\right|\right)}{2\left(b^{4}c^{3}-3\,ab^{3}c^{2}d+3\,a^{2}b^{2}cd^{2}-a^{3}bd^{3}\right)}-\frac{b^{2}d\log\left(\left|dx^{2}+c\right|\right)}{2\left(b^{3}c^{3}d-3\,ab^{2}c^{2}d^{2}+3\,a^{2}bcd^{3}-a^{3}d^{4}\right)}+\frac{3\,b^{2}c^{2}-4\,abcd+a^{2}d^{2}+2\left(b^{2}cd-abd^{2}\right)x^{2}}{4\left(dx^{2}+c\right)^{2}(bc-ad)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3) - 1/2*b^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4) + 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d -
 a*b*d^2)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3)
```

maple [A] time = 0.01, size = 176, normalized size = 1.80

$$-\frac{a^{2}d^{2}}{4\,(ad-bc)^{3}\left(d\,x^{2}+c\right)^{2}}+\frac{abcd}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)^{2}}-\frac{b^{2}c^{2}}{4\,(ad-bc)^{3}\left(d\,x^{2}+c\right)^{2}}+\frac{abd}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)}-\frac{b^{2}c}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)}-\frac{b^{2}\ln\left(b\,x^{2}+a\right)}{2\,(ad-bc)^{3}}+\frac{b^{2}\ln\left(d\,x^{2}+c\right)}{2\,(ad-bc)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b^2/(a*d-b*c)^3*ln(b*x^2+a)-1/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*a^2+1/2*d/
(a*d-b*c)^3/(d*x^2+c)^2*a*b*c-1/4/(a*d-b*c)^3/(d*x^2+c)^2*b^2*c^2+1/2/(a*d-
b*c)^3*ln(d*x^2+c)*b^2+1/2*d/(a*d-b*c)^3*b/(d*x^2+c)*a-1/2/(a*d-b*c)^3*b^2/
(d*x^2+c)*c
```

maxima [B] time = 1.06, size = 211, normalized size = 2.15

$$\frac{b^{2}\log\left(bx^{2}+a\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{b^{2}\log\left(dx^{2}+c\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{2\,bdx^{2}+3\,bc-ad}{4\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)x^{4}+2\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*b^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/4*(2*b*d*x^2 + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^
2*c*d^3)*x^2)
```

mupad [B] time = 0.35, size = 340, normalized size = 3.47

$$\frac{a^2\,d^2+3\,b^2\,c^2+b^2\,c^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+b^2\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-2\,a\,b\,d^2\,x^2+2\,b^2\,c\,d\,x^2-4\,a\,b\,c\,d+b^2\,c\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}}{-4\,a^3\,c^2\,d^3-8\,a^3\,c\,d^4\,x^2-4\,a^3\,d^5\,x^4+12\,a^2\,b\,c^3\,d^2+24\,a^2\,b\,c^2\,d^3\,x^2+12\,a^2\,b\,c\,d^4\,x^4-12\,a\,b^2\,c^4\,d-24\,a\,b^2\,c^3\,d^2\,x^2-12\,a\,b^2\,c^2\,d^3\,x^4+4\,b^3\,c^5+8\,b^3\,c^4\,d\,x^2+4\,b^3\,c^3\,d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] (a^2*d^2 + 3*b^2*c^2 + b^2*c^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*
x^2 + b*c*x^2))*4i + b^2*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*
d*x^2 + b*c*x^2))*4i - 2*a*b*d^2*x^2 + 2*b^2*c*d*x^2 - 4*a*b*c*d + b^2*c*d*
x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i)/(4*b^3*
c^5 - 4*a^3*c^2*d^3 - 4*a^3*d^5*x^4 + 12*a^2*b*c^3*d^2 - 8*a^3*c*d^4*x^2 +
8*b^3*c^4*d*x^2 + 4*b^3*c^3*d^2*x^4 - 12*a*b^2*c^4*d + 12*a^2*b*c*d^4*x^4 -
 24*a*b^2*c^3*d^2*x^2 + 24*a^2*b*c^2*d^3*x^2 - 12*a*b^2*c^2*d^3*x^4)
```

sympy [B] time = 2.87, size = 391, normalized size = 3.99

$$\frac{b^2\log\left(x^2+\frac{-\frac{a^4b^2d^4}{(ad-bc)^3}+\frac{4a^3b^3cd^3}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2}{(ad-bc)^3}+\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d-\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{2\,(ad-bc)^3}-\frac{b^2\log\left(x^2+\frac{\frac{a^4b^2d^4}{(ad-bc)^3}-\frac{4a^3b^3cd^3}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2}{(ad-bc)^3}-\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d+\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{2\,(ad-bc)^3}+\frac{-ad+3bc+2bdx^2}{4a^2c^2d^2-8abc^3d+4b^2c^4+x^4\left(4a^2d^4-8abcd^3+4b^2c^2d^2\right)+x^2\left(8a^2cd^3-16abc^2d^2+8b^2c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] b**2*log(x**2 + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d -
 b*c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*
c)**3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(2*(a*d -
 b*c)**3) - b**2*log(x**2 + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*
d**3/(a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*
d/(a*d - b*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d)
)/(2*(a*d - b*c)**3) + (-a*d + 3*b*c + 2*b*d*x**2)/(4*a**2*c**2*d**2 - 8*a*
b*c**3*d + 4*b**2*c**4 + x**4*(4*a**2*d**4 - 8*a*b*c*d**3 + 4*b**2*c**2*d**
2) + x**2*(8*a**2*c*d**3 - 16*a*b*c**2*d**2 + 8*b**2*c**3*d))
```

3.256 $\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad}$$

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*c
- a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-3ad)x^{2}}{\left(a+bx^2\right)\left(c+dx^2\right)}}{8c^2(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^3\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3}-\frac{\left(d\left(15b^2c^2-\right.\right.}{}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\left(15b^{2}\right.}{}$$

Mathematica [A] time = 0.24, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)^3}+\frac{dx(3ad-7bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{2dx}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-
```

```
(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 2.64, size = 1585, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
```

```
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

giac [A] time = 0.37, size = 217, normalized size = 1.36

$$\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}-\frac{\left(15\,b^{2}c^{2}d-10\,abcd^{2}+3\,a^{2}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\sqrt{cd}}-\frac{7\,bcd^{2}x^{3}-3\,ad^{3}x^{3}+9\,bc^{2}dx-5\,acd^{2}x}{8\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

maple [B] time = 0.01, size = 310, normalized size = 1.94

$$\frac{3a^{2}d^{4}x^{3}}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}c^{2}}-\frac{5ab\,d^{3}x^{3}}{4\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}c}+\frac{7b^{2}d^{2}x^{3}}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{5a^{2}d^{3}x}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}c}-\frac{7ab\,d^{2}x}{4\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{9b^{2}cdx}{8\left(ad-bc\right)^{3}\left(dx^{2}+c\right)^{2}}+\frac{3a^{2}d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{3}\sqrt{cd}\,c^{2}}-\frac{5ab\,d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^{3}\sqrt{cd}\,c}-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^{3}\sqrt{ab}}+\frac{15b^{2}d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{3}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -b^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+3/8*d^4/(a*d-b*c)^3/
(d*x^2+c)^2/c^2*x^3*a^2-5/4*d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x^3*a*b+7/8*d^2/(
a*d-b*c)^3/(d*x^2+c)^2*b^2*x^3+5/8*d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x*a^2-7/4*
d^2/(a*d-b*c)^3/(d*x^2+c)^2*a*b*x+9/8*d/(a*d-b*c)^3/(d*x^2+c)^2*b^2*c*x+3/8
*d^3/(a*d-b*c)^3/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2-5/4*d^2/(a*d
-b*c)^3/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b+15/8*d/(a*d-b*c)^3/(c*d
)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [B] time = 2.33, size = 277, normalized size = 1.73

$$\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}-\frac{\left(15\,b^{2}c^{2}d-10\,abcd^{2}+3\,a^{2}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\sqrt{cd}}-\frac{\left(7\,bcd^{2}-3\,ad^{3}\right)x^{3}+\left(9\,bc^{2}d-5\,acd^{2}\right)x}{8\left(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}+\left(b^{2}c^{4}d^{2}-2\,abc^{3}d^{3}+a^{2}c^{2}d^{4}\right)x^{4}+2\left(b^{2}c^{5}d-2\,abc^{4}d^{2}+a^{2}c^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2*c^6
 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*
x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

mupad [B] time = 2.37, size = 6033, normalized size = 37.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 +
 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 -
 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8
+ a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-
a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^
4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 30
40*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*
d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1
280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7
*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*
(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*
d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(
a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b^5)^(1/2)*(
(x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
+ 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^
2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 176
0*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^
6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d
^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b
^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6
*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^
3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d
^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*
b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d
- 3*a^3*b*c*d^2)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 -
51*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^
```

```
4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + ((-
a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*
a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*
c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-a*b^5)^(1/2)*((256*b^10*
c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5
 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800
*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5
*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -
 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^
3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 23
04*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*
d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 -
 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^
3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2
*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7
*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32
*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7
*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*
b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^
5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(
64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*
b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-a*b^5)^(1/2)*(256
*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c
^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8
 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3
*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*
a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))
))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b^5)^(
1/2)*1i)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2) - (atan(((
((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a
^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^
3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 68
16*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^
2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*
d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-c^5*
d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b
^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^
7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))
/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 +
a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*
d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3
+ 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2
 - 10*a*b*c*d)*1i)/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c
^7*d)) + (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3
```

```
*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5
*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + (((256*b^10*c^10*d^2 - 1760*a*
b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^
6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 +
 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^
2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) +
 (x*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2
- 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*
a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2
*c^4*d^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(
b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d
)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^
3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 +
15*b^2*c^2 - 10*a*b*c*d)*1i)/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 -
 3*a*b^2*c^7*d)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 - 5
1*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4
*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (((x
*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 +
190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*
b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 +
 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*
a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c
^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2
 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-c^5*d)^
(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b^8*
c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d
^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(5
12*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 + a^4
*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*d)^
(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3
*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 -
10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d))
 - (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c
*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 +
 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + (((256*b^10*c^10*d^2 - 1760*a*b^9*c^
9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6
- 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^
8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*
c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-
c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280
*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^
5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d
^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^
8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-
c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*
```

```
d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2
*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*
c^7*d))))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*1i)/(8*(b^3*
c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.257 $\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=149

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\log(}{ac^3}$$

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{d\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}-\frac{d(2bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\left(c+dx^2\right)^2(bc-ad)}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -d/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(2*c^2*(b*c - a*d)^2
*(c + d*x^2)) + Log[x]/(a*c^3) - (b^3*Log[a + b*x^2])/(2*a*(b*c - a*d)^3) +
 (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^3)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{1}{ac^3x}+\frac{b^4}{a(-bc+ad)^3(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)^3}+\frac{d^2(2bc-}{c^2(bc-ad)^2(}\right.\right.$$

$$= -\frac{d}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(2bc-ad)}{2c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\log(x)}{ac^3}-\frac{b^3\log\left(a+bx^2\right)}{2a(bc-ad)^3}+$$

Mathematica [A] time = 0.27, size = 141, normalized size = 0.95

$$\frac{\frac{d\left(\frac{c\left(a^2d^2\left(3c+2dx^2\right)-2abcd\left(4c+3dx^2\right)+b^2c^2\left(5c+4dx^2\right)\right)}{\left(c+dx^2\right)^2}-2\left(a^2d^2-3abcd+3b^2c^2\right)\log\left(c+dx^2\right)\right)}{c^3}+\frac{2b^3\log\left(a+bx^2\right)}{a}}{4(ad-bc)^3}+\frac{\log(x)}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Log[x]/(a*c^3) + ((2*b^3*Log[a + b*x^2])/a + (d*((c*(a^2*d^2*(3*c + 2*d*x^2
) - 2*a*b*c*d*(4*c + 3*d*x^2) + b^2*c^2*(5*c + 4*d*x^2)))/(c + d*x^2)^2 - 2
*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2]))/c^3)/(4*(-(b*c) + a*d)^
3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 8.89, size = 520, normalized size = 3.49

$$-\frac{5\,ab^2c^4d-8\,a^2bc^3d^2+3\,a^3c^2d^3+2\left(2\,ab^2c^3d^2-3\,a^2bc^2d^3+a^3cd^4\right)x^2+2\left(b^3c^3d^2x^4+2\,b^3c^4dx^2+b^3c^5\right)\log\left(bx^2+a\right)-2\left(3\,ab^2c^4d-3\,a^2bc^3d^2+a^3c^2d^3+\left(3\,ab^2c^2d^3-3\,a^2bcd^4+a^3d^5\right)x^4+2\left(3\,ab^2c^3d^2-3\,a^2bc^2d^3+a^3cd^4\right)x^2\right)\log\left(dx^2+c\right)-4\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3+\left(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5\right)x^4+2\left(b^3c^4d-3\,ab^2c^3d^2+3\,a^2bc^2d^3-a^3cd^4\right)x^2\right)\log(x)}{4\left(ab^3c^8-3\,a^2b^2c^7d+3\,a^3bc^6d^2-a^4c^5d^3+\left(ab^3c^6d^2-3\,a^2b^2c^5d^3+3\,a^3bc^4d^4-a^4c^3d^5\right)x^4+2\left(ab^3c^7d-3\,a^2b^2c^6d^2+3\,a^3bc^5d^3-a^4c^4d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(5*a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 3*a^3*c^2*d^3 + 2*(2*a*b^2*c^3*d^2
- 3*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2 + 2*(b^3*c^3*d^2*x^4 + 2*b^3*c^4*d*x^2 +
 b^3*c^5)*log(b*x^2 + a) - 2*(3*a*b^2*c^4*d - 3*a^2*b*c^3*d^2 + a^3*c^2*d^3
 + (3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 + a^3*d^5)*x^4 + 2*(3*a*b^2*c^3*d^2 - 3
*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2)*log(d*x^2 + c) - 4*(b^3*c^5 - 3*a*b^2*c^4*
d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*
b*c*d^4 - a^3*d^5)*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 -
 a^3*c*d^4)*x^2)*log(x))/(a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a
^4*c^5*d^3 + (a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3
*d^5)*x^4 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*
d^4)*x^2)
```

giac [B] time = 0.43, size = 315, normalized size = 2.11

$$-\frac{b^{4} \log\left(\left|bx^{2} + a\right|\right)}{2 \left(ab^{4}c^{3} - 3\,a^{2}b^{3}c^{2}d + 3\,a^{3}b^{2}cd^{2} - a^{4}bd^{3}\right)} + \frac{\left(3\,b^{2}c^{2}d^{2} - 3\,abcd^{3} + a^{2}d^{4}\right) \log\left(\left|dx^{2} + c\right|\right)}{2 \left(b^{3}c^{6}d - 3\,ab^{2}c^{5}d^{2} + 3\,a^{2}bc^{4}d^{3} - a^{3}c^{3}d^{4}\right)} - \frac{9\,b^{2}c^{2}d^{3}x^{4} - 9\,abcd^{4}x^{4} + 3\,a^{2}d^{5}x^{4} + 22\,b^{2}c^{3}d^{2}x^{2} - 24\,abc^{2}d^{3}x^{2} + 8\,a^{2}cd^{4}x^{2} + 14\,b^{2}c^{4}d - 17\,abc^{3}d^{2} + 6\,a^{2}c^{2}d^{3}}{4 \left(b^{3}c^{6} - 3\,ab^{2}c^{5}d + 3\,a^{2}bc^{4}d^{2} - a^{3}c^{3}d^{3}\right)\left(dx^{2} + c\right)^{2}} + \frac{\log\left(x^{2}\right)}{2\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^4*log(abs(b*x^2 + a))/(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
 - a^4*b*d^3) + 1/2*(3*b^2*c^2*d^2 - 3*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 +
 c))/(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/4*(9
*b^2*c^2*d^3*x^4 - 9*a*b*c*d^4*x^4 + 3*a^2*d^5*x^4 + 22*b^2*c^3*d^2*x^2 - 2
4*a*b*c^2*d^3*x^2 + 8*a^2*c*d^4*x^2 + 14*b^2*c^4*d - 17*a*b*c^3*d^2 + 6*a^2
*c^2*d^3)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2
 + c)^2) + 1/2*log(x^2)/(a*c^3)
```

maple [B] time = 0.02, size = 286, normalized size = 1.92

$$\frac{a^{2}d^{3}}{4\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c} - \frac{ab\,d^{2}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}} + \frac{b^{2}cd}{4\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}} + \frac{a^{2}d^{3}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)c^{2}} - \frac{a^{2}d^{3}\ln\left(d\,x^{2}+c\right)}{2\left(ad-bc\right)^{3}c^{3}} - \frac{3ab\,d^{2}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)c} + \frac{3ab\,d^{2}\ln\left(d\,x^{2}+c\right)}{2\left(ad-bc\right)^{3}c^{2}} + \frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2\left(ad-bc\right)^{3}a} - \frac{3b^{2}d\ln\left(d\,x^{2}+c\right)}{2\left(ad-bc\right)^{3}c} + \frac{b^{2}d}{\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)} + \frac{\ln(x)}{a\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^3/a/(a*d-b*c)^3*ln(b*x^2+a)+1/4*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*a^2-1/2
*d^2/(a*d-b*c)^3/(d*x^2+c)^2*a*b+1/4*d*c/(a*d-b*c)^3/(d*x^2+c)^2*b^2-1/2*d^
3/c^3/(a*d-b*c)^3*ln(d*x^2+c)*a^2+3/2*d^2/c^2/(a*d-b*c)^3*ln(d*x^2+c)*a*b-3
/2*d/c/(a*d-b*c)^3*ln(d*x^2+c)*b^2+1/2*d^3/c^2/(a*d-b*c)^3/(d*x^2+c)*a^2-3/
2*d^2/c/(a*d-b*c)^3/(d*x^2+c)*a*b+d/(a*d-b*c)^3/(d*x^2+c)*b^2+ln(x)/a/c^3
```

maxima [A] time = 1.25, size = 278, normalized size = 1.87

$$-\frac{b^{3} \log\left(bx^{2} + a\right)}{2 \left(ab^{3}c^{3} - 3\,a^{2}b^{2}c^{2}d + 3\,a^{3}bcd^{2} - a^{4}d^{3}\right)} + \frac{\left(3\,b^{2}c^{2}d - 3\,abcd^{2} + a^{2}d^{3}\right) \log\left(dx^{2} + c\right)}{2 \left(b^{3}c^{6} - 3\,ab^{2}c^{5}d + 3\,a^{2}bc^{4}d^{2} - a^{3}c^{3}d^{3}\right)} - \frac{5\,bc^{2}d - 3\,acd^{2} + 2\left(2\,bcd^{2} - ad^{3}\right)x^{2}}{4\left(b^{2}c^{6} - 2\,abc^{5}d + a^{2}c^{4}d^{2} + \left(b^{2}c^{4}d^{2} - 2\,abc^{3}d^{3} + a^{2}c^{2}d^{4}\right)x^{4} + 2\left(b^{2}c^{5}d - 2\,abc^{4}d^{2} + a^{2}c^{3}d^{3}\right)x^{2}\right)} + \frac{\log\left(x^{2}\right)}{2\,ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(b*x^2 + a)/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*
d^3) + 1/2*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/(b^3*c^6 -
3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/4*(5*b*c^2*d - 3*a*c*d^2
 + 2*(2*b*c*d^2 - a*d^3)*x^2)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c
^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 +
a^2*c^3*d^3)*x^2) + 1/2*log(x^2)/(a*c^3)
```

mupad [**B**] time = 1.40, size = 246, normalized size = 1.65

$$\frac{\frac{3\,a\,d^2-5\,b\,c\,d}{4\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{d^2\,x^2\,\left(a\,d-2\,b\,c\right)}{2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2+2\,c\,d\,x^2+d^2\,x^4}+\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4\,d^3-6\,a^3\,b\,c\,d^2+6\,a^2\,b^2\,c^2\,d-2\,a\,b^3\,c^3}+\frac{\ln(x)}{a\,c^3}+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^3-3\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{-2\,a^3\,c^3\,d^3+6\,a^2\,b\,c^4\,d^2-6\,a\,b^2\,c^5\,d+2\,b^3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((3*a*d^2 - 5*b*c*d)/(4*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d^2*x^2*(a*d
- 2*b*c))/(2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 + 2*c*d*x
^2) + (b^3*log(a + b*x^2))/(2*a^4*d^3 - 2*a*b^3*c^3 + 6*a^2*b^2*c^2*d - 6*a
^3*b*c*d^2) + log(x)/(a*c^3) + (log(c + d*x^2)*(a^2*d^3 + 3*b^2*c^2*d - 3*a
*b*c*d^2))/(2*b^3*c^6 - 2*a^3*c^3*d^3 + 6*a^2*b*c^4*d^2 - 6*a*b^2*c^5*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.258 $\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=211

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}-\frac{d(9bc-5ad)}{8c^2x\left(c+dx^2\right)(bc-}$$

Rubi [A] time = 0.31, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{15a^2d^2-27abcd+8b^2c^2}{8ac^3x(bc-ad)^2}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3}-\frac{d(9bc-5ad)}{8c^2x\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(8*b^2*c^2 - 27*a*b*c*d + 15*a^2*d^2)/(8*a*c^3*(b*c - a*d)^2*x) - d/(4*c*(
b*c - a*d)*x*(c + d*x^2)^2) - (d*(9*b*c - 5*a*d))/(8*c^2*(b*c - a*d)^2*x*(c
 + d*x^2)) - (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^3)
+ (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(8*c^(7/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= -\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-5ad-5bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)} \\ &= -\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-27abcd+15a^2d^2-3bd(9b}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}}{8c^2(bc-ad)^2} \\ &= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}-\frac{\int 8}{} \\ &= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}-\frac{b^4 \int}{a(b} \\ &= -\frac{\frac{8b^2c}{a}-27bd+\frac{15ad^2}{c}}{8c^2(bc-ad)^2x}-\frac{d}{4c(bc-ad)x\left(c+dx^2\right)^2}-\frac{d(9bc-5ad)}{8c^2(bc-ad)^2x\left(c+dx^2\right)}-\frac{b^{7/2}}{a^3}\end{aligned}$$

Mathematica [A] time = 0.38, size = 172, normalized size = 0.82

$$\frac{1}{8}\left(\frac{8b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}+\frac{d^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}+\frac{d^2x(11bc-7ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{2d^2x}{c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{8}{ac^3x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a*c^3*x) + (2*d^2*x)/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(11*b*c -
7*a*d)*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (8*b^(7/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(3/2)*(-(b*c) + a*d)^3) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d + 1
5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^3))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 4.43, size = 1991, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^3*d^2 - 16*a^3*c^2*d^3 + 2
*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 15*a^3*d^5)*x^4 + 2*(
16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 + 8*
(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 + 2*a
*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*
a^3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 +
 (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*log((d*
x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d
^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^
2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a
^3*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b
*c^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d
^4 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3
- 25*a^3*c*d^4)*x^2 - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5)*x^5
 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*b^2*c
^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ 4*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 +
3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3
*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*
c^6*d^2 - a^4*c^5*d^3)*x), -1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^
3*d^2 - 16*a^3*c^2*d^3 + 2*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d
^4 - 15*a^3*d^5)*x^4 + 2*(16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^
3 - 25*a^3*c*d^4)*x^2 + 16*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*
sqrt(b/a)*arctan(x*sqrt(b/a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^
3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (
35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3
 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2
+ 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3
*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d + 24*a^2*b*c
^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4
 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 -
25*a^3*c*d^4)*x^2 + 8*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(
b/a)*arctan(x*sqrt(b/a)) - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 15*a^3*d^5
)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^3 + (35*a*
b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d
```

```
/c)))/((a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*
x^5 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x
^3 + (a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6*d^2 - a^4*c^5*d^3)*x)]
```

giac [A] time = 0.33, size = 236, normalized size = 1.12

$$-\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\sqrt{ab}} + \frac{\left(35\,b^2c^2d^2 - 42\,abcd^3 + 15\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3\right)\sqrt{cd}} + \frac{11\,bcd^3x^3 - 7\,ad^4x^3 + 13\,bc^2d^2x - 9\,acd^3x}{8\left(b^2c^5 - 2\,abc^4d + a^2c^3d^2\right)\left(dx^2 + c\right)^2} - \frac{1}{ac^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) + 1/8*(11*b*c*d^3*x^3 - 7*a*d^4*x^3 + 13*b*c^2*d^2*x - 9*a*c*d
^3*x)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 1/(a*c^3*x)
```

maple [A] time = 0.02, size = 335, normalized size = 1.59

$$-\frac{7a^2d^5x^3}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2c^3} + \frac{9ab\,d^4x^3}{4\left(ad-bc\right)^3\left(d\,x^2+c\right)^2c^2} - \frac{11b^2d^3x^3}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2c} - \frac{9a^2d^4x}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2c^2} + \frac{11ab\,d^3x}{4\left(ad-bc\right)^3\left(d\,x^2+c\right)^2c} - \frac{13b^2d^2x}{8\left(ad-bc\right)^3\left(d\,x^2+c\right)^2} - \frac{15a^2d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^3\sqrt{cd}\;c^3} + \frac{21ab\,d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^3\sqrt{cd}\;c^2} + \frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^3\sqrt{ab}\;a} - \frac{35b^2d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^3\sqrt{cd}\;c} - \frac{1}{a\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/a*b^4/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-7/8*d^5/c^3/(a*d-
b*c)^3/(d*x^2+c)^2*x^3*a^2+9/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*b-11/8
*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2-9/8*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*
a^2*x+11/4*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*a*b*x-13/8*d^2/(a*d-b*c)^3/(d*x^2+
c)^2*b^2*x-15/8*d^4/c^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a
^2+21/4*d^3/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b-35/8*
d^2/c/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1/a/c^3/x
```

maxima [A] time = 2.54, size = 352, normalized size = 1.67

$$-\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)\sqrt{ab}} + \frac{\left(35\,b^2c^2d^2 - 42\,abcd^3 + 15\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3\right)\sqrt{cd}} - \frac{8\,b^2c^4 - 16\,abc^3d + 8\,a^2c^2d^2 + \left(8\,b^2c^2d^2 - 27\,abcd^3 + 15\,a^2d^4\right)x^4 + \left(16\,b^2c^3d - 45\,abc^2d^2 + 25\,a^2cd^3\right)x^2}{8\left(\left(ab^2c^5d^2 - 2\,a^2bc^4d^3 + a^3c^3d^4\right)x^5 + 2\left(ab^2c^6d - 2\,a^2bc^5d^2 + a^3c^4d^3\right)x^3 + \left(ab^2c^7 - 2\,a^2bc^6d + a^3c^5d^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
```

```
)*sqrt(c*d)) - 1/8*(8*b^2*c^4 - 16*a*b*c^3*d + 8*a^2*c^2*d^2 + (8*b^2*c^2*d
^2 - 27*a*b*c*d^3 + 15*a^2*d^4)*x^4 + (16*b^2*c^3*d - 45*a*b*c^2*d^2 + 25*a
^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^5 + 2*(a*
b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^3 + (a*b^2*c^7 - 2*a^2*b*c^6*d
 + a^3*c^5*d^2)*x)
```

mupad [B] time = 1.30, size = 738, normalized size = 3.50

$$-\frac{\frac{1}{a\,c}+\frac{x^4\,\left(15\,a^2\,d^4-27\,a\,b\,c\,d^3+8\,b^2\,c^2\,d^2\right)}{8\,a\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(25\,a^2\,d^3-45\,a\,b\,c\,d^2+16\,b^2\,c^2\,d\right)}{8\,a\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x+2\,c\,d\,x^3+d^2\,x^5}-\frac{\mathrm{atan}\left(\frac{b\,c^7\,x\,{\left(-a^3\,b^7\right)}^{3/2}\,64{}\mathrm{i}+a^{10}\,b\,d^7\,x\,\sqrt{-a^3\,b^7}\,225{}\mathrm{i}+a^6\,b^5\,c^4\,d^3\,x\,\sqrt{-a^3\,b^7}\,1225{}\mathrm{i}-a^7\,b^4\,c^3\,d^4\,x\,\sqrt{-a^3\,b^7}\,2940{}\mathrm{i}+a^8\,b^3\,c^2\,d^5\,x\,\sqrt{-a^3\,b^7}\,2814{}\mathrm{i}-a^9\,b^2\,c\,d^6\,x\,\sqrt{-a^3\,b^7}\,1260{}\mathrm{i}}{a^3\,b^7\,\left(2940\,a^6\,c^3\,d^4-1225\,a^5\,b\,c^4\,d^3+64\,a^2\,b^4\,c^7\right)-225\,a^{12}\,b^4\,d^7+1260\,a^{11}\,b^5\,c\,d^6-2814\,a^{10}\,b^6\,c^2\,d^5}\right)\,\sqrt{-a^3\,b^7}\,1{}\mathrm{i}}{a^6\,d^3-3\,a^5\,b\,c\,d^2+3\,a^4\,b^2\,c^2\,d-a^3\,b^3\,c^3}-\frac{\mathrm{atan}\left(\frac{a^7\,d^5\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,225{}\mathrm{i}+b^7\,c^{14}\,d\,x\,\sqrt{-c^7\,d^3}\,64{}\mathrm{i}-a^4\,b^3\,c^3\,d^2\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,2940{}\mathrm{i}+a^5\,b^2\,c^2\,d^3\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,2814{}\mathrm{i}-a^6\,b\,c\,d^4\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,1260{}\mathrm{i}+a^3\,b^4\,c^4\,d\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,1225{}\mathrm{i}}{225\,a^7\,c^{11}\,d^9-1260\,a^6\,b\,c^{12}\,d^8+2814\,a^5\,b^2\,c^{13}\,d^7-2940\,a^4\,b^3\,c^{14}\,d^6+1225\,a^3\,b^4\,c^{15}\,d^5-64\,b^7\,c^{18}\,d^2}\right)\,\sqrt{-c^7\,d^3}\,\left(15\,a^2\,d^2-42\,a\,b\,c\,d+35\,b^2\,c^2\right)\,1{}\mathrm{i}}{8\,\left(-a^3\,c^7\,d^3+3\,a^2\,b\,c^8\,d^2-3\,a\,b^2\,c^9\,d+b^3\,c^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] - (1/(a*c) + (x^4*(15*a^2*d^4 + 8*b^2*c^2*d^2 - 27*a*b*c*d^3))/(8*a*c^3*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(25*a^2*d^3 + 16*b^2*c^2*d - 45*a*b*c*
d^2))/(8*a*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x + d^2*x^5 + 2*c*d*x
^3) - (atan((b*c^7*x*(-a^3*b^7)^(3/2)*64i + a^10*b*d^7*x*(-a^3*b^7)^(1/2)*2
25i + a^6*b^5*c^4*d^3*x*(-a^3*b^7)^(1/2)*1225i - a^7*b^4*c^3*d^4*x*(-a^3*b^
7)^(1/2)*2940i + a^8*b^3*c^2*d^5*x*(-a^3*b^7)^(1/2)*2814i - a^9*b^2*c*d^6*x
*(-a^3*b^7)^(1/2)*1260i)/(a^3*b^7*(64*a^2*b^4*c^7 + 2940*a^6*c^3*d^4 - 1225
*a^5*b*c^4*d^3) - 225*a^12*b^4*d^7 + 1260*a^11*b^5*c*d^6 - 2814*a^10*b^6*c^
2*d^5))*(-a^3*b^7)^(1/2)*1i)/(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a
^5*b*c*d^2) - (atan((a^7*d^5*x*(-c^7*d^3)^(3/2)*225i + b^7*c^14*d*x*(-c^7*d
^3)^(1/2)*64i - a^4*b^3*c^3*d^2*x*(-c^7*d^3)^(3/2)*2940i + a^5*b^2*c^2*d^3*
x*(-c^7*d^3)^(3/2)*2814i - a^6*b*c*d^4*x*(-c^7*d^3)^(3/2)*1260i + a^3*b^4*c
^4*d*x*(-c^7*d^3)^(3/2)*1225i)/(225*a^7*c^11*d^9 - 64*b^7*c^18*d^2 - 1260*a
^6*b*c^12*d^8 + 1225*a^3*b^4*c^15*d^5 - 2940*a^4*b^3*c^14*d^6 + 2814*a^5*b^
2*c^13*d^7))*(-c^7*d^3)^(1/2)*(15*a^2*d^2 + 35*b^2*c^2 - 42*a*b*c*d)*1i)/(8
*(b^3*c^10 - a^3*c^7*d^3 + 3*a^2*b*c^8*d^2 - 3*a*b^2*c^9*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.259 $\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=178

$$\frac{b^4\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}-\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{}{4c^2\left(c+\right.}$$

Rubi [A] time = 0.21, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^3}+\frac{b^4\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}-\frac{\log(x)(3ad+bc)}{a^2c^4}+\frac{d^2(3bc-2ad)}{2c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2}{4c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{1}{2ac^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/(2*a*c^3*x^2) + d^2/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(3*b*c - 2*
a*d))/(2*c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d)*Log[x])/(a^2*c^4)
+ (b^4*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3) - (d^2*(6*b^2*c^2 - 8*a*b*c*d
+ 3*a^2*d^2)*Log[c + d*x^2])/(2*c^4*(b*c - a*d)^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ac^3x^2}+\frac{-bc-3ad}{a^2c^4x}-\frac{b^5}{a^2(-bc+ad)^3(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right.\right.\\
&= -\frac{1}{2ac^3x^2}+\frac{d^2}{4c^2(bc-ad)\left(c+dx^2\right)^2}+\frac{d^2(3bc-2ad)}{2c^3(bc-ad)^2\left(c+dx^2\right)}-\frac{(bc+3ad)\log(}{a^2c^4}
\end{aligned}
$$

Mathematica [A]   time = 0.42, size = 171, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2b^4\log\left(a+bx^2\right)}{a^2(ad-bc)^3}-\frac{2d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{c^4(bc-ad)^3}-\frac{4\log(x)(3ad+bc)}{a^2c^4}+\frac{2d^2(3bc-2ad)}{c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2}{c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{2}{ac^3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a*c^3*x^2) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (2*d^2*(3*b*c - 2*a
*d))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (4*(b*c + 3*a*d)*Log[x])/(a^2*c^4) -
 (2*b^4*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)^3) - (2*d^2*(6*b^2*c^2 - 8*a*b*
c*d + 3*a^2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^3))/4
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B]   time = 18.49, size = 640, normalized size = 3.60

$$-\frac{2\,ab^3c^6-6\,a^2b^2c^5d+6\,a^3bc^4d^2-2\,a^4c^3d^3+2\left(ab^3c^4d^2-6\,a^2b^2c^3d^3+8\,a^3bc^2d^4-3\,a^4cd^5\right)x^4+\left(4\,ab^3c^5d-19\,a^2b^2c^4d^2+24\,a^3bc^3d^3-9\,a^4c^2d^4\right)x^2-2\left(b^4c^4d^2x^6+2\,b^4c^5dx^4+b^4c^6x^2\right)\log\left(bx^2+a\right)+2\left(\left(6\,a^2b^2c^2d^4-8\,a^3bcd^5+3\,a^4d^6\right)x^6+2\left(6\,a^2b^2c^3d^3-8\,a^3bc^2d^4+3\,a^4cd^5\right)x^4+\left(6\,a^2b^2c^4d^2-8\,a^3bc^3d^3+3\,a^4c^2d^4\right)x^2\right)\log\left(dx^2+c\right)+4\left(\left(b^4c^4d^2-6\,a^2b^2c^2d^4+8\,a^3bcd^5-3\,a^4d^6\right)x^6+2\left(b^4c^5d-6\,a^2b^2c^3d^3+8\,a^3bc^2d^4-3\,a^4cd^5\right)x^4+\left(b^4c^6-6\,a^2b^2c^4d^2+8\,a^3bc^3d^3-3\,a^4c^2d^4\right)x^2\right)\log(x)}{4\left(\left(a^2b^3c^7d^2-3\,a^3b^2c^6d^3+3\,a^4bc^5d^4-a^5c^4d^5\right)x^6+2\left(a^2b^3c^8d-3\,a^3b^2c^7d^2+3\,a^4bc^6d^3-a^5c^5d^4\right)x^4+\left(a^2b^3c^9-3\,a^3b^2c^8d+3\,a^4bc^7d^2-a^5c^6d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b^3*c^6 - 6*a^2*b^2*c^5*d + 6*a^3*b*c^4*d^2 - 2*a^4*c^3*d^3 + 2*(
a*b^3*c^4*d^2 - 6*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (4
*a*b^3*c^5*d - 19*a^2*b^2*c^4*d^2 + 24*a^3*b*c^3*d^3 - 9*a^4*c^2*d^4)*x^2 -
 2*(b^4*c^4*d^2*x^6 + 2*b^4*c^5*d*x^4 + b^4*c^6*x^2)*log(b*x^2 + a) + 2*((6
*a^2*b^2*c^2*d^4 - 8*a^3*b*c*d^5 + 3*a^4*d^6)*x^6 + 2*(6*a^2*b^2*c^3*d^3 -
8*a^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^4 + (6*a^2*b^2*c^4*d^2 - 8*a^3*b*c^3*d^3 +
 3*a^4*c^2*d^4)*x^2)*log(d*x^2 + c) + 4*((b^4*c^4*d^2 - 6*a^2*b^2*c^2*d^4 +
 8*a^3*b*c*d^5 - 3*a^4*d^6)*x^6 + 2*(b^4*c^5*d - 6*a^2*b^2*c^3*d^3 + 8*a^3*
b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (b^4*c^6 - 6*a^2*b^2*c^4*d^2 + 8*a^3*b*c^3*d
^3 - 3*a^4*c^2*d^4)*x^2)*log(x))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*
a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^6 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3
*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^4 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*
b*c^7*d^2 - a^5*c^6*d^3)*x^2)
```

giac [**B**] time = 0.34, size = 357, normalized size = 2.01

$$\frac{b^{5}\log\left(\left|bx^{2}+a\right|\right)}{2\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}\right)}-\frac{\left(6\,b^{2}c^{2}d^{3}-8\,abcd^{4}+3\,a^{2}d^{5}\right)\log\left(\left|dx^{2}+c\right|\right)}{2\left(b^{3}c^{7}d-3\,ab^{2}c^{6}d^{2}+3\,a^{2}bc^{5}d^{3}-a^{3}c^{4}d^{4}\right)}+\frac{18\,b^{2}c^{2}d^{4}x^{4}-24\,abcd^{5}x^{4}+9\,a^{2}d^{6}x^{4}+42\,b^{2}c^{3}d^{3}x^{2}-58\,abc^{2}d^{4}x^{2}+22\,a^{2}cd^{5}x^{2}+25\,b^{2}c^{4}d^{2}-36\,abc^{3}d^{3}+14\,a^{2}c^{2}d^{4}}{4\left(b^{3}c^{7}-3\,ab^{2}c^{6}d+3\,a^{2}bc^{5}d^{2}-a^{3}c^{4}d^{3}\right)\left(dx^{2}+c\right)^{2}}-\frac{(bc+3\,ad)\log\left(x^{2}\right)}{2\,a^{2}c^{4}}+\frac{bcx^{2}+3\,adx^{2}-ac}{2\,a^{2}c^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^
2 - a^5*b*d^3) - 1/2*(6*b^2*c^2*d^3 - 8*a*b*c*d^4 + 3*a^2*d^5)*log(abs(d*x^
2 + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4) + 1/4
*(18*b^2*c^2*d^4*x^4 - 24*a*b*c*d^5*x^4 + 9*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^
2 - 58*a*b*c^2*d^4*x^2 + 22*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 36*a*b*c^3*d^3
 + 14*a^2*c^2*d^4)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*(d*x^2 + c)^2) - 1/2*(b*c + 3*a*d)*log(x^2)/(a^2*c^4) + 1/2*(b*c*x^2 + 3
*a*d*x^2 - a*c)/(a^2*c^4*x^2)
```

maple [**A**] time = 0.02, size = 322, normalized size = 1.81

$$-\frac{a^{2}d^{4}}{4\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{ab\,d^{3}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c}-\frac{b^{2}d^{2}}{4\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}}-\frac{a^{2}d^{4}}{\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)c^{3}}+\frac{3a^{2}d^{4}\ln\left(d\,x^{2}+c\right)}{2\left(ad-bc\right)^{3}c^{4}}+\frac{5ab\,d^{3}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)c^{2}}-\frac{4ab\,d^{3}\ln\left(d\,x^{2}+c\right)}{\left(ad-bc\right)^{3}c^{3}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2\left(ad-bc\right)^{3}a^{2}}-\frac{3b^{2}d^{2}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)c}+\frac{3b^{2}d^{2}\ln\left(d\,x^{2}+c\right)}{\left(ad-bc\right)^{3}c^{2}}-\frac{3d\ln(x)}{a\,c^{4}}-\frac{b\ln(x)}{a^{2}c^{3}}-\frac{1}{2a\,c^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b^4/a^2/(a*d-b*c)^3*ln(b*x^2+a)-1/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*a^
2+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*a*b-1/4*d^2/(a*d-b*c)^3/(d*x^2+c)^2*b^2
+3/2*d^4/c^4/(a*d-b*c)^3*ln(d*x^2+c)*a^2-4*d^3/c^3/(a*d-b*c)^3*ln(d*x^2+c)*
a*b+3*d^2/c^2/(a*d-b*c)^3*ln(d*x^2+c)*b^2-d^4/c^3/(a*d-b*c)^3/(d*x^2+c)*a^2
+5/2*d^3/c^2/(a*d-b*c)^3/(d*x^2+c)*a*b-3/2*d^2/c/(a*d-b*c)^3/(d*x^2+c)*b^2-
1/2/a/c^3/x^2-3/a/c^4*ln(x)*d-1/a^2/c^3*ln(x)*b
```

maxima [**B**] time = 1.28, size = 364, normalized size = 2.04

$$\frac{b^4\log\left(bx^2+a\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)}-\frac{\left(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4\right)\log\left(dx^2+c\right)}{2\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3\right)}-\frac{2\,b^2c^4-4\,abc^3d+2\,a^2c^2d^2+2\left(b^2c^2d^2-5\,abcd^3+3\,a^2d^4\right)x^4+\left(4\,b^2c^3d-15\,abc^2d^2+9\,a^2cd^3\right)x^2}{4\left(\left(ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4\right)x^6+2\left(ab^2c^6d-2\,a^2bc^5d^2+a^3c^4d^3\right)x^4+\left(ab^2c^7-2\,a^2bc^6d+a^3c^5d^2\right)x^2\right)}-\frac{(bc+3\,ad)\log\left(x^2\right)}{2\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5
*d^3) - 1/2*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*log(d*x^2 + c)/(b^3*c
^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3) - 1/4*(2*b^2*c^4 - 4*a*
b*c^3*d + 2*a^2*c^2*d^2 + 2*(b^2*c^2*d^2 - 5*a*b*c*d^3 + 3*a^2*d^4)*x^4 + (
4*b^2*c^3*d - 15*a*b*c^2*d^2 + 9*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*
c^4*d^3 + a^3*c^3*d^4)*x^6 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3
)*x^4 + (a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*x^2) - 1/2*(b*c + 3*a*d)*
log(x^2)/(a^2*c^4)
```

mupad [**B**] time = 1.50, size = 314, normalized size = 1.76

$$-\frac{\frac{1}{2\,a\,c}+\frac{x^4\,\left(3\,a^2\,d^4-5\,a\,b\,c\,d^3+b^2\,c^2\,d^2\right)}{2\,a\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(9\,a^2\,d^3-15\,a\,b\,c\,d^2+4\,b^2\,c^2\,d\right)}{4\,a\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x^2+2\,c\,d\,x^4+d^2\,x^6}-\frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d^4-8\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{-2\,a^3\,c^4\,d^3+6\,a^2\,b\,c^5\,d^2-6\,a\,b^2\,c^6\,d+2\,b^3\,c^7}-\frac{b^4\,\ln\left(b\,x^2+a\right)}{2\,\left(a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3\right)}-\frac{\ln(x)\,\left(3\,a\,d+b\,c\right)}{a^2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] - (1/(2*a*c) + (x^4*(3*a^2*d^4 + b^2*c^2*d^2 - 5*a*b*c*d^3))/(2*a*c^3*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(9*a^2*d^3 + 4*b^2*c^2*d - 15*a*b*c*d^2)
)/(4*a*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x^2 + d^2*x^6 + 2*c*d*x^4
) - (log(c + d*x^2)*(3*a^2*d^4 + 6*b^2*c^2*d^2 - 8*a*b*c*d^3))/(2*b^3*c^7 -
 2*a^3*c^4*d^3 + 6*a^2*b*c^5*d^2 - 6*a*b^2*c^6*d) - (b^4*log(a + b*x^2))/(2
*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (log(x)*(3*a*
d + b*c))/(a^2*c^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.260 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=270

$$\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{35a^3d^3-55a^2bcd^2+}{8a^2c^4x(bc}$$

Rubi [A] time = 0.43, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{35a^2d^2-55abcd+8b^2c^2}{24ac^3x^3(bc-ad)^2}+\frac{-55a^2bcd^2+35a^3d^3+8ab^2c^2d+8b^3c^3}{8a^2c^4x(bc-ad)^2}-\frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}+\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^3}-\frac{d(11bc-7ad)}{8c^2x^3\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(8*b^2*c^2 - 55*a*b*c*d + 35*a^2*d^2)/(24*a*c^3*(b*c - a*d)^2*x^3) + (8*b^
3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(8*a^2*c^4*(b*c - a*d)
^2*x) - d/(4*c*(b*c - a*d)*x^3*(c + d*x^2)^2) - (d*(11*b*c - 7*a*d))/(8*c^2
*(b*c - a*d)^2*x^3*(c + d*x^2)) + (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
(5/2)*(b*c - a*d)^3) - (d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = -\frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2} + \frac{\int \frac{4bc-7ad-7bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2} - \frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{\int \frac{8b^2c^2-55abcd+35a^2d^2-5bd}{x^4\left(a+bx^2\right)\big(c+dx}}{8c^2(bc-ad}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} - \frac{d}{4c(bc-ad)x^3\left(c+dx^2\right)^2} - \frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3\left(c+dx^2\right)} - \int$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} + \frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{d}{4c(bc-ad)x^3\big(c}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} + \frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{d}{4c(bc-ad)x^3\big(c}$$

$$= -\frac{\frac{8b^2c}{a}-55bd+\frac{35ad^2}{c}}{24c^2(bc-ad)^2x^3} + \frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{d}{4c(bc-ad)x^3\big(c}$$

Mathematica [A] time = 0.43, size = 196, normalized size = 0.73

$$-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3} - \frac{d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3} + \frac{3ad+bc}{a^2c^4x} - \frac{d^3x(15bc-11ad)}{8c^4\left(c+dx^2\right)(bc-ad)^2} - \frac{d^3x}{4c^3\left(c+dx^2\right)^2(bc-ad)} - \frac{1}{3ac^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/3*1/(a*c^3*x^3) + (b*c + 3*a*d)/(a^2*c^4*x) - (d^3*x)/(4*c^3*(b*c - a*d)
*(c + d*x^2)^2) - (d^3*(15*b*c - 11*a*d)*x)/(8*c^4*(b*c - a*d)^2*(c + d*x^2
)) - (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) - (d^
(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
8*c^(9/2)*(b*c - a*d)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)^3), x]
```

fricas [B] time = 12.31, size = 2397, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^2 - 16*a^4*c^3*d^3
 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35*a^4*d^6)*x^6
 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450*a^3*b*c^2*
d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d
^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 24*(b^4*c^4*d^2*x^7 + 2*b^4*c^
5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*
a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^
2 - 90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*s
qrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*
b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4
*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^
7*d^2 - a^5*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b
*c^4*d^2 - 8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b
*c*d^5 - 35*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^
3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5
*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 3*((63*a^
2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 -
90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d
^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 12*(b^4*c^4*d^2*x
^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a
) - a)/(b*x^2 + a)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^
4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d
^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 -
a^5*c^6*d^3)*x^3), -1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^
2 - 16*a^4*c^3*d^3 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5
 - 35*a^4*d^6)*x^6 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^
3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d
- 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 - 48*(b^4*c^4*
d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3
*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*
b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) -
```

```
 c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 -
 a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3
- a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5
*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5*d + 24*a^3*b*c^4*d^2 -
8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35
*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450
*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2
*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 - 24*(b^4*c^4*d^2*x^7
+ 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*((63*a^2
*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*a^2*b^2*c^3*d^3 - 9
0*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d^2 - 90*a^3*b*c^3*d^
3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^3*c^7*d^2 -
 3*a^3*b^2*c^6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d
- 3*a^3*b^2*c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3
*a^3*b^2*c^8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^3)]
```

giac [A] time = 0.31, size = 256, normalized size = 0.95

$$\frac{b^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^{2}b^{3}c^{3}-3\,a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-a^{5}d^{3}\right)\sqrt{ab}}-\frac{\left(63\,b^{2}c^{2}d^{3}-90\,abcd^{4}+35\,a^{2}d^{5}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{3}c^{7}-3\,ab^{2}c^{6}d+3\,a^{2}bc^{5}d^{2}-a^{3}c^{4}d^{3}\right)\sqrt{cd}}-\frac{15\,bcd^{4}x^{3}-11\,ad^{5}x^{3}+17\,bc^{2}d^{3}x-13\,acd^{4}x}{8\left(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}\right)\left(dx^{2}+c\right)^{2}}+\frac{3\,bcx^{2}+9\,adx^{2}-ac}{3\,a^{2}c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*arc
tan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*sqrt(c*d)) - 1/8*(15*b*c*d^4*x^3 - 11*a*d^5*x^3 + 17*b*c^2*d^3*x - 13*a*
c*d^4*x)/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 1/3*(3*b*c
*x^2 + 9*a*d*x^2 - a*c)/(a^2*c^4*x^3)
```

maple [A] time = 0.02, size = 362, normalized size = 1.34

$$\frac{11a^{2}d^{6}x^{3}}{8\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c^{4}}-\frac{13ab\,d^{5}x^{3}}{4\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c^{3}}+\frac{15b^{2}d^{4}x^{3}}{8\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{13a^{2}d^{5}x}{8\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c^{3}}-\frac{15ab\,d^{4}x}{4\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{17b^{2}d^{3}x}{8\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)^{2}c}+\frac{35a^{2}d^{5}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{3}\sqrt{cd}\,c^{4}}-\frac{45ab\,d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^{3}\sqrt{cd}\,c^{3}}-\frac{b^{5}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(ad-bc\right)^{3}\sqrt{ab}\,a^{2}}+\frac{63b^{2}d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{3}\sqrt{cd}\,c^{2}}+\frac{3d}{a\,c^{4}x}+\frac{b}{a^{2}c^{3}x}-\frac{1}{3a\,c^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/a^2*b^5/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+11/8*d^6/c^4/(
a*d-b*c)^3/(d*x^2+c)^2*x^3*a^2-13/4*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^3*a*b
+15/8*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+13/8*d^5/c^3/(a*d-b*c)^3/(d*x
^2+c)^2*a^2*x-15/4*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*a*b*x+17/8*d^3/c/(a*d-b*
c)^3/(d*x^2+c)^2*b^2*x+35/8*d^5/c^4/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^
(1/2)*d*x)*a^2-45/4*d^4/c^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*
```

```
x)*a*b+63/8*d^3/c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1
/3/a/c^3/x^3+3/a/c^4/x*d+1/a^2/c^3/x*b
```

maxima [A]   time = 2.58, size = 440, normalized size = 1.63

$$\frac{b^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\sqrt{ab}}-\frac{\left(63\,b^2c^2d^3-90\,abcd^4+35\,a^2d^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3\right)\sqrt{cd}}-\frac{8\,ab^2c^5-16\,a^2bc^4d+8\,a^3c^3d^2-3\left(8\,b^3c^3d^2+8\,ab^2c^2d^3-55\,a^2bcd^4+35\,a^3d^5\right)x^6-\left(48\,b^3c^4d+40\,ab^2c^3d^2-275\,a^2bc^2d^3+175\,a^3cd^4\right)x^4-8\left(3\,b^3c^5+ab^2c^4d-11\,a^2bc^3d^2+7\,a^3c^2d^3\right)x^2}{24\left(\left(a^2b^2c^6d^2-2\,a^3bc^5d^3+a^4c^4d^4\right)x^7+2\left(a^2b^2c^7d-2\,a^3bc^6d^2+a^4c^5d^3\right)x^5+\left(a^2b^2c^8-2\,a^3bc^7d+a^4c^6d^2\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*arc
tan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^
3)*sqrt(c*d)) - 1/24*(8*a*b^2*c^5 - 16*a^2*b*c^4*d + 8*a^3*c^3*d^2 - 3*(8*b
^3*c^3*d^2 + 8*a*b^2*c^2*d^3 - 55*a^2*b*c*d^4 + 35*a^3*d^5)*x^6 - (48*b^3*c
^4*d + 40*a*b^2*c^3*d^2 - 275*a^2*b*c^2*d^3 + 175*a^3*c*d^4)*x^4 - 8*(3*b^3
*c^5 + a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 7*a^3*c^2*d^3)*x^2)/((a^2*b^2*c^6*d
^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^7 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^
2 + a^4*c^5*d^3)*x^5 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^3)
```

mupad [B]   time = 1.42, size = 785, normalized size = 2.91

$$\frac{\frac{x^2\,(7\,a\,d+3\,b\,c)}{3\,a^2\,c^2}-\frac{1}{3\,a\,c}+\frac{x^4\,\left(175\,a^3\,d^4-275\,a^2\,b\,c\,d^3+40\,a\,b^2\,c^2\,d^2+48\,b^3\,c^3\,d\right)}{24\,a^2\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^6\,\left(35\,a^3\,d^5-55\,a^2\,b\,c\,d^4+8\,a\,b^2\,c^2\,d^3+8\,b^3\,c^3\,d^2\right)}{8\,a^2\,c^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x^3+2\,c\,d\,x^5+d^2\,x^7}+\frac{\mathrm{atan}\left(\frac{b\,c^9\,x\,{\left(-a^5\,b^9\right)}^{3/2}\,64{}\mathrm{i}+a^{14}\,b\,d^9\,x\,\sqrt{-a^5\,b^9}\,1225{}\mathrm{i}+a^{10}\,b^5\,c^4\,d^5\,x\,\sqrt{-a^5\,b^9}\,3969{}\mathrm{i}-a^{11}\,b^4\,c^3\,d^6\,x\,\sqrt{-a^5\,b^9}\,11340{}\mathrm{i}+a^{12}\,b^3\,c^2\,d^7\,x\,\sqrt{-a^5\,b^9}\,12510{}\mathrm{i}-a^{13}\,b^2\,c\,d^8\,x\,\sqrt{-a^5\,b^9}\,6300{}\mathrm{i}}{-1225\,a^{17}\,b^5\,d^9+6300\,a^{16}\,b^6\,c\,d^8-12510\,a^{15}\,b^7\,c^2\,d^7+11340\,a^{14}\,b^8\,c^3\,d^6-3969\,a^{13}\,b^9\,c^4\,d^5+64\,a^8\,b^{14}\,c^9}\right)\,\sqrt{-a^5\,b^9}\,1{}\mathrm{i}}{a^8\,d^3-3\,a^7\,b\,c\,d^2+3\,a^6\,b^2\,c^2\,d-a^5\,b^3\,c^3}+\frac{\mathrm{atan}\left(\frac{a^9\,d^5\,x\,{\left(-c^9\,d^5\right)}^{3/2}\,1225{}\mathrm{i}+b^9\,c^{18}\,d\,x\,\sqrt{-c^9\,d^5}\,64{}\mathrm{i}-a^6\,b^3\,c^3\,d^2\,x\,{\left(-c^9\,d^5\right)}^{3/2}\,11340{}\mathrm{i}+a^7\,b^2\,c^2\,d^3\,x\,{\left(-c^9\,d^5\right)}^{3/2}\,12510{}\mathrm{i}-a^8\,b\,c\,d^4\,x\,{\left(-c^9\,d^5\right)}^{3/2}\,6300{}\mathrm{i}+a^5\,b^4\,c^4\,d\,x\,{\left(-c^9\,d^5\right)}^{3/2}\,3969{}\mathrm{i}}{1225\,a^9\,c^{14}\,d^{12}-6300\,a^8\,b\,c^{15}\,d^{11}+12510\,a^7\,b^2\,c^{16}\,d^{10}-11340\,a^6\,b^3\,c^{17}\,d^9+3969\,a^5\,b^4\,c^{18}\,d^8-64\,b^9\,c^{23}\,d^3}\right)\,\sqrt{-c^9\,d^5}\,\left(35\,a^2\,d^2-90\,a\,b\,c\,d+63\,b^2\,c^2\right)\,1{}\mathrm{i}}{8\,\left(-a^3\,c^9\,d^3+3\,a^2\,b\,c^{10}\,d^2-3\,a\,b^2\,c^{11}\,d+b^3\,c^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*d + 3*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^4*(175*a^3*d^4 + 48*b^3
*c^3*d + 40*a*b^2*c^2*d^2 - 275*a^2*b*c*d^3))/(24*a^2*c^3*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) + (x^6*(35*a^3*d^5 + 8*b^3*c^3*d^2 + 8*a*b^2*c^2*d^3 - 55*a
^2*b*c*d^4))/(8*a^2*c^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x^3 + d^2*x^
7 + 2*c*d*x^5) + (atan((b*c^9*x*(-a^5*b^9)^(3/2)*64i + a^14*b*d^9*x*(-a^5*b
^9)^(1/2)*1225i + a^10*b^5*c^4*d^5*x*(-a^5*b^9)^(1/2)*3969i - a^11*b^4*c^3*
d^6*x*(-a^5*b^9)^(1/2)*11340i + a^12*b^3*c^2*d^7*x*(-a^5*b^9)^(1/2)*12510i
- a^13*b^2*c*d^8*x*(-a^5*b^9)^(1/2)*6300i)/(64*a^8*b^14*c^9 - 1225*a^17*b^5
*d^9 + 6300*a^16*b^6*c*d^8 - 3969*a^13*b^9*c^4*d^5 + 11340*a^14*b^8*c^3*d^6
 - 12510*a^15*b^7*c^2*d^7))*(-a^5*b^9)^(1/2)*1i)/(a^8*d^3 - a^5*b^3*c^3 + 3
*a^6*b^2*c^2*d - 3*a^7*b*c*d^2) + (atan((a^9*d^5*x*(-c^9*d^5)^(3/2)*1225i +
 b^9*c^18*d*x*(-c^9*d^5)^(1/2)*64i - a^6*b^3*c^3*d^2*x*(-c^9*d^5)^(3/2)*113
40i + a^7*b^2*c^2*d^3*x*(-c^9*d^5)^(3/2)*12510i - a^8*b*c*d^4*x*(-c^9*d^5)^
(3/2)*6300i + a^5*b^4*c^4*d*x*(-c^9*d^5)^(3/2)*3969i)/(1225*a^9*c^14*d^12 -
 64*b^9*c^23*d^3 - 6300*a^8*b*c^15*d^11 + 3969*a^5*b^4*c^18*d^8 - 11340*a^6
*b^3*c^17*d^9 + 12510*a^7*b^2*c^16*d^10))*(-c^9*d^5)^(1/2)*(35*a^2*d^2 + 63
*b^2*c^2 - 90*a*b*c*d)*1i)/(8*(b^3*c^12 - a^3*c^9*d^3 + 3*a^2*b*c^10*d^2 -
3*a*b^2*c^11*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.261 $\int \frac{x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx, x, x^2\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{4+x}\,dx, x, x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[x/((1 + x^2)*(4 + x^2)), x]
```

fricas [A] time = 0.96, size = 17, normalized size = 0.81

$$-\frac{1}{6}\,\log\left(x^2+4\right)+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

giac [A] time = 0.28, size = 17, normalized size = 0.81

$$-\frac{1}{6}\,\log\left(x^2+4\right)+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

maple [A]   time = 0.01, size = 18, normalized size = 0.86

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+4),x)
```

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

maxima [A]   time = 0.95, size = 17, normalized size = 0.81

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

mupad [B]   time = 0.15, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}\left(\frac{3\,x^2}{5\,x^2+8}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x^2 + 4)),x)
```

```
[Out] atanh((3*x^2)/(5*x^2 + 8))/3
```

sympy [A]   time = 0.11, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

3.262 $\int \frac{x^4\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{ax(bc-ad)}{2b^3\left(a+bx^2\right)}+\frac{x(bc-2ad)}{b^3}+\frac{dx^3}{3b^2}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 1153, 205}

$$\frac{ax(bc-ad)}{2b^3\left(a+bx^2\right)}+\frac{x(bc-2ad)}{b^3}-\frac{\sqrt{a}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + (a*(b*c - a*d)*x)/(2*b^3*(a + b*x
^2)) - (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)}-\frac{\int \frac{a(bc-ad)-2b(bc-ad)x^2-2b^2dx^4}{a+bx^2}\,dx}{2b^3}\\
&= \frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)}-\frac{\int \left(-2(bc-2ad)-2bdx^2+\frac{3abc-5a^2d}{a+bx^2}\right)dx}{2b^3}\\
&= \frac{(bc-2ad)x}{b^3}+\frac{dx^3}{3b^2}+\frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)}-\frac{(a(3bc-5ad))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= \frac{(bc-2ad)x}{b^3}+\frac{dx^3}{3b^2}+\frac{a(bc-ad)x}{2b^3\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 89, normalized size = 1.02

$$
\frac{x\left(abc-a^2d\right)}{2b^3\left(a+bx^2\right)}+\frac{\sqrt{a}\,(5ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(bc-2ad)}{b^3}+\frac{dx^3}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + ((a*b*c - a^2*d)*x)/(2*b^3*(a + b
*x^2)) + (Sqrt[a]*(-3*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^4\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.95, size = 240, normalized size = 2.76

$$\left[\frac{4\,b^2dx^5+4\left(3\,b^2c-5\,abd\right)x^3-3\left(3\,abc-5\,a^2d+\left(3\,b^2c-5\,abd\right)x^2\right)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\left(3\,abc-5\,a^2d\right)x}{12\left(b^4x^2+ab^3\right)},\ \frac{2\,b^2dx^5+2\left(3\,b^2c-5\,abd\right)x^3-3\left(3\,abc-5\,a^2d+\left(3\,b^2c-5\,abd\right)x^2\right)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\left(3\,abc-5\,a^2d\right)x}{6\left(b^4x^2+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*d*x^5 + 4*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*
b^2*c - 5*a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2
+ a)) + 6*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3), 1/6*(2*b^2*d*x^5 + 2*(3
*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*b^2*c - 5*a*b*d)*x^2)*sqr
t(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3)
]
```

giac [A] time = 0.39, size = 88, normalized size = 1.01

$$-\frac{\left(3\,abc-5\,a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{abcx-a^2dx}{2\left(bx^2+a\right)b^3}+\frac{b^4dx^3+3\,b^4cx-6\,ab^3dx}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c - 5*a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(a*b*c
*x - a^2*d*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d*x^3 + 3*b^4*c*x - 6*a*b^3*d*x)
/b^6
```

maple [A] time = 0.01, size = 105, normalized size = 1.21

$$\frac{d\,x^3}{3b^2}-\frac{a^2dx}{2\left(b\,x^2+a\right)b^3}+\frac{5a^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}+\frac{acx}{2\left(b\,x^2+a\right)b^2}-\frac{3ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}-\frac{2adx}{b^3}+\frac{cx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d*x^3/b^2-2/b^3*a*d*x+1/b^2*c*x-1/2*a^2/b^3*x/(b*x^2+a)*d+1/2*a/b^2*x/(
b*x^2+a)*c+5/2*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-3/2*a/b^2/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c
```

maxima [A] time = 2.24, size = 84, normalized size = 0.97

$$\frac{\left(abc-a^2d\right)x}{2\left(b^4x^2+ab^3\right)}-\frac{\left(3\,abc-5\,a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{bdx^3+3\,(bc-2\,ad)x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*c - a^2*d)*x/(b^4*x^2 + a*b^3) - 1/2*(3*a*b*c - 5*a^2*d)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(b*d*x^3 + 3*(b*c - 2*a*d)*x)/b^3
```

mupad [B] time = 0.08, size = 104, normalized size = 1.20

$$x\left(\frac{c}{b^2}-\frac{2\,a\,d}{b^3}\right)+\frac{d\,x^3}{3\,b^2}-\frac{x\left(\frac{a^2\,d}{2}-\frac{a\,b\,c}{2}\right)}{b^4\,x^2+a\,b^3}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(5\,a\,d-3\,b\,c)}{5\,a^2\,d-3\,a\,b\,c}\right)\,(5\,a\,d-3\,b\,c)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x*(c/b^2 - (2*a*d)/b^3) + (d*x^3)/(3*b^2) - (x*((a^2*d)/2 - (a*b*c)/2))/(a*
b^3 + b^4*x^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(5*a*d - 3*b*c))/(5*a^2*d
 - 3*a*b*c))*(5*a*d - 3*b*c))/(2*b^(7/2))
```

sympy [A] time = 0.63, size = 129, normalized size = 1.48

$$x\left(-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x\left(-a^2d+abc\right)}{2ab^3+2b^4x^2}-\frac{\sqrt{-\frac{a}{b^7}}\,(5ad-3bc)\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{\sqrt{-\frac{a}{b^7}}\,(5ad-3bc)\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{dx^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d/b**3 + c/b**2) + x*(-a**2*d + a*b*c)/(2*a*b**3 + 2*b**4*x**2) - s
qrt(-a/b**7)*(5*a*d - 3*b*c)*log(-b**3*sqrt(-a/b**7) + x)/4 + sqrt(-a/b**7)
*(5*a*d - 3*b*c)*log(b**3*sqrt(-a/b**7) + x)/4 + d*x**3/(3*b**2)
```

3.263 $\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=60

$$\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}+\frac{dx^2}{2b^2}$$

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a(bc-ad)}{2b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{2b^3}+\frac{dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x^2)/(2*b^2) + (a*(b*c - a*d))/(2*b^3*(a + b*x^2)) + ((b*c - 2*a*d)*Log[
a + b*x^2])/(2*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(c+dx)}{(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d}{b^2}+\frac{a(-bc+ad)}{b^2(a+bx)^2}+\frac{bc-2ad}{b^2(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{dx^2}{2b^2}+\frac{a(bc-ad)}{2b^3\left(a+bx^2\right)}+\frac{(bc-2ad)\log\left(a+bx^2\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.83

$$\frac{\frac{a(bc-ad)}{a+bx^2}+(bc-2ad)\log\left(a+bx^2\right)+bdx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (b*d*x^2 + (a*(b*c - a*d))/(a + b*x^2) + (b*c - 2*a*d)*Log[a + b*x^2])/(2*b
^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 1.01, size = 78, normalized size = 1.30

$$\frac{b^2dx^4+abdx^2+abc-a^2d+\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^4 + a*b*d*x^2 + a*b*c - a^2*d + (a*b*c - 2*a^2*d + (b^2*c - 2*
a*b*d)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.31, size = 90, normalized size = 1.50

$$\frac{\frac{\left(bx^2+a\right)d}{b^2} - \frac{(bc-2\,ad)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^2} + \frac{\frac{ab^2c}{bx^2+a}-\frac{a^2bd}{bx^2+a}}{b^3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*((b*x^2 + a)*d/b^2 - (b*c - 2*a*d)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^2 + (a*b^2*c/(b*x^2 + a) - a^2*b*d/(b*x^2 + a))/b^3)/b
```

maple [A] time = 0.01, size = 74, normalized size = 1.23

$$\frac{d\,x^2}{2b^2} - \frac{a^2d}{2\left(b\,x^2+a\right)b^3} + \frac{ac}{2\left(b\,x^2+a\right)b^2} - \frac{ad\ln\left(b\,x^2+a\right)}{b^3} + \frac{c\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d*x^2/b^2-1/2/b^3*a^2/(b*x^2+a)*d+1/2/(b*x^2+a)*a/b^2*c-1/b^3*ln(b*x^2+
a)*a*d+1/2/b^2*c*ln(b*x^2+a)
```

maxima [A] time = 0.99, size = 59, normalized size = 0.98

$$\frac{dx^2}{2\,b^2} + \frac{abc-a^2d}{2\left(b^4x^2+ab^3\right)} + \frac{(bc-2\,ad)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2/b^2 + 1/2*(a*b*c - a^2*d)/(b^4*x^2 + a*b^3) + 1/2*(b*c - 2*a*d)*l
og(b*x^2 + a)/b^3
```

mupad [B] time = 0.08, size = 63, normalized size = 1.05

$$\frac{d\,x^2}{2\,b^2} - \frac{\ln\left(b\,x^2+a\right)\,(2\,a\,d-b\,c)}{2\,b^3} - \frac{a^2\,d-a\,b\,c}{2\,b\,\left(b^3\,x^2+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (d*x^2)/(2*b^2) - (log(a + b*x^2)*(2*a*d - b*c))/(2*b^3) - (a^2*d - a*b*c)/
(2*b*(a*b^2 + b^3*x^2))
```

sympy [A] time = 0.56, size = 56, normalized size = 0.93

$$\frac{-a^2d + abc}{2ab^3 + 2b^4x^2} + \frac{dx^2}{2b^2} - \frac{(2ad - bc)\log\left(a + bx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] (-a**2*d + a*b*c)/(2*a*b**3 + 2*b**4*x**2) + d*x**2/(2*b**2) - (2*a*d - b*c
)*log(a + b*x**2)/(2*b**3)
```

3.264 $\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=67

$$\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(bc-ad)}{2b^2\left(a+bx^2\right)}+\frac{dx}{b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 388, 205}

$$-\frac{x(bc-ad)}{2b^2\left(a+bx^2\right)}+\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) + ((b*c - 3*a*d)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
```

```
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{(bc-ad)x}{2b^2\left(a+bx^2\right)} - \frac{\int \frac{-bc+ad-2bdx^2}{a+bx^2}\,dx}{2b^2} \\
&= \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2\left(a+bx^2\right)} + \frac{(bc-3ad)\int \frac{1}{a+bx^2}\,dx}{2b^2} \\
&= \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2\left(a+bx^2\right)} + \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 68, normalized size = 1.01

$$
-\frac{(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}} - \frac{x(bc-ad)}{2b^2\left(a+bx^2\right)} + \frac{dx}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) - ((-(b*c) + 3*a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.92, size = 202, normalized size = 3.01

$$
\left[\frac{4\,ab^2dx^3 + \left(abc - 3\,a^2d + \left(b^2c - 3\,abd\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right) - 2\left(ab^2c - 3\,a^2bd\right)x}{4\left(ab^4x^2 + a^2b^3\right)}, \frac{2\,ab^2dx^3 + \left(abc - 3\,a^2d + \left(b^2c - 3\,abd\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - \left(ab^2c - 3\,a^2bd\right)x}{2\left(ab^4x^2 + a^2b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-a*b)*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - 3*a^2*b*d)*x)/
(a*b^4*x^2 + a^2*b^3), 1/2*(2*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a
*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (a*b^2*c - 3*a^2*b*d)*x)/(a*b^
4*x^2 + a^2*b^3)]
```

giac [A] time = 0.30, size = 58, normalized size = 0.87

$$\frac{dx}{b^2} + \frac{(bc - 3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2} - \frac{bcx - adx}{2\left(bx^2 + a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(b*
c*x - a*d*x)/((b*x^2 + a)*b^2)
```

maple [A] time = 0.01, size = 82, normalized size = 1.22

$$\frac{adx}{2\left(b\,x^2 + a\right)b^2} - \frac{3ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2} - \frac{cx}{2\left(b\,x^2 + a\right)b} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b} + \frac{dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] d*x/b^2+1/2/b^2*x/(b*x^2+a)*a*d-1/2/(b*x^2+a)/b*c*x-3/2/b^2/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*a*d+1/2/(a*b)^(1/2)/b*c*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.32, size = 60, normalized size = 0.90

$$-\frac{(bc - ad)x}{2\left(b^3x^2 + ab^2\right)} + \frac{dx}{b^2} + \frac{(bc - 3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)*x/(b^3*x^2 + a*b^2) + d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 0.18, size = 59, normalized size = 0.88

$$\frac{x\left(\frac{a\,d}{2}-\frac{b\,c}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{d\,x}{b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,a\,d-b\,c\right)}{2\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (x*((a*d)/2 - (b*c)/2))/(a*b^2 + b^3*x^2) + (d*x)/b^2 - (atan((b^(1/2)*x)/a
^(1/2))*(3*a*d - b*c))/(2*a^(1/2)*b^(5/2))
```

sympy [A] time = 0.53, size = 114, normalized size = 1.70

$$\frac{x\left(ad-bc\right)}{2ab^2+2b^3x^2}+\frac{\sqrt{-\frac{1}{ab^5}}\,\left(3ad-bc\right)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^5}}\,\left(3ad-bc\right)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}+\frac{dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(3*a*d - b*c)*lo
g(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(3*a*d - b*c)*log(a*
b**2*sqrt(-1/(a*b**5)) + x)/4 + d*x/b**2
```

3.265 $\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=41

$$\frac{ad-bc}{2b^2(a+bx^2)}+\frac{d\log(a+bx^2)}{2b^2}$$

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{d\log(a+bx^2)}{2b^2}-\frac{bc-ad}{2b^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -(b*c - a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx}{(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{bc-ad}{b(a+bx)^2}+\frac{d}{b(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{bc-ad}{2b^2\left(a+bx^2\right)}+\frac{d\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{ad-bc}{2b^2\left(a+bx^2\right)}+\frac{d\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (-(b*c) + a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^2\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^2))/(a + b*x^2)^2, x]
```

fricas [A] time = 0.85, size = 45, normalized size = 1.10

$$-\frac{bc-ad-\left(bdx^2+ad\right)\log\left(bx^2+a\right)}{2\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d - (b*d*x^2 + a*d)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.34, size = 65, normalized size = 1.59

$$-\frac{d\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2\,b}-\frac{c}{2\left(bx^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
 - 1/2*c/((b*x^2 + a)*b)
```

maple [A] time = 0.01, size = 47, normalized size = 1.15

$$\frac{ad}{2\left(b\,x^2+a\right)b^2}-\frac{c}{2\left(b\,x^2+a\right)b}+\frac{d\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/2/b^2/(b*x^2+a)*a*d-1/2/(b*x^2+a)/b*c+1/2*d*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.07, size = 40, normalized size = 0.98

$$-\frac{bc-ad}{2\left(b^3x^2+ab^2\right)}+\frac{d\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*c - a*d)/(b^3*x^2 + a*b^2) + 1/2*d*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.15, size = 37, normalized size = 0.90

$$\frac{d\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a\,d-b\,c}{2\,b^2\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (d*log(a + b*x^2))/(2*b^2) + (a*d - b*c)/(2*b^2*(a + b*x^2))
```

sympy [**A**] time = 0.36, size = 36, normalized size = 0.88

$$\frac{ad-bc}{2ab^2+2b^3x^2}+\frac{d\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] (a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + d*log(a + b*x**2)/(2*b**2)
```

3.266 $\int \frac{c+dx^2}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 205}

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2ab}\\ &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A]   time = 0.04, size = 63, normalized size = 1.00

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(ad-bc)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(a + b*x^2)^2, x]
```

fricas [A]   time = 0.80, size = 181, normalized size = 2.87

$$\left[-\frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\left(ab^2c-a^2bd\right)x}{4\left(a^2b^3x^2+a^3b^2\right)},\ \frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(ab^2c-a^2bd\right)x}{2\left(a^2b^3x^2+a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
```

```
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.40, size = 57, normalized size = 0.90

$$\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}+\frac{bcx-adx}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

maple [A] time = 0.01, size = 68, normalized size = 1.08

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}-\frac{(ad-bc)\,x}{2\left(b\,x^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*
d+1/2/(a*b)^(1/2)/a*c*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.39, size = 57, normalized size = 0.90

$$\frac{(bc-ad)x}{2\left(ab^2x^2+a^2b\right)}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.18, size = 51, normalized size = 0.81

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d+b\,c\right)}{2\,a^{3/2}\,b^{3/2}}-\frac{x\,\left(a\,d-b\,c\right)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*c
))/(2*a*b*(a + b*x^2))
```

sympy [B] time = 0.40, size = 112, normalized size = 1.78

$$\frac{x(-ad+bc)}{2a^2b+2ab^2x^2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad+bc)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad+bc)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

3.267 $\int \frac{c+dx^2}{x\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=51

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{c\log\left(a+bx^2\right)}{2a^2}+\frac{c\log(x)}{a^2}+\frac{bc-ad}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)/(2*a*b*(a + b*x^2)) + (c*Log[x])/a^2 - (c*Log[a + b*x^2])/(2*a^
2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{c+dx}{x(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{c}{a^2x}+\frac{-bc+ad}{a(a+bx)^2}-\frac{bc}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{bc-ad}{2ab\left(a+bx^2\right)}+\frac{c\log(x)}{a^2}-\frac{c\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{a(bc-ad)}{b\left(a+bx^2\right)}-c\log\left(a+bx^2\right)+2c\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(b*(a + b*x^2)) + 2*c*Log[x] - c*Log[a + b*x^2])/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{x\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x*(a + b*x^2)^2), x]
```

fricas [A] time = 0.84, size = 71, normalized size = 1.39

$$\frac{abc-a^2d-\left(b^2cx^2+abc\right)\log\left(bx^2+a\right)+2\left(b^2cx^2+abc\right)\log(x)}{2\left(a^2b^2x^2+a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d - (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) + 2*(b^2*c*x^2 + a*
b*c)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

giac [A] time = 0.32, size = 63, normalized size = 1.24

$$\frac{c\log\left(x^2\right)}{2\,a^2} - \frac{c\log\left(\left|bx^2+a\right|\right)}{2\,a^2} + \frac{b^2cx^2+2\,abc-a^2d}{2\left(bx^2+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b^2*c*x^2 + 2*a*b
*c - a^2*d)/((b*x^2 + a)*a^2*b)
```

maple [A] time = 0.01, size = 53, normalized size = 1.04

$$\frac{c}{2\left(b\,x^2+a\right)a} + \frac{c\ln(x)}{a^2} - \frac{c\ln\left(b\,x^2+a\right)}{2a^2} - \frac{d}{2\left(b\,x^2+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2+a)*d+1/2/(b*x^2+a)/a*c-1/2/a^2*c*ln(b*x^2+a)+1/a^2*c*ln(x)
```

maxima [A] time = 1.07, size = 51, normalized size = 1.00

$$\frac{bc-ad}{2\left(ab^2x^2+a^2b\right)} - \frac{c\log\left(bx^2+a\right)}{2\,a^2} + \frac{c\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)/(a*b^2*x^2 + a^2*b) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(
x^2)/a^2
```

mupad [B] time = 0.06, size = 47, normalized size = 0.92

$$\frac{c\,\ln(x)}{a^2} - \frac{c\,\ln\left(b\,x^2+a\right)}{2\,a^2} - \frac{a\,d-b\,c}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (c*log(x))/a^2 - (c*log(a + b*x^2))/(2*a^2) - (a*d - b*c)/(2*a*b*(a + b*x^2
))
```

sympy [**A**] time = 0.41, size = 46, normalized size = 0.90

$$\frac{-ad+bc}{2a^2b+2ab^2x^2}+\frac{c\log\left(x\right)}{a^2}-\frac{c\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x/(b*x**2+a)**2,x)
```

```
[Out] (-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) + c*log(x)/a**2 - c*log(a/b + x**2)
/(2*a**2)
```

3.268 $\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{x(bc-ad)}{2a^2\left(a+bx^2\right)}-\frac{c}{a^2x}$$

Rubi [A] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {456, 453, 205}

$$-\frac{x(bc-ad)}{2a^2\left(a+bx^2\right)}-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) - ((b*c - a*d)*x)/(2*a^2*(a + b*x^2)) - ((3*b*c - a*d)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)}-\frac{1}{2}\int \frac{-\frac{2c}{a}+\frac{(bc-ad)x^2}{a^2}}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{c}{a^2x}-\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)}-\frac{(3bc-ad)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{c}{a^2x}-\frac{(bc-ad)x}{2a^2\left(a+bx^2\right)}-\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.99

$$\frac{(ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{x(ad-bc)}{2a^2\left(a+bx^2\right)}-\frac{c}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + ((-(b*c) + a*d)*x)/(2*a^2*(a + b*x^2)) + ((-3*b*c + a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 0.95, size = 214, normalized size = 3.01

$$\left[-\frac{4\,a^2bc+2\left(3\,ab^2c-a^2bd\right)x^2-\left(\left(3\,b^2c-abd\right)x^3+\left(3\,abc-a^2d\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^3b^2x^3+a^4bx\right)},-\frac{2\,a^2bc+\left(3\,ab^2c-a^2bd\right)x^2+\left(\left(3\,b^2c-abd\right)x^3+\left(3\,abc-a^2d\right)x\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^3b^2x^3+a^4bx\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b*c + 2*(3*a*b^2*c - a^2*b*d)*x^2 - ((3*b^2*c - a*b*d)*x^3 + (
3*a*b*c - a^2*d)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*a^2*b*c + (3*a*b^2*c - a^2*b*d)*x^2 + (
(3*b^2*c - a*b*d)*x^3 + (3*a*b*c - a^2*d)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
))/(a^3*b^2*x^3 + a^4*b*x)]
```

giac [A] time = 0.35, size = 64, normalized size = 0.90

$$-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{3\,bcx^2-adx^2+2\,ac}{2\,\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 -
 a*d*x^2 + 2*a*c)/((b*x^3 + a*x)*a^2)
```

maple [A] time = 0.01, size = 85, normalized size = 1.20

$$\frac{dx}{2\left(b\,x^2+a\right)a}+\frac{d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\;a}-\frac{bcx}{2\left(b\,x^2+a\right)a^2}-\frac{3bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\;a^2}-\frac{c}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a*x/(b*x^2+a)*d-1/2/(b*x^2+a)/a^2*b*c*x+1/2/a/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x)*d-3/2/(a*b)^(1/2)/a^2*b*c*arctan(1/(a*b)^(1/2)*b*x)-1/a^2*c/x
```

maxima [A] time = 2.23, size = 65, normalized size = 0.92

$$-\frac{(3\,bc-ad)x^2+2\,ac}{2\,\left(a^2bx^3+a^3x\right)}-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b*c - a*d)*x^2 + 2*a*c)/(a^2*b*x^3 + a^3*x) - 1/2*(3*b*c - a*d)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [**B**] time = 0.19, size = 61, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a\,d-3\,b\,c)}{2\,a^{5/2}\,\sqrt{b}}-\frac{\frac{c}{a}-\frac{x^2\,(a\,d-3\,b\,c)}{2\,a^2}}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d - 3*b*c))/(2*a^(5/2)*b^(1/2)) - (c/a - (x^2
*(a*d - 3*b*c))/(2*a^2))/(a*x + b*x^3)
```

sympy [**A**] time = 0.49, size = 114, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b}}\,(ad-3bc)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4}+\frac{-2ac+x^2\,(ad-3bc)}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b))*(a*d - 3*b*c)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt(
-1/(a**5*b))*(a*d - 3*b*c)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*a*c + x*
*2*(a*d - 3*b*c))/(2*a**3*x + 2*a**2*b*x**3)
```

3.269 $\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=76

$$\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{bc-ad}{2a^2\left(a+bx^2\right)}-\frac{c}{2a^2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{bc-ad}{2a^2\left(a+bx^2\right)}+\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2bc-ad)}{a^3}-\frac{c}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c/(2*a^2*x^2) - (b*c - a*d)/(2*a^2*(a + b*x^2)) - ((2*b*c - a*d)*Log[x])/a
^3 + ((2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c}{a^2x^2}+\frac{-2bc+ad}{a^3x}-\frac{b(-bc+ad)}{a^2(a+bx)^2}-\frac{b(-2bc+ad)}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{c}{2a^2x^2}-\frac{bc-ad}{2a^2\left(a+bx^2\right)}-\frac{(2bc-ad)\log(x)}{a^3}+\frac{(2bc-ad)\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.84

$$\frac{\frac{a(ad-bc)}{a+bx^2}+(2bc-ad)\log\left(a+bx^2\right)+2\log(x)(ad-2bc)-\frac{ac}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c)/x^2) + (a*(-(b*c) + a*d))/(a + b*x^2) + 2*(-2*b*c + a*d)*Log[x] +
(2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x^3*(a + b*x^2)^2), x]
```

fricas [A] time = 0.85, size = 122, normalized size = 1.61

$$-\frac{a^2c+\left(2\,abc-a^2d\right)x^2-\left(\left(2\,b^2c-abd\right)x^4+\left(2\,abc-a^2d\right)x^2\right)\log\left(bx^2+a\right)+2\left(\left(2\,b^2c-abd\right)x^4+\left(2\,abc-a^2d\right)x^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*c + (2*a*b*c - a^2*d)*x^2 - ((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a
^2*d)*x^2)*log(b*x^2 + a) + 2*((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a^2*d)*x^
2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.29, size = 84, normalized size = 1.11

$$-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}-\frac{2\,bcx^2-adx^2+ac}{2\left(bx^4+ax^2\right)a^2}+\frac{\left(2\,b^2c-abd\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c - a*d)*log(x^2)/a^3 - 1/2*(2*b*c*x^2 - a*d*x^2 + a*c)/((b*x^4 +
 a*x^2)*a^2) + 1/2*(2*b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^3*b)
```

maple [A] time = 0.02, size = 86, normalized size = 1.13

$$\frac{d}{2\left(b\,x^2+a\right)a}-\frac{bc}{2\left(b\,x^2+a\right)a^2}+\frac{d\ln(x)}{a^2}-\frac{d\ln\left(b\,x^2+a\right)}{2a^2}-\frac{2bc\ln(x)}{a^3}+\frac{bc\ln\left(b\,x^2+a\right)}{a^3}-\frac{c}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^3/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)*d-1/2/(b*x^2+a)/a^2*b*c-1/2/a^2*ln(b*x^2+a)*d+1/a^3*b*c*ln(
b*x^2+a)-1/2/a^2*c/x^2+1/a^2*ln(x)*d-2/a^3*b*c*ln(x)
```

maxima [A] time = 1.00, size = 78, normalized size = 1.03

$$-\frac{(2\,bc-ad)x^2+ac}{2\left(a^2bx^4+a^3x^2\right)}+\frac{(2\,bc-ad)\log\left(bx^2+a\right)}{2\,a^3}-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*b*c - a*d)*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + 1/2*(2*b*c - a*d)*lo
g(b*x^2 + a)/a^3 - 1/2*(2*b*c - a*d)*log(x^2)/a^3
```

mupad [B] time = 0.11, size = 74, normalized size = 0.97

$$\frac{\ln(x)\,\left(a\,d-2\,b\,c\right)}{a^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-2\,b\,c\right)}{2\,a^3}-\frac{\frac{c}{2\,a}-\frac{x^2\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{b\,x^4+a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(x)*(a*d - 2*b*c))/a^3 - (log(a + b*x^2)*(a*d - 2*b*c))/(2*a^3) - (c/(2
*a) - (x^2*(a*d - 2*b*c))/(2*a^2))/(a*x^2 + b*x^4)
```

sympy [**A**] time = 0.87, size = 70, normalized size = 0.92

$$\frac{-ac + x^2 (ad - 2bc)}{2a^3x^2 + 2a^2bx^4} + \frac{(ad - 2bc)\log(x)}{a^3} - \frac{(ad - 2bc)\log\left(\frac{a}{b} + x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*c + x**2*(a*d - 2*b*c))/(2*a**3*x**2 + 2*a**2*b*x**4) + (a*d - 2*b*c)*l
og(x)/a**3 - (a*d - 2*b*c)*log(a/b + x**2)/(2*a**3)
```

3.270 $\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{b}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{bx(bc-ad)}{2a^3\left(a+bx^2\right)}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {456, 1261, 205}

$$\frac{bx(bc-ad)}{2a^3\left(a+bx^2\right)}+\frac{2bc-ad}{a^3x}+\frac{\sqrt{b}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -c/(3*a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (b*(b*c - a*d)*x)/(2*a^3*(a + b*x^
2)) + (Sqrt[b]*(5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

```
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx &= \frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \frac{-\frac{2c}{ab}+\frac{2(bc-ad)x^2}{a^2b}-\frac{(bc-ad)x^4}{a^3}}{x^4\left(a+bx^2\right)}\,dx\\ &= \frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}-\frac{1}{2}b\int \left(-\frac{2c}{a^2bx^4}-\frac{2(-2bc+ad)}{a^3bx^2}+\frac{-5bc+3ad}{a^3\left(a+bx^2\right)}\right)dx\\ &= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}+\frac{(b(5bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a^3}\\ &= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3\left(a+bx^2\right)}+\frac{\sqrt{b}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 1.00

$$-\frac{\sqrt{b}\,(3ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{bx(ad-bc)}{2a^3\left(a+bx^2\right)}+\frac{2bc-ad}{a^3x}-\frac{c}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - (b*(-(b*c) + a*d)*x)/(2*a^3*(a +
 b*x^2)) - (Sqrt[b]*(-5*b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2
))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)/(x^4*(a + b*x^2)^2), x]
```

fricas [A] time = 1.02, size = 250, normalized size = 2.78

$$\left[\frac{6\left(5\,b^{2}c-3\,abd\right)x^{4}-4\,a^{2}c+4\left(5\,abc-3\,a^{2}d\right)x^{2}-3\left(\left(5\,b^{2}c-3\,abd\right)x^{5}+\left(5\,abc-3\,a^{2}d\right)x^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{12\left(a^{3}bx^{5}+a^{4}x^{3}\right)}, \frac{3\left(5\,b^{2}c-3\,abd\right)x^{4}-2\,a^{2}c+2\left(5\,abc-3\,a^{2}d\right)x^{2}+3\left(\left(5\,b^{2}c-3\,abd\right)x^{5}+\left(5\,abc-3\,a^{2}d\right)x^{3}\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{6\left(a^{3}bx^{5}+a^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(5*b^2*c - 3*a*b*d)*x^4 - 4*a^2*c + 4*(5*a*b*c - 3*a^2*d)*x^2 - 3*
((5*b^2*c - 3*a*b*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), 1/6*(3*(5*b^2*c
 - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2 + 3*((5*b^2*c - 3*a*b
*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3*b*x^
5 + a^4*x^3)]
```

giac [A] time = 0.35, size = 86, normalized size = 0.96

$$\frac{\left(5\,b^{2}c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}+\frac{b^{2}cx-abdx}{2\left(bx^{2}+a\right)a^{3}}+\frac{6\,bcx^{2}-3\,adx^{2}-ac}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*(b^2*c*
x - a*b*d*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3
)
```

maple [A] time = 0.01, size = 110, normalized size = 1.22

$$-\frac{bdx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{2}}+\frac{b^{2}cx}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{2}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^{3}}-\frac{d}{a^{2}x}+\frac{2bc}{a^{3}x}-\frac{c}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2*b*x/(b*x^2+a)*d+1/2/a^3*b^2*x/(b*x^2+a)*c-3/2/a^2*b/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
*c-1/3*c/a^2/x^3-1/a^2/x*d+2/a^3/x*b*c
```

maxima [A] time = 2.36, size = 93, normalized size = 1.03

$$\frac{3\left(5\,b^{2}c-3\,abd\right)x^{4}-2\,a^{2}c+2\left(5\,abc-3\,a^{2}d\right)x^{2}}{6\left(a^{3}bx^{5}+a^{4}x^{3}\right)}+\frac{\left(5\,b^{2}c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^2*c - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2)/(a^3*
b*x^5 + a^4*x^3) + 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^3)
```

mupad [B] time = 0.20, size = 84, normalized size = 0.93

$$-\frac{\frac{c}{3\,a}+\frac{x^2\,(3\,a\,d-5\,b\,c)}{3\,a^2}+\frac{b\,x^4\,(3\,a\,d-5\,b\,c)}{2\,a^3}}{b\,x^5+a\,x^3}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,a\,d-5\,b\,c)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) + (b*x^4*(3*a*d - 5*b*c))/(2*a^3
))/(a*x^3 + b*x^5) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(3*a*d - 5*b*c))/(2
*a^(7/2))
```

sympy [B] time = 0.59, size = 184, normalized size = 2.04

$$\frac{\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}-\frac{\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}\,(3ad-5bc)}{3abd-5b^2c}+x\right)}{4}+\frac{-2a^2c+x^4\left(-9abd+15b^2c\right)+x^2\left(-6a^2d+10abc\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(-a**4*sqrt(-b/a**7)*(3*a*d - 5*b*c)/(3*a*
b*d - 5*b**2*c) + x)/4 - sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(a**4*sqrt(-b/a**
7)*(3*a*d - 5*b*c)/(3*a*b*d - 5*b**2*c) + x)/4 + (-2*a**2*c + x**4*(-9*a*b*
d + 15*b**2*c) + x**2*(-6*a**2*d + 10*a*b*c))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.271 $\int \frac{x^4\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{a}\,(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x^5(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x^5}{5b^2}$$

Rubi **[A]** time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 302, 205}

$$\frac{x^5(bc-ad)^2}{2ab^2\left(a+bx^2\right)}-\frac{x^3(3bc-7ad)(bc-ad)}{6ab^3}+\frac{x(3bc-7ad)(bc-ad)}{2b^4}-\frac{\sqrt{a}\,(3bc-7ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*c - 7*a*d)*(b*c - a*d)*x)/(2*b^4) - ((3*b*c - 7*a*d)*(b*c - a*d)*x^3)
/(6*a*b^3) + (d^2*x^5)/(5*b^2) + ((b*c - a*d)^2*x^5)/(2*a*b^2*(a + b*x^2))
- (Sqrt[a]*(3*b*c - 7*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9
/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{\int \frac{x^4\left(-2b^2c^2+5(bc-ad)^2-2abd^2x^2\right)}{a+bx^2}\,dx}{2ab^2}$$

$$= \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{((3bc-7ad)(bc-ad))\int \frac{x^4}{a+bx^2}\,dx}{2ab^2}$$

$$= \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{((3bc-7ad)(bc-ad))\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{2ab^2}$$

$$= \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{(a(3bc-7ad}{}$$

$$= \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2\left(a+bx^2\right)} - \frac{\sqrt{a}\,(3bc-7a}{}$$

Mathematica [A] time = 0.09, size = 138, normalized size = 0.95

$$-\frac{\sqrt{a}\left(7a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{9/2}} + \frac{x\left(3a^2d^2-4abcd+b^2c^2\right)}{b^4} + \frac{ax(bc-ad)^2}{2b^4\left(a+bx^2\right)} + \frac{2dx^3(bc-ad)}{3b^3} + \frac{d^2x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d*(b*c - a*d)*x^3)/(3*b^3) +
 (d^2*x^5)/(5*b^2) + (a*(b*c - a*d)^2*x)/(2*b^4*(a + b*x^2)) - (Sqrt[a]*(3*
b^2*c^2 - 10*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.96, size = 400, normalized size = 2.76

$$\left[\frac{12\,b^3d^2x^7+4\left(10\,b^3cd-7\,ab^2d^2\right)x^5+20\left(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2\right)x^3+15\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2+\left(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2\right)x}{60\left(b^5x^2+ab^4\right)}, \frac{6\,b^3d^2x^7+2\left(10\,b^3cd-7\,ab^2d^2\right)x^5+10\left(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2\right)x^3-15\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2+\left(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+15\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2\right)x}{30\left(b^5x^2+ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*d^2*x^7 + 4*(10*b^3*c*d - 7*a*b^2*d^2)*x^5 + 20*(3*b^3*c^2 -
10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^
2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2
*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2)*x)/(b^5*x^2 + a*b^4), 1/30*(6*b^3*d^2*x^7 + 2*(10*b^3*c*d - 7*a*b^2*d^
2)*x^5 + 10*(3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 - 15*(3*a*b^2*c^2
- 10*a^2*b*c*d + 7*a^3*d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*
sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2)*x)/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.45, size = 156, normalized size = 1.08

$$-\frac{\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4}+\frac{ab^2c^2x-2\,a^2bcdx+a^3d^2x}{2\left(bx^2+a\right)b^4}+\frac{3\,b^8d^2x^5+10\,b^8cdx^3-10\,ab^7d^2x^3+15\,b^8c^2x-60\,ab^7cdx+45\,a^2b^6d^2x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) + 1/2*(a*b^2*c^2*x - 2*a^2*b*c*d*x + a^3*d^2*x)/((b*x^2 + a)*b^4)
+ 1/15*(3*b^8*d^2*x^5 + 10*b^8*c*d*x^3 - 10*a*b^7*d^2*x^3 + 15*b^8*c^2*x -
60*a*b^7*c*d*x + 45*a^2*b^6*d^2*x)/b^10
```

maple [A] time = 0.01, size = 196, normalized size = 1.35

$$\frac{d^2x^5}{5b^2}-\frac{2a\,d^2x^3}{3b^3}+\frac{2cd\,x^3}{3b^2}+\frac{a^3d^2x}{2\left(b\,x^2+a\right)b^4}-\frac{7a^3d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^4}-\frac{a^2cdx}{\left(b\,x^2+a\right)b^3}+\frac{5a^2cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^3}+\frac{a\,c^2x}{2\left(b\,x^2+a\right)b^2}-\frac{3a\,c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}+\frac{3a^2d^2x}{b^4}-\frac{4acdx}{b^3}+\frac{c^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^2*x^5/b^2-2/3/b^3*x^3*a*d^2+2/3/b^2*x^3*c*d+3/b^4*a^2*d^2*x-4/b^3*a*c
*d*x+1/b^2*c^2*x+1/2*a^3/b^4*x/(b*x^2+a)*d^2-a^2/b^3*x/(b*x^2+a)*c*d+1/2*a/
b^2*x/(b*x^2+a)*c^2-7/2*a^3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+5
*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d-3/2*a/b^2/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.42, size = 149, normalized size = 1.03

$$\frac{\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)x}{2\left(b^{5}x^{2}+ab^{4}\right)}-\frac{\left(3\,ab^{2}c^{2}-10\,a^{2}bcd+7\,a^{3}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}+\frac{3\,b^{2}d^{2}x^{5}+10\left(b^{2}cd-abd^{2}\right)x^{3}+15\left(b^{2}c^{2}-4\,abcd+3\,a^{2}d^{2}\right)x}{15\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x/(b^5*x^2 + a*b^4) - 1/2*(3*a*b^2*
c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/1
5*(3*b^2*d^2*x^5 + 10*(b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 4*a*b*c*d + 3
*a^2*d^2)*x)/b^4
```

mupad [B] time = 0.17, size = 200, normalized size = 1.38

$$x\left(\frac{c^{2}}{b^{2}}+\frac{2\,a\left(\frac{2\,a\,d^{2}}{b^{3}}-\frac{2\,c\,d}{b^{2}}\right)}{b}-\frac{a^{2}\,d^{2}}{b^{4}}\right)-x^{3}\left(\frac{2\,a\,d^{2}}{3\,b^{3}}-\frac{2\,c\,d}{3\,b^{2}}\right)+\frac{d^{2}\,x^{5}}{5\,b^{2}}+\frac{x\left(\frac{a^{3}\,d^{2}}{2}-a^{2}\,b\,c\,d+\frac{a\,b^{2}\,c^{2}}{2}\right)}{b^{5}\,x^{2}+a\,b^{4}}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)\,(7\,a\,d-3\,b\,c)}{7\,a^{3}\,d^{2}-10\,a^{2}\,b\,c\,d+3\,a\,b^{2}\,c^{2}}\right)(a\,d-b\,c)\,(7\,a\,d-3\,b\,c)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] x*(c^2/b^2 + (2*a*((2*a*d^2)/b^3 - (2*c*d)/b^2))/b - (a^2*d^2)/b^4) - x^3*(
(2*a*d^2)/(3*b^3) - (2*c*d)/(3*b^2)) + (d^2*x^5)/(5*b^2) + (x*((a^3*d^2)/2
+ (a*b^2*c^2)/2 - a^2*b*c*d))/(a*b^4 + b^5*x^2) - (a^(1/2)*atan((a^(1/2)*b^
(1/2)*x*(a*d - b*c)*(7*a*d - 3*b*c))/(7*a^3*d^2 + 3*a*b^2*c^2 - 10*a^2*b*c*
d))*(a*d - b*c)*(7*a*d - 3*b*c))/(2*b^(9/2))
```

sympy [B] time = 0.98, size = 286, normalized size = 1.97

$$x^{3}\left(-\frac{2ad^{2}}{3b^{3}}+\frac{2cd}{3b^{2}}\right)+x\left(\frac{3a^{2}d^{2}}{b^{4}}-\frac{4acd}{b^{3}}+\frac{c^{2}}{b^{2}}\right)+\frac{x\left(a^{3}d^{2}-2a^{2}bcd+ab^{2}c^{2}\right)}{2ab^{4}+2b^{5}x^{2}}+\frac{\sqrt{-\frac{a}{b^{9}}}\,(ad-bc)\,(7ad-3bc)\log\left(-\frac{b^{4}\sqrt{-\frac{a}{b^{9}}}\,(ad-bc)(7ad-3bc)}{7a^{2}d^{2}-10abcd+3b^{2}c^{2}}+x\right)}{4}-\frac{\sqrt{-\frac{a}{b^{9}}}\,(ad-bc)\,(7ad-3bc)\log\left(\frac{b^{4}\sqrt{-\frac{a}{b^{9}}}\,(ad-bc)(7ad-3bc)}{7a^{2}d^{2}-10abcd+3b^{2}c^{2}}+x\right)}{4}+\frac{d^{2}x^{5}}{5b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**2/(3*b**3) + 2*c*d/(3*b**2)) + x*(3*a**2*d**2/b**4 - 4*a*c*d/
b**3 + c**2/b**2) + x*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4 +
2*b**5*x**2) + sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(-b**4*sqrt(-a/
b**9)*(a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2)
+ x)/4 - sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(b**4*sqrt(-a/b**9)*(
a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2) + x)/4
+ d**2*x**5/(5*b**2)
```

3.272 $\int \frac{x^3\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=88

$$\frac{a(bc-ad)^2}{2b^4\left(a+bx^2\right)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{dx^2(bc-ad)}{b^3}+\frac{d^2x^4}{4b^2}$$

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{dx^2(bc-ad)}{b^3}+\frac{a(bc-ad)^2}{2b^4\left(a+bx^2\right)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}+\frac{d^2x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(4*b^2) + (a*(b*c - a*d)^2)/(2*b^4*(a +
 b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/(2*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x(c+dx)^2}{(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{2d(bc-ad)}{b^3}+\frac{d^2x}{b^2}-\frac{a(-bc+ad)^2}{b^3(a+bx)^2}+\frac{(bc-3ad)(bc-ad)}{b^3(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{d(bc-ad)x^2}{b^3}+\frac{d^2x^4}{4b^2}+\frac{a(bc-ad)^2}{2b^4\left(a+bx^2\right)}+\frac{(bc-3ad)(bc-ad)\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.99

$$
\frac{2\left(3a^2d^2-4abcd+b^2c^2\right)\log\left(a+bx^2\right)+4bdx^2(bc-ad)+\frac{2a(bc-ad)^2}{a+bx^2}+b^2d^2x^4}{4b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (4*b*d*(b*c - a*d)*x^2 + b^2*d^2*x^4 + (2*a*(b*c - a*d)^2)/(a + b*x^2) + 2*
(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Log[a + b*x^2])/(4*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 1.00, size = 160, normalized size = 1.82

$$
\frac{b^3d^2x^6+2\,ab^2c^2-4\,a^2bcd+2\,a^3d^2+\left(4\,b^3cd-3\,ab^2d^2\right)x^4+4\left(ab^2cd-a^2bd^2\right)x^2+2\left(ab^2c^2-4\,a^2bcd+3\,a^3d^2+\left(b^3c^2-4\,ab^2cd+3\,a^2bd^2\right)x^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^2*x^6 + 2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (4*b^3*c*d - 3*a
*b^2*d^2)*x^4 + 4*(a*b^2*c*d - a^2*b*d^2)*x^2 + 2*(a*b^2*c^2 - 4*a^2*b*c*d
```

```
+ 3*a^3*d^2 + (b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x^2)*log(b*x^2 + a))/(b
^5*x^2 + a*b^4)
```

giac [A] time = 0.28, size = 163, normalized size = 1.85

$$\frac{\frac{\left(bx^2+a\right)^2\left(d^2+\frac{2\left(2\,b^2cd-3\,abd^2\right)}{\left(bx^2+a\right)b}\right)}{b^3}-\frac{2\left(b^2c^2-4\,abcd+3\,a^2d^2\right)\log\left(\frac{\left|bx^2+a\right|}{\left(bx^2+a\right)^2|b|}\right)}{b^3}+\frac{2\left(\frac{ab^4c^2}{bx^2+a}-\frac{2\,a^2b^3cd}{bx^2+a}+\frac{a^3b^2d^2}{bx^2+a}\right)}{b^5}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)^2*(d^2 + 2*(2*b^2*c*d - 3*a*b*d^2)/((b*x^2 + a)*b))/b^3 -
2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b
)))/b^3 + 2*(a*b^4*c^2/(b*x^2 + a) - 2*a^2*b^3*c*d/(b*x^2 + a) + a^3*b^2*d^
2/(b*x^2 + a))/b^5)/b
```

maple [A] time = 0.01, size = 142, normalized size = 1.61

$$\frac{d^2x^4}{4b^2}-\frac{a\,d^2x^2}{b^3}+\frac{cd\,x^2}{b^2}+\frac{a^3d^2}{2\left(b\,x^2+a\right)b^4}-\frac{a^2cd}{\left(b\,x^2+a\right)b^3}+\frac{3a^2d^2\ln\left(b\,x^2+a\right)}{2b^4}+\frac{a\,c^2}{2\left(b\,x^2+a\right)b^2}-\frac{2acd\ln\left(b\,x^2+a\right)}{b^3}+\frac{c^2\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^2*x^4/b^2-d^2/b^3*a*x^2+d/b^2*c*x^2+1/2/b^4*a^3/(b*x^2+a)*d^2-1/b^3*a
^2/(b*x^2+a)*d*c+1/2/b^2*a/(b*x^2+a)*c^2+3/2/b^4*ln(b*x^2+a)*a^2*d^2-2/b^3*
ln(b*x^2+a)*a*d*c+1/2/b^2*ln(b*x^2+a)*c^2
```

maxima [A] time = 1.00, size = 107, normalized size = 1.22

$$\frac{ab^2c^2-2\,a^2bcd+a^3d^2}{2\left(b^5x^2+ab^4\right)}+\frac{bd^2x^4+4\left(bcd-ad^2\right)x^2}{4\,b^3}+\frac{\left(b^2c^2-4\,abcd+3\,a^2d^2\right)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b^5*x^2 + a*b^4) + 1/4*(b*d^2*x^4
+ 4*(b*c*d - a*d^2)*x^2)/b^3 + 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(b*
x^2 + a)/b^4
```

mupad [B] time = 0.07, size = 112, normalized size = 1.27

$$\frac{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}-x^2\,\left(\frac{a\,d^2}{b^3}-\frac{c\,d}{b^2}\right)+\frac{d^2\,x^4}{4\,b^2}+\frac{\ln\left(b\,x^2+a\right)\,\left(3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] (a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)/(2*b*(a*b^3 + b^4*x^2)) - x^2*((a*d^2)/
b^3 - (c*d)/b^2) + (d^2*x^4)/(4*b^2) + (log(a + b*x^2)*(3*a^2*d^2 + b^2*c^2
 - 4*a*b*c*d))/(2*b^4)
```

sympy [A] time = 0.93, size = 99, normalized size = 1.12

$$x^{2}\left(-\frac{ad^{2}}{b^{3}}+\frac{cd}{b^{2}}\right)+\frac{a^{3}d^{2}-2a^{2}bcd+ab^{2}c^{2}}{2ab^{4}+2b^{5}x^{2}}+\frac{d^{2}x^{4}}{4b^{2}}+\frac{(ad-bc)\,(3ad-bc)\log\left(a+bx^{2}\right)}{2b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x**2*(-a*d**2/b**3 + c*d/b**2) + (a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(
2*a*b**4 + 2*b**5*x**2) + d**2*x**4/(4*b**2) + (a*d - b*c)*(3*a*d - b*c)*lo
g(a + b*x**2)/(2*b**4)
```

3.273 $\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=116

$$\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{x^3(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x^3}{3b^2}$$

Rubi [**A**] time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {463, 459, 321, 205}

$$\frac{x^3(bc-ad)^2}{2ab^2\left(a+bx^2\right)}-\frac{x(bc-5ad)(bc-ad)}{2ab^3}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}+\frac{d^2x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] -((b*c - 5*a*d)*(b*c - a*d)*x)/(2*a*b^3) + (d^2*x^3)/(3*b^2) + ((b*c - a*d)
^2*x^3)/(2*a*b^2*(a + b*x^2)) + ((b*c - 5*a*d)*(b*c - a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*Sqrt[a]*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}-\frac{\int \frac{x^2\left(b^2c^2-6abcd+3a^2d^2-2abd^2x^2\right)}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}-\frac{((bc-5ad)(bc-ad))\int \frac{x^2}{a+bx^2}\,dx}{2ab^2}\\
&= -\frac{(bc-5ad)(bc-ad)x}{2ab^3}+\frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}+\frac{((bc-5ad)(bc-ad))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= -\frac{(bc-5ad)(bc-ad)x}{2ab^3}+\frac{d^2x^3}{3b^2}+\frac{(bc-ad)^2x^3}{2ab^2\left(a+bx^2\right)}+\frac{(bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}
\end{aligned}$$

Mathematica [**A**] time = 0.07, size = 105, normalized size = 0.91

$$\frac{\left(5a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{7/2}}-\frac{x(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{2dx(bc-ad)}{b^3}+\frac{d^2x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (2*d*(b*c - a*d)*x)/b^3 + (d^2*x^3)/(3*b^2) - ((b*c - a*d)^2*x)/(2*b^3*(a +
 b*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*Sqrt[a]*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.78, size = 342, normalized size = 2.95

$$\left[\frac{4\,ab^3d^2x^5+4\left(6\,ab^3cd-5\,a^2b^2d^2\right)x^3-3\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-6\left(ab^3c^2-6\,a^2b^2cd+5\,a^3bd^2\right)x}{12\left(ab^5x^2+a^2b^4\right)},\ \frac{2\,ab^3d^2x^5+2\left(6\,ab^3cd-5\,a^2b^2d^2\right)x^3+3\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+\left(b^3c^2-6\,ab^2cd+5\,a^2bd^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-3\left(ab^3c^2-6\,a^2b^2cd+5\,a^3bd^2\right)x}{6\left(ab^5x^2+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a*b^3*d^2*x^5 + 4*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 - 3*(a*b^2*c^2
 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a*b^3*c^2 - 6*a^
2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4), 1/6*(2*a*b^3*d^2*x^5 + 2
*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 + 3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2
 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) - 3*(a*b^3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4)]
```

giac [A] time = 0.36, size = 114, normalized size = 0.98

$$\frac{\left(b^2c^2-6\,abcd+5\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(bx^2+a\right)b^3}+\frac{b^4d^2x^3+6\,b^4cdx-6\,ab^3d^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d
^2*x^3 + 6*b^4*c*d*x - 6*a*b^3*d^2*x)/b^6
```

maple [A] time = 0.01, size = 156, normalized size = 1.34

$$\frac{d^2x^3}{3b^2}-\frac{a^2d^2x}{2\left(b\,x^2+a\right)b^3}+\frac{5a^2d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}+\frac{acdx}{\left(b\,x^2+a\right)b^2}-\frac{3acd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}-\frac{c^2x}{2\left(b\,x^2+a\right)b}+\frac{c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}-\frac{2a\,d^2x}{b^3}+\frac{2cdx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^2*x^3/b^2-2*d^2/b^3*a*x+2*d/b^2*c*x-1/2/b^3*x/(b*x^2+a)*a^2*d^2+1/b^2
*x/(b*x^2+a)*a*c*d-1/2/b*x/(b*x^2+a)*c^2+5/2/b^3/(a*b)^(1/2)*arctan(1/(a*b)
^(1/2)*b*x)*a^2*d^2-3/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*c*d+1/2/b
/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.41, size = 109, normalized size = 0.94

$$-\frac{\left(b^{2} c^{2} - 2\, abcd + a^{2} d^{2}\right) x}{2 \left(b^{4} x^{2} + a b^{3}\right)} + \frac{\left(b^{2} c^{2} - 6\, abcd + 5\, a^{2} d^{2}\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \sqrt{ab}\, b^{3}} + \frac{b d^{2} x^{3} + 6 \left(bcd - a d^{2}\right) x}{3 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(b^4*x^2 + a*b^3) + 1/2*(b^2*c^2 - 6
*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(b*d^2*x^
3 + 6*(b*c*d - a*d^2)*x)/b^3
```

mupad [B] time = 0.18, size = 148, normalized size = 1.28

$$\frac{d^{2} x^{3}}{3 b^{2}} - \frac{x \left(\frac{a^{2} d^{2}}{2} - a b c d + \frac{b^{2} c^{2}}{2}\right)}{b^{4} x^{2} + a b^{3}} - x \left(\frac{2 a d^{2}}{b^{3}} - \frac{2 c d}{b^{2}}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\, x (a d - b c) (5 a d - b c)}{\sqrt{a} \left(5 a^{2} d^{2} - 6 a b c d + b^{2} c^{2}\right)}\right) (a d - b c)\, (5 a d - b c)}{2 \sqrt{a}\, b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x^3)/(3*b^2) - (x*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(a*b^3 + b^4*
x^2) - x*((2*a*d^2)/b^3 - (2*c*d)/b^2) + (atan((b^(1/2)*x*(a*d - b*c)*(5*a*
d - b*c))/(a^(1/2)*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(5*a*d -
 b*c))/(2*a^(1/2)*b^(7/2))
```

sympy [B] time = 0.86, size = 246, normalized size = 2.12

$$x \left(- \frac{2ad^{2}}{b^{3}} + \frac{2cd}{b^{2}}\right) + \frac{x \left(-a^{2} d^{2} + 2abcd - b^{2} c^{2}\right)}{2ab^{3} + 2b^{4} x^{2}} - \frac{\sqrt{- \frac{1}{ab^{7}}} (ad - bc) (5ad - bc) \log\left(- \frac{ab^{3} \sqrt{- \frac{1}{ab^{7}}} (ad-bc)(5ad-bc)}{5a^{2}d^{2}-6abcd+b^{2}c^{2}} + x\right)}{4} + \frac{\sqrt{- \frac{1}{ab^{7}}} (ad - bc) (5ad - bc) \log\left(\frac{ab^{3} \sqrt{- \frac{1}{ab^{7}}} (ad-bc)(5ad-bc)}{5a^{2}d^{2}-6abcd+b^{2}c^{2}} + x\right)}{4} + \frac{d^{2} x^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**2/b**3 + 2*c*d/b**2) + x*(-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(2
*a*b**3 + 2*b**4*x**2) - sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(-a
*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d
+ b**2*c**2) + x)/4 + sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(a*b**
3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b*
*2*c**2) + x)/4 + d**2*x**3/(3*b**2)
```

3.274 $\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=61

$$-\frac{(bc-ad)^2}{2b^3(a+bx^2)}+\frac{d(bc-ad)\log(a+bx^2)}{b^3}+\frac{d^2x^2}{2b^2}$$

Rubi [**A**] time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$-\frac{(bc-ad)^2}{2b^3(a+bx^2)}+\frac{d(bc-ad)\log(a+bx^2)}{b^3}+\frac{d^2x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x^2)/(2*b^2) - (b*c - a*d)^2/(2*b^3*(a + b*x^2)) + (d*(b*c - a*d)*Log[
a + b*x^2])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^2}{(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{d^2x^2}{2b^2}-\frac{(bc-ad)^2}{2b^3\left(a+bx^2\right)}+\frac{d(bc-ad)\log\left(a+bx^2\right)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx^2}+2d(bc-ad)\log\left(a+bx^2\right)+bd^2x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] (b*d^2*x^2 - (b*c - a*d)^2/(a + b*x^2) + 2*d*(b*c - a*d)*Log[a + b*x^2])/(2
*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^2)^2)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.55, size = 101, normalized size = 1.66

$$\frac{b^2d^2x^4+abd^2x^2-b^2c^2+2\,abcd-a^2d^2+2\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^4 + a*b*d^2*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d
 - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.34, size = 111, normalized size = 1.82

$$\frac{\left(bx^2+a\right)d^2}{2\,b^3}-\frac{\left(bcd-ad^2\right)\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b^3}-\frac{\frac{b^3c^2}{bx^2+a}-\frac{2\,ab^2cd}{bx^2+a}+\frac{a^2bd^2}{bx^2+a}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + a)*d^2/b^3 - (b*c*d - a*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2
*abs(b)))/b^3 - 1/2*(b^3*c^2/(b*x^2 + a) - 2*a*b^2*c*d/(b*x^2 + a) + a^2*b*
d^2/(b*x^2 + a))/b^4
```

maple [A] time = 0.01, size = 97, normalized size = 1.59

$$\frac{d^2x^2}{2b^2}-\frac{a^2d^2}{2\left(b\,x^2+a\right)b^3}+\frac{acd}{\left(b\,x^2+a\right)b^2}-\frac{a\,d^2\ln\left(b\,x^2+a\right)}{b^3}-\frac{c^2}{2\left(b\,x^2+a\right)b}+\frac{cd\ln\left(b\,x^2+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d^2*x^2/b^2-1/2/b^3/(b*x^2+a)*a^2*d^2+1/b^2/(b*x^2+a)*a*d*c-1/2/b/(b*x^
2+a)*c^2-1/b^3*ln(b*x^2+a)*d^2*a+1/b^2*ln(b*x^2+a)*d*c
```

maxima [A] time = 1.04, size = 73, normalized size = 1.20

$$\frac{d^2x^2}{2\,b^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(b^4x^2+ab^3\right)}+\frac{\left(bcd-ad^2\right)\log\left(bx^2+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^2/b^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x^2 + a*b^3) + (
b*c*d - a*d^2)*log(b*x^2 + a)/b^3
```

mupad [B] time = 0.26, size = 77, normalized size = 1.26

$$\frac{d^2\,x^2}{2\,b^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{2\,b\,\left(b^3\,x^2+a\,b^2\right)}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d^2-b\,c\,d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x^2)/(2*b^2) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*b*(a*b^2 + b^3*x^2))
 - (log(a + b*x^2)*(a*d^2 - b*c*d))/b^3
```

sympy [A] time = 0.76, size = 68, normalized size = 1.11

$$\frac{-a^2d^2+2abcd-b^2c^2}{2ab^3+2b^4x^2}+\frac{d^2x^2}{2b^2}-\frac{d\,(ad-bc)\log\left(a+bx^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] (-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) + d**2*x**2/(
2*b**2) - d*(a*d - b*c)*log(a + b*x**2)/b**3
```

3.275 $\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{b^2\left(a+bx^2\right)^2}\right)dx\\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{b^2}\\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 88, normalized size = 1.07

$$
\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(a + b*x^2)^2, x]
```

fricas [A] time = 0.93, size = 297, normalized size = 3.62

$$
\left[\frac{4\,a^2b^2d^2x^3+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(ab^3c^2-2\,a^2b^2cd+3\,a^3bd^2\right)x}{4\left(a^2b^4x^2+a^3b^3\right)},\ \frac{2\,a^2b^2d^2x^3+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(ab^3c^2-2\,a^2b^2cd+3\,a^3bd^2\right)x}{2\left(a^2b^4x^2+a^3b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 +
 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^
2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]
```

giac [A] time = 0.29, size = 94, normalized size = 1.15

$$\frac{d^2x}{b^2} + \frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(bx^2 + a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

maple [A] time = 0.01, size = 129, normalized size = 1.57

$$\frac{a\,d^2x}{2\left(b\,x^2 + a\right)b^2} - \frac{3a\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2} + \frac{c^2x}{2\left(b\,x^2 + a\right)a} + \frac{c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a} - \frac{cdx}{\left(b\,x^2 + a\right)b} + \frac{cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] d^2*x/b^2+1/2/b^2*x*a/(b*x^2+a)*d^2-1/b*x/(b*x^2+a)*c*d+1/2*x/a/(b*x^2+a)*c
^2-3/2/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+1/b/(a*b)^(1/2)*arct
an(1/(a*b)^(1/2)*b*x)*c*d+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2
```

maxima [A] time = 2.47, size = 95, normalized size = 1.16

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{2\left(ab^3x^2 + a^2b^2\right)} + \frac{d^2x}{b^2} + \frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 + 1
/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2
)
```

mupad [B] time = 0.28, size = 124, normalized size = 1.51

$$\frac{d^2\,x}{b^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a\,\left(b^3\,x^2+a\,b^2\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+b\,c\right)}{\sqrt{a}\,\left(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2)) +
 (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^2
+ 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

sympy [B] time = 0.71, size = 236, normalized size = 2.88

$$\frac{x\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2a^{2}b^{2}+2ab^{3}x^{2}}+\frac{\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)\,(3ad+bc)\log\left(-\frac{a^{2}b^{2}\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)(3ad+bc)}{3a^{2}d^{2}-2abcd-b^{2}c^{2}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)\,(3ad+bc)\log\left(\frac{a^{2}b^{2}\sqrt{-\frac{1}{a^{3}b^{5}}}\,(ad-bc)(3ad+bc)}{3a^{2}d^{2}-2abcd-b^{2}c^{2}}+x\right)}{4}+\frac{d^{2}x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

3.276 $\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log(x)}{a^2}+\frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (b*c - a*d)^2/(2*a*b^2*(a + b*x^2)) + (c^2*Log[x])/a^2 - ((c^2/a^2 - d^2/b^
2)*Log[a + b*x^2])/2
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c^2}{a^2x}-\frac{(-bc+ad)^2}{ab(a+bx)^2}+\frac{-b^2c^2+a^2d^2}{a^2b(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{c^2\log(x)}{a^2}-\frac{1}{2}\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.04

$$\frac{\frac{(ad-bc)\left(\left(a+bx^2\right)(ad+bc)\log\left(a+bx^2\right)+a(ad-bc)\right)}{b^2\left(a+bx^2\right)}+2c^2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^2*Log[x] + ((-(b*c) + a*d)*(a*(-(b*c) + a*d) + (b*c + a*d)*(a + b*x^2)
*Log[a + b*x^2]))/(b^2*(a + b*x^2)))/(2*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x*(a + b*x^2)^2), x]
```

fricas [A] time = 0.89, size = 117, normalized size = 1.75

$$\frac{ab^2c^2-2\,a^2bcd+a^3d^2-\left(ab^2c^2-a^3d^2+\left(b^3c^2-a^2bd^2\right)x^2\right)\log\left(bx^2+a\right)+2\left(b^3c^2x^2+ab^2c^2\right)\log(x)}{2\left(a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 - (a*b^2*c^2 - a^3*d^2 + (b^3*c^2 -
a^2*b*d^2)*x^2)*log(b*x^2 + a) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*log(x))/(a^2*b
^3*x^2 + a^3*b^2)
```

giac [A] time = 0.34, size = 99, normalized size = 1.48

$$\frac{c^2\log\left(x^2\right)}{2\,a^2}-\frac{\left(b^2c^2-a^2d^2\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b^2}+\frac{b^2c^2x^2-a^2d^2x^2+2\,abc^2-2\,a^2cd}{2\left(bx^2+a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 - 1/2*(b^2*c^2 - a^2*d^2)*log(abs(b*x^2 + a))/(a^2*b^2
) + 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((b*x^2 + a)*a^
2*b)
```

maple [A] time = 0.01, size = 94, normalized size = 1.40

$$\frac{a\,d^2}{2\left(b\,x^2+a\right)b^2}+\frac{c^2}{2\left(b\,x^2+a\right)a}+\frac{c^2\ln(x)}{a^2}-\frac{c^2\ln\left(b\,x^2+a\right)}{2a^2}-\frac{cd}{\left(b\,x^2+a\right)b}+\frac{d^2\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)*d^2-1/b/(b*x^2+a)*d*c+1/2/a/(b*x^2+a)*c^2+1/2/b^2*ln(b*
x^2+a)*d^2-1/2/a^2*ln(b*x^2+a)*c^2+c^2*ln(x)/a^2
```

maxima [A] time = 1.07, size = 86, normalized size = 1.28

$$\frac{c^2\log\left(x^2\right)}{2\,a^2}+\frac{b^2c^2-2\,abcd+a^2d^2}{2\left(ab^3x^2+a^2b^2\right)}-\frac{\left(b^2c^2-a^2d^2\right)\log\left(bx^2+a\right)}{2\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a*b^3*x^2 + a^2
*b^2) - 1/2*(b^2*c^2 - a^2*d^2)*log(b*x^2 + a)/(a^2*b^2)
```

mupad [B] time = 0.31, size = 80, normalized size = 1.19

$$\frac{c^2\,\ln(x)}{a^2}+\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{2\,a\,b^2\,\left(b\,x^2+a\right)}+\frac{\ln\left(b\,x^2+a\right)\,\left(a^2\,d^2-b^2\,c^2\right)}{2\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x*(a + b*x^2)^2),x)
```

```
[Out] (c^2*log(x))/a^2 + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*a*b^2*(a + b*x^2)) +
(log(a + b*x^2)*(a^2*d^2 - b^2*c^2))/(2*a^2*b^2)
```

sympy [A] time = 1.24, size = 80, normalized size = 1.19

$$\frac{a^2d^2 - 2abcd + b^2c^2}{2a^2b^2 + 2ab^3x^2} + \frac{c^2\log{(x)}}{a^2} + \frac{(ad - bc)(ad + bc)\log{\left(\frac{a}{b} + x^2\right)}}{2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x/(b*x**2+a)**2,x)
```

```
[Out] (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + c**2*lo
g(x)/a**2 + (a*d - b*c)*(a*d + b*c)*log(a/b + x**2)/(2*a**2*b**2)
```

3.277 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=103

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2a\left(a+bx^2\right)}-\frac{c^2}{ax\left(a+bx^2\right)}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {462, 385, 205}

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}-\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2a\left(a+bx^2\right)}-\frac{c^2}{ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2/(a*x*(a + b*x^2))) - (((3*b*c^2)/a - 2*c*d + (a*d^2)/b)*x)/(2*a*(a +
b*x^2)) - ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2
)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
```

```
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{c^2}{ax\left(a+bx^2\right)}+\frac{\int \frac{-c(3bc-2ad)+ad^2x^2}{\left(a+bx^2\right)^2}\,dx}{a}\\ &= -\frac{c^2}{ax\left(a+bx^2\right)}-\frac{\left(\frac{d^2}{b}+\frac{c(3bc-2ad)}{a^2}\right)x}{2\left(a+bx^2\right)}-\frac{((bc-ad)(3bc+ad))\int \frac{1}{a+bx^2}\,dx}{2a^2b}\\ &= -\frac{c^2}{ax\left(a+bx^2\right)}-\frac{\left(\frac{d^2}{b}+\frac{c(3bc-2ad)}{a^2}\right)x}{2\left(a+bx^2\right)}-\frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 91, normalized size = 0.88

$$-\frac{x(ad-bc)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{a^2x}+\frac{\left(a^2d^2+2abcd-3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2/(a^2*x)) - ((-(b*c) + a*d)^2*x)/(2*a^2*b*(a + b*x^2)) + ((-3*b^2*c^2
+ 2*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 0.98, size = 308, normalized size = 2.99

$$\left[-\frac{4\,a^2b^2c^2 + 2\left(3\,ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x^2 - \left(\left(3\,b^3c^2 - 2\,ab^2cd - a^2bd^2\right)x^3 + \left(3\,ab^2c^2 - 2\,a^2bcd - a^3d^2\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right)}{4\left(a^3b^3x^3 + a^4b^2x\right)}, -\frac{2\,a^2b^2c^2 + \left(3\,ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x^2 + \left(\left(3\,b^3c^2 - 2\,ab^2cd - a^2bd^2\right)x^3 + \left(3\,ab^2c^2 - 2\,a^2bcd - a^3d^2\right)x\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^3b^3x^3 + a^4b^2x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b^2*c^2 + 2*(3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 - ((
3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2*a^2*b*c*d - a^3
*d^2)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3
*x^3 + a^4*b^2*x), -1/2*(2*a^2*b^2*c^2 + (3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3
*b*d^2)*x^2 + ((3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2
*a^2*b*c*d - a^3*d^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^3 + a^
4*b^2*x)]
```

giac [A] time = 0.33, size = 103, normalized size = 1.00

$$-\frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b} - \frac{3\,b^2c^2x^2 - 2\,abcdx^2 + a^2d^2x^2 + 2\,abc^2}{2\left(bx^3 + ax\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2
*b) - 1/2*(3*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + 2*a*b*c^2)/((b*x^3
 + a*x)*a^2*b)
```

maple [A] time = 0.01, size = 131, normalized size = 1.27

$$\frac{cdx}{\left(b\,x^2 + a\right)a} + \frac{cd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,\,a} - \frac{b\,c^2x}{2\left(b\,x^2 + a\right)a^2} - \frac{3b\,c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\,a^2} - \frac{d^2x}{2\left(b\,x^2 + a\right)b} + \frac{d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\,b} - \frac{c^2}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^2/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2+a)*d^2+1/a*x/(b*x^2+a)*c*d-1/2/a^2*b*x/(b*x^2+a)*c^2+1/2/b/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2+1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x)*c*d-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2-c^2/a^2/x
```

maxima [A] time = 2.29, size = 101, normalized size = 0.98

$$-\frac{2\,abc^2 + \left(3\,b^2c^2 - 2\,abcd + a^2d^2\right)x^2}{2\left(a^2b^2x^3 + a^3bx\right)} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*c^2 + (3*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^3 + a^3
*b*x) - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^2*b)
```

mupad [B] time = 0.14, size = 128, normalized size = 1.24

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{a}\,\left(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2\right)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,a^{5/2}\,b^{3/2}}-\frac{\frac{c^2}{a}+\frac{x^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{2\,a^2\,b}}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(a^(1/2)*(a^2*d^2 - 3*b^2*c^2 +
 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*a^(5/2)*b^(3/2)) - (c^2/a + (x^
2*(a^2*d^2 + 3*b^2*c^2 - 2*a*b*c*d))/(2*a^2*b))/(a*x + b*x^3)
```

sympy [B] time = 0.87, size = 238, normalized size = 2.31

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}+\frac{-2abc^2+x^2\left(-a^2d^2+2abcd-3b^2c^2\right)}{2a^3bx+2a^2b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**2/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b
**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)/
4 + sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5
*b**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x
)/4 + (-2*a*b*c**2 + x**2*(-a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2))/(2*a**3*b
*x + 2*a**2*b**2*x**3)
```

3.278 $\int \frac{\left(c+dx^2\right)^2}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=80

$$\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{c^2}{2a^2x^2}$$

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}+\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}-\frac{2c\log(x)(bc-ad)}{a^3}-\frac{c^2}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c^2/(2*a^2*x^2) - (b*c - a*d)^2/(2*a^2*b*(a + b*x^2)) - (2*c*(b*c - a*d)*L
og[x])/a^3 + (c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{c^2}{a^2x^2}+\frac{2c(-bc+ad)}{a^3x}+\frac{(-bc+ad)^2}{a^2(a+bx)^2}-\frac{2bc(-bc+ad)}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{c^2}{2a^2x^2}-\frac{(bc-ad)^2}{2a^2b\left(a+bx^2\right)}-\frac{2c(bc-ad)\log(x)}{a^3}+\frac{c(bc-ad)\log\left(a+bx^2\right)}{a^3}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.90

$$
-\frac{\frac{a(bc-ad)^2}{b\left(a+bx^2\right)}-2c(bc-ad)\log\left(a+bx^2\right)+4c\log(x)(bc-ad)+\frac{ac^2}{x^2}}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*((a*c^2)/x^2 + (a*(b*c - a*d)^2)/(b*(a + b*x^2)) + 4*c*(b*c - a*d)*Log
[x] - 2*c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(c+dx^2\right)^2}{x^3\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x^3*(a + b*x^2)^2), x]
```

fricas [B] time = 0.90, size = 159, normalized size = 1.99

$$
-\frac{a^2bc^2+\left(2\,ab^2c^2-2\,a^2bcd+a^3d^2\right)x^2-2\left(\left(b^3c^2-ab^2cd\right)x^4+\left(ab^2c^2-a^2bcd\right)x^2\right)\log\left(bx^2+a\right)+4\left(\left(b^3c^2-ab^2cd\right)x^4+\left(ab^2c^2-a^2bcd\right)x^2\right)\log(x)}{2\left(a^3b^2x^4+a^4bx^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2 - 2*((b^3*c^2 -
 a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(b*x^2 + a) + 4*((b^3*c^2
 - a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(x))/(a^3*b^2*x^4 + a^4
*b*x^2)
```

giac [A] time = 0.34, size = 109, normalized size = 1.36

$$-\frac{\left(bc^2-acd\right)\log\left(x^2\right)}{a^3}+\frac{\left(b^2c^2-abcd\right)\log\left(\left|bx^2+a\right|\right)}{a^3b}-\frac{2\,b^2c^2x^2-2\,abcdx^2+a^2d^2x^2+abc^2}{2\left(bx^4+ax^2\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(b*c^2 - a*c*d)*log(x^2)/a^3 + (b^2*c^2 - a*b*c*d)*log(abs(b*x^2 + a))/(a^
3*b) - 1/2*(2*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + a*b*c^2)/((b*x^4
+ a*x^2)*a^2*b)
```

maple [A] time = 0.01, size = 114, normalized size = 1.42

$$\frac{cd}{\left(b\,x^2+a\right)a}-\frac{b\,c^2}{2\left(b\,x^2+a\right)a^2}+\frac{2cd\ln(x)}{a^2}-\frac{cd\ln\left(b\,x^2+a\right)}{a^2}-\frac{2b\,c^2\ln(x)}{a^3}+\frac{b\,c^2\ln\left(b\,x^2+a\right)}{a^3}-\frac{d^2}{2\left(b\,x^2+a\right)b}-\frac{c^2}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b/(b*x^2+a)*d^2+1/a/(b*x^2+a)*d*c-1/2/a^2/(b*x^2+a)*c^2*b-1/a^2*c*ln(b
*x^2+a)*d+1/a^3*c^2*ln(b*x^2+a)*b-1/2*c^2/a^2/x^2+2*c/a^2*ln(x)*d-2*c^2/a^3
*ln(x)*b
```

maxima [A] time = 1.00, size = 100, normalized size = 1.25

$$-\frac{abc^2+\left(2\,b^2c^2-2\,abcd+a^2d^2\right)x^2}{2\left(a^2b^2x^4+a^3bx^2\right)}+\frac{\left(bc^2-acd\right)\log\left(bx^2+a\right)}{a^3}-\frac{\left(bc^2-acd\right)\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c^2 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^4 + a^3*b
*x^2) + (b*c^2 - a*c*d)*log(b*x^2 + a)/a^3 - (b*c^2 - a*c*d)*log(x^2)/a^3
```

mupad [B] time = 0.20, size = 100, normalized size = 1.25

$$\frac{\ln\left(b\,x^2+a\right)\,\left(b\,c^2-a\,c\,d\right)}{a^3}-\frac{\frac{c^2}{2\,a}+\frac{x^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{2\,a^2\,b}}{b\,x^4+a\,x^2}-\frac{\ln(x)\,\left(2\,b\,c^2-2\,a\,c\,d\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(b*c^2 - a*c*d))/a^3 - (c^2/(2*a) + (x^2*(a^2*d^2 + 2*b^2*c
^2 - 2*a*b*c*d))/(2*a^2*b))/(a*x^2 + b*x^4) - (log(x)*(2*b*c^2 - 2*a*c*d))/
a^3
```

sympy [A] time = 1.36, size = 92, normalized size = 1.15

$$\frac{-abc^2 + x^2\left(-a^2d^2 + 2abcd - 2b^2c^2\right)}{2a^3bx^2 + 2a^2b^2x^4} + \frac{2c\,(ad - bc)\log{(x)}}{a^3} - \frac{c\,(ad - bc)\log{\left(\frac{a}{b} + x^2\right)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*b*c**2 + x**2*(-a**2*d**2 + 2*a*b*c*d - 2*b**2*c**2))/(2*a**3*b*x**2 +
2*a**2*b**2*x**4) + 2*c*(a*d - b*c)*log(x)/a**3 - c*(a*d - b*c)*log(a/b + x
**2)/a**3
```

3.279 $\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=127

$$\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}+\frac{c(5bc-6ad)}{3a^3x}+\frac{x\left(3a^2d^2-6abcd+5b^2c^2\right)}{6a^3\left(a+bx^2\right)}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.14, antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {462, 456, 453, 205}

$$\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{6a\left(a+bx^2\right)}+\frac{c(5bc-6ad)}{3a^3x}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (c*(5*b*c - 6*a*d))/(3*a^3*x) - c^2/(3*a*x^3*(a + b*x^2)) + ((3*d^2 + (b*c*
(5*b*c - 6*a*d))/a^2)*x)/(6*a*(a + b*x^2)) + ((b*c - a*d)*(5*b*c - a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
```

```
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}\,dx &= -\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\int \frac{-c(5bc-6ad)+3ad^2x^2}{x^2\left(a+bx^2\right)^2}\,dx}{3a}\\
&= -\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a\left(a+bx^2\right)}-\frac{\int \frac{\frac{2c(5bc-6ad)}{a}-\left(\frac{5b^2c^2}{a^2}-\frac{6bcd}{a}+3d^2\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{6a}\\
&= \frac{c(5bc-6ad)}{3a^3x}-\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a\left(a+bx^2\right)}+\frac{((bc-ad)(5bc-ad))\int \frac{1}{a+bx^2}\,dx}{2a^3}\\
&= \frac{c(5bc-6ad)}{3a^3x}-\frac{c^2}{3ax^3\left(a+bx^2\right)}+\frac{\left(3d^2+\frac{bc(5bc-6ad)}{a^2}\right)x}{6a\left(a+bx^2\right)}+\frac{(bc-ad)(5bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.84

$$\frac{x(ad-bc)^2}{2a^3\left(a+bx^2\right)}-\frac{2c(ad-bc)}{a^3x}-\frac{c^2}{3a^2x^3}+\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c^2/(a^2*x^3) - (2*c*(-(b*c) + a*d))/(a^3*x) + ((-(b*c) + a*d)^2*x)/(2
*a^3*(a + b*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2)*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^2/(x^4*(a + b*x^2)^2), x]
```

fricas [A] time = 0.91, size = 356, normalized size = 2.80

$$\left[-\frac{4\,a^3bc^2-6\left(5\,ab^3c^2-6\,a^2b^2cd+a^3bd^2\right)x^4-4\left(5\,a^2b^2c^2-6\,a^3bcd\right)x^2+3\left(\left(5\,b^3c^2-6\,ab^2cd+a^2bd^2\right)x^5+\left(5\,ab^2c^2-6\,a^2bcd+a^3d^2\right)x^3\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{12\left(a^4b^2x^5+a^5bx^3\right)},\ -\frac{2\,a^3bc^2-3\left(5\,ab^3c^2-6\,a^2b^2cd+a^3bd^2\right)x^4-2\left(5\,a^2b^2c^2-6\,a^3bcd\right)x^2-3\left(\left(5\,b^3c^2-6\,ab^2cd+a^2bd^2\right)x^5+\left(5\,ab^2c^2-6\,a^2bcd+a^3d^2\right)x^3\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{6\left(a^4b^2x^5+a^5bx^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b*c^2 - 6*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 4*(
5*a^2*b^2*c^2 - 6*a^3*b*c*d)*x^2 + 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)
*x^5 + (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^2*x^5 + a^5*b*x^3), -1/6*(2*a^3*b*c
^2 - 3*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6
*a^3*b*c*d)*x^2 - 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^5 + (5*a*b^2*c
^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^2*
x^5 + a^5*b*x^3)]
```

giac [A] time = 0.41, size = 112, normalized size = 0.88

$$\frac{\left(5\,b^2c^2-6\,abcd+a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}+\frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\left(bx^2+a\right)a^3}+\frac{6\,bc^2x^2-6\,acdx^2-ac^2}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c
^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^3*x^3)
```

maple [A] time = 0.01, size = 161, normalized size = 1.27

$$\frac{d^2x}{2\left(b x^2+a\right)a}+\frac{d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\ a}-\frac{bcdx}{\left(b x^2+a\right)a^2}-\frac{3bcd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^2}+\frac{b^2c^2x}{2\left(b x^2+a\right)a^3}+\frac{5b^2c^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\ a^3}-\frac{2cd}{a^2x}+\frac{2b c^2}{a^3x}-\frac{c^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^4/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a*x/(b*x^2+a)*d^2-1/a^2*x/(b*x^2+a)*b*c*d+1/2/a^3*x/(b*x^2+a)*b^2*c^2+1
/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^2-3/a^2/(a*b)^(1/2)*arctan(1/(
a*b)^(1/2)*b*x)*b*c*d+5/2/a^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b^2*c^2
-1/3*c^2/a^2/x^3-2*c/a^2/x*d+2*c^2/a^3/x*b
```

maxima [A] time = 2.44, size = 118, normalized size = 0.93

$$\frac{3\left(5\,b^2c^2-6\,abcd+a^2d^2\right)x^4-2\,a^2c^2+2\left(5\,abc^2-6\,a^2cd\right)x^2}{6\left(a^3bx^5+a^4x^3\right)}+\frac{\left(5\,b^2c^2-6\,abcd+a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 6
*a^2*c*d)*x^2)/(a^3*b*x^5 + a^4*x^3) + 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2
)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.25, size = 146, normalized size = 1.15

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(a\,d-5\,b\,c\right)}{\sqrt{a}\,\left(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d-5\,b\,c\right)}{2\,a^{7/2}\,\sqrt{b}}-\frac{\frac{c^2}{3\,a}-\frac{x^4\,\left(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2\right)}{2\,a^3}+\frac{c\,x^2\,\left(6\,a\,d-5\,b\,c\right)}{3\,a^2}}{b\,x^5+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x*(a*d - b*c)*(a*d - 5*b*c))/(a^(1/2)*(a^2*d^2 + 5*b^2*c^2 -
 6*a*b*c*d)))*(a*d - b*c)*(a*d - 5*b*c))/(2*a^(7/2)*b^(1/2)) - (c^2/(3*a) -
 (x^4*(a^2*d^2 + 5*b^2*c^2 - 6*a*b*c*d))/(2*a^3) + (c*x^2*(6*a*d - 5*b*c))/
(3*a^2))/(a*x^3 + b*x^5)
```

sympy [B] time = 1.00, size = 248, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{a^7b}}\,(ad-5bc)\,(ad-bc)\log\left(-\frac{a^4\sqrt{-\frac{1}{a^7b}}\,(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b}}\,(ad-5bc)\,(ad-bc)\log\left(\frac{a^4\sqrt{-\frac{1}{a^7b}}\,(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}+\frac{-2a^2c^2+x^4\left(3a^2d^2-18abcd+15b^2c^2\right)+x^2\left(-12a^2cd+10abc^2\right)}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(-a**4*sqrt(-1/(a**7*b))*(a
*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt
(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(a**4*sqrt(-1/(a**7*b))*(a*d - 5
*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + (-2*a**2*c
**2 + x**4*(3*a**2*d**2 - 18*a*b*c*d + 15*b**2*c**2) + x**2*(-12*a**2*c*d +
 10*a*b*c**2))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.280 $\int \frac{x^4\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

<u>**Optimal**</u>. Leaf size=169

$$\frac{dx^3\left(3a^2d^2-7abcd+5b^2c^2\right)}{2b^4}-\frac{3\sqrt{a}\,(bc-3ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}+\frac{3d^2x^5(7bc-3ad}{10b^3}$$

<u>**Rubi**</u> **[A]** time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {467, 570, 205}

$$\frac{dx^3\left(3a^2d^2-7abcd+5b^2c^2\right)}{2b^4}+\frac{3d^2x^5(7bc-3ad)}{10b^3}+\frac{3x(bc-3ad)(bc-ad)^2}{2b^5}-\frac{3\sqrt{a}\,(bc-3ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{9d^3x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*(b*c - 3*a*d)*(b*c - a*d)^2*x)/(2*b^5) + (d*(5*b^2*c^2 - 7*a*b*c*d + 3*a
^2*d^2)*x^3)/(2*b^4) + (3*d^2*(7*b*c - 3*a*d)*x^5)/(10*b^3) + (9*d^3*x^7)/(
14*b^2) - (x^3*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 3*a*d)*
(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

<u>Rule 205</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 467</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

<u>Rule 570</u>

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
```

```
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{\int \frac{x^2\left(c+dx^2\right)^2\left(3c+9dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{\int \Bigg(\frac{3(bc-3ad)(bc-ad)^2}{b^4}+\frac{3d\left(5b^2c^2-7abcd+3a^2d^2\right)x^2}{b^3}+\frac{3d^2(7bc-3ad)x^4}{b^2}+\frac{9d^3x^6}{b}+ 3\left(-ab^3\right.}{2b}$$

$$= \frac{3(bc-3ad)(bc-ad)^2x}{2b^5}+\frac{d\left(5b^2c^2-7abcd+3a^2d^2\right)x^3}{2b^4}+\frac{3d^2(7bc-3ad)x^5}{10b^3}+\frac{9d^3x^7}{14b^2}-\frac{x^3\,(}{2b}$$

$$= \frac{3(bc-3ad)(bc-ad)^2x}{2b^5}+\frac{d\left(5b^2c^2-7abcd+3a^2d^2\right)x^3}{2b^4}+\frac{3d^2(7bc-3ad)x^5}{10b^3}+\frac{9d^3x^7}{14b^2}-\frac{x^3\,(}{2b}$$

Mathematica [A] time = 0.08, size = 151, normalized size = 0.89

$$\frac{3\sqrt{a}\,(bc-ad)^2(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{ax(bc-ad)^3}{2b^5\left(a+bx^2\right)}+\frac{x(bc-4ad)(bc-ad)^2}{b^5}+\frac{dx^3(bc-ad)^2}{b^4}+\frac{d^2x^5(3bc-2ad)}{5b^3}+\frac{d^3x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - 4*a*d)*(b*c - a*d)^2*x)/b^5 + (d*(b*c - a*d)^2*x^3)/b^4 + (d^2*(3*b
*c - 2*a*d)*x^5)/(5*b^3) + (d^3*x^7)/(7*b^2) + (a*(b*c - a*d)^3*x)/(2*b^5*(
a + b*x^2)) + (3*Sqrt[a]*(b*c - a*d)^2*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*b^(11/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.95, size = 580, normalized size = 3.43

$$\left[\frac{20\,b^{4}d^{3}x^{9}+12\left(7\,b^{4}cd^{2}-3\,ab^{3}d^{3}\right)x^{7}+28\left(5\,b^{4}c^{2}d-7\,ab^{3}cd^{2}+3\,a^{2}b^{2}d^{3}\right)x^{5}+140\left(b^{4}c^{3}-5\,ab^{3}c^{2}d+7\,a^{2}b^{2}cd^{2}-3\,a^{3}bd^{3}\right)x^{3}-105\left(ab^{3}c^{3}-5\,a^{2}b^{2}c^{2}d+7\,a^{3}bcd^{2}-3\,a^{4}d^{3}+\left(b^{4}c^{3}-5\,ab^{3}c^{2}d+7\,a^{2}b^{2}cd^{2}-3\,a^{3}bd^{3}\right)x^{2}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)+210\left(ab^{3}c^{3}-5\,a^{2}b^{2}c^{2}d+7\,a^{3}bcd^{2}-3\,a^{4}d^{3}\right)x}{140\left(b^{6}x^{2}+ab^{5}\right)},\ \frac{10\,b^{4}d^{3}x^{9}+6\left(7\,b^{4}cd^{2}-3\,ab^{3}d^{3}\right)x^{7}+14\left(5\,b^{4}c^{2}d-7\,ab^{3}cd^{2}+3\,a^{2}b^{2}d^{3}\right)x^{5}+70\left(b^{4}c^{3}-5\,ab^{3}c^{2}d+7\,a^{2}b^{2}cd^{2}-3\,a^{3}bd^{3}\right)x^{3}-105\left(ab^{3}c^{3}-5\,a^{2}b^{2}c^{2}d+7\,a^{3}bcd^{2}-3\,a^{4}d^{3}+\left(b^{4}c^{3}-5\,ab^{3}c^{2}d+7\,a^{2}b^{2}cd^{2}-3\,a^{3}bd^{3}\right)x^{2}\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\left(ab^{3}c^{3}-5\,a^{2}b^{2}c^{2}d+7\,a^{3}bcd^{2}-3\,a^{4}d^{3}\right)x}{70\left(b^{6}x^{2}+ab^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*d^3*x^9 + 12*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 28*(5*b^4*c^2
*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 140*(b^4*c^3 - 5*a*b^3*c^2*d + 7*
a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3
*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b
*d^3)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210
*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*
b^5), 1/70*(10*b^4*d^3*x^9 + 6*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 14*(5*b^4*
c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 70*(b^4*c^3 - 5*a*b^3*c^2*d +
7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a
^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3
*b*d^3)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(a*b^3*c^3 - 5*a^2*b^2
*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5)]
```

giac [A] time = 0.39, size = 241, normalized size = 1.43

$$-\frac{3\left(ab^{3}c^{3}-5\,a^{2}b^{2}c^{2}d+7\,a^{3}bcd^{2}-3\,a^{4}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{5}}+\frac{ab^{3}c^{3}x-3\,a^{2}b^{2}c^{2}dx+3\,a^{3}bcd^{2}x-a^{4}d^{3}x}{2\left(bx^{2}+a\right)b^{5}}+\frac{5\,b^{12}d^{3}x^{7}+21\,b^{12}cd^{2}x^{5}-14\,ab^{11}d^{3}x^{5}+35\,b^{12}c^{2}dx^{3}-70\,ab^{11}cd^{2}x^{3}+35\,a^{2}b^{10}d^{3}x^{3}+35\,b^{12}c^{3}x-210\,ab^{11}c^{2}dx+315\,a^{2}b^{10}cd^{2}x-140\,a^{3}b^{9}d^{3}x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c^3*x - 3*a^2*b^2*c^2*d*x + 3*a^3*b*
c*d^2*x - a^4*d^3*x)/((b*x^2 + a)*b^5) + 1/35*(5*b^12*d^3*x^7 + 21*b^12*c*d
^2*x^5 - 14*a*b^11*d^3*x^5 + 35*b^12*c^2*d*x^3 - 70*a*b^11*c*d^2*x^3 + 35*a
^2*b^10*d^3*x^3 + 35*b^12*c^3*x - 210*a*b^11*c^2*d*x + 315*a^2*b^10*c*d^2*x
 - 140*a^3*b^9*d^3*x)/b^14
```

maple [B] time = 0.01, size = 302, normalized size = 1.79

$$\frac{d^{3}x^{7}}{7b^{2}}-\frac{2a\,d^{3}x^{5}}{5b^{3}}+\frac{3c\,d^{2}x^{5}}{5b^{2}}+\frac{a^{2}d^{3}x^{3}}{b^{4}}-\frac{2ac\,d^{2}x^{3}}{b^{3}}+\frac{c^{2}d\,x^{3}}{b^{2}}-\frac{a^{4}d^{3}x}{2\left(b\,x^{2}+a\right)b^{5}}+\frac{9a^{4}d^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{5}}+\frac{3a^{3}c\,d^{2}x}{2\left(b\,x^{2}+a\right)b^{4}}-\frac{21a^{3}c\,d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{4}}-\frac{3a^{2}c^{2}dx}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{15a^{2}c^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{3}}+\frac{a\,c^{3}x}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3a\,c^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^{2}}-\frac{4a^{3}d^{3}x}{b^{5}}+\frac{9a^{2}c\,d^{2}x}{b^{4}}-\frac{6a\,c^{2}dx}{b^{3}}+\frac{c^{3}x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/7*d^3*x^7/b^2-2/5/b^3*x^5*a*d^3+3/5/b^2*x^5*c*d^2+1/b^4*x^3*a^2*d^3-2/b^3
*x^3*a*c*d^2+1/b^2*x^3*c^2*d-4/b^5*a^3*d^3*x+9/b^4*a^2*c*d^2*x-6/b^3*a*c^2*
d*x+1/b^2*c^3*x-1/2*a^4/b^5*x/(b*x^2+a)*d^3+3/2*a^3/b^4*x/(b*x^2+a)*c*d^2-3
/2*a^2/b^3*x/(b*x^2+a)*c^2*d+1/2*a/b^2*x/(b*x^2+a)*c^3+9/2*a^4/b^5/(a*b)^(1
/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3-21/2*a^3/b^4/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)*c*d^2+15/2*a^2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d-3/
2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.63, size = 228, normalized size = 1.35

$$\frac{\left(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3\right)x}{2\left(b^6x^2 + ab^5\right)} - \frac{3\left(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5} + \frac{5\,b^3d^3x^7 + 7\left(3\,b^3cd^2 - 2\,ab^2d^3\right)x^5 + 35\left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x^3 + 35\left(b^3c^3 - 6\,ab^2c^2d + 9\,a^2bcd^2 - 4\,a^3d^3\right)x}{35\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x/(b^6*x^2 + a*
b^5) - 3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/35*(5*b^3*d^3*x^7 + 7*(3*b^3*c*d^2 - 2*
a*b^2*d^3)*x^5 + 35*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 35*(b^3*c
^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5
```

mupad [B] time = 0.20, size = 328, normalized size = 1.94

$$x\left(\frac{c^3}{b^2} - \frac{2\,a\left(\frac{3\,c^2\,d}{b^2} + \frac{2\,a\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right)}{b} - \frac{a^2\,d^3}{b^4}\right)}{b} + \frac{a^2\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right)}{b^2}\right) - x^5\left(\frac{2\,a\,d^3}{5\,b^3} - \frac{3\,c\,d^2}{5\,b^2}\right) + x^3\left(\frac{c^2\,d}{b^2} + \frac{2\,a\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right)}{3\,b} - \frac{a^2\,d^3}{3\,b^4}\right) - \frac{x\left(\frac{a^4\,d^3}{2} - \frac{3\,a^3\,b\,c\,d^2}{2} + \frac{3\,a^2\,b^2\,c^2\,d}{2} - \frac{a\,b^3\,c^3}{2}\right)}{b^6\,x^2 + a\,b^5} + \frac{d^3\,x^7}{7\,b^2} + \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d-b\,c\right)}{3\,a^4\,d^3-7\,a^3\,b\,c\,d^2+5\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d-b\,c\right)}{2\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x*(c^3/b^2 - (2*a*((3*c^2*d)/b^2 + (2*a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b
- (a^2*d^3)/b^4))/b + (a^2*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b^2) - x^5*((2*
a*d^3)/(5*b^3) - (3*c*d^2)/(5*b^2)) + x^3*((c^2*d)/b^2 + (2*a*((2*a*d^3)/b^
3 - (3*c*d^2)/b^2))/(3*b) - (a^2*d^3)/(3*b^4)) - (x*((a^4*d^3)/2 - (a*b^3*c
^3)/2 + (3*a^2*b^2*c^2*d)/2 - (3*a^3*b*c*d^2)/2))/(a*b^5 + b^6*x^2) + (d^3*
x^7)/(7*b^2) + (3*a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d - b*c)^2*(3*a*d - b*
c))/(3*a^4*d^3 - a*b^3*c^3 + 5*a^2*b^2*c^2*d - 7*a^3*b*c*d^2))*(a*d - b*c)^
2*(3*a*d - b*c))/(2*b^(11/2))
```

sympy [B] time = 1.36, size = 389, normalized size = 2.30

$$x^5\left(-\frac{2ad^3}{5b^3} + \frac{3cd^2}{5b^2}\right) + x^3\left(\frac{a^2d^3}{b^4} - \frac{2acd^2}{b^3} + \frac{c^2d}{b^2}\right) + x\left(-\frac{4a^3d^3}{b^5} + \frac{9a^2cd^2}{b^4} - \frac{6ac^2d}{b^3} + \frac{c^3}{b^2}\right) + \frac{x\left(-a^4d^3 + 3a^3bcd^2 - 3a^2b^2c^2d + ab^3c^3\right)}{2ab^5 + 2b^6x^2} - \frac{3\sqrt{-\frac{a}{b^{11}}}\,(ad - bc)^2\,(3ad - bc)\log\left(-\frac{3b^5\sqrt{-\frac{a}{b^{11}}}\,(ad-bc)^2(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3} + x\right)}{4} + \frac{3\sqrt{-\frac{a}{b^{11}}}\,(ad - bc)^2\,(3ad - bc)\log\left(\frac{3b^5\sqrt{-\frac{a}{b^{11}}}\,(ad-bc)^2(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3} + x\right)}{4} + \frac{d^3x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*d**3/(5*b**3) + 3*c*d**2/(5*b**2)) + x**3*(a**2*d**3/b**4 - 2*a*
c*d**2/b**3 + c**2*d/b**2) + x*(-4*a**3*d**3/b**5 + 9*a**2*c*d**2/b**4 - 6*
a*c**2*d/b**3 + c**3/b**2) + x*(-a**4*d**3 + 3*a**3*b*c*d**2 - 3*a**2*b**2*
c**2*d + a*b**3*c**3)/(2*a*b**5 + 2*b**6*x**2) - 3*sqrt(-a/b**11)*(a*d - b*
c)**2*(3*a*d - b*c)*log(-3*b**5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)
/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 3*b**3*c**3) + x)/4 +
 3*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)*log(3*b**5*sqrt(-a/b**11)*(a
*d - b*c)**2*(3*a*d - b*c)/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2
*d - 3*b**3*c**3) + x)/4 + d**3*x**7/(7*b**2)
```

3.281 $\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=117

$$\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{d^3x^6}{6b^2}$$

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{d^2x^4(3bc-2ad)}{4b^3}+\frac{3dx^2(bc-ad)^2}{2b^4}+\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}+\frac{d^3x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x^2)/(2*b^4) + (d^2*(3*b*c - 2*a*d)*x^4)/(4*b^3) + (d^3*
x^6)/(6*b^2) + (a*(b*c - a*d)^3)/(2*b^5*(a + b*x^2)) + ((b*c - 4*a*d)*(b*c
- a*d)^2*Log[a + b*x^2])/(2*b^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+dx)^3}{(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{3d(bc-ad)^2}{b^4}+\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{b^2}+\frac{a(-bc+ad)^3}{b^4(a+bx)^2}+\frac{(bc-4ad)(bc-a}{b^4(a+bx)}\right.\right.\\
&= \frac{3d(bc-ad)^2x^2}{2b^4}+\frac{d^2(3bc-2ad)x^4}{4b^3}+\frac{d^3x^6}{6b^2}+\frac{a(bc-ad)^3}{2b^5\left(a+bx^2\right)}+\frac{(bc-4ad)(bc-ad)^2\log\left(a\right.}{2b^5}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 106, normalized size = 0.91

$$
\frac{3b^2d^2x^4(3bc-2ad)+18bdx^2(bc-ad)^2-\frac{6a(ad-bc)^3}{a+bx^2}+6(bc-4ad)(bc-ad)^2\log\left(a+bx^2\right)+2b^3d^3x^6}{12b^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (18*b*d*(b*c - a*d)^2*x^2 + 3*b^2*d^2*(3*b*c - 2*a*d)*x^4 + 2*b^3*d^3*x^6 -
 (6*a*(-(b*c) + a*d)^3)/(a + b*x^2) + 6*(b*c - 4*a*d)*(b*c - a*d)^2*Log[a +
 b*x^2])/(12*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^3\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2, x]
```

fricas [B] time = 0.83, size = 254, normalized size = 2.17

$$
\frac{2\,b^4d^3x^8+6\,ab^3c^3-18\,a^2b^2c^2d+18\,a^3bcd^2-6\,a^4d^3+\left(9\,b^4cd^2-4\,ab^3d^3\right)x^6+3\left(6\,b^4c^2d-9\,ab^3cd^2+4\,a^2b^2d^3\right)x^4+18\left(ab^3c^2d-2\,a^2b^2cd^2+a^3bd^3\right)x^2+6\left(ab^3c^3-6\,a^2b^2c^2d+9\,a^3bcd^2-4\,a^4d^3+\left(b^4c^3-6\,ab^3c^2d+9\,a^2b^2cd^2-4\,a^3bd^3\right)x^2\right)\log\left(bx^2+a\right)}{12\left(b^6x^2+ab^5\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^4*d^3*x^8 + 6*a*b^3*c^3 - 18*a^2*b^2*c^2*d + 18*a^3*b*c*d^2 - 6*a
^4*d^3 + (9*b^4*c*d^2 - 4*a*b^3*d^3)*x^6 + 3*(6*b^4*c^2*d - 9*a*b^3*c*d^2 +
 4*a^2*b^2*d^3)*x^4 + 18*(a*b^3*c^2*d - 2*a^2*b^2*c*d^2 + a^3*b*d^3)*x^2 +
6*(a*b^3*c^3 - 6*a^2*b^2*c^2*d + 9*a^3*b*c*d^2 - 4*a^4*d^3 + (b^4*c^3 - 6*a
*b^3*c^2*d + 9*a^2*b^2*c*d^2 - 4*a^3*b*d^3)*x^2)*log(b*x^2 + a))/(b^6*x^2 +
 a*b^5)
```

giac **[B]** time = 0.37, size = 249, normalized size = 2.13

$$\frac{\frac{\left(2\,d^{3}+\frac{3\left(3\,b^{2}cd^{2}-4\,abd^{3}\right)}{\left(bx^{2}+a\right)b}+\frac{18\left(b^{4}c^{2}d-3\,ab^{3}cd^{2}+2\,a^{2}b^{2}d^{3}\right)}{\left(bx^{2}+a\right)^{2}b^{2}}\right)\left(bx^{2}+a\right)^{3}}{b^{4}}-\frac{6\left(b^{3}c^{3}-6\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}-4\,a^{3}d^{3}\right)\log\left(\frac{\left|bx^{2}+a\right|}{\left(bx^{2}+a\right)^{2}|b|}\right)}{b^{4}}+\frac{6\left(\frac{ab^{6}c^{3}}{bx^{2}+a}-\frac{3\,a^{2}b^{5}c^{2}d}{bx^{2}+a}+\frac{3\,a^{3}b^{4}cd^{2}}{bx^{2}+a}-\frac{a^{4}b^{3}d^{3}}{bx^{2}+a}\right)}{b^{7}}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*((2*d^3 + 3*(3*b^2*c*d^2 - 4*a*b*d^3)/((b*x^2 + a)*b) + 18*(b^4*c^2*d
 - 3*a*b^3*c*d^2 + 2*a^2*b^2*d^3)/((b*x^2 + a)^2*b^2))*(b*x^2 + a)^3/b^4 - 6
*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*log(abs(b*x^2 + a)/(
(b*x^2 + a)^2*abs(b)))/b^4 + 6*(a*b^6*c^3/(b*x^2 + a) - 3*a^2*b^5*c^2*d/(b*
x^2 + a) + 3*a^3*b^4*c*d^2/(b*x^2 + a) - a^4*b^3*d^3/(b*x^2 + a))/b^7)/b
```

maple **[B]** time = 0.01, size = 229, normalized size = 1.96

$$\frac{d^{3}x^{6}}{6b^{2}}-\frac{a\,d^{3}x^{4}}{2b^{3}}+\frac{3c\,d^{2}x^{4}}{4b^{2}}+\frac{3a^{2}d^{3}x^{2}}{2b^{4}}-\frac{3ac\,d^{2}x^{2}}{b^{3}}+\frac{3c^{2}d\,x^{2}}{2b^{2}}-\frac{a^{4}d^{3}}{2\left(b\,x^{2}+a\right)b^{5}}+\frac{3a^{3}c\,d^{2}}{2\left(b\,x^{2}+a\right)b^{4}}-\frac{2a^{3}d^{3}\ln\left(b\,x^{2}+a\right)}{b^{5}}-\frac{3a^{2}c^{2}d}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{9a^{2}c\,d^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}+\frac{a\,c^{3}}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3a\,c^{2}d\ln\left(b\,x^{2}+a\right)}{b^{3}}+\frac{c^{3}\ln\left(b\,x^{2}+a\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/6*d^3*x^6/b^2-1/2*d^3/b^3*x^4*a+3/4*d^2/b^2*x^4*c+3/2*d^3/b^4*x^2*a^2-3*d
^2/b^3*x^2*a*c+3/2*d/b^2*x^2*c^2-1/2/b^5*a^4/(b*x^2+a)*d^3+3/2/b^4*a^3/(b*x
^2+a)*d^2*c-3/2/b^3*a^2/(b*x^2+a)*d*c^2+1/2/b^2*a/(b*x^2+a)*c^3-2/b^5*ln(b*
x^2+a)*a^3*d^3+9/2/b^4*ln(b*x^2+a)*a^2*d^2*c-3/b^3*ln(b*x^2+a)*a*d*c^2+1/2/
b^2*ln(b*x^2+a)*c^3
```

maxima **[A]** time = 1.03, size = 174, normalized size = 1.49

$$\frac{ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}}{2\left(b^{6}x^{2}+ab^{5}\right)}+\frac{2\,b^{2}d^{3}x^{6}+3\left(3\,b^{2}cd^{2}-2\,abd^{3}\right)x^{4}+18\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)x^{2}}{12\,b^{4}}+\frac{\left(b^{3}c^{3}-6\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}-4\,a^{3}d^{3}\right)\log\left(bx^{2}+a\right)}{2\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)/(b^6*x^2 + a*b^
5) + 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 18*(b^2*c^2*d
```

```
- 2*a*b*c*d^2 + a^2*d^3)*x^2)/b^4 + 1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*
c*d^2 - 4*a^3*d^3)*log(b*x^2 + a)/b^5
```

mupad [B] time = 0.17, size = 194, normalized size = 1.66

$$x^2\left(\frac{3\,c^2\,d}{2\,b^2}+\frac{a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{2\,b^4}\right)-x^4\,\left(\frac{a\,d^3}{2\,b^3}-\frac{3\,c\,d^2}{4\,b^2}\right)-\frac{\ln\left(b\,x^2+a\right)\,\left(4\,a^3\,d^3-9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,b^5}-\frac{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}+\frac{d^3\,x^6}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^2*((3*c^2*d)/(2*b^2) + (a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b - (a^2*d^3)/
(2*b^4)) - x^4*((a*d^3)/(2*b^3) - (3*c*d^2)/(4*b^2)) - (log(a + b*x^2)*(4*a
^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(2*b^5) - (a^4*d^3 - a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)/(2*b*(a*b^4 + b^5*x^2)) + (d^3*x^
6)/(6*b^2)
```

sympy [A] time = 1.34, size = 163, normalized size = 1.39

$$x^{4}\left(-\frac{ad^{3}}{2b^{3}}+\frac{3cd^{2}}{4b^{2}}\right)+x^{2}\left(\frac{3a^{2}d^{3}}{2b^{4}}-\frac{3acd^{2}}{b^{3}}+\frac{3c^{2}d}{2b^{2}}\right)+\frac{-a^{4}d^{3}+3a^{3}bcd^{2}-3a^{2}b^{2}c^{2}d+ab^{3}c^{3}}{2ab^{5}+2b^{6}x^{2}}+\frac{d^{3}x^{6}}{6b^{2}}-\frac{(ad-bc)^{2}\,(4ad-bc)\log\left(a+bx^{2}\right)}{2b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**4*(-a*d**3/(2*b**3) + 3*c*d**2/(4*b**2)) + x**2*(3*a**2*d**3/(2*b**4) -
3*a*c*d**2/b**3 + 3*c**2*d/(2*b**2)) + (-a**4*d**3 + 3*a**3*b*c*d**2 - 3*a*
*2*b**2*c**2*d + a*b**3*c**3)/(2*a*b**5 + 2*b**6*x**2) + d**3*x**6/(6*b**2)
 - (a*d - b*c)**2*(4*a*d - b*c)*log(a + b*x**2)/(2*b**5)
```

3.282 $\int \frac{x^2\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{dx\left(105a^2d^2-190abcd+81b^2c^2\right)}{30b^4}+\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{9/2}}+\frac{dx\left(c+dx^2\right)(33bc-35ad)}{30b^3}-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+$$

Rubi [A] time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {467, 528, 388, 205}

$$\frac{dx\left(105a^2d^2-190abcd+81b^2c^2\right)}{30b^4}+\frac{dx\left(c+dx^2\right)(33bc-35ad)}{30b^3}+\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{9/2}}-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(81*b^2*c^2 - 190*a*b*c*d + 105*a^2*d^2)*x)/(30*b^4) + (d*(33*b*c - 35*a
*d)*x*(c + d*x^2))/(30*b^3) + (7*d*x*(c + d*x^2)^2)/(10*b^2) - (x*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = -\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{\int \frac{\left(c+dx^2\right)^2\left(c+7dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= \frac{7dx\left(c+dx^2\right)^2}{10b^2}-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{\int \frac{\left(c+dx^2\right)\left(c(5bc-7ad)+d(33bc-35ad)x^2\right)}{a+bx^2}\,dx}{10b^2}$$

$$= \frac{d(33bc-35ad)x\left(c+dx^2\right)}{30b^3}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}-\frac{x\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{\int \frac{c\left(15b^2c^2-54abcd+35a^2d^2\right)+d\left(8\right.}{a+bx}}{30b}$$

$$= \frac{d\left(81b^2c^2-190abcd+105a^2d^2\right)x}{30b^4}+\frac{d(33bc-35ad)x\left(c+dx^2\right)}{30b^3}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}-\frac{x\left(c\right.}{2b\left(a\right.}$$

$$= \frac{d\left(81b^2c^2-190abcd+105a^2d^2\right)x}{30b^4}+\frac{d(33bc-35ad)x\left(c+dx^2\right)}{30b^3}+\frac{7dx\left(c+dx^2\right)^2}{10b^2}-\frac{x\left(c\right.}{2b\left(a\right.}$$

Mathematica **[A]** time = 0.07, size = 125, normalized size = 0.85

$$\frac{(bc-7ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{9/2}}-\frac{x(bc-ad)^3}{2b^4\left(a+bx^2\right)}+\frac{3dx(bc-ad)^2}{b^4}+\frac{d^2x^3(3bc-2ad)}{3b^3}+\frac{d^3x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (3*d*(b*c - a*d)^2*x)/b^4 + (d^2*(3*b*c - 2*a*d)*x^3)/(3*b^3) + (d^3*x^5)/(
5*b^2) - ((b*c - a*d)^3*x)/(2*b^4*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*d)
^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx^2\right)^3}{\left(a + bx^2\right)^2}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2, x]
```

fricas [A] time = 0.96, size = 508, normalized size = 3.46

$$\left[\frac{12\,ab^4d^3x^7 + 4\left(15\,ab^4cd^2 - 7\,a^2b^3d^3\right)x^5 + 20\left(9\,ab^4c^2d - 15\,a^2b^3cd^2 + 7\,a^3b^2d^3\right)x^3 + 15\left(ab^3c^3 - 9\,a^2b^2c^2d + 15\,a^3bcd^2 - 7\,a^4d^3 + \left(b^4c^3 - 9\,ab^3c^2d + 15\,a^2b^2cd^2 - 7\,a^3bd^3\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2 + 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 30\left(ab^4c^3 - 9\,a^2b^3c^2d + 15\,a^3b^2cd^2 - 7\,a^4bd^3\right)x}{60\left(ab^6x^2 + a^2b^5\right)}, \frac{6\,ab^4d^3x^7 + 2\left(15\,ab^4cd^2 - 7\,a^2b^3d^3\right)x^5 + 10\left(9\,ab^4c^2d - 15\,a^2b^3cd^2 + 7\,a^3b^2d^3\right)x^3 + 15\left(ab^3c^3 - 9\,a^2b^2c^2d + 15\,a^3bcd^2 - 7\,a^4d^3 + \left(b^4c^3 - 9\,ab^3c^2d + 15\,a^2b^2cd^2 - 7\,a^3bd^3\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - 15\left(ab^4c^3 - 9\,a^2b^3c^2d + 15\,a^3b^2cd^2 - 7\,a^4bd^3\right)x}{30\left(ab^6x^2 + a^2b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*d^3*x^7 + 4*(15*a*b^4*c*d^2 - 7*a^2*b^3*d^3)*x^5 + 20*(9*a*
b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x^3 + 15*(a*b^3*c^3 - 9*a^2*b
^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^2*b
^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)) - 30*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^4*b*d
^3)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*d^3*x^7 + 2*(15*a*b^4*c*d^2 - 7
*a^2*b^3*d^3)*x^5 + 10*(9*a*b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x
^3 + 15*(a*b^3*c^3 - 9*a^2*b^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^
3 - 9*a*b^3*c^2*d + 15*a^2*b^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) - 15*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^4*
b*d^3)*x)/(a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.36, size = 184, normalized size = 1.25

$$\frac{\left(b^3c^3 - 9\,ab^2c^2d + 15\,a^2bcd^2 - 7\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4} - \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{2\left(bx^2 + a\right)b^4} + \frac{3\,b^8d^3x^5 + 15\,b^8cd^2x^3 - 10\,ab^7d^3x^3 + 45\,b^8c^2dx - 90\,ab^7cd^2x + 45\,a^2b^6d^3x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^4) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
```

```
- a^3*d^3*x)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*d^3*x^5 + 15*b^8*c*d^2*x^3 - 1
0*a*b^7*d^3*x^3 + 45*b^8*c^2*d*x - 90*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)/b^1
0
```

maple [A] time = 0.01, size = 247, normalized size = 1.68

$$\frac{d^3x^5}{5b^2}-\frac{2a\,d^3x^3}{3b^3}+\frac{c\,d^2x^3}{b^2}+\frac{a^3d^3x}{2\left(b\,x^2+a\right)b^4}-\frac{7a^3d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^4}-\frac{3a^2c\,d^2x}{2\left(b\,x^2+a\right)b^3}+\frac{15a^2c\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^3}+\frac{3a\,c^2dx}{2\left(b\,x^2+a\right)b^2}-\frac{9a\,c^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2}-\frac{c^3x}{2\left(b\,x^2+a\right)b}+\frac{c^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b}+\frac{3a^2d^3x}{b^4}-\frac{6ac\,d^2x}{b^3}+\frac{3c^2dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^3/b^2*x^5-2/3*d^3/b^3*x^3*a+d^2/b^2*x^3*c+3*d^3/b^4*a^2*x-6*d^2/b^3*a
*c*x+3*d/b^2*c^2*x+1/2/b^4*x/(b*x^2+a)*a^3*d^3-3/2/b^3*x/(b*x^2+a)*a^2*c*d^
2+3/2/b^2*x/(b*x^2+a)*a*c^2*d-1/2/b*x/(b*x^2+a)*c^3-7/2/b^4/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*a^3*d^3+15/2/b^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x)*a^2*c*d^2-9/2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*c^2*d+1/2/b/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.29, size = 176, normalized size = 1.20

$$-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x}{2\left(b^5x^2+ab^4\right)}+\frac{\left(b^3c^3-9\,ab^2c^2d+15\,a^2bcd^2-7\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b^4}+\frac{3\,b^2d^3x^5+5\left(3\,b^2cd^2-2\,abd^3\right)x^3+45\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(b^5*x^2 + a*b^4
) + 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*b^4) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - 2*a*b*d^
3)*x^3 + 45*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)/b^4
```

mupad [B] time = 0.07, size = 232, normalized size = 1.58

$$x\left(\frac{3\,c^2\,d}{b^2}+\frac{2\,a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{b^4}\right)-x^3\,\left(\frac{2\,a\,d^3}{3\,b^3}-\frac{c\,d^2}{b^2}\right)+\frac{x\,\left(\frac{a^3\,d^3}{2}-\frac{3\,a^2\,b\,c\,d^2}{2}+\frac{3\,a\,b^2\,c^2\,d}{2}-\frac{b^3\,c^3}{2}\right)}{b^5\,x^2+a\,b^4}+\frac{d^3\,x^5}{5\,b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d-b\,c\right)}{\sqrt{a}\,\left(7\,a^3\,d^3-15\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d-b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d-b\,c\right)}{2\,\sqrt{a}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x*((3*c^2*d)/b^2 + (2*a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b - (a^2*d^3)/b^4)
 - x^3*((2*a*d^3)/(3*b^3) - (c*d^2)/b^2) + (x*((a^3*d^3)/2 - (b^3*c^3)/2 +
(3*a*b^2*c^2*d)/2 - (3*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) + (d^3*x^5)/(5*b^
2) - (atan((b^(1/2)*x*(a*d - b*c)^2*(7*a*d - b*c))/(a^(1/2)*(7*a^3*d^3 - b^
```

```
3*c^3 + 9*a*b^2*c^2*d - 15*a^2*b*c*d^2)))*(a*d - b*c)^2*(7*a*d - b*c))/(2*a
^(1/2)*b^(9/2))
```

sympy [B] time = 1.22, size = 338, normalized size = 2.30

$$x^{3}\left(-\frac{2ad^{3}}{3b^{3}}+\frac{cd^{2}}{b^{2}}\right)+x\left(\frac{3a^{2}d^{3}}{b^{4}}-\frac{6acd^{2}}{b^{3}}+\frac{3c^{2}d}{b^{2}}\right)+\frac{x\left(a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}\right)}{2ab^{4}+2b^{5}x^{2}}+\frac{\sqrt{-\frac{1}{ab^{9}}}\,(ad-bc)^{2}\,(7ad-bc)\log\left(-\frac{ab^{4}\sqrt{-\frac{1}{ab^{9}}}\,(ad-bc)^{2}(7ad-bc)}{7a^{3}d^{3}-15a^{2}bcd^{2}+9ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^{9}}}\,(ad-bc)^{2}\,(7ad-bc)\log\left(\frac{ab^{4}\sqrt{-\frac{1}{ab^{9}}}\,(ad-bc)^{2}(7ad-bc)}{7a^{3}d^{3}-15a^{2}bcd^{2}+9ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}{4}+\frac{d^{3}x^{5}}{5b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**3/(3*b**3) + c*d**2/b**2) + x*(3*a**2*d**3/b**4 - 6*a*c*d**2/
b**3 + 3*c**2*d/b**2) + x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d -
b**3*c**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a
*d - b*c)*log(-a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/(7*a**
3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b**3*c**3) + x)/4 - sqrt(-1/(
a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b
*c)**2*(7*a*d - b*c)/(7*a**3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b*
*3*c**3) + x)/4 + d**3*x**5/(5*b**2)
```

3.283 $\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}$$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{d^2x^2(3bc-2ad)}{2b^3}-\frac{(bc-ad)^3}{2b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) - (b*c - a*d)^3/(2*b^
4*(a + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/(2*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^3}{(a+bx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx, x, x^2\right)\\
&= \frac{d^2(3bc-2ad)x^2}{2b^3}+\frac{d^3x^4}{4b^2}-\frac{(bc-ad)^3}{2b^4\left(a+bx^2\right)}+\frac{3d(bc-ad)^2\log\left(a+bx^2\right)}{2b^4}
\end{aligned}
$$

Mathematica [**A**] time = 0.05, size = 127, normalized size = 1.44

$$
\frac{3\left(a^2d^3-2abcd^2+b^2c^2d\right)\log\left(a+bx^2\right)}{2b^4}+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2b^4\left(a+bx^2\right)}+\frac{d^2x^2(3bc-2ad)}{2b^3}+\frac{d^3x^4}{4b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) + (-(b^3*c^3) + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)/(2*b^4*(a + b*x^2)) + (3*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*Log[a + b*x^2])/(2*b^4)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(x*(c + d*x^2)^3)/(a + b*x^2)^2, x]
```

fricas [**B**] time = 0.99, size = 181, normalized size = 2.06

$$
\frac{b^3d^3x^6-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^4+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x^2+6\left(ab^2c^2d-2\,a^2bcd^2+a^3d^3+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^2+ab^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*d^3*x^6 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^4 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x^2 + 6*(
a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*
d^3)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

giac [B] time = 0.47, size = 183, normalized size = 2.08

$$\frac{\left(d^{3}+\frac{6\left(b^{2}cd^{2}-abd^{3}\right)}{\left(bx^{2}+a\right)b}\right)\left(bx^{2}+a\right)^{2}}{4\,b^{4}}-\frac{3\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\log\left(\frac{\left|bx^{2}+a\right|}{\left(bx^{2}+a\right)^{2}\left|b\right|}\right)}{2\,b^{4}}-\frac{\frac{b^{5}c^{3}}{bx^{2}+a}-\frac{3\,ab^{4}c^{2}d}{bx^{2}+a}+\frac{3\,a^{2}b^{3}cd^{2}}{bx^{2}+a}-\frac{a^{3}b^{2}d^{3}}{bx^{2}+a}}{2\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x^2 + a)*b))*(b*x^2 + a)^2/b^4 - 3/2
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(
b)))/b^4 - 1/2*(b^5*c^3/(b*x^2 + a) - 3*a*b^4*c^2*d/(b*x^2 + a) + 3*a^2*b^3
*c*d^2/(b*x^2 + a) - a^3*b^2*d^3/(b*x^2 + a))/b^6
```

maple [B] time = 0.01, size = 168, normalized size = 1.91

$$\frac{d^{3}x^{4}}{4b^{2}}-\frac{a\,d^{3}x^{2}}{b^{3}}+\frac{3c\,d^{2}x^{2}}{2b^{2}}+\frac{a^{3}d^{3}}{2\left(b\,x^{2}+a\right)b^{4}}-\frac{3a^{2}c\,d^{2}}{2\left(b\,x^{2}+a\right)b^{3}}+\frac{3a^{2}d^{3}\ln\left(b\,x^{2}+a\right)}{2b^{4}}+\frac{3a\,c^{2}d}{2\left(b\,x^{2}+a\right)b^{2}}-\frac{3ac\,d^{2}\ln\left(b\,x^{2}+a\right)}{b^{3}}-\frac{c^{3}}{2\left(b\,x^{2}+a\right)b}+\frac{3c^{2}d\ln\left(b\,x^{2}+a\right)}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-d^3/b^3*a*x^2+3/2*d^2/b^2*c*x^2+1/2/b^4/(b*x^2+a)*a^3*d^3-3
/2/b^3/(b*x^2+a)*a^2*d^2*c+3/2/b^2/(b*x^2+a)*a*d*c^2-1/2/b/(b*x^2+a)*c^3+3/
2/b^4*ln(b*x^2+a)*d^3*a^2-3/b^3*ln(b*x^2+a)*d^2*a*c+3/2/b^2*ln(b*x^2+a)*d*c
^2
```

maxima [A] time = 1.00, size = 124, normalized size = 1.41

$$-\frac{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}{2\left(b^{5}x^{2}+ab^{4}\right)}+\frac{bd^{3}x^{4}+2\left(3\,bcd^{2}-2\,ad^{3}\right)x^{2}}{4\,b^{3}}+\frac{3\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\log\left(bx^{2}+a\right)}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x^2 + a*b^4)
+ 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - 2*a*d^3)*x^2)/b^3 + 3/2*(b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*log(b*x^2 + a)/b^4
```

mupad [B] time = 0.18, size = 130, normalized size = 1.48

$$\frac{\ln\left(b\,x^2+a\right)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{2\,b^4}-x^2\,\left(\frac{a\,d^3}{b^3}-\frac{3\,c\,d^2}{2\,b^2}\right)+\frac{d^3\,x^4}{4\,b^2}+\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] (log(a + b*x^2)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/(2*b^4) - x^2*((a*
d^3)/b^3 - (3*c*d^2)/(2*b^2)) + (d^3*x^4)/(4*b^2) + (a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2)/(2*b*(a*b^3 + b^4*x^2))
```

sympy [A] time = 1.16, size = 112, normalized size = 1.27

$$x^2\left(-\frac{ad^3}{b^3}+\frac{3cd^2}{2b^2}\right)+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2ab^4+2b^5x^2}+\frac{d^3x^4}{4b^2}+\frac{3d\,(ad-bc)^2\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x**2*(-a*d**3/b**3 + 3*c*d**2/(2*b**2)) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*
a*b**2*c**2*d - b**3*c**3)/(2*a*b**4 + 2*b**5*x**2) + d**3*x**4/(4*b**2) +
3*d*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4)
```

3.284 $\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=106

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {390, 385, 205}

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3\left(a+bx^2\right)^2}\right)dx\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{\left(a+bx^2\right)^2}\,dx}{b^3}\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{\left((bc-ad)^2(bc+5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3}\\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 1.00

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(a + b*x^2)^2, x]
```

fricas [B] time = 0.97, size = 442, normalized size = 4.17

$$\left[\frac{4\,a^2b^3d^3x^5+4\left(9\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^3-3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\,a^3bd^3\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\left(ab^4c^3-3\,a^2b^3c^2d+9\,a^3b^2cd^2-5\,a^4bd^3\right)x}{12\left(a^2b^5x^2+a^3b^4\right)},\ \frac{2\,a^2b^3d^3x^5+2\left(9\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^3+3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\,a^3bd^3\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\left(ab^4c^3-3\,a^2b^3c^2d+9\,a^3b^2cd^2-5\,a^4bd^3\right)x}{6\left(a^2b^5x^2+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b
^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c
^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2
 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a
^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*
b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*
d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d
+ 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

giac [A] time = 0.36, size = 152, normalized size = 1.43

$$\frac{\left(b^{3} c^{3} + 3\, a b^{2} c^{2} d - 9\, a^{2} b c d^{2} + 5\, a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2\, \sqrt{a b}\, a b^{3}} + \frac{b^{3} c^{3} x - 3\, a b^{2} c^{2} d x + 3\, a^{2} b c d^{2} x - a^{3} d^{3} x}{2\, \left(b x^{2} + a\right) a b^{3}} + \frac{b^{4} d^{3} x^{3} + 9\, b^{4} c d^{2} x - 6\, a b^{3} d^{3} x}{3\, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

maple [B] time = 0.01, size = 205, normalized size = 1.93

$$\frac{d^{3} x^{3}}{3 b^{2}} - \frac{a^{2} d^{3} x}{2 \left(b\, x^{2} + a\right) b^{3}} + \frac{5 a^{2} d^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\; b^{3}} + \frac{3 a c\, d^{2} x}{2 \left(b\, x^{2} + a\right) b^{2}} - \frac{9 a c\, d^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\; b^{2}} + \frac{c^{3} x}{2 \left(b\, x^{2} + a\right) a} + \frac{c^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\; a} - \frac{3 c^{2} d x}{2 \left(b\, x^{2} + a\right) b} + \frac{3 c^{2} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}\; b} - \frac{2 a\, d^{3} x}{b^{3}} + \frac{3 c\, d^{2} x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^3*x^3/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/2/b^3*x*a^2/(b*x^2+a)*d^3+3/2
/b^2*x*a/(b*x^2+a)*c*d^2-3/2/b*x/(b*x^2+a)*c^2*d+1/2*x/a/(b*x^2+a)*c^3+5/2/
b^3*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3-9/2/b^2*a/(a*b)^(1/2)*arc
tan(1/(a*b)^(1/2)*b*x)*c*d^2+3/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^
2*d+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^3
```

maxima [A] time = 2.35, size = 147, normalized size = 1.39

$$\frac{\left(b^{3} c^{3} - 3\, a b^{2} c^{2} d + 3\, a^{2} b c d^{2} - a^{3} d^{3}\right) x}{2\, \left(a b^{4} x^{2} + a^{2} b^{3}\right)} + \frac{b d^{3} x^{3} + 3\, \left(3\, b c d^{2} - 2\, a d^{3}\right) x}{3\, b^{3}} + \frac{\left(b^{3} c^{3} + 3\, a b^{2} c^{2} d - 9\, a^{2} b c d^{2} + 5\, a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2\, \sqrt{a b}\, a b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2*
b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3*a
*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b^3)
```

mupad [B] time = 0.19, size = 182, normalized size = 1.72

$$\frac{d^3\,x^3}{3\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{\sqrt{a}\,\left(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(5\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

```
[Out] (d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/
2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^2*c
^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2))
```

sympy [B] time = 1.06, size = 314, normalized size = 2.96

$$x \left(- \frac{2 a d^{3}}{b^{3}} + \frac{3 c d^{2}}{b^{2}}\right) + \frac{x \left(- a^{3} d^{3} + 3 a^{2} b c d^{2} - 3 a b^{2} c^{2} d + b^{3} c^{3}\right)}{2 a^{2} b^{3} + 2 a b^{4} x^{2}} - \frac{\sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right) \log{\left(- \frac{a^{2} b^{3} \sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right)}{5 a^{3} d^{3} - 9 a^{2} b c d^{2} + 3 a b^{2} c^{2} d + b^{3} c^{3}} + x \right)}}{4} + \frac{\sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right) \log{\left(\frac{a^{2} b^{3} \sqrt{- \frac{1}{a^{3} b^{7}}} \left(a d - b c\right)^{2} \left(5 a d + b c\right)}{5 a^{3} d^{3} - 9 a^{2} b c d^{2} + 3 a b^{2} c^{2} d + b^{3} c^{3}} + x \right)}}{4} + \frac{d^{3} x^{3}}{3 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7
))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d -
b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b*
*3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(a**
2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a
**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b**2)
```

3.285 $\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=88

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{2a^2b^3}+\frac{c^3\log(x)}{a^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (d^3*x^2)/(2*b^2) + (b*c - a*d)^3/(2*a*b^3*(a + b*x^2)) + (c^3*Log[x])/a^2
- ((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(2*a^2*b^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(c+dx)^3}{x(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{d^3}{b^2}+\frac{c^3}{a^2x}+\frac{(-bc+ad)^3}{ab^2(a+bx)^2}-\frac{(-bc+ad)^2(bc+2ad)}{a^2b^2(a+bx)}\right)dx, x, x^2\right)\\ &= \frac{d^3x^2}{2b^2}+\frac{(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{c^3\log(x)}{a^2}-\frac{(bc-ad)^2(bc+2ad)\log\left(a+bx^2\right)}{2a^2b^3}\end{aligned}$$

Mathematica [**A**] time = 0.10, size = 111, normalized size = 1.26

$$\frac{\frac{\frac{a\left(-a^3d^3+a^2bd^2\left(3c+dx^2\right)+ab^2\left(d^3x^4-3c^2d\right)+b^3c^3\right)}{a+bx^2}-(bc-ad)^2(2ad+bc)\log\left(a+bx^2\right)}{b^3}+2c^3\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] (2*c^3*Log[x] + ((a*(b^3*c^3 - a^3*d^3 + a^2*b*d^2*(3*c + d*x^2) + a*b^2*(-
3*c^2*d + d^3*x^4)))/(a + b*x^2) - (b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^
2])/b^3)/(2*a^2)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x*(a + b*x^2)^2), x]
```

fricas [**B**] time = 0.85, size = 178, normalized size = 2.02

$$\frac{a^2b^2d^3x^4+a^3bd^3x^2+ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3-\left(ab^3c^3-3\,a^3bcd^2+2\,a^4d^3+\left(b^4c^3-3\,a^2b^2cd^2+2\,a^3bd^3\right)x^2\right)\log\left(bx^2+a\right)+2\left(b^4c^3x^2+ab^3c^3\right)\log(x)}{2\left(a^2b^4x^2+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b^2*d^3*x^4 + a^3*b*d^3*x^2 + a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*
b*c*d^2 - a^4*d^3 - (a*b^3*c^3 - 3*a^3*b*c*d^2 + 2*a^4*d^3 + (b^4*c^3 - 3*a
^2*b^2*c*d^2 + 2*a^3*b*d^3)*x^2)*log(b*x^2 + a) + 2*(b^4*c^3*x^2 + a*b^3*c^
3)*log(x))/(a^2*b^4*x^2 + a^3*b^3)
```

giac [A] time = 0.41, size = 150, normalized size = 1.70

$$\frac{d^3x^2}{2\,b^2}+\frac{c^3\log\left(x^2\right)}{2\,a^2}-\frac{\left(b^3c^3-3\,a^2bcd^2+2\,a^3d^3\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b^3}+\frac{b^4c^3x^2-3\,a^2b^2cd^2x^2+2\,a^3bd^3x^2+2\,ab^3c^3-3\,a^2b^2c^2d+a^4d^3}{2\,\left(bx^2+a\right)a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2 + 2*a
^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^3) + 1/2*(b^4*c^3*x^2 - 3*a^2*b^2*c*d^2*
x^2 + 2*a^3*b*d^3*x^2 + 2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + a^4*d^3)/((b*x^2 +
a)*a^2*b^3)
```

maple [A] time = 0.02, size = 146, normalized size = 1.66

$$\frac{d^3x^2}{2b^2}-\frac{a^2d^3}{2\left(b\,x^2+a\right)b^3}+\frac{3ac\,d^2}{2\left(b\,x^2+a\right)b^2}-\frac{a\,d^3\ln\left(b\,x^2+a\right)}{b^3}+\frac{c^3}{2\left(b\,x^2+a\right)a}+\frac{c^3\ln(x)}{a^2}-\frac{c^3\ln\left(b\,x^2+a\right)}{2a^2}-\frac{3c^2d}{2\left(b\,x^2+a\right)b}+\frac{3c\,d^2\ln\left(b\,x^2+a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2*d^3*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)*d^3+3/2*a/b^2/(b*x^2+a)*d^2*c-3/2/b/(
b*x^2+a)*d*c^2+1/2/a/(b*x^2+a)*c^3-a/b^3*ln(b*x^2+a)*d^3+3/2/b^2*ln(b*x^2+a
)*d^2*c-1/2/a^2*ln(b*x^2+a)*c^3+c^3*ln(x)/a^2
```

maxima [A] time = 0.99, size = 122, normalized size = 1.39

$$\frac{d^3x^2}{2\,b^2}+\frac{c^3\log\left(x^2\right)}{2\,a^2}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\,\left(ab^4x^2+a^2b^3\right)}-\frac{\left(b^3c^3-3\,a^2bcd^2+2\,a^3d^3\right)\log\left(bx^2+a\right)}{2\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)/(a*b^4*x^2 + a^2*b^3) - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2
+ 2*a^3*d^3)*log(b*x^2 + a)/(a^2*b^3)
```

mupad [B] time = 0.12, size = 122, normalized size = 1.39

$$\frac{d^3\,x^2}{2\,b^2}+\frac{c^3\,\ln(x)}{a^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(2\,a^3\,d^3-3\,a^2\,b\,c\,d^2+b^3\,c^3\right)}{2\,a^2\,b^3}-\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{2\,a\,b\,\left(b^3\,x^2+a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x*(a + b*x^2)^2),x)
```

```
[Out] (d^3*x^2)/(2*b^2) + (c^3*log(x))/a^2 - (log(a + b*x^2)*(2*a^3*d^3 + b^3*c^3
 - 3*a^2*b*c*d^2))/(2*a^2*b^3) - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2)/(2*a*b*(a*b^2 + b^3*x^2))
```

sympy [A] time = 2.33, size = 110, normalized size = 1.25

$$\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{2a^2b^3+2ab^4x^2}+\frac{d^3x^2}{2b^2}+\frac{c^3\log(x)}{a^2}-\frac{(ad-bc)^2(2ad+bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x/(b*x**2+a)**2,x)
```

```
[Out] (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 +
 2*a*b**4*x**2) + d**3*x**2/(2*b**2) + c**3*log(x)/a**2 - (a*d - b*c)**2*(2
*a*d + b*c)*log(a/b + x**2)/(2*a**2*b**3)
```

3.286 $\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{3(bc-ad)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx\left(a+bx^2\right)}$$

Rubi [**A**] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {468, 570, 205}

$$-\frac{3(bc-ad)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}-\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2x(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(3*b*c - a*d))/(2*a^2*b*x) - (d^2*(b*c - 3*a*d)*x)/(2*a*b^2) + ((b*c
- a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + a*d)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
```

```
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)}-\frac{\int \frac{\left(c+dx^2\right)\left(-c(3bc-ad)+d(bc-3ad)x^2\right)}{x^2\left(a+bx^2\right)}\,dx}{2ab}\\
&= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)}-\frac{\int \left(\frac{d^2(bc-3ad)}{b}+\frac{c^2(-3bc+ad)}{ax^2}+\frac{3(-bc+ad)^2(bc+ad)}{ab\left(a+bx^2\right)}\right)dx}{2ab}\\
&= -\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2(bc-3ad)x}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)}-\frac{\left(3(bc-ad)^2(bc+ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b^2}\\
&= -\frac{c^2(3bc-ad)}{2a^2bx}-\frac{d^2(bc-3ad)x}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2abx\left(a+bx^2\right)}-\frac{3(bc-ad)^2(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 94, normalized size = 0.72

$$-\frac{3(ad-bc)^2(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}+\frac{x(ad-bc)^3}{2a^2b^2\left(a+bx^2\right)}-\frac{c^3}{a^2x}+\frac{d^3x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c^3/(a^2*x)) + (d^3*x)/b^2 + ((-(b*c) + a*d)^3*x)/(2*a^2*b^2*(a + b*x^2))
 - (3*(-(b*c) + a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*
b^(5/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x^2*(a + b*x^2)^2), x]
```

fricas [A] time = 0.58, size = 412, normalized size = 3.15

$$\left[\frac{4\,a^3b^2d^3x^4 - 4\,a^2b^3c^3 - 6\left(ab^4c^3 - a^2b^3c^2d + a^3b^2cd^2 - a^4bd^3\right)x^2 - 3\left(\left(b^4c^3 - ab^3c^2d - a^2b^2cd^2 + a^3bd^3\right)x^3 + \left(ab^3c^3 - a^2b^2c^2d - a^3bcd^2 + a^4d^3\right)x\right)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^3b^4x^3 + a^4b^3x\right)}, \frac{2\,a^3b^2d^3x^4 - 2\,a^2b^3c^3 - 3\left(ab^4c^3 - a^2b^3c^2d + a^3b^2cd^2 - a^4bd^3\right)x^2 - 3\left(\left(b^4c^3 - ab^3c^2d - a^2b^2cd^2 + a^3bd^3\right)x^3 + \left(ab^3c^3 - a^2b^2c^2d - a^3bcd^2 + a^4d^3\right)x\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^3b^4x^3 + a^4b^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*d^3*x^4 - 4*a^2*b^3*c^3 - 6*(a*b^4*c^3 - a^2*b^3*c^2*d + a^
3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 +
a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*sqr
t(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b
^3*x), 1/2*(2*a^3*b^2*d^3*x^4 - 2*a^2*b^3*c^3 - 3*(a*b^4*c^3 - a^2*b^3*c^2*
d + a^3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*
d^2 + a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*
x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

giac [A] time = 0.34, size = 143, normalized size = 1.09

$$\frac{d^3x}{b^2} - \frac{3\left(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2b^2} - \frac{3\,b^3c^3x^2 - 3\,ab^2c^2dx^2 + 3\,a^2bcd^2x^2 - a^3d^3x^2 + 2\,ab^2c^3}{2\left(bx^3 + ax\right)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^3*x/b^2 - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3
*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x^3 + a*x)*a^2*b^2)
```

maple [A] time = 0.02, size = 189, normalized size = 1.44

$$\frac{a\,d^3x}{2\left(b\,x^2+a\right)b^2} - \frac{3a\,d^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b^2} + \frac{3c^2dx}{2\left(b\,x^2+a\right)a} + \frac{3c^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a} - \frac{b\,c^3x}{2\left(b\,x^2+a\right)a^2} - \frac{3b\,c^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^2} - \frac{3c\,d^2x}{2\left(b\,x^2+a\right)b} + \frac{3c\,d^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;b} + \frac{d^3x}{b^2} - \frac{c^3}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^2/(b*x^2+a)^2,x)
```

```
[Out] d^3/b^2*x+1/2*a/b^2*x/(b*x^2+a)*d^3-3/2/b*x/(b*x^2+a)*c*d^2+3/2/a*x/(b*x^2+
a)*c^2*d-1/2/a^2*b*x/(b*x^2+a)*c^3-3/2*a/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/
2)*b*x)*d^3+3/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^2+3/2/a/(a*b)^(
```

```
1/2)*arctan(1/(a*b)^(1/2)*b*x)*c^2*d-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x)*c^3-c^3/a^2/x
```

maxima [A] time = 2.32, size = 140, normalized size = 1.07

$$\frac{d^{3} x}{b^{2}} - \frac{2 \, a b^{2} c^{3} + {\left(3 \, b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right)} x^{2}}{2 \, {\left(a^{2} b^{3} x^{3} + a^{3} b^{2} x\right)}} - \frac{3 \, {\left(b^{3} c^{3} - a b^{2} c^{2} d - a^{2} b c d^{2} + a^{3} d^{3}\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, \sqrt{a b} a^{2} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] d^3*x/b^2 - 1/2*(2*a*b^2*c^3 + (3*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*x^2)/(a^2*b^3*x^3 + a^3*b^2*x) - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2
*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

mupad [B] time = 0.22, size = 173, normalized size = 1.32

$$\frac{\frac{x^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-3\,b^3\,c^3\right)}{2\,a^2}-\frac{b^2\,c^3}{a}}{b^3\,x^3+a\,b^2\,x}+\frac{d^3\,x}{b^2}-\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{b}\,x\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^2}{\sqrt{a}\,\left(3\,a^3\,d^3-3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+3\,b^3\,c^3\right)}\right)\,\left(a\,d+b\,c\right)\,{\left(a\,d-b\,c\right)}^2}{2\,a^{5/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^2*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(a^3*d^3 - 3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2) - (b^2
*c^3)/a)/(b^3*x^3 + a*b^2*x) + (d^3*x)/b^2 - (3*atan((3*b^(1/2)*x*(a*d + b*
c)*(a*d - b*c)^2)/(a^(1/2)*(3*a^3*d^3 + 3*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)))*(a*d + b*c)*(a*d - b*c)^2)/(2*a^(5/2)*b^(5/2))
```

sympy [B] time = 1.54, size = 309, normalized size = 2.36

$$\frac{3 \sqrt{- \frac{1}{a^{5} b^{5}}} \left(a d - b c\right)^{2} \left(a d + b c\right) \log{\left(- \frac{3 a^{3} b^{2} \sqrt{- \frac{1}{a^{5} b^{5}}} \left(a d - b c\right)^{2} \left(a d + b c\right)}{3 a^{3} d^{3} - 3 a^{2} b c d^{2} - 3 a b^{2} c^{2} d + 3 b^{3} c^{3}} + x \right)}}{4} - \frac{3 \sqrt{- \frac{1}{a^{5} b^{5}}} \left(a d - b c\right)^{2} \left(a d + b c\right) \log{\left(\frac{3 a^{3} b^{2} \sqrt{- \frac{1}{a^{5} b^{5}}} \left(a d - b c\right)^{2} \left(a d + b c\right)}{3 a^{3} d^{3} - 3 a^{2} b c d^{2} - 3 a b^{2} c^{2} d + 3 b^{3} c^{3}} + x \right)}}{4} + \frac{- 2 a b^{2} c^{3} + x^{2} \left(a^{3} d^{3} - 3 a^{2} b c d^{2} + 3 a b^{2} c^{2} d - 3 b^{3} c^{3}\right)}{2 a^{3} b^{2} x + 2 a^{2} b^{3} x^{3}} + \frac{d^{3} x}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**2/(b*x**2+a)**2,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)*log(-3*a**3*b**2*sqrt(-1/
(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*
a*b**2*c**2*d + 3*b**3*c**3) + x)/4 - 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2
*(a*d + b*c)*log(3*a**3*b**2*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c
)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + 3*b**3*c**3) + x)/4 +
(-2*a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 3*b
**3*c**3))/(2*a**3*b**2*x + 2*a**2*b**3*x**3) + d**3*x/b**2
```

3.287 $\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}-\frac{c^3}{2a^2x^2}$$

Rubi [A] time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}+\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{2a^3b^2}-\frac{c^2\log(x)(2bc-3ad)}{a^3}-\frac{c^3}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -c^3/(2*a^2*x^2) - (b*c - a*d)^3/(2*a^2*b^2*(a + b*x^2)) - (c^2*(2*b*c - 3*
a*d)*Log[x])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/(2*a^3*b^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{c^3}{a^2x^2}+\frac{c^2(-2bc+3ad)}{a^3x}-\frac{(-bc+ad)^3}{a^2b(a+bx)^2}+\frac{(-bc+ad)^2(2bc+ad)}{a^3b(a+bx)}\right)dx, x, x^2\right)\\ &= -\frac{c^3}{2a^2x^2}-\frac{(bc-ad)^3}{2a^2b^2\left(a+bx^2\right)}-\frac{c^2(2bc-3ad)\log(x)}{a^3}+\frac{(bc-ad)^2(2bc+ad)\log\left(a+bx^2\right)}{2a^3b^2}\end{aligned}$$

Mathematica [**A**] time = 0.10, size = 87, normalized size = 0.89

$$\frac{\frac{a(ad-bc)^3}{b^2\left(a+bx^2\right)}+\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{b^2}+2c^2\log(x)(3ad-2bc)-\frac{ac^3}{x^2}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*c^3)/x^2) + (a*(-(b*c) + a*d)^3)/(b^2*(a + b*x^2)) + 2*c^2*(-2*b*c +
3*a*d)*Log[x] + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/b^2)/(2*a^3)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x^3*(a + b*x^2)^2), x]
```

fricas [**B**] time = 0.89, size = 209, normalized size = 2.13

$$-\frac{a^2b^2c^3+\left(2\,ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)x^2-\left(\left(2\,b^4c^3-3\,ab^3c^2d+a^3bd^3\right)x^4+\left(2\,ab^3c^3-3\,a^2b^2c^2d+a^4d^3\right)x^2\right)\log\left(bx^2+a\right)+2\left(\left(2\,b^4c^3-3\,ab^3c^2d\right)x^4+\left(2\,ab^3c^3-3\,a^2b^2c^2d\right)x^2\right)\log(x)}{2\left(a^3b^3x^4+a^4b^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - 3*a^
```

```
2*b^2*c^2*d + a^4*d^3)*x^2)*log(b*x^2 + a) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d)
*x^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)*log(x))/(a^3*b^3*x^4 + a^4*b^2*
x^2)
```

giac [A] time = 0.29, size = 157, normalized size = 1.60

$$-\frac{\left(2\,bc^{3}-3\,ac^{2}d\right)\log\left(x^{2}\right)}{2\,a^{3}}+\frac{\left(2\,b^{3}c^{3}-3\,ab^{2}c^{2}d+a^{3}d^{3}\right)\log\left(\left|bx^{2}+a\right|\right)}{2\,a^{3}b^{2}}-\frac{a^{2}bd^{3}x^{4}+4\,b^{3}c^{3}x^{2}-6\,ab^{2}c^{2}dx^{2}+6\,a^{2}bcd^{2}x^{2}-a^{3}d^{3}x^{2}+2\,ab^{2}c^{3}}{4\left(bx^{4}+ax^{2}\right)a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/2*(2*b^3*c^3 - 3*a*b^2*c^2*d +
a^3*d^3)*log(abs(b*x^2 + a))/(a^3*b^2) - 1/4*(a^2*b*d^3*x^4 + 4*b^3*c^3*x^2
 - 6*a*b^2*c^2*d*x^2 + 6*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x
^4 + a*x^2)*a^2*b^2)
```

maple [A] time = 0.02, size = 156, normalized size = 1.59

$$\frac{a\,d^{3}}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{3c^{2}d}{2\left(b\,x^{2}+a\right)a}-\frac{b\,c^{3}}{2\left(b\,x^{2}+a\right)a^{2}}+\frac{3c^{2}d\ln(x)}{a^{2}}-\frac{3c^{2}d\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{2b\,c^{3}\ln(x)}{a^{3}}+\frac{b\,c^{3}\ln\left(b\,x^{2}+a\right)}{a^{3}}-\frac{3c\,d^{2}}{2\left(b\,x^{2}+a\right)b}+\frac{d^{3}\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{c^{3}}{2a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^3/(b*x^2+a)^2,x)
```

```
[Out] 1/2*a/b^2/(b*x^2+a)*d^3-3/2/b/(b*x^2+a)*d^2*c+3/2/a/(b*x^2+a)*d*c^2-1/2/a^2
*b/(b*x^2+a)*c^3+1/2/b^2*ln(b*x^2+a)*d^3-3/2/a^2*ln(b*x^2+a)*d*c^2+1/a^3*b*
ln(b*x^2+a)*c^3-1/2*c^3/a^2/x^2+3*c^2/a^2*ln(x)*d-2*c^3/a^3*ln(x)*b
```

maxima [A] time = 1.10, size = 141, normalized size = 1.44

$$-\frac{ab^{2}c^{3}+\left(2\,b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)x^{2}}{2\left(a^{2}b^{3}x^{4}+a^{3}b^{2}x^{2}\right)}-\frac{\left(2\,bc^{3}-3\,ac^{2}d\right)\log\left(x^{2}\right)}{2\,a^{3}}+\frac{\left(2\,b^{3}c^{3}-3\,ab^{2}c^{2}d+a^{3}d^{3}\right)\log\left(bx^{2}+a\right)}{2\,a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^2*c^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2
)/(a^2*b^3*x^4 + a^3*b^2*x^2) - 1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/
2*(2*b^3*c^3 - 3*a*b^2*c^2*d + a^3*d^3)*log(b*x^2 + a)/(a^3*b^2)
```

mupad [B] time = 0.27, size = 135, normalized size = 1.38

$$\frac{\ln\left(b\,x^{2}+a\right)\,\left(a^{3}\,d^{3}-3\,a\,b^{2}\,c^{2}\,d+2\,b^{3}\,c^{3}\right)}{2\,a^{3}\,b^{2}}-\frac{\ln(x)\,\left(2\,b\,c^{3}-3\,a\,c^{2}\,d\right)}{a^{3}}-\frac{\frac{c^{3}}{2\,a}-\frac{x^{2}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-2\,b^{3}\,c^{3}\right)}{2\,a^{2}\,b^{2}}}{b\,x^{4}+a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (log(a + b*x^2)*(a^3*d^3 + 2*b^3*c^3 - 3*a*b^2*c^2*d))/(2*a^3*b^2) - (log(x
)*(2*b*c^3 - 3*a*c^2*d))/a^3 - (c^3/(2*a) - (x^2*(a^3*d^3 - 2*b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2*b^2))/(a*x^2 + b*x^4)
```

sympy [A] time = 3.18, size = 128, normalized size = 1.31

$$\frac{-ab^2c^3 + x^2\left(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - 2b^3c^3\right)}{2a^3b^2x^2 + 2a^2b^3x^4} + \frac{c^2(3ad - 2bc)\log(x)}{a^3} + \frac{(ad - bc)^2(ad + 2bc)\log\left(\frac{a}{b} + x^2\right)}{2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**3/(b*x**2+a)**2,x)
```

```
[Out] (-a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 2*b**
3*c**3))/(2*a**3*b**2*x**2 + 2*a**2*b**3*x**4) + c**2*(3*a*d - 2*b*c)*log(x
)/a**3 + (a*d - b*c)**2*(a*d + 2*b*c)*log(a/b + x**2)/(2*a**3*b**2)
```

3.288 $\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{(bc-ad)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {468, 570, 205}

$$\frac{c\left(2a^2d^2-9abcd+5b^2c^2\right)}{2a^3bx}+\frac{(bc-ad)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(5*b*c - 3*a*d))/(6*a^2*b*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^
2))/(2*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^3*(a + b*x^2)) + ((b
*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{\left(c+dx^2\right)\left(-c(5bc-3ad)-d(bc+ad)x^2\right)}{x^4\left(a+bx^2\right)}\,dx}{2ab}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^3\left(a+bx^2\right)} - \frac{\int \left(\frac{c^2(-5bc+3ad)}{ax^4}+\frac{c\left(5b^2c^2-9abcd+2a^2d^2\right)}{a^2x^2}-\frac{(-bc+ad)^2(5bc+ad)}{a^2\left(a+bx^2\right)}\right)dx}{2ab}$$

$$= -\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c\left(5b^2c^2-9abcd+2a^2d^2\right)}{2a^3bx}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^3\left(a+bx^2\right)}+\frac{\left((bc-ad)^2(5bc+ad\right.}{2a^3b}$$

$$= -\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c\left(5b^2c^2-9abcd+2a^2d^2\right)}{2a^3bx}+\frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^3\left(a+bx^2\right)}+\frac{(bc-ad)^2(5bc+ad)}{2a^{7/2}b^{3/}}$$

Mathematica [A] time = 0.07, size = 109, normalized size = 0.74

$$\frac{(ad-bc)^2(ad+5bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}-\frac{c^2(3ad-2bc)}{a^3x}-\frac{x(ad-bc)^3}{2a^3b\left(a+bx^2\right)}-\frac{c^3}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c^3/(a^2*x^3) - (c^2*(-2*b*c + 3*a*d))/(a^3*x) - ((-(b*c) + a*d)^3*x)/
(2*a^3*b*(a + b*x^2)) + ((-(b*c) + a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^2)^3/(x^4*(a + b*x^2)^2), x]
```

fricas [A] time = 0.68, size = 458, normalized size = 3.12

$$\left[-\frac{4\,a^{3}b^{2}c^{3}-6\left(5\,ab^{4}c^{3}-9\,a^{2}b^{3}c^{2}d+3\,a^{3}b^{2}cd^{2}-a^{4}bd^{3}\right)x^{4}-4\left(5\,a^{2}b^{3}c^{3}-9\,a^{3}b^{2}c^{2}d\right)x^{2}+3\left(\left(5\,b^{4}c^{3}-9\,ab^{3}c^{2}d+3\,a^{2}b^{2}cd^{2}+a^{3}bd^{3}\right)x^{5}+\left(5\,ab^{3}c^{3}-9\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}+a^{4}d^{3}\right)x^{3}\right)\sqrt{-ab}\log\left(\frac{bx^{2}-2\,\sqrt{-ab}x-a}{bx^{2}+a}\right)}{12\left(a^{4}b^{3}x^{5}+a^{5}b^{2}x^{3}\right)},\ -\frac{2\,a^{3}b^{2}c^{3}-3\left(5\,ab^{4}c^{3}-9\,a^{2}b^{3}c^{2}d+3\,a^{3}b^{2}cd^{2}-a^{4}bd^{3}\right)x^{4}-2\left(5\,a^{2}b^{3}c^{3}-9\,a^{3}b^{2}c^{2}d\right)x^{2}-3\left(\left(5\,b^{4}c^{3}-9\,ab^{3}c^{2}d+3\,a^{2}b^{2}cd^{2}+a^{3}bd^{3}\right)x^{5}+\left(5\,ab^{3}c^{3}-9\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}+a^{4}d^{3}\right)x^{3}\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{6\left(a^{4}b^{3}x^{5}+a^{5}b^{2}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c^3 - 6*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
- a^4*b*d^3)*x^4 - 4*(5*a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 + 3*((5*b^4*c^3
- 9*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 + a^4*d^3)*x^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3), -1/6*(2*a^3*b^2*c^3 -
3*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x^4 - 2*(5*
a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 - 3*((5*b^4*c^3 - 9*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
 + a^4*d^3)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^
3)]
```

giac [A] time = 0.30, size = 150, normalized size = 1.02

$$\frac{\left(5\,b^{3}c^{3}-9\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}b}+\frac{b^{3}c^{3}x-3\,ab^{2}c^{2}dx+3\,a^{2}bcd^{2}x-a^{3}d^{3}x}{2\left(bx^{2}+a\right)a^{3}b}+\frac{6\,bc^{3}x^{2}-9\,ac^{2}dx^{2}-ac^{3}}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^3*b) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a^3*b) + 1/3*(6*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^
3)/(a^3*x^3)
```

maple [A] time = 0.02, size = 209, normalized size = 1.42

$$\frac{3c\,d^{2}x}{2\left(b\,x^{2}+a\right)a}+\frac{3c\,d^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{3b\,c^{2}dx}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{9b\,c^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{2}}+\frac{b^{2}c^{3}x}{2\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{2}c^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{3}}-\frac{d^{3}x}{2\left(b\,x^{2}+a\right)b}+\frac{d^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,b}-\frac{3c^{2}d}{a^{2}x}+\frac{2b\,c^{3}}{a^{3}x}-\frac{c^{3}}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^4/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x/(b*x^2+a)*d^3+3/2/a*x/(b*x^2+a)*c*d^2-3/2/a^2*b*x/(b*x^2+a)*c^2*d+
1/2/a^3*b^2*x/(b*x^2+a)*c^3+1/2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d^3
+3/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c*d^2-9/2/a^2*b/(a*b)^(1/2)*ar
```

```
ctan(1/(a*b)^(1/2)*b*x)*c^2*d+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x)*c^3-1/3*c^3/a^2/x^3-3*c^2/a^2/x*d+2*c^3/a^3/x*b
```

maxima [A] time = 2.40, size = 159, normalized size = 1.08

$$-\frac{2\,a^2bc^3 - 3\left(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)x^4 - 2\left(5\,ab^2c^3 - 9\,a^2bc^2d\right)x^2}{6\left(a^3b^2x^5 + a^4bx^3\right)} + \frac{\left(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*b*c^3 - 3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*x^4 - 2*(5*a*b^2*c^3 - 9*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2
*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^3*b)
```

mupad [B] time = 0.26, size = 183, normalized size = 1.24

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(a\,d+5\,b\,c\right)}{\sqrt{a}\,\left(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(a\,d+5\,b\,c\right)}{2\,a^{7/2}\,b^{3/2}}-\frac{\frac{c^3}{3\,a}+\frac{x^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}{2\,a^3\,b}+\frac{c^2\,x^2\,\left(9\,a\,d-5\,b\,c\right)}{3\,a^2}}{b\,x^5+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(a^(1/2)*(a^3*d^3 + 5*b^3*c^3
 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*a^(7/2)
*b^(3/2)) - (c^3/(3*a) + (x^4*(a^3*d^3 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*
b*c*d^2))/(2*a^3*b) + (c^2*x^2*(9*a*d - 5*b*c))/(3*a^2))/(a*x^3 + b*x^5)
```

sympy [B] time = 1.89, size = 321, normalized size = 2.18

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(-\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{-2a^2bc^3+x^4\left(-3a^3d^3+9a^2bcd^2-27ab^2c^2d+15b^3c^3\right)+x^2\left(-18a^2bc^2d+10ab^2c^3\right)}{6a^4bx^3+6a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-a**4*b*sqrt(-1/(a**
7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b*
*2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d
+ 5*b*c)*log(a**4*b*sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3
*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 + (-2*a**2*
b*c**3 + x**4*(-3*a**3*d**3 + 9*a**2*b*c*d**2 - 27*a*b**2*c**2*d + 15*b**3*
c**3) + x**2*(-18*a**2*b*c**2*d + 10*a*b**2*c**3))/(6*a**4*b*x**3 + 6*a**3*
b**2*x**5)
```

3.289 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{a}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {470, 522, 205}

$$-\frac{\sqrt{a}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d)^2) + (c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[d]*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{ax}{2b(bc-ad)\left(a+bx^2\right)}-\frac{\int \frac{ac+(-2bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2b(bc-ad)}\\ &= \frac{ax}{2b(bc-ad)\left(a+bx^2\right)}+\frac{c^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2}-\frac{(a(3bc-ad))\int \frac{1}{a+bx^2}\,dx}{2b(bc-ad)^2}\\ &= \frac{ax}{2b(bc-ad)\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.15, size = 95, normalized size = 0.87

$$\frac{\frac{\sqrt{a}\,(ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{ax(bc-ad)}{b\left(a+bx^2\right)}+\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] ((a*(b*c - a*d)*x)/(b*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(3/2) + (2*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])/(
2*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 1.10, size = 726, normalized size = 6.66

$$\left[-\frac{\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\left(b^2cx^2+abc\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\left(abc-a^2d\right)x}{4\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)},\ -\frac{\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-\left(b^2cx^2+abc\right)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-\left(abc-a^2d\right)x}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)},\ \frac{4\left(b^2cx^2+abc\right)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\left(abc-a^2d\right)x}{4\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)},\ -\frac{\left(3\,abc-a^2d+\left(3\,b^2c-abd\right)x^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\left(b^2cx^2+abc\right)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-\left(abc-a^2d\right)x}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*
x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(a*b*c - a^2*d)*x)/(a*b^3*c^2
 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), -
1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/
b)/a) - (b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(
d*x^2 + c)) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (
b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*(4*(b^2*c*x^2 + a*b*c)*sqrt(
c/d)*arctan(d*x*sqrt(c/d)/c) - (3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sq
rt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*(a*b*c - a^2*d
)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*x^2), -1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*ar
ctan(b*x*sqrt(a/b)/a) - 2*(b^2*c*x^2 + a*b*c)*sqrt(c/d)*arctan(d*x*sqrt(c/d
)/c) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2)]
```

giac [A] time = 0.30, size = 122, normalized size = 1.12

$$\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}-\frac{\left(3\,abc-a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{ab}}+\frac{ax}{2\left(b^2c-abd\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) - 1/2
*(3*a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^
2)*sqrt(a*b)) + 1/2*a*x/((b^2*c - a*b*d)*(b*x^2 + a))
```

maple [A] time = 0.01, size = 144, normalized size = 1.32

$$-\frac{a^2dx}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)b}+\frac{a^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^2\sqrt{ab}\;b}+\frac{acx}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)}-\frac{3ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^2\sqrt{ab}}+\frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad-bc\right)^2\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*a^2/(a*d-b*c)^2/b*x/(b*x^2+a)*d+1/2*a/(a*d-b*c)^2*x/(b*x^2+a)*c+1/2*a^
2/(a*d-b*c)^2/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-3/2*a/(a*d-b*c)^2/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(1
/(c*d)^(1/2)*d*x)
```

maxima [A] time = 2.43, size = 133, normalized size = 1.22

$$\frac{c^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{cd}} + \frac{ax}{2\left(ab^2c - a^2bd + \left(b^3c - ab^2d\right)x^2\right)} - \frac{\left(3\,abc - a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*a*x/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2) - 1/2*(3*a*b*c - a^2*d)*ar
ctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(a*b))
```

mupad [B] time = 1.06, size = 3558, normalized size = 32.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^
4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3
*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) - (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 16*b
^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*
a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2
*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) - (x*(a^4*d^5
+ 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^2*b*d^2
 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2) -
((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 +
 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^
2*c*d^2 - 3*a*b^3*c^2*d)) + (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 16*b^8*c^5*
d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5
*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*
a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + (x*(a^4*d^5 + 4*b^4
*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^2*b*d^2 - 2*a*
b^2*c*d)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(((a^3*c
^2*d^3)/2 - (5*a^2*b*c^3*d^2)/2 + 3*a*b^2*c^4*d)/(b^4*c^3 - a^3*b*d^3 + 3*a
^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + ((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5
*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2
*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) - (x*(-c^3*d)^(1/
```

```
2)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 +
 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^
2*c*d)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a
*b*c*d^2)) - (x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4)
)/(4*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2))/(a^2*d^3 + b^2*c
^2*d - 2*a*b*c*d^2) + ((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6
 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3
- a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) + (x*(-c^3*d)^(1/2)*(16*a^5
*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^
6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^
2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))
 + (x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*
c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a
*b*c*d^2)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2) - (atan(
((((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(2*(b^3*
c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) - ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5
*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(b
^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) - (x*(-a*b^3)^(1/2)*(
a*d - 3*b*c)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b
^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 + a^2*b^3*
d^2 - 2*a*b^4*c*d)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*b*c))/(4*
(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*c)*1i)/(4
*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)) + (((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a
^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) +
((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 1
2*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d
^2 - 3*a*b^3*c^2*d) + (x*(-a*b^3)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7 + 16*
b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32
*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)*(b^3*c^2 + a^2*
b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c
*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*c)*1i)/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*
c*d)))/(((a^3*c^2*d^3)/2 - (5*a^2*b*c^3*d^2)/2 + 3*a*b^2*c^4*d)/(b^4*c^3 -
a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) - (((x*(a^4*d^5 + 4*b^4*c^4*d
+ 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d
)) - ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^
3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^
2*c*d^2 - 3*a*b^3*c^2*d) - (x*(-a*b^3)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7
+ 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4
 + 32*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)*(b^3*c^2 +
 a^2*b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*
b^4*c*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^
4*c*d)) + (((x*(a^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))
/(2*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)) + ((-a*b^3)^(1/2)*((2*a*b^6*c^5*d^
2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^
2*d^5)/(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + (x*(-a*b^3
```

```
)^(1/2)*(a*d - 3*b*c)*(16*a^5*b^3*d^7 + 16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 -
 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 + 32*a^3*b^5*c^2*d^5))/(8*(b^5*c^2 +
 a^2*b^3*d^2 - 2*a*b^4*c*d)*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)))*(a*d - 3*
b*c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)))*(-a*b^3)^(1/2)*(a*d - 3*b*
c))/(4*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d))))*(-a*b^3)^(1/2)*(a*d - 3*b*c
)*1i)/(2*(b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d)) - (a*x)/(2*b*(a + b*x^2)*(a
*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.290 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=74

$$\frac{a}{2b\left(a+bx^2\right)(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a}{2b\left(a+bx^2\right)(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2}+\frac{bc}{(bc-ad)^2(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)}\right)dx, x,\right.\\ &= \frac{a}{2b(bc-ad)\left(a+bx^2\right)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.00

$$\frac{a}{2b\left(a+bx^2\right)(bc-ad)}+\frac{c\log\left(a+bx^2\right)}{2(bc-ad)^2}-\frac{c\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (c
*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 0.82, size = 117, normalized size = 1.58

$$\frac{abc-a^2d+\left(b^2cx^2+abc\right)\log\left(bx^2+a\right)-\left(b^2cx^2+abc\right)\log\left(dx^2+c\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*c - a^2*d + (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) - (b^2*c*x^2 + a*b*
c)*log(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*
b^3*c*d + a^2*b^2*d^2)*x^2)
```

giac [A] time = 0.38, size = 92, normalized size = 1.24

$$-\frac{\frac{b^2c\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{ab}{\left(b^2c-abd\right)\left(bx^2+a\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2) - a*b/((b^2*c - a*b*d)*(b*x^2 + a)))/b
```

maple [A] time = 0.01, size = 95, normalized size = 1.28

$$-\frac{a^2d}{2\,(ad-bc)^2\left(b\,x^2+a\right)b}+\frac{ac}{2\,(ad-bc)^2\left(b\,x^2+a\right)}+\frac{c\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^2}-\frac{c\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2/(a*d-b*c)^2*a^2/b/(b*x^2+a)*d+1/2/(a*d-b*c)^2*a/(b*x^2+a)*c+1/2/(a*d-b
*c)^2*c*ln(b*x^2+a)-1/2*c/(a*d-b*c)^2*ln(d*x^2+c)
```

maxima [A] time = 1.08, size = 105, normalized size = 1.42

$$\frac{c\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{c\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{a}{2\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*log(d*x^2 + c)
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a/(a*b^2*c - a^2*b*d + (b^3*c - a*b^
2*d)*x^2)
```

mupad [B] time = 0.31, size = 172, normalized size = 2.32

$$\frac{a\,\left(b\,c+b\,c\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)-a^2\,d+b^2\,c\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^3\,b\,d^2-4\,a^2\,b^2\,c\,d+2\,a^2\,b^2\,d^2\,x^2+2\,a\,b^3\,c^2-4\,a\,b^3\,c\,d\,x^2+2\,b^4\,c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (a*(b*c + b*c*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2
i) - a^2*d + b^2*c*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*
c*x^2))*2i)/(2*a*b^3*c^2 + 2*a^3*b*d^2 + 2*b^4*c^2*x^2 + 2*a^2*b^2*d^2*x^2
- 4*a^2*b^2*c*d - 4*a*b^3*c*d*x^2)
```

sympy **[B]** time = 1.91, size = 253, normalized size = 3.42

$$-\frac{a}{2a^2bd-2ab^2c+x^2\left(2ab^2d-2b^3c\right)}-\frac{c\log\left(x^2+\frac{-\frac{a^3cd^3}{(ad-bc)^2}+\frac{3a^2bc^2d^2}{(ad-bc)^2}-\frac{3ab^2c^3d}{(ad-bc)^2}+acd+\frac{b^3c^4}{(ad-bc)^2}+bc^2}{2bcd}\right)}{2\left(ad-bc\right)^2}+\frac{c\log\left(x^2+\frac{\frac{a^3cd^3}{(ad-bc)^2}-\frac{3a^2bc^2d^2}{(ad-bc)^2}+\frac{3ab^2c^3d}{(ad-bc)^2}+acd-\frac{b^3c^4}{(ad-bc)^2}+bc^2}{2bcd}\right)}{2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -a/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - c*log(x**2 +
(-a**3*c*d**3/(a*d - b*c)**2 + 3*a**2*b*c**2*d**2/(a*d - b*c)**2 - 3*a*b**2
*c**3*d/(a*d - b*c)**2 + a*c*d + b**3*c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*
d))/(2*(a*d - b*c)**2) + c*log(x**2 + (a**3*c*d**3/(a*d - b*c)**2 - 3*a**2*
b*c**2*d**2/(a*d - b*c)**2 + 3*a*b**2*c**3*d/(a*d - b*c)**2 + a*c*d - b**3*
c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*d))/(2*(a*d - b*c)**2)
```

3.291 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=104

$$-\frac{x}{2\left(a+bx^2\right)(bc-ad)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(bc-ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {471, 522, 205}

$$-\frac{x}{2\left(a+bx^2\right)(bc-ad)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(bc-ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -x/(2*(b*c - a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*Sqrt[a]*Sqrt[b]*(b*c - a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(b*c - a*d)^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= -\frac{x}{2(bc-ad)\left(a+bx^2\right)}+\frac{\int \frac{c-dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2(bc-ad)}\\ &= -\frac{x}{2(bc-ad)\left(a+bx^2\right)}-\frac{(cd)\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2}+\frac{(bc+ad)\int \frac{1}{a+bx^2}\,dx}{2(bc-ad)^2}\\ &= -\frac{x}{2(bc-ad)\left(a+bx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(bc-ad)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 104, normalized size = 1.00

$$\frac{x}{2\left(a+bx^2\right)(ad-bc)}+\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,(ad-bc)^2}-\frac{\sqrt{c}\,\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] x/(2*(-(b*c) + a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[a]*Sqrt[b]*(-(b*c) + a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(b*c - a*d)^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 1.02, size = 704, normalized size = 6.77

$$\left[-\frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\left(ab^2x^2+a^2b\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)+2\left(ab^2c-a^2bd\right)x}{4\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2\right)}, \frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(ab^2x^2+a^2b\right)\sqrt{-cd}\,\log\left(\frac{dx^2-2\sqrt{-cd}\,x-c}{dx^2+c}\right)-\left(ab^2c-a^2bd\right)x}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2\right)}, -\frac{4\left(ab^2x^2+a^2b\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)+\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(ab^2c-a^2bd\right)x}{4\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2\right)}, \frac{\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-2\left(ab^2x^2+a^2b\right)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{c}\right)-\left(ab^2c-a^2bd\right)x}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2+\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 -
2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 -
2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2),
 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
 + (a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 +
 c)) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a
*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2), -1/4*(4*(a*b^2*x^2 + a^2*b)*s
qrt(c*d)*arctan(sqrt(c*d)*x/c) + (a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(a*b^2*c - a^2*b*d
)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d
+ a^3*b^2*d^2)*x^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) - 2*(a*b^2*x^2 + a^2*b)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2)]
```

giac [A] time = 0.43, size = 110, normalized size = 1.06

$$-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{x}{2\left(bx^2+a\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*x/((b*x^2 + a)*(b*c - a*d))
```

maple [A] time = 0.01, size = 134, normalized size = 1.29

$$\frac{adx}{2\,(ad-bc)^2\left(b\,x^2+a\right)}+\frac{ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ad-bc)^2\sqrt{ab}}-\frac{bcx}{2\,(ad-bc)^2\left(b\,x^2+a\right)}+\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ad-bc)^2\sqrt{ab}}-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^2\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2/(a*d-b*c)^2*x/(b*x^2+a)*a*d-1/2/(a*d-b*c)^2*x/(b*x^2+a)*b*c+1/2/(a*d-b*
c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d+1/2/(a*d-b*c)^2/(a*b)^(1/2)*
```

```
arctan(1/(a*b)^(1/2)*b*x)*b*c-c*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(1/(c*d)^(1
/2)*d*x)
```

maxima [A] time = 2.40, size = 119, normalized size = 1.14

$$-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ab}}-\frac{x}{2\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*x/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

mupad [B] time = 0.89, size = 3153, normalized size = 30.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] x/(2*(a + b*x^2)*(a*d - b*c)) + (atan((((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6
*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b
^3*c^2*d^5)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-
c*d)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3
*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x*(a^2*b*d^5 + 5*b^3
*c^2*d^3 + 2*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d) - ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5*d^2 - 8
*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/
(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-c*d)^(1/2)*(
16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*
a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2
)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2
*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2
- 2*a*b*c*d))/((((b^2*c^2*d^3)/2 + (a*b*c*d^4)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2) + ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5*d^2 -
8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)
/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-c*d)^(1/2)*
(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32
*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^
2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x*(a^2*b*d^5 + 5*b^3*c^2*d^3 +
2*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 -
```

```
2*a*b*c*d) + ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3
 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(2*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-c*d)^(1/2)*(16*a^5*b^2*d^7
 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^
4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)) + (x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))
/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(-
c*d)^(1/2)*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - (atan((((-a*b)^(1/2)*((x*(
a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) - ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 +
12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2) - (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*c^5
*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^
4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^2 + a^3*b*d^2 - 2*a
^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*(a*
d + b*c)*1i)/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)) + ((-a*b)^(1/2)*((
x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)) + ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6
 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2) + (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b^2*d^7 + 16*b^7*
c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3
*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^2 + a^3*b*d^2 -
2*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*
(a*d + b*c)*1i)/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))/(((b^2*c^2*d^3
)/2 + (a*b*c*d^4)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) -
((-a*b)^(1/2)*((x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) - ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 +
 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b
^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*
c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^
2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2
*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d)) +
((-a*b)^(1/2)*((x*(a^2*b*d^5 + 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) + ((-a*b)^(1/2)*((2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 +
 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*c^2*d^5)/(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (x*(-a*b)^(1/2)*(a*d + b*c)*(16*a^5*b
^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*
c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a*b^3*c^
2 + a^3*b*d^2 - 2*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2
*a^2*b^2*c*d)))*(a*d + b*c))/(4*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d))))*
(-a*b)^(1/2)*(a*d + b*c)*1i)/(2*(a*b^3*c^2 + a^3*b*d^2 - 2*a^2*b^2*c*d))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.292 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 44}

$$-\frac{1}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x^2)) - (d*Log[a + b*x^2])/(2*(b*c - a*d)^2) + (d*
Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx, x, x\\
&= -\frac{1}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.03, size = 66, normalized size = 0.94

$$\frac{d\left(a+bx^2\right)\log\left(c+dx^2\right)-d\left(a+bx^2\right)\log\left(a+bx^2\right)+ad-bc}{2\left(a+bx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x^2)*Log[a + b*x^2] + d*(a + b*x^2)*Log[c + d*x^2]
)/(2*(b*c - a*d)^2*(a + b*x^2))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [**A**] time = 0.57, size = 103, normalized size = 1.47

$$-\frac{bc-ad+\left(bdx^2+ad\right)\log\left(bx^2+a\right)-\left(bdx^2+ad\right)\log\left(dx^2+c\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b*c - a*d + (b*d*x^2 + a*d)*log(b*x^2 + a) - (b*d*x^2 + a*d)*log(d*x^
2 + c))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b
*d^2)*x^2)
```

giac [A] time = 0.31, size = 85, normalized size = 1.21

$$\frac{bd\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}-\frac{b}{2\left(b^2c-abd\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*b*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*b^2*
c*d + a^2*b*d^2) - 1/2*b/((b^2*c - a*b*d)*(b*x^2 + a))
```

maple [A] time = 0.01, size = 90, normalized size = 1.29

$$\frac{ad}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)}-\frac{bc}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)}-\frac{d\ln\left(b\,x^2+a\right)}{2\left(ad-bc\right)^2}+\frac{d\ln\left(d\,x^2+c\right)}{2\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2/(a*d-b*c)^2/(b*x^2+a)*a*d-1/2*b/(a*d-b*c)^2/(b*x^2+a)*c-1/2/(a*d-b*c)^2
*ln(b*x^2+a)*d+1/2*d/(a*d-b*c)^2*ln(d*x^2+c)
```

maxima [A] time = 1.09, size = 99, normalized size = 1.41

$$-\frac{d\log\left(bx^2+a\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{d\log\left(dx^2+c\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{1}{2\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*d*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^
2)
```

mupad [B] time = 0.32, size = 161, normalized size = 2.30

$$-\frac{b\,c-a\,\left(d-d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}\right)+b\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1{}\mathrm{i}-b\,c\,x^2\,1{}\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2{}\mathrm{i}}{2\,a^3\,d^2-4\,a^2\,b\,c\,d+2\,a^2\,b\,d^2\,x^2+2\,a\,b^2\,c^2-4\,a\,b^2\,c\,d\,x^2+2\,b^3\,c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] -(b*c - a*(d - d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2)
)*2i) + b*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))
*2i)/(2*a^3*d^2 + 2*a*b^2*c^2 + 2*b^3*c^2*x^2 + 2*a^2*b*d^2*x^2 - 4*a^2*b*c
*d - 4*a*b^2*c*d*x^2)
```

sympy [B] time = 1.81, size = 248, normalized size = 3.54

$$\frac{d\log\left(x^2+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\,(ad-bc)^2}-\frac{d\log\left(x^2+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2\,(ad-bc)^2}+\frac{1}{2a^2d-2abc+x^2\left(2abd-2b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d*log(x**2 + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 -
3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b
*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2) - d*log(x**2 + (a**3*d**4/(a*d - b*c)*
*2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a
*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2)
+ 1/(2*a**2*d - 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c))
```

3.293 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Rubi [**A**] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 205}

$$\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}-\frac{\int \frac{-bc+2ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}\\
&= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2}+\frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2}\\
&= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\sqrt{b}\,(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 109, normalized size = 1.01

$$
-\frac{\sqrt{b}\,(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(ad-bc)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^2}-\frac{bx}{2a\left(a+bx^2\right)(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 0.92, size = 699, normalized size = 6.47

$$
\left[-\frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\,\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(b^2c-abd\right)x}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)},\ \frac{4\left(abdx^2+a^2d\right)\sqrt{\frac{d}{c}}\,\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+2\left(b^2c-abd\right)x}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)},\ \frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\,\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+\left(b^2c-abd\right)x}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)},\ \frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\left(abdx^2+a^2d\right)\sqrt{\frac{d}{c}}\,\arctan\left(x\sqrt{\frac{d}{c}}\right)+\left(b^2c-abd\right)x}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^
2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1
/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d
+ (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c -
3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*
x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*
b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)]
```

giac [A] time = 0.38, size = 121, normalized size = 1.12

$$\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{cd}}+\frac{\left(b^2c-3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{ab}}+\frac{bx}{2\left(abc-a^2d\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

maple [A] time = 0.01, size = 144, normalized size = 1.33

$$\frac{b^2cx}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)a}+\frac{b^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^2\sqrt{ab}\;a}-\frac{bdx}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)}-\frac{3bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^2\sqrt{ab}}+\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad-bc\right)^2\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*b/(a*d-b*c)^2*x/(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^2*x/a/(b*x^2+a)*c-3/2*b/
(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+1/2*b^2/(a*d-b*c)^2/a/(
```

```
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(1
/(c*d)^(1/2)*d*x)
```

maxima [A] time = 2.42, size = 132, normalized size = 1.22

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{cd}} + \frac{bx}{2\left(a^2bc - a^3d + \left(ab^2c - a^2bd\right)x^2\right)} + \frac{\left(b^2c - 3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*ar
ctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
```

mupad [B] time = 1.01, size = 3649, normalized size = 33.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*
a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7
- 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*
d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c
^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)
^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^
2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4
*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-
a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*
c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d
)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
)*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b^3*d^5)/2 - (b^4*c
*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - ((-a^3
*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))
/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7 - 2*a*b^7*c^5
*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*
```

```
b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - (x
*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b
^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))
/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d))))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)^(1/2)*(3*a*d -
b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c
*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-a^3*b)^(1/2)*(3*
a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3
*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*
b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/
2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))))/(4*(a^5*d^2 +
 a^3*b^2*c^2 - 2*a^4*b*c*d))))*(-a^3*b)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^5*d^2
 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - (atan((((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 -
 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d
^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2
*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5
))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^
2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2
*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d) - ((-c*d^3)^(1/2)*((((4*a^
6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^
3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2
 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) +
 (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))/(((3*a*b^3*d^5
)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^
6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(1
6*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3
+ 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3
 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c
*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c*d^2 - 2
*a*b*c^2*d) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*
b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^
(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*
```

```
c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*
(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c
*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2)*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d
) - (b*x)/(2*a*(a + b*x^2)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.294 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=99

$$-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}+\frac{b}{2a\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}+\frac{b}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] b/(2*a*(b*c - a*d)*(a + b*x^2)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^2])/(2*a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(2*c*(b*c - a*d)^2)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\Bigg(\int \Bigg(\frac{1}{a^2cx}+\frac{b^2}{a(-bc+ad)(a+bx)^2}+\frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c(bc-ad)^2}\\
&= \frac{b}{2a(bc-ad)\left(a+bx^2\right)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^2}-\frac{d^2\log\left(c+dx^2\right)}{2c(bc-ad)^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.12, size = 97, normalized size = 0.98

$$
\frac{2\log(x)-\frac{a\left(ad^2\left(a+bx^2\right)\log\left(c+dx^2\right)+bc(ad-bc)\right)+bc\left(a+bx^2\right)(bc-2ad)\log\left(a+bx^2\right)}{\left(a+bx^2\right)(bc-ad)^2}}{2a^2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (2*Log[x] - (b*c*(b*c - 2*a*d)*(a + b*x^2)*Log[a + b*x^2] + a*(b*c*(-(b*c)
+ a*d) + a*d^2*(a + b*x^2)*Log[c + d*x^2]))/((b*c - a*d)^2*(a + b*x^2)))/(2
*a^2*c)
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [**B**] time = 2.62, size = 218, normalized size = 2.20

$$
\frac{ab^2c^2-a^2bcd-\left(ab^2c^2-2\,a^2bcd+\left(b^3c^2-2\,ab^2cd\right)x^2\right)\log\left(bx^2+a\right)-\left(a^2bd^2x^2+a^3d^2\right)\log\left(dx^2+c\right)+2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^2\right)\log(x)}{2\left(a^3b^2c^3-2\,a^4bc^2d+a^5cd^2+\left(a^2b^3c^3-2\,a^3b^2c^2d+a^4bcd^2\right)x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^2*c^2 - a^2*b*c*d - (a*b^2*c^2 - 2*a^2*b*c*d + (b^3*c^2 - 2*a*b^2*
c*d)*x^2)*log(b*x^2 + a) - (a^2*b*d^2*x^2 + a^3*d^2)*log(d*x^2 + c) + 2*(a*
b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*
log(x))/(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2 + (a^2*b^3*c^3 - 2*a^3*b^2
*c^2*d + a^4*b*c*d^2)*x^2)
```

giac [A] time = 0.30, size = 183, normalized size = 1.85

$$-\frac{d^3\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)}-\frac{\left(b^3c-2\,ab^2d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}+\frac{b^3cx^2-2\,ab^2dx^2+2\,ab^2c-3\,a^2bd}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\left(bx^2+a\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3) - 1/2*
(b^3*c - 2*a*b^2*d)*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2) + 1/2*(b^3*c*x^2 - 2*a*b^2*d*x^2 + 2*a*b^2*c - 3*a^2*b*d)/((a^2*b^2*
c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)) + 1/2*log(x^2)/(a^2*c)
```

maple [A] time = 0.02, size = 139, normalized size = 1.40

$$\frac{b^2c}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)a}+\frac{bd\ln\left(b\,x^2+a\right)}{\left(ad-bc\right)^2a}-\frac{b^2c\ln\left(b\,x^2+a\right)}{2\left(ad-bc\right)^2a^2}-\frac{bd}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)}-\frac{d^2\ln\left(d\,x^2+c\right)}{2\left(ad-bc\right)^2c}+\frac{\ln(x)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*b/(a*d-b*c)^2/(b*x^2+a)*d+1/2*b^2/a/(a*d-b*c)^2/(b*x^2+a)*c+b/a/(a*d-b
*c)^2*ln(b*x^2+a)*d-1/2*b^2/a^2/(a*d-b*c)^2*ln(b*x^2+a)*c-1/2*d^2/c/(a*d-b*
c)^2*ln(d*x^2+c)+ln(x)/a^2/c
```

maxima [A] time = 1.12, size = 137, normalized size = 1.38

$$-\frac{d^2\log\left(dx^2+c\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}-\frac{\left(b^2c-2\,abd\right)\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}+\frac{b}{2\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d^2*log(d*x^2 + c)/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - 1/2*(b^2*c -
2*a*b*d)*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/2*b/(a^2*
b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*log(x^2)/(a^2*c)
```

mupad **[B]** time = 0.73, size = 127, normalized size = 1.28

$$\frac{\ln(x)}{a^2\,c}-\frac{d^2\,\ln\left(d\,x^2+c\right)}{2\,a^2\,c\,d^2-4\,a\,b\,c^2\,d+2\,b^2\,c^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(b^2\,c-2\,a\,b\,d\right)}{2\,a^4\,d^2-4\,a^3\,b\,c\,d+2\,a^2\,b^2\,c^2}-\frac{b}{2\,a\,\left(b\,x^2+a\right)\,\left(a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] log(x)/(a^2*c) - (d^2*log(c + d*x^2))/(2*b^2*c^3 + 2*a^2*c*d^2 - 4*a*b*c^2*
d) - (log(a + b*x^2)*(b^2*c - 2*a*b*d))/(2*a^4*d^2 + 2*a^2*b^2*c^2 - 4*a^3*
b*c*d) - b/(2*a*(a + b*x^2)*(a*d - b*c))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.295 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.21, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$-\frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{3bc-2ad}{2a^2cx(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(3*b*c - 2*a*d)/(2*a^2*c*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x*(a + b*x^2)
) - (b^(3/2)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c -
 a*d)^2) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{\int \frac{-3bc+2ad-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} + \frac{\int \frac{-3b^2c^2+2abcd+2a^2d^2-bd(3bc-2ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a^2c(bc-ad)}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{d^3\int \frac{1}{c+dx^2}\,dx}{c(bc-ad)^2} - \frac{\left(b^2(3bc-5ad)\right)\int}{2a^2(bc-ad)^2}$$

$$= -\frac{3bc-2ad}{2a^2c(bc-ad)x} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)} - \frac{b^{3/2}(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2} - \frac{d^{5/2}}{c^{3/2}}$$

Mathematica [A] time = 0.18, size = 123, normalized size = 0.85

$$\frac{b^{3/2}(5ad-3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(ad-bc)^2} + \frac{b^2x}{2a^2\left(a+bx^2\right)(ad-bc)} - \frac{1}{a^2cx} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(1/(a^2*c*x)) + (b^2*x)/(2*a^2*(-(b*c) + a*d)*(a + b*x^2)) + (b^(3/2)*(-3*
b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(-(b*c) + a*d)^2) - (d
^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 1.63, size = 1003, normalized size = 6.97

$$\left[-\frac{4\,ab^2c^2 - 8\,a^2bcd + 4\,a^3d^2 + 2\left(3\,b^3c^2 - 5\,ab^2cd + 2\,a^2bd^2\right)x^2 + \left(\left(3\,b^3c^2 - 5\,ab^2cd\right)x^3 + \left(3\,ab^2c^2 - 5\,a^2bcd\right)x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right) - 2\left(a^2bd^2x^3 + a^3d^2x\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 - 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right)}{4\left(\left(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2\right)x^3 + \left(a^3b^2c^3 - 2\,a^4bc^2d + a^5cd^2\right)x\right)}, -\frac{4\,ab^2c^2 - 8\,a^2bcd + 4\,a^3d^2 + 2\left(3\,b^3c^2 - 5\,ab^2cd + 2\,a^2bd^2\right)x^2 + 4\left(a^2bd^2x^3 + a^3d^2x\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right) + \left(\left(3\,b^3c^2 - 5\,ab^2cd\right)x^3 + \left(3\,ab^2c^2 - 5\,a^2bcd\right)x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 + 2\,ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{4\left(\left(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2\right)x^3 + \left(a^3b^2c^3 - 2\,a^4bc^2d + a^5cd^2\right)x\right)}, -\frac{2\,ab^2c^2 - 4\,a^2bcd + 2\,a^3d^2 + \left(3\,b^3c^2 - 5\,ab^2cd + 2\,a^2bd^2\right)x^2 + \left(\left(3\,b^3c^2 - 5\,ab^2cd\right)x^3 + \left(3\,ab^2c^2 - 5\,a^2bcd\right)x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) - \left(a^2bd^2x^3 + a^3d^2x\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2 - 2\,cx\sqrt{-\frac{d}{c}} - c}{dx^2 + c}\right)}{2\left(\left(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2\right)x^3 + \left(a^3b^2c^3 - 2\,a^4bc^2d + a^5cd^2\right)x\right)}, -\frac{2\,ab^2c^2 - 4\,a^2bcd + 2\,a^3d^2 + \left(3\,b^3c^2 - 5\,ab^2cd + 2\,a^2bd^2\right)x^2 + \left(\left(3\,b^3c^2 - 5\,ab^2cd\right)x^3 + \left(3\,ab^2c^2 - 5\,a^2bcd\right)x\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\left(a^2bd^2x^3 + a^3d^2x\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{2\left(\left(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2\right)x^3 + \left(a^3b^2c^3 - 2\,a^4bc^2d + a^5cd^2\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d +
 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b
*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a^
2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x
^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3
 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d
^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + 4*(a^2*b*d^2*x^3 + a^3
*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3
*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2
*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a
^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^
2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 (a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2
*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a
^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^
2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) +
 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^3*c^3
 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*
c*d^2)*x)]
```

giac [A] time = 0.37, size = 164, normalized size = 1.14

$$-\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\sqrt{cd}} - \frac{\left(3\,b^3c - 5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2\right)\sqrt{ab}} - \frac{3\,b^2cx^2 - 2\,abdx^2 + 2\,abc - 2\,a^2d}{2\left(a^2bc^2 - a^3cd\right)\left(bx^3 + ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
- 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c
*d + a^4*d^2)*sqrt(a*b)) - 1/2*(3*b^2*c*x^2 - 2*a*b*d*x^2 + 2*a*b*c - 2*a^2
*d)/((a^2*b*c^2 - a^3*c*d)*(b*x^3 + a*x))
```

maple [A] time = 0.02, size = 169, normalized size = 1.17

$$\frac{b^{2}dx}{2\left(ad-bc\right)^{2}\left(b\,x^{2}+a\right)a}+\frac{5b^{2}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{2}\sqrt{ab}\;a}-\frac{b^{3}cx}{2\left(ad-bc\right)^{2}\left(b\,x^{2}+a\right)a^{2}}-\frac{3b^{3}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{2}\sqrt{ab}\;a^{2}}-\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad-bc\right)^{2}\sqrt{cd}\;c}-\frac{1}{a^{2}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*b^2/a/(a*d-b*c)^2*x/(b*x^2+a)*d-1/2*b^3/a^2/(a*d-b*c)^2*x/(b*x^2+a)*c+5
/2*b^2/a/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-3/2*b^3/a^2/(a
*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/c*d^3/(a*d-b*c)^2/(c*d)
^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/a^2/c/x
```

maxima [A] time = 2.26, size = 178, normalized size = 1.24

$$-\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)\sqrt{cd}}-\frac{\left(3\,b^{3}c-5\,ab^{2}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{ab}}-\frac{2\,abc-2\,a^{2}d+\left(3\,b^{2}c-2\,abd\right)x^{2}}{2\left(\left(a^{2}b^{2}c^{2}-a^{3}bcd\right)x^{3}+\left(a^{3}bc^{2}-a^{4}cd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
- 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c
*d + a^4*d^2)*sqrt(a*b)) - 1/2*(2*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*x^2
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^3 + (a^3*b*c^2 - a^4*c*d)*x)
```

mupad [B] time = 1.00, size = 2400, normalized size = 16.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((a^5*d*x*(-c^3*d^5)^(3/2)*4i + b^5*c^8*d*x*(-c^3*d^5)^(1/2)*9i + a^2*
b^3*c^6*d^3*x*(-c^3*d^5)^(1/2)*25i - a*b^4*c^7*d^2*x*(-c^3*d^5)^(1/2)*30i)/
(4*a^5*c^5*d^8 - 9*b^5*c^10*d^3 + 30*a*b^4*c^9*d^4 - 25*a^2*b^3*c^8*d^5))*(
```

```
-c^3*d^5)^(1/2)*1i)/(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d) - (1/(a*c) - (x^2
*(3*b^2*c - 2*a*b*d))/(2*a^2*c*(a*d - b*c)))/(a*x + b*x^3) - (atan((((x*(14
4*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 2784*a^9
*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^12*b^4*
c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(1280*a
^9*b^9*c^11*d^3 - 192*a^8*b^10*c^12*d^2 - 3520*a^10*b^8*c^10*d^4 + 4992*a^1
1*b^7*c^9*d^5 - 3520*a^12*b^6*c^8*d^6 + 512*a^13*b^5*c^7*d^7 + 960*a^14*b^4
*c^6*d^8 - 640*a^15*b^3*c^5*d^9 + 128*a^16*b^2*c^4*d^10 + (x*(5*a*d - 3*b*c
)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2 - 1536*a^11*b^9*c^12*d^3 + 3584*
a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5 + 3584*a^15*b^5*c^8*d^7 - 3584*a
^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 256*a^18*b^2*c^5*d^10))/(4*(a^7*d
^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)
))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b
*c*d)) + ((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^
8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8
 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d - 3*b*c)*(-a^5*b^
3)^(1/2)*(192*a^8*b^10*c^12*d^2 - 1280*a^9*b^9*c^11*d^3 + 3520*a^10*b^8*c^1
0*d^4 - 4992*a^11*b^7*c^9*d^5 + 3520*a^12*b^6*c^8*d^6 - 512*a^13*b^5*c^7*d^
7 - 960*a^14*b^4*c^6*d^8 + 640*a^15*b^3*c^5*d^9 - 128*a^16*b^2*c^4*d^10 + (
x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2 - 1536*a^11*b^9*
c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5 + 3584*a^15*b^5*
c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 256*a^18*b^2*c^5*
d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*d^2 + a^5*b^2*c^
2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7*d^2 + a^5*b
^2*c^2 - 2*a^6*b*c*d)))/(((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 +
 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*
a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d -
 3*b*c)*(-a^5*b^3)^(1/2)*(192*a^8*b^10*c^12*d^2 - 1280*a^9*b^9*c^11*d^3 + 3
520*a^10*b^8*c^10*d^4 - 4992*a^11*b^7*c^9*d^5 + 3520*a^12*b^6*c^8*d^6 - 512
*a^13*b^5*c^7*d^7 - 960*a^14*b^4*c^6*d^8 + 640*a^15*b^3*c^5*d^9 - 128*a^16*
b^2*c^4*d^10 + (x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2
- 1536*a^11*b^9*c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5
+ 3584*a^15*b^5*c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 2
56*a^18*b^2*c^5*d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*
d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2))/(4*(a^
7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)) - ((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*
b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c
^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10
) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(1280*a^9*b^9*c^11*d^3 - 192*a^8*b^10
*c^12*d^2 - 3520*a^10*b^8*c^10*d^4 + 4992*a^11*b^7*c^9*d^5 - 3520*a^12*b^6*
c^8*d^6 + 512*a^13*b^5*c^7*d^7 + 960*a^14*b^4*c^6*d^8 - 640*a^15*b^3*c^5*d^
9 + 128*a^16*b^2*c^4*d^10 + (x*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b
^10*c^13*d^2 - 1536*a^11*b^9*c^12*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*
b^7*c^10*d^5 + 3584*a^15*b^5*c^8*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^
3*c^6*d^9 - 256*a^18*b^2*c^5*d^10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d
```

```
))))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^
(1/2))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d)) + 144*a^6*b^8*c^7*d^5 - 62
4*a^7*b^7*c^6*d^6 + 976*a^8*b^6*c^5*d^7 - 656*a^9*b^5*c^4*d^8 + 160*a^10*b^
4*c^3*d^9))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(2*(a^7*d^2 + a^5*b^2*c^2
- 2*a^6*b*c*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.296 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=126

$$\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{b^2}{2a^2\left(a+bx^2\right)(bc-ad)}-\frac{1}{2a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}$$

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{b^2}{2a^2\left(a+bx^2\right)(bc-ad)}+\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^2}-\frac{\log(x)(ad+2bc)}{a^3c^2}-\frac{1}{2a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{2c^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(2*a^2*c*x^2) - b^2/(2*a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c + a*d)*Log
[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(2*a^3*(b*c - a*d)^2)
 + (d^3*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\Bigg(\int \frac{1}{x^2(a+bx)^2(c+dx)}\,dx, x, x^2\Bigg) \\
&= \frac{1}{2}\operatorname{Subst}\Bigg(\int \Bigg(\frac{1}{a^2cx^2}+\frac{-2bc-ad}{a^3c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)^2}-\frac{b^3(-2bc+3ad)}{a^3(-bc+ad)^2(a+b} \\
&= -\frac{1}{2a^2cx^2}-\frac{b^2}{2a^2(bc-ad)\left(a+bx^2\right)}-\frac{(2bc+ad)\log(x)}{a^3c^2}+\frac{b^2(2bc-3ad)\log\Big(a+b}{2a^3(bc-ad)^2}
\end{aligned}
$$

Mathematica [**A**] time = 0.16, size = 119, normalized size = 0.94

$$
\frac{1}{2}\left(\frac{b^2(2bc-3ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^2}-\frac{2\log(x)(ad+2bc)}{a^3c^2}+\frac{b^2}{a^2\left(a+bx^2\right)(ad-bc)}-\frac{1}{a^2cx^2}+\frac{d^3\log\left(c+dx^2\right)}{c^2(bc-ad)^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^2)) + b^2/(a^2*(-(b*c) + a*d)*(a + b*x^2)) - (2*(2*b*c + a*d)
*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^
2) + (d^3*Log[c + d*x^2])/(c^2*(b*c - a*d)^2))/2
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [**B**] time = 5.58, size = 303, normalized size = 2.40

$$
-\frac{a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+\left(2\,ab^3c^3-3\,a^2b^2c^2d+a^3bcd^2\right)x^2-\left(\left(2\,b^4c^3-3\,ab^3c^2d\right)x^4+\left(2\,ab^3c^3-3\,a^2b^2c^2d\right)x^2\right)\log\left(bx^2+a\right)-\left(a^3bd^3x^4+a^4d^3x^2\right)\log\left(dx^2+c\right)+2\left(\left(2\,b^4c^3-3\,ab^3c^2d+a^3bd^3\right)x^4+\left(2\,ab^3c^3-3\,a^2b^2c^2d+a^4d^3\right)x^2\right)\log(x)}{2\left(\left(a^3b^3c^4-2\,a^4b^2c^3d+a^5bc^2d^2\right)x^4+\left(a^4b^2c^4-2\,a^5bc^3d+a^6c^2d^2\right)x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^3 - 3*a^2*b^2*c^
2*d + a^3*b*c*d^2)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d)*x^4 + (2*a*b^3*c^3 -
```

```
3*a^2*b^2*c^2*d)*x^2)*log(b*x^2 + a) - (a^3*b*d^3*x^4 + a^4*d^3*x^2)*log(d*
x^2 + c) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 -
3*a^2*b^2*c^2*d + a^4*d^3)*x^2)*log(x))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a
^5*b*c^2*d^2)*x^4 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^2)
```

giac [B] time = 0.35, size = 257, normalized size = 2.04

$$\frac{d^{4} \log\left(\left|dx^{2} + c\right|\right)}{2\left(b^{2} c^{4} d - 2\,abc^{3} d^{2} + a^{2} c^{2} d^{3}\right)} + \frac{\left(2\,b^{4} c - 3\,ab^{3} d\right)\log\left(\left|bx^{2} + a\right|\right)}{2\left(a^{3} b^{3} c^{2} - 2\,a^{4} b^{2} cd + a^{5} bd^{2}\right)} + \frac{a^{2} bd^{3} x^{4} - 4\,b^{3} c^{3} x^{2} + 6\,ab^{2} c^{2} dx^{2} - 2\,a^{2} bcd^{2} x^{2} + a^{3} d^{3} x^{2} - 2\,ab^{2} c^{3} + 4\,a^{2} bc^{2} d - 2\,a^{3} cd^{2}}{4\left(a^{2} b^{2} c^{4} - 2\,a^{3} bc^{3} d + a^{4} c^{2} d^{2}\right)\left(bx^{4} + ax^{2}\right)} - \frac{(2\,bc + ad)\log\left(x^{2}\right)}{2\,a^{3} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3) + 1/2
*(2*b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a
^5*b*d^2) + 1/4*(a^2*b*d^3*x^4 - 4*b^3*c^3*x^2 + 6*a*b^2*c^2*d*x^2 - 2*a^2*
b*c*d^2*x^2 + a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*x^4 + a*x^2)) - 1/2*(2*b*c + a*
d)*log(x^2)/(a^3*c^2)
```

maple [A] time = 0.02, size = 170, normalized size = 1.35

$$\frac{b^{2} d}{2\left(ad - bc\right)^{2}\left(b\,x^{2} + a\right)a} - \frac{b^{3} c}{2\left(ad - bc\right)^{2}\left(b\,x^{2} + a\right)a^{2}} - \frac{3 b^{2} d \ln\left(b\,x^{2} + a\right)}{2\left(ad - bc\right)^{2} a^{2}} + \frac{b^{3} c \ln\left(b\,x^{2} + a\right)}{\left(ad - bc\right)^{2} a^{3}} + \frac{d^{3} \ln\left(d\,x^{2} + c\right)}{2\left(ad - bc\right)^{2} c^{2}} - \frac{d \ln(x)}{a^{2} c^{2}} - \frac{2 b \ln(x)}{a^{3} c} - \frac{1}{2 a^{2} c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*b^2/a/(a*d-b*c)^2/(b*x^2+a)*d-1/2*b^3/a^2/(a*d-b*c)^2/(b*x^2+a)*c-3/2*b
^2/a^2/(a*d-b*c)^2*ln(b*x^2+a)*d+b^3/a^3/(a*d-b*c)^2*ln(b*x^2+a)*c+1/2*d^3/
c^2/(a*d-b*c)^2*ln(d*x^2+c)-1/2/a^2/c/x^2-1/a^2/c^2*ln(x)*d-2/a^3/c*ln(x)*b
```

maxima [A] time = 1.13, size = 189, normalized size = 1.50

$$\frac{d^{3} \log\left(dx^{2} + c\right)}{2\left(b^{2} c^{4} - 2\,abc^{3} d + a^{2} c^{2} d^{2}\right)} + \frac{\left(2\,b^{3} c - 3\,ab^{2} d\right)\log\left(bx^{2} + a\right)}{2\left(a^{3} b^{2} c^{2} - 2\,a^{4} bcd + a^{5} d^{2}\right)} - \frac{abc - a^{2} d + \left(2\,b^{2} c - abd\right)x^{2}}{2\left(\left(a^{2} b^{2} c^{2} - a^{3} bcd\right)x^{4} + \left(a^{3} bc^{2} - a^{4} cd\right)x^{2}\right)} - \frac{(2\,bc + ad)\log\left(x^{2}\right)}{2\,a^{3} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*d^3*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + 1/2*(2*b^3*c
 - 3*a*b^2*d)*log(b*x^2 + a)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2) - 1/2*(a
*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)/((a^2*b^2*c^2 - a^3*b*c*d)*x^4 + (a^3
*b*c^2 - a^4*c*d)*x^2) - 1/2*(2*b*c + a*d)*log(x^2)/(a^3*c^2)
```

mupad [**B**] time = 1.02, size = 171, normalized size = 1.36

$$\frac{\ln\left(b\,x^2+a\right)\,\left(2\,b^3\,c-3\,a\,b^2\,d\right)}{2\,a^5\,d^2-4\,a^4\,b\,c\,d+2\,a^3\,b^2\,c^2}-\frac{\frac{1}{2\,a\,c}-\frac{x^2\,\left(2\,b^2\,c-a\,b\,d\right)}{2\,a^2\,c\,(a\,d-b\,c)}}{b\,x^4+a\,x^2}+\frac{d^3\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}-\frac{\ln(x)\,\left(a\,d+2\,b\,c\right)}{a^3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(2*b^3*c - 3*a*b^2*d))/(2*a^5*d^2 + 2*a^3*b^2*c^2 - 4*a^4*b
*c*d) - (1/(2*a*c) - (x^2*(2*b^2*c - a*b*d))/(2*a^2*c*(a*d - b*c)))/(a*x^2
+ b*x^4) + (d^3*log(c + d*x^2))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) -
 (log(x)*(a*d + 2*b*c))/(a^3*c^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.297 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=189

$$\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-}$$

Rubi [**A**] time = 0.28, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$\frac{-2a^2d^2-2abcd+5b^2c^2}{2a^3c^2x(bc-ad)}+\frac{b^{5/2}(5bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}-\frac{5bc-2ad}{6a^2cx^3(bc-ad)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(5*b*c - 2*a*d)/(6*a^2*c*(b*c - a*d)*x^3) + (5*b^2*c^2 - 2*a*b*c*d - 2*a^2
*d^2)/(2*a^3*c^2*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^
(5/2)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*(b*c - a*d)^2
) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} - \frac{\int \frac{-5bc+2ad-5bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{\int \frac{-3\left(5b^2c^2-2abcd-2a^2d^2\right)-3bd(5bc-2ad)x^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}}{6a^2c(bc-ad)}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} - \int -3\left(5b^3\right.$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{d^4\int \frac{1}{c+d}}{c^2(bc-}$$

$$= -\frac{5bc-2ad}{6a^2c(bc-ad)x^3} + \frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{b^{5/2}(5bc}{2}$$

Mathematica [A] time = 0.29, size = 142, normalized size = 0.75

$$-\frac{b^{5/2}(7ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(ad-bc)^2} - \frac{b^3x}{2a^3\left(a+bx^2\right)(ad-bc)} + \frac{ad+2bc}{a^3c^2x} - \frac{1}{3a^2cx^3} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/3*1/(a^2*c*x^3) + (2*b*c + a*d)/(a^3*c^2*x) - (b^3*x)/(2*a^3*(-(b*c) + a
*d)*(a + b*x^2)) - (b^(5/2)*(-5*b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
2*a^(7/2)*(-(b*c) + a*d)^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2
)*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 3.48, size = 1281, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^2*c^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^
3*c^2*d + 2*a^3*b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2
 + 3*a^4*d^3)*x^2 + 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a
^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) - 6*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5
+ (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3), -1/12*(4*a^2*b^2*c^3 -
8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x
^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 12*(
a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*((5*b^4*c^3
- 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log(
(b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*
d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3),
-1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*(5*b^4*c^3 - 7*a*b^3*
c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 +
 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2
*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*(a^3*b*d^3*x^5 + a^4*d^3
*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^3
*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d
+ a^6*c^2*d^2)*x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*
(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*
c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 +
 (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 6*(a^
```

```
3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^3*b^3*c^4 - 2
*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^
2*d^2)*x^3)]
```

giac [A] time = 0.38, size = 165, normalized size = 0.87

$$\frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)\sqrt{cd}} + \frac{b^3x}{2\left(a^3bc - a^4d\right)\left(bx^2 + a\right)} + \frac{\left(5\,b^4c - 7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2\right)\sqrt{ab}} + \frac{6\,bcx^2 + 3\,adx^2 - ac}{3\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d))
 + 1/2*b^3*x/((a^3*b*c - a^4*d)*(b*x^2 + a)) + 1/2*(5*b^4*c - 7*a*b^3*d)*ar
ctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(a*b)) + 1/3
*(6*b*c*x^2 + 3*a*d*x^2 - a*c)/(a^3*c^2*x^3)
```

maple [A] time = 0.02, size = 191, normalized size = 1.01

$$-\frac{b^3dx}{2\left(ad - bc\right)^2\left(b\,x^2 + a\right)a^2} - \frac{7b^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad - bc\right)^2\sqrt{ab}\;a^2} + \frac{b^4cx}{2\left(ad - bc\right)^2\left(b\,x^2 + a\right)a^3} + \frac{5b^4c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad - bc\right)^2\sqrt{ab}\;a^3} + \frac{d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad - bc\right)^2\sqrt{cd}\;c^2} + \frac{d}{a^2c^2x} + \frac{2b}{a^3cx} - \frac{1}{3a^2c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*b^3/a^2/(a*d-b*c)^2*x/(b*x^2+a)*d+1/2*b^4/a^3/(a*d-b*c)^2*x/(b*x^2+a)*
c-7/2*b^3/a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+5/2*b^4/a
^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+1/c^2*d^4/(a*d-b*c)^
2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/3/a^2/c/x^3+1/a^2/c^2/x*d+2/a^3/c
/x*b
```

maxima [A] time = 2.46, size = 236, normalized size = 1.25

$$\frac{d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^2c^4 - 2\,abc^3d + a^2c^2d^2\right)\sqrt{cd}} + \frac{\left(5\,b^4c - 7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^3b^2c^2 - 2\,a^4bcd + a^5d^2\right)\sqrt{ab}} - \frac{2\,a^2bc^2 - 2\,a^3cd - 3\left(5\,b^3c^2 - 2\,ab^2cd - 2\,a^2bd^2\right)x^4 - 2\left(5\,ab^2c^2 - 2\,a^2bcd - 3\,a^3d^2\right)x^2}{6\left(\left(a^3b^2c^3 - a^4bc^2d\right)x^5 + \left(a^4bc^3 - a^5c^2d\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d))
 + 1/2*(5*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2)*sqrt(a*b)) - 1/6*(2*a^2*b*c^2 - 2*a^3*c*d - 3*(5*b^3*c^2 - 2
*a*b^2*c*d - 2*a^2*b*d^2)*x^4 - 2*(5*a*b^2*c^2 - 2*a^2*b*c*d - 3*a^3*d^2)*x
^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^5 + (a^4*b*c^3 - a^5*c^2*d)*x^3)
```

mupad [B] time = 1.23, size = 4654, normalized size = 24.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] ((x^2*(3*a*d + 5*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^4*(2*a^2*b*d^2 - 5*b^3*
c^2 + 2*a*b^2*c*d))/(2*a^3*c^2*(a*d - b*c)))/(a*x^3 + b*x^5) - (atan(((((x*
(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5
- 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 +
 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a
^18*b^3*c^6*d^12))/2 + ((-c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^15*b^1
0*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^
7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^
3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6
*d)) - 160*a^12*b^11*c^17*d^2 + 1024*a^13*b^10*c^16*d^3 - 2720*a^14*b^9*c^1
5*d^4 + 3840*a^15*b^8*c^14*d^5 - 3104*a^16*b^7*c^13*d^6 + 1600*a^17*b^6*c^1
2*d^7 - 864*a^18*b^5*c^11*d^8 + 640*a^19*b^4*c^10*d^9 - 320*a^20*b^3*c^9*d^
10 + 64*a^21*b^2*c^8*d^11))/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^
5*d^7)^(1/2)*1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d) + (((x*(400*a^9*b^12
*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b
^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*
c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d
^12))/2 + ((-c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^15*b^10*c^18*d^2 -
1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 +
3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 -
256*a^23*b^2*c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)) + 160*a^
12*b^11*c^17*d^2 - 1024*a^13*b^10*c^16*d^3 + 2720*a^14*b^9*c^15*d^4 - 3840*
a^15*b^8*c^14*d^5 + 3104*a^16*b^7*c^13*d^6 - 1600*a^17*b^6*c^12*d^7 + 864*a
^18*b^5*c^11*d^8 - 640*a^19*b^4*c^10*d^9 + 320*a^20*b^3*c^9*d^10 - 64*a^21*
b^2*c^8*d^11))/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^5*d^7)^(1/2)*
1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d))/((((x*(400*a^9*b^12*c^15*d^3 - 2
320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 +
 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192
*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12))/2 + ((-
c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^15*b^10*c^18*d^2 - 1536*a^16*b^9
*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 + 3584*a^20*b^5
*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 - 256*a^23*b^2*
c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)) - 160*a^12*b^11*c^17*
d^2 + 1024*a^13*b^10*c^16*d^3 - 2720*a^14*b^9*c^15*d^4 + 3840*a^15*b^8*c^14
*d^5 - 3104*a^16*b^7*c^13*d^6 + 1600*a^17*b^6*c^12*d^7 - 864*a^18*b^5*c^11*
d^8 + 640*a^19*b^4*c^10*d^9 - 320*a^20*b^3*c^9*d^10 + 64*a^21*b^2*c^8*d^11)
)/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^5*d^7)^(1/2))/(b^2*c^7 + a
^2*c^5*d^2 - 2*a*b*c^6*d) - (((x*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^
```

```
14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c
^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^
10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12))/2 + ((-c^5*d^7)^(1/2)*(
(x*(-c^5*d^7)^(1/2)*(256*a^15*b^10*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584
*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584
*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(
b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d)) + 160*a^12*b^11*c^17*d^2 - 1024*a^13*
b^10*c^16*d^3 + 2720*a^14*b^9*c^15*d^4 - 3840*a^15*b^8*c^14*d^5 + 3104*a^16
*b^7*c^13*d^6 - 1600*a^17*b^6*c^12*d^7 + 864*a^18*b^5*c^11*d^8 - 640*a^19*b
^4*c^10*d^9 + 320*a^20*b^3*c^9*d^10 - 64*a^21*b^2*c^8*d^11))/(2*(b^2*c^7 +
a^2*c^5*d^2 - 2*a*b*c^6*d)))*(-c^5*d^7)^(1/2))/(b^2*c^7 + a^2*c^5*d^2 - 2*a
*b*c^6*d) - 400*a^9*b^10*c^11*d^6 + 1520*a^10*b^9*c^10*d^7 - 1904*a^11*b^8*
c^9*d^8 + 624*a^12*b^7*c^8*d^9 + 384*a^13*b^6*c^7*d^10 - 224*a^14*b^5*c^6*d
^11))*(-c^5*d^7)^(1/2)*1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d) + (atan(((
(7*a*d - 5*b*c)*(x*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*
a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*
a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*
b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12) + ((7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(2
048*a^13*b^10*c^16*d^3 - 320*a^12*b^11*c^17*d^2 - 5440*a^14*b^9*c^15*d^4 +
7680*a^15*b^8*c^14*d^5 - 6208*a^16*b^7*c^13*d^6 + 3200*a^17*b^6*c^12*d^7 -
1728*a^18*b^5*c^11*d^8 + 1280*a^19*b^4*c^10*d^9 - 640*a^20*b^3*c^9*d^10 + 1
28*a^21*b^2*c^8*d^11 + (x*(7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(256*a^15*b^10*c
^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c
^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c
^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)
)))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)))*(-a^7*b^5)^(1/2)*1i)/(4*(a^9
*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)) + ((7*a*d - 5*b*c)*(x*(400*a^9*b^12*c^15
*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^
12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d
^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12)
+ ((7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(320*a^12*b^11*c^17*d^2 - 2048*a^13*b^1
0*c^16*d^3 + 5440*a^14*b^9*c^15*d^4 - 7680*a^15*b^8*c^14*d^5 + 6208*a^16*b^
7*c^13*d^6 - 3200*a^17*b^6*c^12*d^7 + 1728*a^18*b^5*c^11*d^8 - 1280*a^19*b^
4*c^10*d^9 + 640*a^20*b^3*c^9*d^10 - 128*a^21*b^2*c^8*d^11 + (x*(7*a*d - 5*
b*c)*(-a^7*b^5)^(1/2)*(256*a^15*b^10*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 35
84*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 35
84*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4
*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d))))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8
*b*c*d)))*(-a^7*b^5)^(1/2)*1i)/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)))/(
400*a^9*b^10*c^11*d^6 - 1520*a^10*b^9*c^10*d^7 + 1904*a^11*b^8*c^9*d^8 - 62
4*a^12*b^7*c^8*d^9 - 384*a^13*b^6*c^7*d^10 + 224*a^14*b^5*c^6*d^11 - ((7*a*
d - 5*b*c)*(x*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*
b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*
b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c
^7*d^11 - 64*a^18*b^3*c^6*d^12) + ((7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(2048*a
```

```
^13*b^10*c^16*d^3 - 320*a^12*b^11*c^17*d^2 - 5440*a^14*b^9*c^15*d^4 + 7680*
a^15*b^8*c^14*d^5 - 6208*a^16*b^7*c^13*d^6 + 3200*a^17*b^6*c^12*d^7 - 1728*
a^18*b^5*c^11*d^8 + 1280*a^19*b^4*c^10*d^9 - 640*a^20*b^3*c^9*d^10 + 128*a^
21*b^2*c^8*d^11 + (x*(7*a*d - 5*b*c)*(-a^7*b^5)^(1/2)*(256*a^15*b^10*c^18*d
^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d
^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d
^9 - 256*a^23*b^2*c^10*d^10))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d))))/(
4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d)))*(-a^7*b^5)^(1/2))/(4*(a^9*d^2 + a
^7*b^2*c^2 - 2*a^8*b*c*d)) + ((7*a*d - 5*b*c)*(x*(400*a^9*b^12*c^15*d^3 - 2
320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^5 - 6112*a^12*b^9*c^12*d^6 +
 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^8 + 64*a^15*b^6*c^9*d^9 - 192
*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 - 64*a^18*b^3*c^6*d^12) + ((7*a*
d - 5*b*c)*(-a^7*b^5)^(1/2)*(320*a^12*b^11*c^17*d^2 - 2048*a^13*b^10*c^16*d
^3 + 5440*a^14*b^9*c^15*d^4 - 7680*a^15*b^8*c^14*d^5 + 6208*a^16*b^7*c^13*d
^6 - 3200*a^17*b^6*c^12*d^7 + 1728*a^18*b^5*c^11*d^8 - 1280*a^19*b^4*c^10*d
^9 + 640*a^20*b^3*c^9*d^10 - 128*a^21*b^2*c^8*d^11 + (x*(7*a*d - 5*b*c)*(-a
^7*b^5)^(1/2)*(256*a^15*b^10*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*
b^8*c^16*d^4 - 3584*a^18*b^7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*
b^4*c^12*d^8 + 1536*a^22*b^3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(a^9*d^
2 + a^7*b^2*c^2 - 2*a^8*b*c*d))))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d))
)*(-a^7*b^5)^(1/2))/(4*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d))))*(7*a*d - 5*
b*c)*(-a^7*b^5)^(1/2)*1i)/(2*(a^9*d^2 + a^7*b^2*c^2 - 2*a^8*b*c*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.298 $\int \frac{1}{x^5\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=160

$$-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}+\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{d^4\log}{2c^3(}$$

Rubi [A] time = 0.19, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}+\frac{b^3}{2a^3\left(a+bx^2\right)(bc-ad)}-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}+\frac{ad+2bc}{2a^3c^2x^2}-\frac{1}{4a^2cx^4}-\frac{d^4\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(4*a^2*c*x^4) + (2*b*c + a*d)/(2*a^3*c^2*x^2) + b^3/(2*a^3*(b*c - a*d)*(
a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3) - (b^3*(
3*b*c - 4*a*d)*Log[a + b*x^2])/(2*a^4*(b*c - a*d)^2) - (d^4*Log[c + d*x^2])
/(2*c^3*(b*c - a*d)^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)^2(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2cx^3}+\frac{-2bc-ad}{a^3c^2x^2}+\frac{3b^2c^2+2abcd+a^2d^2}{a^4c^3x}+\frac{b^4}{a^3(-bc+ad)(a+b\right.\right.\\
&= -\frac{1}{4a^2cx^4}+\frac{2bc+ad}{2a^3c^2x^2}+\frac{b^3}{2a^3(bc-ad)\left(a+bx^2\right)}+\frac{\left(3b^2c^2+2abcd+a^2d^2\right)\log(x)}{a^4c^3}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 155, normalized size = 0.97

$$
\frac{1}{4}\left(\frac{2b^3(4ad-3bc)\log\left(a+bx^2\right)}{a^4(bc-ad)^2}-\frac{2b^3}{a^3\left(a+bx^2\right)(ad-bc)}+\frac{2ad+4bc}{a^3c^2x^2}-\frac{1}{a^2cx^4}+\frac{4\log(x)\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3}-\frac{2d^4\log\left(c+dx^2\right)}{c^3(bc-ad)^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-(1/(a^2*c*x^4)) + (4*b*c + 2*a*d)/(a^3*c^2*x^2) - (2*b^3)/(a^3*(-(b*c) +
a*d)*(a + b*x^2)) + (4*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3)
+ (2*b^3*(-3*b*c + 4*a*d)*Log[a + b*x^2])/(a^4*(b*c - a*d)^2) - (2*d^4*Log[
c + d*x^2])/(c^3*(b*c - a*d)^2))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^5\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [B] time = 12.77, size = 356, normalized size = 2.22

$$
-\frac{a^3b^2c^4-2\,a^4bc^3d+a^5c^2d^2-2\left(3\,ab^4c^4-4\,a^2b^3c^3d+a^4bcd^3\right)x^4-\left(3\,a^2b^3c^4-4\,a^3b^2c^3d-a^4bc^2d^2+2\,a^5cd^3\right)x^2+2\left(\left(3\,b^5c^4-4\,ab^4c^3d\right)x^6+\left(3\,ab^4c^4-4\,a^2b^3c^3d\right)x^4\right)\log\left(bx^2+a\right)+2\left(a^4bd^4x^6+a^5d^4x^4\right)\log\left(dx^2+c\right)-4\left(\left(3\,b^5c^4-4\,ab^4c^3d+a^4bd^4\right)x^6+\left(3\,ab^4c^4-4\,a^2b^3c^3d+a^5d^4\right)x^4\right)\log(x)}{4\left(\left(a^4b^3c^5-2\,a^5b^2c^4d+a^6bc^3d^2\right)x^6+\left(a^5b^2c^5-2\,a^6bc^4d+a^7c^3d^2\right)x^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 - 2*(3*a*b^4*c^4 - 4*a^2*b^
3*c^3*d + a^4*b*c*d^3)*x^4 - (3*a^2*b^3*c^4 - 4*a^3*b^2*c^3*d - a^4*b*c^2*d
```

```
^2 + 2*a^5*c*d^3)*x^2 + 2*((3*b^5*c^4 - 4*a*b^4*c^3*d)*x^6 + (3*a*b^4*c^4 -
 4*a^2*b^3*c^3*d)*x^4)*log(b*x^2 + a) + 2*(a^4*b*d^4*x^6 + a^5*d^4*x^4)*log
(d*x^2 + c) - 4*((3*b^5*c^4 - 4*a*b^4*c^3*d + a^4*b*d^4)*x^6 + (3*a*b^4*c^4
 - 4*a^2*b^3*c^3*d + a^5*d^4)*x^4)*log(x))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d
+ a^6*b*c^3*d^2)*x^6 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^4)
```

giac [A] time = 0.39, size = 281, normalized size = 1.76

$$-\frac{d^{5} \log \left(\left|dx^{2} + c\right|\right)}{2 \left(b^{2} c^{5} d - 2 \, a b c^{4} d^{2} + a^{2} c^{3} d^{3}\right)} - \frac{\left(3 \, b^{5} c - 4 \, a b^{4} d\right) \log \left(\left|bx^{2} + a\right|\right)}{2 \left(a^{4} b^{3} c^{2} - 2 \, a^{5} b^{2} c d + a^{6} b d^{2}\right)} + \frac{3 \, b^{5} c x^{2} - 4 \, a b^{4} d x^{2} + 4 \, a b^{4} c - 5 \, a^{2} b^{3} d}{2 \left(a^{4} b^{2} c^{2} - 2 \, a^{5} b c d + a^{6} d^{2}\right) \left(bx^{2} + a\right)} + \frac{\left(3 \, b^{2} c^{2} + 2 \, a b c d + a^{2} d^{2}\right) \log \left(x^{2}\right)}{2 \, a^{4} c^{3}} - \frac{9 \, b^{2} c^{2} x^{4} + 6 \, a b c d x^{4} + 3 \, a^{2} d^{2} x^{4} - 4 \, a b c^{2} x^{2} - 2 \, a^{2} c d x^{2} + a^{2} c^{2}}{4 \, a^{4} c^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*d^5*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3) - 1/
2*(3*b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^4*b^3*c^2 - 2*a^5*b^2*c*d +
a^6*b*d^2) + 1/2*(3*b^5*c*x^2 - 4*a*b^4*d*x^2 + 4*a*b^4*c - 5*a^2*b^3*d)/((
a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*(b*x^2 + a)) + 1/2*(3*b^2*c^2 + 2*a*b*
c*d + a^2*d^2)*log(x^2)/(a^4*c^3) - 1/4*(9*b^2*c^2*x^4 + 6*a*b*c*d*x^4 + 3*
a^2*d^2*x^4 - 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^4*c^3*x^4)
```

maple [A] time = 0.02, size = 209, normalized size = 1.31

$$-\frac{b^{3} d}{2 \left(a d - b c\right)^{2} \left(b \,x^{2}+a \right) a^{2}}+\frac{b^{4} c}{2 \left(a d - b c\right)^{2} \left(b \,x^{2}+a \right) a^{3}}+\frac{2 b^{3} d \ln \left(b \,x^{2}+a \right)}{\left(a d - b c\right)^{2} a^{3}}-\frac{3 b^{4} c \ln \left(b \,x^{2}+a \right)}{2 \left(a d - b c\right)^{2} a^{4}}-\frac{d^{4} \ln \left(d \,x^{2}+c \right)}{2 \left(a d - b c\right)^{2} c^{3}}+\frac{d^{2} \ln \left(x \right)}{a^{2} c^{3}}+\frac{2 b d \ln \left(x \right)}{a^{3} c^{2}}+\frac{3 b^{2} \ln \left(x \right)}{a^{4} c}+\frac{d}{2 a^{2} c^{2} x^{2}}+\frac{b}{a^{3} c \,x^{2}}-\frac{1}{4 a^{2} c \,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] -1/2*b^3/a^2/(a*d-b*c)^2/(b*x^2+a)*d+1/2*b^4/a^3/(a*d-b*c)^2/(b*x^2+a)*c+2*
b^3/a^3/(a*d-b*c)^2*ln(b*x^2+a)*d-3/2*b^4/a^4/(a*d-b*c)^2*ln(b*x^2+a)*c-1/2
*d^4/c^3/(a*d-b*c)^2*ln(d*x^2+c)-1/4/a^2/c/x^4+1/2/a^2/c^2/x^2*d+1/a^3/c/x^
2*b+1/a^2/c^3*ln(x)*d^2+2/a^3/c^2*ln(x)*b*d+3/a^4/c*ln(x)*b^2
```

maxima [A] time = 1.12, size = 258, normalized size = 1.61

$$-\frac{d^{4} \log \left(dx^{2} + c\right)}{2 \left(b^{2} c^{5} - 2 \, a b c^{4} d + a^{2} c^{3} d^{2}\right)} - \frac{\left(3 \, b^{4} c - 4 \, a b^{3} d\right) \log \left(bx^{2} + a\right)}{2 \left(a^{4} b^{2} c^{2} - 2 \, a^{5} b c d + a^{6} d^{2}\right)} - \frac{a^{2} b c^{2} - a^{3} c d - 2 \left(3 \, b^{3} c^{2} - a b^{2} c d - a^{2} b d^{2}\right) x^{4} - \left(3 \, a b^{2} c^{2} - a^{2} b c d - 2 \, a^{3} d^{2}\right) x^{2}}{4 \left(\left(a^{3} b^{2} c^{3} - a^{4} b c^{2} d\right) x^{6} + \left(a^{4} b c^{3} - a^{5} c^{2} d\right) x^{4}\right)} + \frac{\left(3 \, b^{2} c^{2} + 2 \, a b c d + a^{2} d^{2}\right) \log \left(x^{2}\right)}{2 \, a^{4} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/2*d^4*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(3*b^4*
c - 4*a*b^3*d)*log(b*x^2 + a)/(a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2) - 1/4*(
a^2*b*c^2 - a^3*c*d - 2*(3*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^4 - (3*a*b^2*
c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b
```

```
*c^3 - a^5*c^2*d)*x^4) + 1/2*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(x^2)/(a^
4*c^3)
```

mupad [**B**] time = 1.11, size = 217, normalized size = 1.36

$$\frac{\frac{x^2\,(2\,a\,d+3\,b\,c)}{4\,a^2\,c^2}-\frac{1}{4\,a\,c}+\frac{x^4\,\left(a^2\,b\,d^2+a\,b^2\,c\,d-3\,b^3\,c^2\right)}{2\,a^3\,c^2\,(a\,d-b\,c)}}{b\,x^6+a\,x^4}-\frac{\ln\left(b\,x^2+a\right)\,\left(3\,b^4\,c-4\,a\,b^3\,d\right)}{2\,a^6\,d^2-4\,a^5\,b\,c\,d+2\,a^4\,b^2\,c^2}-\frac{d^4\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^3\,d^2-2\,a\,b\,c^4\,d+b^2\,c^5\right)}+\frac{\ln(x)\,\left(a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{a^4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] ((x^2*(2*a*d + 3*b*c))/(4*a^2*c^2) - 1/(4*a*c) + (x^4*(a^2*b*d^2 - 3*b^3*c^
2 + a*b^2*c*d))/(2*a^3*c^2*(a*d - b*c)))/(a*x^4 + b*x^6) - (log(a + b*x^2)*
(3*b^4*c - 4*a*b^3*d))/(2*a^6*d^2 + 2*a^4*b^2*c^2 - 4*a^5*b*c*d) - (d^4*log
(c + d*x^2))/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) + (log(x)*(a^2*d^2 +
 3*b^2*c^2 + 2*a*b*c*d))/(a^4*c^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.299 $\int \frac{1}{x^6\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=250

$$-\frac{b^{7/2}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}+\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}$$

Rubi [A] time = 0.41, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {472, 583, 522, 205}

$$\frac{-2a^2d^2-2abcd+7b^2c^2}{6a^3c^2x^3(bc-ad)}-\frac{-2a^2bcd^2-2a^3d^3-2ab^2c^2d+7b^3c^3}{2a^4c^3x(bc-ad)}-\frac{b^{7/2}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2}-\frac{7bc-2ad}{10a^2cx^5(bc-ad)}-\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(7*b*c - 2*a*d)/(10*a^2*c*(b*c - a*d)*x^5) + (7*b^2*c^2 - 2*a*b*c*d - 2*a^
2*d^2)/(6*a^3*c^2*(b*c - a*d)*x^3) - (7*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c
*d^2 - 2*a^3*d^3)/(2*a^4*c^3*(b*c - a*d)*x) + b/(2*a*(b*c - a*d)*x^5*(a + b
*x^2)) - (b^(7/2)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(
b*c - a*d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^
2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{b}{2a(bc-ad)x^5\left(a+bx^2\right)} - \frac{\int \frac{-7bc+2ad-7bdx^2}{x^6\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{2a(bc-ad)}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{b}{2a(bc-ad)x^5\left(a+bx^2\right)} + \frac{\int \frac{-5\left(7b^2c^2-2abcd-2a^2d^2\right)-5bd(7bc-2a}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}}{10a^2c(bc-ad)}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} + \frac{b}{2a(bc-ad)x^5\left(a+bx^2\right)} - \int \frac{-15}{}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3}{2a^4c^3(bc-ad)x}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3}{2a^4c^3(bc-ad)x}$$

$$= -\frac{7bc-2ad}{10a^2c(bc-ad)x^5} + \frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3}{2a^4c^3(bc-ad)x}$$

Mathematica [A] time = 0.31, size = 179, normalized size = 0.72

$$\frac{b^{7/2}(9ad-7bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}(ad-bc)^2} + \frac{b^4x}{2a^4\left(a+bx^2\right)(ad-bc)} + \frac{ad+2bc}{3a^3c^2x^3} - \frac{1}{5a^2cx^5} + \frac{-a^2d^2-2abcd-3b^2c^2}{a^4c^3x} - \frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/5*1/(a^2*c*x^5) + (2*b*c + a*d)/(3*a^3*c^2*x^3) + (-3*b^2*c^2 - 2*a*b*c*
d - a^2*d^2)/(a^4*c^3*x) + (b^4*x)/(2*a^4*(-(b*c) + a*d)*(a + b*x^2)) + (b^
(7/2)*(-7*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(-(b*c) + a*
d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^6*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [A] time = 6.93, size = 1489, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/60*(12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 -
9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*
b*c*d^3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2
*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^
4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) - 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*
sqrt(-d/c) - c)/(d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d
^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/60*(12*a^3*b
^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 - 9*a*b^4*c^3*d +
2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*
d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d
^3)*x^2 + 60*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*sqrt(d/c)) +
15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^4*b^3*c^5 -
 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*
c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*c^3*d + 6*a^5*c^2*d^2 + 15*(
7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 10*(7*a*b^4*c^4 - 9*a^2*b^3*
c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d -
 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 +
 (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 15*(a
```

```
^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(
d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^
2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*
c^3*d + 6*a^5*c^2*d^2 + 15*(7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 +
10*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2
*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^
5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*
sqrt(d/c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2
*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5)]
```

giac [A] time = 0.39, size = 207, normalized size = 0.83

$$-\frac{d^{5} \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^{2} c^{5} - 2\, abc^{4} d + a^{2} c^{3} d^{2}\right)\sqrt{cd}} - \frac{b^{4} x}{2\,\left(a^{4} bc - a^{5} d\right)\left(bx^{2} + a\right)} - \frac{\left(7\, b^{5} c - 9\, ab^{4} d\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\left(a^{4} b^{2} c^{2} - 2\, a^{5} bcd + a^{6} d^{2}\right)\sqrt{ab}} - \frac{45\, b^{2} c^{2} x^{4} + 30\, abcdx^{4} + 15\, a^{2} d^{2} x^{4} - 10\, abc^{2} x^{2} - 5\, a^{2} cdx^{2} + 3\, a^{2} c^{2}}{15\, a^{4} c^{3} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)
) - 1/2*b^4*x/((a^4*b*c - a^5*d)*(b*x^2 + a)) - 1/2*(7*b^5*c - 9*a*b^4*d)*a
rctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(a*b)) - 1/
15*(45*b^2*c^2*x^4 + 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 - 10*a*b*c^2*x^2 - 5*a
^2*c*d*x^2 + 3*a^2*c^2)/(a^4*c^3*x^5)
```

maple [A] time = 0.02, size = 234, normalized size = 0.94

$$\frac{b^{4} dx}{2\left(ad-bc\right)^{2}\left(b\,x^{2}+a\right) a^{3}}+\frac{9 b^{4} d \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{2}\sqrt{ab}\, a^{3}}-\frac{b^{5} cx}{2\left(ad-bc\right)^{2}\left(b\,x^{2}+a\right) a^{4}}-\frac{7 b^{5} c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{2}\sqrt{ab}\, a^{4}}-\frac{d^{5} \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(ad-bc\right)^{2}\sqrt{cd}\, c^{3}}-\frac{d^{2}}{a^{2} c^{3} x}-\frac{2 bd}{a^{3} c^{2} x}-\frac{3 b^{2}}{a^{4} cx}+\frac{d}{3 a^{2} c^{2} x^{3}}+\frac{2 b}{3 a^{3} c\, x^{3}}-\frac{1}{5 a^{2} c\, x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*b^4/a^3/(a*d-b*c)^2*x/(b*x^2+a)*d-1/2*b^5/a^4/(a*d-b*c)^2*x/(b*x^2+a)*c
+9/2*b^4/a^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-7/2*b^5/a^
4/(a*d-b*c)^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/c^3*d^5/(a*d-b*c)^2
/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)-1/5/a^2/c/x^5+1/3/a^2/c^2/x^3*d+2/3/
a^3/c/x^3*b-1/a^2/c^3/x*d^2-2/a^3/c^2/x*b*d-3/a^4/c/x*b^2
```

maxima [A] time = 2.41, size = 303, normalized size = 1.21

$$-\frac{d^{5} \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^{2} c^{5} - 2\, abc^{4} d + a^{2} c^{3} d^{2}\right)\sqrt{cd}} - \frac{\left(7\, b^{5} c - 9\, ab^{4} d\right) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\left(a^{4} b^{2} c^{2} - 2\, a^{5} bcd + a^{6} d^{2}\right)\sqrt{ab}} - \frac{6\, a^{3} bc^{3} - 6\, a^{4} c^{2} d + 15\,\left(7\, b^{4} c^{3} - 2\, ab^{3} c^{2} d - 2\, a^{2} b^{2} cd^{2} - 2\, a^{3} bd^{3}\right)x^{6} + 10\,\left(7\, ab^{3} c^{3} - 2\, a^{2} b^{2} c^{2} d - 2\, a^{3} bcd^{2} - 3\, a^{4} d^{3}\right)x^{4} - 2\,\left(7\, a^{2} b^{2} c^{3} - 2\, a^{3} bc^{2} d - 5\, a^{4} cd^{2}\right)x^{2}}{30\,\left(\left(a^{4} b^{2} c^{4} - a^{5} bc^{3} d\right)x^{7} + \left(a^{5} bc^{4} - a^{6} c^{3} d\right)x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)
) - 1/2*(7*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b
*c*d + a^6*d^2)*sqrt(a*b)) - 1/30*(6*a^3*b*c^3 - 6*a^4*c^2*d + 15*(7*b^4*c^
3 - 2*a*b^3*c^2*d - 2*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^6 + 10*(7*a*b^3*c^3 -
2*a^2*b^2*c^2*d - 2*a^3*b*c*d^2 - 3*a^4*d^3)*x^4 - 2*(7*a^2*b^2*c^3 - 2*a^3
*b*c^2*d - 5*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*c^3*d)*x^7 + (a^5*b*c^4
- a^6*c^3*d)*x^5)
```

mupad [B] time = 1.28, size = 2737, normalized size = 10.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (atan((a^9*d*x*(-c^7*d^9)^(3/2)*4i + b^9*c^16*d*x*(-c^7*d^9)^(1/2)*49i + a^
2*b^7*c^14*d^3*x*(-c^7*d^9)^(1/2)*81i - a*b^8*c^15*d^2*x*(-c^7*d^9)^(1/2)*1
26i)/(4*a^9*c^11*d^14 - 49*b^9*c^20*d^5 + 126*a*b^8*c^19*d^6 - 81*a^2*b^7*c
^18*d^7))*(-c^7*d^9)^(1/2)*1i)/(b^2*c^9 + a^2*c^7*d^2 - 2*a*b*c^8*d) - (1/(
5*a*c) - (x^2*(5*a*d + 7*b*c))/(15*a^2*c^2) + (x^4*(3*a^2*d^2 + 7*b^2*c^2 +
 5*a*b*c*d))/(3*a^3*c^3) + (x^6*(2*a^3*b*d^3 - 7*b^4*c^3 + 2*a^2*b^2*c*d^2
+ 2*a*b^3*c^2*d))/(2*a^4*c^3*(a*d - b*c)))/(a*x^5 + b*x^7) - (atan((((x*(78
4*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12*c^18*d^5 -
10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*b^9*c^15*d^8
 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^4*c^10*d^13
- 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(2816*a^17*b^11
*c^21*d^3 - 448*a^16*b^12*c^22*d^2 - 7360*a^18*b^10*c^20*d^4 + 10240*a^19*b
^9*c^19*d^5 - 8000*a^20*b^8*c^18*d^6 + 3200*a^21*b^7*c^17*d^7 + 64*a^22*b^6
*c^16*d^8 - 1280*a^23*b^5*c^15*d^9 + 1280*a^24*b^4*c^14*d^10 - 640*a^25*b^3
*c^13*d^11 + 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(
256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*b^8*c^21*d^4 -
3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*b^4*c^17*d^8 +
1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d^2 + a^9*b^2*c^
2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)))*(9*a*d -
7*b*c)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)) + (
(x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12*c^18*
d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*b^9*c^
15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^4*c^10
*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(448*a^16
*b^12*c^22*d^2 - 2816*a^17*b^11*c^21*d^3 + 7360*a^18*b^10*c^20*d^4 - 10240*
a^19*b^9*c^19*d^5 + 8000*a^20*b^8*c^18*d^6 - 3200*a^21*b^7*c^17*d^7 - 64*a^
22*b^6*c^16*d^8 + 1280*a^23*b^5*c^15*d^9 - 1280*a^24*b^4*c^14*d^10 + 640*a^
25*b^3*c^13*d^11 - 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a^9*b^7)^(
1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*b^8*c^21*
```

```
d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*b^4*c^17*
d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d^2 + a^9*
b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)))*(9*
a*d - 7*b*c)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d
)))/(((x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12
*c^18*d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*
b^9*c^15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^
4*c^10*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(44
8*a^16*b^12*c^22*d^2 - 2816*a^17*b^11*c^21*d^3 + 7360*a^18*b^10*c^20*d^4 -
10240*a^19*b^9*c^19*d^5 + 8000*a^20*b^8*c^18*d^6 - 3200*a^21*b^7*c^17*d^7 -
 64*a^22*b^6*c^16*d^8 + 1280*a^23*b^5*c^15*d^9 - 1280*a^24*b^4*c^14*d^10 +
640*a^25*b^3*c^13*d^11 - 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a^9*
b^7)^(1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*b^8
*c^21*d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*b^4
*c^17*d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d^2
+ a^9*b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d)
))*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*
c*d)) - ((x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b
^12*c^18*d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^
17*b^9*c^15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22
*b^4*c^10*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*
(2816*a^17*b^11*c^21*d^3 - 448*a^16*b^12*c^22*d^2 - 7360*a^18*b^10*c^20*d^4
 + 10240*a^19*b^9*c^19*d^5 - 8000*a^20*b^8*c^18*d^6 + 3200*a^21*b^7*c^17*d^
7 + 64*a^22*b^6*c^16*d^8 - 1280*a^23*b^5*c^15*d^9 + 1280*a^24*b^4*c^14*d^10
 - 640*a^25*b^3*c^13*d^11 + 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a
^9*b^7)^(1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22*
b^8*c^21*d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^26*
b^4*c^17*d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^11*d
^2 + a^9*b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c
*d)))*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10
*b*c*d)) + 784*a^12*b^12*c^15*d^7 - 2800*a^13*b^11*c^14*d^8 + 3312*a^14*b^1
0*c^13*d^9 - 1296*a^15*b^9*c^12*d^10 + 224*a^16*b^8*c^11*d^11 - 512*a^17*b^
7*c^10*d^12 + 288*a^18*b^6*c^9*d^13))*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*1i)/
(2*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.300 $\int \frac{1}{x^7\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=210

$$\frac{b^4(4bc-5ad)\log\left(a+bx^2\right)}{2a^5(bc-ad)^2}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{4a^3c^2x^4}-\frac{1}{6a^2cx^6}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(a^3d^3+2\right.}{}$$

Rubi [A] time = 0.25, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}-\frac{\log(x)\left(2a^2bcd^2+a^3d^3+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}-\frac{b^4}{2a^4\left(a+bx^2\right)(bc-ad)}+\frac{b^4(4bc-5ad)\log\left(a+bx^2\right)}{2a^5(bc-ad)^2}+\frac{ad+2bc}{4a^3c^2x^4}-\frac{1}{6a^2cx^6}+\frac{d^5\log\left(c+dx^2\right)}{2c^4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -1/(6*a^2*c*x^6) + (2*b*c + a*d)/(4*a^3*c^2*x^4) - (3*b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)/(2*a^4*c^3*x^2) - b^4/(2*a^4*(b*c - a*d)*(a + b*x^2)) - ((4*b^3*c
^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) + (b^4*(4*b
*c - 5*a*d)*Log[a + b*x^2])/(2*a^5*(b*c - a*d)^2) + (d^5*Log[c + d*x^2])/(2
*c^4*(b*c - a*d)^2)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(a+bx)^2(c+dx)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{a^2cx^4}+\frac{-2bc-ad}{a^3c^2x^3}+\frac{3b^2c^2+2abcd+a^2d^2}{a^4c^3x^2}+\frac{-4b^3c^3-3ab^2c^2d-}{a^5c^4}\right.\right.\\ &= -\frac{1}{6a^2cx^6}+\frac{2bc+ad}{4a^3c^2x^4}-\frac{3b^2c^2+2abcd+a^2d^2}{2a^4c^3x^2}-\frac{b^4}{2a^4(bc-ad)\left(a+bx^2\right)}-\frac{\left(4b^3c^3\right.}{}\end{aligned}$$

Mathematica [A] time = 0.28, size = 202, normalized size = 0.96

$$\frac{1}{12}\left(\frac{6b^4(4bc-5ad)\log\left(a+bx^2\right)}{a^5(bc-ad)^2}+\frac{6b^4}{a^4\left(a+bx^2\right)(ad-bc)}+\frac{3ad+6bc}{a^3c^2x^4}-\frac{2}{a^2cx^6}-\frac{6\left(a^2d^2+2abcd+3b^2c^2\right)}{a^4c^3x^2}-\frac{12\log(x)\left(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3\right)}{a^5c^4}+\frac{6d^5\log\left(c+dx^2\right)}{c^4(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (-2/(a^2*c*x^6) + (6*b*c + 3*a*d)/(a^3*c^2*x^4) - (6*(3*b^2*c^2 + 2*a*b*c*d
 + a^2*d^2))/(a^4*c^3*x^2) + (6*b^4)/(a^4*(-(b*c) + a*d)*(a + b*x^2)) - (12
*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4) +
(6*b^4*(4*b*c - 5*a*d)*Log[a + b*x^2])/(a^5*(b*c - a*d)^2) + (6*d^5*Log[c +
 d*x^2])/(c^4*(b*c - a*d)^2))/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^7*(a + b*x^2)^2*(c + d*x^2)), x]
```

fricas [B] time = 19.48, size = 410, normalized size = 1.95

$$-\frac{2\,a^4b^2c^5-4\,a^5bc^4d+2\,a^6c^3d^2+6\left(4\,ab^5c^5-5\,a^2b^4c^4d+a^5bcd^4\right)x^6+3\left(4\,a^2b^4c^5-5\,a^3b^3c^4d-a^5bc^2d^3+2\,a^6cd^4\right)x^4-\left(4\,a^3b^3c^5-5\,a^4b^2c^4d-2\,a^5bc^3d^2+3\,a^6c^2d^3\right)x^2-6\left(\left(4\,b^6c^5-5\,ab^5c^4d\right)x^8+\left(4\,ab^5c^5-5\,a^2b^4c^4d\right)x^6\right)\log\left(bx^2+a\right)-6\left(a^5bd^5x^8+a^6d^5x^6\right)\log\left(dx^2+c\right)+12\left(\left(4\,b^6c^5-5\,ab^5c^4d+a^5bd^5\right)x^8+\left(4\,ab^5c^5-5\,a^2b^4c^4d+a^6d^5\right)x^6\right)\log(x)}{12\left(\left(a^5b^3c^6-2\,a^6b^2c^5d+a^7bc^4d^2\right)x^8+\left(a^6b^2c^6-2\,a^7bc^5d+a^8c^4d^2\right)x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/12*(2*a^4*b^2*c^5 - 4*a^5*b*c^4*d + 2*a^6*c^3*d^2 + 6*(4*a*b^5*c^5 - 5*a
^2*b^4*c^4*d + a^5*b*c*d^4)*x^6 + 3*(4*a^2*b^4*c^5 - 5*a^3*b^3*c^4*d - a^5*
```

```
b*c^2*d^3 + 2*a^6*c*d^4)*x^4 - (4*a^3*b^3*c^5 - 5*a^4*b^2*c^4*d - 2*a^5*b*c
^3*d^2 + 3*a^6*c^2*d^3)*x^2 - 6*((4*b^6*c^5 - 5*a*b^5*c^4*d)*x^8 + (4*a*b^5
*c^5 - 5*a^2*b^4*c^4*d)*x^6)*log(b*x^2 + a) - 6*(a^5*b*d^5*x^8 + a^6*d^5*x^
6)*log(d*x^2 + c) + 12*((4*b^6*c^5 - 5*a*b^5*c^4*d + a^5*b*d^5)*x^8 + (4*a*
b^5*c^5 - 5*a^2*b^4*c^4*d + a^6*d^5)*x^6)*log(x))/((a^5*b^3*c^6 - 2*a^6*b^2
*c^5*d + a^7*b*c^4*d^2)*x^8 + (a^6*b^2*c^6 - 2*a^7*b*c^5*d + a^8*c^4*d^2)*x
^6)
```

giac [A] time = 0.40, size = 354, normalized size = 1.69

$$\frac{d^6\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^6d-2\,abc^5d^2+a^2c^4d^3\right)}+\frac{\left(4\,b^6c-5\,ab^5d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^5b^3c^2-2\,a^6b^2cd+a^7bd^2\right)}-\frac{4\,b^6cx^2-5\,ab^5dx^2+5\,ab^5c-6\,a^2b^4d}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)\left(bx^2+a\right)}-\frac{\left(4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2\,a^5c^4}+\frac{44\,b^3c^3x^6+33\,ab^2c^2dx^6+22\,a^2bcd^2x^6+11\,a^3d^3x^6-18\,ab^2c^3x^4-12\,a^2bc^2dx^4-6\,a^3cd^2x^4+6\,a^2bc^3x^2+3\,a^3c^2dx^2-2\,a^3c^3}{12\,a^5c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*d^6*log(abs(d*x^2 + c))/(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3) + 1/2
*(4*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^5*b^3*c^2 - 2*a^6*b^2*c*d + a
^7*b*d^2) - 1/2*(4*b^6*c*x^2 - 5*a*b^5*d*x^2 + 5*a*b^5*c - 6*a^2*b^4*d)/((a
^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*(b*x^2 + a)) - 1/2*(4*b^3*c^3 + 3*a*b^2
*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4) + 1/12*(44*b^3*c^3*x^6
 + 33*a*b^2*c^2*d*x^6 + 22*a^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 18*a*b^2*c^3*
x^4 - 12*a^2*b*c^2*d*x^4 - 6*a^3*c*d^2*x^4 + 6*a^2*b*c^3*x^2 + 3*a^3*c^2*d*
x^2 - 2*a^3*c^3)/(a^5*c^4*x^6)
```

maple [A] time = 0.03, size = 268, normalized size = 1.28

$$\frac{b^4d}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)a^3}-\frac{b^5c}{2\left(ad-bc\right)^2\left(b\,x^2+a\right)a^4}-\frac{5b^4d\ln\left(b\,x^2+a\right)}{2\left(ad-bc\right)^2a^4}+\frac{2b^5c\ln\left(b\,x^2+a\right)}{\left(ad-bc\right)^2a^5}+\frac{d^5\ln\left(d\,x^2+c\right)}{2\left(ad-bc\right)^2c^4}-\frac{d^3\ln(x)}{a^2c^4}-\frac{2b\,d^2\ln(x)}{a^3c^3}-\frac{3b^2d\ln(x)}{a^4c^2}-\frac{4b^3\ln(x)}{a^5c}-\frac{d^2}{2a^2c^3x^2}-\frac{bd}{a^3c^2x^2}-\frac{3b^2}{2a^4c\,x^2}+\frac{d}{4a^2c^2x^4}+\frac{b}{2a^3c\,x^4}-\frac{1}{6a^2c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/2*b^4/a^3/(a*d-b*c)^2/(b*x^2+a)*d-1/2*b^5/a^4/(a*d-b*c)^2/(b*x^2+a)*c-5/2
*b^4/a^4/(a*d-b*c)^2*ln(b*x^2+a)*d+2*b^5/a^5/(a*d-b*c)^2*ln(b*x^2+a)*c+1/2*
d^5/c^4/(a*d-b*c)^2*ln(d*x^2+c)-1/6/a^2/c/x^6+1/4/a^2/c^2/x^4*d+1/2/a^3/c/x
^4*b-1/2/a^2/c^3/x^2*d^2-1/a^3/c^2/x^2*b*d-3/2/a^4/c/x^2*b^2-1/a^2/c^4*ln(x
)*d^3-2/a^3/c^3*ln(x)*d^2*b-3/a^4/c^2*ln(x)*d*b^2-4/a^5/c*ln(x)*b^3
```

maxima [A] time = 1.17, size = 339, normalized size = 1.61

$$\frac{d^5\log\left(dx^2+c\right)}{2\left(b^2c^6-2\,abc^5d+a^2c^4d^2\right)}+\frac{\left(4\,b^5c-5\,ab^4d\right)\log\left(bx^2+a\right)}{2\left(a^5b^2c^2-2\,a^6bcd+a^7d^2\right)}-\frac{2\,a^3bc^3-2\,a^4c^2d+6\left(4\,b^4c^3-ab^3c^2d-a^2b^2cd^2-a^3bd^3\right)x^6+3\left(4\,ab^3c^3-a^2b^2c^2d-a^3bcd^2-2\,a^4d^3\right)x^4-\left(4\,a^2b^2c^3-a^3bc^2d-3\,a^4cd^2\right)x^2}{12\left(\left(a^4b^2c^4-a^5bc^3d\right)x^8+\left(a^5bc^4-a^6c^3d\right)x^6\right)}-\frac{\left(4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3\right)\log\left(x^2\right)}{2\,a^5c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*d^5*log(d*x^2 + c)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2) + 1/2*(4*b^5*c
 - 5*a*b^4*d)*log(b*x^2 + a)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2) - 1/12*(
2*a^3*b*c^3 - 2*a^4*c^2*d + 6*(4*b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 - a^
3*b*d^3)*x^6 + 3*(4*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 - 2*a^4*d^3)*x^
4 - (4*a^2*b^2*c^3 - a^3*b*c^2*d - 3*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*
c^3*d)*x^8 + (a^5*b*c^4 - a^6*c^3*d)*x^6) - 1/2*(4*b^3*c^3 + 3*a*b^2*c^2*d
+ 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4)
```

mupad [**B**] time = 1.23, size = 278, normalized size = 1.32

$$\frac{\ln\left(b\,x^2+a\right)\,\left(4\,b^5\,c-5\,a\,b^4\,d\right)}{2\,a^7\,d^2-4\,a^6\,b\,c\,d+2\,a^5\,b^2\,c^2}-\frac{\frac{1}{6\,a\,c}-\frac{x^2\,\left(3\,a\,d+4\,b\,c\right)}{12\,a^2\,c^2}+\frac{x^4\,\left(2\,a^2\,d^2+3\,a\,b\,c\,d+4\,b^2\,c^2\right)}{4\,a^3\,c^3}+\frac{x^6\,\left(a^3\,b\,d^3+a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d-4\,b^4\,c^3\right)}{2\,a^4\,c^3\,\left(a\,d-b\,c\right)}}{b\,x^8+a\,x^6}+\frac{d^5\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}-\frac{\ln\left(x\right)\,\left(a^3\,d^3+2\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+4\,b^3\,c^3\right)}{a^5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x)
```

```
[Out] (log(a + b*x^2)*(4*b^5*c - 5*a*b^4*d))/(2*a^7*d^2 + 2*a^5*b^2*c^2 - 4*a^6*b
*c*d) - (1/(6*a*c) - (x^2*(3*a*d + 4*b*c))/(12*a^2*c^2) + (x^4*(2*a^2*d^2 +
 4*b^2*c^2 + 3*a*b*c*d))/(4*a^3*c^3) + (x^6*(a^3*b*d^3 - 4*b^4*c^3 + a^2*b^
2*c*d^2 + a*b^3*c^2*d))/(2*a^4*c^3*(a*d - b*c)))/(a*x^6 + b*x^8) + (d^5*log
(c + d*x^2))/(2*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d)) - (log(x)*(a^3*d^3 +
 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2))/(a^5*c^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.301 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{x(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\,(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)^3}+\frac{\sqrt{c}\,(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)^3}$$

Rubi [A] time = 0.16, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 527, 522, 205}

$$\frac{x(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\,(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}\,(bc-ad)^3}+\frac{\sqrt{c}\,(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*c + a*d)*x)/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*x)/(2*b*(b*c - a*d)*(a
 + b*x^2)*(c + d*x^2)) - (Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[b]*(b*c - a*d)^3) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*Sqrt[d]*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx=\frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{ac+(-2bc-ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2b(bc-ad)}$$

$$=\frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)}+\frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{4abc^2-2bc(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{4bc(bc-ad)^2}$$

$$=\frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)}+\frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{(a(3bc+ad))\int \frac{1}{a+bx^2}}{2(bc-ad)^3}$$

$$=\frac{(bc+ad)x}{2b(bc-ad)^2\left(c+dx^2\right)}+\frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\sqrt{a}\,(3bc+ad)\tan^{-1}}{2\sqrt{b}\,(bc-ad)^3}$$

Mathematica [A] time = 0.21, size = 133, normalized size = 0.82

$$\frac{1}{2}\left(\frac{ax}{\left(a+bx^2\right)(bc-ad)^2}+\frac{cx}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{a}\,(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,(ad-bc)^3}+\frac{\sqrt{c}\,(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*x)/((b*c - a*d)^2*(a + b*x^2)) + (c*x)/((b*c - a*d)^2*(c + d*x^2)) + (S
qrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(-(b*c) + a*d)^3
```

```
) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d
)^3))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 1.56, size = 1407, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 +
a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3
*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2
*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3
*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2
 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*
d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - 2*((3*b^2*c*d + a*b*d
^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^
2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*
x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^
2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 +
3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 + 2*((b^2*c*d + 3*a*b*d^2)
*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c/
d)*arctan(d*x*sqrt(c/d)/c) - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c
*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b
^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
a^4*d^4)*x^2), 1/2*((b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 +
3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(a/b)*arct
an(b*x*sqrt(a/b)/a) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b
^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 2*
```

```
(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4
*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 +
(b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

giac [A] time = 0.34, size = 198, normalized size = 1.22

$$-\frac{\left(3\,abc + a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} + \frac{\left(bc^2 + 3\,acd\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{cd}} + \frac{bcx^3 + adx^3 + 2\,acx}{2\left(bdx^4 + bcx^2 + adx^2 + ac\right)\left(b^2c^2 - 2\,abcd + a^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c + a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/2*(b*c^2 + 3*a*c*d)*arctan(d*x/sqrt(c
*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1/2*
(b*c*x^3 + a*d*x^3 + 2*a*c*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))
```

maple [A] time = 0.02, size = 222, normalized size = 1.37

$$\frac{a^2dx}{2\left(ad-bc\right)^3\left(b\,x^2+a\right)}+\frac{a^2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^3\sqrt{ab}}-\frac{abcx}{2\left(ad-bc\right)^3\left(b\,x^2+a\right)}+\frac{3abc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^3\sqrt{ab}}+\frac{acdx}{2\left(ad-bc\right)^3\left(d\,x^2+c\right)}-\frac{3acd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^3\sqrt{cd}}-\frac{b\,c^2x}{2\left(ad-bc\right)^3\left(d\,x^2+c\right)}-\frac{b\,c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^3\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*a^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*a/(a*d-b*c)^3*x/(b*x^2+a)*b*c+1/2*a^2
/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+3/2*a/(a*d-b*c)^3/(a*b
)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*b*c+1/2*c/(a*d-b*c)^3*x/(d*x^2+c)*a*d-1/2
*c^2/(a*d-b*c)^3*x/(d*x^2+c)*b-3/2*c/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)
^(1/2)*d*x)*a*d-1/2*c^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b
```

maxima [A] time = 2.54, size = 249, normalized size = 1.54

$$-\frac{\left(3\,abc + a^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{ab}} + \frac{\left(bc^2 + 3\,acd\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\sqrt{cd}} + \frac{(bc + ad)x^3 + 2\,acx}{2\left(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + \left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x^4 + \left(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*b*c + a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/2*(b*c^2 + 3*a*c*d)*arctan(d*x/sqrt(c
*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1/2*
((b*c + a*d)*x^3 + 2*a*c*x)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c
```

```
^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^
2 + a^3*d^3)*x^2)
```

mupad [B] time = 1.84, size = 5395, normalized size = 33.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x^3*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (a*c*x)/(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/(a*c + x^2*(a*d + b*c) + b*d*x^4) - (atan((((-c*d)^(
1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^
2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)) - ((-c*d)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a
^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d
^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-c*d
)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 -
 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c
^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*
a^2*b*c*d^3)))*(3*a*d + b*c)*1i)/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2
- 3*a^2*b*c*d^3)) + ((-c*d)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*
d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + ((-c*d)^(1/2)*((4*a*b^8*c^7
*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b
^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5) + (x*(-c*d)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9
*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a
^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b
^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*
c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3
*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)))*(3*a*d + b*c)*1i)/(4*(a^3*d^4 -
 b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)))/(((13*a^2*b^3*c^3*d^2)/4 +
(13*a^3*b^2*c^2*d^3)/4 + (3*a*b^4*c^4*d)/4 + (3*a^4*b*c*d^4)/4)/(a^6*d^6 +
b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*
a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-c*d)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6
*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c
^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - ((-c*d)^(1/2)*((
4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4
 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
```

```
*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-c*d)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d
^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5
*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(
a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 +
 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^
3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)))*(3*a*d + b*c))/(4*(a
^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + ((-c*d)^(1/2)*((x*
(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2
*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3)) + ((-c*d)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^
6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a
^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-c*d)^(1/2)*(
3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b
^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 +
144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c
*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d
^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*
d^3))))*(-c*d)^(1/2)*(3*a*d + b*c)*1i)/(2*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^
2*d^2 - 3*a^2*b*c*d^3)) - (atan((((-a*b)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d +
 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4
*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - ((-a*b)^(1/2)*
((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d
^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-a*b)^(1/2)*(a*d + 3*b*c)*(16*a^7*b^2
*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c
^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8
*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*(a^4*d^4 + b^4*c^4
 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(
b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)))*(a*d + 3*b*c)*1i)/
(4*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) + ((-a*b)^(1/2)
*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^
3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3)) + ((-a*b)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b
^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 -
 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-a*b)^(1
/2)*(a*d + 3*b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*
a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d
^6 + 144*a^5*b^4*c^2*d^7))/(8*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*
b^3*c^2*d)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b
*c*d^3)))*(a*d + 3*b*c))/(4*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^
3*c^2*d)))*(a*d + 3*b*c)*1i)/(4*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*
```

```
a*b^3*c^2*d)))/(((13*a^2*b^3*c^3*d^2)/4 + (13*a^3*b^2*c^2*d^3)/4 + (3*a*b^4
*c^4*d)/4 + (3*a^4*b*c*d^4)/4)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((
-a*b)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4
+ 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*
c^3*d - 4*a^3*b*c*d^3)) - ((-a*b)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8
 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^
4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (
x*(-a*b)^(1/2)*(a*d + 3*b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*c^
6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80*a^
4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c
*d^2 - 3*a*b^3*c^2*d)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*
d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d
^2 - 3*a*b^3*c^2*d)))*(a*d + 3*b*c))/(4*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*
d^2 - 3*a*b^3*c^2*d)) + ((-a*b)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*
c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*
a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + ((-a*b)^(1/2)*((4*a*b^8
*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a
^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^
6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*
c^5*d - 6*a^5*b*c*d^5) + (x*(-a*b)^(1/2)*(a*d + 3*b*c)*(16*a^7*b^2*d^9 + 16
*b^9*c^7*d^2 - 80*a*b^8*c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 -
80*a^3*b^6*c^4*d^5 - 80*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(b^4*c^3
 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)*(a^4*d^4 + b^4*c^4 + 6*a^2*
b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(b^4*c^3 -
 a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)))*(a*d + 3*b*c))/(4*(b^4*c^3
- a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d))))*(-a*b)^(1/2)*(a*d + 3*b*c
)*1i)/(2*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.302 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=107

$$\frac{a}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{c}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{(ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{(ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{c}{2\left(c+dx^2\right)(bc-ad)^2}+\frac{(ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{(ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] a/(2*(b*c - a*d)^2*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2)) + ((b*c +
 a*d)*Log[a + b*x^2])/(2*(b*c - a*d)^3) - ((b*c + a*d)*Log[c + d*x^2])/(2*(
b*c - a*d)^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{ab}{(bc-ad)^2(a+bx)^2}+\frac{b(bc+ad)}{(bc-ad)^3(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)^2}-\frac{}{(b}\right.\right.\\ &= \frac{a}{2(bc-ad)^2\left(a+bx^2\right)}+\frac{c}{2(bc-ad)^2\left(c+dx^2\right)}+\frac{(bc+ad)\log\left(a+bx^2\right)}{2(bc-ad)^3}-\frac{(bc+ad}{2(}\end{aligned}$$

Mathematica [A] time = 0.10, size = 86, normalized size = 0.80

$$\frac{\frac{a(bc-ad)}{a+bx^2}+\frac{c(bc-ad)}{c+dx^2}+(ad+bc)\log\left(a+bx^2\right)-(ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d))/(c + d*x^2) + (b*c + a*d)*Lo
g[a + b*x^2] - (b*c + a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 1.40, size = 296, normalized size = 2.77

$$\frac{2\,abc^2-2\,a^2cd+\left(b^2c^2-a^2d^2\right)x^2+\left(\left(b^2cd+abd^2\right)x^4+abc^2+a^2cd+\left(b^2c^2+2\,abcd+a^2d^2\right)x^2\right)\log\left(bx^2+a\right)-\left(\left(b^2cd+abd^2\right)x^4+abc^2+a^2cd+\left(b^2c^2+2\,abcd+a^2d^2\right)x^2\right)\log\left(dx^2+c\right)}{2\left(ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x^4+\left(b^4c^4-2\,ab^3c^3d+2\,a^3bcd^3-a^4d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*c^2 - 2*a^2*c*d + (b^2*c^2 - a^2*d^2)*x^2 + ((b^2*c*d + a*b*d^2)
*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*x^2)*log(b*x^2 +
```

```
a) - ((b^2*c*d + a*b*d^2)*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d +
 a^2*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d
^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4
)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

giac [A] time = 0.39, size = 178, normalized size = 1.66

$$\frac{\frac{ab^3}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\left(bx^2+a\right)}-\frac{\left(b^3c+ab^2d\right)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^2cd}{(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b*x^2 + a)) - (b^3*c + a
*b^2*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^2*c*d/((b*c - a*d)^3*(b*c/(b*x^2 +
 a) - a*d/(b*x^2 + a) + d)))/b
```

maple [A] time = 0.02, size = 188, normalized size = 1.76

$$\frac{a^2d}{2\,(ad-bc)^3\left(b\,x^2+a\right)}-\frac{abc}{2\,(ad-bc)^3\left(b\,x^2+a\right)}+\frac{acd}{2\,(ad-bc)^3\left(d\,x^2+c\right)}-\frac{ad\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^3}+\frac{ad\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^3}-\frac{b\,c^2}{2\,(ad-bc)^3\left(d\,x^2+c\right)}-\frac{bc\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^3}+\frac{bc\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2/(a*d-b*c)^3*a^2/(b*x^2+a)*d-1/2*b/(a*d-b*c)^3*a/(b*x^2+a)*c-1/2/(a*d-b*
c)^3*ln(b*x^2+a)*a*d-1/2*b/(a*d-b*c)^3*ln(b*x^2+a)*c+1/2*d/(a*d-b*c)^3*ln(d
*x^2+c)*a+1/2/(a*d-b*c)^3*ln(d*x^2+c)*b*c+1/2*d/(a*d-b*c)^3*c/(d*x^2+c)*a-1
/2/(a*d-b*c)^3*c^2/(d*x^2+c)*b
```

maxima [B] time = 1.27, size = 228, normalized size = 2.13

$$\frac{(bc+ad)\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}-\frac{(bc+ad)\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{(bc+ad)x^2+2\,ac}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c + a*d)*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3) - 1/2*(b*c + a*d)*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3) + 1/2*((b*c + a*d)*x^2 + 2*a*c)/(a*b^2*c^3 - 2*a^2*b*c^2*
d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*
b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

mupad [B] time = 0.53, size = 522, normalized size = 4.88

$$\frac{b^2\,c^2\,x^2-a^2\,d^2\,x^2+2\,a\,b\,c^2-2\,a^2\,c\,d+a^2\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+b^2\,c^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a\,b\,c^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a^2\,c\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a\,b\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+b^2\,c\,d\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+a\,b\,c\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}}{-2\,a^4\,c\,d^3-2\,a^4\,d^4\,x^2+6\,a^3\,b\,c^2\,d^2+4\,a^3\,b\,c\,d^3\,x^2-2\,a^3\,b\,d^4\,x^4-6\,a^2\,b^2\,c^3\,d+6\,a^2\,b^2\,c\,d^3\,x^4+2\,a\,b^3\,c^4-4\,a\,b^3\,c^3\,d\,x^2-6\,a\,b^3\,c^2\,d^2\,x^4+2\,b^4\,c^4\,x^2+2\,b^4\,c^3\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] (b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d + a^2*d^2*x^2*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + b^2*c^2*x^2*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*c^2*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a^2*c*d*atan((a*d*x^
2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*d^2*x^4*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + b^2*c*d*x^4*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*c*d*x^2*atan((
a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i)/(2*a*b^3*c^4 - 2*
a^4*c*d^3 - 2*a^4*d^4*x^2 + 2*b^4*c^4*x^2 - 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d
^2 - 2*a^3*b*d^4*x^4 + 2*b^4*c^3*d*x^4 - 4*a*b^3*c^3*d*x^2 + 4*a^3*b*c*d^3*
x^2 - 6*a*b^3*c^2*d^2*x^4 + 6*a^2*b^2*c*d^3*x^4)
```

sympy [B] time = 4.03, size = 507, normalized size = 4.74

$$\frac{2ac+x^2\left(ad+bc\right)}{2a^3cd^2-4a^2bc^2d+2ab^2c^3+x^4\left(2a^2bd^3-4ab^2cd^2+2b^3c^2d\right)+x^2\left(2a^3d^3-2a^2bcd^2-2ab^2c^2d+2b^3c^3\right)}+\frac{\left(ad+bc\right)\log{\left(x^2+\frac{-\frac{a^4d^4(ad+bc)}{(ad-bc)^3}+\frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3}-\frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3}+a^2d^2+\frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3}+2abcd-\frac{b^4c^4(ad+bc)}{(ad-bc)^3}+b^2c^2}{2abd^2+2b^2cd}\right)}}{2\left(ad-bc\right)^3}-\frac{\left(ad+bc\right)\log{\left(x^2+\frac{\frac{a^4d^4(ad+bc)}{(ad-bc)^3}-\frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3}+\frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3}+a^2d^2-\frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3}+2abcd+\frac{b^4c^4(ad+bc)}{(ad-bc)^3}+b^2c^2}{2abd^2+2b^2cd}\right)}}{2\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] (2*a*c + x**2*(a*d + b*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*d + 2*a*b**2*c**3
 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*d*
*3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*c**3)) + (a*d + b*c)*log(x*
*2 + (-a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 + 4*a**3*b*c*d**3*(a*d + b*c)/(
a*d - b*c)**3 - 6*a**2*b**2*c**2*d**2*(a*d + b*c)/(a*d - b*c)**3 + a**2*d**
2 + 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 + 2*a*b*c*d - b**4*c**4*(a*d
 + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 + 2*b**2*c*d))/(2*(a*d - b*
c)**3) - (a*d + b*c)*log(x**2 + (a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 - 4*a
**3*b*c*d**3*(a*d + b*c)/(a*d - b*c)**3 + 6*a**2*b**2*c**2*d**2*(a*d + b*c)
/(a*d - b*c)**3 + a**2*d**2 - 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 +
2*a*b*c*d + b**4*c**4*(a*d + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 +
 2*b**2*c*d))/(2*(a*d - b*c)**3)
```

3.303 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{b}\,(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\,(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)^3}$$

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {471, 527, 522, 205}

$$-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{b}\,(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)^3}-\frac{\sqrt{d}\,(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*x)/((b*c - a*d)^2*(c + d*x^2))) - x/(2*(b*c - a*d)*(a + b*x^2)*(c + d*
x^2)) + (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c
 - a*d)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]
*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\int \frac{c-3dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2(bc-ad)}\\ &= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\int \frac{2c(bc+ad)-4bcdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{4c(bc-ad)^2}\\ &= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{(d(3bc+ad))\int \frac{1}{c+dx^2}\,dx}{2(bc-ad)^3} -\\ &= -\frac{dx}{(bc-ad)^2\left(c+dx^2\right)}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\sqrt{b}\,(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{2\sqrt{a}\,(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.17, size = 137, normalized size = 0.93

$$\frac{1}{2}\left(-\frac{bx}{\left(a+bx^2\right)(bc-ad)^2}-\frac{dx}{\left(c+dx^2\right)(bc-ad)^2}-\frac{\sqrt{b}\,(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)^3}-\frac{\sqrt{d}\,(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-((b*x)/((b*c - a*d)^2*(a + b*x^2))) - (d*x)/((b*c - a*d)^2*(c + d*x^2)) -
 (Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d
```

```
)^3) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c -
a*d)^3))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 1.95, size = 1387, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3
*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2
+ a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 +
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 -
3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^
2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c
*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + 2*((3*b^2*c*d + a*b
*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sq
rt(d/c)*arctan(x*sqrt(d/c)) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*
c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*
sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*
b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*
a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 -
 a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 - 2*((b^2*c*d + 3*a*b*d^2)*
x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a
)*arctan(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d +
(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^
3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^
2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d
^4)*x^2), -1/2*(2*(b^2*c*d - a*b*d^2)*x^3 - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*
b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(b/a)*arctan
(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c^2 - a
```

```
^2*d^2)*x)/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^
4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2
*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)]
```

giac [A] time = 0.36, size = 196, normalized size = 1.33

$$\frac{\left(b^{2}c+3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}-\frac{\left(3\,bcd+ad^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{cd}}-\frac{2\,bdx^{3}+bcx+adx}{2\,\left(bdx^{4}+bcx^{2}+adx^{2}+ac\right)\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c + 3*a*b*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) - 1/2*(3*b*c*d + a*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) - 1/2*(
2*b*d*x^3 + b*c*x + a*d*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2 -
2*a*b*c*d + a^2*d^2))
```

maple [A] time = 0.02, size = 222, normalized size = 1.51

$$-\frac{abdx}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)}-\frac{3abd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{3}\sqrt{ab}}-\frac{a\,d^{2}x}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)}+\frac{a\,d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{3}\sqrt{cd}}+\frac{b^{2}cx}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)}-\frac{b^{2}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{3}\sqrt{ab}}+\frac{bcdx}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)}+\frac{3bcd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^{3}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b/(a*d-b*c)^3*x/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3*x/(b*x^2+a)*c-3/2*b/
(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d-1/2*b^2/(a*d-b*c)^3/(
a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*a+1/
2*d/(a*d-b*c)^3*x/(d*x^2+c)*b*c+1/2*d^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c
*d)^(1/2)*d*x)*a+3/2*d/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b*
c
```

maxima [A] time = 2.42, size = 249, normalized size = 1.69

$$\frac{\left(b^{2}c+3\,abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{ab}}-\frac{\left(3\,bcd+ad^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{cd}}-\frac{2\,bdx^{3}+\left(bc+ad\right)x}{2\,\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{4}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c + 3*a*b*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) - 1/2*(3*b*c*d + a*d^2)*arctan(d*x/sqrt(c*
d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) - 1/2*(
2*b*d*x^3 + (b*c + a*d)*x)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^
```

```
2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2
 + a^3*d^3)*x^2)
```

mupad [**B**] time = 1.67, size = 5236, normalized size = 35.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] (atan((((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) - ((
-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^3
*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18*
a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-a*b)^(1/2)*
(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^
3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72
*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^
2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))
)*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)
))*(3*a*d + b*c)*1i)/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*
d^2)) + ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + (
(-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^
3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18
*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-a*b)^(1/2)
*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b
^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 7
2*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d
^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2
)))*(3*a*d + b*c)*1i)/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c
*d^2)))/(((3*a^2*b^3*d^5)/2 + (3*b^5*c^2*d^3)/2 + 5*a*b^4*c*d^4)/(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^
2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3) - ((-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a
*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 -
 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c
^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*
c*d^5) - (x*(-a*b)^(1/2)*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*
a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5
 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2
```

```
*b^2*c^2*d - 3*a^3*b*c*d^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b
^2*c^2*d - 3*a^3*b*c*d^2)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*
b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*
d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*
d - 4*a^3*b*c*d^3) + ((-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b
^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 1
0*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*
d^5) + (x*(-a*b)^(1/2)*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*
b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 -
40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b
^2*c^2*d - 3*a^3*b*c*d^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*
c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2
*c^2*d - 3*a^3*b*c*d^2)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^
2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b)^(1/2)*(3*a*d + b*c)*1i)/(2*(a^4*d^3 - a*
b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((x*(a*d + b*c))/(2*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*c +
 x^2*(a*d + b*c) + b*d*x^4) + (atan((((-c*d)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b
^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b
^3*c^3*d - 4*a^3*b*c*d^3) - ((-c*d)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 -
 10*a*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*
d^5 - 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*
b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a
^5*b*c*d^5) - (x*(-c*d)^(1/2)*(a*d + 3*b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2
- 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4
*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(b^3*c^4 - a^3*c*d^3 +
3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4
*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(b^3*c^4 - a^3*c*d^3 + 3*
a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(a*d + 3*b*c)*1i)/(4*(b^3*c^4 - a^3*c*d^3
+ 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + ((-c*d)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*
b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*
b^3*c^3*d - 4*a^3*b*c*d^3) + ((-c*d)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2
- 10*a*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4
*d^5 - 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2
*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*
a^5*b*c*d^5) + (x*(-c*d)^(1/2)*(a*d + 3*b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2
 - 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^
4*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*(b^3*c^4 - a^3*c*d^3 +
 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(b^3*c^4 - a^3*c*d^3 + 3
*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(a*d + 3*b*c)*1i)/(4*(b^3*c^4 - a^3*c*d^3
 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))/(((3*a^2*b^3*d^5)/2 + (3*b^5*c^2*d^3)
/2 + 5*a*b^4*c*d^4)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-c*d)^(1/2)
```

```
*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 +
6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) - ((-c*d)^(1/2)*((2*a^7*
b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*
c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^
6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*
d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-c*d)^(1/2)*(a*d + 3*b*c)*(8*a^7
*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7
*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(
4*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^4*d^4 + b^4*c^
4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*
(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(a*d + 3*b*c))/(4
*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + ((-c*d)^(1/2)*(
(x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*
a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + ((-c*d)^(1/2)*((2*a^7*b^
2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^
5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (x*(-c*d)^(1/2)*(a*d + 3*b*c)*(8*a^7*b
^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c
^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*d^6 + 72*a^5*b^4*c^2*d^7))/(4*
(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(a*d + 3*b*c))/(4*(b
^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(a*d + 3*b*c))/(4*(
b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d))))*(-c*d)^(1/2)*(a*d
 + 3*b*c)*1i)/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.304 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=92

$$-\frac{b}{2\left(a+bx^2\right)(bc-ad)^2}-\frac{d}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{bd\log\left(a+bx^2\right)}{(bc-ad)^3}+\frac{bd\log\left(c+dx^2\right)}{(bc-ad)^3}$$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 44}

$$-\frac{b}{2\left(a+bx^2\right)(bc-ad)^2}-\frac{d}{2\left(c+dx^2\right)(bc-ad)^2}-\frac{bd\log\left(a+bx^2\right)}{(bc-ad)^3}+\frac{bd\log\left(c+dx^2\right)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x^2)) - d/(2*(b*c - a*d)^2*(c + d*x^2)) - (b*d*L
og[a + b*x^2])/(b*c - a*d)^3 + (b*d*Log[c + d*x^2])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\right.\right.\\ &= -\frac{b}{2(bc-ad)^2\left(a+bx^2\right)}-\frac{d}{2(bc-ad)^2\left(c+dx^2\right)}-\frac{bd\log\left(a+bx^2\right)}{(bc-ad)^3}+\end{aligned}$$

Mathematica [A]   time = 0.08, size = 77, normalized size = 0.84

$$\frac{\frac{b(ad-bc)}{a+bx^2}+\frac{d(ad-bc)}{c+dx^2}-2bd\log\left(a+bx^2\right)+2bd\log\left(c+dx^2\right)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x^2) + (d*(-(b*c) + a*d))/(c + d*x^2) - 2*b*d*Lo
g[a + b*x^2] + 2*b*d*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B]   time = 1.05, size = 253, normalized size = 2.75

$$-\frac{b^2c^2-a^2d^2+2\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)\log\left(bx^2+a\right)-2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)\log\left(dx^2+c\right)}{2\left(ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4\right)x^4+\left(b^4c^4-2\,ab^3c^3d+2\,a^3bcd^3-a^4d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + a*b*c*d +
```

```
(b^2*c*d + a*b*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a
^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 -
 a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

giac [A] time = 0.37, size = 163, normalized size = 1.77

$$\frac{b^2 d \log\left(\left|\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^3}{2\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)\left(bx^2 + a\right)} + \frac{bd^2}{2\,(bc - ad)^3\left(\frac{bc}{bx^2+a} - \frac{ad}{bx^2+a} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*(b*x^2 + a)) + 1/2*b*d^2/((b*c - a*d)^3*(b*c/(b*x^2 + a) - a*d/(b*
x^2 + a) + d))
```

maple [A] time = 0.02, size = 143, normalized size = 1.55

$$-\frac{abd}{2\,(ad-bc)^3\left(b\,x^2+a\right)} - \frac{a\,d^2}{2\,(ad-bc)^3\left(d\,x^2+c\right)} + \frac{b^2c}{2\,(ad-bc)^3\left(b\,x^2+a\right)} + \frac{bcd}{2\,(ad-bc)^3\left(d\,x^2+c\right)} + \frac{bd\ln\left(b\,x^2+a\right)}{(ad-bc)^3} - \frac{bd\ln\left(d\,x^2+c\right)}{(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b/(a*d-b*c)^3/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3/(b*x^2+a)*c+b/(a*d-b*c
)^3*ln(b*x^2+a)*d-d/(a*d-b*c)^3*b*ln(d*x^2+c)-1/2*d^2/(a*d-b*c)^3/(d*x^2+c)
*a+1/2*d/(a*d-b*c)^3/(d*x^2+c)*b*c
```

maxima [B] time = 1.06, size = 215, normalized size = 2.34

$$-\frac{bd\log\left(bx^2+a\right)}{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3} + \frac{bd\log\left(dx^2+c\right)}{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3} - \frac{2\,bdx^2 + bc + ad}{2\left(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + \left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x^4 + \left(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -b*d*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + b
*d*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2
*(2*b*d*x^2 + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*
d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
 a^3*d^3)*x^2)
```

mupad [B] time = 0.29, size = 378, normalized size = 4.11

$$-\frac{b^2\,c^2-a^2\,d^2+b^2\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-2\,a\,b\,d^2\,x^2+2\,b^2\,c\,d\,x^2+a\,b\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+b^2\,c\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,c\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}}{-2\,a^4\,c\,d^3-2\,a^4\,d^4\,x^2+6\,a^3\,b\,c^2\,d^2+4\,a^3\,b\,c\,d^3\,x^2-2\,a^3\,b\,d^4\,x^4-6\,a^2\,b^2\,c^3\,d+6\,a^2\,b^2\,c\,d^3\,x^4+2\,a\,b^3\,c^4-4\,a\,b^3\,c^3\,d\,x^2-6\,a\,b^3\,c^2\,d^2\,x^4+2\,b^4\,c^4\,x^2+2\,b^4\,c^3\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] -(b^2*c^2 - a^2*d^2 + b^2*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a
*d*x^2 + b*c*x^2))*4i - 2*a*b*d^2*x^2 + 2*b^2*c*d*x^2 + a*b*d^2*x^2*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + b^2*c*d*x^2*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*c*d*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i)/(2*a*b^3*c^4 - 2*a
^4*c*d^3 - 2*a^4*d^4*x^2 + 2*b^4*c^4*x^2 - 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^
2 - 2*a^3*b*d^4*x^4 + 2*b^4*c^3*d*x^4 - 4*a*b^3*c^3*d*x^2 + 4*a^3*b*c*d^3*x
^2 - 6*a*b^3*c^2*d^2*x^4 + 6*a^2*b^2*c*d^3*x^4)
```

sympy [B] time = 2.76, size = 410, normalized size = 4.46

$$-\frac{bd\log\left(x^2+\frac{-\frac{a^4bd^5}{(ad-bc)^3}+\frac{4a^3b^2cd^4}{(ad-bc)^3}-\frac{6a^2b^3c^2d^3}{(ad-bc)^3}+\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2-\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}+\frac{bd\log\left(x^2+\frac{\frac{a^4bd^5}{(ad-bc)^3}-\frac{4a^3b^2cd^4}{(ad-bc)^3}+\frac{6a^2b^3c^2d^3}{(ad-bc)^3}-\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2+\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}+\frac{-ad-bc-2bdx^2}{2a^3cd^2-4a^2bc^2d+2ab^2c^3+x^4\left(2a^2bd^3-4ab^2cd^2+2b^3c^2d\right)+x^2\left(2a^3d^3-2a^2bcd^2-2ab^2c^2d+2b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] -b*d*log(x**2 + (-a**4*b*d**5/(a*d - b*c)**3 + 4*a**3*b**2*c*d**4/(a*d - b*
c)**3 - 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 4*a*b**4*c**3*d**2/(a*d - b*
c)**3 + a*b*d**2 - b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b**2*d**2))/(a
*d - b*c)**3 + b*d*log(x**2 + (a**4*b*d**5/(a*d - b*c)**3 - 4*a**3*b**2*c*d
**4/(a*d - b*c)**3 + 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 4*a*b**4*c**3*d
**2/(a*d - b*c)**3 + a*b*d**2 + b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b
**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x**2)/(2*a**3*c*d**2 - 4*a*
*2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**
3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*
c**3))
```

3.305 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad}$$

Rubi [A] time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{dx(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{-bc+2ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)} \\
&= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{-2\left(b^2c^2-4abcd+a^2d^2\right)-}{\left(a+bx^2\right)\left(c+d\right.}}{4ac(bc-a} \\
&= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\left(b^2(bc-5ad)\right)\int \frac{1}{a+b}}{2a(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{b^{3/2}(bc-5ad)\tan^{-1}}{2a^{3/2}(bc-ad)^3}
\end{aligned}
$$

Mathematica [A] time = 0.33, size = 136, normalized size = 0.81

$$
\frac{1}{2}\left(\frac{b^{3/2}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}+\frac{x(bc-ad)\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 3.06, size = 1681, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
```

```
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

giac [**A**] time = 0.45, size = 232, normalized size = 1.39

$$\frac{\left(b^{3} c - 5 \, a b^{2} d\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, {\left(a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + 3 \, a^{3} b c d^{2} - a^{4} d^{3}\right)} \sqrt{a b}} + \frac{\left(5 \, b c d^{2} - a d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \, {\left(b^{3} c^{4} - 3 \, a b^{2} c^{3} d + 3 \, a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right)} \sqrt{c d}} + \frac{b^{2} c d x^{3} + a b d^{2} x^{3} + b^{2} c^{2} x + a^{2} d^{2} x}{2 \, {\left(a b^{2} c^{3} - 2 \, a^{2} b c^{2} d + a^{3} c d^{2}\right)} {\left(b d x^{4} + b c x^{2} + a d x^{2} + a c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

maple [**A**] time = 0.02, size = 238, normalized size = 1.43

$$\frac{a \, d^{3} x}{2 \left(a d - b c\right)^{3} \left(d \, x^{2} + c\right) c} + \frac{a \, d^{3} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{c d} \; c} - \frac{b^{3} c x}{2 \left(a d - b c\right)^{3} \left(b \, x^{2} + a\right) a} - \frac{b^{3} c \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{a b} \; a} + \frac{b^{2} d x}{2 \left(a d - b c\right)^{3} \left(b \, x^{2} + a\right)} + \frac{5 b^{2} d \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{a b}} - \frac{b \, d^{2} x}{2 \left(a d - b c\right)^{3} \left(d \, x^{2} + c\right)} - \frac{5 b \, d^{2} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \left(a d - b c\right)^{3} \sqrt{c d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3*x/a/(b*x^2+a)*c+5/2*b
^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-1/2*b^3/(a*d-b*c)^3/
a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+1/2*d^3/(a*d-b*c)^3/c*x/(d*x^2+c)
*a-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+c)*b+1/2*d^3/(a*d-b*c)^3/c/(c*d)^(1/2)*arct
an(1/(c*d)^(1/2)*d*x)*a-5/2*d^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2
)*d*x)*b
```

maxima [**B**] time = 2.55, size = 294, normalized size = 1.76

$$\frac{{\left(b^{3} c - 5 \, a b^{2} d\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, {\left(a b^{3} c^{3} - 3 \, a^{2} b^{2} c^{2} d + 3 \, a^{3} b c d^{2} - a^{4} d^{3}\right)} \sqrt{a b}} + \frac{{\left(5 \, b c d^{2} - a d^{3}\right)} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 \, {\left(b^{3} c^{4} - 3 \, a b^{2} c^{3} d + 3 \, a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right)} \sqrt{c d}} + \frac{{\left(b^{2} c d + a b d^{2}\right)} x^{3} + {\left(b^{2} c^{2} + a^{2} d^{2}\right)} x}{2 \, {\left(a^{2} b^{2} c^{4} - 2 \, a^{3} b c^{3} d + a^{4} c^{2} d^{2} + {\left(a b^{3} c^{3} d - 2 \, a^{2} b^{2} c^{2} d^{2} + a^{3} b c d^{3}\right)} x^{4} + {\left(a b^{3} c^{4} - a^{2} b^{2} c^{3} d - a^{3} b c^{2} d^{2} + a^{4} c d^{3}\right)} x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
```

```
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d
^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

mupad [B] time = 2.07, size = 6183, normalized size = 37.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3
*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*
c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 -
 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*
a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2
 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7
*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 -
 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b
*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) -
(x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3
+ 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b
^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^
3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3
*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3
)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*
a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 1
0*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^
3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 +
 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c
^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 2
0*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c
^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x
*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 +
144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4
*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*
c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b
^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^
(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*
d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d -
 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^6*c^2*d
^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d
- 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
```

```
4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*
d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d
- 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c
*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220
*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^
2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*
a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(5*a*d - b*
c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*
c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 8
0*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4
*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a
^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 +
 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a
^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10
 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220*a^5*
b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^2*d^9
)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b
^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(5*a*d - b*c)*(-
a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*c^7*d
^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 80*a^8
*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c
^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*
b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^
3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(
5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2)) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^
3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d
^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*
c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*
a^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^
3*d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d -
6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*
d^4) - (x*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c
^8*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 14
4*a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 -
 a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4
- 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-
c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d
)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*
c^4*d^2 - 3*a*b^2*c^5*d)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*
d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4
```

```
 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^
9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^
4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*
d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*
a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^
4) + (x*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8
*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*
a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a
^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 -
4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^
3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))
)*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^
4*d^2 - 3*a*b^2*c^5*d)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^
6*c^2*d^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5
*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6
*b^2*c^4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3
*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3
*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^
9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^
5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8
*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^
5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(a*d
 - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a
^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*
d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a^3*c^3*d^3 +
 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^
5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)
)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*(a*d - 5*b
*c)*(-c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2
*c^5*d)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*
c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*
d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2
*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 2
20*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*
c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 1
5*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(a*d - 5*
b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^
7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 -
 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a^3*c^3*d^3 + 3*a^
2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d -
 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2))/(4*
(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*(a*d - 5*b*c)*(
-c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*
d))))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*
b*c^4*d^2 - 3*a*b^2*c^5*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.306 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^3}+\frac{d^2}{2c\left(c+dx^2\right)(bc-ad)}$$

Rubi [A] time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^3}+\frac{\log(x)}{a^2c^2}+\frac{b^2}{2a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{2c^2(bc-ad)^3}+\frac{d^2}{2c\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] b^2/(2*a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c + d*x^2)) +
 Log[x]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^3
) - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(2*c^2*(b*c - a*d)^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{1}{a^2c^2x} - \frac{b^3}{a(-bc+ad)^2(a+bx)^2} - \frac{b^3(-bc+3ad)}{a^2(-bc+ad)^3(a+bx)} - \frac{}{c(bc-}\right.\right. \\
&= \frac{b^2}{2a(bc-ad)^2\left(a+bx^2\right)} + \frac{d^2}{2c(bc-ad)^2\left(c+dx^2\right)} + \frac{\log(x)}{a^2c^2} - \frac{b^2(bc-3ad)\log\left(a\right.}{2a^2(bc-ad)}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 133, normalized size = 0.94

$$\frac{1}{2}\left(\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{a^2(ad-bc)^3} + \frac{2\log(x)}{a^2c^2} + \frac{b^2}{a\left(a+bx^2\right)(bc-ad)^2} + \frac{d^2(ad-3bc)\log\left(c+dx^2\right)}{c^2(bc-ad)^3} + \frac{d^2}{c\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (b^2/(a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^2)) + (2
*Log[x])/(a^2*c^2) + (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)
^3) + (d^2*(-3*b*c + a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^3))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 11.97, size = 540, normalized size = 3.83

$$\frac{ab^3c^4 - a^2b^2c^3d + a^3bc^2d^2 - a^4cd^3 + \left(ab^3c^3d - a^3bcd^3\right)x^2 - \left(ab^3c^4 - 3\,a^2b^2c^3d + \left(b^4c^3d - 3\,ab^3c^2d^2\right)x^4 + \left(b^4c^4 - 2\,ab^3c^3d - 3\,a^2b^2c^2d^2\right)x^2\right)\log\left(bx^2+a\right) - \left(3\,a^3bc^2d^2 - a^4cd^3 + \left(3\,a^2b^2cd^3 - a^3bd^4\right)x^4 + \left(3\,a^2b^2c^2d^2 + 2\,a^3bcd^3 - a^4d^4\right)x^2\right)\log\left(dx^2+c\right) + 2\left(ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + \left(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4\right)x^4 + \left(b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4\right)x^2\right)\log(x)}{2\left(a^3b^3c^6 - 3\,a^4b^2c^5d + 3\,a^5bc^4d^2 - a^6c^3d^3 + \left(a^2b^4c^5d - 3\,a^3b^3c^4d^2 + 3\,a^4b^2c^3d^3 - a^5bc^2d^4\right)x^4 + \left(a^2b^4c^6 - 2\,a^3b^3c^5d + 2\,a^5bc^3d^3 - a^6c^2d^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*b^3*c^4 - a^2*b^2*c^3*d + a^3*b*c^2*d^2 - a^4*c*d^3 + (a*b^3*c^3*d -
 a^3*b*c*d^3)*x^2 - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2
```

```
*d^2)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2)*x^2)*log(b*x^2 +
a) - (3*a^3*b*c^2*d^2 - a^4*c*d^3 + (3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (3*
a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(d*x^2 + c) + 2*(a*b^3*c
^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^
3*b*c*d^3 - a^4*d^4)*x^2)*log(x))/(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*
c^4*d^2 - a^6*c^3*d^3 + (a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*
d^3 - a^5*b*c^2*d^4)*x^4 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3
 - a^6*c^2*d^4)*x^2)
```

giac [B] time = 0.39, size = 321, normalized size = 2.28

$$-\frac{\left(b^{4} c - 3 \, a b^{3} d\right) \log\left({\left| b x^{2} + a \right|}\right)}{2 \, \left(a^{2} b^{4} c^{3} - 3 \, a^{3} b^{3} c^{2} d + 3 \, a^{4} b^{2} c d^{2} - a^{5} b d^{3}\right)} - \frac{\left(3 \, b c d^{3} - a d^{4}\right) \log\left({\left| d x^{2} + c \right|}\right)}{2 \, \left(b^{3} c^{5} d - 3 \, a b^{2} c^{4} d^{2} + 3 \, a^{2} b c^{3} d^{3} - a^{3} c^{2} d^{4}\right)} + \frac{b^{3} c^{2} d x^{4} - 2 \, a b^{2} c d^{2} x^{4} + a^{2} b d^{3} x^{4} + b^{3} c^{3} x^{2} + a b^{2} c^{2} d x^{2} + a^{2} b c d^{2} x^{2} + a^{3} d^{3} x^{2} + 3 \, a b^{2} c^{3} - 2 \, a^{2} b c^{2} d + 3 \, a^{3} c d^{2}}{4 \, \left(a^{2} b^{2} c^{4} - 2 \, a^{3} b c^{3} d + a^{4} c^{2} d^{2}\right)\left(b d x^{4} + b c x^{2} + a d x^{2} + a c\right)} + \frac{\log\left(x^{2}\right)}{2 \, a^{2} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d
 + 3*a^4*b^2*c*d^2 - a^5*b*d^3) - 1/2*(3*b*c*d^3 - a*d^4)*log(abs(d*x^2 + c
))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4) + 1/4*(b^3
*c^2*d*x^4 - 2*a*b^2*c*d^2*x^4 + a^2*b*d^3*x^4 + b^3*c^3*x^2 + a*b^2*c^2*d*
x^2 + a^2*b*c*d^2*x^2 + a^3*d^3*x^2 + 3*a*b^2*c^3 - 2*a^2*b*c^2*d + 3*a^3*c
*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^4 + b*c*x^2 + a*d
*x^2 + a*c)) + 1/2*log(x^2)/(a^2*c^2)
```

maple [A] time = 0.03, size = 225, normalized size = 1.60

$$\frac{a \,d^{3}}{2 \left(a d -b c \right)^{3} \left(d \,x^{2}+c \right) c}-\frac{a \,d^{3} \ln \left(d \,x^{2}+c \right)}{2 \left(a d -b c \right)^{3} c^{2}}-\frac{b^{3} c}{2 \left(a d -b c \right)^{3} \left(b \,x^{2}+a \right) a}-\frac{3 b^{2} d \ln \left(b \,x^{2}+a \right)}{2 \left(a d -b c \right)^{3} a}+\frac{b^{3} c \ln \left(b \,x^{2}+a \right)}{2 \left(a d -b c \right)^{3} a^{2}}+\frac{b^{2} d}{2 \left(a d -b c \right)^{3} \left(b \,x^{2}+a \right)}+\frac{3 b \,d^{2} \ln \left(d \,x^{2}+c \right)}{2 \left(a d -b c \right)^{3} c}-\frac{b \,d^{2}}{2 \left(a d -b c \right)^{3} \left(d \,x^{2}+c \right)}+\frac{\ln \left(x \right)}{a^{2} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^3/(b*x^2+a)*d-1/2*b^3/a/(a*d-b*c)^3/(b*x^2+a)*c-3/2*b^2/a
/(a*d-b*c)^3*ln(b*x^2+a)*d+1/2*b^3/a^2/(a*d-b*c)^3*ln(b*x^2+a)*c-1/2*d^3/c^
2/(a*d-b*c)^3*ln(d*x^2+c)*a+3/2*d^2/c/(a*d-b*c)^3*ln(d*x^2+c)*b+1/2*d^3/c/(
a*d-b*c)^3/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3/(d*x^2+c)*b+ln(x)/a^2/c^2
```

maxima [B] time = 1.20, size = 295, normalized size = 2.09

$$-\frac{{\left(b^{3} c - 3 \, a b^{2} d\right)} \log\left(b x^{2} + a\right)}{2 \, {\left(a^{2} b^{3} c^{3} - 3 \, a^{3} b^{2} c^{2} d + 3 \, a^{4} b c d^{2} - a^{5} d^{3}\right)}} - \frac{{\left(3 \, b c d^{2} - a d^{3}\right)} \log\left(d x^{2} + c\right)}{2 \, {\left(b^{3} c^{5} - 3 \, a b^{2} c^{4} d + 3 \, a^{2} b c^{3} d^{2} - a^{3} c^{2} d^{3}\right)}} + \frac{b^{2} c^{2} + a^{2} d^{2} + {\left(b^{2} c d + a b d^{2}\right)} x^{2}}{2 \, {\left(a^{2} b^{2} c^{4} - 2 \, a^{3} b c^{3} d + a^{4} c^{2} d^{2} + {\left(a b^{3} c^{3} d - 2 \, a^{2} b^{2} c^{2} d^{2} + a^{3} b c d^{3}\right)} x^{4} + {\left(a b^{3} c^{4} - a^{2} b^{2} c^{3} d - a^{3} b c^{2} d^{2} + a^{4} c d^{3}\right)} x^{2}\right)}} + \frac{\log\left(x^{2}\right)}{2 \, a^{2} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - 3*a*b^2*d)*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*
a^4*b*c*d^2 - a^5*d^3) - 1/2*(3*b*c*d^2 - a*d^3)*log(d*x^2 + c)/(b^3*c^5 -
3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/2*(b^2*c^2 + a^2*d^2 + (
b^2*c*d + a*b*d^2)*x^2)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3
*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d
- a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/2*log(x^2)/(a^2*c^2)
```

mupad [B] time = 1.44, size = 193, normalized size = 1.37

$$\frac{\frac{a^2\,d^2+b^2\,c^2}{2\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x^2\,\left(a\,d+b\,c\right)}{2\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c}+\frac{\ln\left(x\right)}{a^2\,c^2}-\frac{b^2\,\ln\left(b\,x^2+a\right)\,\left(3\,a\,d-b\,c\right)}{2\,a^2\,{\left(a\,d-b\,c\right)}^3}-\frac{d^2\,\ln\left(d\,x^2+c\right)\,\left(a\,d-3\,b\,c\right)}{2\,c^2\,{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2)/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^2*(a*
d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*c) +
 b*d*x^4) + log(x)/(a^2*c^2) - (b^2*log(a + b*x^2)*(3*a*d - b*c))/(2*a^2*(a
*d - b*c)^3) - (d^2*log(c + d*x^2)*(a*d - 3*b*c))/(2*c^2*(a*d - b*c)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.307 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{d^{5/2}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc}$$

Rubi [A] time = 0.31, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{3a^2d^2-4abcd+3b^2c^2}{2a^2c^2x(bc-ad)^2}-\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{d^{5/2}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(3*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*x) + (d*(b*c
+ a*d))/(2*a*c*(b*c - a*d)^2*x*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x*(a + b*x
^2)*(c + d*x^2)) - (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(2*c^(5/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-3bc+2ad-5bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)} \\
&= \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2(3b^2c^2-4abcd+3}{x^2(a+bx}}{4ac(b} \\
&= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)(c} \\
&= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)(c} \\
&= -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)(c}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 158, normalized size = 0.72

$$
\frac{1}{2}\left(\frac{b^{5/2}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3} - \frac{b^3x}{a^2\left(a+bx^2\right)(bc-ad)^2} - \frac{2}{a^2c^2x} + \frac{d^{5/2}(3ad-7bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3} - \frac{d^3x}{c^2\left(c+dx^2\right)(bc-ad)^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x) - (b^3*x)/(a^2*(b*c - a*d)^2*(a + b*x^2)) - (d^3*x)/(c^2*(b
*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(a^(5/2)*(-(b*c) + a*d)^3) + (d^(5/2)*(-7*b*c + 3*a*d)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [B] time = 6.15, size = 2113, normalized size = 9.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*
(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*
b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 - ((3*b^4*c^3*d -
7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3
+ (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/
a) - a)/(b*x^2 + a)) - ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^
2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)
*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^4*c^5*
d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c
^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 -
3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x), -1/4*(4*a*b^3*c^4 - 12
*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*(3*b^4*c^3*d - 7*a*b^3*
c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*b^4*c^4 - 5*a*b^3*c^3*d
 + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5
+ (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 -
3*a^4*c*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - ((3*b^4*c^3*d - 7*a*b^3*c^2
*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*
c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*
c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d
^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x
), -1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 +
2*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(
3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((3*b^4*c^3*
d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*
x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((
7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3
*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(-d/c)*log((d*x^2 -
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3
*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*
a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b
*c^4*d^2 - a^6*c^3*d^3)*x), -1/2*(2*a*b^3*c^4 - 6*a^2*b^2*c^3*d + 6*a^3*b*c
^2*d^2 - 2*a^4*c*d^3 + (3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3
*a^3*b*d^4)*x^4 + (3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x
^2 + ((3*b^4*c^3*d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*
a^2*b^2*c^2*d^2)*x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(b/a)*arctan(
x*sqrt(b/a)) + ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 +
```

```
4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(d/
c)*arctan(x*sqrt(d/c)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3
*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^
3 - a^6*c^2*d^4)*x^3 + (a^3*b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a
^6*c^3*d^3)*x)]
```

giac [A]   time = 0.41, size = 321, normalized size = 1.47

$$-\frac{\left(3\,b^4c-7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\sqrt{ab}}-\frac{\left(7\,bcd^3-3\,ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\sqrt{cd}}-\frac{3\,b^3c^2dx^4-4\,ab^2cd^2x^4+3\,a^2bd^3x^4+3\,b^3c^3x^2-2\,ab^2c^2dx^2-2\,a^2bcd^2x^2+3\,a^3d^3x^2+2\,ab^2c^3-4\,a^2bc^2d+2\,a^3cd^2}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(bdx^5+bcx^3+adx^3+acx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*
c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - 3*a*d^4)*arc
tan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^
3)*sqrt(c*d)) - 1/2*(3*b^3*c^2*d*x^4 - 4*a*b^2*c*d^2*x^4 + 3*a^2*b*d^3*x^4
+ 3*b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 - 2*a^2*b*c*d^2*x^2 + 3*a^3*d^3*x^2 + 2
*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a
^4*c^2*d^2)*(b*d*x^5 + b*c*x^3 + a*d*x^3 + a*c*x))
```

maple [A]   time = 0.02, size = 261, normalized size = 1.20

$$-\frac{a\,d^4x}{2\,(ad-bc)^3\left(d\,x^2+c\right)c^2}-\frac{3a\,d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(ad-bc)^3\,\sqrt{cd}\,c^2}-\frac{b^3dx}{2\,(ad-bc)^3\left(b\,x^2+a\right)a}-\frac{7b^3d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ad-bc)^3\,\sqrt{ab}\,a}+\frac{b^4cx}{2\,(ad-bc)^3\left(b\,x^2+a\right)a^2}+\frac{3b^4c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ad-bc)^3\,\sqrt{ab}\,a^2}+\frac{b\,d^3x}{2\,(ad-bc)^3\left(d\,x^2+c\right)c}+\frac{7b\,d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(ad-bc)^3\,\sqrt{cd}\,c}-\frac{1}{a^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b^3/a/(a*d-b*c)^3*x/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3*x/(b*x^2+a)*c-
7/2*b^3/a/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+3/2*b^4/a^2/(
a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-1/2*d^4/c^2/(a*d-b*c)^3*
x/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3*x/(d*x^2+c)*b-3/2*d^4/c^2/(a*d-b*c)^3/(
c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a+7/2*d^3/c/(a*d-b*c)^3/(c*d)^(1/2)*ar
ctan(1/(c*d)^(1/2)*d*x)*b-1/a^2/c^2/x
```

maxima [A]   time = 2.50, size = 378, normalized size = 1.73

$$-\frac{\left(3\,b^4c-7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\sqrt{ab}}-\frac{\left(7\,bcd^3-3\,ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\sqrt{cd}}-\frac{2\,ab^2c^3-4\,a^2bc^2d+2\,a^3cd^2+\left(3\,b^3c^2d-4\,ab^2cd^2+3\,a^2bd^3\right)x^4+\left(3\,b^3c^3-2\,ab^2c^2d-2\,a^2bcd^2+3\,a^3d^3\right)x^2}{2\left(\left(a^2b^3c^4d-2\,a^3b^2c^3d^2+a^4bc^2d^3\right)x^5+\left(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3\right)x^3+\left(a^3b^2c^5-2\,a^4bc^4d+a^5c^3d^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*
c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - 3*a*d^4)*arc
```

```
tan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^
3)*sqrt(c*d)) - 1/2*(2*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2 + (3*b^3*c^2
*d - 4*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^4 + (3*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*
b*c*d^2 + 3*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d
^3)*x^5 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3 +
 (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x)
```

mupad [B] time = 1.69, size = 3747, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] - (1/(a*c) + (x^2*(3*a^3*d^3 + 3*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c*d^2))/
(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4*(3*a^2*d^2 + 3*b^2*c
^2 - 4*a*b*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^3*(a*d + b
*c) + a*c*x + b*d*x^5) - (atan((a^7*d^3*x*(-c^5*d^5)^(3/2)*9i + b^7*c^12*d*
x*(-c^5*d^5)^(1/2)*9i + a^2*b^5*c^10*d^3*x*(-c^5*d^5)^(1/2)*49i - a^6*b*c*d
^2*x*(-c^5*d^5)^(3/2)*42i + a^5*b^2*c^2*d*x*(-c^5*d^5)^(3/2)*49i - a*b^6*c^
11*d^2*x*(-c^5*d^5)^(1/2)*42i)/(9*a^7*c^8*d^10 - 9*b^7*c^15*d^3 + 42*a*b^6*
c^14*d^4 - 42*a^6*b*c^9*d^9 - 49*a^2*b^5*c^13*d^5 + 49*a^5*b^2*c^10*d^8))*(
3*a*d - 7*b*c)*(-c^5*d^5)^(1/2)*1i)/(2*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6
*d^2 - 3*a*b^2*c^7*d)) - (atan((((7*a*d - 3*b*c)*(x*(144*a^6*b^15*c^18*d^3
- 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^12*c^15*d^6
 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c^13*d^8 + 31872*a^12*b^9*c^1
2*d^9 - 32000*a^13*b^8*c^11*d^10 + 28144*a^14*b^7*c^10*d^11 - 17664*a^15*b^
6*c^9*d^12 + 6976*a^16*b^5*c^8*d^13 - 1536*a^17*b^4*c^7*d^14 + 144*a^18*b^3
*c^6*d^15) - ((7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(192*a^8*b^15*c^21*d^2 - 217
6*a^9*b^14*c^20*d^3 + 10944*a^10*b^13*c^19*d^4 - 31808*a^11*b^12*c^18*d^5 +
 57600*a^12*b^11*c^17*d^6 - 62784*a^13*b^10*c^16*d^7 + 28032*a^14*b^9*c^15*
d^8 + 28032*a^15*b^8*c^14*d^9 - 62784*a^16*b^7*c^13*d^10 + 57600*a^17*b^6*c
^12*d^11 - 31808*a^18*b^5*c^11*d^12 + 10944*a^19*b^4*c^10*d^13 - 2176*a^20*
b^3*c^9*d^14 + 192*a^21*b^2*c^8*d^15 - (x*(7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*
(256*a^10*b^15*c^23*d^2 - 2816*a^11*b^14*c^22*d^3 + 13824*a^12*b^13*c^21*d^
4 - 39424*a^13*b^12*c^20*d^5 + 70400*a^14*b^11*c^19*d^6 - 76032*a^15*b^10*c
^18*d^7 + 33792*a^16*b^9*c^17*d^8 + 33792*a^17*b^8*c^16*d^9 - 76032*a^18*b^
7*c^15*d^10 + 70400*a^19*b^6*c^14*d^11 - 39424*a^20*b^5*c^13*d^12 + 13824*a
^21*b^4*c^12*d^13 - 2816*a^22*b^3*c^11*d^14 + 256*a^23*b^2*c^10*d^15))/(4*(
a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))))/(4*(a^8*d^3 - a
^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b^5)^(1/2)*1i)/(4*(a^
8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)) + ((7*a*d - 3*b*c)*
(x*(144*a^6*b^15*c^18*d^3 - 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5
 - 17664*a^9*b^12*c^15*d^6 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c^1
3*d^8 + 31872*a^12*b^9*c^12*d^9 - 32000*a^13*b^8*c^11*d^10 + 28144*a^14*b^7
```

```
*c^10*d^11 - 17664*a^15*b^6*c^9*d^12 + 6976*a^16*b^5*c^8*d^13 - 1536*a^17*b
^4*c^7*d^14 + 144*a^18*b^3*c^6*d^15) + ((7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(1
92*a^8*b^15*c^21*d^2 - 2176*a^9*b^14*c^20*d^3 + 10944*a^10*b^13*c^19*d^4 -
31808*a^11*b^12*c^18*d^5 + 57600*a^12*b^11*c^17*d^6 - 62784*a^13*b^10*c^16*
d^7 + 28032*a^14*b^9*c^15*d^8 + 28032*a^15*b^8*c^14*d^9 - 62784*a^16*b^7*c^
13*d^10 + 57600*a^17*b^6*c^12*d^11 - 31808*a^18*b^5*c^11*d^12 + 10944*a^19*
b^4*c^10*d^13 - 2176*a^20*b^3*c^9*d^14 + 192*a^21*b^2*c^8*d^15 + (x*(7*a*d
- 3*b*c)*(-a^5*b^5)^(1/2)*(256*a^10*b^15*c^23*d^2 - 2816*a^11*b^14*c^22*d^3
 + 13824*a^12*b^13*c^21*d^4 - 39424*a^13*b^12*c^20*d^5 + 70400*a^14*b^11*c^
19*d^6 - 76032*a^15*b^10*c^18*d^7 + 33792*a^16*b^9*c^17*d^8 + 33792*a^17*b^
8*c^16*d^9 - 76032*a^18*b^7*c^15*d^10 + 70400*a^19*b^6*c^14*d^11 - 39424*a^
20*b^5*c^13*d^12 + 13824*a^21*b^4*c^12*d^13 - 2816*a^22*b^3*c^11*d^14 + 256
*a^23*b^2*c^10*d^15))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b
*c*d^2))))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(
-a^5*b^5)^(1/2)*1i)/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c
*d^2)))/(((7*a*d - 3*b*c)*(x*(144*a^6*b^15*c^18*d^3 - 1536*a^7*b^14*c^17*d^
4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^12*c^15*d^6 + 28144*a^10*b^11*c^14
*d^7 - 32000*a^11*b^10*c^13*d^8 + 31872*a^12*b^9*c^12*d^9 - 32000*a^13*b^8*
c^11*d^10 + 28144*a^14*b^7*c^10*d^11 - 17664*a^15*b^6*c^9*d^12 + 6976*a^16*
b^5*c^8*d^13 - 1536*a^17*b^4*c^7*d^14 + 144*a^18*b^3*c^6*d^15) + ((7*a*d -
3*b*c)*(-a^5*b^5)^(1/2)*(192*a^8*b^15*c^21*d^2 - 2176*a^9*b^14*c^20*d^3 + 1
0944*a^10*b^13*c^19*d^4 - 31808*a^11*b^12*c^18*d^5 + 57600*a^12*b^11*c^17*d
^6 - 62784*a^13*b^10*c^16*d^7 + 28032*a^14*b^9*c^15*d^8 + 28032*a^15*b^8*c^
14*d^9 - 62784*a^16*b^7*c^13*d^10 + 57600*a^17*b^6*c^12*d^11 - 31808*a^18*b
^5*c^11*d^12 + 10944*a^19*b^4*c^10*d^13 - 2176*a^20*b^3*c^9*d^14 + 192*a^21
*b^2*c^8*d^15 + (x*(7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(256*a^10*b^15*c^23*d^2
 - 2816*a^11*b^14*c^22*d^3 + 13824*a^12*b^13*c^21*d^4 - 39424*a^13*b^12*c^2
0*d^5 + 70400*a^14*b^11*c^19*d^6 - 76032*a^15*b^10*c^18*d^7 + 33792*a^16*b^
9*c^17*d^8 + 33792*a^17*b^8*c^16*d^9 - 76032*a^18*b^7*c^15*d^10 + 70400*a^1
9*b^6*c^14*d^11 - 39424*a^20*b^5*c^13*d^12 + 13824*a^21*b^4*c^12*d^13 - 281
6*a^22*b^3*c^11*d^14 + 256*a^23*b^2*c^10*d^15))/(4*(a^8*d^3 - a^5*b^3*c^3 +
 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*
c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b^5)^(1/2))/(4*(a^8*d^3 - a^5*b^3*c^3 + 3*a^
6*b^2*c^2*d - 3*a^7*b*c*d^2)) - ((7*a*d - 3*b*c)*(x*(144*a^6*b^15*c^18*d^3
- 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^12*c^15*d^6
 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c^13*d^8 + 31872*a^12*b^9*c^1
2*d^9 - 32000*a^13*b^8*c^11*d^10 + 28144*a^14*b^7*c^10*d^11 - 17664*a^15*b^
6*c^9*d^12 + 6976*a^16*b^5*c^8*d^13 - 1536*a^17*b^4*c^7*d^14 + 144*a^18*b^3
*c^6*d^15) - ((7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(192*a^8*b^15*c^21*d^2 - 217
6*a^9*b^14*c^20*d^3 + 10944*a^10*b^13*c^19*d^4 - 31808*a^11*b^12*c^18*d^5 +
 57600*a^12*b^11*c^17*d^6 - 62784*a^13*b^10*c^16*d^7 + 28032*a^14*b^9*c^15*
d^8 + 28032*a^15*b^8*c^14*d^9 - 62784*a^16*b^7*c^13*d^10 + 57600*a^17*b^6*c
^12*d^11 - 31808*a^18*b^5*c^11*d^12 + 10944*a^19*b^4*c^10*d^13 - 2176*a^20*
b^3*c^9*d^14 + 192*a^21*b^2*c^8*d^15 - (x*(7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*
(256*a^10*b^15*c^23*d^2 - 2816*a^11*b^14*c^22*d^3 + 13824*a^12*b^13*c^21*d^
```

```
4 - 39424*a^13*b^12*c^20*d^5 + 70400*a^14*b^11*c^19*d^6 - 76032*a^15*b^10*c
^18*d^7 + 33792*a^16*b^9*c^17*d^8 + 33792*a^17*b^8*c^16*d^9 - 76032*a^18*b^
7*c^15*d^10 + 70400*a^19*b^6*c^14*d^11 - 39424*a^20*b^5*c^13*d^12 + 13824*a
^21*b^4*c^12*d^13 - 2816*a^22*b^3*c^11*d^14 + 256*a^23*b^2*c^10*d^15))/(4*(
a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))))/(4*(a^8*d^3 - a
^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b^5)^(1/2))/(4*(a^8*d
^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)) + 504*a^6*b^13*c^14*d^
5 - 4080*a^7*b^12*c^13*d^6 + 14144*a^8*b^11*c^12*d^7 - 27920*a^9*b^10*c^11*
d^8 + 34704*a^10*b^9*c^10*d^9 - 27920*a^11*b^8*c^9*d^10 + 14144*a^12*b^7*c^
8*d^11 - 4080*a^13*b^6*c^7*d^12 + 504*a^14*b^5*c^6*d^13))*(7*a*d - 3*b*c)*(
-a^5*b^5)^(1/2)*1i)/(2*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c
*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.308 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=156

$$\frac{b^3(bc-2ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{b^3}{2a^2\left(a+bx^2\right)(bc-ad)^2}-\frac{1}{2a^2c^2x^2}+\frac{d^3(2bc-ad)\log\left(c+dx^2\right)}{c^3(bc-ad)^3}-$$

Rubi [A] time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{b^3}{2a^2\left(a+bx^2\right)(bc-ad)^2}+\frac{b^3(bc-2ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^3}-\frac{2\log(x)(ad+bc)}{a^3c^3}-\frac{1}{2a^2c^2x^2}-\frac{d^3}{2c^2\left(c+dx^2\right)(bc-ad)^2}+\frac{d^3(2bc-ad)\log\left(c+dx^2\right)}{c^3(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/(2*a^2*c^2*x^2) - b^3/(2*a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(2*c^2*(b*
c - a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x])/(a^3*c^3) + (b^3*(b*c - 2*
a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (d^3*(2*b*c - a*d)*Log[c + d*x^2
])/(c^3*(b*c - a*d)^3)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2c^2x^2} - \frac{2(bc+ad)}{a^3c^3x} + \frac{b^4}{a^2(-bc+ad)^2(a+bx)^2} + \frac{2b^4(-bc+2}{a^3(-bc+ad)^3(}\right.\right.$$

$$= -\frac{1}{2a^2c^2x^2} - \frac{b^3}{2a^2(bc-ad)^2\left(a+bx^2\right)} - \frac{d^3}{2c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{2(bc+ad)\,\text{lo}}{a^3c^3}$$

Mathematica [**A**] time = 0.22, size = 157, normalized size = 1.01

$$\frac{1}{2}\left(\frac{2b^3(2ad-bc)\log\left(a+bx^2\right)}{a^3(ad-bc)^3} - \frac{4\log(x)(ad+bc)}{a^3c^3} - \frac{b^3}{a^2\left(a+bx^2\right)(bc-ad)^2} - \frac{1}{a^2c^2x^2} + \frac{2d^3(2bc-ad)\log\left(c+dx^2\right)}{c^3(bc-ad)^3} - \frac{d^3}{c^2\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-(1/(a^2*c^2*x^2)) - b^3/(a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(c^2*(b*c -
 a*d)^2*(c + d*x^2)) - (4*(b*c + a*d)*Log[x])/(a^3*c^3) + (2*b^3*(-(b*c) +
2*a*d)*Log[a + b*x^2])/(a^3*(-(b*c) + a*d)^3) + (2*d^3*(2*b*c - a*d)*Log[c
+ d*x^2])/(c^3*(b*c - a*d)^3))/2
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas [**B**] time = 25.06, size = 667, normalized size = 4.28

$$-\frac{a^2b^3c^5 - 3a^3b^2c^4d + 3a^4bc^3d^2 - a^5c^2d^3 + 2\left(ab^4c^4d - 2a^2b^3c^3d^2 + 2a^3b^2c^2d^3 - a^4bcd^4\right)x^4 + \left(2ab^4c^5 - 3a^2b^3c^4d + 3a^4bc^2d^3 - 2a^5cd^4\right)x^2 - 2\left(\left(b^5c^4d - 2ab^4c^3d^2\right)x^6 + \left(b^5c^5 - ab^4c^4d - 2a^2b^3c^3d^2\right)x^4 + \left(ab^4c^5 - 2a^2b^3c^4d\right)x^2\right)\log\left(bx^2+a\right) - 2\left(\left(2a^3b^2cd^4 - a^4bd^5\right)x^6 + \left(2a^3b^2c^2d^3 + a^4bcd^4 - a^5d^5\right)x^4 + \left(2a^4bc^2d^3 - a^5cd^4\right)x^2\right)\log\left(dx^2+c\right) + 4\left(\left(b^5c^4d - 2ab^4c^3d^2 + 2a^3b^2cd^4 - a^4bd^5\right)x^6 + \left(b^5c^5 - ab^4c^4d - 2a^2b^3c^3d^2 + 2a^3b^2c^2d^3 + a^4bcd^4 - a^5d^5\right)x^4 + \left(ab^4c^5 - 2a^2b^3c^4d + 2a^4bc^2d^3 - a^5cd^4\right)x^2\right)\log(x)}{2\left(\left(a^3b^4c^6d - 3a^4b^3c^5d^2 + 3a^5b^2c^4d^3 - a^6bc^3d^4\right)x^6 + \left(a^3b^4c^7 - 2a^4b^3c^6d + 2a^6bc^4d^3 - a^7c^3d^4\right)x^4 + \left(a^4b^3c^7 - 3a^5b^2c^6d + 3a^6bc^5d^2 - a^7c^4d^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + 2*(a*
b^4*c^4*d - 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (2*a
```

```
*b^4*c^5 - 3*a^2*b^3*c^4*d + 3*a^4*b*c^2*d^3 - 2*a^5*c*d^4)*x^2 - 2*((b^5*c
^4*d - 2*a*b^4*c^3*d^2)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2)*x
^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d)*x^2)*log(b*x^2 + a) - 2*((2*a^3*b^2*c*d^
4 - a^4*b*d^5)*x^6 + (2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (2*a
^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(d*x^2 + c) + 4*((b^5*c^4*d - 2*a*b^4*c^3
*d^2 + 2*a^3*b^2*c*d^4 - a^4*b*d^5)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^
3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (a*b^4*c^5 - 2
*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(x))/((a^3*b^4*c^6*d
- 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^6 + (a^3*b^4*c^7
 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^4 + (a^4*b^3*c^7 - 3*
a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^2)
```

giac [B] time = 0.39, size = 333, normalized size = 2.13

$$\frac{\left(b^{5}c-2\,ab^{4}d\right)\log\left(\left|bx^{2}+a\right|\right)}{a^{3}b^{4}c^{3}-3\,a^{4}b^{3}c^{2}d+3\,a^{5}b^{2}cd^{2}-a^{6}bd^{3}}+\frac{\left(2\,bcd^{4}-ad^{5}\right)\log\left(\left|dx^{2}+c\right|\right)}{b^{3}c^{6}d-3\,ab^{2}c^{5}d^{2}+3\,a^{2}bc^{4}d^{3}-a^{3}c^{3}d^{4}}-\frac{2\,b^{3}c^{2}dx^{4}-2\,ab^{2}cd^{2}x^{4}+2\,a^{2}bd^{3}x^{4}+2\,b^{3}c^{3}x^{2}-ab^{2}c^{2}dx^{2}-a^{2}bcd^{2}x^{2}+2\,a^{3}d^{3}x^{2}+ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}}{2\left(a^{2}b^{2}c^{4}-2\,a^{3}bc^{3}d+a^{4}c^{2}d^{2}\right)\left(bdx^{6}+bcx^{4}+adx^{4}+acx^{2}\right)}-\frac{(bc+ad)\log\left(x^{2}\right)}{a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (b^5*c - 2*a*b^4*d)*log(abs(b*x^2 + a))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*
a^5*b^2*c*d^2 - a^6*b*d^3) + (2*b*c*d^4 - a*d^5)*log(abs(d*x^2 + c))/(b^3*c
^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/2*(2*b^3*c^2*d*
x^4 - 2*a*b^2*c*d^2*x^4 + 2*a^2*b*d^3*x^4 + 2*b^3*c^3*x^2 - a*b^2*c^2*d*x^2
 - a^2*b*c*d^2*x^2 + 2*a^3*d^3*x^2 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)
/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^6 + b*c*x^4 + a*d*x^4
+ a*c*x^2)) - (b*c + a*d)*log(x^2)/(a^3*c^3)
```

maple [A] time = 0.03, size = 254, normalized size = 1.63

$$-\frac{a\,d^{4}}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)c^{2}}+\frac{a\,d^{4}\ln\left(d\,x^{2}+c\right)}{(ad-bc)^{3}\,c^{3}}-\frac{b^{3}d}{2\,(ad-bc)^{3}\left(b\,x^{2}+a\right)a}+\frac{b^{4}c}{2\,(ad-bc)^{3}\left(b\,x^{2}+a\right)a^{2}}+\frac{2b^{3}d\ln\left(b\,x^{2}+a\right)}{(ad-bc)^{3}\,a^{2}}-\frac{b^{4}c\ln\left(b\,x^{2}+a\right)}{(ad-bc)^{3}\,a^{3}}+\frac{b\,d^{3}}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)c}-\frac{2b\,d^{3}\ln\left(d\,x^{2}+c\right)}{(ad-bc)^{3}\,c^{2}}-\frac{2d\ln(x)}{a^{2}c^{3}}-\frac{2b\ln(x)}{a^{3}c^{2}}-\frac{1}{2a^{2}c^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b^3/a/(a*d-b*c)^3/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3/(b*x^2+a)*c+2*b^
3/a^2/(a*d-b*c)^3*ln(b*x^2+a)*d-b^4/a^3/(a*d-b*c)^3*ln(b*x^2+a)*c+d^4/c^3/(
a*d-b*c)^3*ln(d*x^2+c)*a-2*d^3/c^2/(a*d-b*c)^3*ln(d*x^2+c)*b-1/2*d^4/c^2/(a
*d-b*c)^3/(d*x^2+c)*a+1/2*d^3/c/(a*d-b*c)^3/(d*x^2+c)*b-1/2/a^2/c^2/x^2-2/a
^2/c^3*ln(x)*d-2/a^3/c^2*ln(x)*b
```

maxima [B] time = 1.21, size = 381, normalized size = 2.44

$$\frac{\left(b^{4}c-2\,ab^{3}d\right)\log\left(bx^{2}+a\right)}{a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}}+\frac{\left(2\,bcd^{3}-ad^{4}\right)\log\left(dx^{2}+c\right)}{b^{3}c^{6}-3\,ab^{2}c^{5}d+3\,a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3}}-\frac{ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+2\left(b^{3}c^{2}d-ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{4}+\left(2\,b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+2\,a^{3}d^{3}\right)x^{2}}{2\left(\left(a^{2}b^{3}c^{4}d-2\,a^{3}b^{2}c^{3}d^{2}+a^{4}bc^{2}d^{3}\right)x^{6}+\left(a^{2}b^{3}c^{5}-a^{3}b^{2}c^{4}d-a^{4}bc^{3}d^{2}+a^{5}c^{2}d^{3}\right)x^{4}+\left(a^{3}b^{2}c^{5}-2\,a^{4}bc^{4}d+a^{5}c^{3}d^{2}\right)x^{2}\right)}-\frac{(bc+ad)\log\left(x^{2}\right)}{a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] (b^4*c - 2*a*b^3*d)*log(b*x^2 + a)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b
*c*d^2 - a^6*d^3) + (2*b*c*d^3 - a*d^4)*log(d*x^2 + c)/(b^3*c^6 - 3*a*b^2*c
^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^
3*c*d^2 + 2*(b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (2*b^3*c^3 - a*b^2*
c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 +
 a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^
2*d^3)*x^4 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x^2) - (b*c + a*d)
*log(x^2)/(a^3*c^3)
```

mupad [**B**] time = 1.64, size = 313, normalized size = 2.01

$$-\frac{\frac{1}{2\,a\,c}+\frac{x^4\,\left(a^2\,b\,d^3-a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}{a^2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(a\,d+b\,c\right)\,\left(2\,a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2\right)}{2\,a^2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^4+a\,c\,x^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(b^4\,c-2\,a\,b^3\,d\right)}{a^6\,d^3-3\,a^5\,b\,c\,d^2+3\,a^4\,b^2\,c^2\,d-a^3\,b^3\,c^3}-\frac{\ln\left(d\,x^2+c\right)\,\left(a\,d^4-2\,b\,c\,d^3\right)}{-a^3\,c^3\,d^3+3\,a^2\,b\,c^4\,d^2-3\,a\,b^2\,c^5\,d+b^3\,c^6}-\frac{\ln(x)\,\left(2\,a\,d+2\,b\,c\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] - (1/(2*a*c) + (x^4*(a^2*b*d^3 + b^3*c^2*d - a*b^2*c*d^2))/(a^2*c^2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(a*d + b*c)*(2*a^2*d^2 + 2*b^2*c^2 - 3*a*b
*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^4*(a*d + b*c) + a*c*
x^2 + b*d*x^6) - (log(a + b*x^2)*(b^4*c - 2*a*b^3*d))/(a^6*d^3 - a^3*b^3*c^
3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2) - (log(c + d*x^2)*(a*d^4 - 2*b*c*d^3))
/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d) - (log(x)*(2*a*d
 + 2*b*c))/(a^3*c^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.309 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=271

$$\frac{b^{7/2}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3}-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{d^{7/2}(9bc-5ad)\tan^{-1}\big(}{2c^{7/2}(bc-ad)^3}$$

Rubi [A] time = 0.45, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$-\frac{5a^2d^2-4abcd+5b^2c^2}{6a^2c^2x^3(bc-ad)^2}+\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{2a^3c^3x(bc-ad)^2}+\frac{b^{7/2}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3}+\frac{d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^3\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(5*b^2*c^2 - 4*a*b*c*d + 5*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((b*c
+ a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*x) + (
d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)) + b/(2*a*(b*c - a*d)*x
^3*(a + b*x^2)*(c + d*x^2)) + (b^(7/2)*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(7/2)*(b*c - a*d)^3) + (d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*c^(7/2)*(b*c - a*d)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-5bc+2ad-7bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\int \frac{-2\left(5b^2c^2-4abcd}{x^4\left(a\right.}}{4ac}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+}{2ac(bc-ad)^2x^3}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+}{2ac(bc-ad)^2x^3}$$

$$= -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)\left(5b^2c^2-9abcd+5a^2d^2\right)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+}{2ac(bc-ad)^2x^3}$$

Mathematica [A] time = 0.39, size = 178, normalized size = 0.66

$$\frac{1}{6}\left(\frac{3b^{7/2}(9ad-5bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(ad-bc)^3} + \frac{3b^4x}{a^3\left(a+bx^2\right)(bc-ad)^2} + \frac{12(ad+bc)}{a^3c^3x} - \frac{2}{a^2c^2x^3} + \frac{3d^{7/2}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3} + \frac{3d^4x}{c^3\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-2/(a^2*c^2*x^3) + (12*(b*c + a*d))/(a^3*c^3*x) + (3*b^4*x)/(a^3*(b*c - a*
d)^2*(a + b*x^2)) + (3*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (3*b^(7/2)*
(-5*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c) + a*d)^3) +
(3*d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d
)^3))/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2), x]
```

fricas **[B]** time = 13.75, size = 2457, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3
 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 -
2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 +
17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*
b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^
5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c
^4*d)*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*(
(9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 -
5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*log((d*x^2
 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2
+ 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d +
 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^
6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d +
12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3
*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3
*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^
4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 6*((9*a^3*b^2*
c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^4 - 5*a^5*d^5)*
x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) -
3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2
*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d
^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*
d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3
*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d
 + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*
b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a
*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 6*((5*b^5*c
^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^
2)*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a))
 - 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*
d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*log(
```

```
(d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^
5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c
^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d
+ 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/6*(2*a^2*b^3*c^5 - 6*a^3*b^2*c^4*
d + 6*a^4*b*c^3*d^2 - 2*a^5*c^2*d^3 - 3*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*
a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - (15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^
3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 10*(a*b
^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4
*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)
*x^5 + (5*a*b^4*c^5 - 9*a^2*b^3*c^4*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c*d^
4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arctan(
x*sqrt(d/c)))/((a^3*b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6
*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^
3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3
)*x^3)]
```

giac [A] time = 0.35, size = 275, normalized size = 1.01

$$\frac{\left(5\,b^5c-9\,ab^4d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3\right)\sqrt{ab}}+\frac{\left(9\,bcd^4-5\,ad^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)\sqrt{cd}}+\frac{b^4c^3dx^3+a^3bd^4x^3+b^4c^4x+a^4d^4x}{2\left(a^3b^2c^5-2\,a^4bc^4d+a^5c^3d^2\right)\left(bdx^4+bcx^2+adx^2+ac\right)}+\frac{6\,bcx^2+6\,adx^2-ac}{3\,a^3c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b)) + 1/2*(9*b*c*d^4 - 5*a*d^5)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) + 1/2*(b^4*c^3*d*x^3 + a^3*b*d^4*x^3 + b^4*c^4*x + a^4*d^4*x)/
((a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 +
 a*c)) + 1/3*(6*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^3*c^3*x^3)
```

maple [A] time = 0.02, size = 285, normalized size = 1.05

$$\frac{a\,d^5x}{2\left(ad-bc\right)^3\left(d\,x^2+c\right)c^3}+\frac{5a\,d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^3\sqrt{cd}\,c^3}+\frac{b^4dx}{2\left(ad-bc\right)^3\left(b\,x^2+a\right)a^2}+\frac{9b^4d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^3\sqrt{ab}\,a^2}-\frac{b^5cx}{2\left(ad-bc\right)^3\left(b\,x^2+a\right)a^3}-\frac{5b^5c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^3\sqrt{ab}\,a^3}-\frac{b\,d^4x}{2\left(ad-bc\right)^3\left(d\,x^2+c\right)c^2}-\frac{9b\,d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(ad-bc\right)^3\sqrt{cd}\,c^2}+\frac{2d}{a^2c^3x}+\frac{2b}{a^3c^2x}-\frac{1}{3a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^4/a^2/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*b^5/a^3/(a*d-b*c)^3*x/(b*x^2+a)*c
+9/2*b^4/a^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-5/2*b^5/a^
3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+1/2*d^5/c^3/(a*d-b*c)
^3*x/(d*x^2+c)*a-1/2*d^4/c^2/(a*d-b*c)^3*x/(d*x^2+c)*b+5/2*d^5/c^3/(a*d-b*c
)^3/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a-9/2*d^4/c^2/(a*d-b*c)^3/(c*d)^(
```

```
1/2)*arctan(1/(c*d)^(1/2)*d*x)*b-1/3/a^2/c^2/x^3+2/a^2/c^3/x*d+2/a^3/c^2/x*
b
```

maxima [**A**] time = 2.68, size = 460, normalized size = 1.70

$$\frac{\left(5\,b^5c-9\,ab^4d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3\right)\sqrt{ab}}+\frac{\left(9\,bcd^4-5\,ad^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)\sqrt{cd}}-\frac{2\,a^2b^2c^4-4\,a^3bc^3d+2\,a^4c^2d^2-3\left(5\,b^4c^3d-4\,ab^3c^2d^2-4\,a^2b^2cd^3+5\,a^3bd^4\right)x^6-\left(15\,b^4c^4-2\,ab^3c^3d-20\,a^2b^2c^2d^2-2\,a^3bcd^3+15\,a^4d^4\right)x^4-10\left(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)x^2}{6\left(\left(a^3b^3c^5d-2\,a^4b^2c^4d^2+a^5bc^3d^3\right)x^7+\left(a^3b^3c^6-a^4b^2c^5d-a^5bc^4d^2+a^6c^3d^3\right)x^5+\left(a^4b^2c^6-2\,a^5bc^5d+a^6c^4d^2\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(5*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b)) + 1/2*(9*b*c*d^4 - 5*a*d^5)*arct
an(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3
)*sqrt(c*d)) - 1/6*(2*a^2*b^2*c^4 - 4*a^3*b*c^3*d + 2*a^4*c^2*d^2 - 3*(5*b^
4*c^3*d - 4*a*b^3*c^2*d^2 - 4*a^2*b^2*c*d^3 + 5*a^3*b*d^4)*x^6 - (15*b^4*c^
4 - 2*a*b^3*c^3*d - 20*a^2*b^2*c^2*d^2 - 2*a^3*b*c*d^3 + 15*a^4*d^4)*x^4 -
10*(a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)/((a^3*b^3*c
^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^7 + (a^3*b^3*c^6 - a^4*b^2*c^5*
d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^5 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
^4*d^2)*x^3)
```

mupad [**B**] time = 1.88, size = 3978, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] (atan((a^9*d^3*x*(-c^7*d^7)^(3/2)*25i + b^9*c^16*d*x*(-c^7*d^7)^(1/2)*25i +
 a^2*b^7*c^14*d^3*x*(-c^7*d^7)^(1/2)*81i - a^8*b*c*d^2*x*(-c^7*d^7)^(3/2)*9
0i + a^7*b^2*c^2*d*x*(-c^7*d^7)^(3/2)*81i - a*b^8*c^15*d^2*x*(-c^7*d^7)^(1/
2)*90i)/(25*a^9*c^11*d^13 - 25*b^9*c^20*d^4 + 90*a*b^8*c^19*d^5 - 90*a^8*b*
c^12*d^12 - 81*a^2*b^7*c^18*d^6 + 81*a^7*b^2*c^13*d^11))*(5*a*d - 9*b*c)*(-
c^7*d^7)^(1/2)*1i)/(2*(b^3*c^10 - a^3*c^7*d^3 + 3*a^2*b*c^8*d^2 - 3*a*b^2*c
^9*d)) - (1/(3*a*c) - (5*x^2*(a*d + b*c))/(3*a^2*c^2) + (x^4*(20*a^2*b^2*c^
2*d^2 - 15*b^4*c^4 - 15*a^4*d^4 + 2*a*b^3*c^3*d + 2*a^3*b*c*d^3))/(6*a^3*c^
3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x^6*(5*a^3*d^3 + 5*b^3*c^3 - 4*a*
b^2*c^2*d - 4*a^2*b*c*d^2))/(2*a^3*c^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x
^5*(a*d + b*c) + a*c*x^3 + b*d*x^7) + (atan((((x*(400*a^9*b^17*c^23*d^3 - 3
840*a^10*b^16*c^22*d^4 + 15936*a^11*b^15*c^21*d^5 - 37376*a^12*b^14*c^20*d^
6 + 54240*a^13*b^13*c^19*d^7 - 49920*a^14*b^12*c^18*d^8 + 29776*a^15*b^11*c
^17*d^9 - 18432*a^16*b^10*c^16*d^10 + 29776*a^17*b^9*c^15*d^11 - 49920*a^18
*b^8*c^14*d^12 + 54240*a^19*b^7*c^13*d^13 - 37376*a^20*b^6*c^12*d^14 + 1593
6*a^21*b^5*c^11*d^15 - 3840*a^22*b^4*c^10*d^16 + 400*a^23*b^3*c^9*d^17) - (
(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(320*a^12*b^16*c^26*d^2 - 3456*a^13*b^15*c
^25*d^3 + 16704*a^14*b^14*c^24*d^4 - 47616*a^15*b^13*c^23*d^5 + 89280*a^16*
```

```
b^12*c^22*d^6 - 118400*a^17*b^11*c^21*d^7 + 123072*a^18*b^10*c^20*d^8 - 119
808*a^19*b^9*c^19*d^9 + 123072*a^20*b^8*c^18*d^10 - 118400*a^21*b^7*c^17*d^
11 + 89280*a^22*b^6*c^16*d^12 - 47616*a^23*b^5*c^15*d^13 + 16704*a^24*b^4*c
^14*d^14 - 3456*a^25*b^3*c^13*d^15 + 320*a^26*b^2*c^12*d^16 - (x*(9*a*d - 5
*b*c)*(-a^7*b^7)^(1/2)*(256*a^15*b^15*c^28*d^2 - 2816*a^16*b^14*c^27*d^3 +
13824*a^17*b^13*c^26*d^4 - 39424*a^18*b^12*c^25*d^5 + 70400*a^19*b^11*c^24*
d^6 - 76032*a^20*b^10*c^23*d^7 + 33792*a^21*b^9*c^22*d^8 + 33792*a^22*b^8*c
^21*d^9 - 76032*a^23*b^7*c^20*d^10 + 70400*a^24*b^6*c^19*d^11 - 39424*a^25*
b^5*c^18*d^12 + 13824*a^26*b^4*c^17*d^13 - 2816*a^27*b^3*c^16*d^14 + 256*a^
28*b^2*c^15*d^15))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c
*d^2))))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2)))*(9
*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*1i)/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c
^2*d - 3*a^9*b*c*d^2)) + ((x*(400*a^9*b^17*c^23*d^3 - 3840*a^10*b^16*c^22*d
^4 + 15936*a^11*b^15*c^21*d^5 - 37376*a^12*b^14*c^20*d^6 + 54240*a^13*b^13*
c^19*d^7 - 49920*a^14*b^12*c^18*d^8 + 29776*a^15*b^11*c^17*d^9 - 18432*a^16
*b^10*c^16*d^10 + 29776*a^17*b^9*c^15*d^11 - 49920*a^18*b^8*c^14*d^12 + 542
40*a^19*b^7*c^13*d^13 - 37376*a^20*b^6*c^12*d^14 + 15936*a^21*b^5*c^11*d^15
 - 3840*a^22*b^4*c^10*d^16 + 400*a^23*b^3*c^9*d^17) + ((9*a*d - 5*b*c)*(-a^
7*b^7)^(1/2)*(320*a^12*b^16*c^26*d^2 - 3456*a^13*b^15*c^25*d^3 + 16704*a^14
*b^14*c^24*d^4 - 47616*a^15*b^13*c^23*d^5 + 89280*a^16*b^12*c^22*d^6 - 1184
00*a^17*b^11*c^21*d^7 + 123072*a^18*b^10*c^20*d^8 - 119808*a^19*b^9*c^19*d^
9 + 123072*a^20*b^8*c^18*d^10 - 118400*a^21*b^7*c^17*d^11 + 89280*a^22*b^6*
c^16*d^12 - 47616*a^23*b^5*c^15*d^13 + 16704*a^24*b^4*c^14*d^14 - 3456*a^25
*b^3*c^13*d^15 + 320*a^26*b^2*c^12*d^16 + (x*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/
2)*(256*a^15*b^15*c^28*d^2 - 2816*a^16*b^14*c^27*d^3 + 13824*a^17*b^13*c^26
*d^4 - 39424*a^18*b^12*c^25*d^5 + 70400*a^19*b^11*c^24*d^6 - 76032*a^20*b^1
0*c^23*d^7 + 33792*a^21*b^9*c^22*d^8 + 33792*a^22*b^8*c^21*d^9 - 76032*a^23
*b^7*c^20*d^10 + 70400*a^24*b^6*c^19*d^11 - 39424*a^25*b^5*c^18*d^12 + 1382
4*a^26*b^4*c^17*d^13 - 2816*a^27*b^3*c^16*d^14 + 256*a^28*b^2*c^15*d^15))/(
4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2))))/(4*(a^10*d^
3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2)))*(9*a*d - 5*b*c)*(-a^7*
b^7)^(1/2)*1i)/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2
)))/(((x*(400*a^9*b^17*c^23*d^3 - 3840*a^10*b^16*c^22*d^4 + 15936*a^11*b^15
*c^21*d^5 - 37376*a^12*b^14*c^20*d^6 + 54240*a^13*b^13*c^19*d^7 - 49920*a^1
4*b^12*c^18*d^8 + 29776*a^15*b^11*c^17*d^9 - 18432*a^16*b^10*c^16*d^10 + 29
776*a^17*b^9*c^15*d^11 - 49920*a^18*b^8*c^14*d^12 + 54240*a^19*b^7*c^13*d^1
3 - 37376*a^20*b^6*c^12*d^14 + 15936*a^21*b^5*c^11*d^15 - 3840*a^22*b^4*c^1
0*d^16 + 400*a^23*b^3*c^9*d^17) + ((9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(320*a^
12*b^16*c^26*d^2 - 3456*a^13*b^15*c^25*d^3 + 16704*a^14*b^14*c^24*d^4 - 476
16*a^15*b^13*c^23*d^5 + 89280*a^16*b^12*c^22*d^6 - 118400*a^17*b^11*c^21*d^
7 + 123072*a^18*b^10*c^20*d^8 - 119808*a^19*b^9*c^19*d^9 + 123072*a^20*b^8*
c^18*d^10 - 118400*a^21*b^7*c^17*d^11 + 89280*a^22*b^6*c^16*d^12 - 47616*a^
23*b^5*c^15*d^13 + 16704*a^24*b^4*c^14*d^14 - 3456*a^25*b^3*c^13*d^15 + 320
*a^26*b^2*c^12*d^16 + (x*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(256*a^15*b^15*c^
28*d^2 - 2816*a^16*b^14*c^27*d^3 + 13824*a^17*b^13*c^26*d^4 - 39424*a^18*b^
```

```
12*c^25*d^5 + 70400*a^19*b^11*c^24*d^6 - 76032*a^20*b^10*c^23*d^7 + 33792*a
^21*b^9*c^22*d^8 + 33792*a^22*b^8*c^21*d^9 - 76032*a^23*b^7*c^20*d^10 + 704
00*a^24*b^6*c^19*d^11 - 39424*a^25*b^5*c^18*d^12 + 13824*a^26*b^4*c^17*d^13
 - 2816*a^27*b^3*c^16*d^14 + 256*a^28*b^2*c^15*d^15))/(4*(a^10*d^3 - a^7*b^
3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2))))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*
a^8*b^2*c^2*d - 3*a^9*b*c*d^2)))*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2))/(4*(a^10
*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b*c*d^2)) - ((x*(400*a^9*b^17*
c^23*d^3 - 3840*a^10*b^16*c^22*d^4 + 15936*a^11*b^15*c^21*d^5 - 37376*a^12*
b^14*c^20*d^6 + 54240*a^13*b^13*c^19*d^7 - 49920*a^14*b^12*c^18*d^8 + 29776
*a^15*b^11*c^17*d^9 - 18432*a^16*b^10*c^16*d^10 + 29776*a^17*b^9*c^15*d^11
- 49920*a^18*b^8*c^14*d^12 + 54240*a^19*b^7*c^13*d^13 - 37376*a^20*b^6*c^12
*d^14 + 15936*a^21*b^5*c^11*d^15 - 3840*a^22*b^4*c^10*d^16 + 400*a^23*b^3*c
^9*d^17) - ((9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(320*a^12*b^16*c^26*d^2 - 3456
*a^13*b^15*c^25*d^3 + 16704*a^14*b^14*c^24*d^4 - 47616*a^15*b^13*c^23*d^5 +
 89280*a^16*b^12*c^22*d^6 - 118400*a^17*b^11*c^21*d^7 + 123072*a^18*b^10*c^
20*d^8 - 119808*a^19*b^9*c^19*d^9 + 123072*a^20*b^8*c^18*d^10 - 118400*a^21
*b^7*c^17*d^11 + 89280*a^22*b^6*c^16*d^12 - 47616*a^23*b^5*c^15*d^13 + 1670
4*a^24*b^4*c^14*d^14 - 3456*a^25*b^3*c^13*d^15 + 320*a^26*b^2*c^12*d^16 - (
x*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(256*a^15*b^15*c^28*d^2 - 2816*a^16*b^14
*c^27*d^3 + 13824*a^17*b^13*c^26*d^4 - 39424*a^18*b^12*c^25*d^5 + 70400*a^1
9*b^11*c^24*d^6 - 76032*a^20*b^10*c^23*d^7 + 33792*a^21*b^9*c^22*d^8 + 3379
2*a^22*b^8*c^21*d^9 - 76032*a^23*b^7*c^20*d^10 + 70400*a^24*b^6*c^19*d^11 -
 39424*a^25*b^5*c^18*d^12 + 13824*a^26*b^4*c^17*d^13 - 2816*a^27*b^3*c^16*d
^14 + 256*a^28*b^2*c^15*d^15))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d
 - 3*a^9*b*c*d^2))))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b^2*c^2*d - 3*a^9*b
*c*d^2)))*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2))/(4*(a^10*d^3 - a^7*b^3*c^3 + 3*
a^8*b^2*c^2*d - 3*a^9*b*c*d^2)) + 1800*a^9*b^15*c^18*d^6 - 12880*a^10*b^14*
c^17*d^7 + 37272*a^11*b^13*c^16*d^8 - 52536*a^12*b^12*c^15*d^9 + 26344*a^13
*b^11*c^14*d^10 + 26344*a^14*b^10*c^13*d^11 - 52536*a^15*b^9*c^12*d^12 + 37
272*a^16*b^8*c^11*d^13 - 12880*a^17*b^7*c^10*d^14 + 1800*a^18*b^6*c^9*d^15)
)*(9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*1i)/(2*(a^10*d^3 - a^7*b^3*c^3 + 3*a^8*b
^2*c^2*d - 3*a^9*b*c*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.310 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=207

$$\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,\sqrt{d}\,(bc-ad)^4}+\frac{3x(3ad+bc)}{8\left(c+dx^2\right)(bc-ad)^3}+\frac{x(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(b}$$

Rubi [A] time = 0.28, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {470, 527, 522, 205}

$$\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{c}\,\sqrt{d}\,(bc-ad)^4}+\frac{3x(3ad+bc)}{8\left(c+dx^2\right)(bc-ad)^3}+\frac{x(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{3\sqrt{a}\,\sqrt{b}\,(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*x)/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*x)/(2*b*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)^2) + (3*(b*c + 3*a*d)*x)/(8*(b*c - a*d)^3*(c + d*
x^2)) - (3*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*(b*c
 - a*d)^4) + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(8*Sqrt[c]*Sqrt[d]*(b*c - a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{ac+(-2bc-3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2b(bc-ad)}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{6abc^2-6bc(bc+2ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}}{8bc(bc-ad)^2}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3 (c+dx}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3 (c+dx}$$

$$= \frac{(bc+2ad)x}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3 (c+dx}$$

Mathematica [A] time = 0.36, size = 166, normalized size = 0.80

$$\frac{\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}} + \frac{2cx(bc-ad)^2}{\left(c+dx^2\right)^2} + \frac{4abx(bc-ad)}{a+bx^2} + \frac{x(5ad+3bc)(bc-ad)}{c+dx^2} - 12\sqrt{a}\,\sqrt{b}\,(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((4*a*b*(b*c - a*d)*x)/(a + b*x^2) + (2*c*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
((b*c - a*d)*(3*b*c + 5*a*d)*x)/(c + d*x^2) - 12*Sqrt[a]*Sqrt[b]*(b*c + a*d
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(8*(b*c - a*d)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 2.90, size = 2859, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^4
*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 + 12*(a*b*c^4*d +
 a^2*c^3*d^2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d
^3 + a^2*c*d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 3*(a*b^2*c^4 + 6*a^2
*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2
*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8
*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 -
 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d - 2*a^2*b*c^3*d^2 - a^
3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3 - 4*a^4*
b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^3*c^3*d^
5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4*c^5*d^3
 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*d^7)*x^4
 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4*d^4 - 7
*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3
 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 -
5*a^3*c*d^4)*x^3 + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^
2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^
2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*
a^3*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 6*(a*b*c^4*d + a^2*c^3*d^
2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*
```

```
d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(-a*b)*log(
(b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 3*(3*a*b^2*c^4*d - 2*a^2*b*c^3*
d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3
- 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^
3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4
*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*
d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4
*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/16*(6*(b^3*c^3*d^2 + 2*a*b^
2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b
*c^2*d^3 - 5*a^3*c*d^4)*x^3 - 24*(a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 +
a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + (b^2*c^4
*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 +
 a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4
)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqr
t(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d -
2*a^2*b*c^3*d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*
b^2*c^5*d^3 - 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^
4 + 6*a^2*b^3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d
^2 - 7*a*b^4*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*
d^6 + a^5*c*d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8
*a^3*b^2*c^4*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d
^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2
- 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 - 12*(a*b*c^4*d + a^2*c^3*d^2 + (b^2*c
^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^2*d^3 + a^2*c*d^4)*x^4 +
 (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^
2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3
+ a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)
*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 3*(3*a*b^2*c^4*d - 2*a^2*b*c^3*d^2
- a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^3 - 4*
a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2*b^3*c^
3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a*b^4*c^5
*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a^5*c*d^7)
*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b^2*c^4*d^4
 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2)]
```

giac [A] time = 0.41, size = 301, normalized size = 1.45

$$\frac{abx}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(bx^{2}+a\right)}-\frac{3\left(ab^{2}c+a^{2}bd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)\sqrt{ab}}+\frac{3\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)\sqrt{cd}}+\frac{3\,bcdx^{3}+5\,ad^{2}x^{3}+5\,bc^{2}x+3\,acdx}{8\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*a*b*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a))
 - 3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2 +
6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*b*c*d*x^3 + 5*
a*d^2*x^3 + 5*b*c^2*x + 3*a*c*d*x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*(d*x^2 + c)^2)
```

maple [**B**]   time = 0.02, size = 388, normalized size = 1.87

$$-\frac{5a^2d^3x^3}{8\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}+\frac{abc\,d^2x^3}{4\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}+\frac{3b^2c^2d\,x^3}{8\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}-\frac{3a^2c\,d^2x}{8\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}-\frac{ab\,c^2dx}{4\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}+\frac{5b^2c^3x}{8\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}-\frac{a^2bdx}{2\left(ad-bc\right)^4\left(b\,x^2+a\right)}-\frac{3a^2bd\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^4\sqrt{ab}}+\frac{3a^2d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^4\sqrt{cd}}+\frac{a\,b^2cx}{2\left(ad-bc\right)^4\left(b\,x^2+a\right)}-\frac{3a\,b^2c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^4\sqrt{ab}}+\frac{9abcd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^4\sqrt{cd}}+\frac{3b^2c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^4\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*a^2*b/(a*d-b*c)^4*x/(b*x^2+a)*d+1/2*a*b^2/(a*d-b*c)^4*x/(b*x^2+a)*c-3/
2*a^2*b/(a*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-3/2*a*b^2/(a*d-
b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-5/8/(a*d-b*c)^4/(d*x^2+c)^2*
x^3*a^2*d^3+1/4/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a*b*c*d^2+3/8/(a*d-b*c)^4/(d*x^
2+c)^2*x^3*b^2*c^2*d-3/8/(a*d-b*c)^4/(d*x^2+c)^2*a^2*c*d^2*x-1/4/(a*d-b*c)^
4/(d*x^2+c)^2*a*b*c^2*d*x+5/8/(a*d-b*c)^4/(d*x^2+c)^2*b^2*c^3*x+3/8/(a*d-b*
c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2*d^2+9/4/(a*d-b*c)^4/(c*d)^(1
/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b*c*d+3/8/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/
(c*d)^(1/2)*d*x)*b^2*c^2
```

maxima [**B**]   time = 2.54, size = 443, normalized size = 2.14

$$-\frac{3\left(ab^2c+a^2bd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{ab}}+\frac{3\left(b^2c^2+6\,abcd+a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{cd}}+\frac{3\left(b^2cd+3\,abd^2\right)x^5+\left(5\,b^2c^2+14\,abcd+5\,a^2d^2\right)x^3+3\left(3\,abc^2+a^2cd\right)x}{8\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^6+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*(b^2*c*d + 3*a*b
*d^2)*x^5 + (5*b^2*c^2 + 14*a*b*c*d + 5*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + a^2*c
*d)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*
c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d
 - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*
c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

mupad [**B**]   time = 2.93, size = 7515, normalized size = 36.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan(((((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^
4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)
) - (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*
b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8
*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 +
 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^
6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36
*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) - (3*x*(-a*b)^(1/2)*(a*d
+ b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^
8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c
^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9
))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*
d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a*d + b*c))/(4*(a^4*d^4 + b
^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(-a*b)^(1/2)*
(a*d + b*c)*3i)/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)) + (((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2
+ 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2
*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*
a^5*b*c*d^5)) + (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11*c^9*d^2)/
2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9*c^7*d^4 -
 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7 + 42*a^7*b
^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 8
4*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*
c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*x*(-a*b)
^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d
^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 35
84*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7
*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a*d + b*c))/(4*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))*(
-a*b)^(1/2)*(a*d + b*c)*3i)/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a
*b^3*c^3*d - 4*a^3*b*c*d^3)))/(((81*a^5*b^3*d^5)/64 + (297*a^4*b^4*c*d^4)/3
2 + (135*a^2*b^6*c^3*d^2)/32 + (189*a^3*b^5*c^2*d^3)/16 + (27*a*b^7*c^4*d)/
64)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*
b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7
 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) - (3*((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 +
 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
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 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) - (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 +
(9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 11
4*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6
*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*
a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^
4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c
*d^8) - (3*x*(-a*b)^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2
- 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a
^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5
*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d
^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)
))*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
 4*a^3*b*c*d^3)))*(-a*b)^(1/2)*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b
^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*((x*(9*b^7*c^4*d + 153*a^
4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(
32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) + (3*(-a*b)^(1/2)*(((3*a^10*b^2
*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*
d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 -
147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^
9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126
*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d
- 9*a^8*b*c*d^8) + (3*x*(-a*b)^(1/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^
11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d
^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7
168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^
2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*
b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a
^5*b*c*d^5)))*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*
b^3*c^3*d - 4*a^3*b*c*d^3)))*(-a*b)^(1/2)*(a*d + b*c))/(4*(a^4*d^4 + b^4*c^
4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))))*(-a*b)^(1/2)*(a*d
 + b*c)*3i)/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a
^3*b*c*d^3)) - ((3*x^5*(3*a*b*d^2 + b^2*c*d))/(8*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2)) + (3*x*(3*a*b*c^2 + a^2*c*d))/(8*(a*d - b*c)*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^3*(5*a^2*d^2 + 5*b^2*c^2 + 14*a*b*c*d))/
(8*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*
c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan(((((x*(9*b^7*c^4*d + 153*
a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))
/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) - (3*(-c*d)^(1/2)*(((3*a^10*b
^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^
8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6
- 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 -
b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 1
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26*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*
d - 9*a^8*b*c*d^8) - (3*x*(-c*d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*(256
*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^1
0 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 35
84*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(512*(a^
4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3
)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)
)/(16*(a^4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^
2*c^3*d^3)))*(-c*d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*3i)/(16*(a^4*c*d^
5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3)) + (
((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4
+ 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^
3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) + (3*(
-c*d)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^
10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5
 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b
^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 -
126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2
*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*x*(-c*d)^(1/2)*(a^2*d^2 + b^
2*c^2 + 6*a*b*c*d)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d
^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 35
84*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7
*b^4*c^2*d^9))/(512*(a^4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*
d^4 + 6*a^2*b^2*c^3*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a^2*d^2
 + b^2*c^2 + 6*a*b*c*d))/(16*(a^4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a
^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3)))*(-c*d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b
*c*d)*3i)/(16*(a^4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 +
6*a^2*b^2*c^3*d^3)))/(((81*a^5*b^3*d^5)/64 + (297*a^4*b^4*c*d^4)/32 + (135*
a^2*b^6*c^3*d^2)/32 + (189*a^3*b^5*c^2*d^3)/16 + (27*a*b^7*c^4*d)/64)/(a^9*
d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d
^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^
8*c^8*d - 9*a^8*b*c*d^8) - (3*((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^
6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^
6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*
c^5*d - 6*a^5*b*c*d^5)) - (3*(-c*d)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11
*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9
*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7
+ 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c
^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 8
4*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) - (
3*x*(-c*d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*(256*a^9*b^2*d^11 + 256*b^
11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d
^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7
```

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168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(512*(a^4*c*d^5 + b^4*c^5*d -
4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3)*(a^6*d^6 + b^6*c^6 +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5
*d - 6*a^5*b*c*d^5)))*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(16*(a^4*c*d^5 + b^4
*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3)))*(-c*d)^(1
/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(16*(a^4*c*d^5 + b^4*c^5*d - 4*a*b^3*c
^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3)) + (3*((x*(9*b^7*c^4*d + 153*
a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))
/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)) + (3*(-c*d)^(1/2)*(((3*a^10*b
^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^
8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6
- 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 -
b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 1
26*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*
d - 9*a^8*b*c*d^8) + (3*x*(-c*d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*(256
*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d^3 - 1792*a^8*b^3*c*d^1
0 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 3584*a^4*b^7*c^5*d^6 + 35
84*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7*b^4*c^2*d^9))/(512*(a^
4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3
)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)
)/(16*(a^4*c*d^5 + b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^
2*c^3*d^3)))*(-c*d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(16*(a^4*c*d^5 +
 b^4*c^5*d - 4*a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3))))*(-c*
d)^(1/2)*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*3i)/(8*(a^4*c*d^5 + b^4*c^5*d - 4*
a*b^3*c^4*d^2 - 4*a^3*b*c^2*d^4 + 6*a^2*b^2*c^3*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.311 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=142

$$\frac{ab}{2\left(a+bx^2\right)(bc-ad)^3}+\frac{ad+bc}{2\left(c+dx^2\right)(bc-ad)^3}+\frac{c}{4\left(c+dx^2\right)^2(bc-ad)^2}+\frac{b(2ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^4}-\frac{b(2ad+bc)\,\mathrm{l}}{2(bc-}$$

Rubi [A] time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{ab}{2\left(a+bx^2\right)(bc-ad)^3}+\frac{ad+bc}{2\left(c+dx^2\right)(bc-ad)^3}+\frac{c}{4\left(c+dx^2\right)^2(bc-ad)^2}+\frac{b(2ad+bc)\log\left(a+bx^2\right)}{2(bc-ad)^4}-\frac{b(2ad+bc)\log\left(c+dx^2\right)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (a*b)/(2*(b*c - a*d)^3*(a + b*x^2)) + c/(4*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*c + a*d)/(2*(b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*a*d)*Log[a + b*x^2])
/(2*(b*c - a*d)^4) - (b*(b*c + 2*a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{ab^2}{(bc-ad)^3(a+bx)^2} + \frac{b^2(bc+2ad)}{(bc-ad)^4(a+bx)} - \frac{cd}{(bc-ad)^2(c+dx)^3} - (b\right.\right.$$

$$= \frac{ab}{2(bc-ad)^3\left(a+bx^2\right)} + \frac{c}{4(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bc+ad}{2(bc-ad)^3\left(c+dx^2\right)} + \frac{b(bc+2a}{2(}$$

Mathematica [A] time = 0.11, size = 121, normalized size = 0.85

$$\frac{\frac{c(bc-ad)^2}{\left(c+dx^2\right)^2} + \frac{2ab(bc-ad)}{a+bx^2} + \frac{2(ad+bc)(bc-ad)}{c+dx^2} + 2b(2ad+bc)\log\left(a+bx^2\right) - 2b(2ad+bc)\log\left(c+dx^2\right)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((2*a*b*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d)^2)/(c + d*x^2)^2 + (2*(b*
c - a*d)*(b*c + a*d))/(c + d*x^2) + 2*b*(b*c + 2*a*d)*Log[a + b*x^2] - 2*b*
(b*c + 2*a*d)*Log[c + d*x^2])/(4*(b*c - a*d)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 1.18, size = 598, normalized size = 4.21

$$\frac{5\,ab^2c^3 - 4\,a^2bc^2d - a^3cd^2 + 2\left(b^3c^2d + ab^2cd^2 - 2\,a^2bd^3\right)x^4 + \left(3\,b^3c^3 + 4\,ab^2c^2d - 5\,a^2bcd^2 - 2\,a^3d^3\right)x^2 + 2\left(\left(b^3cd^2 + 2\,ab^2d^3\right)x^6 + ab^2c^3 + 2\,a^2bc^2d + \left(2\,b^3c^2d + 5\,ab^2cd^2 + 2\,a^2bd^3\right)x^4 + \left(b^3c^3 + 4\,ab^2c^2d + 4\,a^2bcd^2\right)x^2\right)\log\left(bx^2 + a\right) - 2\left(\left(b^3cd^2 + 2\,ab^2d^3\right)x^6 + ab^2c^3 + 2\,a^2bc^2d + \left(2\,b^3c^2d + 5\,ab^2cd^2 + 2\,a^2bd^3\right)x^4 + \left(b^3c^3 + 4\,ab^2c^2d + 4\,a^2bcd^2\right)x^2\right)\log\left(dx^2 + c\right)}{4\left(ab^4c^6 - 4\,a^2b^3c^5d + 6\,a^3b^2c^4d^2 - 4\,a^4bc^3d^3 + a^5c^2d^4 + \left(b^5c^4d^2 - 4\,ab^4c^3d^3 + 6\,a^2b^3c^2d^4 - 4\,a^3b^2cd^5 + a^4bd^6\right)x^6 + \left(2\,b^5c^5d - 7\,ab^4c^4d^2 + 8\,a^2b^3c^3d^3 - 2\,a^3b^2c^2d^4 - 2\,a^4bcd^5 + a^5d^6\right)x^4 + \left(b^5c^6 - 2\,ab^4c^5d - 2\,a^2b^3c^4d^2 + 8\,a^3b^2c^3d^3 - 7\,a^4bc^2d^4 + 2\,a^5cd^5\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a*b^2*c^3 - 4*a^2*b*c^2*d - a^3*c*d^2 + 2*(b^3*c^2*d + a*b^2*c*d^2 -
 2*a^2*b*d^3)*x^4 + (3*b^3*c^3 + 4*a*b^2*c^2*d - 5*a^2*b*c*d^2 - 2*a^3*d^3)
*x^2 + 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*c^3 + 2*a^2*b*c^2*d + (2*b^
3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^3*c^3 + 4*a*b^2*c^2*d + 4*a
^2*b*c*d^2)*x^2)*log(b*x^2 + a) - 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*
c^3 + 2*a^2*b*c^2*d + (2*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^
3*c^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c^6 - 4*
a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^
4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*
x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^
4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4
*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

giac [B] time = 0.49, size = 267, normalized size = 1.88

$$\frac{\frac{2\,ab^5}{\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)\left(bx^2+a\right)}-\frac{2\left(b^4c+2\,ab^3d\right)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{3\,b^3cd^2+2\,ab^2d^3+\frac{2\left(2\,b^5c^2d-ab^4cd^2-a^2b^3d^3\right)}{\left(bx^2+a\right)b}}{(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a*b^5/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*
x^2 + a)) - 2*(b^4*c + 2*a*b^3*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a)
 + d))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4) - (3*b^3*c*d^2 + 2*a*b^2*d^3 + 2*(2*b^5*c^2*d - a*b^4*c*d^2 - a^2*b
^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b*x^2 + a) - a*d/(b*x^2 + a)
+ d)^2))/b
```

maple [B] time = 0.02, size = 283, normalized size = 1.99

$$\frac{a^2c\,d^2}{4\,(ad-bc)^4\left(d\,x^2+c\right)^2}-\frac{ab\,c^2d}{2\,(ad-bc)^4\left(d\,x^2+c\right)^2}+\frac{b^2c^3}{4\,(ad-bc)^4\left(d\,x^2+c\right)^2}-\frac{a^2bd}{2\,(ad-bc)^4\left(b\,x^2+a\right)}-\frac{a^2d^2}{2\,(ad-bc)^4\left(d\,x^2+c\right)}+\frac{a\,b^2c}{2\,(ad-bc)^4\left(b\,x^2+a\right)}+\frac{abd\ln\left(b\,x^2+a\right)}{(ad-bc)^4}-\frac{abd\ln\left(d\,x^2+c\right)}{(ad-bc)^4}+\frac{b^2c^2}{2\,(ad-bc)^4\left(d\,x^2+c\right)}+\frac{b^2c\ln\left(b\,x^2+a\right)}{2\,(ad-bc)^4}-\frac{b^2c\ln\left(d\,x^2+c\right)}{2\,(ad-bc)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b/(a*d-b*c)^4*a^2/(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^4*a/(b*x^2+a)*c+b/(a*d
-b*c)^4*ln(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^4*ln(b*x^2+a)*c+1/4*d^2/(a*d-b*c)
^4*c/(d*x^2+c)^2*a^2-1/2*d/(a*d-b*c)^4*c^2/(d*x^2+c)^2*a*b+1/4/(a*d-b*c)^4*
c^3/(d*x^2+c)^2*b^2-d/(a*d-b*c)^4*b*ln(d*x^2+c)*a-1/2/(a*d-b*c)^4*b^2*ln(d*
x^2+c)*c-1/2*d^2/(a*d-b*c)^4/(d*x^2+c)*a^2+1/2/(a*d-b*c)^4/(d*x^2+c)*b^2*c^
2
```

maxima [B] time = 1.23, size = 415, normalized size = 2.92

$$\frac{\left(b^{2} c + 2 \, a b d\right) \log \left(b x^{2} + a\right)}{2 \, \left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} - \frac{\left(b^{2} c + 2 \, a b d\right) \log \left(d x^{2} + c\right)}{2 \, \left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} + \frac{2 \, \left(b^{2} c d + 2 \, a b d^{2}\right) x^{4} + 5 \, a b c^{2} + a^{2} c d + \left(3 \, b^{2} c^{2} + 7 \, a b c d + 2 \, a^{2} d^{2}\right) x^{2}}{4 \, \left(a b^{3} c^{5} - 3 \, a^{2} b^{2} c^{4} d + 3 \, a^{3} b c^{3} d^{2} - a^{4} c^{2} d^{3} + \left(b^{4} c^{3} d^{2} - 3 \, a b^{3} c^{2} d^{3} + 3 \, a^{2} b^{2} c d^{4} - a^{3} b d^{5}\right) x^{6} + \left(2 \, b^{4} c^{4} d - 5 \, a b^{3} c^{3} d^{2} + 3 \, a^{2} b^{2} c^{2} d^{3} + a^{3} b c d^{4} - a^{4} d^{5}\right) x^{4} + \left(b^{4} c^{5} - a b^{3} c^{4} d - 3 \, a^{2} b^{2} c^{3} d^{2} + 5 \, a^{3} b c^{2} d^{3} - 2 \, a^{4} c d^{4}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c + 2*a*b*d)*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c
^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(b^2*c + 2*a*b*d)*log(d*x^2 + c)/(b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/4
*(2*(b^2*c*d + 2*a*b*d^2)*x^4 + 5*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 7*a*b*c*
d + 2*a^2*d^2)*x^2)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^
2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 +
 (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5
)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^
4*c*d^4)*x^2)
```

mupad [B] time = 0.70, size = 926, normalized size = 6.52

$$\frac{5\,a\,b^2\,c^3-a^3\,c\,d^2-2\,a^3\,d^3\,x^2+3\,b^3\,c^3\,x^2-4\,a^2\,b\,d^3\,x^4+2\,b^3\,c^2\,d\,x^4+a\,b^2\,c^3\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-4\,a^2\,b\,c^2\,d+b^3\,c^3\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a^2\,b\,d^3\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+a\,b^2\,d^3\,x^6\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+b^3\,c^2\,d\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+b^3\,c\,d^2\,x^6\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+4\,a\,b^2\,c^2\,d\,x^2-5\,a^2\,b\,c\,d^2\,x^2+2\,a\,b^2\,c\,d^2\,x^4+a^2\,b\,c^2\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,8\mathrm{i}+a\,b^2\,c^2\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,16\mathrm{i}+a^2\,b\,c\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,16\mathrm{i}+a\,b^2\,c\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,20\mathrm{i}}{4\,a^5\,c^2\,d^4+8\,a^5\,c\,d^5\,x^2+4\,a^5\,d^6\,x^4-16\,a^4\,b\,c^3\,d^3-28\,a^4\,b\,c^2\,d^4\,x^2-8\,a^4\,b\,c\,d^5\,x^4+4\,a^4\,b\,d^6\,x^6+24\,a^3\,b^2\,c^4\,d^2+32\,a^3\,b^2\,c^3\,d^3\,x^2-8\,a^3\,b^2\,c^2\,d^4\,x^4-16\,a^3\,b^2\,c\,d^5\,x^6-16\,a^2\,b^3\,c^5\,d-8\,a^2\,b^3\,c^4\,d^2\,x^2+32\,a^2\,b^3\,c^3\,d^3\,x^4+24\,a^2\,b^3\,c^2\,d^4\,x^6+4\,a\,b^4\,c^6-8\,a\,b^4\,c^5\,d\,x^2-28\,a\,b^4\,c^4\,d^2\,x^4-16\,a\,b^4\,c^3\,d^3\,x^6+4\,b^5\,c^6\,x^2+8\,b^5\,c^5\,d\,x^4+4\,b^5\,c^4\,d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (5*a*b^2*c^3 - a^3*c*d^2 - 2*a^3*d^3*x^2 + 3*b^3*c^3*x^2 - 4*a^2*b*d^3*x^4
+ 2*b^3*c^2*d*x^4 + a*b^2*c^3*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x
^2 + b*c*x^2))*4i - 4*a^2*b*c^2*d + b^3*c^3*x^2*atan((a*d*x^2*1i - b*c*x^2*
1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a^2*b*d^3*x^4*atan((a*d*x^2*1i - b*c*
x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + a*b^2*d^3*x^6*atan((a*d*x^2*1i -
b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + b^3*c^2*d*x^4*atan((a*d*x^2*1
i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + b^3*c*d^2*x^6*atan((a*d*x
^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + 4*a*b^2*c^2*d*x^2 - 5
*a^2*b*c*d^2*x^2 + 2*a*b^2*c*d^2*x^4 + a^2*b*c^2*d*atan((a*d*x^2*1i - b*c*x
^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + a*b^2*c^2*d*x^2*atan((a*d*x^2*1i -
 b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*16i + a^2*b*c*d^2*x^2*atan((a*d*x
^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*16i + a*b^2*c*d^2*x^4*atan
((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*20i)/(4*a*b^4*c^6 +
 4*a^5*c^2*d^4 + 4*b^5*c^6*x^2 + 4*a^5*d^6*x^4 - 16*a^2*b^3*c^5*d - 16*a^4*
b*c^3*d^3 + 4*a^4*b*d^6*x^6 + 8*a^5*c*d^5*x^2 + 8*b^5*c^5*d*x^4 + 24*a^3*b^
2*c^4*d^2 + 4*b^5*c^4*d^2*x^6 - 8*a^2*b^3*c^4*d^2*x^2 + 32*a^3*b^2*c^3*d^3*
x^2 + 32*a^2*b^3*c^3*d^3*x^4 - 8*a^3*b^2*c^2*d^4*x^4 + 24*a^2*b^3*c^2*d^4*x
^6 - 8*a*b^4*c^5*d*x^2 - 8*a^4*b*c*d^5*x^4 - 28*a^4*b*c^2*d^4*x^2 - 28*a*b^
4*c^4*d^2*x^4 - 16*a*b^4*c^3*d^3*x^6 - 16*a^3*b^2*c*d^5*x^6)
```

sympy [B] time = 7.95, size = 784, normalized size = 5.52

$$-\frac{b\left(2ad+bc\right)\log\left(x^2+\frac{-\frac{a^5bd^5(2ad+bc)}{(ad-bc)^4}+\frac{5a^4b^2cd^4(2ad+bc)}{(ad-bc)^4}-\frac{10a^3b^3c^2d^3(2ad+bc)}{(ad-bc)^4}+\frac{10a^2b^4c^3d^2(2ad+bc)}{(ad-bc)^4}+2a^2bd^2-\frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4}+3ab^2cd+\frac{b^6c^5(2ad+bc)}{(ad-bc)^4}+b^3c^2}{4ab^2d^2+2b^3cd}\right)}{2\left(ad-bc\right)^4}+\frac{b\left(2ad+bc\right)\log\left(x^2+\frac{\frac{a^5bd^5(2ad+bc)}{(ad-bc)^4}-\frac{5a^4b^2cd^4(2ad+bc)}{(ad-bc)^4}+\frac{10a^3b^3c^2d^3(2ad+bc)}{(ad-bc)^4}-\frac{10a^2b^4c^3d^2(2ad+bc)}{(ad-bc)^4}+2a^2bd^2+\frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4}+3ab^2cd-\frac{b^6c^5(2ad+bc)}{(ad-bc)^4}+b^3c^2}{4ab^2d^2+2b^3cd}\right)}{2\left(ad-bc\right)^4}+\frac{-a^2cd-5abc^2+x^4\left(-4abd^2-2b^2cd\right)+x^2\left(-2a^2d^2-7abcd-3b^2c^2\right)}{4a^4c^2d^3-12a^3bc^3d^2+12a^2b^2c^4d-4ab^3c^5+x^6\left(4a^3bd^5-12a^2b^2cd^4+12ab^3c^2d^3-4b^4c^3d^2\right)+x^4\left(4a^4d^5-4a^3bcd^4-12a^2b^2c^2d^3+20ab^3c^3d^2-8b^4c^4d\right)+x^2\left(8a^4cd^4-20a^3bc^2d^3+12a^2b^2c^3d^2+4ab^3c^4d-4b^4c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -b*(2*a*d + b*c)*log(x**2 + (-a**5*b*d**5*(2*a*d + b*c)/(a*d - b*c)**4 + 5*
a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**3*b**3*c**2*d**3*(2*a
*d + b*c)/(a*d - b*c)**4 + 10*a**2*b**4*c**3*d**2*(2*a*d + b*c)/(a*d - b*c)
**4 + 2*a**2*b*d**2 - 5*a*b**5*c**4*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*a*b*
*2*c*d + b**6*c**5*(2*a*d + b*c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2*d**2
 + 2*b**3*c*d))/(2*(a*d - b*c)**4) + b*(2*a*d + b*c)*log(x**2 + (a**5*b*d**
5*(2*a*d + b*c)/(a*d - b*c)**4 - 5*a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*
c)**4 + 10*a**3*b**3*c**2*d**3*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**2*b**4*
c**3*d**2*(2*a*d + b*c)/(a*d - b*c)**4 + 2*a**2*b*d**2 + 5*a*b**5*c**4*d*(2
*a*d + b*c)/(a*d - b*c)**4 + 3*a*b**2*c*d - b**6*c**5*(2*a*d + b*c)/(a*d -
b*c)**4 + b**3*c**2)/(4*a*b**2*d**2 + 2*b**3*c*d))/(2*(a*d - b*c)**4) + (-a
**2*c*d - 5*a*b*c**2 + x**4*(-4*a*b*d**2 - 2*b**2*c*d) + x**2*(-2*a**2*d**2
 - 7*a*b*c*d - 3*b**2*c**2))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a
**2*b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4
 + 12*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c
*d**4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**
2*(8*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*
c**4*d - 4*b**4*c**5))
```

3.312 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3d}{4\left(c+dx^2\right)^2}$$

Rubi [A] time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {471, 527, 522, 205}

$$-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}+\frac{b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)^4}-\frac{dx(ad+11bc)}{8c\left(c+dx^2\right)(bc-ad)^3}-\frac{3dx}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x)/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x/(2*(b*c - a*d)*(a + b*x^2)*(c
+ d*x^2)^2) - (d*(11*b*c + a*d)*x)/(8*c*(b*c - a*d)^3*(c + d*x^2)) + (b^(3/
2)*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)^4) - (
Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8
*c^(3/2)*(b*c - a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{\int \frac{c-5dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2(bc-ad)}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}+\frac{\int \frac{2c(2bc+ad)-18bcdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,d}{8c(bc-ad)^2}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}-\frac{d(11bc+ad)x}{8c(bc-ad)^3 (c+dx}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}-\frac{d(11bc+ad)x}{8c(bc-ad)^3 (c+dx}$$

$$= -\frac{3dx}{4(bc-ad)^2\left(c+dx^2\right)^2}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2}-\frac{d(11bc+ad)x}{8c(bc-ad)^3 (c+dx}$$

Mathematica [A] time = 0.41, size = 171, normalized size = 0.86

$$\frac{\frac{\sqrt{d}\left(a^2d^2-10abcd-15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{4b^{3/2}(5ad+bc)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{4b^2x(bc-ad)}{a+bx^2}+\frac{dx(ad-bc)(ad+7bc)}{c\left(c+dx^2\right)}-\frac{2dx(bc-ad)^2}{\left(c+dx^2\right)^2}}{8(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^2*(b*c - a*d)*x)/(a + b*x^2) - (2*d*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
 (d*(-(b*c) + a*d)*(7*b*c + a*d)*x)/(c*(c + d*x^2)) + (4*b^(3/2)*(b*c + 5*a
*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (Sqrt[d]*(-15*b^2*c^2 - 10*a*b*c
*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(8*(b*c - a*d)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 4.60, size = 2891, normalized size = 14.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*b^3*c^3
*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2
*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c^2*
d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2
)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*a*b^2*c^
4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b
*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4
 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt
(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a
*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d
+ 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^
4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b
^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*
b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 +
 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c
^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2
 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*
d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 3
5*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3
*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2
*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^
```

```
3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10
*a^2*b*c^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) + (4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4
*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4
+ (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 +
a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a
^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*
d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5
)*x^2), -1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*
b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 8*(a*b^2*c^4
+ 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b
^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d
^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (15*a*b^2*c^4 + 10*a^2*b*c^3*d - a
^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 + (30*b^3*c^
3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3*c^4 + 40*a*
b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*
c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*
c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 -
4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*
c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5
*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)
*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7
*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2), -1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^
3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3
*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^
6 + (2*b^3*c^3*d + 11*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b
^2*c^3*d + 10*a^2*b*c^2*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (15*a*b^2
*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^
2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*
x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*s
qrt(d/c)*arctan(x*sqrt(d/c)) + (4*b^3*c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^
2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*
b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^
4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 +
 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 +
 (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b
*c^3*d^4 + 2*a^5*c^2*d^5)*x^2)]
```

giac [A] time = 0.36, size = 317, normalized size = 1.58

$$-\frac{b^{2} x}{2 \left(b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}\right) \left(b x^{2} + a\right)} + \frac{\left(b^{3} c + 5 a b^{2} d\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(b^{4} c^{4} - 4 a b^{3} c^{3} d + 6 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + a^{4} d^{4}\right) \sqrt{a b}} - \frac{\left(15 b^{2} c^{2} d + 10 a b c d^{2} - a^{2} d^{3}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 \left(b^{4} c^{5} - 4 a b^{3} c^{4} d + 6 a^{2} b^{2} c^{3} d^{2} - 4 a^{3} b c^{2} d^{3} + a^{4} c d^{4}\right) \sqrt{c d}} - \frac{7 b c d^{2} x^{3} + a d^{3} x^{3} + 9 b c^{2} d x - a c d^{2} x}{8 \left(b^{3} c^{4} - 3 a b^{2} c^{3} d + 3 a^{2} b c^{2} d^{2} - a^{3} c d^{3}\right) \left(d x^{2} + c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)
) + 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2*c^
2*d + 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4
*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*(7*b
*c*d^2*x^3 + a*d^3*x^3 + 9*b*c^2*d*x - a*c*d^2*x)/((b^3*c^4 - 3*a*b^2*c^3*d
 + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

maple [B] time = 0.02, size = 391, normalized size = 1.96

$$\frac{a^{2} d^{4} x^{3}}{8 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2} c}+\frac{3 a b \,d^{3} x^{3}}{4 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}-\frac{7 b^{2} c \,d^{2} x^{3}}{8 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}-\frac{a^{2} d^{3} x}{8 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}+\frac{5 a b c \,d^{2} x}{4 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}-\frac{9 b^{2} c^{2} d x}{8 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}+\frac{a^{2} d^{3} \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{8 \left(a d -b c \right)^{4} \sqrt{c d}\, c}+\frac{a \,b^{2} d x}{2 \left(a d -b c \right)^{4} \left(b \,x^{2}+a \right)}+\frac{5 a \,b^{2} d \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(a d -b c \right)^{4} \sqrt{a b}}-\frac{5 a b \,d^{2} \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{4 \left(a d -b c \right)^{4} \sqrt{c d}}-\frac{b^{3} c x}{2 \left(a d -b c \right)^{4} \left(b \,x^{2}+a \right)}+\frac{b^{3} c \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \left(a d -b c \right)^{4} \sqrt{a b}}-\frac{15 b^{2} c d \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{8 \left(a d -b c \right)^{4} \sqrt{c d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^4*x/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x/(b*x^2+a)*c+5/2*b
^2/(a*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*a*d+1/2*b^3/(a*d-b*c)^
4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+1/8*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c
*x^3*a^2+3/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a*b-7/8*d^2/(a*d-b*c)^4/(d*x^2
+c)^2*x^3*b^2*c+5/4*d^2/(a*d-b*c)^4/(d*x^2+c)^2*a*b*c*x-9/8*d/(a*d-b*c)^4/(
d*x^2+c)^2*b^2*c^2*x-1/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a^2*x+1/8*d^3/(a*d-b*c
)^4/c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2-5/4*d^2/(a*d-b*c)^4/(c*d)^(
1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b-15/8*d/(a*d-b*c)^4*c/(c*d)^(1/2)*arctan(
1/(c*d)^(1/2)*d*x)*b^2
```

maxima [B] time = 2.63, size = 473, normalized size = 2.36

$$\frac{{\left(b^{3} c + 5 \, a b^{2} d\right)} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{a b}} - \frac{{\left(15 \, b^{2} c^{2} d + 10 \, a b c d^{2} - a^{2} d^{3}\right)} \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 \, {\left(b^{4} c^{5} - 4 \, a b^{3} c^{4} d + 6 \, a^{2} b^{2} c^{3} d^{2} - 4 \, a^{3} b c^{2} d^{3} + a^{4} c d^{4}\right)} \sqrt{c d}} - \frac{{\left(11 \, b^{2} c d^{2} + a b d^{3}\right)} x^{5} + {\left(17 \, b^{2} c^{2} d + 6 \, a b c d^{2} + a^{2} d^{3}\right)} x^{3} + {\left(4 \, b^{2} c^{3} + 9 \, a b c^{2} d - a^{2} c d^{2}\right)} x}{8 \, {\left(a b^{3} c^{6} - 3 \, a^{2} b^{2} c^{5} d + 3 \, a^{3} b c^{4} d^{2} - a^{4} c^{3} d^{3} + {\left(b^{4} c^{4} d^{2} - 3 \, a b^{3} c^{3} d^{3} + 3 \, a^{2} b^{2} c^{2} d^{4} - a^{3} b c d^{5}\right)} x^{6} + {\left(2 \, b^{4} c^{5} d - 5 \, a b^{3} c^{4} d^{2} + 3 \, a^{2} b^{2} c^{3} d^{3} + a^{3} b c^{2} d^{4} - a^{4} c d^{5}\right)} x^{4} + {\left(b^{4} c^{6} - a b^{3} c^{5} d - 3 \, a^{2} b^{2} c^{4} d^{2} + 5 \, a^{3} b c^{3} d^{3} - 2 \, a^{4} c^{2} d^{4}\right)} x^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d
+ 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4*d +
 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*((11*b^2
*c*d^2 + a*b*d^3)*x^5 + (17*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (4*b^2
*c^3 + 9*a*b*c^2*d - a^2*c*d^2)*x)/(a*b^3*c^6 - 3*a^2*b^2*c^5*d + 3*a^3*b*c
^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2*b^2*c^2*d^4 -
 a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^2*c^3*d^3 + a^
3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*c^4*d^2 +
 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2)
```

mupad [B] time = 2.77, size = 7929, normalized size = 39.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] ((x^5*(11*b^2*c*d^2 + a*b*d^3))/(8*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) + (x*(4*b^2*c^2 - a^2*d^2 + 9*a*b*c*d))/(8*(a*d - b*c)*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(a^2*d^2 + 17*b^2*c^2 + 6*a*b*c*d))/(8
*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*
c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan((((-a*b^3)^(1/2)*(5*a*d +
 b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*c
*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 +
 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7
*d)) - (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 +
(39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a^
5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3*
d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8 +
 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b^
4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) - (x*
(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 1792*a*b^10*c^10*d^3 + 51
20*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^7*d^6 + 3584*a^5
*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9 - 1792*a^8*b^3*c
^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*
d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3
*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b
^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b
^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))*1i)/(4*(a^5*d^4 + a*b^4*c^4
 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)) + ((-a*b^3)^(1/2)*
(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a
^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c
^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a
*b^5*c^7*d)) + (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^
12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6
+ 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*
b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c
^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 12
6*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*
d) + (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 1792*a*b^10*c^10*
d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^7*d^6 +
3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9 - 1792*a
^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b^4*c^4 - 4*a^2*
b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^6*c^2*d^6 - 6*a
^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4
 - 6*a*b^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a^5*d^4 + a*b^4*c^4 -
 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))*1i)/(4*(a^5*d^4 + a
*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))/(((165*b^
```

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8*c^4*d^3)/64 - (5*a^4*b^4*d^7)/64 + (475*a*b^7*c^3*d^4)/32 - (3*a^3*b^5*c*
d^6)/32 + (39*a^2*b^6*c^2*d^5)/4)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8
 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*
b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) - (
(-a*b^3)^(1/2)*(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6
*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*
d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^
2*c^4*d^4 - 6*a*b^5*c^7*d)) - (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 -
 (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a
^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^
4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*
d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b
^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7
 - 9*a*b^8*c^10*d) - (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 17
92*a*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4
*b^7*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c
^4*d^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b
^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^
6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*
a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a^5*d^
4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))))/(4*
(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)
) + ((-a*b^3)^(1/2)*(5*a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*
a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6
*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a
^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3
)/2 - (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 -
126*a^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b
^5*c^4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9
*c^2*d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*
a^4*b^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4
*d^7 - 9*a*b^8*c^10*d) + (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2
 - 1792*a*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 358
4*a^4*b^7*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*
b^4*c^4*d^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4
+ a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8
 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
+ 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-a*b^3)^(1/2)*(5*a*d + b*c))/(4*(a
^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)))
)/(4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c
*d^3))))*(-a*b^3)^(1/2)*(5*a*d + b*c)*1i)/(2*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b
^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)) + (atan(((((x*(a^4*b^3*d^7 +
 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^5*c^2*d
^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20
*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (((2*b^12*c^11*d^
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2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*d^4)/2
+ 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*
b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d
^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84
*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c
^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) - (x*(-c^3*d)^(1/2)*(15*b^2*c
^2 - a^2*d^2 + 10*a*b*c*d)*(256*b^11*c^11*d^2 - 1792*a*b^10*c^10*d^3 + 5120
*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^7*d^6 + 3584*a^5*b
^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9 - 1792*a^8*b^3*c^3
*d^10 + 256*a^9*b^2*c^2*d^11))/(512*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^
3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3
*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b
^5*c^7*d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d))/(16*(b^4*c^
7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)))*(-
c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d)*1i)/(16*(b^4*c^7 + a^4*c^3
*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)) + (((x*(a^4*b^
3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^6 + 470*a^2*b^
5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d
^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (((2*b^12*
c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 + (39*a^2*b^10*c^9*
d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a^5*b^7*c^6*d^7 - 2
31*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3*d^10 + (17*a^9*b^
3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8 + 36*a^2*b^7*c^9*d
^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^
6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) + (x*(-c^3*d)^(1/2)*(1
5*b^2*c^2 - a^2*d^2 + 10*a*b*c*d)*(256*b^11*c^11*d^2 - 1792*a*b^10*c^10*d^3
 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^7*d^6 + 358
4*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9 - 1792*a^8*
b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(512*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b
*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(b^6*c^8 + a^6*c^2*d^6 - 6*a^
5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4
- 6*a*b^5*c^7*d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d))/(16*
(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*
d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d)*1i)/(16*(b^4*c^7 +
a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)))/(((165
*b^8*c^4*d^3)/64 - (5*a^4*b^4*d^7)/64 + (475*a*b^7*c^3*d^4)/32 - (3*a^3*b^5
*c*d^6)/32 + (39*a^2*b^6*c^2*d^5)/4)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*
d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a
^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d)
- (((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*c*d^
6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15
*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)
) - (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 + (39
*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a^5*b
^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3*d^1
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0 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8 + 36
*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b^4*c
^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) - (x*(-c
^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d)*(256*b^11*c^11*d^2 - 1792*a
*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7
*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d
^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(512*(b^4*c^7 + a^4*c^3
*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(b^6*c^8 + a^6*
c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^
4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*
a*b*c*d))/(16*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2
- 4*a*b^3*c^6*d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d))/(16*
(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*
d)) + (((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3*b^4*
c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5
+ 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^
7*d)) + (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d^12)/2 +
 (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^6 + 231*a
^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a^8*b^4*c^3
*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8*b*c^3*d^8
+ 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4 - 126*a^5*b
^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8*c^10*d) + (x
*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d)*(256*b^11*c^11*d^2 - 17
92*a*b^10*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4
*b^7*c^7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c
^4*d^9 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(512*(b^4*c^7 + a^4
*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(b^6*c^8 +
a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 1
5*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 +
 10*a*b*c*d))/(16*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*
d^2 - 4*a*b^3*c^6*d)))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d))/
(16*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*
c^6*d))))*(-c^3*d)^(1/2)*(15*b^2*c^2 - a^2*d^2 + 10*a*b*c*d)*1i)/(8*(b^4*c^
7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.313 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=126

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 44}

$$-\frac{b^2}{2\left(a+bx^2\right)(bc-ad)^3}-\frac{3b^2d\log\left(a+bx^2\right)}{2(bc-ad)^4}+\frac{3b^2d\log\left(c+dx^2\right)}{2(bc-ad)^4}-\frac{bd}{\left(c+dx^2\right)(bc-ad)^3}-\frac{d}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x^2)) - d/(4*(b*c - a*d)^2*(c + d*x^2)^2) - (b
*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^2])/(2*(b*c - a*d)^4
) + (3*b^2*d*Log[c + d*x^2])/(2*(b*c - a*d)^4)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2} - \frac{3b^3d}{(bc-ad)^4(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)^3} + \frac{}{(bc}\right.\right.$$

$$= -\frac{b^2}{2(bc-ad)^3\left(a+bx^2\right)} - \frac{d}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{bd}{(bc-ad)^3\left(c+dx^2\right)} - \frac{3b^2d\log}{2(bc-}$$

Mathematica [A] time = 0.13, size = 107, normalized size = 0.85

$$-\frac{\frac{2b^2(bc-ad)}{a+bx^2} + 6b^2d\log\left(a+bx^2\right) + \frac{4bd(bc-ad)}{c+dx^2} + \frac{d(bc-ad)^2}{\left(c+dx^2\right)^2} - 6b^2d\log\left(c+dx^2\right)}{4(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/4*((2*b^2*(b*c - a*d))/(a + b*x^2) + (d*(b*c - a*d)^2)/(c + d*x^2)^2 + (
4*b*d*(b*c - a*d))/(c + d*x^2) + 6*b^2*d*Log[a + b*x^2] - 6*b^2*d*Log[c + d
*x^2])/(b*c - a*d)^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 0.93, size = 507, normalized size = 4.02

$$-\frac{2\,b^3c^3 + 3\,ab^2c^2d - 6\,a^2bcd^2 + a^3d^3 + 6\left(b^3cd^2 - ab^2d^3\right)x^4 + 3\left(3\,b^3c^2d - 2\,ab^2cd^2 - a^2bd^3\right)x^2 + 6\left(b^3d^3x^6 + ab^2c^2d + \left(2\,b^3cd^2 + ab^2d^3\right)x^4 + \left(b^3c^2d + 2\,ab^2cd^2\right)x^2\right)\log\left(bx^2 + a\right) - 6\left(b^3d^3x^6 + ab^2c^2d + \left(2\,b^3cd^2 + ab^2d^3\right)x^4 + \left(b^3c^2d + 2\,ab^2cd^2\right)x^2\right)\log\left(dx^2 + c\right)}{4\left(ab^4c^6 - 4\,a^2b^3c^5d + 6\,a^3b^2c^4d^2 - 4\,a^4bc^3d^3 + a^5c^2d^4 + \left(b^5c^4d^2 - 4\,ab^4c^3d^3 + 6\,a^2b^3c^2d^4 - 4\,a^3b^2cd^5 + a^4bd^6\right)x^6 + \left(2\,b^5c^5d - 7\,ab^4c^4d^2 + 8\,a^2b^3c^3d^3 - 2\,a^3b^2c^2d^4 - 2\,a^4bcd^5 + a^5d^6\right)x^4 + \left(b^5c^6 - 2\,ab^4c^5d - 2\,a^2b^3c^4d^2 + 8\,a^3b^2c^3d^3 - 7\,a^4bc^2d^4 + 2\,a^5cd^5\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^4 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x^2 + 6*(b^3*d
```

```
^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2
*c*d^2)*x^2)*log(b*x^2 + a) - 6*(b^3*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 +
 a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c
^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 +
(b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*
b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2
*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*
b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

giac [**A**] time = 0.43, size = 229, normalized size = 1.82

$$\frac{3\,b^3d\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)}-\frac{b^5}{2\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)\left(bx^2+a\right)}+\frac{5\,b^2d^3+\frac{6\left(b^4cd^2-ab^3d^3\right)}{\left(bx^2+a\right)b}}{4\left(bc-ad\right)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/2*b^3*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^5*c^4 - 4*a*b^
4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/2*b^5/((b^6*
c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x^2 + a)) + 1/4*(5*
b^2*d^3 + 6*(b^4*c*d^2 - a*b^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b
*x^2 + a) - a*d/(b*x^2 + a) + d)^2)
```

maple [**A**] time = 0.02, size = 234, normalized size = 1.86

$$-\frac{a^2d^3}{4\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}+\frac{abc\,d^2}{2\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}-\frac{b^2c^2d}{4\left(ad-bc\right)^4\left(d\,x^2+c\right)^2}+\frac{a\,b^2d}{2\left(ad-bc\right)^4\left(b\,x^2+a\right)}+\frac{ab\,d^2}{\left(ad-bc\right)^4\left(d\,x^2+c\right)}-\frac{b^3c}{2\left(ad-bc\right)^4\left(b\,x^2+a\right)}-\frac{b^2cd}{\left(ad-bc\right)^4\left(d\,x^2+c\right)}-\frac{3b^2d\ln\left(b\,x^2+a\right)}{2\left(ad-bc\right)^4}+\frac{3b^2d\ln\left(d\,x^2+c\right)}{2\left(ad-bc\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^4/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4/(b*x^2+a)*c-3/2*b^2/(
a*d-b*c)^4*ln(b*x^2+a)*d-1/4*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a^2+1/2*d^2/(a*d-b
*c)^4/(d*x^2+c)^2*a*b*c-1/4*d/(a*d-b*c)^4/(d*x^2+c)^2*b^2*c^2+3/2*d/(a*d-b*
c)^4*ln(d*x^2+c)*b^2+d^2/(a*d-b*c)^4*b/(d*x^2+c)*a-d/(a*d-b*c)^4*b^2/(d*x^2
+c)*c
```

maxima [**B**] time = 1.26, size = 394, normalized size = 3.13

$$-\frac{3\,b^2d\log\left(bx^2+a\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}+\frac{3\,b^2d\log\left(dx^2+c\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)}-\frac{6\,b^2d^2x^4+2\,b^2c^2+5\,abcd-a^2d^2+3\left(3\,b^2cd+abd^2\right)x^2}{4\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^6+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b^2*d*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4) + 3/2*b^2*d*log(d*x^2 + c)/(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/4*(6*b^2*d^2*x^4 + 2*b^2
*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x^2)/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

mupad [**B**] time = 0.59, size = 707, normalized size = 5.61

$$-\frac{a^3 d^3 + 2 b^3 c^3 - 3 a^2 b d^3 x^2 - 6 a b^2 d^3 x^4 + 9 b^3 c^2 d x^2 + 6 b^3 c d^2 x^4 + 3 a b^2 c^2 d - 6 a^2 b c d^2 + b^3 d^3 x^6 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 12i + a b^2 d^3 x^4 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 12i + b^3 c^2 d x^2 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 12i + b^3 c d^2 x^4 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 24i - 6 a b^2 c d^2 x^2 + a b^2 c^2 d \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 12i + a b^2 c d^2 x^2 \operatorname{atan}\left(\frac{a d x^2 1i - b c x^2 1i}{2 a c + a d x^2 + b c x^2}\right) 24i}{4 a^5 c^2 d^4 + 8 a^5 c d^5 x^2 + 4 a^5 d^6 x^4 - 16 a^4 b c^3 d^3 - 28 a^4 b c^2 d^4 x^2 - 8 a^4 b c d^5 x^4 + 4 a^4 b d^6 x^6 + 24 a^3 b^2 c^4 d^2 + 32 a^3 b^2 c^3 d^3 x^2 - 8 a^3 b^2 c^2 d^4 x^4 - 16 a^3 b^2 c d^5 x^6 - 16 a^2 b^3 c^5 d - 8 a^2 b^3 c^4 d^2 x^2 + 32 a^2 b^3 c^3 d^3 x^4 + 24 a^2 b^3 c^2 d^4 x^6 + 4 a b^4 c^6 - 8 a b^4 c^5 d x^2 - 28 a b^4 c^4 d^2 x^4 - 16 a b^4 c^3 d^3 x^6 + 4 b^5 c^6 x^2 + 8 b^5 c^5 d x^4 + 4 b^5 c^4 d^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] -(a^3*d^3 + 2*b^3*c^3 - 3*a^2*b*d^3*x^2 - 6*a*b^2*d^3*x^4 + 9*b^3*c^2*d*x^2
 + 6*b^3*c*d^2*x^4 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + b^3*d^3*x^6*atan((a*d*
x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + a*b^2*d^3*x^4*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + b^3*c^2*d*x^2*
atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + b^3*c*d^2
*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*24i - 6*a*
b^2*c*d^2*x^2 + a*b^2*c^2*d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2
 + b*c*x^2))*12i + a*b^2*c*d^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c +
a*d*x^2 + b*c*x^2))*24i)/(4*a*b^4*c^6 + 4*a^5*c^2*d^4 + 4*b^5*c^6*x^2 + 4*a
^5*d^6*x^4 - 16*a^2*b^3*c^5*d - 16*a^4*b*c^3*d^3 + 4*a^4*b*d^6*x^6 + 8*a^5*
c*d^5*x^2 + 8*b^5*c^5*d*x^4 + 24*a^3*b^2*c^4*d^2 + 4*b^5*c^4*d^2*x^6 - 8*a^
2*b^3*c^4*d^2*x^2 + 32*a^3*b^2*c^3*d^3*x^2 + 32*a^2*b^3*c^3*d^3*x^4 - 8*a^3
*b^2*c^2*d^4*x^4 + 24*a^2*b^3*c^2*d^4*x^6 - 8*a*b^4*c^5*d*x^2 - 8*a^4*b*c*d
^5*x^4 - 28*a^4*b*c^2*d^4*x^2 - 28*a*b^4*c^4*d^2*x^4 - 16*a*b^4*c^3*d^3*x^6
 - 16*a^3*b^2*c*d^5*x^6)
```

sympy [**B**] time = 10.06, size = 643, normalized size = 5.10

$$\frac{3b^2 d \log\left(x^2 + \frac{-\frac{3a^5b^2d^6}{(ad-bc)^4} + \frac{15a^4b^3cd^5}{(ad-bc)^4} - \frac{30a^3b^4c^2d^4}{(ad-bc)^4} + \frac{30a^2b^5c^3d^3}{(ad-bc)^4} - \frac{15ab^6c^4d^2}{(ad-bc)^4} + 3ab^2d^2 + \frac{3b^7c^5d}{(ad-bc)^4} + 3b^3cd}{6b^3d^2}\right)}{2\left(ad-bc\right)^4} - \frac{3b^2 d \log\left(x^2 + \frac{\frac{3a^5b^2d^6}{(ad-bc)^4} - \frac{15a^4b^3cd^5}{(ad-bc)^4} + \frac{30a^3b^4c^2d^4}{(ad-bc)^4} - \frac{30a^2b^5c^3d^3}{(ad-bc)^4} + \frac{15ab^6c^4d^2}{(ad-bc)^4} + 3ab^2d^2 - \frac{3b^7c^5d}{(ad-bc)^4} + 3b^3cd}{6b^3d^2}\right)}{2\left(ad-bc\right)^4} + \frac{-a^2d^2 + 5abcd + 2b^2c^2 + 6b^2d^2x^4 + x^2\left(3abd^2 + 9b^2cd\right)}{4a^4c^2d^3 - 12a^3bc^3d^2 + 12a^2b^2c^4d - 4ab^3c^5 + x^6\left(4a^3bd^5 - 12a^2b^2cd^4 + 12ab^3c^2d^3 - 4b^4c^3d^2\right) + x^4\left(4a^4d^5 - 4a^3bcd^4 - 12a^2b^2c^2d^3 + 20ab^3c^3d^2 - 8b^4c^4d\right) + x^2\left(8a^4cd^4 - 20a^3bc^2d^3 + 12a^2b^2c^3d^2 + 4ab^3c^4d - 4b^4c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] 3*b**2*d*log(x**2 + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5
/(a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3
*d**3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 +
 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*c)**
4) - 3*b**2*d*log(x**2 + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*
```

```
d**5/(a*d - b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*
c**3*d**3/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d*
*2 - 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*
c)**4) + (-a**2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**4 + x**2*(3
*a*b*d**2 + 9*b**2*c*d))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2 + 12*a**2*
b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2*c*d**4 + 1
2*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*a**3*b*c*d**
4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4*d) + x**2*(8
*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 + 4*a*b**3*c**4
*d - 4*b**4*c**5))
```

3.314 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=230

$$\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{}{2a\left(a+bx^2\right.}$$

Rubi [A] time = 0.30, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {414, 527, 522, 205}

$$\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(2b^2c^2-8abcd+3a^2}{\left(a+bx^2\right)(}}{8ac(bc}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+}{8ac^2(bc-ad)^3(c}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+}{8ac^2(bc-ad)^3(c}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+}{8ac^2(bc-ad)^3(c}$$

Mathematica [A] time = 0.41, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(\frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4} + \frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4} - \frac{4b^3x}{a\left(a+bx^2\right)(ad-bc)^3} + \frac{d^2x(11bc-3ad)}{c^2\left(c+dx^2\right)(bc-ad)^3} + \frac{2d^2x}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 8.74, size = 3239, normalized size = 14.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
 (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
```

```
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

giac [A] time = 0.44, size = 332, normalized size = 1.44

$$\frac{b^{3}x}{2\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)\left(bx^{2}+a\right)}+\frac{\left(b^{4}c-7\,ab^{3}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^{4}c^{4}-4\,a^{2}b^{3}c^{3}d+6\,a^{3}b^{2}c^{2}d^{2}-4\,a^{4}bcd^{3}+a^{5}d^{4}\right)\sqrt{ab}}+\frac{\left(35\,b^{2}c^{2}d^{2}-14\,abcd^{3}+3\,a^{2}d^{4}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{6}-4\,ab^{3}c^{5}d+6\,a^{2}b^{2}c^{4}d^{2}-4\,a^{3}bc^{3}d^{3}+a^{4}c^{2}d^{4}\right)\sqrt{cd}}+\frac{11\,bcd^{3}x^{3}-3\,ad^{4}x^{3}+13\,bc^{2}d^{2}x-5\,acd^{3}x}{8\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.02, size = 403, normalized size = 1.75

$$\frac{3a^{2}d^{5}x^{3}}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{2}}-\frac{7ab\,d^{4}x^{3}}{4\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c}+\frac{11b^{2}d^{3}x^{3}}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}}+\frac{5a^{2}d^{4}x}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c}-\frac{9ab\,d^{3}x}{4\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}}+\frac{13b^{2}c\,d^{2}x}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}}+\frac{3a^{2}d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{4}\sqrt{cd}\,c^{2}}-\frac{7ab\,d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^{4}\sqrt{cd}\,c}+\frac{b^{4}cx}{2\left(ad-bc\right)^{4}\left(b\,x^{2}+a\right)a}+\frac{b^{4}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{4}\sqrt{ab}\,a}-\frac{b^{3}dx}{2\left(ad-bc\right)^{4}\left(b\,x^{2}+a\right)}-\frac{7b^{3}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{4}\sqrt{ab}}+\frac{35b^{2}d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{4}\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b^3/(a*d-b*c)^4*x/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x/(b*x^2+a)*c-7/2*
b^3/(a*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+1/2*b^4/(a*d-b*c)^4
/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+3/8*d^5/(a*d-b*c)^4/(d*x^2+c)^2/
c^2*x^3*a^2-7/4*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^3*a*b+11/8*d^3/(a*d-b*c)^4/
(d*x^2+c)^2*b^2*x^3+5/8*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x*a^2-9/4*d^3/(a*d-b*
c)^4/(d*x^2+c)^2*a*b*x+13/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*b^2*c*x+3/8*d^4/(a*
d-b*c)^4/c^2/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2-7/4*d^3/(a*d-b*c)^4/
c/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b+35/8*d^2/(a*d-b*c)^4/(c*d)^(1/2
)*arctan(1/(c*d)^(1/2)*d*x)*b^2
```

maxima [B] time = 2.68, size = 529, normalized size = 2.30

$$\frac{\left(b^{4}c-7\,ab^{3}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^{4}c^{4}-4\,a^{2}b^{3}c^{3}d+6\,a^{3}b^{2}c^{2}d^{2}-4\,a^{4}bcd^{3}+a^{5}d^{4}\right)\sqrt{ab}}+\frac{\left(35\,b^{2}c^{2}d^{2}-14\,abcd^{3}+3\,a^{2}d^{4}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{6}-4\,ab^{3}c^{5}d+6\,a^{2}b^{2}c^{4}d^{2}-4\,a^{3}bc^{3}d^{3}+a^{4}c^{2}d^{4}\right)\sqrt{cd}}+\frac{\left(4\,b^{3}c^{2}d^{2}+11\,ab^{2}cd^{3}-3\,a^{2}bd^{4}\right)x^{5}+\left(8\,b^{3}c^{3}d+13\,ab^{2}c^{2}d^{2}+6\,a^{2}bcd^{3}-3\,a^{3}d^{4}\right)x^{3}+\left(4\,b^{3}c^{4}+13\,a^{2}bc^{2}d^{2}-5\,a^{3}cd^{3}\right)x}{8\left(a^{2}b^{3}c^{7}-3\,a^{3}b^{2}c^{6}d+3\,a^{4}bc^{5}d^{2}-a^{5}c^{4}d^{3}+\left(ab^{4}c^{5}d^{2}-3\,a^{2}b^{3}c^{4}d^{3}+3\,a^{3}b^{2}c^{3}d^{4}-a^{4}bc^{2}d^{5}\right)x^{6}+\left(2\,ab^{4}c^{6}d-5\,a^{2}b^{3}c^{5}d^{2}+3\,a^{3}b^{2}c^{4}d^{3}+a^{4}bc^{3}d^{4}-a^{5}c^{2}d^{5}\right)x^{4}+\left(ab^{4}c^{7}-a^{2}b^{3}c^{6}d-3\,a^{3}b^{2}c^{5}d^{2}+5\,a^{4}bc^{4}d^{3}-2\,a^{5}c^{3}d^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*d
 + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^
2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3
*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/
```

```
8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d + 13*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5
*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c
^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*
b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d
^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

mupad [B] time = 3.00, size = 8649, normalized size = 37.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^
4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7)
)/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*
a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^13*c
^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*
c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7
*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 +
(35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^
4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*
b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d
^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^
11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^
7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 +
256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a
^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9
*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2
*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^
4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d))
)*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 +
 a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) + (((x
*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2
009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*
b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8
*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + (((2*a*b^13*c^13*d^2 - 2
8*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*
a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^
3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a
^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^
```

```
3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9
*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(
256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 71
68*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^
8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^
2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*
d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*
b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))
*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4
*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3
)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 + a^4*c^5*d^
4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))/(((63*a^5*b^5*d^
9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)
/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 -
 a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 -
 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b
^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 2
24*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^
3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c
^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b
^2*c^6*d^4)) - (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*
c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^
8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^
10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13
)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 3
6*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^
4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*
(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^1
0*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c
^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9
 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5
*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 +
 a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*
a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^
2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b
^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2
 - 4*a*b^3*c^8*d)) + (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d
^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^
4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b
*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) +
 (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 -
(987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 11
97*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b
^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*
```

```
c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11
*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84
*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 3
5*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5
120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*
a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b
^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b
*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 -
 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^
3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b
*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4
*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16
*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8
*d))))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(8*(b^4*c
^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) -
((x^5*(4*b^3*c^2*d^2 - 3*a^2*b*d^4 + 11*a*b^2*c*d^3))/(8*a*c^2*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*b^3*c^3 - 5*a^3*d^3 + 13*a
^2*b*c*d^2))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(
8*b^3*c^3 - 3*a^3*d^3 + 13*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(8*a*c^2*(a*d - b*
c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(
a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3
 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^
6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b
^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a
^6*b^2*c^6*d^4)) - ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b^13*c^13*d^2 - 28
*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
+ 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a
^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3
*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^
3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3
 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*
b^2*c^6*d^7) - (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*b^11*c^13*d^2 - 1
792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 358
4*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*
b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(128*(a^7*d^
4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2
*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^
8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^7*d^4 + a^3*b^4*c
^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)))*(7*a*d - b*c)*(
-a^3*b^5)^(1/2)*1i)/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 - 4*a^6*b*c*d^3)) + (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b
^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6
+ 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d -
 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6
*d^4)) + ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b^13*c^13*d^2 - 28*a^2*b^12*
```

```
c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*
b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6
*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)
/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12
*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6
*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^
7) + (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^
10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*
c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^
9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(128*(a^7*d^4 + a^3*b^
4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2*b^6*c^10
+ a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20
*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4
*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)))*(7*a*d - b*c)*(-a^3*b^5)^
(1/2)*1i)/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 -
 4*a^6*b*c*d^3)))/(((63*a^5*b^5*d^9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9
*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^
3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10
*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^
4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(
9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 200
9*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^
6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d
^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - ((7*a*d - b*c)*(-a^3*b^5)^
(1/2)*((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/
2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7
+ 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^
10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*
b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*
c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5
+ 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(7*a*d - b*c)*(-a^3*b^5)^(1
/2)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4
 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 71
68*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^
11*b^2*c^4*d^11))/(128*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6
*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d
^4))))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*
a^6*b*c*d^3)))*(7*a*d - b*c)*(-a^3*b^5)^(1/2))/(4*(a^7*d^4 + a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)) + (((x*(9*a^6*b^3*d^9
 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4
*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*
c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b
^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b
^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*
```

```
b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^
7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^
11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^
11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84
*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*
c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*
b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b
^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^
7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^
11))/(128*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*
a^6*b*c*d^3)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^
5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^
7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3))
)*(7*a*d - b*c)*(-a^3*b^5)^(1/2))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3))))*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*1
i)/(2*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.315 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=192

$$-\frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4}-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a\left(a+bx^2\right)(bc-ad)^3}+\frac{d^2(}{2c^2\left(c+\right.}$$

Rubi [A] time = 0.24, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$-\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}-\frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4}+\frac{\log(x)}{a^2c^3}+\frac{b^3}{2a\left(a+bx^2\right)(bc-ad)^3}+\frac{d^2(3bc-ad)}{2c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2}{4c\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] b^3/(2*a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(4*c*(b*c - a*d)^2*(c + d*x^2)^2)
 + (d^2*(3*b*c - a*d))/(2*c^2*(b*c - a*d)^3*(c + d*x^2)) + Log[x]/(a^2*c^3)
 - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(2*a^2*(b*c - a*d)^4) - (d^2*(6*b^2*c
^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(2*c^3*(b*c - a*d)^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(\frac{1}{a^2c^3x} + \frac{b^4}{a(-bc+ad)^3(a+bx)^2} + \frac{b^4(-bc+4ad)}{a^2(-bc+ad)^4(a+bx)} -$$

$$= \frac{b^3}{2a(bc-ad)^3\left(a+bx^2\right)} + \frac{d^2}{4c(bc-ad)^2\left(c+dx^2\right)^2} + \frac{d^2(3bc-ad)}{2c^2(bc-ad)^3\left(c+dx^2\right)} +$$

Mathematica [A] time = 0.27, size = 187, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2b^3(4ad-bc)\log\left(a+bx^2\right)}{a^2(bc-ad)^4} - \frac{2d^2\left(a^2d^2-4abcd+6b^2c^2\right)\log\left(c+dx^2\right)}{c^3(bc-ad)^4} + \frac{4\log(x)}{a^2c^3} - \frac{2b^3}{a\left(a+bx^2\right)(ad-bc)^3} + \frac{2d^2(3bc-ad)}{c^2\left(c+dx^2\right)(bc-ad)^3} + \frac{d^2}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-2*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^
2)^2) + (2*d^2*(3*b*c - a*d))/(c^2*(b*c - a*d)^3*(c + d*x^2)) + (4*Log[x])/
(a^2*c^3) + (2*b^3*(-(b*c) + 4*a*d)*Log[a + b*x^2])/(a^2*(b*c - a*d)^4) - (
2*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d)^4)
)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 30.22, size = 1058, normalized size = 5.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^4*c^6 - 2*a^2*b^3*c^5*d + 7*a^3*b^2*c^4*d^2 - 10*a^4*b*c^3*d^3 +
 3*a^5*c^2*d^4 + 2*(a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^3 - 4*a^3*b^2*c^2*d^4 +
 a^4*b*c*d^5)*x^4 + (4*a*b^4*c^5*d + 3*a^2*b^3*c^4*d^2 - 4*a^3*b^2*c^3*d^3
- 5*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2 - 2*(a*b^4*c^6 - 4*a^2*b^3*c^5*d + (b^
5*c^4*d^2 - 4*a*b^4*c^3*d^3)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 - 4*a^2*b
^3*c^3*d^3)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 8*a^2*b^3*c^4*d^2)*x^2)*log(b*
x^2 + a) - 2*(6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (6*a^2*b^
3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (12*a^2*b^3*c^3*d^3 - 2*a^3*
b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (6*a^2*b^3*c^4*d^2 + 8*a^3*b^2
*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(d*x^2 + c) + 4*(a*b^4*c^
6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (
b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b
*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*
c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b
^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(x)
)/(a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^
7*c^5*d^4 + (a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^
5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^6 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 +
 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^4
 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 -
 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^2)
```

giac [B] time = 0.36, size = 470, normalized size = 2.45

$$-\frac{\left(b^{5} c - 4 \, a b^{4} d\right) \log\left({\left| b x^{2} + a \right|}\right)}{2 \, {\left(a^{2} b^{5} c^{4} - 4 \, a^{3} b^{4} c^{3} d + 6 \, a^{4} b^{3} c^{2} d^{2} - 4 \, a^{5} b^{2} c d^{3} + a^{6} b d^{4}\right)}} - \frac{{\left(6 \, b^{2} c^{2} d^{3} - 4 \, a b c d^{4} + a^{2} d^{5}\right)} \log\left({\left| d x^{2} + c \right|}\right)}{2 \, {\left(b^{4} c^{7} d - 4 \, a b^{3} c^{6} d^{2} + 6 \, a^{2} b^{2} c^{5} d^{3} - 4 \, a^{3} b c^{4} d^{4} + a^{4} c^{3} d^{5}\right)}} + \frac{b^{5} c x^{2} - 4 \, a b^{4} d x^{2} + 2 \, a b^{4} c - 5 \, a^{2} b^{3} d}{2 \, {\left(a^{2} b^{4} c^{4} - 4 \, a^{3} b^{3} c^{3} d + 6 \, a^{4} b^{2} c^{2} d^{2} - 4 \, a^{5} b c d^{3} + a^{6} d^{4}\right)} {\left(b x^{2} + a\right)}} + \frac{18 \, b^{2} c^{2} d^{4} x^{4} - 12 \, a b c d^{5} x^{4} + 3 \, a^{2} d^{6} x^{4} + 42 \, b^{2} c^{3} d^{3} x^{2} - 32 \, a b c^{2} d^{4} x^{2} + 8 \, a^{2} c d^{5} x^{2} + 25 \, b^{2} c^{4} d^{2} - 22 \, a b c^{3} d^{3} + 6 \, a^{2} c^{2} d^{4}}{4 \, {\left(b^{4} c^{7} - 4 \, a b^{3} c^{6} d + 6 \, a^{2} b^{2} c^{5} d^{2} - 4 \, a^{3} b c^{4} d^{3} + a^{4} c^{3} d^{4}\right)} {\left(d x^{2} + c\right)}^{2}} + \frac{\log\left(x^{2}\right)}{2 \, a^{2} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d
 + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4) - 1/2*(6*b^2*c^2*d^3 -
4*a*b*c*d^4 + a^2*d^5)*log(abs(d*x^2 + c))/(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5) + 1/2*(b^5*c*x^2 - 4*a*b^
4*d*x^2 + 2*a*b^4*c - 5*a^2*b^3*d)/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*
b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*(b*x^2 + a)) + 1/4*(18*b^2*c^2*d^4*x
^4 - 12*a*b*c*d^5*x^4 + 3*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^2 - 32*a*b*c^2*d^4
*x^2 + 8*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 22*a*b*c^3*d^3 + 6*a^2*c^2*d^4)/(
(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^
4)*(d*x^2 + c)^2) + 1/2*log(x^2)/(a^2*c^3)
```

maple [B] time = 0.03, size = 374, normalized size = 1.95

$$\frac{a^{2} d^{4}}{4 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2} c}-\frac{a b \,d^{3}}{2 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}+\frac{b^{2} c \,d^{2}}{4 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)^{2}}+\frac{a^{2} d^{4}}{2 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right) c^{2}}-\frac{a^{2} d^{4} \ln \left(d \,x^{2}+c \right)}{2 \left(a d -b c \right)^{4} c^{3}}-\frac{2 a b \,d^{3}}{\left(a d -b c \right)^{4} \left(d \,x^{2}+c \right) c}+\frac{2 a b \,d^{3} \ln \left(d \,x^{2}+c \right)}{\left(a d -b c \right)^{4} c^{2}}+\frac{b^{4} c}{2 \left(a d -b c \right)^{4} \left(b \,x^{2}+a \right) a}+\frac{2 b^{3} d \ln \left(b \,x^{2}+a \right)}{\left(a d -b c \right)^{4} a}-\frac{b^{4} c \ln \left(b \,x^{2}+a \right)}{2 \left(a d -b c \right)^{4} a^{2}}-\frac{b^{3} d}{2 \left(a d -b c \right)^{4} \left(b \,x^{2}+a \right)}-\frac{3 b^{2} d^{2} \ln \left(d \,x^{2}+c \right)}{\left(a d -b c \right)^{4} c}+\frac{3 b^{2} d^{2}}{2 \left(a d -b c \right)^{4} \left(d \,x^{2}+c \right)}+\frac{\ln \left(x \right)}{a^{2} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b^3/(a*d-b*c)^4/(b*x^2+a)*d+1/2*b^4/a/(a*d-b*c)^4/(b*x^2+a)*c+2*b^3/a/
(a*d-b*c)^4*ln(b*x^2+a)*d-1/2*b^4/a^2/(a*d-b*c)^4*ln(b*x^2+a)*c+1/4*d^4/c/(
a*d-b*c)^4/(d*x^2+c)^2*a^2-1/2*d^3/(a*d-b*c)^4/(d*x^2+c)^2*a*b+1/4*d^2*c/(a
*d-b*c)^4/(d*x^2+c)^2*b^2-1/2*d^4/c^3/(a*d-b*c)^4*ln(d*x^2+c)*a^2+2*d^3/c^2
/(a*d-b*c)^4*ln(d*x^2+c)*a*b-3*d^2/c/(a*d-b*c)^4*ln(d*x^2+c)*b^2+1/2*d^4/c^
2/(a*d-b*c)^4/(d*x^2+c)*a^2-2*d^3/c/(a*d-b*c)^4/(d*x^2+c)*a*b+3/2*d^2/(a*d-
b*c)^4/(d*x^2+c)*b^2+ln(x)/a^2/c^3
```

maxima [B] time = 1.34, size = 527, normalized size = 2.74

$$-\frac{\left(b^{4}c - 4\,ab^{3}d\right)\log\left(bx^{2} + a\right)}{2\left(a^{2}b^{4}c^{4} - 4\,a^{3}b^{3}c^{3}d + 6\,a^{4}b^{2}c^{2}d^{2} - 4\,a^{5}bcd^{3} + a^{6}d^{4}\right)} - \frac{\left(6\,b^{2}c^{2}d^{2} - 4\,abcd^{3} + a^{2}d^{4}\right)\log\left(dx^{2} + c\right)}{2\left(b^{4}c^{7} - 4\,ab^{3}c^{6}d + 6\,a^{2}b^{2}c^{5}d^{2} - 4\,a^{3}bc^{4}d^{3} + a^{4}c^{3}d^{4}\right)} + \frac{2\,b^{3}c^{4} + 7\,a^{2}bc^{2}d^{2} - 3\,a^{3}cd^{3} + 2\left(b^{3}c^{2}d^{2} + 3\,ab^{2}cd^{3} - a^{2}bd^{4}\right)x^{4} + \left(4\,b^{3}c^{3}d + 7\,ab^{2}c^{2}d^{2} + 3\,a^{2}bcd^{3} - 2\,a^{3}d^{4}\right)x^{2}}{4\left(a^{2}b^{3}c^{7} - 3\,a^{3}b^{2}c^{6}d + 3\,a^{4}bc^{5}d^{2} - a^{5}c^{4}d^{3} + \left(ab^{4}c^{5}d^{2} - 3\,a^{2}b^{3}c^{4}d^{3} + 3\,a^{3}b^{2}c^{3}d^{4} - a^{4}bc^{2}d^{5}\right)x^{6} + \left(2\,ab^{4}c^{6}d - 5\,a^{2}b^{3}c^{5}d^{2} + 3\,a^{3}b^{2}c^{4}d^{3} + a^{4}bc^{3}d^{4} - a^{5}c^{2}d^{5}\right)x^{4} + \left(ab^{4}c^{7} - a^{2}b^{3}c^{6}d - 3\,a^{3}b^{2}c^{5}d^{2} + 5\,a^{4}bc^{4}d^{3} - 2\,a^{5}c^{3}d^{4}\right)x^{2}\right)} + \frac{\log\left(x^{2}\right)}{2\,a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^4*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*
a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/2*(6*b^2*c^2*d^2 - 4*a*b*c*d
^3 + a^2*d^4)*log(d*x^2 + c)/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 -
 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) + 1/4*(2*b^3*c^4 + 7*a^2*b*c^2*d^2 - 3*a^3*
c*d^3 + 2*(b^3*c^2*d^2 + 3*a*b^2*c*d^3 - a^2*b*d^4)*x^4 + (4*b^3*c^3*d + 7*
a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 2*a^3*d^4)*x^2)/(a^2*b^3*c^7 - 3*a^3*b^2*c^
6*d + 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 +
3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2
 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*
b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 1/2
*log(x^2)/(a^2*c^3)
```

mupad [B] time = 2.07, size = 472, normalized size = 2.46

$$\frac{\ln(x)}{a^{2}\,c^{3}} - \frac{\ln\left(b\,x^{2} + a\right)\,\left(b^{4}\,c - 4\,a\,b^{3}\,d\right)}{2\,a^{6}\,d^{4} - 8\,a^{5}\,b\,c\,d^{3} + 12\,a^{4}\,b^{2}\,c^{2}\,d^{2} - 8\,a^{3}\,b^{3}\,c^{3}\,d + 2\,a^{2}\,b^{4}\,c^{4}} - \frac{\ln\left(d\,x^{2} + c\right)\,\left(a^{2}\,d^{4} - 4\,a\,b\,c\,d^{3} + 6\,b^{2}\,c^{2}\,d^{2}\right)}{2\,a^{4}\,c^{3}\,d^{4} - 8\,a^{3}\,b\,c^{4}\,d^{3} + 12\,a^{2}\,b^{2}\,c^{5}\,d^{2} - 8\,a\,b^{3}\,c^{6}\,d + 2\,b^{4}\,c^{7}} - \frac{\frac{-3\,a^{3}\,d^{3}+7\,a^{2}\,b\,c\,d^{2}+2\,b^{3}\,c^{3}}{4\,a\,c\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)} + \frac{x^{2}\,\left(-2\,a^{3}\,d^{4}+3\,a^{2}\,b\,c\,d^{3}+7\,a\,b^{2}\,c^{2}\,d^{2}+4\,b^{3}\,c^{3}\,d\right)}{4\,a\,c^{2}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)} + \frac{b\,d^{2}\,x^{4}\,\left(-a^{2}\,d^{2}+3\,a\,b\,c\,d+b^{2}\,c^{2}\right)}{2\,a\,c^{2}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}}{a\,c^{2} + x^{2}\,\left(b\,c^{2} + 2\,a\,d\,c\right) + x^{4}\,\left(a\,d^{2} + 2\,b\,c\,d\right) + b\,d^{2}\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] log(x)/(a^2*c^3) - (log(a + b*x^2)*(b^4*c - 4*a*b^3*d))/(2*a^6*d^4 + 2*a^2*
b^4*c^4 - 8*a^3*b^3*c^3*d + 12*a^4*b^2*c^2*d^2 - 8*a^5*b*c*d^3) - (log(c +
d*x^2)*(a^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/(2*b^4*c^7 + 2*a^4*c^3*d^4
- 8*a^3*b*c^4*d^3 + 12*a^2*b^2*c^5*d^2 - 8*a*b^3*c^6*d) - ((2*b^3*c^3 - 3*a
^3*d^3 + 7*a^2*b*c*d^2)/(4*a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)) + (x^2*(4*b^3*c^3*d - 2*a^3*d^4 + 7*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)
)/(4*a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^
4*(b^2*c^2 - a^2*d^2 + 3*a*b*c*d))/(2*a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
```

```
2*d - 3*a^2*b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*
d) + b*d^2*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.316 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=297

$$-\frac{3b^{7/2}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^4}+\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}-3($$

Rubi [A] time = 0.50, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{8ac^2x\left(c+dx^2\right)(bc-ad)^3}-\frac{3(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{8a^2c^3x(bc-ad)^3}-\frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}-\frac{3b^{7/2}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^4}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*(2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c*d + 5*a^2*d^2))/(8*a^2*c^3*(b*c - a*
d)^3*x) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x*(c + d*x^2)^2) + b/(2*a*
(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b^2*c^2 + 13*a*b*c*d - 5*a
^2*d^2))/(8*a*c^2*(b*c - a*d)^3*x*(c + d*x^2)) - (3*b^(7/2)*(b*c - 3*a*d)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*
c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*(b*c
- a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-3bc+2ad-7bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(6b^2c^2-8ab\right.}{x^2(}}{8}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2+13\right.}{8ac^2(bc-ad}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{}{2a(bc-}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{}{2a(bc-}$$

$$= -\frac{3(2bc-ad)\left(2b^2c^2-3abcd+5a^2d^2\right)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x\left(c+dx^2\right)^2} + \frac{}{2a(bc-}$$

Mathematica [A] time = 0.42, size = 210, normalized size = 0.71

$$\frac{1}{8}\left(\frac{12b^{7/2}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} + \frac{4b^4x}{a^2\left(a+bx^2\right)(ad-bc)^3} - \frac{3d^{5/2}\left(5a^2d^2-18abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^4} - \frac{8}{a^2c^3x} + \frac{d^3x(7ad-15bc)}{c^3\left(c+dx^2\right)(bc-ad)^3} - \frac{2d^3x}{c^2\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x) + (4*b^4*x)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (2*d^3*x)/
(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-15*b*c + 7*a*d)*x)/(c^3*(b*c - a
*d)^3*(c + d*x^2)) + (12*b^(7/2)*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^4))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 19.65, size = 3753, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3
*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^
2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4*c^
4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d
^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*
c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4
*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (
b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5
*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((21*
a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3
 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^
2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2
*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^2 - 2*c
*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*
a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d
 - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*
d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2
 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9
- 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x),
-1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b*c^3*d^
3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^
4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c^5*d - 64*a*b^4*c^4*d^2
+ 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^
4 + (12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3
- 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^
2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a
^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31
*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3
 + 5*a^5*c^2*d^4)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 6*((b^5*c^4*d^2 - 3*a*
b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5
+ (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*
c^5*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2
*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 +
```

```
a^6*b*c^3*d^6)*x^7 + (2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d
^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9
- 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4
 + 2*a^7*c^4*d^5)*x^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2
- 4*a^6*b*c^6*d^3 + a^7*c^5*d^4)*x), -1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d
 + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^3*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^
2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)
*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2
*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*d^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*
d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d
^5)*x^2 + 24*((b^5*c^4*d^2 - 3*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*
c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d
^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*
c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3
*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*
a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log((d*x^
2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d
^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (2*a^2*b^
5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6
*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*
c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3 + (a^3*b
^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a^7*c^5*d^
4)*x), -1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^4*d^2 - 32*a^4*b
*c^3*d^3 + 8*a^5*c^2*d^4 + 3*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3
*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + (24*b^5*c^5*d - 64*a*b^4*c
^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a^5*
d^6)*x^4 + (12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3*b^2*c
^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3*a*b^4*
c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*x^5 + (b
^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2*b^3*c^5*
d)*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c
*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*
b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^
4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 + (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 +
5*a^5*c^2*d^4)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^2*b^5*c^7*d^2 - 4*a^3*
b^4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 +
(2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^
4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2
*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x^3
 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3 + a
^7*c^5*d^4)*x)]
```

giac [A] time = 0.50, size = 430, normalized size = 1.45

$$-\frac{3\left(b^{5}c-3\,ab^{4}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{2}b^{4}c^{4}-4\,a^{3}b^{3}c^{3}d+6\,a^{4}b^{2}c^{2}d^{2}-4\,a^{5}bcd^{3}+a^{6}d^{4}\right)\sqrt{ab}}-\frac{3\left(21\,b^{2}c^{2}d^{3}-18\,abcd^{4}+5\,a^{2}d^{5}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{7}-4\,ab^{3}c^{6}d+6\,a^{2}b^{2}c^{5}d^{2}-4\,a^{3}bc^{4}d^{3}+a^{4}c^{3}d^{4}\right)\sqrt{cd}}-\frac{3\,b^{4}c^{3}x^{2}-6\,ab^{3}c^{2}dx^{2}+6\,a^{2}b^{2}cd^{2}x^{2}-2\,a^{3}bd^{3}x^{2}+2\,ab^{3}c^{3}-6\,a^{2}b^{2}c^{2}d+6\,a^{3}bcd^{2}-2\,a^{4}d^{3}}{2\left(a^{2}b^{3}c^{6}-3\,a^{3}b^{2}c^{5}d+3\,a^{4}bc^{4}d^{2}-a^{5}c^{3}d^{3}\right)\left(bx^{3}+ax\right)}-\frac{15\,bcd^{4}x^{3}-7\,ad^{5}x^{3}+17\,bc^{2}d^{3}x-9\,acd^{4}x}{8\left(b^{3}c^{6}-3\,ab^{2}c^{5}d+3\,a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3}\right)\left(dx^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^
3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b^2
*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4*a*
b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d)) -
 1/2*(3*b^4*c^3*x^2 - 6*a*b^3*c^2*d*x^2 + 6*a^2*b^2*c*d^2*x^2 - 2*a^3*b*d^3
*x^2 + 2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3)/((a^2*b^3
*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(b*x^3 + a*x)) - 1/
8*(15*b*c*d^4*x^3 - 7*a*d^5*x^3 + 17*b*c^2*d^3*x - 9*a*c*d^4*x)/((b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.02, size = 428, normalized size = 1.44

$$-\frac{7a^{2}d^{6}x^{3}}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{3}}+\frac{11ab\,d^{5}x^{3}}{4\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{2}}-\frac{15b^{2}d^{4}x^{3}}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c}-\frac{9a^{2}d^{5}x}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{13ab\,d^{4}x}{4\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c}-\frac{17b^{2}d^{3}x}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}}-\frac{15a^{2}d^{5}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{4}\sqrt{cd}\;c^{3}}+\frac{27ab\,d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^{4}\sqrt{cd}\;c^{2}}+\frac{b^{4}dx}{2\left(ad-bc\right)^{4}\left(b\,x^{2}+a\right)a}+\frac{9b^{4}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{4}\sqrt{ab}\;a}-\frac{b^{5}cx}{2\left(ad-bc\right)^{4}\left(b\,x^{2}+a\right)a^{2}}-\frac{3b^{5}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{4}\sqrt{ab}\;a^{2}}-\frac{63b^{2}d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{4}\sqrt{cd}\;c}-\frac{1}{a^{2}c^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^4/a/(a*d-b*c)^4*x/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x/(b*x^2+a)*c+9
/2*b^4/a/(a*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d-3/2*b^5/a^2/(a
*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c-7/8*d^6/c^3/(a*d-b*c)^4/(
d*x^2+c)^2*x^3*a^2+11/4*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a*b-15/8*d^4/c/
(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2-9/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*a^2*x+1
3/4*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*a*b*x-17/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*b^
2*x-15/8*d^5/c^3/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a^2+27/4
*d^4/c^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*b-63/8*d^3/c/(
a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1/a^2/c^3/x
```

maxima [B] time = 2.75, size = 639, normalized size = 2.15

$$-\frac{3\left(b^{5}c-3\,ab^{4}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{2}b^{4}c^{4}-4\,a^{3}b^{3}c^{3}d+6\,a^{4}b^{2}c^{2}d^{2}-4\,a^{5}bcd^{3}+a^{6}d^{4}\right)\sqrt{ab}}-\frac{3\left(21\,b^{2}c^{2}d^{3}-18\,abcd^{4}+5\,a^{2}d^{5}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{7}-4\,ab^{3}c^{6}d+6\,a^{2}b^{2}c^{5}d^{2}-4\,a^{3}bc^{4}d^{3}+a^{4}c^{3}d^{4}\right)\sqrt{cd}}-\frac{8\,ab^{3}c^{5}-24\,a^{2}b^{2}c^{4}d+24\,a^{3}bc^{3}d^{2}-8\,a^{4}c^{2}d^{3}+3\left(4\,b^{4}c^{3}d^{2}-8\,ab^{3}c^{2}d^{3}+13\,a^{2}b^{2}cd^{4}-5\,a^{3}bd^{5}\right)x^{6}+\left(24\,b^{4}c^{4}d-40\,ab^{3}c^{3}d^{2}+41\,a^{2}b^{2}c^{2}d^{3}+14\,a^{3}bcd^{4}-15\,a^{4}d^{5}\right)x^{4}+\left(12\,b^{4}c^{5}-8\,ab^{3}c^{4}d-24\,a^{2}b^{2}c^{3}d^{2}+57\,a^{3}bc^{2}d^{3}-25\,a^{4}cd^{4}\right)x^{2}}{8\left(\left(a^{2}b^{4}c^{6}d^{2}-3\,a^{3}b^{3}c^{5}d^{3}+3\,a^{4}b^{2}c^{4}d^{4}-a^{5}bc^{3}d^{5}\right)x^{7}+\left(2\,a^{2}b^{4}c^{7}d-5\,a^{3}b^{3}c^{6}d^{2}+3\,a^{4}b^{2}c^{5}d^{3}+a^{5}bc^{4}d^{4}-a^{6}c^{3}d^{5}\right)x^{5}+\left(a^{2}b^{4}c^{8}-a^{3}b^{3}c^{7}d-3\,a^{4}b^{2}c^{6}d^{2}+5\,a^{5}bc^{5}d^{3}-2\,a^{6}c^{4}d^{4}\right)x^{3}+\left(a^{3}b^{3}c^{8}-3\,a^{4}b^{2}c^{7}d+3\,a^{5}bc^{6}d^{2}-a^{6}c^{5}d^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^
3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b^2
```

```
*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4*a*
b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d)) -
 1/8*(8*a*b^3*c^5 - 24*a^2*b^2*c^4*d + 24*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3
*(4*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 + 13*a^2*b^2*c*d^4 - 5*a^3*b*d^5)*x^6 + (
24*b^4*c^4*d - 40*a*b^3*c^3*d^2 + 41*a^2*b^2*c^2*d^3 + 14*a^3*b*c*d^4 - 15*
a^4*d^5)*x^4 + (12*b^4*c^5 - 8*a*b^3*c^4*d - 24*a^2*b^2*c^3*d^2 + 57*a^3*b*
c^2*d^3 - 25*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*
b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^7 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3
*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^5 + (a^2*b^4*c^8 - a^3*b^
3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^3 + (a^3*b
^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3)*x)
```

mupad [B] time = 2.80, size = 5060, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan((a^9*d^5*x*(-c^7*d^5)^(3/2)*25i + b^9*c^16*d*x*(-c^7*d^5)^(1/2)*16i -
 a^6*b^3*c^3*d^2*x*(-c^7*d^5)^(3/2)*756i + a^7*b^2*c^2*d^3*x*(-c^7*d^5)^(3/
2)*534i + a^2*b^7*c^14*d^3*x*(-c^7*d^5)^(1/2)*144i - a^8*b*c*d^4*x*(-c^7*d^
5)^(3/2)*180i + a^5*b^4*c^4*d*x*(-c^7*d^5)^(3/2)*441i - a*b^8*c^15*d^2*x*(-
c^7*d^5)^(1/2)*96i)/(25*a^9*c^11*d^12 - 16*b^9*c^20*d^3 + 96*a*b^8*c^19*d^4
 - 180*a^8*b*c^12*d^11 - 144*a^2*b^7*c^18*d^5 + 441*a^5*b^4*c^15*d^8 - 756*
a^6*b^3*c^14*d^9 + 534*a^7*b^2*c^13*d^10))*(-c^7*d^5)^(1/2)*(5*a^2*d^2 + 21
*b^2*c^2 - 18*a*b*c*d)*3i)/(8*(b^4*c^11 + a^4*c^7*d^4 - 4*a^3*b*c^8*d^3 + 6
*a^2*b^2*c^9*d^2 - 4*a*b^3*c^10*d)) - (atan((((x*(147456*a^6*b^20*c^26*d^3
- 2211840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^24*d^5 - 56180736*a^9*b^1
7*c^23*d^6 + 144737280*a^10*b^16*c^22*d^7 - 285078528*a^11*b^15*c^21*d^8 +
505018368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*c^19*d^10 + 1434332160*a
^14*b^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12 + 1999835136*a^16*b^10*
c^16*d^13 - 1581355008*a^17*b^9*c^15*d^14 + 938843136*a^18*b^8*c^14*d^15 -
412314624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c^12*d^17 - 28145664*a^21
*b^5*c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 230400*a^23*b^3*c^9*d^20) + (
3*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(3145728*a^9*b^19*c^29*d^3 - 196608*a^8*b^
20*c^30*d^2 - 23003136*a^10*b^18*c^28*d^4 + 101203968*a^11*b^17*c^27*d^5 -
294961152*a^12*b^16*c^26*d^6 + 582500352*a^13*b^15*c^25*d^7 - 729071616*a^1
4*b^14*c^24*d^8 + 339296256*a^15*b^13*c^23*d^9 + 766132224*a^16*b^12*c^22*d
^10 - 2185936896*a^17*b^11*c^21*d^11 + 3127787520*a^18*b^10*c^20*d^12 - 308
4337152*a^19*b^9*c^19*d^13 + 2249834496*a^20*b^8*c^18*d^14 - 1236221952*a^2
1*b^7*c^17*d^15 + 508674048*a^22*b^6*c^16*d^16 - 152715264*a^23*b^5*c^15*d^
17 + 31703040*a^24*b^4*c^14*d^18 - 4079616*a^25*b^3*c^13*d^19 + 245760*a^26
*b^2*c^12*d^20 + (3*x*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(262144*a^10*b^20*c^33
*d^2 - 4194304*a^11*b^19*c^32*d^3 + 31195136*a^12*b^18*c^31*d^4 - 142606336
```

```
*a^13*b^17*c^30*d^5 + 445644800*a^14*b^16*c^29*d^6 - 998244352*a^15*b^15*c^
28*d^7 + 1622147072*a^16*b^14*c^27*d^8 - 1853882368*a^17*b^13*c^26*d^9 + 12
74544128*a^18*b^12*c^25*d^10 - 1274544128*a^20*b^10*c^23*d^12 + 1853882368*
a^21*b^9*c^22*d^13 - 1622147072*a^22*b^8*c^21*d^14 + 998244352*a^23*b^7*c^2
0*d^15 - 445644800*a^24*b^6*c^19*d^16 + 142606336*a^25*b^5*c^18*d^17 - 3119
5136*a^26*b^4*c^17*d^18 + 4194304*a^27*b^3*c^16*d^19 - 262144*a^28*b^2*c^15
*d^20))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4
*a^8*b*c*d^3))))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^
2*d^2 - 4*a^8*b*c*d^3)))*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*3i)/(4*(a^9*d^4 + a
^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) + ((x*(1
47456*a^6*b^20*c^26*d^3 - 2211840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^2
4*d^5 - 56180736*a^9*b^17*c^23*d^6 + 144737280*a^10*b^16*c^22*d^7 - 2850785
28*a^11*b^15*c^21*d^8 + 505018368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*
c^19*d^10 + 1434332160*a^14*b^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12
 + 1999835136*a^16*b^10*c^16*d^13 - 1581355008*a^17*b^9*c^15*d^14 + 9388431
36*a^18*b^8*c^14*d^15 - 412314624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c
^12*d^17 - 28145664*a^21*b^5*c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 23040
0*a^23*b^3*c^9*d^20) + (3*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(196608*a^8*b^20*c
^30*d^2 - 3145728*a^9*b^19*c^29*d^3 + 23003136*a^10*b^18*c^28*d^4 - 1012039
68*a^11*b^17*c^27*d^5 + 294961152*a^12*b^16*c^26*d^6 - 582500352*a^13*b^15*
c^25*d^7 + 729071616*a^14*b^14*c^24*d^8 - 339296256*a^15*b^13*c^23*d^9 - 76
6132224*a^16*b^12*c^22*d^10 + 2185936896*a^17*b^11*c^21*d^11 - 3127787520*a
^18*b^10*c^20*d^12 + 3084337152*a^19*b^9*c^19*d^13 - 2249834496*a^20*b^8*c^
18*d^14 + 1236221952*a^21*b^7*c^17*d^15 - 508674048*a^22*b^6*c^16*d^16 + 15
2715264*a^23*b^5*c^15*d^17 - 31703040*a^24*b^4*c^14*d^18 + 4079616*a^25*b^3
*c^13*d^19 - 245760*a^26*b^2*c^12*d^20 + (3*x*(3*a*d - b*c)*(-a^5*b^7)^(1/2
)*(262144*a^10*b^20*c^33*d^2 - 4194304*a^11*b^19*c^32*d^3 + 31195136*a^12*b
^18*c^31*d^4 - 142606336*a^13*b^17*c^30*d^5 + 445644800*a^14*b^16*c^29*d^6
- 998244352*a^15*b^15*c^28*d^7 + 1622147072*a^16*b^14*c^27*d^8 - 1853882368
*a^17*b^13*c^26*d^9 + 1274544128*a^18*b^12*c^25*d^10 - 1274544128*a^20*b^10
*c^23*d^12 + 1853882368*a^21*b^9*c^22*d^13 - 1622147072*a^22*b^8*c^21*d^14
+ 998244352*a^23*b^7*c^20*d^15 - 445644800*a^24*b^6*c^19*d^16 + 142606336*a
^25*b^5*c^18*d^17 - 31195136*a^26*b^4*c^17*d^18 + 4194304*a^27*b^3*c^16*d^1
9 - 262144*a^28*b^2*c^15*d^20))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d
 + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3))))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*
b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(3*a*d - b*c)*(-a^5*b^7)^(
1/2)*3i)/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
4*a^8*b*c*d^3)))/(1161216*a^6*b^18*c^21*d^5 - 13768704*a^7*b^17*c^20*d^6 +
74221056*a^8*b^16*c^19*d^7 - 244574208*a^9*b^15*c^18*d^8 + 551397888*a^10*b
^14*c^17*d^9 - 893251584*a^11*b^13*c^16*d^10 + 1058724864*a^12*b^12*c^15*d^
11 - 918245376*a^13*b^11*c^14*d^12 + 575106048*a^14*b^10*c^13*d^13 - 252868
608*a^15*b^9*c^12*d^14 + 74055168*a^16*b^8*c^11*d^15 - 12994560*a^17*b^7*c^
10*d^16 + 1036800*a^18*b^6*c^9*d^17 - (3*(x*(147456*a^6*b^20*c^26*d^3 - 221
1840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^24*d^5 - 56180736*a^9*b^17*c^2
3*d^6 + 144737280*a^10*b^16*c^22*d^7 - 285078528*a^11*b^15*c^21*d^8 + 50501
```

```
8368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*c^19*d^10 + 1434332160*a^14*b
^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12 + 1999835136*a^16*b^10*c^16*
d^13 - 1581355008*a^17*b^9*c^15*d^14 + 938843136*a^18*b^8*c^14*d^15 - 41231
4624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c^12*d^17 - 28145664*a^21*b^5*
c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 230400*a^23*b^3*c^9*d^20) + (3*(3*
a*d - b*c)*(-a^5*b^7)^(1/2)*(3145728*a^9*b^19*c^29*d^3 - 196608*a^8*b^20*c^
30*d^2 - 23003136*a^10*b^18*c^28*d^4 + 101203968*a^11*b^17*c^27*d^5 - 29496
1152*a^12*b^16*c^26*d^6 + 582500352*a^13*b^15*c^25*d^7 - 729071616*a^14*b^1
4*c^24*d^8 + 339296256*a^15*b^13*c^23*d^9 + 766132224*a^16*b^12*c^22*d^10 -
 2185936896*a^17*b^11*c^21*d^11 + 3127787520*a^18*b^10*c^20*d^12 - 30843371
52*a^19*b^9*c^19*d^13 + 2249834496*a^20*b^8*c^18*d^14 - 1236221952*a^21*b^7
*c^17*d^15 + 508674048*a^22*b^6*c^16*d^16 - 152715264*a^23*b^5*c^15*d^17 +
31703040*a^24*b^4*c^14*d^18 - 4079616*a^25*b^3*c^13*d^19 + 245760*a^26*b^2*
c^12*d^20 + (3*x*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(262144*a^10*b^20*c^33*d^2
- 4194304*a^11*b^19*c^32*d^3 + 31195136*a^12*b^18*c^31*d^4 - 142606336*a^13
*b^17*c^30*d^5 + 445644800*a^14*b^16*c^29*d^6 - 998244352*a^15*b^15*c^28*d^
7 + 1622147072*a^16*b^14*c^27*d^8 - 1853882368*a^17*b^13*c^26*d^9 + 1274544
128*a^18*b^12*c^25*d^10 - 1274544128*a^20*b^10*c^23*d^12 + 1853882368*a^21*
b^9*c^22*d^13 - 1622147072*a^22*b^8*c^21*d^14 + 998244352*a^23*b^7*c^20*d^1
5 - 445644800*a^24*b^6*c^19*d^16 + 142606336*a^25*b^5*c^18*d^17 - 31195136*
a^26*b^4*c^17*d^18 + 4194304*a^27*b^3*c^16*d^19 - 262144*a^28*b^2*c^15*d^20
))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*
b*c*d^3))))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2
 - 4*a^8*b*c*d^3)))*(3*a*d - b*c)*(-a^5*b^7)^(1/2))/(4*(a^9*d^4 + a^5*b^4*c
^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) + (3*(x*(147456*
a^6*b^20*c^26*d^3 - 2211840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^24*d^5
- 56180736*a^9*b^17*c^23*d^6 + 144737280*a^10*b^16*c^22*d^7 - 285078528*a^1
1*b^15*c^21*d^8 + 505018368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*c^19*d
^10 + 1434332160*a^14*b^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12 + 199
9835136*a^16*b^10*c^16*d^13 - 1581355008*a^17*b^9*c^15*d^14 + 938843136*a^1
8*b^8*c^14*d^15 - 412314624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c^12*d^
17 - 28145664*a^21*b^5*c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 230400*a^23
*b^3*c^9*d^20) + (3*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(196608*a^8*b^20*c^30*d^
2 - 3145728*a^9*b^19*c^29*d^3 + 23003136*a^10*b^18*c^28*d^4 - 101203968*a^1
1*b^17*c^27*d^5 + 294961152*a^12*b^16*c^26*d^6 - 582500352*a^13*b^15*c^25*d
^7 + 729071616*a^14*b^14*c^24*d^8 - 339296256*a^15*b^13*c^23*d^9 - 76613222
4*a^16*b^12*c^22*d^10 + 2185936896*a^17*b^11*c^21*d^11 - 3127787520*a^18*b^
10*c^20*d^12 + 3084337152*a^19*b^9*c^19*d^13 - 2249834496*a^20*b^8*c^18*d^1
4 + 1236221952*a^21*b^7*c^17*d^15 - 508674048*a^22*b^6*c^16*d^16 + 15271526
4*a^23*b^5*c^15*d^17 - 31703040*a^24*b^4*c^14*d^18 + 4079616*a^25*b^3*c^13*
d^19 - 245760*a^26*b^2*c^12*d^20 + (3*x*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(262
144*a^10*b^20*c^33*d^2 - 4194304*a^11*b^19*c^32*d^3 + 31195136*a^12*b^18*c^
31*d^4 - 142606336*a^13*b^17*c^30*d^5 + 445644800*a^14*b^16*c^29*d^6 - 9982
44352*a^15*b^15*c^28*d^7 + 1622147072*a^16*b^14*c^27*d^8 - 1853882368*a^17*
b^13*c^26*d^9 + 1274544128*a^18*b^12*c^25*d^10 - 1274544128*a^20*b^10*c^23*
```

```
d^12 + 1853882368*a^21*b^9*c^22*d^13 - 1622147072*a^22*b^8*c^21*d^14 + 9982
44352*a^23*b^7*c^20*d^15 - 445644800*a^24*b^6*c^19*d^16 + 142606336*a^25*b^
5*c^18*d^17 - 31195136*a^26*b^4*c^17*d^18 + 4194304*a^27*b^3*c^16*d^19 - 26
2144*a^28*b^2*c^15*d^20))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a
^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3))))/(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(3*a*d - b*c)*(-a^5*b^7)^(1/2))/
(4*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3))))*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*3i)/(2*(a^9*d^4 + a^5*b^4*c^4 - 4*a
^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) - (1/(a*c) + (3*x^6*(5*a
^3*b*d^5 - 4*b^4*c^3*d^2 + 8*a*b^3*c^2*d^3 - 13*a^2*b^2*c*d^4))/(8*a^2*c^3*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^2*(25*a^4*d^4 - 1
2*b^4*c^4 + 24*a^2*b^2*c^2*d^2 + 8*a*b^3*c^3*d - 57*a^3*b*c*d^3))/(8*a^2*c^
2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^4*(24*b^4*c^4 - 15*a^
4*d^4 + 41*a^2*b^2*c^2*d^2 - 40*a*b^3*c^3*d + 14*a^3*b*c*d^3))/(8*a^2*c^3*(
a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^3*(b*c^2 + 2*a*c*d) + x^5*(
a*d^2 + 2*b*c*d) + b*d^2*x^7 + a*c^2*x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.317 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=215

$$\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c\right.}{2c^4(bc-ad)^4}$$

Rubi [A] time = 0.29, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 88}

$$\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{2c^4(bc-ad)^4}-\frac{b^4}{2a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{2a^3(bc-ad)^4}-\frac{\log(x)(3ad+2bc)}{a^3c^4}-\frac{1}{2a^2c^3x^2}-\frac{d^3(2bc-ad)}{c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3}{4c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/(2*a^2*c^3*x^2) - b^4/(2*a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/(4*c^2*(b*
c - a*d)^2*(c + d*x^2)^2) - (d^3*(2*b*c - a*d))/(c^3*(b*c - a*d)^3*(c + d*x
^2)) - ((2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (b^4*(2*b*c - 5*a*d)*Log[a + b*
x^2])/(2*a^3*(b*c - a*d)^4) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Lo
g[c + d*x^2])/(2*c^4*(b*c - a*d)^4)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^3}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\Bigg(\int\bigg(\frac{1}{a^2c^3x^2} + \frac{-2bc-3ad}{a^3c^4x} - \frac{b^5}{a^2(-bc+ad)^3(a+bx)^2} - \frac{b^5(-2bc+5a}{a^3(-bc+ad)^4(a}$$

$$= -\frac{1}{2a^2c^3x^2} - \frac{b^4}{2a^2(bc-ad)^3\left(a+bx^2\right)} - \frac{d^3}{4c^2(bc-ad)^2\left(c+dx^2\right)^2} - \frac{d^3(2bc-}{c^3(bc-ad)^3\,(c}$$

Mathematica [A] time = 0.32, size = 208, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2b^4(2bc-5ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^4} - \frac{4\log(x)(3ad+2bc)}{a^3c^4} + \frac{2b^4}{a^2\left(a+bx^2\right)(ad-bc)^3} + \frac{2d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\log\left(c+dx^2\right)}{c^4(bc-ad)^4} - \frac{2}{a^2c^3x^2} + \frac{4d^3(ad-2bc)}{c^3\left(c+dx^2\right)(bc-ad)^3} - \frac{d^3}{c^2\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-2/(a^2*c^3*x^2) + (2*b^4)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - d^3/(c^2*(
b*c - a*d)^2*(c + d*x^2)^2) + (4*d^3*(-2*b*c + a*d))/(c^3*(b*c - a*d)^3*(c
+ d*x^2)) - (4*(2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (2*b^4*(2*b*c - 5*a*d)*L
og[a + b*x^2])/(a^3*(b*c - a*d)^4) + (2*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^
2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^4))/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 72.33, size = 1227, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b^4*c^7 - 8*a^3*b^3*c^6*d + 12*a^4*b^2*c^5*d^2 - 8*a^5*b*c^4*d^
3 + 2*a^6*c^3*d^4 + 2*(2*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 10*a^3*b^3*c^3
```

```
*d^4 - 10*a^4*b^2*c^2*d^5 + 3*a^5*b*c*d^6)*x^6 + (8*a*b^5*c^6*d - 18*a^2*b^
4*c^5*d^2 + 25*a^3*b^3*c^4*d^3 - 10*a^4*b^2*c^3*d^4 - 11*a^5*b*c^2*d^5 + 6*
a^6*c*d^6)*x^4 + (4*a*b^5*c^7 - 6*a^2*b^4*c^6*d - 4*a^3*b^3*c^5*d^2 + 25*a^
4*b^2*c^4*d^3 - 28*a^5*b*c^3*d^4 + 9*a^6*c^2*d^5)*x^2 - 2*((2*b^6*c^5*d^2 -
 5*a*b^5*c^4*d^3)*x^8 + (4*b^6*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3)
*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*c^5*d^2)*x^4 + (2*a*b^5*c^7 -
5*a^2*b^4*c^6*d)*x^2)*log(b*x^2 + a) - 2*((10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*
c*d^6 + 3*a^5*b*d^7)*x^8 + (20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^5 - 4*a^5
*b*c*d^6 + 3*a^6*d^7)*x^6 + (10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a
^5*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (10*a^4*b^2*c^4*d^3 - 10*a^5*b*c^3*d^4 +
3*a^6*c^2*d^5)*x^2)*log(d*x^2 + c) + 4*((2*b^6*c^5*d^2 - 5*a*b^5*c^4*d^3 +
10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (4*b^6*c^6*d - 8
*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d^
5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*
c^5*d^2 + 10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a^5*b*c^2*d^5 + 6*a^
6*c*d^6)*x^4 + (2*a*b^5*c^7 - 5*a^2*b^4*c^6*d + 10*a^4*b^2*c^4*d^3 - 10*a^5
*b*c^3*d^4 + 3*a^6*c^2*d^5)*x^2)*log(x))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*
d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^8 + (2*a^3*b
^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^
7*b*c^5*d^5 + a^8*c^4*d^6)*x^6 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^
3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^4 + (a^4
*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6
*d^4)*x^2)
```

giac [B] time = 0.45, size = 638, normalized size = 2.97

$$\frac{\left(2\,b^{6}c-5\,ab^{5}d\right)\log\left(\left|bx^{2}+a\right|\right)}{2\left(a^{3}b^{5}c^{4}-4\,a^{4}b^{4}c^{3}d+6\,a^{5}b^{3}c^{2}d^{2}-4\,a^{6}b^{2}cd^{3}+a^{7}bd^{4}\right)}+\frac{\left(10\,b^{2}c^{2}d^{4}-10\,abcd^{5}+3\,a^{2}d^{6}\right)\log\left(\left|dx^{2}+c\right|\right)}{2\left(b^{4}c^{8}d-4\,ab^{3}c^{7}d^{2}+6\,a^{2}b^{2}c^{6}d^{3}-4\,a^{3}bc^{5}d^{4}+a^{4}c^{4}d^{5}\right)}+\frac{10\,a^{2}b^{3}c^{2}d^{3}x^{4}-10\,a^{3}b^{2}cd^{4}x^{4}+3\,a^{4}bd^{5}x^{4}-4\,b^{5}c^{5}x^{2}+10\,ab^{4}c^{4}dx^{2}-12\,a^{2}b^{3}c^{3}d^{2}x^{2}+18\,a^{3}b^{2}c^{2}d^{3}x^{2}-12\,a^{4}bcd^{4}x^{2}+3\,a^{5}d^{5}x^{2}-2\,ab^{4}c^{5}+8\,a^{2}b^{3}c^{4}d-12\,a^{3}b^{2}c^{3}d^{2}+8\,a^{4}bc^{2}d^{3}-2\,a^{5}cd^{4}}{4\left(a^{2}b^{4}c^{8}-4\,a^{3}b^{3}c^{7}d+6\,a^{4}b^{2}c^{6}d^{2}-4\,a^{5}bc^{5}d^{3}+a^{6}c^{4}d^{4}\right)\left(bx^{4}+ax^{2}\right)}-\frac{30\,b^{2}c^{2}d^{5}x^{4}-30\,abcd^{6}x^{4}+9\,a^{2}d^{7}x^{4}+68\,b^{2}c^{3}d^{4}x^{2}-72\,abc^{2}d^{5}x^{2}+22\,a^{2}cd^{6}x^{2}+39\,b^{2}c^{4}d^{3}-44\,abc^{3}d^{4}+14\,a^{2}c^{2}d^{5}}{4\left(b^{4}c^{8}-4\,ab^{3}c^{7}d+6\,a^{2}b^{2}c^{6}d^{2}-4\,a^{3}bc^{5}d^{3}+a^{4}c^{4}d^{4}\right)\left(dx^{2}+c\right)^{2}}-\frac{\left(2\,bc+3\,ad\right)\log\left(x^{2}\right)}{2\,a^{3}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*
d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4) + 1/2*(10*b^2*c^2*d^4
- 10*a*b*c*d^5 + 3*a^2*d^6)*log(abs(d*x^2 + c))/(b^4*c^8*d - 4*a*b^3*c^7*d^
2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5) + 1/4*(10*a^2*b^3*c^
2*d^3*x^4 - 10*a^3*b^2*c*d^4*x^4 + 3*a^4*b*d^5*x^4 - 4*b^5*c^5*x^2 + 10*a*b
^4*c^4*d*x^2 - 12*a^2*b^3*c^3*d^2*x^2 + 18*a^3*b^2*c^2*d^3*x^2 - 12*a^4*b*c
*d^4*x^2 + 3*a^5*d^5*x^2 - 2*a*b^4*c^5 + 8*a^2*b^3*c^4*d - 12*a^3*b^2*c^3*d
^2 + 8*a^4*b*c^2*d^3 - 2*a^5*c*d^4)/((a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4
*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4)*(b*x^4 + a*x^2)) - 1/4*(30*b^
2*c^2*d^5*x^4 - 30*a*b*c*d^6*x^4 + 9*a^2*d^7*x^4 + 68*b^2*c^3*d^4*x^2 - 72*
a*b*c^2*d^5*x^2 + 22*a^2*c*d^6*x^2 + 39*b^2*c^4*d^3 - 44*a*b*c^3*d^4 + 14*a
^2*c^2*d^5)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3
 + a^4*c^4*d^4)*(d*x^2 + c)^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

maple [A] time = 0.03, size = 405, normalized size = 1.88

$$-\frac{a^{2} d^{5}}{4 (ad-bc)^{4} \left(d x^{2}+c\right)^{2} c^{2}}+\frac{ab\, d^{4}}{2 (ad-bc)^{4} \left(d x^{2}+c\right)^{2} c}-\frac{b^{2} d^{3}}{4 (ad-bc)^{4} \left(d x^{2}+c\right)^{2}}-\frac{a^{2} d^{5}}{(ad-bc)^{4} \left(d x^{2}+c\right) c^{3}}+\frac{3 a^{2} d^{5} \ln\left(d x^{2}+c\right)}{2 (ad-bc)^{4} c^{4}}+\frac{3ab\, d^{4}}{(ad-bc)^{4} \left(d x^{2}+c\right) c^{2}}-\frac{5ab\, d^{4} \ln\left(d x^{2}+c\right)}{(ad-bc)^{4} c^{3}}+\frac{b^{4} d}{2 (ad-bc)^{4} \left(b x^{2}+a\right) a}-\frac{b^{5} c}{2 (ad-bc)^{4} \left(b x^{2}+a\right) a^{2}}-\frac{5 b^{4} d \ln\left(b x^{2}+a\right)}{2 (ad-bc)^{4} a^{2}}+\frac{b^{5} c \ln\left(b x^{2}+a\right)}{(ad-bc)^{4} a^{3}}-\frac{2 b^{2} d^{3}}{(ad-bc)^{4} \left(d x^{2}+c\right) c}+\frac{5 b^{2} d^{3} \ln\left(d x^{2}+c\right)}{(ad-bc)^{4} c^{2}}-\frac{3d \ln(x)}{a^{2} c^{4}}-\frac{2b \ln(x)}{a^{3} c^{3}}-\frac{1}{2 a^{2} c^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^4/a/(a*d-b*c)^4/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4/(b*x^2+a)*c-5/2*b
^4/a^2/(a*d-b*c)^4*ln(b*x^2+a)*d+b^5/a^3/(a*d-b*c)^4*ln(b*x^2+a)*c-1/4*d^5/
c^2/(a*d-b*c)^4/(d*x^2+c)^2*a^2+1/2*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*a*b-1/4*d
^3/(a*d-b*c)^4/(d*x^2+c)^2*b^2+3/2*d^5/c^4/(a*d-b*c)^4*ln(d*x^2+c)*a^2-5*d^
4/c^3/(a*d-b*c)^4*ln(d*x^2+c)*a*b+5*d^3/c^2/(a*d-b*c)^4*ln(d*x^2+c)*b^2-d^5
/c^3/(a*d-b*c)^4/(d*x^2+c)*a^2+3*d^4/c^2/(a*d-b*c)^4/(d*x^2+c)*a*b-2*d^3/c/
(a*d-b*c)^4/(d*x^2+c)*b^2-1/2/a^2/c^3/x^2-3/a^2/c^4*ln(x)*d-2/a^3/c^3*ln(x)
*b
```

maxima [B] time = 1.48, size = 651, normalized size = 3.03

$$\frac{\left(2 b^{5} c-5 ab^{4} d\right) \log\left(bx^{2}+a\right)}{2 \left(a^{3} b^{4} c^{4}-4 a^{4} b^{3} c^{3} d+6 a^{5} b^{2} c^{2} d^{2}-4 a^{6} bcd^{3}+a^{7} d^{4}\right)}+\frac{\left(10 b^{2} c^{2} d^{3}-10 abcd^{4}+3 a^{2} d^{5}\right) \log\left(dx^{2}+c\right)}{2 \left(b^{4} c^{8}-4 ab^{3} c^{7} d+6 a^{2} b^{2} c^{6} d^{2}-4 a^{3} bc^{5} d^{3}+a^{4} c^{4} d^{4}\right)}-\frac{2 ab^{3} c^{5}-6 a^{2} b^{2} c^{4} d+6 a^{3} bc^{3} d^{2}-2 a^{4} c^{2} d^{3}+2 \left(2 b^{4} c^{3} d^{2}-3 ab^{3} c^{2} d^{3}+7 a^{2} b^{2} cd^{4}-3 a^{3} bd^{5}\right) x^{6}+\left(8 b^{4} c^{4} d-10 ab^{3} c^{3} d^{2}+15 a^{2} b^{2} c^{2} d^{3}+5 a^{3} bcd^{4}-6 a^{4} d^{5}\right) x^{4}+\left(4 b^{4} c^{5}-2 ab^{3} c^{4} d-6 a^{2} b^{2} c^{3} d^{2}+19 a^{3} bc^{2} d^{3}-9 a^{4} cd^{4}\right) x^{2}}{4 \left(\left(a^{2} b^{4} c^{6} d^{2}-3 a^{3} b^{3} c^{5} d^{3}+3 a^{4} b^{2} c^{4} d^{4}-a^{5} bc^{3} d^{5}\right) x^{8}+\left(2 a^{2} b^{4} c^{7} d-5 a^{3} b^{3} c^{6} d^{2}+3 a^{4} b^{2} c^{5} d^{3}+a^{5} bc^{4} d^{4}-a^{6} c^{3} d^{5}\right) x^{6}+\left(a^{2} b^{4} c^{8}-a^{3} b^{3} c^{7} d-3 a^{4} b^{2} c^{6} d^{2}+5 a^{5} bc^{5} d^{3}-2 a^{6} c^{4} d^{4}\right) x^{4}+\left(a^{3} b^{3} c^{8}-3 a^{4} b^{2} c^{7} d+3 a^{5} bc^{6} d^{2}-a^{6} c^{5} d^{3}\right) x^{2}\right)}-\frac{(2 bc+3 ad) \log\left(x^{2}\right)}{2 a^{3} c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^5*c - 5*a*b^4*d)*log(b*x^2 + a)/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6
*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4) + 1/2*(10*b^2*c^2*d^3 - 10*a*b*
c*d^4 + 3*a^2*d^5)*log(d*x^2 + c)/(b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*
d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4) - 1/4*(2*a*b^3*c^5 - 6*a^2*b^2*c^4*d +
 6*a^3*b*c^3*d^2 - 2*a^4*c^2*d^3 + 2*(2*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 7*a
^2*b^2*c*d^4 - 3*a^3*b*d^5)*x^6 + (8*b^4*c^4*d - 10*a*b^3*c^3*d^2 + 15*a^2*
b^2*c^2*d^3 + 5*a^3*b*c*d^4 - 6*a^4*d^5)*x^4 + (4*b^4*c^5 - 2*a*b^3*c^4*d -
 6*a^2*b^2*c^3*d^2 + 19*a^3*b*c^2*d^3 - 9*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2
 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^8 + (2*a^2*b^4*
c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5
)*x^6 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3
- 2*a^6*c^4*d^4)*x^4 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a
^6*c^5*d^3)*x^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

mupad [B] time = 2.42, size = 549, normalized size = 2.55

$$\frac{\ln\left(b\,x^{2}+a\right)\,\left(2\,b^{5}\,c-5\,a\,b^{4}\,d\right)}{2\,a^{7}\,d^{4}-8\,a^{6}\,b\,c\,d^{3}+12\,a^{5}\,b^{2}\,c^{2}\,d^{2}-8\,a^{4}\,b^{3}\,c^{3}\,d+2\,a^{3}\,b^{4}\,c^{4}}-\frac{\frac{1}{2\,a\,c}-\frac{x^{4}\,\left(-6\,a^{4}\,d^{5}+5\,a^{3}\,b\,c\,d^{4}+15\,a^{2}\,b^{2}\,c^{2}\,d^{3}-10\,a\,b^{3}\,c^{3}\,d^{2}+8\,b^{4}\,c^{4}\,d\right)}{4\,a^{2}\,c^{3}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}+\frac{x^{2}\,\left(9\,a^{4}\,d^{4}-19\,a^{3}\,b\,c\,d^{3}+6\,a^{2}\,b^{2}\,c^{2}\,d^{2}+2\,a\,b^{3}\,c^{3}\,d-4\,b^{4}\,c^{4}\right)}{4\,a^{2}\,c^{2}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}+\frac{b\,d^{2}\,x^{6}\,\left(3\,a^{3}\,d^{3}-7\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-2\,b^{3}\,c^{3}\right)}{2\,a^{2}\,c^{3}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}}{x^{4}\,\left(b\,c^{2}+2\,a\,d\,c\right)+x^{6}\,\left(a\,d^{2}+2\,b\,c\,d\right)+a\,c^{2}\,x^{2}+b\,d^{2}\,x^{8}}+\frac{\ln\left(d\,x^{2}+c\right)\,\left(3\,a^{2}\,d^{5}-10\,a\,b\,c\,d^{4}+10\,b^{2}\,c^{2}\,d^{3}\right)}{2\,a^{4}\,c^{4}\,d^{4}-8\,a^{3}\,b\,c^{5}\,d^{3}+12\,a^{2}\,b^{2}\,c^{6}\,d^{2}-8\,a\,b^{3}\,c^{7}\,d+2\,b^{4}\,c^{8}}-\frac{\ln(x)\,\left(3\,a\,d+2\,b\,c\right)}{a^{3}\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (log(a + b*x^2)*(2*b^5*c - 5*a*b^4*d))/(2*a^7*d^4 + 2*a^3*b^4*c^4 - 8*a^4*b
^3*c^3*d + 12*a^5*b^2*c^2*d^2 - 8*a^6*b*c*d^3) - (1/(2*a*c) - (x^4*(8*b^4*c
^4*d - 6*a^4*d^5 - 10*a*b^3*c^3*d^2 + 15*a^2*b^2*c^2*d^3 + 5*a^3*b*c*d^4))/
(4*a^2*c^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^2*(9*a
^4*d^4 - 4*b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 2*a*b^3*c^3*d - 19*a^3*b*c*d^3))/(
4*a^2*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^6
*(3*a^3*d^3 - 2*b^3*c^3 + 3*a*b^2*c^2*d - 7*a^2*b*c*d^2))/(2*a^2*c^3*(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(x^4*(b*c^2 + 2*a*c*d) + x^
6*(a*d^2 + 2*b*c*d) + a*c^2*x^2 + b*d^2*x^8) + (log(c + d*x^2)*(3*a^2*d^5 +
 10*b^2*c^2*d^3 - 10*a*b*c*d^4))/(2*b^4*c^8 + 2*a^4*c^4*d^4 - 8*a^3*b*c^5*d
^3 + 12*a^2*b^2*c^6*d^2 - 8*a*b^3*c^7*d) - (log(x)*(3*a*d + 2*b*c))/(a^3*c^
4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.318 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=377

$$\frac{b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^4}+\frac{d\left(-7a^2d^2+15abcd+4b^2c^2\right)}{8ac^2x^3\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^4}-\;-3$$

Rubi [A] time = 0.69, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {472, 579, 583, 522, 205}

$$\frac{d\left(-7a^2d^2+15abcd+4b^2c^2\right)}{8ac^2x^3\left(c+dx^2\right)(bc-ad)^3}-\frac{75a^2bcd^2-35a^3d^3-24ab^2c^2d+20b^3c^3}{24a^2c^3x^3(bc-ad)^3}+\frac{-24a^2b^2c^2d^2+75a^3bcd^3-35a^4d^4-24ab^3c^3d+20b^4c^4}{8a^3c^4x(bc-ad)^3}+\frac{d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^4}+\frac{b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^4}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d(ad+2bc)}{4acx^3\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(20*b^3*c^3 - 24*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 35*a^3*d^3)/(24*a^2*c^3*(b
*c - a*d)^3*x^3) + (20*b^4*c^4 - 24*a*b^3*c^3*d - 24*a^2*b^2*c^2*d^2 + 75*a
^3*b*c*d^3 - 35*a^4*d^4)/(8*a^3*c^4*(b*c - a*d)^3*x) + (d*(2*b*c + a*d))/(4
*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^2) + b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*
(c + d*x^2)^2) + (d*(4*b^2*c^2 + 15*a*b*c*d - 7*a^2*d^2))/(8*a*c^2*(b*c - a
*d)^3*x^3*(c + d*x^2)) + (b^(9/2)*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(7/2)*(b*c - a*d)^4) + (d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^
2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(9/2)*(b*c - a*d)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-5bc+2ad-9bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(10b^2c^2-8\right.}{x^4}}{8}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(4b^2c^2+15\right.}{8ac^2(bc-a}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2} + \frac{}{2a(bc}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+}{8a^3c^4(bc-ad)^3x}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+}{8a^3c^4(bc-ad)^3x}$$

$$= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3} + \frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+}{8a^3c^4(bc-ad)^3x}$$

Mathematica [A] time = 0.46, size = 230, normalized size = 0.61

$$\frac{1}{24}\left(\frac{12b^{9/2}(5bc-11ad)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)^4} - \frac{12b^5x}{a^3\left(a+bx^2\right)(ad-bc)^3} + \frac{72ad+48bc}{a^3c^4x} + \frac{3d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{9/2}(bc-ad)^4} - \frac{8}{a^2c^3x^3} + \frac{3d^4x(19bc-11ad)}{c^4\left(c+dx^2\right)(bc-ad)^3} + \frac{6d^4x}{c^3\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-8/(a^2*c^3*x^3) + (48*b*c + 72*a*d)/(a^3*c^4*x) - (12*b^5*x)/(a^3*(-(b*c)
 + a*d)^3*(a + b*x^2)) + (6*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)^2) + (3*d
^4*(19*b*c - 11*a*d)*x)/(c^4*(b*c - a*d)^3*(c + d*x^2)) + (12*b^(9/2)*(5*b*
c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d)^4) + (3*d^(7/
2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^
(9/2)*(b*c - a*d)^4))/24
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3), x]
```

fricas [B] time = 47.78, size = 4225, normalized size = 11.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c
^4*d^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3
*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 224*a
*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d
^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d -
184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b
*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*
b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 +
12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d
^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^
2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6
 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*
b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*
a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4
 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*
b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8
*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 +
 (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 -
7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*
a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7
 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4
 - 3*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*
c*d^6 + 35*a^5*b*d^7)*x^8 - (120*b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4
*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 10
5*a^6*d^7)*x^6 - (60*b^6*c^7 - 52*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a
^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x
```

```
^4 - 8*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4
*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 3*((99*a^3*b^3*c^2*d^5 - 110
*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2
*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*
c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110
*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 6*((5
*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 - 1
1*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x^5
 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt
(-b/a) - a)/(b*x^2 + a)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3
*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a^
4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 + a
^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2 + 8*a
^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*b^4*c^10 - 4*a
^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*d^4)*x^3), -1/
48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*c^4*d
^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b^3*c^2
*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 224*a*b^5
*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c^2*d^5 -
 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6*d - 184*
a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494*a^5*b*c^2
*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d + 2*a^3*b^3*
c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5)*x^2 - 24*(
(5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*b^5*c^5*d^2 -
 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*b^4*c^5*d^2)*x
^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) -
3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b
^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a
^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5
 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^
4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9
+ (2*a^3*b^5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*
d^4 - 2*a^7*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d
- 2*a^5*b^3*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*
x^5 + (a^4*b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3
 + a^8*c^6*d^4)*x^3), -1/24*(8*a^2*b^4*c^7 - 32*a^3*b^3*c^6*d + 48*a^4*b^2*
c^5*d^2 - 32*a^5*b*c^4*d^3 + 8*a^6*c^3*d^4 - 3*(20*b^6*c^5*d^2 - 44*a*b^5*c
^4*d^3 + 99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - (120*
b^6*c^6*d - 224*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 -
253*a^4*b^2*c^2*d^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - (60*b^6*c^7 - 52
*a*b^5*c^6*d - 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*
d^4 - 494*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 8*(5*a*b^5*c^7 - 13*a^2*b^4*
c^6*d + 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c
^2*d^5)*x^2 - 12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d -
```

```
17*a*b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*
a^2*b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(b/a)*arct
an(x*sqrt(b/a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7
)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*d^5 - 40*a^5*b*c*d^6 + 35*a^
6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*c^3*d^4 - 185*a^5*b*c^2*d^5 +
 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 110*a^5*b*c^3*d^4 + 35*a^6*c^2*d
^5)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d
^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^
5*c^9*d - 7*a^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7
*b*c^5*d^5 + a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3
*c^8*d^2 + 8*a^6*b^2*c^7*d^3 - 7*a^7*b*c^6*d^4 + 2*a^8*c^5*d^5)*x^5 + (a^4*
b^4*c^10 - 4*a^5*b^3*c^9*d + 6*a^6*b^2*c^8*d^2 - 4*a^7*b*c^7*d^3 + a^8*c^6*
d^4)*x^3)]
```

giac [**A**] time = 0.41, size = 367, normalized size = 0.97

$$\frac{b^{5}x}{2\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}\right)\left(bx^{2}+a\right)}+\frac{\left(5\,b^{6}c-11\,ab^{5}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{3}b^{4}c^{4}-4\,a^{4}b^{3}c^{3}d+6\,a^{5}b^{2}c^{2}d^{2}-4\,a^{6}bcd^{3}+a^{7}d^{4}\right)\sqrt{ab}}+\frac{\left(99\,b^{2}c^{2}d^{4}-110\,abcd^{5}+35\,a^{2}d^{6}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{8}-4\,ab^{3}c^{7}d+6\,a^{2}b^{2}c^{6}d^{2}-4\,a^{3}bc^{5}d^{3}+a^{4}c^{4}d^{4}\right)\sqrt{cd}}+\frac{19\,bcd^{5}x^{3}-11\,ad^{6}x^{3}+21\,bc^{2}d^{4}x-13\,acd^{5}x}{8\left(b^{3}c^{7}-3\,ab^{2}c^{6}d+3\,a^{2}bc^{5}d^{2}-a^{3}c^{4}d^{3}\right)\left(dx^{2}+c\right)^{2}}+\frac{6\,bcx^{2}+9\,adx^{2}-ac}{3\,a^{3}c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(b*x^2
 + a)) + 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 - 4
*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b)) +
1/8*(99*b^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b
^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)
*sqrt(c*d)) + 1/8*(19*b*c*d^5*x^3 - 11*a*d^6*x^3 + 21*b*c^2*d^4*x - 13*a*c*
d^5*x)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x^2 +
c)^2) + 1/3*(6*b*c*x^2 + 9*a*d*x^2 - a*c)/(a^3*c^4*x^3)
```

maple [**A**] time = 0.03, size = 455, normalized size = 1.21

$$\frac{11a^{2}d^{7}x^{3}}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{4}}-\frac{15ab\,d^{6}x^{3}}{4\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{3}}+\frac{19b^{2}d^{5}x^{3}}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{13a^{2}d^{6}x}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{3}}-\frac{17ab\,d^{5}x}{4\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{21b^{2}d^{4}x}{8\left(ad-bc\right)^{4}\left(d\,x^{2}+c\right)^{2}c}+\frac{35a^{2}d^{6}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{4}\sqrt{cd}\,c^{4}}-\frac{55ab\,d^{5}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{4\left(ad-bc\right)^{4}\sqrt{cd}\,c^{3}}-\frac{b^{5}dx}{2\left(ad-bc\right)^{4}\left(b\,x^{2}+a\right)a^{2}}-\frac{11b^{5}d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{4}\sqrt{ab}\,a^{2}}+\frac{b^{6}cx}{2\left(ad-bc\right)^{4}\left(b\,x^{2}+a\right)a^{3}}+\frac{5b^{6}c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ad-bc\right)^{4}\sqrt{ab}\,a^{3}}+\frac{99b^{2}d^{4}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(ad-bc\right)^{4}\sqrt{cd}\,c^{2}}+\frac{3d}{a^{2}c^{4}x}+\frac{2b}{a^{3}c^{3}x}-\frac{1}{3a^{2}c^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b^5/a^2/(a*d-b*c)^4*x/(b*x^2+a)*d+1/2*b^6/a^3/(a*d-b*c)^4*x/(b*x^2+a)*
c-11/2*b^5/a^2/(a*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*d+5/2*b^6/
a^3/(a*d-b*c)^4/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)*c+11/8*d^7/c^4/(a*d-b
*c)^4/(d*x^2+c)^2*x^3*a^2-15/4*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*a*b+19/8
*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2+13/8*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)
^2*a^2*x-17/4*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*a*b*x+21/8*d^4/c/(a*d-b*c)^4/
(d*x^2+c)^2*b^2*x+35/8*d^6/c^4/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)
```

```
*d*x)*a^2-55/4*d^5/c^3/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*a*
b+99/8*d^4/c^2/(a*d-b*c)^4/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x)*b^2-1/3/a^
2/c^3/x^3+3/a^2/c^4/x*d+2/a^3/c^3/x*b
```

maxima [B] time = 2.78, size = 738, normalized size = 1.96

$$\frac{\left(5\,b^{6}c-11\,ab^{5}d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(a^{3}b^{4}c^{4}-4\,a^{4}b^{3}c^{3}d+6\,a^{5}b^{2}c^{2}d^{2}-4\,a^{6}bcd^{3}+a^{7}d^{4}\right)\sqrt{ab}}+\frac{\left(99\,b^{2}c^{2}d^{4}-110\,abcd^{5}+35\,a^{2}d^{6}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^{4}c^{8}-4\,ab^{3}c^{7}d+6\,a^{2}b^{2}c^{6}d^{2}-4\,a^{3}bc^{5}d^{3}+a^{4}c^{4}d^{4}\right)\sqrt{cd}}-\frac{8\,a^{2}b^{3}c^{6}-24\,a^{3}b^{2}c^{5}d+24\,a^{4}bc^{4}d^{2}-8\,a^{5}c^{3}d^{3}-3\left(20\,b^{5}c^{4}d^{2}-24\,ab^{4}c^{3}d^{3}-24\,a^{2}b^{3}c^{2}d^{4}+75\,a^{3}b^{2}cd^{5}-35\,a^{4}bd^{6}\right)x^{8}-\left(120\,b^{5}c^{5}d-104\,ab^{4}c^{4}d^{2}-192\,a^{2}b^{3}c^{3}d^{3}+303\,a^{3}b^{2}c^{2}d^{4}+50\,a^{4}bcd^{5}-105\,a^{5}d^{6}\right)x^{6}-\left(60\,b^{5}c^{6}+8\,ab^{4}c^{5}d-176\,a^{2}b^{3}c^{4}d^{2}+319\,a^{4}bc^{2}d^{4}-175\,a^{5}cd^{5}\right)x^{4}-8\left(5\,ab^{4}c^{6}-8\,a^{2}b^{3}c^{5}d-6\,a^{3}b^{2}c^{4}d^{2}+16\,a^{4}bc^{3}d^{3}-7\,a^{5}c^{2}d^{4}\right)x^{2}}{24\left(\left(a^{3}b^{4}c^{7}d^{2}-3\,a^{4}b^{3}c^{6}d^{3}+3\,a^{5}b^{2}c^{5}d^{4}-a^{6}bc^{4}d^{5}\right)x^{9}+\left(2\,a^{3}b^{4}c^{8}d-5\,a^{4}b^{3}c^{7}d^{2}+3\,a^{5}b^{2}c^{6}d^{3}+a^{6}bc^{5}d^{4}-a^{7}c^{4}d^{5}\right)x^{7}+\left(a^{3}b^{4}c^{9}-a^{4}b^{3}c^{8}d-3\,a^{5}b^{2}c^{7}d^{2}+5\,a^{6}bc^{6}d^{3}-2\,a^{7}c^{5}d^{4}\right)x^{5}+\left(a^{4}b^{3}c^{9}-3\,a^{5}b^{2}c^{8}d+3\,a^{6}bc^{7}d^{2}-a^{7}c^{6}d^{3}\right)x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 - 4*a^4*b^3*
c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b)) + 1/8*(99*b
^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b^4*c^8 -
4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(c*d
)) - 1/24*(8*a^2*b^3*c^6 - 24*a^3*b^2*c^5*d + 24*a^4*b*c^4*d^2 - 8*a^5*c^3*
d^3 - 3*(20*b^5*c^4*d^2 - 24*a*b^4*c^3*d^3 - 24*a^2*b^3*c^2*d^4 + 75*a^3*b^
2*c*d^5 - 35*a^4*b*d^6)*x^8 - (120*b^5*c^5*d - 104*a*b^4*c^4*d^2 - 192*a^2*
b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 + 50*a^4*b*c*d^5 - 105*a^5*d^6)*x^6 - (60
*b^5*c^6 + 8*a*b^4*c^5*d - 176*a^2*b^3*c^4*d^2 + 319*a^4*b*c^2*d^4 - 175*a^
5*c*d^5)*x^4 - 8*(5*a*b^4*c^6 - 8*a^2*b^3*c^5*d - 6*a^3*b^2*c^4*d^2 + 16*a^
4*b*c^3*d^3 - 7*a^5*c^2*d^4)*x^2)/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3
*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^9 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^
2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^7 + (a^3*b^4*c^9 - a
^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^5 + (
a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3)
```

mupad [B] time = 2.29, size = 1161, normalized size = 3.08

$$\frac{\frac{x^{2}\,(7\,a\,d+5\,b\,c)}{3\,a^{2}\,c^{2}}-\frac{1}{3\,a\,c}+\frac{x^{8}\left(35\,a^{4}\,b\,d^{6}-75\,a^{3}\,b^{2}\,c\,d^{5}+24\,a^{2}\,b^{3}\,c^{2}\,d^{4}+24\,a\,b^{4}\,c^{3}\,d^{3}-20\,b^{5}\,c^{4}\,d^{2}\right)}{8\,a^{3}\,c^{4}\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}-\frac{x^{4}\left(-175\,a^{5}\,d^{5}+319\,a^{4}\,b\,c\,d^{4}-176\,a^{2}\,b^{3}\,c^{3}\,d^{2}+8\,a\,b^{4}\,c^{4}\,d+60\,b^{5}\,c^{5}\right)}{24\,a^{3}\,c^{3}\,(a\,d-b\,c)\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}+\frac{d\,x^{6}\left(105\,a^{5}\,d^{5}-50\,a^{4}\,b\,c\,d^{4}-303\,a^{3}\,b^{2}\,c^{2}\,d^{3}+192\,a^{2}\,b^{3}\,c^{3}\,d^{2}+104\,a\,b^{4}\,c^{4}\,d-120\,b^{5}\,c^{5}\right)}{24\,a^{3}\,c^{4}\,(a\,d-b\,c)\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}}{x^{5}\left(b\,c^{2}+2\,a\,d\,c\right)+x^{7}\left(a\,d^{2}+2\,b\,c\,d\right)+a\,c^{2}\,x^{3}+b\,d^{2}\,x^{9}}+\frac{\mathrm{atan}\left(\frac{b^{3}\,c^{11}\,x\,{\left(-a^{7}\,b^{9}\right)}^{3/2}\,400{}\mathrm{i}+a^{18}\,b\,d^{11}\,x\,\sqrt{-a^{7}\,b^{9}}\,1225{}\mathrm{i}+a^{14}\,b^{5}\,c^{4}\,d^{7}\,x\,\sqrt{-a^{7}\,b^{9}}\,9801{}\mathrm{i}-a^{15}\,b^{4}\,c^{3}\,d^{8}\,x\,\sqrt{-a^{7}\,b^{9}}\,21780{}\mathrm{i}+a^{16}\,b^{3}\,c^{2}\,d^{9}\,x\,\sqrt{-a^{7}\,b^{9}}\,19030{}\mathrm{i}-a\,b^{2}\,c^{10}\,d\,x\,{\left(-a^{7}\,b^{9}\right)}^{3/2}\,1760{}\mathrm{i}+a^{2}\,b\,c^{9}\,d^{2}\,x\,{\left(-a^{7}\,b^{9}\right)}^{3/2}\,1936{}\mathrm{i}-a^{17}\,b^{2}\,c\,d^{10}\,x\,\sqrt{-a^{7}\,b^{9}}\,7700{}\mathrm{i}}{-1225\,a^{22}\,b^{5}\,d^{11}+7700\,a^{21}\,b^{6}\,c\,d^{10}-19030\,a^{20}\,b^{7}\,c^{2}\,d^{9}+21780\,a^{19}\,b^{8}\,c^{3}\,d^{8}-9801\,a^{18}\,b^{9}\,c^{4}\,d^{7}+1936\,a^{13}\,b^{14}\,c^{9}\,d^{2}-1760\,a^{12}\,b^{15}\,c^{10}\,d+400\,a^{11}\,b^{16}\,c^{11}}\right)\,(11\,a\,d-5\,b\,c)\,\sqrt{-a^{7}\,b^{9}}\,1{}\mathrm{i}}{2\left(a^{11}\,d^{4}-4\,a^{10}\,b\,c\,d^{3}+6\,a^{9}\,b^{2}\,c^{2}\,d^{2}-4\,a^{8}\,b^{3}\,c^{3}\,d+a^{7}\,b^{4}\,c^{4}\right)}-\frac{\mathrm{atan}\left(\frac{a^{11}\,d^{5}\,x\,{\left(-c^{9}\,d^{7}\right)}^{3/2}\,1225{}\mathrm{i}+b^{11}\,c^{20}\,d\,x\,\sqrt{-c^{9}\,d^{7}}\,400{}\mathrm{i}-a^{8}\,b^{3}\,c^{3}\,d^{2}\,x\,{\left(-c^{9}\,d^{7}\right)}^{3/2}\,21780{}\mathrm{i}+a^{9}\,b^{2}\,c^{2}\,d^{3}\,x\,{\left(-c^{9}\,d^{7}\right)}^{3/2}\,19030{}\mathrm{i}+a^{2}\,b^{9}\,c^{18}\,d^{3}\,x\,\sqrt{-c^{9}\,d^{7}}\,1936{}\mathrm{i}-a^{10}\,b\,c\,d^{4}\,x\,{\left(-c^{9}\,d^{7}\right)}^{3/2}\,7700{}\mathrm{i}+a^{7}\,b^{4}\,c^{4}\,d\,x\,{\left(-c^{9}\,d^{7}\right)}^{3/2}\,9801{}\mathrm{i}-a\,b^{10}\,c^{19}\,d^{2}\,x\,\sqrt{-c^{9}\,d^{7}}\,1760{}\mathrm{i}}{1225\,a^{11}\,c^{14}\,d^{15}-7700\,a^{10}\,b\,c^{15}\,d^{14}+19030\,a^{9}\,b^{2}\,c^{16}\,d^{13}-21780\,a^{8}\,b^{3}\,c^{17}\,d^{12}+9801\,a^{7}\,b^{4}\,c^{18}\,d^{11}-1936\,a^{2}\,b^{9}\,c^{23}\,d^{6}+1760\,a\,b^{10}\,c^{24}\,d^{5}-400\,b^{11}\,c^{25}\,d^{4}}\right)\,\sqrt{-c^{9}\,d^{7}}\,\left(35\,a^{2}\,d^{2}-110\,a\,b\,c\,d+99\,b^{2}\,c^{2}\right)\,1{}\mathrm{i}}{8\left(a^{4}\,c^{9}\,d^{4}-4\,a^{3}\,b\,c^{10}\,d^{3}+6\,a^{2}\,b^{2}\,c^{11}\,d^{2}-4\,a\,b^{3}\,c^{12}\,d+b^{4}\,c^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*d + 5*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^8*(35*a^4*b*d^6 - 20*b^
5*c^4*d^2 + 24*a*b^4*c^3*d^3 - 75*a^3*b^2*c*d^5 + 24*a^2*b^3*c^2*d^4))/(8*a
^3*c^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x^4*(60*b^5*
c^5 - 175*a^5*d^5 - 176*a^2*b^3*c^3*d^2 + 8*a*b^4*c^4*d + 319*a^4*b*c*d^4))
/(24*a^3*c^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^6*(105*a^5
*d^5 - 120*b^5*c^5 + 192*a^2*b^3*c^3*d^2 - 303*a^3*b^2*c^2*d^3 + 104*a*b^4*
c^4*d - 50*a^4*b*c*d^4))/(24*a^3*c^4*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b
*c*d)))/(x^5*(b*c^2 + 2*a*c*d) + x^7*(a*d^2 + 2*b*c*d) + a*c^2*x^3 + b*d^2*
x^9) + (atan((b^3*c^11*x*(-a^7*b^9)^(3/2)*400i + a^18*b*d^11*x*(-a^7*b^9)^(
1/2)*1225i + a^14*b^5*c^4*d^7*x*(-a^7*b^9)^(1/2)*9801i - a^15*b^4*c^3*d^8*x
*(-a^7*b^9)^(1/2)*21780i + a^16*b^3*c^2*d^9*x*(-a^7*b^9)^(1/2)*19030i - a*b
```

```
^2*c^10*d*x*(-a^7*b^9)^(3/2)*1760i + a^2*b*c^9*d^2*x*(-a^7*b^9)^(3/2)*1936i
 - a^17*b^2*c*d^10*x*(-a^7*b^9)^(1/2)*7700i)/(400*a^11*b^16*c^11 - 1225*a^2
2*b^5*d^11 - 1760*a^12*b^15*c^10*d + 7700*a^21*b^6*c*d^10 + 1936*a^13*b^14*
c^9*d^2 - 9801*a^18*b^9*c^4*d^7 + 21780*a^19*b^8*c^3*d^8 - 19030*a^20*b^7*c
^2*d^9))*(11*a*d - 5*b*c)*(-a^7*b^9)^(1/2)*1i)/(2*(a^11*d^4 + a^7*b^4*c^4 -
 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3)) - (atan((a^11*d^5*x
*(-c^9*d^7)^(3/2)*1225i + b^11*c^20*d*x*(-c^9*d^7)^(1/2)*400i - a^8*b^3*c^3
*d^2*x*(-c^9*d^7)^(3/2)*21780i + a^9*b^2*c^2*d^3*x*(-c^9*d^7)^(3/2)*19030i
+ a^2*b^9*c^18*d^3*x*(-c^9*d^7)^(1/2)*1936i - a^10*b*c*d^4*x*(-c^9*d^7)^(3/
2)*7700i + a^7*b^4*c^4*d*x*(-c^9*d^7)^(3/2)*9801i - a*b^10*c^19*d^2*x*(-c^9
*d^7)^(1/2)*1760i)/(1225*a^11*c^14*d^15 - 400*b^11*c^25*d^4 + 1760*a*b^10*c
^24*d^5 - 7700*a^10*b*c^15*d^14 - 1936*a^2*b^9*c^23*d^6 + 9801*a^7*b^4*c^18
*d^11 - 21780*a^8*b^3*c^17*d^12 + 19030*a^9*b^2*c^16*d^13))*(-c^9*d^7)^(1/2
)*(35*a^2*d^2 + 99*b^2*c^2 - 110*a*b*c*d)*1i)/(8*(b^4*c^13 + a^4*c^9*d^4 -
4*a^3*b*c^10*d^3 + 6*a^2*b^2*c^11*d^2 - 4*a*b^3*c^12*d))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.319 $\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx$

Optimal. Leaf size=96

$$\frac{a^3Ax^{m+1}}{m+1}+\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{a^2x^{m+3}(aB+3Ab)}{m+3}+\frac{a^3Ax^{m+1}}{m+1}+\frac{b^2x^{m+7}(3aB+Ab)}{m+7}+\frac{3abx^{m+5}(aB+Ab)}{m+5}+\frac{b^3Bx^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m) + (3*a*b*
(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(7 + m))/(7 + m) + (b
^3*B*x^(9 + m))/(9 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx &= \int \left(a^3Ax^m+a^2(3Ab+aB)x^{2+m}+3ab(Ab+aB)x^{4+m}+b^2(Ab+3aB)x^{6+m}+b\right.\\ &= \frac{a^3Ax^{1+m}}{1+m}+\frac{a^2(3Ab+aB)x^{3+m}}{3+m}+\frac{3ab(Ab+aB)x^{5+m}}{5+m}+\frac{b^2(Ab+3aB)x^{7+m}}{7+m}+\end{aligned}$$

Mathematica [A] time = 0.12, size = 89, normalized size = 0.93

$$x^{m+1}\left(\frac{a^3A}{m+1}+\frac{a^2x^2(aB+3Ab)}{m+3}+\frac{b^2x^6(3aB+Ab)}{m+7}+\frac{3abx^4(aB+Ab)}{m+5}+\frac{b^3Bx^8}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a^3*A)/(1 + m) + (a^2*(3*A*b + a*B)*x^2)/(3 + m) + (3*a*b*(A*b
+ a*B)*x^4)/(5 + m) + (b^2*(A*b + 3*a*B)*x^6)/(7 + m) + (b^3*B*x^8)/(9 + m)
)
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^3*(A + B*x^2), x]
```

fricas [B] time = 1.26, size = 379, normalized size = 3.95

$$\frac{\left(\left(Bb^3m^4+16\,Bb^3m^3+86\,Bb^3m^2+176\,Bb^3m+105\,Bb^3\right)x^9+\left(\left(3\,Bab^2+Ab^3\right)m^4+405\,Bab^2+135\,Ab^3+18\left(3\,Bab^2+Ab^3\right)m^3+104\left(3\,Bab^2+Ab^3\right)m^2+222\left(3\,Bab^2+Ab^3\right)m\right)x^7+3\left(\left(Ba^2b+Aab^2\right)m^4+189\,Ba^2b+189\,Aab^2+20\left(Ba^2b+Aab^2\right)m^3+130\left(Ba^2b+Aab^2\right)m^2+300\left(Ba^2b+Aab^2\right)m\right)x^5+\left(\left(Ba^3+3\,Aa^2b\right)m^4+315\,Ba^3+945\,Aa^2b+22\left(Ba^3+3\,Aa^2b\right)m^3+164\left(Ba^3+3\,Aa^2b\right)m^2+458\left(Ba^3+3\,Aa^2b\right)m\right)x^3+\left(Aa^3m^4+24\,Aa^3m^3+206\,Aa^3m^2+744\,Aa^3m+945\,Aa^3\right)x\right)x^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b^3*m^4 + 16*B*b^3*m^3 + 86*B*b^3*m^2 + 176*B*b^3*m + 105*B*b^3)*x^9 +
((3*B*a*b^2 + A*b^3)*m^4 + 405*B*a*b^2 + 135*A*b^3 + 18*(3*B*a*b^2 + A*b^3)
*m^3 + 104*(3*B*a*b^2 + A*b^3)*m^2 + 222*(3*B*a*b^2 + A*b^3)*m)*x^7 + 3*((B
*a^2*b + A*a*b^2)*m^4 + 189*B*a^2*b + 189*A*a*b^2 + 20*(B*a^2*b + A*a*b^2)*
m^3 + 130*(B*a^2*b + A*a*b^2)*m^2 + 300*(B*a^2*b + A*a*b^2)*m)*x^5 + ((B*a^
3 + 3*A*a^2*b)*m^4 + 315*B*a^3 + 945*A*a^2*b + 22*(B*a^3 + 3*A*a^2*b)*m^3 +
 164*(B*a^3 + 3*A*a^2*b)*m^2 + 458*(B*a^3 + 3*A*a^2*b)*m)*x^3 + (A*a^3*m^4
+ 24*A*a^3*m^3 + 206*A*a^3*m^2 + 744*A*a^3*m + 945*A*a^3)*x)*x^m/(m^5 + 25*
m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [B] time = 0.44, size = 593, normalized size = 6.18

$$\frac{Bb^3m^4x^9x^m+16\,Bb^3m^3x^9x^m+3\,Bab^2m^4x^7x^m+Ab^3m^4x^7x^m+86\,Bb^3m^2x^9x^m+54\,Bab^2m^3x^7x^m+18\,Ab^3m^3x^7x^m+176\,Bb^3mx^9x^m+3\,Ba^2bm^4x^5x^m+3\,Aab^2m^4x^5x^m+312\,Bab^2m^2x^7x^m+104\,Ab^3m^2x^7x^m+105\,Bb^3x^9x^m+60\,Ba^2bm^3x^5x^m+60\,Aab^2m^3x^5x^m+666\,Bab^2mx^7x^m+222\,Ab^3mx^7x^m+Ba^3m^4x^3x^m+3\,Aa^2bm^4x^3x^m+390\,Ba^2bm^2x^5x^m+390\,Aab^2m^2x^5x^m+405\,Bab^2x^7x^m+135\,Ab^3x^7x^m+22\,Ba^3m^3x^3x^m+66\,Aa^2bm^3x^3x^m+900\,Ba^2bmx^5x^m+900\,Aab^2mx^5x^m+Aa^3m^4xx^m+164\,Ba^3m^2x^3x^m+492\,Aa^2bm^2x^3x^m+567\,Ba^2bx^5x^m+567\,Aab^2x^5x^m+24\,Aa^3m^3xx^m+458\,Ba^3mx^3x^m+1374\,Aa^2bmx^3x^m+206\,Aa^3m^2xx^m+315\,Ba^3x^3x^m+945\,Aa^2bx^3x^m+744\,Aa^3mxx^m+945\,Aa^3xx^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b^3*m^4*x^9*x^m + 16*B*b^3*m^3*x^9*x^m + 3*B*a*b^2*m^4*x^7*x^m + A*b^3*m
^4*x^7*x^m + 86*B*b^3*m^2*x^9*x^m + 54*B*a*b^2*m^3*x^7*x^m + 18*A*b^3*m^3*x
^7*x^m + 176*B*b^3*m*x^9*x^m + 3*B*a^2*b*m^4*x^5*x^m + 3*A*a*b^2*m^4*x^5*x^
m + 312*B*a*b^2*m^2*x^7*x^m + 104*A*b^3*m^2*x^7*x^m + 105*B*b^3*x^9*x^m + 6
0*B*a^2*b*m^3*x^5*x^m + 60*A*a*b^2*m^3*x^5*x^m + 666*B*a*b^2*m*x^7*x^m + 22
```

```
2*A*b^3*m*x^7*x^m + B*a^3*m^4*x^3*x^m + 3*A*a^2*b*m^4*x^3*x^m + 390*B*a^2*b
*m^2*x^5*x^m + 390*A*a*b^2*m^2*x^5*x^m + 405*B*a*b^2*x^7*x^m + 135*A*b^3*x^
7*x^m + 22*B*a^3*m^3*x^3*x^m + 66*A*a^2*b*m^3*x^3*x^m + 900*B*a^2*b*m*x^5*x
^m + 900*A*a*b^2*m*x^5*x^m + A*a^3*m^4*x*x^m + 164*B*a^3*m^2*x^3*x^m + 492*
A*a^2*b*m^2*x^3*x^m + 567*B*a^2*b*x^5*x^m + 567*A*a*b^2*x^5*x^m + 24*A*a^3*
m^3*x*x^m + 458*B*a^3*m*x^3*x^m + 1374*A*a^2*b*m*x^3*x^m + 206*A*a^3*m^2*x*
x^m + 315*B*a^3*x^3*x^m + 945*A*a^2*b*x^3*x^m + 744*A*a^3*m*x*x^m + 945*A*a
^3*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.02, size = 474, normalized size = 4.94

$$\frac{\left(B b^{3} m^{4} x^{8}+16 B b^{3} m^{3} x^{8}+A b^{3} m^{4} x^{6}+3 B a b^{2} m^{4} x^{6}+86 B b^{3} m^{2} x^{8}+18 A b^{3} m^{3} x^{6}+54 B a b^{2} m^{3} x^{6}+176 B b^{3} m x^{8}+3 A a b^{2} m^{4} x^{4}+104 A b^{3} m^{2} x^{6}+3 B a^{2} b m^{4} x^{4}+312 B a b^{2} m^{2} x^{6}+105 B b^{3} x^{8}+60 A a b^{2} m^{3} x^{4}+222 A b^{3} m x^{6}+60 B a^{2} b m^{3} x^{4}+666 B a b^{2} m x^{6}+3 A a^{2} b m^{4} x^{2}+390 A a b^{2} m^{2} x^{4}+135 A b^{3} x^{6}+B a^{3} m^{4} x^{2}+390 B a^{2} b m^{2} x^{4}+405 B a b^{2} x^{6}+66 A a^{2} b m^{3} x^{2}+900 A a b^{2} m x^{4}+22 B a^{3} m^{3} x^{2}+900 B a^{2} b m x^{4}+A a^{3} m^{4}+492 A a^{2} b m^{2} x^{2}+567 A a b^{2} x^{4}+164 B a^{3} m^{2} x^{2}+567 B a^{2} b x^{4}+24 A a^{3} m^{3}+1374 A a^{2} b m x^{2}+458 B a^{3} m x^{2}+206 A a^{3} m^{2}+945 A a^{2} b x^{2}+315 B a^{3} x^{2}+744 A a^{3} m+945 A a^{3}\right) x^{m+1}}{\left(m+9\right) \left(m+7\right) \left(m+5\right) \left(m+3\right) \left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] x^(1+m)*(B*b^3*m^4*x^8+16*B*b^3*m^3*x^8+A*b^3*m^4*x^6+3*B*a*b^2*m^4*x^6+86*
B*b^3*m^2*x^8+18*A*b^3*m^3*x^6+54*B*a*b^2*m^3*x^6+176*B*b^3*m*x^8+3*A*a*b^2
*m^4*x^4+104*A*b^3*m^2*x^6+3*B*a^2*b*m^4*x^4+312*B*a*b^2*m^2*x^6+105*B*b^3*
x^8+60*A*a*b^2*m^3*x^4+222*A*b^3*m*x^6+60*B*a^2*b*m^3*x^4+666*B*a*b^2*m*x^6
+3*A*a^2*b*m^4*x^2+390*A*a*b^2*m^2*x^4+135*A*b^3*x^6+B*a^3*m^4*x^2+390*B*a^
2*b*m^2*x^4+405*B*a*b^2*x^6+66*A*a^2*b*m^3*x^2+900*A*a*b^2*m*x^4+22*B*a^3*m
^3*x^2+900*B*a^2*b*m*x^4+A*a^3*m^4+492*A*a^2*b*m^2*x^2+567*A*a*b^2*x^4+164*
B*a^3*m^2*x^2+567*B*a^2*b*x^4+24*A*a^3*m^3+1374*A*a^2*b*m*x^2+458*B*a^3*m*x
^2+206*A*a^3*m^2+945*A*a^2*b*x^2+315*B*a^3*x^2+744*A*a^3*m+945*A*a^3)/(9+m)
/(7+m)/(5+m)/(3+m)/(1+m)
```

maxima [A] time = 1.07, size = 129, normalized size = 1.34

$$\frac{Bb^{3}x^{m+9}}{m+9}+\frac{3\,Bab^{2}x^{m+7}}{m+7}+\frac{Ab^{3}x^{m+7}}{m+7}+\frac{3\,Ba^{2}bx^{m+5}}{m+5}+\frac{3\,Aab^{2}x^{m+5}}{m+5}+\frac{Ba^{3}x^{m+3}}{m+3}+\frac{3\,Aa^{2}bx^{m+3}}{m+3}+\frac{Aa^{3}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] B*b^3*x^(m + 9)/(m + 9) + 3*B*a*b^2*x^(m + 7)/(m + 7) + A*b^3*x^(m + 7)/(m
+ 7) + 3*B*a^2*b*x^(m + 5)/(m + 5) + 3*A*a*b^2*x^(m + 5)/(m + 5) + B*a^3*x^
(m + 3)/(m + 3) + 3*A*a^2*b*x^(m + 3)/(m + 3) + A*a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.50, size = 289, normalized size = 3.01

$$\frac{A\,a^3\,x\,x^m\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{B\,b^3\,x^m\,x^9\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{a^2\,x^m\,x^3\,\left(3\,A\,b+B\,a\right)\,\left(m^4+22\,m^3+164\,m^2+458\,m+315\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{b^2\,x^m\,x^7\,\left(A\,b+3\,B\,a\right)\,\left(m^4+18\,m^3+104\,m^2+222\,m+135\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}+\frac{3\,a\,b\,x^m\,x^5\,\left(A\,b+B\,a\right)\,\left(m^4+20\,m^3+130\,m^2+300\,m+189\right)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] (A*a^3*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 23
0*m^3 + 25*m^4 + m^5 + 945) + (B*b^3*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m^4
 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^2*x^m*x^3*(
3*A*b + B*a)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 23
0*m^3 + 25*m^4 + m^5 + 945) + (b^2*x^m*x^7*(A*b + 3*B*a)*(222*m + 104*m^2 +
 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (
3*a*b*x^m*x^5*(A*b + B*a)*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m +
 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
```

sympy [A] time = 3.08, size = 2069, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**3/(8*x**8) - A*a**2*b/(2*x**6) - 3*A*a*b**2/(4*x**4) - A*b
**3/(2*x**2) - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B*a*b**2/(2*x**2)
+ B*b**3*log(x), Eq(m, -9)), (-A*a**3/(6*x**6) - 3*A*a**2*b/(4*x**4) - 3*A*
a*b**2/(2*x**2) + A*b**3*log(x) - B*a**3/(4*x**4) - 3*B*a**2*b/(2*x**2) + 3
*B*a*b**2*log(x) + B*b**3*x**2/2, Eq(m, -7)), (-A*a**3/(4*x**4) - 3*A*a**2*
b/(2*x**2) + 3*A*a*b**2*log(x) + A*b**3*x**2/2 - B*a**3/(2*x**2) + 3*B*a**2
*b*log(x) + 3*B*a*b**2*x**2/2 + B*b**3*x**4/4, Eq(m, -5)), (-A*a**3/(2*x**2
) + 3*A*a**2*b*log(x) + 3*A*a*b**2*x**2/2 + A*b**3*x**4/4 + B*a**3*log(x) +
 3*B*a**2*b*x**2/2 + 3*B*a*b**2*x**4/4 + B*b**3*x**6/6, Eq(m, -3)), (A*a**3
*log(x) + 3*A*a**2*b*x**2/2 + 3*A*a*b**2*x**4/4 + A*b**3*x**6/6 + B*a**3*x*
*2/2 + 3*B*a**2*b*x**4/4 + B*a*b**2*x**6/2 + B*b**3*x**8/8, Eq(m, -1)), (A*
a**3*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24
*A*a**3*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 206*A*a**3*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 744*A*a**3*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 945*A*a**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 3*A*a**2*b*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 66*A*a**2*b*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 492*A*a**2*b*m**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 1374*A*a**2*b*m*x**3*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 945*A*a**2*b*x**3*x**m/(m**5 + 25*m**4
+ 230*m**3 + 950*m**2 + 1689*m + 945) + 3*A*a*b**2*m**4*x**5*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 60*A*a*b**2*m**3*x**5*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*A*a*b**2*m**2*x*
*5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*A*a*b**
2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 567*A
*a*b**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + A
*b**3*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
```

```
+ 18*A*b**3*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
 945) + 104*A*b**3*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1
689*m + 945) + 222*A*b**3*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 135*A*b**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + B*a**3*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 22*B*a**3*m**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 164*B*a**3*m**2*x**3*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 458*B*a**3*m*x**3*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 315*B*a**3*x**3*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3*B*a**2*b*m**4*x**5*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 60*B*a**2*b*m**3*x**5*x**
m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 390*B*a**2*b*m**2
*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 900*B*a*
*2*b*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 56
7*B*a**2*b*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
+ 3*B*a*b**2*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 54*B*a*b**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 312*B*a*b**2*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 9
50*m**2 + 1689*m + 945) + 666*B*a*b**2*m*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 405*B*a*b**2*x**7*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + B*b**3*m**4*x**9*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 16*B*b**3*m**3*x**9*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*B*b**3*m**2*x**9*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*B*b**3*m*x**9*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 105*B*b**3*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

3.320 $\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)\, dx$

Optimal. Leaf size=71

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

Rubi [**A**] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) + (b*(A*b +
 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right)\, dx &= \int \left(a^2Ax^m+a(2Ab+aB)x^{2+m}+b(Ab+2aB)x^{4+m}+b^2Bx^{6+m}\right) dx\\ &= \frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{3+m}}{3+m}+\frac{b(Ab+2aB)x^{5+m}}{5+m}+\frac{b^2Bx^{7+m}}{7+m}\end{aligned}$$

Mathematica [**A**] time = 0.05, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2A}{m+1}+\frac{bx^4(2aB+Ab)}{m+5}+\frac{ax^2(aB+2Ab)}{m+3}+\frac{b^2Bx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^2)/(3 + m) + (b*(A*b + 2*a*
B)*x^4)/(5 + m) + (b^2*B*x^6)/(7 + m))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^2*(A + B*x^2), x]
```

fricas [B] time = 0.93, size = 215, normalized size = 3.03

$$\frac{\left(\left(Bb^2m^3+9\,Bb^2m^2+23\,Bb^2m+15\,Bb^2\right)x^7+\left(\left(2\,Bab+Ab^2\right)m^3+42\,Bab+21\,Ab^2+11\,\left(2\,Bab+Ab^2\right)m^2+31\,\left(2\,Bab+Ab^2\right)m\right)x^5+\left(\left(Ba^2+2\,Aab\right)m^3+35\,Ba^2+70\,Aab+13\,\left(Ba^2+2\,Aab\right)m^2+47\,\left(Ba^2+2\,Aab\right)m\right)x^3+\left(Aa^2m^3+15\,Aa^2m^2+71\,Aa^2m+105\,Aa^2\right)x\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 9*B*b^2*m^2 + 23*B*b^2*m + 15*B*b^2)*x^7 + ((2*B*a*b + A*b^2)
*m^3 + 42*B*a*b + 21*A*b^2 + 11*(2*B*a*b + A*b^2)*m^2 + 31*(2*B*a*b + A*b^2
)*m)*x^5 + ((B*a^2 + 2*A*a*b)*m^3 + 35*B*a^2 + 70*A*a*b + 13*(B*a^2 + 2*A*a
*b)*m^2 + 47*(B*a^2 + 2*A*a*b)*m)*x^3 + (A*a^2*m^3 + 15*A*a^2*m^2 + 71*A*a^
2*m + 105*A*a^2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

giac [B] time = 0.45, size = 332, normalized size = 4.68

$$\frac{Bb^2m^3x^7x^m+9\,Bb^2m^2x^7x^m+2\,Babm^3x^5x^m+Ab^2m^3x^5x^m+23\,Bb^2mx^7x^m+22\,Babm^2x^5x^m+11\,Ab^2m^2x^5x^m+15\,Bb^2x^7x^m+Ba^2m^3x^3x^m+2\,Aabm^3x^3x^m+62\,Babmx^5x^m+31\,Ab^2mx^5x^m+13\,Ba^2m^2x^3x^m+26\,Aabm^2x^3x^m+42\,Babx^5x^m+21\,Ab^2x^5x^m+Aa^2m^3xx^m+47\,Ba^2mx^3x^m+94\,Aabmx^3x^m+15\,Aa^2m^2xx^m+35\,Ba^2x^3x^m+70\,Aabx^3x^m+71\,Aa^2mxx^m+105\,Aa^2xx^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^7*x^m + 9*B*b^2*m^2*x^7*x^m + 2*B*a*b*m^3*x^5*x^m + A*b^2*m^3*
x^5*x^m + 23*B*b^2*m*x^7*x^m + 22*B*a*b*m^2*x^5*x^m + 11*A*b^2*m^2*x^5*x^m
+ 15*B*b^2*x^7*x^m + B*a^2*m^3*x^3*x^m + 2*A*a*b*m^3*x^3*x^m + 62*B*a*b*m*x
^5*x^m + 31*A*b^2*m*x^5*x^m + 13*B*a^2*m^2*x^3*x^m + 26*A*a*b*m^2*x^3*x^m +
 42*B*a*b*x^5*x^m + 21*A*b^2*x^5*x^m + A*a^2*m^3*x*x^m + 47*B*a^2*m*x^3*x^m
 + 94*A*a*b*m*x^3*x^m + 15*A*a^2*m^2*x*x^m + 35*B*a^2*x^3*x^m + 70*A*a*b*x^
3*x^m + 71*A*a^2*m*x*x^m + 105*A*a^2*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)
```

maple [B] time = 0.01, size = 262, normalized size = 3.69

$$\frac{\left(B\,b^2m^3x^6+9B\,b^2m^2x^6+A\,b^2m^3x^4+2Bab\,m^3x^4+23B\,b^2m\,x^6+11A\,b^2m^2x^4+22Bab\,m^2x^4+15B\,b^2x^6+2Aab\,m^3x^2+31A\,b^2m\,x^4+B\,a^2m^3x^2+62Babm\,x^4+26Aab\,m^2x^2+21A\,b^2x^4+13B\,a^2m^2x^2+42Bab\,x^4+A\,a^2m^3+94Aabm\,x^2+47B\,a^2m\,x^2+15A\,a^2m^2+70Aab\,x^2+35B\,a^2x^2+71A\,a^2m+105a^2A\right)x^{m+1}}{(m+7)\,(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] x^(m+1)*(B*b^2*m^3*x^6+9*B*b^2*m^2*x^6+A*b^2*m^3*x^4+2*B*a*b*m^3*x^4+23*B*b
^2*m*x^6+11*A*b^2*m^2*x^4+22*B*a*b*m^2*x^4+15*B*b^2*x^6+2*A*a*b*m^3*x^2+31*
A*b^2*m*x^4+B*a^2*m^3*x^2+62*B*a*b*m*x^4+26*A*a*b*m^2*x^2+21*A*b^2*x^4+13*B
*a^2*m^2*x^2+42*B*a*b*x^4+A*a^2*m^3+94*A*a*b*m*x^2+47*B*a^2*m*x^2+15*A*a^2*
m^2+70*A*a*b*x^2+35*B*a^2*x^2+71*A*a^2*m+105*A*a^2)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [**A**] time = 1.01, size = 91, normalized size = 1.28

$$\frac{Bb^2x^{m+7}}{m+7}+\frac{2\,Babx^{m+5}}{m+5}+\frac{Ab^2x^{m+5}}{m+5}+\frac{Ba^2x^{m+3}}{m+3}+\frac{2\,Aabx^{m+3}}{m+3}+\frac{Aa^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] B*b^2*x^(m + 7)/(m + 7) + 2*B*a*b*x^(m + 5)/(m + 5) + A*b^2*x^(m + 5)/(m +
5) + B*a^2*x^(m + 3)/(m + 3) + 2*A*a*b*x^(m + 3)/(m + 3) + A*a^2*x^(m + 1)/
(m + 1)
```

mupad [**B**] time = 0.34, size = 177, normalized size = 2.49

$$x^m\left(\frac{B\,b^2\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{A\,a^2\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a\,x^3\,\left(2\,A\,b+B\,a\right)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b\,x^5\,\left(A\,b+2\,B\,a\right)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^m*((B*b^2*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 +
 105) + (A*a^2*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^
4 + 105) + (a*x^3*(2*A*b + B*a)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2
 + 16*m^3 + m^4 + 105) + (b*x^5*(A*b + 2*B*a)*(31*m + 11*m^2 + m^3 + 21))/(
176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

sympy [**A**] time = 1.79, size = 1044, normalized size = 14.70

$$\begin{cases} -\frac{Aa^2}{6x^6}-\frac{Aab}{2x^4}-\frac{Ab^2}{2x^2}-\frac{Ba^2}{4x^4}-\frac{Bab}{x^2}+Bb^2\log(x) & \text{for}\ m=-7 \\ -\frac{Aa^2}{4x^4}-\frac{Aab}{x^2}+Ab^2\log(x)-\frac{Ba^2}{2x^2}+2Bab\log(x)+\frac{Bb^2x^2}{2} & \text{for}\ m=-5 \\ -\frac{Aa^2}{2x^2}+2Aab\log(x)+\frac{Ab^2x^2}{2}+Ba^2\log(x)+Babx^2+\frac{Bb^2x^4}{4} & \text{for}\ m=-3 \\ Aa^2\log(x)+Aabx^2+\frac{Ab^2x^4}{4}+\frac{Ba^2x^2}{2}+\frac{Babx^4}{2}+\frac{Bb^2x^6}{6} & \text{for}\ m=-1 \\ \frac{Aa^2m^3xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{15Aa^2m^2xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{71Aa^2mxx^m}{m^4+16m^3+86m^2+176m+105}+\frac{105Aa^2xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{2Aabm^3x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{26Aabm^2x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{94Aabmx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{70Aabx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{Ab^2m^3x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{11Ab^2m^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{31Ab^2mx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{21Ab^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{Ba^2m^3x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{13Ba^2m^2x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{47Ba^2mx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{35Ba^2x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{2Babm^3x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{22Babm^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{62Babmx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{42Babx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{Bb^2m^3x^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{9Bb^2m^2x^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{23Bb^2mx^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{15Bb^2x^7x^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2) - B*a**2/(4*
x**4) - B*a*b/x**2 + B*b**2*log(x), Eq(m, -7)), (-A*a**2/(4*x**4) - A*a*b/x
```

```
**2 + A*b**2*log(x) - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*b**2*x**2/2, Eq(
m, -5)), (-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2 + B*a**2*log(x)
 + B*a*b*x**2 + B*b**2*x**4/4, Eq(m, -3)), (A*a**2*log(x) + A*a*b*x**2 + A*
b**2*x**4/4 + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2*x**6/6, Eq(m, -1)), (A*
a**2*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*a**2*m**2*
x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*A*a**2*m*x*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 105*A*a**2*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 2*A*a*b*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 1
76*m + 105) + 26*A*a*b*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + 94*A*a*b*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*A*
a*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*b**2*m**3*x**5*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*b**2*m**2*x**5*x**m/(m*
*4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*A*b**2*m*x**5*x**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 21*A*b**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + B*a**2*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
 105) + 13*B*a**2*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 47*B*a**2*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*B*a**2
*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*B*a*b*m**3*x**5*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 22*B*a*b*m**2*x**5*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 62*B*a*b*m*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 42*B*a*b*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + B*b**2*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 9*B*b**2*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*B*
b**2*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*B*b**2*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

3.321 $\int x^m \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=45

$$\frac{x^{m+3}(aB+Ab)}{m+3}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+5}}{m+5}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{x^{m+3}(aB+Ab)}{m+3}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+5}}{m+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*x^(5 + m))
/(5 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aAx^m+(Ab+aB)x^{2+m}+bBx^{4+m}\right)\,dx\\ &= \frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{3+m}}{3+m}+\frac{bBx^{5+m}}{5+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.93

$$x^{m+1}\left(\frac{x^2(aB+Ab)}{m+3}+\frac{aA}{m+1}+\frac{bBx^4}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^2)/(3 + m) + (b*B*x^4)/(5 + m))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)*(A + B*x^2), x]
```

fricas [B] time = 0.77, size = 92, normalized size = 2.04

$$\frac{\left(\left(Bbm^2+4\,Bbm+3\,Bb\right)x^5+\left((Ba+Ab)m^2+5\,Ba+5\,Ab+6\,(Ba+Ab)m\right)x^3+\left(Aam^2+8\,Aam+15\,Aa\right)x\right)x^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 4*B*b*m + 3*B*b)*x^5 + ((B*a + A*b)*m^2 + 5*B*a + 5*A*b + 6*(B*
a + A*b)*m)*x^3 + (A*a*m^2 + 8*A*a*m + 15*A*a)*x)*x^m/(m^3 + 9*m^2 + 23*m +
 15)
```

giac [B] time = 0.36, size = 143, normalized size = 3.18

$$\frac{Bbm^2x^5x^m+4\,Bbmx^5x^m+Bam^2x^3x^m+Abm^2x^3x^m+3\,Bbx^5x^m+6\,Bamx^3x^m+6\,Abmx^3x^m+Aam^2xx^m+5\,Bax^3x^m+5\,Abx^3x^m+8\,Aamxx^m+15\,Aaxx^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^5*x^m + 4*B*b*m*x^5*x^m + B*a*m^2*x^3*x^m + A*b*m^2*x^3*x^m + 3*
B*b*x^5*x^m + 6*B*a*m*x^3*x^m + 6*A*b*m*x^3*x^m + A*a*m^2*x*x^m + 5*B*a*x^3
*x^m + 5*A*b*x^3*x^m + 8*A*a*m*x*x^m + 15*A*a*x*x^m)/(m^3 + 9*m^2 + 23*m +
15)
```

maple [B] time = 0.00, size = 110, normalized size = 2.44

$$\frac{\left(Bb\,m^2x^4+4Bbm\,x^4+Ab\,m^2x^2+Ba\,m^2x^2+3Bb\,x^4+6Abm\,x^2+6Bam\,x^2+Aa\,m^2+5Ab\,x^2+5Ba\,x^2+8Aam+15Aa\right)x^{m+1}}{(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] x^(m+1)*(B*b*m^2*x^4+4*B*b*m*x^4+A*b*m^2*x^2+B*a*m^2*x^2+3*B*b*x^4+6*A*b*m*
x^2+6*B*a*m*x^2+A*a*m^2+5*A*b*x^2+5*B*a*x^2+8*A*a*m+15*A*a)/(m+5)/(m+3)/(m+
1)
```

maxima [A] time = 1.06, size = 53, normalized size = 1.18

$$\frac{Bbx^{m+5}}{m+5}+\frac{Bax^{m+3}}{m+3}+\frac{Abx^{m+3}}{m+3}+\frac{Aax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] B*b*x^(m + 5)/(m + 5) + B*a*x^(m + 3)/(m + 3) + A*b*x^(m + 3)/(m + 3) + A*a
*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.30, size = 95, normalized size = 2.11

$$x^m\,\left(\frac{x^3\,\left(A\,b+B\,a\right)\,\left(m^2+6\,m+5\right)}{m^3+9\,m^2+23\,m+15}+\frac{B\,b\,x^5\,\left(m^2+4\,m+3\right)}{m^3+9\,m^2+23\,m+15}+\frac{A\,a\,x\,\left(m^2+8\,m+15\right)}{m^3+9\,m^2+23\,m+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] x^m*((x^3*(A*b + B*a)*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (B*b*x^5
*(4*m + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (A*a*x*(8*m + m^2 + 15))/(23*
m + 9*m^2 + m^3 + 15))
```

sympy [A] time = 0.94, size = 410, normalized size = 9.11

$$\begin{cases} - \frac{A a}{4 x^{4}} - \frac{A b}{2 x^{2}} - \frac{B a}{2 x^{2}} + B b \log{\left(x \right)} & \text{for}\: m = -5 \\- \frac{A a}{2 x^{2}} + A b \log{\left(x \right)} + B a \log{\left(x \right)} + \frac{B b x^{2}}{2} & \text{for}\: m = -3 \\A a \log{\left(x \right)} + \frac{A b x^{2}}{2} + \frac{B a x^{2}}{2} + \frac{B b x^{4}}{4} & \text{for}\: m = -1 \\\frac{A a m^{2} x x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{8 A a m x x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{15 A a x x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{A b m^{2} x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{6 A b m x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{5 A b x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{B a m^{2} x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{6 B a m x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{5 B a x^{3} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{B b m^{2} x^{5} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{4 B b m x^{5} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} + \frac{3 B b x^{5} x^{m}}{m^{3} + 9 m^{2} + 23 m + 15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] Piecewise((-A*a/(4*x**4) - A*b/(2*x**2) - B*a/(2*x**2) + B*b*log(x), Eq(m,
-5)), (-A*a/(2*x**2) + A*b*log(x) + B*a*log(x) + B*b*x**2/2, Eq(m, -3)), (A
*a*log(x) + A*b*x**2/2 + B*a*x**2/2 + B*b*x**4/4, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 9*m**2 + 23*m + 15) + 8*A*a*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15
) + 15*A*a*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + A*b*m**2*x**3*x**m/(m**3 +
9*m**2 + 23*m + 15) + 6*A*b*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*A*b
*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*a*m**2*x**3*x**m/(m**3 + 9*m**2
```

```
+ 23*m + 15) + 6*B*a*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*B*a*x**3*x
**m/(m**3 + 9*m**2 + 23*m + 15) + B*b*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m
+ 15) + 4*B*b*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*B*b*x**5*x**m/(m*
*3 + 9*m**2 + 23*m + 15), True))
```

3.322 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=151

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad}{m+9}$$

Rubi [A] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc)}{m+9}+\frac{b^2d^3x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(3 + m) + (
c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m) + (d*(3*b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^(9 + m))/(
9 + m) + (b^2*d^3*x^(11 + m))/(11 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int\left(a^2c^3x^m+ac^2(2bc+3ad)x^{2+m}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{4+m}+d\left(3b^2c^2\right.\right.\\ &= \frac{a^2c^3x^{1+m}}{1+m}+\frac{ac^2(2bc+3ad)x^{3+m}}{3+m}+\frac{c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5+m}}{5+m}+\frac{d\left(3b^2c^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.10, size = 141, normalized size = 0.93

$$x^m\left(\frac{dx^7\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{cx^5\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{a^2c^3x}{m+1}+\frac{ac^2x^3(3ad+2bc)}{m+3}+\frac{bd^2x^9(2ad+3bc)}{m+9}+\frac{b^2d^3x^{11}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] x^m*((a^2*c^3*x)/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^3)/(3 + m) + (c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^5)/(5 + m) + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^7)/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^9)/(9 + m) + (b^2*d^3*x^11)/(11
 + m))
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^2*(c + d*x^2)^3, x]
```

fricas [B] time = 1.21, size = 773, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] ((b^2*d^3*m^5 + 25*b^2*d^3*m^4 + 230*b^2*d^3*m^3 + 950*b^2*d^3*m^2 + 1689*b
^2*d^3*m + 945*b^2*d^3)*x^11 + ((3*b^2*c*d^2 + 2*a*b*d^3)*m^5 + 3465*b^2*c*
d^2 + 2310*a*b*d^3 + 27*(3*b^2*c*d^2 + 2*a*b*d^3)*m^4 + 262*(3*b^2*c*d^2 +
2*a*b*d^3)*m^3 + 1122*(3*b^2*c*d^2 + 2*a*b*d^3)*m^2 + 2041*(3*b^2*c*d^2 + 2
*a*b*d^3)*m)*x^9 + ((3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^5 + 4455*b^2*c^
2*d + 8910*a*b*c*d^2 + 1485*a^2*d^3 + 29*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d
^3)*m^4 + 302*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^3 + 1366*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*m^2 + 2577*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*
m)*x^7 + ((b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^5 + 2079*b^2*c^3 + 12474*
a*b*c^2*d + 6237*a^2*c*d^2 + 31*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^4 +
 350*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^3 + 1730*(b^2*c^3 + 6*a*b*c^2*
d + 3*a^2*c*d^2)*m^2 + 3489*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m)*x^5 +
((2*a*b*c^3 + 3*a^2*c^2*d)*m^5 + 6930*a*b*c^3 + 10395*a^2*c^2*d + 33*(2*a*b
*c^3 + 3*a^2*c^2*d)*m^4 + 406*(2*a*b*c^3 + 3*a^2*c^2*d)*m^3 + 2262*(2*a*b*c
^3 + 3*a^2*c^2*d)*m^2 + 5353*(2*a*b*c^3 + 3*a^2*c^2*d)*m)*x^3 + (a^2*c^3*m^
5 + 35*a^2*c^3*m^4 + 470*a^2*c^3*m^3 + 3010*a^2*c^3*m^2 + 9129*a^2*c^3*m +
10395*a^2*c^3)*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 1952
4*m + 10395)
```

giac [B] time = 0.59, size = 1192, normalized size = 7.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (b^2*d^3*m^5*x^11*x^m + 25*b^2*d^3*m^4*x^11*x^m + 3*b^2*c*d^2*m^5*x^9*x^m +
 2*a*b*d^3*m^5*x^9*x^m + 230*b^2*d^3*m^3*x^11*x^m + 81*b^2*c*d^2*m^4*x^9*x^
m + 54*a*b*d^3*m^4*x^9*x^m + 950*b^2*d^3*m^2*x^11*x^m + 3*b^2*c^2*d*m^5*x^7
*x^m + 6*a*b*c*d^2*m^5*x^7*x^m + a^2*d^3*m^5*x^7*x^m + 786*b^2*c*d^2*m^3*x^
9*x^m + 524*a*b*d^3*m^3*x^9*x^m + 1689*b^2*d^3*m*x^11*x^m + 87*b^2*c^2*d*m^
4*x^7*x^m + 174*a*b*c*d^2*m^4*x^7*x^m + 29*a^2*d^3*m^4*x^7*x^m + 3366*b^2*c
*d^2*m^2*x^9*x^m + 2244*a*b*d^3*m^2*x^9*x^m + 945*b^2*d^3*x^11*x^m + b^2*c^
3*m^5*x^5*x^m + 6*a*b*c^2*d*m^5*x^5*x^m + 3*a^2*c*d^2*m^5*x^5*x^m + 906*b^2
*c^2*d*m^3*x^7*x^m + 1812*a*b*c*d^2*m^3*x^7*x^m + 302*a^2*d^3*m^3*x^7*x^m +
 6123*b^2*c*d^2*m*x^9*x^m + 4082*a*b*d^3*m*x^9*x^m + 31*b^2*c^3*m^4*x^5*x^m
 + 186*a*b*c^2*d*m^4*x^5*x^m + 93*a^2*c*d^2*m^4*x^5*x^m + 4098*b^2*c^2*d*m^
2*x^7*x^m + 8196*a*b*c*d^2*m^2*x^7*x^m + 1366*a^2*d^3*m^2*x^7*x^m + 3465*b^
2*c*d^2*x^9*x^m + 2310*a*b*d^3*x^9*x^m + 2*a*b*c^3*m^5*x^3*x^m + 3*a^2*c^2*
d*m^5*x^3*x^m + 350*b^2*c^3*m^3*x^5*x^m + 2100*a*b*c^2*d*m^3*x^5*x^m + 1050
*a^2*c*d^2*m^3*x^5*x^m + 7731*b^2*c^2*d*m*x^7*x^m + 15462*a*b*c*d^2*m*x^7*x
^m + 2577*a^2*d^3*m*x^7*x^m + 66*a*b*c^3*m^4*x^3*x^m + 99*a^2*c^2*d*m^4*x^3
*x^m + 1730*b^2*c^3*m^2*x^5*x^m + 10380*a*b*c^2*d*m^2*x^5*x^m + 5190*a^2*c*
d^2*m^2*x^5*x^m + 4455*b^2*c^2*d*x^7*x^m + 8910*a*b*c*d^2*x^7*x^m + 1485*a^
2*d^3*x^7*x^m + a^2*c^3*m^5*x*x^m + 812*a*b*c^3*m^3*x^3*x^m + 1218*a^2*c^2*
d*m^3*x^3*x^m + 3489*b^2*c^3*m*x^5*x^m + 20934*a*b*c^2*d*m*x^5*x^m + 10467*
a^2*c*d^2*m*x^5*x^m + 35*a^2*c^3*m^4*x*x^m + 4524*a*b*c^3*m^2*x^3*x^m + 678
6*a^2*c^2*d*m^2*x^3*x^m + 2079*b^2*c^3*x^5*x^m + 12474*a*b*c^2*d*x^5*x^m +
6237*a^2*c*d^2*x^5*x^m + 470*a^2*c^3*m^3*x*x^m + 10706*a*b*c^3*m*x^3*x^m +
16059*a^2*c^2*d*m*x^3*x^m + 3010*a^2*c^3*m^2*x*x^m + 6930*a*b*c^3*x^3*x^m +
 10395*a^2*c^2*d*x^3*x^m + 9129*a^2*c^3*m*x*x^m + 10395*a^2*c^3*x*x^m)/(m^6
 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

maple [B] time = 0.01, size = 976, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] x^(m+1)*(b^2*d^3*m^5*x^10+25*b^2*d^3*m^4*x^10+2*a*b*d^3*m^5*x^8+3*b^2*c*d^2
*m^5*x^8+230*b^2*d^3*m^3*x^10+54*a*b*d^3*m^4*x^8+81*b^2*c*d^2*m^4*x^8+950*b
^2*d^3*m^2*x^10+a^2*d^3*m^5*x^6+6*a*b*c*d^2*m^5*x^6+524*a*b*d^3*m^3*x^8+3*b
^2*c^2*d*m^5*x^6+786*b^2*c*d^2*m^3*x^8+1689*b^2*d^3*m*x^10+29*a^2*d^3*m^4*x
^6+174*a*b*c*d^2*m^4*x^6+2244*a*b*d^3*m^2*x^8+87*b^2*c^2*d*m^4*x^6+3366*b^2
*c*d^2*m^2*x^8+945*b^2*d^3*x^10+3*a^2*c*d^2*m^5*x^4+302*a^2*d^3*m^3*x^6+6*a
*b*c^2*d*m^5*x^4+1812*a*b*c*d^2*m^3*x^6+4082*a*b*d^3*m*x^8+b^2*c^3*m^5*x^4+
```

```
906*b^2*c^2*d*m^3*x^6+6123*b^2*c*d^2*m*x^8+93*a^2*c*d^2*m^4*x^4+1366*a^2*d^
3*m^2*x^6+186*a*b*c^2*d*m^4*x^4+8196*a*b*c*d^2*m^2*x^6+2310*a*b*d^3*x^8+31*
b^2*c^3*m^4*x^4+4098*b^2*c^2*d*m^2*x^6+3465*b^2*c*d^2*x^8+3*a^2*c^2*d*m^5*x
^2+1050*a^2*c*d^2*m^3*x^4+2577*a^2*d^3*m*x^6+2*a*b*c^3*m^5*x^2+2100*a*b*c^2
*d*m^3*x^4+15462*a*b*c*d^2*m*x^6+350*b^2*c^3*m^3*x^4+7731*b^2*c^2*d*m*x^6+9
9*a^2*c^2*d*m^4*x^2+5190*a^2*c*d^2*m^2*x^4+1485*a^2*d^3*x^6+66*a*b*c^3*m^4*
x^2+10380*a*b*c^2*d*m^2*x^4+8910*a*b*c*d^2*x^6+1730*b^2*c^3*m^2*x^4+4455*b^
2*c^2*d*x^6+a^2*c^3*m^5+1218*a^2*c^2*d*m^3*x^2+10467*a^2*c*d^2*m*x^4+812*a*
b*c^3*m^3*x^2+20934*a*b*c^2*d*m*x^4+3489*b^2*c^3*m*x^4+35*a^2*c^3*m^4+6786*
a^2*c^2*d*m^2*x^2+6237*a^2*c*d^2*x^4+4524*a*b*c^3*m^2*x^2+12474*a*b*c^2*d*x
^4+2079*b^2*c^3*x^4+470*a^2*c^3*m^3+16059*a^2*c^2*d*m*x^2+10706*a*b*c^3*m*x
^2+3010*a^2*c^3*m^2+10395*a^2*c^2*d*x^2+6930*a*b*c^3*x^2+9129*a^2*c^3*m+103
95*a^2*c^3)/(11+m)/(m+9)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 1.11, size = 215, normalized size = 1.42

$$\frac{b^{2}d^{3}x^{m+11}}{m+11}+\frac{3\,b^{2}cd^{2}x^{m+9}}{m+9}+\frac{2\,abd^{3}x^{m+9}}{m+9}+\frac{3\,b^{2}c^{2}dx^{m+7}}{m+7}+\frac{6\,abcd^{2}x^{m+7}}{m+7}+\frac{a^{2}d^{3}x^{m+7}}{m+7}+\frac{b^{2}c^{3}x^{m+5}}{m+5}+\frac{6\,abc^{2}dx^{m+5}}{m+5}+\frac{3\,a^{2}cd^{2}x^{m+5}}{m+5}+\frac{2\,abc^{3}x^{m+3}}{m+3}+\frac{3\,a^{2}c^{2}dx^{m+3}}{m+3}+\frac{a^{2}c^{3}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*d^3*x^(m + 11)/(m + 11) + 3*b^2*c*d^2*x^(m + 9)/(m + 9) + 2*a*b*d^3*x^(
m + 9)/(m + 9) + 3*b^2*c^2*d*x^(m + 7)/(m + 7) + 6*a*b*c*d^2*x^(m + 7)/(m +
 7) + a^2*d^3*x^(m + 7)/(m + 7) + b^2*c^3*x^(m + 5)/(m + 5) + 6*a*b*c^2*d*x
^(m + 5)/(m + 5) + 3*a^2*c*d^2*x^(m + 5)/(m + 5) + 2*a*b*c^3*x^(m + 3)/(m +
 3) + 3*a^2*c^2*d*x^(m + 3)/(m + 3) + a^2*c^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.66, size = 443, normalized size = 2.93

$$\frac{a^{2}\,c^{3}\,x\,x^{m}\,\left(m^{5}+35\,m^{4}+470\,m^{3}+3010\,m^{2}+9129\,m+10395\right)}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}+\frac{c\,x^{m}\,x^{5}\,\left(3\,a^{2}\,d^{2}+6\,a\,b\,c\,d+b^{2}\,c^{2}\right)\,\left(m^{5}+31\,m^{4}+350\,m^{3}+1730\,m^{2}+3489\,m+2079\right)}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}+\frac{d\,x^{m}\,x^{7}\,\left(a^{2}\,d^{2}+6\,a\,b\,c\,d+3\,b^{2}\,c^{2}\right)\,\left(m^{5}+29\,m^{4}+302\,m^{3}+1366\,m^{2}+2577\,m+1485\right)}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}+\frac{b^{2}\,d^{3}\,x^{m}\,x^{11}\,\left(m^{5}+25\,m^{4}+230\,m^{3}+950\,m^{2}+1689\,m+945\right)}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}+\frac{a\,c^{2}\,x^{m}\,x^{3}\,\left(3\,a\,d+2\,b\,c\right)\,\left(m^{5}+33\,m^{4}+406\,m^{3}+2262\,m^{2}+5353\,m+3465\right)}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}+\frac{b\,d^{2}\,x^{m}\,x^{9}\,\left(2\,a\,d+3\,b\,c\right)\,\left(m^{5}+27\,m^{4}+262\,m^{3}+1122\,m^{2}+2041\,m+1155\right)}{m^{6}+36\,m^{5}+505\,m^{4}+3480\,m^{3}+12139\,m^{2}+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] (a^2*c^3*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524
*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (c*x^m*x^5*(3
*a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 + m^5
 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
 + (d*x^m*x^7*(a^2*d^2 + 3*b^2*c^2 + 6*a*b*c*d)*(2577*m + 1366*m^2 + 302*m^
3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^
5 + m^6 + 10395) + (b^2*d^3*x^m*x^11*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 +
 m^5 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 103
95) + (a*c^2*x^m*x^3*(3*a*d + 2*b*c)*(5353*m + 2262*m^2 + 406*m^3 + 33*m^4
+ m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 1
```

```
0395) + (b*d^2*x^m*x^9*(2*a*d + 3*b*c)*(2041*m + 1122*m^2 + 262*m^3 + 27*m^
4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 +
 10395)
```

sympy [A] time = 5.32, size = 4345, normalized size = 28.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((-a**2*c**3/(10*x**10) - 3*a**2*c**2*d/(8*x**8) - a**2*c*d**2/(2*
x**6) - a**2*d**3/(4*x**4) - a*b*c**3/(4*x**8) - a*b*c**2*d/x**6 - 3*a*b*c*
d**2/(2*x**4) - a*b*d**3/x**2 - b**2*c**3/(6*x**6) - 3*b**2*c**2*d/(4*x**4)
 - 3*b**2*c*d**2/(2*x**2) + b**2*d**3*log(x), Eq(m, -11)), (-a**2*c**3/(8*x
**8) - a**2*c**2*d/(2*x**6) - 3*a**2*c*d**2/(4*x**4) - a**2*d**3/(2*x**2) -
 a*b*c**3/(3*x**6) - 3*a*b*c**2*d/(2*x**4) - 3*a*b*c*d**2/x**2 + 2*a*b*d**3
*log(x) - b**2*c**3/(4*x**4) - 3*b**2*c**2*d/(2*x**2) + 3*b**2*c*d**2*log(x
) + b**2*d**3*x**2/2, Eq(m, -9)), (-a**2*c**3/(6*x**6) - 3*a**2*c**2*d/(4*x
**4) - 3*a**2*c*d**2/(2*x**2) + a**2*d**3*log(x) - a*b*c**3/(2*x**4) - 3*a*
b*c**2*d/x**2 + 6*a*b*c*d**2*log(x) + a*b*d**3*x**2 - b**2*c**3/(2*x**2) +
3*b**2*c**2*d*log(x) + 3*b**2*c*d**2*x**2/2 + b**2*d**3*x**4/4, Eq(m, -7)),
 (-a**2*c**3/(4*x**4) - 3*a**2*c**2*d/(2*x**2) + 3*a**2*c*d**2*log(x) + a**
2*d**3*x**2/2 - a*b*c**3/x**2 + 6*a*b*c**2*d*log(x) + 3*a*b*c*d**2*x**2 + a
*b*d**3*x**4/2 + b**2*c**3*log(x) + 3*b**2*c**2*d*x**2/2 + 3*b**2*c*d**2*x*
*4/4 + b**2*d**3*x**6/6, Eq(m, -5)), (-a**2*c**3/(2*x**2) + 3*a**2*c**2*d*l
og(x) + 3*a**2*c*d**2*x**2/2 + a**2*d**3*x**4/4 + 2*a*b*c**3*log(x) + 3*a*b
*c**2*d*x**2 + 3*a*b*c*d**2*x**4/2 + a*b*d**3*x**6/3 + b**2*c**3*x**2/2 + 3
*b**2*c**2*d*x**4/4 + b**2*c*d**2*x**6/2 + b**2*d**3*x**8/8, Eq(m, -3)), (a
**2*c**3*log(x) + 3*a**2*c**2*d*x**2/2 + 3*a**2*c*d**2*x**4/4 + a**2*d**3*x
**6/6 + a*b*c**3*x**2 + 3*a*b*c**2*d*x**4/2 + a*b*c*d**2*x**6 + a*b*d**3*x*
*8/4 + b**2*c**3*x**4/4 + b**2*c**2*d*x**6/2 + 3*b**2*c*d**2*x**8/8 + b**2*
d**3*x**10/10, Eq(m, -1)), (a**2*c**3*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 35*a**2*c**3*m**4*x*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*
a**2*c**3*m**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 3010*a**2*c**3*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*c**3*m*x*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a*
*2*c**3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 3*a**2*c**2*d*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 99*a**2*c**2*d*m**4*x**3*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1218*a*
*2*c**2*d*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**
2 + 19524*m + 10395) + 6786*a**2*c**2*d*m**2*x**3*x**m/(m**6 + 36*m**5 + 50
```

```
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 16059*a**2*c**2*d*m*x*
*3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 103
95) + 10395*a**2*c**2*d*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 3*a**2*c*d**2*m**5*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 93*a**2*c*d**2*m
**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 1050*a**2*c*d**2*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 5190*a**2*c*d**2*m**2*x**5*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 104
67*a**2*c*d**2*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 6237*a**2*c*d**2*x**5*x**m/(m**6 + 36*m**5 + 505*m
**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + a**2*d**3*m**5*x**7*x**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 29
*a**2*d**3*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 302*a**2*d**3*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1366*a**2*d**3*m**2*x**7
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2577*a**2*d**3*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 1485*a**2*d**3*x**7*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*b*c**3*m**5*x**3*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 66*a*b*c**3*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 812*a*b*c**3*m**3*x**3*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4524*a*b*c**3*m**2*x**3
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 10706*a*b*c**3*m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 6930*a*b*c**3*x**3*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6*a*b*c**2*d*m**5*x**5*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 186*a*b*c**2*d*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 2100*a*b*c**2*d*m**3*x**5*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10380*a*b*c**2*d
*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 20934*a*b*c**2*d*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 12474*a*b*c**2*d*x**5*x**m/(m**6 +
 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6*a*b*c*d
**2*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19
524*m + 10395) + 174*a*b*c*d**2*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1812*a*b*c*d**2*m**3*x**7*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8
196*a*b*c*d**2*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 15462*a*b*c*d**2*m*x**7*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8910*a*b*c*d**2*x**7
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2*a*b*d**3*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
```

```
9*m**2 + 19524*m + 10395) + 54*a*b*d**3*m**4*x**9*x**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 524*a*b*d**3*m**3*x**9
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 2244*a*b*d**3*m**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 4082*a*b*d**3*m*x**9*x**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2310*a*b*d**3*x**9*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ b**2*c**3*m**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 31*b**2*c**3*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 350*b**2*c**3*m**3*x**5*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 1730*b**2*c**3*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 3489*b**2*c**3*m*x**5*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2*c**3*x**5
*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395
) + 3*b**2*c**2*d*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 87*b**2*c**2*d*m**4*x**7*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 906*b**2*c**2*d*
m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 4098*b**2*c**2*d*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3
480*m**3 + 12139*m**2 + 19524*m + 10395) + 7731*b**2*c**2*d*m*x**7*x**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4455*
b**2*c**2*d*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 3*b**2*c*d**2*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 81*b**2*c*d**2*m**4*x**9*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
786*b**2*c*d**2*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 3366*b**2*c*d**2*m**2*x**9*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6123*b**2*c*d**2
*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 3465*b**2*c*d**2*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**
3 + 12139*m**2 + 19524*m + 10395) + b**2*d**3*m**5*x**11*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 25*b**2*d**3*m*
*4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 230*b**2*d**3*m**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 950*b**2*d**3*m**2*x**11*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1689*b*
*2*d**3*m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
19524*m + 10395) + 945*b**2*d**3*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

3.323 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=109

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 + m) + ((b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d*(b*c + a*d)*x^(7 +
m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx &= \int \left(a^2c^2x^m+2ac(bc+ad)x^{2+m}+\left(b^2c^2+4abcd+a^2d^2\right)x^{4+m}+2bd(bc+ad)x^{6}\right.\\ &= \frac{a^2c^2x^{1+m}}{1+m}+\frac{2ac(bc+ad)x^{3+m}}{3+m}+\frac{\left(b^2c^2+4abcd+a^2d^2\right)x^{5+m}}{5+m}+\frac{2bd(bc+ad)x^{7}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.93

$$x^m\left(\frac{x^5\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x}{m+1}+\frac{2bdx^7(ad+bc)}{m+7}+\frac{2acx^3(ad+bc)}{m+3}+\frac{b^2d^2x^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] x^m*((a^2*c^2*x)/(1 + m) + (2*a*c*(b*c + a*d)*x^3)/(3 + m) + ((b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^5)/(5 + m) + (2*b*d*(b*c + a*d)*x^7)/(7 + m) + (b^2*d^
2*x^9)/(9 + m))
```

IntegrateAlgebraic [**F**] time = 0.07, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^2*(c + d*x^2)^2, x]
```

fricas [**B**] time = 1.04, size = 442, normalized size = 4.06

$$\frac{\left(\left(b^2d^2m^4+16\,b^2d^2m^3+86\,b^2d^2m^2+176\,b^2d^2m+105\,b^2d^2\right)x^9+2\left(\left(b^2cd+abd^2\right)m^4+135\,b^2cd+135\,abd^2+18\left(b^2cd+abd^2\right)m^3+104\left(b^2cd+abd^2\right)m^2+222\left(b^2cd+abd^2\right)m\right)x^7+\left(\left(b^2c^2+4\,abcd+a^2d^2\right)m^4+189\,b^2c^2+756\,abcd+189\,a^2d^2+20\left(b^2c^2+4\,abcd+a^2d^2\right)m^3+130\left(b^2c^2+4\,abcd+a^2d^2\right)m^2+300\left(b^2c^2+4\,abcd+a^2d^2\right)m\right)x^5+2\left(\left(abc^2+a^2cd\right)m^4+315\,abc^2+315\,a^2cd+22\left(abc^2+a^2cd\right)m^3+164\left(abc^2+a^2cd\right)m^2+458\left(abc^2+a^2cd\right)m\right)x^3+\left(a^2c^2m^4+24\,a^2c^2m^3+206\,a^2c^2m^2+744\,a^2c^2m+945\,a^2c^2\right)x\right)x^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*d^2*m^4 + 16*b^2*d^2*m^3 + 86*b^2*d^2*m^2 + 176*b^2*d^2*m + 105*b^2*d
^2)*x^9 + 2*((b^2*c*d + a*b*d^2)*m^4 + 135*b^2*c*d + 135*a*b*d^2 + 18*(b^2*
c*d + a*b*d^2)*m^3 + 104*(b^2*c*d + a*b*d^2)*m^2 + 222*(b^2*c*d + a*b*d^2)*
m)*x^7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^4 + 189*b^2*c^2 + 756*a*b*c*d +
 189*a^2*d^2 + 20*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^3 + 130*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*m^2 + 300*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m)*x^5 + 2*((a*b
*c^2 + a^2*c*d)*m^4 + 315*a*b*c^2 + 315*a^2*c*d + 22*(a*b*c^2 + a^2*c*d)*m^
3 + 164*(a*b*c^2 + a^2*c*d)*m^2 + 458*(a*b*c^2 + a^2*c*d)*m)*x^3 + (a^2*c^2
*m^4 + 24*a^2*c^2*m^3 + 206*a^2*c^2*m^2 + 744*a^2*c^2*m + 945*a^2*c^2)*x)*x
^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

giac [**B**] time = 0.38, size = 703, normalized size = 6.45

$$\frac{b^2d^2m^4x^9x^m+16\,b^2d^2m^3x^9x^m+2\,b^2cdm^4x^7x^m+2\,abd^2m^4x^7x^m+86\,b^2d^2m^2x^9x^m+36\,b^2cdm^3x^7x^m+36\,abd^2m^3x^7x^m+176\,b^2d^2mx^9x^m+b^2c^2m^4x^5x^m+4\,abcdm^4x^5x^m+a^2d^2m^4x^5x^m+208\,b^2cdm^2x^7x^m+208\,abd^2m^2x^7x^m+105\,b^2d^2x^9x^m+20\,b^2c^2m^3x^5x^m+80\,abcdm^3x^5x^m+20\,a^2d^2m^3x^5x^m+444\,b^2cdmx^7x^m+444\,abd^2mx^7x^m+2\,abc^2m^4x^3x^m+2\,a^2cdm^4x^3x^m+130\,b^2c^2m^2x^5x^m+520\,abcdm^2x^5x^m+130\,a^2d^2m^2x^5x^m+270\,b^2cdx^7x^m+270\,abd^2x^7x^m+44\,abc^2m^3x^3x^m+44\,a^2cdm^3x^3x^m+300\,b^2c^2mx^5x^m+1200\,abcdmx^5x^m+300\,a^2d^2mx^5x^m+a^2c^2m^4xx^m+328\,abc^2m^2x^3x^m+328\,a^2cdm^2x^3x^m+189\,b^2c^2x^5x^m+756\,abcdx^5x^m+189\,a^2d^2x^5x^m+24\,a^2c^2m^3xx^m+916\,abc^2mx^3x^m+916\,a^2cdmx^3x^m+206\,a^2c^2m^2xx^m+630\,abc^2x^3x^m+630\,a^2cdx^3x^m+744\,a^2c^2mxx^m+945\,a^2c^2xx^m}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*m^4*x^9*x^m + 16*b^2*d^2*m^3*x^9*x^m + 2*b^2*c*d*m^4*x^7*x^m + 2*a
*b*d^2*m^4*x^7*x^m + 86*b^2*d^2*m^2*x^9*x^m + 36*b^2*c*d*m^3*x^7*x^m + 36*a
*b*d^2*m^3*x^7*x^m + 176*b^2*d^2*m*x^9*x^m + b^2*c^2*m^4*x^5*x^m + 4*a*b*c*
d*m^4*x^5*x^m + a^2*d^2*m^4*x^5*x^m + 208*b^2*c*d*m^2*x^7*x^m + 208*a*b*d^2
```

```
*m^2*x^7*x^m + 105*b^2*d^2*x^9*x^m + 20*b^2*c^2*m^3*x^5*x^m + 80*a*b*c*d*m^
3*x^5*x^m + 20*a^2*d^2*m^3*x^5*x^m + 444*b^2*c*d*m*x^7*x^m + 444*a*b*d^2*m*
x^7*x^m + 2*a*b*c^2*m^4*x^3*x^m + 2*a^2*c*d*m^4*x^3*x^m + 130*b^2*c^2*m^2*x
^5*x^m + 520*a*b*c*d*m^2*x^5*x^m + 130*a^2*d^2*m^2*x^5*x^m + 270*b^2*c*d*x^
7*x^m + 270*a*b*d^2*x^7*x^m + 44*a*b*c^2*m^3*x^3*x^m + 44*a^2*c*d*m^3*x^3*x
^m + 300*b^2*c^2*m*x^5*x^m + 1200*a*b*c*d*m*x^5*x^m + 300*a^2*d^2*m*x^5*x^m
 + a^2*c^2*m^4*x*x^m + 328*a*b*c^2*m^2*x^3*x^m + 328*a^2*c*d*m^2*x^3*x^m +
189*b^2*c^2*x^5*x^m + 756*a*b*c*d*x^5*x^m + 189*a^2*d^2*x^5*x^m + 24*a^2*c^
2*m^3*x*x^m + 916*a*b*c^2*m*x^3*x^m + 916*a^2*c*d*m*x^3*x^m + 206*a^2*c^2*m
^2*x*x^m + 630*a*b*c^2*x^3*x^m + 630*a^2*c*d*x^3*x^m + 744*a^2*c^2*m*x*x^m
+ 945*a^2*c^2*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

maple [B] time = 0.01, size = 569, normalized size = 5.22

$$\frac{\left(b^{2} d^{2} m^{4} x^{8}+16 b^{2} d^{2} m^{3} x^{8}+2 a b d^{2} m^{4} x^{6}+2 b^{2} c d m^{4} x^{6}+86 b^{2} d^{2} m^{2} x^{8}+36 a b d^{2} m^{3} x^{6}+36 b^{2} c d m^{3} x^{6}+176 b^{2} d^{2} m x^{8}+a^{2} d^{2} m^{4} x^{4}+4 a b c d m^{4} x^{4}+208 a b d^{2} m^{2} x^{6}+b^{2} c^{2} m^{4} x^{4}+208 b^{2} c d m^{2} x^{6}+105 b^{2} d^{2} x^{8}+20 a^{2} d^{2} m^{3} x^{4}+80 a b c d m^{3} x^{4}+444 a b d^{2} m x^{6}+20 b^{2} c^{2} m^{3} x^{4}+444 b^{2} c d m x^{6}+2 a^{2} c d m^{4} x^{2}+130 a^{2} d^{2} m^{2} x^{4}+2 a b c^{2} m^{4} x^{2}+520 a b c d m^{2} x^{4}+270 a b d^{2} x^{6}+130 b^{2} c^{2} m^{2} x^{4}+270 b^{2} c d x^{6}+44 a^{2} c d m^{3} x^{2}+300 a^{2} d^{2} m x^{4}+44 a b c^{2} m^{3} x^{2}+1200 a b c d m x^{4}+300 b^{2} c^{2} m x^{4}+a^{2} c^{2} m^{4}+328 a^{2} c d m^{2} x^{2}+189 a^{2} d^{2} x^{4}+328 a b c^{2} m^{2} x^{2}+756 a b c d x^{4}+189 b^{2} c^{2} x^{4}+24 a^{2} c^{2} m^{3}+916 a^{2} c d m x^{2}+916 a b c^{2} m x^{2}+206 a^{2} c^{2} m^{2}+630 a^{2} c d x^{2}+630 a b c^{2} x^{2}+744 a^{2} c^{2} m+945 a^{2} c^{2}\right) x^{m+1}}{(m+9)(m+7)(m+5)(m+3)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] x^(m+1)*(b^2*d^2*m^4*x^8+16*b^2*d^2*m^3*x^8+2*a*b*d^2*m^4*x^6+2*b^2*c*d*m^4
*x^6+86*b^2*d^2*m^2*x^8+36*a*b*d^2*m^3*x^6+36*b^2*c*d*m^3*x^6+176*b^2*d^2*m
*x^8+a^2*d^2*m^4*x^4+4*a*b*c*d*m^4*x^4+208*a*b*d^2*m^2*x^6+b^2*c^2*m^4*x^4+
208*b^2*c*d*m^2*x^6+105*b^2*d^2*x^8+20*a^2*d^2*m^3*x^4+80*a*b*c*d*m^3*x^4+4
44*a*b*d^2*m*x^6+20*b^2*c^2*m^3*x^4+444*b^2*c*d*m*x^6+2*a^2*c*d*m^4*x^2+130
*a^2*d^2*m^2*x^4+2*a*b*c^2*m^4*x^2+520*a*b*c*d*m^2*x^4+270*a*b*d^2*x^6+130*
b^2*c^2*m^2*x^4+270*b^2*c*d*x^6+44*a^2*c*d*m^3*x^2+300*a^2*d^2*m*x^4+44*a*b
*c^2*m^3*x^2+1200*a*b*c*d*m*x^4+300*b^2*c^2*m*x^4+a^2*c^2*m^4+328*a^2*c*d*m
^2*x^2+189*a^2*d^2*x^4+328*a*b*c^2*m^2*x^2+756*a*b*c*d*x^4+189*b^2*c^2*x^4+
24*a^2*c^2*m^3+916*a^2*c*d*m*x^2+916*a*b*c^2*m*x^2+206*a^2*c^2*m^2+630*a^2*
c*d*x^2+630*a*b*c^2*x^2+744*a^2*c^2*m+945*a^2*c^2)/(m+9)/(m+7)/(m+5)/(m+3)/
(m+1)
```

maxima [A] time = 1.02, size = 153, normalized size = 1.40

$$\frac{b^{2} d^{2} x^{m+9}}{m+9}+\frac{2 \, b^{2} c d x^{m+7}}{m+7}+\frac{2 \, a b d^{2} x^{m+7}}{m+7}+\frac{b^{2} c^{2} x^{m+5}}{m+5}+\frac{4 \, a b c d x^{m+5}}{m+5}+\frac{a^{2} d^{2} x^{m+5}}{m+5}+\frac{2 \, a b c^{2} x^{m+3}}{m+3}+\frac{2 \, a^{2} c d x^{m+3}}{m+3}+\frac{a^{2} c^{2} x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*d^2*x^(m + 9)/(m + 9) + 2*b^2*c*d*x^(m + 7)/(m + 7) + 2*a*b*d^2*x^(m +
7)/(m + 7) + b^2*c^2*x^(m + 5)/(m + 5) + 4*a*b*c*d*x^(m + 5)/(m + 5) + a^2*
d^2*x^(m + 5)/(m + 5) + 2*a*b*c^2*x^(m + 3)/(m + 3) + 2*a^2*c*d*x^(m + 3)/(
m + 3) + a^2*c^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.47, size = 302, normalized size = 2.77

$$\frac{x^m x^5 \left(a^2 d^2 + 4 a b c d + b^2 c^2\right) \left(m^4 + 20 m^3 + 130 m^2 + 300 m + 189\right)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} + \frac{b^2 d^2 x^m x^9 \left(m^4 + 16 m^3 + 86 m^2 + 176 m + 105\right)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} + \frac{a^2 c^2 x x^m \left(m^4 + 24 m^3 + 206 m^2 + 744 m + 945\right)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} + \frac{2 a c x^m x^3 (a d + b c) \left(m^4 + 22 m^3 + 164 m^2 + 458 m + 315\right)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} + \frac{2 b d x^m x^7 (a d + b c) \left(m^4 + 18 m^3 + 104 m^2 + 222 m + 135\right)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] (x^m*x^5*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d)*(300*m + 130*m^2 + 20*m^3 + m^4 +
189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (b^2*d^2*x^m*x^9*
(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4
 + m^5 + 945) + (a^2*c^2*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(168
9*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*a*c*x^m*x^3*(a*d + b*c)*
(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 + 25*m^
4 + m^5 + 945) + (2*b*d*x^m*x^7*(a*d + b*c)*(222*m + 104*m^2 + 18*m^3 + m^4
 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
```

sympy [A] time = 3.19, size = 2363, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((-a**2*c**2/(8*x**8) - a**2*c*d/(3*x**6) - a**2*d**2/(4*x**4) - a
*b*c**2/(3*x**6) - a*b*c*d/x**4 - a*b*d**2/x**2 - b**2*c**2/(4*x**4) - b**2
*c*d/x**2 + b**2*d**2*log(x), Eq(m, -9)), (-a**2*c**2/(6*x**6) - a**2*c*d/(
2*x**4) - a**2*d**2/(2*x**2) - a*b*c**2/(2*x**4) - 2*a*b*c*d/x**2 + 2*a*b*d
**2*log(x) - b**2*c**2/(2*x**2) + 2*b**2*c*d*log(x) + b**2*d**2*x**2/2, Eq(
m, -7)), (-a**2*c**2/(4*x**4) - a**2*c*d/x**2 + a**2*d**2*log(x) - a*b*c**2
/x**2 + 4*a*b*c*d*log(x) + a*b*d**2*x**2 + b**2*c**2*log(x) + b**2*c*d*x**2
 + b**2*d**2*x**4/4, Eq(m, -5)), (-a**2*c**2/(2*x**2) + 2*a**2*c*d*log(x) +
 a**2*d**2*x**2/2 + 2*a*b*c**2*log(x) + 2*a*b*c*d*x**2 + a*b*d**2*x**4/2 +
b**2*c**2*x**2/2 + b**2*c*d*x**4/2 + b**2*d**2*x**6/6, Eq(m, -3)), (a**2*c*
*2*log(x) + a**2*c*d*x**2 + a**2*d**2*x**4/4 + a*b*c**2*x**2 + a*b*c*d*x**4
 + a*b*d**2*x**6/3 + b**2*c**2*x**4/4 + b**2*c*d*x**6/3 + b**2*d**2*x**8/8,
 Eq(m, -1)), (a**2*c**2*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 24*a**2*c**2*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 206*a**2*c**2*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 744*a**2*c**2*m*x*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 945*a**2*c**2*x*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 2*a**2*c*d*m**4*x**3*x**m/(m**5 + 25*m*
*4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a**2*c*d*m**3*x**3*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a**2*c*d*m**2*x**3*x
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a**2*c*d*m*
x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a**2*
```

```
c*d*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + a**2*
d**2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
 20*a**2*d**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 130*a**2*d**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**
2 + 1689*m + 945) + 300*a**2*d**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 +
950*m**2 + 1689*m + 945) + 189*a**2*d**2*x**5*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 2*a*b*c**2*m**4*x**3*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a*b*c**2*m**3*x**3*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*c**2*m**2*x**3*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 916*a*b*c**2*m*x**3
*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 630*a*b*c**2*
x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a*b*c*d
*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 80*
a*b*c*d*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945
) + 520*a*b*c*d*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 1200*a*b*c*d*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 756*a*b*c*d*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 2*a*b*d**2*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 36*a*b*d**2*m**3*x**7*x**m/(m**5 + 25*m**4 + 23
0*m**3 + 950*m**2 + 1689*m + 945) + 208*a*b*d**2*m**2*x**7*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*a*b*d**2*m*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*a*b*d**2*x**7*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*c**2*m**4*x**5*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 20*b**2*c**2*m
**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 130*b
**2*c**2*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
5) + 300*b**2*c**2*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945) + 189*b**2*c**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 2*b**2*c*d*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945) + 36*b**2*c*d*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 208*b**2*c*d*m**2*x**7*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*b**2*c*d*m*x**7*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 270*b**2*c*d*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*d**2*m**4*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*b**2*d**2*m**3*
x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**2*d
**2*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
176*b**2*d**2*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
945) + 105*b**2*d**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689
*m + 945), True))
```

3.324 $\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=71

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{a^2cx^{m+1}}{m+1}+\frac{ax^{m+3}(ad+2bc)}{m+3}+\frac{bx^{m+5}(2ad+bc)}{m+5}+\frac{b^2dx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) + (b*(b*c +
 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx &= \int \left(a^2cx^m+a(2bc+ad)x^{2+m}+b(bc+2ad)x^{4+m}+b^2dx^{6+m}\right) \, dx\\ &= \frac{a^2cx^{1+m}}{1+m}+\frac{a(2bc+ad)x^{3+m}}{3+m}+\frac{b(bc+2ad)x^{5+m}}{5+m}+\frac{b^2dx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2c}{m+1}+\frac{bx^4(2ad+bc)}{m+5}+\frac{ax^2(ad+2bc)}{m+3}+\frac{b^2dx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] x^(1 + m)*((a^2*c)/(1 + m) + (a*(2*b*c + a*d)*x^2)/(3 + m) + (b*(b*c + 2*a*
d)*x^4)/(5 + m) + (b^2*d*x^6)/(7 + m))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x^2)^2*(c + d*x^2), x]
```

fricas [B] time = 1.03, size = 215, normalized size = 3.03

$$\frac{\left(\left(b^2dm^3+9\,b^2dm^2+23\,b^2dm+15\,b^2d\right)x^7+\left(\left(b^2c+2\,abd\right)m^3+21\,b^2c+42\,abd+11\,\left(b^2c+2\,abd\right)m^2+31\,\left(b^2c+2\,abd\right)m\right)x^5+\left(\left(2\,abc+a^2d\right)m^3+70\,abc+35\,a^2d+13\,\left(2\,abc+a^2d\right)m^2+47\,\left(2\,abc+a^2d\right)m\right)x^3+\left(a^2cm^3+15\,a^2cm^2+71\,a^2cm+105\,a^2c\right)x\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] ((b^2*d*m^3 + 9*b^2*d*m^2 + 23*b^2*d*m + 15*b^2*d)*x^7 + ((b^2*c + 2*a*b*d)
*m^3 + 21*b^2*c + 42*a*b*d + 11*(b^2*c + 2*a*b*d)*m^2 + 31*(b^2*c + 2*a*b*d
)*m)*x^5 + ((2*a*b*c + a^2*d)*m^3 + 70*a*b*c + 35*a^2*d + 13*(2*a*b*c + a^2
*d)*m^2 + 47*(2*a*b*c + a^2*d)*m)*x^3 + (a^2*c*m^3 + 15*a^2*c*m^2 + 71*a^2*
c*m + 105*a^2*c)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

giac [B] time = 0.43, size = 332, normalized size = 4.68

$$\frac{b^2dm^3x^7x^m+9\,b^2dm^2x^7x^m+b^2cm^3x^5x^m+2\,abdm^3x^5x^m+23\,b^2dmx^7x^m+11\,b^2cm^2x^5x^m+22\,abdm^2x^5x^m+15\,b^2dx^7x^m+2\,abcm^3x^3x^m+a^2dm^3x^3x^m+31\,b^2cmx^5x^m+62\,abdmx^5x^m+26\,abcm^2x^3x^m+13\,a^2dm^2x^3x^m+21\,b^2cx^5x^m+42\,abdx^5x^m+a^2cm^3xx^m+94\,abcmx^3x^m+47\,a^2dmx^3x^m+15\,a^2cm^2xx^m+70\,abcx^3x^m+35\,a^2dx^3x^m+71\,a^2cmxx^m+105\,a^2cxx^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*d*m^3*x^7*x^m + 9*b^2*d*m^2*x^7*x^m + b^2*c*m^3*x^5*x^m + 2*a*b*d*m^3*
x^5*x^m + 23*b^2*d*m*x^7*x^m + 11*b^2*c*m^2*x^5*x^m + 22*a*b*d*m^2*x^5*x^m
+ 15*b^2*d*x^7*x^m + 2*a*b*c*m^3*x^3*x^m + a^2*d*m^3*x^3*x^m + 31*b^2*c*m*x
^5*x^m + 62*a*b*d*m*x^5*x^m + 26*a*b*c*m^2*x^3*x^m + 13*a^2*d*m^2*x^3*x^m +
 21*b^2*c*x^5*x^m + 42*a*b*d*x^5*x^m + a^2*c*m^3*x*x^m + 94*a*b*c*m*x^3*x^m
 + 47*a^2*d*m*x^3*x^m + 15*a^2*c*m^2*x*x^m + 70*a*b*c*x^3*x^m + 35*a^2*d*x^
3*x^m + 71*a^2*c*m*x*x^m + 105*a^2*c*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)
```

maple [B] time = 0.01, size = 262, normalized size = 3.69

$$\frac{\left(b^2d\,m^3x^6+9b^2d\,m^2x^6+2abd\,m^3x^4+b^2c\,m^3x^4+23b^2dm\,x^6+22abd\,m^2x^4+11b^2c\,m^2x^4+15b^2d\,x^6+a^2d\,m^3x^2+2abc\,m^3x^2+62abdm\,x^4+31b^2cm\,x^4+13a^2d\,m^2x^2+26abc\,m^2x^2+42abd\,x^4+21b^2c\,x^4+a^2c\,m^3+47a^2dm\,x^2+94abcm\,x^2+15a^2c\,m^2+35a^2d\,x^2+70abc\,x^2+71a^2cm+105a^2c\right)x^{m+1}}{(m+7)\,(m+5)\,(m+3)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] x^(m+1)*(b^2*d*m^3*x^6+9*b^2*d*m^2*x^6+2*a*b*d*m^3*x^4+b^2*c*m^3*x^4+23*b^2
*d*m*x^6+22*a*b*d*m^2*x^4+11*b^2*c*m^2*x^4+15*b^2*d*x^6+a^2*d*m^3*x^2+2*a*b
*c*m^3*x^2+62*a*b*d*m*x^4+31*b^2*c*m*x^4+13*a^2*d*m^2*x^2+26*a*b*c*m^2*x^2+
42*a*b*d*x^4+21*b^2*c*x^4+a^2*c*m^3+47*a^2*d*m*x^2+94*a*b*c*m*x^2+15*a^2*c*
m^2+35*a^2*d*x^2+70*a*b*c*x^2+71*a^2*c*m+105*a^2*c)/(m+7)/(m+5)/(m+3)/(m+1)
```

maxima [A] time = 0.95, size = 91, normalized size = 1.28

$$\frac{b^2dx^{m+7}}{m+7}+\frac{b^2cx^{m+5}}{m+5}+\frac{2\,abdx^{m+5}}{m+5}+\frac{2\,abcx^{m+3}}{m+3}+\frac{a^2dx^{m+3}}{m+3}+\frac{a^2cx^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b^2*d*x^(m + 7)/(m + 7) + b^2*c*x^(m + 5)/(m + 5) + 2*a*b*d*x^(m + 5)/(m +
5) + 2*a*b*c*x^(m + 3)/(m + 3) + a^2*d*x^(m + 3)/(m + 3) + a^2*c*x^(m + 1)/
(m + 1)
```

mupad [B] time = 0.36, size = 177, normalized size = 2.49

$$x^m\,\left(\frac{a\,x^3\,\left(a\,d+2\,b\,c\right)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b\,x^5\,\left(2\,a\,d+b\,c\right)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^2\,d\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a^2\,c\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^m*((a*x^3*(a*d + 2*b*c)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105) + (b*x^5*(2*a*d + b*c)*(31*m + 11*m^2 + m^3 + 21))/(176*m
+ 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*d*x^7*(23*m + 9*m^2 + m^3 + 15))/(176
*m + 86*m^2 + 16*m^3 + m^4 + 105) + (a^2*c*x*(71*m + 15*m^2 + m^3 + 105))/(
176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

sympy [A] time = 1.76, size = 1044, normalized size = 14.70

$$\begin{cases} -\frac{a^2c}{6x^6}-\frac{a^2d}{4x^4}-\frac{abc}{2x^4}-\frac{abd}{x^2}-\frac{b^2c}{2x^2}+b^2d\log{(x)} & \text{for}\: m=-7 \\ -\frac{a^2c}{4x^4}-\frac{a^2d}{2x^2}-\frac{abc}{x^2}+2abd\log{(x)}+b^2c\log{(x)}+\frac{b^2dx^2}{2} & \text{for}\: m=-5 \\ -\frac{a^2c}{2x^2}+a^2d\log{(x)}+2abc\log{(x)}+abdx^2+\frac{b^2cx^2}{2}+\frac{b^2dx^4}{4} & \text{for}\: m=-3 \\ a^2c\log{(x)}+\frac{a^2dx^2}{2}+abcx^2+\frac{abdx^4}{2}+\frac{b^2cx^4}{4}+\frac{b^2dx^6}{6} & \text{for}\: m=-1 \\ \frac{a^2cm^3xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{15a^2cm^2xx^m}{m^4+16m^3+86m^2+176m+105}+\frac{71a^2cmxx^m}{m^4+16m^3+86m^2+176m+105}+\frac{105a^2cxx^m}{m^4+16m^3+86m^2+176m+105}+\frac{a^2dm^3x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{13a^2dm^2x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{47a^2dmx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{35a^2dx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{2abcm^3x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{26abcm^2x^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{94abcmx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{70abcx^3x^m}{m^4+16m^3+86m^2+176m+105}+\frac{2abdm^3x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{22abdm^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{62abdmx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{42abdx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{b^2cm^3x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{11b^2cm^2x^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{31b^2cmx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{21b^2cx^5x^m}{m^4+16m^3+86m^2+176m+105}+\frac{b^2dm^3x^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{9b^2dm^2x^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{23b^2dmx^7x^m}{m^4+16m^3+86m^2+176m+105}+\frac{15b^2dx^7x^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] Piecewise((-a**2*c/(6*x**6) - a**2*d/(4*x**4) - a*b*c/(2*x**4) - a*b*d/x**2
 - b**2*c/(2*x**2) + b**2*d*log(x), Eq(m, -7)), (-a**2*c/(4*x**4) - a**2*d/
```

```
(2*x**2) - a*b*c/x**2 + 2*a*b*d*log(x) + b**2*c*log(x) + b**2*d*x**2/2, Eq(
m, -5)), (-a**2*c/(2*x**2) + a**2*d*log(x) + 2*a*b*c*log(x) + a*b*d*x**2 +
b**2*c*x**2/2 + b**2*d*x**4/4, Eq(m, -3)), (a**2*c*log(x) + a**2*d*x**2/2 +
 a*b*c*x**2 + a*b*d*x**4/2 + b**2*c*x**4/4 + b**2*d*x**6/6, Eq(m, -1)), (a*
*2*c*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*a**2*c*m**2*
x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*a**2*c*m*x*x**m/(m**4
+ 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**2*c*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + a**2*d*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + 13*a**2*d*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + 47*a**2*d*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a
**2*d*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*a*b*c*m**3*x**
3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*a*b*c*m**2*x**3*x**m/(
m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*a*b*c*m*x**3*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 70*a*b*c*x**3*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + 2*a*b*d*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 22*a*b*d*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 62*a*b*d*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 42*a*b*d*x
**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**2*c*m**3*x**5*x**m/(
m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b**2*c*m**2*x**5*x**m/(m**4 +
16*m**3 + 86*m**2 + 176*m + 105) + 31*b**2*c*m*x**5*x**m/(m**4 + 16*m**3 +
86*m**2 + 176*m + 105) + 21*b**2*c*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + b**2*d*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105)
 + 9*b**2*d*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*b*
*2*d*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*b**2*d*x**7*
x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

3.325 $\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{17}bBx^{17/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aAx^{7/2}+(Ab+aB)x^{11/2}+bBx^{15/2}\right)\,dx\\ &= \frac{2}{9}aAx^{9/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{9/2}\left(153x^2(aB+Ab)+221aA+117bBx^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(221*a*A + 153*(A*b + a*B)*x^2 + 117*b*B*x^4))/1989
```

IntegrateAlgebraic [**A**] time = 0.02, size = 41, normalized size = 1.05

$$\frac{2\left(221aAx^{9/2}+153aBx^{13/2}+153Abx^{13/2}+117bBx^{17/2}\right)}{1989}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*(221*a*A*x^(9/2) + 153*A*b*x^(13/2) + 153*a*B*x^(13/2) + 117*b*B*x^(17/2
)))/1989
```

fricas [**A**] time = 1.02, size = 32, normalized size = 0.82

$$\frac{2}{1989}\left(117\,Bbx^{8}+153\,(Ba+Ab)x^{6}+221\,Aax^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*B*b*x^8 + 153*(B*a + A*b)*x^6 + 221*A*a*x^4)*sqrt(x)
```

giac [**A**] time = 0.35, size = 29, normalized size = 0.74

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{13}\,Bax^{\frac{13}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/9*A*a*x^(9/2)
```

maple [**A**] time = 0.01, size = 32, normalized size = 0.82

$$\frac{2\left(117Bb\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)x^{\frac{9}{2}}}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/1989*x^(9/2)*(117*B*b*x^4+153*A*b*x^2+153*B*a*x^2+221*A*a)
```

maxima [**A**] time = 1.05, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/9*A*a*x^(9/2)
```

mupad [B] time = 0.20, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(221\,A\,a+153\,A\,b\,x^2+153\,B\,a\,x^2+117\,B\,b\,x^4\right)}{1989}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(9/2)*(221*A*a + 153*A*b*x^2 + 153*B*a*x^2 + 117*B*b*x^4))/1989
```

sympy [A] time = 10.21, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(17/2
)/17
```

3.326 $\int x^{5/2} \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=39

$$\frac{2}{11} x^{11/2} (aB+Ab)+\frac{2}{7} aAx^{7/2}+\frac{2}{15} bBx^{15/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{11} x^{11/2} (aB+Ab)+\frac{2}{7} aAx^{7/2}+\frac{2}{15} bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2} \left(a+bx^2\right) \left(A+Bx^2\right) \, dx &= \int \left(aAx^{5/2}+(Ab+aB)x^{9/2}+bBx^{13/2}\right) \, dx\\ &= \frac{2}{7} aAx^{7/2}+\frac{2}{11} (Ab+aB)x^{11/2}+\frac{2}{15} bBx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{2x^{7/2} \left(105x^2 (aB+Ab)+165aA+77bBx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(7/2)*(165*a*A + 105*(A*b + a*B)*x^2 + 77*b*B*x^4))/1155
```

IntegrateAlgebraic [A]   time = 0.02, size = 41, normalized size = 1.05

$$\frac{2\left(165aAx^{7/2}+105aBx^{11/2}+105Abx^{11/2}+77bBx^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*(165*a*A*x^(7/2) + 105*A*b*x^(11/2) + 105*a*B*x^(11/2) + 77*b*B*x^(15/2)
))/1155
```

fricas [A]   time = 0.71, size = 32, normalized size = 0.82

$$\frac{2}{1155}\left(77\,Bbx^7+105\,(Ba+Ab)x^5+165\,Aax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b*x^7 + 105*(B*a + A*b)*x^5 + 165*A*a*x^3)*sqrt(x)
```

giac [A]   time = 0.32, size = 29, normalized size = 0.74

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/7*A*a*x^(7/2)
```

maple [A]   time = 0.01, size = 32, normalized size = 0.82

$$\frac{2\left(77Bb\,x^4+105Ab\,x^2+105Ba\,x^2+165Aa\right)x^{\frac{7}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/1155*x^(7/2)*(77*B*b*x^4+105*A*b*x^2+105*B*a*x^2+165*A*a)
```

maxima [A]   time = 1.06, size = 27, normalized size = 0.69

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/7*A*a*x^(7/2)
```

mupad [B] time = 0.19, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(165\,A\,a+105\,A\,b\,x^2+105\,B\,a\,x^2+77\,B\,b\,x^4\right)}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(7/2)*(165*A*a + 105*A*b*x^2 + 105*B*a*x^2 + 77*B*b*x^4))/1155
```

sympy [A] time = 5.30, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(15/2
)/15
```

3.327 $\int x^{3/2} \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{13}bBx^{13/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aAx^{3/2}+(Ab+aB)x^{7/2}+bBx^{11/2}\right)\,dx\\ &= \frac{2}{5}aAx^{5/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{2}{585}x^{5/2}\left(65x^2(aB+Ab)+117aA+45bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(117*a*A + 65*(A*b + a*B)*x^2 + 45*b*B*x^4))/585
```

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.05

$$\frac{2}{585}\left(117aAx^{5/2}+65aBx^{9/2}+65Abx^{9/2}+45bBx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*(117*a*A*x^(5/2) + 65*A*b*x^(9/2) + 65*a*B*x^(9/2) + 45*b*B*x^(13/2)))/5
85
```

fricas [A] time = 0.99, size = 32, normalized size = 0.82

$$\frac{2}{585}\left(45\,Bbx^{6}+65\,(Ba+Ab)x^{4}+117\,Aax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b*x^6 + 65*(B*a + A*b)*x^4 + 117*A*a*x^2)*sqrt(x)
```

giac [A] time = 0.34, size = 29, normalized size = 0.74

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/5*A*a*x^(5/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.82

$$\frac{2\left(45Bb\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)x^{\frac{5}{2}}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)*(B*x^2+A),x)
```

```
[Out] 2/585*x^(5/2)*(45*B*b*x^4+65*A*b*x^2+65*B*a*x^2+117*A*a)
```

maxima [A] time = 1.09, size = 27, normalized size = 0.69

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/13*B*b*x^(13/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/5*A*a*x^(5/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(117\,A\,a+65\,A\,b\,x^2+65\,B\,a\,x^2+45\,B\,b\,x^4\right)}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(5/2)*(117*A*a + 65*A*b*x^2 + 65*B*a*x^2 + 45*B*b*x^4))/585
```

sympy [A] time = 2.41, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)*(B*x**2+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(13/2)/13
```

3.328 $\int \sqrt{x}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{5/2}+bBx^{9/2}\right)dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{2}{231}x^{3/2}\left(33x^2(aB+Ab)+77aA+21bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(77*a*A + 33*(A*b + a*B)*x^2 + 21*b*B*x^4))/231
```

IntegrateAlgebraic [A]   time = 0.02, size = 41, normalized size = 1.05

$$\frac{2}{231}\left(77aAx^{3/2}+33aBx^{7/2}+33Abx^{7/2}+21bBx^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

```
[Out] (2*(77*a*A*x^(3/2) + 33*A*b*x^(7/2) + 33*a*B*x^(7/2) + 21*b*B*x^(11/2)))/23
1
```

fricas [A]   time = 1.02, size = 30, normalized size = 0.77

$$\frac{2}{231}\left(21\,Bbx^{5}+33\,(Ba+Ab)x^{3}+77\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b*x^5 + 33*(B*a + A*b)*x^3 + 77*A*a*x)*sqrt(x)
```

giac [A]   time = 0.30, size = 29, normalized size = 0.74

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{7}\,Bax^{\frac{7}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2/3*A*a*x^(3/2)
```

maple [A]   time = 0.00, size = 32, normalized size = 0.82

$$\frac{2\left(21Bb\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa\right)x^{\frac{3}{2}}}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/231*x^(3/2)*(21*B*b*x^4+33*A*b*x^2+33*B*a*x^2+77*A*a)
```

maxima [A]   time = 0.95, size = 27, normalized size = 0.69

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b*x^(11/2) + 2/7*(B*a + A*b)*x^(7/2) + 2/3*A*a*x^(3/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(77\,A\,a+33\,A\,b\,x^2+33\,B\,a\,x^2+21\,B\,b\,x^4\right)}{231}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(a + b*x^2),x)
```

```
[Out] (2*x^(3/2)*(77*A*a + 33*A*b*x^2 + 33*B*a*x^2 + 21*B*b*x^4))/231
```

sympy [A] time = 1.99, size = 37, normalized size = 0.95

$$\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{11}{2}}}{11}+\frac{2x^{\frac{7}{2}}\,(Ab+Ba)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*B*b*x**(11/2)/11 + 2*x**(7/2)*(A*b + B*a)/7
```

3.329 $\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)\left(A+Bx^2\right)}{\sqrt{x}}\,dx &= \int \left(\frac{aA}{\sqrt{x}}+(Ab+aB)x^{3/2}+bBx^{7/2}\right)dx\\ &= 2aA\sqrt{x}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{9}bBx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$\frac{2}{45}\sqrt{x}\left(9x^2(aB+Ab)+45aA+5bBx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a*A + 9*(A*b + a*B)*x^2 + 5*b*B*x^4))/45
```

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.11

$$\frac{2}{45}\left(45aA\sqrt{x}+9aBx^{5/2}+9Abx^{5/2}+5bBx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
[Out] (2*(45*a*A*Sqrt[x] + 9*A*b*x^(5/2) + 9*a*B*x^(5/2) + 5*b*B*x^(9/2)))/45
```

fricas [A] time = 0.83, size = 29, normalized size = 0.78

$$\frac{2}{45}\left(5\,Bbx^4+9\,(Ba+Ab)x^2+45\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
[Out] 2/45*(5*B*b*x^4 + 9*(B*a + A*b)*x^2 + 45*A*a)*sqrt(x)
```

giac [A] time = 0.31, size = 29, normalized size = 0.78

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="giac")
[Out] 2/9*B*b*x^(9/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) + 2*A*a*sqrt(x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$\frac{2\left(5Bb\,x^4+9Ab\,x^2+9Ba\,x^2+45Aa\right)\sqrt{x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(1/2),x)
[Out] 2/45*x^(1/2)*(5*B*b*x^4+9*A*b*x^2+9*B*a*x^2+45*A*a)
```

maxima [A] time = 1.06, size = 27, normalized size = 0.73

$$\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/5*(B*a + A*b)*x^(5/2) + 2*A*a*sqrt(x)
```

mupad [B] time = 0.19, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(45\,A\,a+9\,A\,b\,x^2+9\,B\,a\,x^2+5\,B\,b\,x^4\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(45*A*a + 9*A*b*x^2 + 9*B*a*x^2 + 5*B*b*x^4))/45
```

sympy [A] time = 0.78, size = 44, normalized size = 1.19

$$2Aa\sqrt{x} + \frac{2Abx^{\frac{5}{2}}}{5} + \frac{2Bax^{\frac{5}{2}}}{5} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(9/2)/9
```

3.330 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)\sqrt{x}+bBx^{5/2}\right)dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$\frac{2\left(-21aA+7aBx^2+7Abx^2+3bBx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-21*a*A + 7*A*b*x^2 + 7*a*B*x^2 + 3*b*B*x^4))/(21*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.95

$$\frac{2\left(-21aA+7aBx^2+7Abx^2+3bBx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-21*a*A + 7*A*b*x^2 + 7*a*B*x^2 + 3*b*B*x^4))/(21*Sqrt[x])
```

fricas [A] time = 1.01, size = 29, normalized size = 0.78

$$\frac{2\left(3\,Bbx^4+7\,(Ba+Ab)x^2-21\,Aa\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*B*b*x^4 + 7*(B*a + A*b)*x^2 - 21*A*a)/sqrt(x)
```

giac [A] time = 0.35, size = 29, normalized size = 0.78

$$\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{3}\,Bax^{\frac{3}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b*x^(7/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2*A*a/sqrt(x)
```

maple [A] time = 0.01, size = 32, normalized size = 0.86

$$-\frac{2\left(-3Bb\,x^4-7Ab\,x^2-7Ba\,x^2+21Aa\right)}{21\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/21*(-3*B*b*x^4-7*A*b*x^2-7*B*a*x^2+21*A*a)/x^(1/2)
```

maxima [A] time = 1.10, size = 27, normalized size = 0.73

$$\frac{2}{7}Bbx^{\frac{7}{2}} + \frac{2}{3}(Ba + Ab)x^{\frac{3}{2}} - \frac{2Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b*x^(7/2) + 2/3*(B*a + A*b)*x^(3/2) - 2*A*a/sqrt(x)
```

mupad [B] time = 0.19, size = 31, normalized size = 0.84

$$\frac{14\,A\,b\,x^2 - 42\,A\,a + 14\,B\,a\,x^2 + 6\,B\,b\,x^4}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(3/2),x)
```

```
[Out] (14*A*b*x^2 - 42*A*a + 14*B*a*x^2 + 6*B*b*x^4)/(21*x^(1/2))
```

sympy [A] time = 0.99, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(7/2)/7
```

3.331 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$2\sqrt{x}\,(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$2\sqrt{x}\,(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{3/2}\right)dx\\ &= -\frac{2aA}{3x^{3/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{5}bBx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.97

$$\frac{2\left(3bx^2\left(5A+Bx^2\right)-5a\left(A-3Bx^2\right)\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-5*a*(A - 3*B*x^2) + 3*b*x^2*(5*A + B*x^2)))/(15*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.95

$$\frac{2\left(-5aA+15aBx^2+15Abx^2+3bBx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
[Out] (2*(-5*a*A + 15*A*b*x^2 + 15*a*B*x^2 + 3*b*B*x^4))/(15*x^(3/2))
```

fricas [A] time = 1.04, size = 29, normalized size = 0.78

$$\frac{2\left(3\,Bbx^4+15\,(Ba+Ab)x^2-5\,Aa\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
[Out] 2/15*(3*B*b*x^4 + 15*(B*a + A*b)*x^2 - 5*A*a)/x^(3/2)
```

giac [A] time = 0.33, size = 29, normalized size = 0.78

$$\frac{2}{5}\,Bbx^{\frac{5}{2}}+2\,Ba\sqrt{x}+2\,Ab\sqrt{x}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="giac")
[Out] 2/5*B*b*x^(5/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/3*A*a/x^(3/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$-\frac{2\left(-3Bb\,x^4-15Ab\,x^2-15Ba\,x^2+5Aa\right)}{15x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(5/2),x)
[Out] -2/15*(-3*B*b*x^4-15*A*b*x^2-15*B*a*x^2+5*A*a)/x^(3/2)
```

maxima [A] time = 1.03, size = 27, normalized size = 0.73

$$\frac{2}{5}\,Bbx^{\frac{5}{2}}+2\,(Ba+Ab)\sqrt{x}-\frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*B*b*x^(5/2) + 2*(B*a + A*b)*sqrt(x) - 2/3*A*a/x^(3/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.84

$$\frac{30\,A\,b\,x^2-10\,A\,a+30\,B\,a\,x^2+6\,B\,b\,x^4}{15\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(5/2),x)
```

```
[Out] (30*A*b*x^2 - 10*A*a + 30*B*a*x^2 + 6*B*b*x^4)/(15*x^(3/2))
```

sympy [A] time = 1.19, size = 42, normalized size = 1.14

$$-\frac{2Aa}{3x^{\frac{3}{2}}}+2Ab\sqrt{x}+2Ba\sqrt{x}+\frac{2Bbx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(5/2)/5
```

3.332 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/3
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx &= \int \left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{x^{3/2}}+bB\sqrt{x}\right)dx\\ &= -\frac{2aA}{5x^{5/2}}-\frac{2(Ab+aB)}{\sqrt{x}}+\frac{2}{3}bBx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.97

$$\frac{10bx^2\left(Bx^2-3A\right)-6a\left(A+5Bx^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (10*b*x^2*(-3*A + B*x^2) - 6*a*(A + 5*B*x^2))/(15*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.95

$$\frac{2\left(-3aA-15aBx^2-15Abx^2+5bBx^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
[Out] (2*(-3*a*A - 15*A*b*x^2 - 15*a*B*x^2 + 5*b*B*x^4))/(15*x^(5/2))
```

fricas [A] time = 0.85, size = 29, normalized size = 0.78

$$\frac{2\left(5\,Bbx^4-15\,(Ba+Ab)x^2-3\,Aa\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(5*B*b*x^4 - 15*(B*a + A*b)*x^2 - 3*A*a)/x^(5/2)
```

giac [A] time = 0.37, size = 31, normalized size = 0.84

$$\frac{2}{3}\,Bbx^{\frac{3}{2}}-\frac{2\left(5\,Bax^2+5\,Abx^2+Aa\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="giac")
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*B*a*x^2 + 5*A*b*x^2 + A*a)/x^(5/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.86

$$-\frac{2\left(-5Bb\,x^4+15Ab\,x^2+15Ba\,x^2+3Aa\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(B*x^2+A)/x^(7/2),x)
[Out] -2/15*(-5*B*b*x^4+15*A*b*x^2+15*B*a*x^2+3*A*a)/x^(5/2)
```

maxima [A] time = 1.04, size = 29, normalized size = 0.78

$$\frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\left(5\,(Ba + Ab)x^2 + Aa\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*b*x^(3/2) - 2/5*(5*(B*a + A*b)*x^2 + A*a)/x^(5/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.84

$$-\frac{6\,A\,a + 30\,A\,b\,x^2 + 30\,B\,a\,x^2 - 10\,B\,b\,x^4}{15\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2))/x^(7/2),x)
```

```
[Out] -(6*A*a + 30*A*b*x^2 + 30*B*a*x^2 - 10*B*b*x^4)/(15*x^(5/2))
```

sympy [A] time = 1.76, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}} - \frac{2Ab}{\sqrt{x}} - \frac{2Ba}{\sqrt{x}} + \frac{2Bbx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a/(5*x**(5/2)) - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + 2*B*b*x**(3/2)/3
```

3.333 $\int x^{7/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{17}bx^{17/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)dx &= \int \left(a^2Ax^{7/2}+a(2Ab+aB)x^{11/2}+b(Ab+2aB)x^{15/2}+b^2Bx^{19/2}\right)dx\\ &= \frac{2}{9}a^2Ax^{9/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547a^2A+819bx^4(2aB+Ab)+1071ax^2(aB+2Ab)+663b^2Bx^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(9/2)*(1547*a^2*A + 1071*a*(2*A*b + a*B)*x^2 + 819*b*(A*b + 2*a*B)*x^4
 + 663*b^2*B*x^6))/13923
```

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 1.10

$$\frac{2\left(1547a^2Ax^{9/2}+1071a^2Bx^{13/2}+2142aAbx^{13/2}+1638abBx^{17/2}+819Ab^2x^{17/2}+663b^2Bx^{21/2}\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*(1547*a^2*A*x^(9/2) + 2142*a*A*b*x^(13/2) + 1071*a^2*B*x^(13/2) + 819*A*
b^2*x^(17/2) + 1638*a*b*B*x^(17/2) + 663*b^2*B*x^(21/2)))/13923
```

fricas [A] time = 1.07, size = 56, normalized size = 0.89

$$\frac{2}{13923}\left(663\,Bb^2x^{10}+819\left(2\,Bab+Ab^2\right)x^8+1547\,Aa^2x^4+1071\left(Ba^2+2\,Aab\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*B*b^2*x^10 + 819*(2*B*a*b + A*b^2)*x^8 + 1547*A*a^2*x^4 + 1071
*(B*a^2 + 2*A*a*b)*x^6)*sqrt(x)
```

giac [A] time = 0.28, size = 53, normalized size = 0.84

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/9*A*a^2*x^(9/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(663B\,b^2x^6+819A\,b^2x^4+1638Bab\,x^4+2142Aab\,x^2+1071B\,a^2x^2+1547a^2A\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/13923*x^(9/2)*(663*B*b^2*x^6+819*A*b^2*x^4+1638*B*a*b*x^4+2142*A*a*b*x^2+
1071*B*a^2*x^2+1547*A*a^2)
```

maxima [A] time = 1.16, size = 51, normalized size = 0.81

$$\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{2}{17}\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}+\frac{2}{13}\left(Ba^2+2\,Aab\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/9*A*a^2*x^(9/2) +
 2/13*(B*a^2 + 2*A*a*b)*x^(13/2)
```

mupad [B] time = 0.21, size = 51, normalized size = 0.81

$$x^{13/2}\,\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{17/2}\,\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b)
/17) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(21/2))/21
```

sympy [A] time = 19.43, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(21/2)/21
```

3.334 $\int x^{5/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)dx &= \int \left(a^2Ax^{5/2}+a(2Ab+aB)x^{9/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{17/2}\right)dx\\ &= \frac{2}{7}a^2Ax^{7/2}+\frac{2}{11}a(2Ab+aB)x^{11/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{19}b^2Bx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 1.10

$$\frac{2\left(3135a^2Ax^{7/2}+1995a^2Bx^{11/2}+3990aAbx^{11/2}+2926abBx^{15/2}+1463Ab^2x^{15/2}+1155b^2Bx^{19/2}\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*(3135*a^2*A*x^(7/2) + 3990*a*A*b*x^(11/2) + 1995*a^2*B*x^(11/2) + 1463*A
*b^2*x^(15/2) + 2926*a*b*B*x^(15/2) + 1155*b^2*B*x^(19/2)))/21945
```

fricas [A] time = 1.07, size = 56, normalized size = 0.89

$$\frac{2}{21945}\left(1155\,Bb^2x^9+1463\left(2\,Bab+Ab^2\right)x^7+3135\,Aa^2x^3+1995\left(Ba^2+2\,Aab\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^2*x^9 + 1463*(2*B*a*b + A*b^2)*x^7 + 3135*A*a^2*x^3 + 199
5*(B*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

giac [A] time = 0.29, size = 53, normalized size = 0.84

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{15}\,Babx^{\frac{15}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/7*A*a^2*x^(7/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(1155B\,b^2x^6+1463A\,b^2x^4+2926Bab\,x^4+3990Aab\,x^2+1995B\,a^2x^2+3135a^2A\right)x^{\frac{7}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*B*b^2*x^6+1463*A*b^2*x^4+2926*B*a*b*x^4+3990*A*a*b*x^
2+1995*B*a^2*x^2+3135*A*a^2)
```

maxima [A] time = 1.10, size = 51, normalized size = 0.81

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{2}{15}\left(2\,Bab+Ab^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}+\frac{2}{11}\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/7*A*a^2*x^(7/2) +
 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\,\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{15/2}\,\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)
/15) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(19/2))/19
```

sympy [A] time = 10.85, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(15/2)/15 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(19/2)/19
```

3.335 $\int x^{3/2} \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx &= \int \left(a^2Ax^{3/2}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{11/2}+b^2Bx^{15/2}\right)\,dx\\ &= \frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989a^2A+765bx^4(2aB+Ab)+1105ax^2(aB+2Ab)+585b^2Bx^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(5/2)*(1989*a^2*A + 1105*a*(2*A*b + a*B)*x^2 + 765*b*(A*b + 2*a*B)*x^4
 + 585*b^2*B*x^6))/9945
```

IntegrateAlgebraic [**A**] time = 0.03, size = 69, normalized size = 1.10

$$\frac{2\left(1989a^2Ax^{5/2}+1105a^2Bx^{9/2}+2210aAbx^{9/2}+1530abBx^{13/2}+765Ab^2x^{13/2}+585b^2Bx^{17/2}\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*(1989*a^2*A*x^(5/2) + 2210*a*A*b*x^(9/2) + 1105*a^2*B*x^(9/2) + 765*A*b^
2*x^(13/2) + 1530*a*b*B*x^(13/2) + 585*b^2*B*x^(17/2)))/9945
```

fricas [**A**] time = 1.33, size = 56, normalized size = 0.89

$$\frac{2}{9945}\left(585\,Bb^2x^8+765\left(2\,Bab+Ab^2\right)x^6+1989\,Aa^2x^2+1105\left(Ba^2+2\,Aab\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*B*b^2*x^8 + 765*(2*B*a*b + A*b^2)*x^6 + 1989*A*a^2*x^2 + 1105*(
B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

giac [**A**] time = 0.31, size = 53, normalized size = 0.84

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{9}\,Ba^2x^{\frac{9}{2}}+\frac{4}{9}\,Aabx^{\frac{9}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/5*A*a^2*x^(5/2)
```

maple [**A**] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(585B\,b^2x^6+765A\,b^2x^4+1530Bab\,x^4+2210Aab\,x^2+1105B\,a^2x^2+1989a^2A\right)x^{\frac{5}{2}}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x)
```

```
[Out] 2/9945*x^(5/2)*(585*B*b^2*x^6+765*A*b^2*x^4+1530*B*a*b*x^4+2210*A*a*b*x^2+1
105*B*a^2*x^2+1989*A*a^2)
```

maxima [A] time = 1.08, size = 51, normalized size = 0.81

$$\frac{2}{17}\,Bb^2x^{\frac{17}{2}} + \frac{2}{13}\left(2\,Bab + Ab^2\right)x^{\frac{13}{2}} + \frac{2}{5}\,Aa^2x^{\frac{5}{2}} + \frac{2}{9}\left(Ba^2 + 2\,Aab\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/5*A*a^2*x^(5/2) +
 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/13
) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(17/2))/17
```

sympy [A] time = 5.71, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{13}{2}}}{13} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{13}{2}}}{13} + \frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(17/2)/17
```

3.336 $\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^
(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx&=\int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{5/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{13/2}\right)\,dx\\&=\frac{2}{3}a^2Ax^{3/2}+\frac{2}{7}a(2Ab+aB)x^{7/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385a^2A+105bx^4(2aB+Ab)+165ax^2(aB+2Ab)+77b^2Bx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*A + 165*a*(2*A*b + a*B)*x^2 + 105*b*(A*b + 2*a*B)*x^4 +
 77*b^2*B*x^6))/1155
```

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 1.10

$$\frac{2\left(385a^2Ax^{3/2}+165a^2Bx^{7/2}+330aAbx^{7/2}+210abBx^{11/2}+105Ab^2x^{11/2}+77b^2Bx^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

```
[Out] (2*(385*a^2*A*x^(3/2) + 330*a*A*b*x^(7/2) + 165*a^2*B*x^(7/2) + 105*A*b^2*x
^(11/2) + 210*a*b*B*x^(11/2) + 77*b^2*B*x^(15/2)))/1155
```

fricas [A] time = 1.17, size = 54, normalized size = 0.86

$$\frac{2}{1155}\left(77\,Bb^2x^7+105\left(2\,Bab+Ab^2\right)x^5+385\,Aa^2x+165\left(Ba^2+2\,Aab\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^2*x^7 + 105*(2*B*a*b + A*b^2)*x^5 + 385*A*a^2*x + 165*(B*a^2
 + 2*A*a*b)*x^3)*sqrt(x)
```

giac [A] time = 0.31, size = 53, normalized size = 0.84

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2/3*A*a^2*x^(3/2)
```

maple [A] time = 0.00, size = 56, normalized size = 0.89

$$\frac{2\left(77B\,b^2x^6+105A\,b^2x^4+210Bab\,x^4+330Aab\,x^2+165B\,a^2x^2+385a^2A\right)x^{\frac{3}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*B*b^2*x^6+105*A*b^2*x^4+210*B*a*b*x^4+330*A*a*b*x^2+165*
B*a^2*x^2+385*A*a^2)
```

maxima [A] time = 1.05, size = 51, normalized size = 0.81

$$\frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{11}\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/3*A*a^2*x^(3/2) +
 2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

```
[Out] x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/11
) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(15/2))/15
```

sympy [A] time = 2.53, size = 66, normalized size = 1.05

$$\frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(Ab^2+2Bab\right)}{11}+\frac{2x^{\frac{7}{2}}\left(2Aab+Ba^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(15/2)/15 + 2*x**(11/2)*(A*b**2 + 2*B*a*b
)/11 + 2*x**(7/2)*(2*A*a*b + B*a**2)/7
```

3.337 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(9/2
))/9 + (2*b^2*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2A}{\sqrt{x}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{7/2}+b^2Bx^{11/2}\right)dx\\ &= 2a^2A\sqrt{x}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.87

$$\frac{2}{585}\sqrt{x}\left(585a^2A+65bx^4(2aB+Ab)+117ax^2(aB+2Ab)+45b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(585*a^2*A + 117*a*(2*A*b + a*B)*x^2 + 65*b*(A*b + 2*a*B)*x^4 +
45*b^2*B*x^6))/585
```

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.13

$$\frac{2}{585}\left(585a^2A\sqrt{x}+117a^2Bx^{5/2}+234aAbx^{5/2}+130abBx^{9/2}+65Ab^2x^{9/2}+45b^2Bx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*(585*a^2*A*Sqrt[x] + 234*a*A*b*x^(5/2) + 117*a^2*B*x^(5/2) + 65*A*b^2*x^
(9/2) + 130*a*b*B*x^(9/2) + 45*b^2*B*x^(13/2)))/585
```

fricas [A] time = 1.34, size = 53, normalized size = 0.87

$$\frac{2}{585}\left(45\,Bb^2x^6+65\left(2\,Bab+Ab^2\right)x^4+585\,Aa^2+117\left(Ba^2+2\,Aab\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^2*x^6 + 65*(2*B*a*b + A*b^2)*x^4 + 585*A*a^2 + 117*(B*a^2 + 2
*A*a*b)*x^2)*sqrt(x)
```

giac [A] time = 0.26, size = 53, normalized size = 0.87

$$\frac{2}{13}\,Bb^2x^{\frac{13}{2}}+\frac{4}{9}\,Babx^{\frac{9}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{2}{5}\,Ba^2x^{\frac{5}{2}}+\frac{4}{5}\,Aabx^{\frac{5}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/5*B*a^2*x^(
5/2) + 4/5*A*a*b*x^(5/2) + 2*A*a^2*sqrt(x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$\frac{2\left(45B\,b^2x^6+65A\,b^2x^4+130Bab\,x^4+234Aab\,x^2+117B\,a^2x^2+585a^2A\right)\sqrt{x}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/585*x^(1/2)*(45*B*b^2*x^6+65*A*b^2*x^4+130*B*a*b*x^4+234*A*a*b*x^2+117*B*
a^2*x^2+585*A*a^2)
```

maxima [A] time = 0.96, size = 51, normalized size = 0.84

$$\frac{2}{13}\,Bb^{2}x^{\frac{13}{2}}+\frac{2}{9}\left(2\,Bab+Ab^{2}\right)x^{\frac{9}{2}}+2\,Aa^{2}\sqrt{x}+\frac{2}{5}\left(Ba^{2}+2\,Aab\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2*A*a^2*sqrt(x) + 2/5
*(B*a^2 + 2*A*a*b)*x^(5/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{5/2}\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{9/2}\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,b^2\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(1/2),x)
```

```
[Out] x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) +
 2*A*a^2*x^(1/2) + (2*B*b^2*x^(13/2))/13
```

sympy [A] time = 2.11, size = 78, normalized size = 1.28

$$2Aa^{2}\sqrt{x}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^{2}x^{\frac{9}{2}}}{9}+\frac{2Ba^{2}x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^{2}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*x**(
5/2)/5 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(13/2)/13
```

3.338 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^{3/2}}\,dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{5/2}+b^2Bx^{9/2}\right)dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{11}b^2Bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.98

$$\frac{-154a^2\left(3A-Bx^2\right)+44abx^2\left(7A+3Bx^2\right)+6b^2x^4\left(11A+7Bx^2\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-154*a^2*(3*A - B*x^2) + 44*a*b*x^2*(7*A + 3*B*x^2) + 6*b^2*x^4*(11*A + 7*
B*x^2))/(231*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-231a^2A+77a^2Bx^2+154aAbx^2+66abBx^4+33Ab^2x^4+21b^2Bx^6\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-231*a^2*A + 154*a*A*b*x^2 + 77*a^2*B*x^2 + 33*A*b^2*x^4 + 66*a*b*B*x^4
 + 21*b^2*B*x^6))/(231*Sqrt[x])
```

fricas [A] time = 1.17, size = 53, normalized size = 0.87

$$\frac{2\left(21\,Bb^2x^6+33\left(2\,Bab+Ab^2\right)x^4-231\,Aa^2+77\left(Ba^2+2\,Aab\right)x^2\right)}{231\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*B*b^2*x^6 + 33*(2*B*a*b + A*b^2)*x^4 - 231*A*a^2 + 77*(B*a^2 + 2*
A*a*b)*x^2)/sqrt(x)
```

giac [A] time = 0.36, size = 53, normalized size = 0.87

$$\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{4}{7}\,Babx^{\frac{7}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+\frac{2}{3}\,Ba^2x^{\frac{3}{2}}+\frac{4}{3}\,Aabx^{\frac{3}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2*A*a^2/sqrt(x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$-\frac{2\left(-21B\,b^2x^6-33A\,b^2x^4-66Bab\,x^4-154Aab\,x^2-77B\,a^2x^2+231a^2A\right)}{231\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/231*(-21*B*b^2*x^6-33*A*b^2*x^4-66*B*a*b*x^4-154*A*a*b*x^2-77*B*a^2*x^2+
231*A*a^2)/x^(1/2)
```

maxima [A] time = 1.03, size = 51, normalized size = 0.84

$$\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{2}{7}\left(2\,Bab+Ab^2\right)x^{\frac{7}{2}}-\frac{2\,Aa^2}{\sqrt{x}}+\frac{2}{3}\left(Ba^2+2\,Aab\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^2*x^(11/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) - 2*A*a^2/sqrt(x) + 2/3
*(B*a^2 + 2*A*a*b)*x^(3/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{3/2}\left(\frac{2\,B\,a^2}{3}+\frac{4\,A\,b\,a}{3}\right)+x^{7/2}\left(\frac{2\,A\,b^2}{7}+\frac{4\,B\,a\,b}{7}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(3/2),x)
```

```
[Out] x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) -
 (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(11/2))/11
```

sympy [A] time = 2.42, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{7}{2}}}{7}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*x**
(3/2)/3 + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(11/2)/11
```

3.339 $\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{5/2}} \, dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
5/2))/5 + (2*b^2*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^{5/2}}\,dx &= \int \left(\frac{a^2A}{x^{5/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{3/2}+b^2Bx^{7/2}\right)dx\\ &= -\frac{2a^2A}{3x^{3/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{5}b(Ab+2aB)x^{5/2}+\frac{2}{9}b^2Bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{-30a^2\left(A-3Bx^2\right)+36abx^2\left(5A+Bx^2\right)+2b^2x^4\left(9A+5Bx^2\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-30*a^2*(A - 3*B*x^2) + 36*a*b*x^2*(5*A + B*x^2) + 2*b^2*x^4*(9*A + 5*B*x^
2))/(45*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-15a^2A+45a^2Bx^2+90aAbx^2+18abBx^4+9Ab^2x^4+5b^2Bx^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*A + 90*a*A*b*x^2 + 45*a^2*B*x^2 + 9*A*b^2*x^4 + 18*a*b*B*x^4 +
5*b^2*B*x^6))/(45*x^(3/2))
```

fricas [A] time = 1.08, size = 53, normalized size = 0.87

$$\frac{2\left(5\,Bb^2x^6+9\left(2\,Bab+Ab^2\right)x^4-15\,Aa^2+45\left(Ba^2+2\,Aab\right)x^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*B*b^2*x^6 + 9*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 45*(B*a^2 + 2*A*a*
b)*x^2)/x^(3/2)
```

giac [A] time = 0.41, size = 53, normalized size = 0.87

$$\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{4}{5}\,Babx^{\frac{5}{2}}+\frac{2}{5}\,Ab^2x^{\frac{5}{2}}+2\,Ba^2\sqrt{x}+4\,Aab\sqrt{x}-\frac{2\,Aa^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2*B*a^2*sqrt(x)
 + 4*A*a*b*sqrt(x) - 2/3*A*a^2/x^(3/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$-\frac{2\left(-5B\,b^2x^6-9A\,b^2x^4-18Bab\,x^4-90Aab\,x^2-45B\,a^2x^2+15a^2A\right)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/45*(-5*B*b^2*x^6-9*A*b^2*x^4-18*B*a*b*x^4-90*A*a*b*x^2-45*B*a^2*x^2+15*A
*a^2)/x^(3/2)
```

maxima [A] time = 0.99, size = 51, normalized size = 0.84

$$\frac{2}{9}\,Bb^{2}x^{\frac{9}{2}}+\frac{2}{5}\left(2\,Bab+Ab^{2}\right)x^{\frac{5}{2}}-\frac{2\,Aa^{2}}{3\,x^{\frac{3}{2}}}+2\left(Ba^{2}+2\,Aab\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b^2*x^(9/2) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) - 2/3*A*a^2/x^(3/2) + 2*(
B*a^2 + 2*A*a*b)*sqrt(x)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2\,B\,a^2+4\,A\,b\,a\right)+x^{5/2}\,\left(\frac{2\,A\,b^2}{5}+\frac{4\,B\,a\,b}{5}\right)-\frac{2\,A\,a^2}{3\,x^{3/2}}+\frac{2\,B\,b^2\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(5/2),x)
```

```
[Out] x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(5/2)*((2*A*b^2)/5 + (4*B*a*b)/5) - (2*A*a^
2)/(3*x^(3/2)) + (2*B*b^2*x^(9/2))/9
```

sympy [A] time = 2.96, size = 76, normalized size = 1.25

$$-\frac{2Aa^{2}}{3x^{\frac{3}{2}}}+4Aab\sqrt{x}+\frac{2Ab^{2}x^{\frac{5}{2}}}{5}+2Ba^{2}\sqrt{x}+\frac{4Babx^{\frac{5}{2}}}{5}+\frac{2Bb^{2}x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(5/2)/5 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(9/2)/9
```

3.340 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2Bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b + 2*a*B)*x
^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{x^{3/2}}+b(Ab+2aB)\sqrt{x}+b^2Bx^{5/2}\right)dx\\ &= -\frac{2a^2A}{5x^{5/2}}-\frac{2a(2Ab+aB)}{\sqrt{x}}+\frac{2}{3}b(Ab+2aB)x^{3/2}+\frac{2}{7}b^2Bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{-42a^2\left(A+5Bx^2\right)+140abx^2\left(Bx^2-3A\right)+10b^2x^4\left(7A+3Bx^2\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (140*a*b*x^2*(-3*A + B*x^2) + 10*b^2*x^4*(7*A + 3*B*x^2) - 42*a^2*(A + 5*B*
x^2))/(105*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-21a^2A-105a^2Bx^2-210aAbx^2+70abBx^4+35Ab^2x^4+15b^2Bx^6\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(-21*a^2*A - 210*a*A*b*x^2 - 105*a^2*B*x^2 + 35*A*b^2*x^4 + 70*a*b*B*x^4
 + 15*b^2*B*x^6))/(105*x^(5/2))
```

fricas [A] time = 1.05, size = 53, normalized size = 0.87

$$\frac{2\left(15\,Bb^2x^6+35\left(2\,Bab+Ab^2\right)x^4-21\,Aa^2-105\left(Ba^2+2\,Aab\right)x^2\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 - 21*A*a^2 - 105*(B*a^2 + 2*
A*a*b)*x^2)/x^(5/2)
```

giac [A] time = 0.40, size = 55, normalized size = 0.90

$$\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{4}{3}\,Babx^{\frac{3}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}-\frac{2\left(5\,Ba^2x^2+10\,Aabx^2+Aa^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) - 2/5*(5*B*a^2*x^
2 + 10*A*a*b*x^2 + A*a^2)/x^(5/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$-\frac{2\left(-15B\,b^2x^6-35A\,b^2x^4-70Bab\,x^4+210Aab\,x^2+105B\,a^2x^2+21a^2A\right)}{105x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/105*(-15*B*b^2*x^6-35*A*b^2*x^4-70*B*a*b*x^4+210*A*a*b*x^2+105*B*a^2*x^2
+21*A*a^2)/x^(5/2)
```

maxima [A] time = 0.99, size = 53, normalized size = 0.87

$$\frac{2}{7}\,Bb^2x^{\frac{7}{2}}+\frac{2}{3}\left(2\,Bab+Ab^2\right)x^{\frac{3}{2}}-\frac{2\left(Aa^2+5\left(Ba^2+2\,Aab\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*B*b^2*x^(7/2) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) - 2/5*(A*a^2 + 5*(B*a^2 +
 2*A*a*b)*x^2)/x^(5/2)
```

mupad [B] time = 0.25, size = 55, normalized size = 0.90

$$-\frac{210\,B\,a^2\,x^2+42\,A\,a^2-140\,B\,a\,b\,x^4+420\,A\,a\,b\,x^2-30\,B\,b^2\,x^6-70\,A\,b^2\,x^4}{105\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^2)/x^(7/2),x)
```

```
[Out] -(42*A*a^2 + 210*B*a^2*x^2 - 70*A*b^2*x^4 - 30*B*b^2*x^6 + 420*A*a*b*x^2 -
140*B*a*b*x^4)/(105*x^(5/2))
```

sympy [A] time = 4.01, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}}-\frac{4Aab}{\sqrt{x}}+\frac{2Ab^2x^{\frac{3}{2}}}{3}-\frac{2Ba^2}{\sqrt{x}}+\frac{4Babx^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*x**(3/2)/3 - 2*B*a**2/s
qrt(x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(7/2)/7
```

3.341 $\int x^{7/2} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.045, Rules used = {448}

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx &= \int \left(a^3Ax^{7/2}+a^2(3Ab+aB)x^{11/2}+3ab(Ab+aB)x^{15/2}+b^2(Ab+3aB)x^{19/2}+\right.\\ &= \frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{21}b^2(Ab+3aB)x\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/25
```

IntegrateAlgebraic [A] time = 0.05, size = 97, normalized size = 1.14

$$\frac{2\left(38675a^3Ax^{9/2}+26775a^3Bx^{13/2}+80325a^2Abx^{13/2}+61425a^2bBx^{17/2}+61425aAb^2x^{17/2}+49725ab^2Bx^{21/2}+16575Ab^3x^{21/2}+13923b^3Bx^{25/2}\right)}{348075}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*(38675*a^3*A*x^(9/2) + 80325*a^2*A*b*x^(13/2) + 26775*a^3*B*x^(13/2) + 6
1425*a*A*b^2*x^(17/2) + 61425*a^2*b*B*x^(17/2) + 16575*A*b^3*x^(21/2) + 497
25*a*b^2*B*x^(21/2) + 13923*b^3*B*x^(25/2)))/348075
```

fricas [A] time = 1.23, size = 78, normalized size = 0.92

$$\frac{2}{348075}\left(13923\,Bb^3x^{12}+16575\left(3\,Bab^2+Ab^3\right)x^{10}+61425\left(Ba^2b+Aab^2\right)x^8+38675\,Aa^3x^4+26775\left(Ba^3+3\,Aa^2b\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*B*b^3*x^12 + 16575*(3*B*a*b^2 + A*b^3)*x^10 + 61425*(B*a^2*
b + A*a*b^2)*x^8 + 38675*A*a^3*x^4 + 26775*(B*a^3 + 3*A*a^2*b)*x^6)*sqrt(x)
```

giac [A] time = 0.37, size = 77, normalized size = 0.91

$$\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{7}\,Bab^2x^{\frac{21}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{6}{17}\,Ba^2bx^{\frac{17}{2}}+\frac{6}{17}\,Aab^2x^{\frac{17}{2}}+\frac{2}{13}\,Ba^3x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2bx^{\frac{13}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 6/17*B*a
^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b*
x^(13/2) + 2/9*A*a^3*x^(9/2)
```

maple [A] time = 0.01, size = 80, normalized size = 0.94

$$\frac{2\left(13923B\,b^3x^8+16575x^6A\,b^3+49725Ba\,b^2x^6+61425x^4Aa\,b^2+61425x^4B\,a^2b+80325A\,a^2b\,x^2+26775B\,a^3x^2+38675A\,a^3\right)x^{\frac{9}{2}}}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/348075*x^(9/2)*(13923*B*b^3*x^8+16575*A*b^3*x^6+49725*B*a*b^2*x^6+61425*A
*a*b^2*x^4+61425*B*a^2*b*x^4+80325*A*a^2*b*x^2+26775*B*a^3*x^2+38675*A*a^3)
```

maxima [A] time = 1.14, size = 73, normalized size = 0.86

$$\frac{2}{25}Bb^3x^{\frac{25}{2}}+\frac{2}{21}\left(3Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{9}Aa^3x^{\frac{9}{2}}+\frac{2}{13}\left(Ba^3+3Aa^2b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

mupad [B] time = 0.04, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{25/2}}{25}+\frac{6\,a\,b\,x^{17/2}\,\left(A\,b+B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*
b^2)/7) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(17/2)*(A*
b + B*a))/17
```

sympy [A] time = 34.25, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(21/2)/21 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(17/2)/17 + 2*
B*a*b**2*x**(21/2)/7 + 2*B*b**3*x**(25/2)/25
```

3.342 $\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*B
)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx &= \int \left(a^3Ax^{5/2}+a^2(3Ab+aB)x^{9/2}+3ab(Ab+aB)x^{13/2}+b^2(Ab+3aB)x^{17/2}+b^3\right.\\ &= \frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2(3Ab+aB)x^{11/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*B
)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.14

$$\frac{2\left(24035a^3Ax^{7/2}+15295a^3Bx^{11/2}+45885a^2Abx^{11/2}+33649a^2bBx^{15/2}+33649aAb^2x^{15/2}+26565ab^2Bx^{19/2}+8855Ab^3x^{19/2}+7315b^3Bx^{23/2}\right)}{168245}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*(24035*a^3*A*x^(7/2) + 45885*a^2*A*b*x^(11/2) + 15295*a^3*B*x^(11/2) + 3
3649*a*A*b^2*x^(15/2) + 33649*a^2*b*B*x^(15/2) + 8855*A*b^3*x^(19/2) + 2656
5*a*b^2*B*x^(19/2) + 7315*b^3*B*x^(23/2)))/168245
```

fricas [A] time = 0.79, size = 78, normalized size = 0.92

$$\frac{2}{168245}\left(7315\,Bb^3x^{11}+8855\left(3\,Bab^2+Ab^3\right)x^9+33649\left(Ba^2b+Aab^2\right)x^7+24035\,Aa^3x^3+15295\left(Ba^3+3\,Aa^2b\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*B*b^3*x^11 + 8855*(3*B*a*b^2 + A*b^3)*x^9 + 33649*(B*a^2*b +
 A*a*b^2)*x^7 + 24035*A*a^3*x^3 + 15295*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

giac [A] time = 0.36, size = 77, normalized size = 0.91

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{19}\,Bab^2x^{\frac{19}{2}}+\frac{2}{19}\,Ab^3x^{\frac{19}{2}}+\frac{2}{5}\,Ba^2bx^{\frac{15}{2}}+\frac{2}{5}\,Aab^2x^{\frac{15}{2}}+\frac{2}{11}\,Ba^3x^{\frac{11}{2}}+\frac{6}{11}\,Aa^2bx^{\frac{11}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 2/5*B*a
^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b*x
^(11/2) + 2/7*A*a^3*x^(7/2)
```

maple [A] time = 0.01, size = 80, normalized size = 0.94

$$\frac{2\left(7315B\,b^3x^8+8855x^6A\,b^3+26565Ba\,b^2x^6+33649x^4Aa\,b^2+33649x^4B\,a^2b+45885A\,a^2b\,x^2+15295B\,a^3x^2+24035A\,a^3\right)x^{\frac{7}{2}}}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/168245*x^(7/2)*(7315*B*b^3*x^8+8855*A*b^3*x^6+26565*B*a*b^2*x^6+33649*A*a
*b^2*x^4+33649*B*a^2*b*x^4+45885*A*a^2*b*x^2+15295*B*a^3*x^2+24035*A*a^3)
```

maxima [A] time = 1.04, size = 73, normalized size = 0.86

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,Bab^2+Ab^3\right)x^{\frac{19}{2}}+\frac{2}{5}\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2}{11}\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{23/2}}{23}+\frac{2\,a\,b\,x^{15/2}\,\left(A\,b+B\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*
b^2)/19) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(23/2))/23 + (2*a*b*x^(15/2)*(A
*b + B*a))/5
```

sympy [A] time = 20.63, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{19}{2}}}{19}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{6Bab^2x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(11/2)/11 + 2*A*a*b**2*x**(15/2)/5 + 2*
A*b**3*x**(19/2)/19 + 2*B*a**3*x**(11/2)/11 + 2*B*a**2*b*x**(15/2)/5 + 6*B*
a*b**2*x**(19/2)/19 + 2*B*b**3*x**(23/2)/23
```

3.343 $\int x^{3/2} \left(a+bx^2\right)^3 \left(A+Bx^2\right) dx$

Optimal. Leaf size=85

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{5}a^3Ax^{5/2}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)*
x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx &= \int \left(a^3Ax^{3/2}+a^2(3Ab+aB)x^{7/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^{15/2}+\right.\\ &= \frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/}\end{aligned}$$

Mathematica [A] time = 0.04, size = 85, normalized size = 1.00

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)*
x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.14

$$\frac{2\left(13923a^3Ax^{5/2}+7735a^3Bx^{9/2}+23205a^2Abx^{9/2}+16065a^2bBx^{13/2}+16065aAb^2x^{13/2}+12285ab^2Bx^{17/2}+4095Ab^3x^{17/2}+3315b^3Bx^{21/2}\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*(13923*a^3*A*x^(5/2) + 23205*a^2*A*b*x^(9/2) + 7735*a^3*B*x^(9/2) + 1606
5*a*A*b^2*x^(13/2) + 16065*a^2*b*B*x^(13/2) + 4095*A*b^3*x^(17/2) + 12285*a
*b^2*B*x^(17/2) + 3315*b^3*B*x^(21/2)))/69615
```

fricas [A] time = 1.28, size = 78, normalized size = 0.92

$$\frac{2}{69615}\left(3315\,Bb^3x^{10}+4095\left(3\,Bab^2+Ab^3\right)x^8+16065\left(Ba^2b+Aab^2\right)x^6+13923\,Aa^3x^2+7735\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*B*b^3*x^10 + 4095*(3*B*a*b^2 + A*b^3)*x^8 + 16065*(B*a^2*b +
A*a*b^2)*x^6 + 13923*A*a^3*x^2 + 7735*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

giac [A] time = 0.38, size = 77, normalized size = 0.91

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{6}{17}\,Bab^2x^{\frac{17}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{6}{13}\,Ba^2bx^{\frac{13}{2}}+\frac{6}{13}\,Aab^2x^{\frac{13}{2}}+\frac{2}{9}\,Ba^3x^{\frac{9}{2}}+\frac{2}{3}\,Aa^2bx^{\frac{9}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^
(9/2) + 2/5*A*a^3*x^(5/2)
```

maple [A] time = 0.01, size = 80, normalized size = 0.94

$$\frac{2\left(3315B\,b^3x^8+4095x^6A\,b^3+12285Ba\,b^2x^6+16065x^4Aa\,b^2+16065x^4B\,a^2b+23205A\,a^2b\,x^2+7735B\,a^3x^2+13923A\,a^3\right)x^{\frac{5}{2}}}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x)
```

```
[Out] 2/69615*x^(5/2)*(3315*B*b^3*x^8+4095*A*b^3*x^6+12285*B*a*b^2*x^6+16065*A*a*
b^2*x^4+16065*B*a^2*b*x^4+23205*A*a^2*b*x^2+7735*B*a^3*x^2+13923*A*a^3)
```

maxima [A] time = 1.09, size = 73, normalized size = 0.86

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}}+\frac{2}{17}\left(3\,Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

mupad [B] time = 0.05, size = 69, normalized size = 0.81

$$x^{9/2}\,\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{17/2}\,\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{21/2}}{21}+\frac{6\,a\,b\,x^{13/2}\,\left(A\,b+B\,a\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^2
)/17) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(21/2))/21 + (6*a*b*x^(13/2)*(A*b
+ B*a))/13
```

sympy [A] time = 11.40, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a*b**2*x**(13/2)/13 + 2*A
*b**3*x**(17/2)/17 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*
b**2*x**(17/2)/17 + 2*B*b**3*x**(21/2)/21
```

3.344 $\int \sqrt{x} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{3}a^3Ax^{3/2}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*
x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x} \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx &= \int \left(a^3A\sqrt{x}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x^{13/2}+b^3\right.\\ &= \frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.84

$$\frac{2x^{3/2}\left(7315a^3A+3135a^2x^2(aB+3Ab)+1463b^2x^6(3aB+Ab)+5985abx^4(aB+Ab)+1155b^3Bx^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*x^(3/2)*(7315*a^3*A + 3135*a^2*(3*A*b + a*B)*x^2 + 5985*a*b*(A*b + a*B)*
x^4 + 1463*b^2*(A*b + 3*a*B)*x^6 + 1155*b^3*B*x^8))/21945
```

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.14

$$\frac{2\left(7315a^3Ax^{3/2}+3135a^3Bx^{7/2}+9405a^2Abx^{7/2}+5985a^2bBx^{11/2}+5985aAb^2x^{11/2}+4389ab^2Bx^{15/2}+1463Ab^3x^{15/2}+1155b^3Bx^{19/2}\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

```
[Out] (2*(7315*a^3*A*x^(3/2) + 9405*a^2*A*b*x^(7/2) + 3135*a^3*B*x^(7/2) + 5985*a
*A*b^2*x^(11/2) + 5985*a^2*b*B*x^(11/2) + 1463*A*b^3*x^(15/2) + 4389*a*b^2*
B*x^(15/2) + 1155*b^3*B*x^(19/2)))/21945
```

fricas [A] time = 0.89, size = 76, normalized size = 0.89

$$\frac{2}{21945}\left(1155\,Bb^3x^9+1463\left(3\,Bab^2+Ab^3\right)x^7+5985\left(Ba^2b+Aab^2\right)x^5+7315\,Aa^3x+3135\left(Ba^3+3\,Aa^2b\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*B*b^3*x^9 + 1463*(3*B*a*b^2 + A*b^3)*x^7 + 5985*(B*a^2*b + A*
a*b^2)*x^5 + 7315*A*a^3*x + 3135*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

giac [A] time = 0.40, size = 77, normalized size = 0.91

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{2}{5}\,Bab^2x^{\frac{15}{2}}+\frac{2}{15}\,Ab^3x^{\frac{15}{2}}+\frac{6}{11}\,Ba^2bx^{\frac{11}{2}}+\frac{6}{11}\,Aab^2x^{\frac{11}{2}}+\frac{2}{7}\,Ba^3x^{\frac{7}{2}}+\frac{6}{7}\,Aa^2bx^{\frac{7}{2}}+\frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 6/11*B*a
^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^(
7/2) + 2/3*A*a^3*x^(3/2)
```

maple [A] time = 0.01, size = 80, normalized size = 0.94

$$\frac{2\left(1155B\,b^3x^8+1463x^6A\,b^3+4389Ba\,b^2x^6+5985x^4Aa\,b^2+5985x^4B\,a^2b+9405A\,a^2b\,x^2+3135B\,a^3x^2+7315A\,a^3\right)x^{\frac{3}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x)
```

```
[Out] 2/21945*x^(3/2)*(1155*B*b^3*x^8+1463*A*b^3*x^6+4389*B*a*b^2*x^6+5985*A*a*b^
2*x^4+5985*B*a^2*b*x^4+9405*A*a^2*b*x^2+3135*B*a^3*x^2+7315*A*a^3)
```

maxima [A] time = 1.17, size = 73, normalized size = 0.86

$$\frac{2}{19}Bb^3x^{\frac{19}{2}}+\frac{2}{15}\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}}+\frac{6}{11}\left(Ba^2b+Aab^2\right)x^{\frac{11}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}+\frac{2}{7}\left(Ba^3+3\,Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{19/2}}{19}+\frac{6\,a\,b\,x^{11/2}\,\left(A\,b+B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

```
[Out] x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2
)/5) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(11/2)*(A*b +
 B*a))/11
```

sympy [A] time = 3.34, size = 95, normalized size = 1.12

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Bb^3x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{15}{2}}\left(Ab^3+3Bab^2\right)}{15}+\frac{2x^{\frac{11}{2}}\left(3Aab^2+3Ba^2b\right)}{11}+\frac{2x^{\frac{7}{2}}\left(3Aa^2b+Ba^3\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(19/2)/19 + 2*x**(15/2)*(A*b**3 + 3*B*a*b
**2)/15 + 2*x**(11/2)*(3*A*a*b**2 + 3*B*a**2*b)/11 + 2*x**(7/2)*(3*A*a**2*b
 + B*a**3)/7
```

3.345 $\int \frac{\left(a+bx^2\right)^3\left(A+Bx^2\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=83

$$2a^3A\sqrt{x}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b + a*B)*x^(9
/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3\left(A+Bx^2\right)}{\sqrt{x}}\,dx &= \int \left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{11/2}+b^3Bx^{15/2}\right.\\ &= 2a^3A\sqrt{x}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{17}\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.86

$$\frac{2\sqrt{x}\left(3315a^3A+663a^2x^2(aB+3Ab)+255b^2x^6(3aB+Ab)+1105abx^4(aB+Ab)+195b^3Bx^8\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3315*a^3*A + 663*a^2*(3*A*b + a*B)*x^2 + 1105*a*b*(A*b + a*B)*x
^4 + 255*b^2*(A*b + 3*a*B)*x^6 + 195*b^3*B*x^8))/3315
```

IntegrateAlgebraic [A] time = 0.04, size = 97, normalized size = 1.17

$$\frac{2\left(3315a^3A\sqrt{x}+663a^3Bx^{5/2}+1989a^2Abx^{5/2}+1105a^2bBx^{9/2}+1105aAb^2x^{9/2}+765ab^2Bx^{13/2}+255Ab^3x^{13/2}+195b^3Bx^{17/2}\right)}{3315}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

```
[Out] (2*(3315*a^3*A*Sqrt[x] + 1989*a^2*A*b*x^(5/2) + 663*a^3*B*x^(5/2) + 1105*a*
A*b^2*x^(9/2) + 1105*a^2*b*B*x^(9/2) + 255*A*b^3*x^(13/2) + 765*a*b^2*B*x^(
13/2) + 195*b^3*B*x^(17/2)))/3315
```

fricas [A] time = 0.76, size = 75, normalized size = 0.90

$$\frac{2}{3315}\left(195\,Bb^3x^8+255\left(3\,Bab^2+Ab^3\right)x^6+1105\left(Ba^2b+Aab^2\right)x^4+3315\,Aa^3+663\left(Ba^3+3\,Aa^2b\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*B*b^3*x^8 + 255*(3*B*a*b^2 + A*b^3)*x^6 + 1105*(B*a^2*b + A*a*b
^2)*x^4 + 3315*A*a^3 + 663*(B*a^3 + 3*A*a^2*b)*x^2)*sqrt(x)
```

giac [A] time = 0.25, size = 77, normalized size = 0.93

$$\frac{2}{17}\,Bb^3x^{\frac{17}{2}}+\frac{6}{13}\,Bab^2x^{\frac{13}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}+\frac{2}{3}\,Ba^2bx^{\frac{9}{2}}+\frac{2}{3}\,Aab^2x^{\frac{9}{2}}+\frac{2}{5}\,Ba^3x^{\frac{5}{2}}+\frac{6}{5}\,Aa^2bx^{\frac{5}{2}}+2\,Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 2/3*B*a
^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5/2
) + 2*A*a^3*sqrt(x)
```

maple [A] time = 0.01, size = 80, normalized size = 0.96

$$\frac{2\left(195B\,b^3x^8+255x^6A\,b^3+765Ba\,b^2x^6+1105x^4Aa\,b^2+1105x^4B\,a^2b+1989A\,a^2b\,x^2+663B\,a^3x^2+3315A\,a^3\right)\sqrt{x}}{3315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x)
```

```
[Out] 2/3315*x^(1/2)*(195*B*b^3*x^8+255*A*b^3*x^6+765*B*a*b^2*x^6+1105*A*a*b^2*x^
4+1105*B*a^2*b*x^4+1989*A*a^2*b*x^2+663*B*a^3*x^2+3315*A*a^3)
```

maxima [A] time = 1.06, size = 73, normalized size = 0.88

$$\frac{2}{17}Bb^3x^{\frac{17}{2}}+\frac{2}{13}\left(3Bab^2+Ab^3\right)x^{\frac{13}{2}}+\frac{2}{3}\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}}+2Aa^3\sqrt{x}+\frac{2}{5}\left(Ba^3+3Aa^2b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^3*x^(17/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) + 2*A*a^3*sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{5/2}\left(\frac{2Ba^3}{5}+\frac{6Aba^2}{5}\right)+x^{13/2}\left(\frac{2Ab^3}{13}+\frac{6Bab^2}{13}\right)+2Aa^3\sqrt{x}+\frac{2Bb^3x^{17/2}}{17}+\frac{2abx^{9/2}(Ab+Ba)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(1/2),x)
```

```
[Out] x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2
)/13) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(17/2))/17 + (2*a*b*x^(9/2)*(A*b + B*a
))/3
```

sympy [A] time = 4.82, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{2Ba^3x^{\frac{5}{2}}}{5}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{2Bb^3x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 2*A*a*b**2*x**(9/2)/3 + 2*A*b**3
*x**(13/2)/13 + 2*B*a**3*x**(5/2)/5 + 2*B*a**2*b*x**(9/2)/3 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(17/2)/17
```

3.346 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{11}b^2x^{11/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{15}b^3Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx &= \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{5/2}+b^2(Ab+3aB)x^{9/2}+b^3Bx^{13/2}\right) \\ &= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{11}b^2(Ab+3aB)x^{11/2}+\frac{2}{15}b^3\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.98

$$\frac{-770a^3(3A-Bx^2)+330a^2bx^2(7A+3Bx^2)+90ab^2x^4(11A+7Bx^2)+14b^3x^6(15A+11Bx^2)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (-770*a^3*(3*A - B*x^2) + 330*a^2*b*x^2*(7*A + 3*B*x^2) + 90*a*b^2*x^4*(11*
A + 7*B*x^2) + 14*b^3*x^6*(15*A + 11*B*x^2))/(1155*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.05, size = 83, normalized size = 1.00

$$\frac{2\left(-1155a^3A+385a^3Bx^2+1155a^2Abx^2+495a^2bBx^4+495aAb^2x^4+315ab^2Bx^6+105Ab^3x^6+77b^3Bx^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

```
[Out] (2*(-1155*a^3*A + 1155*a^2*A*b*x^2 + 385*a^3*B*x^2 + 495*a*A*b^2*x^4 + 495*
a^2*b*B*x^4 + 105*A*b^3*x^6 + 315*a*b^2*B*x^6 + 77*b^3*B*x^8))/(1155*Sqrt[x
])
```

fricas [A] time = 1.03, size = 75, normalized size = 0.90

$$\frac{2\left(77\,Bb^3x^8+105\left(3\,Bab^2+Ab^3\right)x^6+495\left(Ba^2b+Aab^2\right)x^4-1155\,Aa^3+385\left(Ba^3+3\,Aa^2b\right)x^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*B*b^3*x^8 + 105*(3*B*a*b^2 + A*b^3)*x^6 + 495*(B*a^2*b + A*a*b^2
)*x^4 - 1155*A*a^3 + 385*(B*a^3 + 3*A*a^2*b)*x^2)/sqrt(x)
```

giac [A] time = 0.30, size = 77, normalized size = 0.93

$$\frac{2}{15}\,Bb^3x^{\frac{15}{2}}+\frac{6}{11}\,Bab^2x^{\frac{11}{2}}+\frac{2}{11}\,Ab^3x^{\frac{11}{2}}+\frac{6}{7}\,Ba^2bx^{\frac{7}{2}}+\frac{6}{7}\,Aab^2x^{\frac{7}{2}}+\frac{2}{3}\,Ba^3x^{\frac{3}{2}}+2\,Aa^2bx^{\frac{3}{2}}-\frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2)
- 2*A*a^3/sqrt(x)
```

maple [A] time = 0.01, size = 80, normalized size = 0.96

$$-\frac{2\left(-77B\,b^3x^8-105x^6A\,b^3-315Ba\,b^2x^6-495x^4Aa\,b^2-495x^4B\,a^2b-1155A\,a^2b\,x^2-385B\,a^3x^2+1155A\,a^3\right)}{1155\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x)
```

```
[Out] -2/1155*(-77*B*b^3*x^8-105*A*b^3*x^6-315*B*a*b^2*x^6-495*A*a*b^2*x^4-495*B*
a^2*b*x^4-1155*A*a^2*b*x^2-385*B*a^3*x^2+1155*A*a^3)/x^(1/2)
```

maxima [A] time = 0.96, size = 73, normalized size = 0.88

$$\frac{2}{15} Bb^3 x^{\frac{15}{2}} + \frac{2}{11} \left(3 Bab^2 + Ab^3\right) x^{\frac{11}{2}} + \frac{6}{7} \left(Ba^2 b + Aab^2\right) x^{\frac{7}{2}} - \frac{2 Aa^3}{\sqrt{x}} + \frac{2}{3} \left(Ba^3 + 3 Aa^2 b\right) x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^3*x^(15/2) + 2/11*(3*B*a*b^2 + A*b^3)*x^(11/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2*A*a^3/sqrt(x) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{3/2} \left(\frac{2 B a^3}{3} + 2 A b a^2\right) + x^{11/2} \left(\frac{2 A b^3}{11} + \frac{6 B a b^2}{11}\right) - \frac{2 A a^3}{\sqrt{x}} + \frac{2 B b^3 x^{15/2}}{15} + \frac{6 a b x^{7/2} (A b + B a)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(3/2),x)
```

```
[Out] x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(11/2)*((2*A*b^3)/11 + (6*B*a*b^2)/11
) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(15/2))/15 + (6*a*b*x^(7/2)*(A*b + B*a))
/7
```

sympy [A] time = 5.33, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{\sqrt{x}} + 2Aa^2bx^{\frac{3}{2}} + \frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{6Ba^2bx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{11}{2}}}{11} + \frac{2Bb^3x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b**3*
x**(11/2)/11 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x**
(11/2)/11 + 2*B*b**3*x**(15/2)/15
```

3.347 $\int \frac{\left(a+bx^2\right)^3\left(A+Bx^2\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{3x^{3/2}}+2a^2\sqrt{x}\,(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2\sqrt{x}\,(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3\left(A+Bx^2\right)}{x^{5/2}}\,dx &= \int \left(\frac{a^3A}{x^{5/2}}+\frac{a^2(3Ab+aB)}{\sqrt{x}}+3ab(Ab+aB)x^{3/2}+b^2(Ab+3aB)x^{7/2}+b^3Bx^{11/2}\right)dx\\ &= -\frac{2a^3A}{3x^{3/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{5}ab(Ab+aB)x^{5/2}+\frac{2}{9}b^2(Ab+3aB)x^{9/2}+\frac{2}{13}b^3B\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.94

$$\frac{-390a^3\left(A-3Bx^2\right)+702a^2bx^2\left(5A+Bx^2\right)+78ab^2x^4\left(9A+5Bx^2\right)+10b^3x^6\left(13A+9Bx^2\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (-390*a^3*(A - 3*B*x^2) + 702*a^2*b*x^2*(5*A + B*x^2) + 78*a*b^2*x^4*(9*A +
 5*B*x^2) + 10*b^3*x^6*(13*A + 9*B*x^2))/(585*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 83, normalized size = 1.00

$$\frac{2\left(-195a^3A+585a^3Bx^2+1755a^2Abx^2+351a^2bBx^4+351aAb^2x^4+195ab^2Bx^6+65Ab^3x^6+45b^3Bx^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

```
[Out] (2*(-195*a^3*A + 1755*a^2*A*b*x^2 + 585*a^3*B*x^2 + 351*a*A*b^2*x^4 + 351*a
^2*b*B*x^4 + 65*A*b^3*x^6 + 195*a*b^2*B*x^6 + 45*b^3*B*x^8))/(585*x^(3/2))
```

fricas [A] time = 1.02, size = 75, normalized size = 0.90

$$\frac{2\left(45\,Bb^3x^8+65\left(3\,Bab^2+Ab^3\right)x^6+351\left(Ba^2b+Aab^2\right)x^4-195\,Aa^3+585\left(Ba^3+3\,Aa^2b\right)x^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*B*b^3*x^8 + 65*(3*B*a*b^2 + A*b^3)*x^6 + 351*(B*a^2*b + A*a*b^2)*
x^4 - 195*A*a^3 + 585*(B*a^3 + 3*A*a^2*b)*x^2)/x^(3/2)
```

giac [A] time = 0.41, size = 77, normalized size = 0.93

$$\frac{2}{13}\,Bb^3x^{\frac{13}{2}}+\frac{2}{3}\,Bab^2x^{\frac{9}{2}}+\frac{2}{9}\,Ab^3x^{\frac{9}{2}}+\frac{6}{5}\,Ba^2bx^{\frac{5}{2}}+\frac{6}{5}\,Aab^2x^{\frac{5}{2}}+2\,Ba^3\sqrt{x}+6\,Aa^2b\sqrt{x}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 6/5*B*a^2*b
*x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2/3*
A*a^3/x^(3/2)
```

maple [A] time = 0.02, size = 80, normalized size = 0.96

$$-\frac{2\left(-45B\,b^3x^8-65x^6A\,b^3-195Ba\,b^2x^6-351x^4Aa\,b^2-351x^4B\,a^2b-1755A\,a^2b\,x^2-585B\,a^3x^2+195A\,a^3\right)}{585x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x)
```

```
[Out] -2/585*(-45*B*b^3*x^8-65*A*b^3*x^6-195*B*a*b^2*x^6-351*A*a*b^2*x^4-351*B*a^
2*b*x^4-1755*A*a^2*b*x^2-585*B*a^3*x^2+195*A*a^3)/x^(3/2)
```

maxima [A] time = 1.15, size = 73, normalized size = 0.88

$$\frac{2}{13}\,Bb^3x^{\frac{13}{2}}+\frac{2}{9}\left(3\,Bab^2+Ab^3\right)x^{\frac{9}{2}}+\frac{6}{5}\left(Ba^2b+Aab^2\right)x^{\frac{5}{2}}-\frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}+2\left(Ba^3+3\,Aa^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^3*x^(13/2) + 2/9*(3*B*a*b^2 + A*b^3)*x^(9/2) + 6/5*(B*a^2*b + A*a*
b^2)*x^(5/2) - 2/3*A*a^3/x^(3/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$\sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)+x^{9/2}\,\left(\frac{2\,A\,b^3}{9}+\frac{2\,B\,a\,b^2}{3}\right)-\frac{2\,A\,a^3}{3\,x^{3/2}}+\frac{2\,B\,b^3\,x^{13/2}}{13}+\frac{6\,a\,b\,x^{5/2}\,\left(A\,b+B\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(5/2),x)
```

```
[Out] x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(9/2)*((2*A*b^3)/9 + (2*B*a*b^2)/3) - (2*
A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(13/2))/13 + (6*a*b*x^(5/2)*(A*b + B*a))/5
```

sympy [A] time = 6.06, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}}+6Aa^2b\sqrt{x}+\frac{6Aab^2x^{\frac{5}{2}}}{5}+\frac{2Ab^3x^{\frac{9}{2}}}{9}+2Ba^3\sqrt{x}+\frac{6Ba^2bx^{\frac{5}{2}}}{5}+\frac{2Bab^2x^{\frac{9}{2}}}{3}+\frac{2Bb^3x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(5/2)/5 + 2*A*b
**3*x**(9/2)/9 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(5/2)/5 + 2*B*a*b**2*x**(
9/2)/3 + 2*B*b**3*x**(13/2)/13
```

3.348 $\int \frac{\left(a+bx^2\right)^3\left(A+Bx^2\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2(aB+3Ab)}{\sqrt{x}}-\frac{2a^3A}{5x^{5/2}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A*b + a*B)*
x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^3\left(A+Bx^2\right)}{x^{7/2}}\,dx&=\int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(3Ab+aB)}{x^{3/2}}+3ab(Ab+aB)\sqrt{x}+b^2(Ab+3aB)x^{5/2}+b^3Bx^{9/2}\right)dx\\&=-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(3Ab+aB)}{\sqrt{x}}+2ab(Ab+aB)x^{3/2}+\frac{2}{7}b^2(Ab+3aB)x^{7/2}+\frac{2}{11}b^3Bx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.96

$$\frac{2\left(-77a^3\left(A+5Bx^2\right)+385a^2bx^2\left(Bx^2-3A\right)+55ab^2x^4\left(7A+3Bx^2\right)+5b^3x^6\left(11A+7Bx^2\right)\right)}{385x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(385*a^2*b*x^2*(-3*A + B*x^2) + 55*a*b^2*x^4*(7*A + 3*B*x^2) - 77*a^3*(A
 + 5*B*x^2) + 5*b^3*x^6*(11*A + 7*B*x^2)))/(385*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 83, normalized size = 1.02

$$\frac{2\left(-77a^3A-385a^3Bx^2-1155a^2Abx^2+385a^2bBx^4+385aAb^2x^4+165ab^2Bx^6+55Ab^3x^6+35b^3Bx^8\right)}{385x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

```
[Out] (2*(-77*a^3*A - 1155*a^2*A*b*x^2 - 385*a^3*B*x^2 + 385*a*A*b^2*x^4 + 385*a^
2*b*B*x^4 + 55*A*b^3*x^6 + 165*a*b^2*B*x^6 + 35*b^3*B*x^8))/(385*x^(5/2))
```

fricas [A] time = 1.18, size = 75, normalized size = 0.93

$$\frac{2\left(35\,Bb^3x^8+55\left(3\,Bab^2+Ab^3\right)x^6+385\left(Ba^2b+Aab^2\right)x^4-77\,Aa^3-385\left(Ba^3+3\,Aa^2b\right)x^2\right)}{385\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*B*b^3*x^8 + 55*(3*B*a*b^2 + A*b^3)*x^6 + 385*(B*a^2*b + A*a*b^2)*
x^4 - 77*A*a^3 - 385*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

giac [A] time = 0.29, size = 79, normalized size = 0.98

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{6}{7}\,Bab^2x^{\frac{7}{2}}+\frac{2}{7}\,Ab^3x^{\frac{7}{2}}+2\,Ba^2bx^{\frac{3}{2}}+2\,Aab^2x^{\frac{3}{2}}-\frac{2\left(5\,Ba^3x^2+15\,Aa^2bx^2+Aa^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 2*B*a^2*b*x
^(3/2) + 2*A*a*b^2*x^(3/2) - 2/5*(5*B*a^3*x^2 + 15*A*a^2*b*x^2 + A*a^3)/x^(
5/2)
```

maple [A] time = 0.01, size = 80, normalized size = 0.99

$$-\frac{2\left(-35B\,b^3x^8-55x^6A\,b^3-165Ba\,b^2x^6-385x^4Aa\,b^2-385x^4B\,a^2b+1155A\,a^2b\,x^2+385B\,a^3x^2+77A\,a^3\right)}{385x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x)
```

```
[Out] -2/385*(-35*B*b^3*x^8-55*A*b^3*x^6-165*B*a*b^2*x^6-385*A*a*b^2*x^4-385*B*a^
2*b*x^4+1155*A*a^2*b*x^2+385*B*a^3*x^2+77*A*a^3)/x^(5/2)
```

maxima [A] time = 1.01, size = 75, normalized size = 0.93

$$\frac{2}{11}\,Bb^3x^{\frac{11}{2}}+\frac{2}{7}\left(3\,Bab^2+Ab^3\right)x^{\frac{7}{2}}+2\left(Ba^2b+Aab^2\right)x^{\frac{3}{2}}-\frac{2\left(Aa^3+5\left(Ba^3+3\,Aa^2b\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*B*b^3*x^(11/2) + 2/7*(3*B*a*b^2 + A*b^3)*x^(7/2) + 2*(B*a^2*b + A*a*b^
2)*x^(3/2) - 2/5*(A*a^3 + 5*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

mupad [B] time = 0.06, size = 72, normalized size = 0.89

$$x^{7/2}\left(\frac{2\,A\,b^3}{7}+\frac{6\,B\,a\,b^2}{7}\right)-\frac{\frac{2\,A\,a^3}{5}+x^2\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)}{x^{5/2}}+\frac{2\,B\,b^3\,x^{11/2}}{11}+2\,a\,b\,x^{3/2}\,\left(A\,b+B\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^3)/x^(7/2),x)
```

```
[Out] x^(7/2)*((2*A*b^3)/7 + (6*B*a*b^2)/7) - ((2*A*a^3)/5 + x^2*(2*B*a^3 + 6*A*a
^2*b))/x^(5/2) + (2*B*b^3*x^(11/2))/11 + 2*a*b*x^(3/2)*(A*b + B*a)
```

sympy [A] time = 8.68, size = 107, normalized size = 1.32

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}}-\frac{6Aa^2b}{\sqrt{x}}+2Aab^2x^{\frac{3}{2}}+\frac{2Ab^3x^{\frac{7}{2}}}{7}-\frac{2Ba^3}{\sqrt{x}}+2Ba^2bx^{\frac{3}{2}}+\frac{6Bab^2x^{\frac{7}{2}}}{7}+\frac{2Bb^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(B*x**2+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) - 6*A*a**2*b/sqrt(x) + 2*A*a*b**2*x**(3/2) + 2*A*b**
3*x**(7/2)/7 - 2*B*a**3/sqrt(x) + 2*B*a**2*b*x**(3/2) + 6*B*a*b**2*x**(7/2)
/7 + 2*B*b**3*x**(11/2)/11
```

3.349 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=276

$$-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}-\frac{a^{5/4}(Ab}{}$$

Rubi [A] time = 0.26, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}-\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{13/4}}+\frac{2x^{5/2}(Ab-aB)}{5b^2}-\frac{2a\sqrt{x}(Ab-aB)}{b^3}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[x])/b^3 + (2*(A*b - a*B)*x^(5/2))/(5*b^2) + (2*B*x^(
9/2))/(9*b) - (a^(5/4)*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(13/4)) + (a^(5/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(13/4)) - (a^(5/4)*(A*b - a*B)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(13/4)) + (a^(
5/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{2Bx^{9/2}}{9b} - \frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^2}\,dx}{9b} \\
&= \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{(a(Ab-aB))\int \frac{x^{3/2}}{a+bx^2}\,dx}{b^2} \\
&= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^3} \\
&= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(2a^2(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \right.}{b^3} \\
&= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^{3/2}(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, \right.}{b^3} \\
&= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} + \frac{\left(a^{3/2}(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\right.}{2b^{7/2}} \\
&= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{a^{5/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,b^{13/4}} \\
&= -\frac{2a(Ab-aB)\sqrt{x}}{b^3} + \frac{2(Ab-aB)x^{5/2}}{5b^2} + \frac{2Bx^{9/2}}{9b} - \frac{a^{5/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{13/4}} +
\end{aligned}$$

Mathematica [A] time = 0.29, size = 227, normalized size = 0.82

$$\frac{\frac{45\sqrt{2}a^{5/4}(aB-Ab)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)\right)}{\sqrt[4]{b}}+\frac{90\sqrt{2}a^{5/4}(aB-Ab)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{\sqrt[4]{b}}+72bx^{5/2}(Ab-aB)+360a\sqrt{x}(aB-Ab)+40b^2Bx^{9/2}}{180b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (360*a*(-(A*b) + a*B)*Sqrt[x] + 72*b*(A*b - a*B)*x^(5/2) + 40*b^2*B*x^(9/2)
 + (90*Sqrt[2]*a^(5/4)*(-(A*b) + a*B)*(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
```

```
/a^(1/4)] - ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]))/b^(1/4) + (45*S
qrt[2]*a^(5/4)*(-(A*b) + a*B)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
))/b^(1/4))/(180*b^3)
```

IntegrateAlgebraic [A]   time = 0.29, size = 181, normalized size = 0.66

$$\frac{\left(a^{9/4}B-a^{5/4}Ab\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{13/4}}-\frac{\left(a^{9/4}B-a^{5/4}Ab\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{13/4}}+\frac{2\sqrt{x}\,\left(45a^2B-45aAb-9abBx^2+9Ab^2x^2+5b^2Bx^4\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(-45*a*A*b + 45*a^2*B + 9*A*b^2*x^2 - 9*a*b*B*x^2 + 5*b^2*B*x^4)
)/(45*b^3) + ((-(a^(5/4)*A*b) + a^(9/4)*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(13/4)) - ((-(a^(5/4)*A*b) + a^
(9/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(Sqrt[2]*b^(13/4))
```

fricas [B]   time = 0.99, size = 714, normalized size = 2.59

$$\frac{180\,b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6\sqrt{-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}}+\left(B^2a^4-2\,ABa^3b+A^2a^2b^2\right)x}\,b^{10}\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{3}{4}}+\left(Ba^2b^{10}-Aab^{11}\right)\sqrt{x}\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{3}{4}}}{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}\right)+45\,b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}-\left(Ba^2-Aab\right)\sqrt{x}\right)-45\,b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(-b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}-\left(Ba^2-Aab\right)\sqrt{x}\right)+4\left(5\,Bb^2x^4+45\,Ba^2-45\,Aab-9\left(Bab-Ab^2\right)x^2\right)\sqrt{x}}{90\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/90*(180*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*
b^3 + A^4*a^5*b^4)/b^13)^(1/4)*arctan((sqrt(b^6*sqrt(-(B^4*a^9 - 4*A*B^3*a^
8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13) + (B^2*a^4 -
 2*A*B*a^3*b + A^2*a^2*b^2)*x)*b^10*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*
a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(3/4) + (B*a^2*b^10 - A*a*b^
11)*sqrt(x)*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^
3 + A^4*a^5*b^4)/b^13)^(3/4))/(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2
- 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)) + 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A
^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(b^3*(-(B^4*
a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^
13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) - 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b +
6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(-b^3*(-(
B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4
)/b^13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) + 4*(5*B*b^2*x^4 + 45*B*a^2 - 45*A
*a*b - 9*(B*a*b - A*b^2)*x^2)*sqrt(x))/b^3
```

giac [A]   time = 0.37, size = 298, normalized size = 1.08

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba^2-\left(ab^3\right)^{\frac{1}{4}}Aab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba^2-\left(ab^3\right)^{\frac{1}{4}}Aab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba^2-\left(ab^3\right)^{\frac{1}{4}}Aab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba^2-\left(ab^3\right)^{\frac{1}{4}}Aab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4}+\frac{2\left(5\,Bb^8x^{\frac{9}{2}}-9\,Bab^7x^{\frac{5}{2}}+9\,Ab^8x^{\frac{5}{2}}+45\,Ba^2b^6\sqrt{x}-45\,Aab^7\sqrt{x}\right)}{45\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*((a*b^3)^
(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)
^(1/4)*A*a*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 + 1/4*sq
rt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*log(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/b^4 + 2/45*(5*B*b^8*x^(9/2) - 9*B*a*b^7*x^(5/2) +
9*A*b^8*x^(5/2) + 45*B*a^2*b^6*sqrt(x) - 45*A*a*b^7*sqrt(x))/b^9
```

maple [A] time = 0.02, size = 330, normalized size = 1.20

$$\frac{2B\,x^{\frac{9}{2}}}{9b}+\frac{2A\,x^{\frac{5}{2}}}{5b}-\frac{2Ba\,x^{\frac{5}{2}}}{5b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;Aa\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;Aa\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;Aa\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;B\,a^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;B\,a^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;B\,a^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{3}}-\frac{2Aa\sqrt{x}}{b^{2}}+\frac{2B\,a^{2}\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/9*B*x^(9/2)/b+2/5/b*A*x^(5/2)-2/5/b^2*B*x^(5/2)*a-2/b^2*a*A*x^(1/2)+2/b^3
*a^2*B*x^(1/2)+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b
)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))-1/2*a^2/b^3*(a/b)^(1/
4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a^2/b^3*(a/b)^(1/4)*
2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4*a^2/b^3*(a/b)^(1/4)*2^(
1/2)*B*ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))
```

maxima [A] time = 2.49, size = 259, normalized size = 0.94

$$-\frac{\left(\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a^{2}}{4\,b^{3}}+\frac{2\left(5\,Bb^{2}x^{\frac{9}{2}}-9\left(Bab-Ab^{2}\right)x^{\frac{5}{2}}+45\left(Ba^{2}-Aab\right)\sqrt{x}\right)}{45\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(
2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
```

```
a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a^2/b^3 + 2/45*(5*B*b^2*x^(9/2)
- 9*(B*a*b - A*b^2)*x^(5/2) + 45*(B*a^2 - A*a*b)*sqrt(x))/b^3
```

mupad [B] time = 0.38, size = 788, normalized size = 2.86

$$x^{5/2}\left(\frac{2\,A}{5\,b}-\frac{2\,B\,a}{5\,b^2}\right)+\frac{2\,B\,x^{9/2}}{9\,b}-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)\,1{}\mathrm{i}}{2\,b^{13/4}}\right)(A\,b-B\,a)}{2\,b^{13/4}}+\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}+\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)\,1{}\mathrm{i}}{2\,b^{13/4}}\right)(A\,b-B\,a)}{2\,b^{13/4}}}{\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)\,1{}\mathrm{i}}{2\,b^{13/4}}\right)(A\,b-B\,a)\,1{}\mathrm{i}}{2\,b^{13/4}}-\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}+\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)\,1{}\mathrm{i}}{2\,b^{13/4}}\right)(A\,b-B\,a)\,1{}\mathrm{i}}{2\,b^{13/4}}}\right)(A\,b-B\,a)}{b^{13/4}}-\frac{a\,\sqrt{x}\,\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)}{b}-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)}{2\,b^{13/4}}\right)(A\,b-B\,a)\,1{}\mathrm{i}}{2\,b^{13/4}}+\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}+\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)}{2\,b^{13/4}}\right)(A\,b-B\,a)\,1{}\mathrm{i}}{2\,b^{13/4}}}{\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)}{2\,b^{13/4}}\right)(A\,b-B\,a)}{2\,b^{13/4}}-\frac{(-a)^{5/4}\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}+\frac{(-a)^{5/4}\,(A\,b-B\,a)\,\left(32\,B\,a^4-32\,A\,a^3\,b\right)}{2\,b^{13/4}}\right)(A\,b-B\,a)}{2\,b^{13/4}}}\right)(A\,b-B\,a)\,1{}\mathrm{i}}{b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^(5/2)*((2*A)/(5*b) - (2*B*a)/(5*b^2)) + (2*B*x^(9/2))/(9*b) - ((-a)^(5/4)
*atan((((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3
- ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b))/(2*b^(13/4)))*(A*b - B*a
)*1i)/(2*b^(13/4)) + ((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*
B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b))/(2*b^(13/4
)))*(A*b - B*a)*1i)/(2*b^(13/4)))/(((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*
a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*
b))/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)) - ((-a)^(5/4)*((16*x^(1/2)*(B^2
*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4
- 32*A*a^3*b))/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4))))*(A*b - B*a)*1i)/b^
(13/4) - ((-a)^(5/4)*atan((((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2
- 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b)*1i)/(
2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)) + ((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6
+ A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*
A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)))/(((-a)^(5/4)*((16*x^(
1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(
32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a)*1i)/(2*b^(13/4)) - ((-
a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5
/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a)*1i)/(
2*b^(13/4))))*(A*b - B*a))/b^(13/4) - (a*x^(1/2)*((2*A)/b - (2*B*a)/b^2))/b
```

sympy [A] time = 127.30, size = 434, normalized size = 1.57

$$\begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}\right) & \text{for}\ a=0\wedge b=0\\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{13}{2}}}{13}}{a} & \text{for}\ b=0\\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} & \text{for}\ a=0\\ -\frac{\sqrt[4]{-1}\,Aa^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}+\frac{\sqrt[4]{-1}\,Aa^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}-\frac{\sqrt[4]{-1}\,Aa^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^2}-\frac{2Aa\sqrt{x}}{b^2}+\frac{2Ax^{\frac{5}{2}}}{5b}+\frac{\sqrt[4]{-1}\,Ba^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^3}-\frac{\sqrt[4]{-1}\,Ba^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^3}+\frac{\sqrt[4]{-1}\,Ba^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^3}+\frac{2Ba^2\sqrt{x}}{b^3}-\frac{2Bax^{\frac{5}{2}}}{5b^2}+\frac{2Bx^{\frac{9}{2}}}{9b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(9/2)/9 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(
9/2)/9)/b, Eq(a, 0)), (-(-1)**(1/4)*A*a**(5/4)*(1/b)**(1/4)*log(-(-1)**(1/4
)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + (-1)**(1/4)*A*a**(5/4)*(1/b)*
*(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) - (-1)**(1
/4)*A*a**(5/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)
))/b**2 - 2*A*a*sqrt(x)/b**2 + 2*A*x**(5/2)/(5*b) + (-1)**(1/4)*B*a**(9/4)*
(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**3) - (
-1)**(1/4)*B*a**(9/4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
sqrt(x))/(2*b**3) + (-1)**(1/4)*B*a**(9/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sq
rt(x)/(a**(1/4)*(1/b)**(1/4)))/b**3 + 2*B*a**2*sqrt(x)/b**3 - 2*B*a*x**(5/2
)/(5*b**2) + 2*B*x**(9/2)/(9*b), True))
```

3.350 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=257

$$-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{11/4}}+a^{3/4}(Ab-$$

Rubi [A] time = 0.21, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{11/4}}+\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{11/4}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(7/2))/(7*b) + (a^(3/4)*(A*b - a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(11/4)) - (a^(3
/4)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(
11/4)) - (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4)) + (a^(3/4)*(A*b - a*B)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{2Bx^{7/2}}{7b}-\frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^2}\,dx}{7b}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}-\frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}-\frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}+\frac{(a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^{5/2}}-(a(Ab-aB))\,\mathrm{Su}\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}-\frac{(a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^3}-(a(Ab-a\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}-\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{11/4}}+a^{3/4}(Ab-\\
&= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}+\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{a^{3/4}(Ab-aB)\tan^{-1}\left(1\right.}{\sqrt{2}\,b^{11/4}}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 110, normalized size = 0.43

$$
\frac{2b^{3/4}x^{3/2}\left(-7aB+7Ab+3bBx^2\right)-21(-a)^{3/4}(aB-Ab)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{-a}}\right)+21(-a)^{3/4}(aB-Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{-a}}\right)}{21b^{11/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*b^(3/4)*x^(3/2)*(7*A*b - 7*a*B + 3*b*B*x^2) - 21*(-a)^(3/4)*(-(A*b) + a*
B)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + 21*(-a)^(3/4)*(-(A*b) + a*B)*ArcT
anh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(21*b^(11/4))
```

IntegrateAlgebraic [A] time = 0.20, size = 160, normalized size = 0.62

$$
-\frac{\left(a^{7/4}B-a^{3/4}Ab\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{11/4}}-\frac{\left(a^{7/4}B-a^{3/4}Ab\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{11/4}}+\frac{2x^{3/2}\left(-7aB+7Ab+3bBx^2\right)}{21b^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(7*A*b - 7*a*B + 3*b*B*x^2))/(21*b^2) - ((-(a^(3/4)*A*b) + a^(7/
4)*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqr
t[2]*b^(11/4)) - ((-(a^(3/4)*A*b) + a^(7/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(11/4))
```

fricas [B] time = 0.93, size = 899, normalized size = 3.50

$$\frac{84\,b^{2}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^{6}a^{10}-6\,AB^{5}a^{9}b+15\,A^{2}B^{4}a^{8}b^{2}-20\,A^{3}B^{3}a^{7}b^{3}+15\,A^{4}B^{2}a^{6}b^{4}-6\,A^{5}Ba^{5}b^{5}+A^{6}a^{4}b^{6}\right)x-\left(B^{4}a^{7}b^{5}-4\,AB^{3}a^{6}b^{6}+6\,A^{2}B^{2}a^{5}b^{7}-4\,A^{3}Ba^{4}b^{8}+A^{4}a^{3}b^{9}\right)\sqrt{-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}}}\,b^{3}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{1}{4}}+\left(B^{3}a^{5}b^{3}-3\,AB^{2}a^{4}b^{4}+3\,A^{2}Ba^{3}b^{5}-A^{3}a^{2}b^{6}\right)\sqrt{x}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{1}{4}}}{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}\right)-21\,b^{2}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^{8}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{3}{4}}-\left(B^{3}a^{5}-3\,AB^{2}a^{4}b+3\,A^{2}Ba^{3}b^{2}-A^{3}a^{2}b^{3}\right)\sqrt{x}\right)+21\,b^{2}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{1}{4}}\log\left(-b^{8}\left(-\frac{B^{4}a^{7}-4\,AB^{3}a^{6}b+6\,A^{2}B^{2}a^{5}b^{2}-4\,A^{3}Ba^{4}b^{3}+A^{4}a^{3}b^{4}}{b^{11}}\right)^{\frac{3}{4}}-\left(B^{3}a^{5}-3\,AB^{2}a^{4}b+3\,A^{2}Ba^{3}b^{2}-A^{3}a^{2}b^{3}\right)\sqrt{x}\right)+4\left(3\,Bbx^{3}-7\,(Ba-Ab)x\right)\sqrt{x}}{42\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/42*(84*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b
^3 + A^4*a^3*b^4)/b^11)^(1/4)*arctan((sqrt((B^6*a^10 - 6*A*B^5*a^9*b + 15*A
^2*B^4*a^8*b^2 - 20*A^3*B^3*a^7*b^3 + 15*A^4*B^2*a^6*b^4 - 6*A^5*B*a^5*b^5
+ A^6*a^4*b^6)*x - (B^4*a^7*b^5 - 4*A*B^3*a^6*b^6 + 6*A^2*B^2*a^5*b^7 - 4*A
^3*B*a^4*b^8 + A^4*a^3*b^9)*sqrt(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*
b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11))*b^3*(-(B^4*a^7 - 4*A*B^3*a^6*b
+ 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4) + (B^3*a^5
*b^3 - 3*A*B^2*a^4*b^4 + 3*A^2*B*a^3*b^5 - A^3*a^2*b^6)*sqrt(x)*(-(B^4*a^7
- 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^
(1/4))/(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4
*a^3*b^4)) - 21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*
B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6
*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 -
3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^3)*sqrt(x)) + 21*b^2*(-(B^4*a^7
 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)
^(1/4)*log(-b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^
4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b
^2 - A^3*a^2*b^3)*sqrt(x)) + 4*(3*B*b*x^3 - 7*(B*a - A*b)*x)*sqrt(x))/b^2
```

giac [A] time = 0.33, size = 264, normalized size = 1.03

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^{5}}+\frac{2\left(3\,Bb^{6}x^{\frac{7}{2}}-7\,Bab^{5}x^{\frac{3}{2}}+7\,Ab^{6}x^{\frac{3}{2}}\right)}{21\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 + 1/2*sqrt(2)*((a*b^3)^(3/4)
```

```
*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3
)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/b^5 + 2/21*(3*B*b^6*x^(7/2) - 7*B*a*b^5*x^(3/2) + 7*A*b^6*x^(3/2))/
b^7
```

maple [A] time = 0.01, size = 308, normalized size = 1.20

$$\frac{2B\,x^{\frac{7}{2}}}{7b}+\frac{2A\,x^{\frac{3}{2}}}{3b}-\frac{2Ba\,x^{\frac{3}{2}}}{3b^{2}}-\frac{\sqrt{2}\,Aa\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,Aa\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,Aa\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{\sqrt{2}\,B\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{\sqrt{2}\,B\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{\sqrt{2}\,B\,a^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/7*B*x^(7/2)/b+2/3/b*x^(3/2)*A-2/3/b^2*x^(3/2)*B*a-1/4*a/b^2/(a/b)^(1/4)*2
^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2)))-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)-1)+1/4*a^2/b^3/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*2^(1/2)
*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*a^2/
b^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a^2/b^3
/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.41, size = 214, normalized size = 0.83

$$\frac{\left(Ba^{2}-Aab\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b^{2}}+\frac{2\left(3\,Bbx^{\frac{7}{2}}-7\,(Ba-Ab)x^{\frac{3}{2}}\right)}{21\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*a^2 - A*a*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sq
rt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*
b^(3/4)))/b^2 + 2/21*(3*B*b*x^(7/2) - 7*(B*a - A*b)*x^(3/2))/b^2
```

mupad [B] time = 0.18, size = 92, normalized size = 0.36

$$x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)+\frac{2\,B\,x^{7/2}}{7\,b}+\frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)}{b^{11/4}}+\frac{{\left(-a\right)}^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)\,1{}\mathrm{i}}{b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(7/2))/(7*b) + ((-a)^(3/4)
*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/b^(11/4) + ((-a)^(3/4)*ata
n((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*(A*b - B*a)*1i)/b^(11/4)
```

sympy [A] time = 54.69, size = 502, normalized size = 1.95

$$\begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}\right) & \text{for}\ a=0\wedge b=0 \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} & \text{for}\ b=0 \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} & \text{for}\ a=0 \\ \frac{(-1)^{\frac{3}{4}}Aa^{\frac{3}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}-\frac{(-1)^{\frac{3}{4}}Aa^{\frac{3}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}+\frac{(-1)^{\frac{3}{4}}Aa^{\frac{3}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b}-\frac{2(-1)^{\frac{3}{4}}Aa^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^{2}\sqrt[4]{\frac{1}{b}}}+\frac{2Ax^{\frac{3}{2}}}{3b}-\frac{(-1)^{\frac{3}{4}}Ba^{\frac{7}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{2}}+\frac{(-1)^{\frac{3}{4}}Ba^{\frac{7}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^{2}}-\frac{(-1)^{\frac{3}{4}}Ba^{\frac{7}{4}}\left(\frac{1}{b}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^{2}}+\frac{2(-1)^{\frac{3}{4}}Ba^{\frac{7}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^{3}\sqrt[4]{\frac{1}{b}}}-\frac{2Bax^{\frac{3}{2}}}{3b^{2}}+\frac{2Bx^{\frac{7}{2}}}{7b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2
*A*x**(7/2)/7 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(
7/2)/7)/b, Eq(a, 0)), ((-1)**(3/4)*A*a**(3/4)*(1/b)**(3/4)*log(-(-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) - (-1)**(3/4)*A*a**(3/4)*(1/b)**(3/
4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + (-1)**(3/4)*A*a
**(3/4)*(1/b)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b -
2*(-1)**(3/4)*A*a**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/
(b**2*(1/b)**(1/4)) + 2*A*x**(3/2)/(3*b) - (-1)**(3/4)*B*a**(7/4)*(1/b)**(3
/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + (-1)**(3/4
)*B*a**(7/4)*(1/b)**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/
(2*b**2) - (-1)**(3/4)*B*a**(7/4)*(1/b)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a*
*(1/4)*(1/b)**(1/4)))/b**2 + 2*(-1)**(3/4)*B*a**(7/4)*atan((-1)**(3/4)*sqrt
(x)/(a**(1/4)*(1/b)**(1/4)))/(b**3*(1/b)**(1/4)) - 2*B*a*x**(3/2)/(3*b**2)
+ 2*B*x**(7/2)/(7*b), True))
```

3.351 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=255

$$\frac{\sqrt[4]{a}\,(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{a}\,(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\sqrt[4]{a}\,(Ab-a$$

Rubi **[A]** time = 0.20, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2\sqrt{x}\,(Ab-aB)}{b^2}+\frac{\sqrt[4]{a}\,(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{a}\,(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}}+\frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}}-\frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{9/4}}+\frac{2Bx^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(5/2))/(5*b) + (a^(1/4)*(A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4)) - (a^(1/4)*(
A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(9/4))
 + (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(2*Sqrt[2]*b^(9/4)) - (a^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^2}\,dx}{5b} \\
&= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^2} \\
&= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{\left(\sqrt{a}\,(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^2} - \left(\sqrt{a}\,(Ab-a \right. \\
&= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} - \frac{\left(\sqrt{a}\,(Ab-aB)\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{5/2}} - \left(\sqrt{a}\,(Ab \right. \\
&= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} + \frac{\sqrt[4]{a}\,(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{9/4}} - \sqrt[4]{a}\,(Ab- \\
&= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{5/2}}{5b} + \frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{9/4}} - \frac{\sqrt[4]{a}\,(Ab-aB)\tan^{-1}\left(1 \right.}{\sqrt{2}\,b^{9/4}}
\end{aligned}
$$

Mathematica [**A**]   time = 0.20, size = 208, normalized size = 0.82

$$
\frac{40\sqrt{x}\,(Ab-aB) + \frac{5\sqrt{2}\,\sqrt[4]{a}\,(Ab-aB)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)\right)}{\sqrt[4]{b}} + \frac{10\sqrt{2}\,\sqrt[4]{a}\,(Ab-aB)\left(\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)-\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{\sqrt[4]{b}} + 8bBx^{5/2}}{20b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (40*(A*b - a*B)*Sqrt[x] + 8*b*B*x^(5/2) + (10*Sqrt[2]*a^(1/4)*(A*b - a*B)*(
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)]))/b^(1/4) + (5*Sqrt[2]*a^(1/4)*(A*b - a*B)*(Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/b^(1/4))/(20*b^2)
```

IntegrateAlgebraic [A] time = 0.19, size = 158, normalized size = 0.62

$$-\frac{\left(a^{5/4}B-\sqrt[4]{a}\,Ab\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,b^{9/4}}+\frac{\left(a^{5/4}B-\sqrt[4]{a}\,Ab\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{9/4}}+\frac{2\sqrt{x}\,\left(-5aB+5Ab+bBx^2\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(5*A*b - 5*a*B + b*B*x^2))/(5*b^2) - ((-(a^(1/4)*A*b) + a^(5/4)*
B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2
]*b^(9/4)) + ((-(a^(1/4)*A*b) + a^(5/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(9/4))
```

fricas [B] time = 1.29, size = 660, normalized size = 2.59

$$-\frac{20\,b^{2}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^{4}\sqrt{-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}}+\left(B^{2}a^{2}-2\,ABab+A^{2}b^{2}\right)x}\,b^{7}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{3}{4}}+\left(Bab^{7}-Ab^{8}\right)\sqrt{x}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{3}{4}}}{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}\right)+5\,b^{2}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{1}{4}}\log\left(b^{2}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-5\,b^{2}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{1}{4}}\log\left(-b^{2}\left(-\frac{B^{4}a^{5}-4\,AB^{3}a^{4}b+6\,A^{2}B^{2}a^{3}b^{2}-4\,A^{3}Ba^{2}b^{3}+A^{4}ab^{4}}{b^{9}}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-4\left(Bbx^{2}-5\,Ba+5\,Ab\right)\sqrt{x}}{10\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*
b^3 + A^4*a*b^4)/b^9)^(1/4)*arctan((sqrt(b^4*sqrt(-(B^4*a^5 - 4*A*B^3*a^4*b
 + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9) + (B^2*a^2 - 2*A*B
*a*b + A^2*b^2)*x)*b^7*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A
^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(3/4) + (B*a*b^7 - A*b^8)*sqrt(x)*(-(B^4*a^5
 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(3
/4))/(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a
*b^4)) + 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2
*b^3 + A^4*a*b^4)/b^9)^(1/4)*log(b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2
*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) -
 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A
^4*a*b^4)/b^9)^(1/4)*log(-b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^
2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) - 4*(B*b
*x^2 - 5*B*a + 5*A*b)*sqrt(x))/b^2
```

giac [A] time = 0.41, size = 263, normalized size = 1.03

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^{3}}+\frac{2\left(Bb^{4}x^{\frac{5}{2}}-5\,Bab^{3}\sqrt{x}+5\,Ab^{4}\sqrt{x}\right)}{5\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*((a*b^3)^(1/4)
*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*((a*b^3
)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/b^3 + 2/5*(B*b^4*x^(5/2) - 5*B*a*b^3*sqrt(x) + 5*A*b^4*sqrt(x))/b^5
```

maple [A] time = 0.01, size = 299, normalized size = 1.17

$$\frac{2Bx^{\frac{5}{2}}}{5b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ba\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}+\frac{2A\sqrt{x}}{b}-\frac{2Ba\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a),x)
```

```
[Out] 2/5*B*x^(5/2)/b+2/b*A*x^(1/2)-2/b^2*B*a*x^(1/2)-1/4/b*(a/b)^(1/4)*2^(1/2)*A
*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2)))-1/2/b*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)-1/2/b*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+
1/4/b^2*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)
)/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*a+1/2/b^2*(a/b)^(1/4)*2^(1/2
)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a+1/2/b^2*(a/b)^(1/4)*2^(1/2)*B*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a
```

maxima [A] time = 2.24, size = 235, normalized size = 0.92

$$\frac{\left(\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{4\,b^{2}}+\frac{2\left(Bbx^{\frac{5}{2}}-5\,(Ba-Ab)\sqrt{x}\right)}{5\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^2 + 2/5*(B*b*x^(5/2) - 5*(B*a
 - A*b)*sqrt(x))/b^2
```

mupad [B] time = 0.38, size = 789, normalized size = 3.09

$$\sqrt{x}\left(\frac{2A}{b}-\frac{2Ba}{b^2}\right)+\frac{2Bx^{5/2}}{5b}-\frac{(-a)^{1/4}\operatorname{atan}\left(\frac{\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}-\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)}{2b^{9/4}}\right)1i}{2b^{9/4}}+\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}+\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)}{2b^{9/4}}\right)1i}{2b^{9/4}}}{\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}-\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)}{2b^{9/4}}\right)}{2b^{9/4}}-\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}+\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)}{2b^{9/4}}\right)}{2b^{9/4}}}\right)(Ab-Ba)\,1i}{b^{9/4}}-\frac{(-a)^{1/4}\operatorname{atan}\left(\frac{\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}-\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)\,1i}{2b^{9/4}}\right)}{2b^{9/4}}+\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}+\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)\,1i}{2b^{9/4}}\right)}{2b^{9/4}}}{\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}-\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)\,1i}{2b^{9/4}}\right)1i}{2b^{9/4}}-\frac{(-a)^{1/4}(Ab-Ba)\left(\frac{16\sqrt{x}\left(A^2a^2b^2-2ABa^3b+B^2a^4\right)}{b}+\frac{(-a)^{1/4}\left(32Aa^2b^2-32Ba^3b\right)(Ab-Ba)\,1i}{2b^{9/4}}\right)1i}{2b^{9/4}}}\right)(Ab-Ba)}{b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(5/2))/(5*b) - ((-a)^(1/4)*atan(((
(-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/
b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a))/(2*b^(9/4)))*1i)/(
2*b^(9/4)) + ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 -
2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a))/(2*b
^(9/4)))*1i)/(2*b^(9/4)))/(((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 +
A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*
b - B*a))/(2*b^(9/4))))/(2*b^(9/4)) - ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*
(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B
*a^3*b)*(A*b - B*a))/(2*b^(9/4))))/(2*b^(9/4))))*(A*b - B*a)*1i)/b^(9/4) -
((-a)^(1/4)*atan((((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b
^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a)*
1i)/(2*b^(9/4))))/(2*b^(9/4)) + ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a
^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b
)*(A*b - B*a)*1i)/(2*b^(9/4))))/(2*b^(9/4)))/(((-a)^(1/4)*(A*b - B*a)*((16*
x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^
2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9/4)))*1i)/(2*b^(9/4)) - ((-a)^(1/4)*
(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(
1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9/4)))*1i)/(2*b^(9/4
))))*(A*b - B*a))/b^(9/4)
```

sympy [A] time = 16.60, size = 393, normalized size = 1.54

$$\begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) & \text{for } a=0\wedge b=0\\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} & \text{for } b=0\\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} & \text{for } a=0\\ \frac{\sqrt[4]{-1}A\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}-\frac{\sqrt[4]{-1}A\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}+\frac{\sqrt[4]{-1}A\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b}+\frac{2A\sqrt{x}}{b}-\frac{\sqrt[4]{-1}Ba^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}+\frac{\sqrt[4]{-1}Ba^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}-\frac{\sqrt[4]{-1}Ba^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^2}-\frac{2Ba\sqrt{x}}{b^2}+\frac{2Bx^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A*
x**(5/2)/5 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/
b, Eq(a, 0)), ((-1)**(1/4)*A*a**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4
)*(1/b)**(1/4) + sqrt(x))/(2*b) - (-1)**(1/4)*A*a**(1/4)*(1/b)**(1/4)*log((
```

```
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + (-1)**(1/4)*A*a**(1/4)*
(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b + 2*A*sqrt
(x)/b - (-1)**(1/4)*B*a**(5/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)
**(1/4) + sqrt(x))/(2*b**2) + (-1)**(1/4)*B*a**(5/4)*(1/b)**(1/4)*log((-1)*
*(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) - (-1)**(1/4)*B*a**(5/4)*(
1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b**2 - 2*B*a*
sqrt(x)/b**2 + 2*B*x**(5/2)/(5*b), True))
```

3.352 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{a+bx^2}\,dx$

Optimal. Leaf size=237

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\tan^{-1}(}{\sqrt{2}\sqrt[4]{a}}$$

Rubi [A] time = 0.18, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((A*b - a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(7/4)) -
 ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*a^(1/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\left(A+Bx^2\right)}{a+bx^2}\,dx &= \frac{2Bx^{3/2}}{3b}-\frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int\frac{\sqrt{x}}{a+bx^2}\,dx}{3b}\\
&= \frac{2Bx^{3/2}}{3b}-\frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\mathrm{Subst}\left(\int\frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{3b}\\
&= \frac{2Bx^{3/2}}{3b}-\frac{(Ab-aB)\,\mathrm{Subst}\left(\int\frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{b^{3/2}}+\frac{(Ab-aB)\,\mathrm{Subst}\Bigl(\int\frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx,x,}{b^{3/2}}\\
&= \frac{2Bx^{3/2}}{3b}+\frac{(Ab-aB)\,\mathrm{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{2b^2}+\frac{(Ab-aB)\,\mathrm{Subst}\Bigl(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}}}{2b^2}\\
&= \frac{2Bx^{3/2}}{3b}+\frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}\,b^{7/4}}-\frac{(Ab-aB)\log\Bigl(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{}}{2\sqrt{2}\sqrt[4]{a}\,b^{7/4}}\\
&= \frac{2Bx^{3/2}}{3b}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{(Ab-a}{}
\end{aligned}$$

Mathematica [**A**]   time = 0.07, size = 95, normalized size = 0.40

$$\frac{3(Ab-aB)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+(3aB-3Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2\sqrt[4]{-a}\,b^{3/4}Bx^{3/2}}{3\sqrt[4]{-a}\,b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*(-a)^(1/4)*b^(3/4)*B*x^(3/2) + 3*(A*b - a*B)*ArcTan[(b^(1/4)*Sqrt[x])/(-
a)^(1/4)] + (-3*A*b + 3*a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(3*(-a)
^(1/4)*b^(7/4))
```

IntegrateAlgebraic [**A**]   time = 0.19, size = 135, normalized size = 0.57

$$\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{7/4}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((-(A*b) + a*B)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(
1/4)*b^(7/4))
```

fricas [B] time = 1.13, size = 834, normalized size = 3.52

$$\frac{4\,Bx^{\frac{3}{2}} - 12\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6a^6-6\,AB^5a^5b+15\,A^2B^4a^4b^2-20\,A^3B^3a^3b^3+15\,A^4B^2a^2b^4-6\,A^5Bab^5+A^6b^6\right)x-\left(B^4a^5b^3-4\,AB^3a^4b^4+6\,A^2B^2a^3b^5-4\,A^3Ba^2b^6+A^4ab^7\right)\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}}}\,b^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}+\left(B^3a^3b^2-3\,AB^2a^2b^3+3\,A^2Bab^4-A^3b^5\right)\sqrt{x}\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}}{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}\right)+3\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{3}{4}}-\left(B^3a^3-3\,AB^2a^2b+3\,A^2Bab^2-A^3b^3\right)\sqrt{x}\right)-3\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}\log\left(-ab^5\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{3}{4}}-\left(B^3a^3-3\,AB^2a^2b+3\,A^2Bab^2-A^3b^3\right)\sqrt{x}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*B*x^(3/2) - 12*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*arctan((sqrt((B^6*a^6 - 6*A*B^5*a^5*b
 + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a
*b^5 + A^6*b^6)*x - (B^4*a^5*b^3 - 4*A*B^3*a^4*b^4 + 6*A^2*B^2*a^3*b^5 - 4*
A^3*B*a^2*b^6 + A^4*a*b^7)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b
^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7)))*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*
A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4) + (B^3*a^3*b^2 -
3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5)*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3
*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4))/(B^4*a^4
- 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)) + 3*b*(-(B^
4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7
))^(1/4)*log(a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B
*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2
 - A^3*b^3)*sqrt(x)) - 3*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(-a*b^5*(-(B^4*a^4 - 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^
3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)))/b
```

giac [A] time = 0.42, size = 251, normalized size = 1.06

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 1/2*
sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2
```

```
)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(3/4
)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
```

maple [A] time = 0.01, size = 280, normalized size = 1.18

$$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,Ba\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a),x)
```

```
[Out] 2/3*B*x^(3/2)/b+1/4/b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/b/(a/b)^(1
/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/b/(a/b)^(1/4)*2^(1/
2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/b^2/(a/b)^(1/4)*2^(1/2)*B*ln
((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2)))*a-1/2/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)*a-1/2/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)*a
```

maxima [A] time = 2.33, size = 194, normalized size = 0.82

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{(Ba-Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 1/4*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) -
sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(1/4)*b^(3/4)))/b
```

mupad [B] time = 0.15, size = 71, normalized size = 0.30

$$\frac{2\,B\,x^{3/2}}{3\,b}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)}{{\left(-a\right)}^{1/4}\,b^{7/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(A\,b-B\,a\right)}{{\left(-a\right)}^{1/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(a + b*x^2),x)
```

```
[Out] (2*B*x^(3/2))/(3*b) + (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a
)^(1/4)*b^(7/4)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^
(1/4)*b^(7/4))
```

sympy [A] time = 6.76, size = 459, normalized size = 1.94

$$\begin{cases} \tilde{\infty} \left(- \frac{2 A}{\sqrt{x}} + \frac{2 B x^{\frac{3}{2}}}{3}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{\frac{2 A x^{\frac{3}{2}}}{3} + \frac{2 B x^{\frac{7}{2}}}{7}}{a} & \text{for}\: b = 0 \\ \frac{- \frac{2 A}{\sqrt{x}} + \frac{2 B x^{\frac{3}{2}}}{3}}{b} & \text{for}\: a = 0 \\ - \frac{\left(-1\right)^{\frac{3}{4}} A \left(\frac{1}{b}\right)^{\frac{3}{4}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 \sqrt[4]{a}} + \frac{\left(-1\right)^{\frac{3}{4}} A \left(\frac{1}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 \sqrt[4]{a}} - \frac{\left(-1\right)^{\frac{3}{4}} A \left(\frac{1}{b}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{\sqrt[4]{a}} + \frac{2 \left(-1\right)^{\frac{3}{4}} A \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{\sqrt[4]{a} b \sqrt[4]{\frac{1}{b}}} + \frac{\left(-1\right)^{\frac{3}{4}} B a^{\frac{3}{4}} \left(\frac{1}{b}\right)^{\frac{3}{4}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 b} - \frac{\left(-1\right)^{\frac{3}{4}} B a^{\frac{3}{4}} \left(\frac{1}{b}\right)^{\frac{3}{4}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 b} + \frac{\left(-1\right)^{\frac{3}{4}} B a^{\frac{3}{4}} \left(\frac{1}{b}\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{b} - \frac{2 \left(-1\right)^{\frac{3}{4}} B a^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{b^{2} \sqrt[4]{\frac{1}{b}}} + \frac{2 B x^{\frac{3}{2}}}{3 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(3/2)/3 + 2*B*x**(7/2)/7)/a, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*x**(3/2)/3
)/b, Eq(a, 0)), (-(-1)**(3/4)*A*(1/b)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b
)**(1/4) + sqrt(x))/(2*a**(1/4)) + (-1)**(3/4)*A*(1/b)**(3/4)*log((-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)) - (-1)**(3/4)*A*(1/b)**(3/
4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(1/4) + 2*(-1)**(3/
4)*A*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(1/4)*b*(1/b)**(
1/4)) + (-1)**(3/4)*B*a**(3/4)*(1/b)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)
**(1/4) + sqrt(x))/(2*b) - (-1)**(3/4)*B*a**(3/4)*(1/b)**(3/4)*log((-1)**(1
/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + (-1)**(3/4)*B*a**(3/4)*(1/b)**
(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b - 2*(-1)**(3/4)*B
*a**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**2*(1/b)**(1
/4)) + 2*B*x**(3/2)/(3*b), True))
```

3.353 $\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=235

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(Ab-aB)\tan^{-}}{\sqrt{2}\,a^{3}}$$

Rubi [A] time = 0.18, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}-\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*a^(3/4)*b^(5/4)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((A*b - a*B)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) + ((A
*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(3/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,b} + \frac{(Ab-aB)\,\text{Subst}\Big(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x,}{\sqrt{a}\,b}$$

$$= \frac{2B\sqrt{x}}{b} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\,b^{3/2}} + \frac{(Ab-aB)\,\text{Subst}\Bigg(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+}}{2\sqrt{a}\,b^{3/2}}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(Ab-aB)\log\Big(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}}{2\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{(Ab-aB)}{}$$

Mathematica [A] time = 0.13, size = 166, normalized size = 0.71

$$\frac{(aB-Ab)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}}+\frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*B*Sqrt[x])/b + ((-(A*b) + a*B)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^(3/4)*b^(5/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 134, normalized size = 0.57

$$\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{2B\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*B*Sqrt[x])/b + ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((-(A*b) + a*B)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)
*b^(5/4))
```

fricas [B] time = 0.82, size = 645, normalized size = 2.74

$$\frac{4\,b\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^{2}b^{2}\sqrt{-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}}+\left(B^{2}a^{2}-2\,ABab+A^{2}b^{2}\right)x}\,a^{2}b^{4}\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{3}{4}}+\left(Ba^{3}b^{4}-Aa^{2}b^{5}\right)\sqrt{x}\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{3}{4}}}{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}\right)+b\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-b\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{3}b^{5}}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)+4\,B\sqrt{x}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^5))^(1/4)*arctan((sqrt(a^2*b^2*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*
b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5)) + (B^2*a^2 - 2*
A*B*a*b + A^2*b^2)*x)*a^2*b^4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(3/4) + (B*a^3*b^4 - A*a^2*b^5)*sqr
t(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^5))^(3/4))/(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)) + b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b
+ 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*
b)*sqrt(x)) - b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*
b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2
*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt
(x)) + 4*B*sqrt(x))/b
```

giac [A] time = 0.42, size = 251, normalized size = 1.07

$$\frac{2\,B\sqrt{x}}{b}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/2*sq
rt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*((a*b^3)^(1/4)*
B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
```

```
a*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
```

maple [A] time = 0.01, size = 277, normalized size = 1.18

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}+\frac{2B\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 2*B*x^(1/2)/b+1/2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1)+1/4*(a/b)^(1/4)/a*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1
/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*(a/b)^(1/4)/a*2^(1/2)
*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/b*(a/b)^(1/4)*2^(1/2)*B*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/b*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))
)-1/2/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
```

maxima [A] time = 2.36, size = 218, normalized size = 0.93

$$\frac{2\,B\sqrt{x}}{b}-\frac{\frac{2\,\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/b - 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))) + sqrt(2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

mupad [B] time = 0.24, size = 739, normalized size = 3.14

$$\frac{2\,B\,\sqrt{x}}{b}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)}{2\,(-a)^{3/4}\,b^{5/4}}\right)1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)+\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)}{2\,(-a)^{3/4}\,b^{5/4}}\right)1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)}{2\,(-a)^{3/4}\,b^{5/4}}\right)}{2\,(-a)^{3/4}\,b^{5/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)+\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)}{2\,(-a)^{3/4}\,b^{5/4}}\right)}{2\,(-a)^{3/4}\,b^{5/4}}}\right)(A\,b-B\,a)\,1\mathrm{i}}{(-a)^{3/4}\,b^{5/4}}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)\,1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}\right)}{2\,(-a)^{3/4}\,b^{5/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)+\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)\,1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}\right)}{2\,(-a)^{3/4}\,b^{5/4}}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)\,1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}\right)1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)+\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)\,1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}\right)1\mathrm{i}}{2\,(-a)^{3/4}\,b^{5/4}}}\right)(A\,b-B\,a)}{(-a)^{3/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] (2*B*x^(1/2))/b - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b -
 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a))/(2*(-a)^(3/4)*b^
(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16
*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a))/(2*(
-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4)))/(((A*b - B*a)*(x^(1/2)*(16*
A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b
- B*a))/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)) - ((A*b - B*a)*(x^(
1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b
^3)*(A*b - B*a))/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4))))*(A*b - B
*a)*1i)/((-a)^(3/4)*b^(5/4)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 1
6*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a)*1i)/
(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*(x^(1/2)*(16
*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b
 - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)))/(((A*b - B*a)*
(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A
*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4))
- ((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B
*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^
(3/4)*b^(5/4))))*(A*b - B*a))/((-a)^(3/4)*b^(5/4))
```

sympy [**A**] time = 6.50, size = 355, normalized size = 1.51

$$\begin{cases} \tilde{\infty} \left(- \frac{2 A}{3 x^{\frac{3}{2}}} + 2 B \sqrt{x}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 A \sqrt{x} + \frac{2 B x^{\frac{5}{2}}}{5}}{a} & \text{for}\: b = 0 \\ \frac{- \frac{2 A}{3 x^{\frac{3}{2}}} + 2 B \sqrt{x}}{b} & \text{for}\: a = 0 \\ - \frac{\sqrt[4]{-1} A \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} A \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{3}{4}}} - \frac{\sqrt[4]{-1} A \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{a^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} B \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 b} - \frac{\sqrt[4]{-1} B \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 b} + \frac{\sqrt[4]{-1} B \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{b} + \frac{2 B \sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((2
*A*sqrt(x) + 2*B*x**(5/2)/5)/a, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x
))/b, Eq(a, 0)), (-(-1)**(1/4)*A*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/
b)**(1/4) + sqrt(x))/(2*a**(3/4)) + (-1)**(1/4)*A*(1/b)**(1/4)*log((-1)**(1
/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) - (-1)**(1/4)*A*(1/b)**(1
/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4) + (-1)**(1/4
)*B*a**(1/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))
/(2*b) - (-1)**(1/4)*B*a**(1/4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)
**(1/4) + sqrt(x))/(2*b) + (-1)**(1/4)*B*a**(1/4)*(1/b)**(1/4)*atan((-1)**(
3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/b + 2*B*sqrt(x)/b, True))
```

3.354 $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=235

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{(Ab-aB)\tan^{-}}{\sqrt{2}\,a}$$

Rubi [A] time = 0.18, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {453, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((A*b - a*B)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(3/4)) +
((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(5/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a\sqrt{b}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x,\right.}{a\sqrt{b}}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2ab} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+}\right.}{2ab}$$

$$= -\frac{2A}{a\sqrt{x}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{3/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\,a^{5/4}b^{3/4}}$$

$$= -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}} - \frac{(Ab-aB)}{}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.31

$$\frac{\frac{(aB-Ab)\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}\,b^{3/4}}-\frac{2A}{\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((-2*A)/Sqrt[x] + ((-(A*b) + a*B)*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + A
rcTanh[(a*b^(1/4)*Sqrt[x])/(-a)^(5/4)]))/((-a)^(1/4)*b^(3/4)))/a
```

IntegrateAlgebraic [A] time = 0.19, size = 135, normalized size = 0.57

$$-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{5/4}b^{3/4}}-\frac{2A}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) - ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((-(A*b) + a*B)*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(5
/4)*b^(3/4))
```

fricas [B] time = 1.13, size = 843, normalized size = 3.59

$$\frac{4\,ax\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^{6}a^{6}-6\,AB^{5}a^{5}b+15\,A^{2}B^{4}a^{4}b^{2}-20\,A^{3}B^{3}a^{3}b^{3}+15\,A^{4}B^{2}a^{2}b^{4}-6\,A^{5}Bab^{5}+A^{6}b^{6}\right)x-\left(B^{4}a^{7}b-4\,AB^{3}a^{6}b^{2}+6\,A^{2}B^{2}a^{5}b^{3}-4\,A^{3}Ba^{4}b^{4}+A^{4}a^{3}b^{5}\right)\sqrt{-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}}}\,ab\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{1}{4}}+\left(B^{3}a^{4}b-3\,AB^{2}a^{3}b^{2}+3\,A^{2}Ba^{2}b^{3}-A^{3}ab^{4}\right)\sqrt{x}\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{1}{4}}}{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}\right)-ax\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{1}{4}}\log\left(a^{4}b^{2}\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{3}{4}}-\left(B^{3}a^{3}-3\,AB^{2}a^{2}b+3\,A^{2}Bab^{2}-A^{3}b^{3}\right)\sqrt{x}\right)+ax\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{1}{4}}\log\left(-a^{4}b^{2}\left(-\frac{B^{4}a^{4}-4\,AB^{3}a^{3}b+6\,A^{2}B^{2}a^{2}b^{2}-4\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{3}}\right)^{\frac{3}{4}}-\left(B^{3}a^{3}-3\,AB^{2}a^{2}b+3\,A^{2}Bab^{2}-A^{3}b^{3}\right)\sqrt{x}\right)-4\,A\sqrt{x}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a^5*b^3))^(1/4)*arctan((sqrt((B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)*x - (B^4*a^7*b - 4*A*B^3*a^6*b^2 + 6*A^2*B^2*a^5*b^3 - 4*A^3*B*a^4*b^4
 + A^4*a^3*b^5)*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^3)))*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a
^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4) + (B^3*a^4*b - 3*A*B^2*a
^3*b^2 + 3*A^2*B*a^2*b^3 - A^3*a*b^4)*sqrt(x)*(-(B^4*a^4 - 4*A*B^3*a^3*b +
6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4))/(B^4*a^4 - 4
*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)) - a*x*(-(B^4*a
^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3)
)^(1/4)*log(a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*
b^2 - A^3*b^3)*sqrt(x)) + a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-(B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4)
- (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - 4*A*sqrt(x
))/(a*x)
```

giac [A] time = 0.37, size = 251, normalized size = 1.07

$$-\frac{2\,A}{a\sqrt{x}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{2}b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{2}b^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{2}b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1
```

```
/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)
^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a^2*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

maple [A] time = 0.01, size = 277, normalized size = 1.18

$$-\frac{\sqrt{2}\, A \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, A \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, A \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}+\frac{\sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, B \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}-\frac{2 A}{a \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/a/(a
/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/b/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)-1)+1/4/b/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/b/(a/b)
^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-2*A/a/x^(1/2)
```

maxima [A] time = 2.42, size = 194, normalized size = 0.83

$$\frac{(Ba-Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\, x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\, x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4a}-\frac{2A}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)))/a - 2*A/(a*sqrt(x))
```

mupad [B] time = 0.16, size = 71, normalized size = 0.30

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{5/4}\,b^{3/4}}-\frac{2\,A}{a\,\sqrt{x}}-\frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{5/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^(5/4)*b^(3/4)) - (2*
A)/(a*x^(1/2)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^(5
/4)*b^(3/4))
```

sympy [A] time = 19.84, size = 206, normalized size = 0.88

$$A\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0\\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0\\ -\frac{2}{a\sqrt{x}} & \text{for}\: b = 0\\ -\frac{2}{a\sqrt{x}}+\frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise}\end{cases}\right)+2B\operatorname{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(64t^3ab^2+\sqrt{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a),x)
```

```
[Out] A*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a,
0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(
1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4)) - (-1)**(3/
4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4
)) - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(5/4
)*(1/b)**(1/4)), True)) + 2*B*RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*l
og(64*_t**3*a*b**2 + sqrt(x))))
```

3.355 $\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=237

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\tan^{-1}(}{\sqrt{2}\,a^{7/}}$$

Rubi [A] time = 0.18, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {453, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) - ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((A*b - a*B)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(1/4))
- ((A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(7/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{3a}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}} - \frac{(Ab-aB)\,\text{Subst}(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x,}{a^{3/2}}$$

$$= -\frac{2A}{3ax^{3/2}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}\sqrt{b}} - \frac{(Ab-aB)\,\text{Subst}(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}}}{2a^{3/2}\sqrt{b}}$$

$$= -\frac{2A}{3ax^{3/2}} + \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{(Ab-aB)\log(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}}$$

$$= -\frac{2A}{3ax^{3/2}} + \frac{(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{(Ab-aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{(Ab-a}{}$$

Mathematica [A] time = 0.14, size = 168, normalized size = 0.71

$$\frac{(Ab-aB)\left(\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)}{2\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + ((A*b - a*B)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^(7/4)*b^(1/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 136, normalized size = 0.57

$$-\frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}} + \frac{(aB-Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{7/4}\sqrt[4]{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((-(A*b) + a*B)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^
(7/4)*b^(1/4))
```

fricas [B] time = 1.41, size = 653, normalized size = 2.76

$$-\frac{12\,ax^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^4\sqrt{-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}}+\left(B^2a^2-2\,ABab+A^2b^2\right)x}\,a^5b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{3}{4}}+\left(Ba^6b-Aa^5b^2\right)\sqrt{x}\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{3}{4}}}{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}\right)+3\,ax^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)-3\,ax^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}-(Ba-Ab)\sqrt{x}\right)+4\,A\sqrt{x}}{6\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4)/(a^7*b))^(1/4)*arctan((sqrt(a^4*sqrt(-(B^4*a^4 - 4*A*B^3*a^3*
b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b)) + (B^2*a^2 - 2*A*
B*a*b + A^2*b^2)*x)*a^5*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(3/4) + (B*a^6*b - A*a^5*b^2)*sqrt(x)*(-(
B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7
*b))^(3/4))/(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
A^4*b^4)) + 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4)*log(a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(
x)) - 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a^7*b))^(1/4)*log(-a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2
*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(x)) +
 4*A*sqrt(x))/(a*x^2)
```

giac [A] time = 0.36, size = 251, normalized size = 1.06

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/2*sqrt(2)*((a*b^3)^(
1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*
sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1
/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 1/4*sqr
```

```
t(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^2*b) - 2/3*A/(a*x^(3/2))
```

maple [A] time = 0.01, size = 280, normalized size = 1.18

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}-\frac{2A}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/4/a^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b-1/2/a^2*(a/b)^(1/4)*2^(1/
2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b-1/2/a^2*(a/b)^(1/4)*2^(1/2)*A*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b+1/4/a*(a/b)^(1/4)*2^(1/2)*B*ln((x+(
a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2)))+1/2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
+1/2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2/3*A/a/
x^(3/2)
```

maxima [A] time = 2.37, size = 218, normalized size = 0.92

$$\frac{\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,a}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a - 2/3*A/(a*x^(3/2))
```

mupad [B] time = 0.38, size = 811, normalized size = 3.42

$$-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,(-a)^{7/4}\,b^{1/4}}\right)1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,(-a)^{7/4}\,b^{1/4}}\right)1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,(-a)^{7/4}\,b^{1/4}}\right)}{2\,(-a)^{7/4}\,b^{1/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,(-a)^{7/4}\,b^{1/4}}\right)}{2\,(-a)^{7/4}\,b^{1/4}}}\right)\,(A\,b-B\,a)\,1\mathrm{i}}{(-a)^{7/4}\,b^{1/4}}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}\right)}{2\,(-a)^{7/4}\,b^{1/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}\right)}{2\,(-a)^{7/4}\,b^{1/4}}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}\right)1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)+\frac{(A\,b-B\,a)\,\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)\,1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}\right)1\mathrm{i}}{2\,(-a)^{7/4}\,b^{1/4}}}\right)\,(A\,b-B\,a)}{(-a)^{7/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] - (2*A)/(3*a*x^(3/2)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B
^2*a^5*b^3 - 32*A*B*a^4*b^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/
(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4)) + ((A*b - B*a)*(x^(1/2)*
(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5
*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4)))/(
((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) -
((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4)*b^(1/4))))/(2*(-a
)^(7/4)*b^(1/4)) - ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 -
 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4
)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4))))*(A*b - B*a)*1i)/((-a)^(7/4)*b^(1/4))
- (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^
4*b^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/
4))))/(2*(-a)^(7/4)*b^(1/4)) + ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B
^2*a^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1
i)/(2*(-a)^(7/4)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4)))/(((A*b - B*a)*(x^(1/2)*
(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) - ((A*b - B*a)*(32*A*a^5
*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4))
 - ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4)
 + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/4)))*1
i)/(2*(-a)^(7/4)*b^(1/4))))*(A*b - B*a))/((-a)^(7/4)*b^(1/4))
```

sympy [A] time = 31.26, size = 364, normalized size = 1.54

$$\begin{cases} \tilde{\infty} \left(- \frac{2A}{7 x^{\frac{7}{2}}} - \frac{2B}{3 x^{\frac{3}{2}}}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{- \frac{2A}{7 x^{\frac{7}{2}}} - \frac{2B}{3 x^{\frac{3}{2}}}}{b} & \text{for}\: a = 0 \\ \frac{- \frac{2A}{3 x^{\frac{3}{2}}} + 2B \sqrt{x}}{a} & \text{for}\: b = 0 \\ - \frac{2A}{3 a x^{\frac{3}{2}}} + \frac{\sqrt[4]{-1} A b \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{7}{4}}} - \frac{\sqrt[4]{-1} A b \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{7}{4}}} + \frac{\sqrt[4]{-1} A b \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{a^{\frac{7}{4}}} - \frac{\sqrt[4]{-1} B \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} B \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{2 a^{\frac{3}{4}}} - \frac{\sqrt[4]{-1} B \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + (-1)**(1/4)*A*b*(1/b)*
*(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) - (-1
)**(1/4)*A*b*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/
(2*a**(7/4)) + (-1)**(1/4)*A*b*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1
/4)*(1/b)**(1/4)))/a**(7/4) - (-1)**(1/4)*B*(1/b)**(1/4)*log(-(-1)**(1/4)*a
**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) + (-1)**(1/4)*B*(1/b)**(1/4)*l
og((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) - (-1)**(1/4)*
B*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4),
True))
```

3.356 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=255

$$\frac{\sqrt[4]{b}\,(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{\sqrt[4]{b}\,(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{\sqrt[4]{b}\,(Ab-}{}$$

Rubi [A] time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2(Ab-aB)}{a^2\sqrt{x}}+\frac{\sqrt[4]{b}\,(Ab-aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{\sqrt[4]{b}\,(Ab-aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}}-\frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}}+\frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}}-\frac{2A}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + (2*(A*b - a*B))/(a^2*Sqrt[x]) - (b^(1/4)*(A*b - a*B)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)) + (b^(1/4
)*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/
4)) + (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)) - (b^(1/4)*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)}\,dx &= -\frac{2A}{5ax^{5/2}} - \frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{5a} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(b(Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^2} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(2b(Ab-aB))\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\left(\sqrt{b}\,(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2} + \frac{\left(\sqrt{b}\,(Ab-aB)\right.}{} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2} + \frac{(Ab-aB)\,\text{Subs}}{} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} + \frac{\sqrt[4]{b}\,(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}} - \frac{\sqrt[4]{b}\,(Ab-a}{} \\
&= -\frac{2A}{5ax^{5/2}} + \frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}} + \frac{\sqrt[4]{b}\,(Ab-aB)\tan^{-1}\left(1+\right.}{\sqrt{2}\,a^{9/4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.18

$$
-\frac{2\left(5x^2(aB-Ab)\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\frac{bx^2}{a}\right)+aA\right)}{5a^2x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*A + 5*(-(A*b) + a*B)*x^2*Hypergeometric2F1[-1/4, 1, 3/4, -((b*x^2)/a
)]))/(5*a^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 160, normalized size = 0.63

$$\frac{\left(a\sqrt[4]{b}B-Ab^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{9/4}}+\frac{\left(a\sqrt[4]{b}B-Ab^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{9/4}}-\frac{2\left(aA+5aBx^2-5Abx^2\right)}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*A - 5*A*b*x^2 + 5*a*B*x^2))/(5*a^2*x^(5/2)) + ((-(A*b^(5/4)) + a*b^(
1/4)*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(S
qrt[2]*a^(9/4)) + ((-(A*b^(5/4)) + a*b^(1/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(9/4))
```

fricas [B] time = 1.04, size = 883, normalized size = 3.46

$$-\frac{20a^2x^3\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(B^6a^6b^2-6AB^5a^5b^3+15A^2B^4a^4b^4-20A^3B^3a^3b^5+15A^4B^2a^2b^6-6A^5Bab^7+A^6b^8\right)x-\left(B^4a^9b-4AB^3a^8b^2+6A^2B^2a^7b^3-4A^3Ba^6b^4+A^4a^5b^5\right)\sqrt{-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}}}\,a^2\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}+\left(B^3a^5b-3AB^2a^4b^2+3A^2Ba^3b^3-A^3a^2b^4\right)\sqrt{x}\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}}{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}\right)-5a^2x^3\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{3}{4}}-\left(B^3a^3b-3AB^2a^2b^2+3A^2Bab^3-A^3b^4\right)\sqrt{x}\right)+5a^2x^3\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}\log\left(-a^7\left(-\frac{B^4a^4b-4AB^3a^3b^2+6A^2B^2a^2b^3-4A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{3}{4}}-\left(B^3a^3b-3AB^2a^2b^2+3A^2Bab^3-A^3b^4\right)\sqrt{x}\right)+4\left(5(Ba-Ab)x^2+Aa\right)\sqrt{x}}{10a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/10*(20*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^
3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*arctan((sqrt((B^6*a^6*b^2 - 6*A*B^5*a^5*b^3
 + 15*A^2*B^4*a^4*b^4 - 20*A^3*B^3*a^3*b^5 + 15*A^4*B^2*a^2*b^6 - 6*A^5*B*a
*b^7 + A^6*b^8)*x - (B^4*a^9*b - 4*A*B^3*a^8*b^2 + 6*A^2*B^2*a^7*b^3 - 4*A^
3*B*a^6*b^4 + A^4*a^5*b^5)*sqrt(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9))*a^2*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2
+ 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4) + (B^3*a^5*b - 3*
A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3 - A^3*a^2*b^4)*sqrt(x)*(-(B^4*a^4*b - 4*A*B
^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4))/(B^4*
a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)) - 5
*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4
 + A^4*b^5)/a^9)^(1/4)*log(a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2
 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x)) + 5*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3
*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(-a^7*(-(
B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/
a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x
)) + 4*(5*(B*a - A*b)*x^2 + A*a)*sqrt(x))/(a^2*x^3)
```

giac [A] time = 0.35, size = 268, normalized size = 1.05

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^3b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^3b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^3b^2}-\frac{2\left(5Bax^2-5Abx^2+Aa\right)}{5a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) - 1/2*sqrt(2)*((a*b^3
)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^
3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) -
1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 2/5*(5*B*a*x^2 - 5*A*b*x^2 + A*a)/(a
^2*x^(5/2))
```

maple [A] time = 0.02, size = 299, normalized size = 1.17

$$\frac{\sqrt{2}\, A b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{\sqrt{2}\, A b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{\sqrt{2}\, A b \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{\sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, B \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}+\frac{2 A b}{a^{2} \sqrt{x}}-\frac{2 B}{a \sqrt{x}}-\frac{2 A}{5 a\, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a),x)
```

```
[Out] 1/2/a^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b+1/2/a
^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b+1/4/a^2/(a
/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b-1/2/a/(a/b)^(1/4)*2^(1/2)*B*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)-1/4/a/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*2^(1/2)*
x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-2/5*A/a/x
^(5/2)+2/a^2/x^(1/2)*A*b-2/a/x^(1/2)*B
```

maxima [A] time = 2.38, size = 213, normalized size = 0.84

$$-\frac{\left(B a b - A b^{2}\right) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4 a^{2}}-\frac{2 \left(5 (B a-A b) x^{2}+A a\right)}{5 a^{2} x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(B*a*b - A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt
(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(s
```

```
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + s
qrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)))/a^2 - 2/5*(5*(B*a - A*b)*x^2 + A*a)/(a^2*x^(5/2))
```

mupad [B] time = 0.31, size = 90, normalized size = 0.35

$$\frac{(-b)^{1/4}\operatorname{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)(A\,b-B\,a)}{a^{9/4}}-\frac{\frac{2\,A}{5\,a}-\frac{2\,x^2\,(A\,b-B\,a)}{a^2}}{x^{5/2}}-\frac{(-b)^{1/4}\operatorname{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)(A\,b-B\,a)}{a^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4))*(A*b - B*a))/a^(9/4) - ((2*A
)/(5*a) - (2*x^2*(A*b - B*a))/a^2)/x^(5/2) - ((-b)^(1/4)*atanh(((-b)^(1/4)*
x^(1/2))/a^(1/4))*(A*b - B*a))/a^(9/4)
```

sympy [A] time = 124.89, size = 366, normalized size = 1.44

$$A\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0\\-\frac{2}{9bx^{\frac{9}{2}}} & \text{for}\: a = 0\\-\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\: b = 0\\-\frac{2}{5ax^{\frac{5}{2}}}+\frac{2b}{a^2\sqrt{x}}-\frac{(-1)^{\frac{3}{4}}b\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}b\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}b\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise}\end{cases}\right)+B\left(\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0\\-\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0\\-\frac{2}{a\sqrt{x}} & \text{for}\: b = 0\\-\frac{2}{a\sqrt{x}}+\frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}}-\frac{(-1)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a),x)
```

```
[Out] A*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a,
0)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(a**2*sqrt(x))
 - (-1)**(3/4)*b*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(9
/4)*(1/b)**(1/4)) + (-1)**(3/4)*b*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + s
qrt(x))/(2*a**(9/4)*(1/b)**(1/4)) + (-1)**(3/4)*b*atan((-1)**(3/4)*sqrt(x)/
(a**(1/4)*(1/b)**(1/4)))/(a**(9/4)*(1/b)**(1/4)), True)) + B*Piecewise((zoo
/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt
(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1
/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4)) - (-1)**(3/4)*log((-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4)) - (-1)**(3/4)
*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(5/4)*(1/b)**(1/4)),
 True))
```

3.357 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{13/4}}+$$

Rubi [A] time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = $\{457, 321, 329, 211, 1165, 628, 1162, 617, 204\}$

$$-\frac{x^{5/2}(5Ab-9aB)}{10ab^2}+\frac{\sqrt{x}\,(5Ab-9aB)}{2b^3}+\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{13/4}}+\frac{\sqrt[4]{a}\,(5Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{13/4}}-\frac{\sqrt[4]{a}\,(5Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,b^{13/4}}+\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((5*A*b - 9*a*B)*Sqrt[x])/(2*b^3) - ((5*A*b - 9*a*B)*x^(5/2))/(10*a*b^2) +
((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) + (a^(1/4)*(5*A*b - 9*a*B)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5
*A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(
13/4)) + (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4)) - (a^(1/4)*(5*A*b - 9*a*B)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx &= \frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}+\frac{\left(-\frac{5Ab}{2}+\frac{9aB}{2}\right)\int\frac{x^{7/2}}{a+bx^2}\,dx}{2ab}\\
&= -\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}+\frac{(5Ab-9aB)\int\frac{x^{3/2}}{a+bx^2}\,dx}{4b^2}\\
&= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}-\frac{(a(5Ab-9aB))\int\frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^3}\\
&= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}-\frac{(a(5Ab-9aB))\,\text{Subst}\left(\int\frac{1}{a+bx^4}\right.}{2b^3}\\
&= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}-\frac{\left(\sqrt{a}\,(5Ab-9aB)\right)\text{Subst}\left(\int\frac{\sqrt{a}-}{a+}\right.}{4b^3}\\
&= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}-\frac{\left(\sqrt{a}\,(5Ab-9aB)\right)\text{Subst}\left(\int\frac{}{\frac{\sqrt{a}}{\sqrt{b}}-}\right.}{8b^{7/2}}\\
&= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}+\frac{\sqrt[4]{a}\,(5Ab-9aB)\log\left(\sqrt{a}-\sqrt{2}\right.}{8\sqrt{2}\,b^{13/4}}\\
&= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\left(a+bx^2\right)}+\frac{\sqrt[4]{a}\,(5Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}\,b^{13/4}}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 385, normalized size = 1.24

$$\frac{-45\sqrt{2}a^{5/4}B\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+45\sqrt{2}a^{5/4}B\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\frac{40a^2\sqrt[4]{b}B\sqrt{x}}{a+bx^2}+\frac{40aAb^{5/4}\sqrt{x}}{a+bx^2}-10\sqrt{2}\sqrt[4]{a}(9aB-5Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+10\sqrt{2}\sqrt[4]{a}(9aB-5Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+25\sqrt{2}\sqrt[4]{a}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-25\sqrt{2}\sqrt[4]{a}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-320a\sqrt[4]{b}B\sqrt{x}+160Ab^{5/4}\sqrt{x}+32b^{5/4}Bx^{5/2}}{80b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (160*A*b^(5/4)*Sqrt[x] - 320*a*b^(1/4)*B*Sqrt[x] + 32*b^(5/4)*B*x^(5/2) + (
40*a*A*b^(5/4)*Sqrt[x])/(a + b*x^2) - (40*a^2*b^(1/4)*B*Sqrt[x])/(a + b*x^2
) - 10*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] + 10*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)] + 25*Sqrt[2]*a^(1/4)*A*b*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 45*Sqrt[2]*a^(5/4)*B*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 25*Sqrt[2]*a^(1/4)*A*b*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]*a^(5/4)*B*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(80*b^(13/4))
```

IntegrateAlgebraic [A] time = 0.62, size = 208, normalized size = 0.67

$$-\frac{\left(9a^{5/4}B-5\sqrt[4]{a}\,Ab\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{\left(9a^{5/4}B-5\sqrt[4]{a}\,Ab\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,b^{13/4}}+\frac{-45a^2B\sqrt{x}+25aAb\sqrt{x}-36abBx^{5/2}+20Ab^2x^{5/2}+4b^2Bx^{9/2}}{10b^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (25*a*A*b*Sqrt[x] - 45*a^2*B*Sqrt[x] + 20*A*b^2*x^(5/2) - 36*a*b*B*x^(5/2)
+ 4*b^2*B*x^(9/2))/(10*b^3*(a + b*x^2)) - ((-5*a^(1/4)*A*b + 9*a^(5/4)*B)*A
rcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*
b^(13/4)) + ((-5*a^(1/4)*A*b + 9*a^(5/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*b^(13/4))
```

fricas [B] time = 1.61, size = 748, normalized size = 2.41

$$-\frac{20\left(b^4x^2+ab^3\right)\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{b^6\sqrt{-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}}+\left(81\,B^2a^2-90\,ABab+25\,A^2b^2\right)x}\,b^{10}\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{3}{4}}+\left(9\,Bab^{10}-5\,Ab^{11}\right)\sqrt{x}\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{3}{4}}}{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}\right)+5\left(b^4x^2+ab^3\right)\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}-\left(9\,Ba-5\,Ab\right)\sqrt{x}\right)-5\left(b^4x^2+ab^3\right)\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(-b^3\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}-\left(9\,Ba-5\,Ab\right)\sqrt{x}\right)-4\left(4\,Bb^2x^4-45\,Ba^2+25\,Aab-4\left(9\,Bab-5\,Ab^2\right)x^2\right)\sqrt{x}}{40\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2
*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*arctan((sqrt
(b^6*sqrt(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500
*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13) + (81*B^2*a^2 - 90*A*B*a*b + 25*A^2*b
^2)*x)*b^10*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4
500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(3/4) + (9*B*a*b^10 - 5*A*b^11)*sq
rt(x)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^
3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(3/4))/(6561*B^4*a^5 - 14580*A*B^3*a^4*b
 + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)) + 5*(b^4*x^
2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 45
00*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(b^3*(-(6561*B^4*a^5 - 145
80*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4
```

```
)/b^13)^(1/4) - (9*B*a - 5*A*b)*sqrt(x)) - 5*(b^4*x^2 + a*b^3)*(-(6561*B^4*
a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*
A^4*a*b^4)/b^13)^(1/4)*log(-b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150
*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a
 - 5*A*b)*sqrt(x)) - 4*(4*B*b^2*x^4 - 45*B*a^2 + 25*A*a*b - 4*(9*B*a*b - 5*
A*b^2)*x^2)*sqrt(x))/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.51, size = 298, normalized size = 0.96

$$\frac{\sqrt{2}\left(9\left(ab^{3}\right)^{\frac{1}{4}}Ba-5\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}}+\frac{\sqrt{2}\left(9\left(ab^{3}\right)^{\frac{1}{4}}Ba-5\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}}+\frac{\sqrt{2}\left(9\left(ab^{3}\right)^{\frac{1}{4}}Ba-5\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^{4}}-\frac{\sqrt{2}\left(9\left(ab^{3}\right)^{\frac{1}{4}}Ba-5\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^{4}}-\frac{Ba^{2}\sqrt{x}-Aab\sqrt{x}}{2\left(bx^{2}+a\right)b^{3}}+\frac{2\left(Bb^{8}x^{\frac{5}{2}}-10\,Bab^{7}\sqrt{x}+5\,Ab^{8}\sqrt{x}\right)}{5\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/8*sqrt(2)*(9*(a*b^3)
^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
- 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/16*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^
3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/16*s
qrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/b^4 - 1/2*(B*a^2*sqrt(x) - A*a*b*sqrt(x))/((b*x^2
 + a)*b^3) + 2/5*(B*b^8*x^(5/2) - 10*B*a*b^7*sqrt(x) + 5*A*b^8*sqrt(x))/b^1
0
```

maple [A] time = 0.02, size = 339, normalized size = 1.09

$$\frac{2Bx^{\frac{5}{2}}}{5b^{2}}+\frac{Aa\sqrt{x}}{2\left(bx^{2}+a\right)b^{2}}-\frac{Ba^{2}\sqrt{x}}{2\left(bx^{2}+a\right)b^{3}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^{2}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ba\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^{3}}+\frac{2A\sqrt{x}}{b^{2}}-\frac{4Ba\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2/5/b^2*B*x^(5/2)+2/b^2*A*x^(1/2)-4/b^3*B*a*x^(1/2)+1/2*a/b^2*x^(1/2)/(b*x^
2+a)*A-1/2*a^2/b^3*x^(1/2)/(b*x^2+a)*B-5/8/b^2*(a/b)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/b^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)-5/16/b^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+9/
8*a/b^3*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*a/b
^3*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+9/16*a/b^3*(
a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b
)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))
```

maxima [A] time = 2.46, size = 271, normalized size = 0.87

$$-\frac{\left(Ba^2-Aab\right)\sqrt{x}}{2\left(b^4x^2+ab^3\right)}+\frac{\left(\frac{2\sqrt{2}(9Ba-5Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(9Ba-5Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(9Ba-5Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(9Ba-5Ab)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{16b^3}+\frac{2\left(Bbx^{\frac{5}{2}}-5(2Ba-Ab)\sqrt{x}\right)}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a^2 - A*a*b)*sqrt(x)/(b^4*x^2 + a*b^3) + 1/16*(2*sqrt(2)*(9*B*a - 5
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a - 5*A
*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - 5*A*b)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4
)) - sqrt(2)*(9*B*a - 5*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^3 + 2/5*(B*b*x^(5/2) - 5*(2*B*a - A*b)
*sqrt(x))/b^3
```

mupad [B] time = 0.37, size = 823, normalized size = 2.65

$$\sqrt{x}\left(\frac{2A}{b^2}-\frac{4Ba}{b^3}\right)+\frac{2Bx^{5/2}}{5b^2}-\frac{\sqrt{x}\left(\frac{Ba^2}{2}-\frac{Aab}{2}\right)}{b^4x^2+ab^3}+\frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)}{8b^{13/4}}\right)(5Ab-9Ba)\,1i}{8b^{13/4}}+\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}+\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)}{8b^{13/4}}\right)(5Ab-9Ba)\,1i}{8b^{13/4}}}{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)}{8b^{13/4}}\right)(5Ab-9Ba)}{8b^{13/4}}-\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}+\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)}{8b^{13/4}}\right)(5Ab-9Ba)}{8b^{13/4}}}\right)(5Ab-9Ba)\,1i}{4b^{13/4}}+\frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)1i}{8b^{13/4}}\right)(5Ab-9Ba)}{8b^{13/4}}+\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}+\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)1i}{8b^{13/4}}\right)(5Ab-9Ba)}{8b^{13/4}}}{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)1i}{8b^{13/4}}\right)(5Ab-9Ba)\,1i}{8b^{13/4}}-\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}+\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)1i}{8b^{13/4}}\right)(5Ab-9Ba)\,1i}{8b^{13/4}}}\right)(5Ab-9Ba)}{4b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] x^(1/2)*((2*A)/b^2 - (4*B*a)/b^3) + (2*B*x^(5/2))/(5*b^2) - (x^(1/2)*((B*a^
2)/2 - (A*a*b)/2))/(a*b^3 + b^4*x^2) + ((-a)^(1/4)*atan((((-a)^(1/4)*((x^(1
/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b
- 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a)*1i)/(8*b^(1
3/4)) + ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))
/b^3 + ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(
5*A*b - 9*B*a)*1i)/(8*b^(13/4)))/(((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^
2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40
*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(13/4)) - ((-a)^(1/4)*((x^(1
/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 + ((-a)^(1/4)*(5*A*b
- 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(13/4
))))*(5*A*b - 9*B*a)*1i)/(4*b^(13/4)) + ((-a)^(1/4)*atan((((-a)^(1/4)*((x^(
1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b
 - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(
13/4)) + ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b)
)/b^3 + ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)
))*(5*A*b - 9*B*a))/(8*b^(13/4)))/(((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A
```

```
^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 4
0*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a)*1i)/(8*b^(13/4)) - ((-a)^(1/4)
*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 + ((-a)^(1/4)*
(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a)*1
i)/(8*b^(13/4))))*(5*A*b - 9*B*a))/(4*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.358 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{11/4}}-\frac{(3Ab-7aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{11/4}}-\frac{(3Ab-7aB)\,t}{4\sqrt{}}$$

Rubi [A] time = 0.22, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}(3Ab-7aB)}{6ab^2}+\frac{(3Ab-7aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{11/4}}-\frac{(3Ab-7aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{11/4}}-\frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{11/4}}+\frac{(3Ab-7aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{11/4}}+\frac{x^{7/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((3*A*b - 7*a*B)*x^(3/2))/(6*a*b^2) + ((A*b - a*B)*x^(7/2))/(2*a*b*(a + b*
x^2)) - ((3*A*b - 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*
Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) + ((3*A*b - 7*a*B)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/
4)) - ((3*A*b - 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{\left(-\frac{3Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4b^2}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} - \frac{(3Ab-7aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4b^{5/2}} +$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8b^3} +$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} + \frac{(3Ab-7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{11/4}} -$$

$$= -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab\left(a+bx^2\right)} - \frac{(3Ab-7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{11/4}} +$$

Mathematica [C] time = 0.20, size = 136, normalized size = 0.47

$$\frac{3(Ab-2aB)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+(6aB-3Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+2\sqrt[4]{-a}\,b^{3/4}Bx^{3/2}}{3\sqrt[4]{-a}\,b^{11/4}}+\frac{2x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3ab^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (2*(-a)^(1/4)*b^(3/4)*B*x^(3/2) + 3*(A*b - 2*a*B)*ArcTan[(b^(1/4)*Sqrt[x])/
(-a)^(1/4)] + (-3*A*b + 6*a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(3*(-
a)^(1/4)*b^(11/4)) + (2*(-(A*b) + a*B)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/
4, -((b*x^2)/a)])/(3*a*b^2)
```

IntegrateAlgebraic [A] time = 0.67, size = 167, normalized size = 0.58

$$\frac{(7aB-3Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}+\frac{(7aB-3Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{11/4}}+\frac{x^{3/2}\left(7aB-3Ab+4bBx^2\right)}{6b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (x^(3/2)*(-3*A*b + 7*a*B + 4*b*B*x^2))/(6*b^2*(a + b*x^2)) + ((-3*A*b + 7*a
*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqr
t[2]*a^(1/4)*b^(11/4)) + ((-3*A*b + 7*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(1/4)*b^(11/4))
```

fricas [B] time = 1.45, size = 925, normalized size = 3.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B
^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*arctan((sqrt((11
7649*B^6*a^6 - 302526*A*B^5*a^5*b + 324135*A^2*B^4*a^4*b^2 - 185220*A^3*B^3
*a^3*b^3 + 59535*A^4*B^2*a^2*b^4 - 10206*A^5*B*a*b^5 + 729*A^6*b^6)*x - (24
01*B^4*a^5*b^5 - 4116*A*B^3*a^4*b^6 + 2646*A^2*B^2*a^3*b^7 - 756*A^3*B*a^2*
b^8 + 81*A^4*a*b^9)*sqrt(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a
^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11)))*b^3*(-(2401*B^4*a^4 - 411
6*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^1
1))^(1/4) + (343*B^3*a^3*b^3 - 441*A*B^2*a^2*b^4 + 189*A^2*B*a*b^5 - 27*A^3
*b^6)*sqrt(x)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 7
56*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4))/(2401*B^4*a^4 - 4116*A*B^3*a^
3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)) - 3*(b^3*x^2 +
a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*
B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(a*b^8*(-(2401*B^4*a^4 - 4116*A*B^
3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3
/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*sqrt(x
)) + 3*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*
a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(-a*b^8*(-(2401*
```

```
B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^
4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 -
 27*A^3*b^3)*sqrt(x)) - 4*(4*B*b*x^3 + (7*B*a - 3*A*b)*x)*sqrt(x))/(b^3*x^2
 + a*b^2)
```

giac [A] time = 0.42, size = 283, normalized size = 0.98

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^{2}}+\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\left(bx^{2}+a\right)b^{2}}-\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba-3\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^{5}}-\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba-3\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^{5}}+\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba-3\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^{5}}-\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{3}{4}}Ba-3\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 + 1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*b^2) - 1/8
*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) - 1/8*sqrt(2)*(7*(a*b^3
)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*
(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
 - 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sq
rt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
```

maple [A] time = 0.02, size = 317, normalized size = 1.10

$$-\frac{A\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b}+\frac{Ba\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}+\frac{3\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{7\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}-\frac{7\sqrt{2}\,Ba\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}-\frac{7\sqrt{2}\,Ba\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/b^2-1/2/b*x^(3/2)/(b*x^2+a)*A+1/2/b^2*x^(3/2)/(b*x^2+a)*B*a-7
/16/b^3/(a/b)^(1/4)*2^(1/2)*B*a*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-7/8/b^3/(a/b)^(1/4)*2^(1/2
)*B*a*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-7/8/b^3/(a/b)^(1/4)*2^(1/2)*B*a
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*A*ln((x
-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2)))+3/8/b^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+3/8/b^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.39, size = 223, normalized size = 0.77

$$\frac{(Ba-Ab)x^{\frac{3}{2}}}{2\left(b^{3}x^{2}+ab^{2}\right)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,b^{2}}-\frac{(7\,Ba-3\,Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)*x^(3/2)/(b^3*x^2 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/16*(7*B*a
 - 3*A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/
b^2
```

mupad [B] time = 0.20, size = 106, normalized size = 0.37

$$\frac{2\,B\,x^{3/2}}{3\,b^2}-\frac{x^{3/2}\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(3\,A\,b-7\,B\,a\right)}{4\,{\left(-a\right)}^{1/4}\,b^{11/4}}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,\left(3\,A\,b-7\,B\,a\right)\,1\mathrm{i}}{4\,{\left(-a\right)}^{1/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (
atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(3*A*b - 7*B*a))/(4*(-a)^(1/4)*b^(11/4))
 + (atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*(3*A*b - 7*B*a)*1i)/(4*(-a)^(1/4)
*b^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.359 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{(Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\tan}{4\sqrt{2}}$$

Rubi **[A]** time = 0.21, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(Ab-5aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{\sqrt{x}\,(Ab-5aB)}{2ab^2}+\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -((A*b - 5*a*B)*Sqrt[x])/(2*a*b^2) + ((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*x^
2)) - ((A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt
[2]*a^(3/4)*b^(9/4)) + ((A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(9/4)) - ((A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(9/4)) + ((A*b
 - 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sq
rt[2]*a^(3/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{\left(-\frac{Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b^2}$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4\sqrt{a}\,b^2} + (Ab-5$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab-5aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8\sqrt{a}\,b^{5/2}} + (Ab$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}b^{9/4}} + (Ab$$

$$= -\frac{(Ab-5aB)\sqrt{x}}{2ab^2} + \frac{(Ab-aB)x^{5/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}} + \frac{(Ab-5aB)\tan^{-1}}{4\sqrt{2}\,a}$$

Mathematica [A] time = 0.37, size = 353, normalized size = 1.24

$$\frac{\frac{2\sqrt{2}(5aB-Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2}(Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}} - \frac{\sqrt{2}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}} + \frac{\sqrt{2}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}} - \frac{8Ab^{5/4}\sqrt{x}}{a+bx^2} + \frac{8a\sqrt[4]{b}B\sqrt{x}}{a+bx^2} + 5\sqrt{2}\sqrt[4]{a}B\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) - 5\sqrt{2}\sqrt[4]{a}B\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right) + 32\sqrt[4]{b}B\sqrt{x}}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (32*b^(1/4)*B*Sqrt[x] - (8*A*b^(5/4)*Sqrt[x])/(a + b*x^2) + (8*a*b^(1/4)*B*
Sqrt[x])/(a + b*x^2) + (2*Sqrt[2]*(-(A*b) + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1
```

```
/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*(A*b - 5*a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (Sqrt[2]*A*b*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + 5*Sqrt[2]*a^(1/4)*B*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (Sqrt[2]*A*b*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) - 5*Sqrt[2]*a^(
1/4)*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*b^(9
/4))
```

IntegrateAlgebraic [A] time = 0.65, size = 167, normalized size = 0.59

$$\frac{(5aB-Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(5aB-Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{\sqrt{x}\left(5aB-Ab+4bBx^2\right)}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[x]*(-(A*b) + 5*a*B + 4*b*B*x^2))/(2*b^2*(a + b*x^2)) + ((-(A*b) + 5*a
*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqr
t[2]*a^(3/4)*b^(9/4)) - ((-(A*b) + 5*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

fricas [B] time = 1.01, size = 725, normalized size = 2.55

$$\frac{4\left(b^3x^2+ab^2\right)\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^4\sqrt{-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}}+\left(25B^2a^2-10ABab+A^2b^2\right)x}\,a^2b^7\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{3}{4}}+\left(5Ba^3b^7-Aa^2b^8\right)\sqrt{x}\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{3}{4}}}{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}\right)+\left(b^3x^2+ab^2\right)\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}-(5Ba-Ab)\sqrt{x}\right)-\left(b^3x^2+ab^2\right)\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{625B^4a^4-500AB^3a^3b+150A^2B^2a^2b^2-20A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}-(5Ba-Ab)\sqrt{x}\right)+4\left(4Bbx^2+5Ba-Ab\right)\sqrt{x}}{8\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2
*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*arctan((sqrt(a^2*b^4*sqrt
(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^9)) + (25*B^2*a^2 - 10*A*B*a*b + A^2*b^2)*x)*a^2*b^7*(-(625*
B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)
/(a^3*b^9))^(3/4) + (5*B*a^3*b^7 - A*a^2*b^8)*sqrt(x)*(-(625*B^4*a^4 - 500*
A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(3
/4))/(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3
+ A^4*b^4)) + (b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*
B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(a*b^2*(-(625*B
^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/
(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) - (b^3*x^2 + a*b^2)*(-(625*B^4*a^
4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*
b^9))^(1/4)*log(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b
```

```
^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) +
4*(4*B*b*x^2 + 5*B*a - A*b)*sqrt(x))/(b^3*x^2 + a*b^2)
```

giac [**A**] time = 0.43, size = 283, normalized size = 1.00

$$\frac{2\,B\sqrt{x}}{b^2}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3}+\frac{\sqrt{2}\left(5\,\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/
8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/16*sqrt(2)*(5*(a*b^
3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a*b^3) + 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*lo
g(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/2*(B*a*sqrt(x)
- A*b*sqrt(x))/((b*x^2 + a)*b^2)
```

maple [**A**] time = 0.02, size = 323, normalized size = 1.14

$$-\frac{A\sqrt{x}}{2\left(b\,x^2+a\right)b}+\frac{Ba\sqrt{x}}{2\left(b\,x^2+a\right)b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^2}+\frac{2B\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x)
```

```
[Out] 2*B/b^2*x^(1/2)-1/2/b*x^(1/2)/(b*x^2+a)*A+1/2/b^2*x^(1/2)/(b*x^2+a)*B*a+1/8
/b*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/b*(a/b
)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/16/b*(a/b)^(1/4
)/a*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-5/8/b^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)+1)-5/8/b^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)-1)-5/16/b^2*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*2^(1/2)*x
^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))
```

maxima [**A**] time = 2.49, size = 250, normalized size = 0.88

$$\frac{(Ba-Ab)\sqrt{x}}{2\left(b^3x^2+ab^2\right)}+\frac{2\,B\sqrt{x}}{b^2}-\frac{\frac{2\,\sqrt{2}\,(5\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(5\,Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(5\,Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a - A*b)*sqrt(x)/(b^3*x^2 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/16*(2*sqrt(
2)*(5*B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(
5*B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*B*a
 - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)
*b^(1/4)) - sqrt(2)*(5*B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

mupad [B] time = 0.37, size = 744, normalized size = 2.62

$$\frac{2\,B\,\sqrt{x}}{b^2}-\frac{\sqrt{x}\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}+\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}+\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}}{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,(-a)^{3/4}\,b^{9/4}}\right)}{8\,(-a)^{3/4}\,b^{9/4}}-\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,(-a)^{3/4}\,b^{9/4}}\right)}{8\,(-a)^{3/4}\,b^{9/4}}}\right)(A\,b-5\,B\,a)\,1\mathrm{i}}{4\,(-a)^{3/4}\,b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}\right)}{8\,(-a)^{3/4}\,b^{9/4}}+\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}\right)}{8\,(-a)^{3/4}\,b^{9/4}}}{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}-\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\,\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B\,a)\,\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}}\right)(A\,b-5\,B\,a)}{4\,(-a)^{3/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (atan
((((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b - ((A*b -
 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)
*b^(9/4)) + ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b
 + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*
(-a)^(3/4)*b^(9/4)))/(((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A
*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)
)))/(8*(-a)^(3/4)*b^(9/4)) - ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2
 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*
b^(9/4))))/(8*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a)*1i)/(4*(-a)^(3/4)*b^(9/4)
) + (atan((((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b
- ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(3/4)*b^(9/4))))/(8*(
-a)^(3/4)*b^(9/4)) + ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*
B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(3/4)*b^(9/
4))))/(8*(-a)^(3/4)*b^(9/4)))/(((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a
^2 - 10*A*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(
3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)*b^(9/4)) - ((A*b - 5*B*a)*((x^(1/2)*(A^2*b
^2 + 25*B^2*a^2 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*
1i)/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a))/(4*
(-a)^(3/4)*b^(9/4))
```

sympy [A] time = 84.94, size = 984, normalized size = 3.46

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b^{2}} & \text{for}\: a = 0 \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{a^{2}} & \text{for}\: b = 0 \\ -\frac{\sqrt[4]{-1}Aa^{\frac{5}{4}}b\sqrt[4]{\frac{1}{b}}\log{\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{\sqrt[4]{-1}Aa^{\frac{5}{4}}b\sqrt[4]{\frac{1}{b}}\log{\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}-\frac{2\sqrt[4]{-1}Aa^{\frac{5}{4}}b\sqrt[4]{\frac{1}{b}}\operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}-\frac{\sqrt[4]{-1}A\sqrt[4]{a}b^{2}x^{2}\sqrt[4]{\frac{1}{b}}\log{\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{\sqrt[4]{-1}A\sqrt[4]{a}b^{2}x^{2}\sqrt[4]{\frac{1}{b}}\log{\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}-\frac{2\sqrt[4]{-1}A\sqrt[4]{a}b^{2}x^{2}\sqrt[4]{\frac{1}{b}}\operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}-\frac{4Aab\sqrt{x}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{5\sqrt[4]{-1}Ba^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}\log{\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}-\frac{5\sqrt[4]{-1}Ba^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}\log{\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{10\sqrt[4]{-1}Ba^{\frac{9}{4}}\sqrt[4]{\frac{1}{b}}\operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{5\sqrt[4]{-1}Ba^{\frac{5}{4}}bx^{2}\sqrt[4]{\frac{1}{b}}\log{\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}-\frac{5\sqrt[4]{-1}Ba^{\frac{5}{4}}bx^{2}\sqrt[4]{\frac{1}{b}}\log{\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{10\sqrt[4]{-1}Ba^{\frac{5}{4}}bx^{2}\sqrt[4]{\frac{1}{b}}\operatorname{atan}{\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{20Ba^{2}\sqrt{x}}{8a^{2}b^{2}+8ab^{3}x^{2}}+\frac{16Babx^{\frac{5}{2}}}{8a^{2}b^{2}+8ab^{3}x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(3*x**(3/2)) + 2*B*sqrt(x))/b**2, Eq(a, 0)), ((2*A*x**(5/2)/5 + 2*B*x**
(9/2)/9)/a**2, Eq(b, 0)), (-(-1)**(1/4)*A*a**(5/4)*b*(1/b)**(1/4)*log(-(-1)
**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b**2 + 8*a*b**3*x**2) + (-
1)**(1/4)*A*a**(5/4)*b*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
 sqrt(x))/(8*a**2*b**2 + 8*a*b**3*x**2) - 2*(-1)**(1/4)*A*a**(5/4)*b*(1/b)*
*(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b**2 + 8*a
*b**3*x**2) - (-1)**(1/4)*A*a**(1/4)*b**2*x**2*(1/b)**(1/4)*log(-(-1)**(1/4
)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b**2 + 8*a*b**3*x**2) + (-1)**(1
/4)*A*a**(1/4)*b**2*x**2*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4)
 + sqrt(x))/(8*a**2*b**2 + 8*a*b**3*x**2) - 2*(-1)**(1/4)*A*a**(1/4)*b**2*x
**2*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*
b**2 + 8*a*b**3*x**2) - 4*A*a*b*sqrt(x)/(8*a**2*b**2 + 8*a*b**3*x**2) + 5*(
-1)**(1/4)*B*a**(9/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
 sqrt(x))/(8*a**2*b**2 + 8*a*b**3*x**2) - 5*(-1)**(1/4)*B*a**(9/4)*(1/b)**(
1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b**2 + 8*a*b*
*3*x**2) + 10*(-1)**(1/4)*B*a**(9/4)*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/
(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b**2 + 8*a*b**3*x**2) + 5*(-1)**(1/4)*B*a*
*(5/4)*b*x**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x)
)/(8*a**2*b**2 + 8*a*b**3*x**2) - 5*(-1)**(1/4)*B*a**(5/4)*b*x**2*(1/b)**(1
/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**2*b**2 + 8*a*b**
3*x**2) + 10*(-1)**(1/4)*B*a**(5/4)*b*x**2*(1/b)**(1/4)*atan((-1)**(3/4)*sq
rt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**2*b**2 + 8*a*b**3*x**2) + 20*B*a**2*sq
rt(x)/(8*a**2*b**2 + 8*a*b**3*x**2) + 16*B*a*b*x**(5/2)/(8*a**2*b**2 + 8*a*
b**3*x**2), True))
```

3.360 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=261

$$\frac{(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\,\mathrm{ta}}{4\sqrt{2}}$$

Rubi [A] time = 0.18, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(3aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}}-\frac{(3aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((A*b + 3*a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(7/4))
 + ((A*b + 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(5/4)*b^(7/4)) - ((A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{2ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{Ab}{2}+\frac{3aB}{2}\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4ab^{3/2}} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+}{a+b}\right.}{4ab^{3/2}}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab+3aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8ab^2} + \frac{(Ab+3aB)\,\text{Subst}\left(\int\right.}{8}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} + \frac{(Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{7/4}} - \frac{(Ab+3aB)\log\left(\sqrt{a}+\right.}{8\sqrt{2}}$$

$$= \frac{(Ab-aB)x^{3/2}}{2ab\left(a+bx^2\right)} - \frac{(Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}} + \frac{(Ab+3aB)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}} +$$

Mathematica [C] time = 0.12, size = 95, normalized size = 0.36

$$\frac{2x^{3/2}(Ab-aB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^2b} + \frac{B\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{\sqrt[4]{-a}\,b^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] (B*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b^(1/4)*Sqrt[x])/(-a)
^(5/4)]))/((-a)^(1/4)*b^(7/4)) + (2*(A*b - a*B)*x^(3/2)*Hypergeometric2F1[3
/4, 2, 7/4, -((b*x^2)/a)])/(3*a^2*b)
```

IntegrateAlgebraic [A] time = 0.57, size = 160, normalized size = 0.61

$$-\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}} - \frac{(3aB+Ab)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{5/4}b^{7/4}} - \frac{x^{3/2}(aB-Ab)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(A*b) + a*B)*x^(3/2))/(a*b*(a + b*x^2)) - ((A*b + 3*a*B)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(5/4)*b
^(7/4)) - ((A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
 + Sqrt[b]*x)])/(4*Sqrt[2]*a^(5/4)*b^(7/4))
```

fricas **[B]** time = 1.21, size = 912, normalized size = 3.49

$$-\frac{4\,(Ba-Ab)x^{\frac{3}{2}}+4\left(ab^{2}x^{2}+a^{2}b\right)\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{\left(729\,B^{6}a^{6}+1458\,AB^{5}a^{5}b+1215\,A^{2}B^{4}a^{4}b^{2}+540\,A^{3}B^{3}a^{3}b^{3}+135\,A^{4}B^{2}a^{2}b^{4}+18\,A^{5}Bab^{5}+A^{6}b^{6}\right)x-\left(81\,B^{4}a^{7}b^{3}+108\,AB^{3}a^{6}b^{4}+54\,A^{2}B^{2}a^{5}b^{5}+12\,A^{3}Ba^{4}b^{6}+A^{4}a^{3}b^{7}\right)\sqrt{-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}}}\,ab^{2}\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{1}{4}}-\left(27\,B^{3}a^{4}b^{2}+27\,AB^{2}a^{3}b^{3}+9\,A^{2}Ba^{2}b^{4}+A^{3}ab^{5}\right)\sqrt{x}\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{1}{4}}}{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}\right)-\left(ab^{2}x^{2}+a^{2}b\right)\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{1}{4}}\log\left(a^{4}b^{5}\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{3}{4}}+\left(27\,B^{3}a^{3}+27\,AB^{2}a^{2}b+9\,A^{2}Bab^{2}+A^{3}b^{3}\right)\sqrt{x}\right)+\left(ab^{2}x^{2}+a^{2}b\right)\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{1}{4}}\log\left(-a^{4}b^{5}\left(-\frac{81\,B^{4}a^{4}+108\,AB^{3}a^{3}b+54\,A^{2}B^{2}a^{2}b^{2}+12\,A^{3}Bab^{3}+A^{4}b^{4}}{a^{5}b^{7}}\right)^{\frac{3}{4}}+\left(27\,B^{3}a^{3}+27\,AB^{2}a^{2}b+9\,A^{2}Bab^{2}+A^{3}b^{3}\right)\sqrt{x}\right)}{8\left(ab^{2}x^{2}+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(B*a - A*b)*x^(3/2) + 4*(a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B
^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*
arctan((sqrt((729*B^6*a^6 + 1458*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2 + 540*A
^3*B^3*a^3*b^3 + 135*A^4*B^2*a^2*b^4 + 18*A^5*B*a*b^5 + A^6*b^6)*x - (81*B^
4*a^7*b^3 + 108*A*B^3*a^6*b^4 + 54*A^2*B^2*a^5*b^5 + 12*A^3*B*a^4*b^6 + A^4
*a^3*b^7)*sqrt(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)/(a^5*b^7)))*a*b^2*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4) - (27*B^3*a^4*
b^2 + 27*A*B^2*a^3*b^3 + 9*A^2*B*a^2*b^4 + A^3*a*b^5)*sqrt(x)*(-(81*B^4*a^4
 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^
7))^(1/4))/(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*
b^3 + A^4*b^4)) - (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(a^4*b^5*(-
(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b
^3)*sqrt(x)) + (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2
*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-(8
1*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4
)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3
)*sqrt(x)))/(a*b^2*x^2 + a^2*b)
```

giac **[A]** time = 0.37, size = 273, normalized size = 1.05

$$-\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\left(bx^{2}+a\right)ab}+\frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba+\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{4}}+\frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba+\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{4}}-\frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba+\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{2}b^{4}}+\frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba+\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{2}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*a*b) + 1/8*sqrt(2)*(3*(a*b^3)
^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2
*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 1/8*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^
3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(
1/4))/(a^2*b^4) - 1/16*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*lo
g(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 1/16*sqrt(2)*(3*
(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^4)
```

maple [A] time = 0.02, size = 305, normalized size = 1.17

$$\frac{\sqrt{2}\, A \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b}+\frac{\sqrt{2}\, A \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b}+\frac{\sqrt{2}\, A \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} a b}+\frac{3 \sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}+\frac{3 \sqrt{2}\, B \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}+\frac{3 \sqrt{2}\, B \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16 \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}+\frac{\left(A b - B a\right) x^{\frac{3}{2}}}{2 \left(b x^{2}+a\right) a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)+1/8/a/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/a/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/b
^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/b^2/(a/b
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/b^2/(a/b)^(1/4
)*2^(1/2)*B*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2)))
```

maxima [A] time = 2.28, size = 217, normalized size = 0.83

$$-\frac{(Ba-Ab)x^{\frac{3}{2}}}{2\left(ab^{2}x^{2}+a^{2}b\right)}+\frac{(3\,Ba+Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*x^(3/2)/(a*b^2*x^2 + a^2*b) + 1/16*(3*B*a + A*b)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^
(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

mupad [B] time = 0.31, size = 91, normalized size = 0.35

$$\frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b+3\,B\,a)}{4\,(-a)^{5/4}\,b^{7/4}}-\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b+3\,B\,a)}{4\,(-a)^{5/4}\,b^{7/4}}+\frac{x^{3/2}\,(A\,b-B\,a)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

```
[Out] (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 3*B*a))/(4*(-a)^(5/4)*b^(7/4))
- (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 3*B*a))/(4*(-a)^(5/4)*b^(7/4))
 + (x^(3/2)*(A*b - B*a))/(2*a*b*(a + b*x^2))
```

sympy [A] time = 32.13, size = 162, normalized size = 0.62

$$\frac{2Ax^{\frac{3}{2}}}{4a^2+4abx^2}+2A\operatorname{RootSum}\left(65536t^4a^5b^3+1,\left(t\mapsto t\log\left(4096t^3a^4b^2+\sqrt{x}\right)\right)\right)-\frac{2Bax^{\frac{3}{2}}}{4a^2b+4ab^2x^2}-\frac{2Ba\operatorname{RootSum}\left(65536t^4a^5b^3+1,\left(t\mapsto t\log\left(4096t^3a^4b^2+\sqrt{x}\right)\right)\right)}{b}+\frac{2B\operatorname{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(64t^3ab^2+\sqrt{x}\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] 2*A*x**(3/2)/(4*a**2 + 4*a*b*x**2) + 2*A*RootSum(65536*_t**4*a**5*b**3 + 1,
 Lambda(_t, _t*log(4096*_t**3*a**4*b**2 + sqrt(x)))) - 2*B*a*x**(3/2)/(4*a*
*2*b + 4*a*b**2*x**2) - 2*B*a*RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t,
 _t*log(4096*_t**3*a**4*b**2 + sqrt(x))))/b + 2*B*RootSum(256*_t**4*a*b**3
+ 1, Lambda(_t, _t*log(64*_t**3*a*b**2 + sqrt(x))))/b
```

3.361 $\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\,t}{4}$$

Rubi [A] time = 0.18, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{\sqrt{x}\,(Ab-aB)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(5/4))
 - ((3*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{3Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{\left(\frac{3Ab}{2}+\frac{aB}{2}\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{3/2}b} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{\sqrt{a}+}{a+b}\right.}{4a^{3/2}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} + \frac{(3Ab+aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{3/2}b^{3/2}} + \frac{(3Ab+aB)\,\text{Subst}\left(\int\right.}{8a^{3}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} - \frac{(3Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\log\left(\sqrt{a}+\right.}{8\sqrt{2}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{2ab\left(a+bx^2\right)} - \frac{(3Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}} -$$

Mathematica [A] time = 0.23, size = 203, normalized size = 0.78

$$\frac{\frac{(aB+3Ab)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)\right)}{a^{7/4}\sqrt[4]{b}}-32B\sqrt{x}}{48b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (-32*B*Sqrt[x] + ((3*A*b + a*B)*(8*a^(3/4)*b^(1/4)*Sqrt[x] - 3*Sqrt[2]*(a +
 b*x^2)*(2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])))/(a^(7/4)*b^(1/4)))/(48*b*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.58, size = 160, normalized size = 0.61

$$-\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{\sqrt{x}\,(aB-Ab)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] -1/2*((-(A*b) + a*B)*Sqrt[x])/(a*b*(a + b*x^2)) - ((3*A*b + a*B)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(7/4)*b
^(5/4)) + ((3*A*b + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
 + Sqrt[b]*x)])/(4*Sqrt[2]*a^(7/4)*b^(5/4))
```

fricas [B] time = 1.15, size = 717, normalized size = 2.75

$$\frac{4\left(ab^2x^2+a^2b\right)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^4b^2\sqrt{-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}}+\left(B^2a^2+6\,ABab+9\,A^2b^2\right)x}\,a^5b^4\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{3}{4}}-\left(Ba^6b^4+3\,Aa^5b^5\right)\sqrt{x}\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{3}{4}}}{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}\right)+\left(ab^2x^2+a^2b\right)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}+(Ba+3\,Ab)\sqrt{x}\right)-\left(ab^2x^2+a^2b\right)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(-a^2b\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}+(Ba+3\,Ab)\sqrt{x}\right)-4\,(Ba-Ab)\sqrt{x}}{8\left(ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*arctan((sqrt(a^4*b^2*sqrt
(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^5)) + (B^2*a^2 + 6*A*B*a*b + 9*A^2*b^2)*x)*a^5*b^4*(-(B^4*a^4
+ 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*
b^5))^(3/4) - (B*a^6*b^4 + 3*A*a^5*b^5)*sqrt(x)*(-(B^4*a^4 + 12*A*B^3*a^3*b
 + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(3/4))/(B^
4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)
) + (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(B^4*a^4 + 12*A*
B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(
1/4) + (B*a + 3*A*b)*sqrt(x)) - (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a
^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*
log(-a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b
^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*sqrt(x)) - 4*(B*a - A*b)*
sqrt(x))/(a*b^2*x^2 + a^2*b)
```

giac [A] time = 0.38, size = 273, normalized size = 1.05

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^2}-\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/8*sqrt(2)*((a*b^
3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3
*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b
^2) - 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 1/2*(B*a*sqrt(x) - A*b*sqrt
(x))/((b*x^2 + a)*a*b)
```

maple [A] time = 0.01, size = 305, normalized size = 1.17

$$\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}+\frac{(Ab-Ba)\sqrt{x}}{2\left(b\,x^{2}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 1/2*(A*b-B*a)*x^(1/2)/a/b/(b*x^2+a)+3/8/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/8/a
/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/a/b*(a/b
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/16/a/b*(a/b)^(1/4
)*2^(1/2)*B*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2)))
```

maxima [A] time = 2.46, size = 241, normalized size = 0.92

$$-\frac{(Ba-Ab)\sqrt{x}}{2\left(ab^{2}x^{2}+a^{2}b\right)}+\frac{\frac{2\sqrt{2}\,(Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba+3\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba+3\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba+3\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(B*a - A*b)*sqrt(x)/(a*b^2*x^2 + a^2*b) + 1/16*(2*sqrt(2)*(B*a + 3*A*b
)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 3*A*b)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 3*A*b)*log(sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqr
t(2)*(B*a + 3*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [B] time = 0.43, size = 750, normalized size = 2.87

$$\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}\right)}{8\,(-a)^{7/4}\,b^{5/4}}+\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}\right)}{8\,(-a)^{7/4}\,b^{5/4}}}{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}\right)1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}-\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}\right)1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}}\right)\,(3\,A\,b+B\,a)}{4\,(-a)^{7/4}\,b^{5/4}}+\frac{\sqrt{x}\,\left(A\,b-B\,a\right)}{2\,a\,b\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,(-a)^{7/4}\,b^{5/4}}\right)1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}+\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,(-a)^{7/4}\,b^{5/4}}\right)1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}}{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,(-a)^{7/4}\,b^{5/4}}\right)}{8\,(-a)^{7/4}\,b^{5/4}}-\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,(-a)^{7/4}\,b^{5/4}}\right)}{8\,(-a)^{7/4}\,b^{5/4}}}\right)\,(3\,A\,b+B\,a)\,1\mathrm{i}}{4\,(-a)^{7/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] (atan((((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2
- ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a
)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*
a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7/4)*b^(5/4))
)*1i)/(8*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^
2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7
/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)) - ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3
 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(
8*(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4))))*(3*A*b + B*a)*1i)/(4*(-a)^
(7/4)*b^(5/4)) + (atan((((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6
*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2)*1i)/(8*(-a)^(7/4)*
b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B
^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2)*1i)/(8
*(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a)*((x^(1/2)*(9
*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a
*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a)^(7/4)*b^(5/4)) - ((3*A*b + B*
a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(2
4*A*b^3 + 8*B*a*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a)^(7/4)*b^(5/4))
))*(3*A*b + B*a))/(4*(-a)^(7/4)*b^(5/4)) + (x^(1/2)*(A*b - B*a))/(2*a*b*(a
+ b*x^2))
```

sympy [A] time = 78.54, size = 959, normalized size = 3.67

$$\begin{cases} \tilde{\infty} \left(- \frac{2 A}{7 x^{\frac{7}{2}}} - \frac{2 B}{3 x^{\frac{3}{2}}}\right) & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2 A \sqrt{x} + \frac{2 B x^{\frac{5}{2}}}{5}}{a^{2}} & \text{for}\: b = 0 \\ \frac{- \frac{2 A}{7 x^{\frac{7}{2}}} - \frac{2 B}{3 x^{\frac{3}{2}}}}{b^{2}} & \text{for}\: a = 0 \\ - \frac{3 \sqrt[4]{-1} A a^{\frac{5}{4}} b \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} + \frac{3 \sqrt[4]{-1} A a^{\frac{5}{4}} b \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{6 \sqrt[4]{-1} A a^{\frac{5}{4}} b \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{3 \sqrt[4]{-1} A \sqrt[4]{a} b^{2} x^{2} \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} + \frac{3 \sqrt[4]{-1} A \sqrt[4]{a} b^{2} x^{2} \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{6 \sqrt[4]{-1} A \sqrt[4]{a} b^{2} x^{2} \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} + \frac{4 A a b \sqrt{x}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{\sqrt[4]{-1} B a^{\frac{9}{4}} \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} + \frac{\sqrt[4]{-1} B a^{\frac{9}{4}} \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{2 \sqrt[4]{-1} B a^{\frac{9}{4}} \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{\sqrt[4]{-1} B a^{\frac{5}{4}} b x^{2} \sqrt[4]{\frac{1}{b}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} + \frac{\sqrt[4]{-1} B a^{\frac{5}{4}} b x^{2} \sqrt[4]{\frac{1}{b}} \log{\left(\sqrt[4]{-1} \sqrt[4]{a} \sqrt[4]{\frac{1}{b}} + \sqrt{x} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{2 \sqrt[4]{-1} B a^{\frac{5}{4}} b x^{2} \sqrt[4]{\frac{1}{b}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{a} \sqrt[4]{\frac{1}{b}}} \right)}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} - \frac{4 B a^{2} \sqrt{x}}{8 a^{3} b + 8 a^{2} b^{2} x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2)) - 2*
B/(3*x**(3/2)))/b**2, Eq(a, 0)), (-3*(-1)**(1/4)*A*a**(5/4)*b*(1/b)**(1/4)*
log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3*b + 8*a**2*b**2*x
**2) + 3*(-1)**(1/4)*A*a**(5/4)*b*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/
b)**(1/4) + sqrt(x))/(8*a**3*b + 8*a**2*b**2*x**2) - 6*(-1)**(1/4)*A*a**(5/
```

```
4)*b*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3
*b + 8*a**2*b**2*x**2) - 3*(-1)**(1/4)*A*a**(1/4)*b**2*x**2*(1/b)**(1/4)*lo
g(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3*b + 8*a**2*b**2*x**
2) + 3*(-1)**(1/4)*A*a**(1/4)*b**2*x**2*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/
4)*(1/b)**(1/4) + sqrt(x))/(8*a**3*b + 8*a**2*b**2*x**2) - 6*(-1)**(1/4)*A*
a**(1/4)*b**2*x**2*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(
1/4)))/(8*a**3*b + 8*a**2*b**2*x**2) + 4*A*a*b*sqrt(x)/(8*a**3*b + 8*a**2*b
**2*x**2) - (-1)**(1/4)*B*a**(9/4)*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(
1/b)**(1/4) + sqrt(x))/(8*a**3*b + 8*a**2*b**2*x**2) + (-1)**(1/4)*B*a**(9/
4)*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3*b
+ 8*a**2*b**2*x**2) - 2*(-1)**(1/4)*B*a**(9/4)*(1/b)**(1/4)*atan((-1)**(3/4
)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3*b + 8*a**2*b**2*x**2) - (-1)**(1
/4)*B*a**(5/4)*b*x**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) +
 sqrt(x))/(8*a**3*b + 8*a**2*b**2*x**2) + (-1)**(1/4)*B*a**(5/4)*b*x**2*(1/
b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(8*a**3*b + 8*a*
*2*b**2*x**2) - 2*(-1)**(1/4)*B*a**(5/4)*b*x**2*(1/b)**(1/4)*atan((-1)**(3/
4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(8*a**3*b + 8*a**2*b**2*x**2) - 4*B*a**
2*sqrt(x)/(8*a**3*b + 8*a**2*b**2*x**2), True))
```

3.362 $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(5Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\tan}{4\sqrt{2}}$$

Rubi [A] time = 0.21, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(5Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -(5*A*b - a*B)/(2*a^2*b*Sqrt[x]) + (A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x^2))
+ ((5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*
a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - ((5*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(3/4)) + ((5*A*b -
 a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2
]*a^(9/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx &= \frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}+\frac{\left(\frac{5Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{2ab}\\
&= -\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}-\frac{(5Ab-aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^2}\\
&= -\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}-\frac{(5Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{2a^2}\\
&= -\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}+\frac{(5Ab-aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{4a^2\sqrt{b}}-(5Ab-aB\\
&= -\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}-\frac{(5Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{8a^2b}-(5Ab\\
&= -\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}-\frac{(5Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{3/4}}+(5Ab\\
&= -\frac{5Ab-aB}{2a^2b\sqrt{x}}+\frac{Ab-aB}{2ab\sqrt{x}\,\left(a+bx^2\right)}+\frac{(5Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{(5Ab-aB)\tan^{-1}\left(}{4\sqrt{2}\,a^{9/4}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.22, size = 117, normalized size = 0.40

$$
\frac{2x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)+3A\left((-a)^{3/4}\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-(-a)^{3/4}\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)-\frac{2a}{\sqrt{x}}\right)}{3a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (3*A*((-2*a)/Sqrt[x] + (-a)^(3/4)*b^(1/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/
4)] - (-a)^(3/4)*b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)]) + 2*(-(A*b)
 + a*B)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^3)
```

IntegrateAlgebraic [**A**] time = 0.62, size = 167, normalized size = 0.58

$$-\frac{(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}-\frac{(aB-5Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{9/4}b^{3/4}}+\frac{-4aA+aBx^2-5Abx^2}{2a^2\sqrt{x}\,\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a*A - 5*A*b*x^2 + a*B*x^2)/(2*a^2*Sqrt[x]*(a + b*x^2)) - ((-5*A*b + a*B
)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[
2]*a^(9/4)*b^(3/4)) - ((-5*A*b + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(9/4)*b^(3/4))
```

fricas [**B**] time = 1.05, size = 920, normalized size = 3.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^
2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*arctan((sqrt((B^6*a^6 -
 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 - 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2
*a^2*b^4 - 18750*A^5*B*a*b^5 + 15625*A^6*b^6)*x - (B^4*a^9*b - 20*A*B^3*a^8
*b^2 + 150*A^2*B^2*a^7*b^3 - 500*A^3*B*a^6*b^4 + 625*A^4*a^5*b^5)*sqrt(-(B^
4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^
4)/(a^9*b^3)))*a^2*b*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 50
0*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4) + (B^3*a^5*b - 15*A*B^2*a^4*b
^2 + 75*A^2*B*a^3*b^3 - 125*A^3*a^2*b^4)*sqrt(x)*(-(B^4*a^4 - 20*A*B^3*a^3*
b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4))/
(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4
*b^4)) - (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*
b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(a^7*b^2*(-(B^4*a^
4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(
a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)
*sqrt(x)) + (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a
^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-(B^
4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^
```

```
4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*
b^3)*sqrt(x)) + 4*((B*a - 5*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^3 + a^3*x)
```

giac [A] time = 0.46, size = 278, normalized size = 0.96

$$\frac{Bax^2 - 5\,Abx^2 - 4\,Aa}{2\left(bx^{\frac{5}{2}} + a\sqrt{x}\right)a^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - 5\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*x^2 - 5*A*b*x^2 - 4*A*a)/((b*x^(5/2) + a*sqrt(x))*a^2) + 1/8*sqrt(
2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*
B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/(a^3*b^3) - 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^
(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/1
6*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

maple [A] time = 0.02, size = 323, normalized size = 1.12

$$-\frac{Ab\,x^{\frac{3}{2}}}{2\left(b\,x^2 + a\right)a^2} + \frac{B\,x^{\frac{3}{2}}}{2\left(b\,x^2 + a\right)a} - \frac{5\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2} - \frac{5\sqrt{2}\;A\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2} - \frac{5\sqrt{2}\;A\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^2} + \frac{\sqrt{2}\;B\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\;B\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\;B\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} - \frac{2A}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2*x^(3/2)/(b*x^2+a)*A*b+1/2/a*x^(3/2)/(b*x^2+a)*B-5/16/a^2/(a/b)^(1/
4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
2^(1/2)*x^(1/2)+(a/b)^(1/2)))-5/8/a^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)-5/8/a^2/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)-1)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/8/a/b/(a/
b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/8/a/b/(a/b)^(1/4
)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2*A/a^2/x^(1/2)
```

maxima [A] time = 2.43, size = 222, normalized size = 0.77

$$\frac{(Ba - 5\,Ab)x^2 - 4\,Aa}{2\left(a^2bx^{\frac{5}{2}} + a^3\sqrt{x}\right)} + \frac{(Ba - 5\,Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}} + \frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}} - \frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x} + \sqrt{b}\,x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x} + \sqrt{b}\,x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*a - 5*A*b)*x^2 - 4*A*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) + 1/16*(B*a -
 5*A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^
2
```

mupad [**B**] time = 0.32, size = 104, normalized size = 0.36

$$\frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(5\,A\,b-B\,a)}{4\,(-a)^{9/4}\,b^{3/4}}-\frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(5\,A\,b-B\,a)}{4\,(-a)^{9/4}\,b^{3/4}}-\frac{\frac{2\,A}{a}+\frac{x^2\,(5\,A\,b-B\,a)}{2\,a^2}}{a\,\sqrt{x}+b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b - B*a))/(4*(-a)^(9/4)*b^(3/4))
- (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b - B*a))/(4*(-a)^(9/4)*b^(3/4))
 - ((2*A)/a + (x^2*(5*A*b - B*a))/(2*a^2))/(a*x^(1/2) + b*x^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.363 $\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{(7Ab-3aB)\,t}{4\sqrt{\ }}$$

Rubi [A] time = 0.21, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{(7Ab-3aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}-\frac{(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -(7*A*b - 3*a*B)/(6*a^2*b*x^(3/2)) + (A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x^2)
) + ((7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt
[2]*a^(11/4)*b^(1/4)) - ((7*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) + ((7*A*b - 3*a*B)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
- ((7*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(11/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} + \frac{\left(\frac{7Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2a^2}$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{4a^{5/2}} - (7Ab -$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} - \frac{(7Ab-3aB)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{8a^{5/2}\sqrt{b}} - (7A$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} + \frac{(7Ab-3aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{11/4}\sqrt[4]{b}} - (7A$$

$$= -\frac{7Ab-3aB}{6a^2bx^{3/2}} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)} + \frac{(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}} - \frac{(7Ab-3aB)\tan}{4\sqrt{2}\,a}$$

Mathematica [A] time = 0.39, size = 355, normalized size = 1.23

$$\frac{-\frac{24a^{3/4}Ab\sqrt{x}}{a+bx^2}-\frac{32a^{3/4}A}{x^{3/2}}+\frac{24a^{7/4}B\sqrt{x}}{a+bx^2}+21\sqrt{2}\,Ab^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-21\sqrt{2}\,Ab^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+\frac{6\sqrt{2}\,(7Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\,(7Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}-\frac{9\sqrt{2}\,aB\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}}+\frac{9\sqrt{2}\,aB\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{b}}}{48a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] ((-32*a^(3/4)*A)/x^(3/2) - (24*a^(3/4)*A*b*Sqrt[x])/(a + b*x^2) + (24*a^(7/
4)*B*Sqrt[x])/(a + b*x^2) + (6*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*
```

```
b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (6*Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + 21*Sqrt[2]*A*b^(3/4)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (9*Sqrt[2]*a*B*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) - 21*Sqrt[2]*A
*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (9*Sq
rt[2]*a*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/
4))/(48*a^(11/4))
```

IntegrateAlgebraic [A] time = 0.61, size = 170, normalized size = 0.59

$$-\frac{(3aB-7Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{(3aB-7Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{11/4}\sqrt[4]{b}}+\frac{-4aA+3aBx^2-7Abx^2}{6a^2x^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a*A - 7*A*b*x^2 + 3*a*B*x^2)/(6*a^2*x^(3/2)*(a + b*x^2)) - ((-7*A*b + 3
*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*S
qrt[2]*a^(11/4)*b^(1/4)) + ((-7*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(11/4)*b^(1/4))
```

fricas [B] time = 1.60, size = 741, normalized size = 2.56

$$-\frac{12\left(a^2bx^4+a^3x^2\right)\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^6\sqrt{-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}}+\left(9B^2a^2-42ABab+49A^2b^2\right)x}\,a^8b\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{3}{4}}+\left(3Ba^9b-7Aa^8b^2\right)\sqrt{x}\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{3}{4}}}{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}\right)+3\left(a^2bx^4+a^3x^2\right)\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}-(3Ba-7Ab)\sqrt{x}\right)-3\left(a^2bx^4+a^3x^2\right)\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}\log\left(-a^3\left(-\frac{81B^4a^4-756AB^3a^3b+2646A^2B^2a^2b^2-4116A^3Bab^3+2401A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}-(3Ba-7Ab)\sqrt{x}\right)-4\left((3Ba-7Ab)x^2-4Aa\right)\sqrt{x}}{24\left(a^2bx^4+a^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*
B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*arctan((sqrt
(a^6*sqrt(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*
B*a*b^3 + 2401*A^4*b^4)/(a^11*b)) + (9*B^2*a^2 - 42*A*B*a*b + 49*A^2*b^2)*x
)*a^8*b*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B
*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(3/4) + (3*B*a^9*b - 7*A*a^8*b^2)*sqrt(x)*
(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 +
 2401*A^4*b^4)/(a^11*b))^(3/4))/(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^
2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)) + 3*(a^2*b*x^4 + a^3*x^2)*(-(
81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 24
01*A^4*b^4)/(a^11*b))^(1/4)*log(a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*
A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a
 - 7*A*b)*sqrt(x)) - 3*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*
b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)
*log(-a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3
```

```
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) - 4*((3
*B*a - 7*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x^2)
```

giac [A] time = 0.50, size = 283, normalized size = 0.98

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}b}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{3}b}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{3}b}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^{3}b}+\frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\left(bx^{2}+a\right)a^{2}}-\frac{2A}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/8*sqrt(2)*(3*(a*
b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a -
 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3
*b) - 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/2*(B*a*sqrt(x) - A*b*sqrt
(x))/((b*x^2 + a)*a^2) - 2/3*A/(a^2*x^(3/2))
```

maple [A] time = 0.02, size = 317, normalized size = 1.10

$$-\frac{Ab\sqrt{x}}{2\left(bx^{2}+a\right)a^{2}}+\frac{B\sqrt{x}}{2\left(bx^{2}+a\right)a}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^{2}}-\frac{2A}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2*x^(1/2)/(b*x^2+a)*A*b+1/2/a*x^(1/2)/(b*x^2+a)*B-7/8/a^3*(a/b)^(1/4
)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b-7/16/a^3*(a/b)^(1/4)*2^
(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/
2)*x^(1/2)+(a/b)^(1/2)))*b-7/8/a^3*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)*b+3/8/a^2*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)*2^(1/2)*x^(
1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/a^2*(a/b
)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-2/3*A/a^2/x^(3/2)
```

maxima [A] time = 2.71, size = 251, normalized size = 0.87

$$\frac{(3\,Ba-7\,Ab)x^{2}-4\,Aa}{6\left(a^{2}bx^{\frac{7}{2}}+a^{3}x^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\,(3\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(3\,Ba-7\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(3\,Ba-7\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,Ba-7\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((3*B*a - 7*A*b)*x^2 - 4*A*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) + 1/16*(2*s
qrt(2)*(3*B*a - 7*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqr
t(2)*(3*B*a - 7*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2
)*(3*B*a - 7*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*B*a - 7*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

mupad [B] time = 0.47, size = 859, normalized size = 2.97

$$-\frac{\frac{2A}{3a}+\frac{x^2\,(7Ab-3Ba)}{6a^2}}{a\,x^{3/2}+b\,x^{7/2}}-\frac{\mathrm{atan}\left(\frac{\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)-\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)}{8\,(-a)^{11/4}\,b^{1/4}}\right)1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}+\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)+\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)}{8\,(-a)^{11/4}\,b^{1/4}}\right)1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}}{\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)-\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)}{8\,(-a)^{11/4}\,b^{1/4}}\right)}{8\,(-a)^{11/4}\,b^{1/4}}-\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)+\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)}{8\,(-a)^{11/4}\,b^{1/4}}\right)}{8\,(-a)^{11/4}\,b^{1/4}}}\right)(7Ab-3Ba)\,1\mathrm{i}}{4\,(-a)^{11/4}\,b^{1/4}}-\frac{\mathrm{atan}\left(\frac{\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)-\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)\,1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}\right)}{8\,(-a)^{11/4}\,b^{1/4}}+\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)+\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)\,1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}\right)}{8\,(-a)^{11/4}\,b^{1/4}}}{\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)-\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)\,1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}\right)1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}-\frac{(7Ab-3Ba)\left(\sqrt{x}\,(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3)+\frac{(7Ab-3Ba)\,(1792Aa^9b^4-768Ba^{10}b^3)\,1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}\right)1\mathrm{i}}{8\,(-a)^{11/4}\,b^{1/4}}}\right)(7Ab-3Ba)}{4\,(-a)^{11/4}\,b^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] - ((2*A)/(3*a) + (x^2*(7*A*b - 3*B*a))/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) - (
atan((((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*
A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^
(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4)*b^(1/4)) + ((7*A*b - 3*B*a)*(x^(1/2)*(1
568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(1
792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4
)*b^(1/4)))/(((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3
- 1344*A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(
8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/4)*b^(1/4)) - ((7*A*b - 3*B*a)*(x^(1/2
)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a
)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/
4)*b^(1/4))))*(7*A*b - 3*B*a)*1i)/(4*(-a)^(11/4)*b^(1/4)) - (atan((((7*A*b
- 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) -
 ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1
/4))))/(8*(-a)^(11/4)*b^(1/4)) + ((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^
5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(1792*A*a^9*b^4
- 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/4)*b^(1/4)))/((
(7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7
*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/
4)*b^(1/4)))*1i)/(8*(-a)^(11/4)*b^(1/4)) - ((7*A*b - 3*B*a)*(x^(1/2)*(1568*
A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(1792*
A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4)
*b^(1/4))))*(7*A*b - 3*B*a))/(4*(-a)^(11/4)*b^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.364 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-$$

Rubi [A] time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(9Ab-5aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(9Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(9Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{13/4}}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -(9*A*b - 5*a*B)/(10*a^2*b*x^(5/2)) + (9*A*b - 5*a*B)/(2*a^3*Sqrt[x]) + (A*
b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) - (b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)) + (b^(1/4)*(9*A*b
- 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)
) + (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)) - (b^(1/4)*(9*A*b - 5*a*B)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\left(\frac{9Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} - \frac{(9Ab-5aB)\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{4a^2}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{(b(9Ab-5aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{4a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{(b(9Ab-5aB))\,\mathrm{Subst}\Big(\int \frac{x^2}{a+bx^4}\,dx, x,}{2a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\left(\sqrt{b}\,(9Ab-5aB)\right)\mathrm{Subst}\Big(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}}{4a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{(9Ab-5aB)\,\mathrm{Subst}\Bigg(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}}{8a^3}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\sqrt[4]{b}\,(9Ab-5aB)\log\Big(\sqrt{a}-\sqrt{2}\sqrt[4]{a}}{8\sqrt{2}\,a^{13/4}}$$

$$= -\frac{9Ab-5aB}{10a^2bx^{5/2}} + \frac{9Ab-5aB}{2a^3\sqrt{x}} + \frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\sqrt[4]{b}\,(9Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}}$$

Mathematica [C] time = 0.43, size = 151, normalized size = 0.49

$$\frac{2bx^{3/2}(Ab-aB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4} + \frac{4Ab-2aB}{a^3\sqrt{x}} - \frac{2A}{5a^2x^{5/2}} + \frac{\sqrt[4]{b}\,(aB-2Ab)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{13/4}} + \frac{\sqrt[4]{b}\,(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-2*A)/(5*a^2*x^(5/2)) + (4*A*b - 2*a*B)/(a^3*Sqrt[x]) + (b^(1/4)*(-2*A*b +
 a*B)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(13/4) + (b^(1/4)*(2*A*b -
 a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(13/4) + (2*b*(A*b - a*B)
*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^4)
```

IntegrateAlgebraic [**A**] time = 0.59, size = 200, normalized size = 0.65

$$\frac{\left(5a\sqrt[4]{b}\,B-9Ab^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{\left(5a\sqrt[4]{b}\,B-9Ab^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{13/4}}+\frac{-4a^2A-20a^2Bx^2+36aAbx^2-25abBx^4+45Ab^2x^4}{10a^3x^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a^2*A + 36*a*A*b*x^2 - 20*a^2*B*x^2 + 45*A*b^2*x^4 - 25*a*b*B*x^4)/(10*
a^3*x^(5/2)*(a + b*x^2)) + ((-9*A*b^(5/4) + 5*a*b^(1/4)*B)*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(13/4)) + ((-
9*A*b^(5/4) + 5*a*b^(1/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqr
t[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(13/4))
```

fricas [**B**] time = 1.44, size = 974, normalized size = 3.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 121
50*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*arctan((
sqrt((15625*B^6*a^6*b^2 - 168750*A*B^5*a^5*b^3 + 759375*A^2*B^4*a^4*b^4 - 1
822500*A^3*B^3*a^3*b^5 + 2460375*A^4*B^2*a^2*b^6 - 1771470*A^5*B*a*b^7 + 53
1441*A^6*b^8)*x - (625*B^4*a^11*b - 4500*A*B^3*a^10*b^2 + 12150*A^2*B^2*a^9
*b^3 - 14580*A^3*B*a^8*b^4 + 6561*A^4*a^7*b^5)*sqrt(-(625*B^4*a^4*b - 4500*
A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a
^13))*a^3*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 1
4580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4) + (125*B^3*a^6*b - 675*A*B^2*a
^5*b^2 + 1215*A^2*B*a^4*b^3 - 729*A^3*a^3*b^4)*sqrt(x)*(-(625*B^4*a^4*b - 4
500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^
5)/a^13)^(1/4))/(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3
 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)) - 5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*
a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 65
61*A^4*b^5)/a^13)^(1/4)*log(a^10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12
150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*
B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) +
```

```
5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B
^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(-a^10*(-(625
*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4
 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^
2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) + 4*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2
+ 4*(5*B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

giac [A] time = 0.54, size = 303, normalized size = 0.98

$$-\frac{Babx^{\frac{3}{2}} - Ab^{2}x^{\frac{3}{2}}}{2\left(bx^{2} + a\right)a^{3}} - \frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba - 9\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{4}b^{2}} - \frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba - 9\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^{4}b^{2}} + \frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba - 9\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16a^{4}b^{2}} - \frac{\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba - 9\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16a^{4}b^{2}} - \frac{2\left(5Bax^{2} - 10Abx^{2} + Aa\right)}{5a^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b*x^(3/2) - A*b^2*x^(3/2))/((b*x^2 + a)*a^3) - 1/8*sqrt(2)*(5*(a*
b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4)*B*a -
 9*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/
4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*s
qrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 2/5*(5*B*a*x^2 - 10*A*b*x^2 + A*a)/(a
^3*x^(5/2))
```

maple [A] time = 0.02, size = 339, normalized size = 1.09

$$\frac{Ab^{2}x^{\frac{3}{2}}}{2\left(bx^{2} + a\right)a^{3}} - \frac{Bbx^{\frac{3}{2}}}{2\left(bx^{2} + a\right)a^{2}} + \frac{9\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}} + \frac{9\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}} + \frac{9\sqrt{2}\,Ab\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}} - \frac{5\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}} - \frac{5\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}} - \frac{5\sqrt{2}\,B\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}} + \frac{4Ab}{a^{3}\sqrt{x}} - \frac{2B}{a^{2}\sqrt{x}} - \frac{2A}{5a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2/a^3*b^2*x^(3/2)/(b*x^2+a)*A-1/2/a^2*b*x^(3/2)/(b*x^2+a)*B+9/16/a^3*b/(a
/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+9/8/a^3*b/(a/b)^(1/4)*2^(1/2)*A*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8/a^3*b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)-5/16/a^2/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-
5/8/a^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/a^2
/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2/5*A/a^2/x^(5
/2)+4/a^3/x^(1/2)*A*b-2/a^2/x^(1/2)*B
```

maxima [A] time = 2.31, size = 250, normalized size = 0.81

$$-\frac{5\left(5\,Bab-9\,Ab^2\right)x^4+4\,Aa^2+4\left(5\,Ba^2-9\,Aab\right)x^2}{10\left(a^3bx^{\frac{9}{2}}+a^4x^{\frac{5}{2}}\right)}-\frac{\left(5\,Bab-9\,Ab^2\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2 + 4*(5*B*a^2 - 9*A*a*b)*x^2)/(a^
3*b*x^(9/2) + a^4*x^(5/2)) - 1/16*(5*B*a*b - 9*A*b^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x
) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

mupad [B] time = 0.18, size = 121, normalized size = 0.39

$$\frac{\frac{2\,x^2\,(9\,A\,b-5\,B\,a)}{5\,a^2}-\frac{2\,A}{5\,a}+\frac{b\,x^4\,(9\,A\,b-5\,B\,a)}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}}+\frac{{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{13/4}}-\frac{{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x)
```

```
[Out] ((2*x^2*(9*A*b - 5*B*a))/(5*a^2) - (2*A)/(5*a) + (b*x^4*(9*A*b - 5*B*a))/(2
*a^3))/(a*x^(5/2) + b*x^(9/2)) + ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1
/4))*(9*A*b - 5*B*a))/(4*a^(13/4)) - ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))
/a^(1/4))*(9*A*b - 5*B*a))/(4*a^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.365 $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=316

$$-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}-\frac{5(Ab-9a}{3}$$

Rubi [A] time = 0.24, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 288, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{5(Ab-9aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}-\frac{5(Ab-9aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{3/4}b^{13/4}}+\frac{x^{5/2}(Ab-9aB)}{16ab^2\left(a+bx^2\right)}-\frac{5\sqrt{x}(Ab-9aB)}{16ab^3}+\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-5*(A*b - 9*a*B)*Sqrt[x])/(16*a*b^3) + ((A*b - a*B)*x^(9/2))/(4*a*b*(a + b
*x^2)^2) + ((A*b - 9*a*B)*x^(5/2))/(16*a*b^2*(a + b*x^2)) - (5*(A*b - 9*a*B
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(3/4)*b^(13/
4)) + (5*(A*b - 9*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*S
qrt[2]*a^(3/4)*b^(13/4)) - (5*(A*b - 9*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(3/4)*b^(13/4)) + (5*(A*b - 9*a*
B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*
a^(3/4)*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx=\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{\left(-\frac{Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{(5(Ab-9aB))\int \frac{x^{3/2}}{a+bx^2}\,dx}{32ab^2}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32b^3}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\,\text{Subst}\bigg(\int \frac{1}{a+bx^4}\,d}{16b^3}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\,\text{Subst}\bigg(\int \frac{\sqrt{a}-\sqrt{b}\,}{a+bx^4}}{32\sqrt{a}\,b^3}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}+\frac{(5(Ab-9aB))\,\text{Subst}\Bigg(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}}{}}}{64\sqrt{a}\,b^{7/2}}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{5(Ab-9aB)\log\big(\sqrt{a}-\sqrt{2}\sqrt[4]{a}}{64\sqrt{2}\,a^{3/4}b^{13/4}}$$

$$=-\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\left(a+bx^2\right)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\left(a+bx^2\right)}-\frac{5(Ab-9aB)\tan^{-1}\Bigg(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}}{32\sqrt{2}\,a^{3/4}b^{13/4}}$$

Mathematica [A] time = 0.43, size = 402, normalized size = 1.27

$$\frac{\frac{10\sqrt{2}(9aB-Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{10\sqrt{2}(Ab-9aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{5\sqrt{2}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{5\sqrt{2}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{32a^2\sqrt[4]{b}B\sqrt{x}}{\left(a+bx^2\right)^2}+\frac{32aAb^{5/4}\sqrt{x}}{\left(a+bx^2\right)^2}-\frac{72Ab^{5/4}\sqrt{x}}{a+bx^2}+\frac{136a\sqrt[4]{b}B\sqrt{x}}{a+bx^2}+45\sqrt{2}\sqrt[4]{a}B\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-45\sqrt{2}\sqrt[4]{a}B\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+256\sqrt[4]{b}B\sqrt{x}}{128b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (256*b^(1/4)*B*Sqrt[x] + (32*a*A*b^(5/4)*Sqrt[x])/(a + b*x^2)^2 - (32*a^2*b
^(1/4)*B*Sqrt[x])/(a + b*x^2)^2 - (72*A*b^(5/4)*Sqrt[x])/(a + b*x^2) + (136
```

```
*a*b^(1/4)*B*Sqrt[x]])/(a + b*x^2) + (10*Sqrt[2]*(-(A*b) + 9*a*B)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (10*Sqrt[2]*(A*b - 9*a*B)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (5*Sqrt[2]*A*b*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + 45*Sqrt[2]
*a^(1/4)*B*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (5*
Sqrt[2]*A*b*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(
3/4) - 45*Sqrt[2]*a^(1/4)*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(128*b^(13/4))
```

IntegrateAlgebraic [A] time = 0.72, size = 189, normalized size = 0.60

$$\frac{5(9aB-Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{32\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{5(9aB-Ab)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{\sqrt{x}\left(45a^2B-5aAb+81abBx^2-9Ab^2x^2+32b^2Bx^4\right)}{16b^3\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (Sqrt[x]*(-5*a*A*b + 45*a^2*B - 9*A*b^2*x^2 + 81*a*b*B*x^2 + 32*b^2*B*x^4))
/(16*b^3*(a + b*x^2)^2) + (5*(-(A*b) + 9*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(3/4)*b^(13/4)) - (5*(-(A
*b) + 9*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/(32*Sqrt[2]*a^(3/4)*b^(13/4))
```

fricas [B] time = 1.17, size = 793, normalized size = 2.51

$$\frac{20\left(b^5x^4+2ab^4x^2+a^2b^3\right)\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2b^6\sqrt{-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}}+\left(81B^2a^2-18ABab+A^2b^2\right)x}\,a^2b^{10}\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{3}{4}}+\left(9Ba^3b^{10}-Aa^2b^{11}\right)\sqrt{x}\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{3}{4}}}{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}\right)+5\left(b^5x^4+2ab^4x^2+a^2b^3\right)\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(5ab^3\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}-5(9Ba-Ab)\sqrt{x}\right)-5\left(b^5x^4+2ab^4x^2+a^2b^3\right)\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(-5ab^3\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}-5(9Ba-Ab)\sqrt{x}\right)+4\left(32Bb^2x^4+45Ba^2-5Aab+9\left(9Bab-Ab^2\right)x^2\right)\sqrt{x}}{64\left(b^5x^4+2ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3
*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*arct
an((sqrt(a^2*b^6*sqrt(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b
^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13)) + (81*B^2*a^2 - 18*A*B*a*b + A^2
*b^2)*x)*a^2*b^10*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2
- 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(3/4) + (9*B*a^3*b^10 - A*a^2*b^11)
*sqrt(x)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*
B*a*b^3 + A^4*b^4)/(a^3*b^13))^(3/4))/(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 48
6*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)) + 5*(b^5*x^4 + 2*a*b^4*x^2 +
 a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3
*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log(5*a*b^3*(-(6561*B^4*a^4 - 2916*A*
B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/
4) - 5*(9*B*a - A*b)*sqrt(x)) - 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561
*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^
```

```
4)/(a^3*b^13))^(1/4)*log(-5*a*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*
A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a -
A*b)*sqrt(x)) + 4*(32*B*b^2*x^4 + 45*B*a^2 - 5*A*a*b + 9*(9*B*a*b - A*b^2)*
x^2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [**A**] time = 0.41, size = 304, normalized size = 0.96

$$\frac{2\,B\sqrt{x}}{b^{3}}-\frac{5\,\sqrt{2}\left(9\,\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^{4}}-\frac{5\,\sqrt{2}\left(9\,\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^{4}}-\frac{5\,\sqrt{2}\left(9\,\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^{4}}+\frac{5\,\sqrt{2}\left(9\,\left(ab^{3}\right)^{\frac{1}{4}}Ba-\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^{4}}+\frac{17\,Babx^{\frac{5}{2}}-9\,Ab^{2}x^{\frac{5}{2}}+13\,Ba^{2}\sqrt{x}-5\,Aab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^3 - 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5
/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5/128*sqrt(2)*(9*(a
*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/(a*b^4) + 5/128*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) + 1/16*(17*B*a*
b*x^(5/2) - 9*A*b^2*x^(5/2) + 13*B*a^2*sqrt(x) - 5*A*a*b*sqrt(x))/((b*x^2 +
 a)^2*b^3)
```

maple [**A**] time = 0.02, size = 363, normalized size = 1.15

$$-\frac{9A\,x^{\frac{5}{2}}}{16\left(b\,x^{2}+a\right)^{2}b}+\frac{17Ba\,x^{\frac{5}{2}}}{16\left(b\,x^{2}+a\right)^{2}b^{2}}-\frac{5Aa\sqrt{x}}{16\left(b\,x^{2}+a\right)^{2}b^{2}}+\frac{13B\,a^{2}\sqrt{x}}{16\left(b\,x^{2}+a\right)^{2}b^{3}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a\,b^{2}}-\frac{45\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64b^{3}}-\frac{45\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64b^{3}}-\frac{45\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128b^{3}}+\frac{2B\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*B/b^3*x^(1/2)-9/16/b/(b*x^2+a)^2*x^(5/2)*A+17/16/b^2/(b*x^2+a)^2*x^(5/2)*
a*B-5/16/b^2/(b*x^2+a)^2*A*x^(1/2)*a+13/16/b^3/(b*x^2+a)^2*B*x^(1/2)*a^2+5/
64/b^2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+5/64/b
^2*(a/b)^(1/4)/a*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+5/128/b^2*
(a/b)^(1/4)/a*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(
a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-45/64/b^3*(a/b)^(1/4)*2^(1/2)*B*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-45/64/b^3*(a/b)^(1/4)*2^(1/2)*B*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-45/128/b^3*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1
/2)))
```

maxima [A] time = 2.47, size = 283, normalized size = 0.90

$$\frac{\left(17\,Bab-9\,Ab^2\right)x^{\frac{5}{2}}+\left(13\,Ba^2-5\,Aab\right)\sqrt{x}}{16\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}+\frac{2\,B\sqrt{x}}{b^3}-\frac{5\left(\frac{2\,\sqrt{2}\,(9\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(9\,Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(9\,Ba-Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(9\,Ba-Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((17*B*a*b - 9*A*b^2)*x^(5/2) + (13*B*a^2 - 5*A*a*b)*sqrt(x))/(b^5*x^4
 + 2*a*b^4*x^2 + a^2*b^3) + 2*B*sqrt(x)/b^3 - 5/128*(2*sqrt(2)*(9*B*a - A*b
)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a - A*b)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - A*b)*log(sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqr
t(2)*(9*B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(3/4)*b^(1/4)))/b^3
```

mupad [B] time = 0.40, size = 760, normalized size = 2.41

$$\frac{\sqrt{x}\,\left(\frac{13\,B\,a^2}{16}-\frac{5\,A\,a\,b}{16}\right)-x^{5/2}\,\left(\frac{9\,A\,b^2}{16}-\frac{17\,B\,a\,b}{16}\right)}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{2\,B\,\sqrt{x}}{b^3}-\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{5\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}+\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{5\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}}{\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{5\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}-\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{5\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}}\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{32\,(-a)^{3/4}\,b^{13/4}}-\frac{5\,\mathrm{atan}\left(\frac{\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}+\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}}{\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}-\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\,\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}}\right)(A\,b-9\,B\,a)}{32\,(-a)^{3/4}\,b^{13/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (x^(1/2)*((13*B*a^2)/16 - (5*A*a*b)/16) - x^(5/2)*((9*A*b^2)/16 - (17*B*a*b
)/16))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (2*B*x^(1/2))/b^3 - (atan((((A*b
 - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - (5*(
45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(
3/4)*b^(13/4)) + ((A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B
*a*b))/(64*b^3) + (5*(45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(
13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4)))/((5*(A*b - 9*B*a)*((25*x^(1/2)*(A^2*b
^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - (5*(45*B*a^2 - 5*A*a*b)*(A*b - 9*
B*a))/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13/4)) - (5*(A*b - 9*B*a
)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) + (5*(45*B*a^2
 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13/
4))))*(A*b - 9*B*a)*5i)/(32*(-a)^(3/4)*b^(13/4)) - (5*atan(((5*(A*b - 9*B*a
)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - ((45*B*a^2 -
 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13
/4)) + (5*(A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(
64*b^3) + ((45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4)))
```

```
)/(64*(-a)^(3/4)*b^(13/4)))/(((A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*
a^2 - 18*A*B*a*b))/(64*b^3) - ((45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(
-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4)) - ((A*b - 9*B*a)*((25*x^(
1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) + ((45*B*a^2 - 5*A*a*b)*
(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4))))*
(A*b - 9*B*a))/(32*(-a)^(3/4)*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.366 $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=293

$$\frac{3(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab}{32}$$

Rubi [A] time = 0.22, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{3(7aB+Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{11/4}}+\frac{3(7aB+Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{5/4}b^{11/4}}-\frac{x^{3/2}(7aB+Ab)}{16ab^2\left(a+bx^2\right)}+\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) - ((A*b + 7*a*B)*x^(3/2))/(16*a
*b^2*(a + b*x^2)) - (3*(A*b + 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) + (3*(A*b + 7*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(5/4)*b^(11/4)) - (3*(A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} + \frac{\left(\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)} + \frac{(3(Ab+7aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)} + \frac{(3(Ab+7aB))\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)} - \frac{(3(Ab+7aB))\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32ab^{5/2}} + (3(A$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)} + \frac{(3(Ab+7aB))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64ab^3} +$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)} + \frac{3(Ab+7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{5/4}b^{11/4}} - 3$$

$$= \frac{(Ab-aB)x^{7/2}}{4ab\left(a+bx^2\right)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2\left(a+bx^2\right)} - \frac{3(Ab+7aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}} + 3(Ab+7aB)\,\mathrm{t}$$

Mathematica [C] time = 0.21, size = 137, normalized size = 0.47

$$\frac{2b^{3/4}x^{3/2}(Ab-2aB)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)+2b^{3/4}x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)+3(-a)^{7/4}B\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)+\tanh^{-1}\left(\frac{a\sqrt[4]{b}\sqrt{x}}{(-a)^{5/4}}\right)\right)}{3a^2b^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (3*(-a)^(7/4)*B*(ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)] + ArcTanh[(a*b^(1/4)*
Sqrt[x])/(-a)^(5/4)]) + 2*b^(3/4)*(A*b - 2*a*B)*x^(3/2)*Hypergeometric2F1[3
/4, 2, 7/4, -((b*x^2)/a)] + 2*b^(3/4)*(-(A*b) + a*B)*x^(3/2)*Hypergeometric
2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^2*b^(11/4))
```

IntegrateAlgebraic [A] time = 0.73, size = 180, normalized size = 0.61

$$-\frac{3(7aB+Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{3(7aB+Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{x^{3/2}\left(7a^2B+aAb+11abBx^2-3Ab^2x^2\right)}{16ab^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/16*(x^(3/2)*(a*A*b + 7*a^2*B - 3*A*b^2*x^2 + 11*a*b*B*x^2))/(a*b^2*(a +
b*x^2)^2) - (3*(A*b + 7*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(5/4)*b^(11/4)) - (3*(A*b + 7*a*B)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sqrt[2]*a^(5
/4)*b^(11/4))
```

fricas [B] time = 1.31, size = 990, normalized size = 3.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^
3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)
*arctan((sqrt((117649*B^6*a^6 + 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
+ 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 + 42*A^5*B*a*b^5 + A^6*b^6)*x
- (2401*B^4*a^7*b^5 + 1372*A*B^3*a^6*b^6 + 294*A^2*B^2*a^5*b^7 + 28*A^3*B*a
^4*b^8 + A^4*a^3*b^9)*sqrt(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*
a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)))*a*b^3*(-(2401*B^4*a^4 + 13
72*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11)
)^(1/4) - (343*B^3*a^4*b^3 + 147*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 + A^3*a*b
^6)*sqrt(x)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A
^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4))/(2401*B^4*a^4 + 1372*A*B^3*a^3*b +
 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)) - 3*(a*b^4*x^4 + 2*a^2*b^
3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 +
 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(27*a^4*b^8*(-(2401*B^4*a^4
 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*
b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3
```

```
)*sqrt(x)) + 3*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372
*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^
(1/4)*log(-27*a^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*
b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A
*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) + 4*((11*B*a*b - 3*A*b^2)*x
^3 + (7*B*a^2 + A*a*b)*x)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

giac [A] time = 0.47, size = 293, normalized size = 1.00

$$-\frac{11\,Babx^{\frac{7}{2}} - 3\,Ab^2x^{\frac{7}{2}} + 7\,Ba^2x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{16\left(bx^2+a\right)^2ab^2} + \frac{3\sqrt{2}\left(7\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^5} + \frac{3\sqrt{2}\left(7\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^5} - \frac{3\sqrt{2}\left(7\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^2b^5} + \frac{3\sqrt{2}\left(7\left(ab^3\right)^{\frac{3}{4}}Ba + \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*B*a*b*x^(7/2) - 3*A*b^2*x^(7/2) + 7*B*a^2*x^(3/2) + A*a*b*x^(3/2)
)/((b*x^2 + a)^2*a*b^2) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
2*b^5) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 3/128*s
qrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^2*b^5) + 3/128*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*
b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5)
```

maple [A] time = 0.02, size = 325, normalized size = 1.11

$$\frac{3\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2} + \frac{3\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2} + \frac{3\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^2} + \frac{21\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}b^3} + \frac{21\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}b^3} + \frac{21\sqrt{2}\,B\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}b^3} + \frac{\frac{(3Ab-11Ba)x^{\frac{7}{2}}}{16ab} - \frac{(Ab+7Ba)x^{\frac{3}{2}}}{16b^2}}{\left(b\,x^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(3*A*b-11*B*a)/a/b*x^(7/2)-1/32*(A*b+7*B*a)/b^2*x^(3/2))/(b*x^2+a)^
2+3/64/b^2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/
64/b^2/a/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/128/
b^2/a/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/
(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+21/64/b^3/(a/b)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+21/64/b^3/(a/b)^(1/4)*2^(1/2)*B*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+21/128/b^3/(a/b)^(1/4)*2^(1/2)*B*ln((x-(
a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2)))
```

maxima [A] time = 2.37, size = 251, normalized size = 0.86

$$-\frac{\left(11\,Bab-3\,Ab^2\right)x^{\frac{7}{2}}+\left(7\,Ba^2+Aab\right)x^{\frac{3}{2}}}{16\,\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)}+\frac{3\,(7\,Ba+Ab)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*B*a*b - 3*A*b^2)*x^(7/2) + (7*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^4
 + 2*a^2*b^3*x^2 + a^3*b^2) + 3/128*(7*B*a + A*b)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(
1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt
(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^2)
```

mupad [B] time = 0.19, size = 122, normalized size = 0.42

$$\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(A\,b+7\,B\,a\right)}{32\,{(-a)}^{5/4}\,b^{11/4}}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(A\,b+7\,B\,a\right)}{32\,{(-a)}^{5/4}\,b^{11/4}}-\frac{\frac{x^{3/2}\,\left(A\,b+7\,B\,a\right)}{16\,b^2}-\frac{x^{7/2}\,\left(3\,A\,b-11\,B\,a\right)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 7*B*a))/(32*(-a)^(5/4)*b^(11/
4)) - (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 7*B*a))/(32*(-a)^(5/4)*b
^(11/4)) - ((x^(3/2)*(A*b + 7*B*a))/(16*b^2) - (x^(7/2)*(3*A*b - 11*B*a))/(
16*a*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.367 $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=298

$$-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(5aB+3A}{}$$

Rubi **[A]** time = 0.21, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 288, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(5aB+3Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(5aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{\sqrt{x}\,(5aB+3Ab)}{16ab^2\left(a+bx^2\right)}+\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*Sqrt[x])/(16
*a*b^2*(a + b*x^2)) - ((3*A*b + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) - ((3*A*b + 5*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}+\frac{\left(\frac{3Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)}+\frac{(3Ab+5aB)\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32ab^2}\\
&= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)}+\frac{(3Ab+5aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16ab^2}\\
&= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)}+\frac{(3Ab+5aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{3/2}b^2}+(3Ab\\
&= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)}+\frac{(3Ab+5aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64a^{3/2}b^{5/2}}+\\
&= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)}-\frac{(3Ab+5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{7/4}b^{9/4}}+\\
&= \frac{(Ab-aB)x^{5/2}}{4ab\left(a+bx^2\right)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\left(a+bx^2\right)}-\frac{(3Ab+5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(3Ab+5aB)}{32}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 389, normalized size = 1.31

$$\frac{-\frac{2\sqrt{2}(5aB+3Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt{2}(5aB+3Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}}-\frac{3\sqrt{2}Ab\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{3\sqrt{2}Ab\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{5\sqrt{2}B\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{5\sqrt{2}B\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{8Ab^{5/4}\sqrt{x}}{a^2+abx^2}-\frac{32Ab^{5/4}\sqrt{x}}{\left(a+bx^2\right)^2}-\frac{72\sqrt[4]{b}B\sqrt{x}}{a+bx^2}+\frac{32a\sqrt[4]{b}B\sqrt{x}}{\left(a+bx^2\right)^2}}{128b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((-32*A*b^(5/4)*Sqrt[x])/(a + b*x^2)^2 + (32*a*b^(1/4)*B*Sqrt[x])/(a + b*x^
2)^2 - (72*b^(1/4)*B*Sqrt[x])/(a + b*x^2) + (8*A*b^(5/4)*Sqrt[x])/(a^2 + a*
b*x^2) - (2*Sqrt[2]*(3*A*b + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)])/a^(7/4) + (2*Sqrt[2]*(3*A*b + 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*A*b*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) - (5*Sqrt[2]*B*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (3*Sqrt[2]*A*b*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (5*Sqrt[2]*B*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4))/(128*b^(9/4))
```

IntegrateAlgebraic [**A**] time = 0.74, size = 191, normalized size = 0.64

$$-\frac{(5aB+3Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(5aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{-5a^2B\sqrt{x}-3aAb\sqrt{x}-9abBx^{5/2}+Ab^2x^{5/2}}{16ab^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (-3*a*A*b*Sqrt[x] - 5*a^2*B*Sqrt[x] + A*b^2*x^(5/2) - 9*a*b*B*x^(5/2))/(16*
a*b^2*(a + b*x^2)^2) - ((3*A*b + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(7/4)*b^(9/4)) + ((3*A*b + 5*a*
B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sq
rt[2]*a^(7/4)*b^(9/4))
```

fricas [**B**] time = 1.20, size = 806, normalized size = 2.70

$$\frac{4\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^4b^4\sqrt{-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}}+\left(25B^2a^2+30ABab+9A^2b^2\right)x}\,a^5b^7\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{3}{4}}-\left(5Ba^6b^7+3Aa^5b^8\right)\sqrt{x}\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{3}{4}}}{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}\right)+\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}\log\left(a^2b^2\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}+(5Ba+3Ab)\sqrt{x}\right)-\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}\log\left(-a^2b^2\left(-\frac{625B^4a^4+1500AB^3a^3b+1350A^2B^2a^2b^2+540A^3Bab^3+81A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}+(5Ba+3Ab)\sqrt{x}\right)-4\left(5Ba^2+3Aab+\left(9Bab-Ab^2\right)x^2\right)\sqrt{x}}{64\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a
^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4
)*arctan((sqrt(a^4*b^4*sqrt(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2
*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9)) + (25*B^2*a^2 + 30*A*B*
a*b + 9*A^2*b^2)*x)*a^5*b^7*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^
2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(3/4) - (5*B*a^6*b^7 +
 3*A*a^5*b^8)*sqrt(x)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*
b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(3/4))/(625*B^4*a^4 + 1500*A
*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)) + (a*b^4
*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^
2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(a^2*b^2*
```

```
(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3
+ 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) - (a*b^4*x^4 + 2*
a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2
*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(625*B
^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4
*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) - 4*(5*B*a^2 + 3*A*a*b +
(9*B*a*b - A*b^2)*x^2)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

giac [A] time = 0.52, size = 298, normalized size = 1.00

$$\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{3}}+\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{2}b^{3}}+\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{3}}-\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{1}{4}}Ba+3\,\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{2}b^{3}}-\frac{9\,Babx^{\frac{5}{2}}-Ab^{2}x^{\frac{5}{2}}+5\,Ba^{2}\sqrt{x}+3\,Aab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/64*sqrt(2)*(5
*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/128*sqrt(2)*(5*(a*b^3)^(1/4
)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^2*b^3) - 1/128*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*lo
g(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) - 1/16*(9*B*a*b*x
^(5/2) - A*b^2*x^(5/2) + 5*B*a^2*sqrt(x) + 3*A*a*b*sqrt(x))/((b*x^2 + a)^2*
a*b^2)
```

maple [A] time = 0.02, size = 334, normalized size = 1.12

$$\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^{2}b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a\,b^{2}}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a\,b^{2}}+\frac{\frac{(Ab-9Ba)x^{\frac{5}{2}}}{16ab}-\frac{(3Ab+5Ba)\sqrt{x}}{16b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(A*b-9*B*a)/a/b*x^(5/2)-1/32*(3*A*b+5*B*a)/b^2*x^(1/2))/(b*x^2+a)^2
+3/64/b/a^2*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/1
28/b/a^2*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/64/b/a^2*(a/b)^(1/4)*2^(1
/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+5/64/b^2/a*(a/b)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+5/128/b^2/a*(a/b)^(1/4)*2^(1/2)*B*l
n((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2)))+5/64/b^2/a*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)
```

maxima [A] time = 2.38, size = 280, normalized size = 0.94

$$-\frac{\left(9\,Bab-Ab^{2}\right)x^{\frac{5}{2}}+\left(5\,Ba^{2}+3\,Aab\right)\sqrt{x}}{16\left(ab^{4}x^{4}+2\,a^{2}b^{3}x^{2}+a^{3}b^{2}\right)}+\frac{\frac{2\sqrt{2}(5\,Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}(5\,Ba+3\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}(5\,Ba+3\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(5\,Ba+3\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{128\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((9*B*a*b - A*b^2)*x^(5/2) + (5*B*a^2 + 3*A*a*b)*sqrt(x))/(a*b^4*x^4
+ 2*a^2*b^3*x^2 + a^3*b^2) + 1/128*(2*sqrt(2)*(5*B*a + 3*A*b)*arctan(1/2*sq
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(5*B*a + 3*A*b)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*B*a + 3*A*b)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*B*a
 + 3*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)))/(a*b^2)
```

mupad [B] time = 0.47, size = 799, normalized size = 2.68

$$-\frac{\frac{\sqrt{x}\,(3\,A\,b+5\,B\,a)}{16\,b^{2}}-\frac{x^{5/2}\,(A\,b-9\,B\,a)}{16\,a\,b}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}-\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)}{64\,(-a)^{7/4}\,b^{9/4}}\right)1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}+\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}+\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)}{64\,(-a)^{7/4}\,b^{9/4}}\right)1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}}{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}-\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}-\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}+\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}}\right)(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{32\,(-a)^{7/4}\,b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}-\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}+\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}+\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}}{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}-\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}\right)1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}-\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\,(9\,A^{2}\,b^{2}+30\,A\,B\,a\,b+25\,B^{2}\,a^{2})}{64\,a^{2}\,b}+\frac{(3\,A\,b^{2}+5\,B\,a\,b)\,(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}\right)1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}}\right)(3\,A\,b+5\,B\,a)}{32\,(-a)^{7/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] (atan((((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(6
4*a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4)))*1
i)/(64*(-a)^(7/4)*b^(9/4)) + ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2
*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*
(-a)^(7/4)*b^(9/4)))*1i)/(64*(-a)^(7/4)*b^(9/4)))/(((3*A*b + 5*B*a)*((x^(1/
2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) - ((3*A*b^2 + 5*B*a*b)
*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)) - ((3*A
*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + (
(3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/
4)*b^(9/4))))*(3*A*b + 5*B*a)*1i)/(32*(-a)^(7/4)*b^(9/4)) - ((x^(1/2)*(3*A*
b + 5*B*a))/(16*b^2) - (x^(5/2)*(A*b - 9*B*a))/(16*a*b))/(a^2 + b^2*x^4 + 2
*a*b*x^2) + (atan((((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*
A*B*a*b))/(64*a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7
/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)) + ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*
b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b +
5*B*a)*1i)/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)))/(((3*A*b + 5*
B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) - ((3*A*b^
```

```
2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7/4)*b^(9/4)))*1i)/(64*(-a)^(7/4
)*b^(9/4)) - ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*
b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7/4)*b^
(9/4)))*1i)/(64*(-a)^(7/4)*b^(9/4))))*(3*A*b + 5*B*a))/(32*(-a)^(7/4)*b^(9/
4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=298

$$\frac{(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)}{32}$$

Rubi [A] time = 0.22, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{(3aB+5Ab)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(3aB+5Ab)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{x^{3/2}(3aB+5Ab)}{16a^2b\left(a+bx^2\right)}+\frac{x^{3/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*x^(3/2))/(16
*a^2*b*(a + b*x^2)) - ((5*A*b + 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a*B)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) + ((5*A*b + 3*a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(9/4)*b^(7/4)) - ((5*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx = \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{\left(\frac{5Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{16a^2b}$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} - \frac{(5Ab+3aB)\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{32a^2b^{3/2}} + (5Ab$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{64a^2b^2} + (5$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} + \frac{(5Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{9/4}b^{7/4}} - (5$$

$$= \frac{(Ab-aB)x^{3/2}}{4ab\left(a+bx^2\right)^2} + \frac{(5Ab+3aB)x^{3/2}}{16a^2b\left(a+bx^2\right)} - \frac{(5Ab+3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}} + \frac{(5Ab+3aB)\,\mathrm{ta}}{32\sqrt{}}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.21

$$\frac{2x^{3/2}\left((Ab-aB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)+aB\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{3a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] (2*x^(3/2)*(a*B*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)] + (A*b - a*B)*
Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)]))/(3*a^3*b)
```

IntegrateAlgebraic [A] time = 0.72, size = 182, normalized size = 0.61

$$-\frac{(3aB+5Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(3aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{x^{3/2}\left(a^2B-9aAb-3abBx^2-5Ab^2x^2\right)}{16a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

```
[Out] -1/16*(x^(3/2)*(-9*a*A*b + a^2*B - 5*A*b^2*x^2 - 3*a*b*B*x^2))/(a^2*b*(a +
b*x^2)^2) - ((5*A*b + 3*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(9/4)*b^(7/4)) - ((5*A*b + 3*a*B)*ArcTanh[
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sqrt[2]*a^(9/
4)*b^(7/4))
```

fricas [B] time = 0.87, size = 1005, normalized size = 3.37

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^
3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/
4)*arctan((sqrt((729*B^6*a^6 + 7290*A*B^5*a^5*b + 30375*A^2*B^4*a^4*b^2 + 6
7500*A^3*B^3*a^3*b^3 + 84375*A^4*B^2*a^2*b^4 + 56250*A^5*B*a*b^5 + 15625*A^
6*b^6)*x - (81*B^4*a^9*b^3 + 540*A*B^3*a^8*b^4 + 1350*A^2*B^2*a^7*b^5 + 150
0*A^3*B*a^6*b^6 + 625*A^4*a^5*b^7)*sqrt(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 13
50*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7)))*a^2*b^2*(-
(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 6
25*A^4*b^4)/(a^9*b^7))^(1/4) - (27*B^3*a^5*b^2 + 135*A*B^2*a^4*b^3 + 225*A^
2*B*a^3*b^4 + 125*A^3*a^2*b^5)*sqrt(x)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 13
50*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4))/(81*
B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A
^4*b^4)) - (a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*
a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(
1/4)*log(a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1
500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a
^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x)) + (a^2*b^3*x^4 + 2*a^3*b^2*x
^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A
```

```
^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(81*B^4*a^4 + 540
*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*
b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3
)*sqrt(x)) - 4*((3*B*a*b + 5*A*b^2)*x^3 - (B*a^2 - 9*A*a*b)*x)*sqrt(x))/(a^
2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

giac [A] time = 0.52, size = 298, normalized size = 1.00

$$\frac{3\,Babx^{\frac{7}{2}} + 5\,Ab^{2}x^{\frac{7}{2}} - Ba^{2}x^{\frac{3}{2}} + 9\,Aabx^{\frac{3}{2}}}{16\left(bx^{2} + a\right)^{2}a^{2}b} + \frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + 5\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{3}b^{4}} + \frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + 5\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{3}b^{4}} - \frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + 5\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^{3}b^{4}} + \frac{\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}Ba + 5\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^{3}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*B*a*b*x^(7/2) + 5*A*b^2*x^(7/2) - B*a^2*x^(3/2) + 9*A*a*b*x^(3/2))/
((b*x^2 + a)^2*a^2*b) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)
*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
3*b^4) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) - 1/128
*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/128*sqrt(2)*(3*(a*b^3)^(3/4)*B*a +
 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
3*b^4)
```

maple [A] time = 0.02, size = 335, normalized size = 1.12

$$\frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b} + \frac{5\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b} + \frac{5\sqrt{2}\,A\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}b} + \frac{3\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^{2}} + \frac{3\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^{2}} + \frac{3\sqrt{2}\,B\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a\,b^{2}} + \frac{\frac{(5Ab+3Ba)x^{\frac{7}{2}}}{16a^{2}} + \frac{(9Ab-Ba)x^{\frac{3}{2}}}{16ab}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(5*A*b+3*B*a)/a^2*x^(7/2)+1/32*(9*A*b-B*a)/a/b*x^(3/2))/(b*x^2+a)^2
+5/64/a^2/b/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+5/1
28/a^2/b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+5/64/a^2/b/(a/b)^(1/4)*2^(1
/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/64/a/b^2/(a/b)^(1/4)*2^(1/2)*
B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/128/a/b^2/(a/b)^(1/4)*2^(1/2)*B*l
n((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2)))+3/64/a/b^2/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)
```

maxima [**A**] time = 2.43, size = 253, normalized size = 0.85

$$\frac{\left(3\,Bab+5\,Ab^{2}\right)x^{\frac{7}{2}}-\left(Ba^{2}-9\,Aab\right)x^{\frac{3}{2}}}{16\left(a^{2}b^{3}x^{4}+2\,a^{3}b^{2}x^{2}+a^{4}b\right)}+\frac{\left(3\,Ba+5\,Ab\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*B*a*b + 5*A*b^2)*x^(7/2) - (B*a^2 - 9*A*a*b)*x^(3/2))/(a^2*b^3*x^4
 + 2*a^3*b^2*x^2 + a^4*b) + 1/128*(3*B*a + 5*A*b)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(
1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt
(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b)
```

mupad [**B**] time = 0.32, size = 124, normalized size = 0.42

$$\frac{\frac{x^{7/2}\,\left(5\,A\,b+3\,B\,a\right)}{16\,a^2}+\frac{x^{3/2}\,\left(9\,A\,b-B\,a\right)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(5\,A\,b+3\,B\,a\right)}{32\,{\left(-a\right)}^{9/4}\,b^{7/4}}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)\,\left(5\,A\,b+3\,B\,a\right)}{32\,{\left(-a\right)}^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

```
[Out] ((x^(7/2)*(5*A*b + 3*B*a))/(16*a^2) + (x^(3/2)*(9*A*b - B*a))/(16*a*b))/(a^
2 + b^2*x^4 + 2*a*b*x^2) + (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b + 3*B
*a))/(32*(-a)^(9/4)*b^(7/4)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b
+ 3*B*a))/(32*(-a)^(9/4)*b^(7/4))
```

sympy [**A**] time = 152.84, size = 299, normalized size = 1.00

$$\frac{18Aax^{\frac{3}{2}}}{32a^{4}+64a^{3}bx^{2}+32a^{2}b^{2}x^{4}}+\frac{10Abx^{\frac{7}{2}}}{32a^{4}+64a^{3}bx^{2}+32a^{2}b^{2}x^{4}}+2A\operatorname{RootSum}\left(268435456t^{4}a^{9}b^{3}+625,\left(t\mapsto t\log\left(\frac{2097152t^{3}a^{7}b^{2}}{125}+\sqrt{x}\right)\right)\right)-\frac{18Ba^{2}x^{\frac{3}{2}}}{32a^{4}b+64a^{3}b^{2}x^{2}+32a^{2}b^{3}x^{4}}-\frac{10Bax^{\frac{7}{2}}}{32a^{4}+64a^{3}bx^{2}+32a^{2}b^{2}x^{4}}-\frac{2Ba\operatorname{RootSum}\left(268435456t^{4}a^{9}b^{3}+625,\left(t\mapsto t\log\left(\frac{2097152t^{3}a^{7}b^{2}}{125}+\sqrt{x}\right)\right)\right)}{b}+\frac{2Bx^{\frac{3}{2}}}{4a^{2}b+4ab^{2}x^{2}}+\frac{2B\operatorname{RootSum}\left(65536t^{4}a^{5}b^{3}+1,\left(t\mapsto t\log\left(4096t^{3}a^{4}b^{2}+\sqrt{x}\right)\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] 18*A*a*x**(3/2)/(32*a**4 + 64*a**3*b*x**2 + 32*a**2*b**2*x**4) + 10*A*b*x**
(7/2)/(32*a**4 + 64*a**3*b*x**2 + 32*a**2*b**2*x**4) + 2*A*RootSum(26843545
6*_t**4*a**9*b**3 + 625, Lambda(_t, _t*log(2097152*_t**3*a**7*b**2/125 + sq
```

```
rt(x)))) - 18*B*a**2*x**(3/2)/(32*a**4*b + 64*a**3*b**2*x**2 + 32*a**2*b**3
*x**4) - 10*B*a*x**(7/2)/(32*a**4 + 64*a**3*b*x**2 + 32*a**2*b**2*x**4) - 2
*B*a*RootSum(268435456*_t**4*a**9*b**3 + 625, Lambda(_t, _t*log(2097152*_t*
*3*a**7*b**2/125 + sqrt(x))))/b + 2*B*x**(3/2)/(4*a**2*b + 4*a*b**2*x**2) +
 2*B*RootSum(65536*_t**4*a**5*b**3 + 1, Lambda(_t, _t*log(4096*_t**3*a**4*b
**2 + sqrt(x))))/b
```

3.369 $\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=293

$$-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}-3(aB+7A$$

Rubi [A] time = 0.21, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(aB+7Ab)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{3(aB+7Ab)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{\sqrt{x}\,(aB+7Ab)}{16a^2b\left(a+bx^2\right)}+\frac{\sqrt{x}\,(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*Sqrt[x])/(16*a
^2*b*(a + b*x^2)) - (3*(7*A*b + a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) - (3*(7*A*b + a
*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]
*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{\left(\frac{7Ab}{2}+\frac{aB}{2}\right)\int\frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)}+\frac{(3(7Ab+aB))\int\frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)}+\frac{(3(7Ab+aB))\,\mathrm{Subst}\left(\int\frac{1}{a+bx^4}\,dx,x,\sqrt{x}\right)}{16a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)}+\frac{(3(7Ab+aB))\,\mathrm{Subst}\left(\int\frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,x,\sqrt{x}\right)}{32a^{5/2}b}+\frac{(3(7}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)}+\frac{(3(7Ab+aB))\,\mathrm{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,x,\sqrt{x}\right)}{64a^{5/2}b^{3/2}}+$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)}-\frac{3(7Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{4ab\left(a+bx^2\right)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\left(a+bx^2\right)}-\frac{3(7Ab+aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(7Ab+aB)\,\mathrm{t}}{32\sqrt{}}$$

Mathematica [A] time = 0.28, size = 230, normalized size = 0.78

$$\frac{\frac{(aB+7Ab)\left(7\left(a+bx^2\right)\left(8a^{3/4}\sqrt[4]{b}\sqrt{x}-3\sqrt{2}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)\right)\right)+32a^{7/4}\sqrt[4]{b}\sqrt{x}\right)}{a^{11/4}\sqrt[4]{b}}-256B\sqrt{x}}{896b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] (-256*B*Sqrt[x] + ((7*A*b + a*B)*(32*a^(7/4)*b^(1/4)*Sqrt[x] + 7*(a + b*x^2
)*(8*a^(3/4)*b^(1/4)*Sqrt[x] - 3*Sqrt[2]*(a + b*x^2)*(2*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))))/(a^(11/4)*b^(1/4)))/(8
96*b*(a + b*x^2)^2)
```

IntegrateAlgebraic [**A**] time = 0.70, size = 181, normalized size = 0.62

$$-\frac{3(aB+7Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{3(aB+7Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{\sqrt{x}\left(3a^2B-11aAb-abBx^2-7Ab^2x^2\right)}{16a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

```
[Out] -1/16*(Sqrt[x]*(-11*a*A*b + 3*a^2*B - 7*A*b^2*x^2 - a*b*B*x^2))/(a^2*b*(a +
 b*x^2)^2) - (3*(7*A*b + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(11/4)*b^(5/4)) + (3*(7*A*b + a*B)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(32*Sqrt[2]*a^(
11/4)*b^(5/4))
```

fricas [**B**] time = 0.61, size = 793, normalized size = 2.71

$$\frac{12\left(a^2b^3x^4+2a^3b^2x^2+a^4b\right)\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^6b^2\sqrt{-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}}+\left(B^2a^2+14ABab+49A^2b^2\right)x}\,a^8b^4\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{3}{4}}-\left(Ba^9b^4+7Aa^8b^5\right)\sqrt{x}\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{3}{4}}}{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}\right)+3\left(a^2b^3x^4+2a^3b^2x^2+a^4b\right)\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(3a^3b\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}+3(Ba+7Ab)\sqrt{x}\right)-3\left(a^2b^3x^4+2a^3b^2x^2+a^4b\right)\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(-3a^3b\left(-\frac{B^4a^4+28AB^3a^3b+294A^2B^2a^2b^2+1372A^3Bab^3+2401A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}+3(Ba+7Ab)\sqrt{x}\right)-4\left(3Ba^2-11Aab-\left(Bab+7Ab^2\right)x^2\right)\sqrt{x}}{64\left(a^2b^3x^4+2a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b
+ 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*
arctan((sqrt(a^6*b^2*sqrt(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2
+ 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5)) + (B^2*a^2 + 14*A*B*a*b + 49
*A^2*b^2)*x)*a^8*b^4*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 13
72*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(3/4) - (B*a^9*b^4 + 7*A*a^8*b^5
)*sqrt(x)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*
b^3 + 2401*A^4*b^4)/(a^11*b^5))^(3/4))/(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*
B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)) + 3*(a^2*b^3*x^4 + 2*a^3*b^
2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3
*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*log(3*a^3*b*(-(B^4*a^4 + 28*A*B^
3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5)
)^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 3*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b
)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 24
```

```
01*A^4*b^4)/(a^11*b^5))^(1/4)*log(-3*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 29
4*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*
(B*a + 7*A*b)*sqrt(x)) - 4*(3*B*a^2 - 11*A*a*b - (B*a*b + 7*A*b^2)*x^2)*sqr
t(x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

giac [**A**] time = 0.44, size = 293, normalized size = 1.00

$$\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{3}b^{2}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{3}b^{2}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{3}b^{2}}-\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}Ba+7\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{3}b^{2}}+\frac{Babx^{\frac{5}{2}}+7\,Ab^{2}x^{\frac{5}{2}}-3\,Ba^{2}\sqrt{x}+11\,Aab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/64*sqrt(2)*((a*
b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a
+ 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
3*b^2) - 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 1/16*(B*a*b*x^(5/2) + 7
*A*b^2*x^(5/2) - 3*B*a^2*sqrt(x) + 11*A*a*b*sqrt(x))/((b*x^2 + a)^2*a^2*b)
```

maple [**A**] time = 0.02, size = 325, normalized size = 1.11

$$\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,A\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{2}b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^{2}b}+\frac{\frac{(7Ab+Ba)x^{\frac{5}{2}}}{16a^{2}}+\frac{(11Ab-3Ba)\sqrt{x}}{16ab}}{\left(b\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2*(1/32*(7*A*b+B*a)/a^2*x^(5/2)+1/32*(11*A*b-3*B*a)/a/b*x^(1/2))/(b*x^2+a)^
2+21/128/a^3*(a/b)^(1/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^
(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+21/64/a^3*(a/b)^(1/4)*2
^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+21/64/a^3*(a/b)^(1/4)*2^(1/2
)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/128/a^2/b*(a/b)^(1/4)*2^(1/2)*B
*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2)))+3/64/a^2/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+3/64/a^2/b*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)
```

maxima [**A**] time = 2.33, size = 276, normalized size = 0.94

$$\frac{\left(Bab+7\,Ab^{2}\right)x^{\frac{5}{2}}-\left(3\,Ba^{2}-11\,Aab\right)\sqrt{x}}{16\left(a^{2}b^{3}x^{4}+2\,a^{3}b^{2}x^{2}+a^{4}b\right)}+\frac{3\left(\frac{2\sqrt{2}\,(Ba+7\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(Ba+7\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(Ba+7\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(Ba+7\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*((B*a*b + 7*A*b^2)*x^(5/2) - (3*B*a^2 - 11*A*a*b)*sqrt(x))/(a^2*b^3*x^
4 + 2*a^3*b^2*x^2 + a^4*b) + 3/128*(2*sqrt(2)*(B*a + 7*A*b)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 7*A*b)*arctan(-1/2*sqrt(2)
*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 7*A*b)*log(sqrt(2)*a^(1/4)*b^(1
/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a + 7*A*b
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1
/4)))/(a^2*b)
```

mupad [B] time = 0.48, size = 780, normalized size = 2.66

$$\frac{\frac{x^{5/2}\,(7\,A\,b+B\,a)}{16\,a^2}+\frac{\sqrt{x}\,(11\,A\,b-3\,B\,a)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{9\,(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)}{64\,(-a)^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,(-a)^{11/4}\,b^{5/4}}+\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}+\frac{9\,(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)}{64\,(-a)^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,(-a)^{11/4}\,b^{5/4}}}{\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{9\,(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)}{64\,(-a)^{15/4}\,b^{5/4}}\right)}{64\,(-a)^{11/4}\,b^{5/4}}-\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}+\frac{9\,(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)}{64\,(-a)^{15/4}\,b^{5/4}}\right)}{64\,(-a)^{11/4}\,b^{5/4}}}\right)(7\,A\,b+B\,a)\,3\mathrm{i}}{32\,(-a)^{11/4}\,b^{5/4}}-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)9\mathrm{i}}{64\,(-a)^{15/4}\,b^{5/4}}\right)}{64\,(-a)^{11/4}\,b^{5/4}}+\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}+\frac{(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)9\mathrm{i}}{64\,(-a)^{15/4}\,b^{5/4}}\right)}{64\,(-a)^{11/4}\,b^{5/4}}}{\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)9\mathrm{i}}{64\,(-a)^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,(-a)^{11/4}\,b^{5/4}}-\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}+\frac{(7\,A\,b+B\,a)\left(7\,A\,b^3+B\,a\,b^2\right)9\mathrm{i}}{64\,(-a)^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,(-a)^{11/4}\,b^{5/4}}}\right)(7\,A\,b+B\,a)}{32\,(-a)^{11/4}\,b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(1/2)*(a + b*x^2)^3),x)
```

```
[Out] ((x^(5/2)*(7*A*b + B*a))/(16*a^2) + (x^(1/2)*(11*A*b - 3*B*a))/(16*a*b))/(a
^2 + b^2*x^4 + 2*a*b*x^2) - (atan((((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 +
 B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) - (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2)
)/(64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(5/4)) + ((7*A*b + B*a)*(
(9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + (9*(7*A*b +
B*a)*(7*A*b^3 + B*a*b^2))/(64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(
5/4)))/((3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2
))/(64*a^4) - (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2))/(64*(-a)^(15/4)*b^(5/4)
)))/(64*(-a)^(11/4)*b^(5/4)) - (3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B
^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2))/
(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4))))*(7*A*b + B*a)*3i)/(32
*(-a)^(11/4)*b^(5/4)) - (3*atan(((3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 +
 B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) - ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9
i)/(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4)) + (3*(7*A*b + B*a)*(
(9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + ((7*A*b + B*
a)*(7*A*b^3 + B*a*b^2)*9i)/(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/
4)))/(((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(
64*a^4) - ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9i)/(64*(-a)^(15/4)*b^(5/4)))*
3i)/(64*(-a)^(11/4)*b^(5/4)) - ((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2
*a^2*b + 14*A*B*a*b^2))/(64*a^4) + ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9i)/(
64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(5/4))))*(7*A*b + B*a))/(32*
(-a)^(11/4)*b^(5/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.370 $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=322

$$-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB}{32}$$

Rubi [A] time = 0.23, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5(9Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}+\frac{5(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}a^{13/4}b^{3/4}}+\frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)}-\frac{5(9Ab-aB)}{16a^3b\sqrt{x}}+\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-5*(9*A*b - a*B))/(16*a^3*b*Sqrt[x]) + (A*b - a*B)/(4*a*b*Sqrt[x]*(a + b*x
^2)^2) + (9*A*b - a*B)/(16*a^2*b*Sqrt[x]*(a + b*x^2)) + (5*(9*A*b - a*B)*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(13/4)*b^(3/4))
- (5*(9*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[
2]*a^(13/4)*b^(3/4)) - (5*(9*A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(13/4)*b^(3/4)) + (5*(9*A*b - a*B)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(1
3/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{\left(\frac{9Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} + \frac{(5(9Ab-aB))\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\int \frac{\sqrt{x}}{a+bx^2}\,dx}{32a^3}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\,\text{Subst}\left(\int \frac{x}{a+}\right.}{16a^3}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} + \frac{(5(9Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt{}}{}\right.}{32a^3\sqrt{b}}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{(5(9Ab-aB))\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{}}{\sqrt{}}}\right.}{64a^3}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} - \frac{5(9Ab-aB)\log\left(\sqrt{a}-\sqrt{}\right.}{64\sqrt{2}\,a^{13/4}}$$

$$= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}} + \frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2} + \frac{9Ab-aB}{16a^2b\sqrt{x}\left(a+bx^2\right)} + \frac{5(9Ab-aB)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,a^{13/4}b^{3/4}}$$

Mathematica [C] time = 0.18, size = 147, normalized size = 0.46

$$\frac{2x^{3/2}(aB-Ab)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4} - \frac{2Abx^{3/2}\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^4} - \frac{2A}{a^3\sqrt{x}} + \frac{A\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{13/4}} + \frac{aA\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*A)/(a^3*Sqrt[x]) + (A*b^(1/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a
)^(13/4) + (a*A*b^(1/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(17/4)
- (2*A*b*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^4) + (2
*(-(A*b) + a*B)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^
4)
```

IntegrateAlgebraic [A] time = 0.73, size = 190, normalized size = 0.59

$$-\frac{5(aB-9Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{32\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{5(aB-9Ab)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{-32a^2A+9a^2Bx^2-81aAbx^2+5abBx^4-45Ab^2x^4}{16a^3\sqrt{x}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

```
[Out] (-32*a^2*A - 81*a*A*b*x^2 + 9*a^2*B*x^2 - 45*A*b^2*x^4 + 5*a*b*B*x^4)/(16*a
^3*Sqrt[x]*(a + b*x^2)^2) - (5*(-9*A*b + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(13/4)*b^(3/4)) - (5*(-9*
A*b + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/(32*Sqrt[2]*a^(13/4)*b^(3/4))
```

fricas [B] time = 0.82, size = 988, normalized size = 3.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b +
486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*ar
ctan((sqrt((B^6*a^6 - 54*A*B^5*a^5*b + 1215*A^2*B^4*a^4*b^2 - 14580*A^3*B^3
*a^3*b^3 + 98415*A^4*B^2*a^2*b^4 - 354294*A^5*B*a*b^5 + 531441*A^6*b^6)*x -
 (B^4*a^11*b - 36*A*B^3*a^10*b^2 + 486*A^2*B^2*a^9*b^3 - 2916*A^3*B*a^8*b^4
 + 6561*A^4*a^7*b^5)*sqrt(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2
- 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3)))*a^3*b*(-(B^4*a^4 - 36*A*B^3
*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))
^(1/4) + (B^3*a^6*b - 27*A*B^2*a^5*b^2 + 243*A^2*B*a^4*b^3 - 729*A^3*a^3*b^
4)*sqrt(x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a
*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4))/(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2
*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)) - 5*(a^3*b^2*x^5 + 2*a^4*b
*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*
B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*log(125*a^10*b^2*(-(B^4*a^4 - 36*
A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*
b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3
)*sqrt(x)) + 5*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^
```

```
3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1
/4)*log(-125*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2
916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2
*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) + 4*(5*(B*a*b - 9*A*b^2)*x
^4 - 32*A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^
3 + a^5*x)
```

giac [A] time = 0.41, size = 300, normalized size = 0.93

$$-\frac{2\,A}{a^{3}\sqrt{x}}+\frac{5\,Babx^{\frac{7}{2}}-13\,Ab^{2}x^{\frac{7}{2}}+9\,Ba^{2}x^{\frac{3}{2}}-17\,Aabx^{\frac{3}{2}}}{16\,\left(bx^{2}+a\right)^{2}a^{3}}+\frac{5\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-9\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b^{3}}+\frac{5\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-9\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b^{3}}-\frac{5\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-9\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b^{3}}+\frac{5\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}Ba-9\,\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/16*(5*B*a*b*x^(7/2) - 13*A*b^2*x^(7/2) + 9*B*a^2*x^(
3/2) - 17*A*a*b*x^(3/2))/((b*x^2 + a)^2*a^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*
B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(
3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(a^4*b^3) - 5/128*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) + 5/128*sqrt(2)*((a*b
^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^4*b^3)
```

maple [A] time = 0.02, size = 363, normalized size = 1.13

$$-\frac{13 A\,b^{2} x^{\frac{7}{2}}}{16 \left(b\,x^{2}+a\right)^{2} a^{3}}+\frac{5 B b\,x^{\frac{7}{2}}}{16 \left(b\,x^{2}+a\right)^{2} a^{2}}-\frac{17 A b\,x^{\frac{3}{2}}}{16 \left(b\,x^{2}+a\right)^{2} a^{2}}+\frac{9 B\,x^{\frac{3}{2}}}{16 \left(b\,x^{2}+a\right)^{2} a}-\frac{45 \sqrt{2}\,A \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{45 \sqrt{2}\,A \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{45 \sqrt{2}\,A \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}+\frac{5 \sqrt{2}\,B \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2} b}+\frac{5 \sqrt{2}\,B \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2} b}+\frac{5 \sqrt{2}\,B \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2} b}-\frac{2 A}{a^{3} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x)
```

```
[Out] -13/16/a^3/(b*x^2+a)^2*x^(7/2)*b^2*A+5/16/a^2/(b*x^2+a)^2*x^(7/2)*b*B-17/16
/a^2/(b*x^2+a)^2*A*x^(3/2)*b+9/16/a/(b*x^2+a)^2*B*x^(3/2)-45/128/a^3/(a/b)^
(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-45/64/a^3/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)-45/64/a^3/(a/b)^(1/4)*2^(1/2)*A*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)+5/128/a^2/b/(a/b)^(1/4)*2^(1/2)*B*ln((x-(a/b)^(1/4)*
2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+5
/64/a^2/b/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+5/64/
a^2/b/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-2*A/a^3/x
^(1/2)
```

maxima [A] time = 2.59, size = 255, normalized size = 0.79

$$\frac{5\left(Bab-9\,Ab^{2}\right)x^{4}-32\,Aa^{2}+9\left(Ba^{2}-9\,Aab\right)x^{2}}{16\left(a^{3}b^{2}x^{\frac{9}{2}}+2\,a^{4}bx^{\frac{5}{2}}+a^{5}\sqrt{x}\right)}+\frac{5\,(Ba-9\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(5*(B*a*b - 9*A*b^2)*x^4 - 32*A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)/(a^3*b^
2*x^(9/2) + 2*a^4*b*x^(5/2) + a^5*sqrt(x)) + 5/128*(B*a - 9*A*b)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(
1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

mupad [B] time = 0.19, size = 133, normalized size = 0.41

$$\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(9\,A\,b-B\,a\right)}{32\,{(-a)}^{13/4}\,b^{3/4}}-\frac{\frac{2\,A}{a}+\frac{9\,x^2\,\left(9\,A\,b-B\,a\right)}{16\,a^2}+\frac{5\,b\,x^4\,\left(9\,A\,b-B\,a\right)}{16\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{9/2}+2\,a\,b\,x^{5/2}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{{(-a)}^{1/4}}\right)\,\left(9\,A\,b-B\,a\right)}{32\,{(-a)}^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x)
```

```
[Out] (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(9*A*b - B*a))/(32*(-a)^(13/4)*b^(3/4
)) - ((2*A)/a + (9*x^2*(9*A*b - B*a))/(16*a^2) + (5*b*x^4*(9*A*b - B*a))/(1
6*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*x^(5/2)) - (5*atanh((b^(1/4)*x^(
1/2))/(-a)^(1/4))*(9*A*b - B*a))/(32*(-a)^(13/4)*b^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.371 $\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=322

$$\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{7(11Ab}{}$$

Rubi [A] time = 0.24, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{7(11Ab-3aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{7(11Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{7(11Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-7*(11*A*b - 3*a*B))/(48*a^3*b*x^(3/2)) + (A*b - a*B)/(4*a*b*x^(3/2)*(a +
b*x^2)^2) + (11*A*b - 3*a*B)/(16*a^2*b*x^(3/2)*(a + b*x^2)) + (7*(11*A*b -
3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(15/4)*
b^(1/4)) - (7*(11*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) + (7*(11*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(15/4)*b^(1/4)) - (7*
(11*A*b - 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(64*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{\left(\frac{11Ab}{2}-\frac{3aB}{2}\right)\int\frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}+\frac{(7(11Ab-3aB))\int\frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$=-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{(7(11Ab-3aB))\int\frac{1}{\sqrt{x}\left(a+b\right.}}{32a^3}$$

$$=-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{(7(11Ab-3aB))\,\text{Subst}\left(\int\right.}{16a^3}$$

$$=-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{(7(11Ab-3aB))\,\text{Subst}\left(\int\right.}{32a^{7/2}}$$

$$=-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}-\frac{(7(11Ab-3aB))\,\text{Subst}\left(\int\right.}{64a^{7/}}$$

$$=-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}+\frac{7(11Ab-3aB)\log\left(\sqrt{a}-\right.}{64\sqrt{2}\,a^{1}}$$

$$=-\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}\left(a+bx^2\right)}+\frac{7(11Ab-3aB)\tan^{-1}\left(1-\right.}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}$$

Mathematica [A] time = 0.41, size = 400, normalized size = 1.24

$$\frac{-\frac{96a^{7/4}Ab\sqrt{x}}{\left(a+bx^2\right)^2}-\frac{360a^{3/4}Ab\sqrt{x}}{a+bx^2}-\frac{256a^{3/4}A}{x^{3/2}}+\frac{96a^{11/4}B\sqrt{x}}{\left(a+bx^2\right)^2}+\frac{168a^{7/4}B\sqrt{x}}{a+bx^2}+231\sqrt{2}Ab^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-231\sqrt{2}Ab^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+\frac{42\sqrt{2}(11Ab-3aB)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{42\sqrt{2}(11Ab-3aB)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}-\frac{63\sqrt{2}aB\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{63\sqrt{2}aB\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}}{384a^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] ((-256*a^(3/4)*A)/x^(3/2) - (96*a^(7/4)*A*b*Sqrt[x])/(a + b*x^2)^2 + (96*a^
(11/4)*B*Sqrt[x])/(a + b*x^2)^2 - (360*a^(3/4)*A*b*Sqrt[x])/(a + b*x^2) + (
```

```
168*a^(7/4)*B*Sqrt[x]])/(a + b*x^2) + (42*Sqrt[2]*(11*A*b - 3*a*B)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (42*Sqrt[2]*(11*A*b - 3*a*B
)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + 231*Sqrt[2]*A*b^
(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (63*Sqrt
[2]*a*B*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)
 - 231*Sqrt[2]*A*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x] + (63*Sqrt[2]*a*B*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/b^(1/4))/(384*a^(15/4))
```

IntegrateAlgebraic [A] time = 0.62, size = 192, normalized size = 0.60

$$-\frac{7(3aB-11Ab)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{7(3aB-11Ab)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{-32a^2A+33a^2Bx^2-121aAbx^2+21abBx^4-77Ab^2x^4}{48a^3x^{3/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

```
[Out] (-32*a^2*A - 121*a*A*b*x^2 + 33*a^2*B*x^2 - 77*A*b^2*x^4 + 21*a*b*B*x^4)/(4
8*a^3*x^(3/2)*(a + b*x^2)^2) - (7*(-11*A*b + 3*a*B)*ArcTan[(Sqrt[a] - Sqrt[
b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(15/4)*b^(1/4)) + (
7*(-11*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sq
rt[b]*x)])/(32*Sqrt[2]*a^(15/4)*b^(1/4))
```

fricas [B] time = 1.14, size = 809, normalized size = 2.51

$$-\frac{84\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^8\sqrt{-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}}+\left(9\,B^2a^2-66\,ABab+121\,A^2b^2\right)x}\,a^{11}b\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{3}{4}}+\left(3\,Ba^{12}b-11\,Aa^{11}b^2\right)\sqrt{x}\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{3}{4}}}{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}\right)+21\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{1}{4}}\log\left(7\,a^4\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{1}{4}}-7\left(3\,Ba-11\,Ab\right)\sqrt{x}\right)-21\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{1}{4}}\log\left(-7\,a^4\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{1}{4}}-7\left(3\,Ba-11\,Ab\right)\sqrt{x}\right)-4\left(7\left(3\,Bab-11\,Ab^2\right)x^4-32\,Aa^2+11\left(3\,Ba^2-11\,Aab\right)x^2\right)\sqrt{x}}{192\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(84*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B^3
*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)
)^(1/4)*arctan((sqrt(a^8*sqrt(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^
2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b)) + (9*B^2*a^2 - 66*
A*B*a*b + 121*A^2*b^2)*x)*a^11*b*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^
2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(3/4) + (3*B*a
^12*b - 11*A*a^11*b^2)*sqrt(x)*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*
B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(3/4))/(81*B^4*a
^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^
4*b^4)) + 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B
^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*
b))^(1/4)*log(7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2
 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b)*
sqrt(x)) - 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*
```

```
B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15
*b))^(1/4)*log(-7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b
^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b
)*sqrt(x)) - 4*(7*(3*B*a*b - 11*A*b^2)*x^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*
a*b)*x^2)*sqrt(x))/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
```

giac [A] time = 0.45, size = 304, normalized size = 0.94

$$\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b}+\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^{4}b}+\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b}-\frac{7\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}Ba-11\left(ab^{3}\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^{4}b}-\frac{2\,A}{3\,a^{3}x^{\frac{3}{2}}}+\frac{7\,Babx^{\frac{5}{2}}-15\,Ab^{2}x^{\frac{5}{2}}+11\,Ba^{2}\sqrt{x}-19\,Aab\sqrt{x}}{16\left(bx^{2}+a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 7/64*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/64*sqrt(2)*(3*
(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*
B*a - 11*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^4*b) - 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 2/3*A/(a^3*x^(3/2))
 + 1/16*(7*B*a*b*x^(5/2) - 15*A*b^2*x^(5/2) + 11*B*a^2*sqrt(x) - 19*A*a*b*s
qrt(x))/((b*x^2 + a)^2*a^3)
```

maple [A] time = 0.02, size = 357, normalized size = 1.11

$$-\frac{15A\,b^{2}x^{\frac{5}{2}}}{16\left(b\,x^{2}+a\right)^{2}a^{3}}+\frac{7Bb\,x^{\frac{5}{2}}}{16\left(b\,x^{2}+a\right)^{2}a^{2}}-\frac{19Ab\sqrt{x}}{16\left(b\,x^{2}+a\right)^{2}a^{2}}+\frac{11B\sqrt{x}}{16\left(b\,x^{2}+a\right)^{2}a}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{4}}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{4}}-\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,Ab\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^{4}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64a^{3}}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,B\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128a^{3}}-\frac{2A}{3a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x)
```

```
[Out] -15/16/a^3/(b*x^2+a)^2*x^(5/2)*b^2*A+7/16/a^2/(b*x^2+a)^2*x^(5/2)*b*B-19/16
/a^2/(b*x^2+a)^2*A*x^(1/2)*b+11/16/a/(b*x^2+a)^2*B*x^(1/2)-77/64/a^4*(a/b)^
(1/4)*2^(1/2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b-77/128/a^4*(a/b)^(1
/4)*2^(1/2)*A*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b-77/64/a^4*(a/b)^(1/4)*2^(1/2)*A*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)*b+21/64/a^3*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)+21/128/a^3*(a/b)^(1/4)*2^(1/2)*B*ln((x+(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+
21/64/a^3*(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-2/3*A
/a^3/x^(3/2)
```

maxima [A] time = 2.43, size = 285, normalized size = 0.89

$$\frac{7\left(3\,Bab-11\,Ab^{2}\right)x^{4}-32\,Aa^{2}+11\left(3\,Ba^{2}-11\,Aab\right)x^{2}}{48\left(a^{3}b^{2}x^{\frac{11}{2}}+2\,a^{4}bx^{\frac{7}{2}}+a^{5}x^{\frac{3}{2}}\right)}+\frac{7\left(\frac{2\,\sqrt{2}\,(3\,Ba-11\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(3\,Ba-11\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(3\,Ba-11\,Ab)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(3\,Ba-11\,Ab)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(7*(3*B*a*b - 11*A*b^2)*x^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*a*b)*x^2)/
(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7/2) + a^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*B*
a - 11*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*B*a
 - 11*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*B*a -
 11*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4
)*b^(1/4)) - sqrt(2)*(3*B*a - 11*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^3
```

mupad [B] time = 0.53, size = 888, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x)
```

```
[Out] - ((2*A)/(3*a) + (11*x^2*(11*A*b - 3*B*a))/(48*a^2) + (7*b*x^4*(11*A*b - 3*
B*a))/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11/2) + 2*a*b*x^(7/2)) - (atan((((11*
A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 529858
56*A*B*a^10*b^4) - (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^
14*b^3))/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a)^(15/4)*b^(1/4)) + ((11*A*b
- 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A
*B*a^10*b^4) + (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b
^3))/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a)^(15/4)*b^(1/4)))/((7*(11*A*b -
3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B
*a^10*b^4) - (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3
))/(64*(-a)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4)) - (7*(11*A*b - 3*B*a
)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10
*b^4) + (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3))/(6
4*(-a)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4))))*(11*A*b - 3*B*a)*7i)/(3
2*(-a)^(15/4)*b^(1/4)) - (7*atan(((7*(11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^
2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) - ((11*A*b - 3*B*
a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4))
))/(64*(-a)^(15/4)*b^(1/4)) + (7*(11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^
```

```
9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) + ((11*A*b - 3*B*a)*(
80740352*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4))))/(
64*(-a)^(15/4)*b^(1/4)))/(((11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5
+ 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) - ((11*A*b - 3*B*a)*(807403
52*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*
(-a)^(15/4)*b^(1/4)) - ((11*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7
225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) + ((11*A*b - 3*B*a)*(80740352*
A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a
)^(15/4)*b^(1/4))))*(11*A*b - 3*B*a))/(32*(-a)^(15/4)*b^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.372 $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=343

$$\frac{9\sqrt[4]{b}\,(13Ab-5aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{9\sqrt[4]{b}\,(13Ab-5aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-$$

Rubi [A] time = 0.26, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{9\sqrt[4]{b}\,(13Ab-5aB)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{9\sqrt[4]{b}\,(13Ab-5aB)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{64\sqrt{2}\,a^{17/4}}-\frac{9\sqrt[4]{b}\,(13Ab-5aB)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{9\sqrt[4]{b}\,(13Ab-5aB)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{32\sqrt{2}\,a^{17/4}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-9*(13*A*b - 5*a*B))/(80*a^3*b*x^(5/2)) + (9*(13*A*b - 5*a*B))/(16*a^4*Sqr
t[x]) + (A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + (13*A*b - 5*a*B)/(16*a^
2*b*x^(5/2)*(a + b*x^2)) - (9*b^(1/4)*(13*A*b - 5*a*B)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) + (9*b^(1/4)*(13*A*b - 5*a
*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(32*Sqrt[2]*a^(17/4)) +
(9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4)) - (9*b^(1/4)*(13*A*b - 5*a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{\left(\frac{13Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} + \frac{(9(13Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx}{32a^2b}$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} -$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} +$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} +$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} -$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} +$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} +$$

$$= -\frac{9(13Ab-5aB)}{80a^3bx^{5/2}} + \frac{9(13Ab-5aB)}{16a^4\sqrt{x}} + \frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2} + \frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)} -$$

Mathematica [C] time = 0.47, size = 189, normalized size = 0.55

$$-\frac{2bx^{3/2}(aB-2Ab)\,{}_2F_1\left(\frac{3}{4},2;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5} + \frac{2bx^{3/2}(Ab-aB)\,{}_2F_1\left(\frac{3}{4},3;\frac{7}{4};-\frac{bx^2}{a}\right)}{3a^5} + \frac{6Ab-2aB}{a^4\sqrt{x}} - \frac{2A}{5a^3x^{5/2}} + \frac{\sqrt[4]{b}\,(3Ab-aB)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{17/4}} + \frac{\sqrt[4]{b}\,(aB-3Ab)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{-a}}\right)}{(-a)^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-2*A)/(5*a^3*x^(5/2)) + (6*A*b - 2*a*B)/(a^4*Sqrt[x]) + (b^(1/4)*(3*A*b -
a*B)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(17/4) + (b^(1/4)*(-3*A*b +
 a*B)*ArcTanh[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/(-a)^(17/4) - (2*b*(-2*A*b + a
*B)*x^(3/2)*Hypergeometric2F1[3/4, 2, 7/4, -((b*x^2)/a)])/(3*a^5) + (2*b*(A
*b - a*B)*x^(3/2)*Hypergeometric2F1[3/4, 3, 7/4, -((b*x^2)/a)])/(3*a^5)
```

IntegrateAlgebraic [A] time = 0.64, size = 224, normalized size = 0.65

$$\frac{9\left(5a\sqrt[4]{b}\,B-13Ab^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{9\left(5a\sqrt[4]{b}\,B-13Ab^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{32\sqrt{2}\,a^{17/4}}+\frac{-32a^3A-160a^3Bx^2+416a^2Abx^2-405a^2bBx^4+1053aAb^2x^4-225ab^2Bx^6+585Ab^3x^6}{80a^4x^{5/2}\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

```
[Out] (-32*a^3*A + 416*a^2*A*b*x^2 - 160*a^3*B*x^2 + 1053*a*A*b^2*x^4 - 405*a^2*b
*B*x^4 + 585*A*b^3*x^6 - 225*a*b^2*B*x^6)/(80*a^4*x^(5/2)*(a + b*x^2)^2) +
(9*(-13*A*b^(5/4) + 5*a*b^(1/4)*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])])/(32*Sqrt[2]*a^(17/4)) + (9*(-13*A*b^(5/4) + 5*a*b^
(1/4)*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(32*Sqrt[2]*a^(17/4))
```

fricas [B] time = 1.47, size = 1043, normalized size = 3.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(180*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A
*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a
^17)^(1/4)*arctan((sqrt((15625*B^6*a^6*b^2 - 243750*A*B^5*a^5*b^3 + 1584375
*A^2*B^4*a^4*b^4 - 5492500*A^3*B^3*a^3*b^5 + 10710375*A^4*B^2*a^2*b^6 - 111
38790*A^5*B*a*b^7 + 4826809*A^6*b^8)*x - (625*B^4*a^13*b - 6500*A*B^3*a^12*
b^2 + 25350*A^2*B^2*a^11*b^3 - 43940*A^3*B*a^10*b^4 + 28561*A^4*a^9*b^5)*sq
rt(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3
*B*a*b^4 + 28561*A^4*b^5)/a^17))*a^4*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2
+ 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4) +
(125*B^3*a^7*b - 975*A*B^2*a^6*b^2 + 2535*A^2*B*a^5*b^3 - 2197*A^3*a^4*b^4)
*sqrt(x)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43
940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4))/(625*B^4*a^4*b - 6500*A*B^3*a
^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)) - 45*(
a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2
+ 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*lo
```

```
g(729*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 -
43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 975*A
*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) + 45*(a^4*b^2*x^7
+ 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*
B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(1/4)*log(-729*a^13*
(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B
*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*a^3*b - 975*A*B^2*a^2*b^
2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) + 4*(45*(5*B*a*b^2 - 13*A*b^3
)*x^6 + 81*(5*B*a^2*b - 13*A*a*b^2)*x^4 + 32*A*a^3 + 32*(5*B*a^3 - 13*A*a^2
*b)*x^2)*sqrt(x))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

giac [A] time = 0.54, size = 326, normalized size = 0.95

$$-\frac{9\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba-13\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{5}b^{2}}-\frac{9\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba-13\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64a^{5}b^{2}}+\frac{9\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba-13\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^{5}b^{2}}-\frac{9\sqrt{2}\left(5\left(ab^{3}\right)^{\frac{3}{4}}Ba-13\left(ab^{3}\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128a^{5}b^{2}}-\frac{13Bab^{2}x^{\frac{7}{2}}-21Ab^{3}x^{\frac{7}{2}}+17Ba^{2}bx^{\frac{3}{2}}-25Aab^{2}x^{\frac{3}{2}}}{16\left(bx^{2}+a\right)^{2}a^{4}}-\frac{2\left(5Bax^{2}-15Abx^{2}+Aa\right)}{5a^{4}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -9/64*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(
2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) - 9/64*sqrt(2)*
(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 9/128*sqrt(2)*(5*(a*b^3)^(
3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(a^5*b^2) - 9/128*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*
b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) - 1/16*(13*B
*a*b^2*x^(7/2) - 21*A*b^3*x^(7/2) + 17*B*a^2*b*x^(3/2) - 25*A*a*b^2*x^(3/2)
)/((b*x^2 + a)^2*a^4) - 2/5*(5*B*a*x^2 - 15*A*b*x^2 + A*a)/(a^4*x^(5/2))
```

maple [A] time = 0.03, size = 381, normalized size = 1.11

$$\frac{21Ab^{3}x^{\frac{7}{2}}}{16\left(bx^{2}+a\right)^{2}a^{4}}-\frac{13Bb^{2}x^{\frac{7}{2}}}{16\left(bx^{2}+a\right)^{2}a^{3}}+\frac{25Ab^{2}x^{\frac{3}{2}}}{16\left(bx^{2}+a\right)^{2}a^{3}}-\frac{17Bbx^{\frac{3}{2}}}{16\left(bx^{2}+a\right)^{2}a^{2}}+\frac{117\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{4}}+\frac{117\sqrt{2}\,Ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{4}}+\frac{117\sqrt{2}\,Ab\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{4}}-\frac{45\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{45\sqrt{2}\,B\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{45\sqrt{2}\,B\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{128\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{6Ab}{a^{4}\sqrt{x}}-\frac{2B}{a^{3}\sqrt{x}}-\frac{2A}{5a^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x)
```

```
[Out] 21/16/a^4*b^3/(b*x^2+a)^2*x^(7/2)*A-13/16/a^3*b^2/(b*x^2+a)^2*x^(7/2)*B+25/
16/a^3*b^2/(b*x^2+a)^2*A*x^(3/2)-17/16/a^2*b/(b*x^2+a)^2*B*x^(3/2)+117/128/
a^4*b/(a/b)^(1/4)*2^(1/2)*A*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/
(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+117/64/a^4*b/(a/b)^(1/4)*2^(1/
2)*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+117/64/a^4*b/(a/b)^(1/4)*2^(1/2)
*A*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-45/128/a^3/(a/b)^(1/4)*2^(1/2)*B*l
n((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2)))-45/64/a^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*
```

```
x^(1/2)+1)-45/64/a^3/(a/b)^(1/4)*2^(1/2)*B*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)-2/5*A/a^3/x^(5/2)+6/a^4/x^(1/2)*A*b-2/a^3/x^(1/2)*B
```

maxima [**A**] time = 2.46, size = 285, normalized size = 0.83

$$-\frac{45\left(5\,Bab^{2}-13\,Ab^{3}\right)x^{6}+81\left(5\,Ba^{2}b-13\,Aab^{2}\right)x^{4}+32\,Aa^{3}+32\left(5\,Ba^{3}-13\,Aa^{2}b\right)x^{2}}{80\left(a^{4}b^{2}x^{\frac{13}{2}}+2\,a^{5}bx^{\frac{9}{2}}+a^{6}x^{\frac{5}{2}}\right)}-\frac{9\left(5\,Bab-13\,Ab^{2}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{128\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/80*(45*(5*B*a*b^2 - 13*A*b^3)*x^6 + 81*(5*B*a^2*b - 13*A*a*b^2)*x^4 + 32
*A*a^3 + 32*(5*B*a^3 - 13*A*a^2*b)*x^2)/(a^4*b^2*x^(13/2) + 2*a^5*b*x^(9/2)
 + a^6*x^(5/2)) - 9/128*(5*B*a*b - 13*A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*
sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(
b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

mupad [**B**] time = 0.35, size = 152, normalized size = 0.44

$$\frac{\frac{2\,x^{2}\,(13\,A\,b-5\,B\,a)}{5\,a^{2}}-\frac{2\,A}{5\,a}+\frac{9\,b^{2}\,x^{6}\,(13\,A\,b-5\,B\,a)}{16\,a^{4}}+\frac{81\,b\,x^{4}\,(13\,A\,b-5\,B\,a)}{80\,a^{3}}}{a^{2}\,x^{5/2}+b^{2}\,x^{13/2}+2\,a\,b\,x^{9/2}}+\frac{9\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)(13\,A\,b-5\,B\,a)}{32\,a^{17/4}}-\frac{9\,(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)(13\,A\,b-5\,B\,a)}{32\,a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x)
```

```
[Out] ((2*x^2*(13*A*b - 5*B*a))/(5*a^2) - (2*A)/(5*a) + (9*b^2*x^6*(13*A*b - 5*B*
a))/(16*a^4) + (81*b*x^4*(13*A*b - 5*B*a))/(80*a^3))/(a^2*x^(5/2) + b^2*x^(
13/2) + 2*a*b*x^(9/2)) + (9*(-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4))*(
13*A*b - 5*B*a))/(32*a^(17/4)) - (9*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a
^(1/4))*(13*A*b - 5*B*a))/(32*a^(17/4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

3.373 $\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c + 2*a*d)*
x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^{7/2}+a(2bc+ad)x^{11/2}+b(bc+2ad)x^{15/2}+b^2dx^{19/2}\right)dx\\&=\frac{2}{9}a^2cx^{9/2}+\frac{2}{13}a(2bc+ad)x^{13/2}+\frac{2}{17}b(bc+2ad)x^{17/2}+\frac{2}{21}b^2dx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{9/2}\left(1547a^2c+819bx^4(2ad+bc)+1071ax^2(ad+2bc)+663b^2dx^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(9/2)*(1547*a^2*c + 1071*a*(2*b*c + a*d)*x^2 + 819*b*(b*c + 2*a*d)*x^4
 + 663*b^2*d*x^6))/13923
```

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 1.10

$$\frac{2\left(1547a^2cx^{9/2}+1071a^2dx^{13/2}+2142abcx^{13/2}+1638abdx^{17/2}+819b^2cx^{17/2}+663b^2dx^{21/2}\right)}{13923}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*(1547*a^2*c*x^(9/2) + 2142*a*b*c*x^(13/2) + 1071*a^2*d*x^(13/2) + 819*b^
2*c*x^(17/2) + 1638*a*b*d*x^(17/2) + 663*b^2*d*x^(21/2)))/13923
```

fricas [A] time = 1.05, size = 56, normalized size = 0.89

$$\frac{2}{13923}\left(663\,b^2dx^{10}+819\left(b^2c+2\,abd\right)x^8+1547\,a^2cx^4+1071\left(2\,abc+a^2d\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^2*d*x^10 + 819*(b^2*c + 2*a*b*d)*x^8 + 1547*a^2*c*x^4 + 1071
*(2*a*b*c + a^2*d)*x^6)*sqrt(x)
```

giac [A] time = 0.40, size = 53, normalized size = 0.84

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}}+\frac{2}{17}\,b^2cx^{\frac{17}{2}}+\frac{4}{17}\,abdx^{\frac{17}{2}}+\frac{4}{13}\,abcx^{\frac{13}{2}}+\frac{2}{13}\,a^2dx^{\frac{13}{2}}+\frac{2}{9}\,a^2cx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*b^2*c*x^(17/2) + 4/17*a*b*d*x^(17/2) + 4/13*a*b*
c*x^(13/2) + 2/13*a^2*d*x^(13/2) + 2/9*a^2*c*x^(9/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c\right)x^{\frac{9}{2}}}{13923}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/13923*x^(9/2)*(663*b^2*d*x^6+1638*a*b*d*x^4+819*b^2*c*x^4+1071*a^2*d*x^2+
2142*a*b*c*x^2+1547*a^2*c)
```

maxima [A] time = 1.00, size = 51, normalized size = 0.81

$$\frac{2}{21}\,b^2dx^{\frac{21}{2}}+\frac{2}{17}\left(b^2c+2\,abd\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2cx^{\frac{9}{2}}+\frac{2}{13}\left(2\,abc+a^2d\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d*x^(21/2) + 2/17*(b^2*c + 2*a*b*d)*x^(17/2) + 2/9*a^2*c*x^(9/2) +
 2/13*(2*a*b*c + a^2*d)*x^(13/2)
```

mupad [B] time = 0.06, size = 51, normalized size = 0.81

$$x^{13/2}\,\left(\frac{2\,d\,a^2}{13}+\frac{4\,b\,c\,a}{13}\right)+x^{17/2}\,\left(\frac{2\,c\,b^2}{17}+\frac{4\,a\,d\,b}{17}\right)+\frac{2\,a^2\,c\,x^{9/2}}{9}+\frac{2\,b^2\,d\,x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(13/2)*((2*a^2*d)/13 + (4*a*b*c)/13) + x^(17/2)*((2*b^2*c)/17 + (4*a*b*d)
/17) + (2*a^2*c*x^(9/2))/9 + (2*b^2*d*x^(21/2))/21
```

sympy [A] time = 19.90, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{9}{2}}}{9}+\frac{2a^2dx^{\frac{13}{2}}}{13}+\frac{4abcx^{\frac{13}{2}}}{13}+\frac{4abdx^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{17}{2}}}{17}+\frac{2b^2dx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(9/2)/9 + 2*a**2*d*x**(13/2)/13 + 4*a*b*c*x**(13/2)/13 + 4*a*b*
d*x**(17/2)/17 + 2*b**2*c*x**(17/2)/17 + 2*b**2*d*x**(21/2)/21
```

3.374 $\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)*
x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^{5/2}+a(2bc+ad)x^{9/2}+b(bc+2ad)x^{13/2}+b^2dx^{17/2}\right)\,dx\\&=\frac{2}{7}a^2cx^{7/2}+\frac{2}{11}a(2bc+ad)x^{11/2}+\frac{2}{15}b(bc+2ad)x^{15/2}+\frac{2}{19}b^2dx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.00

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)*
x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.10

$$\frac{2\left(3135a^2cx^{7/2}+1995a^2dx^{11/2}+3990abcx^{11/2}+2926abdx^{15/2}+1463b^2cx^{15/2}+1155b^2dx^{19/2}\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*(3135*a^2*c*x^(7/2) + 3990*a*b*c*x^(11/2) + 1995*a^2*d*x^(11/2) + 1463*b
^2*c*x^(15/2) + 2926*a*b*d*x^(15/2) + 1155*b^2*d*x^(19/2)))/21945
```

fricas [A] time = 0.85, size = 56, normalized size = 0.89

$$\frac{2}{21945}\left(1155\,b^2dx^9+1463\left(b^2c+2\,abd\right)x^7+3135\,a^2cx^3+1995\left(2\,abc+a^2d\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d*x^9 + 1463*(b^2*c + 2*a*b*d)*x^7 + 3135*a^2*c*x^3 + 199
5*(2*a*b*c + a^2*d)*x^5)*sqrt(x)
```

giac [A] time = 0.36, size = 53, normalized size = 0.84

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\,b^2cx^{\frac{15}{2}}+\frac{4}{15}\,abdx^{\frac{15}{2}}+\frac{4}{11}\,abcx^{\frac{11}{2}}+\frac{2}{11}\,a^2dx^{\frac{11}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*b^2*c*x^(15/2) + 4/15*a*b*d*x^(15/2) + 4/11*a*b*
c*x^(11/2) + 2/11*a^2*d*x^(11/2) + 2/7*a^2*c*x^(7/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c\right)x^{\frac{7}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/21945*x^(7/2)*(1155*b^2*d*x^6+2926*a*b*d*x^4+1463*b^2*c*x^4+1995*a^2*d*x^
2+3990*a*b*c*x^2+3135*a^2*c)
```

maxima [A] time = 1.03, size = 51, normalized size = 0.81

$$\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\left(b^2c+2\,abd\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}+\frac{2}{11}\left(2\,abc+a^2d\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d*x^(19/2) + 2/15*(b^2*c + 2*a*b*d)*x^(15/2) + 2/7*a^2*c*x^(7/2) +
 2/11*(2*a*b*c + a^2*d)*x^(11/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,d\,a^2}{11}+\frac{4\,b\,c\,a}{11}\right)+x^{15/2}\left(\frac{2\,c\,b^2}{15}+\frac{4\,a\,d\,b}{15}\right)+\frac{2\,a^2\,c\,x^{7/2}}{7}+\frac{2\,b^2\,d\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(11/2)*((2*a^2*d)/11 + (4*a*b*c)/11) + x^(15/2)*((2*b^2*c)/15 + (4*a*b*d)
/15) + (2*a^2*c*x^(7/2))/7 + (2*b^2*d*x^(19/2))/19
```

sympy [A] time = 11.15, size = 80, normalized size = 1.27

$$\frac{2a^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{11}{2}}}{11}+\frac{4abcx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{15}{2}}}{15}+\frac{2b^2cx^{\frac{15}{2}}}{15}+\frac{2b^2dx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(7/2)/7 + 2*a**2*d*x**(11/2)/11 + 4*a*b*c*x**(11/2)/11 + 4*a*b*
d*x**(15/2)/15 + 2*b**2*c*x**(15/2)/15 + 2*b**2*d*x**(19/2)/19
```

3.375 $\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c + 2*a*d)*x^
(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx&=\int\left(a^2cx^{3/2}+a(2bc+ad)x^{7/2}+b(bc+2ad)x^{11/2}+b^2dx^{15/2}\right)dx\\&=\frac{2}{5}a^2cx^{5/2}+\frac{2}{9}a(2bc+ad)x^{9/2}+\frac{2}{13}b(bc+2ad)x^{13/2}+\frac{2}{17}b^2dx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(1989a^2c+765bx^4(2ad+bc)+1105ax^2(ad+2bc)+585b^2dx^6\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(5/2)*(1989*a^2*c + 1105*a*(2*b*c + a*d)*x^2 + 765*b*(b*c + 2*a*d)*x^4
 + 585*b^2*d*x^6))/9945
```

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.10

$$\frac{2\left(1989a^2cx^{5/2}+1105a^2dx^{9/2}+2210abcx^{9/2}+1530abdx^{13/2}+765b^2cx^{13/2}+585b^2dx^{17/2}\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*(1989*a^2*c*x^(5/2) + 2210*a*b*c*x^(9/2) + 1105*a^2*d*x^(9/2) + 765*b^2*
c*x^(13/2) + 1530*a*b*d*x^(13/2) + 585*b^2*d*x^(17/2)))/9945
```

fricas [A] time = 1.16, size = 56, normalized size = 0.89

$$\frac{2}{9945}\left(585\,b^2dx^8+765\left(b^2c+2\,abd\right)x^6+1989\,a^2cx^2+1105\left(2\,abc+a^2d\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d*x^8 + 765*(b^2*c + 2*a*b*d)*x^6 + 1989*a^2*c*x^2 + 1105*(
2*a*b*c + a^2*d)*x^4)*sqrt(x)
```

giac [A] time = 0.32, size = 53, normalized size = 0.84

$$\frac{2}{17}\,b^2dx^{\frac{17}{2}}+\frac{2}{13}\,b^2cx^{\frac{13}{2}}+\frac{4}{13}\,abdx^{\frac{13}{2}}+\frac{4}{9}\,abcx^{\frac{9}{2}}+\frac{2}{9}\,a^2dx^{\frac{9}{2}}+\frac{2}{5}\,a^2cx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*b^2*c*x^(13/2) + 4/13*a*b*d*x^(13/2) + 4/9*a*b*c
*x^(9/2) + 2/9*a^2*d*x^(9/2) + 2/5*a^2*c*x^(5/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(585b^2d\,x^6+1530abd\,x^4+765b^2c\,x^4+1105a^2d\,x^2+2210abc\,x^2+1989a^2c\right)x^{\frac{5}{2}}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 2/9945*x^(5/2)*(585*b^2*d*x^6+1530*a*b*d*x^4+765*b^2*c*x^4+1105*a^2*d*x^2+2
210*a*b*c*x^2+1989*a^2*c)
```

maxima [A] time = 0.96, size = 51, normalized size = 0.81

$$\frac{2}{17}\,b^{2}dx^{\frac{17}{2}}+\frac{2}{13}\left(b^{2}c+2\,abd\right)x^{\frac{13}{2}}+\frac{2}{5}\,a^{2}cx^{\frac{5}{2}}+\frac{2}{9}\left(2\,abc+a^{2}d\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d*x^(17/2) + 2/13*(b^2*c + 2*a*b*d)*x^(13/2) + 2/5*a^2*c*x^(5/2) +
 2/9*(2*a*b*c + a^2*d)*x^(9/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,d\,a^{2}}{9}+\frac{4\,b\,c\,a}{9}\right)+x^{13/2}\left(\frac{2\,c\,b^{2}}{13}+\frac{4\,a\,d\,b}{13}\right)+\frac{2\,a^{2}\,c\,x^{5/2}}{5}+\frac{2\,b^{2}\,d\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(9/2)*((2*a^2*d)/9 + (4*a*b*c)/9) + x^(13/2)*((2*b^2*c)/13 + (4*a*b*d)/13
) + (2*a^2*c*x^(5/2))/5 + (2*b^2*d*x^(17/2))/17
```

sympy [A] time = 5.89, size = 80, normalized size = 1.27

$$\frac{2a^{2}cx^{\frac{5}{2}}}{5}+\frac{2a^{2}dx^{\frac{9}{2}}}{9}+\frac{4abcx^{\frac{9}{2}}}{9}+\frac{4abdx^{\frac{13}{2}}}{13}+\frac{2b^{2}cx^{\frac{13}{2}}}{13}+\frac{2b^{2}dx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] 2*a**2*c*x**(5/2)/5 + 2*a**2*d*x**(9/2)/9 + 4*a*b*c*x**(9/2)/9 + 4*a*b*d*x*
*(13/2)/13 + 2*b**2*c*x**(13/2)/13 + 2*b**2*d*x**(17/2)/17
```

3.376 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c + 2*a*d)*x^
(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx &= \int \left(a^2c\sqrt{x}+a(2bc+ad)x^{5/2}+b(bc+2ad)x^{9/2}+b^2dx^{13/2}\right)dx\\ &= \frac{2}{3}a^2cx^{3/2}+\frac{2}{7}a(2bc+ad)x^{7/2}+\frac{2}{11}b(bc+2ad)x^{11/2}+\frac{2}{15}b^2dx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{2x^{3/2}\left(385a^2c+105bx^4(2ad+bc)+165ax^2(ad+2bc)+77b^2dx^6\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*a^2*c + 165*a*(2*b*c + a*d)*x^2 + 105*b*(b*c + 2*a*d)*x^4 +
 77*b^2*d*x^6))/1155
```

IntegrateAlgebraic [A] time = 0.04, size = 69, normalized size = 1.10

$$\frac{2\left(385a^2cx^{3/2}+165a^2dx^{7/2}+330abcx^{7/2}+210abdx^{11/2}+105b^2cx^{11/2}+77b^2dx^{15/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] (2*(385*a^2*c*x^(3/2) + 330*a*b*c*x^(7/2) + 165*a^2*d*x^(7/2) + 105*b^2*c*x
^(11/2) + 210*a*b*d*x^(11/2) + 77*b^2*d*x^(15/2)))/1155
```

fricas [A] time = 0.76, size = 54, normalized size = 0.86

$$\frac{2}{1155}\left(77\,b^2dx^7+105\left(b^2c+2\,abd\right)x^5+385\,a^2cx+165\left(2\,abc+a^2d\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d*x^7 + 105*(b^2*c + 2*a*b*d)*x^5 + 385*a^2*c*x + 165*(2*a*b
*c + a^2*d)*x^3)*sqrt(x)
```

giac [A] time = 0.33, size = 53, normalized size = 0.84

$$\frac{2}{15}\,b^2dx^{\frac{15}{2}}+\frac{2}{11}\,b^2cx^{\frac{11}{2}}+\frac{4}{11}\,abdx^{\frac{11}{2}}+\frac{4}{7}\,abcx^{\frac{7}{2}}+\frac{2}{7}\,a^2dx^{\frac{7}{2}}+\frac{2}{3}\,a^2cx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*b^2*c*x^(11/2) + 4/11*a*b*d*x^(11/2) + 4/7*a*b*c
*x^(7/2) + 2/7*a^2*d*x^(7/2) + 2/3*a^2*c*x^(3/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.89

$$\frac{2\left(77b^2d\,x^6+210abd\,x^4+105b^2c\,x^4+165a^2d\,x^2+330abc\,x^2+385a^2c\right)x^{\frac{3}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x)
```

```
[Out] 2/1155*x^(3/2)*(77*b^2*d*x^6+210*a*b*d*x^4+105*b^2*c*x^4+165*a^2*d*x^2+330*
a*b*c*x^2+385*a^2*c)
```

maxima [A] time = 1.00, size = 51, normalized size = 0.81

$$\frac{2}{15}\,b^2dx^{\frac{15}{2}}+\frac{2}{11}\left(b^2c+2\,abd\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^2cx^{\frac{3}{2}}+\frac{2}{7}\left(2\,abc+a^2d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d*x^(15/2) + 2/11*(b^2*c + 2*a*b*d)*x^(11/2) + 2/3*a^2*c*x^(3/2) +
 2/7*(2*a*b*c + a^2*d)*x^(7/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{7/2}\left(\frac{2\,d\,a^2}{7}+\frac{4\,b\,c\,a}{7}\right)+x^{11/2}\left(\frac{2\,c\,b^2}{11}+\frac{4\,a\,d\,b}{11}\right)+\frac{2\,a^2\,c\,x^{3/2}}{3}+\frac{2\,b^2\,d\,x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

```
[Out] x^(7/2)*((2*a^2*d)/7 + (4*a*b*c)/7) + x^(11/2)*((2*b^2*c)/11 + (4*a*b*d)/11
) + (2*a^2*c*x^(3/2))/3 + (2*b^2*d*x^(15/2))/15
```

sympy [A] time = 2.57, size = 66, normalized size = 1.05

$$\frac{2a^2cx^{\frac{3}{2}}}{3}+\frac{2b^2dx^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(2abd+b^2c\right)}{11}+\frac{2x^{\frac{7}{2}}\left(a^2d+2abc\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)*x**(1/2),x)
```

```
[Out] 2*a**2*c*x**(3/2)/3 + 2*b**2*d*x**(15/2)/15 + 2*x**(11/2)*(2*a*b*d + b**2*c
)/11 + 2*x**(7/2)*(a**2*d + 2*a*b*c)/7
```

3.377 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] 2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a*d)*x^(9/2
))/9 + (2*b^2*d*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c}{\sqrt{x}}+a(2bc+ad)x^{3/2}+b(bc+2ad)x^{7/2}+b^2dx^{11/2}\right)dx\\ &= 2a^2c\sqrt{x}+\frac{2}{5}a(2bc+ad)x^{5/2}+\frac{2}{9}b(bc+2ad)x^{9/2}+\frac{2}{13}b^2dx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.87

$$\frac{2}{585}\sqrt{x}\left(585a^2c+65bx^4(2ad+bc)+117ax^2(ad+2bc)+45b^2dx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(585*a^2*c + 117*a*(2*b*c + a*d)*x^2 + 65*b*(b*c + 2*a*d)*x^4 +
45*b^2*d*x^6))/585
```

IntegrateAlgebraic [A] time = 0.03, size = 69, normalized size = 1.13

$$\frac{2}{585}\left(585a^2c\sqrt{x}+117a^2dx^{5/2}+234abcx^{5/2}+130abdx^{9/2}+65b^2cx^{9/2}+45b^2dx^{13/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

```
[Out] (2*(585*a^2*c*Sqrt[x] + 234*a*b*c*x^(5/2) + 117*a^2*d*x^(5/2) + 65*b^2*c*x^
(9/2) + 130*a*b*d*x^(9/2) + 45*b^2*d*x^(13/2)))/585
```

fricas [A] time = 1.27, size = 53, normalized size = 0.87

$$\frac{2}{585}\left(45\,b^2dx^6+65\left(b^2c+2\,abd\right)x^4+585\,a^2c+117\left(2\,abc+a^2d\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 65*(b^2*c + 2*a*b*d)*x^4 + 585*a^2*c + 117*(2*a*b*c +
 a^2*d)*x^2)*sqrt(x)
```

giac [A] time = 0.28, size = 53, normalized size = 0.87

$$\frac{2}{13}\,b^2dx^{\frac{13}{2}}+\frac{2}{9}\,b^2cx^{\frac{9}{2}}+\frac{4}{9}\,abdx^{\frac{9}{2}}+\frac{4}{5}\,abcx^{\frac{5}{2}}+\frac{2}{5}\,a^2dx^{\frac{5}{2}}+2\,a^2c\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*b^2*c*x^(9/2) + 4/9*a*b*d*x^(9/2) + 4/5*a*b*c*x^(
5/2) + 2/5*a^2*d*x^(5/2) + 2*a^2*c*sqrt(x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$\frac{2\left(45b^2d\,x^6+130abd\,x^4+65b^2c\,x^4+117a^2d\,x^2+234abc\,x^2+585a^2c\right)\sqrt{x}}{585}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x)
```

```
[Out] 2/585*x^(1/2)*(45*b^2*d*x^6+130*a*b*d*x^4+65*b^2*c*x^4+117*a^2*d*x^2+234*a*
b*c*x^2+585*a^2*c)
```

maxima [A] time = 1.00, size = 51, normalized size = 0.84

$$\frac{2}{13}\,b^2dx^{\frac{13}{2}}+\frac{2}{9}\left(b^2c+2\,abd\right)x^{\frac{9}{2}}+2\,a^2c\sqrt{x}+\frac{2}{5}\left(2\,abc+a^2d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d*x^(13/2) + 2/9*(b^2*c + 2*a*b*d)*x^(9/2) + 2*a^2*c*sqrt(x) + 2/5
*(2*a*b*c + a^2*d)*x^(5/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{5/2}\left(\frac{2\,d\,a^2}{5}+\frac{4\,b\,c\,a}{5}\right)+x^{9/2}\left(\frac{2\,c\,b^2}{9}+\frac{4\,a\,d\,b}{9}\right)+2\,a^2\,c\,\sqrt{x}+\frac{2\,b^2\,d\,x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(1/2),x)
```

```
[Out] x^(5/2)*((2*a^2*d)/5 + (4*a*b*c)/5) + x^(9/2)*((2*b^2*c)/9 + (4*a*b*d)/9) +
 2*a^2*c*x^(1/2) + (2*b^2*d*x^(13/2))/13
```

sympy [A] time = 2.12, size = 78, normalized size = 1.28

$$2a^2c\sqrt{x}+\frac{2a^2dx^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{5}{2}}}{5}+\frac{4abdx^{\frac{9}{2}}}{9}+\frac{2b^2cx^{\frac{9}{2}}}{9}+\frac{2b^2dx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(1/2),x)
```

```
[Out] 2*a**2*c*sqrt(x) + 2*a**2*d*x**(5/2)/5 + 4*a*b*c*x**(5/2)/5 + 4*a*b*d*x**(9
/2)/9 + 2*b**2*c*x**(9/2)/9 + 2*b**2*d*x**(13/2)/13
```

3.378 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c + 2*a*d)*x^(
7/2))/7 + (2*b^2*d*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^{3/2}}\,dx&=\int \left(\frac{a^2c}{x^{3/2}}+a(2bc+ad)\sqrt{x}+b(bc+2ad)x^{5/2}+b^2dx^{9/2}\right)dx\\&=-\frac{2a^2c}{\sqrt{x}}+\frac{2}{3}a(2bc+ad)x^{3/2}+\frac{2}{7}b(bc+2ad)x^{7/2}+\frac{2}{11}b^2dx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 0.98

$$\frac{-154a^2\left(3c-dx^2\right)+44abx^2\left(7c+3dx^2\right)+6b^2x^4\left(11c+7dx^2\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (-154*a^2*(3*c - d*x^2) + 44*a*b*x^2*(7*c + 3*d*x^2) + 6*b^2*x^4*(11*c + 7*
d*x^2))/(231*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-231a^2c+77a^2dx^2+154abcx^2+66abdx^4+33b^2cx^4+21b^2dx^6\right)}{231\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

```
[Out] (2*(-231*a^2*c + 154*a*b*c*x^2 + 77*a^2*d*x^2 + 33*b^2*c*x^4 + 66*a*b*d*x^4
 + 21*b^2*d*x^6))/(231*Sqrt[x])
```

fricas [A] time = 1.08, size = 53, normalized size = 0.87

$$\frac{2\left(21\,b^2dx^6+33\left(b^2c+2\,abd\right)x^4-231\,a^2c+77\left(2\,abc+a^2d\right)x^2\right)}{231\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*d*x^6 + 33*(b^2*c + 2*a*b*d)*x^4 - 231*a^2*c + 77*(2*a*b*c +
a^2*d)*x^2)/sqrt(x)
```

giac [A] time = 0.27, size = 53, normalized size = 0.87

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}}+\frac{2}{7}\,b^2cx^{\frac{7}{2}}+\frac{4}{7}\,abdx^{\frac{7}{2}}+\frac{4}{3}\,abcx^{\frac{3}{2}}+\frac{2}{3}\,a^2dx^{\frac{3}{2}}-\frac{2\,a^2c}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*b^2*c*x^(7/2) + 4/7*a*b*d*x^(7/2) + 4/3*a*b*c*x^(
3/2) + 2/3*a^2*d*x^(3/2) - 2*a^2*c/sqrt(x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$-\frac{2\left(-21b^2d\,x^6-66abd\,x^4-33b^2c\,x^4-77a^2d\,x^2-154abc\,x^2+231a^2c\right)}{231\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x)
```

```
[Out] -2/231*(-21*b^2*d*x^6-66*a*b*d*x^4-33*b^2*c*x^4-77*a^2*d*x^2-154*a*b*c*x^2+
231*a^2*c)/x^(1/2)
```

maxima [A] time = 1.04, size = 51, normalized size = 0.84

$$\frac{2}{11}\,b^2dx^{\frac{11}{2}}+\frac{2}{7}\left(b^2c+2\,abd\right)x^{\frac{7}{2}}-\frac{2\,a^2c}{\sqrt{x}}+\frac{2}{3}\left(2\,abc+a^2d\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d*x^(11/2) + 2/7*(b^2*c + 2*a*b*d)*x^(7/2) - 2*a^2*c/sqrt(x) + 2/3
*(2*a*b*c + a^2*d)*x^(3/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{3/2}\left(\frac{2\,d\,a^2}{3}+\frac{4\,b\,c\,a}{3}\right)+x^{7/2}\left(\frac{2\,c\,b^2}{7}+\frac{4\,a\,d\,b}{7}\right)-\frac{2\,a^2\,c}{\sqrt{x}}+\frac{2\,b^2\,d\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x)
```

```
[Out] x^(3/2)*((2*a^2*d)/3 + (4*a*b*c)/3) + x^(7/2)*((2*b^2*c)/7 + (4*a*b*d)/7) -
 (2*a^2*c)/x^(1/2) + (2*b^2*d*x^(11/2))/11
```

sympy [A] time = 2.31, size = 78, normalized size = 1.28

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2a^2dx^{\frac{3}{2}}}{3}+\frac{4abcx^{\frac{3}{2}}}{3}+\frac{4abdx^{\frac{7}{2}}}{7}+\frac{2b^2cx^{\frac{7}{2}}}{7}+\frac{2b^2dx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(3/2),x)
```

```
[Out] -2*a**2*c/sqrt(x) + 2*a**2*d*x**(3/2)/3 + 4*a*b*c*x**(3/2)/3 + 4*a*b*d*x**(
7/2)/7 + 2*b**2*c*x**(7/2)/7 + 2*b**2*d*x**(11/2)/11
```

3.379 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}\,(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}\,(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c + 2*a*d)*x^(
5/2))/5 + (2*b^2*d*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^{5/2}}\,dx &= \int \left(\frac{a^2c}{x^{5/2}}+\frac{a(2bc+ad)}{\sqrt{x}}+b(bc+2ad)x^{3/2}+b^2dx^{7/2}\right)dx\\ &= -\frac{2a^2c}{3x^{3/2}}+2a(2bc+ad)\sqrt{x}+\frac{2}{5}b(bc+2ad)x^{5/2}+\frac{2}{9}b^2dx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{-30a^2\left(c-3dx^2\right)+36abx^2\left(5c+dx^2\right)+2b^2x^4\left(9c+5dx^2\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (-30*a^2*(c - 3*d*x^2) + 36*a*b*x^2*(5*c + d*x^2) + 2*b^2*x^4*(9*c + 5*d*x^
2))/(45*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-15a^2c+45a^2dx^2+90abcx^2+18abdx^4+9b^2cx^4+5b^2dx^6\right)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

```
[Out] (2*(-15*a^2*c + 90*a*b*c*x^2 + 45*a^2*d*x^2 + 9*b^2*c*x^4 + 18*a*b*d*x^4 +
5*b^2*d*x^6))/(45*x^(3/2))
```

fricas [A] time = 0.82, size = 53, normalized size = 0.87

$$\frac{2\left(5\,b^2dx^6+9\left(b^2c+2\,abd\right)x^4-15\,a^2c+45\left(2\,abc+a^2d\right)x^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*d*x^6 + 9*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 45*(2*a*b*c + a^2*
d)*x^2)/x^(3/2)
```

giac [A] time = 0.33, size = 53, normalized size = 0.87

$$\frac{2}{9}\,b^2dx^{\frac{9}{2}}+\frac{2}{5}\,b^2cx^{\frac{5}{2}}+\frac{4}{5}\,abdx^{\frac{5}{2}}+4\,abc\sqrt{x}+2\,a^2d\sqrt{x}-\frac{2\,a^2c}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*b^2*c*x^(5/2) + 4/5*a*b*d*x^(5/2) + 4*a*b*c*sqrt(x)
 + 2*a^2*d*sqrt(x) - 2/3*a^2*c/x^(3/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$-\frac{2\left(-5b^2d\,x^6-18abd\,x^4-9b^2c\,x^4-45a^2d\,x^2-90abc\,x^2+15a^2c\right)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x)
```

```
[Out] -2/45*(-5*b^2*d*x^6-18*a*b*d*x^4-9*b^2*c*x^4-45*a^2*d*x^2-90*a*b*c*x^2+15*a
^2*c)/x^(3/2)
```

maxima **[A]** time = 1.10, size = 51, normalized size = 0.84

$$\frac{2}{9}\,b^{2}dx^{\frac{9}{2}}+\frac{2}{5}\left(b^{2}c+2\,abd\right)x^{\frac{5}{2}}-\frac{2\,a^{2}c}{3\,x^{\frac{3}{2}}}+2\left(2\,abc+a^{2}d\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^2*d*x^(9/2) + 2/5*(b^2*c + 2*a*b*d)*x^(5/2) - 2/3*a^2*c/x^(3/2) + 2*(
2*a*b*c + a^2*d)*sqrt(x)
```

mupad **[B]** time = 0.05, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2\,d\,a^{2}+4\,b\,c\,a\right)+x^{5/2}\,\left(\frac{2\,c\,b^{2}}{5}+\frac{4\,a\,d\,b}{5}\right)-\frac{2\,a^{2}\,c}{3\,x^{3/2}}+\frac{2\,b^{2}\,d\,x^{9/2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x)
```

```
[Out] x^(1/2)*(2*a^2*d + 4*a*b*c) + x^(5/2)*((2*b^2*c)/5 + (4*a*b*d)/5) - (2*a^2*
c)/(3*x^(3/2)) + (2*b^2*d*x^(9/2))/9
```

sympy **[A]** time = 2.76, size = 76, normalized size = 1.25

$$-\frac{2a^{2}c}{3x^{\frac{3}{2}}}+2a^{2}d\sqrt{x}+4abc\sqrt{x}+\frac{4abdx^{\frac{5}{2}}}{5}+\frac{2b^{2}cx^{\frac{5}{2}}}{5}+\frac{2b^{2}dx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(5/2),x)
```

```
[Out] -2*a**2*c/(3*x**(3/2)) + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt(x) + 4*a*b*d*x**(5
/2)/5 + 2*b**2*c*x**(5/2)/5 + 2*b**2*d*x**(9/2)/9
```

3.380 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c + 2*a*d)*x
^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx &= \int \left(\frac{a^2c}{x^{7/2}}+\frac{a(2bc+ad)}{x^{3/2}}+b(bc+2ad)\sqrt{x}+b^2dx^{5/2}\right)dx\\ &= -\frac{2a^2c}{5x^{5/2}}-\frac{2a(2bc+ad)}{\sqrt{x}}+\frac{2}{3}b(bc+2ad)x^{3/2}+\frac{2}{7}b^2dx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{-42a^2\left(c+5dx^2\right)+140abx^2\left(dx^2-3c\right)+10b^2x^4\left(7c+3dx^2\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (140*a*b*x^2*(-3*c + d*x^2) + 10*b^2*x^4*(7*c + 3*d*x^2) - 42*a^2*(c + 5*d*
x^2))/(105*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 59, normalized size = 0.97

$$\frac{2\left(-21a^2c-105a^2dx^2-210abcx^2+70abdx^4+35b^2cx^4+15b^2dx^6\right)}{105x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

```
[Out] (2*(-21*a^2*c - 210*a*b*c*x^2 - 105*a^2*d*x^2 + 35*b^2*c*x^4 + 70*a*b*d*x^4
 + 15*b^2*d*x^6))/(105*x^(5/2))
```

fricas [A] time = 0.78, size = 53, normalized size = 0.87

$$\frac{2\left(15\,b^2dx^6+35\left(b^2c+2\,abd\right)x^4-21\,a^2c-105\left(2\,abc+a^2d\right)x^2\right)}{105\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d*x^6 + 35*(b^2*c + 2*a*b*d)*x^4 - 21*a^2*c - 105*(2*a*b*c +
a^2*d)*x^2)/x^(5/2)
```

giac [A] time = 0.35, size = 55, normalized size = 0.90

$$\frac{2}{7}\,b^2dx^{\frac{7}{2}}+\frac{2}{3}\,b^2cx^{\frac{3}{2}}+\frac{4}{3}\,abdx^{\frac{3}{2}}-\frac{2\left(10\,abcx^2+5\,a^2dx^2+a^2c\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*b^2*c*x^(3/2) + 4/3*a*b*d*x^(3/2) - 2/5*(10*a*b*c*x
^2 + 5*a^2*d*x^2 + a^2*c)/x^(5/2)
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$-\frac{2\left(-15b^2d\,x^6-70abd\,x^4-35b^2c\,x^4+105a^2d\,x^2+210abc\,x^2+21a^2c\right)}{105x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x)
```

```
[Out] -2/105*(-15*b^2*d*x^6-70*a*b*d*x^4-35*b^2*c*x^4+105*a^2*d*x^2+210*a*b*c*x^2
+21*a^2*c)/x^(5/2)
```

maxima [A] time = 1.08, size = 53, normalized size = 0.87

$$\frac{2}{7}\,b^2dx^{\frac{7}{2}}+\frac{2}{3}\left(b^2c+2\,abd\right)x^{\frac{3}{2}}-\frac{2\left(a^2c+5\left(2\,abc+a^2d\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/7*b^2*d*x^(7/2) + 2/3*(b^2*c + 2*a*b*d)*x^(3/2) - 2/5*(a^2*c + 5*(2*a*b*c
 + a^2*d)*x^2)/x^(5/2)
```

mupad [B] time = 0.05, size = 55, normalized size = 0.90

$$-\frac{210\,d\,a^2\,x^2+42\,c\,a^2-140\,d\,a\,b\,x^4+420\,c\,a\,b\,x^2-30\,d\,b^2\,x^6-70\,c\,b^2\,x^4}{105\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x)
```

```
[Out] -(42*a^2*c + 210*a^2*d*x^2 - 70*b^2*c*x^4 - 30*b^2*d*x^6 + 420*a*b*c*x^2 -
140*a*b*d*x^4)/(105*x^(5/2))
```

sympy [A] time = 3.83, size = 76, normalized size = 1.25

$$-\frac{2a^2c}{5x^{\frac{5}{2}}}-\frac{2a^2d}{\sqrt{x}}-\frac{4abc}{\sqrt{x}}+\frac{4abdx^{\frac{3}{2}}}{3}+\frac{2b^2cx^{\frac{3}{2}}}{3}+\frac{2b^2dx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)/x**(7/2),x)
```

```
[Out] -2*a**2*c/(5*x**(5/2)) - 2*a**2*d/sqrt(x) - 4*a*b*c/sqrt(x) + 4*a*b*d*x**(3
/2)/3 + 2*b**2*c*x**(3/2)/3 + 2*b**2*d*x**(7/2)/7
```

3.381 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {448}

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2*d
^2*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \Big(a^2c^2x^{7/2}+2ac(bc+ad)x^{11/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+2bd(bc+ad)x\\ &= \frac{2}{9}a^2c^2x^{9/2}+\frac{4}{13}ac(bc+ad)x^{13/2}+\frac{2}{17}\left(b^2c^2+4abcd+a^2d^2\right)x^{17/2}+\frac{4}{21}bd(bc+\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.00

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2*d
^2*x^(25/2))/25
```

IntegrateAlgebraic [A] time = 0.05, size = 116, normalized size = 1.20

$$\frac{2\left(38675a^2c^2x^{9/2}+53550a^2cdx^{13/2}+20475a^2d^2x^{17/2}+53550abc^2x^{13/2}+81900abcdx^{17/2}+33150abd^2x^{21/2}+20475b^2c^2x^{17/2}+33150b^2cdx^{21/2}+13923b^2d^2x^{25/2}\right)}{348075}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*(38675*a^2*c^2*x^(9/2) + 53550*a*b*c^2*x^(13/2) + 53550*a^2*c*d*x^(13/2)
 + 20475*b^2*c^2*x^(17/2) + 81900*a*b*c*d*x^(17/2) + 20475*a^2*d^2*x^(17/2)
 + 33150*b^2*c*d*x^(21/2) + 33150*a*b*d^2*x^(21/2) + 13923*b^2*d^2*x^(25/2)
))/348075
```

fricas [A] time = 1.04, size = 90, normalized size = 0.93

$$\frac{2}{348075}\left(13923\,b^2d^2x^{12}+33150\left(b^2cd+abd^2\right)x^{10}+20475\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+38675\,a^2c^2x^4+53550\left(abc^2+a^2cd\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^2*x^12 + 33150*(b^2*c*d + a*b*d^2)*x^10 + 20475*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 38675*a^2*c^2*x^4 + 53550*(a*b*c^2 + a^2*c
*d)*x^6)*sqrt(x)
```

giac [A] time = 0.34, size = 94, normalized size = 0.97

$$\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\,b^2cdx^{\frac{21}{2}}+\frac{4}{21}\,abd^2x^{\frac{21}{2}}+\frac{2}{17}\,b^2c^2x^{\frac{17}{2}}+\frac{8}{17}\,abcdx^{\frac{17}{2}}+\frac{2}{17}\,a^2d^2x^{\frac{17}{2}}+\frac{4}{13}\,abc^2x^{\frac{13}{2}}+\frac{4}{13}\,a^2cdx^{\frac{13}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*b^2*c*d*x^(21/2) + 4/21*a*b*d^2*x^(21/2) + 2/1
7*b^2*c^2*x^(17/2) + 8/17*a*b*c*d*x^(17/2) + 2/17*a^2*d^2*x^(17/2) + 4/13*a
*b*c^2*x^(13/2) + 4/13*a^2*c*d*x^(13/2) + 2/9*a^2*c^2*x^(9/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.00

$$\frac{2\left(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900abcd\,x^4+20475b^2c^2x^4+53550a^2cd\,x^2+53550ab\,c^2x^2+38675a^2c^2\right)x^{\frac{9}{2}}}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/348075*x^(9/2)*(13923*b^2*d^2*x^8+33150*a*b*d^2*x^6+33150*b^2*c*d*x^6+204
75*a^2*d^2*x^4+81900*a*b*c*d*x^4+20475*b^2*c^2*x^4+53550*a^2*c*d*x^2+53550*
a*b*c^2*x^2+38675*a^2*c^2)
```

maxima [A] time = 1.02, size = 85, normalized size = 0.88

$$\frac{2}{25}\,b^{2}d^{2}x^{\frac{25}{2}}+\frac{4}{21}\left(b^{2}cd+abd^{2}\right)x^{\frac{21}{2}}+\frac{2}{17}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^{2}c^{2}x^{\frac{9}{2}}+\frac{4}{13}\left(abc^{2}+a^{2}cd\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^2*x^(25/2) + 4/21*(b^2*c*d + a*b*d^2)*x^(21/2) + 2/17*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(17/2) + 2/9*a^2*c^2*x^(9/2) + 4/13*(a*b*c^2 + a^2*
c*d)*x^(13/2)
```

mupad [B] time = 0.05, size = 78, normalized size = 0.80

$$x^{17/2}\left(\frac{2\,a^2\,d^2}{17}+\frac{8\,a\,b\,c\,d}{17}+\frac{2\,b^2\,c^2}{17}\right)+\frac{2\,a^2\,c^2\,x^{9/2}}{9}+\frac{2\,b^2\,d^2\,x^{25/2}}{25}+\frac{4\,a\,c\,x^{13/2}\,\left(a\,d+b\,c\right)}{13}+\frac{4\,b\,d\,x^{21/2}\,\left(a\,d+b\,c\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(17/2)*((2*a^2*d^2)/17 + (2*b^2*c^2)/17 + (8*a*b*c*d)/17) + (2*a^2*c^2*x^
(9/2))/9 + (2*b^2*d^2*x^(25/2))/25 + (4*a*c*x^(13/2)*(a*d + b*c))/13 + (4*b
*d*x^(21/2)*(a*d + b*c))/21
```

sympy [A] time = 31.91, size = 136, normalized size = 1.40

$$\frac{2a^{2}c^{2}x^{\frac{9}{2}}}{9}+\frac{4a^{2}cdx^{\frac{13}{2}}}{13}+\frac{2a^{2}d^{2}x^{\frac{17}{2}}}{17}+\frac{4abc^{2}x^{\frac{13}{2}}}{13}+\frac{8abcdx^{\frac{17}{2}}}{17}+\frac{4abd^{2}x^{\frac{21}{2}}}{21}+\frac{2b^{2}c^{2}x^{\frac{17}{2}}}{17}+\frac{4b^{2}cdx^{\frac{21}{2}}}{21}+\frac{2b^{2}d^{2}x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(9/2)/9 + 4*a**2*c*d*x**(13/2)/13 + 2*a**2*d**2*x**(17/2)/17
 + 4*a*b*c**2*x**(13/2)/13 + 8*a*b*c*d*x**(17/2)/17 + 4*a*b*d**2*x**(21/2)/
21 + 2*b**2*c**2*x**(17/2)/17 + 4*b**2*c*d*x**(21/2)/21 + 2*b**2*d**2*x**(2
5/2)/25
```

3.382 $\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=97

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2*d
^2*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int\left(a^2c^2x^{5/2}+2ac(bc+ad)x^{9/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{13/2}+2bd(bc+ad)x\right.\\ &= \frac{2}{7}a^2c^2x^{7/2}+\frac{4}{11}ac(bc+ad)x^{11/2}+\frac{2}{15}\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+\frac{4}{19}bd(bc+\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.00

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2*d
^2*x^(23/2))/23
```

IntegrateAlgebraic [A] time = 0.05, size = 116, normalized size = 1.20

$$\frac{2\left(72105a^2c^2x^{7/2}+91770a^2cdx^{11/2}+33649a^2d^2x^{15/2}+91770abc^2x^{11/2}+134596abcdx^{15/2}+53130abd^2x^{19/2}+33649b^2c^2x^{15/2}+53130b^2cdx^{19/2}+21945b^2d^2x^{23/2}\right)}{504735}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*(72105*a^2*c^2*x^(7/2) + 91770*a*b*c^2*x^(11/2) + 91770*a^2*c*d*x^(11/2)
 + 33649*b^2*c^2*x^(15/2) + 134596*a*b*c*d*x^(15/2) + 33649*a^2*d^2*x^(15/2
) + 53130*b^2*c*d*x^(19/2) + 53130*a*b*d^2*x^(19/2) + 21945*b^2*d^2*x^(23/2
)))/504735
```

fricas [A] time = 0.89, size = 90, normalized size = 0.93

$$\frac{2}{504735}\left(21945\,b^2d^2x^{11}+53130\left(b^2cd+abd^2\right)x^9+33649\left(b^2c^2+4\,abcd+a^2d^2\right)x^7+72105\,a^2c^2x^3+91770\left(abc^2+a^2cd\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^2*x^11 + 53130*(b^2*c*d + a*b*d^2)*x^9 + 33649*(b^2*c
^2 + 4*a*b*c*d + a^2*d^2)*x^7 + 72105*a^2*c^2*x^3 + 91770*(a*b*c^2 + a^2*c*
d)*x^5)*sqrt(x)
```

giac [A] time = 0.36, size = 94, normalized size = 0.97

$$\frac{2}{23}\,b^2d^2x^{\frac{23}{2}}+\frac{4}{19}\,b^2cdx^{\frac{19}{2}}+\frac{4}{19}\,abd^2x^{\frac{19}{2}}+\frac{2}{15}\,b^2c^2x^{\frac{15}{2}}+\frac{8}{15}\,abcdx^{\frac{15}{2}}+\frac{2}{15}\,a^2d^2x^{\frac{15}{2}}+\frac{4}{11}\,abc^2x^{\frac{11}{2}}+\frac{4}{11}\,a^2cdx^{\frac{11}{2}}+\frac{2}{7}\,a^2c^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*b^2*c*d*x^(19/2) + 4/19*a*b*d^2*x^(19/2) + 2/1
5*b^2*c^2*x^(15/2) + 8/15*a*b*c*d*x^(15/2) + 2/15*a^2*d^2*x^(15/2) + 4/11*a
*b*c^2*x^(11/2) + 4/11*a^2*c*d*x^(11/2) + 2/7*a^2*c^2*x^(7/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.00

$$\frac{2\left(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596abcd\,x^4+33649b^2c^2x^4+91770a^2cd\,x^2+91770ab\,c^2x^2+72105a^2c^2\right)x^{\frac{7}{2}}}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*b^2*d^2*x^8+53130*a*b*d^2*x^6+53130*b^2*c*d*x^6+336
49*a^2*d^2*x^4+134596*a*b*c*d*x^4+33649*b^2*c^2*x^4+91770*a^2*c*d*x^2+91770
*a*b*c^2*x^2+72105*a^2*c^2)
```

maxima [A] time = 1.17, size = 85, normalized size = 0.88

$$\frac{2}{23}\,b^2d^2x^{\frac{23}{2}}+\frac{4}{19}\left(b^2cd+abd^2\right)x^{\frac{19}{2}}+\frac{2}{15}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^2c^2x^{\frac{7}{2}}+\frac{4}{11}\left(abc^2+a^2cd\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^2*x^(23/2) + 4/19*(b^2*c*d + a*b*d^2)*x^(19/2) + 2/15*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(15/2) + 2/7*a^2*c^2*x^(7/2) + 4/11*(a*b*c^2 + a^2*
c*d)*x^(11/2)
```

mupad [B] time = 0.03, size = 78, normalized size = 0.80

$$x^{15/2}\left(\frac{2\,a^2\,d^2}{15}+\frac{8\,a\,b\,c\,d}{15}+\frac{2\,b^2\,c^2}{15}\right)+\frac{2\,a^2\,c^2\,x^{7/2}}{7}+\frac{2\,b^2\,d^2\,x^{23/2}}{23}+\frac{4\,a\,c\,x^{11/2}\,\left(a\,d+b\,c\right)}{11}+\frac{4\,b\,d\,x^{19/2}\,\left(a\,d+b\,c\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(15/2)*((2*a^2*d^2)/15 + (2*b^2*c^2)/15 + (8*a*b*c*d)/15) + (2*a^2*c^2*x^
(7/2))/7 + (2*b^2*d^2*x^(23/2))/23 + (4*a*c*x^(11/2)*(a*d + b*c))/11 + (4*b
*d*x^(19/2)*(a*d + b*c))/19
```

sympy [A] time = 20.28, size = 136, normalized size = 1.40

$$\frac{2a^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cdx^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{15}{2}}}{15}+\frac{4abc^2x^{\frac{11}{2}}}{11}+\frac{8abcdx^{\frac{15}{2}}}{15}+\frac{4abd^2x^{\frac{19}{2}}}{19}+\frac{2b^2c^2x^{\frac{15}{2}}}{15}+\frac{4b^2cdx^{\frac{19}{2}}}{19}+\frac{2b^2d^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(7/2)/7 + 4*a**2*c*d*x**(11/2)/11 + 2*a**2*d**2*x**(15/2)/15
 + 4*a*b*c**2*x**(11/2)/11 + 8*a*b*c*d*x**(15/2)/15 + 4*a*b*d**2*x**(19/2)/
19 + 2*b**2*c**2*x**(15/2)/15 + 4*b**2*c*d*x**(19/2)/19 + 2*b**2*d**2*x**(2
3/2)/23
```

3.383 $\int x^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d^2
*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2x^{3/2}+2ac(bc+ad)x^{7/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2}+2bd(bc+ad)x^1\right.\\ &= \frac{2}{5}a^2c^2x^{5/2}+\frac{4}{9}ac(bc+ad)x^{9/2}+\frac{2}{13}\left(b^2c^2+4abcd+a^2d^2\right)x^{13/2}+\frac{4}{17}bd(bc+ad\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.00

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d^2
*x^(21/2))/21
```

IntegrateAlgebraic [A] time = 0.06, size = 116, normalized size = 1.20

$$\frac{2\left(13923a^2c^2x^{5/2}+15470a^2cdx^{9/2}+5355a^2d^2x^{13/2}+15470abc^2x^{9/2}+21420abcdx^{13/2}+8190abd^2x^{17/2}+5355b^2c^2x^{13/2}+8190b^2cdx^{17/2}+3315b^2d^2x^{21/2}\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*(13923*a^2*c^2*x^(5/2) + 15470*a*b*c^2*x^(9/2) + 15470*a^2*c*d*x^(9/2) +
 5355*b^2*c^2*x^(13/2) + 21420*a*b*c*d*x^(13/2) + 5355*a^2*d^2*x^(13/2) + 8
190*b^2*c*d*x^(17/2) + 8190*a*b*d^2*x^(17/2) + 3315*b^2*d^2*x^(21/2)))/6961
5
```

fricas [A] time = 1.27, size = 90, normalized size = 0.93

$$\frac{2}{69615}\left(3315\,b^2d^2x^{10}+8190\left(b^2cd+abd^2\right)x^8+5355\left(b^2c^2+4\,abcd+a^2d^2\right)x^6+13923\,a^2c^2x^2+15470\left(abc^2+a^2cd\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^2*x^10 + 8190*(b^2*c*d + a*b*d^2)*x^8 + 5355*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^6 + 13923*a^2*c^2*x^2 + 15470*(a*b*c^2 + a^2*c*d)*x
^4)*sqrt(x)
```

giac [A] time = 0.44, size = 94, normalized size = 0.97

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}}+\frac{4}{17}\,b^2cdx^{\frac{17}{2}}+\frac{4}{17}\,abd^2x^{\frac{17}{2}}+\frac{2}{13}\,b^2c^2x^{\frac{13}{2}}+\frac{8}{13}\,abcdx^{\frac{13}{2}}+\frac{2}{13}\,a^2d^2x^{\frac{13}{2}}+\frac{4}{9}\,abc^2x^{\frac{9}{2}}+\frac{4}{9}\,a^2cdx^{\frac{9}{2}}+\frac{2}{5}\,a^2c^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*b^2*c*d*x^(17/2) + 4/17*a*b*d^2*x^(17/2) + 2/1
3*b^2*c^2*x^(13/2) + 8/13*a*b*c*d*x^(13/2) + 2/13*a^2*d^2*x^(13/2) + 4/9*a*
b*c^2*x^(9/2) + 4/9*a^2*c*d*x^(9/2) + 2/5*a^2*c^2*x^(5/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.00

$$\frac{2\left(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420abcd\,x^4+5355b^2c^2x^4+15470a^2cd\,x^2+15470ab\,c^2x^2+13923a^2c^2\right)x^{\frac{5}{2}}}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*b^2*d^2*x^8+8190*a*b*d^2*x^6+8190*b^2*c*d*x^6+5355*a^
2*d^2*x^4+21420*a*b*c*d*x^4+5355*b^2*c^2*x^4+15470*a^2*c*d*x^2+15470*a*b*c^
2*x^2+13923*a^2*c^2)
```

maxima [A] time = 1.09, size = 85, normalized size = 0.88

$$\frac{2}{21}\,b^2d^2x^{\frac{21}{2}} + \frac{4}{17}\left(b^2cd + abd^2\right)x^{\frac{17}{2}} + \frac{2}{13}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{13}{2}} + \frac{2}{5}\,a^2c^2x^{\frac{5}{2}} + \frac{4}{9}\left(abc^2 + a^2cd\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^2*x^(21/2) + 4/17*(b^2*c*d + a*b*d^2)*x^(17/2) + 2/13*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(13/2) + 2/5*a^2*c^2*x^(5/2) + 4/9*(a*b*c^2 + a^2*c
*d)*x^(9/2)
```

mupad [B] time = 0.03, size = 78, normalized size = 0.80

$$x^{13/2}\left(\frac{2\,a^2\,d^2}{13}+\frac{8\,a\,b\,c\,d}{13}+\frac{2\,b^2\,c^2}{13}\right)+\frac{2\,a^2\,c^2\,x^{5/2}}{5}+\frac{2\,b^2\,d^2\,x^{21/2}}{21}+\frac{4\,a\,c\,x^{9/2}\,\left(a\,d+b\,c\right)}{9}+\frac{4\,b\,d\,x^{17/2}\,\left(a\,d+b\,c\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(13/2)*((2*a^2*d^2)/13 + (2*b^2*c^2)/13 + (8*a*b*c*d)/13) + (2*a^2*c^2*x^
(5/2))/5 + (2*b^2*d^2*x^(21/2))/21 + (4*a*c*x^(9/2)*(a*d + b*c))/9 + (4*b*d
*x^(17/2)*(a*d + b*c))/17
```

sympy [A] time = 10.87, size = 136, normalized size = 1.40

$$\frac{2a^2c^2x^{\frac{5}{2}}}{5}+\frac{4a^2cdx^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{13}{2}}}{13}+\frac{4abc^2x^{\frac{9}{2}}}{9}+\frac{8abcdx^{\frac{13}{2}}}{13}+\frac{4abd^2x^{\frac{17}{2}}}{17}+\frac{2b^2c^2x^{\frac{13}{2}}}{13}+\frac{4b^2cdx^{\frac{17}{2}}}{17}+\frac{2b^2d^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a**2*c**2*x**(5/2)/5 + 4*a**2*c*d*x**(9/2)/9 + 2*a**2*d**2*x**(13/2)/13 +
 4*a*b*c**2*x**(9/2)/9 + 8*a*b*c*d*x**(13/2)/13 + 4*a*b*d**2*x**(17/2)/17 +
 2*b**2*c**2*x**(13/2)/13 + 4*b**2*c*d*x**(17/2)/17 + 2*b**2*d**2*x**(21/2)
/21
```

3.384 $\int \sqrt{x} \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=97

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15/2))/15 + (2*b^2*d^2
*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx &= \int \left(a^2c^2\sqrt{x}+2ac(bc+ad)x^{5/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+2bd(bc+ad)x^1\right.\\ &= \frac{2}{3}a^2c^2x^{3/2}+\frac{4}{7}ac(bc+ad)x^{7/2}+\frac{2}{11}\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2}+\frac{4}{15}bd(bc+a\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.86

$$\frac{2x^{3/2}\left(1995x^4\left(a^2d^2+4abcd+b^2c^2\right)+7315a^2c^2+2926bdx^6(ad+bc)+6270acx^2(ad+bc)+1155b^2d^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2*c^2 + 6270*a*c*(b*c + a*d)*x^2 + 1995*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 + 2926*b*d*(b*c + a*d)*x^6 + 1155*b^2*d^2*x^8))/21945
```

IntegrateAlgebraic [A] time = 0.05, size = 116, normalized size = 1.20

$$\frac{2\left(7315a^2c^2x^{3/2}+6270a^2cdx^{7/2}+1995a^2d^2x^{11/2}+6270abc^2x^{7/2}+7980abcdx^{11/2}+2926abd^2x^{15/2}+1995b^2c^2x^{11/2}+2926b^2cdx^{15/2}+1155b^2d^2x^{19/2}\right)}{21945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] (2*(7315*a^2*c^2*x^(3/2) + 6270*a*b*c^2*x^(7/2) + 6270*a^2*c*d*x^(7/2) + 19
95*b^2*c^2*x^(11/2) + 7980*a*b*c*d*x^(11/2) + 1995*a^2*d^2*x^(11/2) + 2926*
b^2*c*d*x^(15/2) + 2926*a*b*d^2*x^(15/2) + 1155*b^2*d^2*x^(19/2)))/21945
```

fricas [A] time = 0.69, size = 88, normalized size = 0.91

$$\frac{2}{21945}\left(1155\,b^2d^2x^9+2926\left(b^2cd+abd^2\right)x^7+1995\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+7315\,a^2c^2x+6270\left(abc^2+a^2cd\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^2*x^9 + 2926*(b^2*c*d + a*b*d^2)*x^7 + 1995*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^5 + 7315*a^2*c^2*x + 6270*(a*b*c^2 + a^2*c*d)*x^3)*s
qrt(x)
```

giac [A] time = 0.42, size = 94, normalized size = 0.97

$$\frac{2}{19}\,b^2d^2x^{\frac{19}{2}}+\frac{4}{15}\,b^2cdx^{\frac{15}{2}}+\frac{4}{15}\,abd^2x^{\frac{15}{2}}+\frac{2}{11}\,b^2c^2x^{\frac{11}{2}}+\frac{8}{11}\,abcdx^{\frac{11}{2}}+\frac{2}{11}\,a^2d^2x^{\frac{11}{2}}+\frac{4}{7}\,abc^2x^{\frac{7}{2}}+\frac{4}{7}\,a^2cdx^{\frac{7}{2}}+\frac{2}{3}\,a^2c^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*b^2*c*d*x^(15/2) + 4/15*a*b*d^2*x^(15/2) + 2/1
1*b^2*c^2*x^(11/2) + 8/11*a*b*c*d*x^(11/2) + 2/11*a^2*d^2*x^(11/2) + 4/7*a*
b*c^2*x^(7/2) + 4/7*a^2*c*d*x^(7/2) + 2/3*a^2*c^2*x^(3/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.00

$$\frac{2\left(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980abcd\,x^4+1995b^2c^2x^4+6270a^2cd\,x^2+6270ab\,c^2x^2+7315a^2c^2\right)x^{\frac{3}{2}}}{21945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x)
```

```
[Out] 2/21945*x^(3/2)*(1155*b^2*d^2*x^8+2926*a*b*d^2*x^6+2926*b^2*c*d*x^6+1995*a^
2*d^2*x^4+7980*a*b*c*d*x^4+1995*b^2*c^2*x^4+6270*a^2*c*d*x^2+6270*a*b*c^2*x
^2+7315*a^2*c^2)
```

maxima [**A**] time = 1.06, size = 85, normalized size = 0.88

$$\frac{2}{19}\,b^2d^2x^{\frac{19}{2}} + \frac{4}{15}\left(b^2cd + abd^2\right)x^{\frac{15}{2}} + \frac{2}{11}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{11}{2}} + \frac{2}{3}\,a^2c^2x^{\frac{3}{2}} + \frac{4}{7}\left(abc^2 + a^2cd\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^2*x^(19/2) + 4/15*(b^2*c*d + a*b*d^2)*x^(15/2) + 2/11*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(11/2) + 2/3*a^2*c^2*x^(3/2) + 4/7*(a*b*c^2 + a^2*c
*d)*x^(7/2)
```

mupad [**B**] time = 0.03, size = 78, normalized size = 0.80

$$x^{11/2}\left(\frac{2\,a^2\,d^2}{11} + \frac{8\,a\,b\,c\,d}{11} + \frac{2\,b^2\,c^2}{11}\right) + \frac{2\,a^2\,c^2\,x^{3/2}}{3} + \frac{2\,b^2\,d^2\,x^{19/2}}{19} + \frac{4\,a\,c\,x^{7/2}\,\left(a\,d+b\,c\right)}{7} + \frac{4\,b\,d\,x^{15/2}\,\left(a\,d+b\,c\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^(11/2)*((2*a^2*d^2)/11 + (2*b^2*c^2)/11 + (8*a*b*c*d)/11) + (2*a^2*c^2*x^
(3/2))/3 + (2*b^2*d^2*x^(19/2))/19 + (4*a*c*x^(7/2)*(a*d + b*c))/7 + (4*b*d
*x^(15/2)*(a*d + b*c))/15
```

sympy [**A**] time = 3.38, size = 110, normalized size = 1.13

$$\frac{2a^2c^2x^{\frac{3}{2}}}{3} + \frac{2b^2d^2x^{\frac{19}{2}}}{19} + \frac{2x^{\frac{15}{2}}\left(2abd^2 + 2b^2cd\right)}{15} + \frac{2x^{\frac{11}{2}}\left(a^2d^2 + 4abcd + b^2c^2\right)}{11} + \frac{2x^{\frac{7}{2}}\left(2a^2cd + 2abc^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2*x**(1/2),x)
```

```
[Out] 2*a**2*c**2*x**(3/2)/3 + 2*b**2*d**2*x**(19/2)/19 + 2*x**(15/2)*(2*a*b*d**2
 + 2*b**2*c*d)/15 + 2*x**(11/2)*(a**2*d**2 + 4*a*b*c*d + b**2*c**2)/11 + 2*
x**(7/2)*(2*a**2*c*d + 2*a*b*c**2)/7
```

3.385 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Rubi [**A**] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(17
/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c^2}{\sqrt{x}}+2ac(bc+ad)x^{3/2}+\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+2bd(bc+ad)x^{11/2}+b^2c\right.\\ &= 2a^2c^2\sqrt{x}+\frac{4}{5}ac(bc+ad)x^{5/2}+\frac{2}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2}+\frac{4}{13}bd(bc+ad)x^{13/2}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 95, normalized size = 1.00

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(17
/2))/17
```

IntegrateAlgebraic [A] time = 0.06, size = 116, normalized size = 1.22

$$\frac{2\left(9945a^2c^2\sqrt{x}+3978a^2cdx^{5/2}+1105a^2d^2x^{9/2}+3978abc^2x^{5/2}+4420abcdx^{9/2}+1530abd^2x^{13/2}+1105b^2c^2x^{9/2}+1530b^2cdx^{13/2}+585b^2d^2x^{17/2}\right)}{9945}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

```
[Out] (2*(9945*a^2*c^2*Sqrt[x] + 3978*a*b*c^2*x^(5/2) + 3978*a^2*c*d*x^(5/2) + 11
05*b^2*c^2*x^(9/2) + 4420*a*b*c*d*x^(9/2) + 1105*a^2*d^2*x^(9/2) + 1530*b^2
*c*d*x^(13/2) + 1530*a*b*d^2*x^(13/2) + 585*b^2*d^2*x^(17/2)))/9945
```

fricas [A] time = 0.96, size = 87, normalized size = 0.92

$$\frac{2}{9945}\left(585\,b^2d^2x^8+1530\left(b^2cd+abd^2\right)x^6+1105\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+9945\,a^2c^2+3978\left(abc^2+a^2cd\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^2*x^8 + 1530*(b^2*c*d + a*b*d^2)*x^6 + 1105*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^4 + 9945*a^2*c^2 + 3978*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(
x)
```

giac [A] time = 0.31, size = 94, normalized size = 0.99

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}}+\frac{4}{13}\,b^2cdx^{\frac{13}{2}}+\frac{4}{13}\,abd^2x^{\frac{13}{2}}+\frac{2}{9}\,b^2c^2x^{\frac{9}{2}}+\frac{8}{9}\,abcdx^{\frac{9}{2}}+\frac{2}{9}\,a^2d^2x^{\frac{9}{2}}+\frac{4}{5}\,abc^2x^{\frac{5}{2}}+\frac{4}{5}\,a^2cdx^{\frac{5}{2}}+2\,a^2c^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*b^2*c*d*x^(13/2) + 4/13*a*b*d^2*x^(13/2) + 2/9
*b^2*c^2*x^(9/2) + 8/9*a*b*c*d*x^(9/2) + 2/9*a^2*d^2*x^(9/2) + 4/5*a*b*c^2*
x^(5/2) + 4/5*a^2*c*d*x^(5/2) + 2*a^2*c^2*sqrt(x)
```

maple [A] time = 0.01, size = 97, normalized size = 1.02

$$\frac{2\left(585b^2d^2x^8+1530ab\,d^2x^6+1530b^2cd\,x^6+1105a^2d^2x^4+4420abcd\,x^4+1105b^2c^2x^4+3978a^2cd\,x^2+3978ab\,c^2x^2+9945a^2c^2\right)\sqrt{x}}{9945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*b^2*d^2*x^8+1530*a*b*d^2*x^6+1530*b^2*c*d*x^6+1105*a^2*
d^2*x^4+4420*a*b*c*d*x^4+1105*b^2*c^2*x^4+3978*a^2*c*d*x^2+3978*a*b*c^2*x^2
+9945*a^2*c^2)
```

maxima [A] time = 0.95, size = 85, normalized size = 0.89

$$\frac{2}{17}\,b^2d^2x^{\frac{17}{2}}+\frac{4}{13}\left(b^2cd+abd^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{9}{2}}+2\,a^2c^2\sqrt{x}+\frac{4}{5}\left(abc^2+a^2cd\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^2*x^(17/2) + 4/13*(b^2*c*d + a*b*d^2)*x^(13/2) + 2/9*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^(9/2) + 2*a^2*c^2*sqrt(x) + 4/5*(a*b*c^2 + a^2*c*d)*
x^(5/2)
```

mupad [B] time = 0.03, size = 78, normalized size = 0.82

$$x^{9/2}\left(\frac{2\,a^2\,d^2}{9}+\frac{8\,a\,b\,c\,d}{9}+\frac{2\,b^2\,c^2}{9}\right)+2\,a^2\,c^2\,\sqrt{x}+\frac{2\,b^2\,d^2\,x^{17/2}}{17}+\frac{4\,a\,c\,x^{5/2}\,\left(a\,d+b\,c\right)}{5}+\frac{4\,b\,d\,x^{13/2}\,\left(a\,d+b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(1/2),x)
```

```
[Out] x^(9/2)*((2*a^2*d^2)/9 + (2*b^2*c^2)/9 + (8*a*b*c*d)/9) + 2*a^2*c^2*x^(1/2)
 + (2*b^2*d^2*x^(17/2))/17 + (4*a*c*x^(5/2)*(a*d + b*c))/5 + (4*b*d*x^(13/2
)*(a*d + b*c))/13
```

sympy [A] time = 4.78, size = 134, normalized size = 1.41

$$2a^2c^2\sqrt{x}+\frac{4a^2cdx^{\frac{5}{2}}}{5}+\frac{2a^2d^2x^{\frac{9}{2}}}{9}+\frac{4abc^2x^{\frac{5}{2}}}{5}+\frac{8abcdx^{\frac{9}{2}}}{9}+\frac{4abd^2x^{\frac{13}{2}}}{13}+\frac{2b^2c^2x^{\frac{9}{2}}}{9}+\frac{4b^2cdx^{\frac{13}{2}}}{13}+\frac{2b^2d^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] 2*a**2*c**2*sqrt(x) + 4*a**2*c*d*x**(5/2)/5 + 2*a**2*d**2*x**(9/2)/9 + 4*a*
b*c**2*x**(5/2)/5 + 8*a*b*c*d*x**(9/2)/9 + 4*a*b*d**2*x**(13/2)/13 + 2*b**2
*c**2*x**(9/2)/9 + 4*b**2*c*d*x**(13/2)/13 + 2*b**2*d**2*x**(17/2)/17
```

3.386 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{7}x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2))/11 + (2*b^2*d^2*x^
(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{3/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{3/2}}+2ac(bc+ad)\sqrt{x}+\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+2bd(bc+ad)x^{9/2}+b^2\right.\\ &= -\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{3}ac(bc+ad)x^{3/2}+\frac{2}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2}+\frac{4}{11}bd(bc+ad)x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.87

$$\frac{2\left(165x^4\left(a^2d^2+4abcd+b^2c^2\right)-1155a^2c^2+210bdx^6(ad+bc)+770acx^2(ad+bc)+77b^2d^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2*c^2 + 770*a*c*(b*c + a*d)*x^2 + 165*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 + 210*b*d*(b*c + a*d)*x^6 + 77*b^2*d^2*x^8))/(1155*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.05, size = 100, normalized size = 1.05

$$\frac{2\left(-1155a^2c^2+770a^2cdx^2+165a^2d^2x^4+770abc^2x^2+660abcdx^4+210abd^2x^6+165b^2c^2x^4+210b^2cdx^6+77b^2d^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2*c^2 + 770*a*b*c^2*x^2 + 770*a^2*c*d*x^2 + 165*b^2*c^2*x^4 + 6
60*a*b*c*d*x^4 + 165*a^2*d^2*x^4 + 210*b^2*c*d*x^6 + 210*a*b*d^2*x^6 + 77*b
^2*d^2*x^8))/(1155*Sqrt[x])
```

fricas [A] time = 1.05, size = 87, normalized size = 0.92

$$\frac{2\left(77\,b^2d^2x^8+210\left(b^2cd+abd^2\right)x^6+165\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-1155\,a^2c^2+770\left(abc^2+a^2cd\right)x^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^8 + 210*(b^2*c*d + a*b*d^2)*x^6 + 165*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 - 1155*a^2*c^2 + 770*(a*b*c^2 + a^2*c*d)*x^2)/sqrt(x)
```

giac [A] time = 0.39, size = 94, normalized size = 0.99

$$\frac{2}{15}\,b^2d^2x^{\frac{15}{2}}+\frac{4}{11}\,b^2cdx^{\frac{11}{2}}+\frac{4}{11}\,abd^2x^{\frac{11}{2}}+\frac{2}{7}\,b^2c^2x^{\frac{7}{2}}+\frac{8}{7}\,abcdx^{\frac{7}{2}}+\frac{2}{7}\,a^2d^2x^{\frac{7}{2}}+\frac{4}{3}\,abc^2x^{\frac{3}{2}}+\frac{4}{3}\,a^2cdx^{\frac{3}{2}}-\frac{2\,a^2c^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*b^2*c*d*x^(11/2) + 4/11*a*b*d^2*x^(11/2) + 2/7
*b^2*c^2*x^(7/2) + 8/7*a*b*c*d*x^(7/2) + 2/7*a^2*d^2*x^(7/2) + 4/3*a*b*c^2*
x^(3/2) + 4/3*a^2*c*d*x^(3/2) - 2*a^2*c^2/sqrt(x)
```

maple [A] time = 0.01, size = 97, normalized size = 1.02

$$-\frac{2\left(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660abcd\,x^4-165b^2c^2x^4-770a^2cd\,x^2-770ab\,c^2x^2+1155a^2c^2\right)}{1155\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*b^2*d^2*x^8-210*a*b*d^2*x^6-210*b^2*c*d*x^6-165*a^2*d^2*x^4-66
0*a*b*c*d*x^4-165*b^2*c^2*x^4-770*a^2*c*d*x^2-770*a*b*c^2*x^2+1155*a^2*c^2)
/x^(1/2)
```

maxima [A] time = 1.03, size = 85, normalized size = 0.89

$$\frac{2}{15}\,b^2d^2x^{\frac{15}{2}}+\frac{4}{11}\left(b^2cd+abd^2\right)x^{\frac{11}{2}}+\frac{2}{7}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{7}{2}}-\frac{2\,a^2c^2}{\sqrt{x}}+\frac{4}{3}\left(abc^2+a^2cd\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^2*x^(15/2) + 4/11*(b^2*c*d + a*b*d^2)*x^(11/2) + 2/7*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^(7/2) - 2*a^2*c^2/sqrt(x) + 4/3*(a*b*c^2 + a^2*c*d)*
x^(3/2)
```

mupad [B] time = 0.03, size = 78, normalized size = 0.82

$$x^{7/2}\left(\frac{2\,a^2\,d^2}{7}+\frac{8\,a\,b\,c\,d}{7}+\frac{2\,b^2\,c^2}{7}\right)-\frac{2\,a^2\,c^2}{\sqrt{x}}+\frac{2\,b^2\,d^2\,x^{15/2}}{15}+\frac{4\,a\,c\,x^{3/2}\,\left(a\,d+b\,c\right)}{3}+\frac{4\,b\,d\,x^{11/2}\,\left(a\,d+b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x)
```

```
[Out] x^(7/2)*((2*a^2*d^2)/7 + (2*b^2*c^2)/7 + (8*a*b*c*d)/7) - (2*a^2*c^2)/x^(1/
2) + (2*b^2*d^2*x^(15/2))/15 + (4*a*c*x^(3/2)*(a*d + b*c))/3 + (4*b*d*x^(11
/2)*(a*d + b*c))/11
```

sympy [A] time = 5.26, size = 134, normalized size = 1.41

$$-\frac{2a^2c^2}{\sqrt{x}}+\frac{4a^2cdx^{\frac{3}{2}}}{3}+\frac{2a^2d^2x^{\frac{7}{2}}}{7}+\frac{4abc^2x^{\frac{3}{2}}}{3}+\frac{8abcdx^{\frac{7}{2}}}{7}+\frac{4abd^2x^{\frac{11}{2}}}{11}+\frac{2b^2c^2x^{\frac{7}{2}}}{7}+\frac{4b^2cdx^{\frac{11}{2}}}{11}+\frac{2b^2d^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(3/2),x)
```

```
[Out] -2*a**2*c**2/sqrt(x) + 4*a**2*c*d*x**(3/2)/3 + 2*a**2*d**2*x**(7/2)/7 + 4*a
*b*c**2*x**(3/2)/3 + 8*a*b*c*d*x**(7/2)/7 + 4*a*b*d**2*x**(11/2)/11 + 2*b**
2*c**2*x**(7/2)/7 + 4*b**2*c*d*x**(11/2)/11 + 2*b**2*d**2*x**(15/2)/15
```

3.387 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}\,(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{5}x^{5/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}\,(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))/9 + (2*b^2*d^2*x^(1
3/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2c^2}{x^{5/2}}+\frac{2ac(bc+ad)}{\sqrt{x}}+\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+2bd(bc+ad)x^{7/2}+b^2d^2x^{11}\right.\\ &= -\frac{2a^2c^2}{3x^{3/2}}+4ac(bc+ad)\sqrt{x}+\frac{2}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2}+\frac{4}{9}bd(bc+ad)x^{9/2}+\frac{2}{13}\end{aligned}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.87

$$\frac{2\left(117x^4\left(a^2d^2+4abcd+b^2c^2\right)-195a^2c^2+130bdx^6(ad+bc)+1170acx^2(ad+bc)+45b^2d^2x^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (2*(-195*a^2*c^2 + 1170*a*c*(b*c + a*d)*x^2 + 117*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 + 130*b*d*(b*c + a*d)*x^6 + 45*b^2*d^2*x^8))/(585*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 100, normalized size = 1.05

$$\frac{2\left(-195a^2c^2+1170a^2cdx^2+117a^2d^2x^4+1170abc^2x^2+468abcdx^4+130abd^2x^6+117b^2c^2x^4+130b^2cdx^6+45b^2d^2x^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

```
[Out] (2*(-195*a^2*c^2 + 1170*a*b*c^2*x^2 + 1170*a^2*c*d*x^2 + 117*b^2*c^2*x^4 +
468*a*b*c*d*x^4 + 117*a^2*d^2*x^4 + 130*b^2*c*d*x^6 + 130*a*b*d^2*x^6 + 45*
b^2*d^2*x^8))/(585*x^(3/2))
```

fricas [A] time = 0.87, size = 87, normalized size = 0.92

$$\frac{2\left(45\,b^2d^2x^8+130\left(b^2cd+abd^2\right)x^6+117\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-195\,a^2c^2+1170\left(abc^2+a^2cd\right)x^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d^2*x^8 + 130*(b^2*c*d + a*b*d^2)*x^6 + 117*(b^2*c^2 + 4*a*b*
c*d + a^2*d^2)*x^4 - 195*a^2*c^2 + 1170*(a*b*c^2 + a^2*c*d)*x^2)/x^(3/2)
```

giac [A] time = 0.37, size = 94, normalized size = 0.99

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}}+\frac{4}{9}\,b^2cdx^{\frac{9}{2}}+\frac{4}{9}\,abd^2x^{\frac{9}{2}}+\frac{2}{5}\,b^2c^2x^{\frac{5}{2}}+\frac{8}{5}\,abcdx^{\frac{5}{2}}+\frac{2}{5}\,a^2d^2x^{\frac{5}{2}}+4\,abc^2\sqrt{x}+4\,a^2cd\sqrt{x}-\frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*b^2*c*d*x^(9/2) + 4/9*a*b*d^2*x^(9/2) + 2/5*b^2
*c^2*x^(5/2) + 8/5*a*b*c*d*x^(5/2) + 2/5*a^2*d^2*x^(5/2) + 4*a*b*c^2*sqrt(x
) + 4*a^2*c*d*sqrt(x) - 2/3*a^2*c^2/x^(3/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.02

$$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468abcd\,x^4-117b^2c^2x^4-1170a^2cd\,x^2-1170ab\,c^2x^2+195a^2c^2\right)}{585x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*b^2*d^2*x^8-130*a*b*d^2*x^6-130*b^2*c*d*x^6-117*a^2*d^2*x^4-468
*a*b*c*d*x^4-117*b^2*c^2*x^4-1170*a^2*c*d*x^2-1170*a*b*c^2*x^2+195*a^2*c^2)
/x^(3/2)
```

maxima [A] time = 1.01, size = 85, normalized size = 0.89

$$\frac{2}{13}\,b^2d^2x^{\frac{13}{2}} + \frac{4}{9}\left(b^2cd + abd^2\right)x^{\frac{9}{2}} + \frac{2}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{5}{2}} - \frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}} + 4\left(abc^2 + a^2cd\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*b^2*d^2*x^(13/2) + 4/9*(b^2*c*d + a*b*d^2)*x^(9/2) + 2/5*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(5/2) - 2/3*a^2*c^2/x^(3/2) + 4*(a*b*c^2 + a^2*c*d)*sq
rt(x)
```

mupad [B] time = 0.03, size = 78, normalized size = 0.82

$$x^{5/2}\left(\frac{2\,a^2\,d^2}{5} + \frac{8\,a\,b\,c\,d}{5} + \frac{2\,b^2\,c^2}{5}\right) - \frac{2\,a^2\,c^2}{3\,x^{3/2}} + \frac{2\,b^2\,d^2\,x^{13/2}}{13} + 4\,a\,c\,\sqrt{x}\,\left(a\,d + b\,c\right) + \frac{4\,b\,d\,x^{9/2}\,\left(a\,d + b\,c\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x)
```

```
[Out] x^(5/2)*((2*a^2*d^2)/5 + (2*b^2*c^2)/5 + (8*a*b*c*d)/5) - (2*a^2*c^2)/(3*x^
(3/2)) + (2*b^2*d^2*x^(13/2))/13 + 4*a*c*x^(1/2)*(a*d + b*c) + (4*b*d*x^(9/
2)*(a*d + b*c))/9
```

sympy [A] time = 6.07, size = 133, normalized size = 1.40

$$-\frac{2a^2c^2}{3x^{\frac{3}{2}}} + 4a^2cd\sqrt{x} + \frac{2a^2d^2x^{\frac{5}{2}}}{5} + 4abc^2\sqrt{x} + \frac{8abcdx^{\frac{5}{2}}}{5} + \frac{4abd^2x^{\frac{9}{2}}}{9} + \frac{2b^2c^2x^{\frac{5}{2}}}{5} + \frac{4b^2cdx^{\frac{9}{2}}}{9} + \frac{2b^2d^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(5/2),x)
```

```
[Out] -2*a**2*c**2/(3*x**(3/2)) + 4*a**2*c*d*sqrt(x) + 2*a**2*d**2*x**(5/2)/5 + 4
*a*b*c**2*sqrt(x) + 8*a*b*c*d*x**(5/2)/5 + 4*a*b*d**2*x**(9/2)/9 + 2*b**2*c
**2*x**(5/2)/5 + 4*b**2*c*d*x**(9/2)/9 + 2*b**2*d**2*x**(13/2)/13
```

3.388 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{3}x^{3/2}\left(a^2d^2+4abcd+b^2c^2\right)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2))/7 + (2*b^2*d^2*x^
(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x^{7/2}}\,dx &= \int \Bigg(\frac{a^2c^2}{x^{7/2}}+\frac{2ac(bc+ad)}{x^{3/2}}+\left(b^2c^2+4abcd+a^2d^2\right)\sqrt{x}+2bd(bc+ad)x^{5/2}+b^2d^2x\\ &= -\frac{2a^2c^2}{5x^{5/2}}-\frac{4ac(bc+ad)}{\sqrt{x}}+\frac{2}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2}+\frac{4}{7}bd(bc+ad)x^{7/2}+\frac{2}{11}b\end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.87

$$\frac{2\left(385x^4\left(a^2d^2+4abcd+b^2c^2\right)-231a^2c^2+330bdx^6(ad+bc)-2310acx^2(ad+bc)+105b^2d^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^2 - 2310*a*c*(b*c + a*d)*x^2 + 385*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 + 330*b*d*(b*c + a*d)*x^6 + 105*b^2*d^2*x^8))/(1155*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 100, normalized size = 1.05

$$\frac{2\left(-231a^2c^2-2310a^2cdx^2+385a^2d^2x^4-2310abc^2x^2+1540abcdx^4+330abd^2x^6+385b^2c^2x^4+330b^2cdx^6+105b^2d^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^2 - 2310*a*b*c^2*x^2 - 2310*a^2*c*d*x^2 + 385*b^2*c^2*x^4 +
1540*a*b*c*d*x^4 + 385*a^2*d^2*x^4 + 330*b^2*c*d*x^6 + 330*a*b*d^2*x^6 + 10
5*b^2*d^2*x^8))/(1155*x^(5/2))
```

fricas [A] time = 1.09, size = 87, normalized size = 0.92

$$\frac{2\left(105\,b^2d^2x^8+330\left(b^2cd+abd^2\right)x^6+385\left(b^2c^2+4\,abcd+a^2d^2\right)x^4-231\,a^2c^2-2310\left(abc^2+a^2cd\right)x^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^2*d^2*x^8 + 330*(b^2*c*d + a*b*d^2)*x^6 + 385*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 - 231*a^2*c^2 - 2310*(a*b*c^2 + a^2*c*d)*x^2)/x^(5/2)
```

giac [A] time = 0.43, size = 96, normalized size = 1.01

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}}+\frac{4}{7}\,b^2cdx^{\frac{7}{2}}+\frac{4}{7}\,abd^2x^{\frac{7}{2}}+\frac{2}{3}\,b^2c^2x^{\frac{3}{2}}+\frac{8}{3}\,abcdx^{\frac{3}{2}}+\frac{2}{3}\,a^2d^2x^{\frac{3}{2}}-\frac{2\left(10\,abc^2x^2+10\,a^2cdx^2+a^2c^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*b^2*c*d*x^(7/2) + 4/7*a*b*d^2*x^(7/2) + 2/3*b^2
*c^2*x^(3/2) + 8/3*a*b*c*d*x^(3/2) + 2/3*a^2*d^2*x^(3/2) - 2/5*(10*a*b*c^2*
x^2 + 10*a^2*c*d*x^2 + a^2*c^2)/x^(5/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.02

$$-\frac{2\left(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540abcd\,x^4-385b^2c^2x^4+2310a^2cd\,x^2+2310ab\,c^2x^2+231a^2c^2\right)}{1155x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*b^2*d^2*x^8-330*a*b*d^2*x^6-330*b^2*c*d*x^6-385*a^2*d^2*x^4-1
540*a*b*c*d*x^4-385*b^2*c^2*x^4+2310*a^2*c*d*x^2+2310*a*b*c^2*x^2+231*a^2*c
^2)/x^(5/2)
```

maxima [A] time = 1.19, size = 87, normalized size = 0.92

$$\frac{2}{11}\,b^2d^2x^{\frac{11}{2}}+\frac{4}{7}\left(b^2cd+abd^2\right)x^{\frac{7}{2}}+\frac{2}{3}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{3}{2}}-\frac{2\left(a^2c^2+10\left(abc^2+a^2cd\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*b^2*d^2*x^(11/2) + 4/7*(b^2*c*d + a*b*d^2)*x^(7/2) + 2/3*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(3/2) - 2/5*(a^2*c^2 + 10*(a*b*c^2 + a^2*c*d)*x^2)/x^(
5/2)
```

mupad [B] time = 0.06, size = 86, normalized size = 0.91

$$x^{3/2}\left(\frac{2\,a^2\,d^2}{3}+\frac{8\,a\,b\,c\,d}{3}+\frac{2\,b^2\,c^2}{3}\right)-\frac{x^2\,\left(4\,d\,a^2\,c+4\,b\,a\,c^2\right)+\frac{2\,a^2\,c^2}{5}}{x^{5/2}}+\frac{2\,b^2\,d^2\,x^{11/2}}{11}+\frac{4\,b\,d\,x^{7/2}\,\left(a\,d+b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x)
```

```
[Out] x^(3/2)*((2*a^2*d^2)/3 + (2*b^2*c^2)/3 + (8*a*b*c*d)/3) - (x^2*(4*a*b*c^2 +
 4*a^2*c*d) + (2*a^2*c^2)/5)/x^(5/2) + (2*b^2*d^2*x^(11/2))/11 + (4*b*d*x^(
7/2)*(a*d + b*c))/7
```

sympy [A] time = 8.04, size = 133, normalized size = 1.40

$$-\frac{2a^2c^2}{5x^{\frac{5}{2}}}-\frac{4a^2cd}{\sqrt{x}}+\frac{2a^2d^2x^{\frac{3}{2}}}{3}-\frac{4abc^2}{\sqrt{x}}+\frac{8abcdx^{\frac{3}{2}}}{3}+\frac{4abd^2x^{\frac{7}{2}}}{7}+\frac{2b^2c^2x^{\frac{3}{2}}}{3}+\frac{4b^2cdx^{\frac{7}{2}}}{7}+\frac{2b^2d^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(7/2),x)
```

```
[Out] -2*a**2*c**2/(5*x**(5/2)) - 4*a**2*c*d/sqrt(x) + 2*a**2*d**2*x**(3/2)/3 - 4
*a*b*c**2/sqrt(x) + 8*a*b*c*d*x**(3/2)/3 + 4*a*b*d**2*x**(7/2)/7 + 2*b**2*c
**2*x**(3/2)/3 + 4*b**2*c*d*x**(7/2)/7 + 2*b**2*d**2*x**(11/2)/11
```

3.389 $\int x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=139

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}$$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3*x^
(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{7/2}+ac^2(2bc+3ad)x^{11/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+d\left(3b^2c^2\right.\right.\\ &= \frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2(2bc+3ad)x^{13/2}+\frac{2}{17}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{17/2}+\frac{2}{21}d\left(\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 139, normalized size = 1.00

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3*x^
(29/2))/29
```

IntegrateAlgebraic [A] time = 0.07, size = 163, normalized size = 1.17

$$\frac{2\left(1121575a^2c^3x^{9/2}+2329425a^2c^2dx^{13/2}+1781325a^2cd^2x^{17/2}+480675a^2d^3x^{21/2}+1552950abc^3x^{13/2}+3562650abc^2dx^{17/2}+2884050abcd^2x^{21/2}+807534abd^3x^{25/2}+593775b^2c^3x^{17/2}+1442025b^2c^2dx^{21/2}+1211301b^2cd^2x^{25/2}+348075b^2d^3x^{29/2}\right)}{10094175}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*(1121575*a^2*c^3*x^(9/2) + 1552950*a*b*c^3*x^(13/2) + 2329425*a^2*c^2*d*
x^(13/2) + 593775*b^2*c^3*x^(17/2) + 3562650*a*b*c^2*d*x^(17/2) + 1781325*a
^2*c*d^2*x^(17/2) + 1442025*b^2*c^2*d*x^(21/2) + 2884050*a*b*c*d^2*x^(21/2)
 + 480675*a^2*d^3*x^(21/2) + 1211301*b^2*c*d^2*x^(25/2) + 807534*a*b*d^3*x^
(25/2) + 348075*b^2*d^3*x^(29/2)))/10094175
```

fricas [A] time = 0.87, size = 132, normalized size = 0.95

$$\frac{2}{10094175}\left(348075\,b^2d^3x^{14}+403767\left(3\,b^2cd^2+2\,abd^3\right)x^{12}+480675\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+1121575\,a^2c^3x^4+593775\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^8+776475\left(2\,abc^3+3\,a^2c^2d\right)x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/10094175*(348075*b^2*d^3*x^14 + 403767*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 4
80675*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1121575*a^2*c^3*x^4 + 59
3775*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^8 + 776475*(2*a*b*c^3 + 3*a^2*
c^2*d)*x^6)*sqrt(x)
```

giac [A] time = 0.35, size = 135, normalized size = 0.97

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}}+\frac{6}{25}\,b^2cd^2x^{\frac{25}{2}}+\frac{4}{25}\,abd^3x^{\frac{25}{2}}+\frac{2}{7}\,b^2c^2dx^{\frac{21}{2}}+\frac{4}{7}\,abcd^2x^{\frac{21}{2}}+\frac{2}{21}\,a^2d^3x^{\frac{21}{2}}+\frac{2}{17}\,b^2c^3x^{\frac{17}{2}}+\frac{12}{17}\,abc^2dx^{\frac{17}{2}}+\frac{6}{17}\,a^2cd^2x^{\frac{17}{2}}+\frac{4}{13}\,abc^3x^{\frac{13}{2}}+\frac{6}{13}\,a^2c^2dx^{\frac{13}{2}}+\frac{2}{9}\,a^2c^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 6/25*b^2*c*d^2*x^(25/2) + 4/25*a*b*d^3*x^(25/2) + 2
/7*b^2*c^2*d*x^(21/2) + 4/7*a*b*c*d^2*x^(21/2) + 2/21*a^2*d^3*x^(21/2) + 2/
17*b^2*c^3*x^(17/2) + 12/17*a*b*c^2*d*x^(17/2) + 6/17*a^2*c*d^2*x^(17/2) +
4/13*a*b*c^3*x^(13/2) + 6/13*a^2*c^2*d*x^(13/2) + 2/9*a^2*c^3*x^(9/2)
```

maple [A] time = 0.01, size = 138, normalized size = 0.99

$$\frac{2\left(348075b^2d^3x^{10}+807534ab\,d^3x^8+1211301b^2c\,d^2x^8+480675a^2d^3x^6+2884050abc\,d^2x^6+1442025b^2c^2d\,x^6+1781325a^2c\,d^2x^4+3562650ab\,c^2d\,x^4+593775b^2c^3x^4+2329425a^2c^2d\,x^2+1552950ab\,c^3x^2+1121575a^2c^3\right)x^{\frac{9}{2}}}{10094175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/10094175*x^(9/2)*(348075*b^2*d^3*x^10+807534*a*b*d^3*x^8+1211301*b^2*c*d^
2*x^8+480675*a^2*d^3*x^6+2884050*a*b*c*d^2*x^6+1442025*b^2*c^2*d*x^6+178132
5*a^2*c*d^2*x^4+3562650*a*b*c^2*d*x^4+593775*b^2*c^3*x^4+2329425*a^2*c^2*d*
x^2+1552950*a*b*c^3*x^2+1121575*a^2*c^3)
```

maxima [A] time = 1.06, size = 127, normalized size = 0.91

$$\frac{2}{29}\,b^2d^3x^{\frac{29}{2}} + \frac{2}{25}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{25}{2}} + \frac{2}{21}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{21}{2}} + \frac{2}{9}\,a^2c^3x^{\frac{9}{2}} + \frac{2}{17}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{17}{2}} + \frac{2}{13}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/29*b^2*d^3*x^(29/2) + 2/25*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(25/2) + 2/21*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(21/2) + 2/9*a^2*c^3*x^(9/2) + 2/17*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(17/2) + 2/13*(2*a*b*c^3 + 3*a^2*c^2*d
)*x^(13/2)
```

mupad [B] time = 0.21, size = 119, normalized size = 0.86

$$x^{17/2}\left(\frac{6\,a^2\,c\,d^2}{17} + \frac{12\,a\,b\,c^2\,d}{17} + \frac{2\,b^2\,c^3}{17}\right) + x^{21/2}\left(\frac{2\,a^2\,d^3}{21} + \frac{4\,a\,b\,c\,d^2}{7} + \frac{2\,b^2\,c^2\,d}{7}\right) + \frac{2\,a^2\,c^3\,x^{9/2}}{9} + \frac{2\,b^2\,d^3\,x^{29/2}}{29} + \frac{2\,a\,c^2\,x^{13/2}\,\left(3\,a\,d + 2\,b\,c\right)}{13} + \frac{2\,b\,d^2\,x^{25/2}\,\left(2\,a\,d + 3\,b\,c\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(17/2)*((2*b^2*c^3)/17 + (6*a^2*c*d^2)/17 + (12*a*b*c^2*d)/17) + x^(21/2)
*((2*a^2*d^3)/21 + (2*b^2*c^2*d)/7 + (4*a*b*c*d^2)/7) + (2*a^2*c^3*x^(9/2))
/9 + (2*b^2*d^3*x^(29/2))/29 + (2*a*c^2*x^(13/2)*(3*a*d + 2*b*c))/13 + (2*b
*d^2*x^(25/2)*(2*a*d + 3*b*c))/25
```

sympy [A] time = 53.08, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{9}{2}}}{9} + \frac{6a^2c^2dx^{\frac{13}{2}}}{13} + \frac{6a^2cd^2x^{\frac{17}{2}}}{17} + \frac{2a^2d^3x^{\frac{21}{2}}}{21} + \frac{4abc^3x^{\frac{13}{2}}}{13} + \frac{12abc^2dx^{\frac{17}{2}}}{17} + \frac{4abcd^2x^{\frac{21}{2}}}{7} + \frac{4abd^3x^{\frac{25}{2}}}{25} + \frac{2b^2c^3x^{\frac{17}{2}}}{17} + \frac{2b^2c^2dx^{\frac{21}{2}}}{7} + \frac{6b^2cd^2x^{\frac{25}{2}}}{25} + \frac{2b^2d^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(9/2)/9 + 6*a**2*c**2*d*x**(13/2)/13 + 6*a**2*c*d**2*x**(17/
2)/17 + 2*a**2*d**3*x**(21/2)/21 + 4*a*b*c**3*x**(13/2)/13 + 12*a*b*c**2*d*
x**(17/2)/17 + 4*a*b*c*d**2*x**(21/2)/7 + 4*a*b*d**3*x**(25/2)/25 + 2*b**2*
c**3*x**(17/2)/17 + 2*b**2*c**2*d*x**(21/2)/7 + 6*b**2*c*d**2*x**(25/2)/25
+ 2*b**2*d**3*x**(29/2)/29
```

3.390 $\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23}$$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3*x^
(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx&=\int \left(a^2c^3x^{5/2}+ac^2(2bc+3ad)x^{9/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+d\left(3b^2c^2\right.\right.\\&=\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2(2bc+3ad)x^{11/2}+\frac{2}{15}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+\frac{2}{19}d\end{aligned}$$

Mathematica [A] time = 0.04, size = 139, normalized size = 1.00

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3*x^
(27/2))/27
```

IntegrateAlgebraic [A] time = 0.07, size = 163, normalized size = 1.17

$$\frac{2\left(648945a^2c^3x^{7/2}+1238895a^2c^2dx^{11/2}+908523a^2cd^2x^{15/2}+239085a^2d^3x^{19/2}+825930abc^3x^{11/2}+1817046abc^2dx^{15/2}+1434510abcd^2x^{19/2}+395010abd^3x^{23/2}+302841b^2c^3x^{15/2}+717255b^2c^2dx^{19/2}+592515b^2cd^2x^{23/2}+168245b^2d^3x^{27/2}\right)}{4542615}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*(648945*a^2*c^3*x^(7/2) + 825930*a*b*c^3*x^(11/2) + 1238895*a^2*c^2*d*x^
(11/2) + 302841*b^2*c^3*x^(15/2) + 1817046*a*b*c^2*d*x^(15/2) + 908523*a^2*
c*d^2*x^(15/2) + 717255*b^2*c^2*d*x^(19/2) + 1434510*a*b*c*d^2*x^(19/2) + 2
39085*a^2*d^3*x^(19/2) + 592515*b^2*c*d^2*x^(23/2) + 395010*a*b*d^3*x^(23/2
) + 168245*b^2*d^3*x^(27/2)))/4542615
```

fricas [A] time = 0.77, size = 132, normalized size = 0.95

$$\frac{2}{4542615}\left(168245\,b^2d^3x^{13}+197505\left(3\,b^2cd^2+2\,abd^3\right)x^{11}+239085\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^9+648945\,a^2c^3x^3+302841\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+412965\left(2\,abc^3+3\,a^2c^2d\right)x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*b^2*d^3*x^13 + 197505*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 23
9085*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 648945*a^2*c^3*x^3 + 30284
1*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 + 412965*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^5)*sqrt(x)
```

giac [A] time = 0.44, size = 135, normalized size = 0.97

$$\frac{2}{27}\,b^2d^3x^{\frac{27}{2}}+\frac{6}{23}\,b^2cd^2x^{\frac{23}{2}}+\frac{4}{23}\,abd^3x^{\frac{23}{2}}+\frac{6}{19}\,b^2c^2dx^{\frac{19}{2}}+\frac{12}{19}\,abcd^2x^{\frac{19}{2}}+\frac{2}{19}\,a^2d^3x^{\frac{19}{2}}+\frac{2}{15}\,b^2c^3x^{\frac{15}{2}}+\frac{4}{5}\,abc^2dx^{\frac{15}{2}}+\frac{2}{5}\,a^2cd^2x^{\frac{15}{2}}+\frac{4}{11}\,abc^3x^{\frac{11}{2}}+\frac{6}{11}\,a^2c^2dx^{\frac{11}{2}}+\frac{2}{7}\,a^2c^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 6/23*b^2*c*d^2*x^(23/2) + 4/23*a*b*d^3*x^(23/2) + 6
/19*b^2*c^2*d*x^(19/2) + 12/19*a*b*c*d^2*x^(19/2) + 2/19*a^2*d^3*x^(19/2) +
 2/15*b^2*c^3*x^(15/2) + 4/5*a*b*c^2*d*x^(15/2) + 2/5*a^2*c*d^2*x^(15/2) +
4/11*a*b*c^3*x^(11/2) + 6/11*a^2*c^2*d*x^(11/2) + 2/7*a^2*c^3*x^(7/2)
```

maple [A] time = 0.01, size = 138, normalized size = 0.99

$$\frac{2\left(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510abc\,d^2x^6+717255b^2c^2d\,x^6+908523a^2c\,d^2x^4+1817046ab\,c^2d\,x^4+302841b^2c^3x^4+1238895a^2c^2d\,x^2+825930ab\,c^3x^2+648945a^2c^3\right)x^{\frac{7}{2}}}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/4542615*x^(7/2)*(168245*b^2*d^3*x^10+395010*a*b*d^3*x^8+592515*b^2*c*d^2*
x^8+239085*a^2*d^3*x^6+1434510*a*b*c*d^2*x^6+717255*b^2*c^2*d*x^6+908523*a^
2*c*d^2*x^4+1817046*a*b*c^2*d*x^4+302841*b^2*c^3*x^4+1238895*a^2*c^2*d*x^2+
825930*a*b*c^3*x^2+648945*a^2*c^3)
```

maxima [A] time = 1.06, size = 127, normalized size = 0.91

$$\frac{2}{27}\,b^2d^3x^{\frac{27}{2}}+\frac{2}{23}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{23}{2}}+\frac{2}{19}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{19}{2}}+\frac{2}{7}\,a^2c^3x^{\frac{7}{2}}+\frac{2}{15}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/27*b^2*d^3*x^(27/2) + 2/23*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(23/2) + 2/19*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(19/2) + 2/7*a^2*c^3*x^(7/2) + 2/15*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(15/2) + 2/11*(2*a*b*c^3 + 3*a^2*c^2*d
)*x^(11/2)
```

mupad [B] time = 0.04, size = 119, normalized size = 0.86

$$x^{15/2}\left(\frac{2\,a^2\,c\,d^2}{5}+\frac{4\,a\,b\,c^2\,d}{5}+\frac{2\,b^2\,c^3}{15}\right)+x^{19/2}\left(\frac{2\,a^2\,d^3}{19}+\frac{12\,a\,b\,c\,d^2}{19}+\frac{6\,b^2\,c^2\,d}{19}\right)+\frac{2\,a^2\,c^3\,x^{7/2}}{7}+\frac{2\,b^2\,d^3\,x^{27/2}}{27}+\frac{2\,a\,c^2\,x^{11/2}\,\left(3\,a\,d+2\,b\,c\right)}{11}+\frac{2\,b\,d^2\,x^{23/2}\,\left(2\,a\,d+3\,b\,c\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(15/2)*((2*b^2*c^3)/15 + (2*a^2*c*d^2)/5 + (4*a*b*c^2*d)/5) + x^(19/2)*((
2*a^2*d^3)/19 + (6*b^2*c^2*d)/19 + (12*a*b*c*d^2)/19) + (2*a^2*c^3*x^(7/2))
/7 + (2*b^2*d^3*x^(27/2))/27 + (2*a*c^2*x^(11/2)*(3*a*d + 2*b*c))/11 + (2*b
*d^2*x^(23/2)*(2*a*d + 3*b*c))/23
```

sympy [A] time = 33.45, size = 192, normalized size = 1.38

$$\frac{2a^2c^3x^{\frac{7}{2}}}{7}+\frac{6a^2c^2dx^{\frac{11}{2}}}{11}+\frac{2a^2cd^2x^{\frac{15}{2}}}{5}+\frac{2a^2d^3x^{\frac{19}{2}}}{19}+\frac{4abc^3x^{\frac{11}{2}}}{11}+\frac{4abc^2dx^{\frac{15}{2}}}{5}+\frac{12abcd^2x^{\frac{19}{2}}}{19}+\frac{4abd^3x^{\frac{23}{2}}}{23}+\frac{2b^2c^3x^{\frac{15}{2}}}{15}+\frac{6b^2c^2dx^{\frac{19}{2}}}{19}+\frac{6b^2cd^2x^{\frac{23}{2}}}{23}+\frac{2b^2d^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(7/2)/7 + 6*a**2*c**2*d*x**(11/2)/11 + 2*a**2*c*d**2*x**(15/
2)/5 + 2*a**2*d**3*x**(19/2)/19 + 4*a*b*c**3*x**(11/2)/11 + 4*a*b*c**2*d*x*
*(15/2)/5 + 12*a*b*c*d**2*x**(19/2)/19 + 4*a*b*d**3*x**(23/2)/23 + 2*b**2*c
**3*x**(15/2)/15 + 6*b**2*c**2*d*x**(19/2)/19 + 6*b**2*c*d**2*x**(23/2)/23
+ 2*b**2*d**3*x**(27/2)/27
```

3.391 $\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2$$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x^(2
5/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \left(a^2c^3x^{3/2}+ac^2(2bc+3ad)x^{7/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+d\left(3b^2c^2+\right.\right.\\ &= \frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2(2bc+3ad)x^{9/2}+\frac{2}{13}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+\frac{2}{17}d\left(3b\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 139, normalized size = 1.00

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x^(2
5/2))/25
```

IntegrateAlgebraic [A] time = 0.07, size = 163, normalized size = 1.17

$$\frac{2\left(69615a^2c^3x^{5/2}+116025a^2c^2dx^{9/2}+80325a^2cd^2x^{13/2}+20475a^2d^3x^{17/2}+77350abc^3x^{9/2}+160650abc^2dx^{13/2}+122850abcd^2x^{17/2}+33150abd^3x^{21/2}+26775b^2c^3x^{13/2}+61425b^2c^2dx^{17/2}+49725b^2cd^2x^{21/2}+13923b^2d^3x^{25/2}\right)}{348075}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*(69615*a^2*c^3*x^(5/2) + 77350*a*b*c^3*x^(9/2) + 116025*a^2*c^2*d*x^(9/2
) + 26775*b^2*c^3*x^(13/2) + 160650*a*b*c^2*d*x^(13/2) + 80325*a^2*c*d^2*x^
(13/2) + 61425*b^2*c^2*d*x^(17/2) + 122850*a*b*c*d^2*x^(17/2) + 20475*a^2*d
^3*x^(17/2) + 49725*b^2*c*d^2*x^(21/2) + 33150*a*b*d^3*x^(21/2) + 13923*b^2
*d^3*x^(25/2)))/348075
```

fricas [A] time = 0.75, size = 132, normalized size = 0.95

$$\frac{2}{348075}\left(13923\,b^2d^3x^{12}+16575\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+20475\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+69615\,a^2c^3x^2+26775\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+38675\left(2\,abc^3+3\,a^2c^2d\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^2*d^3*x^12 + 16575*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 20475
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 69615*a^2*c^3*x^2 + 26775*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^6 + 38675*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
)*sqrt(x)
```

giac [A] time = 0.31, size = 135, normalized size = 0.97

$$\frac{2}{25}\,b^2d^3x^{\frac{25}{2}}+\frac{2}{7}\,b^2cd^2x^{\frac{21}{2}}+\frac{4}{21}\,abd^3x^{\frac{21}{2}}+\frac{6}{17}\,b^2c^2dx^{\frac{17}{2}}+\frac{12}{17}\,abcd^2x^{\frac{17}{2}}+\frac{2}{17}\,a^2d^3x^{\frac{17}{2}}+\frac{2}{13}\,b^2c^3x^{\frac{13}{2}}+\frac{12}{13}\,abc^2dx^{\frac{13}{2}}+\frac{6}{13}\,a^2cd^2x^{\frac{13}{2}}+\frac{4}{9}\,abc^3x^{\frac{9}{2}}+\frac{2}{3}\,a^2c^2dx^{\frac{9}{2}}+\frac{2}{5}\,a^2c^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/7*b^2*c*d^2*x^(21/2) + 4/21*a*b*d^3*x^(21/2) + 6/
17*b^2*c^2*d*x^(17/2) + 12/17*a*b*c*d^2*x^(17/2) + 2/17*a^2*d^3*x^(17/2) +
2/13*b^2*c^3*x^(13/2) + 12/13*a*b*c^2*d*x^(13/2) + 6/13*a^2*c*d^2*x^(13/2)
+ 4/9*a*b*c^3*x^(9/2) + 2/3*a^2*c^2*d*x^(9/2) + 2/5*a^2*c^3*x^(5/2)
```

maple [A] time = 0.01, size = 138, normalized size = 0.99

$$\frac{2\left(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850abc\,d^2x^6+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+160650ab\,c^2d\,x^4+26775b^2c^3x^4+116025a^2c^2d\,x^2+77350ab\,c^3x^2+69615a^2c^3\right)x^{\frac{5}{2}}}{348075}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 2/348075*x^(5/2)*(13923*b^2*d^3*x^10+33150*a*b*d^3*x^8+49725*b^2*c*d^2*x^8+
20475*a^2*d^3*x^6+122850*a*b*c*d^2*x^6+61425*b^2*c^2*d*x^6+80325*a^2*c*d^2*
x^4+160650*a*b*c^2*d*x^4+26775*b^2*c^3*x^4+116025*a^2*c^2*d*x^2+77350*a*b*c
^3*x^2+69615*a^2*c^3)
```

maxima [A] time = 1.09, size = 127, normalized size = 0.91

$$\frac{2}{25}\,b^{2}d^{3}x^{\frac{25}{2}}+\frac{2}{21}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{\frac{21}{2}}+\frac{2}{17}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{17}{2}}+\frac{2}{5}\,a^{2}c^{3}x^{\frac{5}{2}}+\frac{2}{13}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{\frac{13}{2}}+\frac{2}{9}\left(2\,abc^{3}+3\,a^{2}c^{2}d\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/25*b^2*d^3*x^(25/2) + 2/21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(21/2) + 2/17*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(17/2) + 2/5*a^2*c^3*x^(5/2) + 2/13*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(13/2) + 2/9*(2*a*b*c^3 + 3*a^2*c^2*d)
*x^(9/2)
```

mupad [B] time = 0.04, size = 119, normalized size = 0.86

$$x^{13/2}\left(\frac{6\,a^2\,c\,d^2}{13}+\frac{12\,a\,b\,c^2\,d}{13}+\frac{2\,b^2\,c^3}{13}\right)+x^{17/2}\left(\frac{2\,a^2\,d^3}{17}+\frac{12\,a\,b\,c\,d^2}{17}+\frac{6\,b^2\,c^2\,d}{17}\right)+\frac{2\,a^2\,c^3\,x^{5/2}}{5}+\frac{2\,b^2\,d^3\,x^{25/2}}{25}+\frac{2\,a\,c^2\,x^{9/2}\,\left(3\,a\,d+2\,b\,c\right)}{9}+\frac{2\,b\,d^2\,x^{21/2}\,\left(2\,a\,d+3\,b\,c\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(13/2)*((2*b^2*c^3)/13 + (6*a^2*c*d^2)/13 + (12*a*b*c^2*d)/13) + x^(17/2)
*((2*a^2*d^3)/17 + (6*b^2*c^2*d)/17 + (12*a*b*c*d^2)/17) + (2*a^2*c^3*x^(5/
2))/5 + (2*b^2*d^3*x^(25/2))/25 + (2*a*c^2*x^(9/2)*(3*a*d + 2*b*c))/9 + (2*
b*d^2*x^(21/2)*(2*a*d + 3*b*c))/21
```

sympy [A] time = 19.98, size = 192, normalized size = 1.38

$$\frac{2a^{2}c^{3}x^{\frac{5}{2}}}{5}+\frac{2a^{2}c^{2}dx^{\frac{9}{2}}}{3}+\frac{6a^{2}cd^{2}x^{\frac{13}{2}}}{13}+\frac{2a^{2}d^{3}x^{\frac{17}{2}}}{17}+\frac{4abc^{3}x^{\frac{9}{2}}}{9}+\frac{12abc^{2}dx^{\frac{13}{2}}}{13}+\frac{12abcd^{2}x^{\frac{17}{2}}}{17}+\frac{4abd^{3}x^{\frac{21}{2}}}{21}+\frac{2b^{2}c^{3}x^{\frac{13}{2}}}{13}+\frac{6b^{2}c^{2}dx^{\frac{17}{2}}}{17}+\frac{2b^{2}cd^{2}x^{\frac{21}{2}}}{7}+\frac{2b^{2}d^{3}x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] 2*a**2*c**3*x**(5/2)/5 + 2*a**2*c**2*d*x**(9/2)/3 + 6*a**2*c*d**2*x**(13/2)
/13 + 2*a**2*d**3*x**(17/2)/17 + 4*a*b*c**3*x**(9/2)/9 + 12*a*b*c**2*d*x**(
13/2)/13 + 12*a*b*c*d**2*x**(17/2)/17 + 4*a*b*d**3*x**(21/2)/21 + 2*b**2*c*
*3*x**(13/2)/13 + 6*b**2*c**2*d*x**(17/2)/17 + 2*b**2*c*d**2*x**(21/2)/7 +
2*b**2*d**3*x**(25/2)/25
```

3.392 $\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$

Optimal. Leaf size=139

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}($$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x^(2
3/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \int \Big(a^2c^3\sqrt{x}+ac^2(2bc+3ad)x^{5/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+d\Big(3b^2c^2+\\ &= \frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2(2bc+3ad)x^{7/2}+\frac{2}{11}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2}+\frac{2}{15}d\Big(3\end{aligned}$$

Mathematica [A] time = 0.04, size = 139, normalized size = 1.00

$$\frac{2}{15}dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{11}cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x^(2
3/2))/23
```

IntegrateAlgebraic [A] time = 0.07, size = 163, normalized size = 1.17

$$\frac{2\left(168245a^2c^3x^{3/2}+216315a^2c^2dx^{7/2}+137655a^2cd^2x^{11/2}+33649a^2d^3x^{15/2}+144210abc^3x^{7/2}+275310abc^2dx^{11/2}+201894abcd^2x^{15/2}+53130abd^3x^{19/2}+45885b^2c^3x^{11/2}+100947b^2c^2dx^{15/2}+79695b^2cd^2x^{19/2}+21945b^2d^3x^{23/2}\right)}{504735}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] (2*(168245*a^2*c^3*x^(3/2) + 144210*a*b*c^3*x^(7/2) + 216315*a^2*c^2*d*x^(7
/2) + 45885*b^2*c^3*x^(11/2) + 275310*a*b*c^2*d*x^(11/2) + 137655*a^2*c*d^2
*x^(11/2) + 100947*b^2*c^2*d*x^(15/2) + 201894*a*b*c*d^2*x^(15/2) + 33649*a
^2*d^3*x^(15/2) + 79695*b^2*c*d^2*x^(19/2) + 53130*a*b*d^3*x^(19/2) + 21945
*b^2*d^3*x^(23/2)))/504735
```

fricas [A] time = 0.85, size = 130, normalized size = 0.94

$$\frac{2}{504735}\left(21945\,b^2d^3x^{11}+26565\left(3\,b^2cd^2+2\,abd^3\right)x^9+33649\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+168245\,a^2c^3x+45885\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+72105\left(2\,abc^3+3\,a^2c^2d\right)x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*b^2*d^3*x^11 + 26565*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 33649*
(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + 168245*a^2*c^3*x + 45885*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 72105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3)*
sqrt(x)
```

giac [A] time = 0.38, size = 135, normalized size = 0.97

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}}+\frac{6}{19}\,b^2cd^2x^{\frac{19}{2}}+\frac{4}{19}\,abd^3x^{\frac{19}{2}}+\frac{2}{5}\,b^2c^2dx^{\frac{15}{2}}+\frac{4}{5}\,abcd^2x^{\frac{15}{2}}+\frac{2}{15}\,a^2d^3x^{\frac{15}{2}}+\frac{2}{11}\,b^2c^3x^{\frac{11}{2}}+\frac{12}{11}\,abc^2dx^{\frac{11}{2}}+\frac{6}{11}\,a^2cd^2x^{\frac{11}{2}}+\frac{4}{7}\,abc^3x^{\frac{7}{2}}+\frac{6}{7}\,a^2c^2dx^{\frac{7}{2}}+\frac{2}{3}\,a^2c^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 6/19*b^2*c*d^2*x^(19/2) + 4/19*a*b*d^3*x^(19/2) + 2
/5*b^2*c^2*d*x^(15/2) + 4/5*a*b*c*d^2*x^(15/2) + 2/15*a^2*d^3*x^(15/2) + 2/
11*b^2*c^3*x^(11/2) + 12/11*a*b*c^2*d*x^(11/2) + 6/11*a^2*c*d^2*x^(11/2) +
4/7*a*b*c^3*x^(7/2) + 6/7*a^2*c^2*d*x^(7/2) + 2/3*a^2*c^3*x^(3/2)
```

maple [A] time = 0.01, size = 138, normalized size = 0.99

$$\frac{2\left(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894abc\,d^2x^6+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+275310ab\,c^2d\,x^4+45885b^2c^3x^4+216315a^2c^2d\,x^2+144210ab\,c^3x^2+168245a^2c^3\right)x^{\frac{3}{2}}}{504735}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x)
```

```
[Out] 2/504735*x^(3/2)*(21945*b^2*d^3*x^10+53130*a*b*d^3*x^8+79695*b^2*c*d^2*x^8+
33649*a^2*d^3*x^6+201894*a*b*c*d^2*x^6+100947*b^2*c^2*d*x^6+137655*a^2*c*d^
2*x^4+275310*a*b*c^2*d*x^4+45885*b^2*c^3*x^4+216315*a^2*c^2*d*x^2+144210*a*
b*c^3*x^2+168245*a^2*c^3)
```

maxima [A] time = 1.03, size = 127, normalized size = 0.91

$$\frac{2}{23}\,b^2d^3x^{\frac{23}{2}}+\frac{2}{19}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{19}{2}}+\frac{2}{15}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{15}{2}}+\frac{2}{3}\,a^2c^3x^{\frac{3}{2}}+\frac{2}{11}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{11}{2}}+\frac{2}{7}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/23*b^2*d^3*x^(23/2) + 2/19*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(19/2) + 2/15*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(15/2) + 2/3*a^2*c^3*x^(3/2) + 2/11*(b^
2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(11/2) + 2/7*(2*a*b*c^3 + 3*a^2*c^2*d)
*x^(7/2)
```

mupad [B] time = 0.04, size = 119, normalized size = 0.86

$$x^{11/2}\left(\frac{6\,a^2\,c\,d^2}{11}+\frac{12\,a\,b\,c^2\,d}{11}+\frac{2\,b^2\,c^3}{11}\right)+x^{15/2}\left(\frac{2\,a^2\,d^3}{15}+\frac{4\,a\,b\,c\,d^2}{5}+\frac{2\,b^2\,c^2\,d}{5}\right)+\frac{2\,a^2\,c^3\,x^{3/2}}{3}+\frac{2\,b^2\,d^3\,x^{23/2}}{23}+\frac{2\,a\,c^2\,x^{7/2}\,\left(3\,a\,d+2\,b\,c\right)}{7}+\frac{2\,b\,d^2\,x^{19/2}\,\left(2\,a\,d+3\,b\,c\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^(11/2)*((2*b^2*c^3)/11 + (6*a^2*c*d^2)/11 + (12*a*b*c^2*d)/11) + x^(15/2)
*((2*a^2*d^3)/15 + (2*b^2*c^2*d)/5 + (4*a*b*c*d^2)/5) + (2*a^2*c^3*x^(3/2))
/3 + (2*b^2*d^3*x^(23/2))/23 + (2*a*c^2*x^(7/2)*(3*a*d + 2*b*c))/7 + (2*b*d
^2*x^(19/2)*(2*a*d + 3*b*c))/19
```

sympy [A] time = 4.36, size = 155, normalized size = 1.12

$$\frac{2a^2c^3x^{\frac{3}{2}}}{3}+\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}\left(2abd^3+3b^2cd^2\right)}{19}+\frac{2x^{\frac{15}{2}}\left(a^2d^3+6abcd^2+3b^2c^2d\right)}{15}+\frac{2x^{\frac{11}{2}}\left(3a^2cd^2+6abc^2d+b^2c^3\right)}{11}+\frac{2x^{\frac{7}{2}}\left(3a^2c^2d+2abc^3\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3*x**(1/2),x)
```

```
[Out] 2*a**2*c**3*x**(3/2)/3 + 2*b**2*d**3*x**(23/2)/23 + 2*x**(19/2)*(2*a*b*d**3
 + 3*b**2*c*d**2)/19 + 2*x**(15/2)*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*
d)/15 + 2*x**(11/2)*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3)/11 + 2*x**(7
/2)*(3*a**2*c**2*d + 2*a*b*c**3)/7
```

3.393 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad$$

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2))/
21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^2c^3}{\sqrt{x}}+ac^2(2bc+3ad)x^{3/2}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+d\left(3b^2c^2+6abcd\right.\right.\\ &= 2a^2c^3\sqrt{x}+\frac{2}{5}ac^2(2bc+3ad)x^{5/2}+\frac{2}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}+\frac{2}{13}d\left(3b^2c^2+\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 137, normalized size = 1.00

$$\frac{2}{13}dx^{13/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{9}cx^{9/2}\left(3a^2d^2+6abcd+b^2c^2\right)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] 2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 + 6
*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x
^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2))/
21
```

IntegrateAlgebraic [A] time = 0.07, size = 163, normalized size = 1.19

$$\frac{2\left(69615a^2c^3\sqrt{x}+41769a^2c^2dx^{5/2}+23205a^2cd^2x^{9/2}+5355a^2d^3x^{13/2}+27846abc^3x^{5/2}+46410abc^2dx^{9/2}+32130abcd^2x^{13/2}+8190abd^3x^{17/2}+7735b^2c^3x^{9/2}+16065b^2c^2dx^{13/2}+12285b^2cd^2x^{17/2}+3315b^2d^3x^{21/2}\right)}{69615}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

```
[Out] (2*(69615*a^2*c^3*Sqrt[x] + 27846*a*b*c^3*x^(5/2) + 41769*a^2*c^2*d*x^(5/2)
 + 7735*b^2*c^3*x^(9/2) + 46410*a*b*c^2*d*x^(9/2) + 23205*a^2*c*d^2*x^(9/2)
 + 16065*b^2*c^2*d*x^(13/2) + 32130*a*b*c*d^2*x^(13/2) + 5355*a^2*d^3*x^(13
/2) + 12285*b^2*c*d^2*x^(17/2) + 8190*a*b*d^3*x^(17/2) + 3315*b^2*d^3*x^(21
/2)))/69615
```

fricas [A] time = 0.82, size = 129, normalized size = 0.94

$$\frac{2}{69615}\left(3315\,b^2d^3x^{10}+4095\left(3\,b^2cd^2+2\,abd^3\right)x^8+5355\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6+69615\,a^2c^3+7735\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+13923\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^2*d^3*x^10 + 4095*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 5355*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + 69615*a^2*c^3 + 7735*(b^2*c^3 + 6*a
*b*c^2*d + 3*a^2*c*d^2)*x^4 + 13923*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt(x)
```

giac [A] time = 0.29, size = 135, normalized size = 0.99

$$\frac{2}{21}\,b^2d^3x^{\frac{21}{2}}+\frac{6}{17}\,b^2cd^2x^{\frac{17}{2}}+\frac{4}{17}\,abd^3x^{\frac{17}{2}}+\frac{6}{13}\,b^2c^2dx^{\frac{13}{2}}+\frac{12}{13}\,abcd^2x^{\frac{13}{2}}+\frac{2}{13}\,a^2d^3x^{\frac{13}{2}}+\frac{2}{9}\,b^2c^3x^{\frac{9}{2}}+\frac{4}{3}\,abc^2dx^{\frac{9}{2}}+\frac{2}{3}\,a^2cd^2x^{\frac{9}{2}}+\frac{4}{5}\,abc^3x^{\frac{5}{2}}+\frac{6}{5}\,a^2c^2dx^{\frac{5}{2}}+2\,a^2c^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 6/17*b^2*c*d^2*x^(17/2) + 4/17*a*b*d^3*x^(17/2) + 6
/13*b^2*c^2*d*x^(13/2) + 12/13*a*b*c*d^2*x^(13/2) + 2/13*a^2*d^3*x^(13/2) +
 2/9*b^2*c^3*x^(9/2) + 4/3*a*b*c^2*d*x^(9/2) + 2/3*a^2*c*d^2*x^(9/2) + 4/5*
a*b*c^3*x^(5/2) + 6/5*a^2*c^2*d*x^(5/2) + 2*a^2*c^3*sqrt(x)
```

maple [A] time = 0.01, size = 138, normalized size = 1.01

$$\frac{2\left(3315b^2d^3x^{10}+8190ab\,d^3x^8+12285b^2c\,d^2x^8+5355a^2d^3x^6+32130abc\,d^2x^6+16065b^2c^2d\,x^6+23205a^2c\,d^2x^4+46410ab\,c^2d\,x^4+7735b^2c^3x^4+41769a^2c^2d\,x^2+27846ab\,c^3x^2+69615a^2c^3\right)\sqrt{x}}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 2/69615*x^(1/2)*(3315*b^2*d^3*x^10+8190*a*b*d^3*x^8+12285*b^2*c*d^2*x^8+535
5*a^2*d^3*x^6+32130*a*b*c*d^2*x^6+16065*b^2*c^2*d*x^6+23205*a^2*c*d^2*x^4+4
6410*a*b*c^2*d*x^4+7735*b^2*c^3*x^4+41769*a^2*c^2*d*x^2+27846*a*b*c^3*x^2+6
9615*a^2*c^3)
```

maxima [A] time = 1.08, size = 127, normalized size = 0.93

$$\frac{2}{21}\,b^{2}d^{3}x^{\frac{21}{2}} + \frac{2}{17}\left(3\,b^{2}cd^{2} + 2\,abd^{3}\right)x^{\frac{17}{2}} + \frac{2}{13}\left(3\,b^{2}c^{2}d + 6\,abcd^{2} + a^{2}d^{3}\right)x^{\frac{13}{2}} + 2\,a^{2}c^{3}\sqrt{x} + \frac{2}{9}\left(b^{2}c^{3} + 6\,abc^{2}d + 3\,a^{2}cd^{2}\right)x^{\frac{9}{2}} + \frac{2}{5}\left(2\,abc^{3} + 3\,a^{2}c^{2}d\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*b^2*d^3*x^(21/2) + 2/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(17/2) + 2/13*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(13/2) + 2*a^2*c^3*sqrt(x) + 2/9*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(9/2) + 2/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(
5/2)
```

mupad [B] time = 0.04, size = 119, normalized size = 0.87

$$x^{9/2}\left(\frac{2\,a^2\,c\,d^2}{3}+\frac{4\,a\,b\,c^2\,d}{3}+\frac{2\,b^2\,c^3}{9}\right)+x^{13/2}\left(\frac{2\,a^2\,d^3}{13}+\frac{12\,a\,b\,c\,d^2}{13}+\frac{6\,b^2\,c^2\,d}{13}\right)+2\,a^2\,c^3\,\sqrt{x}+\frac{2\,b^2\,d^3\,x^{21/2}}{21}+\frac{2\,a\,c^2\,x^{5/2}\,\left(3\,a\,d+2\,b\,c\right)}{5}+\frac{2\,b\,d^2\,x^{17/2}\,\left(2\,a\,d+3\,b\,c\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(1/2),x)
```

```
[Out] x^(9/2)*((2*b^2*c^3)/9 + (2*a^2*c*d^2)/3 + (4*a*b*c^2*d)/3) + x^(13/2)*((2*
a^2*d^3)/13 + (6*b^2*c^2*d)/13 + (12*a*b*c*d^2)/13) + 2*a^2*c^3*x^(1/2) + (
2*b^2*d^3*x^(21/2))/21 + (2*a*c^2*x^(5/2)*(3*a*d + 2*b*c))/5 + (2*b*d^2*x^(
17/2)*(2*a*d + 3*b*c))/17
```

sympy [A] time = 9.68, size = 190, normalized size = 1.39

$$2a^{2}c^{3}\sqrt{x} + \frac{6a^{2}c^{2}dx^{\frac{5}{2}}}{5} + \frac{2a^{2}cd^{2}x^{\frac{9}{2}}}{3} + \frac{2a^{2}d^{3}x^{\frac{13}{2}}}{13} + \frac{4abc^{3}x^{\frac{5}{2}}}{5} + \frac{4abc^{2}dx^{\frac{9}{2}}}{3} + \frac{12abcd^{2}x^{\frac{13}{2}}}{13} + \frac{4abd^{3}x^{\frac{17}{2}}}{17} + \frac{2b^{2}c^{3}x^{\frac{9}{2}}}{9} + \frac{6b^{2}c^{2}dx^{\frac{13}{2}}}{13} + \frac{6b^{2}cd^{2}x^{\frac{17}{2}}}{17} + \frac{2b^{2}d^{3}x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] 2*a**2*c**3*sqrt(x) + 6*a**2*c**2*d*x**(5/2)/5 + 2*a**2*c*d**2*x**(9/2)/3 +
 2*a**2*d**3*x**(13/2)/13 + 4*a*b*c**3*x**(5/2)/5 + 4*a*b*c**2*d*x**(9/2)/3
 + 12*a*b*c*d**2*x**(13/2)/13 + 4*a*b*d**3*x**(17/2)/17 + 2*b**2*c**3*x**(9
/2)/9 + 6*b**2*c**2*d*x**(13/2)/13 + 6*b**2*c*d**2*x**(17/2)/17 + 2*b**2*d*
*3*x**(21/2)/21
```

3.394 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3$$

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(19/2
))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{3/2}}\,dx=\int \Bigg(\frac{a^2c^3}{x^{3/2}}+ac^2(2bc+3ad)\sqrt{x}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5/2}+d\Big(3b^2c^2+6abcd$$

$$=-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2(2bc+3ad)x^{3/2}+\frac{2}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2}+\frac{2}{11}d\Big(3b^2c^2+6$$

Mathematica [A] time = 0.05, size = 137, normalized size = 1.00

$$\frac{2}{11}dx^{11/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{7}cx^{7/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(19/2
))/19
```

IntegrateAlgebraic [A] time = 0.06, size = 141, normalized size = 1.03

$$\frac{2\left(-21945a^2c^3+21945a^2c^2dx^2+9405a^2cd^2x^4+1995a^2d^3x^6+14630abc^3x^2+18810abc^2dx^4+11970abcd^2x^6+2926abd^3x^8+3135b^2c^3x^4+5985b^2c^2dx^6+4389b^2cd^2x^8+1155b^2d^3x^{10}\right)}{21945\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

```
[Out] (2*(-21945*a^2*c^3 + 14630*a*b*c^3*x^2 + 21945*a^2*c^2*d*x^2 + 3135*b^2*c^3
*x^4 + 18810*a*b*c^2*d*x^4 + 9405*a^2*c*d^2*x^4 + 5985*b^2*c^2*d*x^6 + 1197
0*a*b*c*d^2*x^6 + 1995*a^2*d^3*x^6 + 4389*b^2*c*d^2*x^8 + 2926*a*b*d^3*x^8
+ 1155*b^2*d^3*x^10))/(21945*Sqrt[x])
```

fricas [A] time = 1.07, size = 129, normalized size = 0.94

$$\frac{2\left(1155\,b^2d^3x^{10}+1463\left(3\,b^2cd^2+2\,abd^3\right)x^8+1995\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-21945\,a^2c^3+3135\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+7315\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)}{21945\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^2*d^3*x^10 + 1463*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1995*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 21945*a^2*c^3 + 3135*(b^2*c^3 + 6*a
*b*c^2*d + 3*a^2*c*d^2)*x^4 + 7315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/sqrt(x)
```

giac [A] time = 0.31, size = 135, normalized size = 0.99

$$\frac{2}{19}\,b^2d^3x^{\frac{19}{2}}+\frac{2}{5}\,b^2cd^2x^{\frac{15}{2}}+\frac{4}{15}\,abd^3x^{\frac{15}{2}}+\frac{6}{11}\,b^2c^2dx^{\frac{11}{2}}+\frac{12}{11}\,abcd^2x^{\frac{11}{2}}+\frac{2}{11}\,a^2d^3x^{\frac{11}{2}}+\frac{2}{7}\,b^2c^3x^{\frac{7}{2}}+\frac{12}{7}\,abc^2dx^{\frac{7}{2}}+\frac{6}{7}\,a^2cd^2x^{\frac{7}{2}}+\frac{4}{3}\,abc^3x^{\frac{3}{2}}+2\,a^2c^2dx^{\frac{3}{2}}-\frac{2\,a^2c^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/5*b^2*c*d^2*x^(15/2) + 4/15*a*b*d^3*x^(15/2) + 6/
11*b^2*c^2*d*x^(11/2) + 12/11*a*b*c*d^2*x^(11/2) + 2/11*a^2*d^3*x^(11/2) +
2/7*b^2*c^3*x^(7/2) + 12/7*a*b*c^2*d*x^(7/2) + 6/7*a^2*c*d^2*x^(7/2) + 4/3*
a*b*c^3*x^(3/2) + 2*a^2*c^2*d*x^(3/2) - 2*a^2*c^3/sqrt(x)
```

maple [A] time = 0.01, size = 138, normalized size = 1.01

$$-\frac{2\left(-1155b^2d^3x^{10}-2926ab\,d^3x^8-4389b^2c\,d^2x^8-1995a^2d^3x^6-11970abc\,d^2x^6-5985b^2c^2d\,x^6-9405a^2c\,d^2x^4-18810ab\,c^2d\,x^4-3135b^2c^3x^4-21945a^2c^2d\,x^2-14630ab\,c^3x^2+21945a^2c^3\right)}{21945\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x)
```

```
[Out] -2/21945*(-1155*b^2*d^3*x^10-2926*a*b*d^3*x^8-4389*b^2*c*d^2*x^8-1995*a^2*d
^3*x^6-11970*a*b*c*d^2*x^6-5985*b^2*c^2*d*x^6-9405*a^2*c*d^2*x^4-18810*a*b*
c^2*d*x^4-3135*b^2*c^3*x^4-21945*a^2*c^2*d*x^2-14630*a*b*c^3*x^2+21945*a^2*
c^3)/x^(1/2)
```

maxima [A] time = 1.11, size = 127, normalized size = 0.93

$$\frac{2}{19}\,b^2d^3x^{\frac{19}{2}}+\frac{2}{15}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{15}{2}}+\frac{2}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{11}{2}}-\frac{2\,a^2c^3}{\sqrt{x}}+\frac{2}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{7}{2}}+\frac{2}{3}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/19*b^2*d^3*x^(19/2) + 2/15*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(15/2) + 2/11*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(11/2) - 2*a^2*c^3/sqrt(x) + 2/7*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(7/2) + 2/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^(
3/2)
```

mupad [B] time = 0.04, size = 119, normalized size = 0.87

$$x^{7/2}\left(\frac{6\,a^2\,c\,d^2}{7}+\frac{12\,a\,b\,c^2\,d}{7}+\frac{2\,b^2\,c^3}{7}\right)+x^{11/2}\left(\frac{2\,a^2\,d^3}{11}+\frac{12\,a\,b\,c\,d^2}{11}+\frac{6\,b^2\,c^2\,d}{11}\right)-\frac{2\,a^2\,c^3}{\sqrt{x}}+\frac{2\,b^2\,d^3\,x^{19/2}}{19}+\frac{2\,a\,c^2\,x^{3/2}\,\left(3\,a\,d+2\,b\,c\right)}{3}+\frac{2\,b\,d^2\,x^{15/2}\,\left(2\,a\,d+3\,b\,c\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x)
```

```
[Out] x^(7/2)*((2*b^2*c^3)/7 + (6*a^2*c*d^2)/7 + (12*a*b*c^2*d)/7) + x^(11/2)*((2
*a^2*d^3)/11 + (6*b^2*c^2*d)/11 + (12*a*b*c*d^2)/11) - (2*a^2*c^3)/x^(1/2)
+ (2*b^2*d^3*x^(19/2))/19 + (2*a*c^2*x^(3/2)*(3*a*d + 2*b*c))/3 + (2*b*d^2*
x^(15/2)*(2*a*d + 3*b*c))/15
```

sympy [A] time = 10.56, size = 189, normalized size = 1.38

$$-\frac{2a^2c^3}{\sqrt{x}}+2a^2c^2dx^{\frac{3}{2}}+\frac{6a^2cd^2x^{\frac{7}{2}}}{7}+\frac{2a^2d^3x^{\frac{11}{2}}}{11}+\frac{4abc^3x^{\frac{3}{2}}}{3}+\frac{12abc^2dx^{\frac{7}{2}}}{7}+\frac{12abcd^2x^{\frac{11}{2}}}{11}+\frac{4abd^3x^{\frac{15}{2}}}{15}+\frac{2b^2c^3x^{\frac{7}{2}}}{7}+\frac{6b^2c^2dx^{\frac{11}{2}}}{11}+\frac{2b^2cd^2x^{\frac{15}{2}}}{5}+\frac{2b^2d^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(3/2),x)
```

```
[Out] -2*a**2*c**3/sqrt(x) + 2*a**2*c**2*d*x**(3/2) + 6*a**2*c*d**2*x**(7/2)/7 +
2*a**2*d**3*x**(11/2)/11 + 4*a*b*c**3*x**(3/2)/3 + 12*a*b*c**2*d*x**(7/2)/7
 + 12*a*b*c*d**2*x**(11/2)/11 + 4*a*b*d**3*x**(15/2)/15 + 2*b**2*c**3*x**(7
/2)/7 + 6*b**2*c**2*d*x**(11/2)/11 + 2*b**2*c*d**2*x**(15/2)/5 + 2*b**2*d**
3*x**(19/2)/19
```

3.395 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}\,(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc$$

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}\,(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/2))
/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int\frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{5/2}}\,dx&=\int\left(\frac{a^2c^3}{x^{5/2}}+\frac{ac^2(2bc+3ad)}{\sqrt{x}}+c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2}+d\left(3b^2c^2+6abcd+\right.\right.\\&=-\frac{2a^2c^3}{3x^{3/2}}+2ac^2(2bc+3ad)\sqrt{x}+\frac{2}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{5/2}+\frac{2}{9}d\left(3b^2c^2+6a\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 137, normalized size = 1.00

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}\,(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
+ 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/2))
/17
```

IntegrateAlgebraic [A] time = 0.09, size = 141, normalized size = 1.03

$$\frac{2\left(-3315a^2c^3+29835a^2c^2dx^2+5967a^2cd^2x^4+1105a^2d^3x^6+19890abc^3x^2+11934abc^2dx^4+6630abcd^2x^6+1530abd^3x^8+1989b^2c^3x^4+3315b^2c^2dx^6+2295b^2cd^2x^8+585b^2d^3x^{10}\right)}{9945x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

```
[Out] (2*(-3315*a^2*c^3 + 19890*a*b*c^3*x^2 + 29835*a^2*c^2*d*x^2 + 1989*b^2*c^3*
x^4 + 11934*a*b*c^2*d*x^4 + 5967*a^2*c*d^2*x^4 + 3315*b^2*c^2*d*x^6 + 6630*
a*b*c*d^2*x^6 + 1105*a^2*d^3*x^6 + 2295*b^2*c*d^2*x^8 + 1530*a*b*d^3*x^8 +
585*b^2*d^3*x^10))/(9945*x^(3/2))
```

fricas [A] time = 1.28, size = 129, normalized size = 0.94

$$\frac{2\left(585\,b^2d^3x^{10}+765\left(3\,b^2cd^2+2\,abd^3\right)x^8+1105\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-3315\,a^2c^3+1989\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4+9945\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)}{9945\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*b^2*d^3*x^10 + 765*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1105*(3*b^2*
c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 3315*a^2*c^3 + 1989*(b^2*c^3 + 6*a*b*c
^2*d + 3*a^2*c*d^2)*x^4 + 9945*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(3/2)
```

giac [A] time = 0.45, size = 135, normalized size = 0.99

$$\frac{2}{17}\,b^2d^3x^{\frac{17}{2}}+\frac{6}{13}\,b^2cd^2x^{\frac{13}{2}}+\frac{4}{13}\,abd^3x^{\frac{13}{2}}+\frac{2}{3}\,b^2c^2dx^{\frac{9}{2}}+\frac{4}{3}\,abcd^2x^{\frac{9}{2}}+\frac{2}{9}\,a^2d^3x^{\frac{9}{2}}+\frac{2}{5}\,b^2c^3x^{\frac{5}{2}}+\frac{12}{5}\,abc^2dx^{\frac{5}{2}}+\frac{6}{5}\,a^2cd^2x^{\frac{5}{2}}+4\,abc^3\sqrt{x}+6\,a^2c^2d\sqrt{x}-\frac{2\,a^2c^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 6/13*b^2*c*d^2*x^(13/2) + 4/13*a*b*d^3*x^(13/2) + 2
/3*b^2*c^2*d*x^(9/2) + 4/3*a*b*c*d^2*x^(9/2) + 2/9*a^2*d^3*x^(9/2) + 2/5*b^
2*c^3*x^(5/2) + 12/5*a*b*c^2*d*x^(5/2) + 6/5*a^2*c*d^2*x^(5/2) + 4*a*b*c^3*
sqrt(x) + 6*a^2*c^2*d*sqrt(x) - 2/3*a^2*c^3/x^(3/2)
```

maple [A] time = 0.01, size = 138, normalized size = 1.01

$$-\frac{2 \left(-585 b^{2} d^{3} x^{10}-1530 a b \,d^{3} x^{8}-2295 b^{2} c \,d^{2} x^{8}-1105 a^{2} d^{3} x^{6}-6630 a b c \,d^{2} x^{6}-3315 b^{2} c^{2} d \,x^{6}-5967 a^{2} c \,d^{2} x^{4}-11934 a b \,c^{2} d \,x^{4}-1989 b^{2} c^{3} x^{4}-29835 a^{2} c^{2} d \,x^{2}-19890 a b \,c^{3} x^{2}+3315 a^{2} c^{3}\right)}{9945 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x)
```

```
[Out] -2/9945*(-585*b^2*d^3*x^10-1530*a*b*d^3*x^8-2295*b^2*c*d^2*x^8-1105*a^2*d^3
*x^6-6630*a*b*c*d^2*x^6-3315*b^2*c^2*d*x^6-5967*a^2*c*d^2*x^4-11934*a*b*c^2
*d*x^4-1989*b^2*c^3*x^4-29835*a^2*c^2*d*x^2-19890*a*b*c^3*x^2+3315*a^2*c^3)
/x^(3/2)
```

maxima [A] time = 1.12, size = 127, normalized size = 0.93

$$\frac{2}{17} \, b^{2} d^{3} x^{\frac{17}{2}} + \frac{2}{13} \, {\left(3 \, b^{2} c d^{2} + 2 \, a b d^{3}\right)} x^{\frac{13}{2}} + \frac{2}{9} \, {\left(3 \, b^{2} c^{2} d + 6 \, a b c d^{2} + a^{2} d^{3}\right)} x^{\frac{9}{2}} - \frac{2 \, a^{2} c^{3}}{3 \, x^{\frac{3}{2}}} + \frac{2}{5} \, {\left(b^{2} c^{3} + 6 \, a b c^{2} d + 3 \, a^{2} c d^{2}\right)} x^{\frac{5}{2}} + 2 \, {\left(2 \, a b c^{3} + 3 \, a^{2} c^{2} d\right)} \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/17*b^2*d^3*x^(17/2) + 2/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(13/2) + 2/9*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(9/2) - 2/3*a^2*c^3/x^(3/2) + 2/5*(b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(5/2) + 2*(2*a*b*c^3 + 3*a^2*c^2*d)*sqrt(
x)
```

mupad [B] time = 0.04, size = 119, normalized size = 0.87

$$x^{5/2}\,\left(\frac{6\,a^2\,c\,d^2}{5}+\frac{12\,a\,b\,c^2\,d}{5}+\frac{2\,b^2\,c^3}{5}\right)+x^{9/2}\,\left(\frac{2\,a^2\,d^3}{9}+\frac{4\,a\,b\,c\,d^2}{3}+\frac{2\,b^2\,c^2\,d}{3}\right)-\frac{2\,a^2\,c^3}{3\,x^{3/2}}+\frac{2\,b^2\,d^3\,x^{17/2}}{17}+2\,a\,c^2\,\sqrt{x}\,\left(3\,a\,d+2\,b\,c\right)+\frac{2\,b\,d^2\,x^{13/2}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x)
```

```
[Out] x^(5/2)*((2*b^2*c^3)/5 + (6*a^2*c*d^2)/5 + (12*a*b*c^2*d)/5) + x^(9/2)*((2*
a^2*d^3)/9 + (2*b^2*c^2*d)/3 + (4*a*b*c*d^2)/3) - (2*a^2*c^3)/(3*x^(3/2)) +
 (2*b^2*d^3*x^(17/2))/17 + 2*a*c^2*x^(1/2)*(3*a*d + 2*b*c) + (2*b*d^2*x^(13
/2)*(2*a*d + 3*b*c))/13
```

sympy [A] time = 12.05, size = 189, normalized size = 1.38

$$- \frac{2 a^{2} c^{3}}{3 x^{\frac{3}{2}}} + 6 a^{2} c^{2} d \sqrt{x} + \frac{6 a^{2} c d^{2} x^{\frac{5}{2}}}{5} + \frac{2 a^{2} d^{3} x^{\frac{9}{2}}}{9} + 4 a b c^{3} \sqrt{x} + \frac{12 a b c^{2} d x^{\frac{5}{2}}}{5} + \frac{4 a b c d^{2} x^{\frac{9}{2}}}{3} + \frac{4 a b d^{3} x^{\frac{13}{2}}}{13} + \frac{2 b^{2} c^{3} x^{\frac{5}{2}}}{5} + \frac{2 b^{2} c^{2} d x^{\frac{9}{2}}}{3} + \frac{6 b^{2} c d^{2} x^{\frac{13}{2}}}{13} + \frac{2 b^{2} d^{3} x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(5/2),x)
```

```
[Out] -2*a**2*c**3/(3*x**(3/2)) + 6*a**2*c**2*d*sqrt(x) + 6*a**2*c*d**2*x**(5/2)/
5 + 2*a**2*d**3*x**(9/2)/9 + 4*a*b*c**3*sqrt(x) + 12*a*b*c**2*d*x**(5/2)/5
+ 4*a*b*c*d**2*x**(9/2)/3 + 4*a*b*d**3*x**(13/2)/13 + 2*b**2*c**3*x**(5/2)/
5 + 2*b**2*c**2*d*x**(9/2)/3 + 6*b**2*c*d**2*x**(13/2)/13 + 2*b**2*d**3*x**
(17/2)/17
```

3.396 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)-$$

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {448}

$$\frac{2}{7}dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{3}cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}b^2d^3x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(11/2))/11 + (2*b^2*d^3*x^(15/2
))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^{7/2}}\,dx &= \int \left(\frac{a^2c^3}{x^{7/2}}+\frac{ac^2(2bc+3ad)}{x^{3/2}}+c\left(b^2c^2+6abcd+3a^2d^2\right)\sqrt{x}+d\left(3b^2c^2+6abcd+\right.\right.\\ &= -\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(2bc+3ad)}{\sqrt{x}}+\frac{2}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2}+\frac{2}{7}d\left(3b^2c^2+6abc\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 121, normalized size = 0.88

$$\frac{2\left(165dx^6\left(a^2d^2+6abcd+3b^2c^2\right)+385cx^4\left(3a^2d^2+6abcd+b^2c^2\right)-231a^2c^3-1155ac^2x^2(3ad+2bc)+105bd^2x^8(2ad+3bc)+77b^2d^3x^{10}\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^3 - 1155*a*c^2*(2*b*c + 3*a*d)*x^2 + 385*c*(b^2*c^2 + 6*a*b*
c*d + 3*a^2*d^2)*x^4 + 165*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 105*b*
d^2*(3*b*c + 2*a*d)*x^8 + 77*b^2*d^3*x^10))/(1155*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 141, normalized size = 1.03

$$\frac{2\left(-231a^2c^3-3465a^2c^2dx^2+1155a^2cd^2x^4+165a^2d^3x^6-2310abc^3x^2+2310abc^2dx^4+990abcd^2x^6+210abd^3x^8+385b^2c^3x^4+495b^2c^2dx^6+315b^2cd^2x^8+77b^2d^3x^{10}\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

```
[Out] (2*(-231*a^2*c^3 - 2310*a*b*c^3*x^2 - 3465*a^2*c^2*d*x^2 + 385*b^2*c^3*x^4
+ 2310*a*b*c^2*d*x^4 + 1155*a^2*c*d^2*x^4 + 495*b^2*c^2*d*x^6 + 990*a*b*c*d
^2*x^6 + 165*a^2*d^3*x^6 + 315*b^2*c*d^2*x^8 + 210*a*b*d^3*x^8 + 77*b^2*d^3
*x^10))/(1155*x^(5/2))
```

fricas [A] time = 1.03, size = 129, normalized size = 0.94

$$\frac{2\left(77\,b^2d^3x^{10}+105\left(3\,b^2cd^2+2\,abd^3\right)x^8+165\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^6-231\,a^2c^3+385\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^4-1155\left(2\,abc^3+3\,a^2c^2d\right)x^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^3*x^10 + 105*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 165*(3*b^2*c^
2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 231*a^2*c^3 + 385*(b^2*c^3 + 6*a*b*c^2*d
 + 3*a^2*c*d^2)*x^4 - 1155*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

giac [A] time = 0.34, size = 137, normalized size = 1.00

$$\frac{2}{15}\,b^2d^3x^{\frac{15}{2}}+\frac{6}{11}\,b^2cd^2x^{\frac{11}{2}}+\frac{4}{11}\,abd^3x^{\frac{11}{2}}+\frac{6}{7}\,b^2c^2dx^{\frac{7}{2}}+\frac{12}{7}\,abcd^2x^{\frac{7}{2}}+\frac{2}{7}\,a^2d^3x^{\frac{7}{2}}+\frac{2}{3}\,b^2c^3x^{\frac{3}{2}}+4\,abc^2dx^{\frac{3}{2}}+2\,a^2cd^2x^{\frac{3}{2}}-\frac{2\left(10\,abc^3x^2+15\,a^2c^2dx^2+a^2c^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 6/11*b^2*c*d^2*x^(11/2) + 4/11*a*b*d^3*x^(11/2) + 6
/7*b^2*c^2*d*x^(7/2) + 12/7*a*b*c*d^2*x^(7/2) + 2/7*a^2*d^3*x^(7/2) + 2/3*b
^2*c^3*x^(3/2) + 4*a*b*c^2*d*x^(3/2) + 2*a^2*c*d^2*x^(3/2) - 2/5*(10*a*b*c^
3*x^2 + 15*a^2*c^2*d*x^2 + a^2*c^3)/x^(5/2)
```

maple [A]   time = 0.01, size = 138, normalized size = 1.01

$$-\frac{2\left(-77b^{2}d^{3}x^{10}-210ab\,d^{3}x^{8}-315b^{2}c\,d^{2}x^{8}-165a^{2}d^{3}x^{6}-990abc\,d^{2}x^{6}-495b^{2}c^{2}d\,x^{6}-1155a^{2}c\,d^{2}x^{4}-2310ab\,c^{2}d\,x^{4}-385b^{2}c^{3}x^{4}+3465a^{2}c^{2}d\,x^{2}+2310ab\,c^{3}x^{2}+231a^{2}c^{3}\right)}{1155x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x)
```

```
[Out] -2/1155*(-77*b^2*d^3*x^10-210*a*b*d^3*x^8-315*b^2*c*d^2*x^8-165*a^2*d^3*x^6
-990*a*b*c*d^2*x^6-495*b^2*c^2*d*x^6-1155*a^2*c*d^2*x^4-2310*a*b*c^2*d*x^4-
385*b^2*c^3*x^4+3465*a^2*c^2*d*x^2+2310*a*b*c^3*x^2+231*a^2*c^3)/x^(5/2)
```

maxima [A]   time = 1.08, size = 129, normalized size = 0.94

$$\frac{2}{15}\,b^{2}d^{3}x^{\frac{15}{2}}+\frac{2}{11}\left(3\,b^{2}cd^{2}+2\,abd^{3}\right)x^{\frac{11}{2}}+\frac{2}{7}\left(3\,b^{2}c^{2}d+6\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{7}{2}}+\frac{2}{3}\left(b^{2}c^{3}+6\,abc^{2}d+3\,a^{2}cd^{2}\right)x^{\frac{3}{2}}-\frac{2\left(a^{2}c^{3}+5\left(2\,abc^{3}+3\,a^{2}c^{2}d\right)x^{2}\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*b^2*d^3*x^(15/2) + 2/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(11/2) + 2/7*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(7/2) + 2/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^(3/2) - 2/5*(a^2*c^3 + 5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

mupad [B]   time = 0.04, size = 125, normalized size = 0.91

$$x^{3/2}\,\left(2\,a^2\,c\,d^2+4\,a\,b\,c^2\,d+\frac{2\,b^2\,c^3}{3}\right)+x^{7/2}\,\left(\frac{2\,a^2\,d^3}{7}+\frac{12\,a\,b\,c\,d^2}{7}+\frac{6\,b^2\,c^2\,d}{7}\right)-\frac{x^2\,\left(6\,d\,a^2\,c^2+4\,b\,a\,c^3\right)+\frac{2\,a^2\,c^3}{5}}{x^{5/2}}+\frac{2\,b^2\,d^3\,x^{15/2}}{15}+\frac{2\,b\,d^2\,x^{11/2}\,\left(2\,a\,d+3\,b\,c\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x)
```

```
[Out] x^(3/2)*((2*b^2*c^3)/3 + 2*a^2*c*d^2 + 4*a*b*c^2*d) + x^(7/2)*((2*a^2*d^3)/
7 + (6*b^2*c^2*d)/7 + (12*a*b*c*d^2)/7) - (x^2*(6*a^2*c^2*d + 4*a*b*c^3) +
(2*a^2*c^3)/5)/x^(5/2) + (2*b^2*d^3*x^(15/2))/15 + (2*b*d^2*x^(11/2)*(2*a*d
 + 3*b*c))/11
```

sympy [A]   time = 15.27, size = 185, normalized size = 1.35

$$-\frac{2a^{2}c^{3}}{5x^{\frac{5}{2}}}-\frac{6a^{2}c^{2}d}{\sqrt{x}}+2a^{2}cd^{2}x^{\frac{3}{2}}+\frac{2a^{2}d^{3}x^{\frac{7}{2}}}{7}-\frac{4abc^{3}}{\sqrt{x}}+4abc^{2}dx^{\frac{3}{2}}+\frac{12abcd^{2}x^{\frac{7}{2}}}{7}+\frac{4abd^{3}x^{\frac{11}{2}}}{11}+\frac{2b^{2}c^{3}x^{\frac{3}{2}}}{3}+\frac{6b^{2}c^{2}dx^{\frac{7}{2}}}{7}+\frac{6b^{2}cd^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{2}d^{3}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(7/2),x)
```

```
[Out] -2*a**2*c**3/(5*x**(5/2)) - 6*a**2*c**2*d/sqrt(x) + 2*a**2*c*d**2*x**(3/2)
+ 2*a**2*d**3*x**(7/2)/7 - 4*a*b*c**3/sqrt(x) + 4*a*b*c**2*d*x**(3/2) + 12*
a*b*c*d**2*x**(7/2)/7 + 4*a*b*d**3*x**(11/2)/11 + 2*b**2*c**3*x**(3/2)/3 +
6*b**2*c**2*d*x**(7/2)/7 + 6*b**2*c*d**2*x**(11/2)/11 + 2*b**2*d**3*x**(15/
2)/15
```

3.397 $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=311

$$-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{17/4}}-\frac{c^{5/4}(bc-$$

Rubi [A] time = 0.31, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{17/4}}-\frac{2bx^{9/2}(bc-2ad)}{9d^2}+\frac{2x^{5/2}(bc-ad)^2}{5d^3}-\frac{2c\sqrt{x}(bc-ad)^2}{d^4}+\frac{2b^2x^{13/2}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d^3) - (2*b
*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*d) - (c^(5/4)*(b*c -
 a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)) +
 (c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqr
t[2]*d^(17/4)) - (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx &= \int \left(-\frac{b(bc-2ad)x^{7/2}}{d^2}+\frac{b^2x^{11/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^{7/2}}{d^2\left(c+dx^2\right)}\right)dx \\
&= -\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{(bc-ad)^2\int \frac{x^{7/2}}{c+dx^2}\,dx}{d^2} \\
&= \frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{\left(c(bc-ad)^2\right)\int \frac{x^{3/2}}{c+dx^2}\,dx}{d^3} \\
&= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(c^2(bc-ad)^2\right)\int -}{d^4} \\
&= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(2c^2(bc-ad)^2\right)\mathrm{Su}}{} \\
&= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(c^{3/2}(bc-ad)^2\right)\mathrm{Su}}{} \\
&= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}+\frac{\left(c^{3/2}(bc-ad)^2\right)\mathrm{Su}}{} \\
&= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad)^2\log\left(\right.}{} \\
&= -\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3}-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}}{\sqrt{2}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 299, normalized size = 0.96

$$
\frac{-585\sqrt{2}\,c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+585\sqrt{2}\,c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-1170\sqrt{2}\,c^{5/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+1170\sqrt{2}\,c^{5/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-520bd^{9/4}x^{9/2}(bc-2ad)+936d^{5/4}x^{5/2}(bc-ad)^2-4680c\sqrt[4]{d}\sqrt{x}(bc-ad)^2+360b^2d^{13/4}x^{13/2}}{2340d^{17/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]

[Out] (-4680*c*d^(1/4)*(b*c - a*d)^2*Sqrt[x] + 936*d^(5/4)*(b*c - a*d)^2*x^(5/2)
- 520*b*d^(9/4)*(b*c - 2*a*d)*x^(9/2) + 360*b^2*d^(13/4)*x^(13/2) - 1170*Sq
```

```
rt[2]*c^(5/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] +
 1170*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)] - 585*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x] + 585*Sqrt[2]*c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2340*d^(17/4))
```

IntegrateAlgebraic [A] time = 0.26, size = 238, normalized size = 0.77

$$\frac{2\sqrt{x}\left(-585a^2cd^2+117a^2d^3x^2+1170abc^2d-234abcd^2x^2+130abd^3x^4-585b^2c^3+117b^2c^2dx^2-65b^2cd^2x^4+45b^2d^3x^6\right)}{585d^4}-\frac{c^{5/4}(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*Sqrt[x]*(-585*b^2*c^3 + 1170*a*b*c^2*d - 585*a^2*c*d^2 + 117*b^2*c^2*d*x
^2 - 234*a*b*c*d^2*x^2 + 117*a^2*d^3*x^2 - 65*b^2*c*d^2*x^4 + 130*a*b*d^3*x
^4 + 45*b^2*d^3*x^6))/(585*d^4) - (c^(5/4)*(b*c - a*d)^2*ArcTan[(c^(1/4)/(S
qrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*d^(17/4
)) + (c^(5/4)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(17/4))
```

fricas [B] time = 1.41, size = 1334, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*arctan((sqrt(d^8*sqrt(-(b^8
*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4
*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 +
a^8*c^5*d^8)/d^17) + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b
*c^3*d^3 + a^4*c^2*d^4)*x)*d^13*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c
^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 2
8*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(3/4) - (b^2*c^3*d
^13 - 2*a*b*c^2*d^14 + a^2*c*d^15)*sqrt(x)*(-(b^8*c^13 - 8*a*b^7*c^12*d + 2
8*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*
c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(3/4))/
(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70
*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8)) + 585*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*
d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^
6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(d^4*(-(b^8*c
```

```
^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b
^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^
8*c^5*d^8)/d^17)^(1/4) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) - 585
*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d
^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(-d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d +
 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^
3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)
 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) + 4*(45*b^2*d^3*x^6 - 585*b
^2*c^3 + 1170*a*b*c^2*d - 585*a^2*c*d^2 - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^4 +
117*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/d^4
```

giac [**A**] time = 0.46, size = 436, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{3}-2\left(cd^{3}\right)^{\frac{1}{4}}abc^{2}d+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}cd^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{5}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{3}-2\left(cd^{3}\right)^{\frac{1}{4}}abc^{2}d+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}cd^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{5}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{3}-2\left(cd^{3}\right)^{\frac{1}{4}}abc^{2}d+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}cd^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{5}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{3}-2\left(cd^{3}\right)^{\frac{1}{4}}abc^{2}d+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}cd^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{5}}+\frac{2\left(45b^{2}d^{12}x^{\frac{13}{2}}-65b^{2}cd^{11}x^{\frac{9}{2}}+130abd^{12}x^{\frac{9}{2}}+117b^{2}c^{2}d^{10}x^{\frac{5}{2}}-234abcd^{11}x^{\frac{5}{2}}+117a^{2}d^{12}x^{\frac{5}{2}}-585b^{2}c^{3}d^{9}\sqrt{x}+1170abc^{2}d^{10}\sqrt{x}-585a^{2}cd^{11}\sqrt{x}\right)}{585d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(1
/4)*a^2*c*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(
1/4))/d^5 + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d
+ (c*d^3)^(1/4)*a^2*c*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/d^5 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/
4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/d^5 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*
b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/d^5 + 2/585*(45*b^2*d^12*x^(13/2) - 65*b^2*c*d^11*x^(9/2) + 130*a
*b*d^12*x^(9/2) + 117*b^2*c^2*d^10*x^(5/2) - 234*a*b*c*d^11*x^(5/2) + 117*a
^2*d^12*x^(5/2) - 585*b^2*c^3*d^9*sqrt(x) + 1170*a*b*c^2*d^10*sqrt(x) - 585
*a^2*c*d^11*sqrt(x))/d^13
```

maple [**B**] time = 0.02, size = 545, normalized size = 1.75

$$\frac{2b^{2}x^{\frac{13}{2}}}{13d}+\frac{4abx^{\frac{9}{2}}}{9d}-\frac{2b^{2}cx^{\frac{9}{2}}}{9d^{2}}+\frac{2a^{2}x^{\frac{5}{2}}}{5d}-\frac{4abcx^{\frac{5}{2}}}{5d^{2}}+\frac{2b^{2}c^{2}x^{\frac{5}{2}}}{5d^{3}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}c\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2d^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}c\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2d^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}c\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4d^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc^{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{d^{3}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc^{2}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{d^{3}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2d^{3}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{3}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2d^{4}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{3}\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2d^{4}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{3}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4d^{4}}-\frac{2a^{2}c\sqrt{x}}{d^{2}}+\frac{4abc^{2}\sqrt{x}}{d^{3}}-\frac{2b^{2}c^{3}\sqrt{x}}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/13*b^2*x^(13/2)/d+4/9/d*x^(9/2)*a*b-2/9/d^2*x^(9/2)*b^2*c+2/5/d*x^(5/2)*a
^2-4/5/d^2*x^(5/2)*a*b*c+2/5/d^3*x^(5/2)*b^2*c^2-2/d^2*a^2*c*x^(1/2)+4/d^3*
a*b*c^2*x^(1/2)-2/d^4*b^2*c^3*x^(1/2)+1/4*c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))*a^2-1/2*c^2/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(
```

```
1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*a*b+1/4*c^3/
d^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))*b^2+1/2*c/d^2*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-c^2/d^3*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2*c^3/d^4*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/2*c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-c^2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)*a*b+1/2*c^3/d^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)*b^2
```

maxima [A] time = 2.44, size = 360, normalized size = 1.16

$$\frac{\left(\frac{2 \sqrt{2} \left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{2 \sqrt{2} \left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{\sqrt{2} \left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}-\frac{\sqrt{2} \left(b^{2} c^{2}-2 \, abcd+a^{2} d^{2}\right) \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}\right) c^{2}}{4 \, d^{4}}+\frac{2 \, \left(45 \, b^{2} d^{3} x^{\frac{13}{2}}-65 \, \left(b^{2} c d^{2}-2 \, a b d^{3}\right) x^{\frac{9}{2}}+117 \, \left(b^{2} c^{2} d-2 \, a b c d^{2}+a^{2} d^{3}\right) x^{\frac{5}{2}}-585 \, \left(b^{2} c^{3}-2 \, a b c^{2} d+a^{2} c d^{2}\right) \sqrt{x}\right)}{585 \, d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/
(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))
 - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c^2/d^4 + 2/585*(45*b^2*d^3*
x^(13/2) - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^(9/2) + 117*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*x^(5/2) - 585*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x))/d^4
```

mupad [B] time = 0.38, size = 1202, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^(5/2)*((2*a^2)/(5*d) + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/(5*d)) - x^(9/2)*(
(2*b^2*c)/(9*d^2) - (4*a*b)/(9*d)) + (2*b^2*x^(13/2))/(13*d) - (c*x^(1/2)*(
(2*a^2)/d + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/d))/d + ((-c)^(5/4)*atan((((-c)
^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3
+ 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - (16*(-c)^(5/4)*(a*d - b*c)^2*(b
^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4))*1i)/(2*d^(17/4)) + ((-c)^(5/
4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*
a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c
^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4))*1i)/(2*d^(17/4)))/(((-c)^(5/4)*(
```

```
a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*
b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 +
 a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4)))/(2*d^(17/4)) - ((-c)^(5/4)*(a*d - b
*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6
*d^2 - 4*a*b^3*c^7*d))/d^5 + (16*(-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^
3*d^2 - 2*a*b*c^4*d))/d^(21/4)))/(2*d^(17/4))))*(a*d - b*c)^2*1i)/d^(17/4)
+ ((-c)^(5/4)*atan((((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c
^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - ((-c)^
(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4)))/(
2*d^(17/4)) + ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4
 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + ((-c)^(5/4)*
(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4)))/(2*d^(1
7/4)))/(((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a
^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - ((-c)^(5/4)*(a*d -
 b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4))*1i)/(2*d^(17/4
)) - ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*
b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + ((-c)^(5/4)*(a*d - b*
c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)*16i)/d^(21/4))*1i)/(2*d^(17/4)))
)*(a*d - b*c)^2)/d^(17/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.398 $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=290

$$-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{15/4}}+\frac{c^{3/4}(bc-a}{}$$

Rubi [A] time = 0.25, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{15/4}}-\frac{2bx^{7/2}(bc-2ad)}{7d^2}+\frac{2x^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2x^{11/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(7*d^2) + (
2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*
d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]
*d^(15/4)) + (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{5/2}}{d^2}+\frac{b^2x^{9/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^{5/2}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{(bc-ad)^2\int\frac{x^{5/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{\left(c(bc-ad)^2\right)\int\frac{\sqrt{x}}{c+dx^2}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{\left(2c(bc-ad)^2\right)\mathrm{Subst}\left(\int\frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{\left(c(bc-ad)^2\right)\mathrm{Subst}\bigl(\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{}}{d^{7/2}}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{\left(c(bc-ad)^2\right)\mathrm{Subst}\Biggl(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,}{2d^4}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}-\frac{c^{3/4}(bc-ad)^2\log\bigl(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+}{2\sqrt{2}\,d^{15/4}}$$

$$= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{15/4}}-\frac{c^{3/}}{}$$

Mathematica [A] time = 0.11, size = 276, normalized size = 0.95

$$\frac{-231\sqrt{2}c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+231\sqrt{2}c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+462\sqrt{2}c^{3/4}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-462\sqrt{2}c^{3/4}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-264bd^{7/4}x^{7/2}(bc-2ad)+616d^{3/4}x^{3/2}(bc-ad)^2+168b^2d^{11/4}x^{11/2}}{924d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (616*d^(3/4)*(b*c - a*d)^2*x^(3/2) - 264*b*d^(7/4)*(b*c - 2*a*d)*x^(7/2) +
168*b^2*d^(11/4)*x^(11/2) + 462*Sqrt[2]*c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 462*Sqrt[2]*c^(3/4)*(b*c - a*d)^2*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 231*Sqrt[2]*c^(3/4)*(b*c - a*d)^2
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + 231*Sqrt[2]*c
```

```
^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(924*d^(15/4))
```

IntegrateAlgebraic [A] time = 0.24, size = 198, normalized size = 0.68

$$\frac{2x^{3/2}\left(77a^2d^2-154abcd+66abd^2x^2+77b^2c^2-33b^2cdx^2+21b^2d^2x^4\right)}{231d^3}+\frac{c^{3/4}(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*x^(3/2)*(77*b^2*c^2 - 154*a*b*c*d + 77*a^2*d^2 - 33*b^2*c*d*x^2 + 66*a*b
*d^2*x^2 + 21*b^2*d^2*x^4))/(231*d^3) + (c^(3/4)*(b*c - a*d)^2*ArcTan[(c^(1
/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*d
^(15/4)) + (c^(3/4)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])
/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(15/4))
```

fricas [B] time = 1.21, size = 1701, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/462*(924*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*arctan((sqrt((b^12*c^16 - 12*a*
b^11*c^15*d + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^1
2*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 +
 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a^11
*b*c^5*d^11 + a^12*c^4*d^12)*x - (b^8*c^11*d^7 - 8*a*b^7*c^10*d^8 + 28*a^2*
b^6*c^9*d^9 - 56*a^3*b^5*c^8*d^10 + 70*a^4*b^4*c^7*d^11 - 56*a^5*b^3*c^6*d^
12 + 28*a^6*b^2*c^5*d^13 - 8*a^7*b*c^4*d^14 + a^8*c^3*d^15)*sqrt(-(b^8*c^11
 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^
7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3
*d^8)/d^15))*d^4*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3
*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6
 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4) - (b^6*c^8*d^4 - 6*a*b^5*c^7*
d^5 + 15*a^2*b^4*c^6*d^6 - 20*a^3*b^3*c^5*d^7 + 15*a^4*b^2*c^4*d^8 - 6*a^5*
b*c^3*d^9 + a^6*c^2*d^10)*sqrt(x)*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6
*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 2
8*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4))/(b^8*c^11 -
 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*
d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d
```

```
^8)) - 231*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b
^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 -
 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(d^11*(-(b^8*c^11 - 8*a*b^7*
c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*
a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)
^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 231*d^3*(
-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*
a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7
 + a^8*c^3*d^8)/d^15)^(1/4)*log(-d^11*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2
*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5
 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(3/4) + (b^6*c
^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c
^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 4*(21*b^2*d^2*x^5 - 33*(
b^2*c*d - 2*a*b*d^2)*x^3 + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)*sqrt(x))/d
^3
```

giac [A] time = 0.46, size = 385, normalized size = 1.33

$$-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{6}} - \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{6}} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4d^{6}} - \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4d^{6}} + \frac{2\left(21b^{2}d^{10}x^{\frac{11}{2}} - 33b^{2}cd^{9}x^{\frac{7}{2}} + 66abd^{10}x^{\frac{7}{2}} + 77b^{2}c^{2}d^{8}x^{\frac{3}{2}} - 154abcd^{9}x^{\frac{3}{2}} + 77a^{2}d^{10}x^{\frac{3}{2}}\right)}{231d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4
))/d^6 - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/d^6 + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*
c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^6 - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^6 +
 2/231*(21*b^2*d^10*x^(11/2) - 33*b^2*c*d^9*x^(7/2) + 66*a*b*d^10*x^(7/2) +
 77*b^2*c^2*d^8*x^(3/2) - 154*a*b*c*d^9*x^(3/2) + 77*a^2*d^10*x^(3/2))/d^11
```

maple [B] time = 0.01, size = 504, normalized size = 1.74

$$\frac{2b^{2}x^{\frac{11}{2}}}{11d} + \frac{4ab\,x^{\frac{7}{2}}}{7d} - \frac{2b^{2}c\,x^{\frac{7}{2}}}{7d^{2}} + \frac{2a^{2}x^{\frac{3}{2}}}{3d} - \frac{4abc\,x^{\frac{3}{2}}}{3d^{2}} + \frac{2b^{2}c^{2}x^{\frac{3}{2}}}{3d^{3}} - \frac{\sqrt{2}\,a^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}} - \frac{\sqrt{2}\,a^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}} - \frac{\sqrt{2}\,a^{2}c\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}} + \frac{\sqrt{2}\,abc^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}} + \frac{\sqrt{2}\,abc^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}} + \frac{\sqrt{2}\,abc^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}} - \frac{\sqrt{2}\,b^{2}c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{4}} - \frac{\sqrt{2}\,b^{2}c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{4}} - \frac{\sqrt{2}\,b^{2}c^{3}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/11*b^2*x^(11/2)/d+4/7/d*x^(7/2)*a*b-2/7/d^2*x^(7/2)*b^2*c+2/3/d*x^(3/2)*a
^2-4/3/d^2*x^(3/2)*a*b*c+2/3/d^3*x^(3/2)*b^2*c^2-1/2*c/d^2/(c/d)^(1/4)*2^(1
```

```
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+c^2/d^3/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-1/2*c^3/d^4/(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+c^2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-1/2*c^3/d^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4*c/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1
/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)
))*a^2+1/2*c^2/d^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-1/4*c^3/d^4/(c/
d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1
/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.45, size = 263, normalized size = 0.91

$$-\frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}} + \frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}} - \frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,d^3} + \frac{2\left(21\,b^2d^2x^{\frac{11}{2}} - 33\left(b^2cd - 2\,abd^2\right)x^{\frac{7}{2}} + 77\left(b^2c^2 - 2\,abcd + a^2d^2\right)x^{\frac{3}{2}}\right)}{231\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt
(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt
(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(1/4)*d^(3/4)))/d^3 + 2/231*(21*b^2*d^2*x^(11/2) - 33*(b^2*c
*d - 2*a*b*d^2)*x^(7/2) + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2))/d^3
```

mupad [B] time = 0.31, size = 435, normalized size = 1.50

$$x^{3/2}\left(\frac{2\,a^2}{3\,d}+\frac{c\,\left(\frac{2\,b^2\,c}{d^2}-\frac{4\,a\,b}{d}\right)}{3\,d}\right)-x^{7/2}\,\left(\frac{2\,b^2\,c}{7\,d^2}-\frac{4\,a\,b}{7\,d}\right)+\frac{2\,b^2\,x^{11/2}}{11\,d}-\frac{{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{3/4}\,d^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(a^4\,c^3\,d^4-4\,a^3\,b\,c^4\,d^3+6\,a^2\,b^2\,c^5\,d^2-4\,a\,b^3\,c^6\,d+b^4\,c^7\right)}{a^6\,c^4\,d^6-6\,a^5\,b\,c^5\,d^5+15\,a^4\,b^2\,c^6\,d^4-20\,a^3\,b^3\,c^7\,d^3+15\,a^2\,b^4\,c^8\,d^2-6\,a\,b^5\,c^9\,d+b^6\,c^{10}}\right)\,{\left(a\,d-b\,c\right)}^2}{d^{15/4}}-\frac{{\left(-c\right)}^{3/4}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{3/4}\,d^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(a^4\,c^3\,d^4-4\,a^3\,b\,c^4\,d^3+6\,a^2\,b^2\,c^5\,d^2-4\,a\,b^3\,c^6\,d+b^4\,c^7\right)\,1{}\mathrm{i}}{a^6\,c^4\,d^6-6\,a^5\,b\,c^5\,d^5+15\,a^4\,b^2\,c^6\,d^4-20\,a^3\,b^3\,c^7\,d^3+15\,a^2\,b^4\,c^8\,d^2-6\,a\,b^5\,c^9\,d+b^6\,c^{10}}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{d^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^(3/2)*((2*a^2)/(3*d) + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/(3*d)) - x^(7/2)*(
(2*b^2*c)/(7*d^2) - (4*a*b)/(7*d)) + (2*b^2*x^(11/2))/(11*d) - ((-c)^(3/4)*
atan(((-c)^(3/4)*d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^7 + a^4*c^3*d^4 - 4*a
^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))/(b^6*c^10 + a^6*c^4*d^6
- 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^
6*d^4 - 6*a*b^5*c^9*d))*(a*d - b*c)^2)/d^(15/4) - ((-c)^(3/4)*atan(((-c)^(3
```

```
/4)*d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3
+ 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*1i)/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*
c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*
a*b^5*c^9*d))*(a*d - b*c)^2*1i)/d^(15/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.399 $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{13/4}}+\frac{\sqrt[4]{c}\,(bc-a}{}$$

Rubi [A] time = 0.24, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2\sqrt{x}\,(bc-ad)^2}{d^3}+\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{13/4}}+\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{13/4}}+\frac{2b^2x^{9/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^2) + (2*b^
2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)^2*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13
/4)) - (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)x^{3/2}}{d^2}+\frac{b^2x^{7/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)x^{3/2}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{(bc-ad)^2\int\frac{x^{3/2}}{c+dx^2}\,dx}{d^2}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(c(bc-ad)^2\right)\int\frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(2c(bc-ad)^2\right)\mathrm{Subst}\,(\int\frac{1}{c+dx^4}\,dx,x,\sqrt{x}}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(\sqrt{c}\,(bc-ad)^2\right)\mathrm{Subst}\,(\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,}{d^3}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}-\frac{\left(\sqrt{c}\,(bc-ad)^2\right)\mathrm{Subst}\,(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}}{2d^{7/2}}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}\,(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{2\sqrt{2}\,d^{13/4}}$$

$$= \frac{2(bc-ad)^2\sqrt{x}}{d^3}-\frac{2b(bc-2ad)x^{5/2}}{5d^2}+\frac{2b^2x^{9/2}}{9d}+\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{13/4}}-$$

Mathematica [A] time = 0.11, size = 276, normalized size = 0.96

$$\frac{-72bd^{5/4}x^{5/2}(bc-2ad)+360\sqrt[4]{d}\sqrt{x}\,(bc-ad)^2+45\sqrt{2}\sqrt[4]{c}\,(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-45\sqrt{2}\sqrt[4]{c}\,(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+90\sqrt{2}\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-90\sqrt{2}\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+40b^2d^{9/4}x^{9/2}}{180d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (360*d^(1/4)*(b*c - a*d)^2*Sqrt[x] - 72*b*d^(5/4)*(b*c - 2*a*d)*x^(5/2) + 4
0*b^2*d^(9/4)*x^(9/2) + 90*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)] - 90*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 45*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 45*Sqrt[2]*c^(1/4)*(
```

```
b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(1
80*d^(13/4))
```

IntegrateAlgebraic [**A**] time = 0.23, size = 199, normalized size = 0.69

$$\frac{2\sqrt{x}\,\left(45a^2d^2-90abcd+18abd^2x^2+45b^2c^2-9b^2cdx^2+5b^2d^2x^4\right)}{45d^3}+\frac{\sqrt[4]{c}\,(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\,\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\,\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,d^{13/4}}-\frac{\sqrt[4]{c}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*Sqrt[x]*(45*b^2*c^2 - 90*a*b*c*d + 45*a^2*d^2 - 9*b^2*c*d*x^2 + 18*a*b*d
^2*x^2 + 5*b^2*d^2*x^4))/(45*d^3) + (c^(1/4)*(b*c - a*d)^2*ArcTan[(c^(1/4)/
(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*d^(13
/4)) - (c^(1/4)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sq
rt[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(13/4))
```

fricas [**B**] time = 1.37, size = 1268, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/90*(180*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5
*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8
*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*arctan((sqrt(d^6*sqrt(-(b^8*c^9 - 8
*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d
^13) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4)*x)*d^10*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^
6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^
7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(3/4) - (b^2*c^2*d^10 - 2*a*b*c*d^11 + a^2*d
^12)*sqrt(x)*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c
^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a
^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(3/4))/(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^
6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 +
28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)) + 45*d^3*(-(b^8*c^9 - 8*
a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4
- 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^
13)^(1/4)*log(d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*
b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6
- 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*sqrt(x)) - 45*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^
```

```
3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^
6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*log(-d^3*(-(b^8*c^9 - 8*a*b^7*
c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a
^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1
/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(5*b^2*d^2*x^4 + 45*b^2*
c^2 - 90*a*b*c*d + 45*a^2*d^2 - 9*(b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(x))/d^3
```

giac [A] time = 0.46, size = 385, normalized size = 1.34

$$-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{4}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4d^{4}}+\frac{2\left(5b^{2}d^{8}x^{\frac{9}{2}}-9b^{2}cd^{7}x^{\frac{5}{2}}+18abd^{8}x^{\frac{5}{2}}+45b^{2}c^{2}d^{6}\sqrt{x}-90abcd^{7}\sqrt{x}+45a^{2}d^{8}\sqrt{x}\right)}{45d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4
))/d^4 - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/d^4 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*
c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^4 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*
d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/d^4 +
 2/45*(5*b^2*d^8*x^(9/2) - 9*b^2*c*d^7*x^(5/2) + 18*a*b*d^8*x^(5/2) + 45*b^
2*c^2*d^6*sqrt(x) - 90*a*b*c*d^7*sqrt(x) + 45*a^2*d^8*sqrt(x))/d^9
```

maple [B] time = 0.01, size = 495, normalized size = 1.72

$$\frac{2b^{2}x^{\frac{9}{2}}}{9d}+\frac{4ab\,x^{\frac{5}{2}}}{5d}-\frac{2b^{2}c\,x^{\frac{5}{2}}}{5d^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{d^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{d^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2d^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2d^{3}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2d^{3}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4d^{3}}+\frac{2a^{2}\sqrt{x}}{d}-\frac{4abc\sqrt{x}}{d^{2}}+\frac{2b^{2}c^{2}\sqrt{x}}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 2/9*b^2*x^(9/2)/d+4/5/d*x^(5/2)*a*b-2/5/d^2*x^(5/2)*b^2*c+2/d*a^2*x^(1/2)-4
/d^2*a*b*c*x^(1/2)+2/d^3*b^2*c^2*x^(1/2)-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)
^(1/2)))*a^2+1/2/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+
(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b*c-1/4/d^3*(c/
d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1
/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2*c^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a*b*c-1/2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)+1)*b^2*c^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)*a^2+1/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
```

```
x^(1/2)-1)*a*b*c-1/2/d^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)-1)*b^2*c^2
```

maxima [A] time = 2.43, size = 324, normalized size = 1.12

$$-\frac{\left(\frac{2\,\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{4\,d^{3}}+\frac{2\left(5\,b^{2}d^{2}x^{\frac{9}{2}}-9\left(b^{2}cd-2\,abd^{2}\right)x^{\frac{5}{2}}+45\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{x}\right)}{45\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)
) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sq
rt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/d^3 + 2/45*(5*b^2*d^2*x^(
9/2) - 9*(b^2*c*d - 2*a*b*d^2)*x^(5/2) + 45*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*sqrt(x))/d^3
```

mupad [B] time = 0.35, size = 1175, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] x^(1/2)*((2*a^2)/d + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/d) - x^(5/2)*((2*b^2*c
)/(5*d^2) - (4*a*b)/(5*d)) + (2*b^2*x^(9/2))/(9*d) - ((-c)^(1/4)*atan((((-c
)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3
+ 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b
^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4)))*1i)/d^(13/4) + ((-c)
^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 +
 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^
2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4)))*1i)/d^(13/4))/(((-c)^
(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 +
6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2
*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4))))/d^(13/4) - ((-c)^(1/4
)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^
2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4
 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4))))/d^(13/4)))*(a*d - b*c)^2*
1i)/d^(13/4) - ((-c)^(1/4)*atan((((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4
```

```
*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/
d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d
)*1i)/(2*d^(13/4))))/d^(13/4) + ((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*
c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d
^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d)
*1i)/(2*d^(13/4))))/d^(13/4))/(((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c
^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^
3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d)*
1i)/(2*d^(13/4)))*1i)/d^(13/4) - ((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4
*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/
d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d
)*1i)/(2*d^(13/4)))*1i)/d^(13/4)))*(a*d - b*c)^2)/d^(13/4)
```

sympy [A] time = 53.53, size = 656, normalized size = 2.28

$$\begin{cases} \tilde{\infty} \left(2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}\right) & \text{for } c = 0 \wedge d = 0 \\ \frac{\frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}}{c} & \text{for } d = 0 \\ \frac{2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}}{d} & \text{for } c = 0 \\ \frac{\sqrt[4]{-1}\, a^2 \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \log\left(-\sqrt[4]{-1}\sqrt[4]{c}\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2d} - \frac{\sqrt[4]{-1}\, a^2 \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \log\left(\sqrt[4]{-1}\sqrt[4]{c}\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2d} + \frac{\sqrt[4]{-1}\, a^2 \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{c}\sqrt[4]{\frac{1}{d}}}\right)}{d} + \frac{2a^2\sqrt{x}}{d} - \frac{\sqrt[4]{-1}\, abc^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}} \log\left(-\sqrt[4]{-1}\sqrt[4]{c}\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{d^2} + \frac{\sqrt[4]{-1}\, abc^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}} \log\left(\sqrt[4]{-1}\sqrt[4]{c}\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{d^2} - \frac{2\sqrt[4]{-1}\, abc^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{c}\sqrt[4]{\frac{1}{d}}}\right)}{d^2} - \frac{4abc\sqrt{x}}{d^2} + \frac{4abx^{\frac{5}{2}}}{5d} + \frac{\sqrt[4]{-1}\, b^2c^{\frac{9}{4}} \sqrt[4]{\frac{1}{d}} \log\left(-\sqrt[4]{-1}\sqrt[4]{c}\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2d^3} - \frac{\sqrt[4]{-1}\, b^2c^{\frac{9}{4}} \sqrt[4]{\frac{1}{d}} \log\left(\sqrt[4]{-1}\sqrt[4]{c}\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2d^3} + \frac{\sqrt[4]{-1}\, b^2c^{\frac{9}{4}} \sqrt[4]{\frac{1}{d}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{c}\sqrt[4]{\frac{1}{d}}}\right)}{d^3} + \frac{2b^2c^2\sqrt{x}}{d^3} - \frac{2b^2cx^{\frac{5}{2}}}{5d^2} + \frac{2b^2x^{\frac{9}{2}}}{9d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9), Eq(
c, 0) & Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2
)/13)/c, Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
)/d, Eq(c, 0)), ((-1)**(1/4)*a**2*c**(1/4)*(1/d)**(1/4)*log(-(-1)**(1/4)*c*
*(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d) - (-1)**(1/4)*a**2*c**(1/4)*(1/d)**(1/
4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d) + (-1)**(1/4)*a**
2*c**(1/4)*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/d
 + 2*a**2*sqrt(x)/d - (-1)**(1/4)*a*b*c**(5/4)*(1/d)**(1/4)*log(-(-1)**(1/4
)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/d**2 + (-1)**(1/4)*a*b*c**(5/4)*(1/d)**(
1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/d**2 - 2*(-1)**(1/4)*
a*b*c**(5/4)*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))
/d**2 - 4*a*b*c*sqrt(x)/d**2 + 4*a*b*x**(5/2)/(5*d) + (-1)**(1/4)*b**2*c**(
9/4)*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**3
) - (-1)**(1/4)*b**2*c**(9/4)*(1/d)**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**
(1/4) + sqrt(x))/(2*d**3) + (-1)**(1/4)*b**2*c**(9/4)*(1/d)**(1/4)*atan((-1
)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/d**3 + 2*b**2*c**2*sqrt(x)/d**3 -
 2*b**2*c*x**(5/2)/(5*d**2) + 2*b**2*x**(9/2)/(9*d), True))
```

3.400 $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{c+dx^2}\,dx$

Optimal. Leaf size=268

$$\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2\tan^{-1}\Big(1}{\sqrt{2}\sqrt[4]{c}\,d^{1}}$$

Rubi [A] time = 0.23, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{2b^2x^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11
/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \int \left(-\frac{b(bc-2ad)\sqrt{x}}{d^2}+\frac{b^2x^{5/2}}{d}+\frac{\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{x}}{d^2\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\int\frac{\sqrt{x}}{c+dx^2}\,dx}{d^2}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int\frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^{5/2}}+\frac{(bc-ad)^2\,\text{Subs}}{}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2d^3}+\frac{(bc-ad)^2}{}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}+\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2}{}$$

$$= -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\right.}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}$$

Mathematica [A] time = 0.12, size = 249, normalized size = 0.93

$$\frac{-56b\sqrt[4]{c}\,d^{3/4}x^{3/2}(bc-2ad)+21\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-21\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-42\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+42\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+24b^2\sqrt[4]{c}\,d^{7/4}x^{7/2}}{84\sqrt[4]{c}\,d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (-56*b*c^(1/4)*d^(3/4)*(b*c - 2*a*d)*x^(3/2) + 24*b^2*c^(1/4)*d^(7/4)*x^(7/
2) - 42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
 + 42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] +
 21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x] - 21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(84*c^(1/4)*d^(11/4))
```

IntegrateAlgebraic [A] time = 0.21, size = 163, normalized size = 0.61

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\sqrt[4]{c}\,d^{11/4}}+\frac{2bx^{3/2}\left(14ad-7bc+3bdx^2\right)}{21d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

```
[Out] (2*b*x^(3/2)*(-7*b*c + 14*a*d + 3*b*d*x^2))/(21*d^2) - ((b*c - a*d)^2*ArcTa
n[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sq
rt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(1/4)*d^(11/4))
```

fricas [B] time = 1.10, size = 1629, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/42*(84*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)*x - (b^8*c^9*d^5 - 8*a*b^7*c^8*d^6 + 28*a^2*b^6*c^7*d^7 - 56*
a^3*b^5*c^6*d^8 + 70*a^4*b^4*c^5*d^9 - 56*a^5*b^3*c^4*d^10 + 28*a^6*b^2*c^3
*d^11 - 8*a^7*b*c^2*d^12 + a^8*c*d^13)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*
a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*
d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11)))*d^3*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c*d^11))^(1/4) - (b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20
*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8 + a^6*d^9)*sqrt(x)*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c*d^11))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*
a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*
d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(c*d^8
```

```
*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8)/(c*d^11))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)
) + 21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(-c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 4*(3*b^2*d*x^3 - 7*(b^2*c
 - 2*a*b*d)*x)*sqrt(x))/d^2
```

giac [**A**] time = 0.54, size = 361, normalized size = 1.35

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{5}} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{5}} - \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,cd^{5}} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{3}{4}} abcd + \left(cd^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,cd^{5}} + \frac{2\left(3\,b^{2}d^{6}x^{\frac{7}{2}} - 7\,b^{2}cd^{5}x^{\frac{3}{2}} + 14\,abd^{6}x^{\frac{3}{2}}\right)}{21\,d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c*d^5) + 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c*d^5) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/
4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c*d^5) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*
b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c*d^5) + 2/21*(3*b^2*d^6*x^(7/2) - 7*b^2*c*d^5*x^(3/2) + 14*a*b*d^6*
x^(3/2))/d^7
```

maple [**B**] time = 0.01, size = 461, normalized size = 1.72

$$\frac{2b^{2}x^{\frac{7}{2}}}{7d} + \frac{4ab\,x^{\frac{3}{2}}}{3d} - \frac{2b^{2}c\,x^{\frac{3}{2}}}{3d^{2}} + \frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}} d} + \frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}} d} + \frac{\sqrt{2}\,a^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}} d} - \frac{\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}} - \frac{\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}} - \frac{\sqrt{2}\,abc\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}} + \frac{\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}} d^{3}} + \frac{\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}} d^{3}} + \frac{\sqrt{2}\,b^{2}c^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x)
```

```
[Out] 2/7*b^2*x^(7/2)/d+4/3*b/d*x^(3/2)*a-2/3*b^2/d^2*x^(3/2)*c+1/4/d/(c/d)^(1/4)
*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1
/2)*x^(1/2)+(c/d)^(1/2)))*a^2-1/2/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)
*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*
a*b*c+1/4/d^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
```

```
1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2*c^2+1/2/d/(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/d^2/(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*c+1/2/d^3/(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2*c^2+1/2/d/(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-1/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*c+1/2/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)*b^2*c^2
```

maxima [A] time = 2.51, size = 229, normalized size = 0.85

$$\frac{\left(b^{2} c^{2}-2\,abcd+a^{2} d^{2}\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,d^{2}}+\frac{2\left(3\,b^{2} d x^{\frac{7}{2}}-7\left(b^{2} c-2\,abd\right)x^{\frac{3}{2}}\right)}{21\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*s
qrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
- sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)))/d^2 + 2/21*(3*b^2*d*x^(7/2) - 7*(b^2*c - 2*a*b*d)
*x^(3/2))/d^2
```

mupad [B] time = 0.18, size = 390, normalized size = 1.46

$$\frac{2\,b^{2}\,x^{7/2}}{7\,d}-x^{3/2}\left(\frac{2\,b^{2}\,c}{3\,d^{2}}-\frac{4\,a\,b}{3\,d}\right)+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^{2}\,\left(a^{4}\,c\,d^{4}-4\,a^{3}\,b\,c^{2}\,d^{3}+6\,a^{2}\,b^{2}\,c^{3}\,d^{2}-4\,a\,b^{3}\,c^{4}\,d+b^{4}\,c^{5}\right)}{{\left(-c\right)}^{1/4}\,\left(a^{6}\,c\,d^{6}-6\,a^{5}\,b\,c^{2}\,d^{5}+15\,a^{4}\,b^{2}\,c^{3}\,d^{4}-20\,a^{3}\,b^{3}\,c^{4}\,d^{3}+15\,a^{2}\,b^{4}\,c^{5}\,d^{2}-6\,a\,b^{5}\,c^{6}\,d+b^{6}\,c^{7}\right)}\right)\,{\left(a\,d-b\,c\right)}^{2}}{{\left(-c\right)}^{1/4}\,d^{11/4}}+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^{2}\,\left(a^{4}\,c\,d^{4}-4\,a^{3}\,b\,c^{2}\,d^{3}+6\,a^{2}\,b^{2}\,c^{3}\,d^{2}-4\,a\,b^{3}\,c^{4}\,d+b^{4}\,c^{5}\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}\,\left(a^{6}\,c\,d^{6}-6\,a^{5}\,b\,c^{2}\,d^{5}+15\,a^{4}\,b^{2}\,c^{3}\,d^{4}-20\,a^{3}\,b^{3}\,c^{4}\,d^{3}+15\,a^{2}\,b^{4}\,c^{5}\,d^{2}-6\,a\,b^{5}\,c^{6}\,d+b^{6}\,c^{7}\right)}\right)\,{\left(a\,d-b\,c\right)}^{2}\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}\,d^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

```
[Out] (2*b^2*x^(7/2))/(7*d) - x^(3/2)*((2*b^2*c)/(3*d^2) - (4*a*b)/(3*d)) + (atan
((d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^5 + a^4*c*d^4 - 4*a^3*b*c^2*d^3 + 6*
a^2*b^2*c^3*d^2 - 4*a*b^3*c^4*d))/((-c)^(1/4)*(b^6*c^7 + a^6*c*d^6 - 6*a^5*
b*c^2*d^5 + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 -
6*a*b^5*c^6*d)))*(a*d - b*c)^2)/((-c)^(1/4)*d^(11/4)) + (atan((d^(1/4)*x^(1
/2)*(a*d - b*c)^2*(b^4*c^5 + a^4*c*d^4 - 4*a^3*b*c^2*d^3 + 6*a^2*b^2*c^3*d^
2 - 4*a*b^3*c^4*d)*1i)/((-c)^(1/4)*(b^6*c^7 + a^6*c*d^6 - 6*a^5*b*c^2*d^5 +
 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a*b^5*c^6
*d)))*(a*d - b*c)^2*1i)/((-c)^(1/4)*d^(11/4))
```

sympy [A] time = 11.40, size = 87, normalized size = 0.32

$$\frac{4abx^{\frac{3}{2}}}{3d}-\frac{2b^2cx^{\frac{3}{2}}}{3d^2}+\frac{2b^2x^{\frac{7}{2}}}{7d}+\frac{2\left(ad-bc\right)^2\operatorname{RootSum}\left(256t^4cd^3+1,\left(t\mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c),x)
```

```
[Out] 4*a*b*x**(3/2)/(3*d) - 2*b**2*c*x**(3/2)/(3*d**2) + 2*b**2*x**(7/2)/(7*d) +
 2*(a*d - b*c)**2*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t**3*
c*d**2 + sqrt(x))))/d**2
```

3.401 $\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=266

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}}{\sqrt{2}c^{3}}$$

Rubi [A] time = 0.21, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {461, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{3/4}d^{9/4}}-\frac{2b\sqrt{x}(bc-2ad)}{d^2}+\frac{2b^2x^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c - a*d)^2*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) + (
(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/
4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2\sqrt{x}}+\frac{b^2x^{3/2}}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\sqrt{x}\left(c+dx^2\right)}\right)dx$$

$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}+\frac{(bc-ad)^2\int\frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{d^2}$$

$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}+\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int\frac{1}{c+dx^4}\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{\sqrt{c}\,d^2}+(bc-ad)^2\,\text{Sub}$$

$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{c}\,d^{5/2}}+(bc-ad)^2$$

$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+(bc-ad)^2$$

$$= -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\right.}{\sqrt{2}\,c^{3/4}d^{9/4}}$$

Mathematica [A] time = 0.12, size = 249, normalized size = 0.94

$$\frac{-40bc^{3/4}\sqrt[4]{d}\sqrt{x}(bc-2ad)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-10\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+10\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+8b^2c^{3/4}d^{5/4}x^{5/2}}{20c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*Sqrt[x] + 8*b^2*c^(3/4)*d^(5/4)*x^(5/2
) - 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
+ 10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] -
5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x] + 5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(20*c^(3/4)*d^(9/4))
```

IntegrateAlgebraic [**A**] time = 0.20, size = 161, normalized size = 0.61

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{2b\sqrt{x}\left(10ad-5bc+bdx^2\right)}{5d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

```
[Out] (2*b*Sqrt[x]*(-5*b*c + 10*a*d + b*d*x^2))/(5*d^2) - ((b*c - a*d)^2*ArcTan[(
c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[
2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(3/4)*d^(9/4))
```

fricas [**B**] time = 1.28, size = 1245, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan((sqrt(c^2*d^4*sqrt(-(b^8*c^8
- 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c
^3*d^9)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*x)*c^2*d^7*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4) - (b^2*c^4*d^7 - 2*a*b*c^3*d^8
 + a^2*c^2*d^9)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(3/4))/(b^8*c^8 - 8*a*b^7*c^7*d
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 5*d^2*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*sqrt(x)) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c
```

```
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 4*(b^2*d*x^2
 - 5*b^2*c + 10*a*b*d)*sqrt(x))/d^2
```

giac [A] time = 0.40, size = 360, normalized size = 1.35

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{3}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^{3}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^{3}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\left(cd^{3}\right)^{\frac{1}{4}}abcd+\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^{3}}+\frac{2\left(b^{2}d^{4}x^{\frac{5}{2}}-5\,b^{2}cd^{3}\sqrt{x}+10\,abd^{4}\sqrt{x}\right)}{5\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c*d^3) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c*d^3) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c*d^3) + 2/5*(b^2*d^4*x^(5/2) - 5*b^2*c*d^3*sqrt(x) + 10*a*b*d^4*sqr
t(x))/d^5
```

maple [B] time = 0.01, size = 452, normalized size = 1.70

$$\frac{2b^{2}x^{\frac{5}{2}}}{5d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4c}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2d^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2d^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4d^{2}}+\frac{4ab\sqrt{x}}{d}-\frac{2b^{2}c\sqrt{x}}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x)
```

```
[Out] 2/5*b^2*x^(5/2)/d+4*b/d*a*x^(1/2)-2*b^2/d^2*c*x^(1/2)+1/4*(c/d)^(1/4)/c*2^(
1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*
x^(1/2)+(c/d)^(1/2)))*a^2-1/2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/
2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+1/
4/d^2*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/
(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+1/2*(c/d)^(1/4)/c*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/d*(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*a^2-1/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a*b+1/2/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)-1)*b^2
```

maxima [A] time = 2.41, size = 291, normalized size = 1.09

$$\frac{2\left(b^{2}dx^{\frac{5}{2}}-5\left(b^{2}c-2\,abd\right)\sqrt{x}\right)}{5\,d^{2}}+\frac{\frac{2\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{4\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^2*d*x^(5/2) - 5*(b^2*c - 2*a*b*d)*sqrt(x))/d^2 + 1/4*(2*sqrt(2)*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2
*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1
/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(
c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*
d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/d^2
```

mupad [B] time = 0.37, size = 1107, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)),x)
```

```
[Out] (2*b^2*x^(5/2))/(5*d) - x^(1/2)*((2*b^2*c)/d^2 - (4*a*b)/d) + (atan(((((8*x
^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d
^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*
(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) + (((8*x^(1/2)*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d
+ ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3
/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)))/((((8*x^(1/2)*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d
 - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3/4)*d^(
9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)) - (((8*x^(1/2)*(a^4*d^4 + b^4*c^
4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*
(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3/4)*d^(9/4)))*(a*
d - b*c)^2)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) +
 (atan(((((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
 - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b
*c^2*d^2)*1i)/(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)) +
 (((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^
3*b*c*d^3))/d + ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^
2)*1i)/(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)))/((((8*x
```

```
^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d
^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2)*1i)/
(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) - (((8*x^(1/
2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))
/d + ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2)*1i)/(2*(
-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2)
/((-c)^(3/4)*d^(9/4))
```

sympy [A] time = 16.76, size = 597, normalized size = 2.24

$$\begin{cases} \tilde{\infty} \left(- \frac{2 a^{2}}{3 x^{\frac{3}{2}}} + 4 a b \sqrt{x} + \frac{2 b^{2} x^{\frac{5}{2}}}{5}\right) & \text{for}\: c = 0 \wedge d = 0 \\ \frac{- \frac{2 a^{2}}{3 x^{\frac{3}{2}}} + 4 a b \sqrt{x} + \frac{2 b^{2} x^{\frac{5}{2}}}{5}}{d} & \text{for}\: c = 0 \\ \frac{2 a^{2} \sqrt{x} + \frac{4 a b x^{\frac{5}{2}}}{5} + \frac{2 b^{2} x^{\frac{9}{2}}}{9}}{c} & \text{for}\: d = 0 \\ - \frac{\sqrt[4]{-1} a^{2} \sqrt[4]{\frac{1}{d}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 c^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} a^{2} \sqrt[4]{\frac{1}{d}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 c^{\frac{3}{4}}} - \frac{\sqrt[4]{-1} a^{2} \sqrt[4]{\frac{1}{d}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{c^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} a b \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{d} - \frac{\sqrt[4]{-1} a b \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{d} + \frac{2 \sqrt[4]{-1} a b \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{d} + \frac{4 a b \sqrt{x}}{d} - \frac{\sqrt[4]{-1} b^{2} c^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 d^{2}} + \frac{\sqrt[4]{-1} b^{2} c^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 d^{2}} - \frac{\sqrt[4]{-1} b^{2} c^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{d^{2}} - \frac{2 b^{2} c \sqrt{x}}{d^{2}} + \frac{2 b^{2} x^{\frac{5}{2}}}{5 d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5),
Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5
/2)/5)/d, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/
9)/c, Eq(d, 0)), (-(-1)**(1/4)*a**2*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*
(1/d)**(1/4) + sqrt(x))/(2*c**(3/4)) + (-1)**(1/4)*a**2*(1/d)**(1/4)*log((-
1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(3/4)) - (-1)**(1/4)*a**2*
(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/c**(3/4) + (
-1)**(1/4)*a*b*c**(1/4)*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4)
 + sqrt(x))/d - (-1)**(1/4)*a*b*c**(1/4)*(1/d)**(1/4)*log((-1)**(1/4)*c**(1
/4)*(1/d)**(1/4) + sqrt(x))/d + 2*(-1)**(1/4)*a*b*c**(1/4)*(1/d)**(1/4)*ata
n((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/d + 4*a*b*sqrt(x)/d - (-1)**
(1/4)*b**2*c**(5/4)*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + s
qrt(x))/(2*d**2) + (-1)**(1/4)*b**2*c**(5/4)*(1/d)**(1/4)*log((-1)**(1/4)*c
**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d**2) - (-1)**(1/4)*b**2*c**(5/4)*(1/d)*
*(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/d**2 - 2*b**2*c*sq
rt(x)/d**2 + 2*b**2*x**(5/2)/(5*d), True))
```

3.402 $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{c\sqrt{x}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+(bc-ad)^2$$

Rubi [A] time = 0.27, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2a^2}{c\sqrt{x}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{2b^2x^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]) + (2*b^2*x^(3/2))/(3*d) + ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4)) - ((b*c - a*d)^
2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*d^(7/4))
- ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*c^(5/4)*d^(7/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*d^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)}\,dx = -\frac{2a^2}{c\sqrt{x}}+\frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-ad)+\frac{1}{2}b^2cx^2\right)}{c+dx^2}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{(bc-ad)^2\int \frac{\sqrt{x}}{c+dx^2}\,dx}{cd}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{cd}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{cd^{3/2}}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}+\sqrt{d}\,x}{c+dx^4}\right.}{cd^{3/2}}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2cd^2}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}}\right.}{2cd}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{}\right.}{2\sqrt{2}\,c^{5}}$$

$$= -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}-\frac{(b}{}$$

Mathematica [A] time = 0.13, size = 261, normalized size = 1.00

$$\frac{-24a^2\sqrt[4]{c}\,d^{7/4}-3\sqrt{2}\sqrt{x}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+3\sqrt{2}\sqrt{x}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+6\sqrt{2}\sqrt{x}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-6\sqrt{2}\sqrt{x}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+8b^2c^{5/4}d^{3/4}x^2}{12c^{5/4}d^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

```
[Out] (-24*a^2*c^(1/4)*d^(7/4) + 8*b^2*c^(5/4)*d^(3/4)*x^2 + 6*Sqrt[2]*(b*c - a*d
)^2*Sqrt[x]*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 6*Sqrt[2]*(b*c
- a*d)^2*Sqrt[x]*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 3*Sqrt[2]*
(b*c - a*d)^2*Sqrt[x]*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
```

```
d]*x] + 3*Sqrt[2]*(b*c - a*d)^2*Sqrt[x]*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(12*c^(5/4)*d^(7/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.22, size = 162, normalized size = 0.62

$$\frac{2\left(b^2cx^2-3a^2d\right)}{3cd\sqrt{x}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}+\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{5/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

```
[Out] (2*(-3*a^2*d + b^2*c*x^2))/(3*c*d*Sqrt[x]) + ((b*c - a*d)^2*ArcTan[(c^(1/4)
/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(5
/4)*d^(7/4)) + ((b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sq
rt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(5/4)*d^(7/4))
```

fricas [B] time = 0.96, size = 1636, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*b^11
*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8
*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)*x - (b^8*c^11*d^3 - 8*a*b^7*c^10*d^4 + 28*a^2*b^6*c^9*d^5 -
56*a^3*b^5*c^8*d^6 + 70*a^4*b^4*c^7*d^7 - 56*a^5*b^3*c^6*d^8 + 28*a^6*b^2*c
^5*d^9 - 8*a^7*b*c^4*d^10 + a^8*c^3*d^11)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7)))*c*d^2*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c^5*d^7))^(1/4) - (b^6*c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d
^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)
*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 3*c*d*x*(-(b^8*c^8 - 8*a*b^7*
```

```
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(
1/4)*log(c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 +
 a^6*d^6)*sqrt(x)) + 3*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*log(-c^4*d^5*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 4*(
b^2*c*x^2 - 3*a^2*d)*sqrt(x))/(c*d*x)
```

giac [**A**] time = 0.48, size = 344, normalized size = 1.32

$$\frac{2\,b^{2}x^{\frac{3}{2}}}{3\,d}-\frac{2\,a^{2}}{c\sqrt{x}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^{2}d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^{2}d^{4}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^{2}d^{4}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^{2}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2
- 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/2*sqrt(2)*((c*d^3)^
(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) + 1/4*sq
rt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*
d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) - 1/4*sqrt(
2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4)
```

maple [**B**] time = 0.02, size = 439, normalized size = 1.69

$$\frac{2b^{2}x^{\frac{3}{2}}}{3d}-\frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,a^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}}d}+\frac{\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}}d}+\frac{\sqrt{2}\,ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d}-\frac{\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}-\frac{\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}-\frac{\sqrt{2}\,b^{2}c\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}-\frac{2a^{2}}{c\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x)
```

```
[Out] 2/3*b^2*x^(3/2)/d-1/2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)-1)*a^2+1/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*
b-1/2*c/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1
/4/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+
```

```
(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+1/2/d/(c/d)^(1/4)*2^(1/2)*ln(
(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+
(c/d)^(1/2)))*a*b-1/4*c/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x
^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-1/2/c/
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/d/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-1/2*c/d^2/(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2*a^2/c/x^(1/2)
```

maxima [A] time = 2.38, size = 223, normalized size = 0.86

$$\frac{2\,b^{2}x^{\frac{3}{2}}}{3\,d}-\frac{2\,a^{2}}{c\sqrt{x}}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sq
rt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/
4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d)
```

mupad [B] time = 0.34, size = 416, normalized size = 1.60

$$\frac{2\,b^2\,x^{3/2}}{3\,d}-\frac{2\,a^2}{c\,\sqrt{x}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^4\,c^4\,d^9-64\,a^3\,b\,c^5\,d^8+96\,a^2\,b^2\,c^6\,d^7-64\,a\,b^3\,c^7\,d^6+16\,b^4\,c^8\,d^5\right)}{{\left(-c\right)}^{5/4}\,d^{7/4}\,\left(16\,a^6\,c^3\,d^9-96\,a^5\,b\,c^4\,d^8+240\,a^4\,b^2\,c^5\,d^7-320\,a^3\,b^3\,c^6\,d^6+240\,a^2\,b^4\,c^7\,d^5-96\,a\,b^5\,c^8\,d^4+16\,b^6\,c^9\,d^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{5/4}\,d^{7/4}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^4\,c^4\,d^9-64\,a^3\,b\,c^5\,d^8+96\,a^2\,b^2\,c^6\,d^7-64\,a\,b^3\,c^7\,d^6+16\,b^4\,c^8\,d^5\right)\,1{}\mathrm{i}}{{\left(-c\right)}^{5/4}\,d^{7/4}\,\left(16\,a^6\,c^3\,d^9-96\,a^5\,b\,c^4\,d^8+240\,a^4\,b^2\,c^5\,d^7-320\,a^3\,b^3\,c^6\,d^6+240\,a^2\,b^4\,c^7\,d^5-96\,a\,b^5\,c^8\,d^4+16\,b^6\,c^9\,d^3\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,1{}\mathrm{i}}{{\left(-c\right)}^{5/4}\,d^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x)
```

```
[Out] (2*b^2*x^(3/2))/(3*d) - (2*a^2)/(c*x^(1/2)) - (atan((x^(1/2)*(a*d - b*c)^2*
(16*a^4*c^4*d^9 + 16*b^4*c^8*d^5 - 64*a*b^3*c^7*d^6 - 64*a^3*b*c^5*d^8 + 96
*a^2*b^2*c^6*d^7))/((-c)^(5/4)*d^(7/4)*(16*a^6*c^3*d^9 + 16*b^6*c^9*d^3 - 9
6*a*b^5*c^8*d^4 - 96*a^5*b*c^4*d^8 + 240*a^2*b^4*c^7*d^5 - 320*a^3*b^3*c^6*
d^6 + 240*a^4*b^2*c^5*d^7)))*(a*d - b*c)^2)/((-c)^(5/4)*d^(7/4)) - (atan((x
^(1/2)*(a*d - b*c)^2*(16*a^4*c^4*d^9 + 16*b^4*c^8*d^5 - 64*a*b^3*c^7*d^6 -
64*a^3*b*c^5*d^8 + 96*a^2*b^2*c^6*d^7)*1i)/((-c)^(5/4)*d^(7/4)*(16*a^6*c^3*
d^9 + 16*b^6*c^9*d^3 - 96*a*b^5*c^8*d^4 - 96*a^5*b*c^4*d^8 + 240*a^2*b^4*c^
7*d^5 - 320*a^3*b^3*c^6*d^6 + 240*a^4*b^2*c^5*d^7)))*(a*d - b*c)^2*1i)/((-c
)^(5/4)*d^(7/4))
```

sympy [A] time = 36.72, size = 394, normalized size = 1.52

$$a^{2}\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: c = 0 \wedge d = 0 \\ - \frac{2}{c \sqrt{x}} & \text{for}\: d = 0 \\ - \frac{2}{5 d x^{\frac{5}{2}}} & \text{for}\: c = 0 \\ - \frac{2}{c \sqrt{x}} + \frac{\left(-1\right)^{\frac{3}{4}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 c^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}}} - \frac{\left(-1\right)^{\frac{3}{4}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 c^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}}} - \frac{\left(-1\right)^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{c^{\frac{5}{4}} \sqrt[4]{\frac{1}{d}}} & \text{otherwise} \end{cases}\right) + 4 a b \operatorname{RootSum} {\left(256 t^{4} c d^{3} + 1, \left(t \mapsto t \log{\left(64 t^{3} c d^{2} + \sqrt{x} \right)} \right)\right)} + b^{2}\left(\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: c = 0 \wedge d = 0 \\ \frac{2 x^{\frac{7}{2}}}{7 c} & \text{for}\: d = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 d} & \text{for}\: c = 0 \\ \frac{\left(-1\right)^{\frac{3}{4}} c^{\frac{3}{4}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 d^{2} \sqrt[4]{\frac{1}{d}}} - \frac{\left(-1\right)^{\frac{3}{4}} c^{\frac{3}{4}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 d^{2} \sqrt[4]{\frac{1}{d}}} - \frac{\left(-1\right)^{\frac{3}{4}} c^{\frac{3}{4}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{d^{2} \sqrt[4]{\frac{1}{d}}} + \frac{2 x^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c),x)
```

```
[Out] a**2*Piecewise((zoo/x**(5/2), Eq(c, 0) & Eq(d, 0)), (-2/(c*sqrt(x)), Eq(d,
0)), (-2/(5*d*x**(5/2)), Eq(c, 0)), (-2/(c*sqrt(x)) + (-1)**(3/4)*log(-(-1)
**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*(1/d)**(1/4)) - (-1)**
(3/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*(1/d)**(
1/4)) - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(c**(
5/4)*(1/d)**(1/4)), True)) + 4*a*b*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t,
 _t*log(64*_t**3*c*d**2 + sqrt(x)))) + b**2*Piecewise((zoo*x**(3/2), Eq(c,
0) & Eq(d, 0)), (2*x**(7/2)/(7*c), Eq(d, 0)), (2*x**(3/2)/(3*d), Eq(c, 0)),
 ((-1)**(3/4)*c**(3/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2
*d**2*(1/d)**(1/4)) - (-1)**(3/4)*c**(3/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**
(1/4) + sqrt(x))/(2*d**2*(1/d)**(1/4)) - (-1)**(3/4)*c**(3/4)*atan((-1)**(3
/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(d**2*(1/d)**(1/4)) + 2*x**(3/2)/(3*d)
, True))
```

3.403 $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(bc-ad}{}$$

Rubi [**A**] time = 0.26, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 459, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2a^2}{3cx^{3/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{2b^2\sqrt{x}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)) + (2*b^2*Sqrt[x])/d + ((b*c - a*d)^2*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*
ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*d^(5/4)) +
((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)}\,dx &= -\frac{2a^2}{3cx^{3/2}}+\frac{2\int \frac{\frac{3}{2}a(2bc-ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\left(c+dx^2\right)}\,dx}{3c}\\
&= -\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}-\frac{(bc-ad)^2\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{cd}\\
&= -\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}-\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx,x,\sqrt{x}\right)}{cd}\\
&= -\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{c^{3/2}d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}+}{c+d}\right.}{c^{3/2}d}\\
&= -\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2c^{3/2}d^{3/2}}-\frac{(bc-ad)^2\,\text{Subst}\left(\int\right.}{2c}\\
&= -\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}+\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\log\left(\sqrt{c}+\right.}{2\sqrt{2}}\\
&= -\frac{2a^2}{3cx^{3/2}}+\frac{2b^2\sqrt{x}}{d}+\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}+
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 261, normalized size = 1.00

$$
\frac{-8a^2c^{3/4}d^{5/4}+3\sqrt{2}x^{3/2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-3\sqrt{2}x^{3/2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+6\sqrt{2}x^{3/2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-6\sqrt{2}x^{3/2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+24b^2c^{7/4}\sqrt[4]{d}\,x^2}{12c^{7/4}d^{5/4}x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (-8*a^2*c^(3/4)*d^(5/4) + 24*b^2*c^(7/4)*d^(1/4)*x^2 + 6*Sqrt[2]*(b*c - a*d
)^2*x^(3/2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 6*Sqrt[2]*(b*c
- a*d)^2*x^(3/2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 3*Sqrt[2]*
(b*c - a*d)^2*x^(3/2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
```

```
d]*x] - 3*Sqrt[2]*(b*c - a*d)^2*x^(3/2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(12*c^(7/4)*d^(5/4)*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 164, normalized size = 0.63

$$\frac{2\left(3b^2cx^2-a^2d\right)}{3cdx^{3/2}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

```
[Out] (2*(-(a^2*d) + 3*b^2*c*x^2))/(3*c*d*x^(3/2)) + ((b*c - a*d)^2*ArcTan[(c^(1/
4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^
(7/4)*d^(5/4)) - ((b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(
Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(7/4)*d^(5/4))
```

fricas [B] time = 0.73, size = 1253, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/6*(12*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*arctan((sqrt(c^4*d^2*sqrt(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^7*d^5)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*x)*c^5*d^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(3/4) - (b^2*c^7*d^4 - 2*a*b*c^6
*d^5 + a^2*c^5*d^6)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(3/4))/(b^8*c^8 - 8*a*b^7*c
^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^
5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 3*c*d*x^2*
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt(x)) - 3*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a
```

```
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(-c^
2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) -
4*(3*b^2*c*x^2 - a^2*d)*sqrt(x))/(c*d*x^2)
```

giac [**A**] time = 0.40, size = 344, normalized size = 1.32

$$\frac{2\,b^2\sqrt{x}}{d}-\frac{2\,a^2}{3\,cx^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^2d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2-2\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
- 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/2*sqrt(2)*((c*d^3)^
(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/4*sq
rt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*
d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2) + 1/4*sqrt(
2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2)
```

maple [**B**] time = 0.01, size = 439, normalized size = 1.69

$$-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2c^2}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2c^2}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4c^2}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2c}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2d}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4d}+\frac{2b^2\sqrt{x}}{d}-\frac{2a^2}{3c\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x)
```

```
[Out] 2*b^2*x^(1/2)/d-1/2/c^2*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1)*a^2+1/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*
a*b-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-1/4
/c^2*d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+1/2/c*(c/d)^(1/4)*2^(1/2)*l
n((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2
)+(c/d)^(1/2)))*a*b-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(
1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-1/2/c^2*
d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/c*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-1/2/d*(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2/3*a^2/c/x^(3/2)
```

maxima [A] time = 2.41, size = 286, normalized size = 1.10

$$\frac{2\,b^{2}\sqrt{x}}{d}-\frac{2\,a^{2}}{3\,cx^{\frac{3}{2}}}-\frac{\frac{2\,\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{4\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4
)*d^(1/4)))/(c*d)
```

mupad [B] time = 0.38, size = 1201, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x)
```

```
[Out] (2*b^2*x^(1/2))/d - (2*a^2)/(3*c*x^(3/2)) - (atan(((((x^(1/2)*(16*a^4*c^3*d
^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5
*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8
))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)) + (((x^(1
/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9
 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7
 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d
^(5/4)))/((((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 -
 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9
 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2)
/((-c)^(7/4)*d^(5/4)) - (((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a
*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(
16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*
(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4))))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)
) - (atan(((((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7
- 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^
9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c
)^2)/((-c)^(7/4)*d^(5/4)) + (((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 -
64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)
```

```
^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(
5/4)))*(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4)))/((((x^(1/2)*(16*a^4*c^3*d^10 +
16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))
/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/
(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)) - (((x^(1/2)
*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 +
96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 -
32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d
^(5/4))))*(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4))
```

sympy [A]   time = 31.82, size = 576, normalized size = 2.22

$$\begin{cases} \tilde{\infty} \left(- \frac{2 a^{2}}{7 x^{\frac{7}{2}}} - \frac{4 a b}{3 x^{\frac{3}{2}}} + 2 b^{2} \sqrt{x}\right) & \text{for}\: c = 0 \wedge d = 0 \\ \frac{- \frac{2 a^{2}}{3 x^{\frac{3}{2}}} + 4 a b \sqrt{x} + \frac{2 b^{2} x^{\frac{5}{2}}}{5}}{c} & \text{for}\: d = 0 \\ \frac{- \frac{2 a^{2}}{7 x^{\frac{7}{2}}} - \frac{4 a b}{3 x^{\frac{3}{2}}} + 2 b^{2} \sqrt{x}}{d} & \text{for}\: c = 0 \\ - \frac{2 a^{2}}{3 c x^{\frac{3}{2}}} + \frac{\sqrt[4]{-1} a^{2} d \sqrt[4]{\frac{1}{d}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 c^{\frac{7}{4}}} - \frac{\sqrt[4]{-1} a^{2} d \sqrt[4]{\frac{1}{d}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 c^{\frac{7}{4}}} + \frac{\sqrt[4]{-1} a^{2} d \sqrt[4]{\frac{1}{d}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{c^{\frac{7}{4}}} - \frac{\sqrt[4]{-1} a b \sqrt[4]{\frac{1}{d}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{c^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} a b \sqrt[4]{\frac{1}{d}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{c^{\frac{3}{4}}} - \frac{2 \sqrt[4]{-1} a b \sqrt[4]{\frac{1}{d}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{c^{\frac{3}{4}}} + \frac{\sqrt[4]{-1} b^{2} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \log{\left(- \sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 d} - \frac{\sqrt[4]{-1} b^{2} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \log{\left(\sqrt[4]{-1} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} + \sqrt{x} \right)}}{2 d} + \frac{\sqrt[4]{-1} b^{2} \sqrt[4]{c} \sqrt[4]{\frac{1}{d}} \operatorname{atan}{\left(\frac{\left(-1\right)^{\frac{3}{4}} \sqrt{x}}{\sqrt[4]{c} \sqrt[4]{\frac{1}{d}}} \right)}}{d} + \frac{2 b^{2} \sqrt{x}}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x))
, Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**
(5/2)/5)/c, Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2
*sqrt(x))/d, Eq(c, 0)), (-2*a**2/(3*c*x**(3/2)) + (-1)**(1/4)*a**2*d*(1/d)*
*(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(7/4)) - (-1
)**(1/4)*a**2*d*(1/d)**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x
))/(2*c**(7/4)) + (-1)**(1/4)*a**2*d*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/
(c**(1/4)*(1/d)**(1/4)))/c**(7/4) - (-1)**(1/4)*a*b*(1/d)**(1/4)*log(-(-1)*
*(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/c**(3/4) + (-1)**(1/4)*a*b*(1/d)**(
1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/c**(3/4) - 2*(-1)**(1
/4)*a*b*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/c**(
3/4) + (-1)**(1/4)*b**2*c**(1/4)*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/
d)**(1/4) + sqrt(x))/(2*d) - (-1)**(1/4)*b**2*c**(1/4)*(1/d)**(1/4)*log((-1
)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*d) + (-1)**(1/4)*b**2*c**(1/4)
*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/d + 2*b**2*
sqrt(x)/d, True))
```

3.404 $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=267

$$-\frac{2a^2}{5cx^{5/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-(bc-ad)^2$$

Rubi [A] time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 453, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2a^2}{5cx^{5/2}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(5*c*x^(5/2)) - (2*a*(2*b*c - a*d))/(c^2*Sqrt[x]) - ((b*c - a*d)^2
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(9/4)*d^(3/4)) +
 ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(
9/4)*d^(3/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*d^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)}\,dx = -\frac{2a^2}{5cx^{5/2}}+\frac{2\int \frac{\frac{5}{2}a(2bc-ad)+\frac{5}{2}b^2cx^2}{x^{3/2}\left(c+dx^2\right)}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}+\frac{(bc-ad)^2\int \frac{\sqrt{x}}{c+dx^2}\,dx}{c^2}$$

$$= -\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}+\frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{c^2}$$

$$= -\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}-\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx,x,\sqrt{x}\right)}{c^2\sqrt{d}}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \right.}{c^2\sqrt{}}$$

$$= -\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx,x,\sqrt{x}\right)}{2c^2d}+\frac{(bc-ad)^2\,\text{Subst}}{}$$

$$= -\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}+\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\log\left(\right.}{2}$$

$$= -\frac{2a^2}{5cx^{5/2}}-\frac{2a(2bc-ad)}{c^2\sqrt{x}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}d^{3/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{}}{\sqrt[4]{c}}\right.}{\sqrt{2}\,c^{9/4}d^{3/4}}$$

Mathematica [A] time = 0.12, size = 254, normalized size = 0.95

$$\frac{-\frac{8a^2c^{5/4}}{x^{5/2}}+\frac{5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{3/4}}-\frac{5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{3/4}}-\frac{10\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{3/4}}+\frac{10\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{3/4}}+\frac{40a\sqrt[4]{c}(ad-2bc)}{\sqrt{x}}}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a^2*c^(5/4))/x^(5/2) + (40*a*c^(1/4)*(-2*b*c + a*d))/Sqrt[x] - (10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) +
```

```
 (10*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d
^(3/4) + (5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/d^(3/4) - (5*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(20*c^(9/4))
```

IntegrateAlgebraic [A] time = 0.21, size = 165, normalized size = 0.62

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}c^{9/4}d^{3/4}}-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}c^{9/4}d^{3/4}}-\frac{2a\left(ac-5adx^2+10bcx^2\right)}{5c^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a*(a*c + 10*b*c*x^2 - 5*a*d*x^2))/(5*c^2*x^(5/2)) - ((b*c - a*d)^2*ArcT
an[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(S
qrt[2]*c^(9/4)*d^(3/4)) - ((b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(9/4)*d^(3/4))
```

fricas [B] time = 1.22, size = 1648, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*arctan((sqrt((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)*x - (b^8*c^13*d - 8*a*b^7*c^12*d^2 + 28*a^2*b^6*c^11*d^3
 - 56*a^3*b^5*c^10*d^4 + 70*a^4*b^4*c^9*d^5 - 56*a^5*b^3*c^8*d^6 + 28*a^6*b
^2*c^7*d^7 - 8*a^7*b*c^6*d^8 + a^8*c^5*d^9)*sqrt(-(b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3)))*c^2*d*
(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^9*d^3))^(1/4) - (b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d
^3 - 20*a^3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^
7)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6
```

```
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) - 5*c^2*x^3*(-(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3
))^(1/4)*log(c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a
^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d
^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d +
 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d
^5 + a^6*d^6)*sqrt(x)) + 5*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*log(-c^7*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x))
 + 4*(a^2*c + 5*(2*a*b*c - a^2*d)*x^2)*sqrt(x))/(c^2*x^3)
```

giac **[A]** time = 0.47, size = 353, normalized size = 1.32

$$-\frac{2\left(10\,abcx^2-5\,a^2dx^2+a^2c\right)}{5\,c^2x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^3d^3}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{3}{4}}b^2c^2-2\left(cd^3\right)^{\frac{3}{4}}abcd+\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -2/5*(10*a*b*c*x^2 - 5*a^2*d*x^2 + a^2*c)/(c^2*x^(5/2)) + 1/2*sqrt(2)*((c*d
^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan
(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/2
*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a
^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/
(c^3*d^3) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^
3*d^3) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*
d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*
d^3)
```

maple **[B]** time = 0.02, size = 452, normalized size = 1.69

$$\frac{\sqrt{2}\,a^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}c^2}+\frac{\sqrt{2}\,a^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}c^2}+\frac{\sqrt{2}\,a^2d\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}c^2}-\frac{\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d}+\frac{\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}d}+\frac{\sqrt{2}\,b^2\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d}+\frac{2a^2d}{c^2\sqrt{x}}-\frac{4ab}{c\sqrt{x}}-\frac{2a^2}{5c\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x)
```

```
[Out] 1/4/c^2*d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)
)/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-1/2/c/(c/d)^(1/4)*2^(1/2
```

```
)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(
1/2)+(c/d)^(1/2)))*a*b+1/4/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*
x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+1/2/c
^2*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-1/c/(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+1/2/d/(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+1/2/c^2*d/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-1/c/(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+1/2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-2/5*a^2/c/x^(5/2)+2*a^2/c^2/x^(1/2)*d-4*a/c
/x^(1/2)*b
```

maxima [A] time = 2.47, size = 229, normalized size = 0.86

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,c^2}-\frac{2\left(a^2c+5\left(2\,abc-a^2d\right)x^2\right)}{5\,c^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*s
qrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
- sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)))/c^2 - 2/5*(a^2*c + 5*(2*a*b*c - a^2*d)*x^2)/(c^2*
x^(5/2))
```

mupad [B] time = 0.34, size = 417, normalized size = 1.56

$$\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^4\,c^7\,d^6-64\,a^3\,b\,c^8\,d^5+96\,a^2\,b^2\,c^9\,d^4-64\,a\,b^3\,c^{10}\,d^3+16\,b^4\,c^{11}\,d^2\right)}{{\left(-c\right)}^{9/4}\,d^{3/4}\,\left(16\,a^6\,c^5\,d^7-96\,a^5\,b\,c^6\,d^6+240\,a^4\,b^2\,c^7\,d^5-320\,a^3\,b^3\,c^8\,d^4+240\,a^2\,b^4\,c^9\,d^3-96\,a\,b^5\,c^{10}\,d^2+16\,b^6\,c^{11}\,d\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{9/4}\,d^{3/4}}-\frac{\frac{2\,a^2}{5\,c}-\frac{2\,a\,x^2\,\left(a\,d-2\,b\,c\right)}{c^2}}{x^{5/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^4\,c^7\,d^6-64\,a^3\,b\,c^8\,d^5+96\,a^2\,b^2\,c^9\,d^4-64\,a\,b^3\,c^{10}\,d^3+16\,b^4\,c^{11}\,d^2\right)}{{\left(-c\right)}^{9/4}\,d^{3/4}\,\left(16\,a^6\,c^5\,d^7-96\,a^5\,b\,c^6\,d^6+240\,a^4\,b^2\,c^7\,d^5-320\,a^3\,b^3\,c^8\,d^4+240\,a^2\,b^4\,c^9\,d^3-96\,a\,b^5\,c^{10}\,d^2+16\,b^6\,c^{11}\,d\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{{\left(-c\right)}^{9/4}\,d^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x)
```

```
[Out] (atan((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^7*d^6 + 16*b^4*c^11*d^2 - 64*a*b^3*c
^10*d^3 - 64*a^3*b*c^8*d^5 + 96*a^2*b^2*c^9*d^4))/((-c)^(9/4)*d^(3/4)*(16*b
^6*c^11*d + 16*a^6*c^5*d^7 - 96*a*b^5*c^10*d^2 - 96*a^5*b*c^6*d^6 + 240*a^2
*b^4*c^9*d^3 - 320*a^3*b^3*c^8*d^4 + 240*a^4*b^2*c^7*d^5)))*(a*d - b*c)^2)/
((-c)^(9/4)*d^(3/4)) - ((2*a^2)/(5*c) - (2*a*x^2*(a*d - 2*b*c))/c^2)/x^(5/2
) - (atanh((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^7*d^6 + 16*b^4*c^11*d^2 - 64*a*
b^3*c^10*d^3 - 64*a^3*b*c^8*d^5 + 96*a^2*b^2*c^9*d^4))/((-c)^(9/4)*d^(3/4)*
```

```
(16*b^6*c^11*d + 16*a^6*c^5*d^7 - 96*a*b^5*c^10*d^2 - 96*a^5*b*c^6*d^6 + 24
0*a^2*b^4*c^9*d^3 - 320*a^3*b^3*c^8*d^4 + 240*a^4*b^2*c^7*d^5)))*(a*d - b*c
)^2)/((-c)^(9/4)*d^(3/4))
```

sympy [A] time = 125.08, size = 406, normalized size = 1.52

$$a^2\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: c = 0 \wedge d = 0 \\ -\frac{2}{9dx^{\frac{9}{2}}} & \text{for}\: c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for}\: d = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} + \frac{2d}{c^2\sqrt{x}} - \frac{(-1)^{\frac{3}{4}}d\log\left(-\sqrt[4]{-1}\,\sqrt[4]{c}\,\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2c^{\frac{9}{4}}\sqrt[4]{\frac{1}{d}}} + \frac{(-1)^{\frac{3}{4}}d\log\left(\sqrt[4]{-1}\,\sqrt[4]{c}\,\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2c^{\frac{9}{4}}\sqrt[4]{\frac{1}{d}}} + \frac{(-1)^{\frac{3}{4}}d\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{c}\,\sqrt[4]{\frac{1}{d}}}\right)}{c^{\frac{9}{4}}\sqrt[4]{\frac{1}{d}}} & \text{otherwise} \end{cases}\right) + 2ab\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: c = 0 \wedge d = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for}\: d = 0 \\ -\frac{2}{5dx^{\frac{5}{2}}} & \text{for}\: c = 0 \\ -\frac{2}{c\sqrt{x}} + \frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{c}\,\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2c^{\frac{5}{4}}\sqrt[4]{\frac{1}{d}}} - \frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{c}\,\sqrt[4]{\frac{1}{d}} + \sqrt{x}\right)}{2c^{\frac{5}{4}}\sqrt[4]{\frac{1}{d}}} - \frac{(-1)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{c}\,\sqrt[4]{\frac{1}{d}}}\right)}{c^{\frac{5}{4}}\sqrt[4]{\frac{1}{d}}} & \text{otherwise} \end{cases}\right) + 2b^2\operatorname{RootSum}\left(256t^4cd^3 + 1, \left(t \mapsto t\log\left(64t^3cd^2 + \sqrt{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c),x)
```

```
[Out] a**2*Piecewise((zoo/x**(9/2), Eq(c, 0) & Eq(d, 0)), (-2/(9*d*x**(9/2)), Eq(
c, 0)), (-2/(5*c*x**(5/2)), Eq(d, 0)), (-2/(5*c*x**(5/2)) + 2*d/(c**2*sqrt(
x)) - (-1)**(3/4)*d*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c*
*(9/4)*(1/d)**(1/4)) + (-1)**(3/4)*d*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4)
+ sqrt(x))/(2*c**(9/4)*(1/d)**(1/4)) + (-1)**(3/4)*d*atan((-1)**(3/4)*sqrt(
x)/(c**(1/4)*(1/d)**(1/4)))/(c**(9/4)*(1/d)**(1/4)), True)) + 2*a*b*Piecewi
se((zoo/x**(5/2), Eq(c, 0) & Eq(d, 0)), (-2/(c*sqrt(x)), Eq(d, 0)), (-2/(5*
d*x**(5/2)), Eq(c, 0)), (-2/(c*sqrt(x)) + (-1)**(3/4)*log(-(-1)**(1/4)*c**(
1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*(1/d)**(1/4)) - (-1)**(3/4)*log((-
1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(2*c**(5/4)*(1/d)**(1/4)) - (-1)
**(3/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(c**(5/4)*(1/d)**
(1/4)), True)) + 2*b**2*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*
_t**3*c*d**2 + sqrt(x))))
```

3.405 $\int \frac{\left(a+bx^2\right)^2}{x^{9/2}\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=269

$$-\frac{2a^2}{7cx^{7/2}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}}-\frac{(bc-ad}{}$$

Rubi [A] time = 0.28, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 453, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2a^2}{7cx^{7/2}}-\frac{2a(2bc-ad)}{3c^2x^{3/2}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a^2)/(7*c*x^(7/2)) - (2*a*(2*b*c - a*d))/(3*c^2*x^(3/2)) - ((b*c - a*d)
^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(11/4)*d^(1/4)
) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*
c^(11/4)*d^(1/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(11/4)*d^(1
/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{9/2}\left(c+dx^2\right)}\,dx = -\frac{2a^2}{7cx^{7/2}} + \frac{2\int \frac{\frac{7}{2}a(2bc-ad)+\frac{7}{2}b^2cx^2}{x^{5/2}\left(c+dx^2\right)}\,dx}{7c}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{c^2}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^2}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c^{5/2}} + \frac{(bc-ad)^2\operatorname{Subst}\Big(\int}{c}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{2c^{5/2}\sqrt{d}} + \frac{(bc-ad)^2\operatorname{Sub}}{}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\log\Big(}{}$$

$$= -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\tan^{-1}\Big(1+\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}$$

Mathematica [**A**] time = 0.13, size = 254, normalized size = 0.94

$$\frac{-\frac{24a^2c^{7/4}}{x^{7/2}}+\frac{56ac^{3/4}(ad-2bc)}{x^{3/2}}-\frac{21\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{\sqrt[4]{d}}+\frac{21\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{\sqrt[4]{d}}-\frac{42\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt[4]{d}}+\frac{42\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt[4]{d}}}{84c^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] ((-24*a^2*c^(7/4))/x^(7/2) + (56*a*c^(3/4)*(-2*b*c + a*d))/x^(3/2) - (42*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(1/4)
```

```
+ (42*Sqrt[2]*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
d^(1/4) - (21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/d^(1/4) + (21*Sqrt[2]*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(1/4))/(84*c^(11/4))
```

IntegrateAlgebraic [A] time = 0.20, size = 165, normalized size = 0.61

$$-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}+\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{11/4}\sqrt[4]{d}}-\frac{2a\left(3ac-7adx^2+14bcx^2\right)}{21c^2x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

```
[Out] (-2*a*(3*a*c + 14*b*c*x^2 - 7*a*d*x^2))/(21*c^2*x^(7/2)) - ((b*c - a*d)^2*A
rcTan[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])
/(Sqrt[2]*c^(11/4)*d^(1/4)) + ((b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(11/4)*d^(1/4))
```

fricas [B] time = 1.34, size = 1252, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*arctan((sqrt(c^6*sqrt(-(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d
^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
11*d)) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*x)*c^8*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(3/4) - (b^2*c^10*d - 2*a*b*c^9*d^2 + a^2
*c^8*d^3)*sqrt(x)*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(3/4))/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)) + 21*c^2*x^4*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(
c^11*d))^(1/4)*log(c^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
```

```
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sqrt(x)) - 21*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(-c^3*(-(b^8*c^8 -
 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d
^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^
11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 4*(3*a^2*c + 7*(2
*a*b*c - a^2*d)*x^2)*sqrt(x))/(c^2*x^4)
```

giac [**A**] time = 0.48, size = 354, normalized size = 1.32

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^{3}d} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2c^{3}d} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4c^{3}d} - \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4c^{3}d} - \frac{2\left(14abcx^{2} - 7a^{2}dx^{2} + 3a^{2}c\right)}{21c^{2}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c^3*d) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x)
)/(c/d)^(1/4))/(c^3*d) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/
4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c^3*d) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*
b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(c^3*d) - 2/21*(14*a*b*c*x^2 - 7*a^2*d*x^2 + 3*a^2*c)/(c^2*x^(7/2))
```

maple [**B**] time = 0.02, size = 461, normalized size = 1.71

$$\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2c^{3}} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2c^{3}} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4c^{3}} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{c^{2}} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{c^{2}} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abd\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{2c^{2}} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4c} + \frac{2a^{2}d}{3c^{2}x^{\frac{3}{2}}} - \frac{4ab}{3c\,x^{\frac{3}{2}}} - \frac{2a^{2}}{7c\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x)
```

```
[Out] 1/2/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2*d^2-1
/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b*d+1/2/c*
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/4/c^3*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4
)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2*d^2-1/2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))*a*b*d+1/4/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+1/2/c^3*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2-1/c^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d+1/2/c*(c/d)^(1/4)*
```

```
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2/7*a^2/c/x^(7/2)+2/3*a^2
/c^2/x^(3/2)*d-4/3*a/c/x^(3/2)*b
```

maxima [A] time = 2.36, size = 293, normalized size = 1.09

$$\frac{\frac{2\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{4\,c^2}-\frac{2\left(3\,a^2c+7\left(2\,abc-a^2d\right)x^2\right)}{21\,c^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/
(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))
 - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/c^2 - 2/21*(3*a^2*c + 7*(2*a
*b*c - a^2*d)*x^2)/(c^2*x^(7/2))
```

mupad [B] time = 0.44, size = 1209, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x)
```

```
[Out] (atan(((((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64
*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 +
16*b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2*
1i)/((-c)^(11/4)*d^(1/4)) + (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 -
64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)
^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(
1/4)))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4)))/((((x^(1/2)*(16*a^4*c^6*d^7
 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d
^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4
))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d^(1/4)) - (((x^(1/
2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6
+ 96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3
 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d
^(1/4))))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4)) - ((2*a^2)/(7*c) - (2*a*x
^2*(a*d - 2*b*c))/(3*c^2))/x^(7/2) + (atan(((((x^(1/2)*(16*a^4*c^6*d^7 + 16
*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/
2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4)*1i)
```

```
/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d^(1/4)) + (((x^(1/2)
*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 +
96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 -
 32*a*b*c^10*d^4)*1i)/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*
d^(1/4)))/((((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4
- 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^
5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4)*1i)/(2*(-c)^(11/4)*d^(1/4)))*(a*d -
b*c)^2*1i)/((-c)^(11/4)*d^(1/4)) - (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10
*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d
 - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4)*1i)/(2*(-c)^
(11/4)*d^(1/4)))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4))))*(a*d - b*c)^2)/(
(-c)^(11/4)*d^(1/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(9/2)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.406 $\int \frac{\left(c+dx^2\right)^2}{x^{11/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=288

$$-\frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}+\sqrt[4]{b}\,(bc-a$$

Rubi [A] time = 0.32, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 453, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}-\frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{13/4}}-\frac{2c^2}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^2)/(9*a*x^(9/2)) + (2*c*(b*c - 2*a*d))/(5*a^2*x^(5/2)) - (2*(b*c - a*
d)^2)/(a^3*Sqrt[x]) + (b^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)) - (b^(1/4)*(b*c - a*d)^
2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(13/4)) + (b^(1/4)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^2}{x^{11/2}\left(a+bx^2\right)}\,dx = -\frac{2c^2}{9ax^{9/2}} + \frac{2\int \frac{-\frac{9}{2}c(bc-2ad)+\frac{9}{2}ad^2x^2}{x^{7/2}\left(a+bx^2\right)}\,dx}{9a}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} + \frac{(bc-ad)^2\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx}{a^2}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{\left(b(bc-ad)^2\right)\int \frac{\sqrt{x}}{a+bx^2}\,dx}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{\left(2b(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^3}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} + \frac{\left(\sqrt{b}\,(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^3} -$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^3} \quad (- -$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} - \frac{\sqrt[4]{b}\,(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}}$$

$$= -\frac{2c^2}{9ax^{9/2}} + \frac{2c(bc-2ad)}{5a^2x^{5/2}} - \frac{2(bc-ad)^2}{a^3\sqrt{x}} + \frac{\sqrt[4]{b}\,(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}} - \sqrt[4]{b}\,(bc-a$$

Mathematica [C] time = 0.21, size = 101, normalized size = 0.35

$$-\frac{2\left(a\left(a^2\left(5c^2+18cdx^2+45d^2x^4\right)-9abcx^2\left(c+10dx^2\right)+45b^2c^2x^4\right)+15bx^6(bc-ad)^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{45a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 10*d*x^2) + a^2*(5*c^2 + 18*c*d*x
^2 + 45*d^2*x^4)) + 15*b*(b*c - a*d)^2*x^6*Hypergeometric2F1[3/4, 1, 7/4, -
((b*x^2)/a)]))/(45*a^4*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 204, normalized size = 0.71

$$\frac{\sqrt[4]{b}\,(ad-bc)^2\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{13/4}}+\frac{\sqrt[4]{b}\,(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{13/4}}-\frac{2\left(5a^2c^2+18a^2cdx^2+45a^2d^2x^4-9abc^2x^2-90abcdx^4+45b^2c^2x^4\right)}{45a^3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(5*a^2*c^2 - 9*a*b*c^2*x^2 + 18*a^2*c*d*x^2 + 45*b^2*c^2*x^4 - 90*a*b*c
*d*x^4 + 45*a^2*d^2*x^4))/(45*a^3*x^(9/2)) + (b^(1/4)*(-(b*c) + a*d)^2*ArcT
an[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(S
qrt[2]*a^(13/4)) + (b^(1/4)*(-(b*c) + a*d)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(13/4))
```

fricas [B] time = 1.47, size = 1686, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/90*(180*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*
b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6
- 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*arctan((sqrt((b^14*c^12 - 12*a*b
^13*c^11*d + 66*a^2*b^12*c^10*d^2 - 220*a^3*b^11*c^9*d^3 + 495*a^4*b^10*c^8
*d^4 - 792*a^5*b^9*c^7*d^5 + 924*a^6*b^8*c^6*d^6 - 792*a^7*b^7*c^5*d^7 + 49
5*a^8*b^6*c^4*d^8 - 220*a^9*b^5*c^3*d^9 + 66*a^10*b^4*c^2*d^10 - 12*a^11*b^
3*c*d^11 + a^12*b^2*d^12)*x - (a^7*b^9*c^8 - 8*a^8*b^8*c^7*d + 28*a^9*b^7*c
^6*d^2 - 56*a^10*b^6*c^5*d^3 + 70*a^11*b^5*c^4*d^4 - 56*a^12*b^4*c^3*d^5 +
28*a^13*b^3*c^2*d^6 - 8*a^14*b^2*c*d^7 + a^15*b*d^8)*sqrt(-(b^9*c^8 - 8*a*b
^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 5
```

```
6*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)
)*a^3*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3
+ 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*
c*d^7 + a^8*b*d^8)/a^13)^(1/4) - (a^3*b^7*c^6 - 6*a^4*b^6*c^5*d + 15*a^5*b^
5*c^4*d^2 - 20*a^6*b^4*c^3*d^3 + 15*a^7*b^3*c^2*d^4 - 6*a^8*b^2*c*d^5 + a^9
*b*d^6)*sqrt(x)*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^
6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 -
8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2
*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5
 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)) - 45*a^3*x^5*(-(b^9*c
^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c
^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*
d^8)/a^13)^(1/4)*log(a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 -
 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*
c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5
*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b
^2*c*d^5 + a^6*b*d^6)*sqrt(x)) + 45*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28
*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3
*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*log(-a
^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 +
70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*
d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^
2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*
sqrt(x)) - 4*(45*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 5*a^2*c^2 - 9*(a*b*c
^2 - 2*a^2*c*d)*x^2)*sqrt(x))/(a^3*x^5)
```

giac [**A**]   time = 0.47, size = 390, normalized size = 1.35

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(ab^{3}\right)^{\frac{3}{4}} abcd + \left(ab^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{4}b^{2}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(ab^{3}\right)^{\frac{3}{4}} abcd + \left(ab^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{4}b^{2}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(ab^{3}\right)^{\frac{3}{4}} abcd + \left(ab^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^{4}b^{2}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}} b^{2}c^{2} - 2\left(ab^{3}\right)^{\frac{3}{4}} abcd + \left(ab^{3}\right)^{\frac{3}{4}} a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^{4}b^{2}} - \frac{2\left(45\,b^{2}c^{2}x^{4} - 90\,abcdx^{4} + 45\,a^{2}d^{2}x^{4} - 9\,abc^{2}x^{2} + 18\,a^{2}cdx^{2} + 5\,a^{2}c^{2}\right)}{45\,a^{3}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4
))/(a^4*b^2) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d
 + (a*b^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3
)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a^4*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(
3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
+ sqrt(a/b))/(a^4*b^2) - 2/45*(45*b^2*c^2*x^4 - 90*a*b*c*d*x^4 + 45*a^2*d^2
*x^4 - 9*a*b*c^2*x^2 + 18*a^2*c*d*x^2 + 5*a^2*c^2)/(a^3*x^(9/2))
```

maple [B] time = 0.02, size = 495, normalized size = 1.72

$$-\frac{\sqrt{2}\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,bcd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,bcd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,bcd\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{\sqrt{2}\,b^{2}c^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}-\frac{2d^{2}}{a\sqrt{x}}+\frac{4bcd}{a^{2}\sqrt{x}}-\frac{2b^{2}c^{2}}{a^{3}\sqrt{x}}-\frac{4cd}{5a\,x^{\frac{5}{2}}}+\frac{2b\,c^{2}}{5a^{2}x^{\frac{5}{2}}}-\frac{2c^{2}}{9a\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x)
```

```
[Out] -1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^2+1/a^2/
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b*c*d-1/2/a^3/(a/
b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^2*c^2-1/4/a/(a/b)^
(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^2+1/2/a^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2)))*b*c*d-1/4/a^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b^2*c^2-1/2/a/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^2+1/a^2/(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b*c*d-1/2/a^3/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^2*c^2-2/9*c^2/a/x^(9/2)-2/a/x^
(1/2)*d^2+4/a^2/x^(1/2)*b*c*d-2/a^3/x^(1/2)*b^2*c^2-4/5*c/a/x^(5/2)*d+2/5*c
^2/a^2/x^(5/2)*b
```

maxima [A] time = 2.60, size = 263, normalized size = 0.91

$$-\frac{\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^{3}}-\frac{2\left(45\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{4}+5\,a^{2}c^{2}-9\left(abc^{2}-2\,a^{2}cd\right)x^{2}\right)}{45\,a^{3}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqr
t(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt
(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3 - 2/45*(45*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x^4 + 5*a^2*c^2 - 9*(a*b*c^2 - 2*a^2*c*d)*x^2)/(a^3*x^(9/2))
```

mupad [B] time = 0.35, size = 451, normalized size = 1.57

$$\frac{{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^{14}\,b^4\,d^4-64\,a^{13}\,b^5\,c\,d^3+96\,a^{12}\,b^6\,c^2\,d^2-64\,a^{11}\,b^7\,c^3\,d+16\,a^{10}\,b^8\,c^4\right)}{a^{13/4}\,\left(16\,a^{13}\,b^4\,d^6-96\,a^{12}\,b^5\,c\,d^5+240\,a^{11}\,b^6\,c^2\,d^4-320\,a^{10}\,b^7\,c^3\,d^3+240\,a^9\,b^8\,c^4\,d^2-96\,a^8\,b^9\,c^5\,d+16\,a^7\,b^{10}\,c^6\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{13/4}}-\frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,\sqrt{x}\,{\left(a\,d-b\,c\right)}^2\,\left(16\,a^{14}\,b^4\,d^4-64\,a^{13}\,b^5\,c\,d^3+96\,a^{12}\,b^6\,c^2\,d^2-64\,a^{11}\,b^7\,c^3\,d+16\,a^{10}\,b^8\,c^4\right)}{a^{13/4}\,\left(16\,a^{13}\,b^4\,d^6-96\,a^{12}\,b^5\,c\,d^5+240\,a^{11}\,b^6\,c^2\,d^4-320\,a^{10}\,b^7\,c^3\,d^3+240\,a^9\,b^8\,c^4\,d^2-96\,a^8\,b^9\,c^5\,d+16\,a^7\,b^{10}\,c^6\right)}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{13/4}}-\frac{\frac{2\,c^2}{9\,a}+\frac{2\,x^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{a^3}+\frac{2\,c\,x^2\,\left(2\,a\,d-b\,c\right)}{5\,a^2}}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2)*(a*d - b*c)^2*(16*a^10*b^8*c^4 + 16*a
^14*b^4*d^4 - 64*a^11*b^7*c^3*d - 64*a^13*b^5*c*d^3 + 96*a^12*b^6*c^2*d^2))
/(a^(13/4)*(16*a^7*b^10*c^6 + 16*a^13*b^4*d^6 - 96*a^8*b^9*c^5*d - 96*a^12*
b^5*c*d^5 + 240*a^9*b^8*c^4*d^2 - 320*a^10*b^7*c^3*d^3 + 240*a^11*b^6*c^2*d
^4)))*(a*d - b*c)^2)/a^(13/4) - ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2)*(a*d -
 b*c)^2*(16*a^10*b^8*c^4 + 16*a^14*b^4*d^4 - 64*a^11*b^7*c^3*d - 64*a^13*b^
5*c*d^3 + 96*a^12*b^6*c^2*d^2))/(a^(13/4)*(16*a^7*b^10*c^6 + 16*a^13*b^4*d^
6 - 96*a^8*b^9*c^5*d - 96*a^12*b^5*c*d^5 + 240*a^9*b^8*c^4*d^2 - 320*a^10*b
^7*c^3*d^3 + 240*a^11*b^6*c^2*d^4)))*(a*d - b*c)^2)/a^(13/4) - ((2*c^2)/(9*
a) + (2*x^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/a^3 + (2*c*x^2*(2*a*d - b*c))/
(5*a^2))/x^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2/x**(11/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.407 $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=375

$$\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{17/4}}-\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{}\right.}{8\sqrt{2}\,d^{17/4}}$$

Rubi [A] time = 0.44, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^{5/2}(13bc-5ad)(bc-ad)}{10cd^3}+\frac{\sqrt{x}\,(13bc-5ad)(bc-ad)}{2d^4}+\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{17/4}}-\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{17/4}}+\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{17/4}}-\frac{\sqrt[4]{c}\,(13bc-5ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{17/4}}+\frac{2b^2x^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] ((13*b*c - 5*a*d)*(b*c - a*d)*Sqrt[x])/(2*d^4) - ((13*b*c - 5*a*d)*(b*c - a
*d)*x^(5/2))/(10*c*d^3) + (2*b^2*x^(9/2))/(9*d^2) + ((b*c - a*d)^2*x^(9/2))
/(2*c*d^2*(c + d*x^2)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*b*c
- 5*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt
[2]*d^(17/4)) + (c^(1/4)*(13*b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(17/4)) - (c^(1/4)*(13*
b*c - 5*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*d^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^{7/2}\left(\frac{1}{2}(3bc-5ad)(3bc-ad)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - \frac{((13bc-5ad)(bc-ad))\int \frac{x^{7/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} + \frac{((13bc-5ad)(bc-ad))\int \frac{x^{3/2}}{c+dx^2}}{4d^3}$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - (c(13$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - (c(13$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - \left(\sqrt{c}\right.$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} - \left(\sqrt{c}\right.$$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} + \sqrt[4]{c}\,($$

$$= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4} - \frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3} + \frac{2b^2x^{9/2}}{9d^2} + \frac{(bc-ad)^2x^{9/2}}{2cd^2\left(c+dx^2\right)} + \sqrt[4]{c}\,($$

Mathematica [A] time = 0.36, size = 372, normalized size = 0.99

$$\frac{1440\sqrt[4]{d}\sqrt{x}\left(a^2d^2-4abcd+3b^2c^2\right)+45\sqrt{2}\sqrt[4]{c}\left(5a^2d^2-18abcd+13b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-45\sqrt{2}\sqrt[4]{c}\left(5a^2d^2-18abcd+13b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+90\sqrt{2}\sqrt[4]{c}\left(5a^2d^2-18abcd+13b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-90\sqrt{2}\sqrt[4]{c}\left(5a^2d^2-18abcd+13b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-576bd^{5/4}x^{5/2}(bc-ad)+\frac{360c\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{c+dx^2}+160b^2d^{9/4}x^{9/2}}{720d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (1440*d^(1/4)*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[x] - 576*b*d^(5/4)*(b*
c - a*d)*x^(5/2) + 160*b^2*d^(9/4)*x^(9/2) + (360*c*d^(1/4)*(b*c - a*d)^2*S
qrt[x])/(c + d*x^2) + 90*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 90*Sqrt[2]*c^(1/4)*(13*
b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)] + 45*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - 45*Sqrt[2]*c^(1/4)*(13*b^
2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(720*d^(17/4))
```

IntegrateAlgebraic [A] time = 0.78, size = 284, normalized size = 0.76

$$\frac{\left(5a^2\sqrt[4]{c}\,d^2-18abc^{5/4}d+13b^2c^{9/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,d^{17/4}}-\frac{\left(5a^2\sqrt[4]{c}\,d^2-18abc^{5/4}d+13b^2c^{9/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,d^{17/4}}+\frac{\sqrt{x}\left(225a^2cd^2+180a^2d^3x^2-810abc^2d-648abcd^2x^2+72abd^3x^4+585b^2c^3+468b^2c^2dx^2-52b^2cd^2x^4+20b^2d^3x^6\right)}{90d^4\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[x]*(585*b^2*c^3 - 810*a*b*c^2*d + 225*a^2*c*d^2 + 468*b^2*c^2*d*x^2 -
 648*a*b*c*d^2*x^2 + 180*a^2*d^3*x^2 - 52*b^2*c*d^2*x^4 + 72*a*b*d^3*x^4 +
20*b^2*d^3*x^6))/(90*d^4*(c + d*x^2)) + ((13*b^2*c^(9/4) - 18*a*b*c^(5/4)*d
 + 5*a^2*c^(1/4)*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x])])/(4*Sqrt[2]*d^(17/4)) - ((13*b^2*c^(9/4) - 18*a*b*c^(5/4)*d + 5*
a^2*c^(1/4)*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[
d]*x)])/(4*Sqrt[2]*d^(17/4))
```

fricas [B] time = 0.77, size = 1373, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(180*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 37247
6*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 18684
0*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*
d^8)/d^17)^(1/4)*arctan((sqrt(d^8*sqrt(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d
 + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4
 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 62
5*a^8*c*d^8)/d^17) + (169*b^4*c^4 - 468*a*b^3*c^3*d + 454*a^2*b^2*c^2*d^2 -
 180*a^3*b*c*d^3 + 25*a^4*d^4)*x)*d^13*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*
d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^
4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 6
25*a^8*c*d^8)/d^17)^(3/4) - (13*b^2*c^2*d^13 - 18*a*b*c*d^14 + 5*a^2*d^15)*
sqrt(x)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 48
5784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55
```

```
100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(3/4))/(285
61*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c
^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^
3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)) + 45*(d^5*x^2 + c*d^4)*(-(2856
1*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^
6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3
*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*log(d^4*(-(28561*b^8
*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3
 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6
- 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*
d + 5*a^2*d^2)*sqrt(x)) - 45*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*
b^7*c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^
4*c^5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2
*d^7 + 625*a^8*c*d^8)/d^17)^(1/4)*log(-d^4*(-(28561*b^8*c^9 - 158184*a*b^7*
c^8*d + 372476*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^
5*d^4 - 186840*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7
 + 625*a^8*c*d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*sqrt(
x)) - 4*(20*b^2*d^3*x^6 + 585*b^2*c^3 - 810*a*b*c^2*d + 225*a^2*c*d^2 - 4*(
13*b^2*c*d^2 - 18*a*b*d^3)*x^4 + 36*(13*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^
3)*x^2)*sqrt(x))/(d^5*x^2 + c*d^4)
```

giac [A] time = 0.39, size = 440, normalized size = 1.17

$$-\frac{\sqrt{2}\left(13\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8d^{5}}-\frac{\sqrt{2}\left(13\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8d^{5}}-\frac{\sqrt{2}\left(13\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16d^{5}}+\frac{\sqrt{2}\left(13\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-18\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16d^{5}}+\frac{b^{2}c^{3}\sqrt{x}-2abc^{2}d\sqrt{x}+a^{2}cd^{2}\sqrt{x}}{2\left(dx^{2}+c\right)d^{4}}+\frac{2\left(5b^{2}d^{16}x^{\frac{9}{2}}-18b^{2}cd^{15}x^{\frac{5}{2}}+18abd^{16}x^{\frac{5}{2}}+135b^{2}c^{2}d^{14}\sqrt{x}-180abcd^{15}\sqrt{x}+45a^{2}d^{16}\sqrt{x}\right)}{45d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/d^5 - 1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b
*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
2*sqrt(x))/(c/d)^(1/4))/d^5 - 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(
c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/d^5 + 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*
d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/d^5 + 1/2*(b^2*c^3*sqrt(x) - 2*a*b*c^2*d*sqrt(x) + a^2
*c*d^2*sqrt(x))/((d*x^2 + c)*d^4) + 2/45*(5*b^2*d^16*x^(9/2) - 18*b^2*c*d^1
5*x^(5/2) + 18*a*b*d^16*x^(5/2) + 135*b^2*c^2*d^14*sqrt(x) - 180*a*b*c*d^15
*sqrt(x) + 45*a^2*d^16*sqrt(x))/d^18
```

maple [A] time = 0.02, size = 563, normalized size = 1.50

$$\frac{2b^{2}x^{\frac{9}{2}}}{9d^{2}}+\frac{4abx^{\frac{5}{2}}}{5d^{2}}-\frac{4b^{2}cx^{\frac{5}{2}}}{5d^{3}}+\frac{a^{2}c\sqrt{x}}{2\left(dx^{2}+c\right)d^{2}}-\frac{abc^{2}\sqrt{x}}{\left(dx^{2}+c\right)d^{3}}+\frac{b^{2}c^{3}\sqrt{x}}{2\left(dx^{2}+c\right)d^{4}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8d^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8d^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16d^{2}}+\frac{9\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4d^{3}}+\frac{9\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4d^{3}}+\frac{9\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,abc\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8d^{3}}-\frac{13\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8d^{4}}-\frac{13\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8d^{4}}-\frac{13\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16d^{4}}+\frac{2a^{2}\sqrt{x}}{d^{2}}-\frac{8abc\sqrt{x}}{d^{3}}+\frac{6b^{2}c^{2}\sqrt{x}}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/9*b^2*x^(9/2)/d^2+4/5/d^2*x^(5/2)*a*b-4/5/d^3*x^(5/2)*b^2*c+2/d^2*a^2*x^(
1/2)-8/d^3*a*b*c*x^(1/2)+6/d^4*b^2*c^2*x^(1/2)+1/2*c/d^2*x^(1/2)/(d*x^2+c)*
a^2-c^2/d^3*x^(1/2)/(d*x^2+c)*a*b+1/2*c^3/d^4*x^(1/2)/(d*x^2+c)*b^2-5/8/d^2
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+9/4*c/d^3*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-13/8*c^2/d^4*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-5/16/d^2*(c/d)^
(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)
*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+9/8*c/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d
)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
1/2)))*a*b-13/16*c^2/d^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-5/8/d^2*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+9/4*c/d^3*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-13/8*c^2/d^4*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [**A**] time = 2.47, size = 377, normalized size = 1.01

$$\frac{\left(b^{2}c^{3} - 2\,abc^{2}d + a^{2}cd^{2}\right)\sqrt{x}}{2\left(d^{5}x^{2} + cd^{4}\right)} - \frac{\left(\frac{2\,\sqrt{2}\left(13\,b^{2}c^{2} - 18\,abcd + 5\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}} + 2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}\left(13\,b^{2}c^{2} - 18\,abcd + 5\,a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}} - 2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}\left(13\,b^{2}c^{2} - 18\,abcd + 5\,a^{2}d^{2}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x} + \sqrt{d}x + \sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}} - \frac{\sqrt{2}\left(13\,b^{2}c^{2} - 18\,abcd + 5\,a^{2}d^{2}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x} + \sqrt{d}x + \sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{16\,d^{4}} + \frac{2\left(5\,b^{2}d^{2}x^{\frac{9}{2}} - 18\left(b^{2}cd - abd^{2}\right)x^{\frac{5}{2}} + 45\left(3\,b^{2}c^{2} - 4\,abcd + a^{2}d^{2}\right)\sqrt{x}\right)}{45\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)/(d^5*x^2 + c*d^4) - 1/16*(2
*sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + 2*sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d +
5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3
/4)*d^(1/4)) - sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/d^4 + 2/
45*(5*b^2*d^2*x^(9/2) - 18*(b^2*c*d - a*b*d^2)*x^(5/2) + 45*(3*b^2*c^2 - 4*
a*b*c*d + a^2*d^2)*sqrt(x))/d^4
```

mupad [**B**] time = 0.44, size = 1367, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] x^(1/2)*((2*a^2)/d^2 + (2*c*((4*b^2*c)/d^3 - (4*a*b)/d^2))/d - (2*b^2*c^2)/
d^4) - x^(5/2)*((4*b^2*c)/(5*d^3) - (4*a*b)/(5*d^2)) + (x^(1/2)*((b^2*c^3)/
2 + (a^2*c*d^2)/2 - a*b*c^2*d))/(c*d^4 + d^5*x^2) + (2*b^2*x^(9/2))/(9*d^2)
 + ((-c)^(1/4)*atan((((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 -
180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 + ((-c)^(1/
4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)
)/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(8*d^(17/4)) + ((-c)^(1/4)*((x
^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*
d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*
b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*
b*c)*1i)/(8*d^(17/4)))/(((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4
 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 + ((-c)^
(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3
*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c))/(8*d^(17/4)) - ((-c)^(1/4)*((x
^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*
d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*
b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*
b*c))/(8*d^(17/4))))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(4*d^(17/4)) - ((-c)^
(1/4)*atan((((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b
*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d -
 b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d^(2
1/4))*(a*d - b*c)*(5*a*d - 13*b*c))/(8*d^(17/4)) + ((-c)^(1/4)*((x^(1/2)*(1
69*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468
*a*b^3*c^5*d))/d^5 + ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 +
 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c))/
(8*d^(17/4)))/(((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^
3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*
d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d
^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(8*d^(17/4)) - ((-c)^(1/4)*((x^(1
/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2
 - 468*a*b^3*c^5*d))/d^5 + ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2
*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d)*1i)/d^(21/4))*(a*d - b*c)*(5*a*d - 13*
b*c)*1i)/(8*d^(17/4))))*(a*d - b*c)*(5*a*d - 13*b*c))/(4*d^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.408 $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=346

$$\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{c}\,d^{15/4}}-\frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\right.}{8\sqrt{2}\sqrt[4]{c}\,d^{15/4}}$$

Rubi [A] time = 0.32, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 459, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{x^{3/2}(11bc-3ad)(bc-ad)}{6cd^3}+\frac{(11bc-3ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{c}\,d^{15/4}}-\frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{c}\,d^{15/4}}-\frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}\,d^{15/4}}+\frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}\,d^{15/4}}+\frac{2b^2x^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((11*b*c - 3*a*d)*(b*c - a*d)*x^(3/2))/(6*c*d^3) + (2*b^2*x^(7/2))/(7*d^2)
 + ((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) - ((11*b*c - 3*a*d)*(b*c -
 a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(
15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) + ((11*b*c - 3*a*d)*(b*c - a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)
*d^(15/4)) - ((11*b*c - 3*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^{5/2}\left(\frac{1}{2}\left(-4a^2d^2+7(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{((11bc-3ad)(bc-ad))\int \frac{x^{5/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{((11bc-3ad)(bc-ad))\int \frac{\sqrt{x}}{c+dx^2}}{4d^3}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{((11bc-3ad)(bc-ad))\,\text{Subst}\left(\right.}{2d^3}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{((11bc-3ad)(bc-ad))\,\text{Subst}\left(\right.}{4d^{7/2}}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{((11bc-3ad)(bc-ad))\,\text{Subst}\left(\right.}{8d^4}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} + \frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{c}\right.}{8\sqrt{2}\sqrt[4]{c}\,d}$$

$$= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3} + \frac{2b^2x^{7/2}}{7d^2} + \frac{(bc-ad)^2x^{7/2}}{2cd^2\left(c+dx^2\right)} - \frac{(11bc-3ad)(bc-ad)\tan^{-1}\left(1\right.}{4\sqrt{2}\sqrt[4]{c}\,d^{15/4}}$$

Mathematica [A] time = 0.19, size = 337, normalized size = 0.97

$$\frac{\frac{21\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}} - \frac{21\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{c}} - \frac{42\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2}\left(3a^2d^2-14abcd+11b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt[4]{c}} - 448bd^{3/4}x^{3/2}(bc-ad) - \frac{168d^{3/4}x^{3/2}(bc-ad)^2}{c+dx^2} + 96b^2d^{7/4}x^{7/2}}{336d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-448*b*d^(3/4)*(b*c - a*d)*x^(3/2) + 96*b^2*d^(7/4)*x^(7/2) - (168*d^(3/4)
*(b*c - a*d)^2*x^(3/2))/(c + d*x^2) - (42*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d
+ 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(1/4) + (42*S
```

```
qrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/c^(1/4) + (21*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(1/4) - (21
*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(1/4))/(336*d^(15/4))
```

IntegrateAlgebraic [A] time = 0.74, size = 233, normalized size = 0.67

$$-\frac{\left(3a^2d^2-14abcd+11b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}-\frac{\left(3a^2d^2-14abcd+11b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,d^{15/4}}+\frac{x^{3/2}\left(-21a^2d^2+98abcd+56abd^2x^2-77b^2c^2-44b^2cdx^2+12b^2d^2x^4\right)}{42d^3\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (x^(3/2)*(-77*b^2*c^2 + 98*a*b*c*d - 21*a^2*d^2 - 44*b^2*c*d*x^2 + 56*a*b*d
^2*x^2 + 12*b^2*d^2*x^4))/(42*d^3*(c + d*x^2)) - ((11*b^2*c^2 - 14*a*b*c*d
+ 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])
])/(4*Sqrt[2]*c^(1/4)*d^(15/4)) - ((11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Ar
cTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*
c^(1/4)*d^(15/4))
```

fricas [B] time = 0.84, size = 1733, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(84*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*
a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a
^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*
d^15))^(1/4)*arctan((sqrt((1771561*b^12*c^12 - 13528284*a*b^11*c^11*d + 459
43458*a^2*b^10*c^10*d^2 - 91492940*a^3*b^9*c^9*d^3 + 118659255*a^4*b^8*c^8*
d^4 - 105323064*a^5*b^7*c^7*d^5 + 65490076*a^6*b^6*c^6*d^6 - 28724472*a^7*b
^5*c^5*d^7 + 8825895*a^8*b^4*c^4*d^8 - 1855980*a^9*b^3*c^3*d^9 + 254178*a^1
0*b^2*c^2*d^10 - 20412*a^11*b*c*d^11 + 729*a^12*d^12)*x - (14641*b^8*c^9*d^
7 - 74536*a*b^7*c^8*d^8 + 158268*a^2*b^6*c^7*d^9 - 181720*a^3*b^5*c^6*d^10
+ 122566*a^4*b^4*c^5*d^11 - 49560*a^5*b^3*c^4*d^12 + 11772*a^6*b^2*c^3*d^13
 - 1512*a^7*b*c^2*d^14 + 81*a^8*c*d^15)*sqrt(-(14641*b^8*c^8 - 74536*a*b^7*
c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^
4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 +
81*a^8*d^8)/(c*d^15)))*d^4*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^
2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5
*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^
15))^(1/4) - (1331*b^6*c^6*d^4 - 5082*a*b^5*c^5*d^5 + 7557*a^2*b^4*c^4*d^6
- 5516*a^3*b^3*c^3*d^7 + 2061*a^4*b^2*c^2*d^8 - 378*a^5*b*c*d^9 + 27*a^6*d^
```

```
10)*sqrt(x)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 -
 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 +
11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4))/(14
641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c
^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2
*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)) - 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*
c^8 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 +
 122566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1
512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(c*d^11*(-(14641*b^8*c^8 -
 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 1225
66*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a
^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d
 + 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378
*a^5*b*c*d^5 + 27*a^6*d^6)*sqrt(x)) + 21*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8
 - 74536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 12
2566*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512
*a^7*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(-c*d^11*(-(14641*b^8*c^8 - 7
4536*a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566
*a^4*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7
*b*c*d^7 + 81*a^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d +
 7557*a^2*b^4*c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a
^5*b*c*d^5 + 27*a^6*d^6)*sqrt(x)) - 4*(12*b^2*d^2*x^5 - 4*(11*b^2*c*d - 14*
a*b*d^2)*x^3 - 7*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(d^4*x^2
 + c*d^3)
```

giac [**A**] time = 0.48, size = 413, normalized size = 1.19

$$-\frac{b^{2}c^{2}x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^{2}d^{2}x^{\frac{3}{2}}}{2\left(dx^{2}+c\right)d^{3}}+\frac{\sqrt{2}\left(11\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-14\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+3\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^{6}}+\frac{\sqrt{2}\left(11\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-14\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+3\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^{6}}-\frac{\sqrt{2}\left(11\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-14\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+3\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^{6}}+\frac{\sqrt{2}\left(11\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-14\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+3\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^{6}}+\frac{2\left(3\,b^{2}d^{12}x^{\frac{7}{2}}-14\,b^{2}cd^{11}x^{\frac{3}{2}}+14\,abd^{12}x^{\frac{3}{2}}\right)}{21\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*d
^3) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*
(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(c*d^6) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^
(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^6) - 1/16*sqrt(2)*(11*(c*d^3)^(3/4)*b
^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 1/16*sqrt(2)*(11*(c*d^3)^(3/4)
*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 2/21*(3*b^2*d^12*x^(7/2) -
14*b^2*c*d^11*x^(3/2) + 14*a*b*d^12*x^(3/2))/d^14
```

maple [A] time = 0.02, size = 523, normalized size = 1.51

$$\frac{2b^2x^{\frac{7}{2}}}{7d^2}-\frac{a^2x^{\frac{3}{2}}}{2\left(dx^2+c\right)d}+\frac{abcx^{\frac{3}{2}}}{\left(dx^2+c\right)d^2}-\frac{b^2c^2x^{\frac{3}{2}}}{2\left(dx^2+c\right)d^3}+\frac{4abx^{\frac{3}{2}}}{3d^2}-\frac{4b^2cx^{\frac{3}{2}}}{3d^3}+\frac{3\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^2}+\frac{3\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^2}+\frac{3\sqrt{2}\,a^2\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(\frac{c}{d}\right)^{\frac{1}{4}}d^2}-\frac{7\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^3}-\frac{7\sqrt{2}\,abc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^3}-\frac{7\sqrt{2}\,abc\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^3}+\frac{11\sqrt{2}\,b^2c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^4}+\frac{11\sqrt{2}\,b^2c^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^4}+\frac{11\sqrt{2}\,b^2c^2\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(\frac{c}{d}\right)^{\frac{1}{4}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/7*b^2*x^(7/2)/d^2+4/3*b/d^2*x^(3/2)*a-4/3*b^2/d^3*x^(3/2)*c-1/2/d*x^(3/2)
/(d*x^2+c)*a^2+1/d^2*x^(3/2)/(d*x^2+c)*a*b*c-1/2/d^3*x^(3/2)/(d*x^2+c)*b^2*
c^2-7/8/d^3/(c/d)^(1/4)*2^(1/2)*a*b*c*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/
d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-7/4/d^3/(c/d)^(1/4)*
2^(1/2)*a*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-7/4/d^3/(c/d)^(1/4)*2^(
1/2)*a*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+11/16/d^4/(c/d)^(1/4)*2^(1
/2)*b^2*c^2*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2
^(1/2)*x^(1/2)+(c/d)^(1/2)))+11/8/d^4/(c/d)^(1/4)*2^(1/2)*b^2*c^2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)+11/8/d^4/(c/d)^(1/4)*2^(1/2)*b^2*c^2*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)+3/16/d^2/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(
1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2
)))+3/8/d^2/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3
/8/d^2/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.42, size = 272, normalized size = 0.79

$$-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x^{\frac{3}{2}}}{2\left(d^4x^2+cd^3\right)}+\frac{\left(11\,b^2c^2-14\,abcd+3\,a^2d^2\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,d^3}+\frac{2\left(3\,b^2dx^{\frac{7}{2}}-14\left(b^2c-abd\right)x^{\frac{3}{2}}\right)}{21\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2)/(d^4*x^2 + c*d^3) + 1/16*(11*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt
(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - s
qrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*
d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(
c))/(c^(1/4)*d^(3/4)))/d^3 + 2/21*(3*b^2*d*x^(7/2) - 14*(b^2*c - a*b*d)*x^(
3/2))/d^3
```

mupad [B] time = 0.22, size = 160, normalized size = 0.46

$$\frac{2\,b^2\,x^{7/2}}{7\,d^2}-\frac{x^{3/2}\,\left(\frac{a^2\,d^2}{2}-a\,b\,c\,d+\frac{b^2\,c^2}{2}\right)}{d^4\,x^2+c\,d^3}-x^{3/2}\,\left(\frac{4\,b^2\,c}{3\,d^3}-\frac{4\,a\,b}{3\,d^2}\right)+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d-11\,b\,c\right)}{4\,{\left(-c\right)}^{1/4}\,d^{15/4}}+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(3\,a\,d-11\,b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-c\right)}^{1/4}\,d^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (2*b^2*x^(7/2))/(7*d^2) - (x^(3/2)*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(
c*d^3 + d^4*x^2) - x^(3/2)*((4*b^2*c)/(3*d^3) - (4*a*b)/(3*d^2)) + (atan((d
^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(3*a*d - 11*b*c))/(4*(-c)^(1/4)*d^(
15/4)) + (atan((d^(1/4)*x^(1/2)*1i)/(-c)^(1/4))*(a*d - b*c)*(3*a*d - 11*b*c
)*1i)/(4*(-c)^(1/4)*d^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.409 $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}d^{13/4}}$$

Rubi [**A**] time = 0.33, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 459, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}d^{13/4}}-\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{(bc-ad)(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{\sqrt{x}\,(bc-ad)(9bc-ad)}{2cd^3}+\frac{2b^2x^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*(9*b*c - a*d)*Sqrt[x])/(2*c*d^3) + (2*b^2*x^(5/2))/(5*d^2) +
((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(9*b*c - a*d)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(13/4))
 + ((b*c - a*d)*(9*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*d^(13/4)) - ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4)) +
 ((b*c - a*d)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{x^{3/2}\left(\frac{1}{2}\left(-4a^2d^2+5(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(9bc-ad))\int \frac{x^{3/2}}{c+dx^2}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}}{4d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\,\text{Subst}(\int \frac{1}{c+}}{2d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\,\text{Subst}(\int \frac{\sqrt{c}}{c}}{4\sqrt{c}\,d^3}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(9bc-ad))\,\text{Subst}(\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}}}{8\sqrt{c}\,d^{7/2}}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(9bc-ad)\log(\sqrt{c}-\sqrt{2}}{8\sqrt{2}\,c^{3/4}d^{13/4}}$$

$$= -\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3} + \frac{2b^2x^{5/2}}{5d^2} + \frac{(bc-ad)^2x^{5/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(9bc-ad)\tan^{-1}(1-\frac{\sqrt{2}}{}}{4\sqrt{2}\,c^{3/4}d^{13/4}}$$

Mathematica [A] time = 0.18, size = 333, normalized size = 0.96

$$\frac{-\frac{5\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}+\frac{5\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}}-\frac{10\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{3/4}}+\frac{10\sqrt{2}\left(a^2d^2-10abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{3/4}}-\frac{40\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{c+dx^2}-320b\sqrt[4]{d}\sqrt{x}(bc-ad)+32b^2d^{5/4}x^{5/2}}{80d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (-320*b*d^(1/4)*(b*c - a*d)*Sqrt[x] + 32*b^2*d^(5/4)*x^(5/2) - (40*d^(1/4)*
(b*c - a*d)^2*Sqrt[x])/(c + d*x^2) - (10*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d +
a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4) + (10*Sqrt[
```

```
2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/c^(3/4) - (5*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(3/4) + (5*Sqrt[2]*(9*
b^2*c^2 - 10*a*b*c*d + a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/c^(3/4))/(80*d^(13/4))
```

IntegrateAlgebraic [A]   time = 0.71, size = 231, normalized size = 0.67

$$\frac{\sqrt{x}\left(-5a^2d^2+50abcd+40abd^2x^2-45b^2c^2-36b^2cdx^2+4b^2d^2x^4\right)}{10d^3\left(c+dx^2\right)}-\frac{\left(a^2d^2-10abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}+\frac{\left(a^2d^2-10abcd+9b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{3/4}d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[x]*(-45*b^2*c^2 + 50*a*b*c*d - 5*a^2*d^2 - 36*b^2*c*d*x^2 + 40*a*b*d^
2*x^2 + 4*b^2*d^2*x^4))/(10*d^3*(c + d*x^2)) - ((9*b^2*c^2 - 10*a*b*c*d + a
^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4
*Sqrt[2]*c^(3/4)*d^(13/4)) + ((9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTanh[(S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(3/4)*
d^(13/4))
```

fricas [B]   time = 1.16, size = 1334, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(20*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*
b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*
c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)
*arctan((sqrt(c^2*d^6*sqrt(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b
^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c
^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13)) + (81*
b^4*c^4 - 180*a*b^3*c^3*d + 118*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + a^4*d^4)
*x)*c^2*d^10*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 -
45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*
a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(3/4) - (9*b^2*c^4*
d^10 - 10*a*b*c^3*d^11 + a^2*c^2*d^12)*sqrt(x)*(-(6561*b^8*c^8 - 29160*a*b^
7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4
*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^
8)/(c^3*d^13))^(3/4))/(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6
*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5
 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)) + 5*(d^4*x^2 + c*d^3)*(
-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*
```

```
c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^
6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*log(c*d^3*(-(6561*b^8*c^8 -
 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*
a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d
^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x
)) - 5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^
6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^
3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*l
og(-c*d^3*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 457
20*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6
*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 1
0*a*b*c*d + a^2*d^2)*sqrt(x)) + 4*(4*b^2*d^2*x^4 - 45*b^2*c^2 + 50*a*b*c*d
- 5*a^2*d^2 - 4*(9*b^2*c*d - 10*a*b*d^2)*x^2)*sqrt(x))/(d^4*x^2 + c*d^3)
```

giac [A] time = 0.47, size = 408, normalized size = 1.18

$$\frac{\sqrt{2}\left(9\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8cd^4}+\frac{\sqrt{2}\left(9\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8cd^4}+\frac{\sqrt{2}\left(9\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16cd^4}-\frac{\sqrt{2}\left(9\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\left(cd^3\right)^{\frac{1}{4}}abcd+\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16cd^4}-\frac{b^2c^2\sqrt{x}-2abcd\sqrt{x}+a^2d^2\sqrt{x}}{2\left(dx^2+c\right)d^3}+\frac{2\left(b^2d^8x^{\frac{5}{2}}-10b^2cd^7\sqrt{x}+10abd^8\sqrt{x}\right)}{5d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1
/4))/(c*d^4) + 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*
c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(c*d^4) + 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(
c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c*d^4) - 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c
*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c*d^4) - 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^
2*d^2*sqrt(x))/((d*x^2 + c)*d^3) + 2/5*(b^2*d^8*x^(5/2) - 10*b^2*c*d^7*sqrt
(x) + 10*a*b*d^8*sqrt(x))/d^10
```

maple [A] time = 0.02, size = 523, normalized size = 1.51

$$\frac{2b^2x^{\frac{5}{2}}}{5d^2}-\frac{a^2\sqrt{x}}{2\left(dx^2+c\right)d}+\frac{abc\sqrt{x}}{\left(dx^2+c\right)d^2}-\frac{b^2c^2\sqrt{x}}{2\left(dx^2+c\right)d^3}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16cd}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4d^2}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4d^2}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8d^2}+\frac{9\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8d^3}+\frac{9\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8d^3}+\frac{9\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16d^3}+\frac{4ab\sqrt{x}}{d^2}-\frac{4b^2c\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 2/5*b^2*x^(5/2)/d^2+4*b/d^2*a*x^(1/2)-4*b^2/d^3*c*x^(1/2)-1/2/d*x^(1/2)/(d*
x^2+c)*a^2+1/d^2*x^(1/2)/(d*x^2+c)*a*b*c-1/2/d^3*x^(1/2)/(d*x^2+c)*b^2*c^2+
1/8/d*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-5/4/d
^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+9/8/d^3*(c
```

```
/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+1/16/d*(c/d)^
(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-5/8/d^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d
)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
1/2)))*a*b+9/16/d^3*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+1/8/d*(c/d)^
(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-5/4/d^2*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+9/8/d^3*(c/d)^(1/4)*c*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [A] time = 2.45, size = 336, normalized size = 0.97

$$-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{x}}{2\left(d^{4}x^{2}+cd^{3}\right)}+\frac{2\left(b^{2}dx^{\frac{5}{2}}-10\left(b^{2}c-abd\right)\sqrt{x}\right)}{5\,d^{3}}+\frac{\frac{2\sqrt{2}\left(9\,b^{2}c^{2}-10\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(9\,b^{2}c^{2}-10\,abcd+a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(9\,b^{2}c^{2}-10\,abcd+a^{2}d^{2}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(9\,b^{2}c^{2}-10\,abcd+a^{2}d^{2}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)/(d^4*x^2 + c*d^3) + 2/5*(b^2*d
*x^(5/2) - 10*(b^2*c - a*b*d)*sqrt(x))/d^3 + 1/16*(2*sqrt(2)*(9*b^2*c^2 - 1
0*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d
)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(
2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*
d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*s
qrt(d))) + sqrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*d
^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(9*b^2*c^
2 - 10*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(3/4)*d^(1/4)))/d^3
```

mupad [B] time = 0.26, size = 1238, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (2*b^2*x^(5/2))/(5*d^2) - (x^(1/2)*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(
c*d^3 + d^4*x^2) - x^(1/2)*((4*b^2*c)/d^3 - (4*a*b)/d^2) + (atan(((((x^(1/2
)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*
c*d^3))/d^3 - ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b
*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-c)^(3/
4)*d^(13/4)) + (((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180
*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^
3 + 8*a^2*c*d^2 - 80*a*b*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d
- 9*b*c)*1i)/(8*(-c)^(3/4)*d^(13/4)))/((((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 1
```

```
18*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 - ((a*d - b*c)*
(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d))/(8*(-c)^(3/4)*d^(1
3/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/4)) - (((x^(1/2)*(a^4
*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3)
)/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d
))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/
4))))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(4*(-c)^(3/4)*d^(13/4)) + (atan(((((x^(
1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3
*b*c*d^3))/d^3 - ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*
a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^
(3/4)*d^(13/4)) + (((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 -
180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2
*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)
*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/4)))/((((x^(1/2)*(a^4*d^4 + 81*b^4*c^4
+ 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 - ((a*d - b*
c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d)*1i)/(8*(-c)^(3/4
)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-c)^(3/4)*d^(13/4)) - (((x^(
1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3
*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*
a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-
c)^(3/4)*d^(13/4))))*(a*d - b*c)*(a*d - 9*b*c))/(4*(-c)^(3/4)*d^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.410 $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right.}{8\sqrt{2}\,c^{5/4}d^{11/4}}$$

Rubi **[A]** time = 0.28, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 459, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bc-ad)(ad+7bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{(bc-ad)(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}-\frac{(bc-ad)(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{2b^2x^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (2*b^2*x^(3/2))/(3*d^2) + ((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d*x^2)) + (
(b*c - a*d)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4
*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*b*c + a*d)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) - ((b*c - a*d)*(7*
b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(5/4)*d^(11/4)) + ((b*c - a*d)*(7*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*d^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{\sqrt{x}\left(\frac{1}{2}\left(-4a^2d^2+3(bc-ad)^2\right)-2b^2cdx^2\right)}{c+dx^2}\,dx}{2cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\int \frac{\sqrt{x}}{c+dx^2}\,dx}{4cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{2cd^2}$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} + \frac{((bc-ad)(7bc+ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{4cd^{5/2}} - ((bc-$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(7bc+ad))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{8cd^3} - ((($$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(7bc+ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}d^{11/4}} + (bc$$

$$= \frac{2b^2x^{3/2}}{3d^2} + \frac{(bc-ad)^2x^{3/2}}{2cd^2\left(c+dx^2\right)} + \frac{(bc-ad)(7bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}} - \frac{(bc-ad)(7bc+}{4}$$

Mathematica [A] time = 0.18, size = 319, normalized size = 1.03

$$\frac{-\frac{3\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{5/4}}+\frac{3\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{5/4}}+\frac{6\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}}-\frac{6\sqrt{2}\left(-a^2d^2-6abcd+7b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}}+\frac{24d^{3/4}x^{3/2}(bc-ad)^2}{c\left(c+dx^2\right)}+32b^2d^{3/4}x^{3/2}}{48d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (32*b^2*d^(3/4)*x^(3/2) + (24*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c*(c + d*x^2)
) + (6*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/c^(5/4) - (6*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (3*Sqrt[2]*(7*b^2
*c^2 - 6*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/c^(5/4) + (3*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(48*d^(11/4))
```

IntegrateAlgebraic [A] time = 0.79, size = 215, normalized size = 0.69

$$\frac{x^{3/2}\left(3a^2d^2-6abcd+7b^2c^2+4b^2cdx^2\right)}{6cd^2\left(c+dx^2\right)}+\frac{\left(-a^2d^2-6abcd+7b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}+\frac{\left(-a^2d^2-6abcd+7b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{5/4}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

```
[Out] (x^(3/2)*(7*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2 + 4*b^2*c*d*x^2))/(6*c*d^2*(c +
 d*x^2)) + ((7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(5/4)*d^(11/4)) + ((7*b^2*
c^2 - 6*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[
c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(5/4)*d^(11/4))
```

fricas [B] time = 1.46, size = 1723, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a
^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*
c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)
*arctan((sqrt((117649*b^12*c^12 - 605052*a*b^11*c^11*d + 1195698*a^2*b^10*c
^10*d^2 - 1049580*a^3*b^9*c^9*d^3 + 247695*a^4*b^8*c^8*d^4 + 184968*a^5*b^7
*c^7*d^5 - 73604*a^6*b^6*c^6*d^6 - 26424*a^7*b^5*c^5*d^7 + 5055*a^8*b^4*c^4
*d^8 + 3060*a^9*b^3*c^3*d^9 + 498*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^
12*d^12)*x - (2401*b^8*c^11*d^5 - 8232*a*b^7*c^10*d^6 + 9212*a^2*b^6*c^9*d^
7 - 2520*a^3*b^5*c^8*d^8 - 1434*a^4*b^4*c^7*d^9 + 360*a^5*b^3*c^6*d^10 + 18
8*a^6*b^2*c^5*d^11 + 24*a^7*b*c^4*d^12 + a^8*c^3*d^13)*sqrt(-(2401*b^8*c^8
- 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4
*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 +
 a^8*d^8)/(c^5*d^11)))*c*d^3*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*
b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3
*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4) +
(343*b^6*c^7*d^3 - 882*a*b^5*c^6*d^4 + 609*a^2*b^4*c^5*d^5 + 36*a^3*b^3*c^4
```

```
*d^6 - 87*a^4*b^2*c^3*d^7 - 18*a^5*b*c^2*d^8 - a^6*c*d^9)*sqrt(x)*(-(2401*b
^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1
434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*
c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4))/(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212
*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^
3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)) - 3*(c*d^3*x^2
 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 252
0*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^
2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)*log(c^4*d^8*(-(2401
*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 -
 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*
b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (343*b^6*c^6 - 882*a*b^5*c^5*d + 609
*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5
 - a^6*d^6)*sqrt(x)) + 3*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7
*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4
 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c
^5*d^11))^(1/4)*log(-c^4*d^8*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*
b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3
*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) -
(343*b^6*c^6 - 882*a*b^5*c^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 -
 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x)) - 4*(4*b^2*c*d*x^3
 + (7*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2)*x)*sqrt(x))/(c*d^3*x^2 + c^2*d^2)
```

giac [**A**] time = 0.46, size = 388, normalized size = 1.25

$$\frac{2\,b^{2}x^{\frac{3}{2}}}{3\,d^{2}}+\frac{b^{2}c^{2}x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^{2}d^{2}x^{\frac{3}{2}}}{2\left(dx^{2}+c\right)cd^{2}}-\frac{\sqrt{2}\left(7\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{2}d^{5}}-\frac{\sqrt{2}\left(7\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{2}d^{5}}+\frac{\sqrt{2}\left(7\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{2}d^{5}}-\frac{\sqrt{2}\left(7\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{2}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^2 + 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^
(3/2))/((d*x^2 + c)*c*d^2) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^
3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b
^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) + 1/16*sqrt(2)*
(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)
*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5) - 1/16*sqrt(2)*
(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)
*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5)
```

maple [**B**] time = 0.02, size = 499, normalized size = 1.61

$$\frac{a^{2}x^{\frac{3}{2}}}{2\left(d\,x^{2}+c\right)c}-\frac{ab\,x^{\frac{3}{2}}}{\left(d\,x^{2}+c\right)d}+\frac{b^{2}c\,x^{\frac{3}{2}}}{2\left(d\,x^{2}+c\right)d^{2}}+\frac{2b^{2}x^{\frac{3}{2}}}{3d^{2}}+\frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}cd}+\frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}cd}+\frac{\sqrt{2}\,a^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(\frac{c}{d}\right)^{\frac{1}{4}}cd}+\frac{3\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}+\frac{3\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}+\frac{3\sqrt{2}\,ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}-\frac{7\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}}-\frac{7\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}}-\frac{7\sqrt{2}\,b^{2}c\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 2/3*b^2*x^(3/2)/d^2+1/2/c*x^(3/2)/(d*x^2+c)*a^2-1/d*x^(3/2)/(d*x^2+c)*a*b+1
/2/d^2*c*x^(3/2)/(d*x^2+c)*b^2+1/16/d/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))
)*a^2+3/8/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-7/16/d^3*c/(c/d)^(1/
4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^
(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+1/8/d/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)*a^2+3/4/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)*a*b-7/8/d^3*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)*b^2+1/8/d/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)*a^2+3/4/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*a*b-7/8/d^3*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1
)*b^2
```

maxima [A] time = 2.51, size = 258, normalized size = 0.83

$$\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{\frac{3}{2}}}{2\left(cd^{3}x^{2}+c^{2}d^{2}\right)}+\frac{2\,b^{2}x^{\frac{3}{2}}}{3\,d^{2}}-\frac{\left(7\,b^{2}c^{2}-6\,abcd-a^{2}d^{2}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2)/(c*d^3*x^2 + c^2*d^2) + 2/3*b^2
*x^(3/2)/d^2 - 1/16*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*
sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x
) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d^2)
```

mupad [B] time = 0.40, size = 137, normalized size = 0.44

$$\frac{2\,b^{2}\,x^{3/2}}{3\,d^{2}}+\frac{x^{3/2}\,\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}{2\,c\,\left(d^{3}\,x^{2}+c\,d^{2}\right)}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(a\,d+7\,b\,c)}{4\,(-c)^{5/4}\,d^{11/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(a\,d+7\,b\,c)\,1\mathrm{i}}{4\,(-c)^{5/4}\,d^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

```
[Out] (2*b^2*x^(3/2))/(3*d^2) + (x^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c
*d^2 + d^3*x^2)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(a*d + 7
*b*c))/(4*(-c)^(5/4)*d^(11/4)) - (atan((d^(1/4)*x^(1/2)*1i)/(-c)^(1/4))*(a*
d - b*c)*(a*d + 7*b*c)*1i)/(4*(-c)^(5/4)*d^(11/4))
```

sympy [A] time = 42.06, size = 173, normalized size = 0.56

$$\frac{4ab\operatorname{RootSum}\left(256t^4cd^3+1,\left(t\mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d}-\frac{4b^2c\operatorname{RootSum}\left(256t^4cd^3+1,\left(t\mapsto t\log\left(64t^3cd^2+\sqrt{x}\right)\right)\right)}{d^2}+\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{2x^{\frac{3}{2}}(ad-bc)^2}{4c^2d^2+4cd^3x^2}+\frac{2(ad-bc)^2\operatorname{RootSum}\left(65536t^4c^5d^3+1,\left(t\mapsto t\log\left(4096t^3c^4d^2+\sqrt{x}\right)\right)\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] 4*a*b*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t**3*c*d**2 + sqr
t(x))))/d - 4*b**2*c*RootSum(256*_t**4*c*d**3 + 1, Lambda(_t, _t*log(64*_t*
*3*c*d**2 + sqrt(x))))/d**2 + 2*b**2*x**(3/2)/(3*d**2) + 2*x**(3/2)*(a*d -
b*c)**2/(4*c**2*d**2 + 4*c*d**3*x**2) + 2*(a*d - b*c)**2*RootSum(65536*_t**
4*c**5*d**3 + 1, Lambda(_t, _t*log(4096*_t**3*c**4*d**2 + sqrt(x))))/d**2
```

3.411 $\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=312

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}$$

Rubi **[A]** time = 0.34, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = $\{463, 459, 329, 211, 1165, 628, 1162, 617, 204\}$

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}+\frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}+\frac{\sqrt{x}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{2b^2\sqrt{x}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (2*b^2*Sqrt[x])/d^2 + ((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2)) + ((b*c
 - a*d)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*
c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*S
qrt[2]*c^(7/4)*d^(9/4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)^2}\,dx = \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} - \frac{\int \frac{\frac{1}{2}(bc-3ad)(bc+ad)-2b^2cdx^2}{\sqrt{x}\left(c+dx^2\right)}\,dx}{2cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(5bc+3ad))\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{4cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(5bc+3ad))\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{2cd^2}$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(5bc+3ad))\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{4c^{3/2}d^2} - ((bc -$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(5bc+3ad))\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx, x, \sqrt{x}\right)}{8c^{3/2}d^{5/2}} - ((\ $$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} + \frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}} - (bc$$

$$= \frac{2b^2\sqrt{x}}{d^2} + \frac{(bc-ad)^2\sqrt{x}}{2cd^2\left(c+dx^2\right)} + \frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}} - (bc-ad)(5bc+3$$

Mathematica [A] time = 0.18, size = 318, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{7/4}} - \frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{7/4}} + \frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{7/4}} + \frac{8\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{c\left(c+dx^2\right)} + 32b^2\sqrt[4]{d}\sqrt{x}}{16d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (32*b^2*d^(1/4)*Sqrt[x] + (8*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c*(c + d*x^2))
 + (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/
```

```
4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^
2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/c^(7/4) - (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(16*d^(9/4
))
```

IntegrateAlgebraic **[A]** time = 0.78, size = 214, normalized size = 0.69

$$\frac{\sqrt{x}\left(a^2d^2-2abcd+5b^2c^2+4b^2cdx^2\right)}{2cd^2\left(c+dx^2\right)}+\frac{\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{\left(-3a^2d^2-2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{7/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

```
[Out] (Sqrt[x]*(5*b^2*c^2 - 2*a*b*c*d + a^2*d^2 + 4*b^2*c*d*x^2))/(2*c*d^2*(c + d
*x^2)) + ((5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(7/4)*d^(9/4)) - ((5*b^2*c
^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(7/4)*d^(9/4))
```

fricas **[B]** time = 1.15, size = 1341, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^
6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^
5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*ar
ctan((sqrt(c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*
d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 32
4*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*b^4*c^4
 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)*x)*c^5*
d^7*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*
c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 +
 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4) + (5*b^2*c^7*d^7 - 2*a*b*c^
6*d^8 - 3*a^2*c^5*d^9)*sqrt(x)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*
b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4))
/(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d
^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*
b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^
```

```
4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^
8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2
*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3
*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)
 - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (c*d^3*x^2 + c^2*d^2)*(-(
625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3
 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^
7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 1000
*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4
*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8
*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) + 4*(
4*b^2*c*d*x^2 + 5*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x))/(c*d^3*x^2 + c^2*
d^2)
```

giac [A] time = 0.50, size = 388, normalized size = 1.24

$$\frac{2\,b^{2}\sqrt{x}}{d^{2}}-\frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{2}d^{3}}-\frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{2}d^{3}}-\frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{2}d^{3}}+\frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-2\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-3\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{2}d^{3}}+\frac{b^{2}c^{2}\sqrt{x}-2\,abcd\sqrt{x}+a^{2}d^{2}\sqrt{x}}{2\left(dx^{2}+c\right)cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d^2 - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*
a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
+ 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
- 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c
*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*lo
g(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/16*sqrt(2)*(5*
(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*
log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/2*(b^2*c^2*
sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c*d^2)
```

maple [B] time = 0.02, size = 496, normalized size = 1.59

$$\frac{a^{2}\sqrt{x}}{2\left(d\,x^{2}+c\right)c}-\frac{ab\sqrt{x}}{\left(d\,x^{2}+c\right)d}+\frac{b^{2}c\sqrt{x}}{2\left(d\,x^{2}+c\right)d^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8c^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8c^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16c^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8cd}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8d^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8d^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16d^{2}}+\frac{2b^{2}\sqrt{x}}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 2*b^2*x^(1/2)/d^2+1/2/c*x^(1/2)/(d*x^2+c)*a^2-1/d*x^(1/2)/(d*x^2+c)*a*b+1/2
/d^2*c*x^(1/2)/(d*x^2+c)*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)
*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*
a^2+1/8/d/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/
2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-5/16/d^2*(c/d)^(1/4)*2
```

```
^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2
)*x^(1/2)+(c/d)^(1/2)))*b^2+3/8/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)*a^2+1/4/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)*a*b-5/8/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*b^2+3/8/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1)*a^2+1/4/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a
*b-5/8/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2
```

maxima [A]   time = 2.58, size = 327, normalized size = 1.05

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\sqrt{x}}{2\left(cd^3x^2 + c^2d^2\right)} + \frac{2\,b^2\sqrt{x}}{d^2} - \frac{\frac{2\,\sqrt{2}\left(5\,b^2c^2-2\,abcd-3\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}} + \frac{2\,\sqrt{2}\left(5\,b^2c^2-2\,abcd-3\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}} + \frac{\sqrt{2}\left(5\,b^2c^2-2\,abcd-3\,a^2d^2\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}} - \frac{\sqrt{2}\left(5\,b^2c^2-2\,abcd-3\,a^2d^2\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)/(c*d^3*x^2 + c^2*d^2) + 2*b^2*s
qrt(x)/d^2 - 1/16*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a
^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2
- 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*lo
g(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))
)/(c*d^2)
```

mupad [B]   time = 0.46, size = 1267, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)^2),x)
```

```
[Out] (2*b^2*x^(1/2))/d^2 + (x^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2
 + d^3*x^2)) + (atan(((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d
^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*
c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d
 - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) + (((x^(1/2)*(9*a^4*d^4
+ 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*
d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2
))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^
(9/4)))/((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3
*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^
3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*
```

```
d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4)) - (((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 2
6*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c
)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)
*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4))))*(a*d - b*c
)*(3*a*d + 5*b*c)*1i)/(4*(-c)^(7/4)*d^(9/4)) + (atan(((((x^(1/2)*(9*a^4*d^4
 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2
*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^
2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d
^(9/4)) + (((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^
3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d
^3 - 40*b^2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(
3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4)))/((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4
 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d -
 b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c
)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) -
(((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d +
12*a^3*b*c*d^3))/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^
2*c^2*d + 16*a*b*c*d^2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*
b*c)*1i)/(8*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c))/(4*(-c)^(7/4
)*d^(9/4))
```

sympy [A] time = 79.53, size = 1574, normalized size = 5.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x))
, Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**
2*sqrt(x))/d**2, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x*
*(9/2)/9)/c**2, Eq(d, 0)), (-3*(-1)**(1/4)*a**2*c**(5/4)*d**2*(1/d)**(1/4)*
log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(8*c**3*d**2 + 8*c**2*d**
3*x**2) + 3*(-1)**(1/4)*a**2*c**(5/4)*d**2*(1/d)**(1/4)*log((-1)**(1/4)*c**
(1/4)*(1/d)**(1/4) + sqrt(x))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 6*(-1)**(1
/4)*a**2*c**(5/4)*d**2*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d
)**(1/4)))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 3*(-1)**(1/4)*a**2*c**(1/4)*d
**3*x**2*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(8*
c**3*d**2 + 8*c**2*d**3*x**2) + 3*(-1)**(1/4)*a**2*c**(1/4)*d**3*x**2*(1/d)
**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(8*c**3*d**2 + 8*c
**2*d**3*x**2) - 6*(-1)**(1/4)*a**2*c**(1/4)*d**3*x**2*(1/d)**(1/4)*atan((-
1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(8*c**3*d**2 + 8*c**2*d**3*x**2)
 + 4*a**2*c*d**2*sqrt(x)/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 2*(-1)**(1/4)*a
*b*c**(9/4)*d*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x)
)/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 2*(-1)**(1/4)*a*b*c**(9/4)*d*(1/d)**(1
```

```
/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(8*c**3*d**2 + 8*c**2*
d**3*x**2) - 4*(-1)**(1/4)*a*b*c**(9/4)*d*(1/d)**(1/4)*atan((-1)**(3/4)*sqr
t(x)/(c**(1/4)*(1/d)**(1/4)))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 2*(-1)**(1
/4)*a*b*c**(5/4)*d**2*x**2*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**(1
/4) + sqrt(x))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 2*(-1)**(1/4)*a*b*c**(5/4
)*d**2*x**2*(1/d)**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(
8*c**3*d**2 + 8*c**2*d**3*x**2) - 4*(-1)**(1/4)*a*b*c**(5/4)*d**2*x**2*(1/d
)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(8*c**3*d**2 + 8
*c**2*d**3*x**2) - 8*a*b*c**2*d*sqrt(x)/(8*c**3*d**2 + 8*c**2*d**3*x**2) +
5*(-1)**(1/4)*b**2*c**(13/4)*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(1/4)*(1/d)**
(1/4) + sqrt(x))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 5*(-1)**(1/4)*b**2*c**(
13/4)*(1/d)**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4) + sqrt(x))/(8*c**3
*d**2 + 8*c**2*d**3*x**2) + 10*(-1)**(1/4)*b**2*c**(13/4)*(1/d)**(1/4)*atan
((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(8*c**3*d**2 + 8*c**2*d**3*x*
*2) + 5*(-1)**(1/4)*b**2*c**(9/4)*d*x**2*(1/d)**(1/4)*log(-(-1)**(1/4)*c**(
1/4)*(1/d)**(1/4) + sqrt(x))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 5*(-1)**(1/
4)*b**2*c**(9/4)*d*x**2*(1/d)**(1/4)*log((-1)**(1/4)*c**(1/4)*(1/d)**(1/4)
+ sqrt(x))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 10*(-1)**(1/4)*b**2*c**(9/4)*
d*x**2*(1/d)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(c**(1/4)*(1/d)**(1/4)))/(8*c*
*3*d**2 + 8*c**2*d**3*x**2) + 20*b**2*c**3*sqrt(x)/(8*c**3*d**2 + 8*c**2*d*
*3*x**2) + 16*b**2*c**2*d*x**(5/2)/(8*c**3*d**2 + 8*c**2*d**3*x**2), True))
```

3.412 $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{x^{3/2}\left(5a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}d^{7/4}}-(bc-a$$

Rubi [A] time = 0.33, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}\left(5a^2d^2-2abcd+b^2c^2\right)}{2c^2d\left(c+dx^2\right)}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)}+\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}d^{7/4}}-\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}+\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) - ((b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^(3/
2))/(2*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) + ((b*c - a*d)*(3*
b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9
/4)*d^(7/4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*d^(7/4)) - ((b*c - a*d)*(3*
b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8
*Sqrt[2]*c^(9/4)*d^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^2}\,dx = -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-5ad)+\frac{1}{2}b^2cx^2\right)}{\left(c+dx^2\right)^2}\,dx}{c}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x^{3/2}}{2c\left(c+dx^2\right)} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{x}}{c+dx^2}\,dx}{4c^2d}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x^{3/2}}{2c\left(c+dx^2\right)} + \frac{((bc-ad)(3bc+5ad))\,\mathrm{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \right.}{2c^2d}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x^{3/2}}{2c\left(c+dx^2\right)} - \frac{((bc-ad)(3bc+5ad))\,\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx, \right.}{4c^2d^{3/2}}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x^{3/2}}{2c\left(c+dx^2\right)} + \frac{((bc-ad)(3bc+5ad))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x}\right.}{8c^2d^2}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x^{3/2}}{2c\left(c+dx^2\right)} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\right.}{8\sqrt{2}\,c^{9/4}d^{7/4}}$$

$$= -\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)} + \frac{\left(2ab-\frac{b^2c}{d}-\frac{5a^2d}{c}\right)x^{3/2}}{2c\left(c+dx^2\right)} - \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}} +$$

Mathematica [A] time = 0.20, size = 317, normalized size = 0.95

$$\frac{\frac{\sqrt{2}\left(-5a^2d^2+2abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{7/4}}+\frac{\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{7/4}}+\frac{2\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{7/4}}+\frac{2\sqrt{2}\left(-5a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{7/4}}-\frac{32a^2\sqrt[4]{c}}{\sqrt{x}}-\frac{8\sqrt[4]{c}\,x^{3/2}(bc-ad)^2}{d\left(c+dx^2\right)}}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(1/4))/Sqrt[x] - (8*c^(1/4)*(b*c - a*d)^2*x^(3/2))/(d*(c + d*x^
2)) + (2*Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a
^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (Sqrt[2]*(
3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/d^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4))/(16*c
^(9/4))
```

IntegrateAlgebraic [A] time = 0.76, size = 221, normalized size = 0.66

$$\frac{-4a^2cd-5a^2d^2x^2+2abcdx^2-b^2c^2x^2}{2c^2d\sqrt{x}\left(c+dx^2\right)}-\frac{\left(-5a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}-\frac{\left(-5a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (-4*a^2*c*d - b^2*c^2*x^2 + 2*a*b*c*d*x^2 - 5*a^2*d^2*x^2)/(2*c^2*d*Sqrt[x]
*(c + d*x^2)) - ((3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt
[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(9/4)*d^(7/4)) - ((
3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(9/4)*d^(7/4))
```

fricas [B] time = 1.44, size = 1739, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*
arctan((sqrt((729*b^12*c^12 + 2916*a*b^11*c^11*d - 2430*a^2*b^10*c^10*d^2 -
 19980*a^3*b^9*c^9*d^3 + 135*a^4*b^8*c^8*d^4 + 59976*a^5*b^7*c^7*d^5 + 6364
*a^6*b^6*c^6*d^6 - 99960*a^7*b^5*c^5*d^7 + 375*a^8*b^4*c^4*d^8 + 92500*a^9*
b^3*c^3*d^9 - 18750*a^10*b^2*c^2*d^10 - 37500*a^11*b*c*d^11 + 15625*a^12*d^
12)*x - (81*b^8*c^13*d^3 + 216*a*b^7*c^12*d^4 - 324*a^2*b^6*c^11*d^5 - 984*
a^3*b^5*c^10*d^6 + 646*a^4*b^4*c^9*d^7 + 1640*a^5*b^3*c^8*d^8 - 900*a^6*b^2
*c^7*d^9 - 1000*a^7*b*c^6*d^10 + 625*a^8*c^5*d^11)*sqrt(-(81*b^8*c^8 + 216*
a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d
^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^
```

```
8*d^8)/(c^9*d^7)))*c^2*d^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^
6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 -
900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4) + (2
7*b^6*c^8*d^2 + 54*a*b^5*c^7*d^3 - 99*a^2*b^4*c^6*d^4 - 172*a^3*b^3*c^5*d^5
 + 165*a^4*b^2*c^4*d^6 + 150*a^5*b*c^3*d^7 - 125*a^6*c^2*d^8)*sqrt(x)*(-(81
*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 64
6*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4))/(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)) - (c^2*d^
2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 98
4*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^
2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(c^7*d^5*(-
(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 +
 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^
7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d -
99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*
c*d^5 - 125*a^6*d^6)*sqrt(x)) + (c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216
*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(c^9*d^7))^(1/4)*log(-c^7*d^5*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324
*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3
*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))
^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^
3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*sqrt(x)) - 4*(
4*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(x))/(c^2*d^2*x^3 +
c^3*d*x)
```

giac [A] time = 0.53, size = 389, normalized size = 1.17

$$-\frac{b^2c^2x^2-2\,abcdx^2+5\,a^2d^2x^2+4\,a^2cd}{2\left(dx^{\frac{5}{2}}+c\sqrt{x}\right)c^2d}+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+2\,\left(cd^3\right)^{\frac{3}{4}}abcd-5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4}+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+2\,\left(cd^3\right)^{\frac{3}{4}}abcd-5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4}-\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+2\,\left(cd^3\right)^{\frac{3}{4}}abcd-5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^3d^4}+\frac{\sqrt{2}\left(3\,\left(cd^3\right)^{\frac{3}{4}}b^2c^2+2\,\left(cd^3\right)^{\frac{3}{4}}abcd-5\,\left(cd^3\right)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 5*a^2*d^2*x^2 + 4*a^2*c*d)/((d*x^(5/2)
+ c*sqrt(x))*c^2*d) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4
)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^
2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) - 1/16*sqrt(2)*(3*
(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*
log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4) + 1/16*sqrt(2)*(
3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2
)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4)
```

maple [A] time = 0.02, size = 495, normalized size = 1.49

$$-\frac{a^{2} d\, x^{\frac{3}{2}}}{2 \left(d\, x^{2}+c\right) c^{2}}+\frac{a b\, x^{\frac{3}{2}}}{\left(d\, x^{2}+c\right) c}-\frac{b^{2} x^{\frac{3}{2}}}{2 \left(d\, x^{2}+c\right) d}-\frac{5 \sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}-\frac{5 \sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}-\frac{5 \sqrt{2}\, a^{2} \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{\sqrt{2}\, a b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4 \left(\frac{c}{d}\right)^{\frac{1}{4}} c d}+\frac{\sqrt{2}\, a b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4 \left(\frac{c}{d}\right)^{\frac{1}{4}} c d}+\frac{\sqrt{2}\, a b \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8 \left(\frac{c}{d}\right)^{\frac{1}{4}} c d}+\frac{3 \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}}+\frac{3 \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}}+\frac{3 \sqrt{2}\, b^{2} \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}}-\frac{2 a^{2}}{c^{2} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x^(3/2)/(d*x^2+c)*a^2+1/c*x^(3/2)/(d*x^2+c)*a*b-1/2/d*x^(3/2)/(d
*x^2+c)*b^2-5/8/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)*a^2+1/4/c/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*
b+3/8/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-5/1
6/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+1/8/c/d/(c/d)^(1/4)*2^(1/2)*
ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/
2)+(c/d)^(1/2)))*a*b+3/16/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-5/8/
c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+1/4/c/d/(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/8/d^2/(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2*a^2/c^2/x^(1/2)
```

maxima [A] time = 2.38, size = 260, normalized size = 0.78

$$-\frac{4 a^{2} c d+\left(b^{2} c^{2}-2 a b c d+5 a^{2} d^{2}\right) x^{2}}{2 \left(c^{2} d^{2} x^{\frac{5}{2}}+c^{3} d \sqrt{x}\right)}+\frac{\left(3 b^{2} c^{2}+2 a b c d-5 a^{2} d^{2}\right) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}-\frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{16 c^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2)/(c^2*d^2*x^(5/2) +
 c^3*d*sqrt(x)) + 1/16*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sq
rt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt
(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*s
qrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^2*d)
```

mupad [B] time = 0.39, size = 138, normalized size = 0.41

$$\frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)}{4\,{\left(-c\right)}^{9/4}\,d^{7/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a\,d+3\,b\,c\right)}{4\,{\left(-c\right)}^{9/4}\,d^{7/4}}-\frac{\frac{2\,a^2}{c}+\frac{x^2\,\left(5\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d}}{c\,\sqrt{x}+d\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x)
```

```
[Out] (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(5*a*d + 3*b*c))/(4*(-c)^(
9/4)*d^(7/4)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(5*a*d + 3*
b*c))/(4*(-c)^(9/4)*d^(7/4)) - ((2*a^2)/c + (x^2*(5*a^2*d^2 + b^2*c^2 - 2*a
*b*c*d))/(2*c^2*d))/(c*x^(1/2) + d*x^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.413 $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt{x}\left(7a^2d^2-6abcd+3b^2c^2\right)}{6c^2d\left(c+dx^2\right)}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}d^{5/4}}+(bc-$$

Rubi [A] time = 0.34, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {462, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}\left(7a^2d^2-6abcd+3b^2c^2\right)}{6c^2d\left(c+dx^2\right)}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}-\frac{(bc-ad)(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}d^{5/4}}-\frac{(bc-ad)(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{(bc-ad)(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) - ((3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*S
qrt[x])/(6*c^2*d*(c + d*x^2)) - ((b*c - a*d)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*
(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(
11/4)*d^(5/4)) - ((b*c - a*d)*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*d^(5/4)) + ((b*c - a*d)*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*
Sqrt[2]*c^(11/4)*d^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^2}\,dx &= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{2\int \frac{\frac{1}{2}a(6bc-7ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\left(c+dx^2\right)^2}\,dx}{3c}\\
&= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{\left(6ab-\frac{3b^2c}{d}-\frac{7a^2d}{c}\right)\sqrt{x}}{6c\left(c+dx^2\right)}+\frac{((bc-ad)(bc+7ad))\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{4c^2d}\\
&= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{\left(6ab-\frac{3b^2c}{d}-\frac{7a^2d}{c}\right)\sqrt{x}}{6c\left(c+dx^2\right)}+\frac{((bc-ad)(bc+7ad))\,\mathrm{Subst}\left(\int \frac{1}{c+dx^4}\,dx,x\right.}{2c^2d}\\
&= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{\left(6ab-\frac{3b^2c}{d}-\frac{7a^2d}{c}\right)\sqrt{x}}{6c\left(c+dx^2\right)}+\frac{((bc-ad)(bc+7ad))\,\mathrm{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,d\right.}{4c^{5/2}d}\\
&= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{\left(6ab-\frac{3b^2c}{d}-\frac{7a^2d}{c}\right)\sqrt{x}}{6c\left(c+dx^2\right)}+\frac{((bc-ad)(bc+7ad))\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}}\right.}{8c^{5/2}d^{3/2}}\\
&= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{\left(6ab-\frac{3b^2c}{d}-\frac{7a^2d}{c}\right)\sqrt{x}}{6c\left(c+dx^2\right)}-\frac{(bc-ad)(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\right.}{8\sqrt{2}\,c^{11/4}d^{5/4}}\\
&= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}+\frac{\left(6ab-\frac{3b^2c}{d}-\frac{7a^2d}{c}\right)\sqrt{x}}{6c\left(c+dx^2\right)}-\frac{(bc-ad)(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 315, normalized size = 0.95

$$
\frac{-\frac{3\sqrt{2}\left(-7a^2d^2+6abcd+b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{5/4}}+\frac{3\sqrt{2}\left(-7a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{5/4}}-\frac{6\sqrt{2}\left(-7a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{5/4}}+\frac{6\sqrt{2}\left(-7a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{5/4}}-\frac{32a^2c^{3/4}}{x^{3/2}}-\frac{24c^{3/4}\sqrt{x}\,(bc-ad)^2}{d\left(c+dx^2\right)}}{48c^{11/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(3/4))/x^(3/2) - (24*c^(3/4)*(b*c - a*d)^2*Sqrt[x])/(d*(c + d*x
^2)) - (6*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (6*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) - (3*Sqrt[2]*(b^
2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/d^(5/4) + (3*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4))/(48*c^(11/
4))
```

IntegrateAlgebraic [A] time = 0.74, size = 219, normalized size = 0.66

$$\frac{-4a^2cd-7a^2d^2x^2+6abcdx^2-3b^2c^2x^2}{6c^2dx^{3/2}\left(c+dx^2\right)}-\frac{\left(-7a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}+\frac{\left(-7a^2d^2+6abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (-4*a^2*c*d - 3*b^2*c^2*x^2 + 6*a*b*c*d*x^2 - 7*a^2*d^2*x^2)/(6*c^2*d*x^(3/
2)*(c + d*x^2)) - ((b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt
[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(11/4)*d^(5/4)) + (
(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])
/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(11/4)*d^(5/4))
```

fricas [B] time = 1.38, size = 1340, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b
^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*
d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(
1/4)*arctan((sqrt(c^6*d^2*sqrt(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6
*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 +
9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)) + (b^4*
c^4 + 12*a*b^3*c^3*d + 22*a^2*b^2*c^2*d^2 - 84*a^3*b*c*d^3 + 49*a^4*d^4)*x)
*c^8*d^4*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^
5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6
- 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(3/4) + (b^2*c^10*d^4 + 6*a*
b*c^9*d^5 - 7*a^2*c^8*d^6)*sqrt(x)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^
6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d
^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(3
/4))/(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3
```

```
- 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232
*a^7*b*c*d^7 + 2401*a^8*d^8)) + 3*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24
*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4
*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 240
1*a^8*d^8)/(c^11*d^5))^(1/4)*log(c^3*d*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^
2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c
^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5)
)^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)) - 3*(c^2*d^2*x^4 + c^3
*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5
*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 -
 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4)*log(-c^3*d*(-(b^8*c^8 +
 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*
c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 +
2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)
) + 4*(4*a^2*c*d + (3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2)*sqrt(x))/(c^2*d
^2*x^4 + c^3*d*x^2)
```

giac [**A**]   time = 0.44, size = 384, normalized size = 1.16

$$-\frac{2\,a^{2}}{3\,c^{2}x^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-7\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{3}d^{2}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-7\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{3}d^{2}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-7\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{3}d^{2}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-7\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{3}d^{2}}-\frac{b^{2}c^{2}\sqrt{x}-2\,abcd\sqrt{x}+a^{2}d^{2}\sqrt{x}}{2\left(dx^{2}+c\right)c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^2*x^(3/2)) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/
4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/16*sqrt(2)*((c*
d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*log
(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/16*sqrt(2)*((c*
d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/2*(b^2*c^2*sqr
t(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c^2*d)
```

maple [**A**]   time = 0.02, size = 498, normalized size = 1.50

$$-\frac{a^{2}d\sqrt{x}}{2\left(d\,x^{2}+c\right)c^{2}}+\frac{ab\sqrt{x}}{\left(d\,x^{2}+c\right)c}-\frac{b^{2}\sqrt{x}}{2\left(d\,x^{2}+c\right)d}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8c^{3}}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8c^{3}}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16c^{3}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4c^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4c^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8c^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16cd}-\frac{2a^{2}}{3c^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/2/c^2*d*x^(1/2)/(d*x^2+c)*a^2+1/c*x^(1/2)/(d*x^2+c)*a*b-1/2/d*x^(1/2)/(d
*x^2+c)*b^2-7/8/c^3*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
```

```
)+1)*a^2+3/4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*
a*b+1/8/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-7
/8/c^3*d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/4/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+1/8/c/d*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-7/16/c^3*d*(c/
d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1
/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+3/8/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/
d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^
(1/2)))*a*b+1/16/c/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+
(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-2/3*a^2/c^2/x
^(3/2)
```

maxima [A] time = 2.43, size = 326, normalized size = 0.98

$$-\frac{4\,a^2cd+\left(3\,b^2c^2-6\,abcd+7\,a^2d^2\right)x^2}{6\left(c^2d^2x^{\frac{7}{2}}+c^3dx^{\frac{3}{2}}\right)}+\frac{\frac{2\,\sqrt{2}\left(b^2c^2+6\,abcd-7\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(b^2c^2+6\,abcd-7\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(b^2c^2+6\,abcd-7\,a^2d^2\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^2c^2+6\,abcd-7\,a^2d^2\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(4*a^2*c*d + (3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2)/(c^2*d^2*x^(7/2)
 + c^3*d*x^(3/2)) + 1/16*(2*sqrt(2)*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sq
rt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 + 6*a*b*c*d -
7*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2
 + 6*a*b*c*d - 7*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*log
(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))
/(c^2*d)
```

mupad [B] time = 0.56, size = 1340, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x)
```

```
[Out] (atan((((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 -
 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) - ((a*d - b*c)*(7*a*d + b*c)*(25
6*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5
/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/4)) + ((x^(1/2)*(15
68*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9
+ 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 179
2*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7
```

```
*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/4)))/(((x^(1/2)*(1568*a^4*c^6*d^10 + 32
*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^
8) - ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536
*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)
^(11/4)*d^(5/4)) - ((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b
^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*
d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)
^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)^(11/4)*d^(5/4))))*(a*d
 - b*c)*(7*a*d + b*c)*1i)/(4*(-c)^(11/4)*d^(5/4)) - ((2*a^2)/(3*c) + (x^2*(
7*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d))/(6*c^2*d))/(c*x^(3/2) + d*x^(7/2)) + (a
tan((((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2
688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) - ((a*d - b*c)*(7*a*d + b*c)*(256*
b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8)*1i)/(8*(-c)^(11/4)*d^(
5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)^(11/4)*d^(5/4)) + ((x^(1/2)*(1568
*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 +
704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*
a^2*c^9*d^9 + 1536*a*b*c^10*d^8)*1i)/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(
7*a*d + b*c))/(8*(-c)^(11/4)*d^(5/4)))/(((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b
^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8)
 - ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a
*b*c^10*d^8)*1i)/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*
(-c)^(11/4)*d^(5/4)) - ((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384
*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(
7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8)*1i)/
(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/
4))))*(a*d - b*c)*(7*a*d + b*c))/(4*(-c)^(11/4)*d^(5/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.414 $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=363

$$-\frac{9a^2d^2-10abcd+5b^2c^2}{10c^2d\sqrt{x}\left(c+dx^2\right)}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}d^{3/4}}-\frac{(bc-9ad)(}{}$$

Rubi [A] time = 0.38, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 457, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{9a^2d^2-10abcd+5b^2c^2}{10c^2d\sqrt{x}\left(c+dx^2\right)}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}+\frac{(bc-9ad)(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}d^{3/4}}-\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{13/4}d^{3/4}}+\frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] ((b*c - 9*a*d)*(b*c - a*d))/(2*c^3*d*Sqrt[x]) - (2*a^2)/(5*c*x^(5/2)*(c + d
*x^2)) - (5*b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)/(10*c^2*d*Sqrt[x]*(c + d*x^2)
) - ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a*d)*(b*c - a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) + ((b*c - 9*a
*d)*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(8*Sqrt[2]*c^(13/4)*d^(3/4)) - ((b*c - 9*a*d)*(b*c - a*d)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*d^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^2}\,dx = -\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{2\int \frac{\frac{1}{2}a(10bc-9ad)+\frac{5}{2}b^2cx^2}{x^{3/2}\left(c+dx^2\right)^2}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} - \frac{((bc-9ad)(bc-ad))\int \frac{1}{x^{3/2}\left(c+dx^2\right)}\,dx}{4c^2d}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} + \frac{((bc-9ad)(bc-ad))\int \frac{}{c+}}{4c^3}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} + \frac{((bc-9ad)(bc-ad))\,\mathrm{Sub}}{2c^3}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} - \frac{((bc-9ad)(bc-ad))\,\mathrm{Sub}}{4c^3}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} + \frac{((bc-9ad)(bc-ad))\,\mathrm{Sub}}{}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} + \frac{(bc-9ad)(bc-ad)\log\left(\right.}{8\sqrt{2}}$$

$$= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)} + \frac{10ab-\frac{5b^2c}{d}-\frac{9a^2d}{c}}{10c\sqrt{x}\left(c+dx^2\right)} - \frac{(bc-9ad)(bc-ad)\tan^{-1}}{4\sqrt{2}\,c^{13/4}d^{3/}}$$

Mathematica [A] time = 0.20, size = 333, normalized size = 0.92

$$\frac{\frac{5\sqrt{2}\left(9a^2d^2-10abcd+b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{5\sqrt{2}\left(9a^2d^2-10abcd+b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{3/4}}-\frac{10\sqrt{2}\left(9a^2d^2-10abcd+b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{3/4}}+\frac{10\sqrt{2}\left(9a^2d^2-10abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{3/4}}-\frac{32a^2c^{5/4}}{x^{5/2}}+\frac{40\sqrt[4]{c}x^{3/2}(bc-ad)^2}{c+dx^2}+\frac{320a\sqrt[4]{c}(ad-bc)}{\sqrt{x}}}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] ((-32*a^2*c^(5/4))/x^(5/2) + (320*a*c^(1/4)*(-(b*c) + a*d))/Sqrt[x] + (40*c
^(1/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2) - (10*Sqrt[2]*(b^2*c^2 - 10*a*b*c
```

```
*d + 9*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) + (1
0*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/d^(3/4) + (5*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4) - (5*Sqrt
[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(80*c^(13/4))
```

IntegrateAlgebraic [A] time = 0.76, size = 237, normalized size = 0.65

$$-\frac{\left(9a^2d^2-10abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{13/4}d^{3/4}}-\frac{\left(9a^2d^2-10abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{13/4}d^{3/4}}+\frac{-4a^2c^2+36a^2cdx^2+45a^2d^2x^4-40abc^2x^2-50abcdx^4+5b^2c^2x^4}{10c^3x^{5/2}\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

```
[Out] (-4*a^2*c^2 - 40*a*b*c^2*x^2 + 36*a^2*c*d*x^2 + 5*b^2*c^2*x^4 - 50*a*b*c*d*
x^4 + 45*a^2*d^2*x^4)/(10*c^3*x^(5/2)*(c + d*x^2)) - ((b^2*c^2 - 10*a*b*c*d
 + 9*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)])/(4*Sqrt[2]*c^(13/4)*d^(3/4)) - ((b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*ArcT
anh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^
(13/4)*d^(3/4))
```

fricas [B] time = 1.56, size = 1737, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(20*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c
^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d
^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^
(1/4)*arctan((sqrt((b^12*c^12 - 60*a*b^11*c^11*d + 1554*a^2*b^10*c^10*d^2 -
 22700*a^3*b^9*c^9*d^3 + 205215*a^4*b^8*c^8*d^4 - 1188600*a^5*b^7*c^7*d^5 +
 4443580*a^6*b^6*c^6*d^6 - 10697400*a^7*b^5*c^5*d^7 + 16622415*a^8*b^4*c^4*
d^8 - 16548300*a^9*b^3*c^3*d^9 + 10195794*a^10*b^2*c^2*d^10 - 3542940*a^11*
b*c*d^11 + 531441*a^12*d^12)*x - (b^8*c^15*d - 40*a*b^7*c^14*d^2 + 636*a^2*
b^6*c^13*d^3 - 5080*a^3*b^5*c^12*d^4 + 21286*a^4*b^4*c^11*d^5 - 45720*a^5*b
^3*c^10*d^6 + 51516*a^6*b^2*c^9*d^7 - 29160*a^7*b*c^8*d^8 + 6561*a^8*c^7*d^
9)*sqrt(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5
*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^
6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3)))*c^3*d*(-(b^8*c^8 - 40*a*
b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*
d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6
561*a^8*d^8)/(c^13*d^3))^(1/4) - (b^6*c^9*d - 30*a*b^5*c^8*d^2 + 327*a^2*b^
```

```
4*c^7*d^3 - 1540*a^3*b^3*c^6*d^4 + 2943*a^4*b^2*c^5*d^5 - 2430*a^5*b*c^4*d^
6 + 729*a^6*c^3*d^7)*sqrt(x)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*
d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5
+ 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1/
4))/(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3
+ 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 2
9160*a^7*b*c*d^7 + 6561*a^8*d^8)) - 5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40
*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c
^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7
+ 6561*a^8*d^8)/(c^13*d^3))^(1/4)*log(c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d
+ 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 4572
0*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^
8)/(c^13*d^3))^(3/4) + (b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 15
40*a^3*b^3*c^3*d^3 + 2943*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)
*sqrt(x)) + 5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b
^6*c^6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c
^3*d^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^
3))^(1/4)*log(-c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 -
 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 515
16*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(3/4) +
(b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 29
43*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)*sqrt(x)) - 4*(5*(b^2*c
^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 4*a^2*c^2 - 4*(10*a*b*c^2 - 9*a^2*c*d)*x
^2)*sqrt(x))/(c^3*d*x^5 + c^4*x^3)
```

giac [**A**] time = 0.50, size = 401, normalized size = 1.10

$$\frac{b^{2}c^{2}x^{\frac{3}{2}}-2\,abcdx^{\frac{3}{2}}+a^{2}d^{2}x^{\frac{3}{2}}}{2\left(dx^{2}+c\right)c^{3}}-\frac{2\left(10\,abcx^{2}-10\,a^{2}dx^{2}+a^{2}c\right)}{5\,c^{3}x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+9\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{4}d^{3}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+9\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^{4}d^{3}}-\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+9\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{4}d^{3}}+\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+9\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^{4}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*c^
3) - 2/5*(10*a*b*c*x^2 - 10*a^2*d*x^2 + a^2*c)/(c^3*x^(5/2)) + 1/8*sqrt(2)*
((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2
)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^
3) + 1/8*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d
^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c
/d)^(1/4))/(c^4*d^3) - 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/
4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(c^4*d^3) + 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3
/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c^4*d^3)
```

maple [A] time = 0.02, size = 524, normalized size = 1.44

$$\frac{a^{2}d^{2}x^{\frac{3}{2}}}{2\left(d\,x^{2}+c\right)c^{3}}-\frac{abd\,x^{\frac{3}{2}}}{\left(d\,x^{2}+c\right)c^{2}}+\frac{b^{2}x^{\frac{3}{2}}}{2\left(d\,x^{2}+c\right)c}+\frac{9\sqrt{2}\;a^{2}d\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{3}}+\frac{9\sqrt{2}\;a^{2}d\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{3}}+\frac{9\sqrt{2}\;a^{2}d\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{3}}-\frac{5\sqrt{2}\;ab\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{5\sqrt{2}\;ab\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{5\sqrt{2}\;ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}+\frac{\sqrt{2}\;b^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}cd}+\frac{\sqrt{2}\;b^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{c}{d}\right)^{\frac{1}{4}}cd}+\frac{\sqrt{2}\;b^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(\frac{c}{d}\right)^{\frac{1}{4}}cd}+\frac{4a^{2}d}{c^{3}\sqrt{x}}-\frac{4ab}{c^{2}\sqrt{x}}-\frac{2a^{2}}{5c^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/2/c^3*x^(3/2)/(d*x^2+c)*a^2*d^2-1/c^2*x^(3/2)/(d*x^2+c)*a*b*d+1/2/c*x^(3/
2)/(d*x^2+c)*b^2+9/16/c^3*d/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*2^(1/
2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+9/8/c^
3*d/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+9/8/c^3*d
/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-5/8/c^2/(c/d
)^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-5/4/c^2/(c/d)^(1/4)*2^(1/2)*a*b*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-5/4/c^2/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)+1/16/c/d/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))
)+1/8/c/d/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/8
/c/d/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5*a^2/
c^2/x^(5/2)+4*a^2/c^3/x^(1/2)*d-4*a/c^2/x^(1/2)*b
```

maxima [A] time = 2.38, size = 275, normalized size = 0.76

$$\frac{5\left(b^{2}c^{2}-10\,abcd+9\,a^{2}d^{2}\right)x^{4}-4\,a^{2}c^{2}-4\left(10\,abc^{2}-9\,a^{2}cd\right)x^{2}}{10\left(c^{3}dx^{\frac{9}{2}}+c^{4}x^{\frac{5}{2}}\right)}+\frac{\left(b^{2}c^{2}-10\,abcd+9\,a^{2}d^{2}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(5*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 4*a^2*c^2 - 4*(10*a*b*c^2
- 9*a^2*c*d)*x^2)/(c^3*d*x^(9/2) + c^4*x^(5/2)) + 1/16*(b^2*c^2 - 10*a*b*c*
d + 9*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)
*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)
*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/
4)))/c^3
```

mupad [B] time = 0.23, size = 152, normalized size = 0.42

$$\frac{\frac{x^4\,\left(9\,a^2\,d^2-10\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^3}-\frac{2\,a^2}{5\,c}+\frac{2\,a\,x^2\,\left(9\,a\,d-10\,b\,c\right)}{5\,c^2}}{c\,x^{5/2}+d\,x^{9/2}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(9\,a\,d-b\,c\right)}{4\,{\left(-c\right)}^{13/4}\,d^{3/4}}+\frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(9\,a\,d-b\,c\right)}{4\,{\left(-c\right)}^{13/4}\,d^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x)
```

```
[Out] ((x^4*(9*a^2*d^2 + b^2*c^2 - 10*a*b*c*d))/(2*c^3) - (2*a^2)/(5*c) + (2*a*x^
2*(9*a*d - 10*b*c))/(5*c^2))/(c*x^(5/2) + d*x^(9/2)) - (atan((d^(1/4)*x^(1/
2))/(-c)^(1/4))*(a*d - b*c)*(9*a*d - b*c))/(4*(-c)^(13/4)*d^(3/4)) + (atanh
((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(9*a*d - b*c))/(4*(-c)^(13/4)*d^
(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.415 $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=440

$$-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}+\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\right.}{64\sqrt{2}\,c^{3/4}d^{17/4}}$$

Rubi [A] time = 0.38, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{x^{5/2}\left(5a^2d^2-90abcd+117b^2c^2\right)}{80c^2d^3}-\frac{\sqrt{x}\left(5a^2d^2-90abcd+117b^2c^2\right)}{16cd^4}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}\,c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}\,c^{3/4}d^{17/4}}-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{3/4}d^{17/4}}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Sqrt[x])/(16*c*d^4) + ((117*b^2*c^
2 - 90*a*b*c*d + 5*a^2*d^2)*x^(5/2))/(80*c^2*d^3) + ((b*c - a*d)^2*x^(9/2))
/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(17*b*c - a*d)*x^(9/2))/(16*c^2*d^2
*(c + d*x^2)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 -
 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32
*Sqrt[2]*c^(3/4)*d^(17/4)) - ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d
^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^{7/2}\left(\frac{1}{2}\left(-8a^2d^2+9(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\int \frac{x^{7/2}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\left(c+dx^2\right)} - \left(117b^2\right.$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}$$

$$= -\frac{\left(117b^2c^2-90abcd+5a^2d^2\right)\sqrt{x}}{16cd^4} + \frac{\left(117b^2c^2-90abcd+5a^2d^2\right)x^{5/2}}{80c^2d^3} + \frac{(bc-ad)^2x^{9/2}}{4cd^2\left(c+dx^2\right)^2}$$

Mathematica [A] time = 0.47, size = 383, normalized size = 0.87

$$\frac{-\frac{40\sqrt[4]{d}\sqrt{x}\left(9a^2d^2-34abcd+25b^2c^2\right)}{c+dx^2} - \frac{5\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}} + \frac{5\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}} - \frac{10\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{3/4}} + \frac{10\sqrt{2}\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{3/4}} + \frac{160c\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{\left(c+dx^2\right)^2} - 1280b\sqrt[4]{d}\sqrt{x}(3bc-2ad) + 256b^2d^{5/4}x^{5/2}}{640d^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (-1280*b*d^(1/4)*(3*b*c - 2*a*d)*Sqrt[x] + 256*b^2*d^(5/4)*x^(5/2) + (160*c
*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 - (40*d^(1/4)*(25*b^2*c^2 - 3
4*a*b*c*d + 9*a^2*d^2)*Sqrt[x])/(c + d*x^2) - (10*Sqrt[2]*(117*b^2*c^2 - 90
*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4
) + (10*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/c^(3/4) - (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d +
 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(
3/4) + (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(3/4))/(640*d^(17/4))
```

IntegrateAlgebraic [A] time = 0.91, size = 272, normalized size = 0.62

$$-\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}\,c^{3/4}d^{17/4}}+\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{3/4}d^{17/4}}+\frac{\sqrt{x}\left(-25a^2cd^2-45a^2d^3x^2+450abc^2d+810abcd^2x^2+320abd^3x^4-585b^2c^3-1053b^2c^2dx^2-416b^2cd^2x^4+32b^2d^3x^6\right)}{80d^4\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[x]*(-585*b^2*c^3 + 450*a*b*c^2*d - 25*a^2*c*d^2 - 1053*b^2*c^2*d*x^2
+ 810*a*b*c*d^2*x^2 - 45*a^2*d^3*x^2 - 416*b^2*c*d^2*x^4 + 320*a*b*d^3*x^4
+ 32*b^2*d^3*x^6))/(80*d^4*(c + d*x^2)^2) - ((117*b^2*c^2 - 90*a*b*c*d + 5*
a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(
32*Sqrt[2]*c^(3/4)*d^(17/4)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcT
anh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c
^(3/4)*d^(17/4))
```

fricas [B] time = 1.44, size = 1427, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(20*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124
525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6
 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*arctan((sqrt(c^2*d^8*
sqrt(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^
2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^
3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3
*d^17)) + (13689*b^4*c^4 - 21060*a*b^3*c^3*d + 9270*a^2*b^2*c^2*d^2 - 900*a
^3*b*c*d^3 + 25*a^4*d^4)*x)*c^2*d^13*(-(187388721*b^8*c^8 - 576580680*a*b^7
*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*
a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 4500
```

```
0*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(3/4) - (117*b^2*c^4*d^13 - 90*a*b
*c^3*d^14 + 5*a^2*c^2*d^15)*sqrt(x)*(-(187388721*b^8*c^8 - 576580680*a*b^7*
c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000
*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(3/4))/(187388721*b^8*c^8 - 5765806
80*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 12
4525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^
6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)) + 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 -
 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c
^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^
17))^(1/4)*log(c*d^4*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 6973176
60*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4
- 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 +
625*a^8*d^8)/(c^3*d^17))^(1/4) + (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqr
t(x)) - 5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124
525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6
 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*log(-c*d^4*(-(1873887
21*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*
a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 12
73500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)
+ (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) + 4*(32*b^2*d^3*x^6 - 585
*b^2*c^3 + 450*a*b*c^2*d - 25*a^2*c*d^2 - 32*(13*b^2*c*d^2 - 10*a*b*d^3)*x^
4 - 9*(117*b^2*c^2*d - 90*a*b*c*d^2 + 5*a^2*d^3)*x^2)*sqrt(x))/(d^6*x^4 + 2
*c*d^5*x^2 + c^2*d^4)
```

giac [A] time = 0.44, size = 451, normalized size = 1.02

$$\frac{\sqrt{2}\left(117\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-90\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^{5}}+\frac{\sqrt{2}\left(117\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-90\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^{5}}+\frac{\sqrt{2}\left(117\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-90\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,cd^{5}}-\frac{\sqrt{2}\left(117\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}-90\left(cd^{3}\right)^{\frac{1}{4}}abcd+5\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,cd^{5}}-\frac{25\,b^{2}c^{2}dx^{\frac{5}{2}}-34\,abcd^{2}x^{\frac{5}{2}}+9\,a^{2}d^{3}x^{\frac{5}{2}}+21\,b^{2}c^{3}\sqrt{x}-26\,abc^{2}d\sqrt{x}+5\,a^{2}cd^{2}\sqrt{x}}{16\left(dx^{2}+c\right)^{2}d^{4}}+\frac{2\left(b^{2}d^{12}x^{\frac{5}{2}}-15\,b^{2}cd^{11}\sqrt{x}+10\,abd^{12}\sqrt{x}\right)}{5\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d
^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(c*d^5) + 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1
/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^5) + 1/128*sqrt(2)*(117*(c*d^3)^(1/4)*b
^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/128*sqrt(2)*(117*(c*d^3)^(1/
4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/16*(25*b^2*c^2*d*x^(5/2
) - 34*a*b*c*d^2*x^(5/2) + 9*a^2*d^3*x^(5/2) + 21*b^2*c^3*sqrt(x) - 26*a*b*
```

```
c^2*d*sqrt(x) + 5*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*d^4) + 2/5*(b^2*d^12*x^
(5/2) - 15*b^2*c*d^11*sqrt(x) + 10*a*b*d^12*sqrt(x))/d^15
```

maple [A] time = 0.02, size = 590, normalized size = 1.34

$$-\frac{9a^2x^{\frac{5}{2}}}{16\left(d\,x^2+c\right)^2 d}+\frac{17abc\,x^{\frac{5}{2}}}{8\left(d\,x^2+c\right)^2 d^2}-\frac{25b^2c^2x^{\frac{5}{2}}}{16\left(d\,x^2+c\right)^2 d^3}-\frac{5a^2c\sqrt{x}}{16\left(d\,x^2+c\right)^2 d^2}+\frac{13ab\,c^2\sqrt{x}}{8\left(d\,x^2+c\right)^2 d^3}-\frac{21b^2c^3\sqrt{x}}{16\left(d\,x^2+c\right)^2 d^4}+\frac{2b^2x^{\frac{5}{2}}}{5d^3}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c\,d^2}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c\,d^2}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128c\,d^2}-\frac{45\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32d^3}-\frac{45\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32d^3}-\frac{45\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64d^3}+\frac{117\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64d^4}+\frac{117\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64d^4}+\frac{117\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128d^4}+\frac{4ab\sqrt{x}}{d^3}-\frac{6b^2c\sqrt{x}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2/5*b^2/d^3*x^(5/2)+4*b/d^3*a*x^(1/2)-6*b^2/d^4*c*x^(1/2)-9/16/d/(d*x^2+c)^
2*x^(5/2)*a^2+17/8/d^2/(d*x^2+c)^2*x^(5/2)*a*b*c-25/16/d^3/(d*x^2+c)^2*x^(5
/2)*b^2*c^2-5/16/d^2/(d*x^2+c)^2*x^(1/2)*a^2*c+13/8/d^3/(d*x^2+c)^2*x^(1/2)
*a*b*c^2-21/16/d^4/(d*x^2+c)^2*x^(1/2)*b^2*c^3+5/64/d^2*(c/d)^(1/4)/c*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-45/32/d^3*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+117/64/d^4*(c/d)^(1/4)*c*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/d^2*(c/d)^(1/4)/c*2^(1/2)*l
n((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2
)+(c/d)^(1/2)))*a^2-45/64/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+117/
128/d^4*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)
)/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+5/64/d^2*(c/d)^(1/4)/c*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-45/32/d^3*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+117/64/d^4*(c/d)^(1/4)*c*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [A] time = 2.44, size = 389, normalized size = 0.88

$$-\frac{\left(25\,b^2c^2d-34\,abcd^2+9\,a^2d^3\right)x^{\frac{5}{2}}+\left(21\,b^2c^3-26\,abc^2d+5\,a^2cd^2\right)\sqrt{x}}{16\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}+\frac{2\left(b^2dx^{\frac{5}{2}}-5\left(3\,b^2c-2\,abd\right)\sqrt{x}\right)}{5\,d^4}+\frac{\frac{2\,\sqrt{2}\left(117\,b^2c^2-90\,abcd+5\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(117\,b^2c^2-90\,abcd+5\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(117\,b^2c^2-90\,abcd+5\,a^2d^2\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(117\,b^2c^2-90\,abcd+5\,a^2d^2\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((25*b^2*c^2*d - 34*a*b*c*d^2 + 9*a^2*d^3)*x^(5/2) + (21*b^2*c^3 - 26
*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(x))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) + 2/5*
(b^2*d*x^(5/2) - 5*(3*b^2*c - 2*a*b*d)*sqrt(x))/d^4 + 1/128*(2*sqrt(2)*(117
*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(
d))) + 2*sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)
*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4
)) - sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^
(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^4
```

mupad [B] time = 0.48, size = 1426, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (2*b^2*x^(5/2))/(5*d^3) - (x^(1/2)*((21*b^2*c^3)/16 + (5*a^2*c*d^2)/16 - (1
3*a*b*c^2*d)/8) + x^(5/2)*((9*a^2*d^3)/16 + (25*b^2*c^2*d)/16 - (17*a*b*c*d
^2)/8))/(c^2*d^4 + d^6*x^4 + 2*c*d^5*x^2) - x^(1/2)*((6*b^2*c)/d^4 - (4*a*b
)/d^3) + (atan(((((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d
^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2
*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/
4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(64*(-c)^(3/4)*d^(
17/4)) + (((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21
060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 -
90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(2
1/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(64*(-c)^(3/4)*d^(17/4)))
/((((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b
^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*
c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(21/4)))*
(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4)) - (((x^(1/
2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d -
 900*a^3*b*c*d^3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*
b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2
 + 117*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4))))*(5*a^2*d^2 + 117*b
^2*c^2 - 90*a*b*c*d)*1i)/(32*(-c)^(3/4)*d^(17/4)) + (atan(((((x^(1/2)*(25*a
^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3
*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3
+ 5*a^2*c*d^2 - 90*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 11
7*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4)) + (((x^(1/2)*(25*a^4*d^4
+ 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^
3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2
*c*d^2 - 90*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c
^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4)))/((((x^(1/2)*(25*a^4*d^4 + 13689
*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64
*d^5) - ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2
- 90*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90
*a*b*c*d)*1i)/(64*(-c)^(3/4)*d^(17/4)) - (((x^(1/2)*(25*a^4*d^4 + 13689*b^4
*c^4 + 9270*a^2*b^2*c^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5
) + ((5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90
*a*b*c^2*d)*1i)/(64*(-c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b
*c*d)*1i)/(64*(-c)^(3/4)*d^(17/4))))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)
)/(32*(-c)^(3/4)*d^(17/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.416 $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=401

$$-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{}\right.}{64\sqrt{2}\,c^{5/4}d^{15/4}}$$

Rubi [A] time = 0.34, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 457, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{5/4}d^{15/4}}-\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{5/4}d^{15/4}}-\frac{x^{3/2}\left(\frac{3a^2d}{c}+42ab-\frac{77b^2c}{d}\right)}{48cd^2}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((42*a*b - (77*b^2*c)/d + (3*a^2*d)/c)*x^(3/2))/(48*c*d^2) + ((b*c - a*d)^
2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(15*b*c + a*d)*x^(7/2))/(
16*c^2*d^2*(c + d*x^2)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b
^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) - ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(5/4)*d^(15/4)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^{5/2}\left(\frac{1}{2}\left(-8a^2d^2+7(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)\int \frac{x^{5/2}}{c+dx^2}dx}{32c^2d^2}$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - (77b^2c^2 -$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - (77b^2c^2 -$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} + (77b^2c^2 -$$

$$(77b^2c^2 -$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} -$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} - (77b^2c^2 -$$

$$= \frac{\left(77b^2c^2-42abcd-3a^2d^2\right)x^{3/2}}{48c^2d^3} + \frac{(bc-ad)^2x^{7/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2\left(c+dx^2\right)} + (77b^2c^2 -$$

Mathematica [A] time = 0.24, size = 363, normalized size = 0.91

$$\frac{\frac{24d^{3/4}x^{3/2}\left(3a^2d^2-22abcd+19b^2c^2\right)}{c\left(c+dx^2\right)} - \frac{3\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}} + \frac{3\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}} + \frac{6\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{6\sqrt{2}\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}} - \frac{96d^{3/4}x^{3/2}(bc-ad)^2}{\left(c+dx^2\right)^2} + 256b^2d^{3/4}x^{3/2}}{384d^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (256*b^2*d^(3/4)*x^(3/2) - (96*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2
 + (24*d^(3/4)*(19*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*x^(3/2))/(c*(c + d*x^2
)) + (6*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (6*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3
*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (3*Sqrt[
2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4) + (3*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3
*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/
4))/(384*d^(15/4))
```

IntegrateAlgebraic [A] time = 1.02, size = 254, normalized size = 0.63

$$\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{5/4}d^{15/4}}+\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{5/4}d^{15/4}}+\frac{x^{3/2}\left(-3a^2cd^2+9a^2d^3x^2-42abc^2d-66abcd^2x^2+77b^2c^3+121b^2c^2dx^2+32b^2cd^2x^4\right)}{48cd^3\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (x^(3/2)*(77*b^2*c^3 - 42*a*b*c^2*d - 3*a^2*c*d^2 + 121*b^2*c^2*d*x^2 - 66*
a*b*c*d^2*x^2 + 9*a^2*d^3*x^2 + 32*b^2*c*d^2*x^4))/(48*c*d^3*(c + d*x^2)^2)
 + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(5/4)*d^(15/4)) + ((77*b^2*c^
2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(5/4)*d^(15/4))
```

fricas [B] time = 1.24, size = 1813, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(12*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c^8 - 7669
7544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 45
36*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*arctan((sqrt((208422380089*b
^12*c^12 - 682109607564*a*b^11*c^11*d + 881427350034*a^2*b^10*c^10*d^2 - 54
3593843100*a^3*b^9*c^9*d^3 + 136525986135*a^4*b^8*c^8*d^4 + 8334677736*a^5*
b^7*c^7*d^5 - 7849956996*a^6*b^6*c^6*d^6 - 324727704*a^7*b^5*c^5*d^7 + 2072
41335*a^8*b^4*c^4*d^8 + 32148900*a^9*b^3*c^3*d^9 + 2030994*a^10*b^2*c^2*d^1
0 + 61236*a^11*b*c*d^11 + 729*a^12*d^12)*x - (35153041*b^8*c^11*d^7 - 76697
544*a*b^7*c^10*d^8 + 57274140*a^2*b^6*c^9*d^9 - 13854456*a^3*b^5*c^8*d^10 -
 1457946*a^4*b^4*c^7*d^11 + 539784*a^5*b^3*c^6*d^12 + 86940*a^6*b^2*c^5*d^1
3 + 4536*a^7*b*c^4*d^14 + 81*a^8*c^3*d^15)*sqrt(-(35153041*b^8*c^8 - 766975
44*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457
946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536
```

```
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15)))*c*d^4*(-(35153041*b^8*c^8 - 7669754
4*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14579
46*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4) + (456533*b^6*c^7*d^4 - 747054*
a*b^5*c^6*d^5 + 354123*a^2*b^4*c^5*d^6 - 15876*a^3*b^3*c^4*d^7 - 13797*a^4*
b^2*c^3*d^8 - 1134*a^5*b*c^2*d^9 - 27*a^6*c*d^10)*sqrt(x)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5
*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4))/(35153041*b^8*c^8
- 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^
3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^
6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)) - 3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^
3)*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 -
13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5
 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)
*log(c^4*d^11*(-(35153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6
*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*
b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d
^15))^(3/4) - (456533*b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2
 - 15876*a^3*b^3*c^3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^
6*d^6)*sqrt(x)) + 3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c
^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5
*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2
*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(-c^4*d^11*(-(35
153041*b^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456
*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940
*a^6*b^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(3/4) - (4565
33*b^6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^
3*d^3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) - 4
*(32*b^2*c*d^2*x^5 + (121*b^2*c^2*d - 66*a*b*c*d^2 + 9*a^2*d^3)*x^3 + (77*b
^2*c^3 - 42*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2
 + c^3*d^3)
```

giac [A] time = 0.47, size = 427, normalized size = 1.06

$$\frac{2\,b^{2}x^{\frac{3}{2}}}{3\,d^{3}}+\frac{19\,b^{2}c^{2}dx^{\frac{7}{2}}-22\,abcd^{2}x^{\frac{7}{2}}+3\,a^{2}d^{3}x^{\frac{7}{2}}+15\,b^{2}c^{3}x^{\frac{3}{2}}-14\,abc^{2}dx^{\frac{3}{2}}-a^{2}cd^{2}x^{\frac{3}{2}}}{16\left(dx^{2}+c\right)^{2}cd^{3}}-\frac{\sqrt{2}\left(77\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-42\left(cd^{3}\right)^{\frac{3}{4}}abcd-3\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{2}d^{6}}-\frac{\sqrt{2}\left(77\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-42\left(cd^{3}\right)^{\frac{3}{4}}abcd-3\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{2}d^{6}}+\frac{\sqrt{2}\left(77\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-42\left(cd^{3}\right)^{\frac{3}{4}}abcd-3\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{2}d^{6}}-\frac{\sqrt{2}\left(77\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}-42\left(cd^{3}\right)^{\frac{3}{4}}abcd-3\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{2}d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2/3*b^2*x^(3/2)/d^3 + 1/16*(19*b^2*c^2*d*x^(7/2) - 22*a*b*c*d^2*x^(7/2) + 3
*a^2*d^3*x^(7/2) + 15*b^2*c^3*x^(3/2) - 14*a*b*c^2*d*x^(3/2) - a^2*c*d^2*x^
(3/2))/((d*x^2 + c)^2*c*d^3) - 1/64*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*
(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2
```

```
)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) - 1/64*sqrt(2)*(77*(c*d^3
)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) + 1
/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^
3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^6
) - 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*
(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c
^2*d^6)
```

maple [A] time = 0.03, size = 562, normalized size = 1.40

$$\frac{3a^2x^{\frac{7}{2}}}{16\left(dx^2+c\right)^2c}-\frac{11ab\,x^{\frac{7}{2}}}{8\left(dx^2+c\right)^2d}+\frac{19b^2c\,x^{\frac{7}{2}}}{16\left(dx^2+c\right)^2d^2}-\frac{a^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2d}-\frac{7abc\,x^{\frac{3}{2}}}{8\left(dx^2+c\right)^2d^2}+\frac{15b^2c^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2d^3}+\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{3\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c\,d^2}+\frac{3\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c\,d^2}+\frac{3\sqrt{2}\,a^2\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128\left(\frac{c}{d}\right)^{\frac{1}{4}}c\,d^2}+\frac{21\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32\left(\frac{c}{d}\right)^{\frac{1}{4}}d^3}+\frac{21\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32\left(\frac{c}{d}\right)^{\frac{1}{4}}d^3}+\frac{21\sqrt{2}\,ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}d^3}-\frac{77\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}d^4}-\frac{77\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}d^4}-\frac{77\sqrt{2}\,b^2c\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128\left(\frac{c}{d}\right)^{\frac{1}{4}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2/3*b^2/d^3*x^(3/2)+3/16/(d*x^2+c)^2/c*x^(7/2)*a^2-11/8/d/(d*x^2+c)^2*x^(7/
2)*a*b+19/16/d^2/(d*x^2+c)^2*c*x^(7/2)*b^2-1/16/d/(d*x^2+c)^2*x^(3/2)*a^2-7
/8/d^2/(d*x^2+c)^2*x^(3/2)*a*b*c+15/16/d^3/(d*x^2+c)^2*x^(3/2)*b^2*c^2+3/64
/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+21/32/
d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-77/64/d^4
*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64/d^2/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+21/32/d^3/(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-77/64/d^4*c/(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/d^2/c/(c/d
)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+21/64/d^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)
^(1/2)))*a*b-77/128/d^4*c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(
1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.54, size = 306, normalized size = 0.76

$$\frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3}+\frac{\left(19\,b^2c^2d-22\,abcd^2+3\,a^2d^3\right)x^{\frac{7}{2}}+\left(15\,b^2c^3-14\,abc^2d-a^2cd^2\right)x^{\frac{3}{2}}}{16\left(cd^5x^4+2\,c^2d^4x^2+c^3d^3\right)}-\frac{\left(77\,b^2c^2-42\,abcd-3\,a^2d^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x^(3/2)/d^3 + 1/16*((19*b^2*c^2*d - 22*a*b*c*d^2 + 3*a^2*d^3)*x^(7/
2) + (15*b^2*c^3 - 14*a*b*c^2*d - a^2*c*d^2)*x^(3/2))/(c*d^5*x^4 + 2*c^2*d^
4*x^2 + c^3*d^3) - 1/128*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*(2*sqrt(2)*a
rctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c
```

```
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/
4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d^3)
```

mupad [B] time = 0.42, size = 197, normalized size = 0.49

$$\frac{2\,b^2\,x^{3/2}}{3\,d^3}-\frac{x^{3/2}\,\left(\frac{a^2\,d^2}{16}+\frac{7\,a\,b\,c\,d}{8}-\frac{15\,b^2\,c^2}{16}\right)-\frac{x^{7/2}\,\left(3\,a^2\,d^3-22\,a\,b\,c\,d^2+19\,b^2\,c^2\,d\right)}{16\,c}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(3\,a^2\,d^2+42\,a\,b\,c\,d-77\,b^2\,c^2\right)}{32\,{\left(-c\right)}^{5/4}\,d^{15/4}}-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-c\right)}^{1/4}}\right)\,\left(3\,a^2\,d^2+42\,a\,b\,c\,d-77\,b^2\,c^2\right)\,1{}\mathrm{i}}{32\,{\left(-c\right)}^{5/4}\,d^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (2*b^2*x^(3/2))/(3*d^3) - (x^(3/2)*((a^2*d^2)/16 - (15*b^2*c^2)/16 + (7*a*b
*c*d)/8) - (x^(7/2)*(3*a^2*d^3 + 19*b^2*c^2*d - 22*a*b*c*d^2))/(16*c))/(c^2
*d^3 + d^5*x^4 + 2*c*d^4*x^2) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*a^2*
d^2 - 77*b^2*c^2 + 42*a*b*c*d))/(32*(-c)^(5/4)*d^(15/4)) - (atan((d^(1/4)*x
^(1/2)*1i)/(-c)^(1/4))*(3*a^2*d^2 - 77*b^2*c^2 + 42*a*b*c*d)*1i)/(32*(-c)^(
5/4)*d^(15/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.417 $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=402

$$\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\right.}{64\sqrt{2}\,c^{7/4}d^{13/4}}$$

Rubi [A] time = 0.33, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {463, 457, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}d^{13/4}}+\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\sqrt{x}\left(\frac{3a^2d}{c}+10ab-\frac{45b^2c}{d}\right)}{16cd^2}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -((10*a*b - (45*b^2*c)/d + (3*a^2*d)/c)*Sqrt[x])/(16*c*d^2) + ((b*c - a*d)^
2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(13*b*c + 3*a*d)*x^(5/2))
/(16*c^2*d^2*(c + d*x^2)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((45
*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^
2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*
c^(7/4)*d^(13/4)) - ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{x^{3/2}\left(\frac{1}{2}\left(-8a^2d^2+5(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\int \frac{x^{3/2}}{c+dx^2}dx}{32c^2d^2}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2\right.}{}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2\right.}{}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2\right.}{}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(45b^2c^2\right.}{}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(45b^2c^2\right.}{}$$

$$= \frac{\left(45b^2c^2-10abcd-3a^2d^2\right)\sqrt{x}}{16c^2d^3} + \frac{(bc-ad)^2x^{5/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(45b^2c^2\right.}{}$$

Mathematica [A] time = 0.39, size = 361, normalized size = 0.90

$$\frac{\frac{8\sqrt[4]{d}\sqrt{x}\left(a^2d^2-18abcd+17b^2c^2\right)}{c\left(c+dx^2\right)} + \frac{\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} - \frac{\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}} + \frac{2\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{2\sqrt{2}\left(-3a^2d^2-10abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{7/4}} - \frac{32\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{\left(c+dx^2\right)^2} + 256b^2\sqrt[4]{d}\sqrt{x}}{128d^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (256*b^2*d^(1/4)*Sqrt[x] - (32*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2
 + (8*d^(1/4)*(17*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*Sqrt[x])/(c*(c + d*x^2))
+ (2*Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (2*Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^
2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) + (Sqrt[2]*(4
5*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/c^(7/4) - (Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^
2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(12
8*d^(13/4))
```

IntegrateAlgebraic [A] time = 1.10, size = 253, normalized size = 0.63

$$\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{7/4}d^{13/4}}-\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{7/4}d^{13/4}}+\frac{\sqrt{x}\left(-3a^2cd^2+a^2d^3x^2-10abc^2d-18abcd^2x^2+45b^2c^3+81b^2c^2dx^2+32b^2cd^2x^4\right)}{16cd^3\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[x]*(45*b^2*c^3 - 10*a*b*c^2*d - 3*a^2*c*d^2 + 81*b^2*c^2*d*x^2 - 18*a
*b*c*d^2*x^2 + a^2*d^3*x^2 + 32*b^2*c*d^2*x^4))/(16*c*d^3*(c + d*x^2)^2) +
((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(7/4)*d^(13/4)) - ((45*b^2*c^2 -
 10*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c]
 + Sqrt[d]*x)])/(32*Sqrt[2]*c^(7/4)*d^(13/4))
```

fricas [B] time = 1.29, size = 1420, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*
a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b
^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7
 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*arctan((sqrt(c^4*d^6*sqrt(-(4100625*b^8*c^
8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 -
 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080
*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13)) + (2025*b^4*c^4 - 900*a*b^3*c^3*d -
170*a^2*b^2*c^2*d^2 + 60*a^3*b*c*d^3 + 9*a^4*d^4)*x)*c^5*d^10*(-(4100625*b^
8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d
^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 +
1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(3/4) + (45*b^2*c^7*d^10 - 10*a*
b*c^6*d^11 - 3*a^2*c^5*d^12)*sqrt(x)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7
*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8
```

```
*d^8)/(c^7*d^13))^(3/4))/(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^
2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*
b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)) + (c*d^
5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4)*log(c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 3
6600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)
/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (c*d^
5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4)*log(-c^2*d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d +
 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 -
36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8
)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*sqrt(x)) + 4*(3
2*b^2*c*d^2*x^4 + 45*b^2*c^3 - 10*a*b*c^2*d - 3*a^2*c*d^2 + (81*b^2*c^2*d -
 18*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3
)
```

giac [A]   time = 0.43, size = 426, normalized size = 1.06

$$\frac{2\,b^2\sqrt{x}}{d^3}-\frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\,\left(cd^3\right)^{\frac{1}{4}}abcd-3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4}-\frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\,\left(cd^3\right)^{\frac{1}{4}}abcd-3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4}-\frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\,\left(cd^3\right)^{\frac{1}{4}}abcd-3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^2d^4}+\frac{\sqrt{2}\left(45\,\left(cd^3\right)^{\frac{1}{4}}b^2c^2-10\,\left(cd^3\right)^{\frac{1}{4}}abcd-3\,\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^2d^4}+\frac{17\,b^2c^2dx^{\frac{5}{2}}-18\,abcd^2x^{\frac{5}{2}}+a^2d^3x^{\frac{5}{2}}+13\,b^2c^3\sqrt{x}-10\,abc^2d\sqrt{x}-3\,a^2cd^2\sqrt{x}}{16\left(dx^2+c\right)^2cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x)/d^3 - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/
4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2
*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/128*sqrt(2
)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^
2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1/128*s
qrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/
4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) + 1
/16*(17*b^2*c^2*d*x^(5/2) - 18*a*b*c*d^2*x^(5/2) + a^2*d^3*x^(5/2) + 13*b^2
*c^3*sqrt(x) - 10*a*b*c^2*d*sqrt(x) - 3*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c
*d^3)
```

maple [A]   time = 0.02, size = 568, normalized size = 1.41

$$\frac{a^2x^{\frac{5}{2}}}{16\left(d\,x^2+c\right)^2c}-\frac{9ab\,x^{\frac{5}{2}}}{8\left(d\,x^2+c\right)^2d}+\frac{17b^2c\,x^{\frac{5}{2}}}{16\left(d\,x^2+c\right)^2d^2}-\frac{3a^2\sqrt{x}}{16\left(d\,x^2+c\right)^2d}-\frac{5abc\sqrt{x}}{8\left(d\,x^2+c\right)^2d^2}+\frac{13b^2c^2\sqrt{x}}{16\left(d\,x^2+c\right)^2d^3}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c^2d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c^2d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128c^2d}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32c\,d^2}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32c\,d^2}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64c\,d^2}-\frac{45\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64d^3}-\frac{45\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64d^3}-\frac{45\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128d^3}+\frac{2b^2\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 2*b^2/d^3*x^(1/2)+1/16/(d*x^2+c)^2/c*x^(5/2)*a^2-9/8/d/(d*x^2+c)^2*x^(5/2)*
a*b+17/16/d^2/(d*x^2+c)^2*c*x^(5/2)*b^2-3/16/d/(d*x^2+c)^2*x^(1/2)*a^2-5/8/
d^2/(d*x^2+c)^2*x^(1/2)*a*b*c+13/16/d^3/(d*x^2+c)^2*x^(1/2)*b^2*c^2+3/64/d/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+5/32/d^2/
c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-45/64/d^3*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/d/c^2*(c
/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(
1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+5/64/d^2/c*(c/d)^(1/4)*2^(1/2)*ln((x
+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c
/d)^(1/2)))*a*b-45/128/d^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^
(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+3/64/d/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+5/32/d^2/
c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-45/64/d^3*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [**A**] time = 2.44, size = 374, normalized size = 0.93

$$\frac{\left(17\,b^{2}c^{2}d-18\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{5}{2}}+\left(13\,b^{2}c^{3}-10\,abc^{2}d-3\,a^{2}cd^{2}\right)\sqrt{x}}{16\left(cd^{5}x^{4}+2\,c^{2}d^{4}x^{2}+c^{3}d^{3}\right)}+\frac{2\,b^{2}\sqrt{x}}{d^{3}}-\frac{\frac{2\,\sqrt{2}\left(45\,b^{2}c^{2}-10\,abcd-3\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(45\,b^{2}c^{2}-10\,abcd-3\,a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(45\,b^{2}c^{2}-10\,abcd-3\,a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(45\,b^{2}c^{2}-10\,abcd-3\,a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,cd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((17*b^2*c^2*d - 18*a*b*c*d^2 + a^2*d^3)*x^(5/2) + (13*b^2*c^3 - 10*a*
b*c^2*d - 3*a^2*c*d^2)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3) + 2*b
^2*sqrt(x)/d^3 - 1/128*(2*sqrt(2)*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(45*b^2*c^2 - 10*a*b*
c*d - 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*s
qrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(4
5*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(45*b^2*c^2 - 10*a*b*c*d -
3*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
3/4)*d^(1/4)))/(c*d^3)
```

mupad [**B**] time = 0.44, size = 1236, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (2*b^2*x^(1/2))/d^3 - (x^(1/2)*((3*a^2*d^2)/16 - (13*b^2*c^2)/16 + (5*a*b*c
*d)/8) - (x^(5/2)*(a^2*d^3 + 17*b^2*c^2*d - 18*a*b*c*d^2))/(16*c))/(c^2*d^3
 + d^5*x^4 + 2*c*d^4*x^2) + (atan(((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^
2/(64*(-c)^(7/4)*d^(13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b
^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 -
45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)) - (((3*a^2*d^2 - 45*b
^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 202
5*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^
2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)))
/((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4)*d^(13/4)) - (x^(
1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60
*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c
)^(7/4)*d^(13/4)) + (((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4
)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 90
0*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10
*a*b*c*d))/(64*(-c)^(7/4)*d^(13/4))))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)
*1i)/(32*(-c)^(7/4)*d^(13/4)) + (atan((((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*
c*d)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 -
170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a
^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c)^(7/4)*d^(13/4)) - ((((3*a^2*d^2
 - 45*b^2*c^2 + 10*a*b*c*d)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^
4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d
^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c)^(7/4)*d^
(13/4)))/(((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2*1i)/(64*(-c)^(7/4)*d^(
13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b
^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*
c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)) + ((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d
)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170
*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*
d^2 - 45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4))))*(3*a^2*d^2 -
45*b^2*c^2 + 10*a*b*c*d))/(32*(-c)^(7/4)*d^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.418 $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=364

$$\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\right.}{64\sqrt{2}\,c^{9/4}d^{11/4}}$$

Rubi [A] time = 0.29, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 457, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{9/4}d^{11/4}}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{9/4}d^{11/4}}-\frac{x^{3/2}(5ad+11bc)(bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(11*b*c + 5*
a*d)*x^(3/2))/(16*c^2*d^2*(c + d*x^2)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(1
1/4)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) + ((21*b^2*c^2 + 6*a*b*c*d
+ 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(6
4*Sqrt[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*d^
(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{\sqrt{x}\left(\frac{1}{2}\left(-8a^2d^2+3(bc-ad)^2\right)-4b^2cdx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\int \frac{\sqrt{x}}{c+dx^2}\,dx}{32c^2d^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\right.}{16c^2d^2}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{\sqrt{c}-}{c+d}\right.}{32c^2d^{5/2}}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\text{Subst}\left(\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}-}\right.}{64c^2d^3}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\log\left(\sqrt{c}-\sqrt{2}\right.}{64\sqrt{2}\,c^{9/4}d^{11/4}}$$

$$= \frac{(bc-ad)^2x^{3/2}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(21b^2c^2+6abcd+5a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{32\sqrt{2}\,c^{9/4}d^{11/4}}$$

Mathematica [A] time = 0.21, size = 339, normalized size = 0.93

$$\frac{\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-2\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\left(5a^2d^2+6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-\frac{8\sqrt[4]{c}d^{3/4}x^{3/2}\left(-5a^2d^2-6abcd+11b^2c^2\right)}{c+dx^2}+\frac{32c^{5/4}d^{3/4}x^{3/2}(bc-ad)^2}{\left(c+dx^2\right)^2}}{128c^{9/4}d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] ((32*c^(5/4)*d^(3/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 - (8*c^(1/4)*d^(3
/4)*(11*b^2*c^2 - 6*a*b*c*d - 5*a^2*d^2)*x^(3/2))/(c + d*x^2) - 2*Sqrt[2]*(
21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)] + 2*Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)] + Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*(21*b^2
*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(128*c^(9/4)*d^(11/4))
```

IntegrateAlgebraic [A] time = 0.87, size = 242, normalized size = 0.66

$$-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{9/4}d^{11/4}}-\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{9/4}d^{11/4}}-\frac{x^{3/2}\left(-9a^2cd^2-5a^2d^3x^2+2abc^2d-6abcd^2x^2+7b^2c^3+11b^2c^2dx^2\right)}{16c^2d^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/16*(x^(3/2)*(7*b^2*c^3 + 2*a*b*c^2*d - 9*a^2*c*d^2 + 11*b^2*c^2*d*x^2 -
6*a*b*c*d^2*x^2 - 5*a^2*d^3*x^2))/(c^2*d^2*(c + d*x^2)^2) - ((21*b^2*c^2 +
6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x])])/(32*Sqrt[2]*c^(9/4)*d^(11/4)) - ((21*b^2*c^2 + 6*a*b*c*d + 5*a
^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(
32*Sqrt[2]*c^(9/4)*d^(11/4))
```

fricas [B] time = 1.58, size = 1811, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264
*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4
*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c
*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*arctan((sqrt((85766121*b^12*c^12 + 14
7027636*a*b^11*c^11*d + 227542770*a^2*b^10*c^10*d^2 + 215040420*a^3*b^9*c^9
*d^3 + 181522215*a^4*b^8*c^8*d^4 + 112905576*a^5*b^7*c^7*d^5 + 63002556*a^6
*b^6*c^6*d^6 + 26882280*a^7*b^5*c^5*d^7 + 10290375*a^8*b^4*c^4*d^8 + 290250
```

```
0*a^9*b^3*c^3*d^9 + 731250*a^10*b^2*c^2*d^10 + 112500*a^11*b*c*d^11 + 15625
*a^12*d^12)*x - (194481*b^8*c^13*d^5 + 222264*a*b^7*c^12*d^6 + 280476*a^2*b
^6*c^11*d^7 + 176904*a^3*b^5*c^10*d^8 + 112806*a^4*b^4*c^9*d^9 + 42120*a^5*
b^3*c^8*d^10 + 15900*a^6*b^2*c^7*d^11 + 3000*a^7*b*c^6*d^12 + 625*a^8*c^5*d
^13)*sqrt(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 +
176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 1
5900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11)))*c^2*d^3
*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a
^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6
*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4) - (9261*b^
6*c^8*d^3 + 7938*a*b^5*c^7*d^4 + 8883*a^2*b^4*c^6*d^5 + 3996*a^3*b^3*c^5*d^
6 + 2115*a^4*b^2*c^4*d^7 + 450*a^5*b*c^3*d^8 + 125*a^6*c^2*d^9)*sqrt(x)*(-(
194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b
^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2
*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4))/(194481*b^8*c
^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 +
 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3
000*a^7*b*c*d^7 + 625*a^8*d^8)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(
-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3
*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b
^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*log(c^7*d^8*
(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^
3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*
b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(3/4) + (9261*b^6
*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 211
5*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*d^6)*sqrt(x)) + (c^2*d^4*x^4
+ 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*
a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a
^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c
^9*d^11))^(1/4)*log(-c^7*d^8*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 28047
6*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120
*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/
(c^9*d^11))^(3/4) + (9261*b^6*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2
 + 3996*a^3*b^3*c^3*d^3 + 2115*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*
d^6)*sqrt(x)) + 4*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^3 + (7*b^2*c^
3 + 2*a*b*c^2*d - 9*a^2*c*d^2)*x)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c
^4*d^2)
```

giac [A] time = 0.52, size = 416, normalized size = 1.14

$$-\frac{11\,b^{2}c^{2}dx^{\frac{7}{2}}-6\,abcd^{2}x^{\frac{7}{2}}-5\,a^{2}d^{3}x^{\frac{7}{2}}+7\,b^{2}c^{3}x^{\frac{3}{2}}+2\,abc^{2}dx^{\frac{3}{2}}-9\,a^{2}cd^{2}x^{\frac{3}{2}}}{16\left(dx^{2}+c\right)^{2}c^{2}d^{2}}+\frac{\sqrt{2}\left(21\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{3}d^{5}}+\frac{\sqrt{2}\left(21\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{3}d^{5}}-\frac{\sqrt{2}\left(21\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{3}d^{5}}+\frac{\sqrt{2}\left(21\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+6\,\left(cd^{3}\right)^{\frac{3}{4}}abcd+5\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{3}d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*b^2*c^2*d*x^(7/2) - 6*a*b*c*d^2*x^(7/2) - 5*a^2*d^3*x^(7/2) + 7*b
^2*c^3*x^(3/2) + 2*a*b*c^2*d*x^(3/2) - 9*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^2*
c^2*d^2) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d
 + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(c^3*d^5) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(
c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^5) - 1/128*sqrt(2)*(21*(c*d^
3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5) + 1/128*sqrt(2)*(21*(
c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*l
og(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5)
```

maple [A] time = 0.02, size = 514, normalized size = 1.41

$$\frac{5\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}d}+\frac{5\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}d}+\frac{5\sqrt{2}\,a^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}d}+\frac{3\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32\left(\frac{c}{d}\right)^{\frac{1}{4}}c\,d^{2}}+\frac{3\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32\left(\frac{c}{d}\right)^{\frac{1}{4}}c\,d^{2}}+\frac{3\sqrt{2}\,ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c\,d^{2}}+\frac{21\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}}+\frac{21\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}}+\frac{21\sqrt{2}\,b^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{3}}+\frac{\frac{\left(5a^{2}d^{2}+6abcd-11b^{2}c^{2}\right)x^{\frac{7}{2}}}{16c^{2}d}+\frac{\left(9a^{2}d^{2}-2abcd-7b^{2}c^{2}\right)x^{\frac{3}{2}}}{16c\,d^{2}}}{\left(d\,x^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] 2*(1/32*(5*a^2*d^2+6*a*b*c*d-11*b^2*c^2)/c^2/d*x^(7/2)+1/32*(9*a^2*d^2-2*a*
b*c*d-7*b^2*c^2)/c/d^2*x^(3/2))/(d*x^2+c)^2+5/64/d/c^2/(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/32/d^2/c/(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+21/64/d^3/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/64/d/c^2/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/32/d^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+21/64/d^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128/d/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(
1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2
)))*a^2+3/64/d^2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+21/128/d^3/(c/d
)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.54, size = 297, normalized size = 0.82

$$-\frac{\left(11\,b^{2}c^{2}d-6\,abcd^{2}-5\,a^{2}d^{3}\right)x^{\frac{7}{2}}+\left(7\,b^{2}c^{3}+2\,abc^{2}d-9\,a^{2}cd^{2}\right)x^{\frac{3}{2}}}{16\left(c^{2}d^{4}x^{4}+2\,c^{3}d^{3}x^{2}+c^{4}d^{2}\right)}+\frac{\left(21\,b^{2}c^{2}+6\,abcd+5\,a^{2}d^{2}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^(7/2) + (7*b^2*c^3 + 2*a*
b*c^2*d - 9*a^2*c*d^2)*x^(3/2))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1
```

```
/128*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/
(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^2*d^2)
```

mupad [B] time = 0.40, size = 184, normalized size = 0.51

$$\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(5\,a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2\right)}{32\,{\left(-c\right)}^{9/4}\,d^{11/4}}-\frac{\frac{x^{3/2}\,\left(-9\,a^2\,d^2+2\,a\,b\,c\,d+7\,b^2\,c^2\right)}{16\,c\,d^2}-\frac{x^{7/2}\,\left(5\,a^2\,d^2+6\,a\,b\,c\,d-11\,b^2\,c^2\right)}{16\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}-\frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(5\,a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2\right)}{32\,{\left(-c\right)}^{9/4}\,d^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(5*a^2*d^2 + 21*b^2*c^2 + 6*a*b*c*d))/(
32*(-c)^(9/4)*d^(11/4)) - ((x^(3/2)*(7*b^2*c^2 - 9*a^2*d^2 + 2*a*b*c*d))/(1
6*c*d^2) - (x^(7/2)*(5*a^2*d^2 - 11*b^2*c^2 + 6*a*b*c*d))/(16*c^2*d))/(c^2
+ d^2*x^4 + 2*c*d*x^2) - (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(5*a^2*d^2 +
21*b^2*c^2 + 6*a*b*c*d))/(32*(-c)^(9/4)*d^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.419 $\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=364

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\right.}{64\sqrt{2}\,c^{11/4}d^{9/4}}$$

Rubi [A] time = 0.30, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {463, 457, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{11/4}d^{9/4}}-\frac{\sqrt{x}\,(7ad+9bc)(bc-ad)}{16c^2d^2\left(c+dx^2\right)}+\frac{\sqrt{x}\,(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) - ((b*c - a*d)*(9*b*c + 7*a
*d)*Sqrt[x])/(16*c^2*d^2*(c + d*x^2)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9
/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d +
21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64
*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*d^
(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)^3}\,dx = \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{\int \frac{\frac{1}{2}\left(-8a^2d^2+(bc-ad)^2\right)-4b^2cdx^2}{\sqrt{x}\left(c+dx^2\right)^2}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{1}{\sqrt{x}\left(c+dx^2\right)}\,dx}{32c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\operatorname{Subst}\left(\int \frac{1}{c+dx^4}\,dx\right.}{16c^2d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{c}-\sqrt{d}\,x}{c+dx^4}\right.}{32c^{5/2}d^2}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\left(c+dx^2\right)} + \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}}{}}\right.}{64c^{5/2}d^{5/2}}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\right.}{64\sqrt{2}\,c^{11/4}d^{9/4}}$$

$$= \frac{(bc-ad)^2\sqrt{x}}{4cd^2\left(c+dx^2\right)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\left(c+dx^2\right)} - \frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{32\sqrt{2}\,c^{11/4}d^{9/4}}$$

Mathematica [A] time = 0.20, size = 339, normalized size = 0.93

$$\frac{-\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-2\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-\frac{8c^{3/4}\sqrt[4]{d}\sqrt{x}\left(-7a^2d^2-2abcd+9b^2c^2\right)}{c+dx^2}+\frac{32c^{7/4}\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{\left(c+dx^2\right)^2}}{128c^{11/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 - (8*c^(3/4)*d^(1
/4)*(9*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*Sqrt[x])/(c + d*x^2) - 2*Sqrt[2]*(5
*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)] + 2*Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Lo
g[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(128*c^(11/4)*d^(9/4))
```

IntegrateAlgebraic [A] time = 0.85, size = 242, normalized size = 0.66

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{11/4}d^{9/4}}-\frac{\sqrt{x}\left(-11a^2cd^2-7a^2d^3x^2+6abc^2d-2abcd^2x^2+5b^2c^3+9b^2c^2dx^2\right)}{16c^2d^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(Sqrt[x]*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2 + 9*b^2*c^2*d*x^2 -
2*a*b*c*d^2*x^2 - 7*a^2*d^3*x^2))/(c^2*d^2*(c + d*x^2)^2) - ((5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x])])/(32*Sqrt[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a
^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(
32*Sqrt[2]*c^(11/4)*d^(9/4))
```

fricas [B] time = 1.56, size = 1416, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*arctan((sqrt(c^6*d^4*sqrt(-(625*b^8*c^
8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 1128
06*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 2222
64*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d
```

```
 + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x)*c^8*d^7*(-(625*b
^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 +
 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 +
 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4) - (5*b^2*c^10*d^7 +
 6*a*b*c^9*d^8 + 21*a^2*c^8*d^9)*sqrt(x)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d
+ 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 +
176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1944
81*a^8*d^8)/(c^11*d^9))^(3/4))/(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*
b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b
^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8))
 + (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*
d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
+ 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 19
4481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7
*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4
 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 1
94481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*sqr
t(x)) - (c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2*(-(625*b^8*c^8 + 3000*a*b
^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c
^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d
^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^
2)*sqrt(x)) - 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2 + (9*b^2*c^2*d - 2*
a*b*c*d^2 - 7*a^2*d^3)*x^2)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2
)
```

giac [A] time = 0.53, size = 416, normalized size = 1.14

$$\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{3}d^{3}}+\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{3}d^{3}}+\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{3}d^{3}}-\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{3}d^{3}}-\frac{9\,b^{2}c^{2}dx^{\frac{5}{2}}-2\,abcd^{2}x^{\frac{5}{2}}-7\,a^{2}d^{3}x^{\frac{5}{2}}+5\,b^{2}c^{3}\sqrt{x}+6\,abc^{2}d\sqrt{x}-11\,a^{2}cd^{2}\sqrt{x}}{16\left(dx^{2}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)
^(1/4))/(c^3*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)
*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2
*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt
(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(1/4)
*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/16*(9*b^2*c^2*d*x^(5/2)
```

```
 - 2*a*b*c*d^2*x^(5/2) - 7*a^2*d^3*x^(5/2) + 5*b^2*c^3*sqrt(x) + 6*a*b*c^2*
d*sqrt(x) - 11*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c^2*d^2)
```

maple [A] time = 0.02, size = 514, normalized size = 1.41

$$\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128c^{3}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64c^{2}d}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c\,d^{2}}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c\,d^{2}}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128c\,d^{2}}+\frac{\frac{\left(7a^{2}d^{2}+2abcd-9b^{2}c^{2}\right)x^{\frac{5}{2}}}{16c^{2}d}+\frac{\left(11a^{2}d^{2}-6abcd-5b^{2}c^{2}\right)\sqrt{x}}{16c\,d^{2}}}{\left(d\,x^{2}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] 2*(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^(5/2)+1/32*(11*a^2*d^2-6*a*
b*c*d-5*b^2*c^2)/c/d^2*x^(1/2))/(d*x^2+c)^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+3/32/c^2/d*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+5/64/c/d^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+5/128/c/d
^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+3/32/c^2/d*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+5/64/c/d^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [A] time = 2.43, size = 366, normalized size = 1.01

$$-\frac{\left(9\,b^{2}c^{2}d-2\,abcd^{2}-7\,a^{2}d^{3}\right)x^{\frac{5}{2}}+\left(5\,b^{2}c^{3}+6\,abc^{2}d-11\,a^{2}cd^{2}\right)\sqrt{x}}{16\left(c^{2}d^{4}x^{4}+2\,c^{3}d^{3}x^{2}+c^{4}d^{2}\right)}+\frac{\frac{2\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^(5/2) + (5*b^2*c^3 + 6*a*b
*c^2*d - 11*a^2*c*d^2)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1
/128*(2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*
sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d
 + 21*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(-sqrt(2
)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^
2)
```

mupad [B] time = 0.61, size = 1419, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)^3),x)
```

```
[Out] - ((x^(1/2)*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(16*c*d^2) - (x^(5/2)*(7*
a^2*d^2 - 9*b^2*c^2 + 2*a*b*c*d))/(16*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2) -
 (atan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d +
 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^
4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(2
1*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - ((((21*a^
2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(6
4*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c
^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2
*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2
 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(
9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3
*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)
)/(64*(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^
2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(44
1*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c
*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^
(9/4))))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(32*(-c)^(11/4)*d^(9/4))
- (atan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d
+ 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^
4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d)
)*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4)) - ((((21*a
^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i
)/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b
^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5
*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^
2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)
*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a
*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*
c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)
*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) + (x
^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 2
52*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(
-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(32*(-c)^(11/4)
*d^(9/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
[Out] Timed out
```

3.420 $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=399

$$-\frac{x^{3/2}\left(9a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}d^{7/4}}-$$

Rubi [A] time = 0.39, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 457, 290, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}\left(9a^2d^2-2abcd+b^2c^2\right)}{4c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}d^{7/4}}-\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}d^{7/4}}-\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{13/4}d^{7/4}}+\frac{\left(5ad(2bc-9ad)+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{13/4}d^{7/4}}+\frac{x^{3/2}\left(\frac{5a(2bc-9ad)}{c^2}+\frac{3b^2}{d}\right)}{16c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) - ((b^2*c^2 - 2*a*b*c*d + 9*a^2*d^2)*x^(
3/2))/(4*c^2*d*(c + d*x^2)^2) + (((3*b^2)/d + (5*a*(2*b*c - 9*a*d))/c^2)*x^
(3/2))/(16*c*(c + d*x^2)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^
2*c^2 + 5*a*d*(2*b*c - 9*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) + ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/
4)*d^(7/4)) - ((3*b^2*c^2 + 5*a*d*(2*b*c - 9*a*d))*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*d^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{2\int \frac{\sqrt{x}\left(\frac{1}{2}a(2bc-9ad)+\frac{1}{2}b^2cx^2\right)}{\left(c+dx^2\right)^3}\,dx}{c}$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{1}{8}\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)\int \frac{\sqrt{x}}{\left(c+dx^2\right)^2}\,dx$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right.}{32c}$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right.}{}$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)}-\frac{\left(3b^2c^2+10abcd\right.}{}$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)}+\frac{\left(3b^2c^2+10abcd\right.}{}$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)}+\frac{\left(3b^2c^2+10abcd\right.}{}$$

$$=-\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}+\frac{\left(2ab-\frac{b^2c}{d}-\frac{9a^2d}{c}\right)x^{3/2}}{4c\left(c+dx^2\right)^2}+\frac{\left(\frac{3b^2}{d}+\frac{5a(2bc-9ad)}{c^2}\right)x^{3/2}}{16c\left(c+dx^2\right)}-\frac{\left(3b^2c^2+10abcd\right.}{}$$

Mathematica [A] time = 0.41, size = 364, normalized size = 0.91

$$\frac{\frac{8\sqrt[4]{c}\,x^{3/2}\left(-13a^2d^2+10abcd+3b^2c^2\right)}{d\left(c+dx^2\right)}+\frac{\sqrt{2}\left(-45a^2d^2+10abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}}+\frac{\sqrt{2}\left(45a^2d^2-10abcd-3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{7/4}}+\frac{2\sqrt{2}\left(45a^2d^2-10abcd-3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{7/4}}+\frac{2\sqrt{2}\left(-45a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{7/4}}-\frac{256a^2\sqrt[4]{c}}{\sqrt{x}}-\frac{32c^{5/4}x^{3/2}(bc-ad)^2}{d\left(c+dx^2\right)^2}}{128c^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(1/4))/Sqrt[x] - (32*c^(5/4)*(b*c - a*d)^2*x^(3/2))/(d*(c + d*
x^2)^2) + (8*c^(1/4)*(3*b^2*c^2 + 10*a*b*c*d - 13*a^2*d^2)*x^(3/2))/(d*(c +
 d*x^2)) + (2*Sqrt[2]*(-3*b^2*c^2 - 10*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 10*a*b*c
*d - 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (
Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 10*a*b*c*d
+ 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d
^(7/4))/(128*c^(13/4))
```

IntegrateAlgebraic [A] time = 1.00, size = 260, normalized size = 0.65

$$-\frac{\left(-45a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{13/4}d^{7/4}}-\frac{\left(-45a^2d^2+10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{13/4}d^{7/4}}+\frac{-32a^2c^2d-81a^2cd^2x^2-45a^2d^3x^4+18abc^2dx^2+10abcd^2x^4-b^2c^3x^2+3b^2c^2dx^4}{16c^3d\sqrt{x}\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*a^2*c^2*d - b^2*c^3*x^2 + 18*a*b*c^2*d*x^2 - 81*a^2*c*d^2*x^2 + 3*b^2*
c^2*d*x^4 + 10*a*b*c*d^2*x^4 - 45*a^2*d^3*x^4)/(16*c^3*d*Sqrt[x]*(c + d*x^2
)^2) - ((3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(13/4)*d^(7/4)) - ((3*b^2
*c^2 + 10*a*b*c*d - 45*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(
Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(13/4)*d^(7/4))
```

fricas [B] time = 1.84, size = 1819, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(4*(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*
c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4
 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 +
4100625*a^8*d^8)/(c^13*d^7))^(1/4)*arctan((sqrt((729*b^12*c^12 + 14580*a*b^
11*c^11*d + 55890*a^2*b^10*c^10*d^2 - 553500*a^3*b^9*c^9*d^3 - 3479625*a^4*
b^8*c^8*d^4 + 10305000*a^5*b^7*c^7*d^5 + 75317500*a^6*b^6*c^6*d^6 - 1545750
00*a^7*b^5*c^5*d^7 - 782915625*a^8*b^4*c^4*d^8 + 1868062500*a^9*b^3*c^3*d^9
 + 2829431250*a^10*b^2*c^2*d^10 - 11071687500*a^11*b*c*d^11 + 8303765625*a^
12*d^12)*x - (81*b^8*c^15*d^3 + 1080*a*b^7*c^14*d^4 + 540*a^2*b^6*c^13*d^5
- 36600*a^3*b^5*c^12*d^6 - 42650*a^4*b^4*c^11*d^7 + 549000*a^5*b^3*c^10*d^8
 + 121500*a^6*b^2*c^9*d^9 - 3645000*a^7*b*c^8*d^10 + 4100625*a^8*c^7*d^11)*
sqrt(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*
c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c
^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7)))*c^3*d^2*(-(81*
```

```
b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 -
42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3
645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4) + (27*b^6*c^9*d^2 +
 270*a*b^5*c^8*d^3 - 315*a^2*b^4*c^7*d^4 - 7100*a^3*b^3*c^6*d^5 + 4725*a^4*
b^2*c^5*d^6 + 60750*a^5*b*c^4*d^7 - 91125*a^6*c^3*d^8)*sqrt(x)*(-(81*b^8*c^
8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*
a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000
*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4))/(81*b^8*c^8 + 1080*a*b^7
*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 +
 4100625*a^8*d^8)) - (c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8
+ 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^
4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a
^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*log(c^10*d^5*(-(81*b^8*c^8
+ 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^
4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a
^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) - (27*b^6*c^6 + 270*a*b^5*c
^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3 + 4725*a^4*b^2*c^2*d^4 +
60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) + (c^3*d^3*x^5 + 2*c^4*d^2*x^3
+ c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a
^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^
6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*lo
g(-c^10*d^5*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*
a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a
^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) -
 (27*b^6*c^6 + 270*a*b^5*c^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3
 + 4725*a^4*b^2*c^2*d^4 + 60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) - 4*(
32*a^2*c^2*d - (3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3)*x^4 + (b^2*c^3 - 1
8*a*b*c^2*d + 81*a^2*c*d^2)*x^2)*sqrt(x))/(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^
5*d*x)
```

giac [A] time = 0.54, size = 427, normalized size = 1.07

$$-\frac{2\,a^{2}}{c^{3}\sqrt{x}}+\frac{3\,b^{2}c^{2}dx^{\frac{7}{2}}+10\,abcd^{2}x^{\frac{7}{2}}-13\,a^{2}d^{3}x^{\frac{7}{2}}-b^{2}c^{3}x^{\frac{3}{2}}+18\,abc^{2}dx^{\frac{3}{2}}-17\,a^{2}cd^{2}x^{\frac{3}{2}}}{16\,\left(dx^{2}+c\right)^{2}c^{3}d}+\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-45\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{4}d^{4}}+\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-45\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{4}d^{4}}-\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-45\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{4}d^{4}}+\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2}+10\,\left(cd^{3}\right)^{\frac{3}{4}}abcd-45\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{4}d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2/(c^3*sqrt(x)) + 1/16*(3*b^2*c^2*d*x^(7/2) + 10*a*b*c*d^2*x^(7/2) - 1
3*a^2*d^3*x^(7/2) - b^2*c^3*x^(3/2) + 18*a*b*c^2*d*x^(3/2) - 17*a^2*c*d^2*x
^(3/2))/((d*x^2 + c)^2*c^3*d) + 1/64*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*
(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(
2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) + 1/64*sqrt(2)*(3*(c*d^3
)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arct
```

```
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) -
1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d
^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^
4) + 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45
*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
c^4*d^4)
```

maple [A] time = 0.02, size = 568, normalized size = 1.42

$$-\frac{13 a^{2} d^{2} x^{\frac{7}{2}}}{16 \left(d x^{2}+c\right)^{2} c^{3}}+\frac{5 a b d x^{\frac{7}{2}}}{8 \left(d x^{2}+c\right)^{2} c^{2}}+\frac{3 b^{2} x^{\frac{7}{2}}}{16 \left(d x^{2}+c\right)^{2} c}-\frac{17 a^{2} d x^{\frac{3}{2}}}{16 \left(d x^{2}+c\right)^{2} c^{2}}+\frac{9 a b x^{\frac{3}{2}}}{8 \left(d x^{2}+c\right)^{2} c}-\frac{b^{2} x^{\frac{3}{2}}}{16 \left(d x^{2}+c\right)^{2} d}-\frac{45 \sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{3}}-\frac{45 \sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{3}}-\frac{45 \sqrt{2}\, a^{2} \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{3}}+\frac{5 \sqrt{2}\, a b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2} d}+\frac{5 \sqrt{2}\, a b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2} d}+\frac{5 \sqrt{2}\, a b \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64 \left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2} d}+\frac{3 \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64 \left(\frac{c}{d}\right)^{\frac{1}{4}} c d^{2}}+\frac{3 \sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64 \left(\frac{c}{d}\right)^{\frac{1}{4}} c d^{2}}+\frac{3 \sqrt{2}\, b^{2} \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128 \left(\frac{c}{d}\right)^{\frac{1}{4}} c d^{2}}-\frac{2 a^{2}}{c^{3} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] -13/16/c^3/(d*x^2+c)^2*x^(7/2)*a^2*d^2+5/8/c^2/(d*x^2+c)^2*x^(7/2)*a*b*d+3/
16/c/(d*x^2+c)^2*x^(7/2)*b^2-17/16/c^2/(d*x^2+c)^2*d*x^(3/2)*a^2+9/8/c/(d*x
^2+c)^2*x^(3/2)*a*b-1/16/(d*x^2+c)^2/d*x^(3/2)*b^2-45/64/c^3/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+5/32/c^2/d/(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/64/c/d^2/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-45/64/c^3/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+5/32/c^2/d/(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/64/c/d^2/(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-45/128/c^3/(c/d)^(1/4)*2^(1/2)*ln((x-(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))*a^2+5/64/c^2/d/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1
/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+3/128/c/d
^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-2*a^2/c^3/x^(1/2)
```

maxima [A] time = 2.52, size = 307, normalized size = 0.77

$$-\frac{32 \, a^{2} c^{2} d - {\left(3 \, b^{2} c^{2} d + 10 \, a b c d^{2} - 45 \, a^{2} d^{3}\right)} x^{4} + {\left(b^{2} c^{3} - 18 \, a b c^{2} d + 81 \, a^{2} c d^{2}\right)} x^{2}}{16 \, {\left(c^{3} d^{3} x^{\frac{9}{2}} + 2 \, c^{4} d^{2} x^{\frac{5}{2}} + c^{5} d \sqrt{x}\right)}} + \frac{{\left(3 \, b^{2} c^{2} + 10 \, a b c d - 45 \, a^{2} d^{2}\right)} {\left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} {\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} + 2 \, \sqrt{d} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} + \frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} {\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} - 2 \, \sqrt{d} \sqrt{x}\right)}}{2 \, \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}}{128 \, c^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(32*a^2*c^2*d - (3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3)*x^4 + (b^2*
c^3 - 18*a*b*c^2*d + 81*a^2*c*d^2)*x^2)/(c^3*d^3*x^(9/2) + 2*c^4*d^2*x^(5/2
) + c^5*d*sqrt(x)) + 1/128*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt
(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sq
```

```
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(
1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^3*d)
```

mupad [**B**] time = 0.40, size = 192, normalized size = 0.48

$$\frac{\operatorname{atanh}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\left(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{32\,(-c)^{13/4}\,d^{7/4}}-\frac{\operatorname{atan}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\left(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{32\,(-c)^{13/4}\,d^{7/4}}-\frac{\frac{2\,a^2}{c}-\frac{x^4\,\left(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{16\,c^3}+\frac{x^2\,\left(81\,a^2\,d^2-18\,a\,b\,c\,d+b^2\,c^2\right)}{16\,c^2\,d}}{c^2\,\sqrt{x}+d^2\,x^{9/2}+2\,c\,d\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x)
```

```
[Out] (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*b^2*c^2 - 45*a^2*d^2 + 10*a*b*c*d))
/(32*(-c)^(13/4)*d^(7/4)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*b^2*c^2
- 45*a^2*d^2 + 10*a*b*c*d))/(32*(-c)^(13/4)*d^(7/4)) - ((2*a^2)/c - (x^4*(3
*b^2*c^2 - 45*a^2*d^2 + 10*a*b*c*d))/(16*c^3) + (x^2*(81*a^2*d^2 + b^2*c^2
- 18*a*b*c*d))/(16*c^2*d))/(c^2*x^(1/2) + d^2*x^(9/2) + 2*c*d*x^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.421 $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\sqrt{x}\,\left(11a^2d^2-6abcd+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}d^{5/4}}$$

Rubi [A] time = 0.41, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {462, 457, 290, 329, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\sqrt{x}\,\left(11a^2d^2-6abcd+3b^2c^2\right)}{12c^2d\left(c+dx^2\right)^2}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}d^{5/4}}-\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{15/4}d^{5/4}}+\frac{\left(7ad(6bc-11ad)+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{15/4}d^{5/4}}+\frac{\sqrt{x}\,\left(\frac{7a(6bc-11ad)}{c^2}+\frac{3b^2}{d}\right)}{48c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) - ((3*b^2*c^2 - 6*a*b*c*d + 11*a^2*d^2
)*Sqrt[x])/(12*c^2*d*(c + d*x^2)^2) + (((3*b^2)/d + (7*a*(6*b*c - 11*a*d))/
c^2)*Sqrt[x])/(48*c*(c + d*x^2)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Ar
cTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4))
+ ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) - ((3*b^2*c^2 + 7*a*d*(6*b*c - 11
*a*d))*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt
[2]*c^(15/4)*d^(5/4)) + ((3*b^2*c^2 + 7*a*d*(6*b*c - 11*a*d))*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*d^(5/4)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx = -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{2\int \frac{\frac{1}{2}a(6bc-11ad)+\frac{3}{2}b^2cx^2}{\sqrt{x}\left(c+dx^2\right)^3}\,dx}{3c}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{1}{24}\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\int \frac{1}{\sqrt{x}\,(c+$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-}{c^2}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-}{c^2}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-}{c^2}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)} + \frac{\left(3b^2c^2+42a\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)} - \frac{\left(3b^2c^2+42a\right.}{}$$

$$= -\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2} + \frac{\left(6ab-\frac{3b^2c}{d}-\frac{11a^2d}{c}\right)\sqrt{x}}{12c\left(c+dx^2\right)^2} + \frac{\left(\frac{3b^2}{d}+\frac{7a(6bc-11ad)}{c^2}\right)\sqrt{x}}{48c\left(c+dx^2\right)} - \frac{\left(3b^2c^2+42a\right.}{}$$

Mathematica [A] time = 0.26, size = 365, normalized size = 0.91

$$\frac{\frac{3\sqrt{2}\left(77a^2d^2-42abcd-3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{3\sqrt{2}\left(-77a^2d^2+42abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{d^{5/4}}+\frac{6\sqrt{2}\left(77a^2d^2-42abcd-3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{5/4}}+\frac{6\sqrt{2}\left(-77a^2d^2+42abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{5/4}}+\frac{24c^{3/4}\sqrt{x}\left(-15a^2d^2+14abcd+b^2c^2\right)}{d\left(c+dx^2\right)}-\frac{256a^2c^{3/4}}{x^{3/2}}-\frac{96c^{7/4}\sqrt{x}(bc-ad)^2}{d\left(c+dx^2\right)^2}}{384c^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(3/4))/x^(3/2) - (96*c^(7/4)*(b*c - a*d)^2*Sqrt[x])/(d*(c + d*
x^2)^2) + (24*c^(3/4)*(b^2*c^2 + 14*a*b*c*d - 15*a^2*d^2)*Sqrt[x])/(d*(c +
d*x^2)) + (6*Sqrt[2]*(-3*b^2*c^2 - 42*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (6*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c*
d - 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) + (3
*Sqrt[2]*(-3*b^2*c^2 - 42*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4) + (3*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c
*d - 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/d^(5/4))/(384*c^(15/4))
```

IntegrateAlgebraic [A] time = 0.97, size = 260, normalized size = 0.65

$$-\frac{\left(-77a^2d^2+42abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{15/4}d^{5/4}}+\frac{\left(-77a^2d^2+42abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{15/4}d^{5/4}}+\frac{-32a^2c^2d-121a^2cd^2x^2-77a^2d^3x^4+66abc^2dx^2+42abcd^2x^4-9b^2c^3x^2+3b^2c^2dx^4}{48c^3dx^{3/2}\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*a^2*c^2*d - 9*b^2*c^3*x^2 + 66*a*b*c^2*d*x^2 - 121*a^2*c*d^2*x^2 + 3*b
^2*c^2*d*x^4 + 42*a*b*c*d^2*x^4 - 77*a^2*d^3*x^4)/(48*c^3*d*x^(3/2)*(c + d*
x^2)^2) - ((3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*
x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(15/4)*d^(5/4)) + ((3*
b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(15/4)*d^(5/4))
```

fricas [B] time = 1.58, size = 1433, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(12*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a
*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b
^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544
*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*arctan((sqrt(c^8*d^2*sqr
t(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*
c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6
*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5)) + (9*b^
4*c^4 + 252*a*b^3*c^3*d + 1302*a^2*b^2*c^2*d^2 - 6468*a^3*b*c*d^3 + 5929*a^
4*d^4)*x)*c^11*d^4*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2
 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*
d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(
c^15*d^5))^(3/4) + (3*b^2*c^13*d^4 + 42*a*b*c^12*d^5 - 77*a^2*c^11*d^6)*sqr
t(x)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*
b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140
```

```
*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(3/
4))/(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5
*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^
6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)) + 3*(c^3*d^3*x^6
+ 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b
^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^
5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*
a^8*d^8)/(c^15*d^5))^(1/4)*log(c^4*d*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 869
40*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 138
54456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 3
5153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*
sqrt(x)) - 3*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536
*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4
*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 766975
44*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*log(-c^4*d*(-(81*b^8*c
^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6
 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2
+ 42*a*b*c*d - 77*a^2*d^2)*sqrt(x)) + 4*(32*a^2*c^2*d - (3*b^2*c^2*d + 42*a
*b*c*d^2 - 77*a^2*d^3)*x^4 + (9*b^2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2
)*sqrt(x))/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)
```

giac [A] time = 0.49, size = 426, normalized size = 1.06

$$-\frac{2\,a^{2}}{3\,c^{3}x^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+42\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-77\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{4}d^{2}}+\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+42\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-77\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{4}d^{2}}+\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+42\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-77\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{4}d^{2}}-\frac{\sqrt{2}\left(3\,\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+42\,\left(cd^{3}\right)^{\frac{1}{4}}abcd-77\,\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^{4}d^{2}}+\frac{b^{2}c^{2}dx^{\frac{5}{2}}+14\,abcd^{2}x^{\frac{5}{2}}-15\,a^{2}d^{3}x^{\frac{5}{2}}-3\,b^{2}c^{3}\sqrt{x}+22\,abc^{2}d\sqrt{x}-19\,a^{2}cd^{2}\sqrt{x}}{16\left(dx^{2}+c\right)^{2}c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -2/3*a^2/(c^3*x^(3/2)) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)
^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d
)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)
*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/128*s
qrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/
4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2) - 1/
128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3
)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^2
) + 1/16*(b^2*c^2*d*x^(5/2) + 14*a*b*c*d^2*x^(5/2) - 15*a^2*d^3*x^(5/2) - 3
*b^2*c^3*sqrt(x) + 22*a*b*c^2*d*sqrt(x) - 19*a^2*c*d^2*sqrt(x))/((d*x^2 + c
)^2*c^3*d)
```

maple [A] time = 0.02, size = 562, normalized size = 1.40

$$-\frac{15a^{2}d^{2}x^{\frac{5}{2}}}{16\left(d\,x^{2}+c\right)^{2}c^{3}}+\frac{7abd\,x^{\frac{5}{2}}}{8\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{b^{2}x^{\frac{5}{2}}}{16\left(d\,x^{2}+c\right)^{2}c}-\frac{19a^{2}d\sqrt{x}}{16\left(d\,x^{2}+c\right)^{2}c^{2}}+\frac{11ab\sqrt{x}}{8\left(d\,x^{2}+c\right)^{2}c}-\frac{3b^{2}\sqrt{x}}{16\left(d\,x^{2}+c\right)^{2}d}-\frac{77\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c^{4}}-\frac{77\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c^{4}}-\frac{77\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128c^{4}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64c^{3}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128c^{2}d}-\frac{2a^{2}}{3c^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] -15/16/c^3/(d*x^2+c)^2*x^(5/2)*a^2*d^2+7/8/c^2/(d*x^2+c)^2*x^(5/2)*a*b*d+1/
16/c/(d*x^2+c)^2*x^(5/2)*b^2-19/16/c^2/(d*x^2+c)^2*d*x^(1/2)*a^2+11/8/c/(d*
x^2+c)^2*x^(1/2)*a*b-3/16/(d*x^2+c)^2/d*x^(1/2)*b^2-77/64/c^4*d*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+21/32/c^3*(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+3/64/c^2/d*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/128/c^4*d*(c/d)^(1/4)*2^(1
/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x
^(1/2)+(c/d)^(1/2)))*a^2+21/64/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^
(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b
+3/128/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1
/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-77/64/c^4*d*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+21/32/c^3*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+3/64/c^2/d*(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-2/3*a^2/c^3/x^(3/2)
```

maxima [A]   time = 2.44, size = 377, normalized size = 0.94

$$-\frac{32\,a^{2}c^{2}d-\left(3\,b^{2}c^{2}d+42\,abcd^{2}-77\,a^{2}d^{3}\right)x^{4}+\left(9\,b^{2}c^{3}-66\,abc^{2}d+121\,a^{2}cd^{2}\right)x^{2}}{48\left(c^{3}d^{3}x^{\frac{11}{2}}+2\,c^{4}d^{2}x^{\frac{7}{2}}+c^{5}dx^{\frac{3}{2}}\right)}+\frac{\frac{2\,\sqrt{2}\left(3\,b^{2}c^{2}+42\,abcd-77\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(3\,b^{2}c^{2}+42\,abcd-77\,a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(3\,b^{2}c^{2}+42\,abcd-77\,a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(3\,b^{2}c^{2}+42\,abcd-77\,a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\,c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(32*a^2*c^2*d - (3*b^2*c^2*d + 42*a*b*c*d^2 - 77*a^2*d^3)*x^4 + (9*b^
2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2)/(c^3*d^3*x^(11/2) + 2*c^4*d^2*x^
(7/2) + c^5*d*x^(3/2)) + 1/128*(2*sqrt(2)*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*
d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b^2*c^2 +
42*a*b*c*d - 77*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sq
rt(2)*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b^2*c^2 + 42*a*b
*c*d - 77*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(
c))/(c^(3/4)*d^(1/4)))/(c^3*d)
```

mupad [B]   time = 0.64, size = 1508, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x)
```

```
[Out] (atan((((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b
^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3*b
^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13
*d^9 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^
2*d^2 + 42*a*b*c*d)*1i)/(64*(-c)^(15/4)*d^(5/4)) + ((x^(1/2)*(97140736*a^4*
c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c
^10*d^9 + 21331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*
d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8))/
(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i)/(64*(-c
)^(15/4)*d^(5/4)))/(((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6
+ 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11
*d^8) - ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 8074
0352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^
2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*(-c)^(15/4)*d^(5/4)) - ((x^(1/2)*(971
40736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 1059717
12*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 +
 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c
^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/
(64*(-c)^(15/4)*d^(5/4))))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i)/(32*(-
c)^(15/4)*d^(5/4)) - ((2*a^2)/(3*c) - (x^4*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b
*c*d))/(48*c^3) + (x^2*(121*a^2*d^2 + 9*b^2*c^2 - 66*a*b*c*d))/(48*c^2*d))/
(c^2*x^(3/2) + d^2*x^(11/2) + 2*c*d*x^(7/2)) + (atan((((x^(1/2)*(97140736*a
^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*
b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b
*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8
)*1i)/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*
(-c)^(15/4)*d^(5/4)) + ((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d
^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c
^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 8
0740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8)*1i)/(64*(-c)^(15/4)*d^(5/4)))
*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*(-c)^(15/4)*d^(5/4)))/(((x^(1/2
)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 1
05971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3*b^2*c^2 - 77*a^2
*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192
*a*b*c^14*d^8)*1i)/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a
*b*c*d)*1i)/(64*(-c)^(15/4)*d^(5/4)) - ((x^(1/2)*(97140736*a^4*c^9*d^10 + 1
47456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21
331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*
b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8)*1i)/(64*(-c)^
(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i)/(64*(-c)^(15/4)*
d^(5/4))))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(32*(-c)^(15/4)*d^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.422 $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=439

$$-\frac{13a^2d^2-10abcd+5b^2c^2}{20c^2d\sqrt{x}\left(c+dx^2\right)^2}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{17/4}d^{3/4}}-$$

Rubi [A] time = 0.45, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {462, 457, 290, 325, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{13a^2d^2-10abcd+5b^2c^2}{20c^2d\sqrt{x}\left(c+dx^2\right)^2}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{17/4}d^{3/4}}-\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{17/4}d^{3/4}}-\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{17/4}d^{3/4}}+\frac{\left(5b^2c^2-9ad(10bc-13ad)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{17/4}d^{3/4}}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}+\frac{5b^2c^2-9ad(10bc-13ad)}{16c^4d\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] (5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))/(16*c^4*d*Sqrt[x]) - (2*a^2)/(5*c*x^(
5/2)*(c + d*x^2)^2) - (5*b^2*c^2 - 10*a*b*c*d + 13*a^2*d^2)/(20*c^2*d*Sqrt[
x]*(c + d*x^2)^2) - ((5*b^2)/d - (9*a*(10*b*c - 13*a*d))/c^2)/(80*c*Sqrt[x]
*(c + d*x^2)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*
a*d*(10*b*c - 13*a*d))*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*d^(3/4)) + ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(
3/4)) - ((5*b^2*c^2 - 9*a*d*(10*b*c - 13*a*d))*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*d^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
```

```
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^3}\,dx = -\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{2\int \frac{\frac{1}{2}a(10bc-13ad)+\frac{5}{2}b^2cx^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx}{5c}$$

$$= -\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}+\frac{1}{40}\left(-\frac{5b^2}{d}+\frac{9a(10bc-13ad)}{c^2}\right)\int \frac{1}{x^{3/2}\left(c+dx\right.}$$

$$= -\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}-\frac{\left(\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}\right)\int \frac{}{x^{3/2}}}{32c}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}+\frac{\left(5b^2c^2\right.}{}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}+\frac{\left(5b^2c^2\right.}{}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}-\frac{\left(5b^2c^2\right.}{}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}+\frac{\left(5b^2c^2\right.}{}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}+\frac{\left(5b^2c^2\right.}{}$$

$$= \frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{16c^2\sqrt{x}}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}+\frac{10ab-\frac{5b^2c}{d}-\frac{13a^2d}{c}}{20c\sqrt{x}\left(c+dx^2\right)^2}-\frac{\frac{5b^2}{d}-\frac{9a(10bc-13ad)}{c^2}}{80c\sqrt{x}\left(c+dx^2\right)}-\frac{\left(5b^2c^2\right.}{}$$

Mathematica [A] time = 0.39, size = 382, normalized size = 0.87

$$\frac{\frac{40\sqrt[4]{c}\,x^{3/2}\left(21a^2d^2-26abcd+5b^2c^2\right)}{c+dx^2}+\frac{5\sqrt{2}\left(117a^2d^2-90abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{3/4}}-\frac{5\sqrt{2}\left(117a^2d^2-90abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{3/4}}-\frac{10\sqrt{2}\left(117a^2d^2-90abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{3/4}}+\frac{10\sqrt{2}\left(117a^2d^2-90abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{3/4}}-\frac{256a^2c^{5/4}}{x^{5/2}}+\frac{160c^{5/4}x^{3/2}(bc-ad)^2}{\left(c+dx^2\right)^2}+\frac{1280a\sqrt[4]{c}\,(3ad-2bc)}{\sqrt{x}}}{640c^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] ((-256*a^2*c^(5/4))/x^(5/2) + (1280*a*c^(1/4)*(-2*b*c + 3*a*d))/Sqrt[x] + (
160*c^(5/4)*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 + (40*c^(1/4)*(5*b^2*c^2 -
 26*a*b*c*d + 21*a^2*d^2)*x^(3/2))/(c + d*x^2) - (10*Sqrt[2]*(5*b^2*c^2 - 9
0*a*b*c*d + 117*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(
3/4) + (10*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(3/4) + (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d
+ 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
d^(3/4) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(3/4))/(640*c^(17/4))
```

IntegrateAlgebraic [A] time = 0.98, size = 278, normalized size = 0.63

$$-\frac{\left(117a^2d^2-90abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{17/4}d^{3/4}}-\frac{\left(117a^2d^2-90abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{17/4}d^{3/4}}+\frac{-32a^2c^3+416a^2c^2dx^2+1053a^2cd^2x^4+585a^2d^3x^6-320abc^3x^2-810abc^2dx^4-450abcd^2x^6+45b^2c^3x^4+25b^2c^2dx^6}{80c^4x^{5/2}\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*a^2*c^3 - 320*a*b*c^3*x^2 + 416*a^2*c^2*d*x^2 + 45*b^2*c^3*x^4 - 810*a
*b*c^2*d*x^4 + 1053*a^2*c*d^2*x^4 + 25*b^2*c^2*d*x^6 - 450*a*b*c*d^2*x^6 +
585*a^2*d^3*x^6)/(80*c^4*x^(5/2)*(c + d*x^2)^2) - ((5*b^2*c^2 - 90*a*b*c*d
+ 117*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
]])/(32*Sqrt[2]*c^(17/4)*d^(3/4)) - ((5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqr
t[2]*c^(17/4)*d^(3/4))
```

fricas [B] time = 2.01, size = 1832, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] -1/320*(20*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b
^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 57
6580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*arctan((sqrt((15
625*b^12*c^12 - 1687500*a*b^11*c^11*d + 78131250*a^2*b^10*c^10*d^2 - 201993
7500*a^3*b^9*c^9*d^3 + 31839834375*a^4*b^8*c^8*d^4 - 314326575000*a^5*b^7*c
^7*d^5 + 1936382557500*a^6*b^6*c^6*d^6 - 7355241855000*a^7*b^5*c^5*d^7 + 17
434219710375*a^8*b^4*c^4*d^8 - 25881265273500*a^9*b^3*c^3*d^9 + 23425464012
210*a^10*b^2*c^2*d^10 - 11839219392780*a^11*b*c*d^11 + 2565164201769*a^12*d
^12)*x - (625*b^8*c^17*d - 45000*a*b^7*c^16*d^2 + 1273500*a^2*b^6*c^15*d^3
- 17739000*a^3*b^5*c^14*d^4 + 124525350*a^4*b^4*c^13*d^5 - 415092600*a^5*b^
```

```
3*c^12*d^6 + 697317660*a^6*b^2*c^11*d^7 - 576580680*a^7*b*c^10*d^8 + 187388
721*a^8*c^9*d^9)*sqrt(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c
^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a
^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388
721*a^8*d^8)/(c^17*d^3)))*c^4*d*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350
0*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 -
415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d
^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4) - (125*b^6*c^10*d - 6750*a*b^5*c^
9*d^2 + 130275*a^2*b^4*c^8*d^3 - 1044900*a^3*b^3*c^7*d^4 + 3048435*a^4*b^2*
c^6*d^5 - 3696030*a^5*b*c^5*d^6 + 1601613*a^6*c^4*d^7)*sqrt(x)*(-(625*b^8*c
^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3
 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^
2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4))/(
625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^
5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 6973176
60*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)) - 5*(c^4*d
^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350
0*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 -
415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d
^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(c^13*d^2*(-(625*b^8*c^8 - 450
00*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 12452
5350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^
6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4) + (125*b^6
*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^3
+ 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d^6)*sqrt(x))
 + 5*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7
*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4
*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 57658068
0*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(-c^13*d^2*(-(625*b
^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5
*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^
6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4
) + (125*b^6*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*
b^3*c^3*d^3 + 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d
^6)*sqrt(x)) - 4*(5*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 32*a^2
*c^3 + 9*(5*b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 32*(10*a*b*c^3 -
13*a^2*c^2*d)*x^2)*sqrt(x))/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

giac [A] time = 0.53, size = 444, normalized size = 1.01

$$\frac{5\,b^{2}c^{2}dx^{\frac{7}{2}} - 26\,abcd^{2}x^{\frac{7}{2}} + 21\,a^{2}d^{3}x^{\frac{7}{2}} + 9\,b^{2}c^{3}x^{\frac{3}{2}} - 34\,abc^{2}dx^{\frac{3}{2}} + 25\,a^{2}cd^{2}x^{\frac{3}{2}}}{16\,{\left(dx^{2} + c\right)}^{2}c^{4}} - \frac{2\,{\left(10\,abcx^{2} - 15\,a^{2}dx^{2} + a^{2}c\right)}}{5\,c^{4}x^{\frac{5}{2}}} + \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2} - 90\,\left(cd^{3}\right)^{\frac{3}{4}}abcd + 117\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{5}d^{3}} + \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2} - 90\,\left(cd^{3}\right)^{\frac{3}{4}}abcd + 117\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^{5}d^{3}} - \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2} - 90\,\left(cd^{3}\right)^{\frac{3}{4}}abcd + 117\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^{5}d^{3}} + \frac{\sqrt{2}\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}b^{2}c^{2} - 90\,\left(cd^{3}\right)^{\frac{3}{4}}abcd + 117\,\left(cd^{3}\right)^{\frac{3}{4}}a^{2}d^{2}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^{5}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*b^2*c^2*d*x^(7/2) - 26*a*b*c*d^2*x^(7/2) + 21*a^2*d^3*x^(7/2) + 9*b
^2*c^3*x^(3/2) - 34*a*b*c^2*d*x^(3/2) + 25*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^
2*c^4) - 2/5*(10*a*b*c*x^2 - 15*a^2*d*x^2 + a^2*c)/(c^4*x^(5/2)) + 1/64*sqr
t(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(c^5*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*
c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
 2*sqrt(x))/(c/d)^(1/4))/(c^5*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2
 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(3/4)*b
^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3)
```

maple [A] time = 0.03, size = 590, normalized size = 1.34

$$\frac{21a^2d^3x^{\frac{7}{2}}}{16\left(dx^2+c\right)^2c^4}-\frac{13ab\,d^2x^{\frac{7}{2}}}{8\left(dx^2+c\right)^2c^3}+\frac{5b^2d\,x^{\frac{7}{2}}}{16\left(dx^2+c\right)^2c^2}+\frac{25a^2d^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2c^3}-\frac{17abd\,x^{\frac{3}{2}}}{8\left(dx^2+c\right)^2c^2}+\frac{9b^2x^{\frac{3}{2}}}{16\left(dx^2+c\right)^2c}+\frac{117\sqrt{2}\,a^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^4}+\frac{117\sqrt{2}\,a^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^4}+\frac{117\sqrt{2}\,a^2d\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128\left(\frac{c}{d}\right)^{\frac{1}{4}}c^4}-\frac{45\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{32\left(\frac{c}{d}\right)^{\frac{1}{4}}c^3}-\frac{45\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{32\left(\frac{c}{d}\right)^{\frac{1}{4}}c^3}-\frac{45\sqrt{2}\,ab\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^3}+\frac{5\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^2d}+\frac{5\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64\left(\frac{c}{d}\right)^{\frac{1}{4}}c^2d}+\frac{5\sqrt{2}\,b^2\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{128\left(\frac{c}{d}\right)^{\frac{1}{4}}c^2d}+\frac{6a^2d}{c^4\sqrt{x}}-\frac{4ab}{c^3\sqrt{x}}-\frac{2a^2}{5c^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x)
```

```
[Out] 21/16/c^4/(d*x^2+c)^2*x^(7/2)*a^2*d^3-13/8/c^3/(d*x^2+c)^2*x^(7/2)*a*b*d^2+
5/16/c^2/(d*x^2+c)^2*x^(7/2)*b^2*d+25/16/c^3/(d*x^2+c)^2*x^(3/2)*a^2*d^2-17
/8/c^2/(d*x^2+c)^2*x^(3/2)*a*b*d+9/16/c/(d*x^2+c)^2*x^(3/2)*b^2+117/128/c^4
*d/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(
x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+117/64/c^4*d/(c/d)^(1/4)*2^(1/2
)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+117/64/c^4*d/(c/d)^(1/4)*2^(1/2
)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-45/64/c^3/(c/d)^(1/4)*2^(1/2)*a
*b*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^
(1/2)+(c/d)^(1/2)))-45/32/c^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)-45/32/c^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)-1)+5/128/c^2/d/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+5/64/c^2
/d/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+5/64/c^2/d
/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5*a^2/c^3/
x^(5/2)+6*a^2/c^4/x^(1/2)*d-4*a/c^3/x^(1/2)*b
```

maxima [A] time = 2.41, size = 324, normalized size = 0.74

$$\frac{5\left(5b^2c^2d-90\,abcd^2+117\,a^2d^3\right)x^6-32\,a^2c^3+9\left(5\,b^2c^3-90\,abc^2d+117\,a^2cd^2\right)x^4-32\left(10\,abc^3-13\,a^2c^2d\right)x^2}{80\left(c^4d^2x^{\frac{13}{2}}+2\,c^5dx^{\frac{9}{2}}+c^6x^{\frac{5}{2}}\right)}+\frac{\left(5\,b^2c^2-90\,abcd+117\,a^2d^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/80*(5*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 32*a^2*c^3 + 9*(5*
b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 32*(10*a*b*c^3 - 13*a^2*c^2*d
)*x^2)/(c^4*d^2*x^(13/2) + 2*c^5*d*x^(9/2) + c^6*x^(5/2)) + 1/128*(5*b^2*c^
2 - 90*a*b*c*d + 117*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4
)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d)
)*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt
(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(
3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))
/(c^(1/4)*d^(3/4)))/c^4
```

mupad [B] time = 0.24, size = 208, normalized size = 0.47

$$\frac{\frac{9\,x^4\,\left(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2\right)}{80\,c^3}-\frac{2\,a^2}{5\,c}+\frac{2\,a\,x^2\,\left(13\,a\,d-10\,b\,c\right)}{5\,c^2}+\frac{d\,x^6\,\left(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2\right)}{16\,c^4}}{c^2\,x^{5/2}+d^2\,x^{13/2}+2\,c\,d\,x^{9/2}}+\frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2\right)}{32\,{\left(-c\right)}^{17/4}\,d^{3/4}}-\frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{{\left(-c\right)}^{1/4}}\right)\,\left(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2\right)}{32\,{\left(-c\right)}^{17/4}\,d^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x)
```

```
[Out] ((9*x^4*(117*a^2*d^2 + 5*b^2*c^2 - 90*a*b*c*d))/(80*c^3) - (2*a^2)/(5*c) +
(2*a*x^2*(13*a*d - 10*b*c))/(5*c^2) + (d*x^6*(117*a^2*d^2 + 5*b^2*c^2 - 90*
a*b*c*d))/(16*c^4))/(c^2*x^(5/2) + d^2*x^(13/2) + 2*c*d*x^(9/2)) + (atan((d
^(1/4)*x^(1/2))/(-c)^(1/4))*(117*a^2*d^2 + 5*b^2*c^2 - 90*a*b*c*d))/(32*(-c
)^(17/4)*d^(3/4)) - (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(117*a^2*d^2 + 5*b
^2*c^2 - 90*a*b*c*d))/(32*(-c)^(17/4)*d^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.423 $\int \frac{x^{5/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=328

$$-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{19/4}}+$$

Rubi [A] time = 0.29, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2dx^{7/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{7b^3}-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{19/4}}-\frac{a^{3/4}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{19/4}}+\frac{2d^2x^{11/2}(3bc-ad)}{11b^2}+\frac{2x^{3/2}(bc-ad)^3}{3b^4}+\frac{2d^3x^{15/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b^2) + (2*d^3*x^(15/2
))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4)) +
(a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(2*Sqrt[2]*b^(19/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{b^3}+\frac{d^2(3bc-ad)x^{9/2}}{b^2}+\frac{d^3x^{13/2}}{b}+\frac{(b^3c^3-3ab^2c^2d+3a^2}{b^3(a+b}\right.$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}+\frac{(bc-ad)^3\int\frac{x^{5/2}}{a+bx^2}\,dx}{b^3}$$

$$=\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}-\frac{(a(}{}$$

$$=\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}-\frac{(2a}{}$$

$$=\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}+\frac{(a(}{}$$

$$=\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}-\frac{(a(}{}$$

$$=\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}-\frac{a^{3/4}}{}$$

$$=\frac{2(bc-ad)^3x^{3/2}}{3b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{7/2}}{7b^3}+\frac{2d^2(3bc-ad)x^{11/2}}{11b^2}+\frac{2d^3x^{15/2}}{15b}+\frac{a^{3/4}}{}$$

Mathematica [C] time = 0.39, size = 132, normalized size = 0.40

$$\frac{2x^{3/2}\left(-385a^3d^3+165a^2bd^2\left(7c+dx^2\right)-15ab^2d\left(77c^2+33cdx^2+7d^2x^4\right)-385(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)+b^3\left(385c^3+495c^2dx^2+315cd^2x^4+77d^3x^6\right)\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(-385*a^3*d^3 + 165*a^2*b*d^2*(7*c + d*x^2) - 15*a*b^2*d*(77*c^2
 + 33*c*d*x^2 + 7*d^2*x^4) + b^3*(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 +
 77*d^3*x^6) - 385*(b*c - a*d)^3*Hypergeometric2F1[3/4, 1, 7/4, -((b*x^2)/a
)]))/(1155*b^4)
```

IntegrateAlgebraic [**A**] time = 0.28, size = 255, normalized size = 0.78

$$-\frac{a^{3/4}(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}b^{19/4}}-\frac{a^{3/4}(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{19/4}}+\frac{2x^{3/2}\left(-385a^3d^3+1155a^2bcd^2+165a^2bd^3x^2-1155ab^2c^2d-495ab^2cd^2x^2-105ab^2d^3x^4+385b^3c^3+495b^3c^2dx^2+315b^3cd^2x^4+77b^3d^3x^6\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(385*b^3*c^3 - 1155*a*b^2*c^2*d + 1155*a^2*b*c*d^2 - 385*a^3*d^3
 + 495*b^3*c^2*d*x^2 - 495*a*b^2*c*d^2*x^2 + 165*a^2*b*d^3*x^2 + 315*b^3*c*
d^2*x^4 - 105*a*b^2*d^3*x^4 + 77*b^3*d^3*x^6))/(1155*b^4) - (a^(3/4)*(-(b*c
) + a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)
))/Sqrt[x]])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(19/4))
```

fricas [**B**] time = 1.66, size = 2528, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2310*(4620*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*
d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924
*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b
^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/
4)*arctan((sqrt((a^4*b^18*c^18 - 18*a^5*b^17*c^17*d + 153*a^6*b^16*c^16*d^2
 - 816*a^7*b^15*c^15*d^3 + 3060*a^8*b^14*c^14*d^4 - 8568*a^9*b^13*c^13*d^5
+ 18564*a^10*b^12*c^12*d^6 - 31824*a^11*b^11*c^11*d^7 + 43758*a^12*b^10*c^1
0*d^8 - 48620*a^13*b^9*c^9*d^9 + 43758*a^14*b^8*c^8*d^10 - 31824*a^15*b^7*c
^7*d^11 + 18564*a^16*b^6*c^6*d^12 - 8568*a^17*b^5*c^5*d^13 + 3060*a^18*b^4*
c^4*d^14 - 816*a^19*b^3*c^3*d^15 + 153*a^20*b^2*c^2*d^16 - 18*a^21*b*c*d^17
 + a^22*d^18)*x - (a^3*b^21*c^12 - 12*a^4*b^20*c^11*d + 66*a^5*b^19*c^10*d^
2 - 220*a^6*b^18*c^9*d^3 + 495*a^7*b^17*c^8*d^4 - 792*a^8*b^16*c^7*d^5 + 92
4*a^9*b^15*c^6*d^6 - 792*a^10*b^14*c^5*d^7 + 495*a^11*b^13*c^4*d^8 - 220*a^
12*b^12*c^3*d^9 + 66*a^13*b^11*c^2*d^10 - 12*a^14*b^10*c*d^11 + a^15*b^9*d^
12)*sqrt(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*
a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c
^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9
 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19))*b^5*(-(a^3*b
^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3
+ 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^1
0*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c
^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4) + (a^2*b^14*c^9 - 9*a^3
*b^13*c^8*d + 36*a^4*b^12*c^7*d^2 - 84*a^5*b^11*c^6*d^3 + 126*a^6*b^10*c^5*
```

```
d^4 - 126*a^7*b^9*c^4*d^5 + 84*a^8*b^8*c^3*d^6 - 36*a^9*b^7*c^2*d^7 + 9*a^1
0*b^6*c*d^8 - a^11*b^5*d^9)*sqrt(x)*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d +
 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8
*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^
4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^
15*d^12)/b^19)^(1/4))/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^1
0*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 9
24*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12
*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)) - 1155
*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6
*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*
d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 +
66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*log(b^14*(
-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c
^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 -
792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^1
3*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 -
 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^
5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a
^10*b*c*d^8 - a^11*d^9)*sqrt(x)) + 1155*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*
c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 -
 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^1
1*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d
^11 + a^15*d^12)/b^19)^(1/4)*log(-b^14*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*
d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*
a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4
*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 +
 a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d
^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^
8*b^3*c^3*d^6 - 36*a^9*b^2*c^2*d^7 + 9*a^10*b*c*d^8 - a^11*d^9)*sqrt(x)) -
4*(77*b^3*d^3*x^7 + 105*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 165*(3*b^3*c^2*d -
3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*x)*sqrt(x))/b^4
```

giac [**B**] time = 0.52, size = 531, normalized size = 1.62

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{7}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{7}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{7}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{7}}+\frac{2\left(77b^{14}d^{3}x^{\frac{15}{2}}+315b^{14}cd^{2}x^{\frac{11}{2}}-105ab^{13}d^{3}x^{\frac{11}{2}}+495b^{14}c^{2}dx^{\frac{7}{2}}-495ab^{13}cd^{2}x^{\frac{7}{2}}+165a^{2}b^{12}d^{3}x^{\frac{7}{2}}+385b^{14}c^{3}x^{\frac{3}{2}}-1155ab^{13}c^{2}dx^{\frac{3}{2}}+1155a^{2}b^{12}cd^{2}x^{\frac{3}{2}}-385a^{3}b^{11}d^{3}x^{\frac{3}{2}}\right)}{1155b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^7 - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c
^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3
```

```
/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/b^7 + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/b^7 - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*
(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3
*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^7 + 2/1155*(77*b^
14*d^3*x^(15/2) + 315*b^14*c*d^2*x^(11/2) - 105*a*b^13*d^3*x^(11/2) + 495*b
^14*c^2*d*x^(7/2) - 495*a*b^13*c*d^2*x^(7/2) + 165*a^2*b^12*d^3*x^(7/2) + 3
85*b^14*c^3*x^(3/2) - 1155*a*b^13*c^2*d*x^(3/2) + 1155*a^2*b^12*c*d^2*x^(3/
2) - 385*a^3*b^11*d^3*x^(3/2))/b^15
```

maple [B] time = 0.01, size = 721, normalized size = 2.20

$$\frac{2d^{3}x^{\frac{15}{2}}}{15b}-\frac{2a\,d^{3}x^{\frac{11}{2}}}{11b^{2}}+\frac{6c\,d^{2}x^{\frac{11}{2}}}{11b}+\frac{2a^{2}d^{3}x^{\frac{7}{2}}}{7b^{3}}-\frac{6ac\,d^{2}x^{\frac{7}{2}}}{7b^{2}}+\frac{6c^{2}d\,x^{\frac{7}{2}}}{7b}-\frac{2a^{3}d^{3}x^{\frac{3}{2}}}{3b^{4}}+\frac{2a^{2}c\,d^{2}x^{\frac{3}{2}}}{b^{3}}-\frac{2a\,c^{2}d\,x^{\frac{3}{2}}}{b^{2}}+\frac{2c^{3}x^{\frac{3}{2}}}{3b}+\frac{\sqrt{2}\,a^{4}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{5}}+\frac{\sqrt{2}\,a^{4}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{5}}+\frac{\sqrt{2}\,a^{4}d^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{5}}-\frac{3\sqrt{2}\,a^{3}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}}-\frac{3\sqrt{2}\,a^{3}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}}-\frac{3\sqrt{2}\,a^{3}c\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}}+\frac{3\sqrt{2}\,a^{2}c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{3\sqrt{2}\,a^{2}c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{3\sqrt{2}\,a^{2}c^{2}d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}-\frac{\sqrt{2}\,a\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,a\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,a\,c^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 2/15*d^3*x^(15/2)/b-2/11/b^2*x^(11/2)*a*d^3+6/11/b*x^(11/2)*c*d^2+2/7/b^3*x
^(7/2)*a^2*d^3-6/7/b^2*x^(7/2)*a*c*d^2+6/7/b*x^(7/2)*c^2*d-2/3/b^4*x^(3/2)*
a^3*d^3+2/b^3*x^(3/2)*a^2*c*d^2-2/b^2*x^(3/2)*a*c^2*d+2/3/b*x^(3/2)*c^3+1/2
*a^4/b^5/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-3/2*
a^3/b^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2+3/2
*a^2/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-1/
2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/4*a
^4/b^5/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(
x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3-3/4*a^3/b^4/(a/b)^(1/4)*2^(
1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*
x^(1/2)+(a/b)^(1/2)))*c*d^2+3/4*a^2/b^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))
)*c^2*d-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3+1/2*a^4/b^5/(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-3/2*a^3/b^4/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+3/2*a^2/b^3/(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d-1/2*a/b^2/(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3
```

maxima [A] time = 2.47, size = 331, normalized size = 1.01

$$-\frac{\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b^{4}}+\frac{2\left(77\,b^{3}d^{3}x^{\frac{15}{2}}+105\left(3\,b^{3}cd^{2}-ab^{2}d^{3}\right)x^{\frac{11}{2}}+165\left(3\,b^{3}c^{2}d-3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{\frac{7}{2}}+385\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)x^{\frac{3}{2}}\right)}{1155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(s
qrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 + 2/1155*(77*b^3*d^3
*x^(15/2) + 105*(3*b^3*c*d^2 - a*b^2*d^3)*x^(11/2) + 165*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x^(7/2) + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*x^(3/2))/b^4
```

mupad [B] time = 0.37, size = 634, normalized size = 1.93

$$x^{3/2}\left(\frac{2c^3}{3b}-\frac{a\left(\frac{6c^2d}{b}+\frac{a\left(\frac{2ad^3}{b^2}-\frac{6cd^2}{b}\right)}{b}\right)}{3b}\right)-x^{11/2}\left(\frac{2ad^3}{11b^2}-\frac{6cd^2}{11b}\right)+x^{7/2}\left(\frac{6c^2d}{7b}+\frac{a\left(\frac{2ad^3}{b^2}-\frac{6cd^2}{b}\right)}{7b}\right)+\frac{2d^3x^{15/2}}{15b}+\frac{(-a)^{3/4}\operatorname{atan}\left(\frac{(-a)^{3/4}b^{1/4}\sqrt{x}\,(ad-bc)^3\left(a^9d^6-6a^8bcd^5+15a^7b^2c^2d^4-20a^6b^3c^3d^3+15a^5b^4c^4d^2-6a^4b^5c^5d+a^3b^6c^6\right)}{a^{13}d^9-9a^{12}bcd^8+36a^{11}b^2c^2d^7-84a^{10}b^3c^3d^6+126a^9b^4c^4d^5-126a^8b^5c^5d^4+84a^7b^6c^6d^3-36a^6b^7c^7d^2+9a^5b^8c^8d-a^4b^9c^9}\right)(ad-bc)^3}{b^{19/4}}+\frac{(-a)^{3/4}\operatorname{atan}\left(\frac{(-a)^{3/4}b^{1/4}\sqrt{x}\,(ad-bc)^3\left(a^9d^6-6a^8bcd^5+15a^7b^2c^2d^4-20a^6b^3c^3d^3+15a^5b^4c^4d^2-6a^4b^5c^5d+a^3b^6c^6\right)1\mathrm{i}}{a^{13}d^9-9a^{12}bcd^8+36a^{11}b^2c^2d^7-84a^{10}b^3c^3d^6+126a^9b^4c^4d^5-126a^8b^5c^5d^4+84a^7b^6c^6d^3-36a^6b^7c^7d^2+9a^5b^8c^8d-a^4b^9c^9}\right)(ad-bc)^3\,1\mathrm{i}}{b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^(3/2)*((2*c^3)/(3*b) - (a*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b)
)/b))/(3*b)) - x^(11/2)*((2*a*d^3)/(11*b^2) - (6*c*d^2)/(11*b)) + x^(7/2)*(
(6*c^2*d)/(7*b) + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(7*b)) + (2*d^3*x^(15/2
))/(15*b) + ((-a)^(3/4)*atan(((-a)^(3/4)*b^(1/4)*x^(1/2)*(a*d - b*c)^3*(a^9
*d^6 + a^3*b^6*c^6 - 6*a^4*b^5*c^5*d + 15*a^5*b^4*c^4*d^2 - 20*a^6*b^3*c^3*
d^3 + 15*a^7*b^2*c^2*d^4 - 6*a^8*b*c*d^5))/(a^13*d^9 - a^4*b^9*c^9 + 9*a^5*
b^8*c^8*d - 36*a^6*b^7*c^7*d^2 + 84*a^7*b^6*c^6*d^3 - 126*a^8*b^5*c^5*d^4 +
 126*a^9*b^4*c^4*d^5 - 84*a^10*b^3*c^3*d^6 + 36*a^11*b^2*c^2*d^7 - 9*a^12*b
*c*d^8))*(a*d - b*c)^3)/b^(19/4) + ((-a)^(3/4)*atan(((-a)^(3/4)*b^(1/4)*x^(
1/2)*(a*d - b*c)^3*(a^9*d^6 + a^3*b^6*c^6 - 6*a^4*b^5*c^5*d + 15*a^5*b^4*c^
4*d^2 - 20*a^6*b^3*c^3*d^3 + 15*a^7*b^2*c^2*d^4 - 6*a^8*b*c*d^5)*1i)/(a^13*
d^9 - a^4*b^9*c^9 + 9*a^5*b^8*c^8*d - 36*a^6*b^7*c^7*d^2 + 84*a^7*b^6*c^6*d
^3 - 126*a^8*b^5*c^5*d^4 + 126*a^9*b^4*c^4*d^5 - 84*a^10*b^3*c^3*d^6 + 36*a
^11*b^2*c^2*d^7 - 9*a^12*b*c*d^8))*(a*d - b*c)^3*1i)/b^(19/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.424 $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=326

$$\frac{2dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{\sqrt[4]{a}\,(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{17/4}}-\frac{\sqrt[4]{a}\,(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\right.}{2\sqrt{2}\,b^{17/4}}$$

Rubi [A] time = 0.28, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {461, 321, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{5b^3}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2\sqrt{x}\,(bc-ad)^3}{b^4}+\frac{\sqrt[4]{a}\,(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{17/4}}-\frac{\sqrt[4]{a}\,(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{17/4}}+\frac{\sqrt[4]{a}\,(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{17/4}}-\frac{\sqrt[4]{a}\,(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{17/4}}+\frac{2d^3x^{13/2}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(5
/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) + (2*d^3*x^(13/2))/(13
*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) - (a^(1/
4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{b^3}+\frac{d^2(3bc-ad)x^{7/2}}{b^2}+\frac{d^3x^{11/2}}{b}+\frac{\left(b^3c^3-3ab^2c^2d+3a^2b}{b^3\left(a+bx\right.}\right.\\
&= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{(bc-ad)^3\int\frac{x^{3/2}}{a+bx^2}\,dx}{b^3}\\
&= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(a(bc\right.}{}\\
&= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(2a(b\right.}{}\\
&= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(\sqrt{a}\,(\right.}{}\\
&= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}-\frac{\left(\sqrt{a}\,(\right.}{}\\
&= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{\sqrt[4]{a}\,(b}{}\\
&= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{5/2}}{5b^3}+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{\sqrt[4]{a}\,(b}{}
\end{aligned}
$$

Mathematica [C] time = 0.40, size = 133, normalized size = 0.41

$$\frac{2\sqrt{x}\left(-585a^3d^3+117a^2bd^2\left(15c+dx^2\right)-13ab^2d\left(135c^2+27cdx^2+5d^2x^4\right)-585(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)+3b^3\left(195c^3+117c^2dx^2+65cd^2x^4+15d^3x^6\right)\right)}{585b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(-585*a^3*d^3 + 117*a^2*b*d^2*(15*c + d*x^2) - 13*a*b^2*d*(135*c
^2 + 27*c*d*x^2 + 5*d^2*x^4) + 3*b^3*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^
4 + 15*d^3*x^6) - 585*(b*c - a*d)^3*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2
)/a)]))/(585*b^4)
```

IntegrateAlgebraic [A] time = 0.27, size = 254, normalized size = 0.78

$$\frac{2\sqrt{x}\left(-585a^3d^3+1755a^2bcd^2+117a^2bd^3x^2-1755ab^2c^2d-351ab^2cd^2x^2-65ab^2d^3x^4+585b^3c^3+351b^3c^2dx^2+195b^3cd^2x^4+45b^3d^3x^6\right)}{585b^4}-\frac{\sqrt[4]{a}\,(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{17/4}}+\frac{\sqrt[4]{a}\,(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*Sqrt[x]*(585*b^3*c^3 - 1755*a*b^2*c^2*d + 1755*a^2*b*c*d^2 - 585*a^3*d^3
 + 351*b^3*c^2*d*x^2 - 351*a*b^2*c*d^2*x^2 + 117*a^2*b*d^3*x^2 + 195*b^3*c*
d^2*x^4 - 65*a*b^2*d^3*x^4 + 45*b^3*d^3*x^6))/(585*b^4) - (a^(1/4)*(-(b*c)
+ a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))
/Sqrt[x]])/(Sqrt[2]*b^(17/4)) + (a^(1/4)*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(17/4))
```

fricas [B] time = 1.49, size = 1898, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2
 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^
7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^
3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*ar
ctan((sqrt(b^8*sqrt(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d
^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*
a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*
c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*b^13*(-(a*b^12*c^12 - 12*a^2*b^11*c^
11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 7
92*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^
4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11
+ a^13*d^12)/b^17)^(3/4) + (b^16*c^3 - 3*a*b^15*c^2*d + 3*a^2*b^14*c*d^2 -
a^3*b^13*d^3)*sqrt(x)*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^1
0*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 9
24*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b
^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(3/
4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*
c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 -
 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11
*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)) + 585*b^4*(-(a*b^12*c^12 - 1
2*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^
```

```
8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7
 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a
^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*log(b^4*(-(a*b^12*c^12 - 12*a^2*b^11*
c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 -
 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*
b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^1
1 + a^13*d^12)/b^17)^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(x)) - 585*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
+ 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^1
0*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^
(1/4)*log(-b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 -
 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*
b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*
d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4) - (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 4*(45*b^3*d^3*
x^6 + 585*b^3*c^3 - 1755*a*b^2*c^2*d + 1755*a^2*b*c*d^2 - 585*a^3*d^3 + 65*
(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 117*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^
3)*x^2)*sqrt(x))/b^4
```

giac [**B**]   time = 0.45, size = 531, normalized size = 1.63

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^{5}}+\frac{2\left(45b^{12}d^{3}x^{\frac{13}{2}}+195b^{12}cd^{2}x^{\frac{9}{2}}-65ab^{11}d^{3}x^{\frac{9}{2}}+351b^{12}c^{2}dx^{\frac{5}{2}}-351ab^{11}cd^{2}x^{\frac{5}{2}}+117a^{2}b^{10}d^{3}x^{\frac{5}{2}}+585b^{12}c^{3}\sqrt{x}-1755ab^{11}c^{2}d\sqrt{x}+1755a^{2}b^{10}cd^{2}\sqrt{x}-585a^{3}b^{9}d^{3}\sqrt{x}\right)}{585b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c
^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1
/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
 + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*
(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3
*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 2/585*(45*b^1
2*d^3*x^(13/2) + 195*b^12*c*d^2*x^(9/2) - 65*a*b^11*d^3*x^(9/2) + 351*b^12*
c^2*d*x^(5/2) - 351*a*b^11*c*d^2*x^(5/2) + 117*a^2*b^10*d^3*x^(5/2) + 585*b
^12*c^3*sqrt(x) - 1755*a*b^11*c^2*d*sqrt(x) + 1755*a^2*b^10*c*d^2*sqrt(x) -
 585*a^3*b^9*d^3*sqrt(x))/b^13
```

maple [**B**]   time = 0.01, size = 712, normalized size = 2.18

$$\frac{2d^{3}x^{\frac{13}{2}}}{13b}-\frac{2a\,d^{3}x^{\frac{9}{2}}}{9b^{2}}+\frac{2c\,d^{2}x^{\frac{9}{2}}}{3b}+\frac{2a^{2}d^{3}x^{\frac{5}{2}}}{5b^{3}}-\frac{6ac\,d^{2}x^{\frac{5}{2}}}{5b^{2}}+\frac{6c^{2}d\,x^{\frac{5}{2}}}{5b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{3}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{4}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{3}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{4}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{3}d^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{4}}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{3}}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{3}}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}c\,d^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,c^{2}d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}-\frac{2a^{3}d^{3}\sqrt{x}}{b^{4}}+\frac{6a^{2}c\,d^{2}\sqrt{x}}{b^{3}}-\frac{6a\,c^{2}d\sqrt{x}}{b^{2}}+\frac{2c^{3}\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 2/13*d^3*x^(13/2)/b-2/9/b^2*x^(9/2)*a*d^3+2/3/b*x^(9/2)*c*d^2+2/5/b^3*x^(5/
2)*a^2*d^3-6/5/b^2*x^(5/2)*a*c*d^2+6/5/b*x^(5/2)*c^2*d-2/b^4*a^3*d^3*x^(1/2
)+6/b^3*a^2*c*d^2*x^(1/2)-6/b^2*a*c^2*d*x^(1/2)+2/b*c^3*x^(1/2)+1/4/b^4*(a/
b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1
/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*a^3*d^3-3/4/b^3*(a/b)^(1/4)*2^(1/2)*ln((x
+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2)))*a^2*c*d^2+3/4/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)
*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*a*c^2*d-
1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x
-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3+1/2/b^4*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^3*d^3-3/2/b^3*(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^2*c*d^2+3/2/b^2*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a*c^2*d-1/2/b*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+1/2/b^4*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a^3*d^3-3/2/b^3*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a^2*c*d^2+3/2/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a*c^2*d-1/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3
```

maxima [A] time = 2.42, size = 437, normalized size = 1.34

$$-\frac{\left(\frac{2\,\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{4\,b^{4}}+\frac{2\left(45\,b^{3}d^{3}x^{\frac{13}{2}}+65\left(3\,b^{3}cd^{2}-ab^{2}d^{3}\right)x^{\frac{9}{2}}+117\left(3\,b^{3}c^{2}d-3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{\frac{5}{2}}+585\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{x}\right)}{585\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^4 + 2/585*(45*b^3*
d^3*x^(13/2) + 65*(3*b^3*c*d^2 - a*b^2*d^3)*x^(9/2) + 117*(3*b^3*c^2*d - 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^(5/2) + 585*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(x))/b^4
```

mupad [B] time = 0.39, size = 1564, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^(1/2)*((2*c^3)/b - (a*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/b)
)/b) - x^(9/2)*((2*a*d^3)/(9*b^2) - (2*c*d^2)/(3*b)) + x^(5/2)*((6*c^2*d)/(
5*b) + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(5*b)) + (2*d^3*x^(13/2))/(13*b) +
 ((-a)^(1/4)*atan((((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*
b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 -
6*a^7*b*c*d^5))/b^5 - (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4))
 + ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a
^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/
b^5 + (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d
 - 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4)))/(((-a)^(1/4)*(
(16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 -
 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 - (16*(-a)^(
1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
))/b^(21/4))*(a*d - b*c)^3)/(2*b^(17/4)) - ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^
6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3
 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 + (16*(-a)^(1/4)*(a*d - b*c)^3*
(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/b^(21/4))*(a*d -
 b*c)^3)/(2*b^(17/4))))*(a*d - b*c)^3*1i)/b^(17/4) + ((-a)^(1/4)*atan((((-a
)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*
c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 - (
(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b
*c*d^2)*16i)/b^(21/4))*(a*d - b*c)^3)/(2*b^(17/4)) + ((-a)^(1/4)*((16*x^(1/
2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b
^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 + ((-a)^(1/4)*(a*d -
b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)*16i)/b^(21
/4))*(a*d - b*c)^3)/(2*b^(17/4)))/(((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*
b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^
6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 - ((-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 -
a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)*16i)/b^(21/4))*(a*d - b*c)^3
*1i)/(2*b^(17/4)) - ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3
*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 -
 6*a^7*b*c*d^5))/b^5 + ((-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)*16i)/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4
))))*(a*d - b*c)^3)/b^(17/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.425 $\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{a+bx^2}\,dx$

Optimal. Leaf size=306

$$\frac{2dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

Rubi **[A]** time = 0.25, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 297, 1162, 617, 204, 1165, 628}

$$\frac{2dx^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{2d^3x^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*(3*b*c - a
*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c - a*d)^3*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - a*
d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/
4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{a+bx^2}\,dx = \int\left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3}+\frac{d^2(3bc-ad)x^{5/2}}{b^2}+\frac{d^3x^{9/2}}{b}+\frac{\left(b^3c^3-3ab^2c^2d+3a^2bc\right.}{b^3\left(a+bx^2\right.}\right.$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}+\frac{(bc-ad)^3\int\frac{\sqrt{x}}{a+bx^2}\,dx}{b^3}$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}+\frac{\left(2(bc-ad)^3\right)\mathrm{Subst}\left(\int\right.}{b^3}$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}-\frac{(bc-ad)^3\,\mathrm{Subst}\left(\int\frac{\sqrt{a}-}{a+}\right.}{b^{7/2}}$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}+\frac{(bc-ad)^3\,\mathrm{Subst}\left(\int\frac{}{\frac{\sqrt{a}}{\sqrt{b}}-}\right.}{2b^4}$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}+\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{a}}$$

$$=\frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2}+\frac{2d^3x^{11/2}}{11b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\sqrt[4]{a}\,b^{15/4}}$$

Mathematica [C] time = 0.38, size = 97, normalized size = 0.32

$$\frac{2x^{3/2}\left(ad\left(77a^2d^2-33abd\left(7c+dx^2\right)+3b^2\left(77c^2+33cdx^2+7d^2x^4\right)\right)+77(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{231ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*x^(3/2)*(a*d*(77*a^2*d^2 - 33*a*b*d*(7*c + d*x^2) + 3*b^2*(77*c^2 + 33*c
*d*x^2 + 7*d^2*x^4)) + 77*(b*c - a*d)^3*Hypergeometric2F1[3/4, 1, 7/4, -((b
*x^2)/a)]))/(231*a*b^3)
```

IntegrateAlgebraic [A] time = 0.25, size = 199, normalized size = 0.65

$$\frac{2dx^{3/2}\left(77a^2d^2-231abcd-33abd^2x^2+231b^2c^2+99b^2cdx^2+21b^2d^2x^4\right)}{231b^3}+\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{15/4}}+\frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{15/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

```
[Out] (2*d*x^(3/2)*(231*b^2*c^2 - 231*a*b*c*d + 77*a^2*d^2 + 99*b^2*c*d*x^2 - 33*
a*b*d^2*x^2 + 21*b^2*d^2*x^4))/(231*b^3) + ((-(b*c) + a*d)^3*ArcTan[(a^(1/4
)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(
1/4)*b^(15/4)) + ((-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(1/4)*b^(15/4))
```

fricas [B] time = 1.60, size = 2441, normalized size = 7.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/462*(924*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*arcta
n((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^1
5*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^
12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9
*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^
12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^1
5*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)*x -
(a*b^19*c^12 - 12*a^2*b^18*c^11*d + 66*a^3*b^17*c^10*d^2 - 220*a^4*b^16*c^9
*d^3 + 495*a^5*b^15*c^8*d^4 - 792*a^6*b^14*c^7*d^5 + 924*a^7*b^13*c^6*d^6 -
 792*a^8*b^12*c^5*d^7 + 495*a^9*b^11*c^4*d^8 - 220*a^10*b^10*c^3*d^9 + 66*a
^11*b^9*c^2*d^10 - 12*a^12*b^8*c*d^11 + a^13*b^7*d^12)*sqrt(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
*b*c*d^11 + a^12*d^12)/(a*b^15)))*b^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a*b^15))^(1/4) + (b^13*c^9 - 9*a*b^12*c^8*d + 36*a^2*b^11*c^7*d^2 - 84*a
^3*b^10*c^6*d^3 + 126*a^4*b^9*c^5*d^4 - 126*a^5*b^8*c^4*d^5 + 84*a^6*b^7*c^
3*d^6 - 36*a^7*b^6*c^2*d^7 + 9*a^8*b^5*c*d^8 - a^9*b^4*d^9)*sqrt(x)*(-(b^12
*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*
c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4))/(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
```

```
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)) - 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4
)*log(a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(3/4) - (b^9*
c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5
*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 +
9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a*b^15))^(1/4)*log(-a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(a*b^15))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^
6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6
- 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 4*(21*b^2*d^3*x^
5 + 33*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 77*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^
3)*x)*sqrt(x))/b^3
```

giac [B] time = 0.49, size = 490, normalized size = 1.60

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{6}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{6}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^{6}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^{6}}+\frac{2\left(21\,b^{10}d^{3}x^{\frac{11}{2}}+99\,b^{10}cd^{2}x^{\frac{7}{2}}-33\,ab^{9}d^{3}x^{\frac{7}{2}}+231\,b^{10}c^{2}dx^{\frac{3}{2}}-231\,ab^{9}cd^{2}x^{\frac{3}{2}}+77\,a^{2}b^{8}d^{3}x^{\frac{3}{2}}\right)}{231\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3
)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^6) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^
3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)
^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a*b^6) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^
2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b
^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3
)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6)
+ 2/231*(21*b^10*d^3*x^(11/2) + 99*b^10*c*d^2*x^(7/2) - 33*a*b^9*d^3*x^(7/2
```

```
) + 231*b^10*c^2*d*x^(3/2) - 231*a*b^9*c*d^2*x^(3/2) + 77*a^2*b^8*d^3*x^(3/
2))/b^11
```

maple [B] time = 0.01, size = 659, normalized size = 2.15

$$\frac{2d^{3}x^{\frac{11}{2}}}{11b}-\frac{2a\,d^{3}x^{\frac{7}{2}}}{7b^{2}}+\frac{6c\,d^{2}x^{\frac{7}{2}}}{7b}+\frac{2a^{2}d^{3}x^{\frac{3}{2}}}{3b^{3}}-\frac{2ac\,d^{2}x^{\frac{3}{2}}}{b^{2}}+\frac{2c^{2}d\,x^{\frac{3}{2}}}{b}-\frac{\sqrt{2}\,a^{3}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}}-\frac{\sqrt{2}\,a^{3}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}}-\frac{\sqrt{2}\,a^{3}d^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}}+\frac{3\sqrt{2}\,a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{3\sqrt{2}\,a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{3\sqrt{2}\,a^{2}c\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}-\frac{3\sqrt{2}\,a\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{3\sqrt{2}\,a\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{3\sqrt{2}\,a\,c^{2}d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,c^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x)
```

```
[Out] 2/11*d^3*x^(11/2)/b-2/7*d^3/b^2*x^(7/2)*a+6/7*d^2/b*x^(7/2)*c+2/3*d^3/b^3*x
^(3/2)*a^2-2*d^2/b^2*x^(3/2)*a*c+2*d/b*x^(3/2)*c^2-1/2/b^4/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^3*d^3+3/2/b^3/(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a^2*c*d^2-3/2/b^2/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*a*c^2*d+1/2/b/(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-1/2/b^4/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a^3*d^3+3/2/b^3/(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a^2*c*d^2-3/2/b^2/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*a*c^2*d+1/2/b/(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3-1/4/b^4/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2)))*a^3*d^3+3/4/b^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*a^2*c*d^2-3/4/b^2
/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b
)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*a*c^2*d+1/4/b/(a/b)^(1/4)*2^(1/2)*ln(
(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2)))*c^3
```

maxima [A] time = 2.42, size = 282, normalized size = 0.92

$$\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,b^{3}}+\frac{2\left(21\,b^{2}d^{3}x^{\frac{11}{2}}+33\left(3\,b^{2}cd^{2}-abd^{3}\right)x^{\frac{7}{2}}+77\left(3\,b^{2}c^{2}d-3\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{3}{2}}\right)}{231\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*arctan(1
/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a
)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 2/231*(21*b^2*d^3*x^(11
```

```
/2) + 33*(3*b^2*c*d^2 - a*b*d^3)*x^(7/2) + 77*(3*b^2*c^2*d - 3*a*b*c*d^2 +
a^2*d^3)*x^(3/2))/b^3
```

mupad [**B**] time = 0.16, size = 574, normalized size = 1.88

$$x^{3/2}\left(\frac{2\,c^2\,d}{b}+\frac{a\,\left(\frac{2\,a\,d^3}{b^2}-\frac{6\,c\,d^2}{b}\right)}{3\,b}\right)-x^{7/2}\,\left(\frac{2\,a\,d^3}{7\,b^2}-\frac{6\,c\,d^2}{7\,b}\right)+\frac{2\,d^3\,x^{11/2}}{11\,b}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,\left(a\,d-b\,c\right)^3\,\left(a^7\,d^6-6\,a^6\,b\,c\,d^5+15\,a^5\,b^2\,c^2\,d^4-20\,a^4\,b^3\,c^3\,d^3+15\,a^3\,b^4\,c^4\,d^2-6\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6\right)}{{\left(-a\right)}^{1/4}\,\left(a^{10}\,d^9-9\,a^9\,b\,c\,d^8+36\,a^8\,b^2\,c^2\,d^7-84\,a^7\,b^3\,c^3\,d^6+126\,a^6\,b^4\,c^4\,d^5-126\,a^5\,b^5\,c^5\,d^4+84\,a^4\,b^6\,c^6\,d^3-36\,a^3\,b^7\,c^7\,d^2+9\,a^2\,b^8\,c^8\,d-a\,b^9\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^3}{{\left(-a\right)}^{1/4}\,b^{15/4}}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,\left(a\,d-b\,c\right)^3\,\left(a^7\,d^6-6\,a^6\,b\,c\,d^5+15\,a^5\,b^2\,c^2\,d^4-20\,a^4\,b^3\,c^3\,d^3+15\,a^3\,b^4\,c^4\,d^2-6\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\left(a^{10}\,d^9-9\,a^9\,b\,c\,d^8+36\,a^8\,b^2\,c^2\,d^7-84\,a^7\,b^3\,c^3\,d^6+126\,a^6\,b^4\,c^4\,d^5-126\,a^5\,b^5\,c^5\,d^4+84\,a^4\,b^6\,c^6\,d^3-36\,a^3\,b^7\,c^7\,d^2+9\,a^2\,b^8\,c^8\,d-a\,b^9\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^3\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

```
[Out] x^(3/2)*((2*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(3*b)) - x^(7/2)*(
(2*a*d^3)/(7*b^2) - (6*c*d^2)/(7*b)) + (2*d^3*x^(11/2))/(11*b) - (atan((b^(
1/4)*x^(1/2)*(a*d - b*c)^3*(a^7*d^6 + a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*
b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5))/((-
a)^(1/4)*(a^10*d^9 - a*b^9*c^9 + 9*a^2*b^8*c^8*d - 36*a^3*b^7*c^7*d^2 + 84*
a^4*b^6*c^6*d^3 - 126*a^5*b^5*c^5*d^4 + 126*a^6*b^4*c^4*d^5 - 84*a^7*b^3*c^
3*d^6 + 36*a^8*b^2*c^2*d^7 - 9*a^9*b*c*d^8)))*(a*d - b*c)^3)/((-a)^(1/4)*b^
(15/4)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^3*(a^7*d^6 + a*b^6*c^6 - 6*a^2
*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 -
 6*a^6*b*c*d^5)*1i)/((-a)^(1/4)*(a^10*d^9 - a*b^9*c^9 + 9*a^2*b^8*c^8*d - 3
6*a^3*b^7*c^7*d^2 + 84*a^4*b^6*c^6*d^3 - 126*a^5*b^5*c^5*d^4 + 126*a^6*b^4*
c^4*d^5 - 84*a^7*b^3*c^3*d^6 + 36*a^8*b^2*c^2*d^7 - 9*a^9*b*c*d^8)))*(a*d -
 b*c)^3*1i)/((-a)^(1/4)*b^(15/4))
```

sympy [**A**] time = 93.47, size = 874, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/sqrt(x) + 2*c**2*d*x**(3/2) + 6*c*d**2*x**(7/2)/7 +
 2*d**3*x**(11/2)/11), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(3/2)/3 + 6*c**2*d
*x**(7/2)/7 + 6*c*d**2*x**(11/2)/11 + 2*d**3*x**(15/2)/15)/a, Eq(b, 0)), ((
-2*c**3/sqrt(x) + 2*c**2*d*x**(3/2) + 6*c*d**2*x**(7/2)/7 + 2*d**3*x**(11/2
)/11)/b, Eq(a, 0)), ((-1)**(3/4)*a**(11/4)*d**3*(1/b)**(3/4)*log(-(-1)**(1/
4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**3) - (-1)**(3/4)*a**(11/4)*d**3*(
1/b)**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**3) - (-1
)**(3/4)*a**(11/4)*d**3*(1/b)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/
b)**(1/4)))/b**3 - 3*(-1)**(3/4)*a**(7/4)*c*d**2*(1/b)**(3/4)*log(-(-1)**(1
/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + 3*(-1)**(3/4)*a**(7/4)*c*d*
*2*(1/b)**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) +
 3*(-1)**(3/4)*a**(7/4)*c*d**2*(1/b)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1
/4)*(1/b)**(1/4)))/b**2 + 3*(-1)**(3/4)*a**(3/4)*c**2*d*(1/b)**(3/4)*log(-(
-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) - 3*(-1)**(3/4)*a**(3/4)*
```

```
c**2*d*(1/b)**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b)
- 3*(-1)**(3/4)*a**(3/4)*c**2*d*(1/b)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(
1/4)*(1/b)**(1/4)))/b + 2*a**2*d**3*x**(3/2)/(3*b**3) - 2*a*c*d**2*x**(3/2)
/b**2 - 2*a*d**3*x**(7/2)/(7*b**2) + 2*c**2*d*x**(3/2)/b + 6*c*d**2*x**(7/2
)/(7*b) + 2*d**3*x**(11/2)/(11*b) - (-1)**(3/4)*c**3*(1/b)**(3/4)*log(-(-1)
**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)) + (-1)**(3/4)*c**3*(1
/b)**(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(1/4)) +
(-1)**(3/4)*c**3*(1/b)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/
4)))/a**(1/4), True))
```

3.426 $\int \frac{\left(c+dx^2\right)^3}{\sqrt{x}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=304

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\Big(}{\sqrt{2}\,a^{3/4}}$$

Rubi [A] time = 0.25, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {461, 329, 211, 1165, 628, 1162, 617, 204}

$$\frac{2d\sqrt{x}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{2d^2x^{5/2}(3bc-ad)}{5b^2}+\frac{2d^3x^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b*c - a*d)*
x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) -
((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{x}\left(a+bx^2\right)}\,dx = \int \Bigg(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3\sqrt{x}} + \frac{d^2(3bc-ad)x^{3/2}}{b^2} + \frac{d^3x^{7/2}}{b} + \frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a}{b^3\sqrt{x}\left(a+bx^2\right)}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\int \frac{1}{\sqrt{x}\left(a+bx^2\right)}\,dx}{b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{\left(2(bc-ad)^3\right)\mathrm{Subst}\Big(\int \frac{}{a+}}{b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\,\mathrm{Subst}\Big(\int \frac{\sqrt{a}-\sqrt{}}{a+bx}}{\sqrt{a}\,b^3}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} + \frac{(bc-ad)^3\,\mathrm{Subst}\Bigg(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{}}}{2\sqrt{a}\,b^{7/}}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} - \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\right.}{2\sqrt{2}\,a^{3/4}b^{1}}$$

$$= \frac{2d\left(3b^2c^2-3abcd+a^2d^2\right)\sqrt{x}}{b^3} + \frac{2d^2(3bc-ad)x^{5/2}}{5b^2} + \frac{2d^3x^{9/2}}{9b} - \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}\,a^{3/4}b^{13/4}}$$

Mathematica [C] time = 0.36, size = 96, normalized size = 0.32

$$\frac{2\sqrt{x}\left(ad\left(45a^2d^2-9abd\left(15c+dx^2\right)+b^2\left(135c^2+27cdx^2+5d^2x^4\right)\right)+45(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{45ab^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[x]*(a*d*(45*a^2*d^2 - 9*a*b*d*(15*c + d*x^2) + b^2*(135*c^2 + 27*c*
d*x^2 + 5*d^2*x^4)) + 45*(b*c - a*d)^3*Hypergeometric2F1[1/4, 1, 5/4, -((b*
x^2)/a)]))/(45*a*b^3)
```

IntegrateAlgebraic [A] time = 0.25, size = 200, normalized size = 0.66

$$\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}b^{13/4}} - \frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}b^{13/4}} + \frac{2d\sqrt{x}\left(45a^2d^2-135abcd-9abd^2x^2+135b^2c^2+27b^2cdx^2+5b^2d^2x^4\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

```
[Out] (2*d*Sqrt[x]*(135*b^2*c^2 - 135*a*b*c*d + 45*a^2*d^2 + 27*b^2*c*d*x^2 - 9*a
*b*d^2*x^2 + 5*b^2*d^2*x^4))/(45*b^3) + ((-(b*c) + a*d)^3*ArcTan[(a^(1/4)/(
Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(3/4
)*b^(13/4)) - ((-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)*b^(13/4))
```

fricas [B] time = 1.66, size = 1862, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(180*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*
c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arc
tan((sqrt(a^2*b^6*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) +
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^2*b^10*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^3*b^13))^(3/4) + (a^2*b^13*c^3 - 3*a^3*b^12*c^2*d + 3*a^
4*b^11*c*d^2 - a^5*b^10*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(a^3*b^13))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 45*b^3*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 1
2*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11
```

```
*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*sqrt(x)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*
b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x))
 - 4*(5*b^2*d^3*x^4 + 135*b^2*c^2*d - 135*a*b*c*d^2 + 45*a^2*d^3 + 9*(3*b^2
*c*d^2 - a*b*d^3)*x^2)*sqrt(x))/b^3
```

giac [B] time = 0.43, size = 490, normalized size = 1.61

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^{4}}+\frac{2\left(5\,b^{8}d^{3}x^{\frac{9}{2}}+27\,b^{8}cd^{2}x^{\frac{5}{2}}-9\,ab^{7}d^{3}x^{\frac{5}{2}}+135\,b^{8}c^{2}d\sqrt{x}-135\,ab^{7}cd^{2}\sqrt{x}+45\,a^{2}b^{6}d^{3}\sqrt{x}\right)}{45\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^
3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)
^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)
^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^
2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
+ 2/45*(5*b^8*d^3*x^(9/2) + 27*b^8*c*d^2*x^(5/2) - 9*a*b^7*d^3*x^(5/2) + 13
5*b^8*c^2*d*sqrt(x) - 135*a*b^7*c*d^2*sqrt(x) + 45*a^2*b^6*d^3*sqrt(x))/b^9
```

maple [B] time = 0.01, size = 650, normalized size = 2.14

$$\frac{2d^{3}x^{\frac{9}{2}}}{9b}-\frac{2a\,d^{3}x^{\frac{5}{2}}}{5b^{2}}+\frac{6c\,d^{2}x^{\frac{5}{2}}}{5b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}d^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ac\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ac\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ac\,d^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}+\frac{2a^{2}d^{3}\sqrt{x}}{b^{3}}-\frac{6ac\,d^{2}\sqrt{x}}{b^{2}}+\frac{6c^{2}d\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/9*d^3*x^(9/2)/b-2/5*d^3/b^2*x^(5/2)*a+6/5*d^2/b*x^(5/2)*c+2*d^3/b^3*a^2*x
^(1/2)-6*d^2/b^2*a*c*x^(1/2)+6*d/b*c^2*x^(1/2)-1/2/b^3*(a/b)^(1/4)*a^2*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2-3/2/b*(a/b)^(1/4)*2^(1/2)*arct
```

```
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-1/2/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-3/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)*c^2*d+1/2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)*c^3-1/4/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2
)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3+3/4
/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2-3/4/b*(a/b)^(1/4)*2^(1/2)
*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2)))*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)
*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3
```

maxima [**A**] time = 2.44, size = 390, normalized size = 1.28

$$\frac{2\left(5\,b^2d^3x^{\frac{9}{2}}+9\left(3\,b^2cd^2-abd^3\right)x^{\frac{5}{2}}+45\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)\sqrt{x}\right)}{45\,b^3}+\frac{\frac{2\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*b^2*d^3*x^(9/2) + 9*(3*b^2*c*d^2 - a*b*d^3)*x^(5/2) + 45*(3*b^2*c^2
*d - 3*a*b*c*d^2 + a^2*d^3)*sqrt(x))/b^3 + 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))
/(a^(3/4)*b^(1/4)))/b^3
```

mupad [**B**] time = 0.38, size = 1460, normalized size = 4.80

result too large to display

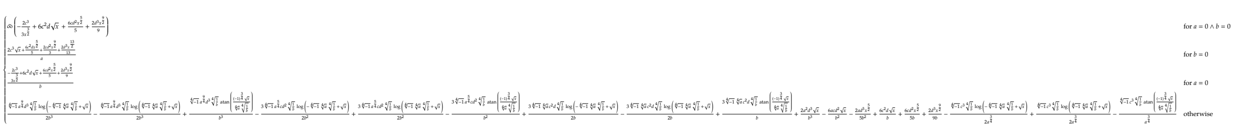
Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(1/2)*(a + b*x^2)),x)
```

```
[Out] x^(1/2)*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/b) - x^(5/2)*((2*a
*d^3)/(5*b^2) - (6*c*d^2)/(5*b)) + (2*d^3*x^(9/2))/(9*b) - (atan(((((8*x^(1
/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b
^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d
^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13
/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) + (((8*x^(1/2)*(a^6*d^6 + b^6
```

```
*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b
^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3
+ 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^
3*1i)/((-a)^(3/4)*b^(13/4)))/((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*
c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b
*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d
 - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(
13/4)) - (((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d
- b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(
2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4))))*(a*d - b*c)^
3*1i)/((-a)^(3/4)*b^(13/4)) - (atan(((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a
^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^
2*c^2*d - 48*a^3*b*c*d^2)*1i)/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)
^(3/4)*b^(13/4)) + (((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^
3 + ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b
*c*d^2)*1i)/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))/
((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3
*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2)*1i)/(2*(-a
)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (((8*x^(1/2)*(
a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(16*a^4*d^3 -
16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2)*1i)/(2*(-a)^(3/4)*b^(13/4
)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4))))*(a*d - b*c)^3)/((-a)^(3/4)*b^
(13/4))
```

sympy [A] time = 71.75, size = 874, normalized size = 2.88



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)
/5 + 2*d**3*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2*c**3*sqrt(x) + 6*c**2*d*
x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(13/2)/13)/a, Eq(b, 0)), ((-2*
c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2
)/9)/b, Eq(a, 0)), ((-1)**(1/4)*a**(9/4)*d**3*(1/b)**(1/4)*log(-(-1)**(1/4)
*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**3) - (-1)**(1/4)*a**(9/4)*d**3*(1/b
)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**3) + (-1)**
```

```
(1/4)*a**(9/4)*d**3*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**
(1/4)))/b**3 - 3*(-1)**(1/4)*a**(5/4)*c*d**2*(1/b)**(1/4)*log(-(-1)**(1/4)*
a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + 3*(-1)**(1/4)*a**(5/4)*c*d**2*(
1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) - 3*(
-1)**(1/4)*a**(5/4)*c*d**2*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*
(1/b)**(1/4)))/b**2 + 3*(-1)**(1/4)*a**(1/4)*c**2*d*(1/b)**(1/4)*log(-(-1)*
*(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) - 3*(-1)**(1/4)*a**(1/4)*c**2
*d*(1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + 3*
(-1)**(1/4)*a**(1/4)*c**2*d*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)
*(1/b)**(1/4)))/b + 2*a**2*d**3*sqrt(x)/b**3 - 6*a*c*d**2*sqrt(x)/b**2 - 2*
a*d**3*x**(5/2)/(5*b**2) + 6*c**2*d*sqrt(x)/b + 6*c*d**2*x**(5/2)/(5*b) + 2
*d**3*x**(9/2)/(9*b) - (-1)**(1/4)*c**3*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1
/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) + (-1)**(1/4)*c**3*(1/b)**(1/4)*lo
g((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) - (-1)**(1/4)*c
**3*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(3/4)
, True))
```

3.427 $\int \frac{\left(c+dx^2\right)^3}{x^{3/2}\left(a+bx^2\right)} \, dx$

Optimal. Leaf size=284

$$-\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \tan^{-1}}{\sqrt{2}a^{5/4}}$$

Rubi [A] time = 0.29, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bc-ad)^3 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}+\frac{(bc-ad)^3 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}-\frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}a^{5/4}b^{11/4}}+\frac{2d^2x^{3/2}(3bc-ad)}{3b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{2d^3x^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(a*Sqrt[x]) + (2*d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (2*d^3*x^(7/
2))/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) +
((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(2*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^{3/2}\left(a+bx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^2\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{c^3}{ax^2}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^6}{b}+\frac{(-bc+ad)^3x^2}{ab^2\left(a+bx^4\right)}\right)dx, x, \sqrt{x}\right)\\
&= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}-\frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab^2}\\
&= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab^{5/2}}-\frac{(bc-a}{}\\
&= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2ab^3}-\frac{(b}{}\\
&= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}b^{11/4}}+\frac{(b}{}\\
&= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\tan^{-}}{\sqrt{2}\,a^{5}}
\end{aligned}
$$

Mathematica [C] time = 0.35, size = 89, normalized size = 0.31

$$
-\frac{2\left(a\left(7a^2d^3x^2-3abd^2x^2\left(7c+dx^2\right)+21b^2c^3\right)+7x^2(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{21a^2b^2\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(21*b^2*c^3 + 7*a^2*d^3*x^2 - 3*a*b*d^2*x^2*(7*c + d*x^2)) + 7*(b*c
- a*d)^3*x^2*Hypergeometric2F1[3/4, 1, 7/4, -((b*x^2)/a)]))/(21*a^2*b^2*Sqr
t[x])
```

IntegrateAlgebraic [A] time = 0.26, size = 190, normalized size = 0.67

$$-\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{5/4}b^{11/4}}-\frac{2\left(7a^2d^3x^2-21abcd^2x^2-3abd^3x^4+21b^2c^3\right)}{21ab^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(21*b^2*c^3 - 21*a*b*c*d^2*x^2 + 7*a^2*d^3*x^2 - 3*a*b*d^3*x^4))/(21*a*
b^2*Sqrt[x]) - ((-(b*c) + a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/
4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(5/4)*b^(11/4)) - ((-(b*c) +
a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(S
qrt[2]*a^(5/4)*b^(11/4))
```

fricas [B] time = 1.55, size = 2442, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/42*(84*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*
arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^
3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a
^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 4862
0*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 185
64*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 81
6*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)
*x - (a^3*b^17*c^12 - 12*a^4*b^16*c^11*d + 66*a^5*b^15*c^10*d^2 - 220*a^6*b
^14*c^9*d^3 + 495*a^7*b^13*c^8*d^4 - 792*a^8*b^12*c^7*d^5 + 924*a^9*b^11*c^
6*d^6 - 792*a^10*b^10*c^5*d^7 + 495*a^11*b^9*c^4*d^8 - 220*a^12*b^8*c^3*d^9
 + 66*a^13*b^7*c^2*d^10 - 12*a^14*b^6*c*d^11 + a^15*b^5*d^12)*sqrt(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11)))*a*b^3*(-(b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(a^5*b^11))^(1/4) + (a*b^12*c^9 - 9*a^2*b^11*c^8*d + 36*a^3*b^
```

```
10*c^7*d^2 - 84*a^4*b^9*c^6*d^3 + 126*a^5*b^8*c^5*d^4 - 126*a^6*b^7*c^4*d^5
 + 84*a^7*b^6*c^3*d^6 - 36*a^8*b^5*c^2*d^7 + 9*a^9*b^4*c*d^8 - a^10*b^3*d^9
)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4))/(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)) - 21*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(a^5*b^11))^(1/4)*log(a^4*b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 21*a*b^2*x*(
-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*log(-a^4*b^8*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8
*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^
5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a
^9*d^9)*sqrt(x)) - 4*(3*a*b*d^3*x^4 - 21*b^2*c^3 + 7*(3*a*b*c*d^2 - a^2*d^3
)*x^2)*sqrt(x))/(a*b^2*x)
```

giac [B] time = 0.46, size = 462, normalized size = 1.63

$$-\frac{2\,c^{3}}{a\sqrt{x}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{2}b^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{2}b^{5}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{2}b^{5}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{2}b^{5}}+\frac{2\left(3\,b^{6}d^{3}x^{\frac{7}{2}}+21\,b^{6}cd^{2}x^{\frac{3}{2}}-7\,ab^{5}d^{3}x^{\frac{3}{2}}\right)}{21\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*c^3/(a*sqrt(x)) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a
*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 1/2*sq
rt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4
```

```
)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c
^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3
/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5) - 1
/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)
^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(a^2*b^5) + 2/21*(3*b^6*d^3*x^(7/2) + 21*b^6*c*d^2*x^(3/
2) - 7*a*b^5*d^3*x^(3/2))/b^7
```

maple [B] time = 0.02, size = 622, normalized size = 2.19

$$\frac{2d^3x^{\frac{7}{2}}}{7b}-\frac{2a\,d^3x^{\frac{3}{2}}}{3b^2}+\frac{2c\,d^2x^{\frac{3}{2}}}{b}+\frac{\sqrt{2}\,a^2d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^3}+\frac{\sqrt{2}\,a^2d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^3}+\frac{\sqrt{2}\,a^2d^3\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^3}-\frac{3\sqrt{2}\,ac\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{3\sqrt{2}\,ac\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{3\sqrt{2}\,ac\,d^2\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^2}-\frac{\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,c^3\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{3\sqrt{2}\,c^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{3\sqrt{2}\,c^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{3\sqrt{2}\,c^2d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{2c^3}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x)
```

```
[Out] 2/7*d^3*x^(7/2)/b-2/3*d^3/b^2*x^(3/2)*a+2*d^2/b*x^(3/2)*c+1/4*a^2/b^3/(a/b)
^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3-3/4*a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/
b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^
(1/2)))*c*d^2+3/4/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d-1/4/a/(a/b)^
(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3+1/2*a^2/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-3/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+3/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)+1)*c^2*d-1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)*c^3+1/2*a^2/b^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)-1)*d^3-3/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)*c*d^2+3/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)*c^2*d-1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1)*c^3-2*c^3/a/x^(1/2)
```

maxima [A] time = 2.40, size = 261, normalized size = 0.92

$$-\frac{2\,c^3}{a\sqrt{x}}+\frac{2\left(3\,bd^3x^{\frac{7}{2}}+7\left(3\,bcd^2-ad^3\right)x^{\frac{3}{2}}\right)}{21\,b^2}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2*c^3/(a*sqrt(x)) + 2/21*(3*b*d^3*x^(7/2) + 7*(3*b*c*d^2 - a*d^3)*x^(3/2))
/b^2 - 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*a
```

```
rctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^2)
```

mupad [B] time = 0.17, size = 580, normalized size = 2.04

$$\frac{2 d^3 x^{7/2}}{7 b} - \frac{2 c^3}{a \sqrt{x}} - x^{3/2}\left(\frac{2 a d^3}{3 b^2} - \frac{2 c d^2}{b}\right) - \frac{\operatorname{atan}\left(\frac{\sqrt{x}\,(a d-b c)^3\left(16 a^{10} b^8 d^6-96 a^9 b^9 c d^5+240 a^8 b^{10} c^2 d^4-320 a^7 b^{11} c^3 d^3+240 a^6 b^{12} c^4 d^2-96 a^5 b^{13} c^5 d+16 a^4 b^{14} c^6\right)}{(-a)^{5/4} b^{11/4}\left(-16 a^{12} b^5 d^9+144 a^{11} b^6 c d^8-576 a^{10} b^7 c^2 d^7+1344 a^9 b^8 c^3 d^6-2016 a^8 b^9 c^4 d^5+2016 a^7 b^{10} c^5 d^4-1344 a^6 b^{11} c^6 d^3+576 a^5 b^{12} c^7 d^2-144 a^4 b^{13} c^8 d+16 a^3 b^{14} c^9\right)}\right)(a d - b c)^3}{(-a)^{5/4}\, b^{11/4}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}\,(a d-b c)^3\left(16 a^{10} b^8 d^6-96 a^9 b^9 c d^5+240 a^8 b^{10} c^2 d^4-320 a^7 b^{11} c^3 d^3+240 a^6 b^{12} c^4 d^2-96 a^5 b^{13} c^5 d+16 a^4 b^{14} c^6\right) 1\mathrm{i}}{(-a)^{5/4} b^{11/4}\left(-16 a^{12} b^5 d^9+144 a^{11} b^6 c d^8-576 a^{10} b^7 c^2 d^7+1344 a^9 b^8 c^3 d^6-2016 a^8 b^9 c^4 d^5+2016 a^7 b^{10} c^5 d^4-1344 a^6 b^{11} c^6 d^3+576 a^5 b^{12} c^7 d^2-144 a^4 b^{13} c^8 d+16 a^3 b^{14} c^9\right)}\right)(a d - b c)^3\, 1\mathrm{i}}{(-a)^{5/4}\, b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(7/2))/(7*b) - (2*c^3)/(a*x^(1/2)) - x^(3/2)*((2*a*d^3)/(3*b^2) -
(2*c*d^2)/b) - (atan((x^(1/2)*(a*d - b*c)^3*(16*a^4*b^14*c^6 + 16*a^10*b^8*
d^6 - 96*a^5*b^13*c^5*d - 96*a^9*b^9*c*d^5 + 240*a^6*b^12*c^4*d^2 - 320*a^7
*b^11*c^3*d^3 + 240*a^8*b^10*c^2*d^4))/((-a)^(5/4)*b^(11/4)*(16*a^3*b^14*c^
9 - 16*a^12*b^5*d^9 - 144*a^4*b^13*c^8*d + 144*a^11*b^6*c*d^8 + 576*a^5*b^1
2*c^7*d^2 - 1344*a^6*b^11*c^6*d^3 + 2016*a^7*b^10*c^5*d^4 - 2016*a^8*b^9*c^
4*d^5 + 1344*a^9*b^8*c^3*d^6 - 576*a^10*b^7*c^2*d^7)))*(a*d - b*c)^3)/((-a)
^(5/4)*b^(11/4)) - (atan((x^(1/2)*(a*d - b*c)^3*(16*a^4*b^14*c^6 + 16*a^10*
b^8*d^6 - 96*a^5*b^13*c^5*d - 96*a^9*b^9*c*d^5 + 240*a^6*b^12*c^4*d^2 - 320
*a^7*b^11*c^3*d^3 + 240*a^8*b^10*c^2*d^4)*1i)/((-a)^(5/4)*b^(11/4)*(16*a^3*
b^14*c^9 - 16*a^12*b^5*d^9 - 144*a^4*b^13*c^8*d + 144*a^11*b^6*c*d^8 + 576*
a^5*b^12*c^7*d^2 - 1344*a^6*b^11*c^6*d^3 + 2016*a^7*b^10*c^5*d^4 - 2016*a^8
*b^9*c^4*d^5 + 1344*a^9*b^8*c^3*d^6 - 576*a^10*b^7*c^2*d^7)))*(a*d - b*c)^3
*1i)/((-a)^(5/4)*b^(11/4))
```

sympy [A] time = 161.47, size = 598, normalized size = 2.11

$$c^3\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{2}{a\sqrt{x}} + \frac{(-1)^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{5}{4}}\sqrt[4]{\frac{1}{b}}} & \text{otherwise} \end{cases}\right) + 6c^2d\operatorname{RootSum}\left(256t^4ab^3 + 1, \left(t \mapsto t\log\left(64t^3ab^2 + \sqrt{x}\right)\right)\right) + 3cd^2\left(\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2b^2\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2b^2\sqrt[4]{\frac{1}{b}}} - \frac{(-1)^{\frac{3}{4}}a^{\frac{3}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^2\sqrt[4]{\frac{1}{b}}} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right) + d^3\left(\begin{cases} \tilde{\infty}x^{\frac{7}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{11}{2}}}{11a} & \text{for } b = 0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } a = 0 \\ -\frac{(-1)^{\frac{3}{4}}a^{\frac{7}{4}}\log\left(-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2b^3\sqrt[4]{\frac{1}{b}}} + \frac{(-1)^{\frac{3}{4}}a^{\frac{7}{4}}\log\left(\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}} + \sqrt{x}\right)}{2b^3\sqrt[4]{\frac{1}{b}}} + \frac{(-1)^{\frac{3}{4}}a^{\frac{7}{4}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\,\sqrt[4]{\frac{1}{b}}}\right)}{b^3\sqrt[4]{\frac{1}{b}}} - \frac{2ax^{\frac{3}{2}}}{3b^2} + \frac{2x^{\frac{7}{2}}}{7b} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a),x)
```

```
[Out] c**3*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(
a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-2/(a*sqrt(x)) + (-1)**(3/4)*log(-(-1)
**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(1/4)) - (-1)**
(3/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(5/4)*(1/b)**(
1/4)) - (-1)**(3/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(a**(
5/4)*(1/b)**(1/4)), True)) + 6*c**2*d*RootSum(256*_t**4*a*b**3 + 1, Lambda(
_t, _t*log(64*_t**3*a*b**2 + sqrt(x)))) + 3*c*d**2*Piecewise((zoo*x**(3/2),
```

```
 Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(
a, 0)), ((-1)**(3/4)*a**(3/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt
(x))/(2*b**2*(1/b)**(1/4)) - (-1)**(3/4)*a**(3/4)*log((-1)**(1/4)*a**(1/4)*
(1/b)**(1/4) + sqrt(x))/(2*b**2*(1/b)**(1/4)) - (-1)**(3/4)*a**(3/4)*atan((
-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/(b**2*(1/b)**(1/4)) + 2*x**(3/2
)/(3*b), True)) + d**3*Piecewise((zoo*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x*
*(11/2)/(11*a), Eq(b, 0)), (2*x**(7/2)/(7*b), Eq(a, 0)), (-(-1)**(3/4)*a**(
7/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**3*(1/b)**(1/4)
) + (-1)**(3/4)*a**(7/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(
2*b**3*(1/b)**(1/4)) + (-1)**(3/4)*a**(7/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1
/4)*(1/b)**(1/4)))/(b**3*(1/b)**(1/4)) - 2*a*x**(3/2)/(3*b**2) + 2*x**(7/2)
/(7*b), True))
```

3.428 $\int \frac{\left(c+dx^2\right)^3}{x^{5/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=284

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1\right.}{\sqrt{2}\,a^{7/4}b}$$

Rubi [A] time = 0.27, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{2d^2\sqrt{x}\,(3bc-ad)}{b^2}-\frac{2c^3}{3ax^{3/2}}+\frac{2d^3x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(3*a*x^(3/2)) + (2*d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (2*d^3*x^(5/2)
)/(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c
 - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sq
rt[2]*a^(7/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^{5/2}\left(a+bx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^4\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{c^3}{ax^4}+\frac{d^3x^4}{b}+\frac{(-bc+ad)^3}{ab^2\left(a+bx^4\right)}\right)dx, x, \sqrt{x}\right)\\
&= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}-\frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{ab^2}\\
&= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}b^2}-(bc-\\
&= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}-\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}b^{5/2}}-\\
&= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}+\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}b^{9/4}}-\\
&= -\frac{2c^3}{3ax^{3/2}}+\frac{2d^2(3bc-ad)\sqrt{x}}{b^2}+\frac{2d^3x^{5/2}}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)^3\tan}{\sqrt{2}}
\end{aligned}
$$

Mathematica [**C**] time = 0.38, size = 89, normalized size = 0.31

$$
-\frac{2\left(a\left(15a^2d^3x^2-3abd^2x^2\left(15c+dx^2\right)+5b^2c^3\right)+15x^2(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{15a^2b^2x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(5*b^2*c^3 + 15*a^2*d^3*x^2 - 3*a*b*d^2*x^2*(15*c + d*x^2)) + 15*(b*
c - a*d)^3*x^2*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2)/a)]))/(15*a^2*b^2*x
^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.25, size = 189, normalized size = 0.67

$$
-\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{2\left(15a^2d^3x^2-45abcd^2x^2-3abd^3x^4+5b^2c^3\right)}{15ab^2x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(5*b^2*c^3 - 45*a*b*c*d^2*x^2 + 15*a^2*d^3*x^2 - 3*a*b*d^3*x^4))/(15*a*
b^2*x^(3/2)) - ((-(b*c) + a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/
4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(7/4)*b^(9/4)) + ((-(b*c) + a
*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sq
rt[2]*a^(7/4)*b^(9/4))
```

fricas **[B]**   time = 1.36, size = 1866, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(60*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*
arctan((sqrt(a^4*b^4*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10
*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 92
4*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3
*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))
+ (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^5*b^7*(-(b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(a^7*b^9))^(3/4) + (a^5*b^10*c^3 - 3*a^6*b^9*c^2*d + 3*a^7
*b^8*c*d^2 - a^8*b^7*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a
^7*b^9))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 15*a*b^2*x^2*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
```

```
^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 15*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(a^7*b^9))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(x)) + 4*(3*a*b*d^3*x^4 - 5*b^2*c^3 + 15*(3*a*b*c*d^2 - a^2*d^3)*x^2)*s
qrt(x))/(a*b^2*x^2)
```

giac [B] time = 0.45, size = 461, normalized size = 1.62

$$-\frac{2c^{3}}{3ax^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{2}b^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{2}b^{3}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{2}b^{3}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{2}b^{3}}+\frac{2\left(b^{4}d^{3}x^{\frac{5}{2}}+15b^{4}cd^{2}\sqrt{x}-5ab^{3}d^{3}\sqrt{x}\right)}{5b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -2/3*c^3/(a*x^(3/2)) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)
*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/2*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3
*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^
(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) +
 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^2*b^3) + 2/5*(b^4*d^3*x^(5/2) + 15*b^4*c*d^2*sqrt(x
) - 5*a*b^3*d^3*sqrt(x))/b^5
```

maple [B] time = 0.02, size = 616, normalized size = 2.17

$$\frac{2d^{3}x^{\frac{5}{2}}}{5b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\,c^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^{2}}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}-\frac{2a\,d^{3}\sqrt{x}}{b^{2}}+\frac{6c\,d^{2}\sqrt{x}}{b}-\frac{2c^{3}}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x)
```

```
[Out] 2/5*d^3*x^(5/2)/b-2*d^3/b^2*a*x^(1/2)+6*d^2/b*c*x^(1/2)+1/2*a/b^2*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-3/2/b*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2+3/2/a*(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-1/2/a^2*b*(a/b)^(1/4)*2^(1/2)*ar
```

```
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*ln((x
+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2)))*d^3-3/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2+3/4/a*(a/b
)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d-1/4/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x+
(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2)))*c^3+1/2*a/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)*d^3-3/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
*c*d^2+3/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*
d-1/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-2
/3*c^3/a/x^(3/2)
```

maxima [A]   time = 2.51, size = 368, normalized size = 1.30

$$-\frac{2\,c^{3}}{3\,ax^{\frac{3}{2}}}+\frac{2\left(bd^{3}x^{\frac{5}{2}}+5\left(3\,bcd^{2}-ad^{3}\right)\sqrt{x}\right)}{5\,b^{2}}-\frac{\frac{2\,\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -2/3*c^3/(a*x^(3/2)) + 2/5*(b*d^3*x^(5/2) + 5*(3*b*c*d^2 - a*d^3)*sqrt(x))/
b^2 - 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2
)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

mupad [B]   time = 0.22, size = 1561, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(5/2))/(5*b) - (2*c^3)/(3*a*x^(3/2)) - x^(1/2)*((2*a*d^3)/b^2 - (6
*c*d^2)/b) - (atan(((((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b
^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3
 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11
*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d
 - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) + (((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b
```

```
^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320
*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^5*b^14*
c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2*(-a)^(7/
4)*b^(9/4)))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)))/((((x^(1/2)*(16*a^3*b^
15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b
^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c
)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*
d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9/4)) - (((x^(1
/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d
^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2
 + ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d +
48*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9
/4))))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) - (atan(((((x^(1/2)*(16*a^3*b
^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*
b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*
c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c
*d^2)*1i)/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9/4)) + (((
x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10
*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4
))/2 + ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*
d + 48*a^7*b^12*c*d^2)*1i)/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/
4)*b^(9/4)))/((((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^
5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240
*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 -
 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2)*1i)/(2*(-a)^(7/4)*b^(9/4)))*(a*d -
b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) - (((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*
d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^
6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^5*b^14*c^3
 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2)*1i)/(2*(-a)^(7/
4)*b^(9/4)))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4))))*(a*d - b*c)^3)/((-a)^
(7/4)*b^(9/4))
```

sympy [A] time = 125.95, size = 828, normalized size = 2.92

$$\begin{cases} \tilde{\infty}\left(-\frac{2c^3}{7x^{\frac{7}{2}}}-\frac{2c^2d}{x^{\frac{3}{2}}}+6cd^2\sqrt{x}+\frac{2d^3x^{\frac{5}{2}}}{5}\right) & \text{for}\ a=0\wedge b=0 \\ \frac{-\frac{2c^3}{3x^{\frac{3}{2}}}+6c^2d\sqrt{x}+\frac{6cd^2x^{\frac{5}{2}}}{5}+\frac{2d^3x^{\frac{9}{2}}}{9}}{a} & \text{for}\ b=0 \\ \frac{-\frac{2c^3}{7x^{\frac{7}{2}}}-\frac{2c^2d}{x^{\frac{3}{2}}}+6cd^2\sqrt{x}+\frac{2d^3x^{\frac{5}{2}}}{5}}{b} & \text{for}\ a=0 \\ -\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}d^3\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}+\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}d^3\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b^2}-\frac{\sqrt[4]{-1}\,a^{\frac{5}{4}}d^3\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b^2}+\frac{3\sqrt[4]{-1}\sqrt[4]{a}\,cd^2\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}-\frac{3\sqrt[4]{-1}\sqrt[4]{a}\,cd^2\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2b}+\frac{3\sqrt[4]{-1}\sqrt[4]{a}\,cd^2\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{b}-\frac{2ad^3\sqrt{x}}{b^2}+\frac{6cd^2\sqrt{x}}{b}+\frac{2d^3x^{\frac{5}{2}}}{5b}-\frac{2c^3}{3ax^{\frac{3}{2}}}-\frac{3\sqrt[4]{-1}\,c^2d\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}}+\frac{3\sqrt[4]{-1}\,c^2d\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{3}{4}}}-\frac{3\sqrt[4]{-1}\,c^2d\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{3}{4}}}+\frac{\sqrt[4]{-1}\,bc^3\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}}-\frac{\sqrt[4]{-1}\,bc^3\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{4}}}+\frac{\sqrt[4]{-1}\,bc^3\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{a^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a),x)
```

```
[Out] Piecewise((zoo*(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x)
 + 2*d**3*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(3*x**(3/2)) + 6*c**
2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9)/a, Eq(b, 0)), ((-2*c
```

```
**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(5/2)/5
)/b, Eq(a, 0)), (-(-1)**(1/4)*a**(5/4)*d**3*(1/b)**(1/4)*log(-(-1)**(1/4)*a
**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) + (-1)**(1/4)*a**(5/4)*d**3*(1/b)*
*(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b**2) - (-1)**(1
/4)*a**(5/4)*d**3*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1
/4)))/b**2 + 3*(-1)**(1/4)*a**(1/4)*c*d**2*(1/b)**(1/4)*log(-(-1)**(1/4)*a*
*(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) - 3*(-1)**(1/4)*a**(1/4)*c*d**2*(1/b)*
*(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*b) + 3*(-1)**(1/
4)*a**(1/4)*c*d**2*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(
1/4)))/b - 2*a*d**3*sqrt(x)/b**2 + 6*c*d**2*sqrt(x)/b + 2*d**3*x**(5/2)/(5*
b) - 2*c**3/(3*a*x**(3/2)) - 3*(-1)**(1/4)*c**2*d*(1/b)**(1/4)*log(-(-1)**(
1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) + 3*(-1)**(1/4)*c**2*d*(
1/b)**(1/4)*log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(3/4)) -
 3*(-1)**(1/4)*c**2*d*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)
**(1/4)))/a**(3/4) + (-1)**(1/4)*b*c**3*(1/b)**(1/4)*log(-(-1)**(1/4)*a**(1
/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) - (-1)**(1/4)*b*c**3*(1/b)**(1/4)*
log((-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + sqrt(x))/(2*a**(7/4)) + (-1)**(1/4)
*b*c**3*(1/b)**(1/4)*atan((-1)**(3/4)*sqrt(x)/(a**(1/4)*(1/b)**(1/4)))/a**(
7/4), True))
```

3.429 $\int \frac{\left(c+dx^2\right)^3}{x^{7/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=283

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\tan^{-1}\Bigl(1}{\sqrt{2}\,a^{9/4}b}$$

Rubi [A] time = 0.28, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{2c^2(bc-3ad)}{a^2\sqrt{x}}-\frac{2c^3}{5ax^{5/2}}+\frac{2d^3x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(5*a*x^(5/2)) + (2*c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (2*d^3*x^(3/
2))/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) - ((b
*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*
Sqrt[2]*a^(9/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{7/2}\left(a+bx^2\right)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^6\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int \left(\frac{c^3}{ax^6} + \frac{c^2(-bc+3ad)}{a^2x^2} + \frac{d^3x^2}{b} - \frac{(-bc+ad)^3x^2}{a^2b\left(a+bx^4\right)}\right)dx, x, \sqrt{x}\right)$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{\left(2(bc-ad)^3\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b}$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} - \frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b^{3/2}} + (bc-ad)$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}} + x^2}\,dx, x, \sqrt{x}\right)}{2a^2b^2} + (bc$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} + \frac{(bc-ad)^3\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x} + \sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}b^{7/4}} - (bc$$

$$= -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} - \frac{(bc-ad)^3\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}} + \frac{(bc-ad)^3\tan^{-1}}{\sqrt{2}\,a^{9/4}}$$

Mathematica [C] time = 0.37, size = 88, normalized size = 0.31

$$-\frac{2\left(a\left(-5a^2d^3x^4 + 3abc^2\left(c+15dx^2\right) - 15b^2c^3x^2\right) - 5x^4(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{15a^3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(-15*b^2*c^3*x^2 - 5*a^2*d^3*x^4 + 3*a*b*c^2*(c + 15*d*x^2)) - 5*(b*
c - a*d)^3*x^4*Hypergeometric2F1[3/4, 1, 7/4, -((b*x^2)/a)]))/(15*a^3*b*x^(
5/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 188, normalized size = 0.66

$$\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{9/4}b^{7/4}}+\frac{2\left(5a^2d^3x^4-3abc^3-45abc^2dx^2+15b^2c^3x^2\right)}{15a^2bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

```
[Out] (2*(-3*a*b*c^3 + 15*b^2*c^3*x^2 - 45*a*b*c^2*d*x^2 + 5*a^2*d^3*x^4))/(15*a^
2*b*x^(5/2)) + ((-(b*c) + a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/
4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(9/4)*b^(7/4)) + ((-(b*c) + a
*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sq
rt[2]*a^(9/4)*b^(7/4))
```

fricas [B] time = 1.47, size = 2451, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(60*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*
arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a^
3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a
^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 4862
0*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 185
64*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 81
6*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18)
*x - (a^5*b^15*c^12 - 12*a^6*b^14*c^11*d + 66*a^7*b^13*c^10*d^2 - 220*a^8*b
^12*c^9*d^3 + 495*a^9*b^11*c^8*d^4 - 792*a^10*b^10*c^7*d^5 + 924*a^11*b^9*c
^6*d^6 - 792*a^12*b^8*c^5*d^7 + 495*a^13*b^7*c^4*d^8 - 220*a^14*b^6*c^3*d^9
 + 66*a^15*b^5*c^2*d^10 - 12*a^16*b^4*c*d^11 + a^17*b^3*d^12)*sqrt(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7)))*a^2*b^2*(-(b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(a^9*b^7))^(1/4) + (a^2*b^11*c^9 - 9*a^3*b^10*c^8*d + 36*a^4*
```

```
b^9*c^7*d^2 - 84*a^5*b^8*c^6*d^3 + 126*a^6*b^7*c^5*d^4 - 126*a^7*b^6*c^4*d^
5 + 84*a^8*b^5*c^3*d^6 - 36*a^9*b^4*c^2*d^7 + 9*a^10*b^3*c*d^8 - a^11*b^2*d
^9)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^
3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6
*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66
*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4))/(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a^11*b*c*d^11 + a^12*d^12)) - 15*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(a^9*b^7))^(1/4)*log(a^7*b^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a
^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3
*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 15*a^2*b*x^
3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(-a^7*b^5*(-(b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^
8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 -
a^9*d^9)*sqrt(x)) + 4*(5*a^2*d^3*x^4 - 3*a*b*c^3 + 15*(b^2*c^3 - 3*a*b*c^2*
d)*x^2)*sqrt(x))/(a^2*b*x^3)
```

giac [B] time = 0.43, size = 455, normalized size = 1.61

$$\frac{2\,d^{3}x^{\frac{3}{2}}}{3\,b}+\frac{2\left(5\,bc^{3}x^{2}-15\,ac^{2}dx^{2}-ac^{3}\right)}{5\,a^{2}x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{3}b^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^{3}b^{4}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{3}b^{4}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^{3}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b + 2/5*(5*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c^3)/(a^2*x^(5/2)
) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a
*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) + 1/2*sqrt(2)*((a*b^3)^(3
```

```
/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^3*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3
/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/4*sqrt(2)*((a*b^
3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^
2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^3*b^4)
```

maple [B]   time = 0.02, size = 616, normalized size = 2.18

$$\frac{2d^{3}x^{\frac{3}{2}}}{3b}-\frac{\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{\sqrt{2}\,a\,d^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{3\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{3\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{3\sqrt{2}\,c^{2}d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,b\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,b\,c^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{3\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{3\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{3\sqrt{2}\,c\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{6c^{2}d}{a\sqrt{x}}+\frac{2b\,c^{3}}{a^{2}\sqrt{x}}-\frac{2c^{3}}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x)
```

```
[Out] 2/3*d^3*x^(3/2)/b-1/2*a/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)*d^3+3/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1)*c*d^2-3/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c
^2*d+1/2/a^2*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^
3-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3+3/4/b/(a/b)^(1/4)*2^(1
/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x
^(1/2)+(a/b)^(1/2)))*c*d^2-3/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d
+1/4/a^2*b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3-1/2*a/b^2/(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/2/b/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2-3/2/a/(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/2/a^2*b/(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-2/5*c^3/a/x^(5/2)-6*c^2/a/x^(1/2)*d+2*c
^3/a^2/x^(1/2)*b
```

maxima [A]   time = 2.42, size = 259, normalized size = 0.92

$$\frac{2\,d^{3}x^{\frac{3}{2}}}{3\,b}+\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,a^{2}b}-\frac{2\left(ac^{3}-5\left(bc^{3}-3\,ac^{2}d\right)x^{2}\right)}{5\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2/3*d^3*x^(3/2)/b + 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
```

```
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b) -
 2/5*(a*c^3 - 5*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x^(5/2))
```

mupad [**B**] time = 0.37, size = 583, normalized size = 2.06

$$\frac{2\,d^3\,x^{3/2}}{3\,b}-\frac{\frac{2\,b\,c^3}{5\,a}+\frac{2\,b\,c^2\,x^2\,(3\,a\,d-b\,c)}{a^2}}{b\,x^{5/2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,\left(16\,a^{13}\,b^5\,d^6-96\,a^{12}\,b^6\,c\,d^5+240\,a^{11}\,b^7\,c^2\,d^4-320\,a^{10}\,b^8\,c^3\,d^3+240\,a^9\,b^9\,c^4\,d^2-96\,a^8\,b^{10}\,c^5\,d+16\,a^7\,b^{11}\,c^6\right)}{(-a)^{9/4}\,b^{7/4}\,\left(-16\,a^{14}\,b^3\,d^9+144\,a^{13}\,b^4\,c\,d^8-576\,a^{12}\,b^5\,c^2\,d^7+1344\,a^{11}\,b^6\,c^3\,d^6-2016\,a^{10}\,b^7\,c^4\,d^5+2016\,a^9\,b^8\,c^5\,d^4-1344\,a^8\,b^9\,c^6\,d^3+576\,a^7\,b^{10}\,c^7\,d^2-144\,a^6\,b^{11}\,c^8\,d+16\,a^5\,b^{12}\,c^9\right)}\right)\,(a\,d-b\,c)^3}{(-a)^{9/4}\,b^{7/4}}+\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,\left(16\,a^{13}\,b^5\,d^6-96\,a^{12}\,b^6\,c\,d^5+240\,a^{11}\,b^7\,c^2\,d^4-320\,a^{10}\,b^8\,c^3\,d^3+240\,a^9\,b^9\,c^4\,d^2-96\,a^8\,b^{10}\,c^5\,d+16\,a^7\,b^{11}\,c^6\right)\,1{}\mathrm{i}}{(-a)^{9/4}\,b^{7/4}\,\left(-16\,a^{14}\,b^3\,d^9+144\,a^{13}\,b^4\,c\,d^8-576\,a^{12}\,b^5\,c^2\,d^7+1344\,a^{11}\,b^6\,c^3\,d^6-2016\,a^{10}\,b^7\,c^4\,d^5+2016\,a^9\,b^8\,c^5\,d^4-1344\,a^8\,b^9\,c^6\,d^3+576\,a^7\,b^{10}\,c^7\,d^2-144\,a^6\,b^{11}\,c^8\,d+16\,a^5\,b^{12}\,c^9\right)}\right)\,(a\,d-b\,c)^3\,1{}\mathrm{i}}{(-a)^{9/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(3/2))/(3*b) - ((2*b*c^3)/(5*a) + (2*b*c^2*x^2*(3*a*d - b*c))/a^2)
/(b*x^(5/2)) + (atan((x^(1/2)*(a*d - b*c)^3*(16*a^7*b^11*c^6 + 16*a^13*b^5*
d^6 - 96*a^8*b^10*c^5*d - 96*a^12*b^6*c*d^5 + 240*a^9*b^9*c^4*d^2 - 320*a^1
0*b^8*c^3*d^3 + 240*a^11*b^7*c^2*d^4))/((-a)^(9/4)*b^(7/4)*(16*a^5*b^12*c^9
 - 16*a^14*b^3*d^9 - 144*a^6*b^11*c^8*d + 144*a^13*b^4*c*d^8 + 576*a^7*b^10
*c^7*d^2 - 1344*a^8*b^9*c^6*d^3 + 2016*a^9*b^8*c^5*d^4 - 2016*a^10*b^7*c^4*
d^5 + 1344*a^11*b^6*c^3*d^6 - 576*a^12*b^5*c^2*d^7)))*(a*d - b*c)^3)/((-a)^
(9/4)*b^(7/4)) + (atan((x^(1/2)*(a*d - b*c)^3*(16*a^7*b^11*c^6 + 16*a^13*b^
5*d^6 - 96*a^8*b^10*c^5*d - 96*a^12*b^6*c*d^5 + 240*a^9*b^9*c^4*d^2 - 320*a
^10*b^8*c^3*d^3 + 240*a^11*b^7*c^2*d^4)*1i)/((-a)^(9/4)*b^(7/4)*(16*a^5*b^1
2*c^9 - 16*a^14*b^3*d^9 - 144*a^6*b^11*c^8*d + 144*a^13*b^4*c*d^8 + 576*a^7
*b^10*c^7*d^2 - 1344*a^8*b^9*c^6*d^3 + 2016*a^9*b^8*c^5*d^4 - 2016*a^10*b^7
*c^4*d^5 + 1344*a^11*b^6*c^3*d^6 - 576*a^12*b^5*c^2*d^7)))*(a*d - b*c)^3*1i
)/((-a)^(9/4)*b^(7/4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.430 $\int \frac{\left(c+dx^2\right)^3}{x^{9/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=283

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\tan^{-1}}{\sqrt{2}\,a^{1}}$$

Rubi [A] time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {466, 461, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}-\frac{2c^3}{7ax^{7/2}}+\frac{2d^3\sqrt{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(7*a*x^(7/2)) + (2*c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) + (2*d^3*Sqr
t[x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(11/4)*b^(5/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) + ((
b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{x^{9/2}\left(a+bx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^8\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{d^3}{b}+\frac{c^3}{ax^8}+\frac{c^2(-bc+3ad)}{a^2x^4}-\frac{(-bc+ad)^3}{a^2b\left(a+bx^4\right)}\right)dx, x, \sqrt{x}\right)\\
&= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2b}\\
&= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{5/2}b}+ (bc-a\\
&= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}+\frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{5/2}b^{3/2}}+ (bc\\
&= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{11/4}b^{5/4}}+ (bc\\
&= -\frac{2c^3}{7ax^{7/2}}+\frac{2c^2(bc-3ad)}{3a^2x^{3/2}}+\frac{2d^3\sqrt{x}}{b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{(bc-ad)^3\tan^{-1}}{\sqrt{2}\,a^{11}}
\end{aligned}
$$

Mathematica [C] time = 0.38, size = 88, normalized size = 0.31

$$
-\frac{2\left(a\left(-21a^2d^3x^4+3abc^2\left(c+7dx^2\right)-7b^2c^3x^2\right)-21x^4(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{21a^3bx^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(-7*b^2*c^3*x^2 - 21*a^2*d^3*x^4 + 3*a*b*c^2*(c + 7*d*x^2)) - 21*(b*
c - a*d)^3*x^4*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^2)/a)]))/(21*a^3*b*x^(
7/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 189, normalized size = 0.67

$$
\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}-\frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{11/4}b^{5/4}}+\frac{2\left(21a^2d^3x^4-3abc^3-21abc^2dx^2+7b^2c^3x^2\right)}{21a^2bx^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

```
[Out] (2*(-3*a*b*c^3 + 7*b^2*c^3*x^2 - 21*a*b*c^2*d*x^2 + 21*a^2*d^3*x^4))/(21*a^
2*b*x^(7/2)) + ((-(b*c) + a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/
4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(11/4)*b^(5/4)) - ((-(b*c) +
a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(S
qrt[2]*a^(11/4)*b^(5/4))
```

fricas [B] time = 1.45, size = 1861, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/42*(84*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4
)*arctan((sqrt(a^6*b^2*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5
)) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^8*b^4*(-(b^12*c^12 - 12*a
*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 49
5*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*
c*d^11 + a^12*d^12)/(a^11*b^5))^(3/4) + (a^8*b^7*c^3 - 3*a^9*b^6*c^2*d + 3*
a^10*b^5*c*d^2 - a^11*b^4*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(a^11*b^5))^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 21*a^2*b*x^4*
(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(a^3*b*(-(b^12*
c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
```

```
12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 21*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^1
1*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^
11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(-a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12)/(a^11*b^5))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3
*d^3)*sqrt(x)) - 4*(21*a^2*d^3*x^4 - 3*a*b*c^3 + 7*(b^2*c^3 - 3*a*b*c^2*d)*
x^2)*sqrt(x))/(a^2*b*x^4)
```

giac [**B**] time = 0.45, size = 455, normalized size = 1.61

$$\frac{2\,d^3\sqrt{x}}{b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2}+\frac{2\left(7\,bc^3x^2-21\,ac^2dx^2-3\,ac^3\right)}{21\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^
2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/2*sqrt(
2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a
^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 1/4*
sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1
/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a^3*b^2) + 2/21*(7*b*c^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(
a^2*x^(7/2))
```

maple [**B**] time = 0.02, size = 622, normalized size = 2.20

$$\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^2\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\,c^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^2}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\,c^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^2}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\,c^2d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^3}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^3}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^3}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4b}+\frac{2d^3\sqrt{x}}{b}-\frac{2c^2d}{a\,x^{\frac{3}{2}}}+\frac{2b\,c^3}{3a^2x^{\frac{3}{2}}}-\frac{2c^3}{7a\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x)
```

```
[Out] 2*d^3*x^(1/2)/b-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3+3/4/a*(a/b)^
(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2-3/4/a^2*b*(a/b)^(1/4)*2^(1/2)*ln((x+(a
```

```
/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)
^(1/2)))*c^2*d+1/4/a^3*b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3-1/2/b*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/2/a*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2-3/2/a^2*b*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/2/a^3*b^2*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-1/2/b*(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/2/a*(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-3/2/a^2*b*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+1/2/a^3*b^2*(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3-2/7*c^3/a/x^(7/2)-2*c^2/a/x^(3/2)*d
+2/3*c^3/a^2/x^(3/2)*b
```

maxima [A] time = 2.59, size = 368, normalized size = 1.30

$$\frac{2\,d^3\sqrt{x}}{b}+\frac{\frac{2\,\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,a^2b}-\frac{2\left(3\,ac^3-7\left(bc^3-3\,ac^2d\right)x^2\right)}{21\,a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 2*d^3*sqrt(x)/b + 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^
(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/
4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*
b) - 2/21*(3*a*c^3 - 7*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x^(7/2))
```

mupad [B] time = 0.25, size = 1564, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x)
```

```
[Out] (2*d^3*x^(1/2))/b - ((2*b*c^3)/(7*a) + (2*b*c^2*x^2*(3*a*d - b*c))/(3*a^2))
/(b*x^(7/2)) + (atan(((((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^
7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d
^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b
^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(
a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) + (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a
```

```
^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2
- 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b
^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)
^(11/4)*b^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)))/((((x^(1/2)*(16*
a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 24
0*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d
 - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*
b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(11/4)*b^(5/4)) -
 (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^1
1*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2
*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*
c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(
11/4)*b^(5/4))))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) + (atan(((((x^(1/2
)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^
5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 -
 ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48
*a^11*b^8*c*d^2)*1i)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(11/4)*b
^(5/4)) + (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d
 - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^1
0*b^8*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*
a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2)*1i)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c
)^3)/((-a)^(11/4)*b^(5/4)))/((((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6
- 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^
9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16
*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2)*1i)/(2*(-a)^(11/4)*b
^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) - (((x^(1/2)*(16*a^6*b^12*
c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^1
0*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 + ((a*d - b*c)^3
*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2
)*1i)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4))))*(a
*d - b*c)^3)/((-a)^(11/4)*b^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.431 $\int \frac{\left(c+dx^2\right)^3}{x^{11/2}\left(a+bx^2\right)} \, dx$

Optimal. Leaf size=303

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\tan^{-1}}{\sqrt{2}\,a^{13/}}$$

Rubi [A] time = 0.29, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{a^3\sqrt{x}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{13/4}b^{3/4}}+\frac{2c^2(bc-3ad)}{5a^2x^{5/2}}-\frac{2c^3}{9ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(9*a*x^(9/2)) + (2*c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) - (2*c*(b^2*
c^2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a*d)^3*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d)^
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(13/4)*b^(3/4))
 - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(2*Sqrt[2]*a^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{11/2}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^{10}\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\Bigg(\int \Bigg(\frac{c^3}{ax^{10}} + \frac{c^2(-bc+3ad)}{a^2x^6} + \frac{c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3x^2} + \frac{(-bc+ad)^3x^2}{a^3\left(a+bx^4\right)}\Bigg)\,dx, x, \sqrt{}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} - \frac{\left(2(bc-ad)^3\right)\text{Subst}\Big(\int \frac{x^2}{a+bx^4}\,dx,}{a^3}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} + \frac{(bc-ad)^3\,\text{Subst}\Big(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx,}{a^3\sqrt{b}}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} - \frac{(bc-ad)^3\,\text{Subst}\Bigg(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x}}{2a^3b}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} - \frac{(bc-ad)^3\log\Big(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}}{2\sqrt{2}\,a^{13/4}b^{3/4}}$$

$$= -\frac{2c^3}{9ax^{9/2}} + \frac{2c^2(bc-3ad)}{5a^2x^{5/2}} - \frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3\sqrt{x}} + \frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{13/4}b^{3/4}}$$

Mathematica [C]   time = 0.38, size = 101, normalized size = 0.33

$$-\frac{2\left(ac\left(a^2\left(5c^2+27cdx^2+135d^2x^4\right)-9abcx^2\left(c+15dx^2\right)+45b^2c^2x^4\right)+15x^6(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{45a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*c*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 15*d*x^2) + a^2*(5*c^2 + 27*c*d
*x^2 + 135*d^2*x^4)) + 15*(b*c - a*d)^3*x^6*Hypergeometric2F1[3/4, 1, 7/4,
-((b*x^2)/a)]))/(45*a^4*x^(9/2))
```

IntegrateAlgebraic [A]   time = 0.26, size = 207, normalized size = 0.68

$$-\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{13/4}b^{3/4}} - \frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{13/4}b^{3/4}} - \frac{2c\left(5a^2c^2+27a^2cdx^2+135a^2d^2x^4-9abc^2x^2-135abcdx^4+45b^2c^2x^4\right)}{45a^3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c*(5*a^2*c^2 - 9*a*b*c^2*x^2 + 27*a^2*c*d*x^2 + 45*b^2*c^2*x^4 - 135*a*
b*c*d*x^4 + 135*a^2*d^2*x^4))/(45*a^3*x^(9/2)) - ((-(b*c) + a*d)^3*ArcTan[(
a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[
2]*a^(13/4)*b^(3/4)) - ((-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(13/4)*b^(3/4))
```

fricas [B] time = 1.76, size = 2459, normalized size = 8.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/90*(180*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)
*arctan((sqrt((b^18*c^18 - 18*a*b^17*c^17*d + 153*a^2*b^16*c^16*d^2 - 816*a
^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*
a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 486
20*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18
564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 8
16*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a^17*b*c*d^17 + a^18*d^18
)*x - (a^7*b^13*c^12 - 12*a^8*b^12*c^11*d + 66*a^9*b^11*c^10*d^2 - 220*a^10
*b^10*c^9*d^3 + 495*a^11*b^9*c^8*d^4 - 792*a^12*b^8*c^7*d^5 + 924*a^13*b^7*
c^6*d^6 - 792*a^14*b^6*c^5*d^7 + 495*a^15*b^5*c^4*d^8 - 220*a^16*b^4*c^3*d^
9 + 66*a^17*b^3*c^2*d^10 - 12*a^18*b^2*c*d^11 + a^19*b*d^12)*sqrt(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3)))*a^3*b*(-(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
 a^12*d^12)/(a^13*b^3))^(1/4) + (a^3*b^10*c^9 - 9*a^4*b^9*c^8*d + 36*a^5*b^
8*c^7*d^2 - 84*a^6*b^7*c^6*d^3 + 126*a^7*b^6*c^5*d^4 - 126*a^8*b^5*c^4*d^5
+ 84*a^9*b^4*c^3*d^6 - 36*a^10*b^3*c^2*d^7 + 9*a^11*b^2*c*d^8 - a^12*b*d^9)
*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b
^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^
6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^
10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4))/(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
```

```
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)) - 45*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(a^13*b^3))^(1/4)*log(a^10*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12
)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) + 45*a^3*x^5*(
-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)*log(-a^10*b^2*(-(b^
12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^
8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a
^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 -
a^9*d^9)*sqrt(x)) + 4*(5*a^2*c^3 + 45*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)
*x^4 - 9*(a*b*c^3 - 3*a^2*c^2*d)*x^2)*sqrt(x))/(a^3*x^5)
```

giac [**B**] time = 0.47, size = 483, normalized size = 1.59

$$-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}b^3c^3-3\left(ab^3\right)^{\frac{3}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{3}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^4b^3}-\frac{2\left(45b^2c^3x^4-135abc^2dx^4+135a^2cd^2x^4-9abc^3x^2+27a^2c^2dx^2+5a^2c^3\right)}{45a^3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3) - 1/2*sqrt(2)*((a*b^3)^(3/4)
*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b
^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(a^4*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)
*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) - 1/4*sqrt(2)*((a*b^3)^
(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
 (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a^4*b^3) - 2/45*(45*b^2*c^3*x^4 - 135*a*b*c^2*d*x^4 + 135*a^2*c*d^2*x^4 - 9
*a*b*c^3*x^2 + 27*a^2*c^2*d*x^2 + 5*a^2*c^3)/(a^3*x^(9/2))
```

maple [B]   time = 0.02, size = 650, normalized size = 2.15

$$-\frac{3\sqrt{2}\, c\, d^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{3\sqrt{2}\, c\, d^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{3\sqrt{2}\, c\, d^{2} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}+\frac{3\sqrt{2}\, b\, c^{2} d \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{3\sqrt{2}\, b\, c^{2} d \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{3\sqrt{2}\, b\, c^{2} d \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{\sqrt{2}\, b^{2} c^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{\sqrt{2}\, b^{2} c^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{\sqrt{2}\, b^{2} c^{3} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}+\frac{\sqrt{2}\, d^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, d^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, d^{3} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} b}-\frac{6 c\, d^{2}}{a \sqrt{x}}+\frac{6 b\, c^{2} d}{a^{2} \sqrt{x}}-\frac{2 b^{2} c^{3}}{a^{3} \sqrt{x}}-\frac{6 c^{2} d}{5 a\, x^{\frac{5}{2}}}+\frac{2 b\, c^{3}}{5 a^{2} x^{\frac{5}{2}}}-\frac{2 c^{3}}{9 a\, x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x)
```

```
[Out] 1/2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-3/2/a/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+3/2/a^2*b/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d-1/2/a^3*b^2/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+1/2/b/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-3/2/a/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2+3/2/a^2*b/(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-1/2/a^3*b^2/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/4/b/(a/b)^(1/4)*2^(1/2)*ln
((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2)))*d^3-3/4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2+3/4/a^2
*b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a
/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d-1/4/a^3*b^2/(a/b)^(1/4)*2^(1/
2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2)))*c^3-2/9*c^3/a/x^(9/2)-6*c/a/x^(1/2)*d^2+6*c^2/a^2/x^(1/
2)*b*d-2*c^3/a^3/x^(1/2)*b^2-6/5*c^2/a/x^(5/2)*d+2/5*c^3/a^2/x^(5/2)*b
```

maxima [A]   time = 2.53, size = 281, normalized size = 0.93

$$-\frac{\left(b^{3} c^{3}-3\, a b^{2} c^{2} d+3\, a^{2} b c d^{2}-a^{3} d^{3}\right) \left(\frac{2\, \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}+2\, \sqrt{b} \sqrt{x}\right)}{2\, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}\, \sqrt{b}}+\frac{2\, \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}}-2\, \sqrt{b} \sqrt{x}\right)}{2\, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}\, \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2}\, a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4\, a^{3}}-\frac{2 \left(5\, a^{2} c^{3}+45 \left(b^{2} c^{3}-3\, a b c^{2} d+3\, a^{2} c d^{2}\right) x^{4}-9 \left(a b c^{3}-3\, a^{2} c^{2} d\right) x^{2}\right)}{45\, a^{3} x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(
a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3 - 2/45*(5*a^2*c^3 + 45*(
b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 9*(a*b*c^3 - 3*a^2*c^2*d)*x^2)/(
a^3*x^(9/2))
```

mupad [B] time = 0.36, size = 591, normalized size = 1.95

$$\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,\left(16\,a^{16}\,b^2\,d^6-96\,a^{15}\,b^3\,c\,d^5+240\,a^{14}\,b^4\,c^2\,d^4-320\,a^{13}\,b^5\,c^3\,d^3+240\,a^{12}\,b^6\,c^4\,d^2-96\,a^{11}\,b^7\,c^5\,d+16\,a^{10}\,b^8\,c^6\right)}{(-a)^{13/4}\,b^{3/4}\,\left(16\,a^{16}\,b\,d^9-144\,a^{15}\,b^2\,c\,d^8+576\,a^{14}\,b^3\,c^2\,d^7-1344\,a^{13}\,b^4\,c^3\,d^6+2016\,a^{12}\,b^5\,c^4\,d^5-2016\,a^{11}\,b^6\,c^5\,d^4+1344\,a^{10}\,b^7\,c^6\,d^3-576\,a^9\,b^8\,c^7\,d^2+144\,a^8\,b^9\,c^8\,d-16\,a^7\,b^{10}\,c^9\right)}\right)\,(a\,d-b\,c)^3}{(-a)^{13/4}\,b^{3/4}}-\frac{\frac{2\,c^3}{9\,a}+\frac{2\,c^2\,x^2\,(3\,a\,d-b\,c)}{5\,a^2}+\frac{2\,c\,x^4\,\left(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2\right)}{a^3}}{x^{9/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,\left(16\,a^{16}\,b^2\,d^6-96\,a^{15}\,b^3\,c\,d^5+240\,a^{14}\,b^4\,c^2\,d^4-320\,a^{13}\,b^5\,c^3\,d^3+240\,a^{12}\,b^6\,c^4\,d^2-96\,a^{11}\,b^7\,c^5\,d+16\,a^{10}\,b^8\,c^6\right)}{(-a)^{13/4}\,b^{3/4}\,\left(16\,a^{16}\,b\,d^9-144\,a^{15}\,b^2\,c\,d^8+576\,a^{14}\,b^3\,c^2\,d^7-1344\,a^{13}\,b^4\,c^3\,d^6+2016\,a^{12}\,b^5\,c^4\,d^5-2016\,a^{11}\,b^6\,c^5\,d^4+1344\,a^{10}\,b^7\,c^6\,d^3-576\,a^9\,b^8\,c^7\,d^2+144\,a^8\,b^9\,c^8\,d-16\,a^7\,b^{10}\,c^9\right)}\right)\,(a\,d-b\,c)^3}{(-a)^{13/4}\,b^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x)
```

```
[Out] (atan((x^(1/2)*(a*d - b*c)^3*(16*a^10*b^8*c^6 + 16*a^16*b^2*d^6 - 96*a^11*b
^7*c^5*d - 96*a^15*b^3*c*d^5 + 240*a^12*b^6*c^4*d^2 - 320*a^13*b^5*c^3*d^3
+ 240*a^14*b^4*c^2*d^4))/((-a)^(13/4)*b^(3/4)*(16*a^16*b*d^9 - 16*a^7*b^10*
c^9 + 144*a^8*b^9*c^8*d - 144*a^15*b^2*c*d^8 - 576*a^9*b^8*c^7*d^2 + 1344*a
^10*b^7*c^6*d^3 - 2016*a^11*b^6*c^5*d^4 + 2016*a^12*b^5*c^4*d^5 - 1344*a^13
*b^4*c^3*d^6 + 576*a^14*b^3*c^2*d^7)))*(a*d - b*c)^3)/((-a)^(13/4)*b^(3/4))
 - ((2*c^3)/(9*a) + (2*c^2*x^2*(3*a*d - b*c))/(5*a^2) + (2*c*x^4*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/a^3)/x^(9/2) - (atanh((x^(1/2)*(a*d - b*c)^3*(16*a
^10*b^8*c^6 + 16*a^16*b^2*d^6 - 96*a^11*b^7*c^5*d - 96*a^15*b^3*c*d^5 + 240
*a^12*b^6*c^4*d^2 - 320*a^13*b^5*c^3*d^3 + 240*a^14*b^4*c^2*d^4))/((-a)^(13
/4)*b^(3/4)*(16*a^16*b*d^9 - 16*a^7*b^10*c^9 + 144*a^8*b^9*c^8*d - 144*a^15
*b^2*c*d^8 - 576*a^9*b^8*c^7*d^2 + 1344*a^10*b^7*c^6*d^3 - 2016*a^11*b^6*c^
5*d^4 + 2016*a^12*b^5*c^4*d^5 - 1344*a^13*b^4*c^3*d^6 + 576*a^14*b^3*c^2*d^
7)))*(a*d - b*c)^3)/((-a)^(13/4)*b^(3/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(11/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.432 $\int \frac{\left(c+dx^2\right)^3}{x^{13/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=305

$$\frac{(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\tan^{-1}\left(\right.}{\sqrt{2}\,a^{15/4}}$$

Rubi [A] time = 0.27, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{3a^3x^{3/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{2c^3}{11ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(11*a*x^(11/2)) + (2*c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c - a*d)^3*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a
*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(15/4)*b^(1
/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(15/4)*b^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{13/2}\left(a+bx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^{12}\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int\left(\frac{c^3}{ax^{12}}+\frac{c^2(-bc+3ad)}{a^2x^8}+\frac{c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3x^4}+\frac{(-bc+ad)^3}{a^3\left(a+bx^4\right)}\right)dx, x, \sqrt{}\right.$$

$$= -\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{3a^3x^{3/2}}-\frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int\frac{1}{a+bx^4}\right.}{a^3}$$

$$= -\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{3a^3x^{3/2}}-\frac{(bc-ad)^3\,\text{Subst}\left(\int\frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\right.}{a^{7/2}}$$

$$= -\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{3a^3x^{3/2}}-\frac{(bc-ad)^3\,\text{Subst}\left(\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}}\right.}{2a^{7/2}\sqrt{b}}$$

$$= -\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{3a^3x^{3/2}}+\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{2\sqrt{2}\,a^{15/4}\sqrt[4]{b}}$$

$$= -\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{3a^3x^{3/2}}+\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}$$

Mathematica [C] time = 0.38, size = 102, normalized size = 0.33

$$-\frac{2\left(ac\left(3a^2\left(7c^2+33cdx^2+77d^2x^4\right)-33abcx^2\left(c+7dx^2\right)+77b^2c^2x^4\right)+231x^6(bc-ad)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{231a^4x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*c*(77*b^2*c^2*x^4 - 33*a*b*c*x^2*(c + 7*d*x^2) + 3*a^2*(7*c^2 + 33*c
*d*x^2 + 77*d^2*x^4)) + 231*(b*c - a*d)^3*x^6*Hypergeometric2F1[1/4, 1, 5/4
, -((b*x^2)/a)]))/(231*a^4*x^(11/2))
```

IntegrateAlgebraic [A] time = 0.25, size = 206, normalized size = 0.68

$$-\frac{(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}+\frac{(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{15/4}\sqrt[4]{b}}-\frac{2c\left(21a^2c^2+99a^2cdx^2+231a^2d^2x^4-33abc^2x^2-231abcdx^4+77b^2c^2x^4\right)}{231a^3x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c*(21*a^2*c^2 - 33*a*b*c^2*x^2 + 99*a^2*c*d*x^2 + 77*b^2*c^2*x^4 - 231*
a*b*c*d*x^4 + 231*a^2*d^2*x^4))/(231*a^3*x^(11/2)) - ((-(b*c) + a*d)^3*ArcT
an[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(S
qrt[2]*a^(15/4)*b^(1/4)) + ((-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(15/4)*b^(1/4))
```

fricas [**B**] time = 1.49, size = 1866, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/462*(924*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*a
rctan((sqrt(a^8*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b)) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x)*a^11*b*(-(b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(a^15*b))^(3/4) + (a^11*b^4*c^3 - 3*a^12*b^3*c^2*d + 3*a^13*b^2*c
*d^2 - a^14*b*d^3)*sqrt(x)*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))
^(3/4))/(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)) + 231*a^3*x^6*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^1
1*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*log(a^4*(-(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
```

```
 a^12*d^12)/(a^15*b))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(x)) - 231*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*
b))^(1/4)*log(-a^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 4*(21*a^2*c
^3 + 77*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 33*(a*b*c^3 - 3*a^2*c^2
*d)*x^2)*sqrt(x))/(a^3*x^6)
```

giac [B] time = 0.51, size = 483, normalized size = 1.58

$$-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{4}b}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{4}b}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{4}b}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{4}b}-\frac{2\left(77b^{2}c^{3}x^{4}-231abc^{2}dx^{4}+231a^{2}cd^{2}x^{4}-33abc^{3}x^{2}+99a^{2}c^{2}dx^{2}+21a^{2}c^{3}\right)}{231a^{3}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b
^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(a^4*b) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b
^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^
3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b)
 - 2/231*(77*b^2*c^3*x^4 - 231*a*b*c^2*d*x^4 + 231*a^2*c*d^2*x^4 - 33*a*b*c
^3*x^2 + 99*a^2*c^2*d*x^2 + 21*a^2*c^3)/(a^3*x^(11/2))
```

maple [B] time = 0.02, size = 659, normalized size = 2.16

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bcd^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^{2}}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bcd^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{2}}-\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bcd^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c^{2}d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{3}c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2a^{4}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{3}c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2a^{4}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{3}c^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4a^{4}}-\frac{2cd^{2}}{a\,x^{\frac{3}{2}}}+\frac{2bc^{2}d}{a^{2}x^{\frac{3}{2}}}-\frac{2b^{2}c^{3}}{3a^{3}x^{\frac{3}{2}}}-\frac{6c^{2}d}{7a\,x^{\frac{7}{2}}}+\frac{2bc^{3}}{7a^{2}x^{\frac{7}{2}}}-\frac{2c^{3}}{11a\,x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x)
```

```
[Out] 1/2/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-3/2/a^2
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b*c*d^2+3/2/a^3*
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^2*c^2*d-1/2/a^4
*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^3*c^3+1/4/a*(a
```

```
/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(
1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3-3/4/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a
/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)
^(1/2)))*b*c*d^2+3/4/a^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b^2*c^2*d-1/4
/a^4*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-
(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b^3*c^3+1/2/a*(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-3/2/a^2*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b*c*d^2+3/2/a^3*(a/b)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^2*c^2*d-1/2/a^4*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^3*c^3-2/11*c^3/a/x^(11/2)-2*c/a/x^(3/2)
*d^2+2*c^2/a^2/x^(3/2)*b*d-2/3*c^3/a^3/x^(3/2)*b^2-6/7*c^2/a/x^(7/2)*d+2/7*
c^3/a^2/x^(7/2)*b
```

maxima [A] time = 2.37, size = 389, normalized size = 1.28

$$-\frac{\frac{2\sqrt{2}\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4a^3}-\frac{2\left(21a^2c^3+77\left(b^2c^3-3abc^2d+3a^2cd^2\right)x^4-33\left(abc^3-3a^2c^2d\right)x^2\right)}{231a^3x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^3 - 2/231*(21*a^2*c^
3 + 77*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 33*(a*b*c^3 - 3*a^2*c^2*
d)*x^2)/(a^3*x^(11/2))
```

mupad [B] time = 0.51, size = 1580, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x)
```

```
[Out] - ((2*c^3)/(11*a) + (2*c^2*x^2*(3*a*d - b*c))/(7*a^2) + (2*c*x^4*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/(3*a^3))/x^(11/2) - (atan(((((x^(1/2)*(16*a^9*b^9*
c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^
7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^
3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^
```

```
2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4)) + (((x
^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*
c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4)
)/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d
 + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15
/4)*b^(1/4)))/((((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c
^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 24
0*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3
 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b
*c)^3)/((-a)^(15/4)*b^(1/4)) - (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6
 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*
b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 -
16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^
(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4))))*(a*d - b*c)^3*1i)/((-a)^(15/
4)*b^(1/4)) - (atan(((((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10
*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^
3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^
3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))
*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4)) + (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^1
5*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 -
320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b
^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-
a)^(15/4)*b^(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4)))/((((x^(1/2)*(16*a
^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*
a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d
- b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b
^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/
4)) - (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96
*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^
5*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14
*b^5*c^2*d + 48*a^15*b^4*c*d^2)*1i)/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*
1i)/((-a)^(15/4)*b^(1/4))))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(13/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.433 $\int \frac{\left(c+dx^2\right)^3}{x^{15/2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{b}\,(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{17/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{17/4}}-\sqrt[4]{b}\,(bc-ad$$

Rubi [A] time = 0.31, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 461, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2c\left(3a^2d^2-3abcd+b^2c^2\right)}{5a^3x^{5/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{\sqrt[4]{b}\,(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{17/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\,a^{17/4}}-\frac{\sqrt[4]{b}\,(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{17/4}}+\frac{\sqrt[4]{b}\,(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{17/4}}-\frac{2c^3}{13ax^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] (-2*c^3)/(13*a*x^(13/2)) + (2*c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2)) - (2*c*(b^
2*c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (2*(b*c - a*d)^3)/(a^4*Sq
rt[x]) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(17/4)) - (b^
(1/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(2*Sqrt[2]*a^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{15/2}\left(a+bx^2\right)}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^{14}\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\Bigg(\int\Bigg(\frac{c^3}{ax^{14}}+\frac{c^2(-bc+3ad)}{a^2x^{10}}+\frac{c\left(b^2c^2-3abcd+3a^2d^2\right)}{a^3x^6}+\frac{(-bc+ad)^3}{a^4x^2}-\frac{b(-bc+}{a^4\,(a+}$$

$$= -\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{\left(2b(bc-ad)^3\right)\mathrm{S}}{}$$

$$= -\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}-\frac{\left(\sqrt{b}\,(bc-ad)^3\right)\mathrm{S}}{}$$

$$= -\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{(bc-ad)^3\,\mathrm{Subs}}{}$$

$$= -\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}+\frac{\sqrt[4]{b}\,(bc-ad)^3\,\mathrm{lo}}{}$$

$$= -\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}}-\frac{2c\left(b^2c^2-3abcd+3a^2d^2\right)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}}-\frac{\sqrt[4]{b}\,(bc-ad)^3\,\mathrm{ta}}{\sqrt{2}}$$

Mathematica [C] time = 0.43, size = 148, normalized size = 0.46

$$-\frac{2\left(a\left(3a^3\left(15c^3+65c^2dx^2+117cd^2x^4+195d^3x^6\right)-13a^2bcx^2\left(5c^2+27cdx^2+135d^2x^4\right)+117ab^2c^2x^4\left(c+15dx^2\right)-585b^3c^3x^6\right)-195bx^8(bc-ad)^3\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{585a^5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(a*(-585*b^3*c^3*x^6 + 117*a*b^2*c^2*x^4*(c + 15*d*x^2) - 13*a^2*b*c*x^
2*(5*c^2 + 27*c*d*x^2 + 135*d^2*x^4) + 3*a^3*(15*c^3 + 65*c^2*d*x^2 + 117*c
*d^2*x^4 + 195*d^3*x^6)) - 195*b*(b*c - a*d)^3*x^8*Hypergeometric2F1[3/4, 1
, 7/4, -((b*x^2)/a)]))/(585*a^5*x^(13/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 262, normalized size = 0.81

$$\frac{\sqrt[4]{b}\,(ad-bc)^3\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{17/4}}+\frac{\sqrt[4]{b}\,(ad-bc)^3\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{17/4}}-\frac{2\left(45a^3c^3+195a^3c^2dx^2+351a^3cd^2x^4+585a^3d^3x^6-65a^2bc^3x^2-351a^2bc^2dx^4-1755a^2bcd^2x^6+117ab^2c^3x^4+1755ab^2c^2dx^6-585b^3c^3x^6\right)}{585a^4x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

```
[Out] (-2*(45*a^3*c^3 - 65*a^2*b*c^3*x^2 + 195*a^3*c^2*d*x^2 + 117*a*b^2*c^3*x^4
- 351*a^2*b*c^2*d*x^4 + 351*a^3*c*d^2*x^4 - 585*b^3*c^3*x^6 + 1755*a*b^2*c^
2*d*x^6 - 1755*a^2*b*c*d^2*x^6 + 585*a^3*d^3*x^6))/(585*a^4*x^(13/2)) + (b^
(1/4)*(-(b*c) + a*d)^3*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqr
t[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(17/4)) + (b^(1/4)*(-(b*c) + a*d)^3*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(
17/4))
```

fricas **[B]** time = 1.81, size = 2512, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/1170*(2340*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2
 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a
^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^
3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4
)*arctan((sqrt((b^20*c^18 - 18*a*b^19*c^17*d + 153*a^2*b^18*c^16*d^2 - 816*
a^3*b^17*c^15*d^3 + 3060*a^4*b^16*c^14*d^4 - 8568*a^5*b^15*c^13*d^5 + 18564
*a^6*b^14*c^12*d^6 - 31824*a^7*b^13*c^11*d^7 + 43758*a^8*b^12*c^10*d^8 - 48
620*a^9*b^11*c^9*d^9 + 43758*a^10*b^10*c^8*d^10 - 31824*a^11*b^9*c^7*d^11 +
 18564*a^12*b^8*c^6*d^12 - 8568*a^13*b^7*c^5*d^13 + 3060*a^14*b^6*c^4*d^14
- 816*a^15*b^5*c^3*d^15 + 153*a^16*b^4*c^2*d^16 - 18*a^17*b^3*c*d^17 + a^18
*b^2*d^18)*x - (a^9*b^13*c^12 - 12*a^10*b^12*c^11*d + 66*a^11*b^11*c^10*d^2
 - 220*a^12*b^10*c^9*d^3 + 495*a^13*b^9*c^8*d^4 - 792*a^14*b^8*c^7*d^5 + 92
4*a^15*b^7*c^6*d^6 - 792*a^16*b^6*c^5*d^7 + 495*a^17*b^5*c^4*d^8 - 220*a^18
*b^4*c^3*d^9 + 66*a^19*b^3*c^2*d^10 - 12*a^20*b^2*c*d^11 + a^21*b*d^12)*sqr
t(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*
d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792
*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*
c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17))*a^4*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4) + (a^4*b^10*c^9 - 9*a^5*b^9*c^8*d + 36
*a^6*b^8*c^7*d^2 - 84*a^7*b^7*c^6*d^3 + 126*a^8*b^6*c^5*d^4 - 126*a^9*b^5*c
^4*d^5 + 84*a^10*b^4*c^3*d^6 - 36*a^11*b^3*c^2*d^7 + 9*a^12*b^2*c*d^8 - a^1
3*b*d^9)*sqrt(x)*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 2
20*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b
^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^
```

```
9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4))/(
b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3
+ 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7
*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*
d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)) - 585*a^4*x^7*(-(b^13*c^12 - 12*a
*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^
8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 4
95*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b
^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(a^13*(-(b^13*c^12 - 12*a*b^12*c^11
*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 79
2*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5
*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11
+ a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2
 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*
b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt(x)) +
585*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^
3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^
6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 6
6*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(-a^
13*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^
9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 7
92*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^
3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*
a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4
 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^7 + 9*a^8*b^
2*c*d^8 - a^9*b*d^9)*sqrt(x)) + 4*(585*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*x^6 - 45*a^3*c^3 - 117*(a*b^2*c^3 - 3*a^2*b*c^2*d + 3*a^3*c
*d^2)*x^4 + 65*(a^2*b*c^3 - 3*a^3*c^2*d)*x^2)*sqrt(x))/(a^4*x^7)
```

giac [B] time = 0.45, size = 536, normalized size = 1.65

$$\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{5}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^{5}b^{2}}-\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{5}b^{2}}+\frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+3\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}-\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^{5}b^{2}}+\frac{2\left(585b^{3}c^{3}x^{6}-1755ab^{2}c^{2}dx^{6}+1755a^{2}bcd^{2}x^{6}-585a^{3}d^{3}x^{6}-117ab^{2}c^{3}x^{4}+351a^{2}bc^{2}dx^{4}-351a^{3}cd^{2}x^{4}+65a^{2}bc^{3}x^{2}-195a^{3}c^{2}dx^{2}-45a^{3}c^{3}\right)}{585a^{4}x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3
)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 1/2*sqrt(2)*((a*b^3)^(3/4)*
b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^
3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(a^5*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*
a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) + 1/4*sqrt(2)*((a*b^3)^(
3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
```

```
(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a
^5*b^2) + 2/585*(585*b^3*c^3*x^6 - 1755*a*b^2*c^2*d*x^6 + 1755*a^2*b*c*d^2*
x^6 - 585*a^3*d^3*x^6 - 117*a*b^2*c^3*x^4 + 351*a^2*b*c^2*d*x^4 - 351*a^3*c
*d^2*x^4 + 65*a^2*b*c^3*x^2 - 195*a^3*c^2*d*x^2 - 45*a^3*c^3)/(a^4*x^(13/2)
)
```

maple [B] time = 0.02, size = 712, normalized size = 2.19

$$-\frac{\sqrt{2}\, d^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, d^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, d^{3} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a}+\frac{3 \sqrt{2}\, b c d^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{3 \sqrt{2}\, b c d^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{3 \sqrt{2}\, b c d^{2} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{3 \sqrt{2}\, b^{2} c^{2} d \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{3 \sqrt{2}\, b^{2} c^{2} d \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}-\frac{3 \sqrt{2}\, b^{2} c^{2} d \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{3}}+\frac{\sqrt{2}\, b^{3} c^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{4}}+\frac{\sqrt{2}\, b^{3} c^{3} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{4}}+\frac{\sqrt{2}\, b^{3} c^{3} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(\frac{a}{b}\right)^{\frac{1}{4}} a^{4}}-\frac{2 d^{3}}{a \sqrt{x}}+\frac{6 b c d^{2}}{a^{2} \sqrt{x}}-\frac{6 b^{2} c^{2} d}{a^{3} \sqrt{x}}+\frac{2 b^{3} c^{3}}{a^{4} \sqrt{x}}-\frac{6 c d^{2}}{5 a x^{\frac{5}{2}}}+\frac{6 b c^{2} d}{5 a^{2} x^{\frac{5}{2}}}-\frac{2 b^{2} c^{3}}{5 a^{3} x^{\frac{5}{2}}}-\frac{2 c^{2} d}{3 a x^{\frac{9}{2}}}+\frac{2 b c^{3}}{9 a^{2} x^{\frac{9}{2}}}-\frac{2 c^{3}}{13 a x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x)
```

```
[Out] -1/2/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/2/a^
2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b*c*d^2-3/2/a^3
/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^2*c^2*d+1/2/a^
4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*b^3*c^3-1/2/a/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/2/a^2/(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b*c*d^2-3/2/a^3/(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^2*c^2*d+1/2/a^4/(a/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*b^3*c^3-1/4/a/(a/b)^(1/
4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3+3/4/a^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))
)*b*c*d^2-3/4/a^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b^2*c^2*d+1/4/a^4/(a
/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(
1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b^3*c^3-2/13*c^3/a/x^(13/2)-2/a/x^(1/2)*
d^3+6/a^2/x^(1/2)*c*d^2*b-6/a^3/x^(1/2)*c^2*d*b^2+2/a^4/x^(1/2)*c^3*b^3-6/5
*c/a/x^(5/2)*d^2+6/5*c^2/a^2/x^(5/2)*b*d-2/5*c^3/a^3/x^(5/2)*b^2-2/3*c^2/a/
x^(9/2)*d+2/9*c^3/a^2/x^(9/2)*b
```

maxima [A] time = 2.42, size = 330, normalized size = 1.02

$$\frac{\left(b^{4} c^{3}-3 a b^{3} c^{2} d+3 a^{2} b^{2} c d^{2}-a^{3} b d^{3}\right) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4 a^{4}}+\frac{2 \left(585 \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) x^{6}-45 a^{3} c^{3}-117 \left(a b^{2} c^{3}-3 a^{2} b c^{2} d+3 a^{3} c d^{2}\right) x^{4}+65 \left(a^{2} b c^{3}-3 a^{3} c^{2} d\right) x^{2}\right)}{585 a^{4} x^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(2*sqrt(2)*arct
an(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
```

```
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4 + 2/585*(585*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^6 - 45*a^3*c^3 - 117*(a*b^2*c
^3 - 3*a^2*b*c^2*d + 3*a^3*c*d^2)*x^4 + 65*(a^2*b*c^3 - 3*a^3*c^2*d)*x^2)/(
a^4*x^(13/2))
```

mupad [B] time = 0.39, size = 639, normalized size = 1.97

$$\frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}\,(a\,d-b\,c)^3\left(16\,a^{19}\,b^4\,d^6-96\,a^{18}\,b^5\,c\,d^5+240\,a^{17}\,b^6\,c^2\,d^4-320\,a^{16}\,b^7\,c^3\,d^3+240\,a^{15}\,b^8\,c^4\,d^2-96\,a^{14}\,b^9\,c^5\,d+16\,a^{13}\,b^{10}\,c^6\right)}{a^{17/4}\left(-16\,a^{18}\,b^4\,d^9+144\,a^{17}\,b^5\,c\,d^8-576\,a^{16}\,b^6\,c^2\,d^7+1344\,a^{15}\,b^7\,c^3\,d^6-2016\,a^{14}\,b^8\,c^4\,d^5+2016\,a^{13}\,b^9\,c^5\,d^4-1344\,a^{12}\,b^{10}\,c^6\,d^3+576\,a^{11}\,b^{11}\,c^7\,d^2-144\,a^{10}\,b^{12}\,c^8\,d+16\,a^9\,b^{13}\,c^9\right)}\right)(a\,d-b\,c)^3}{a^{17/4}}-\frac{\frac{2\,c^3}{13\,a}+\frac{2\,x^6\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{a^4}+\frac{2\,c^2\,x^2\,(3\,a\,d-b\,c)}{9\,a^2}+\frac{2\,c\,x^4\left(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2\right)}{5\,a^3}}{x^{13/2}}-\frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}\,(a\,d-b\,c)^3\left(16\,a^{19}\,b^4\,d^6-96\,a^{18}\,b^5\,c\,d^5+240\,a^{17}\,b^6\,c^2\,d^4-320\,a^{16}\,b^7\,c^3\,d^3+240\,a^{15}\,b^8\,c^4\,d^2-96\,a^{14}\,b^9\,c^5\,d+16\,a^{13}\,b^{10}\,c^6\right)}{a^{17/4}\left(-16\,a^{18}\,b^4\,d^9+144\,a^{17}\,b^5\,c\,d^8-576\,a^{16}\,b^6\,c^2\,d^7+1344\,a^{15}\,b^7\,c^3\,d^6-2016\,a^{14}\,b^8\,c^4\,d^5+2016\,a^{13}\,b^9\,c^5\,d^4-1344\,a^{12}\,b^{10}\,c^6\,d^3+576\,a^{11}\,b^{11}\,c^7\,d^2-144\,a^{10}\,b^{12}\,c^8\,d+16\,a^9\,b^{13}\,c^9\right)}\right)(a\,d-b\,c)^3}{a^{17/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x)
```

```
[Out] ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2)*(a*d - b*c)^3*(16*a^13*b^10*c^6 + 16*a
^19*b^4*d^6 - 96*a^14*b^9*c^5*d - 96*a^18*b^5*c*d^5 + 240*a^15*b^8*c^4*d^2
- 320*a^16*b^7*c^3*d^3 + 240*a^17*b^6*c^2*d^4))/(a^(17/4)*(16*a^9*b^13*c^9
- 16*a^18*b^4*d^9 - 144*a^10*b^12*c^8*d + 144*a^17*b^5*c*d^8 + 576*a^11*b^1
1*c^7*d^2 - 1344*a^12*b^10*c^6*d^3 + 2016*a^13*b^9*c^5*d^4 - 2016*a^14*b^8*
c^4*d^5 + 1344*a^15*b^7*c^3*d^6 - 576*a^16*b^6*c^2*d^7)))*(a*d - b*c)^3)/a^
(17/4) - ((2*c^3)/(13*a) + (2*x^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2))/a^4 + (2*c^2*x^2*(3*a*d - b*c))/(9*a^2) + (2*c*x^4*(3*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d))/(5*a^3))/x^(13/2) - ((-b)^(1/4)*atanh(((-b)^(1/4)*x^
(1/2)*(a*d - b*c)^3*(16*a^13*b^10*c^6 + 16*a^19*b^4*d^6 - 96*a^14*b^9*c^5*d
 - 96*a^18*b^5*c*d^5 + 240*a^15*b^8*c^4*d^2 - 320*a^16*b^7*c^3*d^3 + 240*a^
17*b^6*c^2*d^4))/(a^(17/4)*(16*a^9*b^13*c^9 - 16*a^18*b^4*d^9 - 144*a^10*b^
12*c^8*d + 144*a^17*b^5*c*d^8 + 576*a^11*b^11*c^7*d^2 - 1344*a^12*b^10*c^6*
d^3 + 2016*a^13*b^9*c^5*d^4 - 2016*a^14*b^8*c^4*d^5 + 1344*a^15*b^7*c^3*d^6
 - 576*a^16*b^6*c^2*d^7)))*(a*d - b*c)^3)/a^(17/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(15/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.434 $\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=409

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{21/4}}-\sqrt[4]{a}\,(5bc-1$$

Rubi [A] time = 0.47, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 467, 570, 211, 1165, 628, 1162, 617, 204}

$$\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{10b^4}+\frac{d^2x^{9/2}(39bc-17ad)}{18b^3}+\frac{\sqrt{x}\,(5bc-17ad)(bc-ad)^2}{2b^5}+\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{21/4}}-\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,b^{21/4}}+\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,b^{21/4}}-\frac{\sqrt[4]{a}\,(5bc-17ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,b^{21/4}}-\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{17d^3x^{13/2}}{26b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] ((5*b*c - 17*a*d)*(b*c - a*d)^2*Sqrt[x])/(2*b^5) + (d*(27*b^2*c^2 - 39*a*b*
c*d + 17*a^2*d^2)*x^(5/2))/(10*b^4) + (d^2*(39*b*c - 17*a*d)*x^(9/2))/(18*b
^3) + (17*d^3*x^(13/2))/(26*b^2) - (x^(5/2)*(c + d*x^2)^3)/(2*b*(a + b*x^2)
) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d
)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(21/4)) + (
a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(21/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b
*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*
Sqrt[2]*b^(21/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(c+dx^4\right)^2\left(5c+17dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{x^{5/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\Bigg(\int\Bigg(\frac{(5bc-17ad)(bc-ad)^2}{b^4} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^4}{b^3} + \frac{d^2(39bc-17ad)x^8}{b^2}}{2b}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3}$$

$$= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} + \frac{d\left(27b^2c^2-39abcd+17a^2d^2\right)x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3}$$

Mathematica [C] time = 2.69, size = 419, normalized size = 1.02

$$\frac{35a^2\left(9945a^4\left(64827c^3+194481c^2dx^2+194481cd^2x^4+62651d^3x^6\right)+3978a^3bx^2\left(176389c^3+529167c^2dx^2+541647cd^2x^4+177477d^3x^6\right)+221a^2b^2x^4\left(857691c^3+2417553c^2dx^2+2528145cd^2x^4+846811d^3x^6\right)-9945a\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^3\left(64827c^3+194481c^2dx^2+194481cd^2x^4+62651d^3x^6\right)+a^2bx^2\left(83521c^3+250563c^2dx^2+255555cd^2x^4+83521d^3x^6\right)+ab^2x^4\left(28561c^3+82227c^2dx^2+85683cd^2x^4+28561d^3x^6\right)+b^3x^6\left(2827c^3+6561c^2dx^2+6561cd^2x^4+2187d^3x^6\right)\right)+8704ab^3x^6\left(1609c^3+3423c^2dx^2+3267cd^2x^4+1069d^3x^6\right)-32768b^4x^8\left(c+dx^2\right)^3\right)-98304b^6x^{12}\left(c+dx^2\right)^3\,{}_5F_4\left(2,2,2,2,\frac{13}{4};1,1,1,\frac{29}{4};-\frac{bx^2}{a}\right)}{89107200a^3b^5x^{11/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (35*a^2*(-32768*b^4*x^8*(c + d*x^2)^3 + 8704*a*b^3*x^6*(1609*c^3 + 3423*c^2
*d*x^2 + 3267*c*d^2*x^4 + 1069*d^3*x^6) + 9945*a^4*(64827*c^3 + 194481*c^2*
d*x^2 + 194481*c*d^2*x^4 + 62651*d^3*x^6) + 3978*a^3*b*x^2*(176389*c^3 + 52
9167*c^2*d*x^2 + 541647*c*d^2*x^4 + 177477*d^3*x^6) + 221*a^2*b^2*x^4*(8576
91*c^3 + 2417553*c^2*d*x^2 + 2528145*c*d^2*x^4 + 846811*d^3*x^6) - 9945*a*(
b^3*x^6*(2827*c^3 + 6561*c^2*d*x^2 + 6561*c*d^2*x^4 + 2187*d^3*x^6) + a*b^2
*x^4*(28561*c^3 + 82227*c^2*d*x^2 + 85683*c*d^2*x^4 + 28561*d^3*x^6) + a^3*
(64827*c^3 + 194481*c^2*d*x^2 + 194481*c*d^2*x^4 + 62651*d^3*x^6) + a^2*b*x
^2*(83521*c^3 + 250563*c^2*d*x^2 + 255555*c*d^2*x^4 + 83521*d^3*x^6))*Hyper
geometric2F1[1/4, 1, 5/4, -((b*x^2)/a)]) - 98304*b^6*x^12*(c + d*x^2)^3*Hyp
ergeometricPFQ[{2, 2, 2, 2, 13/4}, {1, 1, 1, 29/4}, -((b*x^2)/a)])/(8910720
0*a^3*b^5*x^(11/2))
```

IntegrateAlgebraic [A] time = 0.58, size = 344, normalized size = 0.84

$$-\frac{\left(17a^{5/4}d-5\sqrt[4]{a}\,bc\right)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,b^{21/4}}+\frac{\left(17a^{5/4}d-5\sqrt[4]{a}\,bc\right)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,b^{21/4}}+\frac{\sqrt{x}\left(-9945a^4d^3+22815a^3bcd^2-7956a^3bd^3x^2-15795a^2b^2c^2d+18252a^2b^2cd^2x^2+884a^2b^2d^3x^4+2925ab^3c^3-12636ab^3c^2dx^2-2028ab^3cd^2x^4-340ab^3d^3x^6+2340b^4c^3x^2+1404b^4c^2dx^4+780b^4cd^2x^6+180b^4d^3x^8\right)}{1170b^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[x]*(2925*a*b^3*c^3 - 15795*a^2*b^2*c^2*d + 22815*a^3*b*c*d^2 - 9945*a
^4*d^3 + 2340*b^4*c^3*x^2 - 12636*a*b^3*c^2*d*x^2 + 18252*a^2*b^2*c*d^2*x^2
 - 7956*a^3*b*d^3*x^2 + 1404*b^4*c^2*d*x^4 - 2028*a*b^3*c*d^2*x^4 + 884*a^2
*b^2*d^3*x^4 + 780*b^4*c*d^2*x^6 - 340*a*b^3*d^3*x^6 + 180*b^4*d^3*x^8))/(1
170*b^5*(a + b*x^2)) - ((-(b*c) + a*d)^2*(-5*a^(1/4)*b*c + 17*a^(5/4)*d)*Ar
cTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*b
^(21/4)) + ((-(b*c) + a*d)^2*(-5*a^(1/4)*b*c + 17*a^(5/4)*d)*ArcTanh[(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*b^(21/4))
```

fricas [B] time = 1.41, size = 2014, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4680*(2340*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d +
 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^
```

```
4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c
^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11
*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*arctan(
(sqrt(b^10*sqrt(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10
*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*
b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 1263145
5*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 -
766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21) + (25*b^6*c^6 - 270*a*b^5*c^5
*d + 1119*a^2*b^4*c^4*d^2 - 2276*a^3*b^3*c^3*d^3 + 2439*a^4*b^2*c^2*d^4 - 1
326*a^5*b*c*d^5 + 289*a^6*d^6)*x)*b^16*(-(625*a*b^12*c^12 - 13500*a^2*b^11*
c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^
8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a
^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168
018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(3/4)
 + (5*b^19*c^3 - 27*a*b^18*c^2*d + 39*a^2*b^17*c*d^2 - 17*a^3*b^16*d^3)*sqr
t(x)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2
- 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^
5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*
c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^1
2*b*c*d^11 + 83521*a^13*d^12)/b^21)^(3/4))/(625*a*b^12*c^12 - 13500*a^2*b^1
1*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*
b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976
*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 31
68018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)) + 585*(b
^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^1
0*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6
*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 126314
55*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 -
 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*log(b^5*(-(625*a*b^12*
c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^
9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^
6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a
^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*
a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*
a^3*d^3)*sqrt(x)) - 585*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^
11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5
*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 1422597
6*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3
168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1
/4)*log(-b^5*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c
^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^
7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*
a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 76
6428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c
^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*sqrt(x)) + 4*(180*b^4*d^3*x^8 + 2925*a*
```

```
b^3*c^3 - 15795*a^2*b^2*c^2*d + 22815*a^3*b*c*d^2 - 9945*a^4*d^3 + 20*(39*b
^4*c*d^2 - 17*a*b^3*d^3)*x^6 + 52*(27*b^4*c^2*d - 39*a*b^3*c*d^2 + 17*a^2*b
^2*d^3)*x^4 + 468*(5*b^4*c^3 - 27*a*b^3*c^2*d + 39*a^2*b^2*c*d^2 - 17*a^3*b
*d^3)*x^2)*sqrt(x))/(b^6*x^2 + a*b^5)
```

giac [**A**] time = 0.49, size = 600, normalized size = 1.47

$$-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}b^3c^3-27\,(ab^3)^{\frac{1}{4}}ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^6}-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}b^3c^3-27\,(ab^3)^{\frac{1}{4}}ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^6}-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}b^3c^3-27\,(ab^3)^{\frac{1}{4}}ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^6}+\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}b^3c^3-27\,(ab^3)^{\frac{1}{4}}ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^6}+\frac{ab^3c^3\sqrt{x}-3\,a^2b^2c^2d\sqrt{x}+3\,a^3bcd^2\sqrt{x}-a^4d^3\sqrt{x}}{2\,(bx^2+a)b^5}+\frac{2\left(45\,b^{24}d^3x^{\frac{13}{2}}+195\,b^{24}cd^2x^{\frac{9}{2}}-130\,ab^{23}d^3x^{\frac{9}{2}}+351\,b^{24}c^2dx^{\frac{5}{2}}-702\,ab^{23}cd^2x^{\frac{5}{2}}+351\,a^2b^{22}d^3x^{\frac{5}{2}}+585\,b^{24}c^3\sqrt{x}-3510\,ab^{23}c^2d\sqrt{x}+5265\,a^2b^{22}cd^2\sqrt{x}-2340\,a^3b^{21}d^3\sqrt{x}\right)}{585\,b^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(
a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^6 - 1/8*sqrt(2)*(5*(a*b^3)^(1
/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 -
 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/b^6 - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)
^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^
3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^6 + 1/16*sqrt(2)*(5*(
a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*
b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/b^6 + 1/2*(a*b^3*c^3*sqrt(x) - 3*a^2*b^2*c^2*d*sqrt(x) + 3*a^3*b
*c*d^2*sqrt(x) - a^4*d^3*sqrt(x))/((b*x^2 + a)*b^5) + 2/585*(45*b^24*d^3*x^
(13/2) + 195*b^24*c*d^2*x^(9/2) - 130*a*b^23*d^3*x^(9/2) + 351*b^24*c^2*d*x
^(5/2) - 702*a*b^23*c*d^2*x^(5/2) + 351*a^2*b^22*d^3*x^(5/2) + 585*b^24*c^3
*sqrt(x) - 3510*a*b^23*c^2*d*sqrt(x) + 5265*a^2*b^22*c*d^2*sqrt(x) - 2340*a
^3*b^21*d^3*sqrt(x))/b^26
```

maple [**B**] time = 0.02, size = 804, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] -39/8*a^2/b^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d
^2+27/8*a/b^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2
*d-39/8*a^2/b^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c
*d^2+27/16*a/b^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d+27/8*a/b^3*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-39/16*a^2/b^4*(
a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2+2/13*d^3*x^(13/2)/b^2+6/5/b^2*x^(
5/2)*c^2*d-8/b^5*a^3*d^3*x^(1/2)+6/5/b^4*x^(5/2)*a^2*d^3+3/2*a^3/b^4*x^(1/2
```

```
)/(b*x^2+a)*c*d^2-3/2*a^2/b^3*x^(1/2)/(b*x^2+a)*c^2*d+17/8*a^3/b^5*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+17/8*a^3/b^5*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+17/16*a^3/b^5*(a/b)^(
1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*
2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3-4/9/b^3*x^(9/2)*a*d^3+2/3/b^2*x^(9/2)*c*d
^2-12/b^3*a*c^2*d*x^(1/2)-1/2*a^4/b^5*x^(1/2)/(b*x^2+a)*d^3+1/2*a/b^2*x^(1/
2)/(b*x^2+a)*c^3-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)*c^3-5/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1)*c^3-5/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)
^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^3+18/b^4*a^2*c*d^2*x
^(1/2)-12/5/b^3*x^(5/2)*a*c*d^2+2/b^2*c^3*x^(1/2)
```

maxima [A]   time = 2.46, size = 499, normalized size = 1.22

$$\frac{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{x}}{2\left(b^6x^2+ab^5\right)}-\frac{\left(\frac{2\sqrt{2}\left(5\,b^3c^3-27\,ab^2c^2d+39\,a^2bcd^2-17\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\left(5\,b^3c^3-27\,ab^2c^2d+39\,a^2bcd^2-17\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(5\,b^3c^3-27\,ab^2c^2d+39\,a^2bcd^2-17\,a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(5\,b^3c^3-27\,ab^2c^2d+39\,a^2bcd^2-17\,a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)a}{16\,b^5}+\frac{2\left(45\,b^3d^3x^{\frac{13}{2}}+65\left(3\,b^3cd^2-2\,ab^2d^3\right)x^{\frac{9}{2}}+351\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^{\frac{5}{2}}+585\left(b^3c^3-6\,ab^2c^2d+9\,a^2bcd^2-4\,a^3d^3\right)\sqrt{x}\right)}{585\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(x)/(b^6*x^
2 + a*b^5) - 1/16*(2*sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 -
 17*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(5*b^
3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*arctan(-1/2*sqrt(2)*(
sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c
*d^2 - 17*a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2
 - 17*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(3/4)*b^(1/4)))*a/b^5 + 2/585*(45*b^3*d^3*x^(13/2) + 65*(3*b^3*c*d^2 - 2
*a*b^2*d^3)*x^(9/2) + 351*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^(5/2) +
 585*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*sqrt(x))/b^5
```

mupad [B]   time = 0.30, size = 1850, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^(1/2)*((2*c^3)/b^2 - (2*a*((6*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2
)/b^2))/b - (2*a^2*d^3)/b^4))/b + (a^2*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/b^2
) - x^(9/2)*((4*a*d^3)/(9*b^3) - (2*c*d^2)/(3*b^2)) + x^(5/2)*((6*c^2*d)/(5
*b^2) + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/(5*b) - (2*a^2*d^3)/(5*b^4))
- (x^(1/2)*((a^4*d^3)/2 - (a*b^3*c^3)/2 + (3*a^2*b^2*c^2*d)/2 - (3*a^3*b*c*
```

```
d^2)/2))/(a*b^5 + b^6*x^2) + (2*d^3*x^(13/2))/(13*b^2) - ((-a)^(1/4)*atan((
((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*
b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 +
 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d - b*c)^2*
(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*
c*d^2))/b^(29/4))*1i)/(8*b^(21/4)) + ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*
b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4
*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d
^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b
^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2))/b^(29/4))*1i)/(8*b^(21/4)))/((
(-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b
^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 +
2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d - b*c)^2*(
17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c
*d^2))/b^(29/4)))/(8*b^(21/4)) - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)
*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4
*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))
/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c
^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2))/b^(29/4)))/(8*b^(21/4))))*(a*d - b
*c)^2*(17*a*d - 5*b*c)*1i)/(4*b^(21/4)) + ((-a)^(1/4)*atan((((-a)^(1/4)*(a*
d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a
^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c
^2*d^4 - 1326*a^7*b*c*d^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c
)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(2
9/4)))/(8*b^(21/4)) + ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*
(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 -
2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a
)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3
*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29/4)))/(8*b^(21/4)))/((((-a)^(1/4)*(a*d
 - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^
3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^
2*d^4 - 1326*a^7*b*c*d^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)
*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29
/4))*1i)/(8*b^(21/4)) - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2
)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2
- 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((
-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a
^3*b^2*c^2*d - 39*a^4*b*c*d^2)*1i)/b^(29/4))*1i)/(8*b^(21/4))))*(a*d - b*c)
^2*(17*a*d - 5*b*c))/(4*b^(21/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.435 $\int \frac{x^{5/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=374

$$\frac{dx^{3/2}\left(5a^2d^2-11abcd+7b^2c^2\right)}{2b^4}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{19/4}}-3(bc-5ad)(bc-ad)^2$$

Rubi [A] time = 0.41, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 467, 570, 297, 1162, 617, 204, 1165, 628}

$$\frac{dx^{3/2}\left(5a^2d^2-11abcd+7b^2c^2\right)}{2b^4}+\frac{3d^2x^{7/2}(11bc-5ad)}{14b^3}+\frac{3(bc-5ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}\,b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{19/4}}+\frac{3(bc-5ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}\,b^{19/4}}-\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{15d^3x^{11/2}}{22b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(7*b^2*c^2 - 11*a*b*c*d + 5*a^2*d^2)*x^(3/2))/(2*b^4) + (3*d^2*(11*b*c -
 5*a*d)*x^(7/2))/(14*b^3) + (15*d^3*x^(11/2))/(22*b^2) - (x^(3/2)*(c + d*x^
2)^3)/(2*b*(a + b*x^2)) - (3*(b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d
)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a
^(1/4)*b^(19/4)) + (3*(b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4)) - (3*(b*c -
 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(1/4)*b^(19/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(c+dx^4\right)^2\left(3c+15dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= -\frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\Big(\int \Big(\frac{3d\left(7b^2c^2-11abcd+5a^2d^2\right)x^2}{b^3} + \frac{3d^2(11bc-5ad)x^6}{b^2} + \frac{15d^3x^{10}}{b} + \frac{3\big(b^3c^3-7ab^2}{}}{2b}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{(3}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} - \frac{(3}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{(3}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{3(}{}$$

$$= \frac{d\left(7b^2c^2-11abcd+5a^2d^2\right)x^{3/2}}{2b^4} + \frac{3d^2(11bc-5ad)x^{7/2}}{14b^3} + \frac{15d^3x^{11/2}}{22b^2} - \frac{x^{3/2}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} - \frac{3(}{}$$

Mathematica [C] time = 2.22, size = 377, normalized size = 1.01

$$\frac{-385a^3\left(130321c^3+390963c^2dx^2+390963cd^2x^4+124561d^3x^6\right)-330a^2bx^2\left(112027c^3+336081c^2dx^2+350865cd^2x^4+114907d^3x^6\right)+385\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\left(a^3\left(130321c^3+390963c^2dx^2+390963cd^2x^4+124561d^3x^6\right)+9a^2bx^2\left(16875c^3+50625c^2dx^2+52033cd^2x^4+16875d^3x^6\right)+3ab^2x^4\left(14641c^3+41235c^2dx^2+43923cd^2x^4+14641d^3x^6\right)+b^3x^6\left(3553c^3+7203c^2dx^2+7203cd^2x^4+2401d^3x^6\right)\right)-45ab^2x^4\left(122993c^3+299987c^2dx^2+322515cd^2x^4+109553d^3x^6\right)-32768b^3x^6\left(c+dx^2\right)^3}{887040ab^4x^{9/2}}-\frac{128bx^{11/2}\left(c+dx^2\right)^3\,{}_5F_4\left(2,2,2,2,\frac{11}{4};1,1,1,\frac{27}{4};-\frac{bx^2}{a}\right)}{72105a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (-32768*b^3*x^6*(c + d*x^2)^3 - 45*a*b^2*x^4*(122993*c^3 + 299987*c^2*d*x^2
 + 322515*c*d^2*x^4 + 109553*d^3*x^6) - 330*a^2*b*x^2*(112027*c^3 + 336081*
c^2*d*x^2 + 350865*c*d^2*x^4 + 114907*d^3*x^6) - 385*a^3*(130321*c^3 + 3909
63*c^2*d*x^2 + 390963*c*d^2*x^4 + 124561*d^3*x^6) + 385*(b^3*x^6*(3553*c^3
+ 7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6) + 3*a*b^2*x^4*(14641*c^3
+ 41235*c^2*d*x^2 + 43923*c*d^2*x^4 + 14641*d^3*x^6) + 9*a^2*b*x^2*(16875*c
^3 + 50625*c^2*d*x^2 + 52033*c*d^2*x^4 + 16875*d^3*x^6) + a^3*(130321*c^3 +
 390963*c^2*d*x^2 + 390963*c*d^2*x^4 + 124561*d^3*x^6))*Hypergeometric2F1[3
/4, 1, 7/4, -((b*x^2)/a)])/(887040*a*b^4*x^(9/2)) - (128*b*x^(11/2)*(c + d*
x^2)^3*HypergeometricPFQ[{2, 2, 2, 2, 11/4}, {1, 1, 1, 27/4}, -((b*x^2)/a)]
)/(72105*a^3)
```

IntegrateAlgebraic [A] time = 0.56, size = 280, normalized size = 0.75

$$\frac{x^{3/2}\left(385a^3d^3-847a^2bcd^2+220a^2bd^3x^2+539ab^2c^2d-484ab^2cd^2x^2-60ab^2d^3x^4-77b^3c^3+308b^3c^2dx^2+132b^3cd^2x^4+28b^3d^3x^6\right)}{154b^4\left(a+bx^2\right)}+\frac{3(5ad-bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{19/4}}+\frac{3(5ad-bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,b^{19/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (x^(3/2)*(-77*b^3*c^3 + 539*a*b^2*c^2*d - 847*a^2*b*c*d^2 + 385*a^3*d^3 + 3
08*b^3*c^2*d*x^2 - 484*a*b^2*c*d^2*x^2 + 220*a^2*b*d^3*x^2 + 132*b^3*c*d^2*
x^4 - 60*a*b^2*d^3*x^4 + 28*b^3*d^3*x^6))/(154*b^4*(a + b*x^2)) + (3*(-(b*c
) + a*d)^2*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(1/4)*b^(19/4)) + (3*(-(b*c) + a*d)^2*(-(b*c
) + 5*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/(4*Sqrt[2]*a^(1/4)*b^(19/4))
```

fricas [B] time = 1.73, size = 2542, normalized size = 6.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/616*(924*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10
*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^
7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d
^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 +
 625*a^12*d^12)/(a*b^19))^(1/4)*arctan((sqrt((b^18*c^18 - 42*a*b^17*c^17*d
+ 801*a^2*b^16*c^16*d^2 - 9200*a^3*b^15*c^15*d^3 + 71220*a^4*b^14*c^14*d^4
- 394392*a^5*b^13*c^13*d^5 + 1619684*a^6*b^12*c^12*d^6 - 5050512*a^7*b^11*c
^11*d^7 + 12147630*a^8*b^10*c^10*d^8 - 22765820*a^9*b^9*c^9*d^9 + 33419166*
```

```
a^10*b^8*c^8*d^10 - 38446992*a^11*b^7*c^7*d^11 + 34503236*a^12*b^6*c^6*d^12
 - 23888280*a^13*b^5*c^5*d^13 + 12508500*a^14*b^4*c^4*d^14 - 4790000*a^15*b
^3*c^3*d^15 + 1265625*a^16*b^2*c^2*d^16 - 206250*a^17*b*c*d^17 + 15625*a^18
*d^18)*x - (a*b^21*c^12 - 28*a^2*b^20*c^11*d + 338*a^3*b^19*c^10*d^2 - 2316
*a^4*b^18*c^9*d^3 + 10015*a^5*b^17*c^8*d^4 - 28856*a^6*b^16*c^7*d^5 + 57148
*a^7*b^15*c^6*d^6 - 78968*a^8*b^14*c^5*d^7 + 76111*a^9*b^13*c^4*d^8 - 50220
*a^10*b^12*c^3*d^9 + 21650*a^11*b^11*c^2*d^10 - 5500*a^12*b^10*c*d^11 + 625
*a^13*b^9*d^12)*sqrt(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2
 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 5
7148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 5022
0*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12
*d^12)/(a*b^19)))*b^5*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d
^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 +
 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50
220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^
12*d^12)/(a*b^19))^(1/4) + (b^14*c^9 - 21*a*b^13*c^8*d + 180*a^2*b^12*c^7*d
^2 - 820*a^3*b^11*c^6*d^3 + 2190*a^4*b^10*c^5*d^4 - 3606*a^5*b^9*c^4*d^5 +
3716*a^6*b^8*c^3*d^6 - 2340*a^7*b^7*c^2*d^7 + 825*a^8*b^6*c*d^8 - 125*a^9*b
^5*d^9)*sqrt(x)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2
316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148
*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^
9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^1
2)/(a*b^19))^(1/4))/(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 -
 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 571
48*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*
a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d
^12)) - 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^1
0*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c
^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*
d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11
+ 625*a^12*d^12)/(a*b^19))^(1/4)*log(27*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^1
1*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4
- 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 7
6111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 55
00*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3/4) - 27*(b^9*c^9 - 21*a*b^8*
c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*d^3 + 2190*a^4*b^5*c^5*d^4 -
3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d^7 + 825*a^
8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28
*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b
^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*
c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^
2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(1/4)*log(-27*a*b^14
*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9
*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^
6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 +
```

```
 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3
/4) - 27*(b^9*c^9 - 21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*
d^3 + 2190*a^4*b^5*c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 -
2340*a^7*b^2*c^2*d^7 + 825*a^8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 4*(28*b^3*
d^3*x^7 + 12*(11*b^3*c*d^2 - 5*a*b^2*d^3)*x^5 + 44*(7*b^3*c^2*d - 11*a*b^2*
c*d^2 + 5*a^2*b*d^3)*x^3 - 77*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5
*a^3*d^3)*x)*sqrt(x))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.60, size = 552, normalized size = 1.48

$$-\frac{b^{3}c^{3}x^{\frac{3}{2}} - 3\,ab^{2}c^{2}dx^{\frac{3}{2}} + 3\,a^{2}bcd^{2}x^{\frac{3}{2}} - a^{3}d^{3}x^{\frac{3}{2}}}{2\left(bx^{2}+a\right)b^{4}} + \frac{3\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} - 7\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} - 5\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^{7}} + \frac{3\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} - 7\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} - 5\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^{7}} - \frac{3\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} - 7\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} - 5\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^{7}} + \frac{3\,\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} - 7\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} - 5\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^{7}} + \frac{2\left(7\,b^{20}d^{3}x^{\frac{11}{2}} + 33\,b^{20}cd^{2}x^{\frac{7}{2}} - 22\,ab^{19}d^{3}x^{\frac{7}{2}} + 77\,b^{20}c^{2}dx^{\frac{3}{2}} - 154\,ab^{19}cd^{2}x^{\frac{3}{2}} + 77\,a^{2}b^{18}d^{3}x^{\frac{3}{2}}\right)}{77\,b^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3
*d^3*x^(3/2))/((b*x^2 + a)*b^4) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a
*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^
3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
*b^7) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d +
11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^7) - 3/16*sqrt(2)*((a*
b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c
*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a*b^7) + 3/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2
*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^7) + 2/77*(7*b^20*d^3*x^(11/2)
 + 33*b^20*c*d^2*x^(7/2) - 22*a*b^19*d^3*x^(7/2) + 77*b^20*c^2*d*x^(3/2) -
154*a*b^19*c*d^2*x^(3/2) + 77*a^2*b^18*d^3*x^(3/2))/b^22
```

maple [B] time = 0.02, size = 748, normalized size = 2.00

$$\frac{2d^{3}x^{\frac{11}{2}}}{11b^{2}} - \frac{4a\,d^{3}x^{\frac{7}{2}}}{7b^{3}} + \frac{6c\,d^{2}x^{\frac{7}{2}}}{7b^{2}} + \frac{a^{3}d^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b^{4}} - \frac{3a^{2}c\,d^{2}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b^{3}} + \frac{3a\,c^{2}d\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b^{2}} - \frac{c^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b} + \frac{2a^{2}d^{3}x^{\frac{3}{2}}}{b^{4}} - \frac{4ac\,d^{2}x^{\frac{3}{2}}}{b^{3}} + \frac{2c^{2}d\,x^{\frac{3}{2}}}{b^{2}} - \frac{15\sqrt{2}\,a^{3}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{5}} - \frac{15\sqrt{2}\,a^{3}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{5}} - \frac{15\sqrt{2}\,a^{3}d^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{5}} + \frac{33\sqrt{2}\,a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}} + \frac{33\sqrt{2}\,a^{2}c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}} + \frac{33\sqrt{2}\,a^{2}c\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}} - \frac{21\sqrt{2}\,a\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}} - \frac{21\sqrt{2}\,a\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}} - \frac{21\sqrt{2}\,a\,c^{2}d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}} + \frac{3\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}} + \frac{3\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}} + \frac{3\sqrt{2}\,c^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 2/11*d^3*x^(11/2)/b^2-4/7*d^3/b^3*x^(7/2)*a+6/7*d^2/b^2*x^(7/2)*c+2*d^3/b^4
*x^(3/2)*a^2-4*d^2/b^3*x^(3/2)*a*c+2*d/b^2*x^(3/2)*c^2+1/2/b^4*x^(3/2)/(b*x
^2+a)*a^3*d^3-3/2/b^3*x^(3/2)/(b*x^2+a)*a^2*c*d^2+3/2/b^2*x^(3/2)/(b*x^2+a)
*a*c^2*d-1/2/b*x^(3/2)/(b*x^2+a)*c^3-15/16/b^5/(a/b)^(1/4)*2^(1/2)*a^3*d^3*
ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2)))-15/8/b^5/(a/b)^(1/4)*2^(1/2)*a^3*d^3*arctan(2^(1/2)/(a/b)^
(1/4)*x^(1/2)+1)-15/8/b^5/(a/b)^(1/4)*2^(1/2)*a^3*d^3*arctan(2^(1/2)/(a/b)^
```

```
(1/4)*x^(1/2)-1)+33/16/b^4/(a/b)^(1/4)*2^(1/2)*a^2*c*d^2*ln((x-(a/b)^(1/4)*
2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3
3/8/b^4/(a/b)^(1/4)*2^(1/2)*a^2*c*d^2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
+33/8/b^4/(a/b)^(1/4)*2^(1/2)*a^2*c*d^2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1)-21/16/b^3/(a/b)^(1/4)*2^(1/2)*a*c^2*d*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-21/8/b^3/(a/b)^(1
/4)*2^(1/2)*a*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-21/8/b^3/(a/b)^(1
/4)*2^(1/2)*a*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/b^2/(a/b)^(1
/4)*2^(1/2)*c^3*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/
4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8/b^2/(a/b)^(1/4)*2^(1/2)*c^3*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8/b^2/(a/b)^(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.36, size = 337, normalized size = 0.90

$$-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^{\frac{3}{2}}}{2\left(b^5x^2+ab^4\right)}+\frac{3\left(b^3c^3-7\,ab^2c^2d+11\,a^2bcd^2-5\,a^3d^3\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,b^4}+\frac{2\left(7\,b^2d^3x^{\frac{11}{2}}+11\left(3\,b^2cd^2-2\,abd^3\right)x^{\frac{7}{2}}+77\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^{\frac{3}{2}}\right)}{77\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^(3/2)/(b^5*x^2 +
 a*b^4) + 3/16*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5*a^3*d^3)*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 + 2/77*(7*b
^2*d^3*x^(11/2) + 11*(3*b^2*c*d^2 - 2*a*b*d^3)*x^(7/2) + 77*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x^(3/2))/b^4
```

mupad [B] time = 0.45, size = 681, normalized size = 1.82

$$x^{3/2}\left(\frac{2\,c^2\,d}{b^2}+\frac{2\,a\left(\frac{4\,a\,d^3}{b^3}-\frac{6\,c\,d^2}{b^2}\right)}{3\,b}-\frac{2\,a^2\,d^3}{3\,b^4}\right)-x^{7/2}\left(\frac{4\,a\,d^3}{7\,b^3}-\frac{6\,c\,d^2}{7\,b^2}\right)+\frac{2\,d^3\,x^{11/2}}{11\,b^2}+\frac{x^{3/2}\left(\frac{a^3\,d^3}{2}-\frac{3\,a^2\,b\,c\,d^2}{2}+\frac{3\,a\,b^2\,c^2\,d}{2}-\frac{b^3\,c^3}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)\,(25\,a^7\,d^6-110\,a^6\,b\,c\,d^5+191\,a^5\,b^2\,c^2\,d^4-164\,a^4\,b^3\,c^3\,d^3+71\,a^3\,b^4\,c^4\,d^2-14\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6)}{(-a)^{1/4}\,(125\,a^{10}\,d^9-825\,a^9\,b\,c\,d^8+2340\,a^8\,b^2\,c^2\,d^7-3716\,a^7\,b^3\,c^3\,d^6+3606\,a^6\,b^4\,c^4\,d^5-2190\,a^5\,b^5\,c^5\,d^4+820\,a^4\,b^6\,c^6\,d^3-180\,a^3\,b^7\,c^7\,d^2+21\,a^2\,b^8\,c^8\,d-a\,b^9\,c^9)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)}{4\,(-a)^{1/4}\,b^{19/4}}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)\,(25\,a^7\,d^6-110\,a^6\,b\,c\,d^5+191\,a^5\,b^2\,c^2\,d^4-164\,a^4\,b^3\,c^3\,d^3+71\,a^3\,b^4\,c^4\,d^2-14\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6)\,1{}\mathrm{i}}{(-a)^{1/4}\,(125\,a^{10}\,d^9-825\,a^9\,b\,c\,d^8+2340\,a^8\,b^2\,c^2\,d^7-3716\,a^7\,b^3\,c^3\,d^6+3606\,a^6\,b^4\,c^4\,d^5-2190\,a^5\,b^5\,c^5\,d^4+820\,a^4\,b^6\,c^6\,d^3-180\,a^3\,b^7\,c^7\,d^2+21\,a^2\,b^8\,c^8\,d-a\,b^9\,c^9)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d-b\,c)\,3{}\mathrm{i}}{4\,(-a)^{1/4}\,b^{19/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^(3/2)*((2*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/(3*b) - (2*a
^2*d^3)/(3*b^4)) - x^(7/2)*((4*a*d^3)/(7*b^3) - (6*c*d^2)/(7*b^2)) + (2*d^3
*x^(11/2))/(11*b^2) + (x^(3/2)*((a^3*d^3)/2 - (b^3*c^3)/2 + (3*a*b^2*c^2*d)
/2 - (3*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) - (3*atan((b^(1/4)*x^(1/2)*(a*d
- b*c)^2*(5*a*d - b*c)*(25*a^7*d^6 + a*b^6*c^6 - 14*a^2*b^5*c^5*d + 71*a^3*
```

```
b^4*c^4*d^2 - 164*a^4*b^3*c^3*d^3 + 191*a^5*b^2*c^2*d^4 - 110*a^6*b*c*d^5))
/((-a)^(1/4)*(125*a^10*d^9 - a*b^9*c^9 + 21*a^2*b^8*c^8*d - 180*a^3*b^7*c^7
*d^2 + 820*a^4*b^6*c^6*d^3 - 2190*a^5*b^5*c^5*d^4 + 3606*a^6*b^4*c^4*d^5 -
3716*a^7*b^3*c^3*d^6 + 2340*a^8*b^2*c^2*d^7 - 825*a^9*b*c*d^8)))*(a*d - b*c
)^2*(5*a*d - b*c))/(4*(-a)^(1/4)*b^(19/4)) - (atan((b^(1/4)*x^(1/2)*(a*d -
b*c)^2*(5*a*d - b*c)*(25*a^7*d^6 + a*b^6*c^6 - 14*a^2*b^5*c^5*d + 71*a^3*b^
4*c^4*d^2 - 164*a^4*b^3*c^3*d^3 + 191*a^5*b^2*c^2*d^4 - 110*a^6*b*c*d^5)*1i
)/((-a)^(1/4)*(125*a^10*d^9 - a*b^9*c^9 + 21*a^2*b^8*c^8*d - 180*a^3*b^7*c^
7*d^2 + 820*a^4*b^6*c^6*d^3 - 2190*a^5*b^5*c^5*d^4 + 3606*a^6*b^4*c^4*d^5 -
 3716*a^7*b^3*c^3*d^6 + 2340*a^8*b^2*c^2*d^7 - 825*a^9*b*c*d^8)))*(a*d - b*
c)^2*(5*a*d - b*c)*3i)/(4*(-a)^(1/4)*b^(19/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.436 $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=386

$$-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{8\sqrt{2}a^{3/4}b^{17/4}}$$

Rubi [A] time = 0.53, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 467, 528, 388, 211, 1165, 628, 1162, 617, 204}

$$\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{90b^4}-\frac{(bc-13ad)(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}-\frac{(bc-13ad)(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{(bc-13ad)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{d\sqrt{x}\left(c+dx^2\right)(113bc-117ad)}{90b^3}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (d*(497*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/(90*b^4) + (d*(113*b
*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(90*b^3) + (13*d*Sqrt[x]*(c + d*x^2)^2)/
(18*b^2) - (Sqrt[x]*(c + d*x^2)^3)/(2*b*(a + b*x^2)) - ((b*c - 13*a*d)*(b*c
 - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)
*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - 13*a*d)*(b*c - a*d)^2
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(
3/4)*b^(17/4)) + ((b*c - 13*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)^2\left(c+13dx^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2b}$$

$$= \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(c(9bc-13ad)+d(113bc-117ad)x^4\right)}{a+bx^4}\,dx, x, \right.}{18b^2}$$

$$= \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+dx^2\right)^2}{18b^2} - \frac{\sqrt{x}\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{c\left(45b^2\right.}{}\right.}{}$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+\right.}{18b^2}$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+\right.}{18b^2}$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+\right.}{18b^2}$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+\right.}{18b^2}$$

$$= \frac{d\left(497b^2c^2-1098abcd+585a^2d^2\right)\sqrt{x}}{90b^4} + \frac{d(113bc-117ad)\sqrt{x}\left(c+dx^2\right)}{90b^3} + \frac{13d\sqrt{x}\left(c+\right.}{18b^2}$$

Mathematica [C] time = 2.69, size = 377, normalized size = 0.98

$$\frac{-585a^3\left(83521c^3+250563c^2dx^2+250563cd^2x^4+78529d^3x^6\right)-234a^2bx^2\left(172447c^3+517341c^2dx^2+543261cd^2x^4+174943d^3x^6\right)+585\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^3\left(83521c^3+250563c^2dx^2+250563cd^2x^4+78529d^3x^6\right)+3a^2bx^2\left(28561c^3+85683c^2dx^2+89139cd^2x^4+28561d^3x^6\right)+9ab^2x^4\left(2187c^3+5921c^2dx^2+6561cd^2x^4+2187d^3x^6\right)+b^3x^6\left(1009c^3+1875c^2dx^2+1875cd^2x^4+625d^3x^6\right)\right)-13ab^2x^4\left(532193c^3+1337379c^2dx^2+1503267cd^2x^4+507233d^3x^6\right)-98304b^3x^6\left(c+dx^2\right)^3}{449280ab^4x^{11/2}}-\frac{128bx^{9/2}\left(c+dx^2\right)^3\,{}_5F_4\left(2,2,2,2,\frac{9}{4};1,1,1,\frac{25}{4};-\frac{bx^2}{a}\right)}{41769a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (-98304*b^3*x^6*(c + d*x^2)^3 - 585*a^3*(83521*c^3 + 250563*c^2*d*x^2 + 250
563*c*d^2*x^4 + 78529*d^3*x^6) - 234*a^2*b*x^2*(172447*c^3 + 517341*c^2*d*x
^2 + 543261*c*d^2*x^4 + 174943*d^3*x^6) - 13*a*b^2*x^4*(532193*c^3 + 133737
9*c^2*d*x^2 + 1503267*c*d^2*x^4 + 507233*d^3*x^6) + 585*(b^3*x^6*(1009*c^3
+ 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6) + 9*a*b^2*x^4*(2187*c^3 +
5921*c^2*d*x^2 + 6561*c*d^2*x^4 + 2187*d^3*x^6) + 3*a^2*b*x^2*(28561*c^3 +
85683*c^2*d*x^2 + 89139*c*d^2*x^4 + 28561*d^3*x^6) + a^3*(83521*c^3 + 25056
3*c^2*d*x^2 + 250563*c*d^2*x^4 + 78529*d^3*x^6))*Hypergeometric2F1[1/4, 1,
5/4, -((b*x^2)/a)])/(449280*a*b^4*x^(11/2)) - (128*b*x^(9/2)*(c + d*x^2)^3*
HypergeometricPFQ[{2, 2, 2, 2, 9/4}, {1, 1, 1, 25/4}, -((b*x^2)/a)])/(41769
*a^3)
```

IntegrateAlgebraic [A] time = 0.54, size = 280, normalized size = 0.73

$$\frac{(13ad-bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{3/4}b^{17/4}}-\frac{(13ad-bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+\frac{\sqrt{x}\left(585a^3d^3-1215a^2bcd^2+468a^2bd^3x^2+675ab^2c^2d-972ab^2cd^2x^2-52ab^2d^3x^4-45b^3c^3+540b^3c^2dx^2+108b^3cd^2x^4+20b^3d^3x^6\right)}{90b^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[x]*(-45*b^3*c^3 + 675*a*b^2*c^2*d - 1215*a^2*b*c*d^2 + 585*a^3*d^3 +
540*b^3*c^2*d*x^2 - 972*a*b^2*c*d^2*x^2 + 468*a^2*b*d^3*x^2 + 108*b^3*c*d^2
*x^4 - 52*a*b^2*d^3*x^4 + 20*b^3*d^3*x^6))/(90*b^4*(a + b*x^2)) + ((-(b*c)
+ a*d)^2*(-(b*c) + 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) - ((-(b*c) + a*d)^2*(-(b*c) +
 13*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(4*Sqrt[2]*a^(3/4)*b^(17/4))
```

fricas [B] time = 1.06, size = 1961, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/360*(180*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^
10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b
^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^
8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276
*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4)*arctan((sqrt(a^2*b^8*sq
rt(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*
c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6
*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*
b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*
d^12)/(a^3*b^17)) + (b^6*c^6 - 30*a*b^5*c^5*d + 279*a^2*b^4*c^4*d^2 - 836*a
^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 702*a^5*b*c*d^5 + 169*a^6*d^6)*x)*a
```

```
^2*b^13*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^
3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a
^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 185364
4*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561
*a^12*d^12)/(a^3*b^17))^(3/4) + (a^2*b^16*c^3 - 15*a^3*b^15*c^2*d + 27*a^4*
b^14*c*d^2 - 13*a^5*b^13*d^3)*sqrt(x)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 145
8*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 5350
32*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 25
20207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10
- 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(3/4))/(b^12*c^12 - 6
0*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a
^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824
*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 8710
26*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)) + 45*(b^5*x
^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1841
2*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 13657
56*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 18
53644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 2
8561*a^12*d^12)/(a^3*b^17))^(1/4)*log(a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d
 + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4
- 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^
7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2
*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^
3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) - 45*(b^5*x^2 +
a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3
*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^
6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644
*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*
a^12*d^12)/(a^3*b^17))^(1/4)*log(-a*b^4*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 53
5032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 +
2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^1
0 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^3 -
15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*sqrt(x)) - 4*(20*b^3*d^3*x^6
- 45*b^3*c^3 + 675*a*b^2*c^2*d - 1215*a^2*b*c*d^2 + 585*a^3*d^3 + 4*(27*b^3
*c*d^2 - 13*a*b^2*d^3)*x^4 + 36*(15*b^3*c^2*d - 27*a*b^2*c*d^2 + 13*a^2*b*d
^3)*x^2)*sqrt(x))/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.50, size = 552, normalized size = 1.43

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} b^3c^3 - 15\left(ab^3\right)^{\frac{1}{4}} ab^2c^2d + 27\left(ab^3\right)^{\frac{1}{4}} a^2bcd^2 - 13\left(ab^3\right)^{\frac{1}{4}} a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^5} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} b^3c^3 - 15\left(ab^3\right)^{\frac{1}{4}} ab^2c^2d + 27\left(ab^3\right)^{\frac{1}{4}} a^2bcd^2 - 13\left(ab^3\right)^{\frac{1}{4}} a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8ab^5} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} b^3c^3 - 15\left(ab^3\right)^{\frac{1}{4}} ab^2c^2d + 27\left(ab^3\right)^{\frac{1}{4}} a^2bcd^2 - 13\left(ab^3\right)^{\frac{1}{4}} a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16ab^5} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}} b^3c^3 - 15\left(ab^3\right)^{\frac{1}{4}} ab^2c^2d + 27\left(ab^3\right)^{\frac{1}{4}} a^2bcd^2 - 13\left(ab^3\right)^{\frac{1}{4}} a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16ab^5} - \frac{b^3c^3\sqrt{x} - 3ab^2c^2d\sqrt{x} + 3a^2bcd^2\sqrt{x} - a^3d^3\sqrt{x}}{2\left(bx^2 + a\right)b^4} + \frac{2\left(5b^{16}d^3x^{\frac{9}{2}} + 27b^{16}cd^2x^{\frac{5}{2}} - 18ab^{15}d^3x^{\frac{5}{2}} + 135b^{16}c^2d\sqrt{x} - 270ab^{15}cd^2\sqrt{x} + 135a^2b^{14}d^3\sqrt{x}\right)}{45b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b
^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 -
13*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3
)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d
^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/16*sqrt(2)
*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a
^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a*b^5) - 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^
2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*b^4) + 2/45*(5*b^16*d^3*x
^(9/2) + 27*b^16*c*d^2*x^(5/2) - 18*a*b^15*d^3*x^(5/2) + 135*b^16*c^2*d*sqr
t(x) - 270*a*b^15*c*d^2*sqrt(x) + 135*a^2*b^14*d^3*sqrt(x))/b^18
```

maple [B]   time = 0.02, size = 748, normalized size = 1.94

$$\frac{2d^3x^{\frac{9}{2}}}{9b^2}-\frac{4a\,d^3x^{\frac{5}{2}}}{5b^3}+\frac{6c\,d^2x^{\frac{5}{2}}}{5b^2}+\frac{a^3d^3\sqrt{x}}{2\left(b\,x^2+a\right)b^4}-\frac{3a^2c\,d^2\sqrt{x}}{2\left(b\,x^2+a\right)b^3}+\frac{3a\,c^2d\sqrt{x}}{2\left(b\,x^2+a\right)b^2}-\frac{c^3\sqrt{x}}{2\left(b\,x^2+a\right)b}-\frac{13\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^4}-\frac{13\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^4}-\frac{13\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a^2d^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^4}+\frac{27\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,c\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^3}+\frac{27\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,c\,d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^3}+\frac{27\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,c\,d^2\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^3}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^2}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^2}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^2}+\frac{6a^2d^3\sqrt{x}}{b^4}-\frac{12a\,c\,d^2\sqrt{x}}{b^3}+\frac{6c^2d\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 2/9*d^3/b^2*x^(9/2)-4/5*d^3/b^3*x^(5/2)*a+6/5*d^2/b^2*x^(5/2)*c+6*d^3/b^4*a
^2*x^(1/2)-12*d^2/b^3*a*c*x^(1/2)+6*d/b^2*c^2*x^(1/2)+1/2/b^4*x^(1/2)/(b*x^
2+a)*a^3*d^3-3/2/b^3*x^(1/2)/(b*x^2+a)*a^2*c*d^2+3/2/b^2*x^(1/2)/(b*x^2+a)*
a*c^2*d-1/2/b*x^(1/2)/(b*x^2+a)*c^3-13/16/b^4*(a/b)^(1/4)*a^2*2^(1/2)*ln((x
+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2)))*d^3+27/16/b^3*(a/b)^(1/4)*a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x
^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2-15/1
6/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x
-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d+1/16/b*(a/b)^(1/4)/a*2^(1/
2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2)))*c^3-13/8/b^4*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)*d^3+27/8/b^3*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)*c*d^2-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)*c^2*d+1/8/b*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)*c^3-13/8/b^4*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)*d^3+27/8/b^3*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1)*c*d^2-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)-1)*c^2*d+1/8/b*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)-1)*c^3
```

maxima [A]   time = 2.35, size = 445, normalized size = 1.15

$$-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{x}}{2\left(b^5x^2+ab^4\right)}+\frac{2\left(5\,b^2d^3x^{\frac{9}{2}}+9\left(3\,b^2cd^2-2\,abd^3\right)x^{\frac{5}{2}}+135\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\sqrt{x}\right)}{45\,b^4}+\frac{\frac{2\sqrt{2}\left(b^3c^3-15\,ab^2c^2d+27\,a^2bcd^2-13\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^3c^3-15\,ab^2c^2d+27\,a^2bcd^2-13\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3-15\,ab^2c^2d+27\,a^2bcd^2-13\,a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^3c^3-15\,ab^2c^2d+27\,a^2bcd^2-13\,a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)/(b^5*x^2 +
 a*b^4) + 2/45*(5*b^2*d^3*x^(9/2) + 9*(3*b^2*c*d^2 - 2*a*b*d^3)*x^(5/2) + 1
35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(x))/b^4 + 1/16*(2*sqrt(2)*(b^3*
c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*arctan(1/2*sqrt(2)*(sqr
t(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^
2 - 13*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^
3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3
- 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

mupad [B] time = 0.45, size = 1691, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] x^(1/2)*((6*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/b - (2*a^2*d
^3)/b^4) - x^(5/2)*((4*a*d^3)/(5*b^3) - (6*c*d^2)/(5*b^2)) + (2*d^3*x^(9/2)
)/(9*b^2) + (x^(1/2)*((a^3*d^3)/2 - (b^3*c^3)/2 + (3*a*b^2*c^2*d)/2 - (3*a^
2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) + (atan(((((x^(1/2)*(169*a^6*d^6 + b^6*c^6
 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*
b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d
^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))
*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4)) + (((x^(1/2)*(169
*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b
^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*
d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2))/((-a
)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4))
)/((((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^
3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a
*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*
a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c))/(8*(-a)^
(3/4)*b^(17/4)) - (((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 -
 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*
d^5))/b^5 - ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*
b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d -
 b*c))/(8*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(4*(-a)^(
3/4)*b^(17/4)) - (atan(((((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4
```

```
*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^
5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 1
5*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*
(13*a*d - b*c))/(8*(-a)^(3/4)*b^(17/4)) + (((x^(1/2)*(169*a^6*d^6 + b^6*c^6
 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*
b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d
^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4
)))*(a*d - b*c)^2*(13*a*d - b*c))/(8*(-a)^(3/4)*b^(17/4)))/((((x^(1/2)*(169
*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b
^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*
d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2)*1i)/(
(-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/
4)) - (((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3
*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 +
((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d -
27*a^3*b*c*d^2)*1i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)
/(8*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^2*(13*a*d - b*c))/(4*(-a)^(3/4)*b^(1
7/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.437 $\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=376

$$\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{(bc-ad)^2(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{8\sqrt{2}\,a^{5/4}b^{15/4}}$$

Rubi [A] time = 0.43, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 297, 1162, 617, 204, 1165, 628}

$$-\frac{dx^{3/2}\left(11a^2d^2-21abcd+6b^2c^2\right)}{6ab^3}+\frac{(bc-ad)^2(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{(bc-ad)^2(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{(bc-ad)^2(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}b^{15/4}}+\frac{(bc-ad)^2(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{d^2x^{7/2}(7bc-11ad)}{14ab^2}+\frac{x^{3/2}\left(c+dx^2\right)^2(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -(d*(6*b^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/(6*a*b^3) - (d^2*(7*b*c
- 11*a*d)*x^(7/2))/(14*a*b^2) + ((b*c - a*d)*x^(3/2)*(c + d*x^2)^2)/(2*a*b*
(a + b*x^2)) - ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) + ((b*c - a*d)^2*(b*c + 11*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*b^(15/
4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*b^(15/4)) - ((b*c - a*d)^2*(b*c + 1
1*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*b^(15/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(c+dx^4\right)\left(-c(bc+3ad)+d(7bc-11ad)x^4\right)}{a+bx^4}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^2}{b^2} + \frac{d^2(7bc-11ad)x^6}{b} - \frac{\left(b^3c^3+9ab^2c^2\right.}{b}\right.\right.}{2ab}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} + \frac{((}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{((}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} + \frac{((}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} + \frac{(b}{}$$

$$= -\frac{d\left(6b^2c^2-21abcd+11a^2d^2\right)x^{3/2}}{6ab^3} - \frac{d^2(7bc-11ad)x^{7/2}}{14ab^2} + \frac{(bc-ad)x^{3/2}\left(c+dx^2\right)^2}{2ab\left(a+bx^2\right)} - \frac{(b}{}$$

Mathematica [C] time = 2.63, size = 355, normalized size = 0.94

$$\frac{95a\left(a\left(77a^2\left(16875c^3+50625c^2dx^2+50625cd^2x^4+15467d^3x^6\right)+22abx^2\left(25931c^3+77793c^2dx^2+87201cd^2x^4+28043d^3x^6\right)+b^2x^4\left(56099c^3+79593c^2dx^2+79593cd^2x^4+26531d^3x^6\right)\right)-77\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\left(a^3\left(16875c^3+50625c^2dx^2+50625cd^2x^4+15467d^3x^6\right)+a^2bx^2\left(14641c^3+43923c^2dx^2+46611cd^2x^4+14641d^3x^6\right)+ab^2x^4\left(2401c^3+6051c^2dx^2+7203cd^2x^4+2401d^3x^6\right)+b^3x^6\left(-101c^3+81c^2dx^2+81cd^2x^4+27d^3x^6\right)\right)\right)-32768b^4x^8\left(c+dx^2\right)^3\,{}_5F_4\left(\frac{7}{4},2,2,2,2;1,1,1,\frac{23}{4};-\frac{bx^2}{a}\right)}{5617920a^3b^3x^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] (95*a*(a*(77*a^2*(16875*c^3 + 50625*c^2*d*x^2 + 50625*c*d^2*x^4 + 15467*d^3
*x^6) + b^2*x^4*(56099*c^3 + 79593*c^2*d*x^2 + 79593*c*d^2*x^4 + 26531*d^3*
x^6) + 22*a*b*x^2*(25931*c^3 + 77793*c^2*d*x^2 + 87201*c*d^2*x^4 + 28043*d^
3*x^6)) - 77*(b^3*x^6*(-101*c^3 + 81*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6)
 + a*b^2*x^4*(2401*c^3 + 6051*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6) +
a^2*b*x^2*(14641*c^3 + 43923*c^2*d*x^2 + 46611*c*d^2*x^4 + 14641*d^3*x^6) +
 a^3*(16875*c^3 + 50625*c^2*d*x^2 + 50625*c*d^2*x^4 + 15467*d^3*x^6))*Hyper
geometric2F1[3/4, 1, 7/4, -((b*x^2)/a)]) - 32768*b^4*x^8*(c + d*x^2)^3*Hype
rgeometricPFQ[{7/4, 2, 2, 2, 2}, {1, 1, 1, 23/4}, -((b*x^2)/a)])/(5617920*a
^3*b^3*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.52, size = 246, normalized size = 0.65

$$-\frac{(11ad+bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{(11ad+bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{5/4}b^{15/4}}-\frac{x^{3/2}\left(77a^3d^3-147a^2bcd^2+44a^2bd^3x^2+63ab^2c^2d-84ab^2cd^2x^2-12ab^2d^3x^4-21b^3c^3\right)}{42ab^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

```
[Out] -1/42*(x^(3/2)*(-21*b^3*c^3 + 63*a*b^2*c^2*d - 147*a^2*b*c*d^2 + 77*a^3*d^3
 - 84*a*b^2*c*d^2*x^2 + 44*a^2*b*d^3*x^2 - 12*a*b^2*d^3*x^4))/(a*b^3*(a + b
*x^2)) - ((-(b*c) + a*d)^2*(b*c + 11*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(5/4)*b^(15/4)) - ((-(b*c) + a
*d)^2*(b*c + 11*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(4*Sqrt[2]*a^(5/4)*b^(15/4))
```

fricas [B] time = 1.39, size = 2531, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/168*(84*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*
b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*
c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*
c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b
*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*arctan((sqrt((b^18*c^18 + 54*a
*b^17*c^17*d + 1089*a^2*b^16*c^16*d^2 + 8976*a^3*b^15*c^15*d^3 + 5940*a^4*b
^14*c^14*d^4 - 279576*a^5*b^13*c^13*d^5 - 338844*a^6*b^12*c^12*d^6 + 600177
6*a^7*b^11*c^11*d^7 - 6412626*a^8*b^10*c^10*d^8 - 62165180*a^9*b^9*c^9*d^9
+ 294333534*a^10*b^8*c^8*d^10 - 671362704*a^11*b^7*c^7*d^11 + 974580036*a^1
```

```
2*b^6*c^6*d^12 - 971334936*a^13*b^5*c^5*d^13 + 678512340*a^14*b^4*c^4*d^14
- 328575984*a^15*b^3*c^3*d^15 + 105546969*a^16*b^2*c^2*d^16 - 20292426*a^17
*b*c*d^17 + 1771561*a^18*d^18)*x - (a^3*b^19*c^12 + 36*a^4*b^18*c^11*d + 40
2*a^5*b^17*c^10*d^2 + 692*a^6*b^16*c^9*d^3 - 10017*a^7*b^15*c^8*d^4 - 5688*
a^8*b^14*c^7*d^5 + 160188*a^9*b^13*c^6*d^6 - 486648*a^10*b^12*c^5*d^7 + 746
703*a^11*b^11*c^4*d^8 - 676588*a^12*b^10*c^3*d^9 + 368082*a^13*b^9*c^2*d^10
 - 111804*a^14*b^8*c*d^11 + 14641*a^15*b^7*d^12)*sqrt(-(b^12*c^12 + 36*a*b^
11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8
*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d
^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*
d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15)))*a*b^4*(-(b^12*c
^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 1001
7*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*
a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*
a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/
4) - (a*b^13*c^9 + 27*a^2*b^12*c^8*d + 180*a^3*b^11*c^7*d^2 - 372*a^4*b^10*
c^6*d^3 - 3186*a^5*b^9*c^5*d^4 + 13194*a^6*b^8*c^4*d^5 - 21372*a^7*b^7*c^3*
d^6 + 17820*a^8*b^6*c^2*d^7 - 7623*a^9*b^5*c*d^8 + 1331*a^10*b^4*d^9)*sqrt(
x)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^
9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d
^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d
^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^
5*b^15))^(1/4))/(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692
*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^
9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^1
2*d^12)) - 21*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a
^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b
^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b
^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^1
1*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*log(a^4*b^11*(-(b^12*c^12 +
 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4
*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b
^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*
b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) +
(b^9*c^9 + 27*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 318
6*a^4*b^5*c^5*d^4 + 13194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a
^7*b^2*c^2*d^7 - 7623*a^8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) + 21*(a*b^4*x^2
+ a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^
3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b
^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b
^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d
^12)/(a^5*b^15))^(1/4)*log(-a^4*b^11*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*
a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*
b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*
```

```
b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^
11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/4) + (b^9*c^9 + 27*a*b^8*c^8*
d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 1319
4*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 - 7623*a^
8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) - 4*(12*a*b^2*d^3*x^5 + 4*(21*a*b^2*c*d^
2 - 11*a^2*b*d^3)*x^3 + 7*(3*b^3*c^3 - 9*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 11*
a^3*d^3)*x)*sqrt(x))/(a*b^4*x^2 + a^2*b^3)
```

giac [A]   time = 0.50, size = 516, normalized size = 1.37

$$\frac{b^{3}c^{3}x^{\frac{3}{2}} - 3\,ab^{2}c^{2}dx^{\frac{3}{2}} + 3\,a^{2}bcd^{2}x^{\frac{3}{2}} - a^{3}d^{3}x^{\frac{3}{2}}}{2\,\left(bx^{2} + a\right)ab^{3}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{6}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{2}b^{6}} - \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^{2}b^{6}} + \frac{\sqrt{2}\left(\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3} + 9\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d - 21\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2} + 11\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^{2}b^{6}} + \frac{2\,\left(3\,b^{12}d^{3}x^{\frac{7}{2}} + 21\,b^{12}cd^{2}x^{\frac{3}{2}} - 14\,ab^{11}d^{3}x^{\frac{3}{2}}\right)}{21\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3*
d^3*x^(3/2))/((b*x^2 + a)*a*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(
a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*
a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/
(a^2*b^6) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*
d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^6) - 1/16*sqrt(
2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*
a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^2*b^6) + 1/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(
3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^6) + 2/21*(3*b^12
*d^3*x^(7/2) + 21*b^12*c*d^2*x^(3/2) - 14*a*b^11*d^3*x^(3/2))/b^14
```

maple [B]   time = 0.02, size = 706, normalized size = 1.88

$$\frac{2d^{3}x^{\frac{7}{2}}}{7b^{2}} - \frac{a^{2}d^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2} + a\right)b^{3}} + \frac{3ac\,d^{2}x^{\frac{3}{2}}}{2\left(b\,x^{2} + a\right)b^{2}} + \frac{c^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2} + a\right)a} - \frac{3c^{2}d\,x^{\frac{3}{2}}}{2\left(b\,x^{2} + a\right)b} - \frac{4a\,d^{3}x^{\frac{3}{2}}}{3b^{3}} + \frac{2c\,d^{2}x^{\frac{3}{2}}}{b^{2}} + \frac{11\sqrt{2}\,a^{2}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}} + \frac{11\sqrt{2}\,a^{2}d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}} + \frac{11\sqrt{2}\,a^{2}d^{3}\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{4}} - \frac{21\sqrt{2}\,ac\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}} - \frac{21\sqrt{2}\,ac\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}} - \frac{21\sqrt{2}\,ac\,d^{2}\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}} + \frac{\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{\sqrt{2}\,c^{3}\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab} + \frac{9\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}} + \frac{9\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}} + \frac{9\sqrt{2}\,c^{2}d\ln\left(\frac{x - \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}{x + \left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x} + \sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 2/7*d^3/b^2*x^(7/2)-4/3*d^3/b^3*x^(3/2)*a+2*d^2/b^2*x^(3/2)*c-1/2/b^3*a^2*x
^(3/2)/(b*x^2+a)*d^3+3/2/b^2*a*x^(3/2)/(b*x^2+a)*c*d^2-3/2/b*x^(3/2)/(b*x^2
+a)*c^2*d+1/2/a*x^(3/2)/(b*x^2+a)*c^3+11/8/b^4*a^2/(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3-21/8/b^3*a/(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+9/8/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+1/8/b/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+11/8/b^4*a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1)*d^3-21/8/b^3*a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
```

```
/(a/b)^(1/4)*x^(1/2)-1)*c*d^2+9/8/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1)*c^2*d+1/8/b/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)-1)*c^3+11/16/b^4*a^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^
3-21/16/b^3*a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(
1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2+9/16/b^2/(a/b)^(1/
4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d+1/16/b/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2)))*c^3
```

maxima [**A**] time = 2.51, size = 309, normalized size = 0.82

$$\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)x^{\frac{3}{2}}}{2\left(ab^{4}x^{2}+a^{2}b^{3}\right)}+\frac{2\left(3\,bd^{3}x^{\frac{7}{2}}+7\left(3\,bcd^{2}-2\,ad^{3}\right)x^{\frac{3}{2}}\right)}{21\,b^{3}}+\frac{\left(b^{3}c^{3}+9\,ab^{2}c^{2}d-21\,a^{2}bcd^{2}+11\,a^{3}d^{3}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^(3/2)/(a*b^4*x^2
+ a^2*b^3) + 2/21*(3*b*d^3*x^(7/2) + 7*(3*b*c*d^2 - 2*a*d^3)*x^(3/2))/b^3 +
 1/16*(b^3*c^3 + 9*a*b^2*c^2*d - 21*a^2*b*c*d^2 + 11*a^3*d^3)*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^3)
```

mupad [**B**] time = 0.23, size = 616, normalized size = 1.64

$$\frac{2\,d^{3}\,x^{7/2}}{7\,b^{2}}-x^{3/2}\,\left(\frac{4\,a\,d^{3}}{3\,b^{3}}-\frac{2\,c\,d^{2}}{b^{2}}\right)-\frac{x^{3/2}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}{2\,a\,\left(b^{4}\,x^{2}+a\,b^{3}\right)}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,\left(a\,d-b\,c\right)^{2}\,\left(11\,a\,d+b\,c\right)\,\left(121\,a^{6}\,d^{6}-462\,a^{5}\,b\,c\,d^{5}+639\,a^{4}\,b^{2}\,c^{2}\,d^{4}-356\,a^{3}\,b^{3}\,c^{3}\,d^{3}+39\,a^{2}\,b^{4}\,c^{4}\,d^{2}+18\,a\,b^{5}\,c^{5}\,d+b^{6}\,c^{6}\right)}{{\left(-a\right)}^{1/4}\,\left(1331\,a^{9}\,d^{9}-7623\,a^{8}\,b\,c\,d^{8}+17820\,a^{7}\,b^{2}\,c^{2}\,d^{7}-21372\,a^{6}\,b^{3}\,c^{3}\,d^{6}+13194\,a^{5}\,b^{4}\,c^{4}\,d^{5}-3186\,a^{4}\,b^{5}\,c^{5}\,d^{4}-372\,a^{3}\,b^{6}\,c^{6}\,d^{3}+180\,a^{2}\,b^{7}\,c^{7}\,d^{2}+27\,a\,b^{8}\,c^{8}\,d+b^{9}\,c^{9}\right)}\right)\,{\left(a\,d-b\,c\right)}^{2}\,\left(11\,a\,d+b\,c\right)}{4\,{\left(-a\right)}^{5/4}\,b^{15/4}}-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,\left(a\,d-b\,c\right)^{2}\,\left(11\,a\,d+b\,c\right)\,\left(121\,a^{6}\,d^{6}-462\,a^{5}\,b\,c\,d^{5}+639\,a^{4}\,b^{2}\,c^{2}\,d^{4}-356\,a^{3}\,b^{3}\,c^{3}\,d^{3}+39\,a^{2}\,b^{4}\,c^{4}\,d^{2}+18\,a\,b^{5}\,c^{5}\,d+b^{6}\,c^{6}\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}\,\left(1331\,a^{9}\,d^{9}-7623\,a^{8}\,b\,c\,d^{8}+17820\,a^{7}\,b^{2}\,c^{2}\,d^{7}-21372\,a^{6}\,b^{3}\,c^{3}\,d^{6}+13194\,a^{5}\,b^{4}\,c^{4}\,d^{5}-3186\,a^{4}\,b^{5}\,c^{5}\,d^{4}-372\,a^{3}\,b^{6}\,c^{6}\,d^{3}+180\,a^{2}\,b^{7}\,c^{7}\,d^{2}+27\,a\,b^{8}\,c^{8}\,d+b^{9}\,c^{9}\right)}\right)\,{\left(a\,d-b\,c\right)}^{2}\,\left(11\,a\,d+b\,c\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{5/4}\,b^{15/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

```
[Out] (2*d^3*x^(7/2))/(7*b^2) - x^(3/2)*((4*a*d^3)/(3*b^3) - (2*c*d^2)/b^2) - (x^
(3/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^
4*x^2)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^2*(11*a*d + b*c)*(121*a^6*d^6
+ b^6*c^6 + 39*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 639*a^4*b^2*c^2*d^4
+ 18*a*b^5*c^5*d - 462*a^5*b*c*d^5))/((-a)^(1/4)*(1331*a^9*d^9 + b^9*c^9 +
180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13194*a^
5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 + 27*a*b^8*c^
```

```
8*d - 7623*a^8*b*c*d^8)))*(a*d - b*c)^2*(11*a*d + b*c))/(4*(-a)^(5/4)*b^(15
/4)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^2*(11*a*d + b*c)*(121*a^6*d^6 + b
^6*c^6 + 39*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 639*a^4*b^2*c^2*d^4 + 1
8*a*b^5*c^5*d - 462*a^5*b*c*d^5)*1i)/((-a)^(1/4)*(1331*a^9*d^9 + b^9*c^9 +
180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13194*a^
5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 + 27*a*b^8*c^
8*d - 7623*a^8*b*c*d^8)))*(a*d - b*c)^2*(11*a*d + b*c)*1i)/(4*(-a)^(5/4)*b^
(15/4))
```

sympy [A] time = 136.46, size = 173, normalized size = 0.46

$$-\frac{4ad^3x^{\frac{3}{2}}}{3b^3}-\frac{2x^{\frac{3}{2}}(ad-bc)^3}{4a^2b^3+4ab^4x^2}+\frac{2cd^2x^{\frac{3}{2}}}{b^2}+\frac{2d^3x^{\frac{7}{2}}}{7b^2}+\frac{6d(ad-bc)^2\operatorname{RootSum}\left(256t^4ab^3+1,\left(t\mapsto t\log\left(64t^3ab^2+\sqrt{x}\right)\right)\right)}{b^3}-\frac{2(ad-bc)^3\operatorname{RootSum}\left(65536t^4a^5b^3+1,\left(t\mapsto t\log\left(4096t^3a^4b^2+\sqrt{x}\right)\right)\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] -4*a*d**3*x**(3/2)/(3*b**3) - 2*x**(3/2)*(a*d - b*c)**3/(4*a**2*b**3 + 4*a*
b**4*x**2) + 2*c*d**2*x**(3/2)/b**2 + 2*d**3*x**(7/2)/(7*b**2) + 6*d*(a*d -
 b*c)**2*RootSum(256*_t**4*a*b**3 + 1, Lambda(_t, _t*log(64*_t**3*a*b**2 +
sqrt(x))))/b**3 - 2*(a*d - b*c)**3*RootSum(65536*_t**4*a**5*b**3 + 1, Lambd
a(_t, _t*log(4096*_t**3*a**4*b**2 + sqrt(x))))/b**3
```

3.438 $\int \frac{\left(c+dx^2\right)^3}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=340

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{8\sqrt{2}a^{7/4}b^{13/4}}$$

Rubi [A] time = 0.39, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {466, 390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}b^{13/4}}+\frac{2d^2\sqrt{x}(3bc-2ad)}{b^3}+\frac{\sqrt{x}(bc-ad)^3}{2ab^3\left(a+bx^2\right)}+\frac{2d^3x^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (2*d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (2*d^3*x^(5/2))/(5*b^2) + ((b*c - a*d
)^3*Sqrt[x])/(2*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(
b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(
4*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
+ (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{x}\left(a+bx^2\right)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\mathrm{Subst}\left(\int\left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\left(a+bx^4\right)^2}\right)dx, x, \sqrt{x}$$

$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{2\,\mathrm{Subst}\left(\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\left(a+bx^2\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\mathrm{Subst}\left(\int \frac{1}{a+bx}\right.}{2ab^3}$$

$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\left(a+bx^2\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-}{a+}\right.}{4a^{3/2}b^3}$$

$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\left(a+bx^2\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}-}\right.}{8a^{3/2}b^{7/2}}$$

$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\left(a+bx^2\right)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{}\right.}{8\sqrt{2}\,a^{7/4}b^{13/4}}$$

$$= \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3\left(a+bx^2\right)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{4\sqrt{2}\,a^{7/4}b^{13/4}}$$

Mathematica [C] time = 2.64, size = 358, normalized size = 1.05

$$\frac{a\left(45a^2\left(28561c^3+85683c^2dx^2+85683cd^2x^4+25105d^3x^6\right)+18abx^2\left(34927c^3+104781c^2dx^2+119181cd^2x^4+36655d^3x^6\right)+b^2x^4\left(50033c^3+98259c^2dx^2+98259cd^2x^4+32753d^3x^6\right)\right)-45\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^3\left(28561c^3+85683c^2dx^2+85683cd^2x^4+25105d^3x^6\right)+9a^2bx^2\left(2187c^3+6561c^2dx^2+7201cd^2x^4+2187d^3x^6\right)+3ab^2x^4\left(625c^3+1491c^2dx^2+1875cd^2x^4+625d^3x^6\right)+b^3x^6\left(-1151c^3+3c^2dx^2+3cd^2x^4+d^3x^6\right)\right)}{34560a^2b^3x^{11/2}}-\frac{128bx^{5/2}\left(c+dx^2\right)^3\,{}_5F_4\left(\frac{5}{4},2,2,2,2;1,1,1,\frac{21}{4};-\frac{bx^2}{a}\right)}{9945a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] (a*(45*a^2*(28561*c^3 + 85683*c^2*d*x^2 + 85683*c*d^2*x^4 + 25105*d^3*x^6)
+ b^2*x^4*(50033*c^3 + 98259*c^2*d*x^2 + 98259*c*d^2*x^4 + 32753*d^3*x^6) +
 18*a*b*x^2*(34927*c^3 + 104781*c^2*d*x^2 + 119181*c*d^2*x^4 + 36655*d^3*x^
6)) - 45*(b^3*x^6*(-1151*c^3 + 3*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 3*a*b
^2*x^4*(625*c^3 + 1491*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6) + 9*a^2*b*
x^2*(2187*c^3 + 6561*c^2*d*x^2 + 7201*c*d^2*x^4 + 2187*d^3*x^6) + a^3*(2856
1*c^3 + 85683*c^2*d*x^2 + 85683*c*d^2*x^4 + 25105*d^3*x^6))*Hypergeometric2
F1[1/4, 1, 5/4, -((b*x^2)/a)])/(34560*a^2*b^3*x^(11/2)) - (128*b*x^(5/2)*(c
 + d*x^2)^3*HypergeometricPFQ[{5/4, 2, 2, 2, 2}, {1, 1, 1, 21/4}, -((b*x^2)
/a)])/(9945*a^3)
```

IntegrateAlgebraic [A] time = 0.51, size = 246, normalized size = 0.72

$$-\frac{3(3ad+bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{7/4}b^{13/4}}+\frac{3(3ad+bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{7/4}b^{13/4}}-\frac{\sqrt{x}\left(45a^3d^3-75a^2bcd^2+36a^2bd^3x^2+15ab^2c^2d-60ab^2cd^2x^2-4ab^2d^3x^4-5b^3c^3\right)}{10ab^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

```
[Out] -1/10*(Sqrt[x]*(-5*b^3*c^3 + 15*a*b^2*c^2*d - 75*a^2*b*c*d^2 + 45*a^3*d^3 -
 60*a*b^2*c*d^2*x^2 + 36*a^2*b*d^3*x^2 - 4*a*b^2*d^3*x^4))/(a*b^3*(a + b*x^
2)) - (3*(-(b*c) + a*d)^2*(b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(-(b*c) + a
*d)^2*(b*c + 3*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sq
rt[b]*x)])/(4*Sqrt[2]*a^(7/4)*b^(13/4))
```

fricas [B] time = 1.37, size = 1944, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(60*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10
*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5
- 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a
^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)
/(a^7*b^13))^(1/4)*arctan((sqrt(a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d
- 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5
*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4
*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 +
81*a^12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 -
4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x)*a^5
*b^10*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^
```

```
9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 3
28*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^1
0*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4) - (a^5
*b^13*c^3 + a^6*b^12*c^2*d - 5*a^7*b^11*c*d^2 + 3*a^8*b^10*d^3)*sqrt(x)*(-(
b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 1
27*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^
5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2
*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(3/4))/(b^12*c^12 + 4
*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^
8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1
039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)) + 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a
*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*
d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 103
9*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^1
1*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4
*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^
8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1
039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a
^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d -
 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) - 15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^1
2 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b
^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^
7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 -
540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(-3*a^2*b^3*(-(b^12*
c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^
4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5
*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10
 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2
*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) + 4*(4*a*b^2*d^3*x^4 + 5*b^3*c
^3 - 15*a*b^2*c^2*d + 75*a^2*b*c*d^2 - 45*a^3*d^3 + 12*(5*a*b^2*c*d^2 - 3*a
^2*b*d^3)*x^2)*sqrt(x))/(a*b^4*x^2 + a^2*b^3)
```

giac [**A**] time = 0.43, size = 511, normalized size = 1.50

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^4}+\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^2b^4}+\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b^4}-\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^2b^4}+\frac{b^3c^3\sqrt{x}-3ab^2c^2d\sqrt{x}+3a^2bcd^2\sqrt{x}-a^3d^3\sqrt{x}}{2\left(bx^2+a\right)ab^3}+\frac{2\left(b^8d^3x^{\frac{5}{2}}+15b^8cd^2\sqrt{x}-10ab^7d^3\sqrt{x}\right)}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^
(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 3/8*sqrt(2)*((a*b^3)^(1/4)*
b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^
3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
```

```
b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a
*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) - 3/16*sqrt(2)*((a*b^3)
^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 +
3*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(a^2*b^4) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sq
rt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a*b^3) + 2/5*(b^8*d^3*x^(5/2) + 15*b^
8*c*d^2*sqrt(x) - 10*a*b^7*d^3*sqrt(x))/b^10
```

maple [B] time = 0.02, size = 697, normalized size = 2.05

$$\frac{2d^{3}x^{\frac{5}{2}}}{5b^{2}}-\frac{a^{2}d^{3}\sqrt{x}}{2\left(b x^{2}+a\right)b^{3}}+\frac{3acd^{2}\sqrt{x}}{2\left(b x^{2}+a\right)b^{2}}+\frac{c^{3}\sqrt{x}}{2\left(b x^{2}+a\right)a}-\frac{3c^{2}d\sqrt{x}}{2\left(b x^{2}+a\right)b}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a d^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^{2}}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^{2}}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c d^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^{2}}-\frac{4a d^{3}\sqrt{x}}{b^{3}}+\frac{6c d^{2}\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/5*d^3*x^(5/2)/b^2-4*d^3/b^3*a*x^(1/2)+6*d^2/b^2*c*x^(1/2)-1/2/b^3*a^2*x^(
1/2)/(b*x^2+a)*d^3+3/2/b^2*a*x^(1/2)/(b*x^2+a)*c*d^2-3/2/b*x^(1/2)/(b*x^2+a
)*c^2*d+1/2/a*x^(1/2)/(b*x^2+a)*c^3+9/16/b^3*a*(a/b)^(1/4)*2^(1/2)*ln((x+(a
/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)
^(1/2)))*d^3-15/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2+3/16/b/a*
(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*ln(
(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2)))*c^3+9/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)+1)*d^3-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)*c*d^2+3/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)*c^2*d+3/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
*c^3+9/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^
3-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2+
3/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+3/8
/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3
```

maxima [A] time = 2.58, size = 412, normalized size = 1.21

$$\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{x}}{2\left(ab^{4}x^{2}+a^{2}b^{3}\right)}+\frac{2\left(bd^{3}x^{\frac{5}{2}}+5\left(3\,bcd^{2}-2\,ad^{3}\right)\sqrt{x}\right)}{5\,b^{3}}+\frac{3\left(\frac{2\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5\,a^{2}bcd^{2}+3\,a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5\,a^{2}bcd^{2}+3\,a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5\,a^{2}bcd^{2}+3\,a^{3}d^{3}\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5\,a^{2}bcd^{2}+3\,a^{3}d^{3}\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{16\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)/(a*b^4*x^2
+ a^2*b^3) + 2/5*(b*d^3*x^(5/2) + 5*(3*b*c*d^2 - 2*a*d^3)*sqrt(x))/b^3 + 3/
```

```
16*(2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/
2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*
a^2*b*c*d^2 + 3*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sq
rt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*
c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3)
```

mupad [B] time = 0.25, size = 1636, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(1/2)*(a + b*x^2)^2),x)
```

```
[Out] (2*d^3*x^(5/2))/(5*b^2) - x^(1/2)*((4*a*d^3)/b^3 - (6*c*d^2)/b^2) - (x^(1/2
)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^
2)) + (atan(((((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*
b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^
3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2
*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c)
*3i)/(8*(-a)^(7/4)*b^(13/4)) + (((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^
4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5
*b*c*d^5))/(a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*
c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*
c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)))/((3*((9*x^(1/2)*(9*a^6*d^6
+ b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*
a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(
72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*
b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*b^(13/4)) - (3*((9*x^
(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4
*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) + (3*(a*d - b*c)^
2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2
))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*b^(1
3/4))))*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(4*(-a)^(7/4)*b^(13/4)) + (3*atan((
(3*((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3
 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) - ((a*d
- b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*
b*c*d^2)*3i)/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^
(7/4)*b^(13/4)) + (3*((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 -
 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/
(a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^
2*c^2*d - 120*a^2*b*c*d^2)*3i)/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*
```

```
d + b*c))/(8*(-a)^(7/4)*b^(13/4)))/((((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a
^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 3
0*a^5*b*c*d^5))/(a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b
^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2)*3i)/(8*(-a)^(7/4)*b^(13/4)))*(a*
d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) - (((9*x^(1/2)*(9*a^6*
d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4
+ 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)
*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2)*3i)/(8*(-a)^(
7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4))))*(
a*d - b*c)^2*(3*a*d + b*c))/(4*(-a)^(7/4)*b^(13/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.439 $\int \frac{\left(c+dx^2\right)^3}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\right.}{8\sqrt{2}\,a^{9/4}b^{11/4}}$$

Rubi [A] time = 0.43, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 297, 1162, 617, 204, 1165, 628}

$$-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}b^{11/4}}+\frac{(bc-ad)^2(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}b^{11/4}}-\frac{(bc-ad)^2(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{9/4}b^{11/4}}-\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}}-\frac{d^2x^{3/2}(3bc-7ad)}{6ab^2}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2ab\sqrt{x}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(5*b*c - a*d))/(2*a^2*b*Sqrt[x]) - (d^2*(3*b*c - 7*a*d)*x^(3/2))/(6*a
*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*Sqrt[x]*(a + b*x^2)) + ((b*c - a
*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) - ((b*c - a*d)^2*(
5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(9/4)*b^(11/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*b^(11/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{3/2}\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^2\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(-c(5bc-ad)+d(3bc-7ad)x^4\right)}{x^2\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-5bc+ad)}{ax^2} + \frac{d^2(3bc-7ad)x^2}{b} + \frac{(-bc+ad)^2(5bc+7ad)x^2}{ab\left(a+bx^4\right)}\right)dx, x,}{2ab}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{\left((bc-ad)^2(5bc+7ad)\right)\text{Su}}{2a^2b^2}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} + \frac{\left((bc-ad)^2(5bc+7ad)\right)\text{Su}}{4a^2}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{\left((bc-ad)^2(5bc+7ad)\right)\text{Su}}{8}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} - \frac{(bc-ad)^2(5bc+7ad)\log\left(\right.}{8\sqrt{2}\,a}$$

$$= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}} - \frac{d^2(3bc-7ad)x^{3/2}}{6ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2ab\sqrt{x}\left(a+bx^2\right)} + \frac{(bc-ad)^2(5bc+7ad)\tan^{-1}}{4\sqrt{2}\,a^{9/4}b^{11}}$$

Mathematica [C] time = 2.39, size = 355, normalized size = 0.96

$$-\frac{32768b^3x^6\left(c+dx^2\right)^3{}_5F_4\left(\frac{3}{4},2,2,2,2;1,1,1,\frac{19}{4};-\frac{bx^2}{a}\right)+55\left(a\left(7a^2\left(14641c^3+43923c^2dx^2+43923cd^2x^4+11953d^3x^6\right)+18abx^2\left(361c^3+1083c^2dx^2+2427cd^2x^4+809d^3x^6\right)-21b^2x^4\left(-1919c^3+3c^2dx^2+3cd^2x^4+d^3x^6\right)\right)-7\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\left(a^3\left(14641c^3+43923c^2dx^2+43923cd^2x^4+11953d^3x^6\right)+3a^2bx^2\left(2401c^3+7203c^2dx^2+8355cd^2x^4+2401d^3x^6\right)+9ab^2x^4\left(27c^3+209c^2dx^2+81cd^2x^4+27d^3x^6\right)+b^3x^6\left(-1919c^3+3c^2dx^2+3cd^2x^4+d^3x^6\right)\right)\right)}{887040a^3b^2x^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/887040*(55*(a*(-21*b^2*x^4*(-1919*c^3 + 3*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*
x^6) + 18*a*b*x^2*(361*c^3 + 1083*c^2*d*x^2 + 2427*c*d^2*x^4 + 809*d^3*x^6)
 + 7*a^2*(14641*c^3 + 43923*c^2*d*x^2 + 43923*c*d^2*x^4 + 11953*d^3*x^6)) -
 7*(b^3*x^6*(-1919*c^3 + 3*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 9*a*b^2*x^4
*(27*c^3 + 209*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) + 3*a^2*b*x^2*(2401*c
^3 + 7203*c^2*d*x^2 + 8355*c*d^2*x^4 + 2401*d^3*x^6) + a^3*(14641*c^3 + 439
23*c^2*d*x^2 + 43923*c*d^2*x^4 + 11953*d^3*x^6))*Hypergeometric2F1[3/4, 1,
7/4, -((b*x^2)/a)]) + 32768*b^3*x^6*(c + d*x^2)^3*HypergeometricPFQ[{3/4, 2
, 2, 2, 2}, {1, 1, 1, 19/4}, -((b*x^2)/a)])/(a^3*b^2*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.57, size = 244, normalized size = 0.66

$$\frac{(7ad+5bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}\,a^{9/4}b^{11/4}}+\frac{(7ad+5bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{9/4}b^{11/4}}+\frac{7a^3d^3x^2-9a^2bcd^2x^2+4a^2bd^3x^4-12ab^2c^3+9ab^2c^2dx^2-15b^3c^3x^2}{6a^2b^2\sqrt{x}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

```
[Out] (-12*a*b^2*c^3 - 15*b^3*c^3*x^2 + 9*a*b^2*c^2*d*x^2 - 9*a^2*b*c*d^2*x^2 + 7
*a^3*d^3*x^2 + 4*a^2*b*d^3*x^4)/(6*a^2*b^2*Sqrt[x]*(a + b*x^2)) + ((-(b*c)
+ a*d)^2*(5*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])])/(4*Sqrt[2]*a^(9/4)*b^(11/4)) + ((-(b*c) + a*d)^2*(5*b*c + 7*
a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*S
qrt[2]*a^(9/4)*b^(11/4))
```

fricas [B] time = 1.27, size = 2547, normalized size = 6.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(12*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d -
3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 287
28*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*
a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^
11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*arctan((sqrt((15625*b^18*c^
18 - 56250*a*b^17*c^17*d - 84375*a^2*b^16*c^16*d^2 + 570000*a^3*b^15*c^15*d
^3 - 211500*a^4*b^14*c^14*d^4 - 2174040*a^5*b^13*c^13*d^5 + 2720004*a^6*b^1
2*c^12*d^6 + 3321072*a^7*b^11*c^11*d^7 - 8368866*a^8*b^10*c^10*d^8 + 640420
*a^9*b^9*c^9*d^9 + 11255310*a^10*b^8*c^8*d^10 - 8509968*a^11*b^7*c^7*d^11 -
 4831644*a^12*b^6*c^6*d^12 + 9537192*a^13*b^5*c^5*d^13 - 3095820*a^14*b^4*c
```

```
^4*d^14 - 2551920*a^15*b^3*c^3*d^15 + 2614689*a^16*b^2*c^2*d^16 - 907578*a^
17*b*c*d^17 + 117649*a^18*d^18)*x - (625*a^5*b^17*c^12 - 1500*a^6*b^16*c^11
*d - 3150*a^7*b^15*c^10*d^2 + 11060*a^8*b^14*c^9*d^3 + 1071*a^9*b^13*c^8*d^
4 - 28728*a^10*b^12*c^7*d^5 + 19068*a^11*b^11*c^6*d^6 + 27144*a^12*b^10*c^5
*d^7 - 37665*a^13*b^9*c^4*d^8 + 2324*a^14*b^8*c^3*d^9 + 19698*a^15*b^7*c^2*
d^10 - 12348*a^16*b^6*c*d^11 + 2401*a^17*b^5*d^12)*sqrt(-(625*b^12*c^12 - 1
500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a
^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*
b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2
*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11)))*a^2*b^3*(-(6
25*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*
c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*
d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9
+ 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11
))^(1/4) - (125*a^2*b^12*c^9 - 225*a^3*b^11*c^8*d - 540*a^4*b^10*c^7*d^2 +
1308*a^5*b^9*c^6*d^3 + 342*a^6*b^8*c^5*d^4 - 2430*a^7*b^7*c^4*d^5 + 1140*a^
8*b^6*c^3*d^6 + 1260*a^9*b^5*c^2*d^7 - 1323*a^10*b^4*c*d^8 + 343*a^11*b^3*d
^9)*sqrt(x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2
+ 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19
068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*
a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12
*d^12)/(a^9*b^11))^(1/4))/(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^
10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*
c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4
*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11
 + 2401*a^12*d^12)) - 3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a
*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^
8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c
^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*
d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(a^7*b^8*
(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*
b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*
c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*
d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*
b^11))^(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*
a^3*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3
*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x))
+ 3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*
a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b
^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*
c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(-a^7*b^8*(-(625*b^12*c^12 -
1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*
a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7
*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^
```

```
2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(3/4) + (125
*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3*b^6*c^6*d^3 + 3
42*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3*c^3*d^6 + 1260*a^7
*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x)) + 4*(4*a^2*b*d^3*x^
4 - 12*a*b^2*c^3 - (15*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 7*a^3*d^3)
*x^2)*sqrt(x))/(a^2*b^3*x^3 + a^3*b^2*x)
```

giac [A] time = 0.50, size = 504, normalized size = 1.37

$$\frac{2\,d^{3}x^{\frac{3}{2}}}{3\,b^{2}}-\frac{5\,b^{3}c^{3}x^{2}-3\,ab^{2}c^{2}dx^{2}+3\,a^{2}bcd^{2}x^{2}-a^{3}d^{3}x^{2}+4\,ab^{2}c^{3}}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^{2}b^{2}}-\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d-9\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+7\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{5}}-\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d-9\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+7\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{5}}+\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d-9\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+7\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{5}}-\frac{\sqrt{2}\left(5\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-3\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d-9\,\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+7\,\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*d^3*x^(3/2)/b^2 - 1/2*(5*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3*a^2*b*c*d^
2*x^2 - a^3*d^3*x^2 + 4*a*b^2*c^3)/((b*x^(5/2) + a*sqrt(x))*a^2*b^2) - 1/8*
sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^
(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^5) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4
)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^3*b^5) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)
^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^5) - 1/16*sqrt(2)*
(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^
2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^3*b^5)
```

maple [B] time = 0.02, size = 682, normalized size = 1.85

$$\frac{a\,d^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b^{2}}+\frac{3c^{2}d\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a}-\frac{b\,c^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a^{2}}-\frac{3c\,d^{2}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b}+\frac{2d^{3}x^{\frac{3}{2}}}{3b^{2}}-\frac{7\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}-\frac{7\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}-\frac{7\sqrt{2}\,a\,d^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{3}}+\frac{3\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{3\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{3\sqrt{2}\,c^{2}d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}-\frac{5\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{5\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{5\sqrt{2}\,c^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{9\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{9\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{9\sqrt{2}\,c\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{2c^{3}}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 2/3*d^3*x^(3/2)/b^2+1/2*a/b^2*x^(3/2)/(b*x^2+a)*d^3-3/2/b*x^(3/2)/(b*x^2+a)
*c*d^2+3/2/a*x^(3/2)/(b*x^2+a)*c^2*d-1/2/a^2*b*x^(3/2)/(b*x^2+a)*c^3-7/16*a
/b^3/(a/b)^(1/4)*2^(1/2)*d^3*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))
/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-7/8*a/b^3/(a/b)^(1/4)*2^(1/2)
*d^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-7/8*a/b^3/(a/b)^(1/4)*2^(1/2)*d^
3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/a/b/(a/b)^(1/4)*2^(1/2)*c^2*d*
ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2)))+3/8/a/b/(a/b)^(1/4)*2^(1/2)*c^2*d*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+3/8/a/b/(a/b)^(1/4)*2^(1/2)*c^2*d*arctan(2^(1/2)/(a/b)^(1/4)*
```

```
x^(1/2)-1)-5/16/a^2/(a/b)^(1/4)*2^(1/2)*c^3*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-5/8/a^2/(a/b)^
(1/4)*2^(1/2)*c^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/a^2/(a/b)^(1/4)
*2^(1/2)*c^3*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+9/16/b^2/(a/b)^(1/4)*2^(
1/2)*c*d^2*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2)))+9/8/b^2/(a/b)^(1/4)*2^(1/2)*c*d^2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+9/8/b^2/(a/b)^(1/4)*2^(1/2)*c*d^2*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)-2*c^3/a^2/x^(1/2)
```

maxima [A] time = 2.48, size = 304, normalized size = 0.83

$$\frac{2\,d^{3}x^{\frac{3}{2}}}{3\,b^{2}} - \frac{4\,ab^{2}c^{3} + \left(5\,b^{3}c^{3} - 3\,ab^{2}c^{2}d + 3\,a^{2}bcd^{2} - a^{3}d^{3}\right)x^{2}}{2\left(a^{2}b^{3}x^{\frac{5}{2}} + a^{3}b^{2}\sqrt{x}\right)} - \frac{\left(5\,b^{3}c^{3} - 3\,ab^{2}c^{2}d - 9\,a^{2}bcd^{2} + 7\,a^{3}d^{3}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2/3*d^3*x^(3/2)/b^2 - 1/2*(4*a*b^2*c^3 + (5*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2)/(a^2*b^3*x^(5/2) + a^3*b^2*sqrt(x)) - 1/16*(5*b^3*
c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 7*a^3*d^3)*(2*sqrt(2)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(
b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b^2)
```

mupad [B] time = 0.41, size = 657, normalized size = 1.79

$$\frac{\frac{x^2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-5b^3c^3\right)}{2a^2}-\frac{2b^2c^3}{a}}{b^3x^{5/2}+ab^2\sqrt{x}}+\frac{2d^3x^{3/2}}{3b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(ad-bc)^2\,(7ad+5bc)\left(1568a^{13}b^8d^6-4032a^{12}b^9cd^5+1248a^{11}b^{10}c^2d^4+3968a^{10}b^{11}c^3d^3-2592a^9b^{12}c^4d^2-960a^8b^{13}c^5d+800a^7b^{14}c^6\right)}{4\,(-a)^{9/4}b^{11/4}\left(2744a^{14}b^5d^9-10584a^{13}b^6cd^8+10080a^{12}b^7c^2d^7+9120a^{11}b^8c^3d^6-19440a^{10}b^9c^4d^5+2736a^9b^{10}c^5d^4+10464a^8b^{11}c^6d^3-4320a^7b^{12}c^7d^2-1800a^6b^{13}c^8d+1000a^5b^{14}c^9\right)}\right)(ad-bc)^2\,(7ad+5bc)}{4\,(-a)^{9/4}\,b^{11/4}}-\frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(ad-bc)^2\,(7ad+5bc)\left(1568a^{13}b^8d^6-4032a^{12}b^9cd^5+1248a^{11}b^{10}c^2d^4+3968a^{10}b^{11}c^3d^3-2592a^9b^{12}c^4d^2-960a^8b^{13}c^5d+800a^7b^{14}c^6\right)1\mathrm{i}}{4\,(-a)^{9/4}b^{11/4}\left(2744a^{14}b^5d^9-10584a^{13}b^6cd^8+10080a^{12}b^7c^2d^7+9120a^{11}b^8c^3d^6-19440a^{10}b^9c^4d^5+2736a^9b^{10}c^5d^4+10464a^8b^{11}c^6d^3-4320a^7b^{12}c^7d^2-1800a^6b^{13}c^8d+1000a^5b^{14}c^9\right)}\right)(ad-bc)^2\,(7ad+5bc)\,1\mathrm{i}}{4\,(-a)^{9/4}\,b^{11/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2) - (2*b
^2*c^3)/a)/(b^3*x^(5/2) + a*b^2*x^(1/2)) + (2*d^3*x^(3/2))/(3*b^2) - (atan(
(x^(1/2)*(a*d - b*c)^2*(7*a*d + 5*b*c)*(800*a^7*b^14*c^6 + 1568*a^13*b^8*d^
6 - 960*a^8*b^13*c^5*d - 4032*a^12*b^9*c*d^5 - 2592*a^9*b^12*c^4*d^2 + 3968
*a^10*b^11*c^3*d^3 + 1248*a^11*b^10*c^2*d^4))/(4*(-a)^(9/4)*b^(11/4)*(1000*
a^5*b^14*c^9 + 2744*a^14*b^5*d^9 - 1800*a^6*b^13*c^8*d - 10584*a^13*b^6*c*d
^8 - 4320*a^7*b^12*c^7*d^2 + 10464*a^8*b^11*c^6*d^3 + 2736*a^9*b^10*c^5*d^4
 - 19440*a^10*b^9*c^4*d^5 + 9120*a^11*b^8*c^3*d^6 + 10080*a^12*b^7*c^2*d^7)
))*(a*d - b*c)^2*(7*a*d + 5*b*c))/(4*(-a)^(9/4)*b^(11/4)) - (atan((x^(1/2)*
```

```
(a*d - b*c)^2*(7*a*d + 5*b*c)*(800*a^7*b^14*c^6 + 1568*a^13*b^8*d^6 - 960*a
^8*b^13*c^5*d - 4032*a^12*b^9*c*d^5 - 2592*a^9*b^12*c^4*d^2 + 3968*a^10*b^1
1*c^3*d^3 + 1248*a^11*b^10*c^2*d^4)*1i)/(4*(-a)^(9/4)*b^(11/4)*(1000*a^5*b^
14*c^9 + 2744*a^14*b^5*d^9 - 1800*a^6*b^13*c^8*d - 10584*a^13*b^6*c*d^8 - 4
320*a^7*b^12*c^7*d^2 + 10464*a^8*b^11*c^6*d^3 + 2736*a^9*b^10*c^5*d^4 - 194
40*a^10*b^9*c^4*d^5 + 9120*a^11*b^8*c^3*d^6 + 10080*a^12*b^7*c^2*d^7)))*(a*
d - b*c)^2*(7*a*d + 5*b*c)*1i)/(4*(-a)^(9/4)*b^(11/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.440 $\int \frac{\left(c+dx^2\right)^3}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=367

$$\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{8\sqrt{2}a^{11/4}b^{9/4}}$$

Rubi [A] time = 0.41, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {466, 468, 570, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}+\frac{(bc-ad)^2(5ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}b^{9/4}}-\frac{(bc-ad)^2(5ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}b^{9/4}}-\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}}-\frac{d^2\sqrt{x}(bc-5ad)}{2ab^2}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(7*b*c - 3*a*d))/(6*a^2*b*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqrt[x])/(2*a
*b^2) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(3/2)*(a + b*x^2)) + ((b*c - a
*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) + ((b*c - a*d)^2*(
7*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*a^(11/4)*b^(9/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{5/2}\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^4\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(-c(7bc-3ad)+d(bc-5ad)x^4\right)}{x^4\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{d^2(bc-5ad)}{b} + \frac{c^2(-7bc+3ad)}{ax^4} + \frac{(-bc+ad)^2(7bc+5ad)}{ab\left(a+bx^4\right)}\right)\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} - \frac{\left((bc-ad)^2(7bc+5ad)\right)\text{Sub}}{2a^2b^2}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} - \frac{\left((bc-ad)^2(7bc+5ad)\right)\text{Sub}}{4a^{5/2}}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} - \frac{\left((bc-ad)^2(7bc+5ad)\right)\text{Sub}}{8a}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} + \frac{(bc-ad)^2(7bc+5ad)\log\left(\right.}{8\sqrt{2}\,a}$$

$$= -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}} - \frac{d^2(bc-5ad)\sqrt{x}}{2ab^2} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{3/2}\left(a+bx^2\right)} + \frac{(bc-ad)^2(7bc+5ad)\tan^{-1}}{4\sqrt{2}\,a^{11/4}b^{9/4}}$$

Mathematica [C] time = 2.67, size = 350, normalized size = 0.95

$$-\frac{32768b^3x^6\left(c+dx^2\right)^3 {}_5F_4\left(\frac{1}{4},2,2,2,2;1,1,1,\frac{17}{4};-\frac{bx^2}{a}\right)+39a\left(15a^2\left(2187c^3+6561c^2dx^2+6561cd^2x^4+1547d^3x^6\right)+6abx^2\left(469c^3+1407c^2dx^2+2367cd^2x^4+789d^3x^6\right)-5b^2x^4\left(-869c^3+81c^2dx^2+81cd^2x^4+27d^3x^6\right)\right)-585\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^3\left(2187c^3+6561c^2dx^2+6561cd^2x^4+1547d^3x^6\right)+a^2bx^2\left(625c^3+1875c^2dx^2+2259cd^2x^4+625d^3x^6\right)+ab^2x^4\left(c^3+1155c^2dx^2+3cd^2x^4+d^3x^6\right)+b^3x^6\left(-869c^3+81c^2dx^2+81cd^2x^4+27d^3x^6\right)\right)}{149760a^3b^2x^{11/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/149760*(39*a*(-5*b^2*x^4*(-869*c^3 + 81*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^
3*x^6) + 6*a*b*x^2*(469*c^3 + 1407*c^2*d*x^2 + 2367*c*d^2*x^4 + 789*d^3*x^6
) + 15*a^2*(2187*c^3 + 6561*c^2*d*x^2 + 6561*c*d^2*x^4 + 1547*d^3*x^6)) - 5
85*(a*b^2*x^4*(c^3 + 1155*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + b^3*x^6*(-86
9*c^3 + 81*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) + a^2*b*x^2*(625*c^3 + 18
75*c^2*d*x^2 + 2259*c*d^2*x^4 + 625*d^3*x^6) + a^3*(2187*c^3 + 6561*c^2*d*x
^2 + 6561*c*d^2*x^4 + 1547*d^3*x^6))*Hypergeometric2F1[1/4, 1, 5/4, -((b*x^
2)/a)] + 32768*b^3*x^6*(c + d*x^2)^3*HypergeometricPFQ[{1/4, 2, 2, 2, 2}, {
1, 1, 1, 17/4}, -((b*x^2)/a)])/(a^3*b^2*x^(11/2))
```

IntegrateAlgebraic [A] time = 0.55, size = 244, normalized size = 0.66

$$\frac{(5ad+7bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{11/4}b^{9/4}}-\frac{(5ad+7bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{11/4}b^{9/4}}+\frac{15a^3d^3x^2-9a^2bcd^2x^2+12a^2bd^3x^4-4ab^2c^3+9ab^2c^2dx^2-7b^3c^3x^2}{6a^2b^2x^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a*b^2*c^3 - 7*b^3*c^3*x^2 + 9*a*b^2*c^2*d*x^2 - 9*a^2*b*c*d^2*x^2 + 15*
a^3*d^3*x^2 + 12*a^2*b*d^3*x^4)/(6*a^2*b^2*x^(3/2)*(a + b*x^2)) + ((-(b*c)
+ a*d)^2*(7*b*c + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])])/(4*Sqrt[2]*a^(11/4)*b^(9/4)) - ((-(b*c) + a*d)^2*(7*b*c + 5*
a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*S
qrt[2]*a^(11/4)*b^(9/4))
```

fricas [B] time = 1.42, size = 1967, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(12*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11
*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4
 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 +
1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 150
0*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*arctan((sqrt(a^6*b^4*sqr
t(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a
^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*
b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*
c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^1
1*b^9)) + (49*b^6*c^6 - 126*a*b^5*c^5*d + 39*a^2*b^4*c^4*d^2 + 124*a^3*b^3*
```

```
c^3*d^3 - 81*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 25*a^6*d^6)*x)*a^8*b^7*(-(2
401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^
9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c
^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d
^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9
))^(3/4) - (7*a^8*b^10*c^3 - 9*a^9*b^9*c^2*d - 3*a^10*b^8*c*d^2 + 5*a^11*b^
7*d^3)*sqrt(x)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^1
0*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^
5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 +
11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a
^12*d^12)/(a^11*b^9))^(3/4))/(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*
a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^
5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^
4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*
d^11 + 625*a^12*d^12)) + 3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 -
12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 3766
5*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a
^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b
^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(a^3
*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 23
24*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*
a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*
b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/
(a^11*b^9))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)
*sqrt(x)) - 3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*
c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8
*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^
7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 -
 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(-a^3*b^2*(-(2401
*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c
^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*
d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9
- 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^
(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(x)) -
4*(12*a^2*b*d^3*x^4 - 4*a*b^2*c^3 - (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 15*a^3*d^3)*x^2)*sqrt(x))/(a^2*b^3*x^4 + a^3*b^2*x^2)
```

giac [**A**] time = 0.48, size = 501, normalized size = 1.37

$$\frac{2\,d^{3}\sqrt{x}}{b^{2}}-\frac{2\,c^{3}}{3\,a^{2}x^{\frac{3}{2}}}-\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-9\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-3\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+5\,\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{3}}-\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-9\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-3\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+5\,\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{3}b^{3}}-\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-9\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-3\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+5\,\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{3}}+\frac{\sqrt{2}\left(7\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-9\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-3\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+5\,\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{3}b^{3}}-\frac{b^{3}c^{3}\sqrt{x}-3\,ab^{2}c^{2}d\sqrt{x}+3\,a^{2}bcd^{2}\sqrt{x}-a^{3}d^{3}\sqrt{x}}{2\left(bx^{2}+a\right)a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 2*d^3*sqrt(x)/b^2 - 2/3*c^3/(a^2*x^(3/2)) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^
```

```
3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^
3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(a^3*b^3) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)
*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 1
/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b
^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 -
9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)
*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) - 1/2
*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3
*sqrt(x))/((b*x^2 + a)*a^2*b^2)
```

maple [B] time = 0.02, size = 682, normalized size = 1.86

$$\frac{a d^3\sqrt{x}}{2\left(b x^2+a\right)b^2}+\frac{3c^2 d\sqrt{x}}{2\left(b x^2+a\right)a}-\frac{b c^3\sqrt{x}}{2\left(b x^2+a\right)a^2}-\frac{3c d^2\sqrt{x}}{2\left(b x^2+a\right)b}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, c d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, c d^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, c d^2\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, c^2 d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^2}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, c^2 d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^2}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, c^2 d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^2}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^3}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^3}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b c^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^3}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, d^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16b^2}+\frac{2d^3\sqrt{x}}{b^2}-\frac{2c^3}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] 2*d^3/b^2*x^(1/2)+1/2*a/b^2*x^(1/2)/(b*x^2+a)*d^3-3/2/b*x^(1/2)/(b*x^2+a)*c
*d^2+3/2/a*x^(1/2)/(b*x^2+a)*c^2*d-1/2/a^2*b*x^(1/2)/(b*x^2+a)*c^3-5/8/b^2*
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/8/a/b*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2+9/8/a^2*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d-7/8/a^3*b*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3-5/8/b^2*(a/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/8/a/b*(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2+9/8/a^2*(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d-7/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3-5/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x+(
a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2)))*d^3+3/16/a/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2+9/16/a^2*
(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d-7/16/a^3*b*(a/b)^(1/4)*2^(1/2)*l
n((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2)))*c^3-2/3*c^3/a^2/x^(3/2)
```

maxima [A] time = 2.47, size = 415, normalized size = 1.13

$$\frac{2\,d^3\sqrt{x}}{b^2}-\frac{4\,ab^2c^3+\left(7\,b^3c^3-9\,ab^2c^2d+9\,a^2bcd^2-3\,a^3d^3\right)x^2}{6\left(a^2b^3x^{\frac{7}{2}}+a^3b^2x^{\frac{3}{2}}\right)}-\frac{\frac{2\sqrt{2}\left(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\left(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(7\,b^3c^3-9\,ab^2c^2d-3\,a^2bcd^2+5\,a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 2*d^3*sqrt(x)/b^2 - 1/6*(4*a*b^2*c^3 + (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 - 3*a^3*d^3)*x^2)/(a^2*b^3*x^(7/2) + a^3*b^2*x^(3/2)) - 1/16*(2*sqrt
(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^
2*b*c*d^2 + 5*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt
(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*
b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

mupad [B] time = 0.47, size = 1759, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x)
```

```
[Out] ((x^2*(3*a^3*d^3 - 7*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(6*a^2) - (2
*b^2*c^3)/(3*a))/(b^3*x^(7/2) + a*b^2*x^(3/2)) + (2*d^3*x^(1/2))/b^2 - (ata
n((((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d -
960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*
a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 12
80*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11
/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4)) + (
(x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*
a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10
*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a
^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11/4)*
b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4)))/(((x^(
1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11
*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^1
1*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*
b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11/4)*b^(9
/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4)) - ((x^(1/2)*(1
568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c
*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d
^4) + ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^
3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-a)^(11/4)*b^(9/4)))*(
a*d - b*c)^2*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4))))*(a*d - b*c)^2*(5*a*
d + 7*b*c)*1i)/(4*(-a)^(11/4)*b^(9/4)) - (atan((((x^(1/2)*(1568*a^6*b^15*c^
6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8
*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) - ((a*d - b
```

```
*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b
^13*c^2*d - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2
*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4)) + ((x^(1/2)*(1568*a^6*b^15*c^6 +
800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^1
3*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) + ((a*d - b*c)^
2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*
c^2*d - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*
a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4)))/(((x^(1/2)*(1568*a^6*b^15*c^6 + 800*
a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^
4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5
*a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*
d - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d
+ 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4)) - ((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a
^12*b^9*d^6 - 4032*a^7*b^14*c^5*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4
*d^2 + 3968*a^9*b^12*c^3*d^3 - 2592*a^10*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*
a*d + 7*b*c)*(1792*a^9*b^14*c^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d
 - 768*a^11*b^12*c*d^2)*1i)/(8*(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d +
 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/4))))*(a*d - b*c)^2*(5*a*d + 7*b*c))/(4*(-a
)^(11/4)*b^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.441 $\int \frac{\left(c+dx^2\right)^3}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=376

$$\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}}-\frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{}\right.}{8\sqrt{2}a^{13/4}b^{7/4}}$$

Rubi [A] time = 0.43, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 468, 570, 297, 1162, 617, 204, 1165, 628}

$$\frac{c\left(2a^2d^2-15abcd+9b^2c^2\right)}{2a^3b\sqrt{x}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}}-\frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}}-\frac{3(bc-ad)^2(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}b^{7/4}}+\frac{3(bc-ad)^2(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{13/4}b^{7/4}}-\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(9*b*c - 5*a*d))/(10*a^2*b*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*b*c*d + 2*
a^2*d^2))/(2*a^3*b*Sqrt[x]) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(5/2)*(a
 + b*x^2)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) + (3*(b*c - a*d)^2*(3*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*b^(7/
4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(b*c - a*d)^2*(3*b*
c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqr
t[2]*a^(13/4)*b^(7/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^3}{x^{7/2}\left(a+bx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{\left(c+dx^4\right)^3}{x^6\left(a+bx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\left(c+dx^4\right)\left(-c(9bc-5ad)-d(bc+3ad)x^4\right)}{x^6\left(a+bx^4\right)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{c^2(-9bc+5ad)}{ax^6} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{a^2x^2} - \frac{3(-bc+ad)^2(3bc+ad)x^2}{a^2\left(a+bx^4\right)}\right)\right.}{2ab}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\left(3(bc-ad)^2(3bc\right.}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{\left(3(bc-ad)^2(3bc\right.}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} + \frac{\left(3(bc-ad)^2(3bc\right.}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} + \frac{3(bc-ad)^2(3bc}{}$$

$$= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}} + \frac{c\left(9b^2c^2-15abcd+2a^2d^2\right)}{2a^3b\sqrt{x}} + \frac{(bc-ad)\left(c+dx^2\right)^2}{2abx^{5/2}\left(a+bx^2\right)} - \frac{3(bc-ad)^2(3bc}{4}$$

Mathematica [C] time = 2.19, size = 353, normalized size = 0.94

$$-\frac{-491520ab^2x^4\left(c+dx^2\right)^3 {}_5F_4\left(-\frac{1}{4},2,2,2,2;1,1,1,\frac{15}{4};-\frac{bx^2}{a}\right) - 77a\left(5a^2\left(2401c^3+7203c^2dx^2+7203cd^2x^4+1249d^3x^6\right)+6abx^2\left(-1423c^3-13485c^2dx^2+915cd^2x^4+305d^3x^6\right)-15b^2x^4\left(-2831c^3+1875c^2dx^2+1875cd^2x^4+625d^3x^6\right)\right)+385\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\frac{bx^2}{a}\right)\left(a^3\left(2401c^3+7203c^2dx^2+7203cd^2x^4+1249d^3x^6\right)+9a^2bx^2\left(27c^3+81c^2dx^2-47cd^2x^4+27d^3x^6\right)+3ab^2x^4\left(c^3+1923c^2dx^2+3cd^2x^4+d^3x^6\right)+b^3x^6\left(-2831c^3+1875c^2dx^2+1875cd^2x^4+625d^3x^6\right)\right)}{887040a^4bx^{9/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/887040*(-77*a*(6*a*b*x^2*(-1423*c^3 - 13485*c^2*d*x^2 + 915*c*d^2*x^4 +
305*d^3*x^6) - 15*b^2*x^4*(-2831*c^3 + 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 62
5*d^3*x^6) + 5*a^2*(2401*c^3 + 7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 1249*d^3*x
^6)) + 385*(3*a*b^2*x^4*(c^3 + 1923*c^2*d*x^2 + 3*c*d^2*x^4 + d^3*x^6) + 9*
a^2*b*x^2*(27*c^3 + 81*c^2*d*x^2 - 47*c*d^2*x^4 + 27*d^3*x^6) + b^3*x^6*(-2
831*c^3 + 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 625*d^3*x^6) + a^3*(2401*c^3 +
7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 1249*d^3*x^6))*Hypergeometric2F1[3/4, 1,
7/4, -((b*x^2)/a)] - 491520*a*b^2*x^4*(c + d*x^2)^3*HypergeometricPFQ[{-1/4
, 2, 2, 2, 2}, {1, 1, 1, 15/4}, -((b*x^2)/a)])/(a^4*b*x^(9/2))
```

IntegrateAlgebraic [A] time = 0.66, size = 255, normalized size = 0.68

$$-\frac{3(ad+3bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{13/4}b^{7/4}}-\frac{3(ad+3bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{13/4}b^{7/4}}+\frac{-5a^3d^3x^4-4a^2bc^3-60a^2bc^2dx^2+15a^2bcd^2x^4+36ab^2c^3x^2-75ab^2c^2dx^4+45b^3c^3x^4}{10a^3bx^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

```
[Out] (-4*a^2*b*c^3 + 36*a*b^2*c^3*x^2 - 60*a^2*b*c^2*d*x^2 + 45*b^3*c^3*x^4 - 75
*a*b^2*c^2*d*x^4 + 15*a^2*b*c*d^2*x^4 - 5*a^3*d^3*x^4)/(10*a^3*b*x^(5/2)*(a
 + b*x^2)) - (3*(-(b*c) + a*d)^2*(3*b*c + a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)
/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(13/4)*b^(7/4)) - (3*(-(b
*c) + a*d)^2*(3*b*c + a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[
a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(13/4)*b^(7/4))
```

fricas [B] time = 1.20, size = 2549, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(60*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a
^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^
4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*
d^12)/(a^13*b^7))^(1/4)*arctan((sqrt((729*b^18*c^18 - 7290*a*b^17*c^17*d +
31833*a^2*b^16*c^16*d^2 - 78192*a^3*b^15*c^15*d^3 + 113940*a^4*b^14*c^14*d^
4 - 88920*a^5*b^13*c^13*d^5 + 10180*a^6*b^12*c^12*d^6 + 46320*a^7*b^11*c^11
*d^7 - 35970*a^8*b^10*c^10*d^8 - 220*a^9*b^9*c^9*d^9 + 12078*a^10*b^8*c^8*d
^10 - 3600*a^11*b^7*c^7*d^11 - 1884*a^12*b^6*c^6*d^12 + 936*a^13*b^5*c^5*d^
13 + 180*a^14*b^4*c^4*d^14 - 112*a^15*b^3*c^3*d^15 - 15*a^16*b^2*c^2*d^16 +
```

```
 6*a^17*b*c*d^17 + a^18*d^18)*x - (81*a^7*b^15*c^12 - 540*a^8*b^14*c^11*d +
 1458*a^9*b^13*c^10*d^2 - 1932*a^10*b^12*c^9*d^3 + 1039*a^11*b^11*c^8*d^4 +
 328*a^12*b^10*c^7*d^5 - 644*a^13*b^9*c^6*d^6 + 136*a^14*b^8*c^5*d^7 + 127*
a^15*b^7*c^4*d^8 - 44*a^16*b^6*c^3*d^9 - 14*a^17*b^5*c^2*d^10 + 4*a^18*b^4*
c*d^11 + a^19*b^3*d^12)*sqrt(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*
b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c
^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 -
44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a
^13*b^7)))*a^3*b^2*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10
*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 -
644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^
3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))
^(1/4) - (27*a^3*b^11*c^9 - 135*a^4*b^10*c^8*d + 252*a^5*b^9*c^7*d^2 - 188*
a^6*b^8*c^6*d^3 - 6*a^7*b^7*c^5*d^4 + 78*a^8*b^6*c^4*d^5 - 20*a^9*b^5*c^3*d
^6 - 12*a^10*b^4*c^2*d^7 + 3*a^11*b^3*c*d^8 + a^12*b^2*d^9)*sqrt(x)*(-(81*b
^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^
3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*
a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^
2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4))/(81*b^12*c^12 - 54
0*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*
b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d
^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^
11*b*c*d^11 + a^12*d^12)) - 15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 -
540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^
4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5
*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*
a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(27*a^10*b^5*(-(81*b^12*c^1
2 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 103
9*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5
*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10
+ 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b
^8*c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 +
78*a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^
8 + a^9*d^9)*sqrt(x)) + 15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*
a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^
8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7
 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11
*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(-27*a^10*b^5*(-(81*b^12*c^12 -
 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a
^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^
5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4
*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b^8*
c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*
a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 +
 a^9*d^9)*sqrt(x)) + 4*(4*a^2*b*c^3 - 5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2
```

```
*b*c*d^2 - a^3*d^3)*x^4 - 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(x))/(a
^3*b^2*x^5 + a^4*b*x^3)
```

giac [A] time = 0.57, size = 505, normalized size = 1.34

$$\frac{b^{3}c^{3}x^{\frac{3}{2}}-3\,ab^{2}c^{2}dx^{\frac{3}{2}}+3\,a^{2}bcd^{2}x^{\frac{3}{2}}-a^{3}d^{3}x^{\frac{3}{2}}}{2\left(bx^{2}+a\right)a^{3}b}+\frac{2\left(10\,bc^{3}x^{2}-15\,ac^{2}dx^{2}-ac^{3}\right)}{5\,a^{3}x^{\frac{5}{2}}}+\frac{3\,\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-5\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}b^{4}}+\frac{3\,\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-5\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}b^{4}}-\frac{3\,\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-5\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{4}b^{4}}+\frac{3\,\sqrt{2}\left(3\,\left(ab^{3}\right)^{\frac{3}{4}}b^{3}c^{3}-5\,\left(ab^{3}\right)^{\frac{3}{4}}ab^{2}c^{2}d+\left(ab^{3}\right)^{\frac{3}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{3}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{4}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3*
d^3*x^(3/2))/((b*x^2 + a)*a^3*b) + 2/5*(10*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c
^3)/(a^3*x^(5/2)) + 3/8*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*
a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) + 3/8*sqr
t(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)
*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) - 3/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3
*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3
/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^4) + 3
/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3
)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^4*b^4)
```

maple [B] time = 0.02, size = 697, normalized size = 1.85

$$\frac{3c\,d^{2}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a}-\frac{3b\,c^{2}d\,x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a^{2}}+\frac{b^{2}c^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)a^{3}}-\frac{d^{3}x^{\frac{3}{2}}}{2\left(b\,x^{2}+a\right)b}+\frac{3\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{3\sqrt{2}\,c\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}+\frac{3\sqrt{2}\,c\,d^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}ab}-\frac{15\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{15\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{15\sqrt{2}\,c^{2}d\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{9\sqrt{2}\,b\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{9\sqrt{2}\,b\,c^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{9\sqrt{2}\,b\,c^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{3}}+\frac{3\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}+\frac{3\sqrt{2}\,d^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(\frac{a}{b}\right)^{\frac{1}{4}}b^{2}}-\frac{6c^{2}d}{a^{2}\sqrt{x}}+\frac{4b\,c^{3}}{a^{3}\sqrt{x}}-\frac{2c^{3}}{5a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x^(3/2)/(b*x^2+a)*d^3+3/2/a*x^(3/2)/(b*x^2+a)*c*d^2-3/2/a^2*b*x^(3/2
)/(b*x^2+a)*c^2*d+1/2/a^3*b^2*x^(3/2)/(b*x^2+a)*c^3+3/8/b^2/(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+3/8/a/b/(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2-15/8/a^2/(a/b)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+9/8/a^3*b/(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+3/8/b^2/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+3/8/a/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-15/8/a^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)*c^2*d+9/8/a^3*b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)*c^3+3/16/b^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d^3
+3/16/a/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)
)/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2-15/16/a^2/(a/b)^(1/4)*
```

```
2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/
2)*x^(1/2)+(a/b)^(1/2)))*c^2*d+9/16/a^3*b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(
1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
)))*c^3-2/5*c^3/a^2/x^(5/2)-6*c^2/a^2/x^(1/2)*d+4*c^3/a^3/x^(1/2)*b
```

maxima [A] time = 2.54, size = 315, normalized size = 0.84

$$-\frac{4\,a^{2}bc^{3}-5\left(9\,b^{3}c^{3}-15\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)x^{4}-12\left(3\,ab^{2}c^{3}-5\,a^{2}bc^{2}d\right)x^{2}}{10\left(a^{3}b^{2}x^{\frac{9}{2}}+a^{4}bx^{\frac{5}{2}}\right)}+\frac{3\left(3\,b^{3}c^{3}-5\,ab^{2}c^{2}d+a^{2}bcd^{2}+a^{3}d^{3}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/10*(4*a^2*b*c^3 - 5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*x^4 - 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^(9/2) + a^4*b*x^(
5/2)) + 3/16*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(
1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*b)
```

mupad [B] time = 0.25, size = 656, normalized size = 1.74

$$\frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{x}\,\left(a\,d-b\,c\right)^2\,\left(a\,d+3\,b\,c\right)\,\left(288\,a^{16}\,b^5\,d^6+576\,a^{15}\,b^6\,c\,d^5-2592\,a^{14}\,b^7\,c^2\,d^4-1152\,a^{13}\,b^8\,c^3\,d^3+8928\,a^{12}\,b^9\,c^4\,d^2-8640\,a^{11}\,b^{10}\,c^5\,d+2592\,a^{10}\,b^{11}\,c^6\right)}{4\,{\left(-a\right)}^{13/4}\,b^{7/4}\,\left(216\,a^{16}\,b^3\,d^9+648\,a^{15}\,b^4\,c\,d^8-2592\,a^{14}\,b^5\,c^2\,d^7-4320\,a^{13}\,b^6\,c^3\,d^6+16848\,a^{12}\,b^7\,c^4\,d^5-1296\,a^{11}\,b^8\,c^5\,d^4-40608\,a^{10}\,b^9\,c^6\,d^3+54432\,a^9\,b^{10}\,c^7\,d^2-29160\,a^8\,b^{11}\,c^8\,d+5832\,a^7\,b^{12}\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(a\,d+3\,b\,c\right)}{4\,{\left(-a\right)}^{13/4}\,b^{7/4}}-\frac{\frac{2\,c^3}{5\,a}+\frac{x^4\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d-9\,b^3\,c^3\right)}{2\,a^3\,b}+\frac{6\,c^2\,x^2\,\left(5\,a\,d-3\,b\,c\right)}{5\,a^2}}{a\,x^{5/2}+b\,x^{9/2}}-\frac{3\,\mathrm{atanh}\left(\frac{3\,\sqrt{x}\,\left(a\,d-b\,c\right)^2\,\left(a\,d+3\,b\,c\right)\,\left(288\,a^{16}\,b^5\,d^6+576\,a^{15}\,b^6\,c\,d^5-2592\,a^{14}\,b^7\,c^2\,d^4-1152\,a^{13}\,b^8\,c^3\,d^3+8928\,a^{12}\,b^9\,c^4\,d^2-8640\,a^{11}\,b^{10}\,c^5\,d+2592\,a^{10}\,b^{11}\,c^6\right)}{4\,{\left(-a\right)}^{13/4}\,b^{7/4}\,\left(216\,a^{16}\,b^3\,d^9+648\,a^{15}\,b^4\,c\,d^8-2592\,a^{14}\,b^5\,c^2\,d^7-4320\,a^{13}\,b^6\,c^3\,d^6+16848\,a^{12}\,b^7\,c^4\,d^5-1296\,a^{11}\,b^8\,c^5\,d^4-40608\,a^{10}\,b^9\,c^6\,d^3+54432\,a^9\,b^{10}\,c^7\,d^2-29160\,a^8\,b^{11}\,c^8\,d+5832\,a^7\,b^{12}\,c^9\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(a\,d+3\,b\,c\right)}{4\,{\left(-a\right)}^{13/4}\,b^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x)
```

```
[Out] (3*atan((3*x^(1/2)*(a*d - b*c)^2*(a*d + 3*b*c)*(2592*a^10*b^11*c^6 + 288*a^
16*b^5*d^6 - 8640*a^11*b^10*c^5*d + 576*a^15*b^6*c*d^5 + 8928*a^12*b^9*c^4*
d^2 - 1152*a^13*b^8*c^3*d^3 - 2592*a^14*b^7*c^2*d^4))/(4*(-a)^(13/4)*b^(7/4
)*(5832*a^7*b^12*c^9 + 216*a^16*b^3*d^9 - 29160*a^8*b^11*c^8*d + 648*a^15*b
^4*c*d^8 + 54432*a^9*b^10*c^7*d^2 - 40608*a^10*b^9*c^6*d^3 - 1296*a^11*b^8*
c^5*d^4 + 16848*a^12*b^7*c^4*d^5 - 4320*a^13*b^6*c^3*d^6 - 2592*a^14*b^5*c^
2*d^7)))*(a*d - b*c)^2*(a*d + 3*b*c))/(4*(-a)^(13/4)*b^(7/4)) - ((2*c^3)/(5
*a) + (x^4*(a^3*d^3 - 9*b^3*c^3 + 15*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^3*b
) + (6*c^2*x^2*(5*a*d - 3*b*c))/(5*a^2))/(a*x^(5/2) + b*x^(9/2)) - (3*atanh
((3*x^(1/2)*(a*d - b*c)^2*(a*d + 3*b*c)*(2592*a^10*b^11*c^6 + 288*a^16*b^5*
d^6 - 8640*a^11*b^10*c^5*d + 576*a^15*b^6*c*d^5 + 8928*a^12*b^9*c^4*d^2 - 1
152*a^13*b^8*c^3*d^3 - 2592*a^14*b^7*c^2*d^4))/(4*(-a)^(13/4)*b^(7/4)*(5832
*a^7*b^12*c^9 + 216*a^16*b^3*d^9 - 29160*a^8*b^11*c^8*d + 648*a^15*b^4*c*d^
```

```
8 + 54432*a^9*b^10*c^7*d^2 - 40608*a^10*b^9*c^6*d^3 - 1296*a^11*b^8*c^5*d^4
 + 16848*a^12*b^7*c^4*d^5 - 4320*a^13*b^6*c^3*d^6 - 2592*a^14*b^5*c^2*d^7))
)*(a*d - b*c)^2*(a*d + 3*b*c))/(4*(-a)^(13/4)*b^(7/4))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.442 $\int \frac{\left(c+dx^2\right)^3}{x^{9/2}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=376

$$-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\right.}{8\sqrt{2}a^{15/4}b^{5/4}}$$

Rubi [A] time = 0.42, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {466, 468, 570, 211, 1165, 628, 1162, 617, 204}

$$\frac{c\left(6a^2d^2-21abcd+11b^2c^2\right)}{6a^3bx^{3/2}}-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}-\frac{(bc-ad)^2(ad+11bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{15/4}b^{5/4}}+\frac{(bc-ad)^2(ad+11bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{15/4}b^{5/4}}-\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] -(c^2*(11*b*c - 7*a*d))/(14*a^2*b*x^(7/2)) + (c*(11*b^2*c^2 - 21*a*b*c*d +
6*a^2*d^2))/(6*a^3*b*x^(3/2)) + ((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^(7/2)*
(a + b*x^2)) - ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(15/4)*b^(5/
4)) - ((b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(15/4)*b^(5/4)) + ((b*c - a*d)^2*(11*b*c
+ a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(15/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{(c+dx^4)^3}{x^8(a+bx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{\mathrm{Subst}\left(\int \frac{(c+dx^4)(-c(11bc-7ad)-d(3bc+ad)x^4)}{x^8(a+bx^4)}\,dx, x, \sqrt{x}\right)}{2ab}$$

$$= \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{\mathrm{Subst}\left(\int \left(\frac{c^2(-11bc+7ad)}{ax^8} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{a^2x^4} - \frac{(-bc+ad)^2(11bc+ad)}{a^2(a+bx^4)}\right)\right.}{2ab}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad)^2(11}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad)^2(11}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} + \frac{((bc-ad)^2(11}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)^2(11b}{}$$

$$= -\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} + \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)^2(11b}{}$$

Mathematica [C] time = 2.40, size = 353, normalized size = 0.94

$$-\frac{-229376ab^2x^4(c+dx^2)^3\,{}_5F_4\left(-\frac{3}{4},2,2,2,2;1,1,1,\frac{13}{4};-\frac{bx^2}{a}\right) - 15a\left(21a^2\left(625c^3+1875c^2dx^2+1875cd^2x^4+241d^3x^6\right)+6abx^2\left(-1195c^3-6657c^2dx^2+2751cd^2x^4+917d^3x^6\right)-7b^2x^4\left(-1823c^3+7203c^2dx^2+7203cd^2x^4+2401d^3x^6\right)\right)+315\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};-\frac{bx^2}{a}\right)\left(a^3\left(625c^3+1875c^2dx^2+1875cd^2x^4+241d^3x^6\right)+3a^2bx^2\left(c^3+3c^2dx^2-1149cd^2x^4+d^3x^6\right)+9ab^2x^4\left(27c^3+977c^2dx^2+81cd^2x^4+27d^3x^6\right)+b^3x^6\left(-1823c^3+7203c^2dx^2+7203cd^2x^4+2401d^3x^6\right)\right)}{241920a^4bx^{11/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] -1/241920*(-15*a*(21*a^2*(625*c^3 + 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 241*d
^3*x^6) + 6*a*b*x^2*(-1195*c^3 - 6657*c^2*d*x^2 + 2751*c*d^2*x^4 + 917*d^3*
x^6) - 7*b^2*x^4*(-1823*c^3 + 7203*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^
6)) + 315*(3*a^2*b*x^2*(c^3 + 3*c^2*d*x^2 - 1149*c*d^2*x^4 + d^3*x^6) + 9*a
*b^2*x^4*(27*c^3 + 977*c^2*d*x^2 + 81*c*d^2*x^4 + 27*d^3*x^6) + a^3*(625*c^
3 + 1875*c^2*d*x^2 + 1875*c*d^2*x^4 + 241*d^3*x^6) + b^3*x^6*(-1823*c^3 + 7
203*c^2*d*x^2 + 7203*c*d^2*x^4 + 2401*d^3*x^6))*Hypergeometric2F1[1/4, 1, 5
/4, -((b*x^2)/a)] - 229376*a*b^2*x^4*(c + d*x^2)^3*HypergeometricPFQ[{-3/4,
 2, 2, 2, 2}, {1, 1, 1, 13/4}, -((b*x^2)/a)])/(a^4*b*x^(11/2))
```

IntegrateAlgebraic [**A**] time = 0.64, size = 255, normalized size = 0.68

$$-\frac{(ad+11bc)(ad-bc)^2\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{(ad+11bc)(ad-bc)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{15/4}b^{5/4}}+\frac{-21a^3d^3x^4-12a^2bc^3-84a^2bc^2dx^2+63a^2bcd^2x^4+44ab^2c^3x^2-147ab^2c^2dx^4+77b^3c^3x^4}{42a^3bx^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

```
[Out] (-12*a^2*b*c^3 + 44*a*b^2*c^3*x^2 - 84*a^2*b*c^2*d*x^2 + 77*b^3*c^3*x^4 - 1
47*a*b^2*c^2*d*x^4 + 63*a^2*b*c*d^2*x^4 - 21*a^3*d^3*x^4)/(42*a^3*b*x^(7/2)
*(a + b*x^2)) - ((-(b*c) + a*d)^2*(11*b*c + a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*
x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(15/4)*b^(5/4)) + ((-(b
*c) + a*d)^2*(11*b*c + a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt
[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(15/4)*b^(5/4))
```

fricas [**B**] time = 0.94, size = 1955, normalized size = 5.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(84*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11
*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8
*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d
^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 +
36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*arctan((sqrt(a^8*b^2*sqrt(-
(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588
*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188
*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b
^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^
5)) + (121*b^6*c^6 - 462*a*b^5*c^5*d + 639*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^
```

```
3*d^3 + 39*a^4*b^2*c^2*d^4 + 18*a^5*b*c*d^5 + a^6*d^6)*x)*a^11*b^4*(-(14641
*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b
^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b
^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3
*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(3
/4) - (11*a^11*b^7*c^3 - 21*a^12*b^6*c^2*d + 9*a^13*b^5*c*d^2 + a^14*b^4*d^
3)*sqrt(x)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10
*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7
*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^
8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d
^12)/(a^15*b^5))^(3/4))/(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^
2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*
a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8
*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^
11 + a^12*d^12)) + 21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804
*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703
*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a
^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2
*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(a^4*b*(-(14
641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^
3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^
6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*
c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))
^(1/4) + (11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) -
 21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 3
68082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 -
 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 1
0017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^1
1*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(-a^4*b*(-(14641*b^12*c^12 - 1
11804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 7
46703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5
688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^1
0*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4) + (11*b^3*
c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) - 4*(12*a^2*b*c^3
- 7*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^3*d^3)*x^4 - 4*(11*a
*b^2*c^3 - 21*a^2*b*c^2*d)*x^2)*sqrt(x))/(a^3*b^2*x^6 + a^4*b*x^4)
```

giac [A] time = 0.46, size = 509, normalized size = 1.35

$$\frac{\sqrt{2}\left(11\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}b^{2}}+\frac{\sqrt{2}\left(11\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^{4}b^{2}}+\frac{\sqrt{2}\left(11\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{4}b^{2}}-\frac{\sqrt{2}\left(11\,\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}-21\,\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d+9\,\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^{4}b^{2}}+\frac{b^{3}c^{3}\sqrt{x}-3\,ab^{2}c^{2}d\sqrt{x}+3\,a^{2}bcd^{2}\sqrt{x}-a^{3}d^{3}\sqrt{x}}{2\left(bx^{2}+a\right)a^{3}b}+\frac{2\left(14\,bc^{3}x^{2}-21\,ac^{2}dx^{2}-3\,ac^{3}\right)}{21\,a^{3}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a
```

```
*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) + 1/8*sqrt(2)*(11*(a*b^3)
^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2
 + (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(
a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*
d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*sqrt
(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1
/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a^4*b^2) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) +
3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a^3*b) + 2/21*(14*b*c
^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(a^3*x^(7/2))
```

maple [B] time = 0.02, size = 706, normalized size = 1.88

$$\frac{3cd^2\sqrt{x}}{2\left(bx^2+a\right)a}-\frac{3bc^2d\sqrt{x}}{2\left(bx^2+a\right)a^2}+\frac{b^2c^3\sqrt{x}}{2\left(bx^2+a\right)a^3}-\frac{d^3\sqrt{x}}{2\left(bx^2+a\right)b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16ab}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^2}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd^2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^2}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd^2\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^2}-\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bc^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^3}-\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bc^2d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^3}-\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bc^2d\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^3}+\frac{11\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8a^4}+\frac{11\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c^3\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8a^4}+\frac{11\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c^3\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16a^4}-\frac{2c^2d}{a^2x^{\frac{3}{2}}}+\frac{4bc^3}{3a^3x^{\frac{3}{2}}}-\frac{2c^3}{7a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/2/b*x^(1/2)/(b*x^2+a)*d^3+3/2/a*x^(1/2)/(b*x^2+a)*c*d^2-3/2/a^2*b*x^(1/2
)/(b*x^2+a)*c^2*d+1/2/a^3*b^2*x^(1/2)/(b*x^2+a)*c^3+1/8/a/b*(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d^3+9/8/a^2*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c*d^2-21/8/a^3*b*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^2*d+11/8/a^4*b^2*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c^3+1/8/a/b*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d^3+9/8/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c*d^2-21/8/a^3*b*(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^2*d+11/8/a^4*b^2*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c^3+1/16/a/b*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b
)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(
1/2)))*d^3+9/16/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c*d^2-21/16/a^3*b*
(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c^2*d+11/16/a^4*b^2*(a/b)^(1/4)*2^(1/2
)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(
1/2)+(a/b)^(1/2)))*c^3-2/7*c^3/a^2/x^(7/2)-2*c^2/a^2/x^(3/2)*d+4/3*c^3/a^3/
x^(3/2)*b
```

maxima [A] time = 2.55, size = 424, normalized size = 1.13

$$-\frac{12\,a^2bc^3-7\left(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2-3\,a^3d^3\right)x^4-4\left(11\,ab^2c^3-21\,a^2bc^2d\right)x^2}{42\left(a^3b^2x^{\frac{11}{2}}+a^4bx^{\frac{7}{2}}\right)}+\frac{\frac{2\sqrt{2}\left(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\left(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/42*(12*a^2*b*c^3 - 7*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^
3*d^3)*x^4 - 4*(11*a*b^2*c^3 - 21*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^(11/2) + a^4
*b*x^(7/2)) + 1/16*(2*sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2
+ a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(11*b^3
*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))) + sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2
+ a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d
^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^
(1/4)))/(a^3*b)
```

mupad [**B**]   time = 0.61, size = 1746, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x)
```

```
[Out] (atan((((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5
*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3
+ 1248*a^13*b^8*c^2*d^4) - ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^
3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^
(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4)) +
 ((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 5
76*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248
*a^13*b^8*c^2*d^4) + ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 25
6*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(15/4)
*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4)))/(((x^(
1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^1
4*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*
b^8*c^2*d^4) - ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16
*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(15/4)*b^(5/
4)))*(a*d - b*c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)) - ((x^(1/2)*(387
2*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d
^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d
^4) + ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3
- 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(15/4)*b^(5/4)))*(a*d
 - b*c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4))))*(a*d - b*c)^2*(a*d + 11
*b*c)*1i)/(4*(-a)^(15/4)*b^(5/4)) - ((2*c^3)/(7*a) + (x^4*(3*a^3*d^3 - 11*b
^3*c^3 + 21*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(6*a^3*b) + (2*c^2*x^2*(21*a*d -
11*b*c))/(21*a^2))/(a*x^(7/2) + b*x^(11/2)) + (atan((((x^(1/2)*(3872*a^9*b^
12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 204
```

```
48*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) - ((
a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a
^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*
c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)) + ((x^(1/2)*(3872*a^9*b^12*c^6
 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 20448*a^1
1*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) + ((a*d -
b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^
9*c^2*d + 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(
a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)))/(((x^(1/2)*(3872*a^9*b^12*c^6 + 32*
a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10
*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) - ((a*d - b*c)^2
*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*
d + 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d +
11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4)) - ((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^1
5*b^6*d^6 - 14784*a^10*b^11*c^5*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^
4*d^2 - 11392*a^12*b^9*c^3*d^3 + 1248*a^13*b^8*c^2*d^4) + ((a*d - b*c)^2*(a
*d + 11*b*c)*(2816*a^13*b^10*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d +
 2304*a^15*b^8*c*d^2)*1i)/(8*(-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*
b*c)*1i)/(8*(-a)^(15/4)*b^(5/4))))*(a*d - b*c)^2*(a*d + 11*b*c))/(4*(-a)^(1
5/4)*b^(5/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.443 $\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=478

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)}+$$

Rubi [A] time = 0.56, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 479, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)}+\frac{a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}b^{7/4}(bc-ad)}-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}+\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{7/4}(bc-ad)}-\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{7/4}(bc-ad)}+\frac{2x^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (a^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) + (c^(7/4)*ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) - (c^(7/4)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(7/4)*(b*c - a*d)) + (
a^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt
[2]*b^(7/4)*(b*c - a*d)) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7/4)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(7/4)*(b*c -
a*d)) + (c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(2*Sqrt[2]*d^(7/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3bd} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2\left(3ac+3(bc+ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{3bd}\\
&= \frac{2x^{3/2}}{3bd} - \frac{2\,\mathrm{Subst}\left(\int \left(\frac{3a^2dx^2}{(-bc+ad)\left(a+bx^4\right)} + \frac{3bc^2x^2}{(bc-ad)\left(c+dx^4\right)}\right) dx, x, \sqrt{x}\right)}{3bd}\\
&= \frac{2x^{3/2}}{3bd} + \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{\left(2c^2\right)\mathrm{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d(bc-ad)}\\
&= \frac{2x^{3/2}}{3bd} - \frac{a^2\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}(bc-ad)} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b^{3/2}(bc-ad)} + \frac{c^2\,\mathrm{Su}}{}\\
&= \frac{2x^{3/2}}{3bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^2(bc-ad)} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{}\right.}{2b^2(bc-ad)}\\
&= \frac{2x^{3/2}}{3bd} + \frac{a^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{7/4}(bc-ad)} - \frac{a^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{2\sqrt{2}\,b^{7/4}(bc-ad)}\\
&= \frac{2x^{3/2}}{3bd} - \frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)} + \frac{a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)} + \frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,d^{7/4}(bc-a}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 411, normalized size = 0.86

$$
\frac{\frac{3\sqrt{2}\,a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{b^{7/4}} - \frac{3\sqrt{2}\,a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{b^{7/4}} - \frac{6\sqrt{2}\,a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{7/4}} + \frac{6\sqrt{2}\,a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{7/4}} - \frac{8ax^{3/2}}{b} - \frac{3\sqrt{2}\,c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{7/4}} + \frac{3\sqrt{2}\,c^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{7/4}} + \frac{6\sqrt{2}\,c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{7/4}} - \frac{6\sqrt{2}\,c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{7/4}} + \frac{8cx^{3/2}}{d}}{12bc-12ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a*x^(3/2))/b + (8*c*x^(3/2))/d - (6*Sqrt[2]*a^(7/4)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (6*Sqrt[2]*a^(7/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(7/4) + (6*Sqrt[2]*c^(7/4)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) - (6*Sqrt[2]*c^(7/4)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(7/4) + (3*Sqrt[2]*a^(7/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) - (3*Sqrt[2]*a^(7/4)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(7/4) - (3*Sq
rt[2]*c^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d
^(7/4) + (3*Sqrt[2]*c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/d^(7/4))/(12*b*c - 12*a*d)
```

IntegrateAlgebraic [A] time = 0.60, size = 282, normalized size = 0.59

$$-\frac{a^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)}-\frac{a^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{7/4}(bc-ad)}-\frac{c^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,d^{7/4}(ad-bc)}-\frac{c^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,d^{7/4}(ad-bc)}+\frac{2x^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*x^(3/2))/(3*b*d) - (a^(7/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)
*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7/4)*A
rcTan[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])
/(Sqrt[2]*d^(7/4)*(-(b*c) + a*d)) - (a^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(7/4)*(b*c - a*d)) - (c^(7/4
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2
]*d^(7/4)*(-(b*c) + a*d))
```

fricas [B] time = 7.23, size = 1422, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*
d^3 + a^4*b^7*d^4))^(1/4)*b*d*arctan(-(sqrt(a^10*x - (a^7*b^5*c^2 - 2*a^8*b
^4*c*d + a^9*b^3*d^2)*sqrt(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*
d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)))*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6
*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*(b^3*c - a*b^2*d)
- (a^5*b^3*c - a^6*b^2*d)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*
d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*sqrt(x))/a^7) - 12*(-c^7/(b^4*c
^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^
```

```
(1/4)*b*d*arctan(-(sqrt(c^10*x - (b^2*c^9*d^3 - 2*a*b*c^8*d^4 + a^2*c^7*d^5
)*sqrt(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*
d^10 + a^4*d^11)))*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9
 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*(b*c*d^2 - a*d^3) - (b*c^6*d^2 - a*c^5
*d^3)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*
d^10 + a^4*d^11))^(1/4)*sqrt(x))/c^7) + 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d
+ 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqr
t(x) + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^1
1*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4)
)^(3/4)) - 3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b
^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x) - (b^8*c^3 - 3*a*b^7*c^2
*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^
2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(3/4)) - 3*(-c^7/(b^4*c^4*d
^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4
)*b*d*log(c^5*sqrt(x) + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^
3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c
*d^10 + a^4*d^11))^(3/4)) + 3*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*
b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*b*d*log(c^5*sqrt(x) - (b^3*
c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4
*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)) + 4
*x^(3/2))/(b*d)
```

giac [A] time = 0.76, size = 476, normalized size = 1.00

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{5}c-\sqrt{2}\,ab^{4}d\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} c \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{4}-\sqrt{2}\,ad^{5}\right)}+\frac{2\,x^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + (a*b^3)^(3/4)*a*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^5*c - sqrt(2)
*a*b^4*d) - (c*d^3)^(3/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - (c*d^3)^(3/4)*c*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c
*d^4 - sqrt(2)*a*d^5) - 1/2*(a*b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + 1/2*(a*b^3)^(3/4)*a*l
og(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c - sqrt(2)*a
*b^4*d) + 1/2*(c*d^3)^(3/4)*c*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/2*(c*d^3)^(3/4)*c*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) + 2/3*
x^(3/2)/(b*d)
```

maple [A] time = 0.02, size = 351, normalized size = 0.73

$$-\frac{\sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(a d -b c \right) \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}-\frac{\sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(a d -b c \right) \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}-\frac{\sqrt{2}\, a^{2} \ln\left(\frac{x -\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x +\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(a d -b c \right) \left(\frac{a}{b}\right)^{\frac{1}{4}} b^{2}}+\frac{\sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2 \left(a d -b c \right) \left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}}+\frac{\sqrt{2}\, c^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2 \left(a d -b c \right) \left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}}+\frac{\sqrt{2}\, c^{2} \ln\left(\frac{x -\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x +\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4 \left(a d -b c \right) \left(\frac{c}{d}\right)^{\frac{1}{4}} d^{2}}+\frac{2 x^{\frac{3}{2}}}{3 b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 2/3*x^(3/2)/b/d-1/4*a^2/b^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-
1/2*a^2/b^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)-1/2*a^2/b^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)+1/4*c^2/d^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/
2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+1/2*c^
2/d^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1
/2*c^2/d^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
-1)
```

maxima [A] time = 2.61, size = 390, normalized size = 0.82

$$\frac{a^{2} \left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \, \sqrt{b} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} + \frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \, \sqrt{b} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4 \, {\left(b^{2} c - a b d\right)}} - \frac{c^{2} \left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} + 2 \, \sqrt{d} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} + \frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} - 2 \, \sqrt{d} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{4 \, {\left(b c d - a d^{2}\right)}} + \frac{2 \, x^{\frac{3}{2}}}{3 \, b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
^2*c - a*b*d) - 1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt
(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*s
qrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*lo
g(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4))
+ sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1
/4)*d^(3/4)))/(b*c*d - a*d^2) + 2/3*x^(3/2)/(b*d)
```

mupad [B] time = 1.93, size = 7892, normalized size = 16.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan(((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c
^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*
a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*
b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^
6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*
b^3*c^3*d^10))/(b^3*d^3) - (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7
 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*
b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*
d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^
3*d^3)) - (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*1i - (-c^7
/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64
*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d
^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d
^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*
b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^1
0))/(b^3*d^3) + (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3
*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^
5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a
^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) + (
256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*1i)/((-c^7/(16*a^4*d^
11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^
10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2
*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4
*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7
 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^
3) - (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 +
96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*
b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*
d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) - (256*x^(1/2)
*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)) + (-c^7/(16*a^4*d^11 + 16*b^4*c^
4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-
c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 -
 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 +
 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*
c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) + (256*x^(1
/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2
*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 +
 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b
^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) + (256*x^(1/2)*(a^5*b^5*c^10
 + a^10*c^5*d^5))/(b^3*d^3)) - (256*(a^7*b^2*c^9 + a^9*c^7*d^2 + a^8*b*c^8*
d))/(b^3*d^3)))*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96
```

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*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*2i + 2*atan(((-c^7/(16*a^4*d^11
+ 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10)
)^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^
2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^
9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 +
48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3)
- (x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*
b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^
8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 6
4*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i + (256*x^(1/2
)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)) - (-c^7/(16*a^4*d^11 + 16*b^4*c
^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((
-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9
- 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4
+ 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5
*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) + (x^(1/2)
*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^
9 - 64*a^3*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 11
2*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*
c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i - (256*x^(1/2)*(a^5*b^5
*c^10 + a^10*c^5*d^5))/(b^3*d^3)))/((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 6
4*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a
^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b
*c*d^10))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b
^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 -
 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) - (x^(1/2)*(-c^7/(16
*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3
*b*c*d^10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*
c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 +
 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i + (256*x^(1/2)*(a^5*b^5*c^10 + a^
10*c^5*d^5))/(b^3*d^3))*1i + (-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3
*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11
 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10
))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*
d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9
*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) + (x^(1/2)*(-c^7/(16*a^4*d^
11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^
10))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7
 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9
*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i - (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*
d^5))/(b^3*d^3))*1i + (256*(a^7*b^2*c^9 + a^9*c^7*d^2 + a^8*b*c^8*d))/(b^3*
d^3)))*(-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*
c^2*d^9 - 64*a^3*b*c*d^10))^(1/4) + atan(((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d
^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*((-a^7
/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64
```

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*a*b^10*c^3*d))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48
*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5
*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) - (256*x^(1/2)
*(-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^
2 - 64*a*b^10*c^3*d))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 11
2*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*
c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^3*d^3)) - (256*x^(1/2)*(a^5*b^5*c^10 +
a^10*c^5*d^5))/(b^3*d^3))*1i - (-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3
*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*((-a^7/(16*b^11*c
^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3
*d))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^
8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a
^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) + (256*x^(1/2)*(-a^7/(16*
b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^
10*c^3*d))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c
^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 +
16*a^9*b^5*c^3*d^11))/(b^3*d^3)) + (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^
5))/(b^3*d^3))*1i)/((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3
+ 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*((-a^7/(16*b^11*c^4 + 16*a^4
*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(3/4)*
((128*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*
a^6*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*
d^9 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) - (256*x^(1/2)*(-a^7/(16*b^11*c^4 +
16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^
(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 12
8*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*
c^3*d^11))/(b^3*d^3)) - (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^
3)) + (-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c
^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*
a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(3/4)*((128*(16*a^3*
b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^
6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*
b^3*c^3*d^10))/(b^3*d^3) + (256*x^(1/2)*(-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4
 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*(16*a^3*
b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*
d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11))/(b^
3*d^3)) + (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)) - (256*(a^
7*b^2*c^9 + a^9*c^7*d^2 + a^8*b*c^8*d))/(b^3*d^3)))*(-a^7/(16*b^11*c^4 + 16
*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1
/4)*2i + 2*atan(((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 9
6*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*((-a^7/(16*b^11*c^4 + 16*a^4*b^
7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(3/4)*((1
28*(16*a^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6
*b^7*c^7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9
 + 16*a^10*b^3*c^3*d^10))/(b^3*d^3) - (x^(1/2)*(-a^7/(16*b^11*c^4 + 16*a^4*
```

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b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*(
16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b
^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^1
1)*256i)/(b^3*d^3))*1i + (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d
^3)) - (-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*
c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64
*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(3/4)*((128*(16*a^3
*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d
^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10
*b^3*c^3*d^10))/(b^3*d^3) + (x^(1/2)*(-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 -
64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*(16*a^3*b^1
1*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8
 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(
b^3*d^3))*1i - (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)))/((-a
^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 -
64*a*b^10*c^3*d))^(1/4)*((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c
*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(3/4)*((128*(16*a^3*b^10*c^10
*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^
7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d
^10))/(b^3*d^3) - (x^(1/2)*(-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8
*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*(16*a^3*b^11*c^9*d^5
- 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7
*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*
1i + (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*1i + (-a^7/(16*
b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^
10*c^3*d))^(1/4)*((-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3 +
96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(3/4)*((128*(16*a^3*b^10*c^10*d^3 -
48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7*d^6 - 16*a^7*b^6*c
^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a^10*b^3*c^3*d^10))/(
b^3*d^3) + (x^(1/2)*(-a^7/(16*b^11*c^4 + 16*a^4*b^7*d^4 - 64*a^3*b^8*c*d^3
+ 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4)*(16*a^3*b^11*c^9*d^5 - 64*a^
4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b^8*c^6*d^8 + 112*a^7*b^7*c^
5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^11)*256i)/(b^3*d^3))*1i - (2
56*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*1i + (256*(a^7*b^2*c^9
 + a^9*c^7*d^2 + a^8*b*c^8*d))/(b^3*d^3)))*(-a^7/(16*b^11*c^4 + 16*a^4*b^7*
d^4 - 64*a^3*b^8*c*d^3 + 96*a^2*b^9*c^2*d^2 - 64*a*b^10*c^3*d))^(1/4) + (2*
x^(3/2))/(3*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.444 $\int \frac{x^{7/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=476

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}+$$

Rubi [A] time = 0.49, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 479, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{5/4}(bc-ad)}-\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{5/4}(bc-ad)}+\frac{2\sqrt{x}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^
(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2
]*b^(5/4)*(b*c - a*d)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(5/4)*(b*c - a*
d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(2*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= \frac{2\sqrt{x}}{bd} - \frac{2\,\text{Subst}\left(\int \frac{ac+(bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{bd}\\
&= \frac{2\sqrt{x}}{bd} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{d(bc-ad)}\\
&= \frac{2\sqrt{x}}{bd} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)} - c^{3/2}\\
&= \frac{2\sqrt{x}}{bd} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}(bc-ad)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \right.}{2b^{3/2}(bc-ad)}\\
&= \frac{2\sqrt{x}}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\right.}{2\sqrt{2}\,b^{5/4}(bc-ad)}\\
&= \frac{2\sqrt{x}}{bd} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,d^{5/4}(bc-ad}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 409, normalized size = 0.86

$$\frac{-\frac{\sqrt{2}\,a^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{b^{5/4}}+\frac{\sqrt{2}\,a^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{b^{5/4}}-\frac{2\sqrt{2}\,a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}\,a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{5/4}}-\frac{8a\sqrt{x}}{b}+\frac{\sqrt{2}\,c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{5/4}}-\frac{\sqrt{2}\,c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{d^{5/4}}+\frac{2\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{d^{5/4}}-\frac{2\sqrt{2}\,c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{d^{5/4}}+\frac{8c\sqrt{x}}{d}}{4bc-4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((-8*a*Sqrt[x])/b + (8*c*Sqrt[x])/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/d^(5/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(5/4) + (Sqrt[2]*
c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/d^(5/4)
 - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/d^(5/4))/(4*b*c - 4*a*d)
```

IntegrateAlgebraic [A] time = 0.57, size = 278, normalized size = 0.58

$$-\frac{a^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{5/4}(bc-ad)}-\frac{c^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}d^{5/4}(ad-bc)}+\frac{c^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}d^{5/4}(ad-bc)}+\frac{2\sqrt{x}}{bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*Sqrt[x])/(b*d) - (a^(5/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x
)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) - (c^(5/4)*Arc
Tan[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(
Sqrt[2]*d^(5/4)*(-(b*c) + a*d)) + (a^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*
ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*
d^(5/4)*(-(b*c) + a*d))
```

fricas [B] time = 2.12, size = 1388, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(-((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c
*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*
a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(a^2*x + (b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*
b^6*c*d^3 + a^4*b^5*d^4))) - (a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2
 - a^4*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*
b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt(x))/a^5) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*
b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arcta
n(-((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^
```

```
4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/
4)*sqrt(c^2*x + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))) - (b^3
*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)*s
qrt(x))/c^5) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b
^6*c*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) + (-a^5/(b^9*c^4 - 4*a*b^8
*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c -
 a*b*d)) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c
*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3
*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b
*d)) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c
*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3
*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2))
 - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8
 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6
 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4
*sqrt(x))/(b*d)
```

giac [A] time = 0.76, size = 476, normalized size = 1.00

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\, b^{3} c - \sqrt{2}\, a b^{2} d} + \frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\, b^{3} c - \sqrt{2}\, a b^{2} d} - \frac{\left(cd^{3}\right)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\, b c d^{2} - \sqrt{2}\, a d^{3}} - \frac{\left(cd^{3}\right)^{\frac{1}{4}} c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\, b c d^{2} - \sqrt{2}\, a d^{3}} + \frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\, b^{3} c - \sqrt{2}\, a b^{2} d\right)} - \frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\, b^{3} c - \sqrt{2}\, a b^{2} d\right)} - \frac{\left(cd^{3}\right)^{\frac{1}{4}} c \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\, b c d^{2} - \sqrt{2}\, a d^{3}\right)} + \frac{\left(cd^{3}\right)^{\frac{1}{4}} c \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\, b c d^{2} - \sqrt{2}\, a d^{3}\right)} + \frac{2\sqrt{x}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)
*a*b^2*d) - (c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - (c*d^3)^(1/4)*c*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c
*d^2 - sqrt(2)*a*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/2*(a*b^3)^(1/4)*a*l
og(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a
*b^2*d) - 1/2*(c*d^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 1/2*(c*d^3)^(1/4)*c*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 2*sq
rt(x)/(b*d)
```

maple [A] time = 0.02, size = 339, normalized size = 0.71

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{2\left(ad - bc\right) b} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{2\left(ad - bc\right) b} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, a \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad - bc\right) b} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{2\left(ad - bc\right) d} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{2\left(ad - bc\right) d} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, c \ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad - bc\right) d} + \frac{2\sqrt{x}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 2*x^(1/2)/b/d-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/
2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/b*
a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/b
*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/4/
d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+1/2/d*c/(a*d-b*c)*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2/d*c/(a*d-b*c)*(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

maxima [**A**] time = 2.59, size = 384, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\left(b^{2}c-abd\right)}-\frac{\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{4\left(bcd-ad^{2}\right)}+\frac{2\sqrt{x}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d
) - 1/4*(2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c
^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sq
rt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(2)*c^
(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(
-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4))/(b*c*d - a
*d^2) + 2*sqrt(x)/(b*d)
```

mupad [**B**] time = 1.60, size = 6428, normalized size = 13.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(
b*d) - (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 +
 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b
^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 +
```

```
 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^
6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) - (256*x^(1/2)*(a^4*b
^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b
^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i - (((512*(a^3*b^6
*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (256*x^(1/2)*(
-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 -
 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b
^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9))
/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*
c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4)
)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7
*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i)/((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 -
a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16
*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/
4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b
^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9
*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^
3*d))^(1/4) - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^
9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c
^3*d))^(1/4) + (((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^
4*d^5))/(b*d) + (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^
6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 -
 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*
c^4*d^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 -
64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + (256*x^(1/
2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 -
 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)))*(-a^5/(16
*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^
8*c^3*d))^(1/4)*2i - 2*atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^
8*d - a^8*b*c^4*d^5))/(b*d) - (x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 -
 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9
*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 4
8*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16
*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/
4)*1i + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4
+ 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))
^(1/4) - (((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5)
)/(b*d) + (x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 +
96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^
8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 +
16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^
3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i - (256*x^(1/2)
*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 6
4*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4))/((((512*(a^3
*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x^(1/2)*(
```

```
-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 -
 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b
^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*
256i)/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2
*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*
c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96
*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i + (((512*(a^3*b^6*c^9 + a^9*c^
3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x^(1/2)*(-a^5/(16*b^9*c^4
+ 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))
^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a
^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-a^
5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64
*a*b^8*c^3*d))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*
(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2
- 64*a*b^8*c^3*d))^(1/4)*1i))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b
^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + atan(((((512*(a^3*
b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (256*x^(1/2
)*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^
7 - 64*a^3*b*c*d^8))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^
5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^
9))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b
^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4) - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d
^4))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*
b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4)*1i - (((512*(a^3*b^6*c^9 + a^9*c^3*d^6
 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (256*x^(1/2)*(-c^5/(16*a^4*d^9 +
 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^
(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^
6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(16*
a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b
*c*d^8))^(1/4) + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-c^5/(16
*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*
b*c*d^8))^(1/4)*1i)/((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^
8*b*c^4*d^5))/(b*d) - (256*x^(1/2)*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*
a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(3/4)*(16*a^3*b^9*c^8
*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^
7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*
d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4) - (256
*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4
*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4) + (((
512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (2
56*x^(1/2)*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b
^2*c^2*d^7 - 64*a^3*b*c*d^8))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^
5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b
^4*c^3*d^9))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 +
 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4) + (256*x^(1/2)*(a^4*b^4*c^8 +
```

```
a^8*c^4*d^4))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6
+ 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4)))*(-c^5/(16*a^4*d^9 + 16*b^4*
c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4)*2i
 - 2*atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^
5))/(b*d) - (x^(1/2)*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6
+ 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*
b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8
+ 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*
a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4)*1i + (256*x^(1/
2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 -
 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4) - (((512*(a
^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x^(1/2)
*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7
 - 64*a^3*b*c*d^8))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5
*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9
)*256i)/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a
^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4)*1i - (256*x^(1/2)*(a^4*b^4*c^8 + a^
8*c^4*d^4))/(b*d))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 +
96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4))/((((512*(a^3*b^6*c^9 + a^9*c^3
*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x^(1/2)*(-c^5/(16*a^4*d^9 +
 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^
(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^
6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-c^5
/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*
a^3*b*c*d^8))^(1/4)*1i + (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(
-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 -
 64*a^3*b*c*d^8))^(1/4)*1i + (((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^
8*d - a^8*b*c^4*d^5))/(b*d) + (x^(1/2)*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 -
 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(3/4)*(16*a^3*b^9
*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 4
8*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9)*256i)/(b*d))*(-c^5/(16*a^4*d^9 + 16
*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/
4)*1i - (256*x^(1/2)*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-c^5/(16*a^4*d^9
+ 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*b^2*c^2*d^7 - 64*a^3*b*c*d^8))
^(1/4)*1i))*(-c^5/(16*a^4*d^9 + 16*b^4*c^4*d^5 - 64*a*b^3*c^3*d^6 + 96*a^2*
b^2*c^2*d^7 - 64*a^3*b*c*d^8))^(1/4) + (2*x^(1/2))/(b*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.445 $\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=463

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}-$$

Rubi [A] time = 0.36, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 481, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,d^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b
*c - a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(3/4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d
)) + (a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(3/4)*(
b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{(2c)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,(bc-ad)} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,(bc-ad)} - \frac{c\,\text{Subst}\Big(\int \frac{\sqrt{c}-}{c+}}{\sqrt{d}\,(b}$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)} - \frac{a^{3/4}}{}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)} + \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)} +$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)} - \frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)} - \frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,d^{3/4}(bc-ad)} +$$

Mathematica [A] time = 0.11, size = 142, normalized size = 0.31

$$\frac{\frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{3/4}} - \frac{(-a)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{-a}}\right)}{b^{3/4}} - \frac{(-c)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{-c}}\right)}{d^{3/4}} + \frac{(-c)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{-c}}\right)}{d^{3/4}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (((-a)^(3/4)*ArcTan[(b^(1/4)*Sqrt[x])/(-a)^(1/4)])/b^(3/4) - ((-c)^(3/4)*Ar
cTan[(d^(1/4)*Sqrt[x])/(-c)^(1/4)])/d^(3/4) - ((-a)^(3/4)*ArcTanh[(b^(1/4)*
Sqrt[x])/(-a)^(1/4)])/b^(3/4) + ((-c)^(3/4)*ArcTanh[(d^(1/4)*Sqrt[x])/(-c)^
(1/4)])/d^(3/4))/(b*c - a*d)
```

IntegrateAlgebraic [A] time = 0.49, size = 263, normalized size = 0.57

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)} + \frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)} + \frac{c^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\,\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\,\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,d^{3/4}(ad-bc)} + \frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,d^{3/4}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(3/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))
/Sqrt[x]])/(Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[(c^(1/4)/(Sqrt[2
]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*d^(3/4)*(-(b
*c) + a*d)) + (a^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTanh[(Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(3/4)*(-(b*c) + a*
d))
```

fricas [B] time = 1.66, size = 1385, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a
^4*b^3*d^4))^(1/4)*arctan(-(sqrt(a^4*x - (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5
*b*d^2)*sqrt(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*
c*d^3 + a^4*b^3*d^4)))*(b^2*c - a*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a
^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4) - (a^2*b^2*c - a^3*b
*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 +
a^4*b^3*d^4))^(1/4)*sqrt(x))/a^3) + 2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4
+ 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(-(sqrt(c^4*x -
 (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3
*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)))*(b*c*d - a*d^2)*(
-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a
^4*d^7))^(1/4) - (b*c^3*d - a*c^2*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4
 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*sqrt(x))/c^3) - 1/2*
(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*
b^3*d^4))^(1/4)*log(a^2*sqrt(x) + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3
*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*
a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*sqrt(x) - (
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4
*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) +
 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d
^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x) + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a
^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*
d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c
^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x
) - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^
4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/
```

```
4))
```

giac [A]   time = 0.69, size = 457, normalized size = 0.99

$$-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{4}c-\sqrt{2}\,ab^{3}d}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{4}c-\sqrt{2}\,ab^{3}d}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{3}-\sqrt{2}\,ad^{4}}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bcd^{3}-\sqrt{2}\,ad^{4}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{4}c-\sqrt{2}\,ab^{3}d\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{4}c-\sqrt{2}\,ab^{3}d\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{3}-\sqrt{2}\,ad^{4}\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bcd^{3}-\sqrt{2}\,ad^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*
b^3*d) + (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + (c*d^3)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 -
sqrt(2)*a*d^4) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sq
rt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1
/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*
b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4)
```

maple [A]   time = 0.01, size = 328, normalized size = 0.71

$$\frac{\sqrt{2}\,a \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\,a \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\,a \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} b}-\frac{\sqrt{2}\,c \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} d}-\frac{\sqrt{2}\,c \arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} d}-\frac{\sqrt{2}\,c \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*a/(a*d-b*c)/b/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a/(a*d-b*c)/b/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4*c/(a*d-b*c)/d/
(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-1/2*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

maxima [A]   time = 2.60, size = 369, normalized size = 0.80

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(bc-ad\right)}+\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*
c - a*d) + 1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2
*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))
/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(
2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(
2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(
3/4)))/(b*c - a*d)
```

mupad [B] time = 1.27, size = 2609, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - 2*atan((2*b^4*c^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4
*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) + 64*a^4*b^4*d^7*x^(1/
2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d
^2 - 64*a*b^6*c^3*d))^(5/4) + 64*b^8*c^4*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16
*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/
4) + 2*a^3*b*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*
d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) + 384*a^2*b^6*c^2*d^5*x^(
1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2
*d^2 - 64*a*b^6*c^3*d))^(5/4) - 256*a*b^7*c^3*d^4*x^(1/2)*(-a^3/(16*b^7*c^4
 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d)
)^(5/4) - 256*a^3*b^5*c*d^6*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64
*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4))/(a^3*d^2 + a*
b^2*c^2 + a^2*b*c*d))*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3
 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) - atan((b^4*c^3*x^(1/2)*(-a^
3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64
*a*b^6*c^3*d))^(1/4)*2i + a^4*b^4*d^7*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^
3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*64i
+ b^8*c^4*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3
 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*64i + a^3*b*d^3*x^(1/2)*(-a^
3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64
*a*b^6*c^3*d))^(1/4)*2i + a^2*b^6*c^2*d^5*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^
```

```
4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*
384i - a*b^7*c^3*d^4*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^
4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*256i - a^3*b^5*c*d^6*
x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*
c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*256i)/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(
-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 -
 64*a*b^6*c^3*d))^(1/4)*2i - 2*atan((2*a^3*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 +
16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(
1/4) + 2*b^3*c^3*d*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^
3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4) + 64*a^4*b^3*d^8*x^(1/2
)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^
5 - 64*a^3*b*c*d^6))^(5/4) + 64*b^7*c^4*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*
b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4
) + 384*a^2*b^5*c^2*d^6*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b
^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4) - 256*a*b^6*c^3*d^
5*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^
2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4) - 256*a^3*b^4*c*d^7*x^(1/2)*(-c^3/(16*a^
4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c
*d^6))^(5/4))/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(16*a^4*d^7 + 16*b^4
*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4) -
 atan((a^3*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^
4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4)*2i + b^3*c^3*d*x^(1/2)*(-c^
3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64
*a^3*b*c*d^6))^(1/4)*2i + a^4*b^3*d^8*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^
4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*64i
+ b^7*c^4*d^4*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4
 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*64i + a^2*b^5*c^2*d^6*x^(1/2
)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^
5 - 64*a^3*b*c*d^6))^(5/4)*384i - a*b^6*c^3*d^5*x^(1/2)*(-c^3/(16*a^4*d^7 +
 16*b^4*c^4*d^3 - 64*a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^
(5/4)*256i - a^3*b^4*c*d^7*x^(1/2)*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*
a*b^3*c^3*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(5/4)*256i)/(b^2*c^3
+ a^2*c*d^2 + a*b*c^2*d))*(-c^3/(16*a^4*d^7 + 16*b^4*c^4*d^3 - 64*a*b^3*c^3
*d^4 + 96*a^2*b^2*c^2*d^5 - 64*a^3*b*c*d^6))^(1/4)*2i
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.446 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right.}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)}$$

Rubi [A] time = 0.36, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 481, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}\,(bc-ad)}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b
*c - a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d
)) - (a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
2*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(1/4)*(
b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)\\ &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}\\ &= -\frac{\sqrt{a}\ \text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{\sqrt{a}\ \text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{c}\ \text{Subst}}{}\\ &= -\frac{\sqrt{a}\ \text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\,(bc-ad)} - \frac{\sqrt{a}\ \text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\,(bc-ad)} +\\ &= \frac{\sqrt[4]{a}\ \log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\ \log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)} - \\ &= \frac{\sqrt[4]{a}\ \tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\ \tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{c}\ \tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)} +\end{aligned}$$

Mathematica [A] time = 0.11, size = 364, normalized size = 0.79

$$\frac{\sqrt[4]{a}\,\sqrt[4]{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\sqrt[4]{a}\,\sqrt[4]{d}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+2\sqrt[4]{a}\,\sqrt[4]{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\,\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)-\sqrt[4]{b}\,\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+\sqrt[4]{b}\,\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-2\sqrt[4]{b}\,\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt[4]{b}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*a^(1/4
)*d^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*b^(1/4)*c^(1/4)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + b^(1/4)*c^(1/4)*Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*b^(1/4)*d^(1/4)*(
b*c - a*d))
```

IntegrateAlgebraic [A] time = 0.46, size = 265, normalized size = 0.57

$$\frac{\sqrt[4]{a}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)}+\frac{\sqrt[4]{c}\,\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\,\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\,\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{d}\,(ad-bc)}-\frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)}-\frac{\sqrt[4]{c}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,\sqrt[4]{d}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))
/Sqrt[x]])/(Sqrt[2]*b^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[(c^(1/4)/(Sqrt[2
]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*d^(1/4)*(-(b
*c) + a*d)) - (a^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)) - (c^(1/4)*ArcTanh[(Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(1/4)*(-(b*c) + a*
d))
```

fricas [B] time = 1.15, size = 1249, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4))^(1/4)*arctan(-((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d
^3)*sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)) + x)*(-a/(b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4) - (b^4*
c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(x)*(-a/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a)
- 2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
a^4*d^5))^(1/4)*arctan(-((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*sqrt((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)) + x)*(-c/(b^4*c^4*d -
4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4) - (b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)*(-c/(b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c
) - 1/2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^
3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-a/(b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4
)*log(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^
```

```
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)
*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4
*d^5))^(1/4) + sqrt(x)) - 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*
c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d -
 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + sq
rt(x))
```

giac [A] time = 0.64, size = 441, normalized size = 0.95

$$-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c-\sqrt{2}abd}-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c-\sqrt{2}abd}+\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd-\sqrt{2}ad^2}+\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd-\sqrt{2}ad^2}-\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*
d) + (c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + (c*d^3)^(1/4)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*
a*d^2) - 1/2*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(
a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/2*(c*d^3)^(
1/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt
(2)*a*d^2) - 1/2*(c*d^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2)
```

maple [A] time = 0.01, size = 304, normalized size = 0.66

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2ad-2bc}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2ad-2bc}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4ad-4bc}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(
1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/(a*d-b*c)*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/(a*d-b*c)*(a/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1
/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x
^(1/2)+(c/d)^(1/2)))-1/2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)-1/2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)-1)
```

maxima [**A**] time = 2.10, size = 367, normalized size = 0.79

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\,(bc-ad)}+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt
(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b*c - a*d) +
 1/4*(2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt
(c)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*c^(1/4)*log(sqrt(2)*c^(1/
4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(1/4)*log(-sq
rt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4))/(b*c - a*d)
```

mupad [**B**] time = 1.42, size = 5963, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] 2*atan(-((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2
*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*
c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9
*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*
d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-a/(16*b^5*c^4 + 16*
a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*
(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 1
63840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 81
92*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3
+ 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(3/4)*1i + 512*a^2*b^6*c^5*d^3 - 51
2*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)
*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5)) + (-a/(16*b^5*c^4 + 16*a^4*b*
d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(
16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^
4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8
192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a
^7*b^4*c^2*d^9) + (-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^
```

```
2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b
^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b
^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5
*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*
d))^(3/4)*1i - 512*a^2*b^6*c^5*d^3 + 512*a^3*b^5*c^4*d^4 + 512*a^4*b^4*c^3*
d^5 - 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*
c^2*d^5)))/((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*
c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b
^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*
b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c
^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-a/(16*b^5*c^4 +
16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/
4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6
 - 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 +
 8192*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d
^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(3/4)*1i + 512*a^2*b^6*c^5*d^3 -
 512*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*1i - x^(1
/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5))*1i - (-a/(16*b^5*c^4 + 16*
a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*
((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 6
4*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d
^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8 +
4096*a^7*b^4*c^2*d^9) + (-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 +
 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152
*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122880
*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-a/(
16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^
4*c^3*d))^(3/4)*1i - 512*a^2*b^6*c^5*d^3 + 512*a^3*b^5*c^4*d^4 + 512*a^4*b^
4*c^3*d^5 - 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^
4*b^3*c^2*d^5))*1i))*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96
*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4) - atan((a^2*d^2*x^(1/2)*1i + b^2*
c^2*x^(1/2)*1i - (b^6*c^6*d*x^(1/2)*16i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*
b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) - (a^2*b^4*c^4*d^3*x^(1/
2)*32i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3
 - 64*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x^(1/2)*32i)/(16*a^4*d^5 + 16*b^4*c^4*
d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) + (a^4*b^2*c^2*
d^5*x^(1/2)*48i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2
*c^2*d^3 - 64*a^3*b*c*d^4) - (a^5*b*c*d^6*x^(1/2)*16i)/(16*a^4*d^5 + 16*b^4
*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) + (a*b^5*c
^5*d^2*x^(1/2)*48i)/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*
b^2*c^2*d^3 - 64*a^3*b*c*d^4))/((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c
^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((c*(32*a^6*b*d^7 + 32
*b^7*c^6*d - 192*a*b^6*c^5*d^2 - 192*a^5*b^2*c*d^6 + 480*a^2*b^5*c^4*d^3 -
640*a^3*b^4*c^3*d^4 + 480*a^4*b^3*c^2*d^5))/(16*a^4*d^5 + 16*b^4*c^4*d - 64
*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4) - 2*b^3*c^3 - 2*a^3*d
```

```
^3 + 2*a*b^2*c^2*d + 2*a^2*b*c*d^2)))*(-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a
*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*2i - atan((a^2*d
^2*x^(1/2)*1i + b^2*c^2*x^(1/2)*1i - (a^6*b*d^6*x^(1/2)*16i)/(16*b^5*c^4 +
16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d) + (a
^2*b^5*c^4*d^2*x^(1/2)*48i)/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 +
 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d) - (a^3*b^4*c^3*d^3*x^(1/2)*32i)/(16*b
^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^
3*d) - (a^4*b^3*c^2*d^4*x^(1/2)*32i)/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^
2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d) - (a*b^6*c^5*d*x^(1/2)*16i)/
(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b
^4*c^3*d) + (a^5*b^2*c*d^5*x^(1/2)*48i)/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3
*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))/((-a/(16*b^5*c^4 + 16*a^
4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((
a*(32*a^6*b*d^7 + 32*b^7*c^6*d - 192*a*b^6*c^5*d^2 - 192*a^5*b^2*c*d^6 + 48
0*a^2*b^5*c^4*d^3 - 640*a^3*b^4*c^3*d^4 + 480*a^4*b^3*c^2*d^5))/(16*b^5*c^4
 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d) -
 2*b^3*c^3 - 2*a^3*d^3 + 2*a*b^2*c^2*d + 2*a^2*b*c*d^2)))*(-a/(16*b^5*c^4 +
 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1
/4)*2i + 2*atan(-((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^
2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64
*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((x^(1/2)*(409
6*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5
*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-c/(16*a^4*
d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4
))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^
6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3
*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3
*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(3/4)*1i + 512*a^2*b^6*c^5
*d^3 - 512*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*1i
- x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5)) + (-c/(16*a^4*d^5 +
16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/
4)*((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3
- 64*a^3*b*c*d^4))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^
6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8
 + 4096*a^7*b^4*c^2*d^9) + (-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^
2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49
152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122
880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-
c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a
^3*b*c*d^4))^(3/4)*1i - 512*a^2*b^6*c^5*d^3 + 512*a^3*b^5*c^4*d^4 + 512*a^4
*b^4*c^3*d^5 - 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256
*a^4*b^3*c^2*d^5)))/((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96
*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((-c/(16*a^4*d^5 + 16*b^4*c^4*d -
 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((x^(1/2)*(
4096*a^2*b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*
```

```
a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-c/(16*a
^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*
d^4))^(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8
*c^6*d^6 - 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*
c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*
b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(3/4)*1i + 512*a^2*b^6*
c^5*d^3 - 512*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*
1i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5))*1i - (-c/(16*a^4*
d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4
))^(1/4)*((-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^
2*d^3 - 64*a^3*b*c*d^4))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*
b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*c
^3*d^8 + 4096*a^7*b^4*c^2*d^9) + (-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*
c^3*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)*(8192*a^2*b^10*c^8*d^
4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5*d^7
 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^2*d^10)*
1i)*(-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3*d^2 + 96*a^2*b^2*c^2*d^3
- 64*a^3*b*c*d^4))^(3/4)*1i - 512*a^2*b^6*c^5*d^3 + 512*a^3*b^5*c^4*d^4 + 5
12*a^4*b^4*c^3*d^5 - 512*a^5*b^3*c^2*d^6)*1i - x^(1/2)*(256*a^2*b^5*c^4*d^3
 + 256*a^4*b^3*c^2*d^5))*1i))*(-c/(16*a^4*d^5 + 16*b^4*c^4*d - 64*a*b^3*c^3
*d^2 + 96*a^2*b^2*c^2*d^3 - 64*a^3*b*c*d^4))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.447 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=463

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Rubi [A] time = 0.36, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 482, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*
(b*c - a*d))) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(1/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c -
a*d)) - (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}\\
&= -\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} + \frac{\sqrt{d}\,\text{Sub}}{}\\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)} - \frac{\text{Subst}\Bigg(\int}{}\\
&= \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)} - \frac{\sqrt[4]{}}{}\\
&= -\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 364, normalized size = 0.79

$$
\frac{\sqrt[4]{b}\,\sqrt[4]{c}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-\sqrt[4]{b}\,\sqrt[4]{c}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-2\sqrt[4]{b}\,\sqrt[4]{c}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\,\sqrt[4]{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)-\sqrt[4]{a}\,\sqrt[4]{d}\,\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+\sqrt[4]{a}\,\sqrt[4]{d}\,\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+2\sqrt[4]{a}\,\sqrt[4]{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)-2\sqrt[4]{a}\,\sqrt[4]{d}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*b^(1/
4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*a^(1/4)*d^(1/4
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - a^(1/4)*d^(1/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + a^(1/4)*d^(1/4)*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*
(b*c - a*d))
```

IntegrateAlgebraic [A] time = 0.50, size = 263, normalized size = 0.57

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}\,(ad-bc)}+\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{c}\,(bc-ad)}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,(ad-bc)}+\frac{\sqrt[4]{d}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\sqrt[4]{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (b^(1/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))
/Sqrt[x]])/(Sqrt[2]*a^(1/4)*(-(b*c) + a*d)) + (d^(1/4)*ArcTan[(c^(1/4)/(Sqr
t[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(1/4)*(
b*c - a*d)) + (b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)])/(Sqrt[2]*a^(1/4)*(-(b*c) + a*d)) + (d^(1/4)*ArcTanh[(Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(1/4)*(b*c - a*
d))
```

fricas [B] time = 1.16, size = 1285, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^
5*d^4))^(1/4)*arctan(-(sqrt(b^2*x - (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)
*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4)))*(b*c - a*d)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^
2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - (b^2*c - a*b*d)*sqrt(x)*(-b/(a*b^4*c^
4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4))/
b) - 2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 +
 a^4*c*d^4))^(1/4)*arctan(-(sqrt(d^2*x - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c
*d^3)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^
3 + a^4*c*d^4)))*(b*c - a*d)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d
^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - (b*c*d - a*d^2)*sqrt(x)*(-d/(b^4
*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1
/4))/d) + 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*
b*c*d^3 + a^5*d^4))^(1/4)*log((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2
- a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c
*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d
+ 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(-(a*b^3*c^3 - 3*a
^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d +
6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-d/
(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)
```

```
)^(1/4)*log((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^
4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(
3/4) + d*sqrt(x)) + 1/2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(-(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*
a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4) + d*sqrt(x))
```

giac [A] time = 0.77, size = 481, normalized size = 1.04

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^{3}c-\sqrt{2}a^{2}b^{2}d}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^{3}c-\sqrt{2}a^{2}b^{2}d}-\frac{\left(cd^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{2}d^{2}-\sqrt{2}acd^{3}}-\frac{\left(cd^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{2}d^{2}-\sqrt{2}acd^{3}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^{3}c-\sqrt{2}a^{2}b^{2}d\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^{3}c-\sqrt{2}a^{2}b^{2}d\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{2}d^{2}-\sqrt{2}acd^{3}\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{2}d^{2}-\sqrt{2}acd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + (a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(
2)*a^2*b^2*d) - (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - (c*d^3)^(3/4)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)
*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)
^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c -
 sqrt(2)*a^2*b^2*d) + 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)
*a*c*d^3)
```

maple [A] time = 0.01, size = 304, normalized size = 0.66

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^
(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/(a*d-b*c)/(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/(a*d-b*c)/(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(
1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*
x^(1/2)+(c/d)^(1/2)))+1/2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
```

```
)^(1/4)*x^(1/2)+1)+1/2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)
```

maxima [A] time = 2.57, size = 369, normalized size = 0.80

$$\frac{b\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\,(bc-ad)}-\frac{d\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c
 - a*d) - 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/
sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2
)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2
)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3
/4)))/(b*c - a*d)
```

mupad [B] time = 1.15, size = 6701, normalized size = 14.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] atan((((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d
^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^
2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*
d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 -
 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9) + 2
048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^
7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) + x^(1/2)*(256*a*b
^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^
3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i - ((-b/(16*a^5*d^4
 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^
(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2
*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*d
```

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^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 -
32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9) + 20
48*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b
^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) - x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b
^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c
^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i)/(((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a
^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16
*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*
c*d^3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^
6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d
^8 - 16384*a^6*b^5*c^2*d^9) + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 614
4*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*
b^5*c^2*d^8) + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5
*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^
3))^(1/4) + ((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2
*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5
*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^
3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^
5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 -
 16384*a^6*b^5*c^2*d^9) + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*
a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) - x^(1/2)*(2
56*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*
a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4) + 256*a*b^5*c*d
^5))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2
 - 64*a^4*b*c*d^3))^(1/4)*2i - 2*atan((((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64
*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(2048*a*b^9*c^
6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*
b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*
d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^
7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9)*1i + 2048*a^6*b^4*c*d^9
- 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144
*a^5*b^5*c^2*d^8)*1i + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b
/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^
4*b*c*d^3))^(1/4) - ((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96
*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b
^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(40
96*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3
*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^
5*c^2*d^9)*1i + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*
d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1
i - x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a
*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4))/
(((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 -
64*a^4*b*c*d^3))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a
*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(
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4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a
^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*
b^5*c^2*d^9)*1i + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*
c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1i + x^(1/2)*(256*a*
b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b
^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i + ((-b/(16*a^5*d^
4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))
^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*
b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*
d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^
7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9)*1i + 2048*a*b^9*c^6*d^4
+ 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a
^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1i - x^(1/2)*(256*a*b^6*c^2*d^5 + 25
6*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^
3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i - 256*a*b^5*c*d^5))*(-b/(16*a^5*d
^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3)
)^(1/4) + atan((((-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2
*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(3/4)*(x^(1/2)*(-d/(16*b^4*c^5 + 16*a^4*c*d
^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*(4096*a
*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8
*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^
2*d^9) + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4
096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) + x^(1/2
)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 -
 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*1i - ((-d/(
16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^
3*c^4*d))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-d/(16*b^4*c^5 + 16*a^4*c*d^
4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*(4096*a*
b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*
c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2
*d^9) + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 +
4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) - x^(1/2)*(256*a*b^6*c^2*d^5 +
 256*a^2*b^5*c*d^6))*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96
*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*1i)/(((-d/(16*b^4*c^5 + 16*a^4*c*
d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(3/4)*(x^(1/
2)*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 -
 64*a*b^3*c^4*d))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*
a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5
*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9) + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c
*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 -
 6144*a^5*b^5*c^2*d^8) + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(
-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*
a*b^3*c^4*d))^(1/4) + ((-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 +
96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(
-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*
```

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a*b^3*c^4*d))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*
b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6
*c^3*d^8 - 16384*a^6*b^5*c^2*d^9) + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d
^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) -
x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-d/(16*b^4*c^5 + 16*a^4*c
*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4) + 256
*a*b^5*c*d^5))*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b
^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*2i - 2*atan((((-d/(16*b^4*c^5 + 16*a^4*
c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(3/4)*(204
8*a*b^9*c^6*d^4 - x^(1/2)*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3
 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*
a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*
b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9)*1i + 2048*a^6*
b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*
d^7 - 6144*a^5*b^5*c^2*d^8)*1i + x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c
*d^6))*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d
^2 - 64*a*b^3*c^4*d))^(1/4) - ((-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^
2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(3/4)*(x^(1/2)*(-d/(16*b^4*c^
5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))
^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 +
 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16
384*a^6*b^5*c^2*d^9)*1i + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^
2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*
c^2*d^8)*1i - x^(1/2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-d/(16*b^4*
c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d
))^(1/4))/(((-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*
c^3*d^2 - 64*a*b^3*c^4*d))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-d/(16*b^4*
c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d
))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5
 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 -
16384*a^6*b^5*c^2*d^9)*1i + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 409
6*a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1i + x^(1/
2)*(256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-d/(16*b^4*c^5 + 16*a^4*c*d^4
- 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*1i + ((-d/
(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b
^3*c^4*d))^(3/4)*(x^(1/2)*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3
 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*(4096*a*b^10*c^7*d^4 + 4096*
a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d^6 - 32768*a^4*
b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9)*1i + 2048*a*b^
9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d^
6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8)*1i - x^(1/2)*(256*a*b^6*c^
2*d^5 + 256*a^2*b^5*c*d^6))*(-d/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d
^3 + 96*a^2*b^2*c^3*d^2 - 64*a*b^3*c^4*d))^(1/4)*1i - 256*a*b^5*c*d^5))*(-d
/(16*b^4*c^5 + 16*a^4*c*d^4 - 64*a^3*b*c^2*d^3 + 96*a^2*b^2*c^3*d^2 - 64*a*
b^3*c^4*d))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.448 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=463

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}+$$

Rubi [A] time = 0.35, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {466, 391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -((b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*
(b*c - a*d))) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c -
a*d)) + (b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4
)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{bc-ad} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{bc-ad}$$

$$= \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,(bc-ad)} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,(bc-ad)} - \frac{d\,\text{Subst}\Big(\int}{\sqrt{\;}}$$

$$= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right.}{2\sqrt{a}\,(bc-ad)}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,\right.}{2\sqrt{2}\,a^{3/4}(bc-ad)}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{\sqrt{2}\,c^{3/4}(bc-ad)}$$

Mathematica [A] time = 0.12, size = 364, normalized size = 0.79

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-b^{3/4}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+b^{3/4}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*b^(3/
4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*a^(3/4)*d^(3/4
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + a^(3/4)*d^(3/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - a^(3/4)*d^(3/4)*Log[Sqrt[c]
+ Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*a^(3/4)*c^(3/4)*
(b*c - a*d))
```

IntegrateAlgebraic [A] time = 0.53, size = 265, normalized size = 0.57

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}(ad-bc)}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}(ad-bc)}+\frac{d^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\,\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\,\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}-\frac{d^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (b^(3/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))
/Sqrt[x]])/(Sqrt[2]*a^(3/4)*(-(b*c) + a*d)) + (d^(3/4)*ArcTan[(c^(1/4)/(Sqr
t[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(3/4)*(
b*c - a*d)) - (b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)*(-(b*c) + a*d)) - (d^(3/4)*ArcTanh[(Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(3/4)*(b*c - a*
d))
```

fricas [B] time = 1.80, size = 1365, normalized size = 2.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
 + a^7*d^4))^(1/4)*arctan(-((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*sqrt(b^2*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))
)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
+ a^7*d^4))^(3/4) - (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*
b*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c
*d^3 + a^7*d^4))^(3/4)*sqrt(x))/b^3) + 2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(-((b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(d^2*x + (b^2*c^4 - 2*
a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5
*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*
a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4) - (b^3*c^5*d - 3*a*
b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
 + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt(x))/d^3)
+ 1/2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*
d^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/
2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3
+ a^7*d^4))^(1/4)*log(b*sqrt(x) - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^
```

```
4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/2*(-
d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^
3*d^4))^(1/4)*log(d*sqrt(x) + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*
d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/2*(-d^3/
(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^
4))^(1/4)*log(d*sqrt(x) - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4))
```

giac [A] time = 0.65, size = 441, normalized size = 0.95

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}abc-\sqrt{2}a^{2}d}+\frac{\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}abc-\sqrt{2}a^{2}d}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{2}-\sqrt{2}acd}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{2}-\sqrt{2}acd}+\frac{\left(ab^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^{2}d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{2}-\sqrt{2}acd\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{2}-\sqrt{2}acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d
) - (c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - (c*d^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a
*c*d) + 1/2*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a
/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(c*d^3)^(1
/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(
2)*a*c*d) + 1/2*(c*d^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d)
```

maple [A] time = 0.01, size = 328, normalized size = 0.71

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)a}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x)
```

```
[Out] -1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b/(a*d-b*c)*(a/
b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)*(a
/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/4*d/(a*d-b*c)*(
c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d
)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+1/2*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

maxima [A]   time = 2.50, size = 371, normalized size = 0.80

$$\frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\,(bc-ad)}-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{4\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b*c -
 a*d) - 1/4*(2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sq
rt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*s
qrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/
sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*lo
g(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/4) - sqrt(2)*
d^(3/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/4)
)/(b*c - a*d)
```

mupad [B]   time = 1.67, size = 8785, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - atan(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*
c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a
^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*
c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6
*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*
d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8
 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^(1/2)*(4096*a^7*b^4*
d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24
576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576
*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 +
96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2
*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2))*1i - (-d^3/(16*b^4*c^7 + 16
*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/
4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*
d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*
b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^
```

```
4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 -
40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 409
60*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 163
84*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^
3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b
^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*
c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) - 512
*b^7*d^7*x^(1/2))*1i)/((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^
3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4
*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(
((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2
 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 4096
0*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*
a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^(1/2)
*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*
b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*
c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a
^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^
8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2)) + (-d^3/(16*
b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3
*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*
a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d
^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a
*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^
9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c
^3*d^9 - 40960*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c
^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^
6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))
*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2
 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*
c*d^7) - 512*b^7*d^7*x^(1/2))))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3
*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*2i - 2*atan(((-d^3
/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*
a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3
+ 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*
c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8
192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a
^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*
b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i + x^(1/2)*(4096*a^7*b^4*d^11 + 409
6*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^
9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c
^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2
*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8*c^2*d^6 -
1024*a*b^7*c*d^7)*1i - 512*b^7*d^7*x^(1/2)) - (-d^3/(16*b^4*c^7 + 16*a^4*c^
3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d
```

```
^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 6
4*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d
^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 819
2*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a
^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*
b^5*c^2*d^10)*1i - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a
*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^
8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c
^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*
d))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8*c^2*d^6 - 1024*a*b^7*c*d^7)*1i + 5
12*b^7*d^7*x^(1/2)))/((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3
 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*
c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((
(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2
 - 64*a*b^3*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960
*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a
^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i + x^(1/
2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^
6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^
7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64
*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4)*1i + 512*a^2*b
^6*d^8 + 512*b^8*c^2*d^6 - 1024*a*b^7*c*d^7)*1i - 512*b^7*d^7*x^(1/2))*1i +
 (-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2
 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^
4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 1
6*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1
/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 7
3728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 7372
8*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i - x^(1/2)*(4096*a^7*b^4*d^11
 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*
a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5
*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a
^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8*c^2*
d^6 - 1024*a*b^7*c*d^7)*1i + 512*b^7*d^7*x^(1/2))*1i))*(-d^3/(16*b^4*c^7 +
16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(
1/4) - atan(((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5
*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 -
 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(((-b^3/(16
*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*
b*c*d^3))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10
*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^
4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^(1/2)*(4096*a^7
*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10
 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 +
24576*a^5*b^6*c^2*d^9))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3
```

```
*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(3/4) - 512*a^2*b^6*d^8 - 512*b^
8*c^2*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2))*1i - (-b^3/(16*a^7*d^4
 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3)
)^(1/4)*((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2
*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64
*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(8192*a*b^11*c
^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d
^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9
- 40960*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4
- 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 122
88*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-b^3/
(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a
^6*b*c*d^3))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7)
- 512*b^7*d^7*x^(1/2))*1i)/((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3
*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*((-b^3/(16*a^7*d^4 + 1
6*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1
/4)*(((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^
2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 -
 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 4
0960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^
(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384
*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4
*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 -
 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(3/4) - 512*a^2*b
^6*d^8 - 512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2)) + (-b^3
/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*
a^6*b*c*d^3))^(1/4)*((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d
+ 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(((-b^3/(16*a^7*d^4 + 16*a^3*
b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(8
192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a
^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*
b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b
^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c
^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*
d^9))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^
2*d^2 - 64*a^6*b*c*d^3))^(3/4) - 512*a^2*b^6*d^8 - 512*b^8*c^2*d^6 + 1024*a
*b^7*c*d^7) - 512*b^7*d^7*x^(1/2))))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 6
4*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*2i - 2*atan((
(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2
- 64*a^6*b*c*d^3))^(1/4)*((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c
^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(((-b^3/(16*a^7*d^4 + 16
*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/
4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73
728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728
*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i + x^(1/2)*(4096*a^7*b^4*d^11
```

```
+ 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a
^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*
b^6*c^2*d^9))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^
5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8*c^2*d
^6 - 1024*a*b^7*c*d^7)*1i - 512*b^7*d^7*x^(1/2)) - (-b^3/(16*a^7*d^4 + 16*a
^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)
*((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^
2 - 64*a^6*b*c*d^3))^(1/4)*(((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^
3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(8192*a*b^11*c^8*d^4
+ 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40
960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960
*a^7*b^5*c^2*d^10)*1i - x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16
384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a
^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-b^3/(16*
a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b
*c*d^3))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8*c^2*d^6 - 1024*a*b^7*c*d^7)*1
i + 512*b^7*d^7*x^(1/2)))/((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*
c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*((-b^3/(16*a^7*d^4 + 16
*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/
4)*(((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2
*d^2 - 64*a^6*b*c*d^3))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 -
40960*a^2*b^10*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40
960*a^5*b^7*c^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i +
x^(1/2)*(4096*a^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 163
84*a^6*b^5*c*d^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a
^4*b^7*c^3*d^8 + 24576*a^5*b^6*c^2*d^9))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4
 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(3/4)*1i + 512*
a^2*b^6*d^8 + 512*b^8*c^2*d^6 - 1024*a*b^7*c*d^7)*1i - 512*b^7*d^7*x^(1/2))
*1i + (-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^
2*d^2 - 64*a^6*b*c*d^3))^(1/4)*((-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4
*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(1/4)*(((-b^3/(16*a^7*d^
4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3
))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^
5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 +
 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10)*1i - x^(1/2)*(4096*a^7*b^4
*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 2
4576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^8 + 2457
6*a^5*b^6*c^2*d^9))*(-b^3/(16*a^7*d^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d +
 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^3))^(3/4)*1i + 512*a^2*b^6*d^8 + 512*b^8
*c^2*d^6 - 1024*a*b^7*c*d^7)*1i + 512*b^7*d^7*x^(1/2))*1i))*(-b^3/(16*a^7*d
^4 + 16*a^3*b^4*c^4 - 64*a^4*b^3*c^3*d + 96*a^5*b^2*c^2*d^2 - 64*a^6*b*c*d^
3))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)/x**(1/2),x)
```

```
[Out] Timed out
```

3.449 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=476

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}-$$

Rubi [A] time = 0.54, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 480, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{2}{ac\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(a*c*Sqrt[x]) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(
5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d
)) - (d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
2*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{ac}$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{2\,\text{Subst}\left(\int \left(-\frac{b^2cx^2}{(bc-ad)\left(a+bx^4\right)} - \frac{ad^2x^2}{(-bc+ad)\left(c+dx^4\right)}\right) dx, x, \sqrt{x}\right)}{ac}$$

$$= -\frac{2}{ac\sqrt{x}} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} + \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x^2}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c(bc-ad)}$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)} - \frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)}$$

$$= -\frac{2}{ac\sqrt{x}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x,\right.}{2a(bc-ad)}$$

$$= -\frac{2}{ac\sqrt{x}} - \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)} + \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\right.}{2\sqrt{2}\,a^{5/4}(bc-ad)}$$

$$= -\frac{2}{ac\sqrt{x}} + \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)} - \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)} - \frac{d^{5/4}\tan^{-1}\left(1-\right.}{\sqrt{2}\,c^{5/4}(b}$$

Mathematica [A] time = 0.23, size = 409, normalized size = 0.86

$$\frac{\frac{\sqrt{2}\,b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{5/4}} - \frac{\sqrt{2}\,b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{5/4}} - \frac{2\sqrt{2}\,b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}\,b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}} + \frac{8b}{a\sqrt{x}} - \frac{\sqrt{2}\,d^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{5/4}} + \frac{\sqrt{2}\,d^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{5/4}} + \frac{2\sqrt{2}\,d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{2\sqrt{2}\,d^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}} - \frac{8d}{c\sqrt{x}}}{4ad-4bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*Sqrt[x]) - (8*d)/(c*Sqrt[x]) - (2*Sqrt[2]*b^(5/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) - (2*Sqrt[2]*d^(5/4)*ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (Sqrt[2]*b^(5/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (Sqrt[2]*b^(5/4)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (Sqrt[
2]*d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5
/4) + (Sqrt[2]*d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/c^(5/4))/(-4*b*c + 4*a*d)
```

IntegrateAlgebraic [A] time = 0.62, size = 280, normalized size = 0.59

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{5/4}(ad-bc)}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{5/4}(ad-bc)}-\frac{d^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{d^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{5/4}(bc-ad)}-\frac{2}{ac\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(a*c*Sqrt[x]) - (b^(5/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)
/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(5/4)*(-(b*c) + a*d)) - (d^(5/4)*A
rcTan[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])
/(Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(5/4)*(-(b*c) + a*d)) - (d^(5/4
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2
]*c^(5/4)*(b*c - a*d))
```

fricas [B] time = 2.49, size = 1421, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(-(sqrt(b^8*x - (a^3*b^7*c^2 - 2*a^4*b^
6*c*d + a^5*b^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c
^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4)))*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d +
6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d) - (a*b^
5*c - a^2*b^4*d)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
```

```
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*sqrt(x))/b^5) - 4*(-d^5/(b^4*c^9 - 4*a*b^3
*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*ar
ctan(-(sqrt(d^8*x - (b^2*c^5*d^5 - 2*a*b*c^4*d^6 + a^2*c^3*d^7)*sqrt(-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
)))*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d) - (b*c^2*d^4 - a*c*d^5)*(-d^5/(b^4*c^9
- 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)
*sqrt(x))/d^5) + (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*sqrt(x) + (a^4*b^3*c^3 - 3*a
^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*
d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c
^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*
a*c*x*log(b^4*sqrt(x) - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^
7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^
2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*sqrt(x) + (b^3*c^7
- 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c
^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + (-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
))^(1/4)*a*c*x*log(d^4*sqrt(x) - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*
b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4*sqrt(x))/(a*c*x)
```

giac [A] time = 0.86, size = 492, normalized size = 1.03

$$-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{2}b^{2}c-\sqrt{2}a^{3}bd\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{3}d-\sqrt{2}ac^{2}d^{2}\right)}-\frac{2}{ac\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - (a*b^3)^(3/4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sq
rt(2)*a^3*b*d) + (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + (c*d^3)^(3/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2
)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*
c - sqrt(2)*a^3*b*d) - 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
x + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/4)*lo
g(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*
a*c^2*d^2) - 2/(a*c*sqrt(x))
```

maple [A] time = 0.02, size = 339, normalized size = 0.71

$$\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,d\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{2}{ac\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*b/a/(a*d-b*c)/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b/a/(a*d-b*c)/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4*d/c/(a*d-b*c)/
(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-1/2*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a/c/x^(1/2)
```

maxima [A] time = 2.54, size = 390, normalized size = 0.82

$$-\frac{b^{2}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(abc-a^{2}d\right)}+\frac{d^{2}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\left(bc^{2}-acd\right)}-\frac{2}{ac\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
a*b*c - a^2*d) + 1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*
sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4))
 + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 2/(a*c*sqrt(x))
```

mupad [B] time = 1.64, size = 6038, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x)

[Out] atan((a^6*b^8*c^9*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c
^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*32i + a^6*b^4*d^5*x^(1/2
)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^
2 - 64*a^8*b*c*d^3))^(1/4)*2i + a^14*c*d^8*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a
^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)
*32i + a^8*b^6*c^7*d^2*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*
b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*192i - a^9*b^5*c^6*
d^3*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*
b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i + a^10*b^4*c^5*d^4*x^(1/2)*(-b^5/
(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a
^8*b*c*d^3))^(5/4)*64i - a^11*b^3*c^4*d^5*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^
5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*
128i + a^12*b^2*c^3*d^6*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6
*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*192i + a^5*b^5*c*d
^4*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b
^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*2i - a^7*b^7*c^8*d*x^(1/2)*(-b^5/(16*a^
9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c
*d^3))^(5/4)*128i - a^13*b*c^2*d^7*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c
^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i)/(
b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b
^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 6
4*a^8*b*c*d^3))^(1/4)*2i + atan((a^9*c^6*d^8*x^(1/2)*(-d^5/(16*b^4*c^9 + 16
*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/
4)*32i + b^5*c^6*d^4*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*
c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*2i + a*b^8*c^14*x^(1/
2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d
^2 - 64*a*b^3*c^8*d))^(5/4)*32i + a^3*b^6*c^12*d^2*x^(1/2)*(-d^5/(16*b^4*c^
9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d
))^(5/4)*192i - a^4*b^5*c^11*d^3*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4
 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*128i + a^
5*b^4*c^10*d^4*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^
3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*64i - a^6*b^3*c^9*d^5*x^(1/
2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d
^2 - 64*a*b^3*c^8*d))^(5/4)*128i + a^7*b^2*c^8*d^6*x^(1/2)*(-d^5/(16*b^4*c^
9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d
))^(5/4)*192i + a*b^4*c^5*d^5*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 -
64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*2i - a^2*b^7
*c^13*d*x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*
a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(5/4)*128i - a^8*b*c^7*d^7*x^(1/2)*(-d^5
/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*
a*b^3*c^8*d))^(5/4)*128i)/(a^4*d^9 + b^4*c^4*d^5 + a*b^3*c^3*d^6 + a^2*b^2*
c^2*d^7 + a^3*b*c*d^8))*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d
^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*2i + 2*atan(((-b^5/(16*a^9
```

```
*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*
d^3))^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*c^11*d^9) - (-b^
5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64
*a^8*b*c*d^3))^(3/4)*(x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b
^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(4096*a^12*b^12*c^20
*d^4 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6 - 16384*a^15*b^9
*c^17*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^9 + 24576*a^18*b
^6*c^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^12*d^12)*1i - 204
8*a^11*b^12*c^19*d^4 + 6144*a^12*b^11*c^18*d^5 - 6144*a^13*b^10*c^17*d^6 +
2048*a^14*b^9*c^16*d^7 + 2048*a^16*b^7*c^14*d^9 - 6144*a^17*b^6*c^13*d^10 +
 6144*a^18*b^5*c^12*d^11 - 2048*a^19*b^4*c^11*d^12)*1i) + (-b^5/(16*a^9*d^4
 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3)
)^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*c^11*d^9) - (-b^5/(1
6*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8
*b*c*d^3))^(3/4)*(x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c
^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(4096*a^12*b^12*c^20*d^4
 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6 - 16384*a^15*b^9*c^1
7*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^9 + 24576*a^18*b^6*c
^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^12*d^12)*1i + 2048*a^
11*b^12*c^19*d^4 - 6144*a^12*b^11*c^18*d^5 + 6144*a^13*b^10*c^17*d^6 - 2048
*a^14*b^9*c^16*d^7 - 2048*a^16*b^7*c^14*d^9 + 6144*a^17*b^6*c^13*d^10 - 614
4*a^18*b^5*c^12*d^11 + 2048*a^19*b^4*c^11*d^12)*1i))/((-b^5/(16*a^9*d^4 + 1
6*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1
/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*c^11*d^9) - (-b^5/(16*a^
9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c
*d^3))^(3/4)*(x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d
 + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(4096*a^12*b^12*c^20*d^4 - 1
6384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6 - 16384*a^15*b^9*c^17*d^
7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^9 + 24576*a^18*b^6*c^14*
d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^12*d^12)*1i - 2048*a^11*b
^12*c^19*d^4 + 6144*a^12*b^11*c^18*d^5 - 6144*a^13*b^10*c^17*d^6 + 2048*a^1
4*b^9*c^16*d^7 + 2048*a^16*b^7*c^14*d^9 - 6144*a^17*b^6*c^13*d^10 + 6144*a^
18*b^5*c^12*d^11 - 2048*a^19*b^4*c^11*d^12)*1i)*1i - (-b^5/(16*a^9*d^4 + 16
*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/
4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*c^11*d^9) - (-b^5/(16*a^9
*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*
d^3))^(3/4)*(x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d
+ 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*(4096*a^12*b^12*c^20*d^4 - 16
384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6 - 16384*a^15*b^9*c^17*d^7
 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^9 + 24576*a^18*b^6*c^14*d
^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^12*d^12)*1i + 2048*a^11*b^
12*c^19*d^4 - 6144*a^12*b^11*c^18*d^5 + 6144*a^13*b^10*c^17*d^6 - 2048*a^14
*b^9*c^16*d^7 - 2048*a^16*b^7*c^14*d^9 + 6144*a^17*b^6*c^13*d^10 - 6144*a^1
8*b^5*c^12*d^11 + 2048*a^19*b^4*c^11*d^12)*1i)*1i))*(-b^5/(16*a^9*d^4 + 16*
a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4
```

```
) + 2*atan(((-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*
b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*
a^12*b^8*c^11*d^9) - (-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3
 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(3/4)*(x^(1/2)*(-d^5/(16*b^4*c^9 +
16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(
1/4)*(4096*a^12*b^12*c^20*d^4 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*
c^18*d^6 - 16384*a^15*b^9*c^17*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^
7*c^15*d^9 + 24576*a^18*b^6*c^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^2
0*b^4*c^12*d^12)*1i - 2048*a^11*b^12*c^19*d^4 + 6144*a^12*b^11*c^18*d^5 - 6
144*a^13*b^10*c^17*d^6 + 2048*a^14*b^9*c^16*d^7 + 2048*a^16*b^7*c^14*d^9 -
6144*a^17*b^6*c^13*d^10 + 6144*a^18*b^5*c^12*d^11 - 2048*a^19*b^4*c^11*d^12
)*1i) + (-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*
c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12
*b^8*c^11*d^9) - (-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96
*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(3/4)*(x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a
^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)
*(4096*a^12*b^12*c^20*d^4 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18
*d^6 - 16384*a^15*b^9*c^17*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^
15*d^9 + 24576*a^18*b^6*c^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^
4*c^12*d^12)*1i + 2048*a^11*b^12*c^19*d^4 - 6144*a^12*b^11*c^18*d^5 + 6144*
a^13*b^10*c^17*d^6 - 2048*a^14*b^9*c^16*d^7 - 2048*a^16*b^7*c^14*d^9 + 6144
*a^17*b^6*c^13*d^10 - 6144*a^18*b^5*c^12*d^11 + 2048*a^19*b^4*c^11*d^12)*1i
))/((-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*
d^2 - 64*a*b^3*c^8*d))^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8
*c^11*d^9) - (-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2
*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(3/4)*(x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c
^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*(40
96*a^12*b^12*c^20*d^4 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6
 - 16384*a^15*b^9*c^17*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d
^9 + 24576*a^18*b^6*c^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^
12*d^12)*1i - 2048*a^11*b^12*c^19*d^4 + 6144*a^12*b^11*c^18*d^5 - 6144*a^13
*b^10*c^17*d^6 + 2048*a^14*b^9*c^16*d^7 + 2048*a^16*b^7*c^14*d^9 - 6144*a^1
7*b^6*c^13*d^10 + 6144*a^18*b^5*c^12*d^11 - 2048*a^19*b^4*c^11*d^12)*1i)*1i
 - (-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d
^2 - 64*a*b^3*c^8*d))^(1/4)*(x^(1/2)*(256*a^11*b^9*c^12*d^8 + 256*a^12*b^8*
c^11*d^9) - (-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*
b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(3/4)*(x^(1/2)*(-d^5/(16*b^4*c^9 + 16*a^4*c^
5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^2 - 64*a*b^3*c^8*d))^(1/4)*(409
6*a^12*b^12*c^20*d^4 - 16384*a^13*b^11*c^19*d^5 + 24576*a^14*b^10*c^18*d^6
- 16384*a^15*b^9*c^17*d^7 + 8192*a^16*b^8*c^16*d^8 - 16384*a^17*b^7*c^15*d^
9 + 24576*a^18*b^6*c^14*d^10 - 16384*a^19*b^5*c^13*d^11 + 4096*a^20*b^4*c^1
2*d^12)*1i + 2048*a^11*b^12*c^19*d^4 - 6144*a^12*b^11*c^18*d^5 + 6144*a^13*
b^10*c^17*d^6 - 2048*a^14*b^9*c^16*d^7 - 2048*a^16*b^7*c^14*d^9 + 6144*a^17
*b^6*c^13*d^10 - 6144*a^18*b^5*c^12*d^11 + 2048*a^19*b^4*c^11*d^12)*1i)*1i)
)*(-d^5/(16*b^4*c^9 + 16*a^4*c^5*d^4 - 64*a^3*b*c^6*d^3 + 96*a^2*b^2*c^7*d^
```

```
2 - 64*a*b^3*c^8*d))^(1/4) - 2/(a*c*x^(1/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.450 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=478

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-$$

Rubi [A] time = 0.48, antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 480, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{2}{3acx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(3*a*c*x^(3/2)) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b
^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c]
- Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(7/4)*(b*c - a
*d)) + (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])
/(2*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2}{3acx^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{3ac}\\
&= -\frac{2}{3acx^{3/2}}-\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a(bc-ad)}+\frac{\left(2d^2\right)\text{Subst}\left(\int \frac{1}{c+dx^4}\,dx, x, \sqrt{x}\right)}{c(bc-ad)}\\
&= -\frac{2}{3acx^{3/2}}-\frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}(bc-ad)}-\frac{b^2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^{3/2}(bc-ad)}-\\
&= -\frac{2}{3acx^{3/2}}-\frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^{3/2}(bc-ad)}-\frac{b^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\right.}{2a^{3/2}(bc-ad)}\\
&= -\frac{2}{3acx^{3/2}}+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\,a^{7/4}(bc-ad}\\
&= -\frac{2}{3acx^{3/2}}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1\right.}{\sqrt{2}\,c^{7/4}(}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 411, normalized size = 0.86

$$\frac{-\frac{3\sqrt{2}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}+\frac{3\sqrt{2}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}}-\frac{6\sqrt{2}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8b}{ax^{3/2}}+\frac{3\sqrt{2}d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}-\frac{3\sqrt{2}d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}+\frac{6\sqrt{2}d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{6\sqrt{2}d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{7/4}}-\frac{8d}{cx^{3/2}}}{12ad-12bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*x^(3/2)) - (8*d)/(c*x^(3/2)) - (6*Sqrt[2]*b^(7/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3*Sqrt[2]*b^(7/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(7/4) + (3
*Sqrt[2]*d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/c^(7/4) - (3*Sqrt[2]*d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/c^(7/4))/(-12*b*c + 12*a*d)
```

IntegrateAlgebraic [A] time = 0.64, size = 280, normalized size = 0.59

$$-\frac{b^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{7/4}(ad-bc)}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{7/4}(ad-bc)}-\frac{d^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}\,x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}+\frac{d^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{\sqrt{2}\,c^{7/4}(bc-ad)}-\frac{2}{3acx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(3*a*c*x^(3/2)) - (b^(7/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*
x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(7/4)*(-(b*c) + a*d)) - (d^(7/4)
*ArcTan[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]
])/(Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(7/4)*(-(b*c) + a*d)) + (d^(7
/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt
[2]*c^(7/4)*(b*c - a*d))
```

fricas [B] time = 15.95, size = 1431, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/6*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b
*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*arctan(-((a^5*b^3*c^3 - 3*a^6*b^2*c^2*d +
 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(b^4*x + (a^4*b^2*c^2 - 2*a
^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^
2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))) - (a^5*b^5*c^3 - 3*a^6*b^4*c^2*d + 3*a
^7*b^3*c*d^2 - a^8*b^2*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^
2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt(x))/b^7) - 12*(-d^7/(b^4
*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4)
)^(1/4)*a*c*x^2*arctan(-((b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c
```

```
^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(3/4)*sqrt(d^4*x + (b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2
)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^
3 + a^4*c^7*d^4))) - (b^3*c^8*d^2 - 3*a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3
*c^5*d^5)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^
8*d^3 + a^4*c^7*d^4))^(3/4)*sqrt(x))/d^7) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^
3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*log
(b^2*sqrt(x) + (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4
*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4
- 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a
*c*x^2*log(b^2*sqrt(x) - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c
^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^
4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4
))^(1/4)*a*c*x^2*log(d^2*sqrt(x) + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2
*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3
*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a
^4*c^7*d^4))^(1/4)*a*c*x^2*log(d^2*sqrt(x) - (-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c
^2*d)) - 4*sqrt(x))/(a*c*x^2)
```

giac [A] time = 0.70, size = 476, normalized size = 1.00

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{2}bc-\sqrt{2}a^{3}d}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{2}bc-\sqrt{2}a^{3}d}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{3}-\sqrt{2}ac^{2}d}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^{3}-\sqrt{2}ac^{2}d}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{2}bc-\sqrt{2}a^{3}d\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{2}bc-\sqrt{2}a^{3}d\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{3}-\sqrt{2}ac^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^{3}-\sqrt{2}ac^{2}d\right)}-\frac{2}{3\,acx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - (a*b^3)^(1/4)*b*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt
(2)*a^3*d) + (c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sq
rt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + (c*d^3)^(1/4)*d*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*
c^3 - sqrt(2)*a*c^2*d) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/2*(a*b^3)^(1/4)*b*
log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2
)*a^3*d) + 1/2*(c*d^3)^(1/4)*d*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/2*(c*d^3)^(1/4)*d*log(-sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 2/3
/(a*c*x^(3/2))
```

maple [A] time = 0.02, size = 351, normalized size = 0.73

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)a^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)c^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)c^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)c^{2}}-\frac{2}{3ac\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] 1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/a^2*b^2/(a*d
-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/a^2*b^2
/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/4/c^
2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)
^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-1/2/c^2*d^2/(a*d-b*c)*
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-1/2/c^2*d^2/(a*d-
b*c)*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/3/a/c/x^(3
/2)
```

maxima [**A**] time = 2.38, size = 396, normalized size = 0.83

$$-\frac{\frac{2\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(abc-a^{2}d\right)}+\frac{\frac{2\sqrt{2}\,d^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\,d^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\,d^{\frac{7}{4}}\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}\,d^{\frac{7}{4}}\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{4\left(bc^{2}-acd\right)}-\frac{2}{3\,acx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(
7/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(
a*b*c - a^2*d) + 1/4*(2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt
(d))) + 2*sqrt(2)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt
(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(
2)*d^(7/4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/
4) - sqrt(2)*d^(7/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/c^(3/4))/(b*c^2 - a*c*d) - 2/3/(a*c*x^(3/2))
```

mupad [**B**] time = 2.40, size = 7540, normalized size = 15.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] 2*atan(((x^(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) - (-d^7/(1
6*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*
b^3*c^10*d))^(1/4)*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3
```

```
 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 -
40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d
^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^1
5*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i + x^(1/2)*(
4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d
^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15
*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5
*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4
- 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/4)*1i + 512*
a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a
^14*b^7*c^9*d^12)*1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^
3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4) + (x^(1/2)*(256*a^9*b^11*c
^11*d^9 + 256*a^11*b^9*c^9*d^11) + (-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64
*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(((-d^7/(16*b
^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3
*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^11*c^20*d^5 + 81920
*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*a^17*b^8*c^17*d^8 - 9
0112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40960*a^20*b^5*c^14*d^1
1 + 8192*a^21*b^4*c^13*d^12)*1i - x^(1/2)*(4096*a^11*b^13*c^20*d^4 - 16384*
a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*a^14*b^10*c^17*d^7 +
4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^10
+ 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^11*
d^13))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*
c^9*d^2 - 64*a*b^3*c^10*d))^(3/4)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*b^1
1*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-d^7/(16*
b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^
3*c^10*d))^(1/4))/((x^(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11)
 - (-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*
d^2 - 64*a*b^3*c^10*d))^(1/4)*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^
3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*
c^21*d^4 - 40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16
*b^9*c^18*d^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a
^19*b^6*c^15*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i
+ x^(1/2)*(4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*
b^11*c^18*d^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^
16*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 163
84*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a
^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/4
)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d
^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a
^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*1i - (x^(1/2)*(
256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) + (-d^7/(16*b^4*c^11 + 16*a^
4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)
*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*
d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^11*c^
```

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20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*a^17*b^
8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40960*a^2
0*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i - x^(1/2)*(4096*a^11*b^13*c^2
0*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*a^14*b^
10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 16384*a^17*
b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12 + 4096*
a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3
 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/4)*1i + 512*a^9*b^12*c^14*d^7
 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9*d^12)*
1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9
*d^2 - 64*a*b^3*c^10*d))^(1/4)*1i))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 6
4*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4) - atan((a^2*
b^5*d^7*x^(1/2)*1i + b^7*c^2*d^5*x^(1/2)*1i - (a^2*b^16*c^11*x^(1/2)*16i)/(
16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a
^10*b*c*d^3) + (a^3*b^15*c^10*d*x^(1/2)*64i)/(16*a^11*d^4 + 16*a^7*b^4*c^4
 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3) - (a^4*b^14*c^9*
d^2*x^(1/2)*96i)/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*
b^2*c^2*d^2 - 64*a^10*b*c*d^3) + (a^5*b^13*c^8*d^3*x^(1/2)*64i)/(16*a^11*d^
4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^
3) - (a^6*b^12*c^7*d^4*x^(1/2)*16i)/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*
b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x^(1/
2)*16i)/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d
^2 - 64*a^10*b*c*d^3) + (a^8*b^10*c^5*d^6*x^(1/2)*64i)/(16*a^11*d^4 + 16*a^
7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3) - (a^9
*b^9*c^4*d^7*x^(1/2)*96i)/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d
 + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x^(1/2)*64i)/(1
6*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^
10*b*c*d^3) - (a^11*b^7*c^2*d^9*x^(1/2)*16i)/(16*a^11*d^4 + 16*a^7*b^4*c^4
 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))/((-b^7/(16*a^11
*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c
*d^3))^(1/4)*((b^7*(32*a^4*b^8*c^12 + 32*a^12*c^4*d^8 - 160*a^5*b^7*c^11*d
 - 160*a^11*b*c^5*d^7 + 320*a^6*b^6*c^10*d^2 - 352*a^7*b^5*c^9*d^3 + 320*a^8
*b^4*c^8*d^4 - 352*a^9*b^3*c^7*d^5 + 320*a^10*b^2*c^6*d^6))/(16*a^11*d^4 +
16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3) +
 2*a^5*b^3*d^8 + 2*b^8*c^5*d^3 - 2*a*b^7*c^4*d^4 - 2*a^4*b^4*c*d^7)))*(-b^7
/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64
*a^10*b*c*d^3))^(1/4)*2i - atan((a^2*b^5*d^7*x^(1/2)*1i + b^7*c^2*d^5*x^(1/
2)*1i - (a^11*c^2*d^16*x^(1/2)*16i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*
b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d) + (a^10*b*c^3*d^15*x^(1/2
)*64i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^
2 - 64*a*b^3*c^10*d) - (a^2*b^9*c^11*d^7*x^(1/2)*16i)/(16*b^4*c^11 + 16*a^4
*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d) + (a^3*
b^8*c^10*d^8*x^(1/2)*64i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3
 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d) - (a^4*b^7*c^9*d^9*x^(1/2)*96i)/(16
*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b
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^3*c^10*d) + (a^5*b^6*c^8*d^10*x^(1/2)*64i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 -
 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d) - (a^6*b^5*c^7*d^
11*x^(1/2)*16i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b
^2*c^9*d^2 - 64*a*b^3*c^10*d) - (a^7*b^4*c^6*d^12*x^(1/2)*16i)/(16*b^4*c^11
 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d
) + (a^8*b^3*c^5*d^13*x^(1/2)*64i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b
*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d) - (a^9*b^2*c^4*d^14*x^(1/2
)*96i)/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^
2 - 64*a*b^3*c^10*d))/((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d
^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*((d^7*(32*a^4*b^8*c^12 +
32*a^12*c^4*d^8 - 160*a^5*b^7*c^11*d - 160*a^11*b*c^5*d^7 + 320*a^6*b^6*c^1
0*d^2 - 352*a^7*b^5*c^9*d^3 + 320*a^8*b^4*c^8*d^4 - 352*a^9*b^3*c^7*d^5 + 3
20*a^10*b^2*c^6*d^6))/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96
*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d) + 2*a^5*b^3*d^8 + 2*b^8*c^5*d^3 - 2*a*b
^7*c^4*d^4 - 2*a^4*b^4*c*d^7)))*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^
3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*2i + 2*atan(((x^
(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) - (-b^7/(16*a^11*d^4
+ 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3)
)^(1/4)*(((-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b
^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*
b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*
a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40
960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i + x^(1/2)*(4096*a^11*b
^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*
a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 1638
4*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12
+ 4096*a^20*b^4*c^11*d^13))*(-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^
3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(3/4)*1i + 512*a^9*b^12*c^
14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9
*d^12)*1i)*(-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*
b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4) + (x^(1/2)*(256*a^9*b^11*c^11*d^9 + 2
56*a^11*b^9*c^9*d^11) + (-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^
3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4)*(((-b^7/(16*a^11*d^4 + 1
6*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(
1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*
c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b
^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^
21*b^4*c^13*d^12)*1i - x^(1/2)*(4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c
^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b
^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^1
8*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-b^
7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 6
4*a^10*b*c*d^3))^(3/4)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8
- 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-b^7/(16*a^11*d^4 +
16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^
```

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(1/4))/((x^(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) - (-b^7/(1
6*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^
10*b*c*d^3))^(1/4)*(((-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d
 + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4)*(8192*a^13*b^12*c^21*d^4 -
40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d
^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^1
5*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i + x^(1/2)*(
4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d
^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15
*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5
*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4
 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(3/4)*1i + 512*
a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a
^14*b^7*c^9*d^12)*1i)*(-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*
d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4)*1i - (x^(1/2)*(256*a^9*b^1
1*c^11*d^9 + 256*a^11*b^9*c^9*d^11) + (-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 -
 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4)*(((-b^7/(1
6*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^
10*b*c*d^3))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^11*c^20*d^5 + 81
920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 81920*a^17*b^8*c^17*d^8
 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b^6*c^15*d^10 - 40960*a^20*b^5*c^14*
d^11 + 8192*a^21*b^4*c^13*d^12)*1i - x^(1/2)*(4096*a^11*b^13*c^20*d^4 - 163
84*a^12*b^12*c^19*d^5 + 24576*a^13*b^11*c^18*d^6 - 16384*a^14*b^10*c^17*d^7
 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^
10 + 24576*a^18*b^6*c^13*d^11 - 16384*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^
11*d^13))*(-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b
^2*c^2*d^2 - 64*a^10*b*c*d^3))^(3/4)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*
b^11*c^13*d^8 - 512*a^13*b^8*c^10*d^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-b^7/(
16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c^3*d + 96*a^9*b^2*c^2*d^2 - 64*a
^10*b*c*d^3))^(1/4)*1i))*(-b^7/(16*a^11*d^4 + 16*a^7*b^4*c^4 - 64*a^8*b^3*c
^3*d + 96*a^9*b^2*c^2*d^2 - 64*a^10*b*c*d^3))^(1/4) - 2/(3*a*c*x^(3/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.451 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$

Optimal. Leaf size=498

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+$$

Rubi [A] time = 0.69, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 480, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{2(ad+bc)}{a^2c^2\sqrt{x}}-\frac{d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{2\sqrt{2}\,c^{9/4}(bc-ad)}+\frac{d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\,c^{9/4}(bc-ad)}-\frac{2}{5acx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] -2/(5*a*c*x^(5/2)) + (2*(b*c + a*d))/(a^2*c^2*Sqrt[x]) - (b^(9/4)*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c -
a*d)) + (d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^
(9/4)*(b*c - a*d)) - (d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*
(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) + (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
```

```
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2\,\text{Subst}\left(\int \frac{-5(bc+ad)-5bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{5ac}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \frac{x^2\left(-5\left(b^2c^2+abcd+a^2d^2\right)-5bd(bc+ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{5a^2c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{2\,\text{Subst}\left(\int \left(-\frac{5b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{5a^2d^3x^2}{(-bc+ad)\left(c+dx^4\right)}\right)dx, x, \sqrt{x}\right)}{5a^2c^2}$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2(bc-ad)} -$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{b^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{a^2(bc-ad)} +$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2a^2(bc-ad)} +$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)} -$$

$$= -\frac{2}{5acx^{5/2}} + \frac{2(bc+ad)}{a^2c^2\sqrt{x}} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)} + \frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)}$$

Mathematica [A] time = 0.28, size = 437, normalized size = 0.88

$$\frac{-\frac{5\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}+\frac{5\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}+\frac{10\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}}-\frac{40b^2}{a^2\sqrt{x}}+\frac{8b}{ax^{5/2}}+\frac{5\sqrt{2}d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}}-\frac{5\sqrt{2}d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}}-\frac{10\sqrt{2}d^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}}+\frac{10\sqrt{2}d^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}}+\frac{40d^2}{c^2\sqrt{x}}-\frac{8d}{cx^{5/2}}}{20ad-20bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((8*b)/(a*x^(5/2)) - (8*d)/(c*x^(5/2)) - (40*b^2)/(a^2*Sqrt[x]) + (40*d^2)/
(c^2*Sqrt[x]) + (10*Sqrt[2]*b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
```

```
(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4) + (5*Sqrt[2]*b^(9/4)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4) + (5*Sqrt[2]*d^(9/4)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(9/4) - (5*Sqrt[2
]*d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(9/
4))/(-20*b*c + 20*a*d)
```

IntegrateAlgebraic [A] time = 0.69, size = 296, normalized size = 0.59

$$\frac{b^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}a^{9/4}(ad-bc)}+\frac{b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{9/4}(ad-bc)}-\frac{2\left(ac-5adx^2-5bcx^2\right)}{5a^2c^2x^{5/2}}+\frac{d^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{d}}-\frac{\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}}{\sqrt{x}}\right)}{\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}c^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (-2*(a*c - 5*b*c*x^2 - 5*a*d*x^2))/(5*a^2*c^2*x^(5/2)) + (b^(9/4)*ArcTan[(a
^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2
]*a^(9/4)*(-(b*c) + a*d)) + (d^(9/4)*ArcTan[(c^(1/4)/(Sqrt[2]*d^(1/4)) - (d
^(1/4)*x)/(Sqrt[2]*c^(1/4)))/Sqrt[x]])/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(
9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqr
t[2]*a^(9/4)*(-(b*c) + a*d)) + (d^(9/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*c^(9/4)*(b*c - a*d))
```

fricas [B] time = 21.82, size = 1484, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/10*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*arctan(-(sqrt(b^14*x - (a^5*b^11*c
^2 - 2*a^6*b^10*c*d + a^7*b^9*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*
d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)))*(-b^9/(a^9*b^4*c^4 -
4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(
a^2*b*c - a^3*d) - (a^2*b^8*c - a^3*b^7*d)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*
c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*sqrt(x))/b^9
) - 20*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10
*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^3*arctan(-(sqrt(d^14*x - (b^2*c^7*d^9
- 2*a*b*c^6*d^10 + a^2*c^5*d^11)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a
^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)))*(-d^9/(b^4*c^13 - 4*a*b
```

```
^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*
c^3 - a*c^2*d) - (b*c^3*d^7 - a*c^2*d^8)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d +
 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*sqrt(x))/d^9)
+ 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c
*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) + (a^7*b^3*c^3 - 3*a^8*
b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d
 + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b
^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)
)^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^
9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^
2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^
12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*
x^3*log(d^7*sqrt(x) + (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7
*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*
d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2
*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^3*log(d^7*sqrt
(x) - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4
*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^
4))^(3/4)) + 4*(5*(b*c + a*d)*x^2 - a*c)*sqrt(x))/(a^2*c^2*x^3)
```

giac [A] time = 0.74, size = 487, normalized size = 0.98

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^{3}bc-\sqrt{2}\,a^{4}d}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^{3}bc-\sqrt{2}\,a^{4}d}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^{4}-\sqrt{2}\,ac^{3}d}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,bc^{4}-\sqrt{2}\,ac^{3}d}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^{3}bc-\sqrt{2}\,a^{4}d\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^{3}bc-\sqrt{2}\,a^{4}d\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^{4}-\sqrt{2}\,ac^{3}d\right)}-\frac{\left(cd^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}\,bc^{4}-\sqrt{2}\,ac^{3}d\right)}+\frac{2\left(5\,bcx^{2}+5\,adx^{2}-ac\right)}{5\,a^{2}c^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a
^4*d) - (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - (c*d^3)^(3/4)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqr
t(2)*a*c^3*d) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/
2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b
*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) + 2/5*(5*b*c*x^2 + 5*a
*d*x^2 - a*c)/(a^2*c^2*x^(5/2))
```

maple [A] time = 0.02, size = 375, normalized size = 0.75

$$-\frac{\sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{\sqrt{2}\, b^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}-\frac{\sqrt{2}\, b^{2} \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} a^{2}}+\frac{\sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{\sqrt{2}\, d^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{\sqrt{2}\, d^{2} \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} c^{2}}+\frac{2d}{a\,c^{2}\sqrt{x}}+\frac{2b}{a^{2}c\sqrt{x}}-\frac{2}{5ac\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] -1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b^2/a^2/(a*
d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^2/a^
2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/4*d
^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+1/2*d^2/c^2/(a*d-b*c)
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/2*d^2/c^2/(a*d
-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c/x^(
5/2)+2/a/c^2/x^(1/2)*d+2/a^2/c/x^(1/2)*b
```

maxima [A] time = 2.55, size = 411, normalized size = 0.83

$$\frac{b^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(a^{2}bc-a^{3}d\right)}-\frac{d^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{4\left(bc^{3}-ac^{2}d\right)}+\frac{2\left(5\,(bc+ad)x^{2}-ac\right)}{5\,a^{2}c^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b*c - a^3*d) - 1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^
(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sq
rt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)
*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)
) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^
(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 2/5*(5*(b*c + a*d)*x^2 - a*c)/(a^2*c^2*
x^(5/2))
```

mupad [B] time = 2.73, size = 4643, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] - 2*atan((32*a^11*b^10*c^13*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 6
4*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 2*a^11*b
```

```
^6*d^9*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96
*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4) + 32*a^21*c^3*d^10*x^(1/2)*(-b^
9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 -
 64*a^12*b*c*d^3))^(5/4) + 2*a^8*b^9*c^3*d^6*x^(1/2)*(-b^9/(16*a^13*d^4 + 1
6*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))
^(1/4) + 192*a^13*b^8*c^11*d^2*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4
- 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a
^14*b^7*c^10*d^3*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*
c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 32*a^15*b^6*c^9*d^4
*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*
b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 32*a^17*b^4*c^7*d^6*x^(1/2)*(-b^9/(
16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64
*a^12*b*c*d^3))^(5/4) - 128*a^18*b^3*c^6*d^7*x^(1/2)*(-b^9/(16*a^13*d^4 + 1
6*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))
^(5/4) + 192*a^19*b^2*c^5*d^8*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 -
 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a^
12*b^9*c^12*d*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3
*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a^20*b*c^4*d^9*x^(
1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*
c^2*d^2 - 64*a^12*b*c*d^3))^(5/4))/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7
+ a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5
 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 -
64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4) - atan((a
^11*b^10*c^13*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3
*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*32i + a^11*b^6*d^9*x^(1/
2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^
2*d^2 - 64*a^12*b*c*d^3))^(1/4)*2i + a^21*c^3*d^10*x^(1/2)*(-b^9/(16*a^13*d
^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c
*d^3))^(5/4)*32i + a^8*b^9*c^3*d^6*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*
c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4)*2i
+ a^13*b^8*c^11*d^2*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b
^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*192i - a^14*b^7*c^
10*d^3*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96
*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*128i + a^15*b^6*c^9*d^4*x^(1/2)
*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*
d^2 - 64*a^12*b*c*d^3))^(5/4)*32i + a^17*b^4*c^7*d^6*x^(1/2)*(-b^9/(16*a^13
*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b
*c*d^3))^(5/4)*32i - a^18*b^3*c^6*d^7*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b
^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*
128i + a^19*b^2*c^5*d^8*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^
10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*192i - a^12*b^
9*c^12*d*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d +
96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4)*128i - a^20*b*c^4*d^9*x^(1/2)
*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*
d^2 - 64*a^12*b*c*d^3))^(5/4)*128i)/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7
```

```
 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^
5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 -
 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4)*2i - (2/
(5*a*c) - (2*x^2*(a*d + b*c))/(a^2*c^2))/x^(5/2) - 2*atan((32*a^3*b^10*c^21
*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b
^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) + 32*a^13*c^11*d^10*x^(1/2)*(-d^9/(16
*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a
*b^3*c^12*d))^(5/4) + 2*b^9*c^11*d^6*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^
9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4) +
 2*a^3*b^6*c^8*d^9*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c
^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4) + 192*a^5*b^8*c^19*
d^2*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^
2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) - 128*a^6*b^7*c^18*d^3*x^(1/2)*(-d
^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2
 - 64*a*b^3*c^12*d))^(5/4) + 32*a^7*b^6*c^17*d^4*x^(1/2)*(-d^9/(16*b^4*c^13
 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*
d))^(5/4) + 32*a^9*b^4*c^15*d^6*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4
 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) - 128*
a^10*b^3*c^14*d^7*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^
10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) + 192*a^11*b^2*c^13*
d^8*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^
2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4) - 128*a^4*b^9*c^20*d*x^(1/2)*(-d^9
/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 -
64*a*b^3*c^12*d))^(5/4) - 128*a^12*b*c^12*d^9*x^(1/2)*(-d^9/(16*b^4*c^13 +
16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d)
)^(5/4))/(a^8*d^16 + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*b
^5*c^5*d^11 + a^4*b^4*c^4*d^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7*
b*c*d^15))*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2
*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4) - atan((a^3*b^10*c^21*x^(1/2)*(-d^9
/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 -
64*a*b^3*c^12*d))^(5/4)*32i + a^13*c^11*d^10*x^(1/2)*(-d^9/(16*b^4*c^13 + 1
6*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))
^(5/4)*32i + b^9*c^11*d^6*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*
a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4)*2i + a^3*b^6
*c^8*d^9*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 +
96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4)*2i + a^5*b^8*c^19*d^2*x^(1/2)
*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*
d^2 - 64*a*b^3*c^12*d))^(5/4)*192i - a^6*b^7*c^18*d^3*x^(1/2)*(-d^9/(16*b^4
*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3
*c^12*d))^(5/4)*128i + a^7*b^6*c^17*d^4*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4
*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4
)*32i + a^9*b^4*c^15*d^6*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a
^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4)*32i - a^10*b^
3*c^14*d^7*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3
+ 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5/4)*128i + a^11*b^2*c^13*d^8*x^
```

```
(1/2)*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*
c^11*d^2 - 64*a*b^3*c^12*d))^(5/4)*192i - a^4*b^9*c^20*d*x^(1/2)*(-d^9/(16*
b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*
b^3*c^12*d))^(5/4)*128i - a^12*b*c^12*d^9*x^(1/2)*(-d^9/(16*b^4*c^13 + 16*a
^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(5
/4)*128i)/(a^8*d^16 + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*
b^5*c^5*d^11 + a^4*b^4*c^4*d^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7
*b*c*d^15))*(-d^9/(16*b^4*c^13 + 16*a^4*c^9*d^4 - 64*a^3*b*c^10*d^3 + 96*a^
2*b^2*c^11*d^2 - 64*a*b^3*c^12*d))^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] Timed out
```

3.452 $\int \frac{x^{11/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=570

$$\frac{a^{9/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}-a^{9/4}$$

Rubi [A] time = 0.85, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 470, 582, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{a^{9/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}+\frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{9/4}(bc-ad)^2}+\frac{c^{5/4}(5bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{9/4}(bc-ad)^2}-\frac{c^{5/4}(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{9/4}(bc-ad)^2}+\frac{\sqrt{x}\,(5bc-4ad)}{2bd^2(bc-ad)}-\frac{cx^{5/2}}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((5*b*c - 4*a*d)*Sqrt[x])/(2*b*d^2*(b*c - a*d)) - (c*x^(5/2))/(2*d*(b*c - a
*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d
)^2) - (c^(5/4)*(5*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)])/(4*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) + (a^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(5/4)*(b*c - a*d)^2) - (a^(9
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*
b^(5/4)*(b*c - a*d)^2) + (c^(5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2) - (c^(
5/4)*(5*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(8*Sqrt[2]*d^(9/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^{12}}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^4\left(5ac+(5bc-4ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\Bigg(\int \frac{ac(5bc-4ad)+\left(5b^2c^2-4abcd-4a^2d^2\right)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}}{2bd^2(bc-ad)}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)^2} - \Big(c^2(5$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{b(bc-ad)^2} - a^{5/2}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^{5/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b^{3/2}(bc-ad)^2}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{9/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)^2}$$

$$= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2} - \frac{a^{9/4}\tan^{-1}\Bigg(}{\sqrt{2}\,b^{5/4}}$$

Mathematica [A] time = 0.36, size = 563, normalized size = 0.99

$$\frac{4\sqrt{2}a^{9/4}d^{9/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}a^{9/4}d^{9/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+8\sqrt{2}a^{9/4}d^{9/4}\left(c+dx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-8\sqrt{2}a^{9/4}d^{9/4}\left(c+dx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)(5bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)(5bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+2\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)(5bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)-2\sqrt{2}b^{5/4}c^{5/4}\left(c+dx^2\right)(5bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+8b^{5/4}c^2\sqrt[4]{d}\sqrt{x}(bc-ad)+32\sqrt[4]{b}\sqrt[4]{d}\sqrt{x}\left(c+dx^2\right)(bc-ad)^2}{16b^{5/4}d^{9/4}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*b^(5/4)*c^2*d^(1/4)*(b*c - a*d)*Sqrt[x] + 32*b^(1/4)*d^(1/4)*(b*c - a*d)
^2*Sqrt[x]*(c + d*x^2) + 8*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*A
```

```
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 2*Sqrt[2]*b^(5/4)*c^(5/4)*(5
*b*c - 9*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 2
*Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(9/4)*d^(9/4)*(c + d*x^2)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt[2]*a^(9/4)*d^(9/4)
*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + S
qrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d)*(c + d*x^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*b^(5/4)*c^(5/4)*(5*b*c - 9*a*d
)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
16*b^(5/4)*d^(9/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 1.20, size = 353, normalized size = 0.62

$$\frac{a^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}-\frac{a^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{5/4}(bc-ad)^2}+\frac{\left(5bc^{9/4}-9ac^{5/4}d\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,d^{9/4}(ad-bc)^2}-\frac{\left(5bc^{9/4}-9ac^{5/4}d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,d^{9/4}(ad-bc)^2}+\frac{\sqrt{x}\left(-4acd-4ad^2x^2+5bc^2+4bcdx^2\right)}{2bd^2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (Sqrt[x]*(5*b*c^2 - 4*a*c*d + 4*b*c*d*x^2 - 4*a*d^2*x^2))/(2*b*d^2*(b*c - a
*d)*(c + d*x^2)) + (a^(9/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)
/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(5/4)*(b*c - a*d)^2) + ((5*b*c^(9/
4) - 9*a*c^(5/4)*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])])/(4*Sqrt[2]*d^(9/4)*(-(b*c) + a*d)^2) - (a^(9/4)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(5/4)*(b*c - a*
d)^2) - ((5*b*c^(9/4) - 9*a*c^(5/4)*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*d^(9/4)*(-(b*c) + a*d)^2)
```

fricas [B] time = 111.54, size = 3393, normalized size = 5.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9
 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^
4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b
^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^
15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)*arctan(((b^6*c^6*d^7 - 6*a*b^5*c^5*d
^8 + 15*a^2*b^4*c^4*d^9 - 20*a^3*b^3*c^3*d^10 + 15*a^4*b^2*c^2*d^11 - 6*a^5
*b*c*d^12 + a^6*d^13)*sqrt((25*b^2*c^4 - 90*a*b*c^3*d + 81*a^2*c^2*d^2)*x +
 (b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d
^8)*sqrt(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a
^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b
```

```
^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^
14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17)))*(-(625*b^4*c^9 - 45
00*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5
*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^
5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 -
8*a^7*b*c*d^16 + a^8*d^17))^(3/4) + (5*b^7*c^8*d^7 - 39*a*b^6*c^7*d^8 + 129
*a^2*b^5*c^6*d^9 - 235*a^3*b^4*c^5*d^10 + 255*a^4*b^3*c^4*d^11 - 165*a^5*b^
2*c^3*d^12 + 59*a^6*b*c^2*d^13 - 9*a^7*c*d^14)*sqrt(x)*(-(625*b^4*c^9 - 450
0*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*
d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5
*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8
*a^7*b*c*d^16 + a^8*d^17))^(3/4))/(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a
^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)) - 16*(-a^9/(b^13*
c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b
^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^
8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*arc
tan(((b^10*c^6 - 6*a*b^9*c^5*d + 15*a^2*b^8*c^4*d^2 - 20*a^3*b^7*c^3*d^3 +
15*a^4*b^6*c^2*d^4 - 6*a^5*b^5*c*d^5 + a^6*b^4*d^6)*(-a^9/(b^13*c^8 - 8*a*b
^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4
- 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))
^(3/4)*sqrt(a^4*x + (b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^
3*c*d^3 + a^4*b^2*d^4)*sqrt(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c
^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28
*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))) - (a^2*b^10*c^6 - 6*a^3
*b^9*c^5*d + 15*a^4*b^8*c^4*d^2 - 20*a^5*b^7*c^3*d^3 + 15*a^6*b^6*c^2*d^4 -
 6*a^7*b^5*c*d^5 + a^8*b^4*d^6)*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b
^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5
 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(3/4)*sqrt(x))/a^9)
 - 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c
^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a
^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 -
a*b*d^4)*x^2)*log(a^2*sqrt(x) + (-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b
^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5
 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)) + 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*
c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 2
8*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*
b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*log(a^2*sqrt(x) - (-a^9/(b^13*c^8 - 8*
a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d
^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^
8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) + (b^2*c^2*d^2 - a*b*c*d^3 +
 (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b
^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7
*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13
 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^
```

```
(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 - 14580*a^3*b
*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c
^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 +
 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)) - (b^2*c^2*d^2 -
a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d +
12150*a^2*b^2*c^7*d^2 - 14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^
9 - 8*a*b^7*c^7*d^10 + 28*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b
^4*c^4*d^13 - 56*a^5*b^3*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 +
a^8*d^17))^(1/4)*log(-(5*b*c^2 - 9*a*c*d)*sqrt(x) - (b^2*c^2*d^2 - 2*a*b*c*
d^3 + a^2*d^4)*(-(625*b^4*c^9 - 4500*a*b^3*c^8*d + 12150*a^2*b^2*c^7*d^2 -
14580*a^3*b*c^6*d^3 + 6561*a^4*c^5*d^4)/(b^8*c^8*d^9 - 8*a*b^7*c^7*d^10 + 2
8*a^2*b^6*c^6*d^11 - 56*a^3*b^5*c^5*d^12 + 70*a^4*b^4*c^4*d^13 - 56*a^5*b^3
*c^3*d^14 + 28*a^6*b^2*c^2*d^15 - 8*a^7*b*c*d^16 + a^8*d^17))^(1/4)) + 4*(5
*b*c^2 - 4*a*c*d + 4*(b*c*d - a*d^2)*x^2)*sqrt(x))/(b^2*c^2*d^2 - a*b*c*d^3
 + (b^2*c*d^3 - a*b*d^4)*x^2)
```

giac [**A**] time = 1.11, size = 718, normalized size = 1.26

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} a^{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} a^{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} a^{2} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a^{2} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}\right)}-\frac{\left(5\left(cd^{3}\right)^{\frac{1}{4}}bc^{2}-9\left(cd^{3}\right)^{\frac{1}{4}}acd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{2}d^{3}-2\sqrt{2}abcd^{4}+\sqrt{2}a^{2}d^{5}\right)}-\frac{\left(5\left(cd^{3}\right)^{\frac{1}{4}}bc^{2}-9\left(cd^{3}\right)^{\frac{1}{4}}acd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{2}d^{3}-2\sqrt{2}abcd^{4}+\sqrt{2}a^{2}d^{5}\right)}-\frac{\left(5\left(cd^{3}\right)^{\frac{1}{4}}bc^{2}-9\left(cd^{3}\right)^{\frac{1}{4}}acd\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{2}d^{3}-2\sqrt{2}abcd^{4}+\sqrt{2}a^{2}d^{5}\right)}+\frac{\left(5\left(cd^{3}\right)^{\frac{1}{4}}bc^{2}-9\left(cd^{3}\right)^{\frac{1}{4}}acd\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{2}d^{3}-2\sqrt{2}abcd^{4}+\sqrt{2}a^{2}d^{5}\right)}+\frac{c^{2}\sqrt{x}}{2\left(bcd^{2}-ad^{3}\right)\left(dx^{2}+c\right)}+\frac{2\sqrt{x}}{bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - (
a*b^3)^(1/4)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/
2*(a*b^3)^(1/4)*a^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*(a*b^3)^(1/4)
*a^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2
*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*
(c*d^3)^(1/4)*a*c*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(
c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) -
 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(
2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) - 1/8*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1
/4)*a*c*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^
2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/8*(5*(c*d^3)^(1/4)*b*c^2
 - 9*(c*d^3)^(1/4)*a*c*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/2*c^2*sq
rt(x)/((b*c*d^2 - a*d^3)*(d*x^2 + c)) + 2*sqrt(x)/(b*d^2)
```

maple [A] time = 0.02, size = 582, normalized size = 1.02

$$-\frac{a c^{2}\sqrt{x}}{2 \left(a d-b c\right)^{2} \left(d x^{2}+c\right) d}+\frac{b c^{3}\sqrt{x}}{2 \left(a d-b c\right)^{2} \left(d x^{2}+c\right) d^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(a d-b c\right)^{2} b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, a^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(a d-b c\right)^{2} b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, a^{2} \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(a d-b c\right)^{2} b}+\frac{9 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a c \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{2} d}+\frac{9 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a c \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{2} d}+\frac{9 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a c \ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(a d-b c\right)^{2} d}-\frac{5 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b c^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{2} d^{2}}-\frac{5 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b c^{2} \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{2} d^{2}}-\frac{5 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b c^{2} \ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(a d-b c\right)^{2} d^{2}}+\frac{2 \sqrt{x}}{b d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 2/b/d^2*x^(1/2)-1/4/b*a^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-
1/2/b*a^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)-1/2/b*a^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)-1/2*c^2/d/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a+1/2*c^3/d^2/(a*d-b*c)^2
*x^(1/2)/(d*x^2+c)*b+9/8*c/d/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a-5/8*c^2/d^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+9/8*c/d/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-5/8*c^2/d^2/(a*d-b*c)^2*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b+9/16*c/d/(a*d-b*c)^2*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4
)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a-5/16*c^2/d^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1
/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x
^(1/2)+(c/d)^(1/2)))*b
```

maxima [A] time = 2.02, size = 494, normalized size = 0.87

$$-\frac{\left(\frac{2 \sqrt{2} (5 bc-9 ad) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{2 \sqrt{2} (5 bc-9 ad) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{\sqrt{2} (5 bc-9 ad) \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}-\frac{\sqrt{2} (5 bc-9 ad) \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}\right) c^{2}}{16 \left(b^{2} c^{2} d^{2}-2 abcd^{3}+a^{2} d^{4}\right)}+\frac{c^{2} \sqrt{x}}{2 \left(bc^{2} d^{2}-acd^{3}+\left(bcd^{3}-ad^{4}\right) x^{2}\right)}-\frac{\frac{2 \sqrt{2} a^{\frac{5}{2}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}}+\frac{2 \sqrt{2} a^{\frac{5}{2}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}}}+\frac{\sqrt{2} a^{\frac{9}{4}} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2} a^{\frac{9}{4}} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4 \left(b^{3} c^{2}-2 ab^{2} cd+a^{2} bd^{2}\right)}+\frac{2 \sqrt{x}}{bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(5*b*c - 9*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + 2*sqrt(2)*(5*b*c - 9*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
 - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))
) + sqrt(2)*(5*b*c - 9*a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b*c - 9*a*d)*log(-sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c^2/(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4) + 1/2*c^2*sqrt(x)/(b*c^2*d^2 - a*c*d^3 + (b*c*d^
3 - a*d^4)*x^2) - 1/4*(2*sqrt(2)*a^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b))
 + 2*sqrt(2)*a^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(9/4)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2
```

```
)*a^(9/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/
4))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 2*sqrt(x)/(b*d^2)
```

mupad [B] time = 3.25, size = 22978, normalized size = 40.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] atan((((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^
11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*
d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 +
16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^
5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 -
128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14
*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*
a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 -
 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*
d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7
*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3
*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) - (2*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8
 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a
^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7
*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b
^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*
a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^6*c^4*d^15 + 5120
*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2
*c*d^7)) - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d +
256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*
b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4
*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (x^(1/2)*(625*a^6*b^8*c^12
 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^
8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b
^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10
 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16
*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896
*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^
7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*1i + ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*
d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 112
0*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*
c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 44
8*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*
b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(640
0*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 -
```

```
1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c
^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^1
1*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a
^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*
c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) + (2
*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6
*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 +
448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40
960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11
 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d
^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c
^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (2*(625*a^4*b^8*c^11 + 576*a
^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d
^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256
*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*
c*d^7)) + (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^1
1*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3
 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5
 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*
d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^
6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^
4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*1i
)/(((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*
c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5
 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*
a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d
^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128
*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^
13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7
*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 15
70816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^1
5 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^
6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^
4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) - (2*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 -
128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*
b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d)
)^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11
*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8
*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^
11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*
d^7)) - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256
*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4
*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^
3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (x^(1/2)*(625*a^6*b^8*c^12 +
1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b
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^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*
c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 +
15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^
13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^
3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c
^2*d^6 - 128*a*b^12*c^7*d))^(1/4) - ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 -
128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*
b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d)
)^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*
b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^
3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*
b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 116531
2*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^1
1 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*
c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^
4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10
 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) + (2*(-a^9
/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 -
 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^
6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^
4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358
400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 -
40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5
 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (2*(625*a^4*b^8*c^11 + 576*a^12*c^
3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6
305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*
b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)
) + (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d -
1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561
*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*
b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 +
15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^
7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 89
6*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)))*(-a^9/
(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 -
896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6
*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*2i + 2*atan((((-a^9/(16*b^13*c^8 +
16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^
5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 -
128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6
*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4
 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x
^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*
c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728
*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 +
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 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^
16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 -
 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^
2*d^9) - ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2
*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c
^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11
*d^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10
*c^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*
b^7*c^5*d^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b
*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i + (2*(625*a^4*b
^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 827
5*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^
3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d
^6 - 3*a^2*b^2*c*d^7))*1i - (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8
- 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 1458
0*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d
^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7
 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^
5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1
120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^1
2*c^7*d))^(1/4) + ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7
+ 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*
a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16
*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896
*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^
7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74
240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^
9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^
9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a
^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*
b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*
d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) + ((-a^9/(16*b^13*c^8 + 16*a
^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^
3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*
a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 + 14
3360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d^12
 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^6*c^4*d
^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^
6 - 3*a^2*b^2*c*d^7))*1i - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a
^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^
8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/
(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i - (x^(1/2
)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b
*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c
^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6
```

```
 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*
c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2
*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c
^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4))/(((-a^9/(16*b^13*c
^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^
10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d
^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^
7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^
4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4
)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*
b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 30
17728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d
^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c
^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*
d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b
^3*c^2*d^9) - ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 44
8*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*
b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13
*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6
*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360
*a^9*b^7*c^5*d^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(
a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i + (2*(625*
a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7
+ 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a
^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*
c^2*d^6 - 3*a^2*b^2*c*d^7))*1i - (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4
*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 -
 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^
6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^
4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a
^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^
3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*
a*b^12*c^7*d))^(1/4)*1i - ((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^
6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^
4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((
-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d
^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 44
8*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*
d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12
*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 269158
4*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 -
 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17
))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*
b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) + ((-a^9/(16*b^13*c^
8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^1
```

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0*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^
6 - 128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*
d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*
c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b
^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^
3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8
- 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^
7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^
5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i -
 (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 144
0*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^
10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6
*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*
a^4*b^3*c^2*d^9))*(-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 +
 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a
^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*1i))*(-a^9/
(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 -
896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6
*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4) + atan(((-(625*b^4*c^9 + 6561*a^4*c
^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4
096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6
*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*
c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4
*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 450
0*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 1
14688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13
- 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^
(1/4)*((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*
b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a
*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*
a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 3276
8*a^7*b*c*d^16))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c
^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^
7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1
570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^
15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c
^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b
^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) - (2*(-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 1
4580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^
17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 22
9376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 +
 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d
^8 - 40960*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c
^8*d^11 + 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^
7*c^5*d^14 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8
```

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- b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) - (2*(625*a^4*b^8*c^11
+ 576*a^12*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^
6*c^9*d^2 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^
5 + 256*a^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a
^2*b^2*c*d^7)) + (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*
b^7*c^11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c
^9*d^3 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c
^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b
^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9))*1i + (-(625*b^4*c^9 + 6561*a^4*c^5*d^4 -
14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d
^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 2
29376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14
+ 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 65
61*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c
^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2
*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*
a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-
(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d
^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*
d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c
^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c
*d^16))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 +
 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^
10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^
10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 1203
2*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 -
6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^
8 + 15*a^4*b^3*c^2*d^9) + (2*(-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*
b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096
*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*
b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a
^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 4096
0*a^4*b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 +
 358400*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^1
4 - 40960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3
*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (2*(625*a^4*b^8*c^11 + 576*a^1
2*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2
 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a
^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*
d^7)) + (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*
d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 +
6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 -
6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^
8 + 15*a^4*b^3*c^2*d^9))*1i)/((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3
*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 409
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6*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3
*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*
a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a^4*c^
5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(40
96*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*
d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c
^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*
c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500
*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 11
4688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 -
 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(
3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a
^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 -
 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^
7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^
5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c
^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^
4*b^3*c^2*d^9) - (2*(-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3
 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*
d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d
^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^
2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^1
2*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a
^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960
*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*
a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8
 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a
^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c
^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (
x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*
a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10
*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c
^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^
4*b^3*c^2*d^9)) - (-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 +
 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^
9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^1
2 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*
d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 1458
0*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17
+ 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 22937
6*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 11
4688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a
^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d
)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6
*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*
```

b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(3/4)*((x^(1/
2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12
*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017728*a^8
*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541
952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 +
4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a
^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^
9) + (2*(-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2
*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*
a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720
*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 327
68*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9 +
143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^7*d
^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^6*c^
4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^
6 - 3*a^2*b^2*c*d^7)) + (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 3875*a^5*
b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*b^5*c^8*d
^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*d^6))/(a^
3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7)) + (x^(1/2)*(625
*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*a^13*b*c^5*d
^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10*b^4*c^8*d^4
+ 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a
^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^
9))))*(-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b
^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*
b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a
^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768
*a^7*b*c*d^16))^(1/4)*2i + 2*atan(((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 1458
0*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17
+ 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 22937
6*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 11
4688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a
^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d
)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6
*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*
b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625
*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 -
4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10
+ 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d
^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^1
6))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384
256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d
^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b
^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^
13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*

```
b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 +
15*a^4*b^3*c^2*d^9) - ((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*
d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c
^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^
5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2
*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*
b^12*c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 35840
0*a^7*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40
960*a^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b*d^8 - b^4*c^3*d^
5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i + (2*(625*a^4*b^8*c^11 + 576*a^1
2*c^3*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2
 - 6305*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a
^10*b^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*
d^7))*1i - (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^
11*d - 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3
 + 6561*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5
 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3
*d^8 + 15*a^4*b^3*c^2*d^9)) + (-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3
*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 409
6*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3
*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*
a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a^4*c^
5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(40
96*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*
d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c
^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*
c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500
*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 11
4688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 -
 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(
3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a
^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 -
 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^
7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^
5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c
^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^
4*b^3*c^2*d^9) + ((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 +
 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^
9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^1
2 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*
d^15 - 32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*
c^10*d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7
*b^9*c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a
^10*b^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b*d^8 - b^4*c^3*d^5 + 3
*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3
```

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*d^8 - 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 63
05*a^7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b
^2*c^5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))
*1i - (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d
- 1440*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 65
61*a^10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*
a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8
+ 15*a^4*b^3*c^2*d^9)))/((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^
6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8
*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*
c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b
^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a^4*c^5*d^4
 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^
8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11
- 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^
14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 +
 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^
3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*
a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 2293
76*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(3/4)*
((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^
13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 3017
728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^1
3 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4
*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^
6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3
*c^2*d^9) - ((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 1215
0*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 3
2768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 2
86720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15
- 32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*
d^9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*
c^7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b
^6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^
3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i + (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8
- 3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^
7*b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^
5*d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i -
 (x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 144
0*a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^
10*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6
*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*
a^4*b^3*c^2*d^9))*1i - (-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*
d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c
^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^
```

```
5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2
*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 -
 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*
d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 -
229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14
 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(1/4)*((-(625*b^4*c^9 + 6
561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*
c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^
2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286720*a^4*b^4*c^4*d^13 - 229376
*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 - 32768*a^7*b*c*d^16))^(3/4)*((
x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^4*b^14*c^13*d^7 + 384256*a^5*b^13
*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2286080*a^7*b^11*c^10*d^10 - 301772
8*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^8*d^12 - 1570816*a^10*b^8*c^7*d^13
+ 541952*a^11*b^7*c^6*d^14 - 74240*a^12*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d
^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6
- 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c
^2*d^9) + ((-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^3 + 12150*
a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8*d^9 - 327
68*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*d^12 + 286
720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c^2*d^15 -
32768*a^7*b*c*d^16))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^
9 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^
7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^6
*c^4*d^15 + 5120*a^11*b^5*c^3*d^16)*2i)/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*
c^2*d^6 - 3*a^2*b^2*c*d^7))*1i - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 -
3875*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7*
b^5*c^8*d^3 + 256*a^8*b^4*c^7*d^4 + 256*a^9*b^3*c^6*d^5 + 256*a^10*b^2*c^5*
d^6))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c^2*d^6 - 3*a^2*b^2*c*d^7))*1i - (
x^(1/2)*(625*a^6*b^8*c^12 + 1296*a^14*c^4*d^8 - 4500*a^7*b^7*c^11*d - 1440*
a^13*b*c^5*d^7 + 12150*a^8*b^6*c^10*d^2 - 14580*a^9*b^5*c^9*d^3 + 6561*a^10
*b^4*c^8*d^4 + 400*a^12*b^2*c^6*d^6))/(a^6*b*d^11 + b^7*c^6*d^5 - 6*a*b^6*c
^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d^7 - 20*a^3*b^4*c^3*d^8 + 15*a^
4*b^3*c^2*d^9))*1i))*(-(625*b^4*c^9 + 6561*a^4*c^5*d^4 - 14580*a^3*b*c^6*d^
3 + 12150*a^2*b^2*c^7*d^2 - 4500*a*b^3*c^8*d)/(4096*a^8*d^17 + 4096*b^8*c^8
*d^9 - 32768*a*b^7*c^7*d^10 + 114688*a^2*b^6*c^6*d^11 - 229376*a^3*b^5*c^5*
d^12 + 286720*a^4*b^4*c^4*d^13 - 229376*a^5*b^3*c^3*d^14 + 114688*a^6*b^2*c
^2*d^15 - 32768*a^7*b*c*d^16))^(1/4) + (2*x^(1/2))/(b*d^2) - (b*c^2*x^(1/2)
)/(2*(b*d^3*x^2 + b*c*d^2)*(a*d - b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.453 $\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=536

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}+$$

Rubi [A] time = 0.59, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 470, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{a^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}+\frac{a^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{7/4}(bc-ad)^2}-\frac{c^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{7/4}(bc-ad)^2}+\frac{c^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{7/4}(bc-ad)^2}-\frac{cx^{3/2}}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*x^(3/2))/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (a^(7/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) -
(c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(7/4)*(b*c - a*d)^2) + (a^(7/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(3
/4)*(b*c - a*d)^2) - (a^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)^2) + (c^(3/4)*(3*b*c - 7*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(
7/4)*(b*c - a*d)^2) - (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(7/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^{10}}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{x^2\left(3ac+(3bc-4ad)x^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\mathrm{Subst}\left(\int \left(-\frac{4a^2dx^2}{(-bc+ad)\left(a+bx^4\right)} + \frac{c(3bc-7ad)x^2}{(bc-ad)\left(c+dx^4\right)}\right)dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(c(3bc-7ad))\,\mathrm{Subst}\left(\right.}{2d(bc-}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^2\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\,(bc-ad)^2} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\right.}{\sqrt{b}\,(bc-ad)^2}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)^2} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+}\right.}{2b(bc}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} + \frac{a^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)^2} - \frac{a^{7/4}\log\left(\sqrt{a}+\right.}{2\sqrt{2}\,b^{3}}$$

$$= -\frac{cx^{3/2}}{2d(bc-ad)\left(c+dx^2\right)} - \frac{a^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2} + \frac{a^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2} - \frac{c^{3}}{}$$

Mathematica [A] time = 0.31, size = 527, normalized size = 0.98

$$\frac{4\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-4\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+8\sqrt{2}a^{7/4}d^{7/4}\left(c+dx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+\sqrt{2}b^{3/4}c^{3/4}\left(c+dx^2\right)(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-\sqrt{2}b^{3/4}c^{3/4}\left(c+dx^2\right)(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-2\sqrt{2}b^{3/4}c^{3/4}\left(c+dx^2\right)(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}b^{3/4}c^{3/4}\left(c+dx^2\right)(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-8b^{3/4}cd^{3/4}x^{3/2}(bc-ad)}{16b^{3/4}d^{7/4}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-8*b^(3/4)*c*d^(3/4)*(b*c - a*d)*x^(3/2) - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(c +
d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*a^(7/4)*d^
(7/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]
```

```
*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sq
rt[x])/c^(1/4)] + 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(c +
 d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt
[2]*a^(7/4)*d^(7/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*b^(3/4)*c^(
3/4)*(3*b*c - 7*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(16*b^(3/4)*d^(7/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 1.05, size = 323, normalized size = 0.60

$$-\frac{a^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{a^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,b^{3/4}(bc-ad)^2}-\frac{\left(3bc^{7/4}-7ac^{3/4}d\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,d^{7/4}(ad-bc)^2}-\frac{\left(3bc^{7/4}-7ac^{3/4}d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,d^{7/4}(ad-bc)^2}+\frac{cx^{3/2}}{2d\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (c*x^(3/2))/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a^(7/4)*ArcTan[(a^(1/4)/(Sq
rt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(3/4)*
(b*c - a*d)^2) - ((3*b*c^(7/4) - 7*a*c^(3/4)*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x
)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/(4*Sqrt[2]*d^(7/4)*(-(b*c) + a*d)^2)
 - (a^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/(Sqrt[2]*b^(3/4)*(b*c - a*d)^2) - ((3*b*c^(7/4) - 7*a*c^(3/4)*d)*ArcTanh[
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*d^(7/4
)*(-(b*c) + a*d)^2)
```

fricas [B] time = 86.41, size = 3524, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*c*x^(3/2) - 4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4
*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a
^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^
5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^1
3 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4)*arctan(((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*sqrt((729*b^6*c^10 - 10206*a*b^5*c^9*d + 59535*a^2*b^4*c^8*d^2 - 1
85220*a^3*b^3*c^7*d^3 + 324135*a^4*b^2*c^6*d^4 - 302526*a^5*b*c^5*d^5 + 117
649*a^6*c^4*d^6)*x - (81*b^8*c^11*d^3 - 1080*a*b^7*c^10*d^4 + 6156*a^2*b^6*
c^9*d^5 - 19560*a^3*b^5*c^8*d^6 + 37846*a^4*b^4*c^7*d^7 - 45640*a^5*b^3*c^6
*d^8 + 33516*a^6*b^2*c^5*d^9 - 13720*a^7*b*c^4*d^10 + 2401*a^8*c^3*d^11)*sq
rt(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d
```

```
^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9
- 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*
b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15)))*(-(81*b^4*c^7 - 756*a*b^3*c^6*d
 + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d
^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^
4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a
^8*d^15))^(1/4) + (27*b^5*c^7*d^2 - 243*a*b^4*c^6*d^3 + 846*a^2*b^3*c^5*d^4
 - 1414*a^3*b^2*c^4*d^5 + 1127*a^4*b*c^3*d^6 - 343*a^5*c^2*d^7)*sqrt(x)*(-(
81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 +
2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*
a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c
^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/4))/(81*b^4*c^7 - 756*a*b^3*c^6*d
+ 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)) + 16*(-a^7
/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*
a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7
 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*arctan((
sqrt(a^10*x - (a^7*b^5*c^4 - 4*a^8*b^4*c^3*d + 6*a^9*b^3*c^2*d^2 - 4*a^10*b
^2*c*d^3 + a^11*b*d^4)*sqrt(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^
6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a
^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8)))*(-a^7/(b^11*c^8 - 8*a*b^1
0*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56
*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/
4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - (a^5*b^3*c^2 - 2*a^6*b^2*c*d + a^7
*b*d^2)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*
c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*
a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*sqrt(x))/a^7) - 4*(-a^7/(b^11*c^8 - 8*a
*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4
- 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))
^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) + (b^8*c
^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c
^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d +
28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c
^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + 4*(-
a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 +
70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*
d^7 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a
^5*sqrt(x) - (b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3
*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8
- 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4
*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*
d^8))^(3/4)) + (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 -
756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*
d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d
^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a
^7*b*c*d^14 + a^8*d^15))^(1/4)*log((b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*
```

```
b^4*c^4*d^7 - 20*a^3*b^3*c^3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^
6*d^11)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b
*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^
6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 2
8*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(3/4) - (27*b^3*c^5 - 189*
a*b^2*c^4*d + 441*a^2*b*c^3*d^2 - 343*a^3*c^2*d^3)*sqrt(x)) - (b*c^2*d - a*
c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(81*b^4*c^7 - 756*a*b^3*c^6*d + 2646*a^2*b
^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4)/(b^8*c^8*d^7 - 8*a*b^7*
c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10 + 70*a^4*b^4*c^4*d^11 -
56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*b*c*d^14 + a^8*d^15))^(1/
4)*log(-(b^6*c^6*d^5 - 6*a*b^5*c^5*d^6 + 15*a^2*b^4*c^4*d^7 - 20*a^3*b^3*c^
3*d^8 + 15*a^4*b^2*c^2*d^9 - 6*a^5*b*c*d^10 + a^6*d^11)*(-(81*b^4*c^7 - 756
*a*b^3*c^6*d + 2646*a^2*b^2*c^5*d^2 - 4116*a^3*b*c^4*d^3 + 2401*a^4*c^3*d^4
)/(b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^10
 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7*
b*c*d^14 + a^8*d^15))^(3/4) - (27*b^3*c^5 - 189*a*b^2*c^4*d + 441*a^2*b*c^3
*d^2 - 343*a^3*c^2*d^3)*sqrt(x)))/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^
2)
```

<u>giac</u> [A] time = 1.24, size = 681, normalized size = 1.27

$$\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{5}c^{2}-2\,\sqrt{2}\,ab^{4}cd+\sqrt{2}\,a^{2}b^{3}d^{2}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,b^{5}c^{2}-2\,\sqrt{2}\,ab^{4}cd+\sqrt{2}\,a^{2}b^{3}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{5}c^{2}-2\,\sqrt{2}\,ab^{4}cd+\sqrt{2}\,a^{2}b^{3}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}} a \log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,b^{5}c^{2}-2\,\sqrt{2}\,ab^{4}cd+\sqrt{2}\,a^{2}b^{3}d^{2}\right)}+\frac{\left(3\,\left(cd^{3}\right)^{\frac{3}{4}} bc-7\,\left(cd^{3}\right)^{\frac{3}{4}} ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^{2}c^{2}d^{4}-2\,\sqrt{2}\,abcd^{5}+\sqrt{2}\,a^{2}d^{6}\right)}+\frac{\left(3\,\left(cd^{3}\right)^{\frac{3}{4}} bc-7\,\left(cd^{3}\right)^{\frac{3}{4}} ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^{2}c^{2}d^{4}-2\,\sqrt{2}\,abcd^{5}+\sqrt{2}\,a^{2}d^{6}\right)}-\frac{\left(3\,\left(cd^{3}\right)^{\frac{3}{4}} bc-7\,\left(cd^{3}\right)^{\frac{3}{4}} ad\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^{2}c^{2}d^{4}-2\,\sqrt{2}\,abcd^{5}+\sqrt{2}\,a^{2}d^{6}\right)}+\frac{\left(3\,\left(cd^{3}\right)^{\frac{3}{4}} bc-7\,\left(cd^{3}\right)^{\frac{3}{4}} ad\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^{2}c^{2}d^{4}-2\,\sqrt{2}\,abcd^{5}+\sqrt{2}\,a^{2}d^{6}\right)}-\frac{cx^{\frac{3}{2}}}{2\left(bcd-ad^{2}\right)\left(dx^{2}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + (a*b
^3)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) - 1/2*(a*
b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*
c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/2*(a*b^3)^(3/4)*a*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*
a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/4*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) + 1/4*(3*(c*d^
3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sq
rt(2)*a^2*d^6) - 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*log(sqrt(2
)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b
*c*d^5 + sqrt(2)*a^2*d^6) + 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)
*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 - 2
*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) - 1/2*c*x^(3/2)/((b*c*d - a*d^2)*(d*x
^2 + c))
```

maple [A] time = 0.02, size = 566, normalized size = 1.06

$$\frac{ac\,x^{\frac{3}{2}}}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)}-\frac{b\,c^{2}x^{\frac{3}{2}}}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)d}+\frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}b}+\frac{\sqrt{2}\,a^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}b}-\frac{7\sqrt{2}\,ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}d}-\frac{7\sqrt{2}\,ac\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}d}-\frac{7\sqrt{2}\,ac\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}d}+\frac{3\sqrt{2}\,b\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}+\frac{3\sqrt{2}\,b\,c^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}+\frac{3\sqrt{2}\,b\,c^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/4*a^2/(a*d-b*c)^2/b/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*a^2/(a*d-b*c
)^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a^2/(a*
d-b*c)^2/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/2*c/
(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x^(3/2)/(d*x^2+c)*b-7
/8*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)*a+3/8*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)*b-7/8*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)*a+3/8*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*c/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*ln((x-(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))*a+3/16*c^2/(a*d-b*c)^2/d^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*
2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b
```

maxima [A] time = 2.53, size = 450, normalized size = 0.84

$$\frac{a^{2}\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}-\frac{cx^{\frac{3}{2}}}{2\left(bc^{2}d-acd^{2}+\left(bcd^{2}-ad^{3}\right)x^{2}\right)}+\frac{\left(3\,bc^{2}-7\,acd\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*x^(3/2)/(b*c^2*d - a*c*d^2 + (b*c*d^2
 - a*d^3)*x^2) + 1/16*(3*b*c^2 - 7*a*c*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/
(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
```

mupad [B] time = 2.53, size = 19871, normalized size = 37.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] 2*atan(((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b
^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d
^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 1
6*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d
^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128
*a*b^10*c^7*d))^(3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 +
74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d
^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7
*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^
4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*
c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21
*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^
3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 112
0*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*
c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^
5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 181
3504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^1
2 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^
3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20
*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (x^(1/2)*(81*a^5*
b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 -
4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^
11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*
d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (-a^7/(16
*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*
a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*
c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 1
28*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7
*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(
3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^
12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b
^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^
11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048
*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*
b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8
- 7*a^6*b*c*d^9) - (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b
^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4
 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(23
```

```
04*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 -
 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*
d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b
^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9
 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6
+ 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^
6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*
d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/
(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*
c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)))/((-a^7/(16*b^11*c^8 + 16*a^
8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 +
 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b
^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7
+ 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^
5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(3/4)*(((864*a^3*b
^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a
^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054
144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 -
 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^1
4)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*
a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9)
+ (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^
2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^
3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*
d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*
c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^
9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143
872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 -
6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*
d^7 - 6*a^5*b*c*d^8))*1i + (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d +
784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9
*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6)*1i)/(a^6*d^9 +
b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 1
5*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (81*a^7*b^6*c^9 - 675*a^8*b^5*c^8*d +
 1372*a^12*b*c^4*d^5 + 1971*a^9*b^4*c^7*d^2 - 2037*a^10*b^3*c^6*d^3 - 392*a
^11*b^2*c^5*d^4)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5
*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6
*b*c*d^9) + (-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a
^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c
^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8
 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c
^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 -
 128*a*b^10*c^7*d))^(3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^
4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^
10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7
```

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*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^1
3*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*
b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7
+ 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) - (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^
8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 +
 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b
^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 16716
8*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 -
 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6
*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^
4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5
- 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8))*1i - (x^(1/2)*(
81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8
*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 -
 672*a^11*b^2*c^4*d^6)*1i)/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^
2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8))))
*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*
d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 44
8*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4) - atan(((-a^7/(16*b^11*c^8 + 1
6*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d
^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128
*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*
d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 89
6*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(3/4)*((864*a^
3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 26700
8*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1
054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^1
1 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*
d^14)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a
^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) +
 (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2
*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3
*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d
^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c
^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9
*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 1438
72*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6
*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d
^7 - 6*a^5*b*c*d^8))*1i + (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 7
84*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*
b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6)*1i)/(a^6*d^9 + b
^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15
*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 -
128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^
7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^
```

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(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^
9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^
5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(3/4)*((864*a^3*b^14*c^14*d^3
- 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*
d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c
^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*
b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)/(a^7*d^10
- b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 -
 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) - (x^(1/2)*(-a^7/
(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 8
96*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b
^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b
^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 122931
2*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 -
1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*
d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 +
 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d
^8))*1i - (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^
7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 14
4*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6)*1i)/(a^6*d^9 + b^6*c^6*d^3 - 6*a
*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7
 - 6*a^5*b*c*d^8)))/((81*a^7*b^6*c^9 - 675*a^8*b^5*c^8*d + 1372*a^12*b*c^4*
d^5 + 1971*a^9*b^4*c^7*d^2 - 2037*a^10*b^3*c^6*d^3 - 392*a^11*b^2*c^5*d^4)/
(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4
*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (-a^7
/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 -
896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*
b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8
 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4
*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d
))^(3/4)*((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12
*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^
8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008
*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2
048*a^14*b^3*c^3*d^14)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b
^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 -
 7*a^6*b*c*d^9) + (x^(1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^
4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4
- 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(230
4*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 -
563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d
^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^
6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9
+ b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 +
 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6
```

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*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d
^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/(
a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c
^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (-a^7/(16*b^11*c^8 + 16*a^8
*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 +
1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^
10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 +
 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5
*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(3/4)*((864*a^3*b^1
4*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6
*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 105414
4*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 1
33952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)
/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^
4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) - (x^(
1/2)*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*
c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5
+ 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 -
29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d
^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*
c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^
12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^
5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 -
6*a^5*b*c*d^8)) - (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*
b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*
d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3
- 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^
2*d^7 - 6*a^5*b*c*d^8))))*(-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4
*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 -
 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*2i -
atan(((-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*
c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*
c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4
*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b
*c*d^14))^(1/4)*((-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 26
46*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 3
2768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286
720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 -
32768*a^7*b*c*d^14))^(3/4)*((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^
4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^
10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7
*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^1
3*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*
c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21
*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (x^(1/2)*(-(81*b^4*c^7 + 2401*a^4*c^3*d
```

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^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8
*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 2
29376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12
+ 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*(2304*a^3*b^14*c^13*
d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*
c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^
9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143
872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 -
6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*
d^7 - 6*a^5*b*c*d^8))*1i + (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d +
784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9
*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6)*1i)/(a^6*d^9 +
b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 1
5*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 41
16*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 +
 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a
^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 11468
8*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*((-(81*b^4*c^7 + 2401*a^4*c
^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096
*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9
 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d
^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(3/4)*((864*a^3*b^14*c
^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^
11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a
^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 1339
52*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)/(a
^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c
^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) - (x^(1/2
)*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*
d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*
d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4
*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d
^14))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*
b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 18135
04*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12
+ 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*
d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a
^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8))*1i - (x^(1/2)*(81*a^5
*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 -
 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a
^11*b^2*c^4*d^6)*1i)/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*
c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)))/((81*a
^7*b^6*c^9 - 675*a^8*b^5*c^8*d + 1372*a^12*b*c^4*d^5 + 1971*a^9*b^4*c^7*d^2
 - 2037*a^10*b^3*c^6*d^3 - 392*a^11*b^2*c^5*d^4)/(a^7*d^10 - b^7*c^7*d^3 +
7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*
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d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (-(81*b^4*c^7 + 2401*a^4*c^3*d^
4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*
d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 22
9376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 +
 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*((-(81*b^4*c^7 + 2401
*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)
/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c
^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3
*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(3/4)*((864*a^3*
b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*
a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 105
4144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11
- 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^
14)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3
*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (
x^(1/2)*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^
2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^
7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b
^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7
*b*c*d^14))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 16716
8*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 -
 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6
*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^
4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5
- 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (x^(1/2)*(81*
a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^
2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 67
2*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*
c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (-(81
*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 7
56*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 1
14688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 -
 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(
1/4)*((-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*
c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*
c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4
*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b
*c*d^14))^(3/4)*((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a
^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 96
8576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 +
 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d
^13 - 2048*a^14*b^3*c^3*d^14)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 2
1*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^
2*d^8 - 7*a^6*b*c*d^9) - (x^(1/2)*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a
^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 409
```

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6*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b
^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^
6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184
*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 +
1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d
^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^
5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5
*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5
*b*c*d^8)) - (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3
*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 +
 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3 - 6*a
*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7
 - 6*a^5*b*c*d^8))))*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3
+ 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7
 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 +
 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^1
3 - 32768*a^7*b*c*d^14))^(1/4)*2i + 2*atan(((-(81*b^4*c^7 + 2401*a^4*c^3*d^
4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*
d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 22
9376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 +
 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*((-(81*b^4*c^7 + 2401
*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)
/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c
^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3
*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(3/4)*(((864*a^3
*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008
*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 10
54144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11
 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d
^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 3
5*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9
) + (x^(1/2)*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a
^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768
*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*
a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 3276
8*a^7*b*c*d^14))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 +
167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*
d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^
7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^
13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4
*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) + (x^(1/2)
*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c
^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5
 - 672*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2
*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) -
```

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(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^
2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^
8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d
^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^1
4))^(1/4)*((-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2
*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a
*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^
4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*
a^7*b*c*d^14))^(3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 7
4592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^
7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*
d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4
*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c
^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*
a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) - (x^(1/2)*(-(81*b^4*c^7 + 2401*a^4*c^3*d^
4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*
d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 22
9376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 +
 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*(2304*a^3*b^14*c^13*d
^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c
^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9
*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 1438
72*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6
*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d
^7 - 6*a^5*b*c*d^8)) - (x^(1/2)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*
a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4
*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^11*b^2*c^4*d^6))/(a^6*d^9 + b^6*c^6
*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b
^2*c^2*d^7 - 6*a^5*b*c*d^8)))/((-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*
b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b
^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*
c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b
^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*((-(81*b^4*c^7 + 2401*a^4*c^3*d^4
- 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^
15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 2293
76*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 1
14688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(3/4)*(((864*a^3*b^14*c^14*d^
3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^1
1*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8
*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^1
2*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7
*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4
*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (x^(1/2)*
(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^
2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^
```

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8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d
^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^1
4))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 167168*a^5*b^
12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^9 - 1813504
*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*c^6*d^12 +
554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13*b^4*c^3*d^
15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d^5 - 20*a^3
*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8))*1i + (x^(1/2)*(81*a^5*b
^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*c^8*d^2 - 4
116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^5 - 672*a^1
1*b^2*c^4*d^6)*1i)/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^
4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)) - (81*a^7
*b^6*c^9 - 675*a^8*b^5*c^8*d + 1372*a^12*b*c^4*d^5 + 1971*a^9*b^4*c^7*d^2 -
 2037*a^10*b^3*c^6*d^3 - 392*a^11*b^2*c^5*d^4)/(a^7*d^10 - b^7*c^7*d^3 + 7*
a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^
7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (-(81*b^4*c^7 + 2401*a^4*c^3*d^4
- 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^
15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 2293
76*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 1
14688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(1/4)*((-(81*b^4*c^7 + 2401*a
^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(
4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^7*d^8 + 114688*a^2*b^6*c^6
*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c^4*d^11 - 229376*a^5*b^3*c
^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c*d^14))^(3/4)*(((864*a^3*b
^14*c^14*d^3 - 12096*a^4*b^13*c^13*d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a
^6*b^11*c^11*d^6 + 617152*a^7*b^10*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054
144*a^9*b^8*c^8*d^9 - 795392*a^10*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 -
 133952*a^12*b^5*c^5*d^12 + 25312*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^1
4)*1i)/(a^7*d^10 - b^7*c^7*d^3 + 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*
a^3*b^4*c^4*d^6 - 35*a^4*b^3*c^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9)
- (x^(1/2)*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2
*b^2*c^5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a
*b^7*c^7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^
4*b^4*c^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*
a^7*b*c*d^14))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*d^6 + 16
7168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^10*c^9*d^
9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328*a^10*b^7*
c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 + 16640*a^13
*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a^2*b^4*c^4*d
^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8))*1i - (x^(1/2
)*(81*a^5*b^8*c^10 - 756*a^6*b^7*c^9*d + 784*a^12*b*c^3*d^7 + 2646*a^7*b^6*
c^8*d^2 - 4116*a^8*b^5*c^7*d^3 + 2401*a^9*b^4*c^6*d^4 + 144*a^10*b^3*c^5*d^
5 - 672*a^11*b^2*c^4*d^6)*1i)/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15
*a^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^8)
)))*(-(81*b^4*c^7 + 2401*a^4*c^3*d^4 - 4116*a^3*b*c^4*d^3 + 2646*a^2*b^2*c^
```

```
5*d^2 - 756*a*b^3*c^6*d)/(4096*a^8*d^15 + 4096*b^8*c^8*d^7 - 32768*a*b^7*c^
7*d^8 + 114688*a^2*b^6*c^6*d^9 - 229376*a^3*b^5*c^5*d^10 + 286720*a^4*b^4*c
^4*d^11 - 229376*a^5*b^3*c^3*d^12 + 114688*a^6*b^2*c^2*d^13 - 32768*a^7*b*c
*d^14))^(1/4) + (c*x^(3/2))/(2*d*(c + d*x^2)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.454 $\int \frac{x^{7/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=532

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}+$$

Rubi [A] time = 0.53, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 470, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{b}\,(bc-ad)^2}-\frac{\sqrt[4]{c}\,(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}\,(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,d^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{c}\,(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}\,(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,d^{5/4}(bc-ad)^2}-\frac{c\sqrt{x}}{2d\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(c*Sqrt[x])/(2*d*(b*c - a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) + (a^(5/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) -
(c^(1/4)*(b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sq
rt[2]*d^(5/4)*(b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)^2) - (a^(5/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(1/4)*
(b*c - a*d)^2) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*b^(1/4)*(b*c - a*d)^2) - (c^(1/4)*(b*c - 5*a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(
b*c - a*d)^2) + (c^(1/4)*(b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(5/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{(a+bx^4)(c+dx^4)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{\text{Subst}\left(\int \frac{ac+(bc-4ad)x^4}{(a+bx^4)(c+dx^4)}\,dx, x, \sqrt{x}\right)}{2d(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(c(bc-5ad))\,\text{Subst}\left(\int\right.}{2d(bc-a}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}}{a+bx^4}\right.}{(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{b}\,(bc-ad)^2} + \frac{a^{3/2}\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}}\right.}{2\sqrt{b}}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2} + \frac{a^{5/4}\log\left(\sqrt{a}+\right.}{2\sqrt{2}\,\sqrt[4]{}}$$

$$= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2} - \frac{\sqrt[4]{}}{}$$

Mathematica [A] time = 0.30, size = 523, normalized size = 0.98

$$\frac{-4\sqrt{2}\,a^{5/4}d^{5/4}(c+dx^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+4\sqrt{2}\,a^{5/4}d^{5/4}(c+dx^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-8\sqrt{2}\,a^{5/4}d^{5/4}(c+dx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)+8\sqrt{2}\,a^{5/4}d^{5/4}(c+dx^2)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,(c+dx^2)(bc-5ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,(c+dx^2)(bc-5ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-2\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,(c+dx^2)(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,(c+dx^2)(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)-8\sqrt[4]{b}\,c\,\sqrt[4]{d}\,\sqrt{x}\,(bc-ad)}{16\sqrt[4]{b}\,d^{5/4}(c+dx^2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-8*b^(1/4)*c*d^(1/4)*(b*c - a*d)*Sqrt[x] - 8*Sqrt[2]*a^(5/4)*d^(5/4)*(c +
d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*a^(5/4)*d^
(5/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]
*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)] + 2*Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*ArcTan[
1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*a^(5/4)*d^(5/4)*(c + d*x
^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4*Sqrt[2]*
a^(5/4)*d^(5/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x] - Sqrt[2]*b^(1/4)*c^(1/4)*(b*c - 5*a*d)*(c + d*x^2)*Log[Sqrt[c]
 - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*b^(1/4)*c^(1/4)*(
b*c - 5*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(16*b^(1/4)*d^(5/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 0.99, size = 320, normalized size = 0.60

$$-\frac{a^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2}+\frac{a^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^2}-\frac{\left(bc^{5/4}-5a\sqrt[4]{c}\,d\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,d^{5/4}(ad-bc)^2}+\frac{\left(bc^{5/4}-5a\sqrt[4]{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,d^{5/4}(ad-bc)^2}+\frac{c\sqrt{x}}{2d\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (c*Sqrt[x])/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a^(5/4)*ArcTan[(a^(1/4)/(Sq
rt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*b^(1/4)*
(b*c - a*d)^2) - ((b*c^(5/4) - 5*a*c^(1/4)*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*d^(5/4)*(-(b*c) + a*d)^2) +
(a^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(Sqrt[2]*b^(1/4)*(b*c - a*d)^2) + ((b*c^(5/4) - 5*a*c^(1/4)*d)*ArcTanh[(Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*d^(5/4)*(-
(b*c) + a*d)^2)
```

fricas [B] time = 17.80, size = 3224, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c
^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^
5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*
c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*
```

```
d^13))^(1/4)*arctan(((b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c^4*d^6 -
20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10)*sqrt((b
^2*c^2 - 10*a*b*c*d + 25*a^2*d^2)*x + (b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^
2*b^2*c^2*d^4 - 4*a^3*b*c*d^5 + a^4*d^6)*sqrt(-(b^4*c^5 - 20*a*b^3*c^4*d +
150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a
*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9
 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13)))
*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 62
5*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b
^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11
 - 8*a^7*b*c*d^12 + a^8*d^13))^(3/4) + (b^7*c^7*d^4 - 11*a*b^6*c^6*d^5 + 45
*a^2*b^5*c^5*d^6 - 95*a^3*b^4*c^4*d^7 + 115*a^4*b^3*c^3*d^8 - 81*a^5*b^2*c^
2*d^9 + 31*a^6*b*c*d^10 - 5*a^7*d^11)*sqrt(x)*(-(b^4*c^5 - 20*a*b^3*c^4*d +
 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*
a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^
9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))
^(3/4))/(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3
 + 625*a^4*c*d^4)) - 16*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2
 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^
3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d
^2 - a*d^3)*x^2)*arctan(((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*(-a^5/
(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4
*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
a^8*b*d^8))^(3/4)*sqrt(a^2*x + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7
*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 2
8*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))) - (a*b^7*c^6 - 6*a^2*b^6
*c^5*d + 15*a^3*b^5*c^4*d^2 - 20*a^4*b^4*c^3*d^3 + 15*a^5*b^3*c^2*d^4 - 6*a
^6*b^2*c*d^5 + a^7*b*d^6)*(-a^5/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d
^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*
b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(3/4)*sqrt(x))/a^5) + (b*c^2*d
 - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^
2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d
^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*
b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)*log(
-(b*c - 5*a*d)*sqrt(x) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 2
0*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b
^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70
*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^
12 + a^8*d^13))^(1/4)) - (b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4
*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c
*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*
d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^
7*b*c*d^12 + a^8*d^13))^(1/4)*log(-(b*c - 5*a*d)*sqrt(x) - (b^2*c^2*d - 2*a
```

```
*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500
*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6
*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 +
28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)) - 4*(-a^5/(b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8))^(1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) + (-a^5
/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^
4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 +
 a^8*b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*(-a^5/(b^9*c^8 - 8*
a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4
- 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8))^(
1/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a*sqrt(x) - (-a^5/(b^9
*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5
*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*
b*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 4*c*sqrt(x))/(b*c^2*d - a*
c*d^2 + (b*c*d^2 - a*d^3)*x^2)
```

giac [A] time = 0.98, size = 669, normalized size = 1.26

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{3}c^{2}-2\sqrt{2}ab^{2}cd+\sqrt{2}a^{2}bd^{2}}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{3}c^{2}-2\sqrt{2}ab^{2}cd+\sqrt{2}a^{2}bd^{2}}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^{3}c^{2}-2\sqrt{2}ab^{2}cd+\sqrt{2}a^{2}bd^{2}\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} a \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^{3}c^{2}-2\sqrt{2}ab^{2}cd+\sqrt{2}a^{2}bd^{2}\right)}+\frac{\left(\left(cd^{3}\right)^{\frac{1}{4}}bc-5\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{2}d^{2}-2\sqrt{2}abcd^{3}+\sqrt{2}a^{2}d^{4}\right)}+\frac{\left(\left(cd^{3}\right)^{\frac{1}{4}}bc-5\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{2}d^{2}-2\sqrt{2}abcd^{3}+\sqrt{2}a^{2}d^{4}\right)}+\frac{\left(\left(cd^{3}\right)^{\frac{1}{4}}bc-5\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{2}d^{2}-2\sqrt{2}abcd^{3}+\sqrt{2}a^{2}d^{4}\right)}-\frac{\left(\left(cd^{3}\right)^{\frac{1}{4}}bc-5\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{2}d^{2}-2\sqrt{2}abcd^{3}+\sqrt{2}a^{2}d^{4}\right)}-\frac{c\sqrt{x}}{2\left(bcd-ad^{2}\right)\left(dx^{2}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + (a*b^3
)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + 1/2*(a*b^3)
^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2
- 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) - 1/2*(a*b^3)^(1/4)*a*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*
c*d + sqrt(2)*a^2*b*d^2) + 1/4*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) + 1/4*((c*d^3)^(1/4)*b*c
 - 5*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x
))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^
4) + 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d
)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(
2)*a^2*d^4) - 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d
^3 + sqrt(2)*a^2*d^4) - 1/2*c*sqrt(x)/((b*c*d - a*d^2)*(d*x^2 + c))
```

maple [A] time = 0.02, size = 533, normalized size = 1.00

$$\frac{ac\sqrt{x}}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)}-\frac{b\,c^{2}\sqrt{x}}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)d}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)^{2}}-\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{2}}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{2}d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{2}d}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bc\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/4*a/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*a/(a*d-b*c)^2*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a/(a*d-b*c)^2*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/2*c/(a*d-b*c)^2*
x^(1/2)/(d*x^2+c)*a-1/2*c^2/(a*d-b*c)^2/d*x^(1/2)/(d*x^2+c)*b-5/8/(a*d-b*c)
^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+1/8*c/(a*d-b
*c)^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-5/16/(a
*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)
)/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a+1/16*c/(a*d-b*c)^2/d*(c/d)
^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4
)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b-5/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+1/8*c/(a*d-b*c)^2/d*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b
```

maxima [A] time = 2.47, size = 468, normalized size = 0.88

$$\frac{\left(\frac{2\sqrt{2}\left(bc-5\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\left(bc-5\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(bc-5\,ad\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(bc-5\,ad\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)c}{16\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)}-\frac{c\sqrt{x}}{2\left(bc^{2}d-acd^{2}+\left(bcd^{2}-ad^{3}\right)x^{2}\right)}+\frac{\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\,a^{\frac{5}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{b^{\frac{1}{4}}}}{4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(b*c - 5*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
+ 2*sqrt(2)*(b*c - 5*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(b*c - 5*a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt
(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c - 5*a*d)*log(-sqrt(2)*c^(1/4)*d^(1/4)
*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/(b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3) - 1/2*c*sqrt(x)/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2) +
 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3
/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)
```

mupad [B] time = 2.48, size = 21485, normalized size = 40.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x)

[Out] atan((((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c
^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 +
 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^
7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 +
 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d
^3) + ((2*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^
7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^
5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 -
 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8
+ 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^1
1 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d
 + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512
*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 1111
04*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 31
7440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13
 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2
*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-
a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 -
896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*
b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)) + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8
*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 -
160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5
*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a
^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c
^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*1i - ((-a^5/(16*b^9*
c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*
c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6
- 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6
*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5)
)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((2*(-a^5/(16*b
^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b
^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d
^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d
^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*
d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^
3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
3*a^2*b*c*d^3) - (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 +
```

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1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 -
 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^1
1 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*
d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3
*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*
a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 +
 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b
^8*c^7*d))^(3/4)) - (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*
c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 +
 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4
*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/
(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*
a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*
c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*1i)/(((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 -
128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^
5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(
1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^
5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*
d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + ((2*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8
 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4
*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d)
)^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*
c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*
c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^
4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(
1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^
6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d
^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c
^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6
*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4
*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b
^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4
 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)) + (
x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6
*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6
))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3
*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*
b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 112
0*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c
^7*d))^(1/4) + ((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*
a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*
c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 -
19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*
c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*
a^2*b*c*d^3) + ((2*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 4
```

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48*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b
^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c
^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9
*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^
6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 -
b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x^(1/2)*(256*a^3*b^14*c^12*
d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d
^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*
d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5
*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2
 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c
*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c
^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 +
 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)) - (x^(1/2)*(a^4*b^9*c^8 - 2
0*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c
^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d
- 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^
2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^
7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*
a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)))*(-a^5/(16
*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3
*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2
*d^6 - 128*a*b^8*c^7*d))^(1/4)*2i + 2*atan((((-a^5/(16*b^9*c^8 + 16*a^8*b*d
^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a
^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*
d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a
^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3
*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (((-a^5/(16*b^9*c^8 + 16*a^8*b*
d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*
a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7
*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^
10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b
^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11
*b^4*c^2*d^13)*2i)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)
+ (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c
^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9
*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11
*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7
 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 +
15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128
*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c
^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4
)*1i)*1i + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 -
 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10
*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
```

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- 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c
^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c
^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 -
 128*a*b^8*c^7*d))^(1/4) - ((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*
c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 -
896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^
3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 +
256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*
c^2*d^2 - 3*a^2*b*c*d^3) - (((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2
*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 -
 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*
a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 2867
20*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143
360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13)*2i
)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x^(1/2)*(256*a
^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a
^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152
*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 4
9664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*
a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^
5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 +
448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*
b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)*1i)*1i - (x^(1/
2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*
d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a
^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8
 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4
*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d)
)^(1/4))/(((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b
^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d
^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^
4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d
^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b
*c*d^3) - (((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*
b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*
d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5
 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^
8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d
^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13)*2i)/(a^3*d^4 - b^3*
c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4
- 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 +
 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10
 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3
*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 1
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5*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6
))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d
^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448
*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)*1i)*1i + (x^(1/2)*(a^4*b^9*c^8 -
 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5
*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*
d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*
c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*
d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 89
6*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*1i + ((-a
^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 8
96*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b
^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d +
131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b
^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (((-
a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 -
896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*
b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b
^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7
*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^
10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13)*2i)/(a^3*d^4 - b^3*c^3*d + 3*a*b^
2*c^2*d^2 - 3*a^2*b*c*d^3) - (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13
*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^
10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10
*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a
^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*
d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-a^5/(16*b
^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b
^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d
^6 - 128*a*b^8*c^7*d))^(3/4)*1i)*1i - (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^
7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160
*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^
5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^
5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*
b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*
d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*1i))*(-a^5/(16*b^9*c^8
+ 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*
d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 12
8*a*b^8*c^7*d))^(1/4) + atan((((((2*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*
c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c
^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*
d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2
*d^11 - 32768*a^7*b*c*d^12))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11
*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^
8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*
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b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*
d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*
d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8
*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c
^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^
4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*
a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096
*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7
 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^1
0 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(3/4) + (2*(a^3*b^8*c^7
- 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^
4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^
2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7
*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4
*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*
c*d^12))^(1/4) + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6
*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 40
0*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^
4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^
5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*
d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6
*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3
*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*1i - ((((2
*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*
a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 1146
88*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 2293
76*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*
(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7
- 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10
 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^
13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x^(1/2)*(25
6*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 2252
8*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369
152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12
- 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d -
 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2
*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150
*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 3276
8*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*
a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768
*a^7*b*c*d^12))^(3/4) + (2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^
5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(
a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*a^
```

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4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a
^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 -
 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10
+ 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4) - (x^(1/2)*(a^4*b^9*
c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^
8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6
*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4
*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d
^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^
5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 +
 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11
 - 32768*a^7*b*c*d^12))^(1/4)*1i)/(((((2*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a
^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*
b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5
*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^
2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4
*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a
^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*
a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2
*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*
c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^1
0*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*
b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^
13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d
^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 +
 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/
(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^
6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^
3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(3/4) + (2*(a^3*b^8
*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a
^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d
^2 - 3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a
^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*
a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^
4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a
^7*b*c*d^12))^(1/4) + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^
7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5
 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b
^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b
^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3
*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^
2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^
5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4) + (((
(2*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 2
0*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 11
```

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4688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 22
9376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4
)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^
7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^
10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*
d^13))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x^(1/2)*(
256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22
528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 3
69152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^1
2 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d
 - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c
^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 1
50*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32
768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 28672
0*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 327
68*a^7*b*c*d^12))^(3/4) + (2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*
c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))
/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*
a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096
*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7
 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^1
0 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4) - (x^(1/2)*(a^4*b^
9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*
a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b
^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a
^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2
*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*
d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8
 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^
11 - 32768*a^7*b*c*d^12))^(1/4)))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^
2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8
*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^
8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d
^11 - 32768*a^7*b*c*d^12))^(1/4)*2i + 2*atan(((((((-(b^4*c^5 + 625*a^4*c*d^
4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^1
3 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 22937
6*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 1146
88*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*(5120*a^3*b^12*c^10*d^5 -
40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 +
 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11
 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13)*2i)/(a^3*d^4 - b^3*c^3
*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 5
12*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 11
1104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 -
317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^
```

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13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a
^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*
(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a
*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 11468
8*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 22937
6*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(3/4)*1
i - (2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*
c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d
 + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*
c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c
^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*
d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2
*d^11 - 32768*a^7*b*c*d^12))^(1/4)*1i - (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*
c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 1
60*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*
c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*
a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*
c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c
^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c
^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*
d^12))^(1/4) - (((((-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2
*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*
b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*
b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7
*b*c*d^12))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360
*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 28672
0*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 51
20*a^11*b^4*c^2*d^13)*2i)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*
c*d^3) - (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5
*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*
a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 1671
68*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(
a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3
*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 50
0*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 40
96*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*
b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6
*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(3/4)*1i - (2*(a^3*b^8*c^7 - 19*a^4*b^
7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 +
 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d
^3))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 -
 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 +
114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 -
229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1
/4)*1i + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 5
```

```
00*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b
^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*
a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096
*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7
 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^1
0 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4))/((((((-(b^4*c^5 +
 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/
(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^
6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^
3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*(5120*a^3*b^1
2*c^10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*
b^9*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9
*b^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13)*2i)/(a^3*
d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14
*c^12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11
*c^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^
8*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^
12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c
^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a
^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c
^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^
7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^
4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d
^12))^(3/4)*1i - (2*(a^3*b^8*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 -
 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4
 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*a^4*c*d^4
 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13
 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376
*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 11468
8*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*1i - (x^(1/2)*(a^4*b^9*c^8
 - 20*a^5*b^8*c^7*d + 150*a^6*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^
5*c^4*d^4 - 160*a^9*b^4*c^3*d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6
*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2
*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 +
 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 -
32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286
720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 3
2768*a^7*b*c*d^12))^(1/4)*1i + (((((-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c
^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^
8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d
^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*
d^11 - 32768*a^7*b*c*d^12))^(1/4)*(5120*a^3*b^12*c^10*d^5 - 40960*a^4*b^11*
c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9*c^7*d^8 + 358400*a^7*b^8
*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b^6*c^4*d^11 - 40960*a^10*b
```

```
^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13)*2i)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^
2*d^2 - 3*a^2*b*c*d^3) - (x^(1/2)*(256*a^3*b^14*c^12*d^4 - 512*a^4*b^13*c^1
1*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c^9*d^7 + 111104*a^7*b^10*c
^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8*c^6*d^10 - 317440*a^10*b^7
*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^12*b^5*c^3*d^13 + 6400*a^13*
b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
- 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(-(b^4*c^5 + 62
5*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(40
96*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d
^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d
^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(3/4)*1i - (2*(a^3*b^8
*c^7 - 19*a^4*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a
^7*b^4*c^3*d^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d
^2 - 3*a^2*b*c*d^3))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a
^2*b^2*c^3*d^2 - 20*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*
a*b^7*c^7*d^6 + 114688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^
4*b^4*c^4*d^9 - 229376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a
^7*b*c*d^12))^(1/4)*1i + (x^(1/2)*(a^4*b^9*c^8 - 20*a^5*b^8*c^7*d + 150*a^6
*b^7*c^6*d^2 - 500*a^7*b^6*c^5*d^3 + 641*a^8*b^5*c^4*d^4 - 160*a^9*b^4*c^3*
d^5 + 400*a^10*b^3*c^2*d^6))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^
2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6))*(
-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 20*a*
b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 114688
*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 229376
*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4)*1i
))*(-(b^4*c^5 + 625*a^4*c*d^4 - 500*a^3*b*c^2*d^3 + 150*a^2*b^2*c^3*d^2 - 2
0*a*b^3*c^4*d)/(4096*a^8*d^13 + 4096*b^8*c^8*d^5 - 32768*a*b^7*c^7*d^6 + 11
4688*a^2*b^6*c^6*d^7 - 229376*a^3*b^5*c^5*d^8 + 286720*a^4*b^4*c^4*d^9 - 22
9376*a^5*b^3*c^3*d^10 + 114688*a^6*b^2*c^2*d^11 - 32768*a^7*b*c*d^12))^(1/4
) + (c*x^(1/2))/(2*d*(c + d*x^2)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.455 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

Optimal. Leaf size=528

$$-\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,(bc-ad)^2}$$

Rubi **[A]** time = 0.59, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 471, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^2}+\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}+\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\sqrt[4]{c}\,d^{3/4}(bc-ad)^2}+\frac{x^{3/2}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((b*c +
 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d
^(3/4)*(b*c - a*d)^2) + ((b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) - (a^(3/4)*b^(1/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) + (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) + ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a
*d)^2) - ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a-bx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \left(\frac{4abx^2}{(bc-ad)\left(a+bx^4\right)} - \frac{(bc+3ad)x^2}{(bc-ad)\left(c+dx^4\right)}\right) dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{x^2}{c+dx^4}\right.}{2(bc-ad)^2}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} + \frac{\left(a\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{\left(a\sqrt{b}\right)\text{Subst}\left(\int \frac{\sqrt{a}+}{a+}\right.}{(bc-a}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}}\right.}{2(bc-ad}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} - \frac{a^{3/4}\sqrt[4]{b}\,\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2} + \frac{a^{3/4}\sqrt[4]{b}\,\log\left(\sqrt{a}\right.}{2}$$

$$= \frac{x^{3/2}}{2(bc-ad)\left(c+dx^2\right)} + \frac{a^{3/4}\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2} - \frac{a^{3/4}\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}$$

Mathematica [A] time = 0.29, size = 522, normalized size = 0.99

$$\frac{-4\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+8\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-8\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+8\sqrt[4]{c}d^{3/4}x^{3/2}(bc-ad)+\sqrt{2}\left(c+dx^2\right)(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-\sqrt{2}\left(c+dx^2\right)(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-2\sqrt{2}\left(c+dx^2\right)(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\left(c+dx^2\right)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{16\sqrt[4]{c}d^{3/4}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*c^(1/4)*d^(3/4)*(b*c - a*d)*x^(3/2) + 8*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*
d^(3/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[
2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(
3/4)*(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
 + 4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(1/4)*d^(3/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Sqrt[2]*(b*c + 3*a*d)*(c + d*x^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] - Sqrt[2]*(b*c
 + 3*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(16*c^(1/4)*d^(3/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 0.97, size = 307, normalized size = 0.58

$$\frac{a^{3/4}\sqrt[4]{b}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}(bc-ad)^2}+\frac{a^{3/4}\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}(bc-ad)^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}+\frac{x^{3/2}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] x^(3/2)/(2*(b*c - a*d)*(c + d*x^2)) + (a^(3/4)*b^(1/4)*ArcTan[(a^(1/4)/(Sqr
t[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*(b*c - a*
d)^2) - ((b*c + 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x])])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2) + (a^(3/4)*b^(1/4)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*(
b*c - a*d)^2) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(S
qrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)^2)
```

fricas [B] time = 32.47, size = 3393, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*
```

```
c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56
*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/
4)*arctan(((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt((b^6*c^6 + 18*a*b^5*c^5
*d + 135*a^2*b^4*c^4*d^2 + 540*a^3*b^3*c^3*d^3 + 1215*a^4*b^2*c^2*d^4 + 145
8*a^5*b*c*d^5 + 729*a^6*d^6)*x - (b^8*c^9*d + 8*a*b^7*c^8*d^2 + 12*a^2*b^6*
c^7*d^3 - 40*a^3*b^5*c^6*d^4 - 74*a^4*b^4*c^5*d^5 + 120*a^5*b^3*c^4*d^6 + 1
08*a^6*b^2*c^3*d^7 - 216*a^7*b*c^2*d^8 + 81*a^8*c*d^9)*sqrt(-(b^4*c^4 + 12*
a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d
^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4
*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8
*c*d^11)))*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d
^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a
^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d
^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4) - (b^5*c^5*d + 7*a*b^4*c^4*d^2 +
 10*a^2*b^3*c^3*d^3 - 18*a^3*b^2*c^2*d^4 - 27*a^4*b*c*d^5 + 27*a^5*d^6)*sqr
t(x)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 8
1*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5
*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8
*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4))/(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^
2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)) - 16*(-a^3*b/(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan((sqrt(a^4*b^2*x - (a^3*b^5*c^4 - 4*
a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*sqrt(-a^3*
b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)))*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4) - (a^2*b^3
*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*sqrt(x))/(a^3*b)) +
 4*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b
*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(a^2*b*sq
rt(x) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*
c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - 4*
(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
+ 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*
d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(a^2*b*sqrt(
x) - (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
```

```
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)) - (b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*
a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*
d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log((b^6*c
^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^4
*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 5
4*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^
8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a
^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4)
 + (b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x)) + (b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c
^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c^8*d^4 + 28*
a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*a^5*b^3*c^4*
d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(1/4)*log(-(b^6*
c^7*d^2 - 6*a*b^5*c^6*d^3 + 15*a^2*b^4*c^5*d^4 - 20*a^3*b^3*c^4*d^5 + 15*a^
4*b^2*c^3*d^6 - 6*a^5*b*c^2*d^7 + a^6*c*d^8)*(-(b^4*c^4 + 12*a*b^3*c^3*d +
54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(b^8*c^9*d^3 - 8*a*b^7*c
^8*d^4 + 28*a^2*b^6*c^7*d^5 - 56*a^3*b^5*c^6*d^6 + 70*a^4*b^4*c^5*d^7 - 56*
a^5*b^3*c^4*d^8 + 28*a^6*b^2*c^3*d^9 - 8*a^7*b*c^2*d^10 + a^8*c*d^11))^(3/4
) + (b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)*sqrt(x)) - 4*x^
(3/2))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

giac [A] time = 0.93, size = 683, normalized size = 1.29

$$\frac{\left(\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{3}d^{3}-2\sqrt{2}abc^{2}d^{4}+\sqrt{2}a^{2}cd^{5}\right)}+\frac{\left(\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{3}d^{3}-2\sqrt{2}abc^{2}d^{4}+\sqrt{2}a^{2}cd^{5}\right)}-\frac{\left(\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{3}d^{3}-2\sqrt{2}abc^{2}d^{4}+\sqrt{2}a^{2}cd^{5}\right)}+\frac{\left(\left(cd^{3}\right)^{\frac{3}{4}}bc+3\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{3}d^{3}-2\sqrt{2}abc^{2}d^{4}+\sqrt{2}a^{2}cd^{5}\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^{4}c^{2}-2\sqrt{2}ab^{3}cd+\sqrt{2}a^{2}b^{2}d^{2}\right)}+\frac{x^{\frac{3}{2}}}{2\left(dx^{2}+c\right)\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c
^2*d^4 + sqrt(2)*a^2*c*d^5) + 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) - 1/8*((c*d^3)^(
3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) + 1
/8*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*
a^2*c*d^5) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*
d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/
(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2)
+ 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
```

```
2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/2*(a*b^3)^(3/4)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^2 - 2*sqr
t(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*x^(3/2)/((d*x^2 + c)*(b*c - a*d
))
```

maple [A] time = 0.02, size = 528, normalized size = 1.00

$$-\frac{a d x^{\frac{3}{2}}}{2 (a d-b c)^{2} \left(d x^{2}+c\right)}+\frac{b c x^{\frac{3}{2}}}{2 (a d-b c)^{2} \left(d x^{2}+c\right)}-\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 (a d-b c)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 (a d-b c)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{3 \sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 (a d-b c)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{3 \sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 (a d-b c)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\, a \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 (a d-b c)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{3 \sqrt{2}\, a \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 (a d-b c)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\, b c \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 (a d-b c)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} d}+\frac{\sqrt{2}\, b c \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 (a d-b c)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} d}+\frac{\sqrt{2}\, b c \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 (a d-b c)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/4*a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*a/(a*d-b*c)^2/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a/(a*d-b*c)^2/
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/2/(a*d-b*c)^2*x
^(3/2)/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*b*c+3/16/(a*d-b*c)^2
/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+3/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)
*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3/8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2
)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+1/16/(a*d-b*c)^2/d/(c/d)^(1/4)*2^
(1/2)*b*c*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(
1/2)*x^(1/2)+(c/d)^(1/2)))+1/8/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*b*c*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+1/8/(a*d-b*c)^2/d/(c/d)^(1/4)*2^(1/2)*b*c*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

maxima [A] time = 2.45, size = 436, normalized size = 0.83

$$-\frac{a b \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4 \left(b^{2} c^{2}-2 a b c d+a^{2} d^{2}\right)}+\frac{(b c+3 a d) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}-\frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{16 \left(b^{2} c^{2}-2 a b c d+a^{2} d^{2}\right)}+\frac{x^{\frac{3}{2}}}{2 \left(b c^{2}-a c d+\left(b c d-a d^{2}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/16*(b*c + 3*a*d)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c
```

```
^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sq
rt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) +
 1/2*x^(3/2)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

mupad [B] time = 2.48, size = 18673, normalized size = 35.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x)

[Out] atan(((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^
4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^
7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 +
 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*
a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*
d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*
b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4
- 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c
*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^1
3*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7
*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a
^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6
*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d
^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 4
48*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b
^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4
)*1i + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 +
54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^
6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*
a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*
c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) -
 (((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 1
3856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 7
4368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 1033
6*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*a^2*b
^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 +
 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c
^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896
*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7)
)^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^1
0*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10
```

```
*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b
^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6
+ b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^
2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^
3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i
- (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^
5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^6 + b
^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b
^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d
^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4))/((a^4
*b^9*c^5*d + 108*a^8*b^5*c*d^5 + 13*a^5*b^8*c^4*d^2 + 63*a^6*b^7*c^3*d^3 +
135*a^7*b^6*c^2*d^4)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c
^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*
c*d^6) + (((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10
*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7
*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^1
0 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 -
21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c
^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 +
16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d
^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*
b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*
b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*
a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 12083
2*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(
a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8
+ 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^
5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(
3/4) + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 +
54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5))/(a^6*d^6 +
 b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2
*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3
*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) + ((
(864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 1385
6*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 7436
8*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a
^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*
c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*
a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8
+ 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^
5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(
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1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d
^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^
7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*
c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b
^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b
^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d
^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4) - (x^(
1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*
c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^
2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*
a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)))*(-(a^3*b)/(16
*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^
4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d
 - 128*a^7*b*c*d^7))^(1/4)*2i - 2*atan((((((864*a^13*b^4*c*d^13 - 32*a^3*b^
14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b
^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^
8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5
*c^2*d^12)*1i)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3
 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)
 + (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*
a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*
c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 +
 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6
 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d
^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^
3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))
*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5
*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 1
28*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4) + (x^(1/2)*(a^3*b^10*c^6*d + 144*a
^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^
4 + 177*a^7*b^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b
)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 11
20*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*
c^7*d - 128*a^7*b*c*d^7))^(1/4) - ((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^1
1*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^
8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*
d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d
^12)*1i)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*
a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^
(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^
```

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5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^
6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*
a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219
136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 1
97120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12
 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a
^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^
3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 +
 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b
^7*c^7*d - 128*a^7*b*c*d^7))^(3/4) - (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5
*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 17
7*a^7*b^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*
a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4
*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d
- 128*a^7*b*c*d^7))^(1/4))/(((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3
+ 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6
- 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 +
256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)*1
i)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^
3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*
(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*
d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 12
8*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^
14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^
6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*
a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 168
96*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(
16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*
a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7
*d - 128*a^7*b*c*d^7))^(3/4)*1i + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*
d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a
^7*b^6*c^2*d^5)*1i)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*
a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4
*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d
- 128*a^7*b*c*d^7))^(1/4) - (a^4*b^9*c^5*d + 108*a^8*b^5*c*d^5 + 13*a^5*b^8
*c^4*d^2 + 63*a^6*b^7*c^3*d^3 + 135*a^7*b^6*c^2*d^4)/(a^7*d^7 - b^7*c^7 - 2
1*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^
2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + ((((864*a^13*b^4*c*d^13 - 32*a^3*b
^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*
b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b
^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^
5*c^2*d^12)*1i)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^
```

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3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6
) - (x^(1/2)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896
*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2
*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14
+ 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^
6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*
d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c
^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)
)*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^
5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 -
128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i - (x^(1/2)*(a^3*b^10*c^6*d + 1
44*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^
3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(
-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d
^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128
*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)))*(-(a^3*b)/(16*a^8*d^8 + 16*b^8*c^8
 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a
^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^
(1/4) + atan(((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13
*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^1
0*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^
4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c
^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*
b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(81*a^4*d^4 + b^4
*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d
^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 11468
8*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 22937
6*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)*(2304*a^13*b^4*c*d^14 +
4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6
- 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^
9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3
*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*
(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c
*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7
*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*
b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(3/4)*1i +
(x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*
b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^6 + b^6
*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b
^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 1
2*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 3276
8*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^
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3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^
6*b^2*c^3*d^9))^(1/4) - (((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 198
4*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 748
16*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a
^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d
^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4
 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^(1/2)*(-(81*a^4
*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(40
96*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^
10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d
^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)*(2304*a^13*b^4
*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^1
2*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*
b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^
11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4
*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*
b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 1
08*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 -
 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 2
86720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(
3/4)*1i - (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2
 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6
*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d
^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c
^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*
d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 -
 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 +
 114688*a^6*b^2*c^3*d^9))^(1/4))/((a^4*b^9*c^5*d + 108*a^8*b^5*c*d^5 + 13*a
^5*b^8*c^4*d^2 + 63*a^6*b^7*c^3*d^3 + 135*a^7*b^6*c^2*d^4)/(a^7*d^7 - b^7*c
^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*
b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (((864*a^13*b^4*c*d^13 - 32*
a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264
*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*
a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^
12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d
^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^
6) + (x^(1/2)*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d
 + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d
^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6
 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9
))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^
10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^1
0*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*
b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
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6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d
^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3
- 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229
376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114
688*a^6*b^2*c^3*d^9))^(3/4) + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6
+ 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b
^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c
^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^1
1 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*
a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*
a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4) + (((864*a^13*b^4*c*d^13 -
 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 4
3264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37
184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 518
4*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c
^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*
c*d^6) - (x^(1/2)*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c
^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c
^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6
*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3
*d^9))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^1
3*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7
*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a
^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6
*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d
^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c
^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*
d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 -
 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 +
 114688*a^6*b^2*c^3*d^9))^(3/4) - (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*
d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a
^7*b^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b
^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c
*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114
688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229
376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)))*(-(81*a^4*d^4 + b^4*
c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^
11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688
*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376
*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)*2i - 2*atan((((((864*a^13
*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^1
2*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9
*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c
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^3*d^11 - 5184*a^12*b^5*c^2*d^12)*1i)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d
^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6
*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2
*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^
3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 2
29376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 1
14688*a^6*b^2*c^3*d^9))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^
4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8
*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^
5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^
5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c
^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^1
1 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*
a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*
a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(3/4) + (x^(1/2)*(a^3*b^10*c^6*d
 + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^
7*c^3*d^4 + 177*a^7*b^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*
(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c
*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7
*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*
b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4) - (((
(864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d^4 - 1385
6*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d^7 + 7436
8*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a
^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)*1i)/(a^7*d^7 - b^7*c^7 - 21*a^2*
b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5
+ 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^(1/2)*(-(81*a^4*d^4 + b^4*c^4 + 54*a^
2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b
^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^
7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^
4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^1
4*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6
*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a
^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 1689
6*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^
4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*
a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10
+ 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7
- 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(3/4) - (x^(1/2)*(a^3*b
^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 1
24*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*
c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b
```

```
*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 10
8*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 -
32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 28
6720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1
/4))/(((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d
^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*d
^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^10
+ 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)*1i)/(a^7*d^7 - b^7*c^7
- 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2
*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(81*a^4*d^4 + b^4*c^
4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11
 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a
^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a
^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)*(2304*a^13*b^4*c*d^14 + 435
2*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 2
19136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 +
 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^
12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(
81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^
3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*
c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4
*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(3/4)*1i + (x^
(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a^4*b^9*c^5*d^2 + 54*a^5*b^8
*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^2*d^5)*1i)/(a^6*d^6 + b^6*c^
6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*
c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a
*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a
*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b
^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b
^2*c^3*d^9))^(1/4) - (a^4*b^9*c^5*d + 108*a^8*b^5*c*d^5 + 13*a^5*b^8*c^4*d^
2 + 63*a^6*b^7*c^3*d^3 + 135*a^7*b^6*c^2*d^4)/(a^7*d^7 - b^7*c^7 - 21*a^2*b
^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 +
 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + ((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^1
1*d^3 + 1984*a^4*b^13*c^10*d^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^
8*d^6 - 74816*a^7*b^10*c^7*d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*
d^9 + 256*a^10*b^7*c^4*d^10 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d
^12)*1i)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*
a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) - (x^
(1/2)*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a
^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 327
68*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 28672
0*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)
*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 33280*a^4*b^13*c^10*d^5 +
 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 + 283136*a^7*b^10*c^7*d^
```

```
8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10 - 120832*a^10*b^7*c^4*
d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2*d^13))/(a^6*d^6 + b^6*c
^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5
*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 12*
a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4096*b^8*c^9*d^3 - 32768*
a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b^6*c^7*d^5 - 229376*a^3*
b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b^3*c^4*d^8 + 114688*a^6*
b^2*c^3*d^9))^(3/4)*1i - (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*
a^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^
2*d^5)*1i)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(81*a^4*d^4 + b^4*c^4
 + 54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11
+ 4096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^
2*b^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^
5*b^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4)))*(-(81*a^4*d^4 + b^4*c^4 +
54*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d + 108*a^3*b*c*d^3)/(4096*a^8*c*d^11 + 4
096*b^8*c^9*d^3 - 32768*a*b^7*c^8*d^4 - 32768*a^7*b*c^2*d^10 + 114688*a^2*b
^6*c^7*d^5 - 229376*a^3*b^5*c^6*d^6 + 286720*a^4*b^4*c^5*d^7 - 229376*a^5*b
^3*c^4*d^8 + 114688*a^6*b^2*c^3*d^9))^(1/4) - x^(3/2)/(2*(c + d*x^2)*(a*d -
 b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.456 $\int \frac{x^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=528

$$\frac{\sqrt[4]{a}\,b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}\,(bc-ad)^2}$$

Rubi [A] time = 0.48, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 471, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\,b^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2}+\frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{(ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{(ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{\sqrt{x}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - (a^(1/4)*b^(3/4)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^2) - ((3*b*c
 + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d
^(1/4)*(b*c - a*d)^2) + ((3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) + (a^(1/4)*b^(3/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c -
a*d)^2) - (a^(1/4)*b^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^2) - ((3*b*c + a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a
*d)^2) + ((3*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(8*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{a-3bx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{(2ab)\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} + \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{1}{c+dx\cdots}\right.}{2(bc-ad)^2\cdots}$$

$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,b\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{\left(\sqrt{a}\,b\right)\text{Subst}\left(\int \frac{\sqrt{a}\cdots}{a\cdots}\right.}{(bc-a\cdots}$$

$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2} - \frac{\left(\sqrt{a}\,\sqrt{b}\right)\text{Su}\cdots}{\cdots}$$

$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^2} - \frac{\sqrt[4]{a}\,b^{3/4}\log\left(\sqrt{\cdots}\right.}{2\cdots}$$

$$= \frac{\sqrt{x}}{2(bc-ad)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^2} - \frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\cdots\right.}{\sqrt{2}\,(bc-ad)^2}$$

Mathematica [A] time = 0.30, size = 522, normalized size = 0.99

$$\frac{4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-4\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)-8\sqrt{2}\,\sqrt[4]{a}\,b^{3/4}c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)+8c^{3/4}\sqrt[4]{d}\,\sqrt{x}\,(bc-ad)-\sqrt{2}\left(c+dx^2\right)(ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+\sqrt{2}\left(c+dx^2\right)(ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-2\sqrt{2}\left(c+dx^2\right)(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\left(c+dx^2\right)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{16c^{3/4}\sqrt[4]{d}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*c^(3/4)*d^(1/4)*(b*c - a*d)*Sqrt[x] + 8*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*
d^(1/4)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 8*Sqrt[
2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(3*b*c + a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*(c + d*x^2)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(
1/4)*(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
 - 4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(3/4)*d^(1/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*(3*b*c + a*d)*(c + d*x^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*(3*b
*c + a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(16*c^(3/4)*d^(1/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 0.92, size = 308, normalized size = 0.58

$$\frac{\sqrt[4]{a}\,b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{\sqrt[4]{a}\,b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,(bc-ad)^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^2}+\frac{\sqrt{x}}{2\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] Sqrt[x]/(2*(b*c - a*d)*(c + d*x^2)) + (a^(1/4)*b^(3/4)*ArcTan[(a^(1/4)/(Sqr
t[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*(b*c - a*
d)^2) - ((3*b*c + a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x])])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2) - (a^(1/4)*b^(3/4)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*(
b*c - a*d)^2) + ((3*b*c + a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(S
qrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^2)
```

fricas [B] time = 13.99, size = 3171, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*
d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^
10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*
a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4
)*arctan(((b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3*c^
```

```
5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7)*sqrt((9*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 -
4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 2
8*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^
6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9)))*(-(81*b^4*c^4
 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^
11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*
b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a
^8*c^3*d^9))^(3/4) - (3*b^7*c^9*d - 17*a*b^6*c^8*d^2 + 39*a^2*b^5*c^7*d^3 -
 45*a^3*b^4*c^6*d^4 + 25*a^4*b^3*c^5*d^5 - 3*a^5*b^2*c^4*d^6 - 3*a^6*b*c^3*
d^7 + a^7*c^2*d^8)*sqrt(x)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^
6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 +
28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(3/4))/(81*b^4*c^4 + 1
08*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)) - 16*(-a*b
^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
 a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*arctan(((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)*sqrt(b^2*x + (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b^3/(b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8))) - (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
+ 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*(-a*b^3/(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(3/4)*s
qrt(x))/(a*b^3)) + (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 10
8*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d
- 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c
^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^
3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c
^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a
^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^
4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8
*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7
*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) - (b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
+ 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9
```

```
*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^
6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) - 4*(-a*b^3/(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^
(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2
*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)) + 4*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*
d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b
^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 -
a*c*d + (b*c*d - a*d^2)*x^2)*log(b*sqrt(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*(-a*b^3/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8))^(1/4)) + 4*sqrt(x))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

giac [**A**]   time = 1.13, size = 655, normalized size = 1.24

$$\frac{\left(3\left(cd^3\right)^{\frac{1}{4}}bc+\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}+\frac{\left(3\left(cd^3\right)^{\frac{1}{4}}bc+\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}+\frac{\left(3\left(cd^3\right)^{\frac{1}{4}}bc+\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}-\frac{\left(3\left(cd^3\right)^{\frac{1}{4}}bc+\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2}-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2}-\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2\right)}+\frac{\sqrt{x}}{2\left(dx^2+c\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2
*d^2 + sqrt(2)*a^2*c*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*
b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/8*(3*(c*d^3)^(1/
4)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) - 1/8*(3*
(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^
3) - (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - (a*b^3)
^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(
sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/2*(a*b^3)^(1/4)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(
2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2
*d^2) + 1/2*sqrt(x)/((d*x^2 + c)*(b*c - a*d))
```

maple [**A**]   time = 0.02, size = 528, normalized size = 1.00

$$-\frac{ad\sqrt{x}}{2(ad-bc)^2\left(dx^2+c\right)}+\frac{bc\sqrt{x}}{2(ad-bc)^2\left(dx^2+c\right)}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8(ad-bc)^2c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8(ad-bc)^2c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ad\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16(ad-bc)^2c}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2(ad-bc)^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2(ad-bc)^2}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8(ad-bc)^2}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8(ad-bc)^2}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4(ad-bc)^2}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/4*b/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b/(a*d-b*c)^2*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b/(a*d-b*c)^2*
(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/2/(a*d-b*c)^2*x
^(1/2)/(d*x^2+c)*a*d+1/2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*b*c+1/16/(a*d-b*c)^2
*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*d+3/16/(a*d-b*c)^2*(c/d)^(1/4)*2^
(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2)))*b+1/8/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)+1)*a*d+3/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+1/8/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*d+3/8/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b
```

maxima [**A**] time = 2.66, size = 461, normalized size = 0.87

$$-\frac{\left(\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)a}{4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}+\frac{\frac{2\sqrt{2}(3\,bc+ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(3\,bc+ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(3\,bc+ad)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(3\,bc+ad)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}+\frac{\sqrt{x}}{2\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)
*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*a/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/16*(2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(
sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(-1/2*sqrt(2
)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqr
t(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(2)*c^(1/4)*d^(
1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b*c + a*
d)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(
1/4)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*sqrt(x)/(b*c^2 - a*c*d + (b*c*
d - a*d^2)*x^2)
```

mupad [**B**] time = 2.36, size = 20689, normalized size = 39.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] - atan(((((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3
*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^(1
/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 304
64*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 1
98656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 -
 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13)
)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2
*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (2*(-(a*b^3)/(16*a^8*d^8 + 16*b
^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 -
 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*
d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12
*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b
^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6
*c^3*d^11 - 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 8
96*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b
^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(16*a^8*d
^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*
c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128
*a^7*b*c*d^7))^(1/4) + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^
2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^
6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*
b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^
6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^
5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i - ((
(2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d
^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - ((x^(1/2)*(256*a
^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^1
4*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*
b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9
*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (2*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 4
48*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b
^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4
)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 +
 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8
 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11
- 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5
*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6
 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(16*a^8*d^8 + 16*b^
8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 -
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896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d
^7))^(1/4) - (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4
*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
- 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2
- 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^
6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i)/((((2*(51*a^4
*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^(1/2)*(256*a^13*b^4*d^
15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5
 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d
^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d
^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6
 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c
^5*d - 6*a^5*b*c*d^5) + (2*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6
*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5
 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(1024*a^
11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^
4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*
a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^1
0*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a*
b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 +
 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b
^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 44
8*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^
3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)
 + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108
*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c
^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*
c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*
b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*
d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) + (((2*(51*a^4*b^7*c*d^5 -
a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - ((x^(1/2)*(256*a^13*b^4*d^15 - 512*a^12
*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5 + 97792*a^4*
b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d^8 - 146944*a
^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a
^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4
*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*
b*c*d^5) - (2*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 89
6*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^
2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*(1024*a^11*b^4*c*d^13
 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^
6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d
^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^1
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2))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a*b^3)/(16*a^8*
d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4
*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 12
8*a^7*b*c*d^7))^(3/4))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6
*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 4
48*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) - (x^(1/2)*(
17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3
*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a
^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*
b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 +
 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b
^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 4
48*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b
^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4
)*2i - 2*atan(((((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 +
189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) -
 ((x^(1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^
4 - 30464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*
d^7 + 198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5
*d^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^
2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + ((-(a*b^3)/(16*a^8*d^8 +
 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*
d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7
*b*c*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3
*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*
a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^
9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6
*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 4
48*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i)*(-(a*b^3
)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 11
20*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*
c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8
*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))
/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^
8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*
a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))
^(1/4) - (((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^
3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^(
1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30
464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 +
198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10
- 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13
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))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-(a*b^3)/(16*a^8*d^8 + 16*b^
8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 -
896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d
^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*
c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^
9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*
c^3*d^11 - 4096*a^10*b^5*c^2*d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 -
 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6
*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i)*(-(a*b^3)/(16*
a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4
*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d
- 128*a^7*b*c*d^7))^(1/4)*1i - (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6
 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 44
8*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^
3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)
)/((((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*
c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) - ((x^(1/2)*(
256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^
3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656
*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 3635
2*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^
6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*
d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + ((-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8
+ 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^
5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(
1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^
5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*
d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^
11 - 4096*a^10*b^5*c^2*d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a
^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c
^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i)*(-(a*b^3)/(16*a^8*d^
8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c
^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*
a^7*b*c*d^7))^(1/4)*1i + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*
a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 +
b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*
a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*
b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*
d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i +
(((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2
```

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*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^(1/2)*(256
*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b
^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^
6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a
^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d
^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - ((-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 4
48*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b
^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4
)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 +
 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8
 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11
- 4096*a^10*b^5*c^2*d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*
b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*
d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4)*1i)*(-(a*b^3)/(16*a^8*d^8 +
 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*
d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7
*b*c*d^7))^(1/4)*1i - (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2
*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6
*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b
^5*c^5*d - 6*a^5*b*c*d^5))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6
*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5
 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4)*1i))*(-(
a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3
 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a
*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) - atan(-(((((x^(1/2)*(256*a^13*b^4*d^1
5 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5
+ 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d^
8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^
11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6
+ 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^
5*d - 6*a^5*b*c*d^5) - (2*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 10
8*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768
*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3
*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6
*b^2*c^5*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744
*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229
376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 409
6*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*
a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a
*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b
^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b
^2*c^5*d^7))^(3/4) - (2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^
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3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3
*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 3276
8*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*
a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4) +
 (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a
^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*
d^5))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a
^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32
768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 28672
0*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)
*1i + ((((x^(1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*
c^11*d^4 - 30464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^
12*c^8*d^7 + 198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*
b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11
*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (2*(-(a^4*d^4 + 8
1*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^
8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 +
114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 -
229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*(1024*a^11*b^4*c*d^
13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*
d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5
*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d
^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*
b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*
c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 11
4688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 22
9376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(3/4) + (2*(51*a^4*b^7*c*d^
5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2
*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^
3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3
 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6
 + 114688*a^6*b^2*c^5*d^7))^(1/4) + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*
c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/
(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b
^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*
c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d
^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d
^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*1i)/(((((x^(1/2)*(256*a^13*b^4*d^15 - 5
12*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5 + 977
92*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d^8 - 1
```

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46944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 +
14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
 6*a^5*b*c*d^5) - (2*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b
^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^
7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*
c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*
c^5*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*
b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a
^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9
*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3
*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*
c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^
8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^
5*d^7))^(3/4) - (2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 1
89*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(
-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*
d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7
*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b
^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4) + (x^(
1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^
10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))
*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*
c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a
^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4
*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4) - ((
((x^(1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4
 - 30464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d
^7 + 198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*
d^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2
*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (2*(-(a^4*d^4 + 81*b^4*c^
4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d
 + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a
^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a
^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 409
6*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 20
0704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 5
7344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12))/(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4
+ 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d +
 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2
*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5
```

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*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(3/4) + (2*(51*a^4*b^7*c*d^5 - a^5*
b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2
 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 -
32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 22937
6*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 11468
8*a^6*b^2*c^5*d^7))^(1/4) + (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 +
81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d
^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9
- 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229
376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114
688*a^6*b^2*c^5*d^7))^(1/4)))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2
+ 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 3
2768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376
*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688
*a^6*b^2*c^5*d^7))^(1/4)*2i - 2*atan(-(((((x^(1/2)*(256*a^13*b^4*d^15 - 512
*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5 + 97792
*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d^8 - 146
944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 + 14
336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^
2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6
*a^5*b*c*d^5) - ((-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c
^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^
10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*
d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*
d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12
*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b
^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6
*c^3*d^11 - 4096*a^10*b^5*c^2*d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
- 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*
c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c
^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8
*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5
*d^7))^(3/4)*1i + (2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 +
 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*
c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a
^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4
*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*1i -
 (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a
^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*
d^5))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a
```

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^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32
768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 28672
0*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)
 + ((((x^(1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^1
1*d^4 - 30464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*
c^8*d^7 + 198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9
*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^
6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + ((-(a^4*d^4 + 81*b^4
*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^1
1*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 11468
8*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 22937
6*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 +
4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 -
 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9
 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12)*
2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^
4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^
11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 1146
88*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 2293
76*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(3/4)*1i - (2*(51*a^4*b^7*c*d
^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^
2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c
^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^
3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^
6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*1i - (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*
b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^
5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b
^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a
^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*
a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c
^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c
^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4))/(((((x^(1/2)*(256*a^13*b^4*d^15 -
512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5 + 97
792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d^8 -
146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 +
 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5) - ((-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^
3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7
*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c
^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c
^5*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b
^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^
```

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6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*
b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b
^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^
7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*
c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*
c^5*d^7))^(3/4)*1i + (2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^
3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3
*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 3276
8*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*
a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*1
i - (x^(1/2)*(17*a^6*b^7*d^7 + 108*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 10
8*a^3*b^10*c^3*d^4 + 198*a^4*b^9*c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*
c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b
*c*d^5))*(-(a^4*d^4 + 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 1
2*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 -
 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 28
6720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1
/4)*1i - ((((x^(1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^
15*c^11*d^4 - 30464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5
*b^12*c^8*d^7 + 198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a
^8*b^9*c^5*d^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a
^11*b^6*c^2*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + ((-(a^4*d^4 +
81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b
^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 +
 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 -
 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*(1024*a^11*b^4*c*d
^13 + 4096*a^2*b^13*c^10*d^4 - 31744*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8
*d^6 - 200704*a^5*b^10*c^7*d^7 + 229376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^
5*d^9 + 57344*a^8*b^7*c^4*d^10 - 4096*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*
d^12)*2i)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 +
 81*b^4*c^4 + 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*
b^8*c^11*d + 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8
+ 114688*a^2*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5
- 229376*a^5*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(3/4)*1i - (2*(51*a^4*b
^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^3*b^8*c^2*d^4))/(a^3*d^
3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a^4*d^4 + 81*b^4*c^4 + 54*
a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d + 4096
*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2*b^6*
c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5*b^3*
c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*1i - (x^(1/2)*(17*a^6*b^7*d^7 + 10
8*a^5*b^8*c*d^6 + 81*a^2*b^11*c^4*d^3 + 108*a^3*b^10*c^3*d^4 + 198*a^4*b^9*
c^2*d^5))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
```

```
*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^4*d^4 + 81*b^4*c^4
+ 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d +
 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2
*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5
*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4)*1i))*(-(a^4*d^4 + 81*b^4*c^4
+ 54*a^2*b^2*c^2*d^2 + 108*a*b^3*c^3*d + 12*a^3*b*c*d^3)/(4096*b^8*c^11*d +
 4096*a^8*c^3*d^9 - 32768*a*b^7*c^10*d^2 - 32768*a^7*b*c^4*d^8 + 114688*a^2
*b^6*c^9*d^3 - 229376*a^3*b^5*c^8*d^4 + 286720*a^4*b^4*c^7*d^5 - 229376*a^5
*b^3*c^6*d^6 + 114688*a^6*b^2*c^5*d^7))^(1/4) - x^(1/2)/(2*(c + d*x^2)*(a*d
 - b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.457 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=536

$$\frac{b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}+$$

Rubi [A] time = 0.60, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 472, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}-\frac{b^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}-\frac{\sqrt[4]{d}\,(5bc-ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}\,(5bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{d}\,(5bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^2}-\frac{\sqrt[4]{d}\,(5bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^2}-\frac{dx^{3/2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x^(3/2))/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) + (b^(5/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^2) +
(d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sq
rt[2]*c^(5/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) + (b^(5/4)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*
(b*c - a*d)^2) - (b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^2) - (d^(1/4)*(5*b*c - a*d)*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(
b*c - a*d)^2) + (d^(1/4)*(5*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\text{Subst}\left(\int \frac{x^2\left(4bc-ad-bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\text{Subst}\left(\int \left(\frac{4b^2cx^2}{(bc-ad)\left(a+bx^4\right)}+\frac{d(-5bc+ad)x^2}{(bc-ad)\left(c+dx^4\right)}\right)dx, x, \sqrt{x}\right)}{2c(bc-ad)}\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2}-\frac{(d(5bc-ad))\,\text{Subst}(}{2c(bc-}\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}-\frac{b^{3/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2}+\frac{b^{3/2}\,\text{Subst}(\int \frac{\sqrt{a}+}{a+bx}}{(bc-ad}\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)^2}+\frac{b\,\text{Subst}(\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+}}{2(bc}\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}-\frac{b^{5/4}\log(\sqrt{a}+}{2\sqrt{2}}\\
&= -\frac{dx^{3/2}}{2c(bc-ad)\left(c+dx^2\right)}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}+\frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^2}+
\end{aligned}$$

Mathematica [A] time = 0.42, size = 523, normalized size = 0.98

$$\frac{4\sqrt{2}\,b^{5/4}c^{5/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-4\sqrt{2}\,b^{5/4}c^{5/4}\left(c+dx^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-8\sqrt{2}\,b^{5/4}c^{5/4}\left(c+dx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)+8\sqrt{2}\,b^{5/4}c^{5/4}\left(c+dx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)+8\sqrt[4]{a}\,\sqrt[4]{c}\,dx^{3/2}(ad-bc)+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{d}\,\left(c+dx^2\right)(ad-5bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{d}\,\left(c+dx^2\right)(5bc-ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)-2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{d}\,\left(c+dx^2\right)(ad-5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{d}\,\left(c+dx^2\right)(ad-5bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{16\sqrt[4]{a}\,c^{5/4}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*a^(1/4)*c^(1/4)*d*(-(b*c) + a*d)*x^(3/2) - 8*Sqrt[2]*b^(5/4)*c^(5/4)*(c
+ d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*b^(5/4)*
c^(5/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[
2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)] + 2*Sqrt[2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 4*Sqrt[2]*b^(5/4)*c^(5/4)*(c +
 d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 4*Sqrt
[2]*b^(5/4)*c^(5/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x] + Sqrt[2]*a^(1/4)*d^(1/4)*(-5*b*c + a*d)*(c + d*x^2)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*a^(1/4)*d^(1
/4)*(5*b*c - a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(16*a^(1/4)*c^(5/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 1.14, size = 323, normalized size = 0.60

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(ad-bc)^2}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(ad-bc)^2}+\frac{\left(5bc\sqrt[4]{d}-ad^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^2}+\frac{\left(5bc\sqrt[4]{d}-ad^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^2}-\frac{dx^{3/2}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)) - (b^(5/4)*ArcTan[(a^(1/4)/(Sq
rt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(1/4)*
(-(b*c) + a*d)^2) + ((5*b*c*d^(1/4) - a*d^(5/4))*ArcTan[(Sqrt[c] - Sqrt[d]*
x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^2) -
(b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(Sqrt[2]*a^(1/4)*(-(b*c) + a*d)^2) + ((5*b*c*d^(1/4) - a*d^(5/4))*ArcTanh[(
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(5/4)
*(b*c - a*d)^2)
```

fricas [B] time = 47.52, size = 3457, normalized size = 6.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*d*x^(3/2) + 4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^
4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^
5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 +
 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6
*d^7 + a^8*c^5*d^8))^(1/4)*arctan(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt
((15625*b^6*c^6*d^2 - 18750*a*b^5*c^5*d^3 + 9375*a^2*b^4*c^4*d^4 - 2500*a^3
*b^3*c^3*d^5 + 375*a^4*b^2*c^2*d^6 - 30*a^5*b*c*d^7 + a^6*d^8)*x - (625*b^8
*c^11*d - 3000*a*b^7*c^10*d^2 + 5900*a^2*b^6*c^9*d^3 - 6120*a^3*b^5*c^8*d^4
 + 3606*a^4*b^4*c^7*d^5 - 1224*a^5*b^3*c^6*d^6 + 236*a^6*b^2*c^5*d^7 - 24*a
^7*b*c^4*d^8 + a^8*c^3*d^9)*sqrt(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*
a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28
*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c
^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)))*(-(625*b^4*c
^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/
(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70
*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^
7 + a^8*c^5*d^8))^(1/4) + (125*b^5*c^6*d - 325*a*b^4*c^5*d^2 + 290*a^2*b^3*
c^4*d^3 - 106*a^3*b^2*c^3*d^4 + 17*a^4*b*c^2*d^5 - a^5*c*d^6)*sqrt(x)*(-(62
5*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^
4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d
^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b
*c^6*d^7 + a^8*c^5*d^8))^(1/4))/(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^
2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5)) + 16*(-b^5/(a*b^8*c^8 - 8*a^2*b^
7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56
*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*
c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*arctan((sqrt(b^8*x - (a*b^9*c^4 -
4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*sqrt(-
b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3
+ 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*
d^7 + a^9*d^8)))*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 -
56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c
^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2) -
(b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d +
28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c
^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*sqrt(x))/b^5)
 - 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c
^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a
^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*lo
g(b^4*sqrt(x) + (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*
b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^
8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*
c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8
))^(3/4)) + 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*
a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*
d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
```

```
)*x^2)*log(b^4*sqrt(x) - (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2
- 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6)*(-b^5/
(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70
*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7
+ a^9*d^8))^(3/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(625*b^4
*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4 + a^4*d^5
)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 +
70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*
d^7 + a^8*c^5*d^8))^(1/4)*log((b^6*c^10 - 6*a*b^5*c^9*d + 15*a^2*b^4*c^8*d^
2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5*d^5 + a^6*c^4*d^6
)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d
^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5
*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 -
8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3*c^3*d - 75*a*b^2*c^2*d^2 +
 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
)*x^2)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*
b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d
^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(1/4)*log(-(b^6*c^10 - 6*a*b^5*c^9*d +
 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a^5*b*c^5
*d^5 + a^6*c^4*d^6)*(-(625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*
d^3 - 20*a^3*b*c*d^4 + a^4*d^5)/(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^1
1*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*
a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8))^(3/4) - (125*b^3*c^3*d -
75*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 - a^3*d^4)*sqrt(x)))/(b*c^3 - a*c^2*d + (
b*c^2*d - a*c*d^2)*x^2)
```

giac [A] time = 1.28, size = 701, normalized size = 1.31

$$-\frac{\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}bc-\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^{2}c^{4}d^{2}-2\,\sqrt{2}\,abc^{3}d^{3}+\sqrt{2}\,a^{2}c^{2}d^{4}\right)}-\frac{\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}bc-\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^{2}c^{4}d^{2}-2\,\sqrt{2}\,abc^{3}d^{3}+\sqrt{2}\,a^{2}c^{2}d^{4}\right)}+\frac{\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}bc-\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^{2}c^{4}d^{2}-2\,\sqrt{2}\,abc^{3}d^{3}+\sqrt{2}\,a^{2}c^{2}d^{4}\right)}-\frac{\left(5\,\left(cd^{3}\right)^{\frac{3}{4}}bc-\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^{2}c^{4}d^{2}-2\,\sqrt{2}\,abc^{3}d^{3}+\sqrt{2}\,a^{2}c^{2}d^{4}\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^{3}c^{2}-2\,\sqrt{2}\,a^{2}b^{2}cd+\sqrt{2}\,a^{3}bd^{2}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^{3}c^{2}-2\,\sqrt{2}\,a^{2}b^{2}cd+\sqrt{2}\,a^{3}bd^{2}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^{3}c^{2}-2\,\sqrt{2}\,a^{2}b^{2}cd+\sqrt{2}\,a^{3}bd^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^{3}c^{2}-2\,\sqrt{2}\,a^{2}b^{2}cd+\sqrt{2}\,a^{3}bd^{2}\right)}-\frac{dx^{\frac{3}{2}}}{2\left(bc^{2}-acd\right)\left(dx^{2}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*
c^3*d^3 + sqrt(2)*a^2*c^2*d^4) - 1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqr
t(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) + 1/8*(5*(c
*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d
^4) - 1/8*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c
/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + s
qrt(2)*a^2*c^2*d^4) + (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
```

```
 + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqr
t(2)*a^3*b*d^2) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2
)*a^3*b*d^2) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) + 1/
2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*
a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) - 1/2*d*x^(3/2)/((b*
c^2 - a*c*d)*(d*x^2 + c))
```

maple [A]   time = 0.02, size = 533, normalized size = 0.99

$$\frac{a d^{2} x^{\frac{3}{2}}}{2 (ad-bc)^{2} \left(d x^{2}+c\right) c}-\frac{b d x^{\frac{3}{2}}}{2 (ad-bc)^{2} \left(d x^{2}+c\right)}+\frac{\sqrt{2}\, a d \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 (ad-bc)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} c}+\frac{\sqrt{2}\, a d \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 (ad-bc)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} c}+\frac{\sqrt{2}\, a d \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 (ad-bc)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}} c}+\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 (ad-bc)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 (ad-bc)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{5 \sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 (ad-bc)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{5 \sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 (ad-bc)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\, b \ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 (ad-bc)^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{5 \sqrt{2}\, b \ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 (ad-bc)^{2} \left(\frac{c}{d}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/4*b/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/
b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*b/(a*d-b*c)^2/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b/(a*d-b*c)^2/(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/2*d^2/(a*d-b*c)^
2/c*x^(3/2)/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*b+1/8*d/(a*d-b*
c)^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-5/8/(a*d
-b*c)^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+1/8*d/(
a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-5/
8/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b+1
/16*d/(a*d-b*c)^2/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(
c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a-5/16/(a*d-b*c)^2
/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d
)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b
```

maxima [A]   time = 2.53, size = 450, normalized size = 0.84

$$\frac{b^{2} \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{4 \left(b^{2} c^{2}-2 a b c d+a^{2} d^{2}\right)}-\frac{d x^{\frac{3}{2}}}{2 \left(b c^{3}-a c^{2} d+\left(b c^{2} d-a c d^{2}\right) x^{2}\right)}-\frac{\left(5 b c d-a d^{2}\right) \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}+\frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}-\frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{16 \left(b^{2} c^{3}-2 a b c^{2} d+a^{2} c d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
```

```
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*d*x^(3/2)/(b*c^3 - a*c^2*d + (b*c^2*d -
 a*c*d^2)*x^2) - 1/16*(5*b*c*d - a*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c
^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)
```

mupad [B] time = 2.39, size = 19453, normalized size = 36.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] 2*atan(((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*
c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5
+ 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b
^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 11
20*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*
c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d
^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^
9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*
c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^
7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c
^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*
a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^
8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^
5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7
))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*
d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12
*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*
b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a
^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*
b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 -
6*a*b^5*c^7*d)) - (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*
d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 + a^6*c^2*d^6 -
 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4
*d^4 - 6*a*b^5*c^7*d)) - (-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7
*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896
*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16
*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4
*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2
```

```
*d^6 - 128*a^8*b*c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 -
 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 +
41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 +
 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^1
2 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^
2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*
b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) - (x^(1/2)*(-b^5/(16*a^9*
d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*
c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6
- 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 327
68*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7
+ 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d
^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c
^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6
*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a
^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c
^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^
8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)))/((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8
- 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*
b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))
^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*
c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5
+ 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*
a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3
*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a
^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 9632
0*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/
(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*
c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/
2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d
^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448
*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13
*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a
^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 7132
16*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 -
137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15
))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b
^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*1i - (x^(1/2)*(4*a^4*b^9*
c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^
10*c^2*d^7)*1i)/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d
^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (-b^5/(16*
a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*
b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*
d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b
```

```
^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4
 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(3/4)*(((3
2*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^
15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*
b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*
a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*
a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^
5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 -
7*a*b^6*c^8*d) - (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^
7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 89
6*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b
^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b
^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*
a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344
576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 -
 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2
*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*1i
 + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b
^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7)*1i)/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3
*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b
^5*c^7*d)) - (5*a^4*b^9*d^8 - 625*a*b^12*c^3*d^5 - 75*a^3*b^10*c*d^7 + 375*
a^2*b^11*c^2*d^6)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7
*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b
^6*c^8*d))))*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*
b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*
d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4) - atan(((-b^5/(16*a^9*d
^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c
^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 -
 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^
7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 89
6*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(3/4)*((32*a^13
*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^1
1*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c
^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^
9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b
^6*c^2*d^14)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2
 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^
8*d) + (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*
a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*
c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d
^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d
^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c
^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^
8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12
*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d
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^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*1i - (x^(1/2
)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^
6 + 10*a^3*b^10*c^2*d^7)*1i)/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*
a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))
 - (-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d
^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448
*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*a*b^8*c^
8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^
5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7
))^(3/4)*((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 +
14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7
+ 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^
10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*
d^13 + 6432*a^11*b^6*c^2*d^14)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 2
1*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^
4*d^5 - 7*a*b^6*c^8*d) - (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^
2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*
d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*(
4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 1210
88*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 -
 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^
11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^
3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5
+ 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^
7*d))*1i + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 1
00*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7)*1i)/(b^6*c^8 + a^6*c^2*d^6 - 6*a
^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4
 - 6*a*b^5*c^7*d)))/((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d +
 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6
*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9
*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5
*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6
 - 128*a^8*b*c*d^7))^(3/4)*((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*
a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280
*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 2199
68*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 3
2000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)/(b^7*c^9 - a^7*c^2*d^7 + 7
*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d
^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-b^5/(16*a^9*d^8 + 16*
a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 +
 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8
*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^
15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*
a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584
704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 +
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 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6
- 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^
4*d^4 - 6*a*b^5*c^7*d)) - (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 -
a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 + a^6*c
^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4
*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2
*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d
^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((
-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 -
 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7
*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(3/4)*((32*a^13*b^4*d^16 - 2048*a*b^16*c^1
2*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10
*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^
7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*
c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)/(b^7*c^9 - a^7
*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a
^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) - (x^(1/2)*(-b^5/(16*a
^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b
^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d
^6 - 128*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 -
32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d
^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^
7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^
7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 +
a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 1
5*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^1
2*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6
*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*
d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (5*a^4*b^9*d^8 - 625*a*b^12*c^
3*d^5 - 75*a^3*b^10*c*d^7 + 375*a^2*b^11*c^2*d^6)/(b^7*c^9 - a^7*c^2*d^7 +
7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*
d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d)))*(-b^5/(16*a^9*d^8 + 16*a*b^8*c^
8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^
5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7
))^(1/4)*2i - atan(((((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b
^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b
^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7
*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a
^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b
*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 2
1*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-(a^4*d^5 + 625*b^4*c^4*d -
500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 +
4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a
^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*
a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^
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13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136
*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 71
3216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12
- 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^
15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3
*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^
4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c
^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 22
9376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 1
14688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(3/4)*1i - (x^(1/2)*(4*a^4*b^9
*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b
^10*c^2*d^7)*1i)/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*
d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5
+ 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)
/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c
^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3
*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4) - (((32*a^13
*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^1
1*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c
^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^
9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b
^6*c^2*d^14)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2
- 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^
8*d) - (x^(1/2)*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^
2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b
*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b
^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*
c^12*d))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15
*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^
5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 58470
4*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 3
4048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 -
6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*
d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*
a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768
*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720
*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*
a*b^7*c^12*d))^(3/4)*1i + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 -
a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7)*1i)/(b^6*c^8 + a^
6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*
a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^
3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5
*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*
d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*
d^6 - 32768*a*b^7*c^12*d))^(1/4))/((((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d
```

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^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^
6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d
^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4
*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)/(b^7*c^9 - a^7*c^
2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*
b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-(a^4*d^5 + 6
25*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4
096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11
*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^
8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4)*(4096*a*b^16*c^
13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^
11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^
11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^
9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*
a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c
^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d
^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d
^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^
6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*
b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(3/4) - (x^(1/2
)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^
6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2
*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-
(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3
*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*
a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 22937
6*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4) + (
((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2
*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a
^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 1834
24*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 64
32*a^11*b^6*c^2*d^14)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5
*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7
*a*b^6*c^8*d) - (x^(1/2)*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 1
50*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32
768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286
720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 327
68*a*b^7*c^12*d))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768
*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 +
486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^1
0 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4
*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c
^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4
*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d
^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^
```

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8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3
 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6
 - 32768*a*b^7*c^12*d))^(3/4) + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*
d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 +
 a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 +
15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3
*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*
c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^
10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c
^7*d^6 - 32768*a*b^7*c^12*d))^(1/4) + (5*a^4*b^9*d^8 - 625*a*b^12*c^3*d^5 -
 75*a^3*b^10*c*d^7 + 375*a^2*b^11*c^2*d^6)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b
*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 2
1*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d)))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3
*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*
c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^
10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c
^7*d^6 - 32768*a*b^7*c^12*d))^(1/4)*2i + 2*atan((((((32*a^13*b^4*d^16 - 204
8*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a
^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784
*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96
320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i
)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^
4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(
1/2)*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 -
 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 +
114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4
- 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1
/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 +
 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*
d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c
^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b
^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3
*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b
^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2
*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*
d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^
9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*
d))^(3/4) - (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 +
100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5
*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 -
 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b
^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*
b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*
b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7
*c^12*d))^(1/4) - ((((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^
```

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5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^
13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*
b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^
10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^
6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4
- 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) - (x^(1/2)*(-(a^4*d^5 + 625*b^4*c^4*d
 - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13
 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 22937
6*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 1146
88*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4)*(4096*a*b^16*c^13*d^4 + 256
*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283
136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 +
 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^
12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2
*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*
a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4
*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^
8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 -
 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5
+ 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(3/4) + (x^(1/2)*(4*a^4*b^9
*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b
^10*c^2*d^7))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2
 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 6
25*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4
096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11
*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^
8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4))/(((((32*a^13*b
^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*
d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8
*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*
c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6
*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^
2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*
c^8*d) + (x^(1/2)*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*
b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7
*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4
*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^
7*c^12*d))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^
15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*
a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584
704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 +
 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6
- 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^
4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 15
0*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 327
```

```
68*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 2867
20*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 3276
8*a*b^7*c^12*d))^(3/4)*1i - (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5
 - a^5*b^8*d^9 + 100*a^2*b^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7)*1i)/(b^6*c^8 +
a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 1
5*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*
c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c
^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^1
0*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^
7*d^6 - 32768*a*b^7*c^12*d))^(1/4) + ((((32*a^13*b^4*d^16 - 2048*a*b^16*c^1
2*d^4 - 704*a^12*b^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10
*d^6 + 41280*a^4*b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^
7*d^9 + 219968*a^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*
c^4*d^12 - 32000*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 -
 a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 +
35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) - (x^(1/2)*(-(a^4*
d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^3*b*c*
d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688*a^2*b
^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 229376*a^5
*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(1/4)*(4096*a*
b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768*a^2*b^15*c^12*d^5 + 121088*a^3*
b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 + 486656*a^5*b^12*c^9*d^8 - 661504
*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d^10 - 584704*a^8*b^9*c^6*d^11 + 34
4576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*c^4*d^13 + 34048*a^11*b^6*c^3*d^14
 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^
2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d))*(
-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2*d^3 - 20*a^
3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*d^7 + 114688
*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^9*d^4 - 2293
76*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*d))^(3/4)*1i
 + (x^(1/2)*(4*a^4*b^9*c*d^8 - 625*a*b^12*c^4*d^5 - a^5*b^8*d^9 + 100*a^2*b
^11*c^3*d^6 + 10*a^3*b^10*c^2*d^7)*1i)/(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^3
*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a*b
^5*c^7*d))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2
*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*
d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^
9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*
d))^(1/4) - (5*a^4*b^9*d^8 - 625*a*b^12*c^3*d^5 - 75*a^3*b^10*c*d^7 + 375*a
^2*b^11*c^2*d^6)/(b^7*c^9 - a^7*c^2*d^7 + 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*
d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^
6*c^8*d)))*(-(a^4*d^5 + 625*b^4*c^4*d - 500*a*b^3*c^3*d^2 + 150*a^2*b^2*c^2
*d^3 - 20*a^3*b*c*d^4)/(4096*b^8*c^13 + 4096*a^8*c^5*d^8 - 32768*a^7*b*c^6*
d^7 + 114688*a^2*b^6*c^11*d^2 - 229376*a^3*b^5*c^10*d^3 + 286720*a^4*b^4*c^
9*d^4 - 229376*a^5*b^3*c^8*d^5 + 114688*a^6*b^2*c^7*d^6 - 32768*a*b^7*c^12*
d))^(1/4) + (d*x^(3/2))/(2*c*(c + d*x^2)*(a*d - b*c))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.458 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=536

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}+$$

Rubi [A] time = 0.53, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {466, 414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d\sqrt{x}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*Sqrt[x])/(2*c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^2) +
(d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*
Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)^2) + (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(
7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\mathrm{Subst}\left(\int \frac{4bc-3ad-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right)}{(bc-ad)^2} - \frac{(d(7bc-3ad))\,\mathrm{Su}}{2c(}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b^2\,\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\,(bc-ad)^2} + \frac{b^2\,\mathrm{Subst}\Big(\int \frac{\sqrt{a}+}{a+b}}{\sqrt{a}\,(bc-}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{a}\,(bc-ad)^2} + \frac{b^{3/2}\,\mathrm{Subst}}{}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\log\Big(\sqrt{a}}{2}$$

$$= -\frac{d\sqrt{x}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^2}$$

Mathematica [A] time = 0.42, size = 526, normalized size = 0.98

$$\frac{8a^{3/4}c^{3/4}d\sqrt{x}(ad-bc)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(3ad-7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-2\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(3ad-7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^2\right)(3ad-7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)-4\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+8\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{16a^{3/4}c^{7/4}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*Sqrt[x] - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c
+ d*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*
c^(7/4)*(c + d*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[
2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*
(c + d*x^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 4*
Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x] + Sqrt[2]*a^(3/4)*d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[2]*a^(3/4)
*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x])/(16*a^(3/4)*c^(7/4)*(b*c - a*d)^2*(c + d*x^2))
```

IntegrateAlgebraic [A] time = 1.13, size = 322, normalized size = 0.60

$$-\frac{b^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}(ad-bc)^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}(ad-bc)^2}+\frac{\left(7bcd^{3/4}-3ad^{7/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{\left(7bcd^{3/4}-3ad^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d\sqrt{x}}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)) - (b^(7/4)*ArcTan[(a^(1/4)/(Sq
rt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(3/4)*
(-(b*c) + a*d)^2) + ((7*b*c*d^(3/4) - 3*a*d^(7/4))*ArcTan[(Sqrt[c] - Sqrt[d
]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^2)
+ (b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/(Sqrt[2]*a^(3/4)*(-(b*c) + a*d)^2) - ((7*b*c*d^(3/4) - 3*a*d^(7/4))*ArcTa
nh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(
7/4)*(b*c - a*d)^2)
```

fricas [B] time = 55.34, size = 3310, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^4*c^4*d^3 - 41
16*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^
8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^
4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7
```

```
 + a^8*c^7*d^8))^(1/4)*arctan(((b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9*
d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*d
^6)*sqrt((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x + (b^4*c^8 - 4*a*b^3
*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^1
2*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*
a^7*b*c^8*d^7 + a^8*c^7*d^8)))*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2
646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^1
4*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*
a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/
4) + (7*b^7*c^12*d - 45*a*b^6*c^11*d^2 + 123*a^2*b^5*c^10*d^3 - 185*a^3*b^4
*c^9*d^4 + 165*a^4*b^3*c^8*d^5 - 87*a^5*b^2*c^7*d^6 + 25*a^6*b*c^6*d^7 - 3*
a^7*c^5*d^8)*sqrt(x)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^
2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a
^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^
10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4))/(2401*
b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 +
 81*a^4*d^7)) + 16*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^
2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b
^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d
- a*c*d^2)*x^2)*arctan(((a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2
 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7
/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 +
 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*
d^7 + a^11*d^8))^(3/4)*sqrt(b^4*x + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*
b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c
^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^
8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))) - (a^2*b^
8*c^6 - 6*a^3*b^7*c^5*d + 15*a^4*b^6*c^4*d^2 - 20*a^5*b^5*c^3*d^3 + 15*a^6*
b^4*c^2*d^4 - 6*a^7*b^3*c*d^5 + a^8*b^2*d^6)*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7
*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*
a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(3/4)*sq
rt(x))/b^7) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 -
 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*
c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a
*c*d^2)*x^2)*log(b^2*sqrt(x) + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^
5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^
5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a
^2*b*c*d + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*
c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28
*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*
c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*
d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b
^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*
```

```
c^2 - 2*a^2*b*c*d + a^3*d^2)) + (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)
*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*
b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56
*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*
c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*sq
rt(x) + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*
b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^1
5 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4
*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^
8*c^7*d^8))^(1/4)) - (b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*(-(2401*b^
4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 8
1*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^
12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8
*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*sqrt(x) - (b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7
*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 -
 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))
^(1/4)) - 4*d*sqrt(x))/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)
```

giac [A] time = 0.97, size = 673, normalized size = 1.26

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^{2}c^{2}-2\sqrt{2}\,a^{2}bcd+\sqrt{2}\,a^{3}d^{2}}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,ab^{2}c^{2}-2\sqrt{2}\,a^{2}bcd+\sqrt{2}\,a^{3}d^{2}}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^{2}c^{2}-2\sqrt{2}\,a^{2}bcd+\sqrt{2}\,a^{3}d^{2}\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,ab^{2}c^{2}-2\sqrt{2}\,a^{2}bcd+\sqrt{2}\,a^{3}d^{2}\right)}-\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^{2}c^{4}-2\sqrt{2}\,abc^{3}d+\sqrt{2}\,a^{2}c^{2}d^{2}\right)}-\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^{2}c^{4}-2\sqrt{2}\,abc^{3}d+\sqrt{2}\,a^{2}c^{2}d^{2}\right)}-\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^{2}c^{4}-2\sqrt{2}\,abc^{3}d+\sqrt{2}\,a^{2}c^{2}d^{2}\right)}+\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^{2}c^{4}-2\sqrt{2}\,abc^{3}d+\sqrt{2}\,a^{2}c^{2}d^{2}\right)}-\frac{d\sqrt{x}}{2\left(bc^{2}-acd\right)\left(dx^{2}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + (a*b^3
)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4)
)/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b^3)
^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^
2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/2*(a*b^3)^(1/4)*b*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*
b*c*d + sqrt(2)*a^3*d^2) - 1/4*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*
arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*
b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/4*(7*(c*d^3)^(1/4)
*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sq
rt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^
2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqr
t(2)*a^2*c^2*d^2) + 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(-sq
rt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b
*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/2*d*sqrt(x)/((b*c^2 - a*c*d)*(d*x^2 + c))
```

maple [A] time = 0.02, size = 566, normalized size = 1.06

$$\frac{a\,d^{2}\sqrt{x}}{2\,(ad-bc)^{2}\left(d\,x^{2}+c\right)c}-\frac{bd\sqrt{x}}{2\,(ad-bc)^{2}\left(d\,x^{2}+c\right)}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{2}\,c^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{2}\,c^{2}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\,(ad-bc)^{2}\,c^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\,(ad-bc)^{2}\,a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\,(ad-bc)^{2}\,a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\,(ad-bc)^{2}\,a}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{2}\,c}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{2}\,c}-\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bd\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\,(ad-bc)^{2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*b^2/(a*d-b*c
)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^2/(a*
d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/2*d^
2/(a*d-b*c)^2/c*x^(1/2)/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*b+3
/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/
2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a-7/16*d/(a*d-
b*c)^2/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))
/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b+3/8*d^2/(a*d-b*c)^2/c^2*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a-7/8*d/(a*d-b*c)^2/
c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b+3/8*d^2/(a*d-
b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a-7/8*
d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b
```

maxima [A] time = 2.42, size = 489, normalized size = 0.91

$$-\frac{d\sqrt{x}}{2\left(bc^{3}-ac^{2}d+\left(bc^{2}d-acd^{2}\right)x^{2}\right)}+\frac{\frac{2\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,b^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}-\frac{\frac{2\sqrt{2}\left(7\,bcd-3\,ad^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\left(7\,bcd-3\,ad^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(7\,bcd-3\,ad^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(7\,bcd-3\,ad^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*d*sqrt(x)/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/4*(2*sqrt(2)
*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(-1
/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(-sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b
*c*d + a^2*d^2) - 1/16*(2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)
*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c
*d - 3*a*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)
```

mupad [B] time = 2.73, size = 21987, normalized size = 41.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] atan(((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6
*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5
 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 + 16*
a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3
 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a
^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c
^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 -
128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^
4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^
(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5
*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3
*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*
d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*
d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8
*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*
c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^
2 - 3*a*b^2*c^6*d) + (x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 +
 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11
*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^
10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*
a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 23
04*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b
^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (
x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^
3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c
^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a
*b^5*c^9*d))*1i - (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d +
 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8
*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^
11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6
*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2
*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2
145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 -
a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11*d^8 + 16*a^
3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 +
 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^1
0*b*c*d^7))^(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7
```

```
*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896
*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a^
2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^
4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728
*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 286
72*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*
a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^1
6*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a
^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 15
11936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^
13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c
^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^
5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*
c^9*d))) - (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*
d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6
 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c
^6*d^4 - 6*a*b^5*c^9*d))*1i)/((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4
*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d
^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(
(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d
^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448
*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11
*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))
/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11
*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b
^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d
^6 - 128*a^10*b*c*d^7))^(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128
*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c
^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/
4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^
6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*
d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^
6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*
c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x^(1/2)*(4096*b^17*c^15*d^4 -
 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^
7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c
^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^
9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 2918
4*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*
a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^
4 - 6*a*b^5*c^9*d))) + (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788
*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 +
 a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 +
15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8
- 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*
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b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7)
)^(1/4)*((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*
b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*
d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 +
 448*b^11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*
c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7
/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 -
896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*
b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*
c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*
a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*
d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^1
2*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6
*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a
^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c
^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x^(1/2)*(4096*b^17*c
^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^1
4*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a
^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1
114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^
15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4
*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b
^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d
^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b
^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c
^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*(-b^7/(16*a^11*d^8 + 16*a^3
*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 +
1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10
*b*c*d^7))^(1/4)*2i + 2*atan(((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4
*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d
^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(
(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d
^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448
*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11
*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))
/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (-b^7/(16*a^11
*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b
^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d
^6 - 128*a^10*b*c*d^7))^(3/4)*(((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a
^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4
*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)
*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6
+ 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^
9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*
d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14)*2i)/(b^3*c^7 - a^3
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*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x^(1/2)*(4096*b^17*c^15*d^4
- 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d
^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*
c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a
^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 291
84*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6
*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d
^4 - 6*a*b^5*c^9*d))*1i)*1i + (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7
 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6
*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7
*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (-b^7/(16*a^11*d^8 + 16*a^3*b
^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 11
20*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b
*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 4
48*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b
^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7
*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a
^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)
- (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6
*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 4
48*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(3/4)*(((-b^7/(16*a^11*d^8 + 16*a^3
*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 +
1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10
*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^
3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 22937
6*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114
688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14)*2i
)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x^(1/2)*(409
6*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832
*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 -
907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*
d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^
6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 +
 a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 +
15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*1i)*1i - (x^(1/2)*(81*a^4*b^9*d^11 + 3
185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^1
1*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2
- 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))/((-b^7/(16*a^1
1*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*
b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*
d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a
^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4
*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)
*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^
8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 -
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 3*a*b^2*c^6*d) - (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d +
 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8
*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(3/4)*(((-b^7/(16*a
^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^
6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^
2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^1
4*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*
c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^
7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*
b^4*c^4*d^14)*2i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)
 + (x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c
^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*
b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580
032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14
+ 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^
17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^
3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*1i)*1i + (x^(1/2)*(81*
a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^1
0 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*
a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))
*1i + (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6
*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5
 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 + 16*
a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3
 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a
^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c
^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 -
128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^
4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^
(3/4)*(((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b
^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d
^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^
5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^
7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d
^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^
5*d^13 + 3072*a^11*b^4*c^4*d^14)*2i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d
^2 - 3*a*b^2*c^6*d) - (x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5
+ 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^1
1*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b
^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432
*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2
304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*
b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*1i)
*1i - (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 -
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 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*
a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^
4 - 6*a*b^5*c^9*d))*1i))*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*
c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 -
896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4) - atan
((((((x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15
*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^
5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 15
80032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^1
4 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*
d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*
a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d) - (2*(-(81*a^4*d^7 +
2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*
d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b
^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^
5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*(28672
*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112
*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329
728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 +
 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*
a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 40
96*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3
*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*
a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(3/4) - (2*(81*a^4*b^7*d^10 + 448*b^
11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8
))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7
 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b
*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^
2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376
*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4) + (
x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^
3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c
^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a
*b^5*c^9*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a
^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768
*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720
*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 3276
8*a*b^7*c^14*d))^(1/4)*1i + ((((x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*
c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4
*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511
936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13
 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3
*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5
+ 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^
```

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9*d) + (2*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*
b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^
7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^
4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a
*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848
*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 22
9376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 +
114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14)
)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7
+ 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*
c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2
*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*
a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(3/4) + (2
*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*
d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a
*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a
^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768
*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720
*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 3276
8*a*b^7*c^14*d))^(1/4) + (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 47
88*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10
 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3
+ 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15
+ 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376
*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114
688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*1i)/(((((x^(1/2)*(4096*b^1
7*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*
b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 90726
4*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12
+ 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4
*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*
c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^
4*b^2*c^6*d^4 - 6*a*b^5*c^9*d) - (2*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116
*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4
096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^
3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688
*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 40
96*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28
672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 -
253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13
 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*
b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^
2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*
a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*
```

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a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768
*a*b^7*c^14*d))^(3/4) - (2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^1
0*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d
^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15
+ 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376
*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114
688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4) + (x^(1/2)*(81*a^4*b^9*d^1
1 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^
2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8
*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*(-(81*a^4*
d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^
3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688
*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229
376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)
- ((((x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15
*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^
5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 15
80032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^1
4 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*
d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*
a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d) + (2*(-(81*a^4*d^7 +
2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*
d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b
^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^
5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*(28672
*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112
*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329
728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 +
 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*
a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 40
96*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3
*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*
a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(3/4) + (2*(81*a^4*b^7*d^10 + 448*b^
11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8
))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7
 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b
*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^
2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376
*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4) + (
x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^
3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c
^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a
*b^5*c^9*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a
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^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768
*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720
*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 3276
8*a*b^7*c^14*d))^(1/4)))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*
d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7
*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*
d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^
9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*2i - 2*atan((((((x^(1/2)*(4096*b^17*c^15
*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c
^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*
b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114
624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15
- 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d) - ((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c
^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*
c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^
12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2
*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^1
4*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*
b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a
^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*
a^11*b^4*c^4*d^14)*2i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c
^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2
*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b
*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b
^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^
7*c^14*d))^(3/4)*1i + (2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*
c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3
 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 411
6*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 +
4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a
^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 11468
8*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*1i - (x^(1/2)*(81*a^4*b^9*d^
11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a
^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^
8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*(-(81*a^4
*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a
^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 11468
8*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 22
9376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)
 + ((((x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^1
5*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a
^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1
580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^
```

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14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2
*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20
*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d) + ((-(81*a^4*d^7 + 2
401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d
^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^
6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5
*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*(28672*
a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*
a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 3297
28*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 2
8672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14)*2i)/(b^3*c^7 - a^3*c^4*d^3
 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 411
6*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 +
4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a
^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 11468
8*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(3/4)*1i - (2*(81*a^4*b^7*d^10 + 4
48*b^11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^
2*d^8))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^
4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*
a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 1146
88*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 2
29376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4
)*1i - (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8
- 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6
*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d
^4 - 6*a*b^5*c^9*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
+ 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8
 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3
+ 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^
6 - 32768*a*b^7*c^14*d))^(1/4))/(((((x^(1/2)*(4096*b^17*c^15*d^4 - 32768*a*
b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 17561
6*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^9*d^10 +
 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b^9*c^7*d^12 + 1114624*a^9*b^8*c^6
*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*a^11*b^6*c^4*d^15 - 29184*a^12*b^
5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5
*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b
^5*c^9*d) - ((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a
^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768
*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720
*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 3276
8*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78
848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 -
 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11
 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^
14)*2i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^
```

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4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*
a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 1146
88*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 2
29376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(3/4
)*1i + (2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*b^10*c^3*d^7 - 675*a
^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*
d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^
8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3
 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d
^6 - 32768*a*b^7*c^14*d))^(1/4)*1i - (x^(1/2)*(81*a^4*b^9*d^11 + 3185*b^13*
c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9
))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*
b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))*(-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(40
96*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*
d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^
10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*1i - ((((x^(1/
2)*(4096*b^17*c^15*d^4 - 32768*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 -
 216832*a^3*b^14*c^12*d^7 + 175616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10
*d^9 - 907264*a^6*b^11*c^9*d^10 + 1511936*a^7*b^10*c^8*d^11 - 1580032*a^8*b
^9*c^7*d^12 + 1114624*a^9*b^8*c^6*d^13 - 530432*a^10*b^7*c^5*d^14 + 163072*
a^11*b^6*c^4*d^15 - 29184*a^12*b^5*c^3*d^16 + 2304*a^13*b^4*c^2*d^17))/(b^6
*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7
*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d) + ((-(81*a^4*d^7 + 2401*b^4*c^4*
d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^
8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 -
 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^
5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^1
3*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^1
1*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c
^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^
5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14)*2i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c
^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*
d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7
*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*
d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^
9*d^6 - 32768*a*b^7*c^14*d))^(3/4)*1i - (2*(81*a^4*b^7*d^10 + 448*b^11*c^4*
d^6 - 2145*a*b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3
*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*(-(81*a^4*d^7 + 2401
*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)
/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c
^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^
3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*b^7*c^14*d))^(1/4)*1i - (x^(1
/2)*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^
```

```
10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d
^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5
*c^9*d))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*d^8 - 32768*a^7
*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d^3 + 286720*a^4
*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9*d^6 - 32768*a*
b^7*c^14*d))^(1/4)*1i))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d
^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(4096*b^8*c^15 + 4096*a^8*c^7*
d^8 - 32768*a^7*b*c^8*d^7 + 114688*a^2*b^6*c^13*d^2 - 229376*a^3*b^5*c^12*d
^3 + 286720*a^4*b^4*c^11*d^4 - 229376*a^5*b^3*c^10*d^5 + 114688*a^6*b^2*c^9
*d^6 - 32768*a*b^7*c^14*d))^(1/4) + (d*x^(1/2))/(2*c*(c + d*x^2)*(a*d - b*c
))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.459 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=570

$$-\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}-$$

Rubi [A] time = 0.75, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}-\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^2}+\frac{d^{5/4}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^2}-\frac{4bc-5ad}{2ac^2\sqrt{x}\,(bc-ad)}-\frac{d}{2c\sqrt{x}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 5*a*d)/(2*a*c^2*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*Sqrt[x]
*(c + d*x^2)) + (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sq
rt[2]*a^(5/4)*(b*c - a*d)^2) - (b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^2) - (d^(5/4)*(9*b*c - 5*a*d)*ArcT
an[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2
) + (d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^2) + (b^(9/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
5/4)*(b*c - a*d)^2) + (d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4
)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) - (d^(5/4
)*(9*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-5ad-5bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(4b^2c^2+4abcd-5a^2d^2+\right.}{\left(a+bx^4\right)\left(c+\right.}\right.}{2ac^2(bc-}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int\left(\frac{4b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{ad^2}{(-b}\right.\right.}{2ac^2(bc-}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x,\right.}{a(bc-ad)^2}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} + \frac{b^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x,\right.}{a(bc-ad)^2}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\right.}{2a(bc-ad)^2}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} - \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,a^{5/4}(bc-ad)^2}$$

$$= -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)\sqrt{x}\left(c+dx^2\right)} + \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^2} -$$

Mathematica [A] time = 2.17, size = 540, normalized size = 0.95

$$\frac{1}{16}\left(-\frac{4\sqrt{2}b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^2}+\frac{4\sqrt{2}b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}(bc-ad)^2}+\frac{8\sqrt{2}b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}(bc-ad)^2}-\frac{8\sqrt{2}b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}(bc-ad)^2}+\frac{\sqrt{2}d^{5/4}(9bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^2}+\frac{\sqrt{2}d^{5/4}(5ad-9bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{9/4}(bc-ad)^2}+\frac{2\sqrt{2}d^{5/4}(5ad-9bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}(bc-ad)^2}+\frac{2\sqrt{2}d^{5/4}(9bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}(bc-ad)^2}+\frac{8d^2x^{3/2}}{c^2\left(c+dx^2\right)(bc-ad)}-\frac{32}{ac^2\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (-32/(a*c^2*Sqrt[x]) + (8*d^2*x^(3/2))/(c^2*(b*c - a*d)*(c + d*x^2)) + (8*S
qrt[2]*b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(b*c
 - a*d)^2) - (8*Sqrt[2]*b^(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(a^(5/4)*(b*c - a*d)^2) + (2*Sqrt[2]*d^(5/4)*(-9*b*c + 5*a*d)*ArcTan[1
- (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^2) + (2*Sqrt[2]*
d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(
9/4)*(b*c - a*d)^2) - (4*Sqrt[2]*b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d)^2) + (4*Sqrt[2]*b^(9/4)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(b*c - a*d
)^2) + (Sqrt[2]*d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5/4)*(-9*b*c
 + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9
/4)*(b*c - a*d)^2))/16
```

IntegrateAlgebraic [A] time = 1.32, size = 352, normalized size = 0.62

$$\frac{b^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{5/4}(ad-bc)^2}+\frac{b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{5/4}(ad-bc)^2}-\frac{\left(9bcd^{5/4}-5ad^{9/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^2}-\frac{\left(9bcd^{5/4}-5ad^{9/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^2}+\frac{-4acd-5ad^2x^2+4bc^2+4bcdx^2}{2ac^2\sqrt{x}\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (4*b*c^2 - 4*a*c*d + 4*b*c*d*x^2 - 5*a*d^2*x^2)/(2*a*c^2*(-(b*c) + a*d)*Sqr
t[x]*(c + d*x^2)) + (b^(9/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x
)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(5/4)*(-(b*c) + a*d)^2) - ((9*b*c
*d^(5/4) - 5*a*d^(9/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2) + (b^(9/4)*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(5/4)*(-(b*c)
+ a*d)^2) - ((9*b*c*d^(5/4) - 5*a*d^(9/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^2)
```

fricas [B] time = 88.69, size = 3630, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^
4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8
+ 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^
5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^
6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4)*arctan(((b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2)*sqrt((531441*b^6*c^6*d^8 - 1771470*a*b^5*c^5*d^9 + 2460375*a^
2*b^4*c^4*d^10 - 1822500*a^3*b^3*c^3*d^11 + 759375*a^4*b^2*c^2*d^12 - 16875
```

```
0*a^5*b*c*d^13 + 15625*a^6*d^14)*x - (6561*b^8*c^13*d^5 - 40824*a*b^7*c^12*
d^6 + 109836*a^2*b^6*c^11*d^7 - 166824*a^3*b^5*c^10*d^8 + 156406*a^4*b^4*c^
9*d^9 - 92680*a^5*b^3*c^8*d^10 + 33900*a^6*b^2*c^7*d^11 - 7000*a^7*b*c^6*d^
12 + 625*a^8*c^5*d^13)*sqrt(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 1215
0*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^1
6*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*
a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8)))*
(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^
3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 -
 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b
^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(1/4) + (729*b^5*c^7*d^4 - 2
673*a*b^4*c^6*d^5 + 3834*a^2*b^3*c^5*d^6 - 2690*a^3*b^2*c^4*d^7 + 925*a^4*b
*c^3*d^8 - 125*a^5*c^2*d^9)*sqrt(x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d
^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*
a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*
d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^
9*d^8))^(1/4))/(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*
d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)) + 16*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*
c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a
^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(
(a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*arctan((sqrt(b^14*
x - (a^3*b^13*c^4 - 4*a^4*b^12*c^3*d + 6*a^5*b^11*c^2*d^2 - 4*a^6*b^10*c*d^
3 + a^7*b^9*d^4)*sqrt(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*
d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^
11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8)))*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^
7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56
*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)
*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - (a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7
*d^2)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^
5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6
- 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*sqrt(x))/b^9) - 4*(-b^9/(a^5*b^8*c^8 -
8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*
d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8
))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b^7*
sqrt(x) + (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*
c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8
 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c
^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*
d^8))^(3/4)) + 4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2
- 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b
^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x
^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b^7*sqrt(x) - (a^4*b^6*c^6 - 6*a^5*b^5*c^
5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*
b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d
^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^1
```

```
1*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(3/4)) - ((a*b*c^3*d - a^2*c^2*
d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d
^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*
a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*
d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^
9*d^8))^(1/4)*log((b^6*c^13 - 6*a*b^5*c^12*d + 15*a^2*b^4*c^11*d^2 - 20*a^3
*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6561
*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d
^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3
*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11
*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4) - (729*b^3*c^3*d^4 - 1215*a*b
^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt(x)) + ((a*b*c^3*d - a^2*c^
2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*(-(6561*b^4*c^4*d^5 - 14580*a*b^3*c^3
*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8 + 625*a^4*d^9)/(b^8*c^17 -
8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^1
3*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*
c^9*d^8))^(1/4)*log(-(b^6*c^13 - 6*a*b^5*c^12*d + 15*a^2*b^4*c^11*d^2 - 20*
a^3*b^3*c^10*d^3 + 15*a^4*b^2*c^9*d^4 - 6*a^5*b*c^8*d^5 + a^6*c^7*d^6)*(-(6
561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*
c*d^8 + 625*a^4*d^9)/(b^8*c^17 - 8*a*b^7*c^16*d + 28*a^2*b^6*c^15*d^2 - 56*
a^3*b^5*c^14*d^3 + 70*a^4*b^4*c^13*d^4 - 56*a^5*b^3*c^12*d^5 + 28*a^6*b^2*c
^11*d^6 - 8*a^7*b*c^10*d^7 + a^8*c^9*d^8))^(3/4) - (729*b^3*c^3*d^4 - 1215*
a*b^2*c^2*d^5 + 675*a^2*b*c*d^6 - 125*a^3*d^7)*sqrt(x)) - 4*(4*b*c^2 - 4*a*
c*d + (4*b*c*d - 5*a*d^2)*x^2)*sqrt(x))/((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a
*b*c^4 - a^2*c^3*d)*x)
```

giac [A] time = 1.17, size = 725, normalized size = 1.27

$$\frac{\left(9 \left(cd^{3}\right)^{\frac{3}{4}} bc - 5 \left(cd^{3}\right)^{\frac{3}{4}} ad\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2} b^{2}c^{5}d - 2\sqrt{2} abc^{4}d^{2} + \sqrt{2} a^{2}c^{3}d^{3}\right)} + \frac{\left(9 \left(cd^{3}\right)^{\frac{3}{4}} bc - 5 \left(cd^{3}\right)^{\frac{3}{4}} ad\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2} b^{2}c^{5}d - 2\sqrt{2} abc^{4}d^{2} + \sqrt{2} a^{2}c^{3}d^{3}\right)} - \frac{\left(9 \left(cd^{3}\right)^{\frac{3}{4}} bc - 5 \left(cd^{3}\right)^{\frac{3}{4}} ad\right) \log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2} b^{2}c^{5}d - 2\sqrt{2} abc^{4}d^{2} + \sqrt{2} a^{2}c^{3}d^{3}\right)} + \frac{\left(9 \left(cd^{3}\right)^{\frac{3}{4}} bc - 5 \left(cd^{3}\right)^{\frac{3}{4}} ad\right) \log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2} b^{2}c^{5}d - 2\sqrt{2} abc^{4}d^{2} + \sqrt{2} a^{2}c^{3}d^{3}\right)} - \frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} + 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2} a^{2}b^{2}c^{2} - 2\sqrt{2} a^{3}bcd + \sqrt{2} a^{4}d^{2}} - \frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2} a^{2}b^{2}c^{2} - 2\sqrt{2} a^{3}bcd + \sqrt{2} a^{4}d^{2}} + \frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2} a^{2}b^{2}c^{2} - 2\sqrt{2} a^{3}bcd + \sqrt{2} a^{4}d^{2}\right)} - \frac{\left(ab^{3}\right)^{\frac{3}{4}} \log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2} a^{2}b^{2}c^{2} - 2\sqrt{2} a^{3}bcd + \sqrt{2} a^{4}d^{2}\right)} - \frac{4\, bcdx^{2} - 5\, ad^{2}x^{2} + 4\, bc^{2} - 4\, acd}{2\left(abc^{3} - a^{2}c^{2}d\right)\left(dx^{\frac{5}{2}} + c\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)
*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c
^4*d^2 + sqrt(2)*a^2*c^3*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*
a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) - 1/8*(9*(c*
d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^
3) + 1/8*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sq
rt(2)*a^2*c^3*d^3) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
+ 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt
```

```
(2)*a^4*d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a
^4*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(a*b^
3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2
*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(4*b*c*d*x^2 - 5*a*d^2*
x^2 + 4*b*c^2 - 4*a*c*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^(5/2) + c*sqrt(x)))
```

maple [A] time = 0.02, size = 582, normalized size = 1.02

$$-\frac{a\,d^{3}x^{\frac{3}{2}}}{2\,(ad-bc)^{2}\left(d\,x^{2}+c\right)c^{2}}+\frac{b\,d^{2}x^{\frac{3}{2}}}{2\,(ad-bc)^{2}\left(d\,x^{2}+c\right)c}-\frac{5\sqrt{2}\;a\,d^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{5\sqrt{2}\;a\,d^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{5\sqrt{2}\;a\,d^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\,(ad-bc)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{\sqrt{2}\;b^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\,(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\;b^{2}\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\,(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\;b^{2}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\,(ad-bc)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{9\sqrt{2}\;bd\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c}+\frac{9\sqrt{2}\;bd\arctan\left(\frac{\sqrt{2}\;\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c}+\frac{9\sqrt{2}\;bd\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\;\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\,(ad-bc)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{2}{a\,c^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/4*b^2/a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b^2/a/(a*d-
b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^2/a/
(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/2*d
^3/c^2/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2*x^(3/2)/(d*x^2
+c)*b-5/16*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*2^(1
/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-5/8*d
^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)-5/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)-1)+9/16*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^(1/4)*2
^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+9/
8*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)+9/8*d/c/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)-1)-2/a/c^2/x^(1/2)
```

maxima [A] time = 2.58, size = 494, normalized size = 0.87

$$-\frac{b^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(ab^{2}c^{2}-2a^{2}bcd+a^{3}d^{2}\right)}+\frac{\left(9bcd^{2}-5ad^{3}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{4}-2abc^{3}d+a^{2}c^{2}d^{2}\right)}-\frac{4bc^{2}-4acd+\left(4bcd-5ad^{2}\right)x^{2}}{2\left(\left(abc^{3}d-a^{2}c^{2}d^{2}\right)x^{\frac{5}{2}}+\left(abc^{4}-a^{2}c^{3}d\right)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
```

```
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/16*(9*b*c*d^2 - 5*a*d^3)*(2*sqrt(2)*
arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqr
t(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
 sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^4 - 2*a*b*c^3*
d + a^2*c^2*d^2) - 1/2*(4*b*c^2 - 4*a*c*d + (4*b*c*d - 5*a*d^2)*x^2)/((a*b*
c^3*d - a^2*c^2*d^2)*x^(5/2) + (a*b*c^4 - a^2*c^3*d)*sqrt(x))
```

mupad [B] time = 4.09, size = 21370, normalized size = 37.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] atan(((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6
*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^
5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b^9/(16*a^13*d^8 + 1
6*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d
^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 1
28*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*
a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^
4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1
/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 352321536
0*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^
21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38
*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12
 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2
241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405
664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108
580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160
*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*
c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25) - 1
6777216*a^11*b^25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13
*b^23*c^40*d^6 + 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38
*d^8 + 21783379968*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 4
8013246464*a^18*b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 2539546
21440*a^20*b^16*c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184
*a^22*b^14*c^31*d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^
24*b^12*c^29*d^17 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^
10*c^27*d^19 - 274688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25
*d^21 - 27505197056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 60
```

```
2931200*a^31*b^5*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25) - x^(1/2)*(323665
92*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c
^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12
- 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863
552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*
b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^
19 + 5120000*a^23*b^10*c^20*d^20))*1i + (-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8
 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9
*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d
^7))^(1/4)*((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a
^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*
c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(1
6*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896
*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b
^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316
480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b
^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39
*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11
+ 539177779200*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 158
9322448896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 258534
8014080*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 179266230
6816*a^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560
*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^
7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24
- 52428800*a^33*b^4*c^23*d^25) + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^
12*b^24*c^41*d^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^3
9*d^7 + 11995709440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 3
1592546304*a^17*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^11 + 10342419
6608*a^19*b^17*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 + 531641663488*
a^21*b^15*c^32*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23
*b^13*c^30*d^16 - 1138334629888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^1
1*c^28*d^18 - 566347431936*a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26
*d^20 - 101363744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 -
5174722560*a^30*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^3
2*b^4*c^21*d^25) - x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^2
2*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11
 + 4269711360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 916597
9648*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19
*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22
*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20))*1i)/((
-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^
2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448
*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b^9/(16*a^13*d^8 + 16*a^5*b
^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 11
20*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12
```

```
*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7
*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 -
 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*(33
554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*
b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40
*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 -
 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 9595
47703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 22410166
27200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 24056640307
20*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*
a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b
^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^
23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25) - 16777216
*a^11*b^25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13*b^23*c
^40*d^6 + 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38*d^8 +
21783379968*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 48013246
464*a^18*b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 253954621440*a
^20*b^16*c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184*a^22*b
^14*c^31*d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^24*b^12
*c^29*d^17 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^10*c^27
*d^19 - 274688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25*d^21 -
 27505197056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 602931200
*a^31*b^5*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25) - x^(1/2)*(32366592*a^12
*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^1
0 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 76643
86048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^1
8*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^
23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 51
20000*a^23*b^10*c^20*d^20)) - (-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6
*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d
^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)
*((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6
*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 +
448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8
 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c
^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6
 - 128*a^12*b*c*d^7))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b
^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d
^7 + 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188
659793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 53917777
9200*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896
*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^
24*b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*
b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*
c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^2
```

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2 - 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800
*a^33*b^4*c^23*d^25) + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^
41*d^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 11
995709440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 31592546304
*a^17*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^11 + 103424196608*a^19*
b^17*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 + 531641663488*a^21*b^15*
c^32*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23*b^13*c^30
*d^16 - 1138334629888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^11*c^28*d^1
8 - 566347431936*a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26*d^20 - 10
1363744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 - 5174722560
*a^30*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21
*d^25) - x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8
 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711
360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*
b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24
*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72
704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20)) - 29859840*a^11*
b^21*c^29*d^9 + 228925440*a^12*b^20*c^28*d^10 - 774144000*a^13*b^19*c^27*d^
11 + 1514700800*a^14*b^18*c^26*d^12 - 1888665600*a^15*b^17*c^25*d^13 + 1555
415040*a^16*b^16*c^24*d^14 - 845578240*a^17*b^15*c^23*d^15 + 292454400*a^18
*b^14*c^22*d^16 - 58368000*a^19*b^13*c^21*d^17 + 5120000*a^20*b^12*c^20*d^1
8))*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c
^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5
 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*2i + 2*atan(((-b^9/(16*a^
13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8
*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c
^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 12
8*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*
c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^
(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 44
8*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b
^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*(33554432*a^12
*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d
^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 1035
00742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 3138178252
80*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 959547703296*a^
21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*
b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^1
2*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c
^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^2
1 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 975175
680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25)*1i - 16777216*a^11*b^
25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13*b^23*c^40*d^6
 + 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38*d^8 + 21783379
968*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 48013246464*a^18
```

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*b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 253954621440*a^20*b^16
*c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184*a^22*b^14*c^31
*d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^24*b^12*c^29*d^
17 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^10*c^27*d^19 -
274688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25*d^21 - 2750519
7056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 602931200*a^31*b^
5*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25)*1i + x^(1/2)*(32366592*a^12*b^21
*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 1
422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7664386048
*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^1
5*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^
17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120000
*a^23*b^10*c^20*d^20)) + (-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*
c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 -
896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b
^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2
- 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a
^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16
*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^
3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 12
8*a^12*b*c*d^7))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c
^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 +
45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 18865979
3920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*
a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22
*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^
13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*
c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*
d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8
506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33
*b^4*c^23*d^25)*1i + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^41
*d^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 1199
5709440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 31592546304*a
^17*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^11 + 103424196608*a^19*b^
17*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 + 531641663488*a^21*b^15*c^
32*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23*b^13*c^30*d
^16 - 1138334629888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^11*c^28*d^18
- 566347431936*a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26*d^20 - 1013
63744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 - 5174722560*a
^30*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21*d
^25)*1i + x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^
8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 426971
1360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17
*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^2
4*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 7
```

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2704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20)))/((-b^9/(16*a^1
3*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*
b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^
2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128
*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c
^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(
3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448
*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^
3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*(33554432*a^12*
b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^
6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 10350
0742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 31381782528
0*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 959547703296*a^2
1*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b
^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^12
*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^
29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21
 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 9751756
80*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25)*1i - 16777216*a^11*b^2
5*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13*b^23*c^40*d^6 +
 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38*d^8 + 217833799
68*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 48013246464*a^18*
b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 253954621440*a^20*b^16*
c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184*a^22*b^14*c^31*
d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^24*b^12*c^29*d^1
7 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^10*c^27*d^19 - 2
74688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25*d^21 - 27505197
056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 602931200*a^31*b^5
*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25)*1i + x^(1/2)*(32366592*a^12*b^21*
c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 14
22057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7664386048*
a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^15
*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^1
7 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120000*
a^23*b^10*c^20*d^20))*1i - (-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^
7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4
- 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((
-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^
2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448
*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 +
16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*
d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 -
128*a^12*b*c*d^7))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24
*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7
+ 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659
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793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 53917777920
0*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^
22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*
b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^1
1*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^2
8*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 -
 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^
33*b^4*c^23*d^25)*1i + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^
41*d^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 11
995709440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 31592546304
*a^17*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^11 + 103424196608*a^19*
b^17*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 + 531641663488*a^21*b^15*
c^32*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23*b^13*c^30
*d^16 - 1138334629888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^11*c^28*d^1
8 - 566347431936*a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26*d^20 - 10
1363744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 - 5174722560
*a^30*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21
*d^25)*1i + x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*
d^8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269
711360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^
17*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c
^24*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 -
 72704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20))*1i + 29859840
*a^11*b^21*c^29*d^9 - 228925440*a^12*b^20*c^28*d^10 + 774144000*a^13*b^19*c
^27*d^11 - 1514700800*a^14*b^18*c^26*d^12 + 1888665600*a^15*b^17*c^25*d^13
- 1555415040*a^16*b^16*c^24*d^14 + 845578240*a^17*b^15*c^23*d^15 - 29245440
0*a^18*b^14*c^22*d^16 + 58368000*a^19*b^13*c^21*d^17 - 5120000*a^20*b^12*c^
20*d^18))*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7
*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^
3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4) - (2/(a*c) + (d*x^2
*(5*a*d - 4*b*c))/(2*a*c^2*(a*d - b*c)))/(c*x^(1/2) + d*x^(5/2)) + atan(((-
(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d
^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10
*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c
^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c
^16*d))^(1/4)*(x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^
32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4
269711360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648
*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^1
4*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^1
8 - 72704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20) - (-(625*a^
4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 45
00*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 +
114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4
 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))
```

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^(3/4)*(x^(1/2)*(-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 1
2150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8
- 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3
+ 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d
^6 - 32768*a*b^7*c^16*d))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^
13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^
41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 +
 188659793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 5391
77779200*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 158932244
8896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 258534801408
0*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a
^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*
b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26
*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 5242
8800*a^33*b^4*c^23*d^25) - 16777216*a^11*b^25*c^42*d^4 + 218103808*a^12*b^2
4*c^41*d^5 - 1308622848*a^13*b^23*c^40*d^6 + 4798283776*a^14*b^22*c^39*d^7
- 11995709440*a^15*b^21*c^38*d^8 + 21783379968*a^16*b^20*c^37*d^9 - 3159254
6304*a^17*b^19*c^36*d^10 + 48013246464*a^18*b^18*c^35*d^11 - 103424196608*a
^19*b^17*c^34*d^12 + 253954621440*a^20*b^16*c^33*d^13 - 531641663488*a^21*b
^15*c^32*d^14 + 875046109184*a^22*b^14*c^31*d^15 - 1125865488384*a^23*b^13*
c^30*d^16 + 1138334629888*a^24*b^12*c^29*d^17 - 906425794560*a^25*b^11*c^28
*d^18 + 566347431936*a^26*b^10*c^27*d^19 - 274688114688*a^27*b^9*c^26*d^20
+ 101363744768*a^28*b^8*c^25*d^21 - 27505197056*a^29*b^7*c^24*d^22 + 517472
2560*a^30*b^6*c^23*d^23 - 602931200*a^31*b^5*c^22*d^24 + 32768000*a^32*b^4*
c^21*d^25))*1i + (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 +
12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8
 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3
 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*
d^6 - 32768*a*b^7*c^16*d))^(1/4)*(x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10
616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14
*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^2
7*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 +
4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800
*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^2
0*d^20) - (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a
^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 3276
8*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 2867
20*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 3
2768*a*b^7*c^16*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 1458
0*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17
+ 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 22937
6*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 11
4688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(33554432*a^12*b^25*c^44
*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267
266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a
```

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^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^
18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^
35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*
d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^1
7 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 +
 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 462212
30080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b
^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25) + 16777216*a^11*b^25*c^42*d^4 -
 218103808*a^12*b^24*c^41*d^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*
a^14*b^22*c^39*d^7 + 11995709440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20
*c^37*d^9 + 31592546304*a^17*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^
11 + 103424196608*a^19*b^17*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 +
531641663488*a^21*b^15*c^32*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 11258
65488384*a^23*b^13*c^30*d^16 - 1138334629888*a^24*b^12*c^29*d^17 + 90642579
4560*a^25*b^11*c^28*d^18 - 566347431936*a^26*b^10*c^27*d^19 + 274688114688*
a^27*b^9*c^26*d^20 - 101363744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7
*c^24*d^22 - 5174722560*a^30*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 -
 32768000*a^32*b^4*c^21*d^25))*1i)/((-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 145
80*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17
 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 2293
76*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 1
14688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(x^(1/2)*(32366592*a^12
*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^1
0 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 76643
86048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^1
8*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^
23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 51
20000*a^23*b^10*c^20*d^20) - (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^
3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096
*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*
b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a
^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^9 + 6561
*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d
^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b
^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^
5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(3355
4432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^
23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d
^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 3
13817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 959547
703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 2241016627
200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405664030720
*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^
27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8
*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23
```

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 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25) - 16777216*a
^11*b^25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13*b^23*c^4
0*d^6 + 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38*d^8 + 21
783379968*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 4801324646
4*a^18*b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 253954621440*a^2
0*b^16*c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184*a^22*b^1
4*c^31*d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^24*b^12*c
^29*d^17 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^10*c^27*d
^19 - 274688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25*d^21 - 2
7505197056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 602931200*a
^31*b^5*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25)) - (-(625*a^4*d^9 + 6561*b
^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8
)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6
*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*
b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(x^(1/2
)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^
13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c
^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14
- 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236
160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^1
1*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20) - (-(625*a^4*d^9 + 6561*b^4*c^4*
d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096
*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d
^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^1
2*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(3/4)*(x^(1/2)*(-(62
5*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7
- 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^
7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13
*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16
*d))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 35
23215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*
a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^
19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^
36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d
^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16
 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 -
1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 17527
9964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^
31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^
25) + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^41*d^5 + 13086228
48*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 11995709440*a^15*b^
21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 31592546304*a^17*b^19*c^36*d
^10 - 48013246464*a^18*b^18*c^35*d^11 + 103424196608*a^19*b^17*c^34*d^12 -
253954621440*a^20*b^16*c^33*d^13 + 531641663488*a^21*b^15*c^32*d^14 - 87504
6109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23*b^13*c^30*d^16 - 113833462
```

9888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^11*c^28*d^18 - 566347431936*
a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26*d^20 - 101363744768*a^28*b
^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 - 5174722560*a^30*b^6*c^23*d^
23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21*d^25)) + 2985984
0*a^11*b^21*c^29*d^9 - 228925440*a^12*b^20*c^28*d^10 + 774144000*a^13*b^19*
c^27*d^11 - 1514700800*a^14*b^18*c^26*d^12 + 1888665600*a^15*b^17*c^25*d^13
- 1555415040*a^16*b^16*c^24*d^14 + 845578240*a^17*b^15*c^23*d^15 - 2924544
00*a^18*b^14*c^22*d^16 + 58368000*a^19*b^13*c^21*d^17 - 5120000*a^20*b^12*c
^20*d^18))*(-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*
a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 327
68*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286
720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 -
32768*a*b^7*c^16*d))^(1/4)*2i + 2*atan(((-(625*a^4*d^9 + 6561*b^4*c^4*d^5 -
14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*
c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 -
229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5
+ 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(x^(1/2)*(32366592*
a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30
*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7
664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552
*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^1
3*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19
+ 5120000*a^23*b^10*c^20*d^20) + (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*
a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 +
4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*
a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 1146
88*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^9 +
6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b
*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a
^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 22937
6*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(
33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^1
4*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^
40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10
- 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 95
9547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 224101
6627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 240566403
0720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 106110858035
2*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29
*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*
d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25)*1i - 167
77216*a^11*b^25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^5 - 1308622848*a^13*b
^23*c^40*d^6 + 4798283776*a^14*b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38*d
^8 + 21783379968*a^16*b^20*c^37*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 480
13246464*a^18*b^18*c^35*d^11 - 103424196608*a^19*b^17*c^34*d^12 + 253954621

```
440*a^20*b^16*c^33*d^13 - 531641663488*a^21*b^15*c^32*d^14 + 875046109184*a
^22*b^14*c^31*d^15 - 1125865488384*a^23*b^13*c^30*d^16 + 1138334629888*a^24
*b^12*c^29*d^17 - 906425794560*a^25*b^11*c^28*d^18 + 566347431936*a^26*b^10
*c^27*d^19 - 274688114688*a^27*b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25*d
^21 - 27505197056*a^29*b^7*c^24*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 6029
31200*a^31*b^5*c^22*d^24 + 32768000*a^32*b^4*c^21*d^25)*1i) + (-(625*a^4*d^
9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a
^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 1146
88*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 2
29376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/
4)*(x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 18
6867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a
^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*
c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16
 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^12*c^22*d^18 - 7270400
0*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20) + (-(625*a^4*d^9 + 656
1*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*
d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*
b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a
^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(3/4)*(x^(
1/2)*(-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^
2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7
*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^
4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*
a*b^7*c^16*d))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^4
3*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45
971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 1886597939
20*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^
20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b
^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13
*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^
30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^
20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 850
6048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b
^4*c^23*d^25)*1i + 16777216*a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^41*d
^5 + 1308622848*a^13*b^23*c^40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 119957
09440*a^15*b^21*c^38*d^8 - 21783379968*a^16*b^20*c^37*d^9 + 31592546304*a^1
7*b^19*c^36*d^10 - 48013246464*a^18*b^18*c^35*d^11 + 103424196608*a^19*b^17
*c^34*d^12 - 253954621440*a^20*b^16*c^33*d^13 + 531641663488*a^21*b^15*c^32
*d^14 - 875046109184*a^22*b^14*c^31*d^15 + 1125865488384*a^23*b^13*c^30*d^1
6 - 1138334629888*a^24*b^12*c^29*d^17 + 906425794560*a^25*b^11*c^28*d^18 -
566347431936*a^26*b^10*c^27*d^19 + 274688114688*a^27*b^9*c^26*d^20 - 101363
744768*a^28*b^8*c^25*d^21 + 27505197056*a^29*b^7*c^24*d^22 - 5174722560*a^3
0*b^6*c^23*d^23 + 602931200*a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21*d^2
5)*1i))/((-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^
```

```
2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768
*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 28672
0*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32
768*a*b^7*c^16*d))^(1/4)*(x^(1/2)*(32366592*a^12*b^21*c^31*d^9 - 10616832*a
^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10 - 1422057472*a^14*b^19*c^
29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7664386048*a^16*b^17*c^27*d^13 +
 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^18*b^15*c^25*d^15 + 44147179
52*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23*d^17 + 465100800*a^21*b^
12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120000*a^23*b^10*c^20*d^20)
+ (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c
^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*
c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b
^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b
^7*c^16*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*
c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a
^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^
5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6
*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 5
03316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23*c^42*d^6 - 15267266560*a
^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20
*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*
d^11 + 539177779200*a^20*b^17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13
+ 1589322448896*a^22*b^15*c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2
585348014080*a^24*b^13*c^32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792
662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 4923693
46560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^
30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*
d^24 - 52428800*a^33*b^4*c^23*d^25)*1i - 16777216*a^11*b^25*c^42*d^4 + 2181
03808*a^12*b^24*c^41*d^5 - 1308622848*a^13*b^23*c^40*d^6 + 4798283776*a^14*
b^22*c^39*d^7 - 11995709440*a^15*b^21*c^38*d^8 + 21783379968*a^16*b^20*c^37
*d^9 - 31592546304*a^17*b^19*c^36*d^10 + 48013246464*a^18*b^18*c^35*d^11 -
103424196608*a^19*b^17*c^34*d^12 + 253954621440*a^20*b^16*c^33*d^13 - 53164
1663488*a^21*b^15*c^32*d^14 + 875046109184*a^22*b^14*c^31*d^15 - 1125865488
384*a^23*b^13*c^30*d^16 + 1138334629888*a^24*b^12*c^29*d^17 - 906425794560*
a^25*b^11*c^28*d^18 + 566347431936*a^26*b^10*c^27*d^19 - 274688114688*a^27*
b^9*c^26*d^20 + 101363744768*a^28*b^8*c^25*d^21 - 27505197056*a^29*b^7*c^24
*d^22 + 5174722560*a^30*b^6*c^23*d^23 - 602931200*a^31*b^5*c^22*d^24 + 3276
8000*a^32*b^4*c^21*d^25)*1i)*1i - (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580
*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 +
 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376
*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114
688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(x^(1/2)*(32366592*a^12*b
^21*c^31*d^9 - 10616832*a^11*b^22*c^32*d^8 + 186867712*a^13*b^20*c^30*d^10
- 1422057472*a^14*b^19*c^29*d^11 + 4269711360*a^15*b^18*c^28*d^12 - 7664386
048*a^16*b^17*c^27*d^13 + 9165979648*a^17*b^16*c^26*d^14 - 7603863552*a^18*
```

```
b^15*c^25*d^15 + 4414717952*a^19*b^14*c^24*d^16 - 1766236160*a^20*b^13*c^23
*d^17 + 465100800*a^21*b^12*c^22*d^18 - 72704000*a^22*b^11*c^21*d^19 + 5120
000*a^23*b^10*c^20*d^20) + (-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*
c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a
^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^
5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6
*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^9 + 6561*b
^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c^2*d^7 - 4500*a^3*b*c*d^8
)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*c^10*d^7 + 114688*a^2*b^6
*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b^4*c^13*d^4 - 229376*a^5*
b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b^7*c^16*d))^(1/4)*(335544
32*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 + 3523215360*a^14*b^23
*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 45971668992*a^16*b^21*c^40*d^8
 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^18*b^19*c^38*d^10 - 313
817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^17*c^36*d^12 - 95954770
3296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*c^34*d^14 - 224101662720
0*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^32*d^16 - 2405664030720*a
^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30*d^18 - 1061108580352*a^27
*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^20 - 175279964160*a^29*b^8*c
^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 8506048512*a^31*b^6*c^25*d^23 +
 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*b^4*c^23*d^25)*1i + 16777216*
a^11*b^25*c^42*d^4 - 218103808*a^12*b^24*c^41*d^5 + 1308622848*a^13*b^23*c^
40*d^6 - 4798283776*a^14*b^22*c^39*d^7 + 11995709440*a^15*b^21*c^38*d^8 - 2
1783379968*a^16*b^20*c^37*d^9 + 31592546304*a^17*b^19*c^36*d^10 - 480132464
64*a^18*b^18*c^35*d^11 + 103424196608*a^19*b^17*c^34*d^12 - 253954621440*a^
20*b^16*c^33*d^13 + 531641663488*a^21*b^15*c^32*d^14 - 875046109184*a^22*b^
14*c^31*d^15 + 1125865488384*a^23*b^13*c^30*d^16 - 1138334629888*a^24*b^12*
c^29*d^17 + 906425794560*a^25*b^11*c^28*d^18 - 566347431936*a^26*b^10*c^27*
d^19 + 274688114688*a^27*b^9*c^26*d^20 - 101363744768*a^28*b^8*c^25*d^21 +
27505197056*a^29*b^7*c^24*d^22 - 5174722560*a^30*b^6*c^23*d^23 + 602931200*
a^31*b^5*c^22*d^24 - 32768000*a^32*b^4*c^21*d^25)*1i)*1i + 29859840*a^11*b^
21*c^29*d^9 - 228925440*a^12*b^20*c^28*d^10 + 774144000*a^13*b^19*c^27*d^11
 - 1514700800*a^14*b^18*c^26*d^12 + 1888665600*a^15*b^17*c^25*d^13 - 155541
5040*a^16*b^16*c^24*d^14 + 845578240*a^17*b^15*c^23*d^15 - 292454400*a^18*b
^14*c^22*d^16 + 58368000*a^19*b^13*c^21*d^17 - 5120000*a^20*b^12*c^20*d^18)
)*(-(625*a^4*d^9 + 6561*b^4*c^4*d^5 - 14580*a*b^3*c^3*d^6 + 12150*a^2*b^2*c
^2*d^7 - 4500*a^3*b*c*d^8)/(4096*b^8*c^17 + 4096*a^8*c^9*d^8 - 32768*a^7*b*
c^10*d^7 + 114688*a^2*b^6*c^15*d^2 - 229376*a^3*b^5*c^14*d^3 + 286720*a^4*b
^4*c^13*d^4 - 229376*a^5*b^3*c^12*d^5 + 114688*a^6*b^2*c^11*d^6 - 32768*a*b
^7*c^16*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
[Out] Timed out
```

3.460 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=570

$$\frac{b^{11/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2}-$$

Rubi [**A**] time = 0.82, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{11/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}+\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}(bc-ad)^2}-\frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^2}+\frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^2}-\frac{4bc-7ad}{6ac^2x^{3/2}(bc-ad)}-\frac{d}{2cx^{3/2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 7*a*d)/(6*a*c^2*(b*c - a*d)*x^(3/2)) - d/(2*c*(b*c - a*d)*x^(3/2)
*(c + d*x^2)) + (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(S
qrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*
d)^2) + (d^(7/4)*(11*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (
b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqr
t[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
 + (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-7ad-7bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{3\left(4b^2c^2+4abcd-7a^2d^2\right)+3}{\left(a+bx^4\right)\left(c+dx\right.}\right.}{6ac^2(bc-}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right.}{a(bc-ad)^2}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \sqrt{\right.}}{a^{3/2}(bc-ad)^2}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} - \frac{b^{5/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\right.}{2a^{3/2}(bc-ad)^2}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.}{2\sqrt{2}\,a^{7/4}(bc-ad)^2}$$

$$= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} - \frac{d}{2c(bc-ad)x^{3/2}\left(c+dx^2\right)} + \frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2} - b$$

Mathematica [A] time = 6.15, size = 602, normalized size = 1.06

$$\frac{b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2} - \frac{b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^2} - \frac{b^{11/4}\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2} - \frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^2} - \frac{d^{7/4}(11bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}(ad-bc)^2} + \frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{11/4}(ad-bc)^2} + \frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}(ad-bc)^2} + \frac{d^{7/4}(11bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{11/4}(ad-bc)^2} + \frac{d^2\sqrt{x}}{2c^2\left(c+dx^2\right)(bc-ad)} - \frac{2}{3ac^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -2/(3*a*c^2*x^(3/2)) + (d^2*Sqrt[x])/(2*c^2*(b*c - a*d)*(c + d*x^2)) - (b^(
11/4)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(
Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1
```

```
/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (d^(7/4)
*(11*b*c - 7*a*d)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*
c^(1/4))])/(4*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^2) + (d^(7/4)*(11*b*c - 7*a*d
)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[
2]*c^(11/4)*(-(b*c) + a*d)^2) + (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (b^(11/4)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4
)*(b*c - a*d)^2) - (d^(7/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^2) + (d^(7
/4)*(11*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(8*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^2)
```

IntegrateAlgebraic [A] time = 1.29, size = 353, normalized size = 0.62

$$\frac{b^{11/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{7/4}(ad-bc)^2}-\frac{b^{11/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{7/4}(ad-bc)^2}-\frac{\left(11bcd^{7/4}-7ad^{11/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^2}+\frac{\left(11bcd^{7/4}-7ad^{11/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^2}+\frac{-4acd-7ad^2x^2+4bc^2+4bcdx^2}{6ac^2x^{3/2}\left(c+dx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (4*b*c^2 - 4*a*c*d + 4*b*c*d*x^2 - 7*a*d^2*x^2)/(6*a*c^2*(-(b*c) + a*d)*x^(
3/2)*(c + d*x^2)) + (b^(11/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*
x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(7/4)*(-(b*c) + a*d)^2) - ((11*b
*c*d^(7/4) - 7*a*d^(11/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2) - (b^(11/4)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(7/4)*(-(
b*c) + a*d)^2) + ((11*b*c*d^(7/4) - 7*a*d^(11/4))*ArcTanh[(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.15, size = 718, normalized size = 1.26

$$-\frac{\left(ab^3\right)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2}-\frac{\left(ab^3\right)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2}-\frac{\left(ab^3\right)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}b^2\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}\,a^2b^2c^2-2\sqrt{2}\,a^3bcd+\sqrt{2}\,a^4d^2\right)}+\frac{\left(11\left(cd^3\right)^{\frac{1}{4}}bcd-7\left(cd^3\right)^{\frac{1}{4}}ad^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^5-2\sqrt{2}\,abc^4d+\sqrt{2}\,a^2c^3d^2\right)}+\frac{\left(11\left(cd^3\right)^{\frac{1}{4}}bcd-7\left(cd^3\right)^{\frac{1}{4}}ad^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}\,b^2c^5-2\sqrt{2}\,abc^4d+\sqrt{2}\,a^2c^3d^2\right)}+\frac{\left(11\left(cd^3\right)^{\frac{1}{4}}bcd-7\left(cd^3\right)^{\frac{1}{4}}ad^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^5-2\sqrt{2}\,abc^4d+\sqrt{2}\,a^2c^3d^2\right)}-\frac{\left(11\left(cd^3\right)^{\frac{1}{4}}bcd-7\left(cd^3\right)^{\frac{1}{4}}ad^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}\,b^2c^5-2\sqrt{2}\,abc^4d+\sqrt{2}\,a^2c^3d^2\right)}+\frac{d^2\sqrt{x}}{2\left(bc^3-ac^2d\right)\left(dx^2+c\right)}-\frac{2}{3\,ac^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - (
a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/
2*(a*b^3)^(1/4)*b^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/2*(a*b^3)^(1/4)
*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2
 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7
*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2)
+ 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)
*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)
^(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2
*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) - 1/8*(11*(c*d^3)^(1/4)*b
*c*d - 7*(c*d^3)^(1/4)*a*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/2*d^
2*sqrt(x)/((b*c^3 - a*c^2*d)*(d*x^2 + c)) - 2/3/(a*c^2*x^(3/2))
```

maple [A] time = 0.02, size = 588, normalized size = 1.03

$$-\frac{a d^{3} \sqrt{x}}{2 \left(a d-b c\right)^{2} \left(d \,x^{2}+c \right) c^{2}}+\frac{b \,d^{2} \sqrt{x}}{2 \left(a d-b c\right)^{2} \left(d \,x^{2}+c \right) c}-\frac{7 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a \,d^{3} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{2} c^{3}}-\frac{7 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a \,d^{3} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{2} c^{3}}-\frac{7 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a \,d^{3} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(a d-b c\right)^{2} c^{3}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{3} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2 \left(a d-b c\right)^{2} a^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{3} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2 \left(a d-b c\right)^{2} a^{2}}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^{3} \ln \left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4 \left(a d-b c\right)^{2} a^{2}}+\frac{11 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b \,d^{2} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{2} c^{2}}+\frac{11 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b \,d^{2} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{2} c^{2}}+\frac{11 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b \,d^{2} \ln \left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(a d-b c\right)^{2} c^{2}}-\frac{2}{3 a \,c^{2} x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] -1/4/a^2*b^3/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2/a^2*b^3/(
a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2/a^
2*b^3/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
-1/2*d^3/c^2/(a*d-b*c)^2*x^(1/2)/(d*x^2+c)*a+1/2*d^2/c/(a*d-b*c)^2*x^(1/2)/
(d*x^2+c)*b-7/8*d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*a+11/8*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)
*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1
/2)+(c/d)^(1/2)))*a+11/16*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/
d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^
(1/2)))*b-7/8*d^3/c^3/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)*a+11/8*d^2/c^2/(a*d-b*c)^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)*b-2/3/a/c^2/x^(3/2)
```

maxima [A] time = 2.49, size = 539, normalized size = 0.95

$$-\frac{\frac{2\sqrt{2}b^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{11}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{11}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(ab^{2}c^{2}-2a^{2}bcd+a^{3}d^{2}\right)}-\frac{4bc^{2}-4acd+\left(4bcd-7ad^{2}\right)x^{2}}{6\left(\left(abc^{3}d-a^{2}c^{2}d^{2}\right)x^{\frac{7}{2}}+\left(abc^{4}-a^{2}c^{3}d\right)x^{\frac{3}{2}}\right)}+\frac{\frac{2\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(11bcd^{2}-7ad^{3}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{2}c^{4}-2abc^{3}d+a^{2}c^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(11/4)*log(
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^
(11/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))
/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - 1/6*(4*b*c^2 - 4*a*c*d + (4*b*c*d -
7*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^(7/2) + (a*b*c^4 - a^2*c^3*d)*x^
(3/2)) + 1/16*(2*sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*arctan(-1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)
*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*log(sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*
c*d^2 - 7*a*d^3)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(3/4)*d^(1/4)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)
```

mupad [B] time = 6.58, size = 27743, normalized size = 48.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] 2*atan(((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*
b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c
^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712
*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^
29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 -
 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 264299
97056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a
^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*
c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) -
 (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6
*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5
+ 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*((-b^11/(16*a^15*d^8 + 16
*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d
```

```
^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 -
128*a^14*b*c*d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^
7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^
4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*
(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a
^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21
*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d
^10 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12
 - 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 -
7600917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 660
3814207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 260456
2120704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 4050691031
04*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*
b^6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^2
5)*1i + x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5
 + 3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 4580179
9680*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^
17*b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^1
8*c^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^
34*d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*
d^16 + 4221965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^1
8 + 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 +
 864890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 844732
82560*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32
*b^5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26))*1i + 11534336*a^9*b^25*c^35
*d^7 - 111149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 123312
5376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*
b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^
28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^
16 - 192702906368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 -
109211664384*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 180622
13120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25
*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24)*1i) + (-b^11/(16*a^15*d^8 + 1
6*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*
d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 -
 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*
a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*
c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^1
4 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^17*c^24*d^16 + 222
56009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22*d^18 + 5629976576
*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12
*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) + (-b^11/(16*a^15*d^8 + 16*a^7*b
^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1
120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^
14*b*c*d^7))^(1/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*
```

```
d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 89
6*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^1
1/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 -
 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*
a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 -
1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896
*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b
^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c
^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 4642121449472*a^22*b^16*c^37
*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800*a^24*b^14*c^35*d^
15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^26*b^12*c^33*d^17
+ 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*b^10*c^31*d^19 + 1
167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c^29*d^21 + 1049582
63296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23 + 2181038080*a^33
*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)*1i - x^(1/2)*(33554432*a^11*
b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a^13*b^24*c^42*d^
6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*c^40*d^8 - 10051
0203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^10 - 18433127219
2*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 336173465600*a^20
*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 2334365057024*a^22*b^
15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 4221965434880*a^24*b^13*
c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 3039679217664*a^26*b^11*c^2
9*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a^28*b^9*c^27*d^2
1 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*c^25*d^23 - 1588
8023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25 - 102760448*a^33
*b^4*c^22*d^26))*1i + 11534336*a^9*b^25*c^35*d^7 - 111149056*a^10*b^24*c^34
*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c^32*d^10 + 1830
010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 - 12820119552*a^
15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 104394047488*a^17*b^1
7*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 192702906368*a^19*b^15*c^2
5*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*a^21*b^13*c^23*d^1
9 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*c^21*d^21 + 422
4417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23 + 39337984*a^26*
b^8*c^18*d^24)*1i))/((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7
*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 8
96*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1
/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168
*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^2
0*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*
d^15 - 26429997056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 -
13398917120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 15699066
88*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11
*c^18*d^22) - (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 44
8*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12
*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*((-b^11/(16*
```

```
a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a
^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b
^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8
 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^
11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c
*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 +
 9126805504*a^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876
224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^1
9*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b
^17*c^38*d^12 - 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15
*c^36*d^14 - 7600917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^
34*d^16 - 6603814207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*
d^18 - 2604562120704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20
 - 405069103104*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 1910
9249024*a^32*b^6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34
*b^4*c^25*d^25)*1i + x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*
b^25*c^43*d^5 + 3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*
d^7 + 45801799680*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 16
3810639872*a^17*b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 6501171
2000*a^19*b^18*c^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640
*a^21*b^16*c^34*d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^
23*b^14*c^32*d^16 + 4221965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*
b^12*c^30*d^18 + 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^1
0*c^28*d^20 + 864890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*
d^22 + 84473282560*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 18
64368128*a^32*b^5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26))*1i + 11534336*
a^9*b^25*c^35*d^7 - 111149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33
*d^9 - 1233125376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 39
1331840*a^14*b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216
*a^16*b^18*c^28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18
*b^16*c^26*d^16 - 192702906368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14
*c^24*d^18 - 109211664384*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*
d^20 - 18062213120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 6
01309184*a^25*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24)*1i)*1i - (-b^11/
(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 8
96*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^
13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*
d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 21688
07424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14037065728*a^
14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^17
*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22*d
^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 261
316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) + (-b^11/(16*a^1
5*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10
*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*
```

c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 -
128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*
b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^
7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a
^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^
3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(67108864*a^13*
b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d
^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 442
364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 182422025
0112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 464212144947
2*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800*a
^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^26
*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*b^
10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c^2
9*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23 +
2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)*1i - x^(1/2)
*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a
^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*
c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^
10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 3
36173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 23343
65057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 42219654
34880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 30396792176
64*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a
^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*
c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25
- 102760448*a^33*b^4*c^22*d^26))*1i + 11534336*a^9*b^25*c^35*d^7 - 11114905
6*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*
c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12
- 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 10439
4047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 1927029063
68*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 10921164384*a^
21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11
*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23
+ 39337984*a^26*b^8*c^18*d^24)*1i)*1i))*(-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^
8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a
^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*
c*d^7))^(1/4) - 2*atan(-((-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3
*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 409
6*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^
3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688
*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*
c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 -
2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 140370657
28*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^16

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*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c
^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20
+ 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) - (-(2401*a
^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 -
 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*
d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^
15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^
18*d))^(1/4)*(((-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 +
 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11
*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16
*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c
^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(67108864*a^13*b^25*c^46*d^4 - 1140850
688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896*a^16*b
^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b^20*c^4
1*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c^39*d^1
1 + 3052916441088*a^21*b^17*c^38*d^12 - 4642121449472*a^22*b^16*c^37*d^13 +
 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800*a^24*b^14*c^35*d^15 + 77
56643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^26*b^12*c^33*d^17 + 46136
00182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*b^10*c^31*d^19 + 11670902
53824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c^29*d^21 + 104958263296*a
^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23 + 2181038080*a^33*b^5*c^
26*d^24 - 117440512*a^34*b^4*c^25*d^25)*1i + x^(1/2)*(33554432*a^11*b^26*c^
44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a^13*b^24*c^42*d^6 - 152
67266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*c^40*d^8 - 100510203904
*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^10 - 184331272192*a^18*
b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 336173465600*a^20*b^17*c
^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 2334365057024*a^22*b^15*c^33
*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 4221965434880*a^24*b^13*c^31*d^
17 - 4009062563840*a^25*b^12*c^30*d^18 + 3039679217664*a^26*b^11*c^29*d^19
- 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a^28*b^9*c^27*d^21 - 313
859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*c^25*d^23 - 15888023552
*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25 - 102760448*a^33*b^4*c^
22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 3557
4*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8
- 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3
+ 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d
^6 - 32768*a*b^7*c^18*d))^(3/4)*1i + 11534336*a^9*b^25*c^35*d^7 - 111149056
*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c
^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 -
 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 104394
047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 19270290636
8*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*a^2
1*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*
c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23 +
 39337984*a^26*b^8*c^18*d^24)*1i) + (-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 -
```

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37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8
*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2
- 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d
^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/2)*(1585971
2*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c
^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13
- 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429
997056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*
a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13
*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22)
+ (-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^
2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*
a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720
*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 327
68*a*b^7*c^18*d))^(1/4)*(((-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^
3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 40
96*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a
^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 11468
8*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(67108864*a^13*b^25*c^46*d^
4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 4581857
6896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^
18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^
18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 4642121449472*a^22*b^16*
c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800*a^24*b^14*c^3
5*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^26*b^12*c^33*d
^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*b^10*c^31*d^19
 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c^29*d^21 + 104
958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23 + 2181038080*
a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)*1i - x^(1/2)*(33554432*a
^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a^13*b^24*c^4
2*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*c^40*d^8 - 1
00510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^10 - 1843312
72192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 336173465600*
a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 2334365057024*a^2
2*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 4221965434880*a^24*b
^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 3039679217664*a^26*b^11
*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a^28*b^9*c^27
*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*c^25*d^23 -
15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25 - 102760448*
a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3
*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^
8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^
5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6
*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4)*1i + 11534336*a^9*b^25*c^35*d^7
- 111149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*
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a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*
c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^
14 - 104394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 -
192702906368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 10921
1664384*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120
*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*
c^19*d^23 + 39337984*a^26*b^8*c^18*d^24)*1i))/((-(2401*a^4*d^11 + 14641*b^4
*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10
)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^
6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5
*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/
2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*
a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20
*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d
^15 - 26429997056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 1
3398917120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 156990668
8*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*
c^18*d^22) - (-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 3
5574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d
^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d
^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^1
3*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(((-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 -
 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^
8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2
 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*
d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(67108864*a^13*b
^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^
6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 4423
64854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220250
112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 4642121449472
*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800*a^
24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^26*
b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*b^1
0*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c^29
*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23 +
2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)*1i + x^(1/2)*
(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a^
13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*c
^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^1
0 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 33
6173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 233436
5057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 422196543
4880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 303967921766
4*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a^
28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*c
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^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25 -
 102760448*a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 3726
8*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^1
9 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 22
9376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 +
 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4)*1i + 11534336*a^9*b^2
5*c^35*d^7 - 111149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 -
1233125376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840
*a^14*b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b
^18*c^28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c
^26*d^16 - 192702906368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d
^18 - 109211664384*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 -
18062213120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 60130918
4*a^25*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24)*1i)*1i - (-(2401*a^4*d^
11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 1509
2*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 +
 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^
4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d)
)^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^1
0 + 600711168*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 634368
0000*a^13*b^20*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^
15*b^18*c^25*d^15 - 26429997056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16
*c^23*d^17 - 13398917120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^
19 - 1569906688*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668
992*a^22*b^11*c^18*d^22) + (-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b
^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4
096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*
a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 1146
88*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(((-(2401*a^4*d^11 + 14641
*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*
d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^
2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376
*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(6
7108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^1
5*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c
^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^1
0 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 -
 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 76
00917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 66038
14207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 26045621
20704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104
*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^
6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)
*1i - x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 +
 3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 458017996
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80*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17
*b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*
c^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34
*d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^
16 + 4221965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18
+ 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 8
64890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282
560*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b
^5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*
c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)
/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6
*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*
b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4)*1i + 11
534336*a^9*b^25*c^35*d^7 - 111149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^
23*c^33*d^9 - 1233125376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^
11 + 391331840*a^14*b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13 + 4659
2393216*a^16*b^18*c^28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 1652971110
40*a^18*b^16*c^26*d^16 - 192702906368*a^19*b^15*c^25*d^17 + 167824392192*a^
20*b^14*c^24*d^18 - 109211664384*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^1
2*c^22*d^20 - 18062213120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d
^22 - 601309184*a^25*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24)*1i)*1i))*
(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*
c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^
7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a
^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768
*a*b^7*c^18*d))^(1/4) - (2/(3*a*c) + (d*x^2*(7*a*d - 4*b*c))/(6*a*c^2*(a*d
- b*c)))/(c*x^(3/2) + d*x^(7/2)) - atan(((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c
^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*
a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b
*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c
^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 +
 6343680000*a^13*b^20*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^14 + 2264801
2800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^17*c^24*d^16 + 22256009216*a^
17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*
c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21
- 19668992*a^22*b^11*c^18*d^22) + (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 12
8*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^
4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7)
)^(1/4)*(11534336*a^9*b^25*c^35*d^7 - (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8
- 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^1
1*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*
d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448
*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*
b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(67108864*a^1
3*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^44
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*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 - 4
42364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1824220
250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 4642121449
472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917708800
*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a^
26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28*
b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*c
^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^23
 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25) - x^(1/2)*
(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a^
13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*c
^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^1
0 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 33
6173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 233436
5057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 422196543
4880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 303967921766
4*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a^
28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*c
^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25 -
 102760448*a^33*b^4*c^22*d^26)) - 111149056*a^10*b^24*c^34*d^8 + 481296384*
a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21*
c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^13
 + 46592393216*a^16*b^18*c^28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 165
297111040*a^18*b^16*c^26*d^16 - 192702906368*a^19*b^15*c^25*d^17 + 16782439
2192*a^20*b^14*c^24*d^18 - 109211664384*a^21*b^13*c^23*d^19 + 52444708864*a
^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^10
*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24))*1
i + (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*
c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d
^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9
*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d
^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 140
37065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 2642999705
6*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*
b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20
*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) - (-b
^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2
 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 44
8*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(11534336*a^9*b^25*c^35*d^7 -
 (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6
*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5
+ 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16
*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d
^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 -
128*a^14*b*c*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^2
```

4*c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7
 + 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 9720
04786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c^39*d^11 + 30529164
41088*a^21*b^17*c^38*d^12 - 4642121449472*a^22*b^16*c^37*d^13 + 63476932280
32*a^23*b^15*c^36*d^14 - 7600917708800*a^24*b^14*c^35*d^15 + 7756643827712*
a^25*b^13*c^34*d^16 - 6603814207488*a^26*b^12*c^33*d^17 + 4613600182272*a^2
7*b^11*c^32*d^18 - 2604562120704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b
^9*c^30*d^20 - 405069103104*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28
*d^22 - 19109249024*a^32*b^6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 11
7440512*a^34*b^4*c^25*d^25) + x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316
480*a^12*b^25*c^43*d^5 + 3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b
^23*c^41*d^7 + 45801799680*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39
*d^9 + 163810639872*a^17*b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11
 + 65011712000*a^19*b^18*c^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148
861808640*a^21*b^16*c^34*d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660
153344*a^23*b^14*c^32*d^16 + 4221965434880*a^24*b^13*c^31*d^17 - 4009062563
840*a^25*b^12*c^30*d^18 + 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544
*a^27*b^10*c^28*d^20 + 864890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*
b^8*c^26*d^22 + 84473282560*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*
d^24 + 1864368128*a^32*b^5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26)) - 111
149056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*
b^22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*
d^12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 -
104394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 19270
2906368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 1092116643
84*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23
*b^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*
d^23 + 39337984*a^26*b^8*c^18*d^24))*1i)/((-b^11/(16*a^15*d^8 + 16*a^7*b^8*
c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120
*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*
b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*
c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12
 + 6343680000*a^13*b^20*c^27*d^13 - 14037065728*a^14*b^19*c^26*d^14 + 226480
12800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^17*c^24*d^16 + 22256009216*a
^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14
*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21
 - 19668992*a^22*b^11*c^18*d^22) + (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 1
28*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b
^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7
))^(1/4)*(11534336*a^9*b^25*c^35*d^7 - (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8
 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^
11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c
d^7))^(3/4)((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 44
8*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12
*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(67108864*a^

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13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^4
4*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 -
442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 182422
0250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 464212144
9472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 760091770880
0*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a
^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28
*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*
c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^2
3 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25) - x^(1/2)
*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a
^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*
c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^
10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 3
36173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 23343
65057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 42219654
34880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 30396792176
64*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a
^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*
c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25
- 102760448*a^33*b^4*c^22*d^26)) - 111149056*a^10*b^24*c^34*d^8 + 481296384
*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c^32*d^10 + 1830010880*a^13*b^21
*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 - 12820119552*a^15*b^19*c^29*d^1
3 + 46592393216*a^16*b^18*c^28*d^14 - 104394047488*a^17*b^17*c^27*d^15 + 16
5297111040*a^18*b^16*c^26*d^16 - 192702906368*a^19*b^15*c^25*d^17 + 1678243
92192*a^20*b^14*c^24*d^18 - 109211664384*a^21*b^13*c^23*d^19 + 52444708864*
a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*c^21*d^21 + 4224417792*a^24*b^1
0*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23 + 39337984*a^26*b^8*c^18*d^24))
- (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^
6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5
 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859712*a^9*b
^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^1
1 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14037
065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*
a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^
15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d
^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) - (-b^1
1/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 -
 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*
a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(11534336*a^9*b^25*c^35*d^7 - (
-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d
^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 +
448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a
^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3
 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 12
```

```
8*a^14*b*c*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*
c^45*d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 +
 162973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004
786176*a^19*b^19*c^40*d^10 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441
088*a^21*b^17*c^38*d^12 - 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032
*a^23*b^15*c^36*d^14 - 7600917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^
25*b^13*c^34*d^16 - 6603814207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*
b^11*c^32*d^18 - 2604562120704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9
*c^30*d^20 - 405069103104*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d
^22 - 19109249024*a^32*b^6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 1174
40512*a^34*b^4*c^25*d^25) + x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 50331648
0*a^12*b^25*c^43*d^5 + 3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^2
3*c^41*d^7 + 45801799680*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d
^9 + 163810639872*a^17*b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 +
65011712000*a^19*b^18*c^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 114886
1808640*a^21*b^16*c^34*d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 354266015
3344*a^23*b^14*c^32*d^16 + 4221965434880*a^24*b^13*c^31*d^17 - 400906256384
0*a^25*b^12*c^30*d^18 + 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a
^27*b^10*c^28*d^20 + 864890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^
8*c^26*d^22 + 84473282560*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^
24 + 1864368128*a^32*b^5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26)) - 11114
9056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^
22*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^
12 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 10
4394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 1927029
06368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384
*a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b
^11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^
23 + 39337984*a^26*b^8*c^18*d^24))))*(-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 -
 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11
*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d
^7))^(1/4)*2i - atan(((-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^
3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a
^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b
^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^
6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/2)*(15859712*a^9*b^24*c^3
1*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11 - 216
8807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14037065728*
a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^16*b^
17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15*c^22
*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^20 + 2
61316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) + (-(2401*a^4*
d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15
092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7
 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*
```

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d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*
d))^(1/4)*(11534336*a^9*b^25*c^35*d^7 - ((-(2401*a^4*d^11 + 14641*b^4*c^4*d
^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(409
6*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17
*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c
^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(67108864*a^
13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^23*c^4
4*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d^8 -
442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 182422
0250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 464212144
9472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 760091770880
0*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207488*a
^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704*a^28
*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30*b^8*
c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^27*d^2
3 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25) - x^(1/2)
*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215360*a
^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*b^22*
c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^38*d^
10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^12 + 3
36173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 + 23343
65057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 42219654
34880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 30396792176
64*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650624*a
^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30*b^7*
c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*d^25
 - 102760448*a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 372
68*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^
19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 2
29376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5
+ 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4) - 111149056*a^10*b^2
4*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c^32*d^10
+ 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 - 12820119
552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 104394047488*a^
17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 192702906368*a^19*b^
15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*a^21*b^13*c^
23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*c^21*d^21
 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23 + 39337984
*a^26*b^8*c^18*d^24))*1i + (-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b
^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4
096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*
a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 1146
88*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/2)*(15859712*a^9*b^2
4*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*d^11
- 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 1403706
```

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5728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 26429997056*a^
16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18*b^15
*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^20*d^2
0 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) - (-(2401
*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9
 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^1
2*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*
c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*
c^18*d))^(1/4)*(11534336*a^9*b^25*c^35*d^7 - ((-(2401*a^4*d^11 + 14641*b^4*
c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)
/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6
*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*
b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(671088
64*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^15*b^2
3*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c^42*d
^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^10 - 1
824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 - 4642
121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 7600917
708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 6603814207
488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 2604562120704
*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104*a^30
*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^6*c^2
7*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25) + x^
(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 3523215
360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*a^15*
b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^20*c^
38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^36*d^1
2 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^14 +
2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16 + 422
1965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 303967
9217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 864890650
624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560*a^30
*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*c^23*
d^25 - 102760448*a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7
 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b
^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^
2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14
*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4) - 111149056*a^1
0*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*c^32*
d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12 - 128
20119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 1043940474
88*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 192702906368*a^
19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*a^21*b^
13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11*c^21
*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23 + 393
```

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37984*a^26*b^8*c^18*d^24))*1i)/((-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 3726
8*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^1
9 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 22
9376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 +
 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/2)*(15859712*a^
9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^29*
d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 - 14
037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 264299970
56*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^18
*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c^2
0*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) + (-
(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^
2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*
b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4
*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a
*b^7*c^18*d))^(1/4)*(11534336*a^9*b^25*c^35*d^7 - ((-(2401*a^4*d^11 + 14641
*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*
d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^
2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376
*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(6
7108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a^1
5*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21*c
^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d^1
0 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12 -
 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 - 76
00917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 66038
14207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 26045621
20704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 405069103104
*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*b^
6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^25)
 - x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 + 35
23215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 45801799680*
a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*b^
20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c^3
6*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*d^
14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^16
+ 4221965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 + 3
039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 8648
90650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 84473282560
*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^5*
c^23*d^25 - 102760448*a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c^4
*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4
096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^
17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3
*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4) - 11114905
```

```
6*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^22*
c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^12
- 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 10439
4047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 1927029063
68*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*a^
21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^11
*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^23
+ 39337984*a^26*b^8*c^18*d^24)) - (-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37
268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c
^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 -
229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5
 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*(x^(1/2)*(15859712*
a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^22*c^2
9*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d^13 -
14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 - 2642999
7056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 13398917120*a^
18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^20*b^13*c
^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^18*d^22) -
(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*
c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^
7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a
^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768
*a*b^7*c^18*d))^(1/4)*(11534336*a^9*b^25*c^35*d^7 - ((-(2401*a^4*d^11 + 146
41*b^4*c^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*
c*d^10)/(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*
a^2*b^6*c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 2293
76*a^5*b^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*
(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*d^5 + 9126805504*a
^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162973876224*a^17*b^21
*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004786176*a^19*b^19*c^40*d
^10 - 1824220250112*a^20*b^18*c^39*d^11 + 3052916441088*a^21*b^17*c^38*d^12
 - 4642121449472*a^22*b^16*c^37*d^13 + 6347693228032*a^23*b^15*c^36*d^14 -
7600917708800*a^24*b^14*c^35*d^15 + 7756643827712*a^25*b^13*c^34*d^16 - 660
3814207488*a^26*b^12*c^33*d^17 + 4613600182272*a^27*b^11*c^32*d^18 - 260456
2120704*a^28*b^10*c^31*d^19 + 1167090253824*a^29*b^9*c^30*d^20 - 4050691031
04*a^30*b^8*c^29*d^21 + 104958263296*a^31*b^7*c^28*d^22 - 19109249024*a^32*
b^6*c^27*d^23 + 2181038080*a^33*b^5*c^26*d^24 - 117440512*a^34*b^4*c^25*d^2
5) + x^(1/2)*(33554432*a^11*b^26*c^44*d^4 - 503316480*a^12*b^25*c^43*d^5 +
3523215360*a^13*b^24*c^42*d^6 - 15267266560*a^14*b^23*c^41*d^7 + 4580179968
0*a^15*b^22*c^40*d^8 - 100510203904*a^16*b^21*c^39*d^9 + 163810639872*a^17*
b^20*c^38*d^10 - 184331272192*a^18*b^19*c^37*d^11 + 65011712000*a^19*b^18*c
^36*d^12 + 336173465600*a^20*b^17*c^35*d^13 - 1148861808640*a^21*b^16*c^34*
d^14 + 2334365057024*a^22*b^15*c^33*d^15 - 3542660153344*a^23*b^14*c^32*d^1
6 + 4221965434880*a^24*b^13*c^31*d^17 - 4009062563840*a^25*b^12*c^30*d^18 +
 3039679217664*a^26*b^11*c^29*d^19 - 1830545260544*a^27*b^10*c^28*d^20 + 86
```

```
4890650624*a^28*b^9*c^27*d^21 - 313859768320*a^29*b^8*c^26*d^22 + 844732825
60*a^30*b^7*c^25*d^23 - 15888023552*a^31*b^6*c^24*d^24 + 1864368128*a^32*b^
5*c^23*d^25 - 102760448*a^33*b^4*c^22*d^26))*(-(2401*a^4*d^11 + 14641*b^4*c
^4*d^7 - 37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/
(4096*b^8*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*
c^17*d^2 - 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b
^3*c^14*d^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(3/4) - 111149
056*a^10*b^24*c^34*d^8 + 481296384*a^11*b^23*c^33*d^9 - 1233125376*a^12*b^2
2*c^32*d^10 + 1830010880*a^13*b^21*c^31*d^11 + 391331840*a^14*b^20*c^30*d^1
2 - 12820119552*a^15*b^19*c^29*d^13 + 46592393216*a^16*b^18*c^28*d^14 - 104
394047488*a^17*b^17*c^27*d^15 + 165297111040*a^18*b^16*c^26*d^16 - 19270290
6368*a^19*b^15*c^25*d^17 + 167824392192*a^20*b^14*c^24*d^18 - 109211664384*
a^21*b^13*c^23*d^19 + 52444708864*a^22*b^12*c^22*d^20 - 18062213120*a^23*b^
11*c^21*d^21 + 4224417792*a^24*b^10*c^20*d^22 - 601309184*a^25*b^9*c^19*d^2
3 + 39337984*a^26*b^8*c^18*d^24))))*(-(2401*a^4*d^11 + 14641*b^4*c^4*d^7 -
37268*a*b^3*c^3*d^8 + 35574*a^2*b^2*c^2*d^9 - 15092*a^3*b*c*d^10)/(4096*b^8
*c^19 + 4096*a^8*c^11*d^8 - 32768*a^7*b*c^12*d^7 + 114688*a^2*b^6*c^17*d^2
- 229376*a^3*b^5*c^16*d^3 + 286720*a^4*b^4*c^15*d^4 - 229376*a^5*b^3*c^14*d
^5 + 114688*a^6*b^2*c^13*d^6 - 32768*a*b^7*c^18*d))^(1/4)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.461 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=618

$$\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^2}+$$

Rubi [A] time = 0.96, antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{-9a^2d^2+4abcd+4b^2c^2}{2a^2c^3\sqrt{x}\,(bc-ad)}+\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^2}+\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^2}-\frac{d^{9/4}(13bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^2}-\frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^2}-\frac{4bc-9ad}{10ac^2x^{5/2}(bc-ad)}-\frac{d}{2cx^{5/2}\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(4*b*c - 9*a*d)/(10*a*c^2*(b*c - a*d)*x^(5/2)) + (4*b^2*c^2 + 4*a*b*c*d -
9*a^2*d^2)/(2*a^2*c^3*(b*c - a*d)*Sqrt[x]) - d/(2*c*(b*c - a*d)*x^(5/2)*(c
+ d*x^2)) - (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[
2]*a^(9/4)*(b*c - a*d)^2) + (b^(13/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2
) - (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (b^(1
3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(9/4)*(b*c - a*d)^2) - (d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) + (
d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{4bc-9ad-9bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx, x, \sqrt{x}\right)}{2c(bc-ad)}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{5\left(4b^2c^2+4abcd-9a^2d^2\right)}{x^2\left(a+bx^4\right)(}\right.}{10ac^2(bc}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{S}}{}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{\text{S}}{}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{(2}{}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{b}{}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{b}{}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} + \frac{b}{}$$

$$= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}} + \frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} - \frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)} - \frac{b}{}$$

Mathematica [A] time = 6.11, size = 621, normalized size = 1.00

$$\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2} - \frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2} + \frac{b^{13/4}\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^2} + \frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^2} + \frac{2(2ad+bc)}{a^2c^3\sqrt{x}} - \frac{d^{9/4}(13bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(ad-bc)^2} + \frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(ad-bc)^2} - \frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(ad-bc)^2} - \frac{d^{9/4}(13bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(ad-bc)^2} - \frac{d^3x^{3/2}}{2c^3\left(c+dx^2\right)(bc-ad)} - \frac{2}{5ac^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -2/(5*a*c^2*x^(5/2)) + (2*(b*c + 2*a*d))/(a^2*c^3*Sqrt[x]) - (d^3*x^(3/2))/
(2*c^3*(b*c - a*d)*(c + d*x^2)) + (b^(13/4)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*
b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^2) + (b^(
13/4)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqr
t[2]*a^(9/4)*(b*c - a*d)^2) - (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[(-(Sqrt[2]*c
^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(13/4)*(-(b*c
) + a*d)^2) - (d^(9/4)*(13*b*c - 9*a*d)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)
*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^2) + (b^(1
3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(9/4)*(b*c - a*d)^2) - (b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^2) - (d^(9/4)*(13*b*c -
 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt
[2]*c^(13/4)*(-(b*c) + a*d)^2) + (d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(-(b*c) + a
*d)^2)
```

IntegrateAlgebraic [A] time = 1.46, size = 410, normalized size = 0.66

$$-\frac{b^{13/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{9/4}(ad-bc)^2}-\frac{b^{13/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{9/4}(ad-bc)^2}+\frac{-4a^2c^2d+36a^2cd^2x^2+45a^2d^3x^4+4abc^3-16abc^2dx^2-20abcd^2x^4-20b^2c^3x^2-20b^2c^2dx^4}{10a^2c^3x^{5/2}\left(c+dx^2\right)(ad-bc)}+\frac{\left(13bcd^{9/4}-9ad^{13/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^2}+\frac{\left(13bcd^{9/4}-9ad^{13/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] (4*a*b*c^3 - 4*a^2*c^2*d - 20*b^2*c^3*x^2 - 16*a*b*c^2*d*x^2 + 36*a^2*c*d^2
*x^2 - 20*b^2*c^2*d*x^4 - 20*a*b*c*d^2*x^4 + 45*a^2*d^3*x^4)/(10*a^2*c^3*(-
(b*c) + a*d)*x^(5/2)*(c + d*x^2)) - (b^(13/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1
/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(9/4)*(-(b*c) +
a*d)^2) + ((13*b*c*d^(9/4) - 9*a*d^(13/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^2) - (b^(1
3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqr
t[2]*a^(9/4)*(-(b*c) + a*d)^2) + ((13*b*c*d^(9/4) - 9*a*d^(13/4))*ArcTanh[(
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(13/4
)*(b*c - a*d)^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A]   time = 1.23, size = 715, normalized size = 1.16

$$-\frac{d^{3}x^{\frac{3}{2}}}{2\left(bc^{4}-ac^{3}d\right)\left(dx^{2}+c\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{3}b^{2}c^{2}-2\sqrt{2}a^{4}bcd+\sqrt{2}a^{5}d^{2}}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^{3}b^{2}c^{2}-2\sqrt{2}a^{4}bcd+\sqrt{2}a^{5}d^{2}}-\frac{\left(ab^{3}\right)^{\frac{3}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{3}b^{2}c^{2}-2\sqrt{2}a^{4}bcd+\sqrt{2}a^{5}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{3}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^{3}b^{2}c^{2}-2\sqrt{2}a^{4}bcd+\sqrt{2}a^{5}d^{2}\right)}-\frac{\left(13\left(cd^{3}\right)^{\frac{3}{4}}bc-9\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{6}-2\sqrt{2}abc^{5}d+\sqrt{2}a^{2}c^{4}d^{2}\right)}-\frac{\left(13\left(cd^{3}\right)^{\frac{3}{4}}bc-9\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^{2}c^{6}-2\sqrt{2}abc^{5}d+\sqrt{2}a^{2}c^{4}d^{2}\right)}+\frac{\left(13\left(cd^{3}\right)^{\frac{3}{4}}bc-9\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{6}-2\sqrt{2}abc^{5}d+\sqrt{2}a^{2}c^{4}d^{2}\right)}-\frac{\left(13\left(cd^{3}\right)^{\frac{3}{4}}bc-9\left(cd^{3}\right)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^{2}c^{6}-2\sqrt{2}abc^{5}d+\sqrt{2}a^{2}c^{4}d^{2}\right)}+\frac{2\left(5bcx^{2}+10adx^{2}-ac\right)}{5a^{2}c^{3}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x^(3/2)/((b*c^4 - a*c^3*d)*(d*x^2 + c)) + (a*b^3)^(3/4)*b*arctan(1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*
c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) + (a*b^3)^(3/4)*b*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*c^2
 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) - 1/2*(a*b^3)^(3/4)*b*log(sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*
b*c*d + sqrt(2)*a^5*d^2) + 1/2*(a*b^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(
1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*
a^5*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt
(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^6 - 2*sqr
t(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^
3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) + 1/8*(
13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*
d^2) - 1/8*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqr
t(2)*a^2*c^4*d^2) + 2/5*(5*b*c*x^2 + 10*a*d*x^2 - a*c)/(a^2*c^3*x^(5/2))
```

maple [A]   time = 0.02, size = 612, normalized size = 0.99

$$\frac{a\,d^{4}x^{\frac{3}{2}}}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)c^{3}}-\frac{b\,d^{3}x^{\frac{3}{2}}}{2\left(ad-bc\right)^{2}\left(d\,x^{2}+c\right)c^{2}}+\frac{9\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{3}}+\frac{9\sqrt{2}\,a\,d^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{3}}+\frac{9\sqrt{2}\,a\,d^{3}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{3}}+\frac{\sqrt{2}\,b^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2\left(ad-bc\right)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,b^{3}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2\left(ad-bc\right)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,b^{3}\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{4\left(ad-bc\right)^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{13\sqrt{2}\,b\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{13\sqrt{2}\,b\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}-\frac{13\sqrt{2}\,b\,d^{2}\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}+\frac{4d}{a\,c^{3}\sqrt{x}}+\frac{2b}{a^{2}c^{2}\sqrt{x}}-\frac{2}{5a\,c^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/4*b^3/a^2/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*b^3/a^2/(a
*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^3
/a^2/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+
1/2*d^4/c^3/(a*d-b*c)^2*x^(3/2)/(d*x^2+c)*a-1/2*d^3/c^2/(a*d-b*c)^2*x^(3/2)
/(d*x^2+c)*b+9/16*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))
)+9/8*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)+9/8*d^3/c^3/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*a*arctan(2^(1/2)/(c/
```

```
d)^(1/4)*x^(1/2)-1)-13/16*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*ln((x-(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))-13/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)-13/8*d^2/c^2/(a*d-b*c)^2/(c/d)^(1/4)*2^(1/2)*b*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c^2/x^(5/2)+4/a/c^3/x^(1/2)*d+2/a^2/c^2
/x^(1/2)*b
```

maxima [A] time = 2.50, size = 551, normalized size = 0.89

$$\frac{b^{4}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)}-\frac{\left(13\,bcd^{3}-9\,ad^{4}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{2}c^{5}-2\,abc^{4}d+a^{2}c^{3}d^{2}\right)}-\frac{4\,abc^{3}-4\,a^{2}c^{2}d-5\left(4\,b^{2}c^{2}d+4\,abcd^{2}-9\,a^{2}d^{3}\right)x^{4}-4\left(5\,b^{2}c^{3}+4\,abc^{2}d-9\,a^{2}cd^{2}\right)x^{2}}{10\left(\left(a^{2}bc^{4}d-a^{3}c^{3}d^{2}\right)x^{\frac{9}{2}}+\left(a^{2}bc^{5}-a^{3}c^{4}d\right)x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/16*(13*b*c*d^3 - 9*a*d^4)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqr
t(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
 + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^
(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^5 - 2*a*b*c^
4*d + a^2*c^3*d^2) - 1/10*(4*a*b*c^3 - 4*a^2*c^2*d - 5*(4*b^2*c^2*d + 4*a*b
*c*d^2 - 9*a^2*d^3)*x^4 - 4*(5*b^2*c^3 + 4*a*b*c^2*d - 9*a^2*c*d^2)*x^2)/((
a^2*b*c^4*d - a^3*c^3*d^2)*x^(9/2) + (a^2*b*c^5 - a^3*c^4*d)*x^(5/2))
```

mupad [B] time = 5.84, size = 17850, normalized size = 28.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] - (2/(5*a*c) - (2*x^2*(9*a*d + 5*b*c))/(5*a^2*c^2) + (d*x^4*(4*b^2*c^2 - 9*
a^2*d^2 + 4*a*b*c*d))/(2*a^2*c^3*(a*d - b*c)))/(c*x^(5/2) + d*x^(9/2)) - 2*
atan((524288*a^3*b^16*c^32*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 7
9092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^
8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2
 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*
```

```
d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 2654208*a^19*c
^16*d^16*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^1
0 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*
c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*
c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b
^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 346112*b^15*c^18*d^6*x^(1/2)*(-(
6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^
2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7
*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^
4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*
a*b^7*c^20*d))^(1/4) - 479232*a*b^14*c^17*d^7*x^(1/2)*(-(6561*a^4*d^13 + 28
561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3
*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 1146
88*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 2
29376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/
4) - 4194304*a^4*b^15*c^31*d*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 -
 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*
b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d
^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^1
6*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 28901376*a^1
8*b*c^17*d^15*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^
3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096
*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3
*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*
a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 165888*a^2*b^13*c^16*d^8*x^
(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a
^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 -
32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 +
286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6
 - 32768*a*b^7*c^20*d))^(1/4) + 3655808*a^3*b^12*c^15*d^9*x^(1/2)*(-(6561*a
^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11
 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^1
4*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*
c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*
c^20*d))^(1/4) - 10123776*a^4*b^11*c^14*d^10*x^(1/2)*(-(6561*a^4*d^13 + 285
61*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*
b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 11468
8*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 22
9376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4
) + 10513152*a^5*b^10*c^13*d^11*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^
9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(40
96*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^1
9*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*
c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) - 4852224*a
^6*b^9*c^12*d^12*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3
```

```
*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4
096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*
a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 1146
88*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) + 839808*a^7*b^8*c^11*d^13
*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 8213
4*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8
 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3
 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*
d^6 - 32768*a*b^7*c^20*d))^(1/4) + 14680064*a^5*b^14*c^30*d^2*x^(1/2)*(-(65
61*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*
d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b
*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*
b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*
b^7*c^20*d))^(5/4) - 29360128*a^6*b^13*c^29*d^3*x^(1/2)*(-(6561*a^4*d^13 +
28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a
^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 11
4688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 -
 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(
5/4) + 36700160*a^7*b^12*c^28*d^4*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*
d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(
4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c
^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^
3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 2936012
8*a^8*b^11*c^27*d^5*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*
b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21
+ 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 2293
76*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 1
14688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 20217856*a^9*b^10*c^2
6*d^6*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 +
 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^1
3*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^1
8*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*
c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 56164352*a^10*b^9*c^25*d^7*x^(1/2)*
(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2
*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*
a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720
*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 327
68*a*b^7*c^20*d))^(5/4) + 219578368*a^11*b^8*c^24*d^8*x^(1/2)*(-(6561*a^4*d
^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 3
7908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^
7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17
*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20
*d))^(5/4) - 546045952*a^12*b^7*c^23*d^9*x^(1/2)*(-(6561*a^4*d^13 + 28561*b
^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*
d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^
```

```
2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376
*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) +
891355136*a^13*b^6*c^22*d^10*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 -
 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*
b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d
^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^1
6*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 995491840*a^
14*b^5*c^21*d^11*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3
*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4
096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*
a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 1146
88*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 770244608*a^15*b^4*c^20*
d^12*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 +
82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13
*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18
*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c
^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) - 407633920*a^16*b^3*c^19*d^13*x^(1/2)
*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^
2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768
*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 28672
0*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32
768*a*b^7*c^20*d))^(5/4) + 141197312*a^17*b^2*c^18*d^14*x^(1/2)*(-(6561*a^4
*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 -
 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*
d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^
17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^
20*d))^(5/4))/(4782969*a^14*d^22 + 562432*b^14*c^14*d^8 - 43264*a*b^13*c^13
*d^9 + 159744*a^2*b^12*c^12*d^10 + 176128*a^3*b^11*c^11*d^11 + 192512*a^4*b
^10*c^10*d^12 + 208896*a^5*b^9*c^9*d^13 + 225280*a^6*b^8*c^8*d^14 + 241664*
a^7*b^7*c^7*d^15 + 258048*a^8*b^6*c^6*d^16 - 62474085*a^9*b^5*c^5*d^17 + 17
8882749*a^10*b^4*c^4*d^18 - 211329810*a^11*b^3*c^3*d^19 + 127191546*a^12*b^
2*c^2*d^20 - 38795193*a^13*b*c*d^21))*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9
- 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096
*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*
d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^
16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) - atan((a^3*b
^16*c^32*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^1
0 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*
c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*
c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b
^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*524288i + a^19*c^16*d^16*x^(1/2)*(
-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*
c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a
^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*
a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 3276
```

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8*a*b^7*c^20*d))^(5/4)*2654208i + b^15*c^18*d^6*x^(1/2)*(-(6561*a^4*d^13 +
28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a
^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 11
4688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 -
 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(
1/4)*346112i - a*b^14*c^17*d^7*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9
 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(409
6*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19
*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c
^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4)*479232i - a^
4*b^15*c^31*d*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^
3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096
*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3
*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*
a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*4194304i - a^18*b*c^17*d^15*x
^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*
a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 -
 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 +
 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^
6 - 32768*a*b^7*c^20*d))^(5/4)*28901376i + a^2*b^13*c^16*d^8*x^(1/2)*(-(656
1*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d
^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*
c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b
^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b
^7*c^20*d))^(1/4)*165888i + a^3*b^12*c^15*d^9*x^(1/2)*(-(6561*a^4*d^13 + 28
561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3
*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 1146
88*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 2
29376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/
4)*3655808i - a^4*b^11*c^14*d^10*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d
^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4
096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^
19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3
*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4)*10123776i
 + a^5*b^10*c^13*d^11*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a
*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21
 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229
376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 +
114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4)*10513152i - a^6*b^9*c^
12*d^12*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10
 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c
^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c
^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^
2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4)*4852224i + a^7*b^8*c^11*d^13*x^(1/2
)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b
```

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^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 3276
8*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 2867
20*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 3
2768*a*b^7*c^20*d))^(1/4)*839808i + a^5*b^14*c^30*d^2*x^(1/2)*(-(6561*a^4*d
^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 3
7908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^
7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17
*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20
*d))^(5/4)*14680064i - a^6*b^13*c^29*d^3*x^(1/2)*(-(6561*a^4*d^13 + 28561*b
^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*
d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^
2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376
*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*29
360128i + a^7*b^12*c^28*d^4*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 -
79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b
^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^
2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16
*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*36700160i - a^8
*b^11*c^27*d^5*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c
^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 409
6*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^
3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688
*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*29360128i + a^9*b^10*c^26*d^
6*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 821
34*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^
8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^
3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15
*d^6 - 32768*a*b^7*c^20*d))^(5/4)*20217856i - a^10*b^9*c^25*d^7*x^(1/2)*(-(
6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^
2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7
*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^
4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*
a*b^7*c^20*d))^(5/4)*56164352i + a^11*b^8*c^24*d^8*x^(1/2)*(-(6561*a^4*d^13
 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 3790
8*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 +
 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^
4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d)
)^(5/4)*219578368i - a^12*b^7*c^23*d^9*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4
*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^
12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*
b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a
^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*5460
45952i + a^13*b^6*c^22*d^10*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 -
79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b
^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^
```

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2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16
*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*891355136i - a^
14*b^5*c^21*d^11*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3
*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4
096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*
a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 1146
88*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*995491840i + a^15*b^4*c^20
*d^12*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 +
 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^1
3*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^1
8*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*
c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4)*770244608i - a^16*b^3*c^19*d^13*x^(1/
2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*
b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 327
68*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286
720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 -
32768*a*b^7*c^20*d))^(5/4)*407633920i + a^17*b^2*c^18*d^14*x^(1/2)*(-(6561*
a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^1
1 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^
14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4
*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7
*c^20*d))^(5/4)*141197312i)/(4782969*a^14*d^22 + 562432*b^14*c^14*d^8 - 432
64*a*b^13*c^13*d^9 + 159744*a^2*b^12*c^12*d^10 + 176128*a^3*b^11*c^11*d^11
+ 192512*a^4*b^10*c^10*d^12 + 208896*a^5*b^9*c^9*d^13 + 225280*a^6*b^8*c^8*
d^14 + 241664*a^7*b^7*c^7*d^15 + 258048*a^8*b^6*c^6*d^16 - 62474085*a^9*b^5
*c^5*d^17 + 178882749*a^10*b^4*c^4*d^18 - 211329810*a^11*b^3*c^3*d^19 + 127
191546*a^12*b^2*c^2*d^20 - 38795193*a^13*b*c*d^21))*(-(6561*a^4*d^13 + 2856
1*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b
*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688
*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229
376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(1/4)
*2i - 2*atan((8192*a^11*b^16*c^21*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*
c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 11
20*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^1
6*b*c*d^7))^(5/4) + 13122*a^15*b^8*d^13*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^
9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^
3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 1
28*a^16*b*c*d^7))^(1/4) + 41472*a^27*c^5*d^16*x^(1/2)*(-b^13/(16*a^17*d^8 +
 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*
c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d
^6 - 128*a^16*b*c*d^7))^(5/4) - 75816*a^14*b^9*c*d^12*x^(1/2)*(-b^13/(16*a^
17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a
^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b
^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4) - 65536*a^12*b^15*c^20*d*x^(1/2)*(-b^
13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^
```

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2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 4
48*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) - 451584*a^26*b*c^6*d^15*x^(
1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b
^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^
3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) + 5408*a^8*b^15*c^7
*d^6*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 44
8*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^1
4*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4) - 7488*a^9*
b^14*c^6*d^7*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^
7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 -
 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4) + 2
592*a^10*b^13*c^5*d^8*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^
10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*
c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^
(1/4) + 57122*a^11*b^12*c^4*d^9*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^
8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120
*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*
b*c*d^7))^(1/4) - 158184*a^12*b^11*c^3*d^10*x^(1/2)*(-b^13/(16*a^17*d^8 + 1
6*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^
5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6
 - 128*a^16*b*c*d^7))^(1/4) + 164268*a^13*b^10*c^2*d^11*x^(1/2)*(-b^13/(16*
a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896
*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15
*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4) + 229376*a^13*b^14*c^19*d^2*x^(1/2)
*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c
^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^
5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) - 458752*a^14*b^13*c^18
*d^3*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 44
8*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^1
4*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) + 573440*a^
15*b^12*c^17*d^4*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^
7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d
^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)
 - 458752*a^16*b^11*c^16*d^5*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 -
 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^
13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c
*d^7))^(5/4) + 315904*a^17*b^10*c^15*d^6*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a
^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d
^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 -
128*a^16*b*c*d^7))^(5/4) - 877568*a^18*b^9*c^14*d^7*x^(1/2)*(-b^13/(16*a^17
*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^1
2*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2
*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) + 3430912*a^19*b^8*c^13*d^8*x^(1/2)*(-b
^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d
^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 +
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448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) - 8531968*a^20*b^7*c^12*d^9
*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^
11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^
3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) + 13927424*a^21
*b^6*c^11*d^10*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*
c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4
 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) -
 15554560*a^22*b^5*c^10*d^11*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 -
 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^
13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c
*d^7))^(5/4) + 12035072*a^23*b^4*c^9*d^12*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*
a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*
d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 -
 128*a^16*b*c*d^7))^(5/4) - 6369280*a^24*b^3*c^8*d^13*x^(1/2)*(-b^13/(16*a^
17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a
^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b
^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4) + 2206208*a^25*b^2*c^7*d^14*x^(1/2)*(
-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6
*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5
 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4))/(256*b^22*c^11 - 6561*a^
11*b^11*d^11 + 24786*a^10*b^12*c*d^10 + 768*a^2*b^20*c^9*d^2 + 1024*a^3*b^1
9*c^8*d^3 + 1280*a^4*b^18*c^7*d^4 + 1536*a^5*b^17*c^6*d^5 + 1792*a^6*b^16*c
^5*d^6 + 2048*a^7*b^15*c^4*d^7 + 2304*a^8*b^14*c^3*d^8 - 26001*a^9*b^13*c^2
*d^9 + 512*a*b^21*c^10*d))*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*
b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4
*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/
4) - atan((a^11*b^16*c^21*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 12
8*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*
b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^
7))^(5/4)*8192i + a^15*b^8*d^13*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^
8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120
*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*
b*c*d^7))^(1/4)*13122i + a^27*c^5*d^16*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9
*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3
 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 12
8*a^16*b*c*d^7))^(5/4)*41472i - a^14*b^9*c*d^12*x^(1/2)*(-b^13/(16*a^17*d^8
 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^
5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2
*d^6 - 128*a^16*b*c*d^7))^(1/4)*75816i - a^12*b^15*c^20*d*x^(1/2)*(-b^13/(1
6*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 8
96*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^
15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*65536i - a^26*b*c^6*d^15*x^(1/2)*
(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^
6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5
 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*451584i + a^8*b^15*c^7*d
```

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^6*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*
a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*
b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4)*5408i - a^9*b
^14*c^6*d^7*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7
*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 -
896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4)*7488
i + a^10*b^13*c^5*d^8*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^
10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*
c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^
(1/4)*2592i + a^11*b^12*c^4*d^9*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^
8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120
*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*
b*c*d^7))^(1/4)*57122i - a^12*b^11*c^3*d^10*x^(1/2)*(-b^13/(16*a^17*d^8 + 1
6*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^
5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6
 - 128*a^16*b*c*d^7))^(1/4)*158184i + a^13*b^10*c^2*d^11*x^(1/2)*(-b^13/(16
*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 89
6*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^1
5*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(1/4)*164268i + a^13*b^14*c^19*d^2*x^(1/
2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6
*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*
d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*229376i - a^14*b^13*c
^18*d^3*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d +
 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*
a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*458752i
+ a^15*b^12*c^17*d^4*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^1
0*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c
^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(
5/4)*573440i - a^16*b^11*c^16*d^5*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*
c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 11
20*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^1
6*b*c*d^7))^(5/4)*458752i + a^17*b^10*c^15*d^6*x^(1/2)*(-b^13/(16*a^17*d^8
+ 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5
*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*
d^6 - 128*a^16*b*c*d^7))^(5/4)*315904i - a^18*b^9*c^14*d^7*x^(1/2)*(-b^13/(
16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 -
896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a
^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*877568i + a^19*b^8*c^13*d^8*x^(1
/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^
6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3
*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*3430912i - a^20*b^7*
c^12*d^9*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d
+ 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896
*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*8531968
i + a^21*b^6*c^11*d^10*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a
```

```
^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4
*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))
^(5/4)*13927424i - a^22*b^5*c^10*d^11*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*
b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3
+ 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128
*a^16*b*c*d^7))^(5/4)*15554560i + a^23*b^4*c^9*d^12*x^(1/2)*(-b^13/(16*a^17
*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^1
2*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2
*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*12035072i - a^24*b^3*c^8*d^13*x^(1/2)*(
-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6
*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5
+ 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*6369280i + a^25*b^2*c^7*d
^14*x^(1/2)*(-b^13/(16*a^17*d^8 + 16*a^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448
*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14
*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 - 128*a^16*b*c*d^7))^(5/4)*2206208i)/(2
56*b^22*c^11 - 6561*a^11*b^11*d^11 + 24786*a^10*b^12*c*d^10 + 768*a^2*b^20*
c^9*d^2 + 1024*a^3*b^19*c^8*d^3 + 1280*a^4*b^18*c^7*d^4 + 1536*a^5*b^17*c^6
*d^5 + 1792*a^6*b^16*c^5*d^6 + 2048*a^7*b^15*c^4*d^7 + 2304*a^8*b^14*c^3*d^
8 - 26001*a^9*b^13*c^2*d^9 + 512*a*b^21*c^10*d))*(-b^13/(16*a^17*d^8 + 16*a
^9*b^8*c^8 - 128*a^10*b^7*c^7*d + 448*a^11*b^6*c^6*d^2 - 896*a^12*b^5*c^5*d
^3 + 1120*a^13*b^4*c^4*d^4 - 896*a^14*b^3*c^3*d^5 + 448*a^15*b^2*c^2*d^6 -
128*a^16*b*c*d^7))^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.462 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=631

$$-\frac{a^{5/4}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\sqrt{2}\right.}{\sqrt{2}(bc-ad)^3}$$

Rubi [A] time = 0.81, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 470, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}-\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}-\frac{a^{5/4}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3}-\frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}(bc-ad)^3}+\frac{\sqrt{x}(bc-9ad)}{16d\left(c+dx^2\right)(bc-ad)^2}-\frac{c\sqrt{x}}{4d\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(c*Sqrt[x])/(4*d*(b*c - a*d)*(c + d*x^2)^2) + ((b*c - 9*a*d)*Sqrt[x])/(16*
d*(b*c - a*d)^2*(c + d*x^2)) - (a^(5/4)*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((3*b^2*c^2 -
30*a*b*c*d - 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*
Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(5
/4)*(b*c - a*d)^3) - (a^(5/4)*b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(5/4)*b^(3/4)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)
^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3) + (
(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{ac+(bc-8ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4d(bc-ad)}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{ac(3bc+5ad)+3bc(bc-9ad)}{\left(a+bx^4\right)\left(c+dx^4\right)}\right)}{16cd(bc-ad)^2}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(2a^2b\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \right)}{(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a^{3/2}b\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, \right)}{(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(a^{3/2}\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}}\right)}{2(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} - \frac{a^{5/4}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\right)}{2\sqrt{2}\,(bc-ad)^3}$$

$$= -\frac{c\sqrt{x}}{4d(bc-ad)\left(c+dx^2\right)^2} + \frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2\left(c+dx^2\right)} - \frac{a^{5/4}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^3} -$$

Mathematica [A] time = 0.54, size = 640, normalized size = 1.01

$$\frac{-32\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+32\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-64\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+64\sqrt{2}a^{5/4}b^{3/4}c^{3/4}d^{5/4}\left(c+dx^2\right)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)-\sqrt{2}\left(c+dx^2\right)^2\left(-5a^2d^2-30abcd+3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)+\sqrt{2}\left(c+dx^2\right)^2\left(-5a^2d^2-30abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)-2\sqrt{2}\left(c+dx^2\right)^2\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)+2\sqrt{2}\left(c+dx^2\right)^2\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)+8c^{3/4}\sqrt[4]{d}\sqrt{x}\left(c+dx^2\right)(bc-9ad)(bc-ad)-32c^{7/4}\sqrt[4]{d}\sqrt{x}(bc-ad)^2}{128c^{3/4}d^{5/4}\left(c+dx^2\right)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*Sqrt[x] + 8*c^(3/4)*d^(1/4)*(b*c - 9*a*d
)*(b*c - a*d)*Sqrt[x]*(c + d*x^2) - 64*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5
```

```
/4)*(c + d*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 64*Sqrt[2
]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*
x^2)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)] + 2*Sqrt[2]*(3*b^2*c^2
 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)] - 32*Sqrt[2]*a^(5/4)*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 32*Sqrt[2]*a^(5/4)
*b^(3/4)*c^(3/4)*d^(5/4)*(c + d*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] - Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c +
d*x^2)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x] + Sqrt[
2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*(c + d*x^2)^2*Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(128*c^(3/4)*d^(5/4)*(b*c - a*d)^3*(
c + d*x^2)^2)
```

IntegrateAlgebraic [A] time = 1.99, size = 378, normalized size = 0.60

$$-\frac{a^{5/4}b^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}\,c^{3/4}d^{5/4}(bc-ad)^3}+\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{3/4}d^{5/4}(bc-ad)^3}+\frac{-5acd\sqrt{x}-9ad^2x^{5/2}-3bc^2\sqrt{x}+bcdx^{5/2}}{16d\left(c+dx^2\right)^2(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-3*b*c^2*Sqrt[x] - 5*a*c*d*Sqrt[x] + b*c*d*x^(5/2) - 9*a*d^2*x^(5/2))/(16*
d*(-(b*c) + a*d)^2*(c + d*x^2)^2) - (a^(5/4)*b^(3/4)*ArcTan[(a^(1/4)/(Sqrt[
2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*(b*c - a*d)
^3) - ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/(32*Sqrt[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3
) + (a^(5/4)*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(Sqrt[2]*(b*c - a*d)^3) + ((3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)
*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt
[2]*c^(3/4)*d^(5/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.57, size = 944, normalized size = 1.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 - s
qrt(2)*a^3*d^3) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
- 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt
(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3
*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*a*log(-sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2
*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(3*(c*d^3)^(1/4)*b^2*c
^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)
*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sq
rt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) + 1/32*(
3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt
(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(
2)*a^3*c*d^5) + 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d -
5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - s
qrt(2)*a^3*c*d^5) - 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*
d - 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^
4 - sqrt(2)*a^3*c*d^5) + 1/16*(b*c*d*x^(5/2) - 9*a*d^2*x^(5/2) - 3*b*c^2*sq
rt(x) - 5*a*c*d*sqrt(x))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(d*x^2 + c)^2
)
```

maple [**A**] time = 0.02, size = 839, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/4*a*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*a*b/(a*d-b*c)
^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*a*b/(a*d-b
*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-9/16/(a*d-b
*c)^3/(d*x^2+c)^2*x^(5/2)*a^2*d^2+5/8/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a*b*c
*d-1/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2*c^2-5/16/(a*d-b*c)^3/(d*x^2+c)^
2*c*d*x^(1/2)*a^2+1/8/(a*d-b*c)^3/(d*x^2+c)^2*c^2*x^(1/2)*a*b+3/16/(a*d-b*c
)^3/(d*x^2+c)^2*c^3/d*x^(1/2)*b^2+5/64/(a*d-b*c)^3*d*(c/d)^(1/4)/c*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+15/32/(a*d-b*c)^3*(c/d)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-3/64/(a*d-b*c)^3/d*(c/d)^(1/
4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+5/64/(a*d-b*c)^3*d*(
c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+15/32/(a*d-b
```

```
*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-3/64/(a
*d-b*c)^3/d*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2
+5/128/(a*d-b*c)^3*d*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2
)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+15/64/(a*d-
b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-3/128/(a*d-b*c)^3/d*(c/d)^(
1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4
)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [**A**] time = 2.64, size = 653, normalized size = 1.03

$$\frac{\left(\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)a^2}{4\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)}+\frac{\left(bcd-9ad^2\right)x^{\frac{5}{2}}-\left(3bc^2+5acd\right)\sqrt{x}}{16\left(b^2c^4d-2abc^3d^2+a^2c^2d^3+\left(b^2c^2d^3-2abcd^4+a^2d^5\right)x^4+2\left(b^2c^3d^2-2abc^2d^3+a^2cd^4\right)x^2\right)}+\frac{\frac{2\sqrt{2}\left(3b^2c^2-30abcd-5a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(3b^2c^2-30abcd-5a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(3b^2c^2-30abcd-5a^2d^2\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(3b^2c^2-30abcd-5a^2d^2\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2
)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*a^2/(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*((b*c*d - 9*a*d^2)
*x^(5/2) - (3*b*c^2 + 5*a*c*d)*sqrt(x))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*
c^2*d^3 + a^2*c*d^4)*x^2) + 1/128*(2*sqrt(2)*(3*b^2*c^2 - 30*a*b*c*d - 5*a^
2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b^2*c^2
- 30*a*b*c*d - 5*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + s
qrt(2)*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b^2*c^2 - 30*a*b
*c*d - 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c
))/(c^(3/4)*d^(1/4)))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)
```

mupad [**B**] time = 4.22, size = 35251, normalized size = 55.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] atan((((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*
c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 -
```

```
 (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a
^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c
^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*
a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^
12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 -
 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 79
20*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*
b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^
17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720
*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10
+ 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9
*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^
14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c
^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*
a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) - (x^(1/2)*(409600*a^
19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 14195
0976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*
c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3
714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^
12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^
15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^
17*b^6*c^2*d^18))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*
a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7
*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
- 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*
b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c
^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d
^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 +
1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4))*(-(
a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b
^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c
^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^
9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*
1i - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d +
 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 +
 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d
^6)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c
^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 +
 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*
b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a
^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7
920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672
*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*
b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - ((((81*a^3*b
^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148
```

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215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*
c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/20
48)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^
5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 -
8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10
*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7
*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8
 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*
a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 1062
40*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8
 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11
*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 133760
0*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 -
6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^
6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 +
28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/2)*(409600*a^19*b^4*d^20 + 1474
56*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14
*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712
768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^1
3*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 +
 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 103612416*a^1
5*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18))/
(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3
- 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6
*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*
d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12
 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*
b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5
*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*
d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4))*(-(a^5*b^3)/(16*a^12*
d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*
a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7
*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*
c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*1i + (x^(1/2)*(81*
a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c
*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c
^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6)*1i)/(4096*(a^1
2*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*
b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d
^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*
a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12
*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^
4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 +
 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192
*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4))/(((((81*a^3*b^13*c^7)/2048 - (6
```

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25*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)
/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (3
86451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8
*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4
*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (
((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a
^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b
^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^
3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1
/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d
^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^1
3*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 591360
0*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^1
4 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*
d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3
*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7
 - 8*a^7*b*c*d^8) - (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d
^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a
^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*
d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 13981
77792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^
10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 7
7070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18))/(4096*(a^12*d^13 +
 b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*
d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792
*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2
*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 +
1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 1267
2*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^
8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*
c^11*d - 192*a^11*b*c*d^11))^(3/4))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^1
2 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 -
12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 792
0*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b
^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*
a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13
*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b
^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b^12*c^12*d - 1
2*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^
8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8
 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a
^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^
10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^
5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 -
3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^1
```

```
1*b*c*d^11))^(1/4) + ((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 -
(3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*
c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2
048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2
 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^
3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^
12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^
4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b
^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^
2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^1
8 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^1
2*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8
*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3
421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*
d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^
8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^
4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(
1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c
^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 20869
93920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^1
4*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12
 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*
a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^1
7 - 17694720*a^17*b^6*c^2*d^18))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11
*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^
5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a
^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*
d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 -
 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 1478
4*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9
*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^
11))^(3/4))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d
^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 +
14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520
*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*
c*d^11))^(1/4) + (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*
b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^
12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^
10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a
^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*
c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 -
 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b
^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^
9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^
6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1
```

```
056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)))*(-(
a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b
^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c
^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^
9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*
2i - ((x^(5/2)*(9*a*d - b*c))/(16*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^(1/
2)*(3*b*c^2 + 5*a*c*d))/(16*d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*
x^4 + 2*c*d*x^2) - 2*atan(((((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/
2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^
5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3
*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*
b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5
- 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/
(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^
3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 -
12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*
a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^1
6*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a
^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 -
6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c
^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^
13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d
^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4
+ 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*
d^8) - (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624
*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*
d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143
488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^
12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 -
 116391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*
b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12
*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*
a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c
^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
- 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b
^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*
c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*
d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 1
92*a^11*b*c*d^11))^(3/4)*1i)*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 105
6*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a
^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b
^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^1
1*d - 192*a^11*b*c*d^11))^(1/4) + (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^
7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^
2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3
```

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*d^5 + 1024380*a^10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^1
1*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d
^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*
a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c
*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2
- 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 147
84*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^
9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d
^11))^(1/4) - (((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*
a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^
2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 +
(997755*a^8*b^8*c^2*d^5)/2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 +
 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*
c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12
 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*
b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5
*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*
d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18
+ 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*
d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b
^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 342
1440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^
15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*
d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*
c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/
2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^1
5*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993
920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*
c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 -
 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^
14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17
- 17694720*a^17*b^6*c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^1
1*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d
^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*
a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c
*d^12)))*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2
- 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 147
84*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^
9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d
^11))^(3/4)*1i)*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^
10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^
5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 -
3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^1
1*b*c*d^11))^(1/4) - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*
a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^
```

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7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 102438
0*a^10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 +
66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*
b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d
^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a
^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^
9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^
6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9
 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4))/
((((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*
d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (26
2899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b
^8*c^2*d^5)/2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c
^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*
a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^
12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 -
 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 79
20*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*
b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^
17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720
*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10
+ 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9
*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^
14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c
^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*
a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) - (x^(1/2)*(409600*a^
19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 14195
0976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*
c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3
714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^
12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^
15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^
17*b^6*c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 +
66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*
b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d
^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a
^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^
9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^
6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9
 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1
i)*(-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520
*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6
*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*
c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^
(1/4)*1i + (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c
```

```
^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5
*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9
*c^2*d^6)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*
b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7
*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 22
0*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)
/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d
^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 -
 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056
*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) + (((((8
1*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048
 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^
6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*
d^5)/2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3
 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2
*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 10
56*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*
a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*
b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^
11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14
*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^
14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 72652
80*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^
13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*
c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2
 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3
*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/2)*(409600*a^19*b^4*
d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^
5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^
8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 37140561
92*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11
*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 10
3612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*
c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*
b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7
*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 22
0*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)
/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d
^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 -
 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056
*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1i)*(-(a
^5*b^3)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^
9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^
6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9
 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*1
```

```
i - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d +
322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 +
658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^
6)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^
10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 +
924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b
^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(a^5*b^3)/(16*a^
12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 79
20*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*
a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b
^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)))*(-(a^5*b^3)/(
16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3
 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 1
2672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a
^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - atan(-((
(-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^
3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^
6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 1677721
6*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1
107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*
b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11
 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*
a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*(((81*a^3*b^13*c^7)
/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*
b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)
/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8
*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^
4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*
c*d^8) + (-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b
^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b
^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17
+ 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4
*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 830472
1920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6
*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 36
90987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(3/4)*(((-(625*a
^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 6493
50*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240
*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c
^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 110729625
6*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11
*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287
555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*
c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*(1280*a^16*b^4*c*d^18 + 896
0*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 -
```

```
 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c
^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*
a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 -
23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8
*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d
^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) - (x^(1/2)*(4
09600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5
 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a
^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*
d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 2595
22560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^
9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 176
94720*a^17*b^6*c^2*d^18))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d
^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792
*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*
c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))
)*1i - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d
 + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3
 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2
*d^6)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10
*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6
 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^
9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(625*a^8*d^8
+ 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*
b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*
c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5
 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b
^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 -
13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*
a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^1
4 + 1107296256*a^10*b^2*c^5*d^15))^(1/4) - ((-(625*a^8*d^8 + 81*b^8*c^8 + 4
8060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 51
3000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^
7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b
^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3
690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*
b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12
 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a
^10*b^2*c^5*d^15))^(1/4)*(((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048
 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^
11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4
)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*
d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5
*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (-(625*a^8*d^8 + 81*b^
8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4
```

```
*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d +
 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 2013
26592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^1
3*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 1328755
5072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5
*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 110
7296256*a^10*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2
*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5
*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^
7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14
*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 369098752
0*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10
*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 83047
21920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*
c^5*d^15))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^
4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 41
53600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*
d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^1
2*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*
a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6
*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^
6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^
3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6
+ 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a
^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9
*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 5089
52576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8
*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18))/(4096
*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220
*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*
c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10
+ 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12))))*1i + (x^(1/2)*(81*a^4*b^15*c^
8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 4806
0*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 32
8680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6)*1i)/(4096*(a^12*d^13 + b^
12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4
 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^
7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^
2*d^11 - 12*a^11*b*c*d^12)))*(-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^
6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^
3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(167
77216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 -
201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b
^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 +
 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a
```

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^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^1
5))^(1/4))/(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a
^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a
^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d
^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b
*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 83
04721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6
*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13
- 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*(((81*
a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 +
 (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*
b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^
5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a
^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d
^7 - 8*a^7*b*c*d^8) + (-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 -
 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 +
 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a
^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 2013265
92*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12
*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 155021
47584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*
c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(3/
4)*(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c
^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c
^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16
777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^1
6 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920
*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9
*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 369098
7520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*(1280*a^16*b^4
*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^
15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 64627
20*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^
12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^
7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 +
b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*
a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8)
- (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*
b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 -
 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a
^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^
8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 1163
91936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c
^3*d^17 - 17694720*a^17*b^6*c^2*d^18))/(4096*(a^12*d^13 + b^12*c^12*d - 12*
a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*
```

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c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^1
1*b*c*d^12)))) - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*
b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^
12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^
10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a
^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*
c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 -
 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(625*a
^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 6493
50*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240
*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c
^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 110729625
6*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11
*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287
555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*
c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4) + ((-(625*a^8*d^8 + 81*b^8*
c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d
^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 1
5000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326
592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*
d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 132875550
72*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c
^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 11072
96256*a^10*b^2*c^5*d^15))^(1/4)*(((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^
7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901
*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*
c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b
^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 -
 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (-(625*a^8*d^8
+ 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*
b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*
c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5
 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b
^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 -
13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*
a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^1
4 + 1107296256*a^10*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 81*b^8*c^8 + 48
060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513
000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7
*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^
11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 36
90987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b
^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12
+ 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^
```

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10*b^2*c^5*d^15))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 10
6240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d
^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^
11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337
600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16
- 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*
b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6
+ 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/2)*(409600*a^19*b^4*d^20 + 14
7456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^
14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 39117
12768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b
^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13
 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 103612416*a
^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18)
)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^
3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a
^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^
3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))) + (x^(1/2)*(81*a^4*b^1
5*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 +
48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4
+ 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b
^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^
4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a
^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c
^2*d^11 - 12*a^11*b*c*d^12)))*(-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c
^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c
^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16
777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 -
 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*
b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10
+ 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*
a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^
15))^(1/4)))*(-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a
^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a
^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d
^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b
*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 83
04721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6
*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13
- 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*2i + 2
*atan(-(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b
^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b
^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17
+ 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4
*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 830472
```

1920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6
*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 36
90987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*((((81*a^3
*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (1
48215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^1
0*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/
2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*
a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*
d^7 - 8*a^7*b*c*d^8) + (-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2
- 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5
+ 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*
a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326
592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^1
2*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502
147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4
*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(3
/4)*(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*
c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*
c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 1
6777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^
16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 830472192
0*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^
9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 36909
87520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*(1280*a^16*b^
4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b
^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462
720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d
^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b
^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 +
b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70
*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8)
- (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4
*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7
- 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*
a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c
^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116
391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*
c^3*d^17 - 17694720*a^17*b^6*c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12*d -
12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*
b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d
^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12
*a^11*b*c*d^12)))*1i) + (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 32
40*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800
*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 102
4380*a^10*b^9*c^2*d^6))/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2

```
 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a
^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^
4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(
-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3
 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6
 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216
*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 11
07296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b
^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11
- 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a
^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4) - ((-(625*a^8*d^8 +
81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^
4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^
7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 -
 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^1
0*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13
287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^
7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14
+ 1107296256*a^10*b^2*c^5*d^15))^(1/4)*((((81*a^3*b^13*c^7)/2048 - (625*a^1
0*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048
+ (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*
a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^5)/2048)*1i)/(a^8*d^9 + b^8*c
^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b
^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (-(
625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 +
 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 -
 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b
^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107
296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8
*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 -
13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9
*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 81*
b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c
^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d
 + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 20
1326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c
^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287
555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b
^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1
107296256*a^10*b^2*c^5*d^15))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c
^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6
*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 72
65280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6
*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b
^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d
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^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*
b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (x^(1/2)*(409600*a^19*b
^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976
*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12
*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 37140
56192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b
^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 -
 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b
^6*c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a
^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*
c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 -
 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*1i) - (x
^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200
*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566
*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6))/(4
096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 -
220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b
^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^
10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*(-(625*a^8*d^8 + 81*b^8*c^8
 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4
+ 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 1500
0*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592
*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7
 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*
a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*
d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 11072962
56*a^10*b^2*c^5*d^15))^(1/4))/(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6
*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3
*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(
16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6
 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^
3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^1
0 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 830472192
0*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*
d^15))^(1/4)*((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^
4*b^12*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)
/2048 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (9
97755*a^8*b^8*c^2*d^5)/2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 2
8*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^
3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (-(625*a^8*d^8 + 81*b^8*c^8 +
 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 +
513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*
a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a
*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 -
 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^
```

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5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^
12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256
*a^10*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^
6*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^
3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(167
77216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 -
201326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b
^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 +
 15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a
^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^1
5))^(1/4)*(1280*a^16*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*
c^13*d^6 + 576000*a^5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a
^7*b^13*c^10*d^9 - 6462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 -
5913600*a^10*b^10*c^7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c
^5*d^14 + 309760*a^13*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^
5*c^2*d^17))/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 -
56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c
^2*d^7 - 8*a^7*b*c*d^8) - (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*
c^16*d^4 + 12058624*a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 71408
0256*a^6*b^17*c^13*d^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15
*c^11*d^9 - 4814143488*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 -
 1398177792*a^11*b^12*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a
^13*b^10*c^6*d^14 - 116391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^
16 + 77070336*a^16*b^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18)*1i)/(4096*(a^
12*d^13 + b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3
*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*
d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66
*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12)))*1i)*1i + (x^(1/2)*(81*a^4*b^15*c^8
 + 26225*a^12*b^7*d^8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060
*a^6*b^13*c^6*d^2 - 307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328
680*a^9*b^10*c^3*d^5 + 1024380*a^10*b^9*c^2*d^6)*1i)/(4096*(a^12*d^13 + b^1
2*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4
+ 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7
*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2
*d^11 - 12*a^11*b*c*d^12)))*(-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6
*d^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3
*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(1677
7216*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 2
01326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^
9*c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 +
15502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^
8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15
))^(1/4) + ((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^
3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^
6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^
```

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17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*
c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 830
4721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*
b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 -
 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*((((81*
a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12*c^6*d)/2048 +
 (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048 - (262899*a^6*
b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (997755*a^8*b^8*c^2*d^
5)/2048)*1i)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 -
56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c
^2*d^7 - 8*a^7*b*c*d^8) + (-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d
^2 - 307800*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d
^5 + 133500*a^6*b^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(167772
16*a^12*c^3*d^17 + 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201
326592*a^11*b*c^4*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*
c^12*d^8 + 8304721920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15
502147584*a^6*b^6*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*
b^4*c^7*d^13 - 3690987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))
^(3/4)*(((-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b
^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b
^2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17
+ 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4
*d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 830472
1920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6
*c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 36
90987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)*(1280*a^16
*b^4*c*d^18 + 8960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^
5*b^15*c^12*d^7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6
462720*a^8*b^12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^
7*d^12 + 3421440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^1
3*b^7*c^4*d^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^
9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 +
 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d
^8) + (x^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*
a^4*b^19*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d
^7 - 2086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 48141434
88*a^9*b^14*c^10*d^10 + 3714056192*a^10*b^13*c^9*d^11 - 1398177792*a^11*b^1
2*c^8*d^12 - 259522560*a^12*b^11*c^7*d^13 + 508952576*a^13*b^10*c^6*d^14 -
116391936*a^14*b^9*c^5*d^15 - 103612416*a^15*b^8*c^4*d^16 + 77070336*a^16*b
^7*c^3*d^17 - 17694720*a^17*b^6*c^2*d^18)*1i)/(4096*(a^12*d^13 + b^12*c^12*
d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a
^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^
5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
 12*a^11*b*c*d^12)))*1i)*1i - (x^(1/2)*(81*a^4*b^15*c^8 + 26225*a^12*b^7*d^
8 - 3240*a^5*b^14*c^7*d + 322200*a^11*b^8*c*d^7 + 48060*a^6*b^13*c^6*d^2 -
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307800*a^7*b^12*c^5*d^3 + 658566*a^8*b^11*c^4*d^4 + 328680*a^9*b^10*c^3*d^5
 + 1024380*a^10*b^9*c^2*d^6)*1i)/(4096*(a^12*d^13 + b^12*c^12*d - 12*a*b^11
*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^
5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a
^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*
d^12)))*(-(625*a^8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^
5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^
2*c^2*d^6 - 3240*a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 +
 16777216*b^12*c^15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*
d^16 + 1107296256*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721
920*a^4*b^8*c^11*d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*
c^9*d^11 - 13287555072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 369
0987520*a^9*b^3*c^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)))*(-(625*a^
8*d^8 + 81*b^8*c^8 + 48060*a^2*b^6*c^6*d^2 - 307800*a^3*b^5*c^5*d^3 + 64935
0*a^4*b^4*c^4*d^4 + 513000*a^5*b^3*c^3*d^5 + 133500*a^6*b^2*c^2*d^6 - 3240*
a*b^7*c^7*d + 15000*a^7*b*c*d^7)/(16777216*a^12*c^3*d^17 + 16777216*b^12*c^
15*d^5 - 201326592*a*b^11*c^14*d^6 - 201326592*a^11*b*c^4*d^16 + 1107296256
*a^2*b^10*c^13*d^7 - 3690987520*a^3*b^9*c^12*d^8 + 8304721920*a^4*b^8*c^11*
d^9 - 13287555072*a^5*b^7*c^10*d^10 + 15502147584*a^6*b^6*c^9*d^11 - 132875
55072*a^7*b^5*c^8*d^12 + 8304721920*a^8*b^4*c^7*d^13 - 3690987520*a^9*b^3*c
^6*d^14 + 1107296256*a^10*b^2*c^5*d^15))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.463 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=628

$$-\frac{a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{\sqrt{2}\,(bc-ad)^3}$$

Rubi [A] time = 0.76, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{5/4}d^{3/4}(bc-ad)^3}+\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{5/4}d^{3/4}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^3}+\frac{x^{3/2}(3ad+5bc)}{16c\left(c+dx^2\right)(bc-ad)^2}+\frac{x^{3/2}}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)^2) + ((5*b*c + 3*a*d)*x^(3/2))/(16*c*(b*
c - a*d)^2*(c + d*x^2)) + (a^(3/4)*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(3/4)*b^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*
b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[
2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) + ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*d^(3/4)*(
b*c - a*d)^3) - (a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) + (a^(3/4)*b^(5/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) +
 ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) - ((5*b^
2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right) \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a-5bx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4(bc-ad)} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a(9bc-ad)-b(5bc+3ad)x\right.}{\left(a+bx^4\right)\left(c+dx^4\right)}\right.}{16c(bc-ad)^2} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int\left(\frac{32ab^2cx^2}{(bc-ad)\left(a+bx^4\right)} - \frac{\left(5b^2c^2\right.}{(b}\right.\right.}{16c(bc-} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(2ab^2\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \sqrt{x}\right.}{(bc-ad)^3} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(ab^{3/2}\right)\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x,\right.}{(bc-ad)^3} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{(ab)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx,\right.}{2(bc-ad)^3} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{a^{3/4}b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{\right.}{2\sqrt{2}(bc-ad)^3} \\
&= \frac{x^{3/2}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} -
\end{aligned}
$$

Mathematica [A] time = 0.75, size = 544, normalized size = 0.87

$$
\frac{-32\sqrt{2}a^{3/4}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+32\sqrt{2}a^{3/4}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+64\sqrt{2}a^{3/4}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-64\sqrt{2}a^{3/4}b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+\frac{\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}d^{3/4}}-\frac{\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}d^{3/4}}-\frac{2\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}d^{3/4}}+\frac{2\sqrt{2}\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}d^{3/4}}+\frac{32x^{3/2}(bc-ad)^2}{\left(c+dx^2\right)^2}+\frac{8x^{3/2}(3ad+5bc)(bc-ad)}{c\left(c+dx^2\right)}}{128(bc-ad)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((32*(b*c - a*d)^2*x^(3/2))/(c + d*x^2)^2 + (8*(b*c - a*d)*(5*b*c + 3*a*d)*
x^(3/2))/(c*(c + d*x^2)) + 64*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - 64*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*d^(3/4)) + (2*Sqrt[
2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(5/4)*d^(3/4)) - 32*Sqrt[2]*a^(3/4)*b^(5/4)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 32*Sqrt[2]*a^(3/4)*b^(5/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + (Sqrt[2]*(5*b^2*c
^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(c^(5/4)*d^(3/4)) - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*
d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(5/4)*d
^(3/4)))/(128*(b*c - a*d)^3)
```

IntegrateAlgebraic [A] time = 1.43, size = 369, normalized size = 0.59

$$\frac{a^{3/4}b^{5/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,(bc-ad)^3}+\frac{a^{3/4}b^{5/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}\,c^{5/4}d^{3/4}(bc-ad)^3}-\frac{\left(-3a^2d^2+30abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{5/4}d^{3/4}(bc-ad)^3}+\frac{x^{3/2}\left(-acd+3ad^2x^2+9bc^2+5bcdx^2\right)}{16c\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (x^(3/2)*(9*b*c^2 - a*c*d + 5*b*c*d*x^2 + 3*a*d^2*x^2))/(16*c*(b*c - a*d)^2
*(c + d*x^2)^2) + (a^(3/4)*b^(5/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(
1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*(b*c - a*d)^3) - ((5*b^2*c^2
+ 30*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(5/4)*d^(3/4)*(b*c - a*d)^3) + (a^(3/4)*b^(5/
4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[
2]*(b*c - a*d)^3) - ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(5/4)*d^(3/4
)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.65, size = 963, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*
a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/
(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 -
sqrt(2)*a^3*c^2*d^6) + 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b
*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*
sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - 1/64*(5*(c*d^3)^(3/4)*b^2*c^
2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)
*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^
4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) + 1/64*(5*(c*d^3)^(3/4)*
b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2
*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - (a*b^3)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b
^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^
3) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d
^2 - sqrt(2)*a^3*b*d^3) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*
b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt
(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/16*(5*b*c*d*x^(7/2) + 3*a*d^2*x^
(7/2) + 9*b*c^2*x^(3/2) - a*c*d*x^(3/2))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*(d*x^2 + c)^2)
```

maple [A] time = 0.02, size = 839, normalized size = 1.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/4*a*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*a*b/(a*d-b*c)^
3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*a*b/(a*d-b*
c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/16/(a*d-b*
c)^3/(d*x^2+c)^2*d^3/c*x^(7/2)*a^2+1/8/(a*d-b*c)^3/(d*x^2+c)^2*d^2*x^(7/2)*
a*b-5/16/(a*d-b*c)^3/(d*x^2+c)^2*d*c*x^(7/2)*b^2-1/16/(a*d-b*c)^3/(d*x^2+c)
^2*x^(3/2)*a^2*d^2+5/8/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a*b*c*d-9/16/(a*d-b*
c)^3/(d*x^2+c)^2*x^(3/2)*b^2*c^2+3/64/(a*d-b*c)^3/c*d/(c/d)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-15/32/(a*d-b*c)^3/(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-5/64/(a*d-b*c)^3*c/d/(c/d)^(1
```

```
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128/(a*d-b*c)^3/c*d
/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d
)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-15/64/(a*d-b*c)^3/(c/d)^(1/4)*2^(
1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*
x^(1/2)+(c/d)^(1/2)))*a*b-5/128/(a*d-b*c)^3*c/d/(c/d)^(1/4)*2^(1/2)*ln((x-(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))*b^2+3/64/(a*d-b*c)^3/c*d/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)*a^2-15/32/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)*a*b-5/64/(a*d-b*c)^3*c/d/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [A] time = 2.61, size = 583, normalized size = 0.93

$$-\frac{ab^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)}+\frac{\left(5b^{2}c^{2}+30abcd-3a^{2}d^{2}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{4}-3ab^{2}c^{3}d+3a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)}+\frac{\left(5bcd+3ad^{2}\right)x^{\frac{7}{2}}+\left(9bc^{2}-acd\right)x^{\frac{3}{2}}}{16\left(b^{2}c^{5}-2abc^{4}d+a^{2}c^{3}d^{2}+\left(b^{2}c^{3}d^{2}-2abc^{2}d^{3}+a^{2}cd^{4}\right)x^{4}+2\left(b^{2}c^{4}d-2abc^{3}d^{2}+a^{2}c^{2}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))
/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/128*(5*b^2*c^2 + 3
0*a*b*c*d - 3*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sq
rt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log
(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) +
 sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3) + 1/16
*((5*b*c*d + 3*a*d^2)*x^(7/2) + (9*b*c^2 - a*c*d)*x^(3/2))/(b^2*c^5 - 2*a*b
*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b
^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

mupad [B] time = 4.52, size = 31866, normalized size = 50.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^
21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/
```

```
2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535
*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*
c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12
)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688
489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c
^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 1
4*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4
*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b
^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^
6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) -
 (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2
 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14
784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a
^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*
d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 2013265
92*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c
^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19
702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 + 2019409920
0*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^1
0*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 -
393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5
*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*
b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c
^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 -
220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5
)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*
d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6
 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 105
6*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4) - (x^(1
/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4
*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*
a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7))/(409
6*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 -
220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6
*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*
d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12
+ 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b
^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*
c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d
^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (((((27*a^20*b^4*d^20
)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*
b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6
)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760
163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*
b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*
```

```
d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (2
08665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c
^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^
12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9
*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8
*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^1
1 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) + (x^(1/2)*(-(a^3*b^5)/(16*a^1
2*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 792
0*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a
^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^
2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4
*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 108960
1536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16
*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10
- 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 154790
78912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*
b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 +
 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14
+ a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c
^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6
- 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10
*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^1
2 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 -
12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 792
0*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b
^11*c^11*d - 192*a^11*b*c*d^11))^(3/4) + (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^
3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^
14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8
*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^12
- 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^
4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^
7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 -
12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^1
0*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^
7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^
8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192
*a^11*b*c*d^11))^(1/4))/((((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)
/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^
5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*
d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1
361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a
^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c
^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 -
(20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 +
a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^
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13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10
*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 +
 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14
*a*b^13*c^15*d) - (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*
a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5
*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4
*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*
d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c
^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630
694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b
^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*
d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 95
80707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^1
5*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 -
 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c
^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^
4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a
^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^1
3*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 -
3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784
*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*
b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^1
1))^(3/4)*1i - (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b
^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^
13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9
*b^10*c^2*d^7)*1i)/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 +
66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^
5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6
*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(
a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b
^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c
^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^
9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)
+ (((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^1
7*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (2
27605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b
^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d
^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 -
(1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^
15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^1
7)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13
*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*
c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9
*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10
 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) + (x^(1
```

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/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 352
0*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^
6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3
*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))
^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4
*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^
7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087
680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11
*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*
d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216
000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d
^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c
^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5
 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^
9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*
a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 +
7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 1267
2*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10
*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1i + (x^(1/2)
*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^
15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7
*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7)*1i)/(409
6*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 -
220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6
*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*
d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12
+ 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b
^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*
c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d
^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) + ((625*a^4*b^16*c^7*d)
/4096 - (945*a^11*b^9*d^8)/4096 + (28215*a^10*b^10*c*d^7)/4096 + (15625*a^5
*b^15*c^6*d^2)/4096 + (145125*a^6*b^14*c^5*d^3)/4096 + (586125*a^7*b^13*c^4
*d^4)/4096 + (810675*a^8*b^12*c^3*d^5)/4096 - (274725*a^9*b^11*c^2*d^6)/409
6)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 -
 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3
003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a
^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b
^2*c^4*d^12 - 14*a*b^13*c^15*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12
+ 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12
672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*
a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^1
1*c^11*d - 192*a^11*b*c*d^11))^(1/4) - atan((((((27*a^20*b^4*d^20)/16 - (11
07*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*
d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (138
2895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^
```

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15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d
^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 +
(744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16
*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2
)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 -
364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 30
03*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^
9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^
2*c^4*d^12 - 14*a*b^13*c^15*d) - (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^
12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*
d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7
 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 1
92*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 1718
6816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*
c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14
511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*
a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^1
1*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 +
 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17
*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^1
2 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*
a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*
c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10
- 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b
^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*
c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*
d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 1
92*a^11*b*c*d^11))^(3/4)*1i - (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*
d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3
+ 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d
^6 - 136260*a^9*b^10*c^2*d^7)*1i)/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^1
1*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^
10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 +
495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^1
1*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d
^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 +
14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520
*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*
c*d^11))^(1/4) - ((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (12
5*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^
15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 -
(2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^
10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*
c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/
2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^
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17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)/(b^14*c^16 + a^14*c^2*d^14
 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001
*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a
^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^
4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*
d) + (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10
*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5
+ 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 35
20*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*
b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201
326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^
17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9
+ 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 + 201940
99200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13
*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^1
6 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18
*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*
a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b
^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^
8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3
*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*
c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*
d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 +
 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1i
+ (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15
000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 +
956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7
)*1i)/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c
^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5
 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^
9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*
a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 +
7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 1267
2*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10
*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4))/(((((27*a^20
*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (3
1893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18
*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/
4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (1117
215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*
b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^1
5)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a
^18*b^6*c^2*d^18)/2)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a
^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^
5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^
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6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^
5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) - (x^(1/2)*(-(a^3*b^5)/(1
6*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3
+ 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12
672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^
10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^1
9*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1
089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7
*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*
d^10 - 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 1
5479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*
a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d
^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*
c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*
b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8
*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66
*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^1
2*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d
^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7
+ 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 19
2*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4) - (x^(1/2)*(81*a^11*b^8*d^9 + 6
25*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a
^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 58308
0*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*
d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 4
95*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b
^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^
10 - 12*a*b^11*c^13*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^
2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b
^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c
^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d
- 192*a^11*b*c*d^11))^(1/4) + ((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d
^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (4448
1*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c
^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2
- (1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (28775
45*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^
10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/
4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)/(b^14*c^16 +
a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^
13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10
*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 +
 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14
*a*b^13*c^15*d) + (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*
a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5
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*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4
*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*
d - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c
^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630
694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b
^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*
d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 95
80707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^1
5*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 -
 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c
^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^
4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a
^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^1
3*d)))*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 -
3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784
*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*
b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^1
1))^(3/4) + (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*
c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*
c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^
10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2
*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*
c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 -
 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(a^3*b^
5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9
*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6
 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 10
56*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - ((62
5*a^4*b^16*c^7*d)/4096 - (945*a^11*b^9*d^8)/4096 + (28215*a^10*b^10*c*d^7)/
4096 + (15625*a^5*b^15*c^6*d^2)/4096 + (145125*a^6*b^14*c^5*d^3)/4096 + (58
6125*a^7*b^13*c^4*d^4)/4096 + (810675*a^8*b^12*c^3*d^5)/4096 - (274725*a^9*
b^11*c^2*d^6)/4096)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^
2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5
*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6
*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5
*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d)))*(-(a^3*b^5)/(16*a^12*d^1
2 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4
*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^
5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2
*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*2i - atan((((((27*a^2
0*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (
31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^1
8*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)
/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (111
7215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13
```

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*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^
15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*
a^18*b^6*c^2*d^18)/2)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*
a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a
^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b
^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c
^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) - (x^(1/2)*(-(81*a^8*d^8
 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a
^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b
^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d
^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2
*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7
- 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072
*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d
^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4)*(147456*a^19*b^4*c*d^20 + 171868
16*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^
15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 1451
1243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^
10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*
c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1
620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b
^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12
- 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^
4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^
7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 -
12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 +
513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 +
48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^1
2*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592
*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d
^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 155021475
84*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^
9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(3/4)
*1i - (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8
+ 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^
4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2
*d^7)*1i)/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^
10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9
*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 22
0*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^
8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*
a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*
b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*
d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^
2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7
```

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 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 1328755507
2*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*
d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4) - ((((27*a^20*b^4*d^20)/16 - (1
107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16
*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (13
82895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b
^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*
d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 +
 (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^1
6*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/
2)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 -
 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3
003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a
^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b
^2*c^4*d^12 - 14*a*b^13*c^15*d) + (x^(1/2)*(-(81*a^8*d^8 + 625*b^8*c^8 + 13
3500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 30
7800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7
*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^
11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 36
90987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b
^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10
+ 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^
10*b^2*c^7*d^13))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^
4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400
*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^
12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 +
 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 95807078
40*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*
c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 47185
92*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^1
1 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 79
2*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4
*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))
*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*
d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d
^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 167772
16*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 +
1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4
*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9
 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520
*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(3/4)*1i + (x^(1/2)*(81*
a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^
7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12
*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7)*1i)/(4096*(b^
12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a
```

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^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*
c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 +
 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 1
33500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 3
07800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^
7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b
^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3
690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*
b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10
 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a
^10*b^2*c^7*d^13))^(1/4))/(((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19
)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a
^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13
*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (
1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*
a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*
c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 -
 (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)/(b^14*c^16 + a^1
4*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*
d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^
6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 10
01*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*
b^13*c^15*d) - (x^(1/2)*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^
2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^
5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216
*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 20132
6592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^
14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502
147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^
4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(
1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b
^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7
 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 1970208768
0*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b
^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^
14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 39321600
0*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^1
9))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^1
2*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 +
 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*
b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 62
5*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^
4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^
7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 -
201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10
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*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 132
87555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*
b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 +
 1107296256*a^10*b^2*c^7*d^13))^(3/4) - (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3
*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^1
4*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*
b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^12 -
 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4
*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7
*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 1
2*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 5
13000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 4
8060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12
*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*
a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^
6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 1550214758
4*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9
*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4)
+ ((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17
*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (22
7605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^
16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^
10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (
1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^1
5*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17
)/2 + (2295*a^18*b^6*c^2*d^18)/2)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^
3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*
d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7
+ 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 36
4*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) + (x^(1/2)*(
-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^
3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6
 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216
*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 11
07296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b
^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 -
 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a
^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4)*(147456*a^19*b^4*c*d
^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536
*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^1
3*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21
851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 1547907891
2*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*
c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 593
75616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^
```

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12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*
d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 79
2*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2
*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^
6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^
3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(
16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4
 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^
3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8
 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 830472192
0*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*
d^13))^(3/4) + (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b
^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^
13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9
*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*
a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b
^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^
8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*
a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 6
49350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15
000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12
*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296
256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^
13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 1328
7555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^
3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4) - ((625*a^4*b^16*c^7*d)/4
096 - (945*a^11*b^9*d^8)/4096 + (28215*a^10*b^10*c*d^7)/4096 + (15625*a^5*b
^15*c^6*d^2)/4096 + (145125*a^6*b^14*c^5*d^3)/4096 + (586125*a^7*b^13*c^4*d
^4)/4096 + (810675*a^8*b^12*c^3*d^5)/4096 - (274725*a^9*b^11*c^2*d^6)/4096)
/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 3
64*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 300
3*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9
*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2
*c^4*d^12 - 14*a*b^13*c^15*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^
6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^
3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(
16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4
 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^
3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8
 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 830472192
0*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*
d^13))^(1/4)*2i + 2*atan((((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19
)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a
^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13
*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (
```

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1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*
a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*
c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 -
 (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 +
 a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c
^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^1
0*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9
+ 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 1
4*a*b^13*c^15*d) - (x^(1/2)*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^
6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^
3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(1677
7216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 2
01326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^
9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 1
5502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^
8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13
))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a
^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*
d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 197020
87680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^
11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^
7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 3932
16000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2
*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10
*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d
^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*
a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^8
+ 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^
4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^
7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^
3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*
b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 -
 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*
a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^
12 + 1107296256*a^10*b^2*c^7*d^13))^(3/4) - (x^(1/2)*(81*a^11*b^8*d^9 + 625
*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5
*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*
a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^
12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495
*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5
*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10
 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2
 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5
 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*
a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326
```

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592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^1
4*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 155021
47584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4
*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1
/4) - (((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21
*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2
- (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a
^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^
10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/
8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (68848
9*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3
*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 14*
a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b
^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7
*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*
d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) + (
x^(1/2)*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b
^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^
2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 +
 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*
d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721
920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c
^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 369
0987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4)*(147456*a^1
9*b^4*c*d^20 + 17186816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1
089601536*a^5*b^18*c^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7
*b^16*c^13*d^8 - 14511243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*
d^10 - 21851799552*a^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 1
5479078912*a^12*b^11*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*
a^14*b^9*c^6*d^15 + 1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d
^17 + 59375616*a^17*b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*
c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*
b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8
*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66
*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 1335
00*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 3078
00*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b
*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11
*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690
987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7
*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 +
8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10
*b^2*c^7*d^13))^(3/4) + (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 59
76*a^10*b^9*c*d^8 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 5386
00*a^6*b^13*c^5*d^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 1
```

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36260*a^9*b^10*c^2*d^7))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d
^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 -
792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b
^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)
))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^
5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2
*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 1677
7216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14
+ 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a
^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d
^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 36909875
20*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4))/((((((27*a^20*b
^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (318
93*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c
^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4
+ (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (111721
5*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^
11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)
/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^1
8*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91
*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*
a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*
b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*
c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) - (x^(1/2)*(-(81*a^8*d^
8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*
a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*
b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*
d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^
2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7
 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 1328755507
2*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*
d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186
816*a^3*b^20*c^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c
^15*d^6 - 3630694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 145
11243264*a^8*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a
^10*b^13*c^10*d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11
*c^8*d^13 + 9580707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 +
1620770816*a^15*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*
b^6*c^3*d^18 - 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12
 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a
^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c
^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 -
 12*a*b^11*c^13*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 +
 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 +
 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^
```

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12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 20132659
2*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*
d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147
584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c
^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(3/4
)*1i - (x^(1/2)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8
 + 15000*a^4*b^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d
^4 + 956550*a^7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^
2*d^7)*1i)/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b
^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^
9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 2
20*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d)))*(-(81*a^8*d
^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350
*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a
*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17
*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a
^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^
7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 132875550
72*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8
*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4) + (((((27*a^20*b^4*d^20)/16 -
(1107*a^19*b^5*c*d^19)/16 + (125*a^3*b^21*c^17*d^3)/16 - (31893*a^4*b^20*c^
16*d^4)/16 + (44481*a^5*b^19*c^15*d^5)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (
1382895*a^7*b^17*c^13*d^7)/4 - (2723535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9
*b^15*c^11*d^9)/2 - (1361943*a^10*b^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^
9*d^11)/8 + (2877545*a^12*b^12*c^8*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2
 + (744837*a^14*b^10*c^6*d^14)/2 - (688489*a^15*b^9*c^5*d^15)/4 + (208665*a
^16*b^8*c^4*d^16)/4 - (20115*a^17*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18
)/2)*1i)/(b^14*c^16 + a^14*c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14
*d^2 - 364*a^3*b^11*c^13*d^3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d
^5 + 3003*a^6*b^8*c^10*d^6 - 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 -
2002*a^9*b^5*c^7*d^9 + 1001*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*
a^12*b^2*c^4*d^12 - 14*a*b^13*c^15*d) + (x^(1/2)*(-(81*a^8*d^8 + 625*b^8*c^
8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^
4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 32
40*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 20132659
2*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^
5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 13287555072
*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10
*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 1107296
256*a^10*b^2*c^7*d^13))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c
^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 3630
694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8*b
^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13*c^10*
d^11 + 20194099200*a^11*b^12*c^9*d^12 - 15479078912*a^12*b^11*c^8*d^13 + 95
80707840*a^13*b^10*c^7*d^14 - 4594335744*a^14*b^9*c^6*d^15 + 1620770816*a^1
```

```
5*b^8*c^5*d^16 - 393216000*a^16*b^7*c^4*d^17 + 59375616*a^17*b^6*c^3*d^18 -
 4718592*a^18*b^5*c^2*d^19))/(4096*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c
^3*d^11 + 66*a^2*b^10*c^12*d^2 - 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^
4 - 792*a^5*b^7*c^9*d^5 + 924*a^6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a
^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^1
3*d))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^
5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2
*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 +
16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d
^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 83047219
20*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^
11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690
987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(3/4)*1i + (x^(1/2
)*(81*a^11*b^8*d^9 + 625*a^3*b^16*c^8*d + 5976*a^10*b^9*c*d^8 + 15000*a^4*b
^15*c^7*d^2 + 133500*a^5*b^14*c^6*d^3 + 538600*a^6*b^13*c^5*d^4 + 956550*a^
7*b^12*c^4*d^5 + 583080*a^8*b^11*c^3*d^6 - 136260*a^9*b^10*c^2*d^7)*1i)/(40
96*(b^12*c^14 + a^12*c^2*d^12 - 12*a^11*b*c^3*d^11 + 66*a^2*b^10*c^12*d^2 -
 220*a^3*b^9*c^11*d^3 + 495*a^4*b^8*c^10*d^4 - 792*a^5*b^7*c^9*d^5 + 924*a^
6*b^6*c^8*d^6 - 792*a^7*b^5*c^7*d^7 + 495*a^8*b^4*c^6*d^8 - 220*a^9*b^3*c^5
*d^9 + 66*a^10*b^2*c^4*d^10 - 12*a*b^11*c^13*d))*(-(81*a^8*d^8 + 625*b^8*c
^8 + 133500*a^2*b^6*c^6*d^2 + 513000*a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d
^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3
240*a^7*b*c*d^7)/(16777216*a^12*c^5*d^15 + 16777216*b^12*c^17*d^3 - 2013265
92*a*b^11*c^16*d^4 - 201326592*a^11*b*c^6*d^14 + 1107296256*a^2*b^10*c^15*d
^5 - 3690987520*a^3*b^9*c^14*d^6 + 8304721920*a^4*b^8*c^13*d^7 - 1328755507
2*a^5*b^7*c^12*d^8 + 15502147584*a^6*b^6*c^11*d^9 - 13287555072*a^7*b^5*c^1
0*d^10 + 8304721920*a^8*b^4*c^9*d^11 - 3690987520*a^9*b^3*c^8*d^12 + 110729
6256*a^10*b^2*c^7*d^13))^(1/4) + ((625*a^4*b^16*c^7*d)/4096 - (945*a^11*b^9
*d^8)/4096 + (28215*a^10*b^10*c*d^7)/4096 + (15625*a^5*b^15*c^6*d^2)/4096 +
 (145125*a^6*b^14*c^5*d^3)/4096 + (586125*a^7*b^13*c^4*d^4)/4096 + (810675*
a^8*b^12*c^3*d^5)/4096 - (274725*a^9*b^11*c^2*d^6)/4096)/(b^14*c^16 + a^14*
c^2*d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^
3 + 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6
- 3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001
*a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^
13*c^15*d)))*(-(81*a^8*d^8 + 625*b^8*c^8 + 133500*a^2*b^6*c^6*d^2 + 513000*
a^3*b^5*c^5*d^3 + 649350*a^4*b^4*c^4*d^4 - 307800*a^5*b^3*c^3*d^5 + 48060*a
^6*b^2*c^2*d^6 + 15000*a*b^7*c^7*d - 3240*a^7*b*c*d^7)/(16777216*a^12*c^5*d
^15 + 16777216*b^12*c^17*d^3 - 201326592*a*b^11*c^16*d^4 - 201326592*a^11*b
*c^6*d^14 + 1107296256*a^2*b^10*c^15*d^5 - 3690987520*a^3*b^9*c^14*d^6 + 83
04721920*a^4*b^8*c^13*d^7 - 13287555072*a^5*b^7*c^12*d^8 + 15502147584*a^6*
b^6*c^11*d^9 - 13287555072*a^7*b^5*c^10*d^10 + 8304721920*a^8*b^4*c^9*d^11
- 3690987520*a^9*b^3*c^8*d^12 + 1107296256*a^10*b^2*c^7*d^13))^(1/4) - ((x^
(3/2)*(a*d - 9*b*c))/(16*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^(7/2)*(3*a
*d + 5*b*c))/(16*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))/(c^2 + d^2*x^4 + 2*c
```

```
*d*x^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.464 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=627

$$-\frac{\big(-3a^2d^2+14abcd+21b^2c^2\big)\log\big(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\big)}{64\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\big(-3a^2d^2+14abcd+21b^2c^2\big)\log\big(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}{64\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}$$

Rubi [A] time = 0.71, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,(bc-ad)^3}+\frac{\sqrt{x}\,(ad+7bc)}{16c\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{x}}{4\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)^2) + ((7*b*c + a*d)*Sqrt[x])/(16*c*(b*c
- a*d)^2*(c + d*x^2)) + (a^(1/4)*b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^2 + 14*a*b
*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2
]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*d^(1/4)*(
b*c - a*d)^3) + (a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) - (a^(1/4)*b^(7/4)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*(b*c - a*d)^3) -
 ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) + ((21*
b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{a-7bx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4(bc-ad)}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\mathrm{Subst}\left(\int \frac{a(11bc-3ad)-3b(7bc+ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\right.}{16c(bc-ad)^2}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(2ab^2\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \sqrt{x}\right.}{(bc-ad)^3}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,b^2\right)\mathrm{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x\right.}{(bc-ad)^3}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,b^{3/2}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x}\right.}{2(bc-ad)^3}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}(bc-ad)^3}$$

$$= \frac{\sqrt{x}}{4(bc-ad)\left(c+dx^2\right)^2} + \frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} -$$

Mathematica [A] time = 0.73, size = 543, normalized size = 0.87

$$\frac{-\frac{\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{7/4}\sqrt[4]{d}}+\frac{\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{7/4}\sqrt[4]{d}}-\frac{2\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{7/4}\sqrt[4]{d}}+\frac{2\sqrt{2}\left(-3a^2d^2+14abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{7/4}\sqrt[4]{d}}+32\sqrt{2}\sqrt[4]{a}\,b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)-32\sqrt{2}\sqrt[4]{a}\,b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)+64\sqrt{2}\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-64\sqrt{2}\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+\frac{32\sqrt{x}(bc-ad)^2}{\left(c+dx^2\right)^2}+\frac{8\sqrt{x}(ad+7bc)(bc-ad)}{c\left(c+dx^2\right)}}{128(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((32*(b*c - a*d)^2*Sqrt[x])/(c + d*x^2)^2 + (8*(b*c - a*d)*(7*b*c + a*d)*Sq
rt[x])/(c*(c + d*x^2)) + 64*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(
```

```
1/4)*Sqrt[x])/a^(1/4)] - 64*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(7/4)*d^(1/4)) + (2*Sqrt[2
]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(7/4)*d^(1/4)) + 32*Sqrt[2]*a^(1/4)*b^(7/4)*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 32*Sqrt[2]*a^(1/4)*b^(7/4)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - (Sqrt[2]*(21*b^2*
c^2 + 14*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(c^(7/4)*d^(1/4)) + (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(7/4)
*d^(1/4)))/(128*(b*c - a*d)^3)
```

IntegrateAlgebraic [A] time = 1.39, size = 369, normalized size = 0.59

$$-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{7/4}\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,b^{7/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,b^{7/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,(bc-ad)^3}+\frac{\sqrt{x}\left(-3acd+ad^2x^2+11bc^2+7bcdx^2\right)}{16c\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (Sqrt[x]*(11*b*c^2 - 3*a*c*d + 7*b*c*d*x^2 + a*d^2*x^2))/(16*c*(b*c - a*d)^
2*(c + d*x^2)^2) + (a^(1/4)*b^(7/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^
(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*(b*c - a*d)^3) - ((21*b^2*c^
2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(7/4)*d^(1/4)*(b*c - a*d)^3) - (a^(1/4)*b^(
7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqr
t[2]*(b*c - a*d)^3) + ((21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(7/4)*d^(
1/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.40, size = 946, normalized size = 1.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 -
sqrt(2)*a^3*d^3) - (a*b^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqr
t(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d +
3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*b*log(-sqrt(2)
*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^
2*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(21*(c*d^3)^(1/4)*b^2
*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d - 3*sq
rt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/32
*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2
*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt
(2)*a^3*c^2*d^4) + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*
d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 -
 sqrt(2)*a^3*c^2*d^4) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a
*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3
*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/16*(7*b*c*d*x^(5/2) + a*d^2*x^(5/2) + 11*b*
c^2*sqrt(x) - 3*a*c*d*sqrt(x))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(d*x^2
+ c)^2)
```

maple [**A**] time = 0.02, size = 848, normalized size = 1.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/4*b^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*b^2/(a*d-b*c)^
3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^2/(a*d-b*
c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/16/(a*d-b*
c)^3/(d*x^2+c)^2*d^3/c*x^(5/2)*a^2+3/8/(a*d-b*c)^3/(d*x^2+c)^2*d^2*x^(5/2)*
a*b-7/16/(a*d-b*c)^3/(d*x^2+c)^2*d*c*x^(5/2)*b^2+7/8/(a*d-b*c)^3/(d*x^2+c)^
2*x^(1/2)*a*b*c*d-11/16/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*b^2*c^2-3/16/(a*d-b
*c)^3/(d*x^2+c)^2*x^(1/2)*a^2*d^2+3/64/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2-7/32/(a*d-b*c)^3/c*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d-21/64/(a*d-b*c)^3*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3/64/(a*d-b*c)^3
/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2*d^2-7/32
/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*
```

```
b*d-21/64/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*b^2+3/128/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2
)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2*d^2
-7/64/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b*d-21/128/(a*d-
b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.61, size = 654, normalized size = 1.04

$$-\frac{\left(\frac{2\,\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\,\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}\right)a}{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{\left(7\,bcd+ad^2\right)x^{\frac{5}{2}}+\left(11\,bc^2-3\,acd\right)\sqrt{x}}{16\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^4+2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x^2\right)}+\frac{\frac{2\,\sqrt{2}\left(21\,b^2c^2+14\,abcd-3\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\,\sqrt{2}\left(21\,b^2c^2+14\,abcd-3\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(21\,b^2c^2+14\,abcd-3\,a^2d^2\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(21\,b^2c^2+14\,abcd-3\,a^2d^2\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(
7/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*a
/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*((7*b*c*d + a*d
^2)*x^(5/2) + (11*b*c^2 - 3*a*c*d)*sqrt(x))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^
3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3)*x^2) + 1/128*(2*sqrt(2)*(21*b^2*c^2 + 14*a*b*c*d -
 3*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(21*b^
2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + sqrt(2)*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(21*b^2*c^2
+ 14*a*b*c*d - 3*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 -
a^3*c*d^3)
```

mupad [B] time = 4.25, size = 36160, normalized size = 57.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] 2*atan(((((((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^
2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^
5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048
```

```
+ (6885*a^7*b^9*c^2*d^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^
7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*
b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d
^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a
^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*
b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c
^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18
*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^1
5*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 495897
6*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*
d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^
7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b
^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d
^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*
b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 1941
01248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^1
8*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 +
 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 98618572
80*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b
^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 +
343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b
^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*1i)/(40
96*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 -
 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a
^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c
^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12
 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*
b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5
*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*
d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1i)*(-(a*b^7)/(16*a^12
*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920
*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^
7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2
*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (x^(1/2)*(81*a^
10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b
^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6
*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^
12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a
^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6
*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9
 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*
b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^
8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d
^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 -
 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) - (((((81*a^9*b^7*d^10)/2048
```

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 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3
*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*
d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)*1i)
/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b
^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 -
 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10
*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7
*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8
 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*
a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 5
06112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14
*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9
*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 -
459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^
16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*
d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b
^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) + (x
^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 10302259
20*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c
^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15*c^13*d^10 -
13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^12 - 552170291
2*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*
c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^6*d^17 - 1274
01984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3
*d^20 + 147456*a^19*b^4*c^2*d^21)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12
*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^
8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*
d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12
*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^
10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^
5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 -
3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^1
1*b*c*d^11))^(3/4)*1i)*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^
10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c
^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d
^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 19
2*a^11*b*c*d^11))^(1/4) + (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10
 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*
d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3
*d^8 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*
b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12
*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 +
495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^1
1*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2
 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14
```

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784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a
^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*
d^11))^(1/4))/((((((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (19
4481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*
b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7
)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b
*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 -
56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16
*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 +
 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 126
72*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^1
0*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2*b^
18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*
a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 +
 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^1
0*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*
a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768
*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6
*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 +
28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(16777216*a^2*b^21*c^19*d^4
 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*
a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^1
4*d^9 + 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9
861857280*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960
*a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*
d^16 + 343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048
*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*
1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^1
4*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5
+ 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^
9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^
12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 79
20*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*
a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b
^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4)*1i)*(-(a*b^7)/(
16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3
 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 1
2672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a
^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*1i - (x^(1
/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 51
8616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 -
 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9)*
1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^1
4*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5
+ 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^
```

9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^
12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 79
20*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*
a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b
^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) + (((((81*a^9*b
^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048
- (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*
a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d
^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d
^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*
b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 +
1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 1267
2*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^
8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*
c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17
*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*
a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10
- 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^
9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^
14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^
12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d
^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7
*c^11*d) + (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d
^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 73358376
96*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15
*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^1
2 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 499384
32*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^
6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728
*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*1i)/(4096*(b^12*c^16 + a^12*
c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3
 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792
*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*
c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 105
6*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a
^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b
^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^1
1*d - 192*a^11*b*c*d^11))^(3/4)*1i)*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12
+ 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12
672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*
a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^1
1*c^11*d - 192*a^11*b*c*d^11))^(1/4)*1i + (x^(1/2)*(81*a^10*b^9*d^11 - 1512
*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 8590
68*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87
192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9)*1i)/(4096*(b^12*c^16 + a^12*

c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3
 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792
*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*
c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 105
6*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a
^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b
^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^1
1*d - 192*a^11*b*c*d^11))^(1/4)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 +
1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 1267
2*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^
8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*
c^11*d - 192*a^11*b*c*d^11))^(1/4) - atan((((((81*a^9*b^7*d^10)/2048 - (143
1*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c
^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/20
48 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)/(b^8*c^12
+ a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3
+ 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^
11*d) - (((-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 -
 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 1478
4*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9
*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^
11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b
^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925
184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*
d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12
*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*
a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7
*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4
- 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(167
77216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19
*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 1
1738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15*c^13*d^10 - 13361086464
*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12
*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 -
484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*
b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147
456*a^19*b^4*c^2*d^21))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^
11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 7
92*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*
b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d
)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*
a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*
b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c
^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(
3/4))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 352

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0*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^
6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3
*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))
^(1/4)*1i + (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*
b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a
^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a
^8*b^11*c^2*d^9)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11
+ 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*
a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4
*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))
*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3
*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6
*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*
d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4
) - ((((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^1
4*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5
)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (68
85*a^7*b^9*c^2*d^8)/2048)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^
2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d
^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d^12 + 16*
b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^
8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d
^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 -
 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95
488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^
7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^1
2*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 19
9936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^1
5 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^
18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a
^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d
^6 - 8*a*b^7*c^11*d) + (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3
*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^
7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486
208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b
^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d
^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960
*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^
19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21))/(4096*(b^12*c^1
6 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9
*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*
d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*
a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c
^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4
- 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7
```

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920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a
*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12
*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^
4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 +
 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192
*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*1i - (x^(1/2)*(81*a^10*b^9*d^11
- 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4
+ 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^
7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9)*1i)/(4096*(b^12*c^16 +
 a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^
13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6
 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^1
0*b^2*c^6*d^10 - 12*a*b^11*c^15*d))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12
 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 1
2672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920
*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^
11*c^11*d - 192*a^11*b*c*d^11))^(1/4))/(((((81*a^9*b^7*d^10)/2048 - (1431*a
^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*
d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048
+ (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)/(b^8*c^12 + a
^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 7
0*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*
d) - (((-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 35
20*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a
^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^
3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11)
)^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16
*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184
*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^1
1 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^
8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^1
5*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*
c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 5
6*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(167772
16*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^
17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 1173
8087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^
9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^
10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484
442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7
*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456
*a^19*b^4*c^2*d^21))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11
+ 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*
a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4
*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))
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*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3
*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6
*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*
d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(3/4
))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a
^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b
^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^
3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1
/4) + (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c
^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^1
4*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^1
1*c^2*d^9))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*
b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c
^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8
 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7
)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*
d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6
 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 105
6*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4) + ((((8
1*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3
)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 +
(18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^
9*c^2*d^8)/2048)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^1
0*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a
^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12
 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 1
2672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920
*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^
11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b
^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 33848
32*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^
10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11
*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288
*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8
*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^
9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*
b^7*c^11*d) + (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^1
8*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 73358
37696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b
^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*
d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 499
38432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8
*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145
728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21))/(4096*(b^12*c^16 + a^12*
c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3
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 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792
*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*
c^6*d^10 - 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 105
6*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a
^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b
^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^1
1*d - 192*a^11*b*c*d^11))^(3/4))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1
056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672
*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8
*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c
^11*d - 192*a^11*b*c*d^11))^(1/4) - (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b
^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4
*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^
7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^12*c^16 + a^12*c^4*d^12
- 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^
4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*
c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10
- 12*a*b^11*c^15*d)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^1
0*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^
7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^
8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192
*a^11*b*c*d^11))^(1/4)))*(-(a*b^7)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*
b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7
*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4
*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d -
192*a^11*b*c*d^11))^(1/4)*2i - atan(((-(81*a^8*d^8 + 194481*b^8*c^8 + 40748
4*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5
*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7
)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^
2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a
^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^
6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 830472192
0*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9
*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 82
32*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^
6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*
d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c
^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304
721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^
6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3
690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a
^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 8
5946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616
*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*
d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a
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^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^
5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 132875550
72*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10
*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(3/4)*(((-(81*a^8*d^8 + 194481*b^8*c
^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4
- 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a
^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^
11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 36
90987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b
^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 +
 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^
10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 +
 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^
14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a
^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13
- 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*
d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^
4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4
*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) -
(x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 103022
5920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17
*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15*c^13*d^10
- 13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^12 - 5521702
912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49938432*a^13*b^1
0*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^6*d^17 - 12
7401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c
^3*d^20 + 147456*a^19*b^4*c^2*d^21))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*
a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8
*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d
^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*
a*b^11*c^15*d))) - ((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (1
94481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4
*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^
7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^
5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*
a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d))*1i - (x^(1/2)*(81*a
^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*
b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^
6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9)*1i)/(4096
*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 2
20*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6
*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7
*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))) - (-(81*a^8*d^8 + 194481*
b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4
*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1
```

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512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592
*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3
 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*
a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*
d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 11072962
56*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b
^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^
3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(1677
7216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201
326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*
c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 155
02147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b
^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))
^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*
b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c
^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 167
77216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12
 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*
a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*
d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 36909875
20*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(3/4)*(((-(81*a^8*d^8
 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a
^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7
*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 -
 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^1
0*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13
287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7
*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10
+ 1107296256*a^10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*
b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384
832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d
^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^1
1*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 7628
8*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^
8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c
^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a
*b^7*c^11*d) + (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^
18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335
837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*
b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11
*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49
938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^
8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 314
5728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21))/(4096*(b^12*c^16 + a^12
*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^
```

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3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 79
2*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2
*c^6*d^10 - 12*a*b^11*c^15*d))) - ((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c
*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/204
8 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*
a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)/(b^8*c^12 + a^8*c^4*d
^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^
4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d))*1i +
 (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^
3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5
*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2
*d^9)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^
10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^1
1*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 -
220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))))/((-(81*a^
8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85
946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*
a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d
^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^
2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5
 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 1328755507
2*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*
d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8
 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 -
1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7
*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11
*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690
987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7
*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8
304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10
*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*
d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 +
 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^
12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*
a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^
4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 1550214758
4*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11
*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(3/4)*
(((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^
5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6
 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*
a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 110
7296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^
8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 -
13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9
```

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*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^18*c^18*d
^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*
c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*
a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^
12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*
c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4
*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2
 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^
2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101
248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*
c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 1
4203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280
*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^1
1*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 34
3080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6
*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21))/(4096*(b
^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*
a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^
6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^
9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))) - ((81*a^9*b^7*d^10)/2048 -
(1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^
13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6
)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)/(b^8*c
^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*
d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^
7*c^11*d)) - (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2
*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*
a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*
a^8*b^11*c^2*d^9))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 +
66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^
5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c
^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))) +
 (-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5
*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6
 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a
^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107
296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8
*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 1
3287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*
b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 19448
1*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c
^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d -
 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 2013265
92*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d
^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 1328755507
```

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2*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^1
2*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 110729
6256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2
*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*
c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16
777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 2
01326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^
9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 1
5502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8
*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11
))^(3/4)*(((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a
^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^
2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d +
16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d
^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 83047219
20*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^
13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 36909
87520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^
18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*
a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 +
 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^1
0*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*
a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768
*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6
*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 +
28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) + (x^(1/2)*(16777216*a^2*b^21*c^19*d^4
 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*
a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^1
4*d^9 + 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9
861857280*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960
*a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*
d^16 + 343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048
*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21))
/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d
^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 9
24*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b
^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))) - ((81*a^9*b^7*d^10
)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (7130
97*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^1
1*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/204
8)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3
*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6
 - 8*a*b^7*c^11*d)) + (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 1
94481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5
+ 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8
```

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 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^
5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4
 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*
a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^
15*d)))))*(-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^
3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2
*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 1
6777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^
12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 830472192
0*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^1
3*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 369098
7520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*2i + 2*atan((
(-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*
d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 +
 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^
12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 11072
96256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*
c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13
287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b
^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481
*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^
4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d -
1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 20132659
2*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^
3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072
*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12
*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296
256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*
b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c
^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(167
77216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 20
1326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9
*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15
502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*
b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11)
)^(3/4)*(((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^
3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2
*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 1
6777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^
12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 830472192
0*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^1
3*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 369098
7520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^1
8*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a
^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 +
```

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4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10
*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a
^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*
a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*
c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 2
8*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(16777216*a^2*b^21*c^19*d^4
 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a
^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14
*d^9 + 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 98
61857280*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*
a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d
^16 + 343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*
a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*1
i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14
*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 +
 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9
*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)))*1i - (((81*a^9*b^
7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 -
 (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a
^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^
8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^
2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b
^2*c^6*d^6 - 8*a*b^7*c^11*d)) + (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*
c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^1
5*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^
12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12
*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^
8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*
d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12
*a*b^11*c^15*d))) - (-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2
 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 83
16*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*
c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^1
1*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 +
 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a
^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^
9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-
(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^
3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 5
18616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12
*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296
256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^
15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 1328
7555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3
*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b
```

```
^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*
d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 15
12*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*
a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3
- 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a
^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d
^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 110729625
6*a^10*b^2*c^9*d^11))^(3/4)*(((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b
^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^
3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(1677
7216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201
326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*
c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 155
02147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b
^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))
^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*
c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*
a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11
 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8
*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15
*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c
^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56
*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) + (x^(1/2)*(1677721
6*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^1
7*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738
087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9
*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^1
0*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 4844
42112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*
c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*
a^19*b^4*c^2*d^21)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^1
1 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 79
2*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b
^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d)
))*1i - (((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*
b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*
d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 +
(6885*a^7*b^9*c^2*d^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7
+ 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^
3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d)) - (x^(1/2)*(81*a^10*b^9*d
^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*
d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^
4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*d^9))/(4096*(b^12*c^16
+ a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c
^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^
```

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6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^
10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))))/((-(81*a^8*d^8 + 194481*b^8*c^8 + 40
7484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*
a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*
d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18
*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 369098752
0*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14
*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 830472
1920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*
c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 +
 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316
*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^
19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*
b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8
304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6
*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9
- 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(8
1*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3
- 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518
616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c
^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 110729625
6*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15
*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 132875
55072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c
^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(3/4)*(((-(81*a^8*d^8 + 194481*b^
8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d
^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 151
2*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a
*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 -
 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^
5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^
8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256
*a^10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^
5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14
*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a^8*b^12*c^12*d^10 - 327782
4*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^
13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c
^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8
*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*
a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d)
 - (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 103
0225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b
^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14203486208*a^8*b^15*c^13*d^
10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280*a^10*b^13*c^11*d^12 - 5521
702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11*c^9*d^14 - 49938432*a^13*
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b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343080960*a^15*b^8*c^6*d^17 -
 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*c^4*d^19 - 3145728*a^18*b^
5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12
 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a
^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5
*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10
 - 12*a*b^11*c^15*d)))*1i - (((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)
/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (
432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b
^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^
8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4
*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d))*1i +
(x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10 + 194481*a^2*b^17*c^8*d^3
 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*d^5 + 610344*a^5*b^14*c^5*
d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3*d^8 + 17532*a^8*b^11*c^2*
d^9)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^1
0*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11
*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 2
20*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))) + (-(81*a^8
*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 859
46*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a
*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^
13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2
*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5
- 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072
*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d
^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8
+ 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1
176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*
b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*
c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 36909
87520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*
c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 83
04721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*
b^2*c^9*d^11))^(1/4)*((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d
^2 + 8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 +
8316*a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^1
2*c^19*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a
^11*b*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4
 + 8304721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584
*a^6*b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*
d^9 - 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(3/4)*(
((-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 + 8232*a^3*b^5*c^5
*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*a^6*b^2*c^2*d^6
+ 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^19*d + 16777216*a
```

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^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b*c^8*d^12 + 1107
296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 8304721920*a^4*b^8
*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*b^6*c^13*d^7 - 1
3287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 - 3690987520*a^9*
b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4)*(8192*a^2*b^18*c^18*d^
4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607168*a^5*b^15*c
^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*d^9 + 4958976*a
^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^10*b^10*c^10*d^1
2 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 + 256512*a^13*b^7*c
^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^17 - 768*a^16*b^4*
c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2
- 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2
*c^6*d^6 - 8*a*b^7*c^11*d) + (x^(1/2)*(16777216*a^2*b^21*c^19*d^4 - 1941012
48*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 - 3328573440*a^5*b^18*c
^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087424*a^7*b^16*c^14*d^9 + 14
203486208*a^8*b^15*c^13*d^10 - 13361086464*a^9*b^14*c^12*d^11 + 9861857280*
a^10*b^13*c^11*d^12 - 5521702912*a^11*b^12*c^10*d^13 + 1989672960*a^12*b^11
*c^9*d^14 - 49938432*a^13*b^10*c^8*d^15 - 484442112*a^14*b^9*c^7*d^16 + 343
080960*a^15*b^8*c^6*d^17 - 127401984*a^16*b^7*c^5*d^18 + 27394048*a^17*b^6*
c^4*d^19 - 3145728*a^18*b^5*c^3*d^20 + 147456*a^19*b^4*c^2*d^21)*1i)/(4096*
(b^12*c^16 + a^12*c^4*d^12 - 12*a^11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 22
0*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*
b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*
d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*b^11*c^15*d))*1i - (((81*a^9*b^7*d^10)/2
048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a^2*b^14*c^7*d^3)/2048 - (713097*
a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*c^5*d^5)/2048 + (18067*a^5*b^11*c
^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/2048 + (6885*a^7*b^9*c^2*d^8)/2048)*
1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^
3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^
6 - 8*a*b^7*c^11*d))*1i - (x^(1/2)*(81*a^10*b^9*d^11 - 1512*a^9*b^10*c*d^10
 + 194481*a^2*b^17*c^8*d^3 + 518616*a^3*b^16*c^7*d^4 + 859068*a^4*b^15*c^6*
d^5 + 610344*a^5*b^14*c^5*d^6 - 14266*a^6*b^13*c^4*d^7 - 87192*a^7*b^12*c^3
*d^8 + 17532*a^8*b^11*c^2*d^9)*1i)/(4096*(b^12*c^16 + a^12*c^4*d^12 - 12*a^
11*b*c^5*d^11 + 66*a^2*b^10*c^14*d^2 - 220*a^3*b^9*c^13*d^3 + 495*a^4*b^8*c
^12*d^4 - 792*a^5*b^7*c^11*d^5 + 924*a^6*b^6*c^10*d^6 - 792*a^7*b^5*c^9*d^7
 + 495*a^8*b^4*c^8*d^8 - 220*a^9*b^3*c^7*d^9 + 66*a^10*b^2*c^6*d^10 - 12*a*
b^11*c^15*d)))))*(-(81*a^8*d^8 + 194481*b^8*c^8 + 407484*a^2*b^6*c^6*d^2 +
8232*a^3*b^5*c^5*d^3 - 85946*a^4*b^4*c^4*d^4 - 1176*a^5*b^3*c^3*d^5 + 8316*
a^6*b^2*c^2*d^6 + 518616*a*b^7*c^7*d - 1512*a^7*b*c*d^7)/(16777216*b^12*c^1
9*d + 16777216*a^12*c^7*d^13 - 201326592*a*b^11*c^18*d^2 - 201326592*a^11*b
*c^8*d^12 + 1107296256*a^2*b^10*c^17*d^3 - 3690987520*a^3*b^9*c^16*d^4 + 83
04721920*a^4*b^8*c^15*d^5 - 13287555072*a^5*b^7*c^14*d^6 + 15502147584*a^6*
b^6*c^13*d^7 - 13287555072*a^7*b^5*c^12*d^8 + 8304721920*a^8*b^4*c^11*d^9 -
 3690987520*a^9*b^3*c^10*d^10 + 1107296256*a^10*b^2*c^9*d^11))^(1/4) - ((x^
(1/2)*(3*a*d - 11*b*c))/(16*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^(5/2)*(
```

```
a*d + 7*b*c))/(16*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))/(c^2 + d^2*x^4 + 2*
c*d*x^2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.465 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\right.}{64\sqrt{2}\,c^{9/4}(bc-ad)^3}$$

Rubi [A] time = 0.82, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 472, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{9/4}(bc-ad)^3}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}+\frac{b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3}-\frac{dx^{3/2}(13bc-5ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(13*b*c - 5*a*d)*x^(3/2))
/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(9/4)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (d^(1/4
)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*
a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqr
t[2]*c^(9/4)*(b*c - a*d)^3) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(9/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*(b
*c - a*d)^3) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] -
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a
*d)^3) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(8bc-5ad-5bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(32b^2c^2-13abcd+5a^2\right.}{\left(a+bx^4\right)\left(c\right.}\right.}{16c^2(bc}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int\left(\frac{32b^3c^2x^2}{(bc-ad)\left(a+bx^4\right)} - \frac{d\left(\right.}{}\right.\right.}{16c^2(}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^2}{a+bx^4}\,dx, x, \right.}{(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{5/2}\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x,\right.}{(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,d\right.}{2(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\right.}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)^3}$$

$$= -\frac{dx^{3/2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\,(bc-ad)^3} +$$

Mathematica [A] time = 0.86, size = 620, normalized size = 0.98

$$\frac{1}{128}\left(\frac{\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{9/4}(ad-bc)^3}+\frac{\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{9/4}(bc-ad)^3}+\frac{2\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{9/4}(bc-ad)^3}-\frac{2\sqrt{2}\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{9/4}(bc-ad)^3}+\frac{32\sqrt{2}\,b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{a}\,(bc-ad)^3}+\frac{32\sqrt{2}\,b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{a}\,(ad-bc)^3}+\frac{64\sqrt{2}\,b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,(ad-bc)^3}-\frac{64\sqrt{2}\,b^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}\,(ad-bc)^3}+\frac{8dx^{3/2}(5ad-13bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{32dx^{3/2}}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-32*d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + (8*d*(-13*b*c + 5*a*d)*x^(
3/2))/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (64*Sqrt[2]*b^(9/4)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)^3) - (64*Sqrt[2]*b^
(9/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d
)^3) + (2*Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^
(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(c^(9/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(9/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)^3) + (3
2*Sqrt[2]*b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d +
 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^
(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2
*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(9/4)*
(b*c - a*d)^3))/128
```

IntegrateAlgebraic [A] time = 1.64, size = 382, normalized size = 0.60

$$\frac{\left(5a^2d^{9/4}-18abcd^{5/4}+45b^2c^2\sqrt[4]{d}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{\left(5a^2d^{9/4}-18abcd^{5/4}+45b^2c^2\sqrt[4]{d}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{b^{9/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\,\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\,\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(ad-bc)^3}+\frac{b^{9/4}\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{a}\,(ad-bc)^3}-\frac{dx^{3/2}\left(-9acd-5ad^2x^2+17bc^2+13bcdx^2\right)}{16c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(d*x^(3/2)*(17*b*c^2 - 9*a*c*d + 13*b*c*d*x^2 - 5*a*d^2*x^2))/(c^2*(b
*c - a*d)^2*(c + d*x^2)^2) + (b^(9/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (
b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(1/4)*(-(b*c) + a*d)^3)
+ ((45*b^2*c^2*d^(1/4) - 18*a*b*c*d^(5/4) + 5*a^2*d^(9/4))*ArcTan[(Sqrt[c]
- Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/(32*Sqrt[2]*c^(9/4)*(b*c -
 a*d)^3) + (b^(9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sq
rt[b]*x)])/(Sqrt[2]*a^(1/4)*(-(b*c) + a*d)^3) + ((45*b^2*c^2*d^(1/4) - 18*a
*b*c*d^(5/4) + 5*a^2*d^(9/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sq
rt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.61, size = 968, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4
- sqrt(2)*a^3*c^3*d^5) - 1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*
a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
 - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 +
 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + 1/64*(45*(c*d^3)^(3/4)*b^
2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqr
t(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^
5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) - 1/64*(45*(c*d^3)^(
3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(-sqr
t(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*
a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + (a*b^3)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a
^4*d^3) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x
))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^
3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqr
t(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d
 + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/16*(13*b*c*d^2*x^(7/2) - 5*
a*d^3*x^(7/2) + 17*b*c^2*d*x^(3/2) - 9*a*c*d^2*x^(3/2))/((b^2*c^4 - 2*a*b*c
^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

maple [A] time = 0.02, size = 855, normalized size = 1.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/4*b^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+
(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b^2/(a*d-b*c)
^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^2/(a*d-b
*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+5/16*d^4/(a
*d-b*c)^3/(d*x^2+c)^2/c^2*x^(7/2)*a^2-9/8*d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x^(
7/2)*a*b+13/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*b^2+9/16*d^3/(a*d-b*c)^3
/(d*x^2+c)^2/c*x^(3/2)*a^2-13/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a*b+17/
```

```
16*d/(a*d-b*c)^3/(d*x^2+c)^2*c*x^(3/2)*b^2+5/64*d^2/(a*d-b*c)^3/c^2/(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-9/32*d/(a*d-b*c)^3/c
/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+45/64/(a*d-b
*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128*d
^2/(a*d-b*c)^3/c^2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c
/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-9/64*d/(a*d-b*c
)^3/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+45/128/(a*d-b*c)^3/(c/d)^(1/
4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^
(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2+5/64*d^2/(a*d-b*c)^3/c^2/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-9/32*d/(a*d-b*c)^3/c/(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b+45/64/(a*d-b*c)^3/(c/d)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
```

maxima [A] time = 2.52, size = 594, normalized size = 0.94

$$\frac{b^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{\left(45\,b^{2}c^{2}d-18\,abcd^{2}+5\,a^{2}d^{3}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)}-\frac{\left(13\,bcd^{2}-5\,ad^{3}\right)x^{\frac{7}{2}}+\left(17\,bc^{2}d-9\,acd^{2}\right)x^{\frac{3}{2}}}{16\left(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}+\left(b^{2}c^{4}d^{2}-2\,abc^{3}d^{3}+a^{2}c^{2}d^{4}\right)x^{4}+2\left(b^{2}c^{5}d-2\,abc^{4}d^{2}+a^{2}c^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/128*(45*b^2*c^2*d - 1
8*a*b*c*d^2 + 5*a^2*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*
sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*l
og(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4))
 + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
1/4)*d^(3/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) -
1/16*((13*b*c*d^2 - 5*a*d^3)*x^(7/2) + (17*b*c^2*d - 9*a*c*d^2)*x^(3/2))/(b
^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2
*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

mupad [B] time = 4.35, size = 32735, normalized size = 51.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^2
2*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/1
6 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*
a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^1
5*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^1
0*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/
2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (28062
3*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d
^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a
^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^
10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*
c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^
10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d)
- (x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 105
6*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a
^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b
^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*
d^11))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140
473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b
^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^
10 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 6608
5978112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344
*a^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9
*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75
104256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*
d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*
c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d
^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 2
20*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a
^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 -
3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784
*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10
*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(3/4) - (x^(1/2
)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487
400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8
+ 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*
d^11))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*
c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d
^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 2
20*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a
^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 -
3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784
*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10
```

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*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4) - ((((204
8*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1
711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5
*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d
^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2
+ (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32
835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^
14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*
d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*
a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13
+ 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2
002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003
*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a
^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) + (x^(1/2)*(-b^
9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10
*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5
+ 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 35
20*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*(1
6777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20
*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 -
22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 5569196851
2*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^
13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10
*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 204
0201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6
*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b
^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*
a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^
6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*
d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a
*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^
9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^
6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 +
1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(3/4) + (x^(1/2)*(625*a^9*b^10
*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^
7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b
^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11))/(4096*(b
^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*
a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^
6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*
d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a
*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^
9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^
6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 +
1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4))/(((((2048*a*b^23*c^20*d
```

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^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21
*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)
/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337
495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^1
0*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^1
2*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^1
7)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (2694
9*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^
22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c
^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^1
5*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d
^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^1
1 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) - (x^(1/2)*(-b^9/(16*a^13*d^12
 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4
*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6
*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3
*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*(16777216*a*b^22*
c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 411
5660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6
*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14
*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13 +
52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 1703713
1776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^
8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 786
4320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12
*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^
3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 79
2*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b
^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 19
2*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^
5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b
^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c
^2*d^10 - 192*a^12*b*c*d^11))^(3/4)*1i - (x^(1/2)*(625*a^9*b^10*d^13 + 4100
625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100
220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9
- 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(4096*(b^12*c^18
+ a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c
^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^
6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*
a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^1
2 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7
920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672
*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11
*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4) + ((((2048*a*b^23*c^20*d^4 + (125
*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6
```

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)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844
557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b
^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^
11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^1
5)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1
110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^
7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1
i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 -
 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3
003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002
*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^
12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) + (x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b
^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*
d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6
- 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 10
56*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*(16777216*a*b^22*c^21*d^4
- 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a
^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^1
6*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^11 + 6
6935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 528074342
40*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 17037131776*a^13
*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^1
8 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 7864320*a^18
*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12
 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a
^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5
*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^
10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^1
1*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8
*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^
7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 -
 192*a^12*b*c*d^11))^(3/4)*1i + (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^1
8*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b
^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*
a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(4096*(b^12*c^18 + a^12*c^
6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 +
 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a
^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*
c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a
^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b
^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*
c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*
d^10 - 192*a^12*b*c*d^11))^(1/4) + ((625*a^8*b^12*d^12)/4096 - (4100625*a*b
^19*c^7*d^5)/4096 - (12375*a^7*b^13*c*d^11)/4096 + (5376375*a^2*b^18*c^6*d^
6)/4096 - (3881925*a^3*b^17*c^5*d^7)/4096 + (1726515*a^4*b^16*c^4*d^8)/4096
```

```
 - (521235*a^5*b^15*c^3*d^9)/4096 + (101925*a^6*b^14*c^2*d^10)/4096)/(b^14*
c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*
b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b
^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*
c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^
8*d^12 - 14*a*b^13*c^19*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2
*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8
*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^
5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^
10 - 192*a^12*b*c*d^11))^(1/4) - atan(((((2048*a*b^23*c^20*d^4 + (125*a^20*
b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 -
 (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^
6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^
13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^1
3)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 -
 (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975
*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*
d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)/(b^14*c
^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b
^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^
8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c
^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8
*d^12 - 14*a*b^13*c^19*d) - (x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 -
 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920
*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^
8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^
2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*(16777216*a*b^22*c^21*d^4 - 20132659
2*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^
18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38
637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600
*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^
12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*
d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470
592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d
^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11
*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^1
4*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7
+ 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*
b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d +
 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 126
72*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a
^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*
b*c*d^11))^(3/4)*1i - (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5
- 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^
7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c
```

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^3*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(4096*(b^12*c^18 + a^12*c^6*d^12 - 1
2*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b
^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^1
1*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 -
 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^
11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4
 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 +
7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192
*a^12*b*c*d^11))^(1/4) - (((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16
 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*
b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^
9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (
14638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (509349
83*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13
*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*
d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (374
5*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)/(b^14*c^20 + a^14*c^6
*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 +
 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3
432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001
*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b
^13*c^19*d) + (x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*
c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d
^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7
+ 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 1
92*a^12*b*c*d^11))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^2
0*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825
629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*
b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13
*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 -
 33731641344*a^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993
600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c
^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a
^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 +
66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^
5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*
c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))
*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10
*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7
*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8
 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(3/
4)*1i + (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^1
1*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^
4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 7110
0*a^7*b^12*c^2*d^11)*1i)/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d
```

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^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 -
792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^
8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^1
7*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^
3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b
^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c
^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11
))^(1/4))/((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b
^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)
/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (934779
9*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b
^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c
^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16
)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280
623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3
*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^1
3*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10
*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^
13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10
*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) -
(x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*
a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6
*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4
*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^
11))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 114047
3856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^1
8*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10
 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 660859
78112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a
^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c
^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 7510
4256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^
22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^
16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5
 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220
*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d))*(-b^9/(16*a^1
3*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 35
20*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a
^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b
^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(3/4) - (x^(1/2)*
(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 448740
0*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 +
1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^
11))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^
16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5
```

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 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220
*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^1
3*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 35
20*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a
^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b
^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4) + (((2048*a
*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711
115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^
19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10
)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (
15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32835
743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*
b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^1
9)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^1
9*b^5*c^2*d^22)/16)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^
2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5
*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^
6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3
*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) + (x^(1/2)*(-b^9/(16*a
^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 -
3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784
*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10
*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*(16777216
*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d
^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 2249352
8064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b
^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12
*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 +
 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216
*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^
20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^1
8 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9
*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*
d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 6
6*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c
^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 +
 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 126
72*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^
11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(3/4) + (x^(1/2)*(625*a^9*b^10*d^13 +
 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 +
 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4
*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11))/(4096*(b^12*c^1
8 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9
*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*
d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 6
```

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6*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c
^12 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 +
 7920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 126
72*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^
11*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4) - ((625*a^8*b^12*d^12)/4096 - (
4100625*a*b^19*c^7*d^5)/4096 - (12375*a^7*b^13*c*d^11)/4096 + (5376375*a^2*
b^18*c^6*d^6)/4096 - (3881925*a^3*b^17*c^5*d^7)/4096 + (1726515*a^4*b^16*c^
4*d^8)/4096 - (521235*a^5*b^15*c^3*d^9)/4096 + (101925*a^6*b^14*c^2*d^10)/4
096)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2
 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 +
 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 20
02*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*
a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d)))*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^1
2 - 192*a^2*b^11*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7
920*a^5*b^8*c^8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672
*a^8*b^5*c^5*d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11
*b^2*c^2*d^10 - 192*a^12*b*c*d^11))^(1/4)*2i + ((x^(3/2)*(9*a*d^2 - 17*b*c*
d))/(16*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^(7/2)*(5*a*d - 13*b*c
))/(16*c*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))/(c^2 + d^2*x^4 + 2*c*d*x^2)
- atan(((-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 575910
0*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359
640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b
^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256
*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*
d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 1328755
5072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^
12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*((
(2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5
+ (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575
*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^
14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12
)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 +
 (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (171763
5*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*
c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2
725*a^19*b^5*c^2*d^22)/16)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13
+ 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2
002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003
*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a
^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) - (x^(1/2)*(-(6
25*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^
6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*
c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 1
6777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^
19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 132875
```

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55072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5
*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 110
7296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*(16777216*a*b^
22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 -
4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*
a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c
^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13
 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 1703
7131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15
*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 -
7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a
^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15
*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 -
 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^1
0*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6
561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 +
1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 -
9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*
a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d
^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 155021475
84*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^1
3*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 20132
6592*a*b^11*c^20*d))^(3/4)*1i - (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^1
8*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b
^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*
a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(4096*(b^12*c^18 + a^12*c^
6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 +
 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a
^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*
c^8*d^10 - 12*a*b^11*c^17*d))) - (-(625*a^8*d^9 + 4100625*b^8*c^8*d - 65610
00*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283
526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000
*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11
*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 +
 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a
^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^
8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592
*a*b^11*c^20*d))^(1/4)*(((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 -
 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^
20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)
/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14
638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983
*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b
^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^
18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*
```

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a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)/(b^14*c^20 + a^14*c^6*d
^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1
001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 343
2*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a
^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^1
3*c^19*d) + (x^(1/2)*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7
*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*
c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8
)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11
 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*
a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*
d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 36909875
20*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20
*d))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 114047
3856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^1
8*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10
 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 660859
78112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a
^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c
^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 7510
4256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^
22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^
16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5
 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220
*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d))*(-(625*a^8*d^
9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3
236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 +
 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a
^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 -
3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5
*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7
 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a
^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(3/4)*1i + (x^(1/2)*(625*a^9*
b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^1
7*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a
^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(
4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2
 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924
*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b
^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d))))/((-(625*a^8*d^9 +
4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 32367
60*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 711
00*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*
c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690
987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7
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*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8
304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*
b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*(((2048*a*b^23*c^20*d^4 + (
125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*
d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (
844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337495*a^
8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^10*b^14
*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*
d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 -
 (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17
*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16
)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 -
364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 30
03*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*
a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^1
2*b^2*c^8*d^12 - 14*a*b^13*c^19*d) - (x^(1/2)*(-(625*a^8*d^9 + 4100625*b^8*
c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c
^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c
^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 2
01326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b
^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 +
15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^
8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^1
0 - 201326592*a*b^11*c^20*d))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a
^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*
d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637
076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^
9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*
c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^1
6 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592
*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21
 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*
c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d
^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 4
95*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^1
1*c^17*d)))*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 57
59100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 -
 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(167772
16*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 110729
6256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c
^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 132
87555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^
3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(3/4
) - (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*
d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^
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15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^
7*b^12*c^2*d^11))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 6
6*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5
*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c
^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))
 + (-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*
b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^
5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^
21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b
^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 -
13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a
^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9
 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*(((2048*
a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (171
1115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b
^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^1
0)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 +
(15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (3283
5743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14
*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^
19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^
19*b^5*c^2*d^22)/16)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a
^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^
5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b
^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^
3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) + (x^(1/2)*(-(625*a^8
*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3
- 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^
6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 1677721
6*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2
 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*
a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*
d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 110729625
6*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*(16777216*a*b^22*c^2
1*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 411566
0800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^
17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^
11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 528
07434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 1703713177
6*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c
^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 786432
0*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^
6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 +
 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a
^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*
```

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c^8*d^10 - 12*a*b^11*c^17*d)))*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000
*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 128352
6*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a
^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b
*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8
304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6
*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8
- 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a
*b^11*c^20*d))^(3/4) + (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5
 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d
^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*
c^3*d^10 + 71100*a^7*b^12*c^2*d^11))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*
a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8
*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*
d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 1
2*a*b^11*c^17*d))) - ((625*a^8*b^12*d^12)/4096 - (4100625*a*b^19*c^7*d^5)/4
096 - (12375*a^7*b^13*c*d^11)/4096 + (5376375*a^2*b^18*c^6*d^6)/4096 - (388
1925*a^3*b^17*c^5*d^7)/4096 + (1726515*a^4*b^16*c^4*d^8)/4096 - (521235*a^5
*b^15*c^3*d^9)/4096 + (101925*a^6*b^14*c^2*d^10)/4096)/(b^14*c^20 + a^14*c^
6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3
+ 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 -
3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 100
1*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*
b^13*c^19*d)))*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 +
 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^
5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(167
77216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 110
7296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^
8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 -
13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9
*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(
1/4)*2i + 2*atan(((-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^
2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4
*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(
16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 +
1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4
*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6
 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*
a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d)
)^(1/4)*((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^2
2*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/1
6 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*
a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^1
5*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^1
0*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/
```

```
2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (28062
3*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d
^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a
^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^
10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*
c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^
10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d)
 - (x^(1/2)*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 575
9100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 -
359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(1677721
6*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296
256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^
17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 1328
7555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3
*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)
*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b
^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8
 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 5569196
8512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10
*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c
^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 +
2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*
b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096
*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 2
20*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6
*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c
^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-(625*a^8*d^9 + 410062
5*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3
*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6
*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^
12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520
*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*
d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721
920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^
11*d^10 - 201326592*a*b^11*c^20*d))^(3/4) - (x^(1/2)*(625*a^9*b^10*d^13 + 4
100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4
100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d
^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11))/(4096*(b^12*c^18
+ a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c
^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^
6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*
a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d))) - (-(625*a^8*d^9 + 4100625*b^8*c^8*
d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d
^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d
^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 20132
```

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6592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c
^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 1550
2147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^
4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 -
201326592*a*b^11*c^20*d))^(1/4)*((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*
d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294
995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18
*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^1
1)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 -
 (50934983*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207
335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*
b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/
2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^2
0 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^1
1*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*
c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^1
1*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d
^12 - 14*a*b^13*c^19*d) + (x^(1/2)*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 656
1000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 12
83526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 90
00*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^
11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3
 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584
*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*
d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 2013265
92*a*b^11*c^20*d))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^2
0*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825
629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*
b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13
*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 -
 33731641344*a^12*b^11*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993
600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c
^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a
^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 +
66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^
5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*
c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))
*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b
^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5
*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^2
1 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^
10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 1
3287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^
7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9
+ 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(3/4) + (x^(1/2
```

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)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487
400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8
+ 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*
d^11))/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*
c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d
^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 2
20*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d))))/((-(625*a^
8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3
 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d
^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 167772
16*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^
2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072
*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14
*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 11072962
56*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*((((2048*a*b^23*c^2
0*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^22*c^19*d^5 + (1711115*a^3*b
^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)/16 + (565575*a^5*b^19*c^16*d
^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (9347799*a^7*b^17*c^14*d^10)/4 + (20
337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*b^15*c^12*d^12)/2 + (15550975*
a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*c^10*d^14)/8 + (32835743*a^12*
b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^16)/2 + (1717635*a^14*b^10*c^7*
d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (280623*a^16*b^8*c^5*d^19)/4 - (2
6949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2
*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^1
2*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*
c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^1
2*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*
d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d) - (x^(1/2)*(-(625*a^8*d^9 +
 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236
760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71
100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12
*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 369
0987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^
7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 +
8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10
*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1/4)*(16777216*a*b^22*c^21*d^4
- 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a
^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^1
6*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 55691968512*a^8*b^15*c^14*d^11 + 6
6935193600*a^9*b^14*c^13*d^12 - 66085978112*a^10*b^13*c^12*d^13 + 528074342
40*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^11*c^10*d^15 + 17037131776*a^13
*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17 + 2040201216*a^15*b^8*c^7*d^1
8 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^17*b^6*c^5*d^20 - 7864320*a^18
*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4096*(b^12*c^18 + a^12*c^6*d^12
 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a
```

```
^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5
*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^
10 - 12*a*b^11*c^17*d)))*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7
*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*
b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c
*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*
d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721
920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c
^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690
987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*
c^20*d))^(3/4)*1i - (x^(1/2)*(625*a^9*b^10*d^13 + 4100625*a*b^18*c^8*d^5 -
9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d^6 + 4100220*a^3*b^16*c^6*d^7
- 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14*c^4*d^9 - 334040*a^6*b^13*c^3
*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(4096*(b^12*c^18 + a^12*c^6*d^12 - 12*
a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8
*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*
d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 1
2*a*b^11*c^17*d))) + (-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7
*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*
c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8
)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11
 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*
a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*
d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 36909875
20*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20
*d))^(1/4)*((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*
b^22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7
)/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (93477
99*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9*
b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^13*
c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*d^1
6)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 + (28
0623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b^6*c^
3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^14 - 1
4*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4
*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b
^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4
*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*
d) + (x^(1/2)*(-(625*a^8*d^9 + 4100625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 +
5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5
 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(1677
7216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 201326592*a^11*b*c^10*d^11 + 1107
296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8
*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 1
3287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*
```

```
b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10 - 201326592*a*b^11*c^20*d))^(1
/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^21*c^20*d^5 + 1140473856*a^
3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7 + 10825629696*a^5*b^18*c^17*
d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637076480*a^7*b^16*c^15*d^10 - 5569
1968512*a^8*b^15*c^14*d^11 + 66935193600*a^9*b^14*c^13*d^12 - 66085978112*a
^10*b^13*c^12*d^13 + 52807434240*a^11*b^12*c^11*d^14 - 33731641344*a^12*b^1
1*c^10*d^15 + 17037131776*a^13*b^10*c^9*d^16 - 6723993600*a^14*b^9*c^8*d^17
 + 2040201216*a^15*b^8*c^7*d^18 - 463470592*a^16*b^7*c^6*d^19 + 75104256*a^
17*b^6*c^5*d^20 - 7864320*a^18*b^5*c^4*d^21 + 409600*a^19*b^4*c^3*d^22))/(4
096*(b^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2
- 220*a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*
a^6*b^6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^
3*c^9*d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d)))*(-(625*a^8*d^9 + 410
0625*b^8*c^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*
a^3*b^5*c^5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*
a^6*b^2*c^2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9
*d^12 - 201326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987
520*a^3*b^9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^
16*d^5 + 15502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304
721920*a^8*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2
*c^11*d^10 - 201326592*a*b^11*c^20*d))^(3/4)*1i + (x^(1/2)*(625*a^9*b^10*d^
13 + 4100625*a*b^18*c^8*d^5 - 9000*a^8*b^11*c*d^12 - 4487400*a^2*b^17*c^7*d
^6 + 4100220*a^3*b^16*c^6*d^7 - 2444184*a^4*b^15*c^5*d^8 + 1099206*a^5*b^14
*c^4*d^9 - 334040*a^6*b^13*c^3*d^10 + 71100*a^7*b^12*c^2*d^11)*1i)/(4096*(b
^12*c^18 + a^12*c^6*d^12 - 12*a^11*b*c^7*d^11 + 66*a^2*b^10*c^16*d^2 - 220*
a^3*b^9*c^15*d^3 + 495*a^4*b^8*c^14*d^4 - 792*a^5*b^7*c^13*d^5 + 924*a^6*b^
6*c^12*d^6 - 792*a^7*b^5*c^11*d^7 + 495*a^8*b^4*c^10*d^8 - 220*a^9*b^3*c^9*
d^9 + 66*a^10*b^2*c^8*d^10 - 12*a*b^11*c^17*d))) + ((625*a^8*b^12*d^12)/409
6 - (4100625*a*b^19*c^7*d^5)/4096 - (12375*a^7*b^13*c*d^11)/4096 + (5376375
*a^2*b^18*c^6*d^6)/4096 - (3881925*a^3*b^17*c^5*d^7)/4096 + (1726515*a^4*b^
16*c^4*d^8)/4096 - (521235*a^5*b^15*c^3*d^9)/4096 + (101925*a^6*b^14*c^2*d^
10)/4096)/(b^14*c^20 + a^14*c^6*d^14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^1
8*d^2 - 364*a^3*b^11*c^17*d^3 + 1001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*
d^5 + 3003*a^6*b^8*c^14*d^6 - 3432*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8
 - 2002*a^9*b^5*c^11*d^9 + 1001*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11
+ 91*a^12*b^2*c^8*d^12 - 14*a*b^13*c^19*d)))*(-(625*a^8*d^9 + 4100625*b^8*c
^8*d - 6561000*a*b^7*c^7*d^2 + 5759100*a^2*b^6*c^6*d^3 - 3236760*a^3*b^5*c^
5*d^4 + 1283526*a^4*b^4*c^4*d^5 - 359640*a^5*b^3*c^3*d^6 + 71100*a^6*b^2*c^
2*d^7 - 9000*a^7*b*c*d^8)/(16777216*b^12*c^21 + 16777216*a^12*c^9*d^12 - 20
1326592*a^11*b*c^10*d^11 + 1107296256*a^2*b^10*c^19*d^2 - 3690987520*a^3*b^
9*c^18*d^3 + 8304721920*a^4*b^8*c^17*d^4 - 13287555072*a^5*b^7*c^16*d^5 + 1
5502147584*a^6*b^6*c^15*d^6 - 13287555072*a^7*b^5*c^14*d^7 + 8304721920*a^8
*b^4*c^13*d^8 - 3690987520*a^9*b^3*c^12*d^9 + 1107296256*a^10*b^2*c^11*d^10
 - 201326592*a*b^11*c^20*d))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.466 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=633

$$-\frac{b^{11/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^3}-\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}$$

Rubi [A] time = 0.83, antiderivative size = 633, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{11/4}(bc-ad)^3}+\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{b^{11/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^3}-\frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}-\frac{d\sqrt{x}(15bc-7ad)}{16c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{d\sqrt{x}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*Sqrt[x])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(15*b*c - 7*a*d)*Sqrt[x])
/(16*c^2*(b*c - a*d)^2*(c + d*x^2)) - (b^(11/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (d^(3
/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 -
 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(3
2*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (b^(11/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(11/4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^
(3/4)*(b*c - a*d)^3) + (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4
)*(b*c - a*d)^3) - (d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/4)*(b
*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-7ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-45abcd+21a}{\left(a+bx^4\right)}\right.}{16c^2(b}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x\right.}{(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^3\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x\right.}{\sqrt{a}\,(bc-ad)^3}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^{5/2}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+}\right.}{2\sqrt{a}\,(bc-ad)}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{2\sqrt{2}\,a^{3/4}(bc-a}$$

$$= -\frac{d\sqrt{x}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\left(c+dx^2\right)} - \frac{b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{3/4}(bc-ad)^3}$$

Mathematica [**A**] time = 0.85, size = 620, normalized size = 0.98

$$\frac{1}{128}\left(\frac{32\sqrt{2}\,b^{11/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}(ad-bc)^3}+\frac{32\sqrt{2}\,b^{11/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}(bc-ad)^3}+\frac{64\sqrt{2}\,b^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}(ad-bc)^3}-\frac{64\sqrt{2}\,b^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}(ad-bc)^3}+\frac{\sqrt{2}\,d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{11/4}(bc-ad)^3}+\frac{\sqrt{2}\,d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{11/4}(ad-bc)^3}+\frac{2\sqrt{2}\,d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{11/4}(bc-ad)^3}-\frac{2\sqrt{2}\,d^{3/4}\left(21a^2d^2-66abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{11/4}(bc-ad)^3}+\frac{8d\sqrt{x}\,(7ad-15bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{32d\sqrt{x}}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-32*d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)^2) + (8*d*(-15*b*c + 7*a*d)*Sqr
t[x])/(c^2*(b*c - a*d)^2*(c + d*x^2)) + (64*Sqrt[2]*b^(11/4)*ArcTan[1 - (Sq
```

```
rt[2]*b^(1/4)*Sqrt[x]])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3) - (64*Sqrt[2]*b
^(11/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a
*d)^3) + (2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) - (2*Sqrt[2
]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(c^(11/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b^(11/4)*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*(-(b*c) + a*
d)^3) + (32*Sqrt[2]*b^(11/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(a^(3/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a
*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(c^(11/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d
+ 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
c^(11/4)*(-(b*c) + a*d)^3))/128
```

IntegrateAlgebraic [A] time = 1.61, size = 383, normalized size = 0.61

$$\frac{b^{11/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}\,x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}\,a^{3/4}(ad-bc)^3}-\frac{b^{11/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{\sqrt{2}\,a^{3/4}(ad-bc)^3}+\frac{\left(21a^2d^{11/4}-66abcd^{7/4}+77b^2c^2d^{3/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{\left(21a^2d^{11/4}-66abcd^{7/4}+77b^2c^2d^{3/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{d\sqrt{x}\left(-11acd-7ad^2x^2+19bc^2+15bcdx^2\right)}{16c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(d*Sqrt[x]*(19*b*c^2 - 11*a*c*d + 15*b*c*d*x^2 - 7*a*d^2*x^2))/(c^2*(
b*c - a*d)^2*(c + d*x^2)^2) + (b^(11/4)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) -
 (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]])/(Sqrt[2]*a^(3/4)*(-(b*c) + a*d)^3
) + ((77*b^2*c^2*d^(3/4) - 66*a*b*c*d^(7/4) + 21*a^2*d^(11/4))*ArcTan[(Sqrt
[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/(32*Sqrt[2]*c^(11/4)*(
b*c - a*d)^3) - (b^(11/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a
] + Sqrt[b]*x)]])/(Sqrt[2]*a^(3/4)*(-(b*c) + a*d)^3) - ((77*b^2*c^2*d^(3/4)
- 66*a*b*c*d^(7/4) + 21*a^2*d^(11/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x])/(Sqrt[c] + Sqrt[d]*x)]])/(32*Sqrt[2]*c^(11/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.46, size = 960, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] (a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d
^2 - sqrt(2)*a^4*d^3) + (a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^
2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(1/4)*b^2*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)
*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(1/
4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3
 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/3
2*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a
^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)
*a^3*c^3*d^3) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d +
 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2
*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*
d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a
^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) + 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(
c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)
^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2
)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/16*(15*b*c*d^2*x^(5/2) - 7*a*d^3
*x^(5/2) + 19*b*c^2*d*sqrt(x) - 11*a*c*d^2*sqrt(x))/((b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

maple [A] time = 0.02, size = 882, normalized size = 1.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] -1/4*b^3/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b^3/(a*d-b*
c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^3/(a
*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+7/16*
d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^(5/2)*a^2-11/8*d^3/(a*d-b*c)^3/(d*x^2+c)^
2/c*x^(5/2)*a*b+15/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2+11/16*d^3/(a*
d-b*c)^3/(d*x^2+c)^2/c*x^(1/2)*a^2-15/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)
*a*b+19/16*d/(a*d-b*c)^3/(d*x^2+c)^2*c*x^(1/2)*b^2+21/64*d^3/(a*d-b*c)^3/c^
3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-33/32*d^2/(
a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*
b+77/64*d/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)*b^2+21/64*d^3/(a*d-b*c)^3/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/
```

```
d)^(1/4)*x^(1/2)-1)*a^2-33/32*d^2/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+77/64*d/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+21/128*d^3/(a*d-b*c)^3/c^3*(c
/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(
1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-33/64*d^2/(a*d-b*c)^3/c^2*(c/d)^(1/4
)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(
1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+77/128*d/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*l
n((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2
)+(c/d)^(1/2)))*b^2
```

maxima [**A**]   time = 2.61, size = 675, normalized size = 1.07

$$-\frac{\left(15\,bcd^{2}-7\,ad^{3}\right)x^{\frac{5}{2}}+\left(19\,bc^{2}d-11\,acd^{2}\right)\sqrt{x}}{16\left(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}+\left(b^{2}c^{4}d^{2}-2\,abc^{3}d^{3}+a^{2}c^{2}d^{4}\right)x^{4}+2\left(b^{2}c^{5}d-2\,abc^{4}d^{2}+a^{2}c^{3}d^{3}\right)x^{2}\right)}+\frac{\frac{2\sqrt{2}b^{3}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^{3}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{11}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{11}{4}}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{\frac{2\sqrt{2}\left(77\,b^{2}c^{2}d-66\,abcd^{2}+21\,a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(77\,b^{2}c^{2}d-66\,abcd^{2}+21\,a^{2}d^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(77\,b^{2}c^{2}d-66\,abcd^{2}+21\,a^{2}d^{3}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(77\,b^{2}c^{2}d-66\,abcd^{2}+21\,a^{2}d^{3}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*((15*b*c*d^2 - 7*a*d^3)*x^(5/2) + (19*b*c^2*d - 11*a*c*d^2)*sqrt(x))/
(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c
^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2) + 1/4*(2*sqr
t(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^3*arcta
n(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(11/4)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(11/4)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/128*(2*sqrt(2)*(77*b^2*c^2*d
- 66*a*b*c*d^2 + 21*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*arctan(-1/2*sqrt(2)*(
sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^
3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1
/4)) - sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*log(-sqrt(2)*c^(1
/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^5 - 3*
a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)
```

mupad [**B**]   time = 4.41, size = 36997, normalized size = 58.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] atan(((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*
a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8
```

```
*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b
^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*
d^11))^(1/4)*((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*
a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c
^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d
^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12
)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 5
6*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2
*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592
*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*
d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 1839726
5920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8
*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*
c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*
d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 2
3871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^
17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))
/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d
^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 9
24*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9
*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) - ((-b^11/(16*a^
15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 -
 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 1478
4*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a
^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(8192*
a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117
952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^
9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9
*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^1
4 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b
^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16
+ a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3
 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^
7*c^15*d))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d +
1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 1267
2*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a
^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14
*b*c*d^11))^(3/4))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c
^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^
4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7
+ 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 -
192*a^14*b*c*d^11))^(1/4)*1i + (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^
19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276
*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^
11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13)*1i)/(4096*(b^
```

```
12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a
^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6
*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*
d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))) - (-b^11/(16*a^15*d^12 +
16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*
b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*
c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^
3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*((((194481*a^8*b
^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (22504
23*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3
*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^1
1*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)/(b^8*c^16 + a^8*c^8*d
^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b
^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) -
 ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256
*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^1
9*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 801
92667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 28998041600
0*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^11
*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10
*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19
- 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a
^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))/(4096*(b^12*c^20 + a^12*c^8*
d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 4
95*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7
*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c
^10*d^10 - 12*a*b^11*c^19*d)) + ((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 -
192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*
a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^1
0*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b
^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b
^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*d^7 + 14279
68*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^13*c^16*d^1
0 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 + 16344064*a^
10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b^8*c^11*d^1
5 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*
c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d
^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a
^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-b^11/(16*a^15*d^
12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520
*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9
*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b
^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(3/4))*(-b^11/(16
*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^
2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 1
```

```
4784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 352
0*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*1i
- (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18
*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424
*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^
12 + 14378364*a^6*b^13*c^2*d^13)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*
a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8
*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*
d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 -
 12*a*b^11*c^19*d))))/((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^
11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^
8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*
d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^1
0 - 192*a^14*b*c*d^11))^(1/4)*((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8
*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86
420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (802710
27*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*
a^6*b^10*c^2*d^12)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2
*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11
*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^2
3*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 35938467
84*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c
^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 -
 181503787008*a^8*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258
621440*a^10*b^13*c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 24996111974
4*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14
*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^2
0 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^
19*b^4*c^4*d^23))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 6
6*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5
*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c
^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))
 - ((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^
5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b
^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4
*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^
11))^(1/4)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^1
7*c^20*d^6 - 1117952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192
*a^6*b^14*c^17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^
11 - 17335296*a^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*
a^11*b^9*c^12*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^1
6 + 501248*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*
d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56
*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*
c^10*d^6 - 8*a*b^7*c^15*d))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a
```

^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b
^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5
*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^
2*d^10 - 192*a^14*b*c*d^11))^(3/4))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12
 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 79
20*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*
a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^1
3*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4) + (x^(1/2)*(194481*a^8*b^11*d^15
 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^
14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a
^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13)
)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*
d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 +
924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^
9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))) + (-b^11/(16*a
^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2
- 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 147
84*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*
a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*((((1
94481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/20
48 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (10
6888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915
667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)/(b^8*c^16
+ a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3
 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^
7*c^15*d) - ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 +
 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a
^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17
*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 +
289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222
688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 14724877516
8*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^
8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 -
132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))/(4096*(b^12*c^20
+ a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c
^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^
6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66
*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) + ((-b^11/(16*a^15*d^12 + 16*a^3*b
^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*
d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6
- 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 +
1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(8192*a*b^19*c^22*d^4 -
90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*
d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^
13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 +

```
16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b
^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 7603
2*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*
a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12
*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-b^11/
(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10
*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5
+ 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 -
3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(3/4))
*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b
^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*
c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^
4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11)
)^(1/4) - (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7 - 13093264
8*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 1
89998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^1
4*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13))/(4096*(b^12*c^20 + a^12*c^8*d^12
- 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^
4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*
c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d
^10 - 12*a*b^11*c^19*d)))))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a
^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b
^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5
*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^
2*d^10 - 192*a^14*b*c*d^11))^(1/4)*2i + 2*atan(((-b^11/(16*a^15*d^12 + 16*a
^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*
c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*
d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^
9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(((((194481*a^8*b^8*
d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*
a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^
13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c
^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^16 + a^8*c^8*
d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*
b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d)
- ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 110729625
6*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^
19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80
192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 2899804160
00*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^1
1*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^1
0*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19
 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*
a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12*c^20 + a^12*
c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3
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 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792
*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b
^2*c^10*d^10 - 12*a*b^11*c^19*d)) + ((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^1
2 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7
920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672
*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^
13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(8192*a*b^19*c^22*d^4 - 90112*a
^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*d^7 + 1
427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^13*c^16
*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 + 1634406
4*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b^8*c^11
*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 76032*a^15*
b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c
^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 -
56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-b^11/(16*a^1
5*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 -
3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784
*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^
12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(3/4)*1i)*(-b
^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*
c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*
d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^
8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1
/4) + (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*
b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 18999
8424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^
3*d^12 + 14378364*a^6*b^13*c^2*d^13))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12
*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^
8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13
*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10
- 12*a*b^11*c^19*d))) - (-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b
^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c
^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5
*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^
10 - 192*a^14*b*c*d^11))^(1/4)*(((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c
^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (
86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (8027
1027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (1212794
1*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 +
28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^
3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^
23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 35
93846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*
b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*
d^11 - 181503787008*a^8*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 -
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352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 24996
1119744*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10*c^10*d^17 - 6771808665
6*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c
^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225
344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9
*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4
- 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*
a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11
*c^19*d)) - ((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d
+ 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12
672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920
*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^
14*b*c*d^11))^(1/4)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 43084
8*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8
 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^1
2*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 -
11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7
*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16
*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14
*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28
*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^1
2 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7
920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672
*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^
13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(3/4)*1i)*(-b^11/(16*a^15*d^12 + 16*a
^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*
c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*
d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^
9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4) - (x^(1/2)*(194481*a
^8*b^11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a
^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 +
 119638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^
13*c^2*d^13))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^
2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7
*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*
d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))))/(
(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^
10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c
^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4
*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))
^(1/4)*(((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^1
5*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^
8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/
2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/204
8)*1i)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56
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*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*
c^10*d^6 - 8*a*b^7*c^15*d) - ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*
a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d
^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265
920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*
b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c
^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d
^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23
871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^1
7*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23)*1
i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18
*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 +
 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a
^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) + ((-b^11/(16*
a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2
 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14
784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520
*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(819
2*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 11
17952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*
d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a
^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d
^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14
*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^1
6 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d
^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*
b^7*c^15*d))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d
+ 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12
672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920
*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^
14*b*c*d^11))^(3/4)*1i)*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b
^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c
^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5
*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^
10 - 192*a^14*b*c*d^11))^(1/4)*1i + (x^(1/2)*(194481*a^8*b^11*d^15 + 412243
37*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 2010
81276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c
^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13)*1i)/(409
6*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 -
220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^
6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*
c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))) + (-b^11/(16*a^15*d^
12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520
*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9
*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b
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^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(((((194481
*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 -
(2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (1068888
69*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a
^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^16 +
 a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3
+ 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7
*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 +
1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^
4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*
d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 2
89980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 3342226
88256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 147248775168
*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8
*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 1
32120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12*c^2
0 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9
*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*
d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 +
66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) - ((-b^11/(16*a^15*d^12 + 16*a^3
*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^
9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^
6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9
+ 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4)*(8192*a*b^19*c^22*d^4
- 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^1
9*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*
b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12
+ 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12
*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 76
032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 -
8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^
12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-b^1
1/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^
10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^
5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8
- 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(3/4
)*1i)*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056*
a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a^8
*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^11*b
^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*b*c*
d^11))^(1/4)*1i - (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7 -
130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6
*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043608
*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13)*1i)/(4096*(b^12*c^20 + a^1
2*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d
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^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 7
92*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10
*b^2*c^10*d^10 - 12*a*b^11*c^19*d)))))*(-b^11/(16*a^15*d^12 + 16*a^3*b^12*c
^12 - 192*a^4*b^11*c^11*d + 1056*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 +
 7920*a^7*b^8*c^8*d^4 - 12672*a^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 126
72*a^10*b^5*c^5*d^7 + 7920*a^11*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*
a^13*b^2*c^2*d^10 - 192*a^14*b*c*d^11))^(1/4) + ((x^(1/2)*(11*a*d^2 - 19*b*
c*d))/(16*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^(5/2)*(7*a*d - 15*b
*c))/(16*c*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))/(c^2 + d^2*x^4 + 2*c*d*x^2
) + atan((((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b
^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*
d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10
)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2
048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a
^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^
10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*
b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7
 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 1839726592
0*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^
15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^1
3*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^1
6 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 2387
1029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*
b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))/(4
096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2
- 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*
a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^
3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) - ((-(194481*a^8*d^
11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6
*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5
*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b
^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 110729625
6*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19
*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 132875
55072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c
^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*(
8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 -
 1117952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^
17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 1733529
6*a^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^1
2*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a
^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*
c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^1
3*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8
*a*b^7*c^15*d))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7
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*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 11918669
4*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2
444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326
592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^
20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502
147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4
*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 2
01326592*a*b^11*c^22*d))^(3/4))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 -
 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^
5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6
*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^
11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 36909
87520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*
c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 83
04721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b
^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*1i + (x^(1/2)*(194481*a^8*b^
11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^
12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 1196
38278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^
2*d^13)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*
b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c
^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^
8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)))*(-(1
94481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116
*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 -
51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)
/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11
 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*
a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*
d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 36909875
20*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22
*d))^(1/4) - ((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*
a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c
^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d
^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12
)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 5
6*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2
*c^10*d^6 - 8*a*b^7*c^15*d) - ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592
*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*
d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 1839726
5920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8
*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*
c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*
d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 2
3871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^
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17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))
/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d
^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 9
24*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9
*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) + ((-(194481*a^8
*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*
c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*
a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(1677721
6*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 110729
6256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c
^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 132
87555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^
3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4
)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^
6 - 1117952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14
*c^17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 1733
5296*a^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*
c^12*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 50124
8*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b
^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*
c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6
 - 8*a*b^7*c^15*d))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*
b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 11918
6694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9
 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201
326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9
*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15
502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*
b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10
 - 201326592*a*b^11*c^22*d))^(3/4))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^
3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5
*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*
a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12
*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 36
90987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b
^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 +
 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^1
0*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*1i - (x^(1/2)*(194481*a^8
*b^11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7
*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 1
19638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13
*c^2*d^13)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a
^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^
7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12
*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))*(
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-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309
116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7
 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^
10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d
^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 83047219
20*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^
17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 36909
87520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c
^22*d))^(1/4))/(((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (347925
93*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^1
4*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^
4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d
^12)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2
 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*
b^2*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326
592*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^
20*d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 1839
7265920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*
a^8*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^
13*c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^
11*d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18
+ 23871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640
*a^17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^2
3))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^1
8*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5
+ 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*
a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) - ((-(194481*
a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b
^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 510436
08*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(1677
7216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 110
7296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^
8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 -
13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9
*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(
1/4)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20
*d^6 - 1117952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b
^14*c^17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 1
7335296*a^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b
^9*c^12*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 50
1248*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))
/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b
^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d
^6 - 8*a*b^7*c^15*d))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712
*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 11
```

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9186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d
^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 -
201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*
b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 +
 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a
^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^
10 - 201326592*a*b^11*c^22*d))^(3/4))*(-(194481*a^8*d^11 + 35153041*b^8*c^8
*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*
b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 143783
64*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a
^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 -
 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^
5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^
7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*
a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4) + (x^(1/2)*(194481*a^8
*b^11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7
*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 1
19638278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13
*c^2*d^13))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*
b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c
^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^
8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)))*(-(1
94481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116
*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 -
51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)
/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11
 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*
a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*
d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 36909875
20*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22
*d))^(1/4) + ((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*
a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c
^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d
^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12
)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 5
6*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2
*c^10*d^6 - 8*a*b^7*c^15*d) - ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592
*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*
d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 1839726
5920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8
*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*
c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*
d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 2
3871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^
17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23))
```

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/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d
^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 9
24*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9
*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) + ((-(194481*a^8
*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*
c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*
a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(1677721
6*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 110729
6256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c
^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 132
87555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^
3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4
)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^
6 - 1117952*a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14
*c^17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 1733
5296*a^9*b^11*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*
c^12*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 50124
8*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b
^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*
c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6
- 8*a*b^7*c^15*d))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*
b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 11918
6694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9
- 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201
326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9
*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15
502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*
b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10
- 201326592*a*b^11*c^22*d))^(3/4))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^
3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5
*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*
a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12
*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 36
90987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b
^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 +
 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^1
0*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4) - (x^(1/2)*(194481*a^8*b^
11*d^15 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^
12*c*d^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 1196
38278*a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^
2*d^13))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^1
0*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15
*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 -
 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d))*(-(1944
81*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^
```

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2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 510
43608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(1
6777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 +
1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4
*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6
 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*
a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d)
)^(1/4)))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d
^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*
b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904
*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^
11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3
 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584
*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*
d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 2013265
92*a*b^11*c^22*d))^(1/4)*2i + 2*atan(((((((194481*a^8*b^8*d^14)/2048 + 1232
*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2
048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048
+ (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (
12127941*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9
*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56
*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) - ((x^(1/2)*(1677
7216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d
^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^19*d^8 - 47535882
24*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^1
6*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*
d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15
- 249961119744*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10*c^10*d^17 - 677
18086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19 - 6245842944*a^1
6*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22
 + 7225344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^1
1*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^
16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7
 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12
*a*b^11*c^19*d)) + ((-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a
*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 1191
86694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9
 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 20
1326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^
9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 1
5502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8
*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10
 - 201326592*a*b^11*c^22*d))^(1/4)*(8192*a*b^19*c^22*d^4 - 90112*a^2*b^18*c
^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*d^7 + 1427968*a^
5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^13*c^16*d^10 + 1
```

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2866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 + 16344064*a^10*b^
10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b^8*c^11*d^15 - 2
033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 76032*a^15*b^5*c^8*d
^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 +
28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^
3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-(194481*a^8*d^11 + 35
153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 -
187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^
3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^2
3 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b
^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 -
13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a
^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9
 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(3/4)*1i)*(-(1
94481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116
*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 -
51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)
/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11
 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*
a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*
d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 36909875
20*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22
*d))^(1/4) + (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7 - 13093
2648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*c^6*d^9
- 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043608*a^5*
b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13))/(4096*(b^12*c^20 + a^12*c^8*d^
12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495
*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b
^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^1
0*d^10 - 12*a*b^11*c^19*d)))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 12
0524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d
^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^
2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*
d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 36909875
20*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^1
8*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 83047
21920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*
c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4) - (((((194481*a^8*b^8*d^14)/204
8 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c
*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^
9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/
2048 + (12127941*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^16 + a^8*c^8*d^8 - 8*a
^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*
d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/
2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256*a^2*b^2
```

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1*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^19*d^8 -
4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80192667648
*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 289980416000*a^9*b^
14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^11*b^12*c^
12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10*c^10*d^
17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19 - 624584
2944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a^18*b^5*
c^5*d^22 + 7225344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12
- 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^
4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*
c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d
^10 - 12*a*b^11*c^19*d)) - ((-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120
524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^
6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2
*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d
^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 369098752
0*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18
*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 830472
1920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c
^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*(8192*a*b^19*c^22*d^4 - 90112*a^
2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*d^7 + 14
27968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^13*c^16*
d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 + 16344064
*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b^8*c^11*
d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 76032*a^15*b
^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^
9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 5
6*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-(194481*a^8*d
^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^
6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^
5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*
b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 11072962
56*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^1
9*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287
555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*
c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(3/4)*
1i)*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 1
93309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^
4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b
*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c
^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 830
4721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b
^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 -
3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b
^11*c^22*d))^(1/4) - (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b^19*c^8*d^7
```

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 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 201081276*a^2*b^17*
c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d^11 - 51043
608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13))/(4096*(b^12*c^20 + a^1
2*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d
^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 7
92*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10
*b^2*c^10*d^10 - 12*a*b^11*c^19*d)))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*
d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b
^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 1437836
4*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^
12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 -
3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5
*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7
 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a
^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4))/((((((194481*a^8*b^8*d
^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/2048 - (2250423*a
^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106888869*a^3*b^1
3*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (38915667*a^5*b^11*c^
3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^16 + a^8*c^8*d
^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b
^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) -
 ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^5 + 1107296256
*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^1
9*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c^17*d^10 + 801
92667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12 + 28998041600
0*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334222688256*a^11
*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10
*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19
- 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a
^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12*c^20 + a^12*c
^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3*b^9*c^17*d^3
+ 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c^14*d^6 - 792*
a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^9 + 66*a^10*b^
2*c^10*d^10 - 12*a*b^11*c^19*d)) + ((-(194481*a^8*d^11 + 35153041*b^8*c^8*d
^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^
5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364
*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^1
2*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3
690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*
b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7
+ 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^
10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*(8192*a*b^19*c^22*d^4 -
90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*a^4*b^16*c^19*
d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 - 5803776*a^7*b^
13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^11*c^14*d^12 +
```

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16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 + 5637888*a^12*b
^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c^9*d^17 - 7603
2*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^8*c^8*d^8 - 8*
a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12
*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d))*(-(1944
81*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^
2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 510
43608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(1
6777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 +
1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4
*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6
 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*
a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d)
)^(3/4)*1i)*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7
*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^
4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 24449
04*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*
a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d
^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 155021475
84*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^1
5*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 20132
6592*a*b^11*c^22*d))^(1/4)*1i + (x^(1/2)*(194481*a^8*b^11*d^15 + 41224337*b
^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d^14 + 20108127
6*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*a^4*b^15*c^4*d
^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13)*1i)/(4096*(b
^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*
a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^
6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11
*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)))*(-(194481*a^8*d^11 + 351
53041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 1
87159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3
*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23
 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^
10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 1
3287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^
7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9
+ 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4) + (((((19
4481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (34792593*a*b^15*c^7*d^7)/204
8 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*b^14*c^6*d^8)/2048 - (106
888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^12*c^4*d^10)/2048 - (389156
67*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*c^2*d^12)/2048)*1i)/(b^8*c^
16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*
d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a
*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4 - 201326592*a*b^22*c^22*d^
5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^3*b^20*c^20*d^7 + 697250611
```

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2*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*d^9 - 18397265920*a^6*b^17*c
^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 181503787008*a^8*b^15*c^15*d^12
 + 289980416000*a^9*b^14*c^14*d^13 - 352258621440*a^10*b^13*c^13*d^14 + 334
222688256*a^11*b^12*c^12*d^15 - 249961119744*a^12*b^11*c^11*d^16 + 14724877
5168*a^13*b^10*c^10*d^17 - 67718086656*a^14*b^9*c^9*d^18 + 23871029248*a^15
*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^20 + 1146224640*a^17*b^6*c^6*d^21
 - 132120576*a^18*b^5*c^5*d^22 + 7225344*a^19*b^4*c^4*d^23)*1i)/(4096*(b^12
*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c^18*d^2 - 220*a^3
*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^5 + 924*a^6*b^6*c
^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 220*a^9*b^3*c^11*d^
9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)) - ((-(194481*a^8*d^11 + 3515
3041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 18
7159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*
d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23
+ 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^1
0*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13
287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7
*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 +
 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*(8192*a*b^
19*c^22*d^4 - 90112*a^2*b^18*c^21*d^5 + 430848*a^3*b^17*c^20*d^6 - 1117952*
a^4*b^16*c^19*d^7 + 1427968*a^5*b^15*c^18*d^8 + 456192*a^6*b^14*c^17*d^9 -
5803776*a^7*b^13*c^16*d^10 + 12866304*a^8*b^12*c^15*d^11 - 17335296*a^9*b^1
1*c^14*d^12 + 16344064*a^10*b^10*c^13*d^13 - 11221760*a^11*b^9*c^12*d^14 +
5637888*a^12*b^8*c^11*d^15 - 2033152*a^13*b^7*c^10*d^16 + 501248*a^14*b^6*c
^9*d^17 - 76032*a^15*b^5*c^8*d^18 + 5376*a^16*b^4*c^7*d^19))/(b^8*c^16 + a^
8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^14*d^2 - 56*a^3*b^5*c^13*d^3 + 7
0*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 + 28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^
15*d))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4
+ 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4
*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^
7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*
b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 +
8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^
6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8
 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*
a*b^11*c^22*d))^(3/4)*1i)*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 12052
4712*a*b^7*c^7*d^4 + 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6
+ 119186694*a^4*b^4*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c
^2*d^9 - 2444904*a^7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^1
2 - 201326592*a^11*b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*
a^3*b^9*c^20*d^3 + 8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d
^5 + 15502147584*a^6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 83047219
20*a^8*b^4*c^15*d^8 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^1
3*d^10 - 201326592*a*b^11*c^22*d))^(1/4)*1i - (x^(1/2)*(194481*a^8*b^11*d^1
5 + 41224337*b^19*c^8*d^7 - 130932648*a*b^18*c^7*d^8 - 2444904*a^7*b^12*c*d
```

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^14 + 201081276*a^2*b^17*c^6*d^9 - 189998424*a^3*b^16*c^5*d^10 + 119638278*
a^4*b^15*c^4*d^11 - 51043608*a^5*b^14*c^3*d^12 + 14378364*a^6*b^13*c^2*d^13
)*1i)/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^11 + 66*a^2*b^10*c
^18*d^2 - 220*a^3*b^9*c^17*d^3 + 495*a^4*b^8*c^16*d^4 - 792*a^5*b^7*c^15*d^
5 + 924*a^6*b^6*c^14*d^6 - 792*a^7*b^5*c^13*d^7 + 495*a^8*b^4*c^12*d^8 - 22
0*a^9*b^3*c^11*d^9 + 66*a^10*b^2*c^10*d^10 - 12*a*b^11*c^19*d)))*(-(194481*
a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4 + 193309116*a^2*b
^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4*c^4*d^7 - 510436
08*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^7*b*c*d^10)/(1677
7216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*b*c^12*d^11 + 110
7296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 + 8304721920*a^4*b^
8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^6*b^6*c^17*d^6 -
13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8 - 3690987520*a^9
*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*a*b^11*c^22*d))^(
1/4)))*(-(194481*a^8*d^11 + 35153041*b^8*c^8*d^3 - 120524712*a*b^7*c^7*d^4
+ 193309116*a^2*b^6*c^6*d^5 - 187159896*a^3*b^5*c^5*d^6 + 119186694*a^4*b^4
*c^4*d^7 - 51043608*a^5*b^3*c^3*d^8 + 14378364*a^6*b^2*c^2*d^9 - 2444904*a^
7*b*c*d^10)/(16777216*b^12*c^23 + 16777216*a^12*c^11*d^12 - 201326592*a^11*
b*c^12*d^11 + 1107296256*a^2*b^10*c^21*d^2 - 3690987520*a^3*b^9*c^20*d^3 +
8304721920*a^4*b^8*c^19*d^4 - 13287555072*a^5*b^7*c^18*d^5 + 15502147584*a^
6*b^6*c^17*d^6 - 13287555072*a^7*b^5*c^16*d^7 + 8304721920*a^8*b^4*c^15*d^8
 - 3690987520*a^9*b^3*c^14*d^9 + 1107296256*a^10*b^2*c^13*d^10 - 201326592*
a*b^11*c^22*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.467 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=681

$$-\frac{b^{13/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^3}-$$

Rubi [A] time = 1.01, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{45a^2d^2-85abcd+32b^2c^2}{16ac^3\sqrt{x}\,(bc-ad)^2}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{13/4}(bc-ad)^3}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{b^{13/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{d(17bc-9ad)}{16c^2\sqrt{x}\,\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4c\sqrt{x}\,\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 85*a*b*c*d + 45*a^2*d^2)/(16*a*c^3*(b*c - a*d)^2*Sqrt[x]) -
d/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) - (d*(17*b*c - 9*a*d))/(16*c^2*(b
*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + (b^(13/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(13/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)
*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 -
 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^(13/
4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a
^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
3/4)*(b*c - a*d)^3) - (d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/
4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
```

```
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-9ad-9bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-}{}\right.}{}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-85bd+\frac{45ad^2}{c}}{16c^2(bc-ad)^2\sqrt{x}} - \frac{d}{4c(bc-ad)\sqrt{x}\left(c+dx^2\right)^2} - \frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}\left(c+d\right.}$$

Mathematica [A] time = 6.17, size = 699, normalized size = 1.03

$$-\frac{b^{13/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(ad-bc)^3}+\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(ad-bc)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(ad-bc)^3}-\frac{d^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(ad-bc)^3}+\frac{d^2x^{3/2}(21bc-13ad)}{16c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{d^2x^{3/2}}{4c^2\left(c+dx^2\right)^2(bc-ad)}-\frac{2}{ac^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -2/(a*c^3*Sqrt[x]) + (d^2*x^(3/2))/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2
*(21*b*c - 13*a*d)*x^(3/2))/(16*c^3*(b*c - a*d)^2*(c + d*x^2)) - (b^(13/4)*
ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2
]*a^(5/4)*(b*c - a*d)^3) - (b^(13/4)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sq
rt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(117*
b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*
Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^3) - (d^(5
/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^
(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^3) -
 (b^(13/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*S
qrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^(13/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(11
7*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^3) + (d^(5/4)*(1
17*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.47, size = 458, normalized size = 0.67

$$-\frac{b^{13/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}a^{5/4}(ad-bc)^3}-\frac{b^{13/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{5/4}(ad-bc)^3}-\frac{\left(45a^2d^{13/4}-130abcd^{9/4}+117b^2c^2d^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{\left(45a^2d^{13/4}-130abcd^{9/4}+117b^2c^2d^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{-32a^2c^2d^2-81a^2cd^3x^2-45a^2d^4x^4+64abc^3d+153abc^2d^2x^2+85abcd^3x^4-32b^2c^4-64b^2c^3dx^2-32b^2c^2d^2x^4}{16ac^3\sqrt{x}\left(c+dx^2\right)^2(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*b^2*c^4 + 64*a*b*c^3*d - 32*a^2*c^2*d^2 - 64*b^2*c^3*d*x^2 + 153*a*b*c
^2*d^2*x^2 - 81*a^2*c*d^3*x^2 - 32*b^2*c^2*d^2*x^4 + 85*a*b*c*d^3*x^4 - 45*
a^2*d^4*x^4)/(16*a*c^3*(-(b*c) + a*d)^2*Sqrt[x]*(c + d*x^2)^2) - (b^(13/4)*
ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]]
)/(Sqrt[2]*a^(5/4)*(-(b*c) + a*d)^3) - ((117*b^2*c^2*d^(5/4) - 130*a*b*c*d^
(9/4) + 45*a^2*d^(13/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x])])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (b^(13/4)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(5/4)*(-(b
*c) + a*d)^3) - ((117*b^2*c^2*d^(5/4) - 130*a*b*c*d^(9/4) + 45*a^2*d^(13/4)
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqr
t[2]*c^(13/4)*(b*c - a*d)^3)
```

fricas **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[A]** time = 1.44, size = 987, normalized size = 1.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*
d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c
^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b^3)^(3/4)*b*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2
)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(3
/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^
3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/
32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 -
sqrt(2)*a^3*c^4*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*
a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 +
3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) - 1/64*(117*(c*d^3)^(3/4)*b^
2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^
6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/64*(117*(c*d^3)^
(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(-
sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)
*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/16*(21*
b*c*d^3*x^(7/2) - 13*a*d^4*x^(7/2) + 25*b*c^2*d^2*x^(3/2) - 17*a*c*d^3*x^(3
/2))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 2/(a*c^3*sqrt(
x))
```

maple **[A]** time = 0.03, size = 900, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/4*b^3/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2*b^3/a/(a*d-b
*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2*b^3/a/(
a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-13/16*
d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a^2+17/8*d^4/c^2/(a*d-b*c)^3/(d*x^2
+c)^2*x^(7/2)*a*b-21/16*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*b^2-17/16*d^4
/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a^2+21/8*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2
*x^(3/2)*a*b-25/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2-45/128*d^3/c^3/(
a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^
(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-45/64*d^3/c^3/(a*d-b*c)
^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-45/64*d^3/
c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)+65/64*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*ln((x-(c/d)^(1/4)*2^(1
/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+65/32
*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)+65/32*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)-1)-117/128*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-
(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/
d)^(1/2)))-117/64*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)+1)-117/64*d/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b^2*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a/c^3/x^(1/2)
```

maxima [A] time = 2.84, size = 668, normalized size = 0.98

$$-\frac{b^{4}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(ab^{3}c^{3}-3a^{2}b^{2}c^{2}d+3a^{3}bcd^{2}-a^{4}d^{3}\right)}+\frac{\left(117b^{2}c^{2}d^{2}-130abcd^{3}+45a^{2}d^{4}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{6}-3ab^{2}c^{5}d+3a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3}\right)}-\frac{32b^{2}c^{4}-64abc^{3}d+32a^{2}c^{2}d^{2}+\left(32b^{2}c^{2}d^{2}-85abcd^{3}+45a^{2}d^{4}\right)x^{4}+\left(64b^{2}c^{3}d-153abc^{2}d^{2}+81a^{2}cd^{3}\right)x^{2}}{16\left(\left(ab^{2}c^{5}d^{2}-2a^{2}bc^{4}d^{3}+a^{3}c^{3}d^{4}\right)x^{\frac{9}{2}}+2\left(ab^{2}c^{6}d-2a^{2}bc^{5}d^{2}+a^{3}c^{4}d^{3}\right)x^{\frac{5}{2}}+\left(ab^{2}c^{7}-2a^{2}bc^{6}d+a^{3}c^{5}d^{2}\right)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-
sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(
a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/128*(117*b^2*c^2
*d^2 - 130*a*b*c*d^3 + 45*a^2*d^4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sq
rt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) -
 sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4
)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
```

```
t(c))/(c^(1/4)*d^(3/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c
^3*d^3) - 1/16*(32*b^2*c^4 - 64*a*b*c^3*d + 32*a^2*c^2*d^2 + (32*b^2*c^2*d^
2 - 85*a*b*c*d^3 + 45*a^2*d^4)*x^4 + (64*b^2*c^3*d - 153*a*b*c^2*d^2 + 81*a
^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^(9/2) + 2
*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^(5/2) + (a*b^2*c^7 - 2*a^2
*b*c^6*d + a^3*c^5*d^2)*sqrt(x))
```

mupad [B] time = 10.85, size = 33717, normalized size = 49.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] atan((a^21*c^16*d^20*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 -
832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c
^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 24798150
0*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*
a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2
- 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a
^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d
^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256
*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*2174327193600i + b^17
*c^20*d^4*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a
*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 15
19673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c
^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d
^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 369098752
0*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20
*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 830472
1920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c
^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*918653239296i + a*b^20*c^36*x^(1
/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 +
 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b
^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 4738
5000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 20132659
2*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22
*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 1550214
7584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c
^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201
326592*a*b^11*c^24*d))^(5/4)*1099511627776i + a*b^16*c^19*d^5*x^(1/2)*(-(41
00625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 16763549
40*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^
9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*
b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*
```

```
c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 83
04721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*
b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 -
 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*
b^11*c^24*d))^(1/4)*10239255576576i - a^2*b^19*c^35*d*x^(1/2)*(-(4100625*a^
8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b
^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 7650
63000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12
)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^1
1 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920
*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19
*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987
520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^2
4*d))^(5/4)*13194139533312i - a^20*b*c^17*d^19*x^(1/2)*(-(4100625*a^8*d^13
+ 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*
d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a
^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(1677
7216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 110
7296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^
8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 -
13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9
*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(
5/4)*38654705664000i - a^2*b^15*c^18*d^6*x^(1/2)*(-(4100625*a^8*d^13 + 1873
88721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 -
1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3
*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b
^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 110729625
6*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21
*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 132875
55072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c
^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*5
2740124835840i + a^3*b^14*c^17*d^7*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*
b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 198916
3800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d
^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^
25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*
b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 -
 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*
a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^
9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*1090764
23639040i - a^4*b^13*c^16*d^8*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c
^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*
a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 +
 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 +
16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*
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c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 1328
7555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b
^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1
107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*130225943347
200i + a^5*b^12*c^15*d^9*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^
5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b
^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 2479
81500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777
216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*
d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 132875550
72*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^
18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 110729
6256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*99593312665600i -
 a^6*b^11*c^14*d^10*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 8
32838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^
5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500
*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a
^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 -
 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^
5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^
7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*
a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*50139168768000i + a^7*
b^10*c^13*d^11*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838
760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8
 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*
b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c
^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690
987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7
*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8
304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*
b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*16251715584000i - a^8*b^9*c
^12*d^12*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*
b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 151
9673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^
2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^
12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520
*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*
d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721
920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^
15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*3105423360000i + a^9*b^8*c^11*d^1
3*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7
*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350
*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11
- 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 20
1326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^
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9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 1
5502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8
*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10
 - 201326592*a*b^11*c^24*d))^(1/4)*268738560000i + a^3*b^18*c^34*d^2*x^(1/2
)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1
676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4
*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 473850
00*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*
a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d
^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 155021475
84*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^1
7*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 20132
6592*a*b^11*c^24*d))^(5/4)*72567767433216i - a^4*b^17*c^33*d^3*x^(1/2)*(-(4
100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354
940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d
^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7
*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b
*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8
304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6
*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8
 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a
*b^11*c^24*d))^(5/4)*241892558110720i + a^5*b^16*c^32*d^4*x^(1/2)*(-(410062
5*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a
^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 -
765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*
d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14
*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 830472
1920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*
c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 369
0987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11
*c^24*d))^(5/4)*558956707577856i - a^6*b^15*c^31*d^5*x^(1/2)*(-(4100625*a^8
*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^
6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 76506
3000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)
/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11
 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*
a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*
d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 36909875
20*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24
*d))^(5/4)*1079857857429504i + a^7*b^14*c^30*d^6*x^(1/2)*(-(4100625*a^8*d^1
3 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^
6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000
*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16
777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1
107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*
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b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6
- 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a
^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))
^(5/4)*2407458018426880i - a^8*b^13*c^29*d^7*x^(1/2)*(-(4100625*a^8*d^13 +
187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^
7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5
*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(167772
16*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 11072
96256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*
c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13
287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b
^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/
4)*6626241184530432i + a^9*b^12*c^28*d^8*x^(1/2)*(-(4100625*a^8*d^13 + 1873
88721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 -
1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3
*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b
^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 110729625
6*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21
*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 132875
55072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c
^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*1
7102710096527360i - a^10*b^11*c^27*d^9*x^(1/2)*(-(4100625*a^8*d^13 + 187388
721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 19
89163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c
^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^1
2*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*
a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d
^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555
072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^1
6*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*353
86201192005632i + a^11*b^10*c^26*d^10*x^(1/2)*(-(4100625*a^8*d^13 + 1873887
21*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 198
9163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^
3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12
*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a
^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^
4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 132875550
72*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16
*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*5700
9634950512640i - a^12*b^9*c^25*d^11*x^(1/2)*(-(4100625*a^8*d^13 + 187388721
*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 19891
63800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*
d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c
^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2
*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4
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- 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072
*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d
^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*718586
85512515584i + a^13*b^8*c^24*d^12*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b
^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163
800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^
10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^2
5 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b
^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 -
13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a
^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9
 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*71386451
710836736i - a^14*b^7*c^23*d^13*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8
*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 198916380
0*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10
 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25
+ 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^1
0*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13
287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7
*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 +
 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*5605860034
2159360i + a^15*b^6*c^22*d^14*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c
^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*
a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 +
 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 +
16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*
c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 1328
7555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b
^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1
107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*346939125934
32576i - a^16*b^5*c^21*d^15*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8
*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^
3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 2
47981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16
777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^
23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 132875
55072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5
*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 110
7296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*16752399678963
712i + a^17*b^4*c^20*d^16*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d
^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*
b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247
981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 1677
7216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23
*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555
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072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c
^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 11072
96256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*6190641306402816
i - a^18*b^3*c^19*d^17*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5
- 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5
*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981
500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 1677721
6*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^
2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072
*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18
*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 11072962
56*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*1693591504158720i +
 a^19*b^2*c^18*d^18*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 8
32838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^
5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500
*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a
^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 -
 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^
5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^
7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*
a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*323711685099520i)/(373
669453125*a^16*d^21 + 1679412953088*b^16*c^16*d^5 - 559804317696*a*b^15*c^1
5*d^6 + 1440180338688*a^2*b^14*c^14*d^7 + 1069458391040*a^3*b^13*c^13*d^8 +
 1465657589760*a^4*b^12*c^12*d^9 - 298323546568125*a^5*b^11*c^11*d^10 + 143
6096821015175*a^6*b^10*c^10*d^11 - 3384298041916875*a^7*b^9*c^9*d^12 + 5036
592389645625*a^8*b^8*c^8*d^13 - 5207312367681250*a^9*b^7*c^7*d^14 + 3905254
606443750*a^10*b^6*c^6*d^15 - 2160273093693750*a^11*b^5*c^5*d^16 + 87950713
9331250*a^12*b^4*c^4*d^17 - 257930235140625*a^13*b^3*c^3*d^18 + 51862552171
875*a^14*b^2*c^2*d^19 - 6435418359375*a^15*b*c*d^20))*(-(4100625*a^8*d^13 +
 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d
^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^
5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777
216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107
296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8
*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 1
3287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*
b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1
/4)*2i + atan((a^6*b^20*c^25*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^1
2 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7
920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 126
72*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*
a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*33554432i + a^14*b^8*d^13*x^(
1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a
^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10
*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*
```

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b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c
*d^11))^(1/4)*8201250i + a^26*c^5*d^20*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^
5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c
^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6
*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d
^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*66355200i + a^5*b^1
7*c^9*d^4*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^
11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4
 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7
 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 -
 192*a^16*b*c*d^11))^(1/4)*28035072i + a^6*b^16*c^8*d^5*x^(1/2)*(-b^13/(16*
a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2
 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 1
4784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 35
20*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*31
2477282i - a^7*b^15*c^7*d^6*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12
 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 79
20*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 1267
2*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a
^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*1609500880i + a^8*b^14*c^6*d^7
*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 10
56*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*
a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a
^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16
*b*c*d^11))^(1/4)*3328748280i - a^9*b^13*c^5*d^8*x^(1/2)*(-b^13/(16*a^17*d^
12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520
*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^
11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14
*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*397418040
0i + a^10*b^12*c^4*d^9*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 19
2*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^
9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^1
2*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b
^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*3039346700i - a^11*b^11*c^3*d^10*x^
(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*
a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^1
0*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13
*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*
c*d^11))^(1/4)*1530126000i + a^12*b^10*c^2*d^11*x^(1/2)*(-b^13/(16*a^17*d^1
2 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*
a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^1
1*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*
b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*495963000i
 + a^8*b^18*c^23*d^2*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*
a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*
```

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b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*
b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2
*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*2214592512i - a^9*b^17*c^22*d^3*x^(1/
2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7
*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b
^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^
4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d
^11))^(5/4)*7381975040i + a^10*b^16*c^21*d^4*x^(1/2)*(-b^13/(16*a^17*d^12 +
 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8
*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b
^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3
*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*17058004992i
- a^11*b^15*c^20*d^5*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*
a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*
b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*
b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2
*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*32954646528i + a^12*b^14*c^19*d^6*x^(
1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a
^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10
*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*
b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c
*d^11))^(5/4)*73469788160i - a^13*b^13*c^18*d^7*x^(1/2)*(-b^13/(16*a^17*d^1
2 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*
a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^1
1*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*
b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*2022168330
24i + a^14*b^12*c^17*d^8*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 -
192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*
a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a
^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15
*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*521933291520i - a^15*b^11*c^16*d^
9*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1
056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672
*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*
a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^1
6*b*c*d^11))^(5/4)*1079901159424i + a^16*b^10*c^15*d^10*x^(1/2)*(-b^13/(16*
a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2
 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 1
4784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 35
20*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*17
39795988480i - a^17*b^9*c^14*d^11*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^1
2*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^
3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6
- 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 +
1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*2192953049088i + a^18*b^
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8*c^13*d^12*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*
c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d
^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d
^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10
 - 192*a^16*b*c*d^11))^(5/4)*2178541617152i - a^19*b^7*c^12*d^13*x^(1/2)*(-
b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10
*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^
7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4
*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))
^(5/4)*1710772715520i + a^20*b^6*c^11*d^14*x^(1/2)*(-b^13/(16*a^17*d^12 + 1
6*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b
^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6
*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c
^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*1058774188032i
- a^21*b^5*c^10*d^15*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*
a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*
b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*
b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2
*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*511242665984i + a^22*b^4*c^9*d^16*x^(
1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a
^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10
*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*
b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c
*d^11))^(5/4)*188923379712i - a^23*b^3*c^8*d^17*x^(1/2)*(-b^13/(16*a^17*d^1
2 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*
a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^1
1*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*
b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*5168431104
0i + a^24*b^2*c^7*d^18*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 19
2*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^
9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^1
2*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b
^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*9878896640i - a^13*b^9*c*d^12*x^(1/
2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7
*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b
^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^
4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d
^11))^(1/4)*94770000i - a^7*b^19*c^24*d*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a
^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*
c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^
6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*
d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(5/4)*402653184i - a^25*
b*c^6*d^19*x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c
^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^
4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^
```

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7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10
- 192*a^16*b*c*d^11))^(5/4)*1179648000i)/(1048576*b^21*c^10 + 4100625*a^10*
b^11*d^10 - 35083125*a^9*b^12*c*d^9 + 6291456*a^2*b^19*c^8*d^2 + 10485760*a
^3*b^18*c^7*d^3 + 15728640*a^4*b^17*c^6*d^4 - 165368625*a^5*b^16*c^5*d^5 +
300032725*a^6*b^15*c^4*d^6 - 264422250*a^7*b^14*c^3*d^7 + 130430250*a^8*b^1
3*c^2*d^8 + 3145728*a*b^20*c^9*d))*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12
- 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 792
0*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672
*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^
15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*2i + 2*atan(((x^(1/2)*(15412443
824768679936*a^11*b^35*c^52*d^8 - 439880110593414266688*a^12*b^34*c^51*d^9 -
 1887306913007783641088*a^13*b^33*c^50*d^10 + 24240068121369040125952*a^14*
b^32*c^49*d^11 - 155426886723407276146688*a^15*b^31*c^48*d^12 + 66196967881
7672344633344*a^16*b^30*c^47*d^13 - 2072522435259453904257024*a^17*b^29*c^4
6*d^14 + 5025620613985914706722816*a^18*b^28*c^45*d^15 - 973973480685060521
0927104*a^19*b^27*c^44*d^16 + 15395587131987386880229376*a^20*b^26*c^43*d^1
7 - 20118464109716534770794496*a^21*b^25*c^42*d^18 + 2192568898069370483454
7712*a^22*b^24*c^41*d^19 - 20031833528060137877536768*a^23*b^23*c^40*d^20 +
 15375655212110710153674752*a^24*b^22*c^39*d^21 - 9908539789204785922572288
*a^25*b^21*c^38*d^22 + 5342151752610266235273216*a^26*b^20*c^37*d^23 - 2393
361740048338255872000*a^27*b^19*c^36*d^24 + 881440288329629213655040*a^28*b
^18*c^35*d^25 - 262552769009553086873600*a^29*b^17*c^34*d^26 + 617372892503
32318105600*a^30*b^16*c^33*d^27 - 11040709176673173504000*a^31*b^15*c^32*d^
28 + 1412353884520710144000*a^32*b^14*c^31*d^29 - 115221946643251200000*a^3
3*b^13*c^30*d^30 + 4508684868648960000*a^34*b^12*c^29*d^31) - (-b^13/(16*a^
17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 -
 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 147
84*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520
*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(3/4)*(x^(
1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a
^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10
*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*
b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c
*d^11))^(1/4)*(18446744073709551616*a^12*b^38*c^68*d^4 - 479615345916448342
016*a^13*b^37*c^67*d^5 + 5995191823955604275200*a^14*b^36*c^66*d^6 - 479615
34591644834201600*a^15*b^35*c^65*d^7 + 276025423003154095538176*a^16*b^34*c
^64*d^8 - 1220386399802376518107136*a^17*b^33*c^63*d^9 + 434188067899918178
5825280*a^18*b^32*c^62*d^10 - 12966583542852067073720320*a^19*b^31*c^61*d^1
1 + 34096448785847177707520000*a^20*b^30*c^60*d^12 - 8340583229325849256787
9680*a^21*b^29*c^59*d^13 + 198753207063509910306160640*a^22*b^28*c^58*d^14
- 466650996519299420897935360*a^23*b^27*c^57*d^15 + 10520562198701980560392
19200*a^24*b^26*c^56*d^16 - 2194458800584304697435750400*a^25*b^25*c^55*d^1
7 + 4119286127977833519707586560*a^26*b^24*c^54*d^18 - 68558304293588787679
93323520*a^27*b^23*c^53*d^19 + 10053745593205095687740456960*a^28*b^22*c^52
*d^20 - 12966109559707844614920601600*a^29*b^21*c^51*d^21 + 147043126501648
```

```
76038740377600*a^30*b^20*c^50*d^22 - 14666481047173052905774120960*a^31*b^1
9*c^49*d^23 + 12864666662378251575193763840*a^32*b^18*c^48*d^24 - 991525421
4005035782929121280*a^33*b^17*c^47*d^25 + 6703228082495101562834124800*a^34
*b^16*c^46*d^26 - 3963723814398261058758246400*a^35*b^15*c^45*d^27 + 204155
2487767277748019527680*a^36*b^14*c^44*d^28 - 910688569282163512962973696*a^
37*b^13*c^43*d^29 + 349132643065901184834338816*a^38*b^12*c^42*d^30 - 11385
9137485172722840371200*a^39*b^11*c^41*d^31 + 31155202813126960689971200*a^4
0*b^10*c^40*d^32 - 7019849261936667709669376*a^41*b^9*c^39*d^33 + 126844980
5817592472928256*a^42*b^8*c^38*d^34 - 176741065216378693222400*a^43*b^7*c^3
7*d^35 + 17829840996752341073920*a^44*b^6*c^36*d^36 - 115922654408516567040
0*a^45*b^5*c^35*d^37 + 36479156981701017600*a^46*b^4*c^34*d^38)*1i - 922337
2036854775808*a^11*b^38*c^65*d^4 + 212137556847659843584*a^12*b^37*c^64*d^5
 - 2333513125324258279424*a^13*b^36*c^63*d^6 + 16334591877269807955968*a^14
*b^35*c^62*d^7 - 81672959386349039779840*a^15*b^34*c^61*d^8 + 3108080596500
00835051520*a^16*b^33*c^60*d^9 - 942943171860407129210880*a^17*b^32*c^59*d^
10 + 2411982412523930344488960*a^18*b^31*c^58*d^11 - 5753067372685321201254
400*a^19*b^30*c^57*d^12 + 14786194741349386435952640*a^20*b^29*c^56*d^13 -
43374839389541821883351040*a^21*b^28*c^55*d^14 + 13129554344989852442820608
0*a^22*b^27*c^54*d^15 - 365631810199400875032576000*a^23*b^26*c^53*d^16 + 8
88615019916519951743057920*a^24*b^25*c^52*d^17 - 18590650885817927342856601
60*a^25*b^24*c^51*d^18 + 3349720497258869063543685120*a^26*b^23*c^50*d^19 -
 5220292063815211666322227200*a^27*b^22*c^49*d^20 + 70676082680641434491342
02880*a^28*b^21*c^48*d^21 - 8342222871228251802477527040*a^29*b^20*c^47*d^2
2 + 8605396720616721741816791040*a^30*b^19*c^46*d^23 - 77675000889790559024
05427200*a^31*b^18*c^45*d^24 + 6135496566696171932913500160*a^32*b^17*c^44*
d^25 - 4236422046382466798589050880*a^33*b^16*c^43*d^26 + 25509806610674854
41771438080*a^34*b^15*c^42*d^27 - 1334575022384247271808040960*a^35*b^14*c^
41*d^28 + 603343239457650202481000448*a^36*b^13*c^40*d^29 - 233967123641003
163353350144*a^37*b^12*c^39*d^30 + 77049527429528415176228864*a^38*b^11*c^3
8*d^31 - 21258749850480450394390528*a^39*b^10*c^37*d^32 + 48238993638199019
75265280*a^40*b^9*c^36*d^33 - 876898617974708020183040*a^41*b^8*c^35*d^34 +
 122811796684756379238400*a^42*b^7*c^34*d^35 - 12444332416601319014400*a^43
*b^6*c^33*d^36 + 812231229670686720000*a^44*b^5*c^32*d^37 - 256494072527585
28000*a^45*b^4*c^31*d^38)*1i)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192
*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9
*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12
*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^
2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4) + (x^(1/2)*(15412443824768679936*a^1
1*b^35*c^52*d^8 - 43988011059341426688*a^12*b^34*c^51*d^9 - 188730691300778
3641088*a^13*b^33*c^50*d^10 + 24240068121369040125952*a^14*b^32*c^49*d^11 -
 155426886723407276146688*a^15*b^31*c^48*d^12 + 661969678817672344633344*a^
16*b^30*c^47*d^13 - 2072522435259453904257024*a^17*b^29*c^46*d^14 + 5025620
613985914706722816*a^18*b^28*c^45*d^15 - 9739734806850605210927104*a^19*b^2
7*c^44*d^16 + 15395587131987386880229376*a^20*b^26*c^43*d^17 - 201184641097
16534770794496*a^21*b^25*c^42*d^18 + 21925688980693704834547712*a^22*b^24*c
```

```
^41*d^19 - 20031833528060137877536768*a^23*b^23*c^40*d^20 + 153756552121107
10153674752*a^24*b^22*c^39*d^21 - 9908539789204785922572288*a^25*b^21*c^38*
d^22 + 5342151752610266235273216*a^26*b^20*c^37*d^23 - 23933617400483382558
72000*a^27*b^19*c^36*d^24 + 881440288329629213655040*a^28*b^18*c^35*d^25 -
262552769009553086873600*a^29*b^17*c^34*d^26 + 61737289250332318105600*a^30
*b^16*c^33*d^27 - 11040709176673173504000*a^31*b^15*c^32*d^28 + 14123538845
20710144000*a^32*b^14*c^31*d^29 - 115221946643251200000*a^33*b^13*c^30*d^30
 + 4508684868648960000*a^34*b^12*c^29*d^31) - (-b^13/(16*a^17*d^12 + 16*a^5
*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^
9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*
d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^
9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(3/4)*(x^(1/2)*(-b^13/(16*
a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2
 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 1
4784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 35
20*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*(1
8446744073709551616*a^12*b^38*c^68*d^4 - 479615345916448342016*a^13*b^37*c^
67*d^5 + 5995191823955604275200*a^14*b^36*c^66*d^6 - 4796153459164483420160
0*a^15*b^35*c^65*d^7 + 276025423003154095538176*a^16*b^34*c^64*d^8 - 122038
6399802376518107136*a^17*b^33*c^63*d^9 + 4341880678999181785825280*a^18*b^3
2*c^62*d^10 - 12966583542852067073720320*a^19*b^31*c^61*d^11 + 340964487858
4717707520000*a^20*b^30*c^60*d^12 - 83405832293258492567879680*a^21*b^29*c
^59*d^13 + 198753207063509910306160640*a^22*b^28*c^58*d^14 - 46665099651929
9420897935360*a^23*b^27*c^57*d^15 + 1052056219870198056039219200*a^24*b^26*
c^56*d^16 - 2194458800584304697435750400*a^25*b^25*c^55*d^17 + 411928612797
7833519707586560*a^26*b^24*c^54*d^18 - 6855830429358878767993323520*a^27*b^
23*c^53*d^19 + 10053745593205095687740456960*a^28*b^22*c^52*d^20 - 12966109
559707844614920601600*a^29*b^21*c^51*d^21 + 14704312650164876038740377600*a
^30*b^20*c^50*d^22 - 14666481047173052905774120960*a^31*b^19*c^49*d^23 + 12
864666662378251575193763840*a^32*b^18*c^48*d^24 - 9915254214005035782929121
280*a^33*b^17*c^47*d^25 + 6703228082495101562834124800*a^34*b^16*c^46*d^26
- 3963723814398261058758246400*a^35*b^15*c^45*d^27 + 2041552487767277748019
527680*a^36*b^14*c^44*d^28 - 910688569282163512962973696*a^37*b^13*c^43*d^2
9 + 349132643065901184834338816*a^38*b^12*c^42*d^30 - 113859137485172722840
371200*a^39*b^11*c^41*d^31 + 31155202813126960689971200*a^40*b^10*c^40*d^32
 - 7019849261936667709669376*a^41*b^9*c^39*d^33 + 1268449805817592472928256
*a^42*b^8*c^38*d^34 - 176741065216378693222400*a^43*b^7*c^37*d^35 + 1782984
0996752341073920*a^44*b^6*c^36*d^36 - 1159226544085165670400*a^45*b^5*c^35*
d^37 + 36479156981701017600*a^46*b^4*c^34*d^38)*1i + 9223372036854775808*a^
11*b^38*c^65*d^4 - 212137556847659843584*a^12*b^37*c^64*d^5 + 2333513125324
258279424*a^13*b^36*c^63*d^6 - 16334591877269807955968*a^14*b^35*c^62*d^7 +
 81672959386349039779840*a^15*b^34*c^61*d^8 - 310808059650000835051520*a^16
*b^33*c^60*d^9 + 942943171860407129210880*a^17*b^32*c^59*d^10 - 24119824125
23930344488960*a^18*b^31*c^58*d^11 + 5753067372685321201254400*a^19*b^30*c^
57*d^12 - 14786194741349386435952640*a^20*b^29*c^56*d^13 + 4337483938954182
```

1883351040*a^21*b^28*c^55*d^14 - 131295543449898524428206080*a^22*b^27*c^54
*d^15 + 365631810199400875032576000*a^23*b^26*c^53*d^16 - 88861501991651995
1743057920*a^24*b^25*c^52*d^17 + 1859065088581792734285660160*a^25*b^24*c^5
1*d^18 - 3349720497258869063543685120*a^26*b^23*c^50*d^19 + 522029206381521
1666322227200*a^27*b^22*c^49*d^20 - 7067608268064143449134202880*a^28*b^21*
c^48*d^21 + 8342222871228251802477527040*a^29*b^20*c^47*d^22 - 860539672061
6721741816791040*a^30*b^19*c^46*d^23 + 7767500088979055902405427200*a^31*b^
18*c^45*d^24 - 6135496566696171932913500160*a^32*b^17*c^44*d^25 + 423642204
6382466798589050880*a^33*b^16*c^43*d^26 - 2550980661067485441771438080*a^34
*b^15*c^42*d^27 + 1334575022384247271808040960*a^35*b^14*c^41*d^28 - 603343
239457650202481000448*a^36*b^13*c^40*d^29 + 233967123641003163353350144*a^3
7*b^12*c^39*d^30 - 77049527429528415176228864*a^38*b^11*c^38*d^31 + 2125874
9850480450394390528*a^39*b^10*c^37*d^32 - 4823899363819901975265280*a^40*b^
9*c^36*d^33 + 876898617974708020183040*a^41*b^8*c^35*d^34 - 122811796684756
379238400*a^42*b^7*c^34*d^35 + 12444332416601319014400*a^43*b^6*c^33*d^36 -
 812231229670686720000*a^44*b^5*c^32*d^37 + 25649407252758528000*a^45*b^4*c
^31*d^38)*1i)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d
 + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 1
2672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7
920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192
*a^16*b*c*d^11))^(1/4))/((x^(1/2)*(15412443824768679936*a^11*b^35*c^52*d^8
- 439880110593414266688*a^12*b^34*c^51*d^9 - 1887306913007783641088*a^13*b^3
3*c^50*d^10 + 24240068121369040125952*a^14*b^32*c^49*d^11 - 155426886723407
276146688*a^15*b^31*c^48*d^12 + 661969678817672344633344*a^16*b^30*c^47*d^1
3 - 2072522435259453904257024*a^17*b^29*c^46*d^14 + 502562061398591470672228
16*a^18*b^28*c^45*d^15 - 9739734806850605210927104*a^19*b^27*c^44*d^16 + 15
395587131987386880229376*a^20*b^26*c^43*d^17 - 20118464109716534770794496*a
^21*b^25*c^42*d^18 + 21925688980693704834547712*a^22*b^24*c^41*d^19 - 20031
833528060137877536768*a^23*b^23*c^40*d^20 + 15375655212110710153674752*a^24
*b^22*c^39*d^21 - 9908539789204785922572288*a^25*b^21*c^38*d^22 + 534215175
2610266235273216*a^26*b^20*c^37*d^23 - 2393361740048338255872000*a^27*b^19*
c^36*d^24 + 881440288329629213655040*a^28*b^18*c^35*d^25 - 2625527690095530
86873600*a^29*b^17*c^34*d^26 + 61737289250332318105600*a^30*b^16*c^33*d^27
- 11040709176673173504000*a^31*b^15*c^32*d^28 + 1412353884520710144000*a^32
*b^14*c^31*d^29 - 115221946643251200000*a^33*b^13*c^30*d^30 + 4508684868648
960000*a^34*b^12*c^29*d^31) - (-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192
*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9
*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12
*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^
2*c^2*d^10 - 192*a^16*b*c*d^11))^(3/4)*(x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a
^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*
c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^
6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*
d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*(18446744073709551
616*a^12*b^38*c^68*d^4 - 479615345916448342016*a^13*b^37*c^67*d^5 + 5995191

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823955604275200*a^14*b^36*c^66*d^6 - 47961534591644834201600*a^15*b^35*c^65
*d^7 + 276025423003154095538176*a^16*b^34*c^64*d^8 - 1220386399802376518107
136*a^17*b^33*c^63*d^9 + 4341880678999181785825280*a^18*b^32*c^62*d^10 - 12
966583542852067073720320*a^19*b^31*c^61*d^11 + 34096448785847177707520000*a
^20*b^30*c^60*d^12 - 83405832293258492567879680*a^21*b^29*c^59*d^13 + 19875
3207063509910306160640*a^22*b^28*c^58*d^14 - 466650996519299420897935360*a^
23*b^27*c^57*d^15 + 1052056219870198056039219200*a^24*b^26*c^56*d^16 - 2194
458800584304697435750400*a^25*b^25*c^55*d^17 + 4119286127977833519707586560
*a^26*b^24*c^54*d^18 - 6855830429358878767993323520*a^27*b^23*c^53*d^19 + 1
0053745593205095687740456960*a^28*b^22*c^52*d^20 - 129661095597078446149206
01600*a^29*b^21*c^51*d^21 + 14704312650164876038740377600*a^30*b^20*c^50*d^
22 - 14666481047173052905774120960*a^31*b^19*c^49*d^23 + 128646666623782515
75193763840*a^32*b^18*c^48*d^24 - 9915254214005035782929121280*a^33*b^17*c^
47*d^25 + 6703228082495101562834124800*a^34*b^16*c^46*d^26 - 39637238143982
61058758246400*a^35*b^15*c^45*d^27 + 2041552487767277748019527680*a^36*b^14
*c^44*d^28 - 910688569282163512962973696*a^37*b^13*c^43*d^29 + 349132643065
901184834338816*a^38*b^12*c^42*d^30 - 113859137485172722840371200*a^39*b^11
*c^41*d^31 + 31155202813126960689971200*a^40*b^10*c^40*d^32 - 7019849261936
667709669376*a^41*b^9*c^39*d^33 + 1268449805817592472928256*a^42*b^8*c^38*d
^34 - 176741065216378693222400*a^43*b^7*c^37*d^35 + 17829840996752341073920
*a^44*b^6*c^36*d^36 - 1159226544085165670400*a^45*b^5*c^35*d^37 + 364791569
81701017600*a^46*b^4*c^34*d^38)*1i - 9223372036854775808*a^11*b^38*c^65*d^4
 + 212137556847659843584*a^12*b^37*c^64*d^5 - 2333513125324258279424*a^13*b
^36*c^63*d^6 + 16334591877269807955968*a^14*b^35*c^62*d^7 - 816729593863490
39779840*a^15*b^34*c^61*d^8 + 310808059650000835051520*a^16*b^33*c^60*d^9 -
 942943171860407129210880*a^17*b^32*c^59*d^10 + 2411982412523930344488960*a
^18*b^31*c^58*d^11 - 5753067372685321201254400*a^19*b^30*c^57*d^12 + 147861
94741349386435952640*a^20*b^29*c^56*d^13 - 43374839389541821883351040*a^21*
b^28*c^55*d^14 + 131295543449898524428206080*a^22*b^27*c^54*d^15 - 36563181
0199400875032576000*a^23*b^26*c^53*d^16 + 888615019916519951743057920*a^24*
b^25*c^52*d^17 - 1859065088581792734285660160*a^25*b^24*c^51*d^18 + 3349720
497258869063543685120*a^26*b^23*c^50*d^19 - 5220292063815211666322227200*a^
27*b^22*c^49*d^20 + 7067608268064143449134202880*a^28*b^21*c^48*d^21 - 8342
222871228251802477527040*a^29*b^20*c^47*d^22 + 8605396720616721741816791040
*a^30*b^19*c^46*d^23 - 7767500088979055902405427200*a^31*b^18*c^45*d^24 + 6
135496566696171932913500160*a^32*b^17*c^44*d^25 - 4236422046382466798589050
880*a^33*b^16*c^43*d^26 + 2550980661067485441771438080*a^34*b^15*c^42*d^27
- 1334575022384247271808040960*a^35*b^14*c^41*d^28 + 603343239457650202481
000448*a^36*b^13*c^40*d^29 - 233967123641003163353350144*a^37*b^12*c^39*d^30
 + 77049527429528415176228864*a^38*b^11*c^38*d^31 - 21258749850480450394390
528*a^39*b^10*c^37*d^32 + 4823899363819901975265280*a^40*b^9*c^36*d^33 - 87
6898617974708020183040*a^41*b^8*c^35*d^34 + 122811796684756379238400*a^42*b
^7*c^34*d^35 - 12444332416601319014400*a^43*b^6*c^33*d^36 + 812231229670686
720000*a^44*b^5*c^32*d^37 - 25649407252758528000*a^45*b^4*c^31*d^38)*1i)*(-
b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10
```

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*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^
7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4
*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))
^(1/4)*1i - (x^(1/2)*(15412443824768679936*a^11*b^35*c^52*d^8 - 43988011059
341426688*a^12*b^34*c^51*d^9 - 18873069130077836 41088*a^13*b^33*c^50*d^10 +
 242400681213690401 25952*a^14*b^32*c^49*d^11 - 1554268867234072761 46688*a^1
5*b^31*c^48*d^12 + 6619696788176723446 33344*a^16*b^30*c^47*d^13 - 207252243
5259453904257024*a^17*b^29*c^46*d^14 + 50256206139859147067 22816*a^18*b^28*
c^45*d^15 - 97397348068506052 10927104*a^19*b^27*c^44*d^16 + 153955871319873
86880229376*a^20*b^26*c^43*d^17 - 20118464109716534770794496*a^21*b^25*c^42
*d^18 + 21925688980693704834547712*a^22*b^24*c^41*d^19 - 200318335280601378
77536768*a^23*b^23*c^40*d^20 + 15375655212110710153674752*a^24*b^22*c^39*d^
21 - 9908539789204785922572288*a^25*b^21*c^38*d^22 + 534215175261026623 5273
216*a^26*b^20*c^37*d^23 - 2393361740048338255872000*a^27*b^19*c^36*d^24 + 8
81440288329629213655040*a^28*b^18*c^35*d^25 - 262552769009553086873600*a^29
*b^17*c^34*d^26 + 61737289250332318105600*a^30*b^16*c^33*d^27 - 11040709176
673173504000*a^31*b^15*c^32*d^28 + 1412353884520710144000*a^32*b^14*c^31*d^
29 - 115221946643251200000*a^33*b^13*c^30*d^30 + 4508684868648960000*a^34*b
^12*c^29*d^31) - (-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^1
1*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4
- 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7
+ 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 -
192*a^16*b*c*d^11))^(3/4)*(x^(1/2)*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12
- 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 792
0*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672
*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^
15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*(18446744073709551616*a^12*b^38
*c^68*d^4 - 479615345916448342016*a^13*b^37*c^67*d^5 + 599519182395560427 52
00*a^14*b^36*c^66*d^6 - 47961534591644834201600*a^15*b^35*c^65*d^7 + 276025
423003154095538176*a^16*b^34*c^64*d^8 - 1220386399802376518107136*a^17*b^33
*c^63*d^9 + 4341880678999181785825280*a^18*b^32*c^62*d^10 - 129665835428520
67073720320*a^19*b^31*c^61*d^11 + 34096448785847177707520000*a^20*b^30*c^60
*d^12 - 83405832293258492567879680*a^21*b^29*c^59*d^13 + 198753207063509910
306160640*a^22*b^28*c^58*d^14 - 466650996519299420897935360*a^23*b^27*c^57*
d^15 + 1052056219870198056039219200*a^24*b^26*c^56*d^16 - 21944588005843046
97435750400*a^25*b^25*c^55*d^17 + 4119286127977833519707586560*a^26*b^24*c^
54*d^18 - 6855830429358878767993323520*a^27*b^23*c^53*d^19 + 10053745593205
095687740456960*a^28*b^22*c^52*d^20 - 12966109559707844614920601600*a^29*b^
21*c^51*d^21 + 14704312650164876038740377600*a^30*b^20*c^50*d^22 - 14666481
047173052905774120960*a^31*b^19*c^49*d^23 + 12864666662378251575193763840*a
^32*b^18*c^48*d^24 - 9915254214005035782929121280*a^33*b^17*c^47*d^25 + 670
3228082495101562834124800*a^34*b^16*c^46*d^26 - 396372381439826105875824640
0*a^35*b^15*c^45*d^27 + 2041552487767277748019527680*a^36*b^14*c^44*d^28 -
910688569282163512962973696*a^37*b^13*c^43*d^29 + 349132643065901184834388
16*a^38*b^12*c^42*d^30 - 113859137485172722840371200*a^39*b^11*c^41*d^31 +
```

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31155202813126960689971200*a^40*b^10*c^40*d^32 - 7019849261936667709669376*
a^41*b^9*c^39*d^33 + 1268449805817592472928256*a^42*b^8*c^38*d^34 - 1767410
65216378693222400*a^43*b^7*c^37*d^35 + 17829840996752341073920*a^44*b^6*c^3
6*d^36 - 1159226544085165670400*a^45*b^5*c^35*d^37 + 36479156981701017600*a
^46*b^4*c^34*d^38)*1i + 9223372036854775808*a^11*b^38*c^65*d^4 - 2121375568
47659843584*a^12*b^37*c^64*d^5 + 2333513125324258279424*a^13*b^36*c^63*d^6
- 16334591877269807955968*a^14*b^35*c^62*d^7 + 81672959386349039779840*a^15
*b^34*c^61*d^8 - 310808059650000835051520*a^16*b^33*c^60*d^9 + 942943171860
407129210880*a^17*b^32*c^59*d^10 - 2411982412523930344488960*a^18*b^31*c^58
*d^11 + 5753067372685321201254400*a^19*b^30*c^57*d^12 - 1478619474134938643
5952640*a^20*b^29*c^56*d^13 + 43374839389541821883351040*a^21*b^28*c^55*d^1
4 - 131295543449898524428206080*a^22*b^27*c^54*d^15 + 365631810199400875032
576000*a^23*b^26*c^53*d^16 - 888615019916519951743057920*a^24*b^25*c^52*d^1
7 + 1859065088581792734285660160*a^25*b^24*c^51*d^18 - 33497204972588690635
43685120*a^26*b^23*c^50*d^19 + 5220292063815211666322227200*a^27*b^22*c^49*
d^20 - 7067608268064143449134202880*a^28*b^21*c^48*d^21 + 83422228712282518
02477527040*a^29*b^20*c^47*d^22 - 8605396720616721741816791040*a^30*b^19*c^
46*d^23 + 7767500088979055902405427200*a^31*b^18*c^45*d^24 - 61354965666961
71932913500160*a^32*b^17*c^44*d^25 + 4236422046382466798589050880*a^33*b^16
*c^43*d^26 - 2550980661067485441771438080*a^34*b^15*c^42*d^27 + 13345750223
84247271808040960*a^35*b^14*c^41*d^28 - 603343239457650202481000448*a^36*b^
13*c^40*d^29 + 233967123641003163353350144*a^37*b^12*c^39*d^30 - 7704952742
9528415176228864*a^38*b^11*c^38*d^31 + 21258749850480450394390528*a^39*b^10
*c^37*d^32 - 4823899363819901975265280*a^40*b^9*c^36*d^33 + 876898617974708
020183040*a^41*b^8*c^35*d^34 - 122811796684756379238400*a^42*b^7*c^34*d^35
+ 12444332416601319014400*a^43*b^6*c^33*d^36 - 812231229670686720000*a^44*b
^5*c^32*d^37 + 25649407252758528000*a^45*b^4*c^31*d^38)*1i)*(-b^13/(16*a^17
*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3
520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784
*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a
^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*d^10 - 192*a^16*b*c*d^11))^(1/4)*1i - 1
49684329919262228480*a^11*b^34*c^48*d^9 + 2374404370680061624320*a^12*b^33*
c^47*d^10 - 17808722627439624192000*a^13*b^32*c^46*d^11 + 83960295795175519
682560*a^14*b^31*c^45*d^12 - 278998813302985850880000*a^15*b^30*c^44*d^13 +
 694438771802419400540160*a^16*b^29*c^43*d^14 - 1342951722708271932375040*a
^17*b^28*c^42*d^15 + 2065391322938120916172800*a^18*b^27*c^41*d^16 - 256421
8746215699966853120*a^19*b^26*c^40*d^17 + 2593338871410901332787200*a^20*b^
25*c^39*d^18 - 2146065846150812380692480*a^21*b^24*c^38*d^19 + 145362544156
9727022366720*a^22*b^23*c^37*d^20 - 802881124954933087436800*a^23*b^22*c^36
*d^21 + 358581985606139180482560*a^24*b^21*c^35*d^22 - 12766036181812531691
5200*a^25*b^20*c^34*d^23 + 35417419405750750412800*a^26*b^19*c^33*d^24 - 73
86837561454362624000*a^27*b^18*c^32*d^25 + 1090533977896255488000*a^28*b^17
*c^31*d^26 - 101695892037304320000*a^29*b^16*c^30*d^27 + 450868486864896000
0*a^30*b^15*c^29*d^28))*(-b^13/(16*a^17*d^12 + 16*a^5*b^12*c^12 - 192*a^6*b
^11*c^11*d + 1056*a^7*b^10*c^10*d^2 - 3520*a^8*b^9*c^9*d^3 + 7920*a^9*b^8*c
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^8*d^4 - 12672*a^10*b^7*c^7*d^5 + 14784*a^11*b^6*c^6*d^6 - 12672*a^12*b^5*c
^5*d^7 + 7920*a^13*b^4*c^4*d^8 - 3520*a^14*b^3*c^3*d^9 + 1056*a^15*b^2*c^2*
d^10 - 192*a^16*b*c*d^11))^(1/4) - (2/(a*c) + (x^2*(81*a^2*d^3 + 64*b^2*c^2
*d - 153*a*b*c*d^2))/(16*a*c*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^
4*(45*a^2*d^2 + 32*b^2*c^2 - 85*a*b*c*d))/(16*a*c^2*(b^2*c^3 + a^2*c*d^2 -
2*a*b*c^2*d)))/(c^2*x^(1/2) + d^2*x^(9/2) + 2*c*d*x^(5/2)) + 2*atan(((-(410
0625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 167635494
0*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9
 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b
*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c
^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 830
4721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b
^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 -
3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b
^11*c^24*d))^(1/4)*(x^(1/2)*(15412443824768679936*a^11*b^35*c^52*d^8 - 4398
8011059341426688*a^12*b^34*c^51*d^9 - 18873069130077836410888*a^13*b^33*c^50
*d^10 + 24240068121369040125952*a^14*b^32*c^49*d^11 - 155426886723407276146
688*a^15*b^31*c^48*d^12 + 661969678817672344633344*a^16*b^30*c^47*d^13 - 20
72522435259453904257024*a^17*b^29*c^46*d^14 + 5025620613985914706722816*a^1
8*b^28*c^45*d^15 - 9739734806850605210927104*a^19*b^27*c^44*d^16 + 15395587
131987386880229376*a^20*b^26*c^43*d^17 - 20118464109716534770794496*a^21*b^
25*c^42*d^18 + 21925688980693704834547712*a^22*b^24*c^41*d^19 - 20031833528
060137877536768*a^23*b^23*c^40*d^20 + 15375655212110710153674752*a^24*b^22*
c^39*d^21 - 9908539789204785922572288*a^25*b^21*c^38*d^22 + 534215175261026
6235273216*a^26*b^20*c^37*d^23 - 2393361740048338255872000*a^27*b^19*c^36*d
^24 + 881440288329629213655040*a^28*b^18*c^35*d^25 - 262552769009553086873
600*a^29*b^17*c^34*d^26 + 61737289250332318105600*a^30*b^16*c^33*d^27 - 1104
0709176673173504000*a^31*b^15*c^32*d^28 + 1412353884520710144000*a^32*b^14*
c^31*d^29 - 115221946643251200000*a^33*b^13*c^30*d^30 + 4508684868648960000
*a^34*b^12*c^29*d^31) - (-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 83283
8760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^
8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6
*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*
c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 369
0987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^
7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 +
8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10
*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(3/4)*(x^(1/2)*(-(4100625*a^8*d^
13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c
^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 76506300
0*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(1
6777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 +
1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4
*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6
 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*
```

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a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d)
)^(1/4)*(18446744073709551616*a^12*b^38*c^68*d^4 - 479615345916448342016*a^
13*b^37*c^67*d^5 + 5995191823955604275200*a^14*b^36*c^66*d^6 - 479615345916
44834201600*a^15*b^35*c^65*d^7 + 276025423003154095538176*a^16*b^34*c^64*d^
8 - 1220386399802376518107136*a^17*b^33*c^63*d^9 + 434188067899918178582528
0*a^18*b^32*c^62*d^10 - 12966583542852067073720320*a^19*b^31*c^61*d^11 + 34
096448785847177707520000*a^20*b^30*c^60*d^12 - 83405832293258492567879680*a
^21*b^29*c^59*d^13 + 198753207063509910306160640*a^22*b^28*c^58*d^14 - 4666
50996519299420897935360*a^23*b^27*c^57*d^15 + 1052056219870198056039219200*
a^24*b^26*c^56*d^16 - 2194458800584304697435750400*a^25*b^25*c^55*d^17 + 41
19286127977833519707586560*a^26*b^24*c^54*d^18 - 685583042935887876799332335
20*a^27*b^23*c^53*d^19 + 10053745593205095687740456960*a^28*b^22*c^52*d^20
 - 12966109559707844614920601600*a^29*b^21*c^51*d^21 + 147043126501648760387
40377600*a^30*b^20*c^50*d^22 - 14666481047173052905774120960*a^31*b^19*c^49
*d^23 + 12864666662378251575193763840*a^32*b^18*c^48*d^24 - 991525421400503
5782929121280*a^33*b^17*c^47*d^25 + 6703228082495101562834124800*a^34*b^16*
c^46*d^26 - 3963723814398261058758246400*a^35*b^15*c^45*d^27 + 204155248776
7277748019527680*a^36*b^14*c^44*d^28 - 910688569282163512962973696*a^37*b^1
3*c^43*d^29 + 349132643065901184834338816*a^38*b^12*c^42*d^30 - 11385913748
5172722840371200*a^39*b^11*c^41*d^31 + 31155202813126960689971200*a^40*b^10
*c^40*d^32 - 7019849261936667709669376*a^41*b^9*c^39*d^33 + 126844980581759
2472928256*a^42*b^8*c^38*d^34 - 176741065216378693222400*a^43*b^7*c^37*d^35
 + 17829840996752341073920*a^44*b^6*c^36*d^36 - 1159226544085165670400*a^45
*b^5*c^35*d^37 + 36479156981701017600*a^46*b^4*c^34*d^38)*1i - 922337203685
4775808*a^11*b^38*c^65*d^4 + 212137556847659843584*a^12*b^37*c^64*d^5 - 233
3513125324258279424*a^13*b^36*c^63*d^6 + 16334591877269807955968*a^14*b^35*
c^62*d^7 - 81672959386349039779840*a^15*b^34*c^61*d^8 + 3108080596500008350
51520*a^16*b^33*c^60*d^9 - 942943171860407129210880*a^17*b^32*c^59*d^10 + 2
411982412523930344488960*a^18*b^31*c^58*d^11 - 5753067372685321201254400*a^
19*b^30*c^57*d^12 + 14786194741349386435952640*a^20*b^29*c^56*d^13 - 433748
39389541821883351040*a^21*b^28*c^55*d^14 + 131295543449898524428206080*a^22
*b^27*c^54*d^15 - 365631810199400875032576000*a^23*b^26*c^53*d^16 + 8886150
19916519951743057920*a^24*b^25*c^52*d^17 - 1859065088581792734285660160*a^2
5*b^24*c^51*d^18 + 3349720497258869063543685120*a^26*b^23*c^50*d^19 - 52202
92063815211666322227200*a^27*b^22*c^49*d^20 + 7067608268064143449134202880*
a^28*b^21*c^48*d^21 - 8342222871228251802477527040*a^29*b^20*c^47*d^22 + 86
05396720616721741816791040*a^30*b^19*c^46*d^23 - 776750008897905590240542720
00*a^31*b^18*c^45*d^24 + 6135496566696171932913500160*a^32*b^17*c^44*d^25 -
 4236422046382466798589050880*a^33*b^16*c^43*d^26 + 255098066106748544177714
38080*a^34*b^15*c^42*d^27 - 1334575022384247271808040960*a^35*b^14*c^41*d^2
8 + 603343239457650202481000448*a^36*b^13*c^40*d^29 - 2339671236410031633533
350144*a^37*b^12*c^39*d^30 + 77049527429528415176228864*a^38*b^11*c^38*d^31
 - 21258749850480450394390528*a^39*b^10*c^37*d^32 + 482389936381990197526528
80*a^40*b^9*c^36*d^33 - 876898617974708020183040*a^41*b^8*c^35*d^34 + 12281
1796684756379238400*a^42*b^7*c^34*d^35 - 12444332416601319014400*a^43*b^6*c
```

```
^33*d^36 + 812231229670686720000*a^44*b^5*c^32*d^37 - 25649407252758528000*
a^45*b^4*c^31*d^38)*1i) + (-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832
838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*
d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a
^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^1
2*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3
690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*
b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7
 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^
10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*(x^(1/2)*(15412443824768
679936*a^11*b^35*c^52*d^8 - 439880110593414266880*a^12*b^34*c^51*d^9 - 18873
0691300778364l088*a^13*b^33*c^50*d^10 + 242400681213690401259552*a^14*b^32*c
^49*d^11 - 1554268867234072761466888*a^15*b^31*c^48*d^12 + 661969678817672344
4633344*a^16*b^30*c^47*d^13 - 20725224352594539042570024*a^17*b^29*c^46*d^14
 + 50256206139859147067228816*a^18*b^28*c^45*d^15 - 97397348068506052109271O
4*a^19*b^27*c^44*d^16 + 153955871319873868802293376*a^20*b^26*c^43*d^17 - 20
1184641097165347707944496*a^21*b^25*c^42*d^18 + 219256889806937048345477712*a
^22*b^24*c^41*d^19 - 200318335280601378775367768*a^23*b^23*c^40*d^20 + 15375
65521211071015367447752*a^24*b^22*c^39*d^21 - 99085397892047859225722288*a^25*
b^21*c^38*d^22 + 53421517526102662352732216*a^26*b^20*c^37*d^23 - 2393361740
0483382558720000*a^27*b^19*c^36*d^24 + 88144028832962921365504O*a^28*b^18*c^
35*d^25 - 2625527690095530868736000*a^29*b^17*c^34*d^26 + 617372892503323181
05600*a^30*b^16*c^33*d^27 - 1104070917667317350400O*a^31*b^15*c^32*d^28 + 1
41235388452071014400O*a^32*b^14*c^31*d^29 - 11522194664325120000O*a^33*b^13
*c^30*d^30 + 4508684868648960000*a^34*b^12*c^29*d^31) - (-(4100625*a^8*d^13
 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6
*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*
a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(167
77216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 11
07296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b
^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 -
 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^
9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^
(3/4)*(x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^
7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 15196
73350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*
d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12
 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a
^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^
5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 830472192
0*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15
*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*(18446744073709551616*a^12*b^38*c^6
8*d^4 - 479615345916448342016*a^13*b^37*c^67*d^5 + 5995191823955604275200*a
^14*b^36*c^66*d^6 - 47961534591644834201600*a^15*b^35*c^65*d^7 + 2760254230
03154095538176*a^16*b^34*c^64*d^8 - 1220386399802376518107136*a^17*b^33*c^6
```

```
3*d^9 + 4341880678999181785825280*a^18*b^32*c^62*d^10 - 1296658354285206707
3720320*a^19*b^31*c^61*d^11 + 34096448785847177707520000*a^20*b^30*c^60*d^1
2 - 83405832293258492567879680*a^21*b^29*c^59*d^13 + 1987532070635099103061
60640*a^22*b^28*c^58*d^14 - 466650996519299420897935360*a^23*b^27*c^57*d^15
 + 1052056219870198056039219200*a^24*b^26*c^56*d^16 - 219445880058430469743
5750400*a^25*b^25*c^55*d^17 + 4119286127977833519707586560*a^26*b^24*c^54*d
^18 - 6855830429358878767993323520*a^27*b^23*c^53*d^19 + 100537455932050956
87740456960*a^28*b^22*c^52*d^20 - 12966109559707844614920601600*a^29*b^21*c
^51*d^21 + 14704312650164876038740377600*a^30*b^20*c^50*d^22 - 146664810471
73052905774120960*a^31*b^19*c^49*d^23 + 12864666662378251575193763840*a^32*
b^18*c^48*d^24 - 9915254214005035782929121280*a^33*b^17*c^47*d^25 + 6703228
082495101562834124800*a^34*b^16*c^46*d^26 - 3963723814398261058758246400*a^
35*b^15*c^45*d^27 + 2041552487767277748019527680*a^36*b^14*c^44*d^28 - 9106
88569282163512962973696*a^37*b^13*c^43*d^29 + 349132643065901184834338816*a
^38*b^12*c^42*d^30 - 113859137485172722840371200*a^39*b^11*c^41*d^31 + 3115
5202813126960689971200*a^40*b^10*c^40*d^32 - 7019849261936667709669376*a^41
*b^9*c^39*d^33 + 1268449805817592472928256*a^42*b^8*c^38*d^34 - 17674106521
6378693222400*a^43*b^7*c^37*d^35 + 17829840996752341073920*a^44*b^6*c^36*d^
36 - 1159226544085165670400*a^45*b^5*c^35*d^37 + 36479156981701017600*a^46*
b^4*c^34*d^38)*1i + 9223372036854775808*a^11*b^38*c^65*d^4 - 21213755684765
9843584*a^12*b^37*c^64*d^5 + 2333513125324258279424*a^13*b^36*c^63*d^6 - 16
334591877269807955968*a^14*b^35*c^62*d^7 + 81672959386349039779840*a^15*b^3
4*c^61*d^8 - 310808059650000835051520*a^16*b^33*c^60*d^9 + 9429431718604071
29210880*a^17*b^32*c^59*d^10 - 2411982412523930344488960*a^18*b^31*c^58*d^1
1 + 5753067372685321201254400*a^19*b^30*c^57*d^12 - 14786194741349386435952
640*a^20*b^29*c^56*d^13 + 43374839389541821883351040*a^21*b^28*c^55*d^14 -
131295543449898524428206080*a^22*b^27*c^54*d^15 + 3656318101994008750325760
00*a^23*b^26*c^53*d^16 - 888615019916519951743057920*a^24*b^25*c^52*d^17 +
1859065088581792734285660160*a^25*b^24*c^51*d^18 - 334972049725886906354368
5120*a^26*b^23*c^50*d^19 + 5220292063815211666322227200*a^27*b^22*c^49*d^20
 - 7067608268064143449134202880*a^28*b^21*c^48*d^21 + 834222287122825180247
7527040*a^29*b^20*c^47*d^22 - 8605396720616721741816791040*a^30*b^19*c^46*d
^23 + 7767500088979055902405427200*a^31*b^18*c^45*d^24 - 613549656669617193
2913500160*a^32*b^17*c^44*d^25 + 4236422046382466798589050880*a^33*b^16*c^4
3*d^26 - 2550980661067485441771438080*a^34*b^15*c^42*d^27 + 133457502238424
7271808040960*a^35*b^14*c^41*d^28 - 603343239457650202481000448*a^36*b^13*c
^40*d^29 + 233967123641003163353350144*a^37*b^12*c^39*d^30 - 77049527429528
415176228864*a^38*b^11*c^38*d^31 + 21258749850480450394390528*a^39*b^10*c^3
7*d^32 - 4823899363819901975265280*a^40*b^9*c^36*d^33 + 8768986179747080201
83040*a^41*b^8*c^35*d^34 - 122811796684756379238400*a^42*b^7*c^34*d^35 + 12
444332416601319014400*a^43*b^6*c^33*d^36 - 812231229670686720000*a^44*b^5*c
^32*d^37 + 25649407252758528000*a^45*b^4*c^31*d^38)*1i))/((-(4100625*a^8*d^
13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c
^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 76506300
0*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(1
```

6777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 +
1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4
*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6
 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*
a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d)
)^(1/4)*(x^(1/2)*(15412443824768679936*a^11*b^35*c^52*d^8 - 439880110593414
26688*a^12*b^34*c^51*d^9 - 188730691300778364 1088*a^13*b^33*c^50*d^10 + 242
400681213690401259 52*a^14*b^32*c^49*d^11 - 155426886723407276146688*a^15*b^
31*c^48*d^12 + 661969678817672344633344*a^16*b^30*c^47*d^13 - 2072522435259
453904257024*a^17*b^29*c^46*d^14 + 5025620613985914706722816*a^18*b^28*c^45
*d^15 - 9739734806850605210927104*a^19*b^27*c^44*d^16 + 1539558713198738688
0229376*a^20*b^26*c^43*d^17 - 20118464109716534770794496*a^21*b^25*c^42*d^1
8 + 21925688980693704834547712*a^22*b^24*c^41*d^19 - 2003183352806013787753
6768*a^23*b^23*c^40*d^20 + 15375655212110710153674752*a^24*b^22*c^39*d^21 -
 9908539789204785922572288*a^25*b^21*c^38*d^22 + 5342151752610266235273216*
a^26*b^20*c^37*d^23 - 2393361740048338255872000*a^27*b^19*c^36*d^24 + 88144
0288329629213655040*a^28*b^18*c^35*d^25 - 262552769009553086873600*a^29*b^1
7*c^34*d^26 + 61737289250332318105600*a^30*b^16*c^33*d^27 - 110407091766731
73504000*a^31*b^15*c^32*d^28 + 1412353884520710144000*a^32*b^14*c^31*d^29 -
 115221946643251200000*a^33*b^13*c^30*d^30 + 4508684868648960000*a^34*b^12*
c^29*d^31) - (-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*
c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673
350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^
11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 -
 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3
*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5
+ 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*
a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d
^10 - 201326592*a*b^11*c^24*d))^(3/4)*(x^(1/2)*(-(4100625*a^8*d^13 + 187388
721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 19
89163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c
^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^1
2*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*
a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d
^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555
072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^1
6*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*(18
446744073709551616*a^12*b^38*c^68*d^4 - 479615345916448342016*a^13*b^37*c^6
7*d^5 + 5995191823955604275200*a^14*b^36*c^66*d^6 - 47961534591644834201600
*a^15*b^35*c^65*d^7 + 276025423003154095538176*a^16*b^34*c^64*d^8 - 1220386
399802376518107136*a^17*b^33*c^63*d^9 + 4341880678999181785825280*a^18*b^32
*c^62*d^10 - 12966583542852067073720320*a^19*b^31*c^61*d^11 + 3409644878584
7177707520000*a^20*b^30*c^60*d^12 - 83405832293258492567879680*a^21*b^29*c^
59*d^13 + 198753207063509910306160640*a^22*b^28*c^58*d^14 - 466650996519299
420897935360*a^23*b^27*c^57*d^15 + 1052056219870198056039219200*a^24*b^26*c

```
^56*d^16 - 21944588005843046974357504OO*a^25*b^25*c^55*d^17 + 4119286127977
833519707586560*a^26*b^24*c^54*d^18 - 6855830429358878767993323520*a^27*b^2
3*c^53*d^19 + 10053745593205095687740456960*a^28*b^22*c^52*d^20 - 129661095
5970784461492060160O*a^29*b^21*c^51*d^21 + 14704312650164876038740377600*a^
30*b^20*c^50*d^22 - 14666481047173052905774120960*a^31*b^19*c^49*d^23 + 128
6466662378251575193763840*a^32*b^18*c^48*d^24 - 99152542140050357829291212
80*a^33*b^17*c^47*d^25 + 6703228082495101562834124800*a^34*b^16*c^46*d^26 -
 3963723814398261058758246400*a^35*b^15*c^45*d^27 + 20415524877672777480195
27680*a^36*b^14*c^44*d^28 - 910688569282163512962973696*a^37*b^13*c^43*d^29
 + 349132643065901184834338816*a^38*b^12*c^42*d^30 - 1138591374851727228403
71200*a^39*b^11*c^41*d^31 + 31155202813126960689971200*a^40*b^10*c^40*d^32
- 7019849261936667709669376*a^41*b^9*c^39*d^33 + 1268449805817592472928256*
a^42*b^8*c^38*d^34 - 176741065216378693222400*a^43*b^7*c^37*d^35 + 17829840
996752341073920*a^44*b^6*c^36*d^36 - 1159226544085165670400*a^45*b^5*c^35*d
^37 + 36479156981701017600*a^46*b^4*c^34*d^38)*1i - 9223372036854775808*a^1
1*b^38*c^65*d^4 + 212137556847659843584*a^12*b^37*c^64*d^5 - 23335131253242
58279424*a^13*b^36*c^63*d^6 + 16334591877269807955968*a^14*b^35*c^62*d^7 -
81672959386349039779840*a^15*b^34*c^61*d^8 + 310808059650000835051520*a^16*
b^33*c^60*d^9 - 942943171860407129210880*a^17*b^32*c^59*d^10 + 241198241252
3930344488960*a^18*b^31*c^58*d^11 - 5753067372685321201254400*a^19*b^30*c^5
7*d^12 + 14786194741349386435952640*a^20*b^29*c^56*d^13 - 43374839389541821
883351040*a^21*b^28*c^55*d^14 + 131295543449898524428206080*a^22*b^27*c^54*
d^15 - 365631810199400875032576000*a^23*b^26*c^53*d^16 + 888615019916519951
743057920*a^24*b^25*c^52*d^17 - 1859065088581792734285660160*a^25*b^24*c^51
*d^18 + 3349720497258869063543685120*a^26*b^23*c^50*d^19 - 5220292063815211
666322227200*a^27*b^22*c^49*d^20 + 7067608268064143449134202880*a^28*b^21*c
^48*d^21 - 8342222871228251802477527040*a^29*b^20*c^47*d^22 + 8605396720616
721741816791040*a^30*b^19*c^46*d^23 - 7767500088979055902405427200*a^31*b^1
8*c^45*d^24 + 6135496566696171932913500160*a^32*b^17*c^44*d^25 - 4236422046
382466798589050880*a^33*b^16*c^43*d^26 + 2550980661067485441771438080*a^34*
b^15*c^42*d^27 - 1334575022384247271808040960*a^35*b^14*c^41*d^28 + 6033432
39457650202481000448*a^36*b^13*c^40*d^29 - 233967123641003163353350144*a^37
*b^12*c^39*d^30 + 77049527429528415176228864*a^38*b^11*c^38*d^31 - 21258749
850480450394390528*a^39*b^10*c^37*d^32 + 4823899363819901975265280*a^40*b^9
*c^36*d^33 - 876898617974708020183040*a^41*b^8*c^35*d^34 + 1228117966847563
79238400*a^42*b^7*c^34*d^35 - 12444332416601319014400*a^43*b^6*c^33*d^36 +
812231229670686720000*a^44*b^5*c^32*d^37 - 25649407252758528000*a^45*b^4*c^
31*d^38)*1i)*1i - (-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a
*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 15
19673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c
^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d
^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 369098752
0*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20
*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 830472
1920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c
```

```
^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*(x^(1/2)*(15412443824768679936*a
^11*b^35*c^52*d^8 - 439880110593414266888*a^12*b^34*c^51*d^9 - 1887306913007
783641088*a^13*b^33*c^50*d^10 + 24240068121369040125952*a^14*b^32*c^49*d^11
 - 155426886723407276146688*a^15*b^31*c^48*d^12 + 661969678817672344633344*
a^16*b^30*c^47*d^13 - 2072522435259453904257024*a^17*b^29*c^46*d^14 + 50256
20613985914706722816*a^18*b^28*c^45*d^15 - 9739734806850605210927104*a^19*b
^27*c^44*d^16 + 15395587131987386880229376*a^20*b^26*c^43*d^17 - 2011846410
9716534770794496*a^21*b^25*c^42*d^18 + 21925688980693704834547712*a^22*b^24
*c^41*d^19 - 20031833528060137877536768*a^23*b^23*c^40*d^20 + 1537565521211
0710153674752*a^24*b^22*c^39*d^21 - 9908539789204785922572288*a^25*b^21*c^3
8*d^22 + 5342151752610266235273216*a^26*b^20*c^37*d^23 - 239336174004833825
5872000*a^27*b^19*c^36*d^24 + 881440288329629213655040*a^28*b^18*c^35*d^25
- 262552769009553086873600*a^29*b^17*c^34*d^26 + 61737289250332318105600*a^
30*b^16*c^33*d^27 - 11040709176673173504000*a^31*b^15*c^32*d^28 + 141235388
4520710144000*a^32*b^14*c^31*d^29 - 115221946643251200000*a^33*b^13*c^30*d^
30 + 4508684868648960000*a^34*b^12*c^29*d^31) - (-(4100625*a^8*d^13 + 18738
8721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1
989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*
c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^
12*c^25 + 16777216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256
*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*
d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 1328755
5072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^
16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(3/4)*(x
^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^
6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^
4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 4
7385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 20132
6592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c
^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 1550
2147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^
4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 -
201326592*a*b^11*c^24*d))^(1/4)*(18446744073709551616*a^12*b^38*c^68*d^4 -
479615345916448342016*a^13*b^37*c^67*d^5 + 5995191823955604275200*a^14*b^36
*c^66*d^6 - 47961534591644834201600*a^15*b^35*c^65*d^7 + 276025423003154095
538176*a^16*b^34*c^64*d^8 - 1220386399802376518107136*a^17*b^33*c^63*d^9 +
4341880678999181785825280*a^18*b^32*c^62*d^10 - 12966583542852067073720320*
a^19*b^31*c^61*d^11 + 34096448785847177707520000*a^20*b^30*c^60*d^12 - 8340
5832293258492567879680*a^21*b^29*c^59*d^13 + 198753207063509910306160640*a^
22*b^28*c^58*d^14 - 466650996519299420897935360*a^23*b^27*c^57*d^15 + 10520
56219870198056039219200*a^24*b^26*c^56*d^16 - 2194458800584304697435750400*
a^25*b^25*c^55*d^17 + 4119286127977833519707586560*a^26*b^24*c^54*d^18 - 68
55830429358878767993323520*a^27*b^23*c^53*d^19 + 10053745593205095687740456
960*a^28*b^22*c^52*d^20 - 12966109559707844614920601600*a^29*b^21*c^51*d^21
 + 14704312650164876038740377600*a^30*b^20*c^50*d^22 - 14666481047173052905
```

```
774120960*a^31*b^19*c^49*d^23 + 12864666662378251575193763840*a^32*b^18*c^4
8*d^24 - 9915254214005035782929121280*a^33*b^17*c^47*d^25 + 670322808249510
1562834124800*a^34*b^16*c^46*d^26 - 3963723814398261058758246400*a^35*b^15*
c^45*d^27 + 2041552487767277748019527680*a^36*b^14*c^44*d^28 - 910688569282
163512962973696*a^37*b^13*c^43*d^29 + 349132643065901184834338816*a^38*b^12
*c^42*d^30 - 113859137485172722840371200*a^39*b^11*c^41*d^31 + 311552028131
26960689971200*a^40*b^10*c^40*d^32 - 7019849261936667709669376*a^41*b^9*c^3
9*d^33 + 1268449805817592472928256*a^42*b^8*c^38*d^34 - 176741065216378693２
22400*a^43*b^7*c^37*d^35 + 17829840996752341073920*a^44*b^6*c^36*d^36 - 115
922654408516567040０*a^45*b^5*c^35*d^37 + 3647915698170101760０*a^46*b^4*c^34
*d^38)*1i + 9223372036854775808*a^11*b^38*c^65*d^4 - 212137556847659843584*
a^12*b^37*c^64*d^5 + 2333513125324258279424*a^13*b^36*c^63*d^6 - 1633459187
7269807955968*a^14*b^35*c^62*d^7 + 81672959386349039779840*a^15*b^34*c^61*d
^8 - 310808059650000835051520*a^16*b^33*c^60*d^9 + 942943171860407129210880
*a^17*b^32*c^59*d^10 - 2411982412523930344488960*a^18*b^31*c^58*d^11 + 5753
067372685321201254400*a^19*b^30*c^57*d^12 - 14786194741349386435952640*a^20
*b^29*c^56*d^13 + 43374839389541821883351040*a^21*b^28*c^55*d^14 - 13129554
3449898524428206080*a^22*b^27*c^54*d^15 + 365631810199400875032576000*a^23*
b^26*c^53*d^16 - 888615019916519951743057920*a^24*b^25*c^52*d^17 + 18590650
88581792734285660160*a^25*b^24*c^51*d^18 - 3349720497258869063543685120*a^2
6*b^23*c^50*d^19 + 5220292063815211666322227200*a^27*b^22*c^49*d^20 - 70676
08268064143449134202880*a^28*b^21*c^48*d^21 + 8342222871228251802477527040*
a^29*b^20*c^47*d^22 - 8605396720616721741816791040*a^30*b^19*c^46*d^23 + 77
67500088979055902405427200*a^31*b^18*c^45*d^24 - 613549656669617193291350０1
60*a^32*b^17*c^44*d^25 + 4236422046382466798589050880*a^33*b^16*c^43*d^26 -
 2550980661067485441771438080*a^34*b^15*c^42*d^27 + 133457502238424727180８0
40960*a^35*b^14*c^41*d^28 - 603343239457650202481000448*a^36*b^13*c^40*d^29
 + 233967123641003163353350144*a^37*b^12*c^39*d^30 - 77049527429528415176２2
8864*a^38*b^11*c^38*d^31 + 21258749850480450394390528*a^39*b^10*c^37*d^32 -
 4823899363819901975265280*a^40*b^9*c^36*d^33 + 876898617974708020183040*a^
41*b^8*c^35*d^34 - 122811796684756379238400*a^42*b^7*c^34*d^35 + 1244433241
6601319014400*a^43*b^6*c^33*d^36 - 812231229670686720000*a^44*b^5*c^32*d^37
 + 25649407252758528000*a^45*b^4*c^31*d^38)*1i)*1i - 1496843299192622２8480*
a^11*b^34*c^48*d^9 + 2374404370680061624320*a^12*b^33*c^47*d^10 - 178087226
2743962419２000*a^13*b^32*c^46*d^11 + 83960295795175519682560*a^14*b^31*c^45
*d^12 - 278998813302985850880000*a^15*b^30*c^44*d^13 + 694438771802419400５4
0160*a^16*b^29*c^43*d^14 - 1342951722708271932375040*a^17*b^28*c^42*d^15 +
2065391322938120916172800*a^18*b^27*c^41*d^16 - 2564218746215699966853120*a
^19*b^26*c^40*d^17 + 2593338871410901332787200*a^20*b^25*c^39*d^18 - 214606
5846150812380692480*a^21*b^24*c^38*d^19 + 1453625441569727022366720*a^22*b^
23*c^37*d^20 - 802881124954933087436800*a^23*b^22*c^36*d^21 + 3585819856061
39180482560*a^24*b^21*c^35*d^22 - 127660361818125316915200*a^25*b^20*c^34*d
^23 + 35417419405750750412800*a^26*b^19*c^33*d^24 - 7386837561454362624000*
a^27*b^18*c^32*d^25 + 1090533977896255488000*a^28*b^17*c^31*d^26 - 10169589
2037304320000*a^29*b^16*c^30*d^27 + 4508684868648960000*a^30*b^15*c^29*d^28
```

```
))*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838760*a*b^7*c^7*d^6 +
1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5*d^8 + 1519673350*a^4*b^
4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 247981500*a^6*b^2*c^2*d^11 - 47385
000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777216*a^12*c^13*d^12 - 201326592
*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d^2 - 3690987520*a^3*b^9*c^22*
d^3 + 8304721920*a^4*b^8*c^21*d^4 - 13287555072*a^5*b^7*c^20*d^5 + 15502147
584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c^18*d^7 + 8304721920*a^8*b^4*c^
17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107296256*a^10*b^2*c^15*d^10 - 2013
26592*a*b^11*c^24*d))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.468 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=681

$$\frac{b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{15/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^3}-$$

Rubi [**A**] time = 0.93, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{77a^2d^2-133abcd+32b^2c^2}{48ac^3x^{3/2}(bc-ad)^2}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{15/4}(bc-ad)^3}-\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{15/4}(bc-ad)^3}+\frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{15/4}(bc-ad)^3}+\frac{b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{15/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{15/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{d(19bc-11ad)}{16c^2x^{3/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{3/2}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 133*a*b*c*d + 77*a^2*d^2)/(48*a*c^3*(b*c - a*d)^2*x^(3/2)) -
 d/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) - (d*(19*b*c - 11*a*d))/(16*c^2*
(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + (b^(15/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/
4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2
 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])
/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(1
5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]
*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(15/4)*(b*c - a*d)^3) + (d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(1
5/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-11ad-11bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{32b^2c^2-13}{}\right.}{}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+d\right.}$$

$$= -\frac{\frac{32b^2c}{a}-133bd+\frac{77ad^2}{c}}{48c^2(bc-ad)^2x^{3/2}} - \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+d\right.}$$

Mathematica [A] time = 6.18, size = 701, normalized size = 1.03

$$\frac{b^{15/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{15/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{15/4}\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{15/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(ad-bc)^3} - \frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(ad-bc)^3} - \frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(ad-bc)^3} - \frac{d^{7/4}\left(77a^2d^2-210abcd+165b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(ad-bc)^3} + \frac{d^2\sqrt{x}(23bc-15ad)}{16c^3\left(c+dx^2\right)(bc-ad)^2} + \frac{d^2\sqrt{x}}{4c^2\left(c+dx^2\right)^2(bc-ad)} - \frac{2}{3ac^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -2/(3*a*c^3*x^(3/2)) + (d^2*Sqrt[x])/(4*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d
^2*(23*b*c - 15*a*d)*Sqrt[x])/(16*c^3*(b*c - a*d)^2*(c + d*x^2)) - (b^(15/4
)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt
[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*
Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(16
5*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4
)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(15/4)*(-(b*c) + a*d)^3) - (d^
(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[(Sqrt[2]*c^(1/4) + 2*
d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(15/4)*(-(b*c) + a*d)^3)
 + (b^(15/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(15/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(
165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(-(b*c) + a*d)^3) - (d^(7/4)*
(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.46, size = 457, normalized size = 0.67

$$-\frac{b^{15/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}a^{7/4}(ad-bc)^3}+\frac{b^{15/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{7/4}(ad-bc)^3}-\frac{\left(77a^2d^{15/4}-210abcd^{11/4}+165b^2c^2d^{7/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{\left(77a^2d^{15/4}-210abcd^{11/4}+165b^2c^2d^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3}+\frac{-32a^2c^2d^2-121a^2cd^3x^2-77a^2d^4x^4+64abc^3d+209abc^2d^2x^2+133abcd^3x^4-32b^2c^4-64b^2c^3dx^2-32b^2c^2d^2x^4}{48ac^3x^{3/2}\left(c+dx^2\right)^2(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*b^2*c^4 + 64*a*b*c^3*d - 32*a^2*c^2*d^2 - 64*b^2*c^3*d*x^2 + 209*a*b*c
^2*d^2*x^2 - 121*a^2*c*d^3*x^2 - 32*b^2*c^2*d^2*x^4 + 133*a*b*c*d^3*x^4 - 7
7*a^2*d^4*x^4)/(48*a*c^3*(-(b*c) + a*d)^2*x^(3/2)*(c + d*x^2)^2) - (b^(15/4
)*ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x
]])/(Sqrt[2]*a^(7/4)*(-(b*c) + a*d)^3) - ((165*b^2*c^2*d^(7/4) - 210*a*b*c*
d^(11/4) + 77*a^2*d^(15/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^3) + (b^(15/4)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(7/4)*(
-(b*c) + a*d)^3) + ((165*b^2*c^2*d^(7/4) - 210*a*b*c*d^(11/4) + 77*a^2*d^(1
5/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32
*Sqrt[2]*c^(15/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.71, size = 995, normalized size = 1.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -(a*b^3)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*
c*d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(1/4)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b
^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(1/4)*b^3
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*
sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b
^3)^(1/4)*b^3*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^
2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d
^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*
(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^
5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d
^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*
c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/64*(165*(c*d^3)^
(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*l
og(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 - 3*sqrt(2
)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) - 1/64*(165*
(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^
2*d^3)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 -
 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1
/16*(23*b*c*d^3*x^(5/2) - 15*a*d^4*x^(5/2) + 27*b*c^2*d^2*sqrt(x) - 19*a*c*
d^3*sqrt(x))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 2/3/(a
*c^3*x^(3/2))
```

maple [A] time = 0.03, size = 906, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 1/4/a^2*b^4/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1/2/a^2*b^4/(a
*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+1/2/a^2
*b^4/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-
```

```
15/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*a^2+19/8*d^4/c^2/(a*d-b*c)^3/
(d*x^2+c)^2*x^(5/2)*a*b-23/16*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(5/2)*b^2-19/
16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*a^2+23/8*d^3/c/(a*d-b*c)^3/(d*x^
2+c)^2*x^(1/2)*a*b-27/16*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(1/2)*b^2-77/64*d^4/
c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a
^2+105/32*d^3/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)+1)*a*b-165/64*d^2/c^2/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2-77/64*d^4/c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+105/32*d^3/c^3/(a*d-b*c)^3*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-165/64*d^2/c^2/(a
*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/
128*d^4/c^4/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/
2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+105/64*d^3
/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-165/128*d^2/c^2/(
a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2
))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-2/3/a/c^3/x^(3/2)
```

maxima [A] time = 2.62, size = 755, normalized size = 1.11

$$-\frac{32\,b^{2}c^{4}-64\,abc^{3}d+32\,a^{2}c^{2}d^{2}+\left(32\,b^{2}c^{2}d^{2}-133\,abcd^{3}+77\,a^{2}d^{4}\right)x^{4}+\left(64\,b^{2}c^{3}d-209\,abc^{2}d^{2}+121\,a^{2}cd^{3}\right)x^{2}}{48\left(\left(ab^{2}c^{5}d^{2}-2\,a^{2}bc^{4}d^{3}+a^{3}c^{3}d^{4}\right)x^{\frac{11}{2}}+2\left(ab^{2}c^{6}d-2\,a^{2}bc^{5}d^{2}+a^{3}c^{4}d^{3}\right)x^{\frac{7}{2}}+\left(ab^{2}c^{7}-2\,a^{2}bc^{6}d+a^{3}c^{5}d^{2}\right)x^{\frac{3}{2}}\right)}-\frac{\frac{2\,\sqrt{2}\,b^{4}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,b^{4}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{15}{4}}\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{15}{4}}\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{4\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)}+\frac{\frac{2\,\sqrt{2}\left(165\,b^{2}c^{2}d^{2}-210\,abcd^{3}+77\,a^{2}d^{4}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(165\,b^{2}c^{2}d^{2}-210\,abcd^{3}+77\,a^{2}d^{4}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(165\,b^{2}c^{2}d^{2}-210\,abcd^{3}+77\,a^{2}d^{4}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(165\,b^{2}c^{2}d^{2}-210\,abcd^{3}+77\,a^{2}d^{4}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^{3}c^{6}-3\,ab^{2}c^{5}d+3\,a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(32*b^2*c^4 - 64*a*b*c^3*d + 32*a^2*c^2*d^2 + (32*b^2*c^2*d^2 - 133*a
*b*c*d^3 + 77*a^2*d^4)*x^4 + (64*b^2*c^3*d - 209*a*b*c^2*d^2 + 121*a^2*c*d^
3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^(11/2) + 2*(a*b^
2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^(7/2) + (a*b^2*c^7 - 2*a^2*b*c^6
*d + a^3*c^5*d^2)*x^(3/2)) - 1/4*(2*sqrt(2)*b^4*arctan(1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^4*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + sqrt(2)*b^(15/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/a^(3/4) - sqrt(2)*b^(15/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/a^(3/4))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
 a^4*d^3) + 1/128*(2*sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)
*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt
(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(165*b^2*c^2*d^2
- 210*a*b*c*d^3 + 77*a^2*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
- 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
 + sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*log(sqrt(2)*c^(1/
4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(165*
b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt
```

```
(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*
a^2*b*c^4*d^2 - a^3*c^3*d^3)
```

mupad [B] time = 10.81, size = 44524, normalized size = 65.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] atan(((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d
^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150
*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^
13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12
- 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^
3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5
 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920
*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*
d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^
8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 1139499
9000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^
3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^
12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256
*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*
d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 1328755
5072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^
18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((
-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 85
87309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^
4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 38
3487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 20132
6592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c
^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 1550
2147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^
4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 -
201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(18446744073709551616*a^11*b^39*c^
68*d^4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*
a^13*b^37*c^66*d^6 - 47961534591644834201600*a^14*b^36*c^65*d^7 + 275778823
901957796659200*a^15*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^
63*d^9 + 4232993998288506144686080*a^17*b^33*c^62*d^10 - 119411640777996540
41845760*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^
12 - 46637619173392487079215104*a^20*b^30*c^59*d^13 + 436116065385578951333
64224*a^21*b^29*c^58*d^14 + 72781112360087761599856640*a^22*b^28*c^57*d^15
- 523234066593179210717593600*a^23*b^27*c^56*d^16 + 172375300102079718474 38
33600*a^24*b^26*c^55*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d^1
```

```
8 + 8727322757849829186700574720*a^26*b^24*c^53*d^19 - 15215326043975142249
374679040*a^27*b^23*c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c^5
1*d^21 - 30231538828274701475145318400*a^29*b^21*c^50*d^22 + 34870163031766
389952882933760*a^30*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*b^
19*c^48*d^24 + 31433146498544749041648926720*a^32*b^18*c^47*d^25 - 24575140
799491012895231180800*a^33*b^17*c^46*d^26 + 16850754961433442876234137600*a
^34*b^16*c^45*d^27 - 10105200492115418262179676160*a^35*b^15*c^44*d^28 + 52
78011312905736232783314944*a^36*b^14*c^43*d^29 - 23872483994059161661698211
84*a^37*b^13*c^42*d^30 + 927828632312674738870681600*a^38*b^12*c^41*d^31 -
306693733103726739901644800*a^39*b^11*c^40*d^32 + 8503807595944604606660608
0*a^40*b^10*c^39*d^33 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 355
1400405635812871372800*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b
^7*c^36*d^36 + 51111802530990496153600*a^44*b^6*c^35*d^37 - 335957723562733
3124096*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) + (-
(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 858
7309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4
*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383
487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326
592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^
24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502
147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4
*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 2
01326592*a*b^11*c^26*d))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4 - 1
06991156275153993728*a^14*b^37*c^70*d^5 + 14978756187852155912192*a^15*b^3
6*c^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7 + 8820160799048623
21508352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d^9
+ 18321125205332103390035968*a^19*b^32*c^65*d^10 - 630215452283771668681195
52*a^20*b^31*c^64*d^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12 -
490713180393588600090918912*a^22*b^29*c^62*d^13 + 1161438545048511890042388
480*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^15
+ 4997541469898172697285754880*a^25*b^26*c^59*d^16 - 9119889428539397211967
979520*a^26*b^25*c^58*d^17 + 15181544306461039744285409280*a^27*b^24*c^57*d
^18 - 22888317982577902576352624640*a^28*b^23*c^56*d^19 + 31049708276802113
763866050560*a^29*b^22*c^55*d^20 - 37706614244767692268335267840*a^30*b^21*
c^54*d^21 + 40833216619792283792163471360*a^31*b^20*c^53*d^22 - 39312168062
751093709382615040*a^32*b^19*c^52*d^23 + 33557805042801128843488788480*a^33
*b^18*c^51*d^24 - 25329188887155786370693201920*a^34*b^17*c^50*d^25 + 16851
463310911481777624186880*a^35*b^16*c^49*d^26 - 9843609097631363291959787520
*a^36*b^15*c^48*d^27 + 5023816147465636127472353280*a^37*b^14*c^47*d^28 - 2
226054577272365612261179392*a^38*b^13*c^46*d^29 + 8494197527189633260773703
68*a^39*b^12*c^45*d^30 - 276172923601113041340465152*a^40*b^11*c^44*d^31 +
75441341408208223215812608*a^41*b^10*c^43*d^32 - 16988052798101408932954112
*a^42*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*c^41*d^34 - 428198
505575496787427328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*b^6*c^
39*d^36 - 2816587235754527162368*a^46*b^5*c^38*d^37 + 88774955854727217152*
```

```
a^47*b^4*c^37*d^38)) - 11889503016258109440*a^9*b^38*c^56*d^7 + 21725364602
4352727040*a^10*b^37*c^55*d^8 - 1879766455667426066432*a^11*b^36*c^54*d^9 +
 10237150327374383939584*a^12*b^35*c^53*d^10 - 37711511320670913953792*a^13
*b^34*c^52*d^11 + 77353208427556875796480*a^14*b^33*c^51*d^12 + 12762723817
2719495249920*a^15*b^32*c^50*d^13 - 2130084466030987427446784*a^16*b^31*c^4
9*d^14 + 11885048527140028256616448*a^17*b^30*c^48*d^15 - 45690531361686842
972831744*a^18*b^29*c^47*d^16 + 135851929384595950057553920*a^19*b^28*c^46*
d^17 - 326376775711477371051704320*a^20*b^27*c^45*d^18 + 648353352496064059
760705536*a^21*b^26*c^44*d^19 - 1080394184249474617790431232*a^22*b^25*c^43
*d^20 + 1524725339928630029153468416*a^23*b^24*c^42*d^21 - 1834102420924176
937716285440*a^24*b^23*c^41*d^22 + 1888062742223171008426147840*a^25*b^22*c
^40*d^23 - 1666588213584359199850102784*a^26*b^21*c^39*d^24 + 1261562453800
014779376467968*a^27*b^20*c^38*d^25 - 817528072151542384572760064*a^28*b^19
*c^37*d^26 + 451847698934426396681830400*a^29*b^18*c^36*d^27 - 211721890947
778234390937600*a^30*b^17*c^35*d^28 + 83366248780838000977248256*a^31*b^16*
c^34*d^29 - 27241266624044306322685952*a^32*b^15*c^33*d^30 + 72575158008605
71589410816*a^33*b^14*c^32*d^31 - 1536699518639901947985920*a^34*b^13*c^31*
d^32 + 248859486128715197317120*a^35*b^12*c^30*d^33 - 289616420421725239377
92*a^36*b^11*c^29*d^34 + 2157438443758953693184*a^37*b^10*c^28*d^35 - 77302
354662372933632*a^38*b^9*c^27*d^36) + x^(1/2)*(30652624963790438400*a^9*b^3
7*c^51*d^9 - 507161613037259980800*a^10*b^36*c^50*d^10 + 477495696955061305
3440*a^11*b^35*c^49*d^11 - 34948190471081762488320*a^12*b^34*c^48*d^12 + 20
8409962786483628670976*a^13*b^33*c^47*d^13 - 990271368055602664177664*a^14*
b^32*c^46*d^14 + 3711631588079120800546816*a^15*b^31*c^45*d^15 - 1105079517
9720929846493184*a^16*b^30*c^44*d^16 + 26487755718620581216649216*a^17*b^29
*c^43*d^17 - 51805174836472540920020992*a^18*b^28*c^42*d^18 + 8361766320914
8864427720704*a^19*b^27*c^41*d^19 - 112350430315654120415952896*a^20*b^26*c
^40*d^20 + 126417217514830317658046464*a^21*b^25*c^39*d^21 - 11953790608112
8203174281216*a^22*b^24*c^38*d^22 + 95089864774620999552335872*a^23*b^23*c^
37*d^23 - 63545506634457987380412416*a^24*b^22*c^36*d^24 + 3552957884614677
4008070144*a^25*b^21*c^35*d^25 - 16501565732136655819636736*a^26*b^20*c^34*
d^26 + 6295808856090071441342464*a^27*b^19*c^33*d^27 - 19408479842499530818
84672*a^28*b^18*c^32*d^28 + 471738031694568778366976*a^29*b^17*c^31*d^29 -
87073083559063809163264*a^30*b^16*c^30*d^30 + 11476570419434950230016*a^31*
b^15*c^29*d^31 - 962765689885917446144*a^32*b^14*c^28*d^32 + 38651177331186
466816*a^33*b^13*c^27*d^33))*1i + (-(35153041*a^8*d^15 + 741200625*b^8*c^8*
d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a
^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12
 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^2
7 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b
^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 -
13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a
^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9
 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(3515
3041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 85873095
```

```
00*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*
d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 38348772
0*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a
^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^
3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 1550214758
4*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19
*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326
592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3
773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*
c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870
125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 167
77216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^2
5*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 1328755
5072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*
c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107
296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(18446
744073709551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12*b^38*c^67*d
^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 - 47961534591644834201600*a^
14*b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8 - 1212936383
169193658286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*a^17*b^33*c^
62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 271048693213330
56471040000*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^20*b^30*c^59*
d^13 + 43611606538557895133364224*a^21*b^29*c^58*d^14 + 727811123600877615
99856640*a^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*b^27*c^56*d^
16 + 1723753001020797184743833600*a^24*b^26*c^55*d^17 - 426943716736587281
4842183680*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^26*b^24*c^53
*d^19 - 15215326043975142249374679040*a^27*b^23*c^52*d^20 + 229626584632465
19625580544000*a^28*b^22*c^51*d^21 - 30231538828274701475145318400*a^29*b^2
1*c^50*d^22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23 - 353167182
38336158489724846080*a^31*b^19*c^48*d^24 + 31433146498544749041648926720*a^
32*b^18*c^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^46*d^26 + 168
50754961433442876234137600*a^34*b^16*c^45*d^27 - 1010520049211541826217967
6160*a^35*b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^14*c^43*d^29
 - 2387248399405916166169821184*a^37*b^13*c^42*d^30 + 92782863231267473887068
1600*a^38*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^11*c^40*d^32
 + 85038075959446046066606080*a^40*b^10*c^39*d^33 - 19409595119210898894356
480*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^37*d^35 - 500
844593983932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153600*a^44*b^6
*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 106807368762718683
136*a^46*b^4*c^33*d^39) - (-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 37
73385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c
^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 18701
25180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 1677
7216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25
*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555
```

072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c
^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 11072
96256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*(368934881474191
03232*a^13*b^38*c^71*d^4 - 1069911156275153993728*a^14*b^37*c^70*d^5 + 1497
875618785215591219 2*a^15*b^36*c^69*d^6 - 134999037738929532960768*a^16*b^35
*c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8 - 446563046345927787
08539392*a^18*b^33*c^66*d^9 + 18321125205332103390035968*a^19*b^32*c^65*d^1
0 - 63021545228377166868119552*a^20*b^31*c^64*d^11 + 187018029382071665408 6
06208*a^21*b^30*c^63*d^12 - 490713180393588600090918912*a^22*b^29*c^62*d^13
 + 1116143854504851189004238848 0*a^23*b^28*c^61*d^14 - 2512974056309066269 89
8833920*a^24*b^27*c^60*d^15 + 4997541469898172697285754880*a^25*b^26*c^59*d
^16 - 9119889428539397211967979520*a^26*b^25*c^58*d^17 + 151815443064610397
44285409280*a^27*b^24*c^57*d^18 - 22888317982577902576352624640*a^28*b^23*c
^56*d^19 + 31049708276802113763866050560*a^29*b^22*c^55*d^20 - 377066142447
67692268335267840*a^30*b^21*c^54*d^21 + 40833216619792283792163471360*a^31*
b^20*c^53*d^22 - 39312168062751093709382615040*a^32*b^19*c^52*d^23 + 335578
05042801128843488788480*a^33*b^18*c^51*d^24 - 25329188887155786370693201920
*a^34*b^17*c^50*d^25 + 16851463310911481777624186880*a^35*b^16*c^49*d^26 -
9843609097631363291959787520*a^36*b^15*c^48*d^27 + 502381614746563612747235
3280*a^37*b^14*c^47*d^28 - 2226054577272365612261179392*a^38*b^13*c^46*d^29
 + 849419752718963326077370368*a^39*b^12*c^45*d^30 - 276172923601113041340 4
65152*a^40*b^11*c^44*d^31 + 75441341408208223215812608*a^41*b^10*c^43*d^32
- 16988052798101408932954112*a^42*b^9*c^42*d^33 + 3070410975444256772063232
*a^43*b^8*c^41*d^34 - 428198505575496787427328*a^44*b^7*c^40*d^35 + 4325415
6088335077998592*a^45*b^6*c^39*d^36 - 2816587235754527162368*a^46*b^5*c^38*
d^37 + 88774955854727217152*a^47*b^4*c^37*d^38)) + 1188950301625810944 0*a^9
*b^38*c^56*d^7 - 217253646024352727040*a^10*b^37*c^55*d^8 + 187976645566742
6066432*a^11*b^36*c^54*d^9 - 10237150327374383939584*a^12*b^35*c^53*d^10 +
37711511320670913953792*a^13*b^34*c^52*d^11 - 77353208427556875796480*a^14*
b^33*c^51*d^12 - 127627238172719495249920*a^15*b^32*c^50*d^13 + 21300844660
3098742744678 4*a^16*b^31*c^49*d^14 - 11885048527140028256616448*a^17*b^30*c
^48*d^15 + 45690531361686842972831744*a^18*b^29*c^47*d^16 - 135851929384595
950057553920*a^19*b^28*c^46*d^17 + 326376775711477371051704320*a^20*b^27*c^
45*d^18 - 648353352496064059760705536*a^21*b^26*c^44*d^19 + 108039418424947
4617790431232*a^22*b^25*c^43*d^20 - 1524725339928630029153468416*a^23*b^24*
c^42*d^21 + 1834102420924176937716285440*a^24*b^23*c^41*d^22 - 188806274222
3171008426147840*a^25*b^22*c^40*d^23 + 1666588213584359199850102784*a^26*b^
21*c^39*d^24 - 1261562453800014779376467968*a^27*b^20*c^38*d^25 + 817528072
15154238457276006 4*a^28*b^19*c^37*d^26 - 451847698934426396681830400*a^29*b
^18*c^36*d^27 + 211721890947778234390937600*a^30*b^17*c^35*d^28 - 833662487
80838000977248256*a^31*b^16*c^34*d^29 + 27241266624044306322685952*a^32*b^1
5*c^33*d^30 - 7257515800860571589410816*a^33*b^14*c^32*d^31 + 1536699518639
901947985920*a^34*b^13*c^31*d^32 - 248859486128715197317120*a^35*b^12*c^30*
d^33 + 28961642042172523937792*a^36*b^11*c^29*d^34 - 2157438443758953693184
*a^37*b^10*c^28*d^35 + 773023546623729336 32*a^38*b^9*c^27*d^36) + x^(1/2)*(

```
30652624963790438400*a^9*b^37*c^51*d^9 - 50716161303725998080O*a^10*b^36*c^
50*d^10 + 477495696955061305344O*a^11*b^35*c^49*d^11 - 3494819047108176248
320*a^12*b^34*c^48*d^12 + 20840996278648362867097G*a^13*b^33*c^47*d^13 - 99
0271368055602664177664*a^14*b^32*c^46*d^14 + 3711631588079120800546816*a^15
*b^31*c^45*d^15 - 11050795179720929846493184*a^16*b^30*c^44*d^16 + 26487755
718620581216649216*a^17*b^29*c^43*d^17 - 51805174836472540920020992*a^18*b^
28*c^42*d^18 + 83617663209148864427720704*a^19*b^27*c^41*d^19 - 11235043031
5654120415952896*a^20*b^26*c^40*d^20 + 126417217514830317658046464*a^21*b^2
5*c^39*d^21 - 119537906081128203174281216*a^22*b^24*c^38*d^22 + 95089864774
620999552335872*a^23*b^23*c^37*d^23 - 63545506634457987380412416*a^24*b^22*
c^36*d^24 + 35529578846146774008070144*a^25*b^21*c^35*d^25 - 16501565732136
655819636736*a^26*b^20*c^34*d^26 + 6295808856090071441342464*a^27*b^19*c^33
*d^27 - 1940847984249953081884672*a^28*b^18*c^32*d^28 + 4717380316945687783
66976*a^29*b^17*c^31*d^29 - 87073083559063809163264*a^30*b^16*c^30*d^30 + 1
1476570419434950230016*a^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b^1
4*c^28*d^32 + 38651177331186466816*a^33*b^13*c^27*d^33))*1i)/((-(35153041*a
^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2
*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 -
 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*
b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*
c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 83
04721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*
b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 -
 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*
b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385
000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^
10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180
*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*
a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2
- 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a
^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d
^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256
*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15
 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^
6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 531766
6200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^1
4)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^
11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 830472192
0*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^2
1*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 369098
7520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^
26*d))^(3/4)*(x^(1/2)*(18446744073709551616*a^11*b^39*c^68*d^4 - 4796153459
16448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*a^13*b^37*c^66*d^6
- 47961534591644834201600*a^14*b^36*c^65*d^7 + 275778823901957796659200*a^1
5*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^63*d^9 + 4232993998
```

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288506144686080*a^17*b^33*c^62*d^10 - 11941164077799654041845760*a^18*b^32*
c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^12 - 46637619173392
487079215104*a^20*b^30*c^59*d^13 + 43611606538557895133364224*a^21*b^29*c^5
8*d^14 + 727811123600877615998566640*a^22*b^28*c^57*d^15 - 52323406659317921
0717593600*a^23*b^27*c^56*d^16 + 1723753001020797184743833600*a^24*b^26*c^5
5*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d^18 + 872732275784982
9186700574720*a^26*b^24*c^53*d^19 - 15215326043975142249374679040*a^27*b^23
*c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c^51*d^21 - 3023153882
8274701475145318400*a^29*b^21*c^50*d^22 + 34870163031766389952882933760*a^3
0*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*b^19*c^48*d^24 + 3143
3146498544749041648926720*a^32*b^18*c^47*d^25 - 24575140799491012895231180
800*a^33*b^17*c^46*d^26 + 16850754961433442876234137600*a^34*b^16*c^45*d^27
 - 10105200492115418262179676160*a^35*b^15*c^44*d^28 + 527801131290573623278
3314944*a^36*b^14*c^43*d^29 - 2387248399405916166169821184*a^37*b^13*c^42*d
^30 + 927828632312674738870681600*a^38*b^12*c^41*d^31 - 3066937331037267399
01644800*a^39*b^11*c^40*d^32 + 85038075959446046066606080*a^40*b^10*c^39*d^
33 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 3551400405635812871372
800*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b^7*c^36*d^36 + 5111
1802530990496153600*a^44*b^6*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^
34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) + (-(35153041*a^8*d^15
 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6
*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666
200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14
)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^1
1 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920
*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21
*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987
520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^2
6*d))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4 - 106991115627515399937
28*a^14*b^37*c^70*d^5 + 1497875618785215591219 2*a^15*b^36*c^69*d^6 - 134999
0377389295329607 68*a^16*b^35*c^68*d^7 + 882016079904862321508352*a^17*b^34*
c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d^9 + 18321125205332103
390035968*a^19*b^32*c^65*d^10 - 63021545228377166868119552*a^20*b^31*c^64*d
^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12 - 49071318039358860000
90918912*a^22*b^29*c^62*d^13 + 1161438545048511890042388480*a^23*b^28*c^61*
d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^15 + 49975414698981726
97285754880*a^25*b^26*c^59*d^16 - 9119889428539397211967979520*a^26*b^25*c^
58*d^17 + 15181544306461039744285409280*a^27*b^24*c^57*d^18 - 2288831798257
790257635262464 0*a^28*b^23*c^56*d^19 + 31049708276802113763866050560*a^29*b
^22*c^55*d^20 - 37706614244767692268335267840*a^30*b^21*c^54*d^21 + 4083321
6619792283792163471360*a^31*b^20*c^53*d^22 - 39312168062751093709382615040*
a^32*b^19*c^52*d^23 + 33557805042801128843488788480*a^33*b^18*c^51*d^24 - 2
5329188887155786370693201920*a^34*b^17*c^50*d^25 + 16851463310911481777624 1
86880*a^35*b^16*c^49*d^26 - 9843609097631363291959787520*a^36*b^15*c^48*d^2
7 + 5023816147465636127472353280*a^37*b^14*c^47*d^28 - 2226054577272365 6122
```

```
61179392*a^38*b^13*c^46*d^29 + 849419752718963326077370368*a^39*b^12*c^45*d
^30 - 276172923601113041340465152*a^40*b^11*c^44*d^31 + 7544134140820822321
5812608*a^41*b^10*c^43*d^32 - 16988052798101408932954112*a^42*b^9*c^42*d^33
 + 3070410975444256772063232*a^43*b^8*c^41*d^34 - 428198505575496787427328*
a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*b^6*c^39*d^36 - 281658723
5754527162368*a^46*b^5*c^38*d^37 + 88774955854727217152*a^47*b^4*c^37*d^38)
) - 1188950301625810944 0*a^9*b^38*c^56*d^7 + 217253646024352727040*a^10*b^3
7*c^55*d^8 - 1879766455667426066432*a^11*b^36*c^54*d^9 + 102371503273743839
39584*a^12*b^35*c^53*d^10 - 37711511320670913953792*a^13*b^34*c^52*d^11 + 7
7353208427556875796480*a^14*b^33*c^51*d^12 + 127627238172719495249920*a^15*
b^32*c^50*d^13 - 2130084466030987427446784*a^16*b^31*c^49*d^14 + 1188504852
7140028256616448*a^17*b^30*c^48*d^15 - 45690531361686842972831744*a^18*b^29
*c^47*d^16 + 135851929384595950057553920*a^19*b^28*c^46*d^17 - 326376775711
477371051704320*a^20*b^27*c^45*d^18 + 648353352496064059760705536*a^21*b^26
*c^44*d^19 - 1080394184249474617790431232*a^22*b^25*c^43*d^20 + 15247253399
28630029153468416*a^23*b^24*c^42*d^21 - 1834102420924176937716285440*a^24*b
^23*c^41*d^22 + 1888062742223171008426147840*a^25*b^22*c^40*d^23 - 16665882
13584359199850102784*a^26*b^21*c^39*d^24 + 1261562453800014779376467968*a^2
7*b^20*c^38*d^25 - 817528072151542384572760064*a^28*b^19*c^37*d^26 + 451847
698934426396681830400*a^29*b^18*c^36*d^27 - 211721890947778234390937600*a^3
0*b^17*c^35*d^28 + 83366248780838000977248256*a^31*b^16*c^34*d^29 - 2724126
6624044306322685952*a^32*b^15*c^33*d^30 + 7257515800860571589410816*a^33*b^
14*c^32*d^31 - 1536699518639901947985920*a^34*b^13*c^31*d^32 + 248859486128
715197317120*a^35*b^12*c^30*d^33 - 28961642042172523937792*a^36*b^11*c^29*d
^34 + 2157438443758953693184*a^37*b^10*c^28*d^35 - 77302354662372933632*a^3
8*b^9*c^27*d^36) + x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 507161
613037259980800*a^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*c^49
*d^11 - 34948190471081762488320*a^12*b^34*c^48*d^12 + 208409962786483628670
976*a^13*b^33*c^47*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14 + 37
11631588079120800546816*a^15*b^31*c^45*d^15 - 11050795179720929846493184*a^
16*b^30*c^44*d^16 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 518051
74836472540920020992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a^19*
b^27*c^41*d^19 - 112350430315654120415952896*a^20*b^26*c^40*d^20 + 12641721
7514830317658046464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a^22*
b^24*c^38*d^22 + 95089864774620999552335872*a^23*b^23*c^37*d^23 - 635455066
34457987380412416*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25*b^2
1*c^35*d^25 - 16501565732136655819636736*a^26*b^20*c^34*d^26 + 629580885609
0071441342464*a^27*b^19*c^33*d^27 - 1940847984249953081884672*a^28*b^18*c^3
2*d^28 + 471738031694568778366976*a^29*b^17*c^31*d^29 - 870730835590638091 6
3264*a^30*b^16*c^30*d^30 + 11476570419434950230016*a^31*b^15*c^29*d^31 - 96
2765689885917446144*a^32*b^14*c^28*d^32 + 38651177331186466816*a^33*b^13*c^
27*d^33)) - (-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7
*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636
798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*
c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15
```

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*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987
520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^
22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304
721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2
*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200
625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 1
1394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*
b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777
216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107
296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8
*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 1
3287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*
b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1
/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^
8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*
a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^1
3 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 -
 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3
*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5
+ 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*
a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d
^10 - 201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(18446744073709551616*a^11*b
^39*c^68*d^4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 599519182395560427
75200*a^13*b^37*c^66*d^6 - 479615345916448342016*a^14*b^36*c^65*d^7 + 275
7788239019577966592000*a^15*b^35*c^64*d^8 - 121293638316919365828608*a^16*b
^34*c^63*d^9 + 42329939982885061446860*a^17*b^33*c^62*d^10 - 119411640777
9965404184576*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c
^60*d^12 - 4663761917339248707921510*a^20*b^30*c^59*d^13 + 43611606538557
895133364224*a^21*b^29*c^58*d^14 + 72781112360087761599856640*a^22*b^28*c^57
*d^15 - 52323406659317921071759360*a^23*b^27*c^56*d^16 + 1723753001020797
184743833600*a^24*b^26*c^55*d^17 - 42694371673658728148421836*a^25*b^25*c^
54*d^18 + 87273227578498291867005747*a^26*b^24*c^53*d^19 - 1521532604397
5142249374679040*a^27*b^23*c^52*d^20 + 22962658463246519625580544000*a^28*b^
22*c^51*d^21 - 30231538828274701475145318400*a^29*b^21*c^50*d^22 + 34870163
03176638995288293376*a^30*b^20*c^49*d^23 - 35316718238336158489724846080*a
^31*b^19*c^48*d^24 + 31433146498544749041648926720*a^32*b^18*c^47*d^25 - 24
57514079949101289523118080*a^33*b^17*c^46*d^26 + 1685075496143344287623413
7600*a^34*b^16*c^45*d^27 - 10105200492115418262179676160*a^35*b^15*c^44*d^2
8 + 5278011312905736232783314944*a^36*b^14*c^43*d^29 - 23872483994059161661
69821184*a^37*b^13*c^42*d^30 + 927828632312674738870681600*a^38*b^12*c^41*d
^31 - 306693733103726739901644800*a^39*b^11*c^40*d^32 + 850380759594460460
6606080*a^40*b^10*c^39*d^33 - 19409595119210898894356480*a^41*b^9*c^38*d^34
 + 3551400405635812871372800*a^42*b^8*c^37*d^35 - 500844593983932480880640*
a^43*b^7*c^36*d^36 + 51111802530990496153600*a^44*b^6*c^35*d^37 - 335957723
5627333124096*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39
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```
) - (-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8
 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a
^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13
 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 -
201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*
b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 +
 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a
^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^
10 - 201326592*a*b^11*c^26*d))^(1/4)*(368934881474191032 32*a^13*b^38*c^71*d
^4 - 106991115627515399 3728*a^14*b^37*c^70*d^5 + 149787561878521559 12192*a^
15*b^36*c^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7 + 8820160799
04862321508352*a^17*b^34*c^67*d^8 - 446563046345927870 8539392*a^18*b^33*c^6
6*d^9 + 1832112520533210339 0035968*a^19*b^32*c^65*d^10 - 630215452283771668
68119552*a^20*b^31*c^64*d^11 + 1870180293820716654 08606208*a^21*b^30*c^63*d
^12 - 4907131803935886000 90918912*a^22*b^29*c^62*d^13 + 116143854504851189 0
042388480*a^23*b^28*c^61*d^14 - 251297405630906626989 8833920*a^24*b^27*c^60
*d^15 + 4997541469898172697 285754880*a^25*b^26*c^59*d^16 - 9119889428539397
21196797 9520*a^26*b^25*c^58*d^17 + 15181544306461039744 285409280*a^27*b^24*
c^57*d^18 - 2288831798257790257635 2624640*a^28*b^23*c^56*d^19 + 31049708276
80211376386605 0560*a^29*b^22*c^55*d^20 - 3770661424476769226833 5267840*a^30
*b^21*c^54*d^21 + 408332166197922837921 63471360*a^31*b^20*c^53*d^22 - 39312
16806275109370938 2615040*a^32*b^19*c^52*d^23 + 33557805042801128843 48878848
0*a^33*b^18*c^51*d^24 - 2532918888715578637 0693201920*a^34*b^17*c^50*d^25 +
 1685146331091148177 7624186880*a^35*b^16*c^49*d^26 - 984360909763136329 1959
787520*a^36*b^15*c^48*d^27 + 50238161474656361274 72353280*a^37*b^14*c^47*d^
28 - 2226054577272365612 261179392*a^38*b^13*c^46*d^29 + 84941975271896332 60
77370368*a^39*b^12*c^45*d^30 - 276172923601113041340465152*a^40*b^11*c^44*d
^31 + 754413414082082232 15812608*a^41*b^10*c^43*d^32 - 169880527981014089 32
954112*a^42*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*c^41*d^34 -
4281985055754967874 27328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*
b^6*c^39*d^36 - 28165872357545271 62368*a^46*b^5*c^38*d^37 + 887749558547272
17152*a^47*b^4*c^37*d^38)) + 1188950301625810944 0*a^9*b^38*c^56*d^7 - 21725
36460243527270 40*a^10*b^37*c^55*d^8 + 1879766455667426066432*a^11*b^36*c^54
*d^9 - 10237150327374383939584*a^12*b^35*c^53*d^10 + 377115113206709139 5379
2*a^13*b^34*c^52*d^11 - 77353208427556875796480*a^14*b^33*c^51*d^12 - 12762
72381727194952 49920*a^15*b^32*c^50*d^13 + 213008446603098742 7446784*a^16*b^
31*c^49*d^14 - 1188504852714002825 6616448*a^17*b^30*c^48*d^15 + 45690531361
686842972831744*a^18*b^29*c^47*d^16 - 135851929384595950057553920*a^19*b^28
*c^46*d^17 + 326376775711477371051704320*a^20*b^27*c^45*d^18 - 648353352496
064059760705536*a^21*b^26*c^44*d^19 + 1080394184249474617790431232*a^22*b^2
5*c^43*d^20 - 1524725339928630029153468416*a^23*b^24*c^42*d^21 + 1834102420
924176937716285440*a^24*b^23*c^41*d^22 - 1888062742223171008426147840*a^25*
b^22*c^40*d^23 + 1666588213584359199850102784*a^26*b^21*c^39*d^24 - 1261562
453800014779376467968*a^27*b^20*c^38*d^25 + 817528072151542384572760064*a^2
8*b^19*c^37*d^26 - 451847698934426396681830400*a^29*b^18*c^36*d^27 + 211721
```

```
890947778234390937600*a^30*b^17*c^35*d^28 - 833662487808380009772482566*a^31
*b^16*c^34*d^29 + 272412666240443063226859522*a^32*b^15*c^33*d^30 - 72575158
00860571589410816*a^33*b^14*c^32*d^31 + 15366995186399019479859200*a^34*b^13
*c^31*d^32 - 24885948612871519731712000*a^35*b^12*c^30*d^33 + 289616420421725
23937792*a^36*b^11*c^29*d^34 - 215743844375895369318400*a^37*b^10*c^28*d^35 +
 7730235466237293363200*a^38*b^9*c^27*d^36) + x^(1/2)*(30652624963790438400*a
^9*b^37*c^51*d^9 - 507161613037259980800*a^10*b^36*c^50*d^10 + 477495696955
0613053440*a^11*b^35*c^49*d^11 - 3494819047108176248832000*a^12*b^34*c^48*d^1
2 + 20840996278648362867097600*a^13*b^33*c^47*d^13 - 99027136805560266417766400
*a^14*b^32*c^46*d^14 + 371163158807912080054681600*a^15*b^31*c^45*d^15 - 1105
07951797209298464931840*a^16*b^30*c^44*d^16 + 264877557186205812166492160*a^1
7*b^29*c^43*d^17 - 518051748364725409200209920*a^18*b^28*c^42*d^18 + 8361766
32091488644277207040*a^19*b^27*c^41*d^19 - 112350430315654120415952896*a^20*
b^26*c^40*d^20 + 126417217514830317658046464*a^21*b^25*c^39*d^21 - 11953790
60811282031742812160*a^22*b^24*c^38*d^22 + 950898647746209995523358720*a^23*b
^23*c^37*d^23 - 635455066344579873804124160*a^24*b^22*c^36*d^24 + 3552957884
614677400807014400*a^25*b^21*c^35*d^25 - 165015657321366558196367360*a^26*b^20
*c^34*d^26 + 629580885609007144134246400*a^27*b^19*c^33*d^27 - 19408479842499
53081884672*a^28*b^18*c^32*d^28 + 471738031694568778366976*a^29*b^17*c^31*d
^29 - 870730835590638091632640*a^30*b^16*c^30*d^30 + 114765704194349502300160
*a^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b^14*c^28*d^32 + 38651177
331186466816*a^33*b^13*c^27*d^33))))*(-(35153041*a^8*d^15 + 741200625*b^8*c
^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 1139499900
0*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d
^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*
c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^
2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4
 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 1328755507
2*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*
d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*2i -
2*atan(((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7
*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 96367981
50*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*
d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^1
2 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*
a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d
^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 83047219
20*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^1
7*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*
b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394
999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*
c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*
b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 11072962
56*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^2
3*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287
```

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555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*
c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*
((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 +
8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*
b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 -
383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201
326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9
*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15
502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*
b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10
- 201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(18446744073709551616*a^11*b^39*
c^68*d^4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 599519182395560427520
0*a^13*b^37*c^66*d^6 - 47961534591644834201600*a^14*b^36*c^65*d^7 + 2757788
23901957796659200*a^15*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*
c^63*d^9 + 4232993998288506144686080*a^17*b^33*c^62*d^10 - 1194116407779965
4041845760*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*
d^12 - 46637619173392487079215104*a^20*b^30*c^59*d^13 + 4361160653855789513
3364224*a^21*b^29*c^58*d^14 + 72781112360087761599856640*a^22*b^28*c^57*d^1
5 - 523234066593179210717593600*a^23*b^27*c^56*d^16 + 172375300102079718474
3833600*a^24*b^26*c^55*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d
^18 + 8727322757849829186700574720*a^26*b^24*c^53*d^19 - 152153260439751422
49374679040*a^27*b^23*c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c
^51*d^21 - 30231538828274701475145318400*a^29*b^21*c^50*d^22 + 348701630317
66389952882933760*a^30*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*
b^19*c^48*d^24 + 31433146498544749041648926720*a^32*b^18*c^47*d^25 - 245751
40799491012895231180800*a^33*b^17*c^46*d^26 + 16850754961433442876234137600
*a^34*b^16*c^45*d^27 - 10105200492115418262179676160*a^35*b^15*c^44*d^28 +
5278011312905736232783314944*a^36*b^14*c^43*d^29 - 238724839940591616616982
1184*a^37*b^13*c^42*d^30 + 927828632312674738870681600*a^38*b^12*c^41*d^31
- 306693733103726739901644800*a^39*b^11*c^40*d^32 + 85038075959446046066606
080*a^40*b^10*c^39*d^33 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 3
551400405635812871372800*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43
*b^7*c^36*d^36 + 51111802530990496153600*a^44*b^6*c^35*d^37 - 3359577235627
333124096*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) -
(-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8
587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b
^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 3
83487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 2013
26592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*
c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 155
02147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b
^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 -
 201326592*a*b^11*c^26*d))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4 -
 1069911156275153993728*a^14*b^37*c^70*d^5 + 14978756187852155912192*a^15*b
^36*c^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7 + 88201607990486
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2321508352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d^
9 + 18321125205332103390035968*a^19*b^32*c^65*d^10 - 630215452283771668681l
9552*a^20*b^31*c^64*d^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12
- 490713180393588600090918912*a^22*b^29*c^62*d^13 + 11614385450485118900423
88480*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^1
5 + 4997541469898172697285754880*a^25*b^26*c^59*d^16 - 911988942853939721l9
67979520*a^26*b^25*c^58*d^17 + 15181544306461039744285409280*a^27*b^24*c^57
*d^18 - 22888317982577902576352624640*a^28*b^23*c^56*d^19 + 310497082768021
13763866050560*a^29*b^22*c^55*d^20 - 37706614244767692268335267840*a^30*b^2
1*c^54*d^21 + 40833216619792283792163471360*a^31*b^20*c^53*d^22 - 393121680
62751093709382615040*a^32*b^19*c^52*d^23 + 33557805042801128843488788480*a^
33*b^18*c^51*d^24 - 25329188887155786370693201920*a^34*b^17*c^50*d^25 + 168
51463310911481777624186880*a^35*b^16*c^49*d^26 - 98436090976313632919597875
20*a^36*b^15*c^48*d^27 + 5023816147465636127472353280*a^37*b^14*c^47*d^28 -
 2226054577272365612261179392*a^38*b^13*c^46*d^29 + 84941975271896332607737
0368*a^39*b^12*c^45*d^30 - 276172923601113041340465152*a^40*b^11*c^44*d^31
+ 75441341408208223215812608*a^41*b^10*c^43*d^32 - 169880527981014089329541
12*a^42*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*c^41*d^34 - 4281
98505575496787427328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*b^6*
c^39*d^36 - 2816587235754527162368*a^46*b^5*c^38*d^37 + 8877495585472721715
2*a^47*b^4*c^37*d^38)*1i)*1i - 11889503016258109440*a^9*b^38*c^56*d^7 + 217
253646024352727040*a^10*b^37*c^55*d^8 - 1879766455667426066432*a^11*b^36*c^
54*d^9 + 10237150327374383939584*a^12*b^35*c^53*d^10 - 37711511320670913953
792*a^13*b^34*c^52*d^11 + 77353208427556875796480*a^14*b^33*c^51*d^12 + 127
627238172719495249920*a^15*b^32*c^50*d^13 - 2130084466030987427446784*a^16*
b^31*c^49*d^14 + 11885048527140028256616448*a^17*b^30*c^48*d^15 - 456905313
61686842972831744*a^18*b^29*c^47*d^16 + 135851929384595950057553920*a^19*b^
28*c^46*d^17 - 326376775711477371051704320*a^20*b^27*c^45*d^18 + 6483533524
96064059760705536*a^21*b^26*c^44*d^19 - 1080394184249474617790431232*a^22*b
^25*c^43*d^20 + 1524725339928630029153468416*a^23*b^24*c^42*d^21 - 18341024
20924176937716285440*a^24*b^23*c^41*d^22 + 1888062742223171008426147840*a^2
5*b^22*c^40*d^23 - 1666588213584359199850102784*a^26*b^21*c^39*d^24 + 12615
62453800014779376467968*a^27*b^20*c^38*d^25 - 817528072151542384572760064*a
^28*b^19*c^37*d^26 + 451847698934426396681830400*a^29*b^18*c^36*d^27 - 2117
21890947778234390937600*a^30*b^17*c^35*d^28 + 83366248780838000977248256*a^
31*b^16*c^34*d^29 - 27241266624044306322685952*a^32*b^15*c^33*d^30 + 725751
5800860571589410816*a^33*b^14*c^32*d^31 - 1536699518639901947985920*a^34*b^
13*c^31*d^32 + 248859486128715197317120*a^35*b^12*c^30*d^33 - 2896164204217
2523937792*a^36*b^11*c^29*d^34 + 2157438443758953693184*a^37*b^10*c^28*d^35
 - 77302354662372933632*a^38*b^9*c^27*d^36)*1i - x^(1/2)*(30652624963790438
400*a^9*b^37*c^51*d^9 - 507161613037259980800*a^10*b^36*c^50*d^10 + 4774956
969550613053440*a^11*b^35*c^49*d^11 - 34948190471081762488320*a^12*b^34*c^4
8*d^12 + 208409962786483628670976*a^13*b^33*c^47*d^13 - 9902713680556026641
77664*a^14*b^32*c^46*d^14 + 3711631588079120800546816*a^15*b^31*c^45*d^15 -
 11050795179720929846493184*a^16*b^30*c^44*d^16 + 2648775571862058121664921
```

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6*a^17*b^29*c^43*d^17 - 5180517483647254092002092*a^18*b^28*c^42*d^18 + 83
617663209148864427720704*a^19*b^27*c^41*d^19 - 112350430315654120415952896*
a^20*b^26*c^40*d^20 + 126417217514830317658046464*a^21*b^25*c^39*d^21 - 119
537906081128203174281216*a^22*b^24*c^38*d^22 + 95089864774620999552335872*a
^23*b^23*c^37*d^23 - 63545506634457987380412416*a^24*b^22*c^36*d^24 + 35529
578846146774008070144*a^25*b^21*c^35*d^25 - 16501565732136655819636736*a^26
*b^20*c^34*d^26 + 6295808856090071441342464*a^27*b^19*c^33*d^27 - 194084798
4249953081884672*a^28*b^18*c^32*d^28 + 471738031694568778366976*a^29*b^17*c
^31*d^29 - 87073083559063809163264*a^30*b^16*c^30*d^30 + 114765704194349502
30016*a^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b^14*c^28*d^32 + 386
51177331186466816*a^33*b^13*c^27*d^33)) + (-(35153041*a^8*d^15 + 741200625*
b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394
999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*
c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*
b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 11072962
56*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^2
3*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287
555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*
c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*
((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 +
8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*
b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 -
383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201
326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9
*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15
502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*
b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10
- 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8
*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*
a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^1
2 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^
27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*
b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 -
 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*
a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^
9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2
)*(18446744073709551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12*b^3
8*c^67*d^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 - 479615345916448342
01600*a^14*b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8 - 12
12936383169193658286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*a^17
*b^33*c^62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 27104869
321333056471040000*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^20*b^
30*c^59*d^13 + 43611606538557895133364224*a^21*b^29*c^58*d^14 + 72781112360
087761599856640*a^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*b^27
*c^56*d^16 + 1723753001020797184743833600*a^24*b^26*c^55*d^17 - 42694371673
```

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65872814842183680*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^26*b
^24*c^53*d^19 - 15215326043975142249374679040*a^27*b^23*c^52*d^20 + 2296265
8463246519625580544000*a^28*b^22*c^51*d^21 - 30231538828274701475145318400*
a^29*b^21*c^50*d^22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23 - 3
5316718238336158489724846080*a^31*b^19*c^48*d^24 + 314331464985447490416489
26720*a^32*b^18*c^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^46*d^
26 + 16850754961433442876234137600*a^34*b^16*c^45*d^27 - 101052004921154182
62179676160*a^35*b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^14*c^
43*d^29 - 2387248399405916166169821184*a^37*b^13*c^42*d^30 + 92782863231267
4738870681600*a^38*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^11*c
^40*d^32 + 85038075959446046066606080*a^40*b^10*c^39*d^33 - 194095951192108
98894356480*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^37*d^
35 - 500844593983932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153600*
a^44*b^6*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 1068073687
62718683136*a^46*b^4*c^33*d^39) + (-(35153041*a^8*d^15 + 741200625*b^8*c^8*
d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a
^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12
 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^2
7 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b
^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 -
13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a
^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9
 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*(3689348
8147419103232*a^13*b^38*c^71*d^4 - 1069911156275153993728*a^14*b^37*c^70*d^
5 + 14978756187852155912192*a^15*b^36*c^69*d^6 - 134999037738929532960768*a
^16*b^35*c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8 - 446563046
3459278708539392*a^18*b^33*c^66*d^9 + 18321125205332103390035968*a^19*b^32*
c^65*d^10 - 63021545228377166868119552*a^20*b^31*c^64*d^11 + 18701802938207
1665408606208*a^21*b^30*c^63*d^12 - 490713180393588600090918912*a^22*b^29*c
^62*d^13 + 1161438545048511890042388480*a^23*b^28*c^61*d^14 - 2512974056309
066269898833920*a^24*b^27*c^60*d^15 + 4997541469898172697285754880*a^25*b^2
6*c^59*d^16 - 9119889428539397211967979520*a^26*b^25*c^58*d^17 + 1518154430
6461039744285409280*a^27*b^24*c^57*d^18 - 22888317982577902576352624640*a^2
8*b^23*c^56*d^19 + 31049708276802113763866050560*a^29*b^22*c^55*d^20 - 3770
6614244767692268335267840*a^30*b^21*c^54*d^21 + 408332166197922837921634713
60*a^31*b^20*c^53*d^22 - 39312168062751093709382615040*a^32*b^19*c^52*d^23
+ 33557805042801128843488788480*a^33*b^18*c^51*d^24 - 253291888871557863706
93201920*a^34*b^17*c^50*d^25 + 16851463310911481777624186880*a^35*b^16*c^49
*d^26 - 9843609097631363291959787520*a^36*b^15*c^48*d^27 + 5023816147465636
127472353280*a^37*b^14*c^47*d^28 - 2226054577272365612261179392*a^38*b^13*c
^46*d^29 + 849419752718963326077370368*a^39*b^12*c^45*d^30 - 27617292360111
3041340465152*a^40*b^11*c^44*d^31 + 75441341408208223215812608*a^41*b^10*c^
43*d^32 - 16988052798101408932954112*a^42*b^9*c^42*d^33 + 30704109754442567
72063232*a^43*b^8*c^41*d^34 - 428198505575496787427328*a^44*b^7*c^40*d^35 +
 43254156088335077998592*a^45*b^6*c^39*d^36 - 2816587235754527162368*a^46*b
```

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^5*c^38*d^37 + 887749558547272171152*a^47*b^4*c^37*d^38)*1i)*1i + 1188950301
6258109440*a^9*b^38*c^56*d^7 - 21725364602435272704 0*a^10*b^37*c^55*d^8 + 1
87976645566742606643 2*a^11*b^36*c^54*d^9 - 102371503273743839395 84*a^12*b^3
5*c^53*d^10 + 37711511320670913953792*a^13*b^34*c^52*d^11 - 773532084275568
75796480*a^14*b^33*c^51*d^12 - 127627238172719495249920*a^15*b^32*c^50*d^13
 + 213008446603098742744 6784*a^16*b^31*c^49*d^14 - 118850485271400282566164
48*a^17*b^30*c^48*d^15 + 456905313616868429728 31744*a^18*b^29*c^47*d^16 - 1
358519293845959500575539 20*a^19*b^28*c^46*d^17 + 326376775711477371051704 32
0*a^20*b^27*c^45*d^18 - 64835335249606405976070 5536*a^21*b^26*c^44*d^19 + 1
080394184249474617790431232*a^22*b^25*c^43*d^20 - 152472533992863002915346 8
416*a^23*b^24*c^42*d^21 + 1834102420924176937716285440*a^24*b^23*c^41*d^22
- 1888062742223171008426147840*a^25*b^22*c^40*d^23 + 16665882135843591998 50
102784*a^26*b^21*c^39*d^24 - 1261562453800014779376467968*a^27*b^20*c^38*d^
25 + 817528072151542384572760064*a^28*b^19*c^37*d^26 - 451847698934426396 68
1830400*a^29*b^18*c^36*d^27 + 211721890947778234390937600*a^30*b^17*c^35*d^
28 - 83366248780838000977248256*a^31*b^16*c^34*d^29 + 27241266624044306322 6
85952*a^32*b^15*c^33*d^30 - 7257515800860571589410816*a^33*b^14*c^32*d^31 +
 1536699518639901947985920*a^34*b^13*c^31*d^32 - 248859486128715197317120*a
^35*b^12*c^30*d^33 + 28961642042172523937792*a^36*b^11*c^29*d^34 - 21574384
43758953693184*a^37*b^10*c^28*d^35 + 77302354662372933632*a^38*b^9*c^27*d^3
6)*1i - x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 50716161303725998
0800*a^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*c^49*d^11 - 349
48190471081762488320*a^12*b^34*c^48*d^12 + 208409962786483628670976*a^13*b^
33*c^47*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14 + 3711631588079
120800546816*a^15*b^31*c^45*d^15 - 11050795179720929846493184*a^16*b^30*c^4
4*d^16 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 51805174836472540
920020992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a^19*b^27*c^41*d
^19 - 112350430315654120415952896*a^20*b^26*c^40*d^20 + 126417217514830317 6
58046464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a^22*b^24*c^38*d
^22 + 95089864774620999552335872*a^23*b^23*c^37*d^23 - 635455066344579873 80
412416*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25*b^21*c^35*d^25
 - 16501565732136655819636736*a^26*b^20*c^34*d^26 + 629580885609007144134 24
64*a^27*b^19*c^33*d^27 - 1940847984249953081884672*a^28*b^18*c^32*d^28 + 47
1738031694568778366976*a^29*b^17*c^31*d^29 - 87073083559063809163264*a^30*b
^16*c^30*d^30 + 11476570419434950230016*a^31*b^15*c^29*d^31 - 962765689885 9
17446144*a^32*b^14*c^28*d^32 + 38651177331186466816*a^33*b^13*c^27*d^33)))/
((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 +
8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*
b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 -
383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201
326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9
*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15
502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*
b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10
- 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8
```

```
*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*
a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^1
2 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^
27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*
b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 -
 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*
a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^
9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(351
53041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309
500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4
*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 3834877
20*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*
a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d
^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 155021475
84*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^1
9*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 20132
6592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(18446744073709551616*a^11*b^39*c^68*d^
4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*a^13*
b^37*c^66*d^6 - 47961534591644834201600*a^14*b^36*c^65*d^7 + 27577882390195
7796659200*a^15*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^63*d^
9 + 4232993998288506144686080*a^17*b^33*c^62*d^10 - 11941164077799654041845
760*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^12 -
46637619173392487079215104*a^20*b^30*c^59*d^13 + 43611606538557895133364224
*a^21*b^29*c^58*d^14 + 72781112360087761599856640*a^22*b^28*c^57*d^15 - 523
234066593179210717593600*a^23*b^27*c^56*d^16 + 1723753001020797184743833600
*a^24*b^26*c^55*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d^18 + 8
727322757849829186700574720*a^26*b^24*c^53*d^19 - 1521532604397514224937467
9040*a^27*b^23*c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c^51*d^2
1 - 30231538828274701475145318400*a^29*b^21*c^50*d^22 + 3487016303176638995
2882933760*a^30*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*b^19*c^
48*d^24 + 31433146498544749041648926720*a^32*b^18*c^47*d^25 - 2457514079949
1012895231180800*a^33*b^17*c^46*d^26 + 16850754961433442876234137600*a^34*b
^16*c^45*d^27 - 10105200492115418262179676160*a^35*b^15*c^44*d^28 + 5278011
312905736232783314944*a^36*b^14*c^43*d^29 - 2387248399405916166169821184*a^
37*b^13*c^42*d^30 + 927828632312674738870681600*a^38*b^12*c^41*d^31 - 30669
3733103726739901644800*a^39*b^11*c^40*d^32 + 85038075959446046066606080*a^4
0*b^10*c^39*d^33 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 35514004
05635812871372800*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b^7*c^
36*d^36 + 51111802530990496153600*a^44*b^6*c^35*d^37 - 33595772356273331240
96*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) - (-(3515
3041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 85873095
00*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*
d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 38348772
0*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a
^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^
```

```
3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 1550214758
4*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19
*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326
592*a*b^11*c^26*d))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4 - 106991
1156275153993728*a^14*b^37*c^70*d^5 + 1497875618785215591 2192*a^15*b^36*c^6
9*d^6 - 13499903773892953296 0768*a^16*b^35*c^68*d^7 + 882016079904862321508
352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d^9 + 183
2112520533210339003 5968*a^19*b^32*c^65*d^10 - 63021545228377166868119552*a^
20*b^31*c^64*d^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12 - 49071
3180393588600090918912*a^22*b^29*c^62*d^13 + 1161438545048511890042388480*a
^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^15 + 499
7541469898172697285754880*a^25*b^26*c^59*d^16 - 911988942853939721196797952
0*a^26*b^25*c^58*d^17 + 15181544306461039744285409280*a^27*b^24*c^57*d^18 -
 22888317982577902576352624640*a^28*b^23*c^56*d^19 + 310497082768021137638 6
6050560*a^29*b^22*c^55*d^20 - 37706614244767692268335267840*a^30*b^21*c^54*
d^21 + 40833216619792283792163471360*a^31*b^20*c^53*d^22 - 3931216806275109
3709382615040*a^32*b^19*c^52*d^23 + 33557805042801128843488788480*a^33*b^18
*c^51*d^24 - 25329188887155786370693201920*a^34*b^17*c^50*d^25 + 1685146331
0911481777624186880*a^35*b^16*c^49*d^26 - 9843609097631363291959787520*a^36
*b^15*c^48*d^27 + 5023816147465636127472353280*a^37*b^14*c^47*d^28 - 222605
4577272365612261179392*a^38*b^13*c^46*d^29 + 849419752718963326077370368*a^
39*b^12*c^45*d^30 - 276172923601113041340465152*a^40*b^11*c^44*d^31 + 75441
341408208223215812608*a^41*b^10*c^43*d^32 - 16988052798101408932954112*a^42
*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*c^41*d^34 - 42819850557
5496787427328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*b^6*c^39*d^
36 - 2816587235754527162368*a^46*b^5*c^38*d^37 + 88774955854727217152*a^47*
b^4*c^37*d^38)*1i)*1i - 1188950301625810944 0*a^9*b^38*c^56*d^7 + 2172536460
24352727040*a^10*b^37*c^55*d^8 - 1879766455667426066432*a^11*b^36*c^54*d^9
+ 10237150327374383939584*a^12*b^35*c^53*d^10 - 37711511320670913953792*a^1
3*b^34*c^52*d^11 + 77353208427556875796480*a^14*b^33*c^51*d^12 + 1276272381
72719495249920*a^15*b^32*c^50*d^13 - 2130084466030987427446784*a^16*b^31*c^
49*d^14 + 11885048527140028256616448*a^17*b^30*c^48*d^15 - 456905313616868 4
2972831744*a^18*b^29*c^47*d^16 + 135851929384595950057553920*a^19*b^28*c^46
*d^17 - 326376775711477371051704320*a^20*b^27*c^45*d^18 + 648353352496064 05
9760705536*a^21*b^26*c^44*d^19 - 1080394184249474617790431232*a^22*b^25*c^4
3*d^20 + 1524725339928630029153468416*a^23*b^24*c^42*d^21 - 18341024209241 7
6937716285440*a^24*b^23*c^41*d^22 + 1888062742223171008426147840*a^25*b^22*
c^40*d^23 - 1666588213584359199850102784*a^26*b^21*c^39*d^24 + 12615624538 0
0014779376467968*a^27*b^20*c^38*d^25 - 817528072151542384572760064*a^28*b^1
9*c^37*d^26 + 451847698934426396681830400*a^29*b^18*c^36*d^27 - 21172189094
7778234390937600*a^30*b^17*c^35*d^28 + 83366248780838000977248256*a^31*b^16
*c^34*d^29 - 27241266624044306322685952*a^32*b^15*c^33*d^30 + 7257515800860
571589410816*a^33*b^14*c^32*d^31 - 1536699518639901947985920*a^34*b^13*c^31
*d^32 + 248859486128715197317120*a^35*b^12*c^30*d^33 - 28961642042172523937
792*a^36*b^11*c^29*d^34 + 2157438443758953693184*a^37*b^10*c^28*d^35 - 7730
```

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2354662372933632*a^38*b^9*c^27*d^36)*1i - x^(1/2)*(30652624963790438400*a^9
*b^37*c^51*d^9 - 507161613037259980800*a^10*b^36*c^50*d^10 + 47749569695506
13053440*a^11*b^35*c^49*d^11 - 34948190471081762488320*a^12*b^34*c^48*d^12
+ 208409962786483628670976*a^13*b^33*c^47*d^13 - 990271368055602664177664*a
^14*b^32*c^46*d^14 + 3711631588079120800546816*a^15*b^31*c^45*d^15 - 110507
95179720929846493184*a^16*b^30*c^44*d^16 + 26487755718620581216649216*a^17*
b^29*c^43*d^17 - 51805174836472540920020992*a^18*b^28*c^42*d^18 + 836176632
09148864427720704*a^19*b^27*c^41*d^19 - 112350430315654120415952896*a^20*b^
26*c^40*d^20 + 126417217514830317658046464*a^21*b^25*c^39*d^21 - 1195379060
81128203174281216*a^22*b^24*c^38*d^22 + 95089864774620999552335872*a^23*b^2
3*c^37*d^23 - 63545506634457987380412416*a^24*b^22*c^36*d^24 + 355295788461
46774008070144*a^25*b^21*c^35*d^25 - 16501565732136655819636736*a^26*b^20*c
^34*d^26 + 6295808856090071441342464*a^27*b^19*c^33*d^27 - 1940847984249953
081884672*a^28*b^18*c^32*d^28 + 471738031694568778366976*a^29*b^17*c^31*d^2
9 - 87073083559063809163264*a^30*b^16*c^30*d^30 + 11476570419434950230016*a
^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b^14*c^28*d^32 + 3865117733
1186466816*a^33*b^13*c^27*d^33))*1i - (-(35153041*a^8*d^15 + 741200625*b^8*
c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 113949990
00*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*
d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12
*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a
^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^
4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 132875550
72*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18
*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(
35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587
309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*
c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 3834
87720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 2013265
92*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^2
4*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 155021
47584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*
c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 20
1326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7
 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*
b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 +
1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 +
 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10
*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 132
87555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*
b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 +
1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(3/4)*(x^(1/2)*(1
8446744073709551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12*b^38*c^
67*d^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 - 479615345916448342016
0*a^14*b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8 - 121293
```

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6383169193658286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*a^17*b^3
3*c^62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 271048693213
33056471040000*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^20*b^30*c
^59*d^13 + 43611606538557895133364224*a^21*b^29*c^58*d^14 + 727811123600877
61599856640*a^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*b^27*c^5
6*d^16 + 1723753001020797184743833600*a^24*b^26*c^55*d^17 - 426943716736587
2814842183680*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^26*b^24*
c^53*d^19 - 15215326043975142249374679040*a^27*b^23*c^52*d^20 + 22962658463
246519625580544000*a^28*b^22*c^51*d^21 - 30231538828274701475145318400*a^29
*b^21*c^50*d^22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23 - 35316
718238336158489724846080*a^31*b^19*c^48*d^24 + 3143314649854474904164892672
0*a^32*b^18*c^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^46*d^26 +
 16850754961433442876234137600*a^34*b^16*c^45*d^27 - 1010520049211541826217
9676160*a^35*b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^14*c^43*d
^29 - 2387248399405916166169821184*a^37*b^13*c^42*d^30 + 927828632312674738
870681600*a^38*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^11*c^40*
d^32 + 85038075959446046066606080*a^40*b^10*c^39*d^33 - 1940959511921089889
4356480*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^37*d^35 -
 500844593983932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153600*a^44
*b^6*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 10680736876271
8683136*a^46*b^4*c^33*d^39) + (-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7
- 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b
^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1
870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 +
16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*
c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 1328
7555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b
^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1
107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*(36893488147
419103232*a^13*b^38*c^71*d^4 - 1069911156275153993728*a^14*b^37*c^70*d^5 +
14978756187852155912192*a^15*b^36*c^69*d^6 - 134999037738929532960768*a^16*
b^35*c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8 - 4465630463459
278708539392*a^18*b^33*c^66*d^9 + 18321125205332103390035968*a^19*b^32*c^65
*d^10 - 63021545228377166868119552*a^20*b^31*c^64*d^11 + 187018029382071665
408606208*a^21*b^30*c^63*d^12 - 490713180393588600090918912*a^22*b^29*c^62*
d^13 + 1161438545048511890042388480*a^23*b^28*c^61*d^14 - 25129740563090662
69898833920*a^24*b^27*c^60*d^15 + 4997541469898172697285754880*a^25*b^26*c^
59*d^16 - 9119889428539397211967979520*a^26*b^25*c^58*d^17 + 15181544306461
039744285409280*a^27*b^24*c^57*d^18 - 22888317982577902576352624640*a^28*b^
23*c^56*d^19 + 31049708276802113763866050560*a^29*b^22*c^55*d^20 - 37706614
244767692268335267840*a^30*b^21*c^54*d^21 + 40833216619792283792163471360*a
^31*b^20*c^53*d^22 - 39312168062751093709382615040*a^32*b^19*c^52*d^23 + 33
557805042801128843488788480*a^33*b^18*c^51*d^24 - 25329188887155786370693201
1920*a^34*b^17*c^50*d^25 + 16851463310911481777624186880*a^35*b^16*c^49*d^2
6 - 9843609097631363291959787520*a^36*b^15*c^48*d^27 + 50238161474656361274
```

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72353280*a^37*b^14*c^47*d^28 - 2226054577272365612261179392*a^38*b^13*c^46*
d^29 + 849419752718963326077370368*a^39*b^12*c^45*d^30 - 276172923601113041
340465152*a^40*b^11*c^44*d^31 + 75441341408208223215812608*a^41*b^10*c^43*d
^32 - 16988052798101408932954112*a^42*b^9*c^42*d^33 + 307041097544425677206
3232*a^43*b^8*c^41*d^34 - 428198505575496787427328*a^44*b^7*c^40*d^35 + 432
54156088335077998592*a^45*b^6*c^39*d^36 - 2816587235754527162368*a^46*b^5*c
^38*d^37 + 88774955854727217152*a^47*b^4*c^37*d^38)*1i)*1i + 11889503016258
109440*a^9*b^38*c^56*d^7 - 217253646024352727040*a^10*b^37*c^55*d^8 + 18797
66455667426066432*a^11*b^36*c^54*d^9 - 10237150327374383939584*a^12*b^35*c^
53*d^10 + 37711511320670913953792*a^13*b^34*c^52*d^11 - 7735320842755687579
6480*a^14*b^33*c^51*d^12 - 127627238172719495249920*a^15*b^32*c^50*d^13 + 2
130084466030987427446784*a^16*b^31*c^49*d^14 - 11885048527140028256616448*a
^17*b^30*c^48*d^15 + 45690531361686842972831744*a^18*b^29*c^47*d^16 - 13585
1929384595950057553920*a^19*b^28*c^46*d^17 + 326376775711477371051704320*a^
20*b^27*c^45*d^18 - 648353352496064059760705536*a^21*b^26*c^44*d^19 + 10803
94184249474617790431232*a^22*b^25*c^43*d^20 - 1524725339928630029153468416*
a^23*b^24*c^42*d^21 + 1834102420924176937716285440*a^24*b^23*c^41*d^22 - 18
8062742223171008426147840*a^25*b^22*c^40*d^23 + 16665882135843591998501027
84*a^26*b^21*c^39*d^24 - 1261562453800014779376467968*a^27*b^20*c^38*d^25 +
 817528072151542384572760064*a^28*b^19*c^37*d^26 - 451847698934426396681830
400*a^29*b^18*c^36*d^27 + 211721890947778234390937600*a^30*b^17*c^35*d^28 -
 83366248780838000977248256*a^31*b^16*c^34*d^29 + 2724126662404430632268595
2*a^32*b^15*c^33*d^30 - 7257515800860571589410816*a^33*b^14*c^32*d^31 + 153
6699518639901947985920*a^34*b^13*c^31*d^32 - 248859486128715197317120*a^35*
b^12*c^30*d^33 + 28961642042172523937792*a^36*b^11*c^29*d^34 - 215743844375
8953693184*a^37*b^10*c^28*d^35 + 77302354662372933632*a^38*b^9*c^27*d^36)*1
i - x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 507161613037259980800
*a^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*c^49*d^11 - 3494819
0471081762488320*a^12*b^34*c^48*d^12 + 208409962786483628670976*a^13*b^33*c
^47*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14 + 37116315880791208
00546816*a^15*b^31*c^45*d^15 - 11050795179720929846493184*a^16*b^30*c^44*d^
16 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 518051748364725409200
20992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a^19*b^27*c^41*d^19
- 112350430315654120415952896*a^20*b^26*c^40*d^20 + 12641721751483031765804
6464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a^22*b^24*c^38*d^22
+ 95089864774620999552335872*a^23*b^23*c^37*d^23 - 635455066344579873804124
16*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25*b^21*c^35*d^25 - 1
6501565732136655819636736*a^26*b^20*c^34*d^26 + 6295808856090071441342464*a
^27*b^19*c^33*d^27 - 1940847984249953081884672*a^28*b^18*c^32*d^28 + 471738
031694568778366976*a^29*b^17*c^31*d^29 - 87073083559063809163264*a^30*b^16*
c^30*d^30 + 11476570419434950230016*a^31*b^15*c^29*d^31 - 96276568988591744
6144*a^32*b^14*c^28*d^32 + 38651177331186466816*a^33*b^13*c^27*d^33))*1i))*
(-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8
587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b
^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 3
```

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83487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 2013
26592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*
c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 155
02147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b
^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 -
 201326592*a*b^11*c^26*d))^(1/4) + atan(((-b^15/(16*a^19*d^12 + 16*a^7*b^12
*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^
3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6
 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 +
 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(x^(1/2)*(3065262496379
0438400*a^9*b^37*c^51*d^9 - 507161613037259980800*a^10*b^36*c^50*d^10 + 477
4956969550613053440*a^11*b^35*c^49*d^11 - 34948190471081762488320*a^12*b^34
*c^48*d^12 + 208409962786483628670976*a^13*b^33*c^47*d^13 - 990271368055602
664177664*a^14*b^32*c^46*d^14 + 3711631588079120800546816*a^15*b^31*c^45*d^
15 - 11050795179720929846493184*a^16*b^30*c^44*d^16 + 264877557186205812166
49216*a^17*b^29*c^43*d^17 - 51805174836472540920020992*a^18*b^28*c^42*d^18
+ 83617663209148864427720704*a^19*b^27*c^41*d^19 - 112350430315654120415952
896*a^20*b^26*c^40*d^20 + 126417217514830317658046464*a^21*b^25*c^39*d^21 -
 119537906081128203174281216*a^22*b^24*c^38*d^22 + 950898647746209995523358
72*a^23*b^23*c^37*d^23 - 63545506634457987380412416*a^24*b^22*c^36*d^24 + 3
5529578846146774008070144*a^25*b^21*c^35*d^25 - 16501565732136655819636736*
a^26*b^20*c^34*d^26 + 6295808856090071441342464*a^27*b^19*c^33*d^27 - 19408
47984249953081884672*a^28*b^18*c^32*d^28 + 471738031694568778366976*a^29*b^
17*c^31*d^29 - 87073083559063809163264*a^30*b^16*c^30*d^30 + 11476570419434
950230016*a^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b^14*c^28*d^32 +
 38651177331186466816*a^33*b^13*c^27*d^33) + (-b^15/(16*a^19*d^12 + 16*a^7*
b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^
9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6
*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d
^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*((x^(1/2)*(18446744
073709551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12*b^38*c^67*d^5
+ 5995191823955604275200*a^13*b^37*c^66*d^6 - 47961534591644834201600*a^14*
b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8 - 1212936383169
193658286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*a^17*b^33*c^62*
d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 2710486932133305647
1040000*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^20*b^30*c^59*d^1
3 + 43611606538557895133364224*a^21*b^29*c^58*d^14 + 7278111236008776159985
6640*a^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*b^27*c^56*d^16
+ 1723753001020797184743833600*a^24*b^26*c^55*d^17 - 4269437167365872814842
183680*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^26*b^24*c^53*d^
19 - 15215326043975142249374679040*a^27*b^23*c^52*d^20 + 229626584632465196
25580544000*a^28*b^22*c^51*d^21 - 30231538828274701475145318400*a^29*b^21*c
^50*d^22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23 - 353167182383
36158489724846080*a^31*b^19*c^48*d^24 + 31433146498544749041648926720*a^32*
b^18*c^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^46*d^26 + 168507
```

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54961433442876234137600*a^34*b^16*c^45*d^27 - 10105200492115418262179676160
*a^35*b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^14*c^43*d^29 - 2
387248399405916166169821184*a^37*b^13*c^42*d^30 + 9278286323126747388706816
00*a^38*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^11*c^40*d^32 +
85038075959446046066606080*a^40*b^10*c^39*d^33 - 19409595119210898894356480
*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^37*d^35 - 500844
593983932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153600*a^44*b^6*c^
35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 106807368762718683136
*a^46*b^4*c^33*d^39) + (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^
11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*
c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*
c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2
*d^10 - 192*a^18*b*c*d^11))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4
- 1069911156275153993728*a^14*b^37*c^70*d^5 + 14978756187852155912192*a^15*
b^36*c^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7 + 8820160799048
62321508352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d
^9 + 18321125205332103390035968*a^19*b^32*c^65*d^10 - 630215452283771668681
19552*a^20*b^31*c^64*d^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12
 - 490713180393588600090918912*a^22*b^29*c^62*d^13 + 1161438545048511890042
388480*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^
15 + 4997541469898172697285754880*a^25*b^26*c^59*d^16 - 9119889428539397211
967979520*a^26*b^25*c^58*d^17 + 15181544306461039744285409280*a^27*b^24*c^5
7*d^18 - 22888317982577902576352624640*a^28*b^23*c^56*d^19 + 31049708276802
113763866050560*a^29*b^22*c^55*d^20 - 37706614244767692268335267840*a^30*b^
21*c^54*d^21 + 40833216619792283792163471360*a^31*b^20*c^53*d^22 - 39312168
062751093709382615040*a^32*b^19*c^52*d^23 + 33557805042801128843488788480*a
^33*b^18*c^51*d^24 - 25329188887155786370693201920*a^34*b^17*c^50*d^25 + 16
851463310911481777624186880*a^35*b^16*c^49*d^26 - 9843609097631363291959787
520*a^36*b^15*c^48*d^27 + 5023816147465636127472353280*a^37*b^14*c^47*d^28
- 2226054577272365612261179392*a^38*b^13*c^46*d^29 + 8494197527189633260773
70368*a^39*b^12*c^45*d^30 - 276172923601113041340465152*a^40*b^11*c^44*d^31
 + 75441341408208223215812608*a^41*b^10*c^43*d^32 - 16988052798101408932954
112*a^42*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*c^41*d^34 - 428
198505575496787427328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*b^6
*c^39*d^36 - 2816587235754527162368*a^46*b^5*c^38*d^37 + 887749558547272171
52*a^47*b^4*c^37*d^38))*(-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b
^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8
*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5
*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^
2*d^10 - 192*a^18*b*c*d^11))^(3/4) - 11889503016258109440*a^9*b^38*c^56*d^7
 + 217253646024352727040*a^10*b^37*c^55*d^8 - 1879766455667426066432*a^11*b
^36*c^54*d^9 + 10237150327374383939584*a^12*b^35*c^53*d^10 - 37711511320670
913953792*a^13*b^34*c^52*d^11 + 77353208427556875796480*a^14*b^33*c^51*d^12
 + 127627238172719495249920*a^15*b^32*c^50*d^13 - 2130084466030987427446784
*a^16*b^31*c^49*d^14 + 11885048527140028256616448*a^17*b^30*c^48*d^15 - 456
```

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90531361686842972831744*a^18*b^29*c^47*d^16 + 135851929384595950057553920*a
^19*b^28*c^46*d^17 - 326376775711477371051704320*a^20*b^27*c^45*d^18 + 6483
53352496064059760705536*a^21*b^26*c^44*d^19 - 1080394184249474617790431232*
a^22*b^25*c^43*d^20 + 1524725339928630029153468416*a^23*b^24*c^42*d^21 - 18
34102420924176937716285440*a^24*b^23*c^41*d^22 + 18880627422231710084261478
40*a^25*b^22*c^40*d^23 - 1666588213584359199850102784*a^26*b^21*c^39*d^24 +
 1261562453800014779376467968*a^27*b^20*c^38*d^25 - 81752807215154238457276
0064*a^28*b^19*c^37*d^26 + 451847698934426396681830400*a^29*b^18*c^36*d^27
 - 211721890947778234390937600*a^30*b^17*c^35*d^28 + 83366248780838000977248
256*a^31*b^16*c^34*d^29 - 27241266624044306322685952*a^32*b^15*c^33*d^30 +
7257515800860571589410816*a^33*b^14*c^32*d^31 - 1536699518639901947985920*a
^34*b^13*c^31*d^32 + 248859486128715197317120*a^35*b^12*c^30*d^33 - 2896164
2042172523937792*a^36*b^11*c^29*d^34 + 2157438443758953693184*a^37*b^10*c^2
8*d^35 - 77302354662372933632*a^38*b^9*c^27*d^36))*1i + (-b^15/(16*a^19*d^1
2 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*
a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a
^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^1
6*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(x^(1/2)
*(30652624963790438400*a^9*b^37*c^51*d^9 - 507161613037259980800*a^10*b^36*
c^50*d^10 + 4774956969550613053440*a^11*b^35*c^49*d^11 - 349481904710817624
88320*a^12*b^34*c^48*d^12 + 208409962786483628670976*a^13*b^33*c^47*d^13 -
990271368055602664177664*a^14*b^32*c^46*d^14 + 3711631588079120800546816*a^
15*b^31*c^45*d^15 - 11050795179720929846493184*a^16*b^30*c^44*d^16 + 264877
55718620581216649216*a^17*b^29*c^43*d^17 - 51805174836472540920020992*a^18*
b^28*c^42*d^18 + 83617663209148864427720704*a^19*b^27*c^41*d^19 - 112350430
315654120415952896*a^20*b^26*c^40*d^20 + 126417217514830317658046464*a^21*b
^25*c^39*d^21 - 119537906081128203174281216*a^22*b^24*c^38*d^22 + 950898647
74620999552335872*a^23*b^23*c^37*d^23 - 63545506634457987380412416*a^24*b^2
2*c^36*d^24 + 35529578846146774008070144*a^25*b^21*c^35*d^25 - 165015657321
36655819636736*a^26*b^20*c^34*d^26 + 6295808856090071441342464*a^27*b^19*c^
33*d^27 - 1940847984249953081884672*a^28*b^18*c^32*d^28 + 47173803169456877
8366976*a^29*b^17*c^31*d^29 - 87073083559063809163264*a^30*b^16*c^30*d^30 +
 11476570419434950230016*a^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b
^14*c^28*d^32 + 38651177331186466816*a^33*b^13*c^27*d^33) + (-b^15/(16*a^19
*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3
520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 147
84*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520
*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*((x^
(1/2)*(18446744073709551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12
*b^38*c^67*d^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 - 47961534591644
834201600*a^14*b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8
 - 1212936383169193658286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*
a^17*b^33*c^62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 2710
4869321333056471040000*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^2
0*b^30*c^59*d^13 + 43611606538557895133364224*a^21*b^29*c^58*d^14 + 7278111
```

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2360087761599856640*a^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*
b^27*c^56*d^16 + 1723753001020797184743833600*a^24*b^26*c^55*d^17 - 4269437
167365872814842183680*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^
26*b^24*c^53*d^19 - 15215326043975142249374679040*a^27*b^23*c^52*d^20 + 229
62658463246519625580544000*a^28*b^22*c^51*d^21 - 30231538828274701475145318
400*a^29*b^21*c^50*d^22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23
 - 35316718238336158489724846080*a^31*b^19*c^48*d^24 + 31433146498544749041
648926720*a^32*b^18*c^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^4
6*d^26 + 16850754961433442876234137600*a^34*b^16*c^45*d^27 - 10105200492115
418262179676160*a^35*b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^1
4*c^43*d^29 - 2387248399405916166169821184*a^37*b^13*c^42*d^30 + 9278286323
12674738870681600*a^38*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^
11*c^40*d^32 + 85038075959446046066606080*a^40*b^10*c^39*d^33 - 19409595119
210898894356480*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^3
7*d^35 - 500844593983932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153
600*a^44*b^6*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 106807
368762718683136*a^46*b^4*c^33*d^39) - (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^
12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 +
 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 -
12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 10
56*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(36893488147419103232*a^13
*b^38*c^71*d^4 - 1069911156275153993728*a^14*b^37*c^70*d^5 + 14978756187852
155912192*a^15*b^36*c^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7
+ 882016079904862321508352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a
^18*b^33*c^66*d^9 + 18321125205332103390035968*a^19*b^32*c^65*d^10 - 630215
45228377166868119552*a^20*b^31*c^64*d^11 + 187018029382071665408606208*a^21
*b^30*c^63*d^12 - 490713180393588600090918912*a^22*b^29*c^62*d^13 + 1161438
545048511890042388480*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^
24*b^27*c^60*d^15 + 4997541469898172697285754880*a^25*b^26*c^59*d^16 - 9119
889428539397211967979520*a^26*b^25*c^58*d^17 + 1518154430646103974428540928
0*a^27*b^24*c^57*d^18 - 22888317982577902576352624640*a^28*b^23*c^56*d^19 +
 31049708276802113763866050560*a^29*b^22*c^55*d^20 - 3770661424476769226833
5267840*a^30*b^21*c^54*d^21 + 40833216619792283792163471360*a^31*b^20*c^53*
d^22 - 39312168062751093709382615040*a^32*b^19*c^52*d^23 + 3355780504280112
8843488788480*a^33*b^18*c^51*d^24 - 25329188887155786370693201920*a^34*b^17
*c^50*d^25 + 16851463310911481777624186880*a^35*b^16*c^49*d^26 - 9843609097
631363291959787520*a^36*b^15*c^48*d^27 + 5023816147465636127472353280*a^37*
b^14*c^47*d^28 - 2226054577272365612261179392*a^38*b^13*c^46*d^29 + 8494197
52718963326077370368*a^39*b^12*c^45*d^30 - 276172923601113041340465152*a^40
*b^11*c^44*d^31 + 75441341408208223215812608*a^41*b^10*c^43*d^32 - 16988052
798101408932954112*a^42*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*
c^41*d^34 - 428198505575496787427328*a^44*b^7*c^40*d^35 + 43254156088335077
998592*a^45*b^6*c^39*d^36 - 2816587235754527162368*a^46*b^5*c^38*d^37 + 887
74955854727217152*a^47*b^4*c^37*d^38))*(-b^15/(16*a^19*d^12 + 16*a^7*b^12*c
^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3
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+ 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 -
 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1
056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(3/4) + 11889503016258109440*a^
9*b^38*c^56*d^7 - 217253646024352727040*a^10*b^37*c^55*d^8 + 18797664556674
26066432*a^11*b^36*c^54*d^9 - 10237150327374383939584*a^12*b^35*c^53*d^10 +
 37711511320670913953792*a^13*b^34*c^52*d^11 - 77353208427556875796480*a^14
*b^33*c^51*d^12 - 127627238172719495249920*a^15*b^32*c^50*d^13 + 2130084466
030987427446784*a^16*b^31*c^49*d^14 - 11885048527140028256616448*a^17*b^30*
c^48*d^15 + 45690531361686842972831744*a^18*b^29*c^47*d^16 - 13585192938459
5950057553920*a^19*b^28*c^46*d^17 + 326376775711477371051704320*a^20*b^27*c
^45*d^18 - 648353352496064059760705536*a^21*b^26*c^44*d^19 + 10803941842494
74617790431232*a^22*b^25*c^43*d^20 - 1524725339928630029153468416*a^23*b^24
*c^42*d^21 + 1834102420924176937716285440*a^24*b^23*c^41*d^22 - 18880627422
23171008426147840*a^25*b^22*c^40*d^23 + 1666588213584359199850102784*a^26*b
^21*c^39*d^24 - 1261562453800014779376467968*a^27*b^20*c^38*d^25 + 81752807
2151542384572760064*a^28*b^19*c^37*d^26 - 451847698934426396681830400*a^29*
b^18*c^36*d^27 + 211721890947778234390937600*a^30*b^17*c^35*d^28 - 83366248
780838000977248256*a^31*b^16*c^34*d^29 + 27241266624044306322685952*a^32*b^
15*c^33*d^30 - 7257515800860571589410816*a^33*b^14*c^32*d^31 + 153669951863
9901947985920*a^34*b^13*c^31*d^32 - 248859486128715197317120*a^35*b^12*c^30
*d^33 + 28961642042172523937792*a^36*b^11*c^29*d^34 - 215743844375895369318
4*a^37*b^10*c^28*d^35 + 77302354662372933632*a^38*b^9*c^27*d^36))*1i)/((-b^
15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c
^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^
7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4
*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))
^(1/4)*(x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 50716161303725998
0800*a^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*c^49*d^11 - 349
48190471081762488320*a^12*b^34*c^48*d^12 + 208409962786483628670976*a^13*b^
33*c^47*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14 + 3711631588079
120800546816*a^15*b^31*c^45*d^15 - 11050795179720929846493184*a^16*b^30*c^4
4*d^16 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 51805174836472540
920020992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a^19*b^27*c^41*d
^19 - 112350430315654120415952896*a^20*b^26*c^40*d^20 + 1264172175148303176
58046464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a^22*b^24*c^38*d
^22 + 95089864774620999552335872*a^23*b^23*c^37*d^23 - 63545506634457987380
412416*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25*b^21*c^35*d^25
 - 16501565732136655819636736*a^26*b^20*c^34*d^26 + 62958088560900714413424
64*a^27*b^19*c^33*d^27 - 1940847984249953081884672*a^28*b^18*c^32*d^28 + 47
1738031694568778366976*a^29*b^17*c^31*d^29 - 87073083559063809163264*a^30*b
^16*c^30*d^30 + 11476570419434950230016*a^31*b^15*c^29*d^31 - 9627656898859
17446144*a^32*b^14*c^28*d^32 + 38651177331186466816*a^33*b^13*c^27*d^33) +
(-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^
10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^
7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4
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*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^
11))^(1/4)*((x^(1/2)*(18446744073709551616*a^11*b^39*c^68*d^4 - 47961534591
6448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 -
 47961534591644834201600*a^14*b^36*c^65*d^7 + 275778823901957796659200*a^15
*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^63*d^9 + 42329939982
88506144686080*a^17*b^33*c^62*d^10 - 11941164077799654041845760*a^18*b^32*c
^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^12 - 466376191733924
87079215104*a^20*b^30*c^59*d^13 + 43611606538557895133364224*a^21*b^29*c^58
*d^14 + 72781112360087761599856640*a^22*b^28*c^57*d^15 - 523234066593179210
717593600*a^23*b^27*c^56*d^16 + 1723753001020797184743833600*a^24*b^26*c^55
*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d^18 + 8727322757849829
186700574720*a^26*b^24*c^53*d^19 - 15215326043975142249374679040*a^27*b^23*
c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c^51*d^21 - 30231538828
274701475145318400*a^29*b^21*c^50*d^22 + 34870163031766389952882933760*a^30
*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*b^19*c^48*d^24 + 31433
146498544749041648926720*a^32*b^18*c^47*d^25 - 245751407994910128952311808
0*a^33*b^17*c^46*d^26 + 16850754961433442876234137600*a^34*b^16*c^45*d^27 -
 10105200492115418262179676160*a^35*b^15*c^44*d^28 + 527801131290573623278
314944*a^36*b^14*c^43*d^29 - 2387248399405916166169821184*a^37*b^13*c^42*d^
30 + 927828632312674738870681600*a^38*b^12*c^41*d^31 - 306693733103726739901
644800*a^39*b^11*c^40*d^32 + 85038075959446046066606080*a^40*b^10*c^39*d^3
3 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 355140040563581287137280
0*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b^7*c^36*d^36 + 51111
802530990496153600*a^44*b^6*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^3
4*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) + (-b^15/(16*a^19*d^12 +
 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^1
0*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13
*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b
^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(3689348814
741910323 2*a^13*b^38*c^71*d^4 - 10699111562751539937 28*a^14*b^37*c^70*d^5 +
 149787561878521559121 92*a^15*b^36*c^69*d^6 - 134999037738929532960768*a^16
*b^35*c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8 - 446563046345
9278708539392*a^18*b^33*c^66*d^9 + 18321125205332103390035968*a^19*b^32*c^6
5*d^10 - 63021545228377166868119552*a^20*b^31*c^64*d^11 + 18701802938207166
5408606208*a^21*b^30*c^63*d^12 - 490713180393588600090918912*a^22*b^29*c^62
*d^13 + 1161438545048511890042388480*a^23*b^28*c^61*d^14 - 2512974056309066
269898833920*a^24*b^27*c^60*d^15 + 4997541469898172697285754880*a^25*b^26*c
^59*d^16 - 9119889428539397211967979520*a^26*b^25*c^58*d^17 + 1518154430646
1039744285409280*a^27*b^24*c^57*d^18 - 22888317982577902576352624640*a^28*b
^23*c^56*d^19 + 31049708276802113763866050560*a^29*b^22*c^55*d^20 - 3770661
4244767692268335267840*a^30*b^21*c^54*d^21 + 40833216619792283792163471360*
a^31*b^20*c^53*d^22 - 39312168062751093709382615040*a^32*b^19*c^52*d^23 + 3
3557805042801128843488788480*a^33*b^18*c^51*d^24 - 253291888871557863706932
01920*a^34*b^17*c^50*d^25 + 16851463310911481777624186880*a^35*b^16*c^49*d^
26 - 9843609097631363291959787520*a^36*b^15*c^48*d^27 + 5023816147465636127
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472353280*a^37*b^14*c^47*d^28 - 2226054577272365612261179392*a^38*b^13*c^46
*d^29 + 849419752718963326077370368*a^39*b^12*c^45*d^30 - 27617292360111304
1340465152*a^40*b^11*c^44*d^31 + 754413414082082232158l2608*a^41*b^10*c^43*
d^32 - 169880527981014089329541l2*a^42*b^9*c^42*d^33 + 30704109754442567720
63232*a^43*b^8*c^41*d^34 - 4281985055754967874273 28*a^44*b^7*c^40*d^35 + 43
254156088335077998592*a^45*b^6*c^39*d^36 - 2816587235754527162368*a^46*b^5*
c^38*d^37 + 88774955854727217152*a^47*b^4*c^37*d^38))*(-b^15/(16*a^19*d^12
+ 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^
10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^1
3*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*
b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(3/4) - 11889503
016258109440*a^9*b^38*c^56*d^7 + 217253646024352727040*a^10*b^37*c^55*d^8 -
 1879766455667426066432*a^11*b^36*c^54*d^9 + 10237150327374383939584*a^12*b
^35*c^53*d^10 - 37711511320670913953792*a^13*b^34*c^52*d^11 + 7735320842755
6875796480*a^14*b^33*c^51*d^12 + 127627238172719495249920*a^15*b^32*c^50*d^
13 - 2130084466030987427446784*a^16*b^31*c^49*d^14 + 11885048527140028256 61
6448*a^17*b^30*c^48*d^15 - 45690531361686842972831744*a^18*b^29*c^47*d^16 +
 135851929384595950057553920*a^19*b^28*c^46*d^17 - 326376775711477371051704
320*a^20*b^27*c^45*d^18 + 648353352496064059760705536*a^21*b^26*c^44*d^19 -
 1080394184249474617790431232*a^22*b^25*c^43*d^20 + 152472533992863002915 34
68416*a^23*b^24*c^42*d^21 - 1834102420924176937716285440*a^24*b^23*c^41*d^2
2 + 1888062742223171008426147840*a^25*b^22*c^40*d^23 - 1666588213584359 1998
50102784*a^26*b^21*c^39*d^24 + 1261562453800014779376467968*a^27*b^20*c^38*
d^25 - 817528072151542384572760064*a^28*b^19*c^37*d^26 + 451847698934426396
681830400*a^29*b^18*c^36*d^27 - 211721890947778234390937600*a^30*b^17*c^35*
d^28 + 83366248780838000977248256*a^31*b^16*c^34*d^29 - 27241266624044306 32
2685952*a^32*b^15*c^33*d^30 + 7257515800860571589410816*a^33*b^14*c^32*d^31
 - 1536699518639901947985920*a^34*b^13*c^31*d^32 + 248859486128715197317120
*a^35*b^12*c^30*d^33 - 28961642042172523937792*a^36*b^11*c^29*d^34 + 215743
8443758953693184*a^37*b^10*c^28*d^35 - 77302354662372933632*a^38*b^9*c^27*d
^36)) - (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 105
6*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672
*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*
a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^1
8*b*c*d^11))^(1/4)*(x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 50716
161303725998080 0*a^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*c^4
9*d^11 - 34948190471081762488320*a^12*b^34*c^48*d^12 + 208409962786483628 67
0976*a^13*b^33*c^47*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14 + 3
711631588079120800546816*a^15*b^31*c^45*d^15 - 11050795179720929846493184*a
^16*b^30*c^44*d^16 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 51805
174836472540920020992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a^19
*b^27*c^41*d^19 - 112350430315654120415952896*a^20*b^26*c^40*d^20 + 1264172
17514830317658046464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a^22
*b^24*c^38*d^22 + 95089864774620999552335872*a^23*b^23*c^37*d^23 - 63545506
63445798738041 2416*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25*b^
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21*c^35*d^25 - 16501565732136655819636736*a^26*b^20*c^34*d^26 + 62958088560
90071441342464*a^27*b^19*c^33*d^27 - 19408479842499530818846 72*a^28*b^18*c^
32*d^28 + 4717380316945687783669 76*a^29*b^17*c^31*d^29 - 870730835590638091
63264*a^30*b^16*c^30*d^30 + 1147657041943495023001 6*a^31*b^15*c^29*d^31 - 9
627656898859174461 44*a^32*b^14*c^28*d^32 + 3865117733118646681 6*a^33*b^13*c
^27*d^33) + (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d +
 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 1
2672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7
920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192
*a^18*b*c*d^11))^(1/4)*((x^(1/2)*(18446744073709551616*a^11*b^39*c^68*d^4 -
 479615345916448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*a^13*b^3
7*c^66*d^6 - 47961534591644834201600*a^14*b^36*c^65*d^7 + 275778823901957796
6659200*a^15*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^63*d^9 +
 4232993998288506144686080*a^17*b^33*c^62*d^10 - 11941164077799654041845760
*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^12 - 466
37619173392487079215104*a^20*b^30*c^59*d^13 + 43611606538557895133364224*a^
21*b^29*c^58*d^14 + 72781112360087761599856640*a^22*b^28*c^57*d^15 - 523234
066593179210717593600*a^23*b^27*c^56*d^16 + 1723753001020797184743833600*a^
24*b^26*c^55*d^17 - 4269437167365872814842183680*a^25*b^25*c^54*d^18 + 8727
322757849829186700574720*a^26*b^24*c^53*d^19 - 152153260439751422493746790 4
0*a^27*b^23*c^52*d^20 + 22962658463246519625580544000*a^28*b^22*c^51*d^21 -
 30231538828274701475145318400*a^29*b^21*c^50*d^22 + 348701630317663899528 8
2933760*a^30*b^20*c^49*d^23 - 35316718238336158489724846080*a^31*b^19*c^48*
d^24 + 31433146498544749041648926720*a^32*b^18*c^47*d^25 - 2457514079949101
2895231180800*a^33*b^17*c^46*d^26 + 16850754961433442876234137600*a^34*b^16
*c^45*d^27 - 10105200492115418262179676160*a^35*b^15*c^44*d^28 + 5278011312
905736232783314944*a^36*b^14*c^43*d^29 - 2387248399405916166169821184*a^37*
b^13*c^42*d^30 + 927828632312674738870681600*a^38*b^12*c^41*d^31 - 30669373
3103726739901644800*a^39*b^11*c^40*d^32 + 85038075959446046066606080*a^40*b
^10*c^39*d^33 - 19409595119210898894356480*a^41*b^9*c^38*d^34 + 35514004056
35812871372800*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b^7*c^36*
d^36 + 51111802530990496153600*a^44*b^6*c^35*d^37 - 3359577235627333124096*
a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) - (-b^15/(16
*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^
2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5
+ 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 -
 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)
*(36893488147419103232*a^13*b^38*c^71*d^4 - 1069911156275153993728*a^14*b^3
7*c^70*d^5 + 14978756187852155912192*a^15*b^36*c^69*d^6 - 13499903773892953
2960768*a^16*b^35*c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8 -
4465630463459278708539392*a^18*b^33*c^66*d^9 + 18321125205332103390035968*a
^19*b^32*c^65*d^10 - 63021545228377166868119552*a^20*b^31*c^64*d^11 + 18701
8029382071665408606208*a^21*b^30*c^63*d^12 - 490713180393588600090918912*a^
22*b^29*c^62*d^13 + 1161438545048511890042388480*a^23*b^28*c^61*d^14 - 2512
974056309066269898833920*a^24*b^27*c^60*d^15 + 4997541469898172697285754880
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*a^25*b^26*c^59*d^16 - 9119889428539397211967979520*a^26*b^25*c^58*d^17 + 1
5181544306461039744285409280*a^27*b^24*c^57*d^18 - 228883179825779025763526
24640*a^28*b^23*c^56*d^19 + 31049708276802113763866050560*a^29*b^22*c^55*d^
20 - 37706614244767692268335267840*a^30*b^21*c^54*d^21 + 408332166197922837
92163471360*a^31*b^20*c^53*d^22 - 39312168062751093709382615040*a^32*b^19*c
^52*d^23 + 33557805042801128843488788480*a^33*b^18*c^51*d^24 - 253291888871
55786370693201920*a^34*b^17*c^50*d^25 + 1685146331091148177624186880*a^35*
b^16*c^49*d^26 - 9843609097631363291959787520*a^36*b^15*c^48*d^27 + 5023816
147465636127472353280*a^37*b^14*c^47*d^28 - 2226054577272365612261179392*a^
38*b^13*c^46*d^29 + 849419752718963326077370368*a^39*b^12*c^45*d^30 - 27617
2923601113041340465152*a^40*b^11*c^44*d^31 + 75441341408208223215812608*a^4
1*b^10*c^43*d^32 - 16988052798101408932954112*a^42*b^9*c^42*d^33 + 30704109
75444256772063232*a^43*b^8*c^41*d^34 - 428198505575496787427328*a^44*b^7*c^
40*d^35 + 43254156088335077998592*a^45*b^6*c^39*d^36 - 28165872357545271623
68*a^46*b^5*c^38*d^37 + 88774955854727217152*a^47*b^4*c^37*d^38))*(-b^15/(1
6*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d
^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5
 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8
- 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(3/4
) + 11889503016258109440*a^9*b^38*c^56*d^7 - 217253646024352727040*a^10*b^3
7*c^55*d^8 + 1879766455667426066432*a^11*b^36*c^54*d^9 - 102371503273743839
39584*a^12*b^35*c^53*d^10 + 37711511320670913953792*a^13*b^34*c^52*d^11 - 7
7353208427556875796480*a^14*b^33*c^51*d^12 - 127627238172719495249920*a^15*
b^32*c^50*d^13 + 2130084466030987427446784*a^16*b^31*c^49*d^14 - 1188504852
7140028256616448*a^17*b^30*c^48*d^15 + 45690531361686842972831744*a^18*b^29
*c^47*d^16 - 135851929384595950057553920*a^19*b^28*c^46*d^17 + 326376775711
477371051704320*a^20*b^27*c^45*d^18 - 648353352496064059760705536*a^21*b^26
*c^44*d^19 + 1080394184249474617790431232*a^22*b^25*c^43*d^20 - 15247253399
28630029153468416*a^23*b^24*c^42*d^21 + 1834102420924176937716285440*a^24*b
^23*c^41*d^22 - 1888062742223171008426147840*a^25*b^22*c^40*d^23 + 16665882
13584359199850102784*a^26*b^21*c^39*d^24 - 1261562453800014779376467968*a^2
7*b^20*c^38*d^25 + 817528072151542384572760064*a^28*b^19*c^37*d^26 - 451847
698934426396681830400*a^29*b^18*c^36*d^27 + 211721890947778234390937600*a^3
0*b^17*c^35*d^28 - 83366248780838000977248256*a^31*b^16*c^34*d^29 + 2724126
6624044306322685952*a^32*b^15*c^33*d^30 - 7257515800860571589410816*a^33*b^
14*c^32*d^31 + 1536699518639901947985920*a^34*b^13*c^31*d^32 - 248859486128
715197317120*a^35*b^12*c^30*d^33 + 28961642042172523937792*a^36*b^11*c^29*d
^34 - 2157438443758953693184*a^37*b^10*c^28*d^35 + 77302354662372933632*a^3
8*b^9*c^27*d^36))))*(-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*
c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8
*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5
*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^
10 - 192*a^18*b*c*d^11))^(1/4)*2i - 2*atan(((-b^15/(16*a^19*d^12 + 16*a^7*b
^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9
*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*
```

d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^
9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(x^(1/2)*(3065262496
3790438400*a^9*b^37*c^51*d^9 - 5071616130372599808000*a^10*b^36*c^50*d^10 +
47749569695506130534400*a^11*b^35*c^49*d^11 - 349481904710817624883200*a^12*b
^34*c^48*d^12 + 2084099627864836286709760*a^13*b^33*c^47*d^13 - 990271368055
6026641776640*a^14*b^32*c^46*d^14 + 37116315880791208005468160*a^15*b^31*c^45
*d^15 - 110507951797209298464931840*a^16*b^30*c^44*d^16 + 264877557186205812
166492160*a^17*b^29*c^43*d^17 - 518051748364725409200209920*a^18*b^28*c^42*d^
18 + 836176632091488644277207040*a^19*b^27*c^41*d^19 - 112350430315654120415
9528960*a^20*b^26*c^40*d^20 + 1264172175148303176580464640*a^21*b^25*c^39*d^2
1 - 1195379060811282031742812160*a^22*b^24*c^38*d^22 + 950898647746209995523
358720*a^23*b^23*c^37*d^23 - 635455066344579873804124160*a^24*b^22*c^36*d^24
+ 355295788461467740080701440*a^25*b^21*c^35*d^25 - 165015657321366558196367
360*a^26*b^20*c^34*d^26 + 62958088560900714413424640*a^27*b^19*c^33*d^27 - 19
408479842499530818846720*a^28*b^18*c^32*d^28 + 4717380316945687783669760*a^29
*b^17*c^31*d^29 - 870730835590638091632640*a^30*b^16*c^30*d^30 + 11476570419
4349502300160*a^31*b^15*c^29*d^31 - 9627656898859174461440*a^32*b^14*c^28*d^3
2 + 386511773311864668160*a^33*b^13*c^27*d^33) - (-b^15/(16*a^19*d^12 + 16*a
^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9
*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*
c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^
3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*((x^(1/2)*(18446
744073709551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12*b^38*c^67*d
^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 - 47961534591644834201600*a^
14*b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8 - 1212936383
169193658286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*a^17*b^33*c^
62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^11 + 271048693213330
56471040000*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^20*b^30*c^59*
d^13 + 43611606538557895133364224*a^21*b^29*c^58*d^14 + 7278111236008776159
9856640*a^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*b^27*c^56*d^
16 + 1723753001020797184743833600*a^24*b^26*c^55*d^17 - 4269437167365872814
842183680*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^26*b^24*c^53
*d^19 - 15215326043975142249374679040*a^27*b^23*c^52*d^20 + 229626584632465
19625580544000*a^28*b^22*c^51*d^21 - 30231538828274701475145318400*a^29*b^2
1*c^50*d^22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23 - 353167182
38336158489724846080*a^31*b^19*c^48*d^24 + 31433146498544749041648926720*a^
32*b^18*c^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^46*d^26 + 168
50754961433442876234137600*a^34*b^16*c^45*d^27 - 10105200492115418262179676
160*a^35*b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^14*c^43*d^29
- 2387248399405916166169821184*a^37*b^13*c^42*d^30 + 92782863231267473887066
81600*a^38*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^11*c^40*d^32
+ 85038075959446046066606080*a^40*b^10*c^39*d^33 - 194095951192108988943566
480*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^37*d^35 - 500
844593983932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153600*a^44*b^6
*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 106807368762718683

```
136*a^46*b^4*c^33*d^39) - (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8
*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b
^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b
^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*
c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(368934881474191032 32*a^13*b^38*c^71*d
^4 - 106991115627515399372 8*a^14*b^37*c^70*d^5 + 1497875618785215591 2192*a^
15*b^36*c^69*d^6 - 13499903773892953296 0768*a^16*b^35*c^68*d^7 + 8820160799
04862321508352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^6
6*d^9 + 1832112520533210339003 5968*a^19*b^32*c^65*d^10 - 630215452283771668
68119552*a^20*b^31*c^64*d^11 + 1870180293820716654086 06208*a^21*b^30*c^63*d
^12 - 490713180393588600090918912*a^22*b^29*c^62*d^13 + 1161438545048511890
042388480*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60
*d^15 + 4997541469898172697285754880*a^25*b^26*c^59*d^16 - 9119889428539397
211967979520*a^26*b^25*c^58*d^17 + 15181544306461039744285409280*a^27*b^24*
c^57*d^18 - 22888317982577902576352624640*a^28*b^23*c^56*d^19 + 31049708276
802113763866050560*a^29*b^22*c^55*d^20 - 37706614244767692268335267840*a^30
*b^21*c^54*d^21 + 40833216619792283792163471360*a^31*b^20*c^53*d^22 - 39312
168062751093709382615040*a^32*b^19*c^52*d^23 + 335578050428011288434887884 8
0*a^33*b^18*c^51*d^24 - 25329188887155786370693201920*a^34*b^17*c^50*d^25 +
 16851463310911481777624186880*a^35*b^16*c^49*d^26 - 9843609097631363291959
787520*a^36*b^15*c^48*d^27 + 5023816147465636127472353280*a^37*b^14*c^47*d^
28 - 2226054577272365612261179392*a^38*b^13*c^46*d^29 + 849419752718963326 0
77370368*a^39*b^12*c^45*d^30 - 276172923601113041340465152*a^40*b^11*c^44*d
^31 + 75441341408208223215812608*a^41*b^10*c^43*d^32 - 16988052798101408932
954112*a^42*b^9*c^42*d^33 + 3070410975444256772063232*a^43*b^8*c^41*d^34 -
428198505575496787427328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*
b^6*c^39*d^36 - 2816587235754527162368*a^46*b^5*c^38*d^37 + 887749558547272
17152*a^47*b^4*c^37*d^38)*1i)*(-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192
*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^
11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^
14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*
b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(3/4)*1i - 11889503016258109440*a^9*b^38
*c^56*d^7 + 217253646024352727040*a^10*b^37*c^55*d^8 - 18797664556674260664
32*a^11*b^36*c^54*d^9 + 10237150327374383939584*a^12*b^35*c^53*d^10 - 37711
511320670913953792*a^13*b^34*c^52*d^11 + 77353208427556875796480*a^14*b^33*
c^51*d^12 + 127627238172719495249920*a^15*b^32*c^50*d^13 - 2130084466030987
427446784*a^16*b^31*c^49*d^14 + 11885048527140028256616448*a^17*b^30*c^48*d
^15 - 45690531361686842972831744*a^18*b^29*c^47*d^16 + 135851929384595950 05
7553920*a^19*b^28*c^46*d^17 - 326376775711477371051704320*a^20*b^27*c^45*d^
18 + 648353352496064059760705536*a^21*b^26*c^44*d^19 - 1080394184249474617 7
90431232*a^22*b^25*c^43*d^20 + 1524725339928630029153468416*a^23*b^24*c^42*
d^21 - 1834102420924176937716285440*a^24*b^23*c^41*d^22 + 1888062742223171 0
08426147840*a^25*b^22*c^40*d^23 - 1666588213584359199850102784*a^26*b^21*c^
39*d^24 + 1261562453800014779376467968*a^27*b^20*c^38*d^25 - 81752807215154
2384572760064*a^28*b^19*c^37*d^26 + 451847698934426396681830400*a^29*b^18*c
```

```
^36*d^27 - 211721890947778234390937600*a^30*b^17*c^35*d^28 + 83366248780838
000977248256*a^31*b^16*c^34*d^29 - 27241266624044306322685952*a^32*b^15*c^3
3*d^30 + 7257515800860571589410816*a^33*b^14*c^32*d^31 - 153669951863990194
7985920*a^34*b^13*c^31*d^32 + 248859486128715197317120*a^35*b^12*c^30*d^33
- 28961642042172523937792*a^36*b^11*c^29*d^34 + 2157438443758953693184*a^37
*b^10*c^28*d^35 - 77302354662372933632*a^38*b^9*c^27*d^36)*1i) + (-b^15/(16
*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^
2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5
+ 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 -
 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)
*(x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 507161613037259980800*a
^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*c^49*d^11 - 349481904
71081762488320*a^12*b^34*c^48*d^12 + 208409962786483628670976*a^13*b^33*c^4
7*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14 + 3711631588079120800
546816*a^15*b^31*c^45*d^15 - 11050795179720929846493184*a^16*b^30*c^44*d^16
 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 51805174836472540920020
992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a^19*b^27*c^41*d^19 -
112350430315654120415952896*a^20*b^26*c^40*d^20 + 126417217514830317658046
464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a^22*b^24*c^38*d^22 +
95089864774620999552335872*a^23*b^23*c^37*d^23 - 63545506634457987380412416
*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25*b^21*c^35*d^25 - 165
01565732136655819636736*a^26*b^20*c^34*d^26 + 6295808856090071441342464*a^2
7*b^19*c^33*d^27 - 1940847984249953081884672*a^28*b^18*c^32*d^28 + 47173803
1694568778366976*a^29*b^17*c^31*d^29 - 87073083559063809163264*a^30*b^16*c^
30*d^30 + 11476570419434950230016*a^31*b^15*c^29*d^31 - 9627656898859174461
44*a^32*b^14*c^28*d^32 + 38651177331186466816*a^33*b^13*c^27*d^33) - (-b^15
/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^1
0*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*
d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d
^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(
1/4)*((x^(1/2)*(18446744073709551616*a^11*b^39*c^68*d^4 - 47961534591644834
2016*a^12*b^38*c^67*d^5 + 5995191823955604275200*a^13*b^37*c^66*d^6 - 47961
534591644834201600*a^14*b^36*c^65*d^7 + 275778823901957796659200*a^15*b^35*
c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^63*d^9 + 42329939982885061
44686080*a^17*b^33*c^62*d^10 - 11941164077799654041845760*a^18*b^32*c^61*d^
11 + 27104869321333056471040000*a^19*b^31*c^60*d^12 - 466376191733924870792
15104*a^20*b^30*c^59*d^13 + 43611606538557895133364224*a^21*b^29*c^58*d^14
+ 72781112360087761599856640*a^22*b^28*c^57*d^15 - 523234066593179210717593
600*a^23*b^27*c^56*d^16 + 1723753001020797184743833600*a^24*b^26*c^55*d^17
- 4269437167365872814842183680*a^25*b^25*c^54*d^18 + 87273227578498291867O0
574720*a^26*b^24*c^53*d^19 - 15215326043975142249374679040*a^27*b^23*c^52*d
^20 + 22962658463246519625580544000*a^28*b^22*c^51*d^21 - 30231538828274701
475145318400*a^29*b^21*c^50*d^22 + 34870163031766389952882933760*a^30*b^20*
c^49*d^23 - 35316718238336158489724846080*a^31*b^19*c^48*d^24 + 31433146498
544749041648926720*a^32*b^18*c^47*d^25 - 24575140799491012895231180800*a^33
```

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*b^17*c^46*d^26 + 16850754961433442876234137600*a^34*b^16*c^45*d^27 - 10105
20049211541826217967616 0*a^35*b^15*c^44*d^28 + 5278011312905736232783314944
*a^36*b^14*c^43*d^29 - 2387248399405916166169821184*a^37*b^13*c^42*d^30 + 9
27828632312674738870681600*a^38*b^12*c^41*d^31 - 306693733103726739901644 80
0*a^39*b^11*c^40*d^32 + 85038075959446046066606080*a^40*b^10*c^39*d^33 - 19
409595119210898894356480*a^41*b^9*c^38*d^34 + 3551400405635812871372800*a^4
2*b^8*c^37*d^35 - 500844593983932480880640*a^43*b^7*c^36*d^36 + 51111802530
990496153600*a^44*b^6*c^35*d^37 - 3359577235627333124096*a^45*b^5*c^34*d^38
 + 106807368762718683136*a^46*b^4*c^33*d^39) + (-b^15/(16*a^19*d^12 + 16*a^
7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*
c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c
^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3
*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(3689348814741910
3232*a^13*b^38*c^71*d^4 - 1069911156275153993728*a^14*b^37*c^70*d^5 + 14978
756187852155912192*a^15*b^36*c^69*d^6 - 134999037738929532960768*a^16*b^35*
c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8 - 446563046345927870
8539392*a^18*b^33*c^66*d^9 + 18321125205332103390035968*a^19*b^32*c^65*d^10
 - 63021545228377166868119552*a^20*b^31*c^64*d^11 + 187018029382071665408 60
6208*a^21*b^30*c^63*d^12 - 490713180393588600090918912*a^22*b^29*c^62*d^13
+ 1161438545048511890042388480*a^23*b^28*c^61*d^14 - 2512974056309066269898
833920*a^24*b^27*c^60*d^15 + 4997541469898172697285754880*a^25*b^26*c^59*d^
16 - 9119889428539397211967979520*a^26*b^25*c^58*d^17 + 1518154430646103974
4285409280*a^27*b^24*c^57*d^18 - 22888317982577902576352624640*a^28*b^23*c^
56*d^19 + 31049708276802113763866050560*a^29*b^22*c^55*d^20 - 3770661424476
7692268335267840*a^30*b^21*c^54*d^21 + 40833216619792283792163471360*a^31*b
^20*c^53*d^22 - 39312168062751093709382615040*a^32*b^19*c^52*d^23 + 3355780
5042801128843488788480*a^33*b^18*c^51*d^24 - 25329188887155786370693201920*
a^34*b^17*c^50*d^25 + 16851463310911481777624186880*a^35*b^16*c^49*d^26 - 9
843609097631363291959787520*a^36*b^15*c^48*d^27 + 5023816147465636127472353
280*a^37*b^14*c^47*d^28 - 2226054577272365612261179392*a^38*b^13*c^46*d^29
+ 849419752718963326077370368*a^39*b^12*c^45*d^30 - 276172923601113041340 46
5152*a^40*b^11*c^44*d^31 + 75441341408208223215812608*a^41*b^10*c^43*d^32 -
 16988052798101408932954112*a^42*b^9*c^42*d^33 + 3070410975444256772063232*
a^43*b^8*c^41*d^34 - 428198505575496787427328*a^44*b^7*c^40*d^35 + 43254156
088335077998592*a^45*b^6*c^39*d^36 - 2816587235754527162368*a^46*b^5*c^38*d
^37 + 88774955854727217152*a^47*b^4*c^37*d^38)*1i)*(-b^15/(16*a^19*d^12 + 1
6*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*
b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b
^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3
*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(3/4)*1i + 11889503
016258109440*a^9*b^38*c^56*d^7 - 217253646024352727040*a^10*b^37*c^55*d^8 +
 1879766455667426066432*a^11*b^36*c^54*d^9 - 10237150327374383939584*a^12*b
^35*c^53*d^10 + 37711511320670913953792*a^13*b^34*c^52*d^11 - 7735320842755
6875796480*a^14*b^33*c^51*d^12 - 12762723817271949524992 0*a^15*b^32*c^50*d^
13 + 2130084466030987427446784*a^16*b^31*c^49*d^14 - 1188504852714002825661
```

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6448*a^17*b^30*c^48*d^15 + 45690531361686842972831744*a^18*b^29*c^47*d^16 -
 135851929384595950057553920*a^19*b^28*c^46*d^17 + 326376775711477371051704
320*a^20*b^27*c^45*d^18 - 648353352496064059760705536*a^21*b^26*c^44*d^19 +
 1080394184249474617790431232*a^22*b^25*c^43*d^20 - 15247253399286300291534
68416*a^23*b^24*c^42*d^21 + 1834102420924176937716285440*a^24*b^23*c^41*d^2
2 - 1888062742223171008426147840*a^25*b^22*c^40*d^23 + 16665882135843591998
50102784*a^26*b^21*c^39*d^24 - 1261562453800014779376467968*a^27*b^20*c^38*
d^25 + 817528072151542384572760064*a^28*b^19*c^37*d^26 - 451847698934426396
681830400*a^29*b^18*c^36*d^27 + 211721890947778234390937600*a^30*b^17*c^35*
d^28 - 83366248780838000977248256*a^31*b^16*c^34*d^29 + 2724126662404430632
2685952*a^32*b^15*c^33*d^30 - 7257515800860571589410816*a^33*b^14*c^32*d^31
 + 1536699518639901947985920*a^34*b^13*c^31*d^32 - 248859486128715197317120
*a^35*b^12*c^30*d^33 + 28961642042172523937792*a^36*b^11*c^29*d^34 - 215743
8443758953693184*a^37*b^10*c^28*d^35 + 77302354662372933632*a^38*b^9*c^27*d
^36)*1i))/((-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d +
1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12
672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 79
20*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*
a^18*b*c*d^11))^(1/4)*(x^(1/2)*(30652624963790438400*a^9*b^37*c^51*d^9 - 50
7161613037259980800*a^10*b^36*c^50*d^10 + 4774956969550613053440*a^11*b^35*
c^49*d^11 - 34948190471081762488320*a^12*b^34*c^48*d^12 + 20840996278648362
8670976*a^13*b^33*c^47*d^13 - 990271368055602664177664*a^14*b^32*c^46*d^14
+ 3711631588079120800546816*a^15*b^31*c^45*d^15 - 1105079517972092984649318
4*a^16*b^30*c^44*d^16 + 26487755718620581216649216*a^17*b^29*c^43*d^17 - 51
805174836472540920020992*a^18*b^28*c^42*d^18 + 83617663209148864427720704*a
^19*b^27*c^41*d^19 - 112350430315654120415952896*a^20*b^26*c^40*d^20 + 1264
17217514830317658046464*a^21*b^25*c^39*d^21 - 119537906081128203174281216*a
^22*b^24*c^38*d^22 + 95089864774620999552335872*a^23*b^23*c^37*d^23 - 63545
506634457987380412416*a^24*b^22*c^36*d^24 + 35529578846146774008070144*a^25
*b^21*c^35*d^25 - 16501565732136655819636736*a^26*b^20*c^34*d^26 + 62958088
56090071441342464*a^27*b^19*c^33*d^27 - 1940847984249953081884672*a^28*b^18
*c^32*d^28 + 471738031694568778366976*a^29*b^17*c^31*d^29 - 870730835590638
09163264*a^30*b^16*c^30*d^30 + 11476570419434950230016*a^31*b^15*c^29*d^31
- 962765689885917446144*a^32*b^14*c^28*d^32 + 38651177331186466816*a^33*b^1
3*c^27*d^33) - (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*
d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4
- 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7
+ 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 -
192*a^18*b*c*d^11))^(1/4)*((x^(1/2)*(18446744073709551616*a^11*b^39*c^68*d^
4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 5995191823955604275200*a^13*
b^37*c^66*d^6 - 47961534591644834201600*a^14*b^36*c^65*d^7 + 27577882390195
7796659200*a^15*b^35*c^64*d^8 - 1212936383169193658286080*a^16*b^34*c^63*d^
9 + 4232993998288506144686080*a^17*b^33*c^62*d^10 - 11941164077799654041845
760*a^18*b^32*c^61*d^11 + 27104869321333056471040000*a^19*b^31*c^60*d^12 -
46637619173392487079215104*a^20*b^30*c^59*d^13 + 43611606538557895133364224
```

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*a^21*b^29*c^58*d^14 + 727811123600877615998566 40*a^22*b^28*c^57*d^15 - 523
2340665931792107175936 00*a^23*b^27*c^56*d^16 + 172375300102079718474383 3600
*a^24*b^26*c^55*d^17 - 42694371673658728148421 83680*a^25*b^25*c^54*d^18 + 8
72732275784982918670057 4720*a^26*b^24*c^53*d^19 - 152153260439751422493746 7
9040*a^27*b^23*c^52*d^20 + 229626584632465196255805 44000*a^28*b^22*c^51*d^2
1 - 302315388282747014751453 18400*a^29*b^21*c^50*d^22 + 348701630317663899 5
2882933760*a^30*b^20*c^49*d^23 - 353167182383361584897248 46080*a^31*b^19*c^
48*d^24 + 314331464985447490416489 26720*a^32*b^18*c^47*d^25 - 245751407994 9
1012895231180800*a^33*b^17*c^46*d^26 + 168507549614334428762341 37600*a^34*b
^16*c^45*d^27 - 101052004921154182621796 76160*a^35*b^15*c^44*d^28 + 527801 1
3129057362327833149 44*a^36*b^14*c^43*d^29 - 238724839940591616616982 1184*a^
37*b^13*c^42*d^30 + 927828632312674738870681 600*a^38*b^12*c^41*d^31 - 3066 9
3733103726739901644 800*a^39*b^11*c^40*d^32 + 850380759594460460666060 80*a^4
0*b^10*c^39*d^33 - 194095951192108988943564 80*a^41*b^9*c^38*d^34 + 3551400 4
056358128713728 00*a^42*b^8*c^37*d^35 - 500844593983932480880640*a^43*b^7*c^
36*d^36 + 511118025309904961536 00*a^44*b^6*c^35*d^37 - 335957723562733312 40
96*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*b^4*c^33*d^39) - (-b^15/
(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10
*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d
^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^
8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1
/4)*(368934881474191032 32*a^13*b^38*c^71*d^4 - 106991115627515399372 8*a^14*
b^37*c^70*d^5 + 149787561878521559121 92*a^15*b^36*c^69*d^6 - 134999037738 92
9532960768*a^16*b^35*c^68*d^7 + 882016079904862321508352*a^17*b^34*c^67*d^8
 - 446563046345927870853939 2*a^18*b^33*c^66*d^9 + 183211252053321033900359 6
8*a^19*b^32*c^65*d^10 - 630215452283771668681195 52*a^20*b^31*c^64*d^11 + 18
701802938207166540860620 8*a^21*b^30*c^63*d^12 - 490713180393588600090918912
*a^22*b^29*c^62*d^13 + 116143854504851189004238 8480*a^23*b^28*c^61*d^14 - 2
512974056309066269898833 920*a^24*b^27*c^60*d^15 + 499754146989817269728575 4
880*a^25*b^26*c^59*d^16 - 911988942853939721196797 9520*a^26*b^25*c^58*d^17
+ 151815443064610397442854 09280*a^27*b^24*c^57*d^18 - 228883179825779025763
52624640*a^28*b^23*c^56*d^19 + 310497082768021137638660 50560*a^29*b^22*c^55
*d^20 - 377066142447676922683352 67840*a^30*b^21*c^54*d^21 + 408332166197922
83792163471360*a^31*b^20*c^53*d^22 - 393121680627510937093826 15040*a^32*b^1
9*c^52*d^23 + 335578050428011288434887 88480*a^33*b^18*c^51*d^24 - 253291888
87155786370693201920*a^34*b^17*c^50*d^25 + 168514633109114817762418 6880*a^
35*b^16*c^49*d^26 - 984360909763136329195978 7520*a^36*b^15*c^48*d^27 + 5023
816147465636127472353280*a^37*b^14*c^47*d^28 - 222605457727236561226117 9392
*a^38*b^13*c^46*d^29 + 849419752718963326077370 368*a^39*b^12*c^45*d^30 - 27
6172923601113041340465152*a^40*b^11*c^44*d^31 + 754413414082082232158126 08*
a^41*b^10*c^43*d^32 - 169880527981014089329541 12*a^42*b^9*c^42*d^33 + 30704
109754442567720632 32*a^43*b^8*c^41*d^34 - 428198505575496787427328*a^44*b^7
*c^40*d^35 + 432541560883350779985 92*a^45*b^6*c^39*d^36 - 281658723575452 71
62368*a^46*b^5*c^38*d^37 + 887749558547272171 52*a^47*b^4*c^37*d^38)*1i)*(-b
^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*
```

```
c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c
^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^
4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11)
)^(3/4)*1i - 1188950301625810944 0*a^9*b^38*c^56*d^7 + 21725364602435272704 0
*a^10*b^37*c^55*d^8 - 187976645566742606643 2*a^11*b^36*c^54*d^9 + 102371503
2737438393958 4*a^12*b^35*c^53*d^10 - 377115113206709139537 92*a^13*b^34*c^52
*d^11 + 773532084275568757964 80*a^14*b^33*c^51*d^12 + 1276272381727194952 49
920*a^15*b^32*c^50*d^13 - 2130084466030987427446784*a^16*b^31*c^49*d^14 + 1
188504852714002825661644 8*a^17*b^30*c^48*d^15 - 4569053136168684297283174 4*
a^18*b^29*c^47*d^16 + 1358519293845959500575539 20*a^19*b^28*c^46*d^17 - 326
376775711477371051704320*a^20*b^27*c^45*d^18 + 6483533524960640597607055 36*
a^21*b^26*c^44*d^19 - 1080394184249474617790431232*a^22*b^25*c^43*d^20 + 15
24725339928630029153468416*a^23*b^24*c^42*d^21 - 183410242092417693771628 54
40*a^24*b^23*c^41*d^22 + 1888062742223171008426147840*a^25*b^22*c^40*d^23 -
 1666588213584359199850102784*a^26*b^21*c^39*d^24 + 126156245380001477937 64
67968*a^27*b^20*c^38*d^25 - 817528072151542384572760064*a^28*b^19*c^37*d^26
 + 451847698934426396681830400*a^29*b^18*c^36*d^27 - 21172189094777823439 09
37600*a^30*b^17*c^35*d^28 + 83366248780838000977248256*a^31*b^16*c^34*d^29
- 27241266624044306322685952*a^32*b^15*c^33*d^30 + 725751580086057158941081
6*a^33*b^14*c^32*d^31 - 1536699518639901947985920*a^34*b^13*c^31*d^32 + 248
859486128715197317120*a^35*b^12*c^30*d^33 - 28961642042172523937792*a^36*b^
11*c^29*d^34 + 2157438443758953693184*a^37*b^10*c^28*d^35 - 77302354662372 9
33632*a^38*b^9*c^27*d^36)*1i)*1i - (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12
- 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 79
20*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 126
72*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*
a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*(x^(1/2)*(306526249637904384 0
0*a^9*b^37*c^51*d^9 - 50716161303725998080 0*a^10*b^36*c^50*d^10 + 477495696
9550613053440*a^11*b^35*c^49*d^11 - 34948190471081762488320*a^12*b^34*c^48*
d^12 + 208409962786483628670976*a^13*b^33*c^47*d^13 - 990271368055602664177
664*a^14*b^32*c^46*d^14 + 3711631588079120800546816*a^15*b^31*c^45*d^15 - 1
1050795179720929846493184*a^16*b^30*c^44*d^16 + 26487755718620581216649216*
a^17*b^29*c^43*d^17 - 51805174836472540920020992*a^18*b^28*c^42*d^18 + 8361
7663209148864427720704*a^19*b^27*c^41*d^19 - 112350430315654120415952896*a^
20*b^26*c^40*d^20 + 126417217514830317658046464*a^21*b^25*c^39*d^21 - 11953
7906081128203174281216*a^22*b^24*c^38*d^22 + 95089864774620999552335872*a^2
3*b^23*c^37*d^23 - 63545506634457987380412416*a^24*b^22*c^36*d^24 + 3552957
8846146774008070144*a^25*b^21*c^35*d^25 - 16501565732136655819636736*a^26*b
^20*c^34*d^26 + 6295808856090071441342464*a^27*b^19*c^33*d^27 - 19408479842
49953081884672*a^28*b^18*c^32*d^28 + 471738031694568778366976*a^29*b^17*c^3
1*d^29 - 87073083559063809163264*a^30*b^16*c^30*d^30 + 114765704194349502 30
016*a^31*b^15*c^29*d^31 - 962765689885917446144*a^32*b^14*c^28*d^32 + 38651
17733118646681 6*a^33*b^13*c^27*d^33) - (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c
^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3
+ 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 -
```

```
 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1
056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4)*((x^(1/2)*(18446744073709
551616*a^11*b^39*c^68*d^4 - 479615345916448342016*a^12*b^38*c^67*d^5 + 5995
191823955604275200*a^13*b^37*c^66*d^6 - 47961534591644834201600*a^14*b^36*c
^65*d^7 + 275778823901957796659200*a^15*b^35*c^64*d^8 - 1212936383169193658
286080*a^16*b^34*c^63*d^9 + 4232993998288506144686080*a^17*b^33*c^62*d^10 -
 11941164077799654041845760*a^18*b^32*c^61*d^11 + 2710486932133305647104000
0*a^19*b^31*c^60*d^12 - 46637619173392487079215104*a^20*b^30*c^59*d^13 + 43
611606538557895133364224*a^21*b^29*c^58*d^14 + 72781112360087761599856640*a
^22*b^28*c^57*d^15 - 523234066593179210717593600*a^23*b^27*c^56*d^16 + 1723
753001020797184743833600*a^24*b^26*c^55*d^17 - 4269437167365872814842183680
*a^25*b^25*c^54*d^18 + 8727322757849829186700574720*a^26*b^24*c^53*d^19 - 1
5215326043975142249374679040*a^27*b^23*c^52*d^20 + 229626584632465196255805
44000*a^28*b^22*c^51*d^21 - 30231538828274701475145318400*a^29*b^21*c^50*d^
22 + 34870163031766389952882933760*a^30*b^20*c^49*d^23 - 353167182383361584
89724846080*a^31*b^19*c^48*d^24 + 31433146498544749041648926720*a^32*b^18*c
^47*d^25 - 24575140799491012895231180800*a^33*b^17*c^46*d^26 + 168507549614
33442876234137600*a^34*b^16*c^45*d^27 - 10105200492115418262179676160*a^35*
b^15*c^44*d^28 + 5278011312905736232783314944*a^36*b^14*c^43*d^29 - 2387248
399405916166169821184*a^37*b^13*c^42*d^30 + 927828632312674738870681600*a^3
8*b^12*c^41*d^31 - 306693733103726739901644800*a^39*b^11*c^40*d^32 + 850380
75959446046066606080*a^40*b^10*c^39*d^33 - 19409595119210898894356480*a^41*
b^9*c^38*d^34 + 3551400405635812871372800*a^42*b^8*c^37*d^35 - 500844593983
932480880640*a^43*b^7*c^36*d^36 + 51111802530990496153600*a^44*b^6*c^35*d^3
7 - 3359577235627333124096*a^45*b^5*c^34*d^38 + 106807368762718683136*a^46*
b^4*c^33*d^39) + (-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^1
1*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^
4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^
7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10
- 192*a^18*b*c*d^11))^(1/4)*(36893488147419103232*a^13*b^38*c^71*d^4 - 1069
911156275153993728*a^14*b^37*c^70*d^5 + 14978756187852155912192*a^15*b^36*c
^69*d^6 - 134999037738929532960768*a^16*b^35*c^68*d^7 + 8820160799048623215
08352*a^17*b^34*c^67*d^8 - 4465630463459278708539392*a^18*b^33*c^66*d^9 + 1
8321125205332103390035968*a^19*b^32*c^65*d^10 - 63021545228377166868119552*
a^20*b^31*c^64*d^11 + 187018029382071665408606208*a^21*b^30*c^63*d^12 - 490
713180393588600090918912*a^22*b^29*c^62*d^13 + 1161438545048511890042388480
*a^23*b^28*c^61*d^14 - 2512974056309066269898833920*a^24*b^27*c^60*d^15 + 4
997541469898172697285754880*a^25*b^26*c^59*d^16 - 9119889428539397211967979
520*a^26*b^25*c^58*d^17 + 15181544306461039744285409280*a^27*b^24*c^57*d^18
 - 22888317982577902576352624640*a^28*b^23*c^56*d^19 + 31049708276802113763
866050560*a^29*b^22*c^55*d^20 - 37706614244767692268335267840*a^30*b^21*c^5
4*d^21 + 40833216619792283792163471360*a^31*b^20*c^53*d^22 - 39312168062751
093709382615040*a^32*b^19*c^52*d^23 + 33557805042801128843488788480*a^33*b^
18*c^51*d^24 - 25329188887155786370693201920*a^34*b^17*c^50*d^25 + 16851463
310911481777624186880*a^35*b^16*c^49*d^26 - 9843609097631363291959787520*a^
```

```
36*b^15*c^48*d^27 + 50238161474656361274723532 80*a^37*b^14*c^47*d^28 - 2226
0545772723656122611793 92*a^38*b^13*c^46*d^29 + 849419752718963326077370368*
a^39*b^12*c^45*d^30 - 2761729236011130413404651 52*a^40*b^11*c^44*d^31 + 754
41341408208223215812608*a^41*b^10*c^43*d^32 - 16988052798101408932954112*a^
42*b^9*c^42*d^33 + 30704109754442567720632 32*a^43*b^8*c^41*d^34 - 428198505
575496787427328*a^44*b^7*c^40*d^35 + 43254156088335077998592*a^45*b^6*c^39*
d^36 - 28165872357545271623 68*a^46*b^5*c^38*d^37 + 88774955854727217152*a^4
7*b^4*c^37*d^38)*1i)*(-b^15/(16*a^19*d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11
*c^11*d + 1056*a^9*b^10*c^10*d^2 - 3520*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^
8*d^4 - 12672*a^12*b^7*c^7*d^5 + 14784*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^
5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d
^10 - 192*a^18*b*c*d^11))^(3/4)*1i + 1188950301625810944 0*a^9*b^38*c^56*d^7
 - 217253646024352727040*a^10*b^37*c^55*d^8 + 1879766455667426066432*a^11*b
^36*c^54*d^9 - 10237150327374383939584*a^12*b^35*c^53*d^10 + 37711511320670
913953792*a^13*b^34*c^52*d^11 - 77353208427556875796480*a^14*b^33*c^51*d^12
 - 127627238172719495249920*a^15*b^32*c^50*d^13 + 2130084466030987427446784
*a^16*b^31*c^49*d^14 - 11885048527140028256616448*a^17*b^30*c^48*d^15 + 456
90531361686842972831744*a^18*b^29*c^47*d^16 - 135851929384595950057553920*a
^19*b^28*c^46*d^17 + 326376775711477371051704320*a^20*b^27*c^45*d^18 - 6483
53352496064059760705536*a^21*b^26*c^44*d^19 + 1080394184249474617790431232*
a^22*b^25*c^43*d^20 - 1524725339928630029153468416*a^23*b^24*c^42*d^21 + 18
34102420924176937716285440*a^24*b^23*c^41*d^22 - 188806274222317100842614 78
40*a^25*b^22*c^40*d^23 + 1666588213584359199850102784*a^26*b^21*c^39*d^24 -
 1261562453800014779376467968*a^27*b^20*c^38*d^25 + 817528072151542384572 76
0064*a^28*b^19*c^37*d^26 - 451847698934426396681830400*a^29*b^18*c^36*d^27
+ 211721890947778234390937600*a^30*b^17*c^35*d^28 - 833662487808380009772 48
256*a^31*b^16*c^34*d^29 + 27241266624044306322685952*a^32*b^15*c^33*d^30 -
7257515800860571589410816*a^33*b^14*c^32*d^31 + 1536699518639901947985920*a
^34*b^13*c^31*d^32 - 248859486128715197317120*a^35*b^12*c^30*d^33 + 2896164
2042172523937792*a^36*b^11*c^29*d^34 - 2157438443758953693184*a^37*b^10*c^2
8*d^35 + 77302354662372933632*a^38*b^9*c^27*d^36)*1i)*1i))*(-b^15/(16*a^19*
d^12 + 16*a^7*b^12*c^12 - 192*a^8*b^11*c^11*d + 1056*a^9*b^10*c^10*d^2 - 35
20*a^10*b^9*c^9*d^3 + 7920*a^11*b^8*c^8*d^4 - 12672*a^12*b^7*c^7*d^5 + 1478
4*a^13*b^6*c^6*d^6 - 12672*a^14*b^5*c^5*d^7 + 7920*a^15*b^4*c^4*d^8 - 3520*
a^16*b^3*c^3*d^9 + 1056*a^17*b^2*c^2*d^10 - 192*a^18*b*c*d^11))^(1/4) - (2/
(3*a*c) + (x^2*(121*a^2*d^3 + 64*b^2*c^2*d - 209*a*b*c*d^2))/(48*a*c*(b^2*c
^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^4*(77*a^2*d^2 + 32*b^2*c^2 - 133*a*
b*c*d))/(48*a*c^2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))/(c^2*x^(3/2) + d^2*
x^(11/2) + 2*c*d*x^(7/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.469 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=743

$$\frac{b^{17/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^3}+-$$

Rubi [A] time = 1.23, antiderivative size = 743, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{117a^2d^2-189abcd+32b^2c^2}{80ac^3x^{5/2}(bc-ad)^2}+\frac{-189a^2bcd^2+117a^3d^3+32ab^2c^2d+32b^3c^3}{16a^2c^4\sqrt{x}(bc-ad)^2}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{17/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{17/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{17/4}(bc-ad)^3}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{17/4}(bc-ad)^3}+\frac{b^{17/4}\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^3}+\frac{b^{17/4}\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{d(21bc-13ad)}{16c^2x^{5/2}\left(c+dx^2\right)(bc-ad)^2}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(32*b^2*c^2 - 189*a*b*c*d + 117*a^2*d^2)/(80*a*c^3*(b*c - a*d)^2*x^(5/2))
+ (32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2 + 117*a^3*d^3)/(16*a^2*c^4
*(b*c - a*d)^2*Sqrt[x]) - d/(4*c*(b*c - a*d)*x^(5/2)*(c + d*x^2)^2) - (d*(2
1*b*c - 13*a*d))/(16*c^2*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) - (b^(17/4)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^3)
 + (b^(17/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(9/4
)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^
3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (b^(17/4)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(
9/4)*(b*c - a*d)^3) - (b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(221*b^2*c^2
- 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 -
 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{8bc-13ad-13bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{4c(bc-ad)}$$

$$= -\frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} - \frac{d(21bc-13ad)}{16c^2(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \text{Subst}\left(\int \frac{32b^2c^2-}{}\right.$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} - \frac{d}{4c(bc-ad)x^{5/2}\left(c+dx^2\right)^2} - \frac{d(21bc-13ad)}{16c^2(bc-ad)^2x^{5/2}\left(c+\right.}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

$$= -\frac{\frac{32b^2c}{a}-189bd+\frac{117ad^2}{c}}{80c^2(bc-ad)^2x^{5/2}} + \frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} - \frac{}{4c(bc}$$

Mathematica [A] time = 6.18, size = 720, normalized size = 0.97

$$\frac{b^{17/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{17/4}\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{17/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(ad-bc)^3}-\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(ad-bc)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(ad-bc)^3}+\frac{d^{9/4}\left(117a^2d^2-306abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(ad-bc)^3}+\frac{2(3ad+bc)}{a^2c^4\sqrt{x}}-\frac{d^3x^{3/2}(29bc-21ad)}{16c^4\left(c+dx^2\right)(bc-ad)^2}-\frac{d^3x^{3/2}}{4c^3\left(c+dx^2\right)^2(bc-ad)}-\frac{2}{5ac^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -2/(5*a*c^3*x^(5/2)) + (2*(b*c + 3*a*d))/(a^2*c^4*Sqrt[x]) - (d^3*x^(3/2))/
(4*c^3*(b*c - a*d)*(c + d*x^2)^2) - (d^3*(29*b*c - 21*a*d)*x^(3/2))/(16*c^4
*(b*c - a*d)^2*(c + d*x^2)) + (b^(17/4)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1
/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(17/4
)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(Sqrt[2]
*a^(9/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2
)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*S
qrt[2]*c^(17/4)*(-(b*c) + a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 1
17*a^2*d^2)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))]
)/(32*Sqrt[2]*c^(17/4)*(-(b*c) + a*d)^3) + (b^(17/4)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (
b^(17/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqr
t[2]*a^(9/4)*(b*c - a*d)^3) + (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2
*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[
2]*c^(17/4)*(-(b*c) + a*d)^3) - (d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a
^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqr
t[2]*c^(17/4)*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.34, size = 546, normalized size = 0.73

$$\frac{b^{17/4}\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{b}}-\frac{\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}}{\sqrt{x}}\right)}{\sqrt{2}a^{9/4}(ad-bc)^3}+\frac{b^{17/4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{9/4}(ad-bc)^3}+\frac{\left(117a^2d^{17/4}-306abcd^{13/4}+221b^2c^2d^{9/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3}+\frac{\left(117a^2d^{17/4}-306abcd^{13/4}+221b^2c^2d^{9/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3}+\frac{-32a^3c^3d^2+416a^3c^2d^3x^2+1053a^3cd^4x^4+585a^3d^5x^6+64a^2bc^4d-672a^2bc^3d^2x^2-1701a^2bc^2d^3x^4-945a^2bcd^4x^6-32ab^2c^5+96ab^2c^4dx^2+288ab^2c^3d^2x^4+160ab^2c^2d^3x^6+160b^3c^5x^2+320b^3c^4dx^4+160b^3c^3d^2x^6}{80a^2c^4x^{5/2}\left(c+dx^2\right)^2(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] (-32*a*b^2*c^5 + 64*a^2*b*c^4*d - 32*a^3*c^3*d^2 + 160*b^3*c^5*x^2 + 96*a*b
^2*c^4*d*x^2 - 672*a^2*b*c^3*d^2*x^2 + 416*a^3*c^2*d^3*x^2 + 320*b^3*c^4*d*
x^4 + 288*a*b^2*c^3*d^2*x^4 - 1701*a^2*b*c^2*d^3*x^4 + 1053*a^3*c*d^4*x^4 +
 160*b^3*c^3*d^2*x^6 + 160*a*b^2*c^2*d^3*x^6 - 945*a^2*b*c*d^4*x^6 + 585*a^
3*d^5*x^6)/(80*a^2*c^4*(-(b*c) + a*d)^2*x^(5/2)*(c + d*x^2)^2) + (b^(17/4)*
ArcTan[(a^(1/4)/(Sqrt[2]*b^(1/4)) - (b^(1/4)*x)/(Sqrt[2]*a^(1/4)))/Sqrt[x]]
)/(Sqrt[2]*a^(9/4)*(-(b*c) + a*d)^3) + ((221*b^2*c^2*d^(9/4) - 306*a*b*c*d^
(13/4) + 117*a^2*d^(17/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^3) + (b^(17/4)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(9/4)*(-
(b*c) + a*d)^3) + ((221*b^2*c^2*d^(9/4) - 306*a*b*c*d^(13/4) + 117*a^2*d^(1
```

```
7/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32
*Sqrt[2]*c^(17/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.86, size = 1000, normalized size = 1.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] (a*b^3)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c
*d^2 - sqrt(2)*a^6*d^3) + (a*b^3)^(3/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^
2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/2*(a*b^3)^(3/4)*b^2*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*s
qrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) + 1/2*(a*b^
3)^(3/4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3
*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^
3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d
^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^
2 - sqrt(2)*a^3*c^5*d^3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3
/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d +
 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) + 1/64*(221*(c*d^3)^(3/4)*b
^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^
7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/64*(221*(c*d^3)^(3
/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(-s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*
b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/16*(29*b*c*d
^4*x^(7/2) - 21*a*d^5*x^(7/2) + 33*b*c^2*d^3*x^(3/2) - 25*a*c*d^4*x^(3/2))/
((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 2/5*(5*b*c*x^2 + 15
*a*d*x^2 - a*c)/(a^2*c^4*x^(5/2))
```

maple [A] time = 0.03, size = 933, normalized size = 1.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] -1/4*b^4/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-1/2*b^4/a^2/(
a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-1/2*b^
4/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
+21/16*d^6/c^4/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*a^2-25/8*d^5/c^3/(a*d-b*c)^3
/(d*x^2+c)^2*x^(7/2)*a*b+29/16*d^4/c^2/(a*d-b*c)^3/(d*x^2+c)^2*x^(7/2)*b^2+
25/16*d^5/c^3/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*a^2-29/8*d^4/c^2/(a*d-b*c)^3/
(d*x^2+c)^2*x^(3/2)*a*b+33/16*d^3/c/(a*d-b*c)^3/(d*x^2+c)^2*x^(3/2)*b^2+117
/128*d^4/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*2^(1/2)*
x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+117/64*d^
4/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)+117/64*d^4/c^4/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)-153/64*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*ln((x
-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c
/d)^(1/2)))-153/32*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*b*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)+1)-153/32*d^3/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*
b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+221/128*d^2/c^2/(a*d-b*c)^3/(c/d)^(
1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1
/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+221/64*d^2/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^
(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+221/64*d^2/c^2/(a*d-b*c)^3/
(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a/c^3/x^(
5/2)+6/a/c^4/x^(1/2)*d+2/a^2/c^3/x^(1/2)*b
```

maxima [A] time = 2.76, size = 756, normalized size = 1.02

$$\frac{b^{5}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{4\left(a^{2}b^{3}c^{3}-3a^{3}b^{2}c^{2}d+3a^{4}bcd^{2}-a^{5}d^{3}\right)}-\frac{\left(221b^{2}c^{2}d^{3}-306abcd^{4}+117a^{2}d^{5}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{3}c^{7}-3ab^{2}c^{6}d+3a^{2}bc^{5}d^{2}-a^{3}c^{4}d^{3}\right)}-\frac{32ab^{2}c^{5}-64a^{2}bc^{4}d+32a^{3}c^{3}d^{2}-5\left(32b^{3}c^{3}d^{2}+32ab^{2}c^{2}d^{3}-189a^{2}bcd^{4}+117a^{3}d^{5}\right)x^{6}-\left(320b^{3}c^{4}d+288ab^{2}c^{3}d^{2}-1701a^{2}bc^{2}d^{3}+1053a^{3}cd^{4}\right)x^{4}-32\left(5b^{3}c^{5}+3ab^{2}c^{4}d-21a^{2}bc^{3}d^{2}+13a^{3}c^{2}d^{3}\right)x^{2}}{80\left(\left(a^{2}b^{2}c^{6}d^{2}-2a^{3}bc^{5}d^{3}+a^{4}c^{4}d^{4}\right)x^{\frac{13}{2}}+2\left(a^{2}b^{2}c^{7}d-2a^{3}bc^{6}d^{2}+a^{4}c^{5}d^{3}\right)x^{\frac{9}{2}}+\left(a^{2}b^{2}c^{8}-2a^{3}bc^{7}d+a^{4}c^{6}d^{2}\right)x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/128*(221*b^2*c^
2*d^3 - 306*a*b*c*d^4 + 117*a^2*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*
```

```
sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
- 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
 - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1
/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3
*c^4*d^3) - 1/80*(32*a*b^2*c^5 - 64*a^2*b*c^4*d + 32*a^3*c^3*d^2 - 5*(32*b^
3*c^3*d^2 + 32*a*b^2*c^2*d^3 - 189*a^2*b*c*d^4 + 117*a^3*d^5)*x^6 - (320*b^
3*c^4*d + 288*a*b^2*c^3*d^2 - 1701*a^2*b*c^2*d^3 + 1053*a^3*c*d^4)*x^4 - 32
*(5*b^3*c^5 + 3*a*b^2*c^4*d - 21*a^2*b*c^3*d^2 + 13*a^3*c^2*d^3)*x^2)/((a^2
*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^(13/2) + 2*(a^2*b^2*c^7*d -
 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^(9/2) + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^
4*c^6*d^2)*x^(5/2))
```

mupad [B] time = 12.55, size = 36917, normalized size = 49.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((2*x^2*(13*a*d + 5*b*c))/(5*a^2*c^2) - 2/(5*a*c) + (x^4*(1053*a^3*d^4 + 32
0*b^3*c^3*d + 288*a*b^2*c^2*d^2 - 1701*a^2*b*c*d^3))/(80*a^2*c^2*(b^2*c^3 +
 a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^6*(117*a^3*d^3 + 32*b^3*c^3 + 32*a*b^2*
c^2*d - 189*a^2*b*c*d^2))/(16*a^2*c^3*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))
/(c^2*x^(5/2) + d^2*x^(13/2) + 2*c*d*x^(9/2)) - atan((a^11*b^22*c^29*x^(1/2
)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^1
1*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^1
4*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17
*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*
c*d^11))^(5/4)*33554432i + a^19*b^10*d^17*x^(1/2)*(-b^17/(16*a^21*d^12 + 16
*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12
*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*
b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^
3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*374777442i +
 a^33*c^7*d^22*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b
^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^
8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^
5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c
^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*448561152i + a^8*b^21*c^11*d^6*x^(1/2)*
(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*
b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*
b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b
^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*
d^11))^(1/4)*100026368i - a^9*b^20*c^10*d^7*x^(1/2)*(-b^17/(16*a^21*d^12 +
16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^
```

```
12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^1
5*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*
b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*276996096i
 + a^10*b^19*c^9*d^8*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*
a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a
^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a
^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19
*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*297676800i + a^11*b^18*c^8*d^9*x^
(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056
*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672
*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*
a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^2
0*b*c*d^11))^(1/4)*4624241570i - a^12*b^17*c^7*d^10*x^(1/2)*(-b^17/(16*a^21
*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 -
 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 1
4784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 35
20*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*26
395336656i + a^13*b^16*c^6*d^11*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*
c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d
^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^
6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9
 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*64982364408i - a^14*b^
15*c^5*d^12*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11
*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c
^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c
^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*
d^10 - 192*a^20*b*c*d^11))^(1/4)*92624356656i + a^15*b^14*c^4*d^13*x^(1/2)*
(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*
b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*
b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b
^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*
d^11))^(1/4)*83665919628i - a^16*b^13*c^3*d^14*x^(1/2)*(-b^17/(16*a^21*d^12
 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520
*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*
a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^
18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*4903642
4112i + a^17*b^12*c^2*d^15*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12
- 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 +
7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 1
2672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 105
6*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*18213050232i + a^13*b^20*c^
27*d^2*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11
*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^
4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^
7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10
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- 192*a^20*b*c*d^11))^(5/4)*2214592512i - a^14*b^19*c^26*d^3*x^(1/2)*(-b^17
/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c
^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^
7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4
*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))
^(5/4)*7381975040i + a^15*b^18*c^25*d^4*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a
^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b
^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^
6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*
c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*16609443840i -
 a^16*b^17*c^24*d^5*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a
^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^
13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^
16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*
b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*26575110144i + a^17*b^16*c^23*d^6*
x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 10
56*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 126
72*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 792
0*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a
^20*b*c*d^11))^(5/4)*32604717056i - a^18*b^15*c^22*d^7*x^(1/2)*(-b^17/(16*a
^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^
2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5
+ 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 -
 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)
*50212110336i + a^19*b^14*c^21*d^8*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^
12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^
9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6
*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d
^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*180183367680i - a^2
0*b^13*c^20*d^9*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*
b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b
^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b
^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*
c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*711482933248i + a^21*b^12*c^19*d^10*x^
(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056
*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672
*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*
a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^2
0*b*c*d^11))^(5/4)*2112400785408i - a^22*b^11*c^18*d^11*x^(1/2)*(-b^17/(16*
a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d
^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5
 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8
- 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4
)*4669808050176i + a^23*b^10*c^17*d^12*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^
9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^
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9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6
*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c
^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*7892313571328i
- a^24*b^9*c^16*d^13*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*
a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a
^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a
^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19
*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*10394916618240i + a^25*b^8*c^15*d
^14*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d
+ 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 -
 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 +
 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 1
92*a^20*b*c*d^11))^(5/4)*10783480283136i - a^26*b^7*c^14*d^15*x^(1/2)*(-b^1
7/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*
c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c
^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^
4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11)
)^(5/4)*8841322299392i + a^27*b^6*c^13*d^16*x^(1/2)*(-b^17/(16*a^21*d^12 +
16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^
12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^1
5*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*
b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*5711010201
600i - a^28*b^5*c^12*d^17*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 -
 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7
920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12
672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056
*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*2876224045056i + a^29*b^4*c^
11*d^18*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^1
1*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d
^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d
^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10
 - 192*a^20*b*c*d^11))^(5/4)*1107358515200i - a^30*b^3*c^10*d^19*x^(1/2)*(-
b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^
10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^
7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4
*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^
11))^(5/4)*315126448128i + a^31*b^2*c^9*d^20*x^(1/2)*(-b^17/(16*a^21*d^12 +
 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a
^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^
15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18
*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*625237032
96i - a^18*b^11*c*d^16*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 19
2*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920
*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672
*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^
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19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*3920748624i - a^12*b^21*c^28*d*
x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 10
56*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 126
72*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 792
0*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a
^20*b*c*d^11))^(5/4)*402653184i - a^32*b*c^8*d^21*x^(1/2)*(-b^17/(16*a^21*d
^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3
520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 147
84*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520
*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*7729
053696i)/(1048576*b^28*c^14 + 187388721*a^14*b^14*d^14 - 1398208149*a^13*b^
15*c*d^13 + 6291456*a^2*b^26*c^12*d^2 + 10485760*a^3*b^25*c^11*d^3 + 157286
40*a^4*b^24*c^10*d^4 + 22020096*a^5*b^23*c^9*d^5 + 29360128*a^6*b^22*c^8*d^
6 + 37748736*a^7*b^21*c^7*d^7 + 47185920*a^8*b^20*c^6*d^8 - 2327771601*a^9*
b^19*c^5*d^9 + 6124562037*a^10*b^18*c^4*d^10 - 7086995370*a^11*b^17*c^3*d^1
1 + 4349734506*a^12*b^16*c^2*d^12 + 3145728*a*b^27*c^13*d))*(-b^17/(16*a^21
*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 -
 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 1
4784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 35
20*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4)*2i
 - 2*atan((1099511627776*a^3*b^22*c^43*x^(1/2)*(-(187388721*a^8*d^17 + 2385
443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d
^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 2451821
2056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^
16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d
^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 83047219
20*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^
23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 36909
87520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c
^28*d))^(5/4) + 14698451828736*a^25*c^21*d^22*x^(1/2)*(-(187388721*a^8*d^17
 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^
6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 -
24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7
*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b
*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8
304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6
*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8
- 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a
*b^11*c^28*d))^(5/4) + 3277664026624*b^21*c^25*d^6*x^(1/2)*(-(187388721*a^8
*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a
^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^
13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196037431
2*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a
^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^
3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550214758
```

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4*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21
*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326
592*a*b^11*c^28*d))^(1/4) + 9754273382400*a^2*b^19*c^23*d^8*x^(1/2)*(-(1873
88721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249
1182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b
^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 -
1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20
1326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^
9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1
5502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8
*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10
 - 201326592*a*b^11*c^28*d))^(1/4) + 151527147765760*a^3*b^18*c^22*d^9*x^(1
/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*
d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 418329
59814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*
c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^1
7*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 369098
7520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c
^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 830
4721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^
2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4) - 864922391543808*a^4*b^17*c^
21*d^10*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 1321168586
4*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d
^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525
116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777
216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*
d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 132875550
72*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^
22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 110729
6256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4) + 212934211692134
4*a^5*b^16*c^20*d^11*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9
 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*
a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d
^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12
*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a
^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^
4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 132875550
72*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20
*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4) - 30
35114918903808*a^6*b^15*c^19*d^12*x^(1/2)*(-(187388721*a^8*d^17 + 238544328
1*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 -
 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*
a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(
16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 +
 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^
```

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4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^
6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520
*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d
))^(1/4) + 2741564854370304*a^7*b^14*c^18*d^13*x^(1/2)*(-(187388721*a^8*d^1
7 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b
^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 -
 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^
7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*
b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 +
8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^
6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8
 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*
a*b^11*c^28*d))^(1/4) - 1606825545302016*a^8*b^13*c^17*d^14*x^(1/2)*(-(1873
88721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249
1182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b
^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 -
1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20
1326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^
9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1
5502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8
*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10
 - 201326592*a*b^11*c^28*d))^(1/4) + 596805230002176*a^9*b^12*c^16*d^15*x^(
1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7
*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832
959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2
*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^
17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 36909
87520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*
c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 83
04721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b
^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4) - 128475090911232*a^10*b^11*
c^15*d^16*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685
864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5
*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 91065
25116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 167
77216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^2
7*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 1328755
5072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*
c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107
296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4) + 1228070721945
6*a^11*b^10*c^14*d^17*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^
9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328
*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*
d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^1
2*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*
```

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a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d
^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555
072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^2
0*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4) + 7
2567767433216*a^5*b^20*c^41*d^2*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*
b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 4
6312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^
5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16
777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1
107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*
b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6
- 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a
^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))
^(5/4) - 241892558110720*a^6*b^19*c^40*d^3*x^(1/2)*(-(187388721*a^8*d^17 +
2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c
^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 245
18212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*
c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^
18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304
721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^
6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3
690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^
11*c^28*d))^(5/4) + 544258255749120*a^7*b^18*c^39*d^4*x^(1/2)*(-(187388721*
a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249118220
4*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4
*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196037
4312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20132659
2*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26
*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550214
7584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c
^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201
326592*a*b^11*c^28*d))^(5/4) - 870813209198592*a^8*b^17*c^38*d^5*x^(1/2)*(-
(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 +
 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*
a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^
15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12
 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a
^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^
5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 830472192
0*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19
*d^10 - 201326592*a*b^11*c^28*d))^(5/4) + 1068391368491008*a^9*b^16*c^37*d^
6*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^
7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 +
41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^
6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^
```

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12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 -
3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5
*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7
 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a
^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) - 1645350431490048*a^10
*b^15*c^36*d^7*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 132
11685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^
5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 +
9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29
+ 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^1
0*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13
287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7
*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 +
 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) + 59042485
92138240*a^11*b^14*c^35*d^8*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*
c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312
178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^
3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(167772
16*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 11072
96256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*
c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13
287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b
^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/
4) - 23313872756670464*a^12*b^13*c^34*d^9*x^(1/2)*(-(187388721*a^8*d^17 + 2
385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^
6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 2451
8212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c
*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^1
8*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 83047
21920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6
*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 36
90987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^1
1*c^28*d))^(5/4) + 69219148936249344*a^13*b^12*c^33*d^10*x^(1/2)*(-(1873887
21*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249118
2204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*
c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196
0374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20132
6592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c
^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550
2147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^
4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 -
201326592*a*b^11*c^28*d))^(5/4) - 153020270188167168*a^14*b^11*c^32*d^11*x^
(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^
7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 4183
2959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^
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2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c
^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690
987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7
*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8
304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*
b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) + 258615331105275904*a^15*b
^10*c^31*d^12*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 1321
1685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5
*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9
106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 +
 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10
*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 132
87555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*
b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 +
1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) - 340620627
746488320*a^16*b^9*c^30*d^13*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8
*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 4631
2178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b
^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777
216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107
296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8
*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 1
3287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*
b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5
/4) + 353353081917800448*a^17*b^8*c^29*d^14*x^(1/2)*(-(187388721*a^8*d^17 +
 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*
c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24
518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b
*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c
^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 830
4721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b
^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 -
3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b
^11*c^28*d))^(5/4) - 289712449106477056*a^18*b^7*c^28*d^15*x^(1/2)*(-(18738
8721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491
182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^
4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1
960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201
326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9
*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15
502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*
b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10
- 201326592*a*b^11*c^28*d))^(5/4) + 187138382286028800*a^19*b^6*c^27*d^16*x
^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c
^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 418
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32959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b
^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*
c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 369
0987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^
7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 +
8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10
*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) - 94248109508395008*a^20*b
^5*c^26*d^17*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211
685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*
c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 91
06525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 +
16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*
c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 1328
7555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b
^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1
107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) + 3628592382
6073600*a^21*b^4*c^25*d^18*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c
^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 463121
78328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3
*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(1677721
6*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 110729
6256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c
^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 132
87555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^
3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4
) - 10326063452258304*a^22*b^3*c^24*d^19*x^(1/2)*(-(187388721*a^8*d^17 + 23
85443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6
*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518
212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*
d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18
*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 830472
1920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*
c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 369
0987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11
*c^28*d))^(5/4) + 2048776709603328*a^23*b^2*c^23*d^20*x^(1/2)*(-(187388721*
a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249118220
4*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4
*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196037
4312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20132659
2*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26
*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550214
7584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c
^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201
326592*a*b^11*c^28*d))^(5/4) - 9076608073728*a*b^20*c^24*d^7*x^(1/2)*(-(187
388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 324
```

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91182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*
b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 -
 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 2
01326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b
^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 +
15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^
8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^1
0 - 201326592*a*b^11*c^28*d))^(1/4) - 13194139533312*a^4*b^21*c^42*d*x^(1/2
)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^
10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959
814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^
2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*
d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 36909875
20*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^2
4*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 83047
21920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*
c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4) - 253265631510528*a^24*b*c^22*d
^21*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*
b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12
+ 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*
a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*
a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2
- 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a
^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d
^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256
*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4))/(30012421606973*a^2
0*d^28 + 11318183591936*b^20*c^20*d^8 - 13059442606080*a*b^19*c^19*d^9 + 99
39358580736*a^2*b^18*c^18*d^10 + 490619273216*a^3*b^17*c^17*d^11 + 25737591
52128*a^4*b^16*c^16*d^12 + 3011845816320*a^5*b^15*c^15*d^13 + 348429218880
*a^6*b^14*c^14*d^14 + 3991098359808*a^7*b^13*c^13*d^15 + 4532264239104*a^8*
b^12*c^12*d^16 - 25743035408641173*a^9*b^11*c^11*d^17 + 172320214465160559*
a^10*b^10*c^10*d^18 - 537854694138813555*a^11*b^9*c^9*d^19 + 10286704896839
26929*a^12*b^8*c^8*d^20 - 1335978873710775378*a^13*b^7*c^7*d^21 + 123502477
0525419510*a^14*b^6*c^6*d^22 - 828236972743874694*a^15*b^5*c^5*d^23 + 40259
0417597719650*a^16*b^4*c^4*d^24 - 138920444110237257*a^17*b^3*c^3*d^25 + 32
396642626079979*a^18*b^2*c^2*d^26 - 4594209085368279*a^19*b*c*d^27))*(-(187
388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 324
91182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*
b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 -
 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 2
01326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b
^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 +
15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^
8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^1
0 - 201326592*a*b^11*c^28*d))^(1/4) - atan((a^3*b^22*c^43*x^(1/2)*(-(187388
```

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721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 324911
82204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4
*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 19
60374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 2013
26592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*
c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 155
02147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b
^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 -
 201326592*a*b^11*c^28*d))^(5/4)*1099511627776i + a^25*c^21*d^22*x^(1/2)*(-
(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 +
 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*
a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^
15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12
 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a
^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^
5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 830472192
0*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19
*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*14698451828736i + b^21*c^25*d^6*x^(
1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7
*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832
959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2
*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^
17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 36909
87520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*
c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 83
04721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b
^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4)*3277664026624i + a^2*b^19*c^
23*d^8*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864
*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^
12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 91065251
16*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 167772
16*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d
^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 1328755507
2*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^2
2*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296
256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4)*9754273382400i + a
^3*b^18*c^22*d^9*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 1
3211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*
b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14
+ 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^2
9 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b
^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 -
13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a
^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9
 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4)*15152714
```

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7765760i - a^4*b^17*c^21*d^10*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^
8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 463
12178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*
b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(1677
7216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 110
7296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^
8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 -
13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9
*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(
1/4)*864922391543808i + a^5*b^16*c^20*d^11*x^(1/2)*(-(187388721*a^8*d^17 +
2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c
^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 245
18212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*
c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^
18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304
721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^
6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3
690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^
11*c^28*d))^(1/4)*2129342116921344i - a^6*b^15*c^19*d^12*x^(1/2)*(-(1873887
21*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249118
2204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*
c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196
0374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20132
6592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c
^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550
2147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^
4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 -
201326592*a*b^11*c^28*d))^(1/4)*3035114918903808i + a^7*b^14*c^18*d^13*x^(1
/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*
d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 418329
59814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*
c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^1
7*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 369098
7520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c
^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 830
4721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^
2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4)*2741564854370304i - a^8*b^13*
c^17*d^14*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685
864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5
*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 91065
25116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 167
77216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^2
7*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 1328755
5072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*
c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107
```

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296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4)*160682554530201
6i + a^9*b^12*c^16*d^15*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*
d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 463121783
28*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^
3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b
^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 110729625
6*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25
*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 132875
55072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c
^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(1/4)*5
96805230002176i - a^10*b^11*c^15*d^16*x^(1/2)*(-(187388721*a^8*d^17 + 23854
43281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^
11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212
056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^1
6)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^
11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 830472192
0*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^2
3*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 369098
7520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^
28*d))^(1/4)*128475090911232i + a^11*b^10*c^14*d^17*x^(1/2)*(-(187388721*a^
8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*
a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d
^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 19603743
12*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*
a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d
^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 155021475
84*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^2
1*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 20132
6592*a*b^11*c^28*d))^(1/4)*12280707219456i + a^5*b^20*c^41*d^2*x^(1/2)*(-(1
87388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3
2491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^
4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15
 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 -
 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3
*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5
+ 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*
a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d
^10 - 201326592*a*b^11*c^28*d))^(5/4)*72567767433216i - a^6*b^19*c^40*d^3*x
^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c
^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 418
32959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b
^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*
c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 369
0987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^
7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 +
```

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8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10
*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*241892558110720i + a^7*b^1
8*c^39*d^4*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 1321168
5864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^
5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106
525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16
777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^
27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 132875
55072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5
*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 110
7296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*54425825574912
0i - a^8*b^17*c^38*d^5*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d
^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 4631217832
8*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3
*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^
12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256
*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*
d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 1328755
5072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^
20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*87
0813209198592i + a^9*b^16*c^37*d^6*x^(1/2)*(-(187388721*a^8*d^17 + 23854432
81*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11
- 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056
*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/
(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11
+ 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a
^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d
^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 369098752
0*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*
d))^(5/4)*1068391368491008i - a^10*b^15*c^36*d^7*x^(1/2)*(-(187388721*a^8*d
^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2
*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13
 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*
a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^1
1*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3
+ 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*
a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d
^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 20132659
2*a*b^11*c^28*d))^(5/4)*1645350431490048i + a^11*b^14*c^35*d^8*x^(1/2)*(-(1
87388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3
2491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^
4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15
 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 -
 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3
*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5
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+ 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*
a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d
^10 - 201326592*a*b^11*c^28*d))^(5/4)*5904248592138240i - a^12*b^13*c^34*d^
9*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^
7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 +
41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^
6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^
12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 -
3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5
*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7
 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a
^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*23313872756670464i + a^
13*b^12*c^33*d^10*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 -
13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3
*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14
 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^
29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*
b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 -
 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*
a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^
9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*6921914
8936249344i - a^14*b^11*c^32*d^11*x^(1/2)*(-(187388721*a^8*d^17 + 238544328
1*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 -
 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*
a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(
16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 +
 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^
4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^
6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520
*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d
))^(5/4)*153020270188167168i + a^15*b^10*c^31*d^12*x^(1/2)*(-(187388721*a^8
*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a
^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^
13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196037431
2*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a
^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^
3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550214758
4*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21
*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326
592*a*b^11*c^28*d))^(5/4)*258615331105275904i - a^16*b^9*c^30*d^13*x^(1/2)*
(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10
 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 4183295981
4*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*
d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^
12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520
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*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*
d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721
920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^
19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*340620627746488320i + a^17*b^8*c^
29*d^14*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 1321168586
4*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d
^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525
116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777
216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*
d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 132875550
72*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^
22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 110729
6256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*35335308191780044
8i - a^18*b^7*c^28*d^15*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*
d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 463121783
28*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^
3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b
^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 110729625
6*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25
*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 132875
55072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c
^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*2
89712449106477056i + a^19*b^6*c^27*d^16*x^(1/2)*(-(187388721*a^8*d^17 + 238
5443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*
d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 245182
12056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d
^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*
d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721
920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c
^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690
987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*
c^28*d))^(5/4)*187138382286028800i - a^20*b^5*c^26*d^17*x^(1/2)*(-(18738872
1*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182
204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c
^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960
374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326
592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^
26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502
147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4
*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 2
01326592*a*b^11*c^28*d))^(5/4)*94248109508395008i + a^21*b^4*c^25*d^18*x^(1
/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*
d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 418329
59814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*
c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^1
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7*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 369098
7520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c
^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 830
4721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^
2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*36285923826073600i - a^22*b^3
*c^24*d^19*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 1321168
5864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^
5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106
525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16
777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^
27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 132875
55072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5
*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 110
7296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)*10326063452258
304i + a^23*b^2*c^23*d^20*x^(1/2)*(-(187388721*a^8*d^17 + 2385443281*b^8*c^
8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^11 - 4631217
8328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 24518212056*a^5*b^3*
c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16)/(16777216
*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^11 + 1107296
256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920*a^4*b^8*c^
25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23*d^6 - 1328
7555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987520*a^9*b^3
*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^28*d))^(5/4)
*2048776709603328i - a*b^20*c^24*d^7*x^(1/2)*(-(187388721*a^8*d^17 + 238544
3281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6*c^6*d^1
1 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 245182120
56*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*b*c*d^16
)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*c^18*d^1
1 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 8304721920
*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*b^6*c^23
*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 - 3690987
520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*b^11*c^2
8*d))^(1/4)*9076608073728i - a^4*b^21*c^42*d*x^(1/2)*(-(187388721*a^8*d^17
+ 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 32491182204*a^2*b^6
*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4*d^13 - 2
4518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 1960374312*a^7*
b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 201326592*a^11*b*
c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26*d^3 + 83
04721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 15502147584*a^6*
b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c^21*d^8 -
 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201326592*a*
b^11*c^28*d))^(5/4)*13194139533312i - a^24*b*c^22*d^21*x^(1/2)*(-(187388721
*a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 324911822
04*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^
4*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 19603
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74312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 2013265
92*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^2
6*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 155021
47584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*
c^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 20
1326592*a*b^11*c^28*d))^(5/4)*253265631510528i)/(300124211606973*a^20*d^28
+ 11318183591936*b^20*c^20*d^8 - 13059442606080*a*b^19*c^19*d^9 + 993935858
0736*a^2*b^18*c^18*d^10 + 490619273216*a^3*b^17*c^17*d^11 + 2573759152128*a
^4*b^16*c^16*d^12 + 3011845816320*a^5*b^15*c^15*d^13 + 3484292218880*a^6*b^
14*c^14*d^14 + 3991098359808*a^7*b^13*c^13*d^15 + 4532264239104*a^8*b^12*c^
12*d^16 - 2574303540864117 3*a^9*b^11*c^11*d^17 + 172320214465160559*a^10*b^
10*c^10*d^18 - 537854694138813555*a^11*b^9*c^9*d^19 + 1028670489683926929*a
^12*b^8*c^8*d^20 - 1335978873710775378*a^13*b^7*c^7*d^21 + 1235024770525419
510*a^14*b^6*c^6*d^22 - 828236972743874694*a^15*b^5*c^5*d^23 + 402590417597
719650*a^16*b^4*c^4*d^24 - 138920444110237257*a^17*b^3*c^3*d^25 + 323966426
26079979*a^18*b^2*c^2*d^26 - 4594209085368279*a^19*b*c*d^27))*(-(187388721*
a^8*d^17 + 2385443281*b^8*c^8*d^9 - 13211685864*a*b^7*c^7*d^10 + 3249118220
4*a^2*b^6*c^6*d^11 - 46312178328*a^3*b^5*c^5*d^12 + 41832959814*a^4*b^4*c^4
*d^13 - 24518212056*a^5*b^3*c^3*d^14 + 9106525116*a^6*b^2*c^2*d^15 - 196037
4312*a^7*b*c*d^16)/(16777216*b^12*c^29 + 16777216*a^12*c^17*d^12 - 20132659
2*a^11*b*c^18*d^11 + 1107296256*a^2*b^10*c^27*d^2 - 3690987520*a^3*b^9*c^26
*d^3 + 8304721920*a^4*b^8*c^25*d^4 - 13287555072*a^5*b^7*c^24*d^5 + 1550214
7584*a^6*b^6*c^23*d^6 - 13287555072*a^7*b^5*c^22*d^7 + 8304721920*a^8*b^4*c
^21*d^8 - 3690987520*a^9*b^3*c^20*d^9 + 1107296256*a^10*b^2*c^19*d^10 - 201
326592*a*b^11*c^28*d))^(1/4)*2i - 2*atan((33554432*a^11*b^22*c^29*x^(1/2)*(
-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b
^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b
^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^
4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d
^11))^(5/4) + 374777442*a^19*b^10*d^17*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^
9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^
9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6
*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c
^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4) + 448561152*a^3
3*c^7*d^22*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*
c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^
8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^
5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d
^10 - 192*a^20*b*c*d^11))^(5/4) + 100026368*a^8*b^21*c^11*d^6*x^(1/2)*(-b^1
7/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*
c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c
^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^
4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11)
)^(1/4) - 276996096*a^9*b^20*c^10*d^7*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9
*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9
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*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*
c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^
3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4) + 297676800*a^10
*b^19*c^9*d^8*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^
11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8
*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5
*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^
2*d^10 - 192*a^20*b*c*d^11))^(1/4) + 4624241570*a^11*b^18*c^8*d^9*x^(1/2)*(
-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b
^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b
^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^
4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d
^11))^(1/4) - 26395336656*a^12*b^17*c^7*d^10*x^(1/2)*(-b^17/(16*a^21*d^12 +
 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a
^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^
15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18
*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4) + 6498236
4408*a^13*b^16*c^6*d^11*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 1
92*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 792
0*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 1267
2*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a
^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4) - 92624356656*a^14*b^15*c^5*d^
12*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d +
 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 -
12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 +
7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 19
2*a^20*b*c*d^11))^(1/4) + 83665919628*a^15*b^14*c^4*d^13*x^(1/2)*(-b^17/(16
*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*
d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^
5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8
 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/
4) - 49036424112*a^16*b^13*c^3*d^14*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b
^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c
^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^
6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*
d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4) + 18213050232*a^17
*b^12*c^2*d^15*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b
^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^
8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^
5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c
^2*d^10 - 192*a^20*b*c*d^11))^(1/4) + 2214592512*a^13*b^20*c^27*d^2*x^(1/2)
*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11
*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14
*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*
b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c
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*d^11))^(5/4) - 7381975040*a^14*b^19*c^26*d^3*x^(1/2)*(-b^17/(16*a^21*d^12
+ 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*
a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a
^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^1
8*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) + 166094
43840*a^15*b^18*c^25*d^4*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 -
192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 79
20*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 126
72*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*
a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) - 26575110144*a^16*b^17*c^24*
d^5*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d
+ 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 -
 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 +
 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 1
92*a^20*b*c*d^11))^(5/4) + 32604717056*a^17*b^16*c^23*d^6*x^(1/2)*(-b^17/(1
6*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10
*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d
^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^
8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5
/4) - 50212110336*a^18*b^15*c^22*d^7*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*
b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*
c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c
^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3
*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) + 180183367680*a^
19*b^14*c^21*d^8*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10
*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*
b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*
b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2
*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) - 711482933248*a^20*b^13*c^20*d^9*x^(
1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*
a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*
a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a
^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20
*b*c*d^11))^(5/4) + 2112400785408*a^21*b^12*c^19*d^10*x^(1/2)*(-b^17/(16*a^
21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2
 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 +
 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 -
3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)
- 4669808050176*a^22*b^11*c^18*d^11*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b
^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c
^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^
6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*
d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) + 7892313571328*a^
23*b^10*c^17*d^12*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^1
0*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13
```

```
*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16
*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^
2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) - 10394916618240*a^24*b^9*c^16*d^13*
x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 10
56*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 126
72*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 792
0*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a
^20*b*c*d^11))^(5/4) + 10783480283136*a^25*b^8*c^15*d^14*x^(1/2)*(-b^17/(16
*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*
d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^
5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8
 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/
4) - 8841322299392*a^26*b^7*c^14*d^15*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9
*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9
*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*
c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^
3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) + 5711010201600*
a^27*b^6*c^13*d^16*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^
10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^1
3*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^1
6*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b
^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) - 2876224045056*a^28*b^5*c^12*d^17*
x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 10
56*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 126
72*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 792
0*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a
^20*b*c*d^11))^(5/4) + 1107358515200*a^29*b^4*c^11*d^18*x^(1/2)*(-b^17/(16*
a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d
^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5
 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8
- 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4
) - 315126448128*a^30*b^3*c^10*d^19*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b
^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c
^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^
6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*
d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) + 62523703296*a^31
*b^2*c^9*d^20*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^
11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8
*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5
*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^
2*d^10 - 192*a^20*b*c*d^11))^(5/4) - 3920748624*a^18*b^11*c*d^16*x^(1/2)*(-
b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^
10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^
7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4
*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^
```

```
11))^(1/4) - 402653184*a^12*b^21*c^28*d*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a
^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b
^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^
6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*
c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4) - 7729053696*a
^32*b*c^8*d^21*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b
^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^
8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^
5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c
^2*d^10 - 192*a^20*b*c*d^11))^(5/4))/(1048576*b^28*c^14 + 187388721*a^14*b^
14*d^14 - 1398208149*a^13*b^15*c*d^13 + 6291456*a^2*b^26*c^12*d^2 + 1048576
0*a^3*b^25*c^11*d^3 + 15728640*a^4*b^24*c^10*d^4 + 22020096*a^5*b^23*c^9*d^
5 + 29360128*a^6*b^22*c^8*d^6 + 37748736*a^7*b^21*c^7*d^7 + 47185920*a^8*b^
20*c^6*d^8 - 2327771601*a^9*b^19*c^5*d^9 + 6124562037*a^10*b^18*c^4*d^10 -
7086995370*a^11*b^17*c^3*d^11 + 4349734506*a^12*b^16*c^2*d^12 + 3145728*a*b
^27*c^13*d))*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d
 + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4
- 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7
+ 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 -
192*a^20*b*c*d^11))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.470 $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=624

$$\frac{a\sqrt{x}}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{x}\,(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{a}\,(3ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-$$

Rubi [A] time = 0.76, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 470, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{a\sqrt{x}}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{\sqrt{x}\,(ad+bc)}{2b\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{a}\,(3ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,(3ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{c}\,(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{c}\,(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{a}\,(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{a}\,(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\sqrt[4]{c}\,(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}+\frac{\sqrt[4]{c}\,(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,\sqrt[4]{d}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b*c + a*d)*Sqrt[x])/(2*b*(b*c - a*d)^2*(c + d*x^2)) + (a*Sqrt[x])/(2*b*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1
/4)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d)^3) + (c^(1/4)*(3*b*
c + 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*d^(1/4
)*(b*c - a*d)^3) + (a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (a^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(8*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) - (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d
)^3) + (c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(8*Sqrt[2]*d^(1/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{ac+(-4bc-3ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{8abc^2-12bc(bc+a}{\left(a+bx^4\right)\left(c+dx^4\right.}\right.}{8bc(bc-ac}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(a(5bc+3ad))\,\text{Subst}\left(\right.}{2(bc-a}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,(5bc+3ad)\right)\text{Subs}}{4(bc}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{a}\,(5bc+3ad)\right)\text{Subs}}{8\sqrt{b}}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,(5bc+3ad)\log\left(\sqrt{\right.}}{8\sqrt{2}\,\sqrt[4]{b}}$$

$$= \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2\left(c+dx^2\right)} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{a}\,(5bc+3ad)\tan^{-1}\left(\right.}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-}$$

Mathematica [A] time = 0.92, size = 585, normalized size = 0.94

$$\frac{1}{16}\left(\frac{8a\sqrt{x}}{\left(a+bx^2\right)(bc-ad)^2}+\frac{8c\sqrt{x}}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{2}\sqrt[4]{a}(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}(bc-ad)^3}-\frac{\sqrt{2}\sqrt[4]{a}(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}(bc-ad)^3}+\frac{\sqrt{2}\sqrt[4]{c}(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}(ad-bc)^3}-\frac{\sqrt{2}\sqrt[4]{c}(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{\sqrt[4]{d}(ad-bc)^3}+\frac{2\sqrt{2}\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}(bc-ad)^3}-\frac{2\sqrt{2}\sqrt[4]{a}(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}(bc-ad)^3}+\frac{2\sqrt{2}\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt[4]{d}(ad-bc)^3}+\frac{2\sqrt{2}\sqrt[4]{c}(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt[4]{d}(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*a*Sqrt[x])/((b*c - a*d)^2*(a + b*x^2)) + (8*c*Sqrt[x])/((b*c - a*d)^2*(
c + d*x^2)) + (2*Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4
```

```
)*Sqrt[x])/a^(1/4)])/(b^(1/4)*(b*c - a*d)^3) - (2*Sqrt[2]*a^(1/4)*(5*b*c +
3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b^(1/4)*(b*c - a*d)^
3) + (2*Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(d^(1/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)
*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(d^(1/4)*(b*c - a*d)^3) + (
Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(b^(1/4)*(b*c - a*d)^3) - (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(b^(1/4)*(b*c
- a*d)^3) + (Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(d^(1/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*c^(1/4)
*(3*b*c + 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
)/(d^(1/4)*(-(b*c) + a*d)^3))/16
```

IntegrateAlgebraic **[A]** time = 1.43, size = 369, normalized size = 0.59

$$\frac{\left(3a^{5/4}d+5\sqrt[4]{a}\,bc\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}-\frac{\left(3a^{5/4}d+5\sqrt[4]{a}\,bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{b}\,(bc-ad)^3}+\frac{\left(5a\sqrt[4]{c}\,d+3bc^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{d}\,(ad-bc)^3}-\frac{\left(5a\sqrt[4]{c}\,d+3bc^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{d}\,(ad-bc)^3}+\frac{\sqrt{x}\,\left(2ac+adx^2+bcx^2\right)}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (Sqrt[x]*(2*a*c + b*c*x^2 + a*d*x^2))/(2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x
^2)) + ((5*a^(1/4)*b*c + 3*a^(5/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])]])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)^3) + ((3*b*c^(5/
4) + 5*a*c^(1/4)*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])]])/(4*Sqrt[2]*d^(1/4)*(-(b*c) + a*d)^3) - ((5*a^(1/4)*b*c + 3*a^(5/4
)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]])/(4*S
qrt[2]*b^(1/4)*(b*c - a*d)^3) - ((3*b*c^(5/4) + 5*a*c^(1/4)*d)*ArcTanh[(Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)]])/(4*Sqrt[2]*d^(1/4)*(-
(b*c) + a*d)^3)
```

fricas **[B]** time = 106.84, size = 5375, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(
625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3
 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a
^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c
^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 +
66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*arctan(-((b
^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 126*a^4
```

```
*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*c^2*d^
7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt((25*b^2*c^2 + 30*a*b*c*d + 9*a^2*d^2)
*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(-(625*a*b^4*c^4 + 1500*a^
2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^
12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a
^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^
5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 -
12*a^11*b^2*c*d^11 + a^12*b*d^12)))*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d +
 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^1
2*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^
4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a
^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c
*d^11 + a^12*b*d^12))^(3/4) - (5*b^11*c^10 - 42*a*b^10*c^9*d + 153*a^2*b^9*
c^8*d^2 - 312*a^3*b^8*c^7*d^3 + 378*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 +
 42*a^6*b^5*c^4*d^6 + 72*a^7*b^4*c^3*d^7 - 63*a^8*b^3*c^2*d^8 + 22*a^9*b^2*
c*d^9 - 3*a^10*b*d^10)*sqrt(x)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350
*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^1
1*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 7
92*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^
5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11
 + a^12*b*d^12))^(3/4))/(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*
c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a
^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c
^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a
^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^
11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*
d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495
*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*
c*d^12 + a^12*d^13))^(1/4)*arctan(-((b^9*c^9*d - 9*a*b^8*c^8*d^2 + 36*a^2*b
^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^5*b^4*c^4*d^6
 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a^9*d^10)*sqrt
((9*b^2*c^2 + 30*a*b*c*d + 25*a^2*d^2)*x + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^
2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a
^6*d^6)*sqrt(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a
^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^
10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d
^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*
a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)))*(
-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3
+ 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 -
 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*
b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d
^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(3/4) - (3*b^10
*c^10*d - 22*a*b^9*c^9*d^2 + 63*a^2*b^8*c^8*d^3 - 72*a^3*b^7*c^7*d^4 - 42*a
```

```
^4*b^6*c^6*d^5 + 252*a^5*b^5*c^5*d^6 - 378*a^6*b^4*c^4*d^7 + 312*a^7*b^3*c^
3*d^8 - 153*a^8*b^2*c^2*d^9 + 42*a^9*b*c*d^10 - 5*a^10*d^11)*sqrt(x)*(-(81*
b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625
*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*
a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c
^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 +
 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13))^(3/4))/(81*b^4*c^5 +
 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^
4)) - (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 +
 a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(6
25*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3
+ 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^
3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^
6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 6
6*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*log((5*b*c +
 3*a*d)*sqrt(x) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(62
5*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 +
 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3
*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6
*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66
*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)) + (a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4
+ (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*a*b^4*c^4 + 1
500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b
^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*
b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d
^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*log((5*b*c + 3*a*d)*sqrt(x)
- (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(625*a*b^4*c^4 + 15
00*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3 + 81*a^5*d^4)/(b^
13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b
^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^
10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)) + (a*b^2*c^3 - 2*a^2*b*c^2*d
 + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b
^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 13
50*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*
a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*
c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^1
1*b*c*d^12 + a^12*d^13))^(1/4)*log((3*b*c + 5*a*d)*sqrt(x) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 135
0*a^2*b^2*c^3*d^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a
*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c
^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
```

```
495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11
*b*c*d^12 + a^12*d^13))^(1/4)) - (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (
b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b
*c*d^2 + a^3*d^3)*x^2)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d
^2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2
+ 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^
5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4
*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^1
2*d^13))^(1/4)*log((3*b*c + 5*a*d)*sqrt(x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*(-(81*b^4*c^5 + 540*a*b^3*c^4*d + 1350*a^2*b^2*c^3*d^
2 + 1500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^12*c^12*d - 12*a*b^11*c^11*d^2 +
 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5
*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*
d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12
*d^13))^(1/4)) + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(x))/(a*b^2*c^3 - 2*a^2*b*
c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3
- a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

giac **[A]** time = 1.30, size = 912, normalized size = 1.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*
c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/4*(5*(a*b^3)^(1/4)
*b*c + 3*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sq
rt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^
2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^3*d - 3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(
2)*a^3*d^4) + 1/4*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a*d)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^3*d -
3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(2)*a^3*d^4) - 1/8*(5
*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^
2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/8*(5*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a
*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*
sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/8*(3
*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(sqrt(2)*b^3*c^3*d - 3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^
2*b*c*d^3 - sqrt(2)*a^3*d^4) - 1/8*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a
*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^3*d -
```

```
3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(2)*a^3*d^4) + 1/2*(b
*c*x^(5/2) + a*d*x^(5/2) + 2*a*c*sqrt(x))/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a
*c)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

maple [A] time = 0.02, size = 740, normalized size = 1.19

$$\frac{a^{2} d \sqrt{x}}{2 (ad-bc)^{3} \left(b x^{2}+a\right)}-\frac{abc\sqrt{x}}{2 (ad-bc)^{3} \left(b x^{2}+a\right)}+\frac{acd\sqrt{x}}{2 (ad-bc)^{3} \left(d x^{2}+c\right)}-\frac{b c^{2} \sqrt{x}}{2 (ad-bc)^{3} \left(d x^{2}+c\right)}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, ad \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8 (ad-bc)^{3}}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, ad \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8 (ad-bc)^{3}}-\frac{5 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, ad \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 (ad-bc)^{3}}-\frac{5 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, ad \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 (ad-bc)^{3}}+\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, ad \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16 (ad-bc)^{3}}-\frac{5 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, ad \ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 (ad-bc)^{3}}+\frac{5 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8 (ad-bc)^{3}}+\frac{5 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8 (ad-bc)^{3}}-\frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 (ad-bc)^{3}}-\frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, bc \arctan\left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 (ad-bc)^{3}}+\frac{5 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, bc \ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16 (ad-bc)^{3}}-\frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, bc \ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 (ad-bc)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*a^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*d-1/2*a/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)
*b*c+3/8*a/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)*d+5/8/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)*b*c+3/8*a/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)*d+5/8/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)*b*c+3/16*a/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d+
5/16/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*b*c+1/2*c/(a*d-b*c)^3
*x^(1/2)/(d*x^2+c)*a*d-1/2*c^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*b-5/8/(a*d-b*c
)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*d-3/8*c/(a*
d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-5/8/(a
*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*d-3/8
*c/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-
5/16/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*d-3/16*c/(a*d-b*c)^
3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/
d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b
```

maxima [A] time = 2.52, size = 617, normalized size = 0.99

$$-\frac{\left(\frac{2 \sqrt{2} (5 bc+3 ad) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}+\frac{2 \sqrt{2} (5 bc+3 ad) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}+\frac{\sqrt{2} (5 bc+3 ad) \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}}-\frac{\sqrt{2} (5 bc+3 ad) \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{1}{4}}}\right) a}{16 \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right)}+\frac{\left(\frac{2 \sqrt{2} (3 bc+5 ad) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{2 \sqrt{2} (3 bc+5 ad) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}}+\frac{\sqrt{2} (3 bc+5 ad) \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}-\frac{\sqrt{2} (3 bc+5 ad) \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}\right) c}{16 \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right)}+\frac{(bc+ad) x^{\frac{5}{2}}+2 ac\sqrt{x}}{2 \left(a b^{2} c^{3}-2 a^{2} b c^{2} d+a^{3} c d^{2}+\left(b^{3} c^{2} d-2 a b^{2} c d^{2}+a^{2} b d^{3}\right) x^{4}+\left(b^{3} c^{3}-a b^{2} c^{2} d-a^{2} b c d^{2}+a^{3} d^{3}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(5*b*c + 3*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*(5*b*c + 3*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + sqrt(2)*(5*b*c + 3*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b*c + 3*a*d)*log(-sqrt(2)*a^(1/4
```

```
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*(2*sqrt(2)*(3*b*c + 5*a*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b*c + 5*a*d)*arct
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b*c + 5*a*d)*log(sqr
t(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqr
t(2)*(3*b*c + 5*a*d)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/(c^(3/4)*d^(1/4)))*c/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3) + 1/2*((b*c + a*d)*x^(5/2) + 2*a*c*sqrt(x))/(a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

mupad [B] time = 3.98, size = 34921, normalized size = 55.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] atan(((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3
*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c
^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b
^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^
7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*
a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*
d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^
12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d
^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7
*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*
c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b
*c*d^12))^(1/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^
4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a
^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*
a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*b^4*c^5 + 625*a^4*
c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*
a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10
*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c
^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^
4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11
*b*c*d^12))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^
16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*
a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^1
0 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11
```

```
*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 -
13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c
^3*d^18 + 102400*a^17*b^4*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^
10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d
^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*
a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^1
1)) + ((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^
3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*
c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*
b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a
^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336
*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(10240*a^2*b^17*c^15*d^4 -
 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^1
2*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8
*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 28
16000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d
^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)))*1i + (x
^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8
*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5
*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d
^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (-(81*b^4*c^5 + 625*a
^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(40
96*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c
^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^
7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8
*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a
^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 +
1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d
- 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4
 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*
d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^
3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(((405*a^2*
b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5
*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b
^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c
^7*d - 8*a^7*b*c*d^7) - (-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3
+ 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*
d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d
^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^
```

```
6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*
c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(3/4)*((x^(1/2)
*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c
^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 3041280
0*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d
^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a
^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16
+ 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c
^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c
^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b
^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(81*b^4*c^5 + 625*
a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4
096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*
c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b
^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^
8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*
a^11*b*c*d^12))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 +
 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^
11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^
9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1
576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d
^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*1i - (x^(1/2)*(2025*a^2*b^11*c^10*
d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d
^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^
9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^12 + b
^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11
*d - 12*a^11*b*c*d^11))))/((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d
^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^
12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^
9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6
*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b
^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81
*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*
a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 2
70336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5
- 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d
^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^
2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b
^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^
5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b
```

```
^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*
a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(
81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 54
0*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 +
 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^
5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5
*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*
c^2*d^11 - 49152*a^11*b*c*d^12))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4
- 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16
*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645
632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9
*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624
*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17
 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19))/(16*(a^12*d^12 +
b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^1
1*d - 12*a^11*b*c*d^11)) + ((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*
d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c
^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c
^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^
6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*
b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(1024
0*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 -
 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c
^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a
^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 +
552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^
17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a
^7*b*c*d^7))) + (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 +
 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 +
115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2
025*a^10*b^3*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) + (-(81*
b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a
*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 27
0336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 -
 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^
8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2
*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a
^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 40
96*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*
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a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 37847
04*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 90
1120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1
/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^
5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d
^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*
a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*
d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) - (-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500
*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 +
4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 90112
0*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 378
4704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 -
901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^
(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 500
1216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^1
3*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*
a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^
13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 13799424*a^1
4*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 10
2400*a^17*b^4*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6
*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(81
*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*
a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 2
70336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5
- 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d
^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^
2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*
b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816
000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^
10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b
^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640
*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*
b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))) - (x^(1/2)*(2025*a^
2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a
^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^
8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11))/(16*(a^12
*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*
b^11*c^11*d - 12*a^11*b*c*d^11)))))*(-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^
3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 409
6*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a
```

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^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 378470
4*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901
120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/
4)*2i - 2*atan(((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a
^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 4915
2*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 202
7520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 -
3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10
 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 +
625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d
)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b
^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a
^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 202752
0*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49
152*a^11*b*c*d^12))^(1/4)*((((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4
 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)
/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)*1i)/(a^8*d^8 + b^8*c^8
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*b^4
*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^
3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 27033
6*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 32
44032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 +
 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^
11 - 49152*a^11*b*c*d^12))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069
056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*
d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^
8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12
+ 24645632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*
b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 106
9056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19)*1i)/(16*(a^12*d^12 + b^1
2*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d
 - 12*a^11*b*c*d^11)) + ((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3
 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12
*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*
d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c
^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3
*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(10240*a
^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 15
76960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10
*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10
*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552
960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17)
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)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4
*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*
b*c*d^7))*1i) - (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 +
 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 +
115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2
025*a^10*b^3*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (-(81*
b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a
*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 27
0336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 -
 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^
8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2
*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a
^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 40
96*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*
a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 37847
04*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 90
1120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1
/4)*((((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d
^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*
d^8 + (405*a^8*b^3*c^2*d^9)/2)*1i)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) - (-(81*b^4*c^5 + 625*a^4*c*d^4 +
 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^
13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 -
901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6
+ 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d
^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^
12))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5
+ 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^1
5*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 932
6592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c
^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 1379942
4*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18
 + 102400*a^17*b^4*c^2*d^19)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))
- ((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^
2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11
*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*
c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b
^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^1
```

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0*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112
640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^
7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^1
1*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 281600
0*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15
- 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*1i) + (x^(1/
2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5
 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8
 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)
)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))))/((-(81*b^4*c^5 + 625*a^4*c*d^
4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12
*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3
 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d
^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^
4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c
*d^12))^(1/4)*((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^
2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152
*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027
520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3
244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10
+ 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((((405*a^2*b^9*c^
8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c
^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2
*d^9)/2)*1i)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7
*d - 8*a^7*b*c*d^7) + (-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 +
1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d
- 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4
 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*
d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^
3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(3/4)*((x^(1/2)*(
102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^1
5*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*
a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^1
1 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^1
2*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 +
5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2
*d^19)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
```

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b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) + ((-(81*b^4*c^5 + 625
*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(
4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10
*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*
b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a
^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152
*a^11*b*c*d^12))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5
 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c
^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a
^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 -
1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*
d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 -
56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c
^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*1i)*1i - (x^(1/2)*(2025*a^2*b^11*c
^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c
^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^
4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^12
 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 49
5*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*
c^11*d - 12*a^11*b*c*d^11))) + (-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c
^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^1
2*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^
9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6
*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a
^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((
-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 +
540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2
 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*
d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c
^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^
2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((((405*a^2*b^9*c^8*d^3)/2 + 1674*
a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a
^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)*1i)/(a^
8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d
^7) - (-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3
*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c
^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b
^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^
7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*
a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(3/4)*((x^(1/2)*(102400*a^2*b^19*
c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424
*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^
9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10
```

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*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 +
 24858624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6
*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19)*1i)/(16*(
a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a*b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(81*b^4*c^5 + 625*a^4*c*d^4 + 150
0*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 +
 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 9011
20*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 37
84704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 -
 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))
^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^1
5*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8 - 304128
0*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11
 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7
*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^1
5*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d
^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7
*c^7*d - 8*a^7*b*c*d^7))*1i)*1i + (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*
a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*
a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^
9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11
*b*c*d^11)))))*(-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^
2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152
*a*b^11*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027
520*a^4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3
244032*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10
+ 270336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4) + ((x^(5/2)*(a*d +
 b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (a*c*x^(1/2))/(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d))/(a*c + x^2*(a*d + b*c) + b*d*x^4) + atan(((-(81*a^5*d^4 +
625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3
)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b
^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*
a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 20275
20*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49
152*a*b^12*c^11*d))^(1/4)*((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3
*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*
d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c
^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^
7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*
b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(((405
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*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 688
4*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*
a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^
5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*
b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c
^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*
b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10
*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*
b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^
9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(3/4)*((x^
(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b
^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30
412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c
^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412
800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*
d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*
b^4*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) + ((-(81*a^5*d^4 +
 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^
3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*
b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032
*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027
520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 4
9152*a*b^12*c^11*d))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*
d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^
13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 13516
80*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^1
3 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*
c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)))*1i + (x^(1/2)*(2025*a^2*b^11*
c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*
c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c
^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^1
2 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^
8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 4
95*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11
*c^11*d - 12*a^11*b*c*d^11))) - (-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^
3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^
12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b
^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a
^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120
*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(
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(-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 +
 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^1
1 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^
8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6
*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b
^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*
a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a
^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^
8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)
- (-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2
 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d
^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*
c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b
^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10
*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17
*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5
*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 +
24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^1
1*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 248
58624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4
*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19))/(16*(a^12*d^
12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c
^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^1
1*c^11*d - 12*a^11*b*c*d^11)) - ((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b
^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a
^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*
b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*
a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 90112
0*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*
(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*
d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b
^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041
280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^
14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c
^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70
*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d
- 8*a^7*b*c*d^7)))*1i - (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c
^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^
6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*
d^10 + 2025*a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)
```

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)))/((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*
d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*
c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b
^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^
7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a
^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*((-(81*a^5*d^4 + 625*a*b^4*c
^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^1
3*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^
2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7
*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*
c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*
c^11*d))^(1/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4
*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^
7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^
6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*a^5*d^4 + 625*a*b^4
*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b
^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*
d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c
^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^
5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^1
2*c^11*d))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^1
6*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a
^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10
 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*
b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 1
3799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^
3*d^18 + 102400*a^17*b^4*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11
)) + ((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2
*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2
*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*
b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a
^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*
a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(10240*a^2*b^17*c^15*d^4 -
112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12
*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*
b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 281
6000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^
15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^
8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b
^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))) + (x^(1/
2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5
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 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8
 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)
)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) + (-(81*a^5*d^4 + 625*a*b^4*c^
4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13
*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2
 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*
d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c
^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c
^11*d))^(1/4)*((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^
3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152
*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 202
7520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 -
3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9
+ 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(((405*a^2*b^9*c^8
*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^
5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*
d^9)/2)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a
^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d -
8*a^7*b*c*d^7) - (-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*
a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 491
52*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2
027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6
- 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^
9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(3/4)*((x^(1/2)*(10240
0*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6
 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b
^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9
326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9
*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 + 50012
16*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19
))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
+ 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7
*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*
d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(81*a^5*d^4 + 625*a*b^4*c
^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^1
3*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^
2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7
*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*
c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*
c^11*d))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960
*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8
- 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*
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c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*
a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 1
0240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))) - (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 1593
0*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 14555
0*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*
a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*
b*c*d^11)))))*(-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3
*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*
a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027
520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3
244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 +
 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*2i - 2*atan(((-(81*
a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a
^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 27
0336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4
 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d
^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2
*d^10 - 49152*a*b^12*c^11*d))^(1/4)*((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a
^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 40
96*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*
a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784
704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 9
01120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1
/4)*((((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d
^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*
d^8 + (405*a^8*b^3*c^2*d^9)/2)*1i)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*a^5*d^4 + 625*a*b^4*c^4 +
 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^
12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 -
901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5
 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*
d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11
*d))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5
 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^1
5*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 932
6592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c
^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 1379942
4*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18
 + 102400*a^17*b^4*c^2*d^19)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c
```

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^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))
+ ((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^
2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*
d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9
*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*
b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^1
0*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112
640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^
7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^1
1*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 281600
0*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15
- 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*1i) - (x^(1/
2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5
 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8
 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11)
)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (-(81*a^5*d^4 + 625*a*b^4*c^
4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13
*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2
 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*
d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c
^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c
^11*d))^(1/4)*((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^
3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152
*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 202
7520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 -
3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9
+ 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*((((405*a^2*b^9*c^
8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c
^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2
*d^9)/2)*1i)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7
*d - 8*a^7*b*c*d^7) - (-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d +
1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12
- 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^
3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6
*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c
^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(3/4)*((x^(1/2)*(
102400*a^2*b^19*c^17*d^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^1
```

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5*d^6 - 13799424*a^5*b^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*
a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^1
1 - 9326592*a^10*b^11*c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^1
2*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 +
5001216*a^15*b^6*c^4*d^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2
*d^19)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(81*a^5*d^4 + 625
*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(
4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11
*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5
*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*
a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152
*a*b^12*c^11*d))^(1/4)*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5
+ 552960*a^4*b^15*c^13*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c
^11*d^8 - 3041280*a^7*b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a
^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 -
1576960*a^12*b^7*c^5*d^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*
d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 -
56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c
^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*1i) + (x^(1/2)*(2025*a^2*b^11*c^10
*d^3 + 15930*a^3*b^10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*
d^6 + 145550*a^6*b^7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d
^9 + 15930*a^9*b^4*c^3*d^10 + 2025*a^10*b^3*c^2*d^11))/(16*(a^12*d^12 + b^1
2*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d
 - 12*a^11*b*c*d^11))))/((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d
 + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^
12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9
*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*
c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^
4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*((-(81*a
^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^
4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270
336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 -
 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^
7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*
d^10 - 49152*a*b^12*c^11*d))^(1/4)*((((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^
8*c^7*d^4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5
*c^4*d^7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)*1i)/(a^8*d^8
+ b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) +
(-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 +
```

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 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^1
1 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^
8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6
*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b
^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d
^4 - 1069056*a^3*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b
^16*c^14*d^7 + 24858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24
645632*a^8*b^13*c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*
c^9*d^12 + 24645632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858
624*a^13*b^8*c^6*d^15 - 13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d
^17 - 1069056*a^16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19)*1i)/(16*(a^12*d
^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^
11*c^11*d - 12*a^11*b*c*d^11)) + ((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*
b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*
a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3
*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704
*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 9011
20*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)
*(10240*a^2*b^17*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13
*d^6 - 1576960*a^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*
b^12*c^10*d^9 + 1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 304
1280*a^10*b^9*c^7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d
^14 + 552960*a^13*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*
c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 7
0*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d
 - 8*a^7*b*c*d^7))*1i)*1i - (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^
10*c^9*d^4 + 56304*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^
7*c^6*d^7 + 115110*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*
c^3*d^10 + 2025*a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d
^11))) + (-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*
c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*
b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a
^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 324403
2*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 2703
36*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*((-(81*a^5*d^4 + 625*a*b
^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096
*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^1
0*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8
*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*
b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b
```

^12*c^11*d))^(1/4)*((((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^4 + (984
3*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^7)/2 + 16
74*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)*1i)/(a^8*d^8 + b^8*c^8 + 28*a
^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) - (-(81*a^5*d^4 +
625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3
)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270336*a^2*b
^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 - 3244032*
a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^7 + 20275
20*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*d^10 - 49
152*a*b^12*c^11*d))^(3/4)*((x^(1/2)*(102400*a^2*b^19*c^17*d^4 - 1069056*a^3
*b^18*c^16*d^5 + 5001216*a^4*b^17*c^15*d^6 - 13799424*a^5*b^16*c^14*d^7 + 2
4858624*a^6*b^15*c^13*d^8 - 30412800*a^7*b^14*c^12*d^9 + 24645632*a^8*b^13*
c^11*d^10 - 9326592*a^9*b^12*c^10*d^11 - 9326592*a^10*b^11*c^9*d^12 + 24645
632*a^11*b^10*c^8*d^13 - 30412800*a^12*b^9*c^7*d^14 + 24858624*a^13*b^8*c^6
*d^15 - 13799424*a^14*b^7*c^5*d^16 + 5001216*a^15*b^6*c^4*d^17 - 1069056*a^
16*b^5*c^3*d^18 + 102400*a^17*b^4*c^2*d^19)*1i)/(16*(a^12*d^12 + b^12*c^12
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a
^11*b*c*d^11)) - ((-(81*a^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350
*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49
152*a^11*b^2*c*d^11 + 270336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 +
2027520*a^4*b^9*c^8*d^4 - 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6
 - 3244032*a^7*b^6*c^5*d^7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d
^9 + 270336*a^10*b^3*c^2*d^10 - 49152*a*b^12*c^11*d))^(1/4)*(10240*a^2*b^17
*c^15*d^4 - 112640*a^3*b^16*c^14*d^5 + 552960*a^4*b^15*c^13*d^6 - 1576960*a
^5*b^14*c^12*d^7 + 2816000*a^6*b^13*c^11*d^8 - 3041280*a^7*b^12*c^10*d^9 +
1351680*a^8*b^11*c^9*d^10 + 1351680*a^9*b^10*c^8*d^11 - 3041280*a^10*b^9*c^
7*d^12 + 2816000*a^11*b^8*c^6*d^13 - 1576960*a^12*b^7*c^5*d^14 + 552960*a^1
3*b^6*c^4*d^15 - 112640*a^14*b^5*c^3*d^16 + 10240*a^15*b^4*c^2*d^17))/(a^8*
d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7
))*1i)*1i + (x^(1/2)*(2025*a^2*b^11*c^10*d^3 + 15930*a^3*b^10*c^9*d^4 + 563
04*a^4*b^9*c^8*d^5 + 115110*a^5*b^8*c^7*d^6 + 145550*a^6*b^7*c^6*d^7 + 1151
10*a^7*b^6*c^5*d^8 + 56304*a^8*b^5*c^4*d^9 + 15930*a^9*b^4*c^3*d^10 + 2025*
a^10*b^3*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))))*(-(81*a
^5*d^4 + 625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^
4*b*c*d^3)/(4096*b^13*c^12 + 4096*a^12*b*d^12 - 49152*a^11*b^2*c*d^11 + 270
336*a^2*b^11*c^10*d^2 - 901120*a^3*b^10*c^9*d^3 + 2027520*a^4*b^9*c^8*d^4 -
 3244032*a^5*b^8*c^7*d^5 + 3784704*a^6*b^7*c^6*d^6 - 3244032*a^7*b^6*c^5*d^
7 + 2027520*a^8*b^5*c^4*d^8 - 901120*a^9*b^4*c^3*d^9 + 270336*a^10*b^3*c^2*

```
d^10 - 49152*a*b^12*c^11*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.471 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=609

$$-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx^{3/2}}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{b}\,(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}-\sqrt[4]{b}\,($$

Rubi [A] time = 0.80, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{dx^{3/2}}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt[4]{b}\,(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}-\frac{\sqrt[4]{b}\,(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}-\frac{\sqrt[4]{d}\,(3ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}+\frac{\sqrt[4]{d}\,(3ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}-\frac{\sqrt[4]{b}\,(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}+\frac{\sqrt[4]{b}\,(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-ad)^3}+\frac{\sqrt[4]{d}\,(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}-\frac{\sqrt[4]{d}\,(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*x^(3/2))/((b*c - a*d)^2*(c + d*x^2))) - x^(3/2)/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) + (b^(1/4)*(3*b*c + 5*a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c -
 a*d)^3) + (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*ArcTan
[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
+ (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^3) - (b^(1/4)*(3*b*c + 5*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4
)*(b*c - a*d)^3) - (d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + (d^(1/4)*(
5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(8*Sqrt[2]*c^(1/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c-5dx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(12c(bc+ad)-8b\right.}{\left(a+bx^4\right)\left(c+dx\right.}\right.}{8c(bc-ad}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int\left(\frac{4bc(3bc+5ad)x^2}{(bc-ad)\left(a+bx^4\right)}\right.\right.}{8c(b}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(5bc+3ad))\,\text{Subst}\left(\int\right.}{2(bc-ad}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(\sqrt{d}\,(5bc+3ad)\right)\text{Subst}}{4(bc-}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(5bc+3ad)\,\text{Subst}\left(\int\right.}{8(bc-}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\sqrt[4]{b}\,(3bc+5ad)\log\left(\sqrt{a}\right.}{8\sqrt{2}\,\sqrt[4]{a}}$$

$$= -\frac{dx^{3/2}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\sqrt[4]{b}\,(3bc+5ad)\tan^{-1}\left(1\right.}{4\sqrt{2}\,\sqrt[4]{a}\,(bc-a}$$

Mathematica [A] time = 0.95, size = 583, normalized size = 0.96

$$\frac{1}{16}\left(-\frac{8bx^{3/2}}{\left(a+bx^2\right)(bc-ad)^2}-\frac{8dx^{3/2}}{\left(c+dx^2\right)(bc-ad)^2}+\frac{\sqrt{2}\,\sqrt[4]{b}\,(5ad+3bc)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{a}\,(bc-ad)^3}+\frac{\sqrt{2}\,\sqrt[4]{b}\,(5ad+3bc)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{\sqrt[4]{a}\,(ad-bc)^3}+\frac{\sqrt{2}\,\sqrt[4]{d}\,(3ad+5bc)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{\sqrt[4]{c}\,(ad-bc)^3}+\frac{\sqrt{2}\,\sqrt[4]{d}\,(3ad+5bc)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{\sqrt[4]{c}\,(bc-ad)^3}+\frac{2\sqrt{2}\,\sqrt[4]{b}\,(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,(ad-bc)^3}-\frac{2\sqrt{2}\,\sqrt[4]{b}\,(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}\,(ad-bc)^3}+\frac{2\sqrt{2}\,\sqrt[4]{d}\,(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt[4]{c}\,(bc-ad)^3}-\frac{2\sqrt{2}\,\sqrt[4]{d}\,(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt[4]{c}\,(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-8*b*x^(3/2))/((b*c - a*d)^2*(a + b*x^2)) - (8*d*x^(3/2))/((b*c - a*d)^2*
(c + d*x^2)) + (2*Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c) + a*d)^3) - (2*Sqrt[2]*b^(1/4)*(3*b*
c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(-(b*c)
+ a*d)^3) + (2*Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)])/(c^(1/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(1/4)*(5*b*c + 3*
a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(1/4)*(b*c - a*d)^3)
 + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*
a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(
-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*
d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x])/(c^(1/4)*(b*c - a*d)^3))/16
```

IntegrateAlgebraic [A] time = 2.05, size = 366, normalized size = 0.60

$$\frac{\left(5a\sqrt[4]{b}\,d+3b^{5/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,(ad-bc)^3}+\frac{\left(5a\sqrt[4]{b}\,d+3b^{5/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{a}\,(ad-bc)^3}+\frac{\left(3ad^{5/4}+5bc\sqrt[4]{d}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}+\frac{\left(3ad^{5/4}+5bc\sqrt[4]{d}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,\sqrt[4]{c}\,(bc-ad)^3}-\frac{x^{3/2}\left(ad+bc+2bdx^2\right)}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(x^(3/2)*(b*c + a*d + 2*b*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*(c + d*x^
2)) + ((3*b^(5/4)*c + 5*a*b^(1/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(1/4)*(-(b*c) + a*d)^3) + ((5*b*c*d
^(1/4) + 3*a*d^(5/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x])])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)^3) + ((3*b^(5/4)*c + 5*a*b^(1/4)
*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sq
rt[2]*a^(1/4)*(-(b*c) + a*d)^3) + ((5*b*c*d^(1/4) + 3*a*d^(5/4))*ArcTanh[(S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(1/4)*
(b*c - a*d)^3)
```

fricas [B] time = 158.69, size = 5814, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(
81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 +
625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 2
20*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^
6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^
```

```
9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(1/4)*arctan(-((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt((729*b^8*c^6 + 7290*
a*b^7*c^5*d + 30375*a^2*b^6*c^4*d^2 + 67500*a^3*b^5*c^3*d^3 + 84375*a^4*b^4
*c^2*d^4 + 56250*a^5*b^3*c*d^5 + 15625*a^6*b^2*d^6)*x - (81*a*b^11*c^10 + 5
4*a^2*b^10*c^9*d - 675*a^3*b^9*c^8*d^2 - 120*a^4*b^8*c^7*d^3 + 2290*a^5*b^7
*c^6*d^4 - 636*a^6*b^6*c^5*d^5 - 3534*a^7*b^5*c^4*d^6 + 2440*a^8*b^4*c^3*d^
7 + 1725*a^9*b^3*c^2*d^8 - 2250*a^10*b^2*c*d^9 + 625*a^11*b*d^10)*sqrt(-(81
*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 62
5*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220
*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*
c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9
+ 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)))*(-(81*b^5*c^4 + 54
0*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/
(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*
d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792
*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2
*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(1/4) - (27*b^7*c^6 + 54*a*b^6*c
^5*d - 99*a^2*b^5*c^4*d^2 - 172*a^3*b^4*c^3*d^3 + 165*a^4*b^3*c^2*d^4 + 150
*a^5*b^2*c*d^5 - 125*a^6*b*d^6)*sqrt(x)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1
350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12
*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8
*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7
+ 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^
12*b*c*d^11 + a^13*d^12))^(1/4))/(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b
^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)) - 4*(a*b^2*c^3 - 2*a^2*b*
c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3
- a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*b^4*c^4*d + 1500*a*b^3*c
^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 -
12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^
8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7
 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^
11*b*c^2*d^11 + a^12*c*d^12))^(1/4)*arctan(-((b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt((15625*b^6*c^6*d^2 + 56250*a*b^5*c^5*d^3 + 84375
*a^2*b^4*c^4*d^4 + 67500*a^3*b^3*c^3*d^5 + 30375*a^4*b^2*c^2*d^6 + 7290*a^5
*b*c*d^7 + 729*a^6*d^8)*x - (625*b^10*c^11*d - 2250*a*b^9*c^10*d^2 + 1725*a
^2*b^8*c^9*d^3 + 2440*a^3*b^7*c^8*d^4 - 3534*a^4*b^6*c^7*d^5 - 636*a^5*b^5*
c^6*d^6 + 2290*a^6*b^4*c^5*d^7 - 120*a^7*b^3*c^4*d^8 - 675*a^8*b^2*c^3*d^9
+ 54*a^9*b*c^2*d^10 + 81*a^10*c*d^11)*sqrt(-(625*b^4*c^4*d + 1500*a*b^3*c^3
*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12
*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*
c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 +
 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11
*b*c^2*d^11 + a^12*c*d^12)))*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a
^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*
d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792
```

```
*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*
c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 +
 a^12*c*d^12))^(1/4) - (125*b^6*c^6*d - 150*a*b^5*c^5*d^2 - 165*a^2*b^4*c^4
*d^3 + 172*a^3*b^3*c^3*d^4 + 99*a^4*b^2*c^2*d^5 - 54*a^5*b*c*d^6 - 27*a^6*d
^7)*sqrt(x)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 +
540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c
^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5
+ 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9
*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1
/4))/(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b
*c*d^4 + 81*a^4*d^5)) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d
 - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 +
a^3*d^3)*x^2)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500
*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*
b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7
*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 22
0*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^
(1/4)*log((a*b^9*c^9 - 9*a^2*b^8*c^8*d + 36*a^3*b^7*c^7*d^2 - 84*a^4*b^6*c^
6*d^3 + 126*a^5*b^5*c^5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a^7*b^3*c^3*d^6 - 36
*a^8*b^2*c^2*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-(81*b^5*c^4 + 540*a*b^4*c^3*
d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12
 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^
5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5
*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 -
12*a^12*b*c*d^11 + a^13*d^12))^(3/4) + (27*b^4*c^3 + 135*a*b^3*c^2*d + 225*
a^2*b^2*c*d^2 + 125*a^3*b*d^3)*sqrt(x)) - (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*
c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*
d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(81*b^5*c^4 + 540*a*b^4*c^3*d + 1350*a^2*
b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*b*d^4)/(a*b^12*c^12 - 12*a^2*b^1
1*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4
 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^
9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d
^11 + a^13*d^12))^(1/4)*log(-(a*b^9*c^9 - 9*a^2*b^8*c^8*d + 36*a^3*b^7*c^7*
d^2 - 84*a^4*b^6*c^6*d^3 + 126*a^5*b^5*c^5*d^4 - 126*a^6*b^4*c^4*d^5 + 84*a
^7*b^3*c^3*d^6 - 36*a^8*b^2*c^2*d^7 + 9*a^9*b*c*d^8 - a^10*d^9)*(-(81*b^5*c
^4 + 540*a*b^4*c^3*d + 1350*a^2*b^3*c^2*d^2 + 1500*a^3*b^2*c*d^3 + 625*a^4*
b*d^4)/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b
^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^
6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a
^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12))^(3/4) + (27*b^4*c^3 + 135
*a*b^3*c^2*d + 225*a^2*b^2*c*d^2 + 125*a^3*b*d^3)*sqrt(x)) - (a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625*b^4*c^4*d + 1500*
a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*
c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495
```

```
*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*
c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10
- 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1/4)*log((b^9*c^10 - 9*a*b^8*c^9*d +
36*a^2*b^7*c^8*d^2 - 84*a^3*b^6*c^7*d^3 + 126*a^4*b^5*c^6*d^4 - 126*a^5*b^4
*c^5*d^5 + 84*a^6*b^3*c^4*d^6 - 36*a^7*b^2*c^3*d^7 + 9*a^8*b*c^2*d^8 - a^9*
c*d^9)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a
^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d
^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924
*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*
c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(3/4) +
 (125*b^3*c^3*d + 225*a*b^2*c^2*d^2 + 135*a^2*b*c*d^3 + 27*a^3*d^4)*sqrt(x)
) + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a
^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(625
*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4 +
81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*
b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*
d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*
a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12))^(1/4)*log(-(b^9*c^10
 - 9*a*b^8*c^9*d + 36*a^2*b^7*c^8*d^2 - 84*a^3*b^6*c^7*d^3 + 126*a^4*b^5*c^
6*d^4 - 126*a^5*b^4*c^5*d^5 + 84*a^6*b^3*c^4*d^6 - 36*a^7*b^2*c^3*d^7 + 9*a
^8*b*c^2*d^8 - a^9*c*d^9)*(-(625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*
b^2*c^2*d^3 + 540*a^3*b*c*d^4 + 81*a^4*d^5)/(b^12*c^13 - 12*a*b^11*c^12*d +
 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^
5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5
*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^
12*c*d^12))^(3/4) + (125*b^3*c^3*d + 225*a*b^2*c^2*d^2 + 135*a^2*b*c*d^3 +
27*a^3*d^4)*sqrt(x)) - 4*(2*b*d*x^3 + (b*c + a*d)*x)*sqrt(x))/(a*b^2*c^3 -
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

giac [B] time = 1.76, size = 952, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b
^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1/4*(3*(a*b^3)^
(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b^4*c^2*d + 3*s
qrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) - 1/4*(5*(c*d^3)^(3/4)*b*c + 3*
(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*
```

```
c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/4*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqr
t(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt
(2)*a^3*c*d^5) - 1/8*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*log(sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b
^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1/8*(3*(a*b^3)^
(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*
d^2 - sqrt(2)*a^4*b^2*d^3) + 1/8*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d
)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^4*d^2 - 3
*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/8
*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt
(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/2*(2*b*d*x^(7/2) + b*c*x^(3/2) +
 a*d*x^(3/2))/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))
```

maple [A]   time = 0.02, size = 740, normalized size = 1.22

$$-\frac{abd\,x^{\frac{3}{2}}}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)}-\frac{a\,d^{2}x^{\frac{3}{2}}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)}+\frac{b^{2}c\,x^{\frac{3}{2}}}{2\left(ad-bc\right)^{3}\left(b\,x^{2}+a\right)}+\frac{bcd\,x^{\frac{3}{2}}}{2\left(ad-bc\right)^{3}\left(d\,x^{2}+c\right)}-\frac{5\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{5\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{3\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{3\sqrt{2}\,ad\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{5\sqrt{2}\,ad\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(ad-bc\right)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{3\sqrt{2}\,ad\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{3\sqrt{2}\,bc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{3\sqrt{2}\,bc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{5\sqrt{2}\,bc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\left(ad-bc\right)^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{5\sqrt{2}\,bc\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\left(ad-bc\right)^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{3\sqrt{2}\,bc\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\left(ad-bc\right)^{3}\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{5\sqrt{2}\,bc\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\left(ad-bc\right)^{3}\left(\frac{c}{d}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3*x^(3/2)/(b*x^2
+a)*c-5/16/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*a*d*ln((x-(a/b)^(1/4)*2^(1/2)*x^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-5/8/(a*d-b*
c)^3/(a/b)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-5/8/(a*d
-b*c)^3/(a/b)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-3/16*
b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)
^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-3/8*b/(a*d-b*c)^3/(a/b
)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)-3/8*b/(a*d-b*c)^3/(
a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-1/2*d^2/(a*d-b*c
)^3*x^(3/2)/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*b*c+5/16/(a*d-b
*c)^3/(c/d)^(1/4)*2^(1/2)*b*c*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)
)/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+5/8/(a*d-b*c)^3/(c/d)^(1/4)*
2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+5/8/(a*d-b*c)^3/(c/d)^(1/
4)*2^(1/2)*b*c*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+3/16*d/(a*d-b*c)^3/(c/
d)^(1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^
(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+3/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+3/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2
)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
```

maxima [**A**] time = 2.53, size = 567, normalized size = 0.93

$$\frac{\left(3\,b^{2}c+5\,abd\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\,\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{\left(5\,bcd+3\,ad^{2}\right)\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\,\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\,\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\,\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{2\,bdx^{\frac{7}{2}}+(bc+ad)x^{\frac{3}{2}}}{2\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{4}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*b^2*c + 5*a*b*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/16*(
5*b*c*d + 3*a*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
+ 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x
))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqr
t(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqr
t(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d
^(3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2*(2*b*d*x
^(7/2) + (b*c + a*d)*x^(3/2))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3
*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*
d^2 + a^3*d^3)*x^2)
```

mupad [**B**] time = 3.97, size = 30956, normalized size = 50.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan((((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*
c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b
^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^
5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032
*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 2703
36*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*(((864*a*b^20*c^17*d^4 +
 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56
640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9
 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*
c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a
^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 320
0*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 + b^14*c^14 +
```

```
91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 200
2*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*
b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*
c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x
^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^
2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^1
1*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*
b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a
^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336
*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 3
6864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6
 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15
*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 310
88640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*
c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*
a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/
(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (x^(1/2)*(5625*a*b^12*c^8*d^5
 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 1
33539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 3
4275*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 -
 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*b^
5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b
^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 2703
36*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3
244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7
 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d
^10 - 49152*a^12*b*c*d^11))^(1/4) - ((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a
^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 40
96*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*
a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 37847
04*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 90
1120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3
/4)*(((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 +
 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^
8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*
c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a
^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 5
6640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1
i)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 +
1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432
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*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*
b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^1
3*d - 14*a^13*b*c*d^13) + (x^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3
*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096
*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^
4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704
*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 9011
20*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4
)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5
 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16
*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141
248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^
8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*
a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 -
 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d
^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*
a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) + (x
^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6
+ 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9
+ 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c
^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d -
12*a^11*b*c*d^11)))*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 13
50*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 -
49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 +
 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^
6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3
*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4))/(((-(81*b^5*
c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4
*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336
*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 324
4032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 +
2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^1
0 - 49152*a^12*b*c*d^11))^(3/4)*(((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^2
0 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d
^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*
c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a
^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 -
220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18
 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^
2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 +
 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*
a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*
```

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b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x^(1/2)*(-(81*b^5*c^
4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c
^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a
^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 32440
32*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 20
27520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10
- 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^2
0 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17
*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 1351680
0*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*
d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*
a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 +
 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^
12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8
*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*
d - 12*a^11*b*c*d^11)))*1i - (x^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c
*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^
5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2
*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*b^5*c^4 + 625*a
^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(40
96*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c
^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^
7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9
*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a
^12*b*c*d^11))^(1/4) + ((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3
+ 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^1
2 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d
^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^
6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3
*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*(((864*a*
b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^1
8*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^
6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 3
90720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*
d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7
*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14
 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10
*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*
d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10
- 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13
*b*c*d^13) + (x^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 +
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1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12
- 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3
 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*
d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c
^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^
20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^
3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5
758976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*
c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 1351
6800*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*
d^16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*
b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*1i + (x^(1/2)*(56
25*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^
3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^
6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11
*b*c*d^11)))*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*
b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a
^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 202752
0*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 324
4032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 +
270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4) - ((16875*a*b^12*c^8
*d^5)/64 + (16875*a^8*b^5*c*d^12)/64 + (131625*a^2*b^11*c^7*d^6)/64 + (4254
75*a^3*b^10*c^6*d^7)/64 + (736745*a^4*b^9*c^5*d^8)/64 + (736745*a^5*b^8*c^4
*d^9)/64 + (425475*a^6*b^7*c^3*d^10)/64 + (131625*a^7*b^6*c^2*d^11)/64)/(a^
14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a
^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b
^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^
4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d -
14*a^13*b*c*d^13)))*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 13
50*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 -
49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 +
 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^
6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3
*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4) - atan((((-(8
1*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540
*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d +
270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4
 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*
d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c
```

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^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*((864*a*b^20*c^17*d^4 + 864*a^17*b^4*
c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c
^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*
b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158
400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^
15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3
*d^18 - 5184*a^16*b^5*c^2*d^19)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d
^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5
+ 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002
*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12
*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x^(1/2)*(-(81*b^5*c
^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*
c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*
a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244
032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2
027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10
 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^
20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^1
7*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 135168
00*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9
*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976
*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17
+ 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b
^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11
*d - 12*a^11*b*c*d^11)))*1i - (x^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*
c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c
^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^
2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*b^5*c^4 + 625*
a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4
096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*
c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b
^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^
9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*
a^12*b*c*d^11))^(1/4) - ((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3
 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^
12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*
d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c
^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^
3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*((864*a*
b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^1
```

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8*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^
6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 3
90720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*
d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7
*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)/(a^14*d^14 + b
^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^1
0*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7
+ 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 36
4*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c
*d^13) + (x^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350
*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49
152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2
027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6
- 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d
^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c
^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^
18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 57589
76*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10
*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800
*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16
 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*
c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*1i + (x^(1/2)*(5625*a
*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^
10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^
7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c
*d^11)))*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*
c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b
^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^
5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032
*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 2703
36*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4))/(((-(81*b^5*c^4 + 625*a
^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(40
96*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c
^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^
7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9
*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a
^12*b*c*d^11))^(3/4)*((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2
*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*
a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 -
```

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158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8
*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*
b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16
*b^5*c^2*d^19)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11
*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^
8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9
 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 1
4*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^
4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13
*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2
 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d
^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^
4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c
*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b
^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293
184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*
d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*
a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^1
5 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^
6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*
d^11))) - (x^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b
^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*
b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11))/(16*(a^12*d
^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^
11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^
2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*
b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b
^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^
7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*
a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(1/4) +
 ((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2
 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^1
1*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c
^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^
5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11
*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*((864*a*b^20*c^17*d^4 + 864*a^1
7*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*
b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 24076
8*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12
 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*
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c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b
^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*
c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^
9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8
- 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 9
1*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) + (x^(1/2)*(-(81
*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*
a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 2
70336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4
- 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d
^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^
2*d^10 - 49152*a^12*b*c*d^11))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^
4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a
^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 +
13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^
12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5
758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4
*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^
12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c
^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^1
1*c^11*d - 12*a^11*b*c*d^11))) + (x^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b
^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^
9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6
*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*b^5*c^4 + 625*
a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4
096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*
c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b
^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 2027520*a^
9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 - 49152*
a^12*b*c*d^11))^(1/4) + ((16875*a*b^12*c^8*d^5)/64 + (16875*a^8*b^5*c*d^12)
/64 + (131625*a^2*b^11*c^7*d^6)/64 + (425475*a^3*b^10*c^6*d^7)/64 + (736745
*a^4*b^9*c^5*d^8)/64 + (736745*a^5*b^8*c^4*d^9)/64 + (425475*a^6*b^7*c^3*d^
10)/64 + (131625*a^7*b^6*c^2*d^11)/64)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12
*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c
^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8
 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 +
91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13))))*(-(81*b^5*c^4
 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^
3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2*b^11*c^11*d + 270336*a^
3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520*a^5*b^8*c^8*d^4 - 324403
2*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 3244032*a^8*b^5*c^5*d^7 + 202
```

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7520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 + 270336*a^11*b^2*c^2*d^10 -
 49152*a^12*b*c*d^11))^(1/4)*2i - ((x^(3/2)*(a*d + b*c))/(2*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d)) + (b*d*x^(7/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*c + x
^2*(a*d + b*c) + b*d*x^4) - atan((((-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b
^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096
*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^
3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 378470
4*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901
120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(3/4
)*((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 32
00*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 +
 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^1
0*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11
*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 5664
0*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)/(a^1
4*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^
4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^
7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4
*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 1
4*a^13*b*c*d^13) - (x^(1/2)*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*
d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c
*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c
^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b
^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9
*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)*(3686
4*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163
264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d
^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8
*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13
+ 13516800*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^
8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488
*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*1i - (x^(1/
2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88
705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88
705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^
12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 1
2*a^11*b*c*d^11)))*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 135
0*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 4
9152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 +
 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^
```

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6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*
d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4) - ((-(81*a^4*d
^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*
c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*
a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 324
4032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 +
2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10
 - 49152*a*b^11*c^12*d))^(3/4)*((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20
- 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7
 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^
11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^1
0*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 22
0800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 -
 5184*a^16*b^5*c^2*d^19)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 36
4*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*
a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^
5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^
2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) + (x^(1/2)*(-(81*a^4*d^5 + 62
5*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/
(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^1
0*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^
5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520
*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 4915
2*a*b^11*c^12*d))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 31
9488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*
d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*a^7*
b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 -
 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*a^12*b
^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 11632
64*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^1
2 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12
*a^11*b*c*d^11)))*1i + (x^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12
+ 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8
+ 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11)
*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*a^4*d^5 + 625*b^4*c^4
*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^1
2*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^
2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8
*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*
c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*
```

```
c^12*d))^(1/4))/(((-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350
*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49
152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 +
2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6
 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d
^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(3/4)*((864*a*b^20*c^
17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*
d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*
c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a
^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 +
369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^
17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)/(a^14*d^14 + b^14*c^1
4 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 -
 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*
a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*
b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13)
- (x^(1/2)*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^
2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^1
1*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520
*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244
032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 27
0336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)*(36864*a*b^20*c^17*d^4
 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15
*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*
b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 +
 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b
^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334
720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^1
9))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (x^(1/2)*(5625*a*b^12*c^8*
d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7
 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10
 + 34275*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d
^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*
a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(8
1*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540
*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 +
270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^
4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6
*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c
^3*d^10 - 49152*a*b^11*c^12*d))^(1/4) + ((-(81*a^4*d^5 + 625*b^4*c^4*d + 15
00*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13
```

+ 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901
120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 +
3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8
 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d)
)^(3/4)*((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^
5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13
*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^
13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 24076
8*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16
+ 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19
)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1
001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*
a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b
^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13
*d - 14*a^13*b*c*d^13) + (x^(1/2)*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^
3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*
a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3
*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704
*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 9011
20*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)
*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5
+ 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*
c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 251412
48*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8
*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a
^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 -
319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) + (x^
(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 +
 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 +
 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c^
12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 1
2*a^11*b*c*d^11)))*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 135
0*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 4
9152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 +
 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^
6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*
d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4) + ((16875*a*b^
12*c^8*d^5)/64 + (16875*a^8*b^5*c*d^12)/64 + (131625*a^2*b^11*c^7*d^6)/64 +
 (425475*a^3*b^10*c^6*d^7)/64 + (736745*a^4*b^9*c^5*d^8)/64 + (736745*a^5*b
^8*c^4*d^9)/64 + (425475*a^6*b^7*c^3*d^10)/64 + (131625*a^7*b^6*c^2*d^11)/6

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4)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 +
1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432
*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*
b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^1
3*d - 14*a^13*b*c*d^13)))*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^
2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d
^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^1
0*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6
*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b
^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)*2i + 2*
atan((((-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^
2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*
c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4
*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*
a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336
*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(3/4)*(((864*a*b^20*c^17*d^4 + 8
64*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 5664
0*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 -
 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^
9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^1
2*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*
a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 + b^14*c^14 + 91
*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*
a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^
6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^
3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x^(
1/2)*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*
d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^
2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b
^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^
7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a
^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)*(36864*a*b^20*c^17*d^4 + 368
64*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 -
 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c
^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088
640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^
7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^
14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(1
6*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*
c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) - (x^(1/2)*(5625*a*b^12*c^8*d^5 +
5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133
539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 342
75*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 2
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20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*a^4*
d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b
*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336
*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 32
44032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 +
 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^1
0 - 49152*a*b^11*c^12*d))^(1/4) - ((-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b
^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096
*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^
3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 378470
4*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901
120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(3/4
)*(((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3
200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8
 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^
10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^1
1*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 566
40*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)
/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 10
01*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a
^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^
4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*
d - 14*a^13*b*c*d^13) + (x^(1/2)*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3
*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a
^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*
b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*
a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 90112
0*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)*
(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 +
 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c
^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 2514124
8*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*
d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^
13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 3
19488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))) + (x^(
1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 +
88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 +
88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11))/(16*(a^12*d^12 + b^12*c^1
2 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
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^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12
*a^11*b*c*d^11)))*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350
*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49
152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 +
2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6
 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d
^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4))/(((-(81*a^4*d^
5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c
*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a
^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244
032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2
027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10
- 49152*a*b^11*c^12*d))^(3/4)*(((864*a*b^20*c^17*d^4 + 864*a^17*b^4*c*d^20
- 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d^6 + 56640*a^4*b^17*c^14*d^7
 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*c^12*d^9 - 240768*a^7*b^14*c^
11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*a^9*b^12*c^9*d^12 - 158400*a^1
0*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14 + 369088*a^12*b^9*c^6*d^15 - 22
0800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4*d^17 + 3200*a^15*b^6*c^3*d^18 -
 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2
- 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3
003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^
9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^
2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b*c*d^13) - (x^(1/2)*(-(81*a^4*d^5
+ 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d
^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2
*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 324403
2*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 202
7520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 -
49152*a*b^11*c^12*d))^(1/4)*(36864*a*b^20*c^17*d^4 + 36864*a^17*b^4*c*d^20
- 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*b^18*c^15*d^6 - 2334720*a^4*b^17*c
^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 5758976*a^6*b^15*c^12*d^9 + 13516800*
a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^10*d^11 + 31088640*a^9*b^12*c^9*d^
12 - 25141248*a^10*b^11*c^8*d^13 + 13516800*a^11*b^10*c^7*d^14 - 5758976*a^
12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^16 - 2334720*a^14*b^7*c^4*d^17 + 1
163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^5*c^2*d^19))/(16*(a^12*d^12 + b^12
*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d
- 12*a^11*b*c*d^11)))*1i - (x^(1/2)*(5625*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d
^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*b^10*c^6*d^7 + 133539*a^4*b^9*c^5*
d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d
^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(81*a^4*d^5 + 625*b^4
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*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096
*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^1
1*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7
*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*
b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b
^11*c^12*d))^(1/4) + ((-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 +
1350*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12
- 49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^
3 + 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7
*d^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c
^4*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(3/4)*(((864*a*b^
20*c^17*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*
c^15*d^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*
b^15*c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390
720*a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^
14 + 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c
^4*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 +
 b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c
^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d^
7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10 -
364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^13*b
*c*d^13) + (x^(1/2)*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 13
50*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 -
49152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3
+ 2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d
^6 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4
*d^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)*(36864*a*b^20
*c^17*d^4 + 36864*a^17*b^4*c*d^20 - 319488*a^2*b^19*c^16*d^5 + 1163264*a^3*
b^18*c^15*d^6 - 2334720*a^4*b^17*c^14*d^7 + 3293184*a^5*b^16*c^13*d^8 - 575
8976*a^6*b^15*c^12*d^9 + 13516800*a^7*b^14*c^11*d^10 - 25141248*a^8*b^13*c^
10*d^11 + 31088640*a^9*b^12*c^9*d^12 - 25141248*a^10*b^11*c^8*d^13 + 135168
00*a^11*b^10*c^7*d^14 - 5758976*a^12*b^9*c^6*d^15 + 3293184*a^13*b^8*c^5*d^
16 - 2334720*a^14*b^7*c^4*d^17 + 1163264*a^15*b^6*c^3*d^18 - 319488*a^16*b^
5*c^2*d^19))/(16*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^
9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^1
0*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*1i + (x^(1/2)*(5625
*a*b^12*c^8*d^5 + 5625*a^8*b^5*c*d^12 + 34275*a^2*b^11*c^7*d^6 + 88705*a^3*
b^10*c^6*d^7 + 133539*a^4*b^9*c^5*d^8 + 133539*a^5*b^8*c^4*d^9 + 88705*a^6*
b^7*c^3*d^10 + 34275*a^7*b^6*c^2*d^11)*1i)/(16*(a^12*d^12 + b^12*c^12 + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b
*c*d^11)))*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350*a^2*b^
2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49152*a^1
```

```
1*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 + 2027520
*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6 - 3244
032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d^9 + 27
0336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4) - ((16875*a*b^12*c^8*d
^5)/64 + (16875*a^8*b^5*c*d^12)/64 + (131625*a^2*b^11*c^7*d^6)/64 + (425475
*a^3*b^10*c^6*d^7)/64 + (736745*a^4*b^9*c^5*d^8)/64 + (736745*a^5*b^8*c^4*d
^9)/64 + (425475*a^6*b^7*c^3*d^10)/64 + (131625*a^7*b^6*c^2*d^11)/64)/(a^14
*d^14 + b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4
*b^10*c^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7
*c^7*d^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*
d^10 - 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14
*a^13*b*c*d^13)))*(-(81*a^4*d^5 + 625*b^4*c^4*d + 1500*a*b^3*c^3*d^2 + 1350
*a^2*b^2*c^2*d^3 + 540*a^3*b*c*d^4)/(4096*b^12*c^13 + 4096*a^12*c*d^12 - 49
152*a^11*b*c^2*d^11 + 270336*a^2*b^10*c^11*d^2 - 901120*a^3*b^9*c^10*d^3 +
2027520*a^4*b^8*c^9*d^4 - 3244032*a^5*b^7*c^8*d^5 + 3784704*a^6*b^6*c^7*d^6
 - 3244032*a^7*b^5*c^6*d^7 + 2027520*a^8*b^4*c^5*d^8 - 901120*a^9*b^3*c^4*d
^9 + 270336*a^10*b^2*c^3*d^10 - 49152*a*b^11*c^12*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.472 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=601

$$-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^3}- b^{3/4}(7a$$

Rubi [A] time = 0.69, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 471, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{3/4}(bc-ad)^3}-\frac{b^{3/4}(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{b^{3/4}(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{3/4}(bc-ad)^3}+\frac{d^{3/4}(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{d^{3/4}(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{d\sqrt{x}}{\left(c+dx^2\right)(bc-ad)^2}-\frac{\sqrt{x}}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -((d*Sqrt[x])/((b*c - a*d)^2*(c + d*x^2))) - Sqrt[x]/(2*(b*c - a*d)*(a + b*
x^2)*(c + d*x^2)) - (b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (b^(3/
4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^3) + (b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d
)^3) + (d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - (d^(3/4)*(7*b*c + a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(3/
4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\left(\int \frac{c-7dx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}\\
&= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\text{Subst}\Big(\int \frac{4c(bc+ad)-24bcdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}}{8c(bc-ad)^2}\\
&= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(7bc+ad))\,\text{Subst}\Big(\int \frac{}{c+}}{2(bc-ad)^3}\\
&= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{(d(7bc+ad))\,\text{Subst}\Big(\int \frac{\sqrt{}}{}}{4\sqrt{c}\,(bc-a}\\
&= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(\sqrt{d}\,(7bc+ad)\right)\text{Subst}\Bigg(\int}{8\sqrt{c}\,(bc}\\
&= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{3/4}(bc+7ad)\log\Big(\sqrt{a}\,-}{8\sqrt{2}\,a^{3/4}(b}\\
&= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{3/4}(bc+7ad)\tan^{-1}\Bigg(1-}{4\sqrt{2}\,a^{3/4}(bc-ad}
\end{aligned}
$$

Mathematica [A] time = 1.03, size = 575, normalized size = 0.96

$$
\frac{1}{16}\left(\frac{\sqrt{2}\,b^{3/4}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}(ad-bc)^3}+\frac{\sqrt{2}\,b^{3/4}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{3/4}(bc-ad)^3}+\frac{2\sqrt{2}\,b^{3/4}(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}(ad-bc)^3}-\frac{2\sqrt{2}\,b^{3/4}(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}(ad-bc)^3}+\frac{\sqrt{2}\,d^{3/4}(ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{3/4}(bc-ad)^3}+\frac{\sqrt{2}\,d^{3/4}(ad+7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{3/4}(ad-bc)^3}+\frac{2\sqrt{2}\,d^{3/4}(ad+7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{3/4}(bc-ad)^3}-\frac{2\sqrt{2}\,d^{3/4}(ad+7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{3/4}(bc-ad)^3}-\frac{8b\sqrt{x}}{\left(a+bx^2\right)(bc-ad)^2}-\frac{8d\sqrt{x}}{\left(c+dx^2\right)(bc-ad)^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((-8*b*Sqrt[x])/((b*c - a*d)^2*(a + b*x^2)) - (8*d*Sqrt[x])/((b*c - a*d)^2*
(c + d*x^2)) + (2*Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
```

```
*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*d)^3) - (2*Sqrt[2]*b^(3/4)*(b*c +
7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*(-(b*c) + a*
d)^3) + (2*Sqrt[2]*d^(3/4)*(7*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)])/(c^(3/4)*(b*c - a*d)^3) - (2*Sqrt[2]*d^(3/4)*(7*b*c + a*d)*Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(3/4)*(b*c - a*d)^3) + (Sqrt
[2]*b^(3/4)*(b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*(b*c - a*d
)^3) + (Sqrt[2]*d^(3/4)*(7*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*(b*c - a*d)^3) + (Sqrt[2]*d^(3/4)*(7*b*c +
a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*(
-(b*c) + a*d)^3))/16
```

IntegrateAlgebraic [A] time = 2.00, size = 362, normalized size = 0.60

$$\frac{\left(7ab^{3/4}d+b^{7/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{3/4}(ad-bc)^3}-\frac{\left(7ab^{3/4}d+b^{7/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{3/4}(ad-bc)^3}+\frac{\left(ad^{7/4}+7bcd^{3/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{\left(ad^{7/4}+7bcd^{3/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{3/4}(bc-ad)^3}-\frac{\sqrt{x}\left(ad+bc+2bdx^2\right)}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(Sqrt[x]*(b*c + a*d + 2*b*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*(c + d*x^
2)) + ((b^(7/4)*c + 7*a*b^(3/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(3/4)*(-(b*c) + a*d)^3) + ((7*b*c*d^(
3/4) + a*d^(7/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x])])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^3) - ((b^(7/4)*c + 7*a*b^(3/4)*d)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*
a^(3/4)*(-(b*c) + a*d)^3) - ((7*b*c*d^(3/4) + a*d^(7/4))*ArcTanh[(Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(3/4)*(b*c - a
*d)^3)
```

fricas [B] time = 130.91, size = 5474, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-
(b^7*c^4 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401
*a^4*b^3*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 -
220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b
^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3
*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*arctan(-
((a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 +
```

```
 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36*a^9*b^
2*c^2*d^7 + 9*a^10*b*c*d^8 - a^11*d^9)*sqrt((b^4*c^2 + 14*a*b^3*c*d + 49*a^
2*b^2*d^2)*x + (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5
*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*sqrt(-(b^7*c^4
 + 28*a*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3
*d^4)/(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*
b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d
^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 6
6*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)))*(-(b^7*c^4 + 28*a*b^6
*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*
b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3
 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^
10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*
c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(3/4) - (a^2*b^11*c^10 - 2*a^3*b^
10*c^9*d - 27*a^4*b^9*c^8*d^2 + 168*a^5*b^8*c^7*d^3 - 462*a^6*b^7*c^6*d^4 +
 756*a^7*b^6*c^5*d^5 - 798*a^8*b^5*c^4*d^6 + 552*a^9*b^4*c^3*d^7 - 243*a^10
*b^3*c^2*d^8 + 62*a^11*b^2*c*d^9 - 7*a^12*b*d^10)*sqrt(x)*(-(b^7*c^4 + 28*a
*b^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(
a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9
*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 79
2*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*
b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(3/4))/(b^7*c^4 + 28*a*b^6*c^
3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)) - 4*(a*
b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c
^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)/(b^12*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*
d^3 + 495*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 7
92*a^7*b^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^
2*c^5*d^10 - 12*a^11*b*c^4*d^11 + a^12*c^3*d^12))^(1/4)*arctan(-((b^9*c^11
- 9*a*b^8*c^10*d + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^
7*d^4 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 + 9*a
^8*b*c^3*d^8 - a^9*c^2*d^9)*sqrt((49*b^2*c^2*d^2 + 14*a*b*c*d^3 + a^2*d^4)*
x + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15
*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(-(2401*b^4*c^4*d^3 +
 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12
*c^15 - 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 49
5*a^4*b^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b
^5*c^8*d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^
10 - 12*a^11*b*c^4*d^11 + a^12*c^3*d^12)))*(-(2401*b^4*c^4*d^3 + 1372*a*b^3
*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*
a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c
^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7
+ 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^1
1*b*c^4*d^11 + a^12*c^3*d^12))^(3/4) - (7*b^10*c^12*d - 62*a*b^9*c^11*d^2 +
```

```
243*a^2*b^8*c^10*d^3 - 552*a^3*b^7*c^9*d^4 + 798*a^4*b^6*c^8*d^5 - 756*a^5
*b^5*c^7*d^6 + 462*a^6*b^4*c^6*d^7 - 168*a^7*b^3*c^5*d^8 + 27*a^8*b^2*c^4*d
^9 + 2*a^9*b*c^3*d^10 - a^10*c^2*d^11)*sqrt(x)*(-(2401*b^4*c^4*d^3 + 1372*a
*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 -
 12*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b
^8*c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*
d^7 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12
*a^11*b*c^4*d^11 + a^12*c^3*d^12))^(3/4))/(2401*b^4*c^4*d^3 + 1372*a*b^3*c^
3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)) - (a*b^2*c^3 - 2*a
^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3
*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(b^7*c^4 + 28*a*b^6*c^3*
d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*b^12*
c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 49
5*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^
5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d
^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*log((b^2*c + 7*a*b*d)*sqrt(x) +
(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-(b^7*c^4 + 28*a*b
^6*c^3*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^
3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d
^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*
a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^
2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)) + (a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(b^7*c^4 + 28*a*b^6*c^3*d + 29
4*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*b^12*c^12 -
 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*
b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*
d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 -
12*a^14*b*c*d^11 + a^15*d^12))^(1/4)*log((b^2*c + 7*a*b*d)*sqrt(x) - (a*b^3
*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-(b^7*c^4 + 28*a*b^6*c^3
*d + 294*a^2*b^5*c^2*d^2 + 1372*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^3*b^12
*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d^3 + 4
95*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792*a^10*b
^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*b^2*c^2*
d^10 - 12*a^14*b*c*d^11 + a^15*d^12))^(1/4)) + (a*b^2*c^3 - 2*a^2*b*c^2*d +
 a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2
*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d
^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*a*b^11
*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^
4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*
a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^
4*d^11 + a^12*c^3*d^12))^(1/4)*log((7*b*c*d + a*d^2)*sqrt(x) + (b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-(2401*b^4*c^4*d^3 + 1372*a*b^
3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12
*a*b^11*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*
```

```
c^11*d^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7
 + 495*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^
11*b*c^4*d^11 + a^12*c^3*d^12))^(1/4)) - (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c
*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d
 - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 2
94*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*a*b^11*c^14*
d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d^4 - 79
2*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495*a^8*b^
4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c^4*d^11
 + a^12*c^3*d^12))^(1/4)*log((7*b*c*d + a*d^2)*sqrt(x) - (b^3*c^4 - 3*a*b^2
*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-(2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*
d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^7)/(b^12*c^15 - 12*a*b^1
1*c^14*d + 66*a^2*b^10*c^13*d^2 - 220*a^3*b^9*c^12*d^3 + 495*a^4*b^8*c^11*d
^4 - 792*a^5*b^7*c^10*d^5 + 924*a^6*b^6*c^9*d^6 - 792*a^7*b^5*c^8*d^7 + 495
*a^8*b^4*c^7*d^8 - 220*a^9*b^3*c^6*d^9 + 66*a^10*b^2*c^5*d^10 - 12*a^11*b*c
^4*d^11 + a^12*c^3*d^12))^(1/4)) + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(x))/(a*b^
2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)
*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

giac [B] time = 1.73, size = 904, normalized size = 1.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2
*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/4*((a*b^3)^(1/4)*b*c
+ 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x)
)/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3
*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/4*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)
*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)
*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*
d^3) - 1/4*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a
*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*d^3) + 1/8*((a*b^3)^(1
/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 -
 sqrt(2)*a^4*d^3) - 1/8*((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(-sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2
*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/8*(7*(c*d^3)^(1/4
)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(
2)*a^3*c*d^3) + 1/8*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(-sqrt(2)*
```

```
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3
*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*d^3) - 1/2*(2*b*d*x^(5/2) + b*
c*sqrt(x) + a*d*sqrt(x))/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))
```

maple [A] time = 0.02, size = 770, normalized size = 1.28

$$-\frac{abd\sqrt{x}}{2\,(ad-bc)^{3}\left(b\,x^{2}+a\right)}-\frac{a\,d^{2}\sqrt{x}}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)}+\frac{b^{2}c\sqrt{x}}{2\,(ad-bc)^{3}\left(b\,x^{2}+a\right)}+\frac{bcd\sqrt{x}}{2\,(ad-bc)^{3}\left(d\,x^{2}+c\right)}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{3}\,c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{3}\,c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^{2}\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\,(ad-bc)^{3}\,c}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{3}\,a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{3}\,a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\,(ad-bc)^{3}\,a}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{3}}+\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8\,(ad-bc)^{3}}+\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8\,(ad-bc)^{3}}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bd\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16\,(ad-bc)^{3}}+\frac{7\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,bd\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16\,(ad-bc)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*a*d+1/2*b^2/(a*d-b*c)^3*x^(1/2)/(b*x^2
+a)*c-7/8*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)-1)*d-1/8*b^2/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)-1)*c-7/16*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d-1
/16*b^2/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)
+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c-7/8*b/(a*d-b*c
)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d-1/8*b^2/(a*
d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c-1/2*
d^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a+1/2*d/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*b*c
+1/8*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)+1)*a+7/8*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*b+1/8*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)*a+7/8*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*b+1/16*d^2/(a*d-b*c)^3*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d)
^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1
/2)))*a+7/16*d/(a*d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^
(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b
```

maxima [A] time = 2.58, size = 620, normalized size = 1.03

$$\frac{\left(\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(bc+7\,ad)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc+7\,ad)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b}{16\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}-\frac{2\,bdx^{\frac{5}{2}}+(bc+ad)\sqrt{x}}{2\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x^{4}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)x^{2}\right)}-\frac{\frac{2\sqrt{2}\left(7\,bcd+ad^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\sqrt{2}\left(7\,bcd+ad^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(7\,bcd+ad^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(7\,bcd+ad^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*(b*c + 7*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*(b*c + 7*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
```

```
(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2*(2*b*d*x^(5/2) + (b*c + a*d)*sqrt(x))/(
a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*
d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2) - 1/16*(2*s
qrt(2)*(7*b*c*d + a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sq
rt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*s
qrt(2)*(7*b*c*d + a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sq
rt(2)*(7*b*c*d + a*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d + a*d^2)*log(-sqrt(2)*c^(1/4)*
d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
```

mupad [B] time = 3.79, size = 34586, normalized size = 57.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan(-(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20
*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^
18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 864870
4*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d
^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^1
1*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 33
7920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^12
*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d
- 12*a^11*b*c*d^11)) + ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3
+ 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^1
2 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d
^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^
6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b
^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192*a
^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*
b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037
504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*
d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^1
1*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*
a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^
5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*
b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*
```

```
d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12
*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c
^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^
6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^
12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4)*1i
- (((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (21
97*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(a^
8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*
d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d
^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d
^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*
c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^
8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^1
0*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336
*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4) - (x^(1/2)*(1225*a^6*b^7*d
^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 759
75*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8
*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 137
2*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4
096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 9011
20*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 37
84704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8
 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11
))^(1/4) + ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^
20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*
b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648
704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9
*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a
^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 +
337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^
12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
- 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8
*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*
d - 12*a^11*b*c*d^11)) - ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^
3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c
^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9
*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*
c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12
*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192
*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^
3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 60
37504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^
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8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a
^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 819
2*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*
c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^
12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9
*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*
b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*
a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4)*1
i + (((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (
2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(
a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c
*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2
*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^1
1*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*
b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a
^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 2703
36*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4) - (x^(1/2)*(1225*a^6*b^7
*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 7
5975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/
(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 49
5*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5
*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1
372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 +
 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 90
1120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 +
3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d
^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^
11))^(1/4))/(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*
b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^
3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 86
48704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c
^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704
*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17
+ 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 +
b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^1
1*d - 12*a^11*b*c*d^11)) + ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*
d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12
*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c
^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^
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6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^
12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(81
92*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*
a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 -
6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*
c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552
*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8
192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^
4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*
b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b
^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^
9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 90112
0*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4)
*1i - (((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 +
 (2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)
/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b
*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c
^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b
^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^
7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032
*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 27
0336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*1i - (x^(1/2)*(1225*a^
6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^1
2 + 75975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^
11)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3
*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^1
5*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*
d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^
7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b
^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^1
4*b*c*d^11))^(1/4) - ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 +
 4096*a*b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 3
37920*a^3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*
d^9 + 8648704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^
8*b^13*c^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 +
 8648704*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c
^4*d^17 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12
*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
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 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*
b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^
3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*
a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a
^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 378470
4*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 9
01120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(
1/4)*(8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4
 + 59392*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^1
1*d^8 - 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a
^8*b^11*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 +
 2455552*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*
d^16 + 8192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 137
2*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4
096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 9011
20*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 37
84704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8
 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11
))^(3/4)*1i + (((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d
^11)/2 + (2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*
d^9)*1i)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d -
 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a
^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 491
52*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 20
27520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 -
 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*
d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*1i - (x^(1/2)*
(1225*a^6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b
^8*c*d^12 + 75975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^
9*c^2*d^11)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401
*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(
4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^
10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^
8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 202752
0*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 4
9152*a^14*b*c*d^11))^(1/4)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c
*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^1
2*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*
c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b
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^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a
^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4) -
atan(-(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c
^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18
*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*
a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^1
2 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*
b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 3379
20*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))/(8*(a^12*d^12 + b^12*c^12
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*
a^11*b*c*d^11)) + ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294
*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 4
9152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 +
2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6
 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^
3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192*a^2*b^
17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*
c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a
^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11
+ 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8
*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*
b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c
^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 +
 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12
 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^
3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6
*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^
3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4) - ((2197
*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^
7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)/(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c
^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6
*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 2703
36*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3
244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7
 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2
*d^10 - 49152*a^14*b*c*d^11))^(1/4)*1i + (x^(1/2)*(1225*a^6*b^7*d^13 + 1225
*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^1
1*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11)*1i)/(8*(a^12*
d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
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+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b
^11*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b
^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3
*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*
b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a
^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 9011
20*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4
) + ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16
*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^
14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6
*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 +
 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^1
0*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*
a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))/(8*(a^12*d^12 + b^12*c^12 +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^1
1*b*c*d^11)) - ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^
2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 4915
2*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 202
7520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 -
3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d
^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192*a^2*b^17*
c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^1
3*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*
b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 1
0070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^
5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5
*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*
d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 29
4*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 -
49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 +
 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^
6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c
^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4) + ((2197*a*
b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c
*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)/(a^8*d^8 + b^8*c^8
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4
+ 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^
3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*
a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244
032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 +
2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^
```

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10 - 49152*a^14*b*c*d^11))^(1/4)*1i + (x^(1/2)*(1225*a^6*b^7*d^13 + 1225*b^
13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^11*c
^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11)*1i)/(8*(a^12*d^1
2 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^
8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 4
95*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11
*c^11*d - 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*
c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^
12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9
*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*
b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*
a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4))/
(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16*d^
5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^14*
d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6*b^
15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 + 52
94080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^10*c
^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*a^1
4*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))/(8*(a^12*d^12 + b^12*c^12 + 66*
a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7
*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
- 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b
*c*d^11)) + ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b
^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a
^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 202752
0*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 324
4032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9
+ 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192*a^2*b^17*c^1
4*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^13*d
^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*b^1
3*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 1007
0016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^5*d
^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5*c^
2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*
a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d -
 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a
^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 491
52*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 20
27520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 -
 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*
d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4) - ((2197*a*b^1
0*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c*d^
10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)/(a^8*d^8 + b^8*c^8 + 2
8*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^
3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2
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401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d
)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5
*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032
*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 202
7520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10
 - 49152*a^14*b*c*d^11))^(1/4) + (x^(1/2)*(1225*a^6*b^7*d^13 + 1225*b^13*c^6
*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^11*c^4*d^9
 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8*(a^12*d^12 + b^12*
c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 7
92*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^
4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d -
 12*a^11*b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 2
94*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 -
 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3
+ 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d
^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*
c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4) - ((((x^(1
/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16*d^5 + 4096*
a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^14*d^7 + 153
600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6*b^15*c^11*d
^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 + 5294080*a^9
*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^10*c^6*d^15 -
 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*a^14*b^7*c^3
*d^18 - 108544*a^15*b^6*c^2*d^19))/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*
c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
+ 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9
*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11))
 - ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^
2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c
^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8
*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10
*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*
a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)*(8192*a^2*b^17*c^14*d^5 - 2
048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^13*d^6 - 6062
08*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*b^13*c^10*d^
9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 10070016*a^9*
b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^5*d^14 - 606
208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5*c^2*d^17))/
(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*
c*d^7))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^
2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^
11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7
*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*
a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270
```

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336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(3/4) + ((2197*a*b^10*c^4*d^7
)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c*d^10)/2 + 9
145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6
*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 2
8*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(b^7*c^4 + 2401*a^4*b
^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a
^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^1
0*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*
c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11
*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a
^14*b*c*d^11))^(1/4) + (x^(1/2)*(1225*a^6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18
186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^11*c^4*d^9 + 71372*
a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8*(a^12*d^12 + b^12*c^12 + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*
b*c*d^11)))*(-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^
5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^
4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520
*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244
032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 +
 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11))^(1/4)))*(-(b^7*c^4 + 2401
*a^4*b^3*d^4 + 1372*a^3*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(
4096*a^15*d^12 + 4096*a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^
10*c^10*d^2 - 901120*a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^
8*b^7*c^7*d^5 + 3784704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 202752
0*a^11*b^4*c^4*d^8 - 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 4
9152*a^14*b*c*d^11))^(1/4)*2i - ((x^(1/2)*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) + (b*d*x^(5/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*c + x^2
*(a*d + b*c) + b*d*x^4) - atan(-(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^2
1*c^17*d^4 + 4096*a*b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*
c^15*d^6 + 337920*a^3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^
5*b^16*c^12*d^9 + 8648704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11
+ 5294080*a^8*b^13*c^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^1
1*c^7*d^14 + 8648704*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 15360
0*a^13*b^8*c^4*d^17 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))
/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^
5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^1
0 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) + ((-(a^4*d^7 + 2401*b^4*c^4*d^3
+ 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^1
5 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 -
 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*
d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c
^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c
```

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^14*d))^(1/4)*(8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*
c^15*d^4 + 59392*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5
*b^14*c^11*d^8 - 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 1
1894784*a^8*b^11*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c
^6*d^13 + 2455552*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13
*b^6*c^3*d^16 + 8192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^
6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*
d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12
*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d
^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c
^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b
^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^
11*c^14*d))^(3/4) - ((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*
b^6*d^11)/2 + (2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8
*c^2*d^9)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70
*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d
- 8*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*
a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49
152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 +
2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d
^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6
*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*1i + (x^(1/2)
*(1225*a^6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*
b^8*c*d^12 + 75975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b
^9*c^2*d^11)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3
*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*
d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*
a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 240
1*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/
(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b
^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032
*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 202
7520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 -
49152*a*b^11*c^14*d))^(1/4) + ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c
^17*d^4 + 4096*a*b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^1
5*d^6 + 337920*a^3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b
^16*c^12*d^9 + 8648704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5
294080*a^8*b^13*c^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c
^7*d^14 + 8648704*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a
^13*b^8*c^4*d^17 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))/(8
*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*
a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c
^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)) - ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1
```

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372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 +
 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 90
1120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5
 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*
d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14
*d))^(1/4)*(8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^1
5*d^4 + 59392*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^
14*c^11*d^8 - 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 1189
4784*a^8*b^11*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*
d^13 + 2455552*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^
6*c^3*d^16 + 8192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3
 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^
15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2
 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10
*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*
c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*
c^14*d))^(3/4) + ((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6
*d^11)/2 + (2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^
2*d^9)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8
*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2
*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152
*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 202
7520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6
 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^
9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*1i + (x^(1/2)*(1
225*a^6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8
*c*d^12 + 75975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*
c^2*d^11)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^
9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^1
0*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b
^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(40
96*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10
*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^
5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 202752
0*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 491
52*a*b^11*c^14*d))^(1/4))/(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17
*d^4 + 4096*a*b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d
^6 + 337920*a^3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16
*c^12*d^9 + 8648704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294
080*a^8*b^13*c^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*
d^14 + 8648704*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13
```

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*b^8*c^4*d^17 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))/(8*(a
^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a*b^11*c^11*d - 12*a^11*b*c*d^11)) + ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372
*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 40
96*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 90112
0*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 +
3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8
 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d)
)^(1/4)*(8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d
^4 + 59392*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*
c^11*d^8 - 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 1189478
4*a^8*b^11*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^1
3 + 2455552*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c
^3*d^16 + 8192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 +
1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15
 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 9
01120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^
5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7
*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^1
4*d))^(3/4) - ((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^
11)/2 + (2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d
^9)/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b
^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^
7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^
2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^
11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 202752
0*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3
244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 +
 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4) + (x^(1/2)*(1225*a^
6*b^7*d^13 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^1
2 + 75975*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^
11))/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^
7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b^4*c^4*d^
3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c
^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2
 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^1
0*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4
*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11
*c^14*d))^(1/4) - ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 40
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96*a*b^20*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 3379
20*a^3*b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9
 + 8648704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b
^13*c^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 86
48704*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*
d^17 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19))/(8*(a^12*d^12
+ b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c
^11*d - 12*a^11*b*c*d^11)) - ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^
3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c
^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9
*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a
^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120
*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*(
8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 5939
2*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8
- 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^1
1*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 24555
52*a^11*b^8*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 +
 8192*a^14*b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^
3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^
12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3
*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 37847
04*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 90
1120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(3/
4) + ((2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (
2197*a^4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)/(a^8*
d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7
))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5
 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*
d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8
*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^
7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a
^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4) + (x^(1/2)*(1225*a^6*b^7*d^1
3 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975
*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8*(
a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*
a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 409
```

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6*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120
*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3
784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8
- 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))
^(1/4)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c
^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*
b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a
^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244
032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 27
0336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*2i + 2*atan(-(((((x^(1
/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16*d^5 + 4096*
a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^14*d^7 + 153
600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6*b^15*c^11*d
^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 + 5294080*a^9
*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^10*c^6*d^15 -
 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*a^14*b^7*c^3
*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^
10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d
^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*
a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d^1
1)) + ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2
*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*
c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4
*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 324403
2*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 2703
36*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*(8192*a^2*b^17*c^14*d^5
- 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^13*d^6 - 6
06208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*b^13*c^10
*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 10070016*a
^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^5*d^14 -
606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5*c^2*d^17
))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7
*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2
*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^1
1*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520
*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 32
44032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 +
270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(3/4)*1i - (((2197*a*b^10*
c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c*d^10
)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(a^8*d^8 + b^8*c^8 +
 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*
c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7 +
 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d
^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a
```

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^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 324
4032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 +
 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^1
0 - 49152*a*b^11*c^14*d))^(1/4) - (x^(1/2)*(1225*a^6*b^7*d^13 + 1225*b^13*c
^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^11*c^4*d
^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8*(a^12*d^12 + b^1
2*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d
 - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 +
 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12
 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d
^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*
c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^
3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4) + ((((x^
(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16*d^5 + 409
6*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^14*d^7 + 1
53600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6*b^15*c^11
*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 + 5294080*a
^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^10*c^6*d^15
 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*a^14*b^7*c
^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c*d
^11)) - ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c
^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*
b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a
^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244
032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 27
0336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*(8192*a^2*b^17*c^14*d^
5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^13*d^6 -
 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*b^13*c^
10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 10070016
*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^5*d^14
 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5*c^2*d^
17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a
^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b
^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a
^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 20275
20*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 -
3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9
+ 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(3/4)*1i + (((2197*a*b^1
0*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c*d^
```

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10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(a^8*d^8 + b^8*c^8
 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7
 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c
*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336
*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3
244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7
 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d
^10 - 49152*a*b^11*c^14*d))^(1/4) - (x^(1/2)*(1225*a^6*b^7*d^13 + 1225*b^13
*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^11*c^4
*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11))/(8*(a^12*d^12 + b
^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^
8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11
*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4
 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^
12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12
*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^
6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*
b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4))/(((((
x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16*d^5 + 4
096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^14*d^7 +
 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6*b^15*c^
11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 + 5294080
*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^10*c^6*d^
15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*a^14*b^7
*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^12*c^12 + 66*a^
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*a^11*b*c
*d^11)) + ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2
*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^1
1*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520
*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 32
44032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 +
270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*(8192*a^2*b^17*c^14*
d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*c^13*d^6
 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a^6*b^13*
c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11 + 100700
16*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8*c^5*d^1
4 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*b^5*c^2*
d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8
*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2
*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152
```

```
*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 202
7520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6
- 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^
9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(3/4)*1i - (((2197*a*b
^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^4*b^7*c*
d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*(-(a^4*d
^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b
*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 2703
36*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 -
 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d
^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5
*d^10 - 49152*a*b^11*c^14*d))^(1/4)*1i - (x^(1/2)*(1225*a^6*b^7*d^13 + 1225
*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975*a^2*b^1
1*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11)*1i)/(8*(a^12*
d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8
*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
+ 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b
^11*c^11*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3
*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^1
2*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*
b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 378470
4*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901
120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4
) - ((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^20*c^16
*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*b^18*c^
14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 8648704*a^6
*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c^9*d^12 +
 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 8648704*a^11*b^1
0*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^17 + 337920*
a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12 + b^12*c^12
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^11*c^11*d - 12*
a^11*b*c*d^11)) - ((-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294
*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 4
9152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 +
 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*
d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^
6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*(8192*a^2*b^
17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15*d^4 + 59392*a^3*b^16*
c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^14*c^11*d^8 - 6037504*a
^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 11894784*a^8*b^11*c^8*d^11
+ 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9*c^6*d^13 + 2455552*a^11*b^8
```

```
*c^5*d^14 - 606208*a^12*b^7*c^4*d^15 + 59392*a^13*b^6*c^3*d^16 + 8192*a^14*
b^5*c^2*d^17))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3
 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c
^7*d - 8*a^7*b*c*d^7))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 +
 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12
 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d
^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*
c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^
3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(3/4)*1i + (((
2197*a*b^10*c^4*d^7)/2 - (7*b^11*c^5*d^6)/2 - (7*a^5*b^6*d^11)/2 + (2197*a^
4*b^7*c*d^10)/2 + 9145*a^2*b^9*c^3*d^8 + 9145*a^3*b^8*c^2*d^9)*1i)/(a^8*d^8
 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))*
(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 +
28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^1
1 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^
11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b
^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10
*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)*1i - (x^(1/2)*(1225*a^6*b^7*d^1
3 + 1225*b^13*c^6*d^7 + 18186*a*b^12*c^5*d^8 + 18186*a^5*b^8*c*d^12 + 75975
*a^2*b^11*c^4*d^9 + 71372*a^3*b^10*c^3*d^10 + 75975*a^4*b^9*c^2*d^11)*1i)/(
8*(a^12*d^12 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495
*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*
c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10
- 12*a*b^11*c^11*d - 12*a^11*b*c*d^11)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 13
72*a*b^3*c^3*d^4 + 294*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 +
4096*a^12*c^3*d^12 - 49152*a^11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901
120*a^3*b^9*c^12*d^3 + 2027520*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5
+ 3784704*a^6*b^6*c^9*d^6 - 3244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d
^8 - 901120*a^9*b^3*c^6*d^9 + 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*
d))^(1/4)))*(-(a^4*d^7 + 2401*b^4*c^4*d^3 + 1372*a*b^3*c^3*d^4 + 294*a^2*b^
2*c^2*d^5 + 28*a^3*b*c*d^6)/(4096*b^12*c^15 + 4096*a^12*c^3*d^12 - 49152*a^
11*b*c^4*d^11 + 270336*a^2*b^10*c^13*d^2 - 901120*a^3*b^9*c^12*d^3 + 202752
0*a^4*b^8*c^11*d^4 - 3244032*a^5*b^7*c^10*d^5 + 3784704*a^6*b^6*c^9*d^6 - 3
244032*a^7*b^5*c^8*d^7 + 2027520*a^8*b^4*c^7*d^8 - 901120*a^9*b^3*c^6*d^9 +
 270336*a^10*b^2*c^5*d^10 - 49152*a*b^11*c^14*d))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=624

$$\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^3}-$$

Rubi [A] time = 0.86, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 472, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{5/4}(bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^3}-\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{b^{5/4}(bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{5/4}(bc-ad)^3}-\frac{d^{5/4}(9bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^3}+\frac{d^{5/4}(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^3}+\frac{dx^{3/2}(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}+\frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x^(3/2))/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x^(3/2))/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(5/4)*(b*c - 9*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) + (b^
(5/4)*(b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) - (d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c
- a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(5/4)*(b
*c - a*d)^3) + (b^(5/4)*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^3) - (b^(5/4)*(b*c -
9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^3) + (d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) - (d^
(5/4)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad-5bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\Bigg(\int \frac{x^2\Big(-4\big(b^2c^2-8ab}{\big(a+}}{8a}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\Bigg(\int\Bigg(-\frac{4b^2c(bc-9ad}{(bc-ad)\big(a+b}}{8a}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(bc-9ad)\right)\text{Subst}\Bigg(}{2a(bc-}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(b^{3/2}(bc-9ad)\right)\text{Subst}}{4a(bc}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{(b(bc-9ad))\,\text{Subst}\Bigg(\int}{8a(b}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^{5/4}(bc-9ad)\log\big(\sqrt{a}}{8\sqrt{2}\,a^{5}}$$

$$= \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{5/4}(bc-9ad)\tan^{-1}\Bigg(1}{4\sqrt{2}\,a^{5/4}(bc-}$$

Mathematica [A] time = 2.09, size = 589, normalized size = 0.94

$$\frac{1}{16}\left(\frac{\sqrt{2}\,b^{5/4}(9ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{5/4}(ad-bc)^3}+\frac{\sqrt{2}\,b^{5/4}(bc-9ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{a^{5/4}(ad-bc)^3}+\frac{2\sqrt{2}\,b^{5/4}(bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}(ad-bc)^3}+\frac{2\sqrt{2}\,b^{5/4}(9ad-bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}(ad-bc)^3}+\frac{8b^2x^{3/2}}{a\left(a+bx^2\right)(bc-ad)^2}+\frac{\sqrt{2}\,d^{5/4}(9bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{5/4}(bc-ad)^3}+\frac{\sqrt{2}\,d^{5/4}(ad-9bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{c^{5/4}(bc-ad)^3}+\frac{2\sqrt{2}\,d^{5/4}(ad-9bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}(bc-ad)^3}+\frac{2\sqrt{2}\,d^{5/4}(9bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}(bc-ad)^3}+\frac{8d^2x^{3/2}}{c\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((8*b^2*x^(3/2))/(a*(b*c - a*d)^2*(a + b*x^2)) + (8*d^2*x^(3/2))/(c*(b*c -
a*d)^2*(c + d*x^2)) + (2*Sqrt[2]*b^(5/4)*(b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-(b*c) + a*d)^3) + (2*Sqrt[2]*b^(5/4)*
(-(b*c) + 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(5/4)*(-
(b*c) + a*d)^3) + (2*Sqrt[2]*d^(5/4)*(-9*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(5/4)*(9*b*c
 - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(5/4)*(b*c - a*d)
^3) + (Sqrt[2]*b^(5/4)*(-(b*c) + 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(a^(5/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*b^(5/4)*(b*c
 - 9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(5
/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(5/4)*(b*c - a*d)^3) + (Sqrt[2]*
d^(5/4)*(-9*b*c + a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(c^(5/4)*(b*c - a*d)^3))/16
```

IntegrateAlgebraic [A] time = 1.67, size = 391, normalized size = 0.63

$$-\frac{\left(9ab^{5/4}d-b^{9/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{5/4}(ad-bc)^3}-\frac{\left(9ab^{5/4}d-b^{9/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{5/4}(ad-bc)^3}+\frac{x^{3/2}\left(a^2d^2+abd^2x^2+b^2c^2+b^2cdx^2\right)}{2ac\left(a+bx^2\right)\left(c+dx^2\right)(ad-bc)^2}-\frac{\left(9bcd^{5/4}-ad^{9/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^3}-\frac{\left(9bcd^{5/4}-ad^{9/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{5/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (x^(3/2)*(b^2*c^2 + a^2*d^2 + b^2*c*d*x^2 + a*b*d^2*x^2))/(2*a*c*(-(b*c) +
a*d)^2*(a + b*x^2)*(c + d*x^2)) - ((-(b^(9/4)*c) + 9*a*b^(5/4)*d)*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(5/4)*
(-(b*c) + a*d)^3) - ((9*b*c*d^(5/4) - a*d^(9/4))*ArcTan[(Sqrt[c] - Sqrt[d]*
x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3) -
((-(b^(9/4)*c) + 9*a*b^(5/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(5/4)*(-(b*c) + a*d)^3) - ((9*b*c*d^(5/
4) - a*d^(9/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d
]*x)])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.64, size = 973, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*((a*b^3)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b
^3*c^2*d + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/4*((a*b^3)^(3/4
)*b*c - 9*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d + 3*sqr
t(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c - (c*d^3
)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3
- sqrt(2)*a^3*c^2*d^4) + 1/4*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3
*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^
2*d^4) - 1/8*((a*b^3)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d
 + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/8*((a*b^3)^(3/4)*b*c -
9*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqr
t(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt
(2)*a^5*b*d^3) - 1/8*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(sqrt(2)*
sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c
^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/8*(9*(c*d^3)^(3
/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3
- sqrt(2)*a^3*c^2*d^4) + 1/2*(b^2*c*d*x^(7/2) + a*b*d^2*x^(7/2) + b^2*c^2*x
^(3/2) + a^2*d^2*x^(3/2))/((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4
 + b*c*x^2 + a*d*x^2 + a*c))
```

maple [A] time = 0.02, size = 778, normalized size = 1.25

$$\frac{a d^{3} x^{\frac{3}{2}}}{2 \left(a d-b c\right)^{3} \left(d x^{2}+c\right) c}-\frac{b^{3} c x^{\frac{3}{2}}}{2 \left(a d-b c\right)^{3} \left(b x^{2}+a\right) a}+\frac{b^{2} d x^{\frac{3}{2}}}{2 \left(a d-b c\right)^{3} \left(b x^{2}+a\right)}-\frac{b d^{2} x^{\frac{3}{2}}}{2 \left(a d-b c\right)^{3} \left(d x^{2}+c\right)}+\frac{\sqrt{2}\, a d^{2} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{c}{d}\right)^{\frac{1}{4}} c}+\frac{\sqrt{2}\, a d^{2} \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{c}{d}\right)^{\frac{1}{4}} c}+\frac{\sqrt{2}\, a d^{2} \ln \left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(a d-b c\right)^{3} \left(\frac{c}{d}\right)^{\frac{1}{4}} c}-\frac{\sqrt{2}\, b^{2} c \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, b^{2} c \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{a}{b}\right)^{\frac{1}{4}} a}-\frac{\sqrt{2}\, b^{2} c \ln \left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16 \left(a d-b c\right)^{3} \left(\frac{a}{b}\right)^{\frac{1}{4}} a}+\frac{9 \sqrt{2}\, b d \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{9 \sqrt{2}\, b d \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{9 \sqrt{2}\, b d \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{9 \sqrt{2}\, b d \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8 \left(a d-b c\right)^{3} \left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{9 \sqrt{2}\, b d \ln \left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{a}{b}}}\right)}{16 \left(a d-b c\right)^{3} \left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{9 \sqrt{2}\, b d \ln \left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, \sqrt{x}+\sqrt{\frac{c}{d}}}\right)}{16 \left(a d-b c\right)^{3} \left(\frac{c}{d}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3/a*x^(3/2)/(b*x^
2+a)*c+9/8*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)-1)*d-1/8*b^2/(a*d-b*c)^3/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)*c+9/16*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d-
1/16*b^2/(a*d-b*c)^3/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2
)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c+9/8*b/(a*d-b*
c)^3/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d-1/8*b^2/(a
*d-b*c)^3/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c+1/2
```

```
*d^3/(a*d-b*c)^3/c*x^(3/2)/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c
)*b+1/8*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*a-9/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)+1)*b+1/8*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*a-9/8*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)-1)*b+1/16*d^2/(a*d-b*c)^3/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)
^(1/2)))*a-9/16*d/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b
```

maxima [**A**] time = 2.48, size = 610, normalized size = 0.98

$$\frac{\left(b^{3}c-9\,ab^{2}d\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)}+\frac{\left(9\,bcd^{2}-ad^{3}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\,\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\,\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{3}c^{4}-3\,ab^{2}c^{3}d+3\,a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)}+\frac{\left(b^{2}cd+abd^{2}\right)x^{\frac{7}{2}}+\left(b^{2}c^{2}+a^{2}d^{2}\right)x^{\frac{3}{2}}}{2\left(a^{2}b^{2}c^{4}-2\,a^{3}bc^{3}d+a^{4}c^{2}d^{2}+\left(ab^{3}c^{3}d-2\,a^{2}b^{2}c^{2}d^{2}+a^{3}bcd^{3}\right)x^{4}+\left(ab^{3}c^{4}-a^{2}b^{2}c^{3}d-a^{3}bc^{2}d^{2}+a^{4}cd^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(b^3*c - 9*a*b^2*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/
16*(9*b*c*d^2 - a*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sq
rt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log
(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) +
 sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3) + 1/2*
((b^2*c*d + a*b*d^2)*x^(7/2) + (b^2*c^2 + a^2*d^2)*x^(3/2))/(a^2*b^2*c^4 -
2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^
3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

mupad [**B**] time = 4.56, size = 32506, normalized size = 52.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan((((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 12
16*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 8
```

```
37408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13
*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784
*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d
^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^
14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 190
40*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15
*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 10
01*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432
*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^1
2*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) - (x^(1/2)*(
-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 3
6*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11
 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^1
3*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b
^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^1
0*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a
^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 974
8480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^
14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 5477
00736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b
^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16
- 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b
^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a
^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66
*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*
b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*
d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*
c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*
b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^
15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b
^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520
*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 4915
2*a*b^11*c^16*d))^(3/4) - (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 362
7*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a
^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13))/(16
*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 +
 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a
^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c
^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b
^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(40
96*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10
*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^
5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027
520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 4
9152*a*b^11*c^16*d))^(1/4) - ((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216
```

```
*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*
a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 +
6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^1
5*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 103
74112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*
c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^
16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^
14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a
^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^
8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b
^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c
^4*d^12) + (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 48
6*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 -
49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3
+ 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^1
1*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3
*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b
^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^
3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 -
116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*
b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^
13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 24919244
8*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*
d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*
b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12
*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^
8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^
7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(
-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 3
6*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11
 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^1
3*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b
^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^1
0*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(3/4) + (x^(1/2)*(81*a^7*b^8*d^15 +
81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^1
3*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^
5*b^10*c^2*d^13))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d -
 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6
*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7
*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10))
)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7
 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d
^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*
c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^
7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*
```

```
a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4))/(((((32*a^19*b^4*d^23 + 32
*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^
21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656
*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^
11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^1
0*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16
 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8
*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2*b^
14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*
b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b
^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*
c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5
*d^11 + 91*a^14*b^2*c^4*d^12) - (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 29
16*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 +
4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901
120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5
+ 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9
*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^1
6*d))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*
c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672
*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*
d^10 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326
144*a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^1
1*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 4
0476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^
3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12
*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*
c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6
 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a
^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 48
6*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 -
49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3
+ 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^1
1*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3
*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(3/4)*1i - (x^(
1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^
9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*
b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13)*1i)/(16*(a^2*b^12*c^14 + a^14*c^2*
d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220
*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^
6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d
^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3
*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^
5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*
c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^
```

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6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 90112
0*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)
- ((3645*a^6*b^9*d^15)/32 + (3645*b^15*c^6*d^9)/32 - (49815*a*b^14*c^5*d^10
)/16 - (49815*a^5*b^10*c*d^14)/16 + (918675*a^2*b^13*c^4*d^11)/32 - (739025
*a^3*b^12*c^3*d^12)/8 + (918675*a^4*b^11*c^2*d^13)/32)/(a^2*b^14*c^16 + a^1
6*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2
 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 +
 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 200
2*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^
14*b^2*c^4*d^12) + ((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^
18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c
^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^
6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^1
2 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11
*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 -
 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3
*d^20 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^
3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^
13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10
*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9
 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) +
 (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*
c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11
*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*
a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 32
44032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 +
 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22*c^19*d
^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^1
7*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*
a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*
d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 54770
0736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^1
0*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 974
8480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^
21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^
3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4
 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^
10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9
+ 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*
d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*
a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 32
44032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^
7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*
d^10 - 49152*a*b^11*c^16*d))^(3/4)*1i + (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15
*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d
```

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^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*
c^2*d^13)*1i)/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*
a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8
*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7
 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-
(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36
*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11
+ 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13
*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^
5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10
*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)))*(-(a^4*d^9 + 6561*b^4*c^4*d^5
 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^
17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2
- 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12
*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^
4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^1
1*c^16*d))^(1/4) - atan(((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^
22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b
^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 65647
68*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^1
1*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112
*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d
^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^
7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a
^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c
^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^1
0*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^
9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12)
- (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2
*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^1
1*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520
*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3
244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9
+ 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22*c^19*
d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^
17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152
*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12
*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 5477
00736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^
10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 97
48480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d
^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c
^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^
4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a
^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9
```

```
 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c
*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336
*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3
244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d
^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7
*d^10 - 49152*a*b^11*c^16*d))^(3/4)*1i - (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^1
5*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*
d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10
*c^2*d^13)*1i)/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12
*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^
8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^
7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(
-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 3
6*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11
 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^1
3*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b
^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^1
0*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4) - (((32*a^19*b^4*d^23 + 32*b^2
3*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c
^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5
*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 +
 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^
13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6
564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4
*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)/(a^2*b^14*c^16
+ a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^1
4*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*
d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8
- 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 +
91*a^14*b^2*c^4*d^12) + (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3
*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^1
2*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*
b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 378470
4*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 9
01120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1
/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5
 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^1
8*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 4
12041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10
*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^
15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*
a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 -
 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^1
1*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3
 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a
```

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^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*
c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^
2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^
11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 202752
0*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 -
3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9
 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(3/4)*1i + (x^(1/2)*(81
*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14
 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3
*d^12 - 80999*a^5*b^10*c^2*d^13)*1i)/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 1
2*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9
*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^
6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*
a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 4
86*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 -
 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3
 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^
11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^
3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4))/(((3645
*a^6*b^9*d^15)/32 + (3645*b^15*c^6*d^9)/32 - (49815*a*b^14*c^5*d^10)/16 - (
49815*a^5*b^10*c*d^14)/16 + (918675*a^2*b^13*c^4*d^11)/32 - (739025*a^3*b^1
2*c^3*d^12)/8 + (918675*a^4*b^11*c^2*d^13)/32)/(a^2*b^14*c^16 + a^16*c^2*d^
14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a
^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^
8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b
^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c
^4*d^12) + (((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 -
1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 +
 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^
13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 44587
84*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8
*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*
a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 1
9040*a^17*b^6*c^2*d^21)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d
 - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001
*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a
^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*
b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) - (x^(1/2)*(-(
a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*
a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 +
 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*
d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5
*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*
b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^1
9*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 97484
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80*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14
*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 547700
736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b^1
2*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 -
116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7
*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2
*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a
^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^
7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^
8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4*c^
4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^
12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15
*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7
*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a
^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*
a*b^11*c^16*d))^(3/4) - (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*
a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3
*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13))/(16*(
a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 6
6*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7
*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6
*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d^9 + 6561*b^4
*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096
*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c
^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*
b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 202752
0*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 491
52*a*b^11*c^16*d))^(1/4) + (((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*
b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3
*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 656
4768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c
^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 103741
12*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6
*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*
b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14
*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11
*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c
^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*
d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12
) + (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b
^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a
^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 20275
20*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 -
 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^
9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22*c^1
```

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9*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*
c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 1167851
52*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^
12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 54
7700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*
b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 -
9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2
*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b
*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*
d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495
*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^4*d
^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b
*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 2703
36*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 -
 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10
*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c
^7*d^10 - 49152*a*b^11*c^16*d))^(3/4) + (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15
*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d
^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*
c^2*d^13))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^1
3*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^
10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 +
495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(a^
4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^
3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 2
70336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^
4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c
^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^
2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)))*(-(a^4*d^9 + 6561*b^4*c^4*d^5 -
2916*a*b^3*c^3*d^6 + 486*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17
+ 4096*a^12*c^5*d^12 - 49152*a^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 9
01120*a^3*b^9*c^14*d^3 + 2027520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^
5 + 3784704*a^6*b^6*c^11*d^6 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c
^9*d^8 - 901120*a^9*b^3*c^8*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c
^16*d))^(1/4)*2i + ((x^(3/2)*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2
 - 2*a*b*c*d)) + (b*d*x^(7/2)*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d)))/(a*c + x^2*(a*d + b*c) + b*d*x^4) - atan(((((32*a^19*b^4*d^23 + 32
*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^
21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656
*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^
11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^1
0*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16
 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8
*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)/(a^2*b^14*c
^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12
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*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c
^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*
d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^1
1 + 91*a^14*b^2*c^4*d^12) - (x^(1/2)*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a
^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096
*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*
a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784
704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8
 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11)
)^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18
*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5
*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10
 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*
a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^
8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476
672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^
20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3
*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11
*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 7
92*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*
b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^
2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 4915
2*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 202
7520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6
 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3
*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(3/4)*1i - (x^(1/2)
*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*
d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11
*c^3*d^12 - 80999*a^5*b^10*c^2*d^13)*1i)/(16*(a^2*b^12*c^14 + a^14*c^2*d^12
 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5
*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^
8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 +
 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3
 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^
12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*
d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6
*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^1
4*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4) - ((
(32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5
*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b
^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 103
31040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c
^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331
040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*
d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6
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*c^2*d^21)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*
c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^1
2*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^
7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10
- 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) + (x^(1/2)*(-(b^9*c^4 + 656
1*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/
(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b
^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a
^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 202
7520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10
- 49152*a^16*b*c*d^11))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22
- 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c
^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192
448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*
c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 -
412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^1
4*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1
486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 +
a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*
d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 9
24*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*
b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*
a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 409
6*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120
*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 378
4704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8
 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11
))^(3/4)*1i + (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6
*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*
d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13)*1i)/(16*(a^2*b^1
2*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b
^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^
9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 -
220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^
4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d
^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2
 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*
d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^
4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16
*b*c*d^11))^(1/4))/(((3645*a^6*b^9*d^15)/32 + (3645*b^15*c^6*d^9)/32 - (498
15*a*b^14*c^5*d^10)/16 - (49815*a^5*b^10*c*d^14)/16 + (918675*a^2*b^13*c^4*
d^11)/32 - (739025*a^3*b^12*c^3*d^12)/8 + (918675*a^4*b^11*c^2*d^13)/32)/(a
^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91
*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*
a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10
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*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^
3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) + (((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4
 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 -
161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14
*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*
a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^1
4 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^1
3*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 16
1664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)/(a^2*b^14*c^16 + a^16*c^2
*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 36
4*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003
*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^1
1*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^
2*c^4*d^12) - (x^(1/2)*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 +
 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12
 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^
3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c
^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*
b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4)*(4096*
a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848
*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8
 - 116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a
^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10
*d^13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 24919
2448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c
^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^
18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d -
 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6
*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7
*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10))
)*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2
 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^1
1*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c
^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12
*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*
a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(3/4) - (x^(1/2)*(81*a^7*b^8*d^15
 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*
b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999
*a^5*b^10*c^2*d^13))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*
d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*
a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*
c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^1
0)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d
^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*
c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^
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8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a
^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 2703
36*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4) + (((32*a^19*b^4*d^23 +
32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*
b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 28426
56*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*
d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a
^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^
16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b
^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)/(a^2*b^14
*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^
12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9
*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^
8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d
^11 + 91*a^14*b^2*c^4*d^12) + (x^(1/2)*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916
*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 40
96*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 90112
0*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 37
84704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^
8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^1
1))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^
18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a
^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^
10 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 60032614
4*a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*
c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 404
76672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*
d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a
^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^
11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 -
 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^1
2*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*
a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49
152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2
027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^
6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c
^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(3/4) + (x^(1/2)*
(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d
^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*
c^3*d^12 - 80999*a^5*b^10*c^2*d^13))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 1
2*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9
*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^
6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*
a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 4
86*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 -
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 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3
+ 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6
*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^
3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4)))*(-(b^9
*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b
^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 27
0336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 -
 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5
*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2
*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4)*2i + 2*atan((((((32*a^19*b^4*d^23 +
 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2
*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842
656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12
*d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*
a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d
^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*
b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2
*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a
^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^
7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b
^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*
c^5*d^11 + 91*a^14*b^2*c^4*d^12) - (x^(1/2)*(-(b^9*c^4 + 6561*a^4*b^5*d^4 -
 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12
 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 -
901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5
 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c
^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*
c*d^11))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^
21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476
672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^
13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600
326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 - 412041216*a^12*
b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17
+ 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6
*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 -
 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b
^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*
d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 6
6*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 +
 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12
 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^
3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c
^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*
b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(3/4) - (x^(
1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^
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9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*
b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13))/(16*(a^2*b^12*c^14 + a^14*c^2*d^1
2 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^
5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c
^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9
+ 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^
3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c
^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9
*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^
6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^
14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4) - (
(((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b
^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4
*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 1
0331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14
*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 103
31040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^
5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b
^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a
^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^
10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*
c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6
*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) + (x^(1/2)*(-(b^9*c^4
 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c
^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336
*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 324
4032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7
 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2
*d^10 - 49152*a^16*b*c*d^11))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c
*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*
b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 +
249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9
*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d
^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 1167851
52*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^
19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c
^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10
*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d
^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220
*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 -
 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12
 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 -
901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5
 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c
^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*
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c*d^11))^(3/4) + (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*
c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c
^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13))/(16*(a^2*b^1
2*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b
^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^
9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 -
220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^
4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d
^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2
 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*
d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^
4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16
*b*c*d^11))^(1/4))/(((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c
^18*d^5 - 1216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*
c^16*d^7 + 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a
^6*b^17*c^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^
12 - 4458784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^1
1*b^12*c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17
- 2842656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^
3*d^20 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a
^3*b^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c
^13*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^1
0*d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^
9 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12)
- (x^(1/2)*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7
*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6
*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*
a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 324
4032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9
+ 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4)*(4096*a*b^22*c^19*
d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^
17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152
*a^6*b^17*c^14*d^9 + 249192448*a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12
*d^11 + 547700736*a^9*b^14*c^11*d^12 - 600326144*a^10*b^13*c^10*d^13 + 5477
00736*a^11*b^12*c^9*d^14 - 412041216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^
10*c^7*d^16 - 116785152*a^14*b^9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 97
48480*a^16*b^7*c^4*d^19 + 1486848*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d
^21))/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c
^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^
4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a
^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4
 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c
^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336
*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 324
4032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7
```

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 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2
*d^10 - 49152*a^16*b*c*d^11))^(3/4)*1i - (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^1
5*c^7*d^8 + 3627*a*b^14*c^6*d^9 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*
d^10 + 339435*a^3*b^12*c^4*d^11 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10
*c^2*d^13)*1i)/(16*(a^2*b^12*c^14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12
*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^
8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^
7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(
-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 3
6*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d
 + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*
d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^
5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^1
5*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4) - ((3645*a^6*b^9*d^15)/32 + (3
645*b^15*c^6*d^9)/32 - (49815*a*b^14*c^5*d^10)/16 - (49815*a^5*b^10*c*d^14)
/16 + (918675*a^2*b^13*c^4*d^11)/32 - (739025*a^3*b^12*c^3*d^12)/8 + (91867
5*a^4*b^11*c^2*d^13)/32)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*
d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 100
1*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*
a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12
*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) + ((((32*a^19
*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1216*a^18*b^5*c*d^22
+ 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 + 837408*a^4*b^19*c^15
*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c^13*d^10 - 10331040*a^
7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 4458784*a^9*b^14*c^10*d^13
 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*c^8*d^15 - 10331040*a^12
*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842656*a^14*b^9*c^5*d^18 + 8
37408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^20 + 19040*a^17*b^6*c^2*d^2
1)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b^13*c^15*d - 14*a^15*b*c^3*
d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13*d^3 + 1001*a^6*b^10*c^12*d^
4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*d^6 - 3432*a^9*b^7*c^9*d^7 +
3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9 + 1001*a^12*b^4*c^6*d^10 - 36
4*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12) + (x^(1/2)*(-(b^9*c^4 + 6561*a^
4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(409
6*a^17*d^12 + 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*
c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*
b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520
*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49
152*a^16*b*c*d^11))^(1/4)*(4096*a*b^22*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 12
2880*a^2*b^21*c^18*d^5 + 1486848*a^3*b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*
d^7 + 40476672*a^5*b^18*c^15*d^8 - 116785152*a^6*b^17*c^14*d^9 + 249192448*
a^7*b^16*c^13*d^10 - 412041216*a^8*b^15*c^12*d^11 + 547700736*a^9*b^14*c^11
*d^12 - 600326144*a^10*b^13*c^10*d^13 + 547700736*a^11*b^12*c^9*d^14 - 4120
41216*a^12*b^11*c^8*d^15 + 249192448*a^13*b^10*c^7*d^16 - 116785152*a^14*b^
9*c^6*d^17 + 40476672*a^15*b^8*c^5*d^18 - 9748480*a^16*b^7*c^4*d^19 + 14868
```

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48*a^17*b^6*c^3*d^20 - 122880*a^18*b^5*c^2*d^21))/(16*(a^2*b^12*c^14 + a^14
*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*c^12*d^2
- 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^5 + 924*a
^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*a^11*b^3*
c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*
b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^
5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8
*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704
*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 9
01120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(
3/4)*1i + (x^(1/2)*(81*a^7*b^8*d^15 + 81*b^15*c^7*d^8 + 3627*a*b^14*c^6*d^9
 + 3627*a^6*b^9*c*d^14 - 80999*a^2*b^13*c^5*d^10 + 339435*a^3*b^12*c^4*d^11
 + 339435*a^4*b^11*c^3*d^12 - 80999*a^5*b^10*c^2*d^13)*1i)/(16*(a^2*b^12*c^
14 + a^14*c^2*d^12 - 12*a^3*b^11*c^13*d - 12*a^13*b*c^3*d^11 + 66*a^4*b^10*
c^12*d^2 - 220*a^5*b^9*c^11*d^3 + 495*a^6*b^8*c^10*d^4 - 792*a^7*b^7*c^9*d^
5 + 924*a^8*b^6*c^8*d^6 - 792*a^9*b^5*c^7*d^7 + 495*a^10*b^4*c^6*d^8 - 220*
a^11*b^3*c^5*d^9 + 66*a^12*b^2*c^4*d^10)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 -
2916*a^3*b^6*c*d^3 + 486*a^2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12
+ 4096*a^5*b^12*c^12 - 49152*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 9
01120*a^8*b^9*c^9*d^3 + 2027520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5
+ 3784704*a^11*b^6*c^6*d^6 - 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^
4*d^8 - 901120*a^14*b^3*c^3*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c
*d^11))^(1/4)))*(-(b^9*c^4 + 6561*a^4*b^5*d^4 - 2916*a^3*b^6*c*d^3 + 486*a^
2*b^7*c^2*d^2 - 36*a*b^8*c^3*d)/(4096*a^17*d^12 + 4096*a^5*b^12*c^12 - 4915
2*a^6*b^11*c^11*d + 270336*a^7*b^10*c^10*d^2 - 901120*a^8*b^9*c^9*d^3 + 202
7520*a^9*b^8*c^8*d^4 - 3244032*a^10*b^7*c^7*d^5 + 3784704*a^11*b^6*c^6*d^6
- 3244032*a^12*b^5*c^5*d^7 + 2027520*a^13*b^4*c^4*d^8 - 901120*a^14*b^3*c^3
*d^9 + 270336*a^15*b^2*c^2*d^10 - 49152*a^16*b*c*d^11))^(1/4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.474 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=628

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3}{}$$

Rubi [A] time = 0.86, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{b\sqrt{x}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d\sqrt{x}\,(ad+bc)}{2ac\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*Sqrt[x])/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*Sqrt[x])/(2*
a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4
)*(11*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2
]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (
b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)
*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-4\left(3b^2c^2\right.}{}\right.}{}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(3bc-11ad)\right)}{2}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^2(3bc-11ad)\right)}{4}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{\left(b^{3/2}(3bc-11ad\right.}{}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{7/4}(3bc-11ad)}{8}$$

$$= \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\left(c+dx^2\right)} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{b^{7/4}(3bc-11ad)}{4\sqrt{2}\,a^{7}}$$

Mathematica [A] time = 6.21, size = 661, normalized size = 1.05

$$\frac{b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(11ad-3bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(11ad-3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^2\sqrt{x}}{2a\left(a+bx^2\right)(ad-bc)^2}+\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(ad-bc)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(ad-bc)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(ad-bc)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(ad-bc)^3}+\frac{d^2\sqrt{x}}{2c\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (b^2*Sqrt[x])/(2*a*(-(b*c) + a*d)^2*(a + b*x^2)) + (d^2*Sqrt[x])/(2*c*(b*c
- a*d)^2*(c + d*x^2)) - (b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4
```

```
)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^
3) - (b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x]
)/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c
- 3*a*d)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))]
)/(4*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(
Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(7/4)
*(-(b*c) + a*d)^3) + (b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/
4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(-(
b*c) + a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.67, size = 391, normalized size = 0.62

$$-\frac{\left(11ab^{7/4}d-3b^{11/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{7/4}(ad-bc)^3}+\frac{\left(11ab^{7/4}d-3b^{11/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{7/4}(ad-bc)^3}+\frac{\sqrt{x}\left(a^2d^2+abd^2x^2+b^2c^2+b^2cdx^2\right)}{2ac\left(a+bx^2\right)\left(c+dx^2\right)(ad-bc)^2}-\frac{\left(11bcd^{7/4}-3ad^{11/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^3}+\frac{\left(11bcd^{7/4}-3ad^{11/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{7/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (Sqrt[x]*(b^2*c^2 + a^2*d^2 + b^2*c*d*x^2 + a*b*d^2*x^2))/(2*a*c*(-(b*c) +
a*d)^2*(a + b*x^2)*(c + d*x^2)) - ((-3*b^(11/4)*c + 11*a*b^(7/4)*d)*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(7/4
)*(-(b*c) + a*d)^3) - ((11*b*c*d^(7/4) - 3*a*d^(11/4))*ArcTan[(Sqrt[c] - Sq
rt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)
^3) + ((-3*b^(11/4)*c + 11*a*b^(7/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(7/4)*(-(b*c) + a*d)^3) + ((11*
b*c*d^(7/4) - 3*a*d^(11/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.29, size = 977, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2
)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/4*(3*(a*b^3)
^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2
*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/4*(11*(c*d^3)^(1/4)*b*c*d
 - 3*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b
*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(
1/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sq
rt(2)*a^3*c^2*d^3) + 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sq
rt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/8*(3*(a*
b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2
)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^
(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*
c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3
) - 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(-sqrt(2)*sqrt(
x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d +
3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/2*(b^2*c*d*x^(5/2) + a*b
*d^2*x^(5/2) + b^2*c^2*sqrt(x) + a^2*d^2*sqrt(x))/((a*b^2*c^3 - 2*a^2*b*c^2
*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

maple [A] time = 0.02, size = 808, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3/a*x^(1/2)/(b*x^
2+a)*c+11/8*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)-1)*d-3/8*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)*c+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*ln((x+(
a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b
)^(1/2)))*d-3/16*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*
2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c
+11/8*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)*d-3/8*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)*c+1/2*d^3/(a*d-b*c)^3/c*x^(1/2)/(d*x^2+c)*a-1/2*d^2/(a*d-b*
c)^3*x^(1/2)/(d*x^2+c)*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x
+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c
```

```
/d)^(1/2)))*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)
*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*
b+3/8*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)*a-11/8*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)*b+3/8*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)*a-11/8*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b
```

<u>maxima</u> [A] time = 2.59, size = 678, normalized size = 1.08

$$\frac{\left(\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-11ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc-11ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^{2}}{16\left(ab^{3}c^{3}-3a^{2}b^{2}c^{2}d+3a^{3}bcd^{2}-a^{4}d^{3}\right)}+\frac{\left(b^{2}cd+abd^{2}\right)x^{\frac{5}{2}}+\left(b^{2}c^{2}+a^{2}d^{2}\right)\sqrt{x}}{2\left(a^{2}b^{2}c^{4}-2a^{3}bc^{3}d+a^{4}c^{2}d^{2}+\left(ab^{3}c^{3}d-2a^{2}b^{2}c^{2}d^{2}+a^{3}bcd^{3}\right)x^{4}+\left(ab^{3}c^{4}-a^{2}b^{2}c^{3}d-a^{3}bc^{2}d^{2}+a^{4}cd^{3}\right)x^{2}\right)}+\frac{\frac{2\sqrt{2}\left(11bcd^{2}-3ad^{3}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(11bcd^{2}-3ad^{3}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(11bcd^{2}-3ad^{3}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(11bcd^{2}-3ad^{3}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{3}c^{4}-3ab^{2}c^{3}d+3a^{2}bc^{2}d^{2}-a^{3}cd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*(3*b*c - 11*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(-sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c
^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/2*((b^2*c*d + a*b*d^2)*
x^(5/2) + (b^2*c^2 + a^2*d^2)*sqrt(x))/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c
^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 -
 a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/16*(2*sqrt(2)*(11*b*c*
d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt
(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11
*b*c*d^2 - 3*a*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d
)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)
*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(-sqrt(2)*c^(
1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

<u>mupad</u> [B] time = 5.07, size = 37332, normalized size = 59.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x^(1/2)*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b
*d*x^(5/2)*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2
*(a*d + b*c) + b*d*x^4) - atan((((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 7127
```

```
04*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d
^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*
a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^1
3*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 56
2982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15
*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19
 - 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*
c^3*d^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 1
2*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9
*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10
*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 +
 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b
^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096
*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*
a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 37
84704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^
8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^1
1))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^
17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 89661
44*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12
*d^11 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144
*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16
+ 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^
19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 2
8*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c
^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a
^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 +
4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901
120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5
+ 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^
4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c
*d^11))^(3/4) + ((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*
c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5
*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c
^2*d^13)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b
^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 159
72*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^1
2 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 -
 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*
d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^
4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18
*b*c*d^11))^(1/4)*1i + (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 14
9094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 -
 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c
```

```
^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12
 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7
*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*
c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d
^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3
*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 40
96*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 90112
0*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 +
3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*
d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d
^11))^(1/4) + ((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d
^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6
*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10
 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408
*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*
c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 2
71552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^
7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a
^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d
 - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^
8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^
5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d
^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^
2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 4
9152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 +
 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6
*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^
3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*(6144*a^
4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230
272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d
^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^
12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14
 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*
c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12
 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2
 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b
^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 653
4*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12
 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d
^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6
*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^1
6*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3/4) - ((
891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7
*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*
b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)/(a^4*b^8*
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c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10
*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^
10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 +
 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*
c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c
^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13
*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120
*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*
1i + (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^
10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*
c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*
a^6*b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^1
5*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 49
5*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^1
1*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c
^6*d^10)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*
a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 -
 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3
 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c
^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*
b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4))/(((((
x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^4*b
^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 2
71552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^9*b
^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^
13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635523
072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*
c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 61726
72*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23))
/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^
11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 7
92*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^
12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^1
1*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 11
88*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11
*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*
c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^1
4*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336
*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*(6144*a^4*b^19*c^19*d^4 -
90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*
d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*
b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12
+ 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^
8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a
^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 -
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8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*
d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81
*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2
- 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*
c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*
b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032
*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 27
0336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3/4) + ((891*a^8*b^7*d^15)/
2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 3150
9*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66
138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)/(a^4*b^8*c^12 + a^12*c^4*d^
8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*
c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(
-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*
d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b
^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a
^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 324
4032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9
+ 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4) + (x^(1/2)*(9801*a
^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*
c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^
4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(
16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11
 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792
*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12
*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*b^11*c^
4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a
*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d +
 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*
d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^
5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^1
7*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4) - ((((x^(1/2)*(36864*a^2*b^23*
c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*
a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d
^9 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 56298291
2*a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13
*c^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 +
 487280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^1
7*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 7
12704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^
16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^
2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 92
4*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^1
3*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^
4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a
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^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^1
0*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b
^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*
a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 491
52*a^18*b*c*d^11))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5
+ 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c
^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456
*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10
*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^1
6*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*
a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^1
1*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^
7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(40
96*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10
*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^
12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027
520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 -
 49152*a^18*b*c*d^11))^(3/4) - ((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2
 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 661
38*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 +
31509*a^6*b^9*c^2*d^13)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8
*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8
*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^
4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)
/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*
b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 324403
2*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 +
2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^
10 - 49152*a^18*b*c*d^11))^(1/4) + (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*
c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^1
5*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 348460
2*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(16*(a^4*b^12*c^16 + a^16
*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2
 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*
a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*
b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 -
15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*
d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^
2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c
^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15
*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a
^18*b*c*d^11))^(1/4)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c
*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7
*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10
```

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*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 378470
4*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 -
901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^
(1/4)*2i + 2*atan((((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c
^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 11043225
6*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15
*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 2272
17408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*
b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^1
7 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^
18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36
864*a^21*b^4*c^2*d^23)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11
*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3
+ 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792
*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b
^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 +
 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*
c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c
^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13
*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120
*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*
(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^
6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^1
4*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 39
87456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*
c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a
^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*
b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*
c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 2
8*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d
^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b
^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b
^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*
a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 90
1120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3
/4)*1i + (((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^
8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10
+ 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^1
3)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 +
 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3
*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972
*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12
+ 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 9
01120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^
5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*
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c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b
*c*d^11))^(1/4) - (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*
a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484
602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^
14 + 1001520*a^6*b^11*c^2*d^15))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^
5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^1
3*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*
a^14*b^2*c^6*d^10)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d
^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b
^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b
^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*
a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 90
1120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1
/4) + ((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 617
2672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^
17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 63552
3072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^
14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^1
5 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512
*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^
20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*
c^2*d^23)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*
a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8
*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9
*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
 - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9
*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*
a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027
520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6
- 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3
*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*(6144*a^4*b^1
9*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a
^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 +
9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^
11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 54
90688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d
^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^
12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56
*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^
6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2
*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49
152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 +
2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*
d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3
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*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3/4)*1i - (((8
91*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*
b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b
^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)*1i)/(a^4*b
^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c
^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28
*a^10*b^2*c^6*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^
3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^
12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^
9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a
^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901
120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/
4) - (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^
10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*
c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*
a^6*b^11*c^2*d^15))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d
 - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a
^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b
^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*
d^10)))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2
*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49
152*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 +
2027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*
d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3
*c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4))/(((((x^(
1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21
*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 2715
52512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^9*b^16
*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^13
- 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635523072
*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7
*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 6172672*
a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23)*1i)
/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^
11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 7
92*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^
12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^1
1*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 11
88*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11
*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*
c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^1
4*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336
*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*(6144*a^4*b^19*c^19*d^4 -
90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*
d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*
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b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12
+ 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^
8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a
^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 -
8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*
d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81
*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2
- 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*
c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*
b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032
*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 27
0336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3/4)*1i + (((891*a^8*b^7*d^
15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 +
31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11
- 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)*1i)/(a^4*b^8*c^12 + a^1
2*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*
a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6
*d^6))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*
b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 491
52*a^8*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2
027520*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d
^6 - 3244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*
c^3*d^9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*1i - (x^(1
/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 14909
4*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c
^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a
^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*
c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*
d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))
*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^
2*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8
*b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520
*a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3
244032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^
9 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4) - ((((x^(1/2)*(3
6864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d
^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a
^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d
^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^13 - 22721
7408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b
^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 +
 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^
6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23)*1i)/(16*(a
^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66
```

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*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*
b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*
c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 +
 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^
10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 27
0336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4
 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c
^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b
^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a
^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5
490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^
13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12 + 91914
24*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d
^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5
*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b
^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 7
0*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*b^11*c
^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*
a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d
+ 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8
*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b
^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^
17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(3/4)*1i - (((891*a^8*b^7*d^15)/2 +
 (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a
^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138
*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)*1i)/(a^4*b^8*c^12 + a^12*c^4*d
^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5
*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*
(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2
*d^2 - 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*
b^11*c^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*
a^11*b^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 32
44032*a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9
 + 270336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)*1i - (x^(1/2)*(98
01*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b
^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 576903
8*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15
)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c
^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^
4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 4
95*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*
b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 -
 1188*a*b^10*c^3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c
^11*d + 270336*a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b
^8*c^8*d^4 - 3244032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*
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a^14*b^5*c^5*d^7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270
336*a^17*b^2*c^2*d^10 - 49152*a^18*b*c*d^11))^(1/4)))*(-(81*b^11*c^4 + 1464
1*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^
3*d)/(4096*a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*
a^9*b^10*c^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 32
44032*a^12*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^
7 + 2027520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^
2*d^10 - 49152*a^18*b*c*d^11))^(1/4) - atan((((((x^(1/2)*(36864*a^2*b^23*c^
21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^
5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9
 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*
a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c
^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 4
87280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*
b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712
704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16
*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2
 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*
a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*
b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7
 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^1
2*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*
d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*
c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^
8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152
*a*b^11*c^18*d))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 +
585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^1
5*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a
^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d
^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*
b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^
19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*
b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 -
 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*
d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096
*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c
^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*
b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 202752
0*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 4
9152*a*b^11*c^18*d))^(3/4) + ((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 -
 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138
*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31
509*a^6*b^9*c^2*d^13)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a
^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d
^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*
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c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(
4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^
10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*
a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 20
27520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10
 - 49152*a*b^11*c^18*d))^(1/4)*1i + (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17
*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^
15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 34846
02*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 +
a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*
d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 +
924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a
^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b
^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^1
7*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^
7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*
a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 491
52*a*b^11*c^18*d))^(1/4) + ((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^
3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 +
110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b
^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^1
2 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 5629829
12*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10
*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 318
99648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d
^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5
*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13
*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a
^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12
*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b
^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704
*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 9
01120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(
1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^
17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^
9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11
 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14
*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585
728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/
(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6
*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^
5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*
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c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*
a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a
^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 378
4704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8
 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d
))^(3/4) - ((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d
^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10
 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^
13)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28
*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^
7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*
b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4
096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 9011
20*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 +
 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11
*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^
18*d))^(1/4)*1i + (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*
a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484
602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^
14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12
*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*
c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*
d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 +
66*a^14*b^2*c^6*d^10)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^
3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^
12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3
*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 37847
04*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 -
 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))
^(1/4))/(((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 +
6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19
*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 63
5523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11
*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*
d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552
512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5
*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b
^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*
a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8
*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9
*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
 + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2
*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*
a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027
520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6
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- 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10
*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)*(6144*a^4*b^1
9*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a
^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 +
9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^
11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 54
90688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d
^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^
12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56
*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^
6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2
*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49
152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 +
2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*
d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*
c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(3/4) + ((891*a
^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*
c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*
c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)/(a^4*b^8*c^12
+ a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2
- 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^
2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534
*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12
- 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^
3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c
^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*
b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4) + (x^
(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149
094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12
 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11
*c^2*d^15))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^
15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c
^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d
^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11
*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*
a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 32
44032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9
 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4) - ((((x^(1/2)*(36
864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^
6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^
7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^
11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217
408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^
```

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11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 +
110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6
*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b
^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6
*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*
c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*
d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 146
41*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*
d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336
*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3
244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d
^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c
^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b
^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 54906
88*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d
^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a
^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15
 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5
*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c
^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^
8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 +
 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*
b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 27
0336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4
 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^
12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b
^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(3/4) - ((891*a^8*b^7*d^15)/2 + (891*b^
15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*
c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^1
0*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b
^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 7
0*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^
11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*
a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11
+ 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15
*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^
5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^
10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4) + (x^(1/2)*(9801*a^8*b^9*d^17
 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 10
01520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*
d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(16*(a^4*b^1
2*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b
^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^
11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^
8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*a^4*d^11 + 14641*b
```

^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10
)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2
*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 32440
32*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 +
 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d
^10 - 49152*a*b^11*c^18*d))^(1/4)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15
972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^
19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2
- 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14
*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^
4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b
^11*c^18*d))^(1/4)*2i + 2*atan((((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 7127
04*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d
^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*
a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^1
3*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 56
2982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15
*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19
- 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*
c^3*d^22 + 36864*a^21*b^4*c^2*d^23)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12
- 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*
b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c
^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^
9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*
b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4
096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 9011
20*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 +
 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11
*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^
18*d))^(1/4)*(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6
*b^17*c^17*d^6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 89
66144*a^9*b^14*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c
^12*d^11 - 3987456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966
144*a^14*b^9*c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^
16 + 585728*a^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4
*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
+ 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^
3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 1597
2*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19
 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 -
901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d
^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*
c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^1
1*c^18*d))^(3/4)*1i + (((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*
a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b

```
^12*c^5*d^10 + 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^
6*b^9*c^2*d^13)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^1
1*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^
4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(40
96*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10
*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^
5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027
520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 -
 49152*a*b^11*c^18*d))^(1/4) - (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*
d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^
6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^
5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(16*(a^4*b^12*c^16 + a^16*c^4
*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 22
0*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10
*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*
c^7*d^9 + 66*a^14*b^2*c^6*d^10)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 1597
2*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19
 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 -
901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d
^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*
c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^1
1*c^18*d))^(1/4) + ((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c
^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 11043225
6*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15
*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 2272
17408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*
b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^1
7 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^
18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36
864*a^21*b^4*c^2*d^23)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11
*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3
+ 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792
*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b
^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 +
 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*
d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^
16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*
b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120
*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)*
(6144*a^4*b^19*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^
6 - 2230272*a^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^1
4*c^14*d^9 + 9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 39
87456*a^12*b^11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*
c^9*d^14 + 5490688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a
```

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^17*b^6*c^6*d^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*
b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*
c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 2
8*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d
^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*
c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^
9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*
a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 90
1120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(3
/4)*1i - (((891*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^
8 - 6210*a^7*b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10
+ 60307*a^4*b^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^1
3)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 +
 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3
*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972
*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19
+ 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 9
01120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^
5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c
^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11
*c^18*d))^(1/4) - (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*
a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484
602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^
14 + 1001520*a^6*b^11*c^2*d^15))/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^
5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^1
3*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*
a^14*b^2*c^6*d^10)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d
^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*
c^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^
9*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*
a^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 90
1120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1
/4))/(((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 617
2672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^
17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 63552
3072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^
14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^1
5 - 635523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512
*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^
20 + 6172672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*
c^2*d^23)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*
a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8
*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9
*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
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 + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2
*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*
a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027
520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6
- 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10
*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)*(6144*a^4*b^1
9*c^19*d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a
^7*b^16*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 +
9191424*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^
11*c^11*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 54
90688*a^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d
^17 - 90112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^
12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56
*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^
6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2
*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49
152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 +
2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*
d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*
c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(3/4)*1i + (((8
91*a^8*b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*
b^8*c*d^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b
^11*c^4*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)*1i)/(a^4*b
^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c
^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28
*a^10*b^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^
8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c
^7*d^12 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9
*c^16*d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a
^6*b^6*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901
120*a^9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/
4)*1i - (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7
*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^
14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 10015
20*a^6*b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*
c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*
a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^
2*c^6*d^10)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 65
34*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^1
2 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*
d^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6
*c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^
9*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4) - (
(((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 712704*a^3*b^22*c^20*d^5 + 6172672*a^
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4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^18*d^7 + 110432256*a^6*b^19*c^17*d^8
- 271552512*a^7*b^18*c^16*d^9 + 487280640*a^8*b^17*c^15*d^10 - 635523072*a^
9*b^16*c^14*d^11 + 562982912*a^10*b^15*c^13*d^12 - 227217408*a^11*b^14*c^12
*d^13 - 227217408*a^12*b^13*c^11*d^14 + 562982912*a^13*b^12*c^10*d^15 - 635
523072*a^14*b^11*c^9*d^16 + 487280640*a^15*b^10*c^8*d^17 - 271552512*a^16*b
^9*c^7*d^18 + 110432256*a^17*b^8*c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 61
72672*a^19*b^6*c^4*d^21 - 712704*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^2
3)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*
c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d
^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 +
495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(
81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^
9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*
c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4
*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 32440
32*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 +
270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)*(6144*a^4*b^19*c^19*
d^4 - 90112*a^5*b^18*c^18*d^5 + 585728*a^6*b^17*c^17*d^6 - 2230272*a^7*b^16
*c^16*d^7 + 5490688*a^8*b^15*c^15*d^8 - 8966144*a^9*b^14*c^14*d^9 + 9191424
*a^10*b^13*c^13*d^10 - 3987456*a^11*b^12*c^12*d^11 - 3987456*a^12*b^11*c^11
*d^12 + 9191424*a^13*b^10*c^10*d^13 - 8966144*a^14*b^9*c^9*d^14 + 5490688*a
^15*b^8*c^8*d^15 - 2230272*a^16*b^7*c^7*d^16 + 585728*a^17*b^6*c^6*d^17 - 9
0112*a^18*b^5*c^5*d^18 + 6144*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*
d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))
*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^1
1*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520
*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3
244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^
9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(3/4)*1i - (((891*a^8*
b^7*d^15)/2 + (891*b^15*c^8*d^7)/2 - 6210*a*b^14*c^7*d^8 - 6210*a^7*b^8*c*d
^14 + 31509*a^2*b^13*c^6*d^9 - 66138*a^3*b^12*c^5*d^10 + 60307*a^4*b^11*c^4
*d^11 - 66138*a^5*b^10*c^3*d^12 + 31509*a^6*b^9*c^2*d^13)*1i)/(a^4*b^8*c^12
 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2
 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b
^2*c^6*d^6))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 653
4*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12
 - 49152*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d
^3 + 2027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*
c^13*d^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9
*b^3*c^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)*1i -
 (x^(1/2)*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 -
 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*
d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*
```

```
b^11*c^2*d^15)*1i)/(16*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d
- 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^
8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^
5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d
^10)))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*
b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 491
52*a^11*b*c^8*d^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2
027520*a^4*b^8*c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d
^6 - 3244032*a^7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c
^10*d^9 + 270336*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)))*(-(81*a^
4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1
188*a^3*b*c*d^10)/(4096*b^12*c^19 + 4096*a^12*c^7*d^12 - 49152*a^11*b*c^8*d
^11 + 270336*a^2*b^10*c^17*d^2 - 901120*a^3*b^9*c^16*d^3 + 2027520*a^4*b^8*
c^15*d^4 - 3244032*a^5*b^7*c^14*d^5 + 3784704*a^6*b^6*c^13*d^6 - 3244032*a^
7*b^5*c^12*d^7 + 2027520*a^8*b^4*c^11*d^8 - 901120*a^9*b^3*c^10*d^9 + 27033
6*a^10*b^2*c^9*d^10 - 49152*a*b^11*c^18*d))^(1/4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.475 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=676

$$-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}(bc-ad)^3}+ b^{9/4}($$

Rubi [A] time = 1.06, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5a^2d^2-8abcd+5b^2c^2}{2a^2c^2\sqrt{x}\,(bc-ad)^2}-\frac{b^{9/4}(5bc-13ad)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{9/4}(bc-ad)^3}+\frac{b^{9/4}(5bc-13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{b^{9/4}(5bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)^3}-\frac{d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^3}-\frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{b}{2a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2ac\sqrt{x}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(5*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)/(2*a^2*c^2*(b*c - a*d)^2*Sqrt[x]) + (d
*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[x]*(c + d*x^2)) + b/(2*a*(b*c - a*d
)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (b^(9/4)*(5*b*c - 13*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (b^(
9/4)*(5*b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(1
3*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^
(9/4)*(b*c - a*d)^3) - (b^(9/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(9
/4)*(5*b*c - 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*
c - a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
```

```
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-9bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)} - \text{Subst}(\int$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

$$= -\frac{5b^2c^2-8abcd+5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\,(a+}$$

Mathematica [A] time = 6.16, size = 674, normalized size = 1.00

$$\frac{b^{9/4}(13ad-5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3} - \frac{b^{9/4}(13ad-5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3} + \frac{b^{9/4}(13ad-5bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3} + \frac{b^{9/4}(13ad-5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3} - \frac{b^3x^{3/2}}{2a^2\left(a+bx^2\right)(ad-bc)^2} - \frac{2}{a^2c^2\sqrt{x}} + \frac{d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(ad-bc)^3} - \frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(ad-bc)^3} + \frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(ad-bc)^3} + \frac{d^{9/4}(13bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(ad-bc)^3} - \frac{d^3x^{3/2}}{2c^2\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -2/(a^2*c^2*Sqrt[x]) - (b^3*x^(3/2))/(2*a^2*(-(b*c) + a*d)^2*(a + b*x^2)) -
 (d^3*x^(3/2))/(2*c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(9/4)*(-5*b*c + 13*a*
d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*S
qrt[2]*a^(9/4)*(b*c - a*d)^3) + (b^(9/4)*(-5*b*c + 13*a*d)*ArcTan[(Sqrt[2]*
a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(9/4)*(b*c -
a*d)^3) + (d^(9/4)*(13*b*c - 5*a*d)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*
Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(9/4)*(-(b*c) + a*d)^3) + (d^(9/4
)*(13*b*c - 5*a*d)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^
(1/4))])/(4*Sqrt[2]*c^(9/4)*(-(b*c) + a*d)^3) + (b^(9/4)*(-5*b*c + 13*a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9
/4)*(b*c - a*d)^3) - (b^(9/4)*(-5*b*c + 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^3) + (d^(9/
4)*(13*b*c - 5*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(8*Sqrt[2]*c^(9/4)*(-(b*c) + a*d)^3) - (d^(9/4)*(13*b*c - 5*a*d)*Log[S
qrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(9/4)*(
-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.40, size = 472, normalized size = 0.70

$$\frac{\left(13ab^{9/4}d-5b^{13/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{9/4}(ad-bc)^3}+\frac{\left(13ab^{9/4}d-5b^{13/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{9/4}(ad-bc)^3}+\frac{-4a^3cd^2-5a^3d^3x^2+8a^2bc^2d+4a^2bcd^2x^2-5a^2bd^3x^4-4ab^2c^3+4ab^2c^2dx^2+8ab^2cd^2x^4-5b^3c^3x^2-5b^3c^2dx^4}{2a^2c^2\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)(ad-bc)^2}+\frac{\left(13bcd^{9/4}-5ad^{13/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^3}+\frac{\left(13bcd^{9/4}-5ad^{13/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-4*a*b^2*c^3 + 8*a^2*b*c^2*d - 4*a^3*c*d^2 - 5*b^3*c^3*x^2 + 4*a*b^2*c^2*d
*x^2 + 4*a^2*b*c*d^2*x^2 - 5*a^3*d^3*x^2 - 5*b^3*c^2*d*x^4 + 8*a*b^2*c*d^2*
x^4 - 5*a^2*b*d^3*x^4)/(2*a^2*c^2*(-(b*c) + a*d)^2*Sqrt[x]*(a + b*x^2)*(c +
 d*x^2)) + ((-5*b^(13/4)*c + 13*a*b^(9/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(9/4)*(-(b*c) + a*d)^3) + (
(13*b*c*d^(9/4) - 5*a*d^(13/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)^3) + ((-5*b^(13/4)*c +
 13*a*b^(9/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)])/(4*Sqrt[2]*a^(9/4)*(-(b*c) + a*d)^3) + ((13*b*c*d^(9/4) - 5*a*d^(13
/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(4*S
qrt[2]*c^(9/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.56, size = 1035, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a
^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(5*(a*b^3)^(3
/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*
sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(13*(c*d^3)^(3/4)*b*c - 5*(c*d
^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 -
sqrt(2)*a^3*c^3*d^3) - 1/4*(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d
^3) + 1/8*(5*(a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d
 + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/8*(5*(a*b^3)^(3/4)*b*c - 13
*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt
(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)
*a^6*d^3) + 1/8*(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d
 + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/8*(13*(c*d^3)^(3/4)*b
*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(
2)*a^3*c^3*d^3) - 1/2*(5*b^3*c^2*d*x^4 - 8*a*b^2*c*d^2*x^4 + 5*a^2*b*d^3*x^
4 + 5*b^3*c^3*x^2 - 4*a*b^2*c^2*d*x^2 - 4*a^2*b*c*d^2*x^2 + 5*a^3*d^3*x^2 +
 4*a*b^2*c^3 - 8*a^2*b*c^2*d + 4*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d +
 a^4*c^2*d^2)*(b*d*x^(9/2) + b*c*x^(5/2) + a*d*x^(5/2) + a*c*sqrt(x)))
```

maple [A] time = 0.03, size = 825, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b^3/a/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3*x^(3/2)/
(b*x^2+a)*c-13/16*b^2/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*ln((x-(a/b)^(1/4)
*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))-
```

```
13/8*b^2/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)-13/8*b^2/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)-1)+5/16*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2)))+5/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)+5/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)-1/2*d^4/c^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a+1/2*d^3/
c/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*b-5/16*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1
/2)*a*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2)))-5/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*a*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-5/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)
*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+13/16*d^2/c/(a*d-b*c)^3/(c/d)^(1/4
)*2^(1/2)*b*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2
^(1/2)*x^(1/2)+(c/d)^(1/2)))+13/8*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)*b*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+13/8*d^2/c/(a*d-b*c)^3/(c/d)^(1/4)*2^(
1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a^2/c^2/x^(1/2)
```

maxima [A] time = 2.93, size = 694, normalized size = 1.03

$$-\frac{\left(5b^{4}c-13ab^{3}d\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(a^{2}b^{3}c^{3}-3a^{3}b^{2}c^{2}d+3a^{4}bcd^{2}-a^{5}d^{3}\right)}-\frac{\left(13bcd^{3}-5ad^{4}\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\left(b^{3}c^{5}-3ab^{2}c^{4}d+3a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)}-\frac{4ab^{2}c^{3}-8a^{2}bc^{2}d+4a^{3}cd^{2}+\left(5b^{3}c^{2}d-8ab^{2}cd^{2}+5a^{2}bd^{3}\right)x^{4}+\left(5b^{3}c^{3}-4ab^{2}c^{2}d-4a^{2}bcd^{2}+5a^{3}d^{3}\right)x^{2}}{2\left(\left(a^{2}b^{3}c^{4}d-2a^{3}b^{2}c^{3}d^{2}+a^{4}bc^{2}d^{3}\right)x^{\frac{9}{2}}+\left(a^{2}b^{3}c^{5}-a^{3}b^{2}c^{4}d-a^{4}bc^{3}d^{2}+a^{5}c^{2}d^{3}\right)x^{\frac{5}{2}}+\left(a^{3}b^{2}c^{5}-2a^{4}bc^{4}d+a^{5}c^{3}d^{2}\right)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(5*b^4*c - 13*a*b^3*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(1/4)*b^(3/4)))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3
) - 1/16*(13*b*c*d^3 - 5*a*d^4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1
/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(
d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*s
qrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sq
rt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d
^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c
))/(c^(1/4)*d^(3/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*
d^3) - 1/2*(4*a*b^2*c^3 - 8*a^2*b*c^2*d + 4*a^3*c*d^2 + (5*b^3*c^2*d - 8*a*
b^2*c*d^2 + 5*a^2*b*d^3)*x^4 + (5*b^3*c^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2 +
 5*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^(9/
2) + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^(5/2) +
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*sqrt(x))
```

mupad [B] time = 12.38, size = 33548, normalized size = 49.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] atan((((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a
^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 -
 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^
3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c
^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*
b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(3/4)*(x^(
1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2
*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 4
9152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3
+ 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^1
5*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^
3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*(52428
800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 24482152448*a^25*b
^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d^7 + 1607834009600*a^27*b^34*c^
53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 + 35303182041088*a^29*b^32*c^51*d
^10 - 120578363097088*a^30*b^31*c^50*d^11 + 342964201062400*a^31*b^30*c^49*
d^12 - 823887134720000*a^32*b^29*c^48*d^13 + 1690057100492800*a^33*b^28*c^4
7*d^14 - 2988135038320640*a^34*b^27*c^46*d^15 + 4595616128696320*a^35*b^26*
c^45*d^16 - 6215915829985280*a^36*b^25*c^44*d^17 + 7509830061260800*a^37*b^
24*c^43*d^18 - 8292025971507200*a^38*b^23*c^42*d^19 + 8624070071418880*a^39
*b^22*c^41*d^20 - 8700497871503360*a^40*b^21*c^40*d^21 + 8624070071418880*a
^41*b^20*c^39*d^22 - 8292025971507200*a^42*b^19*c^38*d^23 + 750983006126080
0*a^43*b^18*c^37*d^24 - 6215915829985280*a^44*b^17*c^36*d^25 + 459561612869
6320*a^45*b^16*c^35*d^26 - 2988135038320640*a^46*b^15*c^34*d^27 + 169005710
0492800*a^47*b^14*c^33*d^28 - 823887134720000*a^48*b^13*c^32*d^29 + 3429642
01062400*a^49*b^12*c^31*d^30 - 120578363097088*a^50*b^11*c^30*d^31 + 353031
82041088*a^51*b^10*c^29*d^32 - 8446069964800*a^52*b^9*c^28*d^33 + 160783400
9600*a^53*b^8*c^27*d^34 - 234134437888*a^54*b^7*c^26*d^35 + 24482152448*a^5
5*b^6*c^25*d^36 - 1635778560*a^56*b^5*c^24*d^37 + 52428800*a^57*b^4*c^23*d^
38) - 32768000*a^21*b^38*c^55*d^4 + 1009254400*a^22*b^37*c^54*d^5 - 1483341
8240*a^23*b^36*c^53*d^6 + 138556735488*a^24*b^35*c^52*d^7 - 924185001984*a^
25*b^34*c^51*d^8 + 4688465362944*a^26*b^33*c^50*d^9 - 18812623126528*a^27*b
^32*c^49*d^10 + 61295191654400*a^28*b^31*c^48*d^11 - 165189260410880*a^29*b
^30*c^47*d^12 + 373165003898880*a^30*b^29*c^46*d^13 - 713540118773760*a^31*
b^28*c^45*d^14 + 1163349301657600*a^32*b^27*c^44*d^15 - 1627141704253440*a^
33*b^26*c^43*d^16 + 1966197351383040*a^34*b^25*c^42*d^17 - 2079216623943680
*a^35*b^24*c^41*d^18 + 1981073955225600*a^36*b^23*c^40*d^19 - 1807512431493
120*a^37*b^22*c^39*d^20 + 1724885956034560*a^38*b^21*c^38*d^21 - 1807512431
493120*a^39*b^20*c^37*d^22 + 1981073955225600*a^40*b^19*c^36*d^23 - 2079216
623943680*a^41*b^18*c^35*d^24 + 1966197351383040*a^42*b^17*c^34*d^25 - 1627
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141704253440*a^43*b^16*c^33*d^26 + 1163349301657600*a^44*b^15*c^32*d^27 - 7
13540118773760*a^45*b^14*c^31*d^28 + 373165003898880*a^46*b^13*c^30*d^29 -
165189260410880*a^47*b^12*c^29*d^30 + 61295191654400*a^48*b^11*c^28*d^31 -
18812623126528*a^49*b^10*c^27*d^32 + 4688465362944*a^50*b^9*c^26*d^33 - 924
185001984*a^51*b^8*c^25*d^34 + 138556735488*a^52*b^7*c^24*d^35 - 1483341824
0*a^53*b^6*c^23*d^36 + 1009254400*a^54*b^5*c^22*d^37 - 32768000*a^55*b^4*c^
21*d^38) + x^(1/2)*(54080000*a^20*b^33*c^43*d^10 - 1361152000*a^21*b^32*c^4
2*d^11 + 16011852800*a^22*b^31*c^41*d^12 - 116736734720*a^23*b^30*c^40*d^13
 + 589861462528*a^24*b^29*c^39*d^14 - 2187899577344*a^25*b^28*c^38*d^15 + 6
149347117056*a^26*b^27*c^37*d^16 - 13298820601344*a^27*b^26*c^36*d^17 + 221
33436343296*a^28*b^25*c^35*d^18 - 27715689750528*a^29*b^24*c^34*d^19 + 2407
7503776768*a^30*b^23*c^33*d^20 - 9645706816512*a^31*b^22*c^32*d^21 - 964570
6816512*a^32*b^21*c^31*d^22 + 24077503776768*a^33*b^20*c^30*d^23 - 27715689
750528*a^34*b^19*c^29*d^24 + 22133436343296*a^35*b^18*c^28*d^25 - 132988206
01344*a^36*b^17*c^27*d^26 + 6149347117056*a^37*b^16*c^26*d^27 - 21878995773
44*a^38*b^15*c^25*d^28 + 589861462528*a^39*b^14*c^24*d^29 - 116736734720*a^
40*b^13*c^23*d^30 + 16011852800*a^41*b^12*c^22*d^31 - 1361152000*a^42*b^11*
c^21*d^32 + 54080000*a^43*b^10*c^20*d^33))*(-(625*a^4*d^13 + 28561*b^4*c^4*
d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4
096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^
10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*
a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 20
27520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^1
0 - 49152*a*b^11*c^20*d))^(1/4)*1i + ((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 -
 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b
^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^
19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b
^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520
*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 4
9152*a*b^11*c^20*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 4
3940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^1
2*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19
*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7
*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a
^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 491
52*a*b^11*c^20*d))^(1/4)*(52428800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^3
7*c^56*d^5 + 24482152448*a^25*b^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d
^7 + 1607834009600*a^27*b^34*c^53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 +
35303182041088*a^29*b^32*c^51*d^10 - 120578363097088*a^30*b^31*c^50*d^11 +
342964201062400*a^31*b^30*c^49*d^12 - 823887134720000*a^32*b^29*c^48*d^13 +
 1690057100492800*a^33*b^28*c^47*d^14 - 2988135038320640*a^34*b^27*c^46*d^1
5 + 4595616128696320*a^35*b^26*c^45*d^16 - 6215915829985280*a^36*b^25*c^44*
d^17 + 7509830061260800*a^37*b^24*c^43*d^18 - 8292025971507200*a^38*b^23*c^
42*d^19 + 8624070071418880*a^39*b^22*c^41*d^20 - 8700497871503360*a^40*b^21
*c^40*d^21 + 8624070071418880*a^41*b^20*c^39*d^22 - 8292025971507200*a^42*b
```

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^19*c^38*d^23 + 7509830061260800*a^43*b^18*c^37*d^24 - 6215915829985280*a^4
4*b^17*c^36*d^25 + 4595616128696320*a^45*b^16*c^35*d^26 - 2988135038320640*
a^46*b^15*c^34*d^27 + 1690057100492800*a^47*b^14*c^33*d^28 - 82388713472000
0*a^48*b^13*c^32*d^29 + 342964201062400*a^49*b^12*c^31*d^30 - 1205783630970
88*a^50*b^11*c^30*d^31 + 35303182041088*a^51*b^10*c^29*d^32 - 8446069964800
*a^52*b^9*c^28*d^33 + 1607834009600*a^53*b^8*c^27*d^34 - 234134437888*a^54*
b^7*c^26*d^35 + 24482152448*a^55*b^6*c^25*d^36 - 1635778560*a^56*b^5*c^24*d
^37 + 52428800*a^57*b^4*c^23*d^38) + 32768000*a^21*b^38*c^55*d^4 - 10092544
00*a^22*b^37*c^54*d^5 + 14833418240*a^23*b^36*c^53*d^6 - 138556735488*a^24*
b^35*c^52*d^7 + 924185001984*a^25*b^34*c^51*d^8 - 4688465362944*a^26*b^33*c
^50*d^9 + 18812623126528*a^27*b^32*c^49*d^10 - 61295191654400*a^28*b^31*c^4
8*d^11 + 165189260410880*a^29*b^30*c^47*d^12 - 373165003898880*a^30*b^29*c^
46*d^13 + 713540118773760*a^31*b^28*c^45*d^14 - 1163349301657600*a^32*b^27*
c^44*d^15 + 1627141704253440*a^33*b^26*c^43*d^16 - 1966197351383040*a^34*b^
25*c^42*d^17 + 2079216623943680*a^35*b^24*c^41*d^18 - 1981073955225600*a^36
*b^23*c^40*d^19 + 1807512431493120*a^37*b^22*c^39*d^20 - 1724885956034560*a
^38*b^21*c^38*d^21 + 1807512431493120*a^39*b^20*c^37*d^22 - 198107395522560
0*a^40*b^19*c^36*d^23 + 2079216623943680*a^41*b^18*c^35*d^24 - 196619735138
3040*a^42*b^17*c^34*d^25 + 1627141704253440*a^43*b^16*c^33*d^26 - 116334930
1657600*a^44*b^15*c^32*d^27 + 713540118773760*a^45*b^14*c^31*d^28 - 3731650
03898880*a^46*b^13*c^30*d^29 + 165189260410880*a^47*b^12*c^29*d^30 - 612951
91654400*a^48*b^11*c^28*d^31 + 18812623126528*a^49*b^10*c^27*d^32 - 4688465
362944*a^50*b^9*c^26*d^33 + 924185001984*a^51*b^8*c^25*d^34 - 138556735488*
a^52*b^7*c^24*d^35 + 14833418240*a^53*b^6*c^23*d^36 - 1009254400*a^54*b^5*c
^22*d^37 + 32768000*a^55*b^4*c^21*d^38) + x^(1/2)*(54080000*a^20*b^33*c^43*
d^10 - 1361152000*a^21*b^32*c^42*d^11 + 16011852800*a^22*b^31*c^41*d^12 - 1
16736734720*a^23*b^30*c^40*d^13 + 589861462528*a^24*b^29*c^39*d^14 - 218789
9577344*a^25*b^28*c^38*d^15 + 6149347117056*a^26*b^27*c^37*d^16 - 132988206
01344*a^27*b^26*c^36*d^17 + 22133436343296*a^28*b^25*c^35*d^18 - 2771568975
0528*a^29*b^24*c^34*d^19 + 24077503776768*a^30*b^23*c^33*d^20 - 96457068165
12*a^31*b^22*c^32*d^21 - 9645706816512*a^32*b^21*c^31*d^22 + 24077503776768
*a^33*b^20*c^30*d^23 - 27715689750528*a^34*b^19*c^29*d^24 + 22133436343296*
a^35*b^18*c^28*d^25 - 13298820601344*a^36*b^17*c^27*d^26 + 6149347117056*a^
37*b^16*c^26*d^27 - 2187899577344*a^38*b^15*c^25*d^28 + 589861462528*a^39*b
^14*c^24*d^29 - 116736734720*a^40*b^13*c^23*d^30 + 16011852800*a^41*b^12*c^
22*d^31 - 1361152000*a^42*b^11*c^21*d^32 + 54080000*a^43*b^10*c^20*d^33))*(
-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c
^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a
^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027
520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6
- 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12
*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*1i)/(((-(625
*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^
11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b
*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a
```

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^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 324
4032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9
+ 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(3/4)*(x^(1/2)*(-(625*a
^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11
 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c
^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4
*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 32440
32*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 +
270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*(52428800*a^23*b^38
*c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 24482152448*a^25*b^36*c^55*d^6
- 234134437888*a^26*b^35*c^54*d^7 + 1607834009600*a^27*b^34*c^53*d^8 - 8446
069964800*a^28*b^33*c^52*d^9 + 35303182041088*a^29*b^32*c^51*d^10 - 1205783
63097088*a^30*b^31*c^50*d^11 + 342964201062400*a^31*b^30*c^49*d^12 - 823887
134720000*a^32*b^29*c^48*d^13 + 1690057100492800*a^33*b^28*c^47*d^14 - 2988
135038320640*a^34*b^27*c^46*d^15 + 4595616128696320*a^35*b^26*c^45*d^16 - 6
215915829985280*a^36*b^25*c^44*d^17 + 7509830061260800*a^37*b^24*c^43*d^18
- 8292025971507200*a^38*b^23*c^42*d^19 + 8624070071418880*a^39*b^22*c^41*d^
20 - 8700497871503360*a^40*b^21*c^40*d^21 + 8624070071418880*a^41*b^20*c^39
*d^22 - 8292025971507200*a^42*b^19*c^38*d^23 + 7509830061260800*a^43*b^18*c
^37*d^24 - 6215915829985280*a^44*b^17*c^36*d^25 + 4595616128696320*a^45*b^1
6*c^35*d^26 - 2988135038320640*a^46*b^15*c^34*d^27 + 1690057100492800*a^47*
b^14*c^33*d^28 - 823887134720000*a^48*b^13*c^32*d^29 + 342964201062400*a^49
*b^12*c^31*d^30 - 120578363097088*a^50*b^11*c^30*d^31 + 35303182041088*a^51
*b^10*c^29*d^32 - 8446069964800*a^52*b^9*c^28*d^33 + 1607834009600*a^53*b^8
*c^27*d^34 - 234134437888*a^54*b^7*c^26*d^35 + 24482152448*a^55*b^6*c^25*d^
36 - 1635778560*a^56*b^5*c^24*d^37 + 52428800*a^57*b^4*c^23*d^38) - 3276800
0*a^21*b^38*c^55*d^4 + 1009254400*a^22*b^37*c^54*d^5 - 14833418240*a^23*b^3
6*c^53*d^6 + 138556735488*a^24*b^35*c^52*d^7 - 924185001984*a^25*b^34*c^51*
d^8 + 4688465362944*a^26*b^33*c^50*d^9 - 18812623126528*a^27*b^32*c^49*d^10
 + 61295191654400*a^28*b^31*c^48*d^11 - 165189260410880*a^29*b^30*c^47*d^12
 + 373165003898880*a^30*b^29*c^46*d^13 - 713540118773760*a^31*b^28*c^45*d^1
4 + 1163349301657600*a^32*b^27*c^44*d^15 - 1627141704253440*a^33*b^26*c^43*
d^16 + 1966197351383040*a^34*b^25*c^42*d^17 - 2079216623943680*a^35*b^24*c^
41*d^18 + 1981073955225600*a^36*b^23*c^40*d^19 - 1807512431493120*a^37*b^22
*c^39*d^20 + 1724885956034560*a^38*b^21*c^38*d^21 - 1807512431493120*a^39*b
^20*c^37*d^22 + 1981073955225600*a^40*b^19*c^36*d^23 - 2079216623943680*a^4
1*b^18*c^35*d^24 + 1966197351383040*a^42*b^17*c^34*d^25 - 1627141704253440*
a^43*b^16*c^33*d^26 + 1163349301657600*a^44*b^15*c^32*d^27 - 71354011877376
0*a^45*b^14*c^31*d^28 + 373165003898880*a^46*b^13*c^30*d^29 - 1651892604108
80*a^47*b^12*c^29*d^30 + 61295191654400*a^48*b^11*c^28*d^31 - 1881262312652
8*a^49*b^10*c^27*d^32 + 4688465362944*a^50*b^9*c^26*d^33 - 924185001984*a^5
1*b^8*c^25*d^34 + 138556735488*a^52*b^7*c^24*d^35 - 14833418240*a^53*b^6*c^
23*d^36 + 1009254400*a^54*b^5*c^22*d^37 - 32768000*a^55*b^4*c^21*d^38) + x^
(1/2)*(54080000*a^20*b^33*c^43*d^10 - 1361152000*a^21*b^32*c^42*d^11 + 1601
1852800*a^22*b^31*c^41*d^12 - 116736734720*a^23*b^30*c^40*d^13 + 5898614625
```

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28*a^24*b^29*c^39*d^14 - 2187899577344*a^25*b^28*c^38*d^15 + 6149347117056*
a^26*b^27*c^37*d^16 - 13298820601344*a^27*b^26*c^36*d^17 + 22133436343296*a
^28*b^25*c^35*d^18 - 27715689750528*a^29*b^24*c^34*d^19 + 24077503776768*a^
30*b^23*c^33*d^20 - 9645706816512*a^31*b^22*c^32*d^21 - 9645706816512*a^32*
b^21*c^31*d^22 + 24077503776768*a^33*b^20*c^30*d^23 - 27715689750528*a^34*b
^19*c^29*d^24 + 22133436343296*a^35*b^18*c^28*d^25 - 13298820601344*a^36*b^
17*c^27*d^26 + 6149347117056*a^37*b^16*c^26*d^27 - 2187899577344*a^38*b^15*
c^25*d^28 + 589861462528*a^39*b^14*c^24*d^29 - 116736734720*a^40*b^13*c^23*
d^30 + 16011852800*a^41*b^12*c^22*d^31 - 1361152000*a^42*b^11*c^21*d^32 + 5
4080000*a^43*b^10*c^20*d^33))*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a
*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21
 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 -
 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*
d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4
*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b
^11*c^20*d))^(1/4) - ((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3
*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*
a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*
a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 37
84704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^
8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20
*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d
^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^
12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^
3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784
704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8
 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d
))^(1/4)*(52428800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 244
82152448*a^25*b^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d^7 + 16078340096
00*a^27*b^34*c^53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 + 35303182041088*a
^29*b^32*c^51*d^10 - 120578363097088*a^30*b^31*c^50*d^11 + 342964201062400*
a^31*b^30*c^49*d^12 - 823887134720000*a^32*b^29*c^48*d^13 + 169005710049280
0*a^33*b^28*c^47*d^14 - 2988135038320640*a^34*b^27*c^46*d^15 + 459561612869
6320*a^35*b^26*c^45*d^16 - 6215915829985280*a^36*b^25*c^44*d^17 + 750983006
1260800*a^37*b^24*c^43*d^18 - 8292025971507200*a^38*b^23*c^42*d^19 + 862407
0071418880*a^39*b^22*c^41*d^20 - 8700497871503360*a^40*b^21*c^40*d^21 + 862
4070071418880*a^41*b^20*c^39*d^22 - 8292025971507200*a^42*b^19*c^38*d^23 +
7509830061260800*a^43*b^18*c^37*d^24 - 6215915829985280*a^44*b^17*c^36*d^25
 + 4595616128696320*a^45*b^16*c^35*d^26 - 2988135038320640*a^46*b^15*c^34*d
^27 + 1690057100492800*a^47*b^14*c^33*d^28 - 823887134720000*a^48*b^13*c^32
*d^29 + 342964201062400*a^49*b^12*c^31*d^30 - 120578363097088*a^50*b^11*c^3
0*d^31 + 35303182041088*a^51*b^10*c^29*d^32 - 8446069964800*a^52*b^9*c^28*d
^33 + 1607834009600*a^53*b^8*c^27*d^34 - 234134437888*a^54*b^7*c^26*d^35 +
24482152448*a^55*b^6*c^25*d^36 - 1635778560*a^56*b^5*c^24*d^37 + 52428800*a
^57*b^4*c^23*d^38) + 32768000*a^21*b^38*c^55*d^4 - 1009254400*a^22*b^37*c^5
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4*d^5 + 14833418240*a^23*b^36*c^53*d^6 - 138556735488*a^24*b^35*c^52*d^7 +
924185001984*a^25*b^34*c^51*d^8 - 4688465362944*a^26*b^33*c^50*d^9 + 188126
23126528*a^27*b^32*c^49*d^10 - 61295191654400*a^28*b^31*c^48*d^11 + 1651892
60410880*a^29*b^30*c^47*d^12 - 373165003898880*a^30*b^29*c^46*d^13 + 713540
118773760*a^31*b^28*c^45*d^14 - 1163349301657600*a^32*b^27*c^44*d^15 + 1627
141704253440*a^33*b^26*c^43*d^16 - 1966197351383040*a^34*b^25*c^42*d^17 + 2
079216623943680*a^35*b^24*c^41*d^18 - 1981073955225600*a^36*b^23*c^40*d^19
+ 1807512431493120*a^37*b^22*c^39*d^20 - 1724885956034560*a^38*b^21*c^38*d^
21 + 1807512431493120*a^39*b^20*c^37*d^22 - 1981073955225600*a^40*b^19*c^36
*d^23 + 2079216623943680*a^41*b^18*c^35*d^24 - 1966197351383040*a^42*b^17*c
^34*d^25 + 1627141704253440*a^43*b^16*c^33*d^26 - 1163349301657600*a^44*b^1
5*c^32*d^27 + 713540118773760*a^45*b^14*c^31*d^28 - 373165003898880*a^46*b^
13*c^30*d^29 + 165189260410880*a^47*b^12*c^29*d^30 - 61295191654400*a^48*b^
11*c^28*d^31 + 18812623126528*a^49*b^10*c^27*d^32 - 4688465362944*a^50*b^9*
c^26*d^33 + 924185001984*a^51*b^8*c^25*d^34 - 138556735488*a^52*b^7*c^24*d^
35 + 14833418240*a^53*b^6*c^23*d^36 - 1009254400*a^54*b^5*c^22*d^37 + 32768
000*a^55*b^4*c^21*d^38) + x^(1/2)*(54080000*a^20*b^33*c^43*d^10 - 136115200
0*a^21*b^32*c^42*d^11 + 16011852800*a^22*b^31*c^41*d^12 - 116736734720*a^23
*b^30*c^40*d^13 + 589861462528*a^24*b^29*c^39*d^14 - 2187899577344*a^25*b^2
8*c^38*d^15 + 6149347117056*a^26*b^27*c^37*d^16 - 13298820601344*a^27*b^26*
c^36*d^17 + 22133436343296*a^28*b^25*c^35*d^18 - 27715689750528*a^29*b^24*c
^34*d^19 + 24077503776768*a^30*b^23*c^33*d^20 - 9645706816512*a^31*b^22*c^3
2*d^21 - 9645706816512*a^32*b^21*c^31*d^22 + 24077503776768*a^33*b^20*c^30*
d^23 - 27715689750528*a^34*b^19*c^29*d^24 + 22133436343296*a^35*b^18*c^28*d
^25 - 13298820601344*a^36*b^17*c^27*d^26 + 6149347117056*a^37*b^16*c^26*d^2
7 - 2187899577344*a^38*b^15*c^25*d^28 + 589861462528*a^39*b^14*c^24*d^29 -
116736734720*a^40*b^13*c^23*d^30 + 16011852800*a^41*b^12*c^22*d^31 - 136115
2000*a^42*b^11*c^21*d^32 + 54080000*a^43*b^10*c^20*d^33))*(-(625*a^4*d^13 +
 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a
^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11
+ 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17
*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^
5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^
10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4) - 175760000*a^20*b^31*c^39*d
^12 + 3507088000*a^21*b^30*c^38*d^13 - 32026300800*a^22*b^29*c^37*d^14 + 17
7335474560*a^23*b^28*c^36*d^15 - 664045775360*a^24*b^27*c^35*d^16 + 1770849
815040*a^25*b^26*c^34*d^17 - 3431196106240*a^26*b^25*c^33*d^18 + 4778178444
800*a^27*b^24*c^32*d^19 - 4440824728320*a^28*b^23*c^31*d^20 + 1838397848320
*a^29*b^22*c^30*d^21 + 1838397848320*a^30*b^21*c^29*d^22 - 4440824728320*a^
31*b^20*c^28*d^23 + 4778178444800*a^32*b^19*c^27*d^24 - 3431196106240*a^33*
b^18*c^26*d^25 + 1770849815040*a^34*b^17*c^25*d^26 - 664045775360*a^35*b^16
*c^24*d^27 + 177335474560*a^36*b^15*c^23*d^28 - 32026300800*a^37*b^14*c^22*
d^29 + 3507088000*a^38*b^13*c^21*d^30 - 175760000*a^39*b^12*c^20*d^31))*(-(
625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2
*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^1
```

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1*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 202752
0*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 -
3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d
^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*2i + 2*atan(((
(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*
c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*
a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 202
7520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6
 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^1
2*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(3/4)*(x^(1/2)*(-
(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^
2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^
11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 20275
20*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 -
 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*
d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*(52428800*a^2
3*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 24482152448*a^25*b^36*c^5
5*d^6 - 234134437888*a^26*b^35*c^54*d^7 + 1607834009600*a^27*b^34*c^53*d^8
- 8446069964800*a^28*b^33*c^52*d^9 + 35303182041088*a^29*b^32*c^51*d^10 - 1
20578363097088*a^30*b^31*c^50*d^11 + 342964201062400*a^31*b^30*c^49*d^12 -
823887134720000*a^32*b^29*c^48*d^13 + 1690057100492800*a^33*b^28*c^47*d^14
- 2988135038320640*a^34*b^27*c^46*d^15 + 4595616128696320*a^35*b^26*c^45*d^
16 - 6215915829985280*a^36*b^25*c^44*d^17 + 7509830061260800*a^37*b^24*c^43
*d^18 - 8292025971507200*a^38*b^23*c^42*d^19 + 8624070071418880*a^39*b^22*c
^41*d^20 - 8700497871503360*a^40*b^21*c^40*d^21 + 8624070071418880*a^41*b^2
0*c^39*d^22 - 8292025971507200*a^42*b^19*c^38*d^23 + 7509830061260800*a^43*
b^18*c^37*d^24 - 6215915829985280*a^44*b^17*c^36*d^25 + 4595616128696320*a^
45*b^16*c^35*d^26 - 2988135038320640*a^46*b^15*c^34*d^27 + 1690057100492800
*a^47*b^14*c^33*d^28 - 823887134720000*a^48*b^13*c^32*d^29 + 34296420106240
0*a^49*b^12*c^31*d^30 - 120578363097088*a^50*b^11*c^30*d^31 + 3530318204108
8*a^51*b^10*c^29*d^32 - 8446069964800*a^52*b^9*c^28*d^33 + 1607834009600*a^
53*b^8*c^27*d^34 - 234134437888*a^54*b^7*c^26*d^35 + 24482152448*a^55*b^6*c
^25*d^36 - 1635778560*a^56*b^5*c^24*d^37 + 52428800*a^57*b^4*c^23*d^38)*1i
- 32768000*a^21*b^38*c^55*d^4 + 1009254400*a^22*b^37*c^54*d^5 - 14833418240
*a^23*b^36*c^53*d^6 + 138556735488*a^24*b^35*c^52*d^7 - 924185001984*a^25*b
^34*c^51*d^8 + 4688465362944*a^26*b^33*c^50*d^9 - 18812623126528*a^27*b^32*
c^49*d^10 + 61295191654400*a^28*b^31*c^48*d^11 - 165189260410880*a^29*b^30*
c^47*d^12 + 373165003898880*a^30*b^29*c^46*d^13 - 713540118773760*a^31*b^28
*c^45*d^14 + 1163349301657600*a^32*b^27*c^44*d^15 - 1627141704253440*a^33*b
^26*c^43*d^16 + 1966197351383040*a^34*b^25*c^42*d^17 - 2079216623943680*a^3
5*b^24*c^41*d^18 + 1981073955225600*a^36*b^23*c^40*d^19 - 1807512431493120*
a^37*b^22*c^39*d^20 + 1724885956034560*a^38*b^21*c^38*d^21 - 18075124314931
20*a^39*b^20*c^37*d^22 + 1981073955225600*a^40*b^19*c^36*d^23 - 20792166239
43680*a^41*b^18*c^35*d^24 + 1966197351383040*a^42*b^17*c^34*d^25 - 16271417
04253440*a^43*b^16*c^33*d^26 + 1163349301657600*a^44*b^15*c^32*d^27 - 71354
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0118773760*a^45*b^14*c^31*d^28 + 373165003898880*a^46*b^13*c^30*d^29 - 1651
89260410880*a^47*b^12*c^29*d^30 + 61295191654400*a^48*b^11*c^28*d^31 - 1881
2623126528*a^49*b^10*c^27*d^32 + 4688465362944*a^50*b^9*c^26*d^33 - 9241850
01984*a^51*b^8*c^25*d^34 + 138556735488*a^52*b^7*c^24*d^35 - 14833418240*a^
53*b^6*c^23*d^36 + 1009254400*a^54*b^5*c^22*d^37 - 32768000*a^55*b^4*c^21*d
^38)*1i - x^(1/2)*(54080000*a^20*b^33*c^43*d^10 - 1361152000*a^21*b^32*c^42
*d^11 + 16011852800*a^22*b^31*c^41*d^12 - 116736734720*a^23*b^30*c^40*d^13
+ 589861462528*a^24*b^29*c^39*d^14 - 2187899577344*a^25*b^28*c^38*d^15 + 61
49347117056*a^26*b^27*c^37*d^16 - 13298820601344*a^27*b^26*c^36*d^17 + 2213
3436343296*a^28*b^25*c^35*d^18 - 27715689750528*a^29*b^24*c^34*d^19 + 24077
503776768*a^30*b^23*c^33*d^20 - 9645706816512*a^31*b^22*c^32*d^21 - 9645706
816512*a^32*b^21*c^31*d^22 + 24077503776768*a^33*b^20*c^30*d^23 - 277156897
50528*a^34*b^19*c^29*d^24 + 22133436343296*a^35*b^18*c^28*d^25 - 1329882060
1344*a^36*b^17*c^27*d^26 + 6149347117056*a^37*b^16*c^26*d^27 - 218789957734
4*a^38*b^15*c^25*d^28 + 589861462528*a^39*b^14*c^24*d^29 - 116736734720*a^4
0*b^13*c^23*d^30 + 16011852800*a^41*b^12*c^22*d^31 - 1361152000*a^42*b^11*c
^21*d^32 + 54080000*a^43*b^10*c^20*d^33))*(-(625*a^4*d^13 + 28561*b^4*c^4*d
^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(40
96*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^1
0*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a
^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 202
7520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10
 - 49152*a*b^11*c^20*d))^(1/4) + ((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 439
40*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*
c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d
^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c
^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8
*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152
*a*b^11*c^20*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940
*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^
21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2
 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^1
6*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b
^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a
*b^11*c^20*d))^(1/4)*(52428800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^
56*d^5 + 24482152448*a^25*b^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d^7 +
 1607834009600*a^27*b^34*c^53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 + 3530
3182041088*a^29*b^32*c^51*d^10 - 120578363097088*a^30*b^31*c^50*d^11 + 3429
64201062400*a^31*b^30*c^49*d^12 - 823887134720000*a^32*b^29*c^48*d^13 + 169
0057100492800*a^33*b^28*c^47*d^14 - 2988135038320640*a^34*b^27*c^46*d^15 +
4595616128696320*a^35*b^26*c^45*d^16 - 6215915829985280*a^36*b^25*c^44*d^17
 + 7509830061260800*a^37*b^24*c^43*d^18 - 8292025971507200*a^38*b^23*c^42*d
^19 + 8624070071418880*a^39*b^22*c^41*d^20 - 8700497871503360*a^40*b^21*c^4
0*d^21 + 8624070071418880*a^41*b^20*c^39*d^22 - 8292025971507200*a^42*b^19*
c^38*d^23 + 7509830061260800*a^43*b^18*c^37*d^24 - 6215915829985280*a^44*b^
```

17*c^36*d^25 + 4595616128696320*a^45*b^16*c^35*d^26 - 2988135038320640*a^46
*b^15*c^34*d^27 + 1690057100492800*a^47*b^14*c^33*d^28 - 823887134720000*a^
48*b^13*c^32*d^29 + 342964201062400*a^49*b^12*c^31*d^30 - 120578363097088*a
^50*b^11*c^30*d^31 + 35303182041088*a^51*b^10*c^29*d^32 - 8446069964800*a^5
2*b^9*c^28*d^33 + 1607834009600*a^53*b^8*c^27*d^34 - 234134437888*a^54*b^7*
c^26*d^35 + 24482152448*a^55*b^6*c^25*d^36 - 1635778560*a^56*b^5*c^24*d^37
+ 52428800*a^57*b^4*c^23*d^38)*1i + 32768000*a^21*b^38*c^55*d^4 - 100925440
0*a^22*b^37*c^54*d^5 + 14833418240*a^23*b^36*c^53*d^6 - 138556735488*a^24*b
^35*c^52*d^7 + 924185001984*a^25*b^34*c^51*d^8 - 4688465362944*a^26*b^33*c^
50*d^9 + 18812623126528*a^27*b^32*c^49*d^10 - 61295191654400*a^28*b^31*c^48
*d^11 + 165189260410880*a^29*b^30*c^47*d^12 - 373165003898880*a^30*b^29*c^4
6*d^13 + 713540118773760*a^31*b^28*c^45*d^14 - 1163349301657600*a^32*b^27*c
^44*d^15 + 1627141704253440*a^33*b^26*c^43*d^16 - 1966197351383040*a^34*b^2
5*c^42*d^17 + 2079216623943680*a^35*b^24*c^41*d^18 - 1981073955225600*a^36*
b^23*c^40*d^19 + 1807512431493120*a^37*b^22*c^39*d^20 - 1724885956034560*a^
38*b^21*c^38*d^21 + 1807512431493120*a^39*b^20*c^37*d^22 - 1981073955225600
*a^40*b^19*c^36*d^23 + 2079216623943680*a^41*b^18*c^35*d^24 - 1966197351383
040*a^42*b^17*c^34*d^25 + 1627141704253440*a^43*b^16*c^33*d^26 - 1163349301
657600*a^44*b^15*c^32*d^27 + 713540118773760*a^45*b^14*c^31*d^28 - 37316500
3898880*a^46*b^13*c^30*d^29 + 165189260410880*a^47*b^12*c^29*d^30 - 6129519
1654400*a^48*b^11*c^28*d^31 + 18812623126528*a^49*b^10*c^27*d^32 - 46884653
62944*a^50*b^9*c^26*d^33 + 924185001984*a^51*b^8*c^25*d^34 - 138556735488*a
^52*b^7*c^24*d^35 + 14833418240*a^53*b^6*c^23*d^36 - 1009254400*a^54*b^5*c^
22*d^37 + 32768000*a^55*b^4*c^21*d^38)*1i - x^(1/2)*(54080000*a^20*b^33*c^4
3*d^10 - 1361152000*a^21*b^32*c^42*d^11 + 16011852800*a^22*b^31*c^41*d^12 -
116736734720*a^23*b^30*c^40*d^13 + 589861462528*a^24*b^29*c^39*d^14 - 2187
899577344*a^25*b^28*c^38*d^15 + 6149347117056*a^26*b^27*c^37*d^16 - 1329882
0601344*a^27*b^26*c^36*d^17 + 22133436343296*a^28*b^25*c^35*d^18 - 27715689
750528*a^29*b^24*c^34*d^19 + 24077503776768*a^30*b^23*c^33*d^20 - 964570681
6512*a^31*b^22*c^32*d^21 - 9645706816512*a^32*b^21*c^31*d^22 + 240775037767
68*a^33*b^20*c^30*d^23 - 27715689750528*a^34*b^19*c^29*d^24 + 2213343634329
6*a^35*b^18*c^28*d^25 - 13298820601344*a^36*b^17*c^27*d^26 + 6149347117056*
a^37*b^16*c^26*d^27 - 2187899577344*a^38*b^15*c^25*d^28 + 589861462528*a^39
*b^14*c^24*d^29 - 116736734720*a^40*b^13*c^23*d^30 + 16011852800*a^41*b^12*
c^22*d^31 - 1361152000*a^42*b^11*c^21*d^32 + 54080000*a^43*b^10*c^20*d^33))
*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2
*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152
*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 20
27520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^
6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^
12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4))/(((-(625*
a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^1
1 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*
c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^
4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244

```
032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 +
 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(3/4)*(x^(1/2)*(-(625*a^
4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11
 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^
10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*
b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 324403
2*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 2
70336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*(52428800*a^23*b^38*
c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 24482152448*a^25*b^36*c^55*d^6 -
 234134437888*a^26*b^35*c^54*d^7 + 1607834009600*a^27*b^34*c^53*d^8 - 84460
69964800*a^28*b^33*c^52*d^9 + 35303182041088*a^29*b^32*c^51*d^10 - 12057836
3097088*a^30*b^31*c^50*d^11 + 342964201062400*a^31*b^30*c^49*d^12 - 8238871
34720000*a^32*b^29*c^48*d^13 + 1690057100492800*a^33*b^28*c^47*d^14 - 29881
35038320640*a^34*b^27*c^46*d^15 + 4595616128696320*a^35*b^26*c^45*d^16 - 62
15915829985280*a^36*b^25*c^44*d^17 + 7509830061260800*a^37*b^24*c^43*d^18 -
 8292025971507200*a^38*b^23*c^42*d^19 + 8624070071418880*a^39*b^22*c^41*d^2
0 - 8700497871503360*a^40*b^21*c^40*d^21 + 8624070071418880*a^41*b^20*c^39*
d^22 - 8292025971507200*a^42*b^19*c^38*d^23 + 7509830061260800*a^43*b^18*c^
37*d^24 - 6215915829985280*a^44*b^17*c^36*d^25 + 4595616128696320*a^45*b^16
*c^35*d^26 - 2988135038320640*a^46*b^15*c^34*d^27 + 1690057100492800*a^47*b
^14*c^33*d^28 - 823887134720000*a^48*b^13*c^32*d^29 + 342964201062400*a^49*
b^12*c^31*d^30 - 120578363097088*a^50*b^11*c^30*d^31 + 35303182041088*a^51*
b^10*c^29*d^32 - 8446069964800*a^52*b^9*c^28*d^33 + 1607834009600*a^53*b^8*
c^27*d^34 - 234134437888*a^54*b^7*c^26*d^35 + 24482152448*a^55*b^6*c^25*d^3
6 - 1635778560*a^56*b^5*c^24*d^37 + 52428800*a^57*b^4*c^23*d^38)*1i - 32768
000*a^21*b^38*c^55*d^4 + 1009254400*a^22*b^37*c^54*d^5 - 14833418240*a^23*b
^36*c^53*d^6 + 138556735488*a^24*b^35*c^52*d^7 - 924185001984*a^25*b^34*c^5
1*d^8 + 4688465362944*a^26*b^33*c^50*d^9 - 18812623126528*a^27*b^32*c^49*d^
10 + 61295191654400*a^28*b^31*c^48*d^11 - 165189260410880*a^29*b^30*c^47*d^
12 + 373165003898880*a^30*b^29*c^46*d^13 - 713540118773760*a^31*b^28*c^45*d
^14 + 1163349301657600*a^32*b^27*c^44*d^15 - 1627141704253440*a^33*b^26*c^4
3*d^16 + 1966197351383040*a^34*b^25*c^42*d^17 - 2079216623943680*a^35*b^24*
c^41*d^18 + 1981073955225600*a^36*b^23*c^40*d^19 - 1807512431493120*a^37*b^
22*c^39*d^20 + 1724885956034560*a^38*b^21*c^38*d^21 - 1807512431493120*a^39
*b^20*c^37*d^22 + 1981073955225600*a^40*b^19*c^36*d^23 - 2079216623943680*a
^41*b^18*c^35*d^24 + 1966197351383040*a^42*b^17*c^34*d^25 - 162714170425344
0*a^43*b^16*c^33*d^26 + 1163349301657600*a^44*b^15*c^32*d^27 - 713540118773
760*a^45*b^14*c^31*d^28 + 373165003898880*a^46*b^13*c^30*d^29 - 16518926041
0880*a^47*b^12*c^29*d^30 + 61295191654400*a^48*b^11*c^28*d^31 - 18812623126
528*a^49*b^10*c^27*d^32 + 4688465362944*a^50*b^9*c^26*d^33 - 924185001984*a
^51*b^8*c^25*d^34 + 138556735488*a^52*b^7*c^24*d^35 - 14833418240*a^53*b^6*
c^23*d^36 + 1009254400*a^54*b^5*c^22*d^37 - 32768000*a^55*b^4*c^21*d^38)*1i
 - x^(1/2)*(54080000*a^20*b^33*c^43*d^10 - 1361152000*a^21*b^32*c^42*d^11 +
 16011852800*a^22*b^31*c^41*d^12 - 116736734720*a^23*b^30*c^40*d^13 + 58986
1462528*a^24*b^29*c^39*d^14 - 2187899577344*a^25*b^28*c^38*d^15 + 614934711
```

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7056*a^26*b^27*c^37*d^16 - 13298820601344*a^27*b^26*c^36*d^17 + 22133436343
296*a^28*b^25*c^35*d^18 - 27715689750528*a^29*b^24*c^34*d^19 + 240775037767
68*a^30*b^23*c^33*d^20 - 9645706816512*a^31*b^22*c^32*d^21 - 9645706816512*
a^32*b^21*c^31*d^22 + 24077503776768*a^33*b^20*c^30*d^23 - 27715689750528*a
^34*b^19*c^29*d^24 + 22133436343296*a^35*b^18*c^28*d^25 - 13298820601344*a^
36*b^17*c^27*d^26 + 6149347117056*a^37*b^16*c^26*d^27 - 2187899577344*a^38*
b^15*c^25*d^28 + 589861462528*a^39*b^14*c^24*d^29 - 116736734720*a^40*b^13*
c^23*d^30 + 16011852800*a^41*b^12*c^22*d^31 - 1361152000*a^42*b^11*c^21*d^3
2 + 54080000*a^43*b^10*c^20*d^33))*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43
940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12
*c^21 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*
d^2 - 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*
c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^
8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 4915
2*a*b^11*c^20*d))^(1/4)*1i - ((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a
*b^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21
 + 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 -
 901120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*
d^5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4
*c^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b
^11*c^20*d))^(3/4)*(x^(1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b
^3*c^3*d^10 + 25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 +
 4096*a^12*c^9*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 9
01120*a^3*b^9*c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^
5 + 3784704*a^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c
^13*d^8 - 901120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^1
1*c^20*d))^(1/4)*(52428800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^56*d
^5 + 24482152448*a^25*b^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d^7 + 160
7834009600*a^27*b^34*c^53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 + 35303182
041088*a^29*b^32*c^51*d^10 - 120578363097088*a^30*b^31*c^50*d^11 + 34296420
1062400*a^31*b^30*c^49*d^12 - 823887134720000*a^32*b^29*c^48*d^13 + 1690057
100492800*a^33*b^28*c^47*d^14 - 2988135038320640*a^34*b^27*c^46*d^15 + 4595
616128696320*a^35*b^26*c^45*d^16 - 6215915829985280*a^36*b^25*c^44*d^17 + 7
509830061260800*a^37*b^24*c^43*d^18 - 8292025971507200*a^38*b^23*c^42*d^19
+ 8624070071418880*a^39*b^22*c^41*d^20 - 8700497871503360*a^40*b^21*c^40*d^
21 + 8624070071418880*a^41*b^20*c^39*d^22 - 8292025971507200*a^42*b^19*c^38
*d^23 + 7509830061260800*a^43*b^18*c^37*d^24 - 6215915829985280*a^44*b^17*c
^36*d^25 + 4595616128696320*a^45*b^16*c^35*d^26 - 2988135038320640*a^46*b^1
5*c^34*d^27 + 1690057100492800*a^47*b^14*c^33*d^28 - 823887134720000*a^48*b
^13*c^32*d^29 + 342964201062400*a^49*b^12*c^31*d^30 - 120578363097088*a^50*
b^11*c^30*d^31 + 35303182041088*a^51*b^10*c^29*d^32 - 8446069964800*a^52*b^
9*c^28*d^33 + 1607834009600*a^53*b^8*c^27*d^34 - 234134437888*a^54*b^7*c^26
*d^35 + 24482152448*a^55*b^6*c^25*d^36 - 1635778560*a^56*b^5*c^24*d^37 + 52
428800*a^57*b^4*c^23*d^38)*1i + 32768000*a^21*b^38*c^55*d^4 - 1009254400*a^
22*b^37*c^54*d^5 + 14833418240*a^23*b^36*c^53*d^6 - 138556735488*a^24*b^35*
```

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c^52*d^7 + 924185001984*a^25*b^34*c^51*d^8 - 4688465362944*a^26*b^33*c^50*d
^9 + 18812623126528*a^27*b^32*c^49*d^10 - 61295191654400*a^28*b^31*c^48*d^1
1 + 165189260410880*a^29*b^30*c^47*d^12 - 373165003898880*a^30*b^29*c^46*d^
13 + 713540118773760*a^31*b^28*c^45*d^14 - 1163349301657600*a^32*b^27*c^44*
d^15 + 1627141704253440*a^33*b^26*c^43*d^16 - 1966197351383040*a^34*b^25*c^
42*d^17 + 2079216623943680*a^35*b^24*c^41*d^18 - 1981073955225600*a^36*b^23
*c^40*d^19 + 1807512431493120*a^37*b^22*c^39*d^20 - 1724885956034560*a^38*b
^21*c^38*d^21 + 1807512431493120*a^39*b^20*c^37*d^22 - 1981073955225600*a^4
0*b^19*c^36*d^23 + 2079216623943680*a^41*b^18*c^35*d^24 - 1966197351383040*
a^42*b^17*c^34*d^25 + 1627141704253440*a^43*b^16*c^33*d^26 - 11633493016576
00*a^44*b^15*c^32*d^27 + 713540118773760*a^45*b^14*c^31*d^28 - 373165003898
880*a^46*b^13*c^30*d^29 + 165189260410880*a^47*b^12*c^29*d^30 - 61295191654
400*a^48*b^11*c^28*d^31 + 18812623126528*a^49*b^10*c^27*d^32 - 468846536294
4*a^50*b^9*c^26*d^33 + 924185001984*a^51*b^8*c^25*d^34 - 138556735488*a^52*
b^7*c^24*d^35 + 14833418240*a^53*b^6*c^23*d^36 - 1009254400*a^54*b^5*c^22*d
^37 + 32768000*a^55*b^4*c^21*d^38)*1i - x^(1/2)*(54080000*a^20*b^33*c^43*d^
10 - 1361152000*a^21*b^32*c^42*d^11 + 16011852800*a^22*b^31*c^41*d^12 - 116
736734720*a^23*b^30*c^40*d^13 + 589861462528*a^24*b^29*c^39*d^14 - 21878995
77344*a^25*b^28*c^38*d^15 + 6149347117056*a^26*b^27*c^37*d^16 - 13298820601
344*a^27*b^26*c^36*d^17 + 22133436343296*a^28*b^25*c^35*d^18 - 277156897505
28*a^29*b^24*c^34*d^19 + 24077503776768*a^30*b^23*c^33*d^20 - 9645706816512
*a^31*b^22*c^32*d^21 - 9645706816512*a^32*b^21*c^31*d^22 + 24077503776768*a
^33*b^20*c^30*d^23 - 27715689750528*a^34*b^19*c^29*d^24 + 22133436343296*a^
35*b^18*c^28*d^25 - 13298820601344*a^36*b^17*c^27*d^26 + 6149347117056*a^37
*b^16*c^26*d^27 - 2187899577344*a^38*b^15*c^25*d^28 + 589861462528*a^39*b^1
4*c^24*d^29 - 116736734720*a^40*b^13*c^23*d^30 + 16011852800*a^41*b^12*c^22
*d^31 - 1361152000*a^42*b^11*c^21*d^32 + 54080000*a^43*b^10*c^20*d^33))*(-(
625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*a^2*b^2*c^2
*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12 - 49152*a^1
1*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18*d^3 + 202752
0*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b^6*c^15*d^6 -
3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120*a^9*b^3*c^12*d
^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/4)*1i + 175760000
*a^20*b^31*c^39*d^12 - 3507088000*a^21*b^30*c^38*d^13 + 32026300800*a^22*b^
29*c^37*d^14 - 177335474560*a^23*b^28*c^36*d^15 + 664045775360*a^24*b^27*c^
35*d^16 - 1770849815040*a^25*b^26*c^34*d^17 + 3431196106240*a^26*b^25*c^33*
d^18 - 4778178444800*a^27*b^24*c^32*d^19 + 4440824728320*a^28*b^23*c^31*d^2
0 - 1838397848320*a^29*b^22*c^30*d^21 - 1838397848320*a^30*b^21*c^29*d^22 +
 4440824728320*a^31*b^20*c^28*d^23 - 4778178444800*a^32*b^19*c^27*d^24 + 34
31196106240*a^33*b^18*c^26*d^25 - 1770849815040*a^34*b^17*c^25*d^26 + 66404
5775360*a^35*b^16*c^24*d^27 - 177335474560*a^36*b^15*c^23*d^28 + 3202630080
0*a^37*b^14*c^22*d^29 - 3507088000*a^38*b^13*c^21*d^30 + 175760000*a^39*b^1
2*c^20*d^31))*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 +
25350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9
*d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*
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c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^
6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 9011
20*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(1/
4) + 2*atan((819200*a^11*b^20*c^23*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*
d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4
096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b
^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032
*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2
027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^1
0 - 49152*a^20*b*c*d^11))^(5/4) + 845000*a^17*b^6*d^15*x^(1/2)*(-(625*b^13*
c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6
500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^
11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b
^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*
a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270
336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) + 819200*a^31*c^3*d^20*
x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*
a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12
 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*
d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^
6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^
18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) - 9
438000*a^16*b^7*c*d^14*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*
a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^1
2 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2
 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^
7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*
b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^
20*b*c*d^11))^(1/4) - 14090240*a^12*b^19*c^22*d*x^(1/2)*(-(625*b^13*c^4 + 2
8561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b
^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d +
270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*
d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^
5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^1
9*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) - 14090240*a^30*b*c^4*d^19*x^(
1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2
*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 4
9152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3
 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c
^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*
b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) + 8450
00*a^8*b^15*c^9*d^6*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3
*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 +
 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 -
901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d
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^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4
*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*
b*c*d^11))^(1/4) - 9438000*a^9*b^14*c^8*d^7*x^(1/2)*(-(625*b^13*c^4 + 28561
*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*
c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 2703
36*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4
 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^
5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^
2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) + 41158200*a^10*b^13*c^7*d^8*x^(1/
2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b
^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 491
52*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 +
 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6
*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^
3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) - 813586
80*a^11*b^12*c^6*d^9*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^
3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12
+ 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 -
 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*
d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^
4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20
*b*c*d^11))^(1/4) + 50890632*a^12*b^11*c^5*d^10*x^(1/2)*(-(625*b^13*c^4 + 2
8561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b
^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d +
270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*
d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^
5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^1
9*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) + 50890632*a^13*b^10*c^4*d^11*
x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*
a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12
- 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*
d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^
6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^
18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) - 8
1358680*a^14*b^9*c^3*d^12*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 439
40*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*
d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*
d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7
*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^
17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152
*a^20*b*c*d^11))^(1/4) + 41158200*a^15*b^8*c^2*d^13*x^(1/2)*(-(625*b^13*c^4
 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500
*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*
d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*
c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^1
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6*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336
*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) + 110723072*a^13*b^18*c^21
*d^2*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 2
5350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*
c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9
*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^
15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 9011
20*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4
) - 527826944*a^14*b^17*c^20*d^3*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^
4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(409
6*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^1
0*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a
^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 202
7520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10
- 49152*a^20*b*c*d^11))^(5/4) + 1708163072*a^15*b^16*c^19*d^4*x^(1/2)*(-(62
5*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*
d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*
b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520
*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3
244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^
9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) - 3975741440*a^1
6*b^15*c^18*d^5*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^1
0*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 409
6*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 9011
20*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 +
 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4
*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*
d^11))^(5/4) + 6877478912*a^17*b^14*c^17*d^6*x^(1/2)*(-(625*b^13*c^4 + 2856
1*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12
*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270
336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4
 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c
^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b
^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) - 9041543168*a^18*b^13*c^16*d^7*x
^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a
^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 -
 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d
^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6
*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^1
8*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) + 93
13648640*a^19*b^12*c^15*d^8*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 4
3940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^2
1*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^1
0*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b
^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*
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a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 491
52*a^20*b*c*d^11))^(5/4) - 8184070144*a^20*b^11*c^14*d^9*x^(1/2)*(-(625*b^1
3*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 -
 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*
c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13
*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 324403
2*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 2
70336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) + 7464878080*a^21*b^1
0*c^13*d^10*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*
d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^
9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a
^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 378
4704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8
 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11
))^(5/4) - 8184070144*a^22*b^9*c^12*d^11*x^(1/2)*(-(625*b^13*c^4 + 28561*a^
4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3
*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*
a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3
244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d
^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c
^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) + 9313648640*a^23*b^8*c^11*d^12*x^(1/
2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b
^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 491
52*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 +
 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6
*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^
3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) - 904154
3168*a^24*b^7*c^10*d^13*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940
*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^
12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^
2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c
^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17
*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a
^20*b*c*d^11))^(5/4) + 6877478912*a^25*b^6*c^9*d^14*x^(1/2)*(-(625*b^13*c^4
 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500
*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*
d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*
c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^1
6*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336
*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) - 3975741440*a^26*b^5*c^8*
d^15*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 2
5350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*
c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9
*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^
15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 9011
```

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20*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4
) + 1708163072*a^27*b^4*c^7*d^16*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^
4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(409
6*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^1
0*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a
^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 202
7520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10
 - 49152*a^20*b*c*d^11))^(5/4) - 527826944*a^28*b^3*c^6*d^17*x^(1/2)*(-(625*
b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^
2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^
11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a
^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 324
4032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9
 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4) + 110723072*a^29*b
^2*c^5*d^18*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*
d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^
9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a
^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 378
4704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8
 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11
))^(5/4))/(78125*b^21*c^13 - 1373125*a^13*b^8*d^13 + 11745500*a^12*b^9*c*d^
12 + 7293750*a^2*b^19*c^11*d^2 - 22537500*a^3*b^18*c^10*d^3 + 34273125*a^4*
b^17*c^9*d^4 - 17203200*a^5*b^16*c^8*d^5 - 8028160*a^6*b^15*c^7*d^6 - 95027
2*a^7*b^14*c^6*d^7 + 4030464*a^8*b^13*c^5*d^8 + 3343923*a^9*b^12*c^4*d^9 +
30329580*a^10*b^11*c^3*d^10 - 33424950*a^11*b^10*c^2*d^11 - 1187500*a*b^20*
c^12*d))*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350
*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12
 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9
*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b
^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a
^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4) +
atan((a^11*b^20*c^23*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^
3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12
+ 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 -
 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*
d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^
4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20
*b*c*d^11))^(5/4)*819200i + a^17*b^6*d^15*x^(1/2)*(-(625*b^13*c^4 + 28561*a
^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^
3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336
*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 -
3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*
d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*
c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4)*845000i + a^31*c^3*d^20*x^(1/2)*(-(6
25*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2
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*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10
*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 202752
0*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 -
3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d
^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*819200i - a^16*
b^7*c*d^14*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d
^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9
*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^
12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784
704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8
- 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11)
)^(1/4)*9438000i - a^12*b^19*c^22*d*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9
*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(
4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*
b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 324403
2*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 +
2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^
10 - 49152*a^20*b*c*d^11))^(5/4)*14090240i - a^30*b*c^4*d^19*x^(1/2)*(-(625
*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d
^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b
^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*
a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 32
44032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9
 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*14090240i + a^8*b
^15*c^9*d^6*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*
d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^
9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a
^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 378
4704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8
 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11
))^(1/4)*845000i - a^9*b^14*c^8*d^7*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9
*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(
4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*
b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 324403
2*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 +
2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^
10 - 49152*a^20*b*c*d^11))^(1/4)*9438000i + a^10*b^13*c^7*d^8*x^(1/2)*(-(62
5*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*
d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*
b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520
*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3
244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^
9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4)*41158200i - a^11
*b^12*c^6*d^9*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*
c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*
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a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120
*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3
784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d
^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^
11))^(1/4)*81358680i + a^12*b^11*c^5*d^10*x^(1/2)*(-(625*b^13*c^4 + 28561*a
^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^
3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336
*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 -
3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*
d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*
c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4)*50890632i + a^13*b^10*c^4*d^11*x^(1/
2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b
^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 491
52*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 +
 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6
*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^
3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4)*50890632
i - a^14*b^9*c^3*d^12*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a
^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12
 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2
- 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7
*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b
^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^2
0*b*c*d^11))^(1/4)*81358680i + a^15*b^8*c^2*d^13*x^(1/2)*(-(625*b^13*c^4 +
28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*
b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d +
 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8
*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b
^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^
19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(1/4)*41158200i + a^13*b^18*c^21*d^
2*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 2535
0*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^1
2 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^
9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*
b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*
a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*1
10723072i - a^14*b^17*c^20*d^3*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4
- 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*
a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*
c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^1
4*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 20275
20*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 -
49152*a^20*b*c*d^11))^(5/4)*527826944i + a^15*b^16*c^19*d^4*x^(1/2)*(-(625*
b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^
2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^
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11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a
^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 324
4032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9
+ 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*1708163072i - a^16
*b^15*c^18*d^5*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10
*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096
*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 90112
0*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 +
3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*
d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d
^11))^(5/4)*3975741440i + a^17*b^14*c^17*d^6*x^(1/2)*(-(625*b^13*c^4 + 2856
1*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12
*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270
336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4
 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c
^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b
^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*6877478912i - a^18*b^13*c^16*d^7*
x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*
a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12
- 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*
d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^
6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^
18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*904
1543168i + a^19*b^12*c^15*d^8*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 -
 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a
^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c
^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14
*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 202752
0*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 4
9152*a^20*b*c*d^11))^(5/4)*9313648640i - a^20*b^11*c^14*d^9*x^(1/2)*(-(625*
b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^
2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^
11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a
^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 324
4032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9
+ 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*8184070144i + a^21
*b^10*c^13*d^10*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^1
0*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 409
6*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 9011
20*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 +
 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4
*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*
d^11))^(5/4)*7464878080i - a^22*b^9*c^12*d^11*x^(1/2)*(-(625*b^13*c^4 + 285
61*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^1
2*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 27
```

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0336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^
4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*
c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*
b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*8184070144i + a^23*b^8*c^11*d^12
*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350
*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12
 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9
*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b
^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a
^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*93
13648640i - a^24*b^7*c^10*d^13*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4
- 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*
a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*
c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^1
4*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 20275
20*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 -
49152*a^20*b*c*d^11))^(5/4)*9041543168i + a^25*b^6*c^9*d^14*x^(1/2)*(-(625*
b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^
2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^
11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a
^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 324
4032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9
 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*6877478912i - a^26
*b^5*c^8*d^15*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*
c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*
a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120
*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3
784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d
^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^
11))^(5/4)*3975741440i + a^27*b^4*c^7*d^16*x^(1/2)*(-(625*b^13*c^4 + 28561*
a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c
^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 27033
6*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 -
 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5
*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2
*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*1708163072i - a^28*b^3*c^6*d^17*x^(
1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 43940*a^3*b^10*c*d^3 + 25350*a^2
*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d^12 + 4096*a^9*b^12*c^12 - 4
9152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d^2 - 901120*a^12*b^9*c^9*d^3
 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*c^7*d^5 + 3784704*a^15*b^6*c
^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^17*b^4*c^4*d^8 - 901120*a^18*
b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*a^20*b*c*d^11))^(5/4)*527826
944i + a^29*b^2*c^5*d^18*x^(1/2)*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 4394
0*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d
^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d
```

```
^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*
c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^1
7*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*
a^20*b*c*d^11))^(5/4)*110723072i)/(78125*b^21*c^13 - 1373125*a^13*b^8*d^13
+ 11745500*a^12*b^9*c*d^12 + 7293750*a^2*b^19*c^11*d^2 - 22537500*a^3*b^18*
c^10*d^3 + 34273125*a^4*b^17*c^9*d^4 - 17203200*a^5*b^16*c^8*d^5 - 8028160*
a^6*b^15*c^7*d^6 - 950272*a^7*b^14*c^6*d^7 + 4030464*a^8*b^13*c^5*d^8 + 334
3923*a^9*b^12*c^4*d^9 + 30329580*a^10*b^11*c^3*d^10 - 33424950*a^11*b^10*c^
2*d^11 - 1187500*a*b^20*c^12*d))*(-(625*b^13*c^4 + 28561*a^4*b^9*d^4 - 4394
0*a^3*b^10*c*d^3 + 25350*a^2*b^11*c^2*d^2 - 6500*a*b^12*c^3*d)/(4096*a^21*d
^12 + 4096*a^9*b^12*c^12 - 49152*a^10*b^11*c^11*d + 270336*a^11*b^10*c^10*d
^2 - 901120*a^12*b^9*c^9*d^3 + 2027520*a^13*b^8*c^8*d^4 - 3244032*a^14*b^7*
c^7*d^5 + 3784704*a^15*b^6*c^6*d^6 - 3244032*a^16*b^5*c^5*d^7 + 2027520*a^1
7*b^4*c^4*d^8 - 901120*a^18*b^3*c^3*d^9 + 270336*a^19*b^2*c^2*d^10 - 49152*
a^20*b*c*d^11))^(1/4)*2i - (2/(a*c) + (x^2*(5*a^3*d^3 + 5*b^3*c^3 - 4*a*b^2
*c^2*d - 4*a^2*b*c*d^2))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d
*x^4*(5*a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)))/(x^(5/2)*(a*d + b*c) + a*c*x^(1/2) + b*d*x^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.476 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=676

$$\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}+$$

Rubi [A] time = 0.97, antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{7a^2d^2-8abcd+7b^2c^2}{6a^2c^2x^{3/2}(bc-ad)^2}+\frac{b^{11/4}(7bc-15ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}+\frac{b^{11/4}(7bc-15ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^{11/4}(7bc-15ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3}+\frac{d^{11/4}(15bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3}+\frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3}-\frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3}+\frac{b}{2ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^{3/2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(7*b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2)/(6*a^2*c^2*(b*c - a*d)^2*x^(3/2)) + (d
*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(3/2)*(c + d*x^2)) + b/(2*a*(b*c - a*d
)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (b^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) - (b
^(11/4)*(7*b*c - 15*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*
Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - (d^(1
1/4)*(15*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqr
t[2]*c^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^
3) - (b^(11/4)*(7*b*c - 15*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7
*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2
]*c^(11/4)*(b*c - a*d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-11bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \text{Subst}\left(\int\right.$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+\right.}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+\right.}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+\right.}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+\right.}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+\right.}$$

$$= -\frac{7b^2c^2-8abcd+7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+\right.}$$

Mathematica [A] time = 6.16, size = 676, normalized size = 1.00

$$-\frac{b^{11/4}(15ad-7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}+\frac{b^{11/4}(15ad-7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3}+\frac{b^{11/4}(15ad-7bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3}+\frac{b^{11/4}(15ad-7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3}-\frac{b^3\sqrt{x}}{2a^2\left(a+bx^2\right)(ad-bc)^2}-\frac{2}{3a^2c^2x^{3/2}}-\frac{d^{11/4}(15bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(ad-bc)^3}+\frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(ad-bc)^3}+\frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(ad-bc)^3}+\frac{d^{11/4}(15bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(ad-bc)^3}-\frac{d^3\sqrt{x}}{2c^2\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -2/(3*a^2*c^2*x^(3/2)) - (b^3*Sqrt[x])/(2*a^2*(-(b*c) + a*d)^2*(a + b*x^2))
 - (d^3*Sqrt[x])/(2*c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(11/4)*(-7*b*c + 15
*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(
4*Sqrt[2]*a^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(-7*b*c + 15*a*d)*ArcTan[(Sqr
t[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(11/4)*(
b*c - a*d)^3) + (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d
^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^3) +
 (d^(11/4)*(15*b*c - 7*a*d)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(S
qrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^3) - (b^(11/4)*(-7*b*c
 + 15*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*S
qrt[2]*a^(11/4)*(b*c - a*d)^3) + (b^(11/4)*(-7*b*c + 15*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*
d)^3) - (d^(11/4)*(15*b*c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^3) + (d^(11/4)*(15*b
*c - 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*
Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.43, size = 472, normalized size = 0.70

$$\frac{\left(15ab^{11/4}d-7b^{15/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{11/4}(ad-bc)^3}-\frac{\left(15ab^{11/4}d-7b^{15/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{11/4}(ad-bc)^3}+\frac{-4a^3cd^2-7a^3d^3x^2+8a^2bc^2d+4a^2bcd^2x^2-7a^2bd^3x^4-4ab^2c^3+4ab^2c^2dx^2+8ab^2cd^2x^4-7b^3c^3x^2-7b^3c^2dx^4}{6a^2c^2x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)(ad-bc)^2}+\frac{\left(15bcd^{11/4}-7ad^{15/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^3}-\frac{\left(15bcd^{11/4}-7ad^{15/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{4\sqrt{2}\,c^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-4*a*b^2*c^3 + 8*a^2*b*c^2*d - 4*a^3*c*d^2 - 7*b^3*c^3*x^2 + 4*a*b^2*c^2*d
*x^2 + 4*a^2*b*c*d^2*x^2 - 7*a^3*d^3*x^2 - 7*b^3*c^2*d*x^4 + 8*a*b^2*c*d^2*
x^4 - 7*a^2*b*d^3*x^4)/(6*a^2*c^2*(-(b*c) + a*d)^2*x^(3/2)*(a + b*x^2)*(c +
 d*x^2)) + ((-7*b^(15/4)*c + 15*a*b^(11/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(11/4)*(-(b*c) + a*d)^3) +
 ((15*b*c*d^(11/4) - 7*a*d^(15/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x])])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3) - ((-7*b^(15/4)
*c + 15*a*b^(11/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
Sqrt[b]*x)])/(4*Sqrt[2]*a^(11/4)*(-(b*c) + a*d)^3) - ((15*b*c*d^(11/4) - 7*
a*d^(15/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)
])/(4*Sqrt[2]*c^(11/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[A]** time = 2.03, size = 1012, normalized size = 1.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqr
t(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(7*(a*b
^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^
2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(15*(c*d^3)^(1/4)*
b*c*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2
)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/4*(15*(c*d^3)^(1/4)*b*c*d^2 - 7*
(c*d^3)^(1/4)*a*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4
*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/8*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)
*a*b^2*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3
*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) +
 1/8*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2
*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/8*(15*(c*d^3)^(1/4)*b*c*d
^2 - 7*(c*d^3)^(1/4)*a*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt
(2)*a^3*c^3*d^3) + 1/8*(15*(c*d^3)^(1/4)*b*c*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*l
og(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(
2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/2*(b^3*
c^2*d*x^(5/2) + a^2*b*d^3*x^(5/2) + b^3*c^3*sqrt(x) + a^3*d^3*sqrt(x))/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c
)) - 2/3/(a^2*c^2*x^(3/2))
```

maple **[A]** time = 0.03, size = 825, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] -1/2*b^3/a/(a*d-b*c)^3*x^(1/2)/(b*x^2+a)*d+1/2*b^4/a^2/(a*d-b*c)^3*x^(1/2)/
(b*x^2+a)*c-15/8*b^3/a^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)*d+7/8*b^4/a^3/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)*c-15/8*b^3/a^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d+7/8*b^4/a^3/(a*d-b*c)^3*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c-15/16*b^3/a^2/(a*d-b*c)^3
```

```
*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b
)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d+7/16*b^4/a^3/(a*d-b*c)^3*(a/b)^(1/4
)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(
1/2)*x^(1/2)+(a/b)^(1/2)))*c-1/2*d^4/c^2/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*a+1/
2*d^3/c/(a*d-b*c)^3*x^(1/2)/(d*x^2+c)*b-7/8*d^4/c^3/(a*d-b*c)^3*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a+15/8*d^3/c^2/(a*d-b*c)^3*(
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b-7/16*d^4/c^3/(a*
d-b*c)^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))
/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a+15/16*d^3/c^2/(a*d-b*c)^3*(
c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^
(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b-7/8*d^4/c^3/(a*d-b*c)^3*(c/d)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a+15/8*d^3/c^2/(a*d-b*c)^3*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b-2/3/a^2/c^2/x^(3/2
)
```

maxima [A] time = 2.69, size = 761, normalized size = 1.13

$$-\frac{\left(\frac{2\sqrt{2}(7bc-15ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(7bc-15ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(7bc-15ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(7bc-15ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^{3}}{16\left(a^{2}b^{3}c^{3}-3a^{3}b^{2}c^{2}d+3a^{4}bcd^{2}-a^{5}d^{3}\right)}-\frac{4ab^{2}c^{3}-8a^{2}bc^{2}d+4a^{3}cd^{2}+\left(7b^{3}c^{2}d-8ab^{2}cd^{2}+7a^{2}bd^{3}\right)x^{4}+\left(7b^{3}c^{3}-4ab^{2}c^{2}d-4a^{2}bcd^{2}+7a^{3}d^{3}\right)x^{2}}{6\left(\left(a^{2}b^{3}c^{4}d-2a^{3}b^{2}c^{3}d^{2}+a^{4}bc^{2}d^{3}\right)x^{\frac{11}{2}}+\left(a^{2}b^{3}c^{5}-a^{3}b^{2}c^{4}d-a^{4}bc^{3}d^{2}+a^{5}c^{2}d^{3}\right)x^{\frac{7}{2}}+\left(a^{3}b^{2}c^{5}-2a^{4}bc^{4}d+a^{5}c^{3}d^{2}\right)x^{\frac{3}{2}}\right)}-\frac{\frac{2\sqrt{2}\left(15bcd^{3}-7ad^{4}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(15bcd^{3}-7ad^{4}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(15bcd^{3}-7ad^{4}\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(15bcd^{3}-7ad^{4}\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\left(b^{3}c^{5}-3ab^{2}c^{4}d+3a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(7*b*c - 15*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*(7*b*c - 15*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + sqrt(2)*(7*b*c - 15*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b*c - 15*a*d)*log(-sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^3/(a^2*b^
3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/6*(4*a*b^2*c^3 - 8*a
^2*b*c^2*d + 4*a^3*c*d^2 + (7*b^3*c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*x^4
+ (7*b^3*c^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2 + 7*a^3*d^3)*x^2)/((a^2*b^3*c^
4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^(11/2) + (a^2*b^3*c^5 - a^3*b^2*
c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^(7/2) + (a^3*b^2*c^5 - 2*a^4*b*c^4*d
 + a^5*c^3*d^2)*x^(3/2)) - 1/16*(2*sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*arctan(1/
2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d
)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*arct
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)
) - sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3)
```

mupad [B] time = 10.59, size = 44436, normalized size = 65.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] atan((((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150
*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^
12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c
^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17
*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120
*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*
(((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*
b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 -
49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^
3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*
c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20
*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(1174
40512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27
*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 370914046771 2*a^29*b^34*
c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^5
3*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^
51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28
*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*
b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 32403938271559680*
a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 4461143707877
3760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 44611437
078773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 324
03938271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25
+ 16064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d
^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^3
4*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^
32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30
*d^33 + 370914046771 2*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35
+ 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 11744051
2*a^59*b^4*c^25*d^38) + x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*
a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^3
6*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^
52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^5
0*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c
^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^2
8*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*
b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^
37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400
*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275554166702
080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085
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570560*a^43*b^18*c^36*d^25 - 882855756431360O*a^44*b^17*c^35*d^26 + 6863546
220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657
695282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 5
58859896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 +
59682471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 28
34667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454
720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4
*c^22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 +
 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b
^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14
*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 378470
4*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 -
901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^
(3/4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 4738
2401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309
120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 272073282805 7
6*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 14959006923161
6*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 3773252781260
80*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 436168221851
648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 11595065421
8240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 3171155516
17024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657
864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 26100858
9107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 7065651
3052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 83989394
63680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 36846375731
2*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b
^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37) + x^(1/2)*(276595200*a^18*b^35
*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^
13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 -
 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 5
5777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 1
60243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 -
250272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 -
 250272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25
- 160243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27
 - 55777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29
- 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 4
96940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304
320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35))*(-(2401*b^15*c^4
+ 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 2058
0*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^1
1*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^
8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a
^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 2703
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36*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*1i - ((-(2401*b^15*c^4 +
 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580
*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11
*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8
*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^
18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 27033
6*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(((-(2401*b^15*c^4 + 5062
5*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^
14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d +
270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*
d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^
5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^2
1*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(117440512*a^25*b^38*c^59*d^4
- 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 53130087
6288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272
*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 303808739540992*
a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 228899597529907
2*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 - 955241025503
2320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^16 - 24054442
827448320*a^38*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 396
85869262602240*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20
- 46346397171056640*a^42*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*
d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*
c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*
b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^
49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 903261116694528*
a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 85777845321728*
a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^5
5*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^
27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38) - x
^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 4531
9454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 283466792960
0*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a
^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*
a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 26576952827576
32*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 68635462205
44000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 97114060
85570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 62755
54166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 22
63049201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 -
 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^2
5 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*
d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^
32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*
c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*
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c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(3/4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37) - x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*1i)/(((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b

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^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*
a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 491
52*a^22*b*c*d^11))^(1/4)*(((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^
3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12
 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2
 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^
7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*
b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^
22*b*c*d^11))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^
58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 +
 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 857
77845321728*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903
261116694528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5
006182506823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15
+ 16064830746132480*a^37*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*
d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*
c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*
b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d^22 - 39685869262602240*
a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c^39*d^24 - 2405444282744
8320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^37*d^26 - 95524102
55032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 22889
95975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303
808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 199
31198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 5313008
76288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58
*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38) + x^(1/2)*(102760448*a^22*b^
39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^
6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14
570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 2000
27983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 131
9333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 -
4599356881633280*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16
 - 8828557564313600*a^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d
^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^4
1*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*
c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^
19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44
*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a
^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d^29 - 131933314167603
2*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30*d^31 - 2000279830528
00*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28*d^33 - 1457042489344
0*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54
*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*
d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4
 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(409
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6*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^
10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*
a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 20
27520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10
 - 49152*a^22*b*c*d^11))^(3/4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*
a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^
34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*
c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c
^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*
c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26
*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^2
4*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^
22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b
^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*
b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39
*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*
b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^1
2*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20
*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37) + x^(
1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 7186
9242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434
048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 2469634886348
8*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 10325155948083
2*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 2135232933048
32*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 263188357892
096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 21352329330
4832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 1032515594
80832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 2469634886
3488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 241225843404
8*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40
*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18
*d^35))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 6615
0*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c
^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*
c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^1
7*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 90112
0*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)
 + ((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^
2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12
 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*
d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^
6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^
20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(((
-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^1
3*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 491
```

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52*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 +
 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6
*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^
3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(1174405
12*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^
36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^5
5*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d
^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*
d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^
49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b^2
6*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 32403938271559680*a^3
9*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 4461143707877376
0*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 44611437078
773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 324039
38271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 1
6064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27
 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d
^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*
d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^
33 + 3709140467712*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 5
4978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a
^59*b^4*c^25*d^38) - x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^2
3*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c
^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*
d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d
^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48
*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c
^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^2
6*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*
b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^
39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275554166702080
*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570
560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220
544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695
282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 5588
59896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 596
82471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 28346
67929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720
*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^
22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66
150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12
*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^
9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a
^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901
120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(3/
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4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 4738240
1024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120
*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a
^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a
^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 377325278126080*
a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 436168221851648
*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 11595065421824
0*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 3171155516170
24*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657864
704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 26100858910
7200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 7065651305
2672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 83989394636
80*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a
^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*
c^19*d^36 + 147517440*a^47*b^8*c^18*d^37) - x^(1/2)*(276595200*a^18*b^35*c^
42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13
- 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 87
51989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 5577
7785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 1602
43801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250
272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 25
0272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 1
60243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 -
55777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 8
751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 4969
40910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320
*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35))*(-(2401*b^15*c^4 + 5
0625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a
*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d
 + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c
^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18
*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*
a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)))*(-(2401*b^15*c^4 + 50625*
a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14
*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 27
0336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^
4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*
c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*
b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*2i + 2*atan((((-(2401*b^15*c^4 +
 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580
*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11
*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8
*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^
18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 27033
6*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(147517440*a^18*b^37*c^47
```

```
*d^8 - ((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 6615
0*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c
^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*
c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^1
7*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 90112
0*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)
*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 5497893683
2*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29
*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^
32*c^53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b
^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^3
5*b^28*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480
*a^37*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 324039382715
59680*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 4461143
7078773760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 44
611437078773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23
 + 32403938271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38
*d^25 + 16064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*
c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^
13*c^34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b
^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^
9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28
*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 11
7440512*a^59*b^4*c^25*d^38)*1i + x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 31
12173568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000
*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^2
7*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29
*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^
31*b^30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280
*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313
600*a^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328
624640*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049
201254400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275
554166702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9
711406085570560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26
+ 6863546220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^
28 + 2657695282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31
*d^30 + 558859896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^2
9*d^32 + 59682471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*
d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 +
 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448
*a^57*b^4*c^22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^1
2*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 40
96*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 90
1120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5
```

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 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c
^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*
c*d^11))^(3/4)*1i - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c
^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*
d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^
14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^
16 - 261008589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d
^18 - 450764657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*
d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33
*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^3
1*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^
29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c
^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c
^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^2
3*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^
34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 1475
17440*a^47*b^8*c^18*d^37)*1i + x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 650
1304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 49694091059
2*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a
^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^
25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a
^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*
a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408
*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 16024380191948
8*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 - 5577778527641
6*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 8751989614592*
a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39
*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^
19*d^34 + 276595200*a^42*b^11*c^18*d^35))*(-(2401*b^15*c^4 + 50625*a^4*b^11
*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/
(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^1
3*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244
032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7
+ 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*
d^10 - 49152*a^22*b*c*d^11))^(1/4) - ((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4
 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(409
6*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^
10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*
a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 20
27520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10
 - 49152*a^22*b*c*d^11))^(1/4)*(147517440*a^18*b^37*c^47*d^8 - ((-(2401*b^1
5*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2
- 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^
11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a
^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 324
```

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4032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9
+ 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(117440512*a^25*b^
38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^
6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 19
931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 3038
08739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 228
8995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 -
9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^1
6 - 24054442827448320*a^38*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^4
5*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^2
2*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 44611437078773760*a^4
3*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 3240393827155968
0*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746
132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182
506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 9032
61116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 857
77845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 37091
40467712*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832
*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^
25*d^38)*1i - x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*
c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7
 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 5
9682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 5
58859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 +
 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^1
5 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*
d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^
42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22
*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b
^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^4
3*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*
a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 26576952827576
32*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 55885989675
0080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 5968247128
0640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 283466792960
0*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b
^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39
))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2
*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 -
 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d
^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6
*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^2
0*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(3/4)*1i -
 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 36846375
7312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 839893946368
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0*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672
*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 26100858910720
0*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 4507646578647
04*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617
024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 11595065421
8240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 4361682218
51648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278
126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 14959006
9231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328
280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309
120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^
45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18
*d^37)*1i - x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c
^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^
14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16
+ 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 +
 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20
+ 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22
 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^2
4 + 213523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^
26 + 103251559480832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^
28 + 24696348863488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30
 + 2412258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 7
1869242368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200
*a^42*b^11*c^18*d^35))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^
12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4
096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 9
01120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^
5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*
c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b
*c*d^11))^(1/4))/(((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c
*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*
a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 90112
0*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 +
3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*
d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d
^11))^(1/4)*(147517440*a^18*b^37*c^47*d^8 - ((-(2401*b^15*c^4 + 50625*a^4*b
^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*
d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*
a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3
244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d
^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c
^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 36574
33088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876288*a^
28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b
```

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^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^
31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*
b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^
36*b^27*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^16 - 240544428274483
20*a^38*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 3968586926
2602240*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 - 46346
397171056640*a^42*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d^22 -
39685869262602240*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c^39*d^
24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^
37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14
*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^
12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^
10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c
^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36
 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38)*1i + x^(1/
2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454
720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^
26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*
b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30
*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a
^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 686354622054400
0*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 971140608557
0560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 627555416
6702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 226304
9201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 890
4303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 -
8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27
 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d
^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30
*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28
*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^3
5 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112
173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*b^15*c^4
 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 205
80*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^
11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b
^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*
a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270
336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(3/4)*1i - 3841073152*a^19*b^
36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44
*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^
13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^1
5 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^
17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d
^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*
```

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d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32
*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^3
0*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^
28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c
^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^
24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*
d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 -
 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37)*1i + x^(1/2)
*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242
368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*
a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^
24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^
26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a
^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 263188357892096*
a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213523293304832
*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 10325155948083
2*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 24696348863488
*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^
38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^1
3*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^3
5))*(-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^
2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12
- 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*
d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^
6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^
20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*1i
+ ((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2
*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 -
 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d
^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6
*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^2
0*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*(147
517440*a^18*b^37*c^47*d^8 - ((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*
a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^
12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d
^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*
c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^1
9*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*
a^22*b*c*d^11))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*
c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7
 + 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 8
5777845321728*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 9
03261116694528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 +
 5006182506823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^1
5 + 16064830746132480*a^37*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^4
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6*d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^2
3*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 - 46346397171056640*a^4
2*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d^22 - 3968586926260224
0*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c^39*d^24 - 24054442827
448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^37*d^26 - 955241
0255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 228
8995975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 3
03808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 1
9931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 53130
0876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^
58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38)*1i - x^(1/2)*(102760448*a^
22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^
55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8
 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 -
 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12
- 1319333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^
14 - 4599356881633280*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45
*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c
^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^2
3*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*
b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^
42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 - 8828557564313600
*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27 - 4599356881633
280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d^29 - 1319333141
676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30*d^31 - 20002798
3052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28*d^33 - 14570424
893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000
*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*
c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*b^15*c^4 + 50625*a^4*b^1
1*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)
/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^
13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 324
4032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7
 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2
*d^10 - 49152*a^22*b*c*d^11))^(3/4)*1i - 3841073152*a^19*b^36*c^46*d^9 + 47
382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 20274743
09120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280
576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231
616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 37732527812
6080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 4361682218
51648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 115950654
218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 31711555
1617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 4507646
57864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008
589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656
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513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 839893
9463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757
312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46
*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37)*1i - x^(1/2)*(276595200*a^18
*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^
40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d
^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^1
7 - 55777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^1
9 - 160243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^
21 - 250272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d
^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*
d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26
*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*
d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^3
1 - 496940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 65
01304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35))*(-(2401*b^15
*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 -
 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^1
1*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^
15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 3244
032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 +
 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4)*1i))*(-(2401*b^15*c
^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 66150*a^2*b^13*c^2*d^2 - 2
0580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*c^12 - 49152*a^12*b^11*
c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^9*c^9*d^3 + 2027520*a^15
*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*a^17*b^6*c^6*d^6 - 324403
2*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 901120*a^20*b^3*c^3*d^9 + 2
70336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^(1/4) + atan(((-(2401*a^4*d
^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 -
20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^
12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*
b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 324403
2*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 2
70336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*((-(2401*a^4*d^15 +
50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*
a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^1
1 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^
19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*
b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*
a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(((-(2401*a^4*d^15 + 50625
*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b
*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 2
70336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^
4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c
^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*
```

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b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(117440512*a^25*b^38*c^59*d^4 -
 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876
288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*
a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 303808739540992*a
^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 2288995975299072
*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 - 9552410255032
320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^16 - 240544428
27448320*a^38*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 3968
5869262602240*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 -
 46346397171056640*a^42*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d
^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c
^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b
^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^4
9*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 903261116694528*a
^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a
^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55
*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^2
7*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38) + x^
(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319
454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600
*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^
28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a
^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 265769528275763
2*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 686354622054
4000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 971140608
5570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 627555
4166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 226
3049201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 -
8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25
 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d
^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^3
2*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c
^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c
^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*
d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3
112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*a^4*d
^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 -
20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^
12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*
b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 324403
2*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 2
70336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(3/4) + 147517440*a^18*b^3
7*c^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^1
0 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 -
8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70
```

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656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 26
1008589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 4
50764657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 -
317115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 +
 115950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24
+ 436168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26
 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^2
8 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^3
0 + 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32
+ 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47
382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a
^47*b^8*c^18*d^37) + x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^
19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^3
2*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c
^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^
35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c
^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*
c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22
*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^2
0*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^1
8*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*
c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21
*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 +
276595200*a^42*b^11*c^18*d^35))*1i - (-(2401*a^4*d^15 + 50625*b^4*c^4*d^11
- 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096
*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10
*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^
5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027
520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10
- 49152*a*b^11*c^22*d))^(1/4)*((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 9450
0*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*
c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*
d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*
c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^
8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 4915
2*a*b^11*c^22*d))^(1/4)*(((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b
^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23
+ 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 -
 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*
d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4
*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b
^11*c^22*d))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^5
8*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 +
3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 8577
7845321728*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 9032
```

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61116694528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 50
06182506823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 +
 16064830746132480*a^37*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d
^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c
^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b
^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d^22 - 39685869262602240*a
^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c^39*d^24 - 24054442827448
320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^37*d^26 - 955241025
5032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 228899
5975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 3038
08739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 1993
1198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 53130087
6288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*
b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38) - x^(1/2)*(102760448*a^22*b^3
9*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6
 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 145
70424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 20002
7983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319
333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4
599356881633280*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16
- 8828557564313600*a^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^
18 - 8904303328624640*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41
*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c
^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^1
9*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*
b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^
46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d^29 - 1319333141676032
*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30*d^31 - 20002798305280
0*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28*d^33 - 14570424893440
*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*
b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d
^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*a^4*d^15 + 50625*b^4*c^4*d^11
- 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096
*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10
*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^
5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027
520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10
- 49152*a*b^11*c^22*d))^(3/4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*a
^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^3
4*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c
^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^
40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c
^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*
c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24
*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^2
```

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2*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^
20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b
^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*
b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b
^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12
*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*
d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37) - x^(1
/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869
242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 24122584340
48*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488
*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 103251559480832
*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 21352329330483
2*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 2631883578920
96*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213523293304
832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 10325155948
0832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 24696348863
488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048
*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*
b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*
d^35))*1i)/((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 +
66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^
11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^
9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*
a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 90
1120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(
1/4)*((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*
a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^1
2 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20
*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^
6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a
^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(
((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b
^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 4
9152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3
+ 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^1
7*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^
3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(11744
0512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*
b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c
^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53
*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^5
1*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*
c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b
^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 32403938271559680*a
^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 44611437078773
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760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 446114370
78773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 3240
3938271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 +
 16064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^
27 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34
*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^3
2*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*
d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 +
 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512
*a^59*b^4*c^25*d^38) + x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a
^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36
*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^5
2*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50
*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^
48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28
*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b
^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^3
7*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400*
a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 62755541667020
80*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 97114060855
70560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 68635462
20544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 26576
95282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 55
8859896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 5
9682471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 283
4667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 453194547
20*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*
c^22*d^39))*(-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 +
66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^
11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^
9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*
a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 90
1120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(
3/4) + 147517440*a^18*b^37*c^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 47382
401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 20274743091
20*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576
*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231616
*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 37732527812608
0*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 4361682218516
48*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 115950654218
240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 31711555161
7024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 4507646578
64704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589
107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513
052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 839893946
```

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3680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312
*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^
9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37) + x^(1/2)*(276595200*a^18*b^35*
c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^1
3 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 -
8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55
777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 16
0243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 2
50272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 -
250272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 -
 160243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27
- 55777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 -
 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 49
6940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 65013043
20*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35)) + (-(2401*a^4*d^15
 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 205
80*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*
d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8
*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a
^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 2703
36*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*((-(2401*a^4*d^15 + 506
25*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3
*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 +
 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*
d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5
*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^1
0*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(((-(2401*a^4*d^15 + 50625*b^
4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*
d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 2703
36*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 -
 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16
*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2
*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 36
57433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876288
*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*a^3
0*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 303808739540992*a^32
*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 2288995975299072*a^
34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 - 9552410255032320
*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^16 - 240544428274
48320*a^38*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 3968586
9262602240*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 - 46
346397171056640*a^42*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d^22
 - 39685869262602240*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c^39
*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16
*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b
```

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^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51
*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53
*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^
8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d
^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38) - x^(1/
2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454
720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^
26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*
b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30
*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a
^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 686354622054400
0*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 971140608557
0560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 627555416
6702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 226304
9201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 890
4303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 -
8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27
 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d
^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30
*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28
*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^3
5 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112
173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*a^4*d^15
 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 205
80*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*
d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8
*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a
^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 2703
36*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(3/4) + 147517440*a^18*b^37*c
^47*d^8 - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 -
 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 839
8939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656
513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 26100
8589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 4507
64657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317
115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 11
5950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 4
36168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 +
377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 +
 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 +
 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2
027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382
401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47
*b^8*c^18*d^37) - x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*
b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c
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^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37
*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*
d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33
*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^3
1*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^
29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c
^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c
^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^2
3*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^
32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276
595200*a^42*b^11*c^18*d^35))))*(-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 9450
0*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*
c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*
d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*
c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^
8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 4915
2*a*b^11*c^22*d))^(1/4)*2i + 2*atan(((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11
- 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096
*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10
*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^
5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027
520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10
- 49152*a*b^11*c^22*d))^(1/4)*((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 9450
0*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*
c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*
d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*
c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^
8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 4915
2*a*b^11*c^22*d))^(1/4)*(147517440*a^18*b^37*c^47*d^8 - ((-(2401*a^4*d^15 +
 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580
*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^
11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c
^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7
*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336
*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(117440512*a^25*b^38*c^59
*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531
300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 199311983
90272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 30380873954
0992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 2288995975
299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 - 9552410
255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^16 - 240
54442827448320*a^38*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^45*d^18
- 39685869262602240*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*
d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*
c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*
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b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*
a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*d^27 + 50061825068236
80*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 90326111669
4528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 8577784532
1728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 370914046771
2*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b
^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38
)*1i + x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^
5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834
667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471
280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 55885989
6750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 265769
5282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 686
3546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 +
9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19
 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d
^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^3
8*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*
c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^
16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47
*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^
49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^
51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*
b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24
*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2
401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^
2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*
a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 202
7520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6
 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^1
4*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(3/4)*1i - 384107
3152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^
21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*
b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b
^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*
b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29
*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^3
1*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^
33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a
^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*
a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616
*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*
a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^4
3*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10
*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37)*
```

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1i + x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^1
2 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 24
12258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696
348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 103251
559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 21352
3293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 2631
88357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213
523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 10
3251559480832*a^34*b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 24
696348863488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 2412
258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 71869242
368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b
^11*c^18*d^35)) - (-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d
^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a
^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*
a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 37
84704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^
8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22
*d))^(1/4)*((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 +
66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^
11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^
9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*
a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 90
1120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(
1/4)*(147517440*a^18*b^37*c^47*d^8 - ((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11
 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(409
6*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^1
0*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a
^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 202
7520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10
 - 49152*a*b^11*c^22*d))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a
^26*b^37*c^58*d^5 + 54978936832*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35
*c^56*d^7 + 3709140467712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^5
4*d^9 + 85777845321728*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^31*c^52
*d^11 + 903261116694528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^
50*d^13 + 5006182506823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^36*b^27
*c^48*d^15 + 16064830746132480*a^37*b^26*c^47*d^16 - 24054442827448320*a^38
*b^25*c^46*d^17 + 32403938271559680*a^39*b^24*c^45*d^18 - 39685869262602240
*a^40*b^23*c^44*d^19 + 44611437078773760*a^41*b^22*c^43*d^20 - 463463971710
56640*a^42*b^21*c^42*d^21 + 44611437078773760*a^43*b^20*c^41*d^22 - 3968586
9262602240*a^44*b^19*c^40*d^23 + 32403938271559680*a^45*b^18*c^39*d^24 - 24
054442827448320*a^46*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^37*d^26
 - 9552410255032320*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d
^28 - 2288995975299072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^12*c^33
*d^30 - 303808739540992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31
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*d^32 - 19931198390272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^3
4 - 531300876288*a^56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657
433088*a^58*b^5*c^26*d^37 + 117440512*a^59*b^4*c^25*d^38)*1i - x^(1/2)*(102
760448*a^22*b^39*c^57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454720*a^2
4*b^37*c^55*d^6 - 422576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35
*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^
51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c
^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a^32*b^2
9*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*
b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 9711406085570560*a^
36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 6275554166702080
*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 2263049201254
400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 8904303328
624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 - 8828557
564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27 - 4599
356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d^29 - 1
319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30*d^31 -
 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28*d^33 -
 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422
576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*
a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*a^4*d^15 + 5062
5*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*
b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 +
270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d
^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*
c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10
*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(3/4)*1i - 3841073152*a^19*b^36*c^46
*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 +
 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27
207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149
590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 37
7325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 4
36168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 +
115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 -
 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25
- 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27
 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^2
9 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31
 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 -
368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 384107
3152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37)*1i - x^(1/2)*(27659
5200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^2
0*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^
31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29
*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27
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*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^2
5*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^
23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b
^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*
b^19*c^26*d^27 - 55777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b
^17*c^24*d^29 - 8751989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15
*c^22*d^31 - 496940910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*
d^33 - 6501304320*a^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35)))/((
-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2
*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 491
52*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 +
2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*
d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*
c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*((-(2401
*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d
^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^1
1*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 202752
0*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 -
3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d
^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(147517440*a^1
8*b^37*c^47*d^8 - ((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*
d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*
a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120
*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3
784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d
^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^2
2*d))^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 +
 54978936832*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140
467712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321
728*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 90326111669
4528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 500618250
6823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 160648
30746132480*a^37*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 3
2403938271559680*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^1
9 + 44611437078773760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^4
2*d^21 + 44611437078773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^1
9*c^40*d^23 + 32403938271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^4
6*b^17*c^38*d^25 + 16064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320
*a^48*b^15*c^36*d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299
072*a^50*b^13*c^34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 30380873954
0992*a^52*b^11*c^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390
272*a^54*b^9*c^30*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 531300876288*a^
56*b^7*c^28*d^35 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^2
6*d^37 + 117440512*a^59*b^4*c^25*d^38)*1i + x^(1/2)*(102760448*a^22*b^39*c^
57*d^4 - 3112173568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 4
```

22576128000*a^25*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^34*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 8904303328624640*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 2263049201254400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 6275554166702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 9711406085570560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 6863546220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 + 2657695282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30 + 558859896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^32 + 59682471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34 + 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 45319454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57*b^4*c^22*d^39))*(-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(3/4)*1i - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12 - 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 - 70656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 - 261008589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 - 450764657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20 - 317115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22 + 115950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^24 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d^30 + 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^32 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 + 47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440*a^47*b^8*c^18*d^37)*1i + x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 6501304320*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^21*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^33*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 - 55

```
777785276416*a^35*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 875
1989614592*a^37*b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 496940
910592*a^39*b^14*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320*a
^41*b^12*c^19*d^34 + 276595200*a^42*b^11*c^18*d^35))*1i + (-(2401*a^4*d^15
+ 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 2058
0*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d
^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*
c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^
7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 27033
6*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*((-(2401*a^4*d^15 + 5062
5*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*
b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 +
270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d
^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*
c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10
*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(147517440*a^18*b^37*c^47*d^8
- ((-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2
*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^12*c^11*d^12 -
 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^
3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c
^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*
b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*d))^(1/4)*(117
440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 54978936832*a^2
7*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467712*a^29*b^34
*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 85777845321728*a^31*b^32*c^
53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694528*a^33*b^30*c
^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 5006182506823680*a^35*b^2
8*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064830746132480*a^37
*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 + 32403938271559680
*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^44*d^19 + 446114370787
73760*a^41*b^22*c^43*d^20 - 46346397171056640*a^42*b^21*c^42*d^21 + 4461143
7078773760*a^43*b^20*c^41*d^22 - 39685869262602240*a^44*b^19*c^40*d^23 + 32
403938271559680*a^45*b^18*c^39*d^24 - 24054442827448320*a^46*b^17*c^38*d^25
 + 16064830746132480*a^47*b^16*c^37*d^26 - 9552410255032320*a^48*b^15*c^36*
d^27 + 5006182506823680*a^49*b^14*c^35*d^28 - 2288995975299072*a^50*b^13*c^
34*d^29 + 903261116694528*a^51*b^12*c^33*d^30 - 303808739540992*a^52*b^11*c
^32*d^31 + 85777845321728*a^53*b^10*c^31*d^32 - 19931198390272*a^54*b^9*c^3
0*d^33 + 3709140467712*a^55*b^8*c^29*d^34 - 531300876288*a^56*b^7*c^28*d^35
 + 54978936832*a^57*b^6*c^27*d^36 - 3657433088*a^58*b^5*c^26*d^37 + 1174405
12*a^59*b^4*c^25*d^38)*1i - x^(1/2)*(102760448*a^22*b^39*c^57*d^4 - 3112173
568*a^23*b^38*c^56*d^5 + 45319454720*a^24*b^37*c^55*d^6 - 422576128000*a^25
*b^36*c^54*d^7 + 2834667929600*a^26*b^35*c^53*d^8 - 14570424893440*a^27*b^3
4*c^52*d^9 + 59682471280640*a^28*b^33*c^51*d^10 - 200027983052800*a^29*b^32
*c^50*d^11 + 558859896750080*a^30*b^31*c^49*d^12 - 1319333141676032*a^31*b^
30*c^48*d^13 + 2657695282757632*a^32*b^29*c^47*d^14 - 4599356881633280*a^33
```

*b^28*c^46*d^15 + 6863546220544000*a^34*b^27*c^45*d^16 - 8828557564313600*a
^35*b^26*c^44*d^17 + 9711406085570560*a^36*b^25*c^43*d^18 - 890430332862464
0*a^37*b^24*c^42*d^19 + 6275554166702080*a^38*b^23*c^41*d^20 - 226304920125
4400*a^39*b^22*c^40*d^21 - 2263049201254400*a^40*b^21*c^39*d^22 + 627555416
6702080*a^41*b^20*c^38*d^23 - 8904303328624640*a^42*b^19*c^37*d^24 + 971140
6085570560*a^43*b^18*c^36*d^25 - 8828557564313600*a^44*b^17*c^35*d^26 + 686
3546220544000*a^45*b^16*c^34*d^27 - 4599356881633280*a^46*b^15*c^33*d^28 +
2657695282757632*a^47*b^14*c^32*d^29 - 1319333141676032*a^48*b^13*c^31*d^30
+ 558859896750080*a^49*b^12*c^30*d^31 - 200027983052800*a^50*b^11*c^29*d^3
2 + 59682471280640*a^51*b^10*c^28*d^33 - 14570424893440*a^52*b^9*c^27*d^34
+ 2834667929600*a^53*b^8*c^26*d^35 - 422576128000*a^54*b^7*c^25*d^36 + 4531
9454720*a^55*b^6*c^24*d^37 - 3112173568*a^56*b^5*c^23*d^38 + 102760448*a^57
*b^4*c^22*d^39))*(-(2401*a^4*d^15 + 50625*b^4*c^4*d^11 - 94500*a*b^3*c^3*d^
12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/(4096*b^12*c^23 + 4096*a^
12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2*b^10*c^21*d^2 - 901120*a
^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 3244032*a^5*b^7*c^18*d^5 + 378
4704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 + 2027520*a^8*b^4*c^15*d^8
- 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*d^10 - 49152*a*b^11*c^22*
d))^(3/4)*1i - 3841073152*a^19*b^36*c^46*d^9 + 47382401024*a^20*b^35*c^45*d
^10 - 368463757312*a^21*b^34*c^44*d^11 + 2027474309120*a^22*b^33*c^43*d^12
- 8398939463680*a^23*b^32*c^42*d^13 + 27207328280576*a^24*b^31*c^41*d^14 -
70656513052672*a^25*b^30*c^40*d^15 + 149590069231616*a^26*b^29*c^39*d^16 -
261008589107200*a^27*b^28*c^38*d^17 + 377325278126080*a^28*b^27*c^37*d^18 -
450764657864704*a^29*b^26*c^36*d^19 + 436168221851648*a^30*b^25*c^35*d^20
- 317115551617024*a^31*b^24*c^34*d^21 + 115950654218240*a^32*b^23*c^33*d^22
+ 115950654218240*a^33*b^22*c^32*d^23 - 317115551617024*a^34*b^21*c^31*d^2
4 + 436168221851648*a^35*b^20*c^30*d^25 - 450764657864704*a^36*b^19*c^29*d^
26 + 377325278126080*a^37*b^18*c^28*d^27 - 261008589107200*a^38*b^17*c^27*d
^28 + 149590069231616*a^39*b^16*c^26*d^29 - 70656513052672*a^40*b^15*c^25*d
^30 + 27207328280576*a^41*b^14*c^24*d^31 - 8398939463680*a^42*b^13*c^23*d^3
2 + 2027474309120*a^43*b^12*c^22*d^33 - 368463757312*a^44*b^11*c^21*d^34 +
47382401024*a^45*b^10*c^20*d^35 - 3841073152*a^46*b^9*c^19*d^36 + 147517440
*a^47*b^8*c^18*d^37)*1i - x^(1/2)*(276595200*a^18*b^35*c^42*d^11 - 65013043
20*a^19*b^34*c^41*d^12 + 71869242368*a^20*b^33*c^40*d^13 - 496940910592*a^2
1*b^32*c^39*d^14 + 2412258434048*a^22*b^31*c^38*d^15 - 8751989614592*a^23*b
^30*c^37*d^16 + 24696348863488*a^24*b^29*c^36*d^17 - 55777785276416*a^25*b^
28*c^35*d^18 + 103251559480832*a^26*b^27*c^34*d^19 - 160243801919488*a^27*b
^26*c^33*d^20 + 213523293304832*a^28*b^25*c^32*d^21 - 250272765841408*a^29*
b^24*c^31*d^22 + 263188357892096*a^30*b^23*c^30*d^23 - 250272765841408*a^31
*b^22*c^29*d^24 + 213523293304832*a^32*b^21*c^28*d^25 - 160243801919488*a^3
3*b^20*c^27*d^26 + 103251559480832*a^34*b^19*c^26*d^27 - 55777785276416*a^3
5*b^18*c^25*d^28 + 24696348863488*a^36*b^17*c^24*d^29 - 8751989614592*a^37*
b^16*c^23*d^30 + 2412258434048*a^38*b^15*c^22*d^31 - 496940910592*a^39*b^14
*c^21*d^32 + 71869242368*a^40*b^13*c^20*d^33 - 6501304320*a^41*b^12*c^19*d^
34 + 276595200*a^42*b^11*c^18*d^35))*1i))*(-(2401*a^4*d^15 + 50625*b^4*c^4*

```
d^11 - 94500*a*b^3*c^3*d^12 + 66150*a^2*b^2*c^2*d^13 - 20580*a^3*b*c*d^14)/
(4096*b^12*c^23 + 4096*a^12*c^11*d^12 - 49152*a^11*b*c^12*d^11 + 270336*a^2
*b^10*c^21*d^2 - 901120*a^3*b^9*c^20*d^3 + 2027520*a^4*b^8*c^19*d^4 - 32440
32*a^5*b^7*c^18*d^5 + 3784704*a^6*b^6*c^17*d^6 - 3244032*a^7*b^5*c^16*d^7 +
 2027520*a^8*b^4*c^15*d^8 - 901120*a^9*b^3*c^14*d^9 + 270336*a^10*b^2*c^13*
d^10 - 49152*a*b^11*c^22*d))^(1/4) - (2/(3*a*c) + (x^2*(7*a^3*d^3 + 7*b^3*c
^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2))/(6*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)) + (b*d*x^4*(7*a^2*d^2 + 7*b^2*c^2 - 8*a*b*c*d))/(6*a^2*c^2*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d)))/(x^(7/2)*(a*d + b*c) + a*c*x^(3/2) + b*d*x^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.477 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=731

$$\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}(bc-ad)^3}-b^{13/4}$$

Rubi [A] time = 1.28, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{9a^2d^2-8abcd+9b^2c^2}{10a^2c^2x^{5/2}(bc-ad)^2}+\frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{2a^3c^3\sqrt{x}\,(bc-ad)^2}+\frac{b^{13/4}(9bc-17ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(9bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{13/4}(bc-ad)^3}+\frac{b^{13/4}(9bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{13/4}(bc-ad)^3}+\frac{d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{8\sqrt{2}\,c^{13/4}(bc-ad)^3}-\frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^3}+\frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{4\sqrt{2}\,c^{13/4}(bc-ad)^3}+\frac{b}{2ax^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}+\frac{d(ad+bc)}{2acx^{5/2}\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -(9*b^2*c^2 - 8*a*b*c*d + 9*a^2*d^2)/(10*a^2*c^2*(b*c - a*d)^2*x^(5/2)) + (
(b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(2*a^3*c^3*(b*c - a*d)^2*
Sqrt[x]) + (d*(b*c + a*d))/(2*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)) + b/(2
*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)) - (b^(13/4)*(9*b*c - 17*a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c -
 a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*
d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) + (b^(13/4)*(9*b*c - 17*a*d)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)
*(b*c - a*d)^3) - (b^(13/4)*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (d^(13/4
)*(17*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*
x])/(8*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(b
*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int \frac{-9bc+4ad-13bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+\right.}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

$$= -\frac{9b^2c^2-8abcd+9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}} + \frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}} + \frac{d(b}{2ac(bc-a}$$

Mathematica [A] time = 6.18, size = 696, normalized size = 0.95

$$-\frac{b^{13/4}(17ad-9bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}+\frac{b^{13/4}(17ad-9bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(17ad-9bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(17ad-9bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3}+\frac{b^4x^{3/2}}{2a^3\left(a+bx^2\right)(ad-bc)^2}+\frac{4(ad+bc)}{a^3c^3\sqrt{x}}-\frac{2}{5a^2c^2x^{5/2}}-\frac{d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(ad-bc)^3}+\frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(ad-bc)^3}-\frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(ad-bc)^3}-\frac{d^{13/4}(17bc-9ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(ad-bc)^3}+\frac{d^4x^{3/2}}{2c^3\left(c+dx^2\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] -2/(5*a^2*c^2*x^(5/2)) + (4*(b*c + a*d))/(a^3*c^3*Sqrt[x]) + (b^4*x^(3/2))/
(2*a^3*(-(b*c) + a*d)^2*(a + b*x^2)) + (d^4*x^(3/2))/(2*c^3*(b*c - a*d)^2*(
c + d*x^2)) - (b^(13/4)*(-9*b*c + 17*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^
(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (b^
(13/4)*(-9*b*c + 17*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt
[2]*a^(1/4))])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a
*d)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*
Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*ArcTan[(Sqr
t[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(4*Sqrt[2]*c^(13/4)*(
-(b*c) + a*d)^3) - (b^(13/4)*(-9*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) + (b^(13
/4)*(-9*b*c + 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^3) - (d^(13/4)*(17*b*c - 9*a*d)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)
*(-(b*c) + a*d)^3) + (d^(13/4)*(17*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(8*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 1.41, size = 564, normalized size = 0.77

$$-\frac{\left(17ab^{13/4}d-9b^{17/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{13/4}(ad-bc)^3}-\frac{\left(17ab^{13/4}d-9b^{17/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{13/4}(ad-bc)^3}+\frac{-4a^4c^2d^2+36a^4cd^3x^2+45a^4d^4x^4+8a^3bc^3d-36a^3bc^2d^2x^2-4a^3bcd^3x^4+45a^3bd^4x^6-4a^2b^2c^4-36a^2b^2c^3dx^2-72a^2b^2c^2d^2x^4-40a^2b^2cd^3x^6+36ab^3c^4x^2-4ab^3c^3dx^4-40ab^3c^2d^2x^6+45b^4c^4x^4+45b^4c^3dx^6}{10a^3c^3x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)(ad-bc)^2}-\frac{\left(17bcd^{13/4}-9ad^{17/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{\left(17bcd^{13/4}-9ad^{17/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (-4*a^2*b^2*c^4 + 8*a^3*b*c^3*d - 4*a^4*c^2*d^2 + 36*a*b^3*c^4*x^2 - 36*a^2
*b^2*c^3*d*x^2 - 36*a^3*b*c^2*d^2*x^2 + 36*a^4*c*d^3*x^2 + 45*b^4*c^4*x^4 -
 4*a*b^3*c^3*d*x^4 - 72*a^2*b^2*c^2*d^2*x^4 - 4*a^3*b*c*d^3*x^4 + 45*a^4*d^
4*x^4 + 45*b^4*c^3*d*x^6 - 40*a*b^3*c^2*d^2*x^6 - 40*a^2*b^2*c*d^3*x^6 + 45
*a^3*b*d^4*x^6)/(10*a^3*c^3*(-(b*c) + a*d)^2*x^(5/2)*(a + b*x^2)*(c + d*x^2
)) - ((-9*b^(17/4)*c + 17*a*b^(13/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/(4*Sqrt[2]*a^(13/4)*(-(b*c) + a*d)^3) - ((17*
b*c*d^(13/4) - 9*a*d^(17/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])]])/(4*Sqrt[2]*c^(13/4)*(b*c - a*d)^3) - ((-9*b^(17/4)*c + 1
7*a*b^(13/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b
]*x)]])/(4*Sqrt[2]*a^(13/4)*(-(b*c) + a*d)^3) - ((17*b*c*d^(13/4) - 9*a*d^(1
7/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)]])/(4*
Sqrt[2]*c^(13/4)*(b*c - a*d)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.96, size = 1015, normalized size = 1.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2
)*a^5*b^2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/4*(9*(a*b^3)
^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2)*a^5*b^2*c^2
*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/4*(17*(c*d^3)^(3/4)*b*c*d
 - 9*(c*d^3)^(3/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b
*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/4*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^(
3/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sq
rt(2)*a^3*c^4*d^3) - 1/8*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^3*c^3 - 3*sq
rt(2)*a^5*b^2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/8*(9*(a*
b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4)
 + x + sqrt(a/b))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2)*a^5*b^2*c^2*d + 3*sqrt(2
)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) - 1/8*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^
(3/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*
c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3
) + 1/8*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^(3/4)*a*d^2)*log(-sqrt(2)*sqrt(
x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d +
3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/2*(b^4*c^3*d*x^(7/2) + a
^3*b*d^4*x^(7/2) + b^4*c^4*x^(3/2) + a^4*d^4*x^(3/2))/((a^3*b^2*c^5 - 2*a^4
*b*c^4*d + a^5*c^3*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)) + 2/5*(10*b*c*
x^2 + 10*a*d*x^2 - a*c)/(a^3*c^3*x^(5/2))
```

maple [A] time = 0.03, size = 849, normalized size = 1.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*b^4/a^2/(a*d-b*c)^3*x^(3/2)/(b*x^2+a)*d-1/2*b^5/a^3/(a*d-b*c)^3*x^(3/2)
/(b*x^2+a)*c+17/16*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*ln((x-(a/b)^(1
/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)
))+17/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+17/8*b^3/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)-9/16*b^4/a^3/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*ln((x
-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a
/b)^(1/2)))-9/8*b^4/a^3/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)+1)-9/8*b^4/a^3/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*c*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+1/2*d^5/c^3/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*a-1
/2*d^4/c^2/(a*d-b*c)^3*x^(3/2)/(d*x^2+c)*b+9/16*d^4/c^3/(a*d-b*c)^3/(c/d)^(
1/4)*2^(1/2)*a*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4
)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+9/8*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/4)*2^(1/2)
*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+9/8*d^4/c^3/(a*d-b*c)^3/(c/d)^(1/4
)*2^(1/2)*a*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-17/16*d^3/c^2/(a*d-b*c)^3
/(c/d)^(1/4)*2^(1/2)*b*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-17/8*d^3/c^2/(a*d-b*c)^3/(c/d)^(1/4
)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-17/8*d^3/c^2/(a*d-b*c)^3/
(c/d)^(1/4)*2^(1/2)*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a^2/c^2/x^(
5/2)+4/a^2/c^3/x^(1/2)*d+4/a^3/c^2/x^(1/2)*b
```

maxima [A] time = 2.75, size = 774, normalized size = 1.06

$$\frac{\left(9 b^{5} c - 17 a b^{4} d\right)\left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} + 2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} - 2 \sqrt{b} \sqrt{x}\right)}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}} - \frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x} + \sqrt{b} x + \sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{16 \left(a^{3} b^{3} c^{3} - 3 a^{4} b^{2} c^{2} d + 3 a^{5} b c d^{2} - a^{6} d^{3}\right)} + \frac{\left(17 b c d^{4} - 9 a d^{5}\right)\left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} + 2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} + \frac{2 \sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} - 2 \sqrt{d} \sqrt{x}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}} - \frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}} + \frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x} + \sqrt{d} x + \sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{16 \left(b^{3} c^{6} - 3 a b^{2} c^{5} d + 3 a^{2} b c^{4} d^{2} - a^{3} c^{3} d^{3}\right)} - \frac{4 a^{2} b^{2} c^{4} - 8 a^{3} b c^{3} d + 4 a^{4} c^{2} d^{2} - 5 \left(9 b^{4} c^{3} d - 8 a b^{3} c^{2} d^{2} - 8 a^{2} b^{2} c d^{3} + 9 a^{3} b d^{4}\right) x^{6} - \left(45 b^{4} c^{4} - 4 a b^{3} c^{3} d - 72 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + 45 a^{4} d^{4}\right) x^{4} - 36 \left(a b^{3} c^{4} - a^{2} b^{2} c^{3} d - a^{3} b c^{2} d^{2} + a^{4} c d^{3}\right) x^{2}}{10 \left(\left(a^{3} b^{3} c^{5} d - 2 a^{4} b^{2} c^{4} d^{2} + a^{5} b c^{3} d^{3}\right) x^{\frac{13}{2}} + \left(a^{3} b^{3} c^{6} - a^{4} b^{2} c^{5} d - a^{5} b c^{4} d^{2} + a^{6} c^{3} d^{3}\right) x^{\frac{9}{2}} + \left(a^{4} b^{2} c^{6} - 2 a^{5} b c^{5} d + a^{6} c^{4} d^{2}\right) x^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(9*b^5*c - 17*a*b^4*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*
sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2
)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(1/4)*b^(3/4)))/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)
 + 1/16*(17*b*c*d^4 - 9*a*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/
4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d
))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sq
rt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqr
t(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^
(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(1/4)*d^(3/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d
```

```
^3) - 1/10*(4*a^2*b^2*c^4 - 8*a^3*b*c^3*d + 4*a^4*c^2*d^2 - 5*(9*b^4*c^3*d
- 8*a*b^3*c^2*d^2 - 8*a^2*b^2*c*d^3 + 9*a^3*b*d^4)*x^6 - (45*b^4*c^4 - 4*a*
b^3*c^3*d - 72*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 45*a^4*d^4)*x^4 - 36*(a*b^
3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)/((a^3*b^3*c^5*d - 2
*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^(13/2) + (a^3*b^3*c^6 - a^4*b^2*c^5*d -
 a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^(9/2) + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*
c^4*d^2)*x^(5/2))
```

mupad [B]   time = 14.27, size = 36571, normalized size = 50.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] 2*atan((2654208*a^16*b^22*c^27*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^
4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(
4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15
*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 32440
32*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 +
 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d
^10 - 49152*a^24*b*c*d^11))^(5/4) + 15169032*a^22*b^8*d^19*x^(1/2)*(-(6561*
b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2
*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^
14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027
520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6
- 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3
*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 2654208*a^3
8*c^5*d^22*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*
c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 409
6*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901
120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5
+ 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^
4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c
*d^11))^(5/4) - 130671792*a^21*b^9*c*d^18*x^(1/2)*(-(6561*b^17*c^4 + 83521*
a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^
16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d +
270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*
d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^
5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^2
3*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 41877504*a^17*b^21*c^26*d*x^
(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 14045
4*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c
^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*
c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^1
```

```
9*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 90112
0*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4)
 - 41877504*a^37*b*c^6*d^21*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 -
 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(409
6*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^
10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*
a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 20
27520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10
 - 49152*a^24*b*c*d^11))^(5/4) + 15169032*a^11*b^19*c^11*d^8*x^(1/2)*(-(656
1*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c
^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*
a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 20
27520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^
6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c
^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 130671792
*a^12*b^18*c^10*d^9*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*
a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d
^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*
d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7
*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^
21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152
*a^24*b*c*d^11))^(1/4) + 450333432*a^13*b^17*c^9*d^10*x^(1/2)*(-(6561*b^17*
c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2
- 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^
11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a
^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 324
4032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9
+ 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 784872864*a^14*b
^16*c^8*d^11*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^1
4*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4
096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 9
01120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^
5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*
c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b
*c*d^11))^(1/4) + 717087608*a^15*b^15*c^7*d^12*x^(1/2)*(-(6561*b^17*c^4 + 8
3521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572
*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11
*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8
*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^
20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 27033
6*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - 264948264*a^16*b^14*c^6
*d^13*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3
 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^1
3*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a
^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 378
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4704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8
 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11
))^(1/4) - 264948264*a^17*b^13*c^5*d^14*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^
4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16
*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 27
0336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^
4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*
c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*
b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 717087608*a^18*b^12*c^4*d^15*x
^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 1404
54*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*
c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9
*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^
19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 9011
20*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4
) - 784872864*a^19*b^11*c^3*d^16*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*
d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)
/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^
15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 324
4032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7
 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2
*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 450333432*a^20*b^10*c^2*d^17*x^(1/2)*
(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*
b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 -
49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^
3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*
c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22
*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) + 304
971776*a^18*b^20*c^25*d^2*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 1
76868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*
a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10
*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^
18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027
520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 -
 49152*a^24*b*c*d^11))^(5/4) - 1359347712*a^19*b^19*c^24*d^3*x^(1/2)*(-(656
1*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c
^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*
a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 20
27520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^
6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c
^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 414479155
2*a^20*b^18*c^23*d^4*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868
*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*
d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10
*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^
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7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a
^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 4915
2*a^24*b*c*d^11))^(5/4) - 9148891136*a^21*b^17*c^22*d^5*x^(1/2)*(-(6561*b^1
7*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^
2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*
b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520
*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3
244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^
9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 15081504768*a^
22*b^16*c^21*d^6*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3
*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12
 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2
 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^
7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*
b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^
24*b*c*d^11))^(5/4) - 18867290112*a^23*b^15*c^20*d^7*x^(1/2)*(-(6561*b^17*c
^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 -
 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^1
1*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^
17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244
032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 +
 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 18014928896*a^24*
b^14*c^19*d^8*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^
14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 +
4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 -
901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d
^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4
*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*
b*c*d^11))^(5/4) - 13171163136*a^25*b^13*c^18*d^9*x^(1/2)*(-(6561*b^17*c^4
+ 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49
572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c
^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*
b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032
*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 27
0336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 7816740864*a^26*b^12
*c^17*d^10*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*
c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 409
6*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901
120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5
+ 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^
4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c
*d^11))^(5/4) - 5554044928*a^27*b^11*c^16*d^11*x^(1/2)*(-(6561*b^17*c^4 + 8
3521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572
*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11
*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8
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*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^
20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 27033
6*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 7816740864*a^28*b^10*c^
15*d^12*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d
^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a
^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120
*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3
784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d
^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^
11))^(5/4) - 13171163136*a^29*b^9*c^14*d^13*x^(1/2)*(-(6561*b^17*c^4 + 8352
1*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*
b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d
 + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^
8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*
b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a
^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 18014928896*a^30*b^8*c^13*
d^14*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3
 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13
*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^
16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784
704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8
 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11)
)^(5/4) - 18867290112*a^31*b^7*c^12*d^15*x^(1/2)*(-(6561*b^17*c^4 + 83521*a
^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^1
6*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 2
70336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d
^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5
*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23
*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 15081504768*a^32*b^6*c^11*d^1
6*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 1
40454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^
12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*
b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704
*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 9
01120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(
5/4) - 9148891136*a^33*b^5*c^10*d^17*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b
^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^
3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 27033
6*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 -
 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5
*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2
*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 4144791552*a^34*b^4*c^9*d^18*x^(1
/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*
a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^1
2 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^
```

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9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*
b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*
a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) -
 1359347712*a^35*b^3*c^8*d^19*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4
 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4
096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*
b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 324403
2*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 +
2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^
10 - 49152*a^24*b*c*d^11))^(5/4) + 304971776*a^36*b^2*c^7*d^20*x^(1/2)*(-(6
561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15
*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 4915
2*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 +
2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*
d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3
*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4))/(3223419
3*a^17*b^11*d^17 - 4782969*b^28*c^17 - 198040140*a^16*b^12*c*d^16 - 1973417
58*a^2*b^26*c^15*d^2 + 384710796*a^3*b^25*c^14*d^3 - 335988081*a^4*b^24*c^1
3*d^4 + 55738368*a^5*b^23*c^12*d^5 + 39223296*a^6*b^22*c^11*d^6 + 24805376*
a^7*b^21*c^10*d^7 + 12484608*a^8*b^20*c^9*d^8 + 2260992*a^9*b^19*c^8*d^9 -
5865472*a^10*b^18*c^7*d^10 - 11894784*a^11*b^17*c^6*d^11 - 15826944*a^12*b^
16*c^5*d^12 + 43224857*a^13*b^15*c^4*d^13 - 308701404*a^14*b^14*c^3*d^14 +
413141310*a^15*b^13*c^2*d^15 + 48892572*a*b^27*c^16*d))*(-(6561*b^17*c^4 +
83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 4957
2*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^1
1*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^
8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a
^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 2703
36*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4) - (2/(5*a*c) - (18*x^2*(
a*d + b*c))/(5*a^2*c^2) + (x^4*(72*a^2*b^2*c^2*d^2 - 45*b^4*c^4 - 45*a^4*d^
4 + 4*a*b^3*c^3*d + 4*a^3*b*c*d^3))/(10*a^3*c^3*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)) - (b*d*x^6*(a*d + b*c)*(9*a^2*d^2 + 9*b^2*c^2 - 17*a*b*c*d))/(2*a^3*c
^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^(9/2)*(a*d + b*c) + a*c*x^(5/2) + b
*d*x^(13/2)) + atan((((-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^
14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 +
4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 -
901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d
^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4
*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*
b*c*d^11))^(3/4)*(x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^
3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^1
2 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^
2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c
^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21
*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a
```

^24*b*c*d^11))^(1/4)*(169869312*a^34*b^40*c^70*d^4 - 5058330624*a^35*b^39*c
^69*d^5 + 72498544640*a^36*b^38*c^68*d^6 - 665979977728*a^37*b^37*c^67*d^7
+ 4405015347200*a^38*b^36*c^66*d^8 - 22343644610560*a^39*b^35*c^65*d^9 + 90
382763294720*a^40*b^34*c^64*d^10 - 299352929075200*a^41*b^33*c^63*d^11 + 82
7049291808768*a^42*b^32*c^62*d^12 - 1932015372861440*a^43*b^31*c^61*d^13 +
3854486254649344*a^44*b^30*c^60*d^14 - 6616635725053952*a^45*b^29*c^59*d^15
 + 9828748597657600*a^46*b^28*c^58*d^16 - 12696914776555520*a^47*b^27*c^57*
d^17 + 14352507102822400*a^48*b^26*c^56*d^18 - 14371219193200640*a^49*b^25*
c^55*d^19 + 13121803937382400*a^50*b^24*c^54*d^20 - 11630118384435200*a^51*
b^23*c^53*d^21 + 10979630865448960*a^52*b^22*c^52*d^22 - 11630118384435200*
a^53*b^21*c^51*d^23 + 13121803937382400*a^54*b^20*c^50*d^24 - 143712191932
00640*a^55*b^19*c^49*d^25 + 14352507102822400*a^56*b^18*c^48*d^26 - 12696914
776555520*a^57*b^17*c^47*d^27 + 9828748597657600*a^58*b^16*c^46*d^28 - 6616
635725053952*a^59*b^15*c^45*d^29 + 3854486254649344*a^60*b^14*c^44*d^30 - 1
932015372861440*a^61*b^13*c^43*d^31 + 827049291808768*a^62*b^12*c^42*d^32 -
 299352929075200*a^63*b^11*c^41*d^33 + 90382763294720*a^64*b^10*c^40*d^34 -
 22343644610560*a^65*b^9*c^39*d^35 + 4405015347200*a^66*b^8*c^38*d^36 - 665
979977728*a^67*b^7*c^37*d^37 + 72498544640*a^68*b^6*c^36*d^38 - 5058330624*
a^69*b^5*c^35*d^39 + 169869312*a^70*b^4*c^34*d^40) + 191102976*a^31*b^41*c^
68*d^4 - 5478285312*a^32*b^40*c^67*d^5 + 75301650432*a^33*b^39*c^66*d^6 - 6
60755972096*a^34*b^38*c^65*d^7 + 4157198565376*a^35*b^37*c^64*d^8 - 1996809
2536832*a^36*b^36*c^63*d^9 + 76124224225280*a^37*b^35*c^62*d^10 - 236401268
359168*a^38*b^34*c^61*d^11 + 609010175442944*a^39*b^33*c^60*d^12 - 13186187
46322944*a^40*b^32*c^59*d^13 + 2422266262192128*a^41*b^31*c^58*d^14 - 38003
65228883968*a^42*b^30*c^57*d^15 + 5115210562535424*a^43*b^29*c^56*d^16 - 59
21595099709440*a^44*b^28*c^55*d^17 + 5899320342609920*a^45*b^27*c^54*d^18 -
 5044901346017280*a^46*b^26*c^53*d^19 + 3659431378944000*a^47*b^25*c^52*d^2
0 - 2131419914567680*a^48*b^24*c^51*d^21 + 688340293386240*a^49*b^23*c^50*d
^22 + 688340293386240*a^50*b^22*c^49*d^23 - 2131419914567680*a^51*b^21*c^48
*d^24 + 3659431378944000*a^52*b^20*c^47*d^25 - 5044901346017280*a^53*b^19*c
^46*d^26 + 5899320342609920*a^54*b^18*c^45*d^27 - 5921595099709440*a^55*b^1
7*c^44*d^28 + 5115210562535424*a^56*b^16*c^43*d^29 - 3800365228883968*a^57*
b^15*c^42*d^30 + 2422266262192128*a^58*b^14*c^41*d^31 - 1318618746322944*a^
59*b^13*c^40*d^32 + 609010175442944*a^60*b^12*c^39*d^33 - 236401268359168*a
^61*b^11*c^38*d^34 + 76124224225280*a^62*b^10*c^37*d^35 - 19968092536832*a^
63*b^9*c^36*d^36 + 4157198565376*a^64*b^8*c^35*d^37 - 660755972096*a^65*b^7
*c^34*d^38 + 75301650432*a^66*b^6*c^33*d^39 - 5478285312*a^67*b^5*c^32*d^40
 + 191102976*a^68*b^4*c^31*d^41) + x^(1/2)*(970818048*a^29*b^37*c^54*d^12 -
 21954447360*a^30*b^36*c^53*d^13 + 234247707648*a^31*b^35*c^52*d^14 - 15681
40904448*a^32*b^34*c^51*d^15 + 7387800533504*a^33*b^33*c^50*d^16 - 26036469
792256*a^34*b^32*c^49*d^17 + 71189396375552*a^35*b^31*c^48*d^18 - 154393382
077440*a^36*b^30*c^47*d^19 + 268607771876352*a^37*b^29*c^46*d^20 - 37459080
0139776*a^38*b^28*c^45*d^21 + 409764654942208*a^39*b^27*c^44*d^22 - 3247871
54001920*a^40*b^26*c^43*d^23 + 124213059109888*a^41*b^25*c^42*d^24 + 124213
059109888*a^42*b^24*c^41*d^25 - 324787154001920*a^43*b^23*c^40*d^26 + 40976

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4654942208*a^44*b^22*c^39*d^27 - 374590800139776*a^45*b^21*c^38*d^28 + 2686
07771876352*a^46*b^20*c^37*d^29 - 154393382077440*a^47*b^19*c^36*d^30 + 711
89396375552*a^48*b^18*c^35*d^31 - 26036469792256*a^49*b^17*c^34*d^32 + 7387
800533504*a^50*b^16*c^33*d^33 - 1568140904448*a^51*b^15*c^32*d^34 + 2342477
07648*a^52*b^14*c^31*d^35 - 21954447360*a^53*b^13*c^30*d^36 + 970818048*a^5
4*b^12*c^29*d^37))*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*
c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 409
6*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901
120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5
+ 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^
4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c
*d^11))^(1/4)*1i - ((-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14
*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 40
96*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 90
1120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5
 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c
^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*
c*d^11))^(3/4)*(191102976*a^31*b^41*c^68*d^4 - x^(1/2)*(-(6561*b^17*c^4 + 8
3521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572
*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11
*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8
*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^
20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 27033
6*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4)*(169869312*a^34*b^40*c^70
*d^4 - 5058330624*a^35*b^39*c^69*d^5 + 72498544640*a^36*b^38*c^68*d^6 - 665
979977728*a^37*b^37*c^67*d^7 + 4405015347200*a^38*b^36*c^66*d^8 - 223436446
10560*a^39*b^35*c^65*d^9 + 90382763294720*a^40*b^34*c^64*d^10 - 29935292907
5200*a^41*b^33*c^63*d^11 + 827049291808768*a^42*b^32*c^62*d^12 - 1932015372
861440*a^43*b^31*c^61*d^13 + 3854486254649344*a^44*b^30*c^60*d^14 - 6616635
725053952*a^45*b^29*c^59*d^15 + 9828748597657600*a^46*b^28*c^58*d^16 - 1269
6914776555520*a^47*b^27*c^57*d^17 + 14352507102822400*a^48*b^26*c^56*d^18 -
 14371219193200640*a^49*b^25*c^55*d^19 + 13121803937382400*a^50*b^24*c^54*d
^20 - 11630118384435200*a^51*b^23*c^53*d^21 + 10979630865448960*a^52*b^22*c
^52*d^22 - 11630118384435200*a^53*b^21*c^51*d^23 + 13121803937382400*a^54*b
^20*c^50*d^24 - 14371219193200640*a^55*b^19*c^49*d^25 + 14352507102822400*a
^56*b^18*c^48*d^26 - 12696914776555520*a^57*b^17*c^47*d^27 + 98287485976576
00*a^58*b^16*c^46*d^28 - 6616635725053952*a^59*b^15*c^45*d^29 + 38544862546
49344*a^60*b^14*c^44*d^30 - 1932015372861440*a^61*b^13*c^43*d^31 + 82704929
1808768*a^62*b^12*c^42*d^32 - 299352929075200*a^63*b^11*c^41*d^33 + 9038276
3294720*a^64*b^10*c^40*d^34 - 22343644610560*a^65*b^9*c^39*d^35 + 440501534
7200*a^66*b^8*c^38*d^36 - 665979977728*a^67*b^7*c^37*d^37 + 72498544640*a^6
8*b^6*c^36*d^38 - 5058330624*a^69*b^5*c^35*d^39 + 169869312*a^70*b^4*c^34*d
^40) - 5478285312*a^32*b^40*c^67*d^5 + 75301650432*a^33*b^39*c^66*d^6 - 660
755972096*a^34*b^38*c^65*d^7 + 4157198565376*a^35*b^37*c^64*d^8 - 199680925
36832*a^36*b^36*c^63*d^9 + 76124224225280*a^37*b^35*c^62*d^10 - 23640126835
```

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9168*a^38*b^34*c^61*d^11 + 609010175442944*a^39*b^33*c^60*d^12 - 1318618746
322944*a^40*b^32*c^59*d^13 + 2422266262192128*a^41*b^31*c^58*d^14 - 3800365
228883968*a^42*b^30*c^57*d^15 + 5115210562535424*a^43*b^29*c^56*d^16 - 5921
595099709440*a^44*b^28*c^55*d^17 + 5899320342609920*a^45*b^27*c^54*d^18 - 5
044901346017280*a^46*b^26*c^53*d^19 + 3659431378944000*a^47*b^25*c^52*d^20
- 2131419914567680*a^48*b^24*c^51*d^21 + 688340293386240*a^49*b^23*c^50*d^2
2 + 688340293386240*a^50*b^22*c^49*d^23 - 2131419914567680*a^51*b^21*c^48*d
^24 + 3659431378944000*a^52*b^20*c^47*d^25 - 5044901346017280*a^53*b^19*c^4
6*d^26 + 5899320342609920*a^54*b^18*c^45*d^27 - 5921595099709440*a^55*b^17*
c^44*d^28 + 5115210562535424*a^56*b^16*c^43*d^29 - 3800365228883968*a^57*b^
15*c^42*d^30 + 2422266262192128*a^58*b^14*c^41*d^31 - 1318618746322944*a^59
*b^13*c^40*d^32 + 609010175442944*a^60*b^12*c^39*d^33 - 236401268359168*a^6
1*b^11*c^38*d^34 + 76124224225280*a^62*b^10*c^37*d^35 - 19968092536832*a^63
*b^9*c^36*d^36 + 4157198565376*a^64*b^8*c^35*d^37 - 660755972096*a^65*b^7*c
^34*d^38 + 75301650432*a^66*b^6*c^33*d^39 - 5478285312*a^67*b^5*c^32*d^40 +
 191102976*a^68*b^4*c^31*d^41) - x^(1/2)*(970818048*a^29*b^37*c^54*d^12 - 2
1954447360*a^30*b^36*c^53*d^13 + 234247707648*a^31*b^35*c^52*d^14 - 1568140
904448*a^32*b^34*c^51*d^15 + 7387800533504*a^33*b^33*c^50*d^16 - 2603646979
2256*a^34*b^32*c^49*d^17 + 71189396375552*a^35*b^31*c^48*d^18 - 15439338207
7440*a^36*b^30*c^47*d^19 + 268607771876352*a^37*b^29*c^46*d^20 - 3745908001
39776*a^38*b^28*c^45*d^21 + 409764654942208*a^39*b^27*c^44*d^22 - 324787154
001920*a^40*b^26*c^43*d^23 + 124213059109888*a^41*b^25*c^42*d^24 + 12421305
9109888*a^42*b^24*c^41*d^25 - 324787154001920*a^43*b^23*c^40*d^26 + 4097646
54942208*a^44*b^22*c^39*d^27 - 374590800139776*a^45*b^21*c^38*d^28 + 268607
771876352*a^46*b^20*c^37*d^29 - 154393382077440*a^47*b^19*c^36*d^30 + 71189
396375552*a^48*b^18*c^35*d^31 - 26036469792256*a^49*b^17*c^34*d^32 + 738780
0533504*a^50*b^16*c^33*d^33 - 1568140904448*a^51*b^15*c^32*d^34 + 234247707
648*a^52*b^14*c^31*d^35 - 21954447360*a^53*b^13*c^30*d^36 + 970818048*a^54*
b^12*c^29*d^37))*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*
d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*
a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 90112
0*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 +
3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*
d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d
^11))^(1/4)*1i)/(((-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c
*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096
*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 9011
20*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 +
 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4
*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*
d^11))^(3/4)*(x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^
14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 +
4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 -
901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d
^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4
```

*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*
b*c*d^11))^(1/4)*(169869312*a^34*b^40*c^70*d^4 - 5058330624*a^35*b^39*c^69*
d^5 + 72498544640*a^36*b^38*c^68*d^6 - 665979977728*a^37*b^37*c^67*d^7 + 44
05015347200*a^38*b^36*c^66*d^8 - 22343644610560*a^39*b^35*c^65*d^9 + 903827
63294720*a^40*b^34*c^64*d^10 - 299352929075200*a^41*b^33*c^63*d^11 + 827049
291808768*a^42*b^32*c^62*d^12 - 1932015372861440*a^43*b^31*c^61*d^13 + 3854
486254649344*a^44*b^30*c^60*d^14 - 6616635725053952*a^45*b^29*c^59*d^15 + 9
828748597657600*a^46*b^28*c^58*d^16 - 12696914776555520*a^47*b^27*c^57*d^17
+ 14352507102822400*a^48*b^26*c^56*d^18 - 14371219193200640*a^49*b^25*c^55
*d^19 + 13121803937382400*a^50*b^24*c^54*d^20 - 11630118384435200*a^51*b^23
*c^53*d^21 + 10979630865448960*a^52*b^22*c^52*d^22 - 11630118384435200*a^53
*b^21*c^51*d^23 + 13121803937382400*a^54*b^20*c^50*d^24 - 14371219193200640
*a^55*b^19*c^49*d^25 + 14352507102822400*a^56*b^18*c^48*d^26 - 126969147765
55520*a^57*b^17*c^47*d^27 + 9828748597657600*a^58*b^16*c^46*d^28 - 66166357
25053952*a^59*b^15*c^45*d^29 + 3854486254649344*a^60*b^14*c^44*d^30 - 19320
15372861440*a^61*b^13*c^43*d^31 + 827049291808768*a^62*b^12*c^42*d^32 - 299
352929075200*a^63*b^11*c^41*d^33 + 90382763294720*a^64*b^10*c^40*d^34 - 223
43644610560*a^65*b^9*c^39*d^35 + 4405015347200*a^66*b^8*c^38*d^36 - 6659799
77728*a^67*b^7*c^37*d^37 + 72498544640*a^68*b^6*c^36*d^38 - 5058330624*a^69
*b^5*c^35*d^39 + 169869312*a^70*b^4*c^34*d^40) + 191102976*a^31*b^41*c^68*d
^4 - 5478285312*a^32*b^40*c^67*d^5 + 75301650432*a^33*b^39*c^66*d^6 - 66075
5972096*a^34*b^38*c^65*d^7 + 4157198565376*a^35*b^37*c^64*d^8 - 19968092536
832*a^36*b^36*c^63*d^9 + 76124224225280*a^37*b^35*c^62*d^10 - 2364012683591
68*a^38*b^34*c^61*d^11 + 609010175442944*a^39*b^33*c^60*d^12 - 131861874632
2944*a^40*b^32*c^59*d^13 + 2422266262192128*a^41*b^31*c^58*d^14 - 380036522
8883968*a^42*b^30*c^57*d^15 + 5115210562535424*a^43*b^29*c^56*d^16 - 592159
5099709440*a^44*b^28*c^55*d^17 + 5899320342609920*a^45*b^27*c^54*d^18 - 504
4901346017280*a^46*b^26*c^53*d^19 + 3659431378944000*a^47*b^25*c^52*d^20 -
2131419914567680*a^48*b^24*c^51*d^21 + 688340293386240*a^49*b^23*c^50*d^22
+ 688340293386240*a^50*b^22*c^49*d^23 - 2131419914567680*a^51*b^21*c^48*d^2
4 + 3659431378944000*a^52*b^20*c^47*d^25 - 5044901346017280*a^53*b^19*c^46*
d^26 + 5899320342609920*a^54*b^18*c^45*d^27 - 5921595099709440*a^55*b^17*c^
44*d^28 + 5115210562535424*a^56*b^16*c^43*d^29 - 3800365228883968*a^57*b^15
*c^42*d^30 + 2422266262192128*a^58*b^14*c^41*d^31 - 1318618746322944*a^59*b
^13*c^40*d^32 + 609010175442944*a^60*b^12*c^39*d^33 - 236401268359168*a^61*
b^11*c^38*d^34 + 76124224225280*a^62*b^10*c^37*d^35 - 19968092536832*a^63*b
^9*c^36*d^36 + 4157198565376*a^64*b^8*c^35*d^37 - 660755972096*a^65*b^7*c^3
4*d^38 + 75301650432*a^66*b^6*c^33*d^39 - 5478285312*a^67*b^5*c^32*d^40 + 1
91102976*a^68*b^4*c^31*d^41) + x^(1/2)*(970818048*a^29*b^37*c^54*d^12 - 219
54447360*a^30*b^36*c^53*d^13 + 234247707648*a^31*b^35*c^52*d^14 - 156814090
4448*a^32*b^34*c^51*d^15 + 7387800533504*a^33*b^33*c^50*d^16 - 260364697922
56*a^34*b^32*c^49*d^17 + 71189396375552*a^35*b^31*c^48*d^18 - 1543933820774
40*a^36*b^30*c^47*d^19 + 268607771876352*a^37*b^29*c^46*d^20 - 374590800139
776*a^38*b^28*c^45*d^21 + 409764654942208*a^39*b^27*c^44*d^22 - 32478715400
1920*a^40*b^26*c^43*d^23 + 124213059109888*a^41*b^25*c^42*d^24 + 1242130591

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09888*a^42*b^24*c^41*d^25 - 324787154001920*a^43*b^23*c^40*d^26 + 409764654
942208*a^44*b^22*c^39*d^27 - 374590800139776*a^45*b^21*c^38*d^28 + 26860777
1876352*a^46*b^20*c^37*d^29 - 154393382077440*a^47*b^19*c^36*d^30 + 7118939
6375552*a^48*b^18*c^35*d^31 - 26036469792256*a^49*b^17*c^34*d^32 + 73878005
33504*a^50*b^16*c^33*d^33 - 1568140904448*a^51*b^15*c^32*d^34 + 23424770764
8*a^52*b^14*c^31*d^35 - 21954447360*a^53*b^13*c^30*d^36 + 970818048*a^54*b^
12*c^29*d^37))*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^
3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^
13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*
a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 37
84704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^
8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^1
1))^(1/4) + ((-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3
+ 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13
*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^
16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784
704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8
- 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11)
)^(3/4)*(191102976*a^31*b^41*c^68*d^4 - x^(1/2)*(-(6561*b^17*c^4 + 83521*a^
4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16
*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 27
0336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^
4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*
c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*
b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4)*(169869312*a^34*b^40*c^70*d^4 -
5058330624*a^35*b^39*c^69*d^5 + 72498544640*a^36*b^38*c^68*d^6 - 6659799777
28*a^37*b^37*c^67*d^7 + 4405015347200*a^38*b^36*c^66*d^8 - 22343644610560*a
^39*b^35*c^65*d^9 + 90382763294720*a^40*b^34*c^64*d^10 - 299352929075200*a^
41*b^33*c^63*d^11 + 827049291808768*a^42*b^32*c^62*d^12 - 1932015372861440*
a^43*b^31*c^61*d^13 + 3854486254649344*a^44*b^30*c^60*d^14 - 66166357250539
52*a^45*b^29*c^59*d^15 + 9828748597657600*a^46*b^28*c^58*d^16 - 12696914776
555520*a^47*b^27*c^57*d^17 + 14352507102822400*a^48*b^26*c^56*d^18 - 143712
19193200640*a^49*b^25*c^55*d^19 + 13121803937382400*a^50*b^24*c^54*d^20 - 1
1630118384435200*a^51*b^23*c^53*d^21 + 10979630865448960*a^52*b^22*c^52*d^2
2 - 11630118384435200*a^53*b^21*c^51*d^23 + 13121803937382400*a^54*b^20*c^5
0*d^24 - 14371219193200640*a^55*b^19*c^49*d^25 + 14352507102822400*a^56*b^1
8*c^48*d^26 - 12696914776555520*a^57*b^17*c^47*d^27 + 9828748597657600*a^58
*b^16*c^46*d^28 - 6616635725053952*a^59*b^15*c^45*d^29 + 3854486254649344*a
^60*b^14*c^44*d^30 - 1932015372861440*a^61*b^13*c^43*d^31 + 827049291808768
*a^62*b^12*c^42*d^32 - 299352929075200*a^63*b^11*c^41*d^33 + 90382763294720
*a^64*b^10*c^40*d^34 - 22343644610560*a^65*b^9*c^39*d^35 + 4405015347200*a^
66*b^8*c^38*d^36 - 665979977728*a^67*b^7*c^37*d^37 + 72498544640*a^68*b^6*c
^36*d^38 - 5058330624*a^69*b^5*c^35*d^39 + 169869312*a^70*b^4*c^34*d^40) -
5478285312*a^32*b^40*c^67*d^5 + 75301650432*a^33*b^39*c^66*d^6 - 6607559720
96*a^34*b^38*c^65*d^7 + 4157198565376*a^35*b^37*c^64*d^8 - 19968092536832*a
```

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^36*b^36*c^63*d^9 + 76124224225280*a^37*b^35*c^62*d^10 - 236401268359168*a^
38*b^34*c^61*d^11 + 609010175442944*a^39*b^33*c^60*d^12 - 1318618746322944*
a^40*b^32*c^59*d^13 + 2422266262192128*a^41*b^31*c^58*d^14 - 38003652288839
68*a^42*b^30*c^57*d^15 + 5115210562535424*a^43*b^29*c^56*d^16 - 59215950997
09440*a^44*b^28*c^55*d^17 + 5899320342609920*a^45*b^27*c^54*d^18 - 50449013
46017280*a^46*b^26*c^53*d^19 + 3659431378944000*a^47*b^25*c^52*d^20 - 21314
19914567680*a^48*b^24*c^51*d^21 + 688340293386240*a^49*b^23*c^50*d^22 + 688
340293386240*a^50*b^22*c^49*d^23 - 2131419914567680*a^51*b^21*c^48*d^24 + 3
659431378944000*a^52*b^20*c^47*d^25 - 5044901346017280*a^53*b^19*c^46*d^26
+ 5899320342609920*a^54*b^18*c^45*d^27 - 5921595099709440*a^55*b^17*c^44*d^
28 + 5115210562535424*a^56*b^16*c^43*d^29 - 3800365228883968*a^57*b^15*c^42
*d^30 + 2422266262192128*a^58*b^14*c^41*d^31 - 1318618746322944*a^59*b^13*c
^40*d^32 + 609010175442944*a^60*b^12*c^39*d^33 - 236401268359168*a^61*b^11*
c^38*d^34 + 76124224225280*a^62*b^10*c^37*d^35 - 19968092536832*a^63*b^9*c^
36*d^36 + 4157198565376*a^64*b^8*c^35*d^37 - 660755972096*a^65*b^7*c^34*d^3
8 + 75301650432*a^66*b^6*c^33*d^39 - 5478285312*a^67*b^5*c^32*d^40 + 191102
976*a^68*b^4*c^31*d^41) - x^(1/2)*(970818048*a^29*b^37*c^54*d^12 - 21954447
360*a^30*b^36*c^53*d^13 + 234247707648*a^31*b^35*c^52*d^14 - 1568140904448*
a^32*b^34*c^51*d^15 + 7387800533504*a^33*b^33*c^50*d^16 - 26036469792256*a^
34*b^32*c^49*d^17 + 71189396375552*a^35*b^31*c^48*d^18 - 154393382077440*a^
36*b^30*c^47*d^19 + 268607771876352*a^37*b^29*c^46*d^20 - 374590800139776*a
^38*b^28*c^45*d^21 + 409764654942208*a^39*b^27*c^44*d^22 - 324787154001920*
a^40*b^26*c^43*d^23 + 124213059109888*a^41*b^25*c^42*d^24 + 124213059109888
*a^42*b^24*c^41*d^25 - 324787154001920*a^43*b^23*c^40*d^26 + 40976465494220
8*a^44*b^22*c^39*d^27 - 374590800139776*a^45*b^21*c^38*d^28 + 2686077718763
52*a^46*b^20*c^37*d^29 - 154393382077440*a^47*b^19*c^36*d^30 + 711893963755
52*a^48*b^18*c^35*d^31 - 26036469792256*a^49*b^17*c^34*d^32 + 7387800533504
*a^50*b^16*c^33*d^33 - 1568140904448*a^51*b^15*c^32*d^34 + 234247707648*a^5
2*b^14*c^31*d^35 - 21954447360*a^53*b^13*c^30*d^36 + 970818048*a^54*b^12*c^
29*d^37))*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 1
40454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^
12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*
b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704
*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 9
01120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(
1/4) + 4125976704*a^29*b^35*c^49*d^15 - 83112811776*a^30*b^34*c^48*d^16 + 7
91027410176*a^31*b^33*c^47*d^17 - 4734885844224*a^32*b^32*c^46*d^18 + 20014
213844608*a^33*b^31*c^45*d^19 - 63580226479104*a^34*b^30*c^44*d^20 + 157689
244277760*a^35*b^29*c^43*d^21 - 313010180862976*a^36*b^28*c^42*d^22 + 50552
4473121024*a^37*b^27*c^41*d^23 - 671337017390592*a^38*b^26*c^40*d^24 + 7374
44677516800*a^39*b^25*c^39*d^25 - 671337017390592*a^40*b^24*c^38*d^26 + 505
524473121024*a^41*b^23*c^37*d^27 - 313010180862976*a^42*b^22*c^36*d^28 + 15
7689244277760*a^43*b^21*c^35*d^29 - 63580226479104*a^44*b^20*c^34*d^30 + 20
014213844608*a^45*b^19*c^33*d^31 - 4734885844224*a^46*b^18*c^32*d^32 + 7910
27410176*a^47*b^17*c^31*d^33 - 83112811776*a^48*b^16*c^30*d^34 + 4125976704
```

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*a^49*b^15*c^29*d^35))*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b
^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 +
 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 -
 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*
d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^
4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24
*b*c*d^11))^(1/4)*2i + 2*atan((2654208*a^5*b^22*c^38*x^(1/2)*(-(6561*a^4*d^
17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 -
 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c
^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4
*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 32440
32*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 +
270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 2654208*a^27*c^16
*d^22*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14
 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^1
2*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^
3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784
704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8
- 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d
))^(5/4) + 15169032*b^19*c^22*d^8*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*
d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16
)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a
^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 324
4032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7
 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^1
5*d^10 - 49152*a*b^11*c^24*d))^(1/4) - 130671792*a*b^18*c^21*d^9*x^(1/2)*(-
(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^
2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49
152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 +
 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19
*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3
*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) - 41877
504*a^6*b^21*c^37*d*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*
a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c
^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d
^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c
^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8
*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152
*a*b^11*c^24*d))^(5/4) - 41877504*a^26*b*c^17*d^21*x^(1/2)*(-(6561*a^4*d^17
 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 4
9572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^1
4*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b
^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032
*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 27
0336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 450333432*a^2*b^17*
```

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c^20*d^10*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*
d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096
*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 90112
0*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 +
3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*
d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^
24*d))^(1/4) - 784872864*a^3*b^16*c^19*d^11*x^(1/2)*(-(6561*a^4*d^17 + 8352
1*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^
3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11
+ 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21
*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^
5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^
10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) + 717087608*a^4*b^15*c^18*d^
12*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 +
140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c
^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b
^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704
*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 9
01120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^
(1/4) - 264948264*a^5*b^14*c^17*d^13*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c
^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d
^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 27033
6*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 -
3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*
d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*
c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) - 264948264*a^6*b^13*c^16*d^14*x^(1
/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*
a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^1
2 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22
*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^
6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a
^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) +
 717087608*a^7*b^12*c^15*d^15*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13
 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4
096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b
^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032
*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2
027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^
10 - 49152*a*b^11*c^24*d))^(1/4) - 784872864*a^8*b^11*c^14*d^16*x^(1/2)*(-(
6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2
*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 491
52*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 +
2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*
d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*
c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) + 450333
```

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432*a^9*b^10*c^13*d^17*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 1768
68*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^1
2*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^2
3*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^
7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*
a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49
152*a*b^11*c^24*d))^(1/4) - 130671792*a^10*b^9*c^12*d^18*x^(1/2)*(-(6561*a^
4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^
15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11
*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520
*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3
244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^
9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) + 15169032*a^11
*b^8*c^11*d^19*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3
*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 +
 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 -
901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d
^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*
c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^
11*c^24*d))^(1/4) + 304971776*a^7*b^20*c^36*d^2*x^(1/2)*(-(6561*a^4*d^17 +
83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 4957
2*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d
^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*
c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^
7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 27033
6*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) - 1359347712*a^8*b^19*c^
35*d^3*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^1
4 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^
12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a
^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 378
4704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8
 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*
d))^(5/4) + 4144791552*a^9*b^18*c^34*d^4*x^(1/2)*(-(6561*a^4*d^17 + 83521*b
^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b
*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 2
70336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^
4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c
^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*
b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) - 9148891136*a^10*b^17*c^33*d^5
*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 14
0454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^1
3*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9
*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a
^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901
120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5
```

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/4) + 15081504768*a^11*b^16*c^32*d^6*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c
^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d
^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 27033
6*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 -
3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*
d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*
c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) - 18867290112*a^12*b^15*c^31*d^7*x^
(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 14045
4*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d
^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^
22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*
b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120
*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)
 + 18014928896*a^13*b^14*c^30*d^8*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*
d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16
)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a
^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 324
4032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7
 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^1
5*d^10 - 49152*a*b^11*c^24*d))^(5/4) - 13171163136*a^14*b^13*c^29*d^9*x^(1/
2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a
^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12
 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*
d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6
*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^
9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) +
7816740864*a^15*b^12*c^28*d^10*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^1
3 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(
4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*
b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 324403
2*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 +
2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d
^10 - 49152*a*b^11*c^24*d))^(5/4) - 5554044928*a^16*b^11*c^27*d^11*x^(1/2)*
(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*
b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 -
49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3
 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^
19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b
^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 781
6740864*a^17*b^10*c^26*d^12*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 -
 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(409
6*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^1
0*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a
^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 202
7520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10
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 - 49152*a*b^11*c^24*d))^(5/4) - 13171163136*a^18*b^9*c^25*d^13*x^(1/2)*(-(
6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2
*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 491
52*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 +
2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*
d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*
c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 180149
28896*a^19*b^8*c^24*d^14*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 17
6868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b
^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c
^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*
b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 202752
0*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 -
49152*a*b^11*c^24*d))^(5/4) - 18867290112*a^20*b^7*c^23*d^15*x^(1/2)*(-(656
1*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^
2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*
a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 202
7520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6
 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^1
6*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 150815047
68*a^21*b^6*c^22*d^16*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 17686
8*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12
*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23
*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7
*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a
^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 491
52*a*b^11*c^24*d))^(5/4) - 9148891136*a^22*b^5*c^21*d^17*x^(1/2)*(-(6561*a^
4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^
15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11
*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520
*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3
244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^
9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 4144791552*a^
23*b^4*c^20*d^18*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b
^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25
 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2
- 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20
*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^
4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*
b^11*c^24*d))^(5/4) - 1359347712*a^24*b^3*c^19*d^19*x^(1/2)*(-(6561*a^4*d^1
7 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 -
49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^
14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*
b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 324403
2*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 2
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70336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4) + 304971776*a^25*b^2
*c^18*d^20*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3
*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 409
6*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 9011
20*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 +
 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17
*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c
^24*d))^(5/4))/(32234193*b^17*c^17*d^11 - 4782969*a^17*d^28 - 198040140*a*b
^16*c^16*d^12 + 413141310*a^2*b^15*c^15*d^13 - 308701404*a^3*b^14*c^14*d^14
 + 43224857*a^4*b^13*c^13*d^15 - 15826944*a^5*b^12*c^12*d^16 - 11894784*a^6
*b^11*c^11*d^17 - 5865472*a^7*b^10*c^10*d^18 + 2260992*a^8*b^9*c^9*d^19 + 1
2484608*a^9*b^8*c^8*d^20 + 24805376*a^10*b^7*c^7*d^21 + 39223296*a^11*b^6*c
^6*d^22 + 55738368*a^12*b^5*c^5*d^23 - 335988081*a^13*b^4*c^4*d^24 + 384710
796*a^14*b^3*c^3*d^25 - 197341758*a^15*b^2*c^2*d^26 + 48892572*a^16*b*c*d^2
7))*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*
a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^1
2 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22
*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^
6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a
^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4) +
 atan((a^5*b^22*c^38*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868
*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*
c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*
d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*
c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^
8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 4915
2*a*b^11*c^24*d))^(5/4)*2654208i + a^27*c^16*d^22*x^(1/2)*(-(6561*a^4*d^17
 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49
572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14
*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^
8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*
a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270
336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*2654208i + b^19*c^22*d
^8*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 +
140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c
^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b
^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704
*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 9
01120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^
(1/4)*15169032i - a*b^18*c^21*d^9*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*
d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16
)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a
^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 324
4032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7
 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^1
```

```
5*d^10 - 49152*a*b^11*c^24*d))^(1/4)*130671792i - a^6*b^21*c^37*d*x^(1/2)*(
-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b
^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 4
9152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3
+ 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^1
9*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^
3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*418775
04i - a^26*b*c^17*d^21*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 1768
68*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^1
2*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^2
3*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^
7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*
a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49
152*a*b^11*c^24*d))^(5/4)*41877504i + a^2*b^17*c^20*d^10*x^(1/2)*(-(6561*a^
4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^
15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11
*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520
*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3
244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^
9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4)*450333432i - a^
3*b^16*c^19*d^11*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b
^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25
 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2
- 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20
*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^
4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*
b^11*c^24*d))^(1/4)*784872864i + a^4*b^15*c^18*d^12*x^(1/2)*(-(6561*a^4*d^1
7 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 -
49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^
14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*
b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 324403
2*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 2
70336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4)*717087608i - a^5*b^1
4*c^17*d^13*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^
3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 40
96*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901
120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5
+ 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^1
7*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*
c^24*d))^(1/4)*264948264i - a^6*b^13*c^16*d^14*x^(1/2)*(-(6561*a^4*d^17 + 8
3521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572
*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^
11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c
^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7
*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336
```

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*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4)*264948264i + a^7*b^12*c^1
5*d^15*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^1
4 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^
12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a
^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 378
4704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8
 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*
d))^(1/4)*717087608i - a^8*b^11*c^14*d^16*x^(1/2)*(-(6561*a^4*d^17 + 83521*
b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*
b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 +
270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d
^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*
c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10
*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4)*784872864i + a^9*b^10*c^13*d^1
7*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 1
40454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^
13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^
9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*
a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 90
1120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(
1/4)*450333432i - a^10*b^9*c^12*d^18*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c
^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d
^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 27033
6*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 -
3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*
d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*
c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4)*130671792i + a^11*b^8*c^11*d^19*x^(
1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454
*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^
12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^2
2*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b
^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*
a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(1/4)*
15169032i + a^7*b^20*c^36*d^2*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13
 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4
096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b
^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032
*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2
027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^
10 - 49152*a*b^11*c^24*d))^(5/4)*304971776i - a^8*b^19*c^35*d^3*x^(1/2)*(-(
6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2
*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 491
52*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 +
2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*
d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*
```

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c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*13593477
12i + a^9*b^18*c^34*d^4*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176
868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^
12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^
23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b
^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520
*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 4
9152*a*b^11*c^24*d))^(5/4)*4144791552i - a^10*b^17*c^33*d^5*x^(1/2)*(-(6561
*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2
*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a
^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027
520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6
- 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16
*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*9148891136i
+ a^11*b^16*c^32*d^6*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868
*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*
c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*
d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*
c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^
8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 4915
2*a*b^11*c^24*d))^(5/4)*15081504768i - a^12*b^15*c^31*d^7*x^(1/2)*(-(6561*a
^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d
^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^1
1*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 202752
0*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 -
3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d
^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*18867290112i +
 a^13*b^14*c^30*d^8*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*
a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c
^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d
^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c
^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8
*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152
*a*b^11*c^24*d))^(5/4)*18014928896i - a^14*b^13*c^29*d^9*x^(1/2)*(-(6561*a^
4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^
15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11
*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520
*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3
244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^
9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*13171163136i +
a^15*b^12*c^28*d^10*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*
a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c
^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d
^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c
^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8
```

```
*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152
*a*b^11*c^24*d))^(5/4)*7816740864i - a^16*b^11*c^27*d^11*x^(1/2)*(-(6561*a^
4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^
15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11
*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520
*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3
244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^
9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*5554044928i + a
^17*b^10*c^26*d^12*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a
*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^
25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^
2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^
20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*
b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*
a*b^11*c^24*d))^(5/4)*7816740864i - a^18*b^9*c^25*d^13*x^(1/2)*(-(6561*a^4*
d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15
 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b
*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a
^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 324
4032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9
+ 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*13171163136i + a^
19*b^8*c^24*d^14*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b
^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25
 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2
- 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20
*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^
4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*
b^11*c^24*d))^(5/4)*18014928896i - a^20*b^7*c^23*d^15*x^(1/2)*(-(6561*a^4*d
^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15
- 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*
c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^
4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244
032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 +
 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*18867290112i + a^2
1*b^6*c^22*d^16*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^
3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25
+ 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 -
 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*
d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4
*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b
^11*c^24*d))^(5/4)*15081504768i - a^22*b^5*c^21*d^17*x^(1/2)*(-(6561*a^4*d^
17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 -
 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c
^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4
*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 32440
```

```
32*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 +
270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*9148891136i + a^23*
b^4*c^20*d^18*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*
c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 +
4096*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 9
01120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^
5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c
^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^1
1*c^24*d))^(5/4)*4144791552i - a^24*b^3*c^19*d^19*x^(1/2)*(-(6561*a^4*d^17
+ 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^14 + 140454*a^2*b^2*c^2*d^15 - 49
572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^12*c^13*d^12 - 49152*a^11*b*c^14
*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a^3*b^9*c^22*d^3 + 2027520*a^4*b^
8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 3784704*a^6*b^6*c^19*d^6 - 3244032*
a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8 - 901120*a^9*b^3*c^16*d^9 + 270
336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*d))^(5/4)*1359347712i + a^25*b^2
*c^18*d^20*x^(1/2)*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3
*d^14 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 409
6*a^12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 9011
20*a^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 +
 3784704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17
*d^8 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c
^24*d))^(5/4)*304971776i)/(32234193*b^17*c^17*d^11 - 4782969*a^17*d^28 - 19
8040140*a*b^16*c^16*d^12 + 413141310*a^2*b^15*c^15*d^13 - 308701404*a^3*b^1
4*c^14*d^14 + 43224857*a^4*b^13*c^13*d^15 - 15826944*a^5*b^12*c^12*d^16 - 1
1894784*a^6*b^11*c^11*d^17 - 5865472*a^7*b^10*c^10*d^18 + 2260992*a^8*b^9*c
^9*d^19 + 12484608*a^9*b^8*c^8*d^20 + 24805376*a^10*b^7*c^7*d^21 + 39223296
*a^11*b^6*c^6*d^22 + 55738368*a^12*b^5*c^5*d^23 - 335988081*a^13*b^4*c^4*d^
24 + 384710796*a^14*b^3*c^3*d^25 - 197341758*a^15*b^2*c^2*d^26 + 48892572*a
^16*b*c*d^27))*(-(6561*a^4*d^17 + 83521*b^4*c^4*d^13 - 176868*a*b^3*c^3*d^1
4 + 140454*a^2*b^2*c^2*d^15 - 49572*a^3*b*c*d^16)/(4096*b^12*c^25 + 4096*a^
12*c^13*d^12 - 49152*a^11*b*c^14*d^11 + 270336*a^2*b^10*c^23*d^2 - 901120*a
^3*b^9*c^22*d^3 + 2027520*a^4*b^8*c^21*d^4 - 3244032*a^5*b^7*c^20*d^5 + 378
4704*a^6*b^6*c^19*d^6 - 3244032*a^7*b^5*c^18*d^7 + 2027520*a^8*b^4*c^17*d^8
 - 901120*a^9*b^3*c^16*d^9 + 270336*a^10*b^2*c^15*d^10 - 49152*a*b^11*c^24*
d))^(1/4)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

3.478 $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=718

$$-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}\right.}{64\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}$$

Rubi [A] time = 1.04, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 470, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\sqrt[4]{a}\,b^{3/4}(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,(bc-ad)^4}-\frac{\sqrt[4]{a}\,b^{3/4}(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,(bc-ad)^4}+\frac{\sqrt[4]{a}\,b^{3/4}(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,(bc-ad)^4}-\frac{\sqrt[4]{a}\,b^{3/4}(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,(bc-ad)^4}+\frac{a\sqrt{x}}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{\sqrt{x}\,(17ad+7bc)}{16\left(c+dx^2\right)(bc-ad)^3}+\frac{\sqrt{x}\,(2ad+bc)}{4b\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((b*c + 2*a*d)*Sqrt[x])/(4*b*(b*c - a*d)^2*(c + d*x^2)^2) + (a*Sqrt[x])/(2*
b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((7*b*c + 17*a*d)*Sqrt[x])/(16*(
b*c - a*d)^3*(c + d*x^2)) + (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/
4)*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2
]*(b*c - a*d)^4) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) +
((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + (a^(1/4)*b^(3/4)*(5
*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
8*Sqrt[2]*(b*c - a*d)^4) - (a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*(b*c - a*d)^4) - ((
21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4) + ((21*b^2
*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^8}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{ac+(-4bc-7ad)x^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2b(bc-ad)}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\Bigl(\int \frac{12abc^2-28bc}{\bigl(a+bx^4\bigr)\bigl(}}{16bc(bc}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\bigl(c+dx}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\bigl(c+dx}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\bigl(c+dx}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\bigl(c+dx}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\bigl(c+dx}$$

$$= \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2\left(c+dx^2\right)^2} + \frac{a\sqrt{x}}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3\bigl(c+dx}$$

Mathematica [A] time = 1.41, size = 604, normalized size = 0.84

$$\frac{-\frac{\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}\sqrt[4]{d}}+\frac{\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{3/4}\sqrt[4]{d}}-\frac{2\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{3/4}\sqrt[4]{d}}+\frac{2\sqrt{2}\left(5a^2d^2+70abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{3/4}\sqrt[4]{d}}+8\sqrt{2}\sqrt[4]{a}b^{3/4}(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-8\sqrt{2}\sqrt[4]{a}b^{3/4}(7ad+5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+16\sqrt{2}\sqrt[4]{a}b^{3/4}(7ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)-16\sqrt{2}\sqrt[4]{a}b^{3/4}(7ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)+\frac{32c\sqrt{x}(bc-ad)^2}{\left(c+dx^2\right)^2}+\frac{64ab\sqrt{x}(bc-ad)}{a+bx^2}+\frac{8\sqrt{x}(9ad+7bc)(bc-ad)}{c+dx^2}}{128(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((64*a*b*(b*c - a*d)*Sqrt[x])/(a + b*x^2) + (32*c*(b*c - a*d)^2*Sqrt[x])/(c
 + d*x^2)^2 + (8*(b*c - a*d)*(7*b*c + 9*a*d)*Sqrt[x])/(c + d*x^2) + 16*Sqrt
[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)] - 16*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)] - (2*Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(c^(3/4)*d^(1/4)) + (2*Sqrt[2
]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x]
)/c^(1/4)])/(c^(3/4)*d^(1/4)) + 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 8*Sqrt[2]*a^(1/
4)*b^(3/4)*(5*b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x] - (Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*d^(1/4)) + (Sqrt[2]*(
21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(c^(3/4)*d^(1/4)))/(128*(b*c - a*d)^4)
```

IntegrateAlgebraic [A] time = 2.43, size = 456, normalized size = 0.64

$$\frac{\left(7\sqrt{2}\,a^{5/4}b^{3/4}d+5\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{8(bc-ad)^4}+\frac{\left(-7\sqrt{2}\,a^{5/4}b^{3/4}d-5\sqrt{2}\,\sqrt[4]{a}\,b^{7/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{8(bc-ad)^4}+\frac{\sqrt{x}\,\left(5a^2cd+9a^2d^2x^2+19abc^2+28abcdx^2+17abd^2x^4+11b^2c^2x^2+7b^2cdx^4\right)}{16\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)^3}-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{3/4}\sqrt[4]{d}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (Sqrt[x]*(19*a*b*c^2 + 5*a^2*c*d + 11*b^2*c^2*x^2 + 28*a*b*c*d*x^2 + 9*a^2*
d^2*x^2 + 7*b^2*c*d*x^4 + 17*a*b*d^2*x^4))/(16*(b*c - a*d)^3*(a + b*x^2)*(c
 + d*x^2)^2) + ((5*Sqrt[2]*a^(1/4)*b^(7/4)*c + 7*Sqrt[2]*a^(5/4)*b^(3/4)*d)
*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(8*(b*c -
 a*d)^4) - ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]
*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c -
a*d)^4) + ((-5*Sqrt[2]*a^(1/4)*b^(7/4)*c - 7*Sqrt[2]*a^(5/4)*b^(3/4)*d)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(8*(b*c - a*
d)^4) + ((21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(3/4)*d^(1/4)*(b*c - a*
d)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.21, size = 1193, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^3*
c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*d^4
) - 1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*
b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4
*d^4) + 1/32*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c
^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/32*(21*(c*d^3)^(1
/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^5*
d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c
^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*
d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 - 4*sq
rt(2)*a*b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqr
t(2)*a^4*d^4) + 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^3*c
^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*d^4)
 + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1
/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*
c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3
*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^
3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4
) + x + sqrt(c/d))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)
*a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/2*a*b*s
qrt(x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)) +
1/16*(7*b*c*d*x^(5/2) + 9*a*d^2*x^(5/2) + 11*b*c^2*sqrt(x) + 5*a*c*d*sqrt(x
))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.03, size = 1066, normalized size = 1.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*a^2*b/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d+1/2*a*b^2/(a*d-b*c)^4*x^(1/2)/(b
*x^2+a)*c-7/8*a*b/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
```

```
)*x^(1/2)+1)*d-5/8*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)*c-7/8*a*b/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)*d-5/8*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)-1)*c-7/16*a*b/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*ln((
x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(
a/b)^(1/2)))*d-5/16*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2
^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c-
9/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a^2*d^3+1/8/(a*d-b*c)^4/(d*x^2+c)^2*x^
(5/2)*a*b*c*d^2+7/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*b^2*c^2*d-5/16/(a*d-b*
c)^4/(d*x^2+c)^2*x^(1/2)*a^2*c*d^2-3/8/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b*
c^2*d+11/16/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*b^2*c^3+5/64/(a*d-b*c)^4*(c/d)^
(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2*d^2+35/32/(a*d-b*
c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b*d+21/64/
(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2
+5/64/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)*a^2*d^2+35/32/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)*a*b*d+21/64/(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)-1)*b^2+5/128/(a*d-b*c)^4*(c/d)^(1/4)/c*2^(1/2)*ln((x+(c/d
)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
1/2)))*a^2*d^2+35/64/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1
/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b*d
+21/128/(a*d-b*c)^4*(c/d)^(1/4)*c*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.65, size = 855, normalized size = 1.19

$$-\frac{\left(\frac{2\,\sqrt{2}\,(5\,bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\,(5\,bc+7\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,(5\,bc+7\,ad)\log\left(\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(5\,bc+7\,ad)\log\left(-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}\,x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)ab}{16\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)}+\frac{\left(7\,b^{2}cd+17\,abd^{2}\right)x^{\frac{9}{2}}+\left(11\,b^{2}c^{2}+28\,abcd+9\,a^{2}d^{2}\right)x^{\frac{5}{2}}+\left(19\,abc^{2}+5\,a^{2}cd\right)\sqrt{x}}{16\left(ab^{3}c^{5}-3\,a^{2}b^{2}c^{4}d+3\,a^{3}bc^{3}d^{2}-a^{4}c^{2}d^{3}+\left(b^{4}c^{3}d^{2}-3\,ab^{3}c^{2}d^{3}+3\,a^{2}b^{2}cd^{4}-a^{3}bd^{5}\right)x^{6}+\left(2\,b^{4}c^{4}d-5\,ab^{3}c^{3}d^{2}+3\,a^{2}b^{2}c^{2}d^{3}+a^{3}bcd^{4}-a^{4}d^{5}\right)x^{4}+\left(b^{4}c^{5}-ab^{3}c^{4}d-3\,a^{2}b^{2}c^{3}d^{2}+5\,a^{3}bc^{2}d^{3}-2\,a^{4}cd^{4}\right)x^{2}\right)}+\frac{\frac{2\,\sqrt{2}\left(21\,b^{2}c^{2}+70\,abcd+5\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{2\,\sqrt{2}\left(21\,b^{2}c^{2}+70\,abcd+5\,a^{2}d^{2}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\,\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\,\sqrt{d}}}\right)}{\sqrt{c}\,\sqrt{\sqrt{c}\,\sqrt{d}}}+\frac{\sqrt{2}\left(21\,b^{2}c^{2}+70\,abcd+5\,a^{2}d^{2}\right)\log\left(\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(21\,b^{2}c^{2}+70\,abcd+5\,a^{2}d^{2}\right)\log\left(-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}\,x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(5*b*c + 7*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*(5*b*c + 7*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + sqrt(2)*(5*b*c + 7*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b*c + 7*a*d)*log(-sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a*b/(b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/16*((7*b^2
*c*d + 17*a*b*d^2)*x^(9/2) + (11*b^2*c^2 + 28*a*b*c*d + 9*a^2*d^2)*x^(5/2)
+ (19*a*b*c^2 + 5*a^2*c*d)*sqrt(x))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
```

```
b*c^2*d^3 - 2*a^4*c*d^4)*x^2) + 1/128*(2*sqrt(2)*(21*b^2*c^2 + 70*a*b*c*d +
 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(21*b^
2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + sqrt(2)*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(21*b^2*c^2
+ 70*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)
```

mupad [B] time = 7.73, size = 48950, normalized size = 68.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] 2*atan(-((((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (9
72405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*
a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*
b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2
*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10
*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d
^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 28
6*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c
*d^12) + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664
200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 +
745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216
*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 2684354
56*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^1
6*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134
351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*
a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c
^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9
395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(3/4)*(((-(625
*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5
*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*
c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d +
16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d
^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 3053453
3120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b
^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^
9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282
879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*
```

```
b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 +
 10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*
d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*
b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 7
7608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^1
2*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16
 + 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^
7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13
 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c
^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7
 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*
a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (x^(1/2)*(655360
0*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 -
5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384
*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20
*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^
11 - 2125492912128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13
 - 4783425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5
154201927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 222988
0750080*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504
*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^
3*d^22 - 810024960*a^21*b^6*c^2*d^23)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 1
53*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 856
8*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 4
3758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 -
31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13
+ 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 -
18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*1i)*(-(625*a^8*d^8 + 194481*b^8*c^8
+ 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^
4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^
7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 2
68435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*
c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 7
3282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 19193135104
0*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c
^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 +
 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*
a^14*b^2*c^5*d^15))^(1/4) + (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*a^1
1*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 5374
50669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*
c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^4*d^9 + 34409
55560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11))/(65536*(a^18*d^18
+ b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14
*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^
11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*
```

```
b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^1
3*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*
b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*(-(625*a^8*d^8 + 1944
81*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150
*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 25930
80*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c
^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 201326592
0*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c
^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 -
191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 19193135104
0*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5
*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 +
2013265920*a^14*b^2*c^5*d^15))^(1/4) - (((((1473515*a^9*b^7*c*d^10)/2048 -
(4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b
^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c
^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^
8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 - 78*a^
2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^
8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*
d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 +
13*a*b^12*c^12*d - 13*a^12*b*c*d^12) + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13
150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^
4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d
+ 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 26843
5456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17
*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282
879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^
7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*
d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 305
34533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14
*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*
c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5
*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c
*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^1
8*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 93952409
60*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11
*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8
+ 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 13435194
5728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b
^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))
^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^
18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6
*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 3
1016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 115315200*a^11*
b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15
```

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 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*
b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*
a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3
*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*
c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9
 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^1
2*b*c*d^12) + (x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 +
419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^
4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^
17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 +
 910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^12 + 3520
229539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 + 5470166
188032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 38679037870
08*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 945071063040*a^1
7*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4
*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23)*1i)/(65
536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3
+ 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6
 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^
9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*
d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d
^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*1i)*(-
(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5
*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*
b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*
d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c
^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 305
34533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a
^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^1
1*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 7
3282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a
^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4) - (x^(1/2)*(3872225
*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78
440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*
c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 59320
52274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9
*c^2*d^11))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3
*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^
6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620
*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 1856
4*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816
*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c
*d^17)))*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664
200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 +
745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216
```

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*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 2684354
56*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^1
6*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134
351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*
a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c
^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9
395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4))/((((((1
473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*
c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^
5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4
096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)*1i)/
(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715
*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b
^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*
d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) + (-(625
*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5
*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*
c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d +
16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d
^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 3053453
3120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b
^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^
9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282
879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*
b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 1944
81*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150
*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 25930
80*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c
^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 201326592
0*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c
^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 -
191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 19193135104
0*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5
*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 +
2013265920*a^14*b^2*c^5*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22
*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a
^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 +
 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^
14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14
 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^1
5*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 14
3360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 -
78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a
^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5
*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^
```

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11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (x^(1/2)*(6553600*a^23*b^4*d^25
 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*
b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*
d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105
331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912
128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13 - 4783425454080
*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^
14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^
11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d
^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024
960*a^21*b^6*c^2*d^23)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^1
6*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13
*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c
^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*
c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4
*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*
d - 18*a^17*b*c*d^17)))*1i)*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*
b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000
*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7
*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15
*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395
240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*
b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*
d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 1343
51945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^
12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^
15))^(1/4)*1i + (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12
 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^
15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 65
51813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^
10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11)*1i)/(65536*(a^18*d^18 + b^18*c^
18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4
 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d
^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d
^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5
*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d
^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*(-(625*a^8*d^8 + 194481*b^8*c^
8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*
c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*
c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 -
 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^1
4*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 -
 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351
040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7
*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12
```

```
 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 201326592
0*a^14*b^2*c^5*d^15))^(1/4) + (((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^1
0*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d
^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2
048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 +
 (6507125*a^8*b^8*c^2*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^
11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5
 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715
*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12
*c^12*d - 13*a^12*b*c*d^12) + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^
2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 73010
00*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a
^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^
15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 93
95240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^
5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^1
2*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 13
4351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*
a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*
d^15))^(3/4)*(((-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 +
 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*
d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16
777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 2
68435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^
13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6
 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 2159227
69920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10
*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^
13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*(1
280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 +
922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^1
5*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a
^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10
*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 549785
60*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d
^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*
c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^1
0*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 +
 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^
10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^1
2) + (x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400
*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^
19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 -
218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 + 91084554
2400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^12 + 3520229539840
```

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*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^
13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b
^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^
6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1
490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23)*1i)/(65536*(a^18
*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^
4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*
a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758
*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 85
68*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153
*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*1i)*(-(625*a^8*
d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3
+ 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d
^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777
216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 +
 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*
a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c
^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 1
91931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 7328287948
8*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c
^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*1i - (x^(1/2)*(3872225*a^12*
b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670
*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^
6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*
a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d
^11)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^
15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b
^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^
9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a
^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^
15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^
17)))*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200
*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745
500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^
16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*
a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d
^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351
945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8
*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*
d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395
240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)))*(-(625*a^
8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^
3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2
*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 167
77216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16
```

```
 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 3053453312
0*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10
*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 -
 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879
488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3
*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4) - atan(-(((((1473515*a^9*b
^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/819
2 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (6
9456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943
363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)/(a^13*d^13 - b^1
3*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4
 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 128
7*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b
^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) + (-(625*a^8*d^8 + 19448
1*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*
a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 259308
0*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^
3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920
*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^
15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 1
91931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040
*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*
c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2
013265920*a^14*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 194481*b^8*c^8 + 131
50620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4
 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d +
 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435
456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*
d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 732828
79488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7
*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d
^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 3053
4533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*
b^2*c^5*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 1446
40*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7
 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16
*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 1
15315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13
*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 -
 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^
3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11
*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 -
 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a
^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c
^12*d - 13*a^12*b*c*d^12) - (x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22
```

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*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 +
31284264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 2108424192
00*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^
19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^
13*d^12 + 3520229539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*
d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16
 + 3867903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 94
5071063040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 4571922432
0*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2
*d^23))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^1
5*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^
12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9
*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^
12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^1
5*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^1
7))))*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200
*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745
500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^
16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*
a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d
^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351
945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8
*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*
d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395
240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*1i - (x^(1/
2)*(3872225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^
10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 203059332
0*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*
d^8 + 5932052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 11433061
65*a^10*b^9*c^2*d^11)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16
*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*
d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^
10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c
^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*
c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d
 - 18*a^17*b*c*d^17)))*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c
^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*
b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*
d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18
*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 939524096
0*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*
c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 +
 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945
728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^
4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^
```

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(1/4) - ((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972
405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^
4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^
10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d
^9)/512)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*
d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1
716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10
*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12)
 + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^
3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500
*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*
c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^1
5*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4
+ 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945
728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^
8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^1
1 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240
960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d
^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 +
 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^
6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 167772
16*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 +
2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a
^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^
13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 19
1931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488
*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^
6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240
*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 -
3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c
^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 7760896
0*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12
*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 236
18560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*
d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^1
3*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4
 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 128
7*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b
^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) + (x^(1/2)*(6553600*a^23
*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 542087
5776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b
^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*
d^9 - 105331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2
125492912128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13 - 4783
425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5154201
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927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 222988075008
0*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504*a^18*
b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22
 - 810024960*a^21*b^6*c^2*d^23))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^
16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^1
3*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*
b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^1
1*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^
14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17
*c^17*d - 18*a^17*b*c*d^17))))*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a
^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301
000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*
a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b
^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9
395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a
^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^
12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 1
34351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120
*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5
*d^15))^(1/4)*1i + (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d
^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4
*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 +
 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^4*d^9 + 3440955560*a^9
*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11)*1i)/(65536*(a^18*d^18 + b^18
*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*
d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^1
1*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^
8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*
c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^
2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*(-(625*a^8*d^8 + 194481*b^8
*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b
^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b
^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^1
7 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*
b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^
5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931
351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*
b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d
^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 201326
5920*a^14*b^2*c^5*d^15))^(1/4))/(((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a
^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7
*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)
/2048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048
 + (6507125*a^8*b^8*c^2*d^9)/512)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11
*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 -
```

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 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a
^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c
^12*d - 13*a^12*b*c*d^12) + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*
b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000
*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7
*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15
*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395
240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*
b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*
d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 1343
51945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^
12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^
15))^(3/4)*(((-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 3
0664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^
5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(1677
7216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268
435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13
*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 +
 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769
920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b
^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13
 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*(128
0*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 92
2880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*
d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9
*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d
^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560
*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^1
8 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^
2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*
d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1
716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10
*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12)
 - (x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400*a
^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19
*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 21
8396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 + 9108455424
00*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^12 + 3520229539840*a
^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^13
*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b^1
2*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*
d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 149
0026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23))/(65536*(a^18*d^18
 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^1
4*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b
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^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10
*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^
13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16
*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17))))*(-(625*a^8*d^8 + 19
4481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 302501
50*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 259
3080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16
*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265
920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12
*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7
- 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351
040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b
^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14
+ 2013265920*a^14*b^2*c^5*d^15))^(1/4) - (x^(1/2)*(3872225*a^12*b^7*d^13 +
120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c
^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534
530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^
4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11))/(6553
6*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 +
3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 -
 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9
+ 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^
12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^1
5 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))*(-(625*a
^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d
^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^
2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16
777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^1
6 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 305345331
20*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^1
0*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9
- 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 7328287
9488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^
3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4) + ((((1473515*a^9*b^7*c*d
^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3
824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (6945679
3*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^
7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)/(a^13*d^13 - b^13*c^13
 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 128
7*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*
b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2
*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) + (-(625*a^8*d^8 + 194481*b^8*
c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^
4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^
7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17
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 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b
^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5
 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 1919313
51040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b
^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^
12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265
920*a^14*b^2*c^5*d^15))^(3/4)*(((-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*
a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 730
1000*a^5*b^3*c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000
*a^7*b*c*d^7)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*
b^15*c^18*d^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 -
9395240960*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*
a^5*b^11*c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c
^12*d^8 + 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 +
134351945728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 3053453312
0*a^12*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^
5*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a^3
*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 803
8400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^13*
d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 1153152
00*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^11*
c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 71936
00*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20
 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 +
 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*
a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4
*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d
 - 13*a^12*b*c*d^12) + (x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c
*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 312842
64960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6
*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^1
5*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^1
2 + 3520229539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 +
 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 386
7903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 94507106
3040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19
*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23)
)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15
*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^1
2*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c
^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6
*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*
c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17))))*
(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b
^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^
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6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^1
9*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b
*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 3
0534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728
*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c
^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 -
 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960
*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4) + (x^(1/2)*(38722
25*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 +
78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^1
4*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 593
2052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b
^9*c^2*d^11))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a
^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*
a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 486
20*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18
564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 8
16*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b
*c*d^17)))*(-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 306
64200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5
+ 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(167772
16*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 26843
5456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c
^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 1
34351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 21592276992
0*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6
*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 -
 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)))*(-(6
25*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 30664200*a^3*b^5*c
^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^3*d^5 + 745500*a^6*b^
2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7)/(16777216*b^16*c^19*d
+ 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d^2 - 268435456*a^15*b*c^4
*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960*a^3*b^13*c^16*d^4 + 30534
533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*c^14*d^6 + 134351945728*a^6
*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8 + 215922769920*a^8*b^8*c^11*
d^9 - 191931351040*a^9*b^7*c^10*d^10 + 134351945728*a^10*b^6*c^9*d^11 - 732
82879488*a^11*b^5*c^8*d^12 + 30534533120*a^12*b^4*c^7*d^13 - 9395240960*a^1
3*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^15))^(1/4)*2i - ((x^(5/2)*(9*a^2
*d^2 + 11*b^2*c^2 + 28*a*b*c*d))/(16*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3
*a^2*b*c*d^2)) + (c*x^(1/2)*(5*a^2*d + 19*a*b*c))/(16*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d*x^(9/2)*(17*a*d + 7*b*c))/(16*(a^3*d
^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c
*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) - atan(((-(625*a*b^7*c^4 + 2401*a^
5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)
/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*
```

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b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32
800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*
d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11
*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491
520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*(
(-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d
^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^
14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891
328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d
^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^
6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 22937
60*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 655
36*a^15*b*c*d^15))^(1/4)*(((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d
^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/204
8 + (11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (
218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (65071
25*a^8*b^8*c^2*d^9)/512)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 28
6*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6
*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^
4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 1
3*a^12*b*c*d^12) + (-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d
 + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c
^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^
12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 468
58240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9
 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b
^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*
a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(3/4)*((x^(1/2)*(6553600*a^23*b^4*d^2
5 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3
*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18
*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 10
5331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 212549291
2128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13 - 478342545408
0*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a
^14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b
^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*
d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 81002
4960*a^21*b^6*c^2*d^23))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*
d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d
^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^1
0*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^
7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c
^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d
 - 18*a^17*b*c*d^17)) + ((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*
c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b
```

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^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a
^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6
- 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^
7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a
^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 6
5536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10
240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6
 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^1
7*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 7760
8960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b
^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 +
23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c
^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 -
b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*
d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 -
1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^1
1*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12)))*1i + (x^(1/2)*(3872
225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 +
 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^
14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 59
32052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*
b^9*c^2*d^11)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 8
16*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18
564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 -
 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11
+ 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14
 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^
17*b*c*d^17))) - (-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d +
 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^1
6 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12
*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858
240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 +
 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4
*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*
b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((-(625*a*b^7*c^4 + 2401*a^5*b^3*
d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096
*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c
^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768
*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 -
46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c
^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^
14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*(((1473
515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8
*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/
1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4096
```

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 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)/(a^13*d
^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^
9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*
d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 +
78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (-(625*a*b^7*
c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3
*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 -
 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*
c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520
*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 -
17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c
^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d
^15))^(3/4)*((x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 4
19430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^4
*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^1
7*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 +
910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^12 + 35202
29539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 + 54701661
88032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 386790378700
8*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 945071063040*a^17
*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*
d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23))/(65536*
(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 30
60*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 3
1824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 +
43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12
 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15
+ 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)) - ((-(625*a
*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 735
0*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*
d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*
b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 527
15520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^
10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*
b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*
b*c*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a
^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 8
038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^1
3*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 11531
5200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^1
1*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 719
3600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^
20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2
 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 171
6*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b
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^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*
d - 13*a^12*b*c*d^12)))*1i - (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*a^
11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 537
450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13
*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^4*d^9 + 3440
955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11)*1i)/(65536*(a^18*d
^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*
b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^
7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a
^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568
*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a
^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17))))/((-(625*a*b^7*c^
4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b
^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2
293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^
11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a
^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17
891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3
*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^1
5))^(1/4)*((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*
a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 49
1520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*
d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^
7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800
768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d
^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c
^15*d - 65536*a^15*b*c*d^15))^(1/4)*(((1473515*a^9*b^7*c*d^10)/2048 - (4375
*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c
^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^
6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/20
48 + (6507125*a^8*b^8*c^2*d^9)/512)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^
11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5
 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715
*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12
*c^12*d - 13*a^12*b*c*d^12) + (-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^
2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 +
4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 745
4720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^1
0*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*
b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 745
4720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^
14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(3/4)*((x^(1/2)*(6553600*a
^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 542
0875776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384*a^
5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^
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16*d^9 - 105331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^11
- 2125492912128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13 - 4
783425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5154
201927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 222988075
0080*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504*a^
18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d
^22 - 810024960*a^21*b^6*c^2*d^23))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2
*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*
b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a
^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*
a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060
*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b
^17*c^17*d - 18*a^17*b*c*d^17)) + ((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 35
00*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^
16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3
+ 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^1
0*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240
*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11
+ 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c
^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*(1280*a^20*b^4*
c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^
20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501
120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*
d^11 - 77608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 12117
2480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*
c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360
*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a
^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a
^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6
*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^
10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12))) + (x^(1/
2)*(3872225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^
10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 203059332
0*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*
d^8 + 5932052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 11433061
65*a^10*b^9*c^2*d^11))/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^
2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5
 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*
d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*
d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4
*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d -
18*a^17*b*c*d^17))) + (-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^
3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^1
6*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4
*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 -
```

46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*
d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^1
2*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 655
36*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((-(625*a*b^7*c^4 + 2401*a^5
*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/
(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b
^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 328
00768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d
^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*
b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 4915
20*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((
(1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^1
4*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*
d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)
/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)/(a
^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a
^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6
*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^
10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (-(625*a
*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 735
0*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*
d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*
b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 527
15520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^
10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*
b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*
b*c*d^15))^(3/4)*((x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^2
4 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 3128426496
0*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^2
1*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^
10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^12 +
3520229539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 + 547
0166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 3867903
787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 945071063040
*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8
*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23))/(6
5536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3
 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^
6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d
^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6
*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*
d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)) - ((-(
625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3
+ 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*
c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328

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*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7
+ 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c
^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*
a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*
a^15*b*c*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144
640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^
7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^1
6*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 +
115315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^1
3*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17
- 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c
^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^1
1*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5
- 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*
a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*
c^12*d - 13*a^12*b*c*d^12))) - (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*
a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 5
37450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^
13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^4*d^9 + 34
40955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11))/(65536*(a^18*d^
18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b
^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7
*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^
10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*
a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^
16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)))))*(-(625*a*b^7*c^4
 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^
5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 22
93760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^1
1*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^
8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 178
91328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*
d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15
))^(1/4)*2i + 2*atan(((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^
3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^1
6*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4
*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 -
46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*
d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^1
2*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 655
36*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((-(625*a*b^7*c^4 + 2401*a^5
*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/
(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b
^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 328
00768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d
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^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*
b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 4915
20*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((
((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^
14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6
*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7
)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)*1
i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 -
715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^
7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c
^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) + (-(
625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3
+ 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*
c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328
*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7
+ 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c
^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*
a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*
a^15*b*c*d^15))^(3/4)*((x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*
c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284
264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^
6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^
15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^
12 + 3520229539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14
+ 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 38
67903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 9450710
63040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^1
9*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23
)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*
c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12
*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b
^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12
*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*
b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)
) + ((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^
4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a
^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 -
17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*
c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^
10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 -
2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d
- 65536*a^15*b*c*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d
^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19
*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920
*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11
```

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*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 9438
2080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c
^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^
18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*
b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*
c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^
8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13
*a*b^12*c^12*d - 13*a^12*b*c*d^12))*1i) - (x^(1/2)*(3872225*a^12*b^7*d^13 +
 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*
c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 461753
4530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c
^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11))/(655
36*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 +
 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6
- 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9
 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d
^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^
15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17))) - (-(62
5*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 +
7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^
14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a
^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 +
52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6
*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^
13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^
15*b*c*d^15))^(1/4)*((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3
*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16
*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*
b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 4
6858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d
^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12
*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 6553
6*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((((1473515*a^9*b^7*c*d^10)/2
048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793
*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*
b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*
c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 -
 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*
a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^
5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d
^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (-(625*a*b^7*c^4 + 2401*a^5*b^
3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(40
96*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13
*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 328007
68*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8
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```
- 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5
*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*
a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(3/4)*((x^(
1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25
*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 -
104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098
560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*
b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^1
6*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c
^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d
^17 - 2229880750080*a^16*b^11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 -
273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496
*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23)*1i)/(65536*(a^18*d^18 + b
^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^
14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*
c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8
*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b
^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2
*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)) - ((-(625*a*b^7*c^4 + 240
1*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*
d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*
a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5
+ 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*
c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*
a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 +
 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/
4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d
^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^1
8*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016
960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13
*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 5
4978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*
c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19
*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^1
0*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*
d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 2
86*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*
c*d^12))*1i) + (x^(1/2)*(3872225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12
+ 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^1
5*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 655
1813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^1
0*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11))/(65536*(a^18*d^18 + b^18*c^18 +
 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8
568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 +
 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10
```

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- 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^1
3 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16
- 18*a*b^17*c^17*d - 18*a^17*b*c*d^17))))/((-(625*a*b^7*c^4 + 2401*a^5*b^3*
d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096
*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c
^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768
*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 -
46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c
^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^
14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((-(625
*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7
350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^1
4*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^
5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 5
2715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*
d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^1
3*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^1
5*b*c*d^15))^(1/4)*((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/
8192 + (972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (
11560479*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (21883
0061*a^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^
8*b^8*c^2*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286
*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*
b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4
*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13
*a^12*b*c*d^12) + (-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d
+ 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^
16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^1
2*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 4685
8240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9
+ 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^
4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a
*b^15*c^15*d - 65536*a^15*b*c*d^15))^(3/4)*((x^(1/2)*(6553600*a^23*b^4*d^25
 + 78643200*a^22*b^5*c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*
b^24*c^20*d^5 + 31284264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*
d^7 + 210842419200*a^6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105
331556352*a^8*b^19*c^15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912
128*a^10*b^17*c^13*d^12 + 3520229539840*a^11*b^16*c^12*d^13 - 4783425454080
*a^12*b^15*c^11*d^14 + 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^
14*b^13*c^9*d^16 + 3867903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^
11*c^7*d^18 + 945071063040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d
^20 + 45719224320*a^19*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024
960*a^21*b^6*c^2*d^23)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^1
6*d^2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13
*d^5 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c
```

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^10*d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*
c^7*d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4
*c^4*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*
d - 18*a^17*b*c*d^17)) + ((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^
6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096
*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720
*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^
6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*
c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720
*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 -
 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 +
10240*a^2*b^22*c^19*d^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d
^6 - 3450880*a^5*b^19*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b
^17*c^14*d^9 + 465920*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77
608960*a^10*b^14*c^11*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^12
*b^12*c^9*d^14 + 94382080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16
+ 23618560*a^15*b^9*c^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7
*c^4*d^19 - 143360*a^18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13
- b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^
9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7
- 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a
^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12))*1i)*1i - (x^(1/2)*
(3872225*a^12*b^7*d^13 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*
d^3 + 78440670*a^3*b^16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a
^5*b^14*c^7*d^6 + 4617534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8
 + 5932052274*a^8*b^11*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*
a^10*b^9*c^2*d^11)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^
2 - 816*a^3*b^15*c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5
 + 18564*a^6*b^12*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*
d^8 - 48620*a^9*b^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*
d^11 + 18564*a^12*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4
*d^14 - 816*a^15*b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d -
18*a^17*b*c*d^17))) + (-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^
3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^1
6*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4
*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 -
46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*
d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^1
2*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 655
36*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((-(625*a*b^7*c^4 + 2401*a^5
*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/
(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*c^14*d^2 - 2293760*a^3*b
^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328*a^5*b^11*c^11*d^5 + 328
00768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7 + 52715520*a^8*b^8*c^8*d
^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c^6*d^10 - 17891328*a^11*
```

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b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*a^13*b^3*c^3*d^13 + 4915
20*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*a^15*b*c*d^15))^(1/4)*((
((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (972405*a^2*b^
14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (11560479*a^4*b^12*c^6
*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a^6*b^10*c^4*d^7
)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8*c^2*d^9)/512)*1
i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 -
715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^
7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c
^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*a^12*b*c*d^12) - (-(
625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*d^3
+ 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b^14*
c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 17891328
*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*d^7
+ 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b^6*c
^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293760*
a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65536*
a^15*b*c*d^15))^(3/4)*((x^(1/2)*(6553600*a^23*b^4*d^25 + 78643200*a^22*b^5*
c*d^24 + 419430400*a^2*b^25*c^21*d^4 - 5420875776*a^3*b^24*c^20*d^5 + 31284
264960*a^4*b^23*c^19*d^6 - 104224784384*a^5*b^22*c^18*d^7 + 210842419200*a^
6*b^21*c^17*d^8 - 218396098560*a^7*b^20*c^16*d^9 - 105331556352*a^8*b^19*c^
15*d^10 + 910845542400*a^9*b^18*c^14*d^11 - 2125492912128*a^10*b^17*c^13*d^
12 + 3520229539840*a^11*b^16*c^12*d^13 - 4783425454080*a^12*b^15*c^11*d^14
+ 5470166188032*a^13*b^14*c^10*d^15 - 5154201927680*a^14*b^13*c^9*d^16 + 38
67903787008*a^15*b^12*c^8*d^17 - 2229880750080*a^16*b^11*c^7*d^18 + 9450710
63040*a^17*b^10*c^6*d^19 - 273892245504*a^18*b^9*c^5*d^20 + 45719224320*a^1
9*b^8*c^4*d^21 - 1490026496*a^20*b^7*c^3*d^22 - 810024960*a^21*b^6*c^2*d^23
)*1i)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*
c^15*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12
*c^12*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b
^9*c^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12
*b^6*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*
b^3*c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17)
) - ((-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^
4*c*d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a
^2*b^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 -
17891328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*
c^9*d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^
10*b^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 -
2293760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d
- 65536*a^15*b*c*d^15))^(1/4)*(1280*a^20*b^4*c*d^22 + 10240*a^2*b^22*c^19*d
^4 - 144640*a^3*b^21*c^18*d^5 + 922880*a^4*b^20*c^17*d^6 - 3450880*a^5*b^19
*c^16*d^7 + 8038400*a^6*b^18*c^15*d^8 - 10501120*a^7*b^17*c^14*d^9 + 465920
*a^8*b^16*c^13*d^10 + 31016960*a^9*b^15*c^12*d^11 - 77608960*a^10*b^14*c^11
*d^12 + 115315200*a^11*b^13*c^10*d^13 - 121172480*a^12*b^12*c^9*d^14 + 9438
```

```
2080*a^13*b^11*c^8*d^15 - 54978560*a^14*b^10*c^7*d^16 + 23618560*a^15*b^9*c
^6*d^17 - 7193600*a^16*b^8*c^5*d^18 + 1423360*a^17*b^7*c^4*d^19 - 143360*a^
18*b^6*c^3*d^20 - 1280*a^19*b^5*c^2*d^21))/(a^13*d^13 - b^13*c^13 - 78*a^2*
b^11*c^11*d^2 + 286*a^3*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*
c^8*d^5 - 1716*a^6*b^7*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^
8 + 715*a^9*b^4*c^4*d^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13
*a*b^12*c^12*d - 13*a^12*b*c*d^12))*1i)*1i + (x^(1/2)*(3872225*a^12*b^7*d^1
3 + 120299550*a^11*b^8*c*d^12 + 4862025*a^2*b^17*c^10*d^3 + 78440670*a^3*b^
16*c^9*d^4 + 537450669*a^4*b^15*c^8*d^5 + 2030593320*a^5*b^14*c^7*d^6 + 461
7534530*a^6*b^13*c^6*d^7 + 6551813940*a^7*b^12*c^5*d^8 + 5932052274*a^8*b^1
1*c^4*d^9 + 3440955560*a^9*b^10*c^3*d^10 + 1143306165*a^10*b^9*c^2*d^11)*1i
)/(65536*(a^18*d^18 + b^18*c^18 + 153*a^2*b^16*c^16*d^2 - 816*a^3*b^15*c^15
*d^3 + 3060*a^4*b^14*c^14*d^4 - 8568*a^5*b^13*c^13*d^5 + 18564*a^6*b^12*c^1
2*d^6 - 31824*a^7*b^11*c^11*d^7 + 43758*a^8*b^10*c^10*d^8 - 48620*a^9*b^9*c
^9*d^9 + 43758*a^10*b^8*c^8*d^10 - 31824*a^11*b^7*c^7*d^11 + 18564*a^12*b^6
*c^6*d^12 - 8568*a^13*b^5*c^5*d^13 + 3060*a^14*b^4*c^4*d^14 - 816*a^15*b^3*
c^3*d^15 + 153*a^16*b^2*c^2*d^16 - 18*a*b^17*c^17*d - 18*a^17*b*c*d^17))))
*(-(625*a*b^7*c^4 + 2401*a^5*b^3*d^4 + 3500*a^2*b^6*c^3*d + 6860*a^4*b^4*c*
d^3 + 7350*a^3*b^5*c^2*d^2)/(4096*a^16*d^16 + 4096*b^16*c^16 + 491520*a^2*b
^14*c^14*d^2 - 2293760*a^3*b^13*c^13*d^3 + 7454720*a^4*b^12*c^12*d^4 - 1789
1328*a^5*b^11*c^11*d^5 + 32800768*a^6*b^10*c^10*d^6 - 46858240*a^7*b^9*c^9*
d^7 + 52715520*a^8*b^8*c^8*d^8 - 46858240*a^9*b^7*c^7*d^9 + 32800768*a^10*b
^6*c^6*d^10 - 17891328*a^11*b^5*c^5*d^11 + 7454720*a^12*b^4*c^4*d^12 - 2293
760*a^13*b^3*c^3*d^13 + 491520*a^14*b^2*c^2*d^14 - 65536*a*b^15*c^15*d - 65
536*a^15*b*c*d^15))^(1/4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.479 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{\right.}{64\sqrt{2}\,c^{5/4}(bc-a}$$

Rubi [A] time = 1.02, antiderivative size = 703, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 471, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{5/4}(bc-ad)^4}+\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{5/4}(bc-ad)^4}-\frac{3\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{5/4}(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3b^{5/4}(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4}+\frac{3b^{5/4}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4}-\frac{3dx^{3/2}(ad+7bc)}{16c\left(c+dx^2\right)(bc-ad)^3}-\frac{x^{3/2}}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{3dx^{3/2}}{4\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*x^(3/2))/(4*(b*c - a*d)^2*(c + d*x^2)^2) - x^(3/2)/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (3*d*(7*b*c + a*d)*x^(3/2))/(16*c*(b*c - a*d)^3*(
c + d*x^2)) - (3*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)^
4) + (3*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15
*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(32*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c
 - a*d)^4) - (3*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(1/4)*(b*c - a*d)^4) - (3*d^(1/4)*(15*b
^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) + (3*d^(1/4)*(15*b^2*c^2
 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x])/(64*Sqrt[2]*c^(5/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c-9dx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(12c(2bc+ad)\right.}{\left(a+bx^4\right)\left(c+\right.}\right.}{16c(bc-}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

$$= -\frac{3dx^{3/2}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{x^{3/2}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3\left(c+dx^2\right.}$$

Mathematica [A]   time = 2.01, size = 604, normalized size = 0.86

$$\frac{\frac{3\sqrt{2}\sqrt[4]{d}\left(a^2d^2-18abcd-15b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{3\sqrt{2}\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{5/4}}+\frac{6\sqrt{2}\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{5/4}}+\frac{6\sqrt{2}\sqrt[4]{d}\left(a^2d^2-18abcd-15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{5/4}}+\frac{24\sqrt{2}b^{5/4}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}-\frac{24\sqrt{2}b^{5/4}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}-\frac{48\sqrt{2}b^{5/4}(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}+\frac{48\sqrt{2}b^{5/4}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}}-\frac{64b^2x^{3/2}(bc-ad)}{a+bx^2}+\frac{8dx^{3/2}(ad-bc)(3ad+13bc)}{c\left(c+dx^2\right)}-\frac{32dx^{3/2}(bc-ad)^2}{\left(c+dx^2\right)^2}}{128(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^2*(b*c - a*d)*x^(3/2))/(a + b*x^2) - (32*d*(b*c - a*d)^2*x^(3/2))/(
c + d*x^2)^2 + (8*d*(-(b*c) + a*d)*(13*b*c + 3*a*d)*x^(3/2))/(c*(c + d*x^2)
) - (48*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/a^(1/4) + (48*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (6*Sqrt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b
*c*d - a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (6
*Sqrt[2]*d^(1/4)*(-15*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*ArcTan[1 + (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)])/c^(5/4) + (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1/4) - (24*Sqrt[2
]*b^(5/4)*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/a^(1/4) + (3*Sqrt[2]*d^(1/4)*(-15*b^2*c^2 - 18*a*b*c*d + a^2*d^2)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4) + (3*Sq
rt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(5/4))/(128*(b*c - a*d)^4)
```

IntegrateAlgebraic [A]   time = 3.36, size = 467, normalized size = 0.66

$$\frac{3\left(-a^2d^{9/4}+18abcd^{5/4}+15b^2c^2\sqrt[4]{d}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4}+\frac{3\left(-a^2d^{9/4}+18abcd^{5/4}+15b^2c^2\sqrt[4]{d}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4}-\frac{x^{3/2}\left(-a^2cd^2+3a^2d^3x^2+17abc^2d+12abcd^2x^2+3abd^3x^4+8b^2c^3+33b^2c^2dx^2+21b^2cd^2x^4\right)}{16c\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)^3}-\frac{3\left(3ab^{5/4}d+b^{9/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}\sqrt[4]{a}(ad-bc)^4}-\frac{3\left(3ab^{5/4}d+b^{9/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}\sqrt[4]{a}(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(x^(3/2)*(8*b^2*c^3 + 17*a*b*c^2*d - a^2*c*d^2 + 33*b^2*c^2*d*x^2 + 1
2*a*b*c*d^2*x^2 + 3*a^2*d^3*x^2 + 21*b^2*c*d^2*x^4 + 3*a*b*d^3*x^4))/(c*(b*
c - a*d)^3*(a + b*x^2)*(c + d*x^2)^2) - (3*(b^(9/4)*c + 3*a*b^(5/4)*d)*ArcT
an[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(
1/4)*(-(b*c) + a*d)^4) + (3*(15*b^2*c^2*d^(1/4) + 18*a*b*c*d^(5/4) - a^2*d^
(9/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32
*Sqrt[2]*c^(5/4)*(b*c - a*d)^4) - (3*(b^(9/4)*c + 3*a*b^(5/4)*d)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(1/4)*
(-(b*c) + a*d)^4) + (3*(15*b^2*c^2*d^(1/4) + 18*a*b*c*d^(5/4) - a^2*d^(9/4)
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqr
t[2]*c^(5/4)*(b*c - a*d)^4)
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.57, size = 1238, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*x^(3/2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^
2 + a)) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(
2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sq
rt(2)*a^5*b*d^4) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4
 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*
c*d^3 + sqrt(2)*a^5*b*d^4) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3
/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3
+ 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6
) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/
4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*
d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/8*((a*b^3)^(3/4)*b
*c + 3*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 -
4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/8*((a*b^3)^(3/4)*b*c + 3*(
a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2
)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(
2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/64*(15*(c*d^3)^(3/4)*b^2*c^2 + 18
*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sq
rt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/
64*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2
*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6*d^
2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c
^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 1/16*(13*b*c*d^2*x^(7/2) + 3*a*d^3*x^(7/2)
+ 17*b*c^2*d*x^(3/2) - a*c*d^2*x^(3/2))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.03, size = 1067, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x^(3/2)/(b*x^
2+a)*c+9/16*b/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*a*d*ln((x-(a/b)^(1/4)*2^(1/2)
*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+9/8*b/(a
*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8
*b/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*a*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1
)+3/16*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+3/8*b^2/(a*d-
b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+3/8*b^2/
(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)+3/1
6*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(7/2)*a^2+5/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2
*x^(7/2)*a*b-13/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(7/2)*b^2-1/16*d^3/(a*d-
b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+9/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a*b*
c-17/16*d/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*b^2*c^2+3/64*d^2/(a*d-b*c)^4/c/(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-27/32*d/(a*d-b*
c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-45/64/(a
*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+3
/64*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)-1)*a^2-27/32*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)*a*b-45/64/(a*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)-1)*b^2+3/128*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*ln((x-(c
/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)
^(1/2)))*a^2-27/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1
/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-4
5/128/(a*d-b*c)^4*c/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(
c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.78, size = 791, normalized size = 1.13

$$\frac{3\left(b^{3}c+3\,ab^{2}d\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)}-\frac{3\left(15\,b^{2}c^{2}d+18\,abcd^{2}-a^{2}d^{3}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{4}c^{5}-4\,ab^{3}c^{4}d+6\,a^{2}b^{2}c^{3}d^{2}-4\,a^{3}bc^{2}d^{3}+a^{4}cd^{4}\right)}-\frac{3\left(7\,b^{2}cd^{2}+abd^{3}\right)x^{\frac{11}{2}}+3\left(11\,b^{2}c^{2}d+4\,abcd^{2}+a^{2}d^{3}\right)x^{\frac{7}{2}}+\left(8\,b^{2}c^{3}+17\,abc^{2}d-a^{2}cd^{2}\right)x^{\frac{3}{2}}}{16\left(ab^{3}c^{6}-3\,a^{2}b^{2}c^{5}d+3\,a^{3}bc^{4}d^{2}-a^{4}c^{3}d^{3}+\left(b^{4}c^{4}d^{2}-3\,ab^{3}c^{3}d^{3}+3\,a^{2}b^{2}c^{2}d^{4}-a^{3}bcd^{5}\right)x^{6}+\left(2\,b^{4}c^{5}d-5\,ab^{3}c^{4}d^{2}+3\,a^{2}b^{2}c^{3}d^{3}+a^{3}bc^{2}d^{4}-a^{4}cd^{5}\right)x^{4}+\left(b^{4}c^{6}-ab^{3}c^{5}d-3\,a^{2}b^{2}c^{4}d^{2}+5\,a^{3}bc^{3}d^{3}-2\,a^{4}c^{2}d^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/16*(b^3*c + 3*a*b^2*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(
1/4)*b^(3/4)))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4) - 3/128*(15*b^2*c^2*d + 18*a*b*c*d^2 - a^2*d^3)*(2*sqrt(2)*arct
```

```
an(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*s
qrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sq
rt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqr
t(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*
sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^5 - 4*a*b^3*c^4*d
+ 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4) - 1/16*(3*(7*b^2*c*d^2 +
 a*b*d^3)*x^(11/2) + 3*(11*b^2*c^2*d + 4*a*b*c*d^2 + a^2*d^3)*x^(7/2) + (8*
b^2*c^3 + 17*a*b*c^2*d - a^2*c*d^2)*x^(3/2))/(a*b^3*c^6 - 3*a^2*b^2*c^5*d +
 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2*b^2
*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^2*c^
3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2
*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2)
```

mupad [B] time = 7.63, size = 44169, normalized size = 62.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*
d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (258711
3*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22
*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d
^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/
2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 -
 (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (1
66924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637
*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^
9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24
)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20
*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^1
7*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7
*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 3527
16*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*
d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^
6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b
^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*
a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6
*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3
*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 +
 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c
^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73
282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040
```

```
*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^
12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 +
 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*
a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^26*c^23*d
^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5 + 612630528*a^3*b
^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 +
 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 89661636608
*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d^12 - 518906707968*a^10*b
^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 - 805866307584*a^12*b^15*c
^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 615914668032*a^14*b^13*c^10*d
^17 + 437990719488*a^15*b^12*c^9*d^18 - 263356153856*a^16*b^11*c^8*d^19 + 1
27919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 1295541862
4*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3
*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18
*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4
*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a
^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758
*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8
568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 15
3*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d
 + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5
*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c
^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 2
68435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^
13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5
 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 2159227
69920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*
b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d
^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 2684354
56*a*b^15*c^20*d))^(3/4) - (9*x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c
^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^
3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 +
718242228*a^6*b^13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^
11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d^1
8 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 30
60*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 3
1824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 +
 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^
12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^1
5 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8
*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b
^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6
*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^
16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*
a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^
```

```
16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 2
15922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728
*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4
*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 2
68435456*a*b^15*c^20*d))^(1/4) - ((((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^2
7)/16 + (1863*a^23*b^5*c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^
25*c^21*d^6)/16 + (2587113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d
^8)/8 + (5105997*a^6*b^22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 -
(93270447*a^8*b^20*c^16*d^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854
835*a^10*b^18*c^14*d^13)/2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033
*a^12*b^16*c^12*d^15)/8 - (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^
14*b^14*c^10*d^17)/4 - (166924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^
12*c^8*d^19)/8 + (8300637*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^
21)/2 + (19723743*a^19*b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 +
(327267*a^21*b^7*c^3*d^24)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23
- a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^1
8*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b
^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930
*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^1
1 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7
*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b
^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^
22*d) + (9*x^(1/2)*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d
^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*
c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d
^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^1
5 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 305345331
20*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^1
0*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8
- 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 7328287
9488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b
^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4
)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c
^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 28274
85184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^2
0*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d
^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 -
 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 6159
14668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 2633561538
56*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*
b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23
 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*
c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3
*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^
6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620
```

```
*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18
564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 8
16*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d))*(-(81*a^
8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d
^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c
^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 1
6777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^1
9*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 7328
2879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a
^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12
*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 3
0534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^
14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(3/4) + (9*x^(1/2)*(729*a^11*b^
8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453150*a^2*b^
17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d^8 + 7545
92274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 + 406721250*a^7*b^12*c
^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13))/(65536*
(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 8
16*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18
564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 -
 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^1
1 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^
14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-
(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6
*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5
*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^
21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^
14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4
- 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 19193135
1040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^
7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^
11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265
920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4))/(((((864*a*b^27*c^
23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16 - 5184*a^2*b^26*
c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^24*c^20*d^7)/16 -
 (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*d^9)/8 + (22410891*
a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/16 + (13320261*a^9*b
^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (279642213*a^11*b^17*
c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (274240863*a^13*b^15*c^1
1*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (166924665*a^15*b^13*c^9*d^
18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*b^11*c^7*d^20)/2 -
(6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*d^22)/16 - (4658715
*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 - (24867*a^22*b^6*c^2
*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^1
9*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^
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16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a
^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 -
352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^
10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b
^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^
2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*a^8*d^9 + 4100625*b^8*c^8
*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c
^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2
*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 -
 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*
b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d
^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 21592
2769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^1
0*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9
*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 26843
5456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^
26 - 167772160*a^2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*
a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18
*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273
093230592*a^9*b^18*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454
848*a^11*b^16*c^13*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a
^13*b^14*c^11*d^16 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b
^12*c^9*d^18 - 263356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7
*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 237
0830336*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5
*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^
2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5
*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*
a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 318
24*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3
060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*
a*b^17*c^19*d))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2
 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^
4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8
)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15
+ 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120
*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*
c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 -
191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 732828794
88*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3
*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(3/4)*
1i - (x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^
9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 5056713
36*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d
^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a
```

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^9*b^10*c^2*d^13)*9i)/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^1
7 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4
- 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^
7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*
d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^
7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*
d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b
^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*
a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a
^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b
*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 3
0534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728
*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c
^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 -
 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 939524096
0*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*
d))^(1/4) + ((((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^
5*c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (25
87113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*
b^22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^
16*d^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^
13)/2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)
/8 - (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4
- (166924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (830
0637*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^1
9*b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*
d^24)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*
a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4
*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280
*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 +
352716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c
^11*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^1
5*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^
18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) + (9*x^(1/2)*(-
(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6
*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5
*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^
21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^
14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4
- 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 19193135
1040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^
7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^
11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265
920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^26*c^
23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5 + 612630528*a
```

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^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c^19*d
^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 8966163
6608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d^12 - 518906707968*a^
10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 - 805866307584*a^12*b^
15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 615914668032*a^14*b^13*c^
10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 263356153856*a^16*b^11*c^8*d^19
 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 129554
18624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6
*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*d^18
- 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060
*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 318
24*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 4
3758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12
 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15
+ 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c
^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5
*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b
^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16
 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^
3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16
*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215
922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a
^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c
^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268
435456*a*b^15*c^20*d))^(3/4)*1i + (x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b
^18*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 2061356
85*a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d
^9 + 718242228*a^6*b^13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a
^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13)*9i)/(65536*(b^18*c^20 + a^18
*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*
d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14
*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^
11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^
6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3
*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 410
0625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 244069
20*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 15
2604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^1
6*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 939
5240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5
*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14
*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 1343
51945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*
a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*
d^14 - 268435456*a*b^15*c^20*d))^(1/4) - ((110716875*a*b^19*c^10*d^5)/26214
```

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4 - (1476225*a^11*b^9*d^15)/262144 + (38677095*a^10*b^10*c*d^14)/131072 + (
789780375*a^2*b^18*c^9*d^6)/131072 + (9357790275*a^3*b^17*c^8*d^7)/262144 +
 (3714477345*a^4*b^16*c^7*d^8)/32768 + (27140987115*a^5*b^15*c^6*d^9)/13107
2 + (14064979725*a^6*b^14*c^5*d^10)/65536 + (14608558575*a^7*b^13*c^4*d^11)
/131072 + (512796825*a^8*b^12*c^3*d^12)/32768 - (1242292545*a^9*b^11*c^2*d^
13)/262144)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*
c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16
*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8
*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 35
2716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10
*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5
*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*
c^4*d^19 - 21*a*b^20*c^22*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000
*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888
486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 58
32*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^
15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3
 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 13435194
5728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b
^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^
10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 93952
40960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c
^20*d))^(1/4) - atan(((((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (186
3*a^23*b^5*c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)
/16 + (2587113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (510
5997*a^6*b^22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^
8*b^20*c^16*d^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^1
8*c^14*d^13)/2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c
^12*d^15)/8 - (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10
*d^17)/4 - (166924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)
/8 + (8300637*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (197
23743*a^19*b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21
*b^7*c^3*d^24)/8 - (24867*a^22*b^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21
+ 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 598
5*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 1
16280*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d
^9 + 352716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*
b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 5426
4*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 13
30*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/
2)*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^
2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 162712
8*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^
16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a
^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17
```

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*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191
931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a
^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^
10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 20
13265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^
26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5 + 612630
528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c
^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 89
661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d^12 - 5189067079
68*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 - 805866307584*a^
12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 615914668032*a^14*b^
13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 263356153856*a^16*b^11*c^8
*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 1
2955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23 + 259522560*a^2
1*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*
d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 +
 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6
- 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^
9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8
*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*
d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*
b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^
3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*
a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5
*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 93952409
60*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11
*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7
+ 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945
728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*
b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14
- 268435456*a*b^15*c^20*d))^(3/4)*1i - (x^(1/2)*(729*a^11*b^8*d^15 + 410062
5*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 20
6135685*a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*
c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841
960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13)*9i)/(65536*(b^18*c^20 +
 a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*
c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12
*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b
^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^
12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^1
5*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9
+ 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 2
4406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6
 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 1677721
6*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2
```

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- 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 7328287948
8*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9
*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 +
 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 3053453
3120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2
*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4) - (((864*a*b^27*c^23*d^4 - (27*
a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (1
32597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5
*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17
*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/16 + (13320261*a^9*b^19*c^15*d^12
)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (279642213*a^11*b^17*c^13*d^14)/8
+ (501573033*a^12*b^16*c^12*d^15)/8 - (274240863*a^13*b^15*c^11*d^16)/4 + (
196146927*a^14*b^14*c^10*d^17)/4 - (166924665*a^15*b^13*c^9*d^18)/8 + (1446
2037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*b^11*c^7*d^20)/2 - (6325749*a^18
*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4
*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 - (24867*a^22*b^6*c^2*d^25)/8)/(b^
21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330
*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 542
64*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8
- 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*
c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*
a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 598
5*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a
*b^20*c^22*d) + (9*x^(1/2)*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b
^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*
a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a
^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b
*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 3
0534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728
*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c
^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 -
 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 939524096
0*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*
d))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^
2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7
 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688
*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^1
8*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^1
3*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^1
6 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 26
3356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 477521510
40*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*
c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(6553
6*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 -
 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 +
```

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18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8
 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d
^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*
d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*
(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b
^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a
^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*
c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*
b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^
4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931
351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*
b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*
d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 20132
65920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(3/4)*1i + (x^(1/2)*(72
9*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 454531
50*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d
^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 + 406721250*a
^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13)
*9i)/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*
c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c
^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^1
0*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11
*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^1
4*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*
c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 3433
5900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 -
 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(1677
7216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 20132
65920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^
12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^
6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 1919313
51040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11
*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^
13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4))/((((86
4*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16 - 518
4*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^24*c^2
0*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*d^9)/8 +
 (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/16 + (133
20261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (279642213
*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (274240863*a^
13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (166924665*a^15*
b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*b^11*c^7
*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*d^22)/16
 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 - (24867*a
^22*b^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*
```

a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349
*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 2
03490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*
d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13
*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349
*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*
a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*a^8*d^9 + 4100625*
b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^
3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*
a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5
*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 93952409
60*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11
*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7
+ 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945
728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*
b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14
- 268435456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b
^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533
725184*a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^
21*c^18*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^1
1 + 273093230592*a^9*b^18*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 72
4629454848*a^11*b^16*c^13*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 7548709
10976*a^13*b^14*c^11*d^16 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488
*a^15*b^12*c^9*d^18 - 263356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b
^10*c^7*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^2
2 - 2370830336*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a
^22*b^5*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 +
153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8
568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 +
43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^1
0 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d
^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^1
6 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*
c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4
*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*
b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^
6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 3053
4533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^
6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13
*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73
282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a
^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))
^(3/4) - (9*x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a
^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 5
05671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13

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*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 1321
8147*a^9*b^10*c^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*
d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d
^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13
*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^
10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5
*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c
^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*
a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 38884
86*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 583
2*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^1
5*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3
+ 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945
728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^
8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^1
0 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 939524
0960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^
20*d))^(1/4) + (((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*
b^5*c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (
2587113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^
6*b^22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*
c^16*d^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*
d^13)/2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^1
5)/8 - (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/
4 - (166924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8
300637*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a
^19*b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^
3*d^24)/8 - (24867*a^22*b^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^
20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b
^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a
^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 35
2716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^1
1*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*
b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18
*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) + (9*x^(1/2)*(-(8
1*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c
^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b
^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21
 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14
*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 -
73282879488*a^5*b^11*c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 1919313510
40*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*
c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11
 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 201326592
0*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d))^(1/4)*(16777216*a*b^26*c^23
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*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5 + 612630528*a^3
*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8
 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^20*c^17*d^10 - 896616366
08*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d^12 - 518906707968*a^10
*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 - 805866307584*a^12*b^15
*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 615914668032*a^14*b^13*c^10
*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 263356153856*a^16*b^11*c^8*d^19 +
 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*b^9*c^6*d^21 + 12955418
624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c
^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*c^20 + a^18*c^2*d^18 -
18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a
^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824
*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 437
58*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 -
 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 +
153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8
*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c
^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2
*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 -
 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*
b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d
^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 21592
2769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^1
0*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9
*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 26843
5456*a*b^15*c^20*d))^(3/4) + (9*x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18
*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*
a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9
+ 718242228*a^6*b^13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*
b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d
^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 +
3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 -
 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9
 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*
d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d
^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*(-(81*a^8*d^9 + 4100625*b
^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3
*b^5*c^5*d^4 + 3888486*a^4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a
^6*b^2*c^2*d^7 - 5832*a^7*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*
d^16 - 268435456*a^15*b*c^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 939524096
0*a^3*b^13*c^18*d^3 + 30534533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*
c^16*d^5 + 134351945728*a^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 +
 215922769920*a^8*b^8*c^13*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 1343519457
28*a^10*b^6*c^11*d^10 - 73282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b
^4*c^9*d^12 - 9395240960*a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 -
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 268435456*a*b^15*c^20*d))^(1/4) + ((110716875*a*b^19*c^10*d^5)/262144 - (1
476225*a^11*b^9*d^15)/262144 + (38677095*a^10*b^10*c*d^14)/131072 + (789780
375*a^2*b^18*c^9*d^6)/131072 + (9357790275*a^3*b^17*c^8*d^7)/262144 + (3714
477345*a^4*b^16*c^7*d^8)/32768 + (27140987115*a^5*b^15*c^6*d^9)/131072 + (1
4064979725*a^6*b^14*c^5*d^10)/65536 + (14608558575*a^7*b^13*c^4*d^11)/13107
2 + (512796825*a^8*b^12*c^3*d^12)/32768 - (1242292545*a^9*b^11*c^2*d^13)/26
2144)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d
^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*
d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*
c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 352716*a
^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^13
+ 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d
^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^
19 - 21*a*b^20*c^22*d)))*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7
*c^7*d^2 + 34335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^
4*b^4*c^4*d^5 - 1627128*a^5*b^3*c^3*d^6 + 152604*a^6*b^2*c^2*d^7 - 5832*a^7
*b*c*d^8)/(16777216*b^16*c^21 + 16777216*a^16*c^5*d^16 - 268435456*a^15*b*c
^6*d^15 + 2013265920*a^2*b^14*c^19*d^2 - 9395240960*a^3*b^13*c^18*d^3 + 305
34533120*a^4*b^12*c^17*d^4 - 73282879488*a^5*b^11*c^16*d^5 + 134351945728*a
^6*b^10*c^15*d^6 - 191931351040*a^7*b^9*c^14*d^7 + 215922769920*a^8*b^8*c^1
3*d^8 - 191931351040*a^9*b^7*c^12*d^9 + 134351945728*a^10*b^6*c^11*d^10 - 7
3282879488*a^11*b^5*c^10*d^11 + 30534533120*a^12*b^4*c^9*d^12 - 9395240960*
a^13*b^3*c^8*d^13 + 2013265920*a^14*b^2*c^7*d^14 - 268435456*a*b^15*c^20*d)
)^(1/4)*2i - ((3*x^(7/2)*(a^2*d^3 + 11*b^2*c^2*d + 4*a*b*c*d^2))/(16*(b^3*c
^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) - (x^(3/2)*(8*b^2*c^2 -
a^2*d^2 + 17*a*b*c*d))/(16*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2)) + (3*b*d*x^(11/2)*(a*d^2 + 7*b*c*d))/(16*(b^3*c^4 - a^3*c*d^3 + 3*a^2*
b*c^2*d^2 - 3*a*b^2*c^3*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 +
2*b*c*d) + b*d^2*x^6) - atan(((-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b
^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a
*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4
*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 3
2800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8
*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328*a^
12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 4
91520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 + 6561
*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)
/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^
14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891
328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d
^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b
^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293
760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(3
/4)*((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/
16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*
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b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*
d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/1
6 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (2
79642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (2742
40863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (1669246
65*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*
b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*
d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 -
(24867*a^22*b^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^2
0 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4
 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16
*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^
11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203
490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15
 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^1
8 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*b^9*c^4 +
6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^
3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^
3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 1
7891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c
^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^
11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 -
2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15)
)^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*
b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 -
 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a
^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*
c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*
d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16
- 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 2633
56153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040
*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^
4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*
(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 8
16*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18
564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 -
 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^1
1 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^
14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*1i
 - (x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*
c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336
*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^1
0 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9
*b^10*c^2*d^13)*9i)/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17
+ 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 -
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8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7
+ 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^
10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*
d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^
16 - 18*a*b^17*c^19*d))) - (-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*
c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^
16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^
13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 3280
0768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^
8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328*a^12*
b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 4915
20*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 + 6561*a^
4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4
096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*
c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328
*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7
+ 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*
c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760
*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(3/4)
*((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16
- 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^2
4*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*d^9
)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/16 +
 (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (2796
42213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (2742408
63*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (166924665*
a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*b^1
1*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*d^2
2)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 - (24
867*a^22*b^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 +
 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 -
20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^
7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*
c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490
*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 +
20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 -
 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) + (9*x^(1/2)*(-(81*b^9*c^4 + 656
1*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d
)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b
^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 1789
1328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*
d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*
b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 229
3760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(
1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^2
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5*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 28
27485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*
b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^1
5*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^1
4 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 6
15914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 2633561
53856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^
18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d
^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^
18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*
a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564
*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48
620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 +
 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14
- 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d)))*1i +
(x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d
^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336*a^
4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 +
 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^
10*c^2*d^13)*9i)/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 1
53*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 856
8*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 4
3758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10
- 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^1
3 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16
- 18*a*b^17*c^19*d))))/((-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d
^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*
c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*
c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 3280076
8*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 -
 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5
*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*
a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 + 6561*a^4*b
^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096
*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^1
4*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^
6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 5
2715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6
*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^
14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(3/4)*((
864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16 - 5
184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^24*c
^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*d^9)/8
 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/16 + (1
3320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (2796422
```

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13*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (274240863*
a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (166924665*a^1
5*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*b^11*c
^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*d^22)/
16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 - (24867
*a^22*b^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 21
0*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 203
49*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 +
 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^1
3*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^
13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 203
49*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 21
0*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*b^9*c^4 + 6561*a
^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(
4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14
*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 1789132
8*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7
 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6
*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 229376
0*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4
)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c
^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 28274
85184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^2
0*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d
^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 -
 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 6159
14668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 2633561538
56*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*
b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23
 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*
c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3
*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^
6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620
*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18
564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 8
16*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d))) - (9*x^(
1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d^14
+ 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336*a^4*b^
15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 + 406
721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c
^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*
b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b
^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^
8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824
*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 306
```

0*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*
b^17*c^19*d))) + (-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 43
74*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 -
65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^
3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b
^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 468582
40*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^
11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^
2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 + 6561*a^4*b^5*d^4
+ 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d
^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 -
2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*
c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520
*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 -
17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*
c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(3/4)*((864*a*b
^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^26)/16 - 5184*a^2
*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*a^4*b^24*c^20*d^7
)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c^18*d^9)/8 + (224
10891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^11)/16 + (13320261
*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2 - (279642213*a^11
*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (274240863*a^13*b^
15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (166924665*a^15*b^13*
c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a^17*b^11*c^7*d^20
)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*c^5*d^22)/16 - (4
658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/8 - (24867*a^22*b
^6*c^2*d^25)/8)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b
^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*
b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490
*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10
- 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*
c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16
*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*
b^2*c^4*d^19 - 21*a*b^20*c^22*d) + (9*x^(1/2)*(-(81*b^9*c^4 + 6561*a^4*b^5*
d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^
17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d
^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b
^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 5271
5520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^
10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*
b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*(1677
7216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772160*a^2*b^25*c^22*d^5
+ 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^20*d^7 - 2827485184*a
^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940786688*a^7*b^20*c^17*
d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a^9*b^18*c^15*d^12 - 5

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18906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^16*c^13*d^14 - 805866
307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^11*d^16 - 61591466803
2*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^18 - 263356153856*a^16
*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 47752151040*a^18*b^9*c^6
*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^20*b^7*c^4*d^23 + 2595
22560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))/(65536*(b^18*c^20 +
a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c
^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*
c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^
9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^1
2*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15
*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19*d))) + (9*x^(1/2)*(7
29*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a^10*b^9*c*d^14 + 45453
150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 505671336*a^4*b^15*c^7*
d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13*c^5*d^10 + 406721250*
a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13218147*a^9*b^10*c^2*d^13
))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^
18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^1
5*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*
c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b
^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*
b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^
19*d))) + ((110716875*a*b^19*c^10*d^5)/262144 - (1476225*a^11*b^9*d^15)/262
144 + (38677095*a^10*b^10*c*d^14)/131072 + (789780375*a^2*b^18*c^9*d^6)/131
072 + (9357790275*a^3*b^17*c^8*d^7)/262144 + (3714477345*a^4*b^16*c^7*d^8)/
32768 + (27140987115*a^5*b^15*c^6*d^9)/131072 + (14064979725*a^6*b^14*c^5*d
^10)/65536 + (14608558575*a^7*b^13*c^4*d^11)/131072 + (512796825*a^8*b^12*c
^3*d^12)/32768 - (1242292545*a^9*b^11*c^2*d^13)/262144)/(b^21*c^23 - a^21*c
^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d
^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17
*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^1
2*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 2939
30*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^1
4 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d
^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d)))*
(-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^
2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^
15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b
^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46
858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d
^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13
*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 6553
6*a^16*b*c*d^15))^(1/4)*2i + 2*atan(((-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 874
8*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 +
 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293
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760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*
d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*
b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 1789
1328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d
^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4
 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8
*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520
*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4
 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^
9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768
*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12
 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^
15))^(3/4)*(((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*
c*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587
113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^
22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16
*d^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13
)/2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8
 - (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 -
(166924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (83006
37*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*
b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^
24)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^
20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b
^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a
^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 35
2716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^1
1*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*
b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18
*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(8
1*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 +
972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d
 + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*
c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 468582
40*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 +
 32800768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4
*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^
16*b*c*d^15))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 16
7772160*a^2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^2
3*c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 +
6940786688*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 2730932305
92*a^9*b^18*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^1
1*b^16*c^13*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^1
4*c^11*d^16 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9
*d^18 - 263356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 -
```

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 47752151040*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336
*a^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^
25))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*
c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c
^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^1
0*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11
*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^1
4*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*
c^19*d))) - (9*x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 36790
2*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7
+ 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b
^13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 1
3218147*a^9*b^10*c^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c
^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^1
6*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c
^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8
*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*
b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^
2*c^4*d^16 - 18*a*b^17*c^19*d))) - (-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*
a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4
096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 229376
0*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^
5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^
8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 178913
28*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^1
3 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 +
 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c
^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a
^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 -
17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*
c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a
^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 -
 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15
))^(3/4)*(((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*
d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (258711
3*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22
*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d
^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/
2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 -
 (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (1
66924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637
*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^
9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24
)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20
*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^1
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7*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7
*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 3527
16*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*
d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^
6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b
^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) + (9*x^(1/2)*(-(81*
b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 97
2*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d +
 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^
12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240
*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 3
2800768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c
^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16
*b*c*d^15))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 1677
72160*a^2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*
c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 69
40786688*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592
*a^9*b^18*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*
b^16*c^13*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*
c^11*d^16 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d
^18 - 263356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 4
7752151040*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a
^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25
))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^
18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^1
5*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*
c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b
^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*
b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^
19*d))) + (9*x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*
a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 +
505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^1
3*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 132
18147*a^9*b^10*c^2*d^13))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3
*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*
d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^1
3*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c
^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^
5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*
c^4*d^16 - 18*a*b^17*c^19*d))))/((-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^
3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 409
6*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*
a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5
+ 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*
c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328
```

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*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13
+ 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 + 6
561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3
*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3
*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17
891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^
9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^1
1*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2
293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))
^(3/4)*(((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*d^
26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587113*
a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22*c
^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d^1
1)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/2
- (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 - (
274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (166
924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637*a
^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^9*
c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24)/
8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b
*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*
c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b
^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716
*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^
12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*
c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3
*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) - (9*x^(1/2)*(-(81*b^
9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*
a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 4
91520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12
*d^4 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a
^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 328
00768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4
*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b
*c*d^15))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 167772
160*a^2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*c^
20*d^7 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 6940
786688*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592*a
^9*b^18*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*b^
16*c^13*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*c^
11*d^16 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d^1
8 - 263356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 477
52151040*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a^2
0*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25))
/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^18
```

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*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^15*
d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*c^
12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b^7
*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*b^
4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^19
*d)))*1i - (x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902*a
^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 + 5
05671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^13
*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 1321
8147*a^9*b^10*c^2*d^13)*9i)/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c
^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^1
6*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c
^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8
*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*
b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^
2*c^4*d^16 - 18*a*b^17*c^19*d))) + (-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*
a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4
096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 229376
0*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^
5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^
8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 178913
28*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^1
3 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)*((-(81*b^9*c^4 +
 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c
^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a
^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 -
17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*
c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a
^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 -
 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15
))^(3/4)*(((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c*
d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (258711
3*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b^22
*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^16*d
^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d^13)/
2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^15)/8 -
 (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)/4 - (1
66924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 + (8300637
*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (19723743*a^19*b^
9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^7*c^3*d^24
)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20
*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^1
7*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7
*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 3527
16*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*
```

```
d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^
6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b
^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) + (9*x^(1/2)*(-(81*
b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*c*d^3 + 4374*a^2*b^7*c^2*d^2 + 97
2*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^16*c^16 - 65536*a^2*b^15*c^15*d +
 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^13*c^13*d^3 + 7454720*a^5*b^12*c^
12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 32800768*a^7*b^10*c^10*d^6 - 46858240
*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^8 - 46858240*a^10*b^7*c^7*d^9 + 3
2800768*a^11*b^6*c^6*d^10 - 17891328*a^12*b^5*c^5*d^11 + 7454720*a^13*b^4*c
^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 491520*a^15*b^2*c^2*d^14 - 65536*a^16
*b*c*d^15))^(1/4)*(16777216*a*b^26*c^23*d^4 + 262144*a^23*b^4*c*d^26 - 1677
72160*a^2*b^25*c^22*d^5 + 612630528*a^3*b^24*c^21*d^6 - 533725184*a^4*b^23*
c^20*d^7 - 2827485184*a^5*b^22*c^19*d^8 + 8081375232*a^6*b^21*c^18*d^9 + 69
40786688*a^7*b^20*c^17*d^10 - 89661636608*a^8*b^19*c^16*d^11 + 273093230592
*a^9*b^18*c^15*d^12 - 518906707968*a^10*b^17*c^14*d^13 + 724629454848*a^11*
b^16*c^13*d^14 - 805866307584*a^12*b^15*c^12*d^15 + 754870910976*a^13*b^14*
c^11*d^16 - 615914668032*a^14*b^13*c^10*d^17 + 437990719488*a^15*b^12*c^9*d
^18 - 263356153856*a^16*b^11*c^8*d^19 + 127919980544*a^17*b^10*c^7*d^20 - 4
7752151040*a^18*b^9*c^6*d^21 + 12955418624*a^19*b^8*c^5*d^22 - 2370830336*a
^20*b^7*c^4*d^23 + 259522560*a^21*b^6*c^3*d^24 - 13631488*a^22*b^5*c^2*d^25
))/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b*c^3*d^17 + 153*a^2*b^16*c^
18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c^16*d^4 - 8568*a^5*b^13*c^1
5*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11*c^13*d^7 + 43758*a^8*b^10*
c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b^8*c^10*d^10 - 31824*a^11*b
^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^13*b^5*c^7*d^13 + 3060*a^14*
b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*b^2*c^4*d^16 - 18*a*b^17*c^
19*d)))*1i + (x^(1/2)*(729*a^11*b^8*d^15 + 4100625*a*b^18*c^10*d^5 + 367902
*a^10*b^9*c*d^14 + 45453150*a^2*b^17*c^9*d^6 + 206135685*a^3*b^16*c^8*d^7 +
 505671336*a^4*b^15*c^7*d^8 + 754592274*a^5*b^14*c^6*d^9 + 718242228*a^6*b^
13*c^5*d^10 + 406721250*a^7*b^12*c^4*d^11 + 89841960*a^8*b^11*c^3*d^12 - 13
218147*a^9*b^10*c^2*d^13)*9i)/(65536*(b^18*c^20 + a^18*c^2*d^18 - 18*a^17*b
*c^3*d^17 + 153*a^2*b^16*c^18*d^2 - 816*a^3*b^15*c^17*d^3 + 3060*a^4*b^14*c
^16*d^4 - 8568*a^5*b^13*c^15*d^5 + 18564*a^6*b^12*c^14*d^6 - 31824*a^7*b^11
*c^13*d^7 + 43758*a^8*b^10*c^12*d^8 - 48620*a^9*b^9*c^11*d^9 + 43758*a^10*b
^8*c^10*d^10 - 31824*a^11*b^7*c^9*d^11 + 18564*a^12*b^6*c^8*d^12 - 8568*a^1
3*b^5*c^7*d^13 + 3060*a^14*b^4*c^6*d^14 - 816*a^15*b^3*c^5*d^15 + 153*a^16*
b^2*c^4*d^16 - 18*a*b^17*c^19*d))) - ((110716875*a*b^19*c^10*d^5)/262144 -
(1476225*a^11*b^9*d^15)/262144 + (38677095*a^10*b^10*c*d^14)/131072 + (7897
80375*a^2*b^18*c^9*d^6)/131072 + (9357790275*a^3*b^17*c^8*d^7)/262144 + (37
14477345*a^4*b^16*c^7*d^8)/32768 + (27140987115*a^5*b^15*c^6*d^9)/131072 +
(14064979725*a^6*b^14*c^5*d^10)/65536 + (14608558575*a^7*b^13*c^4*d^11)/131
072 + (512796825*a^8*b^12*c^3*d^12)/32768 - (1242292545*a^9*b^11*c^2*d^13)/
262144)/(b^21*c^23 - a^21*c^2*d^21 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21
*d^2 - 1330*a^3*b^18*c^20*d^3 + 5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^1
8*d^5 + 54264*a^6*b^15*c^17*d^6 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8*b^1
```

```
3*c^15*d^8 - 293930*a^9*b^12*c^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 352716
*a^11*b^10*c^12*d^11 + 293930*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^1
3 + 116280*a^14*b^7*c^9*d^14 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7
*d^16 - 5985*a^17*b^4*c^6*d^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*
d^19 - 21*a*b^20*c^22*d)))*(-(81*b^9*c^4 + 6561*a^4*b^5*d^4 + 8748*a^3*b^6*
c*d^3 + 4374*a^2*b^7*c^2*d^2 + 972*a*b^8*c^3*d)/(4096*a^17*d^16 + 4096*a*b^
16*c^16 - 65536*a^2*b^15*c^15*d + 491520*a^3*b^14*c^14*d^2 - 2293760*a^4*b^
13*c^13*d^3 + 7454720*a^5*b^12*c^12*d^4 - 17891328*a^6*b^11*c^11*d^5 + 3280
0768*a^7*b^10*c^10*d^6 - 46858240*a^8*b^9*c^9*d^7 + 52715520*a^9*b^8*c^8*d^
8 - 46858240*a^10*b^7*c^7*d^9 + 32800768*a^11*b^6*c^6*d^10 - 17891328*a^12*
b^5*c^5*d^11 + 7454720*a^13*b^4*c^4*d^12 - 2293760*a^14*b^3*c^3*d^13 + 4915
20*a^15*b^2*c^2*d^14 - 65536*a^16*b*c*d^15))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.480 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=703

$$-\frac{b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}-\frac{b^{7/4}(11}{}$$

Rubi [A] time = 1.01, antiderivative size = 703, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 471, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}+\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4}-\frac{b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4}-\frac{b^{7/4}(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4}+\frac{b^{7/4}(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4}-\frac{d\sqrt{x}(ad+23bc)}{16c\left(c+dx^2\right)(bc-ad)^3}-\frac{3d\sqrt{x}}{4\left(c+dx^2\right)^2(bc-ad)^2}-\frac{\sqrt{x}}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*d*Sqrt[x])/(4*(b*c - a*d)^2*(c + d*x^2)^2) - Sqrt[x]/(2*(b*c - a*d)*(a
+ b*x^2)*(c + d*x^2)^2) - (d*(23*b*c + a*d)*Sqrt[x])/(16*c*(b*c - a*d)^3*(c
 + d*x^2)) - (b^(7/4)*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (b^(7/4)*(b*c + 11*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^4)
+ (d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*
c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(32*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a
*d)^4) + (b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(3/4)*(b*c - a*d)^4) + (d^(3/4)*(77*b^2*c^2
+ 22*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4) - (d^(3/4)*(77*b^2*c^2 + 22*a
*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(64*Sqrt[2]*c^(7/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{c-11dx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2(bc-ad)}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{\text{Subst}\left(\int \frac{4c(2bc+ad)-8}{\left(a+bx^4\right)\left(c+\right.}\right.}{16c(bc-}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx\right.}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx\right.}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx\right.}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx\right.}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx\right.}$$

$$= -\frac{3d\sqrt{x}}{4(bc-ad)^2\left(c+dx^2\right)^2} - \frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3\left(c+dx\right.}$$

Mathematica [A] time = 2.39, size = 603, normalized size = 0.86

$$\frac{-\frac{8\sqrt{2}b^{7/4}(11ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{8\sqrt{2}b^{7/4}(11ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{16\sqrt{2}b^{7/4}(11ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{16\sqrt{2}b^{7/4}(11ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}+\frac{\sqrt{2}d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}+\frac{\sqrt{2}d^{3/4}\left(3a^2d^2-22abcd-77b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{c^{7/4}}+\frac{2\sqrt{2}d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{2\sqrt{2}d^{3/4}\left(-3a^2d^2+22abcd+77b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{c^{7/4}}-\frac{64b^2\sqrt{x}(bc-ad)}{a+bx^2}+\frac{8d\sqrt{x}(ad-bc)(ad+15bc)}{c\left(c+dx^2\right)}-\frac{32d\sqrt{x}(bc-ad)^2}{\left(c+dx^2\right)^2}}{128(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-64*b^2*(b*c - a*d)*Sqrt[x])/(a + b*x^2) - (32*d*(b*c - a*d)^2*Sqrt[x])/(
c + d*x^2)^2 + (8*d*(-(b*c) + a*d)*(15*b*c + a*d)*Sqrt[x])/(c*(c + d*x^2))
- (16*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/a^(3/4) + (16*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b
*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) -
(2*Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)])/c^(7/4) - (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4) + (8*Sqrt
[2]*b^(7/4)*(b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/a^(3/4) + (Sqrt[2]*d^(3/4)*(77*b^2*c^2 + 22*a*b*c*d - 3*a^2*d^2
)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4) + (Sq
rt[2]*d^(3/4)*(-77*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/c^(7/4))/(128*(b*c - a*d)^4)
```

IntegrateAlgebraic [A] time = 3.53, size = 465, normalized size = 0.66

$$-\frac{\left(11ab^{7/4}d+b^{11/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{3/4}(ad-bc)^4}+\frac{\left(11ab^{7/4}d+b^{11/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{3/4}(ad-bc)^4}+\frac{\left(-3a^2d^{11/4}+22abcd^{7/4}+77b^2c^2d^{3/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{7/4}(bc-ad)^4}-\frac{\left(-3a^2d^{11/4}+22abcd^{7/4}+77b^2c^2d^{3/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{7/4}(bc-ad)^4}-\frac{\sqrt{x}\left(-3a^2cd^2+a^2d^3x^2+19abc^2d+12abcd^2x^2+abd^3x^4+8b^2c^3+35b^2c^2dx^2+23b^2cd^2x^4\right)}{16c\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/16*(Sqrt[x]*(8*b^2*c^3 + 19*a*b*c^2*d - 3*a^2*c*d^2 + 35*b^2*c^2*d*x^2 +
 12*a*b*c*d^2*x^2 + a^2*d^3*x^2 + 23*b^2*c*d^2*x^4 + a*b*d^3*x^4))/(c*(b*c
- a*d)^3*(a + b*x^2)*(c + d*x^2)^2) - ((b^(11/4)*c + 11*a*b^(7/4)*d)*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(3/
4)*(-(b*c) + a*d)^4) + ((77*b^2*c^2*d^(3/4) + 22*a*b*c*d^(7/4) - 3*a^2*d^(1
1/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*
Sqrt[2]*c^(7/4)*(b*c - a*d)^4) + ((b^(11/4)*c + 11*a*b^(7/4)*d)*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(3/4)*(
-(b*c) + a*d)^4) - ((77*b^2*c^2*d^(3/4) + 22*a*b*c*d^(7/4) - 3*a^2*d^(11/4)
)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqr
t[2]*c^(7/4)*(b*c - a*d)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.32, size = 1217, normalized size = 1.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^
2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a
^5*d^4) + 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4
*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3
+ sqrt(2)*a^5*d^4) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*
c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*
a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) - 1/32*(77
*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*
a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/8*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3
)^(1/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*
b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a
^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/8*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)
*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^4*c^
4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c
*d^3 + sqrt(2)*a^5*d^4) - 1/64*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)
*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*
d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/64*(77*(c*d^3)^(1/
4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*
c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c
^2*d^4) - 1/2*b^2*sqrt(x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*(b*x^2 + a)) - 1/16*(15*b*c*d^2*x^(5/2) + a*d^3*x^(5/2) + 19*b*c^2*d*sq
rt(x) - 3*a*c*d^2*sqrt(x))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^
3*c*d^3)*(d*x^2 + c)^2)
```

maple **[A]** time = 0.03, size = 1094, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^2/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*a*d-1/2*b^3/(a*d-b*c)^4*x^(1/2)/(b*x^
2+a)*c+11/8*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
```

```
x^(1/2)+1)*d+1/8*b^3/(a*d-b*c)^4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)*c+11/8*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)*d+1/8*b^3/(a*d-b*c)^4*(a/b)^(1/4)/a*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c+11/16*b^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*
ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2)))*d+1/16*b^3/(a*d-b*c)^4*(a/b)^(1/4)/a*2^(1/2)*ln((x+(a/b)^(
1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
)))*c+1/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(5/2)*a^2+7/8*d^3/(a*d-b*c)^4/(d
*x^2+c)^2*x^(5/2)*a*b-15/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(5/2)*b^2+11/8*
d^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b*c-19/16*d/(a*d-b*c)^4/(d*x^2+c)^2*x
^(1/2)*b^2*c^2-3/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a^2+3/64*d^3/(a*d-b
*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2-11/
32*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)
+1)*a*b-77/64*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)*b^2+3/64*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)-1)*a^2-11/32*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-77/64*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+3/128*d^3/(a*d-b*c)^4/c^2*(c/d
)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-11/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^
(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2)))*a*b-77/128*d/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d
)^(1/2)))*b^2
```

maxima [A] time = 2.82, size = 889, normalized size = 1.26

$$\frac{\left(\frac{2\sqrt{2}(bc+11ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(bc+11ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(bc+11ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(bc+11ad)\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^2}{16\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)}-\frac{\left(23b^2cd^2+abd^3\right)x^{\frac{9}{2}}+\left(35b^2c^2d+12abcd^2+a^2d^3\right)x^{\frac{5}{2}}+\left(8b^2c^3+19abc^2d-3a^2cd^2\right)\sqrt{x}}{16\left(ab^3c^6-3a^2b^2c^5d+3a^3bc^4d^2-a^4c^3d^3+\left(b^4c^4d^2-3ab^3c^3d^3+3a^2b^2c^2d^4-a^3bcd^5\right)x^6+\left(2b^4c^5d-5ab^3c^4d^2+3a^2b^2c^3d^3+a^3bc^2d^4-a^4cd^5\right)x^4+\left(b^4c^6-ab^3c^5d-3a^2b^2c^4d^2+5a^3bc^3d^3-2a^4c^2d^4\right)x^2\right)}-\frac{\frac{2\sqrt{2}\left(77b^2c^2d+22abcd^2-3a^2d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(77b^2c^2d+22abcd^2-3a^2d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(77b^2c^2d+22abcd^2-3a^2d^3\right)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(77b^2c^2d+22abcd^2-3a^2d^3\right)\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{128\left(b^4c^5-4ab^3c^4d+6a^2b^2c^3d^2-4a^3bc^2d^3+a^4cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*(b*c + 11*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*(b*c + 11*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 sqrt(2)*(b*c + 11*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c + 11*a*d)*log(-sqrt(2)*a^(1/4)*b^(
1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/16*((23*b^2*c*d
^2 + a*b*d^3)*x^(9/2) + (35*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*x^(5/2) + (
8*b^2*c^3 + 19*a*b*c^2*d - 3*a^2*c*d^2)*sqrt(x))/(a*b^3*c^6 - 3*a^2*b^2*c^5
*d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2
*b^2*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^
```

```
2*c^3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2
*b^2*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2) - 1/128*(2*sqrt(2)*(77
*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*
d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*s
qrt(d))) + 2*sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*arctan(-1/2*
sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 -
3*a^2*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3
/4)*d^(1/4)) - sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*log(-sqrt(
2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*c
^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)
```

mupad [B] time = 7.01, size = 50125, normalized size = 71.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] ((x^(1/2)*(8*b^2*c^2 - 3*a^2*d^2 + 19*a*b*c*d))/(16*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^(5/2)*(a^2*d^3 + 35*b^2*c^2*d + 12*a*b*c
*d^2))/(16*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d*x^
(9/2)*(a*d^2 + 23*b*c*d))/(16*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x
^6) + 2*atan(((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^
4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4
*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(
16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2
013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^
4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^1
7*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191
931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*
a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c
^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(
(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a
^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a
^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*
c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*
b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^
4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931
351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*
b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*
d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 201
3265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*((((891*a^9*b^7
*d^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291
```

*a^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b
^13*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c
^4*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/2
56)*1i)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*
d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5
+ 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 71
5*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^1
2*c^16*d) + (-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4
+ 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d
^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16
777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 201
3265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*
b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*
d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 19193
1351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^
11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^1
0*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(3/4)*(((
-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^
2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^
5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c
^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b
^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4
- 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 1919313
51040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b
^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d
^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013
265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(8192*a^2*b^22*c
^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*b^2
0*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 81665
024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^15*
c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^13*d^14 -
122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 16185344*a^
14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^19 -
622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d
^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^
12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 -
1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 128
7*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b
^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(1/2)*(16777216*b^27*c^25*d^4 + 100663
296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c^
22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9 + 1
452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 + 707004
8845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 1128064352
2560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 79712852377
60*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 3274973380608*

```
a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001856*a^16
*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c^7*
d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 84253
0816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*
d^27)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*
b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b
^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^
8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824
*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3
060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*
a*b^17*c^21*d)))*1i) + (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^
7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17
*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 29
87403540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^
12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15))/(65536*(b^18*c^22 + a^18*c^4*d^1
8 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 30
60*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 3
1824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 +
 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*
d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d
^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))) - (-(81*a^8*d^11 + 351530
41*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 141618
4*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*
a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^
7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240
960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^1
1*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7
 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 13435194
5728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12
*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^
14 - 268435456*a*b^15*c^22*d))^(1/4)*((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3
 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*
d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^
9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 26843
5456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c
^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 1
34351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 21592276992
0*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*
c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12
 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456
*a*b^15*c^22*d))^(1/4)*((((891*a^9*b^7*d^15)/8192 + (77*b^16*c^9*d^6)/16 -
(33367697*a*b^15*c^8*d^7)/8192 - (6291*a^8*b^8*c*d^14)/2048 - (107777537*a^
2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^13*c^6*d^9)/1024 - (39606577*a^4*b^1
2*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4*d^11)/4096 + (198309*a^6*b^10*c^3*
d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256)*1i)/(b^13*c^17 - a^13*c^4*d^13 +
```

```
13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4
*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^
6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*
d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) + (-(81*a^8*d^11 + 35153041
*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*
a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^
6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*
d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 939524096
0*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*
c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 +
 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 1343519457
28*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b
^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14
 - 268435456*a*b^15*c^22*d))^(3/4)*(((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3
+ 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d
^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9
 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435
456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^
20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 13
4351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920
*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c
^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12
- 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*
a*b^15*c^22*d))^(1/4)*(8192*a^2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 1425
92*a^3*b^21*c^21*d^6 - 1723648*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^
8 - 32966656*a^6*b^18*c^18*d^9 + 81665024*a^7*b^17*c^17*d^10 - 150731776*a^
8*b^16*c^16*d^11 + 212486144*a^9*b^15*c^15*d^12 - 231069696*a^10*b^14*c^14*
d^13 + 193363456*a^11*b^13*c^13*d^14 - 122330624*a^12*b^12*c^12*d^15 + 5588
3776*a^13*b^11*c^11*d^16 - 16185344*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*
c^9*d^18 + 1205248*a^16*b^8*c^8*d^19 - 622592*a^17*b^7*c^7*d^20 + 145408*a^
18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c
^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b
^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*
c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d
^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) + (x^
(1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*
b^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^
8 - 366041628672*a^5*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 377
0791231488*a^7*b^20*c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 10068131
053568*a^9*b^18*c^16*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 1029675574
8864*a^11*b^16*c^14*d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 542980649779
2*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a
^15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^
10*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23
- 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^2
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2*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27)*1i)/(65536*(b^18*c^22 + a^18*c^
4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3
 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^
6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*
d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*
c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*
c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*1i) - (x^(1/2)*(9801
*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 285714
*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^
10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 11796710
2*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d
^15))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16
*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*
c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^
10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^1
1*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*
a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^
17*c^21*d))))/((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d
^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^
4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/
(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 +
2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a
^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^
17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 19
1931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488
*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*
c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*
((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*
a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*
a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16
*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2
*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d
^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 19193
1351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9
*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12
*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 20
13265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*((((891*a^9*b^
7*d^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (629
1*a^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*
b^13*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*
c^4*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/
256)*1i)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15
*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5
 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 7
15*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^
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12*c^16*d) + (-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4
 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*
d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(1
6777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 20
13265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4
*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17
*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 1919
31351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a
^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^
10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(3/4)*((
(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a
^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a
^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*
c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*
b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^
4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931
351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*
b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*
d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 201
3265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(8192*a^2*b^22*
c^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*b^
20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 8166
5024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^15
*c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^13*d^14
- 122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 16185344*a
^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^19
- 622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*
d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d
^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 -
 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 12
87*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*
b^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(1/2)*(16777216*b^27*c^25*d^4 + 10066
3296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c
^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9 +
1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 + 70700
48845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 112806435
22560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 7971285237
760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 3274973380608
*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001856*a^1
6*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c^7
*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 8425
30816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2
*d^27)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2
*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*
b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a
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^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 3182
4*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 +
3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18
*a*b^17*c^21*d)))*1i)*1i + (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^1
0*d^7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*
b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11
+ 2987403540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^
7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15)*1i)/(65536*(b^18*c^22 + a^18*
c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d
^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*
d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^1
3*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^
6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^
3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))) + (-(81*a^8*d^11 +
 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 -
 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 +
 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*
a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 -
9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*
a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c
^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 1
34351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 305345331
20*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2
*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*((-(81*a^8*d^11 + 35153041*b^8*
c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b
^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2
*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16
- 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3
*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*
d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 2159
22769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^
10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^
11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 26
8435456*a*b^15*c^22*d))^(1/4)*((((891*a^9*b^7*d^15)/8192 + (77*b^16*c^9*d^6
)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291*a^8*b^8*c*d^14)/2048 - (10777
7537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^13*c^6*d^9)/1024 - (39606577*
a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4*d^11)/4096 + (198309*a^6*b^
10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256)*1i)/(b^13*c^17 - a^13*c^4*
d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 +
715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716
*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b
^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) + (-(81*a^8*d^11 + 3
5153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1
416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 1
7820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^
```

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16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 93
95240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^
5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^1
6*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134
351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120
*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c
^9*d^14 - 268435456*a*b^15*c^22*d))^(3/4)*(((-(81*a^8*d^11 + 35153041*b^8*c
^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^
5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*
c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 -
 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*
b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d
^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 21592
2769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^1
0*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^1
1*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268
435456*a*b^15*c^22*d))^(1/4)*(8192*a^2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4
 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*
c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 81665024*a^7*b^17*c^17*d^10 - 15073
1776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^15*c^15*d^12 - 231069696*a^10*b^1
4*c^14*d^13 + 193363456*a^11*b^13*c^13*d^14 - 122330624*a^12*b^12*c^12*d^15
 + 55883776*a^13*b^11*c^11*d^16 - 16185344*a^14*b^10*c^10*d^17 + 1309696*a^
15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^19 - 622592*a^17*b^7*c^7*d^20 + 14
5408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/
(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 28
6*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a
^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^
4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d
) + (x^(1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 18622709
76*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*
c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^1
0 - 3770791231488*a^7*b^20*c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 1
0068131053568*a^9*b^18*c^16*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 102
96755748864*a^11*b^16*c^14*d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 54298
06497792*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^13*c^11*d^18 - 16710689
09568*a^15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c^9*d^20 - 162426519552*
a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^
6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351
744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27)*1i)/(65536*(b^18*c^22 +
a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c
^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*
c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^
9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^
12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^
15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*1i)*1i - (x^(
```

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1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^
8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b
^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12
- 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*
b^11*c^2*d^15)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 +
 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8
568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 +
 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^1
0 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9
*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d
^16 - 18*a*b^17*c^21*d)))))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 4017490
4*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 7872
26*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a
^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*
b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 +
30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 13435194572
8*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*
c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10
- 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240
960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^
22*d))^(1/4) - atan(((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 +
 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^1
6 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^1
3*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a
^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 -
46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*
c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a
^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(((((-(b^11*c^4 + 14641*a^4*
b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096
*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c
^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*
a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7
+ 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6
*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 229376
0*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4
)*(8192*a^2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6
 - 1723648*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^1
8*c^18*d^9 + 81665024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 2
12486144*a^9*b^15*c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a^1
1*b^13*c^13*d^14 - 122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*
d^16 - 16185344*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a
^16*b^8*c^8*d^19 - 622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17
152*a^19*b^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13
+ 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a
^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*
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b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^
7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(1/2)*(16777216*b^27
*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970
170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5
*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*
c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16
*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*
d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^1
7 + 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 +
 654475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 722678
5792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^
7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 235
9296*a^23*b^4*c^2*d^27))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*
d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d
^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15
*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^
12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b
^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2
*c^6*d^16 - 18*a*b^17*c^21*d))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*
b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a
^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a
^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 +
 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8
*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 1789132
8*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13
 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(3/4) + ((891*a^9*b^7*d
^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291*a
^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^1
3*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4
*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256
)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 -
286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716
*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*
b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16
*d))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2
*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^
15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*
a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6
 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^
7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 74547
20*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14
 - 65536*a^18*b*c*d^15))^(1/4)*1i - (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497
*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 543413
2341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*
c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113
```

554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15)*1i)/(65536*(b^18*c^2
2 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^
15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b
^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^
9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 1856
4*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 81
6*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))) - (-(b^11
*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*
b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d +
491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^1
2*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*
a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 +
32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*
c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^1
8*b*c*d^15))^(1/4)*(((((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3
+ 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c
^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c
^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768
*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8
- 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^
5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520
*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(8192*a^2*b^22*c^22*d^5 -
2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*b^20*c^20*d^7
+ 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 81665024*a^7*b^
17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^15*c^15*d^12
- 231069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^13*d^14 - 122330624
*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 16185344*a^14*b^10*c^
10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^19 - 622592*a^
17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d^22 + 768*
a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^
2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b
^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*
c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^1
1 - 13*a*b^12*c^16*d) + (x^(1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b^26
*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7 + 4
3464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9 + 14528764968
96*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 + 7070048845824*a^
8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 11280643522560*a^10*
b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 7971285237760*a^12*b^
15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^13*
c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c^9*
d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 + 117
07613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^21*
b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27))/(65
536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2

```
 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5
+ 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d
^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^1
1*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*
c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d
)))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*
d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^1
5*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a
^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6
- 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7
*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 745472
0*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14
- 65536*a^18*b*c*d^15))^(3/4) + ((891*a^9*b^7*d^15)/8192 + (77*b^16*c^9*d^6
)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291*a^8*b^8*c*d^14)/2048 - (10777
7537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^13*c^6*d^9)/1024 - (39606577*
a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4*d^11)/4096 + (198309*a^6*b^
10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256)/(b^13*c^17 - a^13*c^4*d^13
 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*
a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7
*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c
^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d))*(-(b^11*c^4 + 14641*a^4
*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(409
6*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*
c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328
*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7
 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^
6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 22937
60*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/
4)*1i + (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a
*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295
711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^
14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 1
0537245*a^8*b^11*c^2*d^15)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*
b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*
c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^1
1*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*
b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*
a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^
16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))))/((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5
324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16
+ 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2
293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^
11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*
a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 -
 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*
```

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c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(((((-(b^
11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*
a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d
+ 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c
^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 4685824
0*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9
+ 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^
4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a
^18*b*c*d^15))^(1/4)*(8192*a^2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 14259
2*a^3*b^21*c^21*d^6 - 1723648*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8
 - 32966656*a^6*b^18*c^18*d^9 + 81665024*a^7*b^17*c^17*d^10 - 150731776*a^8
*b^16*c^16*d^11 + 212486144*a^9*b^15*c^15*d^12 - 231069696*a^10*b^14*c^14*d
^13 + 193363456*a^11*b^13*c^13*d^14 - 122330624*a^12*b^12*c^12*d^15 + 55883
776*a^13*b^11*c^11*d^16 - 16185344*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c
^9*d^18 + 1205248*a^16*b^8*c^8*d^19 - 622592*a^17*b^7*c^7*d^20 + 145408*a^1
8*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^
17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^
10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c
^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^
9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(
1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b
^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8
 - 366041628672*a^5*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770
791231488*a^7*b^20*c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 100681310
53568*a^9*b^18*c^16*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 10296755748
864*a^11*b^16*c^14*d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 5429806497792
*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a^
15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^1
0*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 -
 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22
*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27))/(65536*(b^18*c^22 + a^18*c^4*d^
18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3
060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 -
31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9
+ 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10
*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*
d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*(-(b^11*c^4 + 14641*a^4*
b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096
*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c
^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*
a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7
+ 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6
*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 229376
0*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(3/4
) + ((891*a^9*b^7*d^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*
```

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d^7)/8192 - (6291*a^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048
- (83346257*a^3*b^13*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7
338751*a^5*b^11*c^4*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^
7*b^9*c^2*d^13)/256)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a
^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*
b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5
*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^
11 - 13*a*b^12*c^16*d))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^
3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*
c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*
c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 3280076
8*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8
 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b
^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 49152
0*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4) - (x^(1/2)*(9801*a^10*b^9
*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b^10
*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099
206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^1
3*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15))/(65
536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2
 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5
+ 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d
^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^1
1*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*
c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d
))) + (-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^
2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b
^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720
*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^
6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b
^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454
720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^1
4 - 65536*a^18*b*c*d^15))^(1/4)*(((((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*
a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 40
96*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 22937
60*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d
^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11
*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 178
91328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*
d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(8192*a^2*b^2
2*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*
b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 81
665024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^
15*c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^13*d^1
4 - 122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 16185344
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*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^1
9 - 622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^
5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5
*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4
 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 +
1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^1
1*b^2*c^6*d^11 - 13*a*b^12*c^16*d) + (x^(1/2)*(16777216*b^27*c^25*d^4 + 100
663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24
*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9
+ 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 + 707
0048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 1128064
3522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 79712852
37760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 32749733806
08*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001856*a
^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c
^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 84
2530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c
^2*d^27))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*
b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b
^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^
8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824
*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3
060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*
a*b^17*c^21*d)))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726
*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 -
65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^
3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b
^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 4685
8240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*
d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*
b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(3/4) + ((891*a^9*b^7*d^15)/8192 + (77
*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291*a^8*b^8*c*d^14)/
2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^13*c^6*d^9)/1024
 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4*d^11)/4096 + (
198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256)/(b^13*c^17 -
a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^
14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d
^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 2
86*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d))*(-(b^11*c^
4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^1
0*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491
520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d
^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^1
0*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 328
00768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4
```

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*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b
*c*d^15))^(1/4) + (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 8
30454702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*
d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403
540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^
3*d^14 + 10537245*a^8*b^11*c^2*d^15))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 1
8*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^
4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*
a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 4375
8*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12
- 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 +
 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))))*(-(b^11*c^4 + 14641*a^4*b^7*
d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^1
9*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*
d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*
b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52
715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6
*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^
16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*2i
 + 2*atan(((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b
^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*
a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 74
54720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^
10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a
^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 +
 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^
2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(((((-(b^11*c^4 + 14641*a^4*b^7*d^4 +
5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16
 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 -
2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c
^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520
*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10
- 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3
*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(8192*a^
2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648
*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9
 + 81665024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a
^9*b^15*c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^1
3*d^14 - 122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 161
85344*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^
8*d^19 - 622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b
^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*
b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^1
3*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d
^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 7
```

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8*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(1/2)*(16777216*b^27*c^25*d^4
+ 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3
*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20
*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11
+ 7070048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 11
280643522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 797
1285237760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 327497
3380608*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001
856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*
b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24
 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*
b^4*c^2*d^27)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 +
153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 85
68*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 +
43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10
 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*
d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^
16 - 18*a*b^17*c^21*d)))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d
^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16
*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13
*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 328007
68*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^
8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*
b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 4915
20*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(3/4)*1i + (((891*a^9*b^7*d^15
)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291*a^8*
b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^13*c
^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4*d^
11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256)*1
i)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 -
 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 171
6*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9
*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^1
6*d))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^
2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b
^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720
*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^
6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b
^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454
720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^1
4 - 65536*a^18*b*c*d^15))^(1/4) + (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b
^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 54341323
41*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^
6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 11355
4584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15))/(65536*(b^18*c^22 + a
```

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^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^
19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c
^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9
*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^1
2*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^1
5*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))) - (-(b^11*c^4
+ 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*
c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 49152
0*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4
 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*
b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800
768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d
^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c
*d^15))^(1/4)*(((((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 72
6*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 -
 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d
^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*
b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 468
58240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5
*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17
*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(8192*a^2*b^22*c^22*d^5 - 2048*
a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d^6 - 1723648*a^4*b^20*c^20*d^7 + 94
39232*a^5*b^19*c^19*d^8 - 32966656*a^6*b^18*c^18*d^9 + 81665024*a^7*b^17*c^
17*d^10 - 150731776*a^8*b^16*c^16*d^11 + 212486144*a^9*b^15*c^15*d^12 - 231
069696*a^10*b^14*c^14*d^13 + 193363456*a^11*b^13*c^13*d^14 - 122330624*a^12
*b^12*c^12*d^15 + 55883776*a^13*b^11*c^11*d^16 - 16185344*a^14*b^10*c^10*d^
17 + 1309696*a^15*b^9*c^9*d^18 + 1205248*a^16*b^8*c^8*d^19 - 622592*a^17*b^
7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d^22 + 768*a^20*
b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^1
1*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^
12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d
^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 1
3*a*b^12*c^16*d) + (x^(1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b^26*c^24
*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7 + 434645
23776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9 + 1452876496896*a^
6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 + 7070048845824*a^8*b^1
9*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 11280643522560*a^10*b^17*
c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 7971285237760*a^12*b^15*c^
13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^13*c^11*
d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c^9*d^20
- 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 + 11707613
184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^21*b^6*c
^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27)*1i)/(6553
6*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 -
 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 +
```

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18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8
 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*
d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^
8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))
)*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^
2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*
c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7
*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 -
46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c
^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*
a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 -
65536*a^18*b*c*d^15))^(3/4)*1i + (((891*a^9*b^7*d^15)/8192 + (77*b^16*c^9*d
^6)/16 - (33367697*a*b^15*c^8*d^7)/8192 - (6291*a^8*b^8*c*d^14)/2048 - (107
777537*a^2*b^14*c^7*d^8)/2048 - (83346257*a^3*b^13*c^6*d^9)/1024 - (3960657
7*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5*b^11*c^4*d^11)/4096 + (198309*a^6*
b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*d^13)/256)*1i)/(b^13*c^17 - a^13*c^
4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3
+ 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 17
16*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10
*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d))*(-(b^11*c^4 + 146
41*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d
)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5
*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17
891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c
^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a
^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 -
 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15
))^(1/4) - (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 83045470
2*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7
295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5
*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14
+ 10537245*a^8*b^11*c^2*d^15))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*
b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*
c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^1
1*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*
b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*
a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^
16*b^2*c^6*d^16 - 18*a*b^17*c^21*d))))/((-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5
324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16
+ 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2
293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^
11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*
a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 -
 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*
c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(((((-(b^
```

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11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*
a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d
+ 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c
^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 4685824
0*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9
+ 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^
4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a
^18*b*c*d^15))^(1/4)*(8192*a^2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 14259
2*a^3*b^21*c^21*d^6 - 1723648*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8
 - 32966656*a^6*b^18*c^18*d^9 + 81665024*a^7*b^17*c^17*d^10 - 150731776*a^8
*b^16*c^16*d^11 + 212486144*a^9*b^15*c^15*d^12 - 231069696*a^10*b^14*c^14*d
^13 + 193363456*a^11*b^13*c^13*d^14 - 122330624*a^12*b^12*c^12*d^15 + 55883
776*a^13*b^11*c^11*d^16 - 16185344*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c
^9*d^18 + 1205248*a^16*b^8*c^8*d^19 - 622592*a^17*b^7*c^7*d^20 + 145408*a^1
8*b^6*c^6*d^21 - 17152*a^19*b^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^
17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^
10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c
^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^
9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) - (x^(
1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b
^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8
 - 366041628672*a^5*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770
791231488*a^7*b^20*c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 100681310
53568*a^9*b^18*c^16*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 10296755748
864*a^11*b^16*c^14*d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 5429806497792
*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a^
15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^1
0*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 -
 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22
*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27)*1i)/(65536*(b^18*c^22 + a^18*c^4
*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3
+ 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6
 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d
^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c
^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c
^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*(-(b^11*c^4 + 14641*a
^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4
096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^1
4*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 178913
28*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d
^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*
b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 229
3760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(
3/4)*1i + (((891*a^9*b^7*d^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^
15*c^8*d^7)/8192 - (6291*a^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8
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)/2048 - (83346257*a^3*b^13*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/20
48 + (7338751*a^5*b^11*c^4*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (
5265*a^7*b^9*c^2*d^13)/256)*1i)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*
d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4
- 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1
287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11
*b^2*c^6*d^11 - 13*a*b^12*c^16*d))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a
^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 409
6*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 229376
0*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^
5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*
b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 1789
1328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d
^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*1i + (x^(1/2)*
(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 2
85714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c
^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 117
967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*
c^2*d^15)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*
a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a
^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 4375
8*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 3
1824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13
 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 -
 18*a*b^17*c^21*d))) + (-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3
 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c
^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c
^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768
*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8
- 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^
5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520
*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*(((((-(b^11*c^4 + 14641*a^
4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(40
96*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14
*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 1789132
8*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^
7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b
^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293
760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1
/4)*(8192*a^2*b^22*c^22*d^5 - 2048*a*b^23*c^23*d^4 + 142592*a^3*b^21*c^21*d
^6 - 1723648*a^4*b^20*c^20*d^7 + 9439232*a^5*b^19*c^19*d^8 - 32966656*a^6*b
^18*c^18*d^9 + 81665024*a^7*b^17*c^17*d^10 - 150731776*a^8*b^16*c^16*d^11 +
 212486144*a^9*b^15*c^15*d^12 - 231069696*a^10*b^14*c^14*d^13 + 193363456*a
^11*b^13*c^13*d^14 - 122330624*a^12*b^12*c^12*d^15 + 55883776*a^13*b^11*c^1
1*d^16 - 16185344*a^14*b^10*c^10*d^17 + 1309696*a^15*b^9*c^9*d^18 + 1205248
```

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*a^16*b^8*c^8*d^19 - 622592*a^17*b^7*c^7*d^20 + 145408*a^18*b^6*c^6*d^21 -
17152*a^19*b^5*c^5*d^22 + 768*a^20*b^4*c^4*d^23))/(b^13*c^17 - a^13*c^4*d^1
3 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715
*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^
7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*
c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d) + (x^(1/2)*(16777216*b^
27*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 39
70170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a
^5*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^2
0*c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^
16*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^1
4*d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d
^17 + 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19
 + 654475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226
785792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*
b^7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2
359296*a^23*b^4*c^2*d^27)*1i)/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b
*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c
^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11
*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b
^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a
^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^1
6*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324
*a^3*b^8*c*d^3 + 726*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4
096*a^3*b^16*c^16 - 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293
760*a^6*b^13*c^13*d^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*
d^5 + 32800768*a^9*b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^1
1*b^8*c^8*d^8 - 46858240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17
891328*a^14*b^5*c^5*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3
*d^13 + 491520*a^17*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(3/4)*1i + (((891*
a^9*b^7*d^15)/8192 + (77*b^16*c^9*d^6)/16 - (33367697*a*b^15*c^8*d^7)/8192
 - (6291*a^8*b^8*c*d^14)/2048 - (107777537*a^2*b^14*c^7*d^8)/2048 - (8334625
7*a^3*b^13*c^6*d^9)/1024 - (39606577*a^4*b^12*c^5*d^10)/2048 + (7338751*a^5
*b^11*c^4*d^11)/4096 + (198309*a^6*b^10*c^3*d^12)/2048 + (5265*a^7*b^9*c^2*
d^13)/256)*1i)/(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^1
1*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^
12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d
^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 1
3*a*b^12*c^16*d))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 72
6*a^2*b^9*c^2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 -
 65536*a^4*b^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d
^3 + 7454720*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*
b^10*c^10*d^6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 468
58240*a^12*b^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5
*d^11 + 7454720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17
```

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*b^2*c^2*d^14 - 65536*a^18*b*c*d^15))^(1/4)*1i - (x^(1/2)*(9801*a^10*b^9*d^
17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*
d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8099206
482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c
^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15)*1i)/(65
536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2
 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d^5
+ 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^14*d
^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*c^1
1*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*
c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d
)))))*(-(b^11*c^4 + 14641*a^4*b^7*d^4 + 5324*a^3*b^8*c*d^3 + 726*a^2*b^9*c^
2*d^2 + 44*a*b^10*c^3*d)/(4096*a^19*d^16 + 4096*a^3*b^16*c^16 - 65536*a^4*b
^15*c^15*d + 491520*a^5*b^14*c^14*d^2 - 2293760*a^6*b^13*c^13*d^3 + 7454720
*a^7*b^12*c^12*d^4 - 17891328*a^8*b^11*c^11*d^5 + 32800768*a^9*b^10*c^10*d^
6 - 46858240*a^10*b^9*c^9*d^7 + 52715520*a^11*b^8*c^8*d^8 - 46858240*a^12*b
^7*c^7*d^9 + 32800768*a^13*b^6*c^6*d^10 - 17891328*a^14*b^5*c^5*d^11 + 7454
720*a^15*b^4*c^4*d^12 - 2293760*a^16*b^3*c^3*d^13 + 491520*a^17*b^2*c^2*d^1
4 - 65536*a^18*b*c*d^15))^(1/4) - atan(((((891*a^9*b^7*d^15 + 39424*b^16*c^
9*d^6 - 33367697*a*b^15*c^8*d^7 - 25164*a^8*b^8*c*d^14 - 431110148*a^2*b^14
*c^7*d^8 - 666770056*a^3*b^13*c^6*d^9 - 158426308*a^4*b^12*c^5*d^10 + 14677
502*a^5*b^11*c^4*d^11 + 793236*a^6*b^10*c^3*d^12 + 168480*a^7*b^9*c^2*d^13)
/(8192*(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d
^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 +
 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715
*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12
*c^16*d)) + (((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^
4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4
*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(
16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2
013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^
4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^1
7*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191
931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*
a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c
^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(
67108864*a^2*b^22*c^22*d^5 - 16777216*a*b^23*c^23*d^4 + 1168113664*a^3*b^21
*c^21*d^6 - 14120124416*a^4*b^20*c^20*d^7 + 77326188544*a^5*b^19*c^19*d^8 -
 270062845952*a^6*b^18*c^18*d^9 + 668999876608*a^7*b^17*c^17*d^10 - 1234794
708992*a^8*b^16*c^16*d^11 + 1740686491648*a^9*b^15*c^15*d^12 - 189292294963
2*a^10*b^14*c^14*d^13 + 1584033431552*a^11*b^13*c^13*d^14 - 1002132471808*a
^12*b^12*c^12*d^15 + 457799892992*a^13*b^11*c^11*d^16 - 132590338048*a^14*b
^10*c^10*d^17 + 10729029632*a^15*b^9*c^9*d^18 + 9873391616*a^16*b^8*c^8*d^1
9 - 5100273664*a^17*b^7*c^7*d^20 + 1191182336*a^18*b^6*c^6*d^21 - 140509184
*a^19*b^5*c^5*d^22 + 6291456*a^20*b^4*c^4*d^23))/(8192*(b^13*c^17 - a^13*c^
```

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4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3
+ 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 17
16*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10
*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d)) - (x^(1/2)*(16777
216*b^27*c^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^
6 + 3970170880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 36604162
8672*a^5*b^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a
^7*b^20*c^18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b
^18*c^16*d^13 + 11280643522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^
16*c^14*d^15 + 7971285237760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*
c^12*d^17 + 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^1
0*d^19 + 654475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21
+ 7226785792*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536
*a^20*b^7*c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^
26 + 2359296*a^23*b^4*c^2*d^27))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^1
7*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^1
4*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b
^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^1
0*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 856
8*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*
a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d
^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^
5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*
d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268
435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13
*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 +
 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769
920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^
6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^
12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 2684354
56*a*b^15*c^22*d))^(3/4))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*
a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226
*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7
*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*
c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30
534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*
a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^
15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 -
73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 939524096
0*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22
*d))^(1/4) - (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454
702*a*b^18*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 +
 7295711720*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a
^5*b^14*c^5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^1
4 + 10537245*a^8*b^11*c^2*d^15))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^1
```

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7*b*c^5*d^17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^1
4*c^18*d^4 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b
^11*c^15*d^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^1
0*b^8*c^12*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 856
8*a^13*b^5*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*
a^16*b^2*c^6*d^16 - 18*a*b^17*c^21*d)))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d
^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^
5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*
d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268
435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13
*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 +
 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769
920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^
6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^
12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 2684354
56*a*b^15*c^22*d))^(1/4)*1i - (((891*a^9*b^7*d^15 + 39424*b^16*c^9*d^6 - 33
367697*a*b^15*c^8*d^7 - 25164*a^8*b^8*c*d^14 - 431110148*a^2*b^14*c^7*d^8 -
 666770056*a^3*b^13*c^6*d^9 - 158426308*a^4*b^12*c^5*d^10 + 14677502*a^5*b^
11*c^4*d^11 + 793236*a^6*b^10*c^3*d^12 + 168480*a^7*b^9*c^2*d^13)/(8192*(b^
13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a
^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*
b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c
^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d))
 + (((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 117394
20*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 551
76*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b
^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*
a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^1
9*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 19
1931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*
a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c
^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 +
 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(67108864*a
^2*b^22*c^22*d^5 - 16777216*a*b^23*c^23*d^4 + 1168113664*a^3*b^21*c^21*d^6
- 14120124416*a^4*b^20*c^20*d^7 + 77326188544*a^5*b^19*c^19*d^8 - 270062845
952*a^6*b^18*c^18*d^9 + 668999876608*a^7*b^17*c^17*d^10 - 1234794708992*a^8
*b^16*c^16*d^11 + 1740686491648*a^9*b^15*c^15*d^12 - 1892922949632*a^10*b^1
4*c^14*d^13 + 1584033431552*a^11*b^13*c^13*d^14 - 1002132471808*a^12*b^12*c
^12*d^15 + 457799892992*a^13*b^11*c^11*d^16 - 132590338048*a^14*b^10*c^10*d
^17 + 10729029632*a^15*b^9*c^9*d^18 + 9873391616*a^16*b^8*c^8*d^19 - 510027
3664*a^17*b^7*c^7*d^20 + 1191182336*a^18*b^6*c^6*d^21 - 140509184*a^19*b^5*
c^5*d^22 + 6291456*a^20*b^4*c^4*d^23))/(8192*(b^13*c^17 - a^13*c^4*d^13 + 1
3*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*
b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6
*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d
```

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^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d)) + (x^(1/2)*(16777216*b^27*c
^25*d^4 + 100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 397017
0880*a^3*b^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b
^22*c^20*d^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^
18*d^11 + 7070048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d
^13 + 11280643522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^
15 + 7971285237760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17
+ 3274973380608*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 6
54475001856*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 72267857
92*a^18*b^9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*
c^5*d^24 + 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 23592
96*a^23*b^4*c^2*d^27))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^
17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4
 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d
^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12
*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5
*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c
^6*d^16 - 18*a*b^17*c^21*d)))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174
904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 78
7226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376
*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^1
5*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3
+ 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945
728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^
8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^1
0 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 93952
40960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*
c^22*d))^(3/4))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*
d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c
^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)
/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 +
 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*
a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c
^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 1
91931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 7328287948
8*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3
*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)
 + (x^(1/2)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18
*c^9*d^8 - 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 729571172
0*a^3*b^16*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^
5*d^12 - 117967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 105372
45*a^8*b^11*c^2*d^15))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^
17 + 153*a^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4
 - 8568*a^5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d
^7 + 43758*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12
```

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*d^10 - 31824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5
*c^9*d^13 + 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c
^6*d^16 - 18*a*b^17*c^21*d)))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174
904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 78
7226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376
*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^1
5*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3
+ 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945
728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^
8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^1
0 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 93952
40960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*
c^22*d))^(1/4)*1i)/((((891*a^9*b^7*d^15 + 39424*b^16*c^9*d^6 - 33367697*a*b
^15*c^8*d^7 - 25164*a^8*b^8*c*d^14 - 431110148*a^2*b^14*c^7*d^8 - 666770056
*a^3*b^13*c^6*d^9 - 158426308*a^4*b^12*c^5*d^10 + 14677502*a^5*b^11*c^4*d^1
1 + 793236*a^6*b^10*c^3*d^12 + 168480*a^7*b^9*c^2*d^13)/(8192*(b^13*c^17 -
a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^
14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d
^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 2
86*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d)) + (((-(81*
a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6
*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3
*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 +
 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c
^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73
282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040
*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^
14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 +
 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 201326592
0*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(67108864*a^2*b^22*c^
22*d^5 - 16777216*a*b^23*c^23*d^4 + 1168113664*a^3*b^21*c^21*d^6 - 14120124
416*a^4*b^20*c^20*d^7 + 77326188544*a^5*b^19*c^19*d^8 - 270062845952*a^6*b^
18*c^18*d^9 + 668999876608*a^7*b^17*c^17*d^10 - 1234794708992*a^8*b^16*c^16
*d^11 + 1740686491648*a^9*b^15*c^15*d^12 - 1892922949632*a^10*b^14*c^14*d^1
3 + 1584033431552*a^11*b^13*c^13*d^14 - 1002132471808*a^12*b^12*c^12*d^15 +
 457799892992*a^13*b^11*c^11*d^16 - 132590338048*a^14*b^10*c^10*d^17 + 1072
9029632*a^15*b^9*c^9*d^18 + 9873391616*a^16*b^8*c^8*d^19 - 5100273664*a^17*
b^7*c^7*d^20 + 1191182336*a^18*b^6*c^6*d^21 - 140509184*a^19*b^5*c^5*d^22 +
 6291456*a^20*b^4*c^4*d^23))/(8192*(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c
^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d
^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7
+ 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a
^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d)) - (x^(1/2)*(16777216*b^27*c^25*d^4 +
100663296*a*b^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b
^24*c^22*d^7 + 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d
```

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^9 + 1452876496896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 +
7070048845824*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 1128
0643522560*a^10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 79712
85237760*a^12*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 32749733
80608*a^14*b^13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 65447500185
6*a^16*b^11*c^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^
9*c^7*d^22 + 11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 +
 842530816*a^21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^
4*c^2*d^27))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a
^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^
5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758
*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31
824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13
+ 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 -
18*a*b^17*c^21*d))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*
c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b
^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d
^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^
15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533
120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^
10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8
 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 732828
79488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13
*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(
3/4))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 1173
9420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 5
5176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216
*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 201326592
0*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c
^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 -
191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 19193135104
0*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5
*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13
 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4) - (x^(1/2
)*(9801*a^10*b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 -
 285714*a^9*b^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16
*c^7*d^10 + 8099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 1
17967102*a^6*b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^1
1*c^2*d^15))/(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a
^2*b^16*c^20*d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^
5*b^13*c^17*d^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758
*a^8*b^10*c^14*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31
824*a^11*b^7*c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13
+ 3060*a^14*b^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 -
18*a*b^17*c^21*d))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*
```

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c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b
^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d
^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^
15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533
120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^
10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8
 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 732828
79488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13
*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(
1/4) + (((891*a^9*b^7*d^15 + 39424*b^16*c^9*d^6 - 33367697*a*b^15*c^8*d^7 -
 25164*a^8*b^8*c*d^14 - 431110148*a^2*b^14*c^7*d^8 - 666770056*a^3*b^13*c^6
*d^9 - 158426308*a^4*b^12*c^5*d^10 + 14677502*a^5*b^11*c^4*d^11 + 793236*a^
6*b^10*c^3*d^12 + 168480*a^7*b^9*c^2*d^13)/(8192*(b^13*c^17 - a^13*c^4*d^13
 + 13*a^12*b*c^5*d^12 + 78*a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*
a^4*b^9*c^13*d^4 - 1287*a^5*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7
*b^6*c^10*d^7 + 1287*a^8*b^5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c
^7*d^10 - 78*a^11*b^2*c^6*d^11 - 13*a*b^12*c^16*d)) + (((-(81*a^8*d^11 + 35
153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 14
16184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17
820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 + 16777216*a^1
6*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 939
5240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73282879488*a^5
*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16
*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 1343
51945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 + 30534533120*
a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^
9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*(67108864*a^2*b^22*c^22*d^5 - 1677
7216*a*b^23*c^23*d^4 + 1168113664*a^3*b^21*c^21*d^6 - 14120124416*a^4*b^20*
c^20*d^7 + 77326188544*a^5*b^19*c^19*d^8 - 270062845952*a^6*b^18*c^18*d^9 +
 668999876608*a^7*b^17*c^17*d^10 - 1234794708992*a^8*b^16*c^16*d^11 + 17406
86491648*a^9*b^15*c^15*d^12 - 1892922949632*a^10*b^14*c^14*d^13 + 158403343
1552*a^11*b^13*c^13*d^14 - 1002132471808*a^12*b^12*c^12*d^15 + 457799892992
*a^13*b^11*c^11*d^16 - 132590338048*a^14*b^10*c^10*d^17 + 10729029632*a^15*
b^9*c^9*d^18 + 9873391616*a^16*b^8*c^8*d^19 - 5100273664*a^17*b^7*c^7*d^20
+ 1191182336*a^18*b^6*c^6*d^21 - 140509184*a^19*b^5*c^5*d^22 + 6291456*a^20
*b^4*c^4*d^23))/(8192*(b^13*c^17 - a^13*c^4*d^13 + 13*a^12*b*c^5*d^12 + 78*
a^2*b^11*c^15*d^2 - 286*a^3*b^10*c^14*d^3 + 715*a^4*b^9*c^13*d^4 - 1287*a^5
*b^8*c^12*d^5 + 1716*a^6*b^7*c^11*d^6 - 1716*a^7*b^6*c^10*d^7 + 1287*a^8*b^
5*c^9*d^8 - 715*a^9*b^4*c^8*d^9 + 286*a^10*b^3*c^7*d^10 - 78*a^11*b^2*c^6*d
^11 - 13*a*b^12*c^16*d)) + (x^(1/2)*(16777216*b^27*c^25*d^4 + 100663296*a*b
^26*c^24*d^5 - 1862270976*a^2*b^25*c^23*d^6 + 3970170880*a^3*b^24*c^22*d^7
+ 43464523776*a^4*b^23*c^21*d^8 - 366041628672*a^5*b^22*c^20*d^9 + 14528764
96896*a^6*b^21*c^19*d^10 - 3770791231488*a^7*b^20*c^18*d^11 + 7070048845824
*a^8*b^19*c^17*d^12 - 10068131053568*a^9*b^18*c^16*d^13 + 11280643522560*a^
10*b^17*c^15*d^14 - 10296755748864*a^11*b^16*c^14*d^15 + 7971285237760*a^12
```

```
*b^15*c^13*d^16 - 5429806497792*a^13*b^14*c^12*d^17 + 3274973380608*a^14*b^
13*c^11*d^18 - 1671068909568*a^15*b^12*c^10*d^19 + 654475001856*a^16*b^11*c
^9*d^20 - 162426519552*a^17*b^10*c^8*d^21 + 7226785792*a^18*b^9*c^7*d^22 +
11707613184*a^19*b^8*c^6*d^23 - 4677697536*a^20*b^7*c^5*d^24 + 842530816*a^
21*b^6*c^4*d^25 - 72351744*a^22*b^5*c^3*d^26 + 2359296*a^23*b^4*c^2*d^27))/
(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*
d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d
^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^1
4*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*
c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b
^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^2
1*d)))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 117
39420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 +
55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(1677721
6*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 20132659
20*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*
c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 -
 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 1919313510
40*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^
5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^1
3 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(3/4))*(-(81*a
^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6*
c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3*
c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 +
16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c^
21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 732
82879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040*
a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^1
4*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 +
30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 2013265920
*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4) + (x^(1/2)*(9801*a^10*
b^9*d^17 + 35532497*b^19*c^10*d^7 + 830454702*a*b^18*c^9*d^8 - 285714*a^9*b
^10*c*d^16 + 5434132341*a^2*b^17*c^8*d^9 + 7295711720*a^3*b^16*c^7*d^10 + 8
099206482*a^4*b^15*c^6*d^11 + 2987403540*a^5*b^14*c^5*d^12 - 117967102*a^6*
b^13*c^4*d^13 - 113554584*a^7*b^12*c^3*d^14 + 10537245*a^8*b^11*c^2*d^15))/
(65536*(b^18*c^22 + a^18*c^4*d^18 - 18*a^17*b*c^5*d^17 + 153*a^2*b^16*c^20*
d^2 - 816*a^3*b^15*c^19*d^3 + 3060*a^4*b^14*c^18*d^4 - 8568*a^5*b^13*c^17*d
^5 + 18564*a^6*b^12*c^16*d^6 - 31824*a^7*b^11*c^15*d^7 + 43758*a^8*b^10*c^1
4*d^8 - 48620*a^9*b^9*c^13*d^9 + 43758*a^10*b^8*c^12*d^10 - 31824*a^11*b^7*
c^11*d^11 + 18564*a^12*b^6*c^10*d^12 - 8568*a^13*b^5*c^9*d^13 + 3060*a^14*b
^4*c^8*d^14 - 816*a^15*b^3*c^7*d^15 + 153*a^16*b^2*c^6*d^16 - 18*a*b^17*c^2
1*d)))*(-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 117
39420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 +
55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(1677721
6*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 20132659
```

```
20*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*
c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 -
 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 1919313510
40*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^
5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^1
3 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)))*(-(81*
a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*d^4 + 11739420*a^2*b^6
*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*c^4*d^7 + 55176*a^5*b^3
*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^10)/(16777216*b^16*c^23 +
 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^15 + 2013265920*a^2*b^14*c
^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534533120*a^4*b^12*c^19*d^4 - 73
282879488*a^5*b^11*c^18*d^5 + 134351945728*a^6*b^10*c^17*d^6 - 191931351040
*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^15*d^8 - 191931351040*a^9*b^7*c^
14*d^9 + 134351945728*a^10*b^6*c^13*d^10 - 73282879488*a^11*b^5*c^12*d^11 +
 30534533120*a^12*b^4*c^11*d^12 - 9395240960*a^13*b^3*c^10*d^13 + 201326592
0*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c^22*d))^(1/4)*2i
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.481 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=739

$$\frac{b^{9/4}(bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^4}-\frac{b^{9/4}(bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^4}-b^{9/4}(bc-$$

Rubi [A] time = 1.16, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {466, 472, 579, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}(bc-ad)^4}-\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}\,x\right)}{64\sqrt{2}\,c^{9/4}(bc-ad)^4}-\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{9/4}(bc-ad)^4}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{9/4}(bc-ad)^4}+\frac{b^{9/4}(bc-13ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^4}-\frac{b^{9/4}(bc-13ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\,x\right)}{8\sqrt{2}\,a^{5/4}(bc-ad)^4}-\frac{b^{9/4}(bc-13ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)^4}+\frac{b^{9/4}(bc-13ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)^4}+\frac{bx^{3/2}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx^{3/2}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x^(3/2))/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x^(3/2))
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 21*a*b*c*d -
 5*a^2*d^2)*x^(3/2))/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (b^(9/4)*(b*c -
 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*
(b*c - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26
*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sq
rt[2]*c^(9/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d
^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(9/4)*(b*c
 - a*d)^4) + (b^(9/4)*(b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) - (b^(9/4)*(b*c - 1
3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[
2]*a^(5/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^
(9/4)*(b*c - a*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)
*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad-9bdx^4\right)}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \text{Subst}\Big(\int \frac{x^2\big(-4\big(2b^2c}{}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx^{3/2}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+21abcd\right.}{16ac^2(bc-ad)^3}$$

Mathematica [A] time = 6.28, size = 760, normalized size = 1.03

$$-\frac{b^{9/4}(13ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4}+\frac{b^{9/4}(13ad-bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4}-\frac{b^{9/4}(13ad-bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4}-\frac{b^{9/4}(13ad-bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(ad-bc)^4}-\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(ad-bc)^4}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(ad-bc)^4}+\frac{d^{5/4}\left(5a^2d^2-26abcd+117b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(ad-bc)^4}-\frac{b^3x^{3/2}}{2a\left(a+bx^2\right)(ad-bc)^3}+\frac{d^2x^{3/2}(21bc-5ad)}{16c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{d^2x^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/2*(b^3*x^(3/2))/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (d^2*x^(3/2))/(4*c*(b
*c - a*d)^2*(c + d*x^2)^2) + (d^2*(21*b*c - 5*a*d)*x^(3/2))/(16*c^2*(b*c -
a*d)^3*(c + d*x^2)) - (b^(9/4)*(-(b*c) + 13*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4))
 + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4)
 - (b^(9/4)*(-(b*c) + 13*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/
(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) + (d^(5/4)*(117*b^2*c
^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt[x]
)/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(9/4)*(-(b*c) + a*d)^4) + (d^(5/4)*(117
*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt
[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(9/4)*(-(b*c) + a*d)^4) - (b^(9/4)*(
-(b*c) + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(8*Sqrt[2]*a^(5/4)*(b*c - a*d)^4) + (b^(9/4)*(-(b*c) + 13*a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(5/4)*(b*c -
 a*d)^4) + (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(-(b*c) + a
*d)^4) - (d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(9/4)*(-(b*c) + a*d
)^4)
```

IntegrateAlgebraic [A] time = 2.59, size = 510, normalized size = 0.69

$$\frac{\left(13ab^{9/4}d-b^{13/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{5/4}(ad-bc)^4}+\frac{\left(13ab^{9/4}d-b^{13/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{5/4}(ad-bc)^4}-\frac{\left(5a^2d^{13/4}-26abcd^{9/4}+117b^2c^2d^{5/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4}-\frac{\left(5a^2d^{13/4}-26abcd^{9/4}+117b^2c^2d^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4}+\frac{x^{3/2}\left(9a^3cd^3+5a^3d^4x^2-25a^2bc^2d^2-12a^2bcd^3x^2+5a^2bd^4x^4-25ab^2c^2d^2x^2-21ab^2cd^3x^4-8b^3c^4-16b^3c^3dx^2-8b^3c^2d^2x^4\right)}{16ac^2\left(a+bx^2\right)\left(c+dx^2\right)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (x^(3/2)*(-8*b^3*c^4 - 25*a^2*b*c^2*d^2 + 9*a^3*c*d^3 - 16*b^3*c^3*d*x^2 -
25*a*b^2*c^2*d^2*x^2 - 12*a^2*b*c*d^3*x^2 + 5*a^3*d^4*x^2 - 8*b^3*c^2*d^2*x
^4 - 21*a*b^2*c*d^3*x^4 + 5*a^2*b*d^4*x^4))/(16*a*c^2*(-(b*c) + a*d)^3*(a +
 b*x^2)*(c + d*x^2)^2) + ((-(b^(13/4)*c) + 13*a*b^(9/4)*d)*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(5/4)*(-(b*c)
 + a*d)^4) - ((117*b^2*c^2*d^(5/4) - 26*a*b*c*d^(9/4) + 5*a^2*d^(13/4))*Arc
Tan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c
^(9/4)*(b*c - a*d)^4) + ((-(b^(13/4)*c) + 13*a*b^(9/4)*d)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(5/4)*(-(b*c)
 + a*d)^4) - ((117*b^2*c^2*d^(5/4) - 26*a*b*c*d^(9/4) + 5*a^2*d^(13/4))*Arc
Tanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*
c^(9/4)*(b*c - a*d)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.59, size = 1233, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^(3/2)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b
*x^2 + a)) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^4 - 4
*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3
+ sqrt(2)*a^6*d^4) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^
4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5
*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^
(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^
2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d
^5) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3
)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c
^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) - 1/8*((a*b^3)^(3/4
)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*
d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/8*((a*b^3)^(3/4)*b*c - 1
3*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqr
t(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*
sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) - 1/64*(117*(c*d^3)^(3/4)*b^2*c^2 -
26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/
d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*
sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) +
1/64*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4
)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c
^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*
b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) + 1/16*(21*b*c*d^3*x^(7/2) - 5*a*d^4*x^(7/
2) + 25*b*c^2*d^2*x^(3/2) - 9*a*c*d^3*x^(3/2))/((b^3*c^5 - 3*a*b^2*c^4*d +
3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.03, size = 1100, normalized size = 1.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] -1/2*b^3/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x^(3/2)/(b*x
^2+a)*c-13/8*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)*d+1/8*b^3/(a*d-b*c)^4/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)-1)*c-13/16*b^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^
(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/
2)))*d+1/16*b^3/(a*d-b*c)^4/a/(a/b)^(1/4)*2^(1/2)*ln((x-(a/b)^(1/4)*2^(1/2)
*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*c-13/8*b
^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)*d+
1/8*b^3/(a*d-b*c)^4/a/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)*c+5/16*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^(7/2)*a^2-13/8*d^4/(a*d-b*c)^
4/(d*x^2+c)^2/c*x^(7/2)*a*b+21/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*b^2+9
/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(3/2)*a^2-17/8*d^3/(a*d-b*c)^4/(d*x^2+c
)^2*x^(3/2)*a*b+25/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(3/2)*b^2+5/64*d^3/(a
*d-b*c)^4/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2
-13/32*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)*a*b+117/64*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)+1)*b^2+5/64*d^3/(a*d-b*c)^4/c^2/(c/d)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-13/32*d^2/(a*d-b*c)^4/c/(c/d)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+117/64*d/(a*d-b*c)^4/(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2+5/128*d^3/(a*d-b*c)^4/c^
2/(c/d)^(1/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/
d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-13/64*d^2/(a*d-b*c)^4/c/(c/d)^(1
/4)*2^(1/2)*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2
^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+117/128*d/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*
ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/
2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.88, size = 845, normalized size = 1.14

$$\frac{\left(b^{4} c-13 \, a b^{3} d\right) \left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}+2 \, \sqrt{b} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}+\frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}}-2 \, \sqrt{b} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{\sqrt{a} \sqrt{b}} \sqrt{b}}-\frac{\sqrt{2} \log\left(\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \sqrt{x}+\sqrt{b} x+\sqrt{a}\right)}{a^{\frac{1}{4}} b^{\frac{3}{4}}}\right)}{16 \, \left(a b^{4} c^{4}-4 \, a^{2} b^{3} c^{3} d+6 \, a^{3} b^{2} c^{2} d^{2}-4 \, a^{4} b c d^{3}+a^{5} d^{4}\right)}+\frac{\left(117 \, b^{2} c^{2} d^{2}-26 \, a b c d^{3}+5 \, a^{2} d^{4}\right) \left(\frac{2 \, \sqrt{2} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}+2 \, \sqrt{d} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}+\frac{2 \, \sqrt{2} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}-2 \, \sqrt{d} \sqrt{x}\right)}{2 \, \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{\sqrt{c} \sqrt{d}} \sqrt{d}}-\frac{\sqrt{2} \log\left(\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}+\frac{\sqrt{2} \log\left(-\sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} \sqrt{x}+\sqrt{d} x+\sqrt{c}\right)}{c^{\frac{1}{4}} d^{\frac{3}{4}}}\right)}{128 \, \left(b^{4} c^{6}-4 \, a b^{3} c^{5} d+6 \, a^{2} b^{2} c^{4} d^{2}-4 \, a^{3} b c^{3} d^{3}+a^{4} c^{2} d^{4}\right)}+\frac{\left(8 \, b^{3} c^{2} d^{2}+21 \, a b^{2} c d^{3}-5 \, a^{2} b d^{4}\right) x^{\frac{11}{2}}+\left(16 \, b^{3} c^{3} d+25 \, a b^{2} c^{2} d^{2}+12 \, a^{2} b c d^{3}-5 \, a^{3} d^{4}\right) x^{\frac{7}{2}}+\left(8 \, b^{3} c^{4}+25 \, a^{2} b c^{2} d^{2}-9 \, a^{3} c d^{3}\right) x^{\frac{3}{2}}}{16 \, \left(a^{2} b^{3} c^{7}-3 \, a^{3} b^{2} c^{6} d+3 \, a^{4} b c^{5} d^{2}-a^{5} c^{4} d^{3}+\left(a b^{4} c^{5} d^{2}-3 \, a^{2} b^{3} c^{4} d^{3}+3 \, a^{3} b^{2} c^{3} d^{4}-a^{4} b c^{2} d^{5}\right) x^{6}+\left(2 \, a b^{4} c^{6} d-5 \, a^{2} b^{3} c^{5} d^{2}+3 \, a^{3} b^{2} c^{4} d^{3}+a^{4} b c^{3} d^{4}-a^{5} c^{2} d^{5}\right) x^{4}+\left(a b^{4} c^{7}-a^{2} b^{3} c^{6} d-3 \, a^{3} b^{2} c^{5} d^{2}+5 \, a^{4} b c^{4} d^{3}-2 \, a^{5} c^{3} d^{4}\right) x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^4*c - 13*a*b^3*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sq
rt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)
```

```
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^
(1/4)*b^(3/4)))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*
c*d^3 + a^5*d^4) + 1/128*(117*b^2*c^2*d^2 - 26*a*b*c*d^3 + 5*a^2*d^4)*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt
(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4
)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^6 - 4*a*
b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4) + 1/16*((8*b
^3*c^2*d^2 + 21*a*b^2*c*d^3 - 5*a^2*b*d^4)*x^(11/2) + (16*b^3*c^3*d + 25*a*
b^2*c^2*d^2 + 12*a^2*b*c*d^3 - 5*a^3*d^4)*x^(7/2) + (8*b^3*c^4 + 25*a^2*b*c
^2*d^2 - 9*a^3*c*d^3)*x^(3/2))/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5
*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4
 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*
d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3
*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

mupad [B] time = 9.58, size = 45858, normalized size = 62.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] ((x^(7/2)*(16*b^3*c^3*d - 5*a^3*d^4 + 25*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3))/(
16*a*c*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) - (x^(3/2)*
(8*b^3*c^3 - 9*a^3*d^3 + 25*a^2*b*c*d^2))/(16*a*c*(a^3*d^3 - b^3*c^3 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^(11/2)*(8*b^2*c^2 - 5*a^2*d^2 + 21*a
*b*c*d))/(16*a*c*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))/
(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) - atan(
((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 875548
44*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1
264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16
777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 20
13265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4
*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19
*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1919
31351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a
^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^
12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*(
(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8755484
4*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 12
```

64120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(167
77216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 201
3265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*
b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*
d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 19193
1351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a^
11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^1
2*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/4)*((
32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38304*
a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^8 -
(263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 - (111
6788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (471127
4035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (409977
699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (80023416
93*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (331589514
3*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088923057
*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*a^19
*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^9*c^
6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27)/8
- (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b^21*c^27
- a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c
^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*
c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10
*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 3
52716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^1
4*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*
b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21
*b^2*c^8*d^19) - (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 1665677
52*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 72
55846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11
- 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435
456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c
^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 1
34351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 21592276992
0*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*
c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12
- 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 26843545
6*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26
*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618
641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 76566636789
76*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*
a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^
11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^1
3*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15
*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*

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b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^1
0*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^2
4 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 17301504
0*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a
^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*
d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d
^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^
16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^
7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^1
6*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*1i - (x^
(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*c^10
*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 2798689120
5*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16*c^
6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 17210
9080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17)*1i)/(65536*(a^2*b^18*c
^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^1
6*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13
*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*
b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*
a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3
060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))) -
 (-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 875548
44*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1
264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16
777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 20
13265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4
*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19
*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1919
31351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a
^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^
12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*(
(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8755484
4*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 12
64120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(167
77216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 201
3265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*
b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*
d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 19193
1351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a^
11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^1
2*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/4)*((
32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38304*
a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^8 -
(263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 - (111
6788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (471127
```

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4035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (409977
699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (80023416
93*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (331589514
3*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088923057
*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*a^19
*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^9*c^
6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27)/8
- (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b^21*c^27
 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c
^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*
c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10
*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 3
52716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^1
4*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*
b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21
*b^2*c^8*d^19) + (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 1665677
52*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 72
55846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11
- 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435
456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c
^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 1
34351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 21592276992
0*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*
c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12
 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 26843545
6*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26
*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618
641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 76566636789
76*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*
a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^
11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^1
3*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15
*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*
b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^1
0*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^2
4 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 17301504
0*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a
^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*
d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d
^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^
16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^
7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^1
6*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*1i + (x^
(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*c^10
*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 2798689120
```

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5*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16*c^
6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 17210
9080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17)*1i)/(65536*(a^2*b^18*c
^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^1
6*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13
*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*
b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*
a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3
060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))))/
(((1373125*a^10*b^12*d^19)/262144 + (200201625*b^22*c^10*d^9)/262144 - (397
4669595*a*b^21*c^9*d^10)/131072 - (28032875*a^9*b^13*c*d^18)/131072 + (1070
80445745*a^2*b^20*c^8*d^11)/262144 - (64244120525*a^3*b^19*c^7*d^12)/32768
+ (171099678425*a^4*b^18*c^6*d^13)/131072 - (35353616025*a^5*b^17*c^5*d^14)
/65536 + (18119512885*a^6*b^16*c^4*d^15)/131072 - (820327045*a^7*b^15*c^3*d
^16)/32768 + (756189525*a^8*b^14*c^2*d^17)/262144)/(a^2*b^21*c^27 - a^23*c^
6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^25*d^2 -
1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c^22*d^5 +
 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*b^13*c^19
*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 352716*a^13
*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14*d^13 + 1
16280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d
^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d
^19) + (-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 +
87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d
^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^1
2)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^1
5 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 305345331
20*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^1
0*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8
- 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 7328287
9488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*
b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(
1/4)*((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8
7554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^
9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12
)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15
 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 3053453312
0*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10
*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 -
 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879
488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b
^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3
/4)*((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 +
38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*
d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16
```

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- (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (
4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (
409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (80
02341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (331
5895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088
923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (6918151
5*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*
b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^
27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b^2
1*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*
b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7
*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 20349
0*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^
10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b
^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349
*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 21
0*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 1
66567752*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^
8 + 7255846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2
*d^11 - 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 -
268435456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*
b^13*c^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d
^5 + 134351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 21592
2769920*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^1
0*b^6*c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^1
3*d^12 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 26
8435456*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^2
7*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7
 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 76566
63678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 353949694
03392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600
384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 932555516805
12*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 5944894606540
8*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192
*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^
19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c
^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 17
3015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^
24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16
*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*
c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b
^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a
^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 30
60*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))) -
(x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*c
```

```
^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 2798689
1205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16
*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 17
2109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a^2*b^18*c
^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^1
6*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13
*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*
b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*
a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3
060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))) +
 (-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 875548
44*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1
264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16
777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 20
13265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4
*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19
*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1919
31351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a
^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^
12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*(
(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8755484
4*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 12
64120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(167
77216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 201
3265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*
b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*
d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 19193
1351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a^
11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^1
2*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/4)*((
32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38304*
a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^8 -
(263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 - (111
6788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (471127
4035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (409977
699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (80023416
93*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (331589514
3*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088923057
*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*a^19
*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^9*c^
6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27)/8
- (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b^21*c^27
 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c
^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*
c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10
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*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 3
52716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^1
4*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*
b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21
*b^2*c^8*d^19) + (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 1665677
52*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 72
55846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11
- 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435
456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c
^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 1
34351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 21592276992
0*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*
c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12
 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 26843545
6*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26
*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618
641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 76566636789
76*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*
a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^
11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^1
3*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15
*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*
b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^1
0*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^2
4 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 17301504
0*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a
^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*
d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d
^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^
16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^
7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^1
6*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))) + (x^(1/
2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*c^10*d^
9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 27986891205*a
^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16*c^6*d
^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 17210908
0*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a^2*b^18*c^24 +
a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22
*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*
d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c
^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b
^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^
16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))))*(-(62
5*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 87554844*a^2
*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1264120
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*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16777216
*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 20132659
20*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*b^12*
c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*d^6 -
 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1919313510
40*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488*a^11*b^
5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^12*d^1
3 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*2i + 2*
atan(((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8
7554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^
9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12
)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15
 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 3053453312
0*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10
*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 -
 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879
488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b
^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1
/4)*((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 87
554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9
 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)
/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15
+ 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120
*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*
c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 -
191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 732828794
88*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^
3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/
4)*(((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 +
38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*
d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16
- (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (
4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (
409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (80
02341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (331
5895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088
923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (6918151
5*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*
b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^
27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)*1i)/(a^2
*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*
a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349
*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 2
03490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^1
7*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^
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15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 2
0349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18
 - 210*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5
 - 166567752*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^
5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2
*c^2*d^11 - 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^1
6 - 268435456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*
a^3*b^13*c^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^
20*d^5 + 134351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 2
15922769920*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728
*a^10*b^6*c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4
*c^13*d^12 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14
 - 268435456*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2
*b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24
*d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 7
656663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394
969403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 7823827
5600384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551
680512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 594489460
65408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 1858402202
4192*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 255751251558
4*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b
^8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26
 - 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^1
8*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*
b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b
^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^
10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 318
24*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13
 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))
) - (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^
20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 279
86891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*
b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15
 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a^2*b^
18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4
*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*
b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a
^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31
824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13
 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)
)) - (-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 87
554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9
 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)
/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15
```

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+ 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120
*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*
c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 -
191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 732828794
88*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^
3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/
4)*((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 875
54844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9
- 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/
(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 +
 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*
a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c
^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1
91931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 7328287948
8*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3
*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/4
)*(((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 3
8304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d
^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 -
 (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (4
711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (4
09977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (800
2341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (3315
895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (20889
23057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515
*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b
^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^2
7)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)*1i)/(a^2*
b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a
^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*
a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 20
3490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17
*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^1
5*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20
349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 -
 210*a^21*b^2*c^8*d^19) + (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5
- 166567752*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5
*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*
c^2*d^11 - 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16
 - 268435456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a
^3*b^13*c^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^2
0*d^5 + 134351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 21
5922769920*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*
a^10*b^6*c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*
c^13*d^12 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 -
```

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 268435456*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*
b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*
d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 76
56663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 353949
69403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275
600384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 932555516
80512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 5944894606
5408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024
192*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584
*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^
8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 -
 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18
*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b
^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^
13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^1
0*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 3182
4*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 +
 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))
 + (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^2
0*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 2798
6891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b
^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 -
 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a^2*b^1
8*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*
b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b
^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^
10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 318
24*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13
+ 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16))
))/((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 875
54844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9
- 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/
(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 +
 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*
a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c
^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 1
91931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 7328287948
8*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3
*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4
)*((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 8755
4844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 -
 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(
16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10*d^15 +
2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30534533120*a
^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^
```

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19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 19
1931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 - 73282879488
*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*
c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^24*d))^(3/4)
*(((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38
304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^
8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 -
(1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (47
11274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (40
9977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (8002
341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (33158
95143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (208892
3057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*
a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^
9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27
)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)*1i)/(a^2*b
^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^
4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a
^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203
490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*
d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15
*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 203
49*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 -
210*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 -
 166567752*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*
d^8 + 7255846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c
^2*d^11 - 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16
- 268435456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^
3*b^13*c^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20
*d^5 + 134351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215
922769920*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a
^10*b^6*c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c
^13*d^12 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 -
268435456*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b
^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d
^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 765
6663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 3539496
9403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 782382756
00384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 9325555168
0512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065
408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 185840220241
92*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*
a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8
*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 -
173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*
```

```
c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^
16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^1
3*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10
*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824
*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 +
3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*
1i - (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b
^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 27
986891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5
*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15
 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17)*1i)/(65536*(a^
2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153
*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*
a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 437
58*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10
 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*
d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d
^16))) - ((1373125*a^10*b^12*d^19)/262144 + (200201625*b^22*c^10*d^9)/26214
4 - (3974669595*a*b^21*c^9*d^10)/131072 - (28032875*a^9*b^13*c*d^18)/131072
 + (107080445745*a^2*b^20*c^8*d^11)/262144 - (64244120525*a^3*b^19*c^7*d^12
)/32768 + (171099678425*a^4*b^18*c^6*d^13)/131072 - (35353616025*a^5*b^17*c
^5*d^14)/65536 + (18119512885*a^6*b^16*c^4*d^15)/131072 - (820327045*a^7*b^
15*c^3*d^16)/32768 + (756189525*a^8*b^14*c^2*d^17)/262144)/(a^2*b^21*c^27 -
 a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^2
5*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c^
22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*b
^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 352
716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14*
d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^
5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b
^2*c^8*d^19) + (-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^
7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b
^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7
*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*
c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 3
0534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728
*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c
^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 -
 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 93952409
60*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^
24*d))^(1/4)*((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7
*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^
4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*
b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c
^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 30
```

```
534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*
a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^
17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 -
73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 939524096
0*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*c^2
4*d))^(3/4)*(((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^
30)/16 + 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b
^26*c^23*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*
d^10)/16 - (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^
12)/8 - (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^1
4)/16 + (409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16
)/2 + (8002341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)
/8 + (3315895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/
4 + (2088923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 +
 (69181515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778
137*a^21*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b
^7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)
*1i)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^
20 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^
4 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^2
0*d^7 + 203490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12
*b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 -
203490*a^15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12
*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*
c^9*d^18 - 210*a^21*b^2*c^8*d^19) + (x^(1/2)*(-(625*a^8*d^13 + 187388721*b^
8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a
^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 15990
0*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^1
6*c^9*d^16 - 268435456*a^15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 93
95240960*a^3*b^13*c^22*d^3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^
5*b^11*c^20*d^5 + 134351945728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^1
8*d^7 + 215922769920*a^8*b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134
351945728*a^10*b^6*c^15*d^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120
*a^12*b^4*c^13*d^12 - 9395240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c
^11*d^14 - 268435456*a*b^15*c^24*d))^(1/4)*(16777216*a*b^28*c^27*d^4 - 7046
43072*a^2*b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*
b^25*c^24*d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^2
2*d^9 + 7656663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^1
1 + 35394969403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13
+ 78238275600384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 +
 93255551680512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 +
59448946065408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 1
8584022024192*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 255
7512515584*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 1372724920
32*a^21*b^8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*
```

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c^5*d^26 - 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536
*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 +
 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8
568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 +
 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d
^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c
^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c
^8*d^16)))*1i + (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 14
1442353*a*b^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^
9*d^10 + 27986891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 339
6941522*a^5*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^
14*c^4*d^15 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17)*1i)
/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7
*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*
d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^1
7*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8
*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^1
5*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^1
8*b^2*c^8*d^16)))))*(-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b
^7*c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*
a^4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 1300
0*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^
15*b*c^10*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^
3 + 30534533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 1343519
45728*a^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*
b^8*c^17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d
^10 - 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 939
5240960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^
15*c^24*d))^(1/4) - atan(((((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125
*a^26*b^4*c*d^30)/16 + 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 +
 3369600*a^4*b^26*c^23*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*
a^6*b^24*c^21*d^10)/16 - (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^
8*b^22*c^19*d^12)/8 - (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10
*b^20*c^17*d^14)/16 + (409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*
b^18*c^15*d^16)/2 + (8002341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b
^16*c^13*d^18)/8 + (3315895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^
14*c^11*d^20)/4 + (2088923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12
*c^9*d^22)/8 + (69181515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^
24)/2 + (30778137*a^21*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 +
(327093*a^23*b^7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5
*c^2*d^29)/16)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^2
2*b*c^7*d^20 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^
17*c^23*d^4 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^
9*b^14*c^20*d^7 + 203490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 3
52716*a^12*b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^
```

```
15*d^12 - 203490*a^15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^1
7*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330
*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(b^13*c^4 + 28561*
a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d
)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7
*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17
891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9
*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768
*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12
 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^
15))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*d^5 + 118279
37280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618641227776*a^5
*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 7656663678976*a^7*b^22*c
^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*a^9*b^20*c^19
*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^11*b^18*c^17*
d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^13*b^16*c^15*d
^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15*b^14*c^13*d^
18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*b^12*c^11*d^2
0 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^10*c^9*d^22 -
672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24 - 211864780
80*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 173015040*a^24*b^5*c^
4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18
- 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5
*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^
8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 4862
0*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 +
 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^
14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a
^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)
/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*
b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 178
91328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*
c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*
a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12
- 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^1
5))^(3/4)*1i - (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141
442353*a*b^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9
*d^10 + 27986891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396
941522*a^5*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^1
4*c^4*d^15 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17)*1i)/
(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*
d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d
^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17
*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*
c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15
```

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*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18
*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 10
14*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16
 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13
*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a
^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 -
 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5
*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*
a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4) - (((32*b^30*c^27*d^4 - 172
8*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38304*a^2*b^28*c^25*d^6 - 45
9264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^8 - (263413683*a^5*b^25*c^
22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 - (1116788283*a^7*b^23*c^20*
d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (4711274035*a^9*b^21*c^18*d^1
3)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (409977699*a^11*b^19*c^16*d^1
5)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (8002341693*a^13*b^17*c^14*d^17
)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (3315895143*a^15*b^15*c^12*d^19)
/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088923057*a^17*b^13*c^10*d^21)/
8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*a^19*b^11*c^8*d^23)/2 - (1
8239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^9*c^6*d^25)/16 - (5119101*
a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*
d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*
a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^1
8*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b
^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*b^13*c^19*d^8 - 29393
0*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d
^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14*d^13 + 116280*a^16*b
^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a
^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^19) + (x^(1
/2)*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c
^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*
b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 745472
0*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10
*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^1
4*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7
454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*
d^14 - 65536*a^20*b*c*d^15))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^
2*b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^2
4*d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 +
7656663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 3539
4969403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 782382
75600384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 9325555
1680512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946
065408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 185840220
24192*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 25575125155
84*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*
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b^8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26
 - 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^
18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4
*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*
b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a
^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31
824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13
 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)
))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^
2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b
^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720
*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*
d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14
*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 74
54720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d
^14 - 65536*a^20*b*c*d^15))^(3/4)*1i + (x^(1/2)*(105625*a^11*b^10*d^19 + 87
6096*b^21*c^11*d^8 + 141442353*a*b^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 -
 4129947458*a^2*b^19*c^9*d^10 + 27986891205*a^3*b^18*c^8*d^11 - 1891277400*
a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*
d^14 + 769949154*a^7*b^14*c^4*d^15 - 172109080*a^8*b^13*c^3*d^16 + 27361725
*a^9*b^12*c^2*d^17)*1i)/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17
*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^
3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d
^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^
15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b
^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17
*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 -
8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d
^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2
 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^
11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 527
15520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*
d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^1
8*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4))/((
(1373125*a^10*b^12*d^19)/262144 + (200201625*b^22*c^10*d^9)/262144 - (39746
69595*a*b^21*c^9*d^10)/131072 - (28032875*a^9*b^13*c*d^18)/131072 + (107080
445745*a^2*b^20*c^8*d^11)/262144 - (64244120525*a^3*b^19*c^7*d^12)/32768 +
(171099678425*a^4*b^18*c^6*d^13)/131072 - (35353616025*a^5*b^17*c^5*d^14)/6
5536 + (18119512885*a^6*b^16*c^4*d^15)/131072 - (820327045*a^7*b^15*c^3*d^1
6)/32768 + (756189525*a^8*b^14*c^2*d^17)/262144)/(a^2*b^21*c^27 - a^23*c^6*
d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^25*d^2 - 13
30*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c^22*d^5 + 5
4264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*b^13*c^19*d
^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 352716*a^13*b
^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14*d^13 + 116
```

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280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^1
6 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^1
9) + (((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16
+ 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^2
3*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/1
6 - (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 -
 (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 +
 (409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (
8002341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (3
315895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (20
88923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181
515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^2
1*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*
d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)/(a^2*b
^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^
4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a
^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203
490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*
d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15
*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 203
49*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 -
210*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^
3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4
096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293
760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11
*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a
^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 -
17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c
^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)*(16777216*
a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d
^6 - 107105746944*a^4*b^25*c^24*d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513
987174400*a^6*b^23*c^22*d^9 + 7656663678976*a^7*b^22*c^21*d^10 - 1827863946
8544*a^8*b^21*c^20*d^11 + 35394969403392*a^9*b^20*c^19*d^12 - 5709880937676
8*a^10*b^19*c^18*d^13 + 78238275600384*a^11*b^18*c^17*d^14 - 92068449878016
*a^12*b^17*c^16*d^15 + 93255551680512*a^13*b^16*c^15*d^16 - 80877025492992*
a^14*b^15*c^14*d^17 + 59448946065408*a^15*b^14*c^13*d^18 - 36574941151232*a
^16*b^13*c^12*d^19 + 18584022024192*a^17*b^12*c^11*d^20 - 7692575834112*a^1
8*b^11*c^10*d^21 + 2557512515584*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9
*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25
+ 2360868864*a^23*b^6*c^5*d^26 - 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25
*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d
- 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060
*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 318
24*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 +
 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*
```

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d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9
*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3
*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 40
96*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 22937
60*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*
d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^
13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 1
7891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^
3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(3/4) - (x^(1/2)*
(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*c^10*d^9 -
 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 27986891205*a^3*
b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16*c^6*d^13
 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 172109080*a
^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a^2*b^18*c^24 + a^2
0*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^
2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5
 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16
*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*
c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*
b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^
4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*
b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d +
491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^1
2*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 4685824
0*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9
+ 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^
4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a
^20*b*c*d^15))^(1/4) + (((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^
26*b^4*c*d^30)/16 + 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 33
69600*a^4*b^26*c^23*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6
*b^24*c^21*d^10)/16 - (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b
^22*c^19*d^12)/8 - (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^
20*c^17*d^14)/16 + (409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^1
8*c^15*d^16)/2 + (8002341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16
*c^13*d^18)/8 + (3315895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*
c^11*d^20)/4 + (2088923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^
9*d^22)/8 + (69181515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)
/2 + (30778137*a^21*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (32
7093*a^23*b^7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^
2*d^29)/16)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b
*c^7*d^20 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*
c^23*d^4 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b
^14*c^20*d^7 + 203490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 3527
16*a^12*b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*
d^12 - 203490*a^15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b
```

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^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^
20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^19) + (x^(1/2)*(-(b^13*c^4 + 28561*a^4
*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(
4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^
14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891
328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^
9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^
15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 -
2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15)
)^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*d^5 + 118279372
80*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618641227776*a^5*b^
24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 7656663678976*a^7*b^22*c^21
*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*a^9*b^20*c^19*d^
12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^11*b^18*c^17*d^1
4 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^13*b^16*c^15*d^16
 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15*b^14*c^13*d^18
- 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*b^12*c^11*d^20 -
 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^10*c^9*d^22 - 672
468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24 - 21186478080*
a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 173015040*a^24*b^5*c^4*d
^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 1
8*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^
15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b
^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a
^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18
564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14
- 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*
b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4
096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^1
4*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 178913
28*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9
*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^1
5*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2
293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))
^(3/4) + (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353
*a*b^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10
+ 27986891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522
*a^5*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*
d^15 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a
^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 15
3*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568
*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43
758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10
 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11
*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*
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d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^
11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*
a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 74
54720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*
c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240
*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11
 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*
c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 -
8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d
^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2
 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^
11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 527
15520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*
d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^1
8*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)*2i
+ 2*atan((((((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^3
0)/16 + 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^
26*c^23*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d
^10)/16 - (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^1
2)/8 - (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14
)/16 + (409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)
/2 + (8002341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/
8 + (3315895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4
 + (2088923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 +
(69181515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (307781
37*a^21*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^
7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)*
1i)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^2
0 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4
 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20
*d^7 + 203490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*
b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 2
03490*a^15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*
d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c
^9*d^18 - 210*a^21*b^2*c^8*d^19) - (x^(1/2)*(-(b^13*c^4 + 28561*a^4*b^9*d^4
 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^2
1*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*
d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10
*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 +
52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c
^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*
a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)*
(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*d^5 + 11827937280*a^3*b
^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618641227776*a^5*b^24*c^23*
d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 7656663678976*a^7*b^22*c^21*d^10 -
```

18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*a^9*b^20*c^19*d^12 - 570
98809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^11*b^18*c^17*d^14 - 9206
8449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^13*b^16*c^15*d^16 - 80877
025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15*b^14*c^13*d^18 - 365749
41151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*b^12*c^11*d^20 - 7692575
834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^10*c^9*d^22 - 67246856601
6*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24 - 21186478080*a^22*b^7
*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 173015040*a^24*b^5*c^4*d^27 + 65
53600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^
17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*
d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18
*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*
c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14
*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^
17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4
- 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21
*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d
^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*
b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 5
2715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^
6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a
^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(3/4) -
(x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*
c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 279868
91205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^1
6*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 1
72109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17))/(65536*(a^2*b^18*
c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^
16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^1
3*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10
*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824
*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 +
3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*
(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d
^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15
*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^
9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6
- 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^
7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 74547
20*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14
- 65536*a^20*b*c*d^15))^(1/4) - ((((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^
5 - (125*a^26*b^4*c*d^30)/16 + 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^
24*d^7 + 3369600*a^4*b^26*c^23*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (90
3579807*a^6*b^24*c^21*d^10)/16 - (1116788283*a^7*b^23*c^20*d^11)/8 + (19806
89243*a^8*b^22*c^19*d^12)/8 - (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187

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127*a^10*b^20*c^17*d^14)/16 + (409977699*a^11*b^19*c^16*d^15)/2 - (13374998
67*a^12*b^18*c^15*d^16)/2 + (8002341693*a^13*b^17*c^14*d^17)/8 - (834189238
5*a^14*b^16*c^13*d^18)/8 + (3315895143*a^15*b^15*c^12*d^19)/4 - (2079521847
*a^16*b^14*c^11*d^20)/4 + (2088923057*a^17*b^13*c^10*d^21)/8 - (845943917*a
^18*b^12*c^9*d^22)/8 + (69181515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^1
0*c^7*d^24)/2 + (30778137*a^21*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^2
6)/16 + (327093*a^23*b^7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*
a^25*b^5*c^2*d^29)/16)*1i)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^2
6*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 +
 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6
 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*
c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930
*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14
 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10
*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^19) + (x^(1/2)*(-(b^13*
c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*
a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d
 + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c
^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858
240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^
9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*
b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536
*a^20*b*c*d^15))^(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*
d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 6186
41227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 765666367897
6*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*a
^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^1
1*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^13
*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15*
b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*b
^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^10
*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24
 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 173015040
*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a^
20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d
^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^
5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^1
6*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7
*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16
*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c
^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a
*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d +
 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^
12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 468582
40*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9
```

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 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b
^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*
a^20*b*c*d^15))^(3/4) + (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*
d^8 + 141442353*a*b^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2
*b^19*c^9*d^10 + 27986891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^
12 + 3396941522*a^5*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 76994915
4*a^7*b^14*c^4*d^15 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d
^17))/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*
b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*
c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^1
1*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^1
2*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 856
8*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 15
3*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^
3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^1
6*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^1
3*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 3280
0768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8
*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^
16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 4
91520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4))/(((((32*b^30*c^27*d^
4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4*c*d^30)/16 + 38304*a^2*b^28*c^25*d
^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*a^4*b^26*c^23*d^8 - (263413683*a^5*
b^25*c^22*d^9)/16 + (903579807*a^6*b^24*c^21*d^10)/16 - (1116788283*a^7*b^2
3*c^20*d^11)/8 + (1980689243*a^8*b^22*c^19*d^12)/8 - (4711274035*a^9*b^21*c
^18*d^13)/16 + (2530187127*a^10*b^20*c^17*d^14)/16 + (409977699*a^11*b^19*c
^16*d^15)/2 - (1337499867*a^12*b^18*c^15*d^16)/2 + (8002341693*a^13*b^17*c^
14*d^17)/8 - (8341892385*a^14*b^16*c^13*d^18)/8 + (3315895143*a^15*b^15*c^1
2*d^19)/4 - (2079521847*a^16*b^14*c^11*d^20)/4 + (2088923057*a^17*b^13*c^10
*d^21)/8 - (845943917*a^18*b^12*c^9*d^22)/8 + (69181515*a^19*b^11*c^8*d^23)
/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (30778137*a^21*b^9*c^6*d^25)/16 - (5
119101*a^22*b^8*c^5*d^26)/16 + (327093*a^23*b^7*c^4*d^27)/8 - (30645*a^24*b
^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29)/16)*1i)/(a^2*b^21*c^27 - a^23*c^6
*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^25*d^2 - 1
330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c^22*d^5 +
54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*b^13*c^19*
d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 352716*a^13*
b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14*d^13 + 11
6280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b^5*c^11*d^
16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^
19) - (x^(1/2)*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014
*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 -
 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d
^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^1
1*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 4
```

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6858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c
^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^
19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)*(16777216*a*b^28*c^27*d^4 - 7
04643072*a^2*b^27*c^26*d^5 + 11827937280*a^3*b^26*c^25*d^6 - 107105746944*a
^4*b^25*c^24*d^7 + 618641227776*a^5*b^24*c^23*d^8 - 2513987174400*a^6*b^23*
c^22*d^9 + 7656663678976*a^7*b^22*c^21*d^10 - 18278639468544*a^8*b^21*c^20*
d^11 + 35394969403392*a^9*b^20*c^19*d^12 - 57098809376768*a^10*b^19*c^18*d^
13 + 78238275600384*a^11*b^18*c^17*d^14 - 92068449878016*a^12*b^17*c^16*d^1
5 + 93255551680512*a^13*b^16*c^15*d^16 - 80877025492992*a^14*b^15*c^14*d^17
 + 59448946065408*a^15*b^14*c^13*d^18 - 36574941151232*a^16*b^13*c^12*d^19
+ 18584022024192*a^17*b^12*c^11*d^20 - 7692575834112*a^18*b^11*c^10*d^21 +
2557512515584*a^19*b^10*c^9*d^22 - 672468566016*a^20*b^9*c^8*d^23 + 1372724
92032*a^21*b^8*c^7*d^24 - 21186478080*a^22*b^7*c^6*d^25 + 2360868864*a^23*b
^6*c^5*d^26 - 173015040*a^24*b^5*c^4*d^27 + 6553600*a^25*b^4*c^3*d^28))/(65
536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^1
7 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4
- 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^
7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^1
4*d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^
5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^
2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*
a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 -
65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^
3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11
*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46
858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^
5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^1
9*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(3/4)*1i - (x^(1/2)*(105625*a^11*b^1
0*d^19 + 876096*b^21*c^11*d^8 + 141442353*a*b^20*c^10*d^9 - 2213250*a^10*b^
11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10 + 27986891205*a^3*b^18*c^8*d^11 -
1891277400*a^4*b^17*c^7*d^12 + 3396941522*a^5*b^16*c^6*d^13 - 1666839564*a^
6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4*d^15 - 172109080*a^8*b^13*c^3*d^16
 + 27361725*a^9*b^12*c^2*d^17)*1i)/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 -
18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b
^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*
b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*
a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 1
8564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14
 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4
*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(
4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^
14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891
328*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^
9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^
15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 -
```

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2293760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15)
)^(1/4) - ((1373125*a^10*b^12*d^19)/262144 + (200201625*b^22*c^10*d^9)/2621
44 - (3974669595*a*b^21*c^9*d^10)/131072 - (28032875*a^9*b^13*c*d^18)/13107
2 + (107080445745*a^2*b^20*c^8*d^11)/262144 - (64244120525*a^3*b^19*c^7*d^1
2)/32768 + (171099678425*a^4*b^18*c^6*d^13)/131072 - (35353616025*a^5*b^17*
c^5*d^14)/65536 + (18119512885*a^6*b^16*c^4*d^15)/131072 - (820327045*a^7*b
^15*c^3*d^16)/32768 + (756189525*a^8*b^14*c^2*d^17)/262144)/(a^2*b^21*c^27
- a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c^7*d^20 + 210*a^4*b^19*c^
25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^23*d^4 - 20349*a^7*b^16*c
^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^14*c^20*d^7 + 203490*a^10*
b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716*a^12*b^11*c^17*d^10 - 35
2716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^12 - 203490*a^15*b^8*c^14
*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6*c^12*d^15 + 20349*a^18*b
^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20*b^3*c^9*d^18 - 210*a^21*
b^2*c^8*d^19) + ((((32*b^30*c^27*d^4 - 1728*a*b^29*c^26*d^5 - (125*a^26*b^4
*c*d^30)/16 + 38304*a^2*b^28*c^25*d^6 - 459264*a^3*b^27*c^24*d^7 + 3369600*
a^4*b^26*c^23*d^8 - (263413683*a^5*b^25*c^22*d^9)/16 + (903579807*a^6*b^24*
c^21*d^10)/16 - (1116788283*a^7*b^23*c^20*d^11)/8 + (1980689243*a^8*b^22*c^
19*d^12)/8 - (4711274035*a^9*b^21*c^18*d^13)/16 + (2530187127*a^10*b^20*c^1
7*d^14)/16 + (409977699*a^11*b^19*c^16*d^15)/2 - (1337499867*a^12*b^18*c^15
*d^16)/2 + (8002341693*a^13*b^17*c^14*d^17)/8 - (8341892385*a^14*b^16*c^13*
d^18)/8 + (3315895143*a^15*b^15*c^12*d^19)/4 - (2079521847*a^16*b^14*c^11*d
^20)/4 + (2088923057*a^17*b^13*c^10*d^21)/8 - (845943917*a^18*b^12*c^9*d^22
)/8 + (69181515*a^19*b^11*c^8*d^23)/2 - (18239091*a^20*b^10*c^7*d^24)/2 + (
30778137*a^21*b^9*c^6*d^25)/16 - (5119101*a^22*b^8*c^5*d^26)/16 + (327093*a
^23*b^7*c^4*d^27)/8 - (30645*a^24*b^6*c^3*d^28)/8 + (3825*a^25*b^5*c^2*d^29
)/16)*1i)/(a^2*b^21*c^27 - a^23*c^6*d^21 - 21*a^3*b^20*c^26*d + 21*a^22*b*c
^7*d^20 + 210*a^4*b^19*c^25*d^2 - 1330*a^5*b^18*c^24*d^3 + 5985*a^6*b^17*c^
23*d^4 - 20349*a^7*b^16*c^22*d^5 + 54264*a^8*b^15*c^21*d^6 - 116280*a^9*b^1
4*c^20*d^7 + 203490*a^10*b^13*c^19*d^8 - 293930*a^11*b^12*c^18*d^9 + 352716
*a^12*b^11*c^17*d^10 - 352716*a^13*b^10*c^16*d^11 + 293930*a^14*b^9*c^15*d^
12 - 203490*a^15*b^8*c^14*d^13 + 116280*a^16*b^7*c^13*d^14 - 54264*a^17*b^6
*c^12*d^15 + 20349*a^18*b^5*c^11*d^16 - 5985*a^19*b^4*c^10*d^17 + 1330*a^20
*b^3*c^9*d^18 - 210*a^21*b^2*c^8*d^19) + (x^(1/2)*(-(b^13*c^4 + 28561*a^4*b
^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(40
96*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14
*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 1789132
8*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*
d^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15
*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 22
93760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^
(1/4)*(16777216*a*b^28*c^27*d^4 - 704643072*a^2*b^27*c^26*d^5 + 11827937280
*a^3*b^26*c^25*d^6 - 107105746944*a^4*b^25*c^24*d^7 + 618641227776*a^5*b^24
*c^23*d^8 - 2513987174400*a^6*b^23*c^22*d^9 + 7656663678976*a^7*b^22*c^21*d
^10 - 18278639468544*a^8*b^21*c^20*d^11 + 35394969403392*a^9*b^20*c^19*d^12
```

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 - 57098809376768*a^10*b^19*c^18*d^13 + 78238275600384*a^11*b^18*c^17*d^14
- 92068449878016*a^12*b^17*c^16*d^15 + 93255551680512*a^13*b^16*c^15*d^16 -
 80877025492992*a^14*b^15*c^14*d^17 + 59448946065408*a^15*b^14*c^13*d^18 -
36574941151232*a^16*b^13*c^12*d^19 + 18584022024192*a^17*b^12*c^11*d^20 - 7
692575834112*a^18*b^11*c^10*d^21 + 2557512515584*a^19*b^10*c^9*d^22 - 67246
8566016*a^20*b^9*c^8*d^23 + 137272492032*a^21*b^8*c^7*d^24 - 21186478080*a^
22*b^7*c^6*d^25 + 2360868864*a^23*b^6*c^5*d^26 - 173015040*a^24*b^5*c^4*d^2
7 + 6553600*a^25*b^4*c^3*d^28))/(65536*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*
a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17 + 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15
*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 - 8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^1
2*c^18*d^6 - 31824*a^9*b^11*c^17*d^7 + 43758*a^10*b^10*c^16*d^8 - 48620*a^1
1*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*d^10 - 31824*a^13*b^7*c^13*d^11 + 1856
4*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*c^11*d^13 + 3060*a^16*b^4*c^10*d^14 -
816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^
9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(409
6*a^21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*
c^14*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328
*a^10*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d
^7 + 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*
b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 229
3760*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(
3/4)*1i + (x^(1/2)*(105625*a^11*b^10*d^19 + 876096*b^21*c^11*d^8 + 14144235
3*a*b^20*c^10*d^9 - 2213250*a^10*b^11*c*d^18 - 4129947458*a^2*b^19*c^9*d^10
 + 27986891205*a^3*b^18*c^8*d^11 - 1891277400*a^4*b^17*c^7*d^12 + 339694152
2*a^5*b^16*c^6*d^13 - 1666839564*a^6*b^15*c^5*d^14 + 769949154*a^7*b^14*c^4
*d^15 - 172109080*a^8*b^13*c^3*d^16 + 27361725*a^9*b^12*c^2*d^17)*1i)/(6553
6*(a^2*b^18*c^24 + a^20*c^6*d^18 - 18*a^3*b^17*c^23*d - 18*a^19*b*c^7*d^17
+ 153*a^4*b^16*c^22*d^2 - 816*a^5*b^15*c^21*d^3 + 3060*a^6*b^14*c^20*d^4 -
8568*a^7*b^13*c^19*d^5 + 18564*a^8*b^12*c^18*d^6 - 31824*a^9*b^11*c^17*d^7
+ 43758*a^10*b^10*c^16*d^8 - 48620*a^11*b^9*c^15*d^9 + 43758*a^12*b^8*c^14*
d^10 - 31824*a^13*b^7*c^13*d^11 + 18564*a^14*b^6*c^12*d^12 - 8568*a^15*b^5*
c^11*d^13 + 3060*a^16*b^4*c^10*d^14 - 816*a^17*b^3*c^9*d^15 + 153*a^18*b^2*
c^8*d^16)))*(-(b^13*c^4 + 28561*a^4*b^9*d^4 - 8788*a^3*b^10*c*d^3 + 1014*a^
2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^21*d^16 + 4096*a^5*b^16*c^16 - 65
536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14*d^2 - 2293760*a^8*b^13*c^13*d^3
+ 7454720*a^9*b^12*c^12*d^4 - 17891328*a^10*b^11*c^11*d^5 + 32800768*a^11*b
^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 + 52715520*a^13*b^8*c^8*d^8 - 4685
8240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*c^6*d^10 - 17891328*a^16*b^5*c^5*
d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760*a^18*b^3*c^3*d^13 + 491520*a^19*
b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)))*(-(b^13*c^4 + 28561*a^4*b^9*d^
4 - 8788*a^3*b^10*c*d^3 + 1014*a^2*b^11*c^2*d^2 - 52*a*b^12*c^3*d)/(4096*a^
21*d^16 + 4096*a^5*b^16*c^16 - 65536*a^6*b^15*c^15*d + 491520*a^7*b^14*c^14
*d^2 - 2293760*a^8*b^13*c^13*d^3 + 7454720*a^9*b^12*c^12*d^4 - 17891328*a^1
0*b^11*c^11*d^5 + 32800768*a^11*b^10*c^10*d^6 - 46858240*a^12*b^9*c^9*d^7 +
 52715520*a^13*b^8*c^8*d^8 - 46858240*a^14*b^7*c^7*d^9 + 32800768*a^15*b^6*
```

```
c^6*d^10 - 17891328*a^16*b^5*c^5*d^11 + 7454720*a^17*b^4*c^4*d^12 - 2293760
*a^18*b^3*c^3*d^13 + 491520*a^19*b^2*c^2*d^14 - 65536*a^20*b*c*d^15))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.482 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=739

$$-\frac{3b^{11/4}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}-$$

Rubi [A] time = 0.98, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {466, 414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{16ac^2\left(c+dx^2\right)(bc-ad)^3}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}-\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4}+\frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4}-\frac{3b^{11/4}(bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4}-\frac{3b^{11/4}(bc-5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{3b^{11/4}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4}+\frac{b\sqrt{x}}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*Sqrt[x])/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*Sqrt[x])
/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 23*a*b*c*d -
 7*a^2*d^2)*Sqrt[x])/(16*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*b^(11/4)*(b*
c - 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4
)*(b*c - a*d)^4) + (3*b^(11/4)*(b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2
 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(
32*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(11
/4)*(b*c - a*d)^4) - (3*b^(11/4)*(b*c - 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*b^(1
1/4)*(b*c - 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d
 + 7*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(
64*Sqrt[2]*c^(11/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d +
7*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*
Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-11bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-4\left(6b^2c\right.}{}\right.}{}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23ab\right.}{16ac^2(bc-a}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23ab\right.}{16ac^2(bc-a}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23ab\right.}{16ac^2(bc-a}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23ab\right.}{16ac^2(bc-a}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23ab\right.}{16ac^2(bc-a}$$

$$= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2c^2+23ab\right.}{16ac^2(bc-a}$$

Mathematica [A] time = 6.33, size = 760, normalized size = 1.03

$$\frac{3b^{11/4}(5ad-bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4} - \frac{3b^{11/4}(5ad-bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4} - \frac{3b^{11/4}(5ad-bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4} - \frac{3b^{11/4}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4} - \frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(ad-bc)^4} + \frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(ad-bc)^4} + \frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(ad-bc)^4} + \frac{3d^{7/4}\left(7a^2d^2-30abcd+55b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(ad-bc)^4} - \frac{b^3\sqrt{x}}{2a\left(a+bx^2\right)(ad-bc)^3} + \frac{d^2\sqrt{x}(23bc-7ad)}{16c^2\left(c+dx^2\right)(bc-ad)^3} + \frac{d^2\sqrt{x}}{4c\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -1/2*(b^3*Sqrt[x])/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (d^2*Sqrt[x])/(4*c*(b
*c - a*d)^2*(c + d*x^2)^2) + (d^2*(23*b*c - 7*a*d)*Sqrt[x])/(16*c^2*(b*c -
a*d)^3*(c + d*x^2)) - (3*b^(11/4)*(-(b*c) + 5*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4
)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^
4) - (3*b^(11/4)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[
x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) + (3*d^(7/4)*(55*
b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sq
rt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^4) + (3*d^(7
/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/
4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(11/4)*(-(b*c) + a*d)^4) + (3
*b^(11/4)*(-(b*c) + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^4) - (3*b^(11/4)*(-(b*c) + 5*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^
(7/4)*(b*c - a*d)^4) - (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/
4)*(-(b*c) + a*d)^4) + (3*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Log
[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(11/
4)*(-(b*c) + a*d)^4)
```

IntegrateAlgebraic [A] time = 2.47, size = 510, normalized size = 0.69

$$\frac{3\left(5ab^{11/4}d-b^{15/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}\right)}{4\sqrt{2}\,a^{7/4}(ad-bc)^4}-\frac{3\left(5ab^{11/4}d-b^{15/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt{a}+\sqrt{b}\,x}\right)}{4\sqrt{2}\,a^{7/4}(ad-bc)^4}-\frac{3\left(7a^2d^{15/4}-30abcd^{11/4}+55b^2c^2d^{7/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}\,x}{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^4}+\frac{3\left(7a^2d^{15/4}-30abcd^{11/4}+55b^2c^2d^{7/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt{c}+\sqrt{d}\,x}\right)}{32\sqrt{2}\,c^{11/4}(bc-ad)^4}+\frac{\sqrt{x}\left(11a^3cd^3+7a^3d^4x^2-27a^2bc^2d^2-12a^2bcd^3x^2+7a^2bd^4x^4-27ab^2c^2d^2x^2-23ab^2cd^3x^4-8b^3c^4-16b^3c^3dx^2-8b^3c^2d^2x^4\right)}{16ac^2\left(a+bx^2\right)\left(c+dx^2\right)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (Sqrt[x]*(-8*b^3*c^4 - 27*a^2*b*c^2*d^2 + 11*a^3*c*d^3 - 16*b^3*c^3*d*x^2 -
 27*a*b^2*c^2*d^2*x^2 - 12*a^2*b*c*d^3*x^2 + 7*a^3*d^4*x^2 - 8*b^3*c^2*d^2*
x^4 - 23*a*b^2*c*d^3*x^4 + 7*a^2*b*d^4*x^4))/(16*a*c^2*(-(b*c) + a*d)^3*(a
+ b*x^2)*(c + d*x^2)^2) + (3*(-(b^(15/4)*c) + 5*a*b^(11/4)*d)*ArcTan[(Sqrt[
a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*Sqrt[2]*a^(7/4)*(-(b
*c) + a*d)^4) - (3*(55*b^2*c^2*d^(7/4) - 30*a*b*c*d^(11/4) + 7*a^2*d^(15/4)
)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt
[2]*c^(11/4)*(b*c - a*d)^4) - (3*(-(b^(15/4)*c) + 5*a*b^(11/4)*d)*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(7/4)
*(-(b*c) + a*d)^4) + (3*(55*b^2*c^2*d^(7/4) - 30*a*b*c*d^(11/4) + 7*a^2*d^(
15/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(3
2*Sqrt[2]*c^(11/4)*(b*c - a*d)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.53, size = 1253, normalized size = 1.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*sqrt(x)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b
*x^2 + a)) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^
4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c
*d^3 + sqrt(2)*a^6*d^4) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*
d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt
(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*s
qrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d -
30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a
*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*
a^4*c^3*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^
2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a
^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d^4) + 3/8*((a*b
^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)
*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) - 3/8*((a*b^3)^
(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^
4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/64*(55*(c*d^3)
^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*lo
g(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)
*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2
)*a^4*c^3*d^4) - 3/64*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1/4)*a*b*c*
d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(
c/d))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2
- 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d^4) + 1/16*(23*b*c*d^3*x^(5/2)
 - 7*a*d^4*x^(5/2) + 27*b*c^2*d^2*sqrt(x) - 11*a*c*d^3*sqrt(x))/((b^3*c^5 -
 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.03, size = 1124, normalized size = 1.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x)
```

```
[Out] -1/2*b^3/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4/a*x^(1/2)/(b*x
^2+a)*c-15/8*b^3/(a*d-b*c)^4/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x^(1/2)+1)*d+3/8*b^4/(a*d-b*c)^4/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)*c-15/8*b^3/(a*d-b*c)^4/a*(a/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d+3/8*b^4/(a*d-b*c)^4/a^2*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c-15/16*b^3/(a*d-b*c)^4/a*(a/b)^(1/
4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^
(1/2)*x^(1/2)+(a/b)^(1/2)))*d+3/16*b^4/(a*d-b*c)^4/a^2*(a/b)^(1/4)*2^(1/2)*
ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1/2)*x^(1/
2)+(a/b)^(1/2)))*c+7/16*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^(5/2)*a^2-15/8*d^
4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(5/2)*a*b+23/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x
^(5/2)*b^2+11/16*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x^(1/2)*a^2-19/8*d^3/(a*d-b*
c)^4/(d*x^2+c)^2*x^(1/2)*a*b+27/16*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x^(1/2)*b^
2+21/64*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)*a^2-45/32*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)*a*b+165/64*d^2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*b^2+21/64*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2-45/32*d^3/(a*d-b*c)^4/
c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b+165/64*d^
2/(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b
^2+21/128*d^4/(a*d-b*c)^4/c^3*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2-45/6
4*d^3/(a*d-b*c)^4/c^2*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)
+(c/d)^(1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b+165/128*d^2/
(a*d-b*c)^4/c*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(
1/2))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2
```

maxima [A] time = 2.75, size = 951, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/16*(2*sqrt(2)*(b*c - 5*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*(b*c - 5*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*
sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*(b*c - 5*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c - 5*a*d)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^3/(a*b^4*c^4 - 4*a^2*b
^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4) + 1/16*((8*b^3*c^2*
d^2 + 23*a*b^2*c*d^3 - 7*a^2*b*d^4)*x^(9/2) + (16*b^3*c^3*d + 27*a*b^2*c^2*
d^2 + 12*a^2*b*c*d^3 - 7*a^3*d^4)*x^(5/2) + (8*b^3*c^4 + 27*a^2*b*c^2*d^2 -
```

```
 11*a^3*c*d^3)*sqrt(x))/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 -
a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*
b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a
^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^
5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 3/128*(2*sqrt(2)*(55*b^2*c^
2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d
))) + 2*sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(
sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7
*a^2*d^4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/
4)*d^(1/4)) - sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*log(-sqrt
(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*
c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)
```

mupad [B] time = 16.83, size = 150312, normalized size = 203.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] atan(-((((15864057030927974 4*a^62*d^62 + 461689330549653504*b^62*c^62 + 114
3142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^
59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 46901984906438868247 5
1104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346
602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a
^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312
40117103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085 12
0*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12
- 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367
585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^
47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701
324613150775962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575 4
13329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*
c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226
492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144
979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^3
9*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084
025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248 7
1901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^2
7*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28
- 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222
80459690440055062528*a^30*b^32*c^32*d^30 - 5926488746562692758663377064663 4
496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^3
```

```
0*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997
892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527964
95977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b
^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3
3004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822
79521941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088
*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^
41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146
033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142
720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d
^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444
8543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*
a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 +
 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821
717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^1
0*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372229934943254
3100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^5
5 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491226214
4*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 74129824
6991329886208 0*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60
- 33241631799575052288*a*b^61*c^61*d - 1051560351764368588 8*a^61*b*c*d^61)^
(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c
^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^
4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 +
 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 +
5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 +
 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^1
2 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^1
7*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^
15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*
a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 5560978211448
4224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000
583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 16999671
06662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 9791654790
7584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a
^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30
*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^
39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^
31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3
+ 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^
5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^3
6*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^
23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280
*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 238713320
23900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14
```

```
- 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16
*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 3239680774672220160
0*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365
815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21
+ 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^
20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b
^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^3
4*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^3
6*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^
12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a
^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10
 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624
*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4
*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*
(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 +
78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*
a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a
^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15
*b^2*c^10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*
b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^
7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 16169
91223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375
360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032
*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a
^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^
16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^1
8*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*
b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*
c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 +
 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27
*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d
- 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060
*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31
824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9
 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^1
4*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c
^11*d^15 + 153*a^20*b^2*c^10*d^16)) + (3*(((158640570309279744*a^62*d^62 +
461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 2
5023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^5
8*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491039438099
4297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^
7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 1235027598519926616647
2704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^
10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826982427246
1500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^
```

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49*c^49*d^13 + 75441701298170353675853522253440*a^14*b^48*c^48*d^14 - 190680
74318507301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679
131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^4
5*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827683
90639298679139330949120*a^19*b^43*c^43*d^19 + 67959352440643398986749879045
3248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^4
1*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050
8285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962542766
6534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*
c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498
271125182229871738826673618944*a^27*b^35*c^35*d^27 + 2731504644306965670536
2624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^2
9*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30
- 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841
53161669916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327
680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^2
8*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219427
824782390172272112611360768*a^36*b^26*c^26*d^36 - 4568810856096744273528299
5681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b
^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 1
3411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044038
2672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a
^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43
 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105170784005
10775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a
^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 +
 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113
423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12
*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180
30151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^5
3*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839803
35571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c
^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 1179132068271031006
00320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295
363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10
515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b
^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3
 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 4
91512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 320
9681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454
556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4
665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13
 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16
*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^
14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a
```

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^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739
904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 52865985719
80800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183
744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a
^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*
c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476
736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^4
2*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 3
40848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 1
3838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6
- 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35
*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^
22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658155353702
40*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239680
7746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^1
5 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^
15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569
600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886649
4751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22
 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^1
9*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6
*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b
^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2
*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^
5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001
280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*
b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*
c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*
d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16
+ 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 181
168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320
*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8
*d^23 - 14680064*a^24*b^4*c^7*d^24))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 -
 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b
^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7
*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*
c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((15864057030
9279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a
^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329
126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^
5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842
944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12
350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884249
60*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11
 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106
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604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^4
8*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252
00681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140
715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^
44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524
406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030
488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c
^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337
238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385
529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^
36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27
315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595
4658395768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*
a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^
31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647
91541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575
508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*
c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 45688
108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195991
1507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^3
9*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40
- 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521
843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720
*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^4
4 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964
311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a
^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 -
 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79140998232973321566
8467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c
^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453
9500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c
^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401
196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 +
 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b
^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228
8*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^3
1*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 39817366732
80*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a
^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7
*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*
b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^1
1*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200
*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 945449448058
36800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462
901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 -
```

```
 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d
^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^
9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7
*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^2
6 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 21
3454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30
*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 219902
3255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a
^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^
11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312
*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228120721522
68800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247
375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11
+ 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19
*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3887616929606664192
0*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169
296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18
 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^1
2*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640
*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72281207215
2268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 622730
40062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247
1152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 340
84860461056*a^37*b^2*c^13*d^30))^(3/4))*(((158640570309279744*a^62*d^62 + 4
616893305496535O4*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25
0235617157912199168OO*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58
*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994
297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7
 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472
704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^1
0 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461
500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^4
9*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807
4318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134545486791
31136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45
*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839
0639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453
248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41
*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508
285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666
534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c
^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982
71125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362
624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29
*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 -
```

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 592648874656269275866337706466344496*a^31*b^31*c^31*d^31 + 6858659976808415
3161669916447735808*a^32*b^30*c^30*d^32 - 7397419716479154192785863782432 76
80*a^33*b^29*c^29*d^33 + 7396599789228381850891797657550848 0*a^34*b^28*c^28
*d^34 - 6833570476198873825279649597777510 4*a^35*b^27*c^27*d^35 + 582194278
2478239017227211261136076 8*a^36*b^26*c^26*d^36 - 4568810856096744273528299 5
681296384*a^37*b^25*c^25*d^37 + 3300430609963453195991150701314048 0*a^38*b^
24*c^24*d^38 - 2193725581401928227952194112978944 0*a^39*b^23*c^23*d^39 + 13
41128361812078102928086845410508 8*a^40*b^22*c^22*d^40 - 753766357643044038 2
672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437496450252 8*a^
42*b^20*c^20*d^42 - 184528486514603372464593721884672 0*a^43*b^19*c^19*d^43
+ 80224269548729149690512012214272 0*a^44*b^18*c^18*d^44 - 3194105170784005 1
0775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^
46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 +
11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113 4
23033139200*a^49*b^13*c^13*d^49 + 79140998232973321566846713856 0*a^50*b^12*
c^12*d^50 - 17619948573338866382171799552 0*a^51*b^11*c^11*d^51 + 3507361803
015135770796097536 0*a^52*b^10*c^10*d^52 - 619790967453950095474556928 0*a^53
*b^9*c^9*d^53 + 9637222993494325431002726 40*a^54*b^8*c^8*d^54 - 13038398033
5571997403643904*a^55*b^7*c^7*d^55 + 151267326437054011964129 28*a^56*b^6*c^
6*d^56 - 14760095324137349122621 44*a^57*b^5*c^5*d^57 + 11791320682710310060
0320*a^58*b^4*c^4*d^58 - 741298246991329886208 0*a^59*b^3*c^3*d^59 + 3442953
63448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d - 105
1560351764368588 8*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^
31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3
- 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 49
1512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209
681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 74545
56262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 46
65413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13
+ 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*
d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^1
4*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^
19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 300671810237399
04*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 528659857198
0800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183 7
44*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 1687133546496 0*a^
27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c
^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(687194767
36*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42
*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34
0848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13
838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 -
 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*
d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^2
2*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370 24
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0*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807
746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15
 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^1
5*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239005696
00*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494
751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22
- 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19
*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*
c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^
4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*
c^13*d^30))^(1/4) + (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^23*c^12
*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 3444241905*
a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4*b^19*c
^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d^15 - 3
3837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 3774337560*a
^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c^26 + a
^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*
d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d
^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*
c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*
b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a
^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)))*1i -
 (((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 114314278
2440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3
 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a
^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278
587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^5
4*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171
03771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11
*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2685
471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352
253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^4
7*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613
150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551585754133299
20*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d
^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226492432
831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693
568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39
*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233
921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068562487190169
1117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35
*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701
5781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596
90440055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^
31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32
```

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 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283
818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527964959777
5104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^
26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430
6099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219
41129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*
b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3
892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724
645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^4
4*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 +
116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444854369
7499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b
^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79140
9982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995
520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52
 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100 27
2640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15
126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57
*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913 2
98862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324
1631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2)
 - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^
2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 13
2340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 14164
15142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 568562
2110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 54361
79592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26
292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14
 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^1
5*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b
^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a
^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966
720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 16999671066624
00*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a
^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^
3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 1
3200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^1
1*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 3
4084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471
152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62
273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7
 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^3
4*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*
b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 238713320239005
69600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3887
```

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6169296066641920*a^22*b^17*c^28*d^15 + 4130592987707080704O*a^23*b^16*c^27*
d^16 - 3887616929606664192O*a^24*b^15*c^26*d^17 + 3239680774672220160O*a^25
*b^14*c^25*d^18 - 2387133202390056960O*a^26*b^13*c^24*d^19 + 15516365815535
370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433
247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^2
3 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^1
8*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*
c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*
c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*b^8
*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11*b^
9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 + 182
2251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^6*b
^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^15
- 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^4*b
^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6
*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^
8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^
5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c
^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^28*c
^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 + 17
4818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 16169912238
08*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375360*a^
10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032*a^12*
b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a^14*b^
17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^16*b^1
5*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^18*b^13
*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*b^11*c
^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*c^9*d^
24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 + 37407
94880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27*b^4*c
^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a
^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b
^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^
11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 437
58*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12
 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^
15 + 153*a^20*b^2*c^10*d^16)) - (3*(((1586405703092797744*a^62*d^62 + 461689
330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 2502356
1715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58
*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491039438099429772
4928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 22
47504424575830750669045760*a^8*b^54*c^54*d^8 - 1235027598519926616647270400
0*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 2
38022522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826982427246150062
9647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^4
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9*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185
0730136683515006156 8*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136
*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17
 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827683906392
98679139330949120*a^19*b^43*c^43*d^19 + 6795935244064339898674987904532 48*a
^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21
 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050828568
8517650264998543360*a^23*b^39*c^39*d^23 + 58823372387868700896254276665344 0
0*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d
^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498271125
182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053626240 7
1598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33
*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 5926
4887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531616
69916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^
33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34
 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219427824782
390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829956812 9
6384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^
24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 1341128
3618120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044038267251
2877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^
20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 802
242695487291496905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775 2
18487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^
16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 11664
498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033
139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*
d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180301513
57707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*
c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839803355719
97403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^5
6 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*
a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448
368267264*a^60*b^2*c^2*d^60 - 332416317995750522 88*a*b^61*c^61*d - 10515603
517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^
31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 269
37875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 4915120
97931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 320968140
0053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262
416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413
760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 586
96011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15
+ 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^1
4*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^
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12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^
21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*
a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^
25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^
4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^
29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^
7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d -
2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 3408486
04610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 1383845
3347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 2312
99863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 -
 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^3
3*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^1
9*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239680774672
2201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41
305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^2
6*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^
26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886649475173
4497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 192
7498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24
 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*
d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^1
5*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*
d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^5 + 30
19898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001280*a^
8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*b^18*c
^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*c^19*d
^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*d^14 +
 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16 + 7157
99920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 181168766
976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320*a^21*
b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8*d^23
- 14680064*a^24*b^4*c^7*d^24))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^
5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^
18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*
d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d
^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((15864057030927974
4*a^62*d^62 + 461689330549653504*b^62*c^62 + 11431427824409420759 04*a^2*b^6
```

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655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48
*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812
64313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008
*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^1
8 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433
989867498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576
*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^
22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786
870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708
544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^3
6*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046
443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016159546583
95768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b
^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6
8586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419
27858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480
*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d
^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 45688108560
967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195991150701
3140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23
*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537
663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004
374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*
b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31
9410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813
956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^
15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 31964
89115423809113423033139200*a^49*b^13*c^13*d^49 + 79140998232973321566846713
8560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^
51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453950095
4745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^5
4 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412
928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791
3206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3
*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^
61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31
 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3
*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^2
6*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*
c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c
^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20
*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*
b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*
a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032
960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113
```

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445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 -
30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22
 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24
- 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16
871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 21345472
5120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d
^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 219902325555
2*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^3
0*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^2
8*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*
b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*
a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867
248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 1551
6365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*
d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22
*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066
641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238
71332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23
*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*
b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72281207215226880
0*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 622730400623
49312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247115238
3426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 340848604
61056*a^37*b^2*c^13*d^30))^(3/4))*(((158640570309279744*a^62*d^62 + 4616893
30549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25023561
715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*
d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724
928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224
7504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000
*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23
8022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629
647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49
*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807431850
7301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*
a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17
+ 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839063929
8679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^
20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21
+ 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688
517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400
*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^
25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982711251
82229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071
598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*
c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 59264
```

```
887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808415316166
9916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^3
3*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34
- 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823
90172272112611360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296
384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^2
4*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13411283
618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512
877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^2
0*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022
42695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051707840051077521
8487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^1
6*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 116644
98576526727219629743144960*a^48*b^14*c^14*d^48 - 31964891154238091134230331
39200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d
^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803015135
7707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c
^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398033557199
7403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56
 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a
^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442953634483
68267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 105156035
17643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^3
1 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 2693
7875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 49151209
7931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400
053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 74545562624
16384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 46654137
60860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 5869
6011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 +
 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14
*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^1
2*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^2
1*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a
^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^2
5*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4
*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^2
9 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7
*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2
199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34084860
4610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453
347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 23129
9863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 -
1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33
*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19
```

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*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722
201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 413
05929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26
*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^2
6*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734
497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927
498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24
- 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d
^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15
*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d
^30))^(1/4) - (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^23*c^12*d^9 -
 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 3444241905*a^2*b^
21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4*b^19*c^8*d^1
3 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d^15 - 3383715
8700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 3774337560*a^9*b^1
4*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c^26 + a^22*c^
8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 -
816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 1
8564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d
^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^
15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^
4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)))*1i)/((((15
8640570309279744*a^62*d^62 + 4616893305496535504*b^62*c^62 + 11431427824409
42075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392
117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^5
7*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137
521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54
*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714
04688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*
c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663
425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440
*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15
 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775
962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^1
8*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 +
679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318 70
920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^
22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23
+ 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418
993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248719016911756
8*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*
d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104
0901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400
55062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^3
1*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739
```

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74197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508
917976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a
^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^3
6 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963
4531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297
89440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c
^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412
049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464593 7
218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18
*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263
619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444854369749906
0174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^
14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79140998232
9733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^
51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619
7909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a
^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732
643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c
^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862 0
80*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179
9575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 3982
97088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 39
81736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 13234042
4638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 14164151422
46400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 568562211090
4320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 54361795929
66144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873
905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 945
44944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16
 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^
13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^
11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^
22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^2
4*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^
5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*
d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 1320070
3488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32
 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 3408486
0461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383
426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040
062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228
12072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9
+ 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c
^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*
a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3887616929
```

```
6066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 -
 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*
c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240
*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375
867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72
2812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25
 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d
^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d
^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*b^8*d^19
+ 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11*b^9*c*d^
18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 + 182225124
9*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^6*b^14*c^
6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^15 - 6014
89476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^4*b^13*c^
21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*
c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16
*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13
*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^
11)) + ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^28*c^28*d^
5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 + 17481859
0720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 1616991223808*a^8
*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375360*a^10*b^2
1*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032*a^12*b^19*c
^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a^14*b^17*c^1
7*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^16*b^15*c^15
*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^18*b^13*c^13*
d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*b^11*c^11*d^
22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*c^9*d^24 + 1
52202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 + 3740794880*
a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27*b^4*c^4*d^2
9))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*
c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^
22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^1
1*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^1
4*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 856
8*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 1
53*a^20*b^2*c^10*d^16)) + (3*(((158640570309279744*a^62*d^62 + 461689330549
653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 2502356171579
1219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 -
 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a
^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 22475044
24575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*
b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 2380225
2231371417628822208512 0*a^11*b^51*c^51*d^11 + 85112826982427246150062964736
0*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13
```

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 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185073013
66835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*
b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 188
464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827683906392986791
39330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^
42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 216
6299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36498805082856885176 5
0264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24
*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 +
13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229
871738826673618944*a^27*b^35*c^35*d^27 + 2731504644306965670536262407159808
0*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*
d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 5926488746
562692758663377064663449 6*a^31*b^31*c^31*d^31 + 685865997680841531616699164
47735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^2
9*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683
35704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172
272112611360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a
^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^3
8 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 1341128361812
0781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044038267251287759
2576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^2
0*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 802242695
487291496905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487 1
64928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^1
6*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 11664498576
526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200
*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 -
 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180301513577079
60975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^
53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839803355719974036
43904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 14
76009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b
^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267
264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643
685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 4
00891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 269378754
96960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 4915120979312
64*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 320968140005376
0*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384
*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860
160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 586960119
26323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 1216
70839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17
 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^1
```

2*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^1
0*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b
^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6
*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*
d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 2
4461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32
*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 219902
3255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 3408486046105
60*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 1383845334718
8736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 2312998630
88726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 19274
98859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10
 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20
*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239680774672220160
0*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929
877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17
 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^1
3*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886649475173449728
0*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 192749885
9072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231
299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 -
 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28
 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))
^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^5 + 30198988
80*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001280*a^8*b^20
*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*b^18*c^21*d^
10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*c^19*d^12 +
162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*d^14 + 94575
2637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16 + 7157999206
40*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 181168766976*a^
19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320*a^21*b^7*c^
10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8*d^23 - 1468
0064*a^24*b^4*c^7*d^24))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12
*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3
 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 -
1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 2
86*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((158640570309279744*a^62
*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60
*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240
*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910
394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55
*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199
266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52
*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269
824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680

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*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14
- 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134
54548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*
b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36
3482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867
498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*
b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3
649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089
625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^
25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26
 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069
656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595465839576875
0080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^
32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659
9768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586
37824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*
b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 +
58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 45688108560967442
735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195991150701314048
0*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*
d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576
430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964
502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c
^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051
7078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237
787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^1
5*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 31964891154
23809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a
^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3
5073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453950095474556
9280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13
0383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^
56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682
7103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59
+ 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^6
1*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679
477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*
c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26
*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d
^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^
9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*
d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c
^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b
^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000
*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112
```

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995840*a^19*b^12*c^12*d^19 + 5560978211448424*a^20*b^11*c^11*d^20 - 300671
81023739904*a^21*b^10*c^10*d^21 + 1374200058396672O*a^22*b^9*c^9*d^22 - 528
6598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 4521
24225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335
464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a
^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(
68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*
b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41
*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39
*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c
^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b
^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640
*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 1551636581
5535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 +
 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*
c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920
*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320
23900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20
- 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c
^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31
*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*
a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247115238342656
0*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*
a^37*b^2*c^13*d^30))^(3/4))*(((15864057030927974 4*a^62*d^62 + 4616893305496
53504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791
219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 -
4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^
6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442
4575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b
^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23802252
2313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360
*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13
+ 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807431850730136
6835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b
^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884
64041806198255158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139
9330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^4
2*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166
29933394046988554314497969356 8*a^22*b^40*c^40*d^22 - 36498805082856885176502
64998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*
b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1
3517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982711251822298
71738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080
*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d
^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 59264887465
```

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626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808415316166991644
7735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29
*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 6833
5704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823901722
72112611360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^
37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38
 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13411283618120
781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592
576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20
*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954
87291496905120122142720*a^44*b^18*c^18*d^44 - 31941051707840051077521848716
4928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16
*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765
26727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*
a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 -
176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803015135770796
0975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^5
3 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398033557199740364
3904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147
6009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^
4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442953634483682672
64*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 105156035176436
85888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 40
0891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 2693787549
6960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 49151209793126
4*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760
*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*
a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 46654137608601
60*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 5869601192
6323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 12167
0839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17
+ 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12
*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10
*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^
8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*
c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d
^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24
461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*
c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023
255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34084860461056
0*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188
736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 23129986308
8726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 192749
8859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10
- 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*
```

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c^31*d^12 - 2387133202390056960 0*a^20*b^19*c^30*d^13 + 32396807746722201600
*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 413059298
77070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17
+ 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13
*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280
*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859
072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 2312
99863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 -
13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28
- 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^
(1/4) + (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^23*c^12*d^9 - 33073
9200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 3444241905*a^2*b^21*c^1
0*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4*b^19*c^8*d^13 - 73
884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d^15 - 33837158700*a
^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 3774337560*a^9*b^14*c^3*
d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18
 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^
7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a
^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 4
8620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^1
1 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12
*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16))) + (((15864057030
9279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 11431427824409420759 04*a
^2*b^60*c^60*d^2 - 250235617157912199168 00*a^3*b^59*c^59*d^3 + 392117365329
12621748224 0*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^
5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842
944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12
350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884249
60*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11
 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599810 6
604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^4
8*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252
00681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507759628381 40
715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^
44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524
406433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187092008403 0
488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c
^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337
238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899384838 5
529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^
36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27
315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595
4658395768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*
a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^
31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647
```

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915419278586378243276800*a^33*b^29*c^29*d^33 + 73965997892283818508917976575
508480*a^34*b^28*c^28*d^34 - 683357047619887382527964959777751044*a^35*b^27*
c^27*d^35 + 582194278247823901722721126113607688*a^36*b^26*c^26*d^36 - 45688
108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345319599 1
1507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297894440*a^3
9*b^23*c^23*d^39 + 134112836181207810292808684541050888*a^40*b^22*c^22*d^40
- 75376635764304403826725128775925766*a^41*b^21*c^21*d^41 + 3892412049497521
843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464593721884672 0
*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^4
4 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964
3118139562377871 36*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a
^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 -
 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215666
8467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c
^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453
9500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c
^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401
196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 +
 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b
^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228
8*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^3
1*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 39817366732
80*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a
^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7
*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*
b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^1
1*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200
*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 945449448058
36800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462
901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 -
 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d
^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^
9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7
*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^2
6 - 1687133546496 0*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 21
3454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30
*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 219902
3255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a
^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^
11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312
*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228120721522
68800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247
375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11
+ 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19
*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641 92
```

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0*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169
296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18
 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^1
2*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640
*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72281207215
2268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 622730
40062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247
1152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 340
84860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*b^8*d^19 + 2280960
*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11*b^9*c*d^18 + 3227
35104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 + 1822251249*a^4*b^1
6*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^6*b^14*c^6*d^13 -
4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^15 - 601489476*a^9
*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^4*b^13*c^21 - a^17
*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2
- 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 17
16*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 71
5*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)) - ((
9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^28*c^28*d^5 + 51170
50880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 + 174818590720*a^6*
b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 1616991223808*a^8*b^23*c^2
3*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375360*a^10*b^21*c^21*d^
12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032*a^12*b^19*c^19*d^14
+ 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a^14*b^17*c^17*d^16 +
21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^16*b^15*c^15*d^18 + 2
2988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^18*b^13*c^13*d^20 + 10
145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*b^11*c^11*d^22 + 1960
142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*c^9*d^24 + 1522021171
20*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 + 3740794880*a^25*b^6*
c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27*b^4*c^4*d^29))/(6553
6*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17
+ 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 -
8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^
7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^1
6*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^
5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b
^2*c^10*d^16)) - (3*(((15864057030927944*a^62*d^62 + 4616893305496535O4*b^
62*c^62 + 11431427824409420759O4*a^2*b^60*c^60*d^2 - 250235617157912199168O
0*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 46901984
9064388682475110 4*a^5*b^57*c^57*d^5 + 445949103943809942977249 28*a^6*b^56*c
^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830 7
50669045760*a^8*b^54*c^54*d^8 - 123502759851992661664727O4000*a^9*b^53*c^53
*d^9 + 582312401171037714046884 24960*a^10*b^52*c^52*d^10 - 2380225223137141
76288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^
50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417
```

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0129817035367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068126431345454867913113 6*a^16*b^46*c^46
*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 188464041806
198255158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949 1
20*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d
^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 216629933394
0469885543144979693568*a^22*b^40*c^40*d^22 - 364988050828568851765026499854
3360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^3
8*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187
68320685624871901691117568*a^26*b^36*c^36*d^26 - 1949827112518222987173882 6
673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^
34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 48
092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 59264887465626927 58
6633770646634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*
a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^
33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619
88738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823901722721126 11
360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*
c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 21937
2558140192822795219411297894 40*a^39*b^23*c^23*d^39 + 13411283618120781029 28
0868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41
*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 -
1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496
905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^4
5*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 -
38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219
629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^1
3*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 17619948
5733388663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*
a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 9637
22299349432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^5
5*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 14760095324
13734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^
58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*
b^2*c^2*d^60 - 332416317995750522888*a*b^61*c^61*d - 10515603517643685888*a^
61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 4008915763
20*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4
*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^
25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^2
3*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^2
1*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*
b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*
a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 1216708391264
25600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561
503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 +
```

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55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^
21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^
23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25
 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 22
31346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 2446118092
8*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 6
8719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a
^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b
^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12
*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*
a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 19274988590727
16800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 886649
4751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^1
2 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^
18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807
040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968
07746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^
19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^
11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 192749885907271680
0*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088
726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453
347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848
604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(1
6777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^5 + 3019898880*a^6*b^
22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001280*a^8*b^20*c^23*d^8
 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*b^18*c^21*d^10 - 3401
99997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*c^19*d^12 + 162487336
960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*d^14 + 945752637440*a
^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16 + 715799920640*a^17*b
^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 181168766976*a^19*b^9*c^
12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320*a^21*b^7*c^10*d^21 -
 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8*d^23 - 14680064*a^24
*b^4*c^7*d^24))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d +
 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^
8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11
*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b
^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((158640570309279744*a^62*d^62 + 4
61689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25
023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58
*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994
297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7
 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472
704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^1
0 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461
500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^4
```

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9*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807
4318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134545486791
31136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45
*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839
0639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453
248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41
*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508
285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666
534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c
^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982
71125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362
624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29
*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 -
 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808415
3161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586378243276
80*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28
*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278
24782390172272112611360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995
681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^
24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13
411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382
672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^
42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43
+ 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051707840051
0775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^
46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 +
11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 31964891154238091134
23033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*
c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803
0151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53
*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398033
5571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^
6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682710310060
0320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442953
63448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 105
15603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^
31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3
- 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 49
1512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209
681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 74545
56262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 46
65413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13
+ 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*
d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^1
4*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^
```

19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 300671810237399
04*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 528659857198
0800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 4521242251837
44*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^
27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c
^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(687194767
36*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42
*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34
0848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13
838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 -
 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*
d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^2
2*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658155353702 4
0*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807
746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15
 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^1
5*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239005696
00*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494
751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22
- 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19
*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*
c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^
4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*
c^13*d^30))^(3/4))*(((158640570309279744*a^62*d^62 + 4616893305496535 04*b^6
2*c^62 + 114314278244094207590 4*a^2*b^60*c^60*d^2 - 25023561715791219916800
*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849
0643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^
56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583075
0669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*
d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23802252231371417
6288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^5
0*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170
129817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185073013668351500 6
1568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*
d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061
98255158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949 12
0*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^
20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940
469885543144979693568*a^22*b^40*c^40*d^22 - 36498805082856885176502649985 43
360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38
*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876
8320685624871901691117568*a^26*b^36*c^36*d^26 - 194982711251822298717388266
73618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^3
4*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480
92805215322280459690440055062528*a^30*b^32*c^32*d^30 - 5926488746562692758 6

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633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531616699164477358O8*a
^32*b^30*c^30*d^32 - 739741971647915419278586378243276880*a^33*b^29*c^29*d^3
3 + 7396599789228381850891797657550848O*a^34*b^28*c^28*d^34 - 6833570476198
873825279649597777510 4*a^35*b^27*c^27*d^35 + 582194278247823901722721126113
60768*a^36*b^26*c^26*d^36 - 4568810856096744273528299568129 6384*a^37*b^25*c
^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372
5581401928227952194112978944 0*a^39*b^23*c^23*d^39 + 134112836181207810292 80
868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*
b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1
845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954872914969
05120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45
*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3
8606608474448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765267272196
29743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13
*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485
733388663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a
^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372
2299349432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55
*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241
3734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^5
8 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b
^2*c^2*d^60 - 3324163179957505 2288*a*b^61*c^61*d - 10515603517643685888*a^6
1*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 40089157632
0*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*
b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^2
5*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23
*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21
*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b
^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a
^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 12167083912642
5600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 1155615
03891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 5
5609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^2
1 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^2
3 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25
+ 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 223
1346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928
*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68
719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^
38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^
29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*
b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a
^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 192749885907271
6800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494
751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12
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 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^1
8*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 413059298770708070
40*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 3239680
7746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^1
9 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^1
1*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800
*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 2312998630887
26016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 138384533
47188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 3408486
04610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4) - (
9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^23*c^12*d^9 - 330739200*a*b^
22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 3444241905*a^2*b^21*c^10*d^11 -
19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4*b^19*c^8*d^13 - 73884775320
*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d^15 - 33837158700*a^7*b^16*c
^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 3774337560*a^9*b^14*c^3*d^18 + 72
2155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5
*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^
23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*
c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13
*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564
*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 8
16*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)))))*(((158640570309279744*a
^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c
^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482
240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594
910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b
^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985
199266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b
^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128
269824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655
680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^
14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643
13454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^
17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 -
 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989
867498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^
21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22
- 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870
089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544
*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d
^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443
069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595465839576
8750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32
*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858
6599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278
```

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58637824327680*a^33*b^29*c^29*d^33 + 739659978922838185089179765755O8480*a^
34*b^28*c^28*d^34 - 6833570476198873825279649597775104*a^35*b^27*c^27*d^35
 + 5821942782478239017227211261136O768*a^36*b^26*c^26*d^36 - 45688108560967
4427352829956812963 84*a^37*b^25*c^25*d^37 + 330043060996345319599115070131 4
0480*a^38*b^24*c^24*d^38 - 2193725581401928227952194112978944O*a^39*b^23*c^
23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663
576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437 4
964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^1
9*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941
0517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956
237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*
c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 31964891
15423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138 56
0*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51
+ 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453950095474
5569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 -
 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928
*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320
6827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^
59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179957 5052288*a*b^61*
c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 -
679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^
28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c
^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^2
4*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22
*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^
20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^1
8*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^1
5*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960
000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445
112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 300
67181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 -
5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 4
52124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871
335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 2134547251 2
0*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30
)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a
^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c
^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c
^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^2
6*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^1
5*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248
640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 1551636
5815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^1
3 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^
```

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17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641
920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713
32023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^
20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^1
0*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a
^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 622730400623493
12*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247115238342
6560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 340848604610
56*a^37*b^2*c^13*d^30))^(1/4)*2i - ((x^(1/2)*(8*b^3*c^3 - 11*a^3*d^3 + 27*a
^2*b*c*d^2))/(16*a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) +
 (x^(5/2)*(16*b^3*c^3*d - 7*a^3*d^4 + 27*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3))/(
16*a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d*x^(9/2
)*(8*b^2*c^2*d - 7*a^2*d^3 + 23*a*b*c*d^2))/(16*a*c^2*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^
2 + 2*b*c*d) + b*d^2*x^6) + atan(-((-((158640570309279744*a^62*d^62 + 46168
9330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 250235
61715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^5
8*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809942977
24928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2
247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661664727040
00*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 -
238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698242724615006
29647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^
49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 19068074318
507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068126431345454867913113
6*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^1
7 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 363482768390639
298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*
a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^2
1 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36498805082856
88517650264998543360*a^23*b^39*c^39*d^23 + 58823372387868700896254276665344
00*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*
d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 1949827112
5182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053626240
71598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^3
3*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 592
64887465626927586633770646634496*a^31*b^31*c^31*d^31 + 68586599768084153161
669916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a
^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^3
4 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5821942782478
2390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829956812
96384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c
^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 134112
83618120781029280868454105088*a^40*b^22*c^22*d^40 - 75376635764304403826725
12877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b
```

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^20*c^20*d^42 - 18452848651460337246459372188467 20*a^43*b^19*c^19*d^43 + 80
224269548729149690512012214272 0*a^44*b^18*c^18*d^44 - 3194105170784005107 75
218487164928*a^45*b^17*c^17*d^45 + 1162636192259643118139562377871 36*a^46*b
^16*c^16*d^46 - 386066084744485436974990601748 48*a^47*b^15*c^15*d^47 + 1166
449857652672721962974314496 0*a^48*b^14*c^14*d^48 - 3196489115423809113423 03
3139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12
*d^50 - 17619948573338866382171799552 0*a^51*b^11*c^11*d^51 + 35073618030151
357707960975360*a^52*b^10*c^10*d^52 - 61979096745395009547455692 80*a^53*b^9
*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130383980335571
997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^
56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320
*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 34429536344
8368267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d - 1051560
3517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c
^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26
937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512
097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 32096814
00053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 745455626
2416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 466541
3760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58
696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15
 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^
14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b
^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a
^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800
*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a
^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b
^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d
^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a
^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d -
 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848
604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 138384
53347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231
29986308872 6016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8
- 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^
33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^
19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 323968077467
22201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 4
1305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^
26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a
^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 88664947517
34497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 19
27498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^2
4 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17
*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^
```

```
15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13
*d^30))^(1/4)*(((3*(972405*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400
*a*b^19*c^11*d^8 - 15891876*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 -
 1010174976*a^3*b^17*c^9*d^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a
^5*b^15*c^7*d^12 + 5544069624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d
^14 + 1910559474*a^8*b^12*c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 12516638
4*a^10*b^10*c^2*d^17))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c
^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 +
 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 17
16*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286
*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b
^29*c^29*d^4 - 436207616*a^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 -
 36238786560*a^5*b^26*c^26*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249
088*a^7*b^24*c^24*d^9 + 1616991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^
9*b^22*c^22*d^11 + 4939039375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b
^20*c^20*d^13 + 2154962092032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18
*c^18*d^15 - 13996916736000*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*
c^16*d^17 - 25291944624128*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c
^14*d^19 - 16910399832064*a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^
12*d^21 - 4958946590720*a^20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*
d^23 - 618143940608*a^22*b^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28
274851840*a^24*b^7*c^7*d^26 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26
*b^5*c^5*d^28 + 12845056*a^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c
^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 -
 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 +
18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*
d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c
^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b
^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) + (3*(-((1
58640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 11431427824409
42075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 39
2117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^
57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 34660227858713
7521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^5
4*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771
404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51
*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166
3425998106604003655680*a^13*b^49*c^49*d^13 + 75441701298170353675853522534
40*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^1
5 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 9370132461315077
5962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^
18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 +
 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187
0920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a
^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23
```

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 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141
8993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248719016911175
68*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35
*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810
40901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400
055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^
31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 73
974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7396599789228381850
8917976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*
a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^
36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996
34531959911507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129
789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*
c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241
2049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464593
7218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^1
8*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 11626
3619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084744485436974990
60174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c
^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 7914099823
29733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a
^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61
97909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*
a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673
2643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*
c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862
080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332416317
9957505228*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398
297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3
981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 1323404
24638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142
246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110
904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592
966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 2629287
3905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 94
544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^1
6 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c
^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b
^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a
^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^
24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b
^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3
*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 132007
03488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^3
2 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 340848
```

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60461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 247115238
3426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 6227304
0062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722
812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9
 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*
c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600
*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388761692
96066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16
- 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14
*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 1551636581553537024
0*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 443324737
5867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 7
22812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^2
5 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*
d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*
d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^
4 - 335544320*a^5*b^23*c^26*d^5 + 3019898880*a^6*b^22*c^25*d^6 - 1632632832
0*a^7*b^21*c^24*d^7 + 59276001280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19
*c^22*d^9 + 276572405760*a^10*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*
d^11 + 208834396160*a^12*b^16*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13
- 630974316544*a^14*b^14*c^17*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954
476789760*a^16*b^12*c^15*d^16 + 715799920640*a^17*b^11*c^14*d^17 - 41079013
3760*a^18*b^10*c^13*d^18 + 181168766976*a^19*b^9*c^12*d^19 - 60691578880*a^
20*b^8*c^11*d^20 + 15015608320*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9
*d^22 + 283115520*a^23*b^5*c^8*d^23 - 14680064*a^24*b^4*c^7*d^24))/(8192*(a
^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78
*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^
9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^1
2*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b
^2*c^10*d^11)))*(-((158640570309279744*a^62*d^62 + 4616893305496535 04*b^62*
c^62 + 114314278244094207590 4*a^2*b^60*c^60*d^2 - 25023561715791219916800*a
^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 46901984906
43886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56
*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 22475044245758307506
69045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^
9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 2380225223137141762
88222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*
c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012
9817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185073013668351500615
68*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^
16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 188464041806198
255158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*
a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20
 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 216629933394046
9885543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543 36
```

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0*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d
^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683
20685624871901691117568*a^26*b^36*c^36*d^26 - 1949827112518222987173882667З
618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*
c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 48092
805215322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866З
3770646634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^3
2*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33
+ 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887
38252796495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360
768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^2
5*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 21937255
814019282279521941129789440*a^39*b^23*c^23*d^39 + 1341128361812078102928086
8454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^
21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184
5284865146033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905
120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b
^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 386
06608474448543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629
743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c
^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 17619948573
3388663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^5
2*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 9637222
99349432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b
^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 14760095324137
34912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58
- 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2
*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*
b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*
a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^
27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*
c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c
^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c
^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^1
9*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^1
4*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 1216708391264256
00*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503
891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 556
09782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21
- 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23
- 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 -
97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 22313
46216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a
*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 6871
9476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38
```

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*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29
*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^
27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^1
4*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 19274988590727168
00*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 886649475
1734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 -
 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*
c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040
*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968077
46722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19
+ 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*
c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a
^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726
016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347
188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604
610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(3/4))*(-((
158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440
942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 3
92117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b
^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3466022785871
37521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^
54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 5823124011710377
1404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^5
1*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 26854716
63425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703536758535225344
40*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^
15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507
75962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a
^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19
+ 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318
70920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*
a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^2
3 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90840252339214
18993848385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117
568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^3
5*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 37015781
040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904 4
0055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b
^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 7
3974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659978922838185
08917976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104
*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d
^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 33004306099
634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411 2
9789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22
```

```
*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 38924
12049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464459
37218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^
18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 1162
63619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084744485436974 99
060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*
c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409982
329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*
a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 6
197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640
*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 151267
32643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5
*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298246991329886
2080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631
799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 39
8297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 -
3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340
424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 141641514
2246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110
904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 543617959
2966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 262928
73905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 9
4544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^
16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*
c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*
b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*
a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a
^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*
b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^
3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200
703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^
32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084
860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 24711523
83426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 622730
40062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 72
2812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^
9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21
*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 2387133202390056960
0*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169
296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16
 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^1
4*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 155163658155353702
40*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 44332473
75867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 +
722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^
```

25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16
*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14
*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4) + (9*x^(1/2)*(4862025*a^1
2*b^11*d^21 + 15681600*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 8529381
0*a^11*b^12*c*d^20 + 3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c
^9*d^12 + 56099130741*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 6
0509855250*a^6*b^17*c^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*
a^8*b^15*c^4*d^17 - 3774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*
d^19))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21
*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14
*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*
b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*
a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 -
8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15
+ 153*a^20*b^2*c^10*d^16)))*1i - (-((158640570309279744*a^62*d^62 + 4616893
30549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25023561
715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*
d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809942977724
928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224
7504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000
*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23
8022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629
647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49
*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807431850
7301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*
a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17
+ 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839063929
8679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^
20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21
+ 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688
517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400
*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^
25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982711251
82229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071
598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*
c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 59264
887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808415316166
9916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^3
3*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34
- 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823
90172272112611360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296
384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^2
4*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13411283
618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512
877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^2

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0*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022
426954872914969051201221427200*a^44*b^18*c^18*d^44 - 3194105170784005107752
18487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^1
6*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 116644
98576526727219629743144960*a^48*b^14*c^14*d^48 - 319648911542380911342303311
39200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d
^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803015135
7707960975360*a^52*b^10*c^10*d^52 - 619790967453950095474556928 0*a^53*b^9*c
^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398033557199
7403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56
 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a
^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442953634483
68267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d - 105156035
17643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^3
1 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 2693
7875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 49151209
7931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400
053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 74545562624
16384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 46654137
60860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 5869
6011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 -
 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14
*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^1
2*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^2
1*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a
^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^2
5*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4
*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^2
9 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7
*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2
199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34084860
4610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453
347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 23129
9863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 -
1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33
*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19
*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722
201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 413
05929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26
*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^2
6*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734
497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927
498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24
- 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d
^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15
```

```
*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d
^30))^(1/4)*(((3*(972405*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a
*b^19*c^11*d^8 - 15891876*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1
010174976*a^3*b^17*c^9*d^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5
*b^15*c^7*d^12 + 5544069624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^1
4 + 1910559474*a^8*b^12*c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*
a^10*b^10*c^2*d^17))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^2
0*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 7
15*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716
*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a
^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^2
9*c^29*d^4 - 436207616*a^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 3
6238786560*a^5*b^26*c^26*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 61271624908
8*a^7*b^24*c^24*d^9 + 1616991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*
b^22*c^22*d^11 + 4939039375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^2
0*c^20*d^13 + 2154962092032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c
^18*d^15 - 13996916736000*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^
16*d^17 - 25291944624128*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^1
4*d^19 - 16910399832064*a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12
*d^21 - 4958946590720*a^20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^
23 - 618143940608*a^22*b^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 2827
4851840*a^24*b^7*c^7*d^26 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b
^5*c^5*d^28 + 12845056*a^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8
*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 8
16*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18
564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^
8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^1
5*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4
*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) - (3*(-(((158
640570309279744*a^62*d^62 + 4616893305496535 04*b^62*c^62 + 1143142782440942
075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 3921
17365329126217482240*a^4*b^58*c^58*d^4 - 46901984906438868247511 04*a^5*b^57
*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137 5
2176584294 4*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*
d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714 0
4688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c
^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 26854716634
25998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*
a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15
+ 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507759
62838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18
*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 6
79593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318709
20084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^2
2*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 +
```

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 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90840252339214189
93848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248719016911117568
*a^26*b^36*c^36*d^26 - 194982711251822298717388266736189444*a^27*b^35*c^35*d
^27 + 273150464430696567053626240715980800*a^28*b^34*c^34*d^28 - 37015781040
9016159546583957687500080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337706466344496*a^31*b^31
*c^31*d^31 + 685865997680841531616699164477358808*a^32*b^30*c^30*d^32 - 7397
419716479154192785863782432768 0*a^33*b^29*c^29*d^33 + 739659978922838185089
17976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527964959777751 04*a^
35*b^27*c^27*d^35 + 582194278247823901722721126113607 68*a^36*b^26*c^26*d^36
 - 456881085609674427352829956812963 84*a^37*b^25*c^25*d^37 + 33004306099634
531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297 8
9440*a^39*b^23*c^23*d^39 + 134112836181207810292808684541050 88*a^40*b^22*c^
22*d^40 - 753766357643044038267251287759257 6*a^41*b^21*c^21*d^41 + 38924120
494975218430043749645025 28*a^42*b^20*c^20*d^42 - 184528486514603372464593 72
18846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*
c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 1162636
192259643118139562377871 36*a^46*b^16*c^16*d^46 - 386066084744485436974990 60
174848*a^47*b^15*c^15*d^47 + 116644985765267272196297431449 60*a^48*b^14*c^1
4*d^48 - 319648911542380911342303313920 0*a^49*b^13*c^13*d^49 + 791409982329
733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^5
1*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 6197
909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^
54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 151267326
43705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^
5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298246991329886208
0*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799
575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 39829
7088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 398
1736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424
638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 141641514224
6400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904
320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 543617959296
6144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 262928739
05971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 9454
4944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16
+ 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^1
3*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^1
1*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^2
2*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24
*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5
*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d
^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703
488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32
- 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860
```

```
461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 24711523834
26560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 622730400
62349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 72281
2072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 +
 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^
32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a
^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296
066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 -
38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c
^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*
a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 44332473758
67248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722
812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25
+ 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^
27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^
29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4
- 335544320*a^5*b^23*c^26*d^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*
a^7*b^21*c^24*d^7 + 59276001280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c
^22*d^9 + 276572405760*a^10*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^
11 + 208834396160*a^12*b^16*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 -
630974316544*a^14*b^14*c^17*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 95447
6789760*a^16*b^12*c^15*d^16 + 715799920640*a^17*b^11*c^14*d^17 - 4107901337
60*a^18*b^10*c^13*d^18 + 181168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20
*b^8*c^11*d^20 + 15015608320*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d
^22 + 283115520*a^23*b^5*c^8*d^23 - 14680064*a^24*b^4*c^7*d^24))/(8192*(a^4
*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a
^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*
b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*
b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2
*c^10*d^11)))*(-((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^
62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3
*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643
886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d
^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669
045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9
+ 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288
222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^
50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 75441701298
17035367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568
*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16
 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 18846404180619825
5158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^
19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 -
 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 21662993339404698
85543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*
```

```
a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^2
4 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 13517918768320
685624871901691117568*a^26*b^36*c^36*d^26 - 1949827112518222987173882667361
8944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^
34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 4809280
5215322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337
70646634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*
b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 +
73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 68335704761988738
252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823901722721126113607
8*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*
d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 2193725581
4019282279521941129789440*a^39*b^23*c^23*d^39 + 134112836181207810292808684
54105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21
*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 18452
84865146033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905l2
0122142720*a^44*b^18*c^18*d^44 - 3194105170784005107752184871649 28*a^45*b^1
7*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 38606
608474448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765267272196297 4
3144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^1
3*d^49 + 79140998232973321566846713856 0*a^50*b^12*c^12*d^50 - 1761994857333
88663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*
b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299
34943254310027 2640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7
*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734
912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 -
7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c
^2*d^60 - 3324163179957505 2288*a*b^61*c^61*d - 10515603517643685888*a^61*b*
c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^
2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27
*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^
25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^2
3*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^2
1*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*
c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*
b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600
*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 11556150389
1947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609
782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 -
13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 -
1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97
916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346
216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b
^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 687194
76736*a^39*c^11*d^32 - 219902325552*a^8*b^31*c^42*d - 219902325552*a^38*b
```

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*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c
^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27
*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*
b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800
*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 88664947517
34497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 2
3871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^
29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a
^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746
722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 +
15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^
22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^3
0*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 23129986308872601
6*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 1383845334718
8736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 34084860461
0560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(3/4))*(-((15
8640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 + 114314278244094
2075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392
117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^5
7*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137
521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54
*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714
04688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*
c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663
425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440
*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15
 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775
962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^1
8*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 +
679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870
920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^
22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23
+ 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418
993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068562487190169111756
8*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*
d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104
0901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400
55062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^3
1*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739
74197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508
917976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a
^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^3
6 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963
4531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297
89440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c
```

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^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412
0494975218430043749645025 28*a^42*b^20*c^20*d^42 - 184528486514603372464593 7
218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18
*c^18*d^44 - 3194105170784005107752184871649 28*a^45*b^17*c^17*d^45 + 116263
6192259643118139562377871 36*a^46*b^16*c^16*d^46 - 3860660847444854369749906
0174848*a^47*b^15*c^15*d^47 + 1166449857652672721962974314 4960*a^48*b^14*c^
14*d^48 - 319648911542380911342303313 9200*a^49*b^13*c^13*d^49 + 79140998232
973321566846713 8560*a^50*b^12*c^12*d^50 - 1761994857333886638217179955 20*a^
51*b^11*c^11*d^51 + 3507361803015135770796097 5360*a^52*b^10*c^10*d^52 - 619
790967453950095474556928 0*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a
^54*b^8*c^8*d^54 - 1303839803355719974036439 04*a^55*b^7*c^7*d^55 + 15126732
6437054011964129 28*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c
^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298246991329886 20
80*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179
957505228 8*a*b^61*c^61*d - 105156035176436858 88*a^61*b*c*d^61)^(1/2) + 3982
97088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 39
81736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 13234042
4638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 14164151422
46400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 568562211090
4320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 54361795929
66144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 26292873
905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 945
44944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16
 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^
13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^
11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^
22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^2
4*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^
5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*
d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 1320070
3488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32
 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 3408486
0461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383
426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040
062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228
12072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9
+ 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c
^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*
a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3887616929
6066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 -
 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*
c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240
*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375
867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72
2812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25
```

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 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d
^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d
^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4) - (9*x^(1/2)*(4862025*a^12*
b^11*d^21 + 15681600*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*
a^11*b^12*c*d^20 + 3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9
*d^12 + 56099130741*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 605
09855250*a^6*b^17*c^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^
8*b^15*c^4*d^17 - 3774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^
19))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b
*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c
^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^
11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^
14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 85
68*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 +
153*a^20*b^2*c^10*d^16)))*1i)/((-((158640570309279744*a^62*d^62 + 461689330
549653504*b^62*c^62 + 11431427824409420759O4*a^2*b^60*c^60*d^2 - 2502356171
5791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^
4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491039438099429772492
8*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 22475
04424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a
^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 2380
22522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826982427246150062964
7360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d
^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185073
01366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^
16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 +
188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827683906392986
79139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20
*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 +
2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050828568851
7650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a
^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25
 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498271125182
229871738826673618944*a^27*b^35*c^35*d^27 + 2731504644306965670536262407159
8080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^
33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 5926488
7465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531616699
16447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*
b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 -
68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219427824782390
172272112611360768*a^36*b^26*c^26*d^36 - 4568810856096744273528299568129638
4*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*
d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 1341128361
8120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044038267251287
7592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*
```

```
c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 802242
695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105170784005107752184
87164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*
c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 11664498
576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139
200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^5
0 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180301513577
07960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9
*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839803355719974
03643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 -
 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^5
8*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368
267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517
643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31
+ 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 269378
75496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 4915120979
31264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 320968140005
3760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416
384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760
860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 586960
11926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 1
21670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d
^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*
c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*
b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^2
3*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*
b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c
^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29
- 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b
^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 219
9023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 3408486046
10560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 1383845334
7188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 2312998
63088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 19
27498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d
^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b
^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239680774672220
1600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305
929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d
^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*
b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886649475173449
7280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 192749
8859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 -
231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^2
6 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d
```

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^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^3
0))^(1/4)*(((3*(972405*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b
^19*c^11*d^8 - 15891876*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 101
0174976*a^3*b^17*c^9*d^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b
^15*c^7*d^12 + 5544069624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14
+ 1910559474*a^8*b^12*c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^
10*b^10*c^2*d^17))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*
d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715
*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a
^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^1
4*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b^29*
c^29*d^4 - 436207616*a^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 362
38786560*a^5*b^26*c^26*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*
a^7*b^24*c^24*d^9 + 1616991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^
22*c^22*d^11 + 4939039375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*
c^20*d^13 + 2154962092032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^1
8*d^15 - 13996916736000*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16
*d^17 - 25291944624128*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*
d^19 - 16910399832064*a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d
^21 - 4958946590720*a^20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23
 - 618143940608*a^22*b^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 282748
51840*a^24*b^7*c^7*d^26 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5
*c^5*d^28 + 12845056*a^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d
^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816
*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 1856
4*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8
- 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*
d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c
^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) + (3*(-((15864
0570309279744*a^62*d^62 + 4616893305496535 04*b^62*c^62 + 114314278244094207
5904*a^2*b^60*c^60*d^2 - 2502356171579121991 6800*a^3*b^59*c^59*d^3 + 392117
36532912621748 2240*a^4*b^58*c^58*d^4 - 46901984906438868247 51104*a^5*b^57*c
^57*d^5 + 4459491039438099429772 4928*a^6*b^56*c^56*d^6 - 346602278587137521
765842944*a^7*b^55*c^55*d^7 + 22475044245758307506690 45760*a^8*b^54*c^54*d^
8 - 123502759851992661664727 04000*a^9*b^53*c^53*d^9 + 582312401171037714046
88424960*a^10*b^52*c^52*d^10 - 23802252231371417628822 2085120*a^11*b^51*c^5
1*d^11 + 85112826982427246150062 9647360*a^12*b^50*c^50*d^12 - 268547166342 5
998106604003655680*a^13*b^49*c^49*d^13 + 75441701298170353675853522 53440*a^
14*b^48*c^48*d^14 - 1906807431850730136683515 0061568*a^15*b^47*c^47*d^15 +
4392520068126431345454867 9131136*a^16*b^46*c^46*d^16 - 937013246131507759 62
838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413 329920*a^18*b
^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679
59352440643398986749879045 3248*a^20*b^42*c^42*d^20 - 123422649243283187092 0
084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*
b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5
```

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882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993
848385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a
^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^2
7 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090
1615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400550
62528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c
^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741
97164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917
976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35
*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 -
 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453
1959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297894
40*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22
*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049
497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218
846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^
18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619
225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444854369749906017
4848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*
d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79140998232973
3215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*
b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790
9674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54
*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643
705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*
d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*
a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179957
5052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 3982970
88*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 39817
36673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 13234042463
8464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 14164151422464
00*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 568562211090432
0*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 54361795929661
44*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 26292873905
971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 945449
44805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 +
129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*
d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*
c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*
b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b
^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c
^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^2
8 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 1320070348
8*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 -
2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 3408486046
```

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1056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426
560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062
349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228120
72152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4
433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32
*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^2
0*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3887616929606
6641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38
876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^2
5*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^
27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867
248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72281
2072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 +
62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27
 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29
 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 -
335544320*a^5*b^23*c^26*d^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^
7*b^21*c^24*d^7 + 59276001280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^2
2*d^9 + 276572405760*a^10*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11
 + 208834396160*a^12*b^16*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 63
0974316544*a^14*b^14*c^17*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 9544767
89760*a^16*b^12*c^15*d^16 + 715799920640*a^17*b^11*c^14*d^17 - 410790133760
*a^18*b^10*c^13*d^18 + 181168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b
^8*c^11*d^20 + 15015608320*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^2
2 + 283115520*a^23*b^5*c^8*d^23 - 14680064*a^24*b^4*c^7*d^24))/(8192*(a^4*b
^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6
*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^
8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^
5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c
^10*d^11)))*(-((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62
 + 114314278244094207590 4*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b
^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388
6824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6
 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583075066904
5760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 +
58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23802252231371417628822
2085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50
*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817
035367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a
^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 -
 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551
58575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19
*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1
234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885
543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^
```

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23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24
- 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068
5624871901691117568*a^26*b^36*c^36*d^26 - 194982711251822298717388266736189
44*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34
*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052
15322280459690440055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770
646634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^
30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73
965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 6833570476198873825
2796495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*
a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^
37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140
19282279521941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454
105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c
^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284
865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954872914969051201
22142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*
c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660
8474448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765267272196297431
44960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*
d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388
663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^
10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372229934
9432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c
^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491
2262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 74
12982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2
*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*
d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*
b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c
^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25
*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*
d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*
d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^
19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^
17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a
^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 1155615038919
47520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 5560978
2114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13
742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 16
99967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 9791
6547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 223134621
6960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^3
0*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476
736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c
```

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^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^4
0*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c
^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^
25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a
^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734
497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 238
71332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29
*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^2
3*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 3239680774672
2201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15
516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22
*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*
b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*
a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 138384533471887
36*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 3408486046105
60*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(3/4))*(-((1586
40570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 + 114314278244094 20
75904*a^2*b^60*c^60*d^2 - 2502356171579121991680O*a^3*b^59*c^59*d^3 + 39211
7365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388682475 1104*a^5*b^57*
c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 34660227858713752
1765842944*a^7*b^55*c^55*d^7 + 22475044245758307506690457 60*a^8*b^54*c^54*d
^8 - 123502759851992661664727040 00*a^9*b^53*c^53*d^9 + 58231240117103771404
688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^
51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342
5998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a
^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 +
 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962
838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*
b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 67
9593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318709 2
0084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22
*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 +
5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899
3848385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*
a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^
27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409
01615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400 55
062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*
c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 73974
197164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659978922838185089 1
7976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^3
5*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36
- 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345
31959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297 89
440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^2
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2*d^40 - 75376635764304403826725128775592576*a^41*b^21*c^21*d^41 + 389241204
94975218430043749645025 28*a^42*b^20*c^20*d^42 - 184528486514603372464593721
8846720*a^43*b^19*c^19*d^43 + 80224269548729149690512012214 2720*a^44*b^18*c
^18*d^44 - 31941051707840051077521848716 4928*a^45*b^17*c^17*d^45 + 11626361
92259643118139562377871 36*a^46*b^16*c^16*d^46 - 386066084744485436974990601
74848*a^47*b^15*c^15*d^47 + 1166449857652672721962974314 4960*a^48*b^14*c^14
*d^48 - 319648911542380911342303313 9200*a^49*b^13*c^13*d^49 + 7914099823297
3321566846713856 0*a^50*b^12*c^12*d^50 - 1761994857333886638217179955 20*a^51
*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61979
09674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^5
4*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673264
370540119641 2928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5
*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080
*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332416317995
75052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297
088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981
736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 1323404246
38464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246
400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 56856221109043
20*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966
144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 2629287390
5971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544
944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 +
 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13
*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11
*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22
*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*
b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*
c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^
28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 132007034
88*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 -
 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 340848604
61056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 247115238342
6560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 6227304006
2349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812
072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 +
4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^3
2*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^
20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388761692960
66641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 3
8876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^
25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a
^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 443324737586
7248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 7228
12072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 +
```

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 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^2
7 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^2
9 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4) + (9*x^(1/2)*(4862025*a^12*b^
11*d^21 + 15681600*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^
11*b^12*c*d^20 + 3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d
^12 + 56099130741*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509
855250*a^6*b^17*c^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*
b^15*c^4*d^17 - 3774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19
))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c
^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^2
2*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11
*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14
*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568
*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 15
3*a^20*b^2*c^10*d^16))) + (-((158640570309279744*a^62*d^62 + 46168933054965
3504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 250235617157912
19916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4
690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6
*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424
575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^
53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 238022522
313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*
a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 +
 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807431850730136
6835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^
46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 18846
4041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839063929867913
9330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42
*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 21662
99333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050828568851765
0264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b
^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 13
517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 1949827112518222987
1738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*
a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^
29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656
26927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531616699164477
735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*
c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 68335
704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823901722
72112611360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^3
7*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38
- 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 134112836181207
81029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592
576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*
```

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d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 80224269548
7291496905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164
928*a^45*b^17*c^17*d^45 + 1162636192259643118139562377871 36*a^46*b^16*c^16*
d^46 - 386066084744485436974990601 74848*a^47*b^15*c^15*d^47 + 1166449857652
6727219629743144960*a^48*b^14*c^14*d^48 - 31964891154238091134230331392 00*a
^49*b^13*c^13*d^49 + 79140998232973321566846713856 0*a^50*b^12*c^12*d^50 - 1
76199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180301513577079 60
975360*a^52*b^10*c^10*d^52 - 61979096745395009547455692 80*a^53*b^9*c^9*d^53
 + 96372229934943254310027264 0*a^54*b^8*c^8*d^54 - 1303839803355719974036 43
904*a^55*b^7*c^7*d^55 + 151267326437054011964129 28*a^56*b^6*c^6*d^56 - 1476
0095324137349122621 44*a^57*b^5*c^5*d^57 + 11791320682710310060032 0*a^58*b^4
*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267 26
4*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 105156035176436 8
5888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400
891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496
960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264
*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*
a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a
^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 466541376086016
0*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926
323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670
839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 -
 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*
d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*
c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8
*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c
^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^
27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 244
61180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c
^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 21990232
55552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560
*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 138384533471887
36*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088
726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498
859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 -
 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c
^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*
a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 4130592987
7070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 +
 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*
c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*
a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 19274988590
72716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 23129
9863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 1
3838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 -
```

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 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(
1/4)*(((3*(972405*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c
^11*d^8 - 15891876*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 10101749
76*a^3*b^17*c^9*d^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c
^7*d^12 + 5544069624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 191
0559474*a^8*b^12*c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^
10*c^2*d^17))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 1
3*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*
b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b
^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3
*c^11*d^10 - 78*a^15*b^2*c^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^29*c^29*
d^4 - 436207616*a^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786
560*a^5*b^26*c^26*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b
^24*c^24*d^9 + 1616991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^
22*d^11 + 4939039375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*
d^13 + 2154962092032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^1
5 - 13996916736000*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17
 - 25291944624128*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19
- 16910399832064*a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 -
 4958946590720*a^20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 61
8143940608*a^22*b^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840
*a^24*b^7*c^7*d^26 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*
d^28 + 12845056*a^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 -
 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*
b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^1
0*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 486
20*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11
+ 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d
^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) - (3*(-((1586405703
09279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 11431427824409420759 04*
a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 39211736532
9126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d
^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765 84
2944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 1
2350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884 24
960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^1
1 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599810
6604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^
48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 43925
20068126431345454867913113 6*a^16*b^46*c^46*d^16 - 937013246131507759628381 4
0715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c
^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 67959352
4406433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187092008403
0488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*
c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 588233
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7238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899384838
5529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b
^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 2
7315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016159
54658395768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528
*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d
^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 73974197164
791541927858637824327680*a^33*b^29*c^29*d^33 + 7396599789228381850891797657
5508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27
*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 4568
8108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345319599
11507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^
39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40
 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752
1843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464593721884672
0*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^
44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596
4311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*
a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48
- 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 7914099823297332156
68467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*
c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61979096745
39500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*
c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673264370540
1196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57
+ 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*
b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332416317995750522
88*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^
31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673
280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*
a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^
7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9
*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^
11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 2629287390597120
0*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805
836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 12946
2901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18
+ 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*
d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c
^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^
7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^
26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 2
13454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^3
0*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 21990
23255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*
```

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a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a
^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 6227304006234931
2*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152
268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 443324
7375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11
 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^1
9*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388761692960666419
20*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 3887616
9296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^1
8 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^
12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 443324737586724864
0*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 7228120721
52268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273
040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 24
71152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34
084860461056*a^37*b^2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 33554
4320*a^5*b^23*c^26*d^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^2
1*c^24*d^7 + 59276001280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9
 + 276572405760*a^10*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 20
8834396160*a^12*b^16*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 6309743
16544*a^14*b^14*c^17*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760
*a^16*b^12*c^15*d^16 + 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18
*b^10*c^13*d^18 + 181168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^
11*d^20 + 15015608320*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 2
83115520*a^23*b^5*c^8*d^23 - 14680064*a^24*b^4*c^7*d^24))/(8192*(a^4*b^13*c
^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11
*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^1
6*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^1
3*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d
^11)))*(-((158640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 + 11
43142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c
^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 46901984906438868247
51104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 34
6602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*
a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 58231
240117103771404688424960*a^10*b^52*c^52*d^10 - 2380225223137141762882220851
20*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12
 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703536
7585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b
^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 9370
1324613150775962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575
413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43
*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 123422
6492432831870920084030488576*a^21*b^41*c^41*d^21 + 216629933394046988554314
4979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^
```

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39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 908
4025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248
71901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^
27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28
 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 48092805215322
280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337706466З
4496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^
30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7396599
7892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527964
95977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*
b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 +
33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 21937255814019282
279521941129789440*a^39*b^23*c^23*d^39 + 134112836181207810292808684541050 8
8*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d
^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514
6033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142
720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*
d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084744
48543697499060174848*a^47*b^15*c^15*d^47 + 1166449857652672721962974314496 0
*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49
+ 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 17619948573338866382
1717995520*a^51*b^11*c^11*d^51 + 3507361803015135770796097536 0*a^52*b^10*c^
10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432 5
43100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^
55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491226 21
44*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982
46991329886208 0*a^59*b^3*c^3*d^59 + 3442953634483682672 64*a^60*b^2*c^2*d^60
 - 332416317995750522 88*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)
^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*
c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d
^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6
- 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 -
 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10
- 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^
12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^
17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b
^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520
*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 556097821144
84224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 1374200
0583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967
106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 979165479
07584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*
a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^3
0*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a
^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d
```

```
^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3
 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d
^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^
36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b
^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 886649475173449728
0*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332
023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14
 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^1
6*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968077467222016
00*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 1551636
5815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21
 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c
^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*
b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^
34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^
36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(3/4))*(-((158640570
309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 + 114314278244094207590 4
*a^2*b^60*c^60*d^2 - 250235617157912199168OO*a^3*b^59*c^59*d^3 + 3921173653
29126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*
d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3466022785871375217658
42944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 -
12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884 2
4960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^
11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 26854716634259981
06604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b
^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392
5200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838 1
40715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*
c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 6795935
24406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084 0
30488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40
*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 58823
37238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899384 83
85529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*
b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 +
27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161 5
954658395768750080*a^29*b^33*c^33*d^29 + 4809280521532228045969044005506252
8*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*
d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 7397419716
47915419278586378243276 80*a^33*b^29*c^29*d^33 + 739659978922838185089179765
75508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^2
7*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 456
88108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195 9
911507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a
^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^4
```

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0 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 38924120494975
218430043749645025 28*a^42*b^20*c^20*d^42 - 18452848651460337246459372188467
20*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d
^44 - 31941051707840051077521848716492 8*a^45*b^17*c^17*d^45 + 1162636192259
64311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848
*a^47*b^15*c^15*d^47 + 1166449857652672721962974314 4960*a^48*b^14*c^14*d^48
 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215
668467138560*a^50*b^12*c^12*d^50 - 17619948573338866382171799552 0*a^51*b^11
*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674
539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8
*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 151267326437054
01196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57
 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59
*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052
288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a
^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 398173667
3280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464
*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a
^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^
9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a
^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 262928739059712
00*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 9454494480
5836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 1294
62901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18
 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11
*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*
c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c
^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d
^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 +
213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^
30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199
023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056
*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*
a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 622730400623493
12*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 72281207215
2268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 44332
47375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^1
1 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^
19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641
920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 388761
69296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^
18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b
^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 44332473758672486
40*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072
152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 6227
```

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3040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2
471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 3
4084860461056*a^37*b^2*c^13*d^30))^(1/4) - (9*x^(1/2)*(4862025*a^12*b^11*d^
21 + 15681600*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^
12*c*d^20 + 3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 +
 56099130741*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 6050985525
0*a^6*b^17*c^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*
c^4*d^17 - 3774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(6
5536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^
17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4
 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19
*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*
c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17
*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^2
0*b^2*c^10*d^16)))))*(-((158640570309279744*a^62*d^62 + 461689330549653504*
b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916
800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019
8490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56
*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583
0750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^
53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23802252231371
4176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*
b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544
170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807431850730136683515
0061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^
46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418
06198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839063929867913933094
9120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42
*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333
940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998
543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c
^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791
8768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982711251822298717388
26673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*
b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 +
48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 59264887465626927
586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808415316166991644773580
8*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*
d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 6833570476
1988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247823901722721126
11360768*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^2
5*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219
37255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029
280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^
41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42
```

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- 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954872914
96905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a
^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46
- 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765267272
19629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b
^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199
485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803015135770796097536
0*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96
3722299349432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a
^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953
2413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*
d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^6
0*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*
a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 40089157
6320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a
^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*
b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b
^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b
^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^1
2*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 5869601192632320
0*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 12167083912
6425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 1155
61503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19
- 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*
d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*
d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^
25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 -
2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180
928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 +
 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552
*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10
*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^
12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 23129986308872601
6*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 192749885907
2716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866
494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d
^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*
b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 413059298770708
07040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 3239
6807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*
d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*
b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716
800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 2312998630
88726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 138384
53347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 3408
```

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48604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*
2i + 2*atan(((((158640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62
 + 11431427824409420759O4*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b
^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388
6824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6
 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583075066904
5760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 +
58231240117103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222
2085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50
*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817
035367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a
^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 -
 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551
58575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19
*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1
234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885
543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^
23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24
- 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068
5624871901691117568*a^26*b^36*c^36*d^26 - 194982711251822298717388266736189
44*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34
*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052
15322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337770
646634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^
30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73
965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 6833570476198873825
2796495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*
a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^
37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140
19282279521941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454
105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c
^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284
865146033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954872914969051201
22142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*
c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660
8474448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765267272196297431
44960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*
d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388
66382171799552O*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^
10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372229934
9432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c
^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491
2262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 74
12982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2
*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*
```

```
d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*
b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c
^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25
*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*
d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*
d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^
19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^
17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a
^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 1155615038919
47520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 5560978
2114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13
742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 16
99967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 9791
6547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 223134621
6960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 2446118092 8*a*b^3
0*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476
736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c
^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^4
0*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c
^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^
25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a
^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734
497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 238
71332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29
*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^2
3*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968077467 2
2201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15
516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22
*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*
b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*
a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 138384533471887
36*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610 5
60*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972
405*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 1589
1876*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^
9*d^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544
069624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^
12*c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/
(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d
^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 -
 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 +
1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 7
8*a^15*b^2*c^10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 43620761
6*a^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c
^26*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 +
```

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 1616991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939
039375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962
092032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 1399691673
6000*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624
128*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 169103998320
64*a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720
*a^20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^2
2*b^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*
d^26 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 1284505
6*a^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c
^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3
+ 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^
6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^
17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b
^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19
*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) - ((((1586405703092797744*a^62*d^6
2 + 4616893305496535504*b^62*c^62 + 11431427824409420759 04*a^2*b^60*c^60*d^2
 - 2502356171579121991 6800*a^3*b^59*c^59*d^3 + 3921173653291262174 82240*a^4
*b^58*c^58*d^4 - 469019849064388682475 1104*a^5*b^57*c^57*d^5 + 445949103943
80994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^5
5*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661
66472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^5
2*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698242
72461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^1
3*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 19
068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134545 4
8679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45
*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 363482
768390639298679139330949120*a^19*b^43*c^43*d^19 + 6795935244064339898674987
90453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41
*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36498
805082856885176502649985433 60*a^23*b^39*c^39*d^23 + 588233723878687008962 54
27666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b
^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 1
9498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656 7
05362624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080
*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d
^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 68586599768
084153161669916447735808*a^32*b^30*c^30*d^32 - 7397419716479154192785863782
4327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28
*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5821
9427824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352
82995681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^
38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39
 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 75376635764304
```

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40382672512877592576*a^41*b^21*c^21*d^41 + 38924120494975218430043749645025
28*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*
d^43 + 8022426954872914969051201221427 20*a^44*b^18*c^18*d^44 - 319410517078
400510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431181395623778 71
36*a^46*b^16*c^16*d^46 - 386066084744485436974990601 74848*a^47*b^15*c^15*d^
47 + 116644985765267272196297431449 60*a^48*b^14*c^14*d^48 - 319648911542380
9113423033139200*a^49*b^13*c^13*d^49 + 79140998232973321566846713856 0*a^50*
b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 35073
618030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280
*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130383
98033557199740364390 4*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b
^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682710 3
100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 34
4295363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d
- 1051560351764368588 8*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 6794772
48*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28
*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5
 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 -
 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 -
7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11
 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*
d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*
c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^1
7*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 871134451129958
40*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 3006718102
3739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598
571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 45212422
5183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 168713354649
60*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*
b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(6871
9476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31
*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2
 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4
 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*
d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*
c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^1
7*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535
370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 323
96807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28
*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^2
4*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239 0
0569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 88
66494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*
d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8
*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33
```

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*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^
35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37
*b^2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^2
6*d^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 5927
6001280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a
^10*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b
^16*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c
^17*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d
^16 + 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 +
 181168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 1501560
8320*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5
*c^8*d^23 - 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8
*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 28
6*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a
^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^
13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((1586
40570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 + 11431427824409420
75904*a^2*b^60*c^60*d^2 - 250235617157912199168OO*a^3*b^59*c^59*d^3 + 39211
7365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*
c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521
765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d
^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404
688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^
51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342
5998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a
^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 +
 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507759б
2838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*
b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 67
9593524406433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187092
0084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22
*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 +
5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899
3848385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*
a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^
27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409
01615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400550
62528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*
c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 73974
197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7396599789228381850891
7976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^3
5*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36
- 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345
31959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219411297894
40*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^2
```

2*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204
9497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464593721
8846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c
^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 11626361
9225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084744485436974990601
74848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14
*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 7914099823297
33215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51
*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61979
09674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^5
4*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673264
3705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5
*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080
*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332416317995
75052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297
088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981
736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 1323404246
38464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246
400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 56856221109043
20*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966
144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 2629287390
5971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544
944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 -
129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13
*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11
*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22
*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*
b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*
c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^
28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 132007034
88*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 -
2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 340848604
61056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 247115238342
6560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 6227304006
2349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812
072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 +
4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^3
2*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^
20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388761692960
66641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 3
8876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^
25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a
^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 443324737586
7248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 7228
12072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 +

```
 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^2
7 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^2
9 + 34084860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(((158640570309279744*a^6
2*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^6
0*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240
0*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491
0394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^5
5*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 1235027598519
9266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^5
2*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826
9824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599810660400365568
0*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14
 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313
454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17
*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3
63482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989866
7498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21
*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 -
3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008
9625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a
^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^2
6 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430
6965670536262407159808 0*a^28*b^34*c^34*d^28 - 370157810409016159546583957687
50080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c
^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865
99768084153161669916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858
637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34
*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 +
 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 4568810856096744
2735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345319599115070131404
80*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23
*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357
6430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437496
4502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*
c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105
17078400510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431181395623
7787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^
15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115
423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*
a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 +
35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61979096745395009547455
69280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1
30383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a
^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 1179132068
2710310060032 0*a^58*b^4*c^4*d^58 - 74129824699132988620 80*a^59*b^3*c^3*d^59
```

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 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332416317995750522888*a*b^61*c^
61*d - 105156035176436858888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 67
9477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28
*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^2
6*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*
d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d
^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20
*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*
c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*
b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 12946290103296000
0*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 8711344511
2995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067
181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 52
86598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452
124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 1687133
5464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*
a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/
(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8
*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^4
1*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^3
9*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*
c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*
b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 443324737586724864
0*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658
15535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13
+ 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17
*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 3887616929606664192
0*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332
023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20
 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*
c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^3
1*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312
*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 24711523834265
60*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056
*a^37*b^2*c^13*d^30))^(1/4)*1i - (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681
600*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20
+ 3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 560991307
41*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17
*c^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 -
 3774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*
b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a
^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^
9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 437
58*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10
- 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*
```

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d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10
*d^16))) - (((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 +
 11431427824409420759O4*a^2*b^60*c^60*d^2 - 250235617157912199168OO*a^3*b^5
9*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886 8
24751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 -
 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045 7
60*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 58
231240117103771404688424960*a^10*b^52*c^52*d^10 - 23802252231371417628822 20
85120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d
^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703
5367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^1
5*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 9
3701324613150775962838140715008*a^17*b^45*c^45*d^17 + 18846404180619825515 8
575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b
^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 123
4226492432831870920084030488576*a^21*b^41*c^41*d^21 + 21662993339404698855 4
3144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23
*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 -
9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856
24871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944
*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d
^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 48092805215
322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337706 4
6634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30
*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7396
5997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527
96495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^
36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37
 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 21937255814019
28227952194112978944O*a^39*b^23*c^23*d^39 + 134112836181207810292808684541 0
5088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^2
1*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486
5146033724645937218846720*a^43*b^19*c^19*d^43 + 80224269548729149690512012 2
142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^
17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084
74448543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144
960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^
49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 17619948573338866
3821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10
*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 9637222993494
3254310027264O*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7
*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912 2
62144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412
982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d
^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^
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61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 1

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616991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 493903
9375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 215496209
2032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 139969167360
00*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 2529194462412
8*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064
*a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a
^20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*
b^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^
26 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*
a^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^2
5*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 +
3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6
- 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17
*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6
*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b
^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) + ((((158640570309279744*a^62*d^62
+ 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 -
 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b
^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380
994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*
d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166
472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*
d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272
461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*
b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906
8074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134545486
79131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c
^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276
8390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790
453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c
^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880
5082856885176502649985433360*a^23*b^39*c^39*d^23 + 5882337238786870089625427
666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^3
7*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194
98271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705
362624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a
^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^3
0 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808
4153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586378243
27680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c
^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194
27824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352822
995681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38
*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 +
 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440
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382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528
*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^
43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051707840
051077521848716 4928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136
*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47
 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809 1
13423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^
12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361
8030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a
^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398
0335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6
*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103 10
0600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442
95363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d -
105156035176436858 88*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248
*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d
^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 -
 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3
209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 74
54556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 +
 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^
13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^
16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*
b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840
*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 300671810237
39904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 528659857
1980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 4521242251
83744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960
*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^
2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(687194
76736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c
^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 -
 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 -
 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^
6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^
35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*
b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658155353 7
0240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396
807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d
^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*
b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239005
69600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866
494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^
22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c
^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b
```

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^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35
*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b
^2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*
d^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 592760
01280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^1
0*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^1
6*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^1
7*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^1
6 + 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 1
81168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 150156083
20*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c
^8*d^23 - 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d
^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*
a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^1
0*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13
*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((158640
570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075
904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 3921173
65329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^
57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3466022785871375217
65842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8
 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 5823124011710377140468
8424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51
*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 26854716634259
98106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^1
4*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4
3925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507759628
38140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^
44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 6795
93524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318709200
84030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b
^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 58
82337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90840252339214189938
48385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^
26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27
 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 37015781040901
615954658395768750080*a^29*b^33*c^33*d^29 + 4809280521532228045969044005506
2528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^
31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 7397419
7164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659978922838185089179
76575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*
b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 -
45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 33004306099634531
959911507013140480*a^38*b^24*c^24*d^38 - 2193725581401928227952194112978944
0*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*
```

d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 38924120494
97521843004374964502528*a^42*b^20*c^20*d^42 - 18452848651460337246459372188
46720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^1
8*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 1162636192
25964311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174
848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d
^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733
215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b
^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 6197909
674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*
b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 151267326437
05401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d
^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a
^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575
052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 39829708
8*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 398173
6673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638
464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 141641514224640
0*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320
*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 543617959296614
4*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 262928739059
71200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 9454494
4805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 1
29462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d
^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c
^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b
^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^
7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^
5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28
 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488
*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2
199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461
056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 24711523834265
60*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 622730400623
49312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 72281207
2152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 44
33247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*
d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20
*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066
641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 388
76169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25
*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^2
7*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 44332473758672
48640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812
072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 6

2273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27
+ 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29
+ 34084860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(((158640570309279744*a^62*
d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*
d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*
a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103
94380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*
c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992
66166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*
c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698
24272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*
a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 -
 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068126431345
4548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b
^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 363
482768390639298679139330949120*a^19*b^43*c^43*d^19 + 6795935244064339898674
98790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b
^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36
49880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089б
25427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^2
5*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26
- 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696
56705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016159546583957687‍50
080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^3
2*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 68586599
768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586з
7824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b
^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5
8219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427
35282995681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480
*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d
^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 75376635764
30440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437496‍45
02528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^
19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 319410517
078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237‍7
87136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15
*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 319648911542
3809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^
50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 35
073618030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569
280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130
383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^5
6*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827
10310060032‍0*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 +

```
 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61
*d - 105156035176436858888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 6794
77248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c
^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*
d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^
7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9
 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d
^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^
18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^
16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*
a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 871134451129
95840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 3006718
1023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286
598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 45212
4225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 168713354
64960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^
29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(6
8719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b
^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*
d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*
d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^
37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^
24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*
a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815
535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 +
32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c
^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*
a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 2387133202
3900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 -
 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^
21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*
b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a
^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560
*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a
^37*b^2*c^13*d^30))^(1/4)*1i + (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 1568160
0*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 +
3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741
*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c
^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 3
774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^
18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6
*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*
b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758
*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 -
31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^
```

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13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d
^16))))/((((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1
1431427824409420759 04*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*
c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388 6824
751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3
46602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760
*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 5823
1240117103771404688424960*a^10*b^52*c^52*d^10 - 2380225223137141762882220 85
120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^1
2 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703 53
67585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*
b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937
01324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551585 7
5413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^4
3*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342
26492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543 1
44979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b
^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90
84025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068562 4
871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a
^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^2
8 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 4809280521532
2280459690440055062528*a^30*b^32*c^32*d^30 - 5926488746562692758663377064 66
34496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c
^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659
97892283818508917976575508480*a^34*b^28*c^28*d^34 - 6833570476198873825279 6
495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36
*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 +
 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192 8
227952194112978944 0*a^39*b^23*c^23*d^39 + 1341128361812078102928086845410 50
88*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*
d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 18452848651
46033724645937218846720*a^43*b^19*c^19*d^43 + 8022426954872914969051201221 4
2720*a^44*b^18*c^18*d^44 - 31941051707840051077521848716 4928*a^45*b^17*c^17
*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 38606608474
448543697499060174848*a^47*b^15*c^15*d^47 + 1166449857652672721962974314496
0*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49
 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663 8
21717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c
^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432
543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d
^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491226 2
144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298
2469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^6
0 - 3324163179957505 2288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61
```

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)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29
*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*
d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6
 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8
+ 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10
 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d
^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c
^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*
b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 1155615038919475
20*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114
484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 137420
00583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 169996
7106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547
907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960
*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 2446118092 8*a*b^30*c^
30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*
a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*
d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^
3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*
d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c
^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*
b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 886649475173449728
0*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 2387133
2023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^1
4 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^
16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 3239680774672220 1
600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 155163
65815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^2
1 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*
c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32
*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a
^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a
^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*
a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876
*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^
10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 55440696
24*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c
^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(819
2*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12
+ 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 128
7*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287
*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^
15*b^2*c^10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^
3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*
d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 161
```

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6991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 49390393
75360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 21549620920
32*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000
*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*
a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a
^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^2
0*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^
9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26
 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^
27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*
d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 30
60*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 -
31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d
^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c
^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3
*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) - ((((158640570309279744*a^62*d^62 +
461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 2
502356171579121991680*a^3*b^59*c^59*d^3 + 3921173653291262174822400*a^4*b^5
8*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809
94297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^
7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661664 7
2704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^
10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698242724 6
1500629647360*a^12*b^50*c^50*d^12 - 268547166342599810660400365568 0*a^13*b^
49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 190680
74318507301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679
131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^4
5*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827683
90639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790 45
3248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^4
1*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050
8285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962542766
6534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*
c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498
27112518222987173882667361894 4*a^27*b^35*c^35*d^27 + 27315046443069656705 36
2624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^2
9*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30
- 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841
53161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586378243 27
680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^2
8*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219427
824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829 9
5681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b
^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 1
3411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440 38
```

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2672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a
^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43
 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105170784005
10775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a
^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 +
 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113
423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12
*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180
30151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^5
3*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839803
35571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c
^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 1179132068271031006
00320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295
363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10
515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*b
^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3
 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 4
91512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 320
9681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454
556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4
665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^13
 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^16
*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b^
14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*a
^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181023739
904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 52865985719
80800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124225183
744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*a
^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*
c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68719476
736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^4
2*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 3
40848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 1
3838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6
- 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35
*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^
22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658155353702
40*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239680
7746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^1
5 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^
15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569
600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886649
4751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22
 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^1
9*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6
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*c^17*d^26 - 1383845334718873б*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b
^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2
*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^
5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001
280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*
b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*
c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*
d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16
+ 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 181
168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320
*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8
*d^23 - 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d^1
3 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^
7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*
b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b
^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((15864057
0309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 114314278244094207590
4*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365
329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57
*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765
842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 -
 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884
24960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d
^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998
106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*
b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439
25200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838
140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44
*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593
524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084
030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^4
0*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882
337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848
385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26
*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 +
 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161
5954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400550625
28*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31
*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971
64791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976
575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^
27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 45
688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195
9911507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*
a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^
```

```
40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497
521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846
720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*
d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225
964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444854369749906017484
8*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^4
8 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79140998232973321
5668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^1
1*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967
4539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^
8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705
401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^5
7 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^5
9*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505
2288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*
a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 39817366
73280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 13234042463846
4*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*
a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a
^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*
a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971
200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 945449448
05836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129
462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^1
8 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^1
1*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9
*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*
c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*
d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 -
 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a
^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 219
9023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 3408486046105
6*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560
*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349
312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228120721
52268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433
247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^
11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b
^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3887616929606664
1920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876
16929606641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d
^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*
b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248
640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72281207
2152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 622
```

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73040062349312*a^33*b^6*c^17*d^26 - 1383845334718873 6*a^34*b^5*c^16*d^27 +
2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 +
34084860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(((158640570309279744*a^62*d^
62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^
2 - 2502356171579121 9916800*a^3*b^59*c^59*d^3 + 3921173653291262 17482240*a^
4*b^58*c^58*d^4 - 469019849064388682 4751104*a^5*b^57*c^57*d^5 + 44594910394
380994297724928*a^6*b^56*c^56*d^6 - 3466022785871375217658 42944*a^7*b^55*c^
55*d^7 + 224750442457583075066904 5760*a^8*b^54*c^54*d^8 - 1235027598519926 6
166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884 24960*a^10*b^52*c^
52*d^10 - 23802252231371417628822208 5120*a^11*b^51*c^51*d^11 + 851128269824
272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599810660400365 5680*a^
13*b^49*c^49*d^13 + 754417012981703536758535225 3440*a^14*b^48*c^48*d^14 - 1
906807431850730136683515006 1568*a^15*b^47*c^47*d^15 + 439252006812643134545
48679131136*a^16*b^46*c^46*d^16 - 9370132461315077596283814071 5008*a^17*b^4
5*c^45*d^17 + 1884640418061982551585754133 29920*a^18*b^44*c^44*d^18 - 36348
276839063929867913933094 9120*a^19*b^43*c^43*d^19 + 679593524406433989867498
790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^4
1*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649
880508285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962 5
427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*
b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 -
19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656
705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016159546583957687500 8
0*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*
d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976
8084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586378
24327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^2
8*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582
19427824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427 35
282995681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a
^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^3
9 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430
440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437496450 2
528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19
*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051707
8400510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431181395623778 7
136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d
^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423 8
09113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50
*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507
361803015135770796097536 0*a^52*b^10*c^10*d^52 - 619790967453950095474556928
0*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038
3980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*
b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682710
3100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3
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44295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d
 - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477
248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^2
8*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^
5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7
 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 -
 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^1
1 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18
*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16
*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^
17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995
840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 300671810
23739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 528659
8571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 4521242
25183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464
960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29
*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(687
19476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^3
1*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^
2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^
4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37
*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24
*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^
17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 1551636581553
5370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32
396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^2
8*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^
24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239
00569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8
866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21
*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^
8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^3
3*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a
^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^3
7*b^2*c^13*d^30))^(1/4)*1i - (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*
b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 34
44241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a
^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6
*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 377
4337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18
*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b
^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^
13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a
^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31
824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13

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 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^1
6)))*1i + (((158640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 +
1143142782440942075904*a^2*b^60*c^60*d^2 - 250235617157912199168OO*a^3*b^59
*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388682
4751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 -
346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583075066904576
0*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582
31240117103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222O8
5120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^
12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703
5367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15
*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93
701324613150775962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158
57541332992O*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^
43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234
226492432831870920084030488576*a^21*b^41*c^41*d^21 + 216629933394046988554 3
144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*
b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9
084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068562
4871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*
a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^
28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153
22280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337706 46
634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*
c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965
997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527 9
6495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^3
6*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37
+ 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192
82279521941129789440*a^39*b^23*c^23*d^39 + 1341128361812078102928086845410 5
088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21
*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865
146033724645937218846720*a^43*b^19*c^19*d^43 + 80224269548729149690512012 21
42720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^1
7*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847
4448543697499060174848*a^47*b^15*c^15*d^47 + 116644985765267272196297431449
60*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^4
9 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663
821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*
c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372229934943
2543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*
d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 147600953241373491226
2144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 74129
82469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^
60 - 3324163179957505228 8*a*b^61*c^61*d - 105156035176436858 88*a^61*b*c*d^6
```

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1)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^2
9*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27
*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^
6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8
 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^1
0 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*
d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*
c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16
*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 1155615038919475
20*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 5560978211
4484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742
000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 16999
67106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 9791654
7907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 223134621696
0*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c
^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736
*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12
*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d
^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38
*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*
c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16
*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497
280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 238713
32023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^
14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b
^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968077467222
01600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516
365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^
21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9
*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^3
2*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*
a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*
a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405
*a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 1589187
6*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d
^10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069
624*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*
c^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(81
92*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12
 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 12
87*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 128
7*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a
^15*b^2*c^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a
^3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26
*d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 16
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16991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039
375360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092
032*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 1399691673600
0*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128
*a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*
a^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^
20*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b
^9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^2
6 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a
^27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25
*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3
060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 -
 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*
d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*
c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^
3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) + ((((158640570309279744*a^62*d^62 +
 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 -
25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^
58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809
94297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d
^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661664
72704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d
^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698242724
61500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b
^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 19068
074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068126431345454867
9131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^
45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 363482768
390639298679139330949120*a^19*b^43*c^43*d^19 + 6795935244064339898674987904
53248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^
41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36498805
08285688517650264998543360*a^23*b^39*c^39*d^23 + 58823372387868700896254276
66534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37
*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 1949
8271125182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053
62624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^
29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30
 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 68586599768084
153161669916447735808*a^32*b^30*c^30*d^32 - 7397419716479154192785863782432
7680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^
28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5821942
7824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829
95681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*
b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 +
13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 75376635764304403
```

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82672512877592576*a^41*b^21*c^21*d^41 + 38924120494975218430043749645025 28*
a^42*b^20*c^20*d^42 - 18452848651460337246459372188467 20*a^43*b^19*c^19*d^4
3 + 8022426954872914969051201221427 20*a^44*b^18*c^18*d^44 - 319410517078400
510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431181395623778713 6*
a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47
+ 1166449857652672721962974314496 0*a^48*b^14*c^14*d^48 - 319648911542380911
3423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^1
2*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 35073618
0301513577079609753 60*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^
53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130383980
335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*
c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100
600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 34429
5363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d - 1
0515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 679477248*
b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^
3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d^5 -
491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 32
09681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9 - 745
4556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^11 +
4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^18*d^1
3 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^16*c^1
6*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a^17*b
^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 87113445112995840*
a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 3006718102373
9904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 5286598571
980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 45212422518
3744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 16871335464960*
a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2
*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(6871947
6736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^
42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 -
340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 -
13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6
 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^3
5*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b
^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370
240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 323968
07746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^
15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b
^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239005 6
9600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 88664
94751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^2
2 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^
19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^
```

```
6*c^17*d^26 - 1383845334718873 6*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*
b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^
2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d
^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 5927600
1280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10
*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16
*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17
*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16
 + 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 18
1168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 1501560832
0*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^
8*d^23 - 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d^
13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a
^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10
*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*
b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(((1586405
70309279744*a^62*d^62 + 4616893305496535 04*b^62*c^62 + 114314278244094207 59
04*a^2*b^60*c^60*d^2 - 250235617157912199168 00*a^3*b^59*c^59*d^3 + 39211736
5329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^5
7*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3466022785871375 2176
5842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8
- 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046 88
424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*
d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599
8106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14
*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 43
925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962 83
8140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^4
4*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 67959
3524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318709 2008
4030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^
40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 588
2337238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899384
8385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^2
6*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27
+ 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016
15954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596904400550 62
528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^3
1*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 73974197
16479154192785863782432 7680*a^33*b^29*c^29*d^33 + 73965997892283818508917 97
6575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b
^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 4
5688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345319
59911507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440
*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d
```

```
^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949
7521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372464593721884
6720*a^43*b^19*c^19*d^43 + 80224269548729149690512012214272O*a^44*b^18*c^18
*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 11626361922
5964311813956237787136*a^46*b^16*c^16*d^46 - 386066084744485436974990601748
48*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^
48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 7914099823297332
15668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^
11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61979096
74539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b
^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673264370
5401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^
57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^
59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332416317995750
52288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088
*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736
673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 1323404246384
64*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400
*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*
a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144
*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19*d^12 - 2629287390597
1200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944
805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 12
9462901032960000*a^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^
18 - 87113445112995840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^
11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^
9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7
*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5
*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28
- 213454725120*a^29*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*
a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 21
99023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 340848604610
56*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 247115238342656
0*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 6227304006234
9312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072
152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 443
3247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d
^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*
b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388761692960666
41920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 3887
6169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*
d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27
*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 443324737586724
8640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 7228120
72152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62
```

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273040062349312*a^33*b^6*c^17*d^26 - 1383845334718873б*a^34*b^5*c^16*d^27 +
 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 +
 34084860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(((15864057030927974 4*a^62*d
^62 + 4616893305496535 04*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d
^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a
^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491039
4380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c
^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 1235027598519926
6166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c
^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826982
4272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a
^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 -
19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068126431345 4
548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^
45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634
82768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867 49
8790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^
41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364
9880508285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962
5427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25
*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 -
 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 2731504644306965
6705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595465839576875 00
80*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32
*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997
68084153161669916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637
824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^
28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58
219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427 3
5282995681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*
a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^
39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643
0440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437496450
2528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^1
9*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105170
78400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237 78
7136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*
d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423
809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^5
0*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350
73618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453950095474556 92
80*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303
83980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56
*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 1179132068271
03100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 +
```

```
344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*
d - 10515603517643685888*a^61*b*c*d^61)^(1/2) - 398297088*a^31*d^31 - 67947
7248*b^31*c^31 - 400891576320*a^2*b^29*c^29*d^2 + 3981736673280*a^3*b^28*c^
28*d^3 - 26937875496960*a^4*b^27*c^27*d^4 + 132340424638464*a^5*b^26*c^26*d
^5 - 491512097931264*a^6*b^25*c^25*d^6 + 1416415142246400*a^7*b^24*c^24*d^7
 - 3209681400053760*a^8*b^23*c^23*d^8 + 5685622110904320*a^9*b^22*c^22*d^9
- 7454556262416384*a^10*b^21*c^21*d^10 + 5436179592966144*a^11*b^20*c^20*d^
11 + 4665413760860160*a^12*b^19*c^19*d^12 - 26292873905971200*a^13*b^18*c^1
8*d^13 + 58696011926323200*a^14*b^17*c^17*d^14 - 94544944805836800*a^15*b^1
6*c^16*d^15 + 121670839126425600*a^16*b^15*c^15*d^16 - 129462901032960000*a
^17*b^14*c^14*d^17 + 115561503891947520*a^18*b^13*c^13*d^18 - 8711344511299
5840*a^19*b^12*c^12*d^19 + 55609782114484224*a^20*b^11*c^11*d^20 - 30067181
023739904*a^21*b^10*c^10*d^21 + 13742000583966720*a^22*b^9*c^9*d^22 - 52865
98571980800*a^23*b^8*c^8*d^23 + 1699967106662400*a^24*b^7*c^7*d^24 - 452124
225183744*a^25*b^6*c^6*d^25 + 97916547907584*a^26*b^5*c^5*d^26 - 1687133546
4960*a^27*b^4*c^4*d^27 + 2231346216960*a^28*b^3*c^3*d^28 - 213454725120*a^2
9*b^2*c^2*d^29 + 24461180928*a*b^30*c^30*d + 13200703488*a^30*b*c*d^30)/(68
719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^
31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d
^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d
^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^3
7*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^2
4*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a
^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658155
35370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3
2396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^
28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a
^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023
900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 -
8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^2
1*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b
^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^
33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*
a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^
37*b^2*c^13*d^30))^(1/4)*1i + (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600
*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 3
444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*
a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^
6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 37
74337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^1
8*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*
b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b
^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*
a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 3
1824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^1
```

3 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^
16)))*1i))*(((158640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 +
 11431427824409420759O4*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^5
9*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 -
 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583075066904576O*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 58
23124011710377140468842496O*a^10*b^52*c^52*d^10 - 23802252231371417628822208512O*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d
^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703
5367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^1
5*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 9
3701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551585
75413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b
^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 123
4226492432831870920084030488576*a^21*b^41*c^41*d^21 + 216629933394046988554
3144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23
*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 -
9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856
24871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944
*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d
^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 48092805215
322280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337706
46634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30
*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7396
5997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527
96495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^
36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37
 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 21937255814019
282279521941129789440*a^39*b^23*c^23*d^39 + 134112836181207810292808684541O
5088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^2
1*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486
5146033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122
142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^
17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084
74448543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144
960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^
49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 17619948573338866
3821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10
*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 9637222993494
3254310027264O*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7
*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412
98246991329886208O*a^59*b^3*c^3*d^59 + 3442953634483682672640*a^60*b^2*c^2*d
^60 - 33241631799575052288*a*b^61*c^61*d - 1051560351764368588 8*a^61*b*c*d^

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61)^(1/2) - 398297088*a^31*d^31 - 679477248*b^31*c^31 - 400891576320*a^2*b^
29*c^29*d^2 + 3981736673280*a^3*b^28*c^28*d^3 - 26937875496960*a^4*b^27*c^2
7*d^4 + 132340424638464*a^5*b^26*c^26*d^5 - 491512097931264*a^6*b^25*c^25*d
^6 + 1416415142246400*a^7*b^24*c^24*d^7 - 3209681400053760*a^8*b^23*c^23*d^
8 + 5685622110904320*a^9*b^22*c^22*d^9 - 7454556262416384*a^10*b^21*c^21*d^
10 + 5436179592966144*a^11*b^20*c^20*d^11 + 4665413760860160*a^12*b^19*c^19
*d^12 - 26292873905971200*a^13*b^18*c^18*d^13 + 58696011926323200*a^14*b^17
*c^17*d^14 - 94544944805836800*a^15*b^16*c^16*d^15 + 121670839126425600*a^1
6*b^15*c^15*d^16 - 129462901032960000*a^17*b^14*c^14*d^17 + 115561503891947
520*a^18*b^13*c^13*d^18 - 87113445112995840*a^19*b^12*c^12*d^19 + 556097821
14484224*a^20*b^11*c^11*d^20 - 30067181023739904*a^21*b^10*c^10*d^21 + 1374
2000583966720*a^22*b^9*c^9*d^22 - 5286598571980800*a^23*b^8*c^8*d^23 + 1699
967106662400*a^24*b^7*c^7*d^24 - 452124225183744*a^25*b^6*c^6*d^25 + 979165
47907584*a^26*b^5*c^5*d^26 - 16871335464960*a^27*b^4*c^4*d^27 + 22313462169
60*a^28*b^3*c^3*d^28 - 213454725120*a^29*b^2*c^2*d^29 + 2446180928*a*b^30*
c^30*d + 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 6871947673
6*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^1
2*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*
d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^3
8*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25
*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^1
6*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 886649475173449
7280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871
332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d
^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*
b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968077467222
01600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 1551
6365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d
^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^
9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^
32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736
*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560
*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4) + 2*atan(((
-((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782
440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3
+ 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^
5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3466022785
87137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54
*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171O
3771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*
b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 26854
71663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703536758535522
53440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47
*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131
50775962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329 92
```

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0*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^
19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328
31870920084030488576*a^21*b^41*c^41*d^21 + 216629933394046988554314497969 35
68*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*
d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90840252339
21418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248719016 91
117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*
c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 37015
781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804596 9
0440055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^3
1*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32
- 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659978922838
18508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527964959777 75
104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^2
6*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 33004306
099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795219 4
1129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b
^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 38
92412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372 46
4593721884 6720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44
*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 1
16263619225964311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697
499060174848*a^47*b^15*c^15*d^47 + 1166449857652672721962974314496 0*a^48*b^
14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409
982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995 5
20*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52
- 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272
640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 151
26732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*
b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298246991329
8862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241
631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) +
 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2
 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132
340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 141641
5142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622
110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 543617
9592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 262
92873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14
+ 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15
*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^
13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^
20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966 7
20*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 169996710666240
0*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^
```

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26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3
*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13
200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11
*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34
084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 24711
52383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 622
73040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 +
 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34
*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b
^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 2387133202390056
9600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876
169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d
^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*
b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 155163658155353
70240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 44332
47375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23
 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18
*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c
^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c
^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*b^8*
d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11*b^9
*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 + 1822
251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^6*b^
14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^15 -
 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^4*b^
13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*
b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8
*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5
*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^
10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^28*c^
28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 + 174
818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 161699122380
8*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375360*a^1
0*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032*a^12*b
^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a^14*b^1
7*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^16*b^15
*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^18*b^13*
c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*b^11*c^
11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*c^9*d^2
4 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 + 374079
4880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27*b^4*c^
4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^
21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^
14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^1
1*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 4375
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8*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12
- 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^1
5 + 153*a^20*b^2*c^10*d^16)) - ((-((158640570309279744*a^62*d^62 + 46168933
0549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 250235617
15791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d
^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809942977249
28*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2247
504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*
a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 238
022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500629
647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*
d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 19068074318507
301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a
^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 +
 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298
679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^2
0*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 +
 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36498805082856885
17650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*
a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^2
5 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 1949827112518
2229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053626240715
98080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33*c
^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 592648
87465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531616669
916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^33
*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34 -
 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5821942782478239
0172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829956812963
84*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^24
*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 134112836
18120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044038267251287
7592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20
*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 80224
2695487291496905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218
487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16
*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 1166449
8576526727219629743144960*a^48*b^14*c^14*d^48 - 319648911542380911342303313
9200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^
50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357
707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^
9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997
403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56
- 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^
58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 34429536344836
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8267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 1051560351
7643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31
 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937
875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097
931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 32096814000
53760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 745455626241
6384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 466541376
0860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696
011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 -
121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*
d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12
*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21
*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^
23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25
*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*
c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29
 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*
b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 21
99023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604
610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 138384533
47188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299
863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1
927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*
d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*
b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 323968077467222
01600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 4130
5929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*
d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26
*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 88664947517344
97280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 19274
98859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 -
 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^
26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*
d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^
30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^5 + 3019
898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001280*a^8*
b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*b^18*c^2
1*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*c^19*d^1
2 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*d^14 + 9
45752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16 + 715799
920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 18116876697
6*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320*a^21*b^
7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8*d^23 -
14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a
^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c
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^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15
*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*
d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(-((158640570309279
744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b
^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 +
44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*
a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502
75985199266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a
^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8
51128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599810660400
3655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^
48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068
1264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507759628381407150
08*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d
^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 6795935244064
33989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187092008403048857
6*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*
d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 58823372387
86870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899384838552970
8544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c
^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 273150
46443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016159546583
95768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30
*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 +
 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 7397419716479154
1927858637824327680*a^33*b^29*c^29*d^33 + 7396599789228381850891797657550848
0*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27
*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085
60967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345319599115070
13140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^
23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 75
37663576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300
4374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^4
3*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 -
319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 1162636192259643118
13956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*
b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 319
6489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467
138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*
d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500
954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d
^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673264370540119641
2928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117
913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c
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^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*
b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^
31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a
^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b
^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^2
4*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22
*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^
20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^1
3*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 9454494480583680
0*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 1294629010
32960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 871
13445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20
+ 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^
22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^2
4 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 +
16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454
725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c
*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255
552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b
^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b
^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^1
3*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 72281207215226880
0*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 44332473758
67248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15
516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^3
0*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^
22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 388761692960
66641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 2
3871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^
23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^2
9*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268
800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 6227304006
2349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152
383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 3408486
0461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(-((158640570309279744*a^62*d^62 + 4
616893305496535 04*b^62*c^62 + 114314278244094207 5904*a^2*b^60*c^60*d^2 - 25
```

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31136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45
*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839
0639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453
248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41
*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508
285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666
534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c
^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982
71125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069656705362
624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29
*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 -
 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659976808415
3161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586378243276
80*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28
*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278
24782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829995
681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^
24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13
411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382
672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^
42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43
+ 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051707840051
0775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^
46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 +
11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 31964891154238091134
23033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*
c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803
0151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53
*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398033
5571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^
6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682710310060
0320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442953
63448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 105
15603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^
31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3
+ 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 49
1512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209
681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 74545
56262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 46
65413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13
- 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*
d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^1
4*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^
19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 300671810237399
04*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 528659857198
```

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0800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 4521242251837
44*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^
27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c
^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(687194767
36*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42
*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34
0848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13
838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 -
 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*
d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^2
2*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 1551636581553537024
0*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807
746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15
 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^1
5*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239005696
00*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494
751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22
- 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19
*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*
c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^
4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*
c^13*d^30))^(1/4)*1i - (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^23*c
^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 34442419
05*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4*b^1
9*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d^15
- 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 377433756
0*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c^26
+ a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^
24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^2
1*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^
10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^
15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 306
0*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16))) -
 (-((158640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^62 + 11431427
82440942075904*a^2*b^60*c^60*d^2 - 250235617157912199168OO*a^3*b^59*c^59*d^
3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*
a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 34660227
8587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^
54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 58231240117
103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^1
1*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268
5471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981703536758535
2253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^
47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 9370132461
3150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551585754133 29
```

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920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*
d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243
2831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979 69
3568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^3
9*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523
3921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206856248719016
91117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^3
5*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370
15781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804 59
690440055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a
^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^3
2 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7396599789228
3818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382527964959777
75104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c
^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043
06099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795 21
941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40
*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 +
3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528486514603372
4645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^
44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 +
 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066084744485436
97499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*
b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 7914
09982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717 99
5520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^5
2 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100 2
72640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1
5126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^5
7*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298246991 3
298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 332
4163179957505228 8*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2)
 + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d
^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 1
32340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416
415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 56856
22110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436
179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 2
6292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^1
4 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^
15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*
b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*
a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 1374200058396
6720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662
400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*
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a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b
^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d -
13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^
11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 +
34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 247
1152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 6
2273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7
 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^
34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18
*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900
569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388
76169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27
*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^2
5*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 1551636581553
5370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 443
3247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^
23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^
18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5
*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3
*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*b^
8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11*b
^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 + 18
22251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^6*
b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^15
 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^4*
b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^
6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b
^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b
^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*
c^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^28*
c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 + 1
74818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 1616991223
808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375360*a
^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032*a^12
*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a^14*b
^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^16*b^
15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^18*b^1
3*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*b^11*
c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*c^9*d
^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 + 3740
794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27*b^4*
c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*
a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*
b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a
^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43
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758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^1
2 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d
^15 + 153*a^20*b^2*c^10*d^16)) + ((-((158640570309279744*a^62*d^62 + 461689
330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 2502356
1715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58
*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491039438099429772
4928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 22
47504424575830750669045760*a^8*b^54*c^54*d^8 - 1235027598519926616647270400
0*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 2
38022522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826982427246150062
9647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^4
9*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185
07301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136
*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17
 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827683906392
98679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a
^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21
 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050828568
8517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962542766653440
0*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d
^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498271125
182229871738826673618944*a^27*b^35*c^35*d^27 + 2731504644306965670536262407
1598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^33
*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 5926
4887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531616
69916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*a^
33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^34
 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219427824782
390172272112611360768*a^36*b^26*c^26*d^36 - 4568810856096744273528299568129
6384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*c^
24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 1341128
3618120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044038267251
2877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*b^
20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 802
242695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105170784005107752
18487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*b^
16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 11664
498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033
139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*
d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736180301513
57707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*
c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839803355719
97403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^5
6 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*
a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448
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368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603
517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^
31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 269
37875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 4915120
97931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 320968140
0053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262
416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413
760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 586
96011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15
- 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^1
4*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^
12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^
21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*
a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^
25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^
4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^
29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^
7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d -
2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 3408486
04610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 1383845
3347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 2312
99863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 -
 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^3
3*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^1
9*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239680774672
2201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41
305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^2
6*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^
26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886649475173
4497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 192
7498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24
 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*
d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^1
5*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*
d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^5 + 30
19898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001280*a^
8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*b^18*c
^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*c^19*d
^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*d^14 +
 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16 + 7157
99920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 181168766
976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320*a^21*
b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8*d^23
- 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13
*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10
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*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^
15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^1
2*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(-((1586405703092
79744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2
*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 39211736532912
6217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5
+ 44594910394380994297724928*a^6*b^56*c^56*d^6 - 34660227858713752176584294
4*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 1235
0275985199266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960
*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 +
 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 268547166342599810660
4003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*
c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 43925200
681264313454548679131136*a^16*b^46*c^46*d^16 - 9370132461315077596283814071
5008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44
*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 67959352440
6433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187092008403048
8576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^4
0*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723
8786870089625427666534400*a^24*b^38*c^38*d^24 - 908402523392141899384838552
9708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36
*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 2731
5046443069656705362624071598080*a^28*b^34*c^34*d^28 - 370157810409016159546
58395768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^
30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31
 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 73974197164791
541927858637824327680*a^33*b^29*c^29*d^33 + 7396599789228381850891797657550
8480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^
27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 4568810
8560967442735282995681296384*a^37*b^25*c^25*d^37 + 330043060996345319599115
07013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*
b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 -
7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184
3004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a
^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44
- 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431
1813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^4
7*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3
196489115423809113423033139200*a^49*b^13*c^13*d^49 + 7914099823297332156684
67138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^1
1*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 61979096745395
00954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8
*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 1512673264370540119
6412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 1
17913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3
```

```
*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*
a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*
d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280
*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5
*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b
^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^
22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*
b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a
^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805836
800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 12946290
1032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 8
7113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^2
0 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*
d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d
^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26
+ 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 2134
54725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b
*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 21990232
55552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9
*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11
*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a
^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268
800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 443324737
5867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 +
15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c
^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*
a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 3887616929
6066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 -
 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*
c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a
^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 7228120721522
68800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040
062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 24711
52383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084
860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(-((158640570309279744*a^62*d^62 +
 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 -
25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^
58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809
94297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d
^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661664
72704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d
^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698242724
61500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b
^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 19068
074318507301366835150061568*a^15*b^47*c^47*d^15 + 4392520068126431345454867
```

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9131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^
45*d^17 + 18846404180619825515857541332992 0*a^18*b^44*c^44*d^18 - 363482768
390639298679139330949120*a^19*b^43*c^43*d^19 + 6795935244064339898674987904
53248*a^20*b^42*c^42*d^20 - 123422649243283187092008403048857 6*a^21*b^41*c^
41*d^21 + 216629933394046988554314497969356 8*a^22*b^40*c^40*d^22 - 36498805
082856885176502649985433 60*a^23*b^39*c^39*d^23 + 5882337238786870089625427 6
66534400*a^24*b^38*c^38*d^24 - 90840252339214189938483855297085 44*a^25*b^37
*c^37*d^25 + 135179187683206856248719016911175 68*a^26*b^36*c^36*d^26 - 1949
82711251822298717388266736189 44*a^27*b^35*c^35*d^27 + 27315046443069656705 3
62624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^
29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30
 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 68586599768084
1531616699164477358 08*a^32*b^30*c^30*d^32 - 7397419716479154192785863782432
7680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^
28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5821942
782478239017227211261136076 8*a^36*b^26*c^26*d^36 - 4568810856096744273528 29
95681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*
b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 +
13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 75376635764304403
82672512877592576*a^41*b^21*c^21*d^41 + 38924120494975218430043749645025 28*
a^42*b^20*c^20*d^42 - 18452848651460337246459372188467 20*a^43*b^19*c^19*d^4
3 + 80224269548729149690512012214272 0*a^44*b^18*c^18*d^44 - 319410517078400
510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431181395623778713 6*
a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47
+ 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 319648911542380911
3423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^1
2*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 35073618
030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^
53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130383980
335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*
c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100
600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 34429
5363448368267264*a^60*b^2*c^2*d^60 - 3324163179957505228 8*a*b^61*c^61*d - 1
0515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*
b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^
3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 +
491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 32
09681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 745
4556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 -
4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^1
3 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^1
6*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b
^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*
a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 3006718102373
9904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571
```

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980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 45212422518
3744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*
a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2
*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(6871947
6736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^
42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 -
340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 -
13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6
 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^3
5*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b
^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370
240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 323968
07746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^
15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b
^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320239005
69600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 88664
94751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^2
2 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^
19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^
6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*
b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^
2*c^13*d^30))^(1/4)*1i + (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^23
*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 344424
1905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4*b
^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d^1
5 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 3774337
560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c^2
6 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*
c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c
^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^12*
b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 31824*
a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3
060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)))
)/((-((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 114314
2782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*
d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388682475110
4*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602
278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*
b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401
17103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a
^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2
685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585
352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*
c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324
613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061982551585754133
```

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29920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^4
3*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 1234226492
432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979
693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c
^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025
233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351791876832068562487190
1691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b
^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3
7015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 480928052153222804
59690440055062528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496
*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d
^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892
283818508917976575508480*a^34*b^28*c^28*d^34 - 6833570476198873825279649597
7775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26
*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300
4306099634531959911507013140480*a^38*b^24*c^24*d^38 - 219372558140192822795
21941129789440*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^
40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41
+ 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033
724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*
a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45
 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 3860660847444854
3697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^4
8*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79
1409982329733215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717
995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d
^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 96372229934943254310
0272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 +
 15126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a
^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 74129824699
13298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 3
3241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/
2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29
*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 -
 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 14
16415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 568
5622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 54
36179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 +
 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d
^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*
c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^1
8*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 5560978211448422
4*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583
966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 16999671066
62400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 9791654790758
```

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4*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28
*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d
- 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*
c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31
+ 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2
471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 +
 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d
^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*
c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^
18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 238713320239
00569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 3
8876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^
27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a
^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815
535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4
433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*
d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*
c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b
^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b
^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*a^12*
b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876*a^11
*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^10 +
1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 5544069624*a^
6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c^4*d^
15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(8192*(a^
4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*
a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9
*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12
*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^
2*c^10*d^11)) + ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^3*b^2
8*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*d^7 +
 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 16169912
23808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 4939039375360
*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 2154962092032*a^
12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000*a^14
*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*a^16*
b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a^18*b
^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^20*b^1
1*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^9*c^9
*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26 + 37
40794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^27*b^
4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 1
8*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^
8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824
*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 +
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43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d
^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11
*d^15 + 153*a^20*b^2*c^10*d^16)) - ((-((1586405703092797 44*a^62*d^62 + 4616
89330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2 - 25023
56171579121991 6800*a^3*b^59*c^59*d^3 + 39211736532912621 7482240*a^4*b^58*c^
58*d^4 - 469019849064388682 4751104*a^5*b^57*c^57*d^5 + 4459491039438099429 7
724928*a^6*b^56*c^56*d^6 - 34660227858713752 1765842944*a^7*b^55*c^55*d^7 +
2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661 66472704
000*a^9*b^53*c^53*d^9 + 582312401171037714046884 24960*a^10*b^52*c^52*d^10 -
 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269824272461500
629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b^49*c
^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 1906807431
8507301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548679131 1
36*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^
17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36348276839063
9298679139330949120*a^19*b^43*c^43*d^19 + 6795935244064339898674987904 53248
*a^20*b^42*c^42*d^20 - 12342264924328318709200840304 88576*a^21*b^41*c^41*d^
21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3649880508285
688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534
400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37
*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 194982711
25182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053626 24
071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^29*b^
33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30 - 59
264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680841531 6
1669916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824327680*
a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^28*d^
34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 582194278247
82390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829956 81
296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*b^24*
c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 + 13411
283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576430440382672
512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*a^42*
b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^43 + 8
02242695487291496905120122142720*a^44*b^18*c^18*d^44 - 319410517078400510 77
5218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*a^46*
b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47 + 116
64498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809113423 0
33139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^12*c^1
2*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3507361803015
1357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^
9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13038398033557
1997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d
^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682710310060032
0*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 3442953634
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48368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 105156
03517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*
c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 2
6937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 49151
2097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681
400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 74545562
62416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 46654
13760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 5
8696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^1
5 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c
^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*
b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023739904*
a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 528659857198080
0*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*
a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*
b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*
d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*
a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d
- 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 34084
8604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838
453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 23
1299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8
 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c
^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a
^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 32396807746
722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 +
41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c
^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900569600*
a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 8866494751
734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1
927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^
24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^1
7*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c
^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^1
3*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d^5 +
3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 59276001280*
a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10*b^18
*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16*c^19
*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17*d^14
 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16 + 71
5799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 1811687
66976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 15015608320*a^2
1*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^8*d^2
3 - 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d^13 -
13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a^7*b^
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10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*
c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c
^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(-((15864057030
9279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a
^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329
126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^
5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842
944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12
350275985199266166472704000*a^9*b^53*c^53*d^9 + 582312401171037714046884249
60*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11
 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106
604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^4
8*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252
00681264313454548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140
715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^
44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524
406433989867498790453248*a^20*b^42*c^42*d^20 - 123422649243283187092008403 0
488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c
^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337
238786870089625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385
529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^
36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27
315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595
4658395768750080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*
a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^
31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647
91541927858637824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575
508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*
c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 45688
108560967442735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195991
1507013140480*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^3
9*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40
- 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521
843004374964502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720
*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^4
4 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964
311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a
^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 -
 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 79140998232973321566
8467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c
^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453
9500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c
^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401
196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 +
 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b
```

```
^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288
8*a*b^61*c^61*d - 105156035176436858888*a^61*b*c*d^61)^(1/2) + 398297088*a^3
1*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 39817366732
80*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a
^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7
*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*
b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^1
1*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200
*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 945449448058
36800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462
901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 +
 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d
^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^
9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7
*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^2
6 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 21
3454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30
*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 219902
3255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a
^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^
11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312
*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 7228120721522
68800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247
375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11
+ 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19
*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 388761692960664192
0*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169
296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18
 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^1
2*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640
*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 72281207215
2268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 622730
40062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247
1152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 340
84860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(-((158640570309279744*a^62*d^62
 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60*d^2
- 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*
b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 4459491039438
0994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55
*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 1235027598519926616
6472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52
*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 85112826982427
2461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13
*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 190
68074318507301366835150061568*a^15*b^47*c^47*d^15 + 43925200681264313454548
```

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679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*
c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 3634827
68390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790
0453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*
c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 364988
0508285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962542
7666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^
37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19
498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 2731504644306965670
5362624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*
a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^
30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 685865997680
84153161669916447735808*a^32*b^30*c^30*d^32 - 73974197164791541927858637824
327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*
c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 58219
427824782390172272112611360768*a^36*b^26*c^26*d^36 - 4568810856096744273528
2995681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^3
8*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39
+ 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 753766357643044
0382672512877592576*a^41*b^21*c^21*d^41 + 389241204949752184300437496450252
8*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d
^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 3194105170784
00510775218487164928*a^45*b^17*c^17*d^45 + 11626361922596431181395623778713
6*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^4
7 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 3196489115423809
113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b
^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 350736
18030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*
a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 1303839
80335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^
6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 1179132068271031
00600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59 + 344
295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d -
 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 67947724
8*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*
d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5
+ 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 +
3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7
454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11
- 4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d
^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c
^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17
*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 8711344511299584
0*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 30067181023
739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 52865985
```

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71980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 452124225
183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 1687133546496
0*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b
^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719
476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*
c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2
- 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4
- 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d
^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c
^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17
*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 155163658155353
70240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 3239
6807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*
d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24
*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 23871332023900
569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 886
6494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d
^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*
c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*
b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^3
5*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*
b^2*c^13*d^30))^(1/4)*1i - (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 15681600*b^
23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 + 3444
241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 56099130741*a^4
*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*c^6*d
^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 - 37743
37560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b^18*c
^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^6*b^1
6*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9*b^13
*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 43758*a^1
2*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 - 3182
4*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d^13 +
 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*d^16)
))*1i + (-((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1
143142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*
c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824
751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3
46602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760
*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 5823
1240117103771404688424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085
120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^1
2 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035З
67585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*
b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937
01324613150775962838140715008*a^17*b^45*c^45*d^17 + 18846404180619825515857
```

```
5413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^4
3*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342
26492432831870920084030488576*a^21*b^41*c^41*d^21 + 21662993339404698855431
44979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b
^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90
84025233921418993848385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624
871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a
^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^2
8 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 4809280521532
2280459690440055062528*a^30*b^32*c^32*d^30 - 592648874656269275866337706466
34496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c
^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659
97892283818508917976575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796
495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36
*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d^37 +
 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 2193725581401928
2279521941129789440*a^39*b^23*c^23*d^39 + 134112836181207810292808684541050
88*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*
d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 18452848651
46033724645937218846720*a^43*b^19*c^19*d^43 + 80224269548729149690512012214
2720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17
*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 38606608474
448543697499060174848*a^47*b^15*c^15*d^47 + 1166449857652672721962974314496
0*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49
 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 1761994857333886638
21717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c
^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432
543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d
^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 1476009532413734912262
144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 741298
2469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^6
0 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61
)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29
*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*
d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6
 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8
- 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10
 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d
^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c
^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*
b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 11556150389194752
0*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114
484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 137420
00583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 169996
7106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547
```

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907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960
*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^
30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*
a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*
d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^
3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*
d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c
^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*
b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 88664947517344972
80*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 2387133
2023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^1
4 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^
16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201
600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 155163
65815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^2
1 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*
c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32
*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a
^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a
^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)*(((3*(972405*
a^12*b^8*d^19 + 2280960*b^20*c^12*d^7 - 44582400*a*b^19*c^11*d^8 - 15891876
*a^11*b^9*c*d^18 + 322735104*a^2*b^18*c^10*d^9 - 1010174976*a^3*b^17*c^9*d^
10 + 1822251249*a^4*b^16*c^8*d^11 - 4423668876*a^5*b^15*c^7*d^12 + 55440696
24*a^6*b^14*c^6*d^13 - 4056900876*a^7*b^13*c^5*d^14 + 1910559474*a^8*b^12*c
^4*d^15 - 601489476*a^9*b^11*c^3*d^16 + 125166384*a^10*b^10*c^2*d^17))/(819
2*(a^4*b^13*c^21 - a^17*c^8*d^13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12
+ 78*a^6*b^11*c^19*d^2 - 286*a^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 128
7*a^9*b^8*c^16*d^5 + 1716*a^10*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287
*a^12*b^5*c^13*d^8 - 715*a^13*b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^
15*b^2*c^10*d^11)) - ((9*x^(1/2)*(16777216*a^2*b^29*c^29*d^4 - 436207616*a^
3*b^28*c^28*d^5 + 5117050880*a^4*b^27*c^27*d^6 - 36238786560*a^5*b^26*c^26*
d^7 + 174818590720*a^6*b^25*c^25*d^8 - 612716249088*a^7*b^24*c^24*d^9 + 161
6991223808*a^8*b^23*c^23*d^10 - 3258085539840*a^9*b^22*c^22*d^11 + 49390393
75360*a^10*b^21*c^21*d^12 - 5167458811904*a^11*b^20*c^20*d^13 + 21549620920
32*a^12*b^19*c^19*d^14 + 4773749194752*a^13*b^18*c^18*d^15 - 13996916736000
*a^14*b^17*c^17*d^16 + 21965415383040*a^15*b^16*c^16*d^17 - 25291944624128*
a^16*b^15*c^15*d^18 + 22988054331392*a^17*b^14*c^14*d^19 - 16910399832064*a
^18*b^13*c^13*d^20 + 10145615052800*a^19*b^12*c^12*d^21 - 4958946590720*a^2
0*b^11*c^11*d^22 + 1960142962688*a^21*b^10*c^10*d^23 - 618143940608*a^22*b^
9*c^9*d^24 + 152202117120*a^23*b^8*c^8*d^25 - 28274851840*a^24*b^7*c^7*d^26
 + 3740794880*a^25*b^6*c^6*d^27 - 315621376*a^26*b^5*c^5*d^28 + 12845056*a^
27*b^4*c^4*d^29))/(65536*(a^4*b^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*
d - 18*a^21*b*c^9*d^17 + 153*a^6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 30
60*a^8*b^14*c^22*d^4 - 8568*a^9*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 -
31824*a^11*b^11*c^19*d^7 + 43758*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d
```

```
^9 + 43758*a^14*b^8*c^16*d^10 - 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c
^14*d^12 - 8568*a^17*b^5*c^13*d^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3
*c^11*d^15 + 153*a^20*b^2*c^10*d^16)) + ((-((158640570309279744*a^62*d^62 +
 461689330549653504*b^62*c^62 + 11431427824409420759O4*a^2*b^60*c^60*d^2 -
250235617157912199168OO*a^3*b^59*c^59*d^3 + 392117365329126217482240*a^4*b^
58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 445949103943809
94297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55*c^55*d
^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 123502759851992661664
72704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52*c^52*d
^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 8511282698242724
61500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680*a^13*b
^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14 - 19068
074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134545486 7
9131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*b^45*c^
45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 363482768
390639298679139330949120*a^19*b^43*c^43*d^19 + 679593524406433989867498790 4
53248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*b^41*c^
41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 36498805
08285688517650264998543360*a^23*b^39*c^39*d^23 + 588233723878687008962542 76
66534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37
*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26 - 1949
8271125182229871738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053
62624071598080*a^28*b^34*c^34*d^28 - 37015781040901615954658395768750080*a^
29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^32*d^30
 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 68586599768084
153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586378243 2
7680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*b^28*c^
28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 + 5821942
7824782390172272112611360768*a^36*b^26*c^26*d^36 - 456881085609674427352829
95681296384*a^37*b^25*c^25*d^37 + 33004306099634531959911507013140480*a^38*
b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*d^39 +
13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 75376635764304403
82672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964502528*
a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c^19*d^4
3 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 319410517078400
510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237787136*
a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^15*d^47
+ 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 319648911542380911
3423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a^50*b^1
2*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 35073618
030151357707960975360*a^52*b^10*c^10*d^52 - 6197909674539500954745569280*a^
53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 130383980
33557199740364390 4*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^56*b^6*
c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 117913206827103100
600320*a^58*b^4*c^4*d^58 - 74129824691329886208 0*a^59*b^3*c^3*d^59 + 34429
```

5363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^61*d - 1
0515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*
b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^
3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 +
491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 32
09681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 745
4556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 -
4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^1
3 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^1
6*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b
^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112995840*
a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 3006718102373
9904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 5286598571
980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 45212422518
3744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335464960*
a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2
*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(6871947
6736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^
42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 -
340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 -
13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6
 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^3
5*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b
^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 15516365815535370
240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 + 323968
07746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*c^28*d^
15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920*a^24*b
^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 2387133202390056
9600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20 - 88664
94751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c^21*d^2
2 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^
19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^
6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*
b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^
2*c^13*d^30))^(1/4)*(16777216*a^4*b^24*c^27*d^4 - 335544320*a^5*b^23*c^26*d
^5 + 3019898880*a^6*b^22*c^25*d^6 - 16326328320*a^7*b^21*c^24*d^7 + 5927600
1280*a^8*b^20*c^23*d^8 - 151817027584*a^9*b^19*c^22*d^9 + 276572405760*a^10
*b^18*c^21*d^10 - 340199997440*a^11*b^17*c^20*d^11 + 208834396160*a^12*b^16
*c^19*d^12 + 162487336960*a^13*b^15*c^18*d^13 - 630974316544*a^14*b^14*c^17
*d^14 + 945752637440*a^15*b^13*c^16*d^15 - 954476789760*a^16*b^12*c^15*d^16
 + 715799920640*a^17*b^11*c^14*d^17 - 410790133760*a^18*b^10*c^13*d^18 + 18
1168766976*a^19*b^9*c^12*d^19 - 60691578880*a^20*b^8*c^11*d^20 + 1501560832
0*a^21*b^7*c^10*d^21 - 2600468480*a^22*b^6*c^9*d^22 + 283115520*a^23*b^5*c^
8*d^23 - 14680064*a^24*b^4*c^7*d^24)*3i)/(8192*(a^4*b^13*c^21 - a^17*c^8*d^
13 - 13*a^5*b^12*c^20*d + 13*a^16*b*c^9*d^12 + 78*a^6*b^11*c^19*d^2 - 286*a

```
^7*b^10*c^18*d^3 + 715*a^8*b^9*c^17*d^4 - 1287*a^9*b^8*c^16*d^5 + 1716*a^10
*b^7*c^15*d^6 - 1716*a^11*b^6*c^14*d^7 + 1287*a^12*b^5*c^13*d^8 - 715*a^13*
b^4*c^12*d^9 + 286*a^14*b^3*c^11*d^10 - 78*a^15*b^2*c^10*d^11)))*(-((158640
570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075
904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 3921173
65329126217482240*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^
57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 - 3466022785871375217
65842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8
 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 + 5823124011710377140468
8424960*a^10*b^52*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51
*d^11 + 851128269824272461500629647360*a^12*b^50*c^50*d^12 - 26854716634259
98106604003655680*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^1
4*b^48*c^48*d^14 - 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 4
3925200681264313454548679131136*a^16*b^46*c^46*d^16 - 937013246131507759628
38140715008*a^17*b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^
44*c^44*d^18 - 363482768390639298679139330949120*a^19*b^43*c^43*d^19 + 6795
93524406433989867498790453248*a^20*b^42*c^42*d^20 - 12342264924328318709200
84030488576*a^21*b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b
^40*c^40*d^22 - 3649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 58
82337238786870089625427666534400*a^24*b^38*c^38*d^24 - 90840252339214189938
48385529708544*a^25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^
26*b^36*c^36*d^26 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27
 + 27315046443069656705362624071598080*a^28*b^34*c^34*d^28 - 37015781040901
615954658395768750080*a^29*b^33*c^33*d^29 + 4809280521532228045969044005506
2528*a^30*b^32*c^32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^
31*d^31 + 68586599768084153161669916447735808*a^32*b^30*c^30*d^32 - 7397419
7164791541927858637824327680*a^33*b^29*c^29*d^33 + 739659978922838185089179
76575508480*a^34*b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*
b^27*c^27*d^35 + 58219427824782390172272112611360768*a^36*b^26*c^26*d^36 -
45688108560967442735282995681296384*a^37*b^25*c^25*d^37 + 33004306099634531
959911507013140480*a^38*b^24*c^24*d^38 - 2193725581401928227952194112978944
0*a^39*b^23*c^23*d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*
d^40 - 7537663576430440382672512877592576*a^41*b^21*c^21*d^41 + 38924120494
97521843004374964502528*a^42*b^20*c^20*d^42 - 18452848651460337246459372188
46720*a^43*b^19*c^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^1
8*d^44 - 319410517078400510775218487164928*a^45*b^17*c^17*d^45 + 1162636192
25964311813956237787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174
848*a^47*b^15*c^15*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d
^48 - 3196489115423809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733
215668467138560*a^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b
^11*c^11*d^51 + 35073618030151357707960975360*a^52*b^10*c^10*d^52 - 6197909
674539500954745569280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*
b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*c^7*d^55 + 151267326437
0540196412928*a^56*b^6*c^6*d^56 - 14760009532413734912262144*a^57*b^5*c^5*d
^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7412982469913298862080*a
```

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^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575
052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 39829708
8*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 398173
6673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638
464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 141641514224640
0*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320
*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21*d^10 - 543617959296614
4*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 262928739059
71200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 9454494
4805836800*a^15*b^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 1
29462901032960000*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d
^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c
^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b
^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^
7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^
5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28
 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488
*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2
199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461
056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 24711523834265
60*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 622730400623
49312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 72281207
2152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 44
33247375867248640*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*
d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20
*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066
641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 388
76169296066641920*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25
*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^2
7*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^22*d^21 + 44332473758672
48640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812
072152268800*a^31*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 6
2273040062349312*a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27
+ 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29
+ 34084860461056*a^37*b^2*c^13*d^30))^(3/4)*1i)*(-((158640570309279744*a^62
*d^62 + 461689330549653504*b^62*c^62 + 1143142782440942075904*a^2*b^60*c^60
*d^2 - 25023561715791219916800*a^3*b^59*c^59*d^3 + 392117365329126217482240
*a^4*b^58*c^58*d^4 - 4690198490643886824751104*a^5*b^57*c^57*d^5 + 44594910
394380994297724928*a^6*b^56*c^56*d^6 - 346602278587137521765842944*a^7*b^55
*c^55*d^7 + 2247504424575830750669045760*a^8*b^54*c^54*d^8 - 12350275985199
266166472704000*a^9*b^53*c^53*d^9 + 58231240117103771404688424960*a^10*b^52
*c^52*d^10 - 238022522313714176288222085120*a^11*b^51*c^51*d^11 + 851128269
824272461500629647360*a^12*b^50*c^50*d^12 - 2685471663425998106604003655680
*a^13*b^49*c^49*d^13 + 7544170129817035367585352253440*a^14*b^48*c^48*d^14
- 19068074318507301366835150061568*a^15*b^47*c^47*d^15 + 439252006812643134
```

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54548679131136*a^16*b^46*c^46*d^16 - 93701324613150775962838140715008*a^17*
b^45*c^45*d^17 + 188464041806198255158575413329920*a^18*b^44*c^44*d^18 - 36
348276839063929867913933094912O*a^19*b^43*c^43*d^19 + 679593524406433989867
498790453248*a^20*b^42*c^42*d^20 - 1234226492432831870920084030488576*a^21*
b^41*c^41*d^21 + 2166299333940469885543144979693568*a^22*b^40*c^40*d^22 - 3
649880508285688517650264998543360*a^23*b^39*c^39*d^23 + 5882337238786870089
625427666534400*a^24*b^38*c^38*d^24 - 9084025233921418993848385529708544*a^
25*b^37*c^37*d^25 + 13517918768320685624871901691117568*a^26*b^36*c^36*d^26
 - 19498271125182229871738826673618944*a^27*b^35*c^35*d^27 + 27315046443069
656705362624071598080*a^28*b^34*c^34*d^28 - 3701578104090161595465839576875
0080*a^29*b^33*c^33*d^29 + 48092805215322280459690440055062528*a^30*b^32*c^
32*d^30 - 59264887465626927586633770646634496*a^31*b^31*c^31*d^31 + 6858659
9768084153161669916447735808*a^32*b^30*c^30*d^32 - 739741971647915419278586
37824327680*a^33*b^29*c^29*d^33 + 73965997892283818508917976575508480*a^34*
b^28*c^28*d^34 - 68335704761988738252796495977775104*a^35*b^27*c^27*d^35 +
58219427824782390172272112611360768*a^36*b^26*c^26*d^36 - 45688108560967442
735282995681296384*a^37*b^25*c^25*d^37 + 3300430609963453195991150701314048
0*a^38*b^24*c^24*d^38 - 21937255814019282279521941129789440*a^39*b^23*c^23*
d^39 + 13411283618120781029280868454105088*a^40*b^22*c^22*d^40 - 7537663576
430440382672512877592576*a^41*b^21*c^21*d^41 + 3892412049497521843004374964
502528*a^42*b^20*c^20*d^42 - 1845284865146033724645937218846720*a^43*b^19*c
^19*d^43 + 802242695487291496905120122142720*a^44*b^18*c^18*d^44 - 31941051
7078400510775218487164928*a^45*b^17*c^17*d^45 + 116263619225964311813956237
787136*a^46*b^16*c^16*d^46 - 38606608474448543697499060174848*a^47*b^15*c^1
5*d^47 + 11664498576526727219629743144960*a^48*b^14*c^14*d^48 - 31964891154
23809113423033139200*a^49*b^13*c^13*d^49 + 791409982329733215668467138560*a
^50*b^12*c^12*d^50 - 176199485733388663821717995520*a^51*b^11*c^11*d^51 + 3
5073618030151357707960975360*a^52*b^10*c^10*d^52 - 619790967453950095474556
9280*a^53*b^9*c^9*d^53 + 963722299349432543100272640*a^54*b^8*c^8*d^54 - 13
0383980335571997403643904*a^55*b^7*c^7*d^55 + 15126732643705401196412928*a^
56*b^6*c^6*d^56 - 1476009532413734912262144*a^57*b^5*c^5*d^57 + 11791320682
710310060032O*a^58*b^4*c^4*d^58 - 7412982469913298862080*a^59*b^3*c^3*d^59
+ 344295363448368267264*a^60*b^2*c^2*d^60 - 33241631799575052288*a*b^61*c^6
1*d - 10515603517643685888*a^61*b*c*d^61)^(1/2) + 398297088*a^31*d^31 + 679
477248*b^31*c^31 + 400891576320*a^2*b^29*c^29*d^2 - 3981736673280*a^3*b^28*
c^28*d^3 + 26937875496960*a^4*b^27*c^27*d^4 - 132340424638464*a^5*b^26*c^26
*d^5 + 491512097931264*a^6*b^25*c^25*d^6 - 1416415142246400*a^7*b^24*c^24*d
^7 + 3209681400053760*a^8*b^23*c^23*d^8 - 5685622110904320*a^9*b^22*c^22*d^
9 + 7454556262416384*a^10*b^21*c^21*d^10 - 5436179592966144*a^11*b^20*c^20*
d^11 - 4665413760860160*a^12*b^19*c^19*d^12 + 26292873905971200*a^13*b^18*c
^18*d^13 - 58696011926323200*a^14*b^17*c^17*d^14 + 94544944805836800*a^15*b
^16*c^16*d^15 - 121670839126425600*a^16*b^15*c^15*d^16 + 129462901032960000
*a^17*b^14*c^14*d^17 - 115561503891947520*a^18*b^13*c^13*d^18 + 87113445112
995840*a^19*b^12*c^12*d^19 - 55609782114484224*a^20*b^11*c^11*d^20 + 300671
81023739904*a^21*b^10*c^10*d^21 - 13742000583966720*a^22*b^9*c^9*d^22 + 528
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6598571980800*a^23*b^8*c^8*d^23 - 1699967106662400*a^24*b^7*c^7*d^24 + 4521
24225183744*a^25*b^6*c^6*d^25 - 97916547907584*a^26*b^5*c^5*d^26 + 16871335
464960*a^27*b^4*c^4*d^27 - 2231346216960*a^28*b^3*c^3*d^28 + 213454725120*a
^29*b^2*c^2*d^29 - 24461180928*a*b^30*c^30*d - 13200703488*a^30*b*c*d^30)/(
68719476736*a^7*b^32*c^43 + 68719476736*a^39*c^11*d^32 - 2199023255552*a^8*
b^31*c^42*d - 2199023255552*a^38*b*c^12*d^31 + 34084860461056*a^9*b^30*c^41
*d^2 - 340848604610560*a^10*b^29*c^40*d^3 + 2471152383426560*a^11*b^28*c^39
*d^4 - 13838453347188736*a^12*b^27*c^38*d^5 + 62273040062349312*a^13*b^26*c
^37*d^6 - 231299863088726016*a^14*b^25*c^36*d^7 + 722812072152268800*a^15*b
^24*c^35*d^8 - 1927498859072716800*a^16*b^23*c^34*d^9 + 4433247375867248640
*a^17*b^22*c^33*d^10 - 8866494751734497280*a^18*b^21*c^32*d^11 + 1551636581
5535370240*a^19*b^20*c^31*d^12 - 23871332023900569600*a^20*b^19*c^30*d^13 +
 32396807746722201600*a^21*b^18*c^29*d^14 - 38876169296066641920*a^22*b^17*
c^28*d^15 + 41305929877070807040*a^23*b^16*c^27*d^16 - 38876169296066641920
*a^24*b^15*c^26*d^17 + 32396807746722201600*a^25*b^14*c^25*d^18 - 238713320
23900569600*a^26*b^13*c^24*d^19 + 15516365815535370240*a^27*b^12*c^23*d^20
- 8866494751734497280*a^28*b^11*c^22*d^21 + 4433247375867248640*a^29*b^10*c
^21*d^22 - 1927498859072716800*a^30*b^9*c^20*d^23 + 722812072152268800*a^31
*b^8*c^19*d^24 - 231299863088726016*a^32*b^7*c^18*d^25 + 62273040062349312*
a^33*b^6*c^17*d^26 - 13838453347188736*a^34*b^5*c^16*d^27 + 247115238342656
0*a^35*b^4*c^15*d^28 - 340848604610560*a^36*b^3*c^14*d^29 + 34084860461056*
a^37*b^2*c^13*d^30))^(1/4)*1i + (9*x^(1/2)*(4862025*a^12*b^11*d^21 + 156816
00*b^23*c^12*d^9 - 330739200*a*b^22*c^11*d^10 - 85293810*a^11*b^12*c*d^20 +
 3444241905*a^2*b^21*c^10*d^11 - 19611374130*a^3*b^20*c^9*d^12 + 5609913074
1*a^4*b^19*c^8*d^13 - 73884775320*a^5*b^18*c^7*d^14 + 60509855250*a^6*b^17*
c^6*d^15 - 33837158700*a^7*b^16*c^5*d^16 + 13445660610*a^8*b^15*c^4*d^17 -
3774337560*a^9*b^14*c^3*d^18 + 722155581*a^10*b^13*c^2*d^19))/(65536*(a^4*b
^18*c^26 + a^22*c^8*d^18 - 18*a^5*b^17*c^25*d - 18*a^21*b*c^9*d^17 + 153*a^
6*b^16*c^24*d^2 - 816*a^7*b^15*c^23*d^3 + 3060*a^8*b^14*c^22*d^4 - 8568*a^9
*b^13*c^21*d^5 + 18564*a^10*b^12*c^20*d^6 - 31824*a^11*b^11*c^19*d^7 + 4375
8*a^12*b^10*c^18*d^8 - 48620*a^13*b^9*c^17*d^9 + 43758*a^14*b^8*c^16*d^10 -
 31824*a^15*b^7*c^15*d^11 + 18564*a^16*b^6*c^14*d^12 - 8568*a^17*b^5*c^13*d
^13 + 3060*a^18*b^4*c^12*d^14 - 816*a^19*b^3*c^11*d^15 + 153*a^20*b^2*c^10*
d^16)))*1i))*(-((158640570309279744*a^62*d^62 + 4616893305496535O4*b^62*c^6
2 + 11431427824409420759O4*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*
b^59*c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 46901984906438
86824751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^
6 - 346602278587137521765842944*a^7*b^55*c^55*d^7 + 224750442457583075066690
45760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 +
 58231240117103771404688424960*a^10*b^52*c^52*d^10 - 23802252231371417628822
2085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^5
0*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 754417012981
7035367585352253440*a^14*b^48*c^48*d^14 - 19068074318507301366835150061568*
a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d^16
- 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 188464041806198255
```

```
158575413329920*a^18*b^44*c^44*d^18 - 363482768390639298679139330949120*a^1
9*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*d^20 -
1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 216629933394046988
5543144979693568*a^22*b^40*c^40*d^22 - 3649880508285688517650264998543360*a
^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38*c^38*d^24
 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 135179187683206
85624871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871738826673618
944*a^27*b^35*c^35*d^27 + 27315046443069656705362624071598080*a^28*b^34*c^3
4*d^28 - 37015781040901615954658395768750080*a^29*b^33*c^33*d^29 + 48092805
215322280459690440055062528*a^30*b^32*c^32*d^30 - 5926488746562692758663377
0646634496*a^31*b^31*c^31*d^31 + 68586599768084153161669916447735808*a^32*b
^30*c^30*d^32 - 73974197164791541927858637824327680*a^33*b^29*c^29*d^33 + 7
3965997892283818508917976575508480*a^34*b^28*c^28*d^34 - 683357047619887382
52796495977775104*a^35*b^27*c^27*d^35 + 58219427824782390172272112611360768
*a^36*b^26*c^26*d^36 - 45688108560967442735282995681296384*a^37*b^25*c^25*d
^37 + 33004306099634531959911507013140480*a^38*b^24*c^24*d^38 - 21937255814
019282279521941129789440*a^39*b^23*c^23*d^39 + 1341128361812078102928086845
4105088*a^40*b^22*c^22*d^40 - 7537663576430440382672512877592576*a^41*b^21*
c^21*d^41 + 3892412049497521843004374964502528*a^42*b^20*c^20*d^42 - 184528
4865146033724645937218846720*a^43*b^19*c^19*d^43 + 802242695487291496905120
122142720*a^44*b^18*c^18*d^44 - 319410517078400510775218487164928*a^45*b^17
*c^17*d^45 + 116263619225964311813956237787136*a^46*b^16*c^16*d^46 - 386066
08474448543697499060174848*a^47*b^15*c^15*d^47 + 11664498576526727219629743
144960*a^48*b^14*c^14*d^48 - 3196489115423809113423033139200*a^49*b^13*c^13
*d^49 + 791409982329733215668467138560*a^50*b^12*c^12*d^50 - 17619948573338
8663821717995520*a^51*b^11*c^11*d^51 + 35073618030151357707960975360*a^52*b
^10*c^10*d^52 - 6197909674539500954745569280*a^53*b^9*c^9*d^53 + 9637222993
49432543100272640*a^54*b^8*c^8*d^54 - 130383980335571997403643904*a^55*b^7*
c^7*d^55 + 15126732643705401196412928*a^56*b^6*c^6*d^56 - 14760095324137349
12262144*a^57*b^5*c^5*d^57 + 117913206827103100600320*a^58*b^4*c^4*d^58 - 7
412982469913298862080*a^59*b^3*c^3*d^59 + 344295363448368267264*a^60*b^2*c^
2*d^60 - 33241631799575052288*a*b^61*c^61*d - 10515603517643685888*a^61*b*c
*d^61)^(1/2) + 398297088*a^31*d^31 + 679477248*b^31*c^31 + 400891576320*a^2
*b^29*c^29*d^2 - 3981736673280*a^3*b^28*c^28*d^3 + 26937875496960*a^4*b^27*
c^27*d^4 - 132340424638464*a^5*b^26*c^26*d^5 + 491512097931264*a^6*b^25*c^2
5*d^6 - 1416415142246400*a^7*b^24*c^24*d^7 + 3209681400053760*a^8*b^23*c^23
*d^8 - 5685622110904320*a^9*b^22*c^22*d^9 + 7454556262416384*a^10*b^21*c^21
*d^10 - 5436179592966144*a^11*b^20*c^20*d^11 - 4665413760860160*a^12*b^19*c
^19*d^12 + 26292873905971200*a^13*b^18*c^18*d^13 - 58696011926323200*a^14*b
^17*c^17*d^14 + 94544944805836800*a^15*b^16*c^16*d^15 - 121670839126425600*
a^16*b^15*c^15*d^16 + 129462901032960000*a^17*b^14*c^14*d^17 - 115561503891
947520*a^18*b^13*c^13*d^18 + 87113445112995840*a^19*b^12*c^12*d^19 - 556097
82114484224*a^20*b^11*c^11*d^20 + 30067181023739904*a^21*b^10*c^10*d^21 - 1
3742000583966720*a^22*b^9*c^9*d^22 + 5286598571980800*a^23*b^8*c^8*d^23 - 1
699967106662400*a^24*b^7*c^7*d^24 + 452124225183744*a^25*b^6*c^6*d^25 - 979
```

```
16547907584*a^26*b^5*c^5*d^26 + 16871335464960*a^27*b^4*c^4*d^27 - 22313462
16960*a^28*b^3*c^3*d^28 + 213454725120*a^29*b^2*c^2*d^29 - 24461180928*a*b^
30*c^30*d - 13200703488*a^30*b*c*d^30)/(68719476736*a^7*b^32*c^43 + 6871947
6736*a^39*c^11*d^32 - 2199023255552*a^8*b^31*c^42*d - 2199023255552*a^38*b*
c^12*d^31 + 34084860461056*a^9*b^30*c^41*d^2 - 340848604610560*a^10*b^29*c^
40*d^3 + 2471152383426560*a^11*b^28*c^39*d^4 - 13838453347188736*a^12*b^27*
c^38*d^5 + 62273040062349312*a^13*b^26*c^37*d^6 - 231299863088726016*a^14*b
^25*c^36*d^7 + 722812072152268800*a^15*b^24*c^35*d^8 - 1927498859072716800*
a^16*b^23*c^34*d^9 + 4433247375867248640*a^17*b^22*c^33*d^10 - 886649475173
4497280*a^18*b^21*c^32*d^11 + 15516365815535370240*a^19*b^20*c^31*d^12 - 23
871332023900569600*a^20*b^19*c^30*d^13 + 32396807746722201600*a^21*b^18*c^2
9*d^14 - 38876169296066641920*a^22*b^17*c^28*d^15 + 41305929877070807040*a^
23*b^16*c^27*d^16 - 38876169296066641920*a^24*b^15*c^26*d^17 + 323968077467
22201600*a^25*b^14*c^25*d^18 - 23871332023900569600*a^26*b^13*c^24*d^19 + 1
5516365815535370240*a^27*b^12*c^23*d^20 - 8866494751734497280*a^28*b^11*c^2
2*d^21 + 4433247375867248640*a^29*b^10*c^21*d^22 - 1927498859072716800*a^30
*b^9*c^20*d^23 + 722812072152268800*a^31*b^8*c^19*d^24 - 231299863088726016
*a^32*b^7*c^18*d^25 + 62273040062349312*a^33*b^6*c^17*d^26 - 13838453347188
736*a^34*b^5*c^16*d^27 + 2471152383426560*a^35*b^4*c^15*d^28 - 340848604610
560*a^36*b^3*c^14*d^29 + 34084860461056*a^37*b^2*c^13*d^30))^(1/4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.483 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=805

$$\frac{(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}\,a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}\,a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\log\left(\sqrt{b}\,x-\sqrt{2}\sqrt[4]{a}}{8\sqrt{2}\,a^{9/4}(bc-a}$$

Rubi **[A]** time = 1.40, antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{(5bc-17ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{13/4}}{4\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{(5bc-17ad)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{(5bc-17ad)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{13/4}}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)\sqrt{x}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}+\frac{d^{9/4}\left(221b^2c^2-170abdc+45a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}-\frac{40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}}+\frac{d\left(8b^2c^2+25abdc-9a^2d^2\right)}{16ac^2(bc-ad)^3\sqrt{x}\left(dx^2+c\right)}+\frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(dx^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(40*b^3*c^3 - 96*a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(16*a^2*c^3*(
b*c - a*d)^3*Sqrt[x]) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*Sqrt[x]*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*
b^2*c^2 + 25*a*b*c*d - 9*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*Sqrt[x]*(c + d*x
^2)) + (b^(13/4)*(5*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)^4)
+ (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (d^(9/4)*(221
*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - (b^(13/4)*(5*b*c - 17*a*d)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/
4)*(b*c - a*d)^4) + (b^(13/4)*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (d^(9/4
)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) + (d^(9/4)*
(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-5bc+4ad-13bdx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \text{Subst}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2\right.}{16ac^2(b}$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

$$= -\frac{40b^3c^3-96ab^2c^2d+125a^2bcd^2-45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2} +$$

Mathematica [A] time = 6.25, size = 773, normalized size = 0.96

$$\frac{b^{13/4}(17ad-5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^4}-\frac{b^{13/4}(17ad-5bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b^{13/4}(17ad-5bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b^{13/4}(17ad-5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^4}+\frac{b^4x^{3/2}}{2a^2\left(a+bx^2\right)(ad-bc)^3}-\frac{d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(ad-bc)^4}+\frac{d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(ad-bc)^4}-\frac{d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(ad-bc)^4}-\frac{d^{9/4}\left(45a^2d^2-170abcd+221b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(ad-bc)^4}-\frac{2}{a^2c^3\sqrt{x}}-\frac{d^3x^{3/2}(29bc-13ad)}{16c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3x^{3/2}}{4c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -2/(a^2*c^3*Sqrt[x]) + (b^4*x^(3/2))/(2*a^2*(-(b*c) + a*d)^3*(a + b*x^2)) -
 (d^3*x^(3/2))/(4*c^2*(b*c - a*d)^2*(c + d*x^2)^2) - (d^3*(29*b*c - 13*a*d)
*x^(3/2))/(16*c^3*(b*c - a*d)^3*(c + d*x^2)) + (b^(13/4)*(-5*b*c + 17*a*d)*
ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt
[2]*a^(9/4)*(b*c - a*d)^4) + (b^(13/4)*(-5*b*c + 17*a*d)*ArcTan[(Sqrt[2]*a^
(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(9/4)*(b*c - a*
d)^4) - (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[(-(Sqrt[2]
*c^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(13/4)*(-(
b*c) + a*d)^4) - (d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[(
Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(13/
4)*(-(b*c) + a*d)^4) + (b^(13/4)*(-5*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (b^
(13/4)*(-5*b*c + 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(8*Sqrt[2]*a^(9/4)*(b*c - a*d)^4) - (d^(9/4)*(221*b^2*c^2 - 170*a
*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x])/(64*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^4) + (d^(9/4)*(221*b^2*c^2 - 170*
a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x])/(64*Sqrt[2]*c^(13/4)*(-(b*c) + a*d)^4)
```

IntegrateAlgebraic [A] time = 2.54, size = 627, normalized size = 0.78

$$-\frac{\left(17ab^{13/4}d-5b^{17/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{9/4}(ad-bc)^4}-\frac{\left(17ab^{13/4}d-5b^{17/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{9/4}(ad-bc)^4}+\frac{\left(45a^2d^{17/4}-170abcd^{13/4}+221b^2c^2d^{9/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}+\frac{\left(45a^2d^{17/4}-170abcd^{13/4}+221b^2c^2d^{9/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}+\frac{-32a^4c^2d^3-81a^4cd^4x^2-45a^4d^5x^4+96a^3bc^3d^2+193a^3bc^2d^3x^2+44a^3bcd^4x^4-45a^3bd^5x^6-96a^2b^2c^4d-96a^2b^2c^3d^2x^2+129a^2b^2c^2d^3x^4+125a^2b^2cd^4x^6+32ab^3c^5-32ab^3c^4dx^2-160ab^3c^3d^2x^4-96ab^3c^2d^3x^6+40b^4c^5x^2+80b^4c^4dx^4+40b^4c^3d^2x^6}{16a^2c^3\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^2*d^3 + 40*b
^4*c^5*x^2 - 32*a*b^3*c^4*d*x^2 - 96*a^2*b^2*c^3*d^2*x^2 + 193*a^3*b*c^2*d^
3*x^2 - 81*a^4*c*d^4*x^2 + 80*b^4*c^4*d*x^4 - 160*a*b^3*c^3*d^2*x^4 + 129*a
^2*b^2*c^2*d^3*x^4 + 44*a^3*b*c*d^4*x^4 - 45*a^4*d^5*x^4 + 40*b^4*c^3*d^2*x
^6 - 96*a*b^3*c^2*d^3*x^6 + 125*a^2*b^2*c*d^4*x^6 - 45*a^3*b*d^5*x^6)/(16*a
^2*c^3*(-(b*c) + a*d)^3*Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2) - ((-5*b^(17/4)*
c + 17*a*b^(13/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])])/(4*Sqrt[2]*a^(9/4)*(-(b*c) + a*d)^4) + ((221*b^2*c^2*d^(9/4) - 1
70*a*b*c*d^(13/4) + 45*a^2*d^(17/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4) - ((-5*b^(17
/4)*c + 17*a*b^(13/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
```

```
 + Sqrt[b]*x)])/(4*Sqrt[2]*a^(9/4)*(-(b*c) + a*d)^4) + ((221*b^2*c^2*d^(9/4
) - 170*a*b*c*d^(13/4) + 45*a^2*d^(17/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(13/4)*(b*c - a*d)^4)
```

fricas **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[B]** time = 3.05, size = 1333, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(
2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt
(2)*a^7*d^4) - 1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^
4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6
*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)
^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d
)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d
+ 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4
) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3
)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d
^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/8*(5*(a*b^3)^(3/4)*
b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^
2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/8*(5*(a*b^3)^(3/4)*b^2
*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*
d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) + 1/64*(221*(c*d^3)^(3/4)*b^
2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*
d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d
^4) - 1/64*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d
^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(
2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*
```

```
a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) - 1/2*(5*b^4*c^3*x^2 - 12*a*b^3*c^2*d*
x^2 + 12*a^2*b^2*c*d^2*x^2 - 4*a^3*b*d^3*x^2 + 4*a*b^3*c^3 - 12*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 - 4*a^4*d^3)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*
c^4*d^2 - a^5*c^3*d^3)*(b*x^(5/2) + a*sqrt(x))) - 1/16*(29*b*c*d^4*x^(7/2)
- 13*a*d^5*x^(7/2) + 33*b*c^2*d^3*x^(3/2) - 17*a*c*d^4*x^(3/2))/((b^3*c^6 -
 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

maple [A] time = 0.04, size = 1143, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^4/a/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x^(3/2)/(
b*x^2+a)*c+17/16*b^3/a/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*ln((x-(a/b)^(1/4)*
2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))+1
7/8*b^3/a/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)+17/8*b^3/a/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1)-5/16*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*ln((x-(a/b)^(
1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2
)))-5/8*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)-5/8*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1/2)/(
a/b)^(1/4)*x^(1/2)-1)-13/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a^2+21/
8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a*b-29/16*d^4/c/(a*d-b*c)^4/(d*x^
2+c)^2*x^(7/2)*b^2-17/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a^2+25/8*d
^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a*b-33/16*d^3/(a*d-b*c)^4/(d*x^2+c)^2*
x^(3/2)*b^2-45/128*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^
(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/
2)))-45/64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)+1)-45/64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)+85/64*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2
)*a*b*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)
*x^(1/2)+(c/d)^(1/2)))+85/32*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*b*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+85/32*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2
^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-221/128*d^2/c/(a*d-b*c)^4/
(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))-221/64*d^2/c/(a*d-b*c)^4/(c/d)^(1/
4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-221/64*d^2/c/(a*d-b*c)
^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/a^2/c^3/
x^(1/2)
```

maxima [A] time = 3.06, size = 955, normalized size = 1.19

$$-\frac{\left(5\,b^{5}c - 17\,ab^{4}d\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x} + \sqrt{b}x + \sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right)}{16\left(a^{2}b^{4}c^{4} - 4\,a^{3}b^{3}c^{3}d + 6\,a^{4}b^{2}c^{2}d^{2} - 4\,a^{5}bcd^{3} + a^{6}d^{4}\right)} - \frac{\left(221\,b^{2}c^{2}d^{3} - 170\,abcd^{4} + 45\,a^{2}d^{5}\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}} + 2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}} - 2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x} + \sqrt{d}x + \sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x} + \sqrt{d}x + \sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{128\left(b^{4}c^{7} - 4\,ab^{3}c^{6}d + 6\,a^{2}b^{2}c^{5}d^{2} - 4\,a^{3}bc^{4}d^{3} + a^{4}c^{3}d^{4}\right)} - \frac{32\,ab^{3}c^{5} - 96\,a^{2}b^{2}c^{4}d + 96\,a^{3}bc^{3}d^{2} - 32\,a^{4}c^{2}d^{3} + \left(40\,b^{4}c^{3}d^{2} - 96\,ab^{3}c^{2}d^{3} + 125\,a^{2}b^{2}cd^{4} - 45\,a^{3}bd^{5}\right)x^{6} + \left(80\,b^{4}c^{4}d - 160\,ab^{3}c^{3}d^{2} + 129\,a^{2}b^{2}c^{2}d^{3} + 44\,a^{3}bcd^{4} - 45\,a^{4}d^{5}\right)x^{4} + \left(40\,b^{4}c^{5} - 32\,ab^{3}c^{4}d - 96\,a^{2}b^{2}c^{3}d^{2} + 193\,a^{3}bc^{2}d^{3} - 81\,a^{4}cd^{4}\right)x^{2}}{16\left(\left(a^{2}b^{4}c^{6}d^{2} - 3\,a^{3}b^{3}c^{5}d^{3} + 3\,a^{4}b^{2}c^{4}d^{4} - a^{5}bc^{3}d^{5}\right)x^{\frac{13}{2}} + \left(2\,a^{2}b^{4}c^{7}d - 5\,a^{3}b^{3}c^{6}d^{2} + 3\,a^{4}b^{2}c^{5}d^{3} + a^{5}bc^{4}d^{4} - a^{6}c^{3}d^{5}\right)x^{\frac{9}{2}} + \left(a^{2}b^{4}c^{8} - a^{3}b^{3}c^{7}d - 3\,a^{4}b^{2}c^{6}d^{2} + 5\,a^{5}bc^{5}d^{3} - 2\,a^{6}c^{4}d^{4}\right)x^{\frac{5}{2}} + \left(a^{3}b^{3}c^{8} - 3\,a^{4}b^{2}c^{7}d + 3\,a^{5}bc^{6}d^{2} - a^{6}c^{5}d^{3}\right)\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(5*b^5*c - 17*a*b^4*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3
/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(1/4)*b^(3/4)))/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a
^5*b*c*d^3 + a^6*d^4) - 1/128*(221*b^2*c^2*d^3 - 170*a*b*c*d^4 + 45*a^2*d^5
)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x
))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*s
qrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)
*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^7
 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) - 1/1
6*(32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^2*d^3 + (4
0*b^4*c^3*d^2 - 96*a*b^3*c^2*d^3 + 125*a^2*b^2*c*d^4 - 45*a^3*b*d^5)*x^6 +
(80*b^4*c^4*d - 160*a*b^3*c^3*d^2 + 129*a^2*b^2*c^2*d^3 + 44*a^3*b*c*d^4 -
45*a^4*d^5)*x^4 + (40*b^4*c^5 - 32*a*b^3*c^4*d - 96*a^2*b^2*c^3*d^2 + 193*a
^3*b*c^2*d^3 - 81*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3
*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^(13/2) + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c
^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^(9/2) + (a^2*b^
4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4
)*x^(5/2) + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3)
*sqrt(x))
```

mupad [B] time = 24.92, size = 127276, normalized size = 158.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] 2*atan((((-(8398080000*a^33*d^33 - (705277476864000000000*a^66*d^66 + 274877
90694400000000*b^66*c^66 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 8523
959496286928896000000*a^3*b^63*c^63*d^3 + 113031004798163353600000000*a^4*b^6
2*c^62*d^4 - 1154880780847298232975360000*a^5*b^61*c^61*d^5 + 94660933391357
8145788723200*a^6*b^60*c^60*d^6 - 63988382063497445934681292800*a^7*b^59*c^5
9*d^7 + 363943805075927975134589091840*a^8*b^58*c^58*d^8 - 17682391555307866
77574839828480*a^9*b^57*c^57*d^9 + 7425481275746674581907219415040*a^10*b^56*
c^56*d^10 - 27204158429008668904965695078400*a^11*b^55*c^55*d^11 + 876084883
86430107181928939520000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685
```

```
12*a^13*b^53*c^53*d^13 + 632144467425843637996415185059 84*a^14*b^52*c^52*d^
14 - 1431337801106946205058726803537 92*a^15*b^51*c^51*d^15 + 29143271303237
796485395340328960 0*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200
*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^1
8 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793
32590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255080146693645139 96
80*a^21*b^45*c^45*d^21 + 696340844349679345870323761283072 0*a^22*b^44*c^44*
d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814
0153590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189
44169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^4
0*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129
947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012 8
410491118262681 60*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554 88
0*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35
*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807
3329926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056
4698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a
^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d
^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093
9745327887207329057621409792*a^38*b^28*c^28*d^38 - 273939082748055449346653
4979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a
^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d
^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380
5867374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361
710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^4
5*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46
 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217
1953043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355
98848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c
^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965
466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606 1
61715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*
c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836
150036124817025925120 00*a^56*b^10*c^10*d^56 - 134721865560409115491041280 00
00*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3
39421563479651575136256 00000*a^59*b^7*c^7*d^59 + 4295456879982240124108800 0
00*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950
4294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b
^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162727720910848 0
000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 524288
0000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*
c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^2
8*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26
*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24
*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*
```

```
c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b
^20*c^20*d^13 - 5305814389923840 0*a^14*b^19*c^19*d^14 + 150199661741875200*
a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173
068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006
5381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20
+ 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^1
1*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b
^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*
b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b
^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c
^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26
1273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^1
3*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 +
34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 24
71152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 +
62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^
7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c
^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^2
0*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 2387133202390
0569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38
876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^2
9*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^
27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155163658155
35370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 44
33247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d
^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c
^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^
5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^
3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-(8398080
000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 274877906944000000*b^66
*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000
0*a^3*b^63*c^63*d^3 + 113031004798163353600000000*a^4*b^62*c^62*d^4 - 115488
0780847298232975360000*a^5*b^61*c^61*d^5 + 9466093339135781457887232 00*a^6*b
^60*c^60*d^6 - 63988382063497445934681292 80*a^7*b^59*c^59*d^7 + 36394380507
592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b
^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415
842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952
000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d
^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069
4620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600
*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^1
7 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132516
8526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623367881523
2*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d
^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290
```

```
17232408306844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300
735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*
c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92575
964607062084838869289496739840*a^27*b^39*c^39*d^27 + 1299473849307245203884
91615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a
^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^
30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334288
6205619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590
010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*
b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35
+ 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399
654362397243935503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057
621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*
b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40
- 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355
133034582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020
810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b
^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 +
296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808
16245477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132
224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^1
7*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829
95414208640750154437099520*a^51*b^15*c^15*d^51 + 15789654660146705061178096
64163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*
c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604
2795476444797594501120000*a^55*b^11*c^11*d^55 + 694868361500361248170259251
2000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^5
7 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513
625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^6
0 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000
000*a^62*b^4*c^4*d^62 - 26837948400559718400000000*a^63*b^3*c^3*d^63 + 13414
41243847065600000000*a^64*b^2*c^2*d^64 - 16272772091084800000000*a*b^65*c^65*
d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 21
33642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 10699821
3378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 15609364060
56960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 867078777077
7600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 204848128011
30496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1678079
5101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 530
58143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15
 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^
16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*
b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 63043211587399680
0*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 2617739039
36962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825
```

```
496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801
173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 26132
5798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 392233881
6000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c
*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255
552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11
*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13
*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a
^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268
800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443324737
5867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 +
15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c
^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*
a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887616929
6066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 -
 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*
c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a
^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 7228120721522
68800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040
062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 24711
52383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084
860461056*a^39*b^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d
^4 - 12624490475444974387 20*a^24*b^50*c^80*d^5 + 26781213630512448405504*a^
25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 3648418948
406862648705024*a^27*b^47*c^77*d^8 - 28097394584779147947540480*a^28*b^46*c
^76*d^9 + 174448389309337948351627264*a^29*b^45*c^75*d^10 - 897668976119897
481466085376*a^30*b^44*c^74*d^11 + 3905920242884630010868531200*a^31*b^43*c
^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^13 + 473558210687
90227801756139520*a^33*b^41*c^71*d^14 - 134845524585103937061538234368*a^34
*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*c^69*d^16 - 7632
46944716696111818343448576*a^36*b^38*c^68*d^17 + 154156402718081368660763889
53984*a^37*b^37*c^67*d^18 - 2825288027628089174763608473600*a^38*b^36*c^66*
d^19 + 4753041476272000853590444867584*a^39*b^35*c^65*d^20 - 74462634931858
15677622957375488*a^40*b^34*c^64*d^21 + 11045974611794807027964401680384*a^
41*b^33*c^63*d^22 - 15766681100741571532295786987520*a^42*b^32*c^62*d^23 +
21882616570434895907374088847360*a^43*b^31*c^61*d^24 - 29555415901357165913
293077872640*a^44*b^30*c^60*d^25 + 38514249364633213650767204843520*a^45*b^
29*c^59*d^26 - 47767982724772003266224581509120*a^46*b^28*c^58*d^27 + 55618
948537155045120476750807040*a^47*b^27*c^57*d^28 - 601273004136644754796412 1
4156800*a^48*b^26*c^56*d^29 + 59877038998440260638050153922560*a^49*b^25*c^
55*d^30 - 54637051595737047014674020696064*a^50*b^24*c^54*d^31 + 4551962322
3064909005599526617088*a^51*b^23*c^53*d^32 - 345355558165500550852549583831
04*a^52*b^22*c^52*d^33 + 23809729504484309698980012359680*a^53*b^21*c^51*d^
34 - 14885319254535352990241541586944*a^54*b^20*c^50*d^35 + 841964833920295
4390072444583936*a^55*b^19*c^49*d^36 - 4297514831765712413611503124480*a^56
```

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*b^18*c^48*d^37 + 1973123737554130196459440570368*a^57*b^17*c^47*d^38 - 811
770857054497673061303582720*a^58*b^16*c^46*d^39 + 2978583803724393715961880
90368*a^59*b^15*c^45*d^40 - 96910050535770593129744302080*a^60*b^14*c^44*d^
41 + 27758579881177587823480406016*a^61*b^13*c^43*d^42 - 693747450447667210
2869499904*a^62*b^12*c^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^4
1*d^44 - 274100118958300866495381504*a^64*b^10*c^40*d^45 + 4186777846342527
7028466688*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^4
7 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a
^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 36479156981
701017600*a^70*b^4*c^34*d^51)*1i - 18014398509481984000*a^21*b^51*c^78*d^4
+ 778222015609621708800*a^22*b^50*c^77*d^5 - 16199988291606958571520*a^23*b
^49*c^76*d^6 + 216629339029608119402496*a^24*b^48*c^75*d^7 - 20928997043495
01998235648*a^25*b^47*c^74*d^8 + 15576808854093856430358528*a^26*b^46*c^73*
d^9 - 92989305923335928955273216*a^27*b^45*c^72*d^10 + 45771657039050515345
8339840*a^28*b^44*c^71*d^11 - 1895077372829589675098243072*a^29*b^43*c^70*d
^12 + 6699157107174094796222365696*a^30*b^42*c^69*d^13 - 204546083968174670
81213607936*a^31*b^41*c^68*d^14 + 54439663857512808688618831872*a^32*b^40*c
^67*d^15 - 127253623829876322462345461760*a^33*b^39*c^66*d^16 + 26301836032
2301930835307134976*a^34*b^38*c^65*d^17 - 484117148425341461690547437568*a^
35*b^37*c^64*d^18 + 801088032507623116562893897728*a^36*b^36*c^63*d^19 - 12
10191753560658421451373674496*a^37*b^35*c^62*d^20 + 17136621500393119651484
55895040*a^38*b^34*c^61*d^21 - 2368456612874860634985065349120*a^39*b^33*c^
60*d^22 + 3342440882817901253619697582080*a^40*b^32*c^59*d^23 - 49260194192
81526710422764257280*a^41*b^31*c^58*d^24 + 7443043331925522227676535848960*
a^42*b^30*c^57*d^25 - 11053384984245852600223452364800*a^43*b^29*c^56*d^26
+ 15529000135185248373347985653760*a^44*b^28*c^55*d^27 - 201538010268884644
82649904250880*a^45*b^27*c^54*d^28 + 23870821024791437072619829985280*a^46*
b^26*c^53*d^29 - 25662407141873741853910169026560*a^47*b^25*c^52*d^30 + 249
83334964938085602226308382720*a^48*b^24*c^51*d^31 - 22003368361455969032835
868655616*a^49*b^23*c^50*d^32 + 17519758513327663391847122731008*a^50*b^22*
c^49*d^33 - 12601896285489986596049610866688*a^51*b^21*c^48*d^34 + 81796843
90414915120451536551936*a^52*b^20*c^47*d^35 - 47835830811163604549605156454
40*a^53*b^19*c^46*d^36 + 2515171747726250254399514345472*a^54*b^18*c^45*d^3
7 - 1185710361511816082146770026496*a^55*b^17*c^44*d^38 + 49940660461835859
4580969947136*a^56*b^16*c^43*d^39 - 187097254447826761775602204672*a^57*b^1
5*c^42*d^40 + 62002233932522145150727618560*a^58*b^14*c^41*d^41 - 180491158
72947548566748921856*a^59*b^13*c^40*d^42 + 4575187392741408034214903808*a^6
0*b^12*c^39*d^43 - 998642414508019303179091968*a^61*b^11*c^38*d^44 + 184986
735996381058748645376*a^62*b^10*c^37*d^45 - 28520139033328990436720640*a^63
*b^9*c^36*d^46 + 3562072173311951854632960*a^64*b^8*c^35*d^47 - 34637786386
8692037632000*a^65*b^7*c^34*d^48 + 24611841230482125619200*a^66*b^6*c^33*d^
49 - 1137123721538961408000*a^67*b^5*c^32*d^50 + 25649407252758528000*a^68*
b^4*c^31*d^51)*1i + x^(1/2)*(4851701160433680384000*a^21*b^45*c^62*d^11 - 1
342531185305190400000*a^20*b^46*c^63*d^10 - 83128151546809181798400*a^22*b^4
4*c^61*d^12 + 895910897914030472560640*a^23*b^43*c^60*d^13 - 67971299896549
```

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57642481664*a^24*b^42*c^59*d^14 + 3848363054848997163270144O*a^25*b^41*c^58
*d^15 - 167961815050671342785396736*a^26*b^40*c^57*d^16 + 57374801955997860
3695308800*a^27*b^39*c^56*d^17 - 1529836010901462206864424960*a^28*b^38*c^5
5*d^18 + 3075153110865358700094160896*a^29*b^37*c^54*d^19 - 40445110329811
69371925708800*a^30*b^36*c^53*d^20 + 5895906393811028191047843 84*a^31*b^35*c
^52*d^21 + 1457667133433874596965122048O*a^32*b^34*c^51*d^22 - 501491461567
56356561350164480*a^33*b^33*c^50*d^23 + 110550157926715904989065117696*a^34
*b^32*c^49*d^24 - 189331360528461979941957795840*a^35*b^31*c^48*d^25 + 2673
83527373748192433944920064*a^36*b^30*c^47*d^26 - 319821143985825066443750 76
8640*a^37*b^29*c^46*d^27 + 328626898447261055168230195200*a^38*b^28*c^45*d^
28 - 292434560796558751919058714624*a^39*b^27*c^44*d^29 + 226382416482170 29
0892093521920*a^40*b^26*c^43*d^30 - 152776304398053739659930894336*a^41*b^2
5*c^42*d^31 + 89901124622673343064718704640*a^42*b^24*c^41*d^32 - 460625089
64820426479181496320*a^43*b^23*c^40*d^33 + 20486606263737610091045584896*a^
44*b^22*c^39*d^34 - 7870914323775054351244984320*a^45*b^21*c^38*d^35 + 2594
141724382360002965274624*a^46*b^20*c^37*d^36 - 726451024651952784807034880*
a^47*b^19*c^36*d^37 + 170590060365885174888529920*a^48*b^18*c^35*d^38 - 329
86343554204898112307200*a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^5
0*b^16*c^33*d^40 - 613036163719885750272000*a^51*b^15*c^32*d^41 + 532552977
7099820236800 0*a^52*b^14*c^31*d^42 - 29887257926172672 00000*a^53*b^13*c^30*
d^43 + 8143812043997184 0000*a^54*b^12*c^29*d^44))*(-(8398080000*a^33*d^33 -
 (70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 4645656
529679155200000 0*a^2*b^64*c^64*d^2 - 8523959496286928896 00000*a^3*b^63*c^63
*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297
536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 -
6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909
184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 +
742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290086689049 6
569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c
^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467
42584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069462050587268035
3792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50
*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 91675357311
6017703850321517740032*a^18*b^48*c^48*d^18 - 14804725213251685264523823352 3
8144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^4
6*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844
3496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445
32998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^4
2*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622
62545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9257596460706208483 8
869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*
a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d
^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251
79164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039 70
3580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*
```

```
b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34
- 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124
433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935
503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*
b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39
+ 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364
395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912
639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*
b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 -
 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900
743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808162454775635587
89120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c
^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787
56421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829954142086407501
54437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b
^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129
789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279547644479759
4501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*
c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691
8031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*
b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 45597179299
3637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^
4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13414412438470656000
0000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 43883931893
76000000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2
*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^2
9*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*
c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c
^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23
*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12
*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*
a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057
164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 68879
8564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19
- 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^1
2*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b
^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^
25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^
27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29
*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c
^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(6871947
6736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c
^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2
- 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4
- 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d
```

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^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c
^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19
*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 155163658155353
70240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 3239
6807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*
d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26
*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900
569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 886
6494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d
^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*
c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*
b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^3
7*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*
b^2*c^15*d^30))^(1/4) + ((-(8398080000*a^33*d^33 - (70527747686400000000*a^
66*d^66 + 274877906944000000000*b^66*c^66 + 46456565296791552000000*a^2*b^64
*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 113031004798163353
60000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5
+ 946609333913578145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129
280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 1
76823915553078667757483982848*a^9*b^57*c^57*d^9 + 7425481275746674581907219
41504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*
d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082
618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a
^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15
+ 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 53837688933932732
2092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^
18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19
+ 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550
801466936451399680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720
*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d
^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460
407159853772918944169984*a^25*b^41*c^41*d^25 + 6226254579704186675283668534
0344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39
*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177
036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 2431372713606781682
80724887442554880*a^30*b^36*c^36*d^30 - 34711352517916424353692724892794880
0*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34
*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 11365474
00098503091050564698912063488*a^34*b^32*c^32*d^34 - 15786833044632146161337
55020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*
a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*
d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908
27480554493466534979194322944*a^39*b^27*c^27*d^39 + 25581457575927361633598
68236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*
a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*
```

```
d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 84844675
0580244547991361710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639
363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^4
6*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47
 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952
477787555393435598848*a^49*b^17*c^17*d^49 + 1297875642151239082178936230536
8064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^1
5*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318
567145258980606161715200*a^53*b^13*c^13*d^53 + 1297898094790687573306431766
52800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11
*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604
091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*
b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879
982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5
*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971
840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 -
4388393189376000000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65
)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996
090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917
888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 41118923017420
80*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 147939177477877
76*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 225293620110
54080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387
378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150
199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^1
6 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c
^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20
*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 4408131707805696
00*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163
264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 1979265
1594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351
733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 377578967
04000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*
b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 6871
9476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^4
0*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^
29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*
b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a
^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 192749885907271
6800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494
751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12
 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^1
8*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 413059298770708070
40*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239680
7746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^1
```

```
9 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^1
1*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800
*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 2312998630887
26016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 138384533
47188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 3408486
04610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^
(1/2)*(-(8398080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 274877906
94400000000*b^66*c^66 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 8523959
4962869288960000O*a^3*b^63*c^63*d^3 + 1130310047981633536000000O*a^4*b^62*c
^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 94660933391357814
5788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d
^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667775
7483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^5
6*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864
3010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*
a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14
- 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237796
4853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^
17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 -
 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793325
90916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*
a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^2
2 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814015
3590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189441
69984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c
^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947
384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841O
49111826268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a
^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^
31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332
9926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056469
8912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35
*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36
 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974
5327887207329057621409792*a^38*b^28*c^28*d^38 - 273939082748055449346653497
9194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40
*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41
 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380586
7374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710
073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b
^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 -
154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195
3043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355988
48*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16
*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466
```

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0146705061178096641638840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150036124817025925120000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 213364244480 0*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d^4 - 126244904754449743872 0*a^24*b^50*c^80*d^5 + 2678121363
```

```
0512448405504*a^25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*
d^7 + 364841894840686264870502 4*a^27*b^47*c^77*d^8 - 2809739458477914794754
0480*a^28*b^46*c^76*d^9 + 17444838930933794835162726 4*a^29*b^45*c^75*d^10 -
 897668976119897481466085376*a^30*b^44*c^74*d^11 + 390592024288463001086853
1200*a^31*b^43*c^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^1
3 + 47355821068790227801756139520*a^33*b^41*c^71*d^14 - 1348455245851039370
61538234368*a^34*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*
c^69*d^16 - 763246944716696111818343448576*a^36*b^38*c^68*d^17 + 1541564027
180813686607638953984*a^37*b^37*c^67*d^18 - 2825288027628089174763608473600
*a^38*b^36*c^66*d^19 + 4753041476272000853590444867584*a^39*b^35*c^65*d^20
- 7446263493185815677622957375488*a^40*b^34*c^64*d^21 + 1104597461179480702
7964401680384*a^41*b^33*c^63*d^22 - 15766681100741571532295786987520*a^42*b
^32*c^62*d^23 + 21882616570434895907374088847360*a^43*b^31*c^61*d^24 - 2955
5415901357165913293077872640*a^44*b^30*c^60*d^25 + 385142493646332136507672
04843520*a^45*b^29*c^59*d^26 - 47767982724772003266224581509120*a^46*b^28*c
^58*d^27 + 55618948537155045120476750807040*a^47*b^27*c^57*d^28 - 601273004
13664475479641214156800*a^48*b^26*c^56*d^29 + 598770389984402606380501539 22
560*a^49*b^25*c^55*d^30 - 54637051595737047014674020696064*a^50*b^24*c^54*d
^31 + 45519623223064909005599526617088*a^51*b^23*c^53*d^32 - 34535555816550
055085254958383104*a^52*b^22*c^52*d^33 + 23809729504484309698980012359680*a
^53*b^21*c^51*d^34 - 14885319254535352990241541586944*a^54*b^20*c^50*d^35 +
 8419648339202954390072444583936*a^55*b^19*c^49*d^36 - 4297514831765712413 6
11503124480*a^56*b^18*c^48*d^37 + 1973123737554130196459440570368*a^57*b^17
*c^47*d^38 - 811770857054497673061303582720*a^58*b^16*c^46*d^39 + 297858380
372439371596188090368*a^59*b^15*c^45*d^40 - 96910050535770593129744302080*a
^60*b^14*c^44*d^41 + 27758579881177587823480406016*a^61*b^13*c^43*d^42 - 69
37474504476672102869499904*a^62*b^12*c^42*d^43 + 1495682482860276471300096 0
00*a^63*b^11*c^41*d^44 - 274100118958300866495381504*a^64*b^10*c^40*d^45 +
41867778463425277028466688*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a
^66*b^8*c^38*d^47 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 353719920
49308254863360*a^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^
50 - 36479156981701017600*a^70*b^4*c^34*d^51)*1i + 18014398509481984000*a^2
1*b^51*c^78*d^4 - 778222015609621708800*a^22*b^50*c^77*d^5 + 16199988291606
958571520*a^23*b^49*c^76*d^6 - 216629339029608119402496*a^24*b^48*c^75*d^7
+ 2092899704349501998235648*a^25*b^47*c^74*d^8 - 15576808854093856430358528
*a^26*b^46*c^73*d^9 + 92989305923335928955273216*a^27*b^45*c^72*d^10 - 4577
16570390505153458339840*a^28*b^44*c^71*d^11 + 1895077372829589675098243072*
a^29*b^43*c^70*d^12 - 6699157107174094796222365696*a^30*b^42*c^69*d^13 + 20
454608396817467081213607936*a^31*b^41*c^68*d^14 - 544396638575128086886188 3
1872*a^32*b^40*c^67*d^15 + 127253623829876322462345461760*a^33*b^39*c^66*d^
16 - 263018360322301930835307134976*a^34*b^38*c^65*d^17 + 48411714842534146
1690547437568*a^35*b^37*c^64*d^18 - 801088032507623116562893897728*a^36*b^3
6*c^63*d^19 + 1210191753560658421451373674496*a^37*b^35*c^62*d^20 - 1713662
150039311965148455895040*a^38*b^34*c^61*d^21 + 236845661287486063498506534 9
120*a^39*b^33*c^60*d^22 - 3342440882817901253619697582080*a^40*b^32*c^59*d^
```

```
23 + 4926019419281526710422764257280*a^41*b^31*c^58*d^24 - 7443043331925522
227676535848960*a^42*b^30*c^57*d^25 + 11053384984245852600223452364800*a^43
*b^29*c^56*d^26 - 15529000135185248373347985653760*a^44*b^28*c^55*d^27 + 20
153801026888464482649904250880*a^45*b^27*c^54*d^28 - 2387082102479143707261
9829985280*a^46*b^26*c^53*d^29 + 25662407141873741853910169026560*a^47*b^25
*c^52*d^30 - 24983334964938085602226308382720*a^48*b^24*c^51*d^31 + 2200336
8361455969032835868655616*a^49*b^23*c^50*d^32 - 175197585133276633918471227
31008*a^50*b^22*c^49*d^33 + 12601896285489986596049610866688*a^51*b^21*c^48
*d^34 - 8179684390414915120451536551936*a^52*b^20*c^47*d^35 + 4783583081116
360454960515645440*a^53*b^19*c^46*d^36 - 2515171747726250254399514345472*a^
54*b^18*c^45*d^37 + 1185710361511816082146770026496*a^55*b^17*c^44*d^38 - 4
99406604618358594580969947136*a^56*b^16*c^43*d^39 + 18709725444782676177560
2204672*a^57*b^15*c^42*d^40 - 62002233932522145150727618560*a^58*b^14*c^41*
d^41 + 18049115872947548566748921856*a^59*b^13*c^40*d^42 - 4575187392741408
034214903808*a^60*b^12*c^39*d^43 + 998642414508019303179091968*a^61*b^11*c^
38*d^44 - 184986735996381058748645376*a^62*b^10*c^37*d^45 + 285201390333289
90436720640*a^63*b^9*c^36*d^46 - 3562072173311951854632960*a^64*b^8*c^35*d^
47 + 346377863868692037632000*a^65*b^7*c^34*d^48 - 24611841230482125619200*
a^66*b^6*c^33*d^49 + 1137123721538961408000*a^67*b^5*c^32*d^50 - 2564940725
2758528000*a^68*b^4*c^31*d^51)*1i + x^(1/2)*(4851701160433680384000*a^21*b^
45*c^62*d^11 - 134253118530519040000*a^20*b^46*c^63*d^10 - 8312815154680918
1798400*a^22*b^44*c^61*d^12 + 895910897914030472560640*a^23*b^43*c^60*d^13
- 6797129989654957642481664*a^24*b^42*c^59*d^14 + 3848363054848997163270144
0*a^25*b^41*c^58*d^15 - 167961815050671342785396736*a^26*b^40*c^57*d^16 + 5
73748019559978603695308800*a^27*b^39*c^56*d^17 - 15298360109014622068644249
60*a^28*b^38*c^55*d^18 + 3075153110865358700094160896*a^29*b^37*c^54*d^19 -
 4044511032981169371925708800*a^30*b^36*c^53*d^20 + 58959063938110281910478
4384*a^31*b^35*c^52*d^21 + 14576671334338745969651220480*a^32*b^34*c^51*d^2
2 - 50149146156756356561350164480*a^33*b^33*c^50*d^23 + 1105501579267159049
89065117696*a^34*b^32*c^49*d^24 - 189331360528461979941957795840*a^35*b^31*
c^48*d^25 + 267383527373748192433944920064*a^36*b^30*c^47*d^26 - 3198211439
85825066443750768640*a^37*b^29*c^46*d^27 + 328626898447261055168230195200*a
^38*b^28*c^45*d^28 - 292434560796558751919058714624*a^39*b^27*c^44*d^29 + 2
26382416482170290892093521920*a^40*b^26*c^43*d^30 - 15277630439805373965993
0894336*a^41*b^25*c^42*d^31 + 89901124622673343064718704640*a^42*b^24*c^41*
d^32 - 46062508964820426479181496320*a^43*b^23*c^40*d^33 + 2048660626373761
0091045584896*a^44*b^22*c^39*d^34 - 7870914323775054351244984320*a^45*b^21*
c^38*d^35 + 2594141724382360002965274624*a^46*b^20*c^37*d^36 - 726451024651
952784807034880*a^47*b^19*c^36*d^37 + 170590060365885174888529920*a^48*b^18
*c^35*d^38 - 32986343554204898112307200*a^49*b^17*c^34*d^39 + 5118063591384
977873305600*a^50*b^16*c^33*d^40 - 613036163719885750272000*a^51*b^15*c^32*
d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42 - 2988725792617267200000
*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b^12*c^29*d^44))*(-(839808
0000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 27487790694400000000*b^6
6*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 8523959496286928896000
```

```
00*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 11548
807808472982329753600 0*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*
b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 3639438050
7592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*
b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041
5842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864301071819289395
2000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*
d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801106
94620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237796485395340328960
0*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^
17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 14804725213251
68526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623367881 52
32*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*
d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12695869829
017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081401535900391206823 0
0735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41
*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9257
5964607062084838869289496739840*a^27*b^39*c^39*d^27 + 12994738493072452038 8
491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*
a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d
^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 5158333428
86205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332992628044123259
0010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34
*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35
 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244704257539
9654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788720732905
7621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39
*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40
 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 173879220535
5133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380586737468979005302
0810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*
b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 +
 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080
816245477563558789120*a^47*b^19*c^19*d^47 + 739174514721719530430678553581 3
2224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^
17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782
995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809
664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13
*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 317760
42795476444797594501120000*a^55*b^11*c^11*d^55 + 694868361500361248170259 25
12000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^
57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 339421563479651575 1
3625600000*a^59*b^7*c^7*d^59 + 429545687998224012410880000 0*a^60*b^6*c^6*d^
60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 395042949152786350080 0
0000*a^62*b^4*c^4*d^62 - 26837948400559718400000 00*a^63*b^3*c^3*d^63 + 1341
```

```
4412438470656000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65
*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2
133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 1069982
13378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406
056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 86707877707
77600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801
130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 167807
95101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53
058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^1
5 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c
^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19
*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903
936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 5582
5496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 580
1173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 2613
25798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 39223388
16000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*
c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 219902325
5552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^1
1*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^1
3*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*
a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 7228120721522
68800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 44332473
75867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 +
 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*
c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920
*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 388761692
96066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18
- 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12
*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*
a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152
268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 6227304
0062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471
152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 3408
4860461056*a^39*b^2*c^15*d^30))^(1/4))/(((-(8398080000*a^33*d^33 - (7052774
7686400000000*a^66*d^66 + 2748779069440000000*b^66*c^66 + 464565652967915
2000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11
303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^
5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820
6349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b
^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127
5746674581907219415O4*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840
*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12
- 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467425843637
```

```
99641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15
*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5
38376889339327322092190511923200*a^17*b^49*c^49*d^17 + 91675357311601770385
0321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19
*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 -
3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345
8703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*
a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^
24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970
41866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496
739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38
*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243
137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435
36927248927948800*a^31*b^35*c^35*d^31 + 51583334288620561992503970358099968
0*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33
*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683
304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578
392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200
*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28
*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145
757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582
415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400
*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23
*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319705
9864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383
822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^4
7*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48
- 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123
90821789362305368064*a^50*b^16*c^16*d^50 - 47457829954142086407501544370995
20*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*
d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479
068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279547644479759450112000
0*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56
 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691803197496
3609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d
^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 45597179299363710566
4000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 -
2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64
*b^2*c^2*d^64 - 162727720910848000000*a*b^65*c^65*d - 43883931893760000000
00*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 213364244800*a^2*b^31*c^3
1*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^
4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6
- 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 -
 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^1
0 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^2
```

```
1*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19
*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^
16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 68879856484794
3680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 7661592
67095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 -
440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*
d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^
8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^
6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*
d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 -
 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*
b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2
199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 3408486
04610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 1383845
3347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 2312
99863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 -
 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^3
5*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^2
1*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 323968077467
2201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41
305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^2
8*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^
28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 886649475173
4497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 192
7498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24
 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*
d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^1
7*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*
d^30))^(3/4)*(x^(1/2)*(-(8398080000*a^33*d^33 - (70527747686400000000*a^66*
d^66 + 274877906944000000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^
64*d^2 - 85239594962869288960000*a^3*b^63*c^63*d^3 + 113031004798163353600
00000*a^4*b^62*c^62*d^4 - 11548807808472982329753600*a^5*b^61*c^61*d^5 + 9
4660933391357814578872320*a^6*b^60*c^60*d^6 - 639883820634974459346812928
*a^7*b^59*c^59*d^7 + 3639438050759279751345890918400*a^8*b^58*c^58*d^8 - 1768
2391555307866775748398284800*a^9*b^57*c^57*d^9 + 7425481275746674581907219415
04*a^10*b^56*c^56*d^10 - 27204158429008668904965695078400*a^11*b^55*c^55*d^1
1 + 87608488386430107181928939520000*a^12*b^54*c^54*d^12 - 249552350040826187
0704122868531200*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14
*b^52*c^52*d^14 - 14313378011069462050587268035379200*a^15*b^51*c^51*d^15 + 2
9143271303237796485395340328960000*a^16*b^50*c^50*d^16 - 5383768893393273220921
90511923200*a^17*b^49*c^49*d^17 + 91675357311601770385032151774003200*a^18*
b^48*c^48*d^18 - 148047252132516852645238233523814400*a^19*b^47*c^47*d^19 + 2
37012426137933259091623367881523200*a^20*b^46*c^46*d^20 - 394568205038255080146
6936451399680*a^21*b^45*c^45*d^21 + 69634084434967934587032376128307200*a^
22*b^44*c^44*d^22 - 126958698290172324083068445329814400*a^23*b^43*c^43*d^23
```

```
 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460407
159853772918944169984*a^25*b^41*c^41*d^25 + 6226254579704186675283668534034
4320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^
39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036
156654250012841049111826268160*a^29*b^37*c^37*d^29 + 2431372713606781682807
24887442554880*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a
^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^
32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 11365474000
98503091050564698912063488*a^34*b^32*c^32*d^34 - 15786833044632146161337550
20010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^3
6*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^3
7 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274
80554493466534979194322944*a^39*b^27*c^27*d^39 + 25581457575927361633598682
36513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^4
1*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^4
2 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 84844675058
0244547991361710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363
737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b
^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 +
73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952477
787555393435598848*a^49*b^17*c^17*d^49 + 1297875642151239082178936230536806
4*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d
^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567
145258980606161715200*a^53*b^13*c^13*d^53 + 1297898094790687573306431766528
00*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^
55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091
154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8
*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982
240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^
5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840
000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162
727720910848000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(
1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090
880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888
*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*
a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*
a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 225293620110540
80*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378
688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199
661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 +
 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15
*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^
13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*
a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264
708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 1979265159
```

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4874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733
145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 377578967040
00*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^3
2*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 6871947
6736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b
*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*
c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^2
7*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16
*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 192749885907271680
0*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751
734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 -
23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c
^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*
a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239680774
6722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 +
 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c
^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^
32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 2312998630887260
16*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 138384533471
88736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 3408486046
10560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(28823
037615171174400*a^23*b^51*c^81*d^4 - 1262449047544497438720*a^24*b^50*c^80*
d^5 + 26781213630512448405504*a^25*b^49*c^79*d^6 - 366816964670228254425088
*a^26*b^48*c^78*d^7 + 3648418948406862648705024*a^27*b^47*c^77*d^8 - 280973
94584779147947540480*a^28*b^46*c^76*d^9 + 174448389309337948351627264*a^29*
b^45*c^75*d^10 - 897668976119897481466085376*a^30*b^44*c^74*d^11 + 39059202
42884630010868531200*a^31*b^43*c^73*d^12 - 14590896425075765379929735168*a^
32*b^42*c^72*d^13 + 47355821068790227801756139520*a^33*b^41*c^71*d^14 - 134
845524585103937061538234368*a^34*b^40*c^70*d^15 + 3397270967300867141087631
76960*a^35*b^39*c^69*d^16 - 763246944716696111818343448576*a^36*b^38*c^68*d
^17 + 1541564027180813686607638953984*a^37*b^37*c^67*d^18 - 282528802762808
9174763608473600*a^38*b^36*c^66*d^19 + 4753041476272000853590444867584*a^39
*b^35*c^65*d^20 - 7446263493185815677622957375488*a^40*b^34*c^64*d^21 + 110
45974611794807027964401680384*a^41*b^33*c^63*d^22 - 157666811007415715322957
86987520*a^42*b^32*c^62*d^23 + 21882616570434895907374088847360*a^43*b^31*
c^61*d^24 - 29555415901357165913293077872640*a^44*b^30*c^60*d^25 + 38514249
364633213650767204843520*a^45*b^29*c^59*d^26 - 477679827247720032662245815
09120*a^46*b^28*c^58*d^27 + 55618948537155045120476750807040*a^47*b^27*c^57*
d^28 - 60127300413664475479641214156800*a^48*b^26*c^56*d^29 + 5987703899844
0260638050153922560*a^49*b^25*c^55*d^30 - 54637051595737047014674020696064*
a^50*b^24*c^54*d^31 + 45519623223064909005599526617088*a^51*b^23*c^53*d^32
- 34535555816550055085254958383104*a^52*b^22*c^52*d^33 + 238097295044843096
98980012359680*a^53*b^21*c^51*d^34 - 14885319254535352990241541586944*a^54*
b^20*c^50*d^35 + 8419648339202954390072444583936*a^55*b^19*c^49*d^36 - 4297
514831765712413611503124480*a^56*b^18*c^48*d^37 + 197312373755413019645944O
```

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570368*a^57*b^17*c^47*d^38 - 811770857054497673061303582720*a^58*b^16*c^46*
d^39 + 297858380372439371596188090368*a^59*b^15*c^45*d^40 - 969100505357705
93129744302080*a^60*b^14*c^44*d^41 + 27758579881177587823480406016*a^61*b^1
3*c^43*d^42 - 6937474504476672102869499904*a^62*b^12*c^42*d^43 + 1495682482
860276471300096000*a^63*b^11*c^41*d^44 - 274100118958300866495381504*a^64*b
^10*c^40*d^45 + 41867778463425277028466688*a^65*b^9*c^39*d^46 - 51871611309
30763594727424*a^66*b^8*c^38*d^47 + 500879902205011065569280*a^67*b^7*c^37*
d^48 - 35371992049308254863360*a^68*b^6*c^36*d^49 + 1625349105518012006400*
a^69*b^5*c^35*d^50 - 36479156981701017600*a^70*b^4*c^34*d^51)*1i - 18014398
509481984000*a^21*b^51*c^78*d^4 + 778222015609621708800*a^22*b^50*c^77*d^5
- 16199988291606958571520*a^23*b^49*c^76*d^6 + 216629339029608119402496*a^2
4*b^48*c^75*d^7 - 2092899704349501998235648*a^25*b^47*c^74*d^8 + 1557680885
4093856430358528*a^26*b^46*c^73*d^9 - 92989305923335928955273216*a^27*b^45*
c^72*d^10 + 457716570390505153458339840*a^28*b^44*c^71*d^11 - 1895077372829
589675098243072*a^29*b^43*c^70*d^12 + 6699157107174094796222365696*a^30*b^4
2*c^69*d^13 - 20454608396817467081213607936*a^31*b^41*c^68*d^14 + 544396638
57512808688618831872*a^32*b^40*c^67*d^15 - 127253623829876322462345461760*a
^33*b^39*c^66*d^16 + 263018360322301930835307134976*a^34*b^38*c^65*d^17 - 4
84117148425341461690547437568*a^35*b^37*c^64*d^18 + 801088032507623116562893
897728*a^36*b^36*c^63*d^19 - 1210191753560658421451373674496*a^37*b^35*c^6
2*d^20 + 1713662150039311965148455895040*a^38*b^34*c^61*d^21 - 236845661287
4860634985065349120*a^39*b^33*c^60*d^22 + 3342440882817901253619697582080*a
^40*b^32*c^59*d^23 - 4926019419281526710422764257280*a^41*b^31*c^58*d^24 +
7443043331925522227676535848960*a^42*b^30*c^57*d^25 - 110533849842458526002
23452364800*a^43*b^29*c^56*d^26 + 15529000135185248373347985653760*a^44*b^2
8*c^55*d^27 - 20153801026888464482649904250880*a^45*b^27*c^54*d^28 + 238708
21024791437072619829985280*a^46*b^26*c^53*d^29 - 25662407141873741853910169
026560*a^47*b^25*c^52*d^30 + 24983334964938085602226308382720*a^48*b^24*c^5
1*d^31 - 22003368361455969032835868655616*a^49*b^23*c^50*d^32 + 17519758513
327663391847122731008*a^50*b^22*c^49*d^33 - 126018962854899865960496108666
8*a^51*b^21*c^48*d^34 + 8179684390414915120451536551936*a^52*b^20*c^47*d^35
 - 4783583081116360454960515645440*a^53*b^19*c^46*d^36 + 251517174772625025
4399514345472*a^54*b^18*c^45*d^37 - 1185710361511816082146770026496*a^55*b^
17*c^44*d^38 + 499406604618358594580969947136*a^56*b^16*c^43*d^39 - 1870972
54447826761775602204672*a^57*b^15*c^42*d^40 + 62002233932522145150727618560
*a^58*b^14*c^41*d^41 - 18049115872947548566748921856*a^59*b^13*c^40*d^42 +
4575187392741408034214903808*a^60*b^12*c^39*d^43 - 998642414508019303179091
968*a^61*b^11*c^38*d^44 + 184986735996381058748645376*a^62*b^10*c^37*d^45 -
 28520139033328990436720640*a^63*b^9*c^36*d^46 + 3562072173311951854632960*
a^64*b^8*c^35*d^47 - 346377863868692037632000*a^65*b^7*c^34*d^48 + 24611841
230482125619200*a^66*b^6*c^33*d^49 - 1137123721538961408000*a^67*b^5*c^32*d
^50 + 25649407252758528000*a^68*b^4*c^31*d^51)*1i + x^(1/2)*(48517011604336
80384000*a^21*b^45*c^62*d^11 - 134253118530519040000*a^20*b^46*c^63*d^10 -
83128151546809181798400*a^22*b^44*c^61*d^12 + 895910897914030472560640*a^23
*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b^42*c^59*d^14 + 384836305
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48489971632701440*a^25*b^41*c^58*d^15 - 1679618150506713427853967736*a^26*b^
40*c^57*d^16 + 5737480195599786036953088800*a^27*b^39*c^56*d^17 - 1529836010
9014622068644249600*a^28*b^38*c^55*d^18 + 30751531108653587000941608896*a^29*
b^37*c^54*d^19 - 40445110329811693719257088800*a^30*b^36*c^53*d^20 + 5895906
3938110281910478438 4*a^31*b^35*c^52*d^21 + 14576671334338745969651220480*a^
32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b^33*c^50*d^23 + 110
550157926715904989065117696*a^34*b^32*c^49*d^24 - 189331360528461979941957 7
95840*a^35*b^31*c^48*d^25 + 26738352737374819243394492006 4*a^36*b^30*c^47*d
^26 - 31982114398582506644375076864 0*a^37*b^29*c^46*d^27 + 3286268984472610
55168230195200*a^38*b^28*c^45*d^28 - 292434560796558751919058714624*a^39*b^
27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*c^43*d^30 - 1527763
04398053739659930894336*a^41*b^25*c^42*d^31 + 89901124622673343064718704640
*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^43*b^23*c^40*d^33 +
20486606263737610091045584896*a^44*b^22*c^39*d^34 - 787091432377505435124 49
84320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624*a^46*b^20*c^37*d^3
6 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 170590060365885174888
529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*a^49*b^17*c^34*d^39
 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 613036163719885750272000
*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42 - 298872
5792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b^12*c^29*
d^44))*(-(8398080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 27487790
694400000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852395
949628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*
c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578 1
45788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*
d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667 7
57483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^
56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 87608488386
43010718192893952000*a^12*b^54*c^54*d^12 - 249552350040826187070412286853 12
*a^13*b^53*c^53*d^13 + 632144467425843637996415185059 84*a^14*b^52*c^52*d^14
 - 1431337801106946205058726803537 92*a^15*b^51*c^51*d^15 + 2914327130323779
64853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a
^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18
- 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 2370124261379332
590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680
*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^
22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081401
53590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189 44
169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*
c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 12994
7384930724520388491615907348480*a^28*b^38*c^38*d^28 - 17703615665425001284 1
049111826268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*
a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d
^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 7754680733
299262804412325900100567 04*a^33*b^33*c^33*d^33 + 113654740009850309105056 46
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98912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^3
5*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^3
6 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 26989809397
45327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274805544934665349
79194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^4
0*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^4
1 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 12660138058
67374689790053020810084352*a^43*b^23*c^23*d^43 + 84844675058024454799136171
0073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*
b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 -
 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 739174514721719
53043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598
848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^1
6*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 157896546
6014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161
715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^
12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 6948683615
003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000
*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 339
42156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000
*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 395042
94915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3
*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162727720910848000
0000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 52428800
00*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^
30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*
d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d
^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d
^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^
22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^2
0*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^
15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 50441346317306
8800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 7900653
81353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 +
630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*
d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9
*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^
7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5
*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3
*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 2612
73600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*
d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34
084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471
152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62
273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7
```

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+ 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^3
6*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*
b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 238713320239005
69600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 3887
6169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*
d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27
*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535
370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433
247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^2
3 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^2
0*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*
c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*
c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*1i - ((-(8398080000*a
^33*d^33 - (70527747686400000000*a^66*d^66 + 2748779069440000000*b^66*c^66
 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 8523959496286928896000000*a^3
*b^63*c^63*d^3 + 113031004798163353600000000*a^4*b^62*c^62*d^4 - 11548807808
47298232975360000*a^5*b^61*c^61*d^5 + 9466093339135781457887232000*a^6*b^60*c
^60*d^6 - 63988382063497445934681292800*a^7*b^59*c^59*d^7 + 3639438050759279
751345890918400*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c
^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290
0866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a
^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 +
 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801106946205
05872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16
*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 9
16753573116017703850321517740032*a^18*b^48*c^48*d^18 - 14804725213251685264
52382335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^2
0*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 +
 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232
408306844532998144*a^23*b^43*c^43*d^23 + 228294081401535900391206823007354
8*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*
d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9257596460
706208483886928949673984 0*a^27*b^39*c^39*d^27 + 12994738493072452038849161 5
907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b
^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 -
347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 5158333428862056
199250397035809996 80*a^32*b^34*c^34*d^32 - 7754680733299262804412325900100 5
6704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*
c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 204
4085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244704257539965436
2397243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788720732905762140
9792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*
c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 219
8323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 173879220535513303
4582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810 08
```

```
4352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c
^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 29669
2444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245
477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a
^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^4
9 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782995414
208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163
840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*
d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 317760427954
76444797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*
a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 2
29469146918031974963609600000*a^58*b^8*c^8*d^58 - 3394215634796515751362560
0000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 4
55971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a
^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 1341441243
84706560000000*a^64*b^2*c^2*d^64 - 16272772091084800000000*a*b^65*c^65*d - 4
38839318937600000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642
444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 1069982133780
48*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960
*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*
a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496
*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 167807951017
57440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143
899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 30
6575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^
17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*
c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^2
1*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962
560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 5582549611
5836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 580117366
8208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 2613257987
07200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*
a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32
)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a
^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30
*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28
*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b
^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a
^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 44332473758672
48640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516
365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d
^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*
b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 388761692960666
41920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 2387
1332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*
```

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d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b
^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800
*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 6227304006234
9312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383
426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 3408486046
1056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-(8398080000*a^33*d^33 - (7052774
768640000000*a^66*d^66 + 274877906944000000*b^66*c^66 + 4645656529679155
2000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11
303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^
5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820
6349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b
^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127
574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840
*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12
- 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467425843637
99641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15
*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5
38376889339327322092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850
321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19
*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 -
3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345
8703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*
a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^
24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970
41866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496
739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38
*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243
137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435
36927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680
0*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33
*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683
304463214616133755020010061824*a^35*b^31*c^31*d^35 + 20440850601244330725783
92630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200
*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28
*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145
757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582
415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400
*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23
*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319705
9864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383
822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^4
7*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48
- 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123
90821789362305368064*a^50*b^16*c^16*d^50 - 474578299541420864075015443709995
20*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*
```

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d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479
068757330643176652800*a^54*b^12*c^12*d^54 - 317760427954764447975945011200
0*a^55*b^11*c^11*d^55 + 69486836150036124817025925120000*a^56*b^10*c^10*d^56
 - 13472186556040911549104128000000*a^57*b^9*c^9*d^57 + 229469146918031974963
3609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256000000*a^59*b^7*c^7*d
^59 + 42954568799822401241088000000*a^60*b^6*c^6*d^60 - 455971792993637105664
4000000*a^61*b^5*c^5*d^61 + 395042949152786350080000000*a^62*b^4*c^4*d^62 -
26837948400559718400000000*a^63*b^3*c^3*d^63 + 1341441243847065600000000*a^64
*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 438839318937600000000
00*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^3
1*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^
4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6
- 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 -
 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^1
0 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^2
1*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19
*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^
16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 68879856484794
3680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 7661592
67095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 -
440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*
d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^
8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^
6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*
d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 -
 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*
b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2
199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 3408486
04610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 1383845
3347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 2312
99863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 -
 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^3
5*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^2
1*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 323968077467 2
2201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41
305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^2
8*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^
28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 886649475173
4497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 192
7498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24
 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*
d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^1
7*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*
d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d^4 - 1262449047544497438
720*a^24*b^50*c^80*d^5 + 2678121363051244840 5504*a^25*b^49*c^79*d^6 - 36681
```

```
6964670228254425088*a^26*b^48*c^78*d^7 + 3648418948406862648705024*a^27*b^4
7*c^77*d^8 - 2809739458477914794754O480*a^28*b^46*c^76*d^9 + 17444838930933
7948351627264*a^29*b^45*c^75*d^10 - 897668976119897481466085376*a^30*b^44*c
^74*d^11 + 3905920242884630010868531200*a^31*b^43*c^73*d^12 - 1459089642507
5765379929735168*a^32*b^42*c^72*d^13 + 4735582106879022780175613952O*a^33*b
^41*c^71*d^14 - 134845524585103937061538234368*a^34*b^40*c^70*d^15 + 339727
096730086714108763176960*a^35*b^39*c^69*d^16 - 7632469447166961118183434485
76*a^36*b^38*c^68*d^17 + 1541564027180813686607638953984*a^37*b^37*c^67*d^1
8 - 2825288027628089174763608473600*a^38*b^36*c^66*d^19 + 47530414762720008
53590444867584*a^39*b^35*c^65*d^20 - 7446263493185815677622957375488*a^40*b
^34*c^64*d^21 + 11045974611794807027964401680384*a^41*b^33*c^63*d^22 - 1576
6681100741571532295786987520*a^42*b^32*c^62*d^23 + 218826165704348959073740
88847360*a^43*b^31*c^61*d^24 - 29555415901357165913293077872640*a^44*b^30*c
^60*d^25 + 38514249364633213650767204843520*a^45*b^29*c^59*d^26 - 477679827
24772003266224581509120*a^46*b^28*c^58*d^27 + 5561894853715504512047675O807
040*a^47*b^27*c^57*d^28 - 60127300413664475479641214156800*a^48*b^26*c^56*d
^29 + 59877038998440260638050153922560*a^49*b^25*c^55*d^30 - 54637051595737
047014674020696064*a^50*b^24*c^54*d^31 + 45519623223064909005599526617088*a
^51*b^23*c^53*d^32 - 34535555816550055085254958383104*a^52*b^22*c^52*d^33 +
 23809729504484309698980012359680*a^53*b^21*c^51*d^34 - 1488531925453535299
0241541586944*a^54*b^20*c^50*d^35 + 8419648339202954390072444583936*a^55*b^
19*c^49*d^36 - 4297514831765712413611503124480*a^56*b^18*c^48*d^37 + 197312
3737554130196459440570368*a^57*b^17*c^47*d^38 - 811770857054497673061303582
720*a^58*b^16*c^46*d^39 + 297858380372439371596188090368*a^59*b^15*c^45*d^4
0 - 96910050535770593129744302080*a^60*b^14*c^44*d^41 + 2775857988117758782
3480406016*a^61*b^13*c^43*d^42 - 6937474504476672102869499904*a^62*b^12*c^4
2*d^43 + 1495682482860276471300096000*a^63*b^11*c^41*d^44 - 274100118958300
866495381504*a^64*b^10*c^40*d^45 + 41867778463425277028466688*a^65*b^9*c^39
*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^47 + 50087990220501106556
9280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c^36*d^49 + 1625
349105518012006400*a^69*b^5*c^35*d^50 - 36479156981701017600*a^70*b^4*c^34*
d^51)*1i + 18014398509481984000*a^21*b^51*c^78*d^4 - 778222015609621708800*
a^22*b^50*c^77*d^5 + 16199988291606958571520*a^23*b^49*c^76*d^6 - 216629339
029608119402496*a^24*b^48*c^75*d^7 + 2092899704349501998235648*a^25*b^47*c^
74*d^8 - 15576808854093856430358528*a^26*b^46*c^73*d^9 + 929893059233359289
55273216*a^27*b^45*c^72*d^10 - 457716570390505153458339840*a^28*b^44*c^71*d
^11 + 1895077372829589675098243072*a^29*b^43*c^70*d^12 - 669915710717409479
6222365696*a^30*b^42*c^69*d^13 + 20454608396817467081213607936*a^31*b^41*c^
68*d^14 - 54439663857512808688618831872*a^32*b^40*c^67*d^15 + 1272536238298
76322462345461760*a^33*b^39*c^66*d^16 - 263018360322301930835307134976*a^34
*b^38*c^65*d^17 + 484117148425341461690547437568*a^35*b^37*c^64*d^18 - 8010
88032507623116562893897728*a^36*b^36*c^63*d^19 + 12101917535606584214513736
74496*a^37*b^35*c^62*d^20 - 1713662150039311965148455895040*a^38*b^34*c^61*
d^21 + 2368456612874860634985065349120*a^39*b^33*c^60*d^22 - 33424408828179
01253619697582080*a^40*b^32*c^59*d^23 + 4926019419281526710422764257280*a^4
```

```
1*b^31*c^58*d^24 - 7443043331925522227676535848960*a^42*b^30*c^57*d^25 + 11
0533849842458526002234523648 00*a^43*b^29*c^56*d^26 - 15529000135185248373347
985653760*a^44*b^28*c^55*d^27 + 20153801026888464482649904250880*a^45*b^27
*c^54*d^28 - 23870821024791437072619829985280*a^46*b^26*c^53*d^29 + 2566240
7141873741853910169026560*a^47*b^25*c^52*d^30 - 249833349649380856022263083
82720*a^48*b^24*c^51*d^31 + 22003368361455969032835868655616*a^49*b^23*c^50
*d^32 - 17519758513327663391847122731008*a^50*b^22*c^49*d^33 + 126018962854
89986596049610866688*a^51*b^21*c^48*d^34 - 81796843904149151204515365519 36*
a^52*b^20*c^47*d^35 + 47835830811163604549605156454 40*a^53*b^19*c^46*d^36 -
 2515171747726250254399514345472*a^54*b^18*c^45*d^37 + 11857103615118160821
46770026496*a^55*b^17*c^44*d^38 - 499406604618358594580969947136*a^56*b^16*
c^43*d^39 + 187097254447826761775602204672*a^57*b^15*c^42*d^40 - 6200223393
2522145150727618560*a^58*b^14*c^41*d^41 + 18049115872947548566748921856*a^5
9*b^13*c^40*d^42 - 4575187392741408034214903808*a^60*b^12*c^39*d^43 + 99864
2414508019303179091968*a^61*b^11*c^38*d^44 - 184986735996381058748645376*a^
62*b^10*c^37*d^45 + 28520139033328990436720640*a^63*b^9*c^36*d^46 - 3562072
173311951854632960*a^64*b^8*c^35*d^47 + 346377863868692037632000*a^65*b^7*c
^34*d^48 - 24611841230482125619200*a^66*b^6*c^33*d^49 + 1137123721538961408
000*a^67*b^5*c^32*d^50 - 25649407252758528000*a^68*b^4*c^31*d^51)*1i + x^(1
/2)*(485170116043368038400 0*a^21*b^45*c^62*d^11 - 134253118530519040000*a^2
0*b^46*c^63*d^10 - 8312815154680918179840 0*a^22*b^44*c^61*d^12 + 8959108979
14030472560640*a^23*b^43*c^60*d^13 - 679712998965495764248166 4*a^24*b^42*c^
59*d^14 + 38483630548489971632701440*a^25*b^41*c^58*d^15 - 167961815050671 3
42785396736*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^39*c^5
6*d^17 - 1529836010901462206864424960*a^28*b^38*c^55*d^18 + 307515311086535
8700094160896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*b^36*
c^53*d^20 + 589590639381102819104784384*a^31*b^35*c^52*d^21 + 1457667133433
8745969651220480*a^32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b
^33*c^50*d^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24 - 189331
360528461979941957795840*a^35*b^31*c^48*d^25 + 26738352737374819243394492 00
64*a^36*b^30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*c^46*d^27
 + 328626898447261055168230195200*a^38*b^28*c^45*d^28 - 292434560796558751 9
19058714624*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*
c^43*d^30 - 152776304398053739659930894336*a^41*b^25*c^42*d^31 + 8990112462
2673343064718704640*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^4
3*b^23*c^40*d^33 + 20486606263737610091045584896*a^44*b^22*c^39*d^34 - 7870
914323775054351244984320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624
*a^46*b^20*c^37*d^36 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 17
0590060365885174888529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*
a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 61303
6163719885750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14
*c^31*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840
000*a^54*b^12*c^29*d^44))*(-(8398080000*a^33*d^33 - (70527747686400000000*a
^66*d^66 + 27487790694400000000*b^66*c^66 + 464565652967915520000 00*a^2*b^6
4*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335
```

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360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5
 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820634974459346812
9280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 -
176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721
941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55
*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 2495523500408
2618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*
a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15
 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5383768893393273
22092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a
^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19
 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255
0801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345870323761283072
0*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*
d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 3902249846
0407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667528366853
40344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^3
9*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 17
7036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243137271360678168
280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435369272489279488
00*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^3
4*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 1136547
400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133
755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840
*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29
*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 2739390
827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359
868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000
*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24
*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 8484467
50580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319705986478663727463
9363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^
46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^4
7 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 3238737258195
2477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123908217893623053
68064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^
15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 47637131
8567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176
652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^1
1*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 134721865560
4091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58
*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 429545687
9982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^
5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 268379484005597
1840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 -
```

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 1627277209108480000000*a*b^65*c^65*d - 438839318937600000000*a^65*b*c*d^6
5)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 1813499
6090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 46643626691
7888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742
080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787
776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011
054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 383038
7378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 15
0199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^
16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*
c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^2
0*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569
600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 13167616
3264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 197926
51594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 138235
1733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896
704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a
*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 687
19476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^
40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b
^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14
*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*
a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 19274988590727
16800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 886649
4751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^1
2 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^
18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807
040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 323968
07746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^
19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^
11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 192749885907271680
0*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088
726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453
347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848
604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*1i
 + 927185599851397120000*a^20*b^44*c^58*d^12 - 25388837992853929984000*a^21
*b^43*c^57*d^13 + 317358378012506691993600*a^22*b^42*c^56*d^14 - 2373809829
046075554529280*a^23*b^41*c^55*d^15 + 11545284809815729048125440*a^24*b^40*
c^54*d^16 - 35586486107261996158156800*a^25*b^39*c^53*d^17 + 47503987551983
633390632960*a^26*b^38*c^52*d^18 + 160896568335160851531038720*a^27*b^37*c^
51*d^19 - 1289503277949063475180339200*a^28*b^36*c^50*d^20 + 48476955197882
47586575482880*a^29*b^35*c^49*d^21 - 12969376809237608808212070400*a^30*b^3
4*c^48*d^22 + 27198543957428161531839774720*a^31*b^33*c^47*d^23 - 465585326
23156834692403036160*a^32*b^32*c^46*d^24 + 66465033664063788557407354880*a^
33*b^31*c^45*d^25 - 80137164645540595666444615680*a^34*b^30*c^44*d^26 + 822
```

```
41221222993610845821337600*a^35*b^29*c^43*d^27 - 72165140031754207660154552
320*a^36*b^28*c^42*d^28 + 54258643078018614781815029760*a^37*b^27*c^41*d^29
 - 34958604671456258343085015040*a^38*b^26*c^40*d^30 + 19266119383513605759
523880960*a^39*b^25*c^39*d^31 - 9047713278884926997712076800*a^40*b^24*c^38
*d^32 + 3598803321131446378839408640*a^41*b^23*c^37*d^33 - 1201767391129510
053066833920*a^42*b^22*c^36*d^34 + 332745330268979132513648640*a^43*b^21*c^
35*d^35 - 75056967015910052829593600*a^44*b^20*c^34*d^36 + 1344751791353759
4156646400*a^45*b^19*c^33*d^37 - 1841937645534110023680000*a^46*b^18*c^32*d
^38 + 181270486395868151808000*a^47*b^17*c^31*d^39 - 1141943422169382912000
0*a^48*b^16*c^30*d^40 + 346112011869880320000*a^49*b^15*c^29*d^41))*(-(8398
080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 27487790694400000000*b
^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600
0000*a^3*b^63*c^63*d^3 + 113031004798163353600000000*a^4*b^62*c^62*d^4 - 115
488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^
6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380
507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^
9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720
415842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893
952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^5
3*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011
0694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289
600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*
d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132
5168526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623367881
5232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^4
5*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698
29017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081401535900391206823
00735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^
41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92
575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 1299473849307245203
88491615907348480*a^28*b^38*c^38*d^28 - 17703615665425001284104911182626816
0*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36
*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334
2886205619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232
590010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^
34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^
35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575
399654362397243935503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329
057621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^
39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^
40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205
355133034582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053
020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^4
4*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45
 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310
```

```
80816245477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358
132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*
c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457
82995414208640750154437099520*a^51*b^15*c^15*d^51 + 15789654660146705061178
09664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^
13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177
6042795476444797594501120000*a^55*b^11*c^11*d^55 + 694868361500361248170259
2512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*
d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157
513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*
d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008
000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13
4144124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^
65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 +
 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 10699
8213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 15609364
06056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 867078777
0777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 204848128
01130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1678
0795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 -
53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d
^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16
*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^
19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 63043211587399
6800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 2617739
03936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55
825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5
801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 26
1325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 392233
8816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*
b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023
255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a
^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a
^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 6227304006234931
2*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152
268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443324
7375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11
 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^1
9*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066419
20*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887616
9296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^1
8 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^
12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 443324737586724864
0*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 7228120721
52268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273
```

```
040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 24
71152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34
084860461056*a^39*b^2*c^15*d^30))^(1/4) - atan((((-(8398080000*a^33*d^33 -
(70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 46456565
296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*
d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 1154880780847298232975
36000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 6
398838206349744593468129280*a^7*b^59*c^59*d^7 + 363943805075927975134589091
84*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 7
42548127574667458190721941504*a^10*b^56*c^56*d^10 - 27204158429008668904965
69507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^
54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 6321444674
2584363799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353
792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*
d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 916753573116
017703850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238
144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46
*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 6963408443
496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445З
2998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42
*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 6226
2545797041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646070620848388
69289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a
^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^
29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 34711352517
9164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703
580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b
^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 -
 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 20440850601244
33072578392630325411840*a^36*b^30*c^30*d^36 - 24470425753996543623972439355
03155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b
^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 +
 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 21983230073643
959985824159760384000000*a^41*b^25*c^25*d^41 + 17387922053551330345825449126
39590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b
^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 -
523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 2966924446649007
43443383822718074880*a^46*b^20*c^20*d^46 - 15458625383108081624547756355878
9120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^
18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 1297875
6421512390821789362305368064*a^50*b^16*c^16*d^50 - 474578299541420864075015
4437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^
14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 1297
89809479068757330643176652800*a^54*b^12*c^12*d^54 - 31776042795476444797594
501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c
```

```
^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918
031974963609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256000000*a^59*b
^7*c^7*d^59 + 42954568799822401241088000000*a^60*b^6*c^6*d^60 - 455971792993
637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4
*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000
000*a^64*b^2*c^2*d^64 - 16272772091084800000000*a*b^65*c^65*d - 438839318937
6000000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 213364244480*a^2*
b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29
*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c
^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^
25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*
c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*
b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a
^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 3065750780571
64800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798
564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 -
 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12
*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^
10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^2
5*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^2
7*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*
b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^
2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476
736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^
44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 -
 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 -
 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^
6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^
37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*
b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 1551636581553537
0240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396
807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d
^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*
b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 238713320239005
69600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866
494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^
22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c
^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b
^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37
*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b
^2*c^15*d^30))^(3/4)*(x^(1/2)*(-(8398080000*a^33*d^33 - (70527747686400000
00*a^66*d^66 + 274877906944000000*b^66*c^66 + 4645656529679155200000*a^2
*b^64*c^64*d^2 - 85239594962869288960000*a^3*b^63*c^63*d^3 + 1130310047981
633536000000*a^4*b^62*c^62*d^4 - 11548807808472982329753600*a^5*b^61*c^61
*d^5 + 94660933391357814578872320*a^6*b^60*c^60*d^6 - 63988382063497445934
```

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68129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^
8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 74254812757466745819
0721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*
c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 249552350
04082618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505
984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*
d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 538376889339
327322092190511923200*a^17*b^49*c^49*d^17 + 9167535731160177038503215177400
32*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*
d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 39456820503
82550801466936451399680*a^21*b^45*c^45*d^21 + 69634084434967934587032376128
30720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c
^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 390224
98460407159853772918944169984*a^25*b^41*c^41*d^25 + 62262545797041866752836
685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27
*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28
- 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 24313727136067
8168280724887442554880*a^30*b^36*c^36*d^30 - 347113525179164243536927248927
948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34
*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 113
6547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 157868330446321461
6133755020010061824*a^35*b^31*c^31*d^35 + 204408506012443307257839263032541
1840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*
c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 273
9390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 255814575759273616
3359868236513411072*a^40*b^26*c^26*d^40 - 219832300736439599858241597603840
0000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*
c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 848
446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 5231970598647866372
74639363737649152*a^45*b^21*c^21*d^45 + 29669244466490074344338382271807488
0*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19
*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 323873725
81952477787555393435598848*a^49*b^17*c^17*d^49 + 12978756421512390821789362
305368064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^1
5*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 4763
71318567145258980606161715200*a^53*b^13*c^13*d^53 + 12978980947906875733064
3176652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11
*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 13472186
55604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*
a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 42954
56879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^6
1*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 26837948400
55971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^
64 - 1627277209108480000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c
*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 181
```

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34996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 4664362
66917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 411189230
1742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 1479391774
7787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 2252936
2011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 38
30387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14
+ 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^1
7*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b
^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736
*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 44081317078
0569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 1316
76163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19
792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 13
82351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 3775
7896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 1551892480
00*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 +
 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 219902325555
2*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^
12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*
a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726
016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859
072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 88
66494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33
*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^2
3*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 4130592987707
0807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32
396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^2
6*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^3
0*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 19274988590727
16800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 23129986
3088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 1383
8453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 34
0848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4
)*(28823037615171174400*a^23*b^51*c^81*d^4 - 1262449047544497438720*a^24*b^
50*c^80*d^5 + 26781213630512448405504*a^25*b^49*c^79*d^6 - 3668169646702282
54425088*a^26*b^48*c^78*d^7 + 3648418948406862648705024*a^27*b^47*c^77*d^8
- 28097394584779147947540480*a^28*b^46*c^76*d^9 + 1744483893093379483516272
64*a^29*b^45*c^75*d^10 - 897668976119897481466085376*a^30*b^44*c^74*d^11 +
3905920242884630010868531200*a^31*b^43*c^73*d^12 - 1459089642507576537992 97
35168*a^32*b^42*c^72*d^13 + 47355821068790227801756139520*a^33*b^41*c^71*d^
14 - 134845524585103937061538234368*a^34*b^40*c^70*d^15 + 33972709673008671
4108763176960*a^35*b^39*c^69*d^16 - 763246944716696111818343448576*a^36*b^3
8*c^68*d^17 + 1541564027180813686607638953984*a^37*b^37*c^67*d^18 - 2825288
027628089174763608473600*a^38*b^36*c^66*d^19 + 4753041476272000853590444867
584*a^39*b^35*c^65*d^20 - 7446263493185815677622957375488*a^40*b^34*c^64*d^
```

```
21 + 11045974611794807027964401680384*a^41*b^33*c^63*d^22 - 157666811007415
71532295786987520*a^42*b^32*c^62*d^23 + 21882616570434895907374088847360*a^
43*b^31*c^61*d^24 - 29555415901357165913293077872640*a^44*b^30*c^60*d^25 +
38514249364633213650767204843520*a^45*b^29*c^59*d^26 - 47767982724772003266
224581509120*a^46*b^28*c^58*d^27 + 55618948537155045120476750807040*a^47*b^
27*c^57*d^28 - 60127300413664475479641214156800*a^48*b^26*c^56*d^29 + 59877
038998440260638050153922560*a^49*b^25*c^55*d^30 - 5463705159573704701467402
0696064*a^50*b^24*c^54*d^31 + 45519623223064909005599526617088*a^51*b^23*c^
53*d^32 - 34535555816550055085254958383104*a^52*b^22*c^52*d^33 + 2380972950
4484309698980012359680*a^53*b^21*c^51*d^34 - 148853192545353529902415415869
44*a^54*b^20*c^50*d^35 + 8419648339202954390072444583936*a^55*b^19*c^49*d^3
6 - 4297514831765712413611503124480*a^56*b^18*c^48*d^37 + 19731237375541301
96459440570368*a^57*b^17*c^47*d^38 - 811770857054497673061303582720*a^58*b^
16*c^46*d^39 + 297858380372439371596188090368*a^59*b^15*c^45*d^40 - 9691005
0535770593129744302080*a^60*b^14*c^44*d^41 + 27758579881177587823480406016*
a^61*b^13*c^43*d^42 - 6937474504476672102869499904*a^62*b^12*c^42*d^43 + 14
95682482860276471300096000*a^63*b^11*c^41*d^44 - 27410011895830086649538150
4*a^64*b^10*c^40*d^45 + 41867778463425277028466688*a^65*b^9*c^39*d^46 - 518
7161130930763594727424*a^66*b^8*c^38*d^47 + 500879902205011065569280*a^67*b
^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c^36*d^49 + 162534910551801
2006400*a^69*b^5*c^35*d^50 - 36479156981701017600*a^70*b^4*c^34*d^51) - 180
14398509481984000*a^21*b^51*c^78*d^4 + 778222015609621708800*a^22*b^50*c^77
*d^5 - 16199988291606958571520*a^23*b^49*c^76*d^6 + 21662933902960811940249
6*a^24*b^48*c^75*d^7 - 2092899704349501998235648*a^25*b^47*c^74*d^8 + 15576
808854093856430358528*a^26*b^46*c^73*d^9 - 92989305923335928955273216*a^27*
b^45*c^72*d^10 + 457716570390505153458339840*a^28*b^44*c^71*d^11 - 18950773
72829589675098243072*a^29*b^43*c^70*d^12 + 6699157107174094796222365696*a^3
0*b^42*c^69*d^13 - 20454608396817467081213607936*a^31*b^41*c^68*d^14 + 5443
9663857512808688618831872*a^32*b^40*c^67*d^15 - 127253623829876322462345461
760*a^33*b^39*c^66*d^16 + 263018360322301930835307134976*a^34*b^38*c^65*d^1
7 - 484117148425341461690547437568*a^35*b^37*c^64*d^18 + 801088032507623116
562893897728*a^36*b^36*c^63*d^19 - 1210191753560658421451373674496*a^37*b^3
5*c^62*d^20 + 1713662150039311965148455895040*a^38*b^34*c^61*d^21 - 2368456
612874860634985065349120*a^39*b^33*c^60*d^22 + 3342440882817901253619697582
080*a^40*b^32*c^59*d^23 - 4926019419281526710422764257280*a^41*b^31*c^58*d^
24 + 7443043331925522227676535848960*a^42*b^30*c^57*d^25 - 1105338498424585
2600223452364800*a^43*b^29*c^56*d^26 + 15529000135185248373347985653760*a^4
4*b^28*c^55*d^27 - 20153801026888464482649904250880*a^45*b^27*c^54*d^28 + 2
3870821024791437072619829985280*a^46*b^26*c^53*d^29 - 256624071418737418539
10169026560*a^47*b^25*c^52*d^30 + 24983334964938085602226308382720*a^48*b^2
4*c^51*d^31 - 22003368361455969032835868655616*a^49*b^23*c^50*d^32 + 175197
58513327663391847122731008*a^50*b^22*c^49*d^33 - 12601896285489986596049610
866688*a^51*b^21*c^48*d^34 + 8179684390414915120451536551936*a^52*b^20*c^47
*d^35 - 4783583081116360454960515645440*a^53*b^19*c^46*d^36 + 2515171747726
250254399514345472*a^54*b^18*c^45*d^37 - 1185710361511816082146770026496*a^
```

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55*b^17*c^44*d^38 + 499406604618358594580969947136*a^56*b^16*c^43*d^39 - 18
70972544478267617756022046 72*a^57*b^15*c^42*d^40 + 6200223393252214515072 76
18560*a^58*b^14*c^41*d^41 - 1804911587294754856674892 1856*a^59*b^13*c^40*d^
42 + 457518739274140803421490 3808*a^60*b^12*c^39*d^43 - 99864241450801930 31
79091968*a^61*b^11*c^38*d^44 + 18498673599638105874864 5376*a^62*b^10*c^37*d
^45 - 28520139033328990436720640*a^63*b^9*c^36*d^46 + 356207217331195185463
2960*a^64*b^8*c^35*d^47 - 346377863868692037632000*a^65*b^7*c^34*d^48 + 246
1184123048212561920 0*a^66*b^6*c^33*d^49 - 1137123721538961408000*a^67*b^5*c
^32*d^50 + 25649407252758528000*a^68*b^4*c^31*d^51) - x^(1/2)*(485170116043
3680384000*a^21*b^45*c^62*d^11 - 134253118530519040000*a^20*b^46*c^63*d^10
- 8312815154680918179840 0*a^22*b^44*c^61*d^12 + 895910897914030472560640*a^
23*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b^42*c^59*d^14 + 3848363
0548489971632701440*a^25*b^41*c^58*d^15 - 167961815050671342785396736*a^26*
b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^39*c^56*d^17 - 15298360
10901462206864424960*a^28*b^38*c^55*d^18 + 3075153110865358700094160896*a^2
9*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*b^36*c^53*d^20 + 58959
06393811028191047843 84*a^31*b^35*c^52*d^21 + 14576671334338745969651220480*
a^32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b^33*c^50*d^23 + 1
10550157926715904989065117696*a^34*b^32*c^49*d^24 - 189331360528461979941 95
7795840*a^35*b^31*c^48*d^25 + 267383527373748192433944920064*a^36*b^30*c^47
*d^26 - 319821143985825066443750768640*a^37*b^29*c^46*d^27 + 32862689844726
1055168230195200*a^38*b^28*c^45*d^28 - 292434560796558751919058714624*a^39*
b^27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*c^43*d^30 - 15277
6304398053739659930894336*a^41*b^25*c^42*d^31 + 899011246226733430647187046
40*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^43*b^23*c^40*d^33
+ 20486606263737610091045584896*a^44*b^22*c^39*d^34 - 787091432377505435124
4984320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624*a^46*b^20*c^37*d
^36 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 170590060365885174 8
88529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*a^49*b^17*c^34*d^
39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 613036163719885750272 0
00*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42 - 2988
725792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b^12*c^2
9*d^44))*(-(8398080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 274877
906944000000 00*b^66*c^66 + 464565652967915520000 00*a^2*b^64*c^64*d^2 - 8523
959496286928896 00000*a^3*b^63*c^63*d^3 + 113031004798163353600000 00*a^4*b^6
2*c^62*d^4 - 1154880780847298232975360 00*a^5*b^61*c^61*d^5 + 94660933391357
8145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^5
9*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078 66
7757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*
c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883
8643010718192893952000*a^12*b^54*c^54*d^12 - 249552350040826187070412286853
12*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^
14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237
7964853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200
*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^1
```

```
8 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793
32590916233678815232*a^20*b^46*c^46*d^20 - 39456820503825508014669364513996
80*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*
d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814
0153590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189
44169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^4
0*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129
947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 1770361566542500128
41049111826268160*a^29*b^37*c^37*d^29 + 24313727136067816828072488744255488
0*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35
*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807
3329926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056
4698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a
^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d
^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093
9745327887207329057621409792*a^38*b^28*c^28*d^38 - 273939082748055449346653
4979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a
^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d
^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380
5867374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361
710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^4
5*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46
 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217
1953043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355
98848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c
^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965
466014670506117809664163840*a^52*b^14*c^14*d^52 - 4763713185671452589806061
61715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*
c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836
15003612481702592512000*a^56*b^10*c^10*d^56 - 13472186556040911549104128000
00*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3
3942156347965157513625600000*a^59*b^7*c^7*d^59 + 42954568799822401241088000
00*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950
4294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b
^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480
000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 524288
0000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*
c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^2
8*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26
*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24
*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*
c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b
^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*
a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173
068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006
```

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5381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20
+ 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^1
1*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b
^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*
b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b
^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c
^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26
1273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^1
3*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 +
34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 24
71152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 +
62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^
7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c
^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^2
0*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 2387133202390
0569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38
876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^2
9*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^
27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155163658155
35370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 44
33247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d
^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c
^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^
5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^
3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*1i + ((-(8398080000
*a^33*d^33 - (705277476864000000000*a^66*d^66 + 274877906944000000000*b^66*c^
66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a
^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078
084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60
*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592
797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57
*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842
900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000
*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13
 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069462
0505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^
16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 +
 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132516852
6452382335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a
^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21
 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172
32408306844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735
488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^4
1*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92575964
607062084838869289496739840*a^27*b^39*c^39*d^27 + 12994738493072452038849l6
```

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15907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29
*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30
- 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334288620
5619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010
056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^3
2*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2
044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654
362397243935503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621
409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^2
7*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2
198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133
034582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810
084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22
*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296
692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808162
45477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224
*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d
^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829954
14208640750154437099520*a^51*b^15*c^15*d^51 + 15789654660146705061178096641
63840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^1
3*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279
5476444797594501120000*a^55*b^11*c^11*d^55 + 694868361500361248170259251200
0*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 +
 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625
600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 -
 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000
*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13414412
4384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d -
 438839318937600000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 21336
42444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 10699821337
8048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 15609364060569
60*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 867078777077760
0*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 204848128011304
96*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1678079510
1757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 530581
43899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 -
306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*
d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^1
4*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a
^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 2617739039369
62560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496
115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173
668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 26132579
8707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 392233881600
0*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^
```

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32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552
*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^
30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^
28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15
*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800
*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443324737586
7248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 155
16365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32
*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^2
4*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887616929606
6641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23
871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^2
5*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31
*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 7228120721522688
00*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062
349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 24711523
83426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860
461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-(8398080000*a^33*d^33 - (70527
747686400000000*a^66*d^66 + 274877906944000000*b^66*c^66 + 46456565296791
552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 +
11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*
a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 6398838
206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8
*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 7425481
27574667458190721941504*a^10*b^56*c^56*d^10 - 27204158429008668904965695078
40*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^1
2 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 6321444674258436
3799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^
15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 -
 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703
850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^
19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20
- 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 6963408443496793
458703237612830720*a^22*b^44*c^44*d^22 - 1269586982901723240830684453299814
4*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*
d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 6226254579
7041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646070620848388692894
96739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^
38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 2
43137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 34711352517916424
3536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999
680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^
33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 15786
83304463214616133755020010061824*a^35*b^31*c^31*d^35 + 20440850601244330725
78392630325411840*a^36*b^30*c^30*d^36 - 24470425753996543623972439355031552
```

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00*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^
28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 25581
45757592736163359868236513411072*a^40*b^26*c^26*d^40 - 219832300736439599985
824159760384000000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590
400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^
23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 523197
059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443
383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a
^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^4
8 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 1297875642151
2390821789362305368064*a^50*b^16*c^16*d^50 - 474578299541420864075015443709
9520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^1
4*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 1297898094
79068757330643176652800*a^54*b^12*c^12*d^54 - 317760427954764447975945011200
000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^
56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974
963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7
*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105
664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62
- 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^
64*b^2*c^2*d^64 - 162727720910848000000*a*b^65*c^65*d - 438839318937600000
0000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c
^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*
d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^
6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8
 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d
^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c
^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^
19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*
a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847
943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 76615
9267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21
- 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^1
0*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*
c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*
c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^
4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31
 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^
9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d -
 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 34084
8604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838
453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 23
1299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8
 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c
^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a
```

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^21*b^20*c^33*d^12 - 238713320239005696OO*a^22*b^19*c^32*d^13 + 32396807746
722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 +
41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c
^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*
a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751
734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1
927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^
24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^1
9*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c
^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^1
5*d^30))^(1/4)*(288230376151711744OO*a^23*b^51*c^81*d^4 - 12624490475444974
38720*a^24*b^50*c^80*d^5 + 26781213630512448405504*a^25*b^49*c^79*d^6 - 366
816964670228254425088*a^26*b^48*c^78*d^7 + 3648418948406862648705024*a^27*b
^47*c^77*d^8 - 28097394584779147947540480*a^28*b^46*c^76*d^9 + 174448389309
337948351627264*a^29*b^45*c^75*d^10 - 897668976119897481466085376*a^30*b^44
*c^74*d^11 + 3905920242884630010868531200*a^31*b^43*c^73*d^12 - 14590896425
075765379929735168*a^32*b^42*c^72*d^13 + 47355821068790227801756139520*a^33
*b^41*c^71*d^14 - 134845524585103937061538234368*a^34*b^40*c^70*d^15 + 3397
27096730086714108763176960*a^35*b^39*c^69*d^16 - 763246944716696111818343444
8576*a^36*b^38*c^68*d^17 + 1541564027180813686607638953984*a^37*b^37*c^67*d
^18 - 2825288027628089174763608473600*a^38*b^36*c^66*d^19 + 475304147627200
0853590444867584*a^39*b^35*c^65*d^20 - 7446263493185815677622957375488*a^40
*b^34*c^64*d^21 + 11045974611794807027964401680384*a^41*b^33*c^63*d^22 - 15
766681100741571532295786987520*a^42*b^32*c^62*d^23 + 2188261657043489590737
4088847360*a^43*b^31*c^61*d^24 - 29555415901357165913293077872640*a^44*b^30
*c^60*d^25 + 38514249364633213650767204843520*a^45*b^29*c^59*d^26 - 4776798
2724772003266224581509120*a^46*b^28*c^58*d^27 + 556189485371550451204767508
07040*a^47*b^27*c^57*d^28 - 60127300413664475479641214156800*a^48*b^26*c^56
*d^29 + 59877038998440260638050153922560*a^49*b^25*c^55*d^30 - 546370515957
37047014674020696064*a^50*b^24*c^54*d^31 + 45519623223064909005599526617088
*a^51*b^23*c^53*d^32 - 34535555816550055085254958383104*a^52*b^22*c^52*d^33
 + 23809729504484309698980012359680*a^53*b^21*c^51*d^34 - 14885319254535352
990241541586944*a^54*b^20*c^50*d^35 + 8419648339202954390072444583936*a^55*
b^19*c^49*d^36 - 4297514831765712413611503124480*a^56*b^18*c^48*d^37 + 1973
123737554130196459440570368*a^57*b^17*c^47*d^38 - 8117708570544976730613035
82720*a^58*b^16*c^46*d^39 + 297858380372439371596188090368*a^59*b^15*c^45*d
^40 - 96910050535770593129744302080*a^60*b^14*c^44*d^41 + 27758579881177587
823480406016*a^61*b^13*c^43*d^42 - 6937474504476672102869499904*a^62*b^12*c
^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^41*d^44 - 2741001189583
00866495381504*a^64*b^10*c^40*d^45 + 41867778463425277028466688*a^65*b^9*c^
39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^47 + 500879902205011065
569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c^36*d^49 + 16
25349105518012006400*a^69*b^5*c^35*d^50 - 36479156981701017600*a^70*b^4*c^3
4*d^51) + 18014398509481984000*a^21*b^51*c^78*d^4 - 778222015609621708800*a
^22*b^50*c^77*d^5 + 16199988291606958571520*a^23*b^49*c^76*d^6 - 2166293390
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29608119402496*a^24*b^48*c^75*d^7 + 20928997043495019982356480*a^25*b^47*c^7
4*d^8 - 15576808854093856430358528*a^26*b^46*c^73*d^9 + 9298930592333592895
5273216*a^27*b^45*c^72*d^10 - 457716570390505153458339840*a^28*b^44*c^71*d^
11 + 1895077372829589675098243072*a^29*b^43*c^70*d^12 - 6699157107174094796
222365696*a^30*b^42*c^69*d^13 + 20454608396817467081213607936*a^31*b^41*c^6
8*d^14 - 54439663857512808688618831872*a^32*b^40*c^67*d^15 + 12725362382987
6322462345461760*a^33*b^39*c^66*d^16 - 263018360322301930835307134976*a^34*
b^38*c^65*d^17 + 484117148425341461690547437568*a^35*b^37*c^64*d^18 - 80108
8032507623116562893897728*a^36*b^36*c^63*d^19 + 121019175356065842145137367
4496*a^37*b^35*c^62*d^20 - 1713662150039311965148455895040*a^38*b^34*c^61*d
^21 + 2368456612874860634985065349120*a^39*b^33*c^60*d^22 - 334244088281790
1253619697582080*a^40*b^32*c^59*d^23 + 4926019419281526710422764257280*a^41
*b^31*c^58*d^24 - 7443043331925522227676535848960*a^42*b^30*c^57*d^25 + 110
53384984245852600223452364800*a^43*b^29*c^56*d^26 - 15529000135185248373347
985653760*a^44*b^28*c^55*d^27 + 20153801026888464482649904250880*a^45*b^27*
c^54*d^28 - 23870821024791437072619829985280*a^46*b^26*c^53*d^29 + 25662407
141873741853910169026560*a^47*b^25*c^52*d^30 - 2498333496493808560222630838
2720*a^48*b^24*c^51*d^31 + 22003368361455969032835868655616*a^49*b^23*c^50*
d^32 - 17519758513327663391847122731008*a^50*b^22*c^49*d^33 + 1260189628548
9986596049610866688*a^51*b^21*c^48*d^34 - 8179684390414915120451536551936*a
^52*b^20*c^47*d^35 + 4783583081116360454960515645440*a^53*b^19*c^46*d^36 - 
2515171747726250254399514345472*a^54*b^18*c^45*d^37 + 118571036151181608214
6770026496*a^55*b^17*c^44*d^38 - 499406604618358594580969947136*a^56*b^16*c
^43*d^39 + 187097254447826761775602204672*a^57*b^15*c^42*d^40 - 62002233932
522145150727618560*a^58*b^14*c^41*d^41 + 18049115872947548566748921856*a^59
*b^13*c^40*d^42 - 4575187392741408034214903808*a^60*b^12*c^39*d^43 + 998642
414508019303179091968*a^61*b^11*c^38*d^44 - 184986735996381058748645376*a^6
2*b^10*c^37*d^45 + 28520139033328990436720640*a^63*b^9*c^36*d^46 - 35620721
73311951854632960*a^64*b^8*c^35*d^47 + 346377863868692037632000*a^65*b^7*c^
34*d^48 - 24611841230482125619200*a^66*b^6*c^33*d^49 + 11371237215389614080
00*a^67*b^5*c^32*d^50 - 25649407252758528000*a^68*b^4*c^31*d^51) - x^(1/2)*
(4851701160433680384000*a^21*b^45*c^62*d^11 - 134253118530519040000*a^20*b^
46*c^63*d^10 - 83128151546809181798400*a^22*b^44*c^61*d^12 + 89591089791403
0472560640*a^23*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b^42*c^59*d
^14 + 38483630548489971632701440*a^25*b^41*c^58*d^15 - 16796181505067134278
5396736*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^39*c^56*d^
17 - 1529836010901462206864424960*a^28*b^38*c^55*d^18 + 3075153110865358700
094160896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*b^36*c^53
*d^20 + 589590639381102819104784384*a^31*b^35*c^52*d^21 + 14576671334338745
969651220480*a^32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b^33*
c^50*d^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24 - 1893313605
28461979941957795840*a^35*b^31*c^48*d^25 + 267383527373748192433944920064*a
^36*b^30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*c^46*d^27 + 3
28626898447261055168230195200*a^38*b^28*c^45*d^28 - 29243456079655875191905
8714624*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*c^43
```

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*d^30 - 15277630439805373965993089433б*a^41*b^25*c^42*d^31 + 89901124622673
343064718704640*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^43*b^
23*c^40*d^33 + 2048660626373761009104558489б*a^44*b^22*c^39*d^34 - 78709143
23775054351244984320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624*a^4
6*b^20*c^37*d^36 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 170590
060365885174888529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*a^49
*b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 613036163
719885750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14*c^3
1*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840000*
a^54*b^12*c^29*d^44))*(-(8398080000*a^33*d^33 - (70527747686400000000*a^66*
d^66 + 27487790694400000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^
64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 113031004798163353600
00000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 9
46609333913578145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280
*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 1768
23915553078667757483982848*a^9*b^57*c^57*d^9 + 7425481275746674581907219415
04*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^1
1 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618
707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14
*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 2
91432713032377964853953403289600*a^16*b^50*c^50*d^16 - 53837688933932732209
2190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*
b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 2
370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801
466936451399680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^
22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23
 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460407
159853772918944169984*a^25*b^41*c^41*d^25 + 6226254579704186675283668534034
4320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^
39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036
156654250012841049111826268160*a^29*b^37*c^37*d^29 + 2431372713606781682807
24887442554880*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a
^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^
32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 11365474000
98503091050564698912063488*a^34*b^32*c^32*d^34 - 15786833044632146161337550
20010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^3
6*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^3
7 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274
80554493466534979194322944*a^39*b^27*c^27*d^39 + 25581457575927361633598682
36513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^4
1*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^4
2 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 84844675058
0244547991361710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363
737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b
^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 +
```

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73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952477
787555393435598848*a^49*b^17*c^17*d^49 + 1297875642151239082178936230536806
4*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d
^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567
145258980606161715200*a^53*b^13*c^13*d^53 + 1297898094790687573306431766528
00*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^
55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091
154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8
*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982
240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^
5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840
000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162
7277209108480000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(
1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090
880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888
*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*
a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*
a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 225293620110540
80*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378
688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199
661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 +
 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15
*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^
13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*
a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264
708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 1979265159
4874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733
145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 377578967040
00*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^3
2*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 6871947
6736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b
*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*
c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^2
7*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16
*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 192749885907271680
0*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751
734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 -
23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c
^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*
a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239680774
6722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 +
 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c
^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^
32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 2312998630887260
16*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 138384533471
```

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88736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 3408486046
10560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*1i)/((
(-(8398080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 274877906944000
00000*b^66*c^66 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 852395949628
692889600000*a^3*b^63*c^63*d^3 + 113031004798163353600000000*a^4*b^62*c^62*d^
4 - 1154880780847298232975360000*a^5*b^61*c^61*d^5 + 9466093339135781457887
23200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 3
639438050759279751345890918 4*a^8*b^58*c^58*d^8 - 176823915553078667757483982
848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10
 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864301071
8192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b
^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431
33780110694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953
403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^4
9*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 14804
72521325168526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623
3678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b
^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12
695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814015359003
9120682300735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*
a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^
26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947384930
724520388491615907348480*a^28*b^38*c^38*d^28 - 1770361566542500128410491118
26268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^
36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 5
15833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332992628
0441232590010056704*a^33*b^33*c^33*d^33 + 11365474000985030910505646989120 6
3488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*
c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244
7042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788
7207329057621409792*a^38*b^28*c^28*d^38 - 273939082748055449346653497919432
2944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*
c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 173
8792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374 68
979005302081008 4352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053
184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^
21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586
253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195304306
7855358132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^4
9*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50
- 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670
506117809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*
a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54
 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612 4
81702592512000*a^56*b^10*c^10*d^56 - 13472186556040911549104128 00000*a^57*b
```

```
^9*c^9*d^57 + 22946914691803197496360960000*a^58*b^8*c^8*d^58 - 3394215634
796515751362560000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b
^6*c^6*d^60 - 4559717929936371056640000000*a^61*b^5*c^5*d^61 + 3950429491527
863500800000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^
63 + 1341441243847065600000000*a^64*b^2*c^2*d^64 - 162727720910848000000*a*
b^65*c^65*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33
*c^33 + 213364244480*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3
+ 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1
560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 86
70787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20
484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11
 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*
d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18
*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^
17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006538135353
7536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 6304321
15873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 +
261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^
24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d
^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^
28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 +
 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26127360000
0*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 -
2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 340848604
61056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 24711523834
26560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 622730400
62349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 72281
2072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 +
 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^
34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a
^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296
066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 -
38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c
^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*
a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375
867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722
812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25
+ 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^
27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^
29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-(8398080000*a^33*
d^33 - (70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 4
645656529679155200000*a^2*b^64*c^64*d^2 - 85239594962869288960000*a^3*b^6
3*c^63*d^3 + 1130310047981633536000000*a^4*b^62*c^62*d^4 - 115488078084729
82329753600*a^5*b^61*c^61*d^5 + 94660933391357814578872320*a^6*b^60*c^60*
d^6 - 639883820634974459346812928*a^7*b^59*c^59*d^7 + 363943805075927975130
```

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458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*
d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866
890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*
b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632
14446742584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069462050587
2680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^5
0*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 91675
3573116017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132516852645238
2335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^
46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696
3408443496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083
06844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^
24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25
 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062
084838869289496739840*a^27*b^39*c^39*d^27 + 1299473849307245203884916159073
48480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*
c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471
13525179164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334288620561992
5039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704
*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32
*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085
060124433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397
243935503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792
*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27
*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323
007364395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582
544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352
*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*
d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444
664900743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808162454775
63558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*
b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 +
12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829954142086
40750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*
a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53
 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279547644
4797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56
*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946
9146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000
*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 45597
1792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*
b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13414412438470
6560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 43883
93189376000000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 21336424448
00*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a
```

```
^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6
*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*
b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^1
0*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1678079510175744
0*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 530581438992
38400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575
078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 -
 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14
*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^
12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*
a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836
928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208
640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 26132579870720
0*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31
*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(6
8719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*
b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^4
3*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^4
1*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*
c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*
b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443324737586724864
0*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 155163658
15535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13
+ 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17
*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887616929606664192
0*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332
023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20
 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*
c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^3
3*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312
*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 24711523834265
60*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056
*a^39*b^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d^4 - 1262
449047544497438720*a^24*b^50*c^80*d^5 + 26781213630512448405504*a^25*b^49*c
^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 3648418948406862648
705024*a^27*b^47*c^77*d^8 - 28097394584779147947540480*a^28*b^46*c^76*d^9 +
 174448389309337948351627264*a^29*b^45*c^75*d^10 - 897668976119897481466085
376*a^30*b^44*c^74*d^11 + 3905920242884630010868531200*a^31*b^43*c^73*d^12
- 14590896425075765379929735168*a^32*b^42*c^72*d^13 + 473558210687902278017
56139520*a^33*b^41*c^71*d^14 - 134845524585103937061538234368*a^34*b^40*c^7
0*d^15 + 339727096730086714108763176960*a^35*b^39*c^69*d^16 - 7632469447166
96111818343448576*a^36*b^38*c^68*d^17 + 1541564027180813686607638953984*a^3
7*b^37*c^67*d^18 - 2825288027628089174763608473600*a^38*b^36*c^66*d^19 + 47
53041476272000853590444867584*a^39*b^35*c^65*d^20 - 744626349318581567762295
7375488*a^40*b^34*c^64*d^21 + 11045974611794807027964401680384*a^41*b^33*c
```

```
^63*d^22 - 15766681100741571532295786987520*a^42*b^32*c^62*d^23 + 218826165
70434895907374088847360*a^43*b^31*c^61*d^24 - 29555415901357165913293077872
640*a^44*b^30*c^60*d^25 + 38514249364633213650767204843520*a^45*b^29*c^59*d
^26 - 47767982724772003266224581509120*a^46*b^28*c^58*d^27 + 55618948537155
045120476750807040*a^47*b^27*c^57*d^28 - 60127300413664475479641214156800*a
^48*b^26*c^56*d^29 + 59877038998440260638050153922560*a^49*b^25*c^55*d^30 -
 54637051595737047014674020696064*a^50*b^24*c^54*d^31 + 4551962322306490900
5599526617088*a^51*b^23*c^53*d^32 - 34535555816550055085254958383104*a^52*b
^22*c^52*d^33 + 23809729504484309698980012359680*a^53*b^21*c^51*d^34 - 1488
5319254535352990241541586944*a^54*b^20*c^50*d^35 + 841964833920295439007244
4583936*a^55*b^19*c^49*d^36 - 4297514831765712413611503124480*a^56*b^18*c^4
8*d^37 + 1973123737554130196459440570368*a^57*b^17*c^47*d^38 - 811770857054
497673061303582720*a^58*b^16*c^46*d^39 + 297858380372439371596188090368*a^5
9*b^15*c^45*d^40 - 96910050535770593129744302080*a^60*b^14*c^44*d^41 + 2775
8579881177587823480406016*a^61*b^13*c^43*d^42 - 693747450447667210286949990
4*a^62*b^12*c^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^41*d^44 -
274100118958300866495381504*a^64*b^10*c^40*d^45 + 4186777846342527702846668
8*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^47 + 50087
9902205011065569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c
^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 36479156981701017600
*a^70*b^4*c^34*d^51) + 18014398509481984000*a^21*b^51*c^78*d^4 - 7782220156
09621708800*a^22*b^50*c^77*d^5 + 16199988291606958571520*a^23*b^49*c^76*d^6
 - 216629339029608119402496*a^24*b^48*c^75*d^7 + 2092899704349501998235648*
a^25*b^47*c^74*d^8 - 15576808854093856430358528*a^26*b^46*c^73*d^9 + 929893
05923335928955273216*a^27*b^45*c^72*d^10 - 457716570390505153458339840*a^28
*b^44*c^71*d^11 + 1895077372829589675098243072*a^29*b^43*c^70*d^12 - 669915
7107174094796222365696*a^30*b^42*c^69*d^13 + 20454608396817467081213607936*
a^31*b^41*c^68*d^14 - 54439663857512808688618831872*a^32*b^40*c^67*d^15 + 1
27253623829876322462345461760*a^33*b^39*c^66*d^16 - 26301836032230193083530
7134976*a^34*b^38*c^65*d^17 + 484117148425341461690547437568*a^35*b^37*c^64
*d^18 - 801088032507623116562893897728*a^36*b^36*c^63*d^19 + 12101917535606
58421451373674496*a^37*b^35*c^62*d^20 - 1713662150039311965148455895040*a^3
8*b^34*c^61*d^21 + 2368456612874860634985065349120*a^39*b^33*c^60*d^22 - 33
42440882817901253619697582080*a^40*b^32*c^59*d^23 + 49260194192815267104227
64257280*a^41*b^31*c^58*d^24 - 7443043331925522227676535848960*a^42*b^30*c^
57*d^25 + 11053384984245852600223452364800*a^43*b^29*c^56*d^26 - 1552900013
5185248373347985653760*a^44*b^28*c^55*d^27 + 201538010268884644826499042508
80*a^45*b^27*c^54*d^28 - 23870821024791437072619829985280*a^46*b^26*c^53*d^
29 + 25662407141873741853910169026560*a^47*b^25*c^52*d^30 - 249833349649380
85602226308382720*a^48*b^24*c^51*d^31 + 22003368361455969032835868655616*a^
49*b^23*c^50*d^32 - 17519758513327663391847122731008*a^50*b^22*c^49*d^33 +
12601896285489986596049610866688*a^51*b^21*c^48*d^34 - 81796843904149151204
51536551936*a^52*b^20*c^47*d^35 + 4783583081116360454960515645440*a^53*b^19
*c^46*d^36 - 2515171747726250254399514345472*a^54*b^18*c^45*d^37 + 11857103
61511816082146770026496*a^55*b^17*c^44*d^38 - 49940660461835859458096994713
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6*a^56*b^16*c^43*d^39 + 187097254447826761775602204672*a^57*b^15*c^42*d^40
- 62002233932522145150727618560*a^58*b^14*c^41*d^41 + 180491158729475485667
48921856*a^59*b^13*c^40*d^42 - 4575187392741408034214903808*a^60*b^12*c^39*
d^43 + 998642414508019303179091968*a^61*b^11*c^38*d^44 - 184986735996381058
748645376*a^62*b^10*c^37*d^45 + 28520139033328990436720640*a^63*b^9*c^36*d^
46 - 3562072173311951854632960*a^64*b^8*c^35*d^47 + 346377863868692037632000
0*a^65*b^7*c^34*d^48 - 24611841230482125619200*a^66*b^6*c^33*d^49 + 1137123
721538961408000*a^67*b^5*c^32*d^50 - 25649407252758528000*a^68*b^4*c^31*d^5
1) - x^(1/2)*(4851701160433680384000*a^21*b^45*c^62*d^11 - 134253118530519O
40000*a^20*b^46*c^63*d^10 - 83128151546809181798400*a^22*b^44*c^61*d^12 + 8
95910897914030472560640*a^23*b^43*c^60*d^13 - 6797129989654957642481664*a^2
4*b^42*c^59*d^14 + 38483630548489971632701440*a^25*b^41*c^58*d^15 - 1679618
150506713427853967 36*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27
*b^39*c^56*d^17 - 1529836010901462206864424960*a^28*b^38*c^55*d^18 + 307515
3110865358700094160896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a
^30*b^36*c^53*d^20 + 589590639381102819104784384*a^31*b^35*c^52*d^21 + 1457
6671334338745969651220480*a^32*b^34*c^51*d^22 - 501491461567563565613501644
80*a^33*b^33*c^50*d^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24
 - 189331360528461979941957795840*a^35*b^31*c^48*d^25 + 267383527373748192 4
33944920064*a^36*b^30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*
c^46*d^27 + 328626898447261055168230195200*a^38*b^28*c^45*d^28 - 2924345607
96558751919058714624*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a
^40*b^26*c^43*d^30 - 152776304398053739659930894336*a^41*b^25*c^42*d^31 + 8
9901124622673343064718704640*a^42*b^24*c^41*d^32 - 460625089648204264791814
96320*a^43*b^23*c^40*d^33 + 20486606263737610091045584896*a^44*b^22*c^39*d^
34 - 7870914323775054351244984320*a^45*b^21*c^38*d^35 + 2594141724382360002
965274624*a^46*b^20*c^37*d^36 - 726451024651952784807034880*a^47*b^19*c^36*
d^37 + 170590060365885174888529920*a^48*b^18*c^35*d^38 - 329863435542048981
12307200*a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^4
0 - 613036163719885750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*
a^52*b^14*c^31*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 81438120
439971840000*a^54*b^12*c^29*d^44))*(-(8398080000*a^33*d^33 - (7052774768640
0000000*a^66*d^66 + 274877906944000000000*b^66*c^66 + 4645656529679155200000
0*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11303100
479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61
*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820634974
4593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^
58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667
458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*
b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 2495
5235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467425843637996415
18505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*
c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5383768
89339327322092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321 51
7740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*
```

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c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 394568
2050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345870323
7612830720*a^22*b^44*c^44*d^22 - 1269586982901723240830684453298144*a^23*b
^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 3
9022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667
52836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840
*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*
d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243137271
360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435369272
48927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32
*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33
+ 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463
214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630
325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*
b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38
- 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592
736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976
038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*
b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43
+ 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319705986478
6637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718
074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19
*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 3238
7372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123908217
89362305368064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^5
1*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 -
 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479068757
330643176652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55
*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 134
7218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691803197496360960
0000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 +
4295456879982240124108800000*a^60*b^6*c^6*d^60 - 45597179299363710566400000
0*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 268379
4840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c
^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 4388393189376000000000*a^6
5*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2
- 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 46
6436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111
892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793
917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22
529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12
 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*
d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^1
7*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a
^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 7661592670954
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12736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813
170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 -
 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25
 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27
 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 -
 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 15518
9248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c
^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023
255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 3408486046105
60*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 1383845334718
8736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 2312998630
88726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 19274
98859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10
 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20
*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 3239680774672220160
0*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929
877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17
 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^1
3*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 886649475173449728
0*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 192749885
9072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231
299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 -
 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28
 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))
^(1/4) - ((-(8398080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 27487
790694400000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^64*d^2 - 852
39594962869288960000*a^3*b^63*c^63*d^3 + 1130310047981633536000000*a^4*b^
62*c^62*d^4 - 11548807808472982329753600*a^5*b^61*c^61*d^5 + 9466093339135
78145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^
59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 1768239155530786
67757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56
*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 87608488
38643010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685
312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d
^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 2914327130323
77964853953403289600*a^16*b^50*c^50*d^16 - 53837688933932732209219051192320
0*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^
18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 2370124261379
332590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399
680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44
*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081
40153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918
944169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^
40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 12
9947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012
```

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841049111826268160*a^29*b^37*c^37*d^29 + 24313727136067816828072488744255480*a^30*b^36*c^36*d^30 - 34711352517916424353692724892794880 0*a^31*b^35*c^35*d^31 + 51583334288620561992503970358099968 0*a^32*b^34*c^34*d^32 - 77546807332992628044123259001005670 4*a^33*b^33*c^33*d^33 + 11365474000985030910505646989120634 88*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 26989809397453278872073290576214097 92*a^38*b^28*c^28*d^38 - 27393908274805544934665349791943229 44*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 12660138058673746897900530208100843 52*a^43*b^23*c^23*d^43 + 84844675058024454799136171007305318 4*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 739174514721719530430678553581322 24*a^48*b^18*c^18*d^48 - 323873725819524777875553934355988 48*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 157896546601467050611780966416384 0*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162727720910848 0000000*a*b^65*c^65*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^
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13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 +
 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2
471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 +
 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d
^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*
c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^
20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 238713320239
00569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 3
8876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^
29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a
^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815
535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4
433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*
d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*
c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b
^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b
^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-(839808
0000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 27487790694400000000*b^6
6*c^66 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 852395949628692889600
000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 11548
8078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*
b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 3639438050
7592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*
b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041
5842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864301071819289395
2000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*
d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801106
94620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237796485395340328960
0*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^
17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 14804725213251
68526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793325909162336788152
32*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*
d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12695869829
017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814015359003912068230
0735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41
*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9257
5964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388
491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*
a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d
^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 5158333428
86205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332992628044123259
0010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34
*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35
 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244704257539
9654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788720732905
```

```
7621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39
*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40
 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 173879220535
5133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380586737468979005302
0810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*
b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 +
 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080
816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195304306785535813
2224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^
17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782
995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809
664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13
*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 317760
42795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150036124817025925
12000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^
57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3394215634796515751
3625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^
60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950429491527863500800
0000*a^62*b^4*c^4*d^62 - 268379484005597184000000*a^63*b^3*c^3*d^63 + 1341
441243847065600000000*a^64*b^2*c^2*d^64 - 162727720910848000000*a*b^65*c^65
*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2
133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 1069982
13378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406
056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 86707877707
77600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801
130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 167807
95101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53
058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^1
5 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c
^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19
*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 6304321158739968
00*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903
936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 5582
5496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 580
1173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 2613
25798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 39223388
16000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*
c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 219902325
5552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^1
1*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^1
3*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*
a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 72281207215226
8800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 44332473
75867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 +
 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*
```

```
c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920
*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 388761692
96066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18
- 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12
*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*
a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152
268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 6227304
0062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471
152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 3408
4860461056*a^39*b^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*
d^4 - 1262449047544497438720*a^24*b^50*c^80*d^5 + 26781213630512448405504*a
^25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 364841894
8406862648705024*a^27*b^47*c^77*d^8 - 28097394584779147947540480*a^28*b^46*
c^76*d^9 + 174448389309337948351627264*a^29*b^45*c^75*d^10 - 89766897611989
7481466085376*a^30*b^44*c^74*d^11 + 3905920242884630010868531200*a^31*b^43*
c^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^13 + 47355821068
790227801756139520*a^33*b^41*c^71*d^14 - 134845524585103937061538234368*a^3
4*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*c^69*d^16 - 763
246944716696111818343448576*a^36*b^38*c^68*d^17 + 1541564027180813686607638
953984*a^37*b^37*c^67*d^18 - 2825288027628089174763608473600*a^38*b^36*c^66
*d^19 + 4753041476272000853590444867584*a^39*b^35*c^65*d^20 - 7446263493185
815677622957375488*a^40*b^34*c^64*d^21 + 11045974611794807027964401680384*a
^41*b^33*c^63*d^22 - 15766681100741571532295786987520*a^42*b^32*c^62*d^23 +
 21882616570434895907374088847360*a^43*b^31*c^61*d^24 - 2955541590135716591
3293077872640*a^44*b^30*c^60*d^25 + 38514249364633213650767204843520*a^45*b
^29*c^59*d^26 - 47767982724772003266224581509120*a^46*b^28*c^58*d^27 + 5561
8948537155045120476750807040*a^47*b^27*c^57*d^28 - 601273004136644754796412
14156800*a^48*b^26*c^56*d^29 + 59877038998440260638050153922560*a^49*b^25*c
^55*d^30 - 54637051595737047014674020696064*a^50*b^24*c^54*d^31 + 455196232
23064909005599526617088*a^51*b^23*c^53*d^32 - 34535555816550055085254958383
104*a^52*b^22*c^52*d^33 + 23809729504484309698980012359680*a^53*b^21*c^51*d
^34 - 14885319254535352990241541586944*a^54*b^20*c^50*d^35 + 84196483392029
54390072444583936*a^55*b^19*c^49*d^36 - 4297514831765712413611503124480*a^5
6*b^18*c^48*d^37 + 1973123737554130196459440570368*a^57*b^17*c^47*d^38 - 81
1770857054497673061303582720*a^58*b^16*c^46*d^39 + 297858380372439371596188
090368*a^59*b^15*c^45*d^40 - 96910050535770593129744302080*a^60*b^14*c^44*d
^41 + 27758579881177587823480406016*a^61*b^13*c^43*d^42 - 69374745044766721
02869499904*a^62*b^12*c^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^
41*d^44 - 274100118958300866495381504*a^64*b^10*c^40*d^45 + 418677784634252
77028466688*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^
47 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*
a^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 3647915698
1701017600*a^70*b^4*c^34*d^51) - 18014398509481984000*a^21*b^51*c^78*d^4 +
778222015609621708800*a^22*b^50*c^77*d^5 - 16199988291606958571520*a^23*b^4
9*c^76*d^6 + 216629339029608119402496*a^24*b^48*c^75*d^7 - 2092899704349501
```

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998235648*a^25*b^47*c^74*d^8 + 15576808854093856430358528*a^26*b^46*c^73*d^
9 - 92989305923335928955273216*a^27*b^45*c^72*d^10 + 4577165703905051534583
39840*a^28*b^44*c^71*d^11 - 1895077372829589675098243072*a^29*b^43*c^70*d^1
2 + 6699157107174094796222365696*a^30*b^42*c^69*d^13 - 20454608396817467081
213607936*a^31*b^41*c^68*d^14 + 54439663857512808688618831872*a^32*b^40*c^6
7*d^15 - 127253623829876322462345461760*a^33*b^39*c^66*d^16 + 2630183603223
01930835307134976*a^34*b^38*c^65*d^17 - 484117148425341461690547437568*a^35
*b^37*c^64*d^18 + 801088032507623116562893897728*a^36*b^36*c^63*d^19 - 1210
191753560658421451373674496*a^37*b^35*c^62*d^20 + 1713662150039311965148455
895040*a^38*b^34*c^61*d^21 - 2368456612874860634985065349120*a^39*b^33*c^60
*d^22 + 3342440882817901253619697582080*a^40*b^32*c^59*d^23 - 4926019419281
526710422764257280*a^41*b^31*c^58*d^24 + 7443043331925522227676535848960*a^
42*b^30*c^57*d^25 - 11053384984245852600223452364800*a^43*b^29*c^56*d^26 +
15529000135185248373347985653760*a^44*b^28*c^55*d^27 - 20153801026888464482
649904250880*a^45*b^27*c^54*d^28 + 23870821024791437072619829985280*a^46*b^
26*c^53*d^29 - 25662407141873741853910169026560*a^47*b^25*c^52*d^30 + 24983
334964938085602226308382720*a^48*b^24*c^51*d^31 - 2200336836145596903283586
8655616*a^49*b^23*c^50*d^32 + 17519758513327663391847122731008*a^50*b^22*c^
49*d^33 - 12601896285489986596049610866688*a^51*b^21*c^48*d^34 + 8179684390
414915120451536551936*a^52*b^20*c^47*d^35 - 4783583081116360454960515645440
*a^53*b^19*c^46*d^36 + 2515171747726250254399514345472*a^54*b^18*c^45*d^37
- 1185710361511816082146770026496*a^55*b^17*c^44*d^38 + 4994066046183585945
80969947136*a^56*b^16*c^43*d^39 - 187097254447826761775602204672*a^57*b^15*
c^42*d^40 + 62002233932522145150727618560*a^58*b^14*c^41*d^41 - 18049115872
947548566748921856*a^59*b^13*c^40*d^42 + 4575187392741408034214903808*a^60*
b^12*c^39*d^43 - 998642414508019303179091968*a^61*b^11*c^38*d^44 + 18498673
5996381058748645376*a^62*b^10*c^37*d^45 - 28520139033328990436720640*a^63*b
^9*c^36*d^46 + 3562072173311951854632960*a^64*b^8*c^35*d^47 - 3463778638686
92037632000*a^65*b^7*c^34*d^48 + 24611841230482125619200*a^66*b^6*c^33*d^49
 - 1137123721538961408000*a^67*b^5*c^32*d^50 + 25649407252758528000*a^68*b^
4*c^31*d^51) - x^(1/2)*(4851701160433680384000*a^21*b^45*c^62*d^11 - 134253
118530519040000*a^20*b^46*c^63*d^10 - 83128151546809181798400*a^22*b^44*c^6
1*d^12 + 895910897914030472560640*a^23*b^43*c^60*d^13 - 6797129989654957642
481664*a^24*b^42*c^59*d^14 + 38483630548489971632701440*a^25*b^41*c^58*d^15
 - 167961815050671342785396736*a^26*b^40*c^57*d^16 + 5737480195599786036953
08800*a^27*b^39*c^56*d^17 - 1529836010901462206864424960*a^28*b^38*c^55*d^1
8 + 3075153110865358700094160896*a^29*b^37*c^54*d^19 - 40445110329811693719
25708800*a^30*b^36*c^53*d^20 + 589590639381102819104784384*a^31*b^35*c^52*d
^21 + 14576671334338745969651220480*a^32*b^34*c^51*d^22 - 50149146156756356
561350164480*a^33*b^33*c^50*d^23 + 110550157926715904989065117696*a^34*b^32
*c^49*d^24 - 189331360528461979941957795840*a^35*b^31*c^48*d^25 + 267383527
373748192433944920064*a^36*b^30*c^47*d^26 - 319821143985825066443750768640*
a^37*b^29*c^46*d^27 + 328626898447261055168230195200*a^38*b^28*c^45*d^28 -
292434560796558751919058714624*a^39*b^27*c^44*d^29 + 2263824164821702908920
93521920*a^40*b^26*c^43*d^30 - 152776304398053739659930894336*a^41*b^25*c^4
```

```
2*d^31 + 8990112462267334306471870464 0*a^42*b^24*c^41*d^32 - 46062508964820
426479181496320*a^43*b^23*c^40*d^33 + 2048660626373761009104558489 6*a^44*b^
22*c^39*d^34 - 787091432377505435124498432 0*a^45*b^21*c^38*d^35 + 259414172
4382360002965274624*a^46*b^20*c^37*d^36 - 72645102465195278480703488 0*a^47*
b^19*c^36*d^37 + 17059006036588517488852992 0*a^48*b^18*c^35*d^38 - 32986343
5542048981123072 00*a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^1
6*c^33*d^40 - 613036163719885750272000*a^51*b^15*c^32*d^41 + 53255297770998
202368000*a^52*b^14*c^31*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43
+ 81438120439971840000*a^54*b^12*c^29*d^44))*(-(8398080000*a^33*d^33 - (705
27747686400000000*a^66*d^66 + 2748779069440000000*b^66*c^66 + 464565652967
91552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3
+ 11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536 00
0*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 63988
38206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a
^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 74254
8127574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290086689049656950
7840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d
^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584
363799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*
a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16
 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 9167535731160177
03850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*
a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^2
0 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 69634084434967
93458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445329 98
144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^4
2*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 62262545
797041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646070620848388692 8
9496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*
b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 +
 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 347113525179164
243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580 9
99680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*
c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 157
8683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 20440850601244330 7
2578392630325411840*a^36*b^30*c^30*d^36 - 244704257539965436239724393550315
5200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*
c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 255
8145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 219832300736439599
8582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639 59
0400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*
c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 5231
97059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743 44
3383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120
*a^47*b^19*c^19*d^47 + 7391745147217195304306785535813222 4*a^48*b^18*c^18*d
```

^48 - 3238737258195247778755539343559884 8*a^49*b^17*c^17*d^49 + 12978756421
5123908217893623053680 64*a^50*b^16*c^16*d^50 - 47457829954142086407501544 37
099520*a^51*b^15*c^15*d^51 + 15789654660146705061178096641638 40*a^52*b^14*c
^14*d^52 - 47637131856714525898060616171520 0*a^53*b^13*c^13*d^53 + 12978980
94790687573306431766528 00*a^54*b^12*c^12*d^54 - 31776042795476444797594501 1
20000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*
d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 2294691469180319
74963609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256 00000*a^59*b^7*c
^7*d^59 + 42954568799822401241088 00000*a^60*b^6*c^6*d^60 - 4559717929936371
05664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^6
2 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*
a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 4388393189376000
000000*a^65*b*c*d^65)^(1/2) + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31
*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^2
9*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*
d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d
^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23
*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21
*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*
b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 30657507805716480
0*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 6887985648
47943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766
159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^2
1 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c
^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^
8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^
6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*
c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^
31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*
a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d
 - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340
848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 138
38453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 -
231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d
^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22
*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240
*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 323968077
46722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15
 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15
*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 2387133202390056960
0*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 88664947
51734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 -
 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*
d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c
^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4

```
*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c
^15*d^30))^(1/4) + 9271855998513971200000*a^20*b^44*c^58*d^12 - 253888379928
53929984000*a^21*b^43*c^57*d^13 + 317358378012506691993600*a^22*b^42*c^56*d
^14 - 2373809829046075554529280*a^23*b^41*c^55*d^15 + 115452848098157290481
25440*a^24*b^40*c^54*d^16 - 35586486107261996158156800*a^25*b^39*c^53*d^17
+ 47503987551983633390632960*a^26*b^38*c^52*d^18 + 160896568335160851531038
720*a^27*b^37*c^51*d^19 - 1289503277949063475180339200*a^28*b^36*c^50*d^20
+ 4847695519788247586575482880*a^29*b^35*c^49*d^21 - 129693768092376088082 1
2070400*a^30*b^34*c^48*d^22 + 27198543957428161531839774720*a^31*b^33*c^47*
d^23 - 46558532623156834692403036160*a^32*b^32*c^46*d^24 + 6646503366406378
8557407354880*a^33*b^31*c^45*d^25 - 80137164645540595666444615680*a^34*b^30
*c^44*d^26 + 82241221222993610845821337600*a^35*b^29*c^43*d^27 - 7216514003
1754207660154552320*a^36*b^28*c^42*d^28 + 54258643078018614781815029760*a^3
7*b^27*c^41*d^29 - 34958604671456258343085015040*a^38*b^26*c^40*d^30 + 1926
6119383513605759523880960*a^39*b^25*c^39*d^31 - 904771327888492699771207680
0*a^40*b^24*c^38*d^32 + 3598803321131446378839408640*a^41*b^23*c^37*d^33 -
1201767391129510053066833920*a^42*b^22*c^36*d^34 + 332745330268979132513648
640*a^43*b^21*c^35*d^35 - 75056967015910052829593600*a^44*b^20*c^34*d^36 +
13447517913537594156646400*a^45*b^19*c^33*d^37 - 1841937645534110023680000*
a^46*b^18*c^32*d^38 + 181270486395868151808000*a^47*b^17*c^31*d^39 - 114194
34221693829120000*a^48*b^16*c^30*d^40 + 346112011869880320000*a^49*b^15*c^2
9*d^41))*(-(8398080000*a^33*d^33 - (705277476864000000000*a^66*d^66 + 274877
906944000000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^64*d^2 - 8523
959496286928896000000*a^3*b^63*c^63*d^3 + 113031004798163353600000000*a^4*b^6
2*c^62*d^4 - 1154880780847298232975360000*a^5*b^61*c^61*d^5 + 94660933391357
8145788723200*a^6*b^60*c^60*d^6 - 63988382063497445934681292800*a^7*b^59*c^5
9*d^7 + 363943805075927975134589091840*a^8*b^58*c^58*d^8 - 17682391555307866
7757483982848*a^9*b^57*c^57*d^9 + 74254812757466745819072194150400*a^10*b^56*
c^56*d^10 - 272041584290086689049656950784000*a^11*b^55*c^55*d^11 + 876084883
864301071819289395200000*a^12*b^54*c^54*d^12 - 249552350040826187070412286853
12*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^
14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237
796485395340328960 0*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200
*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^1
8 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793
32590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255080146693645139 96
80*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*
d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814
01535900391206823007354 88*a^24*b^42*c^42*d^24 - 3902249846040715985377291 89
44169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^4
0*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129
947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 17703615665425001 28
410491118262681 60*a^29*b^37*c^37*d^29 + 24313727136067816828072488744255488
0*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35
*d^31 + 515833428862056199250397035809996 80*a^32*b^34*c^34*d^32 - 77546807
```

```
3329926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056
4698912063488*a^34*b^32*c^32*d^34 - 15786833044632146161337550200100618240*a
^35*b^31*c^31*d^35 + 20440850601244330725783926303254118840*a^36*b^30*c^30*d
^36 - 24470425753996543623972439355031552000*a^37*b^29*c^29*d^37 + 269898093
97453278872073290576214097920*a^38*b^28*c^28*d^38 - 273939082748055449346653
4979194322944*a^39*b^27*c^27*d^39 + 25581457575927361633598682365134110720*a
^40*b^26*c^26*d^40 - 21983230073643959985824159760384000000*a^41*b^25*c^25*d
^41 + 17387922053551330345825449126395904000*a^42*b^24*c^24*d^42 - 126601380
586737468979005302081008435200*a^43*b^23*c^23*d^43 + 848446750580244547991361
710073053184*a^44*b^22*c^22*d^44 - 52319705986478663727463936373764915200*a^4
5*b^21*c^21*d^45 + 29669244466490074344338382271807488000*a^46*b^20*c^20*d^46
 - 15458625383108081624547756355878912000*a^47*b^19*c^19*d^47 + 7391745147217
19530430678553581322240*a^48*b^18*c^18*d^48 - 3238737258195247778755539343550
98848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c
^16*d^50 - 47457829954142086407501544370995200*a^51*b^15*c^15*d^51 + 1578965
466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606
161715200*a^53*b^13*c^13*d^53 + 12978980947906875733064317665280 0*a^54*b^12*
c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836
15003612481702592512000*a^56*b^10*c^10*d^56 - 134721865560409115491041280000
00*a^57*b^9*c^9*d^57 + 2294691469180319749636096000000*a^58*b^8*c^8*d^58 - 3
394215634796515751362560000 0*a^59*b^7*c^7*d^59 + 42954568799822401241088000
00*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950
429491527863500800000 0*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b
^3*c^3*d^63 + 1341441243847065600000000*a^64*b^2*c^2*d^64 - 162727720910848
0000000*a*b^65*c^65*d - 43883931893760000000000*a^65*b*c*d^65)^(1/2) + 524288
0000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*
c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^2
8*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26
*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24
*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*
c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b
^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*
a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173
068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006
5381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20
+ 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^1
1*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b
^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*
b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b
^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c
^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26
1273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^1
3*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 +
34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 24
71152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 +
```

62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^
7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c
^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^2
0*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 2387133202390
0569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38
876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^2
9*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^
27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155163658155
35370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 44
33247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d
^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c
^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^
5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^
3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*2i + atan(((-((7052
7747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 4645656529679
1552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 +
 11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000
*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883
8206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^
8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548
127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507
840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^
12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467425843
63799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a
^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16
- 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 91675357311601770
3850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a
^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20
 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679
3458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445329981
44*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42
*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457
97041866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289
496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b
^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 +
243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642
43536927248927948800*a^31*b^35*c^35*d^31 + 51583334288620561992503970358099
9680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c
^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578
683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072
578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155
200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c
^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558
145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998
582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590

```
400*a^42*b^24*c^24*d^42 - 12660138058673746897900530208100843 52*a^43*b^23*c
^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319
7059864786637274639363737649152*a^45*b^21*c^21*d^45 + 29669244466490074 3443
383822718074880*a^46*b^20*c^20*d^46 - 15458625383108081624547756355878912 0*
a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^
48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215
12390821789362305368064*a^50*b^16*c^16*d^50 - 474578299541420864075015 44370
99520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^
14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809
47906875733064317665 2800*a^54*b^12*c^12*d^54 - 31776042795476444797594501 12
0000*a^55*b^11*c^11*d^55 + 694868361500361248170259251 2000*a^56*b^10*c^10*d
^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691803197
4963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^
7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 4559717929936371 0
5664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62
 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a
^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 43883931893760000
00000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 +
213364244 4800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998
213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 156093640
6056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770
777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 2048481280
1130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780
795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 5
3058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^
15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*
c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^1
9*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996
800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 26177390
3936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 558
25496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 58
01173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261
325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338
816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b
*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 21990232
55552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^
11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^
13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312
*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 7228120721522
68800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247
375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11
+ 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19
*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 388761692960664192
0*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169
296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18
```

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 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^1
2*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640
*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 72281207215
2268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 622730
40062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 247
1152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 340
84860461056*a^39*b^2*c^15*d^30))^(1/4)*(x^(1/2)*(4851701160433680384000*a^2
1*b^45*c^62*d^11 - 1342531185305190400000*a^20*b^46*c^63*d^10 - 831281515468
09181798400*a^22*b^44*c^61*d^12 + 895910897914030472560640*a^23*b^43*c^60*d
^13 - 6797129989654957642481664*a^24*b^42*c^59*d^14 + 384836305484899716327
01440*a^25*b^41*c^58*d^15 - 167961815050671342785396736*a^26*b^40*c^57*d^16
 + 573748019559978603695308800*a^27*b^39*c^56*d^17 - 1529836010901462206864
424960*a^28*b^38*c^55*d^18 + 3075153110865358700094160896*a^29*b^37*c^54*d^
19 - 4044511032981169371925708800*a^30*b^36*c^53*d^20 + 5895906393811028191
04784384*a^31*b^35*c^52*d^21 + 14576671334338745969651220480*a^32*b^34*c^51
*d^22 - 50149146156756356561350164480*a^33*b^33*c^50*d^23 + 110550157926715
904989065117696*a^34*b^32*c^49*d^24 - 189331360528461979941957795840*a^35*b
^31*c^48*d^25 + 267383527373748192433944920064*a^36*b^30*c^47*d^26 - 319821
143985825066443750768640*a^37*b^29*c^46*d^27 + 3286268984472610551682301952
00*a^38*b^28*c^45*d^28 - 292434560796558751919058714624*a^39*b^27*c^44*d^29
 + 226382416482170290892093521920*a^40*b^26*c^43*d^30 - 1527763043980537396
59930894336*a^41*b^25*c^42*d^31 + 89901124622673343064718704640*a^42*b^24*c
^41*d^32 - 46062508964820426479181496320*a^43*b^23*c^40*d^33 + 204866062637
37610091045584896*a^44*b^22*c^39*d^34 - 7870914323775054351244984320*a^45*b
^21*c^38*d^35 + 2594141724382360002965274624*a^46*b^20*c^37*d^36 - 72645102
4651952784807034880*a^47*b^19*c^36*d^37 + 170590060365885174888529920*a^48*
b^18*c^35*d^38 - 32986343554204898112307200*a^49*b^17*c^34*d^39 + 511806359
1384977873305600*a^50*b^16*c^33*d^40 - 613036163719885750272000*a^51*b^15*c
^32*d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42 - 298872579261726720
0000*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b^12*c^29*d^44) - (-((
70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 464565652
96791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d
^3 + 113031004798163353600000000*a^4*b^62*c^62*d^4 - 1154880780847298232975 3
6000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 63
98838206349744593468129280*a^7*b^59*c^59*d^7 + 363943805075927975134589091 8
4*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 74
2548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290086689049656
9507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^5
4*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 63214446742
584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801106946205058726803537
92*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d
^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 9167535731160
17703850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238 1
44*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*
d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 69634084434
```

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96793458703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532
998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*
c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 62262
545797041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646070620848388 6
9289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^
28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^2
9 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 347113525179
164243536927248927948800*a^31*b^35*c^35*d^31 + 5158333428862056199250397035
80999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^
33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 -
1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 204408506012443
3072578392630325411840*a^36*b^30*c^30*d^36 - 24470425753996543623972439355 0
3155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^
28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 +
2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 219832300736439
5998582415976038400000*a^41*b^25*c^25*d^41 + 173879220535513303458254491263
9590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^
23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 5
23197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 29669244466490074
3443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789
120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^1
8*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 12978756
421512390821789362305368064*a^50*b^16*c^16*d^50 - 474578299541420864075015 4
437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^1
4*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 12978
980947906875733064317665280 0*a^54*b^12*c^12*d^54 - 31776042795476444797594 5
01120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^
10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 2294691469180
31974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^
7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 4559717929936
37105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*
d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000 0
00*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 4388393189376
000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^3
3 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 10
6998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 15609
36406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 867078
7770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 204848
12801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1
6780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13
 - 5305814389923840 0*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^1
8*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b
^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536
*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 63043211587
3996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 2617
```

```
73903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 +
 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26
+ 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 +
 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 392
2338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^
32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199
023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 3408486046105
6*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 247115238342656
0*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 6227304006234
9312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072
152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443
3247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d
^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*
b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 388761692960666
41920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887
6169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*
d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29
*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 443324737586724
8640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 7228120
72152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62
273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 +
 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 +
 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-((70527747686400000000
0*a^66*d^66 + 274877906944000000000*b^66*c^66 + 464565652967915520000000*a^2*
b^64*c^64*d^2 - 8523959496286928896000000*a^3*b^63*c^63*d^3 + 1130310047981
6335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*
d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820634974459346
8129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8
 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190
721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c
^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 2495523500
4082618707041228685312*a^13*b^53*c^53*d^13 + 632144467425843637996415185059
84*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d
^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5383768893393
27322092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740003
2*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d
^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 394568205038
2550801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345870323761283
0720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^
43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 3902249
8460407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667528366
85340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*
b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 -
 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243137271360678
168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435369272489279
```

```
48800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*
c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 1136
547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616
133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411
840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c
^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 2739
390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163
359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400
000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c
^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 8484
46750580244547991361710073053184*a^44*b^22*c^22*d^44 - 5231970598647866372
7463936373764915 2*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880
*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*
d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 3238737258
1952477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123908217893623
05368064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15
*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 47637
1318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479068757330643
176652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*
c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 134721865
5604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a
^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 429545
6879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61
*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 268379484005
5971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^6
4 - 16272772091084800000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*
d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 + 213364244800*a
^2*b^31*c^31*d^2 - 1813499609088 0*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b
^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^2
7*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25
*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^
23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^
12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 5305814389923840
0*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 3065750780
57164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688
798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^1
9 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c
^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23
*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*
a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*
a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^
29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2
*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719
476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31
*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^
```

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2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^
4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39
*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24
*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^
19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 1551636581553
5370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32
396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^3
0*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^
26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 238713320239
00569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8
866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23
*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^
8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^3
5*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a
^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^3
9*b^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d^4 - 12624490
47544497438720*a^24*b^50*c^80*d^5 + 267812136305124484055504*a^25*b^49*c^79*
d^6 - 3668169646702282544250888*a^26*b^48*c^78*d^7 + 364841894840686264870050
24*a^27*b^47*c^77*d^8 - 280973945847791479475404080*a^28*b^46*c^76*d^9 + 174
4483893093379483516272644*a^29*b^45*c^75*d^10 - 8976689761198974814660853776*
a^30*b^44*c^74*d^11 + 39059202428846300108685312000*a^31*b^43*c^73*d^12 - 14
5908964250757653799297351688*a^32*b^42*c^72*d^13 + 47355821068790227801756613
9520*a^33*b^41*c^71*d^14 - 1348455245851039370615382343688*a^34*b^40*c^70*d^
15 + 3397270967300867141087631769600*a^35*b^39*c^69*d^16 - 763246944716696111
1818343448576*a^36*b^38*c^68*d^17 + 15415640271808136866076389539844*a^37*b^
37*c^67*d^18 - 28252880276280891747636084736000*a^38*b^36*c^66*d^19 + 475304
14762720008535904448675844*a^39*b^35*c^65*d^20 - 744626349318581567762295737
5488*a^40*b^34*c^64*d^21 + 110459746117948070279644016803844*a^41*b^33*c^63*
d^22 - 157666811007415715322957869875200*a^42*b^32*c^62*d^23 + 218826165704
348959073740888473600*a^43*b^31*c^61*d^24 - 295554159013571659132930778726400*
a^44*b^30*c^60*d^25 + 385142493646332136507672048435200*a^45*b^29*c^59*d^26
- 477679827247720032662245815091200*a^46*b^28*c^58*d^27 + 556189485371550451
20476750807040*a^47*b^27*c^57*d^28 - 601273004136644754796412141568000*a^48*
b^26*c^56*d^29 + 598770389984402606380501539225600*a^49*b^25*c^55*d^30 - 546
370515957370470146740206960644*a^50*b^24*c^54*d^31 + 455196232230649090055999
526617088*a^51*b^23*c^53*d^32 - 345355558165500550852549583831044*a^52*b^22*
c^52*d^33 + 238097295044843096989800123596800*a^53*b^21*c^51*d^34 - 14885319
254535352990241541586944*a^54*b^20*c^50*d^35 + 841964833920295439007244458 3
936*a^55*b^19*c^49*d^36 - 429751483176571241361150312448000*a^56*b^18*c^48*d^
37 + 197312373755413019645944057036800*a^57*b^17*c^47*d^38 - 811770857054497 6
73061303582720*a^58*b^16*c^46*d^39 + 29785838037243937159618809036800*a^59*b^
15*c^45*d^40 - 96910050535770593129744302080*a^60*b^14*c^44*d^41 + 27758579
881177587823480406016*a^61*b^13*c^43*d^42 - 6937474504476672102869499904*a^
62*b^12*c^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^41*d^44 - 2741
001189583008664953815044*a^64*b^10*c^40*d^45 + 418677784634252770284666688*a^
```

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65*b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^47 + 500879902
205011065569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c^36*
d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 36479156981701017600*a^7
0*b^4*c^34*d^51) - 18014398509481984000*a^21*b^51*c^78*d^4 + 7782201560962
1708800*a^22*b^50*c^77*d^5 - 16199988291606958571520*a^23*b^49*c^76*d^6 + 2
16629339029608119402496*a^24*b^48*c^75*d^7 - 2092899704349501998235648*a^25
*b^47*c^74*d^8 + 15576808854093856430358528*a^26*b^46*c^73*d^9 - 9298930592
3335928955273216*a^27*b^45*c^72*d^10 + 457716570390505153458339840*a^28*b^4
4*c^71*d^11 - 1895077372829589675098243072*a^29*b^43*c^70*d^12 + 6699157107
174094796222365696*a^30*b^42*c^69*d^13 - 20454608396817467081213607936*a^31
*b^41*c^68*d^14 + 54439663857512808688618831872*a^32*b^40*c^67*d^15 - 12725
3623829876322462345461760*a^33*b^39*c^66*d^16 + 263018360322301930835307134
976*a^34*b^38*c^65*d^17 - 484117148425341461690547437568*a^35*b^37*c^64*d^1
8 + 801088032507623116562893897728*a^36*b^36*c^63*d^19 - 121019175356065842
1451373674496*a^37*b^35*c^62*d^20 + 1713662150039311965148455895040*a^38*b^
34*c^61*d^21 - 2368456612874860634985065349120*a^39*b^33*c^60*d^22 + 334244
0882817901253619697582080*a^40*b^32*c^59*d^23 - 492601941928152671042276425
7280*a^41*b^31*c^58*d^24 + 7443043331925522227676535848960*a^42*b^30*c^57*d
^25 - 11053384984245852600223452364800*a^43*b^29*c^56*d^26 + 15529000135185
248373347985653760*a^44*b^28*c^55*d^27 - 20153801026888464482649904250880*a
^45*b^27*c^54*d^28 + 23870821024791437072619829985280*a^46*b^26*c^53*d^29 -
 25662407141873741853910169026560*a^47*b^25*c^52*d^30 + 2498333496493808560
2226308382720*a^48*b^24*c^51*d^31 - 22003368361455969032835868655616*a^49*b
^23*c^50*d^32 + 17519758513327663391847122731008*a^50*b^22*c^49*d^33 - 1260
1896285489986596049610866688*a^51*b^21*c^48*d^34 + 817968439041491512045153
6551936*a^52*b^20*c^47*d^35 - 4783583081116360454960515645440*a^53*b^19*c^4
6*d^36 + 2515171747726250254399514345472*a^54*b^18*c^45*d^37 - 118571036151
1816082146770026496*a^55*b^17*c^44*d^38 + 499406604618358594580969947136*a^
56*b^16*c^43*d^39 - 187097254447826761775602204672*a^57*b^15*c^42*d^40 + 62
002233932522145150727618560*a^58*b^14*c^41*d^41 - 1804911587294754856674892
1856*a^59*b^13*c^40*d^42 + 4575187392741408034214903808*a^60*b^12*c^39*d^43
 - 998642414508019303179091968*a^61*b^11*c^38*d^44 + 1849867359963810587486
45376*a^62*b^10*c^37*d^45 - 28520139033328990436720640*a^63*b^9*c^36*d^46 +
 3562072173311951854632960*a^64*b^8*c^35*d^47 - 346377863868692037632000*a^
65*b^7*c^34*d^48 + 24611841230482125619200*a^66*b^6*c^33*d^49 - 11371237215
38961408000*a^67*b^5*c^32*d^50 + 25649407252758528000*a^68*b^4*c^31*d^51))*
1i + (-((70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 +
46456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^
63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 11548807808472
9823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60
*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 3639438050759279751
3458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57
*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290086
6890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12
*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 63
```

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214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801106946205058
72680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^
50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 9167
53573116017703850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452 3
82335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b
^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 69
63408443496793458703237612830720*a^22*b^44*c^44*d^22 - 1269586982901723240 8
306844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a
^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^2
5 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9257596460706
2084838869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907
348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37
*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 347
113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619 9
25039703580999680*a^32*b^34*c^34*d^32 - 77546807332992628044123259001005670
4*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^3
2*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 204408
5060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244704257539965436239
7243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788720732905762140979
2*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^2
7*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 219832
3007364395998582415976038400000*a^41*b^25*c^25*d^41 + 17387922053551330345 8
2544912639590400*a^42*b^24*c^24*d^42 - 126601380586737468979005302081008435
2*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22
*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 29669244
4664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477
563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48
*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 +
 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782995414208
640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840
*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^5
3 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 317760427954764
44797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^5
6*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 2294
69146918031974963609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256000 0
0*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 4559
71792993637105664000000*a^61*b^5*c^5*d^61 + 3950429491527863500800000 0*a^62
*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384 7
06560000000*a^64*b^2*c^2*d^64 - 162727720910848000000 0*a*b^65*c^65*d - 4388
393189376000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000
*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30
*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^
5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7
 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9
 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22
```

```
*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*
c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15
*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 5044134631730688
00*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381
353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 63
0432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^
22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c
^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*
c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c
^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d
^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273
600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^
32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 3408
4860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 247115
2383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 6227
3040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 +
722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*
d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^
21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569
600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 388761
69296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^
16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b
^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155163658155537
0240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 443324
7375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23
+ 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*
d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^
18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^
16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(x^(1/2)*(4851701160433
680384000*a^21*b^45*c^62*d^11 - 134253118530519040000*a^20*b^46*c^63*d^10 -
 83128151546809181798400*a^22*b^44*c^61*d^12 + 895910897914030472560640*a^2
3*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b^42*c^59*d^14 + 38483630
548489971632701440*a^25*b^41*c^58*d^15 - 167961815050671342785396736*a^26*b
^40*c^57*d^16 + 573748019559978603695308800*a^27*b^39*c^56*d^17 - 152983601
0901462206864424960*a^28*b^38*c^55*d^18 + 3075153110865358700094160896*a^29
*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*b^36*c^53*d^20 + 589590
639381102819104784384*a^31*b^35*c^52*d^21 + 14576671334338745969651220480*a
^32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b^33*c^50*d^23 + 11
0550157926715904989065117696*a^34*b^32*c^49*d^24 - 189331360528461979941957
795840*a^35*b^31*c^48*d^25 + 267383527373748192433944920064*a^36*b^30*c^47*
d^26 - 319821143985825066443750768640*a^37*b^29*c^46*d^27 + 328626898447261
055168230195200*a^38*b^28*c^45*d^28 - 292434560796558751919058714624*a^39*b
^27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*c^43*d^30 - 152776
304398053739659930894336*a^41*b^25*c^42*d^31 + 8990112462267334306471870464
0*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^43*b^23*c^40*d^33 +
```

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 20486606263737610091045584896*a^44*b^22*c^39*d^34 - 7870914323775054351244
984320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624*a^46*b^20*c^37*d^
36 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 17059006036588517488
8529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*a^49*b^17*c^34*d^3
9 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 61303616371988575027200
0*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42 - 29887
25792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b^12*c^29
*d^44) - (-((70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^6
6 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^
3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 1154880780
84729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*
c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 363943805075927
97513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*
c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 27204158429
00866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*
a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13
+ 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620
505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^1
6*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 +
916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526
452382335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^
20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21
+ 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 1269586982901723
2408306844532998144*a^23*b^43*c^43*d^23 + 228294081401535900391206823007354
88*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41
*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646
07062084838869289496739840*a^27*b^39*c^39*d^27 + 12994738493072452038849161
5907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*
b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 -
 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205
619925039703580999680*a^32*b^34*c^34*d^32 - 7754680733299262804412325900100
56704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32
*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 20
44085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 24470425753996543
62397243935503155200*a^37*b^29*c^29*d^37 + 26989809397453278872073290576214
09792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27
*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 21
98323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 17387922053551330
34582544912639590400*a^42*b^24*c^24*d^42 - 12660138058673746897900530208100
84352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*
c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 2966
92444664900743443383822718074880*a^46*b^20*c^20*d^46 - 15458625383108081624
5477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*
a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^
49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 474578299541
```

```
4208640750154437099520*a^51*b^15*c^15*d^51 + 157896546601467050611780966416
3840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13
*d^53 + 1297898094790687573306431766528OO*a^54*b^12*c^12*d^54 - 31776042795
476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150036124817025925120OO
*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 +
229469146918031974963609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256
00000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 -
455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*
a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124
384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d -
4388393189376000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 524288
0000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*
c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^2
8*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26
*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24
*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*
c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b
^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*
a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173
068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006
5381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20
+ 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^1
1*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b
^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*
b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b
^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c
^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26
1273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^1
3*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 +
34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 24
71152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 +
62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^
7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c
^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^2
0*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 2387133202390
0569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38
876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^2
9*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^
27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155163658155
35370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 44
33247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d
^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c
^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^
5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^
3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-((705277
```

```
47686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 464565652967915
52000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 1
130310047981633536000000*a^4*b^62*c^62*d^4 - 1154880780847298232975360000*a
^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 63988382
06349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*
b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 74254812
7574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290086689049656950784
0*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12
 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363
799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^1
5*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 -
538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 9167535731160177038
50321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^1
9*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 -
 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 69634084434967934
58703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144
*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d
^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 62262545797
041866752836685340344320*a^26*b^40*c^40*d^26 - 9257596460706208483886928949
6739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^3
8*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 24
3137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 347113525179164243
536927248927948800*a^31*b^35*c^35*d^31 + 5158333428862056199250397035809996
80*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^3
3*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 157868
3304463214616133755020010061824*a^35*b^31*c^31*d^35 + 204408506012443307257
8392630325411840*a^36*b^30*c^30*d^36 - 244704257539965436239724393550315520
0*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^2
8*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 255814
5757592736163359868236513411072*a^40*b^26*c^26*d^40 - 219832300736439599858
2415976038400000*a^41*b^25*c^25*d^41 + 173879220535513303458254491263959040
0*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^2
3*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 5231970
59864786637274639363737649152*a^45*b^21*c^21*d^45 + 29669244466490074344338
3822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^
47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48
 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 12978756421512
390821789362305368064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099
520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14
*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 12978980947
9068757330643176652800*a^54*b^12*c^12*d^54 - 317760427954764447975945011200
00*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^5
6 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 2294691469180319749
63609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*
d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 4559717929936371056
```

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64000000*a^61*b^5*c^5*d^61 + 395042949152786350080000000*a^62*b^4*c^4*d^62 -
 268379484005597184000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^6
4*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 43883931893760000000
000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 + 21
33642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 10699821
3378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 15609364060
56960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 867078777077
7600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 204848128011
30496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1678079
5101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 530
58143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15
 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^
16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*
b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800
0*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 2617739039
36962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825
496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801
173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 26132
5798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 392233881
6000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c
*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255
552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11
*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13
*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a
^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268
800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443324737
5867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 +
15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c
^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*
a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887616929
6066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 -
 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*
c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a
^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 7228120721522
68800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040
062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 24711
52383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084
860461056*a^39*b^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d
^4 - 1262449047544497438720*a^24*b^50*c^80*d^5 + 26781213630512448405504*a^
25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 3648418948
406862648705024*a^27*b^47*c^77*d^8 - 28097394584779147947540480*a^28*b^46*c
^76*d^9 + 174448389309337948351627264*a^29*b^45*c^75*d^10 - 897668976119897
481466085376*a^30*b^44*c^74*d^11 + 3905920242884630010868531200*a^31*b^43*c
^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^13 + 473558210687
90227801756139520*a^33*b^41*c^71*d^14 - 134845524585103937061538234368*a^34
```

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*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*c^69*d^16 - 7632
469447166961118183434448576*a^36*b^38*c^68*d^17 + 15415640271808136866076389
53984*a^37*b^37*c^67*d^18 - 2825288027628089174763608473600*a^38*b^36*c^66*
d^19 + 4753041476272000853590444867584*a^39*b^35*c^65*d^20 - 74462634931858
15677622957375488*a^40*b^34*c^64*d^21 + 11045974611794807027964401680384*a^
41*b^33*c^63*d^22 - 15766681100741571532295786987520*a^42*b^32*c^62*d^23 +
21882616570434895907374088847360*a^43*b^31*c^61*d^24 - 29555415901357165913
293077872640*a^44*b^30*c^60*d^25 + 38514249364633213650767204843520*a^45*b^
29*c^59*d^26 - 47767982724772003266224581509120*a^46*b^28*c^58*d^27 + 55618
948537155045120476750807040*a^47*b^27*c^57*d^28 - 6012730041366447547964121
4156800*a^48*b^26*c^56*d^29 + 59877038998440260638050153922560*a^49*b^25*c^
55*d^30 - 54637051595737047014674020696064*a^50*b^24*c^54*d^31 + 4551962322
3064909005599526617088*a^51*b^23*c^53*d^32 - 345355558165500550852549583831
04*a^52*b^22*c^52*d^33 + 23809729504484309698980012359680*a^53*b^21*c^51*d^
34 - 14885319254535352990241541586944*a^54*b^20*c^50*d^35 + 841964833920295
4390072444583936*a^55*b^19*c^49*d^36 - 4297514831765712413611503124480*a^56
*b^18*c^48*d^37 + 1973123737554130196459440570368*a^57*b^17*c^47*d^38 - 811
770857054497673061303582720*a^58*b^16*c^46*d^39 + 2978583803724393715961880
90368*a^59*b^15*c^45*d^40 - 96910050535770593129744302080*a^60*b^14*c^44*d^
41 + 27758579881177587823480406016*a^61*b^13*c^43*d^42 - 693747450447667210
2869499904*a^62*b^12*c^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^4
1*d^44 - 274100118958300866495381504*a^64*b^10*c^40*d^45 + 4186777846342527
7028466688*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^4
7 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a
^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 36479156981
701017600*a^70*b^4*c^34*d^51) + 180143985094819840000*a^21*b^51*c^78*d^4 - 7
782220156096217088000*a^22*b^50*c^77*d^5 + 16199988291606958571520*a^23*b^49
*c^76*d^6 - 216629339029608119402496*a^24*b^48*c^75*d^7 + 20928997043495019
98235648*a^25*b^47*c^74*d^8 - 15576808854093856430358528*a^26*b^46*c^73*d^9
 + 92989305923335928955273216*a^27*b^45*c^72*d^10 - 45771657039050515345833
9840*a^28*b^44*c^71*d^11 + 1895077372829589675098243072*a^29*b^43*c^70*d^12
 - 6699157107174094796222365696*a^30*b^42*c^69*d^13 + 204546083968174670812
13607936*a^31*b^41*c^68*d^14 - 54439663857512808688618831872*a^32*b^40*c^67
*d^15 + 127253623829876322462345461760*a^33*b^39*c^66*d^16 - 26301836032230
1930835307134976*a^34*b^38*c^65*d^17 + 484117148425341461690547437568*a^35*
b^37*c^64*d^18 - 801088032507623116562893897728*a^36*b^36*c^63*d^19 + 12101
91753560658421451373674496*a^37*b^35*c^62*d^20 - 17136621500393119651484558
95040*a^38*b^34*c^61*d^21 + 2368456612874860634985065349120*a^39*b^33*c^60*
d^22 - 3342440882817901253619697582080*a^40*b^32*c^59*d^23 + 49260194192815
26710422764257280*a^41*b^31*c^58*d^24 - 7443043331925522227676535848960*a^4
2*b^30*c^57*d^25 + 11053384984245852600223452364800*a^43*b^29*c^56*d^26 - 1
5529000135185248373347985653760*a^44*b^28*c^55*d^27 + 201538010268884644826
49904250880*a^45*b^27*c^54*d^28 - 23870821024791437072619829985280*a^46*b^2
6*c^53*d^29 + 25662407141873741853910169026560*a^47*b^25*c^52*d^30 - 249833
34964938085602226308382720*a^48*b^24*c^51*d^31 + 22003368361455969032835868
```

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655616*a^49*b^23*c^50*d^32 - 17519758513327663391847122731008*a^50*b^22*c^4
9*d^33 + 12601896285489986596049610866688*a^51*b^21*c^48*d^34 - 81796843904
14915120451536551936*a^52*b^20*c^47*d^35 + 4783583081116360454960515645440*
a^53*b^19*c^46*d^36 - 2515171747726250254399514345472*a^54*b^18*c^45*d^37 +
 1185710361511816082146770026496*a^55*b^17*c^44*d^38 - 49940660461835859458
0969947136*a^56*b^16*c^43*d^39 + 187097254447826761775602204672*a^57*b^15*c
^42*d^40 - 62002233932522145150727618560*a^58*b^14*c^41*d^41 + 180491158729
47548566748921856*a^59*b^13*c^40*d^42 - 4575187392741408034214903808*a^60*b
^12*c^39*d^43 + 998642414508019303179091968*a^61*b^11*c^38*d^44 - 184986735
996381058748645376*a^62*b^10*c^37*d^45 + 28520139033328990436720640*a^63*b^
9*c^36*d^46 - 3562072173311951854632960*a^64*b^8*c^35*d^47 + 34637786386869
2037632000*a^65*b^7*c^34*d^48 - 24611841230482125619200*a^66*b^6*c^33*d^49
+ 1137123721538961408000*a^67*b^5*c^32*d^50 - 25649407252758528000*a^68*b^4
*c^31*d^51))*1i)/((-((70527747686400000000*a^66*d^66 + 27487790694400000000
*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889
600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 1
15488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*
a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 363943
80507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*
a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 27
20415842900866890496569507840*a^11*b^55*c^55*d^11 + 87608488386430107181928
93952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c
^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 143133780
110694620505872680353792*a^15*b^51*c^51*d^15 + 2914327130323779648539534032
89600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^4
9*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 1480472521
325168526452382335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678
815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c
^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 1269586
9829017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081401535900391206
82300735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*
b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 -
92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 12994738493072452
0388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268
160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^
36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 515833
342886205619925039703580999680*a^32*b^34*c^34*d^32 - 7754680733299262804412
32590010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*
a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*
d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 24470425
75399654362397243935503155200*a^37*b^29*c^29*d^37 + 26989809397453278872073
29057621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*
a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*
d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 17387922
05355133034582544912639590400*a^42*b^24*c^24*d^42 - 12660138058673746897900
```

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53020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a
^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^
45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 15458625383
1080816245477563558789120*a^47*b^19*c^19*d^47 + 739174514721719530430678553
58132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^1
7*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 474
5782995414208640750154437099520*a^51*b^15*c^15*d^51 + 157896546601467050611
7809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*
b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 31
776042795476444797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702
592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^
9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 339421563479651
57513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^
6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 395042949152786350
08000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 +
134144124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*
c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33
 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*
a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5
*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*
b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*
b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a
^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 38303873786880
00*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 1501996617
41875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504
413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^1
8 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c
^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22
*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 1316761632647086
08*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874
880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 13823517331456
00*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a
^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^
32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736
*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^1
4*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42
*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^
40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^2
5*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^
18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 88664947517344
97280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 2387
1332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*
d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25
*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722
201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155
```

```
16365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*
d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b
^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a
^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 1383845334718873
6*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 34084860461056
0*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(x^(1/2)*(
4851701160433680384000*a^21*b^45*c^62*d^11 - 134253118530519040000*a^20*b^4
6*c^63*d^10 - 83128151546809181798400*a^22*b^44*c^61*d^12 + 895910897914030
472560640*a^23*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b^42*c^59*d^
14 + 38483630548489971632701440*a^25*b^41*c^58*d^15 - 167961815050671342785
396736*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^39*c^56*d^1
7 - 1529836010901462206864424960*a^28*b^38*c^55*d^18 + 30751531108653587000
94160896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*b^36*c^53*
d^20 + 589590639381102819104784384*a^31*b^35*c^52*d^21 + 145766713343387459
69651220480*a^32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b^33*c
^50*d^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24 - 18933136052
8461979941957795840*a^35*b^31*c^48*d^25 + 267383527373748192433944920064*a^
36*b^30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*c^46*d^27 + 32
8626898447261055168230195200*a^38*b^28*c^45*d^28 - 292434560796558751919058
714624*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*c^43*
d^30 - 152776304398053739659930894336*a^41*b^25*c^42*d^31 + 899011246226733
43064718704640*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^43*b^2
3*c^40*d^33 + 20486606263737610091045584896*a^44*b^22*c^39*d^34 - 787091432
3775054351244984320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624*a^46
*b^20*c^37*d^36 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 1705900
60365885174888529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*a^49*
b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 6130361637
19885750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14*c^31
*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840000*a
^54*b^12*c^29*d^44) - (-((70527747686400000000*a^66*d^66 + 2748779069440000
0000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 85239594962869
2889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4
 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723
200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36
394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982
848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10
 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718
192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^
53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 14313
3780110694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953
403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49
*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 148047
2521325168526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623
3678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^
45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 126
```

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95869829017232408306844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039
120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189441699984*a
^25*b^41*c^41*d^25 + 622625457970418667528366853403443320*a^26*b^40*c^40*d^2
6 - 925759646070620848388692894967398400*a^27*b^39*c^39*d^27 + 1299473849307
245203884916159073484800*a^28*b^38*c^38*d^28 - 177036156654250012841049111182
6268160*a^29*b^37*c^37*d^29 + 2431372713606781682807248874425548800*a^30*b^3
6*c^36*d^30 - 3471135251791642435369272489279488000*a^31*b^35*c^35*d^31 + 51
58333428862056199250397035809996800*a^32*b^34*c^34*d^32 - 775468073329926280
44123259001005670400*a^33*b^33*c^33*d^33 + 113654740009850309105056469891206
3488*a^34*b^32*c^32*d^34 - 15786833044632146161337550200100618240*a^35*b^31*c
^31*d^35 + 20440850601244330725783926303254118400*a^36*b^30*c^30*d^36 - 2447
04257539965436239724393550315520000*a^37*b^29*c^29*d^37 + 2698980939745327887
2073290576214097920*a^38*b^28*c^28*d^38 - 273939082748055449346653497919432
2944*a^39*b^27*c^27*d^39 + 25581457575927361633598682365134110720*a^40*b^26*c
^26*d^40 - 21983230073643959985824159760384000000*a^41*b^25*c^25*d^41 + 1738
79220535513303458254491263959040000*a^42*b^24*c^24*d^42 - 1266013805867374689
79005302081008435200*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053
184*a^44*b^22*c^22*d^44 - 52319705986478663727463936373764915200*a^45*b^21*c^2
1*d^45 + 29669244466490074344338382271807488000*a^46*b^20*c^20*d^46 - 1545862
53831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195304306
7855358132224*a^48*b^18*c^18*d^48 - 3238737258195247778755539343559884800*a^49
*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 -
 47457829954142086407501544370995200*a^51*b^15*c^15*d^51 + 1578965466014670
5061178096641638400*a^52*b^14*c^14*d^52 - 4763713185671452589806061617152000*a
^53*b^13*c^13*d^53 + 1297898094790687573306431766528000*a^54*b^12*c^12*d^54
 - 317760427954764447975945011200000*a^55*b^11*c^11*d^55 + 6948683615003612
481702592512000*a^56*b^10*c^10*d^56 - 134721865560409115491041280000000*a^57*b^
9*c^9*d^57 + 2294691469180319749636096000000*a^58*b^8*c^8*d^58 - 33942156347
9651575136256000000*a^59*b^7*c^7*d^59 + 42954568799822401241088000000*a^60*b^
6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278
635008000000*a^62*b^4*c^4*d^62 - 268379484005597184000000*a^63*b^3*c^3*d^6
3 + 1341441243847065600000000*a^64*b^2*c^2*d^64 - 162727720910848000000000*a*b
^65*c^65*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*
d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090
880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888
*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*
a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*
a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 225293620110540
80*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378
688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199
661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 +
 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15
*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^
13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*
a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264
```

```
708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 1979265159
4874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733
145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 377578967040
00*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^3
2*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 6871947
6736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b
*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*
c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^2
7*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16
*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 192749885907271680
0*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751
734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 -
23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c
^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*
a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239680774
6722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 +
 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c
^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^
32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 2312998630887260
16*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 138384533471
88736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 3408486046
10560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/
2)*(-((70527747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 46
456565296791552000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63
*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 1154880780847298
23297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d
^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 363943805075927975134
58909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d
^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 27204158429008668
90496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b
^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 6321
4446742584363799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872
680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50
*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 916753
573116017703850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382
335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^4
6*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 6963
408443496793458703237612830720*a^22*b^44*c^44*d^22 - 1269586982901723240830
6844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^2
4*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25
+ 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646070620
84838869289496739840*a^27*b^39*c^39*d^27 + 12994738493072452038849161590734
8480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c
^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 34711
3525179164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925
```

```
039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*
a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*
d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 20440850
60124433072578392630325411840*a^36*b^30*c^30*d^36 - 24470425753996543623972
43935503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*
a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*
d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 21983230
07364395998582415976038400000*a^41*b^25*c^25*d^41 + 17387922053551330345825
44912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*
a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d
^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 2966924446
64900743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808162454775б
3558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b
^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 1
2978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 474578299541420864
0750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a
^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53
+ 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 31776042795476444
797594501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*
b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 229469
146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*
a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 455971
792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b
^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706
560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 438839
3189376000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b
^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d
^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5
+ 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 +
 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 +
 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d
^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^
20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b
^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800
*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 79006538135
3537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 6304
32115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22
 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9
*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^
7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5
*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^3
0 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 26127360
0000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32
 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 340848
60461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 24711523
```

```
83426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 622730
40062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 72
2812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^
9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21
*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 238713320239005696
0*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169
296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16
 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^1
4*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 155163658155353702
40*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 44332473
75867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 +
722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^
25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18
*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16
*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(288230376151711744400*a^2
3*b^51*c^81*d^4 - 12624490475444497438720*a^24*b^50*c^80*d^5 + 267812136305l
2448405504*a^25*b^49*c^79*d^6 - 3668169646702282544250088*a^26*b^48*c^78*d^7
 + 36484189484068626487050024*a^27*b^47*c^77*d^8 - 280973945847791479475404 8
0*a^28*b^46*c^76*d^9 + 1744483893093379483516272 64*a^29*b^45*c^75*d^10 - 89
76689761198974814660853 76*a^30*b^44*c^74*d^11 + 390592024288463001086853120
0*a^31*b^43*c^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^13 +
 47355821068790227801756139520*a^33*b^41*c^71*d^14 - 134845524585103937061 5
38234368*a^34*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*c^6
9*d^16 - 763246944716696111818343448576*a^36*b^38*c^68*d^17 + 1541564027180
813686607638953984*a^37*b^37*c^67*d^18 - 2825288027628089174763608473600*a^
38*b^36*c^66*d^19 + 4753041476272000853590444867584*a^39*b^35*c^65*d^20 - 7
446263493185815677622957375488*a^40*b^34*c^64*d^21 + 1104597461179480702796
4401680384*a^41*b^33*c^63*d^22 - 15766681100741571532295786987520*a^42*b^32
*c^62*d^23 + 21882616570434895907374088847360*a^43*b^31*c^61*d^24 - 2955541
5901357165913293077872640*a^44*b^30*c^60*d^25 + 385142493646332136507672048
43520*a^45*b^29*c^59*d^26 - 47767982724772003266224581509120*a^46*b^28*c^58
*d^27 + 55618948537155045120476750807040*a^47*b^27*c^57*d^28 - 601273004136
64475479641214156800*a^48*b^26*c^56*d^29 + 59877038998440260638050153922560
*a^49*b^25*c^55*d^30 - 54637051595737047014674020696064*a^50*b^24*c^54*d^31
 + 45519623223064909005599526617088*a^51*b^23*c^53*d^32 - 3453555581655005 5
08525495838 3104*a^52*b^22*c^52*d^33 + 23809729504484309698980012359680*a^53
*b^21*c^51*d^34 - 14885319254535352990241541586944*a^54*b^20*c^50*d^35 + 84
19648339202954390072444583936*a^55*b^19*c^49*d^36 - 429751483176571241361 15
03124480*a^56*b^18*c^48*d^37 + 1973123737554130196459440570368*a^57*b^17*c^
47*d^38 - 811770857054497673061303582720*a^58*b^16*c^46*d^39 + 297858380372
439371596188090368*a^59*b^15*c^45*d^40 - 96910050535770593129744302080*a^60
*b^14*c^44*d^41 + 27758579881177587823480406016*a^61*b^13*c^43*d^42 - 69374
74504476672102869499904*a^62*b^12*c^42*d^43 + 1495682482860276471300096000*
a^63*b^11*c^41*d^44 - 274100118958300866495381504*a^64*b^10*c^40*d^45 + 418
67778463425277028466688*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a^66
```

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*b^8*c^38*d^47 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 353719920493
08254863360*a^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50
- 36479156981701017600*a^70*b^4*c^34*d^51) - 18014398509481984000*a^21*b^51
*c^78*d^4 + 778222015609621708800*a^22*b^50*c^77*d^5 - 16199988291606958571
520*a^23*b^49*c^76*d^6 + 216629339029608119402496*a^24*b^48*c^75*d^7 - 2092
899704349501998235648*a^25*b^47*c^74*d^8 + 15576808854093856430358528*a^26*
b^46*c^73*d^9 - 92989305923335928955273216*a^27*b^45*c^72*d^10 + 4577165703
90505153458339840*a^28*b^44*c^71*d^11 - 1895077372829589675098243072*a^29*b
^43*c^70*d^12 + 6699157107174094796222365696*a^30*b^42*c^69*d^13 - 20454608
396817467081213607936*a^31*b^41*c^68*d^14 + 54439663857512808688618831872*a
^32*b^40*c^67*d^15 - 127253623829876322462345461760*a^33*b^39*c^66*d^16 + 2
63018360322301930835307134976*a^34*b^38*c^65*d^17 - 48411714842534146169054
7437568*a^35*b^37*c^64*d^18 + 801088032507623116562893897728*a^36*b^36*c^63
*d^19 - 1210191753560658421451373674496*a^37*b^35*c^62*d^20 + 1713662150039
311965148455895040*a^38*b^34*c^61*d^21 - 2368456612874860634985065349120*a^
39*b^33*c^60*d^22 + 3342440882817901253619697582080*a^40*b^32*c^59*d^23 - 4
926019419281526710422764257280*a^41*b^31*c^58*d^24 + 7443043331925522227676
535848960*a^42*b^30*c^57*d^25 - 11053384984245852600223452364800*a^43*b^29*
c^56*d^26 + 15529000135185248373347985653760*a^44*b^28*c^55*d^27 - 20153801
026888464482649904250880*a^45*b^27*c^54*d^28 + 2387082102479143707261982998
5280*a^46*b^26*c^53*d^29 - 25662407141873741853910169026560*a^47*b^25*c^52*
d^30 + 24983334964938085602226308382720*a^48*b^24*c^51*d^31 - 2200336836145
5969032835868655616*a^49*b^23*c^50*d^32 + 17519758513327663391847122731008*
a^50*b^22*c^49*d^33 - 12601896285489986596049610866688*a^51*b^21*c^48*d^34
+ 8179684390414915120451536551936*a^52*b^20*c^47*d^35 - 4783583081116360454
960515645440*a^53*b^19*c^46*d^36 + 2515171747726250254399514345472*a^54*b^1
8*c^45*d^37 - 1185710361511816082146770026496*a^55*b^17*c^44*d^38 + 4994066
04618358594580969947136*a^56*b^16*c^43*d^39 - 18709725444782676177560220467
2*a^57*b^15*c^42*d^40 + 62002233932522145150727618560*a^58*b^14*c^41*d^41 -
 18049115872947548566748921856*a^59*b^13*c^40*d^42 + 4575187392741408034214
903808*a^60*b^12*c^39*d^43 - 998642414508019303179091968*a^61*b^11*c^38*d^4
4 + 184986735996381058748645376*a^62*b^10*c^37*d^45 - 285201390333289904367
20640*a^63*b^9*c^36*d^46 + 3562072173311951854632960*a^64*b^8*c^35*d^47 - 3
46377863868692037632000*a^65*b^7*c^34*d^48 + 24611841230482125619200*a^66*b
^6*c^33*d^49 - 1137123721538961408000*a^67*b^5*c^32*d^50 + 2564940725275852
8000*a^68*b^4*c^31*d^51)) - (-((70527747686400000000*a^66*d^66 + 2748779069
4400000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 85239594
9628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^
62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145
788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^
7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757
483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56
*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643
010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a
^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 -
```

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 1431337801106946205058726803537792*a^15*b^51*c^51*d^15 + 291432713032377964
853953403289600*a^16*b^50*c^50*d^16 - 5383768893393273220921905119232000*a^1
7*b^49*c^49*d^17 + 9167535731160177038503215177400032*a^18*b^48*c^48*d^18 -
14804725213251685264523823352381144*a^19*b^47*c^47*d^19 + 237012426137933259
0916233678815232*a^20*b^46*c^46*d^20 - 39456820503825508014669364513999680*a
^21*b^45*c^45*d^21 + 69634084434967934587032376128307200*a^22*b^44*c^44*d^22
 - 126958698290172324083068445329981444*a^23*b^43*c^43*d^23 + 22829408140153
5900391206823007354888*a^24*b^42*c^42*d^24 - 390224984604071598537729189441
69984*a^25*b^41*c^41*d^25 + 622625457970418667528366853403443200*a^26*b^40*c^
40*d^26 - 925759646070620848388692894967398400*a^27*b^39*c^39*d^27 + 1299473
849307245203884916159073484800*a^28*b^38*c^38*d^28 - 17703615665425001284104
9111826268160*a^29*b^37*c^37*d^29 + 2431372713606781682807248874425548800*a^
30*b^36*c^36*d^30 - 3471135251791642435369272489279488000*a^31*b^35*c^35*d^3
1 + 5158333428862056199250397035809996800*a^32*b^34*c^34*d^32 - 775468073329
9262804412325900100567040*a^33*b^33*c^33*d^33 + 113654740009850309105056469
8912063488*a^34*b^32*c^32*d^34 - 15786833044632146161337550200100618224*a^35*
b^31*c^31*d^35 + 20440850601244330725783926303254118400*a^36*b^30*c^30*d^36
- 24470425753996543623972439355031552000*a^37*b^29*c^29*d^37 + 2698980939745
32788720732905762140979200*a^38*b^28*c^28*d^38 - 27393908274805544934665349790
194322944*a^39*b^27*c^27*d^39 + 25581457575927361633598682365134110720*a^40*
b^26*c^26*d^40 - 21983230073643959985824159760384000000*a^41*b^25*c^25*d^41
+ 17387922053551330345825449126395904000*a^42*b^24*c^24*d^42 - 1266013805867
3746897900530208100843520*a^43*b^23*c^23*d^43 + 8484467505802445479913617100
73053184*a^44*b^22*c^22*d^44 - 5231970598647866372746393637376491520*a^45*b^
21*c^21*d^45 + 2966924446649007434433838227180748800*a^46*b^20*c^20*d^46 - 1
545862538310808162454775635587891200*a^47*b^19*c^19*d^47 + 73917451472171953
0430678553581322240*a^48*b^18*c^18*d^48 - 323873725819524777875553934355988
48*a^49*b^17*c^17*d^49 + 129787564215123908217893623053680640*a^50*b^16*c^16*
d^50 - 47457829954142086407501544370995200*a^51*b^15*c^15*d^51 + 15789654660
146705061178096641638400*a^52*b^14*c^14*d^52 - 476371318567145258980606161715
5200*a^53*b^13*c^13*d^53 + 1297898094790687573306431766528000*a^54*b^12*c^12
*d^54 - 317760427954764447975945011200000*a^55*b^11*c^11*d^55 + 694868361500
36124817025925120000*a^56*b^10*c^10*d^56 - 134721865560409115491041280000000*a
^57*b^9*c^9*d^57 + 2294691469180319749636096000000*a^58*b^8*c^8*d^58 - 33942
15634796515751362560000000*a^59*b^7*c^7*d^59 + 42954568799822401241088000000*a
^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294
9152786350080000000*a^62*b^4*c^4*d^62 - 26837948400559718400000000*a^63*b^3*c
^3*d^63 + 1341441243847065600000000*a^64*b^2*c^2*d^64 - 16272772091084800000
00*a*b^65*c^65*d - 43883931893760000000000*a^65*b*c*d^65)^(1/2) + 8398080000
*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134
996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266
917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 41118923017
42080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 147939177477
87776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 225293620
11054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830
```

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387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 +
150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*
d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^1
5*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a
^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 4408131707805
69600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676
163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 1979
2651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382
351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 377578
96704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000
*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 6
8719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*
a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12
*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^
14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 23129986308872601
6*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 192749885907
2716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866
494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d
^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*
b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 413059298770708
07040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239
6807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*
d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*
b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716
800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 2312998630
88726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 138384
53347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 3408
48604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*
(x^(1/2)*(48517011604336803840000*a^21*b^45*c^62*d^11 - 1342531185305190400
0*a^20*b^46*c^63*d^10 - 83128151546809181798400*a^22*b^44*c^61*d^12 + 89591
089791403047256064O*a^23*b^43*c^60*d^13 - 679712998965495764248166 4*a^24*b^
42*c^59*d^14 + 3848363054848997163270144 0*a^25*b^41*c^58*d^15 - 16796181505
0671342785396736*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^3
9*c^56*d^17 - 1529836010901462206864424960*a^28*b^38*c^55*d^18 + 3075153110
865358700094160896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*
b^36*c^53*d^20 + 5895906393811028191047843 84*a^31*b^35*c^52*d^21 + 14576671
334338745969651220480*a^32*b^34*c^51*d^22 - 50149146156756356561350164480*a
^33*b^33*c^50*d^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24 - 1
89331360528461979941957795840*a^35*b^31*c^48*d^25 + 267383527373748192433944
920064*a^36*b^30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*c^46
*d^27 + 328626898447261055168230195200*a^38*b^28*c^45*d^28 - 29243456079655
875191905871462 4*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a^40*
b^26*c^43*d^30 - 152776304398053739659930894336*a^41*b^25*c^42*d^31 + 89901
124622673343064718704640*a^42*b^24*c^41*d^32 - 460625089648204264791814963 2
0*a^43*b^23*c^40*d^33 + 204866062637376100910455848 96*a^44*b^22*c^39*d^34 -
```

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 7870914323775054351244984320*a^45*b^21*c^38*d^35 + 259414172438236000029652
74624*a^46*b^20*c^37*d^36 - 72645102465195278480703488 0*a^47*b^19*c^36*d^37
 + 170590060365885174888529920*a^48*b^18*c^35*d^38 - 329863435542048981123 0
7200*a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 -
613036163719885750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52
*b^14*c^31*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 814381204399
71840000*a^54*b^12*c^29*d^44) - (-((70527747686400000000*a^66*d^66 + 274877
906944000000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 8523
95949628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^6
2*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 94660933391357
8145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^5
9*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 17682391555307866
7757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*
c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883
8643010718192893952000*a^12*b^54*c^54*d^12 - 249552350040826187070412286853
12*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^
14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237
7964853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200
*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^1
8 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793
32590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255080146693645139 96
80*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*
d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814
0153590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189
44169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^4
0*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129
947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012 8
410491118262681 60*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554 88
0*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35
*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807
3329926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056
4698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a
^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d
^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093
9745327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274805544934665 3
4979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a
^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d
^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380
5867374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361
710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^4
5*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46
 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217
1953043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355
98848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c
^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965
```

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466014670506117809664163840*a^52*b^14*c^14*d^52 - 4763713185671452589806061
61715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*
c^12*d^54 - 317760427954764447975945011200000*a^55*b^11*c^11*d^55 + 69486836
150036124817025925120000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800 0
00*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3
39421563479651575136256000 00*a^59*b^7*c^7*d^59 + 42954568799822401241088000
00*a^60*b^6*c^6*d^60 - 4559717929936371056640000 00*a^61*b^5*c^5*d^61 + 3950
429491527863500800 0000*a^62*b^4*c^4*d^62 - 26837948400559718400000 00*a^63*b
^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162727720910848 0
000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 839808
0000*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 1
8134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 46643
6266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892
301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917
747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529
362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 +
3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^1
4 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c
^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18
*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412 7
36*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170
780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 13
1676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 -
19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 -
1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37
757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 15518924
8000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45
 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255
552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*
a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 138384533471887 3
6*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 2312998630887
26016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 19274988
59072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 -
8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^
33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a
^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877
070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 +
32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c
^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a
^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 192749885907
2716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299
863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13
838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 -
340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3
/4)*(x^(1/2)*(-((705277476864000000000*a^66*d^66 + 274877906944000000000*b^66
```

```
*c^66 + 4645656529679155200000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 24470425753996543623972439355031552000*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 17387922053551330345825449126395904000*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 157896546601467050611780964163840*a^52*b^14*c^14*d^52 - 47637131856714525898060616171520 0*a^53*b^13*c^13*d^53 + 12978980947906875733064317665280 0*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150036124817025925120 00*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000
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000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13414
4124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*
d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 52
42880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b
^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28
*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*
c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*
c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b
^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^
13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875
200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 50441346
3173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 7
90065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d
^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11
*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^
24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a
^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^
28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b
^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d
- 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41
*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^3
1 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3
+ 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^
5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^3
8*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^
23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280
*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 238713320
23900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14
- 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16
*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 323968077467222016
0*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365
815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21
+ 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^
22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b
^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^3
6*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^3
8*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(2882303761517
1174400*a^23*b^51*c^81*d^4 - 1262449047544497438720*a^24*b^50*c^80*d^5 + 267
81213630512448405504*a^25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^4
8*c^78*d^7 + 3648418948406862648705024*a^27*b^47*c^77*d^8 - 280973945847791
47947540480*a^28*b^46*c^76*d^9 + 174448389309337948351627264*a^29*b^45*c^75
*d^10 - 897668976119897481466085376*a^30*b^44*c^74*d^11 + 3905920242884630
010868531200*a^31*b^43*c^73*d^12 - 14590896425075765379929735168*a^32*b^42*c
^72*d^13 + 47355821068790227801756139520*a^33*b^41*c^71*d^14 - 134845524585
103937061538234368*a^34*b^40*c^70*d^15 + 339727096730086714108763176960*a^3
```

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5*b^39*c^69*d^16 - 7632469447166961118183434485 76*a^36*b^38*c^68*d^17 + 154
1564027180813686607638953984*a^37*b^37*c^67*d^18 - 282528802762808917476360
8473600*a^38*b^36*c^66*d^19 + 4753041476272000853590444867584*a^39*b^35*c^6
5*d^20 - 7446263493185815677622957375488*a^40*b^34*c^64*d^21 + 110459746117
948070279644016803 84*a^41*b^33*c^63*d^22 - 15766681100741571532295786987520
*a^42*b^32*c^62*d^23 + 21882616570434895907374088847360*a^43*b^31*c^61*d^24
 - 29555415901357165913293077872640*a^44*b^30*c^60*d^25 + 3851424936463321 3
650767204843520*a^45*b^29*c^59*d^26 - 47767982724772003266224581509120*a^46
*b^28*c^58*d^27 + 55618948537155045120476750807040*a^47*b^27*c^57*d^28 - 60
127300413664475479641214156800*a^48*b^26*c^56*d^29 + 5987703899844026063805
0153922560*a^49*b^25*c^55*d^30 - 54637051595737047014674020696064*a^50*b^24
*c^54*d^31 + 45519623223064909005599526617088*a^51*b^23*c^53*d^32 - 3453555
5816550055085254958383104*a^52*b^22*c^52*d^33 + 238097295044843096989800123
59680*a^53*b^21*c^51*d^34 - 14885319254535352990241541586944*a^54*b^20*c^50
*d^35 + 8419648339202954390072444583936*a^55*b^19*c^49*d^36 - 4297514831765
712413611503124480*a^56*b^18*c^48*d^37 + 1973123737554130196459440570368*a^
57*b^17*c^47*d^38 - 811770857054497673061303582720*a^58*b^16*c^46*d^39 + 29
7858380372439371596188090368*a^59*b^15*c^45*d^40 - 969100505357705931297443
02080*a^60*b^14*c^44*d^41 + 27758579881177587823480406016*a^61*b^13*c^43*d^
42 - 6937474504476672102869499904*a^62*b^12*c^42*d^43 + 1495682482860276471
300096000*a^63*b^11*c^41*d^44 - 274100118958300866495381504*a^64*b^10*c^40*
d^45 + 41867778463425277028466688*a^65*b^9*c^39*d^46 - 518716113093076359 47
27424*a^66*b^8*c^38*d^47 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 35
371992049308254863360*a^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*
c^35*d^50 - 36479156981701017600*a^70*b^4*c^34*d^51) + 18014398509481984000
*a^21*b^51*c^78*d^4 - 778222015609621708800*a^22*b^50*c^77*d^5 + 1619998829
1606958571520*a^23*b^49*c^76*d^6 - 216629339029608119402496*a^24*b^48*c^75*
d^7 + 2092899704349501998235648*a^25*b^47*c^74*d^8 - 155768088540938564303 5
8528*a^26*b^46*c^73*d^9 + 92989305923335928955273216*a^27*b^45*c^72*d^10 -
457716570390505153458339840*a^28*b^44*c^71*d^11 + 189507737282958967509824 3
072*a^29*b^43*c^70*d^12 - 6699157107174094796222365696*a^30*b^42*c^69*d^13
+ 20454608396817467081213607936*a^31*b^41*c^68*d^14 - 5443966385751280868 86
18831872*a^32*b^40*c^67*d^15 + 127253623829876322462345461760*a^33*b^39*c^6
6*d^16 - 263018360322301930835307134976*a^34*b^38*c^65*d^17 + 484117148425 3
41461690547437568*a^35*b^37*c^64*d^18 - 801088032507623116562893897728*a^36
*b^36*c^63*d^19 + 1210191753560658421451373674496*a^37*b^35*c^62*d^20 - 171
3662150039311965148455895040*a^38*b^34*c^61*d^21 + 236845661287486063498506
5349120*a^39*b^33*c^60*d^22 - 3342440882817901253619697582080*a^40*b^32*c^5
9*d^23 + 4926019419281526710422764257280*a^41*b^31*c^58*d^24 - 744304333192
5522227676535848960*a^42*b^30*c^57*d^25 + 11053384984245852600223452364800*
a^43*b^29*c^56*d^26 - 15529000135185248373347985653760*a^44*b^28*c^55*d^27
+ 20153801026888464482649904250880*a^45*b^27*c^54*d^28 - 2387082102479143 70
72619829985280*a^46*b^26*c^53*d^29 + 25662407141873741853910169026560*a^47*
b^25*c^52*d^30 - 24983334964938085602226308382720*a^48*b^24*c^51*d^31 + 220
03368361455969032835868655616*a^49*b^23*c^50*d^32 - 17519758513327663391847
```

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122731008*a^50*b^22*c^49*d^33 + 12601896285489986596049610866688*a^51*b^21*
c^48*d^34 - 8179684390414915120451536551936*a^52*b^20*c^47*d^35 + 478358308
1116360454960515645440*a^53*b^19*c^46*d^36 - 251517174772625025439951434547
2*a^54*b^18*c^45*d^37 + 1185710361511816082146770026496*a^55*b^17*c^44*d^38
 - 499406604618358594580969947136*a^56*b^16*c^43*d^39 + 1870972544478267617
75602204672*a^57*b^15*c^42*d^40 - 62002233932522145150727618560*a^58*b^14*c
^41*d^41 + 18049115872947548566748921856*a^59*b^13*c^40*d^42 - 457518739274
1408034214903808*a^60*b^12*c^39*d^43 + 998642414508019303179091968*a^61*b^1
1*c^38*d^44 - 184986735996381058748645376*a^62*b^10*c^37*d^45 + 28520139033
328990436720640*a^63*b^9*c^36*d^46 - 3562072173311951854632960*a^64*b^8*c^3
5*d^47 + 346377863868692037632000*a^65*b^7*c^34*d^48 - 24611841230482125619
200*a^66*b^6*c^33*d^49 + 1137123721538961408000*a^67*b^5*c^32*d^50 - 256494
07252758528000*a^68*b^4*c^31*d^51)) + 927185599851397120000*a^20*b^44*c^58*
d^12 - 25388837992853929984000*a^21*b^43*c^57*d^13 + 3173583780125066919936
00*a^22*b^42*c^56*d^14 - 2373809829046075554529280*a^23*b^41*c^55*d^15 + 11
545284809815729048125440*a^24*b^40*c^54*d^16 - 35586486107261996158156800*a
^25*b^39*c^53*d^17 + 47503987551983633390632960*a^26*b^38*c^52*d^18 + 16089
6568335160851531038720*a^27*b^37*c^51*d^19 - 1289503277949063475180339200*a
^28*b^36*c^50*d^20 + 4847695519788247586575482880*a^29*b^35*c^49*d^21 - 129
69376809237608808212070400*a^30*b^34*c^48*d^22 + 27198543957428161531839774
720*a^31*b^33*c^47*d^23 - 46558532623156834692403036160*a^32*b^32*c^46*d^24
 + 66465033664063788557407354880*a^33*b^31*c^45*d^25 - 80137164645540595666
444615680*a^34*b^30*c^44*d^26 + 82241221222993610845821337600*a^35*b^29*c^4
3*d^27 - 72165140031754207660154552320*a^36*b^28*c^42*d^28 + 54258643078018
614781815029760*a^37*b^27*c^41*d^29 - 34958604671456258343085015040*a^38*b^
26*c^40*d^30 + 19266119383513605759523880960*a^39*b^25*c^39*d^31 - 90477132
78884926997712076800*a^40*b^24*c^38*d^32 + 3598803321131446378839408640*a^4
1*b^23*c^37*d^33 - 1201767391129510053066833920*a^42*b^22*c^36*d^34 + 33274
5330268979132513648640*a^43*b^21*c^35*d^35 - 75056967015910052829593600*a^4
4*b^20*c^34*d^36 + 13447517913537594156646400*a^45*b^19*c^33*d^37 - 1841937
645534110023680000*a^46*b^18*c^32*d^38 + 181270486395868151808000*a^47*b^17
*c^31*d^39 - 11419434221693829120000*a^48*b^16*c^30*d^40 + 3461120118698803
20000*a^49*b^15*c^29*d^41))*(-((70527747686400000000*a^66*d^66 + 2748779069
4400000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 85239594
9628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^
62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145
788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^
7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757
483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56
*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643
010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a
^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 -
 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964
853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^1
7*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 -
```

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1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259
0916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a
^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22
 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 22829408140153
590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189441б
9984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^
40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 1299473
84930724520388491615907348480*a^28*b^38*c^38*d^28 - 17703615665425001284104
9111826268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^
30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^3
1 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329
926280441232590010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698
912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*
b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36
- 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 2698980939745
327887207329057621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979
194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*
b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41
+ 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867
374689790053020810084352*a^43*b^23*c^23*d^43 + 8484467505802445479913617100
73053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^
21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 1
54586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953
043067855358132224*a^48*b^18*c^18*d^48 - 3238737258195247778755539343559884
8*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*
d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 15789654660
14670506117809664163840*a^52*b^14*c^14*d^52 - 47637131856714525898060616171
5200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12
*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 694868361500
3612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a
^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942
156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a
^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294
915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c
^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162727720910848000000
00*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 8398080000
*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134
996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266
917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 41118923017
42080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 147939177477
87776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 225293620
11054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830
387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 +
150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*
d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^1
```

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5*c^15*d^18 + 7900653813535375336*a^19*b^14*c^14*d^19 - 7661592670954127336*a
^20*b^13*c^13*d^20 + 6304321158739968000*a^21*b^12*c^12*d^21 - 4408131707805
69600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676
163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 1979
2651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382
351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 377578
96704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000
*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 6
8719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*
a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12
*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^
14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 23129986308872601
6*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 192749885907
2716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866
494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d
^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*
b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 413059298770708
07040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239
6807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*
d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*
b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716
800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 2312998630
88726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 138384
53347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 3408
48604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*
2i + 2*atan(((-((70527747686400000000*a^66*d^66 + 27487790694400000000*b^66
*c^66 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 852395949628692889600000
0*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 115488
078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b
^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507
592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b
^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415
842900866890496569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952
000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d
^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069
4620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600
*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^1
7 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132516
8526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623367881523
2*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d
^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290
17232408306844532998144*a^23*b^43*c^43*d^23 + 228294081401535900391206823000
735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189441699840*a^25*b^41*
c^41*d^25 + 622625457970418667528366853403443200*a^26*b^40*c^40*d^26 - 92575
9646070620848388692894967398400*a^27*b^39*c^39*d^27 + 12994738493072452038840
```

```
91615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a
^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^
30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334288
6205619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590
010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*
b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35
+ 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399
654362397243935503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057
621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*
b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40
- 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355
133034582544912639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020
810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b
^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 +
296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808
16245477563558789120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132
224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^1
7*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829
95414208640750154437099520*a^51*b^15*c^15*d^51 + 15789654660146705061178096
64163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*
c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604
2795476444797594501120000*a^55*b^11*c^11*d^55 + 694868361500361248170259251
2000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^5
7 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513
625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^6
0 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000
000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13414
4124384706560000000*a^64*b^2*c^2*d^64 - 16272772091084800000000*a*b^65*c^65*
d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 52
42880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b
^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28
*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*
c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*
c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b
^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^
13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875
200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 50441346
3173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 7
90065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d
^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11
*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^
24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a
^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^
28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b
^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d
```

```
- 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41
*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^3
1 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3
+ 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^
5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^3
8*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^
23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280
*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 238713320
23900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14
- 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16
*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 3239680774672220160
0*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365
815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21
+ 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^
22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b
^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^3
6*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^3
8*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(x^(1/2)*(48517
01160433680384000*a^21*b^45*c^62*d^11 - 134253118530519040000*a^20*b^46*c^6
3*d^10 - 83128151546809181798400*a^22*b^44*c^61*d^12 + 89591089791403047256
0640*a^23*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b^42*c^59*d^14 +
38483630548489971632701440*a^25*b^41*c^58*d^15 - 16796181505067134278539673
6*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^39*c^56*d^17 - 1
529836010901462206864424960*a^28*b^38*c^55*d^18 + 3075153110865358700094160
896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a^30*b^36*c^53*d^20
+ 589590639381102819104784384*a^31*b^35*c^52*d^21 + 14576671334338745969651
220480*a^32*b^34*c^51*d^22 - 50149146156756356561350164480*a^33*b^33*c^50*d
^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24 - 1893313605284619
79941957795840*a^35*b^31*c^48*d^25 + 267383527373748192433944920064*a^36*b^
30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*c^46*d^27 + 3286268
98447261055168230195200*a^38*b^28*c^45*d^28 - 29243456079655875191905871462
4*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a^40*b^26*c^43*d^30
- 152776304398053739659930894336*a^41*b^25*c^42*d^31 + 89901124622673343064
718704640*a^42*b^24*c^41*d^32 - 46062508964820426479181496320*a^43*b^23*c^4
0*d^33 + 20486606263737610091045584896*a^44*b^22*c^39*d^34 - 78709143237750
54351244984320*a^45*b^21*c^38*d^35 + 2594141724382360002965274624*a^46*b^20
*c^37*d^36 - 726451024651952784807034880*a^47*b^19*c^36*d^37 + 170590060365
885174888529920*a^48*b^18*c^35*d^38 - 32986343554204898112307200*a^49*b^17*
c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 - 613036163719885
750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42
 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b
^12*c^29*d^44) + (-((70527747686400000000*a^66*d^66 + 274877906944000000000*
b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 8523959496286928896
00000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^62*c^62*d^4 - 11
5488078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a
```

```
^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 3639438
0507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a
^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272
0415842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864301071819289
3952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^
53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801
10694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289
9600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49
*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 14804725213
25168526452382335238144*a^19*b^47*c^47*d^19 + 237012426137933259091623367888
15232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^
45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12695869
829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814015359003912068
2300735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b
^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9
2575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520
388491615907348480*a^28*b^38*c^38*d^28 - 1770361566542500128410491118262681
60*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^3
6*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 5158333
42886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332992628044123
2590010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a
^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d
^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244704257
5399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788720732
9057621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a
^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d
^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 173879220
5355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380586737468979005
3020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^
44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^4
5 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831
080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195304306785535
8132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17
*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745
782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117
809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b
^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 317
76042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150036124817025
92512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9
*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3394215634796515
7513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6
*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950429491527863500
8000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 1
341441243847065600000000*a^64*b^2*c^2*d^64 - 16272772091084800000000*a*b^65*c
^65*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33
```

```
+ 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a
^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*
b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b
^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b
^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^
11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 383038737868800
0*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 15019966174
1875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 5044
13463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18
 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^
13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*
b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 13167616326470860
8*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 197926515948748
80*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 138235173314560
0*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^
30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^3
2*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*
a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14
*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*
d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^4
0*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25
*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^1
8*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 886649475173449
7280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871
332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d
^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*
b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 323968077467222
01600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 1551
6365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d
^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^
9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^
34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736
*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560
*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-
((70527747686400000000*a^66*d^66 + 2748779069440000000*b^66*c^66 + 4645656
529679155200000*a^2*b^64*c^64*d^2 - 85239594962869288960000*a^3*b^63*c^63
*d^3 + 1130310047981633536000000*a^4*b^62*c^62*d^4 - 115488078084729823297
536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 -
6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909
184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 +
742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496
569507840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c
^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467
42584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069462050587268035
3792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50
```

```
*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 91675357311
6017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132516852645238233523
8144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^4
6*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844
3496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445
32998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^4
2*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622
62545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838
869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*
a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d
^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251
79164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334288620561992503970
3580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*
b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34
- 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124
433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935
503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*
b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39
+ 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364
395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912
639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*
b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 -
 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900
743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808162454775635587
89120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c
^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787
56421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829954142086407501
54437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b
^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129
789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279547644479759
4501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*
c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691
8031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*
b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 45597179299
3637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^
4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 13414412438470656000
0000*a^64*b^2*c^2*d^64 - 16272772091084800000000*a*b^65*c^65*d - 43883931893
76000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c
^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 +
106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 156
0936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670
787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 2048
4812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 +
 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^
13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c
```

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^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17
*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 7900653813535375
36*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115
873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 26
1773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24
 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^2
6 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28
 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3
922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*
a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 21
99023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461
056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426
560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062
349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 7228120
72152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4
433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34
*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^2
2*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 3887616929606
6641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38
876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^2
7*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^
29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867
248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 72281
2072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 +
62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27
 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29
 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(288230376151711744000*a^23*b^5
1*c^81*d^4 - 12624490475444497438720*a^24*b^50*c^80*d^5 + 267812136305124484
05504*a^25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 36
48418948406862648705024*a^27*b^47*c^77*d^8 - 28097394584779147947540480*a^2
8*b^46*c^76*d^9 + 174448389309337948351627264*a^29*b^45*c^75*d^10 - 8976689
76119897481466085376*a^30*b^44*c^74*d^11 + 3905920242884630010868531200*a^3
1*b^43*c^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^13 + 4735
5821068790227801756139520*a^33*b^41*c^71*d^14 - 134845524585103937061538234
368*a^34*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*c^69*d^1
6 - 763246944716696111818343448576*a^36*b^38*c^68*d^17 + 154156402718081368
6607638953984*a^37*b^37*c^67*d^18 - 2825288027628089174763608473600*a^38*b^
36*c^66*d^19 + 4753041476272000853590444867584*a^39*b^35*c^65*d^20 - 744626
3493185815677622957375488*a^40*b^34*c^64*d^21 + 110459746117948070279644016
80384*a^41*b^33*c^63*d^22 - 15766681100741571532295786987520*a^42*b^32*c^62
*d^23 + 21882616570434895907374088847360*a^43*b^31*c^61*d^24 - 295554159013
57165913293077872640*a^44*b^30*c^60*d^25 + 38514249364633213650767204843520
*a^45*b^29*c^59*d^26 - 47767982724772003266224581509120*a^46*b^28*c^58*d^27
 + 55618948537155045120476750807040*a^47*b^27*c^57*d^28 - 60127300413664475
479641214156800*a^48*b^26*c^56*d^29 + 59877038998440260638050153922560*a^49
```

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*b^25*c^55*d^30 - 54637051595737047014674020696064*a^50*b^24*c^54*d^31 + 45
519623223064909005599526617088*a^51*b^23*c^53*d^32 - 3453555581655005508525
4958383104*a^52*b^22*c^52*d^33 + 23809729504484309698980012359680*a^53*b^21
*c^51*d^34 - 14885319254535352990241541586944*a^54*b^20*c^50*d^35 + 8419648
339202954390072444583936*a^55*b^19*c^49*d^36 - 4297514831765712413611503124
480*a^56*b^18*c^48*d^37 + 1973123737554130196459440570368*a^57*b^17*c^47*d^
38 - 811770857054497673061303582720*a^58*b^16*c^46*d^39 + 29785838037243937
1596188090368*a^59*b^15*c^45*d^40 - 96910050535770593129744302080*a^60*b^14
*c^44*d^41 + 27758579881177587823480406016*a^61*b^13*c^43*d^42 - 6937474504
476672102869499904*a^62*b^12*c^42*d^43 + 1495682482860276471300096000*a^63*
b^11*c^41*d^44 - 274100118958300866495381504*a^64*b^10*c^40*d^45 + 41867778
463425277028466688*a^65*b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*
c^38*d^47 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 35371992049308254
863360*a^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 364
79156981701017600*a^70*b^4*c^34*d^51)*1i - 18014398509481984000*a^21*b^51*c
^78*d^4 + 778222015609621708800*a^22*b^50*c^77*d^5 - 1619998829160695857152
0*a^23*b^49*c^76*d^6 + 216629339029608119402496*a^24*b^48*c^75*d^7 - 209289
9704349501998235648*a^25*b^47*c^74*d^8 + 15576808854093856430358528*a^26*b^
46*c^73*d^9 - 92989305923335928955273216*a^27*b^45*c^72*d^10 + 457716570390
505153458339840*a^28*b^44*c^71*d^11 - 1895077372829589675098243072*a^29*b^4
3*c^70*d^12 + 6699157107174094796222365696*a^30*b^42*c^69*d^13 - 2045460839
6817467081213607936*a^31*b^41*c^68*d^14 + 54439663857512808688618831872*a^3
2*b^40*c^67*d^15 - 127253623829876322462345461760*a^33*b^39*c^66*d^16 + 263
018360322301930835307134976*a^34*b^38*c^65*d^17 - 4841171484253414616905474
37568*a^35*b^37*c^64*d^18 + 801088032507623116562893897728*a^36*b^36*c^63*d
^19 - 1210191753560658421451373674496*a^37*b^35*c^62*d^20 + 171366215003931
1965148455895040*a^38*b^34*c^61*d^21 - 2368456612874860634985065349120*a^39
*b^33*c^60*d^22 + 3342440882817901253619697582080*a^40*b^32*c^59*d^23 - 492
6019419281526710422764257280*a^41*b^31*c^58*d^24 + 7443043331925552222767653
5848960*a^42*b^30*c^57*d^25 - 11053384984245852600223452364800*a^43*b^29*c^
56*d^26 + 15529000135185248373347985653760*a^44*b^28*c^55*d^27 - 2015380102
6888464482649904250880*a^45*b^27*c^54*d^28 + 238708210247914370726198299852
80*a^46*b^26*c^53*d^29 - 25662407141873741853910169026560*a^47*b^25*c^52*d^
30 + 24983334964938085602226308382720*a^48*b^24*c^51*d^31 - 220033683614559
69032835868655616*a^49*b^23*c^50*d^32 + 17519758513327663391847122731008*a^
50*b^22*c^49*d^33 - 12601896285489986596049610866688*a^51*b^21*c^48*d^34 +
8179684390414915120451536551936*a^52*b^20*c^47*d^35 - 478358308111636045496
0515645440*a^53*b^19*c^46*d^36 + 2515171747726250254399514345472*a^54*b^18*
c^45*d^37 - 1185710361511816082146770026496*a^55*b^17*c^44*d^38 + 499406604
618358594580969947136*a^56*b^16*c^43*d^39 - 187097254447826761775602204672*
a^57*b^15*c^42*d^40 + 62002233932522145150727618560*a^58*b^14*c^41*d^41 - 1
8049115872947548566748921856*a^59*b^13*c^40*d^42 + 457518739274140803421490
3808*a^60*b^12*c^39*d^43 - 998642414508019303179091968*a^61*b^11*c^38*d^44
+ 184986735996381058748645376*a^62*b^10*c^37*d^45 - 28520139033328990436720
640*a^63*b^9*c^36*d^46 + 3562072173311951854632960*a^64*b^8*c^35*d^47 - 346
```

```
3778638686920376320000*a^65*b^7*c^34*d^48 + 24611841230482125619200*a^66*b^6
*c^33*d^49 - 1137123721538961408000*a^67*b^5*c^32*d^50 + 256494072527585280
00*a^68*b^4*c^31*d^51)*1i) + (-((70527747686400000000*a^66*d^66 + 274877906
94400000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^64*d^2 - 8523959
49628692889600000*a^3*b^63*c^63*d^3 + 113031004798163353600000*a^4*b^62*c
^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 94660933391357814
5788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d
^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 17682391555307866775
7483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^5
6*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864
3010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*
a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14
- 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237796
4853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^
17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 -
 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793325
90916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*
a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^2
2 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814015
3590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189441
69984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c
^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947
384930724520388491615907348480*a^28*b^38*c^38*d^28 - 1770361566542500128410
49111826268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a
^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^
31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332
9926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056469
8912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35
*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36
 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974
5327887207329057621409792*a^38*b^28*c^28*d^38 - 273939082748055449346653497
9194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40
*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41
 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380586
7374689790053020810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710
073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b
^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 -
154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195
3043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355988
48*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16
*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466
014670506117809664163840*a^52*b^14*c^14*d^52 - 4763713185671452589806061617
15200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^1
2*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150
03612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*
```

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a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3394
21563479651575136256000000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*
a^60*b^6*c^6*d^60 - 4559717929936371056640000000*a^61*b^5*c^5*d^61 + 3950429
491527863500800000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*
c^3*d^63 + 1341441243847065600000000*a^64*b^2*c^2*d^64 - 1627277209108480000
000*a*b^65*c^65*d - 43883931893760000000000*a^65*b*c*d^65)^(1/2) + 839808000
0*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 1813
4996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 46643626
6917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301
742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747
787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362
011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 383
0387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 +
 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17
*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^
15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*
a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780
569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 13167
6163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 197
92651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 138
2351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757
896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 15518924800
0*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 +
68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552
*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^1
2*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a
^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 2312998630887260
16*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 19274988590
72716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 886
6494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*
d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23
*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070
807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 323
96807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26
*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30
*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 192749885907271
6800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863
088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838
453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340
848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)
*(x^(1/2)*(4851701160433680384000*a^21*b^45*c^62*d^11 - 1342531185305190400
00*a^20*b^46*c^63*d^10 - 83128151546809181798400*a^22*b^44*c^61*d^12 + 8959
10897914030472560640*a^23*b^43*c^60*d^13 - 6797129989654957642481664*a^24*b
^42*c^59*d^14 + 38483630548489971632701440*a^25*b^41*c^58*d^15 - 1679618150
50671342785396736*a^26*b^40*c^57*d^16 + 573748019559978603695308800*a^27*b^
```

```
39*c^56*d^17 - 1529836010901462206864424960*a^28*b^38*c^55*d^18 + 307515311
0865358700094160896*a^29*b^37*c^54*d^19 - 4044511032981169371925708800*a^30
*b^36*c^53*d^20 + 589590639381102819104784384*a^31*b^35*c^52*d^21 + 1457667
1334338745969651220480*a^32*b^34*c^51*d^22 - 50149146156756356561350164480*
a^33*b^33*c^50*d^23 + 110550157926715904989065117696*a^34*b^32*c^49*d^24 -
189331360528461979941957795840*a^35*b^31*c^48*d^25 + 2673835273737481924339
44920064*a^36*b^30*c^47*d^26 - 319821143985825066443750768640*a^37*b^29*c^4
6*d^27 + 328626898447261055168230195200*a^38*b^28*c^45*d^28 - 2924345607965
58751919058714624*a^39*b^27*c^44*d^29 + 226382416482170290892093521920*a^40
*b^26*c^43*d^30 - 152776304398053739659930894336*a^41*b^25*c^42*d^31 + 8990
1124622673343064718704640*a^42*b^24*c^41*d^32 - 460625089648204264791814963
20*a^43*b^23*c^40*d^33 + 20486606263737610091045584896*a^44*b^22*c^39*d^34
- 7870914323775054351244984320*a^45*b^21*c^38*d^35 + 2594141724382360002965
274624*a^46*b^20*c^37*d^36 - 726451024651952784807034880*a^47*b^19*c^36*d^3
7 + 170590060365885174888529920*a^48*b^18*c^35*d^38 - 329863435542048981123
07200*a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^50*b^16*c^33*d^40 -
 613036163719885750272000*a^51*b^15*c^32*d^41 + 53255297770998202368000*a^5
2*b^14*c^31*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^43 + 81438120439
971840000*a^54*b^12*c^29*d^44) + (-((705277476864000000000*a^66*d^66 + 27487
790694400000000*b^66*c^66 + 46456565296791552000000*a^2*b^64*c^64*d^2 - 852
395949628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335360000000*a^4*b^
62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 9466093339135
78145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^
59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 1768239155530786
67757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56
*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 87608488
38643010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685
312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d
^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 2914327130323
77964853953403289600*a^16*b^50*c^50*d^16 - 53837688933932732209219051192320
0*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^
18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 2370124261379
332590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399
680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44
*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081
40153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918
944169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^
40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 12
9947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012
84104911826268160*a^29*b^37*c^37*d^29 + 2431372713606781682807248874425548
80*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^3
5*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 7754680
73329926280441232590010056704*a^33*b^33*c^33*d^33 + 11365474000985030910505
64698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*
a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*
```

```
d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 26989809
39745327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274805544934665
34979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*
a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*
d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 12660138
05867374689790053020810084352*a^43*b^23*c^23*d^43 + 84844675058024454799136
1710073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^
45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^4
6 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 739174514721
71953043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435
598848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*
c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 157896
5466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606
161715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12
*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 6948683
615003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800
000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 -
33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800
000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 395
04294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*
b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162727720910848
0000000*a*b^65*c^65*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 83980
80000*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 -
18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 4664
36266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 411189
2301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 1479391
7747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 2252
9362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 +
 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^
14 + 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*
c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^1
8*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412
736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 44081317
0780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 1
31676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 -
 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 -
 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 3
7757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 1551892
48000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^4
5 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 219902325
5552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560
*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 138384533471887
36*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088
726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498
859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 -
```

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 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c
^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*
a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 4130592987
7070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 +
 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*
c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*
a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 19274988590
72716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 23129
9863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 1
3838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 -
 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(
3/4)*(x^(1/2)*(-((70527747686400000000*a^66*d^66 + 2748779069440000000*b^6
6*c^66 + 464565652967915520000000*a^2*b^64*c^64*d^2 - 8523959496286928896000
00*a^3*b^63*c^63*d^3 + 1130310047981633536000000*a^4*b^62*c^62*d^4 - 11548
8078084729823297536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*
b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*d^7 + 3639438050
7592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*
b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^56*d^10 - 272041
5842900866890496569507840*a^11*b^55*c^55*d^11 + 876084883864301071819289395
2000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*
d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14 - 1431337801106
94620505872680353792*a^15*b^51*c^51*d^15 + 29143271303237796485395340328960
0*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^
17 + 916753573116017703850321517740032*a^18*b^48*c^48*d^18 - 14804725213251
68526452382335238144*a^19*b^47*c^47*d^19 + 23701242613793325909162336788152
32*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*
d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^22 - 12695869829
017232408306844532998144*a^23*b^43*c^43*d^23 + 2282940814015359003912068230
0735488*a^24*b^42*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41
*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*c^40*d^26 - 9257
5964607062084838869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388
491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*
a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d
^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 5158333428
86205619925039703580999680*a^32*b^34*c^34*d^32 - 77546807332992628044123259
0010056704*a^33*b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34
*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35
 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^36 - 244704257539
9654362397243935503155200*a^37*b^29*c^29*d^37 + 269898093974532788720732905
7621409792*a^38*b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39
*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40
 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^41 + 173879220535
5133034582544912639590400*a^42*b^24*c^24*d^42 - 126601380586737468979005302
0810084352*a^43*b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*
b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 +
```

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 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 - 154586253831080
816245477563558789120*a^47*b^19*c^19*d^47 + 7391745147217195304306785535813
2224*a^48*b^18*c^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^
17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^16*d^50 - 4745782
995414208640750154437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809
664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13
*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^12*d^54 - 317760
42795476444797594501120000*a^55*b^11*c^11*d^55 + 69486836150036124817025925
12000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^
57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 3394215634796515751
3625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^
60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 3950429491527863500800
0000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 1341
44124384706560000000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65
*d - 438839318937600000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5
242880000*b^33*c^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*
b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^2
8*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26
*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24
*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*
b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a
^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14 + 15019966174187
5200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 5044134
63173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 +
790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*
d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^1
1*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a
^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*
a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a
^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*
b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d
 - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^4
1*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^
31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3
 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d
^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^
38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b
^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 886649475173449728
0*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332
023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14
 - 38876169296066641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^1
6*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 323968077467222016
00*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 1551636
5815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21
 + 4433247375867248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c
```

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^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*
b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^
36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^
38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4)*(2882303761517
1174400*a^23*b^51*c^81*d^4 - 1262449047544497438720*a^24*b^50*c^80*d^5 + 26
781213630512448405504*a^25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^
48*c^78*d^7 + 3648418948406862648705024*a^27*b^47*c^77*d^8 - 28097394584779
147947540480*a^28*b^46*c^76*d^9 + 174448389309337948351627264*a^29*b^45*c^7
5*d^10 - 897668976119897481466085376*a^30*b^44*c^74*d^11 + 390592024288463
0010868531200*a^31*b^43*c^73*d^12 - 14590896425075765379929735168*a^32*b^42*
c^72*d^13 + 47355821068790227801756139520*a^33*b^41*c^71*d^14 - 13484552458
5103937061538234368*a^34*b^40*c^70*d^15 + 339727096730086714108763176960*a^
35*b^39*c^69*d^16 - 763246944716696111818343448576*a^36*b^38*c^68*d^17 + 15
41564027180813686607638953984*a^37*b^37*c^67*d^18 - 282528802762808917476360
8473600*a^38*b^36*c^66*d^19 + 4753041476272000853590444867584*a^39*b^35*c^
65*d^20 - 7446263493185815677622957375488*a^40*b^34*c^64*d^21 + 11045974611
794807027964401680384*a^41*b^33*c^63*d^22 - 157666811007415715322957869875
20*a^42*b^32*c^62*d^23 + 21882616570434895907374088847360*a^43*b^31*c^61*d^2
4 - 29555415901357165913293077872640*a^44*b^30*c^60*d^25 + 385142493646332
13650767204843520*a^45*b^29*c^59*d^26 - 47767982724772003266224581509120*a^4
6*b^28*c^58*d^27 + 55618948537155045120476750807040*a^47*b^27*c^57*d^28 - 6
0127300413664475479641214156800*a^48*b^26*c^56*d^29 + 598770389984402606380
50153922560*a^49*b^25*c^55*d^30 - 54637051595737047014674020696064*a^50*b^2
4*c^54*d^31 + 45519623223064909005599526617088*a^51*b^23*c^53*d^32 - 345355
55816550055085254958383104*a^52*b^22*c^52*d^33 + 23809729504484309698980012
359680*a^53*b^21*c^51*d^34 - 14885319254535352990241541586944*a^54*b^20*c^5
0*d^35 + 8419648339202954390072444583936*a^55*b^19*c^49*d^36 - 429751483176
5712413611503124480*a^56*b^18*c^48*d^37 + 1973123737554130196459440570368*a
^57*b^17*c^47*d^38 - 811770857054497673061303582720*a^58*b^16*c^46*d^39 + 2
97858380372439371596188090368*a^59*b^15*c^45*d^40 - 96910050535770593129744
302080*a^60*b^14*c^44*d^41 + 27758579881177587823480406016*a^61*b^13*c^43*d
^42 - 6937474504476672102869499904*a^62*b^12*c^42*d^43 + 149568248286027647
1300096000*a^63*b^11*c^41*d^44 - 274100118958300866495381504*a^64*b^10*c^40
*d^45 + 41867778463425277028466688*a^65*b^9*c^39*d^46 - 518716113093076359
4727424*a^66*b^8*c^38*d^47 + 500879902205011065569280*a^67*b^7*c^37*d^48 - 3
5371992049308254863360*a^68*b^6*c^36*d^49 + 1625349105518012006400*a^69*b^5
*c^35*d^50 - 36479156981701017600*a^70*b^4*c^34*d^51)*1i + 1801439850948198
4000*a^21*b^51*c^78*d^4 - 778222015609621708800*a^22*b^50*c^77*d^5 + 161999
8829160695857 1520*a^23*b^49*c^76*d^6 - 216629339029608119402496*a^24*b^48*c
^75*d^7 + 2092899704349501998235648*a^25*b^47*c^74*d^8 - 155768088540938564
30358528*a^26*b^46*c^73*d^9 + 92989305923335928955273216*a^27*b^45*c^72*d^1
0 - 457716570390505153458339840*a^28*b^44*c^71*d^11 + 189507737282958967509
8243072*a^29*b^43*c^70*d^12 - 6699157107174094796222365696*a^30*b^42*c^69*d
^13 + 20454608396817467081213607936*a^31*b^41*c^68*d^14 - 5443966385751280
8688618831872*a^32*b^40*c^67*d^15 + 127253623829876322462345461760*a^33*b^39
```

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*c^66*d^16 - 263018360322301930835307134976*a^34*b^38*c^65*d^17 + 484117148
4253414616905474375 68*a^35*b^37*c^64*d^18 - 801088032507623116562893897728*
a^36*b^36*c^63*d^19 + 1210191753560658421451373674496*a^37*b^35*c^62*d^20 -
 1713662150039311965148455895040*a^38*b^34*c^61*d^21 + 23684566128748606349
85065349120*a^39*b^33*c^60*d^22 - 3342440882817901253619697582080*a^40*b^32
*c^59*d^23 + 4926019419281526710422764257280*a^41*b^31*c^58*d^24 - 74430433
31925522227676535848960*a^42*b^30*c^57*d^25 + 110533849842458526002234523 64
800*a^43*b^29*c^56*d^26 - 15529000135185248373347985653760*a^44*b^28*c^55*d
^27 + 20153801026888464482649904250880*a^45*b^27*c^54*d^28 - 23870821024791
437072619829985280*a^46*b^26*c^53*d^29 + 25662407141873741853910169026560*a
^47*b^25*c^52*d^30 - 24983334964938085602226308382720*a^48*b^24*c^51*d^31 +
 22003368361455969032835868655616*a^49*b^23*c^50*d^32 - 17519758513327663 39
1847122731008*a^50*b^22*c^49*d^33 + 12601896285489986596049610866688*a^51*b
^21*c^48*d^34 - 8179684390414915120451536551936*a^52*b^20*c^47*d^35 + 47835
83081116360454960515645440*a^53*b^19*c^46*d^36 - 2515171747726250254399514 3
45472*a^54*b^18*c^45*d^37 + 1185710361511816082146770026496*a^55*b^17*c^44*
d^38 - 499406604618358594580969947136*a^56*b^16*c^43*d^39 + 187097254447826
761775602204672*a^57*b^15*c^42*d^40 - 62002233932522145150727618560*a^58*b^
14*c^41*d^41 + 18049115872947548566748921856*a^59*b^13*c^40*d^42 - 45751873
92741408034214903808*a^60*b^12*c^39*d^43 + 998642414508019303179091968*a^61
*b^11*c^38*d^44 - 184986735996381058748645376*a^62*b^10*c^37*d^45 + 2852013
9033328990436720640*a^63*b^9*c^36*d^46 - 3562072173311951854632960*a^64*b^8
*c^35*d^47 + 346377863868692037632000*a^65*b^7*c^34*d^48 - 2461184123048212
5619200*a^66*b^6*c^33*d^49 + 1137123721538961408000*a^67*b^5*c^32*d^50 - 25
649407252758528000*a^68*b^4*c^31*d^51)*1i))/((-((705277476864000000000*a^66*
d^66 + 274877906944000000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^
64*d^2 - 8523959496286928896 00000*a^3*b^63*c^63*d^3 + 113031004798163353600
00000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 9
46609333913578145788723200*a^6*b^60*c^60*d^6 - 6398838206349744593468129280
*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 1768
23915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941 5
04*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^1
1 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 249552350040826 18
707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14
*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 2
91432713032377964853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322 09
2190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*
b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 + 2
370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255080 1
466936451399680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^
22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23
 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 39022498460407
159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667528366853403 4
4320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^
39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036
```

```
156654250012841049111826268160*a^29*b^37*c^37*d^29 + 2431372713606781682807
24887442554880*a^30*b^36*c^36*d^30 - 34711352517916424353692724892794880 0*a
^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^
32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 11365474000
98503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755 0
20010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^3
6*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^3
7 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274
80554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868 2
36513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^4
1*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^4
2 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 84844675058
0244547991361710073053184*a^44*b^22*c^22*d^44 - 5231970598647866372746393 63
737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b
^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 +
73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 32387372581952477
787555393435598848*a^49*b^17*c^17*d^49 + 1297875642151239082178936230536806
4*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d
^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567
145258980606161715200*a^53*b^13*c^13*d^53 + 1297898094790687573306431766528
00*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^
55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091
154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8
*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982
240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^
5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840
000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 162
7277209108480000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(
1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*b^31
*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^2
9*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*
d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d
^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23
*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21
*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*
b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 30657507805716480
0*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 6887985648
47943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766
159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^2
1 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c
^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^
8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^
6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*
c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^
31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*
```

```
a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d
 - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340
848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 138
38453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 -
231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d
^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22
*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240
*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 323968077
46722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15
+ 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15
*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 2387133202390056960
0*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 88664947
51734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 -
 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*
d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c
^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4
*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c
^15*d^30))^(1/4)*(x^(1/2)*(4851701160433680384000*a^21*b^45*c^62*d^11 - 134
253118530519040000*a^20*b^46*c^63*d^10 - 83128151546809181798400*a^22*b^44*
c^61*d^12 + 895910897914030472560640*a^23*b^43*c^60*d^13 - 6797129989654957
642481664*a^24*b^42*c^59*d^14 + 38483630548489971632701440*a^25*b^41*c^58*d
^15 - 167961815050671342785396736*a^26*b^40*c^57*d^16 + 5737480195599786036
95308800*a^27*b^39*c^56*d^17 - 1529836010901462206864424960*a^28*b^38*c^55*
d^18 + 3075153110865358700094160896*a^29*b^37*c^54*d^19 - 40445110329811693
71925708800*a^30*b^36*c^53*d^20 + 589590639381102819104784384*a^31*b^35*c^5
2*d^21 + 14576671334338745969651220480*a^32*b^34*c^51*d^22 - 50149146156756
356561350164480*a^33*b^33*c^50*d^23 + 110550157926715904989065117696*a^34*b
^32*c^49*d^24 - 189331360528461979941957795840*a^35*b^31*c^48*d^25 + 267383
527373748192433944920064*a^36*b^30*c^47*d^26 - 319821143985825066437507686
40*a^37*b^29*c^46*d^27 + 328626898447261055168230195200*a^38*b^28*c^45*d^28
 - 292434560796558751919058714624*a^39*b^27*c^44*d^29 + 2263824164821702908
92093521920*a^40*b^26*c^43*d^30 - 152776304398053739659930894336*a^41*b^25*
c^42*d^31 + 89901124622673343064718704640*a^42*b^24*c^41*d^32 - 46062508964
820426479181496320*a^43*b^23*c^40*d^33 + 20486606263737610091045584896*a^44
*b^22*c^39*d^34 - 7870914323775054351244984320*a^45*b^21*c^38*d^35 + 259414
1724382360002965274624*a^46*b^20*c^37*d^36 - 726451024651952784807034880*a^
47*b^19*c^36*d^37 + 170590060365885174888529920*a^48*b^18*c^35*d^38 - 32986
343554204898112307200*a^49*b^17*c^34*d^39 + 5118063591384977873305600*a^50*
b^16*c^33*d^40 - 613036163719885750272000*a^51*b^15*c^32*d^41 + 53255297770
998202368000*a^52*b^14*c^31*d^42 - 2988725792617267200000*a^53*b^13*c^30*d^
43 + 81438120439971840000*a^54*b^12*c^29*d^44) + (-((70527747686400000000*a
^66*d^66 + 274877906944000000000*b^66*c^66 + 4645656529679155200000*a^2*b^6
4*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11303100479816335
360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5
 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820634974459346812
```

```
9280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 -
176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721
941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55
*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 2495523500408
2618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*
a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15
 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5383768893393273
22092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a
^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19
 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255
0801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345870323761283072
0*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*
d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 3902249846
0407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667528366853
40344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^3
9*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 17
7036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243137271360678168
280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435369272489279488
00*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^3
4*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 1136547
400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133
755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840
*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29
*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 2739390
827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359
868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000
*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24
*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 8484467
50580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319705986478663727463
9363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^
46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^47*b^19*c^19*d^4
7 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48 - 3238737258195
2477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123908217893623053
68064*a^50*b^16*c^16*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^
15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*d^52 - 47637131
8567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176
652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^1
1*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 134721865560
4091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58
*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 429545687
9982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^
5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 268379484005597
1840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 -
 1627277209108480000000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^6
5)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 + 2133642444800*a^2*
```

```
b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29
*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c
^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^
25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*
c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*
b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a
^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 3065750780571
64800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798
564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 -
 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12
*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^
10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^2
5*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^2
7*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*
b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^
2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476
736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^
44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 -
 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 -
 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^
6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^
37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*
b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 155163658155353 7
0240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396
807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d
^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*
b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900 5
69600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866
494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^
22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c
^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b
^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37
*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b
^2*c^15*d^30))^(3/4)*(x^(1/2)*(-((70527747686400000000*a^66*d^66 + 27487790
694400000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^64*d^2 - 852395
9496286928896000000*a^3*b^63*c^63*d^3 + 1130310047981633536000000*a^4*b^62*
c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5 + 9466093339135781
4578872320 0*a^6*b^60*c^60*d^6 - 6398838206349744593468129280*a^7*b^59*c^59*
d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 - 176823915553078667 7
57483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721941504*a^10*b^56*c^
56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55*d^11 + 87608488386
43010718192893952000*a^12*b^54*c^54*d^12 - 24955235004082618707041228685312
*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^52*c^52*d^14
 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 291432713032377 9
6485395340328960 0*a^16*b^50*c^50*d^16 - 538376889339327322092190511923200*a
```

```
^17*b^49*c^49*d^17 + 9167535731160177038503215177400032*a^18*b^48*c^48*d^18
- 14804725213251685264523823352381 44*a^19*b^47*c^47*d^19 + 2370124261379332
590916233678815232*a^20*b^46*c^46*d^20 - 3945682050382550801466936451399680
*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720*a^22*b^44*c^44*d^
22 - 12695869829017232408306844532998144*a^23*b^43*c^43*d^23 + 228294081401
53590039120682300735488*a^24*b^42*c^42*d^24 - 390224984604071598537729189 44
169984*a^25*b^41*c^41*d^25 + 62262545797041866752836685340344320*a^26*b^40*
c^40*d^26 - 92575964607062084838869289496739840*a^27*b^39*c^39*d^27 + 12994
7384930724520388491615907348480*a^28*b^38*c^38*d^28 - 177036156654250012841
049111826268160*a^29*b^37*c^37*d^29 + 243137271360678168280724887442554880*
a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d
^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 7754680733
29926280441232590010056704*a^33*b^33*c^33*d^33 + 113654740009850309105056 46
98912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133755020010061824*a^3
5*b^31*c^31*d^35 + 2044085060124433072578392630325411840*a^36*b^30*c^30*d^3
6 - 2447042575399654362397243935503155200*a^37*b^29*c^29*d^37 + 26989809397
45327887207329057621409792*a^38*b^28*c^28*d^38 - 27393908274805544934665349
79194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359868236513411072*a^4
0*b^26*c^26*d^40 - 2198323007364395998582415976038400000*a^41*b^25*c^25*d^4
1 + 1738792205355133034582544912639590400*a^42*b^24*c^24*d^42 - 12660138058
67374689790053020810084352*a^43*b^23*c^23*d^43 + 8484467505802445479913617 1
0073053184*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*
b^21*c^21*d^45 + 296692444664900743443383822718074880*a^46*b^20*c^20*d^46 -
 154586253831080816245477563558789120*a^47*b^19*c^19*d^47 + 739174514721719
53043067855358132224*a^48*b^18*c^18*d^48 - 323873725819524777875553934355 98
848*a^49*b^17*c^17*d^49 + 12978756421512390821789362305368064*a^50*b^16*c^1
6*d^50 - 4745782995414208640750154437099520*a^51*b^15*c^15*d^51 + 157896546
6014670506117809664163840*a^52*b^14*c^14*d^52 - 476371318567145258980606161
715200*a^53*b^13*c^13*d^53 + 129789809479068757330643176652800*a^54*b^12*c^
12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^11*d^55 + 6948683615
003612481702592512000*a^56*b^10*c^10*d^56 - 1347218655604091154910412800000
*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58*b^8*c^8*d^58 - 339
42156347965157513625600000*a^59*b^7*c^7*d^59 + 4295456879982240124108800000
*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^5*c^5*d^61 + 395042
94915278635008000000*a^62*b^4*c^4*d^62 - 2683794840055971840000000*a^63*b^3
*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 - 16272772091084800 0
0000*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^65)^(1/2) + 83980800
00*a^33*d^33 + 5242880000*b^33*c^33 + 213364244480 0*a^2*b^31*c^31*d^2 - 181
34996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29*c^29*d^4 - 4664362
66917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c^27*d^6 - 411189230
1742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^25*d^8 - 1479391774
7787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*c^23*d^10 - 2252936
2011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*b^21*c^21*d^12 + 38
30387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a^14*b^19*c^19*d^14
+ 150199661741875200*a^15*b^18*c^18*d^15 - 306575078057164800*a^16*b^17*c^1
```

```
7*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798564847943680*a^18*b
^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 - 766159267095412736
*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12*d^21 - 44081317078
0569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^10*c^10*d^23 - 1316
76163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^25*b^8*c^8*d^25 - 19
792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^27*b^6*c^6*d^27 - 13
82351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*b^4*c^4*d^29 - 3775
7896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^2*d^31 - 1551892480
00*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 +
 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^44*d - 219902325555
2*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 - 340848604610560*a^
12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 - 13838453347188736*
a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^6 - 231299863088726
016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^37*d^8 - 1927498859
072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*b^22*c^35*d^10 - 88
66494751734497280*a^20*b^21*c^34*d^11 + 15516365815535370240*a^21*b^20*c^33
*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396807746722201600*a^2
3*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d^15 + 4130592987707
0807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*b^15*c^28*d^17 + 32
396807746722201600*a^27*b^14*c^27*d^18 - 23871332023900569600*a^28*b^13*c^2
6*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866494751734497280*a^3
0*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^22 - 19274988590727
16800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c^21*d^24 - 23129986
3088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b^6*c^19*d^26 - 1383
8453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37*b^4*c^17*d^28 - 34
0848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4
)*(28823037615171174400*a^23*b^51*c^81*d^4 - 1262449047544497438720*a^24*b^
50*c^80*d^5 + 26781213630512448405504*a^25*b^49*c^79*d^6 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 3648418948406862648705024*a^27*b^47*c^77*d^8
- 28097394584779147947540480*a^28*b^46*c^76*d^9 + 174448389309337948351627264*a^29*b^45*c^75*d^10 - 897668976119897481466085376*a^30*b^44*c^74*d^11 +
3905920242884630010868531200*a^31*b^43*c^73*d^12 - 14590896425075765379929735168*a^32*b^42*c^72*d^13 + 47355821068790227801756139520*a^33*b^41*c^71*d^
14 - 134845524585103937061538234368*a^34*b^40*c^70*d^15 + 339727096730086714108763176960*a^35*b^39*c^69*d^16 - 763246944716696111818343448576*a^36*b^3
8*c^68*d^17 + 1541564027180813686607638953984*a^37*b^37*c^67*d^18 - 2825288
027628089174763608473600*a^38*b^36*c^66*d^19 + 4753041476272000853590444867
584*a^39*b^35*c^65*d^20 - 7446263493185815677622957375488*a^40*b^34*c^64*d^
21 + 11045974611794807027964401680384*a^41*b^33*c^63*d^22 - 157666811007415
71532295786987520*a^42*b^32*c^62*d^23 + 21882616570434895907374088847360*a^
43*b^31*c^61*d^24 - 29555415901357165913293077872640*a^44*b^30*c^60*d^25 +
38514249364633213650767204843520*a^45*b^29*c^59*d^26 - 4776798272477200326
6224581509120*a^46*b^28*c^58*d^27 + 55618948537155045120476750807040*a^47*b^
27*c^57*d^28 - 60127300413664475479641214156800*a^48*b^26*c^56*d^29 + 59877
038998440260638050153922560*a^49*b^25*c^55*d^30 - 5463705159573704701467402
```

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0696064*a^50*b^24*c^54*d^31 + 45519623223064909005599526617088*a^51*b^23*c^
53*d^32 - 34535555816550055085254958383104*a^52*b^22*c^52*d^33 + 2380972950
4484309698980012359680*a^53*b^21*c^51*d^34 - 148853192545353529902415415869
44*a^54*b^20*c^50*d^35 + 8419648339202954390072444583936*a^55*b^19*c^49*d^3
6 - 4297514831765712413611503124480*a^56*b^18*c^48*d^37 + 19731237375541301
96459440570368*a^57*b^17*c^47*d^38 - 811770857054497673061303582720*a^58*b^
16*c^46*d^39 + 297858380372439371596188090368*a^59*b^15*c^45*d^40 - 9691005
0535770593129744302080*a^60*b^14*c^44*d^41 + 27758579881177587823480406016*
a^61*b^13*c^43*d^42 - 6937474504476672102869499904*a^62*b^12*c^42*d^43 + 14
95682482860276471300096000*a^63*b^11*c^41*d^44 - 274100118958300866495381 50
4*a^64*b^10*c^40*d^45 + 41867778463425277028466688*a^65*b^9*c^39*d^46 - 518
7161130930763594727424*a^66*b^8*c^38*d^47 + 500879902205011065569280*a^67*b
^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c^36*d^49 + 162534910551801
2006400*a^69*b^5*c^35*d^50 - 36479156981701017600*a^70*b^4*c^34*d^51)*1i -
18014398509481984000*a^21*b^51*c^78*d^4 + 778222015609621708800*a^22*b^50*c
^77*d^5 - 16199988291606958571520*a^23*b^49*c^76*d^6 + 2166293390296081194O
2496*a^24*b^48*c^75*d^7 - 2092899704349501998235648*a^25*b^47*c^74*d^8 + 15
576808854093856430358528*a^26*b^46*c^73*d^9 - 92989305923335928955273216*a^
27*b^45*c^72*d^10 + 457716570390505153458339840*a^28*b^44*c^71*d^11 - 18950
77372829589675098243072*a^29*b^43*c^70*d^12 + 6699157107174094796222365696*
a^30*b^42*c^69*d^13 - 20454608396817467081213607936*a^31*b^41*c^68*d^14 + 5
4439663857512808688618831872*a^32*b^40*c^67*d^15 - 127253623829876322462345
461760*a^33*b^39*c^66*d^16 + 263018360322301930835307134976*a^34*b^38*c^65*
d^17 - 484117148425341461690547437568*a^35*b^37*c^64*d^18 + 801088032507623
116562893897728*a^36*b^36*c^63*d^19 - 1210191753560658421451373674496*a^37*
b^35*c^62*d^20 + 1713662150039311965148455895040*a^38*b^34*c^61*d^21 - 2368
456612874860634985065349120*a^39*b^33*c^60*d^22 + 334244088281790125361969 7
582080*a^40*b^32*c^59*d^23 - 4926019419281526710422764257280*a^41*b^31*c^58
*d^24 + 7443043331925522227676535848960*a^42*b^30*c^57*d^25 - 1105338498424
5852600223452364800*a^43*b^29*c^56*d^26 + 15529000135185248373347985653760*
a^44*b^28*c^55*d^27 - 20153801026888464482649904250880*a^45*b^27*c^54*d^28
+ 23870821024791437072619829985280*a^46*b^26*c^53*d^29 - 256624071418737418
53910169026560*a^47*b^25*c^52*d^30 + 24983334964938085602226308382720*a^48*
b^24*c^51*d^31 - 22003368361455969032835868655616*a^49*b^23*c^50*d^32 + 175
19758513327663391847122731008*a^50*b^22*c^49*d^33 - 126018962854899865960 49
610866688*a^51*b^21*c^48*d^34 + 8179684390414915120451536551936*a^52*b^20*c
^47*d^35 - 4783583081116360454960515645440*a^53*b^19*c^46*d^36 + 2515171747
7262502543995143 45472*a^54*b^18*c^45*d^37 - 118571036151181608214677002649 6
*a^55*b^17*c^44*d^38 + 499406604618358594580969947136*a^56*b^16*c^43*d^39 -
 187097254447826761775602204672*a^57*b^15*c^42*d^40 + 62002233932522145150 7
27618560*a^58*b^14*c^41*d^41 - 18049115872947548566748921856*a^59*b^13*c^40
*d^42 + 4575187392741408034214903808*a^60*b^12*c^39*d^43 - 998642414508019 3
03179091968*a^61*b^11*c^38*d^44 + 184986735996381058748645376*a^62*b^10*c^3
7*d^45 - 28520139033328990436720640*a^63*b^9*c^36*d^46 + 356207217331195185
4632960*a^64*b^8*c^35*d^47 - 346377863868692037632000*a^65*b^7*c^34*d^48 +
```

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24611841230482125619200*a^66*b^6*c^33*d^49 - 1137123721538961408000*a^67*b^
5*c^32*d^50 + 25649407252758528000*a^68*b^4*c^31*d^51)*1i)*1i - (-((7052774
7686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 4645656529679155
2000000*a^2*b^64*c^64*d^2 - 852395949628692889600000*a^3*b^63*c^63*d^3 + 11
303100479816335360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^
5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820
6349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b
^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127
574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840
*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12
- 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467425843637
99641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15
*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5
38376889339327322092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850
321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19
*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 -
3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345
8703237612830720*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*
a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^
24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970
41866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496
739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38
*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243
137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251791642435
36927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33
*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683
304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578
392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200
*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28
*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145
757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582
415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400
*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23
*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 - 52319705
9864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383
822718074880*a^46*b^20*c^20*d^46 - 154586253831080816245477563558789120*a^4
7*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d^48
- 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123
90821789362305368064*a^50*b^16*c^16*d^50 - 47457829954142086407501544370995
20*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c^14*
d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129789809479
068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279547644479759450112000
0*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56
 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691803197496
```

```
3609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256000000*a^59*b^7*c^7*d
^59 + 429545687998224012410880000*a^60*b^6*c^6*d^60 - 455971792993637105664
4000000*a^61*b^5*c^5*d^61 + 395042949152786350080000000*a^62*b^4*c^4*d^62 -
2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64
*b^2*c^2*d^64 - 162727720910848000000*a*b^65*c^65*d - 4388393189376000000
00*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 + 213
3642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213
378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 156093640605
6960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777
600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 2048481280113
0496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795
101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 5305
8143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15
- 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^1
6*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b
^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800
*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 26177390393
6962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 558254
96115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 58011
73668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325
798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816
000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*
d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 21990232555
52*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*
b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*
b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^
15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 7228120721522688
00*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375
867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 1
5516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^
32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a
^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296
066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 -
23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c
^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^
31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 72281207215226
8800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 622730400
62349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 247115
2383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 340848
60461056*a^39*b^2*c^15*d^30))^(1/4)*(x^(1/2)*(485170116043368038400 0*a^21*b
^45*c^62*d^11 - 134253118530519040000*a^20*b^46*c^63*d^10 - 831281515468091
81798400*a^22*b^44*c^61*d^12 + 895910897914030472560640*a^23*b^43*c^60*d^13
 - 6797129989654957642481664*a^24*b^42*c^59*d^14 + 384836305484899716327014
40*a^25*b^41*c^58*d^15 - 167961815050671342785396736*a^26*b^40*c^57*d^16 +
573748019559978603695308800*a^27*b^39*c^56*d^17 - 1529836010901462206864424
```

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960*a^28*b^38*c^55*d^18 + 3075153110865358700094160896*a^29*b^37*c^54*d^19
- 4044511032981169371925708800*a^30*b^36*c^53*d^20 + 5895906393811028191047
84384*a^31*b^35*c^52*d^21 + 14576671334338745969651220480*a^32*b^34*c^51*d^
22 - 50149146156756356561350164480*a^33*b^33*c^50*d^23 + 1105501579267159O4
989065117696*a^34*b^32*c^49*d^24 - 189331360528461979941957795840*a^35*b^31
*c^48*d^25 + 267383527373748192433944920064*a^36*b^30*c^47*d^26 - 319821143
985825066443750768640*a^37*b^29*c^46*d^27 + 328626898447261055168230195200*
a^38*b^28*c^45*d^28 - 292434560796558751919058714624*a^39*b^27*c^44*d^29 +
226382416482170290892093521920*a^40*b^26*c^43*d^30 - 152776304398053739659 9
30894336*a^41*b^25*c^42*d^31 + 89901124622673343064718704640*a^42*b^24*c^41
*d^32 - 46062508964820426479181496320*a^43*b^23*c^40*d^33 + 204866062637376
10091045584896*a^44*b^22*c^39*d^34 - 7870914323775054351244984320*a^45*b^21
*c^38*d^35 + 2594141724382360002965274624*a^46*b^20*c^37*d^36 - 72645102465
1952784807034880*a^47*b^19*c^36*d^37 + 170590060365885174888529920*a^48*b^1
8*c^35*d^38 - 32986343554204898112307200*a^49*b^17*c^34*d^39 + 511806359138
4977873305600*a^50*b^16*c^33*d^40 - 613036163719885750272000*a^51*b^15*c^32
*d^41 + 53255297770998202368000*a^52*b^14*c^31*d^42 - 298872579261726720000
0*a^53*b^13*c^30*d^43 + 81438120439971840000*a^54*b^12*c^29*d^44) + (-((705
27747686400000000*a^66*d^66 + 27487790694400000000*b^66*c^66 + 464565652967
91552000000*a^2*b^64*c^64*d^2 - 85239594962869288960000 0*a^3*b^63*c^63*d^3
+ 113031004798163353600000000*a^4*b^62*c^62*d^4 - 115488078084729823297536 00
0*a^5*b^61*c^61*d^5 + 94660933391357814578872320 0*a^6*b^60*c^60*d^6 - 63988
38206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a
^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 + 74254
8127574667458190721941504*a^10*b^56*c^56*d^10 - 272041584290086689049656950
7840*a^11*b^55*c^55*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d
^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584
363799641518505984*a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*
a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16
 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 9167535731160177
03850321517740032*a^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*
a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^2
0 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 69634084434967
93458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445329 98
144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^42*c^4
2*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 62262545
797041866752836685340344320*a^26*b^40*c^40*d^26 - 925759646070620848388692 8
9496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*a^28*
b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d^29 +
 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 347113525179164
243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580 9
99680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*b^33*
c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34 - 157
8683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 204408506012443307
2578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503 15
```

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5200*a^37*b^29*c^29*d^37 + 26989809397453278872073290576214O9792*a^38*b^28*
c^28*d^38 - 27393908274805544934665349791943229444*a^39*b^27*c^27*d^39 + 255
81457575927361633598682365134110 72*a^40*b^26*c^26*d^40 - 2198323007364395 99
8582415976038400000*a^41*b^25*c^25*d^41 + 173879220535513303458254491263959
0400*a^42*b^24*c^24*d^42 - 126601380586737468979005302081OO84352*a^43*b^23*
c^23*d^43 + 84844675058024454799136171007305 3184*a^44*b^22*c^22*d^44 - 5231
9705986478663727463936373764915 2*a^45*b^21*c^21*d^45 + 29669244466490074344
3383822718074880*a^46*b^20*c^20*d^46 - 15458625383108081624547756355878912 0
*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c^18*d
^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 12978756421
5123908217893623053680 64*a^50*b^16*c^16*d^50 - 4745782995414208640750154437
099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b^14*c
^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 12978980
94790687573306431766528 00*a^54*b^12*c^12*d^54 - 317760427954764447975945011
20000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*
d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 2294691469180319
74963609600000*a^58*b^8*c^8*d^58 - 339421563479651575136256 00000*a^59*b^7*c
^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637 1
05664000000*a^61*b^5*c^5*d^61 + 3950429491527863500800000 0*a^62*b^4*c^4*d^6
2 - 2683794840055971840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*
a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 4388393189376000
000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 +
 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 10699
8213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 15609364
06056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 867078777
0777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 204848128
01130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 1678
0795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 -
53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d
^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16
*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^
19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 63043211587399
6800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 2617739
03936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55
825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5
801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 26
1325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 392233
8816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*
b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023
255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a
^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a
^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 6227304006234931
2*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152
268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 443324
7375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11
```

```
 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^1
9*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 388761692960666419
20*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 3887616
9296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^1
8 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^
12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 443324737586724864
0*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 7228120721
52268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273
040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 24
71152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34
084860461056*a^39*b^2*c^15*d^30))^(3/4)*(x^(1/2)*(-((70527747686400000000*a
^66*d^66 + 27487790694400000000*b^66*c^66 + 464565652967915520000000*a^2*b^6
4*c^64*d^2 - 8523959496286928896000000*a^3*b^63*c^63*d^3 + 1130310047981633
5360000000*a^4*b^62*c^62*d^4 - 115488078084729823297536000*a^5*b^61*c^61*d^5
 + 946609333913578145788723200*a^6*b^60*c^60*d^6 - 639883820634974459346812
9280*a^7*b^59*c^59*d^7 + 36394380507592797513458909184*a^8*b^58*c^58*d^8 -
176823915553078667757483982848*a^9*b^57*c^57*d^9 + 742548127574667458190721
941504*a^10*b^56*c^56*d^10 - 2720415842900866890496569507840*a^11*b^55*c^55
*d^11 + 8760848838643010718192893952000*a^12*b^54*c^54*d^12 - 2495523500408
2618707041228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*
a^14*b^52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15
 + 291432713032377964853953403289600*a^16*b^50*c^50*d^16 - 5383768893393273
22092190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a
^18*b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19
 + 2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 394568205038255
0801466936451399680*a^21*b^45*c^45*d^21 + 696340844349679345870323761283072
0*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*
d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 3902249846
0407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667528366853
40344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^27*b^3
9*c^39*d^27 + 129947384930724520388491615907348480*a^28*b^38*c^38*d^28 - 17
7036156654250012841049111826268160*a^29*b^37*c^37*d^29 + 243137271360678168
280724887442554880*a^30*b^36*c^36*d^30 - 347113525179164243536927248927948800*a^31*b^35*c^35*d^31 + 515833342886205619925039703580999680*a^32*b^34*c^3
4*d^32 - 775468073329926280441232590010056704*a^33*b^33*c^33*d^33 + 1136547
400098503091050564698912063488*a^34*b^32*c^32*d^34 - 1578683304463214616133
755020010061824*a^35*b^31*c^31*d^35 + 2044085060124433072578392630325411840
*a^36*b^30*c^30*d^36 - 2447042575399654362397243935503155200*a^37*b^29*c^29
*d^37 + 2698980939745327887207329057621409792*a^38*b^28*c^28*d^38 - 2739390
827480554493466534979194322944*a^39*b^27*c^27*d^39 + 2558145757592736163359
868236513411072*a^40*b^26*c^26*d^40 - 2198323007364395998582415976038400000
*a^41*b^25*c^25*d^41 + 1738792205355133034582544912639590400*a^42*b^24*c^24
*d^42 - 1266013805867374689790053020810084352*a^43*b^23*c^23*d^43 + 8484467
505802445479913617100730531 84*a^44*b^22*c^22*d^44 - 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900743443383822718074880*a^
```

```
46*b^20*c^20*d^46 - 1545862538310808162454775635587891200*a^47*b^19*c^19*d^4
7 + 739174514721719530430678553581322224*a^48*b^18*c^18*d^48 - 3238737258195
2477787555393435598848*a^49*b^17*c^17*d^49 + 129787564215123908217893623053
68064*a^50*b^16*c^16*d^50 - 474578299541420864075015443709952 0*a^51*b^15*c^
15*d^51 + 157896546601467050611780966416384 0*a^52*b^14*c^14*d^52 - 47637131
856714525898060616171520 0*a^53*b^13*c^13*d^53 + 129789809479068757330643176
652800*a^54*b^12*c^12*d^54 - 31776042795476444797594501120000*a^55*b^11*c^1
1*d^55 + 6948683615003612481702592512000*a^56*b^10*c^10*d^56 - 134721865560
4091154910412800000*a^57*b^9*c^9*d^57 + 229469146918031974963609600000*a^58
*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*b^7*c^7*d^59 + 429545687
9982240124108800000*a^60*b^6*c^6*d^60 - 455971792993637105664000000*a^61*b^
5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^4*d^62 - 268379484005597
1840000000*a^63*b^3*c^3*d^63 + 134144124384706560000000*a^64*b^2*c^2*d^64 -
 162727720910848000000 0*a*b^65*c^65*d - 4388393189376000000000*a^65*b*c*d^6
5)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c^33 + 213364244480 0*a^2*
b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 + 106998213378048*a^4*b^29
*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 1560936406056960*a^6*b^27*c
^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670787770777600*a^8*b^25*c^
25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 20484812801130496*a^10*b^23*
c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 + 16780795101757440*a^12*
b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^13 - 53058143899238400*a
^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c^18*d^15 - 3065750780571
64800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17*b^16*c^16*d^17 - 688798
564847943680*a^18*b^15*c^15*d^18 + 790065381353537536*a^19*b^14*c^14*d^19 -
 766159267095412736*a^20*b^13*c^13*d^20 + 630432115873996800*a^21*b^12*c^12
*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 261773903936962560*a^23*b^
10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24 + 55825496115836928*a^2
5*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^26 + 5801173668208640*a^2
7*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28 + 261325798707200*a^29*
b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3922338816000*a^31*b^2*c^
2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*a^32*b*c*d^32)/(68719476
736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 2199023255552*a^10*b^31*c^
44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461056*a^11*b^30*c^43*d^2 -
 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426560*a^13*b^28*c^41*d^4 -
 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062349312*a^15*b^26*c^39*d^
6 - 231299863088726016*a^16*b^25*c^38*d^7 + 722812072152268800*a^17*b^24*c^
37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4433247375867248640*a^19*
b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34*d^11 + 155163658155353 7
0240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^22*b^19*c^32*d^13 + 32396
807746722201600*a^23*b^18*c^31*d^14 - 38876169296066641920*a^24*b^17*c^30*d
^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38876169296066641920*a^26*
b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^27*d^18 - 238713320239005
69600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^29*b^12*c^25*d^20 - 8866
494751734497280*a^30*b^11*c^24*d^21 + 4433247375867248640*a^31*b^10*c^23*d^
22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 722812072152268800*a^33*b^8*c
```

```
^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 + 62273040062349312*a^35*b
^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27 + 2471152383426560*a^37
*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29 + 34084860461056*a^39*b
^2*c^15*d^30))^(1/4)*(28823037615171174400*a^23*b^51*c^81*d^4 - 12624490475
44497438720*a^24*b^50*c^80*d^5 + 26781213630512448405504*a^25*b^49*c^79*d^6
 - 366816964670228254425088*a^26*b^48*c^78*d^7 + 3648418948406862648705024*
a^27*b^47*c^77*d^8 - 28097394584779147947540480*a^28*b^46*c^76*d^9 + 174448
389309337948351627264*a^29*b^45*c^75*d^10 - 897668976119897481466085376*a^3
0*b^44*c^74*d^11 + 3905920242884630010868531200*a^31*b^43*c^73*d^12 - 14590
896425075765379929735168*a^32*b^42*c^72*d^13 + 4735582106879022780175613952
0*a^33*b^41*c^71*d^14 - 134845524585103937061538234368*a^34*b^40*c^70*d^15
+ 339727096730086714108763176960*a^35*b^39*c^69*d^16 - 76324694471669611181
8343448576*a^36*b^38*c^68*d^17 + 1541564027180813686607638953984*a^37*b^37*
c^67*d^18 - 2825288027628089174763608473600*a^38*b^36*c^66*d^19 + 475304147
6272000853590444867584*a^39*b^35*c^65*d^20 - 744626349318581567762295737548
8*a^40*b^34*c^64*d^21 + 11045974611794807027964401680384*a^41*b^33*c^63*d^2
2 - 15766681100741571532295786987520*a^42*b^32*c^62*d^23 + 2188261657043489
5907374088847360*a^43*b^31*c^61*d^24 - 29555415901357165913293077872640*a^4
4*b^30*c^60*d^25 + 38514249364633213650767204843520*a^45*b^29*c^59*d^26 - 4
7767982724772003266224581509120*a^46*b^28*c^58*d^27 + 556189485371550451204
76750807040*a^47*b^27*c^57*d^28 - 60127300413664475479641214156800*a^48*b^2
6*c^56*d^29 + 59877038998440260638050153922560*a^49*b^25*c^55*d^30 - 546370
51595737047014674020696064*a^50*b^24*c^54*d^31 + 45519623223064909005599526
617088*a^51*b^23*c^53*d^32 - 34535555816550055085254958383104*a^52*b^22*c^5
2*d^33 + 23809729504484309698980012359680*a^53*b^21*c^51*d^34 - 14885319254
535352990241541586944*a^54*b^20*c^50*d^35 + 8419648339202954390072444583936
*a^55*b^19*c^49*d^36 - 4297514831765712413611503124480*a^56*b^18*c^48*d^37
+ 1973123737554130196459440570368*a^57*b^17*c^47*d^38 - 8117708570544976730
61303582720*a^58*b^16*c^46*d^39 + 297858380372439371596188090368*a^59*b^15*
c^45*d^40 - 96910050535770593129744302080*a^60*b^14*c^44*d^41 + 27758579881
177587823480406016*a^61*b^13*c^43*d^42 - 6937474504476672102869499904*a^62*
b^12*c^42*d^43 + 1495682482860276471300096000*a^63*b^11*c^41*d^44 - 2741001
18958300866495381504*a^64*b^10*c^40*d^45 + 41867778463425277028466688*a^65*
b^9*c^39*d^46 - 5187161130930763594727424*a^66*b^8*c^38*d^47 + 500879902205
011065569280*a^67*b^7*c^37*d^48 - 35371992049308254863360*a^68*b^6*c^36*d^4
9 + 1625349105518012006400*a^69*b^5*c^35*d^50 - 36479156981701017600*a^70*b
^4*c^34*d^51)*1i + 18014398509481984000*a^21*b^51*c^78*d^4 - 7782201560962
1708800*a^22*b^50*c^77*d^5 + 16199988291606958571520*a^23*b^49*c^76*d^6 - 2
16629339029608119402496*a^24*b^48*c^75*d^7 + 2092899704349501998235648*a^25
*b^47*c^74*d^8 - 15576808854093856430358528*a^26*b^46*c^73*d^9 + 9298930592
3335928955273216*a^27*b^45*c^72*d^10 - 457716570390505153458339840*a^28*b^4
4*c^71*d^11 + 1895077372829589675098243072*a^29*b^43*c^70*d^12 - 6699157107
174094796222365696*a^30*b^42*c^69*d^13 + 20454608396817467081213607936*a^31
*b^41*c^68*d^14 - 54439663857512808688618831872*a^32*b^40*c^67*d^15 + 12725
3623829876322462345461760*a^33*b^39*c^66*d^16 - 263018360322301930835307134
```

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976*a^34*b^38*c^65*d^17 + 484117148425341461690547437568*a^35*b^37*c^64*d^1
8 - 801088032507623116562893897728*a^36*b^36*c^63*d^19 + 121019175356065842
1451373674496*a^37*b^35*c^62*d^20 - 1713662150039311965148455895040*a^38*b^
34*c^61*d^21 + 2368456612874860634985065349120*a^39*b^33*c^60*d^22 - 334244
0882817901253619697582080*a^40*b^32*c^59*d^23 + 492601941928152671042276425
7280*a^41*b^31*c^58*d^24 - 7443043331925522227676535848960*a^42*b^30*c^57*d
^25 + 11053384984245852600223452364800*a^43*b^29*c^56*d^26 - 15529000135185
248373347985653760*a^44*b^28*c^55*d^27 + 20153801026888464482649904250880*a
^45*b^27*c^54*d^28 - 23870821024791437072619829985280*a^46*b^26*c^53*d^29 +
 25662407141873741853910169026560*a^47*b^25*c^52*d^30 - 2498333496493808560
2226308382720*a^48*b^24*c^51*d^31 + 22003368361455969032835868655616*a^49*b
^23*c^50*d^32 - 17519758513327663391847122731008*a^50*b^22*c^49*d^33 + 1260
1896285489986596049610866688*a^51*b^21*c^48*d^34 - 817968439041491512045153
6551936*a^52*b^20*c^47*d^35 + 4783583081116360454960515645440*a^53*b^19*c^4
6*d^36 - 2515171747726250254399514345472*a^54*b^18*c^45*d^37 + 118571036151
1816082146770026496*a^55*b^17*c^44*d^38 - 499406604618358594580969947136*a^
56*b^16*c^43*d^39 + 187097254447826761775602204672*a^57*b^15*c^42*d^40 - 62
002233932522145150727618560*a^58*b^14*c^41*d^41 + 1804911587294754856674892
1856*a^59*b^13*c^40*d^42 - 4575187392741408034214903808*a^60*b^12*c^39*d^43
 + 998642414508019303179091968*a^61*b^11*c^38*d^44 - 1849867359963810587486
45376*a^62*b^10*c^37*d^45 + 28520139033328990436720640*a^63*b^9*c^36*d^46 -
 3562072173311951854632960*a^64*b^8*c^35*d^47 + 346377863868692037632000*a^
65*b^7*c^34*d^48 - 24611841230482125619200*a^66*b^6*c^33*d^49 + 11371237215
38961408000*a^67*b^5*c^32*d^50 - 25649407252758528000*a^68*b^4*c^31*d^51)*1
i)*1i + 9271855998513971200000*a^20*b^44*c^58*d^12 - 25388837992853929984000
*a^21*b^43*c^57*d^13 + 317358378012506691993600*a^22*b^42*c^56*d^14 - 23738
09829046075554529280*a^23*b^41*c^55*d^15 + 11545284809815729048125440*a^24*
b^40*c^54*d^16 - 35586486107261996158156800*a^25*b^39*c^53*d^17 + 475039875
51983633390632960*a^26*b^38*c^52*d^18 + 160896568335160851531038720*a^27*b^
37*c^51*d^19 - 1289503277949063475180339200*a^28*b^36*c^50*d^20 + 484769551
9788247586575482880*a^29*b^35*c^49*d^21 - 12969376809237608808212070400*a^3
0*b^34*c^48*d^22 + 27198543957428161531839774720*a^31*b^33*c^47*d^23 - 4655
8532623156834692403036160*a^32*b^32*c^46*d^24 + 664650336640637885574073548
80*a^33*b^31*c^45*d^25 - 80137164645540595666444615680*a^34*b^30*c^44*d^26
+ 82241221222993610845821337600*a^35*b^29*c^43*d^27 - 721651400317542076601
54552320*a^36*b^28*c^42*d^28 + 54258643078018614781815029760*a^37*b^27*c^41
*d^29 - 34958604671456258343085015040*a^38*b^26*c^40*d^30 + 192661193835136
05759523880960*a^39*b^25*c^39*d^31 - 9047713278884926997712076800*a^40*b^24
*c^38*d^32 + 3598803321131446378839408640*a^41*b^23*c^37*d^33 - 12017673911
29510053066833920*a^42*b^22*c^36*d^34 + 332745330268979132513648640*a^43*b^
21*c^35*d^35 - 75056967015910052829593600*a^44*b^20*c^34*d^36 + 13447517913
537594156646400*a^45*b^19*c^33*d^37 - 1841937645534110023680000*a^46*b^18*c
^32*d^38 + 181270486395868151808000*a^47*b^17*c^31*d^39 - 11419434221693829
120000*a^48*b^16*c^30*d^40 + 346112011869880320000*a^49*b^15*c^29*d^41))*(-
((705277476864000000000*a^66*d^66 + 274877906944000000000*b^66*c^66 + 4645656
```

```
5296791552000000*a^2*b^64*c^64*d^2 - 8523959496286928896000000*a^3*b^63*c^63
*d^3 + 1130310047981633536000000*a^4*b^62*c^62*d^4 - 115488078084729823297
536000*a^5*b^61*c^61*d^5 + 946609333913578145788723200*a^6*b^60*c^60*d^6 -
6398838206349744593468129280*a^7*b^59*c^59*d^7 + 36394380507592797513458909
184*a^8*b^58*c^58*d^8 - 176823915553078667757483982848*a^9*b^57*c^57*d^9 +
742548127574667458190721941504*a^10*b^56*c^56*d^10 - 2720415842900866890496
569507840*a^11*b^55*c^55*d^11 + 87608488386430107181928939520000*a^12*b^54*c
^54*d^12 - 24955235004082618707041228685312*a^13*b^53*c^53*d^13 + 632144467
42584363799641518505984*a^14*b^52*c^52*d^14 - 14313378011069462050587268035
3792*a^15*b^51*c^51*d^15 + 291432713032377964853953403289600*a^16*b^50*c^50
*d^16 - 538376889339327322092190511923200*a^17*b^49*c^49*d^17 + 91675357311
6017703850321517740032*a^18*b^48*c^48*d^18 - 148047252132516852645238233523
8144*a^19*b^47*c^47*d^19 + 2370124261379332590916233678815232*a^20*b^46*c^4
6*d^20 - 3945682050382550801466936451399680*a^21*b^45*c^45*d^21 + 696340844
3496793458703237612830720*a^22*b^44*c^44*d^22 - 126958698290172324083068445
32998144*a^23*b^43*c^43*d^23 + 22829408140153590039120682300735488*a^24*b^4
2*c^42*d^24 - 39022498460407159853772918944169984*a^25*b^41*c^41*d^25 + 622
62545797041866752836685340344320*a^26*b^40*c^40*d^26 - 92575964607062084838
869289496739840*a^27*b^39*c^39*d^27 + 129947384930724520388491615907348480*
a^28*b^38*c^38*d^28 - 177036156654250012841049111826268160*a^29*b^37*c^37*d
^29 + 243137271360678168280724887442554880*a^30*b^36*c^36*d^30 - 3471135251
79164243536927248927948800*a^31*b^35*c^35*d^31 + 51583334288620561992503970
3580999680*a^32*b^34*c^34*d^32 - 775468073329926280441232590010056704*a^33*
b^33*c^33*d^33 + 1136547400098503091050564698912063488*a^34*b^32*c^32*d^34
- 1578683304463214616133755020010061824*a^35*b^31*c^31*d^35 + 2044085060124
433072578392630325411840*a^36*b^30*c^30*d^36 - 2447042575399654362397243935
503155200*a^37*b^29*c^29*d^37 + 2698980939745327887207329057621409792*a^38*
b^28*c^28*d^38 - 2739390827480554493466534979194322944*a^39*b^27*c^27*d^39
+ 2558145757592736163359868236513411072*a^40*b^26*c^26*d^40 - 2198323007364
395998582415976038400000*a^41*b^25*c^25*d^41 + 1738792205355133034582544912
639590400*a^42*b^24*c^24*d^42 - 1266013805867374689790053020810084352*a^43*
b^23*c^23*d^43 + 848446750580244547991361710073053184*a^44*b^22*c^22*d^44 -
 523197059864786637274639363737649152*a^45*b^21*c^21*d^45 + 296692444664900
743443383822718074880*a^46*b^20*c^20*d^46 - 1545862538310808162454775635587
89120*a^47*b^19*c^19*d^47 + 73917451472171953043067855358132224*a^48*b^18*c
^18*d^48 - 32387372581952477787555393435598848*a^49*b^17*c^17*d^49 + 129787
56421512390821789362305368064*a^50*b^16*c^16*d^50 - 47457829954142086407501
54437099520*a^51*b^15*c^15*d^51 + 1578965466014670506117809664163840*a^52*b
^14*c^14*d^52 - 476371318567145258980606161715200*a^53*b^13*c^13*d^53 + 129
789809479068757330643176652800*a^54*b^12*c^12*d^54 - 3177604279547644479759
4501120000*a^55*b^11*c^11*d^55 + 6948683615003612481702592512000*a^56*b^10*
c^10*d^56 - 1347218655604091154910412800000*a^57*b^9*c^9*d^57 + 22946914691
8031974963609600000*a^58*b^8*c^8*d^58 - 33942156347965157513625600000*a^59*
b^7*c^7*d^59 + 4295456879982240124108800000*a^60*b^6*c^6*d^60 - 45597179299
3637105664000000*a^61*b^5*c^5*d^61 + 39504294915278635008000000*a^62*b^4*c^
```

```
4*d^62 - 268379484005597184000000*a^63*b^3*c^3*d^63 + 13414412438470656000
0000*a^64*b^2*c^2*d^64 - 1627277209108480000000*a*b^65*c^65*d - 43883931893
76000000000*a^65*b*c*d^65)^(1/2) + 8398080000*a^33*d^33 + 5242880000*b^33*c
^33 + 2133642444800*a^2*b^31*c^31*d^2 - 18134996090880*a^3*b^30*c^30*d^3 +
106998213378048*a^4*b^29*c^29*d^4 - 466436266917888*a^5*b^28*c^28*d^5 + 156
0936406056960*a^6*b^27*c^27*d^6 - 4111892301742080*a^7*b^26*c^26*d^7 + 8670
787770777600*a^8*b^25*c^25*d^8 - 14793917747787776*a^9*b^24*c^24*d^9 + 2048
4812801130496*a^10*b^23*c^23*d^10 - 22529362011054080*a^11*b^22*c^22*d^11 +
 16780795101757440*a^12*b^21*c^21*d^12 + 3830387378688000*a^13*b^20*c^20*d^
13 - 53058143899238400*a^14*b^19*c^19*d^14 + 150199661741875200*a^15*b^18*c
^18*d^15 - 306575078057164800*a^16*b^17*c^17*d^16 + 504413463173068800*a^17
*b^16*c^16*d^17 - 688798564847943680*a^18*b^15*c^15*d^18 + 7900653813535375
36*a^19*b^14*c^14*d^19 - 766159267095412736*a^20*b^13*c^13*d^20 + 630432115
873996800*a^21*b^12*c^12*d^21 - 440813170780569600*a^22*b^11*c^11*d^22 + 26
1773903936962560*a^23*b^10*c^10*d^23 - 131676163264708608*a^24*b^9*c^9*d^24
 + 55825496115836928*a^25*b^8*c^8*d^25 - 19792651594874880*a^26*b^7*c^7*d^2
6 + 5801173668208640*a^27*b^6*c^6*d^27 - 1382351733145600*a^28*b^5*c^5*d^28
 + 261325798707200*a^29*b^4*c^4*d^29 - 37757896704000*a^30*b^3*c^3*d^30 + 3
922338816000*a^31*b^2*c^2*d^31 - 155189248000*a*b^32*c^32*d - 261273600000*
a^32*b*c*d^32)/(68719476736*a^9*b^32*c^45 + 68719476736*a^41*c^13*d^32 - 21
99023255552*a^10*b^31*c^44*d - 2199023255552*a^40*b*c^14*d^31 + 34084860461
056*a^11*b^30*c^43*d^2 - 340848604610560*a^12*b^29*c^42*d^3 + 2471152383426
560*a^13*b^28*c^41*d^4 - 13838453347188736*a^14*b^27*c^40*d^5 + 62273040062
349312*a^15*b^26*c^39*d^6 - 231299863088726016*a^16*b^25*c^38*d^7 + 7228120
72152268800*a^17*b^24*c^37*d^8 - 1927498859072716800*a^18*b^23*c^36*d^9 + 4
433247375867248640*a^19*b^22*c^35*d^10 - 8866494751734497280*a^20*b^21*c^34
*d^11 + 15516365815535370240*a^21*b^20*c^33*d^12 - 23871332023900569600*a^2
2*b^19*c^32*d^13 + 32396807746722201600*a^23*b^18*c^31*d^14 - 3887616929606
6641920*a^24*b^17*c^30*d^15 + 41305929877070807040*a^25*b^16*c^29*d^16 - 38
876169296066641920*a^26*b^15*c^28*d^17 + 32396807746722201600*a^27*b^14*c^2
7*d^18 - 23871332023900569600*a^28*b^13*c^26*d^19 + 15516365815535370240*a^
29*b^12*c^25*d^20 - 8866494751734497280*a^30*b^11*c^24*d^21 + 4433247375867
248640*a^31*b^10*c^23*d^22 - 1927498859072716800*a^32*b^9*c^22*d^23 + 72281
2072152268800*a^33*b^8*c^21*d^24 - 231299863088726016*a^34*b^7*c^20*d^25 +
62273040062349312*a^35*b^6*c^19*d^26 - 13838453347188736*a^36*b^5*c^18*d^27
 + 2471152383426560*a^37*b^4*c^17*d^28 - 340848604610560*a^38*b^3*c^16*d^29
 + 34084860461056*a^39*b^2*c^15*d^30))^(1/4) - (2/(a*c) + (x^2*(81*a^4*d^4
- 40*b^4*c^4 + 96*a^2*b^2*c^2*d^2 + 32*a*b^3*c^3*d - 193*a^3*b*c*d^3))/(16*
a^2*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^4*(80*b^4
*c^4*d - 45*a^4*d^5 - 160*a*b^3*c^3*d^2 + 129*a^2*b^2*c^2*d^3 + 44*a^3*b*c*
d^4))/(16*a^2*c^2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d))
- (b*d^2*x^6*(45*a^3*d^3 - 40*b^3*c^3 + 96*a*b^2*c^2*d - 125*a^2*b*c*d^2))/
(16*a^2*c^2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))/(x^(5
/2)*(b*c^2 + 2*a*c*d) + x^(9/2)*(a*d^2 + 2*b*c*d) + a*c^2*x^(1/2) + b*d^2*x
^(13/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.484 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=805

$$\frac{(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}\,a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}\,a^{11/4}(bc-ad)^4}+\frac{(7bc-19ad)\log\left(\sqrt{b}\,x-\sqrt{2}\,\sqrt[4]{a}\right.}{8\sqrt{2}\,a^{11/4}(bc-}$$

Rubi [A] time = 1.32, antiderivative size = 805, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {466, 472, 579, 583, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{(7bc-19ad)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)b^{15/4}}{4\sqrt{2}\,a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{15/4}}{4\sqrt{2}\,a^{11/4}(bc-ad)^4}+\frac{(7bc-19ad)\log\left(\sqrt{b}\,x-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}\,a^{11/4}(bc-ad)^4}-\frac{(7bc-19ad)\log\left(\sqrt{b}\,x+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}+\sqrt{a}\right)b^{15/4}}{8\sqrt{2}\,a^{11/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{3/2}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{15/4}(bc-ad)^4}-\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{15/4}(bc-ad)^4}+\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\log\left(\sqrt{d}\,x-\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}\,c^{15/4}(bc-ad)^4}-\frac{d^{11/4}\left(285b^2c^2-266abdc+77a^2d^2\right)\log\left(\sqrt{d}\,x+\sqrt{2}\,\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}\,c^{15/4}(bc-ad)^4}+\frac{d\left(8b^2c^2+27abdc-11a^2d^2\right)}{16ac^2(bc-ad)^3x^{3/2}\left(dx^2+c\right)}-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(dx^2+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -(56*b^3*c^3 - 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(48*a^2*c^3*(
b*c - a*d)^3*x^(3/2)) + (d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(3/2)*(c +
 d*x^2)^2) + b/(2*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*
b^2*c^2 + 27*a*b*c*d - 11*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(3/2)*(c + d*
x^2)) + (b^(15/4)*(7*b*c - 19*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(11/4)*(b*c - a*d)^
4) + (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (d^(11/4)
*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) + (b^(15/4)*(7*b*c - 19*a
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*
a^(11/4)*(b*c - a*d)^4) - (b^(15/4)*(7*b*c - 19*a*d)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) +
(d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) - (
d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-15bdx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \text{Subst}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\left(8b^2\right.}{16ac^2(}$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} +$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} +$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} +$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} +$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} +$$

$$= -\frac{56b^3c^3-96ab^2c^2d+189a^2bcd^2-77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} +$$

Mathematica [A] time = 6.26, size = 775, normalized size = 0.96

$$-\frac{b^{15/4}(19ad-7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b^{15/4}(19ad-7bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b^{15/4}(19ad-7bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b^{15/4}(19ad-7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^4}+\frac{b^4\sqrt{x}}{2a^2\left(a+bx^2\right)(ad-bc)^3}+\frac{d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(ad-bc)^4}-\frac{d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(ad-bc)^4}-\frac{d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(ad-bc)^4}-\frac{d^{11/4}\left(77a^2d^2-266abcd+285b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}\sqrt{x}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(ad-bc)^4}-\frac{2}{3a^2c^3x^{3/2}}-\frac{d^3\sqrt{x}(31bc-15ad)}{16c^3\left(c+dx^2\right)(bc-ad)^3}-\frac{d^3\sqrt{x}}{4c^2\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -2/(3*a^2*c^3*x^(3/2)) + (b^4*Sqrt[x])/(2*a^2*(-(b*c) + a*d)^3*(a + b*x^2))
 - (d^3*Sqrt[x])/(4*c^2*(b*c - a*d)^2*(c + d*x^2)^2) - (d^3*(31*b*c - 15*a*
d)*Sqrt[x])/(16*c^3*(b*c - a*d)^3*(c + d*x^2)) + (b^(15/4)*(-7*b*c + 19*a*d
)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sq
rt[2]*a^(11/4)*(b*c - a*d)^4) + (b^(15/4)*(-7*b*c + 19*a*d)*ArcTan[(Sqrt[2]
*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(11/4)*(b*c
- a*d)^4) - (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTan[(-(Sq
rt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(15/4
)*(-(b*c) + a*d)^4) - (d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Ar
cTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*
c^(15/4)*(-(b*c) + a*d)^4) - (b^(15/4)*(-7*b*c + 19*a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^4
) + (b^(15/4)*(-7*b*c + 19*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(11/4)*(b*c - a*d)^4) + (d^(11/4)*(285*b^2*c^
2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(-(b*c) + a*d)^4) - (d^(11/4)*(285*b^2*
c^2 - 266*a*b*c*d + 77*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(15/4)*(-(b*c) + a*d)^4)
```

IntegrateAlgebraic [A] time = 2.59, size = 627, normalized size = 0.78

$$-\frac{\left(19ab^{15/4}d-7b^{19/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{11/4}(ad-bc)^4}+\frac{\left(19ab^{15/4}d-7b^{19/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{11/4}(ad-bc)^4}+\frac{\left(77a^2d^{19/4}-266abcd^{15/4}+285b^2c^2d^{11/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}-\frac{\left(77a^2d^{19/4}-266abcd^{15/4}+285b^2c^2d^{11/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4}+\frac{-32a^4c^2d^3-121a^4cd^4x^2-77a^4d^5x^4+96a^3bc^3d^2+265a^3bc^2d^3x^2+68a^3bcd^4x^4-77a^3bd^5x^6-96a^2b^2c^4d-96a^2b^2c^3d^2x^2+201a^2b^2c^2d^3x^4+189a^2b^2cd^4x^6+32ab^3c^5-32ab^3c^4dx^2-160ab^3c^3d^2x^4-96ab^3c^2d^3x^6+56b^4c^5x^2+112b^4c^4dx^4+56b^4c^3d^2x^6}{48a^2c^3x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^2*d^3 + 56*b
^4*c^5*x^2 - 32*a*b^3*c^4*d*x^2 - 96*a^2*b^2*c^3*d^2*x^2 + 265*a^3*b*c^2*d^
3*x^2 - 121*a^4*c*d^4*x^2 + 112*b^4*c^4*d*x^4 - 160*a*b^3*c^3*d^2*x^4 + 201
*a^2*b^2*c^2*d^3*x^4 + 68*a^3*b*c*d^4*x^4 - 77*a^4*d^5*x^4 + 56*b^4*c^3*d^2
*x^6 - 96*a*b^3*c^2*d^3*x^6 + 189*a^2*b^2*c*d^4*x^6 - 77*a^3*b*d^5*x^6)/(48
*a^2*c^3*(-(b*c) + a*d)^3*x^(3/2)*(a + b*x^2)*(c + d*x^2)^2) - ((-7*b^(19/4
)*c + 19*a*b^(15/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])]])/(4*Sqrt[2]*a^(11/4)*(-(b*c) + a*d)^4) + ((285*b^2*c^2*d^(11/4)
 - 266*a*b*c*d^(15/4) + 77*a^2*d^(19/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4) + ((-7*b
^(19/4)*c + 19*a*b^(15/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqr
```

```
t[a] + Sqrt[b]*x)])/(4*Sqrt[2]*a^(11/4)*(-(b*c) + a*d)^4) - ((285*b^2*c^2*d
^(11/4) - 266*a*b*c*d^(15/4) + 77*a^2*d^(19/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(15/4)*(b*c - a*d)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.53, size = 1278, normalized size = 1.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^4*sqrt(x)/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)
*(b*x^2 + a)) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*
b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a
^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/
4)*a*b^3*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*
d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^
2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(
1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8
 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*
d^3 + sqrt(2)*a^4*c^4*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d
^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*
c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c
^4*d^4) - 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*log(sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4
*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^
7*d^4) + 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*log(-sqrt(2
)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4
*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^
7*d^4) - 1/64*(285*(c*d^3)^(1/4)*b^2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3
+ 77*(c*d^3)^(1/4)*a^2*d^4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*
sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/64*(285*(c*d^3)^(1/4)*b^2*
c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*log(-sqrt
```

```
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3
*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*
c^4*d^4) - 1/16*(31*b*c*d^4*x^(5/2) - 15*a*d^5*x^(5/2) + 35*b*c^2*d^3*sqrt(
x) - 19*a*c*d^4*sqrt(x))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*
c^3*d^3)*(d*x^2 + c)^2) - 2/3/(a^2*c^3*x^(3/2))
```

maple [A] time = 0.03, size = 1143, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 1/2*b^4/a/(a*d-b*c)^4*x^(1/2)/(b*x^2+a)*d-1/2*b^5/a^2/(a*d-b*c)^4*x^(1/2)/(
b*x^2+a)*c+19/8*b^4/a^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)*d-7/8*b^5/a^3/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)*c+19/8*b^4/a^2/(a*d-b*c)^4*(a/b)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*d-7/8*b^5/a^3/(a*d-b*c)^4*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)*c+19/16*b^4/a^2/(a*d-b*c)^4*
(a/b)^(1/4)*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)
^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2)))*d-7/16*b^5/a^3/(a*d-b*c)^4*(a/b)^(1/4)
*2^(1/2)*ln((x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*2^(1
/2)*x^(1/2)+(a/b)^(1/2)))*c-15/16*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a
^2+23/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(5/2)*a*b-31/16*d^4/c/(a*d-b*c)^4
/(d*x^2+c)^2*x^(5/2)*b^2-19/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a^2+
27/8*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2*x^(1/2)*a*b-35/16*d^3/(a*d-b*c)^4/(d*x^2
+c)^2*x^(1/2)*b^2-77/64*d^5/c^4/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1)*a^2+133/32*d^4/c^3/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*a*b-285/64*d^3/c^2/(a*d-b*c)^4*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)*b^2-77/128*d^5/c^4/(
a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2
))/(x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a^2+133/64*d^4/c^3/(a*d-b*c
)^4*(c/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(
c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*a*b-285/128*d^3/c^2/(a*d-b*c)^4*(c
/d)^(1/4)*2^(1/2)*ln((x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x-(c/d)^(
1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))*b^2-77/64*d^5/c^4/(a*d-b*c)^4*(c/d)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a^2+133/32*d^4/c^3/(a*d-b*c
)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)*a*b-285/64*d^
3/c^2/(a*d-b*c)^4*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)
*b^2-2/3/a^2/c^3/x^(3/2)
```

maxima [A] time = 2.80, size = 1064, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*sqrt(2)*(7*b*c - 19*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 2*sqrt(2)*(7*b*c - 19*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + sqrt(2)*(7*b*c - 19*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b*c - 19*a*d)*log(-sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^4/(a^2*b^
4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/
48*(32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^2*d^3 + (
56*b^4*c^3*d^2 - 96*a*b^3*c^2*d^3 + 189*a^2*b^2*c*d^4 - 77*a^3*b*d^5)*x^6 +
 (112*b^4*c^4*d - 160*a*b^3*c^3*d^2 + 201*a^2*b^2*c^2*d^3 + 68*a^3*b*c*d^4
- 77*a^4*d^5)*x^4 + (56*b^4*c^5 - 32*a*b^3*c^4*d - 96*a^2*b^2*c^3*d^2 + 265
*a^3*b*c^2*d^3 - 121*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3
+ 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^(15/2) + (2*a^2*b^4*c^7*d - 5*a^3*b^
3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^(11/2) + (a^
2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4
*d^4)*x^(7/2) + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*
d^3)*x^(3/2)) - 1/128*(2*sqrt(2)*(285*b^2*c^2*d^3 - 266*a*b*c*d^4 + 77*a^2*
d^5)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(285*b^2*c^2*
d^3 - 266*a*b*c*d^4 + 77*a^2*d^5)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(
d))) + sqrt(2)*(285*b^2*c^2*d^3 - 266*a*b*c*d^4 + 77*a^2*d^5)*log(sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(
285*b^2*c^2*d^3 - 266*a*b*c*d^4 + 77*a^2*d^5)*log(-sqrt(2)*c^(1/4)*d^(1/4)*
sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)
```

mupad [B] time = 19.06, size = 180372, normalized size = 224.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] atan(((x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 - 289253302176664
30894080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13
 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 349820765298262334012129
28*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 +
873231122236416493313064960*a^24*b^42*c^56*d^17 - 3201588318340888739356606
464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19
- 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 625280040368754051508
57986048*a^28*b^38*c^52*d^21 - 133143680796215491474489344000*a^29*b^37*c^5
1*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23 - 4671065777388
```

```
76991145070559232*a^31*b^35*c^49*d^24 + 775321096823109302674935250944*a^32
*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*c^47*d^26 + 162
9690593600095833823295569920*a^34*b^32*c^46*d^27 - 2028143345719314676074795
5761664*a^35*b^31*c^45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^4
4*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30 + 196620485445
7469918399988498432*a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a
^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b^26*c^40*d^33 -
629956523592774331698776113152*a^41*b^25*c^39*d^34 + 3320657643355840042301
53764864*a^42*b^24*c^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^3
7*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^37 - 20757436699772
395749793333248*a^45*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^2
0*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d^40 + 2960843394
24033093684559872*a^48*b^18*c^32*d^41 - 47717950421254308290887680*a^49*b^1
7*c^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*d^43 - 5340378611857
24002336768*a^51*b^15*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^
45 - 872067188534894657536*a^53*b^13*c^27*d^46) + (-(((143986855936*a^35*d^
35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 10675201612
1856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 23907154306007
04*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 188295341785743
36*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110
464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771
167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841
37459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11
2491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^
16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^1
7*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328
*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677
746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4
293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10
*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8
*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b
^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b
^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c
^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 -
(4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 23112188256
1500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*
a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271
*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 1649241388010969
2*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 594157271624297
5*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*
a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^3
2 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084
860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152
383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273
040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7
```

```
22812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d
^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^2
1*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 238713320239005696
00*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616
9296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^1
6 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^
14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370
240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247
375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 +
 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d
^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^2
0*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^1
8*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35
+ 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*
a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^
5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7
*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^
9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616
*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 271920687299
54304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638
022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53
7221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d
^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^
15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 336375933887319244
8*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378
0572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 -
714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27
 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^
29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^3
1 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 -
540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^1
1*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 +
496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4
 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^2
5*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64
512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23
*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*
d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 5657
22720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*
b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d
^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300
*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^2
6 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^
18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-(((143986855936*a^35*d^35 + 4
0282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a
```

```
^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5
*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7
*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^
9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232
*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599
08608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249127
6045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1
074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17
*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*
b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384
896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767
561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25
- 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d
^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6
*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4
*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^3
3 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (45811
79456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a
*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^
25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b
^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*
b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*
b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b
^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 21
99023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461
056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426
560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062
349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228120
72152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4
433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36
*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^2
4*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616929606
6641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38
876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^2
9*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^
31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867
248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281
2072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 +
62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27
 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29
 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 2014
1047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^
32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30
*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*
c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26
```

*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*
b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*
a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762
496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53722161
5798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 +
1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^1
5*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22
*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378057290
5472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022
739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119
692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9
313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 28
2146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930
998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32
*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^
13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201
376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40
*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240
*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*
c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 -
565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*
a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c
^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 +
64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*
b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 20
1376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^2
9 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(6456360425798343065 6*a^25*b^51*c^84*d^
4 - 282235184327756139724 8*a^26*b^50*c^83*d^5 + 601271623082562834923 52*a^2
7*b^49*c^82*d^6 - 831948157724300777881600*a^28*b^48*c^81*d^7 + 84067865581
79361266073600*a^29*b^47*c^80*d^8 - 66144581305899203170402304*a^30*b^46*c^
79*d^9 + 421912670310680329277407232*a^31*b^45*c^78*d^10 - 2243238210521587
022108295168*a^32*b^44*c^77*d^11 + 10145383251984825802817536000*a^33*b^43*
c^76*d^12 - 39641949193820336576213811200*a^34*b^42*c^75*d^13 + 13549409873
5043868075088674816*a^35*b^41*c^74*d^14 - 409284915889091539805067542528*a^
36*b^40*c^73*d^15 + 1102331957384293957070038761472*a^37*b^39*c^72*d^16 - 2
668223165968086459433038643200*a^38*b^38*c^71*d^17 + 5847343583817169075816
733081600*a^39*b^37*c^70*d^18 - 11684105629368324959904469090304*a^40*b^36*
c^69*d^19 + 21435002462698637041098955948032*a^41*b^35*c^68*d^20 - 36343020
410925078321345140359168*a^42*b^34*c^67*d^21 + 572975806876835610307464265 7
2800*a^43*b^33*c^66*d^22 - 84429658980390814235781758976000*a^44*b^32*c^65*
d^23 + 116702744788425677443098849837056*a^45*b^31*c^64*d^24 - 151589903153
597380791972919246848*a^46*b^30*c^63*d^25 + 18500844425978984294365659345 71
52*a^47*b^29*c^62*d^26 - 211756933815433796881181835264000*a^48*b^28*c^61*d
^27 + 226611959433847997212598992896000*a^49*b^27*c^60*d^28 - 2259060314465
65502788593732550656*a^50*b^26*c^59*d^29 + 2089786277491657244300255145492 4

```
8*a^51*b^25*c^58*d^30 - 178726416623100559749866797924352*a^52*b^24*c^57*d^
31 + 140824510781547830729330235801600*a^53*b^23*c^56*d^32 - 10189727059476
4980154443340185600*a^54*b^22*c^55*d^33 + 67499322390719467851063444373504*
a^55*b^21*c^54*d^34 - 40809284384591153062742518136832*a^56*b^20*c^53*d^35
+ 22447282431345050697947118829568*a^57*b^19*c^52*d^36 - 111950426468198932
51483369472000*a^58*b^18*c^51*d^37 + 5042898342903938117430096691200*a^59*b
^17*c^50*d^38 - 2042741359937286689202494242816*a^60*b^16*c^49*d^39 + 74024
9793404633986500581654528*a^61*b^15*c^48*d^40 - 238501265489031484884985577
472*a^62*b^14*c^47*d^41 + 67809805296929472355971891200*a^63*b^13*c^46*d^42
 - 16856343881283213574379929600*a^64*b^12*c^45*d^43 + 36211580663960445400
42543104*a^65*b^11*c^44*d^44 - 662272679138724025500434432*a^66*b^10*c^43*d
^45 + 101087832400064043724832768*a^67*b^9*c^42*d^46 - 12528855636637836430
540800*a^68*b^8*c^41*d^47 + 1211288155777568604160000*a^69*b^7*c^40*d^48 -
85697808358931542573056*a^70*b^6*c^39*d^49 + 3946450310269237198848*a^71*b^
5*c^38*d^50 - 88774955854727217152*a^72*b^4*c^37*d^51) + x^(1/2)*(564931537
25735501824*a^22*b^52*c^81*d^4 - 2396923808077634863104*a^23*b^51*c^80*d^5
+ 49387698492843503910912*a^24*b^50*c^79*d^6 - 658598339056129087111168*a^2
5*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c^77*d^8 - 4811359686
7651945069805568*a^27*b^47*c^76*d^9 + 292502253544635823646834688*a^28*b^46
*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^74*d^11 + 63060035844
09325504378699776*a^30*b^44*c^73*d^12 - 23152095046595175238512672768*a^31*
b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^42*c^71*d^14 - 206784
189076489114265239683072*a^33*b^41*c^70*d^15 + 5110010173907764065745282007
04*a^34*b^40*c^69*d^16 - 1120486424066161848521664233472*a^35*b^39*c^68*d^1
7 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18 - 37948899494273681
42860254707712*a^37*b^37*c^66*d^19 + 5830470252063718134687996051456*a^38*b
^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^35*c^64*d^21 + 87461
84267385996582875203371008*a^40*b^34*c^63*d^22 - 71768719238351983385202193
85856*a^41*b^33*c^62*d^23 + 1365198057841590488549164056576*a^42*b^32*c^61*
d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*d^25 - 2810065405618
0096231365094146048*a^44*b^30*c^59*d^26 + 51280764289348564983994726219776*
a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*a^46*b^28*c^57*d^28
+ 99717561302809906738570708647936*a^47*b^27*c^56*d^29 - 115380588176718582
142644189659136*a^48*b^26*c^55*d^30 + 120101545474959969242488481251328*a^4
9*b^25*c^54*d^31 - 113052494905210552901304563269632*a^50*b^24*c^53*d^32 +
96462689920395704646643948191744*a^51*b^23*c^52*d^33 - 74665519475418951639
228294889472*a^52*b^22*c^51*d^34 + 52413929319422085122116269637632*a^53*b^
21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^20*c^49*d^36 + 19174
031096138345851817803382784*a^55*b^19*c^48*d^37 - 995182746389335697728745
766912*a^56*b^18*c^47*d^38 + 4646728550801039102656464814080*a^57*b^17*c^46
*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*d^40 + 7258109833877
25632884961181696*a^59*b^15*c^44*d^41 - 240265301732777409221605982208*a^60
*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^13*c^42*d^43 - 17809
629928199177184296828928*a^62*b^12*c^41*d^44 + 3907197185884869673284009984
*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64*b^10*c^39*d^46 + 11
```

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3214808531319939527606272*a^65*b^9*c^38*d^47 - 14265899165032610449588224*a
^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*c^36*d^49 - 10050283
3687558254231552*a^68*b^6*c^35*d^50 + 4689814464763011268608*a^69*b^5*c^34*
d^51 - 106807368762718683136*a^70*b^4*c^33*d^52))*(-(((143986855936*a^35*d^
35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 10675201612
1856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 23907154306007
04*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 188295341785743
36*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110
464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771
167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841
37459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11
2491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^
16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^1
7*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328
*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677
746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4
293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10
*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8
*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b
^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b
^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c
^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 -
(4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 23112188256
1500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*
a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271
*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 1649241388010969
2*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 594157271624297
5*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*
a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^3
2 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084
860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152
383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273
040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7
22812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d
^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^2
1*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 238713320239005696
00*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616
9296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^1
6 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^
14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370
240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247
375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 +
 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d
^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^2
0*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^1
```

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8*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35
+ 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*
a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^
5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7
*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^
9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616
*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 271920687299
54304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638
022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53
7221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d
^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^
15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 336375933887319244
8*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378
0572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 -
714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27
 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^
29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^3
1 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 -
540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^1
1*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 +
496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4
 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^2
5*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64
512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23
*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*
d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 5657
22720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*
b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d
^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300
*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^2
6 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^
18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4) + 192609104438451240960*a^18*b^50
*c^68*d^8 - 7086180670911782322176*a^19*b^49*c^67*d^9 + 1250744769136663776
46080*a^20*b^48*c^66*d^10 - 14111528055063183360000000*a^21*b^47*c^65*d^11 +
 11440156274772600537743360*a^22*b^46*c^64*d^12 - 7101975490470375592034304
0*a^23*b^45*c^63*d^13 + 351320863723081970831327232*a^24*b^44*c^62*d^14 - 1
422781934731584726682828800*a^25*b^43*c^61*d^15 + 4808764412319368968195276
800*a^26*b^42*c^60*d^16 - 13753628214096098268020736000*a^27*b^41*c^59*d^17
 + 33604586265646232007931330560*a^28*b^40*c^58*d^18 - 70459004145207625658
058932224*a^29*b^39*c^57*d^19 + 126335924813552658893934428160*a^30*b^38*c^
56*d^20 - 189714420765957587531118673920*a^31*b^37*c^55*d^21 + 221947274468
283773140074496000*a^32*b^36*c^54*d^22 - 142870740343318834154286612480*a^3
3*b^35*c^53*d^23 - 176083118177526399618307325952*a^34*b^34*c^52*d^24 + 895
947027393848326392014438400*a^35*b^33*c^51*d^25 - 2154323340999822995276326
502400*a^36*b^32*c^50*d^26 + 3969865332339043373838394982400*a^37*b^31*c^49
```

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*d^27 - 6147644263312111317325499596800*a^38*b^30*c^48*d^28 + 8260762337957
58018637156319232O*a^39*b^29*c^47*d^29 - 9765601087086458087650885632000*a^
40*b^28*c^46*d^30 + 10223506948306413182866214092800*a^41*b^27*c^45*d^31 -
9508424738292483984119247667200*a^42*b^26*c^44*d^32 + 78668986282545916344O
1331773440*a^43*b^25*c^43*d^33 - 5790724738841488066411751276544*a^44*b^24*
c^42*d^34 + 3789006704063625484256485048320*a^45*b^23*c^41*d^35 - 219999620
5919117948922678476800*a^46*b^22*c^40*d^36 + 113048021505958511282868977664
0*a^47*b^21*c^39*d^37 - 512203696921842163745197916160*a^48*b^20*c^38*d^38
+ 203625309837119046692160667648*a^49*b^19*c^37*d^39 - 70576441632244073218
493644800*a^50*b^18*c^36*d^40 + 21151503372075452883114393600*a^51*b^17*c^3
5*d^41 - 5422672476777259769580748800*a^52*b^16*c^34*d^42 + 117254091349241
4089228451840*a^53*b^15*c^33*d^43 - 209790609112633976926765056*a^54*b^14*c
^32*d^44 + 30239740212369693490544640*a^55*b^13*c^31*d^45 - 337577798099866
6504110080*a^56*b^12*c^30*d^46 + 273981289062762912153600*a^57*b^11*c^29*d^
47 - 14388779197382598328320*a^58*b^10*c^28*d^48 + 367186184646271434752*a^
59*b^9*c^27*d^49))*(-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13
612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644
510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 75404149
07154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420
899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 8261227
2492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80
709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13
 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20
*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b
^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38328508098573721
60*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 65992136
88440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 +
 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^
11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^2
6*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*
a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464
*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a
^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*
c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 +
 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 7005478
2497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904
712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 1631620395
8046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 117608394
41437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 209420692
9053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(6871947673
6*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^4
6*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 -
340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 -
13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6
 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^3
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9*d^8 - 19274988590727168OO*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b
^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370
240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 323968
07746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^
15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b
^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2387133202390056
9600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 88664
94751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^2
2 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^
23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^
6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*
b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^
2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 68060
29991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29282225514
0864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577
216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772
544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130613624622
2848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885
515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 246
8579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 -
 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18
*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*
b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194
816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545
770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24
+ 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9
*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*
c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*
c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^
3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 21
46713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 -
32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a
^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 +
906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24
*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 12
9024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^2
4*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32
*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471
435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31
*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d
^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a
^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 +
 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^3
0)))^(1/4)*1i + (x^(1/2)*(8577124182024781824OO*a^18*b^48*c^62*d^11 - 28925
33021766643089408O*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46
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*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 34982076529826
233401212928*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^
57*d^16 + 873231122236416493313064960*a^24*b^42*c^56*d^17 - 320158831834088
8739356606464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*
c^54*d^19 - 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 62528004036
875405150857986048*a^28*b^38*c^52*d^21 - 133143680796215491474489344000*a^2
9*b^37*c^51*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23 - 467
106577738876991145070559232*a^31*b^35*c^49*d^24 + 7753210968231093026749352
50944*a^32*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*c^47*
d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*d^27 - 20281433457193
14676074795761664*a^35*b^31*c^45*d^28 + 2257905973104023956972306956288*a^3
6*b^30*c^44*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30 + 19
66204854457469918399988498432*a^38*b^28*c^42*d^31 - 15276494060483666212625
68488960*a^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b^26*c^
40*d^33 - 629956523592774331698776113152*a^41*b^25*c^39*d^34 + 332065764335
584004230153764864*a^42*b^24*c^38*d^35 - 152543196968133650922715742208*a^4
3*b^23*c^37*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^37 - 2075
7436699772395749793333248*a^45*b^21*c^35*d^38 + 603782595179703225532022784
0*a^46*b^20*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d^40 +
296084339424033093684559872*a^48*b^18*c^32*d^41 - 4771795042125430829088768
0*a^49*b^17*c^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*d^43 - 534
037861185724002336768*a^51*b^15*c^29*d^44 + 31006369751209579905024*a^52*b^
14*c^28*d^45 - 872067188534894657536*a^53*b^13*c^27*d^46) - (-(((1439868559
36*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1
06752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390
715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829
534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181
2801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9
0502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^1
2 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21
*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^
19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 223657145883607040
0*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913
6181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 -
6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^1
2*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25
*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 63932949751673241
6*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358
144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231
744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*
a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^
34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2
31121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203
904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013
0570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924
```

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13880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415
72716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873
253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^4
3*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^
31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3
 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d
^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^
40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b
^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 886649475173449728
0*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332
023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14
 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^1
6*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 323968077467222016
00*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1551636
5815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21
 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c
^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*
b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^
38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^
40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*
a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376
008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 119535771
5300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089
287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3090640098
5055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 4525137
1385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27
192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14
+ 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19
*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b
^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27406695680908656
64*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 33637593
38873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 -
 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^
10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^
8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*
b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b
^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c
^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(687194
76736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^
16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^2
8*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 336585
6*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^3
8*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 2257
92840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*
b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d
```

^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 34737
3600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b
^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23
+ 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^
6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a
^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-(((143986855936*a^3
5*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1067520
16121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430
600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178
574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181280197
0110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9050274
2771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54
384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14
+ 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^1
9*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18
*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913618173
1328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 672751
8677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23
- 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*
c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27
*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^
29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^
31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b
^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/
4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2311218
82561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732
850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013057063
6271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924138801
09692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415727162
42975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157
362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15
*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 3
4084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 247
1152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 6
2273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7
+ 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^
38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22
*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900
569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388
76169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31
*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^2
9*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1551636581553
5370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443
3247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^
23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^

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22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5
*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3
*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d
^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060
928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 119535771530035
2*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168
*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3090640098505523
2*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 4525137138558
3616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068
729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 5624
5638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16
+ 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^
17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^2
0*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 33637593388731
92448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 21468
83780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^2
5 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*
d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^
6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4
*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^3
3 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*
(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^3
1 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43
*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18
*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9
+ 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*
a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c
^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 -
565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a
^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^
26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 1051
8300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21
*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^
3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(64563604257983430656*a^25*b^
51*c^84*d^4 - 2822351843277561397248*a^26*b^50*c^83*d^5 + 60127162308256283
492352*a^27*b^49*c^82*d^6 - 831948157724300777881600*a^28*b^48*c^81*d^7 + 8
406786558179361266073600*a^29*b^47*c^80*d^8 - 66144581305899203170402304*a^
30*b^46*c^79*d^9 + 421912670310680329277407232*a^31*b^45*c^78*d^10 - 224323
8210521587022108295168*a^32*b^44*c^77*d^11 + 10145383251984825802817536000*
a^33*b^43*c^76*d^12 - 39641949193820336576213811200*a^34*b^42*c^75*d^13 + 1
35494098735043868075088674816*a^35*b^41*c^74*d^14 - 40928491588909153980506
7542528*a^36*b^40*c^73*d^15 + 1102331957384293957070038761472*a^37*b^39*c^7
2*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*d^17 + 584734358381
7169075816733081600*a^39*b^37*c^70*d^18 - 11684105629368324959904469090304*
a^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a^41*b^35*c^68*d^20
```

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- 3634302041092507832134514035916８*a^42*b^34*c^67*d^21 + 572975806876835610
30746426572800*a^43*b^33*c^66*d^22 - 84429658980390814235781758976000*a^44*
b^32*c^65*d^23 + 116702744788425677443098849837056*a^45*b^31*c^64*d^24 - 15
1589903153597380791972919246848*a^46*b^30*c^63*d^25 + 185008444259789842943
656593457152*a^47*b^29*c^62*d^26 - 211756933815433796881181835264000*a^48*b
^28*c^61*d^27 + 226611959433847997212598992896000*a^49*b^27*c^60*d^28 - 225
906031446565502788593732550656*a^50*b^26*c^59*d^29 + 2089786277491657244300
25514549248*a^51*b^25*c^58*d^30 - 178726416623100559749866797924352*a^52*b^
24*c^57*d^31 + 140824510781547830729330235801600*a^53*b^23*c^56*d^32 - 1018
97270594764980154443340185600*a^54*b^22*c^55*d^33 + 67499322390719467851063
444373504*a^55*b^21*c^54*d^34 - 40809284384591153062742518136832*a^56*b^20*
c^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c^52*d^36 - 11195042
646819893251483369472000*a^58*b^18*c^51*d^37 + 5042898342903938117430096691
200*a^59*b^17*c^50*d^38 - 2042741359937286689202494242816*a^60*b^16*c^49*d^
39 + 740249793404633986500581654528*a^61*b^15*c^48*d^40 - 23850126548903148
4884985577472*a^62*b^14*c^47*d^41 + 67809805296929472355971891200*a^63*b^13
*c^46*d^42 - 16856343881283213574379929600*a^64*b^12*c^45*d^43 + 3621158066
396044540042543104*a^65*b^11*c^44*d^44 - 662272679138724025500434432*a^66*b
^10*c^43*d^45 + 101087832400064043724832768*a^67*b^9*c^42*d^46 - 1252885563
6637836430540800*a^68*b^8*c^41*d^47 + 1211288155777568604160000*a^69*b^7*c^
40*d^48 - 85697808358931542573056*a^70*b^6*c^39*d^49 + 39464503102692371988
48*a^71*b^5*c^38*d^50 - 88774955854727217152*a^72*b^4*c^37*d^51) - x^(1/2)*
(56493153725735501824*a^22*b^52*c^81*d^4 - 2396923808077634863104*a^23*b^51
*c^80*d^5 + 49387698492843503910912*a^24*b^50*c^79*d^6 - 658598339056129087
111168*a^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c^77*d^8 -
48113596867651945069805568*a^27*b^47*c^76*d^9 + 292502253544635823646834688
*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^74*d^11 + 6
306003584409325504378699776*a^30*b^44*c^73*d^12 - 2315209504659517523851267
2768*a^31*b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^42*c^71*d^1
4 - 206784189076489114265239683072*a^33*b^41*c^70*d^15 + 511001017390776406
574528200704*a^34*b^40*c^69*d^16 - 1120486424066161848521664233472*a^35*b^3
9*c^68*d^17 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18 - 3794889
949427368142860254707712*a^37*b^37*c^66*d^19 + 5830470252063718134687996051
456*a^38*b^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^35*c^64*d^
21 + 8746184267385996582875203371008*a^40*b^34*c^63*d^22 - 7176871923835198
338520219385856*a^41*b^33*c^62*d^23 + 1365198057841590488549164056576*a^42*
b^32*c^61*d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*d^25 - 281
00654056180096231365094146048*a^44*b^30*c^59*d^26 + 51280764289348564983994
726219776*a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*a^46*b^28*
c^57*d^28 + 99717561302809906738570708647936*a^47*b^27*c^56*d^29 - 11538058
8176718582142644189659136*a^48*b^26*c^55*d^30 + 120101545474959969242488481
251328*a^49*b^25*c^54*d^31 - 113052494905210552901304563269632*a^50*b^24*c^
53*d^32 + 96462689920395704646643948191744*a^51*b^23*c^52*d^33 - 7466551947
5418951639228294889472*a^52*b^22*c^51*d^34 + 524139293194220851221162696376
32*a^53*b^21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^20*c^49*d^
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36 + 19174031096138345851817803382784*a^55*b^19*c^48*d^37 - 995182746389333
5697728745766912*a^56*b^18*c^47*d^38 + 464672855080103910265646481408O*a^57
*b^17*c^46*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*d^40 + 725
810983387725632884961181696*a^59*b^15*c^44*d^41 - 2402653017327774092216059
82208*a^60*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^13*c^42*d^
43 - 17809629928199177184296828928*a^62*b^12*c^41*d^44 + 390719718588486967
3284009984*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64*b^10*c^39
*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^47 - 142658991650326104
49588224*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*c^36*d^49
- 100502833687558254231552*a^68*b^6*c^35*d^50 + 4689814464763011268608*a^69
*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52))*(-(((1439868559
36*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1
06752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390
715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829
534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181
2801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9
0502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^1
2 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21
*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^
19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 223657145883607040
0*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913
6181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 -
6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^1
2*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25
*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 63932949751673241
6*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358
144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231
744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*
a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429426249728*a^34*b*c*d^
34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2
31121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203
904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013
0570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924
13880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415
72716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873
253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^4
3*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^
31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3
 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d
^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^
40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b
^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 886649475173449728
0*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332
023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14
 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^1
```

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6*c^31*d^16 - 3887616929606664192 0*a^28*b^15*c^30*d^17 + 323968077467222016
00*a^29*b^14*c^29*d^18 - 2387133202390056960 0*a^30*b^13*c^28*d^19 + 1551636
5815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21
 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c
^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*
b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^
38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^
40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*
a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376
008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 119535771
5300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089
287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3090640098
5055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 4525137
1385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27
192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14
+ 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19
*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b
^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27406695680908656
64*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 33637593
38873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 -
 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^
10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^
8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*
b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b
^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c
^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(687194
76736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^
16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^2
8*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 336585
6*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^3
8*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 2257
92840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*
b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d
^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 34737
3600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b
^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23
+ 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^
6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a
^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4) + 19260910443845124096 0
*a^18*b^50*c^68*d^8 - 70861806709117823221 76*a^19*b^49*c^67*d^9 + 125074476
913666377646080*a^20*b^48*c^66*d^10 - 1411152805506318336000000*a^21*b^47*c
^65*d^11 + 11440156274772600537743360*a^22*b^46*c^64*d^12 - 710197549047037
55920343040*a^23*b^45*c^63*d^13 + 351320863723081970831327232*a^24*b^44*c^6
2*d^14 - 1422781934731584726682828800*a^25*b^43*c^61*d^15 + 480876441231936
8968195276800*a^26*b^42*c^60*d^16 - 13753628214096098268020736000*a^27*b^41
```

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*c^59*d^17 + 336045862656462320079313305600*a^28*b^40*c^58*d^18 - 7045900414
5207625658058932224*a^29*b^39*c^57*d^19 + 1263359248135526588939344281 60*a^
30*b^38*c^56*d^20 - 1897144207659575875311186739 20*a^31*b^37*c^55*d^21 + 22
19472744682837731400744960 00*a^32*b^36*c^54*d^22 - 142870740343318834154286
612480*a^33*b^35*c^53*d^23 - 176083118177526399618307325952*a^34*b^34*c^52*
d^24 + 895947027393848326392014438400*a^35*b^33*c^51*d^25 - 215432334099982
2995276326502400*a^36*b^32*c^50*d^26 + 396986533233904337383839498240 0*a^37
*b^31*c^49*d^27 - 614764426331211131732549959680 0*a^38*b^30*c^48*d^28 + 826
07623379575801863715631923 20*a^39*b^29*c^47*d^29 - 976560108708645808765088
5632000*a^40*b^28*c^46*d^30 + 1022350694830641318286621409 2800*a^41*b^27*c^
45*d^31 - 950842473829248398411924766720 0*a^42*b^26*c^44*d^32 + 78668986282
54591634401331773440*a^43*b^25*c^43*d^33 - 579072473884148806641175127654 4*
a^44*b^24*c^42*d^34 + 378900670406362548425648504832 0*a^45*b^23*c^41*d^35 -
 2199996205919117948922678476800*a^46*b^22*c^40*d^36 + 11304802150595851128
28689776640*a^47*b^21*c^39*d^37 - 512203696921842163745197916160*a^48*b^20*
c^38*d^38 + 203625309837119046692160667648*a^49*b^19*c^37*d^39 - 7057644163
2244073218493644800*a^50*b^18*c^36*d^40 + 211515033720754528831143936 00*a^5
1*b^17*c^35*d^41 - 542267247677725976958074880 0*a^52*b^16*c^34*d^42 + 11725
409134924140892284518 40*a^53*b^15*c^33*d^43 - 209790609112633976926765056*a
^54*b^14*c^32*d^44 + 302397402123696934905446 40*a^55*b^13*c^31*d^45 - 33757
779809986665041100 80*a^56*b^12*c^30*d^46 + 2739812890627629121536 00*a^57*b^
11*c^29*d^47 - 1438877919738259832832 0*a^58*b^10*c^28*d^48 + 3671861846462 7
1434752*a^59*b^9*c^27*d^49))*(-(((143986855936*a^35*d^35 + 40282095616*b^35
*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^
3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5
+ 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7
+ 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9
 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24
*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22
*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15
*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134
400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850
809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20
+ 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c
^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^
24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666
450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911
340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 1862608
2598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 5642928
49139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 10818619965
44*a*b^34*c^34*d - 429342624972 8*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b
^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12
 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 -
5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 -
16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 -
```

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 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 -
 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(
68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^1
2*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c
^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c
^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^2
6*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^1
9*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248
640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 1551636
5815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^1
3 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^
17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641
920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 238713
32023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^
20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^1
0*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a
^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 622730400623493
12*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 247115238342
6560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 340848604610
56*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^
35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 2
92822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 377
0207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 1891
7210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 413
06136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11
+ 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*
d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c
^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17
*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 191642540492868
6080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 329960
6844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22
 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*
c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^
26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784
*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232
*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a
^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c
^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^
15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^
2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c
^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300
*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37
*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347
373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26
*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*
```

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d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 2257
92840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b
^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 -
 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c
^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^
2*c^17*d^30)))^(1/4)*1i)/((x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^
11 - 28925330217666430894080*a^19*b^47*c^61*d^12 + 465808355868544602210304
*a^20*b^46*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 3498
2076529826233401212928*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^
23*b^43*c^57*d^16 + 873231122236416493313064960*a^24*b^42*c^56*d^17 - 32015
88318340888739356606464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*
a^26*b^40*c^54*d^19 - 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 6
2528004036875405150857986048*a^28*b^38*c^52*d^21 - 133143680796215491474489
344000*a^29*b^37*c^51*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*
d^23 - 467106577738876991145070559232*a^31*b^35*c^49*d^24 + 775321096823109
302674935250944*a^32*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*
b^33*c^47*d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*d^27 - 2028
143345719314676074795761664*a^35*b^31*c^45*d^28 + 2257905973104023956972306
956288*a^36*b^30*c^44*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43
*d^30 + 1966204854457469918399988498432*a^38*b^28*c^42*d^31 - 1527649406048
366621262568488960*a^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^
40*b^26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25*c^39*d^34 + 33
2065764335584004230153764864*a^42*b^24*c^38*d^35 - 152543196968133650922715
742208*a^43*b^23*c^37*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d
^37 - 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 60378259517970322
55320227840*a^46*b^20*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^
33*d^40 + 296084339424033093684559872*a^48*b^18*c^32*d^41 - 477179504212543
08290887680*a^49*b^17*c^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*
d^43 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 310063697512095799050
24*a^52*b^14*c^28*d^45 - 872067188534894657536*a^53*b^13*c^27*d^46) + (-(((
143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^
33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*
d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d
^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*
d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^2
5*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^2
3*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14
*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 4132414539309056
00*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714
58836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 -
 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^
14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^2
3*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 26895850936374
72256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6393294
97516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408
```

```
0636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371
1306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554
295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a
^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^1
2*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22
 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14
 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^1
6 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^
18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^2
0 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719
476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42
*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^2
9*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b
^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^
18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716
800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664947
51734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12
- 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18
*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 4130592987707080704
0*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807
746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19
 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11
*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*
a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 23129986308872
6016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 1383845334
7188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 34084860
4610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71
993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d
^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 -
 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 -
9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 -
30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10
 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23
*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*
c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16
*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035
200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669
568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21
- 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c
^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^
25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 3196647487583662
08*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 3704031833817
9072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615
872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*
a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^3
```

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4)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32
*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 3596
0*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^
6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^2
0*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d
^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 47143
5600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b
^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^
18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024
480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*
c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 9061
92*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^2
8 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-(((143986
855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2
 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 -
2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1
8829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 -
61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10
 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23
*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*
c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^1
6*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714588360
70400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 54813
39136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^2
1 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12
*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*
a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6393294975167
32416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408063667
6358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371130605
1231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914
496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*
c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11
 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 144
2203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 111
50130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16
492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5
941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 49
2873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736
*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^1
6*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44
*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^
42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^2
5*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^
20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664947517344
97280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387
```

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1332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*
d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27
*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722
201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155
16365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*
d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b
^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a
^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 138384533471887З
6*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 34084860461056
0*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427
968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 5
3376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 11953
57715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 941476
7089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 309064
00985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 452
51371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12
- 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d
^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*
c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^
18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090
865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363
759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^
23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^1
0*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^2
7*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a
^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^
31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b
^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68
719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*
b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15
*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 33
65856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23
*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 +
225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a
^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^
31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 3
47373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^
32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d
^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^3
7*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 49
60*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(645636042579834306
56*a^25*b^51*c^84*d^4 - 2822351843277561397248*a^26*b^50*c^83*d^5 + 6012716
230825628349235 2*a^27*b^49*c^82*d^6 - 83194815772430077788160 0*a^28*b^48*c^
81*d^7 + 840678655817936126607360 0*a^29*b^47*c^80*d^8 - 661445813058992031 7
0402304*a^30*b^46*c^79*d^9 + 42191267031068032927740723 2*a^31*b^45*c^78*d^1
```

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0 - 2243238210521587022108295168*a^32*b^44*c^77*d^11 + 10145383251984825802
817536000*a^33*b^43*c^76*d^12 - 39641949193820336576213811200*a^34*b^42*c^7
5*d^13 + 135494098735043868075088674816*a^35*b^41*c^74*d^14 - 4092849158890
91539805067542528*a^36*b^40*c^73*d^15 + 1102331957384293957070038761472*a^3
7*b^39*c^72*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*d^17 + 58
47343583817169075816733081600*a^39*b^37*c^70*d^18 - 11684105629368324959904
469090304*a^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a^41*b^35*
c^68*d^20 - 36343020410925078321345140359168*a^42*b^34*c^67*d^21 + 57297580
687683561030746426572800*a^43*b^33*c^66*d^22 - 8442965898039081423578175897
6000*a^44*b^32*c^65*d^23 + 116702744788425677443098849837056*a^45*b^31*c^64
*d^24 - 151589903153597380791972919246848*a^46*b^30*c^63*d^25 + 18500844425
9789842943656593457152*a^47*b^29*c^62*d^26 - 211756933815433796881181835264
000*a^48*b^28*c^61*d^27 + 226611959433847997212598992896000*a^49*b^27*c^60*
d^28 - 225906031446565502788593732550656*a^50*b^26*c^59*d^29 + 208978627749
165724430025514549248*a^51*b^25*c^58*d^30 - 1787264166231005597498667979243
52*a^52*b^24*c^57*d^31 + 140824510781547830729330235801600*a^53*b^23*c^56*d
^32 - 101897270594764980154443340185600*a^54*b^22*c^55*d^33 + 6749932239071
9467851063444373504*a^55*b^21*c^54*d^34 - 40809284384591153062742518136832*
a^56*b^20*c^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c^52*d^36
- 11195042646819893251483369472000*a^58*b^18*c^51*d^37 + 504289834290393811
7430096691200*a^59*b^17*c^50*d^38 - 2042741359937286689202494242816*a^60*b^
16*c^49*d^39 + 740249793404633986500581654528*a^61*b^15*c^48*d^40 - 2385012
65489031484884985577472*a^62*b^14*c^47*d^41 + 67809805296929472355971891200
*a^63*b^13*c^46*d^42 - 16856343881283213574379929600*a^64*b^12*c^45*d^43 +
3621158066396044540042543104*a^65*b^11*c^44*d^44 - 662272679138724025500434
432*a^66*b^10*c^43*d^45 + 101087832400064043724832768*a^67*b^9*c^42*d^46 -
12528855636637836430540800*a^68*b^8*c^41*d^47 + 1211288155777568604160000*a
^69*b^7*c^40*d^48 - 85697808358931542573056*a^70*b^6*c^39*d^49 + 3946450310
269237198848*a^71*b^5*c^38*d^50 - 88774955854727217152*a^72*b^4*c^37*d^51)
+ x^(1/2)*(56493153725735501824*a^22*b^52*c^81*d^4 - 2396923808077634863104
*a^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b^50*c^79*d^6 - 65859833
9056129087111168*a^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c
^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 + 29250225354463582
3646834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^7
4*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12 - 231520950465951
75238512672768*a^31*b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^4
2*c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^70*d^15 + 51100101
7390776406574528200704*a^34*b^40*c^69*d^16 - 112048642406616184852166423347
2*a^35*b^39*c^68*d^17 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18
 - 3794889949427368142860254707712*a^37*b^37*c^66*d^19 + 583047025206371813
4687996051456*a^38*b^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^
35*c^64*d^21 + 8746184267385996582875203371008*a^40*b^34*c^63*d^22 - 717687
1923835198338520219385856*a^41*b^33*c^62*d^23 + 136519805784159048854916405
6576*a^42*b^32*c^61*d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*
d^25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^26 + 5128076428934
```

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8564983994726219776*a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*
a^46*b^28*c^57*d^28 + 99717561302809906738570708647936*a^47*b^27*c^56*d^29
- 115380588176718582142644189659136*a^48*b^26*c^55*d^30 + 12010154547495996
9242488481251328*a^49*b^25*c^54*d^31 - 113052494905210552901304563269632*a^
50*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*b^23*c^52*d^33 -
74665519475418951639228294889472*a^52*b^22*c^51*d^34 + 52413929319422085122
116269637632*a^53*b^21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^
20*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*c^48*d^37 - 99518
27463893335697728745766912*a^56*b^18*c^47*d^38 + 46467285508010391026564648
14080*a^57*b^17*c^46*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*
d^40 + 725810983387725632884961181696*a^59*b^15*c^44*d^41 - 240265301732777
409221605982208*a^60*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^
13*c^42*d^43 - 17809629928199177184296828928*a^62*b^12*c^41*d^44 + 39071971
85884869673284009984*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64
*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^47 - 14265899
165032610449588224*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*
c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 + 46898144647630112
68608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52))*(-(((
143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^
33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*
d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d
^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*
d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^2
5*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^2
3*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14
*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 4132414539309056
00*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714
58836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 -
 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^
14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^2
3*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 26895850936374
72256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6393294
97516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408
0636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371
1306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554
295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a
^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^1
2*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22
 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14
 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^1
6 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^
18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^2
0 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719
476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42
*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^2
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9*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b
^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^
18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716
800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664947
51734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12
- 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18
*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 413059298770708070 4
0*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807
746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19
 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11
*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*
a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 23129986308872
6016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 1383845334
7188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 34084860
4610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71
993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d
^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 -
 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 -
9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 -
30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10
 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23
*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*
c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16
*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035
200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669
568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21
- 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c
^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^
25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 3196647487583662
08*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 3704031833817
9072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615
872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*
a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^3
4)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32
*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 3596
0*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^
6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^2
0*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d
^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 47143
5600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b
^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^
18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024
480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*
c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 9061
92*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^2
```

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8 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4) + 19260910443
8451240960*a^18*b^50*c^68*d^8 - 7086180670911782322176*a^19*b^49*c^67*d^9 +
 125074476913666377646080*a^20*b^48*c^66*d^10 - 1411152805506318336000000*a
^21*b^47*c^65*d^11 + 11440156274772600537743360*a^22*b^46*c^64*d^12 - 71019
754904703755920343040*a^23*b^45*c^63*d^13 + 351320863723081970831327232*a^2
4*b^44*c^62*d^14 - 1422781934731584726682828800*a^25*b^43*c^61*d^15 + 48087
64412319368968195276800*a^26*b^42*c^60*d^16 - 13753628214096098268020736000
*a^27*b^41*c^59*d^17 + 33604586265646232007931330560*a^28*b^40*c^58*d^18 -
70459004145207625658058932224*a^29*b^39*c^57*d^19 + 126335924813552658893934
4428160*a^30*b^38*c^56*d^20 - 189714420765957587531118673920*a^31*b^37*c^55
*d^21 + 221947274468283773140074496000*a^32*b^36*c^54*d^22 - 14287074034331
8834154286612480*a^33*b^35*c^53*d^23 - 176083118177526399618307325952*a^34*
b^34*c^52*d^24 + 895947027393848326392014438400*a^35*b^33*c^51*d^25 - 21543
23340999822995276326502400*a^36*b^32*c^50*d^26 + 396986533233904337383839949
82400*a^37*b^31*c^49*d^27 - 6147644263312111317325499596800*a^38*b^30*c^48*
d^28 + 8260762337957580186371563192320*a^39*b^29*c^47*d^29 - 97656010870864
58087650885632000*a^40*b^28*c^46*d^30 + 10223506948306413182866214092800*a^
41*b^27*c^45*d^31 - 9508424738292483984119247667200*a^42*b^26*c^44*d^32 + 7
866898628254591634401331773440*a^43*b^25*c^43*d^33 - 579072473884148806641
1751276544*a^44*b^24*c^42*d^34 + 3789006704063625484256485048320*a^45*b^23*c
^41*d^35 - 2199996205919117948922678476800*a^46*b^22*c^40*d^36 + 1130480215
059585112828689776640*a^47*b^21*c^39*d^37 - 512203696921842163745197916160*
a^48*b^20*c^38*d^38 + 203625309837119046692160667648*a^49*b^19*c^37*d^39 -
70576441632244073218493644800*a^50*b^18*c^36*d^40 + 211515033720754528831143
93600*a^51*b^17*c^35*d^41 - 5422672476777259769580748800*a^52*b^16*c^34*d^
42 + 1172540913492414089228451840*a^53*b^15*c^33*d^43 - 209790609112633976926765056*a^54*b^14*c^32*d^44 + 30239740212369693490544640*a^55*b^13*c^31*d^
45 - 3375777980998666504110080*a^56*b^12*c^30*d^46 + 27398128906276291215360
0*a^57*b^11*c^29*d^47 - 14388779197382598328320*a^58*b^10*c^28*d^48 + 3671
86184646271434752*a^59*b^9*c^27*d^49))*(-(((143986855936*a^35*d^35 + 402820
95616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^
32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30
*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28
*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^2
6*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11
*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608
*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 1124912760455
24992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 107444
3231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18
 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*
c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a
^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 429376756114
5810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142
8045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 -
 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29
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 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31
 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1
081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456
161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26
*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^
10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c
^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*
c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*
c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c
^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023
255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a
^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a
^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 622730400623493
12*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281207215
2268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 443324
7375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11
 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^1
9*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388761692960666419
20*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 3887616
9296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^1
8 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^
12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443324737586724864
0*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 7228120721
52268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273
040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 24
71152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34
084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 201410478
08*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^
32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30
*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*
d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26
*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*
c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*
b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a
^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 5372216157980
67200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 19164
25404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^2
0 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13
*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*
a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 71402273983
3225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 11969295
5670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 931304
1299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 2821464
24569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 54093099827
2*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47
```

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 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^
30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a
^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7
+ 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21
*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*
d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 5657
22720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*
b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d
^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512
240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c
^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*
a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 4
96*a^41*b^2*c^17*d^30)))^(1/4) - (x^(1/2)*(857712418202478182400*a^18*b^48*
c^62*d^11 - 28925330217666430894080*a^19*b^47*c^61*d^12 + 46580835586854460
2210304*a^20*b^46*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c^59*d^14
 + 34982076529826233401212928*a^22*b^44*c^58*d^15 - 19581110681554207729778
6880*a^23*b^43*c^57*d^16 + 873231122236416493313064960*a^24*b^42*c^56*d^17
- 3201588318340888739356606464*a^25*b^41*c^55*d^18 + 9904866981547362725832
687616*a^26*b^40*c^54*d^19 - 26475613142538536817178705920*a^27*b^39*c^53*d
^20 + 62528004036875405150857986048*a^28*b^38*c^52*d^21 - 13314368079621549
1474489344000*a^29*b^37*c^51*d^22 + 259595474982835164713400139776*a^30*b^3
6*c^50*d^23 - 467106577738876991145070559232*a^31*b^35*c^49*d^24 + 77532109
6823109302674935250944*a^32*b^34*c^48*d^25 - 11794249438926800592278264012
8*a^33*b^33*c^47*d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*d^27
 - 2028143345719314676074795761664*a^35*b^31*c^45*d^28 + 225790597310402395
6972306956288*a^36*b^30*c^44*d^29 - 2237449183565830435563494178816*a^37*b^
29*c^43*d^30 + 1966204854457469918399988498432*a^38*b^28*c^42*d^31 - 152764
9406048366621262568488960*a^39*b^27*c^41*d^32 + 104640945875852234799512656
2816*a^40*b^26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25*c^39*d^
34 + 332065764335584004230153764864*a^42*b^24*c^38*d^35 - 15254319696813365
0922715742208*a^43*b^23*c^37*d^36 + 60699171433471101739298979840*a^44*b^22
*c^36*d^37 - 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 6037825951
797032255320227840*a^46*b^20*c^34*d^39 - 1473449639082715479512449024*a^47*
b^19*c^33*d^40 + 296084339424033093684559872*a^48*b^18*c^32*d^41 - 47717950
421254308290887680*a^49*b^17*c^31*d^42 + 5931528400797457427988480*a^50*b^1
6*c^30*d^43 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 31006369751209
579905024*a^52*b^14*c^28*d^45 - 872067188534894657536*a^53*b^13*c^27*d^46)
- (-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*
b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^3
1*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29
*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^2
7*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b
^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a
^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 49371585774551
04*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453
```

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930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2
236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16
*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*
b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486
848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585
093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 +
639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28
 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^3
0 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32
+ 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 42934262
49728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b
^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16
*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c
^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*
c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20
*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*
c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47
+ 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 21990232555
52*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a
^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736
*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 23129986308872
6016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 192749885
9072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8
866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^3
5*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^
25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 413059298770
70807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 3
2396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^
28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^
32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072
716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 2312998
63088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 138
38453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 3
40848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/
2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33
*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^3
1*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29
*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27
*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^
25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^
23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^1
4*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111828572
9418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 -
2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^1
```

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4*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23
*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 134479254681873
6128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748
758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 370403
18338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 185565
3025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147
957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*
b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46
*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3
 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*
c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048
800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21
*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13
+ 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390
*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*
c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 -
 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^
34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25
 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c
^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-((
(143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c
^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31
*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*
d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27
*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^
25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^
23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^1
4*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905
600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571
458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19
- 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c
^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^
23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637
472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329
497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 740
80636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 37
11306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 6155
4295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*
a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^
12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^2
2 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^1
4 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^
16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d
^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^
20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 6871
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9476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^4
2*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^
29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*
b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a
^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 192749885907271
6800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494
751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12
 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^1
8*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 413059298770708070
40*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 3239680
7746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^1
9 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^1
1*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800
*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 2312998630887
26016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 138384533
47188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 3408486
04610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 7
1993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*
d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4
- 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 -
 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 -
 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^1
0 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^2
3*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21
*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^1
6*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111828572941803
5200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 274066
9568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21
 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*
c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a
^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366
208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 370403183381
79072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 185565302561
5872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248
*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^
34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 3
2*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 359
60*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d
^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^
20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*
d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 4714
35600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*
b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d
^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 12902
4480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9
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*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906
192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^
28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(64563604257
983430656*a^25*b^51*c^84*d^4 - 2822351843277561397248*a^26*b^50*c^83*d^5 +
60127162308256283492352*a^27*b^49*c^82*d^6 - 831948157724300777881600*a^28*
b^48*c^81*d^7 + 8406786558179361266073600*a^29*b^47*c^80*d^8 - 661445813058
99203170402304*a^30*b^46*c^79*d^9 + 421912670310680329277407232*a^31*b^45*c
^78*d^10 - 2243238210521587022108295168*a^32*b^44*c^77*d^11 + 1014538325198
4825802817536000*a^33*b^43*c^76*d^12 - 39641949193820336576213811200*a^34*b
^42*c^75*d^13 + 135494098735043868075088674816*a^35*b^41*c^74*d^14 - 409284
915889091539805067542528*a^36*b^40*c^73*d^15 + 1102331957384293957070038761
472*a^37*b^39*c^72*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*d^
17 + 5847343583817169075816733081600*a^39*b^37*c^70*d^18 - 1168410562936832
4959904469090304*a^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a^4
1*b^35*c^68*d^20 - 36343020410925078321345140359168*a^42*b^34*c^67*d^21 + 5
7297580687683561030746426572800*a^43*b^33*c^66*d^22 - 844296589803908142357
81758976000*a^44*b^32*c^65*d^23 + 116702744788425677443098849837056*a^45*b^
31*c^64*d^24 - 151589903153597380791972919246848*a^46*b^30*c^63*d^25 + 1850
08444259789842943656593457152*a^47*b^29*c^62*d^26 - 21175693381543379688118
1835264000*a^48*b^28*c^61*d^27 + 226611959433847997212598992896000*a^49*b^2
7*c^60*d^28 - 225906031446565502788593732550656*a^50*b^26*c^59*d^29 + 20897
8627749165724430025514549248*a^51*b^25*c^58*d^30 - 178726416623100559749866
797924352*a^52*b^24*c^57*d^31 + 140824510781547830729330235801600*a^53*b^23
*c^56*d^32 - 101897270594764980154443340185600*a^54*b^22*c^55*d^33 + 674993
22390719467851063444373504*a^55*b^21*c^54*d^34 - 40809284384591153062742518
136832*a^56*b^20*c^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c^5
2*d^36 - 11195042646819893251483369472000*a^58*b^18*c^51*d^37 + 50428983429
03938117430096691200*a^59*b^17*c^50*d^38 - 2042741359937286689202494242816*
a^60*b^16*c^49*d^39 + 740249793404633986500581654528*a^61*b^15*c^48*d^40 -
238501265489031484884985577472*a^62*b^14*c^47*d^41 + 6780980529692947235597
1891200*a^63*b^13*c^46*d^42 - 16856343881283213574379929600*a^64*b^12*c^45*
d^43 + 3621158066396044540042543104*a^65*b^11*c^44*d^44 - 66227267913872402
5500434432*a^66*b^10*c^43*d^45 + 101087832400064043724832768*a^67*b^9*c^42*
d^46 - 12528855636637836430540800*a^68*b^8*c^41*d^47 + 1211288155775686041
60000*a^69*b^7*c^40*d^48 - 85697808358931542573056*a^70*b^6*c^39*d^49 + 394
6450310269237198848*a^71*b^5*c^38*d^50 - 88774955854727217152*a^72*b^4*c^37
*d^51) - x^(1/2)*(56493153725735501824*a^22*b^52*c^81*d^4 - 239692380807763
4863104*a^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b^50*c^79*d^6 - 6
58598339056129087111168*a^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26
*b^48*c^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 + 2925022535
44635823646834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a^29*b
^45*c^74*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12 - 23152095
046595175238512672768*a^31*b^43*c^72*d^13 + 73885584363642186267654881280*a
^32*b^42*c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^70*d^15 + 5
11001017390776406574528200704*a^34*b^40*c^69*d^16 - 11204864240661618485216
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64233472*a^35*b^39*c^68*d^17 + 21861837328424319732409046138888*a^36*b^38*c^
67*d^18 - 37948899494273681428602547077712*a^37*b^37*c^66*d^19 + 58304702520
6371813468799605 1456*a^38*b^36*c^65*d^20 - 78076190336035905304794696253344*
a^39*b^35*c^64*d^21 + 87461842673859965828752033710 08*a^40*b^34*c^63*d^22 -
 71768719238351983385202193858 56*a^41*b^33*c^62*d^23 + 1365198057841590488 5
49164056576*a^42*b^32*c^61*d^24 + 10199723921158867878218460823552*a^43*b^3
1*c^60*d^25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^26 + 512807
64289348564983994726219776*a^45*b^29*c^58*d^27 - 766964769797208743427005 27
124480*a^46*b^28*c^57*d^28 + 99717561302809906738570708647936*a^47*b^27*c^5
6*d^29 - 11538058817671858214264418965 9136*a^48*b^26*c^55*d^30 + 1201015454
74959969242488481251328*a^49*b^25*c^54*d^31 - 113052494905210552901304563 26
9632*a^50*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*b^23*c^52*
d^33 - 74665519475418951639228294889472*a^52*b^22*c^51*d^34 + 5241392931942
208512211626963 7632*a^53*b^21*c^50*d^35 - 33334185869182979296764484386816*
a^54*b^20*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*c^48*d^37
- 99518274638933356977287457669 12*a^56*b^18*c^47*d^38 + 4646728550801039102
65646481408 0*a^57*b^17*c^46*d^39 - 1944469658660080242790338920448*a^58*b^1
6*c^45*d^40 + 725810983387725632884961181696*a^59*b^15*c^44*d^41 - 24026530
1732777409221605982208*a^60*b^14*c^43*d^42 + 70028310560132415015125778432*
a^61*b^13*c^42*d^43 - 17809629928199177184296828928*a^62*b^12*c^41*d^44 + 3
907197185884869673284009984*a^63*b^11*c^40*d^45 - 728569061655967140126130 1
76*a^64*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^47 - 1
4265899165032610449588224*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^
67*b^7*c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 + 4689814464
763011268608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52)
)*(-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 1361205998387 2*a^2*
b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^3
1*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29
*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^2
7*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b
^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a
^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455 1
04*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453
930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2
236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16
*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*
b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486
848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585
093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 +
639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28
 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^3
0 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32
+ 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 42934262
49728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 1584059900062 5*b
^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16
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*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c
^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*
c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20
*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*
c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47
 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 21990232555
52*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a
^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736
*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 23129986308872
6016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 192749885
9072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8
866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^3
5*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^
25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 413059298770
70807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 3
2396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^
28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^
32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072
716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 2312998
63088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 138
38453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 3
40848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/
2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33
*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^3
1*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29
*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27
*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^
25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^
23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^1
4*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 2066207269654528
00*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111828572
9418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 -
2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^1
4*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23
*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 134479254681873
6128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748
758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 370403
18338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 185565
3025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147
957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*
b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46
*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3
 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*
c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048
800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21
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*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13
+ 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390
*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*
c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 -
 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^
34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25
 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c
^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4) + 1926
09104438451240960*a^18*b^50*c^68*d^8 - 708618067091178232176*a^19*b^49*c^6
7*d^9 + 12507447691366637764608O*a^20*b^48*c^66*d^10 - 141115280550631833600
00000*a^21*b^47*c^65*d^11 + 114401562747726005377433360*a^22*b^46*c^64*d^12
- 710197549047037559203430400*a^23*b^45*c^63*d^13 + 351320863723081970831327
232*a^24*b^44*c^62*d^14 - 142278193473158472668282880O*a^25*b^43*c^61*d^15
+ 480876441231936896819527680O*a^26*b^42*c^60*d^16 - 1375362821409609826802
0736000*a^27*b^41*c^59*d^17 + 336045862656462320079313305600*a^28*b^40*c^58*
d^18 - 704590041452076256580589322240*a^29*b^39*c^57*d^19 + 1263359248135526
58893934428160*a^30*b^38*c^56*d^20 - 1897144207659575875311186739200*a^31*b^
37*c^55*d^21 + 2219472744682837731400744960000*a^32*b^36*c^54*d^22 - 1428707
4034331883415428661248O*a^33*b^35*c^53*d^23 - 176083118177526399618307325955
2*a^34*b^34*c^52*d^24 + 89594702739384832639201443840O*a^35*b^33*c^51*d^25
- 215432334099982299527632650240O*a^36*b^32*c^50*d^26 + 3969865332339043373
838394982400*a^37*b^31*c^49*d^27 - 614764426331211131732549959680O*a^38*b^3
0*c^48*d^28 + 8260762337957580186371563192320*a^39*b^29*c^47*d^29 - 9765601
087086458087650885632000*a^40*b^28*c^46*d^30 + 1022350694830641318286621409
2800*a^41*b^27*c^45*d^31 - 950842473829248398411924766720O*a^42*b^26*c^44*d
^32 + 7866898628254591634401331773440*a^43*b^25*c^43*d^33 - 579072473884148
8066411751276544*a^44*b^24*c^42*d^34 + 3789006704063625484256485048320*a^45
*b^23*c^41*d^35 - 219999620591911794892267847680O*a^46*b^22*c^40*d^36 + 113
0480215059585112828689776640*a^47*b^21*c^39*d^37 - 512203696921842163745197
916160*a^48*b^20*c^38*d^38 + 203625309837119046692160667648*a^49*b^19*c^37*
d^39 - 70576441632244073218493644800*a^50*b^18*c^36*d^40 + 2115150337207545
2883114393600*a^51*b^17*c^35*d^41 - 5422672476777259769580748800*a^52*b^16*
c^34*d^42 + 1172540913492414089228451840*a^53*b^15*c^33*d^43 - 209790609112
633976926765056*a^54*b^14*c^32*d^44 + 30239740212369693490544640*a^55*b^13*
c^31*d^45 - 3375777980998666504110080*a^56*b^12*c^30*d^46 + 273981289062762
912153600*a^57*b^11*c^29*d^47 - 14388779197382598328320*a^58*b^10*c^28*d^48
 + 367186184646271434752*a^59*b^9*c^27*d^49))*(-(((143986855936*a^35*d^35 +
 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856
*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a
^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a
^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*
a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 905027427711672
32*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 5438413745
9908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491
276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 +
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 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^
17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^2
0*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 67275186777463
84896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42937
67561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^2
5 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8
*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c
^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c
^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d
^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (458
1179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500
*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*
b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4
*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^
6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^
8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10
*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 -
2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 340848604
61056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 24711523834
26560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 622730400
62349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281
2072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 +
 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^
36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a
^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296
066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 -
38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c
^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*
a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 44332473758
67248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722
812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25
+ 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^
27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^
29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20
141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*
b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^
30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^2
8*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^
26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^1
1*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 2719206872995430
4*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 562456380227
62496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221
615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18
+ 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c
^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^
```

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22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572
905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140
22739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1
19692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 -
 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 -
282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 5409
30998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^
32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*
a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 2
01376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^
40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 645122
40*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^2
0*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14
 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 56572272
0*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13
*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21
+ 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^3
5*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 -
201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d
^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)))*(-(((143986855936*a^35*d^35 + 40282
095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b
^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^3
0*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^2
8*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^
26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^1
1*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599086
8*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045
524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 10744
43231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^1
8 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15
*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*
a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42937675611
45810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14
28045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27
- 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^2
9 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^3
1 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 -
1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (458117945
6161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^2
6*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c
^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*
c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21
*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19
*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*
c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 219902
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3255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*
a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*
a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 622730400623493
12*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281207215
2268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 44332
47375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^1
1 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^
19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641
920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761
69296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^
18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b
^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 44332473758672486
40*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072
152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 6227
3040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2
471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 3
4084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047
808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c
^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^3
0*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28
*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^2
6*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24
*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13
*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*
a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798
067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916
425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^
20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^1
3*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472
*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140227398
33225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1196929
55670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 93130
41299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146
424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 5409309982
72*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^4
7 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b
^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*
a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7
 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^2
1*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35
*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565
722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28
*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*
d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 6451
2240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*
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c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376
*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 +
496*a^41*b^2*c^17*d^30)))^(1/4)*2i - 2*atan(-((x^(1/2)*(8577124182024781824
00*a^18*b^48*c^62*d^11 - 289253302176664308940800*a^19*b^47*c^61*d^12 + 4658
083558685446022103040*a^20*b^46*c^60*d^13 - 47721899383594535532625920*a^21*b
^45*c^59*d^14 + 349820765298262334012129280*a^22*b^44*c^58*d^15 - 1958111068
1554207729778688000*a^23*b^43*c^57*d^16 + 873231122236416493313064960*a^24*b^
42*c^56*d^17 - 3201588318340888739356606464*a^25*b^41*c^55*d^18 + 990486698
1547362725832687616*a^26*b^40*c^54*d^19 - 26475613142538536817178705920*a^2
7*b^39*c^53*d^20 + 62528004036875405150857986048*a^28*b^38*c^52*d^21 - 1331
43680796215491474489344000*a^29*b^37*c^51*d^22 + 259595474982835164713400013
9776*a^30*b^36*c^50*d^23 - 4671065777388769911450705592320*a^31*b^35*c^49*d^
24 + 7753210968231093026749352509440*a^32*b^34*c^48*d^25 - 1179424943892680005
922278264012800*a^33*b^33*c^47*d^26 + 16296905936000958338232955699200*a^34*b
^32*c^46*d^27 - 20281433457193146760747957616640*a^35*b^31*c^45*d^28 + 22579
059731040239569723069562880*a^36*b^30*c^44*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30 + 19662048544574699183999884984320*a^38*b^28*c^42*
d^31 - 15276494060483666212625684889600*a^39*b^27*c^41*d^32 + 10464094587585
22347995126562816*a^40*b^26*c^40*d^33 - 6299565235927743316987761131520*a^41
*b^25*c^39*d^34 + 3320657643355840042301537648640*a^42*b^24*c^38*d^35 - 1525
43196968133650922715742208*a^43*b^23*c^37*d^36 + 606991714334711017392989798
40*a^44*b^22*c^36*d^37 - 20757436699772395749793333248*a^45*b^21*c^35*d^38
 + 6037825951797032255320227840*a^46*b^20*c^34*d^39 - 147344963908271547951
2449024*a^47*b^19*c^33*d^40 + 296084339424033093684559872*a^48*b^18*c^32*d^
41 - 47717950421254308290887680*a^49*b^17*c^31*d^42 + 593152840079745742798
8480*a^50*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 3
1006369751209579905024*a^52*b^14*c^28*d^45 - 872067188534894657536*a^53*b^1
3*c^27*d^46) - (-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 136120
59983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 5856445102
81728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 754041490715
4432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 378344208995
45088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492
445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 807097
71031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4
937158577455104*a^14*b^21*c^21*d^14 + 11249127604552499 2*a^15*b^20*c^20*d^1
5 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*
c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a
^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659921368844
0389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 582
7091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d
^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^
9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28
*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^3
0*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*
b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34
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*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 158
40599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497
084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 50654279047121
40*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046
776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 1176083944143
7688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053
932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^
11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d
- 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 3408
48604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 1383
8453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 2
31299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^
8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*
c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*
a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 3239680774
6722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 +
 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*
c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600
*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 886649475
1734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 -
1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d
^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^
21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*
c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^
17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 680602999
1936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29282225140864
*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*
a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*
a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848
*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515 9
52128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579
288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206
62072696545280 0*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^1
7 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16
*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*
a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 29135457702
72743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13
44792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^2
6 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*
d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*
d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^
32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 214671
3124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a
^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*
b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 9061
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92*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^3
9*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024
480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^
19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^1
5 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 4714356
00*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^1
2*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22
- 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*
b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 359
60*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))
^(1/4)*(((-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 136120599838
72*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*
a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a
^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*
a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696
*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 807097710319
04256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158
577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 41
3241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d
^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^
16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659921368844038963
2*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 582709154
0545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 +
2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*
d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c
^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*
c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^
3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4
293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 158405990
00625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^
11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3
*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^
5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a
^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^
9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^3
2*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199
023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 3408486046
10560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 1383845334
7188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 2312998
63088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 19
27498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d
^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b
^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 323968077467222
01600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305
929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d
```

```
^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*
b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 886649475173449
7280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 192749
8859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 -
231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^2
6 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d
^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^3
0))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a
^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b
^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^
29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^
27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*
b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*
a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727
552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726
965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11
18285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*
d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b
^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 29135457702727434
24*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925
46818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31
9664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 +
 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 +
 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3
0777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 214671312486
4*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^
31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c
^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^1
7*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8
- 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^
22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^3
4*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 60
1080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^2
9*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27
*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 2804
8800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^
22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^3
9*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)
*(64563604257983430656*a^25*b^51*c^84*d^4 - 2822351843277561397248*a^26*b^5
0*c^83*d^5 + 60127162308256283492352*a^27*b^49*c^82*d^6 - 83194815772430077
7881600*a^28*b^48*c^81*d^7 + 8406786558179361266073600*a^29*b^47*c^80*d^8 -
 66144581305899203170402304*a^30*b^46*c^79*d^9 + 42191267031068032927740723
2*a^31*b^45*c^78*d^10 - 2243238210521587022108295168*a^32*b^44*c^77*d^11 +
10145383251984825802817536000*a^33*b^43*c^76*d^12 - 39641949193820336576213
811200*a^34*b^42*c^75*d^13 + 135494098735043868075088674816*a^35*b^41*c^74*
```

d^14 - 40928491588909153980506754252 8*a^36*b^40*c^73*d^15 + 110233195738429
3957070038761472*a^37*b^39*c^72*d^16 - 2668223165968086459433038643200*a^38
*b^38*c^71*d^17 + 5847343583817169075816733081600*a^39*b^37*c^70*d^18 - 116
84105629368324959904469090304*a^40*b^36*c^69*d^19 + 2143500246269863704109 8
955948032*a^41*b^35*c^68*d^20 - 36343020410925078321345140359168*a^42*b^34*
c^67*d^21 + 57297580687683561030746426572800*a^43*b^33*c^66*d^22 - 84429658
980390814235781758976000*a^44*b^32*c^65*d^23 + 116702744788425677443098849 8
37056*a^45*b^31*c^64*d^24 - 151589903153597380791972919246848*a^46*b^30*c^6
3*d^25 + 185008444259789842943656593457152*a^47*b^29*c^62*d^26 - 2117569338
15433796881181835264000*a^48*b^28*c^61*d^27 + 2266119594338479972125989928 9
6000*a^49*b^27*c^60*d^28 - 225906031446565502788593732550656*a^50*b^26*c^59
*d^29 + 208978627749165724430025514549248*a^51*b^25*c^58*d^30 - 17872641662
3100559749866797924352*a^52*b^24*c^57*d^31 + 14082451078154783072933023580 1
600*a^53*b^23*c^56*d^32 - 101897270594764980154443340185600*a^54*b^22*c^55*
d^33 + 67499322390719467851063444373504*a^55*b^21*c^54*d^34 - 4080928438459
1153062742518136832*a^56*b^20*c^53*d^35 + 22447282431345050697947118829568*
a^57*b^19*c^52*d^36 - 11195042646819893251483369472000*a^58*b^18*c^51*d^37
+ 5042898342903938117430096691200*a^59*b^17*c^50*d^38 - 204274135993728668 9
20249424281 6*a^60*b^16*c^49*d^39 + 740249793404633986500581654528*a^61*b^15
*c^48*d^40 - 238501265489031484884985577472*a^62*b^14*c^47*d^41 + 678098052
96929472355971891200*a^63*b^13*c^46*d^42 - 16856343881283213574379929600*a^
64*b^12*c^45*d^43 + 3621158066396044540042543104*a^65*b^11*c^44*d^44 - 6622
72679138724025500434432*a^66*b^10*c^43*d^45 + 101087832400064043724832768*a
^67*b^9*c^42*d^46 - 12528855636637836430540800*a^68*b^8*c^41*d^47 + 1211288
155777568604160000*a^69*b^7*c^40*d^48 - 85697808358931542573056*a^70*b^6*c^
39*d^49 + 3946450310269237198848*a^71*b^5*c^38*d^50 - 88774955854727217152*
a^72*b^4*c^37*d^51)*1i + x^(1/2)*(564931537257355018 24*a^22*b^52*c^81*d^4 -
2396923808077634863104*a^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b
^50*c^79*d^6 - 658598339056129087111168*a^25*b^49*c^78*d^7 + 63911638676343
30475954176*a^26*b^48*c^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*
d^9 + 292502253544635823646834688*a^28*b^46*c^75*d^10 - 14760026454804159 17
311524864*a^29*b^45*c^74*d^11 + 6306003584409325504378699776*a^30*b^44*c^73
*d^12 - 23152095046595175238512672768*a^31*b^43*c^72*d^13 + 738855843636421
86267654881280*a^32*b^42*c^71*d^14 - 206784189076489114265239683072*a^33*b^
41*c^70*d^15 + 511001017390776406574528200704*a^34*b^40*c^69*d^16 - 1120486
424066161848521664233472*a^35*b^39*c^68*d^17 + 218618373284243197324090461 3
888*a^36*b^38*c^67*d^18 - 3794889949427368142860254707712*a^37*b^37*c^66*d^
19 + 5830470252063718134687996051456*a^38*b^36*c^65*d^20 - 7807619033603590
53047946962534 4*a^39*b^35*c^64*d^21 + 8746184267385996582875203371008*a^40*
b^34*c^63*d^22 - 7176871923835198338520219385856*a^41*b^33*c^62*d^23 + 1365
1980578415904885491640565 76*a^42*b^32*c^61*d^24 + 101997239211588678782184 6
0823552*a^43*b^31*c^60*d^25 - 28100654056180096231365094146048*a^44*b^30*c^
59*d^26 + 51280764289348564983994726219776*a^45*b^29*c^58*d^27 - 7669647697
97208743427005271 24480*a^46*b^28*c^57*d^28 + 99717561302809906738570708647 9
36*a^47*b^27*c^56*d^29 - 115380588176718582142644189659136*a^48*b^26*c^55*d

```
^30 + 120101545474959969242488481251328*a^49*b^25*c^54*d^31 - 1130524949052
10552901304563269632*a^50*b^24*c^53*d^32 + 96462689920395704646643948191744
*a^51*b^23*c^52*d^33 - 74665519475418951639228294889472*a^52*b^22*c^51*d^34
 + 52413929319422085122116269637632*a^53*b^21*c^50*d^35 - 33334185869182979
296764484386816*a^54*b^20*c^49*d^36 + 19174031096138345851817803382784*a^55
*b^19*c^48*d^37 - 9951827463893335697728745766912*a^56*b^18*c^47*d^38 + 464
6728550801039102656464814080*a^57*b^17*c^46*d^39 - 194446965866008024279033
8920448*a^58*b^16*c^45*d^40 + 725810983387725632884961181696*a^59*b^15*c^44
*d^41 - 240265301732777409221605982208*a^60*b^14*c^43*d^42 + 70028310560132
415015125778432*a^61*b^13*c^42*d^43 - 17809629928199177184296828928*a^62*b^
12*c^41*d^44 + 3907197185884869673284009984*a^63*b^11*c^40*d^45 - 728569061
655967140126130176*a^64*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b
^9*c^38*d^47 - 14265899165032610449588224*a^66*b^8*c^37*d^48 + 140050916393
5752188329984*a^67*b^7*c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d
^50 + 4689814464763011268608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^7
0*b^4*c^33*d^52))*(-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 136
12059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 5856445
10281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 754041490
7154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 378344208
99545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272
492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 807
09771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13
+ 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*
d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^
18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383285080985737216
0*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659921368
8440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 +
5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^1
1*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26
*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a
^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*
a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^
32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c
^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 +
15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782
497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 50654279047
12140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958
046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 1176083944
1437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929
053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736
*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46
*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 3
40848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 1
3838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6
- 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39
```

```
*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^
22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155163658155353702
40*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 3239680
7746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^1
5 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^
15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2387133202390056
9600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 886649
4751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22
 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^2
3*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6
*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b
^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2
*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 680602
9991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140
864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 37702074535772
16*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 189172104497725
44*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222
848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 403548855
15952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468
579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 -
206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*
d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b
^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 32996068442201948
16*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 29135457
70272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 +
 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*
d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c
^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c
^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3
*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 214
6713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 3
2*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^
14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 9
06192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*
c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129
024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24
*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*
d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 4714
35600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*
b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^
22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^
36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 +
35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30
)))^(3/4)*1i - 192609104438451240960*a^18*b^50*c^68*d^8 + 7086180670911782322176*a^19*b^49*c^67*d^9 - 125074476913666377646080*a^20*b^48*c^66*d^10 + 1
```

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41115280550631833600000 0*a^21*b^47*c^65*d^11 - 11440156274772600537743360*a
^22*b^46*c^64*d^12 + 71019754904703755920343040*a^23*b^45*c^63*d^13 - 35132
086372308197083132723 2*a^24*b^44*c^62*d^14 + 14227819347315847266828 28800*a
^25*b^43*c^61*d^15 - 480876441231936896819527 6800*a^26*b^42*c^60*d^16 + 137
53628214096098268020736000*a^27*b^41*c^59*d^17 - 336045862656462320079313 30
560*a^28*b^40*c^58*d^18 + 704590041452076256580589322 24*a^29*b^39*c^57*d^19
 - 1263359248135526588939344281 60*a^30*b^38*c^56*d^20 + 1897144207659575875
31118673920*a^31*b^37*c^55*d^21 - 2219472744682837731400744960 00*a^32*b^36*
c^54*d^22 + 14287074034331883415428661248 0*a^33*b^35*c^53*d^23 + 1760831181
775263996183073259 52*a^34*b^34*c^52*d^24 - 8959470273938483263920144384 00*a
^35*b^33*c^51*d^25 + 21543233409998229952763265024 00*a^36*b^32*c^50*d^26 -
39698653323390433738383949824 00*a^37*b^31*c^49*d^27 + 6147644263312111317 32
5499596800*a^38*b^30*c^48*d^28 - 826076233795758018637156319232 0*a^39*b^29*
c^47*d^29 + 97656010870864580876508856320 00*a^40*b^28*c^46*d^30 - 102235069
48306413182866214092800*a^41*b^27*c^45*d^31 + 950842473829248398411924766 72
00*a^42*b^26*c^44*d^32 - 78668986282545916344013317734 40*a^43*b^25*c^43*d^3
3 + 579072473884148806641175127654 4*a^44*b^24*c^42*d^34 - 37890067040636254
84256485048320*a^45*b^23*c^41*d^35 + 2199996205919117948922678476 800*a^46*b
^22*c^40*d^36 - 113048021505958511282868977664 0*a^47*b^21*c^39*d^37 + 51220
369692184216374519791616 0*a^48*b^20*c^38*d^38 - 203625309837119046692160667
648*a^49*b^19*c^37*d^39 + 70576441632244073218493644800*a^50*b^18*c^36*d^40
 - 21151503372075452883114393600*a^51*b^17*c^35*d^41 + 5422672476777259769 5
80748800*a^52*b^16*c^34*d^42 - 1172540913492414089228451840*a^53*b^15*c^33*
d^43 + 209790609112633976926765056*a^54*b^14*c^32*d^44 - 30239740212369693 4
90544640*a^55*b^13*c^31*d^45 + 33757779809986665041100 80*a^56*b^12*c^30*d^4
6 - 2739812890627629121536 00*a^57*b^11*c^29*d^47 + 143887791973825983283 20*
a^58*b^10*c^28*d^48 - 367186184646271434752*a^59*b^9*c^27*d^49)*1i)*(-(((14
3986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33
*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^
4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6
 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^
8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*
d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*
c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b
^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600
*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458
836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5
481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14
*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*
b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472
256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497
516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 740806
36676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 37113
06051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 6155429
5914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342624972 8*a^3
```

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4*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*
d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 +
 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 +
 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16
+ 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18
 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20
+ 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 6871947
6736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b
*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*
c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^2
7*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18
*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 192749885907271680
0*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751
734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 -
23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c
^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*
a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 3239680774
6722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 +
 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c
^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^
34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 2312998630887260
16*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 138384533471
88736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 3408486046
10560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 7199
3427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2
 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1
195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 94
14767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30
906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 -
 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d
^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^
21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b
^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111828572941803520
0*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 274066956
8090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 -
3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^1
2*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25
*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208
*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 370403183381790
72*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 185565302561587
2*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^
33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)
/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a
^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*
a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6
```

- 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*
b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^1
1 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 4714356
00*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^1
6*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18
 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 12902448
0*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^
24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192
*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28
- 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4) + (x^(1/2)*(857
712418202478182400*a^18*b^48*c^62*d^11 - 28925330217666430894080*a^19*b^47*
c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13 - 4772189938359453
553262592*a^21*b^45*c^59*d^14 + 34982076529826233401212928*a^22*b^44*c^58*d
^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 + 873231122364164933
13064960*a^24*b^42*c^56*d^17 - 3201588318340888739356606464*a^25*b^41*c^55*
d^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19 - 26475613142538536
817178705920*a^27*b^39*c^53*d^20 + 62528004036875405150857986048*a^28*b^38*
c^52*d^21 - 133143680796215491474489344000*a^29*b^37*c^51*d^22 + 2595954749
82835164713400139776*a^30*b^36*c^50*d^23 - 467106577738876991145070559232*a
^31*b^35*c^49*d^24 + 775321096823109302674935250944*a^32*b^34*c^48*d^25 - 1
179424943892680059222782640128*a^33*b^33*c^47*d^26 + 1629690593600095833823
295569920*a^34*b^32*c^46*d^27 - 2028143345719314676074795761664*a^35*b^31*c
^45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^44*d^29 - 2237449183
565830435563494178816*a^37*b^29*c^43*d^30 + 1966204854457469918399988498432
*a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a^39*b^27*c^41*d^32
+ 1046409458758522347995126562816*a^40*b^26*c^40*d^33 - 6299565235927743316
98776113152*a^41*b^25*c^39*d^34 + 332065764335584004230153764864*a^42*b^24*
c^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^37*d^36 + 6069917143
3471101739298979840*a^44*b^22*c^36*d^37 - 20757436699772395749793333248*a^4
5*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^20*c^34*d^39 - 14734
49639082715479512449024*a^47*b^19*c^33*d^40 + 296084339424033093684559872*a
^48*b^18*c^32*d^41 - 47717950421254308290887680*a^49*b^17*c^31*d^42 + 59315
28400797457427988480*a^50*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^
15*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^45 - 87206718853489
4657536*a^53*b^13*c^27*d^46) + (-(((143986855936*a^35*d^35 + 40282095616*b^
35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*
d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^
5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^
7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d
^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^
24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^
22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^
15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 10744432315961
34400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38328
50809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^2

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0 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13
*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*
a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14280454796
66450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 2393859
11340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626
082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 56429
2849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 108186199
6544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12
*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^
12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13
- 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15
- 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17
 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19
 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)
*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a
^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30
*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28
*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b
^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a
^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 44332473758672
48640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516
365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d
^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*
b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692960666
41920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2387
1332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*
d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b
^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800
*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 6227304006234
9312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383
426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 3408486046
1056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*
c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 +
 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3
770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18
917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4
1306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^1
1 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^2
2*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20
*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^
17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928
686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299
606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^
22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^1
1*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*
```

a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*

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d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^
41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^
24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*
a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815
535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 +
32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c
^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*
a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2387133202
3900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 -
 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^
25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*
b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a
^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560
*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a
^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 +
 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29282
2255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207
453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210
449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130613
6246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40
354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13
 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*
d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^1
8*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080
*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844
220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2
913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11
*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b
^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^2
8*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^3
0*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*
b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*
d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d
^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 -
4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*
d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^1
9*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^1
0 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 3473736
00*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^1
7*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17
 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 22579284
0*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*
c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 336
5856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*
d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^
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17*d^30)))^(1/4)*(6456360425798343065 6*a^25*b^51*c^84*d^4 - 282235184327756
1397248*a^26*b^50*c^83*d^5 + 6012716230825628349 2352*a^27*b^49*c^82*d^6 - 8
319481577243007778816 00*a^28*b^48*c^81*d^7 + 840678655817936126607 3600*a^29
*b^47*c^80*d^8 - 6614458130589920317040 2304*a^30*b^46*c^79*d^9 + 4219126703
106803292774 07232*a^31*b^45*c^78*d^10 - 22432382105215870221082 95168*a^32*b
^44*c^77*d^11 + 1014538325198482580281 7536000*a^33*b^43*c^76*d^12 - 3964194
919382033657621381 1200*a^34*b^42*c^75*d^13 + 135494098735043868075088674816
*a^35*b^41*c^74*d^14 - 409284915889091539805067542528*a^36*b^40*c^73*d^15 +
 1102331957384293957070038761472*a^37*b^39*c^72*d^16 - 266822316596808645 94
33038643200*a^38*b^38*c^71*d^17 + 5847343583817169075816733081600*a^39*b^37
*c^70*d^18 - 11684105629368324959904469090304*a^40*b^36*c^69*d^19 + 2143500
24626986370410989559480 32*a^41*b^35*c^68*d^20 - 363430204109250783213451403
59168*a^42*b^34*c^67*d^21 + 57297580687683561030746426572800*a^43*b^33*c^66
*d^22 - 84429658980390814235781758976000*a^44*b^32*c^65*d^23 + 116702744788
425677443098849837056*a^45*b^31*c^64*d^24 - 151589903153597380791972919246 8
48*a^46*b^30*c^63*d^25 + 185008444259789842943656593457152*a^47*b^29*c^62*d
^26 - 211756933815433796881181835264000*a^48*b^28*c^61*d^27 + 2266119594338
47997212598992896000*a^49*b^27*c^60*d^28 - 2259060314465655027885937325506 5
6*a^50*b^26*c^59*d^29 + 208978627749165724430025514549248*a^51*b^25*c^58*d^
30 - 178726416623100559749866797924352*a^52*b^24*c^57*d^31 + 1408245107815 4
78307293302358016 00*a^53*b^23*c^56*d^32 - 101897270594764980154443340185600
*a^54*b^22*c^55*d^33 + 67499322390719467851063444373504*a^55*b^21*c^54*d^34
 - 40809284384591153062742518136832*a^56*b^20*c^53*d^35 + 2244728243134505 0
697947118829568*a^57*b^19*c^52*d^36 - 11195042646819893251483369472000*a^58
*b^18*c^51*d^37 + 5042898342903938117430096691200*a^59*b^17*c^50*d^38 - 204
2741359937286689202494242816*a^60*b^16*c^49*d^39 + 74024979340463398650058 1
654528*a^61*b^15*c^48*d^40 - 238501265489031484884985577472*a^62*b^14*c^47*
d^41 + 67809805296929472355971891200*a^63*b^13*c^46*d^42 - 168563438812832 1
357437992960 0*a^64*b^12*c^45*d^43 + 3621158066396044540042543104*a^65*b^11*
c^44*d^44 - 662272679138724025500434432*a^66*b^10*c^43*d^45 + 101087832400 0
6404372483276 8*a^67*b^9*c^42*d^46 - 12528855636637836430540800*a^68*b^8*c^4
1*d^47 + 1211288155777568604160000*a^69*b^7*c^40*d^48 - 8569780835893154257
3056*a^70*b^6*c^39*d^49 + 3946450310269237198848*a^71*b^5*c^38*d^50 - 88774
95585472721715 2*a^72*b^4*c^37*d^51)*1i - x^(1/2)*(5649315372573550182 4*a^22
*b^52*c^81*d^4 - 2396923808077634863104*a^23*b^51*c^80*d^5 + 49387698492843
503910912*a^24*b^50*c^79*d^6 - 658598339056129087111168*a^25*b^49*c^78*d^7
+ 6391163867634330475954176*a^26*b^48*c^77*d^8 - 48113596867651945069805568
*a^27*b^47*c^76*d^9 + 292502253544635823646834688*a^28*b^46*c^75*d^10 - 147
6002645480415917311524864*a^29*b^45*c^74*d^11 + 630600358440932550437869977
6*a^30*b^44*c^73*d^12 - 23152095046595175238512672768*a^31*b^43*c^72*d^13 +
 73885584363642186267654881280*a^32*b^42*c^71*d^14 - 206784189076489114265 2
39683072*a^33*b^41*c^70*d^15 + 511001017390776406574528200704*a^34*b^40*c^6
9*d^16 - 1120486424066161848521664233472*a^35*b^39*c^68*d^17 + 21861837328 4
2431973240904613888*a^36*b^38*c^67*d^18 - 3794889949427368142860254707712*a
^37*b^37*c^66*d^19 + 5830470252063718134687996051456*a^38*b^36*c^65*d^20 -
```

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7807619033603590530479469625344*a^39*b^35*c^64*d^21 + 874618426738599658287
5203371008*a^40*b^34*c^63*d^22 - 7176871923835198338520219385856*a^41*b^33*
c^62*d^23 + 1365198057841590488549164056576*a^42*b^32*c^61*d^24 + 101997239
2115886787821846082355 2*a^43*b^31*c^60*d^25 - 281006540561800962313650941 46
048*a^44*b^30*c^59*d^26 + 5128076428934856498399472621977 6*a^45*b^29*c^58*d
^27 - 7669647697972087434270052712448 0*a^46*b^28*c^57*d^28 + 99717561302809
9067385707086479 36*a^47*b^27*c^56*d^29 - 115380588176718582142644189659136*
a^48*b^26*c^55*d^30 + 120101545474959969242488481251328*a^49*b^25*c^54*d^31
 - 113052494905210552901304563269632*a^50*b^24*c^53*d^32 + 9646268992039570
4646643948191744*a^51*b^23*c^52*d^33 - 74665519475418951639228294889472*a^5
2*b^22*c^51*d^34 + 52413929319422085122116269637632*a^53*b^21*c^50*d^35 - 3
3334185869182979296764484386816*a^54*b^20*c^49*d^36 + 191740310961383458518
17803382784*a^55*b^19*c^48*d^37 - 9951827463893335697728745766912*a^56*b^18
*c^47*d^38 + 4646728550801039102656464814080*a^57*b^17*c^46*d^39 - 19444696
58660080242790338920448*a^58*b^16*c^45*d^40 + 725810983387725632884961181 69
6*a^59*b^15*c^44*d^41 - 240265301732777409221605982208*a^60*b^14*c^43*d^42
+ 70028310560132415015125778432*a^61*b^13*c^42*d^43 - 178096299281991771842
96828928*a^62*b^12*c^41*d^44 + 3907197185884869673284009984*a^63*b^11*c^40*
d^45 - 728569061655967140126130176*a^64*b^10*c^39*d^46 + 113214808531319939
527606272*a^65*b^9*c^38*d^47 - 14265899165032610449588224*a^66*b^8*c^37*d^4
8 + 1400509163935752188329984*a^67*b^7*c^36*d^49 - 100502833687558254231552
*a^68*b^6*c^35*d^50 + 4689814464763011268608*a^69*b^5*c^34*d^51 - 106807368
762718683136*a^70*b^4*c^33*d^52))*(-(((143986855936*a^35*d^35 + 40282095616
*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^
32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30
*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28
*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^2
6*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24
*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13
*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992
*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 10744432315
96134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38
32850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*
d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b
^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42937675611458109
44*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14280454
79666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 2393
85911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18
626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 56
4292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 108186
1996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a
^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11
*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^
13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^
15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d
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^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d
^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^
21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 219902325555
2*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b
^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b
^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^1
7*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281207215226880
0*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 44332473758
67248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15
516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^3
4*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^
26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692960
66641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2
3871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^
27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^3
3*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268
800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 6227304006
2349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152
383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 3408486
0461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^
35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^
3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5
+ 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 +
 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9
+ 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*
d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*
c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b
^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200
*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404
928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3
299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13
*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*
b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140227398332252
16*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1196929556701
34784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 93130412994
23232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569
856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b
^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^
43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^
45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b
^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 105
18300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22
*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12
- 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720
*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*
```

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c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 +
 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a
^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d
^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*
b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^
41*b^2*c^17*d^30)))^(3/4)*1i - 19260910443845124096O*a^18*b^50*c^68*d^8 + 7
086180670911782322176*a^19*b^49*c^67*d^9 - 125074476913666377646080*a^20*b^
48*c^66*d^10 + 1411152805506318336000000*a^21*b^47*c^65*d^11 - 114401562747
72600537743360*a^22*b^46*c^64*d^12 + 71019754904703755920343040*a^23*b^45*c
^63*d^13 - 351320863723081970831327232*a^24*b^44*c^62*d^14 + 14227819347315
84726682828800*a^25*b^43*c^61*d^15 - 4808764412319368968195276800*a^26*b^42
*c^60*d^16 + 13753628214096098268020736000*a^27*b^41*c^59*d^17 - 3360458626
5646232007931330560*a^28*b^40*c^58*d^18 + 70459004145207625658058932224*a^2
9*b^39*c^57*d^19 - 126335924813552658893934428160*a^30*b^38*c^56*d^20 + 189
714420765957587531118673920*a^31*b^37*c^55*d^21 - 2219472744682837731400744
96000*a^32*b^36*c^54*d^22 + 142870740343318834154286612480*a^33*b^35*c^53*d
^23 + 176083118177526399618307325952*a^34*b^34*c^52*d^24 - 8959470273938483
26392014438400*a^35*b^33*c^51*d^25 + 2154323340999822995276326502400*a^36*b
^32*c^50*d^26 - 3969865332339043373838394982400*a^37*b^31*c^49*d^27 + 61476
44263312111317325499596800*a^38*b^30*c^48*d^28 - 826076233795758018637156З1
92320*a^39*b^29*c^47*d^29 + 9765601087086458087650885632000*a^40*b^28*c^46*
d^30 - 10223506948306413182866214092800*a^41*b^27*c^45*d^31 + 9508424738292
483984119247667200*a^42*b^26*c^44*d^32 - 7866898628254591634401331773440*a^
43*b^25*c^43*d^33 + 5790724738841488066411751276544*a^44*b^24*c^42*d^34 - 3
789006704063625484256485048320*a^45*b^23*c^41*d^35 + 2199996205919117948922
678476800*a^46*b^22*c^40*d^36 - 1130480215059585112828689776640*a^47*b^21*c
^39*d^37 + 512203696921842163745197916160*a^48*b^20*c^38*d^38 - 20362530983
7119046692160667648*a^49*b^19*c^37*d^39 + 70576441632244073218493644800*a^5
0*b^18*c^36*d^40 - 21151503372075452883114393600*a^51*b^17*c^35*d^41 + 5422
672476777259769580748800*a^52*b^16*c^34*d^42 - 1172540913492414089228451840
*a^53*b^15*c^33*d^43 + 209790609112633976926765056*a^54*b^14*c^32*d^44 - 30
239740212369693490544640*a^55*b^13*c^31*d^45 + 3375777980998666504110080*a^
56*b^12*c^30*d^46 - 273981289062762912153600*a^57*b^11*c^29*d^47 + 14388779
197382598328320*a^58*b^10*c^28*d^48 - 367186184646271434752*a^59*b^9*c^27*d
^49)*1i)*(-(((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 136120599838
72*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*
a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a
^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*
a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696
*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 807097710319
04256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158
577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 41
3241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d
^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^
16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659921368844038963
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2*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 582709154
0545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 +
2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*
d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c
^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*
c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^
3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4
293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 158405990
00625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^
11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3
*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^
5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a
^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^
9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^3
2*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199
023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 3408486046
10560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 1383845334
7188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 2312998
63088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 19
27498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d
^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b
^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 3239680774672220
1600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305
929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d
^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*
b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 886649475173449
7280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 192749
8859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 -
231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^2
6 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d
^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^3
0))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a
^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b
^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^
29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^
27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*
b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*
a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727
552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726
965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11
18285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*
d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b
^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 29135457702727434
24*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925
46818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31
```

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9664748758366208*a^27*b^8*c^8*d^27 - 1196929556701347 84*a^28*b^7*c^7*d^28 +
 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 +
 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3
0777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 214671312486
4*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^
31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c
^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^1
7*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8
- 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^
22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^3
4*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 60
1080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^2
9*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27
*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 2804
8800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^
22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^3
9*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)
)/((x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 - 289253302176664308
94080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13 -
4772189938359453553262592*a^21*b^45*c^59*d^14 + 34982076529826233401212928*
a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 + 873
231122236416493313064960*a^24*b^42*c^56*d^17 - 3201588318340888739356606464
*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19 - 2
6475613142538536817178705920*a^27*b^39*c^53*d^20 + 625280040368754051508579
86048*a^28*b^38*c^52*d^21 - 133143680796215491474489344000*a^29*b^37*c^51*d
^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23 - 467106577388769
91145070559232*a^31*b^35*c^49*d^24 + 775321096823109302674935250944*a^32*b^
34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*c^47*d^26 + 162969
0593600095833823295569920*a^34*b^32*c^46*d^27 - 202814334571931467607479576
1664*a^35*b^31*c^45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^44*d
^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30 + 196620485445746
9918399988498432*a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a^39
*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b^26*c^40*d^33 - 629
956523592774331698776113152*a^41*b^25*c^39*d^34 + 3320657643355840042301537
64864*a^42*b^24*c^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^37*d
^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^37 - 20757436699772395
749793333248*a^45*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^20*c
^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d^40 + 2960843394240
33093684559872*a^48*b^18*c^32*d^41 - 47717950421254308290887680*a^49*b^17*c
^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*d^43 - 5340378611857240
02336768*a^51*b^15*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^45
- 872067188534894657536*a^53*b^13*c^27*d^46) - (-(((143986855936*a^35*d^35
+ 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 10675201612185
6*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*
a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*
```

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a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464
*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167
232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374
59908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249
1276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16
+ 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c
^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^
20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746
384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293
767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^
25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^
8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*
c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*
c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*
d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (45
81179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 23112188256150
0*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2
*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^
4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a
^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a
^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^1
0*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 -
 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860
461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383
426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040
062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228
12072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9
+ 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c
^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*
a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616929
6066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 -
 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*
c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240
*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375
867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72
2812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25
 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d
^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d
^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 2
0141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3
*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b
^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^
28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b
^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^
11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 271920687299543
```

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04*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022
762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53722
1615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18
 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*
c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a
^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378057
2905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714
022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 -
119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29
- 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 -
 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540
930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b
^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496
*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 -
201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c
^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512
240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^
20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^1
4 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 5657227
20*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^1
3*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21
 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^
35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 -
 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*
d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-(((143986855936*a^35*d^35 + 4028
2095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*
b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^
30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^
28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b
^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^
11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599086
08*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249127604
5524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074
443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^
18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^1
5*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896
*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561
145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1
428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27
 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^
29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^
31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 -
 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (45811794
56161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^
26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*
```

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c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23
*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^2
1*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^1
9*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17
*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 21990
23255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056
*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560
*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349
312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228120721
52268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433
247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^
11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b
^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616929606664
1920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876
169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d
^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*
b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248
640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281207
2152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 622
73040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 +
2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 +
34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 2014104
7808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*
c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^
30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^2
8*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^
26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^2
4*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^1
3*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496
*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53722161579
8067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 191
6425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d
^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^
13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378057290547
2*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739
833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692
955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313
041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 28214
6424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998
272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^
47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*
b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376
*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^
7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^
21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^3
```

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5*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 56
5722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^2
8*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28
*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 645
12240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8
*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 20137
6*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 +
 496*a^41*b^2*c^17*d^30)))^(1/4)*(645636042579834306656*a^25*b^51*c^84*d^4 -
 28223518432775613972448*a^26*b^50*c^83*d^5 + 601271623082562834923 52*a^27*b
^49*c^82*d^6 - 83194815772430077788 1600*a^28*b^48*c^81*d^7 + 84067865581793
61266073600*a^29*b^47*c^80*d^8 - 6614458130589920317040 2304*a^30*b^46*c^79*
d^9 + 42191267031068032927 7407232*a^31*b^45*c^78*d^10 - 2243238210521587022
108295168*a^32*b^44*c^77*d^11 + 1014538325198482580281 7536000*a^33*b^43*c^7
6*d^12 - 39641949193820336576213811200*a^34*b^42*c^75*d^13 + 13549409873504
3868075088674816*a^35*b^41*c^74*d^14 - 40928491588909153980506 7542528*a^36*
b^40*c^73*d^15 + 110233195738429395707003 8761472*a^37*b^39*c^72*d^16 - 2668
22316596808645943303 8643200*a^38*b^38*c^71*d^17 + 5847343583817169075816733
081600*a^39*b^37*c^70*d^18 - 11684105629368324959904469090304*a^40*b^36*c^6
9*d^19 + 21435002462698637041098955948032*a^41*b^35*c^68*d^20 - 36343020410
925078321345140359168*a^42*b^34*c^67*d^21 + 572975806876835610307464265728 0
0*a^43*b^33*c^66*d^22 - 84429658980390814235781758976000*a^44*b^32*c^65*d^2
3 + 116702744788425677443098849837056*a^45*b^31*c^64*d^24 - 151589903153597
380791972919246848*a^46*b^30*c^63*d^25 + 185008444259789842943656593457152*
a^47*b^29*c^62*d^26 - 211756933815433796881181835264000*a^48*b^28*c^61*d^27
 + 226611959433847997212598992896000*a^49*b^27*c^60*d^28 - 225906031446565 5
02788593732550656*a^50*b^26*c^59*d^29 + 208978627749165724430025514549248*a
^51*b^25*c^58*d^30 - 178726416623100559749866797924352*a^52*b^24*c^57*d^31
+ 140824510781547830729330235801600*a^53*b^23*c^56*d^32 - 101897270594764 98
0154443340185600*a^54*b^22*c^55*d^33 + 67499322390719467851063444373504*a^5
5*b^21*c^54*d^34 - 40809284384591153062742518136832*a^56*b^20*c^53*d^35 + 2
2447282431345050697947118829568*a^57*b^19*c^52*d^36 - 1119504264681989325 14
83369472000*a^58*b^18*c^51*d^37 + 5042898342903938117430096691200*a^59*b^17
*c^50*d^38 - 2042741359937286689202494242816*a^60*b^16*c^49*d^39 + 74024979
34046339865005816545 28*a^61*b^15*c^48*d^40 - 238501265489031484884985577472
*a^62*b^14*c^47*d^41 + 67809805296929472355971891200*a^63*b^13*c^46*d^42 -
16856343881283213574379929600*a^64*b^12*c^45*d^43 + 362115806639604454004 25
43104*a^65*b^11*c^44*d^44 - 662272679138724025500434432*a^66*b^10*c^43*d^45
 + 10108783240006404372483 2768*a^67*b^9*c^42*d^46 - 125288556366378364305 40
800*a^68*b^8*c^41*d^47 + 1211288155777568604160000*a^69*b^7*c^40*d^48 - 856
97808358931542573056*a^70*b^6*c^39*d^49 + 3946450310269237198848*a^71*b^5*c
^38*d^50 - 88774955854727217152*a^72*b^4*c^37*d^51)*1i + x^(1/2)*(564931537
25735501824*a^22*b^52*c^81*d^4 - 2396923808077634863104*a^23*b^51*c^80*d^5
+ 49387698492843503910912*a^24*b^50*c^79*d^6 - 658598339056129087111168*a^2
5*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c^77*d^8 - 4811359686
7651945069805568*a^27*b^47*c^76*d^9 + 292502253544635823646834688*a^28*b^46
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*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^74*d^11 + 63060035844
09325504378699776*a^30*b^44*c^73*d^12 - 23152095046595175238512672768*a^31*
b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^42*c^71*d^14 - 206784
189076489114265239683072*a^33*b^41*c^70*d^15 + 5110010173907764065745282007
04*a^34*b^40*c^69*d^16 - 1120486424066161848521664233472*a^35*b^39*c^68*d^1
7 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18 - 37948899494273681
42860254707712*a^37*b^37*c^66*d^19 + 5830470252063718134687996051456*a^38*b
^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^35*c^64*d^21 + 87461
84267385996582875203371008*a^40*b^34*c^63*d^22 - 71768719238351983385202193
85856*a^41*b^33*c^62*d^23 + 1365198057841590488549164056576*a^42*b^32*c^61*
d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*d^25 - 2810065405618
0096231365094146048*a^44*b^30*c^59*d^26 + 51280764289348564983994726219776*
a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*a^46*b^28*c^57*d^28
+ 99717561302809906738570708647936*a^47*b^27*c^56*d^29 - 115380588176718582
142644189659136*a^48*b^26*c^55*d^30 + 120101545474959969242488481251328*a^4
9*b^25*c^54*d^31 - 113052494905210552901304563269632*a^50*b^24*c^53*d^32 +
96462689920395704646643948191744*a^51*b^23*c^52*d^33 - 74665519475418951639
228294889472*a^52*b^22*c^51*d^34 + 52413929319422085122116269637632*a^53*b^
21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^20*c^49*d^36 + 19174
031096138345851817803382784*a^55*b^19*c^48*d^37 - 9951827463893335697728745
766912*a^56*b^18*c^47*d^38 + 4646728550801039102656464814080*a^57*b^17*c^46
*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*d^40 + 7258109833877
25632884961181696*a^59*b^15*c^44*d^41 - 240265301732777409221605982208*a^60
*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^13*c^42*d^43 - 17809
629928199177184296828928*a^62*b^12*c^41*d^44 + 3907197185884869673284009984
*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64*b^10*c^39*d^46 + 11
3214808531319939527606272*a^65*b^9*c^38*d^47 - 14265899165032610449588224*a
^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*c^36*d^49 - 10050283
3687558254231552*a^68*b^6*c^35*d^50 + 4689814464763011268608*a^69*b^5*c^34*
d^51 - 106807368762718683136*a^70*b^4*c^33*d^52))*(-(((143986855936*a^35*d^
35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 10675201612
1856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 23907154306007
04*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 188295341785743
36*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110
464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771
167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841
37459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11
2491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^
16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^1
7*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328
*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677
746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4
293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10
*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8
*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b
```

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^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b
^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c
^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 -
(4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 23112188256
1500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*
a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271
*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924138801096
92*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 594157271624297
5*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*
a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^3
2 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084
860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152
383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273
040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7
22812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d
^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^2
1*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 238713320239005696
00*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616
9296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^1
6 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^
14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370
240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247
375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 +
 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d
^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^2
0*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^1
8*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35
+ 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*
a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^
5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7
*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^
9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616
*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 271920687299
54304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638
022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53
7221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d
^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^
15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 336375933887319244
8*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378
0572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 -
714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27
 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^
29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^3
1 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 -
540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^1
```

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1*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 +
496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4
 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^2
5*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64
512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23
*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*
d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 5657
22720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*
b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d
^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300
*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^2
6 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^
18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4)*1i - 192609104438451240960*a^18*b
^50*c^68*d^8 + 7086180670911782322176*a^19*b^49*c^67*d^9 - 1250744769136663
77646080*a^20*b^48*c^66*d^10 + 1411152805506318336000000*a^21*b^47*c^65*d^1
1 - 11440156274772600537743360*a^22*b^46*c^64*d^12 + 7101975490470375592034
3040*a^23*b^45*c^63*d^13 - 351320863723081970831327232*a^24*b^44*c^62*d^14
+ 1422781934731584726682828800*a^25*b^43*c^61*d^15 - 4808764412319368968195
276800*a^26*b^42*c^60*d^16 + 13753628214096098268020736000*a^27*b^41*c^59*d
^17 - 33604586265646232007931330560*a^28*b^40*c^58*d^18 + 70459004145207625
658058932224*a^29*b^39*c^57*d^19 - 126335924813552658893934428160*a^30*b^38
*c^56*d^20 + 189714420765957587531118673920*a^31*b^37*c^55*d^21 - 221947274
468283773140074496000*a^32*b^36*c^54*d^22 + 142870740343318834154286612480*
a^33*b^35*c^53*d^23 + 176083118177526399618307325952*a^34*b^34*c^52*d^24 -
895947027393848326392014438400*a^35*b^33*c^51*d^25 + 2154323340999822995276
326502400*a^36*b^32*c^50*d^26 - 3969865332339043373838394982400*a^37*b^31*c
^49*d^27 + 6147644263312111317325499596800*a^38*b^30*c^48*d^28 - 8260762337
957580186371563192320*a^39*b^29*c^47*d^29 + 9765601087086458087650885632000
*a^40*b^28*c^46*d^30 - 10223506948306413182866214092800*a^41*b^27*c^45*d^31
 + 9508424738292483984119247667200*a^42*b^26*c^44*d^32 - 786689862825459163
4401331773440*a^43*b^25*c^43*d^33 + 5790724738841488066411751276544*a^44*b^
24*c^42*d^34 - 3789006704063625484256485048320*a^45*b^23*c^41*d^35 + 219999
6205919117948922678476800*a^46*b^22*c^40*d^36 - 113048021505958511282868977
6640*a^47*b^21*c^39*d^37 + 512203696921842163745197916160*a^48*b^20*c^38*d^
38 - 203625309837119046692160667648*a^49*b^19*c^37*d^39 + 70576441632244073
218493644800*a^50*b^18*c^36*d^40 - 21151503372075452883114393600*a^51*b^17*
c^35*d^41 + 5422672476777259769580748800*a^52*b^16*c^34*d^42 - 117254091349
2414089228451840*a^53*b^15*c^33*d^43 + 209790609112633976926765056*a^54*b^1
4*c^32*d^44 - 30239740212369693490544640*a^55*b^13*c^31*d^45 + 337577798099
8666504110080*a^56*b^12*c^30*d^46 - 273981289062762912153600*a^57*b^11*c^29
*d^47 + 14388779197382598328320*a^58*b^10*c^28*d^48 - 367186184646271434752
*a^59*b^9*c^27*d^49)*1i)*(-(((143986855936*a^35*d^35 + 40282095616*b^35*c^3
5 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 +
585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 75
40414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37
```

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834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 8
2612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^1
1 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^2
2*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^2
0*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*
a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38328508098
57372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 65
99213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*
d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b
^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14280454796664504
32*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 2393859113402
69568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598
846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 56429284913
9712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a
*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*
d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 7
0054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065
427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 1631
6203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 117
60839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 209
4206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(6871
9476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^
31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*
d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*
d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^
41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^
24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*
a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815
535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 +
32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c
^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*
a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2387133202
3900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 -
 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^
25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*
b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a
^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560
*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a
^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 20141047808*b^35*c^35 +
 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29282
2255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207
453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210
449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130613
6246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40
354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13
```

```
 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*
d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^1
8*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080
*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844
220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2
913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11
*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b
^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^2
8*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^3
0*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*
b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*
d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d
^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 -
4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*
d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^1
9*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^1
0 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 3473736
00*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^1
7*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17
 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 22579284
0*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*
c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 336
5856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*
d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^
17*d^30)))^(1/4)*1i - (x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 -
 28925330217666430894080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^2
0*b^46*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 34982076
529826233401212928*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b
^43*c^57*d^16 + 873231122236416493313064960*a^24*b^42*c^56*d^17 - 320158831
8340888739356606464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26
*b^40*c^54*d^19 - 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 62528
004036875405150857986048*a^28*b^38*c^52*d^21 - 133143680796215491474489344O
00*a^29*b^37*c^51*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23
 - 467106577738876991145070559232*a^31*b^35*c^49*d^24 + 7753210968231093026
74935250944*a^32*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33
*c^47*d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*d^27 - 20281433
45719314676074795761664*a^35*b^31*c^45*d^28 + 22579059731040239569723069562
88*a^36*b^30*c^44*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^3
0 + 1966204854457469918399988498432*a^38*b^28*c^42*d^31 - 15276494060483666
2126256848896O*a^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b
^26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25*c^39*d^34 + 332065
764335584004230153764864*a^42*b^24*c^38*d^35 - 1525431969681336509227157422
08*a^43*b^23*c^37*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^37
- 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 603782595179703225532
0227840*a^46*b^20*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d
```

```
^40 + 296084339424033093684559872*a^48*b^18*c^32*d^41 - 477179504212543082 9
0887680*a^49*b^17*c^31*d^42 + 593152840079745742798848 0*a^50*b^16*c^30*d^43
 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 310063697512095799050 24*a
^52*b^14*c^28*d^45 - 87206718853489465753 6*a^53*b^13*c^27*d^46) + (-(((1439
86855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d
^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4
- 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 -
 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8
- 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^
10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^
23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^2
1*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a
^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 223657145883
6070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548
1339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d
^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^
12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 268958509363747225
6*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 63932949751
6732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636
676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306
051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 615542959
14496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342624972 8*a^34*
b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^
11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1
442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1
1150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 +
16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 +
 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 +
492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 687194767
36*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c
^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^
44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*
c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b
^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*
a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 886649475173
4497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23
871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^3
3*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^
27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 323968077467
22201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1
5516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^2
6*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34
*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016
*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188
736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610
```

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560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 719934
27968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 -
 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 119
5357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414
767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3090
6400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 4
5251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^1
2 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21
*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^1
9*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*
a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27406695680
90865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 33
63759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*
d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b
^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a
^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072
*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*
a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33
*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(
68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^4
2*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^
15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 -
3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^
23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11
+ 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600
*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*
c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 -
 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*
a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24
*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a
^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 -
4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(((-(((1439868559
36*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1
06752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390
715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829
534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181
2801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9
0502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^1
2 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21
*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^
19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 223657145883607040
0*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913
6181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 -
6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^1
2*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25
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*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 63932949751673241
6*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358
144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231
744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*
a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^
34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2
31121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203
904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013
0570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924
13880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415
72716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873
253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^4
3*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^
31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3
 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d
^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^
40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b
^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 886649475173449728
0*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332
023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14
 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^1
6*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 323968077467222016
00*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1551636
5815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21
 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c
^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*
b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^
38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^
40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*
a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376
008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 119535771
5300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089
287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3090640098
5055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 4525137
1385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27
192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14
+ 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19
*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b
^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27406695680908656
64*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 33637593
38873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 -
 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^
10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^
8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*
b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b
```

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^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c
^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(687194
76736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^
16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^2
8*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 336585
6*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^3
8*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 2257
92840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*
b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d
^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 34737
3600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b
^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23
+ 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^
6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a
^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(64563604257983430656*a
^25*b^51*c^84*d^4 - 2822351843277561397248*a^26*b^50*c^83*d^5 + 60127162308
256283492352*a^27*b^49*c^82*d^6 - 831948157724300777881600*a^28*b^48*c^81*d
^7 + 8406786558179361266073600*a^29*b^47*c^80*d^8 - 661445813058992031704 02
304*a^30*b^46*c^79*d^9 + 421912670310680329277407232*a^31*b^45*c^78*d^10 -
2243238210521587022108295168*a^32*b^44*c^77*d^11 + 101453832519848258028175
36000*a^33*b^43*c^76*d^12 - 39641949193820336576213811200*a^34*b^42*c^75*d^
13 + 135494098735043868075088674816*a^35*b^41*c^74*d^14 - 40928491588909153
9805067542528*a^36*b^40*c^73*d^15 + 1102331957384293957070038761472*a^37*b^
39*c^72*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*d^17 + 584734
3583817169075816733081600*a^39*b^37*c^70*d^18 - 116841056293683249599044690
90304*a^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a^41*b^35*c^68
*d^20 - 36343020410925078321345140359168*a^42*b^34*c^67*d^21 + 572975806876
83561030746426572800*a^43*b^33*c^66*d^22 - 84429658980390814235781758976000
*a^44*b^32*c^65*d^23 + 116702744788425677443098849837056*a^45*b^31*c^64*d^2
4 - 151589903153597380791972919246848*a^46*b^30*c^63*d^25 + 185008444259789
842943656593457152*a^47*b^29*c^62*d^26 - 211756933815433796881181835264000*
a^48*b^28*c^61*d^27 + 226611959433847997212598992896000*a^49*b^27*c^60*d^28
 - 225906031446565502788593732550656*a^50*b^26*c^59*d^29 + 208978627749165 7
2443002551454924 8*a^51*b^25*c^58*d^30 - 178726416623100559749866797924352*a
^52*b^24*c^57*d^31 + 140824510781547830729330235801600*a^53*b^23*c^56*d^32
- 101897270594764980154443340185600*a^54*b^22*c^55*d^33 + 67499322390719467
851063444373504*a^55*b^21*c^54*d^34 - 40809284384591153062742518136832*a^56
*b^20*c^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c^52*d^36 - 11
195042646819893251483369472000*a^58*b^18*c^51*d^37 + 5042898342903938117430
096691200*a^59*b^17*c^50*d^38 - 2042741359937286689202494242816*a^60*b^16*c
^49*d^39 + 740249793404633986500581654528*a^61*b^15*c^48*d^40 - 23850126548
9031484884985577472*a^62*b^14*c^47*d^41 + 67809805296929472355971891200*a^6
3*b^13*c^46*d^42 - 16856343881283213574379929600*a^64*b^12*c^45*d^43 + 3621
158066396044540042543104*a^65*b^11*c^44*d^44 - 662272679138724025500434432*
a^66*b^10*c^43*d^45 + 101087832400064043724832768*a^67*b^9*c^42*d^46 - 1252
```

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8855636637836430540800*a^68*b^8*c^41*d^47 + 1211288155777568604160000*a^69*
b^7*c^40*d^48 - 856978083589315425730 56*a^70*b^6*c^39*d^49 + 39464503102692
37198848*a^71*b^5*c^38*d^50 - 887749558547272171 52*a^72*b^4*c^37*d^51)*1i -
 x^(1/2)*(5649315372573550182 4*a^22*b^52*c^81*d^4 - 239692380807763486310 4*
a^23*b^51*c^80*d^5 + 4938769849284350391091 2*a^24*b^50*c^79*d^6 - 658598339
056129087111168*a^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c^
77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 + 292502253544635823
646834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^74
*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12 - 231520950465951 7
5238512672768*a^31*b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^42
*c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^70*d^15 + 511001017
390776406574528200704*a^34*b^40*c^69*d^16 - 1120486424066161848521664233472
*a^35*b^39*c^68*d^17 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18
- 3794889949427368142860254707712*a^37*b^37*c^66*d^19 + 5830470252063718134
687996051456*a^38*b^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^3
5*c^64*d^21 + 8746184267385996582875203371008*a^40*b^34*c^63*d^22 - 7176871
923835198338520219385856*a^41*b^33*c^62*d^23 + 1365198057841590488549164056
576*a^42*b^32*c^61*d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*d
^25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^26 + 51280764289348
564983994726219776*a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*a
^46*b^28*c^57*d^28 + 99717561302809906738570708647936*a^47*b^27*c^56*d^29 -
 115380588176718582142644189659136*a^48*b^26*c^55*d^30 + 120101545474959969
242488481251328*a^49*b^25*c^54*d^31 - 113052494905210552901304563269632*a^5
0*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*b^23*c^52*d^33 - 7
4665519475418951639228294889472*a^52*b^22*c^51*d^34 + 524139293194220851221
16269637632*a^53*b^21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^2
0*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*c^48*d^37 - 995182
7463893335697728745766912*a^56*b^18*c^47*d^38 + 464672855080103910265646481
4080*a^57*b^17*c^46*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*d
^40 + 725810983387725632884961181696*a^59*b^15*c^44*d^41 - 240265301732777 4
09221605982208*a^60*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^1
3*c^42*d^43 - 17809629928199177184296828928*a^62*b^12*c^41*d^44 + 390719718
5884869673284009984*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64*
b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^47 - 142658991
65032610449588224*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*c
^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 + 468981446476301126
8608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52))*(-(((1
43986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^3
3*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d
^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^
6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d
^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25
*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23
*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*
b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 4132414539309056 0
```

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0*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 223657145
8836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 -
5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^1
4*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23
*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 268958509363747
2256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 63932949
7516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080
636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711
306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 615542
95914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342649728*a^
34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12
*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22
+ 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14
+ 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16
 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^1
8 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20
 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 687194
76736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*
b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29
*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^
27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^1
8*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 19274988590727168
00*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 886649475
1734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 -
 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*
c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040
*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 323968077
46722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19
+ 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*
c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a
^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726
016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347
188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604
610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 719
93427968*a^35*d^35 + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^
2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 -
1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9
414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3
0906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10
- 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*
d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c
^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*
b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11182857294180352
00*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27406695
68090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 -
```

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 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^
12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^2
5*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966474875836620
8*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179
072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18556530256158
72*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a
^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34
)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*
a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960
*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6
 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20
*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^
11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435
600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^
16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^1
8 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 1290244
80*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c
^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 90619
2*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28
 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4)*1i - 192609104
438451240960*a^18*b^50*c^68*d^8 + 7086180670911782322176*a^19*b^49*c^67*d^9
 - 125074476913666377646080*a^20*b^48*c^66*d^10 + 1411152805506318336000000
*a^21*b^47*c^65*d^11 - 11440156274772600537743360*a^22*b^46*c^64*d^12 + 710
19754904703755920343040*a^23*b^45*c^63*d^13 - 351320863723081970831327232*a
^24*b^44*c^62*d^14 + 1422781934731584726682828800*a^25*b^43*c^61*d^15 - 480
8764412319368968195276800*a^26*b^42*c^60*d^16 + 137536282140960982680207360
00*a^27*b^41*c^59*d^17 - 33604586265646232007931330560*a^28*b^40*c^58*d^18
+ 70459004145207625658058932224*a^29*b^39*c^57*d^19 - 126335924813552658893
934428160*a^30*b^38*c^56*d^20 + 189714420765957587531118673920*a^31*b^37*c^
55*d^21 - 221947274468283773140074496000*a^32*b^36*c^54*d^22 + 142870740343
318834154286612480*a^33*b^35*c^53*d^23 + 176083118177526399618307325952*a^3
4*b^34*c^52*d^24 - 895947027393848326392014438400*a^35*b^33*c^51*d^25 + 215
4323340999822995276326502400*a^36*b^32*c^50*d^26 - 396986533233904337383839
4982400*a^37*b^31*c^49*d^27 + 6147644263312111317325499596800*a^38*b^30*c^4
8*d^28 - 8260762337957580186371563192320*a^39*b^29*c^47*d^29 + 976560108708
6458087650885632000*a^40*b^28*c^46*d^30 - 10223506948306413182866214092800*
a^41*b^27*c^45*d^31 + 9508424738292483984119247667200*a^42*b^26*c^44*d^32 -
 7866898628254591634401331773440*a^43*b^25*c^43*d^33 + 57907247388414880664
11751276544*a^44*b^24*c^42*d^34 - 3789006704063625484256485048320*a^45*b^23
*c^41*d^35 + 2199996205919117948922678476800*a^46*b^22*c^40*d^36 - 11304802
15059585112828689776640*a^47*b^21*c^39*d^37 + 51220369692184216374519791616
0*a^48*b^20*c^38*d^38 - 203625309837119046692160667648*a^49*b^19*c^37*d^39
+ 70576441632244073218493644800*a^50*b^18*c^36*d^40 - 211515033720754528831
14393600*a^51*b^17*c^35*d^41 + 5422672476777259769580748800*a^52*b^16*c^34*
d^42 - 1172540913492414089228451840*a^53*b^15*c^33*d^43 + 20979060911263397
```

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6926765056*a^54*b^14*c^32*d^44 - 30239740212369693490544640*a^55*b^13*c^31*
d^45 + 33757779809986665041110080*a^56*b^12*c^30*d^46 - 27398128906276291215
3600*a^57*b^11*c^29*d^47 + 14388779197382598328320*a^58*b^10*c^28*d^48 - 36
7186184646271434752*a^59*b^9*c^27*d^49)*1i)*(-(((143986855936*a^35*d^35 + 4
0282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a
^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5
*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7
*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^
9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232
*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599
08608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249127
6045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1
074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17
*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*
b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384
896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767
561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25
- 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d
^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6
*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4
*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^3
3 - 1081861996544*a*b^34*c^34*d - 429426249728*a^34*b*c*d^34)^2/4 - (45811
79456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a
*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^
25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b
^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*
b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*
b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b
^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 21
99023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461
056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426
560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062
349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228120
72152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4
433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36
*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^2
4*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616929606
6641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38
876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^2
9*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^
31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867
248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281
2072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 +
62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27
 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29
```

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 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 2014
1047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^
32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30
*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*
c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26
*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*
b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*
a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762
496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53722161
5798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 +
1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^1
5*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22
*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378057290
5472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022
739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119
692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9
313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 28
2146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930
998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32
*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^
13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201
376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40
*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240
*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*
c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 -
 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*
a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c
^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 +
64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*
b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 20
1376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^2
9 + 496*a^41*b^2*c^17*d^30)))^(1/4)*1i))*(-(((143986855936*a^35*d^35 + 4028
2095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*
b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^
30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^
28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b
^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^
11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599086
08*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249127604
5524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074
443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^
18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^1
5*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896
*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561
145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1
```

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428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27
 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^
29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^
31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 -
 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (45811794
56161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^
26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*
c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23
*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^2
1*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^1
9*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17
*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 21990
23255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056
*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560
*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349
312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228120721
52268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433
247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^
11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b
^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388761692960664
1920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876
169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d
^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*
b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248
640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281207
2152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 622
73040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 +
2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 +
34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 71993427968*a^35*d^35 + 2014104
7808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*
c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^
30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^2
8*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^
26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^2
4*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^1
3*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496
*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 53722161579
8067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 191
6425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d
^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^
13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688378057290547
2*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739
833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692
955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313
041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 28214
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6424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998
272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^
47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*
b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376
*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^
7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^
21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^3
5*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 56
5722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^2
8*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28
*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 645
12240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8
*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 20137
6*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 +
 496*a^41*b^2*c^17*d^30)))^(1/4) + atan(((x^(1/2)*(857712418202478182400*a^
18*b^48*c^62*d^11 - 28925330217666430894080*a^19*b^47*c^61*d^12 + 465808355
868544602210304*a^20*b^46*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c
^59*d^14 + 34982076529826233401212928*a^22*b^44*c^58*d^15 - 195811106815542
077297786880*a^23*b^43*c^57*d^16 + 873231122236416493313064960*a^24*b^42*c^
56*d^17 - 3201588318340888739356606464*a^25*b^41*c^55*d^18 + 99048669815473
62725832687616*a^26*b^40*c^54*d^19 - 26475613142538536817178705920*a^27*b^3
9*c^53*d^20 + 62528004036875405150857986048*a^28*b^38*c^52*d^21 - 133143680
796215491474489344000*a^29*b^37*c^51*d^22 + 259595474982835164713400139776*
a^30*b^36*c^50*d^23 - 467106577738876991145070559232*a^31*b^35*c^49*d^24 +
775321096823109302674935250944*a^32*b^34*c^48*d^25 - 1179424943892680059222
782640128*a^33*b^33*c^47*d^26 + 1629690593600095833823295569920*a^34*b^32*c
^46*d^27 - 2028143345719314676074795761664*a^35*b^31*c^45*d^28 + 2257905973
104023956972306956288*a^36*b^30*c^44*d^29 - 2237449183565830435563494178816
*a^37*b^29*c^43*d^30 + 1966204854457469918399988498432*a^38*b^28*c^42*d^31
- 1527649406048366621262568488960*a^39*b^27*c^41*d^32 + 1046409458758522347
995126562816*a^40*b^26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25
*c^39*d^34 + 332065764335584004230153764864*a^42*b^24*c^38*d^35 - 152543196
968133650922715742208*a^43*b^23*c^37*d^36 + 60699171433471101739298979840*a
^44*b^22*c^36*d^37 - 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 60
37825951797032255320227840*a^46*b^20*c^34*d^39 - 14734496390827154795124490
24*a^47*b^19*c^33*d^40 + 296084339424033093684559872*a^48*b^18*c^32*d^41 -
47717950421254308290887680*a^49*b^17*c^31*d^42 + 5931528400797457427988480*
a^50*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 310063
69751209579905024*a^52*b^14*c^28*d^45 - 872067188534894657536*a^53*b^13*c^2
7*d^46) + (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616
*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^
32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30
*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28
*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^2
6*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24
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*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13
*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992
*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 10744432315
96134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38
32850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*
d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b
^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42937675611458109
44*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14280454
79666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 2393
85911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18
626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 56
4292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 108186
1996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a
^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11
*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^
13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^
15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d
^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d
^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^
21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 219902325555
2*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b
^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b
^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^1
7*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281207215226880
0*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 44332473758
67248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15
516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^3
4*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^
26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692960
66641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2
3871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^
27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^3
3*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268
800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 6227304006
2349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152
383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 3408486
0461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*
a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*
b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b
^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b
^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10
*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128
*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 246857928872
7552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 20662072
6965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1
```

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1182857294180352OO*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16
*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*
b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743
424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792
546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 3
19664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28
+ 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30
+ 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 +
30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 21467131248
64*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b
^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*
c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^
17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8
 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a
^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^
34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 6
01080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^
29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^2
7*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 280
48800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c
^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^
39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4
)*((x^(1/2)*(564931537257355O1824*a^22*b^52*c^81*d^4 - 2396923808077634863l
04*a^23*b^51*c^80*d^5 + 4938769849284350391O912*a^24*b^50*c^79*d^6 - 658598
3390561290871l1168*a^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48
*c^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 + 292502253544635
823646834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c
^74*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12 - 2315209504659
5175238512672768*a^31*b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b
^42*c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^70*d^15 + 511001
017390776406574528200704*a^34*b^40*c^69*d^16 - 1120486424066161848521664233
472*a^35*b^39*c^68*d^17 + 2186183732842431973240904613888*a^36*b^38*c^67*d^
18 - 3794889949427368142860254707712*a^37*b^37*c^66*d^19 + 5830470252063718
134687996051456*a^38*b^36*c^65*d^20 - 7807619033603590530479469625344*a^39*
b^35*c^64*d^21 + 8746184267385996582875203371008*a^40*b^34*c^63*d^22 - 7176
871923835198338520219385856*a^41*b^33*c^62*d^23 + 1365198057841590488549164
056576*a^42*b^32*c^61*d^24 + 10199723921158867878218460823552*a^43*b^31*c^6
0*d^25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^26 + 51280764289
348564983994726219776*a^45*b^29*c^58*d^27 - 7669647697972087434270052712448
0*a^46*b^28*c^57*d^28 + 99717561302809906738570708647936*a^47*b^27*c^56*d^2
9 - 115380588176718582142644189659136*a^48*b^26*c^55*d^30 + 120101545474959
969242488481251328*a^49*b^25*c^54*d^31 - 113052494905210552901304563269632*
a^50*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*b^23*c^52*d^33
- 74665519475418951639228294889472*a^52*b^22*c^51*d^34 + 524139293194220851
22116269637632*a^53*b^21*c^50*d^35 - 33334185869182979296764484386816*a^54*
```

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b^20*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*c^48*d^37 - 995
18274638933356977287457 66912*a^56*b^18*c^47*d^38 + 4646728550801039102656 46
4814080*a^57*b^17*c^46*d^39 - 19444696586600802427903389 20448*a^58*b^16*c^4
5*d^40 + 7258109833877256328849611816 96*a^59*b^15*c^44*d^41 - 240265301 7327
77409221605982208*a^60*b^14*c^43*d^42 + 700283105601324150151257784 32*a^61*
b^13*c^42*d^43 - 1780962992819917718429682 8928*a^62*b^12*c^41*d^44 + 390719
7185884869673284009984*a^63*b^11*c^40*d^45 - 7285690616559671401261301 76*a^
64*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^47 - 142658
99165032610449588224*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^
7*c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 + 468981446476301
1268608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52) + (-
(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 +
 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585
644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 75404
14907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834
420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 8261
2272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 +
 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d
^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c
^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^1
7*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38328508098573
72160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 65992
13688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^2
2 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11
*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*
a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269 5
68*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846
464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 56429284913971
2*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^
34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^2
3 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 7005
4782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427
904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 1631620
3958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 117608
39441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 209420
6929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(6871947
6736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 219902325 5552*a^12*b^31*
c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2
 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4
 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*
d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*
c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^2
1*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535
370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 323
96807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32
```

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*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^2
8*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2387133202390
0569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 88
66494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*
d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8
*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37
*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^
39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41
*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^3
3*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^
4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6
 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8
 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d
^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c
^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^
21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a
^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418
035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740
669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^
21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^1
2*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128
*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 3196647487583
66208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 3704031833
8179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025
615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 307771479572
48*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*
d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d -
 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 3
5960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41
*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*
a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^3
6*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 47
1435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^2
7*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29
*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129
024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b
^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 9
06192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*
d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(645636042
57983430656*a^25*b^51*c^84*d^4 - 282235184327561397248*a^26*b^50*c^83*d^5
+ 601271623082562834923552*a^27*b^49*c^82*d^6 - 831948157724300777881600*a^2
8*b^48*c^81*d^7 + 840678655817936126607360O*a^29*b^47*c^80*d^8 - 6614458130
5899203170402304*a^30*b^46*c^79*d^9 + 4219126703106803292774O7232*a^31*b^45
*c^78*d^10 - 22432382105215870221082951 68*a^32*b^44*c^77*d^11 + 10145383251
984825802817536000*a^33*b^43*c^76*d^12 - 3964194919382033657621381120O*a^34
```

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*b^42*c^75*d^13 + 135494098735043868075088674816*a^35*b^41*c^74*d^14 - 4092
849158890915398050675425 28*a^36*b^40*c^73*d^15 + 1102331957384293957070038761472*a^37*b^39*c^72*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*
d^17 + 5847343583817169075816733081600*a^39*b^37*c^70*d^18 - 11684105629368
324959904469090304*a^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a
^41*b^35*c^68*d^20 - 36343020410925078321345140359168*a^42*b^34*c^67*d^21 +
 57297580687683561030746426572800*a^43*b^33*c^66*d^22 - 844296589803908142 3
5781758976000*a^44*b^32*c^65*d^23 + 116702744788425677443098849837056*a^45*
b^31*c^64*d^24 - 151589903153597380791972919246848*a^46*b^30*c^63*d^25 + 18
5008444259789842943656593457152*a^47*b^29*c^62*d^26 - 211756933815433796881
181835264000*a^48*b^28*c^61*d^27 + 226611959433847997212598992896000*a^49*b
^27*c^60*d^28 - 225906031446565502788593732550656*a^50*b^26*c^59*d^29 + 208
978627749165724430025514549248*a^51*b^25*c^58*d^30 - 178726416623100559749 8
66797924352*a^52*b^24*c^57*d^31 + 140824510781547830729330235801600*a^53*b^
23*c^56*d^32 - 101897270594764980154443340185600*a^54*b^22*c^55*d^33 + 6749
9322390719467851063444373504*a^55*b^21*c^54*d^34 - 408092843845911530627425
18136832*a^56*b^20*c^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c
^52*d^36 - 11195042646819893251483369472000*a^58*b^18*c^51*d^37 + 504289834
2903938117430096691200*a^59*b^17*c^50*d^38 - 204274135993728668920249424281
6*a^60*b^16*c^49*d^39 + 740249793404633986500581654528*a^61*b^15*c^48*d^40
- 238501265489031484884985577472*a^62*b^14*c^47*d^41 + 67809805296929472355
971891200*a^63*b^13*c^46*d^42 - 16856343881283213574379929600*a^64*b^12*c^4
5*d^43 + 3621158066396044540042543104*a^65*b^11*c^44*d^44 - 662272679138724
025500434432*a^66*b^10*c^43*d^45 + 101087832400064043724832768*a^67*b^9*c^4
2*d^46 - 12528855636637836430540800*a^68*b^8*c^41*d^47 + 121128815577756860
4160000*a^69*b^7*c^40*d^48 - 85697808358931542573056*a^70*b^6*c^39*d^49 + 3
946450310269237198848*a^71*b^5*c^38*d^50 - 88774955854727217152*a^72*b^4*c^
37*d^51))*(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616
*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^
32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30
*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28
*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^2
6*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24
*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13
*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992
*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 10744432315
96134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 38
32850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*
d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b
^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42937675611458109
44*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14280454
79666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 2393
85911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18
626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 56
4292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 108186
```

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1996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a
^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11
*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^
13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^
15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d
^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d
^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^
21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 219902325555
2*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b
^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b
^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^1
7*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281207215226880
0*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 44332473758
67248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15
516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^3
4*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^
26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692960
66641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 2
3871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^
27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^3
3*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268
800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 6227304006
2349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152
383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 3408486
0461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*
a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*
b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b
^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b
^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10
*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128
*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 246857928872
7552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 20662072
6965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1
118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16
*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*
b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743
424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792
546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 3
19664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28
+ 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30
+ 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 +
30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124
864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b
^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*
c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^
```

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17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8
 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a
^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^
34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 6
01080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^
29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^2
7*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 280
48800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c
^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^
39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4
) + 1926091044384512409600*a^18*b^50*c^68*d^8 - 708618067091178232217б*a^19*
```

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17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8
 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a
^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^
34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 6
01080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^
29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^2
7*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 280
48800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c
^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^
39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4
) + 192609104438451240960*a^18*b^50*c^68*d^8 - 7086180670911782322176*a^19*
b^49*c^67*d^9 + 125074476913666377646080*a^20*b^48*c^66*d^10 - 141115280550
6318336000000*a^21*b^47*c^65*d^11 + 11440156274772600537743360*a^22*b^46*c^
64*d^12 - 71019754904703755920343040*a^23*b^45*c^63*d^13 + 351320863723081970831327232*a^24*b^44*c^62*d^14 - 1422781934731584726682828800*a^25*b^43*c^
61*d^15 + 4808764412319368968195276800*a^26*b^42*c^60*d^16 - 13753628214096
098268020736000*a^27*b^41*c^59*d^17 + 33604586265646232007931330560*a^28*b^
40*c^58*d^18 - 70459004145207625658058932224*a^29*b^39*c^57*d^19 + 12633592
4813552658893934428160*a^30*b^38*c^56*d^20 - 189714420765957587531118673920
*a^31*b^37*c^55*d^21 + 221947274468283773140074496000*a^32*b^36*c^54*d^22 -
 142870740343318834154286612480*a^33*b^35*c^53*d^23 - 176083118177526399618
307325952*a^34*b^34*c^52*d^24 + 895947027393848326392014438400*a^35*b^33*c^
51*d^25 - 2154323340999822995276326502400*a^36*b^32*c^50*d^26 + 39698653323
39043373838394982400*a^37*b^31*c^49*d^27 - 6147644263312111317325499596800*
a^38*b^30*c^48*d^28 + 8260762337957580186371563192320*a^39*b^29*c^47*d^29 -
 9765601087086458087650885632000*a^40*b^28*c^46*d^30 + 10223506948306413182
866214092800*a^41*b^27*c^45*d^31 - 9508424738292483984119247667200*a^42*b^2
6*c^44*d^32 + 7866898628254591634401331773440*a^43*b^25*c^43*d^33 - 5790724
738841488066411751276544*a^44*b^24*c^42*d^34 + 3789006704063625484256485048
320*a^45*b^23*c^41*d^35 - 2199996205919117948922678476800*a^46*b^22*c^40*d^
36 + 1130480215059585112828689776640*a^47*b^21*c^39*d^37 - 512203696921842163745197916160*a^48*b^20*c^38*d^38 + 203625309837119046692160667648*a^49*b^
19*c^37*d^39 - 70576441632244073218493644800*a^50*b^18*c^36*d^40 + 21151503
372075452883114393600*a^51*b^17*c^35*d^41 - 5422672476777259769580748800*a^
52*b^16*c^34*d^42 + 1172540913492414089228451840*a^53*b^15*c^33*d^43 - 2097
90609112633976926765056*a^54*b^14*c^32*d^44 + 30239740212369693490544640*a^
55*b^13*c^31*d^45 - 3375777980998666504110080*a^56*b^12*c^30*d^46 + 2739812
89062762912153600*a^57*b^11*c^29*d^47 - 14388779197382598328320*a^58*b^10*c
^28*d^48 + 367186184646271434752*a^59*b^9*c^27*d^49))*(-(71993427968*a^35*d
^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2
*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^
31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^2
9*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^
27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*
b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*
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a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455
104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 41324145
3930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 -
2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^1
6*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21
*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 582709154054548
6848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 268958
5093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 +
 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^2
8 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^
30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32
 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426
249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*
b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^1
6*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*
c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22
*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^2
0*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18
*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47
 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255
552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*
a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 1383845334718873
6*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 2312998630887
26016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 19274988
59072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 -
8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^
35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a
^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877
070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 +
32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c
^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a
^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 192749885907
2716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299
863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13
838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 -
340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1
/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060
928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 119535771530035
2*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168
*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3090640098505523
2*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 4525137138558
3616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068
729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 5624
5638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16
+ 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^
```

```
17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^2
0*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 33637593388731
92448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 21468
83780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^2
5 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*
d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^
6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4
*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^3
3 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*
(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^3
1 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43
*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18
*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9
+ 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*
a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c
^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 -
565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a
^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^
26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 1051
8300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21
*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^
3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*1i + (x^(1/2)*(85771241820247
8182400*a^18*b^48*c^62*d^11 - 28925330217666430894080*a^19*b^47*c^61*d^12 +
 465808355868544602210304*a^20*b^46*c^60*d^13 - 4772189938359453553262592*a
^21*b^45*c^59*d^14 + 34982076529826233401212928*a^22*b^44*c^58*d^15 - 19581
1106815542077297786880*a^23*b^43*c^57*d^16 + 873231122236416493313064960*a^
24*b^42*c^56*d^17 - 3201588318340888739356606464*a^25*b^41*c^55*d^18 + 9904
866981547362725832687616*a^26*b^40*c^54*d^19 - 2647561314253853681717870592
0*a^27*b^39*c^53*d^20 + 62528004036875405150857986048*a^28*b^38*c^52*d^21 -
 133143680796215491474489344000*a^29*b^37*c^51*d^22 + 259595474982835164713
400139776*a^30*b^36*c^50*d^23 - 467106577738876991145070559232*a^31*b^35*c^
49*d^24 + 775321096823109302674935250944*a^32*b^34*c^48*d^25 - 117942494389
2680059222782640128*a^33*b^33*c^47*d^26 + 1629690593600095833823295569920*a
^34*b^32*c^46*d^27 - 2028143345719314676074795761664*a^35*b^31*c^45*d^28 +
2257905973104023956972306956288*a^36*b^30*c^44*d^29 - 223744918356583043556
3494178816*a^37*b^29*c^43*d^30 + 1966204854457469918399988498432*a^38*b^28*
c^42*d^31 - 1527649406048366621262568488960*a^39*b^27*c^41*d^32 + 104640945
8758522347995126562816*a^40*b^26*c^40*d^33 - 629956523592774331698776113152
*a^41*b^25*c^39*d^34 + 332065764335584004230153764864*a^42*b^24*c^38*d^35 -
 152543196968133650922715742208*a^43*b^23*c^37*d^36 + 606991714334711017392
98979840*a^44*b^22*c^36*d^37 - 20757436699772395749793333248*a^45*b^21*c^35
*d^38 + 6037825951797032255320227840*a^46*b^20*c^34*d^39 - 1473449639082715
479512449024*a^47*b^19*c^33*d^40 + 296084339424033093684559872*a^48*b^18*c^
32*d^41 - 47717950421254308290887680*a^49*b^17*c^31*d^42 + 5931528400797457
427988480*a^50*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^15*c^29*d^4
```

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4 + 3100636975120957990502 4*a^52*b^14*c^28*d^45 - 872067188534894657536*a^5
```

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4 + 31006369751209579905024*a^52*b^14*c^28*d^45 - 872067188534894657536*a^5
3*b^13*c^27*d^46) + (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 4
0282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a
^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5
*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7
*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^
9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232
*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599
08608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249127
6045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1
074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17
*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*
b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384
896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767
561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25
- 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d
^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6
*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4
*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^3
3 - 1081861996544*a*b^34*c^34*d - 429342624972 8*a^34*b*c*d^34)^2/4 - (45811
79456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a
*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^
25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b
^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*
b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*
b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b
^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 21
99023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461
056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426
560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062
349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228120
72152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4
433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36
*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^2
4*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616929606
6641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38
876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^2
9*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^
31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867
248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281
2072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 +
62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27
 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29
 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806
029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 2928222551
```

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40864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 377020745357
7216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 1891721044977
2544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 413061362462
22848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 4035488
5515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 24
68579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15
- 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^1
8*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19
*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 329960684422019
4816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 291354
5770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24
 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^
9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7
*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5
*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c
^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2
146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 -
 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*
a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 +
 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^2
4*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 1
29024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^
24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^3
2*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 47
1435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^3
1*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*
d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*
a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27
+ 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^
30)))^(1/4)*((x^(1/2)*(56493153725735501824*a^22*b^52*c^81*d^4 - 2396923808
077634863104*a^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b^50*c^79*d^
6 - 658598339056129087111168*a^25*b^49*c^78*d^7 + 6391163867634330475954176
*a^26*b^48*c^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 + 29250
2253544635823646834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a
^29*b^45*c^74*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12 - 231
52095046595175238512672768*a^31*b^43*c^72*d^13 + 73885584363642186267654881
280*a^32*b^42*c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^70*d^1
5 + 511001017390776406574528200704*a^34*b^40*c^69*d^16 - 112048642406616184
8521664233472*a^35*b^39*c^68*d^17 + 2186183732842431973240904613888*a^36*b^
38*c^67*d^18 - 3794889949427368142860254707712*a^37*b^37*c^66*d^19 + 583047
0252063718134687996051456*a^38*b^36*c^65*d^20 - 780761903360359053047946962
5344*a^39*b^35*c^64*d^21 + 8746184267385996582875203371008*a^40*b^34*c^63*d
^22 - 7176871923835198338520219385856*a^41*b^33*c^62*d^23 + 136519805784159
0488549164056576*a^42*b^32*c^61*d^24 + 10199723921158867878218460823552*a^4
3*b^31*c^60*d^25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^26 + 5
```

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1280764289348564983994726219776*a^45*b^29*c^58*d^27 - 766964769797208743427
00527124480*a^46*b^28*c^57*d^28 + 997175613028099067385707086479 36*a^47*b^2
7*c^56*d^29 - 1153805881767185821426441896591 36*a^48*b^26*c^55*d^30 + 12010
154547495996924248848125132 8*a^49*b^25*c^54*d^31 - 113052494905210552901304
563269632*a^50*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*b^23*
c^52*d^33 - 74665519475418951639228294889472*a^52*b^22*c^51*d^34 + 52413929
319422085122116269637632*a^53*b^21*c^50*d^35 - 333341858691829792967644843 8
6816*a^54*b^20*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*c^48*
d^37 - 9951827463893335697728745766912*a^56*b^18*c^47*d^38 + 46467285508010
39102656464814080*a^57*b^17*c^46*d^39 - 1944469658660080242790338920448*a^5
8*b^16*c^45*d^40 + 725810983387725632884961181696*a^59*b^15*c^44*d^41 - 240
265301732777409221605982208*a^60*b^14*c^43*d^42 + 700283105601324150151257 7
8432*a^61*b^13*c^42*d^43 - 17809629928199177184296828928*a^62*b^12*c^41*d^4
4 + 3907197185884869673284009984*a^63*b^11*c^40*d^45 - 7285690616559671401 2
6130176*a^64*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^4
7 - 14265899165032610449588224*a^66*b^8*c^37*d^48 + 1400509163935752188329 9
84*a^67*b^7*c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 + 46898
14464763011268608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*
d^52) - (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b
^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32
*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d
^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d
^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*
d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c
^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b
^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a
^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596
134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832
850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^
20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^1
3*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944
*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479
666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385
911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 1862
6082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 5642
92849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 10818619
96544*a*b^34*c^34*d - 429342624972 8*a^34*b*c*d^34)^2/4 - (4581179456161*a^1
2*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d
^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13
 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15
 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^1
7 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^1
9 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21
)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*
a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^3
```

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0*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^2
8*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*
b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*
a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867
248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 1551
6365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*
d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26
*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066
641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 238
71332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27
*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*
b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281207215226880
0*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 622730400623
49312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 247115238
3426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 340848604
61056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^
2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^
31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^2
9*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^2
7*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b
^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a
^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 24685792887275
52*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 2066207269
65452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111
8285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d
^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^
14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 291354577027274342
4*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 134479254
6818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319
664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 +
37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 +
1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30
777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864
*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^3
1*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^
44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17
*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 -
 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^2
2*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34
*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601
080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29
*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*
d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048
800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^2
2*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39
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*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*
(64563604257983430656*a^25*b^51*c^84*d^4 - 2822351843277561397248*a^26*b^50
*c^83*d^5 + 60127162308256283492352*a^27*b^49*c^82*d^6 - 831948157724300777
881600*a^28*b^48*c^81*d^7 + 8406786558179361266073600*a^29*b^47*c^80*d^8 -
66144581305899203170402304*a^30*b^46*c^79*d^9 + 421912670310680329277407232
*a^31*b^45*c^78*d^10 - 2243238210521587022108295168*a^32*b^44*c^77*d^11 + 1
0145383251984825802817536000*a^33*b^43*c^76*d^12 - 396419491938203365762138
11200*a^34*b^42*c^75*d^13 + 135494098735043868075088674816*a^35*b^41*c^74*d
^14 - 409284915889091539805067542528*a^36*b^40*c^73*d^15 + 1102331957384293
957070038761472*a^37*b^39*c^72*d^16 - 2668223165968086459433038643200*a^38*
b^38*c^71*d^17 + 5847343583817169075816733081600*a^39*b^37*c^70*d^18 - 1168
4105629368324959904469090304*a^40*b^36*c^69*d^19 + 214350024626986370410989
55948032*a^41*b^35*c^68*d^20 - 36343020410925078321345140359168*a^42*b^34*c
^67*d^21 + 57297580687683561030746426572800*a^43*b^33*c^66*d^22 - 844296589
80390814235781758976000*a^44*b^32*c^65*d^23 + 11670274478842567744309884983
7056*a^45*b^31*c^64*d^24 - 151589903153597380791972919246848*a^46*b^30*c^63
*d^25 + 185008444259789842943656593457152*a^47*b^29*c^62*d^26 - 21175693381
5433796881181835264000*a^48*b^28*c^61*d^27 + 226611959433847997212598992896
000*a^49*b^27*c^60*d^28 - 225906031446565502788593732550656*a^50*b^26*c^59*
d^29 + 208978627749165724430025514549248*a^51*b^25*c^58*d^30 - 178726416623
100559749866797924352*a^52*b^24*c^57*d^31 + 1408245107815478307293302358016
00*a^53*b^23*c^56*d^32 - 101897270594764980154443340185600*a^54*b^22*c^55*d
^33 + 67499322390719467851063444373504*a^55*b^21*c^54*d^34 - 40809284384591
153062742518136832*a^56*b^20*c^53*d^35 + 22447282431345050697947118829568*a
^57*b^19*c^52*d^36 - 11195042646819893251483369472000*a^58*b^18*c^51*d^37 +
 5042898342903938117430096691200*a^59*b^17*c^50*d^38 - 20427413599372866892
02494242816*a^60*b^16*c^49*d^39 + 740249793404633986500581654528*a^61*b^15*
c^48*d^40 - 238501265489031484884985577472*a^62*b^14*c^47*d^41 + 6780980529
6929472355971891200*a^63*b^13*c^46*d^42 - 16856343881283213574379929600*a^6
4*b^12*c^45*d^43 + 3621158066396044540042543104*a^65*b^11*c^44*d^44 - 66227
2679138724025500434432*a^66*b^10*c^43*d^45 + 101087832400064043724832768*a^
67*b^9*c^42*d^46 - 12528855636637836430540800*a^68*b^8*c^41*d^47 + 12112881
55777568604160000*a^69*b^7*c^40*d^48 - 85697808358931542573056*a^70*b^6*c^3
9*d^49 + 3946450310269237198848*a^71*b^5*c^38*d^50 - 88774955854727217152*a
^72*b^4*c^37*d^51))*(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 4
0282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a
^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5
*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7
*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^
9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232
*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 543841374599
08608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 11249127
6045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1
074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17
*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*
```

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b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384
896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767
561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25
- 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d
^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6
*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4
*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^3
3 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (45811
79456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a
*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^
25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b
^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*
b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*
b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b
^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 21
99023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461
056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426
560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062
349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 7228120
72152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4
433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36
*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^2
4*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 3887616929606
6641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38
876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^2
9*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^
31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443324737586
7248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 72281
2072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 +
62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27
 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29
 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806
029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 2928222551
40864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 377020745357
7216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 1891721044977
2544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 413061362462
22848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 4035488
5515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 24
68579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15
- 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^1
8*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19
*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 329960684422019
4816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 291354
5770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24
 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^
```

```
9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7
*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5
*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c
^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2
146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 -
 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*
a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 +
 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^2
4*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 1
29024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^
24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^3
2*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 47
1435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^3
1*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*
d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*
a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27
+ 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^
30)))^(3/4) - 19260910443845124096O*a^18*b^50*c^68*d^8 + 708618067091178232
2176*a^19*b^49*c^67*d^9 - 125074476913666377646080*a^20*b^48*c^66*d^10 + 14
1115280550631833600000O*a^21*b^47*c^65*d^11 - 11440156274772600537743360*a^
22*b^46*c^64*d^12 + 71019754904703755920343040*a^23*b^45*c^63*d^13 - 351320
863723081970831327232*a^24*b^44*c^62*d^14 + 1422781934731584726682828800*a^
25*b^43*c^61*d^15 - 4808764412319368968195276800*a^26*b^42*c^60*d^16 + 1375
3628214096098268020736000*a^27*b^41*c^59*d^17 - 336045862656462320079313305
60*a^28*b^40*c^58*d^18 + 70459004145207625658058932224*a^29*b^39*c^57*d^19
- 126335924813552658893934428160*a^30*b^38*c^56*d^20 + 1897144207659575875З
1118673920*a^31*b^37*c^55*d^21 - 2219472744682837731400744960OO*a^32*b^36*c
^54*d^22 + 142870740343318834154286612480*a^33*b^35*c^53*d^23 + 17608311817
75263996183073259 52*a^34*b^34*c^52*d^24 - 8959470273938483263920144384O0*a^
35*b^33*c^51*d^25 + 2154323340999822995276326502400*a^36*b^32*c^50*d^26 - 3
969865332339043373838394982400*a^37*b^31*c^49*d^27 + 614764426331211131732 5
499596800*a^38*b^30*c^48*d^28 - 8260762337957580186371563192320*a^39*b^29*c
^47*d^29 + 9765601087086458087650885632000*a^40*b^28*c^46*d^30 - 1022350694
8306413182866214092800*a^41*b^27*c^45*d^31 + 950842473829248398411924766720
0*a^42*b^26*c^44*d^32 - 7866898628254591634401331773440*a^43*b^25*c^43*d^33
 + 5790724738841488066411751276544*a^44*b^24*c^42*d^34 - 378900670406362548
4256485048320*a^45*b^23*c^41*d^35 + 2199996205919117948922678476800*a^46*b^
22*c^40*d^36 - 1130480215059585112828689776640*a^47*b^21*c^39*d^37 + 512203
696921842163745197916160*a^48*b^20*c^38*d^38 - 203625309837119046692160667 6
48*a^49*b^19*c^37*d^39 + 70576441632244073218493644800*a^50*b^18*c^36*d^40
- 21151503372075452883114393600*a^51*b^17*c^35*d^41 + 5422672476777259769 58
0748800*a^52*b^16*c^34*d^42 - 1172540913492414089228451840*a^53*b^15*c^33*d
^43 + 209790609112633976926765056*a^54*b^14*c^32*d^44 - 302397402123696934 9
0544640*a^55*b^13*c^31*d^45 + 3375777980998666504110080*a^56*b^12*c^30*d^46
 - 273981289062762912153600*a^57*b^11*c^29*d^47 + 14388779197382598328320*a
```

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^58*b^10*c^28*d^48 - 367186184646271434752*a^59*b^9*c^27*d^49))*(-(71993427
968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059
983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281
728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 75404149071544
32*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545
088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 8261227249244
5696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771
031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 493
7158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15
 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^
18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^1
9*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 65992136884403
89632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 58270
91540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^2
4 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*
c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b
^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*
b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^
3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d
 - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840
599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 7005478249708
4*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140
*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 1631620395804677
6*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 117608394414376
88*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 209420692905393
2*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11
*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d -
2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848
604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 138384
53347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231
299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8
 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^
37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^
23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 323968077467
22201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 4
1305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^
30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a
^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 88664947517
34497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 19
27498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^2
4 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21
*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^
19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17
*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 5
3376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 11953
```

```
57715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 941476
7089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 309064
00985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 452
51371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12
- 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d
^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*
c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^
18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090
865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363
759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^
23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^1
0*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^2
7*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a
^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^
31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b
^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68
719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*
b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15
*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 33
65856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23
*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 +
225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a
^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^
31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 3
47373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^
32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d
^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^3
7*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 49
60*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*1i)/((x^(1/2)*(8577
12418202478182400*a^18*b^48*c^62*d^11 - 28925330217666430894080*a^19*b^47*c
^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13 - 47721899383594535
53262592*a^21*b^45*c^59*d^14 + 34982076529826233401212928*a^22*b^44*c^58*d^
15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 + 87323112223641649331
3064960*a^24*b^42*c^56*d^17 - 3201588318340888739356606464*a^25*b^41*c^55*d
^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19 - 264756131425385368
17178705920*a^27*b^39*c^53*d^20 + 62528004036875405150857986048*a^28*b^38*c
^52*d^21 - 13143680796215491474489344000*a^29*b^37*c^51*d^22 + 25959547498
2835164713400139776*a^30*b^36*c^50*d^23 - 467106577738876991145070559232*a^
31*b^35*c^49*d^24 + 775321096823109302674935250944*a^32*b^34*c^48*d^25 - 11
79424943892680059222782640128*a^33*b^33*c^47*d^26 + 16296905936000958338232
95569920*a^34*b^32*c^46*d^27 - 2028143345719314676074795761664*a^35*b^31*c^
45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^44*d^29 - 22374491835
65830435563494178816*a^37*b^29*c^43*d^30 + 1966204854457469918399988498432*
a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a^39*b^27*c^41*d^32 +
 1046409458758522347995126562816*a^40*b^26*c^40*d^33 - 62995652359277433169
```

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8776113152*a^41*b^25*c^39*d^34 + 332065764335584004230153764864*a^42*b^24*c
^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^37*d^36 + 60699171433
471101739298979840*a^44*b^22*c^36*d^37 - 20757436699772395749793333248*a^45
*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^20*c^34*d^39 - 147344
9639082715479512449024*a^47*b^19*c^33*d^40 + 296084339424033093684559872*a^
48*b^18*c^32*d^41 - 47717950421254308290887680*a^49*b^17*c^31*d^42 + 593152
8400797457427988480*a^50*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^1
5*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^45 - 872067188534894
657536*a^53*b^13*c^27*d^46) + (-(71993427968*a^35*d^35 - ((143986855936*a^3
5*d^35 + 4028209561 6*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1067520
16121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430
600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178
574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181280197
0110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9050274
2771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54
384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14
+ 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^1
9*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18
*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913618173
1328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 672751
8677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23
 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*
c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27
*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^
29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^
31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b
^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/
4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2311218
82561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732
850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013057063
6271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924138801
09692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415727162
42975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157
362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15
*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 3
4084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 247
1152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 6
2273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7
 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^
38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22
*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900
569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388
76169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31
*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^2
9*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1551636581553
```

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5370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443
3247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^
23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^
22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5
*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3
*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c
^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 +
292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 37
70207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 189
17210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41
306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11
 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22
*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*
c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^1
7*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 19164254049286
86080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 32996
06844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^2
2 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11
*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a
^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 11969295567013478
4*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 931304129942323
2*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*
a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*
c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c
^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d
^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*
c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 1051830
0*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^3
7*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 34
7373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^2
6*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30
*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225
792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*
b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24
- 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*
c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b
^2*c^17*d^30)))^(1/4)*((x^(1/2)*(5649315372573550l824*a^22*b^52*c^81*d^4 -
2396923808077634863104*a^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b^
50*c^79*d^6 - 658598339056129087111168*a^25*b^49*c^78*d^7 + 639116386763433
0475954176*a^26*b^48*c^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d
^9 + 292502253544635823646834688*a^28*b^46*c^75*d^10 - 14760026454804159173
11524864*a^29*b^45*c^74*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*
d^12 - 23152095046595175238512672768*a^31*b^43*c^72*d^13 + 7388558436364218
6267654881280*a^32*b^42*c^71*d^14 - 206784189076489114265239683072*a^33*b^4
1*c^70*d^15 + 511001017390776406574528200704*a^34*b^40*c^69*d^16 - 11204864
```

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24066161848521664233472*a^35*b^39*c^68*d^17 + 218618373284243197324090461388
88*a^36*b^38*c^67*d^18 - 3794889949427368142860254707712*a^37*b^37*c^66*d^1
9 + 5830470252063718134687996051456*a^38*b^36*c^65*d^20 - 78076190336035905
30479469625344*a^39*b^35*c^64*d^21 + 8746184267385996582875203371008*a^40*b
^34*c^63*d^22 - 7176871923835198338520219385856*a^41*b^33*c^62*d^23 + 13651
98057841590488549164056576*a^42*b^32*c^61*d^24 + 10199723921158867878218460
823552*a^43*b^31*c^60*d^25 - 28100654056180096231365094146048*a^44*b^30*c^5
9*d^26 + 51280764289348564983994726219776*a^45*b^29*c^58*d^27 - 76696476979
720874342700527124480*a^46*b^28*c^57*d^28 + 9971756130280990673857070864793
6*a^47*b^27*c^56*d^29 - 115380588176718582142644189659136*a^48*b^26*c^55*d^
30 + 120101545474959969242488481251328*a^49*b^25*c^54*d^31 - 11305249490521
0552901304563269632*a^50*b^24*c^53*d^32 + 96462689920395704646643948191744*
a^51*b^23*c^52*d^33 - 74665519475418951639228294889472*a^52*b^22*c^51*d^34
+ 52413929319422085122116269637632*a^53*b^21*c^50*d^35 - 333341858691829792
96764484386816*a^54*b^20*c^49*d^36 + 19174031096138345851817803382784*a^55*
b^19*c^48*d^37 - 9951827463893335697728745766912*a^56*b^18*c^47*d^38 + 4646
728550801039102656464814080*a^57*b^17*c^46*d^39 - 1944469658660080242790338
920448*a^58*b^16*c^45*d^40 + 725810983387725632884961181696*a^59*b^15*c^44*
d^41 - 240265301732777409221605982208*a^60*b^14*c^43*d^42 + 700283105601324
15015125778432*a^61*b^13*c^42*d^43 - 17809629928199177184296828928*a^62*b^1
2*c^41*d^44 + 3907197185884869673284009984*a^63*b^11*c^40*d^45 - 7285690616
55967140126130176*a^64*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^
9*c^38*d^47 - 14265899165032610449588224*a^66*b^8*c^37*d^48 + 1400509163935
752188329984*a^67*b^7*c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^
50 + 4689814464763011268608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70
*b^4*c^33*d^52) + (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 402
82095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3
*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b
^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b
^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*
b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a
^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908
608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 1124912760
45524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 107
4443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d
^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^
15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 672751867774638489
6*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 429376756
1145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 -
1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^2
7 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d
^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d
^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33
- 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179
456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b
```

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^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25
*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^2
3*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^
21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^
19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^1
7*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199
023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 3408486046105
6*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 247115238342656
0*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 6227304006234
9312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072
152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 443
3247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d
^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*
b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388761692960666
41920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 3887
6169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*
d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31
*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443324737586724
8640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 7228120
72152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62
273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 +
 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 +
 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 680602
9991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140
864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577
216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772
544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130613624622
2848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 403548855
15952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468
579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 -
206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*
d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b
^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194
816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 29135457
70272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 +
 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*
d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c
^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c
^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3
*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 214
6713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 3
2*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^
14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 9
06192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*
c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129
```

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024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24
*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*
d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 4714
35600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*
b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^
22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^
36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 +
35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30
)))^(1/4)*(645636042579834306566*a^25*b^51*c^84*d^4 - 2822351843277561397248
*a^26*b^50*c^83*d^5 + 60127162308256283492352*a^27*b^49*c^82*d^6 - 83194815
7724300777881600*a^28*b^48*c^81*d^7 + 8406786558179361266073600*a^29*b^47*c
^80*d^8 - 66144581305899203170402304*a^30*b^46*c^79*d^9 + 42191267031068032
9277407232*a^31*b^45*c^78*d^10 - 2243238210521587022108295168*a^32*b^44*c^7
7*d^11 + 10145383251984825802817536000*a^33*b^43*c^76*d^12 - 39641949193820
336576213811200*a^34*b^42*c^75*d^13 + 135494098735043868075088674816*a^35*b
^41*c^74*d^14 - 409284915889091539805067542528*a^36*b^40*c^73*d^15 + 110233
1957384293957070038761472*a^37*b^39*c^72*d^16 - 266822316596808645943303864
3200*a^38*b^38*c^71*d^17 + 5847343583817169075816733081600*a^39*b^37*c^70*d
^18 - 11684105629368324959904469090304*a^40*b^36*c^69*d^19 + 21435002462698
637041098955948032*a^41*b^35*c^68*d^20 - 36343020410925078321345140359168*a
^42*b^34*c^67*d^21 + 57297580687683561030746426572800*a^43*b^33*c^66*d^22 -
 84429658980390814235781758976000*a^44*b^32*c^65*d^23 + 1167027447884256774
43098849837056*a^45*b^31*c^64*d^24 - 151589903153597380791972919246848*a^46
*b^30*c^63*d^25 + 185008444259789842943656593457152*a^47*b^29*c^62*d^26 - 2
11756933815433796881181835264000*a^48*b^28*c^61*d^27 + 22661195943384799721
2598992896000*a^49*b^27*c^60*d^28 - 225906031446565502788593732550656*a^50*
b^26*c^59*d^29 + 208978627749165724430025514549248*a^51*b^25*c^58*d^30 - 17
8726416623100559749866797924352*a^52*b^24*c^57*d^31 + 140824510781547830729
330235801600*a^53*b^23*c^56*d^32 - 101897270594764980154443340185600*a^54*b
^22*c^55*d^33 + 67499322390719467851063444373504*a^55*b^21*c^54*d^34 - 4080
9284384591153062742518136832*a^56*b^20*c^53*d^35 + 224472824313450506979471
18829568*a^57*b^19*c^52*d^36 - 11195042646819893251483369472000*a^58*b^18*c
^51*d^37 + 5042898342903938117430096691200*a^59*b^17*c^50*d^38 - 2042741359
937286689202494242816*a^60*b^16*c^49*d^39 + 740249793404633986500581654528*
a^61*b^15*c^48*d^40 - 238501265489031484884985577472*a^62*b^14*c^47*d^41 +
67809805296929472355971891200*a^63*b^13*c^46*d^42 - 16856343881283213574379
929600*a^64*b^12*c^45*d^43 + 3621158066396044540042543104*a^65*b^11*c^44*d^
44 - 662272679138724025500434432*a^66*b^10*c^43*d^45 + 10108783240006404372
4832768*a^67*b^9*c^42*d^46 - 12528855636637836430540800*a^68*b^8*c^41*d^47
+ 1211288155777568604160000*a^69*b^7*c^40*d^48 - 85697808358931542573056*a^
70*b^6*c^39*d^49 + 3946450310269237198848*a^71*b^5*c^38*d^50 - 887749558547
27217152*a^72*b^4*c^37*d^51))*(-(71993427968*a^35*d^35 - ((143986855936*a^3
5*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1067520
16121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430
600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178
```

```
574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181280197
0110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9050274
2771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54
384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14
+ 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^1
9*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18
*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913618173
1328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 672751
8677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23
 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*
c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27
*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^
29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^
31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b
^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/
4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2311218
82561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732
850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013057063
6271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924138801
09692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415727162
42975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157
362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15
*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 3
4084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 247
1152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 6
2273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7
 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^
38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22
*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900
569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388
76169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31
*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^2
9*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1551636581553
5370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443
3247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^
23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^
22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5
*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3
*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c
^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 +
292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 37
70207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 189
17210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41
306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11
 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22
```

*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4) + 19260910443845124096 0*a^18*b^50*c^68*d^8 - 708618067091178232 2176*a^19*b^49*c^67*d^9 + 1250744769136663776 46080*a^20*b^48*c^66*d^10 - 1411152805506318336000000*a^21*b^47*c^65*d^11 + 11440156274772600537743360*a^22*b^46*c^64*d^12 - 71019754904703755920343040*a^23*b^45*c^63*d^13 + 351320863723081970831327232*a^24*b^44*c^62*d^14 - 1422781934731584726682828800*a^25*b^43*c^61*d^15 + 4808764412319368968195276800*a^26*b^42*c^60*d^16 - 13753628214096098268020736000*a^27*b^41*c^59*d^17 + 33604586265646232007931330560*a^28*b^40*c^58*d^18 - 70459004145207625658058932224*a^29*b^39*c^57*d^19 + 126335924813552658893934428160*a^30*b^38*c^56*d^20 - 1897144207659575875311186739 20*a^31*b^37*c^55*d^21 + 221947274468283773140074496000*a^32*b^36*c^54*d^22 - 142870740343318834154286612480*a^33*b^35*c^53*d^23 - 1760831181775263996183073259 52*a^34*b^34*c^52*d^24 + 8959470273938483263920144 38400*a^35*b^33*c^51*d^25 - 2154323340999822995276326502400*a^36*b^32*c^50*d^26 + 3969865332339043373838394982400*a^37*b^31*c^49*d^27 - 6147644263312111317325499596800*a^38*b^30*c^48*d^28 + 8260762337957580186371563192320*a^39*b^29*c^47*d^29 - 9765601087086458087650885632000*a^40*b^28*c^46*d^30 + 10223506948306413182866214092800*a^41*b^27*c^45*d^31 - 950842473829248398411924766 7200*a^42*b^26*c^44*d^32 + 7866898628254591634401331773440*a^43*b^25*c^43*d^33 - 5790724738841488066411751276544*a^44*b^24*c^42*d^34 + 3789006704063625484256485048320*a^45*b^23*c^41*d^35 - 2199996205919117948922678476800*a^46*b^22*c^40*d^36 + 1130480215059585112828689776640*a^47*b^21*c^39*d^3

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7 - 512203696921842163745197916160*a^48*b^20*c^38*d^38 + 203625309837119046
692160667648*a^49*b^19*c^37*d^39 - 705764416322440732184936448OO*a^50*b^18*
c^36*d^40 + 211515033720754528831143936OO*a^51*b^17*c^35*d^41 - 54226724767
772597695807488OO*a^52*b^16*c^34*d^42 + 117254091349241408922845184O*a^53*b
^15*c^33*d^43 - 209790609112633976926765056*a^54*b^14*c^32*d^44 + 302397402
12369693490544640*a^55*b^13*c^31*d^45 - 3375777980998666504110080*a^56*b^12
*c^30*d^46 + 273981289062762912153600*a^57*b^11*c^29*d^47 - 143887791973825
98328320*a^58*b^10*c^28*d^48 + 367186184646271434752*a^59*b^9*c^27*d^49))*(
-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35
+ 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 58
5644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540
414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 3783
4420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 826
12272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11
+ 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*
d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*
c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^
17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857
372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599
213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^
22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^1
1*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432
*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269
568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 1862608259884
6464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 5642928491397
12*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b
^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^
23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 700
54782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 506542
7904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 163162
03958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760
839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 20942
06929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(687194
76736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31
*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^
2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^
4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41
*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24
*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^
21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 1551636581553
5370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32
396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^3
2*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^
28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 238713320239
00569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8
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866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25
*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^
8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^3
7*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a
^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^4
1*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^
33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d
^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^
6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^
8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*
d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*
c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b
^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*
a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111828572941
8035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 274
0669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d
^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^
12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 134479254681873612
8*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758
366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 370403183
38179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 185565302
5615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957
248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c
*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d
- 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 +
35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^4
1*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800
*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^
36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 4
71435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^
27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^2
9*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 12
9024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*
b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 +
906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19
*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4) - (x^(1/2
)*(857712418202478182400*a^18*b^48*c^62*d^11 - 28925330217666430894080*a^19
*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13 - 4772189938
359453553262592*a^21*b^45*c^59*d^14 + 34982076529826233401212928*a^22*b^44*
c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 + 873231122364
1649313064960*a^24*b^42*c^56*d^17 - 320158831834088873935606464*a^25*b^41
*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19 - 26475613142
538536817178705920*a^27*b^39*c^53*d^20 + 62528004036875405150857986048*a^28
*b^38*c^52*d^21 - 133143680796215491474489344000*a^29*b^37*c^51*d^22 + 2595
95474982835164713400139776*a^30*b^36*c^50*d^23 - 46710657773887699114507055
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9232*a^31*b^35*c^49*d^24 + 7753210968231093026749352509944*a^32*b^34*c^48*d^
25 - 11794249438926800592227826401228*a^33*b^33*c^47*d^26 + 1629690593600095
833823295569920*a^34*b^32*c^46*d^27 - 2028143345719314676074795761664*a^35*
b^31*c^45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^44*d^29 - 2237
449183565830435563494178816*a^37*b^29*c^43*d^30 + 1966204854457469918399988
498432*a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a^39*b^27*c^41
*d^32 + 1046409458758522347995126562816*a^40*b^26*c^40*d^33 - 6299565235927
74331698776113152*a^41*b^25*c^39*d^34 + 332065764335584004230153764864*a^42
*b^24*c^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^37*d^36 + 6069
9171433471101739298979840*a^44*b^22*c^36*d^37 - 207574366997723957497933332
48*a^45*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^20*c^34*d^39 -
 1473449639082715479512449024*a^47*b^19*c^33*d^40 + 296084339424033093684559
9872*a^48*b^18*c^32*d^41 - 47717950421254308290887680*a^49*b^17*c^31*d^42 +
 5931528400797457427988480*a^50*b^16*c^30*d^43 - 534037861185724002336768*a
^51*b^15*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^45 - 87206718
8534894657536*a^53*b^13*c^27*d^46) + (-(71993427968*a^35*d^35 - ((143986855
936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 -
106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 239
0715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1882
9534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 618
12801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 -
90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^
12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^2
1*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b
^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714588360704
00*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 54813391
36181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 -
 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^
12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^2
5*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732
416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408063667635
8144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371130605123
1744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496
*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d
^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 -
231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 144220
3904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 111501
30570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492
413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941
572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 49287
3253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^
43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d
^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^
3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*
d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c
```

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^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*
b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664947517344972
80*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387133
2023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^1
4 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^
16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201
600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155163
65815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^2
1 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*
c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36
*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a
^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a
^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808
*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32
*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d
^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^
7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d
^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^
24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^
22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^1
5*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067
200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425
404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20
+ 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c
^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^
24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140227398332
25216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1196929556
70134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 93130412
99423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424
569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*
a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 +
 a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30
*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^1
6*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 +
10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b
^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^
12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722
720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^
15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^1
9 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 6451224
0*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^2
3*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^
38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496
*a^41*b^2*c^17*d^30)))^(1/4)*((x^(1/2)*(5649315372573501824*a^22*b^52*c^81
*d^4 - 23969238080776348631O4*a^23*b^51*c^80*d^5 + 4938769849284350391O912*
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a^24*b^50*c^79*d^6 - 6585983390561290871111168*a^25*b^49*c^78*d^7 + 63911638
67634330475954176*a^26*b^48*c^77*d^8 - 4811359686765194506980568*a^27*b^47
*c^76*d^9 + 29250225354463582364683468 8*a^28*b^46*c^75*d^10 - 1476002645480
415917311524864*a^29*b^45*c^74*d^11 + 6306003584409325504378699776*a^30*b^4
4*c^73*d^12 - 23152095046595175238512672768*a^31*b^43*c^72*d^13 + 738855843
63642186267654881280*a^32*b^42*c^71*d^14 - 206784189076489114265239683072*a
^33*b^41*c^70*d^15 + 511001017390776406574528200704*a^34*b^40*c^69*d^16 - 1
120486424066161848521664233472*a^35*b^39*c^68*d^17 + 21861837328424319732 40
904613888*a^36*b^38*c^67*d^18 - 3794889949427368142860254707712*a^37*b^37*c
^66*d^19 + 5830470252063718134687996051456*a^38*b^36*c^65*d^20 - 7807619033
603590530479469625344*a^39*b^35*c^64*d^21 + 8746184267385996582875203371008
*a^40*b^34*c^63*d^22 - 7176871923835198338520219385856*a^41*b^33*c^62*d^23
+ 1365198057841590488549164056576*a^42*b^32*c^61*d^24 + 101997239211588678 7
8218460823552*a^43*b^31*c^60*d^25 - 28100654056180096231365094146048*a^44*b
^30*c^59*d^26 + 51280764289348564983994726219776*a^45*b^29*c^58*d^27 - 7669
6476979720874342700527124480*a^46*b^28*c^57*d^28 + 997175613028099067385707
08647936*a^47*b^27*c^56*d^29 - 115380588176718582142644189659136*a^48*b^26*
c^55*d^30 + 120101545474959969242488481251328*a^49*b^25*c^54*d^31 - 1130524
949052105529013045632696 32*a^50*b^24*c^53*d^32 + 964626899203957046466439 48
191744*a^51*b^23*c^52*d^33 - 746655194754189516392282948 89472*a^52*b^22*c^5
1*d^34 + 524139293194220851221162696 37632*a^53*b^21*c^50*d^35 - 33334185869
1829792967644843868 16*a^54*b^20*c^49*d^36 + 19174031096138345851817803382 78
4*a^55*b^19*c^48*d^37 - 99518274638933356977287457669 12*a^56*b^18*c^47*d^38
 + 46467285508010391026564648140 80*a^57*b^17*c^46*d^39 - 19444696586600802 4
2790338920448*a^58*b^16*c^45*d^40 + 72581098338772563288496118169 6*a^59*b^1
5*c^44*d^41 - 2402653017327774092216059822 08*a^60*b^14*c^43*d^42 + 70028310
5601324150151257784 32*a^61*b^13*c^42*d^43 - 1780962992819917718429682892 8*a
^62*b^12*c^41*d^44 + 390719718588486967328400998 4*a^63*b^11*c^40*d^45 - 728
569061655967140126130176*a^64*b^10*c^39*d^46 + 11321480853131993952760627 2*
a^65*b^9*c^38*d^47 - 142658991650326104495882 24*a^66*b^8*c^37*d^48 + 140050
9163935752188329984*a^67*b^7*c^36*d^49 - 1005028336875582542315 52*a^68*b^6*
c^35*d^50 + 4689814464763011268608*a^69*b^5*c^34*d^51 - 1068073687627186831
36*a^70*b^4*c^33*d^52) - (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^3
5 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121
856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 239071543060070
4*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1882953417857433
6*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 618128019701104
64*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 905027427711
67232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 5438413
7459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112
491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^1
6 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17
*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*
a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 67275186777
46384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42
```

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93767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*
d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*
c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^
6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^
4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^
2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (
4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561
500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a
^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*
a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692
*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975
*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a
^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32
 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 340848
60461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 24711523
83426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 622730
40062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72
2812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^
9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21
*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387133202390056960
0*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169
296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16
 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^1
4*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155163658155353702
40*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 44332473
75867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 +
722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^
25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20
*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18
*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 +
 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29282
2255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207
453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210
449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130613
6246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40
354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13
 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*
d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^1
8*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080
*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844
220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2
913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11
*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b
^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^2
8*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^3
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0*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*
b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*
d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d
^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 -
4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*
d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^1
9*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^1
0 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 3473736
00*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^1
7*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17
 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 22579284
0*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*
c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 336
5856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*
d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^
17*d^30)))^(1/4)*(64563604257983430656*a^25*b^51*c^84*d^4 - 282235184327756
1397248*a^26*b^50*c^83*d^5 + 60127162308256283492352*a^27*b^49*c^82*d^6 - 8
31948157724300777881600*a^28*b^48*c^81*d^7 + 8406786558179361266073600*a^29
*b^47*c^80*d^8 - 66144581305899203170402304*a^30*b^46*c^79*d^9 + 4219126703
10680329277407232*a^31*b^45*c^78*d^10 - 2243238210521587022108295168*a^32*b
^44*c^77*d^11 + 10145383251984825802817536000*a^33*b^43*c^76*d^12 - 3964194
9193820336576213811200*a^34*b^42*c^75*d^13 + 135494098735043868075088674816
*a^35*b^41*c^74*d^14 - 409284915889091539805067542528*a^36*b^40*c^73*d^15 +
 1102331957384293957070038761472*a^37*b^39*c^72*d^16 - 26682231659680864594
33038643200*a^38*b^38*c^71*d^17 + 5847343583817169075816733081600*a^39*b^37
*c^70*d^18 - 11684105629368324959904469090304*a^40*b^36*c^69*d^19 + 2143500
2462698637041098955948032*a^41*b^35*c^68*d^20 - 363430204109250783213451403
59168*a^42*b^34*c^67*d^21 + 57297580687683561030746426572800*a^43*b^33*c^66
*d^22 - 84429658980390814235781758976000*a^44*b^32*c^65*d^23 + 116702744788
425677443098849837056*a^45*b^31*c^64*d^24 - 1515899031535973807919729192468
48*a^46*b^30*c^63*d^25 + 185008444259789842943656593457152*a^47*b^29*c^62*d
^26 - 211756933815433796881181835264000*a^48*b^28*c^61*d^27 + 2266119594338
47997212598992896000*a^49*b^27*c^60*d^28 - 22590603144656550278859373255065
6*a^50*b^26*c^59*d^29 + 208978627749165724430025514549248*a^51*b^25*c^58*d^
30 - 178726416623100559749866797924352*a^52*b^24*c^57*d^31 + 14082451078154
7830729330235801600*a^53*b^23*c^56*d^32 - 101897270594764980154443340185600
*a^54*b^22*c^55*d^33 + 67499322390719467851063444373504*a^55*b^21*c^54*d^34
 - 40809284384591153062742518136832*a^56*b^20*c^53*d^35 + 22447282431345050
697947118829568*a^57*b^19*c^52*d^36 - 11195042646819893251483369472000*a^58
*b^18*c^51*d^37 + 5042898342903938117430096691200*a^59*b^17*c^50*d^38 - 204
2741359937286689202494242816*a^60*b^16*c^49*d^39 + 740249793404633986500581
654528*a^61*b^15*c^48*d^40 - 238501265489031484884985577472*a^62*b^14*c^47*
d^41 + 67809805296929472355971891200*a^63*b^13*c^46*d^42 - 1685634388128321
3574379929600*a^64*b^12*c^45*d^43 + 3621158066396044540042543104*a^65*b^11*
c^44*d^44 - 662272679138724025500434432*a^66*b^10*c^43*d^45 + 1010878324000
```

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64043724832768*a^67*b^9*c^42*d^46 - 12528855636637836430540800*a^68*b^8*c^4
1*d^47 + 1211288155777568604160000*a^69*b^7*c^40*d^48 - 8569780835893154257
3056*a^70*b^6*c^39*d^49 + 3946450310269237198848*a^71*b^5*c^38*d^50 - 88774
955854727217152*a^72*b^4*c^37*d^51))*(-(71993427968*a^35*d^35 - ((143986855
936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 -
106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 239
0715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1882
9534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 618
12801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 -
90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^
12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^2
1*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b
^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714588360704
00*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 54813391
36181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 -
 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^
12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^2
5*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6393294975167324
16*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408063667635
8144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371130605123
1744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496
*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d
^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 -
231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 144220
3904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 111501
30570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492
413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941
572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 49287
3253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^
43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d
^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^
3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*
d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c
^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*
b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664947517344972
80*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387133
2023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^1
4 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^
16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201
600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155163
65815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^2
1 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*
c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36
*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a
^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a
```

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^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808
*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32
*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d
^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^
7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d
^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^
24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^
22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^1
5*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067
200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425
404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20
+ 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c
^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^
24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140227398332
25216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1196929556
70134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 93130412
99423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424
569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*
a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 +
 a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30
*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^1
6*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 +
10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b
^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^
12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722
720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^
15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^1
9 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 6451224
0*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^2
3*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^
38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496
*a^41*b^2*c^17*d^30)))^(3/4) - 192609104438451240960*a^18*b^50*c^68*d^8 + 7
086180670911782322176*a^19*b^49*c^67*d^9 - 125074476913666377646080*a^20*b^
48*c^66*d^10 + 1411152805506318336000000*a^21*b^47*c^65*d^11 - 114401562747
72600537743360*a^22*b^46*c^64*d^12 + 71019754904703755920343040*a^23*b^45*c
^63*d^13 - 351320863723081970831327232*a^24*b^44*c^62*d^14 + 14227819347315
84726682828800*a^25*b^43*c^61*d^15 - 4808764412319368968195276800*a^26*b^42
*c^60*d^16 + 13753628214096098268020736000*a^27*b^41*c^59*d^17 - 3360458626
5646232007931330560*a^28*b^40*c^58*d^18 + 70459004145207625658058932224*a^2
9*b^39*c^57*d^19 - 126335924813552658893934428160*a^30*b^38*c^56*d^20 + 189
714420765957587531118673920*a^31*b^37*c^55*d^21 - 2219472744682837731400744
96000*a^32*b^36*c^54*d^22 + 142870740343318834154286612480*a^33*b^35*c^53*d
^23 + 176083118177526399618307325952*a^34*b^34*c^52*d^24 - 8959470273938483
26392014438400*a^35*b^33*c^51*d^25 + 2154323340999822995276326502400*a^36*b
^32*c^50*d^26 - 3969865332339043373838394982400*a^37*b^31*c^49*d^27 + 61476
```

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44263312111317325499596800*a^38*b^30*c^48*d^28 - 82607623379575801863715631
92320*a^39*b^29*c^47*d^29 + 9765601087086458087650885632000*a^40*b^28*c^46*
d^30 - 10223506948306413182866214092800*a^41*b^27*c^45*d^31 + 9508424738292
483984119247667200*a^42*b^26*c^44*d^32 - 7866898628254591634401331773440*a^
43*b^25*c^43*d^33 + 5790724738841488066411751276544*a^44*b^24*c^42*d^34 - 3
789006704063625484256485048320*a^45*b^23*c^41*d^35 + 2199996205919117948922
678476800*a^46*b^22*c^40*d^36 - 1130480215059585112828689776640*a^47*b^21*c
^39*d^37 + 512203696921842163745197916160*a^48*b^20*c^38*d^38 - 20362530983
7119046692160667648*a^49*b^19*c^37*d^39 + 70576441632244073218493644800*a^5
0*b^18*c^36*d^40 - 21151503372075452883114393600*a^51*b^17*c^35*d^41 + 5422
672476777259769580748800*a^52*b^16*c^34*d^42 - 1172540913492414089228451840
*a^53*b^15*c^33*d^43 + 209790609112633976926765056*a^54*b^14*c^32*d^44 - 30
239740212369693490544640*a^55*b^13*c^31*d^45 + 3375777980998666504110080*a^
56*b^12*c^30*d^46 - 273981289062762912153600*a^57*b^11*c^29*d^47 + 14388779
197382598328320*a^58*b^10*c^28*d^48 - 367186184646271434752*a^59*b^9*c^27*d
^49))*(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^3
5*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d
^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5
 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7
 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^
9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^2
4*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^2
2*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^1
5*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 107444323159613
4400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383285
0809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20
 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*
c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a
^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142804547966
6450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 23938591
1340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 186260
82598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292
849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996
544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*
b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^1
2 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 -
 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 -
 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17
- 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19
- 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*
(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^
12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*
c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*
c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^
26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^
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19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 443324737586724
8640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155163
65815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^
13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b
^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692960664
1920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871
332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d
^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^
10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*
a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349
312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 24711523834
26560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461
056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*
b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31
*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*
c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*
c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^2
5*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^1
2*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552
*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965
452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11182
85729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^1
9 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14
*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*
a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925468
18736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966
4748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37
040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18
55653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3077
7147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a
^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*
c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44
*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b
^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 2
8048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*
b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d
^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 60108
0390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b
^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^
20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 2804880
0*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*
d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b
^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)))*
(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35
 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 5
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85644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 754
0414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 378
34420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82
612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11
 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22
*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20
*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a
^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383285080985
7372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659
9213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d
^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^
11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142804547966645043
2*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 23938591134026
9568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 186260825988
46464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139
712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*
b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d
^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70
054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 50654
27904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316
203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 1176
0839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094
206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719
476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^3
1*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d
^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d
^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^4
1*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^2
4*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a
^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155163658155
35370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 3
2396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^
32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a
^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023
900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 -
8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^2
5*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b
^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^
37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*
a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^
41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c
^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*
d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d
^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d
^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25
```

*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23
*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*
b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800
*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11182857294
18035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27
40669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*
d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b
^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925468187361
28*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966474875
8366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318
338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18556530
25615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3077714795
7248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*
c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d
 - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 +
 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^
41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 2804880
0*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c
^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 +
471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a
^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^
29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 1
29024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34
*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 +
 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^1
9*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*2i - 2*a
tan(((x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 - 2892533021766643
0894080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^13
- 4772189938359453553262592*a^21*b^45*c^59*d^14 + 3498207652982623340121292
8*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 + 8
73231122236416493313064960*a^24*b^42*c^56*d^17 - 32015883183408887393566064
64*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*c^54*d^19 -
 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 6252800403687540515085
7986048*a^28*b^38*c^52*d^21 - 133143680796215491474489344000*a^29*b^37*c^51
*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23 - 46710657773887
6991145070559232*a^31*b^35*c^49*d^24 + 775321096823109302674935250944*a^32*
b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*c^47*d^26 + 1629
690593600095833823295569920*a^34*b^32*c^46*d^27 - 2028143345719314676074795
761664*a^35*b^31*c^45*d^28 + 2257905973104023956972306956288*a^36*b^30*c^44
*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30 + 1966204854457
469918399988498432*a^38*b^28*c^42*d^31 - 1527649406048366621262568488960*a^
39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b^26*c^40*d^33 - 6
29956523592774331698776113152*a^41*b^25*c^39*d^34 + 33206576433558400423015
3764864*a^42*b^24*c^38*d^35 - 152543196968133650922715742208*a^43*b^23*c^37
*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^37 - 207574366997723

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95749793333248*a^45*b^21*c^35*d^38 + 6037825951797032255320227840*a^46*b^20
*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d^40 + 29608433942
4033093684559872*a^48*b^18*c^32*d^41 - 477179504212543082908887680*a^49*b^17
*c^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*d^43 - 53403786118572
4002336768*a^51*b^15*c^29*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^4
5 - 872067188534894657536*a^53*b^13*c^27*d^46) - (-(71993427968*a^35*d^35 -
 ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33
*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^
31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^2
9*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^
27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*
c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*
b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a
^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 4132414539309
05600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365
71458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^1
9 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14
*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*
a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 26895850936
37472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6393
29497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7
4080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 +
3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61
554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342624972
8*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*
c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d
^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d
^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*
d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5
*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*
d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68
719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a
^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*
b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^1
6*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016
*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072
716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664
94751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^
12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b
^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 4130592987707080
7040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396
807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d
^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b
^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 19274988590727168
00*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 23129986308
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8726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 1383845
3347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 34084
8604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) +
 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a
^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5
*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*
b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9
*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*
a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 2719206872995
4304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 562456380
22762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537
221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^
18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^1
5*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448
*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780
572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7
14022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27
- 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^2
9 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31
 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 5
40930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11
*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 4
96*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4
- 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25
*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 645
12240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*
b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d
^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 56572
2720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b
^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^
21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*
a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26
 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^1
8*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*((x^(1/2)*(5649315372573550 1824*a^
22*b^52*c^81*d^4 - 23969238080776348 63104*a^23*b^51*c^80*d^5 + 493876984928
43503910912*a^24*b^50*c^79*d^6 - 65859833905612908 7111168*a^25*b^49*c^78*d^
7 + 63911638676343304 75954176*a^26*b^48*c^77*d^8 - 481135968676519450698055
68*a^27*b^47*c^76*d^9 + 292502253544635823646834688*a^28*b^46*c^75*d^10 - 1
476002645480415917311524864*a^29*b^45*c^74*d^11 + 630600358440932550437869 9
776*a^30*b^44*c^73*d^12 - 23152095046595175238512672768*a^31*b^43*c^72*d^13
 + 73885584363642186267654881280*a^32*b^42*c^71*d^14 - 2067841890764891142 6
5239683072*a^33*b^41*c^70*d^15 + 511001017390776406574528200704*a^34*b^40*c
^69*d^16 - 1120486424066161848521664233472*a^35*b^39*c^68*d^17 + 2186183732
842431973240904613888*a^36*b^38*c^67*d^18 - 3794889949427368142860254707712
*a^37*b^37*c^66*d^19 + 5830470252063718134687996051456*a^38*b^36*c^65*d^20
```

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- 7807619033603590530479469625344*a^39*b^35*c^64*d^21 + 8746184267385996582
875203371008*a^40*b^34*c^63*d^22 - 7176871923835198338520219385856*a^41*b^3
3*c^62*d^23 + 13651980578415904885491640565 76*a^42*b^32*c^61*d^24 + 1019972
3921158867878218460823552*a^43*b^31*c^60*d^25 - 281006540561800962313650941
46048*a^44*b^30*c^59*d^26 + 5128076428934856498399472621977 6*a^45*b^29*c^58
*d^27 - 7669647697972087434270052712448 0*a^46*b^28*c^57*d^28 + 997175613028
0990673857070864793 6*a^47*b^27*c^56*d^29 - 115380588176718582142644189659 13
6*a^48*b^26*c^55*d^30 + 1201015454749599692424884812513 28*a^49*b^25*c^54*d^
31 - 1130524949052105529013045632696 32*a^50*b^24*c^53*d^32 + 96462689920395
704646643948191744*a^51*b^23*c^52*d^33 - 746655194754189516392282948894 72*a
^52*b^22*c^51*d^34 + 5241392931942208512211626963763 2*a^53*b^21*c^50*d^35 -
 33334185869182979296764484386816*a^54*b^20*c^49*d^36 + 1917403109613834585
1817803382784*a^55*b^19*c^48*d^37 - 9951827463893335697728745766912*a^56*b^
18*c^47*d^38 + 4646728550801039102656464814080*a^57*b^17*c^46*d^39 - 194446
9658660080242790338920448*a^58*b^16*c^45*d^40 + 725810983387725632884961181
696*a^59*b^15*c^44*d^41 - 240265301732777409221605982208*a^60*b^14*c^43*d^4
2 + 70028310560132415015125778432*a^61*b^13*c^42*d^43 - 178096299281991771 8
4296828928*a^62*b^12*c^41*d^44 + 3907197185884869673284009984*a^63*b^11*c^4
0*d^45 - 728569061655967140126130176*a^64*b^10*c^39*d^46 + 1132148085313199
39527606272*a^65*b^9*c^38*d^47 - 14265899165032610449588224*a^66*b^8*c^37*d
^48 + 1400509163935752188329984*a^67*b^7*c^36*d^49 - 100502833687558254231 5
52*a^68*b^6*c^35*d^50 + 4689814464763011268608*a^69*b^5*c^34*d^51 - 1068073
68762718683136*a^70*b^4*c^33*d^52) - (-(71993427968*a^35*d^35 - ((143986855
936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 -
106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 239
0715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1882
9534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 618
12801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 -
90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^
12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^2
1*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b
^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714588360704
00*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 54813391
36181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 -
 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^
12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^2
5*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732 4
16*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408063667635
8144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371130605123
1744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496
*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342624972 8*a^34*b*c*d
^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 -
231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 144220
3904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 111501
30570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492
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413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941
572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 49287
3253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^
43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d
^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^
3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*
d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c
^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*
b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 886649475173449072
80*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387133
2023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^1
4 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^
16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 3239680774672201
600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155163
65815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^2
1 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*
c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36
*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a
^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a
^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808
*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32
*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d
^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^
7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d
^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^
24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^
22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^1
5*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067
200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425
404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20
+ 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c
^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^
24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140227398332
25216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1196929556
70134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 93130412
99423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424
569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*
a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 +
 a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30
*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^1
6*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 +
10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b
^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^
12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722
720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^
```

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15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^1
9 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 6451224
0*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^2
3*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^
38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496
*a^41*b^2*c^17*d^30)))^(1/4)*(645636042579834306 56*a^25*b^51*c^84*d^4 - 282
2351843277561397248*a^26*b^50*c^83*d^5 + 6012716230825628349 2352*a^27*b^49*
c^82*d^6 - 831948157724300777881600*a^28*b^48*c^81*d^7 + 840678655817936126
6073600*a^29*b^47*c^80*d^8 - 66144581305899203170402304*a^30*b^46*c^79*d^9
+ 421912670310680329277407232*a^31*b^45*c^78*d^10 - 22432382105215870221082
95168*a^32*b^44*c^77*d^11 + 10145383251984825802817536000*a^33*b^43*c^76*d^
12 - 39641949193820336576213811200*a^34*b^42*c^75*d^13 + 135494098735043868
075088674816*a^35*b^41*c^74*d^14 - 409284915889091539805067542528*a^36*b^40
*c^73*d^15 + 1102331957384293957070038761472*a^37*b^39*c^72*d^16 - 26682231
65968086459433038643200*a^38*b^38*c^71*d^17 + 58473435838171690758167330816
00*a^39*b^37*c^70*d^18 - 11684105629368324959904469090304*a^40*b^36*c^69*d^
19 + 21435002462698637041098955948032*a^41*b^35*c^68*d^20 - 363430204109250
78321345140359168*a^42*b^34*c^67*d^21 + 57297580687683561030746426572800*a^
43*b^33*c^66*d^22 - 84429658980390814235781758976000*a^44*b^32*c^65*d^23 +
116702744788425677443098849837056*a^45*b^31*c^64*d^24 - 151589903153597380 7
91972919246848*a^46*b^30*c^63*d^25 + 185008444259789842943656593457152*a^47
*b^29*c^62*d^26 - 211756933815433796881181835264000*a^48*b^28*c^61*d^27 + 2
26611959433847997212598992896000*a^49*b^27*c^60*d^28 - 2259060314465655027 8
8593732550656*a^50*b^26*c^59*d^29 + 208978627749165724430025514549248*a^51*
b^25*c^58*d^30 - 178726416623100559749866797924352*a^52*b^24*c^57*d^31 + 14
0824510781547830729330235801600*a^53*b^23*c^56*d^32 - 101897270594764980154
443340185600*a^54*b^22*c^55*d^33 + 67499322390719467851063444373504*a^55*b^
21*c^54*d^34 - 40809284384591153062742518136832*a^56*b^20*c^53*d^35 + 22447
282431345050697947118829568*a^57*b^19*c^52*d^36 - 111950426468198932514833 6
9472000*a^58*b^18*c^51*d^37 + 5042898342903938117430096691200*a^59*b^17*c^5
0*d^38 - 2042741359937286689202494242816*a^60*b^16*c^49*d^39 + 740249793404
633986500581654528*a^61*b^15*c^48*d^40 - 238501265489031484884985577472*a^6
2*b^14*c^47*d^41 + 67809805296929472355971891200*a^63*b^13*c^46*d^42 - 1685
6343881283213574379929600*a^64*b^12*c^45*d^43 + 362115806639604454004254310
4*a^65*b^11*c^44*d^44 - 662272679138724025500434432*a^66*b^10*c^43*d^45 + 1
01087832400064043724832768*a^67*b^9*c^42*d^46 - 12528855636637836430540800*
a^68*b^8*c^41*d^47 + 1211288155777568604160000*a^69*b^7*c^40*d^48 - 8569780
8358931542573056*a^70*b^6*c^39*d^49 + 394645031026923719884 8*a^71*b^5*c^38*
d^50 - 887749558547272171 52*a^72*b^4*c^37*d^51)*1i)*(-(71993427968*a^35*d^3
5 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b
^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31
*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*
c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27
*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^
25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^
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12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 493715857745510
4*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 4132414539
30905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22
36571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*
d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b
^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 58270915405454868
48*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 26895850
93637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6
39329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28
+ 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30
 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 +
 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342624
9728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 1584059900625*b^
27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*
c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^
9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c
^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*
c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c
^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 +
 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 219902325555
2*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^
14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*
a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726
016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859
072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88
66494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35
*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^2
5*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 413059298770
70807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32
396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^2
8*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^3
2*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 19274988590727
16800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 23129986
3088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 1383
8453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 34
0848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2
) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 5337600806092
8*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*
a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a
^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*
a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 452513713855836
16*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 2719206872
9954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 562456
38022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 +
537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17
```

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*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*
b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192
448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883
780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25
- 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^
27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*
d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d
^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33
- 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a
^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31
+ 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d
^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b
^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 +
64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^
23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^3
3*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 56
5722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^3
0*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26
*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 105183
00*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d
^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*
c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4)*1i + 192609104438451240960*a^18
*b^50*c^68*d^8 - 7086180670911782322176*a^19*b^49*c^67*d^9 + 12507447691366
6377646080*a^20*b^48*c^66*d^10 - 1411152805506318336000000*a^21*b^47*c^65*d
^11 + 11440156274772600537743360*a^22*b^46*c^64*d^12 - 71019754904703755920
343040*a^23*b^45*c^63*d^13 + 351320863723081970831327232*a^24*b^44*c^62*d^1
4 - 1422781934731584726682828800*a^25*b^43*c^61*d^15 + 48087644123193689681
95276800*a^26*b^42*c^60*d^16 - 13753628214096098268020736000*a^27*b^41*c^59
*d^17 + 33604586265646232007931330560*a^28*b^40*c^58*d^18 - 704590041452076
25658058932224*a^29*b^39*c^57*d^19 + 126335924813552658893934428160*a^30*b^
38*c^56*d^20 - 189714420765957587531118673920*a^31*b^37*c^55*d^21 + 2219472
74468283773140074496000*a^32*b^36*c^54*d^22 - 142870740343318834154286612480
0*a^33*b^35*c^53*d^23 - 176083118177526399618307325952*a^34*b^34*c^52*d^24
+ 895947027393848326392014438400*a^35*b^33*c^51*d^25 - 215432334099982299952
76326502400*a^36*b^32*c^50*d^26 + 3969865332339043373838394982400*a^37*b^31
*c^49*d^27 - 6147644263312111317325499596800*a^38*b^30*c^48*d^28 + 82607623
37957580186371563192320*a^39*b^29*c^47*d^29 - 976560108708645808765088563200
00*a^40*b^28*c^46*d^30 + 10223506948306413182866214092800*a^41*b^27*c^45*d^
31 - 9508424738292483984119247667200*a^42*b^26*c^44*d^32 + 7866898628254591
634401331773440*a^43*b^25*c^43*d^33 - 5790724738841488066411751276544*a^44*
b^24*c^42*d^34 + 3789006704063625484256485048320*a^45*b^23*c^41*d^35 - 2199
996205919117948922678476800*a^46*b^22*c^40*d^36 + 11304802150595851128286889
776640*a^47*b^21*c^39*d^37 - 5122036969218421637451979161600*a^48*b^20*c^38*
d^38 + 2036253098371190466921606676480*a^49*b^19*c^37*d^39 - 705764416322440
7321849364480*a^50*b^18*c^36*d^40 + 2115150337207545288311439360*a^51*b^1
```

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7*c^35*d^41 - 5422672476777259769580748800*a^52*b^16*c^34*d^42 + 1172540913
492414089228451840*a^53*b^15*c^33*d^43 - 2097906091126339769267650 56*a^54*b
^14*c^32*d^44 + 3023974021236969349054464 0*a^55*b^13*c^31*d^45 - 3375777980
998666504110080*a^56*b^12*c^30*d^46 + 2739812890627629121536 00*a^57*b^11*c^
29*d^47 - 1438877919738259832832 0*a^58*b^10*c^28*d^48 + 3671861846462714347
52*a^59*b^9*c^27*d^49)*1i)*(-(71993427968*a^35*d^35 - ((143986855936*a^35*d
^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 1067520161
21856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600
704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574
336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 6181280197011
0464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 9050274277
1167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384
137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 1
12491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d
^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^
17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913618173132
8*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 672751867
7746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 -
4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^1
0*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^
8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*
b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*
b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*
c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 -
 (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 2311218825
61500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850
*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 1115013057063627
1*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 164924138801096
92*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 59415727162429
75*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362
*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^
32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 3408
4860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 247115
2383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 6227
3040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 +
722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*
d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^
21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569
600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 388761
69296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^
16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b
^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 1551636581553537
0240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 443324
7375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23
+ 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*
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d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^
20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^
18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35
 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292
822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 37702
07453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 189172
10449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306
136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 +
40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^
13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^2
0*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b
^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686
080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 32996068
44220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 +
 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^
11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26
*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a
^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a
^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^3
2*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^3
4*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15
*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2
- 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^4
2*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a
^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d
^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 34737
3600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b
^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^
17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792
840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^1
0*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3
365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^2
0*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*
c^17*d^30)))^(1/4) + (x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 -
28925330217666430894080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20
*b^46*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 349820765
29826233401212928*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^
43*c^57*d^16 + 873231122236416493313064960*a^24*b^42*c^56*d^17 - 3201588318
340888739356606464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*
b^40*c^54*d^19 - 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 625280
04036875405150857986048*a^28*b^38*c^52*d^21 - 1331436807962154914744893440
0*a^29*b^37*c^51*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23
- 467106577738876991145070559232*a^31*b^35*c^49*d^24 + 7753210968231093026
74935250944*a^32*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*
c^47*d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*d^27 - 202814334
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5719314676074795761664*a^35*b^31*c^45*d^28 + 225790597310402395697230695628
8*a^36*b^30*c^44*d^29 - 22374491835658304355634941788l6*a^37*b^29*c^43*d^30
 + 19662048544574699183999884984 32*a^38*b^28*c^42*d^31 - 152764940604836662
1262568488960*a^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b^
26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25*c^39*d^34 + 3320657
64335584004230153764864*a^42*b^24*c^38*d^35 - 152543196968133650922715742 20
8*a^43*b^23*c^37*d^36 + 606991714334711017392989798 40*a^44*b^22*c^36*d^37 -
 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 6037825951797032255320
227840*a^46*b^20*c^34*d^39 - 1473449639082715479512449024*a^47*b^19*c^33*d^
40 + 296084339424033093684559872*a^48*b^18*c^32*d^41 - 477179504212543082 90
887680*a^49*b^17*c^31*d^42 + 5931528400797457427988480*a^50*b^16*c^30*d^43
 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 31006369751209579905024*a^
52*b^14*c^28*d^45 - 872067188534894657536*a^53*b^13*c^27*d^46) - (-(7199342
7968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 1361205
9983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 58564451028
1728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154
432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 3783442089954
5088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 826122724924
45696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 8070977
1031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 49
37158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15
 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c
^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^
19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440
389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827
091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^
24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9
*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*
b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30
*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b
^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*
d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 1584
0599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 700547824970
84*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 506542790471214
0*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 163162039580467
76*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437
688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 20942069290539
32*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^1
1*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d -
 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 34084
8604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838
453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 23
1299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8
 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c
^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a
```

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^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746
722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 +
41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c
^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*
a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751
734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1
927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^
24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^2
1*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c
^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^1
7*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 -
53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195
357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 94147
67089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906
400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45
251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12
 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*
d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19
*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a
^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 274066956809
0865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 336
3759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d
^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^
10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^
27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*
a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a
^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*
b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(6
8719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42
*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^1
5*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3
365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^2
3*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 +
 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*
a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c
^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 -
347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a
^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*
d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^
37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4
960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*((x^(1/2)*(5649315
3725735501824*a^22*b^52*c^81*d^4 - 239692380807763486310 4*a^23*b^51*c^80*d^
5 + 4938769849284350391091 2*a^24*b^50*c^79*d^6 - 6585983390561290871116 8*a
^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c^77*d^8 - 48113596
86765194506980556 8*a^27*b^47*c^76*d^9 + 29250225354463582364683468 8*a^28*b^
```

```
46*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^74*d^11 + 630600358
4409325504378699776*a^30*b^44*c^73*d^12 - 23152095046595175238512672768*a^3
1*b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^42*c^71*d^14 - 2067
84189076489114265239683072*a^33*b^41*c^70*d^15 + 51100101739077640657452820
0704*a^34*b^40*c^69*d^16 - 1120486424066161848521664233472*a^35*b^39*c^68*d
^17 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18 - 379488994942736
8142860254707712*a^37*b^37*c^66*d^19 + 5830470252063718134687996051456*a^38
*b^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^35*c^64*d^21 + 874
6184267385996582875203371008*a^40*b^34*c^63*d^22 - 717687192383519833852021
9385856*a^41*b^33*c^62*d^23 + 1365198057841590488549164056576*a^42*b^32*c^6
1*d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*d^25 - 28100654056
180096231365094146048*a^44*b^30*c^59*d^26 + 5128076428934856498399472621977
6*a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*a^46*b^28*c^57*d^2
8 + 99717561302809906738570708647936*a^47*b^27*c^56*d^29 - 1153805881767185
82142644189659136*a^48*b^26*c^55*d^30 + 120101545474959969242488481251328*a
^49*b^25*c^54*d^31 - 113052494905210552901304563269632*a^50*b^24*c^53*d^32
+ 96462689920395704646643948191744*a^51*b^23*c^52*d^33 - 746655194754189516
39228294889472*a^52*b^22*c^51*d^34 + 52413929319422085122116269637632*a^53*
b^21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^20*c^49*d^36 + 191
74031096138345851817803382784*a^55*b^19*c^48*d^37 - 99518274638933356977287
45766912*a^56*b^18*c^47*d^38 + 4646728550801039102656464814080*a^57*b^17*c^
46*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*d^40 + 72581098338
7725632884961181696*a^59*b^15*c^44*d^41 - 240265301732777409221605982208*a^
60*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^13*c^42*d^43 - 178
09629928199177184296828928*a^62*b^12*c^41*d^44 + 39071971858848696732840099
84*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64*b^10*c^39*d^46 +
113214808531319939527606272*a^65*b^9*c^38*d^47 - 14265899165032610449588224
*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*c^36*d^49 - 100502
833687558254231552*a^68*b^6*c^35*d^50 + 4689814464763011268608*a^69*b^5*c^3
4*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52) + (-(71993427968*a^35*d^
35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*
b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^3
1*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29
*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^2
7*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b
^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a
^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 49371585774551
04*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453
930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2
236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16
*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*
b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486
848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585
093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 +
639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28
```

```
 + 7408063667635814 4*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^3
0 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32
+ 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 42934262
49728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b
^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16
*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c
^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*
c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20
*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*
c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47
+ 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 21990232555
52*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a
^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736
*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 23129986308872
6016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 192749885
9072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8
866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^3
5*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^
25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877
070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 3
2396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^
28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^
32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072
716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 2312998
63088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 138
38453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 3
40848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/
2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060 9
28*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352
*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*
a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232
*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583
616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 271920687
29954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245
638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 +
 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^1
7*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20
*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 336375933887319
2448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 214688
3780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25
 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d
^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6
*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*
d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33
 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(
```

```
a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31
 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*
d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*
b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 +
 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a
^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^
33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 5
65722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^
30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^2
6*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518
300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*
d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3
*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(645636042579834306 56*a^25*b^5
1*c^84*d^4 - 2822351843277561397248*a^26*b^50*c^83*d^5 + 601271623082562834
92352*a^27*b^49*c^82*d^6 - 831948157724300777881600*a^28*b^48*c^81*d^7 + 84
06786558179361266073600*a^29*b^47*c^80*d^8 - 66144581305899203170402304*a^3
0*b^46*c^79*d^9 + 421912670310680329277407232*a^31*b^45*c^78*d^10 - 2243238
210521587022108295168*a^32*b^44*c^77*d^11 + 10145383251984825802817536000*a
^33*b^43*c^76*d^12 - 39641949193820336576213811200*a^34*b^42*c^75*d^13 + 13
5494098735043868075088674816*a^35*b^41*c^74*d^14 - 409284915889091539805067
542528*a^36*b^40*c^73*d^15 + 1102331957384293957070038761472*a^37*b^39*c^72
*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*d^17 + 5847343583817
169075816733081600*a^39*b^37*c^70*d^18 - 11684105629368324959904469090304*a
^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a^41*b^35*c^68*d^20 -
 36343020410925078321345140359168*a^42*b^34*c^67*d^21 + 5729758068768356103
0746426572800*a^43*b^33*c^66*d^22 - 84429658980390814235781758976000*a^44*b
^32*c^65*d^23 + 116702744788425677443098849837056*a^45*b^31*c^64*d^24 - 151
589903153597380791972919246848*a^46*b^30*c^63*d^25 + 185008444259789842943 6
56593457152*a^47*b^29*c^62*d^26 - 211756933815433796881181835264000*a^48*b^
28*c^61*d^27 + 226611959433847997212598992896000*a^49*b^27*c^60*d^28 - 2259
06031446565502788593732550656*a^50*b^26*c^59*d^29 + 20897862774916572443002
5514549248*a^51*b^25*c^58*d^30 - 178726416623100559749866797924352*a^52*b^2
4*c^57*d^31 + 140824510781547830729330235801600*a^53*b^23*c^56*d^32 - 10189
7270594764980154443340185600*a^54*b^22*c^55*d^33 + 674993223907194678510634
44373504*a^55*b^21*c^54*d^34 - 40809284384591153062742518136832*a^56*b^20*c
^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c^52*d^36 - 111950426
46819893251483369472000*a^58*b^18*c^51*d^37 + 5042898342903938117430096691 2
00*a^59*b^17*c^50*d^38 - 2042741359937286689202494242816*a^60*b^16*c^49*d^3
9 + 740249793404633986500581654528*a^61*b^15*c^48*d^40 - 238501265489031484
884985577472*a^62*b^14*c^47*d^41 + 67809805296929472355971891200*a^63*b^13*
c^46*d^42 - 16856343881283213574379929600*a^64*b^12*c^45*d^43 + 36211580663
96044540042543104*a^65*b^11*c^44*d^44 - 662272679138724025500434432*a^66*b^
10*c^43*d^45 + 101087832400064043724832768*a^67*b^9*c^42*d^46 - 12528855636
637836430540800*a^68*b^8*c^41*d^47 + 1211288155777568604160000*a^69*b^7*c^4
0*d^48 - 85697808358931542573056*a^70*b^6*c^39*d^49 + 394645031026923719884
```

```
8*a^71*b^5*c^38*d^50 - 887749558547272171152*a^72*b^4*c^37*d^51)*1i)*(-(7199
3427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 1361
2059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 58564451
0281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907
154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 3783442089
9545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 826122724
92445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 8070
9771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 +
 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d
^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^1
8*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160
*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688
440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5
827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11
*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*
b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^
28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a
^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^3
2*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^
34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 1
5840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 700547824
97084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 506542790471
2140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 163162039580
46776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441
437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 20942069290
53932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*
a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*
d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 34
0848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13
838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 -
 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*
d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^2
2*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 1551636581553537024
0*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807
746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15
 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^1
5*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 238713320239005696
00*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494
751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22
- 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23
*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*
c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^
4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*
c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2
 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1
```

```
195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 94
14767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30
906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 -
 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d
^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^
21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b
^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 111828572941803520
0*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 274066956
8090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 -
3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^1
2*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25
*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208
*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 370403183381790
72*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 185565302561587
2*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^
33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)
/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a
^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*
a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6
- 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*
b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^1
1 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 4714356
00*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^1
6*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18
 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 12902448
0*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^
24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192
*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28
- 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4)*1i - 1926091044
38451240960*a^18*b^50*c^68*d^8 + 708618067091178232176*a^19*b^49*c^67*d^9
- 1250744769136663776460800*a^20*b^48*c^66*d^10 + 1411152805506318336000000*
a^21*b^47*c^65*d^11 - 11440156274772600537743360*a^22*b^46*c^64*d^12 + 7101
9754904703755920343040*a^23*b^45*c^63*d^13 - 351320863723081970831327232*a^
24*b^44*c^62*d^14 + 1422781934731584726682828800*a^25*b^43*c^61*d^15 - 4808
764412319368968195276800*a^26*b^42*c^60*d^16 + 1375362821409609826802073600
0*a^27*b^41*c^59*d^17 - 33604586265646232007931330560*a^28*b^40*c^58*d^18 +
 70459004145207625658058932224*a^29*b^39*c^57*d^19 - 1263359248135526588939
34428160*a^30*b^38*c^56*d^20 + 189714420765957587531118673920*a^31*b^37*c^5
5*d^21 - 221947274468283773140074496000*a^32*b^36*c^54*d^22 + 1428707403433
18834154286612480*a^33*b^35*c^53*d^23 + 176083118177526399618307325952*a^34
*b^34*c^52*d^24 - 895947027393848326392014438400*a^35*b^33*c^51*d^25 + 2154
323340999822995276326502400*a^36*b^32*c^50*d^26 - 3969865332339043373838394
982400*a^37*b^31*c^49*d^27 + 6147644263312111317325499596800*a^38*b^30*c^48
*d^28 - 8260762337957580186371563192320*a^39*b^29*c^47*d^29 + 9765601087086
458087650885632000*a^40*b^28*c^46*d^30 - 10223506948306413182866214092800*a
```

```
^41*b^27*c^45*d^31 + 9508424738292483984119247667200*a^42*b^26*c^44*d^32 -
7866898628254591634401331773440*a^43*b^25*c^43*d^33 + 579072473884148806641
1751276544*a^44*b^24*c^42*d^34 - 3789006704063625484256485048320*a^45*b^23*
c^41*d^35 + 2199996205919117948922678476800*a^46*b^22*c^40*d^36 - 113048021
5059585112828689776640*a^47*b^21*c^39*d^37 + 512203696921842163745197916160
*a^48*b^20*c^38*d^38 - 203625309837119046692160667648*a^49*b^19*c^37*d^39 +
 70576441632244073218493644800*a^50*b^18*c^36*d^40 - 2115150337207545288311
4393600*a^51*b^17*c^35*d^41 + 5422672476777259769580748800*a^52*b^16*c^34*d
^42 - 1172540913492414089228451840*a^53*b^15*c^33*d^43 + 209790609112633976
926765056*a^54*b^14*c^32*d^44 - 30239740212369693490544640*a^55*b^13*c^31*d
^45 + 3375777980998666504110080*a^56*b^12*c^30*d^46 - 273981289062762912153
600*a^57*b^11*c^29*d^47 + 14388779197382598328320*a^58*b^10*c^28*d^48 - 367
186184646271434752*a^59*b^9*c^27*d^49)*1i)*(-(71993427968*a^35*d^35 - ((143
986855936*a^35*d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*
d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4
 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6
- 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8
 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d
^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c
^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^
21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*
a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 22365714588
36070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 54
81339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*
d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b
^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 26895850936374722
56*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 6393294975
16732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 7408063
6676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 371130
6051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295
914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34
*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d
^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 +
1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 +
11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 +
 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18
+ 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 +
 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476
736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*
c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c
^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27
*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*
b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800
*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 88664947517
34497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2
```

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3871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^
33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a
^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746
722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 +
15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^
26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^3
4*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 23129986308872601
6*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 1383845334718
8736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 34084860461
0560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141
047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^3
2*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*
c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c
^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*
c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b
^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a
^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 562456380227624
96*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615
798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1
916425404928686080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15
*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*
b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905
472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 7140227
39833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 1196
92955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 93
13041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282
146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 5409309
98272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*
c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^1
3*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 2013
76*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*
d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*
a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c
^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 -
565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a
^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^
28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 6
4512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b
^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201
376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29
 + 496*a^41*b^2*c^17*d^30)))^(1/4))/((x^(1/2)*(857712418202478182400*a^18*b
^48*c^62*d^11 - 28925330217666430894080*a^19*b^47*c^61*d^12 + 4658083558685
44602210304*a^20*b^46*c^60*d^13 - 4772189938359453553262592*a^21*b^45*c^59*
d^14 + 34982076529826233401212928*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^16 + 873231122236416493313064960*a^24*b^42*c^56*d
```

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^17 - 32015883183408887393566064 64*a^25*b^41*c^55*d^18 + 990486698154736272
5832687616*a^26*b^40*c^54*d^19 - 26475613142538536817178705920*a^27*b^39*c^
53*d^20 + 62528004036875405150857986048*a^28*b^38*c^52*d^21 - 1331436807962
15491474489344000*a^29*b^37*c^51*d^22 + 25959547498283516471340013977 6*a^30
*b^36*c^50*d^23 - 46710657773887699114507055923 2*a^31*b^35*c^49*d^24 + 7753
210968231093026749352509 44*a^32*b^34*c^48*d^25 - 1179424943892680059222782 6
40128*a^33*b^33*c^47*d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*
d^27 - 2028143345719314676074795761664*a^35*b^31*c^45*d^28 + 22579059731040
23956972306956288*a^36*b^30*c^44*d^29 - 2237449183565830435563494178816*a^3
7*b^29*c^43*d^30 + 1966204854457469918399988498432*a^38*b^28*c^42*d^31 - 15
27649406048366621262568488960*a^39*b^27*c^41*d^32 + 104640945875852234799 51
26562816*a^40*b^26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25*c^3
9*d^34 + 332065764335584004230153764864*a^42*b^24*c^38*d^35 - 1525431969681
33650922715742208*a^43*b^23*c^37*d^36 + 60699171433471101739298979840*a^44*
b^22*c^36*d^37 - 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 603782
5951797032255320227840*a^46*b^20*c^34*d^39 - 1473449639082715479512449024*a
^47*b^19*c^33*d^40 + 296084339424033093684559872*a^48*b^18*c^32*d^41 - 4771
7950421254308290887680*a^49*b^17*c^31*d^42 + 5931528400797457427988480*a^50
*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^15*c^29*d^44 + 3100636975
1209579905024*a^52*b^14*c^28*d^45 - 872067188534894657536*a^53*b^13*c^27*d^
46) - (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 4028209561 6*b^3
5*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d
^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5
 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7
 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^
9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^2
4*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^2
2*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^1
5*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 107444323159613
4400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383285
0809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20
 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*
c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a
^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142804547966
6450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 23938591
1340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 186260
82598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292
849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996
544*a*b^34*c^34*d - 429342624972 8*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*
b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^1
2 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 -
 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 -
 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17
- 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19
- 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*
```

(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^
12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*
c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*
c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^
26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^
19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 443324737586724
8640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155163
65815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^
13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b
^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 3887616929606664
1920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871
332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d
^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^
10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*
a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349
312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 24711523834
26560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461
056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*
b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31
*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*
c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*
c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^2
5*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^1
2*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552
*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965
452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11182
85729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^1
9 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14
*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*
a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925468
18736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966
4748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37
040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18
55653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3077
7147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a
^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*
c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44
*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b
^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 2
8048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*
b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d
^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 60108
0390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b
^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^
20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 2804880

```
0*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*
d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b
^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*((
x^(1/2)*(56493153725735501824*a^22*b^52*c^81*d^4 - 2396923808077634863104*a
^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b^50*c^79*d^6 - 6585983390
56129087111168*a^25*b^49*c^78*d^7 + 6391163867634330475954176*a^26*b^48*c^7
7*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 + 292502253544635823б
46834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524864*a^29*b^45*c^74*
d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12 - 23152095046595175
238512672768*a^31*b^43*c^72*d^13 + 73885584363642186267654881280*a^32*b^42*
c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^70*d^15 + 5110010173
90776406574528200704*a^34*b^40*c^69*d^16 - 1120486424066161848521664233472*
a^35*b^39*c^68*d^17 + 2186183732842431973240904613888*a^36*b^38*c^67*d^18 -
 3794889949427368142860254707712*a^37*b^37*c^66*d^19 + 58304702520637181346
87996051456*a^38*b^36*c^65*d^20 - 7807619033603590530479469625344*a^39*b^35
*c^64*d^21 + 8746184267385996582875203371008*a^40*b^34*c^63*d^22 - 71768719
23835198338520219385856*a^41*b^33*c^62*d^23 + 136519805784159048854916405б5
76*a^42*b^32*c^61*d^24 + 10199723921158867878218460823552*a^43*b^31*c^60*d^
25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^26 + 512807642893485
64983994726219776*a^45*b^29*c^58*d^27 - 76696476979720874342700527124480*a^
46*b^28*c^57*d^28 + 99717561302809906738570708647936*a^47*b^27*c^56*d^29 -
115380588176718582142644189659136*a^48*b^26*c^55*d^30 + 1201015454749599692
42488481251328*a^49*b^25*c^54*d^31 - 113052494905210552901304563269632*a^50
*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*b^23*c^52*d^33 - 74
665519475418951639228294889472*a^52*b^22*c^51*d^34 + 5241392931942208512211
6269637632*a^53*b^21*c^50*d^35 - 33334185869182979296764484386816*a^54*b^20
*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*c^48*d^37 - 9951827
463893335697728745766912*a^56*b^18*c^47*d^38 + 4646728550801039102656464814
080*a^57*b^17*c^46*d^39 - 1944469658660080242790338920448*a^58*b^16*c^45*d^
40 + 725810983387725632884961181696*a^59*b^15*c^44*d^41 - 24026530173277740
9221605982208*a^60*b^14*c^43*d^42 + 70028310560132415015125778432*a^61*b^13
*c^42*d^43 - 17809629928199177184296828928*a^62*b^12*c^41*d^44 + 3907197185
884869673284009984*a^63*b^11*c^40*d^45 - 728569061655967140126130176*a^64*b
^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^38*d^47 - 1426589916
5032610449588224*a^66*b^8*c^37*d^48 + 1400509163935752188329984*a^67*b^7*c^
36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 + 4689814464763011268
608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*c^33*d^52) - (-(719
93427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 136
12059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 5856445
10281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 754041490
7154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 378344208
99545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272
492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 807
09771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13
+ 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*
```

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d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^
18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383285080985737216
0*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659921368
8440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 +
5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^1
1*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26
*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a
^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*
a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^
32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c
^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 +
15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782
497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 50654279047
12140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958
046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 1176083944
1437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929
053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736
*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46
*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 3
40848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 1
3838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6
- 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39
*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^
22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155163658155353702
40*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 3239680
7746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^1
5 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^
15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569
600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 886649
4751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22
 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^2
3*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6
*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b
^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2
*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^
2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 -
1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 9
414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 3
0906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10
- 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*
d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c
^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*
b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27406695
68090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 -
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 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^
12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^2
5*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966474875836620
8*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179
072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18556530256158
72*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a
^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34
)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*
a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960
*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6
 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20
*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^
11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435
600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^
16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^1
8 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 1290244
80*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c
^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 90619
2*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28
 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*(6456360425798
3430656*a^25*b^51*c^84*d^4 - 2822351843277561397248*a^26*b^50*c^83*d^5 + 60
127162308256283492352*a^27*b^49*c^82*d^6 - 831948157724300777881600*a^28*b^
48*c^81*d^7 + 8406786558179361266073600*a^29*b^47*c^80*d^8 - 66144581305899
203170402304*a^30*b^46*c^79*d^9 + 421912670310680329277407232*a^31*b^45*c^7
8*d^10 - 2243238210521587022108295168*a^32*b^44*c^77*d^11 + 101453832519848
25802817536000*a^33*b^43*c^76*d^12 - 39641949193820336576213811200*a^34*b^4
2*c^75*d^13 + 135494098735043868075088674816*a^35*b^41*c^74*d^14 - 40928491
5889091539805067542528*a^36*b^40*c^73*d^15 + 110233195738429395707003876147
2*a^37*b^39*c^72*d^16 - 2668223165968086459433038643200*a^38*b^38*c^71*d^17
 + 5847343583817169075816733081600*a^39*b^37*c^70*d^18 - 116841056293683249
59904469090304*a^40*b^36*c^69*d^19 + 21435002462698637041098955948032*a^41*
b^35*c^68*d^20 - 36343020410925078321345140359168*a^42*b^34*c^67*d^21 + 572
97580687683561030746426572800*a^43*b^33*c^66*d^22 - 844296589803908142357781
758976000*a^44*b^32*c^65*d^23 + 116702744788425677443098849837056*a^45*b^31
*c^64*d^24 - 151589903153597380791972919246848*a^46*b^30*c^63*d^25 + 185008
444259789842943656593457152*a^47*b^29*c^62*d^26 - 2117569338154337968811818
35264000*a^48*b^28*c^61*d^27 + 226611959433847997212598992896000*a^49*b^27*
c^60*d^28 - 225906031446565502788593732550656*a^50*b^26*c^59*d^29 + 2089786
27749165724430025514549248*a^51*b^25*c^58*d^30 - 17872641662310055974986679
7924352*a^52*b^24*c^57*d^31 + 140824510781547830729330235801600*a^53*b^23*c
^56*d^32 - 101897270594764980154443340185600*a^54*b^22*c^55*d^33 + 67499322
390719467851063444373504*a^55*b^21*c^54*d^34 - 4080928438459115306274251813
6832*a^56*b^20*c^53*d^35 + 22447282431345050697947118829568*a^57*b^19*c^52*
d^36 - 11195042646819893251483369472000*a^58*b^18*c^51*d^37 + 5042898342903
938117430096691200*a^59*b^17*c^50*d^38 - 2042741359937286689202494242816*a^
```

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60*b^16*c^49*d^39 + 740249793404633986500581654528*a^61*b^15*c^48*d^40 - 23
85012654890314848849855774 72*a^62*b^14*c^47*d^41 + 678098052969294723559718
91200*a^63*b^13*c^46*d^42 - 168563438812832135743799296 00*a^64*b^12*c^45*d^
43 + 362115806639604454004254 3104*a^65*b^11*c^44*d^44 - 66227267913872402 55
00434432*a^66*b^10*c^43*d^45 + 1010878324000640437248327 68*a^67*b^9*c^42*d^
46 - 125288556366378364305408 00*a^68*b^8*c^41*d^47 + 121128815577756860416 0
000*a^69*b^7*c^40*d^48 - 856978083589315425730 56*a^70*b^6*c^39*d^49 + 39464
503102692371988 48*a^71*b^5*c^38*d^50 - 887749558547272171 52*a^72*b^4*c^37*d
^51)*1i)*(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*
b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^3
2*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*
d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*
d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26
*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*
c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*
b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*
a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 107444323159
6134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383
2850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d
^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^
13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 429376756114581094
4*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142804547
9666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 23938
5911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 186
26082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564
292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861
996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^
12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*
d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^1
3 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^1
5 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^
17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^
19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^2
1)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552
*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^
30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^
28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17
*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800
*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 443324737586
7248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155
16365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34
*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^2
6*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 3887616929606
6641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23
871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^2
```

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7*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33
*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 7228120721522688
00*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062
349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 24711523
83426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860
461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a
^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b
^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^
29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^
27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*
b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*
a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727
552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726
965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11
18285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*
d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b
^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 291354577027274 34
24*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925
46818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31
9664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 +
 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 +
 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3
0777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 214671312486
4*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^
31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c
^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^1
7*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8
- 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^
22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^3
4*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 60
1080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^2
9*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27
*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 2804
8800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^
22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^3
9*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(3/4)
*1i + 19260910443845124096 0*a^18*b^50*c^68*d^8 - 70861806709117823221 76*a^1
9*b^49*c^67*d^9 + 125074476913666377646080*a^20*b^48*c^66*d^10 - 1411152805
506318336000000*a^21*b^47*c^65*d^11 + 11440156274772600537743360*a^22*b^46*
c^64*d^12 - 71019754904703755920343040*a^23*b^45*c^63*d^13 + 35132086372308
1970831327232*a^24*b^44*c^62*d^14 - 1422781934731584726682828800*a^25*b^43*
c^61*d^15 + 4808764412319368968195276800*a^26*b^42*c^60*d^16 - 137536282140
96098268020736000*a^27*b^41*c^59*d^17 + 33604586265646232007931330560*a^28*
b^40*c^58*d^18 - 70459004145207625658058932224*a^29*b^39*c^57*d^19 + 126335
92481355265889393442816 0*a^30*b^38*c^56*d^20 - 1897144207659575875311186739
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20*a^31*b^37*c^55*d^21 + 221947274468283773140074496000*a^32*b^36*c^54*d^22
 - 142870740343318834154286612480*a^33*b^35*c^53*d^23 - 1760831181775263996
18307325952*a^34*b^34*c^52*d^24 + 895947027393848326392014438400*a^35*b^33*
c^51*d^25 - 2154323340999822995276326502400*a^36*b^32*c^50*d^26 + 396986533
2339043373838394982400*a^37*b^31*c^49*d^27 - 614764426331211131732549959680
0*a^38*b^30*c^48*d^28 + 8260762337957580186371563192320*a^39*b^29*c^47*d^29
 - 9765601087086458087650885632000*a^40*b^28*c^46*d^30 + 102235069483064131
82866214092800*a^41*b^27*c^45*d^31 - 9508424738292483984119247667200*a^42*b
^26*c^44*d^32 + 7866898628254591634401331773440*a^43*b^25*c^43*d^33 - 57907
24738841488066411751276544*a^44*b^24*c^42*d^34 + 37890067040636254842564850
48320*a^45*b^23*c^41*d^35 - 2199996205919117948922678476800*a^46*b^22*c^40*
d^36 + 1130480215059585112828689776640*a^47*b^21*c^39*d^37 - 51220369692184
2163745197916160*a^48*b^20*c^38*d^38 + 203625309837119046692160667648*a^49*
b^19*c^37*d^39 - 70576441632244073218493644800*a^50*b^18*c^36*d^40 + 211515
03372075452883114393600*a^51*b^17*c^35*d^41 - 5422672476777259769580748800*
a^52*b^16*c^34*d^42 + 1172540913492414089228451840*a^53*b^15*c^33*d^43 - 20
9790609112633976926765056*a^54*b^14*c^32*d^44 + 30239740212369693490544640*
a^55*b^13*c^31*d^45 - 3375777980998666504110080*a^56*b^12*c^30*d^46 + 27398
1289062762912153600*a^57*b^11*c^29*d^47 - 14388779197382598328320*a^58*b^10
*c^28*d^48 + 367186184646271434752*a^59*b^9*c^27*d^49)*1i)*(-(71993427968*a
^35*d^35 - ((143986855936*a^35*d^35 + 40282095616*b^35*c^35 + 1361205998387
2*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 585644510281728*a
^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 7540414907154432*a^
6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 37834420899545088*a
^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82612272492445696*
a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11 + 8070977103190
4256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22*d^13 + 49371585
77455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20*c^20*d^15 - 413
241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a^17*b^18*c^18*d^
17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 3832850809857372160*a^19*b^1
6*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 6599213688440389632
*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d^22 + 5827091540
545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^11*c^11*d^24 + 2
689585093637472256*a^25*b^10*c^10*d^25 - 1428045479666450432*a^26*b^9*c^9*d
^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^
7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c
^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3
*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 42
93426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d^23 + 1584059900
0625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70054782497084*a^1
1*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*
b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316203958046776*a^5
*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 11760839441437688*a^
7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9
*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32
```

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*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^31*c^46*d - 21990
23255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d^2 - 34084860461
0560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d^4 - 13838453347
188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^41*d^6 - 23129986
3088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^24*c^39*d^8 - 192
7498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a^21*b^22*c^37*d^
10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815535370240*a^23*b^
20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 32396807746722201
600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^32*d^15 + 413059
29877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a^28*b^15*c^30*d^
17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023900569600*a^30*b
^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 - 8866494751734497
280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^25*d^22 - 1927498
859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b^8*c^23*d^24 - 2
31299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^37*b^6*c^21*d^26
 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^
28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30
))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c^33*d^2 - 533760
08060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*d^4 - 1195357715
300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d^6 - 94147670892
87168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d^8 - 30906400985
055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25*d^10 - 45251371
385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23*c^23*d^12 - 271
92068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*b^21*c^21*d^14 +
 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800*a^16*b^19*c^19*
d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 1118285729418035200*a^18*b^
17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 274066956809086566
4*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*d^21 - 336375933
8873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b^12*c^12*d^23 -
2146883780572905472*a^24*b^11*c^11*d^24 + 1344792546818736128*a^25*b^10*c^1
0*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 319664748758366208*a^27*b^8
*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b
^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^
4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^
2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*c*d^34)/(6871947
6736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^1
6*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28
*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856
*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38
*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c^36*d^11 + 22579
2840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 + 471435600*a^25*b
^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^
16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^29*d^18 - 347373
600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^
11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 +
```

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 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6
*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^
40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*1i - (x^(1/2)*(857712418
202478182400*a^18*b^48*c^62*d^11 - 28925330217666430894080*a^19*b^47*c^61*d
^12 + 465808355868544602210304*a^20*b^46*c^60*d^13 - 4772189938359453553262
592*a^21*b^45*c^59*d^14 + 34982076529826233401212928*a^22*b^44*c^58*d^15 -
195811106815542077297786880*a^23*b^43*c^57*d^16 + 873231122236416493313064
960*a^24*b^42*c^56*d^17 - 3201588318340888739356606464*a^25*b^41*c^55*d^18 +
 9904866981547362725832687616*a^26*b^40*c^54*d^19 - 26475613142538536817178
705920*a^27*b^39*c^53*d^20 + 62528004036875405150857986048*a^28*b^38*c^52*d
^21 - 133143680796215491474489344000*a^29*b^37*c^51*d^22 + 2595954749828351
64713400139776*a^30*b^36*c^50*d^23 - 467106577738876991145070559232*a^31*b^
35*c^49*d^24 + 775321096823109302674935250944*a^32*b^34*c^48*d^25 - 1179424
943892680059222782640128*a^33*b^33*c^47*d^26 + 162969059360009583382329556
920*a^34*b^32*c^46*d^27 - 2028143345719314676074795761664*a^35*b^31*c^45*d^
28 + 2257905973104023956972306956288*a^36*b^30*c^44*d^29 - 2237449183565830
435563494178816*a^37*b^29*c^43*d^30 + 1966204854457469918399988498432*a^38*
b^28*c^42*d^31 - 1527649406048366621262568488960*a^39*b^27*c^41*d^32 + 1046
409458758522347995126562816*a^40*b^26*c^40*d^33 - 629956523592774331698776
113152*a^41*b^25*c^39*d^34 + 332065764335584004230153764864*a^42*b^24*c^38*d
^35 - 152543196968133650922715742208*a^43*b^23*c^37*d^36 + 6069917143347110
1739298979840*a^44*b^22*c^36*d^37 - 20757436699772395749793333248*a^45*b^21
*c^35*d^38 + 6037825951797032255320227840*a^46*b^20*c^34*d^39 - 14734496390
82715479512449024*a^47*b^19*c^33*d^40 + 296084339424033093684559872*a^48*b^
18*c^32*d^41 - 47717950421254308290887680*a^49*b^17*c^31*d^42 + 59315284007
97457427988480*a^50*b^16*c^30*d^43 - 534037861185724002336768*a^51*b^15*c^2
9*d^44 + 31006369751209579905024*a^52*b^14*c^28*d^45 - 872067188534894657536
6*a^53*b^13*c^27*d^46) - (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^3
5 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121
856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 239071543060070
4*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1882953417857433
6*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 618128019701104
64*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 905027427711
67232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 5438413
7459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112
491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^1
6 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17
*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*
a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 67275186777
46384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42
93767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*
d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*
c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^
6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^
4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^
```

```
2*d^33 - 1081861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (
4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561
500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a
^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*
a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692
*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975
*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a
^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32
 - 2199023255552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 340848
60461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 24711523
83426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 622730
40062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72
2812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^
9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21
*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387133202390056960
0*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169
296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16
 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^1
4*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155163658155353702
40*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 44332473
75867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 +
722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^
25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20
*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18
*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 +
 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29282
2255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207
453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210
449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130613
6246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40
354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13
 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*
d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^1
8*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080
*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844
220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2
913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11
*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b
^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^2
8*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^3
0*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*
b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*
d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d
^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 -
4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*
```

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d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^1
9*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^1
0 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 3473736
00*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^1
7*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17
 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 22579284
0*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*
c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 336
5856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*
d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^
17*d^30)))^(1/4)*((x^(1/2)*(56493153725735501824*a^22*b^52*c^81*d^4 - 23969
23808077634863104*a^23*b^51*c^80*d^5 + 49387698492843503910912*a^24*b^50*c^
79*d^6 - 658598339056129087111168*a^25*b^49*c^78*d^7 + 63911638676343304759
54176*a^26*b^48*c^77*d^8 - 48113596867651945069805568*a^27*b^47*c^76*d^9 +
292502253544635823646834688*a^28*b^46*c^75*d^10 - 1476002645480415917311524
864*a^29*b^45*c^74*d^11 + 6306003584409325504378699776*a^30*b^44*c^73*d^12
- 23152095046595175238512672768*a^31*b^43*c^72*d^13 + 738855843636421862676
54881280*a^32*b^42*c^71*d^14 - 206784189076489114265239683072*a^33*b^41*c^7
0*d^15 + 511001017390776406574528200704*a^34*b^40*c^69*d^16 - 1120486424066
161848521664233472*a^35*b^39*c^68*d^17 + 2186183732842431973240904613888*a^
36*b^38*c^67*d^18 - 3794889949427368142860254707712*a^37*b^37*c^66*d^19 + 5
830470252063718134687996051456*a^38*b^36*c^65*d^20 - 7807619033603590530479
469625344*a^39*b^35*c^64*d^21 + 8746184267385996582875203371008*a^40*b^34*c
^63*d^22 - 7176871923835198338520219385856*a^41*b^33*c^62*d^23 + 1365198057
841590488549164056576*a^42*b^32*c^61*d^24 + 1019972392115886787821846082355
2*a^43*b^31*c^60*d^25 - 28100654056180096231365094146048*a^44*b^30*c^59*d^2
6 + 51280764289348564983994726219776*a^45*b^29*c^58*d^27 - 7669647697972087
4342700527124480*a^46*b^28*c^57*d^28 + 99717561302809906738570708647936*a^4
7*b^27*c^56*d^29 - 115380588176718582142644189659136*a^48*b^26*c^55*d^30 +
120101545474959969242488481251328*a^49*b^25*c^54*d^31 - 1130524949052105529
01304563269632*a^50*b^24*c^53*d^32 + 96462689920395704646643948191744*a^51*
b^23*c^52*d^33 - 74665519475418951639228294889472*a^52*b^22*c^51*d^34 + 524
13929319422085122116269637632*a^53*b^21*c^50*d^35 - 33334185869182979296764
484386816*a^54*b^20*c^49*d^36 + 19174031096138345851817803382784*a^55*b^19*
c^48*d^37 - 9951827463893335697728745766912*a^56*b^18*c^47*d^38 + 464672855
0801039102656464814080*a^57*b^17*c^46*d^39 - 194446965866008024279033892044
8*a^58*b^16*c^45*d^40 + 725810983387725632884961181696*a^59*b^15*c^44*d^41
- 240265301732777409221605982208*a^60*b^14*c^43*d^42 + 70028310560132415015
125778432*a^61*b^13*c^42*d^43 - 17809629928199177184296828928*a^62*b^12*c^4
1*d^44 + 3907197185884869673284009984*a^63*b^11*c^40*d^45 - 728569061655967
140126130176*a^64*b^10*c^39*d^46 + 113214808531319939527606272*a^65*b^9*c^3
8*d^47 - 14265899165032610449588224*a^66*b^8*c^37*d^48 + 140050916393575218
8329984*a^67*b^7*c^36*d^49 - 100502833687558254231552*a^68*b^6*c^35*d^50 +
4689814464763011268608*a^69*b^5*c^34*d^51 - 106807368762718683136*a^70*b^4*
c^33*d^52) + (-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 40282095
```

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616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32
*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c
^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c
^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*
c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b
^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a
^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524
992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 10744432
31596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 +
 3832850809857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^
15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^2
2*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 42937675611458
10944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 14280
45479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 2
39385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 -
 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 -
 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 108
1861996544*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (458117945616
1*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c
^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10
*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8
*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^
6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^
4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2
*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 219902325
5552*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^1
3*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^1
5*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*
a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 72281207215226
8800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 44332473
75867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 +
 15516365815535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*
c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920
*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692
96066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18
- 23871332023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12
*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*
a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152
268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 6227304
0062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471
152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 3408
4860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 68060299919
36*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a
^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^
6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^
```

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8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a
^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952
128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 246857928
8727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 20662
0726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17
- 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c
^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^
21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272
743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 1344
792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26
+ 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^
28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^
30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32
 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 21467131
24864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^1
2*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^
29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192
*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*
d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 12902448
0*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19
*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15
+ 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600
*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*
c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 -
28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^
7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960
*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(
1/4)*(64563604257983430656*a^25*b^51*c^84*d^4 - 2822351843277561397248*a^26
*b^50*c^83*d^5 + 60127162308256283492352*a^27*b^49*c^82*d^6 - 8319481577243
00777881600*a^28*b^48*c^81*d^7 + 8406786558179361266073600*a^29*b^47*c^80*d
^8 - 66144581305899203170402304*a^30*b^46*c^79*d^9 + 4219126703106803292774
07232*a^31*b^45*c^78*d^10 - 2243238210521587022108295168*a^32*b^44*c^77*d^1
1 + 10145383251984825802817536000*a^33*b^43*c^76*d^12 - 3964194919382033657
6213811200*a^34*b^42*c^75*d^13 + 135494098735043868075088674816*a^35*b^41*c
^74*d^14 - 409284915889091539805067542528*a^36*b^40*c^73*d^15 + 11023319573
84293957070038761472*a^37*b^39*c^72*d^16 - 2668223165968086459433038643200*
a^38*b^38*c^71*d^17 + 5847343583817169075816733081600*a^39*b^37*c^70*d^18 -
 11684105629368324959904469090304*a^40*b^36*c^69*d^19 + 2143500246269863704
1098955948032*a^41*b^35*c^68*d^20 - 36343020410925078321345140359168*a^42*b
^34*c^67*d^21 + 57297580687683561030746426572800*a^43*b^33*c^66*d^22 - 8442
9658980390814235781758976000*a^44*b^32*c^65*d^23 + 116702744788425677443098
849837056*a^45*b^31*c^64*d^24 - 151589903153597380791972919246848*a^46*b^30
*c^63*d^25 + 185008444259789842943656593457152*a^47*b^29*c^62*d^26 - 211756
933815433796881181835264000*a^48*b^28*c^61*d^27 + 2266119594338479972125989
92896000*a^49*b^27*c^60*d^28 - 225906031446565502788593732550656*a^50*b^26*
```

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c^59*d^29 + 208978627749165724430025514549248*a^51*b^25*c^58*d^30 - 1787264
16623100559749866797924352*a^52*b^24*c^57*d^31 + 14082451078154783072933023
5801600*a^53*b^23*c^56*d^32 - 101897270594764980154443340185600*a^54*b^22*c
^55*d^33 + 67499322390719467851063444373504*a^55*b^21*c^54*d^34 - 408092843
84591153062742518136832*a^56*b^20*c^53*d^35 + 224472824313450506979471188829
568*a^57*b^19*c^52*d^36 - 1119504264681989325148336947200 0*a^58*b^18*c^51*d
^37 + 5042898342903938117430096691200*a^59*b^17*c^50*d^38 - 204274135993728
6689202494242816*a^60*b^16*c^49*d^39 + 740249793404633986500581654528*a^61*
b^15*c^48*d^40 - 238501265489031484884985577472*a^62*b^14*c^47*d^41 + 67809
805296929472355971891200*a^63*b^13*c^46*d^42 - 168563438812832135743799296 0
0*a^64*b^12*c^45*d^43 + 3621158066396044540042543104*a^65*b^11*c^44*d^44 -
662272679138724025500434432*a^66*b^10*c^43*d^45 + 101087832400064043724832 7
68*a^67*b^9*c^42*d^46 - 12528855636637836430540800*a^68*b^8*c^41*d^47 + 121
1288155777568604160000*a^69*b^7*c^40*d^48 - 85697808358931542573056*a^70*b^
6*c^39*d^49 + 3946450310269237198848*a^71*b^5*c^38*d^50 - 88774955854727217
152*a^72*b^4*c^37*d^51)*1i)*(-(71993427968*a^35*d^35 - ((143986855936*a^35*
d^35 + 40282095616*b^35*c^35 + 13612059983872*a^2*b^33*c^33*d^2 - 106752016
121856*a^3*b^32*c^32*d^3 + 585644510281728*a^4*b^31*c^31*d^4 - 239071543060
0704*a^5*b^30*c^30*d^5 + 7540414907154432*a^6*b^29*c^29*d^6 - 1882953417857
4336*a^7*b^28*c^28*d^7 + 37834420899545088*a^8*b^27*c^27*d^8 - 618128019701
10464*a^9*b^26*c^26*d^9 + 82612272492445696*a^10*b^25*c^25*d^10 - 905027427
71167232*a^11*b^24*c^24*d^11 + 80709771031904256*a^12*b^23*c^23*d^12 - 5438
4137459908608*a^13*b^22*c^22*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 +
112491276045524992*a^15*b^20*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*
d^16 + 1074443231596134400*a^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b
^17*c^17*d^18 + 3832850809857372160*a^19*b^16*c^16*d^19 - 548133913618173 13
28*a^20*b^15*c^15*d^20 + 6599213688440389632*a^21*b^14*c^14*d^21 - 67275186
77746384896*a^22*b^13*c^13*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 -
 4293767561145810944*a^24*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^
10*d^25 - 1428045479666450432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b
^8*c^8*d^27 - 239385911340269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29
*b^6*c^6*d^29 - 18626082598846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31
*b^4*c^4*d^31 - 564292849139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2
*c^2*d^33 - 1081861996544*a*b^34*c^34*d - 429342624972 8*a^34*b*c*d^34)^2/4
- (4581179456161*a^12*b^15*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882
561500*a*b^26*c^11*d^12 - 70054782497084*a^11*b^16*c*d^22 + 144220390473285
0*a^2*b^25*c^10*d^13 - 5065427904712140*a^3*b^24*c^9*d^14 + 111501305706362
71*a^4*b^23*c^8*d^15 - 16316203958046776*a^5*b^22*c^7*d^16 + 16492413880109
692*a^6*b^21*c^6*d^17 - 11760839441437688*a^7*b^20*c^5*d^18 + 5941572716242
975*a^8*b^19*c^4*d^19 - 2094206929053932*a^9*b^18*c^3*d^20 + 49287325315736
2*a^10*b^17*c^2*d^21)*(68719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d
^32 - 219902325555 2*a^12*b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 340
84860461056*a^13*b^30*c^45*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 24711
52383426560*a^15*b^28*c^43*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 622
73040062349312*a^17*b^26*c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 +
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 722812072152268800*a^19*b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38
*d^9 + 4433247375867248640*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b
^21*c^36*d^11 + 15516365815535370240*a^23*b^20*c^35*d^12 - 2387133202390056
9600*a^24*b^19*c^34*d^13 + 32396807746722201600*a^25*b^18*c^33*d^14 - 38876
169296066641920*a^26*b^17*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d
^16 - 38876169296066641920*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*
b^14*c^29*d^18 - 23871332023900569600*a^30*b^13*c^28*d^19 + 155163658155353
70240*a^31*b^12*c^27*d^20 - 8866494751734497280*a^32*b^11*c^26*d^21 + 44332
47375867248640*a^33*b^10*c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23
 + 722812072152268800*a^35*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22
*d^25 + 62273040062349312*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c
^20*d^27 + 2471152383426560*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c
^18*d^29 + 34084860461056*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^3
5 + 6806029991936*a^2*b^33*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 29
2822255140864*a^4*b^31*c^31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770
207453577216*a^6*b^29*c^29*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917
210449772544*a^8*b^27*c^27*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 4130
6136246222848*a^10*b^25*c^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 +
 40354885515952128*a^12*b^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d
^13 + 2468579288727552*a^14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^
20*d^15 - 206620726965452800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*
b^18*c^18*d^17 - 1118285729418035200*a^18*b^17*c^17*d^18 + 1916425404928686
080*a^19*b^16*c^16*d^19 - 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606
844220194816*a^21*b^14*c^14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22
+ 2913545770272743424*a^23*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c
^11*d^24 + 1344792546818736128*a^25*b^10*c^10*d^25 - 714022739833225216*a^2
6*b^9*c^9*d^26 + 319664748758366208*a^27*b^8*c^8*d^27 - 119692955670134784*
a^28*b^7*c^7*d^28 + 37040318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*
a^30*b^5*c^5*d^30 + 1855653025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^
32*b^3*c^3*d^32 + 30777147957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^
34*d - 2146713124864*a^34*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^1
5*d^32 - 32*a^12*b^31*c^46*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2
 - 4960*a^14*b^29*c^44*d^3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^
42*d^5 + 906192*a^17*b^26*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*
a^19*b^24*c^39*d^8 - 28048800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*
d^10 - 129024480*a^22*b^21*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 3473
73600*a^24*b^19*c^34*d^13 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*
b^17*c^32*d^15 + 601080390*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d
^17 + 471435600*a^29*b^14*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 22579
2840*a^31*b^12*c^27*d^20 - 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^
10*c^25*d^22 - 28048800*a^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 -
3365856*a^36*b^7*c^22*d^25 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^
20*d^27 + 35960*a^39*b^4*c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2
*c^17*d^30)))^(3/4)*1i - 192609104438451240960*a^18*b^50*c^68*d^8 + 7086180
670911782322176*a^19*b^49*c^67*d^9 - 125074476913666377646080*a^20*b^48*c^6
```

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6*d^10 + 14111528055063183360000000*a^21*b^47*c^65*d^11 - 114401562747726005
37743360*a^22*b^46*c^64*d^12 + 710197549047037559203430400*a^23*b^45*c^63*d^
13 - 3513208637230819708313272320*a^24*b^44*c^62*d^14 + 142278193473158472666
82828800*a^25*b^43*c^61*d^15 - 48087644123193689681952768000*a^26*b^42*c^60*
d^16 + 137536282140960982680207360000*a^27*b^41*c^59*d^17 - 3360458626564623
2007931330560*a^28*b^40*c^58*d^18 + 704590041452076256580589322240*a^29*b^39
*c^57*d^19 - 1263359248135526588939344281600*a^30*b^38*c^56*d^20 + 189714420
7659575875311186739200*a^31*b^37*c^55*d^21 - 2219472744682837731400744960000*
a^32*b^36*c^54*d^22 + 1428707403433188341542866124800*a^33*b^35*c^53*d^23 +
1760831181775263996183073259520*a^34*b^34*c^52*d^24 - 8959470273938483263920
14438400*a^35*b^33*c^51*d^25 + 21543233409998229952763265024000*a^36*b^32*c^
50*d^26 - 39698653323390433738383949824000*a^37*b^31*c^49*d^27 + 61476442633
121113173254995968000*a^38*b^30*c^48*d^28 - 82607623379575801863715631923200*
a^39*b^29*c^47*d^29 + 97656010870864580876508856320000*a^40*b^28*c^46*d^30 -
 102235069483064131828662140928000*a^41*b^27*c^45*d^31 + 9508424738292483984
119247667200*a^42*b^26*c^44*d^32 - 78668986282545916344013317734400*a^43*b^2
5*c^43*d^33 + 57907247388414880664117512765440*a^44*b^24*c^42*d^34 - 3789006
7040636254842564850483200*a^45*b^23*c^41*d^35 + 21999962059191179489226784760
800*a^46*b^22*c^40*d^36 - 11304802150595851128286897766400*a^47*b^21*c^39*d^
37 + 5122036969218421637451979161600*a^48*b^20*c^38*d^38 - 20362530983711904
66921606676480*a^49*b^19*c^37*d^39 + 705764416322440732184936448000*a^50*b^18
*c^36*d^40 - 211515033720754528831143936000*a^51*b^17*c^35*d^41 + 5422672476
7772597695807488000*a^52*b^16*c^34*d^42 - 11725409134924140892284518400*a^53*
b^15*c^33*d^43 + 2097906091126339769267650560*a^54*b^14*c^32*d^44 - 30239740
2123696934905446400*a^55*b^13*c^31*d^45 + 33757779809986665041100800*a^56*b^1
2*c^30*d^46 - 2739812890627629121536000*a^57*b^11*c^29*d^47 + 14388779197382
5983283200*a^58*b^10*c^28*d^48 - 3671861846462714347520*a^59*b^9*c^27*d^49)*1
i)*(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 4028209561600*b^35*c
^35 + 136120599838720*a^2*b^33*c^33*d^2 - 1067520161218560*a^3*b^32*c^32*d^3
+ 5856445102817280*a^4*b^31*c^31*d^4 - 23907154306007040*a^5*b^30*c^30*d^5 +
75404149071544320*a^6*b^29*c^29*d^6 - 188295341785743360*a^7*b^28*c^28*d^7 +
378344208995450880*a^8*b^27*c^27*d^8 - 618128019701104640*a^9*b^26*c^26*d^9 +
 826122724924456960*a^10*b^25*c^25*d^10 - 905027427711672320*a^11*b^24*c^24*d
^11 + 807097710319042560*a^12*b^23*c^23*d^12 - 543841374599086080*a^13*b^22*c
^22*d^13 + 49371585774551040*a^14*b^21*c^21*d^14 + 1124912760455249920*a^15*b
^20*c^20*d^15 - 4132414539309056000*a^16*b^19*c^19*d^16 + 107444323159613440
0*a^17*b^18*c^18*d^17 - 22365714588360704000*a^18*b^17*c^17*d^18 + 383285080
9857372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 +
6599213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^1
3*d^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24
*b^11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142804547966645
0432*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 23938591134
0269568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 186260825
98846464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849
139712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544
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*a*b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^1
5*d^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 -
 70054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 50
65427904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16
316203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 1
1760839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2
094206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68
719476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*
b^31*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^4
5*d^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^4
3*d^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*
c^41*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*
b^24*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 443324737586724864
0*a^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 155163658
15535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13
+ 32396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17
*c^32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 388761692960664192
0*a^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332
023900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20
 - 8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*
c^25*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^3
5*b^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312
*a^37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 24711523834265
60*a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056
*a^41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^3
3*c^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^
31*d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^2
9*d^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^2
7*d^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c
^25*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b
^23*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^
14*b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452
800*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11182857
29418035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 -
 2740669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^
14*d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^2
3*b^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925468187
36128*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966474
8758366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040
318338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18556
53025615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3077714
7957248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34
*b*c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^4
6*d - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^
3 + 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26
```

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*c^41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 2804
8800*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^2
1*c^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13
 + 471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 60108039
0*a^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14
*c^29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20
- 129024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a
^34*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^2
5 + 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*
c^19*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4)*1i))*
(-(71993427968*a^35*d^35 - ((143986855936*a^35*d^35 + 4028209561б*b^35*c^35
 + 1361205998387 2*a^2*b^33*c^33*d^2 - 106752016121856*a^3*b^32*c^32*d^3 + 5
8564451028172 8*a^4*b^31*c^31*d^4 - 2390715430600704*a^5*b^30*c^30*d^5 + 754
0414907154432*a^6*b^29*c^29*d^6 - 18829534178574336*a^7*b^28*c^28*d^7 + 378
34420899545088*a^8*b^27*c^27*d^8 - 61812801970110464*a^9*b^26*c^26*d^9 + 82
612272492445696*a^10*b^25*c^25*d^10 - 90502742771167232*a^11*b^24*c^24*d^11
 + 80709771031904256*a^12*b^23*c^23*d^12 - 54384137459908608*a^13*b^22*c^22
*d^13 + 4937158577455104*a^14*b^21*c^21*d^14 + 112491276045524992*a^15*b^20
*c^20*d^15 - 413241453930905600*a^16*b^19*c^19*d^16 + 1074443231596134400*a
^17*b^18*c^18*d^17 - 2236571458836070400*a^18*b^17*c^17*d^18 + 383285080985
7372160*a^19*b^16*c^16*d^19 - 5481339136181731328*a^20*b^15*c^15*d^20 + 659
9213688440389632*a^21*b^14*c^14*d^21 - 6727518677746384896*a^22*b^13*c^13*d
^22 + 5827091540545486848*a^23*b^12*c^12*d^23 - 4293767561145810944*a^24*b^
11*c^11*d^24 + 2689585093637472256*a^25*b^10*c^10*d^25 - 142804547966645043
2*a^26*b^9*c^9*d^26 + 639329497516732416*a^27*b^8*c^8*d^27 - 23938591134026
9568*a^28*b^7*c^7*d^28 + 74080636676358144*a^29*b^6*c^6*d^29 - 186260825988
46464*a^30*b^5*c^5*d^30 + 3711306051231744*a^31*b^4*c^4*d^31 - 564292849139
712*a^32*b^3*c^3*d^32 + 61554295914496*a^33*b^2*c^2*d^33 - 1081861996544*a*
b^34*c^34*d - 4293426249728*a^34*b*c*d^34)^2/4 - (4581179456161*a^12*b^15*d
^23 + 15840599000625*b^27*c^12*d^11 - 231121882561500*a*b^26*c^11*d^12 - 70
054782497084*a^11*b^16*c*d^22 + 1442203904732850*a^2*b^25*c^10*d^13 - 50654
27904712140*a^3*b^24*c^9*d^14 + 11150130570636271*a^4*b^23*c^8*d^15 - 16316
203958046776*a^5*b^22*c^7*d^16 + 16492413880109692*a^6*b^21*c^6*d^17 - 1176
0839441437688*a^7*b^20*c^5*d^18 + 5941572716242975*a^8*b^19*c^4*d^19 - 2094
206929053932*a^9*b^18*c^3*d^20 + 492873253157362*a^10*b^17*c^2*d^21)*(68719
476736*a^11*b^32*c^47 + 68719476736*a^43*c^15*d^32 - 2199023255552*a^12*b^3
1*c^46*d - 2199023255552*a^42*b*c^16*d^31 + 34084860461056*a^13*b^30*c^45*d
^2 - 340848604610560*a^14*b^29*c^44*d^3 + 2471152383426560*a^15*b^28*c^43*d
^4 - 13838453347188736*a^16*b^27*c^42*d^5 + 62273040062349312*a^17*b^26*c^4
1*d^6 - 231299863088726016*a^18*b^25*c^40*d^7 + 722812072152268800*a^19*b^2
4*c^39*d^8 - 1927498859072716800*a^20*b^23*c^38*d^9 + 4433247375867248640*a
^21*b^22*c^37*d^10 - 8866494751734497280*a^22*b^21*c^36*d^11 + 15516365815
535370240*a^23*b^20*c^35*d^12 - 23871332023900569600*a^24*b^19*c^34*d^13 + 3
2396807746722201600*a^25*b^18*c^33*d^14 - 38876169296066641920*a^26*b^17*c^
32*d^15 + 41305929877070807040*a^27*b^16*c^31*d^16 - 38876169296066641920*a
```

```
^28*b^15*c^30*d^17 + 32396807746722201600*a^29*b^14*c^29*d^18 - 23871332023
900569600*a^30*b^13*c^28*d^19 + 15516365815535370240*a^31*b^12*c^27*d^20 -
8866494751734497280*a^32*b^11*c^26*d^21 + 4433247375867248640*a^33*b^10*c^2
5*d^22 - 1927498859072716800*a^34*b^9*c^24*d^23 + 722812072152268800*a^35*b
^8*c^23*d^24 - 231299863088726016*a^36*b^7*c^22*d^25 + 62273040062349312*a^
37*b^6*c^21*d^26 - 13838453347188736*a^38*b^5*c^20*d^27 + 2471152383426560*
a^39*b^4*c^19*d^28 - 340848604610560*a^40*b^3*c^18*d^29 + 34084860461056*a^
41*b^2*c^17*d^30))^(1/2) + 20141047808*b^35*c^35 + 6806029991936*a^2*b^33*c
^33*d^2 - 53376008060928*a^3*b^32*c^32*d^3 + 292822255140864*a^4*b^31*c^31*
d^4 - 1195357715300352*a^5*b^30*c^30*d^5 + 3770207453577216*a^6*b^29*c^29*d
^6 - 9414767089287168*a^7*b^28*c^28*d^7 + 18917210449772544*a^8*b^27*c^27*d
^8 - 30906400985055232*a^9*b^26*c^26*d^9 + 41306136246222848*a^10*b^25*c^25
*d^10 - 45251371385583616*a^11*b^24*c^24*d^11 + 40354885515952128*a^12*b^23
*c^23*d^12 - 27192068729954304*a^13*b^22*c^22*d^13 + 2468579288727552*a^14*
b^21*c^21*d^14 + 56245638022762496*a^15*b^20*c^20*d^15 - 206620726965452800
*a^16*b^19*c^19*d^16 + 537221615798067200*a^17*b^18*c^18*d^17 - 11182857294
18035200*a^18*b^17*c^17*d^18 + 1916425404928686080*a^19*b^16*c^16*d^19 - 27
40669568090865664*a^20*b^15*c^15*d^20 + 3299606844220194816*a^21*b^14*c^14*
d^21 - 3363759338873192448*a^22*b^13*c^13*d^22 + 2913545770272743424*a^23*b
^12*c^12*d^23 - 2146883780572905472*a^24*b^11*c^11*d^24 + 13447925468187361
28*a^25*b^10*c^10*d^25 - 714022739833225216*a^26*b^9*c^9*d^26 + 31966474875
8366208*a^27*b^8*c^8*d^27 - 119692955670134784*a^28*b^7*c^7*d^28 + 37040318
338179072*a^29*b^6*c^6*d^29 - 9313041299423232*a^30*b^5*c^5*d^30 + 18556530
25615872*a^31*b^4*c^4*d^31 - 282146424569856*a^32*b^3*c^3*d^32 + 3077714795
7248*a^33*b^2*c^2*d^33 - 540930998272*a*b^34*c^34*d - 2146713124864*a^34*b*
c*d^34)/(68719476736*(a^11*b^32*c^47 + a^43*c^15*d^32 - 32*a^12*b^31*c^46*d
 - 32*a^42*b*c^16*d^31 + 496*a^13*b^30*c^45*d^2 - 4960*a^14*b^29*c^44*d^3 +
 35960*a^15*b^28*c^43*d^4 - 201376*a^16*b^27*c^42*d^5 + 906192*a^17*b^26*c^
41*d^6 - 3365856*a^18*b^25*c^40*d^7 + 10518300*a^19*b^24*c^39*d^8 - 2804880
0*a^20*b^23*c^38*d^9 + 64512240*a^21*b^22*c^37*d^10 - 129024480*a^22*b^21*c
^36*d^11 + 225792840*a^23*b^20*c^35*d^12 - 347373600*a^24*b^19*c^34*d^13 +
471435600*a^25*b^18*c^33*d^14 - 565722720*a^26*b^17*c^32*d^15 + 601080390*a
^27*b^16*c^31*d^16 - 565722720*a^28*b^15*c^30*d^17 + 471435600*a^29*b^14*c^
29*d^18 - 347373600*a^30*b^13*c^28*d^19 + 225792840*a^31*b^12*c^27*d^20 - 1
29024480*a^32*b^11*c^26*d^21 + 64512240*a^33*b^10*c^25*d^22 - 28048800*a^34
*b^9*c^24*d^23 + 10518300*a^35*b^8*c^23*d^24 - 3365856*a^36*b^7*c^22*d^25 +
 906192*a^37*b^6*c^21*d^26 - 201376*a^38*b^5*c^20*d^27 + 35960*a^39*b^4*c^1
9*d^28 - 4960*a^40*b^3*c^18*d^29 + 496*a^41*b^2*c^17*d^30)))^(1/4) - (2/(3*
a*c) - (x^4*(112*b^4*c^4*d - 77*a^4*d^5 - 160*a*b^3*c^3*d^2 + 201*a^2*b^2*c
^2*d^3 + 68*a^3*b*c*d^4))/(48*a^2*c^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (x^2*(121*a^4*d^4 - 56*b^4*c^4 + 96*a^2*b^2*c^2*d^2 + 32*
a*b^3*c^3*d - 265*a^3*b*c*d^3))/(48*a^2*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2)) + (b*d*x^6*(77*a^3*d^4 - 56*b^3*c^3*d + 96*a*b^2*c^2*
d^2 - 189*a^2*b*c*d^3))/(48*a^2*c^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)))/(x^(7/2)*(b*c^2 + 2*a*c*d) + x^(11/2)*(a*d^2 + 2*b*c*d) + a*
```

```
c^2*x^(3/2) + b*d^2*x^(15/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.485 $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=881

$$-\frac{3(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}\,a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{17/4}}{4\sqrt{2}\,a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\log\left(\sqrt{b}\,x-\sqrt{2}\right.}{8\sqrt{2}\,a^{13/4}(bc}$$

Rubi [A] time = 1.69, antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {466, 472, 579, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}\,a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)b^{17/4}}{4\sqrt{2}\,a^{13/4}(bc-ad)^4}+\frac{3(3bc-7ad)\log\left(\sqrt{b}\,x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}\,a^{13/4}(bc-ad)^4}-\frac{3(3bc-7ad)\log\left(\sqrt{b}\,x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}\,a^{13/4}(bc-ad)^4}+\frac{b}{2a(bc-ad)x^{5/2}\left(bx^2+a\right)\left(dx^2+c\right)^2}-\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}\,c^{17/4}(bc-ad)^4}+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{32\sqrt{2}\,c^{17/4}(bc-ad)^4}+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\log\left(\sqrt{d}\,x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}\,c^{17/4}(bc-ad)^4}-\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\log\left(\sqrt{d}\,x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}\,c^{17/4}(bc-ad)^4}+\frac{3\left(24b^4c^4-32ab^3dc^3-32a^2b^2d^2c^2+87a^3bd^3c-39a^4d^4\right)}{16a^3c^4(bc-ad)^3\sqrt{x}}+\frac{d\left(8b^2c^2+29abdc-13a^2d^2\right)}{16ac^2(bc-ad)^3x^{5/2}\left(dx^2+c\right)}+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(dx^2+c\right)^2}-\frac{3\left(24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (-3*(24*b^3*c^3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3))/(80*a^2*c^
3*(b*c - a*d)^3*x^(5/2)) + (3*(24*b^4*c^4 - 32*a*b^3*c^3*d - 32*a^2*b^2*c^2
*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4))/(16*a^3*c^4*(b*c - a*d)^3*Sqrt[x]) + (
d*(2*b*c + a*d))/(4*a*c*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)^2) + b/(2*a*(b*c
- a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2) + (d*(8*b^2*c^2 + 29*a*b*c*d - 13
*a^2*d^2))/(16*a*c^2*(b*c - a*d)^3*x^(5/2)*(c + d*x^2)) - (3*b^(17/4)*(3*b*
c - 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/
4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119*b
^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126
*a*b*c*d + 39*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(32*S
qrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*
d)^4) - (3*b^(17/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*
b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) - (3*d^(13/4)*(119
*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = 2\,\text{Subst}\left(\int \frac{1}{x^6\left(a+bx^4\right)^2\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)$$

$$= \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\text{Subst}\left(\int \frac{-9bc+4ad-17bdx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)^3}\,dx, x, \sqrt{x}\right)}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} - \text{Subst}$$

$$= \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d\,(8b^2}{16ac^2(}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)^2}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

$$= -\frac{3\left(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3\right)}{80a^2c^3(bc-ad)^3x^{5/2}} + \frac{3\,(24b^4c^4-32ab^3c^3d-32a^2}{16a^3c^4(bc}$$

Mathematica [A] time = 6.27, size = 797, normalized size = 0.90

$$-\frac{x^{3/2}b^5}{2a^3(ad-bc)^3\left(bx^2+a\right)}-\frac{3(7ad-3bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}-\frac{3(7ad-3bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}+\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)b^{17/4}}{4\sqrt{2}a^{13/4}(bc-ad)^4}-\frac{3(7ad-3bc)\log\left(\sqrt{b}x-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3(7ad-3bc)\log\left(\sqrt{b}x+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right)b^{17/4}}{8\sqrt{2}a^{13/4}(bc-ad)^4}+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(ad-bc)^4}+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\tan^{-1}\left(\frac{2\sqrt[4]{d}\sqrt{x}+\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(ad-bc)^4}+\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\log\left(\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{17/4}(ad-bc)^4}-\frac{3d^{13/4}\left(119b^2c^2-126abdc+39a^2d^2\right)\log\left(\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right)}{64\sqrt{2}c^{17/4}(ad-bc)^4}+\frac{2(2bc+3ad)}{a^3c^4\sqrt{x}}+\frac{d^4(37bc-21ad)x^{3/2}}{16c^4(bc-ad)^3\left(dx^2+c\right)}+\frac{d^4x^{3/2}}{4c^3(bc-ad)^2\left(dx^2+c\right)^2}-\frac{2}{5a^2c^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] -2/(5*a^2*c^3*x^(5/2)) + (2*(2*b*c + 3*a*d))/(a^3*c^4*Sqrt[x]) - (b^5*x^(3/
2))/(2*a^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (d^4*x^(3/2))/(4*c^3*(b*c - a*d)
^2*(c + d*x^2)^2) + (d^4*(37*b*c - 21*a*d)*x^(3/2))/(16*c^4*(b*c - a*d)^3*(
c + d*x^2)) - (3*b^(17/4)*(-3*b*c + 7*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b
^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) - (3
*b^(17/4)*(-3*b*c + 7*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sq
rt[2]*a^(1/4))])/(4*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*b^2*
c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[(-(Sqrt[2]*c^(1/4)) + 2*d^(1/4)*Sqrt
[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(17/4)*(-(b*c) + a*d)^4) + (3*d^(13/
4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(
1/4)*Sqrt[x])/(Sqrt[2]*c^(1/4))])/(32*Sqrt[2]*c^(17/4)*(-(b*c) + a*d)^4) -
(3*b^(17/4)*(-3*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(8*Sqrt[2]*a^(13/4)*(b*c - a*d)^4) + (3*b^(17/4)*(-3*b*c + 7*
a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(8*Sqrt[2]
*a^(13/4)*(b*c - a*d)^4) + (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*
d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sqrt[2
]*c^(17/4)*(-(b*c) + a*d)^4) - (3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*
a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(64*Sq
rt[2]*c^(17/4)*(-(b*c) + a*d)^4)
```

IntegrateAlgebraic [A] time = 2.29, size = 753, normalized size = 0.85

$$\frac{3\left(7ab^{17/4}d-3b^{21/4}c\right)\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{4\sqrt{2}a^{13/4}(ad-bc)^4}+\frac{3\left(7ab^{17/4}d-3b^{21/4}c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{13/4}(ad-bc)^4}-\frac{3\left(39a^2d^{21/4}-126abcd^{17/4}+119b^2c^2d^{13/4}\right)\tan^{-1}\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}-\frac{3\left(39a^2d^{21/4}-126abcd^{17/4}+119b^2c^2d^{13/4}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}+\frac{-32a^5c^3d^3+416a^5c^2d^4x^2+1053a^5cd^5x^4+585a^5d^6x^6+96a^4bc^4d^2-960a^4bc^3d^3x^2-1933a^4bc^2d^4x^4-252a^4bcd^5x^6+585a^4bd^6x^8-96a^3b^2c^5d+384a^3b^2c^4d^2x^2-64a^3b^2c^3d^3x^4-1869a^3b^2c^2d^4x^6-1305a^3b^2cd^5x^8+32a^2b^3c^6+448a^2b^3c^5dx^2+1280a^2b^3c^4d^2x^4+1344a^2b^3c^3d^3x^6+480a^2b^3c^2d^4x^8-288ab^4c^6x^2-96ab^4c^5dx^4+672ab^4c^4d^2x^6+480ab^4c^3d^3x^8-360b^5c^6x^4-720b^5c^5dx^6-360b^5c^4d^2x^8}{80a^3c^4x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (32*a^2*b^3*c^6 - 96*a^3*b^2*c^5*d + 96*a^4*b*c^4*d^2 - 32*a^5*c^3*d^3 - 28
8*a*b^4*c^6*x^2 + 448*a^2*b^3*c^5*d*x^2 + 384*a^3*b^2*c^4*d^2*x^2 - 960*a^4
*b*c^3*d^3*x^2 + 416*a^5*c^2*d^4*x^2 - 360*b^5*c^6*x^4 - 96*a*b^4*c^5*d*x^4
 + 1280*a^2*b^3*c^4*d^2*x^4 - 64*a^3*b^2*c^3*d^3*x^4 - 1933*a^4*b*c^2*d^4*x
^4 + 1053*a^5*c*d^5*x^4 - 720*b^5*c^5*d*x^6 + 672*a*b^4*c^4*d^2*x^6 + 1344*
a^2*b^3*c^3*d^3*x^6 - 1869*a^3*b^2*c^2*d^4*x^6 - 252*a^4*b*c*d^5*x^6 + 585*
a^5*d^6*x^6 - 360*b^5*c^4*d^2*x^8 + 480*a*b^4*c^3*d^3*x^8 + 480*a^2*b^3*c^2
*d^4*x^8 - 1305*a^3*b^2*c*d^5*x^8 + 585*a^4*b*d^6*x^8)/(80*a^3*c^4*(-(b*c)
+ a*d)^3*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2) + (3*(-3*b^(21/4)*c + 7*a*b^(17
/4)*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(4*
```

```
Sqrt[2]*a^(13/4)*(-(b*c) + a*d)^4) - (3*(119*b^2*c^2*d^(13/4) - 126*a*b*c*d
^(17/4) + 39*a^2*d^(21/4))*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^4) + (3*(-3*b^(21/4)*c +
7*a*b^(17/4)*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b
]*x)])/(4*Sqrt[2]*a^(13/4)*(-(b*c) + a*d)^4) - (3*(119*b^2*c^2*d^(13/4) - 1
26*a*b*c*d^(17/4) + 39*a^2*d^(21/4))*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x])/(Sqrt[c] + Sqrt[d]*x)])/(32*Sqrt[2]*c^(17/4)*(b*c - a*d)^4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.73, size = 1289, normalized size = 1.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^(3/2)/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
(b*x^2 + a)) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^
4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7
*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*
a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2
 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c
^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*
sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 +
 sqrt(2)*a^4*c^5*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126*(c*d^3)^(3/
4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d +
 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4)
 - 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3
*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) +
 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*log(-sqrt(2)*sqrt(x)
*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*
d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) -
3/64*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3
```

```
)^(3/4)*a^2*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*
b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3
*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 3/64*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126
*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sq
rt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 1/
16*(37*b*c*d^5*x^(7/2) - 21*a*d^6*x^(7/2) + 41*b*c^2*d^4*x^(3/2) - 25*a*c*d
^5*x^(3/2))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d*x
^2 + c)^2) + 2/5*(10*b*c*x^2 + 15*a*d*x^2 - a*c)/(a^3*c^4*x^(5/2))
```

maple [A] time = 0.04, size = 1170, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 6/a^2/c^4/x^(1/2)*d+4/a^3/c^3/x^(1/2)*b+41/16*d^4/c/(a*d-b*c)^4/(d*x^2+c)^2
*x^(3/2)*b^2-1/2*b^5/a^2/(a*d-b*c)^4*x^(3/2)/(b*x^2+a)*d+1/2*b^6/a^3/(a*d-b
*c)^4*x^(3/2)/(b*x^2+a)*c+21/16*d^7/c^4/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*a^2
+37/16*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(7/2)*b^2+25/16*d^6/c^3/(a*d-b*c)^
4/(d*x^2+c)^2*x^(3/2)*a^2-189/64*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a*
b*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(
1/2)+(c/d)^(1/2)))-21/16*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*ln((x-(a
/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)
^(1/2)))-21/8*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)-21/8*b^4/a^2/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*d*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)-1)+9/16*b^5/a^3/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c
*ln((x-(a/b)^(1/4)*2^(1/2)*x^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*2^(1/2)*x^(1
/2)+(a/b)^(1/2)))+9/8*b^5/a^3/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+9/8*b^5/a^3/(a*d-b*c)^4/(a/b)^(1/4)*2^(1/2)*c*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)-29/8*d^6/c^3/(a*d-b*c)^4/(d*x^2+c)^2*x^
(7/2)*a*b-33/8*d^5/c^2/(a*d-b*c)^4/(d*x^2+c)^2*x^(3/2)*a*b+117/128*d^5/c^4/
(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*ln((x-(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)
^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+117/64*d^5/c^4/(a*d-b*
c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+117/64*d
^5/c^4/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*a^2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)-1)+357/128*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*ln((x-(c/d)^(1/4)
*2^(1/2)*x^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*2^(1/2)*x^(1/2)+(c/d)^(1/2)))+
357/64*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)+357/64*d^3/c^2/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*b^2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1)-189/32*d^4/c^3/(a*d-b*c)^4/(c/d)^(1/4)*2^(1/2)*
a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)-189/32*d^4/c^3/(a*d-b*c)^4/(c/d)^
(1/4)*2^(1/2)*a*b*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)-2/5/a^2/c^3/x^(5/2)
```

maxima [A] time = 2.88, size = 1066, normalized size = 1.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/16*(3*b^6*c - 7*a*b^5*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)
*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)))/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4) + 3/128*(119*b^2*c^2*d^4 - 126*a*b*c*d^5 + 39*a^2*d^6)*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))
/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4
)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^8 -
 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4) - 1/80*
(32*a^2*b^3*c^6 - 96*a^3*b^2*c^5*d + 96*a^4*b*c^4*d^2 - 32*a^5*c^3*d^3 - 15
*(24*b^5*c^4*d^2 - 32*a*b^4*c^3*d^3 - 32*a^2*b^3*c^2*d^4 + 87*a^3*b^2*c*d^5
 - 39*a^4*b*d^6)*x^8 - 3*(240*b^5*c^5*d - 224*a*b^4*c^4*d^2 - 448*a^2*b^3*c
^3*d^3 + 623*a^3*b^2*c^2*d^4 + 84*a^4*b*c*d^5 - 195*a^5*d^6)*x^6 - (360*b^5
*c^6 + 96*a*b^4*c^5*d - 1280*a^2*b^3*c^4*d^2 + 64*a^3*b^2*c^3*d^3 + 1933*a^
4*b*c^2*d^4 - 1053*a^5*c*d^5)*x^4 - 32*(9*a*b^4*c^6 - 14*a^2*b^3*c^5*d - 12
*a^3*b^2*c^4*d^2 + 30*a^4*b*c^3*d^3 - 13*a^5*c^2*d^4)*x^2)/((a^3*b^4*c^7*d^
2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^(17/2) + (2*a^
3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c
^4*d^5)*x^(13/2) + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6
*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^(9/2) + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^
6*b*c^7*d^2 - a^7*c^6*d^3)*x^(5/2))
```

mupad [B] time = 22.14, size = 143600, normalized size = 163.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] atan((((((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152
*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a
^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a
^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*
a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 1490305003825397
```

```
76*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023
528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 563
47698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15
- 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20
*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*
b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 1183226127125708
3904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743
319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d
^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*
b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 559713091980460
0320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963
087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 842002
49113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 27334
33701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574
625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*
a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c
^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^1
0*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c
^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*
c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21
*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^1
9*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199
023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 3408486046105
6*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 247115238342656
0*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 6227304006234
9312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072
152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 443
3247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d
^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*
b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 388761692960666
41920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 3887
6169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*
d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33
*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 443324737586724
8640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 7228120
72152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62
273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 +
 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 +
 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772
100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b
^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^3
2*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^3
0*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^
28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^1
1*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 4984537565629440
```

0*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 67718624892
27264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 1752045
58709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 -
1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^1
7*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22
*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372
225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 783
9134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d
^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*
c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b
^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b
^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2
*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68
719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*
b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17
*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 33
65856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23
*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 +
225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a
^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^
33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 3
47373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^
34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d
^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^3
9*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 49
60*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*((((767544
201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^
2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4
- 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6
- 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8
 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27
*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^
25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^
14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 707025204592312
32*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 118050703
5769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 -
6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^
16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a
^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192
158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 +
10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9
*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^
7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*
b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*
b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^3

```
6*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 +
106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437
396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520
578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 9452121
6268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 733706
51908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432
588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(6871
9476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^
31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*
d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*
d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^
43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^
24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*
a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815
535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 +
32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c
^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*
a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 2387133202
3900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 -
 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^
27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*
b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a
^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560
*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a
^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37
- 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 64
9958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718
627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 3616
7749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 745
15250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11
- 69732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*
d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c
^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*
b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 15612153027875389
44*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 59161306
35628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 -
 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*
c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*
a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 27985654599
02300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942
981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 421
00124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 136
6716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 13942
87312896*a*b^36*c^36*d + 1109986998681 6*a^36*b*c*d^36)/(68719476736*(a^13*b
^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496
```

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*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 -
201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c
^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512
240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^
20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^1
4 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 5657227
20*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^1
3*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21
 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^
37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 -
 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*
d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154605056*a^34*b^53*c^94
*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 + 78828702034483948290048*
a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^91*d^7 + 9892540
360265140468187136*a^38*b^49*c^90*d^8 - 73440220164348137346957312*a^39*b^4
8*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*d^10 - 219623725323
4092465387995136*a^41*b^46*c^87*d^11 + 9274296316144595646699012096*a^42*b^
45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c^85*d^13 + 10637653
0102998491281999527936*a^44*b^43*c^84*d^14 - 294921432301504798990377418752
*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^46*b^41*c^82*d^16 -
 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 30724712085399739725
78986360832*a^48*b^39*c^80*d^18 - 5384106777252432871416869683200*a^49*b^38
*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c^78*d^20 - 12376921
822825560832675204300800*a^51*b^36*c^77*d^21 + 167075893904326210567387490
54976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857792*a^53*b^34*c^75*
d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d^24 - 3837839313852
1379212996695293952*a^55*b^32*c^73*d^25 + 54918846093258397577855222415360*
a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a^57*b^30*c^71*d^27
+ 113888426387729629146256565600256*a^58*b^29*c^70*d^28 - 15275410650031254
5531177547595776*a^59*b^28*c^69*d^29 + 189549778508563263438068404715520*a^
60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^61*b^26*c^67*d^31 +
 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 211293730951350565
888869600854016*a^63*b^24*c^65*d^33 + 181575241776706668284956756672512*a^6
4*b^23*c^64*d^34 - 141794149619600448829729705820160*a^65*b^22*c^63*d^35 +
100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 6458112355324399057
2098666889216*a^67*b^20*c^61*d^37 + 37540992634094717640084094451712*a^68*b
^19*c^60*d^38 - 19695179695689601910490494140416*a^69*b^18*c^59*d^39 + 9296
840942046414522746815905792*a^70*b^17*c^58*d^40 - 3933446196282108795457464
434688*a^71*b^16*c^57*d^41 + 1484644864880431945098662510592*a^72*b^15*c^56
*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*d^43 + 14649370730228
9292776429322240*a^74*b^13*c^54*d^44 - 37679005999847399095674077184*a^75*b
^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11*c^52*d^46 - 15765465
23407725355918688256*a^77*b^10*c^51*d^47 + 247744258459119342197932032*a^78
*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b^8*c^49*d^49 + 3133065413
748205302054912*a^80*b^7*c^48*d^50 - 227270011883594899783680*a^81*b^6*c^47
```

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*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52 - 246599101196298878976*
a^83*b^4*c^45*d^53) - 1050599721072989306888*a^31*b^54*c^91*d^4 + 4202398884
291957227520*a^32*b^53*c^90*d^5 - 8145649837385910426009 6*a^33*b^52*c^89*d^
6 + 101947084044860443852 8000*a^34*b^51*c^88*d^7 - 926158518777940552345190
4*a^35*b^50*c^87*d^8 + 650949719443986711450746 88*a^36*b^49*c^86*d^9 - 3684
023954539163231893585 92*a^37*b^48*c^85*d^10 + 1725226316150928144278224896*
a^38*b^47*c^84*d^11 - 6817742452202868128486522880*a^39*b^46*c^83*d^12 + 23
071505195064931052886687744*a^40*b^45*c^82*d^13 - 676140892161236694923319 7
0560*a^41*b^44*c^81*d^14 + 173115025562473785468905324544*a^42*b^43*c^80*d^
15 - 389913831719674713212222177280*a^43*b^42*c^79*d^16 + 77679008891243214
1093966970880*a^44*b^41*c^78*d^17 - 1374611983251272530469308071936*a^45*b^
40*c^77*d^18 + 2167454612994156285048662261760*a^46*b^39*c^76*d^19 - 305033
7310429700535004075917312*a^47*b^38*c^75*d^20 + 382688562287149657050232494
4896*a^48*b^37*c^74*d^21 - 4238713393375513383921726259200*a^49*b^36*c^73*d
^22 + 3984291896345024467843348955136*a^50*b^35*c^72*d^23 - 265197142646459
7412032295206912*a^51*b^34*c^71*d^24 - 479249403658129639733534392320*a^52*
b^33*c^70*d^25 + 6697452529698647734837548417024*a^53*b^32*c^69*d^26 - 1793
1054269995149998277682790400*a^54*b^31*c^68*d^27 + 363117150219056347997847
47335680*a^55*b^30*c^67*d^28 - 63073617076394089001091166371840*a^56*b^29*c
^66*d^29 + 97105565168138147055402127196160*a^57*b^28*c^65*d^30 - 133993666
277013207597272619024384*a^58*b^27*c^64*d^31 + 1664920848331020446958593507
32800*a^59*b^26*c^63*d^32 - 186717161118223967667066928889856*a^60*b^25*c^6
2*d^33 + 189235624153406619951659086774272*a^61*b^24*c^61*d^34 - 1734218252
88151984221422006304768*a^62*b^23*c^60*d^35 + 143715376746696050902973036888
8064*a^63*b^22*c^59*d^36 - 107645128880801788128312132894720*a^64*b^21*c^58
*d^37 + 72802169209714119238549751463936*a^65*b^20*c^57*d^38 - 443896392701
36779232591657041920*a^66*b^19*c^56*d^39 + 24348625105436875280486976454656
*a^67*b^18*c^55*d^40 - 11981145511938522697620070072320*a^68*b^17*c^54*d^41
 + 5269759325089910260644729323520*a^69*b^16*c^53*d^42 - 206247152253002743
3706750214144*a^70*b^15*c^52*d^43 + 714227824367410213467319173120*a^71*b^1
4*c^51*d^44 - 217305373751493983005392764928*a^72*b^13*c^50*d^45 + 57574411
1484335694244416061 44*a^73*b^12*c^49*d^46 - 13133947360733882065354752000*a
^74*b^11*c^48*d^47 + 2542019460242050797665255424*a^75*b^10*c^47*d^48 - 409
310322447365741947650048*a^76*b^9*c^46*d^49 + 53356649691793134232535040*a^
77*b^8*c^45*d^50 - 5410594924578893614546944*a^78*b^7*c^44*d^51 + 400464195
437318897664000*a^79*b^6*c^43*d^52 - 19246289226179889070080*a^80*b^5*c^42*
d^53 + 450813981874483888128*a^81*b^4*c^41*d^54) - x^(1/2)*(119342219331695
731015680*a^30*b^49*c^73*d^13 - 3677615218076424339456*a^29*b^50*c^74*d^12
- 18560134430309724255682 56*a^31*b^48*c^72*d^14 + 184260999964528072584069 1
2*a^32*b^47*c^71*d^15 - 131228123459738637629915136*a^33*b^46*c^70*d^16 + 7
14182072565091774626791424*a^34*b^45*c^69*d^17 - 308823741534848443145728 81
92*a^35*b^44*c^68*d^18 + 10882952503625649640326561792*a^36*b^43*c^67*d^19
- 31757074600474077803581538304*a^37*b^42*c^66*d^20 + 77306011497125960924 9
62750464*a^38*b^41*c^65*d^21 - 156439291025195069838804910080*a^39*b^40*c^6
4*d^22 + 256967446361217518429496410112*a^40*b^39*c^63*d^23 - 3159302665384
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85089912448090112*a^41*b^38*c^62*d^24 + 193264836517334230347779407872*a^42
*b^37*c^61*d^25 + 320732651390132179677984325632*a^43*b^36*c^60*d^26 - 1433
302686817582744983683727360*a^44*b^35*c^59*d^27 + 3214765851097197421262933
065728*a^45*b^34*c^58*d^28 - 5465398361763642490480861642752*a^46*b^33*c^57
*d^29 + 7690728695480443198104101978112*a^47*b^32*c^56*d^30 - 9256447758824
794945376420364288*a^48*b^31*c^55*d^31 + 9672669866587270697877661286400*a^
49*b^30*c^54*d^32 - 8839280066432157154484139589632*a^50*b^29*c^53*d^33 + 7
086822067089169522912760168448*a^51*b^28*c^52*d^34 - 4988522538878293079151
039479808*a^52*b^27*c^51*d^35 + 3079795601090740527825181212672*a^53*b^26*c
^50*d^36 - 1663341919096805892341077377024*a^54*b^25*c^49*d^37 + 7826660388
49476274770105335808*a^55*b^24*c^48*d^38 - 319013552886948801896949743616*a
^56*b^23*c^47*d^39 + 111766668098727585639133347840*a^57*b^22*c^46*d^40 - 3
3312207294098258580851392512*a^58*b^21*c^45*d^41 + 833079130628766161188788
6336*a^59*b^20*c^44*d^42 - 1715502625948903704153292800*a^60*b^19*c^43*d^43
 + 283282946101439324535914496*a^61*b^18*c^42*d^44 - 3606933247058679884572
2624*a^62*b^17*c^41*d^45 + 3324850588931239515783168*a^63*b^16*c^40*d^46 -
197512325498721785610240*a^64*b^15*c^39*d^47 + 5678869390326597943296*a^65*
b^14*c^38*d^48))*((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324
2744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 12999174
35830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 154372555
94213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498
051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030
500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 +
 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*
d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*
c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17
*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312243060557507
7888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322
61271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^
22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b
^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1567826806107753
6768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971
30919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 -
1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^3
1 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^
33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35
 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443
545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 144603505249081
2*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^
2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*
a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292
*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 3890715322816345
5*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 357468305702344
2*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d
^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 340
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84860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 24711
52383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622
73040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 +
 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40
*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b
^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 2387133202390056
9600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876
169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d
^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*
b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 155163658155353
70240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332
47375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23
 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24
*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c
^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c
^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^3
7 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 12402634103
1936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 25438432285163
52*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 186000754165678
08*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815
808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172
889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 498453
7565629440 0*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6
771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16
 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^
19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^2
0*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 87373333813085
79840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 1146
2371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*
d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b
^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152
*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652
160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473
344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 15412348116172
8*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c
*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d
- 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 +
35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^4
3*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800
*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^
38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 4
71435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^
29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^3
1*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 12
9024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*
```

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b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 +
906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21
*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*1i + ((((
(767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*
c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^
33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^
31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c
^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^
27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*
a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 5634769849329
2544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520
459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 11
80507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*
d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*
b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 217433192156964
12672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 2054
8937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*
d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*
b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*
a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 842002491132149
76*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 273343370143334
4*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b
^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*
d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 -
 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 2
9104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 -
94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 -
 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21
- 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23
)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*
a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^3
0*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^2
8*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*
b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*
a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867
248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 1551
6365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*
d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28
*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066
641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 238
71332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29
*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*
b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 72281207215226880
0*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 622730400623
49312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 247115238
```

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3426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 340848604
61056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^3
7*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d
^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5
 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7
 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^
9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^2
6*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^2
4*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15
*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 17520455870952652
8*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302
787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5
916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15
*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^2
4*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538
768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2798
565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 +
 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^3
1 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^3
3 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35
+ 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*
(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^3
1 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45
*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20
*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9
+ 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*
a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c
^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 -
565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a
^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^
28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 1051
8300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23
*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^
3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*((((767544201216*a^3
7*d^37 + 110075314176*b^37*c^37 + 3324274488115 2*a^2*b^35*c^35*d^2 - 248052
682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 50876864
57032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150
833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 1146619
16059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 15
9158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^
12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^
23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^
21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224
*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089
679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 -
```

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17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c
^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*
a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 1017318402
3521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 25
97066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30
 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^
32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^
34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 2219
9739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 1065719475
10161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*
a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*
a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328
*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 7337065190848696
8*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 144325881654023
16*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^
13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d
- 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 3408
48604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 1383
8453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 2
31299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^
8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*
c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*
a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 3239680774
6722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 +
 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*
c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600
*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 886649475
1734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 -
1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d
^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^
23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*
c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^
19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372
440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915
136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 77186277971066
88*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660
928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 7451525019126
9888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511
764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 281
7384924664 6272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 +
 3535126022961 5616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*
d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^
18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 591613063562854195
2*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 108716596
07848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24
```

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- 1027446859607932 1088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*
c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a
^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903
232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 4210012455660
7488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716
672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a
*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 +
 a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30
*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^1
8*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 +
10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b
^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^
12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722
720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^
15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^1
9 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 6451224
0*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^2
5*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^
40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496
*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154605056*a^34*b^53*c^94*d^4 - 389
1110078048108544000*a^35*b^52*c^93*d^5 + 78828702034483948290048*a^36*b^51*
c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^91*d^7 + 98925403602651404
68187136*a^38*b^49*c^90*d^8 - 73440220164348137346957312*a^39*b^48*c^89*d^9
 + 440649383366170539762647040*a^40*b^47*c^88*d^10 - 2196237253234092465387
995136*a^41*b^46*c^87*d^11 + 9274296316144595646699012096*a^42*b^45*c^86*d^
12 - 33677881501046993339969175552*a^43*b^44*c^85*d^13 + 106376530102998491
281999527936*a^44*b^43*c^84*d^14 - 294921432301504798990377418752*a^45*b^42
*c^83*d^15 + 722903045142137525367365173248*a^46*b^41*c^82*d^16 - 157607244
7576504233275626094592*a^47*b^40*c^81*d^17 + 307247120853997397257898636083
2*a^48*b^39*c^80*d^18 - 5384106777252432871416869683200*a^49*b^38*c^79*d^19
 + 8537351598354925496836275830784*a^50*b^37*c^78*d^20 - 123769218228255608
32675204300800*a^51*b^36*c^77*d^21 + 16707589390432621056738749054976*a^52*
b^35*c^76*d^22 - 21667130911214476307455165857792*a^53*b^34*c^75*d^23 + 282
11207618793157944689200988160*a^54*b^33*c^74*d^24 - 383783931385213792129966
95293952*a^55*b^32*c^73*d^25 + 54918846093258397577855222415360*a^56*b^31*
c^72*d^26 - 80082941438212170767896978391040*a^57*b^30*c^71*d^27 + 11388842
6387729629146256565600256*a^58*b^29*c^70*d^28 - 152754106500312545531177547
595776*a^59*b^28*c^69*d^29 + 189549778508563263438068404715520*a^60*b^27*c^
68*d^30 - 215546518234822631781377148715008*a^61*b^26*c^67*d^31 + 223641896
308855873457165036421120*a^62*b^25*c^66*d^32 - 2112937309513505658888696008
54016*a^63*b^24*c^65*d^33 + 181575241776706668284956756672512*a^64*b^23*c^6
4*d^34 - 141794149619600448829729705820160*a^65*b^22*c^63*d^35 + 1005115760
25621687034384100622336*a^66*b^21*c^62*d^36 - 645811235532439905720986668889
216*a^67*b^20*c^61*d^37 + 375409926340947176400840944517 12*a^68*b^19*c^60*d
^38 - 196951796956896019104904941404 16*a^69*b^18*c^59*d^39 + 92968409420464
```

```
14522746815905792*a^70*b^17*c^58*d^40 - 3933446196282108795457464434688*a^7
1*b^16*c^57*d^41 + 1484644864880431945098662510592*a^72*b^15*c^56*d^42 - 49
69938773331195363812777656 32*a^73*b^14*c^55*d^43 + 146493707302289292776429
322240*a^74*b^13*c^54*d^44 - 376790059998473990956740771 84*a^75*b^12*c^53*d
^45 + 8360094623991181223468728320*a^76*b^11*c^52*d^46 - 157654652340772535
5918688256*a^77*b^10*c^51*d^47 + 247744258459119342197932032*a^78*b^9*c^50*
d^48 - 31566136012926195282739200*a^79*b^8*c^49*d^49 + 313306541374820530 20
54912*a^80*b^7*c^48*d^50 - 227270011883594899783680*a^81*b^6*c^47*d^51 + 10
717576321223758970880*a^82*b^5*c^46*d^52 - 246599101196298878976*a^83*b^4*c
^45*d^53) + 1050599721072989306 88*a^31*b^54*c^91*d^4 - 420239888429195722 75
20*a^32*b^53*c^90*d^5 + 8145649837385910426 0096*a^33*b^52*c^89*d^6 - 101947
0840448604438528000*a^34*b^51*c^88*d^7 + 9261585187779405523451904*a^35*b^5
0*c^87*d^8 - 65094971944398671145074688*a^36*b^49*c^86*d^9 + 36840239545391
6323189358592*a^37*b^48*c^85*d^10 - 1725226316150928144278224896*a^38*b^47*
c^84*d^11 + 6817742452202868128486522880*a^39*b^46*c^83*d^12 - 230715051950
64931052886687744*a^40*b^45*c^82*d^13 + 67614089216123669492331970560*a^41*
b^44*c^81*d^14 - 173115025562473785468905324544*a^42*b^43*c^80*d^15 + 38991
3831719674713212222177280*a^43*b^42*c^79*d^16 - 776790088912432141093966970
880*a^44*b^41*c^78*d^17 + 1374611983251272530469308071936*a^45*b^40*c^77*d^
18 - 2167454612994156285048662261760*a^46*b^39*c^76*d^19 + 3050337310429700
535004075917312*a^47*b^38*c^75*d^20 - 3826885622871496570502324944896*a^48*
b^37*c^74*d^21 + 4238713393375513383921726259200*a^49*b^36*c^73*d^22 - 3984
291896345024467843348955136*a^50*b^35*c^72*d^23 + 2651971426464597412032295
206912*a^51*b^34*c^71*d^24 + 479249403658129639733534392320*a^52*b^33*c^70*
d^25 - 6697452529698647734837548417024*a^53*b^32*c^69*d^26 + 17931054269995
149998277682790400*a^54*b^31*c^68*d^27 - 36311715021905634799784747335680*a
^55*b^30*c^67*d^28 + 63073617076394089001091166371840*a^56*b^29*c^66*d^29 -
 97105565168138147055402127196160*a^57*b^28*c^65*d^30 + 133993666277013 2075
97272619024384*a^58*b^27*c^64*d^31 - 166492084833102044695859350732800*a^59
*b^26*c^63*d^32 + 186717161118223967667066928889856*a^60*b^25*c^62*d^33 - 1
89235624153406619951659086774272*a^61*b^24*c^61*d^34 + 17342182528815198422
1422006304768*a^62*b^23*c^60*d^35 - 143715376746696050902973036888064*a^63*
b^22*c^59*d^36 + 107645128880801788128312132894720*a^64*b^21*c^58*d^37 - 72
802169209714119238549751463936*a^65*b^20*c^57*d^38 + 4438963927013677923259
1657041920*a^66*b^19*c^56*d^39 - 24348625105436875280486976454656*a^67*b^18
*c^55*d^40 + 11981145511938522697620070072320*a^68*b^17*c^54*d^41 - 5269759
325089910260644729323520*a^69*b^16*c^53*d^42 + 2062471522530027433706750214
144*a^70*b^15*c^52*d^43 - 714227824367410213467319173120*a^71*b^14*c^51*d^4
4 + 217305373751493983005392764928*a^72*b^13*c^50*d^45 - 575744111484335694
24441606144*a^73*b^12*c^49*d^46 + 13133947360733882065354752000*a^74*b^11*c
^48*d^47 - 2542019460242050797665255424*a^75*b^10*c^47*d^48 + 409310322447 3
65741947650048*a^76*b^9*c^46*d^49 - 53356649691793134232535040*a^77*b^8*c^4
5*d^50 + 5410594924578893614546944*a^78*b^7*c^44*d^51 - 400464195437318897 6
64000*a^79*b^6*c^43*d^52 + 19246289226179889070080*a^80*b^5*c^42*d^53 - 450
813981874483888128*a^81*b^4*c^41*d^54) - x^(1/2)*(119342219331695731015680*
```

```
a^30*b^49*c^73*d^13 - 3677615218076424339456*a^29*b^50*c^74*d^12 - 18560134
43030972425568256*a^31*b^48*c^72*d^14 + 184260999964528072584069l2*a^32*b^4
7*c^71*d^15 - 1312281234597386376299151 36*a^33*b^46*c^70*d^16 + 71418207256
5091774626791424*a^34*b^45*c^69*d^17 - 30882374153484844314572881 92*a^35*b^
44*c^68*d^18 + 10882952503625649640326561792*a^36*b^43*c^67*d^19 - 31757074
6004740778035815383 04*a^37*b^42*c^66*d^20 + 77306011497125960924962750464*a
^38*b^41*c^65*d^21 - 156439291025195069838804910080*a^39*b^40*c^64*d^22 + 2
56967446361217518429496410112*a^40*b^39*c^63*d^23 - 315930266538485089912 44
8090112*a^41*b^38*c^62*d^24 + 193264836517334230347779407872*a^42*b^37*c^61
*d^25 + 320732651390132179677984325632*a^43*b^36*c^60*d^26 - 1433302686817 5
82744983683727360*a^44*b^35*c^59*d^27 + 3214765851097197421262933065728*a^4
5*b^34*c^58*d^28 - 5465398361763642490480861642752*a^46*b^33*c^57*d^29 + 76
907286954804431981041019781 12*a^47*b^32*c^56*d^30 - 925644775882479494537 64
20364288*a^48*b^31*c^55*d^31 + 9672669866587270697877661286400*a^49*b^30*c^
54*d^32 - 8839280066432157154484139589632*a^50*b^29*c^53*d^33 + 7086822067 0
891695229127601684 48*a^51*b^28*c^52*d^34 - 4988522538878293079151039479808*
a^52*b^27*c^51*d^35 + 3079795601090740527825181212672*a^53*b^26*c^50*d^36 -
 1663341919096805892341077377024*a^54*b^25*c^49*d^37 + 782666038849476274 77
0105335808*a^55*b^24*c^48*d^38 - 319013552886948801896949743616*a^56*b^23*c
^47*d^39 + 111766668098727585639133347840*a^57*b^22*c^46*d^40 - 33312207294
098258580851392512*a^58*b^21*c^45*d^41 + 8330791306287661611887886336*a^59*
b^20*c^44*d^42 - 1715502625948903704153292800*a^60*b^19*c^43*d^43 + 2832829
46101439324535914496*a^61*b^18*c^42*d^44 - 36069332470586798845722624*a^62*
b^17*c^41*d^45 + 3324850588931239515783168*a^63*b^16*c^40*d^46 - 1975123254
98721785610240*a^64*b^15*c^39*d^47 + 5678869390326597943296*a^65*b^14*c^38*
d^48))*((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324274488115 2
*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a
^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a
^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*
a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539 7
76*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023
528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 563
47698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15
- 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20
*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*
b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322612712570 8
3904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743
319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d
^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*
b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 559713091980460
0320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963
087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 842002
49113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 27334
33701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574
625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*
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a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c
^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^1
0*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c
^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*
c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21
*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^1
9*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199
023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 3408486046105
6*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 247115238342656
0*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 6227304006234
9312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072
152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 443
3247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d
^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*
b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 388761692960666
41920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 3887
6169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*
d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33
*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 443324737586724
8640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 7228120
72152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62
273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 +
 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 +
 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772
100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b
^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^3
2*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^3
0*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^
28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^1
1*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 4984537565629440
0*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 67718624892
27264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 1752045
58709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 -
1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^1
7*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22
*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372
225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 783
9134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d
^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*
c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b
^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b
^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2
*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68
719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*
b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17
```

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*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 33
65856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23
*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 +
225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a
^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^
33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 3
47373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^
34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d
^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^3
9*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 49
60*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*1i)/((((((767544201
216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 -
 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5
087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 3
7200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 -
114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^
10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*
c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*
b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*
a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 118050703576
9012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 669
2023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*
d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23
*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158
642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 101
7318402352182O672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^
28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c
^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5
*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3
*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d
 - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106
571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396
380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578
391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 9452121626
8814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 733706519
08486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588
165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(6871947
6736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*
c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2
 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4
 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*
d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*
c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^2
3*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535
370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 323
```

96807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34
*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^3
0*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 2387133202390
0569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 88
66494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*
d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8
*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39
*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^
41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43
*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 1
6621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 64995
8717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627
797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 3616774
9025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 745152
50191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 6
9732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^1
3 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22
*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^2
0*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*
a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 59161306356
28541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10
871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^1
3*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^2
6*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 27985654599023
00160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981
543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 421001
24556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 136671
6850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 13942873
12896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32
*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^
15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201
376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42
*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240
*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*
c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 -
565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*
a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c
^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 +
64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*
b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 20
1376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^2
9 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*((((767544201216*a^37*d^37 + 1
10075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*
a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a
^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*

```
a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 11466191605963161
6*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345
778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690
751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 -
13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^
16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19
*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*
a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 1747466676
2617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 -
 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*
c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672
*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 259706627263
0370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 3232371
80229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373
183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 30824
6962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632
*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29
*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*
c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c
^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*
c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22
*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^2
0*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^
49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 21990232
55552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 34084860461056
0*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188
736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 23129986308
8726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 192749
8859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10
- 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*
c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600
*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 413059298
77070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17
+ 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13
*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280
*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859
072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 2312
99863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 -
13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28
- 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^
(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2
*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^
33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^3
1*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^
29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*
```

b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*
a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 2817384924664
6272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 353512602
29615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 590
253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^
19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^1
6*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 108716596078482063
36*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10274468
596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26
- 5086592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^
9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^
7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*
b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b
^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36
*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17
*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2
- 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^4
4*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a
^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d
^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 34737
3600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b
^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^
17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792
840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^1
0*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3
365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^2
2*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*
c^19*d^30)))^(1/4)*(93386641873154605056*a^34*b^53*c^94*d^4 - 389111007804
8108544000*a^35*b^52*c^93*d^5 + 78828702034483948290048*a^36*b^51*c^92*d^6 -
1034672110486845715906560*a^37*b^50*c^91*d^7 + 9892540360265140468187136*a
^38*b^49*c^90*d^8 - 73440220164348137346957312*a^39*b^48*c^89*d^9 + 4406493
83366170539762647040*a^40*b^47*c^88*d^10 - 2196237253234092465387995136*a^4
1*b^46*c^87*d^11 + 9274296316144595646699012096*a^42*b^45*c^86*d^12 - 33677
881501046993339969175552*a^43*b^44*c^85*d^13 + 106376530102998491281999527
936*a^44*b^43*c^84*d^14 - 294921432301504798990377418752*a^45*b^42*c^83*d^15
+ 722903045142137525367365173248*a^46*b^41*c^82*d^16 - 157607244757650423
3275626094592*a^47*b^40*c^81*d^17 + 3072471208539973972578986360832*a^48*b^3
9*c^80*d^18 - 5384106777252432871416869683200*a^49*b^38*c^79*d^19 + 8537351
598354925496836275830784*a^50*b^37*c^78*d^20 - 123769218228255608326752043
00800*a^51*b^36*c^77*d^21 + 16707589390432621056738749054976*a^52*b^35*c^76*
d^22 - 21667130911214476307455165857792*a^53*b^34*c^75*d^23 + 282112076187
93157944689200988160*a^54*b^33*c^74*d^24 - 38378393138521379212996695293952*
a^55*b^32*c^73*d^25 + 54918846093258397577855222415360*a^56*b^31*c^72*d^26
- 80082941438212170767896978391040*a^57*b^30*c^71*d^27 + 113888426387729629
14625656560256*a^58*b^29*c^70*d^28 - 152754106500312545531177547595776*a^5

```
9*b^28*c^69*d^29 + 18954977850856326343806840471552O*a^60*b^27*c^68*d^30 -
2155465182348226317813771487150O8*a^61*b^26*c^67*d^31 + 2236418963088558734
57165036421120*a^62*b^25*c^66*d^32 - 21129373095135056588869600854016*a^63
*b^24*c^65*d^33 + 181575241776706668284956756672512*a^64*b^23*c^64*d^34 - 1
41794149619600448829729705820160*a^65*b^22*c^63*d^35 + 100511576025621687O3
4384100622336*a^66*b^21*c^62*d^36 - 64581123553243990572098666889216*a^67*b
^20*c^61*d^37 + 37540992634094717640084094451712*a^68*b^19*c^60*d^38 - 1969
5179695689601910490494140416*a^69*b^18*c^59*d^39 + 929684094204641452274681
5905792*a^70*b^17*c^58*d^40 - 3933446196282108795457464434688*a^71*b^16*c^5
7*d^41 + 1484644864880431945098662510592*a^72*b^15*c^56*d^42 - 496993877333
119536381277765632*a^73*b^14*c^55*d^43 + 146493707302289292776429322240*a^7
4*b^13*c^54*d^44 - 37679005999847399095674077184*a^75*b^12*c^53*d^45 + 8360
094623991181223468728320*a^76*b^11*c^52*d^46 - 1576546523407725355918688256
*a^77*b^10*c^51*d^47 + 247744258459119342197932032*a^78*b^9*c^50*d^48 - 315
66136012926195282739200*a^79*b^8*c^49*d^49 + 3133065413748205302054912*a^80
*b^7*c^48*d^50 - 227270011883594899783680*a^81*b^6*c^47*d^51 + 107175763212
23758970880*a^82*b^5*c^46*d^52 - 246599101196298878976*a^83*b^4*c^45*d^53)
+ 105059972107298930688*a^31*b^54*c^91*d^4 - 4202398884291957227520*a^32*b^
53*c^90*d^5 + 81456498373859104260096*a^33*b^52*c^89*d^6 - 1019470840448604
438528000*a^34*b^51*c^88*d^7 + 9261585187779405523451904*a^35*b^50*c^87*d^8
 - 65094971944398671145074688*a^36*b^49*c^86*d^9 + 368402395453916323189358
592*a^37*b^48*c^85*d^10 - 1725226316150928144278224896*a^38*b^47*c^84*d^11
+ 6817742452202868128486522880*a^39*b^46*c^83*d^12 - 2307150519506493105288
6687744*a^40*b^45*c^82*d^13 + 67614089216123669492331970560*a^41*b^44*c^81*
d^14 - 173115025562473785468905324544*a^42*b^43*c^80*d^15 + 389913831719674
713212222177280*a^43*b^42*c^79*d^16 - 776790088912432141093966970880*a^44*b
^41*c^78*d^17 + 1374611983251272530469308071936*a^45*b^40*c^77*d^18 - 21674
54612994156285048662261760*a^46*b^39*c^76*d^19 + 30503373104297005350040759
17312*a^47*b^38*c^75*d^20 - 3826885622871496570502324944896*a^48*b^37*c^74*
d^21 + 4238713393375513383921726259200*a^49*b^36*c^73*d^22 - 39842918963450
24467843348955136*a^50*b^35*c^72*d^23 + 2651971426464597412032295206912*a^5
1*b^34*c^71*d^24 + 479249403658129639733534392320*a^52*b^33*c^70*d^25 - 669
7452529698647734837548417024*a^53*b^32*c^69*d^26 + 179310542699951499982776
82790400*a^54*b^31*c^68*d^27 - 36311715021905634799784747335680*a^55*b^30*c
^67*d^28 + 63073617076394089001091166371840*a^56*b^29*c^66*d^29 - 971055651
68138147055402127196160*a^57*b^28*c^65*d^30 + 133993666277013207597272619O2
4384*a^58*b^27*c^64*d^31 - 166492084833102044695859350732800*a^59*b^26*c^63
*d^32 + 186717161118223967667066928889856*a^60*b^25*c^62*d^33 - 18923562415
3406619951659086774272*a^61*b^24*c^61*d^34 + 173421825288151984221422006304
768*a^62*b^23*c^60*d^35 - 143715376746696050902973036888064*a^63*b^22*c^59*
d^36 + 107645128880801788128312132894720*a^64*b^21*c^58*d^37 - 728021692097
14119238549751463936*a^65*b^20*c^57*d^38 + 44389639270136779232591657041920
*a^66*b^19*c^56*d^39 - 24348625105436875280486976454656*a^67*b^18*c^55*d^40
 + 11981145511938522697620070072320*a^68*b^17*c^54*d^41 - 526975932508991O2
60644729323520*a^69*b^16*c^53*d^42 + 2062471522530027433706750214144*a^70*b
```

```
^15*c^52*d^43 - 714227824367410213467319173120*a^71*b^14*c^51*d^44 + 217305
373751493983005392764928*a^72*b^13*c^50*d^45 - 575744111484335694244416061 4
4*a^73*b^12*c^49*d^46 + 131339473607338820653547520 00*a^74*b^11*c^48*d^47 -
 2542019460242050797665255424*a^75*b^10*c^47*d^48 + 409310322447365741947 65
0048*a^76*b^9*c^46*d^49 - 5335664969179313423253504 0*a^77*b^8*c^45*d^50 + 5
41059492457889361454694 4*a^78*b^7*c^44*d^51 - 40046419543731889766400 0*a^79
*b^6*c^43*d^52 + 192462892261798890700 80*a^80*b^5*c^42*d^53 - 4508139818744
83888128*a^81*b^4*c^41*d^54) - x^(1/2)*(11934221933169573101568 0*a^30*b^49*
c^73*d^13 - 3677615218076424339456*a^29*b^50*c^74*d^12 - 185601344303097242
5568256*a^31*b^48*c^72*d^14 + 18426099996452807258406912*a^32*b^47*c^71*d^1
5 - 131228123459738637629915136*a^33*b^46*c^70*d^16 + 714182072565091774626
791424*a^34*b^45*c^69*d^17 - 3088237415348484431457288192*a^35*b^44*c^68*d^
18 + 10882952503625649640326561792*a^36*b^43*c^67*d^19 - 317570746004740778
03581538304*a^37*b^42*c^66*d^20 + 77306011497125960924962750464*a^38*b^41*c
^65*d^21 - 156439291025195069838804910080*a^39*b^40*c^64*d^22 + 25696744636
1217518429496410112*a^40*b^39*c^63*d^23 - 315930266538485089912448090112*a^
41*b^38*c^62*d^24 + 193264836517334230347779407872*a^42*b^37*c^61*d^25 + 32
0732651390132179677984325632*a^43*b^36*c^60*d^26 - 1433302686817582744983 68
3727360*a^44*b^35*c^59*d^27 + 3214765851097197421262933065728*a^45*b^34*c^5
8*d^28 - 5465398361763642490480861642752*a^46*b^33*c^57*d^29 + 769072869548
0443198104101978112*a^47*b^32*c^56*d^30 - 9256447758824794945376420364288*a
^48*b^31*c^55*d^31 + 9672669866587270697877661286400*a^49*b^30*c^54*d^32 -
8839280066432157154484139589632*a^50*b^29*c^53*d^33 + 708682206708916952291
2760168448*a^51*b^28*c^52*d^34 - 4988522538878293079151039479808*a^52*b^27*
c^51*d^35 + 3079795601090740527825181212672*a^53*b^26*c^50*d^36 - 166334191
9096805892341077377024*a^54*b^25*c^49*d^37 + 782666038849476274770105335808
*a^55*b^24*c^48*d^38 - 319013552886948801896949743616*a^56*b^23*c^47*d^39 +
 111766668098727585639133347840*a^57*b^22*c^46*d^40 - 333122072940982585808
51392512*a^58*b^21*c^45*d^41 + 8330791306287661611887886336*a^59*b^20*c^44*
d^42 - 1715502625948903704153292800*a^60*b^19*c^43*d^43 + 283282946101439 32
4535914496*a^61*b^18*c^42*d^44 - 36069332470586798845722624*a^62*b^17*c^41*
d^45 + 3324850588931239515783168*a^63*b^16*c^40*d^46 - 197512325498721785 61
0240*a^64*b^15*c^39*d^47 + 5678869390326597943296*a^65*b^14*c^38*d^48))*(((
(767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*
c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^
33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^
31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c
^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^
27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*
a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 5634769849329
2544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520
459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 11
80507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*
d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*
b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 217433192156964
```

```
12672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 2054
8937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*
d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*
b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*
a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 8420024911321497
6*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 273343370143334
4*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b
^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*
d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 -
 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 2
9104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 -
94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 -
 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21
- 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23
)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*
a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^3
0*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^2
8*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*
b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*
a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867
248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 1551
6365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*
d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28
*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066
641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 238
71332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29
*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*
b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 72281207215226880
0*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 622730400623
49312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 247115238
3426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 340848604
61056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^3
7*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d
^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5
 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7
 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^
9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^2
6*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^2
4*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15
*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 17520455870952652
8*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302
787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5
916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15
*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^2
4*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538
```

```
768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2798
565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 +
 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^3
1 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^3
3 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35
+ 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*
(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^3
1 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45
*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20
*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9
+ 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*
a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c
^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 -
565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a
^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^
28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 1051
8300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23
*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^
3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4) - (((((767544201216*a^37*d^37
 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063
872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 50876864570327
04*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135
616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 1146619160596
31616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 15915865
2345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 9
9690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^1
4 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^2
1*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*
b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 669202308967926
9888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 174746
66762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^
23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b
^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 1017318402352182
0672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 25970662
72630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323
237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 1
7373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 3
08246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 2219973997
3632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 10657194751061*
b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b
^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^
26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b
^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*
b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9
*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^3
```

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2*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199
023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 3408486046
10560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 1383845334
7188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 2312998
63088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 19
27498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d
^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b
^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 3239680774672220
1600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305
929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d
^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*
b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 886649475173449
7280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 192749
8859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 -
231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^2
6 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d
^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^3
0))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576
*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^
4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6
*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^
8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a
^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185
088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 281738492
46646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351
260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 +
 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^1
8*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21
*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 108716596078482
06336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 1027
4468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d
^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^
9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^3
0*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a
^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^
34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*
c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*
c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*
d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27
*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 105183
00*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^
39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 3
47373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^
28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^3
2*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 22
```

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5792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35
*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24
 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5
*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*
b^2*c^19*d^30)))^(3/4)*(x^(1/2)*((((767544201216*a^37*d^37 + 110075314176*b
^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34
*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*
d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30
*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^
28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b
^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*
a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 1354372497845
4528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 35040911
7419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 +
3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^1
7*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^
22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 229247423647
44450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1
5678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^
10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*
b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a
^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*
a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^
35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^3
6)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 -
1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550
655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 636
13900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98
620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 3
8907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 +
3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476
736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*
c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c
^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27
*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*
b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800
*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751
734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 2
3871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^
35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a
^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746
722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 +
15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^
28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^3
6*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 23129986308872601
```

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6*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 1383845334718
8736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 34084860461
0560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037
657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^
2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 +
 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 +
18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 +
 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^1
0 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^2
5*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^2
3*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16
*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506l
12*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 33460115
44839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 +
 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c
^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*
a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 50865920117
60910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298
533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 -
161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32
- 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 -
 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869
986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^
14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b
^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 90619
2*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41
*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 1290244
80*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^1
9*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15
 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 47143560
0*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12
*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 -
 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b
^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 3596
0*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^
(1/4)*(93386641873154605056*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^3
5*b^52*c^93*d^5 + 78828702034483948290048*a^36*b^51*c^92*d^6 - 103467211048
6845715906560*a^37*b^50*c^91*d^7 + 989254036026514046818713 6*a^38*b^49*c^90
*d^8 - 7344022016434813734695731 2*a^39*b^48*c^89*d^9 + 4406493833661705397 6
2647040*a^40*b^47*c^88*d^10 - 21962372532340924653879951 36*a^41*b^46*c^87*d
^11 + 92742963161445956466990120 96*a^42*b^45*c^86*d^12 - 3367788150104699 33
39969175552*a^43*b^44*c^85*d^13 + 106376530102998491281999527936*a^44*b^43*
c^84*d^14 - 294921432301504798990377418752*a^45*b^42*c^83*d^15 + 7229030451
42137525367365173248*a^46*b^41*c^82*d^16 - 1576072447576504233275626094592*
a^47*b^40*c^81*d^17 + 3072471208539973972578986360832*a^48*b^39*c^80*d^18 -
```

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 5384106777252432871416869683200*a^49*b^38*c^79*d^19 + 853735159835492549 68
36275830784*a^50*b^37*c^78*d^20 - 12376921822825560832675204300800*a^51*b^3
6*c^77*d^21 + 16707589390432621056738749054976*a^52*b^35*c^76*d^22 - 216671
30911214476307455165857792*a^53*b^34*c^75*d^23 + 2821120761879315794468920 0
988160*a^54*b^33*c^74*d^24 - 38378393138521379212996695293952*a^55*b^32*c^7
3*d^25 + 54918846093258397577855222415360*a^56*b^31*c^72*d^26 - 80082941438
212170767896978391040*a^57*b^30*c^71*d^27 + 1138884263877296291462565656002
56*a^58*b^29*c^70*d^28 - 152754106500312545531177547595776*a^59*b^28*c^69*d
^29 + 189549778508563263438068404715520*a^60*b^27*c^68*d^30 - 2155465182348
22631781377148715008*a^61*b^26*c^67*d^31 + 2236418963088558734571650364211 2
0*a^62*b^25*c^66*d^32 - 211293730951350565888869600854016*a^63*b^24*c^65*d^
33 + 181575241776706668284956756672512*a^64*b^23*c^64*d^34 - 14179414961960
0448829729705820160*a^65*b^22*c^63*d^35 + 100511576025621687034384100622336
*a^66*b^21*c^62*d^36 - 64581123553243990572098666889216*a^67*b^20*c^61*d^37
 + 37540992634094717640084094451712*a^68*b^19*c^60*d^38 - 19695179695689601
910490494140416*a^69*b^18*c^59*d^39 + 9296840942046414522746815905792*a^70*
b^17*c^58*d^40 - 3933446196282108795457464434688*a^71*b^16*c^57*d^41 + 1484
644864880431945098662510592*a^72*b^15*c^56*d^42 - 496993877333119536381277 7
65632*a^73*b^14*c^55*d^43 + 146493707302289292776429322240*a^74*b^13*c^54*d
^44 - 37679005999847399095674077184*a^75*b^12*c^53*d^45 + 83600946239911812
23468728320*a^76*b^11*c^52*d^46 - 1576546523407725355918688256*a^77*b^10*c^
51*d^47 + 247744258459119342197932032*a^78*b^9*c^50*d^48 - 315661360129261 9
5282739200*a^79*b^8*c^49*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^5
0 - 227270011883594899783680*a^81*b^6*c^47*d^51 + 10717576321223758970880*a
^82*b^5*c^46*d^52 - 246599101196298878976*a^83*b^4*c^45*d^53) - 10505997210
7298930688*a^31*b^54*c^91*d^4 + 4202398884291957227520*a^32*b^53*c^90*d^5 -
 81456498373859104260096*a^33*b^52*c^89*d^6 + 1019470840448604438528000*a^3
4*b^51*c^88*d^7 - 9261585187779405523451904*a^35*b^50*c^87*d^8 + 6509497194
4398671145074688*a^36*b^49*c^86*d^9 - 368402395453916323189358592*a^37*b^48
*c^85*d^10 + 1725226316150928144278224896*a^38*b^47*c^84*d^11 - 68177424522
02868128486522880*a^39*b^46*c^83*d^12 + 23071505195064931052886687744*a^40*
b^45*c^82*d^13 - 67614089216123669492331970560*a^41*b^44*c^81*d^14 + 173115
025562473785468905324544*a^42*b^43*c^80*d^15 - 389913831719674713212222177 2
80*a^43*b^42*c^79*d^16 + 776790088912432141093966970880*a^44*b^41*c^78*d^17
 - 1374611983251272530469308071936*a^45*b^40*c^77*d^18 + 2167454612994156 28
5048662261760*a^46*b^39*c^76*d^19 - 3050337310429700535004075917312*a^47*b^
38*c^75*d^20 + 3826885622871496570502324944896*a^48*b^37*c^74*d^21 - 423871
3393375513383921726259200*a^49*b^36*c^73*d^22 + 398429189634502446784334895
5136*a^50*b^35*c^72*d^23 - 2651971426464597412032295206912*a^51*b^34*c^71*d
^24 - 479249403658129639733534392320*a^52*b^33*c^70*d^25 + 6697452529698647
734837548417024*a^53*b^32*c^69*d^26 - 17931054269995149998277682790400*a^54
*b^31*c^68*d^27 + 36311715021905634799784747335680*a^55*b^30*c^67*d^28 - 63
073617076394089001091166371840*a^56*b^29*c^66*d^29 + 971055651681381470554 0
2127196160*a^57*b^28*c^65*d^30 - 133993666277013207597272619024384*a^58*b^2
7*c^64*d^31 + 166492084833102044695859350732800*a^59*b^26*c^63*d^32 - 18671
```

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71611182239676670669288898856*a^60*b^25*c^62*d^33 + 189235624153406619951659
086774272*a^61*b^24*c^61*d^34 - 1734218252881519842214220063047680*a^62*b^23
*c^60*d^35 + 1437153767466960509029730368880064*a^63*b^22*c^59*d^36 - 107645
12888080178812831213289472O*a^64*b^21*c^58*d^37 + 728021692097141192385497 5
1463936*a^65*b^20*c^57*d^38 - 44389639270136779232591657041920*a^66*b^19*c^
56*d^39 + 24348625105436875280486976454656*a^67*b^18*c^55*d^40 - 1198114551
193852269762007007232O*a^68*b^17*c^54*d^41 + 526975932508991026064472932352
0*a^69*b^16*c^53*d^42 - 2062471522530027433706750214144*a^70*b^15*c^52*d^43
 + 714227824367410213467319173120*a^71*b^14*c^51*d^44 - 217305373751493983O
05392764928*a^72*b^13*c^50*d^45 + 57574411148433569424441606144*a^73*b^12*c
^49*d^46 - 13133947360733882065354752000*a^74*b^11*c^48*d^47 + 254201946024
2050797665255424*a^75*b^10*c^47*d^48 - 409310322447365741947650048*a^76*b^9
*c^46*d^49 + 53356649691793134232535040*a^77*b^8*c^45*d^50 - 54105949245788
93614546944*a^78*b^7*c^44*d^51 + 400464195437318897664000*a^79*b^6*c^43*d^5
2 - 19246289226179889070080*a^80*b^5*c^42*d^53 + 450813981874483888128*a^81
*b^4*c^41*d^54) - x^(1/2)*(119342219331695731015680*a^30*b^49*c^73*d^13 - 3
6776152180764243394456*a^29*b^50*c^74*d^12 - 18560134430309724255682256*a^31*
b^48*c^72*d^14 + 184260999964528072584069l2*a^32*b^47*c^71*d^15 - 131228123
4597386376299151 36*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b
^45*c^69*d^17 - 3088237415348484431457288192*a^35*b^44*c^68*d^18 + 10882952
50362564964032656l792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a
^37*b^42*c^66*d^20 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 15
6439291025195069838804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496
410112*a^40*b^39*c^63*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*
d^24 + 193264836517334230347779407872*a^42*b^37*c^61*d^25 + 320732651390132
179677984325632*a^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*
b^35*c^59*d^27 + 3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 5465
398361763642490480861642752*a^46*b^33*c^57*d^29 + 769072869548044319810410 1
978112*a^47*b^32*c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55
*d^31 + 9672669866587270697877661286400*a^49*b^30*c^54*d^32 - 883928006643 2
1571544841395 89632*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^
51*b^28*c^52*d^34 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 3
079795601090740527825181212672*a^53*b^26*c^50*d^36 - 1663341919096805892341
077377024*a^54*b^25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^
48*d^38 - 319013552886948801896949743616*a^56*b^23*c^47*d^39 + 111766668098
727585639133347840*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58
*b^21*c^45*d^41 + 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 171550
2625948903704153292800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^
61*b^18*c^42*d^44 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 332485
058893123951578316 8*a^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^1
5*c^39*d^47 + 5678869390326597943296*a^65*b^14*c^38*d^48))*((((767544201216
*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 24
8052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087
686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 3720
0150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114
```

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661916059631616*a^9*b^28*c^28*d^9 + 1490305003825397776*a^10*b^27*c^27*d^10
- 1591586523457781776*a^11*b^26*c^26*d^11 + 1394650235283701776*a^12*b^25*c^2
5*d^12 - 996907513125888000*a^13*b^24*c^24*d^13 + 563476984932925544*a^14*b^2
3*c^23*d^14 - 135437249784545528*a^15*b^22*c^22*d^15 - 707025204592312232*a^1
6*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 118050703576901
2224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 669202
3089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^2
1 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^
14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642
176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 101731
84023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28
+ 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*
d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^
5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^
3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d -
22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571
947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380
252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391
916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 9452121626881
4328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 733706519084
86968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165
402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(6871947673
6*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^4
8*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 -
340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 -
13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6
 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^4
1*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b
^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370
240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 323968
07746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^
15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b
^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 238713320239005
69600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 88664
94751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^2
2 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^
25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^
6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*
b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^
2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 1662
1372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 64995871
7915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797
106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 3616774902
5660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 745152501
91269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 6973
```

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2511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 -
 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^
15 + 35351260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c
^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^1
9*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 59161306356285
41952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10871
659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d
^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b
^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 27985654599023001
60*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543
903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 421001245
56607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 136671685
0716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 13942873128
96*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^
49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*
b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376
*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^
7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^
23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^3
7*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 56
5722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^3
0*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30
*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 645
12240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8
*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 20137
6*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 +
 496*a^43*b^2*c^19*d^30)))^(1/4) + 410283947766651090370 56*a^29*b^48*c^68*d
^15 - 1210739885076097825505280*a^30*b^47*c^67*d^16 + 172436287687809497479
24992*a^31*b^46*c^66*d^17 - 158081319004444765483696128*a^32*b^45*c^65*d^18
 + 1049494986915760527133114368*a^33*b^44*c^64*d^19 - 538068304649035443813
6397824*a^34*b^43*c^63*d^20 + 22176160052724101903372255232*a^35*b^42*c^62*
d^21 - 75486313325241636679770439680*a^36*b^41*c^61*d^22 + 2162883756151096
59684325294080*a^37*b^40*c^60*d^23 - 528818181695424054504437317632*a^38*b^
39*c^59*d^24 + 1114222690302433619242395893760*a^39*b^38*c^58*d^25 - 203754
5055293058005529639518208*a^40*b^37*c^57*d^26 + 324991885790433797585082777
6000*a^41*b^36*c^56*d^27 - 4536394700759564584125915463680*a^42*b^35*c^55*d
^28 + 5552435240283931429496420302848*a^43*b^34*c^54*d^29 - 596429082568322
4886861470105600*a^44*b^33*c^53*d^30 + 5621639355410781338712284332032*a^45
*b^32*c^52*d^31 - 4644077108074496901042866749440*a^46*b^31*c^51*d^32 + 335
5360862716129153108295024640*a^47*b^30*c^50*d^33 - 211340528170478221509350
6015232*a^48*b^29*c^49*d^34 + 1155283596049337948225918730240*a^49*b^28*c^4
8*d^35 - 544829519870376944469402451968*a^50*b^27*c^47*d^36 + 2199261720378
99117268712816640*a^51*b^26*c^46*d^37 - 75201916274561138554746961920*a^52*
b^25*c^45*d^38 + 21483948869172056418164932608*a^53*b^24*c^44*d^39 - 503234
6201606164325320359936*a^54*b^23*c^43*d^40 + 941275744618015035796488192*a^
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55*b^22*c^42*d^41 - 135189136301093329947328512*a^56*b^21*c^41*d^42 + 13999
140307267180988203008*a^57*b^20*c^40*d^43 - 930460907799665663016960*a^58*b
^19*c^39*d^44 + 29814064299214639202304*a^59*b^18*c^38*d^45))*((((767544201
216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 -
 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5
087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 3
7200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 -
114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^
10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*
c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*
b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*
a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 118050703576
9012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 669
2023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*
d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23
*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158
642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 101
73184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^
28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c
^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5
*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3
*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d
 - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106
571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396
380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578
391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 9452121626
8814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 733706519
08486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588
165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(6871947
6736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*
c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2
 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4
 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*
d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*
c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^2
3*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535
370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 323
96807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34
*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^3
0*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 2387133202390
0569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 88
66494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*
d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8
*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39
*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^
```

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41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43
*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 1
6621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 64995
8717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627
797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 3616774
9025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 745152
50191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 6
9732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^1
3 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22
*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^2
0*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*
a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 59161306356
28541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10
871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^1
3*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^2
6*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 27985654599023
00160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981
543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 421001
24556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 136671
6850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 13942873
12896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32
*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^
15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201
376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42
*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240
*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*
c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 -
 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*
a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c
^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 +
64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*
b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 20
1376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^2
9 + 496*a^43*b^2*c^19*d^30)))^(1/4)*2i - (2/(5*a*c) - (2*x^2*(13*a*d + 9*b*
c))/(5*a^2*c^2) + (3*x^6*(195*a^5*d^6 - 240*b^5*c^5*d + 224*a*b^4*c^4*d^2 +
 448*a^2*b^3*c^3*d^3 - 623*a^3*b^2*c^2*d^4 - 84*a^4*b*c*d^5))/(80*a^3*c^3*(
b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (x^4*(360*b^5*c^5
 - 1053*a^5*d^5 - 1280*a^2*b^3*c^3*d^2 + 64*a^3*b^2*c^2*d^3 + 96*a*b^4*c^4*
d + 1933*a^4*b*c*d^4))/(80*a^3*c^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2)) + (3*b*d^2*x^8*(39*a^4*d^4 - 24*b^4*c^4 + 32*a^2*b^2*c^2*d^2 +
 32*a*b^3*c^3*d - 87*a^3*b*c*d^3))/(16*a^3*c^3*(b^3*c^4 - a^3*c*d^3 + 3*a^2
*b*c^2*d^2 - 3*a*b^2*c^3*d)))/(x^(9/2)*(b*c^2 + 2*a*c*d) + x^(13/2)*(a*d^2
+ 2*b*c*d) + a*c^2*x^(5/2) + b*d^2*x^(17/2)) + atan((((-(383772100608*a^37*
d^37 + 55037657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37
*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^
```

```
3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5
 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^
7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*
d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26
*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^1
3*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 1354372497845452
8*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 35040911741
9053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312
2430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d
^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*
b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 229247423647444
50048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1567
8268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*
d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8
*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31
*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^3
3*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*
b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^
2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 144
6035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655
952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 636139
00184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620
802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 3890
7153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 357
4683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736
*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^1
8*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46
*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^
44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^2
5*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^
22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 88664947517344
97280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 2387
1332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*
d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29
*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722
201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 155
16365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*
d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b
^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a
^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 1383845334718873
6*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 34084860461056
0*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372
440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915
136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 77186277971066
88*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660
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928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 7451525019126
9888*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511
764185088*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 281
73849246646272*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 -
 35351260229615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*
d^17 - 590253517884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^
18*c^18*d^19 - 3346011544839634944*a^20*b^17*c^17*d^20 + 591613063562854195
2*a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 108716596
07848206336*a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24
+ 10274468596079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*
c^11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a
^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 5039429815439032
32*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 4210012455660
7488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716
672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a
*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 +
 a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30
*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^1
8*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 +
10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b
^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^
12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722
720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^
15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^1
9 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 6451224
0*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^2
5*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^
40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496
*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*(-(383772100608*a^37*d^37 + 550376570
88*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744
881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 129991743583
0272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 1543725559421
3376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 723354980513
21856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 1490305003
82539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139
465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13
 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22
*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^2
0*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888
*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 1183226127
1257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 +
 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*
c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768
*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 559713091
9804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007
```

```
885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 -
84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 -
 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2
788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (364435458
48801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*
b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^
27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*
b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6
*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^
8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^
10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32
- 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 3408486
0461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 247115238
3426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 6227304
0062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722
812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9
 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*
c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600
*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 388761692
96066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16
- 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14
*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 1551636581553537024
0*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 443324737
5867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 7
22812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^2
5 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*
d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*
d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*
c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^3
3*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31
*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^2
9*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c
^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b
^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a
^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 353512602296156
16*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 590253517
884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 3
346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16
*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23
*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079
321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 5086
592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28
 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*
d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^
5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3
```

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*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 1
1099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32
- 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960
*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5
+ 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^
24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 -
129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a
^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^
34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 4
71435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^
33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27
*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856
*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27
 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d
^30)))^(1/4)*(93386641873154605056*a^34*b^53*c^94*d^4 - 3891110078048108544
000*a^35*b^52*c^93*d^5 + 78828702034483948290048*a^36*b^51*c^92*d^6 - 10346
72110486845715906560*a^37*b^50*c^91*d^7 + 9892540360265140468187136*a^38*b^
49*c^90*d^8 - 73440220164348137346957312*a^39*b^48*c^89*d^9 + 4406493833661
70539762647040*a^40*b^47*c^88*d^10 - 2196237253234092465387995136*a^41*b^46
*c^87*d^11 + 9274296316144595646699012096*a^42*b^45*c^86*d^12 - 33677881501
046993339969175552*a^43*b^44*c^85*d^13 + 106376530102998491281999527936*a^4
4*b^43*c^84*d^14 - 294921432301504798990377418752*a^45*b^42*c^83*d^15 + 722
903045142137525367365173248*a^46*b^41*c^82*d^16 - 1576072447576504233275626
094592*a^47*b^40*c^81*d^17 + 3072471208539973972578986360832*a^48*b^39*c^80
*d^18 - 5384106777252432871416869683200*a^49*b^38*c^79*d^19 + 8537351598354
925496836275830784*a^50*b^37*c^78*d^20 - 12376921822825560832675204300800*a
^51*b^36*c^77*d^21 + 16707589390432621056738749054976*a^52*b^35*c^76*d^22 -
 21667130911214476307455165857792*a^53*b^34*c^75*d^23 + 28211207618793157944
689200988160*a^54*b^33*c^74*d^24 - 38378393138521379212996695293952*a^55*b
^32*c^73*d^25 + 54918846093258397577855222415360*a^56*b^31*c^72*d^26 - 8008
2941438212170767896978391040*a^57*b^30*c^71*d^27 + 113888426387729629146256
565600256*a^58*b^29*c^70*d^28 - 152754106500312545531177547595776*a^59*b^28
*c^69*d^29 + 189549778508563263438068404715520*a^60*b^27*c^68*d^30 - 215546
518234822631781377148715008*a^61*b^26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 211293730951350565888869600854016*a^63*b^24*
c^65*d^33 + 181575241776706668284956756672512*a^64*b^23*c^64*d^34 - 1417941
496196004488297297058201600*a^65*b^22*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 64581123553243990572098666889216*a^67*b^20*c^
61*d^37 + 37540992634094717640084094451712*a^68*b^19*c^60*d^38 - 1969517969
5689601910490494140416*a^69*b^18*c^59*d^39 + 929684094204641452274681590579
2*a^70*b^17*c^58*d^40 - 3933446196282108795457464434688*a^71*b^16*c^57*d^41
 + 1484644864880431945098662510592*a^72*b^15*c^56*d^42 - 496993877333119536
381277765632*a^73*b^14*c^55*d^43 + 146493707302289292776429322240*a^74*b^13
*c^54*d^44 - 37679005999847399095674077184*a^75*b^12*c^53*d^45 + 8360094623
991181223468728320*a^76*b^11*c^52*d^46 - 1576546523407725355918688256*a^77*
```

b^10*c^51*d^47 + 2477442584591193421979320 32*a^78*b^9*c^50*d^48 - 315661360
12926195282739200*a^79*b^8*c^49*d^49 + 3133065413748205302054912*a^80*b^7*c
^48*d^50 - 2272700118835948997836 80*a^81*b^6*c^47*d^51 + 107175763212237589
70880*a^82*b^5*c^46*d^52 - 2465991011962988789 76*a^83*b^4*c^45*d^53) - 1050
5997210729893068 8*a^31*b^54*c^91*d^4 + 4202398884291957227520*a^32*b^53*c^9
0*d^5 - 8145649837385910426009 6*a^33*b^52*c^89*d^6 + 101947084044860443852 8
000*a^34*b^51*c^88*d^7 - 926158518777940552345190 4*a^35*b^50*c^87*d^8 + 650
9497194439867114507468 8*a^36*b^49*c^86*d^9 - 368402395453916323189358592*a^
37*b^48*c^85*d^10 + 1725226316150928144278224896*a^38*b^47*c^84*d^11 - 6817
742452202868128486522880*a^39*b^46*c^83*d^12 + 230715051950649310528866877 4
4*a^40*b^45*c^82*d^13 - 67614089216123669492331970560*a^41*b^44*c^81*d^14 +
173115025562473785468905324544*a^42*b^43*c^80*d^15 - 389913831719674713212
222177280*a^43*b^42*c^79*d^16 + 776790088912432141093966970880*a^44*b^41*c^
78*d^17 - 1374611983251272530469308071936*a^45*b^40*c^77*d^18 + 2167454612 9
9415628504866226176 0*a^46*b^39*c^76*d^19 - 3050337310429700535004075917312*
a^47*b^38*c^75*d^20 + 3826885622871496570502324944896*a^48*b^37*c^74*d^21 -
4238713393375513383921726259200*a^49*b^36*c^73*d^22 + 398429189634502446 78
43348955136*a^50*b^35*c^72*d^23 - 2651971426464597412032295206912*a^51*b^34
*c^71*d^24 - 479249403658129639733534392320*a^52*b^33*c^70*d^25 + 669745252
9698647734837548417024*a^53*b^32*c^69*d^26 - 179310542699951499982776827904
00*a^54*b^31*c^68*d^27 + 36311715021905634799784747335680*a^55*b^30*c^67*d^
28 - 63073617076394089001091166371840*a^56*b^29*c^66*d^29 + 971055651681381
47055402127196160*a^57*b^28*c^65*d^30 - 133993666277013207597272619024384*a
^58*b^27*c^64*d^31 + 166492084833102044695859350732800*a^59*b^26*c^63*d^32
- 186717161118223967667066928889856*a^60*b^25*c^62*d^33 + 1892356241534066 1
995165908677427 2*a^61*b^24*c^61*d^34 - 173421825288151984221422006304768*a^
62*b^23*c^60*d^35 + 143715376746696050902973036888064*a^63*b^22*c^59*d^36 -
107645128880801788128312132894720*a^64*b^21*c^58*d^37 + 728021692097141192
38549751463936*a^65*b^20*c^57*d^38 - 44389639270136779232591657041920*a^66*
b^19*c^56*d^39 + 24348625105436875280486976454656*a^67*b^18*c^55*d^40 - 119
81145511938522697620070072320*a^68*b^17*c^54*d^41 + 526975932508991026064 47
29323520*a^69*b^16*c^53*d^42 - 2062471522530027433706750214144*a^70*b^15*c^
52*d^43 + 714227824367410213467319173120*a^71*b^14*c^51*d^44 - 217305373751
49398300539276492 8*a^72*b^13*c^50*d^45 + 57574411148433569424441606144*a^73
*b^12*c^49*d^46 - 13133947360733882065354752000*a^74*b^11*c^48*d^47 + 25420
19460242050797665255424*a^75*b^10*c^47*d^48 - 409310322447365741947650048*a
^76*b^9*c^46*d^49 + 53356649691793134232535040*a^77*b^8*c^45*d^50 - 5410594
924578893614546944*a^78*b^7*c^44*d^51 + 400464195437318897664000*a^79*b^6*c
^43*d^52 - 19246289226179889070080*a^80*b^5*c^42*d^53 + 450813981874483888 1
28*a^81*b^4*c^41*d^54) - x^(1/2)*(1193422193316957310156 80*a^30*b^49*c^73*d
^13 - 367761521807642433945 6*a^29*b^50*c^74*d^12 - 185601344303097242556825
6*a^31*b^48*c^72*d^14 + 1842609999645280725840691 2*a^32*b^47*c^71*d^15 - 13
1228123459738637629915136*a^33*b^46*c^70*d^16 + 714182072565091774626791424
*a^34*b^45*c^69*d^17 - 3088237415348484431457288192*a^35*b^44*c^68*d^18 + 1
0882952503625649640326561792*a^36*b^43*c^67*d^19 - 317570746004740778035815

38304*a^37*b^42*c^66*d^20 + 773060114971259609249627504464*a^38*b^41*c^65*d^
21 - 1564392910251950698388049100800*a^39*b^40*c^64*d^22 + 25696744636121751
8429496410112*a^40*b^39*c^63*d^23 - 3159302665384850899124480900112*a^41*b^3
8*c^62*d^24 + 193264836517334230347779407872*a^42*b^37*c^61*d^25 + 32073265
13901321796779843256320*a^43*b^36*c^60*d^26 - 143330268681758274498368372736
0*a^44*b^35*c^59*d^27 + 3214765851097197421262933065728*a^45*b^34*c^58*d^28
- 5465398361763642490480861642752*a^46*b^33*c^57*d^29 + 769072869548044319
8104101978112*a^47*b^32*c^56*d^30 - 9256447758824794945376420364288*a^48*b^
31*c^55*d^31 + 9672669866587270697877661286400*a^49*b^30*c^54*d^32 - 883928
0066432157154484139589632*a^50*b^29*c^53*d^33 + 708682206708916952291276016
8448*a^51*b^28*c^52*d^34 - 4988522538878293079151039479808*a^52*b^27*c^51*d
^35 + 3079795601090740527825181212672*a^53*b^26*c^50*d^36 - 166334191909680
5892341077377024*a^54*b^25*c^49*d^37 + 782666038849476274770105335808*a^55*
b^24*c^48*d^38 - 319013552886948801896949743616*a^56*b^23*c^47*d^39 + 11176
6668098727585639133347840*a^57*b^22*c^46*d^40 - 333122072940982585808513925
12*a^58*b^21*c^45*d^41 + 8330791306287661611887886336*a^59*b^20*c^44*d^42 -
1715502625948903704153292800*a^60*b^19*c^43*d^43 + 283282946101439324535914
4496*a^61*b^18*c^42*d^44 - 36069332470586798845722624*a^62*b^17*c^41*d^45 +
3324850588931239515783168*a^63*b^16*c^40*d^46 - 197512325498721785610240*a
^64*b^15*c^39*d^47 + 5678869390326597943296*a^65*b^14*c^38*d^48))*(-(383772
100608*a^37*d^37 + 55037657088*b^37*c^37 + ((767544201216*a^37*d^37 + 11007
5314176*b^37*c^37 + 3324274488 1152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*
b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b
^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*
b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^
9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 1591586523457781
76*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 996907513
12588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 1354
3724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 +
350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^1
9*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20
*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617
159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 229
24742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12
*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^2
7*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370
304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 32323718022
9304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 173731837
36946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962
323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^3
6*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^1
2*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^
24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d
^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*
d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5

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*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^
3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 +
 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 219902325555
2*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^
16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*
a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726
016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859
072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 88
66494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37
*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^2
7*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 4130592987707
0807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32
396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^3
0*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^3
4*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 19274988590727
16800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 23129986
3088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 1383
8453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 34
0848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2
) + 16621372440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 +
649958717915136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 77
18627797106688*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36
167749025660928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 7
4515250191269888*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^1
1 + 69732511764185088*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^2
4*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22
*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^1
7*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^19*d^18 + 156121530278753
8944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20*b^17*c^17*d^20 + 591613
0635628541952*a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22
 + 10871659607848206336*a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^1
3*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d^25 - 783913403053876838
4*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^27 - 279856545
9902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 5039
42981543903232*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 4
2100124556607488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1
366716850716672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 139
4287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13
*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 4
96*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4
- 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25
*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 645
12240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*
b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d
^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 56572
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2720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b
^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^
21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*
a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26
 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^2
0*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*1i + ((-(383772100608*a^37*d^37 +
55037657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 +
 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 129
9917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 1543
7255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 723
35498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 1
49030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d
^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*
c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*
b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056
*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605
575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 1
1832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^
15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a
^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061
077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 -
5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^
29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^
6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c
^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2
*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (
36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052
490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 85506559526615
22*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 6361390018439
4735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 986208026593
91292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228
163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057
023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c
^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31
+ 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 +
2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5
+ 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*
d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23
*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a
^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023
900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 -
38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c
^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*
a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 1551636581
5535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 +
```

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4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26
*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7
*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*
b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*
b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*
a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4
*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*
b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8
*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^
10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 697325117641850
88*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 2817384924
6646272*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 353512
60229615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 -
590253517884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18
*d^19 - 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*
b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 1087165960784820
6336*a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274
468596079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^
26 + 5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9
*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30
*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^
32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^3
4*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c
^36*d - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c
^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d
^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*
c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 1051830
0*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^3
9*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 34
7373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^2
8*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32
*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225
792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*
b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24
- 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*
c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b
^2*c^19*d^30)))^(3/4)*(x^(1/2)*(-(383772100608*a^37*d^37 + 55037657088*b^37
*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*
a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^
4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^
6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a
^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14903050038253977
6*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 1394650235
28370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 5634
7698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 -
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 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*
d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b
^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083
904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 217433
19215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^
24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b
^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600
320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 10078859630
87806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 8420024
9113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 273343
3701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 27885746
25792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a
^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^
11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10
*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^
8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c
^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*
c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19
*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 21990
23255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056
*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560
*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349
312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 7228120721
52268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433
247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^
11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b
^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3887616929606664
1920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876
169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d
^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*
b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248
640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 72281207
2152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 622
73040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 +
2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 +
34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^
2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 -
 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 -
18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 -
 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^1
0 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^2
5*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^2
3*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16
*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 5902535178845061
12*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 33460115
```

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44839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 -
 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c
^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*
a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 50865920117
60910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1298
533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 +
161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32
+ 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 +
 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869
986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^
14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b
^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 90619
2*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41
*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 1290244
80*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^1
9*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15
 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 47143560
0*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12
*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 -
 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b
^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 3596
0*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^
(1/4)*(9338664187315460505 6*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^3
5*b^52*c^93*d^5 + 78828702034483948290048*a^36*b^51*c^92*d^6 - 103467211048
6845715906560*a^37*b^50*c^91*d^7 + 9892540360265140468187136*a^38*b^49*c^90
*d^8 - 73440220164348137346957312*a^39*b^48*c^89*d^9 + 44064938336617053976
2647040*a^40*b^47*c^88*d^10 - 2196237253234092465387995136*a^41*b^46*c^87*d
^11 + 9274296316144595646699012096*a^42*b^45*c^86*d^12 - 336778815010469933
39969175552*a^43*b^44*c^85*d^13 + 106376530102998491281999527936*a^44*b^43*
c^84*d^14 - 294921432301504798990377418752*a^45*b^42*c^83*d^15 + 7229030451
42137525367365173248*a^46*b^41*c^82*d^16 - 1576072447576504233275626094592*
a^47*b^40*c^81*d^17 + 3072471208539973972578986360832*a^48*b^39*c^80*d^18 -
 5384106777252432871416869683200*a^49*b^38*c^79*d^19 + 853735159835492549 68
36275830784*a^50*b^37*c^78*d^20 - 12376921822825560832675204300800*a^51*b^3
6*c^77*d^21 + 16707589390432621056738749054976*a^52*b^35*c^76*d^22 - 216671
30911214476307455165857792*a^53*b^34*c^75*d^23 + 28211207618793157944689200
988160*a^54*b^33*c^74*d^24 - 38378393138521379212996695293952*a^55*b^32*c^7
3*d^25 + 54918846093258397577855222415360*a^56*b^31*c^72*d^26 - 80082941438
212170767896978391040*a^57*b^30*c^71*d^27 + 113888426387729629146256565600 2
56*a^58*b^29*c^70*d^28 - 152754106500312545531177547595776*a^59*b^28*c^69*d
^29 + 189549778508563263438068404715520*a^60*b^27*c^68*d^30 - 2155465182348
226317813771487150 08*a^61*b^26*c^67*d^31 + 22364189630885587345716503642112
0*a^62*b^25*c^66*d^32 - 211293730951350565888869600854016*a^63*b^24*c^65*d^
33 + 181575241776706668284956756672512*a^64*b^23*c^64*d^34 - 14179414961960
044882972970582016 0*a^65*b^22*c^63*d^35 + 100511576025621687034384100622336
```

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*a^66*b^21*c^62*d^36 - 645811235532439905720986668892l6*a^67*b^20*c^61*d^37
 + 37540992634094717640084094451712*a^68*b^19*c^60*d^38 - 19695179695689601
910490494140416*a^69*b^18*c^59*d^39 + 92968409420464145227468159057 92*a^70*
b^17*c^58*d^40 - 39334461962821087954574644346 88*a^71*b^16*c^57*d^41 + 1484
6448648804319450986625105 92*a^72*b^15*c^56*d^42 - 4969938773331195363812777
65632*a^73*b^14*c^55*d^43 + 146493707302289292776429322240*a^74*b^13*c^54*d
^44 - 376790059998473990956740771 84*a^75*b^12*c^53*d^45 + 83600946239911812
23468728320*a^76*b^11*c^52*d^46 - 1576546523407725355918688256*a^77*b^10*c^
51*d^47 + 247744258459119342197932032*a^78*b^9*c^50*d^48 - 3156613601292619
5282739200*a^79*b^8*c^49*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^5
0 - 227270011883594899783680*a^81*b^6*c^47*d^51 + 10717576321223758970880*a
^82*b^5*c^46*d^52 - 246599101196298878976*a^83*b^4*c^45*d^53) + 10505997210
7298930688*a^31*b^54*c^91*d^4 - 4202398884291957227520*a^32*b^53*c^90*d^5 +
 81456498373859104260096*a^33*b^52*c^89*d^6 - 1019470840448604438528000*a^3
4*b^51*c^88*d^7 + 9261585187779405523451904*a^35*b^50*c^87*d^8 - 6509497194
4398671145074688*a^36*b^49*c^86*d^9 + 368402395453916323189358592*a^37*b^48
*c^85*d^10 - 1725226316150928144278224896*a^38*b^47*c^84*d^11 + 68177424522
02868128486522880*a^39*b^46*c^83*d^12 - 23071505195064931052886687744*a^40*
b^45*c^82*d^13 + 67614089216123669492331970560*a^41*b^44*c^81*d^14 - 173115
025562473785468905324544*a^42*b^43*c^80*d^15 + 389913831719674713212222 1772
80*a^43*b^42*c^79*d^16 - 776790088912432141093966970880*a^44*b^41*c^78*d^17
 + 1374611983251272530469308071936*a^45*b^40*c^77*d^18 - 216745461299415628
5048662261760*a^46*b^39*c^76*d^19 + 3050337310429700535004075917312*a^47*b^
38*c^75*d^20 - 3826885622871496570502324944896*a^48*b^37*c^74*d^21 + 423871
3393375513383921726259200*a^49*b^36*c^73*d^22 - 398429189634502446784334895
5136*a^50*b^35*c^72*d^23 + 2651971426464597412032295206912*a^51*b^34*c^71*d
^24 + 479249403658129639733534392320*a^52*b^33*c^70*d^25 - 6697452529698647
734837548417024*a^53*b^32*c^69*d^26 + 17931054269995149998277682790400*a^54
*b^31*c^68*d^27 - 36311715021905634799784747335680*a^55*b^30*c^67*d^28 + 63
073617076394089001091166371840*a^56*b^29*c^66*d^29 - 971055651681381470554 0
2127196160*a^57*b^28*c^65*d^30 + 133993666277013207597272619024384*a^58*b^2
7*c^64*d^31 - 166492084833102044695859350732800*a^59*b^26*c^63*d^32 + 18671
7161118223967667066928889856*a^60*b^25*c^62*d^33 - 189235624153406619951659
086774272*a^61*b^24*c^61*d^34 + 173421825288151984221422006304768*a^62*b^23
*c^60*d^35 - 143715376746696050902973036888064*a^63*b^22*c^59*d^36 + 107645
128880801788128312132894720*a^64*b^21*c^58*d^37 - 728021692097141192385497 5
1463936*a^65*b^20*c^57*d^38 + 44389639270136779232591657041920*a^66*b^19*c^
56*d^39 - 24348625105436875280486976454656*a^67*b^18*c^55*d^40 + 1198114551
1938522697620070072320*a^68*b^17*c^54*d^41 - 526975932508991026064472932352
0*a^69*b^16*c^53*d^42 + 2062471522530027433706750214144*a^70*b^15*c^52*d^43
 - 714227824367410213467319173120*a^71*b^14*c^51*d^44 + 21730537375149398 30
05392764928*a^72*b^13*c^50*d^45 - 57574411148433569424441606144*a^73*b^12*c
^49*d^46 + 13133947360733882065354752000*a^74*b^11*c^48*d^47 - 254201946024
2050797665255424*a^75*b^10*c^47*d^48 + 409310322447365741947650048*a^76*b^9
*c^46*d^49 - 53356649691793134232535040*a^77*b^8*c^45*d^50 + 54105949245788
```

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93614546944*a^78*b^7*c^44*d^51 - 4004641954373188976640000*a^79*b^6*c^43*d^5
2 + 19246289226179889070080*a^80*b^5*c^42*d^53 - 450813981874483888128*a^81
*b^4*c^41*d^54) - x^(1/2)*(1193422193316957310156800*a^30*b^49*c^73*d^13 - 3
677615218076424339456*a^29*b^50*c^74*d^12 - 18560134430309724255682560*a^31*
b^48*c^72*d^14 + 18426099996452807258406912*a^32*b^47*c^71*d^15 - 131228123
459738637629915136*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b
^45*c^69*d^17 - 3088237415348484431457288192*a^35*b^44*c^68*d^18 + 10882952
503625649640326561792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a
^37*b^42*c^66*d^20 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 15
6439291025195069838804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496
410112*a^40*b^39*c^63*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*
d^24 + 193264836517334230347779407872*a^42*b^37*c^61*d^25 + 320732651390132
179677984325632*a^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*
b^35*c^59*d^27 + 3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 5465
398361763642490480861642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101
978112*a^47*b^32*c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55
*d^31 + 9672669866587270697877661286400*a^49*b^30*c^54*d^32 - 8839280066432
157154484139589632*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^
51*b^28*c^52*d^34 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 3
079795601090740527825181212672*a^53*b^26*c^50*d^36 - 1663341919096805892341
077377024*a^54*b^25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^
48*d^38 - 319013552886948801896949743616*a^56*b^23*c^47*d^39 + 111766668098
727585639133347840*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58
*b^21*c^45*d^41 + 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 171550
2625948903704153292800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^
61*b^18*c^42*d^44 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 332485
0588931239515783168*a^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^1
5*c^39*d^47 + 5678869390326597943296*a^65*b^14*c^38*d^48))*(-(383772100608*
a^37*d^37 + 55037657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176
*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^
34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^3
2*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^
30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*
c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11
*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 9969075131258880
0*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978
454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409
117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18
+ 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c
^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*
a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 2292474236
4744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 -
 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*
c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^2
9*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320
```

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*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 1737318373694668
8*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*
a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d
^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13
- 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 85
50655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 6
3613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 +
98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 +
 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22
+ 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 687194
76736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*
b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29
*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^
27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^2
0*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 19274988590727168
00*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 886649475
1734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 -
 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*
c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040
*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 323968077
46722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19
+ 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*
c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a
^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726
016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347
188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604
610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 166
21372440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 6499587
17915136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 771862779
7106688*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 361677490
25660928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250
191269888*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 697
32511764185088*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13
+ 28173849246646272*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d
^15 - 3535126029615616*a^16*b^21*c^21*d^16 + 17520455870952628*a^17*b^20*
c^20*d^17 - 59025351788450612*a^18*b^19*c^19*d^18 + 156121530278753894*a^
19*b^18*c^18*d^19 - 334601154483963494*a^20*b^17*c^17*d^20 + 5916130635628
541952*a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 1087
1659607848206336*a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*
d^24 + 10274468596079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*
b^11*c^11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300
160*a^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 50394298154
3903232*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124
556607488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 13667168
50716672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312
```

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896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c
^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15
*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 20137
6*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d
^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a
^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^
37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 5
65722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^
30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^3
0*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64
512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^
8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 2013
76*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29
 + 496*a^43*b^2*c^19*d^30)))^(1/4)*1i)/(((-(383772100608*a^37*d^37 + 5503765
7088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 332427
44881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435
830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594
213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 7233549805
1321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14903050
0382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 1
39465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^
13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^
22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b
^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312243060557507788
88*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261
271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22
 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^1
3*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1567826806107753676
8*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130
919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 10
07885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31
- 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33
 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 -
 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (3644354
5848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*
a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*
b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^
4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a
^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*
a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*
a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^3
2 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084
860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152
383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273
040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 7
```

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22812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d
^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^2
1*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 238713320239005696
00*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3887616
9296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^1
6 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^
14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370
240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247
375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 +
 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d
^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^2
2*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^2
0*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^3
5*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c
^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^
31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c
^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27
*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12
*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272
*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3535126022961
5616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 5902535
17884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 -
 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^
16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^
23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 102744685960
79321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 50
86592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^
28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^
7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*
c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c
^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d -
 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^3
2 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 49
60*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^
5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*
b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10
 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600
*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*
c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 +
 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*
a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^
27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 33658
56*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^
27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19
*d^30)))^(3/4)*(x^(1/2)*(-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 +
```

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((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^3
5*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*
c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*
c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29
*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*
b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 13946502352837017
6*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493
292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 707025
20459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 -
1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^1
8*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^2
1*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 2174331921569
6412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20
548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^1
1*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^2
8*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 100788596308780646
4*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214
976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433
344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a
*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^1
7*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14
 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 -
 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17
- 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19
 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^2
1 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^
23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 219902325555
2*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b
^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b
^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^1
9*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 72281207215226880
0*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 44332473758
67248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15
516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^3
6*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^
28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 388761692960
66641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 2
3871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^
29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^3
5*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268
800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 6227304006
2349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152
383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 3408486
0461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124
026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 254384
```

```
3228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 1860007
5416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 573309
58029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 795
79326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12
- 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*
d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c
^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18
*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 334601154483963
4944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 873733
3381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^2
3 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^
12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 508659201176091033
6*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 12985331363
15185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 1616185
90114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 86865
91868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123
481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*
a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31
*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^4
6*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*
b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 -
28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24
*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*
d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 6010
80390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*
b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d
^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 280488
00*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24
*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*
b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(
93386641873154605056*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*
c^93*d^5 + 78828702034483948290048*a^36*b^51*c^92*d^6 - 1034672110486845715
906560*a^37*b^50*c^91*d^7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 -
73440220164348137346957312*a^39*b^48*c^89*d^9 + 440649383366170539762647040
*a^40*b^47*c^88*d^10 - 2196237253234092465387995136*a^41*b^46*c^87*d^11 + 9
274296316144595646699012096*a^42*b^45*c^86*d^12 - 3367788150104699333996917
5552*a^43*b^44*c^85*d^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^
14 - 294921432301504798990377418752*a^45*b^42*c^83*d^15 + 72290304514213752
5367365173248*a^46*b^41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^
40*c^81*d^17 + 3072471208539973972578986360832*a^48*b^39*c^80*d^18 - 538410
6777252432871416869683200*a^49*b^38*c^79*d^19 + 853735159835492549683627583
0784*a^50*b^37*c^78*d^20 - 12376921822825560832675204300800*a^51*b^36*c^77*
d^21 + 16707589390432621056738749054976*a^52*b^35*c^76*d^22 - 2166713091121
4476307455165857792*a^53*b^34*c^75*d^23 + 28211207618793157944689200988160*
a^54*b^33*c^74*d^24 - 38378393138521379212996695293952*a^55*b^32*c^73*d^25
```

```
+ 54918846093258397577855222415360*a^56*b^31*c^72*d^26 - 800829414382121707
67896978391040*a^57*b^30*c^71*d^27 + 113888426387729629146256565600256*a^58
*b^29*c^70*d^28 - 152754106500312545531177547595776*a^59*b^28*c^69*d^29 + 1
89549778508563263438068404715520*a^60*b^27*c^68*d^30 - 215546518234822631781
377148715008*a^61*b^26*c^67*d^31 + 223641896308855873457165036421120*a^62*
b^25*c^66*d^32 - 211293730951350565888869600854016*a^63*b^24*c^65*d^33 + 18
1575241776706668284956756672512*a^64*b^23*c^64*d^34 - 141794149619600448829
729705820160*a^65*b^22*c^63*d^35 + 100511576025621687034384100622336*a^66*b
^21*c^62*d^36 - 64581123553243990572098666889216*a^67*b^20*c^61*d^37 + 3754
0992634094717640084094451712*a^68*b^19*c^60*d^38 - 196951796956896019104904
94140416*a^69*b^18*c^59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^
58*d^40 - 3933446196282108795457464434688*a^71*b^16*c^57*d^41 + 14846448648
80431945098662510592*a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a
^73*b^14*c^55*d^43 + 146493707302289292776429322240*a^74*b^13*c^54*d^44 - 3
7679005999847399095674077184*a^75*b^12*c^53*d^45 + 836009462399118122346872
8320*a^76*b^11*c^52*d^46 - 1576546523407725355918688256*a^77*b^10*c^51*d^47
 + 247744258459119342197932032*a^78*b^9*c^50*d^48 - 31566136012926195282739
200*a^79*b^8*c^49*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 227
270011883594899783680*a^81*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5
*c^46*d^52 - 246599101196298878976*a^83*b^4*c^45*d^53) + 105059972107298930
688*a^31*b^54*c^91*d^4 - 4202398884291957227520*a^32*b^53*c^90*d^5 + 814564
98373859104260096*a^33*b^52*c^89*d^6 - 1019470840448604438528000*a^34*b^51*
c^88*d^7 + 9261585187779405523451904*a^35*b^50*c^87*d^8 - 65094971944398671
145074688*a^36*b^49*c^86*d^9 + 368402395453916323189358592*a^37*b^48*c^85*d
^10 - 1725226316150928144278224896*a^38*b^47*c^84*d^11 + 681774245220286812
8486522880*a^39*b^46*c^83*d^12 - 23071505195064931052886687744*a^40*b^45*c^
82*d^13 + 67614089216123669492331970560*a^41*b^44*c^81*d^14 - 1731150255624
73785468905324544*a^42*b^43*c^80*d^15 + 389913831719674713212222177280*a^43
*b^42*c^79*d^16 - 776790088912432141093966970880*a^44*b^41*c^78*d^17 + 1374
611983251272530469308071936*a^45*b^40*c^77*d^18 - 2167454612994156285048662
261760*a^46*b^39*c^76*d^19 + 3050337310429700535004075917312*a^47*b^38*c^75
*d^20 - 3826885622871496570502324944896*a^48*b^37*c^74*d^21 + 4238713393375
513383921726259200*a^49*b^36*c^73*d^22 - 3984291896345024467843348955136*a^
50*b^35*c^72*d^23 + 2651971426464597412032295206912*a^51*b^34*c^71*d^24 + 4
79249403658129639733534392320*a^52*b^33*c^70*d^25 - 66974525296986477348375
48417024*a^53*b^32*c^69*d^26 + 17931054269995149998277682790400*a^54*b^31*c
^68*d^27 - 36311715021905634799784747335680*a^55*b^30*c^67*d^28 + 630736170
76394089001091166371840*a^56*b^29*c^66*d^29 - 97105565168138147055402127196
160*a^57*b^28*c^65*d^30 + 133993666277013207597272619024384*a^58*b^27*c^64*
d^31 - 166492084833102044695859350732800*a^59*b^26*c^63*d^32 + 186717161118
223967667066928889856*a^60*b^25*c^62*d^33 - 1892356241534066199516590867742
72*a^61*b^24*c^61*d^34 + 173421825288151984221422006304768*a^62*b^23*c^60*d
^35 - 143715376746696050902973036888064*a^63*b^22*c^59*d^36 + 1076451288808
01788128312132894720*a^64*b^21*c^58*d^37 - 72802169209714119238549751463936
*a^65*b^20*c^57*d^38 + 44389639270136779232591657041920*a^66*b^19*c^56*d^39
```

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 - 24348625105436875280486976454656*a^67*b^18*c^55*d^40 + 11981145511938522
697620070072320*a^68*b^17*c^54*d^41 - 5269759325089910260644729323520*a^69*
b^16*c^53*d^42 + 2062471522530027433706750214144*a^70*b^15*c^52*d^43 - 7142
27824367410213467319173120*a^71*b^14*c^51*d^44 + 21730537375149398300539276
4928*a^72*b^13*c^50*d^45 - 57574411148433569424441606144*a^73*b^12*c^49*d^4
6 + 13133947360733882065354752000*a^74*b^11*c^48*d^47 - 2542019460242050797
665255424*a^75*b^10*c^47*d^48 + 409310322447365741947650048*a^76*b^9*c^46*d
^49 - 53356649691793134232535040*a^77*b^8*c^45*d^50 + 541059492457889361454
6944*a^78*b^7*c^44*d^51 - 400464195437318897664000*a^79*b^6*c^43*d^52 + 192
46289226179889070080*a^80*b^5*c^42*d^53 - 450813981874483888128*a^81*b^4*c^
41*d^54) - x^(1/2)*(119342219331695731015680*a^30*b^49*c^73*d^13 - 36776152
18076424339456*a^29*b^50*c^74*d^12 - 1856013443030972425568256*a^31*b^48*c^
72*d^14 + 18426099996452807258406912*a^32*b^47*c^71*d^15 - 1312281234597386
37629915136*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^6
9*d^17 - 3088237415348484431457288192*a^35*b^44*c^68*d^18 + 108829525036256
49640326561792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^4
2*c^66*d^20 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 156439291
025195069838804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*
a^40*b^39*c^63*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 +
193264836517334230347779407872*a^42*b^37*c^61*d^25 + 3207326513901321796779
84325632*a^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^
59*d^27 + 3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 54653983617
63642490480861642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*
a^47*b^32*c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 +
 9672669866587270697877661286400*a^49*b^30*c^54*d^32 - 88392800664321571544
84139589632*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28
*c^52*d^34 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 30797956
01090740527825181212672*a^53*b^26*c^50*d^36 - 16633419190968058923410773770
24*a^54*b^25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38
 - 319013552886948801896949743616*a^56*b^23*c^47*d^39 + 1117666680987275856
39133347840*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c
^45*d^41 + 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 1715502625948
903704153292800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18
*c^42*d^44 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931
239515783168*a^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*
d^47 + 5678869390326597943296*a^65*b^14*c^38*d^48))*(-(383772100608*a^37*d^
37 + 55037657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c
^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3
+ 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 +
 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7
+ 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^
9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c
^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*
b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*
a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 3504091174190
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53056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 31224
30605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^2
0 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^
15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 2292474236474445O
048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 156782
68061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^
27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c
^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b
^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*
b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^
2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/
4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 14460
35052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 855065595
2661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900
184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 9862080
2659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 389071
53228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 35746
83057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a
^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*
d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d
^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44
*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*
c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22
*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497
280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 238713
32023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^
14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b
^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 323968077467222O
1600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516
365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^
21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9
*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^3
8*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*
a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*
a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 1662137244
0576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 64995871791513
6*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688
*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 3616774902566092
8*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 745152501912698
88*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 6973251176
4185088*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173
849246646272*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3
5351260229615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^
17 - 590253517884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18
*c^18*d^19 - 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*
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a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607
848206336*a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 +
10274468596079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^
11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^2
8*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232
*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 421001245566074
88*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 136671685071667
2*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b
^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a
^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c
^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*
b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10
518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^2
2*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12
 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 56572272
0*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15
*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19
+ 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*
a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*
d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40
*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a
^43*b^2*c^19*d^30)))^(1/4) - ((-(383772100608*a^37*d^37 + 55037657088*b^37*
c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a
^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4
*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6
*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^
8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776
*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 13946502352
8370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347
698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 -
70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d
^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^
18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322612712570839
04*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 2174331
9215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^2
4 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^
11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971309198046003
20*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 100788596308
7806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249
113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433
701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 278857462
5792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^
12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^1
1*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*
d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8
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*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^
6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c
^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*
c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 219902
3255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*
a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*
a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622730400623493
12*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 72281207215
2268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 44332
47375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^1
1 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^
19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641
920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 388761
69296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^
18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b
^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332473758672486
40*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072
152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 6227
3040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2
471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 3
4084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2
 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 -
2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 1
8600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 -
57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10
 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25
*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23
*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3535126022961516*a^16*
b^21*c^21*d^16 + 17520455870952652*a^17*b^20*c^20*d^17 - 59025351788450611
2*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 334601154
4839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 -
8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^
14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a
^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 508659201176
0910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 12985
33136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 1
61618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 +
 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 +
154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 110998699
86816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^1
4*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^
29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192
*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*
d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 12902448
0*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19
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*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15
+ 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600
*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*
c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 -
28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^
7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960
*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(
3/4)*(x^(1/2)*(-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((7675442
01216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2
 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 -
 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 -
 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8
- 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*
d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^2
5*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^1
4*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 7070252045923123
2*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035
769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6
692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^1
6*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^
23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 205489371921
58642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 1
0173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*
d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7
*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b
^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b
^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36
*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 1
0657194751016l*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 5334373
96380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 291045205
78391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216
268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 7337065
1908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 144325
88165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719
476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^3
1*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d
^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d
^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^4
3*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^2
4*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a
^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 155163658155
35370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 3
2396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^
34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a
^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023
```

```
900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 -
8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^2
7*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b
^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^
39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*
a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^
43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 1240263410319
36*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 2543843228516352
*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 18600075416567808
*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 5733095802981580
8*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 7957932617288
9088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 49845375
656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 677
1862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16 +
 175204558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^19
*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20*
b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 8737333381308579
840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 114623
71182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d^
25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^1
0*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*a
^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 16161859011465216
0*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 868659186847334
4*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123481161728*
a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d
^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d -
32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35
960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*
d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a
^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38
*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471
435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29
*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*
d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 1290
24480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^
9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 90
6192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d
^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(9338664187
3154605056*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 +
 78828702034483948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^3
7*b^50*c^91*d^7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 7344022016
4348137346957312*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47
*c^88*d^10 - 2196237253234092465387995136*a^41*b^46*c^87*d^11 + 92742963161
44595646699012096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*
b^44*c^85*d^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 29492
```

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1432301504798990377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173
248*a^46*b^41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^
17 + 3072471208539973972578986360832*a^48*b^39*c^80*d^18 - 5384106777252432
871416869683200*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*
b^37*c^78*d^20 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 167
07589390432621056738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455
165857792*a^53*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*
c^74*d^24 - 38378393138521379212996695293952*a^55*b^32*c^73*d^25 + 54918846
093258397577855222415360*a^56*b^31*c^72*d^26 - 8008294143821217076789697839
1040*a^57*b^30*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70
*d^28 - 152754106500312545531177547595776*a^59*b^28*c^69*d^29 + 18954977850
8563263438068404715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715
008*a^61*b^26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*
d^32 - 211293730951350565888869600854016*a^63*b^24*c^65*d^33 + 181575241776
706668284956756672512*a^64*b^23*c^64*d^34 - 1417941496196004488297297058201
60*a^65*b^22*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d
^36 - 64581123553243990572098666889216*a^67*b^20*c^61*d^37 + 37540992634094
717640084094451712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a
^69*b^18*c^59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 -
3933446196282108795457464434688*a^71*b^16*c^57*d^41 + 148464486488043194509
8662510592*a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c
^55*d^43 + 146493707302289292776429322240*a^74*b^13*c^54*d^44 - 37679005999
847399095674077184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*
b^11*c^52*d^46 - 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 2477442
58459119342197932032*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b
^8*c^49*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 2272700118835
94899783680*a^81*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52
 - 246599101196298878976*a^83*b^4*c^45*d^53) - 105059972107298930688*a^31*b
^54*c^91*d^4 + 4202398884291957227520*a^32*b^53*c^90*d^5 - 8145649837385910
4260096*a^33*b^52*c^89*d^6 + 1019470840448604438528000*a^34*b^51*c^88*d^7 -
 9261585187779405523451904*a^35*b^50*c^87*d^8 + 65094971944398671145074688*
a^36*b^49*c^86*d^9 - 368402395453916323189358592*a^37*b^48*c^85*d^10 + 1725
226316150928144278224896*a^38*b^47*c^84*d^11 - 6817742452202868128486522880
*a^39*b^46*c^83*d^12 + 23071505195064931052886687744*a^40*b^45*c^82*d^13 -
67614089216123669492331970560*a^41*b^44*c^81*d^14 + 17311502556247378546890
5324544*a^42*b^43*c^80*d^15 - 389913831719674713212222177280*a^43*b^42*c^79
*d^16 + 776790088912432141093966970880*a^44*b^41*c^78*d^17 - 13746119832512
72530469308071936*a^45*b^40*c^77*d^18 + 2167454612994156285048662261760*a^4
6*b^39*c^76*d^19 - 3050337310429700535004075917312*a^47*b^38*c^75*d^20 + 38
26885622871496570502324944896*a^48*b^37*c^74*d^21 - 42387133933755133839217
26259200*a^49*b^36*c^73*d^22 + 3984291896345024467843348955136*a^50*b^35*c^
72*d^23 - 2651971426464597412032295206912*a^51*b^34*c^71*d^24 - 47924940365
8129639733534392320*a^52*b^33*c^70*d^25 + 6697452529698647734837548417024*a
^53*b^32*c^69*d^26 - 17931054269995149998277682790400*a^54*b^31*c^68*d^27 +
 36311715021905634799784747335680*a^55*b^30*c^67*d^28 - 6307361707639408900
```

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1091166371840*a^56*b^29*c^66*d^29 + 97105565168138147055402127196160*a^57*b
^28*c^65*d^30 - 133993666277013207597272619024384*a^58*b^27*c^64*d^31 + 166
4920848331020446958593507328 00*a^59*b^26*c^63*d^32 - 1867171611182239676670
66928889856*a^60*b^25*c^62*d^33 + 189235624153406619951659086774272*a^61*b^
24*c^61*d^34 - 173421825288151984221422006304768*a^62*b^23*c^60*d^35 + 1437
15376746696050902973036888064*a^63*b^22*c^59*d^36 - 107645128880801788128831
2132894720*a^64*b^21*c^58*d^37 + 72802169209714119238549751463936*a^65*b^20
*c^57*d^38 - 44389639270136779232591657041920*a^66*b^19*c^56*d^39 + 2434862
5105436875280486976454656*a^67*b^18*c^55*d^40 - 119811455119385226976200700
72320*a^68*b^17*c^54*d^41 + 5269759325089910260644729323520*a^69*b^16*c^53*
d^42 - 2062471522530027433706750214144*a^70*b^15*c^52*d^43 + 71422782436741
0213467319173120*a^71*b^14*c^51*d^44 - 217305373751493983005392764928*a^72*
b^13*c^50*d^45 + 57574411148433569424441606144*a^73*b^12*c^49*d^46 - 131339
47360733882065354752000*a^74*b^11*c^48*d^47 + 2542019460242050797665255424*
a^75*b^10*c^47*d^48 - 409310322447365741947650048*a^76*b^9*c^46*d^49 + 5335
6649691793134232535040*a^77*b^8*c^45*d^50 - 541059492457889361454 6944*a^78*
b^7*c^44*d^51 + 400464195437318897664000*a^79*b^6*c^43*d^52 - 1924628922617
9889070080*a^80*b^5*c^42*d^53 + 450813981874483888128*a^81*b^4*c^41*d^54) -
 x^(1/2)*(1193422193316957310156 80*a^30*b^49*c^73*d^13 - 3677615218076424 33
9456*a^29*b^50*c^74*d^12 - 18560134430309724255682 56*a^31*b^48*c^72*d^14 +
184260999964528072584069 12*a^32*b^47*c^71*d^15 - 131228123459738637629915 13
6*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 - 3
088237415348484431457288192*a^35*b^44*c^68*d^18 + 108829525036256496403265 6
1792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d^2
0 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 1564392910251950698
38804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*
c^63*d^23 - 315930266538485089124480901 12*a^41*b^38*c^62*d^24 + 1932648365
17334230347779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a
^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 +
3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176364249048
0861642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*
c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 967266986
6587270697877661286400*a^49*b^30*c^54*d^32 - 883928006643215715448413958963
2*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34
 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560109074052
7825181212672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^
25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 3190135
52886948801896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347 84
0*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 +
 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 171550262594890370415 32
92800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44
 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 332485058893123951578 31
68*a^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 567
886939032659794329 6*a^65*b^14*c^38*d^48))*(-(383772100608*a^37*d^37 + 55037
657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324
```

```
2744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 12999174
35830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 154372555
94213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498
051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030
500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 +
 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*
d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*
c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17
*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312243060557507
7888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322
61271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^
22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b
^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1567826806107753
6768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971
30919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 -
1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^3
1 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^
33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35
 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443
545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 144603505249081
2*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^
2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*
a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292
*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 3890715322816345
5*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 357468305702344
2*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d
^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 340
84860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 24711
52383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622
73040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 +
 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40
*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b
^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 2387133202390056
9600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876
169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d
^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*
b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535
370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332
47375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23
 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24
*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c
^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c
^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b
^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33
*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*
```

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c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29
*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^
27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^
12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 281738492466462
72*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 35351260229
615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 59025
3517884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19
 - 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*
c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*
a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 1027446859
6079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 +
5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*
d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*
c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^
5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3
*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d
 - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d
^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 -
4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*
d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^2
1*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^1
0 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 3473736
00*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^1
7*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17
 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 22579284
0*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*
c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 336
5856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*
d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^
19*d^30)))^(1/4) + 4102839477666510903705 6*a^29*b^48*c^68*d^15 - 1210739885
076097825505280*a^30*b^47*c^67*d^16 + 17243628768780949747924992*a^31*b^46*
c^66*d^17 - 158081319004444765483696128*a^32*b^45*c^65*d^18 + 1049494986915
760527133114368*a^33*b^44*c^64*d^19 - 5380683046490354438136397824*a^34*b^4
3*c^63*d^20 + 22176160052724101903372255232*a^35*b^42*c^62*d^21 - 754863133
25241636679770439680*a^36*b^41*c^61*d^22 + 216288375615109659684325294080*a
^37*b^40*c^60*d^23 - 528818181695424054504437317632*a^38*b^39*c^59*d^24 + 1
114222690302433619242395893760*a^39*b^38*c^58*d^25 - 2037545055293058005529
639518208*a^40*b^37*c^57*d^26 + 3249918857904337975850827776000*a^41*b^36*c
^56*d^27 - 4536394700759564584125915463680*a^42*b^35*c^55*d^28 + 5552435240
283931429496420302848*a^43*b^34*c^54*d^29 - 5964290825683224886861470105600
*a^44*b^33*c^53*d^30 + 5621639355410781338712284332032*a^45*b^32*c^52*d^31
- 4644077108074496901042866749440*a^46*b^31*c^51*d^32 + 3355360862716129153
108295024640*a^47*b^30*c^50*d^33 - 2113405281704782215093506015232*a^48*b^2
9*c^49*d^34 + 1155283596049337948225918730240*a^49*b^28*c^48*d^35 - 5448295
198703769444694024519 68*a^50*b^27*c^47*d^36 + 2199261720378991172687128166 4
```

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0*a^51*b^26*c^46*d^37 - 75201916274561138554746961920*a^52*b^25*c^45*d^38 +
 21483948869172056418164932608*a^53*b^24*c^44*d^39 - 503234620160616432532O
359936*a^54*b^23*c^43*d^40 + 941275744618015035796488192*a^55*b^22*c^42*d^4
1 - 135189136301093329947328512*a^56*b^21*c^41*d^42 + 139991403072671809882
03008*a^57*b^20*c^40*d^43 - 930460907799665663016960*a^58*b^19*c^39*d^44 +
29814064299214639202304*a^59*b^18*c^38*d^45))*(-(383772100608*a^37*d^37 + 5
5037657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 +
33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299
917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437
255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 7233
5498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14
9030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^
11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c
^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b
^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*
a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 31224306055
75077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11
832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^1
5*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^
24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 156782680610
77536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5
597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^2
9 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6
*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^
4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*
d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (3
6443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 14460350524
90812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 855065595266152
2*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394
735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 9862080265939
1292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 389071532281
63455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 35746830570
23442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^
17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 +
 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2
471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 +
 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d
^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*
c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^
24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 238713320239
00569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3
8876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^
33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a
^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815
535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4
```

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433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*
d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*
c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b
^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b
^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a
^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*
b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b
^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*
b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^1
0*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 6973251176418508
8*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246
646272*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3535126
0229615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 5
90253517884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*
d^19 - 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b
^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206
336*a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 102744
68596079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^2
6 + 5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*
c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*
b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^3
2*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34
*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^
36*d - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^
17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^
2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c
^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300
*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39
*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347
373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28
*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*
d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 2257
92840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b
^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 -
 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c
^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^
2*c^19*d^30)))^(1/4)*2i - 2*atan(((((((767544201216*a^37*d^37 + 11007531417
6*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c
^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^
32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c
^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28
*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^1
1*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 996907513125888
00*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 1354372497
8454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 35040
```

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9117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18
 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*
c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680
*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 229247423
64744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25
- 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10
*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^
29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 32323718022930432
0*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 173731837369466
88*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456
*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*
d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13
 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8
550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 +
63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 +
 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20
+ 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22
 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719
476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44
*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^2
9*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b
^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^
20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716
800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 88664947
51734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12
- 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18
*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 4130592987707080704
0*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807
746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19
 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11
*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*
a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 23129986308872
6016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 1383845334
7188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 34084860
4610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55
037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35
*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^
4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6
 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^
8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*
d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*
c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*
b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a
^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 5902535178845
06112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 33460
```

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11544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^2
1 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^1
4*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 102744685960793210
88*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 50865920
11760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1
298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30
 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^
32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^3
4 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099
869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32
*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^1
6*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 90
6192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c
^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 1290
24480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*
b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d
^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 47143
5600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b
^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^2
2 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^3
8*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 3
5960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)
))^(3/4)*(x^(1/2)*((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 332
42744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917
435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255
594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 7233549
8051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14903
0500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11
+ 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24
*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22
*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^1
7*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 31224306055750
77888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832
261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d
^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*
b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 156782680610775
36768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597
130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 -
 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^
31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d
^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^3
5 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (3644
3545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 14460350524908
12*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a
^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735
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*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 9862080265939129
2*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 389071532281634
55*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 35746830570234
42*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*
d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34
084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471
152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62
273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7
+ 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^4
0*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*
b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 238713320239005
69600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3887
6169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*
d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31
*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535
370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433
247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^2
3 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^2
4*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*
c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*
c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^
37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 1240263410
31936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516
352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567
808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 5733095802981
5808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 7957932617
2889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49845
375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 +
6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^1
6 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c
^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^
20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308
579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 114
62371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12
*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*
b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 129853313631518515
2*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 16161859011465
2160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 868659186847
3344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 1541234811617
28*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*
c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d
 - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 +
 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^
43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 2804880
0*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c
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^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 +
471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a
^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^
31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 1
29024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36
*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 +
 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^2
1*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(9338664
187315460505б*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^
5 + 7882870203448394829004８*a^36*b^51*c^92*d^6 - 1034672110486845715906560*
a^37*b^50*c^91*d^7 + 989254036026514046818713６*a^38*b^49*c^90*d^8 - 7344022
0164348137346957312*a^39*b^48*c^89*d^9 + 44064938336617053976264704０*a^40*b
^47*c^88*d^10 - 219623725323409246538799513６*a^41*b^46*c^87*d^11 + 92742963
16144595646699012096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^
43*b^44*c^85*d^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 29
4921432301504798990377418752*a^45*b^42*c^83*d^15 + 7229030451421375253673６5
173248*a^46*b^41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81
*d^17 + 3072471208539973972578986360832*a^48*b^39*c^80*d^18 - 5384106777252
432871416869683200*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^
50*b^37*c^78*d^20 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 +
16707589390432621056738749054976*a^52*b^35*c^76*d^22 - 216671309112144763０7
455165857792*a^53*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^
33*c^74*d^24 - 38378393138521379212996695293952*a^55*b^32*c^73*d^25 + 54918
846093258397577855222415360*a^56*b^31*c^72*d^26 - 800829414382121707678969７
8391040*a^57*b^30*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c
^70*d^28 - 152754106500312545531177547595776*a^59*b^28*c^69*d^29 + 18954977
8508563263438068404715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148
715008*a^61*b^26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^
66*d^32 - 211293730951350565888869600854016*a^63*b^24*c^65*d^33 + 181575241
776706668284956756672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705８
20160*a^65*b^22*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^6
2*d^36 - 64581123553243990572098666889216*a^67*b^20*c^61*d^37 + 37540992634
094717640084094451712*a^68*b^19*c^60*d^38 - 196951796956896019104904941404１
6*a^69*b^18*c^59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40
 - 3933446196282108795457464434688*a^71*b^16*c^57*d^41 + 148464486488043194
5098662510592*a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^1
4*c^55*d^43 + 146493707302289292776429322240*a^74*b^13*c^54*d^44 - 37679005
999847399095674077184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^
76*b^11*c^52*d^46 - 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 2477
442584591193421979320３2*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^7
9*b^8*c^49*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 2272700118
83594899783680*a^81*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d
^52 - 246599101196298878976*a^83*b^4*c^45*d^53)*1i - 105059972107298930688*
a^31*b^54*c^91*d^4 + 420239888429195722752０*a^32*b^53*c^90*d^5 - 8145649837
3859104260096*a^33*b^52*c^89*d^6 + 1019470840448604438528000*a^34*b^51*c^88
```

*d^7 - 9261585187779405523451904*a^35*b^50*c^87*d^8 + 650949719443986711450
74688*a^36*b^49*c^86*d^9 - 368402395453916323189358592*a^37*b^48*c^85*d^10
+ 1725226316150928144278224896*a^38*b^47*c^84*d^11 - 6817742452202868128486
522880*a^39*b^46*c^83*d^12 + 23071505195064931052886687744*a^40*b^45*c^82*d
^13 - 67614089216123669492331970560*a^41*b^44*c^81*d^14 + 17311502556247378
5468905324544*a^42*b^43*c^80*d^15 - 389913831719674713212222177280*a^43*b^4
2*c^79*d^16 + 776790088912432141093966970880*a^44*b^41*c^78*d^17 - 13746119
83251272530469308071936*a^45*b^40*c^77*d^18 + 21674546129941562850486622617
60*a^46*b^39*c^76*d^19 - 3050337310429700535004075917312*a^47*b^38*c^75*d^2
0 + 3826885622871496570502324944896*a^48*b^37*c^74*d^21 - 42387133933755133
83921726259200*a^49*b^36*c^73*d^22 + 3984291896345024467843348955136*a^50*b
^35*c^72*d^23 - 2651971426464597412032295206912*a^51*b^34*c^71*d^24 - 47924
9403658129639733534392320*a^52*b^33*c^70*d^25 + 669745252969864773483754841
7024*a^53*b^32*c^69*d^26 - 17931054269995149998277682790400*a^54*b^31*c^68*
d^27 + 36311715021905634799784747335680*a^55*b^30*c^67*d^28 - 6307361707639
4089001091166371840*a^56*b^29*c^66*d^29 + 97105565168138147055402127196160*
a^57*b^28*c^65*d^30 - 133993666277013207597272619024384*a^58*b^27*c^64*d^31
 + 166492084833102044695859350732800*a^59*b^26*c^63*d^32 - 1867171611182239
67667066928889856*a^60*b^25*c^62*d^33 + 189235624153406619951659086774272*a
^61*b^24*c^61*d^34 - 173421825288151984221422006304768*a^62*b^23*c^60*d^35
+ 143715376746696050902973036888064*a^63*b^22*c^59*d^36 - 10764512888080178
8128312132894720*a^64*b^21*c^58*d^37 + 72802169209714119238549751463936*a^6
5*b^20*c^57*d^38 - 44389639270136779232591657041920*a^66*b^19*c^56*d^39 + 2
4348625105436875280486976454656*a^67*b^18*c^55*d^40 - 119811455119385226976
20070072320*a^68*b^17*c^54*d^41 + 5269759325089910260644729323520*a^69*b^16
*c^53*d^42 - 2062471522530027433706750214144*a^70*b^15*c^52*d^43 + 71422782
4367410213467319173120*a^71*b^14*c^51*d^44 - 217305373751493983005392764928
*a^72*b^13*c^50*d^45 + 57574411148433569424441606144*a^73*b^12*c^49*d^46 -
13133947360733882065354752000*a^74*b^11*c^48*d^47 + 25420194602420507976652
55424*a^75*b^10*c^47*d^48 - 409310322447365741947650048*a^76*b^9*c^46*d^49
+ 53356649691793134232535040*a^77*b^8*c^45*d^50 - 5410594924578893614546944
*a^78*b^7*c^44*d^51 + 400464195437318897664000*a^79*b^6*c^43*d^52 - 1924628
9226179889070080*a^80*b^5*c^42*d^53 + 450813981874483888128*a^81*b^4*c^41*d
^54)*1i + x^(1/2)*(119342219331695731015680*a^30*b^49*c^73*d^13 - 367761521
8076424339456*a^29*b^50*c^74*d^12 - 1856013443030972425568256*a^31*b^48*c^7
2*d^14 + 1842609996452807258406912*a^32*b^47*c^71*d^15 - 13122812345973863
7629915136*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69
*d^17 - 3088237415348484431457288192*a^35*b^44*c^68*d^18 + 1088295250362564
9640326561792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42
*c^66*d^20 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 1564392910
25195069838804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a
^40*b^39*c^63*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 1
93264836517334230347779407872*a^42*b^37*c^61*d^25 + 32073265139013217967798
4325632*a^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^5
9*d^27 + 3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176

3642490480861642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a
^47*b^32*c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 +
9672669866587270697877661286400*a^49*b^30*c^54*d^32 - 883928006643215715448
4139589632*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*
c^52*d^34 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560
1090740527825181212672*a^53*b^26*c^50*d^36 - 166334191909680589234107737702
4*a^54*b^25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38
- 319013552886948801896949743616*a^56*b^23*c^47*d^39 + 11176666809872758563
9133347840*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^
45*d^41 + 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 17155026259489
03704153292800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*
c^42*d^44 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 33248505889312
39515783168*a^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d
^47 + 5678869390326597943296*a^65*b^14*c^38*d^48))*((((767544201216*a^37*d^
37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 2480526820
63872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 508768645703
2704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 372001508331
35616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 11466191605
9631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158
652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 -
99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d
^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c
^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^1
8*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 66920230896792
69888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 1747
4666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*
d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25
*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521
820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 259706
6272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 3
23237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 +
17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 +
308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739
973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 10657194751016
1*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11
*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*
b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5
*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^
7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a
^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b
^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 21
99023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 34084860
4610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453
347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 23129
9863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 -

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1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39
*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25
*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722
201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 413
05929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32
*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^3
2*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734
497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927
498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24
- 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d
^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21
*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d
^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 166213724405
76*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*
a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a
^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*
a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888
*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 697325117641
85088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 2817384
9246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 353
51260229615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17
 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c
^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^
21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 1087165960784
8206336*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10
274468596079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11
*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*
b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a
^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488
*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*
a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^3
6*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^4
5*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^4
7*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^
27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 1051
8300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*
c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 -
 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*
a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c
^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 +
225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^
35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^
24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b
^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^4
3*b^2*c^19*d^30)))^(1/4) + (((((767544201216*a^37*d^37 + 110075314176*b^37*
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c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3
 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5
+ 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7
 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d
^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*
c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13
*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528
*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419
053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122
430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^
20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b
^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 2292474236474445
0048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678
268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d
^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*
c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*
b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33
*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b
^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2
/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446
035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 85506559
52661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 6361390
0184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 986208
02659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907
153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574
683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*
a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18
*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*
d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^4
4*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25
*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^2
2*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 886649475173449
7280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871
332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d
^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*
b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 323968077467222
01600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 1551
6365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d
^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^
9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^
38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736
*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560
*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 550376570
88*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 +
124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 254
```

```
3843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 1860
0075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 573
30958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 +
79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^
12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^
23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^2
1*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a
^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 334601154483
9634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 873
7333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*
d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25
*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 508659201176091
0336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 12985331
36315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 1616
18590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 86
86591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154
123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 110998699868
16*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b
^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*
c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^
19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8
 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a
^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^
36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 6
01080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^
31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^2
9*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 280
48800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c
^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^
41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4
)*(x^(1/2)*((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324274488
1152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 12999174358302
72*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 154372555942133
76*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321
856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382
539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 13946
5023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 +
 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d
^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*
c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a
^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322612712
57083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 2
1743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^
13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a
^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971309198
```

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04600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 100788
5963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84
200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2
733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 278
8574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848
801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^
28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27
*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^
25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b
^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*
b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10
*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 -
2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 340848604
61056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 24711523834
26560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622730400
62349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 72281
2072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 +
 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^
38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a
^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296
066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 -
38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c
^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*
a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332473758
67248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722
812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25
+ 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^
27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^
29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 38
3772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a
^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5
*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7
*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^
9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088
*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 498453756562
94400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862
489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 175
204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^1
8 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17
*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*
a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 1146237118
2372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 +
 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^
10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*
b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^
```

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31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^
33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35
*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)
/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a
^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*
a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6
 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*
b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^1
1 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 4714356
00*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^1
6*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18
 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 12902448
0*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^
26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192
*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28
 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154
605056*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 + 788
28702034483948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^
50*c^91*d^7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 73440220164348
137346957312*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^8
8*d^10 - 2196237253234092465387995136*a^41*b^46*c^87*d^11 + 927429631614459
5646699012096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44
*c^85*d^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 294921432
301504798990377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*
a^46*b^41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 +
 3072471208539973972578986360832*a^48*b^39*c^80*d^18 - 53841067772524328714
16869683200*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37
*c^78*d^20 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 1670758
9390432621056738749054976*a^52*b^35*c^76*d^22 - 216671309112144763074551658
57792*a^53*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74
*d^24 - 38378393138521379212996695293952*a^55*b^32*c^73*d^25 + 549188460932
58397577855222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040
*a^57*b^30*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^2
8 - 152754106500312545531177547595776*a^59*b^28*c^69*d^29 + 189549778508563
263438068404715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*
a^61*b^26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32
 - 211293730951350565888869600854016*a^63*b^24*c^65*d^33 + 1815752417767066
68284956756672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a
^65*b^22*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36
 - 64581123553243990572098666889216*a^67*b^20*c^61*d^37 + 375409926340947176
40084094451712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a^69*
b^18*c^59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 - 3933
446196282108795457464434688*a^71*b^16*c^57*d^41 + 1484644864880431945098662
510592*a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*
d^43 + 146493707302289292776429322240*a^74*b^13*c^54*d^44 - 376790059998473
```

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99095674077184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11
*c^52*d^46 - 15765465234077253559186882 56*a^77*b^10*c^51*d^47 + 24774425845
9119342197932032*a^78*b^9*c^50*d^48 - 3156613601292619528273920 0*a^79*b^8*c
^49*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 22727001188359489
9783680*a^81*b^6*c^47*d^51 + 107175763212237589708 80*a^82*b^5*c^46*d^52 - 2
465991011962988789 76*a^83*b^4*c^45*d^53)*1i + 10505997210729893068 8*a^31*b^
54*c^91*d^4 - 42023988842919572275 20*a^32*b^53*c^90*d^5 + 8145649837385910 4
260096*a^33*b^52*c^89*d^6 - 10194708404486044385 28000*a^34*b^51*c^88*d^7 +
92615851877794055234519 04*a^35*b^50*c^87*d^8 - 65094971944398671145074688*a
^36*b^49*c^86*d^9 + 368402395453916323189358592*a^37*b^48*c^85*d^10 - 17252
263161509281442782248 96*a^38*b^47*c^84*d^11 + 68177424522028681284865228 80*
a^39*b^46*c^83*d^12 - 230715051950649310528866877 44*a^40*b^45*c^82*d^13 + 6
7614089216123669492331970560*a^41*b^44*c^81*d^14 - 17311502556247378546890 5
324544*a^42*b^43*c^80*d^15 + 389913831719674713212222177280*a^43*b^42*c^79*
d^16 - 776790088912432141093966970880*a^44*b^41*c^78*d^17 + 137461198325127
2530469308071936*a^45*b^40*c^77*d^18 - 2167454612994156285048662261760*a^46
*b^39*c^76*d^19 + 3050337310429700535004075917312*a^47*b^38*c^75*d^20 - 382
6885622871496570502324944896*a^48*b^37*c^74*d^21 + 423871339337551338392172
6259200*a^49*b^36*c^73*d^22 - 3984291896345024467843348955136*a^50*b^35*c^7
2*d^23 + 2651971426464597412032295206912*a^51*b^34*c^71*d^24 + 479249403658
1296397335343923 20*a^52*b^33*c^70*d^25 - 669745252969864773483754841702 4*a^
53*b^32*c^69*d^26 + 179310542699951499982776827904 00*a^54*b^31*c^68*d^27 -
363117150219056347997847473356 80*a^55*b^30*c^67*d^28 + 63073617076394089001
091166371840*a^56*b^29*c^66*d^29 - 97105565168138147055402127196160*a^57*b^
28*c^65*d^30 + 133993666277013207597272619024384*a^58*b^27*c^64*d^31 - 1664
92084833102044695859350732800*a^59*b^26*c^63*d^32 + 186717161118223967667 06
6928889856*a^60*b^25*c^62*d^33 - 189235624153406619951659086774272*a^61*b^2
4*c^61*d^34 + 173421825288151984221422006304768*a^62*b^23*c^60*d^35 - 14371
5376746696050902973036888064*a^63*b^22*c^59*d^36 + 107645128880801788128312
132894720*a^64*b^21*c^58*d^37 - 72802169209714119238549751463936*a^65*b^20*
c^57*d^38 + 44389639270136779232591657041920*a^66*b^19*c^56*d^39 - 24348625
105436875280486976454656*a^67*b^18*c^55*d^40 + 119811455119385226976200700 7
2320*a^68*b^17*c^54*d^41 - 5269759325089910260644729323520*a^69*b^16*c^53*d
^42 + 2062471522530027433706750214144*a^70*b^15*c^52*d^43 - 714227824367410
213467319173120*a^71*b^14*c^51*d^44 + 217305373751493983005392764928*a^72*b
^13*c^50*d^45 - 57574411148433569424441606144*a^73*b^12*c^49*d^46 + 1313394
7360733882065354752000*a^74*b^11*c^48*d^47 - 2542019460242050797665255424*a
^75*b^10*c^47*d^48 + 409310322447365741947650048*a^76*b^9*c^46*d^49 - 53356
649691793134232535040*a^77*b^8*c^45*d^50 + 5410594924578893614546944*a^78*b
^7*c^44*d^51 - 400464195437318897664000*a^79*b^6*c^43*d^52 + 19246289226179
889070080*a^80*b^5*c^42*d^53 - 450813981874483888128*a^81*b^4*c^41*d^54)*1i
 + x^(1/2)*(119342219331695731015680*a^30*b^49*c^73*d^13 - 3677615218076424
339456*a^29*b^50*c^74*d^12 - 18560134430309724255682 56*a^31*b^48*c^72*d^14
+ 18426099996452807258406912*a^32*b^47*c^71*d^15 - 131228123459738637629915
136*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 -
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 3088237415348484431457288192*a^35*b^44*c^68*d^18 + 10882952503625649640326
561792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d
^20 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 1564392910251950б
9838804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^3
9*c^63*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 19326483
6517334230347779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632
*a^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27
+ 3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 5465398361763642490
480861642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^3
2*c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 9672669
866587270697877661286400*a^49*b^30*c^54*d^32 - 8839280066432157154484139589
632*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^
34 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 3079795601090740
527825181212672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*
b^25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 31901
3552886948801896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347
840*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41
 + 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 171550262594890370415
3292800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^
44 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931239515783
3168*a^63*b^16*c^40*d^46 - 1975123254987217856102400*a^64*b^15*c^39*d^47 + 5
678869390326597943296*a^65*b^14*c^38*d^48))*((((767544201216*a^37*d^37 + 11
0075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a
^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^
5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a
^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616
*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 1591586523457
78176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 996907
51312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 1
3543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^1
6 + 35040911741905305б*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*
c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a
^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762
617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 -
22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c
^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*
a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630
370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 32323718
0229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 173731
83736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246
962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*
a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*
c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c
*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^
9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c
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^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*
c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20
*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^4
9 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 219902325
5552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560
*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 138384533471887
36*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088
726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498
859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 -
 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c
^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*
a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 4130592987
7070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 +
 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*
c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*
a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 19274988590
72716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 23129
9863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 1
3838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 -
 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(
1/2) - 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*
b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^3
3*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31
*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^2
9*c^29*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b
^27*c^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a
^12*b^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646
272*a^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 3535126022
9615616*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 5902
53517884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^1
9 + 3346011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16
*c^16*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336
*a^23*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 102744685
96079321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 -
 5086592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9
*d^28 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7
*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b
^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^
3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*
d + 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*
d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 -
 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44
*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^
21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^
10 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373
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600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^
17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^1
7 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 2257928
40*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10
*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 33
65856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22
*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c
^19*d^30)))^(1/4))/((((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 +
33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299
917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437
255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 7233
5498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14
9030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^
11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c
^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b
^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*
a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 31224306055
75077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11
832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^1
5*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^
24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 156782680610
77536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5
597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^2
9 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6
*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^
4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*
d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (3
644354584880l*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 14460350524
90812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 855065595266152
2*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394
735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 9862080265939
1292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 389071532281
63455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 35746830570
23442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^
17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 +
 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2
471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 +
 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d
^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*
c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^
24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 238713320239
00569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3
8876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^
33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a
^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815
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535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4
433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*
d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*
c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b
^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b
^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37
*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 1240263
41031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228
516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416
567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 5733095802
9815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 7957932
6172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49
845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14
 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*
d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^1
9*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944
*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381
308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 +
11462371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c
^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^
27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 129853313631518
5152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 16161859011
4652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 868659186
8473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 1541234811
61728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36
*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^4
8*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^
3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26
*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 2804
8800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^2
1*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13
 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 60108039
0*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14
*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20
- 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a
^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^2
5 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*
c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1
/2)*((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^
2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*
b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*
b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8
*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*
a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528
370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 563476
```

```
98493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 7
0702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^
17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^1
8*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 1183226127125708390
4*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319
215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24
 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^1
1*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 559713091980460032
0*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087
806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 842002491
13214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 27334337
01433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625
792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^1
2*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11
*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d
^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*
d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6
*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^
4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c
^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023
255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a
^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a
^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 6227304006234931
2*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152
268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 443324
7375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11
 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^1
9*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 388761692960666419
20*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 3887616
9296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^1
8 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^
12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 443324737586724864
0*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 7228120721
52268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273
040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 24
71152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34
084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100
608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34
*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c
^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c
^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*
c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b
^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a
^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 67718624892272
64*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 1752045587
```

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09526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 156
1215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d
^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^
15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225
024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 783913
4030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27
 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8
*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*
c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*
c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^
2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(68719
476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c
^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^
28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 33658
56*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^
40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225
792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27
*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*
d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 3473
73600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*
b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23
 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b
^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*
a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154605056*
a^34*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 + 7882870203
4483948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^91
*d^7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 734402201643481373469
57312*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*d^10
- 2196237253234092465387995136*a^41*b^46*c^87*d^11 + 92742963161445956466699
012096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c^85*d
^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 2949214323015047
98990377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^46*b^
41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 307247
1208539973972578986360832*a^48*b^39*c^80*d^18 - 538410677725243287141686968
3200*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c^78*d
^20 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 16707589390432
621056738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857792*a
^53*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d^24 -
 38378393138521379212996695293952*a^55*b^32*c^73*d^25 + 5491884609325839757
7855222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a^57*b
^30*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^28 - 152
754106500312545531177547595776*a^59*b^28*c^69*d^29 + 1895497785085632634380
68404715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^61*b^
26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 2112
93730951350565888869600854016*a^63*b^24*c^65*d^33 + 18157524177670666828495
```

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6756672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a^65*b^2
2*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 64581
123553243990572098666889216*a^67*b^20*c^61*d^37 + 3754099263409471764008409
4451712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a^69*b^18*c^
59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 - 39334461962
82108795457464434688*a^71*b^16*c^57*d^41 + 1484644864880431945098662510592*
a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*d^43 +
146493707302289292776429322240*a^74*b^13*c^54*d^44 - 3767900599984739909567
4077184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11*c^52*d
^46 - 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 247744258459119342
197932032*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b^8*c^49*d^4
9 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 227270011883594899783680
*a^81*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52 - 24659910
1196298878976*a^83*b^4*c^45*d^53)*1i - 105059972107298930688*a^31*b^54*c^91
*d^4 + 4202398884291957227520*a^32*b^53*c^90*d^5 - 81456498373859104260096*
a^33*b^52*c^89*d^6 + 1019470840448604438528000*a^34*b^51*c^88*d^7 - 9261585
187779405523451904*a^35*b^50*c^87*d^8 + 65094971944398671145074688*a^36*b^4
9*c^86*d^9 - 368402395453916323189358592*a^37*b^48*c^85*d^10 + 172522631615
0928144278224896*a^38*b^47*c^84*d^11 - 6817742452202868128486522880*a^39*b^
46*c^83*d^12 + 23071505195064931052886687744*a^40*b^45*c^82*d^13 - 67614089
216123669492331970560*a^41*b^44*c^81*d^14 + 173115025562473785468905324544*
a^42*b^43*c^80*d^15 - 389913831719674713212222177280*a^43*b^42*c^79*d^16 +
776790088912432141093966970880*a^44*b^41*c^78*d^17 - 1374611983251272530469
308071936*a^45*b^40*c^77*d^18 + 2167454612994156285048662261760*a^46*b^39*c
^76*d^19 - 3050337310429700535004075917312*a^47*b^38*c^75*d^20 + 3826885622
871496570502324944896*a^48*b^37*c^74*d^21 - 4238713393375513383921726259200
*a^49*b^36*c^73*d^22 + 3984291896345024467843348955136*a^50*b^35*c^72*d^23
- 2651971426464597412032295206912*a^51*b^34*c^71*d^24 - 4792494036581296397
33534392320*a^52*b^33*c^70*d^25 + 6697452529698647734837548417024*a^53*b^32
*c^69*d^26 - 17931054269995149998277682790400*a^54*b^31*c^68*d^27 + 3631171
5021905634799784747335680*a^55*b^30*c^67*d^28 - 630736170763940890010911663
71840*a^56*b^29*c^66*d^29 + 97105565168138147055402127196160*a^57*b^28*c^65
*d^30 - 133993666277013207597272619024384*a^58*b^27*c^64*d^31 + 16649208483
3102044695859350732800*a^59*b^26*c^63*d^32 - 186717161118223967667066928889
856*a^60*b^25*c^62*d^33 + 189235624153406619951659086774272*a^61*b^24*c^61*
d^34 - 173421825288151984221422006304768*a^62*b^23*c^60*d^35 + 143715376746
696050902973036888064*a^63*b^22*c^59*d^36 - 1076451288808017881283121328947
20*a^64*b^21*c^58*d^37 + 72802169209714119238549751463936*a^65*b^20*c^57*d^
38 - 44389639270136779232591657041920*a^66*b^19*c^56*d^39 + 243486251054368
75280486976454656*a^67*b^18*c^55*d^40 - 11981145511938522697620070072320*a^
68*b^17*c^54*d^41 + 5269759325089910260644729323520*a^69*b^16*c^53*d^42 - 2
062471522530027433706750214144*a^70*b^15*c^52*d^43 + 7142278243674102134673
19173120*a^71*b^14*c^51*d^44 - 217305373751493983005392764928*a^72*b^13*c^5
0*d^45 + 57574411148433569424441606144*a^73*b^12*c^49*d^46 - 13133947360733
882065354752000*a^74*b^11*c^48*d^47 + 2542019460242050797665255424*a^75*b^1
```

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0*c^47*d^48 - 4093103224473657419476500048*a^76*b^9*c^46*d^49 + 533566496917
93134232535040*a^77*b^8*c^45*d^50 - 541059492457889361454694 4*a^78*b^7*c^44
*d^51 + 4004641954373188976640000*a^79*b^6*c^43*d^52 - 19246289226179889070 0
80*a^80*b^5*c^42*d^53 + 450813981874483888128*a^81*b^4*c^41*d^54)*1i + x^(1
/2)*(11934221933169573101568 0*a^30*b^49*c^73*d^13 - 3677615218076424339456*
a^29*b^50*c^74*d^12 - 185601344303097242556825 6*a^31*b^48*c^72*d^14 + 18426
0999964528072584069 12*a^32*b^47*c^71*d^15 - 131228123459738637629915136*a^3
3*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 - 308823
741534848443145728 8192*a^35*b^44*c^68*d^18 + 10882952503625649640326561792*
a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d^20 + 7
7306011497125960924962750464*a^38*b^41*c^65*d^21 - 156439291025195069838804
910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*c^63*
d^23 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 193264836517334
230347779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a^43*b
^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 + 32147
65851097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176364249048086 16
42752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*c^56*
d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 96726698665872
70697877661286400*a^49*b^30*c^54*d^32 - 8839280066432157154484139589632*a^5
0*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34 - 49
88522538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560109074052782 51
81212672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^25*c^
49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 319013552886
948801896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347840*a^5
7*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 + 8330
791306287661611887886336*a^59*b^20*c^44*d^42 - 1715502625948903704153292800
*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44 - 36
069332470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931239515783168*a^
63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 56788693
90326597943296*a^65*b^14*c^38*d^48))*((((767544201216*a^37*d^37 + 110075314
176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34
*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*
c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30
*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^
28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a
^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 9969075131258
8800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724
978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350
409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^
18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^1
7*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 174746667626171596
80*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 2292474
2364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^2
5 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^
10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*
```

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a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304
320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 1737318373694
6688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 3082469623234
56*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*
c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^
13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 +
 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16
+ 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18
 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^2
0 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^
22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 687
19476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^
44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b
^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18
*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*
a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 19274988590727
16800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 886649
4751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^1
2 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^
18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807
040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 323968
07746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^
19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^
11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 192749885907271680
0*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088
726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453
347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848
604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) -
55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 1662137244057б*a^2*b^35*c^
35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*
d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d
^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*
d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^2
7*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^2
5*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^1
4*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616
*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 59025351788
4506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 334
6011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d
^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b
^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 1027446859607932
1088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 508659
2011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 -
 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^
30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*
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d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d
^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 110
99869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 -
32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a
^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 +
906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24
*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 12
9024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^2
6*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34
*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471
435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33
*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d
^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a
^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 +
 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^3
0)))^(1/4)*1i - (((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324
2744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 12999174
35830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 154372555
94213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498
051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030
500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 +
 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*
d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*
c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17
*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312243060557507
7888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322
61271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^
22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b
^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1567826806107753
6768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971
30919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 -
1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^3
1 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^
33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35
 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443
545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 144603505249081
2*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^
2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*
a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292
*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 3890715322816345
5*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 357468305702344
2*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d
^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 340
84860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 24711
52383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622
```

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73040062349312*a^19*b^26*c^43*d^6 - 2312998630887260l6*a^20*b^25*c^42*d^7 +
 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40
*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b
^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 2387133202390056
9600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876
169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d
^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*
b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 155163658155353
70240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332
47375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23
 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24
*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c
^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c
^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^3
7 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 12402634103
1936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 25438432285163
52*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 186000754165678
08*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815
808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172
889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 498453
75656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6
771862489227264*a^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16
 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^
19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^2
0*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21 + 87373333813085
79840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 1146
2371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*
d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b
^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152
*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652
160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473
344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 15412348116172
8*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c
*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d
- 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 +
35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^4
3*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800
*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^
38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 4
71435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^
29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^3
1*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 12
9024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*
b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 +
906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21
```

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*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*
((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^
35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33
*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31
*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^2
9*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10
*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 1394650235283701
76*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 5634769849
3292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702
520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 -
 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^
18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^
21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 217433192156
96412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 2
0548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^
11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^
28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 10078859630878064
64*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 8420024911321
4976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 273343370143
3344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*
a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^
17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^1
4 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15
- 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17
 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^1
9 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^
21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d
^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 21990232555
52*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*
b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*
b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^
19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 7228120721522688
00*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375
867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 1
5516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^
36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a
^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296
066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 -
23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c
^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^
35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 72281207215226
8800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 622730400
62349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 247115
2383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 340848
60461056*a^43*b^2*c^19*d^30))^(1/2) - 55037657088*b^37*c^37 - 383772100608*
```

```
a^37*d^37 - 16621372440576*a^2*b^35*c^35*d^2 + 124026341031936*a^3*b^34*c^3
4*d^3 - 649958717915136*a^4*b^33*c^33*d^4 + 2543843228516352*a^5*b^32*c^32*
d^5 - 7718627797106688*a^6*b^31*c^31*d^6 + 18600075416567808*a^7*b^30*c^30*
d^7 - 36167749025660928*a^8*b^29*c^29*d^8 + 57330958029815808*a^9*b^28*c^28
*d^9 - 74515250191269888*a^10*b^27*c^27*d^10 + 79579326172889088*a^11*b^26*
c^26*d^11 - 69732511764185088*a^12*b^25*c^25*d^12 + 49845375656294400*a^13*
b^24*c^24*d^13 - 28173849246646272*a^14*b^23*c^23*d^14 + 6771862489227264*a
^15*b^22*c^22*d^15 + 35351260229615616*a^16*b^21*c^21*d^16 - 17520455870952
6528*a^17*b^20*c^20*d^17 + 590253517884506112*a^18*b^19*c^19*d^18 - 1561215
302787538944*a^19*b^18*c^18*d^19 + 3346011544839634944*a^20*b^17*c^17*d^20
- 5916130635628541952*a^21*b^16*c^16*d^21 + 8737333381308579840*a^22*b^15*c
^15*d^22 - 10871659607848206336*a^23*b^14*c^14*d^23 + 11462371182372225024*
a^24*b^13*c^13*d^24 - 10274468596079321088*a^25*b^12*c^12*d^25 + 7839134030
538768384*a^26*b^11*c^11*d^26 - 5086592011760910336*a^27*b^10*c^10*d^27 + 2
798565459902300160*a^28*b^9*c^9*d^28 - 1298533136315185152*a^29*b^8*c^8*d^2
9 + 503942981543903232*a^30*b^7*c^7*d^30 - 161618590114652160*a^31*b^6*c^6*
d^31 + 42100124556607488*a^32*b^5*c^5*d^32 - 8686591868473344*a^33*b^4*c^4*
d^33 + 1366716850716672*a^34*b^3*c^3*d^34 - 154123481161728*a^35*b^2*c^2*d^
35 + 1394287312896*a*b^36*c^36*d + 11099869986816*a^36*b*c*d^36)/(687194767
36*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*
d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c
^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a
^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d
^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 2257928
40*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^1
8*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16
 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 34737360
0*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11
*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 1
0518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c
^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42
*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(933866418731546050566*a^34
*b^53*c^94*d^4 - 38911100780481085440000*a^35*b^52*c^93*d^5 + 7882870203448
3948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^91*d^7
 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 7344022016434813734695731
2*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*d^10 - 21
96237253234092465387995136*a^41*b^46*c^87*d^11 + 92742963161445956466990120
96*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c^85*d^13
+ 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 29492143230150479899
0377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^46*b^41*c
^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 3072471208
539973972578986360832*a^48*b^39*c^80*d^18 - 5384106777252432871416869683200
*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c^78*d^20
- 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 167075893904326210
56738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857792*a^53*
```

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b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d^24 - 383
78393138521379212996695293952*a^55*b^32*c^73*d^25 + 54918846093258397577855
222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a^57*b^30*
c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^28 - 1527541
06500312545531177547595776*a^59*b^28*c^69*d^29 + 18954977850856326343806840
4715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^61*b^26*c
^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 21129373
0951350565888869600854016*a^63*b^24*c^65*d^33 + 181575241776706668284956756
672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a^65*b^22*c^
63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 645811235
53243990572098666889216*a^67*b^20*c^61*d^37 + 37540992634094717640084094451
712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a^69*b^18*c^59*d
^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 - 393344619628210
8795457464434688*a^71*b^16*c^57*d^41 + 1484644864880431945098662510592*a^72
*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*d^43 + 1464
93707302289292776429322240*a^74*b^13*c^54*d^44 - 37679005999847399095674077
184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11*c^52*d^46
 - 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 2477442584591193421979
32032*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b^8*c^49*d^49 +
3133065413748205302054912*a^80*b^7*c^48*d^50 - 227270011883594899783680*a^8
1*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52 - 246599101196
298878976*a^83*b^4*c^45*d^53)*1i + 105059972107298930688*a^31*b^54*c^91*d^4
 - 4202398884291957227520*a^32*b^53*c^90*d^5 + 81456498373859104260096*a^33
*b^52*c^89*d^6 - 1019470840448604438528000*a^34*b^51*c^88*d^7 + 92615851877
79405523451904*a^35*b^50*c^87*d^8 - 65094971944398671145074688*a^36*b^49*c^
86*d^9 + 368402395453916323189358592*a^37*b^48*c^85*d^10 - 1725226316150928
144278224896*a^38*b^47*c^84*d^11 + 6817742452202868128486522880*a^39*b^46*c
^83*d^12 - 23071505195064931052886687744*a^40*b^45*c^82*d^13 + 676140892161
23669492331970560*a^41*b^44*c^81*d^14 - 173115025562473785468905324544*a^42
*b^43*c^80*d^15 + 389913831719674713212222177280*a^43*b^42*c^79*d^16 - 7767
90088912432141093966970880*a^44*b^41*c^78*d^17 + 13746119832512725304693080
71936*a^45*b^40*c^77*d^18 - 2167454612994156285048662261760*a^46*b^39*c^76*
d^19 + 3050337310429700535004075917312*a^47*b^38*c^75*d^20 - 38268856228714
96570502324944896*a^48*b^37*c^74*d^21 + 4238713393375513383921726259200*a^4
9*b^36*c^73*d^22 - 3984291896345024467843348955136*a^50*b^35*c^72*d^23 + 26
51971426464597412032295206912*a^51*b^34*c^71*d^24 + 47924940365812963973353
4392320*a^52*b^33*c^70*d^25 - 6697452529698647734837548417024*a^53*b^32*c^6
9*d^26 + 17931054269995149998277682790400*a^54*b^31*c^68*d^27 - 36311715021
905634799784747335680*a^55*b^30*c^67*d^28 + 6307361707639408900109116637184
0*a^56*b^29*c^66*d^29 - 97105565168138147055402127196160*a^57*b^28*c^65*d^3
0 + 133993666277013207597272619024384*a^58*b^27*c^64*d^31 - 166492084833102
044695859350732800*a^59*b^26*c^63*d^32 + 186717161118223967667066928889856*
a^60*b^25*c^62*d^33 - 189235624153406619951659086774272*a^61*b^24*c^61*d^34
 + 173421825288151984221422006304768*a^62*b^23*c^60*d^35 - 1437153767466960
50902973036888064*a^63*b^22*c^59*d^36 + 107645128880801788128312132894720*a
```

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^64*b^21*c^58*d^37 - 72802169209714119238549751463936*a^65*b^20*c^57*d^38 +
 44389639270136779232591657041920*a^66*b^19*c^56*d^39 - 2434862510543687528
0486976454656*a^67*b^18*c^55*d^40 + 11981145511938522697620070072320*a^68*b
^17*c^54*d^41 - 5269759325089910260644729323520*a^69*b^16*c^53*d^42 + 20624
71522530027433706750214144*a^70*b^15*c^52*d^43 - 714227824367410213467319173
3120*a^71*b^14*c^51*d^44 + 217305373751493983005392764928*a^72*b^13*c^50*d^
45 - 57574411148433569424441606144*a^73*b^12*c^49*d^46 + 131339473607338820
65354752000*a^74*b^11*c^48*d^47 - 2542019460242050797665255424*a^75*b^10*c^
47*d^48 + 409310322447365741947650048*a^76*b^9*c^46*d^49 - 533566496917931З
4232535040*a^77*b^8*c^45*d^50 + 5410594924578893614546944*a^78*b^7*c^44*d^5
1 - 400464195437318897664000*a^79*b^6*c^43*d^52 + 19246289226179889070080*a
^80*b^5*c^42*d^53 - 450813981874483888128*a^81*b^4*c^41*d^54)*1i + x^(1/2)*
(1193422193316957310156680*a^30*b^49*c^73*d^13 - 367761521807642433945б*a^29
*b^50*c^74*d^12 - 18560134430309724255568256*a^31*b^48*c^72*d^14 + 184260999
964528072584069l2*a^32*b^47*c^71*d^15 - 1312281234597386376299151З6*a^33*b^
46*c^70*d^16 + 7141820725650917746267914 24*a^34*b^45*c^69*d^17 - 3088237415
348484431457288192*a^35*b^44*c^68*d^18 + 108829525036256496403265617 92*a^36
*b^43*c^67*d^19 - 317570746004740778035815383 04*a^37*b^42*c^66*d^20 + 77306
011497125960924962750464*a^38*b^41*c^65*d^21 - 156439291025195069838804910 0
80*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*c^63*d^23
 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 193264836517334230З
47779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a^43*b^36*
c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 + 321476585
1097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176364249048086164275
2*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*c^56*d^30
 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 967266986658727069
7877661286400*a^49*b^30*c^54*d^32 - 8839280066432157154484139589632*a^50*b^
29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34 - 498852
2538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560109074052782518121
2672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^25*c^49*d
^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 3190135528869488
01896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347840*a^57*b^
22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 + 83307913
06287661611887886336*a^59*b^20*c^44*d^42 - 1715502625948903704153292800*a^6
0*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44 - 360693
32470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931239515783168*a^63*b
^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 567886939032
6597943296*a^65*b^14*c^38*d^48))*((((767544201216*a^37*d^37 + 110075314176*
b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^3
4*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32
*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^3
0*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c
^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*
b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800
*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 135437249784
```

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54528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 3504091
17419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 +
 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^
17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a
^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364
744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 -
15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c
^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29
*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*
a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688
*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a
^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^
36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 -
 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 855
0655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63
613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 9
8620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 +
38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 +
 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 6871947
6736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b
*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*
c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^2
7*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20
*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 192749885907271680
0*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751
734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 -
23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c
^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*
a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 3239680774
6722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 +
 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c
^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^
36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 2312998630887260
16*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 138384533471
88736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 3408486046
10560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) - 5503
7657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*c^35*d
^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^33*d^4
+ 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31*d^6 +
 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^29*d^8
+ 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c^27*d^
10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b^25*c^
25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a^14*b^
23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 3535126022961561 6*a^1
6*b^21*c^21*d^16 - 17520455870952652 8*a^17*b^20*c^20*d^17 + 590253517884506
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112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3346011
544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16*d^21
+ 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23*b^14*
c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10274468596079321088
*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086592011
760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28 - 129
8533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*d^30 -
 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^5*d^32
 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3*d^34
- 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 1109986
9986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a
^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*
b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 9061
92*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^4
1*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024
480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^
19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^1
5 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 4714356
00*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^1
2*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22
- 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*
b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 359
60*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))
^(1/4)*1i + 41028394776665109037056*a^29*b^48*c^68*d^15 - 12107398850760978
25505280*a^30*b^47*c^67*d^16 + 17243628768780949747924992*a^31*b^46*c^66*d^
17 - 15808131900444765483696128*a^32*b^45*c^65*d^18 + 10494949869157605271
33114368*a^33*b^44*c^64*d^19 - 5380683046490354438136397824*a^34*b^43*c^63*
d^20 + 22176160052724101903372255232*a^35*b^42*c^62*d^21 - 7548631332524163
6679770439680*a^36*b^41*c^61*d^22 + 216288375615109659684325294080*a^37*b^4
0*c^60*d^23 - 528818181695424054504437317632*a^38*b^39*c^59*d^24 + 11142226
90302433619242395893760*a^39*b^38*c^58*d^25 - 20375450552930580055296395182
08*a^40*b^37*c^57*d^26 + 3249918857904337975850827776000*a^41*b^36*c^56*d^2
7 - 4536394700759564584125915463680*a^42*b^35*c^55*d^28 + 55524352402839314
29496420302848*a^43*b^34*c^54*d^29 - 5964290825683224886861470105600*a^44*b
^33*c^53*d^30 + 5621639355410781338712284332032*a^45*b^32*c^52*d^31 - 46440
77108074496901042866749440*a^46*b^31*c^51*d^32 + 33553608627161291531082950
24640*a^47*b^30*c^50*d^33 - 2113405281704782215093506015232*a^48*b^29*c^49*
d^34 + 1155283596049337948225918730240*a^49*b^28*c^48*d^35 - 54482951987037
6944469402451968*a^50*b^27*c^47*d^36 + 219926172037899117268712816640*a^51*
b^26*c^46*d^37 - 75201916274561138554746961920*a^52*b^25*c^45*d^38 + 214839
48869172056418164932608*a^53*b^24*c^44*d^39 - 5032346201606164325320359936*
a^54*b^23*c^43*d^40 + 941275744618015035796488192*a^55*b^22*c^42*d^41 - 135
189136301093329947328512*a^56*b^21*c^41*d^42 + 13999140307267180988203008*a
^57*b^20*c^40*d^43 - 930460907799665663016960*a^58*b^19*c^39*d^44 + 2981406
4299214639202304*a^59*b^18*c^38*d^45))*((((767544201216*a^37*d^37 + 1100753
```

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14176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^
34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^3
2*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^
30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*
b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159158652345778176
*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312
588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 135437
24978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 3
50409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*
d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b
^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 1747466676261715
9680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924
742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d
^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*
b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 259706627263037030
4*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 3232371802293
04320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736
946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 30824696232
3456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*
b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*
d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24
 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^1
6 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^
18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d
^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*
d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 6
8719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*
a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16
*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^
18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 23129986308872601
6*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 192749885907
2716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866
494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d
^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*
b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 413059298770708
07040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 3239
6807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*
d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*
b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1927498859072716
800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 - 2312998630
88726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 138384
53347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 3408
48604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2)
- 55037657088*b^37*c^37 - 383772100608*a^37*d^37 - 16621372440576*a^2*b^35*
c^35*d^2 + 124026341031936*a^3*b^34*c^34*d^3 - 649958717915136*a^4*b^33*c^3
```

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3*d^4 + 2543843228516352*a^5*b^32*c^32*d^5 - 7718627797106688*a^6*b^31*c^31
*d^6 + 18600075416567808*a^7*b^30*c^30*d^7 - 36167749025660928*a^8*b^29*c^2
9*d^8 + 57330958029815808*a^9*b^28*c^28*d^9 - 74515250191269888*a^10*b^27*c
^27*d^10 + 79579326172889088*a^11*b^26*c^26*d^11 - 69732511764185088*a^12*b
^25*c^25*d^12 + 49845375656294400*a^13*b^24*c^24*d^13 - 28173849246646272*a
^14*b^23*c^23*d^14 + 6771862489227264*a^15*b^22*c^22*d^15 + 353512602296156
16*a^16*b^21*c^21*d^16 - 175204558709526528*a^17*b^20*c^20*d^17 + 590253517
884506112*a^18*b^19*c^19*d^18 - 1561215302787538944*a^19*b^18*c^18*d^19 + 3
346011544839634944*a^20*b^17*c^17*d^20 - 5916130635628541952*a^21*b^16*c^16
*d^21 + 8737333381308579840*a^22*b^15*c^15*d^22 - 10871659607848206336*a^23
*b^14*c^14*d^23 + 11462371182372225024*a^24*b^13*c^13*d^24 - 10274468596079
321088*a^25*b^12*c^12*d^25 + 7839134030538768384*a^26*b^11*c^11*d^26 - 5086
592011760910336*a^27*b^10*c^10*d^27 + 2798565459902300160*a^28*b^9*c^9*d^28
 - 1298533136315185152*a^29*b^8*c^8*d^29 + 503942981543903232*a^30*b^7*c^7*
d^30 - 161618590114652160*a^31*b^6*c^6*d^31 + 42100124556607488*a^32*b^5*c^
5*d^32 - 8686591868473344*a^33*b^4*c^4*d^33 + 1366716850716672*a^34*b^3*c^3
*d^34 - 154123481161728*a^35*b^2*c^2*d^35 + 1394287312896*a*b^36*c^36*d + 1
1099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32
- 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960
*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5
+ 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^
24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 -
129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a
^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^
34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 4
71435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^
33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27
*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856
*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27
 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d
^30)))^(1/4) - 2*atan((((-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 +
 ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^3
5*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*
c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*
c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29
*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*
b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 13946502352837017
6*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493
292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 707025
20459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 -
1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^1
8*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^2
1*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 2174331921569
6412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20
548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^1
```

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1*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^2
8*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 100788596308780646
4*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214
976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433
344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a
*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^1
7*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14
 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 -
 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17
- 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19
 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^2
1 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^
23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 219902325555
2*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b
^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b
^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^1
9*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 72281207215226880
0*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 44332473758
67248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15
516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^3
6*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^
28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 388761692960
66641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 2
3871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^
29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^3
5*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268
800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 6227304006
2349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152
383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 3408486
0461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124
026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 254384
3228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 1860007
5416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 573309
58029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 795
79326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12
- 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*
d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c
^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18
*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 334601154483963
4944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 873733
3381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^2
3 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^
12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 508659201176091033
6*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 12985331363
15185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 1616185
```

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90114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 86865
91868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123
481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*
a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31
*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^4
6*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*
b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 -
28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24
*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*
d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 6010
80390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*
b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d
^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 280488
00*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24
*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*
b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(
x^(1/2)*(-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((767544201216*
a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248
052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 50876
86457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200
150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 1146
61916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 -
 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25
*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23
*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16
*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012
224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023
089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21
 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^1
4*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 205489371921586421
76*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 1017318
4023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 +
 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d
^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5
*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3
*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 2
2199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 1065719
47510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 5334373963802
52*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 291045205783919
16*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814
328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 7337065190848
6968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 144325881654
02316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736
*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48
*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 3
```

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40848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 1
3838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6
- 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41
*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^
22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 155163658155353702
40*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 3239680
7746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^1
5 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^
15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569
600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 886649
4751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22
 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^2
5*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6
*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b
^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2
*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3
*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b
^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b
^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*
b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 79579326172889088*a
^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 49845375656294
400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 677186248
9227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16 + 17520
4558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^19*d^18
+ 1561215302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20*b^17*c
^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 8737333381308579840*a^
22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 114623711823
72225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d^25 - 7
839134030538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^10*c^10
*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^
8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 161618590114652160*a^31
*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33
*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b
^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(
68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^4
4*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^
17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 -
3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^
23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11
+ 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600
*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*
c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 -
 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*
a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26
*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a
```

```
^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 -
4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(9338664187315460
5056*a^34*b^53*c^94*d^4 - 38911100780481085440000*a^35*b^52*c^93*d^5 + 78828
70203448394829004 8*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50
*c^91*d^7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 7344022016434813
7346957312*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*
d^10 - 2196237253234092465387995136*a^41*b^46*c^87*d^11 + 92742963161445956
46699012096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c
^85*d^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 29492143230
1504798990377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^
46*b^41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 3
072471208539973972578986360832*a^48*b^39*c^80*d^18 - 5384106777252432871416
869683200*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c
^78*d^20 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 167075893
90432621056738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857
792*a^53*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d
^24 - 38378393138521379212996695293952*a^55*b^32*c^73*d^25 + 54918846093258
397577855222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a
^57*b^30*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^28
- 152754106500312545531177547595776*a^59*b^28*c^69*d^29 + 18954977850856326
343806840471552 0*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^
61*b^26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 -
 211293730951350565888869600854016*a^63*b^24*c^65*d^33 + 181575241776706668
284956756672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a^6
5*b^22*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 -
64581123553243990572098666889216*a^67*b^20*c^61*d^37 + 37540992634094717640
084094451712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a^69*b^
18*c^59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 - 393344
6196282108795457464434688*a^71*b^16*c^57*d^41 + 148464486488043194509866251
0592*a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*d^
43 + 146493707302289292776429322240*a^74*b^13*c^54*d^44 - 37679005999847399
095674077184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11*c
^52*d^46 - 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 2477442584591
19342197932032*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b^8*c^4
9*d^49 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 2272700118835948997
83680*a^81*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52 - 246
599101196298878976*a^83*b^4*c^45*d^53)*1i - 105059972107298930688*a^31*b^54
*c^91*d^4 + 4202398884291957227520*a^32*b^53*c^90*d^5 - 8145649837385910426
0096*a^33*b^52*c^89*d^6 + 1019470840448604438528000*a^34*b^51*c^88*d^7 - 92
6158518777940552345190 4*a^35*b^50*c^87*d^8 + 65094971944398671145074688*a^3
6*b^49*c^86*d^9 - 368402395453916323189358592*a^37*b^48*c^85*d^10 + 1725226
316150928144278224896*a^38*b^47*c^84*d^11 - 6817742452202868128486522880*a^
39*b^46*c^83*d^12 + 23071505195064931052886687744*a^40*b^45*c^82*d^13 - 676
14089216123669492331970560*a^41*b^44*c^81*d^14 + 17311502556247378546890532
4544*a^42*b^43*c^80*d^15 - 389913831719674713212222177280*a^43*b^42*c^79*d^
```

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16 + 7767900889124321410939669708880*a^44*b^41*c^78*d^17 - 13746119832512725
30469308071936*a^45*b^40*c^77*d^18 + 2167454612994156285048662261760*a^46*b
^39*c^76*d^19 - 3050337310429700535004075917312*a^47*b^38*c^75*d^20 + 38268
856228714965705023249448896*a^48*b^37*c^74*d^21 - 4238713393375513383921726
259200*a^49*b^36*c^73*d^22 + 3984291896345024467843348955136*a^50*b^35*c^72*
d^23 - 2651971426464597412032295206912*a^51*b^34*c^71*d^24 - 4792494036581
2963973353439232 0*a^52*b^33*c^70*d^25 + 6697452529698647734837548417024*a^53
*b^32*c^69*d^26 - 17931054269995149998277682790400*a^54*b^31*c^68*d^27 + 36
311715021905634799784747335680*a^55*b^30*c^67*d^28 - 630736170763940890010 9
1166371840*a^56*b^29*c^66*d^29 + 97105565168138147055402127196160*a^57*b^28
*c^65*d^30 - 133993666277013207597272619024384*a^58*b^27*c^64*d^31 + 166492
084833102044695859350732800*a^59*b^26*c^63*d^32 - 18671716111822396766706 69
28889856*a^60*b^25*c^62*d^33 + 18923562415340661995165908677 4272*a^61*b^24*
c^61*d^34 - 173421825288151984221422006304768*a^62*b^23*c^60*d^35 + 1437153
76746696050902973036888064*a^63*b^22*c^59*d^36 - 107645128880801788128312 13
2894720*a^64*b^21*c^58*d^37 + 72802169209714119238549751463936*a^65*b^20*c^
57*d^38 - 44389639270136779232591657041920*a^66*b^19*c^56*d^39 + 2434862510
5436875280486976454656*a^67*b^18*c^55*d^40 - 119811455119385226976200700723
20*a^68*b^17*c^54*d^41 + 5269759325089910260644729323520*a^69*b^16*c^53*d^4
2 - 2062471522530027433706750214144*a^70*b^15*c^52*d^43 + 714227824367410 21
3467319173120*a^71*b^14*c^51*d^44 - 217305373751493983005392764928*a^72*b^1
3*c^50*d^45 + 57574411148433569424441606144*a^73*b^12*c^49*d^46 - 131339473
607338820653547520 00*a^74*b^11*c^48*d^47 + 2542019460242050797665255424*a^7
5*b^10*c^47*d^48 - 409310322447365741947650048*a^76*b^9*c^46*d^49 + 5335664
9691793134232535040*a^77*b^8*c^45*d^50 - 5410594924578893614546944*a^78*b^7
*c^44*d^51 + 400464195437318897664000*a^79*b^6*c^43*d^52 - 192462892261798 8
9070080*a^80*b^5*c^42*d^53 + 4508139818744838881 28*a^81*b^4*c^41*d^54)*1i +
 x^(1/2)*(119342219331695731015680*a^30*b^49*c^73*d^13 - 3677615218076424 33
9456*a^29*b^50*c^74*d^12 - 1856013443030972425568256*a^31*b^48*c^72*d^14 +
18426099996452807258406912*a^32*b^47*c^71*d^15 - 131228123459738637629915 13
6*a^33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 - 3
088237415348484431457288192*a^35*b^44*c^68*d^18 + 10882952503625649640326 56
1792*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d^2
0 + 77306011497125960924962750464*a^38*b^41*c^65*d^21 - 1564392910251950698
38804910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*
c^63*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 1932648365
17334230347779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a
^43*b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 +
3214765851097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176364249048
0861642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*
c^56*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 967266986
6587270697877661286400*a^49*b^30*c^54*d^32 - 883928006643215715448413958963
2*a^50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34
 - 4988522538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560109074052
7825181212672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^
```

```
25*c^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 3190135
52886948801896949743616*a^56*b^23*c^47*d^39 + 11176666809872758563913334784
0*a^57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 +
 8330791306287661611887886336*a^59*b^20*c^44*d^42 - 17155026259489037041532
92800*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44
 - 36069332470586798845722624*a^62*b^17*c^41*d^45 + 33248505889312395157831
68*a^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 567
8869390326597943296*a^65*b^14*c^38*d^48))*(-(383772100608*a^37*d^37 + 55037
657088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324
2744881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 12999174
35830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 154372555
94213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498
051321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030
500382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 +
 139465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*
d^13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*
c^22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17
*b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312243060557507
7888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322
61271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^
22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b
^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 1567826806107753
6768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971
30919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 -
1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^3
1 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^
33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35
 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443
545848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 144603505249081
2*a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^
2*b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*
a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292
*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 3890715322816345
5*a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 357468305702344
2*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d
^32 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 340
84860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 24711
52383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622
73040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 +
 722812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40
*d^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b
^21*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 2387133202390056
9600*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876
169296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d
^16 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*
```

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b^14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 155163658155353
70240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332
47375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23
 + 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24
*d^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c
^22*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c
^20*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b
^35*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33
*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*
c^31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29
*c^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^
27*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^
12*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 281738492466462
72*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 35351260229
615616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 59025
3517884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19
 - 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*
c^16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*
a^23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 1027446859
6079321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 +
5086592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*
d^28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*
c^7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^
5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3
*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d
 - 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d
^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 -
4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*
d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^2
1*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^1
0 - 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 3473736
00*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^1
7*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17
 + 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 22579284
0*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*
c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 336
5856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*
d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^
19*d^30)))^(1/4) + ((-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((7
67544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^
35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33
*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31
*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^2
9*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27
*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^
```

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12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 563476984932925
44*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 7070252045
9231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180
507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^
19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^
16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412
672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 205489
37192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^
26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^
9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^
30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*
a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*
a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^3
6*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^
25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 5
33437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 291
04520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94
521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 7
3370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 -
14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*
(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^
14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*
c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*
c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^
26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^
21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 443324737586724
8640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 155163
65815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^
13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b
^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 388761692960664
1920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871
332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d
^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^
10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*
a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349
312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 24711523834
26560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461
056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 1240263
41031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 2543843228
516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 18600075416
567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 5733095802
9815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 7957932
6172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 49
845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14
 - 6771862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*
```

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d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^1
9*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944
*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 8737333381
308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 -
11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c
^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^
27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 12985331363151
85152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 16161859011
4652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 868659186
8473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 1541234811
61728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 1109986998816*a^36
*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^4
8*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^
3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26
*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 2804
8800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^2
1*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13
 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 60108039
0*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14
*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20
- 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a
^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^2
5 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*
c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1
/2)*(-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((767544201216*a^37
*d^37 + 110075314176*b^37*c^37 + 3324274881152*a^2*b^35*c^35*d^2 - 2480526
82063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 508768645
7032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 372001508
33135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 11466191
6059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 159
158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^1
2 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^2
3*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^2
1*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*
a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 66920230896
79269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 1
7474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^
14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a
^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 10173184023
521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 259
7066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30
+ 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^3
2 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^3
4 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199
739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 10657194751
```

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0161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a
^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a
^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*
a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968
*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 1443258816540231
6*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^1
3*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d -
 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 34084
8604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 13838
453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 23
1299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8
 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c
^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a
^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 32396807746
722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 +
41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c
^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*
a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 8866494751
734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 1
927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^
24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^2
3*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c
^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^1
9*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^3
4*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*
c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*
c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28
*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*
b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 49845375656294400*
a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 6771862489227
264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16 + 175204558
709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^19*d^18 + 15
61215302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20*b^17*c^17*
d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b
^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 114623711823722
5024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d^25 - 78391
34030538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^2
7 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^
8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6
*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4
*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c
^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(6871
9476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*
c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b
^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365
```

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856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c
^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 22
5792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^2
7*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33
*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 347
373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34
*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^2
3 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*
b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960
*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154605056
*a^34*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 + 788287020
34483948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^9
1*d^7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 73440220164348137346
957312*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*d^10
 - 2196237253234092465387995136*a^41*b^46*c^87*d^11 + 927429631614459564669
9012096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c^85*
d^13 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 294921432301504
798990377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^46*b
^41*c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 30724
71208539973972578986360832*a^48*b^39*c^80*d^18 - 53841067772524328714168696
83200*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c^78*
d^20 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 1670758939043
2621056738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857792*
a^53*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d^24
- 38378393138521379212996695293952*a^55*b^32*c^73*d^25 + 549188460932583975
77855222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a^57*
b^30*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^28 - 15
2754106500312545531177547595776*a^59*b^28*c^69*d^29 + 189549778508563263438
068404715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^61*b
^26*c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 211
293730951350565888869600854016*a^63*b^24*c^65*d^33 + 1815752417767066682849
56756672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a^65*b^
22*c^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 6458
1123553243990572098666889216*a^67*b^20*c^61*d^37 + 375409926340947176400840
94451712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a^69*b^18*c
^59*d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 - 3933446196
2821087954574644346 88*a^71*b^16*c^57*d^41 + 1484644864880431945098662510592
*a^72*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*d^43 +
 146493707302289292776429322240*a^74*b^13*c^54*d^44 - 376790059998473990956
74077184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11*c^52*
d^46 - 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 24774425845911934
2197932032*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b^8*c^49*d^
49 + 3133065413748205302054912*a^80*b^7*c^48*d^50 - 22727001188359489978368
0*a^81*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52 - 2465991
0119629887 8976*a^83*b^4*c^45*d^53)*1i + 105059972107298930688*a^31*b^54*c^9
```

```
1*d^4 - 4202398884291957227520*a^32*b^53*c^90*d^5 + 81456498373859104260096
*a^33*b^52*c^89*d^6 - 1019470840448604438528000*a^34*b^51*c^88*d^7 + 926158
5187779405523451904*a^35*b^50*c^87*d^8 - 65094971944398671145074688*a^36*b^
49*c^86*d^9 + 368402395453916323189358592*a^37*b^48*c^85*d^10 - 17252263161
50928144278224896*a^38*b^47*c^84*d^11 + 6817742452202868128486522880*a^39*b
^46*c^83*d^12 - 23071505195064931052886687744*a^40*b^45*c^82*d^13 + 6761408
9216123669492331970560*a^41*b^44*c^81*d^14 - 173115025562473785468905324544
*a^42*b^43*c^80*d^15 + 389913831719674713212222177280*a^43*b^42*c^79*d^16 -
 776790088912432141093966970880*a^44*b^41*c^78*d^17 + 137461198325127253046
9308071936*a^45*b^40*c^77*d^18 - 2167454612994156285048662261760*a^46*b^39*
c^76*d^19 + 3050337310429700535004075917312*a^47*b^38*c^75*d^20 - 382688562
2871496570502324944896*a^48*b^37*c^74*d^21 + 423871339337551338392172625920
0*a^49*b^36*c^73*d^22 - 3984291896345024467843348955136*a^50*b^35*c^72*d^23
 + 2651971426464597412032295206912*a^51*b^34*c^71*d^24 + 479249403658129639
733534392320*a^52*b^33*c^70*d^25 - 6697452529698647734837548417024*a^53*b^3
2*c^69*d^26 + 17931054269995149998277682790400*a^54*b^31*c^68*d^27 - 363117
15021905634799784747335680*a^55*b^30*c^67*d^28 + 63073617076394089001091166
371840*a^56*b^29*c^66*d^29 - 97105565168138147055402127196160*a^57*b^28*c^6
5*d^30 + 133993666277013207597272619024384*a^58*b^27*c^64*d^31 - 1664920848
33102044695859350732800*a^59*b^26*c^63*d^32 + 18671716111822396766706692888
9856*a^60*b^25*c^62*d^33 - 189235624153406619951659086774272*a^61*b^24*c^61
*d^34 + 173421825288151984221422006304768*a^62*b^23*c^60*d^35 - 14371537674
6696050902973036888064*a^63*b^22*c^59*d^36 + 107645128880801788128312132894
720*a^64*b^21*c^58*d^37 - 72802169209714119238549751463936*a^65*b^20*c^57*d
^38 + 44389639270136779232591657041920*a^66*b^19*c^56*d^39 - 24348625105436
875280486976454656*a^67*b^18*c^55*d^40 + 11981145511938522697620070072320*a
^68*b^17*c^54*d^41 - 5269759325089910260644729323520*a^69*b^16*c^53*d^42 +
2062471522530027433706750214144*a^70*b^15*c^52*d^43 - 714227824367410213467
319173120*a^71*b^14*c^51*d^44 + 217305373751493983005392764928*a^72*b^13*c^
50*d^45 - 57574411148433569424441606144*a^73*b^12*c^49*d^46 + 1313394736073
3882065354752000*a^74*b^11*c^48*d^47 - 2542019460242050797665255424*a^75*b^
10*c^47*d^48 + 409310322447365741947650048*a^76*b^9*c^46*d^49 - 53356649691
793134232535040*a^77*b^8*c^45*d^50 + 5410594924578893614546944*a^78*b^7*c^4
4*d^51 - 400464195437318897664000*a^79*b^6*c^43*d^52 + 19246289226179889070
080*a^80*b^5*c^42*d^53 - 450813981874483888128*a^81*b^4*c^41*d^54)*1i + x^(
1/2)*(119342219331695731015680*a^30*b^49*c^73*d^13 - 3677615218076424339456
*a^29*b^50*c^74*d^12 - 1856013443030972425568256*a^31*b^48*c^72*d^14 + 1842
6099996452807258406912*a^32*b^47*c^71*d^15 - 131228123459738637629915136*a^
33*b^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 - 30882
37415348484431457288192*a^35*b^44*c^68*d^18 + 10882952503625649640326561792
*a^36*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d^20 +
77306011497125960924962750464*a^38*b^41*c^65*d^21 - 156439291025195069838804
910080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*c^63
*d^23 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 19326483651733
4230347779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a^43*
```

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b^36*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 + 3214
765851097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176364249048086l
642752*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*c^56
*d^30 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 9672669866587
270697877661286400*a^49*b^30*c^54*d^32 - 8839280066432157154484139589632*a^
50*b^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34 - 4
988522538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560109074052782 5
181212672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^25*c
^49*d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 31901355288
6948801896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347840*a^
57*b^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 + 833
0791306287661611887886336*a^59*b^20*c^44*d^42 - 171550262594890370415329280
0*a^60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44 - 3
6069332470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931239515783168*a
^63*b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 5678869
390326597943296*a^65*b^14*c^38*d^48))*(-(383772100608*a^37*d^37 + 550376570
88*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744
881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 129991743583
0272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 1543725559421
3376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 723354980513
21856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 1490305003
82539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139
465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13
 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22
*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^2
0*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888
*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 1183226127
1257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 +
 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*
c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768
*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 559713091
9804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007
885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 -
84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 -
 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2
788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (364435458
48801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*
b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^
27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*
b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6
*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^
8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^
10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32
- 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 3408486
0461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 247115238
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3426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 6227304
0062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722
812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9
 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*
c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600
*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 388761692
96066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16
- 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14
*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 155163658155353702 4
0*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 443324737
5867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 7
22812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^2
5 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*
d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*
d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*
c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^3
3*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31
*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^2
9*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c
^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b
^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a
^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 353512602296156
16*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 590253517
884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 3
346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16
*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23
*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 10274468596079
321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 5086
592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28
 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*
d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^
5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3
*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 1
1099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32
- 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960
*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5
+ 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^
24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 -
129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a
^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^
34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 4
71435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^
33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27
*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856
*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27
```

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 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d
^30)))^(1/4))/(((-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((76754
4201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d
^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4
 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6
 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^
8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^2
7*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b
^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a
^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231
232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 11805070
35769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 -
 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c
^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*
a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 2054893719
2158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 +
 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^
9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b
^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32
*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34
*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^
36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 +
 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 53343
7396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 2910452
0578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 945212
16268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370
651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 1443
2588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(687
19476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b
^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47
*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45
*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c
^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b
^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640
*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 1551636581
5535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 +
 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*
c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920
*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 238713320
23900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20
- 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c
^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37
*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*
a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 247115238342656
0*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*
```

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a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 12402634103
1936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 25438432285163
52*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 186000754165678
08*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 57330958029815
808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 79579326172
889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 498453
75656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 6
771862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16
 + 175204558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^
19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^2
0*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 87373333813085
79840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 1146
2371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*
d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b
^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152
*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 161618590114652
160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 8686591868473
344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 15412348116172
8*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c
*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d
- 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 +
35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^4
3*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800
*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^
38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 4
71435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^
29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^3
1*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 12
9024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*
b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 +
906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21
*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*
(-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((767544201216*a^37*d^3
7 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 24805268206
3872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 5087686457032
704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 3720015083313
5616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 114661916059
631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 1591586
52345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 -
99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^
14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^
21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18
*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 669202308967926
9888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 17474
666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d
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^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*
b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 101731840235218
20672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 2597066
272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 32
3237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 +
17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 +
308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 221997399
73632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161
*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*
b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b
^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*
b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7
*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^
9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^
32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 219
9023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 340848604
610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 138384533
47188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 231299
863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 1
927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*
d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*
b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 323968077467222
01600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 4130
5929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*
d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32
*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 88664947517344
97280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 19274
98859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 -
 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^
26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*
d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^
30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^
34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32
*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30
*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^2
8*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26
*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 49845375656294400*a^13
*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 6771862489227264*
a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16 + 1752045587095
26528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^19*d^18 + 156121
5302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20*b^17*c^17*d^20
 + 5916130635628541952*a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*
c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 11462371182372225024
*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d^25 - 783913403
0538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^27 -
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2798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^
29 - 503942981543903232*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6
*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4
*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d
^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(68719476
736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18
*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*
c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*
a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*
d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 225792
840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^
18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^1
6 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 3473736
00*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^1
1*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 +
10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*
c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^4
2*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154605056*a^3
4*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 + 7882870203448
3948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^91*d^
7 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 734402201643481373469573
12*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*d^10 - 2
196237253234092465387995136*a^41*b^46*c^87*d^11 + 9274296316144595646699012
096*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c^85*d^13
 + 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 2949214323015047989
90377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^46*b^41*
c^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 307247120
8539973972578986360832*a^48*b^39*c^80*d^18 - 538410677725243287141686968320
0*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c^78*d^20
 - 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 16707589390432621
056738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857792*a^53
*b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d^24 - 38
378393138521379212996695293952*a^55*b^32*c^73*d^25 + 5491884609325839757785
5222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a^57*b^30
*c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^28 - 152754
106500312545531177547595776*a^59*b^28*c^69*d^29 + 1895497785085632634380684
04715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^61*b^26*
c^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 2112937
30951350565888869600854016*a^63*b^24*c^65*d^33 + 18157524177670666828495675
6672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a^65*b^22*c
^63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 64581123
553243990572098666889216*a^67*b^20*c^61*d^37 + 3754099263409471764008409445
1712*a^68*b^19*c^60*d^38 - 19695179695689601910490494140416*a^69*b^18*c^59*
d^39 + 9296840942046414522746815905792*a^70*b^17*c^58*d^40 - 39334461962821
08795457464434688*a^71*b^16*c^57*d^41 + 1484644864880431945098662510592*a^7
```

```
2*b^15*c^56*d^42 - 4969938773331195363812777656 32*a^73*b^14*c^55*d^43 + 146
49370730228929277642932224 0*a^74*b^13*c^54*d^44 - 376790059998473990956740 7
7184*a^75*b^12*c^53*d^45 + 836009462399118122346872832 0*a^76*b^11*c^52*d^46
 - 157654652340772535591868825 6*a^77*b^10*c^51*d^47 + 2477442584591193421 97
932032*a^78*b^9*c^50*d^48 - 315661360129261952827392 00*a^79*b^8*c^49*d^49 +
 31330654137482053020549 12*a^80*b^7*c^48*d^50 - 2272700118835948997836 80*a^
81*b^6*c^47*d^51 + 107175763212237589708 80*a^82*b^5*c^46*d^52 - 24659910119
6298878976*a^83*b^4*c^45*d^53)*1i - 10505997210729893068 8*a^31*b^54*c^91*d^
4 + 420239888429195722752 0*a^32*b^53*c^90*d^5 - 814564983738591042600 96*a^3
3*b^52*c^89*d^6 + 101947084044860443852800 0*a^34*b^51*c^88*d^7 - 9261585187
779405523451904*a^35*b^50*c^87*d^8 + 650949719443986711450746 88*a^36*b^49*c
^86*d^9 - 368402395453916323189358592*a^37*b^48*c^85*d^10 + 172522631615092
8144278224896*a^38*b^47*c^84*d^11 - 681774245220286812848652288 0*a^39*b^46*
c^83*d^12 + 230715051950649310528866877 44*a^40*b^45*c^82*d^13 - 67614089216
123669492331970560*a^41*b^44*c^81*d^14 + 173115025562473785468905324544*a^4
2*b^43*c^80*d^15 - 389913831719674713212222177280*a^43*b^42*c^79*d^16 + 776
790088912432141093966970880*a^44*b^41*c^78*d^17 - 137461198325127253046930 8
071936*a^45*b^40*c^77*d^18 + 2167454612994156285048662261760*a^46*b^39*c^76
*d^19 - 3050337310429700535004075917312*a^47*b^38*c^75*d^20 + 3826885622871
496570502324944896*a^48*b^37*c^74*d^21 - 4238713393375513383921726259200*a^
49*b^36*c^73*d^22 + 3984291896345024467843348955136*a^50*b^35*c^72*d^23 - 2
651971426464597412032295206912*a^51*b^34*c^71*d^24 - 479249403658129639733 5
34392320*a^52*b^33*c^70*d^25 + 6697452529698647734837548417024*a^53*b^32*c^
69*d^26 - 17931054269995149998277682790400*a^54*b^31*c^68*d^27 + 3631171502
1905634799784747335680*a^55*b^30*c^67*d^28 - 630736170763940890010911663718
40*a^56*b^29*c^66*d^29 + 97105565168138147055402127196160*a^57*b^28*c^65*d^
30 - 133993666277013207597272619024384*a^58*b^27*c^64*d^31 + 16649208483310
2044695859350732800*a^59*b^26*c^63*d^32 - 186717161118223967667066928889856
*a^60*b^25*c^62*d^33 + 189235624153406619951659086774272*a^61*b^24*c^61*d^3
4 - 173421825288151984221422006304768*a^62*b^23*c^60*d^35 + 143715376746696
050902973036888064*a^63*b^22*c^59*d^36 - 107645128880801788128312132894720*
a^64*b^21*c^58*d^37 + 72802169209714119238549751463936*a^65*b^20*c^57*d^38
- 44389639270136779232591657041920*a^66*b^19*c^56*d^39 + 243486251054368752
8048697645 4656*a^67*b^18*c^55*d^40 - 11981145511938522697620070072320*a^68*
b^17*c^54*d^41 + 5269759325089910260644729323520*a^69*b^16*c^53*d^42 - 2062
471522530027433706750214144*a^70*b^15*c^52*d^43 + 714227824367410213467319 1
73120*a^71*b^14*c^51*d^44 - 217305373751493983005392764928*a^72*b^13*c^50*d
^45 + 57574411148433569424441606144*a^73*b^12*c^49*d^46 - 1313394736073388 2
065354752000*a^74*b^11*c^48*d^47 + 2542019460242050797665255424*a^75*b^10*c
^47*d^48 - 409310322447365741947650048*a^76*b^9*c^46*d^49 + 533566496917931
34232535040*a^77*b^8*c^45*d^50 - 5410594924578893614546944*a^78*b^7*c^44*d^
51 + 400464195437318897664000*a^79*b^6*c^43*d^52 - 19246289226179889070080*
a^80*b^5*c^42*d^53 + 450813981874483888128*a^81*b^4*c^41*d^54)*1i + x^(1/2)
*(1193422193316957310156 80*a^30*b^49*c^73*d^13 - 367761521807642433945 6*a^2
9*b^50*c^74*d^12 - 185601344303097242556825 6*a^31*b^48*c^72*d^14 + 18426099
```

```
996452807258406912*a^32*b^47*c^71*d^15 - 131228123459738637629915136*a^33*b
^46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 - 308823741
5348484431457288192*a^35*b^44*c^68*d^18 + 10882952503625649640326561792*a^3
6*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d^20 + 7730
6011497125960924962750464*a^38*b^41*c^65*d^21 - 156439291025195069838804910
080*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*c^63*d^2
3 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 193264836517334230
347779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a^43*b^36
*c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 + 32147658
51097197421262933065728*a^45*b^34*c^58*d^28 - 54653983617636424904808616427
52*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*c^56*d^3
0 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 96726698665872706
97877661286400*a^49*b^30*c^54*d^32 - 8839280066432157154484139589632*a^50*b
^29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34 - 49885
22538878293079151039479808*a^52*b^27*c^51*d^35 + 30797956010907405278251812
12672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^25*c^49*
d^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 319013552886948
801896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347840*a^57*b
^22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 + 8330791
306287661611887886336*a^59*b^20*c^44*d^42 - 1715502625948903704153292800*a^
60*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44 - 36069
332470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931239515783168*a^63*
b^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 56788693903
26597943296*a^65*b^14*c^38*d^48))*(-(383772100608*a^37*d^37 + 55037657088*b
^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 332427448811
52*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272
*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376
*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 7233549805132185
6*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14903050038253
9776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 1394650
23528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 5
6347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^1
5 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^
20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^1
9*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257
083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 217
43319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13
*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^2
6*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804
600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 10078859
63087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 8420
0249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 273
3433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 27885
74625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (3644354584880
1*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28
```

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*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c
^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25
*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^2
3*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^
21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b
^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 21
99023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461
056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426
560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062
349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 7228120
72152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4
433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38
*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^2
6*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3887616929606
6641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38
876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^3
1*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^
33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867
248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 72281
2072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 +
62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27
 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29
 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35
*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^
4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6
 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^
8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*
d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*
c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*
b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3535126022961561 6*a
^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 5902535178845
06112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 33460
11544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^2
1 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^1
4*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 102744685960793210
88*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 50865920
11760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1
298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30
 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^
32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^3
4 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099
869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32
*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^1
6*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 90
6192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c
```

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^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 1290
24480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*
b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d
^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 47143
5600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b
^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^2
2 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^3
8*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 3
5960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)
))^(1/4)*1i - ((-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((767544
201216*a^37*d^37 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^
2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4
- 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6
- 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8
 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27
*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^
25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^
14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231
232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 118050703
5769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 -
6692023089679269888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^
16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a
^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192
158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 +
10173184023521820672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9
*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^
7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*
b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*
b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^3
6*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 +
106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437
396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520
578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 9452121
6268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 733706
51908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432
588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(6871
9476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^
31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*
d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*
d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^
43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^
24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*
a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815
535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 +
32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c
```

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^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*
a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 2387133202
3900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 -
 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^
27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*
b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a
^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560
*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a
^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124026341031
936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 254384322851635
2*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 1860007541656780
8*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 573309580298158
08*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 795793261728
89088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 4984537
5656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 67
71862489227264*a^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16
+ 175204558709526528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^1
9*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20
*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21 - 873733338130857
9840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 11462
371182372225024*a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d
^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^
10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*
a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30 + 1616185901146521
60*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 86865918684733
44*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123481161728
*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*
d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d -
 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 3
5960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43
*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*
a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^3
8*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 47
1435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^2
9*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31
*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129
024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b
^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 9
06192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*
d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(3/4)*(x^(1/2)*(
-(383772100608*a^37*d^37 + 55037657088*b^37*c^37 + ((767544201216*a^37*d^37
 + 110075314176*b^37*c^37 + 33242744881152*a^2*b^35*c^35*d^2 - 248052682063
872*a^3*b^34*c^34*d^3 + 1299917435830272*a^4*b^33*c^33*d^4 - 50876864570327
04*a^5*b^32*c^32*d^5 + 15437255594213376*a^6*b^31*c^31*d^6 - 37200150833135
616*a^7*b^30*c^30*d^7 + 72335498051321856*a^8*b^29*c^29*d^8 - 1146619160596
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31616*a^9*b^28*c^28*d^9 + 149030500382539776*a^10*b^27*c^27*d^10 - 15915865
2345778176*a^11*b^26*c^26*d^11 + 139465023528370176*a^12*b^25*c^25*d^12 - 9
9690751312588800*a^13*b^24*c^24*d^13 + 56347698493292544*a^14*b^23*c^23*d^1
4 - 13543724978454528*a^15*b^22*c^22*d^15 - 70702520459231232*a^16*b^21*c^2
1*d^16 + 350409117419053056*a^17*b^20*c^20*d^17 - 1180507035769012224*a^18*
b^19*c^19*d^18 + 3122430605575077888*a^19*b^18*c^18*d^19 - 6692023089679269
888*a^20*b^17*c^17*d^20 + 11832261271257083904*a^21*b^16*c^16*d^21 - 174746
66762617159680*a^22*b^15*c^15*d^22 + 21743319215696412672*a^23*b^14*c^14*d^
23 - 22924742364744450048*a^24*b^13*c^13*d^24 + 20548937192158642176*a^25*b
^12*c^12*d^25 - 15678268061077536768*a^26*b^11*c^11*d^26 + 1017318402352182
0672*a^27*b^10*c^10*d^27 - 5597130919804600320*a^28*b^9*c^9*d^28 + 25970662
72630370304*a^29*b^8*c^8*d^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323
237180229304320*a^31*b^6*c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 1
7373183736946688*a^33*b^4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 3
08246962323456*a^35*b^2*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 2219973997
3632*a^36*b*c*d^36)^2/4 - (36443545848801*a^12*b^17*d^25 + 106571947510161*
b^29*c^12*d^13 - 1446035052490812*a*b^28*c^11*d^14 - 533437396380252*a^11*b
^18*c*d^24 + 8550655952661522*a^2*b^27*c^10*d^15 - 29104520578391916*a^3*b^
26*c^9*d^16 + 63613900184394735*a^4*b^25*c^8*d^17 - 94521216268814328*a^5*b
^24*c^7*d^18 + 98620802659391292*a^6*b^23*c^6*d^19 - 73370651908486968*a^7*
b^22*c^5*d^20 + 38907153228163455*a^8*b^21*c^4*d^21 - 14432588165402316*a^9
*b^20*c^3*d^22 + 3574683057023442*a^10*b^19*c^2*d^23)*(68719476736*a^13*b^3
2*c^49 + 68719476736*a^45*c^17*d^32 - 2199023255552*a^14*b^31*c^48*d - 2199
023255552*a^44*b*c^18*d^31 + 34084860461056*a^15*b^30*c^47*d^2 - 3408486046
10560*a^16*b^29*c^46*d^3 + 2471152383426560*a^17*b^28*c^45*d^4 - 1383845334
7188736*a^18*b^27*c^44*d^5 + 62273040062349312*a^19*b^26*c^43*d^6 - 2312998
63088726016*a^20*b^25*c^42*d^7 + 722812072152268800*a^21*b^24*c^41*d^8 - 19
27498859072716800*a^22*b^23*c^40*d^9 + 4433247375867248640*a^23*b^22*c^39*d
^10 - 8866494751734497280*a^24*b^21*c^38*d^11 + 15516365815535370240*a^25*b
^20*c^37*d^12 - 23871332023900569600*a^26*b^19*c^36*d^13 + 323968077467222
01600*a^27*b^18*c^35*d^14 - 38876169296066641920*a^28*b^17*c^34*d^15 + 41305
929877070807040*a^29*b^16*c^33*d^16 - 38876169296066641920*a^30*b^15*c^32*d
^17 + 32396807746722201600*a^31*b^14*c^31*d^18 - 23871332023900569600*a^32*
b^13*c^30*d^19 + 15516365815535370240*a^33*b^12*c^29*d^20 - 886649475173449
7280*a^34*b^11*c^28*d^21 + 4433247375867248640*a^35*b^10*c^27*d^22 - 192749
8859072716800*a^36*b^9*c^26*d^23 + 722812072152268800*a^37*b^8*c^25*d^24 -
231299863088726016*a^38*b^7*c^24*d^25 + 62273040062349312*a^39*b^6*c^23*d^2
6 - 13838453347188736*a^40*b^5*c^22*d^27 + 2471152383426560*a^41*b^4*c^21*d
^28 - 340848604610560*a^42*b^3*c^20*d^29 + 34084860461056*a^43*b^2*c^19*d^3
0))^(1/2) + 16621372440576*a^2*b^35*c^35*d^2 - 124026341031936*a^3*b^34*c^3
4*d^3 + 649958717915136*a^4*b^33*c^33*d^4 - 2543843228516352*a^5*b^32*c^32*
d^5 + 7718627797106688*a^6*b^31*c^31*d^6 - 18600075416567808*a^7*b^30*c^30*
d^7 + 36167749025660928*a^8*b^29*c^29*d^8 - 57330958029815808*a^9*b^28*c^28
*d^9 + 74515250191269888*a^10*b^27*c^27*d^10 - 79579326172889088*a^11*b^26*
c^26*d^11 + 69732511764185088*a^12*b^25*c^25*d^12 - 49845375656294400*a^13*
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b^24*c^24*d^13 + 28173849246646272*a^14*b^23*c^23*d^14 - 6771862489227264*a
^15*b^22*c^22*d^15 - 35351260229615616*a^16*b^21*c^21*d^16 + 17520455870952
6528*a^17*b^20*c^20*d^17 - 590253517884506112*a^18*b^19*c^19*d^18 + 1561215
302787538944*a^19*b^18*c^18*d^19 - 3346011544839634944*a^20*b^17*c^17*d^20
+ 5916130635628541952*a^21*b^16*c^16*d^21 - 8737333381308579840*a^22*b^15*c
^15*d^22 + 10871659607848206336*a^23*b^14*c^14*d^23 - 11462371182372225024*
a^24*b^13*c^13*d^24 + 10274468596079321088*a^25*b^12*c^12*d^25 - 7839134030
538768384*a^26*b^11*c^11*d^26 + 5086592011760910336*a^27*b^10*c^10*d^27 - 2
798565459902300160*a^28*b^9*c^9*d^28 + 1298533136315185152*a^29*b^8*c^8*d^2
9 - 503942981543903232*a^30*b^7*c^7*d^30 + 161618590114652160*a^31*b^6*c^6*
d^31 - 42100124556607488*a^32*b^5*c^5*d^32 + 8686591868473344*a^33*b^4*c^4*
d^33 - 1366716850716672*a^34*b^3*c^3*d^34 + 154123481161728*a^35*b^2*c^2*d^
35 - 1394287312896*a*b^36*c^36*d - 11099869986816*a^36*b*c*d^36)/(687194767
36*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*
d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c
^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a
^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d
^9 + 64512240*a^23*b^22*c^39*d^10 - 129024480*a^24*b^21*c^38*d^11 + 2257928
40*a^25*b^20*c^37*d^12 - 347373600*a^26*b^19*c^36*d^13 + 471435600*a^27*b^1
8*c^35*d^14 - 565722720*a^28*b^17*c^34*d^15 + 601080390*a^29*b^16*c^33*d^16
 - 565722720*a^30*b^15*c^32*d^17 + 471435600*a^31*b^14*c^31*d^18 - 34737360
0*a^32*b^13*c^30*d^19 + 225792840*a^33*b^12*c^29*d^20 - 129024480*a^34*b^11
*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 1
0518300*a^37*b^8*c^25*d^24 - 3365856*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c
^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42
*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30)))^(1/4)*(93386641873154605056*a^34
*b^53*c^94*d^4 - 3891110078048108544000*a^35*b^52*c^93*d^5 + 78828702034483
948290048*a^36*b^51*c^92*d^6 - 1034672110486845715906560*a^37*b^50*c^91*d^7
 + 9892540360265140468187136*a^38*b^49*c^90*d^8 - 7344022016434813734695731
2*a^39*b^48*c^89*d^9 + 440649383366170539762647040*a^40*b^47*c^88*d^10 - 21
96237253234092465387995136*a^41*b^46*c^87*d^11 + 92742963161445956466990120
96*a^42*b^45*c^86*d^12 - 33677881501046993339969175552*a^43*b^44*c^85*d^13
+ 106376530102998491281999527936*a^44*b^43*c^84*d^14 - 29492143230150479899
0377418752*a^45*b^42*c^83*d^15 + 722903045142137525367365173248*a^46*b^41*c
^82*d^16 - 1576072447576504233275626094592*a^47*b^40*c^81*d^17 + 3072471208
539973972578986360832*a^48*b^39*c^80*d^18 - 5384106777252432871416869683200
*a^49*b^38*c^79*d^19 + 8537351598354925496836275830784*a^50*b^37*c^78*d^20
- 12376921822825560832675204300800*a^51*b^36*c^77*d^21 + 167075893904326210
56738749054976*a^52*b^35*c^76*d^22 - 21667130911214476307455165857792*a^53*
b^34*c^75*d^23 + 28211207618793157944689200988160*a^54*b^33*c^74*d^24 - 383
78393138521379212996695293952*a^55*b^32*c^73*d^25 + 54918846093258397577855
222415360*a^56*b^31*c^72*d^26 - 80082941438212170767896978391040*a^57*b^30*
c^71*d^27 + 113888426387729629146256565600256*a^58*b^29*c^70*d^28 - 1527541
06500312545531177547595776*a^59*b^28*c^69*d^29 + 18954977850856326343806840
4715520*a^60*b^27*c^68*d^30 - 215546518234822631781377148715008*a^61*b^26*c
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^67*d^31 + 223641896308855873457165036421120*a^62*b^25*c^66*d^32 - 21129373
0951350565888869600854016*a^63*b^24*c^65*d^33 + 181575241776706668284956756
672512*a^64*b^23*c^64*d^34 - 141794149619600448829729705820160*a^65*b^22*c^
63*d^35 + 100511576025621687034384100622336*a^66*b^21*c^62*d^36 - 645811235
53243990572098666889216*a^67*b^20*c^61*d^37 + 375409926340947176400840944451
712*a^68*b^19*c^60*d^38 - 196951796956896019104904941440416*a^69*b^18*c^59*d
^39 + 92968409420464145227468159057 92*a^70*b^17*c^58*d^40 - 393344619628210
8795457464434688*a^71*b^16*c^57*d^41 + 1484644864880431945098662510592*a^72
*b^15*c^56*d^42 - 496993877333119536381277765632*a^73*b^14*c^55*d^43 + 1464
93707302289292776429322240*a^74*b^13*c^54*d^44 - 376790059998473990956740 77
184*a^75*b^12*c^53*d^45 + 8360094623991181223468728320*a^76*b^11*c^52*d^46
- 1576546523407725355918688256*a^77*b^10*c^51*d^47 + 247744258459119342197 9
32032*a^78*b^9*c^50*d^48 - 31566136012926195282739200*a^79*b^8*c^49*d^49 +
3133065413748205302054912*a^80*b^7*c^48*d^50 - 227270011883594899783680*a^8
1*b^6*c^47*d^51 + 10717576321223758970880*a^82*b^5*c^46*d^52 - 246599101196
298878976*a^83*b^4*c^45*d^53)*1i + 10505997210729893068 8*a^31*b^54*c^91*d^4
 - 4202398884291957227520*a^32*b^53*c^90*d^5 + 81456498373859104260096*a^33
*b^52*c^89*d^6 - 1019470840448604438528000*a^34*b^51*c^88*d^7 + 92615851877
79405523451904*a^35*b^50*c^87*d^8 - 65094971944398671145074688*a^36*b^49*c^
86*d^9 + 368402395453916323189358592*a^37*b^48*c^85*d^10 - 1725226316150928
144278224896*a^38*b^47*c^84*d^11 + 6817742452202868128486522880*a^39*b^46*c
^83*d^12 - 23071505195064931052886687744*a^40*b^45*c^82*d^13 + 676140892161
23669492331970560*a^41*b^44*c^81*d^14 - 173115025562473785468905324544*a^42
*b^43*c^80*d^15 + 389913831719674713212222177280*a^43*b^42*c^79*d^16 - 7767
90088912432141093966970880*a^44*b^41*c^78*d^17 + 137461198325127253046930 80
71936*a^45*b^40*c^77*d^18 - 2167454612994156285048662261760*a^46*b^39*c^76*
d^19 + 3050337310429700535004075917312*a^47*b^38*c^75*d^20 - 3826885622871 4
96570502324944896*a^48*b^37*c^74*d^21 + 4238713393375513383921726259200*a^4
9*b^36*c^73*d^22 - 3984291896345024467843348955136*a^50*b^35*c^72*d^23 + 26
51971426464597412032295206912*a^51*b^34*c^71*d^24 + 479249403658129639733 53
4392320*a^52*b^33*c^70*d^25 - 6697452529698647734837548417024*a^53*b^32*c^6
9*d^26 + 17931054269995149998277682790400*a^54*b^31*c^68*d^27 - 36311715021
905634799784747335680*a^55*b^30*c^67*d^28 + 630736170763940890010911663718 4
0*a^56*b^29*c^66*d^29 - 97105565168138147055402127196160*a^57*b^28*c^65*d^3
0 + 133993666277013207597272619024384*a^58*b^27*c^64*d^31 - 166492084833102
044695859350732800*a^59*b^26*c^63*d^32 + 186717161118223967667066928889856*
a^60*b^25*c^62*d^33 - 189235624153406619951659086774272*a^61*b^24*c^61*d^34
 + 173421825288151984221422006304768*a^62*b^23*c^60*d^35 - 143715376746696 0
509029730368880 64*a^63*b^22*c^59*d^36 + 107645128880801788128312132894720*a
^64*b^21*c^58*d^37 - 72802169209714119238549751463936*a^65*b^20*c^57*d^38 +
 44389639270136779232591657041920*a^66*b^19*c^56*d^39 - 24348625105436875 28
0486976454656*a^67*b^18*c^55*d^40 + 11981145511938522697620070072320*a^68*b
^17*c^54*d^41 - 5269759325089910260644729323520*a^69*b^16*c^53*d^42 + 20624
71522530027433706750214144*a^70*b^15*c^52*d^43 - 714227824367410213467319 17
3120*a^71*b^14*c^51*d^44 + 217305373751493983005392764928*a^72*b^13*c^50*d^
```

```
45 - 5757441114843356942444l606144*a^73*b^12*c^49*d^46 + 131339473607338820
65354752000*a^74*b^11*c^48*d^47 - 254201946024205079766525 5424*a^75*b^10*c^
47*d^48 + 409310322447365741947650048*a^76*b^9*c^46*d^49 - 5335664969179313
4232535040*a^77*b^8*c^45*d^50 + 541059492457889361454 6944*a^78*b^7*c^44*d^5
1 - 400464195437318897664000*a^79*b^6*c^43*d^52 + 19246289226179889070080*a
^80*b^5*c^42*d^53 - 450813981874483888128*a^81*b^4*c^41*d^54)*1i + x^(1/2)*
(119342219331695731015680*a^30*b^49*c^73*d^13 - 3677615218076424339456*a^29
*b^50*c^74*d^12 - 1856013443030972425568256*a^31*b^48*c^72*d^14 + 184260999
96452807258406912*a^32*b^47*c^71*d^15 - 131228123459738637629915136*a^33*b^
46*c^70*d^16 + 714182072565091774626791424*a^34*b^45*c^69*d^17 - 3088237415
348484431457288192*a^35*b^44*c^68*d^18 + 10882952503625649640326561792*a^36
*b^43*c^67*d^19 - 31757074600474077803581538304*a^37*b^42*c^66*d^20 + 77306
011497125960924962750464*a^38*b^41*c^65*d^21 - 156439291025195069838804910 0
80*a^39*b^40*c^64*d^22 + 256967446361217518429496410112*a^40*b^39*c^63*d^23
 - 315930266538485089912448090112*a^41*b^38*c^62*d^24 + 193264836517334230 3
47779407872*a^42*b^37*c^61*d^25 + 320732651390132179677984325632*a^43*b^36*
c^60*d^26 - 1433302686817582744983683727360*a^44*b^35*c^59*d^27 + 321476585
1097197421262933065728*a^45*b^34*c^58*d^28 - 546539836176364249048086164275
2*a^46*b^33*c^57*d^29 + 7690728695480443198104101978112*a^47*b^32*c^56*d^30
 - 9256447758824794945376420364288*a^48*b^31*c^55*d^31 + 967266986658727069
7877661286400*a^49*b^30*c^54*d^32 - 8839280066432157154484139589632*a^50*b^
29*c^53*d^33 + 7086822067089169522912760168448*a^51*b^28*c^52*d^34 - 498852
2538878293079151039479808*a^52*b^27*c^51*d^35 + 307979560109074052782518121
2672*a^53*b^26*c^50*d^36 - 1663341919096805892341077377024*a^54*b^25*c^49*d
^37 + 782666038849476274770105335808*a^55*b^24*c^48*d^38 - 3190135528869488
01896949743616*a^56*b^23*c^47*d^39 + 111766668098727585639133347840*a^57*b^
22*c^46*d^40 - 33312207294098258580851392512*a^58*b^21*c^45*d^41 + 83307913
06287661611887886336*a^59*b^20*c^44*d^42 - 1715502625948903704153292800*a^6
0*b^19*c^43*d^43 + 283282946101439324535914496*a^61*b^18*c^42*d^44 - 360693
32470586798845722624*a^62*b^17*c^41*d^45 + 3324850588931239515783168*a^63*b
^16*c^40*d^46 - 197512325498721785610240*a^64*b^15*c^39*d^47 + 567886939032
6597943296*a^65*b^14*c^38*d^48))*(-(383772100608*a^37*d^37 + 55037657088*b^
37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324274488115
2*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272*
a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594213376*
a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321856
*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 149030500382539
776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 13946502
3528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^13 + 56
347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^22*d^15
 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b^20*c^2
0*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 3122430605575077888*a^19
*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 118322612712570
83904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22 + 2174
3319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^13*c^13*
```

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d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536768*a^26
*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 55971309198046
00320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 100788596
3087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31 - 84200
249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33 - 2733
433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 - 278857
4625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (36443545848801
*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*a*b^28*
c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*b^27*c^
10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^4*b^25*
c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a^6*b^23
*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*a^8*b^2
1*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*a^10*b^
19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^32 - 219
9023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 340848604610
56*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 24711523834265
60*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 622730400623
49312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 72281207
2152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d^9 + 44
33247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^21*c^38*
d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 23871332023900569600*a^26
*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 38876169296066
641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^16 - 388
76169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^14*c^31
*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370240*a^3
3*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 44332473758672
48640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 + 722812
072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d^25 + 6
2273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^22*d^27
+ 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^20*d^29
+ 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 1662137244057б*a^2*b^35*c^35*
d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c^33*d^4
 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^31*d^6
- 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c^29*d^8
 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27*c^27*d
^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12*b^25*c
^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272*a^14*b
^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3535126022961561б*a^
16*b^21*c^21*d^16 + 17520455870952652８*a^17*b^20*c^20*d^17 - 59025351788450
6112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 - 334601
1544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^16*d^21
 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^23*b^14
*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 1027446859607932108
8*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 508659201
1760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^28 + 12
```

```
98533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^7*d^30
+ 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*c^5*d^3
2 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c^3*d^34
 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d - 110998
69986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^32 - 32*
a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 4960*a^16
*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^5 + 906
192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*b^24*c^
41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10 - 12902
4480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600*a^26*b
^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*c^34*d^
15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 + 471435
600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*a^33*b^
12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^27*d^22
 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 3365856*a^38
*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^27 + 35
960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19*d^30))
)^(1/4)*1i + 41028394776665109037056*a^29*b^48*c^68*d^15 - 1210739885076097
825505280*a^30*b^47*c^67*d^16 + 17243628768780949747924992*a^31*b^46*c^66*d
^17 - 158081319004444765483696128*a^32*b^45*c^65*d^18 + 1049494986915760527
133114368*a^33*b^44*c^64*d^19 - 5380683046490354438136397824*a^34*b^43*c^63
*d^20 + 22176160052724101903372255232*a^35*b^42*c^62*d^21 - 754863133252416
36679770439680*a^36*b^41*c^61*d^22 + 216288375615109659684325294080*a^37*b^
40*c^60*d^23 - 528818181695424054504437317632*a^38*b^39*c^59*d^24 + 1114222
690302433619242395893760*a^39*b^38*c^58*d^25 - 2037545055293058005529639518
208*a^40*b^37*c^57*d^26 + 3249918857904337975850827776000*a^41*b^36*c^56*d^
27 - 4536394700759564584125915463680*a^42*b^35*c^55*d^28 + 5552435240283931
429496420302848*a^43*b^34*c^54*d^29 - 5964290825683224886861470105600*a^44*
b^33*c^53*d^30 + 5621639355410781338712284332032*a^45*b^32*c^52*d^31 - 4644
077108074496901042866749440*a^46*b^31*c^51*d^32 + 3355360862716129153108295
024640*a^47*b^30*c^50*d^33 - 2113405281704782215093506015232*a^48*b^29*c^49
*d^34 + 1155283596049337948225918730240*a^49*b^28*c^48*d^35 - 5448295198703
76944469402451968*a^50*b^27*c^47*d^36 + 219926172037899117268712816640*a^51
*b^26*c^46*d^37 - 75201916274561138554746961920*a^52*b^25*c^45*d^38 + 21483
948869172056418164932608*a^53*b^24*c^44*d^39 - 5032346201606164325320359936
*a^54*b^23*c^43*d^40 + 941275744618015035796488192*a^55*b^22*c^42*d^41 - 13
5189136301093329947328512*a^56*b^21*c^41*d^42 + 13999140307267180988203008*
a^57*b^20*c^40*d^43 - 930460907799665663016960*a^58*b^19*c^39*d^44 + 298140
64299214639202304*a^59*b^18*c^38*d^45))*(-(383772100608*a^37*d^37 + 5503765
7088*b^37*c^37 + ((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 332427
44881152*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435
830272*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 15437255594
213376*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 7233549805
1321856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14903050
0382539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 1
```

39465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^
13 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^
22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*b
^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 31224306055750778
88*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832261
271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*d^22
 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^24*b^1
3*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 15678268061077536
768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 - 5597130
919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d^29 - 10
07885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*c^6*d^31
 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^4*c^4*d^33
 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2*c^2*d^35 -
 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)^2/4 - (3644354
5848801*a^12*b^17*d^25 + 106571947510161*b^29*c^12*d^13 - 1446035052490812*
a*b^28*c^11*d^14 - 533437396380252*a^11*b^18*c*d^24 + 8550655952661522*a^2*
b^27*c^10*d^15 - 29104520578391916*a^3*b^26*c^9*d^16 + 63613900184394735*a^
4*b^25*c^8*d^17 - 94521216268814328*a^5*b^24*c^7*d^18 + 98620802659391292*a
^6*b^23*c^6*d^19 - 73370651908486968*a^7*b^22*c^5*d^20 + 38907153228163455*
a^8*b^21*c^4*d^21 - 14432588165402316*a^9*b^20*c^3*d^22 + 3574683057023442*
a^10*b^19*c^2*d^23)*(68719476736*a^13*b^32*c^49 + 68719476736*a^45*c^17*d^3
2 - 2199023255552*a^14*b^31*c^48*d - 2199023255552*a^44*b*c^18*d^31 + 34084
860461056*a^15*b^30*c^47*d^2 - 340848604610560*a^16*b^29*c^46*d^3 + 2471152
383426560*a^17*b^28*c^45*d^4 - 13838453347188736*a^18*b^27*c^44*d^5 + 62273
040062349312*a^19*b^26*c^43*d^6 - 231299863088726016*a^20*b^25*c^42*d^7 + 7
22812072152268800*a^21*b^24*c^41*d^8 - 1927498859072716800*a^22*b^23*c^40*d
^9 + 4433247375867248640*a^23*b^22*c^39*d^10 - 8866494751734497280*a^24*b^2
1*c^38*d^11 + 15516365815535370240*a^25*b^20*c^37*d^12 - 238713320239005696
00*a^26*b^19*c^36*d^13 + 32396807746722201600*a^27*b^18*c^35*d^14 - 3887616
9296066641920*a^28*b^17*c^34*d^15 + 41305929877070807040*a^29*b^16*c^33*d^1
6 - 38876169296066641920*a^30*b^15*c^32*d^17 + 32396807746722201600*a^31*b^
14*c^31*d^18 - 23871332023900569600*a^32*b^13*c^30*d^19 + 15516365815535370
240*a^33*b^12*c^29*d^20 - 8866494751734497280*a^34*b^11*c^28*d^21 + 4433247
375867248640*a^35*b^10*c^27*d^22 - 1927498859072716800*a^36*b^9*c^26*d^23 +
 722812072152268800*a^37*b^8*c^25*d^24 - 231299863088726016*a^38*b^7*c^24*d
^25 + 62273040062349312*a^39*b^6*c^23*d^26 - 13838453347188736*a^40*b^5*c^2
2*d^27 + 2471152383426560*a^41*b^4*c^21*d^28 - 340848604610560*a^42*b^3*c^2
0*d^29 + 34084860461056*a^43*b^2*c^19*d^30))^(1/2) + 16621372440576*a^2*b^3
5*c^35*d^2 - 124026341031936*a^3*b^34*c^34*d^3 + 649958717915136*a^4*b^33*c
^33*d^4 - 2543843228516352*a^5*b^32*c^32*d^5 + 7718627797106688*a^6*b^31*c^
31*d^6 - 18600075416567808*a^7*b^30*c^30*d^7 + 36167749025660928*a^8*b^29*c
^29*d^8 - 57330958029815808*a^9*b^28*c^28*d^9 + 74515250191269888*a^10*b^27
*c^27*d^10 - 79579326172889088*a^11*b^26*c^26*d^11 + 69732511764185088*a^12
*b^25*c^25*d^12 - 49845375656294400*a^13*b^24*c^24*d^13 + 28173849246646272
*a^14*b^23*c^23*d^14 - 6771862489227264*a^15*b^22*c^22*d^15 - 3535126022961

```
5616*a^16*b^21*c^21*d^16 + 175204558709526528*a^17*b^20*c^20*d^17 - 5902535
17884506112*a^18*b^19*c^19*d^18 + 1561215302787538944*a^19*b^18*c^18*d^19 -
 3346011544839634944*a^20*b^17*c^17*d^20 + 5916130635628541952*a^21*b^16*c^
16*d^21 - 8737333381308579840*a^22*b^15*c^15*d^22 + 10871659607848206336*a^
23*b^14*c^14*d^23 - 11462371182372225024*a^24*b^13*c^13*d^24 + 102744685960
79321088*a^25*b^12*c^12*d^25 - 7839134030538768384*a^26*b^11*c^11*d^26 + 50
86592011760910336*a^27*b^10*c^10*d^27 - 2798565459902300160*a^28*b^9*c^9*d^
28 + 1298533136315185152*a^29*b^8*c^8*d^29 - 503942981543903232*a^30*b^7*c^
7*d^30 + 161618590114652160*a^31*b^6*c^6*d^31 - 42100124556607488*a^32*b^5*
c^5*d^32 + 8686591868473344*a^33*b^4*c^4*d^33 - 1366716850716672*a^34*b^3*c
^3*d^34 + 154123481161728*a^35*b^2*c^2*d^35 - 1394287312896*a*b^36*c^36*d -
 11099869986816*a^36*b*c*d^36)/(68719476736*(a^13*b^32*c^49 + a^45*c^17*d^3
2 - 32*a^14*b^31*c^48*d - 32*a^44*b*c^18*d^31 + 496*a^15*b^30*c^47*d^2 - 49
60*a^16*b^29*c^46*d^3 + 35960*a^17*b^28*c^45*d^4 - 201376*a^18*b^27*c^44*d^
5 + 906192*a^19*b^26*c^43*d^6 - 3365856*a^20*b^25*c^42*d^7 + 10518300*a^21*
b^24*c^41*d^8 - 28048800*a^22*b^23*c^40*d^9 + 64512240*a^23*b^22*c^39*d^10
- 129024480*a^24*b^21*c^38*d^11 + 225792840*a^25*b^20*c^37*d^12 - 347373600
*a^26*b^19*c^36*d^13 + 471435600*a^27*b^18*c^35*d^14 - 565722720*a^28*b^17*
c^34*d^15 + 601080390*a^29*b^16*c^33*d^16 - 565722720*a^30*b^15*c^32*d^17 +
 471435600*a^31*b^14*c^31*d^18 - 347373600*a^32*b^13*c^30*d^19 + 225792840*
a^33*b^12*c^29*d^20 - 129024480*a^34*b^11*c^28*d^21 + 64512240*a^35*b^10*c^
27*d^22 - 28048800*a^36*b^9*c^26*d^23 + 10518300*a^37*b^8*c^25*d^24 - 33658
56*a^38*b^7*c^24*d^25 + 906192*a^39*b^6*c^23*d^26 - 201376*a^40*b^5*c^22*d^
27 + 35960*a^41*b^4*c^21*d^28 - 4960*a^42*b^3*c^20*d^29 + 496*a^43*b^2*c^19
*d^30)))^(1/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

3.486 $\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=103

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(5/2))/(5*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7*b^4) + (B*(a + b*x^
2)^(9/2))/(9*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)dx=\frac{1}{2}\text{Subst}\left(\int x^2\sqrt{a+bx}\,(A+Bx)\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+\frac{(Ab-3a}{}\right.\right.$$

$$=\frac{a^2(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{7}}{7b^4}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.73

$$\frac{\left(a+bx^2\right)^{3/2}\left(-16a^3B+24a^2b\left(A+Bx^2\right)-6ab^2x^2\left(6A+5Bx^2\right)+5b^3x^4\left(9A+7Bx^2\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^2) - 6*a*b^2*x^2*(6*A + 5
*B*x^2) + 5*b^3*x^4*(9*A + 7*B*x^2)))/(315*b^4)
```

IntegrateAlgebraic [A] time = 0.05, size = 80, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{3/2}\left(-16a^3B+24a^2Ab+24a^2bBx^2-36aAb^2x^2-30ab^2Bx^4+45Ab^3x^4+35b^3Bx^6\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(24*a^2*A*b - 16*a^3*B - 36*a*A*b^2*x^2 + 24*a^2*b*B*x^2
 + 45*A*b^3*x^4 - 30*a*b^2*B*x^4 + 35*b^3*B*x^6))/(315*b^4)
```

fricas [A] time = 1.37, size = 99, normalized size = 0.96

$$\frac{\left(35\,Bb^4x^8+5\left(Bab^3+9\,Ab^4\right)x^6-16\,Ba^4+24\,Aa^3b-3\left(2\,Ba^2b^2-3\,Aab^3\right)x^4+4\left(2\,Ba^3b-3\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(B*a*b^3 + 9*A*b^4)*x^6 - 16*B*a^4 + 24*A*a^3*b - 3
*(2*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^2)*sqrt(b*x^
2 + a)/b^4
```

giac [A] time = 0.31, size = 104, normalized size = 1.01

$$\frac{35 \left(bx^2+a\right)^{\frac{9}{2}} B - 135 \left(bx^2+a\right)^{\frac{7}{2}} Ba + 189 \left(bx^2+a\right)^{\frac{5}{2}} Ba^2 - 105 \left(bx^2+a\right)^{\frac{3}{2}} Ba^3 + 45 \left(bx^2+a\right)^{\frac{7}{2}} Ab - 126 \left(bx^2+a\right)^{\frac{5}{2}} Aab + 105 \left(bx^2+a\right)^{\frac{3}{2}} Aa^2 b}{315 \, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2)*B - 135*(b*x^2 + a)^(7/2)*B*a + 189*(b*x^2 + a)
^(5/2)*B*a^2 - 105*(b*x^2 + a)^(3/2)*B*a^3 + 45*(b*x^2 + a)^(7/2)*A*b - 126
*(b*x^2 + a)^(5/2)*A*a*b + 105*(b*x^2 + a)^(3/2)*A*a^2*b)/b^4
```

maple [A] time = 0.01, size = 77, normalized size = 0.75

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}} \left(35B\,x^6 b^3 + 45A\,b^3 x^4 - 30Ba\,b^2 x^4 - 36Aa\,b^2 x^2 + 24B\,a^2 b\,x^2 + 24A\,a^2 b - 16B\,a^3\right)}{315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/315*(b*x^2+a)^(3/2)*(35*B*b^3*x^6+45*A*b^3*x^4-30*B*a*b^2*x^4-36*A*a*b^2*
x^2+24*B*a^2*b*x^2+24*A*a^2*b-16*B*a^3)/b^4
```

maxima [A] time = 0.97, size = 132, normalized size = 1.28

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}} Bx^6}{9\,b} - \frac{2 \left(bx^2+a\right)^{\frac{3}{2}} Bax^4}{21\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}} Ax^4}{7\,b} + \frac{8 \left(bx^2+a\right)^{\frac{3}{2}} Ba^2x^2}{105\,b^3} - \frac{4 \left(bx^2+a\right)^{\frac{3}{2}} Aax^2}{35\,b^2} - \frac{16 \left(bx^2+a\right)^{\frac{3}{2}} Ba^3}{315\,b^4} + \frac{8 \left(bx^2+a\right)^{\frac{3}{2}} Aa^2}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(3/2)*B*x^6/b - 2/21*(b*x^2 + a)^(3/2)*B*a*x^4/b^2 + 1/7*(b
*x^2 + a)^(3/2)*A*x^4/b + 8/105*(b*x^2 + a)^(3/2)*B*a^2*x^2/b^3 - 4/35*(b*x
^2 + a)^(3/2)*A*a*x^2/b^2 - 16/315*(b*x^2 + a)^(3/2)*B*a^3/b^4 + 8/105*(b*x
^2 + a)^(3/2)*A*a^2/b^3
```

mupad [B] time = 0.64, size = 96, normalized size = 0.93

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,x^8}{9} - \frac{16\,B\,a^4 - 24\,A\,a^3\,b}{315\,b^4} + \frac{x^6\,\left(45\,A\,b^4 + 5\,B\,a\,b^3\right)}{315\,b^4} - \frac{4\,a^2\,x^2\,\left(3\,A\,b - 2\,B\,a\right)}{315\,b^3} + \frac{a\,x^4\,\left(3\,A\,b - 2\,B\,a\right)}{105\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*x^8)/9 - (16*B*a^4 - 24*A*a^3*b)/(315*b^4) + (x^6*(45
*A*b^4 + 5*B*a*b^3))/(315*b^4) - (4*a^2*x^2*(3*A*b - 2*B*a))/(315*b^3) + (a
*x^4*(3*A*b - 2*B*a))/(105*b^2))
```

sympy [A] time = 2.22, size = 212, normalized size = 2.06

$$\begin{cases} \frac{8Aa^3\sqrt{a+bx^2}}{105b^3} - \frac{4Aa^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{Aax^4\sqrt{a+bx^2}}{35b} + \frac{Ax^6\sqrt{a+bx^2}}{7} - \frac{16Ba^4\sqrt{a+bx^2}}{315b^4} + \frac{8Ba^3x^2\sqrt{a+bx^2}}{315b^3} - \frac{2Ba^2x^4\sqrt{a+bx^2}}{105b^2} + \frac{Bax^6\sqrt{a+bx^2}}{63b} + \frac{Bx^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*A*a**2*x**2*sqrt(a + b*
x**2)/(105*b**2) + A*a*x**4*sqrt(a + b*x**2)/(35*b) + A*x**6*sqrt(a + b*x**
2)/7 - 16*B*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*B*a**3*x**2*sqrt(a + b*x**
2)/(315*b**3) - 2*B*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + B*a*x**6*sqrt(a
 + b*x**2)/(63*b) + B*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*(A*x**6/
6 + B*x**8/8), True))
```

3.487 $\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=155

$$\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}-\frac{a^2x\sqrt{a+bx^2}\,(8Ab-5aB)}{128b^3}+\frac{ax^3\sqrt{a+bx^2}\,(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}\,(8Ab-5a}{48b}$$

Rubi **[A]** time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$-\frac{a^2x\sqrt{a+bx^2}\,(8Ab-5aB)}{128b^3}+\frac{a^3(8Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{ax^3\sqrt{a+bx^2}\,(8Ab-5aB)}{192b^2}+\frac{x^5\sqrt{a+bx^2}\,(8Ab-5aB)}{48b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -(a^2*(8*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(128*b^3) + (a*(8*A*b - 5*a*B)*x^3
*Sqrt[a + b*x^2])/(192*b^2) + ((8*A*b - 5*a*B)*x^5*Sqrt[a + b*x^2])/(48*b)
+ (B*x^5*(a + b*x^2)^(3/2))/(8*b) + (a^3*(8*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b} - \frac{(-8Ab+5aB)\int x^4\sqrt{a+bx^2}\,dx}{8b}$$

$$= \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b} + \frac{(a(8Ab-5aB))\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{48b}$$

$$= \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b} + \frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b} - \left(a^2\right.$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+}}{48b}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+}}{48b}$$

$$= -\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3} + \frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2} + \frac{(8Ab-5aB)x^5\sqrt{a+}}{48b}$$

Mathematica [A] time = 0.28, size = 130, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(15a^3B-2a^2b\left(12A+5Bx^2\right)+8ab^2x^2\left(2A+Bx^2\right)+16b^3x^4\left(4A+3Bx^2\right)\right)-\frac{3a^{5/2}(5aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3*B + 8*a*b^2*x^2*(2*A + B*x^2) + 16*b^3*
x^4*(4*A + 3*B*x^2) - 2*a^2*b*(12*A + 5*B*x^2)) - (3*a^(5/2)*(-8*A*b + 5*a*
B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 127, normalized size = 0.82

$$\frac{\left(5a^4B-8a^3Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{7/2}}+\frac{\sqrt{a+bx^2}\left(15a^3Bx-24a^2Abx-10a^2bBx^3+16aAb^2x^3+8ab^2Bx^5+64Ab^3x^5+48b^3Bx^7\right)}{384b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-24*a^2*A*b*x + 15*a^3*B*x + 16*a*A*b^2*x^3 - 10*a^2*b*B*
x^3 + 64*A*b^3*x^5 + 8*a*b^2*B*x^5 + 48*b^3*B*x^7))/(384*b^3) + ((-8*a^3*A*
b + 5*a^4*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(7/2))
```

fricas [A] time = 1.11, size = 257, normalized size = 1.66

$$\left[-\frac{3\left(5\,Ba^4-8\,Aa^3b\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(48\,Bb^4x^7+8\left(Bab^3+8\,Ab^4\right)x^5-2\left(5\,Ba^2b^2-8\,Aab^3\right)x^3+3\left(5\,Ba^3b-8\,Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{768\,b^4},\ \frac{3\left(5\,Ba^4-8\,Aa^3b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(48\,Bb^4x^7+8\left(Bab^3+8\,Ab^4\right)x^5-2\left(5\,Ba^2b^2-8\,Aab^3\right)x^3+3\left(5\,Ba^3b-8\,Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{384\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^
2 - 8*A*a*b^3)*x^3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1
/384*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
 (48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^2 - 8*A*a*b^3)*x^
3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.39, size = 132, normalized size = 0.85

$$\frac{1}{384}\left(2\left(4\left(6\,Bx^2+\frac{Bab^5+8\,Ab^6}{b^6}\right)x^2-\frac{5\,Ba^2b^4-8\,Aab^5}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^3-8\,Aa^2b^4\right)}{b^6}\right)\sqrt{bx^2+a}\,x+\frac{\left(5\,Ba^4-8\,Aa^3b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*x^2 + (B*a*b^5 + 8*A*b^6)/b^6)*x^2 - (5*B*a^2*b^4 - 8*A*a*
b^5)/b^6)*x^2 + 3*(5*B*a^3*b^3 - 8*A*a^2*b^4)/b^6)*sqrt(b*x^2 + a)*x + 1/12
8*(5*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.02, size = 181, normalized size = 1.17

$$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} B\,x^{5}}{8b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} A\,x^{3}}{6b}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}} Ba\,x^{3}}{48b^{2}}+\frac{A\,a^{3}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{5}{2}}}-\frac{5B\,a^{4}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{7}{2}}}+\frac{\sqrt{b\,x^{2}+a}\;A\,a^{2}x}{16b^{2}}-\frac{5\sqrt{b\,x^{2}+a}\;B\,a^{3}x}{128b^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} Aax}{8b^{2}}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}} B\,a^{2}x}{64b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*B*x^5*(b*x^2+a)^(3/2)/b-5/48*B*a/b^2*x^3*(b*x^2+a)^(3/2)+5/64*B*a^2/b^3
*x*(b*x^2+a)^(3/2)-5/128*B*a^3/b^3*x*(b*x^2+a)^(1/2)-5/128*B*a^4/b^(7/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*A*x^3*(b*x^2+a)^(3/2)/b-1/8*A*a/b^2*x*(b*x^
2+a)^(3/2)+1/16*A*a^2/b^2*x*(b*x^2+a)^(1/2)+1/16*A*a^3/b^(5/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.00, size = 166, normalized size = 1.07

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bx^{5}}{8\,b}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Bax^{3}}{48\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ax^{3}}{6\,b}+\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}x}{64\,b^{3}}-\frac{5\,\sqrt{bx^{2}+a}\,Ba^{3}x}{128\,b^{3}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Aax}{8\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,Aa^{2}x}{16\,b^{2}}-\frac{5\,Ba^{4}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}+\frac{Aa^{3}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(3/2)*B*x^5/b - 5/48*(b*x^2 + a)^(3/2)*B*a*x^3/b^2 + 1/6*(b
*x^2 + a)^(3/2)*A*x^3/b + 5/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^3 - 5/128*sqrt(b
*x^2 + a)*B*a^3*x/b^3 - 1/8*(b*x^2 + a)^(3/2)*A*a*x/b^2 + 1/16*sqrt(b*x^2 +
 a)*A*a^2*x/b^2 - 5/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/16*A*a^3*a
rcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^4*(A + B*x^2)*(a + b*x^2)^(1/2), x)
```

sympy [A] time = 17.65, size = 286, normalized size = 1.85

$$-\frac{Aa^{\frac{5}{2}}x}{16b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{\frac{3}{2}}x^{3}}{48b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5A\sqrt{a}\,x^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}+\frac{Aa^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Abx^{7}}{6\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b^{3}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5Ba^{\frac{5}{2}}x^{3}}{384b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{3}{2}}x^{5}}{192b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{7B\sqrt{a}\,x^{7}}{48\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5Ba^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{Bbx^{9}}{8\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] -A*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - A*a**(3/2)*x**3/(48*b*sqrt(1 +
 b*x**2/a)) + 5*A*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(5/2)) + A*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B
*a**(7/2)*x/(128*b**3*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x**3/(384*b**2*sqr
t(1 + b*x**2/a)) - B*a**(3/2)*x**5/(192*b*sqrt(1 + b*x**2/a)) + 7*B*sqrt(a)
*x**7/(48*sqrt(1 + b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
7/2)) + B*b*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.488 $\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(5/
2))/(5*b^3) + (B*(a + b*x^2)^(7/2))/(7*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)dx &= \frac{1}{2}\,\text{Subst}\left(\int x\sqrt{a+bx}\,(A+Bx)\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\Bigg(\int\bigg(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\bigg)\,dx,\\
&= -\frac{a(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.04, size = 57, normalized size = 0.78

$$
\frac{\left(a+bx^2\right)^{3/2}\left(8a^2B-2ab\left(7A+6Bx^2\right)+3b^2x^2\left(7A+5Bx^2\right)\right)}{105b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(8*a^2*B + 3*b^2*x^2*(7*A + 5*B*x^2) - 2*a*b*(7*A + 6*B*
x^2)))/(105*b^3)
```

IntegrateAlgebraic [**A**] time = 0.04, size = 56, normalized size = 0.77

$$
\frac{\left(a+bx^2\right)^{3/2}\left(8a^2B-14aAb-12abBx^2+21Ab^2x^2+15b^2Bx^4\right)}{105b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^2 - 12*a*b*B*x^2 + 15*
b^2*B*x^4))/(105*b^3)
```

fricas [**A**] time = 1.29, size = 75, normalized size = 1.03

$$
\frac{\left(15\,Bb^3x^6+3\left(Bab^2+7\,Ab^3\right)x^4+8\,Ba^3-14\,Aa^2b-\left(4\,Ba^2b-7\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{105\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 + 3*(B*a*b^2 + 7*A*b^3)*x^4 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 0.30, size = 73, normalized size = 1.00

$$\frac{15\left(bx^2+a\right)^{\frac{7}{2}}B-42\left(bx^2+a\right)^{\frac{5}{2}}Ba+35\left(bx^2+a\right)^{\frac{3}{2}}Ba^2+21\left(bx^2+a\right)^{\frac{5}{2}}Ab-35\left(bx^2+a\right)^{\frac{3}{2}}Aab}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(b*x^2 + a)^(7/2)*B - 42*(b*x^2 + a)^(5/2)*B*a + 35*(b*x^2 + a)^(
3/2)*B*a^2 + 21*(b*x^2 + a)^(5/2)*A*b - 35*(b*x^2 + a)^(3/2)*A*a*b)/b^3
```

maple [A] time = 0.01, size = 53, normalized size = 0.73

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-15B\,b^2x^4-21A\,b^2x^2+12Bab\,x^2+14abA-8a^2B\right)}{105b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(-15*B*b^2*x^4-21*A*b^2*x^2+12*B*a*b*x^2+14*A*a*b-8*
B*a^2)/b^3
```

maxima [A] time = 1.09, size = 90, normalized size = 1.23

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^4}{7\,b}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bax^2}{35\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax^2}{5\,b}+\frac{8\left(bx^2+a\right)^{\frac{3}{2}}Ba^2}{105\,b^3}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Aa}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(3/2)*B*x^4/b - 4/35*(b*x^2 + a)^(3/2)*B*a*x^2/b^2 + 1/5*(b
*x^2 + a)^(3/2)*A*x^2/b + 8/105*(b*x^2 + a)^(3/2)*B*a^2/b^3 - 2/15*(b*x^2 +
 a)^(3/2)*A*a/b^2
```

mupad [B] time = 0.56, size = 76, normalized size = 1.04

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,x^6}{7}+\frac{8\,B\,a^3-14\,A\,a^2\,b}{105\,b^3}+\frac{x^4\,\left(21\,A\,b^3+3\,B\,a\,b^2\right)}{105\,b^3}+\frac{a\,x^2\,\left(7\,A\,b-4\,B\,a\right)}{105\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*x^6)/7 + (8*B*a^3 - 14*A*a^2*b)/(105*b^3) + (x^4*(21*
A*b^3 + 3*B*a*b^2))/(105*b^3) + (a*x^2*(7*A*b - 4*B*a))/(105*b^2))
```

sympy [A] time = 0.95, size = 162, normalized size = 2.22

$$\begin{cases} -\frac{2Aa^2\sqrt{a+bx^2}}{15b^2} + \frac{Aax^2\sqrt{a+bx^2}}{15b} + \frac{Ax^4\sqrt{a+bx^2}}{5} + \frac{8Ba^3\sqrt{a+bx^2}}{105b^3} - \frac{4Ba^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{Bax^4\sqrt{a+bx^2}}{35b} + \frac{Bx^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^4}{4} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a**2*sqrt(a + b*x**2)/(15*b**2) + A*a*x**2*sqrt(a + b*x**2)
/(15*b) + A*x**4*sqrt(a + b*x**2)/5 + 8*B*a**3*sqrt(a + b*x**2)/(105*b**3)
- 4*B*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + B*a*x**4*sqrt(a + b*x**2)/(35
*b) + B*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**6/6),
 True))
```

3.489 $\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=122

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}(2Ab-aB)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$-\frac{a^2(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{ax\sqrt{a+bx^2}(2Ab-aB)}{16b^2}+\frac{x^3\sqrt{a+bx^2}(2Ab-aB)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (a*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b^2) + ((2*A*b - a*B)*x^3*Sqrt[a +
b*x^2])/(8*b) + (B*x^3*(a + b*x^2)^(3/2))/(6*b) - (a^2*(2*A*b - a*B)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)dx = \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{(-6Ab+3aB)\int x^2\sqrt{a+bx^2}\,dx}{6b}$$

$$= \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} + \frac{(a(2Ab-aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b}$$

$$= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2} + \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{\left(a^2(2Ab\right.}{}$$

$$= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2} + \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{\left(a^2(2Ab\right.}{}$$

$$= \frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2} + \frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b} + \frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b} - \frac{a^2(2Ab}{}$$

Mathematica [A] time = 0.22, size = 108, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\left(\frac{3a^{3/2}(aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-3a^2B+2ab\left(3A+Bx^2\right)+4b^2x^2\left(3A+2Bx^2\right)\right)\right)}{48b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-3*a^2*B + 2*a*b*(3*A + B*x^2) + 4*b^2*x^2*(3*
A + 2*B*x^2)) + (3*a^(3/2)*(-2*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[1 + (b*x^2)/a]))/(48*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 103, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-3a^2Bx+6aAbx+2abBx^3+12Ab^2x^3+8b^2Bx^5\right)}{48b^2}+\frac{\left(2a^2Ab-a^3B\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(6*a*A*b*x - 3*a^2*B*x + 12*A*b^2*x^3 + 2*a*b*B*x^3 + 8*b^
2*B*x^5))/(48*b^2) + ((2*a^2*A*b - a^3*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/(16*b^(5/2))
```

fricas [A] time = 0.81, size = 206, normalized size = 1.69

$$\left[-\frac{3\left(Ba^3-2\,Aa^2b\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(8\,Bb^3x^5+2\left(Bab^2+6\,Ab^3\right)x^3-3\left(Ba^2b-2\,Aab^2\right)x\right)\sqrt{bx^2+a}}{96\,b^3},\ -\frac{3\left(Ba^3-2\,Aa^2b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,Bb^3x^5+2\left(Bab^2+6\,Ab^3\right)x^3-3\left(Ba^2b-2\,Aab^2\right)x\right)\sqrt{bx^2+a}}{48\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*a^3 - 2*A*a^2*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 - 3*(B*a^2*b - 2*A*
a*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*(B*a^3 - 2*A*a^2*b)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 -
3*(B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.40, size = 100, normalized size = 0.82

$$\frac{1}{48}\left(2\left(4\,Bx^2+\frac{Bab^3+6\,Ab^4}{b^4}\right)x^2-\frac{3\left(Ba^2b^2-2\,Aab^3\right)}{b^4}\right)\sqrt{bx^2+a}\,x-\frac{\left(Ba^3-2\,Aa^2b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2 + (B*a*b^3 + 6*A*b^4)/b^4)*x^2 - 3*(B*a^2*b^2 - 2*A*a*b^3)
/b^4)*sqrt(b*x^2 + a)*x - 1/16*(B*a^3 - 2*A*a^2*b)*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 139, normalized size = 1.14

$$\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B\,x^3}{6b}-\frac{A\,a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}+\frac{B\,a^3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}}-\frac{\sqrt{b\,x^2+a}\,Aax}{8b}+\frac{\sqrt{b\,x^2+a}\,B\,a^2x}{16b^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Ax}{4b}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Bax}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/8*B*a/b^2*x*(b*x^2+a)^(3/2)+1/16*B*a^2/b^2*x*
(b*x^2+a)^(1/2)+1/16*B*a^3/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*A*x*(b
*x^2+a)^(3/2)/b-1/8*A*a/b*x*(b*x^2+a)^(1/2)-1/8*A*a^2/b^(3/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))
```

maxima [A] time = 1.01, size = 124, normalized size = 1.02

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^3}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax}{4\,b}-\frac{\sqrt{bx^2+a}\,Aax}{8\,b}+\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*B*x^3/b - 1/8*(b*x^2 + a)^(3/2)*B*a*x/b^2 + 1/16*sqrt
(b*x^2 + a)*B*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^2 + a)
*A*a*x/b + 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*A*a^2*arcsinh(b*
x/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int(x^2*(A + B*x^2)*(a + b*x^2)^(1/2), x)
```

sympy [B] time = 12.09, size = 226, normalized size = 1.85

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Bbx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) - B*a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)
*x**3/(48*b*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a))
+ B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(
1 + b*x**2/a))
```

3.490 $\int x\sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (B*(a + b*x^2)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^2}\left(A+Bx^2\right)dx&=\frac{1}{2}\operatorname{Subst}\left(\int\sqrt{a+bx}\,(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx,x,x^2\right)\\&=\frac{(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(-2aB+5Ab+3bBx^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^2))/(15*b^2)
```

IntegrateAlgebraic [**A**] time = 0.03, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{3/2}\left(-2aB+5Ab+3bBx^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^2))/(15*b^2)
```

fricas [**A**] time = 0.96, size = 50, normalized size = 1.09

$$\frac{\left(3\,Bb^2x^4-2\,Ba^2+5\,Aab+\left(Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 - 2*B*a^2 + 5*A*a*b + (B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 +
 a)/b^2
```

giac [**A**] time = 0.31, size = 44, normalized size = 0.96

$$\frac{3\left(bx^2+a\right)^{\frac{5}{2}}B-5\left(bx^2+a\right)^{\frac{3}{2}}Ba+5\left(bx^2+a\right)^{\frac{3}{2}}Ab}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(b*x^2 + a)^(5/2)*B - 5*(b*x^2 + a)^(3/2)*B*a + 5*(b*x^2 + a)^(3/2)
*A*b)/b^2
```

maple [A] time = 0.00, size = 31, normalized size = 0.67

$$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(3Bb\,x^{2}+5Ab-2Ba\right)}{15b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(3/2)*(3*B*b*x^2+5*A*b-2*B*a)/b^2
```

maxima [A] time = 0.98, size = 50, normalized size = 1.09

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bx^{2}}{5\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{3}{2}}Ba}{15\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(3/2)*B*x^2/b - 2/15*(b*x^2 + a)^(3/2)*B*a/b^2 + 1/3*(b*x^2
 + a)^(3/2)*A/b
```

mupad [B] time = 0.54, size = 53, normalized size = 1.15

$$\sqrt{b\,x^{2}+a}\,\left(\frac{B\,x^{4}}{5}-\frac{2\,B\,a^{2}-5\,A\,a\,b}{15\,b^{2}}+\frac{x^{2}\,\left(5\,A\,b^{2}+B\,a\,b\right)}{15\,b^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*x^4)/5 - (2*B*a^2 - 5*A*a*b)/(15*b^2) + (x^2*(5*A*b^2
 + B*a*b))/(15*b^2))
```

sympy [A] time = 0.37, size = 110, normalized size = 2.39

$$\begin{cases} \frac{Aa\sqrt{a+bx^{2}}}{3b}+\frac{Ax^{2}\sqrt{a+bx^{2}}}{3}-\frac{2Ba^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{Bax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{Bx^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\ b\neq 0\\ \sqrt{a}\left(\frac{Ax^{2}}{2}+\frac{Bx^{4}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*a*sqrt(a + b*x**2)/(3*b) + A*x**2*sqrt(a + b*x**2)/3 - 2*B*a**
2*sqrt(a + b*x**2)/(15*b**2) + B*a*x**2*sqrt(a + b*x**2)/(15*b) + B*x**4*sq
rt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*(A*x**2/2 + B*x**4/4), True))
```

3.491 $\int \sqrt{a+bx^2}\left(A+Bx^2\right)dx$

Optimal. Leaf size=87

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4Ab-aB)}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {388, 195, 217, 206}

$$\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}\,(4Ab-aB)}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] ((4*A*b - a*B)*x*Sqrt[a + b*x^2])/(8*b) + (B*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+bx^2}\left(A+Bx^2\right)dx &= \frac{Bx\left(a+bx^2\right)^{3/2}}{4b}-\frac{(-4Ab+aB)\int \sqrt{a+bx^2}\,dx}{4b}\\
&= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}+\frac{(a(4Ab-aB))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}+\frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{8b}\\
&= \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}+\frac{a(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.98

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(B\left(a+2bx^2\right)+4Ab\right)-\frac{\sqrt{a}\,(aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{8b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(4*A*b + B*(a + 2*b*x^2)) - (Sqrt[a]*(-4*A*b +
a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(8*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 77, normalized size = 0.89

$$
\frac{\left(a^2B-4aAb\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{3/2}}+\frac{\sqrt{a+bx^2}\left(aBx+4Abx+2bBx^3\right)}{8b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*A*b*x + a*B*x + 2*b*B*x^3))/(8*b) + ((-4*a*A*b + a^2*B)
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

fricas [A] time = 0.93, size = 155, normalized size = 1.78

$$\left[-\frac{\left(Ba^{2}-4\,Aab\right)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,\left(2\,Bb^{2}x^{3}+\left(Bab+4\,Ab^{2}\right)x\right)\sqrt{bx^{2}+a}}{16\,b^{2}},\ \frac{\left(Ba^{2}-4\,Aab\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(2\,Bb^{2}x^{3}+\left(Bab+4\,Ab^{2}\right)x\right)\sqrt{bx^{2}+a}}{8\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*B*b^2*x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/8*((
B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b^2*x^3
 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.49, size = 69, normalized size = 0.79

$$\frac{1}{8}\left(2\,Bx^{2}+\frac{Bab+4\,Ab^{2}}{b^{2}}\right)\sqrt{bx^{2}+a}\,x+\frac{\left(Ba^{2}-4\,Aab\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*B*x^2 + (B*a*b + 4*A*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/8*(B*a^2 - 4*A*
a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 96, normalized size = 1.10

$$\frac{Aa\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}-\frac{B\,a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^{2}+a}\;Ax}{2}-\frac{\sqrt{b\,x^{2}+a}\;Bax}{8b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bx}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x*(b*x^2+a)^(3/2)/b-1/8*B*a/b*x*(b*x^2+a)^(1/2)-1/8*B*a^2/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*A*x*(b*x^2+a)^(1/2)+1/2*A*a/b^(1/2)*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.12, size = 81, normalized size = 0.93

$$\frac{1}{2}\,\sqrt{bx^{2}+a}\,Ax+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bx}{4\,b}-\frac{\sqrt{bx^{2}+a}\,Bax}{8\,b}-\frac{Ba^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)
*B*a*x/b - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x/s
qrt(a*b))/sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^2+A\right)\,\sqrt{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^(1/2),x)
```

```
[Out] int((A + B*x^2)*(a + b*x^2)^(1/2), x)
```

sympy [A] time = 6.30, size = 144, normalized size = 1.66

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x
**2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a
)*sqrt(1 + b*x**2/a))
```

3.492 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x}\,dx$

Optimal. Leaf size=59

$$A\sqrt{a+bx^2}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$A\sqrt{a+bx^2}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] A*Sqrt[a + b*x^2] + (B*(a + b*x^2)^(3/2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x}\,dx, x, x^2\right)\\
&= \frac{B\left(a+bx^2\right)^{3/2}}{3b} + \frac{1}{2}A\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)\\
&= A\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{3/2}}{3b} + \frac{1}{2}(aA)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= A\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{3/2}}{3b} + \frac{(aA)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= A\sqrt{a+bx^2} + \frac{B\left(a+bx^2\right)^{3/2}}{3b} - \sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\left(B\left(a+bx^2\right)+3Ab\right)}{3b} - \sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*A*b + B*(a + b*x^2)))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

IntegrateAlgebraic [**A**] time = 0.05, size = 59, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\,\left(aB+3Ab+bBx^2\right)}{3b}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(3*A*b + a*B + b*B*x^2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]]
```

fricas [**A**] time = 0.89, size = 123, normalized size = 2.08

$$\left[\frac{3\,A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(Bbx^2+Ba+3\,Ab\right)\sqrt{bx^2+a}}{6\,b},\ \frac{3\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(Bbx^2+Ba+3\,Ab\right)\sqrt{bx^2+a}}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2
*(B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b, 1/3*(3*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + (B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b]
```

giac [**A**] time = 0.41, size = 60, normalized size = 1.02

$$\frac{Aa\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^2+a}\,Ab^3}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] A*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*((b*x^2 + a)^(3/2)*B*b^
2 + 3*sqrt(b*x^2 + a)*A*b^3)/b^3
```

maple [**A**] time = 0.01, size = 57, normalized size = 0.97

$$-A\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}\,A+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b-A*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)*a^(1/2)+A*(
b*x^2+a)^(1/2)
```

maxima [A] time = 0.95, size = 45, normalized size = 0.76

$$-A\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,A+\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -A*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A + 1/3*(b*x^2 +
 a)^(3/2)*B/b
```

mupad [B] time = 0.88, size = 47, normalized size = 0.80

$$A\sqrt{b\,x^2+a}+\frac{B\left(b\,x^2+a\right)^{3/2}}{3\,b}-A\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x,x)
```

```
[Out] A*(a + b*x^2)^(1/2) + (B*(a + b*x^2)^(3/2))/(3*b) - A*a^(1/2)*atanh((a + b*
x^2)^(1/2)/a^(1/2))
```

sympy [A] time = 25.64, size = 76, normalized size = 1.29

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^2}\right)}{2}-\frac{B\left(\begin{cases}-\sqrt{a}\,x^2 & \text{for}\ b=0\\ -\frac{2\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**2))/2 -
 B*Piecewise((-sqrt(a)*x**2, Eq(b, 0)), (-2*(a + b*x**2)**(3/2)/(3*b), True
))/2
```

3.493 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^2}\,dx$

Optimal. Leaf size=84

$$\frac{x\sqrt{a+bx^2}\,(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

Rubi **[A]** time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 195, 217, 206}

$$\frac{x\sqrt{a+bx^2}\,(aB+2Ab)}{2a}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] ((2*A*b + a*B)*x*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(a*x) + ((2
*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^2}\,dx &= -\frac{A\left(a+bx^2\right)^{3/2}}{ax}-\frac{(-2Ab-aB)\int \sqrt{a+bx^2}\,dx}{a}\\
&= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}-\frac{1}{2}(-2Ab-aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}-\frac{1}{2}(-2Ab-aB)\,\text{Subst}\Bigg(\int \frac{1}{1-bx^2}\,dx,x,\\
&= \frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{ax}+\frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 71, normalized size = 0.85

$$
\frac{1}{2}\sqrt{a+bx^2}\left(\frac{(aB+2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{\frac{bx^2}{a}+1}}-\frac{2A}{x}+Bx\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*((-2*A)/x + B*x + ((2*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/2
```

IntegrateAlgebraic [A] time = 0.12, size = 67, normalized size = 0.80

$$
\frac{\sqrt{a+bx^2}\left(Bx^2-2A\right)}{2x}+\frac{(-aB-2Ab)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*A + B*x^2))/(2*x) + ((-2*A*b - a*B)*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

fricas [A] time = 0.95, size = 134, normalized size = 1.60

$$\left[\frac{(Ba + 2\,Ab)\sqrt{b}\,x\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + 2\left(Bbx^2 - 2\,Ab\right)\sqrt{bx^2 + a}}{4\,bx}, -\frac{(Ba + 2\,Ab)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right) - \left(Bbx^2 - 2\,Ab\right)\sqrt{bx^2 + a}}{2\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((B*a + 2*A*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a))/(b*x), -1/2*((B*a + 2*A*b)*sqrt(-
b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a)
)/(b*x)]
```

giac [A] time = 0.42, size = 84, normalized size = 1.00

$$\frac{1}{2}\,\sqrt{bx^2 + a}\,Bx + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a} - \frac{\left(Ba\sqrt{b} + 2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a) - 1/4*(B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

maple [A] time = 0.01, size = 93, normalized size = 1.11

$$A\sqrt{b}\;\ln\left(\sqrt{b}\;x + \sqrt{b\,x^2 + a}\right) + \frac{Ba\ln\left(\sqrt{b}\;x + \sqrt{b\,x^2 + a}\right)}{2\sqrt{b}} + \frac{\sqrt{b\,x^2 + a}\;Abx}{a} + \frac{\sqrt{b\,x^2 + a}\;Bx}{2} - \frac{\left(b\,x^2 + a\right)^{\frac{3}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] 1/2*x*B*(b*x^2+a)^(1/2)+1/2*B*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-A*(b*
x^2+a)^(3/2)/a/x+A*b/a*x*(b*x^2+a)^(1/2)+A*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))
```

maxima [A] time = 1.04, size = 59, normalized size = 0.70

$$\frac{1}{2}\,\sqrt{bx^2 + a}\,Bx + \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} + A\sqrt{b}\;\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2 + a}\,A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + A*sqrt(b
)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*A/x
```

mupad [B] time = 1.26, size = 94, normalized size = 1.12

$$\frac{B\,x\,\sqrt{b\,x^2+a}}{2}-\frac{A\,\sqrt{b\,x^2+a}}{x}+\frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}-\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^2,x)
```

```
[Out] (B*x*(a + b*x^2)^(1/2))/2 - (A*(a + b*x^2)^(1/2))/x + (B*a*log(b^(1/2)*x +
(a + b*x^2)^(1/2)))/(2*b^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(
a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

sympy [A] time = 4.41, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B*a*a
sinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

3.494 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^2}\,(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{a+bx^2}\,(2aB+Ab)}{2a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^2])/(2*a) - (A*(a + b*x^2)^(3/2))/(2*a*x^2) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^2}\,dx, x, x^2\right)\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}+\frac{(Ab+2aB)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^2\right)}{4a}\\ &= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}+\frac{1}{4}(Ab+2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x\right.\\ &= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}+\frac{(Ab+2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.}{2b}\\ &= \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{2ax^2}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.75

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\left(2Bx^2-A\right)}{x^2}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-A + 2*B*x^2))/x^2 - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/Sqrt[a])/2
```

IntegrateAlgebraic [A] time = 0.11, size = 65, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(2Bx^2-A\right)}{2x^2}+\frac{(-2aB-Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-A + 2*B*x^2))/(2*x^2) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

fricas [A] time = 0.86, size = 141, normalized size = 1.68

$$\left[\frac{(2\,Ba+Ab)\sqrt{a}\,x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Bax^2-Aa\right)\sqrt{bx^2+a}}{4\,ax^2},\frac{(2\,Ba+Ab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,Bax^2-Aa\right)\sqrt{bx^2+a}}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*B*a + A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(2*B*a*x^2 - A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*((2*B*a + A*b)
*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x^2 - A*a)*sqrt(b*x
^2 + a))/(a*x^2)]
```

giac [A] time = 0.36, size = 68, normalized size = 0.81

$$\frac{2\,\sqrt{bx^2+a}\,Bb+\frac{\left(2\,Bab+Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^2+a}\,Ab}{x^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(b*x^2 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^2 + a)*A*b/x^2)/b
```

maple [A]   time = 0.01, size = 106, normalized size = 1.26

$$-\frac{Ab\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}}-B\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\frac{\sqrt{b\,x^{2}+a}\,Ab}{2a}+\sqrt{b\,x^{2}+a}\,B-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(3/2)/a/x^2-1/2*A*b/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)+1/2*A*b/a*(b*x^2+a)^(1/2)-B*a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/
2))/x)+B*(b*x^2+a)^(1/2)
```

maxima [A]   time = 1.10, size = 83, normalized size = 0.99

$$-B\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{Ab\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,\sqrt{a}}+\sqrt{bx^{2}+a}\,B+\frac{\sqrt{bx^{2}+a}\,Ab}{2\,a}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}A}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/sqrt(a) + sqrt(b*x^2 + a)*B + 1/2*sqrt(b*x^2 + a)*A*b/a - 1/2*(b*x^2
+ a)^(3/2)*A/(a*x^2)
```

mupad [B]   time = 1.35, size = 68, normalized size = 0.81

$$B\,\sqrt{b\,x^{2}+a}-\frac{A\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}-B\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)-\frac{A\,b\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^3,x)
```

```
[Out] B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/(2*x^2) - B*a^(1/2)*atanh((a +
b*x^2)^(1/2)/a^(1/2)) - (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

sympy [A]   time = 43.70, size = 107, normalized size = 1.27

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{Ab\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}-B\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba}{\sqrt{b}\,x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{B\sqrt{b}\,x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a/(sqrt(b)*x*sqrt(a/(b*x
**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.495 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^4}\,dx$

Optimal. Leaf size=66

$$-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {451, 277, 217, 206}

$$-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}-\frac{B\sqrt{a+bx^2}}{x}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] -((B*Sqrt[a + b*x^2])/x) - (A*(a + b*x^2)^(3/2))/(3*a*x^3) + Sqrt[b]*B*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^4}\,dx &= -\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}+B\int \frac{\sqrt{a+bx^2}}{x^2}\,dx \\
&= -\frac{B\sqrt{a+bx^2}}{x}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}+(bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= -\frac{B\sqrt{a+bx^2}}{x}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}+(bB)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right) \\
&= -\frac{B\sqrt{a+bx^2}}{x}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^3}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.17, size = 81, normalized size = 1.23

$$\frac{\sqrt{a+bx^2}\left(\frac{3\sqrt{a}\,\sqrt{b}\,B\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}-\frac{aA+3aBx^2+Abx^2}{x^3}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-((a*A + A*b*x^2 + 3*a*B*x^2)/x^3) + (3*Sqrt[a]*Sqrt[b]*B
*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(3*a)
```

IntegrateAlgebraic [A] time = 0.12, size = 70, normalized size = 1.06

$$\frac{\sqrt{a+bx^2}\left(-aA-3aBx^2-Abx^2\right)}{3ax^3}-\sqrt{b}\,B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(a*A) - A*b*x^2 - 3*a*B*x^2))/(3*a*x^3) - Sqrt[b]*B*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 0.88, size = 137, normalized size = 2.08

$$\left[\frac{3\,Ba\sqrt{b}\,x^{3}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left((3\,Ba+Ab)x^{2}+Aa\right)\sqrt{bx^{2}+a}}{6\,ax^{3}},-\frac{3\,Ba\sqrt{-b}\,x^{3}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left((3\,Ba+Ab)x^{2}+Aa\right)\sqrt{bx^{2}+a}}{3\,ax^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a))/(a*x^3), -1/3*(3*B*a*sqrt(-b)*x
^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^
2 + a))/(a*x^3)]
```

giac [B] time = 0.52, size = 151, normalized size = 2.29

$$-\frac{1}{2}\,B\sqrt{b}\,\log\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{4}Ba\sqrt{b}+3\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{4}Ab^{\frac{3}{2}}-6\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}Ba^{2}\sqrt{b}+3\,Ba^{3}\sqrt{b}+Aa^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*B*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a*sqrt(b) + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2)
 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 3*B*a^3*sqrt(b) + A*a^
2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 75, normalized size = 1.14

$$B\sqrt{b}\;\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)+\frac{\sqrt{b\,x^{2}+a}\;Bbx}{a}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B}{ax}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A}{3a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -B/a/x*(b*x^2+a)^(3/2)+B*b/a*x*(b*x^2+a)^(1/2)+B*b^(1/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))-1/3*A*(b*x^2+a)^(3/2)/a/x^3
```

maxima [A] time = 1.08, size = 48, normalized size = 0.73

$$B\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\sqrt{bx^{2}+a}\,B}{x}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}A}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*B/x - 1/3*(b*x^2 + a)^(3
/2)*A/(a*x^3)
```

mupad [B] time = 1.47, size = 76, normalized size = 1.15

$$-\frac{B\sqrt{b\,x^2+a}}{x}-\frac{A\left(b\,x^2+a\right)^{3/2}}{3\,a\,x^3}-\frac{B\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^4,x)
```

```
[Out] - (B*(a + b*x^2)^(1/2))/x - (A*(a + b*x^2)^(3/2))/(3*a*x^3) - (B*b^(1/2)*as
in((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(
1/2))
```

sympy [A] time = 3.35, size = 107, normalized size = 1.62

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(3*a) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt(a
)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

3.496 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}\,(Ab-4aB)}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

Rubi **[A]** time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}\,(Ab-4aB)}{8ax^2}-\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a*x^2) - (A*(a + b*x^2)^(3/2))/(4*a*x^4)
 + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^3}\,dx, x, x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{3/2}}{4ax^4} + \frac{\left(-\frac{Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{3/2}}{4ax^4} - \frac{(b(Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{16a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{3/2}}{4ax^4} - \frac{(Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{8a}\\
&= \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{3/2}}{4ax^4} + \frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 1.06

$$\frac{-\left(a+bx^2\right)\left(2a\left(A+2Bx^2\right)+Abx^2\right)-bx^4\sqrt{\frac{bx^2}{a}+1}\,(4aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{8ax^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] (-((a + b*x^2)*(A*b*x^2 + 2*a*(A + 2*B*x^2))) - b*(-(A*b) + 4*a*B)*x^4*Sqrt
[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(8*a*x^4*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.11, size = 79, normalized size = 0.90

$$\frac{\left(Ab^2-4abB\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}+\frac{\sqrt{a+bx^2}\,\left(-2aA-4aBx^2-Abx^2\right)}{8ax^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*A - A*b*x^2 - 4*a*B*x^2))/(8*a*x^4) + ((A*b^2 - 4*a*
b*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

fricas [A] time = 0.82, size = 170, normalized size = 1.93

$$\left[-\frac{\left(4\,Bab-Ab^2\right)\sqrt{a}\,x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\left(2\,Aa^2+\left(4\,Ba^2+Aab\right)x^2\right)\sqrt{bx^2+a}}{16\,a^2x^4},\ \frac{\left(4\,Bab-Ab^2\right)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(2\,Aa^2+\left(4\,Ba^2+Aab\right)x^2\right)\sqrt{bx^2+a}}{8\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - A*b^2)*sqrt(a)*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*
x^4), 1/8*((4*B*a*b - A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a))
- (2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^4)]
```

giac [A] time = 0.35, size = 120, normalized size = 1.36

$$\frac{\frac{\left(4\,Bab^2-Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{4\,\left(bx^2+a\right)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^2+a}\,Ba^2b^2+\left(bx^2+a\right)^{\frac{3}{2}}Ab^3+\sqrt{bx^2+a}\,Aab^3}{ab^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (4
*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 + (b*x^2 + a)^(3/2
)*A*b^3 + sqrt(b*x^2 + a)*A*a*b^3)/(a*b^2*x^4))/b
```

maple [B] time = 0.01, size = 153, normalized size = 1.74

$$\frac{A\,b^2\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{3}{2}}}-\frac{Bb\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2\sqrt{a}}-\frac{\sqrt{b\,x^2+a}\,A\,b^2}{8a^2}+\frac{\sqrt{b\,x^2+a}\,Bb}{2a}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Ab}{8a^2x^2}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}B}{2a\,x^2}-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}A}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4*A*(b*x^2+a)^(3/2)/a/x^4+1/8*A*b/a^2/x^2*(b*x^2+a)^(3/2)+1/8*A*b^2/a^(3
/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/8*A*b^2/a^2*(b*x^2+a)^(1/2)-1/2
*B/a/x^2*(b*x^2+a)^(3/2)-1/2*B*b/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)+1/2*B*b/a*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.05, size = 130, normalized size = 1.48

$$-\frac{Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\sqrt{a}}+\frac{Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}\,Bb}{2\,a}-\frac{\sqrt{bx^2+a}\,Ab^2}{8\,a^2}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{2\,ax^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab}{8\,a^2x^2}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(3/2) + 1/2*sqrt(b*x^2 + a)*B*b/a - 1/8*sqrt(b*x^2 + a)*A*b
^2/a^2 - 1/2*(b*x^2 + a)^(3/2)*B/(a*x^2) + 1/8*(b*x^2 + a)^(3/2)*A*b/(a^2*x
^2) - 1/4*(b*x^2 + a)^(3/2)*A/(a*x^4)
```

mupad [B] time = 1.70, size = 93, normalized size = 1.06

$$\frac{A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{A\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a\,x^4}-\frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^5,x)
```

```
[Out] (A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (B*(a + b*x^2)^(1/2)
)/(2*x^2) - (A*(a + b*x^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(3/2))/(8*a*x^4)
 - (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

sympy [A] time = 143.85, size = 144, normalized size = 1.64

$$-\frac{Aa}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{3A\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{Ab^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Bb\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -A*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)/(8*x**3*sqrt(a/(b*
x**2) + 1)) - A*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + A*b**2*asinh(sqrt(a
)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b*as
inh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a))
```

3.497 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^6}\,dx$

Optimal. Leaf size=53

$$\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.091, Rules used = {453, 264}

$$\frac{\left(a+bx^2\right)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(5*a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(3/2))/(15*
a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^6}\,dx&=-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}-\frac{(2Ab-5aB)\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{5a}\\&=-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^5}+\frac{(2Ab-5aB)\left(a+bx^2\right)^{3/2}}{15a^2x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.75

$$-\frac{\left(a+bx^2\right)^{3/2}\left(3aA+5aBx^2-2Abx^2\right)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] -1/15*((a + b*x^2)^(3/2)*(3*a*A - 2*A*b*x^2 + 5*a*B*x^2))/(a^2*x^5)
```

IntegrateAlgebraic [A] time = 0.13, size = 62, normalized size = 1.17

$$\frac{\sqrt{a+bx^2}\left(-3a^2A-5a^2Bx^2-aAbx^2-5abBx^4+2Ab^2x^4\right)}{15a^2x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*A - a*A*b*x^2 - 5*a^2*B*x^2 + 2*A*b^2*x^4 - 5*a*b*
B*x^4))/(15*a^2*x^5)
```

fricas [A] time = 0.91, size = 55, normalized size = 1.04

$$-\frac{\left(\left(5\,Bab-2\,Ab^2\right)x^4+3\,Aa^2+\left(5\,Ba^2+Aab\right)x^2\right)\sqrt{bx^2+a}}{15\,a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*((5*B*a*b - 2*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + A*a*b)*x^2)*sqrt(b*x^
2 + a)/(a^2*x^5)
```

giac [B] time = 0.43, size = 232, normalized size = 4.38

$$\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{3}{2}}-30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{3}{2}}+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{5}{2}}+20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2b^{\frac{3}{2}}+10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aab^{\frac{5}{2}}-10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3b^{\frac{3}{2}}+10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^2b^{\frac{5}{2}}+5\,Ba^4b^{\frac{3}{2}}-2\,Aa^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(3/2) - 30*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*B*a*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(5/2) + 2
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) + 10*(sqrt(b)*x - sqrt(b*x^
```

```
2 + a))^4*A*a*b^(5/2) - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2) +
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(5/2) + 5*B*a^4*b^(3/2) - 2*A*a^
3*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 37, normalized size = 0.70

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-2Ab\,x^{2}+5Ba\,x^{2}+3Aa\right)}{15a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/15*(b*x^2+a)^(3/2)*(-2*A*b*x^2+5*B*a*x^2+3*A*a)/x^5/a^2
```

maxima [A] time = 1.08, size = 56, normalized size = 1.06

$$-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}B}{3\,ax^{3}}+\frac{2\left(bx^{2}+a\right)^{\frac{3}{2}}Ab}{15\,a^{2}x^{3}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}A}{5\,ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/3*(b*x^2 + a)^(3/2)*B/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b/(a^2*x^3) - 1
/5*(b*x^2 + a)^(3/2)*A/(a*x^5)
```

mupad [B] time = 0.90, size = 97, normalized size = 1.83

$$\frac{\left(A\,b^{2}+B\,a\,b\right)\sqrt{b\,x^{2}+a}}{5\,a^{2}\,x}-\frac{\left(5\,B\,a^{2}+A\,b\,a\right)\sqrt{b\,x^{2}+a}}{15\,a^{2}\,x^{3}}-\frac{A\,\sqrt{b\,x^{2}+a}}{5\,x^{5}}-\frac{b\,\sqrt{b\,x^{2}+a}\,\left(A\,b+8\,B\,a\right)}{15\,a^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^6,x)
```

```
[Out] ((A*b^2 + B*a*b)*(a + b*x^2)^(1/2))/(5*a^2*x) - ((5*B*a^2 + A*a*b)*(a + b*x
^2)^(1/2))/(15*a^2*x^3) - (A*(a + b*x^2)^(1/2))/(5*x^5) - (b*(a + b*x^2)^(1
/2)*(A*b + 8*B*a))/(15*a^2*x)
```

sympy [B] time = 3.23, size = 119, normalized size = 2.25

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{5x^{4}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15ax^{2}}+\frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{2}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/
(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*sqrt(b)*sqrt(
a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a)
```

3.498 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^7}\,dx$

Optimal. Leaf size=120

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}\,(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}\,(Ab-2aB)}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{b^2(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{b\sqrt{a+bx^2}\,(Ab-2aB)}{16a^2x^2}+\frac{\sqrt{a+bx^2}\,(Ab-2aB)}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] ((A*b - 2*a*B)*Sqrt[a + b*x^2])/(8*a*x^4) + (b*(A*b - 2*a*B)*Sqrt[a + b*x^2
])/(16*a^2*x^2) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) - (b^2*(A*b - 2*a*B)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^4}\,dx,x,x^2\right) \\
&= -\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}+\frac{\left(-\frac{3Ab}{2}+3aB\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx,x,x^2\right)}{6a} \\
&= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}-\frac{(b(Ab-2aB))\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{16a} \\
&= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4}+\frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}+\frac{\left(b^2(Ab-2aB)\right)}{} \\
&= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4}+\frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}+\frac{(b(Ab-2aB))\,\mathrm{S}}{} \\
&= \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4}+\frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2}-\frac{A\left(a+bx^2\right)^{3/2}}{6ax^6}-\frac{b^2(Ab-2aB)\,\mathrm{ta}}{16a}
\end{aligned}$$

Mathematica [**C**] time = 0.02, size = 61, normalized size = 0.51

$$-\frac{\left(a+bx^2\right)^{3/2}\left(a^3A+b^2x^6(2aB-Ab)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx^2}{a}+1\right)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] -1/6*((a + b*x^2)^(3/2)*(a^3*A + b^2*(-(A*b) + 2*a*B)*x^6*Hypergeometric2F1
[3/2, 3, 5/2, 1 + (b*x^2)/a]))/(a^4*x^6)
```

IntegrateAlgebraic [**A**] time = 0.17, size = 104, normalized size = 0.87

$$\frac{\left(2ab^2B-Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}+\frac{\sqrt{a+bx^2}\left(-8a^2A-12a^2Bx^2-2aAbx^2-6abBx^4+3Ab^2x^4\right)}{48a^2x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 2*a*A*b*x^2 - 12*a^2*B*x^2 + 3*A*b^2*x^4 - 6*a
*b*B*x^4))/(48*a^2*x^6) + ((-(A*b^3) + 2*a*b^2*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*a^(5/2))
```

fricas [A]   time = 0.91, size = 221, normalized size = 1.84

$$\left[-\frac{3\left(2\,Bab^{2}-Ab^{3}\right)\sqrt{a}\,x^{6}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(3\left(2\,Ba^{2}b-Aab^{2}\right)x^{4}+8\,Aa^{3}+2\left(6\,Ba^{3}+Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{96\,a^{3}x^{6}},-\frac{3\left(2\,Bab^{2}-Ab^{3}\right)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(3\left(2\,Ba^{2}b-Aab^{2}\right)x^{4}+8\,Aa^{3}+2\left(6\,Ba^{3}+Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{48\,a^{3}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*(2*B*a^2*b - A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3
+ A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6), -1/48*(3*(2*B*a*b^2 - A*b^3)*sq
rt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - A*a*b^2)*x^4
+ 8*A*a^3 + 2*(6*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

giac [A]   time = 0.31, size = 140, normalized size = 1.17

$$-\frac{\frac{3\left(2\,Bab^{3}-Ab^{4}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{6\left(bx^{2}+a\right)^{\frac{5}{2}}Bab^{3}-6\,\sqrt{bx^{2}+a}\,Ba^{3}b^{3}-3\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{4}+8\left(bx^{2}+a\right)^{\frac{3}{2}}Aab^{4}+3\,\sqrt{bx^{2}+a}\,Aa^{2}b^{4}}{a^{2}b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(2*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (6*(b*x^2 + a)^(5/2)*B*a*b^3 - 6*sqrt(b*x^2 + a)*B*a^3*b^3 - 3*(b*x^2 +
 a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b^4 + 3*sqrt(b*x^2 + a)*A*a^2*b^4
)/(a^2*b^3*x^6))/b
```

maple [A]   time = 0.01, size = 197, normalized size = 1.64

$$-\frac{A\,b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{5}{2}}}+\frac{B\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{3}{2}}}+\frac{\sqrt{b\,x^{2}+a}\,A\,b^{3}}{16a^{3}}-\frac{\sqrt{b\,x^{2}+a}\,B\,b^{2}}{8a^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{2}}{16a^{3}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bb}{8a^{2}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Ab}{8a^{2}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B}{4a\,x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/4*B/a/x^4*(b*x^2+a)^(3/2)+1/8*B*b/a^2/x^2*(b*x^2+a)^(3/2)+1/8*B*b^2/a^(3
/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/8*B*b^2/a^2*(b*x^2+a)^(1/2)-1/6
*A*(b*x^2+a)^(3/2)/a/x^6+1/8*A*b/a^2/x^4*(b*x^2+a)^(3/2)-1/16*A*b^2/a^3/x^2
```

```
*(b*x^2+a)^(3/2)-1/16*A*b^3/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+1
/16*A*b^3/a^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.13, size = 174, normalized size = 1.45

$$\frac{Bb^{2} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8 a^{\frac{3}{2}}} - \frac{Ab^{3} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16 a^{\frac{5}{2}}} - \frac{\sqrt{bx^{2} + a}\, Bb^{2}}{8 a^{2}} + \frac{\sqrt{bx^{2} + a}\, Ab^{3}}{16 a^{3}} + \frac{\left(bx^{2} + a\right)^{\frac{3}{2}} Bb}{8 a^{2} x^{2}} - \frac{\left(bx^{2} + a\right)^{\frac{3}{2}} Ab^{2}}{16 a^{3} x^{2}} - \frac{\left(bx^{2} + a\right)^{\frac{3}{2}} B}{4 a x^{4}} + \frac{\left(bx^{2} + a\right)^{\frac{3}{2}} Ab}{8 a^{2} x^{4}} - \frac{\left(bx^{2} + a\right)^{\frac{3}{2}} A}{6 a x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/16*A*b^3*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(5/2) - 1/8*sqrt(b*x^2 + a)*B*b^2/a^2 + 1/16*sqrt(b*x^2 +
 a)*A*b^3/a^3 + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(3/2
)*A*b^2/(a^3*x^2) - 1/4*(b*x^2 + a)^(3/2)*B/(a*x^4) + 1/8*(b*x^2 + a)^(3/2)
*A*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(3/2)*A/(a*x^6)
```

mupad [B] time = 2.20, size = 134, normalized size = 1.12

$$\frac{B\, b^{2} \operatorname{atanh}\left(\frac{\sqrt{b\, x^{2}+a}}{\sqrt{a}}\right)}{8\, a^{3/2}} - \frac{B\, \sqrt{b\, x^{2} + a}}{8\, x^{4}} - \frac{A\, \sqrt{b\, x^{2} + a}}{16\, x^{6}} - \frac{A\, \left(b\, x^{2} + a\right)^{3/2}}{6\, a\, x^{6}} + \frac{A\, \left(b\, x^{2} + a\right)^{5/2}}{16\, a^{2}\, x^{6}} - \frac{B\, \left(b\, x^{2} + a\right)^{3/2}}{8\, a\, x^{4}} + \frac{A\, b^{3} \operatorname{atan}\left(\frac{\sqrt{b\, x^{2}+a}\, 1\mathrm{i}}{\sqrt{a}}\right) 1\mathrm{i}}{16\, a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^7,x)
```

```
[Out] (A*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x^
2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(1/2))/(16*x^6) + (B*b^2*atanh((a + b*x^
2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) + (A*(a +
b*x^2)^(5/2))/(16*a^2*x^6) - (B*(a + b*x^2)^(3/2))/(8*a*x^4)
```

sympy [B] time = 144.35, size = 226, normalized size = 1.88

$$- \frac{A a}{6 \sqrt{b} x^{7} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{5 A \sqrt{b}}{24 x^{5} \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{A b^{\frac{3}{2}}}{48 a x^{3} \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{A b^{\frac{5}{2}}}{16 a^{2} x \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{A b^{3} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x}\right)}{16 a^{\frac{5}{2}}} - \frac{B a}{4 \sqrt{b} x^{5} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{3 B \sqrt{b}}{8 x^{3} \sqrt{\frac{a}{b x^{2}} + 1}} - \frac{B b^{\frac{3}{2}}}{8 a x \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{B b^{2} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b} x}\right)}{8 a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)/(24*x**5*sqrt(a/(b
*x**2) + 1)) + A*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)/(16
*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5
/2)) - B*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*sqrt(b)/(8*x**3*sqrt
(a/(b*x**2) + 1)) - B*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + B*b**2*asinh(
sqrt(a)/(sqrt(b)*x))/(8*a**(3/2))
```

3.499 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^8}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{105a^3x^3}+\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$-\frac{2b\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{105a^3x^3}+\frac{\left(a+bx^2\right)^{3/2}(4Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(7*a*x^7) + ((4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(35*
a^2*x^5) - (2*b*(4*A*b - 7*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^8}\,dx &= -\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}-\frac{(4Ab-7aB)\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{7a}\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{(4Ab-7aB)\left(a+bx^2\right)^{3/2}}{35a^2x^5}+\frac{(2b(4Ab-7aB))\int \frac{\sqrt{a+bx^2}}{x^4}\,dx}{35a^2}\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{(4Ab-7aB)\left(a+bx^2\right)^{3/2}}{35a^2x^5}-\frac{2b(4Ab-7aB)\left(a+bx^2\right)^{3/2}}{105a^3x^3}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 63, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{3/2}\left(-3a^2\left(5A+7Bx^2\right)+2abx^2\left(6A+7Bx^2\right)-8Ab^2x^4\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(-8*A*b^2*x^4 - 3*a^2*(5*A + 7*B*x^2) + 2*a*b*x^2*(6*A +
 7*B*x^2)))/(105*a^3*x^7)
```

IntegrateAlgebraic [**A**] time = 0.15, size = 86, normalized size = 1.02

$$\frac{\sqrt{a+bx^2}\left(-15a^3A-21a^3Bx^2-3a^2Abx^2-7a^2bBx^4+4aAb^2x^4+14ab^2Bx^6-8Ab^3x^6\right)}{105a^3x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^3*A - 3*a^2*A*b*x^2 - 21*a^3*B*x^2 + 4*a*A*b^2*x^4
- 7*a^2*b*B*x^4 - 8*A*b^3*x^6 + 14*a*b^2*B*x^6))/(105*a^3*x^7)
```

fricas [**A**] time = 0.88, size = 81, normalized size = 0.96

$$\frac{\left(2\left(7\,Bab^2-4\,Ab^3\right)x^6-\left(7\,Ba^2b-4\,Aab^2\right)x^4-15\,Aa^3-3\left(7\,Ba^3+Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{105\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(7*B*a*b^2 - 4*A*b^3)*x^6 - (7*B*a^2*b - 4*A*a*b^2)*x^4 - 15*A*a^3
 - 3*(7*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^3*x^7)
```

giac [B] time = 0.46, size = 288, normalized size = 3.43

$$\frac{4\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Bab^{\frac{5}{2}}+280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ab^{\frac{7}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^2b^{\frac{5}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aab^{\frac{7}{2}}-42\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^3b^{\frac{5}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^2b^{\frac{7}{2}}+49\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^4b^{\frac{5}{2}}-28\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^3b^{\frac{7}{2}}-7Ba^5b^{\frac{5}{2}}+4Aa^4b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*b^(5/2) - 175*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*B*a*b^(5/2) + 280*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(7/2
) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(5/2) + 140*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a*b^(7/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(5
/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(7/2) + 49*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*B*a^4*b^(5/2) - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b
^(7/2) - 7*B*a^5*b^(5/2) + 4*A*a^4*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^7
```

maple [A] time = 0.00, size = 59, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(8A\,b^2x^4-14Bab\,x^4-12Aab\,x^2+21B\,a^2x^2+15a^2A\right)}{105a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x)
```

```
[Out] -1/105*(b*x^2+a)^(3/2)*(8*A*b^2*x^4-14*B*a*b*x^4-12*A*a*b*x^2+21*B*a^2*x^2+
15*A*a^2)/x^7/a^3
```

maxima [A] time = 1.10, size = 96, normalized size = 1.14

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb}{15\,a^2x^3}-\frac{8\left(bx^2+a\right)^{\frac{3}{2}}Ab^2}{105\,a^3x^3}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{5\,ax^5}+\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Ab}{35\,a^2x^5}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^2 + a)^(3/2)*B*b/(a^2*x^3) - 8/105*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x
^3) - 1/5*(b*x^2 + a)^(3/2)*B/(a*x^5) + 4/35*(b*x^2 + a)^(3/2)*A*b/(a^2*x^5
) - 1/7*(b*x^2 + a)^(3/2)*A/(a*x^7)
```

mupad [B] time = 1.09, size = 132, normalized size = 1.57

$$\frac{4\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{B\,\sqrt{b\,x^2+a}}{5\,x^5}-\frac{A\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{B\,b\,\sqrt{b\,x^2+a}}{15\,a\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}+\frac{2\,B\,b^2\,\sqrt{b\,x^2+a}}{15\,a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^8,x)
```

```
[Out] (4*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (B*(a + b*x^2)^(1/2))/(5*x^5) -
 (A*b*(a + b*x^2)^(1/2))/(35*a*x^5) - (B*b*(a + b*x^2)^(1/2))/(15*a*x^3) -
(A*(a + b*x^2)^(1/2))/(7*x^7) - (8*A*b^3*(a + b*x^2)^(1/2))/(105*a^3*x) + (
2*B*b^2*(a + b*x^2)^(1/2))/(15*a^2*x)
```

sympy [B] time = 3.89, size = 442, normalized size = 5.26

$$-\frac{15Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**8,x)
```

```
[Out] -15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a*b
**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*sqrt(b)*sqrt(a/
(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*
b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

3.500 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^9}\,dx$

Optimal. Leaf size=156

$$\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{a+bx^2}\,(5Ab-8aB)}{128a^3x^2}+\frac{b\sqrt{a+bx^2}\,(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}\,(5Ab-8aB)}{48ax^6}-$$

Rubi [A] time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{b^2\sqrt{a+bx^2}\,(5Ab-8aB)}{128a^3x^2}+\frac{b^3(5Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}+\frac{b\sqrt{a+bx^2}\,(5Ab-8aB)}{192a^2x^4}+\frac{\sqrt{a+bx^2}\,(5Ab-8aB)}{48ax^6}-\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] ((5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(48*a*x^6) + (b*(5*A*b - 8*a*B)*Sqrt[a +
b*x^2])/(192*a^2*x^4) - (b^2*(5*A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a^3*x^2)
 - (A*(a + b*x^2)^(3/2))/(8*a*x^8) + (b^3*(5*A*b - 8*a*B)*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^9}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{3/2}}{8ax^8} + \frac{\left(-\frac{5Ab}{2}+4aB\right)\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx, x, x^2\right)}{8a}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} - \frac{A\left(a+bx^2\right)^{3/2}}{8ax^8} - \frac{(b(5Ab-8aB))\,\mathrm{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x\right.}{96a}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{A\left(a+bx^2\right)^{3/2}}{8ax^8} + \frac{\left(b^2(5Ab-8\right.}{}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2} - \frac{A}{}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2} - \frac{A}{}$$

$$= \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2} - \frac{A}{}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.40

$$-\frac{\left(a+bx^2\right)^{3/2}\left(3a^4A+b^3x^8(5Ab-8aB)\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx^2}{a}+1\right)\right)}{24a^5x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] -1/24*((a + b*x^2)^(3/2)*(3*a^4*A + b^3*(5*A*b - 8*a*B)*x^8*Hypergeometric2
F1[3/2, 4, 5/2, 1 + (b*x^2)/a]))/(a^5*x^8)
```

IntegrateAlgebraic [A] time = 0.19, size = 128, normalized size = 0.82

$$\frac{\left(5Ab^4-8ab^3B\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}} + \frac{\sqrt{a+bx^2}\left(-48a^3A-64a^3Bx^2-8a^2Abx^2-16a^2bBx^4+10aAb^2x^4+24ab^2Bx^6-15Ab^3x^6\right)}{384a^3x^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*A - 8*a^2*A*b*x^2 - 64*a^3*B*x^2 + 10*a*A*b^2*x^4
 - 16*a^2*b*B*x^4 - 15*A*b^3*x^6 + 24*a*b^2*B*x^6))/(384*a^3*x^8) + ((5*A*b
^4 - 8*a*b^3*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

fricas [A] time = 1.00, size = 269, normalized size = 1.72

$$\left[-\frac{3\left(8\,Bab^{3}-5\,Ab^{4}\right)\sqrt{a}\,x^{8}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\left(3\left(8\,Ba^{2}b^{2}-5\,Aab^{3}\right)x^{6}-48\,Aa^{4}-2\left(8\,Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{4}-8\left(8\,Ba^{4}+Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{768\,a^{4}x^{8}},\ \frac{3\left(8\,Bab^{3}-5\,Ab^{4}\right)\sqrt{-a}\,x^{8}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(3\left(8\,Ba^{2}b^{2}-5\,Aab^{3}\right)x^{6}-48\,Aa^{4}-2\left(8\,Ba^{3}b-5\,Aa^{2}b^{2}\right)x^{4}-8\left(8\,Ba^{4}+Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{384\,a^{4}x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(
8*B*a^3*b - 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/
(a^4*x^8), 1/384*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/sqrt
(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(8*B*a^3*b -
 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^8)]
```

giac [A] time = 0.45, size = 194, normalized size = 1.24

$$\frac{\frac{3\left(8\,Bab^{4}-5\,Ab^{5}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}}+\frac{24\left(bx^{2}+a\right)^{\frac{7}{2}}Bab^{4}-88\left(bx^{2}+a\right)^{\frac{5}{2}}Ba^{2}b^{4}+40\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{3}b^{4}+24\,\sqrt{bx^{2}+a}\,Ba^{4}b^{4}-15\left(bx^{2}+a\right)^{\frac{7}{2}}Ab^{5}+55\left(bx^{2}+a\right)^{\frac{5}{2}}Aab^{5}-73\left(bx^{2}+a\right)^{\frac{3}{2}}Aa^{2}b^{5}-15\,\sqrt{bx^{2}+a}\,Aa^{3}b^{5}}{a^{3}b^{4}x^{8}}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(3*(8*B*a*b^4 - 5*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 - 88*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 40*(
b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 15*(b*x^2 + a)^
(7/2)*A*b^5 + 55*(b*x^2 + a)^(5/2)*A*a*b^5 - 73*(b*x^2 + a)^(3/2)*A*a^2*b^5
 - 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^3*b^4*x^8))/b
```

maple [A] time = 0.02, size = 239, normalized size = 1.53

$$\frac{5A\,b^{4}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{7}{2}}}-\frac{B\,b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{5}{2}}}-\frac{5\sqrt{b\,x^{2}+a}\,A\,b^{4}}{128a^{4}}+\frac{\sqrt{b\,x^{2}+a}\,B\,b^{3}}{16a^{3}}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{3}}{128a^{4}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,b^{2}}{16a^{3}x^{2}}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{2}}{64a^{3}x^{4}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bb}{8a^{2}x^{4}}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}Ab}{48a^{2}x^{6}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B}{6a\,x^{6}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A}{8a\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*A*(b*x^2+a)^(3/2)/a/x^8+5/48*A*b/a^2/x^6*(b*x^2+a)^(3/2)-5/64*A*b^2/a^
3/x^4*(b*x^2+a)^(3/2)+5/128*A*b^3/a^4/x^2*(b*x^2+a)^(3/2)+5/128*A*b^4/a^(7/
```

```
2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-5/128*A*b^4/a^4*(b*x^2+a)^(1/2)-1/
6*B/a/x^6*(b*x^2+a)^(3/2)+1/8*B*b/a^2/x^4*(b*x^2+a)^(3/2)-1/16*B*b^2/a^3/x^
2*(b*x^2+a)^(3/2)-1/16*B*b^3/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+
1/16*B*b^3/a^3*(b*x^2+a)^(1/2)
```

maxima [**A**] time = 1.17, size = 216, normalized size = 1.38

$$-\frac{Bb^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}}+\frac{5\,Ab^{4}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{7}{2}}}+\frac{\sqrt{bx^{2}+a}\,Bb^{3}}{16\,a^{3}}-\frac{5\,\sqrt{bx^{2}+a}\,Ab^{4}}{128\,a^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bb^{2}}{16\,a^{3}x^{2}}+\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{3}}{128\,a^{4}x^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bb}{8\,a^{2}x^{4}}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{2}}{64\,a^{3}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}B}{6\,ax^{6}}+\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ab}{48\,a^{2}x^{6}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}A}{8\,ax^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/128*A*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(7/2) + 1/16*sqrt(b*x^2 + a)*B*b^3/a^3 - 5/128*sqrt(b*
x^2 + a)*A*b^4/a^4 - 1/16*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^2) + 5/128*(b*x^2
+ a)^(3/2)*A*b^3/(a^4*x^2) + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^2*x^4) - 5/64*(b*
x^2 + a)^(3/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(3/2)*B/(a*x^6) + 5/48*(b*
x^2 + a)^(3/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(3/2)*A/(a*x^8)
```

mupad [**B**] time = 2.68, size = 173, normalized size = 1.11

$$\frac{55\,A\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^2\,x^8}-\frac{B\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{73\,A\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a\,x^8}-\frac{5\,A\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^3\,x^8}-\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a\,x^6}+\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^2\,x^6}-\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{7/2}}+\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^9,x)
```

```
[Out] (B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x^
2)^(1/2))/(16*x^6) - (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a
^(7/2)) - (5*A*(a + b*x^2)^(1/2))/(128*x^8) - (73*A*(a + b*x^2)^(3/2))/(384
*a*x^8) + (55*A*(a + b*x^2)^(5/2))/(384*a^2*x^8) - (5*A*(a + b*x^2)^(7/2))/
(128*a^3*x^8) - (B*(a + b*x^2)^(3/2))/(6*a*x^6) + (B*(a + b*x^2)^(5/2))/(16
*a^2*x^6)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] Timed out
```

3.501 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=117

$$\frac{8b^2\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{315a^4x^3}-\frac{4b\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{105a^3x^5}+\frac{\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{21a^2x^7}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8b^2\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{315a^4x^3}-\frac{4b\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{105a^3x^5}+\frac{\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{21a^2x^7}-\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(3/2))/(9*a*x^9) + ((2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(21*
a^2*x^7) - (4*b*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(105*a^3*x^5) + (8*b^2*(
2*A*b - 3*a*B)*(a + b*x^2)^(3/2))/(315*a^4*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{10}}\,dx &= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}-\frac{(6Ab-9aB)\int \frac{\sqrt{a+bx^2}}{x^8}\,dx}{9a}\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{21a^2x^7}+\frac{(4b(2Ab-3aB))\int \frac{\sqrt{a+bx^2}}{x^6}\,dx}{21a^2}\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{4b(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{105a^3x^5}-(8b^2\\ &= -\frac{A\left(a+bx^2\right)^{3/2}}{9ax^9}+\frac{(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{21a^2x^7}-\frac{4b(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{105a^3x^5}+8b^2(\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.69

$$\frac{\left(a+bx^2\right)^{3/2}\left(-5a^3\left(7A+9Bx^2\right)+6a^2bx^2\left(5A+6Bx^2\right)-24ab^2x^4\left(A+Bx^2\right)+16Ab^3x^6\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(3/2)*(16*A*b^3*x^6 - 24*a*b^2*x^4*(A + B*x^2) + 6*a^2*b*x^2*(
5*A + 6*B*x^2) - 5*a^3*(7*A + 9*B*x^2)))/(315*a^4*x^9)
```

IntegrateAlgebraic [A] time = 0.18, size = 110, normalized size = 0.94

$$\frac{\sqrt{a+bx^2}\left(-35a^4A-45a^4Bx^2-5a^3Abx^2-9a^3bBx^4+6a^2Ab^2x^4+12a^2b^2Bx^6-8aAb^3x^6-24ab^3Bx^8+16Ab^4x^8\right)}{315a^4x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4*A - 5*a^3*A*b*x^2 - 45*a^4*B*x^2 + 6*a^2*A*b^2*x^
4 - 9*a^3*b*B*x^4 - 8*a*A*b^3*x^6 + 12*a^2*b^2*B*x^6 + 16*A*b^4*x^8 - 24*a*
b^3*B*x^8))/(315*a^4*x^9)
```

fricas [A] time = 0.91, size = 105, normalized size = 0.90

$$-\frac{\left(8\left(3\,Bab^3-2\,Ab^4\right)x^8-4\left(3\,Ba^2b^2-2\,Aab^3\right)x^6+35\,Aa^4+3\left(3\,Ba^3b-2\,Aa^2b^2\right)x^4+5\left(9\,Ba^4+Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{315\,a^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(8*(3*B*a*b^3 - 2*A*b^4)*x^8 - 4*(3*B*a^2*b^2 - 2*A*a*b^3)*x^6 + 35*
A*a^4 + 3*(3*B*a^3*b - 2*A*a^2*b^2)*x^4 + 5*(9*B*a^4 + A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^4*x^9)
```

giac [B] time = 0.42, size = 344, normalized size = 2.94

$$\frac{16 \left(210 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{12} B b^{\frac{7}{2}}-315 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{10} B a b^{\frac{7}{2}}+630 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{10} A b^{\frac{9}{2}}+63 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{8} B a^{2} b^{\frac{7}{2}}+378 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{8} A a b^{\frac{9}{2}}-42 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{6} B a^{3} b^{\frac{7}{2}}+168 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{6} A a^{2} b^{\frac{9}{2}}+108 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} B a^{4} b^{\frac{7}{2}}-72 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{4} A a^{3} b^{\frac{9}{2}}-27 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} B a^{5} b^{\frac{7}{2}}+18 \left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2} A a^{4} b^{\frac{9}{2}}+3 B a^{6} b^{\frac{7}{2}}-2 A a^{5} b^{\frac{9}{2}}\right)}{315 \left(\left(\sqrt{b} x-\sqrt{b x^{2}+a}\right)^{2}-a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(7/2) - 315*(sqrt(b)*x - s
qrt(b*x^2 + a))^10*B*a*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(
9/2) + 63*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(7/2) + 378*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*A*a*b^(9/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b
^(7/2) + 168*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(9/2) + 108*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^4*b^(7/2) - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*
a^3*b^(9/2) - 27*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(7/2) + 18*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(9/2) + 3*B*a^6*b^(7/2) - 2*A*a^5*b^(9/2))
/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 83, normalized size = 0.71

$$-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} \left(-16 A \,b^{3} x^{6}+24 B a \,b^{2} x^{6}+24 x^{4} A a \,b^{2}-36 B \,a^{2} b \,x^{4}-30 A \,a^{2} b \,x^{2}+45 B \,a^{3} x^{2}+35 A \,a^{3}\right)}{315 a^{4} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(3/2)*(-16*A*b^3*x^6+24*B*a*b^2*x^6+24*A*a*b^2*x^4-36*B*a^
2*b*x^4-30*A*a^2*b*x^2+45*B*a^3*x^2+35*A*a^3)/x^9/a^4
```

maxima [A] time = 0.99, size = 138, normalized size = 1.18

$$-\frac{8 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} B b^{2}}{105 \, a^{3} x^{3}} + \frac{16 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} A b^{3}}{315 \, a^{4} x^{3}} + \frac{4 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} B b}{35 \, a^{2} x^{5}} - \frac{8 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} A b^{2}}{105 \, a^{3} x^{5}} - \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} B}{7 \, a x^{7}} + \frac{2 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} A b}{21 \, a^{2} x^{7}} - \frac{{\left(b x^{2} + a\right)}^{\frac{3}{2}} A}{9 \, a x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -8/105*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^3) + 16/315*(b*x^2 + a)^(3/2)*A*b^3/(
a^4*x^3) + 4/35*(b*x^2 + a)^(3/2)*B*b/(a^2*x^5) - 8/105*(b*x^2 + a)^(3/2)*A
*b^2/(a^3*x^5) - 1/7*(b*x^2 + a)^(3/2)*B/(a*x^7) + 2/21*(b*x^2 + a)^(3/2)*A
*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)*A/(a*x^9)
```

mupad [B] time = 1.46, size = 174, normalized size = 1.49

$$\frac{2\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5}-\frac{B\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{A\,b\,\sqrt{b\,x^2+a}}{63\,a\,x^7}-\frac{B\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{A\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3}+\frac{16\,A\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}+\frac{4\,B\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{8\,B\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^10,x)
```

```
[Out] (2*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (B*(a + b*x^2)^(1/2))/(7*x^7) -
 (A*b*(a + b*x^2)^(1/2))/(63*a*x^7) - (B*b*(a + b*x^2)^(1/2))/(35*a*x^5) -
(A*(a + b*x^2)^(1/2))/(9*x^9) - (8*A*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3) +
 (16*A*b^4*(a + b*x^2)^(1/2))/(315*a^4*x) + (4*B*b^2*(a + b*x^2)^(1/2))/(10
5*a^2*x^3) - (8*B*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

sympy [B] time = 5.24, size = 957, normalized size = 8.18

$$-\frac{35Aa^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{110Aa^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{114Aa^5b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{40Aa^4b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{5Aa^3b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{30Aa^2b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{40Aab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}+\frac{16Ab^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}-\frac{15Ba^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Ba^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Ba^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Ba^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Bab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Bb^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**10,x)
```

```
[Out] -35*A*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**6*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**5*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**4*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) + 5*A*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*
a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**
12*x**14) + 30*A*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x
**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) +
 40*A*a*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6
*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*A*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*B*a**5*b**(9/2)*sqrt(a/(b*
x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
- 33*B*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a
**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**3*b**(13/2)*x**4*sqrt(a/(b*x
**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
```

```
 3*B*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**
4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*B*a*b**(17/2)*x**8*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*
b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x*
*8 + 105*a**3*b**6*x**10)
```

3.502 $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b^3\sqrt{a+bx^2}\,(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}\,(7Ab-10aB)}{384a^3x^4}+\frac{b\sqrt{a+bx^2}\,(7Ab-1}{480a^2x^6}$$

Rubi [A] time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{b^3\sqrt{a+bx^2}\,(7Ab-10aB)}{256a^4x^2}-\frac{b^2\sqrt{a+bx^2}\,(7Ab-10aB)}{384a^3x^4}-\frac{b^4(7Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{b\sqrt{a+bx^2}\,(7Ab-10aB)}{480a^2x^6}+\frac{\sqrt{a+bx^2}\,(7Ab-10aB)}{80ax^8}-\frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] ((7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(80*a*x^8) + (b*(7*A*b - 10*a*B)*Sqrt[a
+ b*x^2])/(480*a^2*x^6) - (b^2*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(384*a^3*x
^4) + (b^3*(7*A*b - 10*a*B)*Sqrt[a + b*x^2])/(256*a^4*x^2) - (A*(a + b*x^2)
^(3/2))/(10*a*x^10) - (b^4*(7*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(256*a^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}\,\left(A+Bx^2\right)}{x^{11}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^6}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}} + \frac{\left(-\frac{7Ab}{2}+5aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^5}\,dx,x,x^2\right)}{10a}\\
&= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} - \frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}} - \frac{(b(7Ab-10aB))\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx}}\,dx\right.}{160a}\\
&= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{A\left(a+bx^2\right)^{3/2}}{10ax^{10}} + \frac{\left(b^2(7Ab-\right.}{}\\
&= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} -\\
&= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} -\\
&= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} -\\
&= \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8} + \frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4} -
\end{aligned}$$

Mathematica [**C**] time = 0.03, size = 62, normalized size = 0.33

$$-\frac{\left(a+bx^2\right)^{3/2}\left(3a^5A+b^4x^{10}(10aB-7Ab)\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};\frac{bx^2}{a}+1\right)\right)}{30a^6x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] -1/30*((a + b*x^2)^(3/2)*(3*a^5*A + b^4*(-7*A*b + 10*a*B)*x^10*Hypergeometr
ic2F1[3/2, 5, 5/2, 1 + (b*x^2)/a]))/(a^6*x^10)
```

IntegrateAlgebraic [**A**] time = 0.26, size = 152, normalized size = 0.80

$$\frac{\left(10ab^4B-7Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}+\frac{\sqrt{a+bx^2}\left(-384a^4A-480a^4Bx^2-48a^3Abx^2-80a^3bBx^4+56a^2Ab^2x^4+100a^2b^2Bx^6-70aAb^3x^6-150ab^3Bx^8+105Ab^4x^8\right)}{3840a^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-384*a^4*A - 48*a^3*A*b*x^2 - 480*a^4*B*x^2 + 56*a^2*A*b^
2*x^4 - 80*a^3*b*B*x^4 - 70*a*A*b^3*x^6 + 100*a^2*b^2*B*x^6 + 105*A*b^4*x^8
 - 150*a*b^3*B*x^8))/(3840*a^4*x^10) + ((-7*A*b^5 + 10*a*b^4*B)*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]])/(256*a^(9/2))
```

fricas [A] time = 1.09, size = 317, normalized size = 1.68

$$\left[-\frac{15 \, {\left(10 \, B a b^{4} - 7 \, A b^{5}\right)} \sqrt{a} x^{10} \log\left(-\frac{b x^{2} - 2 \, \sqrt{b x^{2} + a} \sqrt{a} + 2 \, a}{x^{2}}\right) + 2 \, {\left(15 \, {\left(10 \, B a^{2} b^{3} - 7 \, A a b^{4}\right)} x^{8} - 10 \, {\left(10 \, B a^{3} b^{2} - 7 \, A a^{2} b^{3}\right)} x^{6} + 384 \, A a^{5} + 8 \, {\left(10 \, B a^{4} b - 7 \, A a^{3} b^{2}\right)} x^{4} + 48 \, {\left(10 \, B a^{5} + A a^{4} b\right)} x^{2}\right)} \sqrt{b x^{2} + a}}{7680 \, a^{5} x^{10}}, -\frac{15 \, {\left(10 \, B a b^{4} - 7 \, A b^{5}\right)} \sqrt{-a} x^{10} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2} + a}}\right) + {\left(15 \, {\left(10 \, B a^{2} b^{3} - 7 \, A a b^{4}\right)} x^{8} - 10 \, {\left(10 \, B a^{3} b^{2} - 7 \, A a^{2} b^{3}\right)} x^{6} + 384 \, A a^{5} + 8 \, {\left(10 \, B a^{4} b - 7 \, A a^{3} b^{2}\right)} x^{4} + 48 \, {\left(10 \, B a^{5} + A a^{4} b\right)} x^{2}\right)} \sqrt{b x^{2} + a}}{3840 \, a^{5} x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 7*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2
 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B
*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4
+ 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^5*x^10), -1/3840*(15*(10
*B*a*b^4 - 7*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(1
0*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*
a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(
b*x^2 + a))/(a^5*x^10)]
```

giac [A] time = 0.34, size = 230, normalized size = 1.22

$$-\frac{\frac{15 \, {\left(10 \, B a b^{5} - 7 \, A b^{6}\right)} \arctan\left(\frac{\sqrt{b x^{2} + a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^{4}} + \frac{150 \, {\left(b x^{2} + a\right)}^{\frac{9}{2}} B a b^{5} - 700 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} B a^{2} b^{5} + 1280 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} B a^{3} b^{5} - 580 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} B a^{4} b^{5} - 150 \, \sqrt{b x^{2} + a} B a^{5} b^{5} - 105 \, {\left(b x^{2} + a\right)}^{\frac{9}{2}} A b^{6} + 490 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} A a b^{6} - 896 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} A a^{2} b^{6} + 790 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} A a^{3} b^{6} + 105 \, \sqrt{b x^{2} + a} A a^{4} b^{6}}{a^{4} b^{5} x^{10}}}{3840 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 7*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^4) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 - 700*(b*x^2 + a)^(7/2)*B*a^2*b^5
+ 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 - 580*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 150*
sqrt(b*x^2 + a)*B*a^5*b^5 - 105*(b*x^2 + a)^(9/2)*A*b^6 + 490*(b*x^2 + a)^(
7/2)*A*a*b^6 - 896*(b*x^2 + a)^(5/2)*A*a^2*b^6 + 790*(b*x^2 + a)^(3/2)*A*a^
3*b^6 + 105*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^4*b^5*x^10))/b
```

maple [A] time = 0.03, size = 281, normalized size = 1.49

$$-\frac{7 A \,b^{5} \ln \left(\frac{2 a +2 \sqrt{b \,x^{2}+a}\, \sqrt{a}}{x}\right)}{256 a^{\frac{9}{2}}}+\frac{5 B \,b^{4} \ln \left(\frac{2 a +2 \sqrt{b \,x^{2}+a}\, \sqrt{a}}{x}\right)}{128 a^{\frac{7}{2}}}+\frac{7 \sqrt{b \,x^{2}+a}\, A \,b^{5}}{256 a^{5}}-\frac{5 \sqrt{b \,x^{2}+a}\, B \,b^{4}}{128 a^{4}}-\frac{7 \left(b \,x^{2}+a \right)^{\frac{3}{2}} A \,b^{4}}{256 a^{5} x^{2}}+\frac{5 \left(b \,x^{2}+a \right)^{\frac{3}{2}} B \,b^{3}}{128 a^{4} x^{2}}+\frac{7 \left(b \,x^{2}+a \right)^{\frac{3}{2}} A \,b^{3}}{128 a^{4} x^{4}}-\frac{5 \left(b \,x^{2}+a \right)^{\frac{3}{2}} B \,b^{2}}{64 a^{3} x^{4}}-\frac{7 \left(b \,x^{2}+a \right)^{\frac{3}{2}} A \,b^{2}}{96 a^{3} x^{6}}+\frac{5 \left(b \,x^{2}+a \right)^{\frac{3}{2}} B b}{48 a^{2} x^{6}}+\frac{7 \left(b \,x^{2}+a \right)^{\frac{3}{2}} A b}{80 a^{2} x^{8}}-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} B}{8 a \,x^{8}}-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} A}{10 a \,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(3/2)/a/x^10+7/80*A*b/a^2/x^8*(b*x^2+a)^(3/2)-7/96*A*b^2/
a^3/x^6*(b*x^2+a)^(3/2)+7/128*A*b^3/a^4/x^4*(b*x^2+a)^(3/2)-7/256*A*b^4/a^5
/x^2*(b*x^2+a)^(3/2)-7/256*A*b^5/a^(9/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))
/x)+7/256*A*b^5/a^5*(b*x^2+a)^(1/2)-1/8*B/a/x^8*(b*x^2+a)^(3/2)+5/48*B*b/a^
2/x^6*(b*x^2+a)^(3/2)-5/64*B*b^2/a^3/x^4*(b*x^2+a)^(3/2)+5/128*B*b^3/a^4/x^
2*(b*x^2+a)^(3/2)+5/128*B*b^4/a^(7/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
-5/128*B*b^4/a^4*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.17, size = 258, normalized size = 1.37

$$\frac{5\,Bb^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{7}{2}}} - \frac{7\,Ab^5\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{9}{2}}} - \frac{5\,\sqrt{bx^2+a}\,Bb^4}{128\,a^4} + \frac{7\,\sqrt{bx^2+a}\,Ab^5}{256\,a^5} + \frac{5\,\left(bx^2+a\right)^{\frac{3}{2}}Bb^3}{128\,a^4x^2} - \frac{7\,\left(bx^2+a\right)^{\frac{3}{2}}Ab^4}{256\,a^5x^2} - \frac{5\,\left(bx^2+a\right)^{\frac{3}{2}}Bb^2}{64\,a^3x^4} + \frac{7\,\left(bx^2+a\right)^{\frac{3}{2}}Ab^3}{128\,a^4x^4} + \frac{5\,\left(bx^2+a\right)^{\frac{3}{2}}Bb}{48\,a^2x^6} - \frac{7\,\left(bx^2+a\right)^{\frac{3}{2}}Ab^2}{96\,a^3x^6} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{8\,ax^8} + \frac{7\,\left(bx^2+a\right)^{\frac{3}{2}}Ab}{80\,a^2x^8} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 5/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 7/256*A*b^5*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(9/2) - 5/128*sqrt(b*x^2 + a)*B*b^4/a^4 + 7/256*sqrt(b
*x^2 + a)*A*b^5/a^5 + 5/128*(b*x^2 + a)^(3/2)*B*b^3/(a^4*x^2) - 7/256*(b*x^
2 + a)^(3/2)*A*b^4/(a^5*x^2) - 5/64*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^4) + 7/1
28*(b*x^2 + a)^(3/2)*A*b^3/(a^4*x^4) + 5/48*(b*x^2 + a)^(3/2)*B*b/(a^2*x^6)
 - 7/96*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(3/2)*B/(a*x^8)
 + 7/80*(b*x^2 + a)^(3/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(3/2)*A/(a*x^10)
```

mupad [B] time = 3.50, size = 209, normalized size = 1.11

$$\frac{7\,A\,{\left(b\,x^2+a\right)}^{5/2}}{30\,a^2\,x^{10}} - \frac{5\,B\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{79\,A\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a\,x^{10}} - \frac{7\,A\,\sqrt{b\,x^2+a}}{256\,x^{10}} - \frac{49\,A\,{\left(b\,x^2+a\right)}^{7/2}}{384\,a^3\,x^{10}} + \frac{7\,A\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^4\,x^{10}} - \frac{73\,B\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a\,x^8} + \frac{55\,B\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^2\,x^8} - \frac{5\,B\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^3\,x^8} + \frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,7{}\mathrm{i}}{256\,a^{9/2}} - \frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^11,x)
```

```
[Out] (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*7i)/(256*a^(9/2)) - (5*B*(a + b
*x^2)^(1/2))/(128*x^8) - (7*A*(a + b*x^2)^(1/2))/(256*x^10) - (B*b^4*atan((
(a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a^(7/2)) - (79*A*(a + b*x^2)^(3/2))
/(384*a*x^10) + (7*A*(a + b*x^2)^(5/2))/(30*a^2*x^10) - (49*A*(a + b*x^2)^(
7/2))/(384*a^3*x^10) + (7*A*(a + b*x^2)^(9/2))/(256*a^4*x^10) - (73*B*(a +
b*x^2)^(3/2))/(384*a*x^8) + (55*B*(a + b*x^2)^(5/2))/(384*a^2*x^8) - (5*B*(
a + b*x^2)^(7/2))/(128*a^3*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Timed out
```

3.503 $\int x^5 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=103

$$\frac{a^2\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2}(Ab-3aB)}{9b^4}-\frac{a\left(a+bx^2\right)^{7/2}(2Ab-3aB)}{7b^4}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^4}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^4}+\frac{\left(a+bx^2\right)^{9/2}(Ab-3aB)}{9b^4}-\frac{a\left(a+bx^2\right)^{7/2}(2Ab-3aB)}{7b^4}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(7/2))/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9*b^4) + (B*(a + b*x^
2)^(11/2))/(11*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}+\frac{(Ab-}{}\right.\right.\\ &= \frac{a^2(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)}{9b^4}\end{aligned}$$

Mathematica [**A**] time = 0.06, size = 78, normalized size = 0.76

$$\frac{\left(a+bx^2\right)^{5/2}\left(-48a^3B+8a^2b\left(11A+15Bx^2\right)-10ab^2x^2\left(22A+21Bx^2\right)+35b^3x^4\left(11A+9Bx^2\right)\right)}{3465b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-48*a^3*B + 35*b^3*x^4*(11*A + 9*B*x^2) + 8*a^2*b*(11*A
 + 15*B*x^2) - 10*a*b^2*x^2*(22*A + 21*B*x^2)))/(3465*b^4)
```

IntegrateAlgebraic [**A**] time = 0.06, size = 80, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{5/2}\left(-48a^3B+88a^2Ab+120a^2bBx^2-220aAb^2x^2-210ab^2Bx^4+385Ab^3x^4+315b^3Bx^6\right)}{3465b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^2 + 120*a^2*b*B*x
^2 + 385*A*b^3*x^4 - 210*a*b^2*B*x^4 + 315*b^3*B*x^6))/(3465*b^4)
```

fricas [**A**] time = 0.85, size = 124, normalized size = 1.20

$$\frac{\left(315\,Bb^5x^{10}+35\left(12\,Bab^4+11\,Ab^5\right)x^8+5\left(3\,Ba^2b^3+110\,Aab^4\right)x^6-48\,Ba^5+88\,Aa^4b-3\left(6\,Ba^3b^2-11\,Aa^2b^3\right)x^4+4\left(6\,Ba^4b-11\,Aa^3b^2\right)x^2\right)\sqrt{bx^2+a}}{3465\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*B*b^5*x^10 + 35*(12*B*a*b^4 + 11*A*b^5)*x^8 + 5*(3*B*a^2*b^3 +
110*A*a*b^4)*x^6 - 48*B*a^5 + 88*A*a^4*b - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x
^4 + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 0.43, size = 104, normalized size = 1.01

$$\frac{315 \left(bx^2+a\right)^{\frac{11}{2}} B - 1155 \left(bx^2+a\right)^{\frac{9}{2}} Ba + 1485 \left(bx^2+a\right)^{\frac{7}{2}} Ba^2 - 693 \left(bx^2+a\right)^{\frac{5}{2}} Ba^3 + 385 \left(bx^2+a\right)^{\frac{9}{2}} Ab - 990 \left(bx^2+a\right)^{\frac{7}{2}} Aab + 693 \left(bx^2+a\right)^{\frac{5}{2}} Aa^2 b}{3465\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(b*x^2 + a)^(11/2)*B - 1155*(b*x^2 + a)^(9/2)*B*a + 1485*(b*x^2
 + a)^(7/2)*B*a^2 - 693*(b*x^2 + a)^(5/2)*B*a^3 + 385*(b*x^2 + a)^(9/2)*A*b
 - 990*(b*x^2 + a)^(7/2)*A*a*b + 693*(b*x^2 + a)^(5/2)*A*a^2*b)/b^4
```

maple [A] time = 0.01, size = 77, normalized size = 0.75

$$\frac{\left(b\,x^2+a\right)^{\frac{5}{2}} \left(315 B\, x^6 b^3 + 385 A\, b^3 x^4 - 210 Ba\, b^2 x^4 - 220 Aa\, b^2 x^2 + 120 B\, a^2 b\, x^2 + 88 A\, a^2 b - 48 B\, a^3\right)}{3465 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/3465*(b*x^2+a)^(5/2)*(315*B*b^3*x^6+385*A*b^3*x^4-210*B*a*b^2*x^4-220*A*a
*b^2*x^2+120*B*a^2*b*x^2+88*A*a^2*b-48*B*a^3)/b^4
```

maxima [A] time = 1.05, size = 132, normalized size = 1.28

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}} Bx^6}{11\, b} - \frac{2 \left(bx^2+a\right)^{\frac{5}{2}} Bax^4}{33\, b^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}} Ax^4}{9\, b} + \frac{8 \left(bx^2+a\right)^{\frac{5}{2}} Ba^2 x^2}{231\, b^3} - \frac{4 \left(bx^2+a\right)^{\frac{5}{2}} Aax^2}{63\, b^2} - \frac{16 \left(bx^2+a\right)^{\frac{5}{2}} Ba^3}{1155\, b^4} + \frac{8 \left(bx^2+a\right)^{\frac{5}{2}} Aa^2}{315\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(5/2)*B*x^6/b - 2/33*(b*x^2 + a)^(5/2)*B*a*x^4/b^2 + 1/9*(
b*x^2 + a)^(5/2)*A*x^4/b + 8/231*(b*x^2 + a)^(5/2)*B*a^2*x^2/b^3 - 4/63*(b*
x^2 + a)^(5/2)*A*a*x^2/b^2 - 16/1155*(b*x^2 + a)^(5/2)*B*a^3/b^4 + 8/315*(b
*x^2 + a)^(5/2)*A*a^2/b^3
```

mupad [B] time = 0.74, size = 117, normalized size = 1.14

$$\sqrt{b\,x^2+a}\,\left(\frac{x^8\,\left(385\,A\,b^5+420\,B\,a\,b^4\right)}{3465\,b^4}-\frac{48\,B\,a^5-88\,A\,a^4\,b}{3465\,b^4}+\frac{B\,b\,x^{10}}{11}+\frac{a^2\,x^4\,\left(11\,A\,b-6\,B\,a\right)}{1155\,b^2}-\frac{4\,a^3\,x^2\,\left(11\,A\,b-6\,B\,a\right)}{3465\,b^3}+\frac{a\,x^6\,\left(110\,A\,b+3\,B\,a\right)}{693\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^8*(385*A*b^5 + 420*B*a*b^4))/(3465*b^4) - (48*B*a^5 -
 88*A*a^4*b)/(3465*b^4) + (B*b*x^10)/11 + (a^2*x^4*(11*A*b - 6*B*a))/(1155*
b^2) - (4*a^3*x^2*(11*A*b - 6*B*a))/(3465*b^3) + (a*x^6*(110*A*b + 3*B*a))/
(693*b))
```

sympy [A] time = 8.36, size = 260, normalized size = 2.52

$$\begin{cases} \frac{8Aa^4\sqrt{a+bx^2}}{315b^3} - \frac{4Aa^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{Aa^2x^4\sqrt{a+bx^2}}{105b} + \frac{10Aax^6\sqrt{a+bx^2}}{63} + \frac{Abx^8\sqrt{a+bx^2}}{9} - \frac{16Ba^5\sqrt{a+bx^2}}{1155b^4} + \frac{8Ba^4x^2\sqrt{a+bx^2}}{1155b^3} - \frac{2Ba^3x^4\sqrt{a+bx^2}}{385b^2} + \frac{Ba^2x^6\sqrt{a+bx^2}}{231b} + \frac{4Bax^8\sqrt{a+bx^2}}{33} + \frac{Bbx^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*A*a**3*x**2*sqrt(a + b*
x**2)/(315*b**2) + A*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*A*a*x**6*sqrt(
a + b*x**2)/63 + A*b*x**8*sqrt(a + b*x**2)/9 - 16*B*a**5*sqrt(a + b*x**2)/(
1155*b**4) + 8*B*a**4*x**2*sqrt(a + b*x**2)/(1155*b**3) - 2*B*a**3*x**4*sqr
t(a + b*x**2)/(385*b**2) + B*a**2*x**6*sqrt(a + b*x**2)/(231*b) + 4*B*a*x**
8*sqrt(a + b*x**2)/33 + B*b*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(3/2)
*(A*x**6/6 + B*x**8/8), True))
```

3.504 $\int x^4 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=188

$$\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}-\frac{3a^3x\sqrt{a+bx^2}\,(2Ab-aB)}{256b^3}+\frac{a^2x^3\sqrt{a+bx^2}\,(2Ab-aB)}{128b^2}+\frac{x^5\left(a+bx^2\right)^{3/2}(2Ab-}{16b}$$

Rubi [**A**] time = 0.10, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$\frac{a^2x^3\sqrt{a+bx^2}\,(2Ab-aB)}{128b^2}-\frac{3a^3x\sqrt{a+bx^2}\,(2Ab-aB)}{256b^3}+\frac{3a^4(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{x^5\left(a+bx^2\right)^{3/2}(2Ab-aB)}{16b}+\frac{ax^5\sqrt{a+bx^2}\,(2Ab-aB)}{32b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (-3*a^3*(2*A*b - a*B)*x*Sqrt[a + b*x^2])/(256*b^3) + (a^2*(2*A*b - a*B)*x^3
*Sqrt[a + b*x^2])/(128*b^2) + (a*(2*A*b - a*B)*x^5*Sqrt[a + b*x^2])/(32*b)
+ ((2*A*b - a*B)*x^5*(a + b*x^2)^(3/2))/(16*b) + (B*x^5*(a + b*x^2)^(5/2))/
(10*b) + (3*a^4*(2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^
(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx &= \frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b} - \frac{(-10Ab+5aB) \int x^4 \left(a+bx^2\right)^{3/2} \, dx}{10b} \\
&= \frac{(2Ab-aB)x^5 \left(a+bx^2\right)^{3/2}}{16b} + \frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b} + \frac{(3a(2Ab-aB)) \int x^4 \sqrt{a+bx^2}}{16b} \\
&= \frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b} + \frac{(2Ab-aB)x^5 \left(a+bx^2\right)^{3/2}}{16b} + \frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b} + \frac{(a}{} \\
&= \frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2} + \frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b} + \frac{(2Ab-aB)x^5 \left(a+bx\right.}{16b} \\
&= -\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3} + \frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2} + \frac{a(2Ab-aB)x^5\sqrt{a}}{32b} \\
&= -\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3} + \frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2} + \frac{a(2Ab-aB)x^5\sqrt{a}}{32b} \\
&= -\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3} + \frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2} + \frac{a(2Ab-aB)x^5\sqrt{a}}{32b}
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 150, normalized size = 0.80

$$
\frac{\sqrt{a+bx^2} \left(\sqrt{b}\, x \left(15a^4B - 10a^3b\left(3A+Bx^2\right) + 4a^2b^2x^2\left(5A+2Bx^2\right) + 16ab^3x^4\left(15A+11Bx^2\right) + 32b^4x^6\left(5A+4Bx^2\right)\right) - \frac{15a^{7/2}(aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{1280b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^4*B - 10*a^3*b*(3*A + B*x^2) + 4*a^2*b^2*
x^2*(5*A + 2*B*x^2) + 32*b^4*x^6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(15*A + 11*
B*x^2)) - (15*a^(7/2)*(-2*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 +
 (b*x^2)/a]))/(1280*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 150, normalized size = 0.80

$$\frac{3\left(a^5B-2a^4Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{256b^{7/2}}+\frac{\sqrt{a+bx^2}\,\left(15a^4Bx-30a^3Abx-10a^3bBx^3+20a^2Ab^2x^3+8a^2b^2Bx^5+240aAb^3x^5+176ab^3Bx^7+160Ab^4x^7+128b^4Bx^9\right)}{1280b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-30*a^3*A*b*x + 15*a^4*B*x + 20*a^2*A*b^2*x^3 - 10*a^3*b*
B*x^3 + 240*a*A*b^3*x^5 + 8*a^2*b^2*B*x^5 + 160*A*b^4*x^7 + 176*a*b^3*B*x^7
 + 128*b^4*B*x^9))/(1280*b^3) + (3*(-2*a^4*A*b + a^5*B)*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/(256*b^(7/2))
```

fricas [A] time = 1.03, size = 299, normalized size = 1.59

$$\left[-\frac{15\left(Ba^5-2\,Aa^4b\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(128\,Bb^5x^9+16\left(11\,Bab^4+10\,Ab^5\right)x^7+8\left(Ba^2b^3+30\,Aab^4\right)x^5-10\left(Ba^3b^2-2\,Aa^2b^3\right)x^3+15\left(Ba^4b-2\,Aa^3b^2\right)x\right)\sqrt{bx^2+a}}{2560\,b^4},\ \frac{15\left(Ba^5-2\,Aa^4b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(128\,Bb^5x^9+16\left(11\,Bab^4+10\,Ab^5\right)x^7+8\left(Ba^2b^3+30\,Aab^4\right)x^5-10\left(Ba^3b^2-2\,Aa^2b^3\right)x^3+15\left(Ba^4b-2\,Aa^3b^2\right)x\right)\sqrt{bx^2+a}}{1280\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(B*a^5 - 2*A*a^4*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(128*B*b^5*x^9 + 16*(11*B*a*b^4 + 10*A*b^5)*x^7 + 8*(B*a^
2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3)*x^3 + 15*(B*a^4*b -
2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/1280*(15*(B*a^5 - 2*A*a^4*b)*sqrt(-
b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (128*B*b^5*x^9 + 16*(11*B*a*b^4 + 1
0*A*b^5)*x^7 + 8*(B*a^2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3
)*x^3 + 15*(B*a^4*b - 2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.42, size = 159, normalized size = 0.85

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,Bbx^2+\frac{11\,Bab^8+10\,Ab^9}{b^8}\right)x^2+\frac{Ba^2b^7+30\,Aab^8}{b^8}\right)x^2-\frac{5\left(Ba^3b^6-2\,Aa^2b^7\right)}{b^8}\right)x^2+\frac{15\left(Ba^4b^5-2\,Aa^3b^6\right)}{b^8}\right)\sqrt{bx^2+a}\,x+\frac{3\left(Ba^5-2\,Aa^4b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*B*b*x^2 + (11*B*a*b^8 + 10*A*b^9)/b^8)*x^2 + (B*a^2*b^7
+ 30*A*a*b^8)/b^8)*x^2 - 5*(B*a^3*b^6 - 2*A*a^2*b^7)/b^8)*x^2 + 15*(B*a^4*b
^5 - 2*A*a^3*b^6)/b^8)*sqrt(b*x^2 + a)*x + 3/256*(B*a^5 - 2*A*a^4*b)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 219, normalized size = 1.16

$$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} B\,x^{5}}{10b}+\frac{3A\,a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{5}{2}}}-\frac{3B\,a^{5}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{256b^{\frac{7}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\,A\,a^{3}x}{128b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} A\,x^{3}}{8b}-\frac{3\sqrt{b\,x^{2}+a}\,B\,a^{4}x}{256b^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} Ba\,x^{3}}{16b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} A\,a^{2}x}{64b^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} B\,a^{3}x}{128b^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} Aax}{16b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} B\,a^{2}x}{32b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/10*B*x^5*(b*x^2+a)^(5/2)/b-1/16*B*a/b^2*x^3*(b*x^2+a)^(5/2)+1/32*B*a^2/b^
3*x*(b*x^2+a)^(5/2)-1/128*B*a^3/b^3*x*(b*x^2+a)^(3/2)-3/256*B*a^4/b^3*x*(b*
x^2+a)^(1/2)-3/256*B*a^5/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/8*A*x^3*(b
*x^2+a)^(5/2)/b-1/16*A*a/b^2*x*(b*x^2+a)^(5/2)+1/64*A*a^2/b^2*x*(b*x^2+a)^(
3/2)+3/128*A*a^3/b^2*x*(b*x^2+a)^(1/2)+3/128*A*a^4/b^(5/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))
```

maxima [A] time = 1.01, size = 204, normalized size = 1.09

$$\frac{{\left(bx^{2}+a\right)}^{\frac{5}{2}} Bx^{5}}{10\,b}-\frac{{\left(bx^{2}+a\right)}^{\frac{5}{2}} Bax^{3}}{16\,b^{2}}+\frac{{\left(bx^{2}+a\right)}^{\frac{5}{2}} Ax^{3}}{8\,b}+\frac{{\left(bx^{2}+a\right)}^{\frac{5}{2}} Ba^{2}x}{32\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{\frac{3}{2}} Ba^{3}x}{128\,b^{3}}-\frac{3\,\sqrt{bx^{2}+a}Ba^{4}x}{256\,b^{3}}-\frac{{\left(bx^{2}+a\right)}^{\frac{5}{2}} Aax}{16\,b^{2}}+\frac{{\left(bx^{2}+a\right)}^{\frac{3}{2}} Aa^{2}x}{64\,b^{2}}+\frac{3\,\sqrt{bx^{2}+a}Aa^{3}x}{128\,b^{2}}-\frac{3\,Ba^{5}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}+\frac{3\,Aa^{4}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(5/2)*B*x^5/b - 1/16*(b*x^2 + a)^(5/2)*B*a*x^3/b^2 + 1/8*(
b*x^2 + a)^(5/2)*A*x^3/b + 1/32*(b*x^2 + a)^(5/2)*B*a^2*x/b^3 - 1/128*(b*x^
2 + a)^(3/2)*B*a^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^3 - 1/16*(b*x^2
+ a)^(5/2)*A*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*A*a^2*x/b^2 + 3/128*sqrt(b*x^
2 + a)*A*a^3*x/b^2 - 3/256*B*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/128*A*a
^4*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^4*(A + B*x^2)*(a + b*x^2)^(3/2), x)
```

sympy [B] time = 51.18, size = 345, normalized size = 1.84

$$-\frac{3Aa^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Aa^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5A\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Ab^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^{\frac{9}{2}}x}{256b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{7}{2}}x^3}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^5}{640b\sqrt{1+\frac{bx^2}{a}}}+\frac{23Ba^{\frac{3}{2}}x^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{19B\sqrt{a}\,bx^9}{80\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{7}{2}}}+\frac{Bb^2x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] -3*A*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - A*a**(5/2)*x**3/(128*b*sqrt
(1 + b*x**2/a)) + 13*A*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*A*sqrt(a)*
b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**
(5/2)) + A*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + 3*B*a**(9/2)*x/(256*b
**3*sqrt(1 + b*x**2/a)) + B*a**(7/2)*x**3/(256*b**2*sqrt(1 + b*x**2/a)) - B
*a**(5/2)*x**5/(640*b*sqrt(1 + b*x**2/a)) + 23*B*a**(3/2)*x**7/(160*sqrt(1
+ b*x**2/a)) + 19*B*sqrt(a)*b*x**9/(80*sqrt(1 + b*x**2/a)) - 3*B*a**5*asinh
(sqrt(b)*x/sqrt(a))/(256*b**(7/2)) + B*b**2*x**11/(10*sqrt(a)*sqrt(1 + b*x*
*2/a))
```

3.505 $\int x^3 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{a\left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^3}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^3}$$

Rubi [**A**] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{7/2} (Ab-2aB)}{7b^3}-\frac{a\left(a+bx^2\right)^{5/2} (Ab-aB)}{5b^3}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(7/
2))/(7*b^3) + (B*(a + b*x^2)^(9/2))/(9*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{3/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{5/2}}{b^2}+\frac{B(a+bx)^{7/}}{b^2}\right.\right.\\ &= -\frac{a(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{5/2}\left(8a^2B-2ab\left(9A+10Bx^2\right)+5b^2x^2\left(9A+7Bx^2\right)\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(8*a^2*B + 5*b^2*x^2*(9*A + 7*B*x^2) - 2*a*b*(9*A + 10*B
*x^2)))/(315*b^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.77

$$\frac{\left(a+bx^2\right)^{5/2}\left(8a^2B-18aAb-20abBx^2+45Ab^2x^2+35b^2Bx^4\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^2 - 20*a*b*B*x^2 + 35*
b^2*B*x^4))/(315*b^3)
```

fricas [A] time = 0.84, size = 99, normalized size = 1.36

$$\frac{\left(35\,Bb^4x^8+5\left(10\,Bab^3+9\,Ab^4\right)x^6+8\,Ba^4-18\,Aa^3b+3\left(Ba^2b^2+24\,Aab^3\right)x^4-\left(4\,Ba^3b-9\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/315*(35*B*b^4*x^8 + 5*(10*B*a*b^3 + 9*A*b^4)*x^6 + 8*B*a^4 - 18*A*a^3*b +
 3*(B*a^2*b^2 + 24*A*a*b^3)*x^4 - (4*B*a^3*b - 9*A*a^2*b^2)*x^2)*sqrt(b*x^2
 + a)/b^3
```

giac [A] time = 0.28, size = 73, normalized size = 1.00

$$\frac{35\left(bx^2+a\right)^{\frac{9}{2}}B-90\left(bx^2+a\right)^{\frac{7}{2}}Ba+63\left(bx^2+a\right)^{\frac{5}{2}}Ba^2+45\left(bx^2+a\right)^{\frac{7}{2}}Ab-63\left(bx^2+a\right)^{\frac{5}{2}}Aab}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/315*(35*(b*x^2 + a)^(9/2)*B - 90*(b*x^2 + a)^(7/2)*B*a + 63*(b*x^2 + a)^(
5/2)*B*a^2 + 45*(b*x^2 + a)^(7/2)*A*b - 63*(b*x^2 + a)^(5/2)*A*a*b)/b^3
```

maple [A] time = 0.01, size = 53, normalized size = 0.73

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(-35B\,b^2x^4-45A\,b^2x^2+20Bab\,x^2+18abA-8a^2B\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(-35*B*b^2*x^4-45*A*b^2*x^2+20*B*a*b*x^2+18*A*a*b-8*
B*a^2)/b^3
```

maxima [A] time = 1.00, size = 90, normalized size = 1.23

$$\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx^4}{9\,b}-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}Bax^2}{63\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ax^2}{7\,b}+\frac{8\left(bx^2+a\right)^{\frac{5}{2}}Ba^2}{315\,b^3}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}Aa}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(5/2)*B*x^4/b - 4/63*(b*x^2 + a)^(5/2)*B*a*x^2/b^2 + 1/7*(b
*x^2 + a)^(5/2)*A*x^2/b + 8/315*(b*x^2 + a)^(5/2)*B*a^2/b^3 - 2/35*(b*x^2 +
 a)^(5/2)*A*a/b^2
```

mupad [B] time = 0.63, size = 96, normalized size = 1.32

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,B\,a^4-18\,A\,a^3\,b}{315\,b^3}+\frac{x^6\,\left(45\,A\,b^4+50\,B\,a\,b^3\right)}{315\,b^3}+\frac{B\,b\,x^8}{9}+\frac{a^2\,x^2\,\left(9\,A\,b-4\,B\,a\right)}{315\,b^2}+\frac{a\,x^4\,\left(24\,A\,b+B\,a\right)}{105\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*B*a^4 - 18*A*a^3*b)/(315*b^3) + (x^6*(45*A*b^4 + 50*B
*a*b^3))/(315*b^3) + (B*b*x^8)/9 + (a^2*x^2*(9*A*b - 4*B*a))/(315*b^2) + (a
*x^4*(24*A*b + B*a))/(105*b))
```

sympy [A] time = 2.72, size = 209, normalized size = 2.86

$$\begin{cases} - \frac{2 A a^{3} \sqrt{a + b x^{2}}}{35 b^{2}} + \frac{A a^{2} x^{2} \sqrt{a + b x^{2}}}{35 b} + \frac{8 A a x^{4} \sqrt{a + b x^{2}}}{35} + \frac{A b x^{6} \sqrt{a + b x^{2}}}{7} + \frac{8 B a^{4} \sqrt{a + b x^{2}}}{315 b^{3}} - \frac{4 B a^{3} x^{2} \sqrt{a + b x^{2}}}{315 b^{2}} + \frac{B a^{2} x^{4} \sqrt{a + b x^{2}}}{105 b} + \frac{10 B a x^{6} \sqrt{a + b x^{2}}}{63} + \frac{B b x^{8} \sqrt{a + b x^{2}}}{9} & \text{for}\: b \neq 0 \\ a^{\frac{3}{2}} \left(\frac{A x^{4}}{4} + \frac{B x^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**3*sqrt(a + b*x**2)/(35*b**2) + A*a**2*x**2*sqrt(a + b*x*
*2)/(35*b) + 8*A*a*x**4*sqrt(a + b*x**2)/35 + A*b*x**6*sqrt(a + b*x**2)/7 +
 8*B*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*B*a**3*x**2*sqrt(a + b*x**2)/(315
*b**2) + B*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*B*a*x**6*sqrt(a + b*x**2
)/63 + B*b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**6
/6), True))
```

3.506 $\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=155

$$-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{a^2x\sqrt{a+bx^2}\,(8Ab-3aB)}{128b^2}+\frac{ax^3\sqrt{a+bx^2}\,(8Ab-3aB)}{64b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8Ab-}{48b}$$

Rubi [A] time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$\frac{a^2x\sqrt{a+bx^2}\,(8Ab-3aB)}{128b^2}-\frac{a^3(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{ax^3\sqrt{a+bx^2}\,(8Ab-3aB)}{64b}+\frac{x^3\left(a+bx^2\right)^{3/2}(8Ab-3aB)}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(128*b^2) + (a*(8*A*b - 3*a*B)*x^3*
Sqrt[a + b*x^2])/(64*b) + ((8*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(48*b) +
(B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a^3*(8*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx &= \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{(-8Ab+3aB)\int x^2\left(a+bx^2\right)^{3/2}dx}{8b}\\
&= \frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{(a(8Ab-3aB))\int x^2\sqrt{a+b}}{16b}\\
&= \frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}\\
&= \frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a\right.}{48b}\\
&= \frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a\right.}{48b}\\
&= \frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3\left(a\right.}{48b}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 130, normalized size = 0.84

$$
\frac{\sqrt{a+bx^2}\left(\frac{3a^{5/2}(3aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\sqrt{b}x\left(-9a^3B+6a^2b\left(4A+Bx^2\right)+8ab^2x^2\left(14A+9Bx^2\right)+16b^3x^4\left(4A+3Bx^2\right)\right)\right)}{384b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-9*a^3*B + 6*a^2*b*(4*A + B*x^2) + 16*b^3*x^4*
(4*A + 3*B*x^2) + 8*a*b^2*x^2*(14*A + 9*B*x^2)) + (3*a^(5/2)*(-8*A*b + 3*a*
B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 127, normalized size = 0.82

$$\frac{\left(8a^3Ab-3a^4B\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{5/2}}+\frac{\sqrt{a+bx^2}\,\left(-9a^3Bx+24a^2Abx+6a^2bBx^3+112aAb^2x^3+72ab^2Bx^5+64Ab^3x^5+48b^3Bx^7\right)}{384b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(24*a^2*A*b*x - 9*a^3*B*x + 112*a*A*b^2*x^3 + 6*a^2*b*B*x^
3 + 64*A*b^3*x^5 + 72*a*b^2*B*x^5 + 48*b^3*B*x^7))/(384*b^2) + ((8*a^3*A*b
- 3*a^4*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(5/2))
```

fricas [A] time = 1.25, size = 260, normalized size = 1.68

$$\left[-\frac{3\left(3\,Ba^4-8\,Aa^3b\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(48\,Bb^4x^7+8\left(9\,Bab^3+8\,Ab^4\right)x^5+2\left(3\,Ba^2b^2+56\,Aab^3\right)x^3-3\left(3\,Ba^3b-8\,Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{768\,b^3},\,-\frac{3\left(3\,Ba^4-8\,Aa^3b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(48\,Bb^4x^7+8\left(9\,Bab^3+8\,Ab^4\right)x^5+2\left(3\,Ba^2b^2+56\,Aab^3\right)x^3-3\left(3\,Ba^3b-8\,Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{384\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*
b^2 + 56*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3
, -1/384*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - (48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*b^2 + 56*A*a*
b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.38, size = 133, normalized size = 0.86

$$\frac{1}{384}\left(2\left(4\left(6\,Bbx^2+\frac{9\,Bab^6+8\,Ab^7}{b^6}\right)x^2+\frac{3\,Ba^2b^5+56\,Aab^6}{b^6}\right)x^2-\frac{3\left(3\,Ba^3b^4-8\,Aa^2b^5\right)}{b^6}\right)\sqrt{bx^2+a}\,x-\frac{\left(3\,Ba^4-8\,Aa^3b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b*x^2 + (9*B*a*b^6 + 8*A*b^7)/b^6)*x^2 + (3*B*a^2*b^5 + 56
*A*a*b^6)/b^6)*x^2 - 3*(3*B*a^3*b^4 - 8*A*a^2*b^5)/b^6)*sqrt(b*x^2 + a)*x -
 1/128*(3*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 177, normalized size = 1.14

$$-\frac{A\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}+\frac{3B\,a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{5}{2}}}-\frac{\sqrt{b\,x^{2}+a}\;A\,a^{2}x}{16b}+\frac{3\sqrt{b\,x^{2}+a}\;B\,a^{3}x}{128b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,x^{3}}{8b}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Aax}{24b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,a^{2}x}{64b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ax}{6b}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bax}{16b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/16*B*a/b^2*x*(b*x^2+a)^(5/2)+1/64*B*a^2/b^2*x
*(b*x^2+a)^(3/2)+3/128*B*a^3/b^2*x*(b*x^2+a)^(1/2)+3/128*B*a^4/b^(5/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))+1/6*A*x*(b*x^2+a)^(5/2)/b-1/24*A*a/b*x*(b*x^2+a)^
(3/2)-1/16*A*a^2/b*x*(b*x^2+a)^(1/2)-1/16*A*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))
```

maxima [A] time = 1.05, size = 162, normalized size = 1.05

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bx^{3}}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bax}{16\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}x}{64\,b^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,Ba^{3}x}{128\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ax}{6\,b}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Aax}{24\,b}-\frac{\sqrt{bx^{2}+a}\,Aa^{2}x}{16\,b}+\frac{3\,Ba^{4}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{Aa^{3}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*B*x^3/b - 1/16*(b*x^2 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*
x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*B*a^3*x/b^2 + 1/6*(b*x^2
 + a)^(5/2)*A*x/b - 1/24*(b*x^2 + a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A
*a^2*x/b + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*A*a^3*arcsinh(
b*x/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int(x^2*(A + B*x^2)*(a + b*x^2)^(3/2), x)
```

sympy [B] time = 26.19, size = 287, normalized size = 1.85

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17Aa^{\frac{3}{2}}x^{3}}{48\sqrt{1+\frac{bx^{2}}{a}}}+\frac{11A\sqrt{a}\,bx^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^{2}x^{7}}{6\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3Ba^{\frac{7}{2}}x}{128b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{\frac{5}{2}}x^{3}}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{13Ba^{\frac{3}{2}}x^{5}}{64\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5B\sqrt{a}\,bx^{7}}{16\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3Ba^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Bb^{2}x^{9}}{8\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) -
3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(
1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b
*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.507 $\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

Rubi [**A**] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (B*(a + b*x^2)^(7/2))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^{3/2}(A+Bx)\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)dx,x,x^2\right)\\&=\frac{(Ab-aB)\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(-2aB+7Ab+5bBx^2\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^2))/(35*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{5/2}\left(-2aB+7Ab+5bBx^2\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^2))/(35*b^2)
```

fricas [A] time = 0.96, size = 73, normalized size = 1.59

$$\frac{\left(5\,Bb^3x^6+\left(8\,Bab^2+7\,Ab^3\right)x^4-2\,Ba^3+7\,Aa^2b+\left(Ba^2b+14\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/35*(5*B*b^3*x^6 + (8*B*a*b^2 + 7*A*b^3)*x^4 - 2*B*a^3 + 7*A*a^2*b + (B*a^
2*b + 14*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 0.43, size = 44, normalized size = 0.96

$$\frac{5\left(bx^2+a\right)^{\frac{7}{2}}B-7\left(bx^2+a\right)^{\frac{5}{2}}Ba+7\left(bx^2+a\right)^{\frac{5}{2}}Ab}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/35*(5*(b*x^2 + a)^(7/2)*B - 7*(b*x^2 + a)^(5/2)*B*a + 7*(b*x^2 + a)^(5/2)
*A*b)/b^2
```

maple [A] time = 0.00, size = 31, normalized size = 0.67

$$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(5Bb\,x^{2}+7Ab-2Ba\right)}{35b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/35*(b*x^2+a)^(5/2)*(5*B*b*x^2+7*A*b-2*B*a)/b^2
```

maxima [A] time = 1.01, size = 50, normalized size = 1.09

$$\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bx^{2}}{7\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}}Ba}{35\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*B*x^2/b - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2 + 1/5*(b*x^2
 + a)^(5/2)*A/b
```

mupad [B] time = 0.56, size = 76, normalized size = 1.65

$$\sqrt{b\,x^{2}+a}\,\left(\frac{x^{4}\,\left(7\,A\,b^{3}+8\,B\,a\,b^{2}\right)}{35\,b^{2}}-\frac{2\,B\,a^{3}-7\,A\,a^{2}\,b}{35\,b^{2}}+\frac{B\,b\,x^{6}}{7}+\frac{a\,x^{2}\,\left(14\,A\,b+B\,a\right)}{35\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^4*(7*A*b^3 + 8*B*a*b^2))/(35*b^2) - (2*B*a^3 - 7*A*a^
2*b)/(35*b^2) + (B*b*x^6)/7 + (a*x^2*(14*A*b + B*a))/(35*b))
```

sympy [A] time = 1.54, size = 158, normalized size = 3.43

$$\begin{cases}\frac{Aa^{2}\sqrt{a+bx^{2}}}{5b}+\frac{2Aax^{2}\sqrt{a+bx^{2}}}{5}+\frac{Abx^{4}\sqrt{a+bx^{2}}}{5}-\frac{2Ba^{3}\sqrt{a+bx^{2}}}{35b^{2}}+\frac{Ba^{2}x^{2}\sqrt{a+bx^{2}}}{35b}+\frac{8Bax^{4}\sqrt{a+bx^{2}}}{35}+\frac{Bbx^{6}\sqrt{a+bx^{2}}}{7} & \text{for } b\neq 0\\ a^{\frac{3}{2}}\left(\frac{Ax^{2}}{2}+\frac{Bx^{4}}{4}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**2*sqrt(a + b*x**2)/(5*b) + 2*A*a*x**2*sqrt(a + b*x**2)/5 +
A*b*x**4*sqrt(a + b*x**2)/5 - 2*B*a**3*sqrt(a + b*x**2)/(35*b**2) + B*a**2*
x**2*sqrt(a + b*x**2)/(35*b) + 8*B*a*x**4*sqrt(a + b*x**2)/35 + B*b*x**6*sq
rt(a + b*x**2)/7, Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**4/4), True))
```

3.508 $\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=118

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}\,(6Ab-aB)}{16b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

Rubi [A] time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {388, 195, 217, 206}

$$\frac{a^2(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6Ab-aB)}{24b}+\frac{ax\sqrt{a+bx^2}\,(6Ab-aB)}{16b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (a*(6*A*b - a*B)*x*Sqrt[a + b*x^2])/(16*b) + ((6*A*b - a*B)*x*(a + b*x^2)^(
3/2))/(24*b) + (B*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*A*b - a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx &= \frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{(-6Ab+aB)\int\left(a+bx^2\right)^{3/2}dx}{6b}\\
&= \frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}+\frac{(a(6Ab-aB))\int\sqrt{a+bx^2}\,dx}{8b}\\
&= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b}+\frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}+\frac{\left(a^2(6A\right.}{}\\
&= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b}+\frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}+\frac{\left(a^2(6A\right.}{}\\
&= \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b}+\frac{(6Ab-aB)x\left(a+bx^2\right)^{3/2}}{24b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}+\frac{a^2(6Ab}{}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 109, normalized size = 0.92

$$
\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(3a^2B+2ab\left(15A+7Bx^2\right)+4b^2x^2\left(3A+2Bx^2\right)\right)-\frac{3a^{3/2}(aB-6Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{48b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(3*a^2*B + 4*b^2*x^2*(3*A + 2*B*x^2) + 2*a*b*(1
5*A + 7*B*x^2)) - (3*a^(3/2)*(-6*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/S
qrt[1 + (b*x^2)/a]))/(48*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 102, normalized size = 0.86

$$
\frac{\sqrt{a+bx^2}\left(3a^2Bx+30aAbx+14abBx^3+12Ab^2x^3+8b^2Bx^5\right)}{48b}+\frac{\left(a^3B-6a^2Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(30*a*A*b*x + 3*a^2*B*x + 12*A*b^2*x^3 + 14*a*b*B*x^3 + 8*
b^2*B*x^5))/(48*b) + ((-6*a^2*A*b + a^3*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(16*b^(3/2))
```

fricas [A] time = 0.90, size = 207, normalized size = 1.75

$$\left[-\frac{3\left(Ba^{3}-6\,Aa^{2}b\right)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left(8\,Bb^{3}x^{5}+2\left(7\,Bab^{2}+6\,Ab^{3}\right)x^{3}+3\left(Ba^{2}b+10\,Aab^{2}\right)x\right)\sqrt{bx^{2}+a}}{96\,b^{2}},\ \frac{3\left(Ba^{3}-6\,Aa^{2}b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(8\,Bb^{3}x^{5}+2\left(7\,Bab^{2}+6\,Ab^{3}\right)x^{3}+3\left(Ba^{2}b+10\,Aab^{2}\right)x\right)\sqrt{bx^{2}+a}}{48\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^3 + 3*(B*a^2*b + 10
*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^
3 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.42, size = 102, normalized size = 0.86

$$\frac{1}{48}\left(2\left(4\,Bbx^{2}+\frac{7\,Bab^{4}+6\,Ab^{5}}{b^{4}}\right)x^{2}+\frac{3\left(Ba^{2}b^{3}+10\,Aab^{4}\right)}{b^{4}}\right)\sqrt{bx^{2}+a}\,x+\frac{\left(Ba^{3}-6\,Aa^{2}b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*b*x^2 + (7*B*a*b^4 + 6*A*b^5)/b^4)*x^2 + 3*(B*a^2*b^3 + 10*A*a
*b^4)/b^4)*sqrt(b*x^2 + a)*x + 1/16*(B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

maple [A] time = 0.01, size = 131, normalized size = 1.11

$$\frac{3A\,a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}-\frac{B\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\,Aax}{8}-\frac{\sqrt{b\,x^{2}+a}\,B\,a^{2}x}{16b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Ax}{4}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bax}{24b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bx}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A),x)
```

```
[Out] 1/6*B*x*(b*x^2+a)^(5/2)/b-1/24*B*a/b*x*(b*x^2+a)^(3/2)-1/16*B*a^2/b*x*(b*x^
2+a)^(1/2)-1/16*B*a^3/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*A*x*(b*x^2+
a)^(3/2)+3/8*A*a*x*(b*x^2+a)^(1/2)+3/8*A*a^2/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))
```

maxima [A] time = 0.96, size = 116, normalized size = 0.98

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}\,Aax+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b}-\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 1/6*(b*x^2 + a)^(5/
2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*x^2 + a)*B*a^2*x/b
- 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*
b))/sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^(3/2),x)
```

```
[Out] int((A + B*x^2)*(a + b*x^2)^(3/2), x)
```

sympy [B] time = 16.54, size = 253, normalized size = 2.14

$$\frac{Aa^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Ab^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11B\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Bb^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**(5/2)*x/(
16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b*x**2/a)) + 11*
B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt(b)*x/sqrt(a))/
(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.509 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=76

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] a*A*Sqrt[a + b*x^2] + (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b)
 - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx,x,x^2\right)\\
&= \frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{2}A\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{2}(aA)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^2\right)\\
&= aA\sqrt{a+bx^2}+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{2}\left(a^2A\right)\text{Subst}\Bigg(\int \frac{1}{x\sqrt{a+bx}}\\
&= aA\sqrt{a+bx^2}+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{\left(a^2A\right)\text{Subst}\Bigg(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,}{b}\\
&= aA\sqrt{a+bx^2}+\frac{1}{3}A\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.00

$$
\frac{1}{3}A\left(a+bx^2\right)^{3/2}+aA\left(\sqrt{a+bx^2}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b) + a*A*(Sqrt[a + b*x^2
] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
```

IntegrateAlgebraic [A] time = 0.06, size = 83, normalized size = 1.09

$$\frac{\sqrt{a+bx^2}\left(3a^2B+20aAb+6abBx^2+5Ab^2x^2+3b^2Bx^4\right)}{15b}-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^2 + 6*a*b*B*x^2 + 3*b^2*B*
x^4))/(15*b) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 0.87, size = 170, normalized size = 2.24

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,Bb^2x^4+3\,Ba^2+20\,Aab+\left(6\,Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{30\,b},\ \frac{15\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\,Bb^2x^4+3\,Ba^2+20\,Aab+\left(6\,Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(3*B*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2
+ a))/b, 1/15*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*B*b^
2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 + a))/b]
```

giac [A] time = 0.35, size = 79, normalized size = 1.04

$$\frac{Aa^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}Bb^4+5\left(bx^2+a\right)^{\frac{3}{2}}Ab^5+15\sqrt{bx^2+a}\,Aab^5}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] A*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(3*(b*x^2 + a)^(5/2)
*B*b^4 + 5*(b*x^2 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^2 + a)*A*a*b^5)/b^5
```

maple [A] time = 0.01, size = 70, normalized size = 0.92

$$-A\,a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^{2}+a}\,Aa+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A}{3}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x,x)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+1/3*A*(b*x^2+a)^(3/2)-A*a^(3/2)*ln((2*a+2*(b*x^2+a)
^(1/2)*a^(1/2))/x)+a*A*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.04, size = 58, normalized size = 0.76

$$-Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}A+\sqrt{bx^{2}+a}\,Aa+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] -A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*A + sqrt(b
*x^2 + a)*A*a + 1/5*(b*x^2 + a)^(5/2)*B/b
```

mupad [B] time = 0.97, size = 60, normalized size = 0.79

$$\frac{A\left(b\,x^{2}+a\right)^{3/2}}{3}+\frac{B\left(b\,x^{2}+a\right)^{5/2}}{5\,b}-A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+A\,a\,\sqrt{b\,x^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x,x)
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b) - A*a^(3/2)*atanh((a
+ b*x^2)^(1/2)/a^(1/2)) + A*a*(a + b*x^2)^(1/2)
```

sympy [A] time = 61.94, size = 71, normalized size = 0.93

$$\frac{Aa^{2}\operatorname{atan}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+Aa\sqrt{a+bx^{2}}+\frac{A\left(a+bx^{2}\right)^{\frac{3}{2}}}{3}+\frac{B\left(a+bx^{2}\right)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x,x)
```

```
[Out] A*a**2*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + A*a*sqrt(a + b*x**2) + A*
(a + b*x**2)**(3/2)/3 + B*(a + b*x**2)**(5/2)/(5*b)
```

3.510 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^2} \, dx$

Optimal. Leaf size=109

$$\frac{x\left(a+bx^2\right)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}\,(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 195, 217, 206}

$$\frac{x\left(a+bx^2\right)^{3/2}(aB+4Ab)}{4a}+\frac{3}{8}x\sqrt{a+bx^2}\,(aB+4Ab)+\frac{3a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-\frac{A\left(a+bx^2\right)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (3*(4*A*b + a*B)*x*Sqrt[a + b*x^2])/8 + ((4*A*b + a*B)*x*(a + b*x^2)^(3/2))
/(4*a) - (A*(a + b*x^2)^(5/2))/(a*x) + (3*a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^2}\,dx = -\frac{A\left(a+bx^2\right)^{5/2}}{ax} - \frac{(-4Ab-aB)\int \left(a+bx^2\right)^{3/2}\,dx}{a}$$

$$= \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{1}{4}(3(4Ab+aB))\int \sqrt{a+bx^2}\,dx$$

$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{1}{8}(3a(4$$

$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{1}{8}(3a(4$$

$$= \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x\left(a+bx^2\right)^{3/2}}{4a} - \frac{A\left(a+bx^2\right)^{5/2}}{ax} + \frac{3a(4A}{}$$

Mathematica [A] time = 0.19, size = 87, normalized size = 0.80

$$\frac{1}{8}\sqrt{a+bx^2}\left(\frac{3\sqrt{a}\,(aB+4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}} - \frac{8aA}{x} + 5aBx + 4Abx + 2bBx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*((-8*a*A)/x + 4*A*b*x + 5*a*B*x + 2*b*B*x^3 + (3*Sqrt[a]*(
4*A*b + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/
8
```

IntegrateAlgebraic [A] time = 0.21, size = 86, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\,\left(-8aA+5aBx^2+4Abx^2+2bBx^4\right)}{8x} - \frac{3\left(a^2B+4aAb\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a*A + 4*A*b*x^2 + 5*a*B*x^2 + 2*b*B*x^4))/(8*x) - (3*(
4*a*A*b + a^2*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

fricas [A] time = 0.71, size = 182, normalized size = 1.67

$$\left[\frac{3\left(Ba^{2}+4\,Aab\right)\sqrt{b}\,x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\left(2\,Bb^{2}x^{4}-8\,Aab+\left(5\,Bab+4\,Ab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,bx},-\frac{3\left(Ba^{2}+4\,Aab\right)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(2\,Bb^{2}x^{4}-8\,Aab+\left(5\,Bab+4\,Ab^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}}{8\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(B*a^2 + 4*A*a*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^2
+ a))/(b*x), -1/8*(3*(B*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - (2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^2 +
a))/(b*x)]
```

giac [A] time = 0.44, size = 114, normalized size = 1.05

$$\frac{2\,Aa^{2}\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a}+\frac{1}{8}\left(2\,Bbx^{2}+\frac{5\,Bab^{2}+4\,Ab^{3}}{b^{2}}\right)\sqrt{bx^{2}+a}\,x-\frac{3\left(Ba^{2}\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*B*b*x^2 + (5
*B*a*b^2 + 4*A*b^3)/b^2)*sqrt(b*x^2 + a)*x - 3/16*(B*a^2*sqrt(b) + 4*A*a*b^
(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b
```

maple [A] time = 0.01, size = 125, normalized size = 1.15

$$\frac{3Aa\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2}+\frac{3B\,a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}+\frac{3\sqrt{b\,x^{2}+a}\,Abx}{2}+\frac{3\sqrt{b\,x^{2}+a}\,Bax}{8}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Abx}{a}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bx}{4}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/4*x*B*(b*x^2+a)^(3/2)+3/8*B*a*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(1/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))-A*(b*x^2+a)^(5/2)/a/x+A*b/a*x*(b*x^2+a)^(3/2)+3/2*A*
b*x*(b*x^2+a)^(1/2)+3/2*A*b^(1/2)*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.14, size = 91, normalized size = 0.83

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx + \frac{3}{8}\sqrt{bx^2+a}\,Bax + \frac{3}{2}\sqrt{bx^2+a}\,Abx + \frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3}{2}\,Aa\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/2*sqrt(b*x^2 + a)
*A*b*x + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/2*A*a*sqrt(b)*arcsinh
(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2)*A/x
```

mupad [B] time = 1.50, size = 80, normalized size = 0.73

$$\frac{B\,x\left(b\,x^2+a\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}} - \frac{A\left(b\,x^2+a\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^2,x)
```

```
[Out] (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
+ 1)^(3/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))
/(x*((b*x^2)/a + 1)^(3/2))
```

sympy [B] time = 12.20, size = 216, normalized size = 1.98

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} + \frac{A\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2} - \frac{A\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}} + \frac{3Aa\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2} + \frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{3B\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{3Ba^2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}} + \frac{Bb^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
+ B*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) +
 3*B*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(
a))/(8*sqrt(b)) + B*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.511 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^3}\,dx$

Optimal. Leaf size=110

$$\frac{\left(a+bx^2\right)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Ab)-\frac{1}{2}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(a+bx^2\right)^{3/2}(2aB+3Ab)}{6a}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Ab)-\frac{1}{2}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((3*A*b + 2*a*B)*(a + b*x^2)^(3/2))/(
6*a) - (A*(a + b*x^2)^(5/2))/(2*a*x^2) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}+\frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^2\right)}{2a}\\
&= \frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}+\frac{1}{4}(3Ab+2aB)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\right.\\
&= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2}+\frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}+\frac{1}{4}(a(3A\\
&= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2}+\frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}+\frac{(a(3Ab}{}\\
&= \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2}+\frac{(3Ab+2aB)\left(a+bx^2\right)^{3/2}}{6a}-\frac{A\left(a+bx^2\right)^{5/2}}{2ax^2}-\frac{1}{2}\sqrt{a}\,(3
\end{aligned}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.73

$$\frac{1}{6}\left(\frac{\sqrt{a+bx^2}\left(-3aA+8aBx^2+6Abx^2+2bBx^4\right)}{x^2}-3\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-3*a*A + 6*A*b*x^2 + 8*a*B*x^2 + 2*b*B*x^4))/x^2 - 3*Sqr
t[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/6
```

IntegrateAlgebraic [A] time = 0.14, size = 85, normalized size = 0.77

$$\frac{1}{2}\left(-2a^{3/2}B-3\sqrt{a}\,Ab\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{\sqrt{a+bx^2}\left(-3aA+8aBx^2+6Abx^2+2bBx^4\right)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a*A + 6*A*b*x^2 + 8*a*B*x^2 + 2*b*B*x^4))/(6*x^2) + ((
-3*Sqrt[a]*A*b - 2*a^(3/2)*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

fricas [A] time = 0.91, size = 167, normalized size = 1.52

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}\,x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Bbx^4+2\,(4\,Ba+3\,Ab)x^2-3\,Aa\right)\sqrt{bx^2+a}}{12\,x^2},\ \frac{3\,(2\,Ba+3\,Ab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,Bbx^4+2\,(4\,Ba+3\,Ab)x^2-3\,Aa\right)\sqrt{bx^2+a}}{6\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*B*a + 3*A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) + 2*(2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 +
a))/x^2, 1/6*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a
)) + (2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 + a))/x^2]
```

giac [A] time = 0.41, size = 103, normalized size = 0.94

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb+6\,\sqrt{bx^2+a}\,Bab+6\,\sqrt{bx^2+a}\,Ab^2-\frac{3\,\sqrt{bx^2+a}\,Aab}{x^2}+\frac{3\left(2\,Ba^2b+3\,Aab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b*x^2 + a)^(3/2)*B*b + 6*sqrt(b*x^2 + a)*B*a*b + 6*sqrt(b*x^2 + a)*
A*b^2 - 3*sqrt(b*x^2 + a)*A*a*b/x^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt
(b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

maple [A] time = 0.01, size = 132, normalized size = 1.20

$$-\frac{3A\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2}-B\,a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\frac{3\sqrt{b\,x^{2}+a}\,Ab}{2}+\sqrt{b\,x^{2}+a}\,Ba+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Ab}{2a}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B}{3}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(5/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(3/2)-3/2*A*b*a^(1/2)*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/2*A*b*(b*x^2+a)^(1/2)+1/3*B*(b*x^2+a)^(
3/2)-B*a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/2)*a
```

maxima [A] time = 1.03, size = 109, normalized size = 0.99

$$-Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{3}{2}\,A\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}B+\sqrt{bx^{2}+a}\,Ba+\frac{3}{2}\,\sqrt{bx^{2}+a}\,Ab+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ab}{2\,a}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] -B*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/2*A*sqrt(a)*b*arcsinh(a/(sqrt(
a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 + a)*B*a + 3/2*sqrt(b*
x^2 + a)*A*b + 1/2*(b*x^2 + a)^(3/2)*A*b/a - 1/2*(b*x^2 + a)^(5/2)*A/(a*x^2
)
```

mupad [B] time = 1.49, size = 94, normalized size = 0.85

$$\frac{B\left(b\,x^{2}+a\right)^{3/2}}{3}-B\,a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+A\,b\,\sqrt{b\,x^{2}+a}+B\,a\,\sqrt{b\,x^{2}+a}-\frac{A\,a\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}-\frac{3\,A\,\sqrt{a}\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^3,x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*b*
(a + b*x^2)^(1/2) + B*a*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2)
 - (3*A*a^(1/2)*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/2
```

sympy [A] time = 71.66, size = 184, normalized size = 1.67

$$-\frac{3A\sqrt{a}\,b\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2} - \frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right) + \frac{Ba^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}} + Bb\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b = 0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(
a/(b*x**2) + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt
(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

3.512 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{(a+bx^2)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 195, 217, 206}

$$-\frac{(a+bx^2)^{3/2}(3aB+2Ab)}{3ax}+\frac{bx\sqrt{a+bx^2}(3aB+2Ab)}{2a}+\frac{1}{2}\sqrt{b}(3aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (b*(2*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/(2*a) - ((2*A*b + 3*a*B)*(a + b*x^2)^
(3/2))/(3*a*x) - (A*(a + b*x^2)^(5/2))/(3*a*x^3) + (Sqrt[b]*(2*A*b + 3*a*B)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^4}\,dx &= -\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}-\frac{(-2Ab-3aB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx}{3a}\\
&= -\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\frac{(b(2Ab+3aB))\int \sqrt{a+bx^2}\,dx}{a}\\
&= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\frac{1}{2}(b(2A\\
&= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\frac{1}{2}(b(2A\\
&= \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a}-\frac{(2Ab+3aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}+\frac{1}{2}\sqrt{b}\,(2
\end{aligned}$$

Mathematica [C] time = 0.08, size = 83, normalized size = 0.70

$$\frac{\sqrt{a+bx^2}\,(-3aB-2Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{3x\sqrt{\frac{bx^2}{a}+1}}-\frac{A\left(a+bx^2\right)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(A*(a + b*x^2)^(5/2))/(a*x^3) + ((-2*A*b - 3*a*B)*Sqrt[a + b*x^2]*Hype
rgeometric2F1[-3/2, -1/2, 1/2, -((b*x^2)/a)])/(3*x*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.23, size = 88, normalized size = 0.74

$$\frac{1}{2}\left(-3a\sqrt{b}\,B-2Ab^{3/2}\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)+\frac{\sqrt{a+bx^2}\,\left(-2aA-6aBx^2-8Abx^2+3bBx^4\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*A - 8*A*b*x^2 - 6*a*B*x^2 + 3*b*B*x^4))/(6*x^3) + ((
-2*A*b^(3/2) - 3*a*Sqrt[b]*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

fricas [A] time = 0.98, size = 166, normalized size = 1.39

$$\left[\frac{3\,(3\,Ba+2\,Ab)\sqrt{b}\,x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(3\,Bbx^4-2\,(3\,Ba+4\,Ab)x^2-2\,Aa\right)\sqrt{bx^2+a}}{12\,x^3},-\frac{3\,(3\,Ba+2\,Ab)\sqrt{-b}\,x^3\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(3\,Bbx^4-2\,(3\,Ba+4\,Ab)x^2-2\,Aa\right)\sqrt{bx^2+a}}{6\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*B*a + 2*A*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/
x^3, -1/6*(3*(3*B*a + 2*A*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3]
```

giac [B] time = 0.52, size = 207, normalized size = 1.74

$$\frac{1}{2}\,\sqrt{bx^2+a}\,Bbx-\frac{1}{4}\,\left(3\,Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,\left(3\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba^2\sqrt{b}+6\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Aab^{\frac{3}{2}}-6\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}-6\,\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aa^2b^{\frac{3}{2}}+3\,Ba^4\sqrt{b}+4\,Aa^3b^{\frac{3}{2}}\right)}{3\,\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b*x - 1/4*(3*B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)*
x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*sqrt
(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2) - 6*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^3*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2
) + 3*B*a^4*sqrt(b) + 4*A*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^3
```

maple [A] time = 0.01, size = 168, normalized size = 1.41

$$A\,b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+\frac{3Ba\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{\sqrt{b\,x^2+a}\,A\,b^2x}{a}+\frac{3\sqrt{b\,x^2+a}\,Bbx}{2}+\frac{2\left(b\,x^2+a\right)^{\frac{3}{2}}A\,b^2x}{3a^2}+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Bbx}{a}-\frac{2\left(b\,x^2+a\right)^{\frac{5}{2}}Ab}{3a^2x}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B}{ax}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x)
```

```
[Out] -B/a/x*(b*x^2+a)^(5/2)+B*b/a*x*(b*x^2+a)^(3/2)+3/2*B*b*x*(b*x^2+a)^(1/2)+3/
2*B*b^(1/2)*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-1/3*A*(b*x^2+a)^(5/2)/a/x^3-2/3
*A*b/a^2/x*(b*x^2+a)^(5/2)+2/3*A*b^2/a^2*x*(b*x^2+a)^(3/2)+A*b^2/a*x*(b*x^2
+a)^(1/2)+A*b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.01, size = 115, normalized size = 0.97

$$\frac{3}{2}\sqrt{bx^2+a}\,Bbx+\frac{\sqrt{bx^2+a}\,Ab^2x}{a}+\frac{3}{2}\,Ba\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+Ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{x}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Ab}{3\,ax}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*B*b*x + sqrt(b*x^2 + a)*A*b^2*x/a + 3/2*B*a*sqrt(b)*arc
sinh(b*x/sqrt(a*b)) + A*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2)*
B/x - 2/3*(b*x^2 + a)^(3/2)*A*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*A/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^4, x)
```

sympy [A] time = 7.75, size = 202, normalized size = 1.70

$$-\frac{A\sqrt{a}\,b}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3}+Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Ab^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{B\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x
**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*asinh(sqrt(b)*x/sqrt(
a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(3/2)/(x*sqrt(1 + b*x**2
/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a
) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
```

3.513 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=115

$$-\frac{(a+bx^2)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{3/2}(4aB+Ab)}{8ax^2}+\frac{3b\sqrt{a+bx^2}(4aB+Ab)}{8a}-\frac{3b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{A(a+bx^2)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (3*b*(A*b + 4*a*B)*Sqrt[a + b*x^2])/(8*a) - ((A*b + 4*a*B)*(a + b*x^2)^(3/2
))/(8*a*x^2) - (A*(a + b*x^2)^(5/2))/(4*a*x^4) - (3*b*(A*b + 4*a*B)*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{5/2}}{4ax^4} + \frac{(Ab+4aB)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, x^2\right)}{8a}$$

$$= -\frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{5/2}}{4ax^4} + \frac{(3b(Ab+4aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,d\right.}{16a}$$

$$= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{5/2}}{4ax^4} + \frac{1}{16}(3b(A$$

$$= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{5/2}}{4ax^4} + \frac{1}{8}(3(Ab$$

$$= \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)\left(a+bx^2\right)^{3/2}}{8ax^2} - \frac{A\left(a+bx^2\right)^{5/2}}{4ax^4} - \frac{3b(Ab+}{}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.51

$$\frac{\left(a+bx^2\right)^{5/2}\left(bx^4(4aB+Ab)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^2}{a}+1\right)-5a^2A\right)}{20a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-5*a^2*A + b*(A*b + 4*a*B)*x^4*Hypergeometric2F1[2, 5/2
, 7/2, 1 + (b*x^2)/a]))/(20*a^3*x^4)
```

IntegrateAlgebraic [A] time = 0.15, size = 83, normalized size = 0.72

$$\frac{\sqrt{a+bx^2}\left(-2aA-4aBx^2-5Abx^2+8bBx^4\right)}{8x^4} - \frac{3\left(4abB+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*A - 5*A*b*x^2 - 4*a*B*x^2 + 8*b*B*x^4))/(8*x^4) - (3
*(A*b^2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

fricas [A] time = 0.94, size = 189, normalized size = 1.64

$$\left[\frac{3\left(4\,Bab+Ab^{2}\right)\sqrt{a}\,x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(8\,Babx^{4}-2\,Aa^{2}-\left(4\,Ba^{2}+5\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{16\,ax^{4}},\frac{3\left(4\,Bab+Ab^{2}\right)\sqrt{-a}\,x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(8\,Babx^{4}-2\,Aa^{2}-\left(4\,Ba^{2}+5\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{8\,ax^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(a) + 2*a)/x^2) + 2*(8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqrt(
b*x^2 + a))/(a*x^4), 1/8*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + (8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqrt(b
*x^2 + a))/(a*x^4)]
```

giac [A] time = 0.47, size = 131, normalized size = 1.14

$$\frac{8\,\sqrt{bx^{2}+a}\,Bb^{2}+\frac{3\left(4\,Bab^{2}+Ab^{3}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{4\left(bx^{2}+a\right)^{\frac{3}{2}}Bab^{2}-4\,\sqrt{bx^{2}+a}\,Ba^{2}b^{2}+5\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{3}-3\,\sqrt{bx^{2}+a}\,Aab^{3}}{b^{2}x^{4}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^2 + a)
/sqrt(-a))/sqrt(-a) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^
2*b^2 + 5*(b*x^2 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^2 + a)*A*a*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 184, normalized size = 1.60

$$-\frac{3A\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8\sqrt{a}}-\frac{3B\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2}+\frac{3\sqrt{b\,x^{2}+a}\,A\,b^{2}}{8a}+\frac{3\sqrt{b\,x^{2}+a}\,Bb}{2}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{2}}{8a^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bb}{2a}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ab}{8a^{2}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B}{2a\,x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/2*B/a/x^2*(b*x^2+a)^(5/2)+1/2*B*b/a*(b*x^2+a)^(3/2)-3/2*B*b*a^(1/2)*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/2*B*b*(b*x^2+a)^(1/2)-1/4*A*(b*x^2+a)^(
5/2)/a/x^4-1/8*A*b/a^2/x^2*(b*x^2+a)^(5/2)+1/8*A*b^2/a^2*(b*x^2+a)^(3/2)-3/
8*A*b^2/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/8*A*b^2/a*(b*x^2+a)
^(1/2)
```

maxima [**A**] time = 1.05, size = 161, normalized size = 1.40

$$-\frac{3}{2}B\sqrt{a}\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{3\,Ab^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{3}{2}\sqrt{bx^{2}+a}\,Bb+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bb}{2\,a}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{2}}{8\,a^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,Ab^{2}}{8\,a}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{2\,ax^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab}{8\,a^{2}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] -3/2*B*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/sqrt(a) + 3/2*sqrt(b*x^2 + a)*B*b + 1/2*(b*x^2 + a)^(3/2)*B*b
/a + 1/8*(b*x^2 + a)^(3/2)*A*b^2/a^2 + 3/8*sqrt(b*x^2 + a)*A*b^2/a - 1/2*(b
*x^2 + a)^(5/2)*B/(a*x^2) - 1/8*(b*x^2 + a)^(5/2)*A*b/(a^2*x^2) - 1/4*(b*x^
2 + a)^(5/2)*A/(a*x^4)
```

mupad [**B**] time = 1.90, size = 104, normalized size = 0.90

$$B\,b\,\sqrt{b\,x^{2}+a}-\frac{5\,A\,\left(b\,x^{2}+a\right)^{3/2}}{8\,x^{4}}-\frac{3\,A\,b^{2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}+\frac{3\,A\,a\,\sqrt{b\,x^{2}+a}}{8\,x^{4}}-\frac{B\,a\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}-\frac{3\,B\,\sqrt{a}\,\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^5,x)
```

```
[Out] B*b*(a + b*x^2)^(1/2) - (5*A*(a + b*x^2)^(3/2))/(8*x^4) - (3*A*b^2*atanh((a
 + b*x^2)^(1/2)/a^(1/2)))/(8*a^(1/2)) + (3*A*a*(a + b*x^2)^(1/2))/(8*x^4) -
 (B*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*B*a^(1/2)*b*atanh((a + b*x^2)^(1/2)/a
^(1/2)))/2
```

sympy [**B**] time = 154.89, size = 216, normalized size = 1.88

$$-\frac{Aa^{2}}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Aa\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{2x}-\frac{Ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Ab^{2}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8\sqrt{a}}-\frac{3B\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Ba\sqrt{b}\,\sqrt{\frac{a}{bx^{2}}+1}}{2x}+\frac{Ba\sqrt{b}}{x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Bb^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -A*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a*sqrt(b)/(8*x**3*sqrt(
a/(b*x**2) + 1)) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)/(8*x*
sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - 3
*B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1
)/(2*x) + B*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)*x/sqrt(a/(b*x**
2) + 1)
```

3.514 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{bB\sqrt{a+bx^2}}{x}-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {451, 277, 217, 206}

$$-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{bB\sqrt{a+bx^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -((b*B*Sqrt[a + b*x^2])/x) - (B*(a + b*x^2)^(3/2))/(3*x^3) - (A*(a + b*x^2)
^(5/2))/(5*a*x^5) + b^(3/2)*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^6}\,dx &= -\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+B\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx\\
&= -\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+(bB)\int \frac{\sqrt{a+bx^2}}{x^2}\,dx\\
&= -\frac{bB\sqrt{a+bx^2}}{x}-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+\left(b^2B\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{bB\sqrt{a+bx^2}}{x}-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+\left(b^2B\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.\\
&= -\frac{bB\sqrt{a+bx^2}}{x}-\frac{B\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}+b^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.04, size = 76, normalized size = 0.88

$$-\frac{A\left(a+bx^2\right)^{5/2}}{5ax^5}-\frac{aB\sqrt{a+bx^2}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(A*(a + b*x^2)^(5/2))/(a*x^5) - (a*B*Sqrt[a + b*x^2]*Hypergeometric2F1
[-3/2, -3/2, -1/2, -((b*x^2)/a)])/(3*x^3*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.21, size = 92, normalized size = 1.07

$$\frac{\sqrt{a+bx^2}\left(-3a^2A-5a^2Bx^2-6aAbx^2-20abBx^4-3Ab^2x^4\right)}{15ax^5}-b^{3/2}B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*A - 6*a*A*b*x^2 - 5*a^2*B*x^2 - 3*A*b^2*x^4 - 20*a
*b*B*x^4))/(15*a*x^5) - b^(3/2)*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 0.82, size = 184, normalized size = 2.14

$$\left[\frac{15\,Bab^{\frac{3}{2}}x^{5}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)-2\left(\left(20\,Bab+3\,Ab^{2}\right)x^{4}+3\,Aa^{2}+\left(5\,Ba^{2}+6\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{30\,ax^{5}},-\frac{15\,Ba\sqrt{-b}\,bx^{5}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(\left(20\,Bab+3\,Ab^{2}\right)x^{4}+3\,Aa^{2}+\left(5\,Ba^{2}+6\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{15\,ax^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*B*a*b^(3/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
 2*((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt(b*x^
2 + a))/(a*x^5), -1/15*(15*B*a*sqrt(-b)*b*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) + ((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt
(b*x^2 + a))/(a*x^5)]
```

giac [B] time = 0.44, size = 236, normalized size = 2.74

$$-\frac{1}{2}\,Bb^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Bab^{\frac{3}{2}}+15\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ba^{2}b^{\frac{3}{2}}+110\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba^{3}b^{\frac{3}{2}}+30\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Aa^{2}b^{\frac{5}{2}}-70\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{4}b^{\frac{3}{2}}+20\,Ba^{5}b^{\frac{3}{2}}+3\,Aa^{4}b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(30*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5
/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 110*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*
b^(5/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) + 20*B*a^5*b^(3/
2) + 3*A*a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 115, normalized size = 1.34

$$B\,b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)+\frac{\sqrt{b\,x^{2}+a}\,B\,b^{2}x}{a}+\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,b^{2}x}{3a^{2}}-\frac{2\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bb}{3a^{2}x}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B}{3a\,x^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A}{5a\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*A*(b*x^2+a)^(5/2)/a/x^5-1/3*B/a/x^3*(b*x^2+a)^(5/2)-2/3*B*b/a^2/x*(b*x
^2+a)^(5/2)+2/3*B*b^2/a^2*x*(b*x^2+a)^(3/2)+B*b^2/a*x*(b*x^2+a)^(1/2)+B*b^(
3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.11, size = 88, normalized size = 1.02

$$\frac{\sqrt{bx^2+a}\,Bb^2x}{a} + Bb^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb}{3\,ax} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{3\,ax^3} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] sqrt(b*x^2 + a)*B*b^2*x/a + B*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2 +
 a)^(3/2)*B*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*B/(a*x^3) - 1/5*(b*x^2 + a)^(5/
2)*A/(a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^6, x)
```

sympy [B] time = 6.74, size = 184, normalized size = 2.14

$$-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{B\sqrt{a}\,b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - \frac{Bb^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - B*sqrt(a)*b/(x*sqrt(
1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqr
t(a/(b*x**2) + 1)/3 + B*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**2*x/(sqrt(
a)*sqrt(1 + b*x**2/a))
```

3.515 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^7}\,dx$

Optimal. Leaf size=120

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{b\sqrt{a+bx^2}\,(Ab-6aB)}{16ax^2}+\frac{\left(a+bx^2\right)^{3/2}(Ab-6aB)}{24ax^4}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b^2(Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-6aB)}{24ax^4}+\frac{b\sqrt{a+bx^2}\,(Ab-6aB)}{16ax^2}-\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (b*(A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a*x^2) + ((A*b - 6*a*B)*(a + b*x^2)^(
3/2))/(24*a*x^4) - (A*(a + b*x^2)^(5/2))/(6*a*x^6) + (b^2*(A*b - 6*a*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^7}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx, x, x^2\right)}{6a}$$

$$= \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} - \frac{(b(Ab-6aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x\right.}{16a}$$

$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} - \frac{\left(b^2(Ab-\right.}{}$$

$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} - \frac{(b(Ab-6}{}$$

$$= \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{5/2}}{6ax^6} + \frac{b^2(Ab-6}{}$$

Mathematica [A] time = 0.08, size = 119, normalized size = 0.99

$$\frac{-\left(a+bx^2\right)\left(4a^2\left(2A+3Bx^2\right)+2abx^2\left(7A+15Bx^2\right)+3Ab^2x^4\right)-3b^2x^6\sqrt{\frac{bx^2}{a}+1}\,(6aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{48ax^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (-((a + b*x^2)*(3*A*b^2*x^4 + 4*a^2*(2*A + 3*B*x^2) + 2*a*b*x^2*(7*A + 15*B
*x^2))) - 3*b^2*(-(A*b) + 6*a*B)*x^6*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (
b*x^2)/a]])/(48*a*x^6*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.19, size = 103, normalized size = 0.86

$$\frac{\left(Ab^3-6ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}+\frac{\sqrt{a+bx^2}\,\left(-8a^2A-12a^2Bx^2-14aAbx^2-30abBx^4-3Ab^2x^4\right)}{48ax^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 14*a*A*b*x^2 - 12*a^2*B*x^2 - 3*A*b^2*x^4 - 30
*a*b*B*x^4))/(48*a*x^6) + ((A*b^3 - 6*a*b^2*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/(16*a^(3/2))
```

fricas [A] time = 0.98, size = 222, normalized size = 1.85

$$\left[-\frac{3\left(6\,Bab^2-Ab^3\right)\sqrt{a}\,x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(10\,Ba^2b+Aab^2\right)x^4+8\,Aa^3+2\left(6\,Ba^3+7\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{96\,a^2x^6},\ \frac{3\left(6\,Bab^2-Ab^3\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(3\left(10\,Ba^2b+Aab^2\right)x^4+8\,Aa^3+2\left(6\,Ba^3+7\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{48\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*(10*B*a^2*b + A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^3
 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6), 1/48*(3*(6*B*a*b^2 - A*b^3)*
sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*(10*B*a^2*b + A*a*b^2)*x
^4 + 8*A*a^3 + 2*(6*B*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

giac [A] time = 0.33, size = 159, normalized size = 1.32

$$\frac{\frac{3\left(6\,Bab^3-Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{30\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-48\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+18\,\sqrt{bx^2+a}\,Ba^3b^3+3\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+8\left(bx^2+a\right)^{\frac{3}{2}}Aab^4-3\,\sqrt{bx^2+a}\,Aa^2b^4}{ab^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) -
 (30*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 18*sqrt(b
*x^2 + a)*B*a^3*b^3 + 3*(b*x^2 + a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b
^4 - 3*sqrt(b*x^2 + a)*A*a^2*b^4)/(a*b^3*x^6))/b
```

maple [B] time = 0.01, size = 233, normalized size = 1.94

$$\frac{A\,b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{3}{2}}}-\frac{3B\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8\sqrt{a}}-\frac{\sqrt{b\,x^{2}+a}\,A\,b^{3}}{16a^{2}}+\frac{3\sqrt{b\,x^{2}+a}\,B\,b^{2}}{8a}-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{3}}{48a^{3}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,b^{2}}{8a^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,b^{2}}{48a^{3}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bb}{8a^{2}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ab}{24a^{2}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B}{4a\,x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(5/2)/a/x^6+1/24*A*b/a^2/x^4*(b*x^2+a)^(5/2)+1/48*A*b^2/a^
3/x^2*(b*x^2+a)^(5/2)-1/48*A*b^3/a^3*(b*x^2+a)^(3/2)+1/16*A*b^3/a^(3/2)*ln(
(2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/16*A*b^3/a^2*(b*x^2+a)^(1/2)-1/4*B/a/x
^4*(b*x^2+a)^(5/2)-1/8*B*b/a^2/x^2*(b*x^2+a)^(5/2)+1/8*B*b^2/a^2*(b*x^2+a)^
(3/2)-3/8*B*b^2/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/8*B*b^2/a*(
b*x^2+a)^(1/2)
```

maxima [B] time = 1.09, size = 210, normalized size = 1.75

$$-\frac{3\,Bb^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,\sqrt{a}}+\frac{Ab^{3}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bb^{2}}{8\,a^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,Bb^{2}}{8\,a}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{3}}{48\,a^{3}}-\frac{\sqrt{bx^{2}+a}\,Ab^{3}}{16\,a^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bb}{8\,a^{2}x^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{2}}{48\,a^{3}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{4\,ax^{4}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab}{24\,a^{2}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{6\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] -3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*A*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(3/2) + 1/8*(b*x^2 + a)^(3/2)*B*b^2/a^2 + 3/8*sqrt(b*x^2
 + a)*B*b^2/a - 1/48*(b*x^2 + a)^(3/2)*A*b^3/a^3 - 1/16*sqrt(b*x^2 + a)*A*b
^3/a^2 - 1/8*(b*x^2 + a)^(5/2)*B*b/(a^2*x^2) + 1/48*(b*x^2 + a)^(5/2)*A*b^2
/(a^3*x^2) - 1/4*(b*x^2 + a)^(5/2)*B/(a*x^4) + 1/24*(b*x^2 + a)^(5/2)*A*b/(
a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)*A/(a*x^6)
```

mupad [B] time = 2.63, size = 130, normalized size = 1.08

$$\frac{A\,a\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{5\,B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}-\frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}+\frac{3\,B\,a\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a\,x^6}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)1{}\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^7,x)
```

```
[Out] (A*a*(a + b*x^2)^(1/2))/(16*x^6) - (5*B*(a + b*x^2)^(3/2))/(8*x^4) - (A*b^3
*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(3/2)) - (3*B*b^2*atanh((a
+ b*x^2)^(1/2)/a^(1/2)))/(8*a^(1/2)) - (A*(a + b*x^2)^(3/2))/(6*x^6) + (3*B
*a*(a + b*x^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(5/2))/(16*a*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**7,x)
```

```
[Out] Timed out
```

3.516 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=53

$$\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{\left(a+bx^2\right)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(7*a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(5/2))/(35*
a^2*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^8}\,dx=-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}-\frac{(2Ab-7aB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx}{7a}$$
$$=-\frac{A\left(a+bx^2\right)^{5/2}}{7ax^7}+\frac{(2Ab-7aB)\left(a+bx^2\right)^{5/2}}{35a^2x^5}$$

Mathematica [**A**] time = 0.02, size = 40, normalized size = 0.75

$$-\frac{\left(a+bx^2\right)^{5/2}\left(5aA+7aBx^2-2Abx^2\right)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -1/35*((a + b*x^2)^(5/2)*(5*a*A - 2*A*b*x^2 + 7*a*B*x^2))/(a^2*x^7)
```

IntegrateAlgebraic [**A**] time = 0.22, size = 86, normalized size = 1.62

$$\frac{\sqrt{a+bx^2}\left(-5a^3A-7a^3Bx^2-8a^2Abx^2-14a^2bBx^4-aAb^2x^4-7ab^2Bx^6+2Ab^3x^6\right)}{35a^2x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-5*a^3*A - 8*a^2*A*b*x^2 - 7*a^3*B*x^2 - a*A*b^2*x^4 - 14
*a^2*b*B*x^4 + 2*A*b^3*x^6 - 7*a*b^2*B*x^6))/(35*a^2*x^7)
```

fricas [**A**] time = 0.96, size = 78, normalized size = 1.47

$$-\frac{\left(\left(7\,Bab^2-2\,Ab^3\right)x^6+\left(14\,Ba^2b+Aab^2\right)x^4+5\,Aa^3+\left(7\,Ba^3+8\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{35\,a^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*((7*B*a*b^2 - 2*A*b^3)*x^6 + (14*B*a^2*b + A*a*b^2)*x^4 + 5*A*a^3 + (
7*B*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^2*x^7)
```

giac [**B**] time = 0.47, size = 344, normalized size = 6.49

$$\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{5}{2}}-70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{5}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{7}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^2b^{\frac{5}{2}}+70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aab^{\frac{7}{2}}-140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^3b^{\frac{5}{2}}+140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^2b^{\frac{7}{2}}+77\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^4b^{\frac{5}{2}}+28\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^3b^{\frac{7}{2}}-14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^5b^{\frac{5}{2}}+14\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^4b^{\frac{7}{2}}+7\,Ba^6b^{\frac{5}{2}}-2\,Aa^5b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(5/2) - 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*B*a*b^(5/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(7/2)
+ 105*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2) + 70*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*A*a*b^(7/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/
2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(7/2) + 77*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*B*a^4*b^(5/2) + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b
^(7/2) - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5/2) + 14*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*A*a^4*b^(7/2) + 7*B*a^6*b^(5/2) - 2*A*a^5*b^(7/2))/((sq
rt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.01, size = 37, normalized size = 0.70

$$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-2Ab\,x^{2}+7Ba\,x^{2}+5Aa\right)}{35a^{2}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x)
```

```
[Out] -1/35*(b*x^2+a)^(5/2)*(-2*A*b*x^2+7*B*a*x^2+5*A*a)/x^7/a^2
```

maxima [A] time = 1.05, size = 56, normalized size = 1.06

$$-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{5\,ax^{5}}+\frac{2\left(bx^{2}+a\right)^{\frac{5}{2}}Ab}{35\,a^{2}x^{5}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{7\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] -1/5*(b*x^2 + a)^(5/2)*B/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*A*b/(a^2*x^5) - 1
/7*(b*x^2 + a)^(5/2)*A/(a*x^7)
```

mupad [B] time = 1.52, size = 128, normalized size = 2.42

$$\frac{2\,A\,b^{3}\,\sqrt{b\,x^{2}+a}}{35\,a^{2}\,x}-\frac{8\,A\,b\,\sqrt{b\,x^{2}+a}}{35\,x^{5}}-\frac{B\,a\,\sqrt{b\,x^{2}+a}}{5\,x^{5}}-\frac{2\,B\,b\,\sqrt{b\,x^{2}+a}}{5\,x^{3}}-\frac{A\,b^{2}\,\sqrt{b\,x^{2}+a}}{35\,a\,x^{3}}-\frac{A\,a\,\sqrt{b\,x^{2}+a}}{7\,x^{7}}-\frac{B\,b^{2}\,\sqrt{b\,x^{2}+a}}{5\,a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^8,x)
```

```
[Out] (2*A*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*A*b*(a + b*x^2)^(1/2))/(35*x^5)
 - (B*a*(a + b*x^2)^(1/2))/(5*x^5) - (2*B*b*(a + b*x^2)^(1/2))/(5*x^3) - (A
*b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (A*a*(a + b*x^2)^(1/2))/(7*x^7) - (B*b
^2*(a + b*x^2)^(1/2))/(5*a*x)
```

sympy [B] time = 6.49, size = 518, normalized size = 9.77

$$-\frac{15Aa^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aa^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Aab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}-\frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2}-\frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**
2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(10
5*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b**(3/2)*s
qrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2)
+ 2*A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a*sqrt(b)*sqrt(a/(b*x**2)
 + 1)/(5*x**4) - 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(5/2)*sq
rt(a/(b*x**2) + 1)/(5*a)
```

3.517 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^9}\,dx$

Optimal. Leaf size=156

$$-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-8aB)}{128a^2x^2}+\frac{\left(a+bx^2\right)^{3/2}(3Ab-8aB)}{48ax^6}+\frac{b\sqrt{a+bx^2}\,(3Ab-8aB}{64ax^4}$$

Rubi [A] time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx^2}\,(3Ab-8aB)}{128a^2x^2}-\frac{b^3(3Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{b\sqrt{a+bx^2}\,(3Ab-8aB)}{64ax^4}+\frac{\left(a+bx^2\right)^{3/2}(3Ab-8aB)}{48ax^6}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (b*(3*A*b - 8*a*B)*Sqrt[a + b*x^2])/(64*a*x^4) + (b^2*(3*A*b - 8*a*B)*Sqrt[
a + b*x^2])/(128*a^2*x^2) + ((3*A*b - 8*a*B)*(a + b*x^2)^(3/2))/(48*a*x^6)
- (A*(a + b*x^2)^(5/2))/(8*a*x^8) - (b^3*(3*A*b - 8*a*B)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^9}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^5}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}+\frac{\left(-\frac{3Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^4}\,dx,x,x^2\right)}{8a}\\
&= \frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}-\frac{(b(3Ab-8aB))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3}\,dx}{32a}\\
&= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{A\left(a+bx^2\right)^{5/2}}{8ax^8}-\left(b^2(3A\right.\\
&= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}\\
&= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}\\
&= \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4}+\frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2}+\frac{(3Ab-8aB)\left(a+bx^2\right)^{3/2}}{48ax^6}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 62, normalized size = 0.40

$$
-\frac{\left(a+bx^2\right)^{5/2}\left(5a^4A+b^3x^8(3Ab-8aB)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx^2}{a}+1\right)\right)}{40a^5x^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] -1/40*((a + b*x^2)^(5/2)*(5*a^4*A + b^3*(3*A*b - 8*a*B)*x^8*Hypergeometric2
F1[5/2, 4, 7/2, 1 + (b*x^2)/a]))/(a^5*x^8)
```

IntegrateAlgebraic [A] time = 0.23, size = 128, normalized size = 0.82

$$
\frac{\left(8ab^3B-3Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}+\frac{\sqrt{a+bx^2}\left(-48a^3A-64a^3Bx^2-72a^2Abx^2-112a^2bBx^4-6aAb^2x^4-24ab^2Bx^6+9Ab^3x^6\right)}{384a^2x^8}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*A - 72*a^2*A*b*x^2 - 64*a^3*B*x^2 - 6*a*A*b^2*x^4
 - 112*a^2*b*B*x^4 + 9*A*b^3*x^6 - 24*a*b^2*B*x^6))/(384*a^2*x^8) + ((-3*A*
b^4 + 8*a*b^3*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

fricas [A] time = 0.71, size = 271, normalized size = 1.74

$$\left[-\frac{3 \left(8 \, B a b^{3} - 3 \, A b^{4}\right) \sqrt{a} x^{8} \log\left(-\frac{b x^{2} - 2 \, \sqrt{b x^{2} + a} \sqrt{a} + 2 \, a}{x^{2}}\right) + 2 \left(3 \left(8 \, B a^{2} b^{2} - 3 \, A a b^{3}\right) x^{6} + 48 \, A a^{4} + 2 \left(56 \, B a^{3} b + 3 \, A a^{2} b^{2}\right) x^{4} + 8 \left(8 \, B a^{4} + 9 \, A a^{3} b\right) x^{2}\right) \sqrt{b x^{2} + a}}{768 \, a^{3} x^{8}}, -\frac{3 \left(8 \, B a b^{3} - 3 \, A b^{4}\right) \sqrt{-a} x^{8} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2} + a}}\right) + \left(3 \left(8 \, B a^{2} b^{2} - 3 \, A a b^{3}\right) x^{6} + 48 \, A a^{4} + 2 \left(56 \, B a^{3} b + 3 \, A a^{2} b^{2}\right) x^{4} + 8 \left(8 \, B a^{4} + 9 \, A a^{3} b\right) x^{2}\right) \sqrt{b x^{2} + a}}{384 \, a^{3} x^{8}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
56*B*a^3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a
))/(a^3*x^8), -1/384*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(56*B*a^
3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^3
*x^8)]
```

giac [A] time = 0.41, size = 194, normalized size = 1.24

$$-\frac{\frac{3 \left(8 \, B a b^{4} - 3 \, A b^{5}\right) \arctan\left(\frac{\sqrt{b x^{2} + a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^{2}} + \frac{24 \left(b x^{2} + a\right)^{\frac{7}{2}} B a b^{4} + 40 \left(b x^{2} + a\right)^{\frac{5}{2}} B a^{2} b^{4} - 88 \left(b x^{2} + a\right)^{\frac{3}{2}} B a^{3} b^{4} + 24 \, \sqrt{b x^{2} + a} B a^{4} b^{4} - 9 \left(b x^{2} + a\right)^{\frac{7}{2}} A b^{5} + 33 \left(b x^{2} + a\right)^{\frac{5}{2}} A a b^{5} + 33 \left(b x^{2} + a\right)^{\frac{3}{2}} A a^{2} b^{5} - 9 \, \sqrt{b x^{2} + a} A a^{3} b^{5}}{a^{2} b^{4} x^{8}}}{384 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] -1/384*(3*(8*B*a*b^4 - 3*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*
a^2) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 + 40*(b*x^2 + a)^(5/2)*B*a^2*b^4 - 88*
(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 9*(b*x^2 + a)^
(7/2)*A*b^5 + 33*(b*x^2 + a)^(5/2)*A*a*b^5 + 33*(b*x^2 + a)^(3/2)*A*a^2*b^5
 - 9*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^2*b^4*x^8))/b
```

maple [B] time = 0.02, size = 275, normalized size = 1.76

$$-\frac{3 A \,b^{4} \ln \left(\frac{2 a +2 \sqrt{b \,x^{2}+a}\, \sqrt{a}}{x}\right)}{128 a^{\frac{5}{2}}}+\frac{B \,b^{3} \ln \left(\frac{2 a +2 \sqrt{b \,x^{2}+a}\, \sqrt{a}}{x}\right)}{16 a^{\frac{3}{2}}}+\frac{3 \sqrt{b \,x^{2}+a}\, A \,b^{4}}{128 a^{3}}-\frac{\sqrt{b \,x^{2}+a}\, B \,b^{3}}{16 a^{2}}+\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} A \,b^{4}}{128 a^{4}}-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}} B \,b^{3}}{48 a^{3}}-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} A \,b^{3}}{128 a^{4} x^{2}}+\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} B \,b^{2}}{48 a^{3} x^{2}}-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} A \,b^{2}}{64 a^{3} x^{4}}+\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} B b}{24 a^{2} x^{4}}+\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} A b}{16 a^{2} x^{6}}-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} B}{6 a \,x^{6}}-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}} A}{8 a \,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x)
```

```
[Out] -1/6*B/a/x^6*(b*x^2+a)^(5/2)+1/24*B*b/a^2/x^4*(b*x^2+a)^(5/2)+1/48*B*b^2/a^
3/x^2*(b*x^2+a)^(5/2)-1/48*B*b^3/a^3*(b*x^2+a)^(3/2)+1/16*B*b^3/a^(3/2)*ln(
```

```
(2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/16*B*b^3/a^2*(b*x^2+a)^(1/2)-1/8*A*(b*
x^2+a)^(5/2)/a/x^8+1/16*A*b/a^2/x^6*(b*x^2+a)^(5/2)-1/64*A*b^2/a^3/x^4*(b*x
^2+a)^(5/2)-1/128*A*b^3/a^4/x^2*(b*x^2+a)^(5/2)+1/128*A*b^4/a^4*(b*x^2+a)^(
3/2)-3/128*A*b^4/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/128*A*b^4/
a^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.12, size = 252, normalized size = 1.62

$$\frac{Bb^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{3}{2}}}-\frac{3\,Ab^{4}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{5}{2}}}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bb^{3}}{48\,a^{3}}-\frac{\sqrt{bx^{2}+a}\,Bb^{3}}{16\,a^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{4}}{128\,a^{4}}+\frac{3\,\sqrt{bx^{2}+a}\,Ab^{4}}{128\,a^{3}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bb^{2}}{48\,a^{3}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{3}}{128\,a^{4}x^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bb}{24\,a^{2}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{2}}{64\,a^{3}x^{4}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{6\,ax^{6}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab}{16\,a^{2}x^{6}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{8\,ax^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/128*A*b^4*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(5/2) - 1/48*(b*x^2 + a)^(3/2)*B*b^3/a^3 - 1/16*sqrt(b*
x^2 + a)*B*b^3/a^2 + 1/128*(b*x^2 + a)^(3/2)*A*b^4/a^4 + 3/128*sqrt(b*x^2 +
 a)*A*b^4/a^3 + 1/48*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^2) - 1/128*(b*x^2 + a)^
(5/2)*A*b^3/(a^4*x^2) + 1/24*(b*x^2 + a)^(5/2)*B*b/(a^2*x^4) - 1/64*(b*x^2
+ a)^(5/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(5/2)*B/(a*x^6) + 1/16*(b*x^2
+ a)^(5/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(5/2)*A/(a*x^8)
```

mupad [B] time = 3.57, size = 169, normalized size = 1.08

$$\frac{3\,A\,a\,\sqrt{b\,x^{2}+a}}{128\,x^{8}}-\frac{B\,\left(b\,x^{2}+a\right)^{3/2}}{6\,x^{6}}-\frac{11\,A\,\left(b\,x^{2}+a\right)^{3/2}}{128\,x^{8}}+\frac{B\,a\,\sqrt{b\,x^{2}+a}}{16\,x^{6}}-\frac{11\,A\,\left(b\,x^{2}+a\right)^{5/2}}{128\,a\,x^{8}}+\frac{3\,A\,\left(b\,x^{2}+a\right)^{7/2}}{128\,a^{2}\,x^{8}}-\frac{B\,\left(b\,x^{2}+a\right)^{5/2}}{16\,a\,x^{6}}+\frac{A\,b^{4}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{128\,a^{5/2}}-\frac{B\,b^{3}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^9,x)
```

```
[Out] (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (B*(a + b*x
^2)^(3/2))/(6*x^6) - (11*A*(a + b*x^2)^(3/2))/(128*x^8) - (B*b^3*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(3/2)) + (3*A*a*(a + b*x^2)^(1/2))/(12
8*x^8) + (B*a*(a + b*x^2)^(1/2))/(16*x^6) - (11*A*(a + b*x^2)^(5/2))/(128*a
*x^8) + (3*A*(a + b*x^2)^(7/2))/(128*a^2*x^8) - (B*(a + b*x^2)^(5/2))/(16*a
*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**9,x)
```

```
[Out] Timed out
```

3.518 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$-\frac{2b\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{315a^3x^5}+\frac{\left(a+bx^2\right)^{5/2}(4Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(5/2))/(9*a*x^9) + ((4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(63*
a^2*x^7) - (2*b*(4*A*b - 9*a*B)*(a + b*x^2)^(5/2))/(315*a^3*x^5)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{10}}\,dx &= -\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}-\frac{(4Ab-9aB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^8}\,dx}{9a}\\ &= -\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}+\frac{(4Ab-9aB)\left(a+bx^2\right)^{5/2}}{63a^2x^7}+\frac{(2b(4Ab-9aB))\int \frac{\left(a+bx^2\right)^{3/2}}{x^6}\,dx}{63a^2}\\ &= -\frac{A\left(a+bx^2\right)^{5/2}}{9ax^9}+\frac{(4Ab-9aB)\left(a+bx^2\right)^{5/2}}{63a^2x^7}-\frac{2b(4Ab-9aB)\left(a+bx^2\right)^{5/2}}{315a^3x^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.75

$$\frac{\left(a+bx^2\right)^{5/2}\left(-5a^2\left(7A+9Bx^2\right)+2abx^2\left(10A+9Bx^2\right)-8Ab^2x^4\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] ((a + b*x^2)^(5/2)*(-8*A*b^2*x^4 - 5*a^2*(7*A + 9*B*x^2) + 2*a*b*x^2*(10*A
+ 9*B*x^2)))/(315*a^3*x^9)
```

IntegrateAlgebraic [A] time = 0.25, size = 110, normalized size = 1.31

$$\frac{\sqrt{a+bx^2}\left(-35a^4A-45a^4Bx^2-50a^3Abx^2-72a^3bBx^4-3a^2Ab^2x^4-9a^2b^2Bx^6+4aAb^3x^6+18ab^3Bx^8-8Ab^4x^8\right)}{315a^3x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-35*a^4*A - 50*a^3*A*b*x^2 - 45*a^4*B*x^2 - 3*a^2*A*b^2*x
^4 - 72*a^3*b*B*x^4 + 4*a*A*b^3*x^6 - 9*a^2*b^2*B*x^6 - 8*A*b^4*x^8 + 18*a*
b^3*B*x^8))/(315*a^3*x^9)
```

fricas [A] time = 1.00, size = 105, normalized size = 1.25

$$\frac{\left(2\left(9\,Bab^3-4\,Ab^4\right)x^8-\left(9\,Ba^2b^2-4\,Aab^3\right)x^6-35\,Aa^4-3\left(24\,Ba^3b+Aa^2b^2\right)x^4-5\left(9\,Ba^4+10\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{315\,a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(9*B*a*b^3 - 4*A*b^4)*x^8 - (9*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 35*A*a
^4 - 3*(24*B*a^3*b + A*a^2*b^2)*x^4 - 5*(9*B*a^4 + 10*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^3*x^9)
```

giac [**B**] time = 0.49, size = 400, normalized size = 4.76

$$\frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{7}{2}}+840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{7}{2}}+1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Aab^{\frac{9}{2}}-819\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^3b^{\frac{7}{2}}+1764\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aa^2b^{\frac{9}{2}}+441\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^4b^{\frac{7}{2}}+504\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^3b^{\frac{9}{2}}-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^5b^{\frac{7}{2}}+144\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^4b^{\frac{9}{2}}+81\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^6b^{\frac{7}{2}}-36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^5b^{\frac{9}{2}}-9Ba^7b^{\frac{7}{2}}+4Aa^6b^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*b^(7/2) - 315*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*B*a*b^(7/2) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(9
/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(7/2) + 1260*(sqrt(b)*x
- sqrt(b*x^2 + a))^10*A*a*b^(9/2) - 819*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a
^3*b^(7/2) + 1764*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(9/2) + 441*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(7/2) + 504*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*A*a^3*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(7/2) + 144*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(9/2) + 81*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*B*a^6*b^(7/2) - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(9/2) - 9
*B*a^7*b^(7/2) + 4*A*a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

maple [**A**] time = 0.01, size = 59, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}\left(8A\,b^2x^4-18Bab\,x^4-20Aab\,x^2+45B\,a^2x^2+35a^2A\right)}{315a^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x)
```

```
[Out] -1/315*(b*x^2+a)^(5/2)*(8*A*b^2*x^4-18*B*a*b*x^4-20*A*a*b*x^2+45*B*a^2*x^2+
35*A*a^2)/x^9/a^3
```

maxima [**A**] time = 1.22, size = 96, normalized size = 1.14

$$\frac{2\left(bx^2+a\right)^{\frac{5}{2}}Bb}{35\,a^2x^5}-\frac{8\left(bx^2+a\right)^{\frac{5}{2}}Ab^2}{315\,a^3x^5}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{7\,ax^7}+\frac{4\left(bx^2+a\right)^{\frac{5}{2}}Ab}{63\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] 2/35*(b*x^2 + a)^(5/2)*B*b/(a^2*x^5) - 8/315*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x
^5) - 1/7*(b*x^2 + a)^(5/2)*B/(a*x^7) + 4/63*(b*x^2 + a)^(5/2)*A*b/(a^2*x^7
) - 1/9*(b*x^2 + a)^(5/2)*A/(a*x^9)
```

mupad [B] time = 2.13, size = 170, normalized size = 2.02

$$\frac{4\,A\,b^3\,\sqrt{b\,x^2+a}}{315\,a^2\,x^3}-\frac{10\,A\,b\,\sqrt{b\,x^2+a}}{63\,x^7}-\frac{B\,a\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{8\,B\,b\,\sqrt{b\,x^2+a}}{35\,x^5}-\frac{A\,b^2\,\sqrt{b\,x^2+a}}{105\,a\,x^5}-\frac{A\,a\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,A\,b^4\,\sqrt{b\,x^2+a}}{315\,a^3\,x}-\frac{B\,b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3}+\frac{2\,B\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^10,x)
```

```
[Out] (4*A*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*A*b*(a + b*x^2)^(1/2))/(63*
x^7) - (B*a*(a + b*x^2)^(1/2))/(7*x^7) - (8*B*b*(a + b*x^2)^(1/2))/(35*x^5)
 - (A*b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (A*a*(a + b*x^2)^(1/2))/(9*x^9)
- (8*A*b^4*(a + b*x^2)^(1/2))/(315*a^3*x) - (B*b^2*(a + b*x^2)^(1/2))/(35*a
*x^3) + (2*B*b^3*(a + b*x^2)^(1/2))/(35*a^2*x)
```

sympy [B] time = 7.15, size = 1408, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**4*b**(27/2)
*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**4*b**(13/2)*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) + 30*A*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 17*A*
a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) - 3*A*a**2*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a*b**(33/2)*x
**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*
a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 12*A*a*b**(19/2)*x**8*sqrt(a/(b*
```

```
x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
- 8*A*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b
**5*x**8 + 105*a**3*b**6*x**10) - 15*B*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**5*
b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**
8 + 105*a**3*b**6*x**10) - 17*B*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(1
05*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**3*b*
*(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8
+ 105*a**3*b**6*x**10) - 12*B*a**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105
*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a*b**(19/
2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(5/2)*
sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a*
*2)
```

3.519 $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{11}}\,dx$

Optimal. Leaf size=184

$$\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}-\frac{3b^3\sqrt{a+bx^2}\,(Ab-2aB)}{256a^3x^2}+\frac{b^2\sqrt{a+bx^2}\,(Ab-2aB)}{128a^2x^4}+\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{16ax^8}+$$

Rubi [A] time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{3b^3\sqrt{a+bx^2}\,(Ab-2aB)}{256a^3x^2}+\frac{b^2\sqrt{a+bx^2}\,(Ab-2aB)}{128a^2x^4}+\frac{3b^4(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}+\frac{b\sqrt{a+bx^2}\,(Ab-2aB)}{32ax^6}+\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{16ax^8}-\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(32*a*x^6) + (b^2*(A*b - 2*a*B)*Sqrt[a +
b*x^2])/(128*a^2*x^4) - (3*b^3*(A*b - 2*a*B)*Sqrt[a + b*x^2])/(256*a^3*x^2)
 + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/(16*a*x^8) - (A*(a + b*x^2)^(5/2))/(10
*a*x^10) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(7
/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^{11}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}} + \frac{\left(-\frac{5Ab}{2}+5aB\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= \frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8} - \frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}} - \frac{(3b(Ab-2aB))\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^4}\,dx,\right.}{32a}$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8} - \frac{A\left(a+bx^2\right)^{5/2}}{10ax^{10}} - \left(b^2(Ab-\right.$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} + \frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{16ax^8} -$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2} +$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2} +$$

$$= \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6} + \frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2} +$$

Mathematica [C] time = 0.04, size = 61, normalized size = 0.33

$$-\frac{\left(a+bx^2\right)^{5/2}\left(a^5A+b^4x^{10}(2aB-Ab)\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx^2}{a}+1\right)\right)}{10a^6x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] -1/10*((a + b*x^2)^(5/2)*(a^5*A + b^4*(-(A*b) + 2*a*B)*x^10*Hypergeometric2
F1[5/2, 5, 7/2, 1 + (b*x^2)/a]))/(a^6*x^10)
```

IntegrateAlgebraic [A] time = 0.26, size = 152, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(-128a^4A-160a^4Bx^2-176a^3Abx^2-240a^3bBx^4-8a^2Ab^2x^4-20a^2b^2Bx^6+10aAb^3x^6+30ab^3Bx^8-15Ab^4x^8\right)}{1280a^3x^{10}}-\frac{3\left(2ab^4B-Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-128*a^4*A - 176*a^3*A*b*x^2 - 160*a^4*B*x^2 - 8*a^2*A*b^
2*x^4 - 240*a^3*b*B*x^4 + 10*a*A*b^3*x^6 - 20*a^2*b^2*B*x^6 - 15*A*b^4*x^8
+ 30*a*b^3*B*x^8))/(1280*a^3*x^10) - (3*(-(A*b^5) + 2*a*b^4*B)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/(256*a^(7/2))
```

fricas [A] time = 1.08, size = 317, normalized size = 1.72

$$\left[-\frac{15\left(2Bab^4-Ab^5\right)\sqrt{a}x^{10}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(15\left(2Ba^2b^3-Aab^4\right)x^8-10\left(2Ba^3b^2-Aa^2b^3\right)x^6-128Aa^5-8\left(30Ba^4b+Aa^3b^2\right)x^4-16\left(10Ba^5+11Aa^4b\right)x^2\right)\sqrt{bx^2+a}}{2560a^4x^{10}}, \frac{15\left(2Bab^4-Ab^5\right)\sqrt{-a}x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15\left(2Ba^2b^3-Aab^4\right)x^8-10\left(2Ba^3b^2-Aa^2b^3\right)x^6-128Aa^5-8\left(30Ba^4b+Aa^3b^2\right)x^4-16\left(10Ba^5+11Aa^4b\right)x^2\right)\sqrt{bx^2+a}}{1280a^4x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(2*B*a*b^4 - A*b^5)*sqrt(a)*x^10*log(-(b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(a) + 2*a)/x^2) - 2*(15*(2*B*a^2*b^3 - A*a*b^4)*x^8 - 10*(2*B*a^3*b^
2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*b + A*a^3*b^2)*x^4 - 16*(10*B*
a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^10), 1/1280*(15*(2*B*a*b^4 -
 A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(2*B*a^2*b^3 -
 A*a*b^4)*x^8 - 10*(2*B*a^3*b^2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*
b + A*a^3*b^2)*x^4 - 16*(10*B*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*
x^10)]
```

giac [A] time = 0.48, size = 212, normalized size = 1.15

$$\frac{\frac{15\left(2Bab^5-Ab^6\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{30\left(bx^2+a\right)^{\frac{9}{2}}Bab^5-140\left(bx^2+a\right)^{\frac{7}{2}}Ba^2b^5+140\left(bx^2+a\right)^{\frac{3}{2}}Ba^4b^5-30\sqrt{bx^2+a}Ba^5b^5-15\left(bx^2+a\right)^{\frac{9}{2}}Ab^6+70\left(bx^2+a\right)^{\frac{7}{2}}Aab^6-128\left(bx^2+a\right)^{\frac{5}{2}}Aa^2b^6-70\left(bx^2+a\right)^{\frac{3}{2}}Aa^3b^6+15\sqrt{bx^2+a}Aa^4b^6}{a^3b^5x^{10}}}{1280b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/1280*(15*(2*B*a*b^5 - A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (30*(b*x^2 + a)^(9/2)*B*a*b^5 - 140*(b*x^2 + a)^(7/2)*B*a^2*b^5 + 140
*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 30*sqrt(b*x^2 + a)*B*a^5*b^5 - 15*(b*x^2 + a
)^(9/2)*A*b^6 + 70*(b*x^2 + a)^(7/2)*A*a*b^6 - 128*(b*x^2 + a)^(5/2)*A*a^2*
b^6 - 70*(b*x^2 + a)^(3/2)*A*a^3*b^6 + 15*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^3*b
^5*x^10))/b
```

maple [B] time = 0.04, size = 317, normalized size = 1.72

$$\frac{3Ab^5\ln\left(\frac{2a+2\sqrt{bx^2+a}\sqrt{a}}{x}\right)}{256a^{\frac{7}{2}}}-\frac{3Bb^4\ln\left(\frac{2a+2\sqrt{bx^2+a}\sqrt{a}}{x}\right)}{128a^{\frac{5}{2}}}-\frac{3\sqrt{bx^2+a}\,Ab^5}{256a^4}+\frac{3\sqrt{bx^2+a}\,Bb^4}{128a^3}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab^5}{256a^5}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bb^4}{128a^4}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^4}{256a^5x^2}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bb^3}{128a^4x^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^3}{128a^4x^4}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bb^2}{64a^3x^4}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^2}{32a^3x^6}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bb}{16a^2x^6}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab}{16a^2x^8}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{8ax^8}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{10ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(5/2)/a/x^10+1/16*A*b/a^2/x^8*(b*x^2+a)^(5/2)-1/32*A*b^2/
a^3/x^6*(b*x^2+a)^(5/2)+1/128*A*b^3/a^4/x^4*(b*x^2+a)^(5/2)+1/256*A*b^4/a^5
/x^2*(b*x^2+a)^(5/2)-1/256*A*b^5/a^5*(b*x^2+a)^(3/2)+3/256*A*b^5/a^(7/2)*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-3/256*A*b^5/a^4*(b*x^2+a)^(1/2)-1/8*B/a
/x^8*(b*x^2+a)^(5/2)+1/16*B*b/a^2/x^6*(b*x^2+a)^(5/2)-1/64*B*b^2/a^3/x^4*(b
*x^2+a)^(5/2)-1/128*B*b^3/a^4/x^2*(b*x^2+a)^(5/2)+1/128*B*b^4/a^4*(b*x^2+a)
^(3/2)-3/128*B*b^4/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/128*B*b^
4/a^3*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.20, size = 294, normalized size = 1.60

$$-\frac{3 Bb^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128 a^{\frac{5}{2}}} + \frac{3 Ab^5 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256 a^{\frac{7}{2}}} + \frac{\left(bx^2 + a\right)^{\frac{3}{2}} Bb^4}{128 a^4} + \frac{3 \sqrt{bx^2 + a} Bb^4}{128 a^3} - \frac{\left(bx^2 + a\right)^{\frac{3}{2}} Ab^5}{256 a^5} - \frac{3 \sqrt{bx^2 + a} Ab^5}{256 a^4} - \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Bb^3}{128 a^4 x^2} + \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Ab^4}{256 a^5 x^2} - \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Bb^2}{64 a^3 x^4} + \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Ab^3}{128 a^4 x^4} + \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Bb}{16 a^2 x^6} - \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Ab^2}{32 a^3 x^6} - \frac{\left(bx^2 + a\right)^{\frac{5}{2}} B}{8 ax^8} + \frac{\left(bx^2 + a\right)^{\frac{5}{2}} Ab}{16 a^2 x^8} - \frac{\left(bx^2 + a\right)^{\frac{5}{2}} A}{10 ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] -3/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/256*A*b^5*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) + 1/128*(b*x^2 + a)^(3/2)*B*b^4/a^4 + 3/128*sqr
t(b*x^2 + a)*B*b^4/a^3 - 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^5 - 3/256*sqrt(b*x
^2 + a)*A*b^5/a^4 - 1/128*(b*x^2 + a)^(5/2)*B*b^3/(a^4*x^2) + 1/256*(b*x^2
+ a)^(5/2)*A*b^4/(a^5*x^2) - 1/64*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^4) + 1/128
*(b*x^2 + a)^(5/2)*A*b^3/(a^4*x^4) + 1/16*(b*x^2 + a)^(5/2)*B*b/(a^2*x^6) -
 1/32*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(5/2)*B/(a*x^8) +
 1/16*(b*x^2 + a)^(5/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(5/2)*A/(a*x^10)
```

mupad [B] time = 4.89, size = 205, normalized size = 1.11

$$\frac{3 A a \sqrt{b x^2 + a}}{256 x^{10}} - \frac{11 B \left(b x^2 + a\right)^{3/2}}{128 x^8} - \frac{7 A \left(b x^2 + a\right)^{3/2}}{128 x^{10}} + \frac{3 B a \sqrt{b x^2 + a}}{128 x^8} - \frac{A \left(b x^2 + a\right)^{5/2}}{10 a x^{10}} + \frac{7 A \left(b x^2 + a\right)^{7/2}}{128 a^2 x^{10}} - \frac{3 A \left(b x^2 + a\right)^{9/2}}{256 a^3 x^{10}} - \frac{11 B \left(b x^2 + a\right)^{5/2}}{128 a x^8} + \frac{3 B \left(b x^2 + a\right)^{7/2}}{128 a^2 x^8} - \frac{A b^5 \operatorname{atan}\left(\frac{\sqrt{b x^2+a}\, 1\mathrm{i}}{\sqrt{a}}\right) 3\mathrm{i}}{256 a^{7/2}} + \frac{B b^4 \operatorname{atan}\left(\frac{\sqrt{b x^2+a}\, 1\mathrm{i}}{\sqrt{a}}\right) 3\mathrm{i}}{128 a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^11,x)
```

```
[Out] (B*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*B*(a +
b*x^2)^(3/2))/(128*x^8) - (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(
256*a^(7/2)) - (7*A*(a + b*x^2)^(3/2))/(128*x^10) + (3*A*a*(a + b*x^2)^(1/2
))/(256*x^10) + (3*B*a*(a + b*x^2)^(1/2))/(128*x^8) - (A*(a + b*x^2)^(5/2))
/(10*a*x^10) + (7*A*(a + b*x^2)^(7/2))/(128*a^2*x^10) - (3*A*(a + b*x^2)^(9
/2))/(256*a^3*x^10) - (11*B*(a + b*x^2)^(5/2))/(128*a*x^8) + (3*B*(a + b*x^
2)^(7/2))/(128*a^2*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

3.520 $\int x^5 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=103

$$\frac{a^2\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^4}+\frac{\left(a+bx^2\right)^{11/2}(Ab-3aB)}{11b^4}-\frac{a\left(a+bx^2\right)^{9/2}(2Ab-3aB)}{9b^4}+\frac{B\left(a+bx^2\right)^{13/2}}{13b^4}$$

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^4}+\frac{\left(a+bx^2\right)^{11/2}(Ab-3aB)}{11b^4}-\frac{a\left(a+bx^2\right)^{9/2}(2Ab-3aB)}{9b^4}+\frac{B\left(a+bx^2\right)^{13/2}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2
)^(9/2))/(9*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(11/2))/(11*b^4) + (B*(a + b*
x^2)^(13/2))/(13*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)^{5/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^{5/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{7/2}}{b^3}+\frac{(Ab-}{}\right.\right.\\ &= \frac{a^2(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{9/2}}{9b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)}{11b^4}\end{aligned}$$

Mathematica [**A**] time = 0.07, size = 78, normalized size = 0.76

$$\frac{\left(a+bx^2\right)^{7/2}\left(-48a^3B+8a^2b\left(13A+21Bx^2\right)-14ab^2x^2\left(26A+27Bx^2\right)+63b^3x^4\left(13A+11Bx^2\right)\right)}{9009b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-48*a^3*B + 63*b^3*x^4*(13*A + 11*B*x^2) + 8*a^2*b*(13*
A + 21*B*x^2) - 14*a*b^2*x^2*(26*A + 27*B*x^2)))/(9009*b^4)
```

IntegrateAlgebraic [**A**] time = 0.06, size = 80, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{7/2}\left(-48a^3B+104a^2Ab+168a^2bBx^2-364aAb^2x^2-378ab^2Bx^4+819Ab^3x^4+693b^3Bx^6\right)}{9009b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(104*a^2*A*b - 48*a^3*B - 364*a*A*b^2*x^2 + 168*a^2*b*B*
x^2 + 819*A*b^3*x^4 - 378*a*b^2*B*x^4 + 693*b^3*B*x^6))/(9009*b^4)
```

fricas [**A**] time = 0.68, size = 147, normalized size = 1.43

$$\frac{\left(693\,Bb^6x^{12}+63\left(27\,Bab^5+13\,Ab^6\right)x^{10}+7\left(159\,Ba^2b^4+299\,Aab^5\right)x^8-48\,Ba^6+104\,Aa^5b+\left(15\,Ba^3b^3+1469\,Aa^2b^4\right)x^6-3\left(6\,Ba^4b^2-13\,Aa^3b^3\right)x^4+4\left(6\,Ba^5b-13\,Aa^4b^2\right)x^2\right)\sqrt{bx^2+a}}{9009\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9009*(693*B*b^6*x^12 + 63*(27*B*a*b^5 + 13*A*b^6)*x^10 + 7*(159*B*a^2*b^4
 + 299*A*a*b^5)*x^8 - 48*B*a^6 + 104*A*a^5*b + (15*B*a^3*b^3 + 1469*A*a^2*b
```

```
^4)*x^6 - 3*(6*B*a^4*b^2 - 13*A*a^3*b^3)*x^4 + 4*(6*B*a^5*b - 13*A*a^4*b^2)
*x^2)*sqrt(b*x^2 + a)/b^4
```

giac [A] time = 0.34, size = 104, normalized size = 1.01

$$\frac{693\left(bx^2+a\right)^{\frac{13}{2}}B-2457\left(bx^2+a\right)^{\frac{11}{2}}Ba+3003\left(bx^2+a\right)^{\frac{9}{2}}Ba^2-1287\left(bx^2+a\right)^{\frac{7}{2}}Ba^3+819\left(bx^2+a\right)^{\frac{11}{2}}Ab-2002\left(bx^2+a\right)^{\frac{9}{2}}Aab+1287\left(bx^2+a\right)^{\frac{7}{2}}Aa^2b}{9009\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9009*(693*(b*x^2 + a)^(13/2)*B - 2457*(b*x^2 + a)^(11/2)*B*a + 3003*(b*x^
2 + a)^(9/2)*B*a^2 - 1287*(b*x^2 + a)^(7/2)*B*a^3 + 819*(b*x^2 + a)^(11/2)*
A*b - 2002*(b*x^2 + a)^(9/2)*A*a*b + 1287*(b*x^2 + a)^(7/2)*A*a^2*b)/b^4
```

maple [A] time = 0.01, size = 77, normalized size = 0.75

$$\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(693B\,x^6b^3+819A\,b^3x^4-378Ba\,b^2x^4-364Aa\,b^2x^2+168B\,a^2b\,x^2+104A\,a^2b-48B\,a^3\right)}{9009b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/9009*(b*x^2+a)^(7/2)*(693*B*b^3*x^6+819*A*b^3*x^4-378*B*a*b^2*x^4-364*A*a
*b^2*x^2+168*B*a^2*b*x^2+104*A*a^2*b-48*B*a^3)/b^4
```

maxima [A] time = 1.12, size = 132, normalized size = 1.28

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^6}{13\,b}-\frac{6\left(bx^2+a\right)^{\frac{7}{2}}Bax^4}{143\,b^2}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax^4}{11\,b}+\frac{8\left(bx^2+a\right)^{\frac{7}{2}}Ba^2x^2}{429\,b^3}-\frac{4\left(bx^2+a\right)^{\frac{7}{2}}Aax^2}{99\,b^2}-\frac{16\left(bx^2+a\right)^{\frac{7}{2}}Ba^3}{3003\,b^4}+\frac{8\left(bx^2+a\right)^{\frac{7}{2}}Aa^2}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/13*(b*x^2 + a)^(7/2)*B*x^6/b - 6/143*(b*x^2 + a)^(7/2)*B*a*x^4/b^2 + 1/11
*(b*x^2 + a)^(7/2)*A*x^4/b + 8/429*(b*x^2 + a)^(7/2)*B*a^2*x^2/b^3 - 4/99*(
b*x^2 + a)^(7/2)*A*a*x^2/b^2 - 16/3003*(b*x^2 + a)^(7/2)*B*a^3/b^4 + 8/693*
(b*x^2 + a)^(7/2)*A*a^2/b^3
```

mupad [B] time = 0.82, size = 136, normalized size = 1.32

$$\sqrt{b\,x^2+a}\,\left(\frac{B\,b^2\,x^{12}}{13}-\frac{48\,B\,a^6-104\,A\,a^5\,b}{9009\,b^4}+\frac{x^{10}\,\left(819\,A\,b^6+1701\,B\,a\,b^5\right)}{9009\,b^4}+\frac{a\,x^8\,\left(299\,A\,b+159\,B\,a\right)}{1287}+\frac{a^3\,x^4\,\left(13\,A\,b-6\,B\,a\right)}{3003\,b^2}-\frac{4\,a^4\,x^2\,\left(13\,A\,b-6\,B\,a\right)}{9009\,b^3}+\frac{a^2\,x^6\,\left(1469\,A\,b+15\,B\,a\right)}{9009\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((B*b^2*x^12)/13 - (48*B*a^6 - 104*A*a^5*b)/(9009*b^4) +
(x^10*(819*A*b^6 + 1701*B*a*b^5))/(9009*b^4) + (a*x^8*(299*A*b + 159*B*a))/
1287 + (a^3*x^4*(13*A*b - 6*B*a))/(3003*b^2) - (4*a^4*x^2*(13*A*b - 6*B*a))
/(9009*b^3) + (a^2*x^6*(1469*A*b + 15*B*a))/(9009*b))
```

sympy **[A]** time = 10.42, size = 313, normalized size = 3.04

$$\begin{cases} \frac{8Aa^{5}\sqrt{a+bx^{2}}}{693b^{3}} - \frac{4Aa^{4}x^{2}\sqrt{a+bx^{2}}}{693b^{2}} + \frac{Aa^{3}x^{4}\sqrt{a+bx^{2}}}{231b} + \frac{113Aa^{2}x^{6}\sqrt{a+bx^{2}}}{693} + \frac{23Aabx^{8}\sqrt{a+bx^{2}}}{99} + \frac{Ab^{2}x^{10}\sqrt{a+bx^{2}}}{11} - \frac{16Ba^{6}\sqrt{a+bx^{2}}}{3003b^{4}} + \frac{8Ba^{5}x^{2}\sqrt{a+bx^{2}}}{3003b^{3}} - \frac{2Ba^{4}x^{4}\sqrt{a+bx^{2}}}{1001b^{2}} + \frac{5Ba^{3}x^{6}\sqrt{a+bx^{2}}}{3003b} + \frac{53Ba^{2}x^{8}\sqrt{a+bx^{2}}}{429} + \frac{27Babx^{10}\sqrt{a+bx^{2}}}{143} + \frac{Bb^{2}x^{12}\sqrt{a+bx^{2}}}{13} & \text{for}\: b \neq 0 \\ a^{\frac{5}{2}} \left(\frac{Ax^{6}}{6} + \frac{Bx^{8}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((8*A*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*A*a**4*x**2*sqrt(a + b*
x**2)/(693*b**2) + A*a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*A*a**2*x**6*s
qrt(a + b*x**2)/693 + 23*A*a*b*x**8*sqrt(a + b*x**2)/99 + A*b**2*x**10*sqrt
(a + b*x**2)/11 - 16*B*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*B*a**5*x**2*sq
rt(a + b*x**2)/(3003*b**3) - 2*B*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5
*B*a**3*x**6*sqrt(a + b*x**2)/(3003*b) + 53*B*a**2*x**8*sqrt(a + b*x**2)/42
9 + 27*B*a*b*x**10*sqrt(a + b*x**2)/143 + B*b**2*x**12*sqrt(a + b*x**2)/13,
 Ne(b, 0)), (a**(5/2)*(A*x**6/6 + B*x**8/8), True))
```

3.521 $\int x^4 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx$

Optimal. Leaf size=221

$$\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}-\frac{a^4x\sqrt{a+bx^2}\,(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}\,(12Ab-5aB)}{1536b^2}+\frac{a^2x^5\sqrt{a+bx^2}\,(12}{384b}$$

Rubi [A] time = 0.10, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$-\frac{a^4x\sqrt{a+bx^2}\,(12Ab-5aB)}{1024b^3}+\frac{a^3x^3\sqrt{a+bx^2}\,(12Ab-5aB)}{1536b^2}+\frac{a^5(12Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{a^2x^5\sqrt{a+bx^2}\,(12Ab-5aB)}{384b}+\frac{ax^5\left(a+bx^2\right)^{3/2}(12Ab-5aB)}{192b}+\frac{x^5\left(a+bx^2\right)^{5/2}(12Ab-5aB)}{120b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -(a^4*(12*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(1024*b^3) + (a^3*(12*A*b - 5*a*B
)*x^3*Sqrt[a + b*x^2])/(1536*b^2) + (a^2*(12*A*b - 5*a*B)*x^5*Sqrt[a + b*x^
2])/(384*b) + (a*(12*A*b - 5*a*B)*x^5*(a + b*x^2)^(3/2))/(192*b) + ((12*A*b
 - 5*a*B)*x^5*(a + b*x^2)^(5/2))/(120*b) + (B*x^5*(a + b*x^2)^(7/2))/(12*b)
 + (a^5*(12*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^4 (a+bx^2)^{5/2} (A+Bx^2)\,dx &= \frac{Bx^5 (a+bx^2)^{7/2}}{12b} - \frac{(-12Ab+5aB)\int x^4 (a+bx^2)^{5/2}\,dx}{12b} \\
&= \frac{(12Ab-5aB)x^5 (a+bx^2)^{5/2}}{120b} + \frac{Bx^5 (a+bx^2)^{7/2}}{12b} + \frac{(a(12Ab-5aB))\int x^4 (a+b}{24b} \\
&= \frac{a(12Ab-5aB)x^5 (a+bx^2)^{3/2}}{192b} + \frac{(12Ab-5aB)x^5 (a+bx^2)^{5/2}}{120b} + \frac{Bx^5 (a+bx^2)}{12b} \\
&= \frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b} + \frac{a(12Ab-5aB)x^5 (a+bx^2)^{3/2}}{192b} + \frac{(12Ab-5aB)x}{12} \\
&= \frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2} + \frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b} + \frac{a(12Ab-5aB)x}{192} \\
&= -\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3} + \frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2} + \frac{a^2(12Ab-5aB}{384} \\
&= -\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3} + \frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2} + \frac{a^2(12Ab-5aB}{384} \\
&= -\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3} + \frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2} + \frac{a^2(12Ab-5aB}{384}
\end{aligned}
$$

Mathematica [A] time = 0.45, size = 172, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(75a^5B-10a^4b\left(18A+5Bx^2\right)+40a^3b^2x^2\left(3A+Bx^2\right)+48a^2b^3x^4\left(62A+45Bx^2\right)+64ab^4x^6\left(63A+50Bx^2\right)+256b^5x^8\left(6A+5Bx^2\right)\right)-\frac{15a^{9/2}(5aB-12Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{15360b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(75*a^5*B + 40*a^3*b^2*x^2*(3*A + B*x^2) + 256*
b^5*x^8*(6*A + 5*B*x^2) - 10*a^4*b*(18*A + 5*B*x^2) + 48*a^2*b^3*x^4*(62*A
+ 45*B*x^2) + 64*a*b^4*x^6*(63*A + 50*B*x^2)) - (15*a^(9/2)*(-12*A*b + 5*a*
B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(15360*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.29, size = 175, normalized size = 0.79

$$\frac{\left(5a^6B-12a^5Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{1024b^{7/2}}+\frac{\sqrt{a+bx^2}\left(75a^5Bx-180a^4Abx-50a^4bBx^3+120a^3Ab^2x^3+40a^3b^2Bx^5+2976a^2Ab^3x^5+2160a^2b^3Bx^7+4032aAb^4x^7+3200ab^4Bx^9+1536Ab^5x^9+1280b^5Bx^{11}\right)}{15360b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-180*a^4*A*b*x + 75*a^5*B*x + 120*a^3*A*b^2*x^3 - 50*a^4*
b*B*x^3 + 2976*a^2*A*b^3*x^5 + 40*a^3*b^2*B*x^5 + 4032*a*A*b^4*x^7 + 2160*a
^2*b^3*B*x^7 + 1536*A*b^5*x^9 + 3200*a*b^4*B*x^9 + 1280*b^5*B*x^11))/(15360
*b^3) + ((-12*a^5*A*b + 5*a^6*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1024
*b^(7/2))
```

fricas [A] time = 1.06, size = 355, normalized size = 1.61

$$\left[-\frac{15\left(5Ba^6-12Aa^5b\right)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(1280Bb^6x^{11}+128\left(25Bab^5+12Ab^6\right)x^9+144\left(15Ba^2b^4+28Aab^5\right)x^7+8\left(5Ba^3b^3+372Aa^2b^4\right)x^5-10\left(5Ba^4b^2-12Aa^3b^3\right)x^3+15\left(5Ba^5b-12Aa^4b^2\right)x\right)\sqrt{bx^2+a}}{30720b^4},\ \frac{15\left(5Ba^6-12Aa^5b\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(1280Bb^6x^{11}+128\left(25Bab^5+12Ab^6\right)x^9+144\left(15Ba^2b^4+28Aab^5\right)x^7+8\left(5Ba^3b^3+372Aa^2b^4\right)x^5-10\left(5Ba^4b^2-12Aa^3b^3\right)x^3+15\left(5Ba^5b-12Aa^4b^2\right)x\right)\sqrt{bx^2+a}}{15360b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9 +
144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^5 -
 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)*sqr
t(b*x^2 + a))/b^4, 1/15360*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + (1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9
 + 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^
5 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)*
sqrt(b*x^2 + a))/b^4]
```

giac [A] time = 0.35, size = 195, normalized size = 0.88

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bb^{2}x^{2}+\frac{25\,Bab^{11}+12\,Ab^{12}}{b^{10}}\right)x^{2}+\frac{9\left(15\,Ba^{2}b^{10}+28\,Aab^{11}\right)}{b^{10}}\right)x^{2}+\frac{5\,Ba^{3}b^{9}+372\,Aa^{2}b^{10}}{b^{10}}\right)x^{2}-\frac{5\left(5\,Ba^{4}b^{8}-12\,Aa^{3}b^{9}\right)}{b^{10}}\right)x^{2}+\frac{15\left(5\,Ba^{5}b^{7}-12\,Aa^{4}b^{8}\right)}{b^{10}}\right)\sqrt{bx^{2}+a}\,x+\frac{\left(5\,Ba^{6}-12\,Aa^{5}b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{1024\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*B*b^2*x^2 + (25*B*a*b^11 + 12*A*b^12)/b^10)*x^2 + 9
*(15*B*a^2*b^10 + 28*A*a*b^11)/b^10)*x^2 + (5*B*a^3*b^9 + 372*A*a^2*b^10)/b
^10)*x^2 - 5*(5*B*a^4*b^8 - 12*A*a^3*b^9)/b^10)*x^2 + 15*(5*B*a^5*b^7 - 12*
A*a^4*b^8)/b^10)*sqrt(b*x^2 + a)*x + 1/1024*(5*B*a^6 - 12*A*a^5*b)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.02, size = 257, normalized size = 1.16

$$\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,x^{5}}{12b}+\frac{3A\,a^{5}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{256b^{\frac{5}{2}}}-\frac{5B\,a^{6}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{1024b^{\frac{7}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\;A\,a^{4}x}{256b^{2}}-\frac{5\sqrt{b\,x^{2}+a}\;B\,a^{5}x}{1024b^{3}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,a^{3}x}{128b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A\,x^{3}}{10b}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,a^{4}x}{1536b^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ba\,x^{3}}{24b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,a^{2}x}{160b^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,a^{3}x}{384b^{3}}-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}Aax}{80b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,a^{2}x}{64b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/12*B*x^5*(b*x^2+a)^(7/2)/b-1/24*B*a/b^2*x^3*(b*x^2+a)^(7/2)+1/64*B*a^2/b^
3*x*(b*x^2+a)^(7/2)-1/384*B*a^3/b^3*x*(b*x^2+a)^(5/2)-5/1536*B*a^4/b^3*x*(b
*x^2+a)^(3/2)-5/1024*B*a^5/b^3*x*(b*x^2+a)^(1/2)-5/1024*B*a^6/b^(7/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))+1/10*A*x^3*(b*x^2+a)^(7/2)/b-3/80*A*a/b^2*x*(b*x^2
+a)^(7/2)+1/160*A*a^2/b^2*x*(b*x^2+a)^(5/2)+1/128*A*a^3/b^2*x*(b*x^2+a)^(3/
2)+3/256*A*a^4/b^2*x*(b*x^2+a)^(1/2)+3/256*A*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))
```

maxima [A] time = 1.12, size = 242, normalized size = 1.10

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Bx^{5}}{12\,b}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Bax^{3}}{24\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Ax^{3}}{10\,b}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Ba^{2}x}{64\,b^{3}}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ba^{3}x}{384\,b^{3}}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{4}x}{1536\,b^{3}}-\frac{5\sqrt{bx^{2}+a}\,Ba^{5}x}{1024\,b^{3}}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{2}}Aax}{80\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Aa^{2}x}{160\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Aa^{3}x}{128\,b^{2}}+\frac{3\sqrt{bx^{2}+a}\,Aa^{4}x}{256\,b^{2}}-\frac{5\,Ba^{6}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{3\,Aa^{5}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(7/2)*B*x^5/b - 1/24*(b*x^2 + a)^(7/2)*B*a*x^3/b^2 + 1/10*
(b*x^2 + a)^(7/2)*A*x^3/b + 1/64*(b*x^2 + a)^(7/2)*B*a^2*x/b^3 - 1/384*(b*x
^2 + a)^(5/2)*B*a^3*x/b^3 - 5/1536*(b*x^2 + a)^(3/2)*B*a^4*x/b^3 - 5/1024*s
qrt(b*x^2 + a)*B*a^5*x/b^3 - 3/80*(b*x^2 + a)^(7/2)*A*a*x/b^2 + 1/160*(b*x^
2 + a)^(5/2)*A*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*A*a^3*x/b^2 + 3/256*sqrt
(b*x^2 + a)*A*a^4*x/b^2 - 5/1024*B*a^6*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/2
56*A*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(B x^2 + A\right) \left(b x^2 + a\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^4*(A + B*x^2)*(a + b*x^2)^(5/2), x)
```

sympy **[A]** time = 83.28, size = 405, normalized size = 1.83

$$-\frac{3Aa^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129Aa^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73Aa^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29A\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{Ab^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{11}{2}}x}{1024b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{9}{2}}x^3}{3072b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{7}{2}}x^5}{1536b\sqrt{1+\frac{bx^2}{a}}}+\frac{55Ba^{\frac{5}{2}}x^7}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{67Ba^{\frac{3}{2}}bx^9}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{7B\sqrt{a}\,b^2x^{11}}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^6\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1024b^{\frac{7}{2}}}+\frac{Bb^3x^{13}}{12\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] -3*A*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - A*a**(7/2)*x**3/(256*b*sqrt
(1 + b*x**2/a)) + 129*A*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*A*a**(3
/2)*b*x**7/(160*sqrt(1 + b*x**2/a)) + 29*A*sqrt(a)*b**2*x**9/(80*sqrt(1 + b
*x**2/a)) + 3*A*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + A*b**3*x**11
/(10*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(11/2)*x/(1024*b**3*sqrt(1 + b*x*
*2/a)) + 5*B*a**(9/2)*x**3/(3072*b**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**5
/(1536*b*sqrt(1 + b*x**2/a)) + 55*B*a**(5/2)*x**7/(384*sqrt(1 + b*x**2/a))
+ 67*B*a**(3/2)*b*x**9/(192*sqrt(1 + b*x**2/a)) + 7*B*sqrt(a)*b**2*x**11/(2
4*sqrt(1 + b*x**2/a)) - 5*B*a**6*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(7/2)) +
 B*b**3*x**13/(12*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.522 $\int x^3 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=73

$$\frac{\left(a+bx^2\right)^{9/2} (Ab-2aB)}{9b^3}-\frac{a\left(a+bx^2\right)^{7/2} (Ab-aB)}{7b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

Rubi [**A**] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{9/2} (Ab-2aB)}{9b^3}-\frac{a\left(a+bx^2\right)^{7/2} (Ab-aB)}{7b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(9/
2))/(9*b^3) + (B*(a + b*x^2)^(11/2))/(11*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^{5/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{a(-Ab+aB)(a+bx)^{5/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{7/2}}{b^2}+\frac{B(a+bx)^{9/}}{b^2}\right.\right.\\ &= -\frac{a(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{9/2}}{9b^3}+\frac{B\left(a+bx^2\right)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.78

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2B-2ab\left(11A+14Bx^2\right)+7b^2x^2\left(11A+9Bx^2\right)\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(8*a^2*B + 7*b^2*x^2*(11*A + 9*B*x^2) - 2*a*b*(11*A + 14
*B*x^2)))/(693*b^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.77

$$\frac{\left(a+bx^2\right)^{7/2}\left(8a^2B-22aAb-28abBx^2+77Ab^2x^2+63b^2Bx^4\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-22*a*A*b + 8*a^2*B + 77*A*b^2*x^2 - 28*a*b*B*x^2 + 63*
b^2*B*x^4))/(693*b^3)
```

fricas [A] time = 0.72, size = 122, normalized size = 1.67

$$\frac{\left(63\,Bb^5x^{10}+7\left(23\,Bab^4+11\,Ab^5\right)x^8+\left(113\,Ba^2b^3+209\,Aab^4\right)x^6+8\,Ba^5-22\,Aa^4b+3\left(Ba^3b^2+55\,Aa^2b^3\right)x^4-\left(4\,Ba^4b-11\,Aa^3b^2\right)x^2\right)\sqrt{bx^2+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/693*(63*B*b^5*x^10 + 7*(23*B*a*b^4 + 11*A*b^5)*x^8 + (113*B*a^2*b^3 + 209
*A*a*b^4)*x^6 + 8*B*a^5 - 22*A*a^4*b + 3*(B*a^3*b^2 + 55*A*a^2*b^3)*x^4 - (
4*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

giac [A] time = 0.41, size = 73, normalized size = 1.00

$$\frac{63\left(bx^2+a\right)^{\frac{11}{2}}B-154\left(bx^2+a\right)^{\frac{9}{2}}Ba+99\left(bx^2+a\right)^{\frac{7}{2}}Ba^2+77\left(bx^2+a\right)^{\frac{9}{2}}Ab-99\left(bx^2+a\right)^{\frac{7}{2}}Aab}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/693*(63*(b*x^2 + a)^(11/2)*B - 154*(b*x^2 + a)^(9/2)*B*a + 99*(b*x^2 + a)
^(7/2)*B*a^2 + 77*(b*x^2 + a)^(9/2)*A*b - 99*(b*x^2 + a)^(7/2)*A*a*b)/b^3
```

maple [A] time = 0.01, size = 53, normalized size = 0.73

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-63B\,b^2x^4-77A\,b^2x^2+28Bab\,x^2+22abA-8a^2B\right)}{693b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] -1/693*(b*x^2+a)^(7/2)*(-63*B*b^2*x^4-77*A*b^2*x^2+28*B*a*b*x^2+22*A*a*b-8*
B*a^2)/b^3
```

maxima [A] time = 1.06, size = 90, normalized size = 1.23

$$\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^4}{11\,b}-\frac{4\left(bx^2+a\right)^{\frac{7}{2}}Bax^2}{99\,b^2}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax^2}{9\,b}+\frac{8\left(bx^2+a\right)^{\frac{7}{2}}Ba^2}{693\,b^3}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Aa}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*(b*x^2 + a)^(7/2)*B*x^4/b - 4/99*(b*x^2 + a)^(7/2)*B*a*x^2/b^2 + 1/9*(
b*x^2 + a)^(7/2)*A*x^2/b + 8/693*(b*x^2 + a)^(7/2)*B*a^2/b^3 - 2/63*(b*x^2
+ a)^(7/2)*A*a/b^2
```

mupad [B] time = 0.69, size = 115, normalized size = 1.58

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,B\,a^5-22\,A\,a^4\,b}{693\,b^3}+\frac{B\,b^2\,x^{10}}{11}+\frac{x^8\,\left(77\,A\,b^5+161\,B\,a\,b^4\right)}{693\,b^3}+\frac{a\,x^6\,\left(209\,A\,b+113\,B\,a\right)}{693}+\frac{a^3\,x^2\,\left(11\,A\,b-4\,B\,a\right)}{693\,b^2}+\frac{a^2\,x^4\,\left(55\,A\,b+B\,a\right)}{231\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*B*a^5 - 22*A*a^4*b)/(693*b^3) + (B*b^2*x^10)/11 + (x^
8*(77*A*b^5 + 161*B*a*b^4))/(693*b^3) + (a*x^6*(209*A*b + 113*B*a))/693 + (
a^3*x^2*(11*A*b - 4*B*a))/(693*b^2) + (a^2*x^4*(55*A*b + B*a))/(231*b))
```

sympy [A] time = 8.85, size = 260, normalized size = 3.56

$$\begin{cases} -\frac{2Aa^4\sqrt{a+bx^2}}{63b^2} + \frac{Aa^3x^2\sqrt{a+bx^2}}{63b} + \frac{5Aa^2x^4\sqrt{a+bx^2}}{21} + \frac{19Aabx^6\sqrt{a+bx^2}}{63} + \frac{Ab^2x^8\sqrt{a+bx^2}}{9} + \frac{8Ba^5\sqrt{a+bx^2}}{693b^3} - \frac{4Ba^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{Ba^3x^4\sqrt{a+bx^2}}{231b} + \frac{113Ba^2x^6\sqrt{a+bx^2}}{693} + \frac{23Babx^8\sqrt{a+bx^2}}{99} + \frac{Bb^2x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((-2*A*a**4*sqrt(a + b*x**2)/(63*b**2) + A*a**3*x**2*sqrt(a + b*x*
*2)/(63*b) + 5*A*a**2*x**4*sqrt(a + b*x**2)/21 + 19*A*a*b*x**6*sqrt(a + b*x
**2)/63 + A*b**2*x**8*sqrt(a + b*x**2)/9 + 8*B*a**5*sqrt(a + b*x**2)/(693*b
**3) - 4*B*a**4*x**2*sqrt(a + b*x**2)/(693*b**2) + B*a**3*x**4*sqrt(a + b*x
**2)/(231*b) + 113*B*a**2*x**6*sqrt(a + b*x**2)/693 + 23*B*a*b*x**8*sqrt(a
+ b*x**2)/99 + B*b**2*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(5/2)*(A*x*
*4/4 + B*x**6/6), True))
```

3.523 $\int x^2\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=188

$$-\frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^3x\sqrt{a+bx^2}\,(10Ab-3aB)}{256b^2}+\frac{a^2x^3\sqrt{a+bx^2}\,(10Ab-3aB)}{128b}+\frac{ax^3\left(a+bx^2\right)^{3/2}\,(}{96b}$$

Rubi [A] time = 0.09, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 279, 321, 217, 206}

$$\frac{a^3x\sqrt{a+bx^2}\,(10Ab-3aB)}{256b^2}-\frac{a^4(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{a^2x^3\sqrt{a+bx^2}\,(10Ab-3aB)}{128b}+\frac{ax^3\left(a+bx^2\right)^{3/2}(10Ab-3aB)}{96b}+\frac{x^3\left(a+bx^2\right)^{5/2}(10Ab-3aB)}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(256*b^2) + (a^2*(10*A*b - 3*a*B)*
x^3*Sqrt[a + b*x^2])/(128*b) + (a*(10*A*b - 3*a*B)*x^3*(a + b*x^2)^(3/2))/(
96*b) + ((10*A*b - 3*a*B)*x^3*(a + b*x^2)^(5/2))/(80*b) + (B*x^3*(a + b*x^2
)^(7/2))/(10*b) - (a^4*(10*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(256*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx &= \frac{Bx^3 \left(a+bx^2\right)^{7/2}}{10b} - \frac{(-10Ab+3aB)\int x^2 \left(a+bx^2\right)^{5/2} dx}{10b} \\
&= \frac{(10Ab-3aB)x^3 \left(a+bx^2\right)^{5/2}}{80b} + \frac{Bx^3 \left(a+bx^2\right)^{7/2}}{10b} + \frac{(a(10Ab-3aB))\int x^2 \left(a+\right.}{16b} \\
&= \frac{a(10Ab-3aB)x^3 \left(a+bx^2\right)^{3/2}}{96b} + \frac{(10Ab-3aB)x^3 \left(a+bx^2\right)^{5/2}}{80b} + \frac{Bx^3 \left(a+bx^2\right.}{10b} \\
&= \frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b} + \frac{a(10Ab-3aB)x^3 \left(a+bx^2\right)^{3/2}}{96b} + \frac{(10Ab-3aB}{8} \\
&= \frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2} + \frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b} + \frac{a(10Ab-3aB)x}{96} \\
&= \frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2} + \frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b} + \frac{a(10Ab-3aB)x}{96} \\
&= \frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2} + \frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b} + \frac{a(10Ab-3aB)x}{96}
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 151, normalized size = 0.80

$$
\frac{\sqrt{a+bx^2}\left(\frac{15a^{7/2}(3aB-10Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}} + \sqrt{b}\,x\left(-45a^4B+30a^3b\left(5A+Bx^2\right)+4a^2b^2x^2\left(295A+186Bx^2\right)+16ab^3x^4\left(85A+63Bx^2\right)+96b^4x^6\left(5A+4Bx^2\right)\right)\right)}{3840b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(-45*a^4*B + 30*a^3*b*(5*A + B*x^2) + 96*b^4*x^
6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(85*A + 63*B*x^2) + 4*a^2*b^2*x^2*(295*A +
 186*B*x^2)) + (15*a^(7/2)*(-10*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/
Sqrt[1 + (b*x^2)/a]))/(3840*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 151, normalized size = 0.80

$$\frac{\left(10a^4Ab-3a^5B\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{256b^{5/2}}+\frac{\sqrt{a+bx^2}\,\left(-45a^4Bx+150a^3Abx+30a^3bBx^3+1180a^2Ab^2x^3+744a^2b^2Bx^5+1360aAb^3x^5+1008ab^3Bx^7+480Ab^4x^7+384b^4Bx^9\right)}{3840b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(150*a^3*A*b*x - 45*a^4*B*x + 1180*a^2*A*b^2*x^3 + 30*a^3*
b*B*x^3 + 1360*a*A*b^3*x^5 + 744*a^2*b^2*B*x^5 + 480*A*b^4*x^7 + 1008*a*b^3
*B*x^7 + 384*b^4*B*x^9))/(3840*b^2) + ((10*a^4*A*b - 3*a^5*B)*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(256*b^(5/2))
```

fricas [A] time = 1.14, size = 308, normalized size = 1.64

$$\left[-\frac{15\left(3\,Ba^5-10\,Aa^4b\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(384\,Bb^5x^9+48\left(21\,Bab^4+10\,Ab^5\right)x^7+8\left(93\,Ba^2b^3+170\,Aab^4\right)x^5+10\left(3\,Ba^3b^2+118\,Aa^2b^3\right)x^3-15\left(3\,Ba^4b-10\,Aa^3b^2\right)x\right)\sqrt{bx^2+a}}{7680\,b^3},\,-\frac{15\left(3\,Ba^5-10\,Aa^4b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(384\,Bb^5x^9+48\left(21\,Bab^4+10\,Ab^5\right)x^7+8\left(93\,Ba^2b^3+170\,Aab^4\right)x^5+10\left(3\,Ba^3b^2+118\,Aa^2b^3\right)x^3-15\left(3\,Ba^4b-10\,Aa^3b^2\right)x\right)\sqrt{bx^2+a}}{3840\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(3*B*a^5 - 10*A*a^4*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) - 2*(384*B*b^5*x^9 + 48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(9
3*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*
(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(3*B*a^5 -
10*A*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*B*b^5*x^9 +
48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(93*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3
*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x
^2 + a))/b^3]
```

giac [A] time = 0.41, size = 165, normalized size = 0.88

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,Bb^2x^2+\frac{21\,Bab^9+10\,Ab^{10}}{b^8}\right)x^2+\frac{93\,Ba^2b^8+170\,Aab^9}{b^8}\right)x^2+\frac{5\left(3\,Ba^3b^7+118\,Aa^2b^8\right)}{b^8}\right)x^2-\frac{15\left(3\,Ba^4b^6-10\,Aa^3b^7\right)}{b^8}\right)\sqrt{bx^2+a}\,x-\frac{\left(3\,Ba^5-10\,Aa^4b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*B*b^2*x^2 + (21*B*a*b^9 + 10*A*b^10)/b^8)*x^2 + (93*B*a^
2*b^8 + 170*A*a*b^9)/b^8)*x^2 + 5*(3*B*a^3*b^7 + 118*A*a^2*b^8)/b^8)*x^2 -
15*(3*B*a^4*b^6 - 10*A*a^3*b^7)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(3*B*a^5 - 1
0*A*a^4*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 215, normalized size = 1.14

$$-\frac{5A\,a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}+\frac{3B\,a^{5}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{256b^{\frac{5}{2}}}-\frac{5\sqrt{b\,x^{2}+a}\;A\,a^{3}x}{128b}+\frac{3\sqrt{b\,x^{2}+a}\;B\,a^{4}x}{256b^{2}}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,a^{2}x}{192b}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,a^{3}x}{128b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,x^{3}}{10b}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Aax}{48b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,a^{2}x}{160b^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ax}{8b}-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}Bax}{80b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/10*B*x^3*(b*x^2+a)^(7/2)/b-3/80*B*a/b^2*x*(b*x^2+a)^(7/2)+1/160*B*a^2/b^2
*x*(b*x^2+a)^(5/2)+1/128*B*a^3/b^2*x*(b*x^2+a)^(3/2)+3/256*B*a^4/b^2*x*(b*x
^2+a)^(1/2)+3/256*B*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/8*A*x*(b*x^
2+a)^(7/2)/b-1/48*A*a/b*x*(b*x^2+a)^(5/2)-5/192*A*a^2/b*x*(b*x^2+a)^(3/2)-5
/128*A*a^3/b*x*(b*x^2+a)^(1/2)-5/128*A*a^4/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))
```

maxima [A] time = 1.06, size = 200, normalized size = 1.06

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Bx^{3}}{10\,b}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{2}}Bax}{80\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ba^{2}x}{160\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{3}x}{128\,b^{2}}+\frac{3\sqrt{bx^{2}+a}\,Ba^{4}x}{256\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Ax}{8\,b}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Aax}{48\,b}-\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Aa^{2}x}{192\,b}-\frac{5\sqrt{bx^{2}+a}\,Aa^{3}x}{128\,b}+\frac{3\,Ba^{5}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}-\frac{5\,Aa^{4}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*B*x^3/b - 3/80*(b*x^2 + a)^(7/2)*B*a*x/b^2 + 1/160*(
b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*B*a^3*x/b^2 + 3/256*
sqrt(b*x^2 + a)*B*a^4*x/b^2 + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2 + a
)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2 + a)
*A*a^3*x/b + 3/256*B*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*A*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] int(x^2*(A + B*x^2)*(a + b*x^2)^(5/2), x)
```

sympy [B] time = 59.84, size = 348, normalized size = 1.85

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^3x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}-\frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129Ba^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73Ba^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29B\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{Bb^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) - 3*B*a**(9/2)*x/
(256*b**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a))
+ 129*B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*
sqrt(1 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*
a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqr
t(1 + b*x**2/a))
```

3.524 $\int x \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$

Optimal. Leaf size=46

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (B*(a + b*x^2)^(9/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^{5/2}(A+Bx)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{5/2}}{b}+\frac{B(a+bx)^{7/2}}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(-2aB+9Ab+7bBx^2\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x^2))/(63*b^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{\left(a+bx^2\right)^{7/2}\left(-2aB+9Ab+7bBx^2\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] ((a + b*x^2)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x^2))/(63*b^2)
```

fricas [B] time = 0.91, size = 97, normalized size = 2.11

$$\frac{\left(7\,Bb^4x^8+\left(19\,Bab^3+9\,Ab^4\right)x^6-2\,Ba^4+9\,Aa^3b+3\left(5\,Ba^2b^2+9\,Aab^3\right)x^4+\left(Ba^3b+27\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/63*(7*B*b^4*x^8 + (19*B*a*b^3 + 9*A*b^4)*x^6 - 2*B*a^4 + 9*A*a^3*b + 3*(5
*B*a^2*b^2 + 9*A*a*b^3)*x^4 + (B*a^3*b + 27*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)
/b^2
```

giac [A] time = 0.46, size = 44, normalized size = 0.96

$$\frac{7\left(bx^2+a\right)^{\frac{9}{2}}B-9\left(bx^2+a\right)^{\frac{7}{2}}Ba+9\left(bx^2+a\right)^{\frac{7}{2}}Ab}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/63*(7*(b*x^2 + a)^(9/2)*B - 9*(b*x^2 + a)^(7/2)*B*a + 9*(b*x^2 + a)^(7/2)
*A*b)/b^2
```

maple [A] time = 0.00, size = 31, normalized size = 0.67

$$\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(7Bb\,x^{2}+9Ab-2Ba\right)}{63b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/63*(b*x^2+a)^(7/2)*(7*B*b*x^2+9*A*b-2*B*a)/b^2
```

maxima [A] time = 1.03, size = 50, normalized size = 1.09

$$\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}Bx^{2}}{9\,b}-\frac{2\left(bx^{2}+a\right)^{\frac{7}{2}}Ba}{63\,b^{2}}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}A}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*B*x^2/b - 2/63*(b*x^2 + a)^(7/2)*B*a/b^2 + 1/7*(b*x^2
 + a)^(7/2)*A/b
```

mupad [B] time = 0.67, size = 44, normalized size = 0.96

$$\frac{7\,B\left(b\,x^{2}+a\right)^{9/2}+9\,A\,b\left(b\,x^{2}+a\right)^{7/2}-9\,B\,a\left(b\,x^{2}+a\right)^{7/2}}{63\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] (7*B*(a + b*x^2)^(9/2) + 9*A*b*(a + b*x^2)^(7/2) - 9*B*a*(a + b*x^2)^(7/2))
/(63*b^2)
```

sympy [A] time = 5.66, size = 209, normalized size = 4.54

$$\begin{cases} \frac{Aa^{3}\sqrt{a+bx^{2}}}{7b}+\frac{3Aa^{2}x^{2}\sqrt{a+bx^{2}}}{7}+\frac{3Aabx^{4}\sqrt{a+bx^{2}}}{7}+\frac{Ab^{2}x^{6}\sqrt{a+bx^{2}}}{7}-\frac{2Ba^{4}\sqrt{a+bx^{2}}}{63b^{2}}+\frac{Ba^{3}x^{2}\sqrt{a+bx^{2}}}{63b}+\frac{5Ba^{2}x^{4}\sqrt{a+bx^{2}}}{21}+\frac{19Babx^{6}\sqrt{a+bx^{2}}}{63}+\frac{Bb^{2}x^{8}\sqrt{a+bx^{2}}}{9} & \text{for}\: b \neq 0 \\ a^{\frac{5}{2}}\left(\frac{Ax^{2}}{2}+\frac{Bx^{4}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] Piecewise((A*a**3*sqrt(a + b*x**2)/(7*b) + 3*A*a**2*x**2*sqrt(a + b*x**2)/7
 + 3*A*a*b*x**4*sqrt(a + b*x**2)/7 + A*b**2*x**6*sqrt(a + b*x**2)/7 - 2*B*a
**4*sqrt(a + b*x**2)/(63*b**2) + B*a**3*x**2*sqrt(a + b*x**2)/(63*b) + 5*B*
a**2*x**4*sqrt(a + b*x**2)/21 + 19*B*a*b*x**6*sqrt(a + b*x**2)/63 + B*b**2*
x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**4/4), True))
```

3.525 $\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx$

Optimal. Leaf size=149

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}\,(8Ab-aB)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8Ab-aB)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8Ab}{192b}$$

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {388, 195, 217, 206}

$$\frac{5a^3(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}\,(8Ab-aB)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8Ab-aB)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8Ab-aB)}{192b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*A*b - a*B)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*A*b - a*B)*x*(a + b*x^2)^(5/2))/(48*b) + (B*x*
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)dx = \frac{Bx\left(a+bx^2\right)^{7/2}}{8b} - \frac{(-8Ab+aB)\int \left(a+bx^2\right)^{5/2}dx}{8b}$$

$$= \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx\left(a+bx^2\right)^{7/2}}{8b} + \frac{(5a(8Ab-aB))\int \left(a+bx^2\right)^{3/2}dx}{48b}$$

$$= \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\left(a+bx^2\right)^{5/2}}{48b} + \frac{Bx\left(a+bx^2\right)^{7/2}}{8b} + \big(5a$$

$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\big(a+bx^2}{48b}$$

$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\big(a+bx^2}{48b}$$

$$= \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x\left(a+bx^2\right)^{3/2}}{192b} + \frac{(8Ab-aB)x\big(a+bx^2}{48b}$$

Mathematica [A] time = 0.23, size = 130, normalized size = 0.87

$$\frac{\sqrt{a+bx^2}\left(\sqrt{b}\,x\left(15a^3B+2a^2b\left(132A+59Bx^2\right)+8ab^2x^2\left(26A+17Bx^2\right)+16b^3x^4\left(4A+3Bx^2\right)\right)-\frac{15a^{5/2}(aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[b]*x*(15*a^3*B + 16*b^3*x^4*(4*A + 3*B*x^2) + 8*a*b^
2*x^2*(26*A + 17*B*x^2) + 2*a^2*b*(132*A + 59*B*x^2)) - (15*a^(5/2)*(-8*A*b
 + a*B)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[1 + (b*x^2)/a]))/(384*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 126, normalized size = 0.85

$$\frac{5\left(a^4B-8a^3Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{128b^{3/2}}+\frac{\sqrt{a+bx^2}\left(15a^3Bx+264a^2Abx+118a^2bBx^3+208aAb^2x^3+136ab^2Bx^5+64Ab^3x^5+48b^3Bx^7\right)}{384b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(264*a^2*A*b*x + 15*a^3*B*x + 208*a*A*b^2*x^3 + 118*a^2*b*
B*x^3 + 64*A*b^3*x^5 + 136*a*b^2*B*x^5 + 48*b^3*B*x^7))/(384*b) + (5*(-8*a^
3*A*b + a^4*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

fricas [A] time = 1.09, size = 257, normalized size = 1.72

$$\left[-\frac{15\left(Ba^{4}-8\,Aa^{3}b\right)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left(48\,Bb^{4}x^{7}+8\left(17\,Bab^{3}+8\,Ab^{4}\right)x^{5}+2\left(59\,Ba^{2}b^{2}+104\,Aab^{3}\right)x^{3}+3\left(5\,Ba^{3}b+88\,Aa^{2}b^{2}\right)x\right)\sqrt{bx^{2}+a}}{768\,b^{2}},\ \frac{15\left(Ba^{4}-8\,Aa^{3}b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(48\,Bb^{4}x^{7}+8\left(17\,Bab^{3}+8\,Ab^{4}\right)x^{5}+2\left(59\,Ba^{2}b^{2}+104\,Aab^{3}\right)x^{3}+3\left(5\,Ba^{3}b+88\,Aa^{2}b^{2}\right)x\right)\sqrt{bx^{2}+a}}{384\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2
*b^2 + 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/
b^2, 1/384*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2*b^2 + 104*
A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

giac [A] time = 0.51, size = 134, normalized size = 0.90

$$\frac{1}{384}\left(2\left(4\left(6\,Bb^{2}x^{2}+\frac{17\,Bab^{7}+8\,Ab^{8}}{b^{6}}\right)x^{2}+\frac{59\,Ba^{2}b^{6}+104\,Aab^{7}}{b^{6}}\right)x^{2}+\frac{3\left(5\,Ba^{3}b^{5}+88\,Aa^{2}b^{6}\right)}{b^{6}}\right)\sqrt{bx^{2}+a}\,x+\frac{5\left(Ba^{4}-8\,Aa^{3}b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*B*b^2*x^2 + (17*B*a*b^7 + 8*A*b^8)/b^6)*x^2 + (59*B*a^2*b^6
+ 104*A*a*b^7)/b^6)*x^2 + 3*(5*B*a^3*b^5 + 88*A*a^2*b^6)/b^6)*sqrt(b*x^2 +
a)*x + 5/128*(B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

maple [A] time = 0.01, size = 166, normalized size = 1.11

$$\frac{5A\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}-\frac{5B\,a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}+\frac{5\sqrt{b\,x^{2}+a}\,A\,a^{2}x}{16}-\frac{5\sqrt{b\,x^{2}+a}\,B\,a^{3}x}{128b}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}Aax}{24}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,a^{2}x}{192b}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Ax}{6}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bax}{48b}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}Bx}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A),x)
```

```
[Out] 1/8*B*x*(b*x^2+a)^(7/2)/b-1/48*B*a/b*x*(b*x^2+a)^(5/2)-5/192*B*a^2/b*x*(b*x
^2+a)^(3/2)-5/128*B*a^3/b*x*(b*x^2+a)^(1/2)-5/128*B*a^4/b^(3/2)*ln(b^(1/2)*
```

```
x+(b*x^2+a)^(1/2))+1/6*A*x*(b*x^2+a)^(5/2)+5/24*A*a*x*(b*x^2+a)^(3/2)+5/16*
A*a^2*x*(b*x^2+a)^(1/2)+5/16*A*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.13, size = 151, normalized size = 1.01

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^2+a}\,Aa^2x+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{192\,b}-\frac{5\sqrt{bx^2+a}\,Ba^3x}{128\,b}-\frac{5\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{5\,Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
+ a)*A*a^2*x + 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b
 - 5/192*(b*x^2 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/
128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b)
)/sqrt(b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2)^(5/2),x)
```

```
[Out] int((A + B*x^2)*(a + b*x^2)^(5/2), x)
```

sympy [B] time = 32.84, size = 316, normalized size = 2.12

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Ba^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Ba^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23B\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Bb^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(7/2)*x/(128*b*sqrt(1 + b*
x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**(3/2)*b*
x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 + b*x**2/
a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**9/(8*sqr
t(a)*sqrt(1 + b*x**2/a))
```

3.526 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx$

Optimal. Leaf size=95

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+a^2A\sqrt{a+bx^2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$a^2A\sqrt{a+bx^2}-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] a^2*A*Sqrt[a + b*x^2] + (a*A*(a + b*x^2)^(3/2))/3 + (A*(a + b*x^2)^(5/2))/5
 + (B*(a + b*x^2)^(7/2))/(7*b) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x}\,dx, x, x^2\right)\\
&= \frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{2}A\,\mathrm{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{2}(aA)\,\mathrm{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{2}\left(a^2A\right)\mathrm{Subst}\left(\int \frac{\sqrt{a+}}{x}\right.\\
&= a^2A\sqrt{a+bx^2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{1}{2}\left(a^3A\right)\\
&= a^2A\sqrt{a+bx^2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{\left(a^3A\right)\mathrm{Su}}{}\\
&= a^2A\sqrt{a+bx^2}+\frac{1}{3}aA\left(a+bx^2\right)^{3/2}+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}-a^{5/2}A\,\mathrm{ta}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 88, normalized size = 0.93

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{5}A\left(a+bx^2\right)^{5/2}+\frac{1}{3}aA\left(4a+bx^2\right)\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + (B*(a + b*x^2)^(7/2))/(7*b) + (a*A*Sqrt[a + b*x^2
]*(4*a + b*x^2))/3 - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.08, size = 107, normalized size = 1.13

$$\frac{\sqrt{a+bx^2}\left(15a^3B+161a^2Ab+45a^2bBx^2+77aAb^2x^2+45ab^2Bx^4+21Ab^3x^4+15b^3Bx^6\right)}{105b}-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

```
[Out] (Sqrt[a + b*x^2]*(161*a^2*A*b + 15*a^3*B + 77*a*A*b^2*x^2 + 45*a^2*b*B*x^2
+ 21*A*b^3*x^4 + 45*a*b^2*B*x^4 + 15*b^3*B*x^6))/(105*b) - a^(5/2)*A*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]]
```

fricas [A] time = 1.11, size = 220, normalized size = 2.32

$$\left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(15\,Bb^3x^6+3\left(15\,Bab^2+7\,Ab^3\right)x^4+15\,Ba^3+161\,Aa^2b+\left(45\,Ba^2b+77\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{210\,b},\frac{105\,A\sqrt{-a}\,a^2b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15\,Bb^3x^6+3\left(15\,Bab^2+7\,Ab^3\right)x^4+15\,Ba^3+161\,Aa^2b+\left(45\,Ba^2b+77\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{105\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b
+ (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/b, 1/105*(105*A*sqrt(-a)*
a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*
A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b
*x^2 + a))/b]
```

giac [A] time = 0.37, size = 97, normalized size = 1.02

$$\frac{Aa^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\left(bx^2+a\right)^{\frac{7}{2}}Bb^6+21\left(bx^2+a\right)^{\frac{5}{2}}Ab^7+35\left(bx^2+a\right)^{\frac{3}{2}}Aab^7+105\sqrt{bx^2+a}\,Aa^2b^7}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="giac")
```

```
[Out] A*a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/105*(15*(b*x^2 + a)^(7/
2)*B*b^6 + 21*(b*x^2 + a)^(5/2)*A*b^7 + 35*(b*x^2 + a)^(3/2)*A*a*b^7 + 105*
sqrt(b*x^2 + a)*A*a^2*b^7)/b^7
```

maple [A] time = 0.01, size = 85, normalized size = 0.89

$$-A\,a^{\frac{5}{2}}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}\,A\,a^2+\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}Aa}{3}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A}{5}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x,x)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+1/5*A*(b*x^2+a)^(5/2)+1/3*a*A*(b*x^2+a)^(3/2)-A*a^(
5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+a^2*A*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.06, size = 73, normalized size = 0.77

$$-Aa^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}A+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Aa+\sqrt{bx^2+a}\,Aa^2+\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

```
[Out] -A*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b
*x^2 + a)^(3/2)*A*a + sqrt(b*x^2 + a)*A*a^2 + 1/7*(b*x^2 + a)^(7/2)*B/b
```

mupad [B] time = 1.03, size = 78, normalized size = 0.82

$$\frac{A\left(b\,x^2+a\right)^{5/2}}{5}+A\,a^2\,\sqrt{b\,x^2+a}+\frac{B\left(b\,x^2+a\right)^{7/2}}{7\,b}+\frac{A\,a\left(b\,x^2+a\right)^{3/2}}{3}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\;1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x,x)
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + (B*(a + b*x^2)^(7/2))/(
7*b) + A*a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)
^(3/2))/3
```

sympy [A] time = 82.47, size = 88, normalized size = 0.93

$$\frac{Aa^3\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}+Aa^2\sqrt{a+bx^2}+\frac{Aa\left(a+bx^2\right)^{\frac{3}{2}}}{3}+\frac{A\left(a+bx^2\right)^{\frac{5}{2}}}{5}+\frac{B\left(a+bx^2\right)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x,x)
```

```
[Out] A*a**3*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + A*a**2*sqrt(a + b*x**2) +
 A*a*(a + b*x**2)**(3/2)/3 + A*(a + b*x**2)**(5/2)/5 + B*(a + b*x**2)**(7/2
)/(7*b)
```

3.527 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^2}\,dx$

Optimal. Leaf size=136

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}\,(aB+6A$$

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 195, 217, 206}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{x\left(a+bx^2\right)^{5/2}(aB+6Ab)}{6a}+\frac{5}{24}x\left(a+bx^2\right)^{3/2}(aB+6Ab)+\frac{5}{16}ax\sqrt{a+bx^2}\,(aB+6Ab)-\frac{A\left(a+bx^2\right)^{7/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (5*a*(6*A*b + a*B)*x*Sqrt[a + b*x^2])/16 + (5*(6*A*b + a*B)*x*(a + b*x^2)^(
3/2))/24 + ((6*A*b + a*B)*x*(a + b*x^2)^(5/2))/(6*a) - (A*(a + b*x^2)^(7/2)
)/(a*x) + (5*a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sq
rt[b])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^2}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{ax} - \frac{(-6Ab-aB)\int \left(a+bx^2\right)^{5/2}\,dx}{a}$$

$$= \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a} - \frac{A\left(a+bx^2\right)^{7/2}}{ax} + \frac{1}{6}(5(6Ab+aB))\int \left(a+bx^2\right)^{3/2}\,dx$$

$$= \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x\left(a+bx^2\right)^{5/2}}{6a} - \frac{A\left(a+bx^2\right)^{7/2}}{ax} + \frac{1}{8}($$

$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x(a+b}{6a}$$

$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x(a+b}{6a}$$

$$= \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2} + \frac{5}{24}(6Ab+aB)x\left(a+bx^2\right)^{3/2} + \frac{(6Ab+aB)x(a+b}{6a}$$

Mathematica [A] time = 0.36, size = 125, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(\frac{(aB+6Ab)\left(15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\left(33a^2+26abx^2+8b^2x^4\right)\right)}{\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}-\frac{48A\left(a+bx^2\right)^3}{x}\right)}{48a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*((-48*A*(a + b*x^2)^3)/x + ((6*A*b + a*B)*(Sqrt[b]*x*Sqrt[
1 + (b*x^2)/a]*(33*a^2 + 26*a*b*x^2 + 8*b^2*x^4) + 15*a^(5/2)*ArcSinh[(Sqrt
[b]*x)/Sqrt[a]]))/(Sqrt[b]*Sqrt[1 + (b*x^2)/a])))/(48*a)
```

IntegrateAlgebraic [A] time = 0.25, size = 112, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\left(-48a^2A+33a^2Bx^2+54aAbx^2+26abBx^4+12Ab^2x^4+8b^2Bx^6\right)}{48x}-\frac{5\left(a^3B+6a^2Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^2*A + 54*a*A*b*x^2 + 33*a^2*B*x^2 + 12*A*b^2*x^4 +
26*a*b*B*x^4 + 8*b^2*B*x^6))/(48*x) - (5*(6*a^2*A*b + a^3*B)*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

fricas [A] time = 1.13, size = 236, normalized size = 1.74

$$\left[\frac{15\left(Ba^3+6\,Aa^2b\right)\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(8\,Bb^3x^6+2\left(13\,Bab^2+6\,Ab^3\right)x^4-48\,Aa^2b+3\left(11\,Ba^2b+18\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{96\,bx},-\frac{15\left(Ba^3+6\,Aa^2b\right)\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(8\,Bb^3x^6+2\left(13\,Bab^2+6\,Ab^3\right)x^4-48\,Aa^2b+3\left(11\,Ba^2b+18\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{48\,bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 2*(8*B*b^3*x^6 + 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b +
 3*(11*B*a^2*b + 18*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(b*x), -1/48*(15*(B*a^3
+ 6*A*a^2*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^6 +
 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^2)*x^
2)*sqrt(b*x^2 + a))/(b*x)]
```

giac [A] time = 0.54, size = 146, normalized size = 1.07

$$\frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{48}\left(2\left(4\,Bb^2x^2+\frac{13\,Bab^5+6\,Ab^6}{b^4}\right)x^2+\frac{3\left(11\,Ba^2b^4+18\,Aab^5\right)}{b^4}\right)\sqrt{bx^2+a}\,x-\frac{5\left(Ba^3\sqrt{b}+6\,Aa^2b^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

```
[Out] 2*A*a^3*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/48*(2*(4*B*b^2*x^
2 + (13*B*a*b^5 + 6*A*b^6)/b^4)*x^2 + 3*(11*B*a^2*b^4 + 18*A*a*b^5)/b^4)*sq
rt(b*x^2 + a)*x - 5/32*(B*a^3*sqrt(b) + 6*A*a^2*b^(3/2))*log((sqrt(b)*x - s
qrt(b*x^2 + a))^2)/b
```

maple [A] time = 0.01, size = 158, normalized size = 1.16

$$\frac{15A\,a^2\sqrt{b}\;\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{8}+\frac{5B\,a^3\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}+\frac{15\sqrt{b\,x^2+a}\;Aabx}{8}+\frac{5\sqrt{b\,x^2+a}\;B\,a^2x}{16}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Abx}{4}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}Bax}{24}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Abx}{a}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}Bx}{6}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x)
```

```
[Out] 1/6*x*B*(b*x^2+a)^(5/2)+5/24*B*a*x*(b*x^2+a)^(3/2)+5/16*B*a^2*x*(b*x^2+a)^(
1/2)+5/16*B*a^3/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-A*(b*x^2+a)^(7/2)/a/x
+A*b/a*x*(b*x^2+a)^(5/2)+5/4*A*b*x*(b*x^2+a)^(3/2)+15/8*A*b*a*x*(b*x^2+a)^(
1/2)+15/8*A*b^(1/2)*a^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A]   time = 0.99, size = 124, normalized size = 0.91

$$\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Bx+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Bax+\frac{5}{16}\sqrt{bx^2+a}\,Ba^2x+\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx+\frac{15}{8}\sqrt{bx^2+a}\,Aabx+\frac{5\,Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}+\frac{15}{8}Aa^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
+ a)*B*a^2*x + 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x +
 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 15/8*A*a^2*sqrt(b)*arcsinh(b*x
/sqrt(a*b)) - (b*x^2 + a)^(5/2)*A/x
```

mupad [B]   time = 1.92, size = 80, normalized size = 0.59

$$\frac{B\,x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}-\frac{A\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^2,x)
```

```
[Out] (B*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
+ 1)^(5/2) - (A*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))
/(x*((b*x^2)/a + 1)^(5/2))
```

sympy [B]   time = 24.29, size = 306, normalized size = 2.25

$$-\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+Aa^{\frac{3}{2}}bx\sqrt{1+\frac{bx^2}{a}}-\frac{7Aa^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{15Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{Ab^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Ba^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17B\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Bb^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
```

```
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) + B*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*B*a**(5/
2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a))
 + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)) + 5*B*a**3*asinh(sqrt(b)*
x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.528 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}\,(2a$$

Rubi [**A**] time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{(a+bx^2)^{5/2}(2aB+5Ab)}{10a}+\frac{1}{6}(a+bx^2)^{3/2}(2aB+5Ab)+\frac{1}{2}a\sqrt{a+bx^2}\,(2aB+5Ab)-\frac{A(a+bx^2)^{7/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (a*(5*A*b + 2*a*B)*Sqrt[a + b*x^2])/2 + ((5*A*b + 2*a*B)*(a + b*x^2)^(3/2))
/6 + ((5*A*b + 2*a*B)*(a + b*x^2)^(5/2))/(10*a) - (A*(a + b*x^2)^(7/2))/(2*
a*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{7/2}}{2ax^2}+\frac{\left(\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx,x,x^2\right)}{2a}\\
&= \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^2}+\frac{1}{4}(5Ab+2aB)\,\text{Subst}\left(\int \frac{(a+bx)^{3/}}{x}\right.\\
&= \frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2}+\frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^2}+\frac{1}{4}(a(5\\
&= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2}+\frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2}+\frac{(5Ab+2aB)\left(a+bx^2\right)^{5}}{10a}\\
&= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2}+\frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2}+\frac{(5Ab+2aB)\left(a+bx^2\right)^{5}}{10a}\\
&= \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2}+\frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2}+\frac{(5Ab+2aB)\left(a+bx^2\right)^{5}}{10a}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.78

$$\frac{\sqrt{a+bx^2}\left(a^2\left(46Bx^2-15A\right)+a\left(70Abx^2+22bBx^4\right)+2b^2x^4\left(5A+3Bx^2\right)\right)}{30x^2}-\frac{1}{2}a^{3/2}(2aB+5Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(2*b^2*x^4*(5*A + 3*B*x^2) + a^2*(-15*A + 46*B*x^2) + a*(7
0*A*b*x^2 + 22*b*B*x^4)))/(30*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/2
```

IntegrateAlgebraic [A] time = 0.19, size = 109, normalized size = 0.81

$$\frac{1}{2}\left(-5a^{3/2}Ab-2a^{5/2}B\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{\sqrt{a+bx^2}\left(-15a^2A+46a^2Bx^2+70aAbx^2+22abBx^4+10Ab^2x^4+6b^2Bx^6\right)}{30x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^2*A + 70*a*A*b*x^2 + 46*a^2*B*x^2 + 10*A*b^2*x^4 +
22*a*b*B*x^4 + 6*b^2*B*x^6))/(30*x^2) + ((-5*a^(3/2)*A*b - 2*a^(5/2)*B)*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

fricas [A] time = 1.20, size = 221, normalized size = 1.64

$$\left[\frac{15\left(2Ba^2+5Aab\right)\sqrt{a}x^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(6Bb^2x^6+2\left(11Bab+5Ab^2\right)x^4-15Aa^2+2\left(23Ba^2+35Aab\right)x^2\right)\sqrt{bx^2+a}}{60x^2}, \frac{15\left(2Ba^2+5Aab\right)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(6Bb^2x^6+2\left(11Bab+5Ab^2\right)x^4-15Aa^2+2\left(23Ba^2+35Aab\right)x^2\right)\sqrt{bx^2+a}}{30x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(2*B*a^2 + 5*A*a*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(6*B*b^2*x^6 + 2*(11*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2
 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^2, 1/30*(15*(2*B*a^2 + 5
*A*a*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (6*B*b^2*x^6 + 2*(1
1*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2
 + a))/x^2]
```

giac [A] time = 0.37, size = 139, normalized size = 1.03

$$\frac{6\left(bx^2+a\right)^{\frac{5}{2}}Bb+10\left(bx^2+a\right)^{\frac{3}{2}}Bab+30\sqrt{bx^2+a}Ba^2b+10\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+60\sqrt{bx^2+a}Aab^2-\frac{15\sqrt{bx^2+a}Aa^2b}{x^2}+\frac{15\left(2Ba^3b+5Aa^2b^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{30b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(b*x^2 + a)^(5/2)*B*b + 10*(b*x^2 + a)^(3/2)*B*a*b + 30*sqrt(b*x^2
+ a)*B*a^2*b + 10*(b*x^2 + a)^(3/2)*A*b^2 + 60*sqrt(b*x^2 + a)*A*a*b^2 - 15
*sqrt(b*x^2 + a)*A*a^2*b/x^2 + 15*(2*B*a^3*b + 5*A*a^2*b^2)*arctan(sqrt(b*x
^2 + a)/sqrt(-a))/sqrt(-a))/b
```

maple [A] time = 0.01, size = 161, normalized size = 1.19

$$-\frac{5 A\,a^{\frac{3}{2}} b \ln\left(\frac{2 a+2 \sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2}-B\,a^{\frac{5}{2}} \ln\left(\frac{2 a+2 \sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)+\frac{5 \sqrt{b\,x^{2}+a}\,A a b}{2}+\sqrt{b\,x^{2}+a}\,B\,a^{2}+\frac{5 \left(b\,x^{2}+a\right)^{\frac{3}{2}} A b}{6}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} B a}{3}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} A b}{2 a}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} B}{5}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} A}{2 a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x)
```

```
[Out] -1/2*A*(b*x^2+a)^(7/2)/a/x^2+1/2*A*b/a*(b*x^2+a)^(5/2)+5/6*A*b*(b*x^2+a)^(3
/2)-5/2*A*b*a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/2*A*b*a*(b*x^2+
a)^(1/2)+1/5*B*(b*x^2+a)^(5/2)+1/3*B*a*(b*x^2+a)^(3/2)-B*a^(5/2)*ln((2*a+2*
(b*x^2+a)^(1/2)*a^(1/2))/x)+B*(b*x^2+a)^(1/2)*a^2
```

maxima [A] time = 1.06, size = 138, normalized size = 1.02

$$-B a^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{a b}\,|x|}\right)-\frac{5}{2}\,A a^{\frac{3}{2}} b \operatorname{arsinh}\left(\frac{a}{\sqrt{a b}\,|x|}\right)+\frac{1}{5}\,\left(b x^{2}+a\right)^{\frac{5}{2}} B+\frac{1}{3}\,\left(b x^{2}+a\right)^{\frac{3}{2}} B a+\sqrt{b x^{2}+a}\,B a^{2}+\frac{5}{6}\,\left(b x^{2}+a\right)^{\frac{3}{2}} A b+\frac{\left(b x^{2}+a\right)^{\frac{5}{2}} A b}{2\,a}+\frac{5}{2}\,\sqrt{b x^{2}+a}\,A a b-\frac{\left(b x^{2}+a\right)^{\frac{7}{2}} A}{2\,a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

```
[Out] -B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(
a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(
b*x^2 + a)*B*a^2 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a
+ 5/2*sqrt(b*x^2 + a)*A*a*b - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

mupad [B] time = 1.85, size = 132, normalized size = 0.98

$$\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{5}+B\,a^2\,\sqrt{b\,x^2+a}+B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}+\frac{A\,b\,{\left(b\,x^2+a\right)}^{3/2}}{3}+\frac{B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}+\frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^3,x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (A*b*(a + b*x^2)^(3/2))/3 + (B*a*(a + b*x^2)^(
```

```
3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^(1/2))/(2*x^2) + (
A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2
```

sympy [A] time = 68.24, size = 296, normalized size = 2.19

$$-\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2}-\frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab^2\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Ba^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+2Bab\left(\begin{cases}\frac{\sqrt{a}x^2}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a
**3/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1
) + 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b
), True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*s
qrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/
4, True))
```

3.529 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^4}\,dx$

Optimal. Leaf size=146

$$-\frac{\left(a+bx^2\right)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx\left(a+bx^2\right)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}\,(3aB+4Ab)+\frac{5}{8}a\sqrt{b}\,(3aB+4Ab)\tanh^{-1}$$

Rubi [A] time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 195, 217, 206}

$$-\frac{\left(a+bx^2\right)^{5/2}(3aB+4Ab)}{3ax}+\frac{5bx\left(a+bx^2\right)^{3/2}(3aB+4Ab)}{12a}+\frac{5}{8}bx\sqrt{a+bx^2}\,(3aB+4Ab)+\frac{5}{8}a\sqrt{b}\,(3aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (5*b*(4*A*b + 3*a*B)*x*Sqrt[a + b*x^2])/8 + (5*b*(4*A*b + 3*a*B)*x*(a + b*x
^2)^(3/2))/(12*a) - ((4*A*b + 3*a*B)*(a + b*x^2)^(5/2))/(3*a*x) - (A*(a + b
*x^2)^(7/2))/(3*a*x^3) + (5*a*Sqrt[b]*(4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^4}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}-\frac{(-4Ab-3aB)\int\frac{\left(a+bx^2\right)^{5/2}}{x^2}\,dx}{3a}$$

$$= -\frac{(4Ab+3aB)\left(a+bx^2\right)^{5/2}}{3ax}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}+\frac{(5b(4Ab+3aB))\int\left(a+bx^2\right)^{3/2}}{3a}$$

$$= \frac{5b(4Ab+3aB)x\left(a+bx^2\right)^{3/2}}{12a}-\frac{(4Ab+3aB)\left(a+bx^2\right)^{5/2}}{3ax}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}+\frac{1}{4}($$

$$= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x\left(a+bx^2\right)^{3/2}}{12a}-\frac{(4Ab+3aB)(a+}{3ax}$$

$$= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x\left(a+bx^2\right)^{3/2}}{12a}-\frac{(4Ab+3aB)(a+}{3ax}$$

$$= \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2}+\frac{5b(4Ab+3aB)x\left(a+bx^2\right)^{3/2}}{12a}-\frac{(4Ab+3aB)(a+}{3ax}$$

Mathematica [C] time = 0.04, size = 84, normalized size = 0.58

$$\frac{a\sqrt{a+bx^2}\,(-3aB-4Ab)\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{3x\sqrt{\frac{bx^2}{a}+1}}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] -1/3*(A*(a + b*x^2)^(7/2))/(a*x^3) + (a*(-4*A*b - 3*a*B)*Sqrt[a + b*x^2]*Hy
pergeometric2F1[-5/2, -1/2, 1/2, -((b*x^2)/a)])/(3*x*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.38, size = 115, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(-8a^2A-24a^2Bx^2-56aAbx^2+27abBx^4+12Ab^2x^4+6b^2Bx^6\right)}{24x^3}-\frac{5}{8}\left(3a^2\sqrt{b}B+4aAb^{3/2}\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 56*a*A*b*x^2 - 24*a^2*B*x^2 + 12*A*b^2*x^4 + 2
7*a*b*B*x^4 + 6*b^2*B*x^6))/(24*x^3) - (5*(4*a*A*b^(3/2) + 3*a^2*Sqrt[b]*B)
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/8
```

fricas [A] time = 0.88, size = 220, normalized size = 1.51

$$\left[\frac{15\left(3Ba^2+4Aab\right)\sqrt{b}x^3\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6Bb^2x^6+3\left(9Bab+4Ab^2\right)x^4-8Aa^2-8\left(3Ba^2+7Aab\right)x^2\right)\sqrt{bx^2+a}}{48x^3}, -\frac{15\left(3Ba^2+4Aab\right)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(6Bb^2x^6+3\left(9Bab+4Ab^2\right)x^4-8Aa^2-8\left(3Ba^2+7Aab\right)x^2\right)\sqrt{bx^2+a}}{24x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(3*B*a^2 + 4*A*a*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(6*B*b^2*x^6 + 3*(9*B*a*b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(
3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^3, -1/24*(15*(3*B*a^2 + 4*A*a*b)
*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^6 + 3*(9*B*a*
b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^
3]
```

giac [A] time = 0.47, size = 238, normalized size = 1.63

$$\frac{1}{8}\left(2Bb^2x^2+\frac{9Bab^3+4Ab^4}{b^2}\right)\sqrt{bx^2+a}\,x-\frac{5}{16}\left(3Ba^2\sqrt{b}+4Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^3\sqrt{b}+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^4\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^3b^{\frac{3}{2}}+3Ba^5\sqrt{b}+7Aa^4b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*B*b^2*x^2 + (9*B*a*b^3 + 4*A*b^4)/b^2)*sqrt(b*x^2 + a)*x - 5/16*(3*B
*a^2*sqrt(b) + 4*A*a*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b) + 9*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*A*a^2*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*sqrt(b) - 1
```

```
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(3/2) + 3*B*a^5*sqrt(b) + 7*A*a^4
*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 204, normalized size = 1.40

$$\frac{5Aa\,b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2}+\frac{15B\,a^{2}\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8}+\frac{5\sqrt{b\,x^{2}+a}\,A\,b^{2}x}{2}+\frac{15\sqrt{b\,x^{2}+a}\,Babx}{8}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{2}x}{3a}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bbx}{4}+\frac{4\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,b^{2}x}{3a^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bbx}{a}-\frac{4\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ab}{3a^{2}x}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B}{ax}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A}{3a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x)
```

```
[Out] -B/a/x*(b*x^2+a)^(7/2)+B*b/a*x*(b*x^2+a)^(5/2)+5/4*B*b*x*(b*x^2+a)^(3/2)+15
/8*B*b*a*x*(b*x^2+a)^(1/2)+15/8*B*b^(1/2)*a^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
-1/3*A*(b*x^2+a)^(7/2)/a/x^3-4/3*A*b/a^2/x*(b*x^2+a)^(7/2)+4/3*A*b^2/a^2*x*
(b*x^2+a)^(5/2)+5/3*A*b^2/a*x*(b*x^2+a)^(3/2)+5/2*A*b^2*x*(b*x^2+a)^(1/2)+5
/2*A*b^(3/2)*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.05, size = 151, normalized size = 1.03

$$\frac{5}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}Bbx+\frac{15}{8}\sqrt{bx^{2}+a}\,Babx+\frac{5}{2}\sqrt{bx^{2}+a}\,Ab^{2}x+\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{2}x}{3\,a}+\frac{15}{8}\,Ba^{2}\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+\frac{5}{2}\,Aab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{x}-\frac{4\left(bx^{2}+a\right)^{\frac{5}{2}}Ab}{3\,ax}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}A}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 5/2*sqrt(b*x^2
 + a)*A*b^2*x + 5/3*(b*x^2 + a)^(3/2)*A*b^2*x/a + 15/8*B*a^2*sqrt(b)*arcsin
h(b*x/sqrt(a*b)) + 5/2*A*a*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(5/
2)*B/x - 4/3*(b*x^2 + a)^(5/2)*A*b/(a*x) - 1/3*(b*x^2 + a)^(7/2)*A/(a*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{5/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^4, x)
```

sympy [B] time = 15.71, size = 299, normalized size = 2.05

$$-\frac{2Aa^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^{2}}{a}}}+\frac{A\sqrt{a}\,b^{2}x\sqrt{1+\frac{bx^{2}}{a}}}{2}-\frac{2A\sqrt{a}\,b^{2}x}{\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Aa^{2}\sqrt{b}\,\sqrt{\frac{a}{bx^{2}}+1}}{3x^{2}}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3}+\frac{5Aab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}-\frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^{2}}{a}}}+Ba^{\frac{3}{2}}bx\sqrt{1+\frac{bx^{2}}{a}}-\frac{7Ba^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3B\sqrt{a}\,b^{2}x^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15Ba^{2}\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8}+\frac{Bb^{3}x^{5}}{4\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**4,x)
```

```
[Out] -2*A*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b**2*x*sqrt(1 + b*x**2/a
)/2 - 2*A*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - A*a**2*sqrt(b)*sqrt(a/(b*x**2
) + 1)/(3*x**2) - A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + 5*A*a*b**(3/2)*asin
h(sqrt(b)*x/sqrt(a))/2 - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3/2)*b*x
*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)
*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(
a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.530 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx$

Optimal. Leaf size=143

$$-\frac{(a+bx^2)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b(a+bx^2)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}\,(4aB+3Ab)-\frac{5}{8}\sqrt{a}\,b(4aB+3Ab)\tanh^{-1}\Big($$

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{(a+bx^2)^{5/2}(4aB+3Ab)}{8ax^2}+\frac{5b(a+bx^2)^{3/2}(4aB+3Ab)}{24a}+\frac{5}{8}b\sqrt{a+bx^2}\,(4aB+3Ab)-\frac{5}{8}\sqrt{a}\,b(4aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{A(a+bx^2)^{7/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (5*b*(3*A*b + 4*a*B)*Sqrt[a + b*x^2])/8 + (5*b*(3*A*b + 4*a*B)*(a + b*x^2)^
(3/2))/(24*a) - ((3*A*b + 4*a*B)*(a + b*x^2)^(5/2))/(8*a*x^2) - (A*(a + b*x
^2)^(7/2))/(4*a*x^4) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^3}\,dx,x,x^2\right)\\
&= -\frac{A\left(a+bx^2\right)^{7/2}}{4ax^4}+\frac{\left(\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2}\,dx,x,x^2\right)}{4a}\\
&= -\frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{7/2}}{4ax^4}+\frac{(5b(3Ab+4aB))\,\text{Subst}\left(\int \frac{(a+bx)}{x}\right.}{16a}\\
&= \frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a}-\frac{(3Ab+4aB)\left(a+bx^2\right)^{5/2}}{8ax^2}-\frac{A\left(a+bx^2\right)^{7/2}}{4ax^4}+\frac{1}{16}(\\
&= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2}+\frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a}-\frac{(3Ab+4aB)\left(a+bx\right.}{8ax^2}\\
&= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2}+\frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a}-\frac{(3Ab+4aB)\left(a+bx\right.}{8ax^2}\\
&= \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2}+\frac{5b(3Ab+4aB)\left(a+bx^2\right)^{3/2}}{24a}-\frac{(3Ab+4aB)\left(a+bx\right.}{8ax^2}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.42

$$
\frac{\left(a+bx^2\right)^{7/2}\left(bx^4(4aB+3Ab)\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)-7a^2A\right)}{28a^3x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] ((a + b*x^2)^(7/2)*(-7*a^2*A + b*(3*A*b + 4*a*B)*x^4*Hypergeometric2F1[2, 7
/2, 9/2, 1 + (b*x^2)/a]))/(28*a^3*x^4)
```

IntegrateAlgebraic [A] time = 0.17, size = 112, normalized size = 0.78

$$
\frac{\sqrt{a+bx^2}\left(-6a^2A-12a^2Bx^2-27aAbx^2+56abBx^4+24Ab^2x^4+8b^2Bx^6\right)}{24x^4}-\frac{5}{8}\left(4a^{3/2}bB+3\sqrt{a}\,Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-6*a^2*A - 27*a*A*b*x^2 - 12*a^2*B*x^2 + 24*A*b^2*x^4 + 5
6*a*b*B*x^4 + 8*b^2*B*x^6))/(24*x^4) - (5*(3*Sqrt[a]*A*b^2 + 4*a^(3/2)*b*B)
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

fricas [A] time = 0.94, size = 221, normalized size = 1.55

$$\left[\frac{15\left(4\,Bab+3\,Ab^{2}\right)\sqrt{a}\,x^{4}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(8\,Bb^{2}x^{6}+8\left(7\,Bab+3\,Ab^{2}\right)x^{4}-6\,Aa^{2}-3\left(4\,Ba^{2}+9\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{48\,x^{4}},\ \frac{15\left(4\,Bab+3\,Ab^{2}\right)\sqrt{-a}\,x^{4}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(8\,Bb^{2}x^{6}+8\left(7\,Bab+3\,Ab^{2}\right)x^{4}-6\,Aa^{2}-3\left(4\,Ba^{2}+9\,Aab\right)x^{2}\right)\sqrt{bx^{2}+a}}{24\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(4*B*a*b + 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(8*B*b^2*x^6 + 8*(7*B*a*b + 3*A*b^2)*x^4 - 6*A*a^2 -
 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^4, 1/24*(15*(4*B*a*b + 3*A*b
^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*B*b^2*x^6 + 8*(7*B*a
*b + 3*A*b^2)*x^4 - 6*A*a^2 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x
^4]
```

giac [A] time = 0.35, size = 171, normalized size = 1.20

$$\frac{8\left(bx^{2}+a\right)^{\frac{3}{2}}Bb^{2}+48\,\sqrt{bx^{2}+a}\,Bab^{2}+24\,\sqrt{bx^{2}+a}\,Ab^{3}+\frac{15\left(4\,Ba^{2}b^{2}+3\,Aab^{3}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\left(4\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}b^{2}-4\,\sqrt{bx^{2}+a}\,Ba^{3}b^{2}+9\left(bx^{2}+a\right)^{\frac{3}{2}}Aab^{3}-7\,\sqrt{bx^{2}+a}\,Aa^{2}b^{3}\right)}{b^{2}x^{4}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(b*x^2 + a)^(3/2)*B*b^2 + 48*sqrt(b*x^2 + a)*B*a*b^2 + 24*sqrt(b*x^
2 + a)*A*b^3 + 15*(4*B*a^2*b^2 + 3*A*a*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a)
)/sqrt(-a) - 3*(4*(b*x^2 + a)^(3/2)*B*a^2*b^2 - 4*sqrt(b*x^2 + a)*B*a^3*b^2
 + 9*(b*x^2 + a)^(3/2)*A*a*b^3 - 7*sqrt(b*x^2 + a)*A*a^2*b^3)/(b^2*x^4))/b
```

maple [A] time = 0.01, size = 213, normalized size = 1.49

$$-\frac{15A\sqrt{a}\;b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{8}-\frac{5B\,a^{\frac{3}{2}}b\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\;\sqrt{a}}{x}\right)}{2}+\frac{15\sqrt{b\,x^{2}+a}\;A\,b^{2}}{8}+\frac{5\sqrt{b\,x^{2}+a}\;Bab}{2}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{2}}{8a}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}Bb}{6}+\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,b^{2}}{8a^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}Bb}{2a}-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ab}{8a^{2}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B}{2a\,x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x)
```

```
[Out] -1/2*B/a/x^2*(b*x^2+a)^(7/2)+1/2*B*b/a*(b*x^2+a)^(5/2)+5/6*B*b*(b*x^2+a)^(3
/2)-5/2*B*b*a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/2*B*b*a*(b*x^2+
a)^(1/2)-1/4*A*(b*x^2+a)^(7/2)/a/x^4-3/8*A*b/a^2/x^2*(b*x^2+a)^(7/2)+3/8*A*
b^2/a^2*(b*x^2+a)^(5/2)+5/8*A*b^2/a*(b*x^2+a)^(3/2)-15/8*A*b^2*a^(1/2)*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+15/8*A*b^2*(b*x^2+a)^(1/2)
```

maxima [A] time = 1.06, size = 190, normalized size = 1.33

$$-\frac{5}{2}Ba^{\frac{3}{2}}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{15}{8}A\sqrt{a}\,b^{2}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\left(bx^{2}+a\right)^{\frac{3}{2}}Bb+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Bb}{2\,a}+\frac{5}{2}\sqrt{bx^{2}+a}\,Bab+\frac{15}{8}\sqrt{bx^{2}+a}\,Ab^{2}+\frac{3\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{2}}{8\,a^{2}}+\frac{5\left(bx^{2}+a\right)^{\frac{3}{2}}Ab^{2}}{8\,a}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}B}{2\,ax^{2}}-\frac{3\left(bx^{2}+a\right)^{\frac{7}{2}}Ab}{8\,a^{2}x^{2}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}A}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

```
[Out] -5/2*B*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 15/8*A*sqrt(a)*b^2*arcsinh
(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*B*b + 1/2*(b*x^2 + a)^(5/2)*
B*b/a + 5/2*sqrt(b*x^2 + a)*B*a*b + 15/8*sqrt(b*x^2 + a)*A*b^2 + 3/8*(b*x^2
 + a)^(5/2)*A*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*A*b^2/a - 1/2*(b*x^2 + a)^(7/
2)*B/(a*x^2) - 3/8*(b*x^2 + a)^(7/2)*A*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(7/2)*
A/(a*x^4)
```

mupad [B] time = 2.59, size = 144, normalized size = 1.01

$$A\,b^{2}\,\sqrt{b\,x^{2}+a}+\frac{B\,b\,{\left(b\,x^{2}+a\right)}^{3/2}}{3}+2\,B\,a\,b\,\sqrt{b\,x^{2}+a}+\frac{A\,\sqrt{a}\,b^{2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,15{}\mathrm{i}}{8}-\frac{9\,A\,a\,{\left(b\,x^{2}+a\right)}^{3/2}}{8\,x^{4}}+\frac{7\,A\,a^{2}\,\sqrt{b\,x^{2}+a}}{8\,x^{4}}-\frac{B\,a^{2}\,\sqrt{b\,x^{2}+a}}{2\,x^{2}}+\frac{B\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^{2}+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^5,x)
```

```
[Out] A*b^2*(a + b*x^2)^(1/2) + (B*b*(a + b*x^2)^(3/2))/3 + 2*B*a*b*(a + b*x^2)^(
1/2) + (A*a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*15i)/8 - (9*A*a*
(a + b*x^2)^(3/2))/(8*x^4) + (7*A*a^2*(a + b*x^2)^(1/2))/(8*x^4) - (B*a^2*(
a + b*x^2)^(1/2))/(2*x^2) + (B*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2
))*5i)/2
```

sympy [A] time = 165.19, size = 279, normalized size = 1.95

$$-\frac{15A\sqrt{a}\,b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8}-\frac{Aa^{3}}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Aa^{2}\sqrt{b}}{8x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{x}+\frac{7Aab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{Ab^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ba^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Ba^{2}\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2x}+\frac{2Ba^{2}\sqrt{b}}{x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{2Bab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^{2}}+1}}+Bb^{2}\left(\begin{cases}\frac{\sqrt{a}\,x^{2}}{2} & \text{for}\: b = 0\\\frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**5,x)
```

```
[Out] -15*A*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - A*a**3/(4*sqrt(b)*x**5*sq
rt(a/(b*x**2) + 1)) - 3*A*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - A*a*
b**(3/2)*sqrt(a/(b*x**2) + 1)/x + 7*A*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1))
 + A*b**(5/2)*x/sqrt(a/(b*x**2) + 1) - 5*B*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b
)*x))/2 - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + 2*B*a**2*sqrt(b)/(x*s
qrt(a/(b*x**2) + 1)) + 2*B*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + B*b**2*Piece
wise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

3.531 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^6}\,dx$

Optimal. Leaf size=152

$$\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{b^2x\sqrt{a+bx^2}\,(5aB+2Ab)}{2a}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2Ab)}{3ax}-\frac{\left(a+bx^2\right)^{5/2}(5aB}{15ax^3}$$

Rubi [A] time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 195, 217, 206}

$$\frac{b^2x\sqrt{a+bx^2}\,(5aB+2Ab)}{2a}+\frac{1}{2}b^{3/2}(5aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(5aB+2Ab)}{15ax^3}-\frac{b\left(a+bx^2\right)^{3/2}(5aB+2Ab)}{3ax}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (b^2*(2*A*b + 5*a*B)*x*Sqrt[a + b*x^2])/(2*a) - (b*(2*A*b + 5*a*B)*(a + b*x
^2)^(3/2))/(3*a*x) - ((2*A*b + 5*a*B)*(a + b*x^2)^(5/2))/(15*a*x^3) - (A*(a
 + b*x^2)^(7/2))/(5*a*x^5) + (b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^6}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}-\frac{(-2Ab-5aB)\int \frac{\left(a+bx^2\right)^{5/2}}{x^4}\,dx}{5a}$$

$$= -\frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}+\frac{(b(2Ab+5aB))\int \frac{\left(a+bx^2\right)^{3/2}}{x^2}\,dx}{3a}$$

$$= -\frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}+\frac{\left(b^2\right.}{}$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a}-\frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{(2Ab+5aB)\left(a+bx^2\right.}{15ax^3}$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a}-\frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{(2Ab+5aB)\left(a+bx^2\right.}{15ax^3}$$

$$= \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a}-\frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax}-\frac{(2Ab+5aB)\left(a+bx^2\right.}{15ax^3}$$

Mathematica [C] time = 0.04, size = 84, normalized size = 0.55

$$\frac{a\sqrt{a+bx^2}\,(-5aB-2Ab)\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx^2}{a}\right)}{15x^3\sqrt{\frac{bx^2}{a}+1}}-\frac{A\left(a+bx^2\right)^{7/2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] -1/5*(A*(a + b*x^2)^(7/2))/(a*x^5) + (a*(-2*A*b - 5*a*B)*Sqrt[a + b*x^2]*Hy
pergeometric2F1[-5/2, -3/2, -1/2, -((b*x^2)/a)])/(15*x^3*Sqrt[1 + (b*x^2)/a
])
```

IntegrateAlgebraic [A] time = 0.29, size = 112, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-6a^2A-10a^2Bx^2-22aAbx^2-70abBx^4-46Ab^2x^4+15b^2Bx^6\right)}{30x^5}+\frac{1}{2}\left(-5ab^{3/2}B-2Ab^{5/2}\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-6*a^2*A - 22*a*A*b*x^2 - 10*a^2*B*x^2 - 46*A*b^2*x^4 - 7
0*a*b*B*x^4 + 15*b^2*B*x^6))/(30*x^5) + ((-2*A*b^(5/2) - 5*a*b^(3/2)*B)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

fricas [A] time = 1.12, size = 220, normalized size = 1.45

$$\left[\frac{15\left(5\,Bab+2\,Ab^2\right)\sqrt{b}\,x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\left(15\,Bb^2x^6-2\left(35\,Bab+23\,Ab^2\right)x^4-6\,Aa^2-2\left(5\,Ba^2+11\,Aab\right)x^2\right)\sqrt{bx^2+a}}{60\,x^5},-\frac{15\left(5\,Bab+2\,Ab^2\right)\sqrt{-b}\,x^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(15\,Bb^2x^6-2\left(35\,Bab+23\,Ab^2\right)x^4-6\,Aa^2-2\left(5\,Ba^2+11\,Aab\right)x^2\right)\sqrt{bx^2+a}}{30\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(5*B*a*b + 2*A*b^2)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(15*B*b^2*x^6 - 2*(35*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 -
2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^5, -1/30*(15*(5*B*a*b + 2*A*
b^2)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (15*B*b^2*x^6 - 2*(3
5*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 - 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2
+ a))/x^5]
```

giac [B] time = 0.57, size = 321, normalized size = 2.11

$$\frac{1}{2}\,\sqrt{bx^2+a}\,Bb^2x-\frac{1}{4}\left(5\,Bab^{\frac{3}{2}}+2\,Ab^{\frac{5}{2}}\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\left(45\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Ba^2b^{\frac{3}{2}}+45\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8Aab^{\frac{5}{2}}-150\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Ba^3b^{\frac{3}{2}}-90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6Aa^2b^{\frac{5}{2}}+200\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Ba^4b^{\frac{3}{2}}+140\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4Aa^3b^{\frac{5}{2}}-130\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^5b^{\frac{3}{2}}-70\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Aa^4b^{\frac{5}{2}}+35\,Ba^6b^{\frac{3}{2}}+23\,Aa^5b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*b^2*x - 1/4*(5*B*a*b^(3/2) + 2*A*b^(5/2))*log((sqrt(b
)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*
b^(3/2) + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(5/2) - 150*(sqrt(b)*x -
```

```
 sqrt(b*x^2 + a))^6*B*a^3*b^(3/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^
2*b^(5/2) + 200*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(3/2) + 140*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(5/2) - 130*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*B*a^5*b^(3/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(5/2) + 35*B*a
^6*b^(3/2) + 23*A*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

maple [A] time = 0.01, size = 251, normalized size = 1.65

$$A b^{\frac{5}{2}} \ln\left(\sqrt{b}\, x+\sqrt{b x^{2}+a}\right)+\frac{5 B a b^{\frac{3}{2}} \ln\left(\sqrt{b}\, x+\sqrt{b x^{2}+a}\right)}{2}+\frac{\sqrt{b x^{2}+a}\, A b^{3} x}{a}+\frac{5 \sqrt{b x^{2}+a}\, B b^{2} x}{2}+\frac{2 \left(b x^{2}+a\right)^{\frac{3}{2}} A b^{3} x}{3 a^{2}}+\frac{5 \left(b x^{2}+a\right)^{\frac{3}{2}} B b^{2} x}{3 a}+\frac{8 \left(b x^{2}+a\right)^{\frac{5}{2}} A b^{3} x}{15 a^{3}}+\frac{4 \left(b x^{2}+a\right)^{\frac{5}{2}} B b^{2} x}{3 a^{2}}-\frac{8 \left(b x^{2}+a\right)^{\frac{7}{2}} A b^{2}}{15 a^{3} x}-\frac{4 \left(b x^{2}+a\right)^{\frac{7}{2}} B b}{3 a^{2} x}-\frac{2 \left(b x^{2}+a\right)^{\frac{7}{2}} A b}{15 a^{2} x^{3}}-\frac{\left(b x^{2}+a\right)^{\frac{7}{2}} B}{3 a x^{3}}-\frac{\left(b x^{2}+a\right)^{\frac{7}{2}} A}{5 a x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x)
```

```
[Out] -1/5*A*(b*x^2+a)^(7/2)/a/x^5-2/15*A*b/a^2/x^3*(b*x^2+a)^(7/2)-8/15*A*b^2/a^
3/x*(b*x^2+a)^(7/2)+8/15*A*b^3/a^3*x*(b*x^2+a)^(5/2)+2/3*A*b^3/a^2*x*(b*x^2
+a)^(3/2)+A*b^3/a*x*(b*x^2+a)^(1/2)+A*b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
-1/3*B/a/x^3*(b*x^2+a)^(7/2)-4/3*B*b/a^2/x*(b*x^2+a)^(7/2)+4/3*B*b^2/a^2*x*
(b*x^2+a)^(5/2)+5/3*B*b^2/a*x*(b*x^2+a)^(3/2)+5/2*B*b^2*x*(b*x^2+a)^(1/2)+5
/2*B*b^(3/2)*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.12, size = 198, normalized size = 1.30

$$\frac{5}{2} \sqrt{bx^{2}+a}\, Bb^{2}x+\frac{5 \left(bx^{2}+a\right)^{\frac{3}{2}} Bb^{2}x}{3\, a}+\frac{2 \left(bx^{2}+a\right)^{\frac{3}{2}} Ab^{3}x}{3\, a^{2}}+\frac{\sqrt{bx^{2}+a}\, Ab^{3}x}{a}+\frac{5}{2}\, Bab^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+Ab^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{4 \left(bx^{2}+a\right)^{\frac{5}{2}} Bb}{3\, ax}-\frac{8 \left(bx^{2}+a\right)^{\frac{5}{2}} Ab^{2}}{15\, a^{2}x}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}} B}{3\, ax^{3}}-\frac{2 \left(bx^{2}+a\right)^{\frac{7}{2}} Ab}{15\, a^{2}x^{3}}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}} A}{5\, ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(b*x^2 + a)*B*b^2*x + 5/3*(b*x^2 + a)^(3/2)*B*b^2*x/a + 2/3*(b*x^2
+ a)^(3/2)*A*b^3*x/a^2 + sqrt(b*x^2 + a)*A*b^3*x/a + 5/2*B*a*b^(3/2)*arcsin
h(b*x/sqrt(a*b)) + A*b^(5/2)*arcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5/2)
*B*b/(a*x) - 8/15*(b*x^2 + a)^(5/2)*A*b^2/(a^2*x) - 1/3*(b*x^2 + a)^(7/2)*B
/(a*x^3) - 2/15*(b*x^2 + a)^(7/2)*A*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)*A/(
a*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,{\left(b\,x^2+a\right)}^{5/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^6,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^6, x)
```

sympy [B] time = 11.54, size = 292, normalized size = 1.92

$$-\frac{A\sqrt{a}\,b^2}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{11Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2} - \frac{8Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15} + Ab^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) - \frac{Ab^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{2Ba^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}} + \frac{B\sqrt{a}\,b^2x\sqrt{1+\frac{bx^2}{a}}}{2} - \frac{2B\sqrt{a}\,b^2x}{\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + \frac{5Bab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**6,x)
```

```
[Out] -A*sqrt(a)*b**2/(x*sqrt(1 + b*x**2/a)) - A*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1
)/(5*x**4) - 11*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*A*b**(5/2)*
sqrt(a/(b*x**2) + 1)/15 + A*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - A*b**3*x/(s
qrt(a)*sqrt(1 + b*x**2/a)) - 2*B*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) + B*sqrt
(a)*b**2*x*sqrt(1 + b*x**2/a)/2 - 2*B*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - B
*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*a*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/3 + 5*B*a*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/2
```

3.532 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^7}\,dx$

Optimal. Leaf size=149

$$\frac{5b^2\sqrt{a+bx^2}\,(6aB+Ab)}{16a}-\frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{5b\left(a+bx^2\right)^{3/2}(6aB+Ab)}{48ax^2}-\frac{\left(a+bx^2\right)^{5/2}(6aB+A}{24ax^4}$$

Rubi [A] time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$\frac{5b^2\sqrt{a+bx^2}\,(6aB+Ab)}{16a}-\frac{5b^2(6aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{\left(a+bx^2\right)^{5/2}(6aB+Ab)}{24ax^4}-\frac{5b\left(a+bx^2\right)^{3/2}(6aB+Ab)}{48ax^2}-\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (5*b^2*(A*b + 6*a*B)*Sqrt[a + b*x^2])/(16*a) - (5*b*(A*b + 6*a*B)*(a + b*x^
2)^(3/2))/(48*a*x^2) - ((A*b + 6*a*B)*(a + b*x^2)^(5/2))/(24*a*x^4) - (A*(a
 + b*x^2)^(7/2))/(6*a*x^6) - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{6ax^6} + \frac{(Ab+6aB)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3}\,dx, x, x^2\right)}{12a}$$

$$= -\frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{7/2}}{6ax^6} + \frac{(5b(Ab+6aB))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\right.}{48a}$$

$$= -\frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2} - \frac{(Ab+6aB)\left(a+bx^2\right)^{5/2}}{24ax^4} - \frac{A\left(a+bx^2\right)^{7/2}}{6ax^6} + \left(5b^2\right.$$

$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2} - \frac{(Ab+6aB)\left(a+bx^2\right)^{5}}{24ax^4}$$

$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2} - \frac{(Ab+6aB)\left(a+bx^2\right)^{5}}{24ax^4}$$

$$= \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)\left(a+bx^2\right)^{3/2}}{48ax^2} - \frac{(Ab+6aB)\left(a+bx^2\right)^{5}}{24ax^4}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.41

$$-\frac{\left(a+bx^2\right)^{7/2}\left(7a^3A+b^2x^6(6aB+Ab)\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx^2}{a}+1\right)\right)}{42a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] -1/42*((a + b*x^2)^(7/2)*(7*a^3*A + b^2*(A*b + 6*a*B)*x^6*Hypergeometric2F1
[3, 7/2, 9/2, 1 + (b*x^2)/a]))/(a^4*x^6)
```

IntegrateAlgebraic [A] time = 0.18, size = 109, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\left(-8a^2A-12a^2Bx^2-26aAbx^2-54abBx^4-33Ab^2x^4+48b^2Bx^6\right)}{48x^6} - \frac{5\left(6ab^2B+Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A - 26*a*A*b*x^2 - 12*a^2*B*x^2 - 33*A*b^2*x^4 - 5
4*a*b*B*x^4 + 48*b^2*B*x^6))/(48*x^6) - (5*(A*b^3 + 6*a*b^2*B)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

fricas [A] time = 1.03, size = 241, normalized size = 1.62

$$\left[\frac{15\left(6\,Bab^{2}+Ab^{3}\right)\sqrt{a}\,x^{6}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(48\,Bab^{2}x^{6}-3\left(18\,Ba^{2}b+11\,Aab^{2}\right)x^{4}-8\,Aa^{3}-2\left(6\,Ba^{3}+13\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{96\,ax^{6}},\ \frac{15\left(6\,Bab^{2}+Ab^{3}\right)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(48\,Bab^{2}x^{6}-3\left(18\,Ba^{2}b+11\,Aab^{2}\right)x^{4}-8\,Aa^{3}-2\left(6\,Ba^{3}+13\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{48\,ax^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(6*B*a*b^2 + A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(48*B*a*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 -
8*A*a^3 - 2*(6*B*a^3 + 13*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6), 1/48*(15*
(6*B*a*b^2 + A*b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (48*B*a
*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 + 13*A*a^
2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6)]
```

giac [A] time = 0.36, size = 167, normalized size = 1.12

$$\frac{48\,\sqrt{bx^{2}+a}\,Bb^{3}+\frac{15\left(6\,Bab^{3}+Ab^{4}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{54\left(bx^{2}+a\right)^{\frac{5}{2}}Bab^{3}-96\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}b^{3}+42\,\sqrt{bx^{2}+a}\,Ba^{3}b^{3}+33\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{4}-40\left(bx^{2}+a\right)^{\frac{3}{2}}Aab^{4}+15\,\sqrt{bx^{2}+a}\,Aa^{2}b^{4}}{b^{3}x^{6}}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(b*x^2 + a)*B*b^3 + 15*(6*B*a*b^3 + A*b^4)*arctan(sqrt(b*x^2 +
 a)/sqrt(-a))/sqrt(-a) - (54*(b*x^2 + a)^(5/2)*B*a*b^3 - 96*(b*x^2 + a)^(3/
2)*B*a^2*b^3 + 42*sqrt(b*x^2 + a)*B*a^3*b^3 + 33*(b*x^2 + a)^(5/2)*A*b^4 -
40*(b*x^2 + a)^(3/2)*A*a*b^4 + 15*sqrt(b*x^2 + a)*A*a^2*b^4)/(b^3*x^6))/b
```

maple [B] time = 0.02, size = 266, normalized size = 1.79

$$-\frac{5A\,b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16\sqrt{a}}-\frac{15B\sqrt{a}\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8}+\frac{5\sqrt{b\,x^{2}+a}\,A\,b^{3}}{16a}+\frac{15\sqrt{b\,x^{2}+a}\,B\,b^{2}}{8}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{3}}{48a^{2}}+\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,b^{2}}{8a}+\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,b^{3}}{16a^{3}}+\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,b^{2}}{8a^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A\,b^{2}}{16a^{3}x^{2}}-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}Bb}{8a^{2}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ab}{24a^{2}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B}{4a\,x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A}{6a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x)
```

```
[Out] -1/6*A*(b*x^2+a)^(7/2)/a/x^6-1/24*A*b/a^2/x^4*(b*x^2+a)^(7/2)-1/16*A*b^2/a^
3/x^2*(b*x^2+a)^(7/2)+1/16*A*b^3/a^3*(b*x^2+a)^(5/2)+5/48*A*b^3/a^2*(b*x^2+
a)^(3/2)-5/16*A*b^3/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/16*A*b^
```

```
3/a*(b*x^2+a)^(1/2)-1/4*B/a/x^4*(b*x^2+a)^(7/2)-3/8*B*b/a^2/x^2*(b*x^2+a)^(
7/2)+3/8*B*b^2/a^2*(b*x^2+a)^(5/2)+5/8*B*b^2/a*(b*x^2+a)^(3/2)-15/8*B*b^2*a
^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+15/8*B*b^2*(b*x^2+a)^(1/2)
```

maxima [**A**] time = 1.20, size = 243, normalized size = 1.63

$$-\frac{15}{8}\,B\sqrt{a}\,b^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)-\frac{5\,Ab^3\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{15}{8}\,\sqrt{bx^2+a}\,Bb^2+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}Bb^2}{8\,a^2}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Bb^2}{8\,a}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^3}{16\,a^3}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ab^3}{48\,a^2}+\frac{5\,\sqrt{bx^2+a}\,Ab^3}{16\,a}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}Bb}{8\,a^2x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab^2}{16\,a^3x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{4\,ax^4}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab}{24\,a^2x^4}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

```
[Out] -15/8*B*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) - 5/16*A*b^3*arcsinh(a/(s
qrt(a*b)*abs(x)))/sqrt(a) + 15/8*sqrt(b*x^2 + a)*B*b^2 + 3/8*(b*x^2 + a)^(5
/2)*B*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*B*b^2/a + 1/16*(b*x^2 + a)^(5/2)*A*b^
3/a^3 + 5/48*(b*x^2 + a)^(3/2)*A*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*A*b^3/a - 3
/8*(b*x^2 + a)^(7/2)*B*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(7/2)*A*b^2/(a^3*x^2)
 - 1/4*(b*x^2 + a)^(7/2)*B/(a*x^4) - 1/24*(b*x^2 + a)^(7/2)*A*b/(a^2*x^4) -
 1/6*(b*x^2 + a)^(7/2)*A/(a*x^6)
```

mupad [**B**] time = 3.37, size = 150, normalized size = 1.01

$$B\,b^2\,\sqrt{b\,x^2+a}-\frac{11\,A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,x^6}+\frac{5\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{9\,B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{8\,x^4}-\frac{5\,A\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}+\frac{7\,B\,a^2\,\sqrt{b\,x^2+a}}{8\,x^4}+\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}+\frac{B\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,15{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^7,x)
```

```
[Out] B*b^2*(a + b*x^2)^(1/2) - (11*A*(a + b*x^2)^(5/2))/(16*x^6) + (A*b^3*atan((
(a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) + (B*a^(1/2)*b^2*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*15i)/8 + (5*A*a*(a + b*x^2)^(3/2))/(6*x^6) - (9*
B*a*(a + b*x^2)^(3/2))/(8*x^4) - (5*A*a^2*(a + b*x^2)^(1/2))/(16*x^6) + (7*
B*a^2*(a + b*x^2)^(1/2))/(8*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**7,x)
```

```
[Out] Timed out
```

3.533 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx$

Optimal. Leaf size=108

$$-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}$$

Rubi [A] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {451, 277, 217, 206}

$$-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}-\frac{b^2B\sqrt{a+bx^2}}{x}+b^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -((b^2*B*Sqrt[a + b*x^2])/x) - (b*B*(a + b*x^2)^(3/2))/(3*x^3) - (B*(a + b*
x^2)^(5/2))/(5*x^5) - (A*(a + b*x^2)^(7/2))/(7*a*x^7) + b^(5/2)*B*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^8}\,dx &= -\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+B\int \frac{\left(a+bx^2\right)^{5/2}}{x^6}\,dx \\
&= -\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+(bB)\int \frac{\left(a+bx^2\right)^{3/2}}{x^4}\,dx \\
&= -\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+\left(b^2B\right)\int \frac{\sqrt{a+bx^2}}{x^2}\,dx \\
&= -\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+\left(b^3B\right)\int \\
&= -\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+\left(b^3B\right)\mathrm{Su} \\
&= -\frac{b^2B\sqrt{a+bx^2}}{x}-\frac{bB\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{B\left(a+bx^2\right)^{5/2}}{5x^5}-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}+b^{5/2}B\tan
\end{aligned}$$

Mathematica [C] time = 0.09, size = 78, normalized size = 0.72

$$-\frac{a^2B\sqrt{a+bx^2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};-\frac{bx^2}{a}\right)}{5x^5\sqrt{\frac{bx^2}{a}+1}}-\frac{A\left(a+bx^2\right)^{7/2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] -1/7*(A*(a + b*x^2)^(7/2))/(a*x^7) - (a^2*B*Sqrt[a + b*x^2]*Hypergeometric2
F1[-5/2, -5/2, -3/2, -((b*x^2)/a)])/(5*x^5*Sqrt[1 + (b*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.30, size = 116, normalized size = 1.07

$$\frac{\sqrt{a+bx^2}\left(-15a^3A-21a^3Bx^2-45a^2Abx^2-77a^2bBx^4-45aAb^2x^4-161ab^2Bx^6-15Ab^3x^6\right)}{105ax^7}-b^{5/2}B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^3*A - 45*a^2*A*b*x^2 - 21*a^3*B*x^2 - 45*a*A*b^2*x^
4 - 77*a^2*b*B*x^4 - 15*A*b^3*x^6 - 161*a*b^2*B*x^6))/(105*a*x^7) - b^(5/2)
*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

fricas [A] time = 0.85, size = 234, normalized size = 2.17

$$\left[\frac{105\,Bab^{\frac{5}{2}}x^{7}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)-2\left(\left(161\,Bab^{2}+15\,Ab^{3}\right)x^{6}+\left(77\,Ba^{2}b+45\,Aab^{2}\right)x^{4}+15\,Aa^{3}+3\left(7\,Ba^{3}+15\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{210\,ax^{7}},\ -\frac{105\,Ba\sqrt{-b}\,b^{2}x^{7}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(\left(161\,Bab^{2}+15\,Ab^{3}\right)x^{6}+\left(77\,Ba^{2}b+45\,Aab^{2}\right)x^{4}+15\,Aa^{3}+3\left(7\,Ba^{3}+15\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\,ax^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

```
[Out] [1/210*(105*B*a*b^(5/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((161*B*a*b^2 + 15*A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a
^3 + 3*(7*B*a^3 + 15*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7), -1/105*(105*B*
a*sqrt(-b)*b^2*x^7*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((161*B*a*b^2 + 15*
A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + 15*A*a
^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7)]
```

giac [B] time = 0.58, size = 320, normalized size = 2.96

$$-\frac{1}{2}\,Bb^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}\right)+\frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Bab^{\frac{5}{2}}+105\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Ab^{\frac{7}{2}}-1260\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}Ba^{2}b^{\frac{5}{2}}+2555\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ba^{3}b^{\frac{5}{2}}+525\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Aa^{2}b^{\frac{7}{2}}-3080\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ba^{4}b^{\frac{5}{2}}+2121\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba^{5}b^{\frac{5}{2}}+315\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Aa^{4}b^{\frac{7}{2}}-812\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{6}b^{\frac{5}{2}}+161\,Ba^{7}b^{\frac{5}{2}}+15\,Aa^{6}b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

```
[Out] -1/2*B*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/105*(315*(sqrt(b)*x
 - sqrt(b*x^2 + a))^12*B*a*b^(5/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A
*b^(7/2) - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 2555*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) + 525*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*A*a^2*b^(7/2) - 3080*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 21
21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) + 315*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*A*a^4*b^(7/2) - 812*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^6*b^(5/
2) + 161*B*a^7*b^(5/2) + 15*A*a^6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^7
```

maple [A] time = 0.02, size = 155, normalized size = 1.44

$$B\,b^{\frac{5}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)+\frac{\sqrt{b\,x^{2}+a}\;B\,b^{3}x}{a}+\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,b^{3}x}{3a^{2}}+\frac{8\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,b^{3}x}{15a^{3}}-\frac{8\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,b^{2}}{15a^{3}x}-\frac{2\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,b}{15a^{2}x^{3}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B}{5a\,x^{5}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A}{7a\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x)
```

```
[Out] -1/5*B/a/x^5*(b*x^2+a)^(7/2)-2/15*B*b/a^2/x^3*(b*x^2+a)^(7/2)-8/15*B*b^2/a^
3/x*(b*x^2+a)^(7/2)+8/15*B*b^3/a^3*x*(b*x^2+a)^(5/2)+2/3*B*b^3/a^2*x*(b*x^2
+a)^(3/2)+B*b^3/a*x*(b*x^2+a)^(1/2)+B*b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
-1/7*A*(b*x^2+a)^(7/2)/a/x^7
```

maxima [A] time = 1.09, size = 128, normalized size = 1.19

$$\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bb^3x}{3\,a^2}+\frac{\sqrt{bx^2+a}\,Bb^3x}{a}+Bb^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{8\left(bx^2+a\right)^{\frac{5}{2}}Bb^2}{15\,a^2x}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Bb}{15\,a^2x^3}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{5\,ax^5}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^2 + a)^(3/2)*B*b^3*x/a^2 + sqrt(b*x^2 + a)*B*b^3*x/a + B*b^(5/2)*a
rcsinh(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*B*b^2/(a^2*x) - 2/15*(b*x^2
+ a)^(7/2)*B*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)*B/(a*x^5) - 1/7*(b*x^2 + a
)^(7/2)*A/(a*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^8,x)
```

```
[Out] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^8, x)
```

sympy [B] time = 14.27, size = 592, normalized size = 5.48

$$-\frac{15Aa^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{33Aa^6b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{17Aa^5b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{3Aa^4b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{12Aa^3b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{8Aa^2b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}-\frac{2Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{7Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a}-\frac{B\sqrt{a}\,b^2}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^2\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{11Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2}-\frac{8Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15}+Bb^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bb^3x}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**8,x)
```

```
[Out] -15*A*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**6*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a
**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 3*A*a**4*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*A*a**
3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
```

```
**8 + 105*a**3*b**6*x**10) - 8*A*a**2*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 2*A*a*b**
(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15
*x**2) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a) - B*sqrt(a)*b**2/(x*sqrt(1
+ b*x**2/a)) - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*B*a*b**(3/
2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 +
B*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**3*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

3.534 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^9}\,dx$

Optimal. Leaf size=152

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}\,(Ab-8aB)}{128ax^2}+\frac{\left(a+bx^2\right)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b\left(a+bx^2\right)^{3/2}(Ab-8a}{192ax^4}$$

Rubi [A] time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{5b^3(Ab-8aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}+\frac{5b^2\sqrt{a+bx^2}\,(Ab-8aB)}{128ax^2}+\frac{\left(a+bx^2\right)^{5/2}(Ab-8aB)}{48ax^6}+\frac{5b\left(a+bx^2\right)^{3/2}(Ab-8aB)}{192ax^4}-\frac{A\left(a+bx^2\right)^{7/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (5*b^2*(A*b - 8*a*B)*Sqrt[a + b*x^2])/(128*a*x^2) + (5*b*(A*b - 8*a*B)*(a +
 b*x^2)^(3/2))/(192*a*x^4) + ((A*b - 8*a*B)*(a + b*x^2)^(5/2))/(48*a*x^6) -
 (A*(a + b*x^2)^(7/2))/(8*a*x^8) + (5*b^3*(A*b - 8*a*B)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^9}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{8ax^8} + \frac{\left(-\frac{Ab}{2}+4aB\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^4}\,dx, x, x^2\right)}{8a}$$

$$= \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A\left(a+bx^2\right)^{7/2}}{8ax^8} - \frac{(5b(Ab-8aB))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3}\,dx,\right.}{96a}$$

$$= \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6} - \frac{A\left(a+bx^2\right)^{7/2}}{8ax^8} - \left(5b^2(A\right.$$

$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6}$$

$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6}$$

$$= \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2} + \frac{5b(Ab-8aB)\left(a+bx^2\right)^{3/2}}{192ax^4} + \frac{(Ab-8aB)\left(a+bx^2\right)^{5/2}}{48ax^6}$$

Mathematica [A] time = 0.14, size = 140, normalized size = 0.92

$$\frac{-\left(a+bx^2\right)\left(16a^3\left(3A+4Bx^2\right)+8a^2bx^2\left(17A+26Bx^2\right)+2ab^2x^4\left(59A+132Bx^2\right)+15Ab^3x^6\right)-15b^3x^8\sqrt{\frac{bx^2}{a}+1}\,(8aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{384ax^8\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (-((a + b*x^2)*(15*A*b^3*x^6 + 16*a^3*(3*A + 4*B*x^2) + 8*a^2*b*x^2*(17*A +
 26*B*x^2) + 2*a*b^2*x^4*(59*A + 132*B*x^2))) - 15*b^3*(-(A*b) + 8*a*B)*x^8
*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(384*a*x^8*Sqrt[a + b*x^
2])
```

IntegrateAlgebraic [A] time = 0.22, size = 128, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-48a^3A-64a^3Bx^2-136a^2Abx^2-208a^2bBx^4-118aAb^2x^4-264ab^2Bx^6-15Ab^3x^6\right)}{384ax^8}-\frac{5\left(8ab^3B-Ab^4\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*A - 136*a^2*A*b*x^2 - 64*a^3*B*x^2 - 118*a*A*b^2*
x^4 - 208*a^2*b*B*x^4 - 15*A*b^3*x^6 - 264*a*b^2*B*x^6))/(384*a*x^8) - (5*(
-(A*b^4) + 8*a*b^3*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

fricas [A] time = 0.94, size = 272, normalized size = 1.79

$$\left[-\frac{15\left(8\,Bab^3-Ab^4\right)\sqrt{a}\,x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(88\,Ba^2b^2+5\,Aab^3\right)x^6+48\,Aa^4+2\left(104\,Ba^3b+59\,Aa^2b^2\right)x^4+8\left(8\,Ba^4+17\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{768\,a^2x^8},\ \frac{15\left(8\,Bab^3-Ab^4\right)\sqrt{-a}\,x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(3\left(88\,Ba^2b^2+5\,Aab^3\right)x^6+48\,Aa^4+2\left(104\,Ba^3b+59\,Aa^2b^2\right)x^4+8\left(8\,Ba^4+17\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{384\,a^2x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(8*B*a*b^3 - A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) + 2*(3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
104*B*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2
+ a))/(a^2*x^8), 1/384*(15*(8*B*a*b^3 - A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)
/sqrt(b*x^2 + a)) - (3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(104*B
*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2 + a))
/(a^2*x^8)]
```

giac [A] time = 0.44, size = 195, normalized size = 1.28

$$\frac{\frac{15\left(8\,Bab^4-Ab^5\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{264\left(bx^2+a\right)^{\frac{7}{2}}Bab^4-584\left(bx^2+a\right)^{\frac{5}{2}}Ba^2b^4+440\left(bx^2+a\right)^{\frac{3}{2}}Ba^3b^4-120\sqrt{bx^2+a}\,Ba^4b^4+15\left(bx^2+a\right)^{\frac{7}{2}}Ab^5+73\left(bx^2+a\right)^{\frac{5}{2}}Aab^5-55\left(bx^2+a\right)^{\frac{3}{2}}Aa^2b^5+15\sqrt{bx^2+a}\,Aa^3b^5}{ab^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/384*(15*(8*B*a*b^4 - A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a)
 - (264*(b*x^2 + a)^(7/2)*B*a*b^4 - 584*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 440*(
b*x^2 + a)^(3/2)*B*a^3*b^4 - 120*sqrt(b*x^2 + a)*B*a^4*b^4 + 15*(b*x^2 + a)
^(7/2)*A*b^5 + 73*(b*x^2 + a)^(5/2)*A*a*b^5 - 55*(b*x^2 + a)^(3/2)*A*a^2*b^
5 + 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a*b^4*x^8))/b
```

maple [B] time = 0.02, size = 311, normalized size = 2.05

$$\frac{5A\,b^4\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{3}{2}}}-\frac{5B\,b^3\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{16\sqrt{a}}-\frac{5\sqrt{b\,x^2+a}\;A\,b^4}{128a^2}+\frac{5\sqrt{b\,x^2+a}\;B\,b^3}{16a}-\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}A\,b^4}{384a^3}+\frac{5\left(b\,x^2+a\right)^{\frac{3}{2}}B\,b^3}{48a^2}-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}A\,b^4}{128a^4}+\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}B\,b^3}{16a^3}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A\,b^3}{128a^4x^2}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B\,b^2}{16a^3x^2}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A\,b^2}{192a^3x^4}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}Bb}{24a^2x^4}+\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}Ab}{48a^2x^6}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}B}{6a\,x^6}-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}A}{8a\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x)
```

```
[Out] -1/6*B/a/x^6*(b*x^2+a)^(7/2)-1/24*B*b/a^2/x^4*(b*x^2+a)^(7/2)-1/16*B*b^2/a^
3/x^2*(b*x^2+a)^(7/2)+1/16*B*b^3/a^3*(b*x^2+a)^(5/2)+5/48*B*b^3/a^2*(b*x^2+
a)^(3/2)-5/16*B*b^3/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+5/16*B*b^
3/a*(b*x^2+a)^(1/2)-1/8*A*(b*x^2+a)^(7/2)/a/x^8+1/48*A*b/a^2/x^6*(b*x^2+a)^
(7/2)+1/192*A*b^2/a^3/x^4*(b*x^2+a)^(7/2)+1/128*A*b^3/a^4/x^2*(b*x^2+a)^(7/
2)-1/128*A*b^4/a^4*(b*x^2+a)^(5/2)-5/384*A*b^4/a^3*(b*x^2+a)^(3/2)+5/128*A*
b^4/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-5/128*A*b^4/a^2*(b*x^2+a)
^(1/2)
```

maxima [B] time = 1.17, size = 288, normalized size = 1.89

$$-\frac{5\,Bb^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,\sqrt{a}}+\frac{5\,Ab^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bb^3}{16\,a^3}+\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Bb^3}{48\,a^2}+\frac{5\,\sqrt{bx^2+a}\,Bb^3}{16\,a}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^4}{128\,a^4}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ab^4}{384\,a^3}-\frac{5\,\sqrt{bx^2+a}\,Ab^4}{128\,a^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bb^2}{16\,a^3x^2}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab^3}{128\,a^4x^2}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bb}{24\,a^2x^4}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab^2}{192\,a^3x^4}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{6\,ax^6}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab}{48\,a^2x^6}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

```
[Out] -5/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 5/128*A*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(3/2) + 1/16*(b*x^2 + a)^(5/2)*B*b^3/a^3 + 5/48*(b*x^2
 + a)^(3/2)*B*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*B*b^3/a - 1/128*(b*x^2 + a)^(5
/2)*A*b^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*A*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*A
*b^4/a^2 - 1/16*(b*x^2 + a)^(7/2)*B*b^2/(a^3*x^2) + 1/128*(b*x^2 + a)^(7/2)
*A*b^3/(a^4*x^2) - 1/24*(b*x^2 + a)^(7/2)*B*b/(a^2*x^4) + 1/192*(b*x^2 + a)
^(7/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(7/2)*B/(a*x^6) + 1/48*(b*x^2 + a)
^(7/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)*A/(a*x^8)
```

mupad [B] time = 4.58, size = 169, normalized size = 1.11

$$\frac{55\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{384\,x^8}-\frac{11\,B\,{\left(b\,x^2+a\right)}^{5/2}}{16\,x^6}-\frac{73\,A\,{\left(b\,x^2+a\right)}^{5/2}}{384\,x^8}+\frac{5\,B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{6\,x^6}-\frac{5\,A\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{5\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^8}-\frac{5\,B\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{3/2}}+\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^9,x)
```

```
[Out] (B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*B*(a + b
*x^2)^(5/2))/(16*x^6) - (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(12
8*a^(3/2)) - (73*A*(a + b*x^2)^(5/2))/(384*x^8) + (55*A*a*(a + b*x^2)^(3/2)
)/(384*x^8) + (5*B*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*A*a^2*(a + b*x^2)^(1/2
))/(128*x^8) - (5*A*(a + b*x^2)^(7/2))/(128*a*x^8) - (5*B*a^2*(a + b*x^2)^(
1/2))/(16*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**9,x)
```

```
[Out] Timed out
```

3.535 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{10}}\,dx$

Optimal. Leaf size=53

$$\frac{\left(a+bx^2\right)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{7/2}}{9ax^9}$$

Rubi [**A**] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{\left(a+bx^2\right)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A\left(a+bx^2\right)^{7/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -(A*(a + b*x^2)^(7/2))/(9*a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(7/2))/(63*
a^2*x^7)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{10}}\,dx = -\frac{A\left(a+bx^2\right)^{7/2}}{9ax^9}-\frac{(2Ab-9aB)\int \frac{\left(a+bx^2\right)^{5/2}}{x^8}\,dx}{9a}$$
$$= -\frac{A\left(a+bx^2\right)^{7/2}}{9ax^9}+\frac{(2Ab-9aB)\left(a+bx^2\right)^{7/2}}{63a^2x^7}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.75

$$-\frac{\left(a+bx^2\right)^{7/2}\left(7aA+9aBx^2-2Abx^2\right)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] -1/63*((a + b*x^2)^(7/2)*(7*a*A - 2*A*b*x^2 + 9*a*B*x^2))/(a^2*x^9)
```

IntegrateAlgebraic [B] time = 0.32, size = 110, normalized size = 2.08

$$\frac{\sqrt{a+bx^2}\left(-7a^4A-9a^4Bx^2-19a^3Abx^2-27a^3bBx^4-15a^2Ab^2x^4-27a^2b^2Bx^6-aAb^3x^6-9ab^3Bx^8+2Ab^4x^8\right)}{63a^2x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-7*a^4*A - 19*a^3*A*b*x^2 - 9*a^4*B*x^2 - 15*a^2*A*b^2*x^
4 - 27*a^3*b*B*x^4 - a*A*b^3*x^6 - 27*a^2*b^2*B*x^6 + 2*A*b^4*x^8 - 9*a*b^3
*B*x^8))/(63*a^2*x^9)
```

fricas [B] time = 1.11, size = 102, normalized size = 1.92

$$-\frac{\left(\left(9\,Bab^3-2\,Ab^4\right)x^8+\left(27\,Ba^2b^2+Aab^3\right)x^6+7\,Aa^4+3\left(9\,Ba^3b+5\,Aa^2b^2\right)x^4+\left(9\,Ba^4+19\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{63\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

```
[Out] -1/63*((9*B*a*b^3 - 2*A*b^4)*x^8 + (27*B*a^2*b^2 + A*a*b^3)*x^6 + 7*A*a^4 +
 3*(9*B*a^3*b + 5*A*a^2*b^2)*x^4 + (9*B*a^4 + 19*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a)/(a^2*x^9)
```

giac [B] time = 0.47, size = 456, normalized size = 8.60

$$\frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}Bb^{\frac{7}{2}}-126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bab^{\frac{7}{2}}+126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Ab^{\frac{9}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ba^2b^{\frac{7}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Aab^{\frac{9}{2}}-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^3b^{\frac{7}{2}}+630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Aa^2b^{\frac{9}{2}}+504\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ba^4b^{\frac{7}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Aa^3b^{\frac{9}{2}}-378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Ba^5b^{\frac{7}{2}}+378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}Aa^4b^{\frac{9}{2}}+198\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Ba^6b^{\frac{7}{2}}+54\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}Aa^5b^{\frac{9}{2}}-18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Ba^7b^{\frac{7}{2}}+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}Aa^6b^{\frac{9}{2}}+9Ba^8b^{\frac{7}{2}}-2Aa^7b^{\frac{9}{2}}\right)}{63\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

```
[Out] 2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*b^(7/2) - 126*(sqrt(b)*x - sqrt
(b*x^2 + a))^14*B*a*b^(7/2) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*b^(9/2
) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(7/2) + 210*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*A*a*b^(9/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^3
*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^2*b^(9/2) + 504*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(7/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^
8*A*a^3*b^(9/2) - 378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2) + 378*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^4*b^(9/2) + 198*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^6*b^(7/2) + 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^5*b^(9/2) -
18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^7*b^(7/2) + 18*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*A*a^6*b^(9/2) + 9*B*a^8*b^(7/2) - 2*A*a^7*b^(9/2))/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^9
```

maple [A] time = 0.01, size = 37, normalized size = 0.70

$$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}\left(-2Ab\,x^2+9Ba\,x^2+7Aa\right)}{63a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x)
```

```
[Out] -1/63*(b*x^2+a)^(7/2)*(-2*A*b*x^2+9*B*a*x^2+7*A*a)/x^9/a^2
```

maxima [A] time = 1.05, size = 56, normalized size = 1.06

$$-\frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,ax^7}+\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Ab}{63\,a^2x^7}-\frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{9\,ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

```
[Out] -1/7*(b*x^2 + a)^(7/2)*B/(a*x^7) + 2/63*(b*x^2 + a)^(7/2)*A*b/(a^2*x^7) - 1
/9*(b*x^2 + a)^(7/2)*A/(a*x^9)
```

mupad [B] time = 2.72, size = 170, normalized size = 3.21

$$\frac{2\,A\,b^4\,\sqrt{b\,x^2+a}}{63\,a^2\,x}-\frac{5\,A\,b^2\,\sqrt{b\,x^2+a}}{21\,x^5}-\frac{B\,a^2\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{3\,B\,b^2\,\sqrt{b\,x^2+a}}{7\,x^3}-\frac{A\,b^3\,\sqrt{b\,x^2+a}}{63\,a\,x^3}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{B\,b^3\,\sqrt{b\,x^2+a}}{7\,a\,x}-\frac{19\,A\,a\,b\,\sqrt{b\,x^2+a}}{63\,x^7}-\frac{3\,B\,a\,b\,\sqrt{b\,x^2+a}}{7\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^10,x)
```

```
[Out] (2*A*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*A*b^2*(a + b*x^2)^(1/2))/(21*x^
5) - (B*a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*B*b^2*(a + b*x^2)^(1/2))/(7*x^3
) - (A*b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (A*a^2*(a + b*x^2)^(1/2))/(9*x^9
) - (B*b^3*(a + b*x^2)^(1/2))/(7*a*x) - (19*A*a*b*(a + b*x^2)^(1/2))/(63*x^
7) - (3*B*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

sympy [B] time = 13.95, size = 1489, normalized size = 28.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**10,x)
```

```
[Out] -35*A*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b*
*10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**8*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**7*b**(23/2)*x**4*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b
**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**6*b**(25/2)*x**6*sqrt(a/(b*x**
2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 30*A*a**6*b**(11/2)*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**5*b**(27/2)
*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 66*A*a**5*b**(13/2)*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) + 30*A*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 +
945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 34*A*
a**4*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**3*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) - 6*A*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5
*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 16*A*a**2*b**(33/2
)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 9
45*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 24*A*a**2*b**(19/2)*x**8*sqrt
(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x
**10) - 16*A*a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 2
10*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/
(5*x**4) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + 2*A*b**(9/2)*sqrt(
a/(b*x**2) + 1)/(15*a**2) - 15*B*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**6*b**(11
/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) - 17*B*a**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**
```

```
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**4*b**(15/2
)*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*
a**3*b**6*x**10) - 12*B*a**3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*
b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 8*B*a**2*b**(19/2)*
x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 2*B*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*B*b**(5/
2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - B*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a)
```

3.536 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{11}}\,dx$

Optimal. Leaf size=189

$$-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^3\sqrt{a+bx^2}\,(3Ab-10aB)}{256a^2x^2}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-10aB)}{128ax^4}+\frac{\left(a+bx^2\right)^{5/2}(3Ab-}{80ax^8}$$

Rubi [A] time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$\frac{b^3\sqrt{a+bx^2}\,(3Ab-10aB)}{256a^2x^2}-\frac{b^4(3Ab-10aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}+\frac{b^2\sqrt{a+bx^2}\,(3Ab-10aB)}{128ax^4}+\frac{b\left(a+bx^2\right)^{3/2}(3Ab-10aB)}{96ax^6}+\frac{\left(a+bx^2\right)^{5/2}(3Ab-10aB)}{80ax^8}-\frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (b^2*(3*A*b - 10*a*B)*Sqrt[a + b*x^2])/(128*a*x^4) + (b^3*(3*A*b - 10*a*B)*
Sqrt[a + b*x^2])/(256*a^2*x^2) + (b*(3*A*b - 10*a*B)*(a + b*x^2)^(3/2))/(96
*a*x^6) + ((3*A*b - 10*a*B)*(a + b*x^2)^(5/2))/(80*a*x^8) - (A*(a + b*x^2)^
(7/2))/(10*a*x^10) - (b^4*(3*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(256*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{11}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}(A+Bx)}{x^6}\,dx, x, x^2\right)$$

$$= -\frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}} + \frac{\left(-\frac{3Ab}{2}+5aB\right)\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^5}\,dx, x, x^2\right)}{10a}$$

$$= \frac{(3Ab-10aB)\left(a+bx^2\right)^{5/2}}{80ax^8} - \frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}} - \frac{(b(3Ab-10aB))\operatorname{Subst}\left(\int \frac{(a+bx)^{3/}}{x^4}\right.}{32a}$$

$$= \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)\left(a+bx^2\right)^{5/2}}{80ax^8} - \frac{A\left(a+bx^2\right)^{7/2}}{10ax^{10}} - \frac{\left(b^2\right.}{}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b(3Ab-10aB)\left(a+bx^2\right)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)\left(a+b\right.}{80ax^8}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)\left(a+b\right.}{96ax^6}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)\left(a+b\right.}{96ax^6}$$

$$= \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)\left(a+b\right.}{96ax^6}$$

Mathematica [C] time = 0.03, size = 62, normalized size = 0.33

$$-\frac{\left(a+bx^2\right)^{7/2}\left(7a^5A+b^4x^{10}(10aB-3Ab)\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx^2}{a}+1\right)\right)}{70a^6x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] -1/70*((a + b*x^2)^(7/2)*(7*a^5*A + b^4*(-3*A*b + 10*a*B)*x^10*Hypergeometr
ic2F1[7/2, 5, 9/2, 1 + (b*x^2)/a]))/(a^6*x^10)
```

IntegrateAlgebraic [A] time = 0.30, size = 152, normalized size = 0.80

$$\frac{\left(10ab^4B-3Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}} + \frac{\sqrt{a+bx^2}\left(-384a^4A-480a^4Bx^2-1008a^3Abx^2-1360a^3bBx^4-744a^2Ab^2x^4-1180a^2b^2Bx^6-30aAb^3x^6-150ab^3Bx^8+45Ab^4x^8\right)}{3840a^2x^{10}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-384*a^4*A - 1008*a^3*A*b*x^2 - 480*a^4*B*x^2 - 744*a^2*A
*b^2*x^4 - 1360*a^3*b*B*x^4 - 30*a*A*b^3*x^6 - 1180*a^2*b^2*B*x^6 + 45*A*b^
4*x^8 - 150*a*b^3*B*x^8))/(3840*a^2*x^10) + ((-3*A*b^5 + 10*a*b^4*B)*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(256*a^(5/2))
```

fricas [A] time = 0.98, size = 319, normalized size = 1.69

$$\left[-\frac{15\left(10\,Bab^{4}-3\,Ab^{5}\right)\sqrt{a}\,x^{10}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(15\left(10\,Ba^{2}b^{3}-3\,Aab^{4}\right)x^{8}+10\left(118\,Ba^{3}b^{2}+3\,Aa^{2}b^{3}\right)x^{6}+384\,Aa^{5}+8\left(170\,Ba^{4}b+93\,Aa^{3}b^{2}\right)x^{4}+48\left(10\,Ba^{5}+21\,Aa^{4}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{7680\,a^{3}x^{10}},\ -\frac{15\left(10\,Bab^{4}-3\,Ab^{5}\right)\sqrt{-a}\,x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(15\left(10\,Ba^{2}b^{3}-3\,Aab^{4}\right)x^{8}+10\left(118\,Ba^{3}b^{2}+3\,Aa^{2}b^{3}\right)x^{6}+384\,Aa^{5}+8\left(170\,Ba^{4}b+93\,Aa^{3}b^{2}\right)x^{4}+48\left(10\,Ba^{5}+21\,Aa^{4}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{3840\,a^{3}x^{10}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(10*B*a*b^4 - 3*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2
 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*
B*a^3*b^2 + 3*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x
^4 + 48*(10*B*a^5 + 21*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10), -1/3840*(
15*(10*B*a*b^4 - 3*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*B*a^3*b^2 + 3*A*a^2*b^3)*x^6 +
 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + 21*A*a^4*b
)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

giac [A] time = 0.35, size = 230, normalized size = 1.22

$$-\frac{\frac{15\left(10\,Bab^{5}-3\,Ab^{6}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{150\left(bx^{2}+a\right)^{\frac{9}{2}}Bab^{5}+580\left(bx^{2}+a\right)^{\frac{7}{2}}Ba^{2}b^{5}-1280\left(bx^{2}+a\right)^{\frac{5}{2}}Ba^{3}b^{5}+700\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{4}b^{5}-150\,\sqrt{bx^{2}+a}\,Ba^{5}b^{5}-45\left(bx^{2}+a\right)^{\frac{9}{2}}Ab^{6}+210\left(bx^{2}+a\right)^{\frac{7}{2}}Aab^{6}+384\left(bx^{2}+a\right)^{\frac{5}{2}}Aa^{2}b^{6}-210\left(bx^{2}+a\right)^{\frac{3}{2}}Aa^{3}b^{6}+45\,\sqrt{bx^{2}+a}\,Aa^{4}b^{6}}{a^{2}b^{5}x^{10}}}{3840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

```
[Out] -1/3840*(15*(10*B*a*b^5 - 3*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-
a)*a^2) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 + 580*(b*x^2 + a)^(7/2)*B*a^2*b^5
- 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 + 700*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 150*
sqrt(b*x^2 + a)*B*a^5*b^5 - 45*(b*x^2 + a)^(9/2)*A*b^6 + 210*(b*x^2 + a)^(7
/2)*A*a*b^6 + 384*(b*x^2 + a)^(5/2)*A*a^2*b^6 - 210*(b*x^2 + a)^(3/2)*A*a^3
*b^6 + 45*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^2*b^5*x^10))/b
```

maple [B] time = 0.04, size = 353, normalized size = 1.87

$$-\frac{3A\,b^{5}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{256a^{\frac{5}{2}}}+\frac{5B\,b^{4}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{128a^{\frac{3}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\,A\,b^{5}}{256a^{3}}-\frac{5\sqrt{b\,x^{2}+a}\,B\,b^{4}}{128a^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}A\,b^{5}}{256a^{4}}-\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}}B\,b^{4}}{384a^{3}}+\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{2}}A\,b^{5}}{1280a^{5}}-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}B\,b^{4}}{128a^{4}}-\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}A\,b^{4}}{1280a^{5}x^{2}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,b^{3}}{128a^{4}x^{2}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A\,b^{3}}{640a^{4}x^{4}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B\,b^{2}}{192a^{3}x^{4}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A\,b^{2}}{160a^{3}x^{6}}+\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}Bb}{48a^{2}x^{6}}+\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}}Ab}{80a^{2}x^{8}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}B}{8a\,x^{8}}-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}A}{10a\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x)
```

```
[Out] -1/10*A*(b*x^2+a)^(7/2)/a/x^10+3/80*A*b/a^2/x^8*(b*x^2+a)^(7/2)-1/160*A*b^2
/a^3/x^6*(b*x^2+a)^(7/2)-1/640*A*b^3/a^4/x^4*(b*x^2+a)^(7/2)-3/1280*A*b^4/a
^5/x^2*(b*x^2+a)^(7/2)+3/1280*A*b^5/a^5*(b*x^2+a)^(5/2)+1/256*A*b^5/a^4*(b*
x^2+a)^(3/2)-3/256*A*b^5/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+3/25
6*A*b^5/a^3*(b*x^2+a)^(1/2)-1/8*B/a/x^8*(b*x^2+a)^(7/2)+1/48*B*b/a^2/x^6*(b
*x^2+a)^(7/2)+1/192*B*b^2/a^3/x^4*(b*x^2+a)^(7/2)+1/128*B*b^3/a^4/x^2*(b*x^
2+a)^(7/2)-1/128*B*b^4/a^4*(b*x^2+a)^(5/2)-5/384*B*b^4/a^3*(b*x^2+a)^(3/2)+
5/128*B*b^4/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-5/128*B*b^4/a^2*(
b*x^2+a)^(1/2)
```

maxima [**B**] time = 1.27, size = 330, normalized size = 1.75

$$\frac{5\,Bb^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{3}{2}}} - \frac{3\,Ab^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{256\,a^{\frac{5}{2}}} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Bb^4}{128\,a^4} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Bb^4}{384\,a^3} - \frac{5\sqrt{bx^2+a}\,Bb^4}{128\,a^2} + \frac{3\left(bx^2+a\right)^{\frac{5}{2}}Ab^5}{1280\,a^5} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab^5}{256\,a^4} + \frac{3\sqrt{bx^2+a}\,Ab^5}{256\,a^3} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Bb^3}{128\,a^4x^2} - \frac{3\left(bx^2+a\right)^{\frac{7}{2}}Ab^4}{1280\,a^5x^2} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Bb^2}{192\,a^3x^4} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab^3}{640\,a^4x^4} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}Bb}{48\,a^2x^6} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}Ab^2}{160\,a^3x^6} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{8\,ax^8} + \frac{3\left(bx^2+a\right)^{\frac{7}{2}}Ab}{80\,a^2x^8} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{10\,ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

```
[Out] 5/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/256*A*b^5*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(5/2) - 1/128*(b*x^2 + a)^(5/2)*B*b^4/a^4 - 5/384*(b*x
^2 + a)^(3/2)*B*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*B*b^4/a^2 + 3/1280*(b*x^2 +
 a)^(5/2)*A*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^4 + 3/256*sqrt(b*x^2
+ a)*A*b^5/a^3 + 1/128*(b*x^2 + a)^(7/2)*B*b^3/(a^4*x^2) - 3/1280*(b*x^2 +
a)^(7/2)*A*b^4/(a^5*x^2) + 1/192*(b*x^2 + a)^(7/2)*B*b^2/(a^3*x^4) - 1/640*
(b*x^2 + a)^(7/2)*A*b^3/(a^4*x^4) + 1/48*(b*x^2 + a)^(7/2)*B*b/(a^2*x^6) -
1/160*(b*x^2 + a)^(7/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(7/2)*B/(a*x^8) +
 3/80*(b*x^2 + a)^(7/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(7/2)*A/(a*x^10)
```

mupad [**B**] time = 6.06, size = 205, normalized size = 1.08

$$\frac{7\,A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{128\,x^{10}} - \frac{73\,B\,{\left(b\,x^2+a\right)}^{5/2}}{384\,x^8} - \frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{10\,x^{10}} + \frac{55\,B\,a\,{\left(b\,x^2+a\right)}^{3/2}}{384\,x^8} - \frac{3\,A\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}} - \frac{7\,A\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^{10}} + \frac{3\,A\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^2\,x^{10}} - \frac{5\,B\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{5\,B\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a\,x^8} + \frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,3{}\mathrm{i}}{256\,a^{5/2}} - \frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^11,x)
```

```
[Out] (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (73*B*(a +
b*x^2)^(5/2))/(384*x^8) - (A*(a + b*x^2)^(5/2))/(10*x^10) - (B*b^4*atan(((a
 + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a^(3/2)) + (7*A*a*(a + b*x^2)^(3/2))/
(128*x^10) + (55*B*a*(a + b*x^2)^(3/2))/(384*x^8) - (3*A*a^2*(a + b*x^2)^(1
/2))/(256*x^10) - (7*A*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*A*(a + b*x^2)^(
9/2))/(256*a^2*x^10) - (5*B*a^2*(a + b*x^2)^(1/2))/(128*x^8) - (5*B*(a + b*
x^2)^(7/2))/(128*a*x^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**11,x)
```

```
[Out] Timed out
```

3.537 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{a^2\sqrt{a+bx^2}\,(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

Rubi [**A**] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^2\sqrt{a+bx^2}\,(Ab-aB)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}(Ab-3aB)}{5b^4}-\frac{a\left(a+bx^2\right)^{3/2}(2Ab-3aB)}{3b^4}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (a*(2*A*b - 3*a*B)*(a + b*x^2)^(3/2
))/(3*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4) + (B*(a + b*x^2)^(7/
2))/(7*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}}+\frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3}+\frac{(Ab-3aB)(a+bx)^{3/2}}{b^3}+\frac{B(a+}{}\right.\right.\\
&= \frac{a^2(Ab-aB)\sqrt{a+bx^2}}{b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{B\left(a+bx\right.}{7b^4}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.78

$$
\frac{\sqrt{a+bx^2}\,\left(-48a^3B+8a^2b\left(7A+3Bx^2\right)-2ab^2x^2\left(14A+9Bx^2\right)+3b^3x^4\left(7A+5Bx^2\right)\right)}{105b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^2) + 3*b^3*x^4*(7*A + 5*
B*x^2) - 2*a*b^2*x^2*(14*A + 9*B*x^2)))/(105*b^4)
```

IntegrateAlgebraic [A] time = 0.05, size = 80, normalized size = 0.80

$$
\frac{\sqrt{a+bx^2}\,\left(-48a^3B+56a^2Ab+24a^2bBx^2-28aAb^2x^2-18ab^2Bx^4+21Ab^3x^4+15b^3Bx^6\right)}{105b^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^2 + 24*a^2*b*B*x^2 +
 21*A*b^3*x^4 - 18*a*b^2*B*x^4 + 15*b^3*B*x^6))/(105*b^4)
```

fricas [A] time = 1.02, size = 76, normalized size = 0.76

$$
\frac{\left(15\,Bb^3x^6-3\left(6\,Bab^2-7\,Ab^3\right)x^4-48\,Ba^3+56\,Aa^2b+4\left(6\,Ba^2b-7\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{105\,b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^3*x^6 - 3*(6*B*a*b^2 - 7*A*b^3)*x^4 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

giac [**A**] time = 0.31, size = 101, normalized size = 1.01

$$-\frac{\left(Ba^{3}-Aa^{2}b\right)\sqrt{bx^{2}+a}}{b^{4}}+\frac{15\left(bx^{2}+a\right)^{\frac{7}{2}}B-63\left(bx^{2}+a\right)^{\frac{5}{2}}Ba+105\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}+21\left(bx^{2}+a\right)^{\frac{5}{2}}Ab-70\left(bx^{2}+a\right)^{\frac{3}{2}}Aab}{105\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(B*a^3 - A*a^2*b)*sqrt(b*x^2 + a)/b^4 + 1/105*(15*(b*x^2 + a)^(7/2)*B - 63
*(b*x^2 + a)^(5/2)*B*a + 105*(b*x^2 + a)^(3/2)*B*a^2 + 21*(b*x^2 + a)^(5/2)
*A*b - 70*(b*x^2 + a)^(3/2)*A*a*b)/b^4
```

maple [**A**] time = 0.01, size = 77, normalized size = 0.77

$$\frac{\sqrt{b\,x^{2}+a}\;\left(15B\,x^{6}b^{3}+21A\,b^{3}x^{4}-18Ba\,b^{2}x^{4}-28Aa\,b^{2}x^{2}+24B\,a^{2}b\,x^{2}+56A\,a^{2}b-48B\,a^{3}\right)}{105b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(1/2)*(15*B*b^3*x^6+21*A*b^3*x^4-18*B*a*b^2*x^4-28*A*a*b^2*
x^2+24*B*a^2*b*x^2+56*A*a^2*b-48*B*a^3)/b^4
```

maxima [**A**] time = 0.96, size = 132, normalized size = 1.32

$$\frac{\sqrt{bx^{2}+a}\,Bx^{6}}{7\,b}-\frac{6\,\sqrt{bx^{2}+a}\,Bax^{4}}{35\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,Ax^{4}}{5\,b}+\frac{8\,\sqrt{bx^{2}+a}\,Ba^{2}x^{2}}{35\,b^{3}}-\frac{4\,\sqrt{bx^{2}+a}\,Aax^{2}}{15\,b^{2}}-\frac{16\,\sqrt{bx^{2}+a}\,Ba^{3}}{35\,b^{4}}+\frac{8\,\sqrt{bx^{2}+a}\,Aa^{2}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*B*x^6/b - 6/35*sqrt(b*x^2 + a)*B*a*x^4/b^2 + 1/5*sqrt(b
*x^2 + a)*A*x^4/b + 8/35*sqrt(b*x^2 + a)*B*a^2*x^2/b^3 - 4/15*sqrt(b*x^2 +
a)*A*a*x^2/b^2 - 16/35*sqrt(b*x^2 + a)*B*a^3/b^4 + 8/15*sqrt(b*x^2 + a)*A*a
^2/b^3
```

mupad [**B**] time = 0.67, size = 80, normalized size = 0.80

$$-\sqrt{b\,x^{2}+a}\,\left(\frac{48\,B\,a^{3}-56\,A\,a^{2}\,b}{105\,b^{4}}-\frac{B\,x^{6}}{7\,b}-\frac{x^{4}\,\left(21\,A\,b^{3}-18\,B\,a\,b^{2}\right)}{105\,b^{4}}+\frac{4\,a\,x^{2}\,\left(7\,A\,b-6\,B\,a\right)}{105\,b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] -(a + b*x^2)^(1/2)*((48*B*a^3 - 56*A*a^2*b)/(105*b^4) - (B*x^6)/(7*b) - (x^
4*(21*A*b^3 - 18*B*a*b^2))/(105*b^4) + (4*a*x^2*(7*A*b - 6*B*a))/(105*b^3))
```

sympy **[A]** time = 2.50, size = 172, normalized size = 1.72

$$\begin{cases} \frac{8Aa^2\sqrt{a+bx^2}}{15b^3} - \frac{4Aax^2\sqrt{a+bx^2}}{15b^2} + \frac{Ax^4\sqrt{a+bx^2}}{5b} - \frac{16Ba^3\sqrt{a+bx^2}}{35b^4} + \frac{8Ba^2x^2\sqrt{a+bx^2}}{35b^3} - \frac{6Bax^4\sqrt{a+bx^2}}{35b^2} + \frac{Bx^6\sqrt{a+bx^2}}{7b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(a + b*x**2
)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3*sqrt(a + b*x**2)/(3
5*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*B*a*x**4*sqrt(a + b*
x**2)/(35*b**2) + B*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)), ((A*x**6/6 + B*
x**8/8)/sqrt(a), True))
```

3.538 $\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}-\frac{ax\sqrt{a+bx^2}\,(6Ab-5aB)}{16b^3}+\frac{x^3\sqrt{a+bx^2}\,(6Ab-5aB)}{24b^2}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

Rubi **[A]** time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 321, 217, 206}

$$\frac{a^2(6Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}\,(6Ab-5aB)}{24b^2}-\frac{ax\sqrt{a+bx^2}\,(6Ab-5aB)}{16b^3}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -(a*(6*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*A*b - 5*a*B)*x^3*Sqrt
[a + b*x^2])/(24*b^2) + (B*x^5*Sqrt[a + b*x^2])/(6*b) + (a^2*(6*A*b - 5*a*B
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{Bx^5\sqrt{a+bx^2}}{6b} - \frac{(-6Ab+5aB)\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} - \frac{(a(6Ab-5aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{8b^2}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{\left(a^2(6Ab-5aB)\right)}{16b^3}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{\left(a^2(6Ab-5aB)\right)}{}$$

$$= -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3} + \frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2} + \frac{Bx^5\sqrt{a+bx^2}}{6b} + \frac{a^2(6Ab-5aB)\,t}{16b}$$

Mathematica [A] time = 0.10, size = 100, normalized size = 0.82

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(15a^2B-2ab\left(9A+5Bx^2\right)+4b^2x^2\left(3A+2Bx^2\right)\right)-3a^2(5aB-6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^2*B + 4*b^2*x^2*(3*A + 2*B*x^2) - 2*a*b*(9
*A + 5*B*x^2)) - 3*a^2*(-6*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(48*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 103, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(15a^2Bx-18aAbx-10abBx^3+12Ab^2x^3+8b^2Bx^5\right)}{48b^3} + \frac{\left(5a^3B-6a^2Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-18*a*A*b*x + 15*a^2*B*x + 12*A*b^2*x^3 - 10*a*b*B*x^3 +
8*b^2*B*x^5))/(48*b^3) + ((-6*a^2*A*b + 5*a^3*B)*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(16*b^(7/2))
```

fricas **[A]** time = 0.99, size = 211, normalized size = 1.73

$$\left[-\frac{3\left(5\,Ba^{3}-6\,Aa^{2}b\right)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left(8\,Bb^{3}x^{5}-2\left(5\,Bab^{2}-6\,Ab^{3}\right)x^{3}+3\left(5\,Ba^{2}b-6\,Aab^{2}\right)x\right)\sqrt{bx^{2}+a}}{96\,b^{4}},\ \frac{3\left(5\,Ba^{3}-6\,Aa^{2}b\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(8\,Bb^{3}x^{5}-2\left(5\,Bab^{2}-6\,Ab^{3}\right)x^{3}+3\left(5\,Ba^{2}b-6\,Aab^{2}\right)x\right)\sqrt{bx^{2}+a}}{48\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^3)*x^3 + 3*(5*B*a^2*b
- 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/48*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^
3)*x^3 + 3*(5*B*a^2*b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

giac **[A]** time = 0.45, size = 107, normalized size = 0.88

$$\frac{1}{48}\left(2\left(\frac{4\,Bx^{2}}{b}-\frac{5\,Bab^{3}-6\,Ab^{4}}{b^{5}}\right)x^{2}+\frac{3\left(5\,Ba^{2}b^{2}-6\,Aab^{3}\right)}{b^{5}}\right)\sqrt{bx^{2}+a}\,x+\frac{\left(5\,Ba^{3}-6\,Aa^{2}b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*B*x^2/b - (5*B*a*b^3 - 6*A*b^4)/b^5)*x^2 + 3*(5*B*a^2*b^2 - 6*A*
a*b^3)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple **[A]** time = 0.01, size = 143, normalized size = 1.17

$$\frac{\sqrt{b\,x^{2}+a}\;B\,x^{5}}{6b}+\frac{\sqrt{b\,x^{2}+a}\;A\,x^{3}}{4b}-\frac{5\sqrt{b\,x^{2}+a}\;Ba\,x^{3}}{24b^{2}}+\frac{3A\,a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}-\frac{5B\,a^{3}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{7}{2}}}-\frac{3\sqrt{b\,x^{2}+a}\;Aax}{8b^{2}}+\frac{5\sqrt{b\,x^{2}+a}\;B\,a^{2}x}{16b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^5*(b*x^2+a)^(1/2)/b-5/24*B*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*B*a^2/b^3
*x*(b*x^2+a)^(1/2)-5/16*B*a^3/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*A*x
```

```
^3/b*(b*x^2+a)^(1/2)-3/8*A*a/b^2*x*(b*x^2+a)^(1/2)+3/8*A*a^2/b^(5/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.06, size = 128, normalized size = 1.05

$$\frac{\sqrt{bx^2+a}\,Bx^5}{6\,b}-\frac{5\,\sqrt{bx^2+a}\,Bax^3}{24\,b^2}+\frac{\sqrt{bx^2+a}\,Ax^3}{4\,b}+\frac{5\,\sqrt{bx^2+a}\,Ba^2x}{16\,b^3}-\frac{3\,\sqrt{bx^2+a}\,Aax}{8\,b^2}-\frac{5\,Ba^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}+\frac{3\,Aa^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*B*x^5/b - 5/24*sqrt(b*x^2 + a)*B*a*x^3/b^2 + 1/4*sqrt(b
*x^2 + a)*A*x^3/b + 5/16*sqrt(b*x^2 + a)*B*a^2*x/b^3 - 3/8*sqrt(b*x^2 + a)*
A*a*x/b^2 - 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/8*A*a^2*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(B\,x^2+A\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^4*(A + B*x^2))/(a + b*x^2)^(1/2), x)
```

sympy [B] time = 12.43, size = 235, normalized size = 1.93

$$-\frac{3Aa^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{A\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Ax^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{5}{2}}x}{16b^3\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^{\frac{3}{2}}x^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}\,x^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{Bx^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - A*sqrt(a)*x**3/(8*b*sqrt(1 +
b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + A*x**5/(4*sqr
t(a)*sqrt(1 + b*x**2/a)) + 5*B*a**(5/2)*x/(16*b**3*sqrt(1 + b*x**2/a)) + 5*
B*a**(3/2)*x**3/(48*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**5/(24*b*sqrt(1
+ b*x**2/a)) - 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(7/2)) + B*x**7/(6*
sqrt(a)*sqrt(1 + b*x**2/a))
```

3.539 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=71

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}\,(Ab-aB)}{b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}$$

Rubi [**A**] time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(a+bx^2\right)^{3/2}(Ab-2aB)}{3b^3}-\frac{a\sqrt{a+bx^2}\,(Ab-aB)}{b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3) + ((A*b - 2*a*B)*(a + b*x^2)^(3/2))/
(3*b^3) + (B*(a + b*x^2)^(5/2))/(5*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx,x,x^2\right)\\&=-\frac{a(Ab-aB)\sqrt{a+bx^2}}{b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(8a^2B-2ab\left(5A+2Bx^2\right)+b^2x^2\left(5A+3Bx^2\right)\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^2) + b^2*x^2*(5*A + 3*B*x^2)
))/(15*b^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.79

$$\frac{\sqrt{a+bx^2}\left(8a^2B-10aAb-4abBx^2+5Ab^2x^2+3b^2Bx^4\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^2 - 4*a*b*B*x^2 + 3*b^2*B
*x^4))/(15*b^3)
```

fricas [A] time = 0.85, size = 52, normalized size = 0.73

$$\frac{\left(3\,Bb^2x^4+8\,Ba^2-10\,Aab-\left(4\,Bab-5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^2*x^4 + 8*B*a^2 - 10*A*a*b - (4*B*a*b - 5*A*b^2)*x^2)*sqrt(b*x^
2 + a)/b^3
```

giac [A] time = 0.33, size = 69, normalized size = 0.97

$$\frac{\left(Ba^2 - Aab\right)\sqrt{bx^2 + a}}{b^3} + \frac{3\left(bx^2 + a\right)^{\frac{5}{2}}B - 10\left(bx^2 + a\right)^{\frac{3}{2}}Ba + 5\left(bx^2 + a\right)^{\frac{3}{2}}Ab}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (B*a^2 - A*a*b)*sqrt(b*x^2 + a)/b^3 + 1/15*(3*(b*x^2 + a)^(5/2)*B - 10*(b*x
^2 + a)^(3/2)*B*a + 5*(b*x^2 + a)^(3/2)*A*b)/b^3
```

maple [A] time = 0.01, size = 53, normalized size = 0.75

$$-\frac{\sqrt{b\,x^2 + a}\,\left(-3B\,b^2x^4 - 5A\,b^2x^2 + 4Bab\,x^2 + 10abA - 8a^2B\right)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(-3*B*b^2*x^4-5*A*b^2*x^2+4*B*a*b*x^2+10*A*a*b-8*B*a^
2)/b^3
```

maxima [A] time = 1.02, size = 90, normalized size = 1.27

$$\frac{\sqrt{bx^2 + a}\,Bx^4}{5\,b} - \frac{4\,\sqrt{bx^2 + a}\,Bax^2}{15\,b^2} + \frac{\sqrt{bx^2 + a}\,Ax^2}{3\,b} + \frac{8\,\sqrt{bx^2 + a}\,Ba^2}{15\,b^3} - \frac{2\,\sqrt{bx^2 + a}\,Aa}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*B*x^4/b - 4/15*sqrt(b*x^2 + a)*B*a*x^2/b^2 + 1/3*sqrt(b
*x^2 + a)*A*x^2/b + 8/15*sqrt(b*x^2 + a)*B*a^2/b^3 - 2/3*sqrt(b*x^2 + a)*A*
a/b^2
```

mupad [B] time = 0.60, size = 57, normalized size = 0.80

$$\sqrt{b\,x^2 + a}\,\left(\frac{8\,B\,a^2 - 10\,A\,a\,b}{15\,b^3} + \frac{x^2\,\left(5\,A\,b^2 - 4\,B\,a\,b\right)}{15\,b^3} + \frac{B\,x^4}{5\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*B*a^2 - 10*A*a*b)/(15*b^3) + (x^2*(5*A*b^2 - 4*B*a*b)
)/(15*b^3) + (B*x^4)/(5*b))
```

sympy [A] time = 1.22, size = 121, normalized size = 1.70

$$\begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2} + \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{8Ba^2\sqrt{a+bx^2}}{15b^3} - \frac{4Bax^2\sqrt{a+bx^2}}{15b^2} + \frac{Bx^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x**2)/(3*b)
 + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqrt(a + b*x**2)/(15*b*
*2) + B*x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/sqrt
(a), True))
```

3.540 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4Ab-3aB)}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 321, 217, 206}

$$\frac{x\sqrt{a+bx^2}\,(4Ab-3aB)}{8b^2}-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] ((4*A*b - 3*a*B)*x*Sqrt[a + b*x^2])/(8*b^2) + (B*x^3*Sqrt[a + b*x^2])/(4*b)
 - (a*(4*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{(-4Ab+3aB)\int\frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b}\\ &= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{(a(4Ab-3aB))\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2}\\ &= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{(a(4Ab-3aB))\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b^2}\\ &= \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 74, normalized size = 0.83

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(-3aB+4Ab+2bBx^2\right)+a(3aB-4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(4*A*b - 3*a*B + 2*b*B*x^2) + a*(-4*A*b + 3*a*B)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 79, normalized size = 0.89

$$\frac{\left(4aAb-3a^2B\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{5/2}}+\frac{\sqrt{a+bx^2}\left(-3aBx+4Abx+2bBx^3\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(4*A*b*x - 3*a*B*x + 2*b*B*x^3))/(8*b^2) + ((4*a*A*b - 3*a
^2*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

fricas [A] time = 0.71, size = 162, normalized size = 1.82

$$\left[-\frac{\left(3\,Ba^{2}-4\,Aab\right)\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,\left(2\,Bb^{2}x^{3}-\left(3\,Bab-4\,Ab^{2}\right)x\right)\sqrt{bx^{2}+a}}{16\,b^{3}},-\frac{\left(3\,Ba^{2}-4\,Aab\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(2\,Bb^{2}x^{3}-\left(3\,Bab-4\,Ab^{2}\right)x\right)\sqrt{bx^{2}+a}}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*(2*B*b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1
/8*((3*B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*
b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

giac [A] time = 0.36, size = 75, normalized size = 0.84

$$\frac{1}{8}\,\sqrt{bx^{2}+a}\left(\frac{2\,Bx^{2}}{b}-\frac{3\,Bab-4\,Ab^{2}}{b^{3}}\right)x-\frac{\left(3\,Ba^{2}-4\,Aab\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*B*x^2/b - (3*B*a*b - 4*A*b^2)/b^3)*x - 1/8*(3*B*a^2
- 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 101, normalized size = 1.13

$$\frac{\sqrt{b\,x^{2}+a}\;B\,x^{3}}{4b}-\frac{Aa\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}+\frac{3B\,a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}+\frac{\sqrt{b\,x^{2}+a}\;Ax}{2b}-\frac{3\sqrt{b\,x^{2}+a}\;Bax}{8b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^3*(b*x^2+a)^(1/2)/b-3/8*B*a/b^2*x*(b*x^2+a)^(1/2)+3/8*B*a^2/b^(5/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*A*x/b*(b*x^2+a)^(1/2)-1/2*A*a/b^(3/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.08, size = 86, normalized size = 0.97

$$\frac{\sqrt{bx^{2}+a}\,Bx^{3}}{4\,b}-\frac{3\,\sqrt{bx^{2}+a}\,Bax}{8\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,Ax}{2\,b}+\frac{3\,Ba^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*B*x^3/b - 3/8*sqrt(b*x^2 + a)*B*a*x/b^2 + 1/2*sqrt(b*x^
2 + a)*A*x/b + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*A*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(a + b*x^2)^(1/2), x)
```

sympy [A] time = 7.78, size = 150, normalized size = 1.69

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Aa\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - B*sqrt(a)*x**3/(8*b*sq
rt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + B*x**5
/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

3.541 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{a+bx^2}\,(Ab-aB)}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\sqrt{a+bx^2}\,(Ab-aB)}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

```
[Out] ((A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (B*(a + b*x^2)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx,x,x^2\right)\\ &= \frac{(Ab-aB)\sqrt{a+bx^2}}{b^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 33, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-2aB+3Ab+bBx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(3*A*b - 2*a*B + b*B*x^2))/(3*b^2)
```

IntegrateAlgebraic [**A**] time = 0.03, size = 33, normalized size = 0.77

$$\frac{\sqrt{a+bx^2}\left(-2aB+3Ab+bBx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(3*A*b - 2*a*B + b*B*x^2))/(3*b^2)
```

fricas [**A**] time = 0.83, size = 29, normalized size = 0.67

$$\frac{\left(Bbx^2-2\,Ba+3\,Ab\right)\sqrt{bx^2+a}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/3*(B*b*x^2 - 2*B*a + 3*A*b)*sqrt(b*x^2 + a)/b^2
```

giac [A] time = 0.31, size = 38, normalized size = 0.88

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{3\,b^2}-\frac{\sqrt{bx^2+a}\,(Ba-Ab)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^2 + a)^(3/2)*B/b^2 - sqrt(b*x^2 + a)*(B*a - A*b)/b^2
```

maple [A] time = 0.01, size = 30, normalized size = 0.70

$$\frac{\sqrt{b\,x^2+a}\;\left(Bb\,x^2+3Ab-2Ba\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(B*b*x^2+3*A*b-2*B*a)/b^2
```

maxima [A] time = 1.00, size = 49, normalized size = 1.14

$$\frac{\sqrt{bx^2+a}\,Bx^2}{3\,b}-\frac{2\,\sqrt{bx^2+a}\,Ba}{3\,b^2}+\frac{\sqrt{bx^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*B*x^2/b - 2/3*sqrt(b*x^2 + a)*B*a/b^2 + sqrt(b*x^2 + a)
*A/b
```

mupad [B] time = 0.57, size = 34, normalized size = 0.79

$$\left(\frac{3\,A\,b-2\,B\,a}{3\,b^2}+\frac{B\,x^2}{3\,b}\right)\,\sqrt{b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

```
[Out] ((3*A*b - 2*B*a)/(3*b^2) + (B*x^2)/(3*b))*(a + b*x^2)^(1/2)
```

sympy [A] time = 0.65, size = 70, normalized size = 1.63

$$\begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2) + B*x**2*
sqrt(a + b*x**2)/(3*b), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/sqrt(a), True))
```

3.542 $\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 217, 206}

$$\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx &= \frac{Bx\sqrt{a+bx^2}}{2b}-\frac{(-2Ab+aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{Bx\sqrt{a+bx^2}}{2b}-\frac{(-2Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{Bx\sqrt{a+bx^2}}{2b}+\frac{(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 57, normalized size = 0.98

$$\frac{Bx\sqrt{a+bx^2}}{2b}-\frac{(aB-2Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) - ((-2*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.06, size = 59, normalized size = 1.02

$$\frac{(aB-2Ab)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2])/(2*b) + ((-2*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

fricas [**A**] time = 1.12, size = 110, normalized size = 1.90

$$\left[\frac{2\,\sqrt{bx^2+a}\,Bbx-(Ba-2\,Ab)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{4\,b^2},\ \frac{\sqrt{bx^2+a}\,Bbx+(Ba-2\,Ab)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*B*b*x - (B*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*B*b*x + (B*a - 2*A*b)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

giac [A] time = 0.40, size = 48, normalized size = 0.83

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b}+\frac{(Ba-2\,Ab)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b + 1/2*(B*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

maple [A] time = 0.00, size = 62, normalized size = 1.07

$$\frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{Ba\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{b\,x^2+a}\,Bx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*x*(b*x^2+a)^(1/2)/b-1/2*B*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+A*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

maxima [A] time = 1.16, size = 47, normalized size = 0.81

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b}-\frac{Ba\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{A\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + A*arcs
inh(b*x/sqrt(a*b))/sqrt(b)
```

mupad [B] time = 1.07, size = 86, normalized size = 1.48

$$\begin{cases} \frac{B\,x^3+3\,A\,x}{3\,\sqrt{a}} & \text{if } b=0\\ \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*A*x + B*x^3)/(3*a^(1/2)), b ~= 0, (A*log(b^(1/2)*x + (
a + b*x^2)^(1/2)))/b^(1/2) - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(
2*b^(3/2)) + (B*x*(a + b*x^2)^(1/2))/(2*b))
```

sympy [A] time = 3.31, size = 126, normalized size = 2.17

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\\frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\\frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)
/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2))
```

3.543 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{B\sqrt{a+bx^2}}{b}+\frac{1}{2}A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{B\sqrt{a+bx^2}}{b}+\frac{A\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\
&= \frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.04, size = 43, normalized size = 1.00

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.97, size = 102, normalized size = 2.37

$$\left[\frac{A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}\,Ba}{2\,ab},\ \frac{A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,Ba}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*s
qrt(b*x^2 + a)*B*a)/(a*b), (A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) +
 sqrt(b*x^2 + a)*B*a)/(a*b)]
```

giac [A] time = 0.37, size = 38, normalized size = 0.88

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)*B/b
```

maple [A] time = 0.01, size = 45, normalized size = 1.05

$$-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{\sqrt{b\,x^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(1/2),x)
```

```
[Out] B*(b*x^2+a)^(1/2)/b-A/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.05, size = 33, normalized size = 0.77

$$-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*B/b
```

mupad [B] time = 1.02, size = 35, normalized size = 0.81

$$\frac{B\sqrt{b\,x^2+a}}{b}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*(a + b*x^2)^(1/2))/b - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 13.20, size = 61, normalized size = 1.42

$$\frac{A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^2}{\sqrt{a}} & \text{for}\ b=0\\ -\frac{2\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**2)))/(a*sqrt(-1/a)) - B*Piecewise((-x**2
/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**2)/b, True))/2
```

3.544 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Rubi [**A**] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {451, 217, 206}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{ax}+B\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{A\sqrt{a+bx^2}}{ax}+B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [**A**] time = 0.02, size = 47, normalized size = 1.00

$$
\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqr
t[b]
```

IntegrateAlgebraic [**A**] time = 0.08, size = 50, normalized size = 1.06

$$
-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt
[b]
```

fricas [**A**] time = 0.67, size = 109, normalized size = 2.32

$$
\left[\frac{Ba\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,\sqrt{bx^2+a}\,Ab}{2\,abx}, -\frac{Ba\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}\,Ab}{abx}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqr
t(b*x^2 + a)*A*b)/(a*b*x), -(B*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + sqrt(b*x^2 + a)*A*b)/(a*b*x)]
```

giac **[A]** time = 0.41, size = 58, normalized size = 1.23

$$-\frac{B\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*B*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2*A*sqrt(b)/((sqrt(b)
*x - sqrt(b*x^2 + a))^2 - a)
```

maple **[A]** time = 0.01, size = 41, normalized size = 0.87

$$\frac{B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{\sqrt{b\,x^2+a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] B*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

maxima **[A]** time = 1.09, size = 33, normalized size = 0.70

$$\frac{B\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{\sqrt{bx^2+a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - sqrt(b*x^2 + a)*A/(a*x)
```

mupad **[B]** time = 0.74, size = 40, normalized size = 0.85

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*(a + b*x^2)^(1/2))/(a*x
)
```

sympy **[A]** time = 2.64, size = 99, normalized size = 2.11

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + B*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/
a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a
> 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0
)))
```

3.545 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(2*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{\left(-\frac{Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}\\ &= -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 1.03

$$\frac{1}{2}\left(-\frac{2\left(aB-\frac{Ab}{2}\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^2])/(a*x^2)) - (2*(-1/2*(A*b) + a*B)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/a^(3/2))/2
```

IntegrateAlgebraic [A] time = 0.07, size = 58, normalized size = 1.00

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*a^(3/2))
```

fricas [A] time = 0.89, size = 124, normalized size = 2.14

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}\,Aa}{4\,a^2x^2},\ \frac{(2\,Ba-Ab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,Aa}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) + 2*sqrt(b*x^2 + a)*A*a)/(a^2*x^2), 1/2*((2*B*a - A*b)*sqrt(-a)*x
^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*A*a)/(a^2*x^2)]
```

giac [A] time = 0.37, size = 62, normalized size = 1.07

$$\frac{\frac{\left(2\,Bab-Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\sqrt{bx^2+a}\,Ab}{ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^2 + a)*A*b/(a*x^2))/b
```

maple [A] time = 0.01, size = 79, normalized size = 1.36

$$\frac{Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}-\frac{\sqrt{b\,x^2+a}\,A}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*A*(b*x^2+a)^(1/2)/a/x^2+1/2*A*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-B/a^(1/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.11, size = 56, normalized size = 0.97

$$-\frac{B\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{Ab\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

mupad [B] time = 1.34, size = 60, normalized size = 1.03

$$\frac{A\,b\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}-\frac{B\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (A*(a + b*x^2)^(1/2))/
(2*a*x^2) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 39.42, size = 66, normalized size = 1.14

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}-\frac{B\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.546 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{a+bx^2}\,(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{\sqrt{a+bx^2}\,(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2])/(3*a^2*x
)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{3ax^3}-\frac{(2Ab-3aB)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\ &= -\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}\left(a\left(A+3Bx^2\right)-2Abx^2\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/3*(Sqrt[a + b*x^2]*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/(a^2*x^3)
```

IntegrateAlgebraic [A] time = 0.09, size = 40, normalized size = 0.75

$$\frac{\sqrt{a+bx^2}\left(-aA-3aBx^2+2Abx^2\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(a*A) + 2*A*b*x^2 - 3*a*B*x^2))/(3*a^2*x^3)
```

fricas [A] time = 0.99, size = 34, normalized size = 0.64

$$-\frac{\left((3\,Ba-2\,Ab)x^2+Aa\right)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*B*a - 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

giac [B] time = 0.43, size = 120, normalized size = 2.26

$$\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4B\sqrt{b}-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ab^{\frac{3}{2}}+3\,Ba^2\sqrt{b}-2\,Aab^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(3/2) + 3*B*a
^2*sqrt(b) - 2*A*a*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

maple [A] time = 0.01, size = 36, normalized size = 0.68

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2Ab\,x^{2}+3Ba\,x^{2}+Aa\right)}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(b*x^2+a)^(1/2)*(-2*A*b*x^2+3*B*a*x^2+A*a)/x^3/a^2
```

maxima [A] time = 1.12, size = 56, normalized size = 1.06

$$-\frac{\sqrt{bx^{2}+a}\,B}{ax}+\frac{2\,\sqrt{bx^{2}+a}\,Ab}{3\,a^{2}x}-\frac{\sqrt{bx^{2}+a}\,A}{3\,ax^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x^2 + a)*B/(a*x) + 2/3*sqrt(b*x^2 + a)*A*b/(a^2*x) - 1/3*sqrt(b*x^2
 + a)*A/(a*x^3)
```

mupad [B] time = 0.58, size = 35, normalized size = 0.66

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(A\,a-2\,A\,b\,x^{2}+3\,B\,a\,x^{2}\right)}{3\,a^{2}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(A*a - 2*A*b*x^2 + 3*B*a*x^2))/(3*a^2*x^3)
```

sympy [A] time = 3.87, size = 70, normalized size = 1.32

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}+\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/(b*x**2) +
 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.547 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=90

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}\,(3Ab-4aB)}{8a^2x^2}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^2}\,(3Ab-4aB)}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{A\sqrt{a+bx^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^2*x
^2) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\ &= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(b(3Ab-4aB))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{16a^2}\\ &= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}+\frac{(3Ab-4aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{8a^2}\\ &= -\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 83, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\left(-\frac{2a^2\left(A+2Bx^2\right)}{x^4}+\frac{b(4aB-3Ab)\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\sqrt{\frac{bx^2}{a}+1}}+\frac{3aAb}{x^2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*((3*a*A*b)/x^2 - (2*a^2*(A + 2*B*x^2))/x^4 + (b*(-3*A*b +
4*a*B)*ArcTanh[Sqrt[1 + (b*x^2)/a]])/Sqrt[1 + (b*x^2)/a]))/(8*a^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 80, normalized size = 0.89

$$\frac{\left(4abB-3Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{\sqrt{a+bx^2}\,\left(-2aA-4aBx^2+3Abx^2\right)}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*A + 3*A*b*x^2 - 4*a*B*x^2))/(8*a^2*x^4) + ((-3*A*b^2
 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

fricas [A] time = 1.08, size = 171, normalized size = 1.90

$$\left[-\frac{\left(4\,Bab-3\,Ab^2\right)\sqrt{a}\,x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\left(2\,Aa^2+\left(4\,Ba^2-3\,Aab\right)x^2\right)\sqrt{bx^2+a}}{16\,a^3x^4},\,-\frac{\left(4\,Bab-3\,Ab^2\right)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(2\,Aa^2+\left(4\,Ba^2-3\,Aab\right)x^2\right)\sqrt{bx^2+a}}{8\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(
a^3*x^4), -1/8*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2
 + a)) + (2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^4)]
```

giac [A] time = 0.34, size = 121, normalized size = 1.34

$$-\frac{\frac{\left(4\,Bab^2-3\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{4\,\left(bx^2+a\right)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^2+a}\,Ba^2b^2-3\,\left(bx^2+a\right)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^2+a}\,Aab^3}{a^2b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2)
 + (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 - 3*(b*x^2 +
a)^(3/2)*A*b^3 + 5*sqrt(b*x^2 + a)*A*a*b^3)/(a^2*b^2*x^4))/b
```

maple [A] time = 0.01, size = 119, normalized size = 1.32

$$-\frac{3A\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}+\frac{Bb\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{3\sqrt{b\,x^{2}+a}\,Ab}{8a^{2}x^{2}}-\frac{\sqrt{b\,x^{2}+a}\,B}{2a\,x^{2}}-\frac{\sqrt{b\,x^{2}+a}\,A}{4a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B/a/x^2*(b*x^2+a)^(1/2)+1/2*B*b/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)-1/4*A*(b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*A*b
^2/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.06, size = 96, normalized size = 1.07

$$\frac{Bb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{3\,Ab^{2}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}-\frac{\sqrt{bx^{2}+a}\,B}{2\,ax^{2}}+\frac{3\,\sqrt{bx^{2}+a}\,Ab}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}\,A}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*A*b^2*arcsinh(a/(sqrt(a
*b)*abs(x)))/a^(5/2) - 1/2*sqrt(b*x^2 + a)*B/(a*x^2) + 3/8*sqrt(b*x^2 + a)*
A*b/(a^2*x^2) - 1/4*sqrt(b*x^2 + a)*A/(a*x^4)
```

mupad [B] time = 1.53, size = 99, normalized size = 1.10

$$\frac{3\,A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{5\,A\,\sqrt{b\,x^2+a}}{8\,a\,x^4}-\frac{3\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{B\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (3*A*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*A*(a + b*x^2)^(1/2))/(8*a*x^4) - (
3*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) - (B*(a + b*x^2)^(1/2
))/(2*a*x^2) + (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2))
```

sympy [A] time = 88.38, size = 150, normalized size = 1.67

$$-\frac{A}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{A\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3Ab^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{2ax}+\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sqrt(a/(b*x*
*2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(s
qrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) +
 B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

3.548 $\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2b\sqrt{a+bx^2}\,(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}\,(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$-\frac{2b\sqrt{a+bx^2}\,(4Ab-5aB)}{15a^3x}+\frac{\sqrt{a+bx^2}\,(4Ab-5aB)}{15a^2x^3}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(5*a*x^5) + ((4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^2*
x^3) - (2*b*(4*A*b - 5*a*B)*Sqrt[a + b*x^2])/(15*a^3*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{5ax^5}-\frac{(4Ab-5aB)\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{5a}\\ &= -\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3}+\frac{(2b(4Ab-5aB))\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{15a^2}\\ &= -\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3}-\frac{2b(4Ab-5aB)\sqrt{a+bx^2}}{15a^3x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}\left(a^2\left(3A+5Bx^2\right)-2abx^2\left(2A+5Bx^2\right)+8Ab^2x^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/15*(Sqrt[a + b*x^2]*(8*A*b^2*x^4 - 2*a*b*x^2*(2*A + 5*B*x^2) + a^2*(3*A
+ 5*B*x^2)))/(a^3*x^5)
```

IntegrateAlgebraic [A] time = 0.12, size = 62, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-3a^2A-5a^2Bx^2+4aAbx^2+10abBx^4-8Ab^2x^4\right)}{15a^3x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*a^2*A + 4*a*A*b*x^2 - 5*a^2*B*x^2 - 8*A*b^2*x^4 + 10*a
*b*B*x^4))/(15*a^3*x^5)
```

fricas [A] time = 1.16, size = 58, normalized size = 0.69

$$\frac{\left(2\left(5\,Bab-4\,Ab^2\right)x^4-3\,Aa^2-\left(5\,Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 - (5*B*a^2 - 4*A*a*b)*x^2)*sqrt(b
*x^2 + a)/(a^3*x^5)
```

giac [**B**] time = 0.42, size = 176, normalized size = 2.10

$$\frac{4\left(15\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Bb^{\frac{3}{2}}-35\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Bab^{\frac{3}{2}}+40\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ab^{\frac{5}{2}}+25\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{2}b^{\frac{3}{2}}-20\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aab^{\frac{5}{2}}-5Ba^{3}b^{\frac{3}{2}}+4Aa^{2}b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*b^(3/2) - 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*B*a*b^(3/2) + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(5/2) + 2
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^(3/2) - 20*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a*b^(5/2) - 5*B*a^3*b^(3/2) + 4*A*a^2*b^(5/2))/((sqrt(b)*x - sq
rt(b*x^2 + a))^2 - a)^5
```

maple [**A**] time = 0.01, size = 59, normalized size = 0.70

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(8A\,b^{2}x^{4}-10Bab\,x^{4}-4Aab\,x^{2}+5B\,a^{2}x^{2}+3a^{2}A\right)}{15a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(8*A*b^2*x^4-10*B*a*b*x^4-4*A*a*b*x^2+5*B*a^2*x^2+3*A
*a^2)/x^5/a^3
```

maxima [**A**] time = 1.02, size = 96, normalized size = 1.14

$$\frac{2\,\sqrt{bx^{2}+a}\,Bb}{3\,a^{2}x}-\frac{8\,\sqrt{bx^{2}+a}\,Ab^{2}}{15\,a^{3}x}-\frac{\sqrt{bx^{2}+a}\,B}{3\,ax^{3}}+\frac{4\,\sqrt{bx^{2}+a}\,Ab}{15\,a^{2}x^{3}}-\frac{\sqrt{bx^{2}+a}\,A}{5\,ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^2 + a)*B*b/(a^2*x) - 8/15*sqrt(b*x^2 + a)*A*b^2/(a^3*x) - 1/3*
sqrt(b*x^2 + a)*B/(a*x^3) + 4/15*sqrt(b*x^2 + a)*A*b/(a^2*x^3) - 1/5*sqrt(b
*x^2 + a)*A/(a*x^5)
```

mupad [**B**] time = 0.68, size = 58, normalized size = 0.69

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(5\,B\,a^{2}\,x^{2}+3\,A\,a^{2}-10\,B\,a\,b\,x^{4}-4\,A\,a\,b\,x^{2}+8\,A\,b^{2}\,x^{4}\right)}{15\,a^{3}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(3*A*a^2 + 5*B*a^2*x^2 + 8*A*b^2*x^4 - 4*A*a*b*x^2 - 10
*B*a*b*x^4))/(15*a^3*x^5)
```

sympy [B] time = 3.17, size = 355, normalized size = 4.23

$$-\frac{3Aa^{4}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{2Aa^{3}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{3Aa^{2}b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{12Aab^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{8Ab^{\frac{17}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{15a^{5}b^{4}x^{4}+30a^{4}b^{5}x^{6}+15a^{3}b^{6}x^{8}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{2}}+1}}{3ax^{2}}+\frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*b**(13/
2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*A*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - B*
sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a**2)
```

3.549 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=123

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}\,(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}\,(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{b^2(5Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{b\sqrt{a+bx^2}\,(5Ab-6aB)}{16a^3x^2}+\frac{\sqrt{a+bx^2}\,(5Ab-6aB)}{24a^2x^4}-\frac{A\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^2*
x^4) - (b*(5*A*b - 6*a*B)*Sqrt[a + b*x^2])/(16*a^3*x^2) + (b^2*(5*A*b - 6*a
*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx}}\,dx,x,x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{\left(-\frac{5Ab}{2}+3aB\right)\operatorname{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} + \frac{(b(5Ab-6aB))\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{16a^2}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} - \frac{\left(b^2(5Ab-6aB)\right)\operatorname{Subst}}{32a}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} - \frac{(b(5Ab-6aB))\operatorname{Subst}\Bigg(}{1}$$

$$= -\frac{A\sqrt{a+bx^2}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4} - \frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2} + \frac{b^2(5Ab-6aB)\tanh^{-1}\Big(}{16a^{7/2}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.50

$$-\frac{\sqrt{a+bx^2}\left(a^3A+b^2x^6(6aB-5Ab)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx^2}{a}+1\right)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/6*(Sqrt[a + b*x^2]*(a^3*A + b^2*(-5*A*b + 6*a*B)*x^6*Hypergeometric2F1[1
/2, 3, 3/2, 1 + (b*x^2)/a]))/(a^4*x^6)
```

IntegrateAlgebraic [A]   time = 0.21, size = 104, normalized size = 0.85

$$\frac{\left(5Ab^3-6ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}+\frac{\sqrt{a+bx^2}\,\left(-8a^2A-12a^2Bx^2+10aAbx^2+18abBx^4-15Ab^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2*x^4 + 1
8*a*b*B*x^4))/(48*a^3*x^6) + ((5*A*b^3 - 6*a*b^2*B)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/(16*a^(7/2))
```

fricas [A]   time = 0.74, size = 223, normalized size = 1.81

$$\left[-\frac{3\left(6\,Bab^2-5\,Ab^3\right)\sqrt{a}\,x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\left(3\left(6\,Ba^2b-5\,Aab^2\right)x^4-8\,Aa^3-2\left(6\,Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{96\,a^4x^6},\ \frac{3\left(6\,Bab^2-5\,Ab^3\right)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\left(6\,Ba^2b-5\,Aab^2\right)x^4-8\,Aa^3-2\left(6\,Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{48\,a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*B*a*b^2 - 5*A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) - 2*(3*(6*B*a^2*b - 5*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*
a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6), 1/48*(3*(6*B*a*b^2 - 5*A*
b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(6*B*a^2*b - 5*A*a*
b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6
)]
```

giac [A]   time = 0.38, size = 158, normalized size = 1.28

$$\frac{\frac{3\left(6\,Bab^3-5\,Ab^4\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{18\left(bx^2+a\right)^{\frac{5}{2}}Bab^3-48\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^3+30\,\sqrt{bx^2+a}\,Ba^3b^3-15\left(bx^2+a\right)^{\frac{5}{2}}Ab^4+40\left(bx^2+a\right)^{\frac{3}{2}}Aab^4-33\,\sqrt{bx^2+a}\,Aa^2b^4}{a^3b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(3*(6*B*a*b^3 - 5*A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^
3) + (18*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 30*sq
```

```
rt(b*x^2 + a)*B*a^3*b^3 - 15*(b*x^2 + a)^(5/2)*A*b^4 + 40*(b*x^2 + a)^(3/2)
*A*a*b^4 - 33*sqrt(b*x^2 + a)*A*a^2*b^4)/(a^3*b^3*x^6))/b
```

maple [A] time = 0.02, size = 161, normalized size = 1.31

$$\frac{5Ab^3\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{7}{2}}}-\frac{3Bb^2\ln\left(\frac{2a+2\sqrt{bx^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}-\frac{5\sqrt{bx^2+a}\,Ab^2}{16a^3x^2}+\frac{3\sqrt{bx^2+a}\,Bb}{8a^2x^2}+\frac{5\sqrt{bx^2+a}\,Ab}{24a^2x^4}-\frac{\sqrt{bx^2+a}\,B}{4ax^4}-\frac{\sqrt{bx^2+a}\,A}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*B/a/x^4*(b*x^2+a)^(1/2)+3/8*B*b/a^2/x^2*(b*x^2+a)^(1/2)-3/8*B*b^2/a^(5
/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/6*A*(b*x^2+a)^(1/2)/a/x^6+5/24*
A*b/a^2/x^4*(b*x^2+a)^(1/2)-5/16*A*b^2/a^3/x^2*(b*x^2+a)^(1/2)+5/16*A*b^3/a
^(7/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.00, size = 138, normalized size = 1.12

$$-\frac{3\,Bb^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{5\,Ab^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}}+\frac{3\,\sqrt{bx^2+a}\,Bb}{8\,a^2x^2}-\frac{5\,\sqrt{bx^2+a}\,Ab^2}{16\,a^3x^2}-\frac{\sqrt{bx^2+a}\,B}{4\,ax^4}+\frac{5\,\sqrt{bx^2+a}\,Ab}{24\,a^2x^4}-\frac{\sqrt{bx^2+a}\,A}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/16*A*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(7/2) + 3/8*sqrt(b*x^2 + a)*B*b/(a^2*x^2) - 5/16*sqrt(b*
x^2 + a)*A*b^2/(a^3*x^2) - 1/4*sqrt(b*x^2 + a)*B/(a*x^4) + 5/24*sqrt(b*x^2
+ a)*A*b/(a^2*x^4) - 1/6*sqrt(b*x^2 + a)*A/(a*x^6)
```

mupad [B] time = 1.68, size = 140, normalized size = 1.14

$$\frac{5\,A\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{11\,A\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{5\,A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,B\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^(1/2)),x)
```

```
[Out] (5*A*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (3*B*b^2*atanh((a + b*x^2)^(1/2)/a^(1
/2)))/(8*a^(5/2)) - (11*A*(a + b*x^2)^(1/2))/(16*a*x^6) - (A*b^3*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (5*A*(a + b*x^2)^(5/2))/(16*a
^3*x^6) - (5*B*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*B*(a + b*x^2)^(3/2))/(8*a^
2*x^4)
```

sympy [B] time = 133.68, size = 235, normalized size = 1.91

$$-\frac{A}{6\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{A\sqrt{b}}{24ax^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{3}{2}}}{48a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{5Ab^{\frac{5}{2}}}{16a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5Ab^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{7}{2}}}-\frac{B}{4\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{B\sqrt{b}}{8ax^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3Bb^{\frac{3}{2}}}{8a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3Bb^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(24*a*x**5*sqrt(a/(b*x
**2) + 1)) - 5*A*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2
)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(1
6*a**(7/2)) - B/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*x**3
*sqrt(a/(b*x**2) + 1)) + 3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B
*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2))
```

3.550 $\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$\frac{8b^2\sqrt{a+bx^2}\,(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}\,(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

Rubi [A] time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8b^2\sqrt{a+bx^2}\,(6Ab-7aB)}{105a^4x}-\frac{4b\sqrt{a+bx^2}\,(6Ab-7aB)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,(6Ab-7aB)}{35a^2x^5}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2])/(7*a*x^7) + ((6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(35*a^2*
x^5) - (4*b*(6*A*b - 7*a*B)*Sqrt[a + b*x^2])/(105*a^3*x^3) + (8*b^2*(6*A*b
- 7*a*B)*Sqrt[a + b*x^2])/(105*a^4*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{7ax^7}-\frac{(6Ab-7aB)\int \frac{1}{x^6\sqrt{a+bx^2}}\,dx}{7a}\\ &= -\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5}+\frac{(4b(6Ab-7aB))\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}\\ &= -\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5}-\frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3}-\frac{\left(8b^2(6Ab-7aB)\right)\int}{105a^3}\\ &= -\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5}-\frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3}+\frac{8b^2(6Ab-7aB)\sqrt{a}}{105a^4x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.72

$$\frac{\left(\frac{bx^2}{a}+1\right)\left(3a^2-4abx^2+8b^2x^4\right)(6Ab-7aB)}{105a^3x^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/7*(A*Sqrt[a + b*x^2])/(a*x^7) + ((6*A*b - 7*a*B)*(1 + (b*x^2)/a)*(3*a^2
- 4*a*b*x^2 + 8*b^2*x^4))/(105*a^3*x^5*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.15, size = 86, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\left(-15a^3A-21a^3Bx^2+18a^2Abx^2+28a^2bBx^4-24aAb^2x^4-56ab^2Bx^6+48Ab^3x^6\right)}{105a^4x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-15*a^3*A + 18*a^2*A*b*x^2 - 21*a^3*B*x^2 - 24*a*A*b^2*x^
4 + 28*a^2*b*B*x^4 + 48*A*b^3*x^6 - 56*a*b^2*B*x^6))/(105*a^4*x^7)
```

fricas [A] time = 1.06, size = 82, normalized size = 0.70

$$-\frac{\left(8\left(7\,Bab^2-6\,Ab^3\right)x^6-4\left(7\,Ba^2b-6\,Aab^2\right)x^4+15\,Aa^3+3\left(7\,Ba^3-6\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*(8*(7*B*a*b^2 - 6*A*b^3)*x^6 - 4*(7*B*a^2*b - 6*A*a*b^2)*x^4 + 15*A*
a^3 + 3*(7*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*x^7)
```

giac [B] time = 0.42, size = 232, normalized size = 1.98

$$\frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{7}{2}}+147\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2b^{\frac{5}{2}}-126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aab^{\frac{7}{2}}-49\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3b^{\frac{5}{2}}+42\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^2b^{\frac{7}{2}}+7Ba^4b^{\frac{5}{2}}-6Aa^3b^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(5/2) - 175*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*B*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(7/2)
 + 147*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(5/2) - 126*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*A*a*b^(7/2) - 49*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(5
/2) + 42*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(7/2) + 7*B*a^4*b^(5/2) -
6*A*a^3*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

maple [A] time = 0.01, size = 83, normalized size = 0.71

$$-\frac{\sqrt{b\,x^2+a}\,\left(-48A\,b^3x^6+56Ba\,b^2x^6+24x^4Aa\,b^2-28B\,a^2b\,x^4-18A\,a^2b\,x^2+21B\,a^3x^2+15A\,a^3\right)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/105*(b*x^2+a)^(1/2)*(-48*A*b^3*x^6+56*B*a*b^2*x^6+24*A*a*b^2*x^4-28*B*a^
2*b*x^4-18*A*a^2*b*x^2+21*B*a^3*x^2+15*A*a^3)/x^7/a^4
```

maxima [A] time = 1.10, size = 138, normalized size = 1.18

$$-\frac{8\sqrt{bx^2+a}\,Bb^2}{15\,a^3x}+\frac{16\sqrt{bx^2+a}\,Ab^3}{35\,a^4x}+\frac{4\sqrt{bx^2+a}\,Bb}{15\,a^2x^3}-\frac{8\sqrt{bx^2+a}\,Ab^2}{35\,a^3x^3}-\frac{\sqrt{bx^2+a}\,B}{5\,ax^5}+\frac{6\sqrt{bx^2+a}\,Ab}{35\,a^2x^5}-\frac{\sqrt{bx^2+a}\,A}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -8/15*sqrt(b*x^2 + a)*B*b^2/(a^3*x) + 16/35*sqrt(b*x^2 + a)*A*b^3/(a^4*x) +
 4/15*sqrt(b*x^2 + a)*B*b/(a^2*x^3) - 8/35*sqrt(b*x^2 + a)*A*b^2/(a^3*x^3)
- 1/5*sqrt(b*x^2 + a)*B/(a*x^5) + 6/35*sqrt(b*x^2 + a)*A*b/(a^2*x^5) - 1/7*
sqrt(b*x^2 + a)*A/(a*x^7)
```

mupad [B] time = 0.73, size = 105, normalized size = 0.90

$$\frac{\sqrt{b\,x^2+a}\,\left(6\,A\,b-7\,B\,a\right)}{35\,a^2\,x^5}+\frac{\sqrt{b\,x^2+a}\,\left(48\,A\,b^3-56\,B\,a\,b^2\right)}{105\,a^4\,x}-\frac{\left(24\,A\,b^2-28\,B\,a\,b\right)\,\sqrt{b\,x^2+a}}{105\,a^3\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^8*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(6*A*b - 7*B*a))/(35*a^2*x^5) + ((a + b*x^2)^(1/2)*(48*A
*b^3 - 56*B*a*b^2))/(105*a^4*x) - ((24*A*b^2 - 28*B*a*b)*(a + b*x^2)^(1/2))
/(105*a^3*x^3) - (A*(a + b*x^2)^(1/2))/(7*a*x^7)
```

sympy [B] time = 3.68, size = 819, normalized size = 7.00

$$-\frac{5Aa^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{9Aa^5b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{5Aa^4b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{5Aa^3b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{30Aa^2b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{40Aab^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}+\frac{16Ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{3Ba^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2Ba^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3Ba^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12Bab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{8Bb^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*A*a**6*b**(19/2)*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*A*a**5*b**(21/2)*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*A*a**4*b**(23/2)*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) + 5*A*a**3*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**
7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x*
*12) + 30*A*a**2*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 1
05*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 40*A*a*b
**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 16*A*b**(31/2)*x**12*sqrt
(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*
x**10 + 35*a**4*b**12*x**12) - 3*B*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a
**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*a**3*b**(11/2)
*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3
*b**6*x**8) - 3*B*a**2*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*B*a*b**(15/2)*x**6*sqrt(a/
(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) -
 8*B*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8)
```

3.551 $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{5ax\sqrt{a+bx^2}\,(6Ab-7aB)}{16b^4}+\frac{5x^3\sqrt{a+bx^2}\,(6Ab-7aB)}{24b^3}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{}{6b\sqrt{}}$$

Rubi [A] time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 321, 217, 206}

$$\frac{5a^2(6Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{x^5(6Ab-7aB)}{6b^2\sqrt{a+bx^2}}+\frac{5x^3\sqrt{a+bx^2}\,(6Ab-7aB)}{24b^3}-\frac{5ax\sqrt{a+bx^2}\,(6Ab-7aB)}{16b^4}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((6*A*b - 7*a*B)*x^5)/(6*b^2*Sqrt[a + b*x^2]) + (B*x^7)/(6*b*Sqrt[a + b*x^
2]) - (5*a*(6*A*b - 7*a*B)*x*Sqrt[a + b*x^2])/(16*b^4) + (5*(6*A*b - 7*a*B)
*x^3*Sqrt[a + b*x^2])/(24*b^3) + (5*a^2*(6*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{(-6Ab+7aB)\int \frac{x^6}{\left(a+bx^2\right)^{3/2}}\,dx}{6b}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} + \frac{(5(6Ab-7aB))\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{6b^2}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3} - \frac{(5a(6Ab-7aB))\int \frac{x^2}{\sqrt{a+bx}}}{8b^3}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx}}{24b^3}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx}}{24b^3}$$

$$= -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}} + \frac{Bx^7}{6b\sqrt{a+bx^2}} - \frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4} + \frac{5(6Ab-7aB)x^3\sqrt{a+bx}}{24b^3}$$

Mathematica [A] time = 0.16, size = 131, normalized size = 0.86

$$\frac{\sqrt{b}\,x\left(105a^3B+a^2\left(35bBx^2-90Ab\right)-2ab^2x^2\left(15A+7Bx^2\right)+4b^3x^4\left(3A+2Bx^2\right)\right)-15a^{5/2}\sqrt{\frac{bx^2}{a}+1}\,(7aB-6Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{48b^{9/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(105*a^3*B + 4*b^3*x^4*(3*A + 2*B*x^2) - 2*a*b^2*x^2*(15*A + 7*B
*x^2) + a^2*(-90*A*b + 35*b*B*x^2)) - 15*a^(5/2)*(-6*A*b + 7*a*B)*Sqrt[1 +
(b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(48*b^(9/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [**A**] time = 0.24, size = 127, normalized size = 0.84

$$\frac{5\left(7a^3B-6a^2Ab\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{16b^{9/2}}+\frac{105a^3Bx-90a^2Abx+35a^2bBx^3-30aAb^2x^3-14ab^2Bx^5+12Ab^3x^5+8b^3Bx^7}{48b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-90*a^2*A*b*x + 105*a^3*B*x - 30*a*A*b^2*x^3 + 35*a^2*b*B*x^3 + 12*A*b^3*x
^5 - 14*a*b^2*B*x^5 + 8*b^3*B*x^7)/(48*b^4*Sqrt[a + b*x^2]) + (5*(-6*a^2*A*
b + 7*a^3*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(9/2))
```

fricas [**A**] time = 1.15, size = 325, normalized size = 2.14

$$\left[-\frac{15\left(7\,Ba^4-6\,Aa^3b+\left(7\,Ba^3b-6\,Aa^2b^2\right)x^2\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(8\,Bb^4x^7-2\left(7\,Bab^3-6\,Ab^4\right)x^5+5\left(7\,Ba^2b^2-6\,Aab^3\right)x^3+15\left(7\,Ba^3b-6\,Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{96\left(b^6x^2+ab^5\right)},\ \frac{15\left(7\,Ba^4-6\,Aa^3b+\left(7\,Ba^3b-6\,Aa^2b^2\right)x^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(8\,Bb^4x^7-2\left(7\,Bab^3-6\,Ab^4\right)x^5+5\left(7\,Ba^2b^2-6\,Aab^3\right)x^3+15\left(7\,Ba^3b-6\,Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{48\left(b^6x^2+ab^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*B*b^4*x^7 - 2*(7*B*a*b
^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b^3)*x^3 + 15*(7*B*a^3*b - 6*A*a
^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5), 1/48*(15*(7*B*a^4 - 6*A*a^3*
b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (8*B*b^4*x^7 - 2*(7*B*a*b^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b
^3)*x^3 + 15*(7*B*a^3*b - 6*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5
)]
```

giac [**A**] time = 0.37, size = 136, normalized size = 0.89

$$\frac{\left(\left(2\left(\frac{4\,Bx^2}{b}-\frac{7\,Bab^5-6\,Ab^6}{b^7}\right)x^2+\frac{5\left(7\,Ba^2b^4-6\,Aab^5\right)}{b^7}\right)x^2+\frac{15\left(7\,Ba^3b^3-6\,Aa^2b^4\right)}{b^7}\right)x}{48\,\sqrt{bx^2+a}}+\frac{5\left(7\,Ba^3-6\,Aa^2b\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*B*x^2/b - (7*B*a*b^5 - 6*A*b^6)/b^7)*x^2 + 5*(7*B*a^2*b^4 - 6*A
*a*b^5)/b^7)*x^2 + 15*(7*B*a^3*b^3 - 6*A*a^2*b^4)/b^7)*x/sqrt(b*x^2 + a) +
5/16*(7*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

maple **[A]** time = 0.02, size = 185, normalized size = 1.22

$$\frac{B\,x^{7}}{6\sqrt{b\,x^{2}+a}\;b}+\frac{A\,x^{5}}{4\sqrt{b\,x^{2}+a}\;b}-\frac{7Ba\,x^{5}}{24\sqrt{b\,x^{2}+a}\;b^{2}}-\frac{5Aa\,x^{3}}{8\sqrt{b\,x^{2}+a}\;b^{2}}+\frac{35B\,a^{2}x^{3}}{48\sqrt{b\,x^{2}+a}\;b^{3}}-\frac{15A\,a^{2}x}{8\sqrt{b\,x^{2}+a}\;b^{3}}+\frac{35B\,a^{3}x}{16\sqrt{b\,x^{2}+a}\;b^{4}}+\frac{15A\,a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{7}{2}}}-\frac{35B\,a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/6*B*x^7/b/(b*x^2+a)^(1/2)-7/24*B*a/b^2*x^5/(b*x^2+a)^(1/2)+35/48*B*a^2/b^
3*x^3/(b*x^2+a)^(1/2)+35/16*B*a^3/b^4*x/(b*x^2+a)^(1/2)-35/16*B*a^3/b^(9/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/4*A*x^5/b/(b*x^2+a)^(1/2)-5/8*A*a/b^2*x^3/
(b*x^2+a)^(1/2)-15/8*A*a^2/b^3*x/(b*x^2+a)^(1/2)+15/8*A*a^2/b^(7/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))
```

maxima **[A]** time = 1.06, size = 170, normalized size = 1.12

$$\frac{Bx^{7}}{6\,\sqrt{bx^{2}+a}\,b}-\frac{7\,Bax^{5}}{24\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{Ax^{5}}{4\,\sqrt{bx^{2}+a}\,b}+\frac{35\,Ba^{2}x^{3}}{48\,\sqrt{bx^{2}+a}\,b^{3}}-\frac{5\,Aax^{3}}{8\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{35\,Ba^{3}x}{16\,\sqrt{bx^{2}+a}\,b^{4}}-\frac{15\,Aa^{2}x}{8\,\sqrt{bx^{2}+a}\,b^{3}}-\frac{35\,Ba^{3}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}}+\frac{15\,Aa^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*B*x^7/(sqrt(b*x^2 + a)*b) - 7/24*B*a*x^5/(sqrt(b*x^2 + a)*b^2) + 1/4*A*
x^5/(sqrt(b*x^2 + a)*b) + 35/48*B*a^2*x^3/(sqrt(b*x^2 + a)*b^3) - 5/8*A*a*x
^3/(sqrt(b*x^2 + a)*b^2) + 35/16*B*a^3*x/(sqrt(b*x^2 + a)*b^4) - 15/8*A*a^2
*x/(sqrt(b*x^2 + a)*b^3) - 35/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 15/
8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{6}\,\left(B\,x^{2}+A\right)}{{\left(b\,x^{2}+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^6*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

sympy **[A]** time = 35.61, size = 233, normalized size = 1.53

$$A\left(-\frac{15a^{\frac{3}{2}}x}{8b^{3}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5\sqrt{a}\,x^{3}}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{5}}{4\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(\frac{35a^{\frac{5}{2}}x}{16b^{4}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{35a^{\frac{3}{2}}x^{3}}{48b^{3}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{7\sqrt{a}\,x^{5}}{24b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{35a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}}+\frac{x^{7}}{6\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt
(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*s
qrt(a)*b*sqrt(1 + b*x**2/a))) + B*(35*a**(5/2)*x/(16*b**4*sqrt(1 + b*x**2/a
)) + 35*a**(3/2)*x**3/(48*b**3*sqrt(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24*b**
2*sqrt(1 + b*x**2/a)) - 35*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) + x*
*7/(6*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.552 $\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}\,(2Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}+\frac{\left(a+bx^2\right)^{3/2}(Ab-3aB)}{3b^4}-\frac{a\sqrt{a+bx^2}\,(2Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2])) - (a*(2*A*b - 3*a*B)*Sqrt[a + b*
x^2])/b^4 + ((A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) + (B*(a + b*x^2)^(5/2
))/(5*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right) d \\
&= -\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}}-\frac{a(2Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^4}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.78

$$
\frac{48a^3B-8a^2b\left(5A-3Bx^2\right)-2ab^2x^2\left(10A+3Bx^2\right)+b^3x^4\left(5A+3Bx^2\right)}{15b^4\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (48*a^3*B - 8*a^2*b*(5*A - 3*B*x^2) + b^3*x^4*(5*A + 3*B*x^2) - 2*a*b^2*x^2
*(10*A + 3*B*x^2))/(15*b^4*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 80, normalized size = 0.81

$$
\frac{48a^3B-40a^2Ab+24a^2bBx^2-20aAb^2x^2-6ab^2Bx^4+5Ab^3x^4+3b^3Bx^6}{15b^4\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-40*a^2*A*b + 48*a^3*B - 20*a*A*b^2*x^2 + 24*a^2*b*B*x^2 + 5*A*b^3*x^4 - 6
*a*b^2*B*x^4 + 3*b^3*B*x^6)/(15*b^4*Sqrt[a + b*x^2])
```

fricas [A] time = 0.71, size = 88, normalized size = 0.89

$$
\frac{\left(3\,Bb^3x^6-\left(6\,Bab^2-5\,Ab^3\right)x^4+48\,Ba^3-40\,Aa^2b+4\left(6\,Ba^2b-5\,Aab^2\right)x^2\right)\sqrt{bx^2+a}}{15\left(b^5x^2+ab^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^3*x^6 - (6*B*a*b^2 - 5*A*b^3)*x^4 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^5*x^2 + a*b^4)
```

giac [A] time = 0.34, size = 113, normalized size = 1.14

$$\frac{Ba^3 - Aa^2b}{\sqrt{bx^2+a}\,b^4} + \frac{3\left(bx^2+a\right)^{\frac{5}{2}}Bb^{16} - 15\left(bx^2+a\right)^{\frac{3}{2}}Bab^{16} + 45\sqrt{bx^2+a}\,Ba^2b^{16} + 5\left(bx^2+a\right)^{\frac{3}{2}}Ab^{17} - 30\sqrt{bx^2+a}\,Aab^{17}}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*a^3 - A*a^2*b)/(sqrt(b*x^2 + a)*b^4) + 1/15*(3*(b*x^2 + a)^(5/2)*B*b^16
- 15*(b*x^2 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^2 + a)*B*a^2*b^16 + 5*(b*x^2
+ a)^(3/2)*A*b^17 - 30*sqrt(b*x^2 + a)*A*a*b^17)/b^20
```

maple [A] time = 0.01, size = 77, normalized size = 0.78

$$-\frac{-3B\,x^6b^3 - 5A\,b^3x^4 + 6Ba\,b^2x^4 + 20Aa\,b^2x^2 - 24B\,a^2b\,x^2 + 40A\,a^2b - 48B\,a^3}{15\sqrt{b\,x^2+a}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/15*(-3*B*b^3*x^6-5*A*b^3*x^4+6*B*a*b^2*x^4+20*A*a*b^2*x^2-24*B*a^2*b*x^2
+40*A*a^2*b-48*B*a^3)/(b*x^2+a)^(1/2)/b^4
```

maxima [A] time = 1.09, size = 132, normalized size = 1.33

$$\frac{Bx^6}{5\sqrt{bx^2+a}\,b} - \frac{2\,Bax^4}{5\sqrt{bx^2+a}\,b^2} + \frac{Ax^4}{3\sqrt{bx^2+a}\,b} + \frac{8\,Ba^2x^2}{5\sqrt{bx^2+a}\,b^3} - \frac{4\,Aax^2}{3\sqrt{bx^2+a}\,b^2} + \frac{16\,Ba^3}{5\sqrt{bx^2+a}\,b^4} - \frac{8\,Aa^2}{3\sqrt{bx^2+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*B*x^6/(sqrt(b*x^2 + a)*b) - 2/5*B*a*x^4/(sqrt(b*x^2 + a)*b^2) + 1/3*A*x
^4/(sqrt(b*x^2 + a)*b) + 8/5*B*a^2*x^2/(sqrt(b*x^2 + a)*b^3) - 4/3*A*a*x^2/
(sqrt(b*x^2 + a)*b^2) + 16/5*B*a^3/(sqrt(b*x^2 + a)*b^4) - 8/3*A*a^2/(sqrt(
b*x^2 + a)*b^3)
```

mupad [B] time = 0.82, size = 89, normalized size = 0.90

$$\frac{\frac{B\left(b\,x^2+a\right)^3}{5} + B\,a^3 + \frac{A\,b\left(b\,x^2+a\right)^2}{3} - B\,a\left(b\,x^2+a\right)^2 + 3\,B\,a^2\,\left(b\,x^2+a\right) - A\,a^2\,b - 2\,A\,a\,b\,\left(b\,x^2+a\right)}{b^4\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] ((B*(a + b*x^2)^3)/5 + B*a^3 + (A*b*(a + b*x^2)^2)/3 - B*a*(a + b*x^2)^2 +
3*B*a^2*(a + b*x^2) - A*a^2*b - 2*A*a*b*(a + b*x^2))/(b^4*(a + b*x^2)^(1/2)
)
```

sympy [A] time = 3.64, size = 172, normalized size = 1.74

$$\begin{cases} -\frac{8Aa^{2}}{3b^{3}\sqrt{a+bx^{2}}} - \frac{4Aax^{2}}{3b^{2}\sqrt{a+bx^{2}}} + \frac{Ax^{4}}{3b\sqrt{a+bx^{2}}} + \frac{16Ba^{3}}{5b^{4}\sqrt{a+bx^{2}}} + \frac{8Ba^{2}x^{2}}{5b^{3}\sqrt{a+bx^{2}}} - \frac{2Bax^{4}}{5b^{2}\sqrt{a+bx^{2}}} + \frac{Bx^{6}}{5b\sqrt{a+bx^{2}}} & \text{for}\: b \neq 0 \\ \frac{\frac{Ax^{6}}{6} + \frac{Bx^{8}}{8}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-8*A*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*A*a*x**2/(3*b**2*sqrt(a
+ b*x**2)) + A*x**4/(3*b*sqrt(a + b*x**2)) + 16*B*a**3/(5*b**4*sqrt(a + b*x
**2)) + 8*B*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*B*a*x**4/(5*b**2*sqrt(a
 + b*x**2)) + B*x**6/(5*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8
/8)/a**(3/2), True))
```

3.553 $\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{3x\sqrt{a+bx^2}\,(4Ab-5aB)}{8b^3}-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 321, 217, 206}

$$-\frac{x^3(4Ab-5aB)}{4b^2\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}\,(4Ab-5aB)}{8b^3}-\frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((4*A*b - 5*a*B)*x^3)/(4*b^2*Sqrt[a + b*x^2]) + (B*x^5)/(4*b*Sqrt[a + b*x^
2]) + (3*(4*A*b - 5*a*B)*x*Sqrt[a + b*x^2])/(8*b^3) - (3*a*(4*A*b - 5*a*B)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Bx^5}{4b\sqrt{a+bx^2}} - \frac{(-4Ab+5aB)\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{4b}$$

$$= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}} + \frac{Bx^5}{4b\sqrt{a+bx^2}} + \frac{(3(4Ab-5aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b^2}$$

$$= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}} + \frac{Bx^5}{4b\sqrt{a+bx^2}} + \frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3} - \frac{(3a(4Ab-5aB))\int \frac{1}{\sqrt{a+bx^2}}\,d}{8b^3}$$

$$= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}} + \frac{Bx^5}{4b\sqrt{a+bx^2}} + \frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3} - \frac{(3a(4Ab-5aB))\,\text{Subst}\left(\int\right.}{8b^3}$$

$$= -\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}} + \frac{Bx^5}{4b\sqrt{a+bx^2}} + \frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3} - \frac{3a(4Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a+}}\right.}{8b^{7/2}}$$

Mathematica [A] time = 0.12, size = 108, normalized size = 0.91

$$\frac{3a^{3/2}\sqrt{\frac{bx^2}{a}+1}\,(5aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(-15a^2B+ab\left(12A-5Bx^2\right)+2b^2x^2\left(2A+Bx^2\right)\right)}{8b^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(-15*a^2*B + a*b*(12*A - 5*B*x^2) + 2*b^2*x^2*(2*A + B*x^2)) + 3
*a^(3/2)*(-4*A*b + 5*a*B)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])
/(8*b^(7/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.18, size = 101, normalized size = 0.85

$$\frac{-15a^2Bx+12aAbx-5abBx^3+4Ab^2x^3+2b^2Bx^5}{8b^3\sqrt{a+bx^2}}-\frac{3\left(5a^2B-4aAb\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (12*a*A*b*x - 15*a^2*B*x + 4*A*b^2*x^3 - 5*a*b*B*x^3 + 2*b^2*B*x^5)/(8*b^3*
Sqrt[a + b*x^2]) - (3*(-4*a*A*b + 5*a^2*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(8*b^(7/2))
```

fricas [A] time = 1.08, size = 274, normalized size = 2.30

$$\left[-\frac{3\left(5\,Ba^3-4\,Aa^2b+\left(5\,Ba^2b-4\,Aab^2\right)x^2\right)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(2\,Bb^3x^5-\left(5\,Bab^2-4\,Ab^3\right)x^3-3\left(5\,Ba^2b-4\,Aab^2\right)x\right)\sqrt{bx^2+a}}{16\left(b^5x^2+ab^4\right)},-\frac{3\left(5\,Ba^3-4\,Aa^2b+\left(5\,Ba^2b-4\,Aab^2\right)x^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-\left(2\,Bb^3x^5-\left(5\,Bab^2-4\,Ab^3\right)x^3-3\left(5\,Ba^2b-4\,Aab^2\right)x\right)\sqrt{bx^2+a}}{8\left(b^5x^2+ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(b)*log(-
2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*B*b^3*x^5 - (5*B*a*b^2 -
4*A*b^3)*x^3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b
^4), -1/8*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b^3*x^5 - (5*B*a*b^2 - 4*A*b^3)*x^
3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

giac [A] time = 0.37, size = 104, normalized size = 0.87

$$\frac{\left(\left(\frac{2\,Bx^2}{b}-\frac{5\,Bab^3-4\,Ab^4}{b^5}\right)x^2-\frac{3\left(5\,Ba^2b^2-4\,Aab^3\right)}{b^5}\right)x}{8\,\sqrt{bx^2+a}}-\frac{3\left(5\,Ba^2-4\,Aab\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*B*x^2/b - (5*B*a*b^3 - 4*A*b^4)/b^5)*x^2 - 3*(5*B*a^2*b^2 - 4*A*a*b
^3)/b^5)*x/sqrt(b*x^2 + a) - 3/8*(5*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(7/2)
```

maple [A] time = 0.01, size = 141, normalized size = 1.18

$$\frac{B\,x^{5}}{4\sqrt{b\,x^{2}+a}\;b}+\frac{A\,x^{3}}{2\sqrt{b\,x^{2}+a}\;b}-\frac{5Ba\,x^{3}}{8\sqrt{b\,x^{2}+a}\;b^{2}}+\frac{3Aax}{2\sqrt{b\,x^{2}+a}\;b^{2}}-\frac{15B\,a^{2}x}{8\sqrt{b\,x^{2}+a}\;b^{3}}-\frac{3Aa\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{5}{2}}}+\frac{15B\,a^{2}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*B*x^5/b/(b*x^2+a)^(1/2)-5/8*B*a/b^2*x^3/(b*x^2+a)^(1/2)-15/8*B*a^2/b^3*
x/(b*x^2+a)^(1/2)+15/8*B*a^2/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*A*x^
3/b/(b*x^2+a)^(1/2)+3/2*A*a/b^2*x/(b*x^2+a)^(1/2)-3/2*A*a/b^(5/2)*ln(b^(1/2
)*x+(b*x^2+a)^(1/2))
```

maxima [A] time = 1.15, size = 126, normalized size = 1.06

$$\frac{Bx^{5}}{4\,\sqrt{bx^{2}+a}\,b}-\frac{5\,Bax^{3}}{8\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{Ax^{3}}{2\,\sqrt{bx^{2}+a}\,b}-\frac{15\,Ba^{2}x}{8\,\sqrt{bx^{2}+a}\,b^{3}}+\frac{3\,Aax}{2\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{15\,Ba^{2}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}-\frac{3\,Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*B*x^5/(sqrt(b*x^2 + a)*b) - 5/8*B*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*A*x
^3/(sqrt(b*x^2 + a)*b) - 15/8*B*a^2*x/(sqrt(b*x^2 + a)*b^3) + 3/2*A*a*x/(sq
rt(b*x^2 + a)*b^2) + 15/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*A*a*ar
csinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4}\,\left(B\,x^{2}+A\right)}{{\left(b\,x^{2}+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^4*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

sympy [A] time = 16.63, size = 177, normalized size = 1.49

$$A\left(\frac{3\sqrt{a}\,x}{2b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^{3}}{2\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(-\frac{15a^{\frac{3}{2}}x}{8b^{3}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5\sqrt{a}\,x^{3}}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{15a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{5}}{4\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(-15*a**(3/2)*x/(8
*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*
x**2/a)))
```

3.554 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{a+bx^2}\,(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}$$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\sqrt{a+bx^2}\,(Ab-2aB)}{b^3}+\frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + ((A*b - 2*a*B)*Sqrt[a + b*x^2])/b^3
 + (B*(a + b*x^2)^(3/2))/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}}+\frac{Ab-2aB}{b^2\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^2}\right)dx,x,x^2\right)\\
&= \frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{(Ab-2aB)\sqrt{a+bx^2}}{b^3}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^3}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.82

$$\frac{-8a^2B+a\left(6Ab-4bBx^2\right)+b^2x^2\left(3A+Bx^2\right)}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-8*a^2*B + b^2*x^2*(3*A + B*x^2) + a*(6*A*b - 4*b*B*x^2))/(3*b^3*Sqrt[a +
b*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.82

$$\frac{-8a^2B+6aAb-4abBx^2+3Ab^2x^2+b^2Bx^4}{3b^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (6*a*A*b - 8*a^2*B + 3*A*b^2*x^2 - 4*a*b*B*x^2 + b^2*B*x^4)/(3*b^3*Sqrt[a +
 b*x^2])
```

fricas [A] time = 0.87, size = 63, normalized size = 0.94

$$\frac{\left(Bb^2x^4-8\,Ba^2+6\,Aab-\left(4\,Bab-3\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{3\left(b^4x^2+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^4 - 8*B*a^2 + 6*A*a*b - (4*B*a*b - 3*A*b^2)*x^2)*sqrt(b*x^2 +
a)/(b^4*x^2 + a*b^3)
```

giac [A] time = 0.38, size = 77, normalized size = 1.15

$$-\frac{Ba^2 - Aab}{\sqrt{bx^2 + a}\,b^3} + \frac{\left(bx^2 + a\right)^{\frac{3}{2}}Bb^6 - 6\sqrt{bx^2 + a}\,Bab^6 + 3\sqrt{bx^2 + a}\,Ab^7}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a^2 - A*a*b)/(sqrt(b*x^2 + a)*b^3) + 1/3*((b*x^2 + a)^(3/2)*B*b^6 - 6*s
qrt(b*x^2 + a)*B*a*b^6 + 3*sqrt(b*x^2 + a)*A*b^7)/b^9
```

maple [A] time = 0.00, size = 52, normalized size = 0.78

$$\frac{B\,b^2x^4 + 3A\,b^2x^2 - 4Bab\,x^2 + 6abA - 8a^2B}{3\sqrt{b\,x^2 + a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/3*(B*b^2*x^4+3*A*b^2*x^2-4*B*a*b*x^2+6*A*a*b-8*B*a^2)/(b*x^2+a)^(1/2)/b^3
```

maxima [A] time = 1.04, size = 89, normalized size = 1.33

$$\frac{Bx^4}{3\sqrt{bx^2 + a}\,b} - \frac{4\,Bax^2}{3\sqrt{bx^2 + a}\,b^2} + \frac{Ax^2}{\sqrt{bx^2 + a}\,b} - \frac{8\,Ba^2}{3\sqrt{bx^2 + a}\,b^3} + \frac{2\,Aa}{\sqrt{bx^2 + a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*x^4/(sqrt(b*x^2 + a)*b) - 4/3*B*a*x^2/(sqrt(b*x^2 + a)*b^2) + A*x^2/(
sqrt(b*x^2 + a)*b) - 8/3*B*a^2/(sqrt(b*x^2 + a)*b^3) + 2*A*a/(sqrt(b*x^2 +
a)*b^2)
```

mupad [B] time = 0.68, size = 59, normalized size = 0.88

$$\frac{B\left(b\,x^2 + a\right)^2 - 3\,B\,a^2 + 3\,A\,b\,\left(b\,x^2 + a\right) - 6\,B\,a\,\left(b\,x^2 + a\right) + 3\,A\,a\,b}{3\,b^3\,\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*(a + b*x^2)^2 - 3*B*a^2 + 3*A*b*(a + b*x^2) - 6*B*a*(a + b*x^2) + 3*A*a*
b)/(3*b^3*(a + b*x^2)^(1/2))
```

sympy [A] time = 2.02, size = 117, normalized size = 1.75

$$\begin{cases} \frac{2Aa}{b^{2}\sqrt{a+bx^{2}}} + \frac{Ax^{2}}{b\sqrt{a+bx^{2}}} - \frac{8Ba^{2}}{3b^{3}\sqrt{a+bx^{2}}} - \frac{4Bax^{2}}{3b^{2}\sqrt{a+bx^{2}}} + \frac{Bx^{4}}{3b\sqrt{a+bx^{2}}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^{4}}{4} + \frac{Bx^{6}}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((2*A*a/(b**2*sqrt(a + b*x**2)) + A*x**2/(b*sqrt(a + b*x**2)) - 8*
B*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*B*a*x**2/(3*b**2*sqrt(a + b*x**2)) + B
*x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(3/2), T
rue))
```

3.555 $\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

Rubi **[A]** time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 388, 217, 206}

$$-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}+\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + (B*x*Sqrt[a + b*x^2])/(2*b^2) +
((2*A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
```

```
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}}-\frac{\int \frac{-Ab+aB-bBx^2}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}+\frac{(2Ab-3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}+\frac{(2Ab-3aB)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^2}+\frac{(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 86, normalized size = 1.04

$$\frac{\sqrt{b}\,x\left(3aB-2Ab+bBx^2\right)-\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\,(3aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*(-2*A*b + 3*a*B + b*B*x^2) - Sqrt[a]*(-2*A*b + 3*a*B)*Sqrt[1 + (
b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(5/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.15, size = 75, normalized size = 0.90

$$\frac{(3aB-2Ab)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{5/2}}+\frac{3aBx-2Abx+bBx^3}{2b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-2*A*b*x + 3*a*B*x + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + ((-2*A*b + 3*a*B)*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

fricas [A] time = 0.71, size = 213, normalized size = 2.57

$$\left[-\frac{\left(3\,Ba^{2}-2\,Aab+\left(3\,Bab-2\,Ab^{2}\right)x^{2}\right)\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\left(Bb^{2}x^{3}+\left(3\,Bab-2\,Ab^{2}\right)x\right)\sqrt{bx^{2}+a}}{4\left(b^{4}x^{2}+ab^{3}\right)},\ \frac{\left(3\,Ba^{2}-2\,Aab+\left(3\,Bab-2\,Ab^{2}\right)x^{2}\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(Bb^{2}x^{3}+\left(3\,Bab-2\,Ab^{2}\right)x\right)\sqrt{bx^{2}+a}}{2\left(b^{4}x^{2}+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(B*b^2*x^3 + (3*B*a*b - 2*A*b^2)*x)*s
qrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A
*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b^2*x^3 + (3*B*
a*b - 2*A*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.47, size = 70, normalized size = 0.84

$$\frac{\left(\frac{Bx^{2}}{b}+\frac{3\,Bab-2\,Ab^{2}}{b^{3}}\right)x}{2\,\sqrt{bx^{2}+a}}+\frac{\left(3\,Ba-2\,Ab\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(B*x^2/b + (3*B*a*b - 2*A*b^2)/b^3)*x/sqrt(b*x^2 + a) + 1/2*(3*B*a - 2*
A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 97, normalized size = 1.17

$$\frac{B\,x^{3}}{2\sqrt{b\,x^{2}+a}\;b}-\frac{Ax}{\sqrt{b\,x^{2}+a}\;b}+\frac{3Bax}{2\sqrt{b\,x^{2}+a}\;b^{2}}+\frac{A\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}-\frac{3Ba\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*B*x^3/b/(b*x^2+a)^(1/2)+3/2*B*a/b^2*x/(b*x^2+a)^(1/2)-3/2*B*a/b^(5/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))-A*x/b/(b*x^2+a)^(1/2)+A/b^(3/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))
```

maxima [A] time = 1.01, size = 82, normalized size = 0.99

$$\frac{Bx^3}{2\sqrt{bx^2+a}\,b}+\frac{3\,Bax}{2\sqrt{bx^2+a}\,b^2}-\frac{Ax}{\sqrt{bx^2+a}\,b}-\frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt(b*x^2 + a)*b^2) - A*x/(sqrt
(b*x^2 + a)*b) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + A*arcsinh(b*x/sqr
t(a*b))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

sympy [A] time = 14.26, size = 114, normalized size = 1.37

$$A\left(\frac{\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.556 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{B\sqrt{a+bx^2}}{b^2}-\frac{Ab-aB}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A*b - a*B)/(b^2*Sqrt[a + b*x^2])) + (B*Sqrt[a + b*x^2])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)dx,x,x^2\right)\\ &= -\frac{Ab-aB}{b^2\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^2}\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 30, normalized size = 0.73

$$\frac{2aB-Ab+bBx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
[Out] (-(A*b) + 2*a*B + b*B*x^2)/(b^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [**A**] time = 0.03, size = 30, normalized size = 0.73

$$\frac{2aB-Ab+bBx^2}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
[Out] (-(A*b) + 2*a*B + b*B*x^2)/(b^2*Sqrt[a + b*x^2])
```

fricas [**A**] time = 1.06, size = 40, normalized size = 0.98

$$\frac{\left(Bbx^2+2\,Ba-Ab\right)\sqrt{bx^2+a}}{b^3x^2+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] (B*b*x^2 + 2*B*a - A*b)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

giac [A] time = 0.43, size = 36, normalized size = 0.88

$$\frac{\sqrt{bx^2+a}\,B}{b^2}+\frac{Ba-Ab}{\sqrt{bx^2+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*B/b^2 + (B*a - A*b)/(sqrt(b*x^2 + a)*b^2)
```

maple [A] time = 0.00, size = 30, normalized size = 0.73

$$-\frac{-Bb\,x^2+Ab-2Ba}{\sqrt{b\,x^2+a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -(-B*b*x^2+A*b-2*B*a)/(b*x^2+a)^(1/2)/b^2
```

maxima [A] time = 1.04, size = 49, normalized size = 1.20

$$\frac{Bx^2}{\sqrt{bx^2+a}\,b}+\frac{2\,Ba}{\sqrt{bx^2+a}\,b^2}-\frac{A}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x^2/(sqrt(b*x^2 + a)*b) + 2*B*a/(sqrt(b*x^2 + a)*b^2) - A/(sqrt(b*x^2 + a
)*b)
```

mupad [B] time = 0.60, size = 30, normalized size = 0.73

$$\frac{B\,a-A\,b+B\,\left(b\,x^2+a\right)}{b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*a - A*b + B*(a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 0.68, size = 66, normalized size = 1.61

$$\begin{cases} -\frac{A}{b\sqrt{a+bx^2}} + \frac{2Ba}{b^2\sqrt{a+bx^2}} + \frac{Bx^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2} + \frac{Bx^4}{4}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-A/(b*sqrt(a + b*x**2)) + 2*B*a/(b**2*sqrt(a + b*x**2)) + B*x**2
/(b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/a**(3/2), True))
```

3.557 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 217, 206}

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\ &= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b}\\ &= \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 1.30

$$\frac{a^{3/2}B\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x(Ab-aB)}{ab^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(A*b - a*B)*x + a^(3/2)*B*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/
Sqrt[a]])/(a*b^(3/2)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.09, size = 58, normalized size = 1.07

$$-\frac{x(aB-Ab)}{ab\sqrt{a+bx^2}}-\frac{B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((-(A*b) + a*B)*x)/(a*b*Sqrt[a + b*x^2])) - (B*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/b^(3/2)
```

fricas [A] time = 1.04, size = 168, normalized size = 3.11

$$\left[-\frac{2\left(Bab-Ab^2\right)\sqrt{bx^2+a}\,x-\left(Babx^2+Ba^2\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)}{2\left(ab^3x^2+a^2b^2\right)},-\frac{\left(Bab-Ab^2\right)\sqrt{bx^2+a}\,x+\left(Babx^2+Ba^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)}{ab^3x^2+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(B*a*b - A*b^2)*sqrt(b*x^2 + a)*x - (B*a*b*x^2 + B*a^2)*sqrt(b)*lo
g(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), -((B*
a*b - A*b^2)*sqrt(b*x^2 + a)*x + (B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

giac [A] time = 0.44, size = 51, normalized size = 0.94

$$-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}-\frac{(Ba-Ab)x}{\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (B*a - A*b)*x/(sqrt(b*x
^2 + a)*a*b)
```

maple [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{Ax}{\sqrt{b\,x^2+a}\;a}-\frac{Bx}{\sqrt{b\,x^2+a}\;b}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(3/2),x)
```

```
[Out] -B*x/b/(b*x^2+a)^(1/2)+B/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+A*x/a/(b*x^2
+a)^(1/2)
```

maxima [A] time = 1.01, size = 46, normalized size = 0.85

$$\frac{Ax}{\sqrt{bx^2+a}\,a}-\frac{Bx}{\sqrt{bx^2+a}\,b}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

mupad [B] time = 0.77, size = 53, normalized size = 0.98

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}+\frac{A\,x}{a\,\sqrt{b\,x^2+a}}-\frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (A*x)/(a*(a + b*x^2)^(1/2)
) - (B*x)/(b*(a + b*x^2)^(1/2))
```

sympy [A] time = 10.20, size = 60, normalized size = 1.11

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.558 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [**A**] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{Ab-aB}{ab\sqrt{a+bx^2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{Ab-aB}{ab\sqrt{a+bx^2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.00

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 53, normalized size = 1.00

$$\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^
(3/2)
```

fricas [A] time = 0.95, size = 167, normalized size = 3.15

$$\left[\frac{\left(Ab^{2}x^{2}+Aab\right)\sqrt{a}\log\left(-\frac{bx^{2}-2\sqrt{bx^{2}+a}\sqrt{a}+2a}{x^{2}}\right)-2\left(Ba^{2}-Aab\right)\sqrt{bx^{2}+a}}{2\left(a^{2}b^{2}x^{2}+a^{3}b\right)},\ \frac{\left(Ab^{2}x^{2}+Aab\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(Ba^{2}-Aab\right)\sqrt{bx^{2}+a}}{a^{2}b^{2}x^{2}+a^{3}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b^2*x^2 + A*a*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - 2*(B*a^2 - A*a*b)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b), ((A*
b^2*x^2 + A*a*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*a^2 - A*a*b
)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.29, size = 52, normalized size = 0.98

$$\frac{A\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{Ba-Ab}{\sqrt{bx^{2}+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (B*a - A*b)/(sqrt(b*x^2 +
 a)*a*b)
```

maple [A] time = 0.01, size = 60, normalized size = 1.13

$$-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{b\,x^{2}+a}\,a}-\frac{B}{\sqrt{b\,x^{2}+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(3/2),x)
```

```
[Out] -B/b/(b*x^2+a)^(1/2)+A/a/(b*x^2+a)^(1/2)-A/a^(3/2)*ln((2*a+2*(b*x^2+a)^(1/2
)*a^(1/2))/x)
```

maxima [A]   time = 1.06, size = 48, normalized size = 0.91

$$-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^2+a}\,a}-\frac{B}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt(b*x^2 + a)*a) - B/(sqrt(
b*x^2 + a)*b)
```

mupad [B]   time = 1.06, size = 50, normalized size = 0.94

$$\frac{A}{a\,\sqrt{b\,x^2+a}}-\frac{B}{b\,\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] A/(a*(a + b*x^2)^(1/2)) - B/(b*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(3/2)
```

sympy [A]   time = 23.90, size = 48, normalized size = 0.91

$$\frac{A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{a\sqrt{-a}}-\frac{-Ab+Ba}{ab\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*atan(sqrt(a + b*x**2)/sqrt(-a))/(a*sqrt(-a)) - (-A*b + B*a)/(a*b*sqrt(a +
 b*x**2))
```

3.559 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 191}

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx&=-\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{a}\\&=-\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)x}{a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.77

$$\frac{-aA + aBx^2 - 2Abx^2}{a^2x\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 2*A*b*x^2 + a*B*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 36, normalized size = 0.77

$$\frac{-aA + aBx^2 - 2Abx^2}{a^2x\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 2*A*b*x^2 + a*B*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

fricas [A] time = 1.09, size = 43, normalized size = 0.91

$$\frac{\left((Ba - 2\,Ab)x^2 - Aa\right)\sqrt{bx^2 + a}}{a^2bx^3 + a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] ((B*a - 2*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

giac [A] time = 0.51, size = 57, normalized size = 1.21

$$\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a\right)a} + \frac{(Ba - Ab)x}{\sqrt{bx^2 + a}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + (B*a - A*b)*x/(sqrt
(b*x^2 + a)*a^2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$-\frac{2Ab\,x^{2}-Ba\,x^{2}+Aa}{\sqrt{b\,x^{2}+a}\;a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x)
```

```
[Out] -(2*A*b*x^2-B*a*x^2+A*a)/(b*x^2+a)^(1/2)/x/a^2
```

maxima [A] time = 1.04, size = 51, normalized size = 1.09

$$\frac{Bx}{\sqrt{bx^{2}+a}\,a}-\frac{2\,Abx}{\sqrt{bx^{2}+a}\,a^{2}}-\frac{A}{\sqrt{bx^{2}+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x/(sqrt(b*x^2 + a)*a) - 2*A*b*x/(sqrt(b*x^2 + a)*a^2) - A/(sqrt(b*x^2 + a
)*a*x)
```

mupad [B] time = 0.54, size = 46, normalized size = 0.98

$$-\frac{\sqrt{b\,x^{2}+a}\,\left(\frac{A}{a}-x^{2}\,\left(\frac{B}{a}-\frac{2\,A\,b}{a^{2}}\right)\right)}{b\,x^{3}+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*(A/a - x^2*(B/a - (2*A*b)/a^2)))/(a*x + b*x^3)
```

sympy [A] time = 13.67, size = 68, normalized size = 1.45

$$A\left(-\frac{1}{a\sqrt{b}\,x^{2}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{2\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx^{2}}+1}}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.560 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

Rubi [**A**] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(3*A*b - 2*a*B)/(2*a^2*Sqrt[a + b*x^2]) - A/(2*a*x^2*Sqrt[a + b*x^2]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{A}{2ax^2\sqrt{a+bx^2}}+\frac{\left(-\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}-\frac{(3Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}\\
&= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}-\frac{(3Ab-2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2b}\\
&= -\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica **[C]**   time = 0.02, size = 57, normalized size = 0.66

$$\frac{x^2(2aB-3Ab)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)-aA}{2a^2x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) + (-3*A*b + 2*a*B)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/
a])/(2*a^2*x^2*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.14, size = 77, normalized size = 0.90

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}+\frac{-aA+2aBx^2-3Abx^2}{2a^2x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) - 3*A*b*x^2 + 2*a*B*x^2)/(2*a^2*x^2*Sqrt[a + b*x^2]) + ((3*A*b - 2*
a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

fricas [A] time = 1.22, size = 232, normalized size = 2.70

$$\left[-\frac{\left(\left(2\,Bab-3\,Ab^2\right)x^4+\left(2\,Ba^2-3\,Aab\right)x^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(Aa^2-\left(2\,Ba^2-3\,Aab\right)x^2\right)\sqrt{bx^2+a}}{4\left(a^3bx^4+a^4x^2\right)},\ \frac{\left(\left(2\,Bab-3\,Ab^2\right)x^4+\left(2\,Ba^2-3\,Aab\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(Aa^2-\left(2\,Ba^2-3\,Aab\right)x^2\right)\sqrt{bx^2+a}}{2\left(a^3bx^4+a^4x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*B*a*b - 3*A*b^2)*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(a)*log(-(b*
x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a^2 - (2*B*a^2 - 3*A*a*b
)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2), 1/2*(((2*B*a*b - 3*A*b^2)*x^
4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (A
*a^2 - (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

giac [A] time = 0.35, size = 99, normalized size = 1.15

$$\frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^2}+\frac{2\left(bx^2+a\right)Ba-2\,Ba^2-3\left(bx^2+a\right)Ab+2\,Aab}{2\left(\left(bx^2+a\right)^{\frac{3}{2}}-\sqrt{bx^2+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 3*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*(
2*(b*x^2 + a)*B*a - 2*B*a^2 - 3*(b*x^2 + a)*A*b + 2*A*a*b)/(((b*x^2 + a)^(3
/2) - sqrt(b*x^2 + a)*a)*a^2)
```

maple [A] time = 0.01, size = 109, normalized size = 1.27

$$\frac{3Ab\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}}}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{a^{\frac{3}{2}}}-\frac{3Ab}{2\sqrt{b\,x^2+a}\,a^2}+\frac{B}{\sqrt{b\,x^2+a}\,a}-\frac{A}{2\sqrt{b\,x^2+a}\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(1/2)-3/2*A*b/a^2/(b*x^2+a)^(1/2)+3/2*A*b/a^(5/2)*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+B/a/(b*x^2+a)^(1/2)-B/a^(3/2)*ln((2*a+2
*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.08, size = 86, normalized size = 1.00

$$-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}\,a^2}-\frac{A}{2\,\sqrt{bx^2+a}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - 1/2
*A/(sqrt(b*x^2 + a)*a*x^2)
```

mupad [B] time = 1.44, size = 90, normalized size = 1.05

$$\frac{B}{a\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{3\,A\,b}{2\,a^2\,\sqrt{b\,x^2+a}}-\frac{A}{2\,a\,x^2\,\sqrt{b\,x^2+a}}+\frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - (3
*A*b)/(2*a^2*(a + b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*A*b*at
anh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

sympy [B] time = 41.26, size = 262, normalized size = 3.05

$$A\left(-\frac{1}{2a\sqrt{b}\,x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{2a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(\frac{2a^{3}\sqrt{1+\frac{bx^{2}}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}+\frac{a^{3}\log\left(\frac{bx^{2}}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}+\frac{a^{2}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}-\frac{2a^{2}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(2*a**3*s
qrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*
a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9
/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/
2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(
7/2)*b*x**2))
```

3.561 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{2bx(4Ab-3aB)}{3a^3\sqrt{a+bx^2}}+\frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(3*a*x^3*Sqrt[a + b*x^2]) + (4*A*b - 3*a*B)/(3*a^2*x*Sqrt[a + b*x^2]) +
(2*b*(4*A*b - 3*a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{3ax^3\sqrt{a+bx^2}} - \frac{(4Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a}$$

$$= -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{(2b(4Ab-3aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{2b(4Ab-3aB)x}{3a^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.74

$$\frac{\left(a+2bx^2\right)(4Ab-3aB)}{3a^3x\sqrt{a+bx^2}} - \frac{A}{3ax^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] -1/3*A/(a*x^3*Sqrt[a + b*x^2]) + ((4*A*b - 3*a*B)*(a + 2*b*x^2))/(3*a^3*x*S
qrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.13, size = 62, normalized size = 0.76

$$\frac{-a^2A-3a^2Bx^2+4aAbx^2-6abBx^4+8Ab^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*A) + 4*a*A*b*x^2 - 3*a^2*B*x^2 + 8*A*b^2*x^4 - 6*a*b*B*x^4)/(3*a^3*x
^3*Sqrt[a + b*x^2])
```

fricas [A] time = 0.95, size = 68, normalized size = 0.83

$$-\frac{\left(2\left(3\,Bab-4\,Ab^2\right)x^4+Aa^2+\left(3\,Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3bx^5+a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*B*a*b - 4*A*b^2)*x^4 + A*a^2 + (3*B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x
^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

giac [B] time = 0.57, size = 181, normalized size = 2.21

$$-\frac{\left(Bab - Ab^2\right)x}{\sqrt{bx^2 + a}\,a^3} + \frac{2\left(3\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 Ba\sqrt{b} - 3\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4 Ab^{\frac{3}{2}} - 6\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 Ba^2\sqrt{b} + 12\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 Aab^{\frac{3}{2}} + 3\,Ba^3\sqrt{b} - 5\,Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 - a\right)^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*x/(sqrt(b*x^2 + a)*a^3) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^4*B*a*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 5*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^3*a^2)
```

maple [A] time = 0.01, size = 58, normalized size = 0.71

$$-\frac{-8A\,b^2x^4 + 6Bab\,x^4 - 4Aab\,x^2 + 3B\,a^2x^2 + a^2A}{3\sqrt{b\,x^2 + a}\;a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/3*(-8*A*b^2*x^4+6*B*a*b*x^4-4*A*a*b*x^2+3*B*a^2*x^2+A*a^2)/(b*x^2+a)^(1/
2)/x^3/a^3
```

maxima [A] time = 1.01, size = 92, normalized size = 1.12

$$-\frac{2\,Bbx}{\sqrt{bx^2 + a}\,a^2} + \frac{8\,Ab^2x}{3\,\sqrt{bx^2 + a}\,a^3} - \frac{B}{\sqrt{bx^2 + a}\,ax} + \frac{4\,Ab}{3\,\sqrt{bx^2 + a}\,a^2x} - \frac{A}{3\,\sqrt{bx^2 + a}\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*A*b^2*x/(sqrt(b*x^2 + a)*a^3) - B/(sqr
t(b*x^2 + a)*a*x) + 4/3*A*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3*A/(sqrt(b*x^2 + a
)*a*x^3)
```

mupad [B] time = 0.66, size = 57, normalized size = 0.70

$$-\frac{3\,B\,a^2\,x^2 + A\,a^2 + 6\,B\,a\,b\,x^4 - 4\,A\,a\,b\,x^2 - 8\,A\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x)
```

```
[Out] -(A*a^2 + 3*B*a^2*x^2 - 8*A*b^2*x^4 - 4*A*a*b*x^2 + 6*B*a*b*x^4)/(3*a^3*x^3
*(a + b*x^2)^(1/2))
```

sympy [B] time = 9.99, size = 284, normalized size = 3.46

$$A\left(-\frac{a^{3}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}+\frac{3a^{2}b^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}+\frac{12ab^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}+\frac{8b^{\frac{15}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}x^{2}+6a^{4}b^{5}x^{4}+3a^{3}b^{6}x^{6}}\right)+B\left(-\frac{1}{a\sqrt{b}\,x^{2}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{2\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx^{2}}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b
**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) +
8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4
+ 3*a**3*b**6*x**6)) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt
(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

3.562 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{3b(5Ab-4aB)}{8a^3\sqrt{a+bx^2}}+\frac{5Ab-4aB}{8a^2x^2\sqrt{a+bx^2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{3\sqrt{a+bx^2}\,(5Ab-4aB)}{8a^3x^2}-\frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}}-\frac{3b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{A}{4ax^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(4*a*x^4*Sqrt[a + b*x^2]) - (5*A*b - 4*a*B)/(4*a^2*x^2*Sqrt[a + b*x^2])
+ (3*(5*A*b - 4*a*B)*Sqrt[a + b*x^2])/(8*a^3*x^2) - (3*b*(5*A*b - 4*a*B)*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} + \frac{\left(-\frac{5Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} - \frac{(3(5Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{8a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(3b(5Ab-4aB))\,\text{Subst}}{16a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} + \frac{(3(5Ab-4aB))\,\text{Subst}\Bigg(}{8}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^2}} - \frac{5Ab-4aB}{4a^2x^2\sqrt{a+bx^2}} + \frac{3(5Ab-4aB)\sqrt{a+bx^2}}{8a^3x^2} - \frac{3b(5Ab-4aB)\tanh^{-1}}{8a^{7/2}}$$

Mathematica [C] time = 0.07, size = 60, normalized size = 0.51

$$\frac{bx^4(5Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^2}{a}+1\right)-a^2A}{4a^3x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*A) + b*(5*A*b - 4*a*B)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^
2)/a])/(4*a^3*x^4*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.16, size = 102, normalized size = 0.86

$$\frac{3\left(4abB-5Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}+\frac{-2a^2A-4a^2Bx^2+5aAbx^2-12abBx^4+15Ab^2x^4}{8a^3x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2*A + 5*a*A*b*x^2 - 4*a^2*B*x^2 + 15*A*b^2*x^4 - 12*a*b*B*x^4)/(8*a^3
*x^4*Sqrt[a + b*x^2]) + (3*(-5*A*b^2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(8*a^(7/2))
```

fricas [A] time = 1.05, size = 287, normalized size = 2.43

$$\left[-\frac{3\left(\left(4\,Bab^2-5\,Ab^3\right)x^6+\left(4\,Ba^2b-5\,Aab^2\right)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\left(4\,Ba^2b-5\,Aab^2\right)x^4+2\,Aa^3+\left(4\,Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{16\left(a^4bx^6+a^5x^4\right)},-\frac{3\left(\left(4\,Bab^2-5\,Ab^3\right)x^6+\left(4\,Ba^2b-5\,Aab^2\right)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\left(4\,Ba^2b-5\,Aab^2\right)x^4+2\,Aa^3+\left(4\,Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{8\left(a^4bx^6+a^5x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^2)*x^4)*sqrt(a)
*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(4*B*a^2*b - 5*
A*a*b^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b
*x^6 + a^5*x^4), -1/8*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^
2)*x^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(4*B*a^2*b - 5*A*a*b
^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b*x^6
+ a^5*x^4)]
```

giac [A] time = 0.37, size = 137, normalized size = 1.16

$$-\frac{3\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}-\frac{Bab-Ab^2}{\sqrt{bx^2+a}\,a^3}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^2+a}\,Ba^2b-7\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+9\,\sqrt{bx^2+a}\,Aab^2}{8\,a^3b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
(B*a*b - A*b^2)/(sqrt(b*x^2 + a)*a^3) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*
sqrt(b*x^2 + a)*B*a^2*b - 7*(b*x^2 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^2 + a)*A*a
*b^2)/(a^3*b^2*x^4)
```

maple [A] time = 0.01, size = 153, normalized size = 1.30

$$-\frac{15 A\,b^2 \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{7}{2}}}+\frac{3Bb \ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{5}{2}}}+\frac{15A\,b^2}{8\sqrt{b\,x^2+a}\,a^3}-\frac{3Bb}{2\sqrt{b\,x^2+a}\,a^2}+\frac{5Ab}{8\sqrt{b\,x^2+a}\,a^2x^2}-\frac{B}{2\sqrt{b\,x^2+a}\,a\,x^2}-\frac{A}{4\sqrt{b\,x^2+a}\,a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*B/a/x^2/(b*x^2+a)^(1/2)-3/2*B*b/a^2/(b*x^2+a)^(1/2)+3/2*B*b/a^(5/2)*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/4*A/a/x^4/(b*x^2+a)^(1/2)+5/8*A*b/a^2
/x^2/(b*x^2+a)^(1/2)+15/8*A*b^2/a^3/(b*x^2+a)^(1/2)-15/8*A*b^2/a^(7/2)*ln((
2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.04, size = 130, normalized size = 1.10

$$\frac{3\,Bb\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{15\,Ab^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{7}{2}}}-\frac{3\,Bb}{2\,\sqrt{bx^2+a}\,a^2}+\frac{15\,Ab^2}{8\,\sqrt{bx^2+a}\,a^3}-\frac{B}{2\,\sqrt{bx^2+a}\,ax^2}+\frac{5\,Ab}{8\,\sqrt{bx^2+a}\,a^2x^2}-\frac{A}{4\,\sqrt{bx^2+a}\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 15/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(7/2) - 3/2*B*b/(sqrt(b*x^2 + a)*a^2) + 15/8*A*b^2/(sqrt(b*
x^2 + a)*a^3) - 1/2*B/(sqrt(b*x^2 + a)*a*x^2) + 5/8*A*b/(sqrt(b*x^2 + a)*a^
2*x^2) - 1/4*A/(sqrt(b*x^2 + a)*a*x^4)
```

mupad [B] time = 1.91, size = 134, normalized size = 1.14

$$\frac{15\,A\,b^2}{8\,a^3\,\sqrt{b\,x^2+a}}-\frac{3\,B\,b}{2\,a^2\,\sqrt{b\,x^2+a}}-\frac{15\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{A}{4\,a\,x^4\,\sqrt{b\,x^2+a}}-\frac{B}{2\,a\,x^2\,\sqrt{b\,x^2+a}}+\frac{3\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{5\,A\,b}{8\,a^2\,x^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x)
```

```
[Out] (15*A*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - (3*B*b)/(2*a^2*(a + b*x^2)^(1/2)) -
(15*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - A/(4*a*x^4*(a + b
```

```
*x^2)^(1/2)) - B/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*B*b*atanh((a + b*x^2)^(1/
2)/a^(1/2)))/(2*a^(5/2)) + (5*A*b)/(8*a^2*x^2*(a + b*x^2)^(1/2))
```

sympy [A] time = 84.40, size = 180, normalized size = 1.53

$$A\left(-\frac{1}{4a\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5\sqrt{b}}{8a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{15b^{\frac{3}{2}}}{8a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{15b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{2a\sqrt{b}\,x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{2a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqrt
(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b**2*a
sinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b
*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a
)/(sqrt(b)*x))/(2*a**(5/2)))
```

3.563 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$-\frac{8b^2x(6Ab-5aB)}{15a^4\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^2]) + (6*A*b - 5*a*B)/(15*a^2*x^3*Sqrt[a + b*x^2])
 - (4*b*(6*A*b - 5*a*B))/(15*a^3*x*Sqrt[a + b*x^2]) - (8*b^2*(6*A*b - 5*a*B
)*x)/(15*a^4*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{5ax^5\sqrt{a+bx^2}}-\frac{(6Ab-5aB)\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx}{5a}\\
&= -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}+\frac{(4b(6Ab-5aB))\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{15a^2}\\
&= -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}-\frac{\left(8b^2(6Ab-5aB)\right)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,d}{15a^3}\\
&= -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}}-\frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}-\frac{8b^2(6Ab-5aB)x}{15a^4\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [**A**] time = 0.02, size = 60, normalized size = 0.52

$$\frac{x^2\left(a^2-4abx^2-8b^2x^4\right)(6Ab-5aB)-3a^3A}{15a^4x^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-3*a^3*A + (6*A*b - 5*a*B)*x^2*(a^2 - 4*a*b*x^2 - 8*b^2*x^4))/(15*a^4*x^5*
Sqrt[a + b*x^2])
```

IntegrateAlgebraic [**A**] time = 0.16, size = 86, normalized size = 0.75

$$\frac{-3a^3A-5a^3Bx^2+6a^2Abx^2+20a^2bBx^4-24aAb^2x^4+40ab^2Bx^6-48Ab^3x^6}{15a^4x^5\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-3*a^3*A + 6*a^2*A*b*x^2 - 5*a^3*B*x^2 - 24*a*A*b^2*x^4 + 20*a^2*b*B*x^4 -
 48*A*b^3*x^6 + 40*a*b^2*B*x^6)/(15*a^4*x^5*Sqrt[a + b*x^2])
```

fricas [**A**] time = 1.04, size = 94, normalized size = 0.82

$$\frac{\left(8\left(5\,Bab^2-6\,Ab^3\right)x^6+4\left(5\,Ba^2b-6\,Aab^2\right)x^4-3\,Aa^3-\left(5\,Ba^3-6\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{15\left(a^4bx^7+a^5x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*(5*B*a*b^2 - 6*A*b^3)*x^6 + 4*(5*B*a^2*b - 6*A*a*b^2)*x^4 - 3*A*a^3
 - (5*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b*x^7 + a^5*x^5)
```

giac [B] time = 0.47, size = 294, normalized size = 2.56

$$\frac{\left(Bab^{2}-Ab^{3}\right)x}{\sqrt{bx^{2}+a}\,a^{4}}-\frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Bab^{\frac{3}{2}}-15\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ba^{2}b^{\frac{3}{2}}+90\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Aab^{\frac{5}{2}}+160\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba^{3}b^{\frac{3}{2}}-240\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Aa^{2}b^{\frac{5}{2}}-110\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{4}b^{\frac{3}{2}}+150\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aa^{3}b^{\frac{5}{2}}+25\,Ba^{5}b^{\frac{3}{2}}-33\,Aa^{4}b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^4) - 2/15*(15*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*B*a*b^(3/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 90*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*A*a*b^(5/2) + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 24
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 110*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*B*a^4*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2
) + 25*B*a^5*b^(3/2) - 33*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^5*a^3)
```

maple [A] time = 0.01, size = 83, normalized size = 0.72

$$-\frac{48A\,b^{3}x^{6}-40Ba\,b^{2}x^{6}+24x^{4}Aa\,b^{2}-20B\,a^{2}b\,x^{4}-6A\,a^{2}b\,x^{2}+5B\,a^{3}x^{2}+3A\,a^{3}}{15\sqrt{b\,x^{2}+a}\;a^{4}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/15*(48*A*b^3*x^6-40*B*a*b^2*x^6+24*A*a*b^2*x^4-20*B*a^2*b*x^4-6*A*a^2*b*
x^2+5*B*a^3*x^2+3*A*a^3)/(b*x^2+a)^(1/2)/x^5/a^4
```

maxima [A] time = 1.05, size = 134, normalized size = 1.17

$$\frac{8\,Bb^{2}x}{3\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{16\,Ab^{3}x}{5\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{4\,Bb}{3\,\sqrt{bx^{2}+a}\,a^{2}x}-\frac{8\,Ab^{2}}{5\,\sqrt{bx^{2}+a}\,a^{3}x}-\frac{B}{3\,\sqrt{bx^{2}+a}\,ax^{3}}+\frac{2\,Ab}{5\,\sqrt{bx^{2}+a}\,a^{2}x^{3}}-\frac{A}{5\,\sqrt{bx^{2}+a}\,ax^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*B*b^2*x/(sqrt(b*x^2 + a)*a^3) - 16/5*A*b^3*x/(sqrt(b*x^2 + a)*a^4) + 4/
3*B*b/(sqrt(b*x^2 + a)*a^2*x) - 8/5*A*b^2/(sqrt(b*x^2 + a)*a^3*x) - 1/3*B/(
```

```
sqrt(b*x^2 + a)*a*x^3) + 2/5*A*b/(sqrt(b*x^2 + a)*a^2*x^3) - 1/5*A/(sqrt(b*
x^2 + a)*a*x^5)
```

mupad [B] time = 0.83, size = 82, normalized size = 0.71

$$-\frac{5\,B\,a^3\,x^2+3\,A\,a^3-20\,B\,a^2\,b\,x^4-6\,A\,a^2\,b\,x^2-40\,B\,a\,b^2\,x^6+24\,A\,a\,b^2\,x^4+48\,A\,b^3\,x^6}{15\,a^4\,x^5\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x)
```

```
[Out] -(3*A*a^3 + 5*B*a^3*x^2 + 48*A*b^3*x^6 - 6*A*a^2*b*x^2 + 24*A*a*b^2*x^4 - 2
0*B*a^2*b*x^4 - 40*B*a*b^2*x^6)/(15*a^4*x^5*(a + b*x^2)^(1/2))
```

sympy [B] time = 17.52, size = 593, normalized size = 5.16

$$A\left(-\frac{a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{5a^3b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{30a^2b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{40ab^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}-\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\right)+B\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x
**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt
(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**
8 + 5*a**4*b**12*x**10) - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a*
*7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10
) - 40*a*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b*
*10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**10*sq
rt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x
**8 + 5*a**4*b**12*x**10)) + B*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*s
qrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6
) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**
5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*
b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6))
```

3.564 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b^2(7Ab-6aB)}{16a^4\sqrt{a+bx^2}}-\frac{5b(7Ab-6aB)}{48a^3x^2\sqrt{a+bx^2}}+\frac{7Ab-6aB}{24a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

Rubi [**A**] time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{5b^2(7Ab-6aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{5b\sqrt{a+bx^2}\,(7Ab-6aB)}{16a^4x^2}+\frac{5\sqrt{a+bx^2}\,(7Ab-6aB)}{24a^3x^4}-\frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^2]) - (7*A*b - 6*a*B)/(6*a^2*x^4*Sqrt[a + b*x^2])
+ (5*(7*A*b - 6*a*B)*Sqrt[a + b*x^2])/(24*a^3*x^4) - (5*b*(7*A*b - 6*a*B)*S
qrt[a + b*x^2])/(16*a^4*x^2) + (5*b^2*(7*A*b - 6*a*B)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(16*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4(a+bx)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{A}{6ax^6\sqrt{a+bx^2}} + \frac{\left(-\frac{7Ab}{2}+3aB\right)\text{Subst}\left(\int \frac{1}{x^3(a+bx)^{3/2}}\,dx, x, x^2\right)}{6a}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} - \frac{(5(7Ab-6aB))\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{12a^2}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} + \frac{(5b(7Ab-6aB))\,\text{Subst}}{16}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)\sqrt{a+bx}}{16a^4x^2}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)\sqrt{a+bx}}{16a^4x^2}\\
&= -\frac{A}{6ax^6\sqrt{a+bx^2}} - \frac{7Ab-6aB}{6a^2x^4\sqrt{a+bx^2}} + \frac{5(7Ab-6aB)\sqrt{a+bx^2}}{24a^3x^4} - \frac{5b(7Ab-6aB)\sqrt{a+bx}}{16a^4x^2}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 62, normalized size = 0.41

$$\frac{b^2x^6(6aB-7Ab)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx^2}{a}+1\right)-a^3A}{6a^4x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a^3*A) + b^2*(-7*A*b + 6*a*B)*x^6*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b
*x^2)/a])/(6*a^4*x^6*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.24, size = 128, normalized size = 0.84

$$\frac{-8a^3A-12a^3Bx^2+14a^2Abx^2+30a^2bBx^4-35aAb^2x^4+90ab^2Bx^6-105Ab^3x^6}{48a^4x^6\sqrt{a+bx^2}}-\frac{5\left(6ab^2B-7Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-8*a^3*A + 14*a^2*A*b*x^2 - 12*a^3*B*x^2 - 35*a*A*b^2*x^4 + 30*a^2*b*B*x^4
 - 105*A*b^3*x^6 + 90*a*b^2*B*x^6)/(48*a^4*x^6*Sqrt[a + b*x^2]) - (5*(-7*A*
b^3 + 6*a*b^2*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(9/2))
```

fricas [A] time = 0.74, size = 341, normalized size = 2.23

$$\left[-\frac{15\left(\left(6\,Bab^{3}-7\,Ab^{4}\right)x^{8}+\left(6\,Ba^{2}b^{2}-7\,Aab^{3}\right)x^{6}\right)\sqrt{a}\,\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\left(15\left(6\,Ba^{2}b^{2}-7\,Aab^{3}\right)x^{6}-8\,Aa^{4}+5\left(6\,Ba^{3}b-7\,Aa^{2}b^{2}\right)x^{4}-2\left(6\,Ba^{4}-7\,Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{96\left(a^{5}bx^{8}+a^{6}x^{6}\right)},\ \frac{15\left(\left(6\,Bab^{3}-7\,Ab^{4}\right)x^{8}+\left(6\,Ba^{2}b^{2}-7\,Aab^{3}\right)x^{6}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(15\left(6\,Ba^{2}b^{2}-7\,Aab^{3}\right)x^{6}-8\,Aa^{4}+5\left(6\,Ba^{3}b-7\,Aa^{2}b^{2}\right)x^{4}-2\left(6\,Ba^{4}-7\,Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{48\left(a^{5}bx^{8}+a^{6}x^{6}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((6*B*a*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt
(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*(6*B*a^2*b^
2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4
 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b*x^8 + a^6*x^6), 1/48*(15*((6*B*a
*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt(-a)*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (15*(6*B*a^2*b^2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*
a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a
^5*b*x^8 + a^6*x^6)]
```

giac [A] time = 0.38, size = 180, normalized size = 1.18

$$\frac{5\left(6\,Bab^{2}-7\,Ab^{3}\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^{4}}+\frac{Bab^{2}-Ab^{3}}{\sqrt{bx^{2}+a}\,a^{4}}+\frac{42\left(bx^{2}+a\right)^{\frac{5}{2}}Bab^{2}-96\left(bx^{2}+a\right)^{\frac{3}{2}}Ba^{2}b^{2}+54\,\sqrt{bx^{2}+a}\,Ba^{3}b^{2}-57\left(bx^{2}+a\right)^{\frac{5}{2}}Ab^{3}+136\left(bx^{2}+a\right)^{\frac{3}{2}}Aab^{3}-87\,\sqrt{bx^{2}+a}\,Aa^{2}b^{3}}{48\,a^{4}b^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 5/16*(6*B*a*b^2 - 7*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4)
+ (B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*a^4) + 1/48*(42*(b*x^2 + a)^(5/2)*B*a*
b^2 - 96*(b*x^2 + a)^(3/2)*B*a^2*b^2 + 54*sqrt(b*x^2 + a)*B*a^3*b^2 - 57*(b
*x^2 + a)^(5/2)*A*b^3 + 136*(b*x^2 + a)^(3/2)*A*a*b^3 - 87*sqrt(b*x^2 + a)*
A*a^2*b^3)/(a^4*b^3*x^6)
```

maple [A] time = 0.01, size = 197, normalized size = 1.29

$$\frac{35A\,b^{3}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{16a^{\frac{9}{2}}}-\frac{15B\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{7}{2}}}-\frac{35A\,b^{3}}{16\sqrt{b\,x^{2}+a}\,a^{4}}+\frac{15B\,b^{2}}{8\sqrt{b\,x^{2}+a}\,a^{3}}-\frac{35A\,b^{2}}{48\sqrt{b\,x^{2}+a}\,a^{3}x^{2}}+\frac{5Bb}{8\sqrt{b\,x^{2}+a}\,a^{2}x^{2}}+\frac{7Ab}{24\sqrt{b\,x^{2}+a}\,a^{2}x^{4}}-\frac{B}{4\sqrt{b\,x^{2}+a}\,a\,x^{4}}-\frac{A}{6\sqrt{b\,x^{2}+a}\,a\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/4*B/a/x^4/(b*x^2+a)^(1/2)+5/8*B*b/a^2/x^2/(b*x^2+a)^(1/2)+15/8*B*b^2/a^3
/(b*x^2+a)^(1/2)-15/8*B*b^2/a^(7/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1
/6*A/a/x^6/(b*x^2+a)^(1/2)+7/24*A*b/a^2/x^4/(b*x^2+a)^(1/2)-35/48*A*b^2/a^3
/x^2/(b*x^2+a)^(1/2)-35/16*A*b^3/a^4/(b*x^2+a)^(1/2)+35/16*A*b^3/a^(9/2)*ln
((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.05, size = 174, normalized size = 1.14

$$-\frac{15\,Bb^{2}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{7}{2}}}+\frac{35\,Ab^{3}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{9}{2}}}+\frac{15\,Bb^{2}}{8\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{35\,Ab^{3}}{16\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{5\,Bb}{8\,\sqrt{bx^{2}+a}\,a^{2}x^{2}}-\frac{35\,Ab^{2}}{48\,\sqrt{bx^{2}+a}\,a^{3}x^{2}}-\frac{B}{4\,\sqrt{bx^{2}+a}\,ax^{4}}+\frac{7\,Ab}{24\,\sqrt{bx^{2}+a}\,a^{2}x^{4}}-\frac{A}{6\,\sqrt{bx^{2}+a}\,ax^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -15/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 35/16*A*b^3*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(9/2) + 15/8*B*b^2/(sqrt(b*x^2 + a)*a^3) - 35/16*A*b^3
/(sqrt(b*x^2 + a)*a^4) + 5/8*B*b/(sqrt(b*x^2 + a)*a^2*x^2) - 35/48*A*b^2/(s
qrt(b*x^2 + a)*a^3*x^2) - 1/4*B/(sqrt(b*x^2 + a)*a*x^4) + 7/24*A*b/(sqrt(b*
x^2 + a)*a^2*x^4) - 1/6*A/(sqrt(b*x^2 + a)*a*x^6)
```

mupad [B] time = 2.25, size = 178, normalized size = 1.16

$$\frac{35\,A\,b^{3}\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{16\,a^{9/2}}-\frac{15\,B\,b^{2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,a^{7/2}}-\frac{35\,A\,b^{3}}{16\,a^{4}\,\sqrt{b\,x^{2}+a}}+\frac{15\,B\,b^{2}}{8\,a^{3}\,\sqrt{b\,x^{2}+a}}-\frac{A}{6\,a\,x^{6}\,\sqrt{b\,x^{2}+a}}-\frac{B}{4\,a\,x^{4}\,\sqrt{b\,x^{2}+a}}+\frac{7\,A\,b}{24\,a^{2}\,x^{4}\,\sqrt{b\,x^{2}+a}}+\frac{5\,B\,b}{8\,a^{2}\,x^{2}\,\sqrt{b\,x^{2}+a}}-\frac{35\,A\,b^{2}}{48\,a^{3}\,x^{2}\,\sqrt{b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x)
```

```
[Out] (35*A*b^3*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(16*a^(9/2)) - (15*B*b^2*atanh(
(a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - (35*A*b^3)/(16*a^4*(a + b*x^2)^(1
/2)) + (15*B*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - A/(6*a*x^6*(a + b*x^2)^(1/2))
 - B/(4*a*x^4*(a + b*x^2)^(1/2)) + (7*A*b)/(24*a^2*x^4*(a + b*x^2)^(1/2)) +
 (5*B*b)/(8*a^2*x^2*(a + b*x^2)^(1/2)) - (35*A*b^2)/(48*a^3*x^2*(a + b*x^2)
^(1/2))
```

sympy [A] time = 141.33, size = 236, normalized size = 1.54

$$A\left(-\frac{1}{6a\sqrt{b}\,x^{7}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{7\sqrt{b}}{24a^{2}x^{5}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{35b^{\frac{3}{2}}}{48a^{3}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{35b^{\frac{5}{2}}}{16a^{4}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{35b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{9}{2}}}\right)+B\left(-\frac{1}{4a\sqrt{b}\,x^{5}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{5\sqrt{b}}{8a^{2}x^{3}\sqrt{\frac{a}{bx^{2}}+1}}+\frac{15b^{\frac{3}{2}}}{8a^{3}x\sqrt{\frac{a}{bx^{2}}+1}}-\frac{15b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{7}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + 7*sqrt(b)/(24*a**2*x**5*sqr
t(a/(b*x**2) + 1)) - 35*b**(3/2)/(48*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 35*b
```

```
**(5/2)/(16*a**4*x*sqrt(a/(b*x**2) + 1)) + 35*b**3*asinh(sqrt(a)/(sqrt(b)*x
))/(16*a**(9/2))) + B*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(
b)/(8*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**
2) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2)))
```

3.565 $\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

Rubi [A] time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{16b^3x(8Ab-7aB)}{35a^5\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^2]) + (8*A*b - 7*a*B)/(35*a^2*x^5*Sqrt[a + b*x^2])
 - (2*b*(8*A*b - 7*a*B))/(35*a^3*x^3*Sqrt[a + b*x^2]) + (8*b^2*(8*A*b - 7*a
*B))/(35*a^4*x*Sqrt[a + b*x^2]) + (16*b^3*(8*A*b - 7*a*B)*x)/(35*a^5*Sqrt[a
 + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{7ax^7\sqrt{a+bx^2}}-\frac{(8Ab-7aB)\int \frac{1}{x^6\left(a+bx^2\right)^{3/2}}\,dx}{7a}\\
&= -\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}+\frac{(6b(8Ab-7aB))\int \frac{1}{x^4\left(a+bx^2\right)^{3/2}}\,dx}{35a^2}\\
&= -\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}-\frac{\left(8b^2(8Ab-7aB)\right)\int \frac{1}{x^2\left(a+bx^2\right)^{3/}}}{35a^3}\\
&= -\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}+\frac{\left(16b^3(8Ab\right.}{}\\
&= -\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}}-\frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}+\frac{16b^3(8Ab-}{35a^5\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.48

$$\frac{x^2\left(a^3-2a^2bx^2+8ab^2x^4+16b^3x^6\right)(8Ab-7aB)-5a^4A}{35a^5x^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-5*a^4*A + (8*A*b - 7*a*B)*x^2*(a^3 - 2*a^2*b*x^2 + 8*a*b^2*x^4 + 16*b^3*x
^6))/(35*a^5*x^7*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 110, normalized size = 0.74

$$\frac{-5a^4A-7a^4Bx^2+8a^3Abx^2+14a^3bBx^4-16a^2Ab^2x^4-56a^2b^2Bx^6+64aAb^3x^6-112ab^3Bx^8+128Ab^4x^8}{35a^5x^7\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-5*a^4*A + 8*a^3*A*b*x^2 - 7*a^4*B*x^2 - 16*a^2*A*b^2*x^4 + 14*a^3*b*B*x^4
 + 64*a*A*b^3*x^6 - 56*a^2*b^2*B*x^6 + 128*A*b^4*x^8 - 112*a*b^3*B*x^8)/(35
*a^5*x^7*Sqrt[a + b*x^2])
```

fricas [**A**] time = 1.16, size = 117, normalized size = 0.79

$$-\frac{\left(16\left(7\,Bab^{3}-8\,Ab^{4}\right)x^{8}+8\left(7\,Ba^{2}b^{2}-8\,Aab^{3}\right)x^{6}+5\,Aa^{4}-2\left(7\,Ba^{3}b-8\,Aa^{2}b^{2}\right)x^{4}+\left(7\,Ba^{4}-8\,Aa^{3}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{35\left(a^{5}bx^{9}+a^{6}x^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*(7*B*a*b^3 - 8*A*b^4)*x^8 + 8*(7*B*a^2*b^2 - 8*A*a*b^3)*x^6 + 5*A
*a^4 - 2*(7*B*a^3*b - 8*A*a^2*b^2)*x^4 + (7*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^5*b*x^9 + a^6*x^7)
```

giac [**B**] time = 0.54, size = 407, normalized size = 2.75

$$-\frac{\left(Bab^{3}-Ab^{4}\right)x}{\sqrt{bx^{2}+a}\,a^{5}}+\frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Bab^{\frac{5}{2}}-35\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{12}Ab^{\frac{7}{2}}-280\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}Ba^{2}b^{\frac{5}{2}}+280\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{10}Aab^{\frac{7}{2}}+1015\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ba^{3}b^{\frac{5}{2}}-1015\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Aa^{2}b^{\frac{7}{2}}-1680\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ba^{4}b^{\frac{5}{2}}+2240\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Aa^{3}b^{\frac{7}{2}}+1337\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba^{5}b^{\frac{5}{2}}-1673\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Aa^{4}b^{\frac{7}{2}}-504\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{6}b^{\frac{5}{2}}+616\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aa^{5}b^{\frac{7}{2}}+77\,Ba^{7}b^{\frac{5}{2}}-93\,Aa^{6}b^{\frac{7}{2}}\right)}{35\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{7}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*a*b^3 - A*b^4)*x/(sqrt(b*x^2 + a)*a^5) + 2/35*(35*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*B*a*b^(5/2) - 35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 2
80*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 280*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*A*a*b^(7/2) + 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5
/2) - 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 1680*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a^3*b^(7/2) + 1337*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 1673*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*B*a^6*b^(5/2) + 616*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(7/2) +
77*B*a^7*b^(5/2) - 93*A*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^7*a^4)
```

maple [**A**] time = 0.01, size = 107, normalized size = 0.72

$$-\frac{-128A\,b^{4}x^{8}+112Ba\,b^{3}x^{8}-64Aa\,b^{3}x^{6}+56B\,a^{2}b^{2}x^{6}+16A\,a^{2}b^{2}x^{4}-14B\,a^{3}b\,x^{4}-8A\,a^{3}b\,x^{2}+7B\,a^{4}x^{2}+5A\,a^{4}}{35\sqrt{b\,x^{2}+a}\;a^{5}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*(-128*A*b^4*x^8+112*B*a*b^3*x^8-64*A*a*b^3*x^6+56*B*a^2*b^2*x^6+16*A*
a^2*b^2*x^4-14*B*a^3*b*x^4-8*A*a^3*b*x^2+7*B*a^4*x^2+5*A*a^4)/(b*x^2+a)^(1/
2)/x^7/a^5
```

maxima [**A**] time = 1.06, size = 176, normalized size = 1.19

$$-\frac{16\,Bb^{3}x}{5\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{128\,Ab^{4}x}{35\,\sqrt{bx^{2}+a}\,a^{5}}-\frac{8\,Bb^{2}}{5\,\sqrt{bx^{2}+a}\,a^{3}x}+\frac{64\,Ab^{3}}{35\,\sqrt{bx^{2}+a}\,a^{4}x}+\frac{2\,Bb}{5\,\sqrt{bx^{2}+a}\,a^{2}x^{3}}-\frac{16\,Ab^{2}}{35\,\sqrt{bx^{2}+a}\,a^{3}x^{3}}-\frac{B}{5\,\sqrt{bx^{2}+a}\,ax^{5}}+\frac{8\,Ab}{35\,\sqrt{bx^{2}+a}\,a^{2}x^{5}}-\frac{A}{7\,\sqrt{bx^{2}+a}\,ax^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -16/5*B*b^3*x/(sqrt(b*x^2 + a)*a^4) + 128/35*A*b^4*x/(sqrt(b*x^2 + a)*a^5)
- 8/5*B*b^2/(sqrt(b*x^2 + a)*a^3*x) + 64/35*A*b^3/(sqrt(b*x^2 + a)*a^4*x) +
 2/5*B*b/(sqrt(b*x^2 + a)*a^2*x^3) - 16/35*A*b^2/(sqrt(b*x^2 + a)*a^3*x^3)
- 1/5*B/(sqrt(b*x^2 + a)*a*x^5) + 8/35*A*b/(sqrt(b*x^2 + a)*a^2*x^5) - 1/7*
A/(sqrt(b*x^2 + a)*a*x^7)
```

mupad [**B**] time = 1.07, size = 148, normalized size = 1.00

$$-\frac{x^{2}\left(\frac{58\,A\,b^{4}-42\,B\,a\,b^{3}}{35\,a^{5}}-\frac{2\,b^{3}\,(93\,A\,b-77\,B\,a)}{35\,a^{5}}\right)-\frac{b^{2}\,(93\,A\,b-77\,B\,a)}{35\,a^{4}}}{x\,\sqrt{b\,x^{2}+a}}-\frac{\left(7\,B\,a^{2}-13\,A\,a\,b\right)\,\sqrt{b\,x^{2}+a}}{35\,a^{4}\,x^{5}}-\frac{A\,\sqrt{b\,x^{2}+a}}{7\,a^{2}\,x^{7}}-\frac{b\,\sqrt{b\,x^{2}+a}\,\left(29\,A\,b-21\,B\,a\right)}{35\,a^{4}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x)
```

```
[Out] - (x^2*((58*A*b^4 - 42*B*a*b^3)/(35*a^5) - (2*b^3*(93*A*b - 77*B*a))/(35*a^
5)) - (b^2*(93*A*b - 77*B*a))/(35*a^4))/(x*(a + b*x^2)^(1/2)) - ((7*B*a^2 -
 13*A*a*b)*(a + b*x^2)^(1/2))/(35*a^4*x^5) - (A*(a + b*x^2)^(1/2))/(7*a^2*x
^7) - (b*(a + b*x^2)^(1/2)*(29*A*b - 21*B*a))/(35*a^4*x^3)
```

sympy [**B**] time = 19.08, size = 1030, normalized size = 6.96

$$A\left(-\frac{5a^{7}b^{\frac{33}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}-\frac{7a^{6}b^{\frac{35}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}-\frac{7a^{5}b^{\frac{37}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{35a^{4}b^{\frac{39}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{280a^{3}b^{\frac{41}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{560a^{2}b^{\frac{43}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{448ab^{\frac{45}{2}}x^{12}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}+\frac{128b^{\frac{47}{2}}x^{14}\sqrt{\frac{a}{bx^{2}}+1}}{35a^{9}b^{16}x^{6}+140a^{8}b^{17}x^{8}+210a^{7}b^{18}x^{10}+140a^{6}b^{19}x^{12}+35a^{5}b^{20}x^{14}}\right)+B\left(-\frac{a^{5}b^{\frac{19}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{5a^{3}b^{\frac{23}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{30a^{2}b^{\frac{25}{2}}x^{6}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{40ab^{\frac{27}{2}}x^{8}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}-\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^{2}}+1}}{5a^{7}b^{9}x^{4}+15a^{6}b^{10}x^{6}+15a^{5}b^{11}x^{8}+5a^{4}b^{12}x^{10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-5*a**7*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b*
*17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**1
4) - 7*a**6*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a
**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**2
0*x**14) - 7*a**5*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 +
 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**
5*b**20*x**14) + 35*a**4*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16
*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 +
 35*a**5*b**20*x**14) + 280*a**3*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a*
*9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19
```

```
*x**12 + 35*a**5*b**20*x**14) + 560*a**2*b**(43/2)*x**10*sqrt(a/(b*x**2) +
1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a
**6*b**19*x**12 + 35*a**5*b**20*x**14) + 448*a*b**(45/2)*x**12*sqrt(a/(b*x*
*2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 +
 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 128*b**(47/2)*x**14*sqrt(a/(
b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**
10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14)) + B*(-a**5*b**(19/2)*sqrt
(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**
8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(5*a**
7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10)
 - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*
b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 40*a*b**(27/2)*x**8
*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**1
1*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(5*a
**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**1
0))
```

3.566 $\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{a^3(Ab-aB)}{3b^5\left(a+bx^2\right)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}\,(Ab-2aB)}{b^5}+\frac{\left(a+bx^2\right)^{3/2}(Ab-4aB)}{3b^5}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^5}$$

Rubi [A] time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{a^3(Ab-aB)}{3b^5\left(a+bx^2\right)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a\sqrt{a+bx^2}\,(Ab-2aB)}{b^5}+\frac{\left(a+bx^2\right)^{3/2}(Ab-4aB)}{3b^5}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (a^2*(3*A*b - 4*a*B))/(b^5*Sq
rt[a + b*x^2]) - (3*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^5 + ((A*b - 4*a*B)*(
a + b*x^2)^(3/2))/(3*b^5) + (B*(a + b*x^2)^(5/2))/(5*b^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\Bigg(\int \Bigg(\frac{a^3(-Ab+aB)}{b^4(a+bx)^{5/2}}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^{3/2}}+\frac{3a(-Ab+2aB)}{b^4\sqrt{a+bx}}+\frac{(Ab-4aB)\sqrt{a+bx}}{b^4}\\ &= \frac{a^3(Ab-aB)}{3b^5\left(a+bx^2\right)^{3/2}}-\frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{3a(Ab-2aB)\sqrt{a+bx^2}}{b^5}+\frac{(Ab-4aB)\left(a+bx^2\right)^{3/2}}{3b^5}+\end{aligned}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.77

$$\frac{128a^4B+a^3\left(192bBx^2-80Ab\right)+24a^2b^2x^2\left(2Bx^2-5A\right)-2ab^3x^4\left(15A+4Bx^2\right)+b^4x^6\left(5A+3Bx^2\right)}{15b^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (128*a^4*B + 24*a^2*b^2*x^2*(-5*A + 2*B*x^2) + b^4*x^6*(5*A + 3*B*x^2) - 2*
a*b^3*x^4*(15*A + 4*B*x^2) + a^3*(-80*A*b + 192*b*B*x^2))/(15*b^5*(a + b*x^
2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 104, normalized size = 0.81

$$\frac{128a^4B-80a^3Ab+192a^3bBx^2-120a^2Ab^2x^2+48a^2b^2Bx^4-30aAb^3x^4-8ab^3Bx^6+5Ab^4x^6+3b^4Bx^8}{15b^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-80*a^3*A*b + 128*a^4*B - 120*a^2*A*b^2*x^2 + 192*a^3*b*B*x^2 - 30*a*A*b^3
*x^4 + 48*a^2*b^2*B*x^4 + 5*A*b^4*x^6 - 8*a*b^3*B*x^6 + 3*b^4*B*x^8)/(15*b^
5*(a + b*x^2)^(3/2))
```

fricas [A] time = 1.20, size = 123, normalized size = 0.96

$$\frac{\left(3\,Bb^4x^8-\left(8\,Bab^3-5\,Ab^4\right)x^6+128\,Ba^4-80\,Aa^3b+6\left(8\,Ba^2b^2-5\,Aab^3\right)x^4+24\left(8\,Ba^3b-5\,Aa^2b^2\right)x^2\right)\sqrt{bx^2+a}}{15\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*B*b^4*x^8 - (8*B*a*b^3 - 5*A*b^4)*x^6 + 128*B*a^4 - 80*A*a^3*b + 6*
(8*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 24*(8*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(b*x^
2 + a)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

giac [A] time = 0.36, size = 141, normalized size = 1.10

$$\frac{12\left(bx^2+a\right)Ba^3-Ba^4-9\left(bx^2+a\right)Aa^2b+Aa^3b}{3\left(bx^2+a\right)^{\frac{3}{2}}b^5}+\frac{3\left(bx^2+a\right)^{\frac{5}{2}}Bb^{20}-20\left(bx^2+a\right)^{\frac{3}{2}}Bab^{20}+90\sqrt{bx^2+a}\,Ba^2b^{20}+5\left(bx^2+a\right)^{\frac{3}{2}}Ab^{21}-45\sqrt{bx^2+a}\,Aab^{21}}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(12*(b*x^2 + a)*B*a^3 - B*a^4 - 9*(b*x^2 + a)*A*a^2*b + A*a^3*b)/((b*x^
2 + a)^(3/2)*b^5) + 1/15*(3*(b*x^2 + a)^(5/2)*B*b^20 - 20*(b*x^2 + a)^(3/2)
*B*a*b^20 + 90*sqrt(b*x^2 + a)*B*a^2*b^20 + 5*(b*x^2 + a)^(3/2)*A*b^21 - 45
*sqrt(b*x^2 + a)*A*a*b^21)/b^25
```

maple [A] time = 0.01, size = 101, normalized size = 0.79

$$-\frac{-3x^8B\,b^4-5A\,b^4x^6+8Ba\,b^3x^6+30Aa\,b^3x^4-48B\,a^2b^2x^4+120A\,a^2b^2x^2-192B\,a^3b\,x^2+80A\,a^3b-128B\,a^4}{15\left(b\,x^2+a\right)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/15*(-3*B*b^4*x^8-5*A*b^4*x^6+8*B*a*b^3*x^6+30*A*a*b^3*x^4-48*B*a^2*b^2*x
^4+120*A*a^2*b^2*x^2-192*B*a^3*b*x^2+80*A*a^3*b-128*B*a^4)/(b*x^2+a)^(3/2)/
b^5
```

maxima [A] time = 1.21, size = 174, normalized size = 1.36

$$\frac{Bx^8}{5\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{8\,Bax^6}{15\left(bx^2+a\right)^{\frac{3}{2}}b^2}+\frac{Ax^6}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{16\,Ba^2x^4}{5\left(bx^2+a\right)^{\frac{3}{2}}b^3}-\frac{2\,Aax^4}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}+\frac{64\,Ba^3x^2}{5\left(bx^2+a\right)^{\frac{3}{2}}b^4}-\frac{8\,Aa^2x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{128\,Ba^4}{15\left(bx^2+a\right)^{\frac{3}{2}}b^5}-\frac{16\,Aa^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5*B*x^8/((b*x^2 + a)^(3/2)*b) - 8/15*B*a*x^6/((b*x^2 + a)^(3/2)*b^2) + 1/
3*A*x^6/((b*x^2 + a)^(3/2)*b) + 16/5*B*a^2*x^4/((b*x^2 + a)^(3/2)*b^3) - 2*
A*a*x^4/((b*x^2 + a)^(3/2)*b^2) + 64/5*B*a^3*x^2/((b*x^2 + a)^(3/2)*b^4) -
8*A*a^2*x^2/((b*x^2 + a)^(3/2)*b^3) + 128/15*B*a^4/((b*x^2 + a)^(3/2)*b^5)
- 16/3*A*a^3/((b*x^2 + a)^(3/2)*b^4)
```

mupad [B] time = 1.00, size = 122, normalized size = 0.95

$$\frac{3\,B\,{\left(b\,x^2+a\right)}^4-5\,B\,a^4+90\,B\,a^2\,{\left(b\,x^2+a\right)}^2+5\,A\,b\,{\left(b\,x^2+a\right)}^3-20\,B\,a\,{\left(b\,x^2+a\right)}^3+60\,B\,a^3\,\left(b\,x^2+a\right)+5\,A\,a^3\,b-45\,A\,a\,b\,{\left(b\,x^2+a\right)}^2-45\,A\,a^2\,b\,\left(b\,x^2+a\right)}{15\,b^5\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*B*(a + b*x^2)^4 - 5*B*a^4 + 90*B*a^2*(a + b*x^2)^2 + 5*A*b*(a + b*x^2)^3
 - 20*B*a*(a + b*x^2)^3 + 60*B*a^3*(a + b*x^2) + 5*A*a^3*b - 45*A*a*b*(a +
b*x^2)^2 - 45*A*a^2*b*(a + b*x^2))/(15*b^5*(a + b*x^2)^(3/2))
```

sympy [A] time = 4.21, size = 437, normalized size = 3.41

$$\begin{cases} - \frac{80 A a^{3} b}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} - \frac{120 A a^{2} b^{2} x^{2}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} - \frac{30 A a b^{3} x^{4}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} + \frac{5 A b^{4} x^{6}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} + \frac{128 B a^{4}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} + \frac{192 B a^{3} b x^{2}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} + \frac{48 B a^{2} b^{2} x^{4}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} - \frac{8 B a b^{3} x^{6}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} + \frac{3 B b^{4} x^{8}}{15 a b^{5} \sqrt{a + b x^{2}} + 15 b^{6} x^{2} \sqrt{a + b x^{2}}} & \text{for}\: b \neq 0 \\ \frac{\frac{A x^{8}}{8} + \frac{B x^{10}}{10}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-80*A*a**3*b/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a +
 b*x**2)) - 120*A*a**2*b**2*x**2/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2
*sqrt(a + b*x**2)) - 30*A*a*b**3*x**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6
*x**2*sqrt(a + b*x**2)) + 5*A*b**4*x**6/(15*a*b**5*sqrt(a + b*x**2) + 15*b*
*6*x**2*sqrt(a + b*x**2)) + 128*B*a**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b**
6*x**2*sqrt(a + b*x**2)) + 192*B*a**3*b*x**2/(15*a*b**5*sqrt(a + b*x**2) +
15*b**6*x**2*sqrt(a + b*x**2)) + 48*B*a**2*b**2*x**4/(15*a*b**5*sqrt(a + b*
x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) - 8*B*a*b**3*x**6/(15*a*b**5*sqrt(a
+ b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 3*B*b**4*x**8/(15*a*b**5*sqrt(
a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**8/8 + B*x**
10/10)/a**(5/2), True))
```

3.567 $\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{5x\sqrt{a+bx^2}\,(4Ab-7aB)}{8b^4}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}-\frac{x^5(4Ab-7aB)}{12b^2\left(a+bx^2\right)^{3/2}}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/}}$$

Rubi [A] time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 321, 217, 206}

$$-\frac{x^5(4Ab-7aB)}{12b^2\left(a+bx^2\right)^{3/2}}-\frac{5x^3(4Ab-7aB)}{12b^3\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}\,(4Ab-7aB)}{8b^4}-\frac{5a(4Ab-7aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -((4*A*b - 7*a*B)*x^5)/(12*b^2*(a + b*x^2)^(3/2)) + (B*x^7)/(4*b*(a + b*x^2
)^(3/2)) - (5*(4*A*b - 7*a*B)*x^3)/(12*b^3*Sqrt[a + b*x^2]) + (5*(4*A*b - 7
*a*B)*x*Sqrt[a + b*x^2])/(8*b^4) - (5*a*(4*A*b - 7*a*B)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{(-4Ab+7aB)\int \frac{x^6}{\left(a+bx^2\right)^{5/2}}\,dx}{4b}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} + \frac{(5(4Ab-7aB))\int \frac{x^4}{\left(a+bx^2\right)^{3/2}}\,dx}{12b^2}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{(5(4Ab-7aB))\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4b^3}$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4} - (5$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4} - (5$$

$$= -\frac{(4Ab-7aB)x^5}{12b^2\left(a+bx^2\right)^{3/2}} + \frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}} - \frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}} + \frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4} - 5a$$

Mathematica [A] time = 0.23, size = 139, normalized size = 0.93

$$\frac{x\left(-105a^3B+20a^2b\left(3A-7Bx^2\right)+ab^2x^2\left(80A-21Bx^2\right)+6b^3x^4\left(2A+Bx^2\right)\right)}{24b^4\left(a+bx^2\right)^{3/2}}+\frac{5\sqrt{a}\sqrt{a+bx^2}\,(7aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(-105*a^3*B + a*b^2*x^2*(80*A - 21*B*x^2) + 20*a^2*b*(3*A - 7*B*x^2) + 6
*b^3*x^4*(2*A + B*x^2)))/(24*b^4*(a + b*x^2)^(3/2)) + (5*Sqrt[a]*(-4*A*b +
7*a*B)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2)*Sqrt[1 + (b
*x^2)/a])
```

IntegrateAlgebraic [A] time = 0.28, size = 125, normalized size = 0.84

$$\frac{-105a^3Bx+60a^2Abx-140a^2bBx^3+80aAb^2x^3-21ab^2Bx^5+12Ab^3x^5+6b^3Bx^7}{24b^4\left(a+bx^2\right)^{3/2}}-\frac{5\left(7a^2B-4aAb\right)\log\left(\sqrt{a+bx^2}-\sqrt{b}x\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (60*a^2*A*b*x - 105*a^3*B*x + 80*a*A*b^2*x^3 - 140*a^2*b*B*x^3 + 12*A*b^3*x
^5 - 21*a*b^2*B*x^5 + 6*b^3*B*x^7)/(24*b^4*(a + b*x^2)^(3/2)) - (5*(-4*a*A*
b + 7*a^2*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(9/2))
```

fricas [A] time = 1.29, size = 392, normalized size = 2.63

$$\left[-\frac{15\left(7Ba^4-4Aa^3b+\left(7Ba^2b^2-4Aab^3\right)x^4+2\left(7Ba^3b-4Aa^2b^2\right)x^2\right)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(6Bb^4x^7-3\left(7Bab^3-4Ab^4\right)x^5-20\left(7Ba^2b^2-4Aab^3\right)x^3-15\left(7Ba^3b-4Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{48\left(b^7x^4+2ab^6x^2+a^2b^5\right)},-\frac{15\left(7Ba^4-4Aa^3b+\left(7Ba^2b^2-4Aab^3\right)x^4+2\left(7Ba^3b-4Aa^2b^2\right)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(6Bb^4x^7-3\left(7Bab^3-4Ab^4\right)x^5-20\left(7Ba^2b^2-4Aab^3\right)x^3-15\left(7Ba^3b-4Aa^2b^2\right)x\right)\sqrt{bx^2+a}}{24\left(b^7x^4+2ab^6x^2+a^2b^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*x^4 + 2*(7*B*a^
3*b - 4*A*a^2*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b^2 - 4*A
*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7*x^4 + 2
*a*b^6*x^2 + a^2*b^5), -1/24*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*
a*b^3)*x^4 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b
^2 - 4*A*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7
*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

giac [A] time = 0.58, size = 148, normalized size = 0.99

$$\frac{\left(\left(3\left(\frac{2Bx^2}{b}-\frac{7Ba^2b^5-4Aab^6}{ab^7}\right)x^2-\frac{20\left(7Ba^3b^4-4Aa^2b^5\right)}{ab^7}\right)x^2-\frac{15\left(7Ba^4b^3-4Aa^3b^4\right)}{ab^7}\right)x}{24\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\left(7Ba^2-4Aab\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*B*x^2/b - (7*B*a^2*b^5 - 4*A*a*b^6)/(a*b^7))*x^2 - 20*(7*B*a^3*
b^4 - 4*A*a^2*b^5)/(a*b^7))*x^2 - 15*(7*B*a^4*b^3 - 4*A*a^3*b^4)/(a*b^7))*x
/(b*x^2 + a)^(3/2) - 5/8*(7*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(9/2)
```

maple [A] time = 0.02, size = 181, normalized size = 1.21

$$\frac{Bx^7}{4\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{Ax^5}{2\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{7Bax^5}{8\left(bx^2+a\right)^{\frac{3}{2}}b^2}+\frac{5Aax^3}{6\left(bx^2+a\right)^{\frac{3}{2}}b^2}-\frac{35Ba^2x^3}{24\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{5Aax}{2\sqrt{bx^2+a}\,b^3}-\frac{35Ba^2x}{8\sqrt{bx^2+a}\,b^4}-\frac{5Aa\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2b^{\frac{7}{2}}}+\frac{35Ba^2\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/4*B*x^7/b/(b*x^2+a)^(3/2)-7/8*B*a/b^2*x^5/(b*x^2+a)^(3/2)-35/24*B*a^2/b^3
*x^3/(b*x^2+a)^(3/2)-35/8*B*a^2/b^4*x/(b*x^2+a)^(1/2)+35/8*B*a^2/b^(9/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*A*x^5/b/(b*x^2+a)^(3/2)+5/6*A*a/b^2*x^3/(b*
x^2+a)^(3/2)+5/2*A*a/b^3*x/(b*x^2+a)^(1/2)-5/2*A*a/b^(7/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))
```

maxima [A] time = 1.09, size = 210, normalized size = 1.41

$$\frac{Bx^7}{4\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{7Bax^5}{8\left(bx^2+a\right)^{\frac{3}{2}}b^2}+\frac{Ax^5}{2\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{35Ba^2x\left(\frac{3x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{24b^2}+\frac{5Aax\left(\frac{3x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)}{6b}-\frac{35Ba^2x}{24\sqrt{bx^2+a}\,b^4}+\frac{5Aax}{6\sqrt{bx^2+a}\,b^3}+\frac{35Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{\frac{9}{2}}}-\frac{5Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*B*x^7/((b*x^2 + a)^(3/2)*b) - 7/8*B*a*x^5/((b*x^2 + a)^(3/2)*b^2) + 1/2
*A*x^5/((b*x^2 + a)^(3/2)*b) - 35/24*B*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 5/6*A*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
+ 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*B*a^2*x/(sqrt(b*x^2 + a)*b^4) + 5/
6*A*a*x/(sqrt(b*x^2 + a)*b^3) + 35/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(9/2) -
 5/2*A*a*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6 \left(B x^2 + A\right)}{\left(b x^2 + a\right)^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^6*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 38.82, size = 804, normalized size = 5.40

$$A \left(- \frac{15 a^{\frac{81}{2}} b^{22} \sqrt{1 + \frac{b x^{2}}{a}} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{6 a^{\frac{79}{2}} b^{\frac{51}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 6 a^{\frac{77}{2}} b^{\frac{53}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{15 a^{\frac{79}{2}} b^{23} x^{2} \sqrt{1 + \frac{b x^{2}}{a}} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{6 a^{\frac{79}{2}} b^{\frac{51}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 6 a^{\frac{77}{2}} b^{\frac{53}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{15 a^{40} b^{\frac{45}{2}} x}{6 a^{\frac{79}{2}} b^{\frac{51}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 6 a^{\frac{77}{2}} b^{\frac{53}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{20 a^{39} b^{\frac{47}{2}} x^{3}}{6 a^{\frac{79}{2}} b^{\frac{51}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 6 a^{\frac{77}{2}} b^{\frac{53}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{3 a^{38} b^{\frac{49}{2}} x^{5}}{6 a^{\frac{79}{2}} b^{\frac{51}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 6 a^{\frac{77}{2}} b^{\frac{53}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}}\right) + B \left(\frac{105 a^{\frac{157}{2}} b^{41} \sqrt{1 + \frac{b x^{2}}{a}} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{24 a^{\frac{153}{2}} b^{\frac{91}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 24 a^{\frac{151}{2}} b^{\frac{93}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{105 a^{\frac{155}{2}} b^{42} x^{2} \sqrt{1 + \frac{b x^{2}}{a}} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{24 a^{\frac{153}{2}} b^{\frac{91}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 24 a^{\frac{151}{2}} b^{\frac{93}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{105 a^{78} b^{\frac{83}{2}} x}{24 a^{\frac{153}{2}} b^{\frac{91}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 24 a^{\frac{151}{2}} b^{\frac{93}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{140 a^{77} b^{\frac{85}{2}} x^{3}}{24 a^{\frac{153}{2}} b^{\frac{91}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 24 a^{\frac{151}{2}} b^{\frac{93}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} - \frac{21 a^{76} b^{\frac{87}{2}} x^{5}}{24 a^{\frac{153}{2}} b^{\frac{91}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 24 a^{\frac{151}{2}} b^{\frac{93}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{6 a^{75} b^{\frac{89}{2}} x^{7}}{24 a^{\frac{153}{2}} b^{\frac{91}{2}} \sqrt{1 + \frac{b x^{2}}{a}} + 24 a^{\frac{151}{2}} b^{\frac{93}{2}} x^{2} \sqrt{1 + \frac{b x^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(7
9/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x
**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*s
qrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b
*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**(47
/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*
x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sq
rt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))) + B*(105
*a**(157/2)*b**41*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(24*a**(153/2
)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x*
*2/a)) + 105*a**(155/2)*b**42*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(
a))/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x
**2*sqrt(1 + b*x**2/a)) - 105*a**78*b**(83/2)*x/(24*a**(153/2)*b**(91/2)*sq
rt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 140*a
**77*b**(85/2)*x**3/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(15
1/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 21*a**76*b**(87/2)*x**5/(24*a**(1
53/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 +
b*x**2/a)) + 6*a**75*b**(89/2)*x**7/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**
2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)))
```

3.568 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^2\right)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\,(Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^4}$$

Rubi [**A**] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^2\right)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{\sqrt{a+bx^2}\,(Ab-3aB)}{b^4}+\frac{B\left(a+bx^2\right)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(a^2*(A*b - a*B))/(3*b^4*(a + b*x^2)^(3/2)) + (a*(2*A*b - 3*a*B))/(b^4*Sqr
t[a + b*x^2]) + ((A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (B*(a + b*x^2)^(3/2))
/(3*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}} + \frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}} + \frac{Ab-3aB}{b^3\sqrt{a+bx}} + \frac{B\sqrt{a+bx}}{b^3}\right)dx, x, x^2\right)$$

$$= -\frac{a^2(Ab-aB)}{3b^4\left(a+bx^2\right)^{3/2}} + \frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}} + \frac{(Ab-3aB)\sqrt{a+bx^2}}{b^4} + \frac{B\left(a+bx^2\right)^{3/2}}{3b^4}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 0.75

$$\frac{-16a^3B + 8a^2b\left(A-3Bx^2\right) - 6ab^2x^2\left(Bx^2-2A\right) + b^3x^4\left(3A+Bx^2\right)}{3b^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-16*a^3*B + 8*a^2*b*(A - 3*B*x^2) - 6*a*b^2*x^2*(-2*A + B*x^2) + b^3*x^4*(
3*A + B*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 79, normalized size = 0.81

$$\frac{-16a^3B + 8a^2Ab - 24a^2bBx^2 + 12aAb^2x^2 - 6ab^2Bx^4 + 3Ab^3x^4 + b^3Bx^6}{3b^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (8*a^2*A*b - 16*a^3*B + 12*a*A*b^2*x^2 - 24*a^2*b*B*x^2 + 3*A*b^3*x^4 - 6*a
*b^2*B*x^4 + b^3*B*x^6)/(3*b^4*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.86, size = 98, normalized size = 1.01

$$\frac{\left(Bb^3x^6 - 3\left(2\,Bab^2 - Ab^3\right)x^4 - 16\,Ba^3 + 8\,Aa^2b - 12\left(2\,Ba^2b - Aab^2\right)x^2\right)\sqrt{bx^2+a}}{3\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^3*x^6 - 3*(2*B*a*b^2 - A*b^3)*x^4 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

giac [**A**] time = 0.45, size = 104, normalized size = 1.07

$$-\frac{9\left(bx^2+a\right)Ba^2-Ba^3-6\left(bx^2+a\right)Aab+Aa^2b}{3\left(bx^2+a\right)^{\frac{3}{2}}b^4}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bb^8-9\sqrt{bx^2+a}\,Bab^8+3\sqrt{bx^2+a}\,Ab^9}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(9*(b*x^2 + a)*B*a^2 - B*a^3 - 6*(b*x^2 + a)*A*a*b + A*a^2*b)/((b*x^2
+ a)^(3/2)*b^4) + 1/3*((b*x^2 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^2 + a)*B*a*b^8
+ 3*sqrt(b*x^2 + a)*A*b^9)/b^12
```

maple [**A**] time = 0.01, size = 76, normalized size = 0.78

$$\frac{B\,x^6b^3+3A\,b^3x^4-6Ba\,b^2x^4+12Aa\,b^2x^2-24B\,a^2b\,x^2+8A\,a^2b-16B\,a^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(B*b^3*x^6+3*A*b^3*x^4-6*B*a*b^2*x^4+12*A*a*b^2*x^2-24*B*a^2*b*x^2+8*A*
a^2*b-16*B*a^3)/(b*x^2+a)^(3/2)/b^4
```

maxima [**A**] time = 1.16, size = 131, normalized size = 1.35

$$\frac{Bx^6}{3\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{2\,Bax^4}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}+\frac{Ax^4}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{8\,Ba^2x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b^3}+\frac{4\,Aax^2}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}-\frac{16\,Ba^3}{3\left(bx^2+a\right)^{\frac{3}{2}}b^4}+\frac{8\,Aa^2}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*B*x^6/((b*x^2 + a)^(3/2)*b) - 2*B*a*x^4/((b*x^2 + a)^(3/2)*b^2) + A*x^4
/((b*x^2 + a)^(3/2)*b) - 8*B*a^2*x^2/((b*x^2 + a)^(3/2)*b^3) + 4*A*a*x^2/((
b*x^2 + a)^(3/2)*b^2) - 16/3*B*a^3/((b*x^2 + a)^(3/2)*b^4) + 8/3*A*a^2/((b*
x^2 + a)^(3/2)*b^3)
```

mupad [B] time = 0.86, size = 89, normalized size = 0.92

$$\frac{B\left(b\,x^2+a\right)^3+B\,a^3+3\,A\,b\left(b\,x^2+a\right)^2-9\,B\,a\left(b\,x^2+a\right)^2-9\,B\,a^2\,\left(b\,x^2+a\right)-A\,a^2\,b+6\,A\,a\,b\,\left(b\,x^2+a\right)}{3\,b^4\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*(a + b*x^2)^3 + B*a^3 + 3*A*b*(a + b*x^2)^2 - 9*B*a*(a + b*x^2)^2 - 9*B*
a^2*(a + b*x^2) - A*a^2*b + 6*A*a*b*(a + b*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.99, size = 337, normalized size = 3.47

$$\begin{cases} \frac{8Aa^2b}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{12Aab^2x^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{3Ab^3x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{16Ba^3}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{24Ba^2bx^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{6Bab^2x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{Bb^3x^6}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} & \text{for}\: b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((8*A*a**2*b/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x
**2)) + 12*A*a*b**2*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) + 3*A*b**3*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) - 16*B*a**3/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*x*
*2)) - 24*B*a**2*b*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b
*x**2)) - 6*B*a*b**2*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
 b*x**2)) + B*b**3*x**6/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b
*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/a**(5/2), True))
```

3.569 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {455, 1157, 388, 217, 206}

$$-\frac{x(4Ab-7aB)}{3b^3\sqrt{a+bx^2}}+\frac{ax(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}+\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - ((4*A*b - 7*a*B)*x)/(3*b^3*Sq
rt[a + b*x^2]) + (B*x*Sqrt[a + b*x^2])/(2*b^3) + ((2*A*b - 5*a*B)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{a(Ab-aB)-3b(Ab-aB)x^2-3b^2Bx^4}{\left(a+bx^2\right)^{3/2}}\,dx}{3b^3}$$

$$=\frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}}+\frac{\int \frac{3a(Ab-2aB)+3abBx^2}{\sqrt{a+bx^2}}\,dx}{3ab^3}$$

$$=\frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}+\frac{(2Ab-5aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3}$$

$$=\frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}+\frac{(2Ab-5aB)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+b}}\right.}{2b^3}$$

$$=\frac{a(Ab-aB)x}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}}+\frac{Bx\sqrt{a+bx^2}}{2b^3}+\frac{(2Ab-5aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

Mathematica [A] time = 0.26, size = 116, normalized size = 1.02

$$\frac{\sqrt{b}\,x\left(15a^2B+a\left(20bBx^2-6Ab\right)+b^2x^2\left(3Bx^2-8A\right)\right)-3\sqrt{a}\,\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,(5aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6b^{7/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(15*a^2*B + b^2*x^2*(-8*A + 3*B*x^2) + a*(-6*A*b + 20*b*B*x^2))
- 3*Sqrt[a]*(-2*A*b + 5*a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[
b]*x)/Sqrt[a]])/(6*b^(7/2)*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 98, normalized size = 0.86

$$\frac{15a^2Bx-6aAbx+20abBx^3-8Ab^2x^3+3b^2Bx^5}{6b^3\left(a+bx^2\right)^{3/2}}+\frac{(5aB-2Ab)\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-6*a*A*b*x + 15*a^2*B*x - 8*A*b^2*x^3 + 20*a*b*B*x^3 + 3*b^2*B*x^5)/(6*b^3
*(a + b*x^2)^(3/2)) + ((-2*A*b + 5*a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
)/(2*b^(7/2))
```

fricas [A] time = 0.70, size = 333, normalized size = 2.92

$$\left[-\frac{3\left(\left(5\,Bab^2-2\,Ab^3\right)x^4+5\,Ba^3-2\,Aa^2b+2\left(5\,Ba^2b-2\,Aab^2\right)x^2\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(3\,Bb^3x^5+4\left(5\,Bab^2-2\,Ab^3\right)x^3+3\left(5\,Ba^2b-2\,Aab^2\right)x\right)\sqrt{bx^2+a}}{12\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)},\ \frac{3\left(\left(5\,Bab^2-2\,Ab^3\right)x^4+5\,Ba^3-2\,Aa^2b+2\left(5\,Ba^2b-2\,Aab^2\right)x^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(3\,Bb^3x^5+4\left(5\,Bab^2-2\,Ab^3\right)x^3+3\left(5\,Ba^2b-2\,Aab^2\right)x\right)\sqrt{bx^2+a}}{6\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((5*B*a*b^2 - 2*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b -
 2*A*a*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*(3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)
*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(3*((5*B*a*b^2 - 2
*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^2)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3
)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^
2 + a^2*b^4)]
```

giac [A]   time = 0.50, size = 112, normalized size = 0.98

$$\frac{\left(\left(\frac{3\,Bx^{2}}{b}+\frac{4\left(5\,Ba^{2}b^{3}-2\,Aab^{4}\right)}{ab^{5}}\right)x^{2}+\frac{3\left(5\,Ba^{3}b^{2}-2\,Aa^{2}b^{3}\right)}{ab^{5}}\right)x}{6\left(bx^{2}+a\right)^{\frac{3}{2}}}+\frac{\left(5\,Ba-2\,Ab\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*B*x^2/b + 4*(5*B*a^2*b^3 - 2*A*a*b^4)/(a*b^5))*x^2 + 3*(5*B*a^3*b^2
 - 2*A*a^2*b^3)/(a*b^5))*x/(b*x^2 + a)^(3/2) + 1/2*(5*B*a - 2*A*b)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

maple [A]   time = 0.01, size = 134, normalized size = 1.18

$$\frac{B\,x^{5}}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}-\frac{A\,x^{3}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}+\frac{5Ba\,x^{3}}{6\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{2}}-\frac{Ax}{\sqrt{b\,x^{2}+a}\;b^{2}}+\frac{5Bax}{2\sqrt{b\,x^{2}+a}\;b^{3}}+\frac{A\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{5}{2}}}-\frac{5Ba\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/2*B*x^5/b/(b*x^2+a)^(3/2)+5/6*B*a/b^2*x^3/(b*x^2+a)^(3/2)+5/2*B*a/b^3*x/(
b*x^2+a)^(1/2)-5/2*B*a/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-1/3*A*x^3/b/(b
*x^2+a)^(3/2)-A/b^2*x/(b*x^2+a)^(1/2)+A/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))
```

maxima [A]   time = 1.13, size = 160, normalized size = 1.40

$$\frac{Bx^{5}}{2\left(bx^{2}+a\right)^{\frac{3}{2}}b}-\frac{1}{3}\,Ax\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)+\frac{5\,Bax\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)}{6\,b}+\frac{5\,Bax}{6\,\sqrt{bx^{2}+a}\,b^{3}}-\frac{Ax}{3\,\sqrt{bx^{2}+a}\,b^{2}}-\frac{5\,Ba\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}+\frac{A\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*A*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*
a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/(
(b*x^2 + a)^(3/2)*b^2))/b + 5/6*B*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3*A*x/(sqrt
(b*x^2 + a)*b^2) - 5/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + A*arcsinh(b*x/s
qrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^4*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 20.57, size = 675, normalized size = 5.92

$$A\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^3}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^5}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/
2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**
2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt
(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x
**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*
sqrt(1 + b*x**2/a))) + B*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt
(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(
53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**
(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/
2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**
2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) +
 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6
*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1
 + b*x**2/a)))
```

3.570 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{a(Ab-aB)}{3b^3(a+bx^2)^{3/2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (A*b - 2*a*B)/(b^3*Sqrt[a + b*x
^2]) + (B*Sqrt[a + b*x^2])/b^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}}+\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)dx,x,x^2\right)$$
$$= \frac{a(Ab-aB)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{Ab-2aB}{b^3\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^3}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.79

$$\frac{8a^2B-2ab\left(A-6Bx^2\right)+3b^2x^2\left(Bx^2-A\right)}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (8*a^2*B - 2*a*b*(A - 6*B*x^2) + 3*b^2*x^2*(-A + B*x^2))/(3*b^3*(a + b*x^2)
^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.82

$$\frac{8a^2B-2aAb+12abBx^2-3Ab^2x^2+3b^2Bx^4}{3b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a*A*b + 8*a^2*B - 3*A*b^2*x^2 + 12*a*b*B*x^2 + 3*b^2*B*x^4)/(3*b^3*(a +
 b*x^2)^(3/2))
```

fricas [A] time = 0.86, size = 75, normalized size = 1.10

$$\frac{\left(3\,Bb^2x^4+8\,Ba^2-2\,Aab+3\left(4\,Bab-Ab^2\right)x^2\right)\sqrt{bx^2+a}}{3\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*B*b^2*x^4 + 8*B*a^2 - 2*A*a*b + 3*(4*B*a*b - A*b^2)*x^2)*sqrt(b*x^2
+ a)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [A] time = 0.34, size = 62, normalized size = 0.91

$$\frac{\sqrt{bx^2+a}B}{b^3}+\frac{6\left(bx^2+a\right)Ba-Ba^2-3\left(bx^2+a\right)Ab+Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)*B/b^3 + 1/3*(6*(b*x^2 + a)*B*a - B*a^2 - 3*(b*x^2 + a)*A*b
+ A*a*b)/((b*x^2 + a)^(3/2)*b^3)
```

maple [A] time = 0.01, size = 53, normalized size = 0.78

$$-\frac{-3B\,b^2x^4+3A\,b^2x^2-12Bab\,x^2+2abA-8a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-3*B*b^2*x^4+3*A*b^2*x^2-12*B*a*b*x^2+2*A*a*b-8*B*a^2)/(b*x^2+a)^(3/2
)/b^3
```

maxima [A] time = 1.07, size = 89, normalized size = 1.31

$$\frac{Bx^4}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{4\,Bax^2}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}-\frac{Ax^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{8\,Ba^2}{3\left(bx^2+a\right)^{\frac{3}{2}}b^3}-\frac{2\,Aa}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] B*x^4/((b*x^2 + a)^(3/2)*b) + 4*B*a*x^2/((b*x^2 + a)^(3/2)*b^2) - A*x^2/((b
*x^2 + a)^(3/2)*b) + 8/3*B*a^2/((b*x^2 + a)^(3/2)*b^3) - 2/3*A*a/((b*x^2 +
a)^(3/2)*b^2)
```

mupad [B] time = 0.67, size = 59, normalized size = 0.87

$$\frac{3\,B\left(b\,x^2+a\right)^2-B\,a^2-3\,A\,b\,\left(b\,x^2+a\right)+6\,B\,a\,\left(b\,x^2+a\right)+A\,a\,b}{3\,b^3\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*B*(a + b*x^2)^2 - B*a^2 - 3*A*b*(a + b*x^2) + 6*B*a*(a + b*x^2) + A*a*b)
/(3*b^3*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.97, size = 240, normalized size = 3.53

$$\begin{cases} -\frac{2Aab}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} - \frac{3Ab^2x^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{8Ba^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12Babx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3Bb^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*a*b/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)) - 3*A*b**2*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)) + 8*B*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) +
 12*B*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)) +
 3*B*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)),
Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(5/2), True))
```

3.571 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{x^3(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}$$

Rubi [**A**] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.182, Rules used = {452, 288, 217, 206}

$$\frac{x^3(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) - (B*x)/(b^2*Sqrt[a + b*x^2]) +
 (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}}+\frac{B\int \frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{b}\\ &= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\ &= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\ &= \frac{(Ab-aB)x^3}{3ab\left(a+bx^2\right)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 96, normalized size = 1.25

$$\frac{3a^{3/2}B\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(-3a^2B-4abBx^2+Ab^2x^2\right)}{3ab^{5/2}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - 4*a*b*B*x^2) + 3*a^(3/2)*B*(a + b*x^2)*S
qrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(3*a*b^(5/2)*(a + b*x^2)^(
3/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 75, normalized size = 0.97

$$\frac{-3a^2Bx-4abBx^3+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}-\frac{B\log\left(\sqrt{a+bx^2}-\sqrt{b}\,x\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-3*a^2*B*x + A*b^2*x^3 - 4*a*b*B*x^3)/(3*a*b^2*(a + b*x^2)^(3/2)) - (B*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

fricas [A] time = 0.95, size = 245, normalized size = 3.18

$$\left[\frac{3\left(Bab^2x^4+2\,Ba^2bx^2+Ba^3\right)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\left(3\,Ba^2bx+\left(4\,Bab^2-Ab^3\right)x^3\right)\sqrt{bx^2+a}}{6\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)},-\frac{3\left(Bab^2x^4+2\,Ba^2bx^2+Ba^3\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(3\,Ba^2bx+\left(4\,Bab^2-Ab^3\right)x^3\right)\sqrt{bx^2+a}}{3\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*a*b^2*x^4 + 2*B*a^2*b*x^2 + B*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt
(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3), -1/3*(3*(B*a*b^2*x^4 +
2*B*a^2*b*x^2 + B*a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*a
^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x
^2 + a^3*b^3)]
```

giac [A] time = 0.38, size = 69, normalized size = 0.90

$$-\frac{x\left(\frac{3\,Ba}{b^2}+\frac{\left(4\,Bab^2-Ab^3\right)x^2}{ab^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(3*B*a/b^2 + (4*B*a*b^2 - A*b^3)*x^2/(a*b^3))/(b*x^2 + a)^(3/2) - B*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

maple [A] time = 0.01, size = 92, normalized size = 1.19

$$-\frac{B\,x^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b}-\frac{Ax}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b}+\frac{Ax}{3\sqrt{b\,x^2+a}\;ab}-\frac{Bx}{\sqrt{b\,x^2+a}\;b^2}+\frac{B\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B*x^3/b/(b*x^2+a)^(3/2)-B*x/b^2/(b*x^2+a)^(1/2)+B/b^(5/2)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))-1/3*A/b*x/(b*x^2+a)^(3/2)+1/3*A/a/b*x/(b*x^2+a)^(1/2)
```

maxima [A] time = 1.02, size = 103, normalized size = 1.34

$$-\frac{1}{3}\,Bx\left(\frac{3\,x^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^2+a\right)^{\frac{3}{2}}b^2}\right)-\frac{Bx}{3\,\sqrt{bx^2+a}\,b^2}-\frac{Ax}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{Ax}{3\,\sqrt{bx^2+a}\,ab}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3*
B*x/(sqrt(b*x^2 + a)*b^2) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqrt(b
*x^2 + a)*a*b) + B*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^2*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

sympy [B] time = 14.52, size = 352, normalized size = 4.57

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a*
*(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 +
b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt
(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2
*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 +
b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25
/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*
x**2*sqrt(1 + b*x**2/a)))
```

3.572 $\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=44

$$\frac{aB-Ab}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$-\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^2)^(3/2)) - B/(b^2*Sqrt[a + b*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx,x,x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx,x,x^2\right) \\
&= -\frac{Ab-aB}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B}{b^2\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.77

$$\frac{-2aB-Ab-3bBx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(A*b) - 2*a*B - 3*b*B*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.77

$$\frac{-2aB-Ab-3bBx^2}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(A*b) - 2*a*B - 3*b*B*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 1.03, size = 52, normalized size = 1.18

$$-\frac{\left(3\,Bbx^2+2\,Ba+Ab\right)\sqrt{bx^2+a}}{3\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*B*b*x^2 + 2*B*a + A*b)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2
*b^2)
```

giac [**A**] time = 0.34, size = 32, normalized size = 0.73

$$-\frac{3\left(bx^2+a\right)B-Ba+Ab}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*(b*x^2 + a)*B - B*a + A*b)/((b*x^2 + a)^(3/2)*b^2)
```

maple [**A**] time = 0.00, size = 30, normalized size = 0.68

$$-\frac{3Bb\,x^2+Ab+2Ba}{3\left(b\,x^2+a\right)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(3*B*b*x^2+A*b+2*B*a)/(b*x^2+a)^(3/2)/b^2
```

maxima [**A**] time = 1.02, size = 50, normalized size = 1.14

$$-\frac{Bx^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{2\,Ba}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}-\frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

mupad [**B**] time = 0.54, size = 32, normalized size = 0.73

$$-\frac{A\,b-B\,a+3\,B\,\left(b\,x^2+a\right)}{3\,b^2\,\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

```
[Out] -(A*b - B*a + 3*B*(a + b*x^2))/(3*b^2*(a + b*x^2)^(3/2))
```

sympy [A] time = 1.48, size = 143, normalized size = 3.25

$$\begin{cases} -\frac{Ab}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{2Ba}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{3Bbx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for } b\neq 0\\ \frac{\frac{Ax^{2}}{2}+\frac{Bx^{4}}{4}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-A*b/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
- 2*B*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*B*b*
x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0)),
 ((A*x**2/2 + B*x**4/4)/a**(5/2), True))
```

3.573 $\int \frac{A+Bx^2}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{\left(a+bx^2\right)^{5/2}}\,dx&=\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\&=\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x\left(A+Bx^2\right)}{3a\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.79

$$\frac{x\left(3aA+aBx^2+2Abx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*A + 2*A*b*x^2 + a*B*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 37, normalized size = 0.79

$$\frac{3aAx+aBx^3+2Abx^3}{3a^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a*A*x + 2*A*b*x^3 + a*B*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

fricas [A] time = 1.04, size = 54, normalized size = 1.15

$$\frac{\left((Ba+2\,Ab)x^3+3\,Aax\right)\sqrt{bx^2+a}}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((B*a + 2*A*b)*x^3 + 3*A*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

giac [A] time = 0.40, size = 40, normalized size = 0.85

$$\frac{x\left(\frac{3\,A}{a}+\frac{\left(Bab+2\,Ab^2\right)x^2}{a^2b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*A/a + (B*a*b + 2*A*b^2)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

maple **[A]** time = 0.00, size = 34, normalized size = 0.72

$$\frac{\left(2Ab\,x^2+Ba\,x^2+3Aa\right)x}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*A*b*x^2+B*a*x^2+3*A*a)/(b*x^2+a)^(3/2)/a^2
```

maxima **[A]** time = 1.01, size = 68, normalized size = 1.45

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}\,a^2}+\frac{Ax}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{Bx}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{Bx}{3\,\sqrt{bx^2+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*x/((b
*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
```

mupad **[B]** time = 0.55, size = 33, normalized size = 0.70

$$\frac{3\,A\,a\,x+2\,A\,b\,x^3+B\,a\,x^3}{3\,a^2\left(b\,x^2+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2)^(5/2),x)
```

```
[Out] (3*A*a*x + 2*A*b*x^3 + B*a*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

sympy **[B]** time = 11.43, size = 144, normalized size = 3.06

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

3.574 $\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{A}{a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^2)^(3/2)) + A/(a^2*Sqrt[a + b*x^2]) - (A*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^2\right)}{2a}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{Ab-aB}{3ab\left(a+bx^2\right)^{3/2}}+\frac{A}{a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.85

$$\frac{a(Ab-aB)+3Ab\left(a+bx^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^2}{a}+1\right)}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (a*(A*b - a*B) + 3*A*b*(a + b*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x
^2)/a])/(3*a^2*b*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 69, normalized size = 0.96

$$\frac{a^2(-B)+4aAb+3Ab^2x^2}{3a^2b\left(a+bx^2\right)^{3/2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (4*a*A*b - a^2*B + 3*A*b^2*x^2)/(3*a^2*b*(a + b*x^2)^(3/2)) - (A*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 1.13, size = 241, normalized size = 3.35

$$\left[\frac{3\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(3\,Aab^2x^2-Ba^3+4\,Aa^2b\right)\sqrt{bx^2+a}}{6\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)},\ \frac{3\left(Ab^3x^4+2\,Aab^2x^2+Aa^2b\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(3\,Aab^2x^2-Ba^3+4\,Aa^2b\right)\sqrt{bx^2+a}}{3\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*A*a*b^2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt
(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b), 1/3*(3*(A*b^3*x^4 + 2*A
*a*b^2*x^2 + A*a^2*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*A*a*b^
2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 +
a^5*b)]
```

giac [A] time = 0.31, size = 66, normalized size = 0.92

$$\frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}-\frac{Ba^2-3\left(bx^2+a\right)Ab-Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(B*a^2 - 3*(b*x^2 +
 a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a^2*b)
```

maple [A] time = 0.01, size = 75, normalized size = 1.04

$$\frac{A}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a}-\frac{B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}-\frac{A\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{b\,x^{2}+a}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*B/b/(b*x^2+a)^(3/2)+1/3*A/a/(b*x^2+a)^(3/2)+A/a^2/(b*x^2+a)^(1/2)-A/a^
(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.04, size = 63, normalized size = 0.88

$$-\frac{A\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a^{2}}+\frac{A}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{B}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/
((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x^2 + a)^(3/2)*b)
```

mupad [B] time = 1.11, size = 65, normalized size = 0.90

$$\frac{\frac{A}{3\,a}+\frac{A\left(b\,x^{2}+a\right)}{a^{2}}}{\left(b\,x^{2}+a\right)^{3/2}}-\frac{B}{3\,b\left(b\,x^{2}+a\right)^{3/2}}-\frac{A\,\operatorname{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - B/(3*b*(a + b*x^2)^(3/2
)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/2)
```

sympy [A] time = 42.17, size = 66, normalized size = 0.92

$$\frac{A}{a^{2}\sqrt{a+bx^{2}}}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^{2}}}{\sqrt{-a}}\right)}{a^{2}\sqrt{-a}}-\frac{-Ab+Ba}{3ab\left(a+bx^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A/(a**2*sqrt(a + b*x**2)) + A*atan(sqrt(a + b*x**2)/sqrt(-a))/(a**2*sqrt(-a
)) - (-A*b + B*a)/(3*a*b*(a + b*x**2)**(3/2))
```

3.575 $\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{ax\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 192, 191}

$$-\frac{2x(4Ab-aB)}{3a^3\sqrt{a+bx^2}}-\frac{x(4Ab-aB)}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*x)/(3*a^2*(a + b*x^2)^(3/2))
- (2*(4*A*b - a*B)*x)/(3*a^3*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{(2(4Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^2}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{3/2}}-\frac{(4Ab-aB)x}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{2(4Ab-aB)x}{3a^3\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.78

$$\frac{-3a^2\left(A-Bx^2\right)+2abx^2\left(Bx^2-6A\right)-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 - 3*a^2*(A - B*x^2) + 2*a*b*x^2*(-6*A + B*x^2))/(3*a^3*x*(a +
 b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 62, normalized size = 0.81

$$\frac{-3a^2A+3a^2Bx^2-12aAbx^2+2abBx^4-8Ab^2x^4}{3a^3x\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 12*a*A*b*x^2 + 3*a^2*B*x^2 - 8*A*b^2*x^4 + 2*a*b*B*x^4)/(3*a^3*
x*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.95, size = 77, normalized size = 1.00

$$\frac{\left(2\left(Bab-4\,Ab^2\right)x^4-3\,Aa^2+3\left(Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 + 3*(B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x^
2 + a)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

giac [A] time = 0.42, size = 101, normalized size = 1.31

$$\frac{x\left(\frac{\left(2\,Ba^{3}b^{2}-5\,Aa^{2}b^{3}\right)x^{2}}{a^{5}b}+\frac{3\left(Ba^{4}b-2\,Aa^{3}b^{2}\right)}{a^{5}b}\right)}{3\left(bx^{2}+a\right)^{\frac{3}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((2*B*a^3*b^2 - 5*A*a^2*b^3)*x^2/(a^5*b) + 3*(B*a^4*b - 2*A*a^3*b^2)/
(a^5*b))/(b*x^2 + a)^(3/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a^2)
```

maple [A] time = 0.00, size = 59, normalized size = 0.77

$$-\frac{8A\,b^{2}x^{4}-2Bab\,x^{4}+12Aab\,x^{2}-3B\,a^{2}x^{2}+3a^{2}A}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\,a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(8*A*b^2*x^4-2*B*a*b*x^4+12*A*a*b*x^2-3*B*a^2*x^2+3*A*a^2)/(b*x^2+a)^(
3/2)/x/a^3
```

maxima [A] time = 1.13, size = 85, normalized size = 1.10

$$\frac{2\,Bx}{3\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{Bx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{8\,Abx}{3\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{4\,Abx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{A}{\left(bx^{2}+a\right)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - 8/3*A*b*x/(
sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - A/((b*x^2 + a)^(
3/2)*a*x)
```

mupad [B] time = 0.62, size = 68, normalized size = 0.88

$$\frac{A\,a^2-8\,A\,{\left(b\,x^2+a\right)}^2+B\,a^2\,x^2+4\,A\,a\,\left(b\,x^2+a\right)+2\,B\,a\,x^2\,\left(b\,x^2+a\right)}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A*a^2 - 8*A*(a + b*x^2)^2 + B*a^2*x^2 + 4*A*a*(a + b*x^2) + 2*B*a*x^2*(a +
 b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
```

sympy [B] time = 21.74, size = 265, normalized size = 3.44

$$A\left(-\frac{3a^{2}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{12ab^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{8b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}\right)+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(3*a*x/(3*a**(7
/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x**3/(
3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

3.576 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}+\frac{2aB-5Ab}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}-\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2))
- (5*A*b - 2*a*B)/(2*a^3*Sqrt[a + b*x^2]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} + \frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right)}{4a^2}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,d\right.}{4a^3}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} - \frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{2a^3b}$$

$$= -\frac{5Ab-2aB}{6a^2\left(a+bx^2\right)^{3/2}} - \frac{A}{2ax^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}} + \frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [C] time = 0.04, size = 57, normalized size = 0.50

$$\frac{x^2(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^2}{a}+1\right)-3aA}{6a^2x^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a*A + (-5*A*b + 2*a*B)*x^2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^2)
/a])/(6*a^2*x^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 99, normalized size = 0.88

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{-3a^2A+8a^2Bx^2-20aAbx^2+6abBx^4-15Ab^2x^4}{6a^3x^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A - 20*a*A*b*x^2 + 8*a^2*B*x^2 - 15*A*b^2*x^4 + 6*a*b*B*x^4)/(6*a^3
*x^2*(a + b*x^2)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(2*a^(7/2))
```

fricas [A] time = 1.08, size = 349, normalized size = 3.09

$$\left[-\frac{3\left(\left(2\,Bab^{2}-5\,Ab^{3}\right)x^{6}+2\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{4}+\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{2}\right)\sqrt{a}\,\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\left(3\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{4}-3\,Aa^{3}+4\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{12\left(a^{4}b^{2}x^{6}+2\,a^{5}bx^{4}+a^{6}x^{2}\right)},\ \frac{3\left(\left(2\,Bab^{2}-5\,Ab^{3}\right)x^{6}+2\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{4}+\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{2}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(3\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{4}-3\,Aa^{3}+4\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{6\left(a^{4}b^{2}x^{6}+2\,a^{5}bx^{4}+a^{6}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B
*a^3 - 5*A*a^2*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2
*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), 1/6*(3*((2
*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B*a^3 - 5*A*a^
2*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - 5*A*a
*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^
2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

giac [A] time = 0.36, size = 101, normalized size = 0.89

$$\frac{\left(2\,Ba-5\,Ab\right)\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^{3}}+\frac{3\left(bx^{2}+a\right)Ba+Ba^{2}-6\left(bx^{2}+a\right)Ab-Aab}{3\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}-\frac{\sqrt{bx^{2}+a}\,A}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - 5*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 1/3*(
3*(b*x^2 + a)*B*a + B*a^2 - 6*(b*x^2 + a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a
^3) - 1/2*sqrt(b*x^2 + a)*A/(a^3*x^2)
```

maple [A] time = 0.02, size = 140, normalized size = 1.24

$$-\frac{5Ab}{6\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}+\frac{B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a}+\frac{5Ab\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{7}{2}}}-\frac{B\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{a^{\frac{5}{2}}}-\frac{5Ab}{2\sqrt{b\,x^{2}+a}\,a^{3}}+\frac{B}{\sqrt{b\,x^{2}+a}\,a^{2}}-\frac{A}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/2*A/a/x^2/(b*x^2+a)^(3/2)-5/6*A*b/a^2/(b*x^2+a)^(3/2)-5/2*A*b/a^3/(b*x^2
+a)^(1/2)+5/2*A*b/a^(7/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)+1/3*B/a/(b*
x^2+a)^(3/2)+B/a^2/(b*x^2+a)^(1/2)-B/a^(5/2)*ln((2*a+2*(b*x^2+a)^(1/2)*a^(1
/2))/x)
```

maxima [A] time = 0.95, size = 117, normalized size = 1.04

$$-\frac{B \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{5 \, Ab \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2 \, a^{\frac{7}{2}}} + \frac{B}{\sqrt{bx^2 + a}\, a^2} + \frac{B}{3 \left(bx^2 + a\right)^{\frac{3}{2}} a} - \frac{5 \, Ab}{2 \sqrt{bx^2 + a}\, a^3} - \frac{5 \, Ab}{6 \left(bx^2 + a\right)^{\frac{3}{2}} a^2} - \frac{A}{2 \left(bx^2 + a\right)^{\frac{3}{2}} ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/2*A*b*arcsinh(a/(sqrt(a*b)*abs
(x)))/a^(7/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x^2 + a)^(3/2)*a) - 5/2
*A*b/(sqrt(b*x^2 + a)*a^3) - 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - 1/2*A/((b*x^
2 + a)^(3/2)*a*x^2)
```

mupad [B] time = 1.55, size = 126, normalized size = 1.12

$$\frac{\frac{B}{3\,a} + \frac{B\left(b\,x^2+a\right)}{a^2}}{\left(b\,x^2 + a\right)^{3/2}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{10\,A\,b}{3\,a^2\left(b\,x^2 + a\right)^{3/2}} - \frac{A}{2\,a\,x^2\left(b\,x^2 + a\right)^{3/2}} + \frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}} - \frac{5\,A\,b^2\,x^2}{2\,a^3\left(b\,x^2 + a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(5/2) - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a
+ b*x^2)^(3/2)) + (5*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5
*A*b^2*x^2)/(2*a^3*(a + b*x^2)^(3/2))
```

sympy [B] time = 68.64, size = 1608, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
```

```
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*
a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3
*x**6) - 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x
**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt
(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4
+ 6*a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a*
*(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*
b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*
a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*
x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17
/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2
*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a
)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)
*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 1
8*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

3.577 $\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Rubi [**A**] time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$\frac{8bx(2Ab-aB)}{3a^4\sqrt{a+bx^2}}+\frac{4bx(2Ab-aB)}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(3*a*x^3*(a + b*x^2)^(3/2)) + (2*A*b - a*B)/(a^2*x*(a + b*x^2)^(3/2)) +
(4*b*(2*A*b - a*B)*x)/(3*a^3*(a + b*x^2)^(3/2)) + (8*b*(2*A*b - a*B)*x)/(3*
a^4*Sqrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}-\frac{(6Ab-3aB)\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{3a}\\
&= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{(4b(2Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{a^2}\\
&= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{(8b(2Ab-aB))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a^3}\\
&= -\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{8b(2Ab-aB)x}{3a^4\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 79, normalized size = 0.73

$$
\frac{-a^3\left(A+3Bx^2\right)+6a^2bx^2\left(A-2Bx^2\right)-8ab^2x^4\left(Bx^2-3A\right)+16Ab^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (16*A*b^3*x^6 + 6*a^2*b*x^2*(A - 2*B*x^2) - 8*a*b^2*x^4*(-3*A + B*x^2) - a^
3*(A + 3*B*x^2))/(3*a^4*x^3*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 86, normalized size = 0.80

$$
\frac{-a^3A-3a^3Bx^2+6a^2Abx^2-12a^2bBx^4+24aAb^2x^4-8ab^2Bx^6+16Ab^3x^6}{3a^4x^3\left(a+bx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-(a^3*A) + 6*a^2*A*b*x^2 - 3*a^3*B*x^2 + 24*a*A*b^2*x^4 - 12*a^2*b*B*x^4 +
 16*A*b^3*x^6 - 8*a*b^2*B*x^6)/(3*a^4*x^3*(a + b*x^2)^(3/2))
```

fricas [A] time = 0.72, size = 101, normalized size = 0.94

$$-\frac{\left(8\left(Bab^2-2\,Ab^3\right)x^6+12\left(Ba^2b-2\,Aab^2\right)x^4+Aa^3+3\left(Ba^3-2\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*(B*a*b^2 - 2*A*b^3)*x^6 + 12*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 3*
(B*a^3 - 2*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x
^3)
```

giac [B] time = 0.47, size = 224, normalized size = 2.07

$$-\frac{x\left(\frac{\left(5\,Ba^4b^3-8\,Aa^3b^4\right)x^2}{a^7b}+\frac{3\left(2\,Ba^5b^2-3\,Aa^4b^3\right)}{a^7b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aab^{\frac{3}{2}}+3\,Ba^3\sqrt{b}-8\,Aa^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*B*a^4*b^3 - 8*A*a^3*b^4)*x^2/(a^7*b) + 3*(2*B*a^5*b^2 - 3*A*a^4*
b^3)/(a^7*b))/(b*x^2 + a)^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*
a*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*B*a^2*sqrt(b) + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(
3/2) + 3*B*a^3*sqrt(b) - 8*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
 - a)^3*a^3)
```

maple [A] time = 0.01, size = 82, normalized size = 0.76

$$-\frac{-16A\,b^3x^6+8Ba\,b^2x^6-24x^4Aa\,b^2+12B\,a^2b\,x^4-6A\,a^2b\,x^2+3B\,a^3x^2+A\,a^3}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*(-16*A*b^3*x^6+8*B*a*b^2*x^6-24*A*a*b^2*x^4+12*B*a^2*b*x^4-6*A*a^2*b*x
^2+3*B*a^3*x^2+A*a^3)/(b*x^2+a)^(3/2)/x^3/a^4
```

maxima [A] time = 1.13, size = 128, normalized size = 1.19

$$-\frac{8\,Bbx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,Bbx}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a^2}+\frac{16\,Ab^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ab^2x}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a^3}-\frac{B}{\left(bx^2+a\right)^{\frac{3}{2}}ax}+\frac{2\,Ab}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x}-\frac{A}{3\,\left(bx^2+a\right)^{\frac{3}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 16/3
*A*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*A*b^2*x/((b*x^2 + a)^(3/2)*a^3) - B/((
b*x^2 + a)^(3/2)*a*x) + 2*A*b/((b*x^2 + a)^(3/2)*a^2*x) - 1/3*A/((b*x^2 + a
)^(3/2)*a*x^3)
```

mupad [B] time = 0.76, size = 123, normalized size = 1.14

$$-\frac{16\,A\,\left(b\,x^2+a\right)^3+A\,a^3+B\,a^3\,x^2-24\,A\,a\,\left(b\,x^2+a\right)^2+6\,A\,a^2\,\left(b\,x^2+a\right)-8\,B\,a\,x^2\,\left(b\,x^2+a\right)^2+4\,B\,a^2\,x^2\,\left(b\,x^2+a\right)}{\left(b\,x^2+a\right)^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,\left(b\,x^2+a\right)}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x)
```

```
[Out] -(16*A*(a + b*x^2)^3 + A*a^3 + B*a^3*x^2 - 24*A*a*(a + b*x^2)^2 + 6*A*a^2*(
a + b*x^2) - 8*B*a*x^2*(a + b*x^2)^2 + 4*B*a^2*x^2*(a + b*x^2))/((a + b*x^2
)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

sympy [B] time = 30.02, size = 524, normalized size = 4.85

$$A\left(-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\right)+B\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x*
*4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/
(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3
*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**
9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*a*
b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 +
 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x**2)
 + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b*
```

```
*12*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4
*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

3.578 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{5b(7Ab-4aB)}{8a^4\sqrt{a+bx^2}}+\frac{5b(7Ab-4aB)}{24a^3\left(a+bx^2\right)^{3/2}}+\frac{7Ab-4aB}{8a^2x^2\left(a+bx^2\right)^{3/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{5\sqrt{a+bx^2}\,(7Ab-4aB)}{8a^4x^2}-\frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}}-\frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}}-\frac{5b(7Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(4*a*x^4*(a + b*x^2)^(3/2)) - (7*A*b - 4*a*B)/(12*a^2*x^2*(a + b*x^2)^(3
/2)) - (5*(7*A*b - 4*a*B))/(12*a^3*x^2*Sqrt[a + b*x^2]) + (5*(7*A*b - 4*a*B
)*Sqrt[a + b*x^2])/(8*a^4*x^2) - (5*b*(7*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(8*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} + \frac{\left(-\frac{7Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{(5(7Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)}{24a^2}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} - \frac{(5(7Ab-4aB))\,\text{Subst}\Big(\int}{8a^3}$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a+bx^2}}{8a^4x^2} +$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a+bx^2}}{8a^4x^2} +$$

$$= -\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}} - \frac{7Ab-4aB}{12a^2x^2\left(a+bx^2\right)^{3/2}} - \frac{5(7Ab-4aB)}{12a^3x^2\sqrt{a+bx^2}} + \frac{5(7Ab-4aB)\sqrt{a+bx^2}}{8a^4x^2} -$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.41

$$\frac{bx^4(7Ab-4aB)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx^2}{a}+1\right)-3a^2A}{12a^3x^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*A + b*(7*A*b - 4*a*B)*x^4*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x
^2)/a])/(12*a^3*x^4*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 126, normalized size = 0.86

$$\frac{5\left(4abB-7Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}+\frac{-6a^3A-12a^3Bx^2+21a^2Abx^2-80a^2bBx^4+140aAb^2x^4-60ab^2Bx^6+105Ab^3x^6}{24a^4x^4\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-6*a^3*A + 21*a^2*A*b*x^2 - 12*a^3*B*x^2 + 140*a*A*b^2*x^4 - 80*a^2*b*B*x^
4 + 105*A*b^3*x^6 - 60*a*b^2*B*x^6)/(24*a^4*x^4*(a + b*x^2)^(3/2)) + (5*(-7
*A*b^2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(9/2))
```

fricas [A] time = 1.06, size = 407, normalized size = 2.79

$$\left[-\frac{15\left(\left(4\,Bab^3-7\,Ab^4\right)x^8+2\left(4\,Ba^2b^2-7\,Aab^3\right)x^6+\left(4\,Ba^3b-7\,Aa^2b^2\right)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\left(15\left(4\,Ba^2b^2-7\,Aab^3\right)x^6+6\,Aa^4+20\left(4\,Ba^3b-7\,Aa^2b^2\right)x^4+3\left(4\,Ba^4-7\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{48\left(a^5b^2x^8+2\,a^6bx^6+a^7x^4\right)},-\frac{15\left(\left(4\,Bab^3-7\,Ab^4\right)x^8+2\left(4\,Ba^2b^2-7\,Aab^3\right)x^6+\left(4\,Ba^3b-7\,Aa^2b^2\right)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(15\left(4\,Ba^2b^2-7\,Aab^3\right)x^6+6\,Aa^4+20\left(4\,Ba^3b-7\,Aa^2b^2\right)x^4+3\left(4\,Ba^4-7\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{24\left(a^5b^2x^8+2\,a^6bx^6+a^7x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (
4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^4 + 20*(4*B*a^
3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5
*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4), -1/24*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2
*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(-a)*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^
4 + 20*(4*B*a^3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*
x^2 + a))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)]
```

giac [A] time = 0.41, size = 165, normalized size = 1.13

$$-\frac{5\left(4\,Bab-7\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^4}-\frac{6\left(bx^2+a\right)Bab+Ba^2b-9\left(bx^2+a\right)Ab^2-Aab^2}{3\left(bx^2+a\right)^{\frac{3}{2}}a^4}-\frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^2+a}\,Ba^2b-11\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+13\,\sqrt{bx^2+a}\,Aab^2}{8\,a^4b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*(4*B*a*b - 7*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) -
1/3*(6*(b*x^2 + a)*B*a*b + B*a^2*b - 9*(b*x^2 + a)*A*b^2 - A*a*b^2)/((b*x^2
 + a)^(3/2)*a^4) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*sqrt(b*x^2 + a)*B*a^2
*b - 11*(b*x^2 + a)^(3/2)*A*b^2 + 13*sqrt(b*x^2 + a)*A*a*b^2)/(a^4*b^2*x^4)
```

maple [A] time = 0.01, size = 187, normalized size = 1.28

$$\frac{35A\,b^{2}}{24\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{3}}-\frac{5Bb}{6\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}-\frac{35A\,b^{2}\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{9}{2}}}+\frac{5Bb\ln\left(\frac{2a+2\sqrt{b\,x^{2}+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{7}{2}}}+\frac{35A\,b^{2}}{8\sqrt{b\,x^{2}+a}\,a^{4}}-\frac{5Bb}{2\sqrt{b\,x^{2}+a}\,a^{3}}+\frac{7Ab}{8\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}x^{2}}-\frac{B}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}a\,x^{2}}-\frac{A}{4\left(b\,x^{2}+a\right)^{\frac{3}{2}}a\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/4*A/a/x^4/(b*x^2+a)^(3/2)+7/8*A*b/a^2/x^2/(b*x^2+a)^(3/2)+35/24*A*b^2/a^
3/(b*x^2+a)^(3/2)+35/8*A*b^2/a^4/(b*x^2+a)^(1/2)-35/8*A*b^2/a^(9/2)*ln((2*a
+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-1/2*B/a/x^2/(b*x^2+a)^(3/2)-5/6*B*b/a^2/(b*x
^2+a)^(3/2)-5/2*B*b/a^3/(b*x^2+a)^(1/2)+5/2*B*b/a^(7/2)*ln((2*a+2*(b*x^2+a)
^(1/2)*a^(1/2))/x)
```

maxima [A] time = 1.10, size = 164, normalized size = 1.12

$$\frac{5\,Bb\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{35\,Ab^{2}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{9}{2}}}-\frac{5\,Bb}{2\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{5\,Bb}{6\,\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}}+\frac{35\,Ab^{2}}{8\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{35\,Ab^{2}}{24\,\left(bx^{2}+a\right)^{\frac{3}{2}}a^{3}}-\frac{B}{2\,\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{2}}+\frac{7\,Ab}{8\,\left(bx^{2}+a\right)^{\frac{3}{2}}a^{2}x^{2}}-\frac{A}{4\,\left(bx^{2}+a\right)^{\frac{3}{2}}ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 35/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(9/2) - 5/2*B*b/(sqrt(b*x^2 + a)*a^3) - 5/6*B*b/((b*x^2 + a
)^(3/2)*a^2) + 35/8*A*b^2/(sqrt(b*x^2 + a)*a^4) + 35/24*A*b^2/((b*x^2 + a)^
(3/2)*a^3) - 1/2*B/((b*x^2 + a)^(3/2)*a*x^2) + 7/8*A*b/((b*x^2 + a)^(3/2)*a
^2*x^2) - 1/4*A/((b*x^2 + a)^(3/2)*a*x^4)
```

mupad [B] time = 2.02, size = 176, normalized size = 1.21

$$\frac{35\,A\,b^{2}}{6\,a^{3}\,\left(b\,x^{2}+a\right)^{3/2}}-\frac{10\,B\,b}{3\,a^{2}\,\left(b\,x^{2}+a\right)^{3/2}}-\frac{35\,A\,b^{2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8\,a^{9/2}}-\frac{A}{4\,a\,x^{4}\,\left(b\,x^{2}+a\right)^{3/2}}-\frac{B}{2\,a\,x^{2}\,\left(b\,x^{2}+a\right)^{3/2}}+\frac{5\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}+\frac{7\,A\,b}{8\,a^{2}\,x^{2}\,\left(b\,x^{2}+a\right)^{3/2}}+\frac{35\,A\,b^{3}\,x^{2}}{8\,a^{4}\,\left(b\,x^{2}+a\right)^{3/2}}-\frac{5\,B\,b^{2}\,x^{2}}{2\,a^{3}\,\left(b\,x^{2}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x)
```

```
[Out] (35*A*b^2)/(6*a^3*(a + b*x^2)^(3/2)) - (10*B*b)/(3*a^2*(a + b*x^2)^(3/2)) -
 (35*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(9/2)) - A/(4*a*x^4*(a +
b*x^2)^(3/2)) - B/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*B*b*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/(2*a^(7/2)) + (7*A*b)/(8*a^2*x^2*(a + b*x^2)^(3/2)) + (35*A*b
^3*x^2)/(8*a^4*(a + b*x^2)^(3/2)) - (5*B*b^2*x^2)/(2*a^3*(a + b*x^2)^(3/2))
```

sympy [B] time = 139.24, size = 1323, normalized size = 9.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**(89/2)*b**75/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) +
24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) + 21*a**(87/2)*b**76*x**
2/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153
/2)*x**7*sqrt(a/(b*x**2) + 1)) + 140*a**(85/2)*b**77*x**4/(24*a**(93/2)*b**
(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*
x**2) + 1)) + 105*a**(83/2)*b**78*x**6/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a
/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) - 105*a
**42*b**(155/2)*x**5*sqrt(a/(b*x**2) + 1)*asinh(sqrt(a)/(sqrt(b)*x))/(24*a*
*(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7
*sqrt(a/(b*x**2) + 1)) - 105*a**41*b**(157/2)*x**7*sqrt(a/(b*x**2) + 1)*asi
nh(sqrt(a)/(sqrt(b)*x))/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1)
+ 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1))) + B*(-6*a**17*sqrt(1
+ b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x*
*6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x**2/a)/(12*a**(3
9/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**
3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b
*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**16*b*x**2*
log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a
**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*b**2*x**4*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x**2/a)/(12*a**(39/2)
*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x*
*8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 3
6*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 30*
a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4
 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**14*b**3*x**6*lo
g(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x*
*6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sqrt(1 + b*x**2/a) +
1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a
**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(12*a**(39/2)*x**2 +
 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 3
0*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8))
```

3.579 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4\left(a+bx^2\right)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$-\frac{16b^2x(8Ab-5aB)}{15a^5\sqrt{a+bx^2}}-\frac{8b^2x(8Ab-5aB)}{15a^4\left(a+bx^2\right)^{3/2}}-\frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^2)^(3/2)) + (8*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)^(3
/2)) - (2*b*(8*A*b - 5*a*B))/(5*a^3*x*(a + b*x^2)^(3/2)) - (8*b^2*(8*A*b -
5*a*B)*x)/(15*a^4*(a + b*x^2)^(3/2)) - (16*b^2*(8*A*b - 5*a*B)*x)/(15*a^5*S
qrt[a + b*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} - \frac{(8Ab-5aB)\int \frac{1}{x^4\left(a+bx^2\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} + \frac{(2b(8Ab-5aB))\int \frac{1}{x^2\left(a+bx^2\right)^{5/2}}\,dx}{5a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{\left(8b^2(8Ab-5aB)\right)\int \frac{}{(a+}}{5a^3}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}} - \frac{(16}{}$$

$$= -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}} + \frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}} - \frac{16b}{1}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.49

$$\frac{ax^2\left(a^3-6a^2bx^2-24ab^2x^4-16b^3x^6\right)(8Ab-5aB)-3a^5A}{15a^6x^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^5*A + a*(8*A*b - 5*a*B)*x^2*(a^3 - 6*a^2*b*x^2 - 24*a*b^2*x^4 - 16*b^
3*x^6))/(15*a^6*x^5*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.21, size = 110, normalized size = 0.75

$$\frac{-3a^4A-5a^4Bx^2+8a^3Abx^2+30a^3bBx^4-48a^2Ab^2x^4+120a^2b^2Bx^6-192aAb^3x^6+80ab^3Bx^8-128Ab^4x^8}{15a^5x^5\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^4*A + 8*a^3*A*b*x^2 - 5*a^4*B*x^2 - 48*a^2*A*b^2*x^4 + 30*a^3*b*B*x^4
 - 192*a*A*b^3*x^6 + 120*a^2*b^2*B*x^6 - 128*A*b^4*x^8 + 80*a*b^3*B*x^8)/(1
5*a^5*x^5*(a + b*x^2)^(3/2))
```

fricas [A] time = 1.23, size = 129, normalized size = 0.88

$$\frac{\left(16\left(5\,Bab^3-8\,Ab^4\right)x^8+24\left(5\,Ba^2b^2-8\,Aab^3\right)x^6-3\,Aa^4+6\left(5\,Ba^3b-8\,Aa^2b^2\right)x^4-\left(5\,Ba^4-8\,Aa^3b\right)x^2\right)\sqrt{bx^2+a}}{15\left(a^5b^2x^9+2\,a^6bx^7+a^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(16*(5*B*a*b^3 - 8*A*b^4)*x^8 + 24*(5*B*a^2*b^2 - 8*A*a*b^3)*x^6 - 3*A
*a^4 + 6*(5*B*a^3*b - 8*A*a^2*b^2)*x^4 - (5*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*
x^2 + a)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)
```

giac [B] time = 0.58, size = 336, normalized size = 2.30

$$\frac{x\left(\frac{\left(8\,Ba^5b^4-11\,Aa^4b^5\right)x^2}{a^9b}+\frac{3\left(3\,Ba^6b^3-4\,Aa^5b^4\right)}{a^9b}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}-45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+240\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Aab^{\frac{5}{2}}+250\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^3b^{\frac{3}{2}}-490\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{5}{2}}-170\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^4b^{\frac{3}{2}}+320\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^3b^{\frac{5}{2}}+40\,Ba^5b^{\frac{3}{2}}-73\,Aa^4b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((8*B*a^5*b^4 - 11*A*a^4*b^5)*x^2/(a^9*b) + 3*(3*B*a^6*b^3 - 4*A*a^5*
b^4)/(a^9*b))/(b*x^2 + a)^(3/2) - 2/15*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
B*a*b^(3/2) - 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 150*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a*b^(5/2) + 250*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 490*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 170*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*B*a^4*b^(3/2) + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2) + 40*B
*a^5*b^(3/2) - 73*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a
^4)
```

maple [A] time = 0.01, size = 107, normalized size = 0.73

$$-\frac{128A\,b^4x^8-80Ba\,b^3x^8+192Aa\,b^3x^6-120B\,a^2b^2x^6+48A\,a^2b^2x^4-30B\,a^3b\,x^4-8A\,a^3b\,x^2+5B\,a^4x^2+3A\,a^4}{15\left(b\,x^2+a\right)^{\frac{3}{2}}a^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x)
```

```
[Out] -1/15*(128*A*b^4*x^8-80*B*a*b^3*x^8+192*A*a*b^3*x^6-120*B*a^2*b^2*x^6+48*A*
a^2*b^2*x^4-30*B*a^3*b*x^4-8*A*a^3*b*x^2+5*B*a^4*x^2+3*A*a^4)/(b*x^2+a)^(3/
2)/x^5/a^5
```

maxima [A] time = 1.18, size = 172, normalized size = 1.18

$$\frac{16\,Bb^2x}{3\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Bb^2x}{3\,{\left(bx^2+a\right)}^{\frac{3}{2}}a^3}-\frac{128\,Ab^3x}{15\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Ab^3x}{15\,{\left(bx^2+a\right)}^{\frac{3}{2}}a^4}+\frac{2\,Bb}{{\left(bx^2+a\right)}^{\frac{3}{2}}a^2x}-\frac{16\,Ab^2}{5\,{\left(bx^2+a\right)}^{\frac{3}{2}}a^3x}-\frac{B}{3\,{\left(bx^2+a\right)}^{\frac{3}{2}}ax^3}+\frac{8\,Ab}{15\,{\left(bx^2+a\right)}^{\frac{3}{2}}a^2x^3}-\frac{A}{5\,{\left(bx^2+a\right)}^{\frac{3}{2}}ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/3*B*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*B*b^2*x/((b*x^2 + a)^(3/2)*a^3) -
128/15*A*b^3*x/(sqrt(b*x^2 + a)*a^5) - 64/15*A*b^3*x/((b*x^2 + a)^(3/2)*a^4
) + 2*B*b/((b*x^2 + a)^(3/2)*a^2*x) - 16/5*A*b^2/((b*x^2 + a)^(3/2)*a^3*x)
- 1/3*B/((b*x^2 + a)^(3/2)*a*x^3) + 8/15*A*b/((b*x^2 + a)^(3/2)*a^2*x^3) -
1/5*A/((b*x^2 + a)^(3/2)*a*x^5)
```

mupad [B] time = 1.00, size = 231, normalized size = 1.58

$$\frac{\frac{a\left(\frac{b^2\,\left(73\,A\,b-40\,B\,a\right)}{18\,a^4}+\frac{b^2\,\left(86\,A\,b-35\,B\,a\right)}{30\,a^4}+\frac{a\,\left(\frac{28\,A\,b^4-10\,B\,a\,b^3}{45\,a^5}-\frac{b^3\,\left(86\,A\,b-35\,B\,a\right)}{18\,a^5}\right)}{b}\right)}{b}-\frac{b\,\left(73\,A\,b-40\,B\,a\right)}{30\,a^3}}{x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{x^2\,\left(\frac{28\,A\,b^3-10\,B\,a\,b^2}{15\,a^5}-\frac{2\,b^2\,\left(26\,A\,b-15\,B\,a\right)}{5\,a^5}\right)-\frac{b\,\left(26\,A\,b-15\,B\,a\right)}{5\,a^4}}{x\,\sqrt{b\,x^2+a}}-\frac{\sqrt{b\,x^2+a}\,\left(5\,B\,a^3-14\,A\,a^2\,b\right)}{15\,a^6\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x)
```

```
[Out] ((a*((b^2*(73*A*b - 40*B*a))/(18*a^4) + (b^2*(86*A*b - 35*B*a))/(30*a^4) +
(a*((28*A*b^4 - 10*B*a*b^3)/(45*a^5) - (b^3*(86*A*b - 35*B*a))/(18*a^5)))/b
))/b - (b*(73*A*b - 40*B*a))/(30*a^3))/(x*(a + b*x^2)^(3/2)) + (x^2*((28*A*
b^3 - 10*B*a*b^2)/(15*a^5) - (2*b^2*(26*A*b - 15*B*a))/(5*a^5)) - (b*(26*A*
b - 15*B*a))/(5*a^4))/(x*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(5*B*a^3 -
 14*A*a^2*b))/(15*a^6*x^3) - (A*(a + b*x^2)^(1/2))/(5*a^3*x^5)
```

sympy [B] time = 60.74, size = 944, normalized size = 6.47

$$A\left(-\frac{3a^6b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}+\frac{2a^5b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}-\frac{35a^4b^{\frac{37}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}-\frac{280a^3b^{\frac{39}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}-\frac{560a^2b^{\frac{41}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}-\frac{448ab^{\frac{43}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}-\frac{128b^{\frac{45}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}\right)+B\left(-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}+\frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**6/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**6*b**(33/2)*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b**
17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12) +
 2*a**5*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b
**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12)
 - 35*a**4*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**
8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**
12) - 280*a**3*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60
*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20
*x**12) - 560*a**2*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4
+ 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b
**20*x**12) - 448*a*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**
4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5
*b**20*x**12) - 128*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**
4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5
*b**20*x**12)) + B*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2
+ 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(2
1/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a*
*5*b**11*x**6 + 3*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2)
 + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b*
*12*x**8) + 40*a*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*
a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**
8*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11
*x**6 + 3*a**4*b**12*x**8))
```

3.580 $\int x^5 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=157

$$\frac{\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-}{5d^5}$$

Rubi [**A**] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 88}

$$\frac{\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{2b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-ad)(2bc-ad)}{5d^5}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (2*c*(b*c - a*d)*(2*b*c - a
*d)*(c + d*x^2)^(5/2))/(5*d^5) + ((6*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(c + d*
x^2)^(7/2))/(7*d^5) - (2*b*(2*b*c - a*d)*(c + d*x^2)^(9/2))/(9*d^5) + (b^2*
(c + d*x^2)^(11/2))/(11*d^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{c^2(bc-ad)^2\sqrt{c+dx}}{d^4}+\frac{2c(bc-ad)(-2bc+ad)(c+dx)^{3/2}}{d^4}+\frac{\left(6b^2c^2\right.}{}\right.\right.\\ &= \frac{c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^5}-\frac{2c(bc-ad)(2bc-ad)\left(c+dx^2\right)^{5/2}}{5d^5}+\frac{\left(6b^2c^2-6abcd+\right.}{7}\end{aligned}$$

Mathematica [A] time = 0.13, size = 132, normalized size = 0.84

$$\frac{\left(c+dx^2\right)^{3/2}\left(33a^2d^2\left(8c^2-12cdx^2+15d^2x^4\right)+22abd\left(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6\right)+b^2\left(128c^4-192c^3dx^2+240c^2d^2x^4-280cd^3x^6+315d^4x^8\right)\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(33*a^2*d^2*(8*c^2 - 12*c*d*x^2 + 15*d^2*x^4) + 22*a*b*d
*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6) + b^2*(128*c^4 - 192*
c^3*d*x^2 + 240*c^2*d^2*x^4 - 280*c*d^3*x^6 + 315*d^4*x^8)))/(3465*d^5)
```

IntegrateAlgebraic [A] time = 0.07, size = 152, normalized size = 0.97

$$\frac{\left(c+dx^2\right)^{3/2}\left(264a^2c^2d^2-396a^2cd^3x^2+495a^2d^4x^4-352abc^3d+528abc^2d^2x^2-660abcd^3x^4+770abd^4x^6+128b^2c^4-192b^2c^3dx^2+240b^2c^2d^2x^4-280b^2cd^3x^6+315b^2d^4x^8\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(128*b^2*c^4 - 352*a*b*c^3*d + 264*a^2*c^2*d^2 - 192*b^2
*c^3*d*x^2 + 528*a*b*c^2*d^2*x^2 - 396*a^2*c*d^3*x^2 + 240*b^2*c^2*d^2*x^4
- 660*a*b*c*d^3*x^4 + 495*a^2*d^4*x^4 - 280*b^2*c*d^3*x^6 + 770*a*b*d^4*x^6
 + 315*b^2*d^4*x^8))/(3465*d^5)
```

fricas [A] time = 1.53, size = 179, normalized size = 1.14

$$\frac{\left(315\,b^2d^5x^{10}+35\left(b^2cd^4+22\,abd^5\right)x^8+128\,b^2c^5-352\,abc^4d+264\,a^2c^3d^2-5\left(8\,b^2c^2d^3-22\,abcd^4-99\,a^2d^5\right)x^6+3\left(16\,b^2c^3d^2-44\,abc^2d^3+33\,a^2cd^4\right)x^4-4\left(16\,b^2c^4d-44\,abc^3d^2+33\,a^2c^2d^3\right)x^2\right)\sqrt{dx^2+c}}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 35*(b^2*c*d^4 + 22*a*b*d^5)*x^8 + 128*b^2*c^5 -
352*a*b*c^4*d + 264*a^2*c^3*d^2 - 5*(8*b^2*c^2*d^3 - 22*a*b*c*d^4 - 99*a^2*
```

```
d^5)*x^6 + 3*(16*b^2*c^3*d^2 - 44*a*b*c^2*d^3 + 33*a^2*c*d^4)*x^4 - 4*(16*b
^2*c^4*d - 44*a*b*c^3*d^2 + 33*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^5
```

giac [A] time = 0.29, size = 204, normalized size = 1.30

$$\frac{315 \left(dx^2+c\right)^{\frac{11}{2}} b^2 - 1540 \left(dx^2+c\right)^{\frac{9}{2}} b^2 c + 2970 \left(dx^2+c\right)^{\frac{7}{2}} b^2 c^2 - 2772 \left(dx^2+c\right)^{\frac{5}{2}} b^2 c^3 + 1155 \left(dx^2+c\right)^{\frac{3}{2}} b^2 c^4 + 770 \left(dx^2+c\right)^{\frac{9}{2}} abd - 2970 \left(dx^2+c\right)^{\frac{7}{2}} abcd + 4158 \left(dx^2+c\right)^{\frac{5}{2}} abc^2 d - 2310 \left(dx^2+c\right)^{\frac{3}{2}} abc^3 d + 495 \left(dx^2+c\right)^{\frac{7}{2}} a^2 d^2 - 1386 \left(dx^2+c\right)^{\frac{5}{2}} a^2 c d^2 + 1155 \left(dx^2+c\right)^{\frac{3}{2}} a^2 c^2 d^2}{3465 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(d*x^2 + c)^(11/2)*b^2 - 1540*(d*x^2 + c)^(9/2)*b^2*c + 2970*(d
*x^2 + c)^(7/2)*b^2*c^2 - 2772*(d*x^2 + c)^(5/2)*b^2*c^3 + 1155*(d*x^2 + c)
^(3/2)*b^2*c^4 + 770*(d*x^2 + c)^(9/2)*a*b*d - 2970*(d*x^2 + c)^(7/2)*a*b*c
*d + 4158*(d*x^2 + c)^(5/2)*a*b*c^2*d - 2310*(d*x^2 + c)^(3/2)*a*b*c^3*d +
495*(d*x^2 + c)^(7/2)*a^2*d^2 - 1386*(d*x^2 + c)^(5/2)*a^2*c*d^2 + 1155*(d*
x^2 + c)^(3/2)*a^2*c^2*d^2)/d^5
```

maple [A] time = 0.01, size = 149, normalized size = 0.95

$$\frac{\left(d x^2+c\right)^{\frac{3}{2}} \left(315 b^2 x^8 d^4 + 770 ab\, d^4 x^6 - 280 b^2 c\, d^3 x^6 + 495 a^2 d^4 x^4 - 660 abc\, d^3 x^4 + 240 b^2 c^2 d^2 x^4 - 396 a^2 c\, d^3 x^2 + 528 ab\, c^2 d^2 x^2 - 192 b^2 c^3 d\, x^2 + 264 a^2 c^2 d^2 - 352 ab\, c^3 d + 128 b^2 c^4\right)}{3465 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3465*(d*x^2+c)^(3/2)*(315*b^2*d^4*x^8+770*a*b*d^4*x^6-280*b^2*c*d^3*x^6+4
95*a^2*d^4*x^4-660*a*b*c*d^3*x^4+240*b^2*c^2*d^2*x^4-396*a^2*c*d^3*x^2+528*
a*b*c^2*d^2*x^2-192*b^2*c^3*d*x^2+264*a^2*c^2*d^2-352*a*b*c^3*d+128*b^2*c^4
)/d^5
```

maxima [A] time = 1.08, size = 249, normalized size = 1.59

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}} b^2 x^8}{11\, d} - \frac{8 \left(dx^2+c\right)^{\frac{3}{2}} b^2 c x^6}{99\, d^2} + \frac{2 \left(dx^2+c\right)^{\frac{3}{2}} abx^6}{9\, d} + \frac{16 \left(dx^2+c\right)^{\frac{3}{2}} b^2 c^2 x^4}{231\, d^3} - \frac{4 \left(dx^2+c\right)^{\frac{3}{2}} abcx^4}{21\, d^2} + \frac{\left(dx^2+c\right)^{\frac{3}{2}} a^2 x^4}{7\, d} - \frac{64 \left(dx^2+c\right)^{\frac{3}{2}} b^2 c^3 x^2}{1155\, d^4} + \frac{16 \left(dx^2+c\right)^{\frac{3}{2}} abc^2 x^2}{105\, d^3} - \frac{4 \left(dx^2+c\right)^{\frac{3}{2}} a^2 c x^2}{35\, d^2} + \frac{128 \left(dx^2+c\right)^{\frac{3}{2}} b^2 c^4}{3465\, d^5} - \frac{32 \left(dx^2+c\right)^{\frac{3}{2}} abc^3}{315\, d^4} + \frac{8 \left(dx^2+c\right)^{\frac{3}{2}} a^2 c^2}{105\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*(d*x^2 + c)^(3/2)*b^2*x^8/d - 8/99*(d*x^2 + c)^(3/2)*b^2*c*x^6/d^2 + 2
/9*(d*x^2 + c)^(3/2)*a*b*x^6/d + 16/231*(d*x^2 + c)^(3/2)*b^2*c^2*x^4/d^3 -
 4/21*(d*x^2 + c)^(3/2)*a*b*c*x^4/d^2 + 1/7*(d*x^2 + c)^(3/2)*a^2*x^4/d - 6
4/1155*(d*x^2 + c)^(3/2)*b^2*c^3*x^2/d^4 + 16/105*(d*x^2 + c)^(3/2)*a*b*c^2
*x^2/d^3 - 4/35*(d*x^2 + c)^(3/2)*a^2*c*x^2/d^2 + 128/3465*(d*x^2 + c)^(3/2
)*b^2*c^4/d^5 - 32/315*(d*x^2 + c)^(3/2)*a*b*c^3/d^4 + 8/105*(d*x^2 + c)^(3
/2)*a^2*c^2/d^3
```

mupad [B] time = 0.76, size = 171, normalized size = 1.09

$$\sqrt{d\,x^2+c}\,\left(\frac{264\,a^2\,c^3\,d^2-352\,a\,b\,c^4\,d+128\,b^2\,c^5}{3465\,d^5}+\frac{b^2\,x^{10}}{11}+\frac{x^6\,\left(495\,a^2\,d^5+110\,a\,b\,c\,d^4-40\,b^2\,c^2\,d^3\right)}{3465\,d^5}+\frac{b\,x^8\,(22\,a\,d+b\,c)}{99\,d}+\frac{c\,x^4\,\left(33\,a^2\,d^2-44\,a\,b\,c\,d+16\,b^2\,c^2\right)}{1155\,d^3}-\frac{4\,c^2\,x^2\,\left(33\,a^2\,d^2-44\,a\,b\,c\,d+16\,b^2\,c^2\right)}{3465\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((128*b^2*c^5 + 264*a^2*c^3*d^2 - 352*a*b*c^4*d)/(3465*d^
5) + (b^2*x^10)/11 + (x^6*(495*a^2*d^5 - 40*b^2*c^2*d^3 + 110*a*b*c*d^4))/(
3465*d^5) + (b*x^8*(22*a*d + b*c))/(99*d) + (c*x^4*(33*a^2*d^2 + 16*b^2*c^2
 - 44*a*b*c*d))/(1155*d^3) - (4*c^2*x^2*(33*a^2*d^2 + 16*b^2*c^2 - 44*a*b*c
*d))/(3465*d^4))
```

sympy [A] time = 5.96, size = 389, normalized size = 2.48

$$\begin{cases} \frac{8a^2c^3\sqrt{c+dx^2}}{105d^3}-\frac{4a^2c^2x^2\sqrt{c+dx^2}}{105d^2}+\frac{a^2cx^4\sqrt{c+dx^2}}{35d}+\frac{a^2x^6\sqrt{c+dx^2}}{7}-\frac{32abc^4\sqrt{c+dx^2}}{315d^4}+\frac{16abc^3x^2\sqrt{c+dx^2}}{315d^3}-\frac{4abc^2x^4\sqrt{c+dx^2}}{105d^2}+\frac{2abcx^6\sqrt{c+dx^2}}{63d}+\frac{2abx^8\sqrt{c+dx^2}}{9}+\frac{128b^2c^5\sqrt{c+dx^2}}{3465d^5}-\frac{64b^2c^4x^2\sqrt{c+dx^2}}{3465d^4}+\frac{16b^2c^3x^4\sqrt{c+dx^2}}{1155d^3}-\frac{8b^2c^2x^6\sqrt{c+dx^2}}{693d^2}+\frac{b^2cx^8\sqrt{c+dx^2}}{99d}+\frac{b^2x^{10}\sqrt{c+dx^2}}{11} & \text{for}\: d \neq 0 \\ \sqrt{c}\left(\frac{a^2x^6}{6}+\frac{abx^8}{4}+\frac{b^2x^{10}}{10}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((8*a**2*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*a**2*c**2*x**2*sqrt(
c + d*x**2)/(105*d**2) + a**2*c*x**4*sqrt(c + d*x**2)/(35*d) + a**2*x**6*sq
rt(c + d*x**2)/7 - 32*a*b*c**4*sqrt(c + d*x**2)/(315*d**4) + 16*a*b*c**3*x*
*2*sqrt(c + d*x**2)/(315*d**3) - 4*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d**2
) + 2*a*b*c*x**6*sqrt(c + d*x**2)/(63*d) + 2*a*b*x**8*sqrt(c + d*x**2)/9 +
128*b**2*c**5*sqrt(c + d*x**2)/(3465*d**5) - 64*b**2*c**4*x**2*sqrt(c + d*x
**2)/(3465*d**4) + 16*b**2*c**3*x**4*sqrt(c + d*x**2)/(1155*d**3) - 8*b**2*
c**2*x**6*sqrt(c + d*x**2)/(693*d**2) + b**2*c*x**8*sqrt(c + d*x**2)/(99*d)
 + b**2*x**10*sqrt(c + d*x**2)/11, Ne(d, 0)), (sqrt(c)*(a**2*x**6/6 + a*b*x
**8/4 + b**2*x**10/10), True))
```

3.581 $\int x^3 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\, dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{7/2}(3bc-2ad)}{7d^4}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)(3bc-ad)}{5d^4}-\frac{c\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^4}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(5/2))/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^4) +
 (b^2*(c + d*x^2)^(9/2))/(9*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int x(a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\Bigg(\int\Bigg(-\frac{c(bc-ad)^2\sqrt{c+dx}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{3/2}}{d^3}-\frac{b(3bc-2a}{}\\ &= -\frac{c(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{5/2}}{5d^4}-\frac{b(3bc-2ad)(c+a}{7d^4}\end{aligned}$$

Mathematica [A] time = 0.11, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{3/2}\left(21a^2d^2\left(3dx^2-2c\right)+6abd\left(8c^2-12cdx^2+15d^2x^4\right)+b^2\left(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6\right)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(21*a^2*d^2*(-2*c + 3*d*x^2) + 6*a*b*d*(8*c^2 - 12*c*d*x
^2 + 15*d^2*x^4) + b^2*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6)
))/(315*d^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 111, normalized size = 0.97

$$\frac{\left(c+dx^2\right)^{3/2}\left(-42a^2cd^2+63a^2d^3x^2+48abc^2d-72abcd^2x^2+90abd^3x^4-16b^2c^3+24b^2c^2dx^2-30b^2cd^2x^4+35b^2d^3x^6\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(-16*b^2*c^3 + 48*a*b*c^2*d - 42*a^2*c*d^2 + 24*b^2*c^2*
d*x^2 - 72*a*b*c*d^2*x^2 + 63*a^2*d^3*x^2 - 30*b^2*c*d^2*x^4 + 90*a*b*d^3*x
^4 + 35*b^2*d^3*x^6))/(315*d^4)
```

fricas [A] time = 1.14, size = 140, normalized size = 1.23

$$\frac{\left(35\,b^2d^4x^8+5\left(b^2cd^3+18\,abd^4\right)x^6-16\,b^2c^4+48\,abc^3d-42\,a^2c^2d^2-3\left(2\,b^2c^2d^2-6\,abcd^3-21\,a^2d^4\right)x^4+\left(8\,b^2c^3d-24\,abc^2d^2+21\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 5*(b^2*c*d^3 + 18*a*b*d^4)*x^6 - 16*b^2*c^4 + 48*a*
b*c^3*d - 42*a^2*c^2*d^2 - 3*(2*b^2*c^2*d^2 - 6*a*b*c*d^3 - 21*a^2*d^4)*x^4
 + (8*b^2*c^3*d - 24*a*b*c^2*d^2 + 21*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

giac [A] time = 0.35, size = 150, normalized size = 1.32

$$\frac{35\left(dx^2+c\right)^{\frac{9}{2}}b^2-135\left(dx^2+c\right)^{\frac{7}{2}}b^2c+189\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2-105\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3+90\left(dx^2+c\right)^{\frac{7}{2}}abd-252\left(dx^2+c\right)^{\frac{5}{2}}abcd+210\left(dx^2+c\right)^{\frac{3}{2}}abc^2d+63\left(dx^2+c\right)^{\frac{5}{2}}a^2d^2-105\left(dx^2+c\right)^{\frac{3}{2}}a^2cd^2}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(d*x^2 + c)^(9/2)*b^2 - 135*(d*x^2 + c)^(7/2)*b^2*c + 189*(d*x^2
+ c)^(5/2)*b^2*c^2 - 105*(d*x^2 + c)^(3/2)*b^2*c^3 + 90*(d*x^2 + c)^(7/2)*a
*b*d - 252*(d*x^2 + c)^(5/2)*a*b*c*d + 210*(d*x^2 + c)^(3/2)*a*b*c^2*d + 63
*(d*x^2 + c)^(5/2)*a^2*d^2 - 105*(d*x^2 + c)^(3/2)*a^2*c*d^2)/d^4
```

maple [A] time = 0.01, size = 108, normalized size = 0.95

$$-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(-35b^2x^6d^3-90ab\,d^3x^4+30b^2c\,d^2x^4-63a^2d^3x^2+72abc\,d^2x^2-24b^2c^2d\,x^2+42a^2c\,d^2-48ab\,c^2d+16b^2c^3\right)}{315d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] -1/315*(d*x^2+c)^(3/2)*(-35*b^2*d^3*x^6-90*a*b*d^3*x^4+30*b^2*c*d^2*x^4-63*
a^2*d^3*x^2+72*a*b*c*d^2*x^2-24*b^2*c^2*d*x^2+42*a^2*c*d^2-48*a*b*c^2*d+16*
b^2*c^3)/d^4
```

maxima [A] time = 1.03, size = 181, normalized size = 1.59

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2x^6}{9\,d}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2cx^4}{21\,d^2}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}abx^4}{7\,d}+\frac{8\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2x^2}{105\,d^3}-\frac{8\left(dx^2+c\right)^{\frac{3}{2}}abcx^2}{35\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2x^2}{5\,d}-\frac{16\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3}{315\,d^4}+\frac{16\left(dx^2+c\right)^{\frac{3}{2}}abc^2}{105\,d^3}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}a^2c}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d*x^2 + c)^(3/2)*b^2*x^6/d - 2/21*(d*x^2 + c)^(3/2)*b^2*c*x^4/d^2 + 2/
7*(d*x^2 + c)^(3/2)*a*b*x^4/d + 8/105*(d*x^2 + c)^(3/2)*b^2*c^2*x^2/d^3 - 8
/35*(d*x^2 + c)^(3/2)*a*b*c*x^2/d^2 + 1/5*(d*x^2 + c)^(3/2)*a^2*x^2/d - 16/
315*(d*x^2 + c)^(3/2)*b^2*c^3/d^4 + 16/105*(d*x^2 + c)^(3/2)*a*b*c^2/d^3 -
2/15*(d*x^2 + c)^(3/2)*a^2*c/d^2
```

mupad [B] time = 0.65, size = 137, normalized size = 1.20

$$\sqrt{d\,x^2+c}\,\left(\frac{b^2\,x^8}{9}-\frac{42\,a^2\,c^2\,d^2-48\,a\,b\,c^3\,d+16\,b^2\,c^4}{315\,d^4}+\frac{x^4\,\left(63\,a^2\,d^4+18\,a\,b\,c\,d^3-6\,b^2\,c^2\,d^2\right)}{315\,d^4}+\frac{b\,x^6\,\left(18\,a\,d+b\,c\right)}{63\,d}+\frac{c\,x^2\,\left(21\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2\right)}{315\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((b^2*x^8)/9 - (16*b^2*c^4 + 42*a^2*c^2*d^2 - 48*a*b*c^3*
d)/(315*d^4) + (x^4*(63*a^2*d^4 - 6*b^2*c^2*d^2 + 18*a*b*c*d^3))/(315*d^4)
+ (b*x^6*(18*a*d + b*c))/(63*d) + (c*x^2*(21*a^2*d^2 + 8*b^2*c^2 - 24*a*b*c
*d))/(315*d^3))
```

sympy [A] time = 3.24, size = 308, normalized size = 2.70

$$\begin{cases} - \frac{2a^2c^2\sqrt{c+dx^2}}{15d^2} + \frac{a^2cx^2\sqrt{c+dx^2}}{15d} + \frac{a^2x^4\sqrt{c+dx^2}}{5} + \frac{16abc^3\sqrt{c+dx^2}}{105d^3} - \frac{8abc^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{2abcx^4\sqrt{c+dx^2}}{35d} + \frac{2abx^6\sqrt{c+dx^2}}{7} - \frac{16b^2c^4\sqrt{c+dx^2}}{315d^4} + \frac{8b^2c^3x^2\sqrt{c+dx^2}}{315d^3} - \frac{2b^2c^2x^4\sqrt{c+dx^2}}{105d^2} + \frac{b^2cx^6\sqrt{c+dx^2}}{63d} + \frac{b^2x^8\sqrt{c+dx^2}}{9} & \text{for}\: d \neq 0 \\ \sqrt{c}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c**2*sqrt(c + d*x**2)/(15*d**2) + a**2*c*x**2*sqrt(c + d
*x**2)/(15*d) + a**2*x**4*sqrt(c + d*x**2)/5 + 16*a*b*c**3*sqrt(c + d*x**2)
/(105*d**3) - 8*a*b*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + 2*a*b*c*x**4*sq
rt(c + d*x**2)/(35*d) + 2*a*b*x**6*sqrt(c + d*x**2)/7 - 16*b**2*c**4*sqrt(c
 + d*x**2)/(315*d**4) + 8*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**3) - 2*b*
*2*c**2*x**4*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**6*sqrt(c + d*x**2)/(63
*d) + b**2*x**8*sqrt(c + d*x**2)/9, Ne(d, 0)), (sqrt(c)*(a**2*x**4/4 + a*b*
x**6/3 + b**2*x**8/8), True))
```

3.582 $\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{5/2}(bc-ad)}{5d^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(5
/2))/(5*d^3) + (b^2*(c + d*x^2)^(7/2))/(7*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^2\sqrt{c+dx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right)dx,x,\right.\\
&= \frac{(bc-ad)^2\left(c+dx^2\right)^{3/2}}{3d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{5/2}}{5d^3}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^3}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 67, normalized size = 0.87

$$
\frac{\left(c+dx^2\right)^{3/2}\left(35a^2d^2+14abd\left(3dx^2-2c\right)+b^2\left(8c^2-12cdx^2+15d^2x^4\right)\right)}{105d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x^2) + b^2*(8*c^2 - 1
2*c*d*x^2 + 15*d^2*x^4)))/(105*d^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 0.94

$$
\frac{\left(c+dx^2\right)^{3/2}\left(35a^2d^2-28abcd+42abd^2x^2+8b^2c^2-12b^2cdx^2+15b^2d^2x^4\right)}{105d^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((c + d*x^2)^(3/2)*(8*b^2*c^2 - 28*a*b*c*d + 35*a^2*d^2 - 12*b^2*c*d*x^2 +
42*a*b*d^2*x^2 + 15*b^2*d^2*x^4))/(105*d^3)
```

fricas [A] time = 0.96, size = 103, normalized size = 1.34

$$
\frac{\left(15\,b^2d^3x^6+8\,b^2c^3-28\,abc^2d+35\,a^2cd^2+3\left(b^2cd^2+14\,abd^3\right)x^4-\left(4\,b^2c^2d-14\,abcd^2-35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{105\,d^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^4 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x^2)*sqrt
(d*x^2 + c)/d^3
```

giac [A] time = 0.29, size = 98, normalized size = 1.27

$$\frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2-42\left(dx^2+c\right)^{\frac{5}{2}}b^2c+35\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2+42\left(dx^2+c\right)^{\frac{5}{2}}abd-70\left(dx^2+c\right)^{\frac{3}{2}}abcd+35\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*(d*x^2 + c)^(7/2)*b^2 - 42*(d*x^2 + c)^(5/2)*b^2*c + 35*(d*x^2 +
c)^(3/2)*b^2*c^2 + 42*(d*x^2 + c)^(5/2)*a*b*d - 70*(d*x^2 + c)^(3/2)*a*b*c*
d + 35*(d*x^2 + c)^(3/2)*a^2*d^2)/d^3
```

maple [A] time = 0.01, size = 69, normalized size = 0.90

$$\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(15b^2x^4d^2+42ab\,d^2x^2-12b^2cd\,x^2+35a^2d^2-28abcd+8b^2c^2\right)}{105d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(3/2)*(15*b^2*d^2*x^4+42*a*b*d^2*x^2-12*b^2*c*d*x^2+35*a^2*
d^2-28*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 0.92, size = 115, normalized size = 1.49

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2x^4}{7\,d}-\frac{4\left(dx^2+c\right)^{\frac{3}{2}}b^2cx^2}{35\,d^2}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}abx^2}{5\,d}+\frac{8\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2}{105\,d^3}-\frac{4\left(dx^2+c\right)^{\frac{3}{2}}abc}{15\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(d*x^2 + c)^(3/2)*b^2*x^4/d - 4/35*(d*x^2 + c)^(3/2)*b^2*c*x^2/d^2 + 2/
5*(d*x^2 + c)^(3/2)*a*b*x^2/d + 8/105*(d*x^2 + c)^(3/2)*b^2*c^2/d^3 - 4/15*
(d*x^2 + c)^(3/2)*a*b*c/d^2 + 1/3*(d*x^2 + c)^(3/2)*a^2/d
```

mupad [B] time = 0.63, size = 101, normalized size = 1.31

$$\sqrt{d\,x^2+c}\,\left(\frac{35\,a^2\,c\,d^2-28\,a\,b\,c^2\,d+8\,b^2\,c^3}{105\,d^3}+\frac{b^2\,x^6}{7}+\frac{x^2\,\left(35\,a^2\,d^3+14\,a\,b\,c\,d^2-4\,b^2\,c^2\,d\right)}{105\,d^3}+\frac{b\,x^4\,\left(14\,a\,d+b\,c\right)}{35\,d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((8*b^2*c^3 + 35*a^2*c*d^2 - 28*a*b*c^2*d)/(105*d^3) + (b
^2*x^6)/7 + (x^2*(35*a^2*d^3 - 4*b^2*c^2*d + 14*a*b*c*d^2))/(105*d^3) + (b*
x^4*(14*a*d + b*c))/(35*d))
```

sympy [A] time = 1.60, size = 226, normalized size = 2.94

$$\begin{cases} \frac{a^{2}c\sqrt{c+dx^{2}}}{3d} + \frac{a^{2}x^{2}\sqrt{c+dx^{2}}}{3} - \frac{4abc^{2}\sqrt{c+dx^{2}}}{15d^{2}} + \frac{2abcx^{2}\sqrt{c+dx^{2}}}{15d} + \frac{2abx^{4}\sqrt{c+dx^{2}}}{5} + \frac{8b^{2}c^{3}\sqrt{c+dx^{2}}}{105d^{3}} - \frac{4b^{2}c^{2}x^{2}\sqrt{c+dx^{2}}}{105d^{2}} + \frac{b^{2}cx^{4}\sqrt{c+dx^{2}}}{35d} + \frac{b^{2}x^{6}\sqrt{c+dx^{2}}}{7} & \text{for}\: d \neq 0 \\ \sqrt{c}\left(\frac{a^{2}x^{2}}{2} + \frac{abx^{4}}{2} + \frac{b^{2}x^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*c*sqrt(c + d*x**2)/(3*d) + a**2*x**2*sqrt(c + d*x**2)/3 - 4
*a*b*c**2*sqrt(c + d*x**2)/(15*d**2) + 2*a*b*c*x**2*sqrt(c + d*x**2)/(15*d)
 + 2*a*b*x**4*sqrt(c + d*x**2)/5 + 8*b**2*c**3*sqrt(c + d*x**2)/(105*d**3)
- 4*b**2*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**4*sqrt(c + d*x**
2)/(35*d) + b**2*x**6*sqrt(c + d*x**2)/7, Ne(d, 0)), (sqrt(c)*(a**2*x**2/2
+ a*b*x**4/2 + b**2*x**6/6), True))
```

3.583 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x} \, dx$

Optimal. Leaf size=92

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{b\left(c+dx^2\right)^{3/2}(bc-2ad)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] a^2*Sqrt[c + d*x^2] - (b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (b^2*(c
 + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b(bc-2ad)\sqrt{c+dx}}{d}+\frac{a^2\sqrt{c+dx}}{x}+\frac{b^2(c+dx)^{3/2}}{d}\right)dx,x,x^2\right)\\
&= -\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx,x,x^2\right)\\
&= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{1}{2}\left(a^2c\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+}}\right.\\
&= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx\right.}{d}\\
&= a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}-a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right.
\end{aligned}$$

Mathematica [**A**] time = 0.14, size = 93, normalized size = 1.01

$$a^2\sqrt{c+dx^2}-a^2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b\left(c+dx^2\right)^{3/2}(2ad-bc)}{3d^2}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] a^2*Sqrt[c + d*x^2] + (b*(-(b*c) + 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (b^2
*(c + d*x^2)^(5/2))/(5*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

IntegrateAlgebraic [A] time = 0.09, size = 100, normalized size = 1.09

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2+10abcd+10abd^2x^2-2b^2c^2+b^2cdx^2+3b^2d^2x^4\right)}{15d^2}-a^2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2 + b^2*c*d*x^2 + 10*a
*b*d^2*x^2 + 3*b^2*d^2*x^4))/(15*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]]
```

fricas [A] time = 0.86, size = 207, normalized size = 2.25

$$\left[\frac{15\,a^2\sqrt{c}\,d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+\left(b^2cd+10\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{30\,d^2},\ \frac{15\,a^2\sqrt{-c}\,d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+\left(b^2cd+10\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^
2) + 2*(3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2 + (b^2*c*d + 10
*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2, 1/15*(15*a^2*sqrt(-c)*d^2*arctan(sqrt(
-c)/sqrt(d*x^2 + c)) + (3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2
 + (b^2*c*d + 10*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2]
```

giac [A] time = 0.32, size = 101, normalized size = 1.10

$$\frac{a^2c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^2d^8-5\left(dx^2+c\right)^{\frac{3}{2}}b^2cd^8+10\left(dx^2+c\right)^{\frac{3}{2}}abd^9+15\sqrt{dx^2+c}\,a^2d^{10}}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/15*(3*(d*x^2 + c)^(5/2)
*b^2*d^8 - 5*(d*x^2 + c)^(3/2)*b^2*c*d^8 + 10*(d*x^2 + c)^(3/2)*a*b*d^9 + 1
5*sqrt(d*x^2 + c)*a^2*d^10)/d^10
```

maple [A] time = 0.01, size = 100, normalized size = 1.09

$$-a^{2}\sqrt{c}\;\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\;\sqrt{c}}{x}\right)+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}x^{2}}{5d}+\sqrt{d\,x^{2}+c}\;a^{2}+\frac{2\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab}{3d}-\frac{2\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}c}{15d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] 1/5*b^2*x^2*(d*x^2+c)^(3/2)/d-2/15*b^2*c/d^2*(d*x^2+c)^(3/2)+2/3*a*b*(d*x^2
+c)^(3/2)/d-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)*a^2+a^2*(d*x^2+c)
^(1/2)
```

maxima [A] time = 1.03, size = 88, normalized size = 0.96

$$\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}x^{2}}{5\,d}-a^{2}\sqrt{c}\;\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\sqrt{dx^{2}+c}\,a^{2}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c}{15\,d^{2}}+\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(d*x^2 + c)^(3/2)*b^2*x^2/d - a^2*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x)))
 + sqrt(d*x^2 + c)*a^2 - 2/15*(d*x^2 + c)^(3/2)*b^2*c/d^2 + 2/3*(d*x^2 + c)
^(3/2)*a*b/d
```

mupad [B] time = 0.69, size = 135, normalized size = 1.47

$$\sqrt{d\,x^{2}+c}\;\left(\frac{(a\,d-b\,c)^{2}}{d^{2}}-c\,\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{d^{2}}-\frac{b^{2}\,c}{d^{2}}\right)\right)-\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{3\,d^{2}}-\frac{b^{2}\,c}{3\,d^{2}}\right)\,\left(d\,x^{2}+c\right)^{3/2}+\frac{b^{2}\,\left(d\,x^{2}+c\right)^{5/2}}{5\,d^{2}}+a^{2}\,\sqrt{c}\;\mathrm{atan}\left(\frac{\sqrt{d\,x^{2}+c}\;1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x,x)
```

```
[Out] (c + d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)
/d^2)) - ((2*b^2*c - 2*a*b*d)/(3*d^2) - (b^2*c)/(3*d^2))*(c + d*x^2)^(3/2)
+ a^2*c^(1/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + (b^2*(c + d*x^2)^(5
/2))/(5*d^2)
```

sympy [A] time = 72.46, size = 90, normalized size = 0.98

$$\frac{a^{2}c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+a^{2}\sqrt{c+dx^{2}}+\frac{b^{2}\left(c+dx^{2}\right)^{\frac{5}{2}}}{5d^{2}}+\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}\left(4abd-2b^{2}c\right)}{6d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] a**2*c*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*sqrt(c + d*x**2) + b
**2*(c + d*x**2)**(5/2)/(5*d**2) + (c + d*x**2)**(3/2)*(4*a*b*d - 2*b**2*c)
/(6*d**2)
```

3.584 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}\,(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d}$$

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2}+\frac{a\sqrt{c+dx^2}\,(ad+4bc)}{2c}-\frac{a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (a*(4*b*c + a*d)*Sqrt[c + d*x^2])/(2*c) + (b^2*(c + d*x^2)^(3/2))/(3*d) - (
a^2*(c + d*x^2)^(3/2))/(2*c*x^2) - (a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]])/(2*Sqrt[c])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+ad)+b^2cx\right)\sqrt{c+dx}}{x}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{(a(4bc+ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, x^2\right)}{4c}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{1}{4}(a(4bc+ad))\,\text{Subst}\left(\right.$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} + \frac{(a(4bc+ad))\,\text{Subst}\left(\int\right.}{2}$$

$$= \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c} + \frac{b^2\left(c+dx^2\right)^{3/2}}{3d} - \frac{a^2\left(c+dx^2\right)^{3/2}}{2cx^2} - \frac{a(4bc+ad)\tanh^{-1}\left(\right.}{2\sqrt{c}}$$

Mathematica [A] time = 0.11, size = 87, normalized size = 0.80

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^2}\left(-3a^2d+12abdx^2+2b^2x^2\left(c+dx^2\right)\right)}{dx^2} - \frac{3a(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-3*a^2*d + 12*a*b*d*x^2 + 2*b^2*x^2*(c + d*x^2)))/(d*x^2
) - (3*a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/6
```

IntegrateAlgebraic [A] time = 0.13, size = 94, normalized size = 0.86

$$\frac{\sqrt{c+dx^2}\left(-3a^2d+12abdx^2+2b^2cx^2+2b^2dx^4\right)}{6dx^2} + \frac{\left(a^2(-d)-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*a^2*d + 2*b^2*c*x^2 + 12*a*b*d*x^2 + 2*b^2*d*x^4))/(6*
d*x^2) + ((-4*a*b*c - a^2*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*Sqrt[c])
```

fricas [A] time = 1.60, size = 211, normalized size = 1.94

$$\left[\frac{3\left(4\,abcd + a^{2}d^{2}\right)\sqrt{c}\,x^{2}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right) + 2\left(2\,b^{2}cdx^{4} - 3\,a^{2}cd + 2\left(b^{2}c^{2} + 6\,abcd\right)x^{2}\right)\sqrt{dx^{2} + c}}{12\,cdx^{2}}, \frac{3\left(4\,abcd + a^{2}d^{2}\right)\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right) + \left(2\,b^{2}cdx^{4} - 3\,a^{2}cd + 2\left(b^{2}c^{2} + 6\,abcd\right)x^{2}\right)\sqrt{dx^{2} + c}}{6\,cdx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(4*a*b*c*d + a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(c) + 2*c)/x^2) + 2*(2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^2 + 6*a*b*c*d
)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2), 1/6*(3*(4*a*b*c*d + a^2*d^2)*sqrt(-c)*x^
2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^
2 + 6*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

giac [A] time = 0.38, size = 89, normalized size = 0.82

$$\frac{2\left(dx^{2} + c\right)^{\frac{3}{2}}b^{2} + 12\,\sqrt{dx^{2} + c}\,abd - \frac{3\,\sqrt{dx^{2}+c}\,a^{2}d}{x^{2}} + \frac{3\left(4\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x^2 + c)^(3/2)*b^2 + 12*sqrt(d*x^2 + c)*a*b*d - 3*sqrt(d*x^2 + c)
*a^2*d/x^2 + 3*(4*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(
-c))/d
```

maple [A] time = 0.01, size = 132, normalized size = 1.21

$$-\frac{a^{2}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2\sqrt{c}} - 2ab\sqrt{c}\,\ln\left(\frac{2c + 2\sqrt{d\,x^{2} + c}\,\sqrt{c}}{x}\right) + \frac{\sqrt{d\,x^{2} + c}\,a^{2}d}{2c} + 2\sqrt{d\,x^{2} + c}\,ab + \frac{\left(d\,x^{2} + c\right)^{\frac{3}{2}}b^{2}}{3d} - \frac{\left(d\,x^{2} + c\right)^{\frac{3}{2}}a^{2}}{2c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] 1/3*b^2*(d*x^2+c)^(3/2)/d-1/2*a^2*(d*x^2+c)^(3/2)/c/x^2-1/2*a^2*d/c^(1/2)*l
n((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/2*a^2*d/c*(d*x^2+c)^(1/2)-2*c^(1/2)*
ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)*a*b+2*(d*x^2+c)^(1/2)*a*b
```

maxima [A] time = 1.06, size = 109, normalized size = 1.00

$$-2\,ab\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{a^{2}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,\sqrt{c}}+2\,\sqrt{dx^{2}+c}\,ab+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}}{3\,d}+\frac{\sqrt{dx^{2}+c}\,a^{2}d}{2\,c}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{2\,cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a*b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/sqrt(c) + 2*sqrt(d*x^2 + c)*a*b + 1/3*(d*x^2 + c)^(3/2)*b^2/d +
 1/2*sqrt(d*x^2 + c)*a^2*d/c - 1/2*(d*x^2 + c)^(3/2)*a^2/(c*x^2)
```

mupad [B] time = 1.06, size = 103, normalized size = 0.94

$$\frac{b^{2}\left(d\,x^{2}+c\right)^{3/2}}{3\,d}-\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{d}-\frac{2\,b^{2}\,c}{d}\right)\sqrt{d\,x^{2}+c}-\frac{a^{2}\,\sqrt{d\,x^{2}+c}}{2\,x^{2}}+\frac{a\,\operatorname{atan}\left(\frac{\sqrt{d\,x^{2}+c}\,1\mathrm{i}}{\sqrt{c}}\right)(a\,d+4\,b\,c)\,1\mathrm{i}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^3,x)
```

```
[Out] (b^2*(c + d*x^2)^(3/2))/(3*d) - ((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d)*(c +
d*x^2)^(1/2) - (a^2*(c + d*x^2)^(1/2))/(2*x^2) + (a*atan(((c + d*x^2)^(1/2)
*1i)/c^(1/2))*(a*d + 4*b*c)*1i)/(2*c^(1/2))
```

sympy [A] time = 73.98, size = 148, normalized size = 1.36

$$-\frac{a^{2}\sqrt{d}\,\sqrt{\frac{c}{dx^{2}}+1}}{2x}-\frac{a^{2}d\,\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2\sqrt{c}}-2ab\sqrt{c}\,\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{2abc}{\sqrt{d}\,x\sqrt{\frac{c}{dx^{2}}+1}}+\frac{2ab\sqrt{d}\,x}{\sqrt{\frac{c}{dx^{2}}+1}}+b^{2}\left(\begin{cases}\frac{\sqrt{c}\,x^{2}}{2} & \text{for}\ d=0\\ \frac{\left(c+dx^{2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*asinh(sqrt(c)/(sqrt(d)*x)
)/(2*sqrt(c)) - 2*a*b*sqrt(c)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c/(sqrt(d)
*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + b**2*Piec
ewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True))
```

3.585 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^5}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{\sqrt{c+dx^2}\,\left(ad(8bc-ad)+8b^2c^2\right)}{8c^2}-\frac{\left(ad(8bc-ad)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a\left(c+dx^2\right)^{3/2}(8}{8c^2x^2}$$

Rubi [A] time = 0.16, antiderivative size = 140, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{1}{8}\sqrt{c+dx^2}\left(\frac{ad(8bc-ad)}{c^2}+8b^2\right)-\frac{\left(ad(8bc-ad)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}-\frac{a\left(c+dx^2\right)^{3/2}(8bc-ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] ((8*b^2 + (a*d*(8*b*c - a*d))/c^2)*Sqrt[c + d*x^2])/8 - (a^2*(c + d*x^2)^(3
/2))/(4*c*x^4) - (a*(8*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^2) - ((8*b^2*
c^2 + a*d*(8*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^5}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}+\frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc-ad)+2b^2cx\right)\sqrt{c+dx}}{x^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}-\frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2}+\frac{1}{16}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\text{Subst}\Bigg(\int\\
&= \frac{1}{8}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}-\frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2}+\\
&= \frac{1}{8}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}-\frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2}+\\
&= \frac{1}{8}\left(8b^2+\frac{ad(8bc-ad)}{c^2}\right)\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{4cx^4}-\frac{a(8bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^2}-
\end{aligned}$$

Mathematica [A] time = 0.12, size = 104, normalized size = 0.73

$$\frac{\sqrt{c+dx^2}\left(-a^2\left(2c+dx^2\right)-8abcx^2+8b^2cx^4\right)}{8cx^4}-\frac{\left(-a^2d^2+8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a*b*c*x^2 + 8*b^2*c*x^4 - a^2*(2*c + d*x^2)))/(8*c*x^4
) - ((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8
*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 104, normalized size = 0.73

$$\frac{\left(a^2d^2-8abcd-8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{\sqrt{c+dx^2}\left(-2a^2c-a^2dx^2-8abcx^2+8b^2cx^4\right)}{8cx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c - 8*a*b*c*x^2 - a^2*d*x^2 + 8*b^2*c*x^4))/(8*c*x
^4) + ((-8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(8*c^(3/2))
```

fricas [A] time = 1.27, size = 225, normalized size = 1.57

$$\left[-\frac{\left(8\,b^{2}c^{2}+8\,abcd-a^{2}d^{2}\right)\sqrt{c}\,x^{4}\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\left(8\,b^{2}c^{2}x^{4}-2\,a^{2}c^{2}-\left(8\,abc^{2}+a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{16\,c^{2}x^{4}},\ \frac{\left(8\,b^{2}c^{2}+8\,abcd-a^{2}d^{2}\right)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(8\,b^{2}c^{2}x^{4}-2\,a^{2}c^{2}-\left(8\,abc^{2}+a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{8\,c^{2}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/16*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 + 2*sqrt(
d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*b^2*c^2*x^4 - 2*a^2*c^2 - (8*a*b*c^2
+ a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4), 1/8*((8*b^2*c^2 + 8*a*b*c*d - a
^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c^2*x^4 - 2*
a^2*c^2 - (8*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4)]
```

giac [A] time = 0.36, size = 153, normalized size = 1.07

$$\frac{8\,\sqrt{dx^{2}+c}\,b^{2}d+\frac{\left(8\,b^{2}c^{2}d+8\,abcd^{2}-a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}abcd^{2}-8\,\sqrt{dx^{2}+c}\,abc^{2}d^{2}+\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}+\sqrt{dx^{2}+c}\,a^{2}cd^{3}}{cd^{2}x^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(d*x^2 + c)*b^2*d + (8*b^2*c^2*d + 8*a*b*c*d^2 - a^2*d^3)*arctan
(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 -
8*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3 + sqrt(d*x^2 + c)
*a^2*c*d^3)/(c*d^2*x^4))/d
```

maple [A] time = 0.02, size = 207, normalized size = 1.45

$$\frac{a^{2}d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{8c^{\frac{3}{2}}}-\frac{abd\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{\sqrt{c}}-b^{2}\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)-\frac{\sqrt{d\,x^{2}+c}\,a^{2}d^{2}}{8c^{2}}+\frac{\sqrt{d\,x^{2}+c}\,abd}{c}+\sqrt{d\,x^{2}+c}\,b^{2}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d}{8c^{2}x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab}{c\,x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}}{4c\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(3/2)/c/x^4+1/8*a^2*d/c^2/x^2*(d*x^2+c)^(3/2)+1/8*a^2*d^
2/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/8*a^2*d^2/c^2*(d*x^2+c)^(
1/2)-a*b/c/x^2*(d*x^2+c)^(3/2)-a*b*d/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1
```

```
/2))/x)+a*b*d/c*(d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/
x)*b^2+(d*x^2+c)^(1/2)*b^2
```

maxima [**A**] time = 1.07, size = 173, normalized size = 1.21

$$-b^2\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{abd\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{\sqrt{c}}+\frac{a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8c^{\frac{3}{2}}}+\sqrt{dx^2+c}\,b^2+\frac{\sqrt{dx^2+c}\,abd}{c}-\frac{\sqrt{dx^2+c}\,a^2d^2}{8c^2}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}ab}{cx^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2d}{8c^2x^2}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2}{4cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -b^2*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - a*b*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/sqrt(c) + 1/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + sqrt(d*
x^2 + c)*b^2 + sqrt(d*x^2 + c)*a*b*d/c - 1/8*sqrt(d*x^2 + c)*a^2*d^2/c^2 -
(d*x^2 + c)^(3/2)*a*b/(c*x^2) + 1/8*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^2) - 1/4
*(d*x^2 + c)^(3/2)*a^2/(c*x^4)
```

mupad [**B**] time = 1.33, size = 137, normalized size = 0.96

$$b^2\sqrt{d\,x^2+c}-\frac{\left(\frac{a^2\,d^2}{8}-a\,b\,c\,d\right)\sqrt{d\,x^2+c}+\frac{\left(a^2\,d^2+8\,b\,c\,a\,d\right)\left(d\,x^2+c\right)^{3/2}}{8\,c}}{\left(d\,x^2+c\right)^2-2\,c\,\left(d\,x^2+c\right)+c^2}-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\left(-a^2\,d^2+8\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^5,x)
```

```
[Out] b^2*(c + d*x^2)^(1/2) - (((a^2*d^2)/8 - a*b*c*d)*(c + d*x^2)^(1/2) + ((a^2*
d^2 + 8*a*b*c*d)*(c + d*x^2)^(3/2))/(8*c))/((c + d*x^2)^2 - 2*c*(c + d*x^2)
 + c^2) - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(8*b^2*c^2 - a^2*d^2 + 8*a*b*c*
d))/(8*c^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

3.586 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{c+dx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6}-\frac{a\left(c+dx^2\right)^{3/2}(4}{8c^2x^4}$$

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 47, 63, 208}

$$-\frac{\sqrt{c+dx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16c^2x^2}-\frac{d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}-\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6}-\frac{a\left(c+dx^2\right)^{3/2}(4bc-ad)}{8c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] -((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2])/(16*c^2*x^2) - (a^2*(c
 + d*x^2)^(3/2))/(6*c*x^6) - (a*(4*b*c - a*d)*(c + d*x^2)^(3/2))/(8*c^2*x^4
) - (d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/
(16*c^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2\sqrt{c+dx}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{3}{2}a(4bc-ad)+3b^2cx\right)\sqrt{c+dx}}{x^3}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} + \frac{\left(8b^2c^2-4abcd+a^2d^2\right)\text{Subst}\left(\int \frac{\sqrt{c+}}{x^2}\right.}{16c^2}$$

$$= -\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} +$$

$$= -\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} +$$

$$= -\frac{\left(8b^2c^2-4abcd+a^2d^2\right)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2\left(c+dx^2\right)^{3/2}}{6cx^6} - \frac{a(4bc-ad)\left(c+dx^2\right)^{3/2}}{8c^2x^4} -$$

Mathematica [A] time = 0.17, size = 142, normalized size = 0.95

$$\frac{-3dx^6\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\sqrt{\frac{dx^2}{c}+1}\right)-\left(c+dx^2\right)\left(a^2\left(8c^2+2cdx^2-3d^2x^4\right)+12abcx^2\left(2c+dx^2\right)+24b^2c^2x^4\right)}{48c^2x^6\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] (-((c + d*x^2)*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c + d*x^2) + a^2*(8*c^2 +
2*c*d*x^2 - 3*d^2*x^4))) - 3*d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^6*Sqrt[1
 + (d*x^2)/c]*ArcTanh[Sqrt[1 + (d*x^2)/c]])/(48*c^2*x^6*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.23, size = 135, normalized size = 0.91

$$\frac{\sqrt{c+dx^2}\left(-8a^2c^2-2a^2cdx^2+3a^2d^2x^4-24abc^2x^2-12abcdx^4-24b^2c^2x^4\right)}{48c^2x^6}+\frac{\left(-a^2d^3+4abcd^2-8b^2c^2d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*c^2 - 24*a*b*c^2*x^2 - 2*a^2*c*d*x^2 - 24*b^2*c^2*
x^4 - 12*a*b*c*d*x^4 + 3*a^2*d^2*x^4))/(48*c^2*x^6) + ((-8*b^2*c^2*d + 4*a*
b*c*d^2 - a^2*d^3)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(5/2))
```

fricas [A]   time = 1.81, size = 276, normalized size = 1.85

$$\left[\frac{3\left(8\,b^{2}c^{2}d - 4\,abcd^{2} + a^{2}d^{3}\right)\sqrt{c}\,x^{6}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right) - 2\left(8\,a^{2}c^{3} + 3\left(8\,b^{2}c^{3} + 4\,abc^{2}d - a^{2}cd^{2}\right)x^{4} + 2\left(12\,abc^{3} + a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{96\,c^{3}x^{6}}, \frac{3\left(8\,b^{2}c^{2}d - 4\,abcd^{2} + a^{2}d^{3}\right)\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right) - \left(8\,a^{2}c^{3} + 3\left(8\,b^{2}c^{3} + 4\,abc^{2}d - a^{2}cd^{2}\right)x^{4} + 2\left(12\,abc^{3} + a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,c^{3}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c
^2*d - a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c
^3*x^6), 1/48*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) - (8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c^2*d - a^2*c
*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^6)]
```

giac [A]   time = 0.48, size = 222, normalized size = 1.49

$$\frac{\frac{3\left(8\,b^{2}c^{2}d^{2}-4\,abcd^{3}+a^{2}d^{4}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^{2}} - \frac{24\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}c^{2}d^{2}-48\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{3}d^{2}+24\,\sqrt{dx^{2}+c}\,b^{2}c^{4}d^{2}+12\left(dx^{2}+c\right)^{\frac{5}{2}}abcd^{3}-12\,\sqrt{dx^{2}+c}\,abc^{3}d^{3}-3\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d^{4}+8\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cd^{4}+3\,\sqrt{dx^{2}+c}\,a^{2}c^{2}d^{4}}{c^{2}d^{3}x^{6}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(8*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*arctan(sqrt(d*x^2 + c)/sqrt
(-c))/(sqrt(-c)*c^2) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 + c)^(
3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 12*(d*x^2 + c)^(5/2)*a*
b*c*d^3 - 12*sqrt(d*x^2 + c)*a*b*c^3*d^3 - 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*
(d*x^2 + c)^(3/2)*a^2*c*d^4 + 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c^2*d^3*x^6))
/d
```

maple [B]   time = 0.02, size = 281, normalized size = 1.89

$$-\frac{a^{2}d^{3}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{16c^{\frac{5}{2}}}+\frac{ab\,d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{4c^{\frac{3}{2}}}-\frac{b^{2}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2\sqrt{c}}+\frac{\sqrt{d\,x^{2}+c}\,a^{2}d^{3}}{16c^{3}}-\frac{\sqrt{d\,x^{2}+c}\,ab\,d^{2}}{4c^{2}}+\frac{\sqrt{d\,x^{2}+c}\,b^{2}d}{2c}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{16c^{3}x^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}abd}{4c^{2}x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}}{2c\,x^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d}{8c^{2}x^{4}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab}{2c\,x^{4}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}}{6c\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(3/2)/c/x^6+1/8*a^2*d/c^2/x^4*(d*x^2+c)^(3/2)-1/16*a^2*d
^2/c^3/x^2*(d*x^2+c)^(3/2)-1/16*a^2*d^3/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c
^(1/2))/x)+1/16*a^2*d^3/c^3*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*x^2+c)^(3/2)+1
```

```
/4*a*b*d/c^2/x^2*(d*x^2+c)^(3/2)+1/4*a*b*d^2/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1
/2)*c^(1/2))/x)-1/4*a*b*d^2/c^2*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(3/
2)-1/2*b^2*d/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/2*b^2*d/c*(d*x
^2+c)^(1/2)
```

maxima [A] time = 1.02, size = 247, normalized size = 1.66

$$-\frac{b^{2}d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,\sqrt{c}}+\frac{abd^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,c^{\frac{3}{2}}}-\frac{a^{2}d^{3}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{5}{2}}}+\frac{\sqrt{dx^{2}+c}\,b^{2}d}{2\,c}-\frac{\sqrt{dx^{2}+c}\,abd^{2}}{4\,c^{2}}+\frac{\sqrt{dx^{2}+c}\,a^{2}d^{3}}{16\,c^{3}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}}{2\,cx^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}abd}{4\,c^{2}x^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{16\,c^{3}x^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}ab}{2\,cx^{4}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d}{8\,c^{2}x^{4}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{6\,cx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/4*a*b*d^2*arcsinh(c/(s
qrt(c*d)*abs(x)))/c^(3/2) - 1/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5
/2) + 1/2*sqrt(d*x^2 + c)*b^2*d/c - 1/4*sqrt(d*x^2 + c)*a*b*d^2/c^2 + 1/16*
sqrt(d*x^2 + c)*a^2*d^3/c^3 - 1/2*(d*x^2 + c)^(3/2)*b^2/(c*x^2) + 1/4*(d*x^
2 + c)^(3/2)*a*b*d/(c^2*x^2) - 1/16*(d*x^2 + c)^(3/2)*a^2*d^2/(c^3*x^2) - 1
/2*(d*x^2 + c)^(3/2)*a*b/(c*x^4) + 1/8*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^4) -
1/6*(d*x^2 + c)^(3/2)*a^2/(c*x^6)
```

mupad [B] time = 1.84, size = 193, normalized size = 1.30

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^2\,d^3}{16}-\frac{a\,b\,c\,d^2}{4}+\frac{b^2\,c^2\,d}{2}\right)+\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a^2\,d^3-6\,b^2\,c^2\,d\right)}{6\,c}+\frac{{\left(d\,x^2+c\right)}^{5/2}\,\left(-a^2\,d^3+4\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c^2}}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(a^2\,d^2-4\,a\,b\,c\,d+8\,b^2\,c^2\right)}{16\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^7,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((a^2*d^3)/16 + (b^2*c^2*d)/2 - (a*b*c*d^2)/4) + ((c + d
*x^2)^(3/2)*(a^2*d^3 - 6*b^2*c^2*d))/(6*c) + ((c + d*x^2)^(5/2)*(8*b^2*c^2*
d - a^2*d^3 + 4*a*b*c*d^2))/(16*c^2))/(3*c*(c + d*x^2)^2 - 3*c^2*(c + d*x^2
) - (c + d*x^2)^3 + c^3) - (d*atanh((c + d*x^2)^(1/2)/c^(1/2))*(a^2*d^2 + 8
*b^2*c^2 - 4*a*b*c*d))/(16*c^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**7,x)
```

```
[Out] Timed out
```

3.587 $\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=191

$$-\frac{c^2\left(16a^2d^2+bc(5bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{x^3\sqrt{c+dx^2}\left(16a^2d^2+bc(5bc-16ad)\right)}{64d^2}+\frac{cx\sqrt{c+dx^2}\left(16a^2d^2\right.}{128d}$$

Rubi **[A]** time = 0.19, antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 459, 279, 321, 217, 206}

$$-\frac{c^2\left(16a^2d^2+bc(5bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}+\frac{1}{64}x^3\sqrt{c+dx^2}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)+\frac{cx\sqrt{c+dx^2}\left(16a^2d^2+bc(5bc-16ad)\right)}{128d^3}-\frac{bx^3\left(c+dx^2\right)^{3/2}(5bc-16ad)}{48d^2}+\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (c*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^3) + ((16*
a^2 + (b*c*(5*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/64 - (b*(5*b*c - 16*
a*d)*x^3*(c + d*x^2)^(3/2))/(48*d^2) + (b^2*x^5*(c + d*x^2)^(3/2))/(8*d) -
(c^2*(16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(128*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx &= \frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}+\frac{\int x^2\sqrt{c+dx^2}\left(8a^2d-b(5bc-16ad)x^2\right)dx}{8d}\\ &= -\frac{b(5bc-16ad)x^3\left(c+dx^2\right)^{3/2}}{48d^2}+\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}+\frac{1}{16}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right.\\ &= \frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\frac{b(5bc-16ad)x^3\left(c+dx^2\right)^{3/2}}{48d^2}+\frac{b^2x^5}{}\\ &= \frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\\ &= \frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\\ &= \frac{c\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d}+\frac{1}{64}\left(16a^2+\frac{bc(5bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}-\end{aligned}$$

Mathematica [A] time = 0.16, size = 157, normalized size = 0.82

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\left(48a^2d^2\left(c+2dx^2\right)+16abd\left(-3c^2+2cdx^2+8d^2x^4\right)+b^2\left(15c^3-10c^2dx^2+8cd^2x^4+48d^3x^6\right)\right)-3c^2\left(16a^2d^2-16abcd+5b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{384d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(c + 2*d*x^2) + 16*a*b*d*(-3*c^2 + 2
*c*d*x^2 + 8*d^2*x^4) + b^2*(15*c^3 - 10*c^2*d*x^2 + 8*c*d^2*x^4 + 48*d^3*x
^6)) - 3*c^2*(5*b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c
 + d*x^2]])/(384*d^(7/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 173, normalized size = 0.91

$$\frac{\sqrt{c+dx^2}\left(48a^2cd^2x+96a^2d^3x^3-48abc^2dx+32abcd^2x^3+128abd^3x^5+15b^2c^3x-10b^2c^2dx^3+8b^2cd^2x^5+48b^2d^3x^7\right)}{384d^3}+\frac{\left(16a^2c^2d^2-16abc^3d+5b^2c^4\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{128d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c^3*x - 48*a*b*c^2*d*x + 48*a^2*c*d^2*x - 10*b^2*c
^2*d*x^3 + 32*a*b*c*d^2*x^3 + 96*a^2*d^3*x^3 + 8*b^2*c*d^2*x^5 + 128*a*b*d^
```

```
3*x^5 + 48*b^2*d^3*x^7))/(384*d^3) + ((5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^
2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(128*d^(7/2))
```

fricas [A] time = 1.66, size = 341, normalized size = 1.79

$$\left[\frac{3\left(5\,b^{2}c^{4}-16\,abc^{3}d+16\,a^{2}c^{2}d^{2}\right)\sqrt{d}\,\log\left(-2\,dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)+2\left(48\,b^{2}d^{4}x^{7}+8\left(b^{2}cd^{3}+16\,abd^{4}\right)x^{5}-2\left(5\,b^{2}c^{2}d^{2}-16\,abcd^{3}-48\,a^{2}d^{4}\right)x^{3}+3\left(5\,b^{2}c^{3}d-16\,abc^{2}d^{2}+16\,a^{2}cd^{3}\right)x\right)\sqrt{dx^{2}+c}}{768\,d^{4}},\ \frac{3\left(5\,b^{2}c^{4}-16\,abc^{3}d+16\,a^{2}c^{2}d^{2}\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+\left(48\,b^{2}d^{4}x^{7}+8\left(b^{2}cd^{3}+16\,abd^{4}\right)x^{5}-2\left(5\,b^{2}c^{2}d^{2}-16\,abcd^{3}-48\,a^{2}d^{4}\right)x^{3}+3\left(5\,b^{2}c^{3}d-16\,abc^{2}d^{2}+16\,a^{2}cd^{3}\right)x\right)\sqrt{dx^{2}+c}}{384\,d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*
a*b*d^4)*x^5 - 2*(5*b^2*c^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2
*c^3*d - 16*a*b*c^2*d^2 + 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/384*(3*(
5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) + (48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*a*b*d^4)*x^5 - 2*(5*b^2*c
^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 16*a*b*c^2*d^2 +
 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

giac [A] time = 0.49, size = 174, normalized size = 0.91

$$\frac{1}{384}\left(2\left(4\left(6\,b^{2}x^{2}+\frac{b^{2}cd^{5}+16\,abd^{6}}{d^{6}}\right)x^{2}-\frac{5\,b^{2}c^{2}d^{4}-16\,abcd^{5}-48\,a^{2}d^{6}}{d^{6}}\right)x^{2}+\frac{3\left(5\,b^{2}c^{3}d^{3}-16\,abc^{2}d^{4}+16\,a^{2}cd^{5}\right)}{d^{6}}\right)\sqrt{dx^{2}+c}\,x+\frac{\left(5\,b^{2}c^{4}-16\,abc^{3}d+16\,a^{2}c^{2}d^{2}\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^{2}+c}\right|\right)}{128\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2 + (b^2*c*d^5 + 16*a*b*d^6)/d^6)*x^2 - (5*b^2*c^2*d^4
 - 16*a*b*c*d^5 - 48*a^2*d^6)/d^6)*x^2 + 3*(5*b^2*c^3*d^3 - 16*a*b*c^2*d^4
+ 16*a^2*c*d^5)/d^6)*sqrt(d*x^2 + c)*x + 1/128*(5*b^2*c^4 - 16*a*b*c^3*d +
16*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

maple [A] time = 0.02, size = 259, normalized size = 1.36

$$\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}x^{5}}{8d}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab\,x^{3}}{3d}-\frac{5\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}c\,x^{3}}{48d^{2}}-\frac{a^{2}c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{8d^{\frac{3}{2}}}+\frac{ab\,c^{3}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{8d^{\frac{5}{2}}}-\frac{5b^{2}c^{4}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{128d^{\frac{7}{2}}}-\frac{\sqrt{d\,x^{2}+c}\,a^{2}cx}{8d}+\frac{\sqrt{d\,x^{2}+c}\,ab\,c^{2}x}{8d^{2}}-\frac{5\sqrt{d\,x^{2}+c}\,b^{2}c^{3}x}{128d^{3}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}x}{4d}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}abcx}{4d^{2}}+\frac{5\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}c^{2}x}{64d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*b^2*x^5*(d*x^2+c)^(3/2)/d-5/48*b^2*c/d^2*x^3*(d*x^2+c)^(3/2)+5/64*b^2*c
^2/d^3*x*(d*x^2+c)^(3/2)-5/128*b^2*c^3/d^3*x*(d*x^2+c)^(1/2)-5/128*b^2*c^4/
d^(7/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/3*a*b*x^3*(d*x^2+c)^(3/2)/d-1/4*a*b
*c/d^2*x*(d*x^2+c)^(3/2)+1/8*a*b*c^2/d^2*x*(d*x^2+c)^(1/2)+1/8*a*b*c^3/d^(5
/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))+1/4*a^2*x*(d*x^2+c)^(3/2)/d-1/8*a^2*c/d*x
*(d*x^2+c)^(1/2)-1/8*a^2*c^2/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

maxima [A]   time = 1.11, size = 237, normalized size = 1.24

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2x^5}{8\,d}-\frac{5\left(dx^2+c\right)^{\frac{3}{2}}b^2cx^3}{48\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}abx^3}{3\,d}+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2x}{64\,d^3}-\frac{5\,\sqrt{dx^2+c}\,b^2c^3x}{128\,d^3}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}abcx}{4\,d^2}+\frac{\sqrt{dx^2+c}\,abc^2x}{8\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2x}{4\,d}-\frac{\sqrt{dx^2+c}\,a^2cx}{8\,d}-\frac{5\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{7}{2}}}+\frac{abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}-\frac{a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^2 + c)^(3/2)*b^2*x^5/d - 5/48*(d*x^2 + c)^(3/2)*b^2*c*x^3/d^2 + 1/
3*(d*x^2 + c)^(3/2)*a*b*x^3/d + 5/64*(d*x^2 + c)^(3/2)*b^2*c^2*x/d^3 - 5/12
8*sqrt(d*x^2 + c)*b^2*c^3*x/d^3 - 1/4*(d*x^2 + c)^(3/2)*a*b*c*x/d^2 + 1/8*s
qrt(d*x^2 + c)*a*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a^2*x/d - 1/8*sqrt(d*x
^2 + c)*a^2*c*x/d - 5/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 1/8*a*b*
c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(
3/2)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

sympy [B]   time = 21.91, size = 411, normalized size = 2.15

$$\frac{a^2c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}\,x^3}{8\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{a^2dx^5}{4\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{3}{2}}x^3}{24d\sqrt{1+\frac{dx^2}{c}}}+\frac{5ab\sqrt{c}\,x^5}{12\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^3\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}}+\frac{abdx^7}{3\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{7}{2}}x}{128d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{5}{2}}x^3}{384d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}x^5}{192d\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2\sqrt{c}\,x^7}{48\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^4\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{7}{2}}}+\frac{b^2dx^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*c**(3/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*x**3/(8*sqrt(1 +
d*x**2/c)) - a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(3/2)) + a**2*d*x**5/
(4*sqrt(c)*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)*x/(8*d**2*sqrt(1 + d*x**2/c))
 - a*b*c**(3/2)*x**3/(24*d*sqrt(1 + d*x**2/c)) + 5*a*b*sqrt(c)*x**5/(12*sqr
t(1 + d*x**2/c)) + a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a*b*d*x
**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d**3*sqrt(1 + d
*x**2/c)) + 5*b**2*c**(5/2)*x**3/(384*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(3
/2)*x**5/(192*d*sqrt(1 + d*x**2/c)) + 7*b**2*sqrt(c)*x**7/(48*sqrt(1 + d*x*
*2/c)) - 5*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(7/2)) + b**2*d*x**9/
(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.588 $\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+ bx\,($$

Rubi **[A]** time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2-4abcd+b^2c^2\right)}{16d^2}+\frac{c\left(8a^2d^2-4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}-\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{24d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] ((b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^2) - (b*(3*b*c
- 8*a*d)*x*(c + d*x^2)^(3/2))/(24*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^(3/2)
)/(6*d) + (c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(16*d^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx &= \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}+\frac{\int \sqrt{c+dx^2}\,\left(-a(bc-6ad)-b(3bc-8ad)x^2\right)dx}{6d}\\
&= -\frac{b(3bc-8ad)x\left(c+dx^2\right)^{3/2}}{24d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}+\frac{(b^2c^2-4abcd+8a^2d^2)}{8d^2}\\
&= \frac{\left(b^2c^2-4abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x\left(c+dx^2\right)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c}{6d}\\
&= \frac{\left(b^2c^2-4abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x\left(c+dx^2\right)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c}{6d}\\
&= \frac{\left(b^2c^2-4abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16d^2}-\frac{b(3bc-8ad)x\left(c+dx^2\right)^{3/2}}{24d^2}+\frac{bx(a+bx^2)(c}{6d}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.82

$$\frac{3c\left(8a^2d^2-4abcd+b^2c^2\right)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)+\sqrt{d}\,x\sqrt{c+dx^2}\,\left(24a^2d^2+12abd\left(c+2dx^2\right)+b^2\left(-3c^2+2cdx^2+8d^2x^4\right)\right)}{48d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(c + 2*d*x^2) + b^2*(-3*c
^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*Log[d*
x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*d^(5/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 132, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(24a^2d^2x+12abcdx+24abd^2x^3-3b^2c^2x+2b^2cdx^3+8b^2d^2x^5\right)}{48d^2}+\frac{\left(-8a^2cd^2+4abc^2d-b^2c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*b^2*c^2*x + 12*a*b*c*d*x + 24*a^2*d^2*x + 2*b^2*c*d*x^
3 + 24*a*b*d^2*x^3 + 8*b^2*d^2*x^5))/(48*d^2) + ((-(b^2*c^3) + 4*a*b*c^2*d
- 8*a^2*c*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(16*d^(5/2))
```

fricas [A] time = 1.77, size = 262, normalized size = 1.76

$$\left[\frac{3\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(8\,b^2d^3x^5+2\left(b^2cd^2+12\,abd^3\right)x^3-3\left(b^2c^2d-4\,abcd^2-8\,a^2d^3\right)x\right)\sqrt{dx^2+c}}{96\,d^3},-\frac{3\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(8\,b^2d^3x^5+2\left(b^2cd^2+12\,abd^3\right)x^3-3\left(b^2c^2d-4\,abcd^2-8\,a^2d^3\right)x\right)\sqrt{dx^2+c}}{48\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqr
t(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)
*x^3 - 3*(b^2*c^2*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/
48*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)*x^3 - 3*(b^2*c^2
*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

giac [A] time = 0.38, size = 128, normalized size = 0.86

$$\frac{1}{48}\left(2\left(4\,b^2x^2+\frac{b^2cd^3+12\,abd^4}{d^4}\right)x^2-\frac{3\left(b^2c^2d^2-4\,abcd^3-8\,a^2d^4\right)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{\left(b^2c^3-4\,abc^2d+8\,a^2cd^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2 + (b^2*c*d^3 + 12*a*b*d^4)/d^4)*x^2 - 3*(b^2*c^2*d^2 - 4
*a*b*c*d^3 - 8*a^2*d^4)/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b^2*c^3 - 4*a*b*c^2*
d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

maple [A] time = 0.01, size = 190, normalized size = 1.28

$$\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}b^2x^3}{6d}+\frac{a^2c\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}-\frac{ab\,c^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{4d^{\frac{3}{2}}}+\frac{b^2c^3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{16d^{\frac{5}{2}}}+\frac{\sqrt{d\,x^2+c}\,a^2x}{2}-\frac{\sqrt{d\,x^2+c}\,abcx}{4d}+\frac{\sqrt{d\,x^2+c}\,b^2c^2x}{16d^2}+\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}abx}{2d}-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}b^2cx}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*b^2*x^3*(d*x^2+c)^(3/2)/d-1/8*b^2*c/d^2*x*(d*x^2+c)^(3/2)+1/16*b^2*c^2/
d^2*x*(d*x^2+c)^(1/2)+1/16*b^2*c^3/d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/
2*a*b*x*(d*x^2+c)^(3/2)/d-1/4*a*b*c/d*x*(d*x^2+c)^(1/2)-1/4*a*b*c^2/d^(3/2)
*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/2*a^2*x*(d*x^2+c)^(1/2)+1/2*a^2*c/d^(1/2)*
ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.00, size = 168, normalized size = 1.13

$$\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2x^3}{6\,d}+\frac{1}{2}\sqrt{dx^2+c}\,a^2x-\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2cx}{8\,d^2}+\frac{\sqrt{dx^2+c}\,b^2c^2x}{16\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}abx}{2\,d}-\frac{\sqrt{dx^2+c}\,abcx}{4\,d}+\frac{b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{abc^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{3}{2}}}+\frac{a^2c\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(3/2)*b^2*x^3/d + 1/2*sqrt(d*x^2 + c)*a^2*x - 1/8*(d*x^2 +
c)^(3/2)*b^2*c*x/d^2 + 1/16*sqrt(d*x^2 + c)*b^2*c^2*x/d^2 + 1/2*(d*x^2 + c)
^(3/2)*a*b*x/d - 1/4*sqrt(d*x^2 + c)*a*b*c*x/d + 1/16*b^2*c^3*arcsinh(d*x/s
qrt(c*d))/d^(5/2) - 1/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a^2*c*
arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}\;d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

sympy [B] time = 13.72, size = 291, normalized size = 1.95

$$\frac{a^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}}+\frac{abc^{\frac{3}{2}}x}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}\,x^3}{4\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^2\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4d^{\frac{3}{2}}}+\frac{abdx^5}{2\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x}{16d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}x^3}{48d\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2\sqrt{c}\,x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{5}{2}}}+\frac{b^2dx^7}{6\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/2 + a**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*sq
rt(d)) + a*b*c**(3/2)*x/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*sqrt(c)*x**3/(4*sq
rt(1 + d*x**2/c)) - a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*d**(3/2)) + a*b*d*
x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x/(16*d**2*sqrt(1 + d*x
**2/c)) - b**2*c**(3/2)*x**3/(48*d*sqrt(1 + d*x**2/c)) + 5*b**2*sqrt(c)*x**
5/(24*sqrt(1 + d*x**2/c)) + b**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(5/2)
) + b**2*d*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.589 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}-\frac{\left(b^2c^2-8ad(ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}-\frac{x\sqrt{c+dx^2}\,\left(b^2c^2-8ad(ad+bc)\right)}{8cd}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}$$

Rubi [A] time = 0.09, antiderivative size = 130, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 388, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}-\frac{\left(b^2c^2-8ad(ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}-\frac{1}{8}x\sqrt{c+dx^2}\left(\frac{b^2c}{d}-\frac{8a(ad+bc)}{c}\right)+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -((((b^2*c)/d - (8*a*(b*c + a*d))/c)*x*Sqrt[c + d*x^2])/8 - (a^2*(c + d*x^2)
^(3/2))/(c*x) + (b^2*x*(c + d*x^2)^(3/2))/(4*d) - ((b^2*c^2 - 8*a*d*(b*c +
a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^2}\,dx = -\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{\int\left(2a(bc+ad)+b^2cx^2\right)\sqrt{c+dx^2}\,dx}{c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{1}{4}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)\int\sqrt{c+dx^2}\,dx$$

$$= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{1}{8}\Big(\frac{b^2c^2}{d}$$

$$= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{1}{8}\Big(\frac{b^2c^2}{d}$$

$$= -\frac{1}{8}\left(\frac{b^2c}{d}-\frac{8a(bc+ad)}{c}\right)x\sqrt{c+dx^2}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{3/2}}{4d}-\frac{\Big(\frac{b^2c^2}{d}-}{}$$

Mathematica [A] time = 0.18, size = 99, normalized size = 0.74

$$\frac{\left(8a^2d^2+8abcd-b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{3/2}}+\sqrt{c+dx^2}\left(-\frac{a^2}{x}+abx+\frac{b^2x\left(c+2dx^2\right)}{8d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-(a^2/x) + a*b*x + (b^2*x*(c + 2*d*x^2))/(8*d)) + ((-(b^2*
c^2) + 8*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(3/2
))
```

IntegrateAlgebraic [A] time = 0.19, size = 106, normalized size = 0.80

$$\frac{\left(-8a^2d^2-8abcd+b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{3/2}}+\frac{\sqrt{c+dx^2}\,\left(-8a^2d+8abdx^2+b^2cx^2+2b^2dx^4\right)}{8dx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*d + b^2*c*x^2 + 8*a*b*d*x^2 + 2*b^2*d*x^4))/(8*d*x
) + ((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])
/(8*d^(3/2))
```

fricas [A] time = 1.43, size = 215, normalized size = 1.62

$$\left[-\frac{\left(b^2c^2-8\,abcd-8\,a^2d^2\right)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(2\,b^2d^2x^4-8\,a^2d^2+\left(b^2cd+8\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{16\,d^2x},\ \frac{\left(b^2c^2-8\,abcd-8\,a^2d^2\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(2\,b^2d^2x^4-8\,a^2d^2+\left(b^2cd+8\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{8\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^2*d^2*x^4 - 8*a^2*d^2 + (b^2*c*d + 8*a*b*
d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x), 1/8*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*
sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^4 - 8*a^2*d^2
+ (b^2*c*d + 8*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

giac [A] time = 0.47, size = 126, normalized size = 0.95

$$\frac{2\,a^2c\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{8}\left(2\,b^2x^2+\frac{b^2cd+8\,abd^2}{d^2}\right)\sqrt{dx^2+c}\,x+\frac{\left(b^2c^2\sqrt{d}-8\,abcd^{\frac{3}{2}}-8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/8*(2*b^2*x^2 + (b
^2*c*d + 8*a*b*d^2)/d^2)*sqrt(d*x^2 + c)*x + 1/16*(b^2*c^2*sqrt(d) - 8*a*b*
c*d^(3/2) - 8*a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

maple [A] time = 0.01, size = 163, normalized size = 1.23

$$a^{2}\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+\frac{abc\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}-\frac{b^{2}c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{8d^{\frac{3}{2}}}+\frac{\sqrt{d\,x^{2}+c}\,a^{2}dx}{c}+\sqrt{d\,x^{2}+c}\,abx-\frac{\sqrt{d\,x^{2}+c}\,b^{2}cx}{8d}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}x}{4d}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] 1/4*b^2*x*(d*x^2+c)^(3/2)/d-1/8*b^2*c/d*x*(d*x^2+c)^(1/2)-1/8*b^2*c^2/d^(3/
2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+a*b*x*(d*x^2+c)^(1/2)+a*b*c/d^(1/2)*ln(d^(
1/2)*x+(d*x^2+c)^(1/2))-a^2*(d*x^2+c)^(3/2)/c/x+a^2*d/c*x*(d*x^2+c)^(1/2)+a
^2*d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.08, size = 120, normalized size = 0.90

$$\sqrt{dx^{2}+c}\,abx+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}x}{4\,d}-\frac{\sqrt{dx^{2}+c}\,b^{2}cx}{8\,d}-\frac{b^{2}c^{2}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{abc\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}+a^{2}\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\sqrt{dx^{2}+c}\,a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] sqrt(d*x^2 + c)*a*b*x + 1/4*(d*x^2 + c)^(3/2)*b^2*x/d - 1/8*sqrt(d*x^2 + c)
*b^2*c*x/d - 1/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + a*b*c*arcsinh(d*x
/sqrt(c*d))/sqrt(d) + a^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*
a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{2}+a\right)^{2}\sqrt{d\,x^{2}+c}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^2, x)
```

sympy [A] time = 9.05, size = 219, normalized size = 1.65

$$-\frac{a^{2}\sqrt{c}}{x\sqrt{1+\frac{dx^{2}}{c}}}+a^{2}\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{a^{2}dx}{\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}+ab\sqrt{c}\,x\sqrt{1+\frac{dx^{2}}{c}}+\frac{abc\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{d}}+\frac{b^{2}c^{\frac{3}{2}}x}{8d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3b^{2}\sqrt{c}\,x^{3}}{8\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}c^{2}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{b^{2}dx^{5}}{4\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] -a**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)
) - a**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)*x*sqrt(1 + d*x**2/c
) + a*b*c*asinh(sqrt(d)*x/sqrt(c))/sqrt(d) + b**2*c**(3/2)*x/(8*d*sqrt(1 +
d*x**2/c)) + 3*b**2*sqrt(c)*x**3/(8*sqrt(1 + d*x**2/c)) - b**2*c**2*asinh(s
qrt(d)*x/sqrt(c))/(8*d**(3/2)) + b**2*d*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.590 $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^4}\,dx$

Optimal. Leaf size=111

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}\,(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

Rubi [**A**] time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{bx\sqrt{c+dx^2}\,(4ad+bc)}{2c}+\frac{b(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (b*(b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (a^2*(c + d*x^2)^(3/2))/(3*c*x^
3) - (2*a*b*(c + d*x^2)^(3/2))/(c*x) + (b*(b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^4}\,dx &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}+\frac{\int \frac{\left(6abc+3b^2cx^2\right)\sqrt{c+dx^2}}{x^2}\,dx}{3c}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{(b(bc+4ad))\int \sqrt{c+dx^2}\,dx}{c}\\
&= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{1}{2}(b(bc+4ad))\int\\
&= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{1}{2}(b(bc+4ad))\,\mathrm{Subst}\\
&= \frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{3cx^3}-\frac{2ab\left(c+dx^2\right)^{3/2}}{cx}+\frac{b(bc+4ad)\tanh^{-1}\Big(}{2\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.82

$$\sqrt{c+dx^2}\left(-\frac{a^2}{3x^3}-\frac{a(ad+6bc)}{3cx}+\frac{b^2x}{2}\right)+\frac{b(4ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (-1/3*a^2/x^3 - (a*(6*b*c + a*d))/(3*c*x) + (b^2*x)/2)*Sqrt[c + d*x^2] + (b
*(b*c + 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*Sqrt[d])
```

IntegrateAlgebraic [A] time = 0.21, size = 97, normalized size = 0.87

$$\frac{\sqrt{c+dx^2}\left(-2a^2c-2a^2dx^2-12abcx^2+3b^2cx^4\right)}{6cx^3}+\frac{\left(b^2(-c)-4abd\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c - 12*a*b*c*x^2 - 2*a^2*d*x^2 + 3*b^2*c*x^4))/(6*
c*x^3) + ((-(b^2*c) - 4*a*b*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*Sqrt
[d])
```

fricas [A] time = 1.64, size = 210, normalized size = 1.89

$$\left[\frac{3\left(b^2c^2+4\,abcd\right)\sqrt{d}\,x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(3\,b^2cdx^4-2\,a^2cd-2\left(6\,abcd+a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{12\,cdx^3},-\frac{3\left(b^2c^2+4\,abcd\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(3\,b^2cdx^4-2\,a^2cd-2\left(6\,abcd+a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{6\,cdx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) + 2*(3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c*d + a^2*d^2)*x^
2)*sqrt(d*x^2 + c))/(c*d*x^3), -1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^3*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c
*d + a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c*d*x^3)]
```

giac [B] time = 0.46, size = 188, normalized size = 1.69

$$\frac{1}{2}\,\sqrt{dx^2+c}\,b^2x-\frac{\left(b^2c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{4\,d}+\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+6\,abc^3\sqrt{d}+a^2c^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x - 1/4*(b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*
sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(3/2) - 12*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 6*a*b*c^3*sqrt(d) + a^2*c^2*d^(3/2))/((
sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

maple [A] time = 0.01, size = 122, normalized size = 1.10

$$2ab\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+\frac{b^{2}c\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}+\frac{2\sqrt{d\,x^{2}+c}\,abdx}{c}+\frac{\sqrt{d\,x^{2}+c}\,b^{2}x}{2}-\frac{2\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab}{cx}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}}{3c\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x)
```

```
[Out] 1/2*x*b^2*(d*x^2+c)^(1/2)+1/2*b^2*c/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-2
*a*b*(d*x^2+c)^(3/2)/c/x+2*a*b*d/c*x*(d*x^2+c)^(1/2)+2*a*b*d^(1/2)*ln(d^(1/
2)*x+(d*x^2+c)^(1/2))-1/3*a^2*(d*x^2+c)^(3/2)/c/x^3
```

maxima [A] time = 1.08, size = 86, normalized size = 0.77

$$\frac{1}{2}\sqrt{dx^{2}+c}\,b^{2}x+\frac{b^{2}c\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}+2\,ab\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{2\sqrt{dx^{2}+c}\,ab}{x}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{3\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x + 1/2*b^2*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 2*a*
b*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - 2*sqrt(d*x^2 + c)*a*b/x - 1/3*(d*x^2 + c
)^(3/2)*a^2/(c*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^{2}+a\right)^{2}\sqrt{d\,x^{2}+c}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^4, x)
```

sympy [A] time = 5.46, size = 170, normalized size = 1.53

$$-\frac{a^{2}\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{3x^{2}}-\frac{a^{2}d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{3c}-\frac{2ab\sqrt{c}}{x\sqrt{1+\frac{dx^{2}}{c}}}+2ab\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{2abdx}{\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{b^{2}\sqrt{c}\,x\sqrt{1+\frac{dx^{2}}{c}}}{2}+\frac{b^{2}c\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**4,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2)
 + 1)/(3*c) - 2*a*b*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + 2*a*b*sqrt(d)*asinh(sq
rt(d)*x/sqrt(c)) - 2*a*b*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + b**2*sqrt(c)*x*
sqrt(1 + d*x**2/c)/2 + b**2*c*asinh(sqrt(d)*x/sqrt(c))/(2*sqrt(d))
```

3.591 $\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{x^6}\,dx$

Optimal. Leaf size=103

$$-\frac{a^2 \left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a \left(c+dx^2\right)^{3/2} (5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 451, 277, 217, 206}

$$-\frac{a^2 \left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a \left(c+dx^2\right)^{3/2} (5bc-ad)}{15c^2x^3}-\frac{b^2\sqrt{c+dx^2}}{x}+b^2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -((b^2*Sqrt[c + d*x^2])/x) - (a^2*(c + d*x^2)^(3/2))/(5*c*x^5) - (2*a*(5*b*
c - a*d)*(c + d*x^2)^(3/2))/(15*c^2*x^3) + b^2*Sqrt[d]*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^6}\,dx &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}+\frac{\int \frac{\left(2a(5bc-ad)+5b^2cx^2\right)\sqrt{c+dx^2}}{x^4}\,dx}{5c}\\ &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+b^2\int \frac{\sqrt{c+dx^2}}{x^2}\,dx\\ &= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+\left(b^2d\right)\int \frac{1}{\sqrt{c+dx^2}}\\ &= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+\left(b^2d\right)\mathrm{Subst}\left(\int \frac{}{1-}\right.\\ &= -\frac{b^2\sqrt{c+dx^2}}{x}-\frac{a^2\left(c+dx^2\right)^{3/2}}{5cx^5}-\frac{2a(5bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^3}+b^2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{c+}}\right.\end{aligned}$$

Mathematica [A] time = 0.15, size = 104, normalized size = 1.01

$$b^2\sqrt{d}\,\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)-\frac{\sqrt{c+dx^2}\left(a^2\left(3c^2+cdx^2-2d^2x^4\right)+10abcx^2\left(c+dx^2\right)+15b^2c^2x^4\right)}{15c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c + d*x^2) + a^2*(3*
c^2 + c*d*x^2 - 2*d^2*x^4)))/(c^2*x^5) + b^2*Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt
[c + d*x^2]]
```

IntegrateAlgebraic [A] time = 0.21, size = 113, normalized size = 1.10

$$\frac{\sqrt{c+dx^2}\left(-3a^2c^2-a^2cdx^2+2a^2d^2x^4-10abc^2x^2-10abcdx^4-15b^2c^2x^4\right)}{15c^2x^5}-b^2\sqrt{d}\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*a^2*c^2 - 10*a*b*c^2*x^2 - a^2*c*d*x^2 - 15*b^2*c^2*x^
4 - 10*a*b*c*d*x^4 + 2*a^2*d^2*x^4))/(15*c^2*x^5) - b^2*Sqrt[d]*Log[-(Sqrt[
d]*x) + Sqrt[c + d*x^2]]
```

fricas [A] time = 1.52, size = 221, normalized size = 2.15

$$\left[\frac{15\,b^2c^2\sqrt{d}\,x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\,\left(\left(15\,b^2c^2+10\,abcd-2\,a^2d^2\right)x^4+3\,a^2c^2+\left(10\,abc^2+a^2cd\right)x^2\right)\sqrt{dx^2+c}}{30\,c^2x^5},-\frac{15\,b^2c^2\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(\left(15\,b^2c^2+10\,abcd-2\,a^2d^2\right)x^4+3\,a^2c^2+\left(10\,abc^2+a^2cd\right)x^2\right)\sqrt{dx^2+c}}{15\,c^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b^2*c^2*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) - 2*((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2)*x^4 + 3*a^2*c^2 + (10*a*b*c^2
 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5), -1/15*(15*b^2*c^2*sqrt(-d)*x^5
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2
)*x^4 + 3*a^2*c^2 + (10*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5)]
```

giac [B] time = 0.47, size = 403, normalized size = 3.91

$$-\frac{1}{2}\,b^2\sqrt{d}\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)+\frac{2\left(15\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8b^2c\sqrt{d}+30\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8abd^{\frac{3}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b^2c^2\sqrt{d}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6abcd^{\frac{3}{2}}+30\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6a^2d^{\frac{5}{2}}+90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^2c^3\sqrt{d}+40\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^2d^{\frac{3}{2}}+10\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2cd^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^4\sqrt{d}-20\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^3d^{\frac{3}{2}}+10\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^2d^{\frac{5}{2}}+15\,b^2c^5\sqrt{d}+10\,abc^4d^{\frac{3}{2}}-2\,a^2c^3d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/2*b^2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*b^2*c*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*
b*d^(3/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^2*sqrt(d) - 60*(sqrt(d
)*x - sqrt(d*x^2 + c))^6*a*b*c*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*a^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^3*sqrt(d) + 40*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*d^(3/2) + 10*(sqrt(d)*x - sqrt(d*x^2 +
c))^4*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^4*sqrt(d) -
```

```
20*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^3*d^(3/2) + 10*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a^2*c^2*d^(5/2) + 15*b^2*c^5*sqrt(d) + 10*a*b*c^4*d^(3/2) - 2*
a^2*c^3*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

maple [A] time = 0.02, size = 123, normalized size = 1.19

$$b^2\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+\frac{\sqrt{d\,x^2+c}\,b^2dx}{c}-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}b^2}{cx}+\frac{2\left(d\,x^2+c\right)^{\frac{3}{2}}a^2d}{15c^2x^3}-\frac{2\left(d\,x^2+c\right)^{\frac{3}{2}}ab}{3c\,x^3}-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}a^2}{5c\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(3/2)/c/x^5+2/15*a^2*d/c^2/x^3*(d*x^2+c)^(3/2)-b^2/c/x*(
d*x^2+c)^(3/2)+b^2*d/c*x*(d*x^2+c)^(1/2)+b^2*d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)
^(1/2))-2/3*a*b/c/x^3*(d*x^2+c)^(3/2)
```

maxima [A] time = 1.01, size = 94, normalized size = 0.91

$$b^2\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\sqrt{dx^2+c}\,b^2}{x}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}ab}{3\,cx^3}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}a^2d}{15\,c^2x^3}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*b^2/x - 2/3*(d*x^2 + c
)^(3/2)*a*b/(c*x^3) + 2/15*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^3) - 1/5*(d*x^2 +
 c)^(3/2)*a^2/(c*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^6,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^6, x)
```

sympy [B] time = 5.50, size = 199, normalized size = 1.93

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2}+\frac{2a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}-\frac{b^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}}+b^2\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{b^2dx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**6,x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - a**2*d**(3/2)*sqrt(c/(d*x**2)
 + 1)/(15*c*x**2) + 2*a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - 2*a*b*
sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)
/(3*c) - b**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + b**2*sqrt(d)*asinh(sqrt(d)*x
/sqrt(c)) - b**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c))
```

3.592 $\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^8} \, dx$

Optimal. Leaf size=99

$$-\frac{a^2 \left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{\left(c+dx^2\right)^{3/2} \left(35b^2c^2-4ad(7bc-2ad)\right)}{105c^3x^3}-\frac{2a \left(c+dx^2\right)^{3/2} (7bc-2ad)}{35c^2x^5}$$

Rubi [**A**] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 453, 264}

$$-\frac{\left(c+dx^2\right)^{3/2} \left(8a^2d^2-28abcd+35b^2c^2\right)}{105c^3x^3}-\frac{a^2 \left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{2a \left(c+dx^2\right)^{3/2} (7bc-2ad)}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(7*c*x^7) - (2*a*(7*b*c - 2*a*d)*(c + d*x^2)^(3/2)
)/(35*c^2*x^5) - ((35*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)*(c + d*x^2)^(3/2))/
(105*c^3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^8}\,dx &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}+\frac{\int \frac{\left(2a(7bc-2ad)+7b^2cx^2\right)\sqrt{c+dx^2}}{x^6}\,dx}{7c}\\ &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{2a(7bc-2ad)\left(c+dx^2\right)^{3/2}}{35c^2x^5}-\frac{1}{35}\left(-35b^2+\frac{4ad(7bc-2ad)}{c^2}\right)\int\\ &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{7cx^7}-\frac{2a(7bc-2ad)\left(c+dx^2\right)^{3/2}}{35c^2x^5}-\frac{\left(35b^2-\frac{4ad(7bc-2ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{105cx^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 76, normalized size = 0.77

$$-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(15c^2-12cdx^2+8d^2x^4\right)+14abcx^2\left(3c-2dx^2\right)+35b^2c^2x^4\right)}{105c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] -1/105*((c + d*x^2)^(3/2)*(35*b^2*c^2*x^4 + 14*a*b*c*x^2*(3*c - 2*d*x^2) +
a^2*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4)))/(c^3*x^7)
```

IntegrateAlgebraic [A] time = 0.21, size = 120, normalized size = 1.21

$$\frac{\sqrt{c+dx^2}\left(-15a^2c^3-3a^2c^2dx^2+4a^2cd^2x^4-8a^2d^3x^6-42abc^3x^2-14abc^2dx^4+28abcd^2x^6-35b^2c^3x^4-35b^2c^2dx^6\right)}{105c^3x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*c^3 - 42*a*b*c^3*x^2 - 3*a^2*c^2*d*x^2 - 35*b^2*c
^3*x^4 - 14*a*b*c^2*d*x^4 + 4*a^2*c*d^2*x^4 - 35*b^2*c^2*d*x^6 + 28*a*b*c*d
^2*x^6 - 8*a^2*d^3*x^6))/(105*c^3*x^7)
```

fricas [A] time = 1.43, size = 107, normalized size = 1.08

$$-\frac{\left(\left(35\,b^2c^2d-28\,abcd^2+8\,a^2d^3\right)x^6+15\,a^2c^3+\left(35\,b^2c^3+14\,abc^2d-4\,a^2cd^2\right)x^4+3\left(14\,abc^3+a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{105\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*((35*b^2*c^2*d - 28*a*b*c*d^2 + 8*a^2*d^3)*x^6 + 15*a^2*c^3 + (35*b^
2*c^3 + 14*a*b*c^2*d - 4*a^2*c*d^2)*x^4 + 3*(14*a*b*c^3 + a^2*c^2*d)*x^2)*s
qrt(d*x^2 + c)/(c^3*x^7)
```

giac [B] time = 0.44, size = 490, normalized size = 4.95

$$\frac{2\left(105\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2d^{\frac{3}{2}}-420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2cd^{\frac{3}{2}}+420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abd^{\frac{5}{2}}+665\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}b^2c^2d^{\frac{3}{2}}-700\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}abcd^{\frac{5}{2}}+560\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}a^2d^{\frac{7}{2}}-560\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}b^2c^3d^{\frac{3}{2}}+280\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}abc^2d^{\frac{5}{2}}+280\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}a^2cd^{\frac{7}{2}}+315\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}b^2c^4d^{\frac{3}{2}}-168\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}abc^3d^{\frac{5}{2}}+168\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}a^2c^2d^{\frac{7}{2}}-140\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}b^2c^5d^{\frac{3}{2}}+196\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}abc^4d^{\frac{5}{2}}-56\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}a^2c^3d^{\frac{7}{2}}+35\,b^2c^6d^{\frac{3}{2}}-28\,abc^5d^{\frac{5}{2}}+8\,a^2c^4d^{\frac{7}{2}}\right)}{105\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*d^(3/2) - 420*(sqrt(d)*x -
sqrt(d*x^2 + c))^10*b^2*c*d^(3/2) + 420*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*
b*d^(5/2) + 665*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*d^(3/2) - 700*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(5/2) + 560*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a^2*d^(7/2) - 560*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*d^(3/2) + 280
*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(5/2) + 280*(sqrt(d)*x - sqrt(d*
x^2 + c))^6*a^2*c*d^(7/2) + 315*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*d^(
3/2) - 168*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(5/2) + 168*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a^2*c^2*d^(7/2) - 140*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*b^2*c^5*d^(3/2) + 196*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(5/2) - 56
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(7/2) + 35*b^2*c^6*d^(3/2) - 28*
a*b*c^5*d^(5/2) + 8*a^2*c^4*d^(7/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^
7
```

maple [A] time = 0.01, size = 78, normalized size = 0.79

$$-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(8a^{2}d^{2}x^{4}-28abcd\,x^{4}+35b^{2}c^{2}x^{4}-12a^{2}cd\,x^{2}+42ab\,c^{2}x^{2}+15a^{2}c^{2}\right)}{105c^{3}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x)
```

```
[Out] -1/105*(d*x^2+c)^(3/2)*(8*a^2*d^2*x^4-28*a*b*c*d*x^4+35*b^2*c^2*x^4-12*a^2*
c*d*x^2+42*a*b*c^2*x^2+15*a^2*c^2)/x^7/c^3
```

maxima [A] time = 1.10, size = 124, normalized size = 1.25

$$-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}}{3\,cx^{3}}+\frac{4\left(dx^{2}+c\right)^{\frac{3}{2}}abd}{15\,c^{2}x^{3}}-\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{105\,c^{3}x^{3}}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}ab}{5\,cx^{5}}+\frac{4\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d}{35\,c^{2}x^{5}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{7\,cx^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/3*(d*x^2 + c)^(3/2)*b^2/(c*x^3) + 4/15*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^3)
 - 8/105*(d*x^2 + c)^(3/2)*a^2*d^2/(c^3*x^3) - 2/5*(d*x^2 + c)^(3/2)*a*b/(c
*x^5) + 4/35*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^5) - 1/7*(d*x^2 + c)^(3/2)*a^2/
(c*x^7)
```

mupad [**B**] time = 1.63, size = 181, normalized size = 1.83

$$\frac{4\,a^2\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^3}-\frac{b^2\,\sqrt{d\,x^2+c}}{3\,x^3}-\frac{2\,a\,b\,\sqrt{d\,x^2+c}}{5\,x^5}-\frac{a^2\,\sqrt{d\,x^2+c}}{7\,x^7}-\frac{8\,a^2\,d^3\,\sqrt{d\,x^2+c}}{105\,c^3\,x}-\frac{a^2\,d\,\sqrt{d\,x^2+c}}{35\,c\,x^5}-\frac{b^2\,d\,\sqrt{d\,x^2+c}}{3\,c\,x}+\frac{4\,a\,b\,d^2\,\sqrt{d\,x^2+c}}{15\,c^2\,x}-\frac{2\,a\,b\,d\,\sqrt{d\,x^2+c}}{15\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^8,x)
```

```
[Out] (4*a^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (b^2*(c + d*x^2)^(1/2))/(3*x^
3) - (2*a*b*(c + d*x^2)^(1/2))/(5*x^5) - (a^2*(c + d*x^2)^(1/2))/(7*x^7) -
(8*a^2*d^3*(c + d*x^2)^(1/2))/(105*c^3*x) - (a^2*d*(c + d*x^2)^(1/2))/(35*c
*x^5) - (b^2*d*(c + d*x^2)^(1/2))/(3*c*x) + (4*a*b*d^2*(c + d*x^2)^(1/2))/(
15*c^2*x) - (2*a*b*d*(c + d*x^2)^(1/2))/(15*c*x^3)
```

sympy [**B**] time = 4.51, size = 510, normalized size = 5.15

$$-\frac{15a^2c^5d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{33a^2c^4d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{17a^2c^3d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{3a^2c^2d^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{12a^2cd^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{8a^2d^{\frac{19}{2}}x^{10}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2}+\frac{4abd^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{b^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**8,x)
```

```
[Out] -15*a**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*
d**5*x**8 + 105*c**3*d**6*x**10) - 33*a**2*c**4*d**(11/2)*x**2*sqrt(c/(d*x*
*2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) -
17*a**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*
c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*a**2*c**2*d**(15/2)*x**6*sqrt(c/(
d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10
) - 12*a**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*a**2*d**(19/2)*x**10*sqrt(c/(d*x
**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) -
 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a*b*d**(3/2)*sqrt(c/(d*x**
2) + 1)/(15*c*x**2) + 4*a*b*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - b**2*
sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/
(3*c)
```

3.593 $\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{x^{10}}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{2d\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{315c^4x^3}-\frac{\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{105c^3x^5}-\frac{2a\left(c+dx^2\right)}{21}$$

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 271, 264}

$$-\frac{\left(c+dx^2\right)^{3/2}\left(8a^2d^2-24abcd+21b^2c^2\right)}{105c^3x^5}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{2d\left(c+dx^2\right)^{3/2}\left(21b^2c^2-8ad(3bc-ad)\right)}{315c^4x^3}-\frac{2a\left(c+dx^2\right)^{3/2}(3bc-ad)}{21c^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(9*c*x^9) - (2*a*(3*b*c - a*d)*(c + d*x^2)^(3/2))/
(21*c^2*x^7) - ((21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*(c + d*x^2)^(3/2))/(1
05*c^3*x^5) + (2*d*(21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(c + d*x^2)^(3/2))/(3
15*c^4*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{10}}\,dx &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}+\frac{\int \frac{\left(6a(3bc-ad)+9b^2cx^2\right)\sqrt{c+dx^2}}{x^8}\,dx}{9c}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^7}-\frac{1}{21}\left(-21b^2+\frac{8ad(3bc-ad)}{c^2}\right)\int\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^7}-\frac{\left(21b^2-\frac{8ad(3bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{105cx^5}-\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}-\frac{2a(3bc-ad)\left(c+dx^2\right)^{3/2}}{21c^2x^7}-\frac{\left(21b^2-\frac{8ad(3bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{105cx^5}+
\end{aligned}$$

Mathematica [A] time = 0.15, size = 108, normalized size = 0.76

$$-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6\right)+6abcx^2\left(15c^2-12cdx^2+8d^2x^4\right)+21b^2c^2x^4\left(3c-2dx^2\right)\right)}{315c^4x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] -1/315*((c + d*x^2)^(3/2)*(21*b^2*c^2*x^4*(3*c - 2*d*x^2) + 6*a*b*c*x^2*(15
*c^2 - 12*c*d*x^2 + 8*d^2*x^4) + a^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4
- 16*d^3*x^6)))/(c^4*x^9)
```

IntegrateAlgebraic [A] time = 0.24, size = 161, normalized size = 1.13

$$\frac{\sqrt{c+dx^2}\left(-35a^2c^4-5a^2c^3dx^2+6a^2c^2d^2x^4-8a^2cd^3x^6+16a^2d^4x^8-90abc^4x^2-18abc^3dx^4+24abc^2d^2x^6-48abcd^3x^8-63b^2c^4x^4-21b^2c^3dx^6+42b^2c^2d^2x^8\right)}{315c^4x^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-35*a^2*c^4 - 90*a*b*c^4*x^2 - 5*a^2*c^3*d*x^2 - 63*b^2*c
^4*x^4 - 18*a*b*c^3*d*x^4 + 6*a^2*c^2*d^2*x^4 - 21*b^2*c^3*d*x^6 + 24*a*b*c
^2*d^2*x^6 - 8*a^2*c*d^3*x^6 + 42*b^2*c^2*d^2*x^8 - 48*a*b*c*d^3*x^8 + 16*a
^2*d^4*x^8))/(315*c^4*x^9)
```

fricas [A] time = 2.04, size = 147, normalized size = 1.03

$$\frac{\left(2\left(21\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4\right)x^8-\left(21\,b^2c^3d-24\,abc^2d^2+8\,a^2cd^3\right)x^6-35\,a^2c^4-3\left(21\,b^2c^4+6\,abc^3d-2\,a^2c^2d^2\right)x^4-5\left(18\,abc^4+a^2c^3d\right)x^2\right)\sqrt{dx^2+c}}{315\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(21*b^2*c^2*d^2 - 24*a*b*c*d^3 + 8*a^2*d^4)*x^8 - (21*b^2*c^3*d -
24*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^6 - 35*a^2*c^4 - 3*(21*b^2*c^4 + 6*a*b*c^3*
d - 2*a^2*c^2*d^2)*x^4 - 5*(18*a*b*c^4 + a^2*c^3*d)*x^2)*sqrt(d*x^2 + c)/(c
^4*x^9)
```

giac [B] time = 0.44, size = 579, normalized size = 4.05

$$\frac{4\left(315\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2d^{\frac{5}{2}}-1155\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2cd^{\frac{5}{2}}+1680\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}abd^{\frac{7}{2}}+1575\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2c^2d^{\frac{5}{2}}-2520\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abcd^{\frac{7}{2}}+2520\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}a^2d^{\frac{9}{2}}-1071\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}b^2c^3d^{\frac{5}{2}}+504\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}abc^2d^{\frac{7}{2}}+1512\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}a^2cd^{\frac{9}{2}}+609\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}b^2c^4d^{\frac{5}{2}}-336\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}abc^3d^{\frac{7}{2}}+672\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}a^2c^2d^{\frac{9}{2}}-441\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}b^2c^5d^{\frac{5}{2}}+864\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}abc^4d^{\frac{7}{2}}-288\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}a^2c^3d^{\frac{9}{2}}+189\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}b^2c^6d^{\frac{5}{2}}-216\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}abc^5d^{\frac{7}{2}}+72\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}a^2c^4d^{\frac{9}{2}}-21\,b^2c^7d^{\frac{5}{2}}+24\,abc^6d^{\frac{7}{2}}-8\,a^2c^5d^{\frac{9}{2}}\right)}{315\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(d)*x - sqrt(d*x^2 + c))^14*b^2*d^(5/2) - 1155*(sqrt(d)*x -
 sqrt(d*x^2 + c))^12*b^2*c*d^(5/2) + 1680*(sqrt(d)*x - sqrt(d*x^2 + c))^12*
a*b*d^(7/2) + 1575*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*c^2*d^(5/2) - 2520*
(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c*d^(7/2) + 2520*(sqrt(d)*x - sqrt(d*x
^2 + c))^10*a^2*d^(9/2) - 1071*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*d^(5
/2) + 504*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(7/2) + 1512*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^2*c*d^(9/2) + 609*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^2*c^4*d^(5/2) - 336*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(7/2) + 672*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(9/2) - 441*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*b^2*c^5*d^(5/2) + 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^
(7/2) - 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(9/2) + 189*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*b^2*c^6*d^(5/2) - 216*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*b*c^5*d^(7/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(9/2) - 21
*b^2*c^7*d^(5/2) + 24*a*b*c^6*d^(7/2) - 8*a^2*c^5*d^(9/2))/((sqrt(d)*x - sq
rt(d*x^2 + c))^2 - c)^9
```

maple [A] time = 0.01, size = 117, normalized size = 0.82

$$-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(-16a^2d^3x^6+48abc\,d^2x^6-42b^2c^2d\,x^6+24a^2c\,d^2x^4-72ab\,c^2d\,x^4+63b^2c^3x^4-30a^2c^2d\,x^2+90ab\,c^3x^2+35a^2c^3\right)}{315c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x)
```

```
[Out] -1/315*(d*x^2+c)^(3/2)*(-16*a^2*d^3*x^6+48*a*b*c*d^2*x^6-42*b^2*c^2*d*x^6+2
4*a^2*c*d^2*x^4-72*a*b*c^2*d*x^4+63*b^2*c^3*x^4-30*a^2*c^2*d*x^2+90*a*b*c^3
*x^2+35*a^2*c^3)/x^9/c^4
```

maxima [A] time = 1.16, size = 190, normalized size = 1.33

$$\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d}{15\,c^{2}x^{3}}-\frac{16\left(dx^{2}+c\right)^{\frac{3}{2}}abd^{2}}{105\,c^{3}x^{3}}+\frac{16\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}}{315\,c^{4}x^{3}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}}{5\,cx^{5}}+\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}abd}{35\,c^{2}x^{5}}-\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{105\,c^{3}x^{5}}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}ab}{7\,cx^{7}}+\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d}{21\,c^{2}x^{7}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{9\,cx^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] 2/15*(d*x^2 + c)^(3/2)*b^2*d/(c^2*x^3) - 16/105*(d*x^2 + c)^(3/2)*a*b*d^2/(
c^3*x^3) + 16/315*(d*x^2 + c)^(3/2)*a^2*d^3/(c^4*x^3) - 1/5*(d*x^2 + c)^(3/
2)*b^2/(c*x^5) + 8/35*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^5) - 8/105*(d*x^2 + c)
^(3/2)*a^2*d^2/(c^3*x^5) - 2/7*(d*x^2 + c)^(3/2)*a*b/(c*x^7) + 2/21*(d*x^2
+ c)^(3/2)*a^2*d/(c^2*x^7) - 1/9*(d*x^2 + c)^(3/2)*a^2/(c*x^9)
```

mupad [B] time = 2.29, size = 249, normalized size = 1.74

$$\frac{2\,a^{2}\,d^{2}\,\sqrt{d\,x^{2}+c}}{105\,c^{2}\,x^{5}}-\frac{b^{2}\,\sqrt{d\,x^{2}+c}}{5\,x^{5}}-\frac{2\,a\,b\,\sqrt{d\,x^{2}+c}}{7\,x^{7}}-\frac{a^{2}\,\sqrt{d\,x^{2}+c}}{9\,x^{9}}-\frac{8\,a^{2}\,d^{3}\,\sqrt{d\,x^{2}+c}}{315\,c^{3}\,x^{3}}+\frac{16\,a^{2}\,d^{4}\,\sqrt{d\,x^{2}+c}}{315\,c^{4}\,x}+\frac{2\,b^{2}\,d^{2}\,\sqrt{d\,x^{2}+c}}{15\,c^{2}\,x}-\frac{a^{2}\,d\,\sqrt{d\,x^{2}+c}}{63\,c\,x^{7}}-\frac{b^{2}\,d\,\sqrt{d\,x^{2}+c}}{15\,c\,x^{3}}+\frac{8\,a\,b\,d^{2}\,\sqrt{d\,x^{2}+c}}{105\,c^{2}\,x^{3}}-\frac{16\,a\,b\,d^{3}\,\sqrt{d\,x^{2}+c}}{105\,c^{3}\,x}-\frac{2\,a\,b\,d\,\sqrt{d\,x^{2}+c}}{35\,c\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^10,x)
```

```
[Out] (2*a^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^5) - (b^2*(c + d*x^2)^(1/2))/(5*x^
5) - (2*a*b*(c + d*x^2)^(1/2))/(7*x^7) - (a^2*(c + d*x^2)^(1/2))/(9*x^9) -
(8*a^2*d^3*(c + d*x^2)^(1/2))/(315*c^3*x^3) + (16*a^2*d^4*(c + d*x^2)^(1/2)
)/(315*c^4*x) + (2*b^2*d^2*(c + d*x^2)^(1/2))/(15*c^2*x) - (a^2*d*(c + d*x^
2)^(1/2))/(63*c*x^7) - (b^2*d*(c + d*x^2)^(1/2))/(15*c*x^3) + (8*a*b*d^2*(c
 + d*x^2)^(1/2))/(105*c^2*x^3) - (16*a*b*d^3*(c + d*x^2)^(1/2))/(105*c^3*x)
 - (2*a*b*d*(c + d*x^2)^(1/2))/(35*c*x^5)
```

sympy [B] time = 5.36, size = 1061, normalized size = 7.42

$$-\frac{35a^{2}c^{7}d^{\frac{19}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}-\frac{110a^{2}c^{6}d^{\frac{21}{2}}x^{2}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}-\frac{114a^{2}c^{5}d^{\frac{23}{2}}x^{4}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}-\frac{40a^{2}c^{4}d^{\frac{25}{2}}x^{6}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}+\frac{5a^{2}c^{3}d^{\frac{27}{2}}x^{8}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}+\frac{30a^{2}c^{2}d^{\frac{29}{2}}x^{10}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}+\frac{40a^{2}cd^{\frac{31}{2}}x^{12}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}+\frac{16a^{2}d^{\frac{33}{2}}x^{14}\sqrt{\frac{c}{dx^{2}}+1}}{315c^{7}d^{9}x^{8}+945c^{6}d^{10}x^{10}+945c^{5}d^{11}x^{12}+315c^{4}d^{12}x^{14}}-\frac{30abc^{5}d^{\frac{9}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{66abc^{4}d^{\frac{11}{2}}x^{2}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{34abc^{3}d^{\frac{13}{2}}x^{4}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{6abc^{2}d^{\frac{15}{2}}x^{6}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{24abcd^{\frac{17}{2}}x^{8}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{16abd^{\frac{19}{2}}x^{10}\sqrt{\frac{c}{dx^{2}}+1}}{105c^{5}d^{4}x^{6}+210c^{4}d^{5}x^{8}+105c^{3}d^{6}x^{10}}-\frac{b^{2}\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{5x^{4}}-\frac{b^{2}d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{15cx^{2}}+\frac{2b^{2}d^{\frac{5}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{15c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**10,x)
```

```
[Out] -35*a**2*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 110*a**2*c**6
```

```
*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x
**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 114*a**2*c**5*d**(23/
2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 9
45*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 40*a**2*c**4*d**(25/2)*x**6*s
qrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d
**11*x**12 + 315*c**4*d**12*x**14) + 5*a**2*c**3*d**(27/2)*x**8*sqrt(c/(d*x
**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12
 + 315*c**4*d**12*x**14) + 30*a**2*c**2*d**(29/2)*x**10*sqrt(c/(d*x**2) + 1
)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c
**4*d**12*x**14) + 40*a**2*c*d**(31/2)*x**12*sqrt(c/(d*x**2) + 1)/(315*c**7
*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x
**14) + 16*a**2*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 30*a*
b*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x*
*8 + 105*c**3*d**6*x**10) - 66*a*b*c**4*d**(11/2)*x**2*sqrt(c/(d*x**2) + 1)
/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 34*a*b*c
**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5
*x**8 + 105*c**3*d**6*x**10) - 6*a*b*c**2*d**(15/2)*x**6*sqrt(c/(d*x**2) +
1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 24*a*b
*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*
x**8 + 105*c**3*d**6*x**10) - 16*a*b*d**(19/2)*x**10*sqrt(c/(d*x**2) + 1)/(
105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - b**2*sqrt(
d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - b**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*c
*x**2) + 2*b**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2)
```

3.594 $\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{x^{12}}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{8d^2\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{3465c^5x^3}+\frac{4d\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{1155c^4x^5}-(c+$$

Rubi [A] time = 0.17, antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 271, 264}

$$-\frac{\left(c+dx^2\right)^{3/2}\left(16a^2d^2-44abcd+33b^2c^2\right)}{231c^3x^7}-\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{8d^2\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{3465c^5x^3}+\frac{4d\left(c+dx^2\right)^{3/2}\left(33b^2c^2-4ad(11bc-4ad)\right)}{1155c^4x^5}-\frac{2a\left(c+dx^2\right)^{3/2}(11bc-4ad)}{99c^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -(a^2*(c + d*x^2)^(3/2))/(11*c*x^11) - (2*a*(11*b*c - 4*a*d)*(c + d*x^2)^(3
/2))/(99*c^2*x^9) - ((33*b^2*c^2 - 44*a*b*c*d + 16*a^2*d^2)*(c + d*x^2)^(3/
2))/(231*c^3*x^7) + (4*d*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d*x^2)^
(3/2))/(1155*c^4*x^5) - (8*d^2*(33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(c + d
*x^2)^(3/2))/(3465*c^5*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{12}}\,dx &= -\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}+\frac{\int \frac{\left(2a(11bc-4ad)+11b^2cx^2\right)\sqrt{c+dx^2}}{x^{10}}\,dx}{11c}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{2a(11bc-4ad)\left(c+dx^2\right)^{3/2}}{99c^2x^9}-\frac{1}{33}\left(-33b^2+\frac{4ad(11bc-4ad)}{c^2}\right)\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{2a(11bc-4ad)\left(c+dx^2\right)^{3/2}}{99c^2x^9}-\frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)\left(c+dx^2\right)^{3/}}{231cx^7}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{2a(11bc-4ad)\left(c+dx^2\right)^{3/2}}{99c^2x^9}-\frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)\left(c+dx^2\right)^{3/}}{231cx^7}\\
&= -\frac{a^2\left(c+dx^2\right)^{3/2}}{11cx^{11}}-\frac{2a(11bc-4ad)\left(c+dx^2\right)^{3/2}}{99c^2x^9}-\frac{\left(33b^2-\frac{4ad(11bc-4ad)}{c^2}\right)\left(c+dx^2\right)^{3/}}{231cx^7}
\end{aligned}
$$

Mathematica [A]   time = 0.14, size = 141, normalized size = 0.75

$$
-\frac{\left(c+dx^2\right)^{3/2}\left(a^2\left(315c^4-280c^3dx^2+240c^2d^2x^4-192cd^3x^6+128d^4x^8\right)+22abcx^2\left(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6\right)+33b^2c^2x^4\left(15c^2-12cdx^2+8d^2x^4\right)\right)}{3465c^5x^{11}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] -1/3465*((c + d*x^2)^(3/2)*(33*b^2*c^2*x^4*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4
) + 22*a*b*c*x^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4 - 16*d^3*x^6) + a^2*
(315*c^4 - 280*c^3*d*x^2 + 240*c^2*d^2*x^4 - 192*c*d^3*x^6 + 128*d^4*x^8)))
/(c^5*x^11)
```

IntegrateAlgebraic [A] time = 0.29, size = 202, normalized size = 1.07

$$\frac{\sqrt{c+dx^2}\left(-315a^2c^5-35a^2c^4dx^2+40a^2c^3d^2x^4-48a^2c^2d^3x^6+64a^2cd^4x^8-128a^2d^5x^{10}-770abc^5x^2-110abc^4dx^4+132abc^3d^2x^6-176abc^2d^3x^8+352abcd^4x^{10}-495b^2c^5x^4-99b^2c^4dx^6+132b^2c^3d^2x^8-264b^2c^2d^3x^{10}\right)}{3465c^5x^{11}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-315*a^2*c^5 - 770*a*b*c^5*x^2 - 35*a^2*c^4*d*x^2 - 495*b
^2*c^5*x^4 - 110*a*b*c^4*d*x^4 + 40*a^2*c^3*d^2*x^4 - 99*b^2*c^4*d*x^6 + 13
2*a*b*c^3*d^2*x^6 - 48*a^2*c^2*d^3*x^6 + 132*b^2*c^3*d^2*x^8 - 176*a*b*c^2*
d^3*x^8 + 64*a^2*c*d^4*x^8 - 264*b^2*c^2*d^3*x^10 + 352*a*b*c*d^4*x^10 - 12
8*a^2*d^5*x^10))/(3465*c^5*x^11)
```

fricas [A] time = 2.33, size = 185, normalized size = 0.98

$$-\frac{\left(8\left(33\,b^2c^2d^3-44\,abcd^4+16\,a^2d^5\right)x^{10}-4\left(33\,b^2c^3d^2-44\,abc^2d^3+16\,a^2cd^4\right)x^8+315\,a^2c^5+3\left(33\,b^2c^4d-44\,abc^3d^2+16\,a^2c^2d^3\right)x^6+5\left(99\,b^2c^5+22\,abc^4d-8\,a^2c^3d^2\right)x^4+35\left(22\,abc^5+a^2c^4d\right)x^2\right)\sqrt{dx^2+c}}{3465\,c^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(8*(33*b^2*c^2*d^3 - 44*a*b*c*d^4 + 16*a^2*d^5)*x^10 - 4*(33*b^2*c^
3*d^2 - 44*a*b*c^2*d^3 + 16*a^2*c*d^4)*x^8 + 315*a^2*c^5 + 3*(33*b^2*c^4*d
- 44*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x^6 + 5*(99*b^2*c^5 + 22*a*b*c^4*d - 8*a
^2*c^3*d^2)*x^4 + 35*(22*a*b*c^5 + a^2*c^4*d)*x^2)*sqrt(d*x^2 + c)/(c^5*x^1
1)
```

giac [B] time = 0.42, size = 668, normalized size = 3.53

$$\frac{16\left(2310\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{16}b^2d^{\frac{7}{2}}-8085\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2cd^{\frac{7}{2}}+13860\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}abd^{\frac{9}{2}}+9933\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2c^2d^{\frac{7}{2}}-19404\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}abcd^{\frac{9}{2}}+22176\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}a^2d^{\frac{11}{2}}-5313\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2c^3d^{\frac{7}{2}}+924\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abc^2d^{\frac{9}{2}}+14784\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}a^2cd^{\frac{11}{2}}+2805\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}b^2c^4d^{\frac{7}{2}}-660\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}abc^3d^{\frac{9}{2}}+5280\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}a^2c^2d^{\frac{11}{2}}-3135\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}b^2c^5d^{\frac{7}{2}}+7260\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}abc^4d^{\frac{9}{2}}-2640\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{6}a^2c^3d^{\frac{11}{2}}+1815\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}b^2c^6d^{\frac{7}{2}}-2420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}abc^5d^{\frac{9}{2}}+880\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{4}a^2c^4d^{\frac{11}{2}}-363\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}b^2c^7d^{\frac{7}{2}}+484\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}abc^6d^{\frac{9}{2}}-176\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{2}a^2c^5d^{\frac{11}{2}}+33\,b^2c^8d^{\frac{7}{2}}-44\,abc^7d^{\frac{9}{2}}+16\,a^2c^6d^{\frac{11}{2}}\right)}{3465\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(d)*x - sqrt(d*x^2 + c))^16*b^2*d^(7/2) - 8085*(sqrt(d)*
x - sqrt(d*x^2 + c))^14*b^2*c*d^(7/2) + 13860*(sqrt(d)*x - sqrt(d*x^2 + c))
^14*a*b*d^(9/2) + 9933*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*c^2*d^(7/2) - 1
9404*(sqrt(d)*x - sqrt(d*x^2 + c))^12*a*b*c*d^(9/2) + 22176*(sqrt(d)*x - sq
rt(d*x^2 + c))^12*a^2*d^(11/2) - 5313*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*
c^3*d^(7/2) + 924*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c^2*d^(9/2) + 14784*
(sqrt(d)*x - sqrt(d*x^2 + c))^10*a^2*c*d^(11/2) + 2805*(sqrt(d)*x - sqrt(d*
x^2 + c))^8*b^2*c^4*d^(7/2) - 660*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^3*d
^(9/2) + 5280*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*c^2*d^(11/2) - 3135*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*b^2*c^5*d^(7/2) + 7260*(sqrt(d)*x - sqrt(d*x^2 +
```

```
 c))^6*a*b*c^4*d^(9/2) - 2640*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^3*d^(11
/2) + 1815*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^6*d^(7/2) - 2420*(sqrt(d)*
x - sqrt(d*x^2 + c))^4*a*b*c^5*d^(9/2) + 880*(sqrt(d)*x - sqrt(d*x^2 + c))^
4*a^2*c^4*d^(11/2) - 363*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^7*d^(7/2) +
484*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^6*d^(9/2) - 176*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a^2*c^5*d^(11/2) + 33*b^2*c^8*d^(7/2) - 44*a*b*c^7*d^(9/2) +
 16*a^2*c^6*d^(11/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^11
```

maple [A] time = 0.01, size = 158, normalized size = 0.84

$$-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(128a^{2}d^{4}x^{8}-352abc\,d^{3}x^{8}+264b^{2}c^{2}d^{2}x^{8}-192a^{2}c\,d^{3}x^{6}+528ab\,c^{2}d^{2}x^{6}-396b^{2}c^{3}d\,x^{6}+240a^{2}c^{2}d^{2}x^{4}-660ab\,c^{3}d\,x^{4}+495b^{2}c^{4}x^{4}-280a^{2}c^{3}d\,x^{2}+770ab\,c^{4}x^{2}+315a^{2}c^{4}\right)}{3465c^{5}x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x)
```

```
[Out] -1/3465*(d*x^2+c)^(3/2)*(128*a^2*d^4*x^8-352*a*b*c*d^3*x^8+264*b^2*c^2*d^2*
x^8-192*a^2*c*d^3*x^6+528*a*b*c^2*d^2*x^6-396*b^2*c^3*d*x^6+240*a^2*c^2*d^2
*x^4-660*a*b*c^3*d*x^4+495*b^2*c^4*x^4-280*a^2*c^3*d*x^2+770*a*b*c^4*x^2+31
5*a^2*c^4)/x^11/c^5
```

maxima [A] time = 1.12, size = 258, normalized size = 1.37

$$-\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d^{2}}{105\,c^{3}x^{3}}+\frac{32\left(dx^{2}+c\right)^{\frac{3}{2}}abd^{3}}{315\,c^{4}x^{3}}-\frac{128\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{4}}{3465\,c^{5}x^{3}}+\frac{4\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d}{35\,c^{2}x^{5}}-\frac{16\left(dx^{2}+c\right)^{\frac{3}{2}}abd^{2}}{105\,c^{3}x^{5}}+\frac{64\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}}{1155\,c^{4}x^{5}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}}{7\,cx^{7}}+\frac{4\left(dx^{2}+c\right)^{\frac{3}{2}}abd}{21\,c^{2}x^{7}}-\frac{16\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{231\,c^{3}x^{7}}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}ab}{9\,cx^{9}}+\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d}{99\,c^{2}x^{9}}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{11\,cx^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="maxima")
```

```
[Out] -8/105*(d*x^2 + c)^(3/2)*b^2*d^2/(c^3*x^3) + 32/315*(d*x^2 + c)^(3/2)*a*b*d
^3/(c^4*x^3) - 128/3465*(d*x^2 + c)^(3/2)*a^2*d^4/(c^5*x^3) + 4/35*(d*x^2 +
 c)^(3/2)*b^2*d/(c^2*x^5) - 16/105*(d*x^2 + c)^(3/2)*a*b*d^2/(c^3*x^5) + 64
/1155*(d*x^2 + c)^(3/2)*a^2*d^3/(c^4*x^5) - 1/7*(d*x^2 + c)^(3/2)*b^2/(c*x^
7) + 4/21*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^7) - 16/231*(d*x^2 + c)^(3/2)*a^2*
d^2/(c^3*x^7) - 2/9*(d*x^2 + c)^(3/2)*a*b/(c*x^9) + 8/99*(d*x^2 + c)^(3/2)*
a^2*d/(c^2*x^9) - 1/11*(d*x^2 + c)^(3/2)*a^2/(c*x^11)
```

mupad [B] time = 2.99, size = 317, normalized size = 1.68

$$\frac{8\,a^{2}\,d^{2}\,\sqrt{d\,x^{2}+c}}{693\,c^{2}\,x^{7}}-\frac{b^{2}\,\sqrt{d\,x^{2}+c}}{7\,x^{7}}-\frac{2\,a\,b\,\sqrt{d\,x^{2}+c}}{9\,x^{9}}-\frac{a^{2}\,\sqrt{d\,x^{2}+c}}{11\,x^{11}}-\frac{16\,a^{2}\,d^{3}\,\sqrt{d\,x^{2}+c}}{1155\,c^{3}\,x^{5}}+\frac{64\,a^{2}\,d^{4}\,\sqrt{d\,x^{2}+c}}{3465\,c^{4}\,x^{3}}-\frac{128\,a^{2}\,d^{5}\,\sqrt{d\,x^{2}+c}}{3465\,c^{5}\,x}+\frac{4\,b^{2}\,d^{2}\,\sqrt{d\,x^{2}+c}}{105\,c^{2}\,x^{3}}-\frac{8\,b^{2}\,d^{3}\,\sqrt{d\,x^{2}+c}}{105\,c^{3}\,x}-\frac{a^{2}\,d\,\sqrt{d\,x^{2}+c}}{99\,c\,x^{9}}-\frac{b^{2}\,d\,\sqrt{d\,x^{2}+c}}{35\,c\,x^{5}}+\frac{4\,a\,b\,d^{2}\,\sqrt{d\,x^{2}+c}}{105\,c^{2}\,x^{5}}-\frac{16\,a\,b\,d^{3}\,\sqrt{d\,x^{2}+c}}{315\,c^{3}\,x^{3}}+\frac{32\,a\,b\,d^{4}\,\sqrt{d\,x^{2}+c}}{315\,c^{4}\,x}-\frac{2\,a\,b\,d\,\sqrt{d\,x^{2}+c}}{63\,c\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^12,x)
```

```
[Out] (8*a^2*d^2*(c + d*x^2)^(1/2))/(693*c^2*x^7) - (b^2*(c + d*x^2)^(1/2))/(7*x^
7) - (2*a*b*(c + d*x^2)^(1/2))/(9*x^9) - (a^2*(c + d*x^2)^(1/2))/(11*x^11)
```

```
- (16*a^2*d^3*(c + d*x^2)^(1/2))/(1155*c^3*x^5) + (64*a^2*d^4*(c + d*x^2)^(
1/2))/(3465*c^4*x^3) - (128*a^2*d^5*(c + d*x^2)^(1/2))/(3465*c^5*x) + (4*b^
2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (8*b^2*d^3*(c + d*x^2)^(1/2))/(105
*c^3*x) - (a^2*d*(c + d*x^2)^(1/2))/(99*c*x^9) - (b^2*d*(c + d*x^2)^(1/2))/
(35*c*x^5) + (4*a*b*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^5) - (16*a*b*d^3*(c +
 d*x^2)^(1/2))/(315*c^3*x^3) + (32*a*b*d^4*(c + d*x^2)^(1/2))/(315*c^4*x) -
 (2*a*b*d*(c + d*x^2)^(1/2))/(63*c*x^7)
```

sympy [B] time = 8.28, size = 1856, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**12,x)
```

```
[Out] -315*a**2*c**9*d**(33/2)*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 1386
0*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465
*c**5*d**20*x**18) - 1295*a**2*c**8*d**(35/2)*x**2*sqrt(c/(d*x**2) + 1)/(34
65*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 138
60*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1990*a**2*c**7*d**(37/2)*x**
4*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20
790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 13
58*a**2*c**6*d**(39/2)*x**6*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 1
3860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3
465*c**5*d**20*x**18) - 343*a**2*c**5*d**(41/2)*x**8*sqrt(c/(d*x**2) + 1)/(
3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 1
3860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 35*a**2*c**4*d**(43/2)*x**
10*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 2
80*a**2*c**3*d**(45/2)*x**12*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 +
13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 +
3465*c**5*d**20*x**18) - 560*a**2*c**2*d**(47/2)*x**14*sqrt(c/(d*x**2) + 1)
/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 +
 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 448*a**2*c*d**(49/2)*x**
16*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 2
0790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1
28*a**2*d**(51/2)*x**18*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860
*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*
c**5*d**20*x**18) - 70*a*b*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d*
*9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**1
4) - 220*a*b*c**6*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 +
 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 228*
a*b*c**5*d**(23/2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6
*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 80*a*b*c**4*d
**(25/2)*x**6*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**
```

```
10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 10*a*b*c**3*d**(27/2)*x
**8*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c
**5*d**11*x**12 + 315*c**4*d**12*x**14) + 60*a*b*c**2*d**(29/2)*x**10*sqrt(
c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11
*x**12 + 315*c**4*d**12*x**14) + 80*a*b*c*d**(31/2)*x**12*sqrt(c/(d*x**2) +
 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315
*c**4*d**12*x**14) + 32*a*b*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**7*
d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x*
*14) - 15*b**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210
*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 33*b**2*c**4*d**(11/2)*x**2*sqrt(c
/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**
10) - 17*b**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
+ 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*b**2*c**2*d**(15/2)*x**6*sq
rt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6
*x**10) - 12*b**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6
 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*b**2*d**(19/2)*x**10*sqrt(
c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x*
*10)
```

3.595 $\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=281

$$\frac{c^4\left(24a^2d^2+bc(7bc-24ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{1024d^{9/2}}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1024d^4}+\frac{c^2x^3\sqrt{c+dx^2}\ \big(24a^2d^2}{1536d}$$

Rubi [A] time = 0.27, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 459, 279, 321, 217, 206}

$$\frac{c^2x^3\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1536d^3}-\frac{c^3x\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{1024d^4}+\frac{c^4\left(24a^2d^2+bc(7bc-24ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{1024d^{9/2}}+\frac{1}{192}x^5\left(c+dx^2\right)^{3/2}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)+\frac{cx^5\sqrt{c+dx^2}\left(24a^2d^2+bc(7bc-24ad)\right)}{384d^2}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{120d^2}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -(c^3*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^4) + (
c^2*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(1536*d^3) + (
c*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*x^5*Sqrt[c + d*x^2])/(384*d^2) + ((24
*a^2 + (b*c*(7*b*c - 24*a*d))/d^2)*x^5*(c + d*x^2)^(3/2))/192 - (b*(7*b*c -
 24*a*d)*x^5*(c + d*x^2)^(5/2))/(120*d^2) + (b^2*x^7*(c + d*x^2)^(5/2))/(12
*d) + (c^4*(24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(1024*d^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx &= \frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}+\frac{\int x^4\left(c+dx^2\right)^{3/2}\left(12a^2d-b(7bc-24ad)x^2\right)\,dx}{12d}\\ &= -\frac{b(7bc-24ad)x^5\left(c+dx^2\right)^{5/2}}{120d^2}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}+\frac{1}{24}\left(24a^2+\frac{bc(7bc-24a}{d^2}\right.\\ &= \frac{1}{192}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\left(c+dx^2\right)^{3/2}-\frac{b(7bc-24ad)x^5\left(c+dx^2\right)^{5/2}}{120d^2}+\\ &= \frac{1}{384}c\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\sqrt{c+dx^2}+\frac{1}{192}\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\\ &= \frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{1}{384}c\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^5\sqrt{c+d}\\ &= -\frac{c^3\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2}+\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{}{3}\\ &= -\frac{c^3\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2}+\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{}{3}\\ &= -\frac{c^3\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d^2}+\frac{c^2\left(24a^2+\frac{bc(7bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{1536d}+\frac{}{3}\end{aligned}$$

Mathematica [A] time = 0.21, size = 225, normalized size = 0.80

$$\frac{15c^4\left(24a^2d^2-24abcd+7b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}\,x\sqrt{c+dx^2}\left(120a^2d^2\left(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6\right)+24abd\left(15c^4-10c^3dx^2+8c^2d^2x^4+176cd^3x^6+128d^4x^8\right)+b^2\left(-105c^5+70c^4dx^2-56c^3d^2x^4+48c^2d^3x^6+1664cd^4x^8+1280d^5x^{10}\right)\right)}{15360d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(120*a^2*d^2*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^
4 + 16*d^3*x^6) + 24*a*b*d*(15*c^4 - 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d
^3*x^6 + 128*d^4*x^8) + b^2*(-105*c^5 + 70*c^4*d*x^2 - 56*c^3*d^2*x^4 + 48*
c^2*d^3*x^6 + 1664*c*d^4*x^8 + 1280*d^5*x^10)) + 15*c^4*(7*b^2*c^2 - 24*a*b
*c*d + 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(15360*d^(9/2))
```

IntegrateAlgebraic [A] time = 0.37, size = 255, normalized size = 0.91

$$\frac{\left(-24a^2c^4d^2+24abc^5d-7b^2c^6\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{1024d^{9/2}}+\frac{\sqrt{c+dx^2}\left(-360a^2c^3d^2x+240a^2c^2d^3x^3+2880a^2cd^4x^5+1920a^2d^5x^7+360abc^4dx-240abc^3d^2x^3+192abc^2d^3x^5+4224abcd^4x^7+3072abd^5x^9-105b^2c^5x+70b^2c^4dx^3-56b^2c^3d^2x^5+48b^2c^2d^3x^7+1664b^2cd^4x^9+1280b^2d^5x^{11}\right)}{15360d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-105*b^2*c^5*x + 360*a*b*c^4*d*x - 360*a^2*c^3*d^2*x + 70
*b^2*c^4*d*x^3 - 240*a*b*c^3*d^2*x^3 + 240*a^2*c^2*d^3*x^3 - 56*b^2*c^3*d^2
*x^5 + 192*a*b*c^2*d^3*x^5 + 2880*a^2*c*d^4*x^5 + 48*b^2*c^2*d^3*x^7 + 4224
*a*b*c*d^4*x^7 + 1920*a^2*d^5*x^7 + 1664*b^2*c*d^4*x^9 + 3072*a*b*d^5*x^9 +
 1280*b^2*d^5*x^11))/(15360*d^4) + ((-7*b^2*c^6 + 24*a*b*c^5*d - 24*a^2*c^4
*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(1024*d^(9/2))
```

fricas [A] time = 2.51, size = 494, normalized size = 1.76

$$\left[\frac{15\left(7b^2c^6-24abc^5d+24a^2c^4d^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(1280b^2d^6x^{11}+128\left(13b^2cd^5+24abd^6\right)x^9+48\left(b^2c^2d^4+88abcd^5+40a^2d^6\right)x^7-8\left(7b^2c^3d^3-24abc^2d^4-360a^2cd^5\right)x^5+10\left(7b^2c^4d^2-24abc^3d^3+24a^2c^2d^4\right)x^3-15\left(7b^2c^5d-24abc^4d^2+24a^2c^3d^3\right)x\right)\sqrt{dx^2+c}}{30720d^5},\ -\frac{15\left(7b^2c^6-24abc^5d+24a^2c^4d^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(1280b^2d^6x^{11}+128\left(13b^2cd^5+24abd^6\right)x^9+48\left(b^2c^2d^4+88abcd^5+40a^2d^6\right)x^7-8\left(7b^2c^3d^3-24abc^2d^4-360a^2cd^5\right)x^5+10\left(7b^2c^4d^2-24abc^3d^3+24a^2c^2d^4\right)x^3-15\left(7b^2c^5d-24abc^4d^2+24a^2c^3d^3\right)x\right)\sqrt{dx^2+c}}{15360d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*b^2*c^6 - 24*a*b*c^5*d + 24*a^2*c^4*d^2)*sqrt(d)*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(13*b^2*
c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 88*a*b*c*d^5 + 40*a^2*d^6)*x^7
- 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2*c*d^5)*x^5 + 10*(7*b^2*c^4*d^
2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15*(7*b^2*c^5*d - 24*a*b*c^4*d^2
 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/15360*(15*(7*b^2*c^6 - 24*a*
b*c^5*d + 24*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (12
80*b^2*d^6*x^11 + 128*(13*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 8
8*a*b*c*d^5 + 40*a^2*d^6)*x^7 - 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2
*c*d^5)*x^5 + 10*(7*b^2*c^4*d^2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15
*(7*b^2*c^5*d - 24*a*b*c^4*d^2 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

giac [A] time = 0.49, size = 263, normalized size = 0.94

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10b^2dx^2+\frac{13b^2cd^{10}+24abd^{11}}{d^{10}}\right)x^2+\frac{3\left(b^2c^2d^9+88abcd^{10}+40a^2d^{11}\right)}{d^{10}}\right)x^2-\frac{7b^2c^3d^8-24abc^2d^9-360a^2cd^{10}}{d^{10}}\right)x^2+\frac{5\left(7b^2c^4d^7-24abc^3d^8+24a^2c^2d^9\right)}{d^{10}}\right)x^2-\frac{15\left(7b^2c^5d^6-24abc^4d^7+24a^2c^3d^8\right)}{d^{10}}\right)\sqrt{dx^2+c}\,x-\frac{\left(7b^2c^6-24abc^5d+24a^2c^4d^2\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{1024d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d*x^2 + (13*b^2*c*d^10 + 24*a*b*d^11)/d^10)*x^2
 + 3*(b^2*c^2*d^9 + 88*a*b*c*d^10 + 40*a^2*d^11)/d^10)*x^2 - (7*b^2*c^3*d^8
 - 24*a*b*c^2*d^9 - 360*a^2*c*d^10)/d^10)*x^2 + 5*(7*b^2*c^4*d^7 - 24*a*b*c
^3*d^8 + 24*a^2*c^2*d^9)/d^10)*x^2 - 15*(7*b^2*c^5*d^6 - 24*a*b*c^4*d^7 + 2
4*a^2*c^3*d^8)/d^10)*sqrt(d*x^2 + c)*x - 1/1024*(7*b^2*c^6 - 24*a*b*c^5*d +
 24*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

maple [A] time = 0.02, size = 389, normalized size = 1.38

$$\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2x^7}{12d}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}abx^5}{5d}-\frac{7\left(dx^2+c\right)^{\frac{5}{2}}b^2cx^5}{120d^2}+\frac{3a^2c^4\ln\left(\sqrt{d}\,x+\sqrt{dx^2+c}\right)}{128d^{\frac{5}{2}}}-\frac{3abc^5\ln\left(\sqrt{d}\,x+\sqrt{dx^2+c}\right)}{128d^{\frac{7}{2}}}+\frac{7b^2c^6\ln\left(\sqrt{d}\,x+\sqrt{dx^2+c}\right)}{1024d^{\frac{9}{2}}}+\frac{3\sqrt{dx^2+c}\,a^2c^3x}{128d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2x^3}{8d}-\frac{3\sqrt{dx^2+c}\,abc^4x}{128d^3}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}abcx^3}{8d^2}+\frac{7\sqrt{dx^2+c}\,b^2c^5x}{1024d^4}+\frac{7\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2x^3}{192d^3}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2c^2x}{64d^2}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}abc^3x}{64d^3}+\frac{7\left(dx^2+c\right)^{\frac{3}{2}}b^2c^4x}{1536d^4}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2cx}{16d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}abc^2x}{16d^3}-\frac{7\left(dx^2+c\right)^{\frac{5}{2}}b^2c^3x}{384d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/12*b^2*x^7*(d*x^2+c)^(5/2)/d-7/120*b^2*c/d^2*x^5*(d*x^2+c)^(5/2)+7/192*b^
2*c^2/d^3*x^3*(d*x^2+c)^(5/2)-7/384*b^2*c^3/d^4*x*(d*x^2+c)^(5/2)+7/1536*b^
2*c^4/d^4*x*(d*x^2+c)^(3/2)+7/1024*b^2*c^5/d^4*x*(d*x^2+c)^(1/2)+7/1024*b^2
*c^6/d^(9/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/5*a*b*x^5*(d*x^2+c)^(5/2)/d-1/
8*a*b*c/d^2*x^3*(d*x^2+c)^(5/2)+1/16*a*b*c^2/d^3*x*(d*x^2+c)^(5/2)-1/64*a*b
*c^3/d^3*x*(d*x^2+c)^(3/2)-3/128*a*b*c^4/d^3*x*(d*x^2+c)^(1/2)-3/128*a*b*c^
5/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/8*a^2*x^3*(d*x^2+c)^(5/2)/d-1/16*
a^2*c/d^2*x*(d*x^2+c)^(5/2)+1/64*a^2*c^2/d^2*x*(d*x^2+c)^(3/2)+3/128*a^2*c^
3/d^2*x*(d*x^2+c)^(1/2)+3/128*a^2*c^4/d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.14, size = 367, normalized size = 1.31

$$\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2x^7}{12\,d}-\frac{7\left(dx^2+c\right)^{\frac{5}{2}}b^2cx^5}{120\,d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}abx^5}{5\,d}+\frac{7\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2x^3}{192\,d^3}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}abcx^3}{8\,d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2x^3}{8\,d}-\frac{7\left(dx^2+c\right)^{\frac{5}{2}}b^2c^3x}{384\,d^4}+\frac{7\left(dx^2+c\right)^{\frac{3}{2}}b^2c^4x}{1536\,d^4}+\frac{7\sqrt{dx^2+c}\,b^2c^5x}{1024\,d^4}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}abc^2x}{16\,d^3}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}abc^3x}{64\,d^3}-\frac{3\sqrt{dx^2+c}\,abc^4x}{128\,d^3}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2cx}{16\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2c^2x}{64\,d^2}+\frac{3\sqrt{dx^2+c}\,a^2c^3x}{128\,d^2}+\frac{7\,b^2c^6\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{1024\,d^{\frac{9}{2}}}-\frac{3\,abc^5\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{7}{2}}}+\frac{3\,a^2c^4\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^2 + c)^(5/2)*b^2*x^7/d - 7/120*(d*x^2 + c)^(5/2)*b^2*c*x^5/d^2 +
1/5*(d*x^2 + c)^(5/2)*a*b*x^5/d + 7/192*(d*x^2 + c)^(5/2)*b^2*c^2*x^3/d^3 -
 1/8*(d*x^2 + c)^(5/2)*a*b*c*x^3/d^2 + 1/8*(d*x^2 + c)^(5/2)*a^2*x^3/d - 7/
384*(d*x^2 + c)^(5/2)*b^2*c^3*x/d^4 + 7/1536*(d*x^2 + c)^(3/2)*b^2*c^4*x/d^
4 + 7/1024*sqrt(d*x^2 + c)*b^2*c^5*x/d^4 + 1/16*(d*x^2 + c)^(5/2)*a*b*c^2*x
/d^3 - 1/64*(d*x^2 + c)^(3/2)*a*b*c^3*x/d^3 - 3/128*sqrt(d*x^2 + c)*a*b*c^4
*x/d^3 - 1/16*(d*x^2 + c)^(5/2)*a^2*c*x/d^2 + 1/64*(d*x^2 + c)^(3/2)*a^2*c^
2*x/d^2 + 3/128*sqrt(d*x^2 + c)*a^2*c^3*x/d^2 + 7/1024*b^2*c^6*arcsinh(d*x/
sqrt(c*d))/d^(9/2) - 3/128*a*b*c^5*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/128*a
^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

sympy [B] time = 83.44, size = 598, normalized size = 2.13

$$-\frac{3a^2c^{\frac{7}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^{\frac{5}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{13a^2c^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{dx^2}{c}}}+\frac{5a^2\sqrt{c}\,dx^7}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}}+\frac{a^2d^2x^9}{8\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^{\frac{9}{2}}x}{128d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^{\frac{7}{2}}x^3}{128d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{5}{2}}x^5}{320d\sqrt{1+\frac{dx^2}{c}}}+\frac{23abc^{\frac{3}{2}}x^7}{80\sqrt{1+\frac{dx^2}{c}}}+\frac{19ab\sqrt{c}\,dx^9}{40\sqrt{1+\frac{dx^2}{c}}}-\frac{3abc^5\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{7}{2}}}+\frac{abd^2x^{11}}{5\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{7b^2c^{\frac{11}{2}}x}{1024d^4\sqrt{1+\frac{dx^2}{c}}}-\frac{7b^2c^{\frac{9}{2}}x^3}{3072d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2c^{\frac{7}{2}}x^5}{7680d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x^7}{1920d\sqrt{1+\frac{dx^2}{c}}}+\frac{107b^2c^{\frac{3}{2}}x^9}{960\sqrt{1+\frac{dx^2}{c}}}+\frac{23b^2\sqrt{c}\,dx^{11}}{120\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2c^6\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{1024d^{\frac{9}{2}}}+\frac{b^2d^2x^{13}}{12\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] -3*a**2*c**(7/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - a**2*c**(5/2)*x**3/(128*
d*sqrt(1 + d*x**2/c)) + 13*a**2*c**(3/2)*x**5/(64*sqrt(1 + d*x**2/c)) + 5*a
**2*sqrt(c)*d*x**7/(16*sqrt(1 + d*x**2/c)) + 3*a**2*c**4*asinh(sqrt(d)*x/sq
rt(c))/(128*d**(5/2)) + a**2*d**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c)) + 3*a
*b*c**(9/2)*x/(128*d**3*sqrt(1 + d*x**2/c)) + a*b*c**(7/2)*x**3/(128*d**2*s
qrt(1 + d*x**2/c)) - a*b*c**(5/2)*x**5/(320*d*sqrt(1 + d*x**2/c)) + 23*a*b*
c**(3/2)*x**7/(80*sqrt(1 + d*x**2/c)) + 19*a*b*sqrt(c)*d*x**9/(40*sqrt(1 +
d*x**2/c)) - 3*a*b*c**5*asinh(sqrt(d)*x/sqrt(c))/(128*d**(7/2)) + a*b*d**2*
x**11/(5*sqrt(c)*sqrt(1 + d*x**2/c)) - 7*b**2*c**(11/2)*x/(1024*d**4*sqrt(1
 + d*x**2/c)) - 7*b**2*c**(9/2)*x**3/(3072*d**3*sqrt(1 + d*x**2/c)) + 7*b**
2*c**(7/2)*x**5/(7680*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x**7/(1920*d
*sqrt(1 + d*x**2/c)) + 107*b**2*c**(3/2)*x**9/(960*sqrt(1 + d*x**2/c)) + 23
*b**2*sqrt(c)*d*x**11/(120*sqrt(1 + d*x**2/c)) + 7*b**2*c**6*asinh(sqrt(d)*
x/sqrt(c))/(1024*d**(9/2)) + b**2*d**2*x**13/(12*sqrt(c)*sqrt(1 + d*x**2/c)
)
```

3.596 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^4}$$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(7/2))/(7*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(9/2))/(9*d^4) +
 (b^2*(c + d*x^2)^(11/2))/(11*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x(a+bx)^2(c+dx)^{3/2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int\left(-\frac{c(bc-ad)^2(c+dx)^{3/2}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{5/2}}{d^3}-\frac{b(3b}{}\right.\right.\\ &= -\frac{c(bc-ad)^2\left(c+dx^2\right)^{5/2}}{5d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{7/2}}{7d^4}-\frac{b(3bc-2ad)\left(c\right.}{9d^4}\end{aligned}$$

Mathematica [A] time = 0.17, size = 100, normalized size = 0.88

$$\frac{\left(c+dx^2\right)^{5/2}\left(99a^2d^2\left(5dx^2-2c\right)+22abd\left(8c^2-20cdx^2+35d^2x^4\right)-3b^2\left(16c^3-40c^2dx^2+70cd^2x^4-105d^3x^6\right)\right)}{3465d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(99*a^2*d^2*(-2*c + 5*d*x^2) + 22*a*b*d*(8*c^2 - 20*c*d*
x^2 + 35*d^2*x^4) - 3*b^2*(16*c^3 - 40*c^2*d*x^2 + 70*c*d^2*x^4 - 105*d^3*x
^6)))/(3465*d^4)
```

IntegrateAlgebraic [A] time = 0.07, size = 111, normalized size = 0.97

$$\frac{\left(c+dx^2\right)^{5/2}\left(-198a^2cd^2+495a^2d^3x^2+176abc^2d-440abcd^2x^2+770abd^3x^4-48b^2c^3+120b^2c^2dx^2-210b^2cd^2x^4+315b^2d^3x^6\right)}{3465d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(-48*b^2*c^3 + 176*a*b*c^2*d - 198*a^2*c*d^2 + 120*b^2*c
^2*d*x^2 - 440*a*b*c*d^2*x^2 + 495*a^2*d^3*x^2 - 210*b^2*c*d^2*x^4 + 770*a*
b*d^3*x^4 + 315*b^2*d^3*x^6))/(3465*d^4)
```

fricas [A] time = 1.20, size = 179, normalized size = 1.57

$$\frac{\left(315\,b^2d^5x^{10}+70\left(6\,b^2cd^4+11\,abd^5\right)x^8-48\,b^2c^5+176\,abc^4d-198\,a^2c^3d^2+5\left(3\,b^2c^2d^3+220\,abcd^4+99\,a^2d^5\right)x^6-6\left(3\,b^2c^3d^2-11\,abc^2d^3-132\,a^2cd^4\right)x^4+\left(24\,b^2c^4d-88\,abc^3d^2+99\,a^2c^2d^3\right)x^2\right)\sqrt{dx^2+c}}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^2*d^5*x^10 + 70*(6*b^2*c*d^4 + 11*a*b*d^5)*x^8 - 48*b^2*c^5 +
 176*a*b*c^4*d - 198*a^2*c^3*d^2 + 5*(3*b^2*c^2*d^3 + 220*a*b*c*d^4 + 99*a^
```

```
2*d^5)*x^6 - 6*(3*b^2*c^3*d^2 - 11*a*b*c^2*d^3 - 132*a^2*c*d^4)*x^4 + (24*b
^2*c^4*d - 88*a*b*c^3*d^2 + 99*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

giac [A] time = 0.37, size = 150, normalized size = 1.32

$$\frac{315\left(dx^2+c\right)^{\frac{11}{2}}b^2-1155\left(dx^2+c\right)^{\frac{9}{2}}b^2c+1485\left(dx^2+c\right)^{\frac{7}{2}}b^2c^2-693\left(dx^2+c\right)^{\frac{5}{2}}b^2c^3+770\left(dx^2+c\right)^{\frac{9}{2}}abd-1980\left(dx^2+c\right)^{\frac{7}{2}}abcd+1386\left(dx^2+c\right)^{\frac{5}{2}}abc^2d+495\left(dx^2+c\right)^{\frac{7}{2}}a^2d^2-693\left(dx^2+c\right)^{\frac{5}{2}}a^2cd^2}{3465\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3465*(315*(d*x^2 + c)^(11/2)*b^2 - 1155*(d*x^2 + c)^(9/2)*b^2*c + 1485*(d
*x^2 + c)^(7/2)*b^2*c^2 - 693*(d*x^2 + c)^(5/2)*b^2*c^3 + 770*(d*x^2 + c)^(
9/2)*a*b*d - 1980*(d*x^2 + c)^(7/2)*a*b*c*d + 1386*(d*x^2 + c)^(5/2)*a*b*c^
2*d + 495*(d*x^2 + c)^(7/2)*a^2*d^2 - 693*(d*x^2 + c)^(5/2)*a^2*c*d^2)/d^4
```

maple [A] time = 0.01, size = 108, normalized size = 0.95

$$-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}\left(-315b^2x^6d^3-770ab\,d^3x^4+210b^2c\,d^2x^4-495a^2d^3x^2+440abc\,d^2x^2-120b^2c^2d\,x^2+198a^2c\,d^2-176ab\,c^2d+48b^2c^3\right)}{3465d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3465*(d*x^2+c)^(5/2)*(-315*b^2*d^3*x^6-770*a*b*d^3*x^4+210*b^2*c*d^2*x^4
-495*a^2*d^3*x^2+440*a*b*c*d^2*x^2-120*b^2*c^2*d*x^2+198*a^2*c*d^2-176*a*b*
c^2*d+48*b^2*c^3)/d^4
```

maxima [A] time = 1.11, size = 181, normalized size = 1.59

$$\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2x^6}{11\,d}-\frac{2\left(dx^2+c\right)^{\frac{5}{2}}b^2cx^4}{33\,d^2}+\frac{2\left(dx^2+c\right)^{\frac{5}{2}}abx^4}{9\,d}+\frac{8\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2x^2}{231\,d^3}-\frac{8\left(dx^2+c\right)^{\frac{5}{2}}abcx^2}{63\,d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2x^2}{7\,d}-\frac{16\left(dx^2+c\right)^{\frac{5}{2}}b^2c^3}{1155\,d^4}+\frac{16\left(dx^2+c\right)^{\frac{5}{2}}abc^2}{315\,d^3}-\frac{2\left(dx^2+c\right)^{\frac{5}{2}}a^2c}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(d*x^2 + c)^(5/2)*b^2*x^6/d - 2/33*(d*x^2 + c)^(5/2)*b^2*c*x^4/d^2 + 2
/9*(d*x^2 + c)^(5/2)*a*b*x^4/d + 8/231*(d*x^2 + c)^(5/2)*b^2*c^2*x^2/d^3 -
8/63*(d*x^2 + c)^(5/2)*a*b*c*x^2/d^2 + 1/7*(d*x^2 + c)^(5/2)*a^2*x^2/d - 16
/1155*(d*x^2 + c)^(5/2)*b^2*c^3/d^4 + 16/315*(d*x^2 + c)^(5/2)*a*b*c^2/d^3
- 2/35*(d*x^2 + c)^(5/2)*a^2*c/d^2
```

mupad [B] time = 0.82, size = 170, normalized size = 1.49

$$\sqrt{d\,x^2+c}\,\left(\frac{x^6\,\left(495\,a^2\,d^5+1100\,a\,b\,c\,d^4+15\,b^2\,c^2\,d^3\right)}{3465\,d^4}-\frac{198\,a^2\,c^3\,d^2-176\,a\,b\,c^4\,d+48\,b^2\,c^5}{3465\,d^4}+\frac{2\,b\,x^8\,\left(11\,a\,d+6\,b\,c\right)}{99}+\frac{b^2\,d\,x^{10}}{11}+\frac{2\,c\,x^4\,\left(132\,a^2\,d^2+11\,a\,b\,c\,d-3\,b^2\,c^2\right)}{1155\,d^2}+\frac{c^2\,x^2\,\left(99\,a^2\,d^2-88\,a\,b\,c\,d+24\,b^2\,c^2\right)}{3465\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((x^6*(495*a^2*d^5 + 15*b^2*c^2*d^3 + 1100*a*b*c*d^4))/(3
465*d^4) - (48*b^2*c^5 + 198*a^2*c^3*d^2 - 176*a*b*c^4*d)/(3465*d^4) + (2*b
*x^8*(11*a*d + 6*b*c))/99 + (b^2*d*x^10)/11 + (2*c*x^4*(132*a^2*d^2 - 3*b^2
*c^2 + 11*a*b*c*d))/(1155*d^2) + (c^2*x^2*(99*a^2*d^2 + 24*b^2*c^2 - 88*a*b
*c*d))/(3465*d^3))
```

sympy [A] time = 8.40, size = 384, normalized size = 3.37

$$\begin{cases} - \frac{2 a^{2} c^{3} \sqrt{c + d x^{2}}}{35 d^{2}} + \frac{a^{2} c^{2} x^{2} \sqrt{c + d x^{2}}}{35 d} + \frac{8 a^{2} c x^{4} \sqrt{c + d x^{2}}}{35} + \frac{a^{2} d x^{6} \sqrt{c + d x^{2}}}{7} + \frac{16 a b c^{4} \sqrt{c + d x^{2}}}{315 d^{3}} - \frac{8 a b c^{3} x^{2} \sqrt{c + d x^{2}}}{315 d^{2}} + \frac{2 a b c^{2} x^{4} \sqrt{c + d x^{2}}}{105 d} + \frac{20 a b c x^{6} \sqrt{c + d x^{2}}}{63} + \frac{2 a b d x^{8} \sqrt{c + d x^{2}}}{9} - \frac{16 b^{2} c^{5} \sqrt{c + d x^{2}}}{1155 d^{4}} + \frac{8 b^{2} c^{4} x^{2} \sqrt{c + d x^{2}}}{1155 d^{3}} - \frac{2 b^{2} c^{3} x^{4} \sqrt{c + d x^{2}}}{385 d^{2}} + \frac{b^{2} c^{2} x^{6} \sqrt{c + d x^{2}}}{231 d} + \frac{4 b^{2} c x^{8} \sqrt{c + d x^{2}}}{33} + \frac{b^{2} d x^{10} \sqrt{c + d x^{2}}}{11} & \text{for}\: d \neq 0 \\c^{\frac{3}{2}} \left(\frac{a^{2} x^{4}}{4} + \frac{a b x^{6}}{3} + \frac{b^{2} x^{8}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a**2*c**3*sqrt(c + d*x**2)/(35*d**2) + a**2*c**2*x**2*sqrt(c
+ d*x**2)/(35*d) + 8*a**2*c*x**4*sqrt(c + d*x**2)/35 + a**2*d*x**6*sqrt(c +
 d*x**2)/7 + 16*a*b*c**4*sqrt(c + d*x**2)/(315*d**3) - 8*a*b*c**3*x**2*sqrt
(c + d*x**2)/(315*d**2) + 2*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d) + 20*a*b
*c*x**6*sqrt(c + d*x**2)/63 + 2*a*b*d*x**8*sqrt(c + d*x**2)/9 - 16*b**2*c**
5*sqrt(c + d*x**2)/(1155*d**4) + 8*b**2*c**4*x**2*sqrt(c + d*x**2)/(1155*d*
*3) - 2*b**2*c**3*x**4*sqrt(c + d*x**2)/(385*d**2) + b**2*c**2*x**6*sqrt(c
+ d*x**2)/(231*d) + 4*b**2*c*x**8*sqrt(c + d*x**2)/33 + b**2*d*x**10*sqrt(c
 + d*x**2)/11, Ne(d, 0)), (c**(3/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
), True))
```

3.597 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}\, dx$

Optimal. Leaf size=235

$$-\frac{c^3\left(16a^2d^2+3bc(bc-4ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{c^2x\sqrt{c+dx^2}\,\left(16a^2d^2+3bc(bc-4ad)\right)}{256d^3}+\frac{x^3\left(c+dx^2\right)^{3/2}\left(16a^2d^2 -\right.}{96d^2}$$

Rubi [A] time = 0.22, antiderivative size = 232, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 459, 279, 321, 217, 206}

$$\frac{c^2x\sqrt{c+dx^2}\,\left(16a^2d^2+3bc(bc-4ad)\right)}{256d^3}-\frac{c^3\left(16a^2d^2+3bc(bc-4ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}+\frac{1}{96}x^3\left(c+dx^2\right)^{3/2}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)+\frac{cx^3\sqrt{c+dx^2}\,\left(16a^2d^2+3bc(bc-4ad)\right)}{128d^2}-\frac{bx^3\left(c+dx^2\right)^{5/2}(bc-4ad)}{16d^2}+\frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c^2*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x*Sqrt[c + d*x^2])/(256*d^3) + (c*(
16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*x^3*Sqrt[c + d*x^2])/(128*d^2) + ((16*a^2
 + (3*b*c*(b*c - 4*a*d))/d^2)*x^3*(c + d*x^2)^(3/2))/96 - (b*(b*c - 4*a*d)*
x^3*(c + d*x^2)^(5/2))/(16*d^2) + (b^2*x^5*(c + d*x^2)^(5/2))/(10*d) - (c^3
*(16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
256*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx &= \frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}+\frac{\int x^2\left(c+dx^2\right)^{3/2}\left(10a^2d-5b(bc-4ad)x^2\right)\,dx}{10d} \\
&= -\frac{b(bc-4ad)x^3\left(c+dx^2\right)^{5/2}}{16d^2}+\frac{b^2x^5\left(c+dx^2\right)^{5/2}}{10d}+\frac{1}{16}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right) \\
&= \frac{1}{96}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\left(c+dx^2\right)^{3/2}-\frac{b(bc-4ad)x^3\left(c+dx^2\right)^{5/2}}{16d^2}+\frac{b^2x^5}{} \\
&= \frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{96}\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\left(c+\right. \\
&= \frac{c^2\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x\sqrt{c+dx^2}}{256d}+\frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2} \\
&= \frac{c^2\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x\sqrt{c+dx^2}}{256d}+\frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2} \\
&= \frac{c^2\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x\sqrt{c+dx^2}}{256d}+\frac{1}{128}c\left(16a^2+\frac{3bc(bc-4ad)}{d^2}\right)x^3\sqrt{c+dx^2}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 193, normalized size = 0.82

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\left(80a^2d^2\left(3c^2+14cdx^2+8d^2x^4\right)+60abd\left(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6\right)+3b^2\left(15c^4-10c^3dx^2+8c^2d^2x^4+176cd^3x^6+128d^4x^8\right)\right)-15c^3\left(16a^2d^2-12abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{3840d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4) + 6
0*a*b*d*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) + 3*b^2*(15*c^4
- 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d^3*x^6 + 128*d^4*x^8)) - 15*c^3*(3*
b^2*c^2 - 12*a*b*c*d + 16*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(384
0*d^(7/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 214, normalized size = 0.91

$$\frac{\left(16a^2c^3d^2-12abc^4d+3b^2c^5\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{256d^{7/2}}+\frac{\sqrt{c+dx^2}\left(240a^2c^2d^2x+1120a^2cd^3x^3+640a^2d^4x^5-180abc^3dx+120abc^2d^2x^3+1440abcd^3x^5+960abd^4x^7+45b^2c^4x-30b^2c^3dx^3+24b^2c^2d^2x^5+528b^2cd^3x^7+384b^2d^4x^9\right)}{3840d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(45*b^2*c^4*x - 180*a*b*c^3*d*x + 240*a^2*c^2*d^2*x - 30*b
^2*c^3*d*x^3 + 120*a*b*c^2*d^2*x^3 + 1120*a^2*c*d^3*x^3 + 24*b^2*c^2*d^2*x^
5 + 1440*a*b*c*d^3*x^5 + 640*a^2*d^4*x^5 + 528*b^2*c*d^3*x^7 + 960*a*b*d^4*
x^7 + 384*b^2*d^4*x^9))/(3840*d^3) + ((3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^
3*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(256*d^(7/2))
```

fricas [A] time = 1.87, size = 419, normalized size = 1.78

$$\left[\frac{15\left(3\,b^{2}c^{5}-12\,abc^{4}d+16\,a^{2}c^{3}d^{2}\right)\sqrt{d}\,\log\left(-2\,dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{d}x-c\right)+2\left(384\,b^{2}d^{5}x^{9}+48\left(11\,b^{2}cd^{4}+20\,abd^{5}\right)x^{7}+8\left(3\,b^{2}c^{2}d^{3}+180\,abcd^{4}+80\,a^{2}d^{5}\right)x^{5}-10\left(3\,b^{2}c^{3}d^{2}-12\,abc^{2}d^{3}-112\,a^{2}cd^{4}\right)x^{3}+15\left(3\,b^{2}c^{4}d-12\,abc^{3}d^{2}+16\,a^{2}c^{2}d^{3}\right)x\right)\sqrt{dx^{2}+c}}{7680\,d^{4}},\ \frac{15\left(3\,b^{2}c^{5}-12\,abc^{4}d+16\,a^{2}c^{3}d^{2}\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)+\left(384\,b^{2}d^{5}x^{9}+48\left(11\,b^{2}cd^{4}+20\,abd^{5}\right)x^{7}+8\left(3\,b^{2}c^{2}d^{3}+180\,abcd^{4}+80\,a^{2}d^{5}\right)x^{5}-10\left(3\,b^{2}c^{3}d^{2}-12\,abc^{2}d^{3}-112\,a^{2}cd^{4}\right)x^{3}+15\left(3\,b^{2}c^{4}d-12\,abc^{3}d^{2}+16\,a^{2}c^{2}d^{3}\right)x\right)\sqrt{dx^{2}+c}}{3840\,d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x^
2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(11*b^2*c*d^
4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 -
10*(3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d -
 12*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/3840*(15*(3*b^
2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^
2 + c)) + (384*b^2*d^5*x^9 + 48*(11*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*
c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 - 10*(3*b^2*c^3*d^2 - 12*a*b*c^2*
d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d - 12*a*b*c^3*d^2 + 16*a^2*c^2*d^
3)*x)*sqrt(d*x^2 + c))/d^4]
```

giac [A] time = 0.43, size = 219, normalized size = 0.93

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^{2}dx^{2}+\frac{11\,b^{2}cd^{8}+20\,abd^{9}}{d^{8}}\right)x^{2}+\frac{3\,b^{2}c^{2}d^{7}+180\,abcd^{8}+80\,a^{2}d^{9}}{d^{8}}\right)x^{2}-\frac{5\left(3\,b^{2}c^{3}d^{6}-12\,abc^{2}d^{7}-112\,a^{2}cd^{8}\right)}{d^{8}}\right)x^{2}+\frac{15\left(3\,b^{2}c^{4}d^{5}-12\,abc^{3}d^{6}+16\,a^{2}c^{2}d^{7}\right)}{d^{8}}\right)\sqrt{dx^{2}+c}\,x+\frac{\left(3\,b^{2}c^{5}-12\,abc^{4}d+16\,a^{2}c^{3}d^{2}\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^{2}+c}\right|\right)}{256\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d*x^2 + (11*b^2*c*d^8 + 20*a*b*d^9)/d^8)*x^2 + (3*b^
2*c^2*d^7 + 180*a*b*c*d^8 + 80*a^2*d^9)/d^8)*x^2 - 5*(3*b^2*c^3*d^6 - 12*a*
b*c^2*d^7 - 112*a^2*c*d^8)/d^8)*x^2 + 15*(3*b^2*c^4*d^5 - 12*a*b*c^3*d^6 +
16*a^2*c^2*d^7)/d^8)*sqrt(d*x^2 + c)*x + 1/256*(3*b^2*c^5 - 12*a*b*c^4*d +
16*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

maple [A] time = 0.01, size = 321, normalized size = 1.37

$$\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} b^{2}x^{5}}{10d}-\frac{a^{2}c^{3}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{16d^{\frac{3}{2}}}+\frac{3ab\,c^{4}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{64d^{\frac{5}{2}}}-\frac{3b^{2}c^{5}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{256d^{\frac{7}{2}}}-\frac{\sqrt{d\,x^{2}+c}\,a^{2}c^{2}x}{16d}+\frac{3\sqrt{d\,x^{2}+c}\,ab\,c^{3}x}{64d^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} ab\,x^{3}}{4d}-\frac{3\sqrt{d\,x^{2}+c}\,b^{2}c^{4}x}{256d^{3}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} b^{2}c\,x^{3}}{16d^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}} a^{2}cx}{24d}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}} ab\,c^{2}x}{32d^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}} b^{2}c^{3}x}{128d^{3}}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} a^{2}x}{6d}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} abcx}{8d^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} b^{2}c^{2}x}{32d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/10*b^2*x^5*(d*x^2+c)^(5/2)/d-1/16*b^2*c/d^2*x^3*(d*x^2+c)^(5/2)+1/32*b^2*
c^2/d^3*x*(d*x^2+c)^(5/2)-1/128*b^2*c^3/d^3*x*(d*x^2+c)^(3/2)-3/256*b^2*c^4
/d^3*x*(d*x^2+c)^(1/2)-3/256*b^2*c^5/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+
1/4*a*b*x^3*(d*x^2+c)^(5/2)/d-1/8*a*b*c/d^2*x*(d*x^2+c)^(5/2)+1/32*a*b*c^2/
d^2*x*(d*x^2+c)^(3/2)+3/64*a*b*c^3/d^2*x*(d*x^2+c)^(1/2)+3/64*a*b*c^4/d^(5/
2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/6*a^2*x*(d*x^2+c)^(5/2)/d-1/24*a^2*c/d*x
*(d*x^2+c)^(3/2)-1/16*a^2*c^2/d*x*(d*x^2+c)^(1/2)-1/16*a^2*c^3/d^(3/2)*ln(d
^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.04, size = 299, normalized size = 1.27

$$\frac{\left(dx^2+c\right)^{\frac{5}{2}} b^2 x^5}{10\,d} - \frac{\left(dx^2+c\right)^{\frac{5}{2}} b^2 c x^3}{16\,d^2} + \frac{\left(dx^2+c\right)^{\frac{5}{2}} abx^3}{4\,d} + \frac{\left(dx^2+c\right)^{\frac{5}{2}} b^2 c^2 x}{32\,d^3} - \frac{\left(dx^2+c\right)^{\frac{3}{2}} b^2 c^3 x}{128\,d^3} - \frac{3\,\sqrt{dx^2+c}\,b^2 c^4 x}{256\,d^3} - \frac{\left(dx^2+c\right)^{\frac{5}{2}} abcx}{8\,d^2} + \frac{\left(dx^2+c\right)^{\frac{3}{2}} abc^2 x}{32\,d^2} + \frac{3\,\sqrt{dx^2+c}\,abc^3 x}{64\,d^2} + \frac{\left(dx^2+c\right)^{\frac{5}{2}} a^2 x}{6\,d} - \frac{\left(dx^2+c\right)^{\frac{3}{2}} a^2 cx}{24\,d} - \frac{\sqrt{dx^2+c}\,a^2 c^2 x}{16\,d} - \frac{3\,b^2 c^5 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{256\,d^{\frac{7}{2}}} + \frac{3\,abc^4 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{64\,d^{\frac{5}{2}}} - \frac{a^2 c^3 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x^2 + c)^(5/2)*b^2*x^5/d - 1/16*(d*x^2 + c)^(5/2)*b^2*c*x^3/d^2 + 1
/4*(d*x^2 + c)^(5/2)*a*b*x^3/d + 1/32*(d*x^2 + c)^(5/2)*b^2*c^2*x/d^3 - 1/1
28*(d*x^2 + c)^(3/2)*b^2*c^3*x/d^3 - 3/256*sqrt(d*x^2 + c)*b^2*c^4*x/d^3 -
1/8*(d*x^2 + c)^(5/2)*a*b*c*x/d^2 + 1/32*(d*x^2 + c)^(3/2)*a*b*c^2*x/d^2 +
3/64*sqrt(d*x^2 + c)*a*b*c^3*x/d^2 + 1/6*(d*x^2 + c)^(5/2)*a^2*x/d - 1/24*(
d*x^2 + c)^(3/2)*a^2*c*x/d - 1/16*sqrt(d*x^2 + c)*a^2*c^2*x/d - 3/256*b^2*c
^5*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/64*a*b*c^4*arcsinh(d*x/sqrt(c*d))/d^(
5/2) - 1/16*a^2*c^3*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

sympy [B] time = 53.69, size = 505, normalized size = 2.15

$$\frac{a^2 c^{\frac{5}{2}} x}{16 d \sqrt{1+\frac{dx^2}{c}}} + \frac{17 a^2 c^{\frac{3}{2}} x^3}{48 \sqrt{1+\frac{dx^2}{c}}} + \frac{11 a^2 \sqrt{c}\, d x^5}{24 \sqrt{1+\frac{dx^2}{c}}} - \frac{a^2 c^3 \operatorname{asinh}\left(\frac{\sqrt{d}\, x}{\sqrt{c}}\right)}{16 d^{\frac{3}{2}}} + \frac{a^2 d^2 x^7}{6 \sqrt{c} \sqrt{1+\frac{dx^2}{c}}} - \frac{3 a b c^{\frac{7}{2}} x}{64 d^2 \sqrt{1+\frac{dx^2}{c}}} - \frac{a b c^{\frac{5}{2}} x^3}{64 d \sqrt{1+\frac{dx^2}{c}}} + \frac{13 a b c^{\frac{3}{2}} x^5}{32 \sqrt{1+\frac{dx^2}{c}}} + \frac{5 a b \sqrt{c}\, d x^7}{8 \sqrt{1+\frac{dx^2}{c}}} + \frac{3 a b c^4 \operatorname{asinh}\left(\frac{\sqrt{d}\, x}{\sqrt{c}}\right)}{64 d^{\frac{5}{2}}} + \frac{a b d^2 x^9}{4 \sqrt{c} \sqrt{1+\frac{dx^2}{c}}} + \frac{3 b^2 c^{\frac{9}{2}} x}{256 d^3 \sqrt{1+\frac{dx^2}{c}}} + \frac{b^2 c^{\frac{7}{2}} x^3}{256 d^2 \sqrt{1+\frac{dx^2}{c}}} - \frac{b^2 c^{\frac{5}{2}} x^5}{640 d \sqrt{1+\frac{dx^2}{c}}} + \frac{23 b^2 c^{\frac{3}{2}} x^7}{160 \sqrt{1+\frac{dx^2}{c}}} + \frac{19 b^2 \sqrt{c}\, d x^9}{80 \sqrt{1+\frac{dx^2}{c}}} - \frac{3 b^2 c^5 \operatorname{asinh}\left(\frac{\sqrt{d}\, x}{\sqrt{c}}\right)}{256 d^{\frac{7}{2}}} + \frac{b^2 d^2 x^{11}}{10 \sqrt{c} \sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*a**2*c**(3/2)*x**3/(48*sqrt(
1 + d*x**2/c)) + 11*a**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)) - a**2*c**3
```

```
*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + a**2*d**2*x**7/(6*sqrt(c)*sqrt(1
+ d*x**2/c)) - 3*a*b*c**(7/2)*x/(64*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(5/2)
*x**3/(64*d*sqrt(1 + d*x**2/c)) + 13*a*b*c**(3/2)*x**5/(32*sqrt(1 + d*x**2/
c)) + 5*a*b*sqrt(c)*d*x**7/(8*sqrt(1 + d*x**2/c)) + 3*a*b*c**4*asinh(sqrt(d
)*x/sqrt(c))/(64*d**(5/2)) + a*b*d**2*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) +
 3*b**2*c**(9/2)*x/(256*d**3*sqrt(1 + d*x**2/c)) + b**2*c**(7/2)*x**3/(256*
d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x**5/(640*d*sqrt(1 + d*x**2/c)) +
23*b**2*c**(3/2)*x**7/(160*sqrt(1 + d*x**2/c)) + 19*b**2*sqrt(c)*d*x**9/(80
*sqrt(1 + d*x**2/c)) - 3*b**2*c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(7/2))
+ b**2*d**2*x**11/(10*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.598 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(7
/2))/(7*d^3) + (b^2*(c + d*x^2)^(9/2))/(9*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int(a+bx)^2(c+dx)^{3/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)\right.\\&=\frac{(bc-ad)^2\left(c+dx^2\right)^{5/2}}{5d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{7/2}}{7d^3}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{5/2}\left(63a^2d^2+18abd\left(5dx^2-2c\right)+b^2\left(8c^2-20cdx^2+35d^2x^4\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x^2) + b^2*(8*c^2 - 2
0*c*d*x^2 + 35*d^2*x^4)))/(315*d^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 72, normalized size = 0.94

$$\frac{\left(c+dx^2\right)^{5/2}\left(63a^2d^2-36abcd+90abd^2x^2+8b^2c^2-20b^2cdx^2+35b^2d^2x^4\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] ((c + d*x^2)^(5/2)*(8*b^2*c^2 - 36*a*b*c*d + 63*a^2*d^2 - 20*b^2*c*d*x^2 +
90*a*b*d^2*x^2 + 35*b^2*d^2*x^4))/(315*d^3)
```

fricas [B] time = 1.40, size = 141, normalized size = 1.83

$$\frac{\left(35\,b^2d^4x^8+10\left(5\,b^2cd^3+9\,abd^4\right)x^6+8\,b^2c^4-36\,abc^3d+63\,a^2c^2d^2+3\left(b^2c^2d^2+48\,abcd^3+21\,a^2d^4\right)x^4-2\left(2\,b^2c^3d-9\,abc^2d^2-63\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*d^4*x^8 + 10*(5*b^2*c*d^3 + 9*a*b*d^4)*x^6 + 8*b^2*c^4 - 36*a
*b*c^3*d + 63*a^2*c^2*d^2 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^4
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

giac [A] time = 0.32, size = 98, normalized size = 1.27

$$\frac{35\left(dx^2+c\right)^{\frac{9}{2}}b^2-90\left(dx^2+c\right)^{\frac{7}{2}}b^2c+63\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2+90\left(dx^2+c\right)^{\frac{7}{2}}abd-126\left(dx^2+c\right)^{\frac{5}{2}}abcd+63\left(dx^2+c\right)^{\frac{5}{2}}a^2d^2}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*(d*x^2 + c)^(9/2)*b^2 - 90*(d*x^2 + c)^(7/2)*b^2*c + 63*(d*x^2 +
c)^(5/2)*b^2*c^2 + 90*(d*x^2 + c)^(7/2)*a*b*d - 126*(d*x^2 + c)^(5/2)*a*b*c
*d + 63*(d*x^2 + c)^(5/2)*a^2*d^2)/d^3
```

maple [A] time = 0.01, size = 69, normalized size = 0.90

$$\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}\left(35b^2x^4d^2+90ab\,d^2x^2-20b^2cd\,x^2+63a^2d^2-36abcd+8b^2c^2\right)}{315d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/315*(d*x^2+c)^(5/2)*(35*b^2*d^2*x^4+90*a*b*d^2*x^2-20*b^2*c*d*x^2+63*a^2*
d^2-36*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 1.00, size = 115, normalized size = 1.49

$$\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2x^4}{9\,d}-\frac{4\left(dx^2+c\right)^{\frac{5}{2}}b^2cx^2}{63\,d^2}+\frac{2\left(dx^2+c\right)^{\frac{5}{2}}abx^2}{7\,d}+\frac{8\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2}{315\,d^3}-\frac{4\left(dx^2+c\right)^{\frac{5}{2}}abc}{35\,d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d*x^2 + c)^(5/2)*b^2*x^4/d - 4/63*(d*x^2 + c)^(5/2)*b^2*c*x^2/d^2 + 2/
7*(d*x^2 + c)^(5/2)*a*b*x^2/d + 8/315*(d*x^2 + c)^(5/2)*b^2*c^2/d^3 - 4/35*
(d*x^2 + c)^(5/2)*a*b*c/d^2 + 1/5*(d*x^2 + c)^(5/2)*a^2/d
```

mupad [B] time = 0.73, size = 136, normalized size = 1.77

$$\sqrt{d\,x^2+c}\,\left(\frac{63\,a^2\,c^2\,d^2-36\,a\,b\,c^3\,d+8\,b^2\,c^4}{315\,d^3}+\frac{x^4\,\left(63\,a^2\,d^4+144\,a\,b\,c\,d^3+3\,b^2\,c^2\,d^2\right)}{315\,d^3}+\frac{2\,b\,x^6\,\left(9\,a\,d+5\,b\,c\right)}{63}+\frac{b^2\,d\,x^8}{9}+\frac{2\,c\,x^2\,\left(63\,a^2\,d^2+9\,a\,b\,c\,d-2\,b^2\,c^2\right)}{315\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((8*b^2*c^4 + 63*a^2*c^2*d^2 - 36*a*b*c^3*d)/(315*d^3) +
(x^4*(63*a^2*d^4 + 3*b^2*c^2*d^2 + 144*a*b*c*d^3))/(315*d^3) + (2*b*x^6*(9*
a*d + 5*b*c))/63 + (b^2*d*x^8)/9 + (2*c*x^2*(63*a^2*d^2 - 2*b^2*c^2 + 9*a*b
*c*d))/(315*d^2))
```

sympy [A] time = 3.68, size = 303, normalized size = 3.94

$$\begin{cases} \frac{a^2c^2\sqrt{c+dx^2}}{5d}+\frac{2a^2cx^2\sqrt{c+dx^2}}{5}+\frac{a^2dx^4\sqrt{c+dx^2}}{5}-\frac{4abc^3\sqrt{c+dx^2}}{35d^2}+\frac{2abc^2x^2\sqrt{c+dx^2}}{35d}+\frac{16abcx^4\sqrt{c+dx^2}}{35}+\frac{2abdx^6\sqrt{c+dx^2}}{7}+\frac{8b^2c^4\sqrt{c+dx^2}}{315d^3}-\frac{4b^2c^3x^2\sqrt{c+dx^2}}{315d^2}+\frac{b^2c^2x^4\sqrt{c+dx^2}}{105d}+\frac{10b^2cx^6\sqrt{c+dx^2}}{63}+\frac{b^2dx^8\sqrt{c+dx^2}}{9} & \text{for}\ d\neq 0 \\ c^{\frac{3}{2}}\left(\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((a**2*c**2*sqrt(c + d*x**2)/(5*d) + 2*a**2*c*x**2*sqrt(c + d*x**2
)/5 + a**2*d*x**4*sqrt(c + d*x**2)/5 - 4*a*b*c**3*sqrt(c + d*x**2)/(35*d**2
) + 2*a*b*c**2*x**2*sqrt(c + d*x**2)/(35*d) + 16*a*b*c*x**4*sqrt(c + d*x**2
)/35 + 2*a*b*d*x**6*sqrt(c + d*x**2)/7 + 8*b**2*c**4*sqrt(c + d*x**2)/(315*
d**3) - 4*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**2) + b**2*c**2*x**4*sqrt(
c + d*x**2)/(105*d) + 10*b**2*c*x**6*sqrt(c + d*x**2)/63 + b**2*d*x**8*sqrt
(c + d*x**2)/9, Ne(d, 0)), (c**(3/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/
6), True))
```

3.599 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=196

$$\frac{x\left(c+dx^2\right)^{3/2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{192d^2}+\frac{cx\sqrt{c+dx^2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{128d^2}+\frac{c^2\left(48a^2d^2-16abcd+3b^2c^2\right)}{128d^{5/2}}$$

Rubi [A] time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\left(c+dx^2\right)^{3/2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{192d^2}+\frac{cx\sqrt{c+dx^2}\left(48a^2d^2-16abcd+3b^2c^2\right)}{128d^2}+\frac{c^2\left(48a^2d^2-16abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{5/2}}-\frac{bx\left(c+dx^2\right)^{5/2}(3bc-10ad)}{48d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*Sqrt[c + d*x^2])/(128*d^2) + ((3
*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*x*(c + d*x^2)^(3/2))/(192*d^2) - (b*(3*
b*c - 10*a*d)*x*(c + d*x^2)^(5/2))/(48*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(5/2))/(8*d) + (c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(128*d^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx = \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d} + \frac{\int \left(c+dx^2\right)^{3/2}\left(-a(bc-8ad)-b(3bc-10ad)x^2\right) dx}{8d}$$

$$= -\frac{b(3bc-10ad)x\left(c+dx^2\right)^{5/2}}{48d^2} + \frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d} - (-bc(3bc-10ad) +$$

$$= \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x\left(c+dx^2\right)^{3/2}}{192d^2} - \frac{b(3bc-10ad)x\left(c+dx^2\right)^{5/2}}{48d^2} + bx(a$$

$$= \frac{c\left(3b^2c^2-16abcd+48a^2d^2\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x(c+dx}{192d^2}$$

$$= \frac{c\left(3b^2c^2-16abcd+48a^2d^2\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x(c+dx}{192d^2}$$

$$= \frac{c\left(3b^2c^2-16abcd+48a^2d^2\right)x\sqrt{c+dx^2}}{128d^2} + \frac{\left(3b^2c^2-16abcd+48a^2d^2\right)x(c+dx}{192d^2}$$

Mathematica [A] time = 0.10, size = 159, normalized size = 0.81

$$\frac{3c^2\left(48a^2d^2-16abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}x\sqrt{c+dx^2}\left(48a^2d^2\left(5c+2dx^2\right)+16abd\left(3c^2+14cdx^2+8d^2x^4\right)+b^2\left(-9c^3+6c^2dx^2+72cd^2x^4+48d^3x^6\right)\right)}{384d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(5*c + 2*d*x^2) + 16*a*b*d*(3*c^2 +
14*c*d*x^2 + 8*d^2*x^4) + b^2*(-9*c^3 + 6*c^2*d*x^2 + 72*c*d^2*x^4 + 48*d^3
*x^6)) + 3*c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt
[c + d*x^2]])/(384*d^(5/2))
```

IntegrateAlgebraic [A] time = 0.27, size = 173, normalized size = 0.88

$$\frac{\sqrt{c+dx^2}\,\left(240a^2cd^2x+96a^2d^3x^3+48abc^2dx+224abcd^2x^3+128abd^3x^5-9b^2c^3x+6b^2c^2dx^3+72b^2cd^2x^5+48b^2d^3x^7\right)}{384d^2}+\frac{\left(-48a^2c^2d^2+16abc^3d-3b^2c^4\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{128d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-9*b^2*c^3*x + 48*a*b*c^2*d*x + 240*a^2*c*d^2*x + 6*b^2*c
^2*d*x^3 + 224*a*b*c*d^2*x^3 + 96*a^2*d^3*x^3 + 72*b^2*c*d^2*x^5 + 128*a*b*
d^3*x^5 + 48*b^2*d^3*x^7))/(384*d^2) + ((-3*b^2*c^4 + 16*a*b*c^3*d - 48*a^2
*c^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(128*d^(5/2))
```

fricas [A] time = 1.31, size = 344, normalized size = 1.76

$$\left[\frac{3\left(3b^2c^4-16abc^3d+48a^2c^2d^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(48b^2d^4x^7+8\left(9b^2cd^3+16abd^4\right)x^5+2\left(3b^2c^2d^2+112abcd^3+48a^2d^4\right)x^3-3\left(3b^2c^3d-16abc^2d^2-80a^2cd^3\right)x\right)\sqrt{dx^2+c}}{768d^3},-\frac{3\left(3b^2c^4-16abc^3d+48a^2c^2d^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(48b^2d^4x^7+8\left(9b^2cd^3+16abd^4\right)x^5+2\left(3b^2c^2d^2+112abcd^3+48a^2d^4\right)x^3-3\left(3b^2c^3d-16abc^2d^2-80a^2cd^3\right)x\right)\sqrt{dx^2+c}}{384d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 1
6*a*b*d^4)*x^5 + 2*(3*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*
b^2*c^3*d - 16*a*b*c^2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/384*
(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - (48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 16*a*b*d^4)*x^5 + 2*(3
*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*b^2*c^3*d - 16*a*b*c^
2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

giac [A] time = 0.46, size = 175, normalized size = 0.89

$$\frac{1}{384}\left(2\left(4\left(6b^2dx^2+\frac{9b^2cd^6+16abd^7}{d^6}\right)x^2+\frac{3b^2c^2d^5+112abcd^6+48a^2d^7}{d^6}\right)x^2-\frac{3\left(3b^2c^3d^4-16abc^2d^5-80a^2cd^6\right)}{d^6}\right)\sqrt{dx^2+c}\,x-\frac{\left(3b^2c^4-16abc^3d+48a^2c^2d^2\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{128d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (9*b^2*c*d^6 + 16*a*b*d^7)/d^6)*x^2 + (3*b^2*c^2
*d^5 + 112*a*b*c*d^6 + 48*a^2*d^7)/d^6)*x^2 - 3*(3*b^2*c^3*d^4 - 16*a*b*c^2
*d^5 - 80*a^2*c*d^6)/d^6)*sqrt(d*x^2 + c)*x - 1/128*(3*b^2*c^4 - 16*a*b*c^3
*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

maple [A] time = 0.01, size = 249, normalized size = 1.27

$$\frac{3a^2c^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8\sqrt{d}}-\frac{ab\,c^3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8d^{\frac{3}{2}}}+\frac{3b^2c^4\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{128d^{\frac{5}{2}}}+\frac{3\sqrt{d\,x^2+c}\,a^2cx}{8}-\frac{\sqrt{d\,x^2+c}\,ab\,c^2x}{8d}+\frac{3\sqrt{d\,x^2+c}\,b^2c^3x}{128d^2}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}b^2x^3}{8d}+\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}a^2x}{4}-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}abcx}{12d}+\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}b^2c^2x}{64d^2}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}abx}{3d}-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}b^2cx}{16d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/8*b^2*x^3*(d*x^2+c)^(5/2)/d-1/16*b^2*c/d^2*x*(d*x^2+c)^(5/2)+1/64*b^2*c^2
/d^2*x*(d*x^2+c)^(3/2)+3/128*b^2*c^3/d^2*x*(d*x^2+c)^(1/2)+3/128*b^2*c^4/d^
(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/3*a*b*x*(d*x^2+c)^(5/2)/d-1/12*a*b*c/
d*x*(d*x^2+c)^(3/2)-1/8*a*b*c^2/d*x*(d*x^2+c)^(1/2)-1/8*a*b*c^3/d^(3/2)*ln(
d^(1/2)*x+(d*x^2+c)^(1/2))+1/4*a^2*x*(d*x^2+c)^(3/2)+3/8*a^2*c*x*(d*x^2+c)^
(1/2)+3/8*a^2*c^2/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.14, size = 227, normalized size = 1.16

$$\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2x^3}{8\,d}+\frac{1}{4}\left(dx^2+c\right)^{\frac{3}{2}}a^2x+\frac{3}{8}\sqrt{dx^2+c}\,a^2cx-\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2cx}{16\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2x}{64\,d^2}+\frac{3\sqrt{dx^2+c}\,b^2c^3x}{128\,d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}abx}{3\,d}-\frac{\left(dx^2+c\right)^{\frac{3}{2}}abcx}{12\,d}-\frac{\sqrt{dx^2+c}\,abc^2x}{8\,d}+\frac{3\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}}-\frac{abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{3\,a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^2 + c)^(5/2)*b^2*x^3/d + 1/4*(d*x^2 + c)^(3/2)*a^2*x + 3/8*sqrt(d*
x^2 + c)*a^2*c*x - 1/16*(d*x^2 + c)^(5/2)*b^2*c*x/d^2 + 1/64*(d*x^2 + c)^(3
/2)*b^2*c^2*x/d^2 + 3/128*sqrt(d*x^2 + c)*b^2*c^3*x/d^2 + 1/3*(d*x^2 + c)^(
5/2)*a*b*x/d - 1/12*(d*x^2 + c)^(3/2)*a*b*c*x/d - 1/8*sqrt(d*x^2 + c)*a*b*c
^2*x/d + 3/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*b*c^3*arcsinh
(d*x/sqrt(c*d))/d^(3/2) + 3/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

sympy [B] time = 30.55, size = 440, normalized size = 2.24

$$\frac{a^2c^{\frac{3}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{a^2c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2\sqrt{c}\,dx^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3a^2c^2\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{a^2d^2x^5}{4\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{abc^{\frac{5}{2}}x}{8d\sqrt{1+\frac{dx^2}{c}}}+\frac{17abc^{\frac{3}{2}}x^3}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{11ab\sqrt{c}\,dx^5}{12\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^3\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{3}{2}}}+\frac{abd^2x^7}{3\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{3b^2c^{\frac{7}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{5}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{13b^2c^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2\sqrt{c}\,dx^7}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c^4\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}}+\frac{b^2d^2x^9}{8\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + a**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c
)) + 3*a**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(
d)*x/sqrt(c))/(8*sqrt(d)) + a**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) +
 a*b*c**(5/2)*x/(8*d*sqrt(1 + d*x**2/c)) + 17*a*b*c**(3/2)*x**3/(24*sqrt(1
+ d*x**2/c)) + 11*a*b*sqrt(c)*d*x**5/(12*sqrt(1 + d*x**2/c)) - a*b*c**3*asi
nh(sqrt(d)*x/sqrt(c))/(8*d**(3/2)) + a*b*d**2*x**7/(3*sqrt(c)*sqrt(1 + d*x*
*2/c)) - 3*b**2*c**(7/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(5/2)*x*
*3/(128*d*sqrt(1 + d*x**2/c)) + 13*b**2*c**(3/2)*x**5/(64*sqrt(1 + d*x**2/c
)) + 5*b**2*sqrt(c)*d*x**7/(16*sqrt(1 + d*x**2/c)) + 3*b**2*c**4*asinh(sqrt
(d)*x/sqrt(c))/(128*d**(5/2)) + b**2*d**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c
))
```

3.600 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=111

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

Rubi [A] time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\sqrt{c+dx^2}-\frac{b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] a^2*c*Sqrt[c + d*x^2] + (a^2*(c + d*x^2)^(3/2))/3 - (b*(b*c - 2*a*d)*(c + d
*x^2)^(5/2))/(5*d^2) + (b^2*(c + d*x^2)^(7/2))/(7*d^2) - a^2*c^(3/2)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{b(bc-2ad)(c+dx)^{3/2}}{d}+\frac{a^2(c+dx)^{3/2}}{x}+\frac{b^2(c+dx)^{5/2}}{d}\right)dx,x,x^2\right)\\
&= -\frac{b(bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{1}{2}a^2\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{3}a^2\left(c+dx^2\right)^{3/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{1}{2}\left(a^2c\right)\text{Subst}\Bigg(\int\\
&= a^2c\sqrt{c+dx^2}+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{1}{2}\\
&= a^2c\sqrt{c+dx^2}+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{(a}{}\\
&= a^2c\sqrt{c+dx^2}+\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}-a^2
\end{aligned}$$

Mathematica [A]   time = 0.08, size = 110, normalized size = 0.99

$$\frac{1}{3}a^2\left(c+dx^2\right)^{3/2}+a^2c\left(\sqrt{c+dx^2}-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{b\left(c+dx^2\right)^{5/2}(2ad-bc)}{5d^2}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] (a^2*(c + d*x^2)^(3/2))/3 + (b*(-(b*c) + 2*a*d)*(c + d*x^2)^(5/2))/(5*d^2)
+ (b^2*(c + d*x^2)^(7/2))/(7*d^2) + a^2*c*(Sqrt[c + d*x^2] - Sqrt[c]*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])
```

IntegrateAlgebraic [A]   time = 0.11, size = 140, normalized size = 1.26

$$\frac{\sqrt{c+dx^2}\left(140a^2cd^2+35a^2d^3x^2+42abc^2d+84abcd^2x^2+42abd^3x^4-6b^2c^3+3b^2c^2dx^2+24b^2cd^2x^4+15b^2d^3x^6\right)}{105d^2}-a^2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 3*b^2*c^2*d*x
^2 + 84*a*b*c*d^2*x^2 + 35*a^2*d^3*x^2 + 24*b^2*c*d^2*x^4 + 42*a*b*d^3*x^4
+ 15*b^2*d^3*x^6))/(105*d^2) - a^2*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

fricas [A]   time = 1.41, size = 282, normalized size = 2.54

$$\left[\frac{105\,a^2c^{\frac{3}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\left(4\,b^2cd^2+7\,abd^3\right)x^4+\left(3\,b^2c^2d+84\,abcd^2+35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{210\,d^2},\ \frac{105\,a^2\sqrt{-c}\,cd^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+140\,a^2cd^2+6\left(4\,b^2cd^2+7\,abd^3\right)x^4+\left(3\,b^2c^2d+84\,abcd^2+35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{105\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/210*(105*a^2*c^(3/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 2*(15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6*(4*
b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*x^2)
*sqrt(d*x^2 + c))/d^2, 1/105*(105*a^2*sqrt(-c)*c*d^2*arctan(sqrt(-c)/sqrt(d
*x^2 + c)) + (15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6
*(4*b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*
x^2)*sqrt(d*x^2 + c))/d^2]
```

giac [A]   time = 0.38, size = 121, normalized size = 1.09

$$\frac{a^2c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2d^{12}-21\left(dx^2+c\right)^{\frac{5}{2}}b^2cd^{12}+42\left(dx^2+c\right)^{\frac{5}{2}}abd^{13}+35\left(dx^2+c\right)^{\frac{3}{2}}a^2d^{14}+105\sqrt{dx^2+c}\,a^2cd^{14}}{105\,d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/105*(15*(d*x^2 + c)^(
7/2)*b^2*d^12 - 21*(d*x^2 + c)^(5/2)*b^2*c*d^12 + 42*(d*x^2 + c)^(5/2)*a*b*
d^13 + 35*(d*x^2 + c)^(3/2)*a^2*d^14 + 105*sqrt(d*x^2 + c)*a^2*c*d^14)/d^14
```

maple [A] time = 0.01, size = 115, normalized size = 1.04

$$-a^{2} c^{\frac{3}{2}} \ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)+\sqrt{d\,x^{2}+c}\,a^{2} c+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}} b^{2} x^{2}}{7d}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}} a^{2}}{3}+\frac{2\left(d\,x^{2}+c\right)^{\frac{5}{2}} a b}{5d}-\frac{2\left(d\,x^{2}+c\right)^{\frac{5}{2}} b^{2} c}{35d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x)
```

```
[Out] 1/7*b^2*x^2*(d*x^2+c)^(5/2)/d-2/35*b^2*c/d^2*(d*x^2+c)^(5/2)+2/5*a*b*(d*x^2
+c)^(5/2)/d+1/3*a^2*(d*x^2+c)^(3/2)-a^2*c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c
^(1/2))/x)+a^2*c*(d*x^2+c)^(1/2)
```

maxima [A] time = 1.11, size = 103, normalized size = 0.93

$$\frac{\left(dx^{2}+c\right)^{\frac{5}{2}} b^{2} x^{2}}{7\,d}-a^{2} c^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{1}{3}\left(dx^{2}+c\right)^{\frac{3}{2}} a^{2}+\sqrt{dx^{2}+c}\,a^{2} c-\frac{2\left(dx^{2}+c\right)^{\frac{5}{2}} b^{2} c}{35\,d^{2}}+\frac{2\left(dx^{2}+c\right)^{\frac{5}{2}} a b}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/7*(d*x^2 + c)^(5/2)*b^2*x^2/d - a^2*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x)))
 + 1/3*(d*x^2 + c)^(3/2)*a^2 + sqrt(d*x^2 + c)*a^2*c - 2/35*(d*x^2 + c)^(5/
2)*b^2*c/d^2 + 2/5*(d*x^2 + c)^(5/2)*a*b/d
```

mupad [B] time = 0.70, size = 191, normalized size = 1.72

$$\left(d\,x^2+c\right)^{3/2}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{3\,d^2}-\frac{c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)}{3}\right)-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{5\,d^2}-\frac{b^2\,c}{5\,d^2}\right)\,{\left(d\,x^2+c\right)}^{5/2}+\frac{b^2\,{\left(d\,x^2+c\right)}^{7/2}}{7\,d^2}+c\,\sqrt{d\,x^2+c}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{d^2}-c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)+a^2\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x)
```

```
[Out] (c + d*x^2)^(3/2)*((a*d - b*c)^2/(3*d^2) - (c*((2*b^2*c - 2*a*b*d)/d^2 - (b
^2*c)/d^2))/3) - ((2*b^2*c - 2*a*b*d)/(5*d^2) - (b^2*c)/(5*d^2))*(c + d*x^2
)^(5/2) + a^2*c^(3/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + (b^2*(c + d
```

```
*x^2)^(7/2))/(7*d^2) + c*(c + d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c
 - 2*a*b*d)/d^2 - (b^2*c)/d^2))
```

sympy [A] time = 103.74, size = 109, normalized size = 0.98

$$\frac{a^2c^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + a^2c\sqrt{c+dx^2} + \frac{a^2\left(c+dx^2\right)^{\frac{3}{2}}}{3} + \frac{b^2\left(c+dx^2\right)^{\frac{7}{2}}}{7d^2} + \frac{\left(c+dx^2\right)^{\frac{5}{2}}\left(4abd-2b^2c\right)}{10d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x,x)
```

```
[Out] a**2*c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*c*sqrt(c + d*x**2
) + a**2*(c + d*x**2)**(3/2)/3 + b**2*(c + d*x**2)**(7/2)/(7*d**2) + (c + d
*x**2)**(5/2)*(4*a*b*d - 2*b**2*c)/(10*d**2)
```

3.601 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=175

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}-\frac{c\left(b^2c^2-12ad(2ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\left(c+dx^2\right)^{3/2}\left(b^2c^2-12ad(2ad+bc)\right)}{24cd}-\frac{x\sqrt{c+dx^2}}{}$$

Rubi **[A]** time = 0.12, antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 388, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}-\frac{c\left(b^2c^2-12ad(2ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}-\frac{x\sqrt{c+dx^2}\left(b^2c^2-12ad(2ad+bc)\right)}{16d}-\frac{1}{24}x\left(c+dx^2\right)^{3/2}\left(\frac{b^2c}{d}-\frac{12a(2ad+bc)}{c}\right)+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] -((b^2*c^2 - 12*a*d*(b*c + 2*a*d))*x*Sqrt[c + d*x^2])/(16*d) - (((b^2*c)/d
- (12*a*(b*c + 2*a*d))/c)*x*(c + d*x^2)^(3/2))/24 - (a^2*(c + d*x^2)^(5/2))
/(c*x) + (b^2*x*(c + d*x^2)^(5/2))/(6*d) - (c*(b^2*c^2 - 12*a*d*(b*c + 2*a*
d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^2}\,dx &= -\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}+\frac{\int\left(2a(bc+2ad)+b^2cx^2\right)\left(c+dx^2\right)^{3/2}\,dx}{c}\\
&= -\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(bc+2ad)\right)\int\left(c+dx^2\right)^{3/2}\,dx}{6cd}\\
&= -\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{5/2}}{6d}-\\
&= -\frac{\left(b^2c^2-12ad(bc+2ad)\right)x\sqrt{c+dx^2}}{16d}-\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}\\
&= -\frac{\left(b^2c^2-12ad(bc+2ad)\right)x\sqrt{c+dx^2}}{16d}-\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}\\
&= -\frac{\left(b^2c^2-12ad(bc+2ad)\right)x\sqrt{c+dx^2}}{16d}-\frac{1}{24}\left(\frac{b^2c}{d}-\frac{12a(bc+2ad)}{c}\right)x\left(c+dx^2\right)^{3/2}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 135, normalized size = 0.77

$$
\sqrt{c+dx^2}\left(\frac{x\left(8a^2d^2+20abcd+b^2c^2\right)}{16d}-\frac{a^2c}{x}+\frac{1}{24}bx^3(12ad+7bc)+\frac{1}{6}b^2dx^5\right)-\frac{c\left(-24a^2d^2-12abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{16d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-((a^2*c)/x) + ((b^2*c^2 + 20*a*b*c*d + 8*a^2*d^2)*x)/(16*
d) + (b*(7*b*c + 12*a*d)*x^3)/24 + (b^2*d*x^5)/6) - (c*(b^2*c^2 - 12*a*b*c*
d - 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(16*d^(3/2))
```

IntegrateAlgebraic [A] time = 0.30, size = 147, normalized size = 0.84

$$\frac{\sqrt{c+dx^2}\left(-48a^2cd+24a^2d^2x^2+60abcdx^2+24abd^2x^4+3b^2c^2x^2+14b^2cdx^4+8b^2d^2x^6\right)}{48dx}+\frac{\left(-24a^2cd^2-12abc^2d+b^2c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-48*a^2*c*d + 3*b^2*c^2*x^2 + 60*a*b*c*d*x^2 + 24*a^2*d^2
*x^2 + 14*b^2*c*d*x^4 + 24*a*b*d^2*x^4 + 8*b^2*d^2*x^6))/(48*d*x) + ((b^2*c
^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(16*
d^(3/2))
```

fricas [A] time = 1.36, size = 293, normalized size = 1.67

$$\left[-\frac{3\left(b^2c^3-12\,abc^2d-24\,a^2cd^2\right)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(8\,b^2d^3x^6-48\,a^2cd^2+2\left(7\,b^2cd^2+12\,abd^3\right)x^4+3\left(b^2c^2d+20\,abcd^2+8\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{96\,d^2x},\ \frac{3\left(b^2c^3-12\,abc^2d-24\,a^2cd^2\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(8\,b^2d^3x^6-48\,a^2cd^2+2\left(7\,b^2cd^2+12\,abd^3\right)x^4+3\left(b^2c^2d+20\,abcd^2+8\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{48\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*sqrt(d)*x*log(-2*d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^2*d^3*x^6 - 48*a^2*c*d^2 + 2*(7*b
^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*d^3)*x^2)*
sqrt(d*x^2 + c))/(d^2*x), 1/48*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*s
qrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^6 - 48*a^2*c*d^
2 + 2*(7*b^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*
d^3)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

giac [A] time = 0.50, size = 173, normalized size = 0.99

$$\frac{2\,a^2c^2\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{48}\left(2\left(4\,b^2dx^2+\frac{7\,b^2cd^4+12\,abd^5}{d^4}\right)x^2+\frac{3\left(b^2c^2d^3+20\,abcd^4+8\,a^2d^5\right)}{d^4}\right)\sqrt{dx^2+c}\,x+\frac{\left(b^2c^3\sqrt{d}-12\,abc^2d^{\frac{3}{2}}-24\,a^2cd^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{32\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/48*(2*(4*b^2*d*
x^2 + (7*b^2*c*d^4 + 12*a*b*d^5)/d^4)*x^2 + 3*(b^2*c^2*d^3 + 20*a*b*c*d^4 +
 8*a^2*d^5)/d^4)*sqrt(d*x^2 + c)*x + 1/32*(b^2*c^3*sqrt(d) - 12*a*b*c^2*d^(
3/2) - 24*a^2*c*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

maple [A]   time = 0.01, size = 221, normalized size = 1.26

$$\frac{3a^{2}c\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{2}+\frac{3ab\,c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{4\sqrt{d}}-\frac{b^{2}c^{3}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{16d^{\frac{3}{2}}}+\frac{3\sqrt{d\,x^{2}+c}\,a^{2}dx}{2}+\frac{3\sqrt{d\,x^{2}+c}\,abcx}{4}-\frac{\sqrt{d\,x^{2}+c}\,b^{2}c^{2}x}{16d}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}dx}{c}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}abx}{2}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}cx}{24d}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}b^{2}x}{6d}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x)
```

```
[Out] 1/6*b^2*x*(d*x^2+c)^(5/2)/d-1/24*b^2*c/d*x*(d*x^2+c)^(3/2)-1/16*b^2*c^2/d*x
*(d*x^2+c)^(1/2)-1/16*b^2*c^3/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/2*a*b
*x*(d*x^2+c)^(3/2)+3/4*a*b*c*x*(d*x^2+c)^(1/2)+3/4*a*b*c^2/d^(1/2)*ln(d^(1/
2)*x+(d*x^2+c)^(1/2))-a^2*(d*x^2+c)^(5/2)/c/x+a^2*d/c*x*(d*x^2+c)^(3/2)+3/2
*a^2*d*x*(d*x^2+c)^(1/2)+3/2*a^2*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A]   time = 1.14, size = 178, normalized size = 1.02

$$\frac{1}{2}\left(dx^{2}+c\right)^{\frac{3}{2}}abx+\frac{3}{4}\sqrt{dx^{2}+c}\,abcx+\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}x}{6\,d}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}cx}{24\,d}-\frac{\sqrt{dx^{2}+c}\,b^{2}c^{2}x}{16\,d}+\frac{3}{2}\sqrt{dx^{2}+c}\,a^{2}dx-\frac{b^{2}c^{3}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{3}{2}}}+\frac{3\,abc^{2}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,\sqrt{d}}+\frac{3}{2}\,a^{2}c\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + c)^(3/2)*a*b*x + 3/4*sqrt(d*x^2 + c)*a*b*c*x + 1/6*(d*x^2 + c)
^(5/2)*b^2*x/d - 1/24*(d*x^2 + c)^(3/2)*b^2*c*x/d - 1/16*sqrt(d*x^2 + c)*b^
2*c^2*x/d + 3/2*sqrt(d*x^2 + c)*a^2*d*x - 1/16*b^2*c^3*arcsinh(d*x/sqrt(c*d
))/d^(3/2) + 3/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 3/2*a^2*c*sqrt(d)
*arcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)^(3/2)*a^2/x
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2, x)
```

sympy [B]   time = 22.41, size = 367, normalized size = 2.10

$$-\frac{a^{2}c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^{2}}{c}}}+\frac{a^{2}\sqrt{c}\,dx\sqrt{1+\frac{dx^{2}}{c}}}{2}-\frac{a^{2}\sqrt{c}\,dx}{\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3a^{2}c\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2}+abc^{\frac{3}{2}}x\sqrt{1+\frac{dx^{2}}{c}}+\frac{abc^{\frac{3}{2}}x}{4\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3ab\sqrt{c}\,dx^{3}}{4\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3abc^{2}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4\sqrt{d}}+\frac{abd^{2}x^{5}}{2\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}+\frac{b^{2}c^{\frac{5}{2}}x}{16d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{17b^{2}c^{\frac{3}{2}}x^{3}}{48\sqrt{1+\frac{dx^{2}}{c}}}+\frac{11b^{2}\sqrt{c}\,dx^{5}}{24\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}c^{3}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{3}{2}}}+\frac{b^{2}d^{2}x^{7}}{6\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**2,x)
```

```
[Out] -a**2*c**(3/2)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)
/2 - a**2*sqrt(c)*d*x/sqrt(1 + d*x**2/c) + 3*a**2*c*sqrt(d)*asinh(sqrt(d)*x
/sqrt(c))/2 + a*b*c**(3/2)*x*sqrt(1 + d*x**2/c) + a*b*c**(3/2)*x/(4*sqrt(1
+ d*x**2/c)) + 3*a*b*sqrt(c)*d*x**3/(4*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asi
nh(sqrt(d)*x/sqrt(c))/(4*sqrt(d)) + a*b*d**2*x**5/(2*sqrt(c)*sqrt(1 + d*x**
2/c)) + b**2*c**(5/2)*x/(16*d*sqrt(1 + d*x**2/c)) + 17*b**2*c**(3/2)*x**3/(
48*sqrt(1 + d*x**2/c)) + 11*b**2*sqrt(c)*d*x**5/(24*sqrt(1 + d*x**2/c)) - b
**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16*d**(3/2)) + b**2*d**2*x**7/(6*sqrt(c)
*sqrt(1 + d*x**2/c))
```

3.602 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=136

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2}+\frac{a\left(c+dx^2\right)^{3/2}(3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}\,(3ad+4bc)-\frac{1}{2}a\sqrt{c}\,(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2}{}$$

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2}+\frac{a\left(c+dx^2\right)^{3/2}(3ad+4bc)}{6c}+\frac{1}{2}a\sqrt{c+dx^2}\,(3ad+4bc)-\frac{1}{2}a\sqrt{c}\,(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (a*(4*b*c + 3*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 3*a*d)*(c + d*x^2)^(3/2
))/(6*c) + (b^2*(c + d*x^2)^(5/2))/(5*d) - (a^2*(c + d*x^2)^(5/2))/(2*c*x^2
) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^2}\,dx, x, x^2\right)\\
&= -\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+3ad)+b^2cx\right)(c+dx)^{3/2}}{x}\,dx, x, x^2\right)}{2c}\\
&= \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2} + \frac{(a(4bc+3ad))\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, x^2\right)}{4c}\\
&= \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^2} + \frac{1}{4}(a(4bc+3ad))\\
&= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+\right.}{2c}\\
\\
&= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+\right.}{2c}\\
&= \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2} + \frac{a(4bc+3ad)\left(c+dx^2\right)^{3/2}}{6c} + \frac{b^2\left(c+dx^2\right)^{5/2}}{5d} - \frac{a^2\left(c+\right.}{2c}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 108, normalized size = 0.79

$$
\frac{\sqrt{c+dx^2}\left(-15a^2d\left(c-2dx^2\right)+20abdx^2\left(4c+dx^2\right)+6b^2x^2\left(c+dx^2\right)^2\right)}{30dx^2} - \frac{1}{2}a\sqrt{c}\,(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*d*(c - 2*d*x^2) + 6*b^2*x^2*(c + d*x^2)^2 + 20*a*
b*d*x^2*(4*c + d*x^2)))/(30*d*x^2) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/2
```

IntegrateAlgebraic [A] time = 0.17, size = 135, normalized size = 0.99

$$
\frac{\sqrt{c+dx^2}\left(-15a^2cd+30a^2d^2x^2+80abcdx^2+20abd^2x^4+6b^2c^2x^2+12b^2cdx^4+6b^2d^2x^6\right)}{30dx^2} + \frac{1}{2}\left(-3a^2\sqrt{c}\,d-4abc^{3/2}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*c*d + 6*b^2*c^2*x^2 + 80*a*b*c*d*x^2 + 30*a^2*d^2
*x^2 + 12*b^2*c*d*x^4 + 20*a*b*d^2*x^4 + 6*b^2*d^2*x^6))/(30*d*x^2) + ((-4*
a*b*c^(3/2) - 3*a^2*Sqrt[c]*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

fricas [A] time = 1.63, size = 267, normalized size = 1.96

$$\left[\frac{15\left(4\,abcd+3\,a^2d^2\right)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(6\,b^2d^2x^6+4\left(3\,b^2cd+5\,abd^2\right)x^4-15\,a^2cd+2\left(3\,b^2c^2+40\,abcd+15\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{60\,dx^2},\ \frac{15\left(4\,abcd+3\,a^2d^2\right)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(6\,b^2d^2x^6+4\left(3\,b^2cd+5\,abd^2\right)x^4-15\,a^2cd+2\left(3\,b^2c^2+40\,abcd+15\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{30\,dx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 +
c)*sqrt(c) + 2*c)/x^2) + 2*(6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 -
 15*a^2*c*d + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(d*x^2), 1/30*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqr
t(d*x^2 + c)) + (6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 - 15*a^2*c*d
 + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

giac [A] time = 0.37, size = 126, normalized size = 0.93

$$\frac{6\left(dx^2+c\right)^{\frac{5}{2}}b^2+20\left(dx^2+c\right)^{\frac{3}{2}}abd+60\,\sqrt{dx^2+c}\,abcd+30\,\sqrt{dx^2+c}\,a^2d^2-\frac{15\,\sqrt{dx^2+c}\,a^2cd}{x^2}+\frac{15\left(4\,abc^2d+3\,a^2cd^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*(d*x^2 + c)^(5/2)*b^2 + 20*(d*x^2 + c)^(3/2)*a*b*d + 60*sqrt(d*x^2
+ c)*a*b*c*d + 30*sqrt(d*x^2 + c)*a^2*d^2 - 15*sqrt(d*x^2 + c)*a^2*c*d/x^2
+ 15*(4*a*b*c^2*d + 3*a^2*c*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c))
/d
```

maple [A] time = 0.01, size = 161, normalized size = 1.18

$$-\frac{3a^2\sqrt{c}\,d\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{2}-2ab\,c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)+\frac{3\sqrt{d\,x^2+c}\,a^2d}{2}+2\sqrt{d\,x^2+c}\,abc+\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}a^2d}{2c}+\frac{2\left(d\,x^2+c\right)^{\frac{3}{2}}ab}{3}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}b^2}{5d}-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}a^2}{2c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x)
```

```
[Out] 1/5*b^2*(d*x^2+c)^(5/2)/d-1/2*a^2*(d*x^2+c)^(5/2)/c/x^2+1/2*a^2*d/c*(d*x^2+
c)^(3/2)-3/2*a^2*d*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+3/2*a^2*d*
```

```
(d*x^2+c)^(1/2)+2/3*a*b*(d*x^2+c)^(3/2)-2*a*b*c^(3/2)*ln((2*c+2*(d*x^2+c)^(
1/2)*c^(1/2))/x)+2*a*b*(d*x^2+c)^(1/2)*c
```

maxima [A] time = 1.15, size = 138, normalized size = 1.01

$$-2\,abc^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{3}{2}\,a^2\sqrt{c}\,d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{2}{3}\left(dx^2+c\right)^{\frac{3}{2}}ab+2\,\sqrt{dx^2+c}\,abc+\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2}{5\,d}+\frac{3}{2}\,\sqrt{dx^2+c}\,a^2d+\frac{\left(dx^2+c\right)^{\frac{3}{2}}a^2d}{2\,c}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a*b*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 3/2*a^2*sqrt(c)*d*arcsinh(c/
(sqrt(c*d)*abs(x))) + 2/3*(d*x^2 + c)^(3/2)*a*b + 2*sqrt(d*x^2 + c)*a*b*c +
 1/5*(d*x^2 + c)^(5/2)*b^2/d + 3/2*sqrt(d*x^2 + c)*a^2*d + 1/2*(d*x^2 + c)^
(3/2)*a^2*d/c - 1/2*(d*x^2 + c)^(5/2)*a^2/(c*x^2)
```

mupad [B] time = 1.17, size = 201, normalized size = 1.48

$$\frac{b^2\left(d\,x^2+c\right)^{5/2}}{5\,d}-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{3\,d}-\frac{2\,b^2\,c}{3\,d}\right)\left(d\,x^2+c\right)^{3/2}-\sqrt{d\,x^2+c}\,\left(2\,c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)-\frac{\left(a\,d-b\,c\right)^2}{d}+\frac{b^2\,c^2}{d}\right)-\frac{a^2\,c\,\sqrt{d\,x^2+c}}{2\,x^2}+2\,a\,\mathrm{atan}\left(\frac{2\,a\,\sqrt{d\,x^2+c}\,\left(3\,a\,d+4\,b\,c\right)\,\sqrt{-\frac{c}{16}}}{\frac{3\,d\,a^2\,c}{2}+2\,b\,a\,c^2}\right)\left(3\,a\,d+4\,b\,c\right)\,\sqrt{-\frac{c}{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x)
```

```
[Out] (b^2*(c + d*x^2)^(5/2))/(5*d) - ((2*b^2*c - 2*a*b*d)/(3*d) - (2*b^2*c)/(3*d
))*(c + d*x^2)^(3/2) - (c + d*x^2)^(1/2)*(2*c*((2*b^2*c - 2*a*b*d)/d - (2*b
^2*c)/d) - (a*d - b*c)^2/d + (b^2*c^2)/d) - (a^2*c*(c + d*x^2)^(1/2))/(2*x^
2) + 2*a*atan((2*a*(c + d*x^2)^(1/2)*(3*a*d + 4*b*c)*(-c/16)^(1/2))/(2*a*b*
c^2 + (3*a^2*c*d)/2))*(3*a*d + 4*b*c)*(-c/16)^(1/2)
```

sympy [A] time = 88.86, size = 303, normalized size = 2.23

$$-\frac{3a^2\sqrt{c}\,d\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2}-\frac{a^2c\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{2x}+\frac{a^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}}+\frac{a^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}-2abc^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{2abc^2}{\sqrt{d}\,x\sqrt{\frac{c}{dx^2}+1}}+\frac{2abc\sqrt{d}\,x}{\sqrt{\frac{c}{dx^2}+1}}+2abd\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2c\left(\begin{cases}\frac{\sqrt{c}\,x^2}{2} & \text{for}\ d=0\\ \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+b^2d\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**3,x)
```

```
[Out] -3*a**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c*sqrt(d)*sqrt(c/(d*x
**2) + 1)/(2*x) + a**2*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + a**2*d**(3/2)*x
/sqrt(c/(d*x**2) + 1) - 2*a*b*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c
**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c*sqrt(d)*x/sqrt(c/(d*x**2) +
1) + 2*a*b*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*
d), True)) + b**2*c*Piecewise((sqrt(c)*x**2/2, Eq(d, 0)), ((c + d*x**2)**(3
/2)/(3*d), True)) + b**2*d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) +
c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(
c)*x**4/4, True))
```

3.603 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=184

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{x\left(c+dx^2\right)^{3/2}\left(8ad(ad+3bc)+3b^2c^2\right)}{12c^2}+\frac{x\sqrt{c+dx^2}\left(8ad(ad+3bc)+3b^2c^2\right)}{8c}+\frac{(8ad(ad+3bc)+}{}$$

Rubi [A] time = 0.13, antiderivative size = 181, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{1}{12}x\left(c+dx^2\right)^{3/2}\left(\frac{8ad(ad+3bc)}{c^2}+3b^2\right)+\frac{x\sqrt{c+dx^2}\left(8ad(ad+3bc)+3b^2c^2\right)}{8c}+\frac{\left(8ad(ad+3bc)+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8\sqrt{d}}-\frac{2a\left(c+dx^2\right)^{5/2}(ad+3bc)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + ((3*b^2 + (8*
a*d*(3*b*c + a*d))/c^2)*x*(c + d*x^2)^(3/2))/12 - (a^2*(c + d*x^2)^(5/2))/(
3*c*x^3) - (2*a*(3*b*c + a*d)*(c + d*x^2)^(5/2))/(3*c^2*x) + ((3*b^2*c^2 +
8*a*d*(3*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^4}\,dx &= -\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}+\frac{\int \frac{\left(2a(3bc+ad)+3b^2cx^2\right)\left(c+dx^2\right)^{3/2}}{x^2}\,dx}{3c}\\
&= -\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}-\frac{2a(3bc+ad)\left(c+dx^2\right)^{5/2}}{3c^2x}-\frac{1}{3}\left(-3b^2-\frac{8ad(3bc+ad)}{c^2}\right)\int \big(c\\
&= \frac{1}{12}\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2}-\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}-\frac{2a(3bc+ad)\big(c+d}{3c^2x}\\
&= \frac{1}{8}c\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/}\\
&= \frac{1}{8}c\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/}\\
&= \frac{1}{8}c\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}\left(3b^2+\frac{8ad(3bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 118, normalized size = 0.64

$$\frac{1}{24}\left(\frac{3\left(8a^2d^2+24abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+\frac{\sqrt{c+dx^2}\left(-8a^2c+3bx^4(8ad+5bc)-16ax^2(2ad+3bc)+6b^2dx^6\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*c - 16*a*(3*b*c + 2*a*d)*x^2 + 3*b*(5*b*c + 8*a*d
)*x^4 + 6*b^2*d*x^6))/x^3 + (3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*Log[d*x
 + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/24
```

IntegrateAlgebraic [A]   time = 0.32, size = 122, normalized size = 0.66

$$\frac{\left(-8a^2d^2-24abcd-3b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8\sqrt{d}}+\frac{\sqrt{c+dx^2}\,\left(-8a^2c-32a^2dx^2-48abcx^2+24abdx^4+15b^2cx^4+6b^2dx^6\right)}{24x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*c - 48*a*b*c*x^2 - 32*a^2*d*x^2 + 15*b^2*c*x^4 + 2
4*a*b*d*x^4 + 6*b^2*d*x^6))/(24*x^3) + ((-3*b^2*c^2 - 24*a*b*c*d - 8*a^2*d^
2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*Sqrt[d])
```

fricas [A]   time = 1.58, size = 266, normalized size = 1.45

$$\left[\frac{3\left(3\,b^2c^2+24\,abcd+8\,a^2d^2\right)\sqrt{d}\,x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(6\,b^2d^2x^6+3\left(5\,b^2cd+8\,abd^2\right)x^4-8\,a^2cd-16\left(3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{48\,dx^3},-\frac{3\left(3\,b^2c^2+24\,abcd+8\,a^2d^2\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(6\,b^2d^2x^6+3\left(5\,b^2cd+8\,abd^2\right)x^4-8\,a^2cd-16\left(3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{24\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^3*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^
2)*x^4 - 8*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^
3), -1/24*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^3*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^2)*x^4 - 8
*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

giac [A]   time = 0.46, size = 262, normalized size = 1.42

$$\frac{1}{8}\left(2\,b^2dx^2+\frac{5\,b^2cd^2+8\,abd^3}{d^2}\right)\sqrt{dx^2+c}\,x-\frac{\left(3\,b^2c^2\sqrt{d}+24\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,d}+\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^2\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2cd^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^3\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^2d^{\frac{3}{2}}+3\,abc^4\sqrt{d}+2\,a^2c^3d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d*x^2 + (5*b^2*c*d^2 + 8*a*b*d^3)/d^2)*sqrt(d*x^2 + c)*x - 1/16*
(3*b^2*c^2*sqrt(d) + 24*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x - sqr
```

```
t(d*x^2 + c))^2)/d + 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*sqrt(d)
 + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a*b*c^3*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^2*d^(
3/2) + 3*a*b*c^4*sqrt(d) + 2*a^2*c^3*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c)
)^2 - c)^3
```

maple [A] time = 0.02, size = 241, normalized size = 1.31

$$a^{2}d^{\frac{3}{2}}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+3abc\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+\frac{3b^{2}c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{8\sqrt{d}}+\frac{\sqrt{d\,x^{2}+c}\,a^{2}d^{2}x}{c}+3\sqrt{d\,x^{2}+c}\,abdx+\frac{3\sqrt{d\,x^{2}+c}\,b^{2}cx}{8}+\frac{2\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}x}{3c^{2}}+\frac{2\left(d\,x^{2}+c\right)^{\frac{3}{2}}abdx}{c}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}x}{4}-\frac{2\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}d}{3c^{2}x}-\frac{2\left(d\,x^{2}+c\right)^{\frac{5}{2}}ab}{cx}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}}{3c\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x)
```

```
[Out] 1/4*x*b^2*(d*x^2+c)^(3/2)+3/8*b^2*c*x*(d*x^2+c)^(1/2)+3/8*b^2*c^2/d^(1/2)*l
n(d^(1/2)*x+(d*x^2+c)^(1/2))-2*a*b/c/x*(d*x^2+c)^(5/2)+2*a*b*d/c*x*(d*x^2+c
)^(3/2)+3*a*b*d*x*(d*x^2+c)^(1/2)+3*a*b*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1
/2))-1/3*a^2*(d*x^2+c)^(5/2)/c/x^3-2/3*a^2*d/c^2/x*(d*x^2+c)^(5/2)+2/3*a^2*
d^2/c^2*x*(d*x^2+c)^(3/2)+a^2*d^2/c*x*(d*x^2+c)^(1/2)+a^2*d^(3/2)*ln(d^(1/2
)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.13, size = 177, normalized size = 0.96

$$\frac{1}{4}\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}x+\frac{3}{8}\sqrt{dx^{2}+c}\,b^{2}cx+3\sqrt{dx^{2}+c}\,abdx+\frac{\sqrt{dx^{2}+c}\,a^{2}d^{2}x}{c}+\frac{3\,b^{2}c^{2}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{d}}+3\,abc\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+a^{2}d^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}ab}{x}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d}{3\,cx}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}}{3\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*(d*x^2 + c)^(3/2)*b^2*x + 3/8*sqrt(d*x^2 + c)*b^2*c*x + 3*sqrt(d*x^2 +
c)*a*b*d*x + sqrt(d*x^2 + c)*a^2*d^2*x/c + 3/8*b^2*c^2*arcsinh(d*x/sqrt(c*d
))/sqrt(d) + 3*a*b*c*sqrt(d)*arcsinh(d*x/sqrt(c*d)) + a^2*d^(3/2)*arcsinh(d
*x/sqrt(c*d)) - 2*(d*x^2 + c)^(3/2)*a*b/x - 2/3*(d*x^2 + c)^(3/2)*a^2*d/(c*
x) - 1/3*(d*x^2 + c)^(5/2)*a^2/(c*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4, x)
```

sympy [B] time = 14.13, size = 352, normalized size = 1.91

$$-\frac{a^{2}\sqrt{c}\,d}{x\sqrt{1+\frac{dx^{2}}{c}}}-\frac{a^{2}c\sqrt{d}\,\sqrt{\frac{c}{dx^{2}}+1}}{3x^{2}}-\frac{a^{2}d^{\frac{3}{2}}\sqrt{\frac{c}{dx^{2}}+1}}{3}+a^{2}d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{a^{2}d^{2}x}{\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}-\frac{2abc^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^{2}}{c}}}+ab\sqrt{c}\,dx\sqrt{1+\frac{dx^{2}}{c}}-\frac{2ab\sqrt{c}\,dx}{\sqrt{1+\frac{dx^{2}}{c}}}+3abc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)+\frac{b^{2}c^{\frac{3}{2}}x\sqrt{1+\frac{dx^{2}}{c}}}{2}+\frac{b^{2}c^{\frac{3}{2}}x}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3b^{2}\sqrt{c}\,dx^{3}}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3b^{2}c^{2}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{b^{2}d^{2}x^{5}}{4\sqrt{c}\,\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**4,x)
```

```
[Out] -a**2*sqrt(c)*d/(x*sqrt(1 + d*x**2/c)) - a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1
)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + a**2*d**(3/2)*asinh(sqr
t(d)*x/sqrt(c)) - a**2*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 2*a*b*c**(3/2)
/(x*sqrt(1 + d*x**2/c)) + a*b*sqrt(c)*d*x*sqrt(1 + d*x**2/c) - 2*a*b*sqrt(c
)*d*x/sqrt(1 + d*x**2/c) + 3*a*b*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + b**2*
c**(3/2)*x*sqrt(1 + d*x**2/c)/2 + b**2*c**(3/2)*x/(8*sqrt(1 + d*x**2/c)) +
3*b**2*sqrt(c)*d*x**3/(8*sqrt(1 + d*x**2/c)) + 3*b**2*c**2*asinh(sqrt(d)*x/
sqrt(c))/(8*sqrt(d)) + b**2*d**2*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.604 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=181

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4}+\frac{\left(c+dx^2\right)^{3/2}\left(3ad(ad+8bc)+8b^2c^2\right)}{24c^2}+\frac{\sqrt{c+dx^2}\left(3ad(ad+8bc)+8b^2c^2\right)}{8c}-\frac{\big(3ad(ad+8bc)+8}{}$$

Rubi [A] time = 0.21, antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4}+\frac{1}{24}\left(c+dx^2\right)^{3/2}\left(\frac{3ad(ad+8bc)}{c^2}+8b^2\right)+\frac{\sqrt{c+dx^2}\left(3ad(ad+8bc)+8b^2c^2\right)}{8c}-\frac{\left(3ad(ad+8bc)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}}-\frac{a\left(c+dx^2\right)^{5/2}(ad+8bc)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*Sqrt[c + d*x^2])/(8*c) + ((8*b^2 + (3*a*
d*(8*b*c + a*d))/c^2)*(c + d*x^2)^(3/2))/24 - (a^2*(c + d*x^2)^(5/2))/(4*c*
x^4) - (a*(8*b*c + a*d)*(c + d*x^2)^(5/2))/(8*c^2*x^2) - ((8*b^2*c^2 + 3*a*
d*(8*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^5}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^3}\,dx,x,x^2\right)$$

$$=-\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4}+\frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc+ad)+2b^2cx\right)(c+dx)^{3/2}}{x^2}\,dx,x,x^2\right)}{4c}$$

$$=-\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4}-\frac{a(8bc+ad)\left(c+dx^2\right)^{5/2}}{8c^2x^2}+\frac{1}{16}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\text{Subst}\Bigg($$

$$=\frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}-\frac{a^2\left(c+dx^2\right)^{5/2}}{4cx^4}-\frac{a(8bc+ad)\left(c+dx^2\right.}{8c^2x^2}$$

$$=\frac{1}{8}c\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2}+\frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}-$$

$$=\frac{1}{8}c\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2}+\frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}-$$

$$=\frac{1}{8}c\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\sqrt{c+dx^2}+\frac{1}{24}\left(8b^2+\frac{3ad(8bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}-$$

Mathematica [A] time = 0.11, size = 116, normalized size = 0.64

$$\frac{1}{24}\left(\frac{\sqrt{c+dx^2}\left(-3a^2\left(2c+5dx^2\right)-24abx^2\left(c-2dx^2\right)+8b^2x^4\left(4c+dx^2\right)\right)}{x^4}-\frac{3\left(3a^2d^2+24abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-24*a*b*x^2*(c - 2*d*x^2) + 8*b^2*x^4*(4*c + d*x^2) - 3*
a^2*(2*c + 5*d*x^2)))/x^4 - (3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*ArcTanh
[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/24
```

IntegrateAlgebraic [A] time = 0.19, size = 119, normalized size = 0.66

$$\frac{\left(-3a^2d^2-24abcd-8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{\sqrt{c+dx^2}\left(-6a^2c-15a^2dx^2-24abcx^2+48abdx^4+32b^2cx^4+8b^2dx^6\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-6*a^2*c - 24*a*b*c*x^2 - 15*a^2*d*x^2 + 32*b^2*c*x^4 + 4
8*a*b*d*x^4 + 8*b^2*d*x^6))/(24*x^4) + ((-8*b^2*c^2 - 24*a*b*c*d - 3*a^2*d^
2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*Sqrt[c])
```

fricas [A] time = 1.47, size = 267, normalized size = 1.48

$$\left[\frac{3\left(8\,b^{2}c^{2}+24\,abcd+3\,a^{2}d^{2}\right)\sqrt{c}\,x^{4}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\left(8\,b^{2}cdx^{6}+16\left(2\,b^{2}c^{2}+3\,abcd\right)x^{4}-6\,a^{2}c^{2}-3\left(8\,abc^{2}+5\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,cx^{4}},\ \frac{3\left(8\,b^{2}c^{2}+24\,abcd+3\,a^{2}d^{2}\right)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(8\,b^{2}cdx^{6}+16\left(2\,b^{2}c^{2}+3\,abcd\right)x^{4}-6\,a^{2}c^{2}-3\left(8\,abc^{2}+5\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{24\,cx^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a
*b*c*d)*x^4 - 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(
c*x^4), 1/24*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^4*arctan(sq
rt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a*b*c*d)*x^4 -
 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^4)]
```

giac [A] time = 0.35, size = 182, normalized size = 1.01

$$\frac{8\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d+24\,\sqrt{dx^{2}+c}\,b^{2}cd+48\,\sqrt{dx^{2}+c}\,abd^{2}+\frac{3\left(8\,b^{2}c^{2}d+24\,abcd^{2}+3\,a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{3\left(8\left(dx^{2}+c\right)^{\frac{3}{2}}abcd^{2}-8\,\sqrt{dx^{2}+c}\,abc^{2}d^{2}+5\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}-3\,\sqrt{dx^{2}+c}\,a^{2}cd^{3}\right)}{d^{2}x^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(8*(d*x^2 + c)^(3/2)*b^2*d + 24*sqrt(d*x^2 + c)*b^2*c*d + 48*sqrt(d*x^
2 + c)*a*b*d^2 + 3*(8*b^2*c^2*d + 24*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x
^2 + c)/sqrt(-c))/sqrt(-c) - 3*(8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^
2 + c)*a*b*c^2*d^2 + 5*(d*x^2 + c)^(3/2)*a^2*d^3 - 3*sqrt(d*x^2 + c)*a^2*c*
d^3)/(d^2*x^4))/d
```

maple [A] time = 0.01, size = 256, normalized size = 1.41

$$-\frac{3a^{2}d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{8\sqrt{c}}-3ab\sqrt{c}\,d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)-b^{2}c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)+\frac{3\sqrt{d\,x^{2}+c}\,a^{2}d^{2}}{8c}+3\sqrt{d\,x^{2}+c}\,abd+\sqrt{d\,x^{2}+c}\,b^{2}c+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{8c^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}abd}{c}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}}{3}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}d}{8c^{2}x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}ab}{c\,x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}}{4c\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(5/2)/c/x^4-1/8*a^2*d/c^2/x^2*(d*x^2+c)^(5/2)+1/8*a^2*d^
2/c^2*(d*x^2+c)^(3/2)-3/8*a^2*d^2/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2)
)/x)+3/8*a^2*d^2/c*(d*x^2+c)^(1/2)-a*b/c/x^2*(d*x^2+c)^(5/2)+a*b*d/c*(d*x^2
+c)^(3/2)-3*a*b*d*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+3*a*b*d*(d*
x^2+c)^(1/2)+1/3*b^2*(d*x^2+c)^(3/2)-b^2*c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*
c^(1/2))/x)+b^2*(d*x^2+c)^(1/2)*c
```

maxima [**A**] time = 1.13, size = 222, normalized size = 1.23

$$-b^{2} c^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} |x|}\right) - 3 \, ab\sqrt{c} d \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} |x|}\right) - \frac{3 \, a^{2}d^{2} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} |x|}\right)}{8 \, \sqrt{c}} + \frac{1}{3} \, {\left(dx^{2} + c\right)}^{\frac{3}{2}} b^{2} + \sqrt{dx^{2} + c} b^{2}c + 3 \, \sqrt{dx^{2} + c} abd + \frac{{\left(dx^{2} + c\right)}^{\frac{3}{2}} abd}{c} + \frac{{\left(dx^{2} + c\right)}^{\frac{3}{2}} a^{2}d^{2}}{8 \, c^{2}} + \frac{3 \, \sqrt{dx^{2} + c} a^{2}d^{2}}{8 \, c} - \frac{{\left(dx^{2} + c\right)}^{\frac{5}{2}} ab}{cx^{2}} - \frac{{\left(dx^{2} + c\right)}^{\frac{5}{2}} a^{2}d}{8 \, c^{2}x^{2}} - \frac{{\left(dx^{2} + c\right)}^{\frac{5}{2}} a^{2}}{4 \, cx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -b^2*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 3*a*b*sqrt(c)*d*arcsinh(c/(sqr
t(c*d)*abs(x))) - 3/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/3*(
d*x^2 + c)^(3/2)*b^2 + sqrt(d*x^2 + c)*b^2*c + 3*sqrt(d*x^2 + c)*a*b*d + (d
*x^2 + c)^(3/2)*a*b*d/c + 1/8*(d*x^2 + c)^(3/2)*a^2*d^2/c^2 + 3/8*sqrt(d*x^
2 + c)*a^2*d^2/c - (d*x^2 + c)^(5/2)*a*b/(c*x^2) - 1/8*(d*x^2 + c)^(5/2)*a^
2*d/(c^2*x^2) - 1/4*(d*x^2 + c)^(5/2)*a^2/(c*x^4)
```

mupad [**B**] time = 1.65, size = 208, normalized size = 1.15

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{3\,a^2\,c\,d^2}{8}+b\,a\,c^2\,d\right)-\left(\frac{5\,a^2\,d^2}{8}+b\,c\,a\,d\right)\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(d\,x^2+c\right)}^2-2\,c\,\left(d\,x^2+c\right)+c^2}+\sqrt{d\,x^2+c}\,\left(c\,b^2+2\,a\,d\,b\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{3/2}}{3}+\frac{\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,\left(3\,a^2\,d^2+24\,a\,b\,c\,d+8\,b^2\,c^2\right)\,1{}\mathrm{i}}{4\,\sqrt{c}\,\left(\frac{3\,a^2\,d^2}{4}+6\,a\,b\,c\,d+2\,b^2\,c^2\right)}\right)\,\left(3\,a^2\,d^2+24\,a\,b\,c\,d+8\,b^2\,c^2\right)\,1{}\mathrm{i}}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((3*a^2*c*d^2)/8 + a*b*c^2*d) - ((5*a^2*d^2)/8 + a*b*c*d
)*(c + d*x^2)^(3/2))/((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) + (c + d*x^2)^
(1/2)*(b^2*c + 2*a*b*d) + (b^2*(c + d*x^2)^(3/2))/3 + (atan(((c + d*x^2)^(1
/2)*(3*a^2*d^2 + 8*b^2*c^2 + 24*a*b*c*d)*1i)/(4*c^(1/2)*((3*a^2*d^2)/4 + 2*
b^2*c^2 + 6*a*b*c*d)))*(3*a^2*d^2 + 8*b^2*c^2 + 24*a*b*c*d)*1i)/(8*c^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.605 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=147

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{b\left(c+dx^2\right)^{3/2}(4ad+3bc)}{3cx}+\frac{bdx\sqrt{c+dx^2}\,(4ad+3bc)}{2c}+\frac{1}{2}b\sqrt{d}\,(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)-$$

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {462, 453, 277, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5}-\frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3}-\frac{b\left(c+dx^2\right)^{3/2}(4ad+3bc)}{3cx}+\frac{bdx\sqrt{c+dx^2}\,(4ad+3bc)}{2c}+\frac{1}{2}b\sqrt{d}\,(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] (b*d*(3*b*c + 4*a*d)*x*Sqrt[c + d*x^2])/(2*c) - (b*(3*b*c + 4*a*d)*(c + d*x
^2)^(3/2))/(3*c*x) - (a^2*(c + d*x^2)^(5/2))/(5*c*x^5) - (2*a*b*(c + d*x^2)
^(5/2))/(3*c*x^3) + (b*Sqrt[d]*(3*b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^6}\,dx = -\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5} + \frac{\int \frac{\left(10abc+5b^2cx^2\right)\left(c+dx^2\right)^{3/2}}{x^4}\,dx}{5c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5} - \frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3} + \frac{(b(3bc+4ad))\int \frac{\left(c+dx^2\right)^{3/2}}{x^2}\,dx}{3c}$$

$$= -\frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx} - \frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5} - \frac{2ab\left(c+dx^2\right)^{5/2}}{3cx^3} + \frac{(bd(3bc+4ad)}{}$$

$$= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx} - \frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5} - \frac{2ab\left(c\,-\right.}{3c}$$

$$= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx} - \frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5} - \frac{2ab\left(c\,-\right.}{3c}$$

$$= \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)\left(c+dx^2\right)^{3/2}}{3cx} - \frac{a^2\left(c+dx^2\right)^{5/2}}{5cx^5} - \frac{2ab\left(c\,-\right.}{3c}$$

Mathematica [A] time = 0.11, size = 113, normalized size = 0.77

$$\frac{1}{2}b\sqrt{d}\,(4ad+3bc)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right) - \frac{\sqrt{c+dx^2}\left(6a^2\left(c+dx^2\right)^2+20abcx^2\left(c+4dx^2\right)+15b^2cx^4\left(2c-dx^2\right)\right)}{30cx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] -1/30*(Sqrt[c + d*x^2]*(15*b^2*c*x^4*(2*c - d*x^2) + 6*a^2*(c + d*x^2)^2 +
20*a*b*c*x^2*(c + 4*d*x^2)))/(c*x^5) + (b*Sqrt[d]*(3*b*c + 4*a*d)*Log[d*x +
 Sqrt[d]*Sqrt[c + d*x^2]])/2
```

IntegrateAlgebraic [A] time = 0.34, size = 138, normalized size = 0.94

$$\frac{\sqrt{c+dx^2}\left(-6a^2c^2-12a^2cdx^2-6a^2d^2x^4-20abc^2x^2-80abcdx^4-30b^2c^2x^4+15b^2cdx^6\right)}{30cx^5} + \frac{1}{2}\left(-4abd^{3/2}-3b^2c\sqrt{d}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-6*a^2*c^2 - 20*a*b*c^2*x^2 - 12*a^2*c*d*x^2 - 30*b^2*c^2
*x^4 - 80*a*b*c*d*x^4 - 6*a^2*d^2*x^4 + 15*b^2*c*d*x^6))/(30*c*x^5) + ((-3*
b^2*c*Sqrt[d] - 4*a*b*d^(3/2))*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/2
```

fricas [A] time = 1.56, size = 266, normalized size = 1.81

$$\left[\frac{15\left(3b^{2}c^{2}+4abcd\right)\sqrt{d}x^{5}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+2\left(15b^{2}cdx^{6}-2\left(15b^{2}c^{2}+40abcd+3a^{2}d^{2}\right)x^{4}-6a^{2}c^{2}-4\left(5abc^{2}+3a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{60cx^{5}},-\frac{15\left(3b^{2}c^{2}+4abcd\right)\sqrt{-d}x^{5}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\left(15b^{2}cdx^{6}-2\left(15b^{2}c^{2}+40abcd+3a^{2}d^{2}\right)x^{4}-6a^{2}c^{2}-4\left(5abc^{2}+3a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{30cx^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 +
 c)*sqrt(d)*x - c) + 2*(15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2
*d^2)*x^4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*
x^5), -1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^2)*x^
4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^5)]
```

giac [B] time = 0.53, size = 407, normalized size = 2.77

$$\frac{1}{2}\sqrt{dx^{2}+c}\,b^{2}dx-\frac{1}{4}\left(3b^{2}c\sqrt{d}+4abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}\right)+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}b^{2}c^{2}\sqrt{d}+60\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}abcd^{\frac{3}{2}}+15\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{8}a^{2}d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b^{2}c^{3}\sqrt{d}-180\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}abc^{2}d^{\frac{3}{2}}+90\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{2}c^{4}\sqrt{d}+220\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}abc^{3}d^{\frac{3}{2}}+30\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}c^{2}d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c^{5}\sqrt{d}-140\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abc^{4}d^{\frac{3}{2}}+15b^{2}c^{6}\sqrt{d}+40abc^{5}d^{\frac{3}{2}}+3a^{2}c^{4}d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*d*x - 1/4*(3*b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((sq
rt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^
2*c^2*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 15*(sqrt
(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*b^2*c^3*sqrt(d) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) + 90
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) + 220*(sqrt(d)*x - sqrt(d*
x^2 + c))^4*a*b*c^3*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^
(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) - 140*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) + 15*b^2*c^6*sqrt(d) + 40*a*b*c^5*d^(
3/2) + 3*a^2*c^4*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

maple [A] time = 0.02, size = 203, normalized size = 1.38

$$2abd^{\frac{3}{2}}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+\frac{3b^{2}c\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{2}+\frac{2\sqrt{d\,x^{2}+c}\,ab\,d^{2}x}{c}+\frac{3\sqrt{d\,x^{2}+c}\,b^{2}dx}{2}+\frac{4\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab\,d^{2}x}{3c^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}dx}{c}-\frac{4\left(d\,x^{2}+c\right)^{\frac{5}{2}}abd}{3c^{2}x}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}b^{2}}{cx}-\frac{2\left(d\,x^{2}+c\right)^{\frac{5}{2}}ab}{3c\,x^{3}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}}{5c\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(5/2)/c/x^5-b^2/c/x*(d*x^2+c)^(5/2)+b^2*d/c*x*(d*x^2+c)^
(3/2)+3/2*b^2*d*x*(d*x^2+c)^(1/2)+3/2*b^2*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2+c)^
(1/2))-2/3*a*b*(d*x^2+c)^(5/2)/c/x^3-4/3*a*b*d/c^2/x*(d*x^2+c)^(5/2)+4/3*a*
b*d^2/c^2*x*(d*x^2+c)^(3/2)+2*a*b*d^2/c*x*(d*x^2+c)^(1/2)+2*a*b*d^(3/2)*ln(
d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 1.07, size = 147, normalized size = 1.00

$$\frac{3}{2}\sqrt{dx^2+c}\,b^2dx+\frac{2\sqrt{dx^2+c}\,abd^2x}{c}+\frac{3}{2}b^2c\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+2\,abd^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2}{x}-\frac{4\left(dx^2+c\right)^{\frac{3}{2}}abd}{3\,cx}-\frac{2\left(dx^2+c\right)^{\frac{5}{2}}ab}{3\,cx^3}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(d*x^2 + c)*b^2*d*x + 2*sqrt(d*x^2 + c)*a*b*d^2*x/c + 3/2*b^2*c*sqr
t(d)*arcsinh(d*x/sqrt(c*d)) + 2*a*b*d^(3/2)*arcsinh(d*x/sqrt(c*d)) - (d*x^2
 + c)^(3/2)*b^2/x - 4/3*(d*x^2 + c)^(3/2)*a*b*d/(c*x) - 2/3*(d*x^2 + c)^(5/
2)*a*b/(c*x^3) - 1/5*(d*x^2 + c)^(5/2)*a^2/(c*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6, x)
```

sympy [B] time = 8.79, size = 304, normalized size = 2.07

$$-\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{5x^2}-\frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{5c}-\frac{2ab\sqrt{c}\,d}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+2abd^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{2abd^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2\sqrt{c}\,dx\sqrt{1+\frac{dx^2}{c}}}{2}-\frac{b^2\sqrt{c}\,dx}{\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2c\sqrt{d}\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**6,x)
```

```
[Out] -a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a**2*d**(3/2)*sqrt(c/(d*x
**2) + 1)/(5*x**2) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(5*c) - 2*a*b*sqrt(
c)*d/(x*sqrt(1 + d*x**2/c)) - 2*a*b*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2)
 - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 2*a*b*d**(3/2)*asinh(sqrt(d)*x/s
qrt(c)) - 2*a*b*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)/(x*sqrt
(1 + d*x**2/c)) + b**2*sqrt(c)*d*x*sqrt(1 + d*x**2/c)/2 - b**2*sqrt(c)*d*x/
sqrt(1 + d*x**2/c) + 3*b**2*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/2
```

3.606 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=187

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{3/2}\left(ad(12bc-ad)+24b^2c^2\right)}{48c^2x^2}+\frac{d\sqrt{c+dx^2}\left(ad(12bc-ad)+24b^2c^2\right)}{16c^2}-\frac{d\Big(ad(12bc-ad}{}$$

Rubi [A] time = 0.22, antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 89, 78, 47, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{3/2}\left(\frac{ad(12bc-ad)}{c^2}+24b^2\right)}{48x^2}+\frac{d\sqrt{c+dx^2}\left(ad(12bc-ad)+24b^2c^2\right)}{16c^2}-\frac{d\left(ad(12bc-ad)+24b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}-\frac{a\left(c+dx^2\right)^{5/2}(12bc-ad)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] (d*(24*b^2*c^2 + a*d*(12*b*c - a*d))*Sqrt[c + d*x^2])/(16*c^2) - ((24*b^2 +
 (a*d*(12*b*c - a*d))/c^2)*(c + d*x^2)^(3/2))/(48*x^2) - (a^2*(c + d*x^2)^(
5/2))/(6*c*x^6) - (a*(12*b*c - a*d)*(c + d*x^2)^(5/2))/(24*c^2*x^4) - (d*(2
4*b^2*c^2 + a*d*(12*b*c - a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/
2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^7}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{3/2}}{x^4}\,dx,x,x^2\right)$$

$$=-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}+\frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(12bc-ad)+3b^2cx\right)(c+dx)^{3/2}}{x^3}\,dx,x,x^2\right)}{6c}$$

$$=-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}-\frac{a(12bc-ad)\left(c+dx^2\right)^{5/2}}{24c^2x^4}+\frac{1}{48}\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\text{Subs}$$

$$=-\frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2}-\frac{a^2\left(c+dx^2\right)^{5/2}}{6cx^6}-\frac{a(12bc-ad)\left(c+dx^2\right)^{5/2}}{24c^2x^4}$$

$$=\frac{1}{16}d\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2}-\frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2}-a^2($$

$$=\frac{1}{16}d\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2}-\frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2}-a^2($$

$$=\frac{1}{16}d\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\sqrt{c+dx^2}-\frac{\left(24b^2+\frac{ad(12bc-ad)}{c^2}\right)\left(c+dx^2\right)^{3/2}}{48x^2}-a^2($$

Mathematica [C] time = 0.05, size = 92, normalized size = 0.49

$$-\frac{\left(c+dx^2\right)^{5/2}\left(dx^6\left(a^2d^2-12abcd-24b^2c^2\right)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{dx^2}{c}+1\right)+5ac^2\left(4ac-adx^2+12bcx^2\right)\right)}{120c^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] -1/120*((c + d*x^2)^(5/2)*(5*a*c^2*(4*a*c + 12*b*c*x^2 - a*d*x^2) + d*(-24*
b^2*c^2 - 12*a*b*c*d + a^2*d^2)*x^6*Hypergeometric2F1[2, 5/2, 7/2, 1 + (d*x
^2)/c]))/(c^4*x^6)
```

IntegrateAlgebraic [A] time = 0.24, size = 144, normalized size = 0.77

$$\frac{\sqrt{c+dx^2}\left(-8a^2c^2-14a^2cdx^2-3a^2d^2x^4-24abc^2x^2-60abcdx^4-24b^2c^2x^4+48b^2cdx^6\right)}{48cx^6}+\frac{\left(a^2d^3-12abcd^2-24b^2c^2d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*c^2 - 24*a*b*c^2*x^2 - 14*a^2*c*d*x^2 - 24*b^2*c^2
*x^4 - 60*a*b*c*d*x^4 - 3*a^2*d^2*x^4 + 48*b^2*c*d*x^6))/(48*c*x^6) + ((-24
*b^2*c^2*d - 12*a*b*c*d^2 + a^2*d^3)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*
c^(3/2))
```

fricas [A] time = 1.69, size = 301, normalized size = 1.61

$$\left[-\frac{3\left(24\,b^{2}c^{2}d+12\,abcd^{2}-a^{2}d^{3}\right)\sqrt{c}\,x^{6}\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\left(48\,b^{2}c^{2}dx^{6}-8\,a^{2}c^{3}-3\left(8\,b^{2}c^{3}+20\,abc^{2}d+a^{2}cd^{2}\right)x^{4}-2\left(12\,abc^{3}+7\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{96\,c^{2}x^{6}},\ \frac{3\left(24\,b^{2}c^{2}d+12\,abcd^{2}-a^{2}d^{3}\right)\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(48\,b^{2}c^{2}dx^{6}-8\,a^{2}c^{3}-3\left(8\,b^{2}c^{3}+20\,abc^{2}d+a^{2}cd^{2}\right)x^{4}-2\left(12\,abc^{3}+7\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,c^{2}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 +
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(48*b^2*c^2*d*x^6 - 8*a^2*c^3 -
3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 + 7*a^2*c^2*d)
*x^2)*sqrt(d*x^2 + c))/(c^2*x^6), 1/48*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^
2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (48*b^2*c^2*d*x^6 -
8*a^2*c^3 - 3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 +
7*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^6)]
```

giac [A] time = 0.32, size = 259, normalized size = 1.39

$$\frac{48\,\sqrt{dx^{2}+c}\,b^{2}d^{2}+\frac{3\left(24\,b^{2}c^{2}d^{2}+12\,abcd^{3}-a^{2}d^{4}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}-\frac{24\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}c^{2}d^{2}-48\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{3}d^{2}+24\,\sqrt{dx^{2}+c}\,b^{2}c^{4}d^{2}+60\left(dx^{2}+c\right)^{\frac{5}{2}}abcd^{3}-96\left(dx^{2}+c\right)^{\frac{3}{2}}abc^{2}d^{3}+36\,\sqrt{dx^{2}+c}\,abc^{3}d^{3}+3\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d^{4}+8\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cd^{4}-3\,\sqrt{dx^{2}+c}\,a^{2}c^{2}d^{4}}{cd^{3}x^{6}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(48*sqrt(d*x^2 + c)*b^2*d^2 + 3*(24*b^2*c^2*d^2 + 12*a*b*c*d^3 - a^2*d
^4)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (24*(d*x^2 + c)^(5/2)*b
^2*c^2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*
d^2 + 60*(d*x^2 + c)^(5/2)*a*b*c*d^3 - 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 3
6*sqrt(d*x^2 + c)*a*b*c^3*d^3 + 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*(d*x^2 + c)
^(3/2)*a^2*c*d^4 - 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c*d^3*x^6))/d
```

maple [B] time = 0.02, size = 335, normalized size = 1.79

$$\frac{a^{2}d^{3}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{16c^{\frac{3}{2}}}-\frac{3ab\,d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{4\sqrt{c}}-\frac{3b^{2}\sqrt{c}\,d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2}-\frac{\sqrt{d\,x^{2}+c}\,a^{2}d^{3}}{16c^{2}}+\frac{3\sqrt{d\,x^{2}+c}\,ab\,d^{2}}{4c}+\frac{3\sqrt{d\,x^{2}+c}\,b^{2}d}{2}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}}{48c^{3}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab\,d^{2}}{4c^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}d}{2c}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}d^{2}}{48c^{3}x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}abd}{4c^{2}x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}b^{2}}{2c\,x^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}d}{24c^{2}x^{4}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}ab}{2c\,x^{4}}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}}{6c\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(5/2)/c/x^6+1/24*a^2*d/c^2/x^4*(d*x^2+c)^(5/2)+1/48*a^2*
d^2/c^3/x^2*(d*x^2+c)^(5/2)-1/48*a^2*d^3/c^3*(d*x^2+c)^(3/2)+1/16*a^2*d^3/c
^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/16*a^2*d^3/c^2*(d*x^2+c)^(1/
2)-1/2*a*b/c/x^4*(d*x^2+c)^(5/2)-1/4*a*b*d/c^2/x^2*(d*x^2+c)^(5/2)+1/4*a*b*
d^2/c^2*(d*x^2+c)^(3/2)-3/4*a*b*d^2/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/
2))/x)+3/4*a*b*d^2/c*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(5/2)+1/2*b^2*
d/c*(d*x^2+c)^(3/2)-3/2*b^2*d*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
+3/2*b^2*d*(d*x^2+c)^(1/2)
```

maxima [A] time = 0.95, size = 301, normalized size = 1.61

$$-\frac{3}{2}\,b^{2}\sqrt{c}\,d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{3\,abd^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,\sqrt{c}}+\frac{a^{2}d^{3}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{3}{2}}}+\frac{3}{2}\,\sqrt{dx^{2}+c}\,b^{2}d+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d}{2\,c}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}abd^{2}}{4\,c^{2}}+\frac{3\,\sqrt{dx^{2}+c}\,abd^{2}}{4\,c}-\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}}{48\,c^{3}}-\frac{\sqrt{dx^{2}+c}\,a^{2}d^{3}}{16\,c^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}}{2\,cx^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}abd}{4\,c^{2}x^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d^{2}}{48\,c^{3}x^{2}}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}ab}{2\,cx^{4}}+\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d}{24\,c^{2}x^{4}}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}}{6\,cx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -3/2*b^2*sqrt(c)*d*arcsinh(c/(sqrt(c*d)*abs(x))) - 3/4*a*b*d^2*arcsinh(c/(s
qrt(c*d)*abs(x)))/sqrt(c) + 1/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3
/2) + 3/2*sqrt(d*x^2 + c)*b^2*d + 1/2*(d*x^2 + c)^(3/2)*b^2*d/c + 1/4*(d*x^
2 + c)^(3/2)*a*b*d^2/c^2 + 3/4*sqrt(d*x^2 + c)*a*b*d^2/c - 1/48*(d*x^2 + c)
^(3/2)*a^2*d^3/c^3 - 1/16*sqrt(d*x^2 + c)*a^2*d^3/c^2 - 1/2*(d*x^2 + c)^(5/
2)*b^2/(c*x^2) - 1/4*(d*x^2 + c)^(5/2)*a*b*d/(c^2*x^2) + 1/48*(d*x^2 + c)^(
5/2)*a^2*d^2/(c^3*x^2) - 1/2*(d*x^2 + c)^(5/2)*a*b/(c*x^4) + 1/24*(d*x^2 +
c)^(5/2)*a^2*d/(c^2*x^4) - 1/6*(d*x^2 + c)^(5/2)*a^2/(c*x^6)
```

mupad [B] time = 2.35, size = 215, normalized size = 1.15

$$\frac{\sqrt{d\,x^2+c}\,\left(-\frac{a^2\,c\,d^3}{16}+\frac{3\,a\,b\,c^2\,d^2}{4}+\frac{b^2\,c^3\,d}{2}\right)-{\left(d\,x^2+c\right)}^{3/2}\,\left(-\frac{a^2\,d^3}{6}+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)+\frac{{\left(d\,x^2+c\right)}^{5/2}\,\left(a^2\,d^3+20\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c}}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}+b^2\,d\,\sqrt{d\,x^2+c}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(-a^2\,d^2+12\,a\,b\,c\,d+24\,b^2\,c^2\right)}{16\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((b^2*c^3*d)/2 - (a^2*c*d^3)/16 + (3*a*b*c^2*d^2)/4) - (
c + d*x^2)^(3/2)*(b^2*c^2*d - (a^2*d^3)/6 + 2*a*b*c*d^2) + ((c + d*x^2)^(5/
2)*(a^2*d^3 + 8*b^2*c^2*d + 20*a*b*c*d^2))/(16*c))/(3*c*(c + d*x^2)^2 - 3*c
^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + b^2*d*(c + d*x^2)^(1/2) - (d*atanh(
(c + d*x^2)^(1/2)/c^(1/2))*(24*b^2*c^2 - a^2*d^2 + 12*a*b*c*d))/(16*c^(3/2)
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**7,x)
```

```
[Out] Timed out
```

3.607 $\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx$

Optimal. Leaf size=114

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

Rubi [**A**] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4}+\frac{\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4}-\frac{c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4}+\frac{b^2\left(c+dx^2\right)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] -(c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^4) + ((b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(9/2))/(9*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(11/2))/(11*d^4)
 + (b^2*(c + d*x^2)^(13/2))/(13*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^2(c+dx)^{5/2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{c(bc-ad)^2(c+dx)^{5/2}}{d^3}+\frac{(bc-ad)(3bc-ad)(c+dx)^{7/2}}{d^3}-\frac{b(3bc\right.\right.\\
&= -\frac{c(bc-ad)^2\left(c+dx^2\right)^{7/2}}{7d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{9/2}}{9d^4}-\frac{b(3bc-2ad)\left(c\right.}{11d^4}
\end{aligned}
$$

Mathematica [**A**] time = 0.11, size = 99, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(143a^2d^2\left(7dx^2-2c\right)+26abd\left(8c^2-28cdx^2+63d^2x^4\right)+b^2\left(-48c^3+168c^2dx^2-378cd^2x^4+693d^3x^6\right)\right)}{9009d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(143*a^2*d^2*(-2*c + 7*d*x^2) + 26*a*b*d*(8*c^2 - 28*c*d
*x^2 + 63*d^2*x^4) + b^2*(-48*c^3 + 168*c^2*d*x^2 - 378*c*d^2*x^4 + 693*d^3
*x^6)))/(9009*d^4)
```

IntegrateAlgebraic [**A**] time = 0.07, size = 111, normalized size = 0.97

$$\frac{\left(c+dx^2\right)^{7/2}\left(-286a^2cd^2+1001a^2d^3x^2+208abc^2d-728abcd^2x^2+1638abd^3x^4-48b^2c^3+168b^2c^2dx^2-378b^2cd^2x^4+693b^2d^3x^6\right)}{9009d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(-48*b^2*c^3 + 208*a*b*c^2*d - 286*a^2*c*d^2 + 168*b^2*c
^2*d*x^2 - 728*a*b*c*d^2*x^2 + 1001*a^2*d^3*x^2 - 378*b^2*c*d^2*x^4 + 1638*
a*b*d^3*x^4 + 693*b^2*d^3*x^6))/(9009*d^4)
```

fricas [**B**] time = 1.52, size = 216, normalized size = 1.89

$$\frac{\left(693\,b^2d^6x^{12}+63\left(27\,b^2cd^5+26\,abd^6\right)x^{10}+7\left(159\,b^2c^2d^4+598\,abcd^5+143\,a^2d^6\right)x^8-48\,b^2c^6+208\,abc^5d-286\,a^2c^4d^2+\left(15\,b^2c^3d^3+2938\,abc^2d^4+2717\,a^2cd^5\right)x^6-3\left(6\,b^2c^4d^2-26\,abc^3d^3-715\,a^2c^2d^4\right)x^4+\left(24\,b^2c^5d-104\,abc^4d^2+143\,a^2c^3d^3\right)x^2\right)\sqrt{dx^2+c}}{9009\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9009*(693*b^2*d^6*x^12 + 63*(27*b^2*c*d^5 + 26*a*b*d^6)*x^10 + 7*(159*b^2
*c^2*d^4 + 598*a*b*c*d^5 + 143*a^2*d^6)*x^8 - 48*b^2*c^6 + 208*a*b*c^5*d -
```

```
286*a^2*c^4*d^2 + (15*b^2*c^3*d^3 + 2938*a*b*c^2*d^4 + 2717*a^2*c*d^5)*x^6
- 3*(6*b^2*c^4*d^2 - 26*a*b*c^3*d^3 - 715*a^2*c^2*d^4)*x^4 + (24*b^2*c^5*d
- 104*a*b*c^4*d^2 + 143*a^2*c^3*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

giac [A] time = 0.27, size = 150, normalized size = 1.32

$$\frac{693 \left(dx^2+c\right)^{\frac{13}{2}} b^2 - 2457 \left(dx^2+c\right)^{\frac{11}{2}} b^2 c + 3003 \left(dx^2+c\right)^{\frac{9}{2}} b^2 c^2 - 1287 \left(dx^2+c\right)^{\frac{7}{2}} b^2 c^3 + 1638 \left(dx^2+c\right)^{\frac{11}{2}} abd - 4004 \left(dx^2+c\right)^{\frac{9}{2}} abcd + 2574 \left(dx^2+c\right)^{\frac{7}{2}} abc^2 d + 1001 \left(dx^2+c\right)^{\frac{9}{2}} a^2 d^2 - 1287 \left(dx^2+c\right)^{\frac{7}{2}} a^2 c d^2}{9009\, d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/9009*(693*(d*x^2 + c)^(13/2)*b^2 - 2457*(d*x^2 + c)^(11/2)*b^2*c + 3003*(
d*x^2 + c)^(9/2)*b^2*c^2 - 1287*(d*x^2 + c)^(7/2)*b^2*c^3 + 1638*(d*x^2 + c
)^(11/2)*a*b*d - 4004*(d*x^2 + c)^(9/2)*a*b*c*d + 2574*(d*x^2 + c)^(7/2)*a*
b*c^2*d + 1001*(d*x^2 + c)^(9/2)*a^2*d^2 - 1287*(d*x^2 + c)^(7/2)*a^2*c*d^2
)/d^4
```

maple [A] time = 0.01, size = 108, normalized size = 0.95

$$-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}} \left(-693 b^2 x^6 d^3 - 1638 ab\, d^3 x^4 + 378 b^2 c\, d^2 x^4 - 1001 a^2 d^3 x^2 + 728 abc\, d^2 x^2 - 168 b^2 c^2 d\, x^2 + 286 a^2 c\, d^2 - 208 ab\, c^2 d + 48 b^2 c^3\right)}{9009 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] -1/9009*(d*x^2+c)^(7/2)*(-693*b^2*d^3*x^6-1638*a*b*d^3*x^4+378*b^2*c*d^2*x^
4-1001*a^2*d^3*x^2+728*a*b*c*d^2*x^2-168*b^2*c^2*d*x^2+286*a^2*c*d^2-208*a*
b*c^2*d+48*b^2*c^3)/d^4
```

maxima [A] time = 0.96, size = 181, normalized size = 1.59

$$\frac{\left(dx^2+c\right)^{\frac{7}{2}} b^2 x^6}{13\, d} - \frac{6 \left(dx^2+c\right)^{\frac{7}{2}} b^2 c x^4}{143\, d^2} + \frac{2 \left(dx^2+c\right)^{\frac{7}{2}} abx^4}{11\, d} + \frac{8 \left(dx^2+c\right)^{\frac{7}{2}} b^2 c^2 x^2}{429\, d^3} - \frac{8 \left(dx^2+c\right)^{\frac{7}{2}} abcx^2}{99\, d^2} + \frac{\left(dx^2+c\right)^{\frac{7}{2}} a^2 x^2}{9\, d} - \frac{16 \left(dx^2+c\right)^{\frac{7}{2}} b^2 c^3}{3003\, d^4} + \frac{16 \left(dx^2+c\right)^{\frac{7}{2}} abc^2}{693\, d^3} - \frac{2 \left(dx^2+c\right)^{\frac{7}{2}} a^2 c}{63\, d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/13*(d*x^2 + c)^(7/2)*b^2*x^6/d - 6/143*(d*x^2 + c)^(7/2)*b^2*c*x^4/d^2 +
2/11*(d*x^2 + c)^(7/2)*a*b*x^4/d + 8/429*(d*x^2 + c)^(7/2)*b^2*c^2*x^2/d^3
- 8/99*(d*x^2 + c)^(7/2)*a*b*c*x^2/d^2 + 1/9*(d*x^2 + c)^(7/2)*a^2*x^2/d -
16/3003*(d*x^2 + c)^(7/2)*b^2*c^3/d^4 + 16/693*(d*x^2 + c)^(7/2)*a*b*c^2/d^
3 - 2/63*(d*x^2 + c)^(7/2)*a^2*c/d^2
```

mupad [B] time = 0.92, size = 207, normalized size = 1.82

$$\sqrt{d\,x^2+c}\,\left(\frac{x^8\,\left(1001\,a^2\,d^6+4186\,a\,b\,c\,d^5+1113\,b^2\,c^2\,d^4\right)}{9009\,d^4}-\frac{286\,a^2\,c^4\,d^2-208\,a\,b\,c^5\,d+48\,b^2\,c^6}{9009\,d^4}+\frac{b^2\,d^2\,x^{12}}{13}+\frac{c\,x^6\,\left(2717\,a^2\,d^2+2938\,a\,b\,c\,d+15\,b^2\,c^2\right)}{9009\,d}+\frac{b\,d\,x^{10}\,\left(26\,a\,d+27\,b\,c\right)}{143}+\frac{c^3\,x^2\,\left(143\,a^2\,d^2-104\,a\,b\,c\,d+24\,b^2\,c^2\right)}{9009\,d^3}+\frac{c^2\,x^4\,\left(715\,a^2\,d^2+26\,a\,b\,c\,d-6\,b^2\,c^2\right)}{3003\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((x^8*(1001*a^2*d^6 + 1113*b^2*c^2*d^4 + 4186*a*b*c*d^5))
/(9009*d^4) - (48*b^2*c^6 + 286*a^2*c^4*d^2 - 208*a*b*c^5*d)/(9009*d^4) + (
b^2*d^2*x^12)/13 + (c*x^6*(2717*a^2*d^2 + 15*b^2*c^2 + 2938*a*b*c*d))/(9009
*d) + (b*d*x^10*(26*a*d + 27*b*c))/143 + (c^3*x^2*(143*a^2*d^2 + 24*b^2*c^2
 - 104*a*b*c*d))/(9009*d^3) + (c^2*x^4*(715*a^2*d^2 - 6*b^2*c^2 + 26*a*b*c*
d))/(3003*d^2))
```

sympy [A] time = 19.94, size = 468, normalized size = 4.11

$$\begin{cases} - \frac{2a^2c^4\sqrt{c+dx^2}}{63d^2} + \frac{a^2c^3x^2\sqrt{c+dx^2}}{63d} + \frac{5a^2c^2x^4\sqrt{c+dx^2}}{21} + \frac{19a^2cdx^6\sqrt{c+dx^2}}{63} + \frac{a^2d^2x^8\sqrt{c+dx^2}}{9} + \frac{16abc^5\sqrt{c+dx^2}}{693d^3} - \frac{8abc^4x^2\sqrt{c+dx^2}}{693d^2} + \frac{2abc^3x^4\sqrt{c+dx^2}}{231d} + \frac{226abc^2x^6\sqrt{c+dx^2}}{693} + \frac{46abcdx^8\sqrt{c+dx^2}}{99} + \frac{2abd^2x^{10}\sqrt{c+dx^2}}{11} - \frac{16b^2c^6\sqrt{c+dx^2}}{3003d^4} + \frac{8b^2c^5x^2\sqrt{c+dx^2}}{3003d^3} - \frac{2b^2c^4x^4\sqrt{c+dx^2}}{1001d^2} + \frac{5b^2c^3x^6\sqrt{c+dx^2}}{3003d} + \frac{53b^2c^2x^8\sqrt{c+dx^2}}{429} + \frac{27b^2cdx^{10}\sqrt{c+dx^2}}{143} + \frac{b^2d^2x^{12}\sqrt{c+dx^2}}{13} & \text{for}\: d \neq 0 \\ c^{\frac{5}{2}} \left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c**4*sqrt(c + d*x**2)/(63*d**2) + a**2*c**3*x**2*sqrt(c
+ d*x**2)/(63*d) + 5*a**2*c**2*x**4*sqrt(c + d*x**2)/21 + 19*a**2*c*d*x**6*
sqrt(c + d*x**2)/63 + a**2*d**2*x**8*sqrt(c + d*x**2)/9 + 16*a*b*c**5*sqrt(
c + d*x**2)/(693*d**3) - 8*a*b*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + 2*a*
b*c**3*x**4*sqrt(c + d*x**2)/(231*d) + 226*a*b*c**2*x**6*sqrt(c + d*x**2)/6
93 + 46*a*b*c*d*x**8*sqrt(c + d*x**2)/99 + 2*a*b*d**2*x**10*sqrt(c + d*x**2
)/11 - 16*b**2*c**6*sqrt(c + d*x**2)/(3003*d**4) + 8*b**2*c**5*x**2*sqrt(c
+ d*x**2)/(3003*d**3) - 2*b**2*c**4*x**4*sqrt(c + d*x**2)/(1001*d**2) + 5*b
**2*c**3*x**6*sqrt(c + d*x**2)/(3003*d) + 53*b**2*c**2*x**8*sqrt(c + d*x**2
)/429 + 27*b**2*c*d*x**10*sqrt(c + d*x**2)/143 + b**2*d**2*x**12*sqrt(c + d
*x**2)/13, Ne(d, 0)), (c**(5/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8), T
rue))
```

3.608 $\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=281

$$-\frac{c^4\left(40a^2d^2+bc(5bc-24ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{1024d^{7/2}}+\frac{c^3x\sqrt{c+dx^2}\,\left(40a^2d^2+bc(5bc-24ad)\right)}{1024d^3}+\frac{c^2x^3\sqrt{c+dx^2}\,\left(40a^2}{51}$$

Rubi [A] time = 0.26, antiderivative size = 278, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 459, 279, 321, 217, 206}

$$\frac{c^2x^3\sqrt{c+dx^2}\,\left(40a^2d^2+bc(5bc-24ad)\right)}{512d^2}+\frac{c^3x\sqrt{c+dx^2}\,\left(40a^2d^2+bc(5bc-24ad)\right)}{1024d^3}-\frac{c^4\left(40a^2d^2+bc(5bc-24ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{1024d^{7/2}}+\frac{1}{320}x^3\left(c+dx^2\right)^{5/2}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)+\frac{cx^3\left(c+dx^2\right)^{3/2}\left(40a^2d^2+bc(5bc-24ad)\right)}{384d^2}-\frac{bx^3\left(c+dx^2\right)^{7/2}(5bc-24ad)}{120d^2}+\frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x*Sqrt[c + d*x^2])/(1024*d^3) + (c
^2*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*Sqrt[c + d*x^2])/(512*d^2) + (c*
(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*x^3*(c + d*x^2)^(3/2))/(384*d^2) + ((40
*a^2 + (b*c*(5*b*c - 24*a*d))/d^2)*x^3*(c + d*x^2)^(5/2))/320 - (b*(5*b*c -
 24*a*d)*x^3*(c + d*x^2)^(7/2))/(120*d^2) + (b^2*x^5*(c + d*x^2)^(7/2))/(12
*d) - (c^4*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(1024*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx=\frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}+\frac{\int x^2\left(c+dx^2\right)^{5/2}\left(12a^2d-b(5bc-24ad)x^2\right)\,dx}{12d}$$

$$=-\frac{b(5bc-24ad)x^3\left(c+dx^2\right)^{7/2}}{120d^2}+\frac{b^2x^5\left(c+dx^2\right)^{7/2}}{12d}+\frac{1}{40}\Bigg(40a^2+\frac{bc(5bc-24}{d^2}$$

$$=\frac{1}{320}\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\left(c+dx^2\right)^{5/2}-\frac{b(5bc-24ad)x^3\left(c+dx^2\right)^{7/2}}{120d^2}$$

$$=\frac{1}{384}c\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\left(c+dx^2\right)^{3/2}+\frac{1}{320}\Bigg(40a^2+\frac{bc(5bc-24ad)}{d^2}$$

$$=\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+dx^2}+\frac{1}{384}c\Bigg(40a^2+\frac{bc(5bc-24ad)}{d^2}\Bigg)$$

$$=\frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d}+\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+}$$

$$=\frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d}+\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+}$$

$$=\frac{c^3\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x\sqrt{c+dx^2}}{1024d}+\frac{1}{512}c^2\left(40a^2+\frac{bc(5bc-24ad)}{d^2}\right)x^3\sqrt{c+}$$

Mathematica [A] time = 0.17, size = 226, normalized size = 0.80

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\left(40a^2d^2\left(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6\right)+24abd\left(-15c^4+10c^3dx^2+248c^2d^2x^4+336cd^3x^6+128d^4x^8\right)+5b^2\left(15c^5-10c^4dx^2+8c^3d^2x^4+432c^2d^3x^6+640cd^4x^8+256d^5x^{10}\right)\right)-15c^4\left(40a^2d^2-24abcd+5b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{15360d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(40*a^2*d^2*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*
x^4 + 48*d^3*x^6) + 24*a*b*d*(-15*c^4 + 10*c^3*d*x^2 + 248*c^2*d^2*x^4 + 33
6*c*d^3*x^6 + 128*d^4*x^8) + 5*b^2*(15*c^5 - 10*c^4*d*x^2 + 8*c^3*d^2*x^4 +
 432*c^2*d^3*x^6 + 640*c*d^4*x^8 + 256*d^5*x^10)) - 15*c^4*(5*b^2*c^2 - 24*
a*b*c*d + 40*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(15360*d^(7/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 255, normalized size = 0.91

$$\frac{\left(40a^2c^4d^2-24abc^5d+5b^2c^6\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{1024d^{7/2}}+\frac{\sqrt{c+dx^2}\left(600a^2c^3d^2x+4720a^2c^2d^3x^3+5440a^2cd^4x^5+1920a^2d^5x^7-360abc^4dx+240abc^3d^2x^3+5952abc^2d^3x^5+8064abcd^4x^7+3072abd^5x^9+75b^2c^5x-50b^2c^4dx^3+40b^2c^3d^2x^5+2160b^2c^2d^3x^7+3200b^2cd^4x^9+1280b^2d^5x^{11}\right)}{15360d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(75*b^2*c^5*x - 360*a*b*c^4*d*x + 600*a^2*c^3*d^2*x - 50*b
^2*c^4*d*x^3 + 240*a*b*c^3*d^2*x^3 + 4720*a^2*c^2*d^3*x^3 + 40*b^2*c^3*d^2*
x^5 + 5952*a*b*c^2*d^3*x^5 + 5440*a^2*c*d^4*x^5 + 2160*b^2*c^2*d^3*x^7 + 80
64*a*b*c*d^4*x^7 + 1920*a^2*d^5*x^7 + 3200*b^2*c*d^4*x^9 + 3072*a*b*d^5*x^9
 + 1280*b^2*d^5*x^11))/(15360*d^3) + ((5*b^2*c^6 - 24*a*b*c^5*d + 40*a^2*c^
4*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(1024*d^(7/2))
```

fricas [A]   time = 2.12, size = 495, normalized size = 1.76

$$\left[\frac{15\left(5b^2c^6 - 24abc^5d + 40a^2c^4d^2\right)\sqrt{d}\log\left(-2dx^2 + 2\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2\left(1280b^2d^6x^{11} + 128\left(25b^2cd^5 + 24abd^6\right)x^9 + 48\left(45b^2c^2d^4 + 168abcd^5 + 40a^2d^6\right)x^7 + 8\left(5b^2c^3d^3 + 744abc^2d^4 + 680a^2cd^5\right)x^5 - 10\left(5b^2c^4d^2 - 24abc^3d^3 - 472a^2c^2d^4\right)x^3 + 15\left(5b^2c^5d - 24abc^4d^2 + 40a^2c^3d^3\right)x\right)\sqrt{dx^2+c}}{30720d^4}, \frac{15\left(5b^2c^6 - 24abc^5d + 40a^2c^4d^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) + \left(1280b^2d^6x^{11} + 128\left(25b^2cd^5 + 24abd^6\right)x^9 + 48\left(45b^2c^2d^4 + 168abcd^5 + 40a^2d^6\right)x^7 + 8\left(5b^2c^3d^3 + 744abc^2d^4 + 680a^2cd^5\right)x^5 - 10\left(5b^2c^4d^2 - 24abc^3d^3 - 472a^2c^2d^4\right)x^3 + 15\left(5b^2c^5d - 24abc^4d^2 + 40a^2c^3d^3\right)x\right)\sqrt{dx^2+c}}{15360d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(5*b^2*c^6 - 24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(d)*log(-2*d*x
^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(25*b^2*
c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*
x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c
^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c
^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/15360*(15*(5*b^2*c^6 -
24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
+ (1280*b^2*d^6*x^11 + 128*(25*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2
*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4
 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^
4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 +
 c))/d^4]
```

giac [A]   time = 0.49, size = 265, normalized size = 0.94

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10b^2d^2x^2 + \frac{25b^2cd^{11} + 24abd^{12}}{d^{10}}\right)x^2 + \frac{3\left(45b^2c^2d^{10} + 168abcd^{11} + 40a^2d^{12}\right)}{d^{10}}\right)x^2 + \frac{5b^2c^3d^9 + 744abc^2d^{10} + 680a^2cd^{11}}{d^{10}}\right)x^2 - \frac{5\left(5b^2c^4d^8 - 24abc^3d^9 - 472a^2c^2d^{10}\right)}{d^{10}}\right)x^2 + \frac{15\left(5b^2c^5d^7 - 24abc^4d^8 + 40a^2c^3d^9\right)}{d^{10}}\right)\sqrt{dx^2+c}\,x + \frac{\left(5b^2c^6 - 24abc^5d + 40a^2c^4d^2\right)\log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{1024d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^2*x^2 + (25*b^2*c*d^11 + 24*a*b*d^12)/d^10)*x
^2 + 3*(45*b^2*c^2*d^10 + 168*a*b*c*d^11 + 40*a^2*d^12)/d^10)*x^2 + (5*b^2*
c^3*d^9 + 744*a*b*c^2*d^10 + 680*a^2*c*d^11)/d^10)*x^2 - 5*(5*b^2*c^4*d^8 -
 24*a*b*c^3*d^9 - 472*a^2*c^2*d^10)/d^10)*x^2 + 15*(5*b^2*c^5*d^7 - 24*a*b*
c^4*d^8 + 40*a^2*c^3*d^9)/d^10)*sqrt(d*x^2 + c)*x + 1/1024*(5*b^2*c^6 - 24*
a*b*c^5*d + 40*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

maple [A] time = 0.02, size = 383, normalized size = 1.36

$$\frac{\left(d\,x^2+c\right)^{\frac{7}{2}} b^2x^5}{12d}-\frac{5a^2c^4\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{128d^{\frac{3}{2}}}+\frac{3ab\,c^5\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{128d^{\frac{5}{2}}}-\frac{5b^2c^6\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{1024d^{\frac{7}{2}}}-\frac{5\sqrt{d\,x^2+c}\,a^2c^3x}{128d}+\frac{3\sqrt{d\,x^2+c}\,ab\,c^4x}{128d^2}-\frac{5\sqrt{d\,x^2+c}\,b^2c^5x}{1024d^3}-\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}}a^2c^2x}{192d}+\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}ab\,c^3x}{64d^2}+\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}ab\,x^3}{5d}-\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}}b^2c^4x}{1536d^3}-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}b^2c\,x^3}{24d^2}-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}a^2cx}{48d}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}ab\,c^2x}{80d^2}-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}b^2c^3x}{384d^3}+\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}a^2x}{8d}-\frac{3\left(d\,x^2+c\right)^{\frac{7}{2}}abcx}{40d^2}+\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}b^2c^2x}{64d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/12*b^2*x^5*(d*x^2+c)^(7/2)/d-1/24*b^2*c/d^2*x^3*(d*x^2+c)^(7/2)+1/64*b^2*
c^2/d^3*x*(d*x^2+c)^(7/2)-1/384*b^2*c^3/d^3*x*(d*x^2+c)^(5/2)-5/1536*b^2*c^
4/d^3*x*(d*x^2+c)^(3/2)-5/1024*b^2*c^5/d^3*x*(d*x^2+c)^(1/2)-5/1024*b^2*c^6
/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/5*a*b*x^3*(d*x^2+c)^(7/2)/d-3/40*a
*b*c/d^2*x*(d*x^2+c)^(7/2)+1/80*a*b*c^2/d^2*x*(d*x^2+c)^(5/2)+1/64*a*b*c^3/
d^2*x*(d*x^2+c)^(3/2)+3/128*a*b*c^4/d^2*x*(d*x^2+c)^(1/2)+3/128*a*b*c^5/d^(
5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/8*a^2*x*(d*x^2+c)^(7/2)/d-1/48*a^2*c/d
*x*(d*x^2+c)^(5/2)-5/192*a^2*c^2/d*x*(d*x^2+c)^(3/2)-5/128*a^2*c^3/d*x*(d*x
^2+c)^(1/2)-5/128*a^2*c^4/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 0.97, size = 361, normalized size = 1.28

$$\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2x^5}{12\,d}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2cx^3}{24\,d^2}+\frac{\left(dx^2+c\right)^{\frac{7}{2}}abx^3}{5\,d}+\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2c^2x}{64\,d^3}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2c^3x}{384\,d^3}-\frac{5\left(dx^2+c\right)^{\frac{3}{2}}b^2c^4x}{1536\,d^3}-\frac{5\sqrt{dx^2+c}\,b^2c^5x}{1024\,d^3}-\frac{3\left(dx^2+c\right)^{\frac{7}{2}}abcx}{40\,d^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}abc^2x}{80\,d^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}abc^3x}{64\,d^2}+\frac{3\sqrt{dx^2+c}\,abc^4x}{128\,d^2}+\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2x}{8\,d}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2cx}{48\,d}-\frac{5\left(dx^2+c\right)^{\frac{3}{2}}a^2c^2x}{192\,d}-\frac{5\sqrt{dx^2+c}\,a^2c^3x}{128\,d}-\frac{5\,b^2c^6\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{1024\,d^{\frac{7}{2}}}+\frac{3\,abc^5\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}}-\frac{5\,a^2c^4\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(d*x^2 + c)^(7/2)*b^2*x^5/d - 1/24*(d*x^2 + c)^(7/2)*b^2*c*x^3/d^2 + 1
/5*(d*x^2 + c)^(7/2)*a*b*x^3/d + 1/64*(d*x^2 + c)^(7/2)*b^2*c^2*x/d^3 - 1/3
84*(d*x^2 + c)^(5/2)*b^2*c^3*x/d^3 - 5/1536*(d*x^2 + c)^(3/2)*b^2*c^4*x/d^3
 - 5/1024*sqrt(d*x^2 + c)*b^2*c^5*x/d^3 - 3/40*(d*x^2 + c)^(7/2)*a*b*c*x/d^
2 + 1/80*(d*x^2 + c)^(5/2)*a*b*c^2*x/d^2 + 1/64*(d*x^2 + c)^(3/2)*a*b*c^3*x
/d^2 + 3/128*sqrt(d*x^2 + c)*a*b*c^4*x/d^2 + 1/8*(d*x^2 + c)^(7/2)*a^2*x/d
 - 1/48*(d*x^2 + c)^(5/2)*a^2*c*x/d - 5/192*(d*x^2 + c)^(3/2)*a^2*c^2*x/d -
5/128*sqrt(d*x^2 + c)*a^2*c^3*x/d - 5/1024*b^2*c^6*arcsinh(d*x/sqrt(c*d))/d
^(7/2) + 3/128*a*b*c^5*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 5/128*a^2*c^4*arcsi
nh(d*x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(b\,x^2+a\right)^2\left(d\,x^2+c\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2), x)
```

sympy [**B**] time = 96.02, size = 602, normalized size = 2.14

$$\frac{5a^2c^{\frac{7}{2}}x}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{133a^2c^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{dx^2}{c}}}+\frac{127a^2c^{\frac{3}{2}}dx^5}{192\sqrt{1+\frac{dx^2}{c}}}+\frac{23a^2\sqrt{c}\,d^2x^7}{48\sqrt{1+\frac{dx^2}{c}}}-\frac{5a^2c^4\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{3}{2}}}+\frac{a^2d^3x^9}{8\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}-\frac{3abc^{\frac{9}{2}}x}{128d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{abc^{\frac{7}{2}}x^3}{128d\sqrt{1+\frac{dx^2}{c}}}+\frac{129abc^{\frac{5}{2}}x^5}{320\sqrt{1+\frac{dx^2}{c}}}+\frac{73abc^{\frac{3}{2}}dx^7}{80\sqrt{1+\frac{dx^2}{c}}}+\frac{29ab\sqrt{c}\,d^2x^9}{40\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^5\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{5}{2}}}+\frac{abd^3x^{11}}{5\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{11}{2}}x}{1024d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{9}{2}}x^3}{3072d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2c^{\frac{7}{2}}x^5}{1536d\sqrt{1+\frac{dx^2}{c}}}+\frac{55b^2c^{\frac{5}{2}}x^7}{384\sqrt{1+\frac{dx^2}{c}}}+\frac{67b^2c^{\frac{3}{2}}dx^9}{192\sqrt{1+\frac{dx^2}{c}}}+\frac{7b^2\sqrt{c}\,d^2x^{11}}{24\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^6\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{1024d^{\frac{7}{2}}}+\frac{b^2d^3x^{13}}{12\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] 5*a**2*c**(7/2)*x/(128*d*sqrt(1 + d*x**2/c)) + 133*a**2*c**(5/2)*x**3/(384*
sqrt(1 + d*x**2/c)) + 127*a**2*c**(3/2)*d*x**5/(192*sqrt(1 + d*x**2/c)) + 2
3*a**2*sqrt(c)*d**2*x**7/(48*sqrt(1 + d*x**2/c)) - 5*a**2*c**4*asinh(sqrt(d
)*x/sqrt(c))/(128*d**(3/2)) + a**2*d**3*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
 - 3*a*b*c**(9/2)*x/(128*d**2*sqrt(1 + d*x**2/c)) - a*b*c**(7/2)*x**3/(128*
d*sqrt(1 + d*x**2/c)) + 129*a*b*c**(5/2)*x**5/(320*sqrt(1 + d*x**2/c)) + 73
*a*b*c**(3/2)*d*x**7/(80*sqrt(1 + d*x**2/c)) + 29*a*b*sqrt(c)*d**2*x**9/(40
*sqrt(1 + d*x**2/c)) + 3*a*b*c**5*asinh(sqrt(d)*x/sqrt(c))/(128*d**(5/2)) +
 a*b*d**3*x**11/(5*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(11/2)*x/(1024*d
**3*sqrt(1 + d*x**2/c)) + 5*b**2*c**(9/2)*x**3/(3072*d**2*sqrt(1 + d*x**2/c
)) - b**2*c**(7/2)*x**5/(1536*d*sqrt(1 + d*x**2/c)) + 55*b**2*c**(5/2)*x**7
/(384*sqrt(1 + d*x**2/c)) + 67*b**2*c**(3/2)*d*x**9/(192*sqrt(1 + d*x**2/c)
) + 7*b**2*sqrt(c)*d**2*x**11/(24*sqrt(1 + d*x**2/c)) - 5*b**2*c**6*asinh(s
qrt(d)*x/sqrt(c))/(1024*d**(7/2)) + b**2*d**3*x**13/(12*sqrt(c)*sqrt(1 + d*
x**2/c))
```

3.609 $\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}$$

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^(9
/2))/(9*d^3) + (b^2*(c + d*x^2)^(11/2))/(11*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx&=\frac{1}{2}\text{Subst}\left(\int(a+bx)^2(c+dx)^{5/2}\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)\right.\\&=\frac{(bc-ad)^2\left(c+dx^2\right)^{7/2}}{7d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{9/2}}{9d^3}+\frac{b^2\left(c+dx^2\right)^{11/2}}{11d^3}\end{aligned}$$

Mathematica [**A**] time = 0.05, size = 67, normalized size = 0.87

$$\frac{\left(c+dx^2\right)^{7/2}\left(99a^2d^2+22abd\left(7dx^2-2c\right)+b^2\left(8c^2-28cdx^2+63d^2x^4\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x^2) + b^2*(8*c^2 - 2
8*c*d*x^2 + 63*d^2*x^4)))/(693*d^3)
```

IntegrateAlgebraic [**A**] time = 0.05, size = 72, normalized size = 0.94

$$\frac{\left(c+dx^2\right)^{7/2}\left(99a^2d^2-44abcd+154abd^2x^2+8b^2c^2-28b^2cdx^2+63b^2d^2x^4\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] ((c + d*x^2)^(7/2)*(8*b^2*c^2 - 44*a*b*c*d + 99*a^2*d^2 - 28*b^2*c*d*x^2 +
154*a*b*d^2*x^2 + 63*b^2*d^2*x^4))/(693*d^3)
```

fricas [**B**] time = 1.46, size = 178, normalized size = 2.31

$$\frac{\left(63\,b^2d^5x^{10}+7\left(23\,b^2cd^4+22\,abd^5\right)x^8+8\,b^2c^5-44\,abc^4d+99\,a^2c^3d^2+\left(113\,b^2c^2d^3+418\,abcd^4+99\,a^2d^5\right)x^6+3\left(b^2c^3d^2+110\,abc^2d^3+99\,a^2cd^4\right)x^4-\left(4\,b^2c^4d-22\,abc^3d^2-297\,a^2c^2d^3\right)x^2\right)\sqrt{dx^2+c}}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/693*(63*b^2*d^5*x^10 + 7*(23*b^2*c*d^4 + 22*a*b*d^5)*x^8 + 8*b^2*c^5 - 44
*a*b*c^4*d + 99*a^2*c^3*d^2 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5
)*x^6 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^4 - (4*b^2*c^4*d
 - 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

giac [**A**] time = 0.32, size = 98, normalized size = 1.27

$$\frac{63\left(dx^2+c\right)^{\frac{11}{2}}b^2-154\left(dx^2+c\right)^{\frac{9}{2}}b^2c+99\left(dx^2+c\right)^{\frac{7}{2}}b^2c^2+154\left(dx^2+c\right)^{\frac{9}{2}}abd-198\left(dx^2+c\right)^{\frac{7}{2}}abcd+99\left(dx^2+c\right)^{\frac{7}{2}}a^2d^2}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/693*(63*(d*x^2 + c)^(11/2)*b^2 - 154*(d*x^2 + c)^(9/2)*b^2*c + 99*(d*x^2
+ c)^(7/2)*b^2*c^2 + 154*(d*x^2 + c)^(9/2)*a*b*d - 198*(d*x^2 + c)^(7/2)*a*
b*c*d + 99*(d*x^2 + c)^(7/2)*a^2*d^2)/d^3
```

maple [A]   time = 0.01, size = 69, normalized size = 0.90

$$\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}\left(63b^{2}x^{4}d^{2}+154ab\,d^{2}x^{2}-28b^{2}cd\,x^{2}+99a^{2}d^{2}-44abcd+8b^{2}c^{2}\right)}{693d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/693*(d*x^2+c)^(7/2)*(63*b^2*d^2*x^4+154*a*b*d^2*x^2-28*b^2*c*d*x^2+99*a^2
*d^2-44*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A]   time = 0.89, size = 115, normalized size = 1.49

$$\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}x^{4}}{11\,d}-\frac{4\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}cx^{2}}{99\,d^{2}}+\frac{2\left(dx^{2}+c\right)^{\frac{7}{2}}abx^{2}}{9\,d}+\frac{8\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}c^{2}}{693\,d^{3}}-\frac{4\left(dx^{2}+c\right)^{\frac{7}{2}}abc}{63\,d^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}a^{2}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*(d*x^2 + c)^(7/2)*b^2*x^4/d - 4/99*(d*x^2 + c)^(7/2)*b^2*c*x^2/d^2 + 2
/9*(d*x^2 + c)^(7/2)*a*b*x^2/d + 8/693*(d*x^2 + c)^(7/2)*b^2*c^2/d^3 - 4/63
*(d*x^2 + c)^(7/2)*a*b*c/d^2 + 1/7*(d*x^2 + c)^(7/2)*a^2/d
```

mupad [B]   time = 0.80, size = 98, normalized size = 1.27

$$\frac{d\left(\frac{2\,a\,b\,{\left(d\,x^2+c\right)}^{9/2}}{9}-\frac{2\,a\,b\,c\,{\left(d\,x^2+c\right)}^{7/2}}{7}\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{11/2}}{11}-\frac{2\,b^2\,c\,{\left(d\,x^2+c\right)}^{9/2}}{9}+\frac{a^2\,d^2\,{\left(d\,x^2+c\right)}^{7/2}}{7}+\frac{b^2\,c^2\,{\left(d\,x^2+c\right)}^{7/2}}{7}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] (d*((2*a*b*(c + d*x^2)^(9/2))/9 - (2*a*b*c*(c + d*x^2)^(7/2))/7) + (b^2*(c
+ d*x^2)^(11/2))/11 - (2*b^2*c*(c + d*x^2)^(9/2))/9 + (a^2*d^2*(c + d*x^2)^
(7/2))/7 + (b^2*c^2*(c + d*x^2)^(7/2))/7)/d^3
```

sympy [A]   time = 14.55, size = 384, normalized size = 4.99

$$\begin{cases} \frac{a^{2}c^{3}\sqrt{c+dx^{2}}}{7d}+\frac{3a^{2}c^{2}x^{2}\sqrt{c+dx^{2}}}{7}+\frac{3a^{2}cdx^{4}\sqrt{c+dx^{2}}}{7}+\frac{a^{2}d^{2}x^{6}\sqrt{c+dx^{2}}}{7}-\frac{4abc^{4}\sqrt{c+dx^{2}}}{63d^{2}}+\frac{2abc^{3}x^{2}\sqrt{c+dx^{2}}}{63d}+\frac{10abc^{2}x^{4}\sqrt{c+dx^{2}}}{21}+\frac{38abcdx^{6}\sqrt{c+dx^{2}}}{63}+\frac{2abd^{2}x^{8}\sqrt{c+dx^{2}}}{9}+\frac{8b^{2}c^{5}\sqrt{c+dx^{2}}}{693d^{3}}-\frac{4b^{2}c^{4}x^{2}\sqrt{c+dx^{2}}}{693d^{2}}+\frac{b^{2}c^{3}x^{4}\sqrt{c+dx^{2}}}{231d}+\frac{113b^{2}c^{2}x^{6}\sqrt{c+dx^{2}}}{693}+\frac{23b^{2}cdx^{8}\sqrt{c+dx^{2}}}{99}+\frac{b^{2}d^{2}x^{10}\sqrt{c+dx^{2}}}{11} & \text{for}\: d \neq 0 \\ c^{\frac{5}{2}}\left(\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((a**2*c**3*sqrt(c + d*x**2)/(7*d) + 3*a**2*c**2*x**2*sqrt(c + d*x
**2)/7 + 3*a**2*c*d*x**4*sqrt(c + d*x**2)/7 + a**2*d**2*x**6*sqrt(c + d*x**
2)/7 - 4*a*b*c**4*sqrt(c + d*x**2)/(63*d**2) + 2*a*b*c**3*x**2*sqrt(c + d*x
**2)/(63*d) + 10*a*b*c**2*x**4*sqrt(c + d*x**2)/21 + 38*a*b*c*d*x**6*sqrt(c
 + d*x**2)/63 + 2*a*b*d**2*x**8*sqrt(c + d*x**2)/9 + 8*b**2*c**5*sqrt(c + d
*x**2)/(693*d**3) - 4*b**2*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + b**2*c**
3*x**4*sqrt(c + d*x**2)/(231*d) + 113*b**2*c**2*x**6*sqrt(c + d*x**2)/693 +
 23*b**2*c*d*x**8*sqrt(c + d*x**2)/99 + b**2*d**2*x**10*sqrt(c + d*x**2)/11
, Ne(d, 0)), (c**(5/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6), True))
```

3.610 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}\,dx$

Optimal. Leaf size=240

$$\frac{x\left(c+dx^2\right)^{5/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{480d^2}+\frac{cx\left(c+dx^2\right)^{3/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}\,\left(80a^2d^2-2\right.}{256d^2}$$

Rubi [**A**] time = 0.15, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {416, 388, 195, 217, 206}

$$\frac{x\left(c+dx^2\right)^{5/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{480d^2}+\frac{cx\left(c+dx^2\right)^{3/2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{384d^2}+\frac{c^2x\sqrt{c+dx^2}\left(80a^2d^2-20abcd+3b^2c^2\right)}{256d^2}+\frac{c^3\left(80a^2d^2-20abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{5/2}}-\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{80d^2}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{7/2}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (c^2*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*Sqrt[c + d*x^2])/(256*d^2) + (
c*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(3/2))/(384*d^2) + ((
3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*x*(c + d*x^2)^(5/2))/(480*d^2) - (3*b*
(b*c - 4*a*d)*x*(c + d*x^2)^(7/2))/(80*d^2) + (b*x*(a + b*x^2)*(c + d*x^2)^
(7/2))/(10*d) + (c^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(256*d^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int (a+bx^2)^2 (c+dx^2)^{5/2}\,dx &= \frac{bx(a+bx^2)(c+dx^2)^{7/2}}{10d} + \frac{\int (c+dx^2)^{5/2}(-a(bc-10ad)-3b(bc-4ad)x^2)\,dx}{10d} \\
&= -\frac{3b(bc-4ad)x(c+dx^2)^{7/2}}{80d^2} + \frac{bx(a+bx^2)(c+dx^2)^{7/2}}{10d} - \frac{(8ad(bc-10ad)-3bc(}{} \\
&= \frac{(3b^2c^2-20abcd+80a^2d^2)x(c+dx^2)^{5/2}}{480d^2} - \frac{3b(bc-4ad)x(c+dx^2)^{7/2}}{80d^2} + \frac{bx(a+}{} \\
&= \frac{c(3b^2c^2-20abcd+80a^2d^2)x(c+dx^2)^{3/2}}{384d^2} + \frac{(3b^2c^2-20abcd+80a^2d^2)x(c+d}{480d^2} \\
&= \frac{c^2(3b^2c^2-20abcd+80a^2d^2)x\sqrt{c+dx^2}}{256d^2} + \frac{c(3b^2c^2-20abcd+80a^2d^2)x(c+d}{384d^2} \\
&= \frac{c^2(3b^2c^2-20abcd+80a^2d^2)x\sqrt{c+dx^2}}{256d^2} + \frac{c(3b^2c^2-20abcd+80a^2d^2)x(c+d}{384d^2} \\
&= \frac{c^2(3b^2c^2-20abcd+80a^2d^2)x\sqrt{c+dx^2}}{256d^2} + \frac{c(3b^2c^2-20abcd+80a^2d^2)x(c+d}{384d^2}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 192, normalized size = 0.80

$$\frac{15c^3(80a^2d^2-20abcd+3b^2c^2)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}\,x\sqrt{c+dx^2}\left(80a^2d^2(33c^2+26cdx^2+8d^2x^4)+20abd(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6)+b^2(-45c^4+30c^3dx^2+744c^2d^2x^4+1008cd^3x^6+384d^4x^8)\right)}{3840d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) +
20*a*b*d*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*x^4 + 48*d^3*x^6) + b^2*(-45*c
^4 + 30*c^3*d*x^2 + 744*c^2*d^2*x^4 + 1008*c*d^3*x^6 + 384*d^4*x^8)) + 15*c
^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]
)/(3840*d^(5/2))
```

IntegrateAlgebraic [A]   time = 0.37, size = 214, normalized size = 0.89

$$\frac{\left(-80a^2c^3d^2+20abc^4d-3b^2c^5\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{256d^{5/2}}+\frac{\sqrt{c+dx^2}\left(2640a^2c^2d^2x+2080a^2cd^3x^3+640a^2d^4x^5+300abc^3dx+2360abc^2d^2x^3+2720abcd^3x^5+960abd^4x^7-45b^2c^4x+30b^2c^3dx^3+744b^2c^2d^2x^5+1008b^2cd^3x^7+384b^2d^4x^9\right)}{3840d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-45*b^2*c^4*x + 300*a*b*c^3*d*x + 2640*a^2*c^2*d^2*x + 30
*b^2*c^3*d*x^3 + 2360*a*b*c^2*d^2*x^3 + 2080*a^2*c*d^3*x^3 + 744*b^2*c^2*d^
2*x^5 + 2720*a*b*c*d^3*x^5 + 640*a^2*d^4*x^5 + 1008*b^2*c*d^3*x^7 + 960*a*b
*d^4*x^7 + 384*b^2*d^4*x^9))/(3840*d^2) + ((-3*b^2*c^5 + 20*a*b*c^4*d - 80*
a^2*c^3*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(256*d^(5/2))
```

fricas [A]   time = 1.88, size = 420, normalized size = 1.75

$$\left[\frac{15\left(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(384\,b^2d^5x^9+48\left(21\,b^2cd^4+20\,abd^5\right)x^7+8\left(93\,b^2c^2d^3+340\,abcd^4+80\,a^2d^5\right)x^5+10\left(3\,b^2c^3d^2+236\,abc^2d^3+208\,a^2cd^4\right)x^3-15\left(3\,b^2c^4d-20\,abc^3d^2-176\,a^2c^2d^3\right)x\right)\sqrt{dx^2+c}}{7680\,d^3},-\frac{15\left(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(384\,b^2d^5x^9+48\left(21\,b^2cd^4+20\,abd^5\right)x^7+8\left(93\,b^2c^2d^3+340\,abcd^4+80\,a^2d^5\right)x^5+10\left(3\,b^2c^3d^2+236\,abc^2d^3+208\,a^2cd^4\right)x^3-15\left(3\,b^2c^4d-20\,abc^3d^2-176\,a^2c^2d^3\right)x\right)\sqrt{dx^2+c}}{3840\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(21*b^2*c*d^
4 + 20*a*b*d^5)*x^7 + 8*(93*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 +
 10*(3*b^2*c^3*d^2 + 236*a*b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d
 - 20*a*b*c^3*d^2 - 176*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/3840*(15*(
3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) - (384*b^2*d^5*x^9 + 48*(21*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*(93
*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 + 10*(3*b^2*c^3*d^2 + 236*a*
b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d - 20*a*b*c^3*d^2 - 176*a^2
*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

giac [A]   time = 0.51, size = 221, normalized size = 0.92

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{21\,b^2cd^9+20\,abd^{10}}{d^8}\right)x^2+\frac{93\,b^2c^2d^8+340\,abcd^9+80\,a^2d^{10}}{d^8}\right)x^2+\frac{5\left(3\,b^2c^3d^7+236\,abc^2d^8+208\,a^2cd^9\right)}{d^8}\right)x^2-\frac{15\left(3\,b^2c^4d^6-20\,abc^3d^7-176\,a^2c^2d^8\right)}{d^8}\right)\sqrt{dx^2+c}\,x-\frac{\left(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (21*b^2*c*d^9 + 20*a*b*d^10)/d^8)*x^2 + (9
3*b^2*c^2*d^8 + 340*a*b*c*d^9 + 80*a^2*d^10)/d^8)*x^2 + 5*(3*b^2*c^3*d^7 +
236*a*b*c^2*d^8 + 208*a^2*c*d^9)/d^8)*x^2 - 15*(3*b^2*c^4*d^6 - 20*a*b*c^3*
d^7 - 176*a^2*c^2*d^8)/d^8)*sqrt(d*x^2 + c)*x - 1/256*(3*b^2*c^5 - 20*a*b*c
^4*d + 80*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

maple [A] time = 0.02, size = 308, normalized size = 1.28

$$\frac{5a^{2}c^{3}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{16\sqrt{d}}-\frac{5ab\,c^{4}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{64d^{\frac{3}{2}}}+\frac{3b^{2}c^{5}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{256d^{\frac{5}{2}}}+\frac{5\sqrt{d\,x^{2}+c}\,a^{2}c^{2}x}{16}-\frac{5\sqrt{d\,x^{2}+c}\,ab\,c^{3}x}{64d}+\frac{3\sqrt{d\,x^{2}+c}\,b^{2}c^{4}x}{256d^{2}}+\frac{5\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}cx}{24}-\frac{5\left(d\,x^{2}+c\right)^{\frac{3}{2}}ab\,c^{2}x}{96d}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}c^{3}x}{128d^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}b^{2}x^{3}}{10d}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}x}{6}-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}abcx}{24d}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}b^{2}c^{2}x}{160d^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}abx}{4d}-\frac{3\left(d\,x^{2}+c\right)^{\frac{7}{2}}b^{2}cx}{80d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2),x)
```

```
[Out] 1/10*b^2*x^3*(d*x^2+c)^(7/2)/d-3/80*b^2*c/d^2*x*(d*x^2+c)^(7/2)+1/160*b^2*c
^2/d^2*x*(d*x^2+c)^(5/2)+1/128*b^2*c^3/d^2*x*(d*x^2+c)^(3/2)+3/256*b^2*c^4/
d^2*x*(d*x^2+c)^(1/2)+3/256*b^2*c^5/d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1
/4*a*b*x*(d*x^2+c)^(7/2)/d-1/24*a*b*c/d*x*(d*x^2+c)^(5/2)-5/96*a*b*c^2/d*x*
(d*x^2+c)^(3/2)-5/64*a*b*c^3/d*x*(d*x^2+c)^(1/2)-5/64*a*b*c^4/d^(3/2)*ln(d^
(1/2)*x+(d*x^2+c)^(1/2))+1/6*a^2*x*(d*x^2+c)^(5/2)+5/24*a^2*c*x*(d*x^2+c)^(
3/2)+5/16*a^2*c^2*x*(d*x^2+c)^(1/2)+5/16*a^2*c^3/d^(1/2)*ln(d^(1/2)*x+(d*x^
2+c)^(1/2))
```

maxima [A] time = 0.98, size = 286, normalized size = 1.19

$$\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}x^{3}}{10\,d}+\frac{1}{6}\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}x+\frac{5}{24}\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cx+\frac{5}{16}\sqrt{dx^{2}+c}\,a^{2}c^{2}x-\frac{3\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}cx}{80\,d^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}c^{2}x}{160\,d^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{3}x}{128\,d^{2}}+\frac{3\sqrt{dx^{2}+c}\,b^{2}c^{4}x}{256\,d^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}abx}{4\,d}-\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}abcx}{24\,d}-\frac{5\left(dx^{2}+c\right)^{\frac{3}{2}}abc^{2}x}{96\,d}-\frac{5\sqrt{dx^{2}+c}\,abc^{3}x}{64\,d}+\frac{3\,b^{2}c^{5}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{256\,d^{\frac{5}{2}}}-\frac{5\,abc^{4}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{64\,d^{\frac{3}{2}}}+\frac{5\,a^{2}c^{3}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x^2 + c)^(7/2)*b^2*x^3/d + 1/6*(d*x^2 + c)^(5/2)*a^2*x + 5/24*(d*x^
2 + c)^(3/2)*a^2*c*x + 5/16*sqrt(d*x^2 + c)*a^2*c^2*x - 3/80*(d*x^2 + c)^(7
/2)*b^2*c*x/d^2 + 1/160*(d*x^2 + c)^(5/2)*b^2*c^2*x/d^2 + 1/128*(d*x^2 + c)
^(3/2)*b^2*c^3*x/d^2 + 3/256*sqrt(d*x^2 + c)*b^2*c^4*x/d^2 + 1/4*(d*x^2 + c
)^(7/2)*a*b*x/d - 1/24*(d*x^2 + c)^(5/2)*a*b*c*x/d - 5/96*(d*x^2 + c)^(3/2)
*a*b*c^2*x/d - 5/64*sqrt(d*x^2 + c)*a*b*c^3*x/d + 3/256*b^2*c^5*arcsinh(d*x
/sqrt(c*d))/d^(5/2) - 5/64*a*b*c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 5/16*a^
2*c^3*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

```
[Out] int((a + b*x^2)^2*(c + d*x^2)^(5/2), x)
```

sympy [B] time = 96.68, size = 537, normalized size = 2.24

$$\frac{a^{2}c^{\frac{5}{2}}x\sqrt{1+\frac{dx^{2}}{c}}}{2}+\frac{3a^{2}c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^{2}}{c}}}+\frac{35a^{2}c^{\frac{3}{2}}dx^{3}}{48\sqrt{1+\frac{dx^{2}}{c}}}+\frac{17a^{2}\sqrt{c}\,d^{2}x^{5}}{24\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5a^{2}c^{3}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16\sqrt{d}}+\frac{a^{2}d^{3}x^{7}}{6\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5abc^{\frac{7}{2}}x}{64d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{133abc^{\frac{5}{2}}x^{3}}{192\sqrt{1+\frac{dx^{2}}{c}}}+\frac{127abc^{\frac{3}{2}}dx^{5}}{96\sqrt{1+\frac{dx^{2}}{c}}}+\frac{23ab\sqrt{c}\,d^{2}x^{7}}{24\sqrt{1+\frac{dx^{2}}{c}}}-\frac{5abc^{4}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{64d^{\frac{3}{2}}}+\frac{abd^{3}x^{9}}{4\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{3b^{2}c^{\frac{9}{2}}x}{256d^{2}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}c^{\frac{7}{2}}x^{3}}{256d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{129b^{2}c^{\frac{5}{2}}x^{5}}{640\sqrt{1+\frac{dx^{2}}{c}}}+\frac{73b^{2}c^{\frac{3}{2}}dx^{7}}{160\sqrt{1+\frac{dx^{2}}{c}}}+\frac{29b^{2}\sqrt{c}\,d^{2}x^{9}}{80\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3b^{2}c^{5}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{256d^{\frac{5}{2}}}+\frac{b^{2}d^{3}x^{11}}{10\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

```
[Out] a**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*a**2*c**(5/2)*x/(16*sqrt(1 + d*x**
2/c)) + 35*a**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c)) + 17*a**2*sqrt(c)*d
**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*a**2*c**3*asinh(sqrt(d)*x/sqrt(c))/(16
*sqrt(d)) + a**2*d**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(7/2)*
x/(64*d*sqrt(1 + d*x**2/c)) + 133*a*b*c**(5/2)*x**3/(192*sqrt(1 + d*x**2/c)
) + 127*a*b*c**(3/2)*d*x**5/(96*sqrt(1 + d*x**2/c)) + 23*a*b*sqrt(c)*d**2*x
**7/(24*sqrt(1 + d*x**2/c)) - 5*a*b*c**4*asinh(sqrt(d)*x/sqrt(c))/(64*d**(3
/2)) + a*b*d**3*x**9/(4*sqrt(c)*sqrt(1 + d*x**2/c)) - 3*b**2*c**(9/2)*x/(25
6*d**2*sqrt(1 + d*x**2/c)) - b**2*c**(7/2)*x**3/(256*d*sqrt(1 + d*x**2/c))
+ 129*b**2*c**(5/2)*x**5/(640*sqrt(1 + d*x**2/c)) + 73*b**2*c**(3/2)*d*x**7
/(160*sqrt(1 + d*x**2/c)) + 29*b**2*sqrt(c)*d**2*x**9/(80*sqrt(1 + d*x**2/c
)) + 3*b**2*c**5*asinh(sqrt(d)*x/sqrt(c))/(256*d**(5/2)) + b**2*d**3*x**11/
(10*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.611 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+a^2c^2\sqrt{c+dx^2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}-\frac{b\left(c+dx^2\right)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2\left(c\right.}{9}$$

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$a^2c^2\sqrt{c+dx^2}-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}-\frac{b\left(c+dx^2\right)^{7/2}(bc-2ad)}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] a^2*c^2*Sqrt[c + d*x^2] + (a^2*c*(c + d*x^2)^(3/2))/3 + (a^2*(c + d*x^2)^(5
/2))/5 - (b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) + (b^2*(c + d*x^2)^(9/
2))/(9*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int\left(-\frac{b(bc-2ad)(c+dx)^{5/2}}{d}+\frac{a^2(c+dx)^{5/2}}{x}+\frac{b^2(c+dx)^{7/2}}{d}\right)dx,x,x^2\right.\\
&= -\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}+\frac{1}{2}a^2\,\mathrm{Subst}\left(\int\frac{(c+dx)^{5/2}}{x}\,dx,x,x\right.\\
&= \frac{1}{5}a^2\left(c+dx^2\right)^{5/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}+\frac{1}{2}\left(a^2c\right)\mathrm{Subst}\Bigg(\\
&= \frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}\\
&= a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2}\\
&= a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2}\\
&= a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c\left(c+dx^2\right)^{3/2}+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}-\frac{b(bc-2ad)\left(c+dx^2\right)^{7/2}}{7d^2}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 123, normalized size = 0.93

$$\frac{1}{3}a^2c\left(\sqrt{c+dx^2}\left(4c+dx^2\right)-3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{1}{5}a^2\left(c+dx^2\right)^{5/2}+\frac{b\left(c+dx^2\right)^{7/2}(2ad-bc)}{7d^2}+\frac{b^2\left(c+dx^2\right)^{9/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(c + d*x^2)^(5/2))/5 + (b*(-(b*c) + 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2)
+ (b^2*(c + d*x^2)^(9/2))/(9*d^2) + (a^2*c*(Sqrt[c + d*x^2]*(4*c + d*x^2) -
 3*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/3
```

IntegrateAlgebraic [A] time = 0.12, size = 181, normalized size = 1.37

$$\frac{\sqrt{c+dx^2}\left(483a^2c^2d^2+231a^2cd^3x^2+63a^2d^4x^4+90abc^3d+270abc^2d^2x^2+270abcd^3x^4+90abd^4x^6-10b^2c^4+5b^2c^3dx^2+75b^2c^2d^2x^4+95b^2cd^3x^6+35b^2d^4x^8\right)}{315d^2}-a^2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-10*b^2*c^4 + 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 5*b^2*c^3*
d*x^2 + 270*a*b*c^2*d^2*x^2 + 231*a^2*c*d^3*x^2 + 75*b^2*c^2*d^2*x^4 + 270*
a*b*c*d^3*x^4 + 63*a^2*d^4*x^4 + 95*b^2*c*d^3*x^6 + 90*a*b*d^4*x^6 + 35*b^2
*d^4*x^8))/(315*d^2) - a^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

fricas [A] time = 1.64, size = 360, normalized size = 2.73

$$\left[\frac{315\,a^2c^{\frac{5}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(35\,b^2d^4x^8+5\left(19\,b^2cd^3+18\,abd^4\right)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\left(25\,b^2c^2d^2+90\,abcd^3+21\,a^2d^4\right)x^4+\left(5\,b^2c^3d+270\,abc^2d^2+231\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{630\,d^2},\ \frac{315\,a^2\sqrt{-c}\,c^2d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(35\,b^2d^4x^8+5\left(19\,b^2cd^3+18\,abd^4\right)x^6-10\,b^2c^4+90\,abc^3d+483\,a^2c^2d^2+3\left(25\,b^2c^2d^2+90\,abcd^3+21\,a^2d^4\right)x^4+\left(5\,b^2c^3d+270\,abc^2d^2+231\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{315\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*a^2*c^(5/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) + 2*(35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 +
 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a^2
*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2
 + c))/d^2, 1/315*(315*a^2*sqrt(-c)*c^2*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4 + 90*a
*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a^2*d^4)
*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)
)/d^2]
```

giac [A] time = 0.44, size = 141, normalized size = 1.07

$$\frac{a^2c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{35\left(dx^2+c\right)^{\frac{9}{2}}b^2d^{16}-45\left(dx^2+c\right)^{\frac{7}{2}}b^2cd^{16}+90\left(dx^2+c\right)^{\frac{7}{2}}abd^{17}+63\left(dx^2+c\right)^{\frac{5}{2}}a^2d^{18}+105\left(dx^2+c\right)^{\frac{3}{2}}a^2cd^{18}+315\,\sqrt{dx^2+c}\,a^2c^2d^{18}}{315\,d^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="giac")
```

```
[Out] a^2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/315*(35*(d*x^2 + c)^(
9/2)*b^2*d^16 - 45*(d*x^2 + c)^(7/2)*b^2*c*d^16 + 90*(d*x^2 + c)^(7/2)*a*b*
d^17 + 63*(d*x^2 + c)^(5/2)*a^2*d^18 + 105*(d*x^2 + c)^(3/2)*a^2*c*d^18 + 3
15*sqrt(d*x^2 + c)*a^2*c^2*d^18)/d^18
```

maple [**A**] time = 0.01, size = 132, normalized size = 1.00

$$-a^{2} c^{\frac{5}{2}} \ln \left(\frac{2 c+2 \sqrt{d x^{2}+c}\, \sqrt{c}}{x}\right)+\sqrt{d x^{2}+c}\, a^{2} c^{2}+\frac{\left(d x^{2}+c\right)^{\frac{3}{2}} a^{2} c}{3}+\frac{\left(d x^{2}+c\right)^{\frac{7}{2}} b^{2} x^{2}}{9 d}+\frac{\left(d x^{2}+c\right)^{\frac{5}{2}} a^{2}}{5}+\frac{2 \left(d x^{2}+c\right)^{\frac{7}{2}} a b}{7 d}-\frac{2 \left(d x^{2}+c\right)^{\frac{7}{2}} b^{2} c}{63 d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x)
```

```
[Out] 1/9*b^2*x^2*(d*x^2+c)^(7/2)/d-2/63*b^2*c/d^2*(d*x^2+c)^(7/2)+2/7*a*b*(d*x^2
+c)^(7/2)/d+1/5*a^2*(d*x^2+c)^(5/2)+1/3*a^2*c*(d*x^2+c)^(3/2)-a^2*c^(5/2)*l
n((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+a^2*c^2*(d*x^2+c)^(1/2)
```

maxima [**A**] time = 0.89, size = 120, normalized size = 0.91

$$\frac{\left(dx^{2}+c\right)^{\frac{7}{2}} b^{2} x^{2}}{9 \, d}-a^{2} c^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} \left|x\right|}\right)+\frac{1}{5} \left(dx^{2}+c\right)^{\frac{5}{2}} a^{2}+\frac{1}{3} \left(dx^{2}+c\right)^{\frac{3}{2}} a^{2} c+\sqrt{dx^{2}+c}\, a^{2} c^{2}-\frac{2 \left(dx^{2}+c\right)^{\frac{7}{2}} b^{2} c}{63 \, d^{2}}+\frac{2 \left(dx^{2}+c\right)^{\frac{7}{2}} a b}{7 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/9*(d*x^2 + c)^(7/2)*b^2*x^2/d - a^2*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x)))
 + 1/5*(d*x^2 + c)^(5/2)*a^2 + 1/3*(d*x^2 + c)^(3/2)*a^2*c + sqrt(d*x^2 + c
)*a^2*c^2 - 2/63*(d*x^2 + c)^(7/2)*b^2*c/d^2 + 2/7*(d*x^2 + c)^(7/2)*a*b/d
```

mupad [**B**] time = 0.77, size = 249, normalized size = 1.89

$$\left(d\,x^2+c\right)^{5/2}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{5\,d^2}-\frac{c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)}{5}\right)-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{7\,d^2}-\frac{b^2\,c}{7\,d^2}\right)\,{\left(d\,x^2+c\right)}^{7/2}+c^2\,\sqrt{d\,x^2+c}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{d^2}-c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{9/2}}{9\,d^2}+\frac{c\,{\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{d^2}-c\,\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right)}{3}+a^2\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x)
```

```
[Out] (c + d*x^2)^(5/2)*((a*d - b*c)^2/(5*d^2) - (c*((2*b^2*c - 2*a*b*d)/d^2 - (b
^2*c)/d^2))/5) - ((2*b^2*c - 2*a*b*d)/(7*d^2) - (b^2*c)/(7*d^2))*(c + d*x^2
)^(7/2) + a^2*c^(5/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + c^2*(c + d*
x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2))
```

```
+ (b^2*(c + d*x^2)^(9/2))/(9*d^2) + (c*(c + d*x^2)^(3/2)*((a*d - b*c)^2/d^2
 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2)))/3
```

sympy **[A]** time = 128.71, size = 128, normalized size = 0.97

$$\frac{a^2c^3 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + a^2c^2\sqrt{c+dx^2} + \frac{a^2c\left(c+dx^2\right)^{\frac{3}{2}}}{3} + \frac{a^2\left(c+dx^2\right)^{\frac{5}{2}}}{5} + \frac{b^2\left(c+dx^2\right)^{\frac{9}{2}}}{9d^2} + \frac{\left(c+dx^2\right)^{\frac{7}{2}}\left(4abd-2b^2c\right)}{14d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x,x)
```

```
[Out] a**2*c**3*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + a**2*c**2*sqrt(c + d*x
**2) + a**2*c*(c + d*x**2)**(3/2)/3 + a**2*(c + d*x**2)**(5/2)/5 + b**2*(c
+ d*x**2)**(9/2)/(9*d**2) + (c + d*x**2)**(7/2)*(4*a*b*d - 2*b**2*c)/(14*d*
*2)
```

3.612 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=217

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{cx}-\frac{5c^2\left(b^2c^2-16ad(3ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{x\left(c+dx^2\right)^{5/2}\left(b^2c^2-16ad(3ad+bc)\right)}{48cd}-5x\left(c+\right.$$

Rubi **[A]** time = 0.14, antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 388, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{cx}-\frac{5c^2\left(b^2c^2-16ad(3ad+bc)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}-\frac{5x\left(c+dx^2\right)^{3/2}\left(b^2c^2-16ad(3ad+bc)\right)}{192d}-\frac{5cx\sqrt{c+dx^2}\left(b^2c^2-16ad(3ad+bc)\right)}{128d}-\frac{1}{48}x\left(c+dx^2\right)^{5/2}\left(\frac{b^2c}{d}-\frac{16a(3ad+bc)}{c}\right)+\frac{b^2x\left(c+dx^2\right)^{7/2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] (-5*c*(b^2*c^2 - 16*a*d*(b*c + 3*a*d))*x*Sqrt[c + d*x^2])/(128*d) - (5*(b^2
*c^2 - 16*a*d*(b*c + 3*a*d))*x*(c + d*x^2)^(3/2))/(192*d) - (((b^2*c)/d - (
16*a*(b*c + 3*a*d))/c)*x*(c + d*x^2)^(5/2))/48 - (a^2*(c + d*x^2)^(7/2))/(c
*x) + (b^2*x*(c + d*x^2)^(7/2))/(8*d) - (5*c^2*(b^2*c^2 - 16*a*d*(b*c + 3*a
*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(128*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^2}\,dx &= -\frac{a^2\left(c+dx^2\right)^{7/2}}{cx}+\frac{\int \left(2a(bc+3ad)+b^2cx^2\right)\left(c+dx^2\right)^{5/2}\,dx}{c}\\
&= -\frac{a^2\left(c+dx^2\right)^{7/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{7/2}}{8d}-\frac{\left(b^2c^2-16ad(bc+3ad)\right)\int \left(c+dx^2\right)^{5/2}\,dx}{8cd}\\
&= -\frac{1}{48}\left(\frac{b^2c}{d}-\frac{16a(bc+3ad)}{c}\right)x\left(c+dx^2\right)^{5/2}-\frac{a^2\left(c+dx^2\right)^{7/2}}{cx}+\frac{b^2x\left(c+dx^2\right)^{7/2}}{8d}-\\
&= -\frac{5\left(b^2c^2-16ad(bc+3ad)\right)x\left(c+dx^2\right)^{3/2}}{192d}-\frac{1}{48}\left(\frac{b^2c}{d}-\frac{16a(bc+3ad)}{c}\right)x\left(c+dx^2\right.\\
&= -\frac{5c\left(b^2c^2-16ad(bc+3ad)\right)x\sqrt{c+dx^2}}{128d}-\frac{5\left(b^2c^2-16ad(bc+3ad)\right)x\left(c+dx^2\right)^{3/}}{192d}\\
&= -\frac{5c\left(b^2c^2-16ad(bc+3ad)\right)x\sqrt{c+dx^2}}{128d}-\frac{5\left(b^2c^2-16ad(bc+3ad)\right)x\left(c+dx^2\right)^{3/}}{192d}\\
&= -\frac{5c\left(b^2c^2-16ad(bc+3ad)\right)x\sqrt{c+dx^2}}{128d}-\frac{5\left(b^2c^2-16ad(bc+3ad)\right)x\left(c+dx^2\right)^{3/}}{192d}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 174, normalized size = 0.80

$$\sqrt{c+dx^2}\left(\frac{1}{192}x^3\left(48a^2d^2+208abcd+59b^2c^2\right)+\frac{cx\left(144a^2d^2+176abcd+5b^2c^2\right)}{128d}-\frac{a^2c^2}{x}+\frac{1}{48}bdx^5(16ad+17bc)+\frac{1}{8}b^2d^2x^7\right)-\frac{5c^2\left(-48a^2d^2-16abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{128d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] Sqrt[c + d*x^2]*(-((a^2*c^2)/x) + (c*(5*b^2*c^2 + 176*a*b*c*d + 144*a^2*d^2
)*x)/(128*d) + ((59*b^2*c^2 + 208*a*b*c*d + 48*a^2*d^2)*x^3)/192 + (b*d*(17
*b*c + 16*a*d)*x^5)/48 + (b^2*d^2*x^7)/8) - (5*c^2*(b^2*c^2 - 16*a*b*c*d -
48*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(128*d^(3/2))
```

IntegrateAlgebraic **[A]** time = 0.40, size = 190, normalized size = 0.88

$$\frac{\sqrt{c+dx^2}\left(-384a^2c^2d+432a^2cd^2x^2+96a^2d^3x^4+528abc^2dx^2+416abcd^2x^4+128abd^3x^6+15b^2c^3x^2+118b^2c^2dx^4+136b^2cd^2x^6+48b^2d^3x^8\right)}{384dx}+\frac{5\left(-48a^2c^2d^2-16abc^3d+b^2c^4\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{128d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-384*a^2*c^2*d + 15*b^2*c^3*x^2 + 528*a*b*c^2*d*x^2 + 432
*a^2*c*d^2*x^2 + 118*b^2*c^2*d*x^4 + 416*a*b*c*d^2*x^4 + 96*a^2*d^3*x^4 + 1
36*b^2*c*d^2*x^6 + 128*a*b*d^3*x^6 + 48*b^2*d^3*x^8))/(384*d*x) + (5*(b^2*c
^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(1
28*d^(3/2))
```

fricas **[A]** time = 1.69, size = 375, normalized size = 1.73

$$\left[-\frac{15\left(b^2c^4-16\,abc^3d-48\,a^2c^2d^2\right)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(48\,b^2d^4x^8+8\left(17\,b^2cd^3+16\,abd^4\right)x^6-384\,a^2c^2d^2+2\left(59\,b^2c^2d^2+208\,abcd^3+48\,a^2d^4\right)x^4+3\left(5\,b^2c^3d+176\,abc^2d^2+144\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{768\,d^2x},\ \frac{15\left(b^2c^4-16\,abc^3d-48\,a^2c^2d^2\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(48\,b^2d^4x^8+8\left(17\,b^2cd^3+16\,abd^4\right)x^6-384\,a^2c^2d^2+2\left(59\,b^2c^2d^2+208\,abcd^3+48\,a^2d^4\right)x^4+3\left(5\,b^2c^3d+176\,abc^2d^2+144\,a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{384\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*sqrt(d)*x*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(48*b^2*d^4*x^8 + 8*(17*b^2*c*d^3
+ 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*d^3 + 4
8*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x^2)*sqr
t(d*x^2 + c))/(d^2*x), 1/384*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*
sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (48*b^2*d^4*x^8 + 8*(17*b^2
*c*d^3 + 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*
d^3 + 48*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x
^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

giac **[A]** time = 0.56, size = 219, normalized size = 1.01

$$\frac{2\,a^2c^3\sqrt{d}}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}+\frac{1}{384}\left(2\left(4\left(6\,b^2d^2x^2+\frac{17\,b^2cd^7+16\,abd^8}{d^6}\right)x^2+\frac{59\,b^2c^2d^6+208\,abcd^7+48\,a^2d^8}{d^6}\right)x^2+\frac{3\left(5\,b^2c^3d^5+176\,abc^2d^6+144\,a^2cd^7\right)}{d^6}\right)\sqrt{dx^2+c}\,x+\frac{5\left(b^2c^4\sqrt{d}-16\,abc^3d^{\frac{3}{2}}-48\,a^2c^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{256\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a^2*c^3*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/384*(2*(4*(6*b^
2*d^2*x^2 + (17*b^2*c*d^7 + 16*a*b*d^8)/d^6)*x^2 + (59*b^2*c^2*d^6 + 208*a*
b*c*d^7 + 48*a^2*d^8)/d^6)*x^2 + 3*(5*b^2*c^3*d^5 + 176*a*b*c^2*d^6 + 144*a
^2*c*d^7)/d^6)*sqrt(d*x^2 + c)*x + 5/256*(b^2*c^4*sqrt(d) - 16*a*b*c^3*d^(3
/2) - 48*a^2*c^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^2
```

maple [A]   time = 0.01, size = 278, normalized size = 1.28

$$\frac{15a^2c^2\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8}+\frac{5ab\,c^3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8\sqrt{d}}-\frac{5b^2c^4\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{128d^{\frac{3}{2}}}+\frac{15\sqrt{d\,x^2+c}\,a^2cdx}{8}+\frac{5\sqrt{d\,x^2+c}\,ab\,c^2x}{8}-\frac{5\sqrt{d\,x^2+c}\,b^2c^3x}{128d}+\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}}a^2dx}{4}+\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}}abcx}{12}-\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}}b^2c^2x}{192d}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}a^2dx}{c}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}abx}{3}-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}b^2cx}{48d}+\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}b^2x}{8d}-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}a^2}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x)
```

```
[Out] 1/8*b^2*x*(d*x^2+c)^(7/2)/d-1/48*b^2*c/d*x*(d*x^2+c)^(5/2)-5/192*b^2*c^2/d*
x*(d*x^2+c)^(3/2)-5/128*b^2*c^3/d*x*(d*x^2+c)^(1/2)-5/128*b^2*c^4/d^(3/2)*l
n(d^(1/2)*x+(d*x^2+c)^(1/2))+1/3*a*b*x*(d*x^2+c)^(5/2)+5/12*a*b*c*x*(d*x^2+
c)^(3/2)+5/8*a*b*c^2*x*(d*x^2+c)^(1/2)+5/8*a*b*c^3/d^(1/2)*ln(d^(1/2)*x+(d*
x^2+c)^(1/2))-a^2*(d*x^2+c)^(7/2)/c/x+a^2*d/c*x*(d*x^2+c)^(5/2)+5/4*a^2*d*x
*(d*x^2+c)^(3/2)+15/8*a^2*d*c*x*(d*x^2+c)^(1/2)+15/8*a^2*d^(1/2)*c^2*ln(d^(
1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A]   time = 0.97, size = 235, normalized size = 1.08

$$\frac{1}{3}\left(dx^2+c\right)^{\frac{5}{2}}abx+\frac{5}{12}\left(dx^2+c\right)^{\frac{3}{2}}abcx+\frac{5}{8}\sqrt{dx^2+c}\,abc^2x+\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2x}{8\,d}-\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2cx}{48\,d}-\frac{5\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2x}{192\,d}-\frac{5\sqrt{dx^2+c}\,b^2c^3x}{128\,d}+\frac{5}{4}\left(dx^2+c\right)^{\frac{3}{2}}a^2dx+\frac{15}{8}\sqrt{dx^2+c}\,a^2cdx-\frac{5\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{3}{2}}}+\frac{5\,abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{d}}+\frac{15}{8}a^2c^2\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*(d*x^2 + c)^(5/2)*a*b*x + 5/12*(d*x^2 + c)^(3/2)*a*b*c*x + 5/8*sqrt(d*x
^2 + c)*a*b*c^2*x + 1/8*(d*x^2 + c)^(7/2)*b^2*x/d - 1/48*(d*x^2 + c)^(5/2)*
b^2*c*x/d - 5/192*(d*x^2 + c)^(3/2)*b^2*c^2*x/d - 5/128*sqrt(d*x^2 + c)*b^2
*c^3*x/d + 5/4*(d*x^2 + c)^(3/2)*a^2*d*x + 15/8*sqrt(d*x^2 + c)*a^2*c*d*x -
 5/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 5/8*a*b*c^3*arcsinh(d*x/sqr
t(c*d))/sqrt(d) + 15/8*a^2*c^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)
^(5/2)*a^2/x
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2, x)
```

sympy [B] time = 44.32, size = 496, normalized size = 2.29

$$-\frac{a^{2}c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^{2}}{c}}}+a^{2}c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^{2}}{c}}-\frac{7a^{2}c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3a^{2}\sqrt{c}\,d^{2}x^{3}}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{15a^{2}c^{2}\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8}+\frac{a^{2}d^{3}x^{5}}{4\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+abc^{\frac{5}{2}}x\sqrt{1+\frac{dx^{2}}{c}}+\frac{3abc^{\frac{5}{2}}x}{8\sqrt{1+\frac{dx^{2}}{c}}}+\frac{35abc^{\frac{3}{2}}dx^{3}}{24\sqrt{1+\frac{dx^{2}}{c}}}+\frac{17ab\sqrt{c}\,d^{2}x^{5}}{12\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5abc^{3}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8\sqrt{d}}+\frac{abd^{3}x^{7}}{3\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5b^{2}c^{\frac{7}{2}}x}{128d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{133b^{2}c^{\frac{5}{2}}x^{3}}{384\sqrt{1+\frac{dx^{2}}{c}}}+\frac{127b^{2}c^{\frac{3}{2}}dx^{5}}{192\sqrt{1+\frac{dx^{2}}{c}}}+\frac{23b^{2}\sqrt{c}\,d^{2}x^{7}}{48\sqrt{1+\frac{dx^{2}}{c}}}-\frac{5b^{2}c^{4}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{3}{2}}}+\frac{b^{2}d^{3}x^{9}}{8\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**2,x)
```

```
[Out] -a**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + a**2*c**(3/2)*d*x*sqrt(1 + d*x**2/c
) - 7*a**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*a**2*sqrt(c)*d**2*x**3/(
8*sqrt(1 + d*x**2/c)) + 15*a**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/8 + a
**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + a*b*c**(5/2)*x*sqrt(1 + d*x*
*2/c) + 3*a*b*c**(5/2)*x/(8*sqrt(1 + d*x**2/c)) + 35*a*b*c**(3/2)*d*x**3/(2
4*sqrt(1 + d*x**2/c)) + 17*a*b*sqrt(c)*d**2*x**5/(12*sqrt(1 + d*x**2/c)) +
5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*sqrt(d)) + a*b*d**3*x**7/(3*sqrt(c)*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(7/2)*x/(128*d*sqrt(1 + d*x**2/c)) + 133*b*
*2*c**(5/2)*x**3/(384*sqrt(1 + d*x**2/c)) + 127*b**2*c**(3/2)*d*x**5/(192*s
qrt(1 + d*x**2/c)) + 23*b**2*sqrt(c)*d**2*x**7/(48*sqrt(1 + d*x**2/c)) - 5*
b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**(3/2)) + b**2*d**3*x**9/(8*sqrt(
c)*sqrt(1 + d*x**2/c))
```

3.613 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=162

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2}-\frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{a\left(c+dx^2\right)^{5/2}(5ad+4bc)}{10c}+\frac{1}{6}a\left(c+dx^2\right)^{3/2}(5ad+4bc)+\frac{1}{2}$$

Rubi [A] time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2}-\frac{1}{2}ac^{3/2}(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{a\left(c+dx^2\right)^{5/2}(5ad+4bc)}{10c}+\frac{1}{6}a\left(c+dx^2\right)^{3/2}(5ad+4bc)+\frac{1}{2}ac\sqrt{c+dx^2}(5ad+4bc)+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] (a*c*(4*b*c + 5*a*d)*Sqrt[c + d*x^2])/2 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(3
/2))/6 + (a*(4*b*c + 5*a*d)*(c + d*x^2)^(5/2))/(10*c) + (b^2*(c + d*x^2)^(7
/2))/(7*d) - (a^2*(c + d*x^2)^(7/2))/(2*c*x^2) - (a*c^(3/2)*(4*b*c + 5*a*d)
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(4bc+5ad)+b^2cx\right)(c+dx)^{5/2}}{x}\,dx, x, x^2\right)}{2c}$$

$$= \frac{b^2\left(c+dx^2\right)^{7/2}}{7d} - \frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2} + \frac{(a(4bc+5ad))\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x}\,dx, x, x^2\right)}{4c}$$

$$= \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + \frac{b^2\left(c+dx^2\right)^{7/2}}{7d} - \frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^2} + \frac{1}{4}(a(4bc+5ad))\,\text{S}$$

$$= \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + \frac{b^2\left(c+dx^2\right)^{7/2}}{7d} -$$

$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5}}{10c}$$

$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5}}{10c}$$

$$= \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5}}{10c}$$

Mathematica [A]   time = 0.24, size = 122, normalized size = 0.75

$$-\frac{\frac{a^2\left(c+dx^2\right)^{7/2}}{x^2} + \frac{1}{15}a(5ad+4bc)\left(15c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) - \sqrt{c+dx^2}\left(23c^2+11cdx^2+3d^2x^4\right)\right) - \frac{2b^2c\left(c+dx^2\right)^{7/2}}{7d}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] -1/2*((-2*b^2*c*(c + d*x^2)^(7/2))/(7*d) + (a^2*(c + d*x^2)^(7/2))/x^2 + (a
*(4*b*c + 5*a*d)*(-(Sqrt[c + d*x^2]*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)) + 15
*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/15)/c
```

IntegrateAlgebraic [A]   time = 0.19, size = 176, normalized size = 1.09

$$\frac{\sqrt{c+dx^2}\left(-105a^2c^2d+490a^2cd^2x^2+70a^2d^3x^4+644abc^2dx^2+308abcd^2x^4+84abd^3x^6+30b^2c^3x^2+90b^2c^2dx^4+90b^2cd^2x^6+30b^2d^3x^8\right)}{210dx^2} + \frac{1}{2}\left(-5a^2c^{3/2}d-4abc^{5/2}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-105*a^2*c^2*d + 30*b^2*c^3*x^2 + 644*a*b*c^2*d*x^2 + 490
*a^2*c*d^2*x^2 + 90*b^2*c^2*d*x^4 + 308*a*b*c*d^2*x^4 + 70*a^2*d^3*x^4 + 90
*b^2*c*d^2*x^6 + 84*a*b*d^3*x^6 + 30*b^2*d^3*x^8))/(210*d*x^2) + ((-4*a*b*c
^(5/2) - 5*a^2*c^(3/2)*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

fricas [A] time = 1.58, size = 349, normalized size = 2.15

$$\left[\frac{105\left(4abc^2d+5a^2cd^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(30b^2d^3x^8+6\left(15b^2cd^2+14abd^3\right)x^6-105a^2c^2d+2\left(45b^2c^2d+154abcd^2+35a^2d^3\right)x^4+2\left(15b^2c^3+322abc^2d+245a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{420dx^2},\ \frac{105\left(4abc^2d+5a^2cd^2\right)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(30b^2d^3x^8+6\left(15b^2cd^2+14abd^3\right)x^6-105a^2c^2d+2\left(45b^2c^2d+154abcd^2+35a^2d^3\right)x^4+2\left(15b^2c^3+322abc^2d+245a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{210dx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/420*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*
x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b
*d^3)*x^6 - 105*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x
^4 + 2*(15*b^2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(
d*x^2), 1/210*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)
/sqrt(d*x^2 + c)) + (30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b*d^3)*x^6 - 1
05*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x^4 + 2*(15*b^
2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2)]
```

giac [A] time = 0.50, size = 165, normalized size = 1.02

$$\frac{30\left(dx^2+c\right)^{\frac{7}{2}}b^2+84\left(dx^2+c\right)^{\frac{5}{2}}abd+140\left(dx^2+c\right)^{\frac{3}{2}}abcd+420\sqrt{dx^2+c}\,abc^2d+70\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2+420\sqrt{dx^2+c}\,a^2cd^2-\frac{105\sqrt{dx^2+c}\,a^2c^2d}{x^2}+\frac{105\left(4abc^3d+5a^2c^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{210d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/210*(30*(d*x^2 + c)^(7/2)*b^2 + 84*(d*x^2 + c)^(5/2)*a*b*d + 140*(d*x^2 +
 c)^(3/2)*a*b*c*d + 420*sqrt(d*x^2 + c)*a*b*c^2*d + 70*(d*x^2 + c)^(3/2)*a^
2*d^2 + 420*sqrt(d*x^2 + c)*a^2*c*d^2 - 105*sqrt(d*x^2 + c)*a^2*c^2*d/x^2 +
 105*(4*a*b*c^3*d + 5*a^2*c^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c
))/d
```

maple [A] time = 0.01, size = 193, normalized size = 1.19

$$-\frac{5a^2c^{\frac{3}{2}}d\ln\left(\frac{2c+2\sqrt{dx^2+c}\,\sqrt{c}}{x}\right)}{2}-2abc^{\frac{5}{2}}\ln\left(\frac{2c+2\sqrt{dx^2+c}\,\sqrt{c}}{x}\right)+\frac{5\sqrt{dx^2+c}\,a^2cd}{2}+2\sqrt{dx^2+c}\,abc^2+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}a^2d}{6}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}abc}{3}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2d}{2c}+\frac{2\left(dx^2+c\right)^{\frac{5}{2}}ab}{5}+\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2}{7d}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x)
```

```
[Out] 1/7*b^2*(d*x^2+c)^(7/2)/d-1/2*a^2*(d*x^2+c)^(7/2)/c/x^2+1/2*a^2*d/c*(d*x^2+
c)^(5/2)+5/6*a^2*d*(d*x^2+c)^(3/2)-5/2*a^2*d*c^(3/2)*ln((2*c+2*(d*x^2+c)^(1
/2)*c^(1/2))/x)+5/2*a^2*d*c*(d*x^2+c)^(1/2)+2/5*a*b*(d*x^2+c)^(5/2)+2/3*a*b
*c*(d*x^2+c)^(3/2)-2*a*b*c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+2*a*
b*(d*x^2+c)^(1/2)*c^2
```

maxima [A] time = 0.95, size = 170, normalized size = 1.05

$$-2\,abc^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{5}{2}\,a^{2}c^{\frac{3}{2}}d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\frac{2}{5}\left(dx^{2}+c\right)^{\frac{5}{2}}ab+\frac{2}{3}\left(dx^{2}+c\right)^{\frac{3}{2}}abc+2\sqrt{dx^{2}+c}\,abc^{2}+\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}b^{2}}{7\,d}+\frac{5}{6}\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d+\frac{\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d}{2\,c}+\frac{5}{2}\sqrt{dx^{2}+c}\,a^{2}cd-\frac{\left(dx^{2}+c\right)^{\frac{7}{2}}a^{2}}{2\,cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -2*a*b*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 5/2*a^2*c^(3/2)*d*arcsinh(c/
(sqrt(c*d)*abs(x))) + 2/5*(d*x^2 + c)^(5/2)*a*b + 2/3*(d*x^2 + c)^(3/2)*a*b
*c + 2*sqrt(d*x^2 + c)*a*b*c^2 + 1/7*(d*x^2 + c)^(7/2)*b^2/d + 5/6*(d*x^2 +
 c)^(3/2)*a^2*d + 1/2*(d*x^2 + c)^(5/2)*a^2*d/c + 5/2*sqrt(d*x^2 + c)*a^2*c
*d - 1/2*(d*x^2 + c)^(7/2)*a^2/(c*x^2)
```

mupad [B] time = 1.45, size = 274, normalized size = 1.69

$$\sqrt{d\,x^{2}+c}\,\left(c^{2}\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{d}-\frac{2\,b^{2}\,c}{d}\right)-2\,c\left(2\,c\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{d}-\frac{2\,b^{2}\,c}{d}\right)-\frac{(a\,d-b\,c)^{2}}{d}+\frac{b^{2}\,c^{2}}{d}\right)\right)-\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{5\,d}-\frac{2\,b^{2}\,c}{5\,d}\right)\left(d\,x^{2}+c\right)^{5/2}-\left(d\,x^{2}+c\right)^{3/2}\left(\frac{2\,c\left(\frac{2\,b^{2}\,c-2\,a\,b\,d}{d}-\frac{2\,b^{2}\,c}{d}\right)}{3}-\frac{(a\,d-b\,c)^{2}}{3\,d}+\frac{b^{2}\,c^{2}}{3\,d}\right)+\frac{b^{2}\left(d\,x^{2}+c\right)^{7/2}}{7\,d}-\frac{a^{2}\,c^{2}\,\sqrt{d\,x^{2}+c}}{2\,x^{2}}+\frac{a\,c^{3/2}\operatorname{atan}\left(\frac{\sqrt{d\,x^{2}+c}\,1\mathrm{i}}{\sqrt{c}}\right)(5\,a\,d+4\,b\,c)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x)
```

```
[Out] (c + d*x^2)^(1/2)*(c^2*((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d) - 2*c*(2*c*((2
*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d) - (a*d - b*c)^2/d + (b^2*c^2)/d)) - ((2*
b^2*c - 2*a*b*d)/(5*d) - (2*b^2*c)/(5*d))*(c + d*x^2)^(5/2) - (c + d*x^2)^(
3/2)*((2*c*((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d))/3 - (a*d - b*c)^2/(3*d) +
 (b^2*c^2)/(3*d)) + (b^2*(c + d*x^2)^(7/2))/(7*d) + (a*c^(3/2)*atan(((c + d
*x^2)^(1/2)*1i)/c^(1/2))*(5*a*d + 4*b*c)*1i)/2 - (a^2*c^2*(c + d*x^2)^(1/2)
)/(2*x^2)
```

sympy [A] time = 91.81, size = 518, normalized size = 3.20

$$-\frac{5a^{2}c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)}{2}-\frac{a^{2}c^{2}\sqrt{d}\sqrt{\frac{c}{dx^{2}}+1}}{2x}+\frac{2a^{2}c^{2}\sqrt{d}}{x\sqrt{\frac{c}{dx^{2}}+1}}+\frac{2a^{2}cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^{2}}+1}}+a^{2}d^{2}\left(\begin{cases}\frac{\sqrt{c}\,x^{2}}{2} & \text{for}\ d=0\\\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)-2abc^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\,x}\right)+\frac{2abc^{3}}{\sqrt{d}\,x\sqrt{\frac{c}{dx^{2}}+1}}+\frac{2abc^{2}\sqrt{d}\,x}{\sqrt{\frac{c}{dx^{2}}+1}}+4abcd\left(\begin{cases}\frac{\sqrt{c}\,x^{2}}{2} & \text{for}\ d=0\\\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+2abd^{2}\left(\begin{cases}-\frac{2c^{2}\sqrt{c+dx^{2}}}{15d^{2}}+\frac{cx^{2}\sqrt{c+dx^{2}}}{15d}+\frac{x^{4}\sqrt{c+dx^{2}}}{5} & \text{for}\ d\neq 0\\\frac{\sqrt{c}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)+b^{2}c^{2}\left(\begin{cases}\frac{\sqrt{c}\,x^{2}}{2} & \text{for}\ d=0\\\frac{\left(c+dx^{2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+2b^{2}cd\left(\begin{cases}-\frac{2c^{2}\sqrt{c+dx^{2}}}{15d^{2}}+\frac{cx^{2}\sqrt{c+dx^{2}}}{15d}+\frac{x^{4}\sqrt{c+dx^{2}}}{5} & \text{for}\ d\neq 0\\\frac{\sqrt{c}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)+b^{2}d^{2}\left(\begin{cases}\frac{8c^{3}\sqrt{c+dx^{2}}}{105d^{3}}-\frac{4c^{2}x^{2}\sqrt{c+dx^{2}}}{105d^{2}}+\frac{cx^{4}\sqrt{c+dx^{2}}}{35d}+\frac{x^{6}\sqrt{c+dx^{2}}}{7} & \text{for}\ d\neq 0\\\frac{\sqrt{c}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**3,x)
```

```
[Out] -5*a**2*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c**2*sqrt(d)*sqrt(c/
(d*x**2) + 1)/(2*x) + 2*a**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a**2
*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1) + a**2*d**2*Piecewise((sqrt(c)*x**2/2, E
q(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) - 2*a*b*c**(5/2)*asinh(sqrt(c)
/(sqrt(d)*x)) + 2*a*b*c**3/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c**2*sq
rt(d)*x/sqrt(c/(d*x**2) + 1) + 4*a*b*c*d*Piecewise((sqrt(c)*x**2/2, Eq(d, 0
)), ((c + d*x**2)**(3/2)/(3*d), True)) + 2*a*b*d**2*Piecewise((-2*c**2*sqrt
(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x
**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*c**2*Piecewise((sqrt(c)*x
**2/2, Eq(d, 0)), ((c + d*x**2)**(3/2)/(3*d), True)) + 2*b**2*c*d*Piecewise
((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**
4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**2*d**2*Piecew
ise((8*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*c**2*x**2*sqrt(c + d*x**2)/(105
*d**2) + c*x**4*sqrt(c + d*x**2)/(35*d) + x**6*sqrt(c + d*x**2)/7, Ne(d, 0)
), (sqrt(c)*x**6/6, True))
```

3.614 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=223

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3}+\frac{x\left(c+dx^2\right)^{5/2}\left(4ad(2ad+3bc)+b^2c^2\right)}{6c^2}+\frac{5x\left(c+dx^2\right)^{3/2}\left(4ad(2ad+3bc)+b^2c^2\right)}{24c}+\frac{5}{16}x\sqrt{c+dx^2}$$

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3}+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\left(\frac{4ad(2ad+3bc)}{c^2}+b^2\right)+\frac{5x\left(c+dx^2\right)^{3/2}\left(4ad(2ad+3bc)+b^2c^2\right)}{24c}+\frac{5}{16}x\sqrt{c+dx^2}\left(4ad(2ad+3bc)+b^2c^2\right)+\frac{5c\left(4ad(2ad+3bc)+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16\sqrt{d}}-\frac{2a\left(c+dx^2\right)^{7/2}(2ad+3bc)}{3c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] (5*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*x*Sqrt[c + d*x^2])/16 + (5*(b^2*c^2 +
4*a*d*(3*b*c + 2*a*d))*x*(c + d*x^2)^(3/2))/(24*c) + ((b^2 + (4*a*d*(3*b*c
+ 2*a*d))/c^2)*x*(c + d*x^2)^(5/2))/6 - (a^2*(c + d*x^2)^(7/2))/(3*c*x^3) -
 (2*a*(3*b*c + 2*a*d)*(c + d*x^2)^(7/2))/(3*c^2*x) + (5*c*(b^2*c^2 + 4*a*d*
(3*b*c + 2*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^4}\,dx = -\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3} + \frac{\int \frac{\left(2a(3bc+2ad)+3b^2cx^2\right)\left(c+dx^2\right)^{5/2}}{x^2}\,dx}{3c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)\left(c+dx^2\right)^{7/2}}{3c^2x} + \left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)\int \Big(c+$$

$$= \frac{1}{6}\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x\left(c+dx^2\right)^{5/2} - \frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)\big(c+d}{3c^2x}$$

$$= \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2} + \frac{1}{6}\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x\big(c+dx^2$$

$$= \frac{5}{16}\left(b^2c^2+12abcd+8a^2d^2\right)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x\big(c+dx^2$$

$$= \frac{5}{16}\left(b^2c^2+12abcd+8a^2d^2\right)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x\big(c+dx^2$$

$$= \frac{5}{16}\left(b^2c^2+12abcd+8a^2d^2\right)x\sqrt{c+dx^2} + \frac{5}{24}c\left(b^2+\frac{4ad(3bc+2ad)}{c^2}\right)x\big(c+dx^2$$

Mathematica [A] time = 0.13, size = 155, normalized size = 0.70

$$\frac{1}{48}\left(\frac{15c\left(8a^2d^2+12abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+\frac{\sqrt{c+dx^2}\left(-8a^2\left(2c^2+14cdx^2-3d^2x^4\right)+12abx^2\left(-8c^2+9cdx^2+2d^2x^4\right)+b^2x^4\left(33c^2+26cdx^2+8d^2x^4\right)\right)}{x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] ((Sqrt[c + d*x^2]*(-8*a^2*(2*c^2 + 14*c*d*x^2 - 3*d^2*x^4) + 12*a*b*x^2*(-8
*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + b^2*x^4*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4)))
/x^3 + (15*c*(b^2*c^2 + 12*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c +
d*x^2]])/Sqrt[d])/48
```

IntegrateAlgebraic [A] time = 0.38, size = 165, normalized size = 0.74

$$\frac{\sqrt{c+dx^2}\left(-16a^2c^2-112a^2cdx^2+24a^2d^2x^4-96abc^2x^2+108abcdx^4+24abd^2x^6+33b^2c^2x^4+26b^2cdx^6+8b^2d^2x^8\right)}{48x^3}-\frac{5\left(8a^2cd^2+12abc^2d+b^2c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-16*a^2*c^2 - 96*a*b*c^2*x^2 - 112*a^2*c*d*x^2 + 33*b^2*c
^2*x^4 + 108*a*b*c*d*x^4 + 24*a^2*d^2*x^4 + 26*b^2*c*d*x^6 + 24*a*b*d^2*x^6
 + 8*b^2*d^2*x^8))/(48*x^3) - (5*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*Log
[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(16*Sqrt[d])
```

fricas [A] time = 1.58, size = 346, normalized size = 1.55

$$\left[\frac{15\left(b^2c^3+12\,abc^2d+8\,a^2cd^2\right)\sqrt{d}\,x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(8\,b^2d^3x^8+2\left(13\,b^2cd^2+12\,abd^3\right)x^6-16\,a^2c^2d+3\left(11\,b^2c^2d+36\,abcd^2+8\,a^2d^3\right)x^4-16\left(6\,abc^2d+7\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{96\,dx^3},\ -\frac{15\left(b^2c^3+12\,abc^2d+8\,a^2cd^2\right)\sqrt{-d}\,x^3\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(8\,b^2d^3x^8+2\left(13\,b^2cd^2+12\,abd^3\right)x^6-16\,a^2c^2d+3\left(11\,b^2c^2d+36\,abcd^2+8\,a^2d^3\right)x^4-16\left(6\,abc^2d+7\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{48\,dx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*x^3*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12
*a*b*d^3)*x^6 - 16*a^2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*
x^4 - 16*(6*a*b*c^2*d + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3), -1/48*(
15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)) - (8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12*a*b*d^3)*x^6 - 16*a^
2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*x^4 - 16*(6*a*b*c^2*d
 + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

giac [A] time = 0.60, size = 307, normalized size = 1.38

$$\frac{1}{48}\left(2\left(4\,b^2d^2x^2+\frac{13\,b^2cd^5+12\,abd^6}{d^4}\right)x^2+\frac{3\left(11\,b^2c^2d^4+36\,abcd^5+8\,a^2d^6\right)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{5\left(b^2c^3\sqrt{d}+12\,abc^2d^{\frac{3}{2}}+8\,a^2cd^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{32\,d}+\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^3\sqrt{d}+9\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2c^2d^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^4\sqrt{d}-12\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^3d^{\frac{3}{2}}+6\,abc^5\sqrt{d}+7\,a^2c^4d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*d^2*x^2 + (13*b^2*c*d^5 + 12*a*b*d^6)/d^4)*x^2 + 3*(11*b^2*c
^2*d^4 + 36*a*b*c*d^5 + 8*a^2*d^6)/d^4)*sqrt(d*x^2 + c)*x - 5/32*(b^2*c^3*s
qrt(d) + 12*a*b*c^2*d^(3/2) + 8*a^2*c*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2
+ c))^2)/d + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*sqrt(d) + 9*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 +
 c))^2*a*b*c^4*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(3/2)
 + 6*a*b*c^5*sqrt(d) + 7*a^2*c^4*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2
- c)^3
```

maple [A]   time = 0.02, size = 298, normalized size = 1.34

$$\frac{5a^2 c\, d^{\frac{3}{2}} \ln\left(\sqrt{d}\, x+\sqrt{d\,x^2+c}\right)}{2}+\frac{15ab\,c^2\sqrt{d}\, \ln\left(\sqrt{d}\, x+\sqrt{d\,x^2+c}\right)}{4}+\frac{5b^2c^3\ln\left(\sqrt{d}\, x+\sqrt{d\,x^2+c}\right)}{16\sqrt{d}}+\frac{5\sqrt{d\,x^2+c}\, a^2d^2x}{2}+\frac{15\sqrt{d\,x^2+c}\, abcdx}{4}+\frac{5\sqrt{d\,x^2+c}\, b^2c^2x}{16}+\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}} a^2d^2x}{3c}+\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}} abdx}{2}+\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}} b^2cx}{24}+\frac{4\left(d\,x^2+c\right)^{\frac{5}{2}} a^2d^2x}{3c^2}+\frac{2\left(d\,x^2+c\right)^{\frac{5}{2}} abdx}{c}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}} b^2x}{6}-\frac{4\left(d\,x^2+c\right)^{\frac{7}{2}} a^2d}{3c^2x}-\frac{2\left(d\,x^2+c\right)^{\frac{7}{2}} ab}{cx}-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}} a^2}{3c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x)
```

```
[Out] 1/6*x*b^2*(d*x^2+c)^(5/2)+5/24*b^2*c*x*(d*x^2+c)^(3/2)+5/16*b^2*c^2*x*(d*x^
2+c)^(1/2)+5/16*b^2*c^3/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-2*a*b/c/x*(d*
x^2+c)^(7/2)+2*a*b*d/c*x*(d*x^2+c)^(5/2)+5/2*a*b*d*x*(d*x^2+c)^(3/2)+15/4*a
*b*d*c*x*(d*x^2+c)^(1/2)+15/4*a*b*d^(1/2)*c^2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
-1/3*a^2*(d*x^2+c)^(7/2)/c/x^3-4/3*a^2*d/c^2/x*(d*x^2+c)^(7/2)+4/3*a^2*d^2/
c^2*x*(d*x^2+c)^(5/2)+5/3*a^2*d^2/c*x*(d*x^2+c)^(3/2)+5/2*a^2*d^2*x*(d*x^2+
c)^(1/2)+5/2*a^2*d^(3/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A]   time = 0.99, size = 234, normalized size = 1.05

$$\frac{1}{6}\left(dx^2+c\right)^{\frac{5}{2}}b^2x+\frac{5}{24}\left(dx^2+c\right)^{\frac{3}{2}}b^2cx+\frac{5}{16}\sqrt{dx^2+c}\,b^2c^2x+\frac{5}{2}\left(dx^2+c\right)^{\frac{3}{2}}abdx+\frac{15}{4}\sqrt{dx^2+c}\,abcdx+\frac{5}{2}\sqrt{dx^2+c}\,a^2d^2x+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2x}{3\,c}+\frac{5\,b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,\sqrt{d}}+\frac{15}{4}abc^2\sqrt{d}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+\frac{5}{2}a^2cd^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{2\left(dx^2+c\right)^{\frac{5}{2}}ab}{x}-\frac{4\left(dx^2+c\right)^{\frac{5}{2}}a^2d}{3\,cx}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(5/2)*b^2*x + 5/24*(d*x^2 + c)^(3/2)*b^2*c*x + 5/16*sqrt(d*
x^2 + c)*b^2*c^2*x + 5/2*(d*x^2 + c)^(3/2)*a*b*d*x + 15/4*sqrt(d*x^2 + c)*a
*b*c*d*x + 5/2*sqrt(d*x^2 + c)*a^2*d^2*x + 5/3*(d*x^2 + c)^(3/2)*a^2*d^2*x/
c + 5/16*b^2*c^3*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 15/4*a*b*c^2*sqrt(d)*arcs
inh(d*x/sqrt(c*d)) + 5/2*a^2*c*d^(3/2)*arcsinh(d*x/sqrt(c*d)) - 2*(d*x^2 +
c)^(5/2)*a*b/x - 4/3*(d*x^2 + c)^(5/2)*a^2*d/(c*x) - 1/3*(d*x^2 + c)^(7/2)*
a^2/(c*x^3)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4, x)
```

sympy [**B**] time = 24.42, size = 490, normalized size = 2.20

$$-\frac{2a^2c^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}}+\frac{a^2\sqrt{c}\,d^2x\sqrt{1+\frac{dx^2}{c}}}{2}-\frac{2a^2\sqrt{c}\,d^2x}{\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+\frac{5a^2cd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2}-\frac{2abc^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+2abc^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7abc^{\frac{3}{2}}dx}{4\sqrt{1+\frac{dx^2}{c}}}+\frac{3ab\sqrt{c}\,d^2x^3}{4\sqrt{1+\frac{dx^2}{c}}}+\frac{15abc^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4}+\frac{abd^3x^5}{2\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{b^2c^{\frac{5}{2}}x\sqrt{1+\frac{dx^2}{c}}}{2}+\frac{3b^2c^{\frac{5}{2}}x}{16\sqrt{1+\frac{dx^2}{c}}}+\frac{35b^2c^{\frac{3}{2}}dx^3}{48\sqrt{1+\frac{dx^2}{c}}}+\frac{17b^2\sqrt{c}\,d^2x^5}{24\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^3\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16\sqrt{d}}+\frac{b^2d^3x^7}{6\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**4,x)
```

```
[Out] -2*a**2*c**(3/2)*d/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(c)*d**2*x*sqrt(1 + d*
x**2/c)/2 - 2*a**2*sqrt(c)*d**2*x/sqrt(1 + d*x**2/c) - a**2*c**2*sqrt(d)*sq
rt(c/(d*x**2) + 1)/(3*x**2) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*
*2*c*d**(3/2)*asinh(sqrt(d)*x/sqrt(c))/2 - 2*a*b*c**(5/2)/(x*sqrt(1 + d*x**
2/c)) + 2*a*b*c**(3/2)*d*x*sqrt(1 + d*x**2/c) - 7*a*b*c**(3/2)*d*x/(4*sqrt(
1 + d*x**2/c)) + 3*a*b*sqrt(c)*d**2*x**3/(4*sqrt(1 + d*x**2/c)) + 15*a*b*c*
*2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c))/4 + a*b*d**3*x**5/(2*sqrt(c)*sqrt(1 + d
*x**2/c)) + b**2*c**(5/2)*x*sqrt(1 + d*x**2/c)/2 + 3*b**2*c**(5/2)*x/(16*sq
rt(1 + d*x**2/c)) + 35*b**2*c**(3/2)*d*x**3/(48*sqrt(1 + d*x**2/c)) + 17*b*
*2*sqrt(c)*d**2*x**5/(24*sqrt(1 + d*x**2/c)) + 5*b**2*c**3*asinh(sqrt(d)*x/
sqrt(c))/(16*sqrt(d)) + b**2*d**3*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.615 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=222

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4}+\frac{\left(c+dx^2\right)^{5/2}\left(5ad(3ad+8bc)+8b^2c^2\right)}{40c^2}+\frac{\left(c+dx^2\right)^{3/2}\left(5ad(3ad+8bc)+8b^2c^2\right)}{24c}+\frac{1}{8}\sqrt{c+dx^2}\ (5$$

Rubi [A] time = 0.25, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4}+\frac{1}{40}\left(c+dx^2\right)^{5/2}\left(\frac{5ad(3ad+8bc)}{c^2}+8b^2\right)+\frac{\left(c+dx^2\right)^{3/2}\left(5ad(3ad+8bc)+8b^2c^2\right)}{24c}+\frac{1}{8}\sqrt{c+dx^2}\left(5ad(3ad+8bc)+8b^2c^2\right)-\frac{1}{8}\sqrt{c}\left(5ad(3ad+8bc)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*Sqrt[c + d*x^2])/8 + ((8*b^2*c^2 + 5*a
*d*(8*b*c + 3*a*d))*(c + d*x^2)^(3/2))/(24*c) + ((8*b^2 + (5*a*d*(8*b*c + 3
*a*d))/c^2)*(c + d*x^2)^(5/2))/40 - (a^2*(c + d*x^2)^(7/2))/(4*c*x^4) - (a*
(8*b*c + 3*a*d)*(c + d*x^2)^(7/2))/(8*c^2*x^2) - (Sqrt[c]*(8*b^2*c^2 + 5*a*
d*(8*b*c + 3*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(8bc+3ad)+2b^2cx\right)(c+dx)^{5/2}}{x^2}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4} - \frac{a(8bc+3ad)\left(c+dx^2\right)^{7/2}}{8c^2x^2} + \frac{1}{16}\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\text{Subs}$$

$$= \frac{1}{40}\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{5/2} - \frac{a^2\left(c+dx^2\right)^{7/2}}{4cx^4} - \frac{a(8bc+3ad)\,\Big(c+d}{8c^2x^2}$$

$$= \frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\left(c+dx^2\right)^{3/2} + \frac{1}{40}\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\Big(c+dx^2$$

$$= \frac{1}{8}\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c+dx^2} + \frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\Big(c+dx^2$$

$$= \frac{1}{8}\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c+dx^2} + \frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\Big(c+dx^2$$

$$= \frac{1}{8}\left(8b^2c^2+40abcd+15a^2d^2\right)\sqrt{c+dx^2} + \frac{1}{24}c\left(8b^2+\frac{5ad(8bc+3ad)}{c^2}\right)\Big(c+dx^2$$

Mathematica [A] time = 0.11, size = 153, normalized size = 0.69

$$\frac{\sqrt{c+dx^2}\left(-15a^2\left(2c^2+9cdx^2-8d^2x^4\right)+40abx^2\left(-3c^2+14cdx^2+2d^2x^4\right)+8b^2x^4\left(23c^2+11cdx^2+3d^2x^4\right)\right)}{120x^4} - \frac{1}{8}\sqrt{c}\left(15a^2d^2+40abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) + 40*a*b*x^2*(-3*
c^2 + 14*c*d*x^2 + 2*d^2*x^4) + 8*b^2*x^4*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)
))/(120*x^4) - (Sqrt[c]*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/8
```

IntegrateAlgebraic [A] time = 0.20, size = 166, normalized size = 0.75

$$\frac{1}{8}\left(-15a^2\sqrt{c}\,d^2-40abc^{3/2}d-8b^2c^{5/2}\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) + \frac{\sqrt{c+dx^2}\left(-30a^2c^2-135a^2cdx^2+120a^2d^2x^4-120abc^2x^2+560abcdx^4+80abd^2x^6+184b^2c^2x^4+88b^2cdx^6+24b^2d^2x^8\right)}{120x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-30*a^2*c^2 - 120*a*b*c^2*x^2 - 135*a^2*c*d*x^2 + 184*b^2
*c^2*x^4 + 560*a*b*c*d*x^4 + 120*a^2*d^2*x^4 + 88*b^2*c*d*x^6 + 80*a*b*d^2*
x^6 + 24*b^2*d^2*x^8))/(120*x^4) + ((-8*b^2*c^(5/2) - 40*a*b*c^(3/2)*d - 15
*a^2*Sqrt[c]*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/8
```

fricas [A] time = 1.50, size = 319, normalized size = 1.44

$$\left[\frac{15\left(8\,b^{2}c^{2}+40\,abcd+15\,a^{2}d^{2}\right)\sqrt{c}\,x^{4}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\left(24\,b^{2}d^{2}x^{8}+8\left(11\,b^{2}cd+10\,abd^{2}\right)x^{6}+8\left(23\,b^{2}c^{2}+70\,abcd+15\,a^{2}d^{2}\right)x^{4}-30\,a^{2}c^{2}-15\left(8\,abc^{2}+9\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{240\,x^{4}},\ \frac{15\left(8\,b^{2}c^{2}+40\,abcd+15\,a^{2}d^{2}\right)\sqrt{-c}\,x^{4}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(24\,b^{2}d^{2}x^{8}+8\left(11\,b^{2}cd+10\,abd^{2}\right)x^{6}+8\left(23\,b^{2}c^{2}+70\,abcd+15\,a^{2}d^{2}\right)x^{4}-30\,a^{2}c^{2}-15\left(8\,abc^{2}+9\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{120\,x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(24*b^2*d^2*x^8 + 8*(11*b^2*c*d +
 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^
2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^4, 1/120*(15*(8*b^2*
c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + (24*b^2*d^2*x^8 + 8*(11*b^2*c*d + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*
a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sq
rt(d*x^2 + c))/x^4]
```

giac [A] time = 0.53, size = 242, normalized size = 1.09

$$\frac{24\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}d+40\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}cd+120\,\sqrt{dx^{2}+c}\,b^{2}c^{2}d+80\left(dx^{2}+c\right)^{\frac{3}{2}}abd^{2}+480\,\sqrt{dx^{2}+c}\,abcd^{2}+120\,\sqrt{dx^{2}+c}\,a^{2}d^{3}+\frac{15\left(8\,b^{2}c^{3}d+40\,abc^{2}d^{2}+15\,a^{2}cd^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{15\left(8\left(dx^{2}+c\right)^{\frac{3}{2}}abc^{2}d^{2}-8\,\sqrt{dx^{2}+c}\,abc^{3}d^{2}+9\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cd^{3}-7\,\sqrt{dx^{2}+c}\,a^{2}c^{2}d^{3}\right)}{d^{2}x^{4}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/120*(24*(d*x^2 + c)^(5/2)*b^2*d + 40*(d*x^2 + c)^(3/2)*b^2*c*d + 120*sqrt
(d*x^2 + c)*b^2*c^2*d + 80*(d*x^2 + c)^(3/2)*a*b*d^2 + 480*sqrt(d*x^2 + c)*
a*b*c*d^2 + 120*sqrt(d*x^2 + c)*a^2*d^3 + 15*(8*b^2*c^3*d + 40*a*b*c^2*d^2
+ 15*a^2*c*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - 15*(8*(d*x^2 +
c)^(3/2)*a*b*c^2*d^2 - 8*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 9*(d*x^2 + c)^(3/2)*
a^2*c*d^3 - 7*sqrt(d*x^2 + c)*a^2*c^2*d^3)/(d^2*x^4))/d
```

maple [A] time = 0.02, size = 305, normalized size = 1.37

$$-\frac{15a^{2}\sqrt{c}\,d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{8}-5ab\,c^{\frac{3}{2}}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)-b^{2}c^{\frac{5}{2}}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)+\frac{15\sqrt{d\,x^{2}+c}\,a^{2}d^{2}}{8}+5\sqrt{d\,x^{2}+c}\,abcd+\sqrt{d\,x^{2}+c}\,b^{2}c^{2}+\frac{5\left(d\,x^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}}{8c}+\frac{5\left(d\,x^{2}+c\right)^{\frac{3}{2}}abd}{3}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b^{2}c}{3}+\frac{3\left(d\,x^{2}+c\right)^{\frac{5}{2}}a^{2}d^{2}}{8c^{2}}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}abd}{c}+\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}b^{2}}{5}-\frac{3\left(d\,x^{2}+c\right)^{\frac{7}{2}}a^{2}d}{8c^{2}x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}ab}{c\,x^{2}}-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}a^{2}}{4c\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(7/2)/c/x^4-3/8*a^2*d/c^2/x^2*(d*x^2+c)^(7/2)+3/8*a^2*d^
2/c^2*(d*x^2+c)^(5/2)+5/8*a^2*d^2/c*(d*x^2+c)^(3/2)-15/8*a^2*d^2*c^(1/2)*ln
((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+15/8*a^2*d^2*(d*x^2+c)^(1/2)-a*b/c/x^2*
(d*x^2+c)^(7/2)+a*b*d/c*(d*x^2+c)^(5/2)+5/3*a*b*d*(d*x^2+c)^(3/2)-5*a*b*d*c
^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+5*a*b*d*c*(d*x^2+c)^(1/2)+1/5*
b^2*(d*x^2+c)^(5/2)+1/3*b^2*c*(d*x^2+c)^(3/2)-b^2*c^(5/2)*ln((2*c+2*(d*x^2+
c)^(1/2)*c^(1/2))/x)+b^2*(d*x^2+c)^(1/2)*c^2
```

maxima [A] time = 0.94, size = 271, normalized size = 1.22

$$-b^{2} c^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} \left|x\right|}\right) - 5 \, abc^{\frac{3}{2}} d \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} \left|x\right|}\right) - \frac{15}{8} \, a^{2} \sqrt{c} d^{2} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd} \left|x\right|}\right) + \frac{1}{5} \, {\left(dx^{2} + c\right)}^{\frac{5}{2}} b^{2} + \frac{1}{3} \, {\left(dx^{2} + c\right)}^{\frac{3}{2}} b^{2} c + \sqrt{dx^{2} + c} b^{2} c^{2} + \frac{5}{3} \, {\left(dx^{2} + c\right)}^{\frac{3}{2}} abd + \frac{{\left(dx^{2} + c\right)}^{\frac{5}{2}} abd}{c} + 5 \, \sqrt{dx^{2} + c} abcd + \frac{15}{8} \, \sqrt{dx^{2} + c} a^{2} d^{2} + \frac{3 \, {\left(dx^{2} + c\right)}^{\frac{5}{2}} a^{2} d^{2}}{8 \, c^{2}} + \frac{5 \, {\left(dx^{2} + c\right)}^{\frac{3}{2}} a^{2} d^{2}}{8 \, c} - \frac{{\left(dx^{2} + c\right)}^{\frac{7}{2}} ab}{cx^{2}} - \frac{3 \, {\left(dx^{2} + c\right)}^{\frac{7}{2}} a^{2} d}{8 \, c^{2} x^{2}} - \frac{{\left(dx^{2} + c\right)}^{\frac{7}{2}} a^{2}}{4 \, cx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -b^2*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 5*a*b*c^(3/2)*d*arcsinh(c/(sqr
t(c*d)*abs(x))) - 15/8*a^2*sqrt(c)*d^2*arcsinh(c/(sqrt(c*d)*abs(x))) + 1/5*
(d*x^2 + c)^(5/2)*b^2 + 1/3*(d*x^2 + c)^(3/2)*b^2*c + sqrt(d*x^2 + c)*b^2*c
^2 + 5/3*(d*x^2 + c)^(3/2)*a*b*d + (d*x^2 + c)^(5/2)*a*b*d/c + 5*sqrt(d*x^2
 + c)*a*b*c*d + 15/8*sqrt(d*x^2 + c)*a^2*d^2 + 3/8*(d*x^2 + c)^(5/2)*a^2*d^
2/c^2 + 5/8*(d*x^2 + c)^(3/2)*a^2*d^2/c - (d*x^2 + c)^(7/2)*a*b/(c*x^2) - 3
/8*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^2) - 1/4*(d*x^2 + c)^(7/2)*a^2/(c*x^4)
```

mupad [B] time = 1.88, size = 262, normalized size = 1.18

$${\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)-\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{9\,a^2\,c\,d^2}{8}+b\,a\,c^2\,d\right)-\left(\frac{7\,a^2\,c^2\,d^2}{8}+b\,a\,c^3\,d\right)\,\sqrt{d\,x^2+c}}{{\left(d\,x^2+c\right)}^2-2\,c\,\left(d\,x^2+c\right)+c^2}+\sqrt{d\,x^2+c}\,\left({\left(a\,d-b\,c\right)}^2+3\,c\,\left(c\,b^2+2\,a\,d\,b\right)-3\,b^2\,c^2\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{5/2}}{5}+2\,\mathrm{atan}\left(\frac{2\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{c}{256}}\,\left(15\,a^2\,d^2+40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{\frac{15\,a^2\,c\,d^2}{8}+5\,a\,b\,c^2\,d+b^2\,c^3}\right)\,\sqrt{-\frac{c}{256}}\,\left(15\,a^2\,d^2+40\,a\,b\,c\,d+8\,b^2\,c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x)
```

```
[Out] (c + d*x^2)^(3/2)*((b^2*c)/3 + (2*a*b*d)/3) - ((c + d*x^2)^(3/2)*((9*a^2*c*
d^2)/8 + a*b*c^2*d) - ((7*a^2*c^2*d^2)/8 + a*b*c^3*d)*(c + d*x^2)^(1/2))/((
c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) + (c + d*x^2)^(1/2)*((a*d - b*c)^2 +
3*c*(b^2*c + 2*a*b*d) - 3*b^2*c^2) + (b^2*(c + d*x^2)^(5/2))/5 + 2*atan((2*
(c + d*x^2)^(1/2)*(-c/256)^(1/2)*(15*a^2*d^2 + 8*b^2*c^2 + 40*a*b*c*d))/(b^
2*c^3 + (15*a^2*c*d^2)/8 + 5*a*b*c^2*d))*(-c/256)^(1/2)*(15*a^2*d^2 + 8*b^2
*c^2 + 40*a*b*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**5,x)
```

```
[Out] Timed out
```

3.616 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=228

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{\left(c+dx^2\right)^{5/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{15c^2x}+\frac{dx\left(c+dx^2\right)^{3/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{12c^2}+\frac{dx\sqrt{c+dx}}{}$$

Rubi [A] time = 0.16, antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {462, 453, 277, 195, 217, 206}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{\left(c+dx^2\right)^{5/2}\left(\frac{8ad(ad+5bc)}{c^2}+15b^2\right)}{15x}+\frac{dx\left(c+dx^2\right)^{3/2}\left(8ad(ad+5bc)+15b^2c^2\right)}{12c^2}+\frac{dx\sqrt{c+dx^2}\left(8ad(ad+5bc)+15b^2c^2\right)}{8c}+\frac{1}{8}\sqrt{d}\left(8ad(ad+5bc)+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] (d*(15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*x*Sqrt[c + d*x^2])/(8*c) + (d*(15*b^2
*c^2 + 8*a*d*(5*b*c + a*d))*x*(c + d*x^2)^(3/2))/(12*c^2) - ((15*b^2 + (8*a
*d*(5*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(15*x) - (a^2*(c + d*x^2)^(7/2))/
(5*c*x^5) - (2*a*(5*b*c + a*d)*(c + d*x^2)^(7/2))/(15*c^2*x^3) + (Sqrt[d]*(
15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/8
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^6}\,dx = -\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}+\frac{\int \frac{\left(2a(5bc+ad)+5b^2cx^2\right)\left(c+dx^2\right)^{5/2}}{x^4}\,dx}{5c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{2a(5bc+ad)\left(c+dx^2\right)^{7/2}}{15c^2x^3}-\frac{1}{15}\left(-15b^2-\frac{8ad(5bc+ad)}{c^2}\right)\int$$

$$= -\frac{\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}}{15x}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}-\frac{2a(5bc+ad)\left(c+dx^2\right)^{7/2}}{15c^2x^3}$$

$$= \frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\left(c+dx^2\right)^{3/2}-\frac{\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}}{15x}-$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\,(c+d$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\,(c+d$$

$$= \frac{1}{8}cd\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\sqrt{c+dx^2}+\frac{1}{12}d\left(15b^2+\frac{8ad(5bc+ad)}{c^2}\right)x\,(c+d$$

Mathematica [A] time = 0.17, size = 158, normalized size = 0.69

$$\frac{1}{8}\sqrt{d}\left(8a^2d^2+40abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{c+dx^2}\left(\frac{-23a^2d^2-70abcd-15b^2c^2}{15x}-\frac{a^2c^2}{5x^5}-\frac{ac(11ad+10bc)}{15x^3}+\frac{1}{8}bdx(8ad+9bc)+\frac{1}{4}b^2d^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] Sqrt[c + d*x^2]*(-1/5*(a^2*c^2)/x^5 - (a*c*(10*b*c + 11*a*d))/(15*x^3) + (-
15*b^2*c^2 - 70*a*b*c*d - 23*a^2*d^2)/(15*x) + (b*d*(9*b*c + 8*a*d)*x)/8 +
(b^2*d^2*x^3)/4) + (Sqrt[d]*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*Log[d*x +
 Sqrt[d]*Sqrt[c + d*x^2]])/8
```

IntegrateAlgebraic [A] time = 0.48, size = 169, normalized size = 0.74

$$\frac{1}{8}\left(-8a^2d^{5/2}-40abcd^{3/2}-15b^2c^2\sqrt{d}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)+\frac{\sqrt{c+dx^2}\left(-24a^2c^2-88a^2cdx^2-184a^2d^2x^4-80abc^2x^2-560abcdx^4+120abd^2x^6-120b^2c^2x^4+135b^2cdx^6+30b^2d^2x^8\right)}{120x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-24*a^2*c^2 - 80*a*b*c^2*x^2 - 88*a^2*c*d*x^2 - 120*b^2*c
^2*x^4 - 560*a*b*c*d*x^4 - 184*a^2*d^2*x^4 + 135*b^2*c*d*x^6 + 120*a*b*d^2*
x^6 + 30*b^2*d^2*x^8))/(120*x^5) + ((-15*b^2*c^2*Sqrt[d] - 40*a*b*c*d^(3/2)
 - 8*a^2*d^(5/2))*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/8
```

fricas [A] time = 1.05, size = 318, normalized size = 1.39

$$\left[\frac{15\left(15\,b^2c^2+40\,abcd+8\,a^2d^2\right)\sqrt{d}\,x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(30\,b^2d^2x^8+15\left(9\,b^2cd+8\,abd^2\right)x^6-8\left(15\,b^2c^2+70\,abcd+23\,a^2d^2\right)x^4-24\,a^2c^2-8\left(10\,abc^2+11\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{240\,x^5}, -\frac{15\left(15\,b^2c^2+40\,abcd+8\,a^2d^2\right)\sqrt{-d}\,x^5\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(30\,b^2d^2x^8+15\left(9\,b^2cd+8\,abd^2\right)x^6-8\left(15\,b^2c^2+70\,abcd+23\,a^2d^2\right)x^4-24\,a^2c^2-8\left(10\,abc^2+11\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{120\,x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^5*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a
*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8
*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^5, -1/120*(15*(15*b^2*c^
2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
 - (30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*
b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqr
t(d*x^2 + c))/x^5]
```

giac [B] time = 0.54, size = 510, normalized size = 2.24

$$\frac{1}{8}\left(2\,b^2d^2x^2+\frac{9\,b^2cd^3+8\,abd^4}{d^2}\right)\sqrt{dx^2+c}\,x-\frac{1}{16}\left(15\,b^2c^2\sqrt{d}+40\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)+\frac{2\left(15\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8b^2c^3\sqrt{d}+90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8abc^2d^{\frac{3}{2}}+45\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^8a^2cd^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6b^2c^4\sqrt{d}-300\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6abc^3d^{\frac{3}{2}}-90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^6a^2c^2d^{\frac{5}{2}}+90\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b^2c^5\sqrt{d}+400\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc^4d^{\frac{3}{2}}+140\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2c^3d^{\frac{5}{2}}-60\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^2c^6\sqrt{d}-260\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^5d^{\frac{3}{2}}-70\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2c^4d^{\frac{5}{2}}+15\,b^2c^7\sqrt{d}+70\,abc^6d^{\frac{3}{2}}+23\,a^2c^5d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*d^2*x^2 + (9*b^2*c*d^3 + 8*a*b*d^4)/d^2)*sqrt(d*x^2 + c)*x - 1/1
6*(15*b^2*c^2*sqrt(d) + 40*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x -
sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*sqrt
(d) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(3/2) + 45*(sqrt(d)*x -
sqrt(d*x^2 + c))^8*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c
^4*sqrt(d) - 300*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(3/2) - 90*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c
))^4*b^2*c^5*sqrt(d) + 400*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*d^(3/2)
+ 140*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(5/2) - 60*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*b^2*c^6*sqrt(d) - 260*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c
^5*d^(3/2) - 70*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(5/2) + 15*b^2*c^
7*sqrt(d) + 70*a*b*c^6*d^(3/2) + 23*a^2*c^5*d^(5/2))/((sqrt(d)*x - sqrt(d*x
^2 + c))^2 - c)^5
```

maple [A] time = 0.02, size = 369, normalized size = 1.62

$$a^2d^{\frac{5}{2}}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+5abc\,d^{\frac{3}{2}}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+\frac{15b^2c^2\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8}+\frac{\sqrt{d\,x^2+c}\,a^2d^3x}{c}+5\sqrt{d\,x^2+c}\,ab\,d^2x+\frac{15\sqrt{d\,x^2+c}\,b^2cdx}{8}+\frac{2\left(d\,x^2+c\right)^{\frac{3}{2}}a^2d^3x}{3c^2}+\frac{10\left(d\,x^2+c\right)^{\frac{3}{2}}ab\,d^2x}{3c}+\frac{5\left(d\,x^2+c\right)^{\frac{3}{2}}b^2dx}{4}+\frac{8\left(d\,x^2+c\right)^{\frac{5}{2}}a^2d^3x}{15c^3}+\frac{8\left(d\,x^2+c\right)^{\frac{5}{2}}ab\,d^2x}{3c^2}+\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}b^2dx}{c}-\frac{8\left(d\,x^2+c\right)^{\frac{7}{2}}a^2d^2}{15c^3x}-\frac{8\left(d\,x^2+c\right)^{\frac{7}{2}}abd}{3c^2x}-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}b^2}{cx}-\frac{2\left(d\,x^2+c\right)^{\frac{7}{2}}a^2d}{15c^2x^3}-\frac{2\left(d\,x^2+c\right)^{\frac{7}{2}}ab}{3c\,x^3}-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}a^2}{5c\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x)
```

```
[Out] -1/5*a^2*(d*x^2+c)^(7/2)/c/x^5-2/15*a^2*d/c^2/x^3*(d*x^2+c)^(7/2)-8/15*a^2*
d^2/c^3/x*(d*x^2+c)^(7/2)+8/15*a^2*d^3/c^3*x*(d*x^2+c)^(5/2)+2/3*a^2*d^3/c^
2*x*(d*x^2+c)^(3/2)+a^2*d^3/c*x*(d*x^2+c)^(1/2)+a^2*d^(5/2)*ln(d^(1/2)*x+(d
*x^2+c)^(1/2))-b^2/c/x*(d*x^2+c)^(7/2)+b^2*d/c*x*(d*x^2+c)^(5/2)+5/4*b^2*d*
x*(d*x^2+c)^(3/2)+15/8*b^2*d*c*x*(d*x^2+c)^(1/2)+15/8*b^2*d^(1/2)*c^2*ln(d^
(1/2)*x+(d*x^2+c)^(1/2))-2/3*a*b/c/x^3*(d*x^2+c)^(7/2)-8/3*a*b*d/c^2/x*(d*x
^2+c)^(7/2)+8/3*a*b*d^2/c^2*x*(d*x^2+c)^(5/2)+10/3*a*b*d^2/c*x*(d*x^2+c)^(3
/2)+5*a*b*d^2*x*(d*x^2+c)^(1/2)+5*a*b*d^(3/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2
))
```

maxima [**A**] time = 0.96, size = 285, normalized size = 1.25

$$\frac{5}{4}\left(dx^2+c\right)^{\frac{3}{2}}b^2dx+\frac{15}{8}\sqrt{dx^2+c}\,b^2cdx+5\sqrt{dx^2+c}\,abd^2x+\frac{10\left(dx^2+c\right)^{\frac{3}{2}}abd^2x}{3c}+\frac{2\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3x}{3c^2}+\frac{\sqrt{dx^2+c}\,a^2d^3x}{c}+\frac{15}{8}b^2c^2\sqrt{d}\ \mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+5\,abcd^{\frac{3}{2}}\ \mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)+a^2d^{\frac{5}{2}}\ \mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2}{x}-\frac{8\left(dx^2+c\right)^{\frac{5}{2}}abd}{3\,cx}-\frac{8\left(dx^2+c\right)^{\frac{5}{2}}a^2d^2}{15\,c^2x}-\frac{2\left(dx^2+c\right)^{\frac{7}{2}}ab}{3\,cx^3}-\frac{2\left(dx^2+c\right)^{\frac{7}{2}}a^2d}{15\,c^2x^3}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 5/4*(d*x^2 + c)^(3/2)*b^2*d*x + 15/8*sqrt(d*x^2 + c)*b^2*c*d*x + 5*sqrt(d*x
^2 + c)*a*b*d^2*x + 10/3*(d*x^2 + c)^(3/2)*a*b*d^2*x/c + 2/3*(d*x^2 + c)^(3
/2)*a^2*d^3*x/c^2 + sqrt(d*x^2 + c)*a^2*d^3*x/c + 15/8*b^2*c^2*sqrt(d)*arcs
inh(d*x/sqrt(c*d)) + 5*a*b*c*d^(3/2)*arcsinh(d*x/sqrt(c*d)) + a^2*d^(5/2)*a
rcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)^(5/2)*b^2/x - 8/3*(d*x^2 + c)^(5/2)*a*b
*d/(c*x) - 8/15*(d*x^2 + c)^(5/2)*a^2*d^2/(c^2*x) - 2/3*(d*x^2 + c)^(7/2)*a
*b/(c*x^3) - 2/15*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^3) - 1/5*(d*x^2 + c)^(7/2)
*a^2/(c*x^5)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x)
```

```
[Out] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6, x)
```

sympy [**B**] time = 20.41, size = 474, normalized size = 2.08

$$-\frac{a^2\sqrt{c}\,d^2}{x\sqrt{1+\frac{dx^2}{c}}}-\frac{a^2c^2\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{11a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15x^2}-\frac{8a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15}+a^2d^{\frac{5}{2}}\ \mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{a^2d^3x}{\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}-\frac{4abc^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}}+ab\sqrt{c}\,d^2x\sqrt{1+\frac{dx^2}{c}}-\frac{4ab\sqrt{c}\,d^2x}{\sqrt{1+\frac{dx^2}{c}}}-\frac{2abc^2\sqrt{d}\,\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3}+5abcd^{\frac{3}{2}}\ \mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)-\frac{b^2c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}}+b^2c^{\frac{3}{2}}dx\sqrt{1+\frac{dx^2}{c}}-\frac{7b^2c^{\frac{3}{2}}dx}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{3b^2\sqrt{c}\,d^2x^3}{8\sqrt{1+\frac{dx^2}{c}}}+\frac{15b^2c^2\sqrt{d}\ \mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8}+\frac{b^2d^3x^5}{4\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**6,x)
```

```
[Out] -a**2*sqrt(c)*d**2/(x*sqrt(1 + d*x**2/c)) - a**2*c**2*sqrt(d)*sqrt(c/(d*x**
2) + 1)/(5*x**4) - 11*a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*x**2) - 8*a*
*2*d**(5/2)*sqrt(c/(d*x**2) + 1)/15 + a**2*d**(5/2)*asinh(sqrt(d)*x/sqrt(c)
) - a**2*d**3*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 4*a*b*c**(3/2)*d/(x*sqrt(1 +
 d*x**2/c)) + a*b*sqrt(c)*d**2*x*sqrt(1 + d*x**2/c) - 4*a*b*sqrt(c)*d**2*x/
sqrt(1 + d*x**2/c) - 2*a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a
*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 5*a*b*c*d**(3/2)*asinh(sqrt(d)*x/sqr
t(c)) - b**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) + b**2*c**(3/2)*d*x*sqrt(1 + d
*x**2/c) - 7*b**2*c**(3/2)*d*x/(8*sqrt(1 + d*x**2/c)) + 3*b**2*sqrt(c)*d**2
*x**3/(8*sqrt(1 + d*x**2/c)) + 15*b**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)
)/8 + b**2*d**3*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.617 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=222

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{5/2}\left(ad(ad+12bc)+8b^2c^2\right)}{16c^2x^2}+\frac{5d\left(c+dx^2\right)^{3/2}\left(ad(ad+12bc)+8b^2c^2\right)}{48c^2}+\frac{5d\sqrt{c+dx^2}\;\Big(}{}$$

Rubi [A] time = 0.25, antiderivative size = 219, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 89, 78, 47, 50, 63, 208}

$$-\frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6}-\frac{\left(c+dx^2\right)^{5/2}\left(\frac{ad(ad+12bc)}{c^2}+8b^2\right)}{16x^2}+\frac{5d\left(c+dx^2\right)^{3/2}\left(ad(ad+12bc)+8b^2c^2\right)}{48c^2}+\frac{5d\sqrt{c+dx^2}\left(ad(ad+12bc)+8b^2c^2\right)}{16c}-\frac{5d\left(ad(ad+12bc)+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] (5*d*(8*b^2*c^2 + a*d*(12*b*c + a*d))*Sqrt[c + d*x^2])/(16*c) + (5*d*(8*b^2
*c^2 + a*d*(12*b*c + a*d))*(c + d*x^2)^(3/2))/(48*c^2) - ((8*b^2 + (a*d*(12
*b*c + a*d))/c^2)*(c + d*x^2)^(5/2))/(16*x^2) - (a^2*(c + d*x^2)^(7/2))/(6*
c*x^6) - (a*(12*b*c + a*d)*(c + d*x^2)^(7/2))/(24*c^2*x^4) - (5*d*(8*b^2*c^
2 + a*d*(12*b*c + a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2(c+dx)^{5/2}}{x^4}\,dx,x,x^2\right)$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}a(12bc+ad)+3b^2cx\right)(c+dx)^{5/2}}{x^3}\,dx,x,x^2\right)}{6c}$$

$$= -\frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6} - \frac{a(12bc+ad)\left(c+dx^2\right)^{7/2}}{24c^2x^4} + \frac{1}{16}\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\text{Subst}$$

$$= -\frac{\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}}{16x^2} - \frac{a^2\left(c+dx^2\right)^{7/2}}{6cx^6} - \frac{a(12bc+ad)\left(c+dx^2\right)^{7/2}}{24c^2x^4} +$$

$$= \frac{5}{48}d\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/2} - \frac{\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\left(c+dx^2\right)^{5/2}}{16x^2} - a^2($$

$$= \frac{5}{16}cd\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48}d\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/}$$

$$= \frac{5}{16}cd\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48}d\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/}$$

$$= \frac{5}{16}cd\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\sqrt{c+dx^2} + \frac{5}{48}d\left(8b^2+\frac{ad(12bc+ad)}{c^2}\right)\left(c+dx^2\right)^{3/}$$

Mathematica [C] time = 0.06, size = 92, normalized size = 0.41

$$\frac{\left(c+dx^2\right)^{7/2}\left(3dx^6\left(a^2d^2+12abcd+8b^2c^2\right){}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{dx^2}{c}+1\right)-7ac^2\left(4ac+adx^2+12bcx^2\right)\right)}{168c^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] ((c + d*x^2)^(7/2)*(-7*a*c^2*(4*a*c + 12*b*c*x^2 + a*d*x^2) + 3*d*(8*b^2*c^
2 + 12*a*b*c*d + a^2*d^2)*x^6*Hypergeometric2F1[2, 7/2, 9/2, 1 + (d*x^2)/c]
))/(168*c^4*x^6)
```

IntegrateAlgebraic [A]   time = 0.26, size = 162, normalized size = 0.73

$$\frac{\sqrt{c+dx^2}\left(-8a^2c^2-26a^2cdx^2-33a^2d^2x^4-24abc^2x^2-108abcdx^4+96abd^2x^6-24b^2c^2x^4+112b^2cdx^6+16b^2d^2x^8\right)}{48x^6}-\frac{5\left(a^2d^3+12abcd^2+8b^2c^2d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*c^2 - 24*a*b*c^2*x^2 - 26*a^2*c*d*x^2 - 24*b^2*c^2
*x^4 - 108*a*b*c*d*x^4 - 33*a^2*d^2*x^4 + 112*b^2*c*d*x^6 + 96*a*b*d^2*x^6
+ 16*b^2*d^2*x^8))/(48*x^6) - (5*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*Arc
Tanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*Sqrt[c])
```

fricas [A]   time = 1.70, size = 347, normalized size = 1.56

$$\left[\frac{15\left(8b^2c^2d+12abcd^2+a^2d^3\right)\sqrt{c}x^6\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(16b^2cd^2x^8+16\left(7b^2c^2d+6abcd^2\right)x^6-8a^2c^3-3\left(8b^2c^3+36abc^2d+11a^2cd^2\right)x^4-2\left(12abc^3+13a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{96cx^6}, \frac{15\left(8b^2c^2d+12abcd^2+a^2d^3\right)\sqrt{-c}x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(16b^2cd^2x^8+16\left(7b^2c^2d+6abcd^2\right)x^6-8a^2c^3-3\left(8b^2c^3+36abc^2d+11a^2cd^2\right)x^4-2\left(12abc^3+13a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{48cx^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2
*d + 6*a*b*c*d^2)*x^6 - 8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*
d^2)*x^4 - 2*(12*a*b*c^3 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6), 1/4
8*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) + (16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2*d + 6*a*b*c*d^2)*x^6 -
8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*d^2)*x^4 - 2*(12*a*b*c^3
 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6)]
```

giac [A]   time = 0.47, size = 286, normalized size = 1.29

$$\frac{16\left(dx^2+c\right)^{\frac{3}{2}}b^2d^2+96\sqrt{dx^2+c}\,b^2cd^2+96\sqrt{dx^2+c}\,abd^3+\frac{15\left(8b^2c^2d^2+12abcd^3+a^2d^4\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{24\left(dx^2+c\right)^{\frac{5}{2}}b^2c^2d^2-48\left(dx^2+c\right)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}\,b^2c^4d^2+108\left(dx^2+c\right)^{\frac{5}{2}}abcd^3-192\left(dx^2+c\right)^{\frac{3}{2}}abc^2d^3+84\sqrt{dx^2+c}\,abc^3d^3+33\left(dx^2+c\right)^{\frac{5}{2}}a^2d^4-40\left(dx^2+c\right)^{\frac{3}{2}}a^2cd^4+15\sqrt{dx^2+c}\,a^2c^2d^4}{d^3x^6}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(16*(d*x^2 + c)^(3/2)*b^2*d^2 + 96*sqrt(d*x^2 + c)*b^2*c*d^2 + 96*sqrt
(d*x^2 + c)*a*b*d^3 + 15*(8*b^2*c^2*d^2 + 12*a*b*c*d^3 + a^2*d^4)*arctan(sq
rt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(
d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 108*(d*x^2
+ c)^(5/2)*a*b*c*d^3 - 192*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 84*sqrt(d*x^2 +
c)*a*b*c^3*d^3 + 33*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*
d^4 + 15*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(d^3*x^6))/d
```

maple [A] time = 0.02, size = 387, normalized size = 1.74

$$-\frac{5a^2d^3\ln\left(\frac{2c+2\sqrt{dx^2+c}\,\sqrt{c}}{x}\right)}{16\sqrt{c}}-\frac{15ab\sqrt{c}\,d^2\ln\left(\frac{2c+2\sqrt{dx^2+c}\,\sqrt{c}}{x}\right)}{4}-\frac{5b^2c^{\frac{3}{2}}d\ln\left(\frac{2c+2\sqrt{dx^2+c}\,\sqrt{c}}{x}\right)}{2}+\frac{5\sqrt{dx^2+c}\,a^2d^3}{16c}+\frac{15\sqrt{dx^2+c}\,abd^2}{4}+\frac{5\sqrt{dx^2+c}\,b^2cd}{2}+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3}{48c^2}+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}abd^2}{4c}+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}b^2d}{6}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2d^3}{16c^3}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}abd^2}{4c^2}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2d}{2c}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2d^2}{16c^3x^2}-\frac{3\left(dx^2+c\right)^{\frac{7}{2}}abd}{4c^2x^2}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2}{2cx^2}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2d}{24c^2x^4}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}ab}{2cx^4}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{6cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(7/2)/c/x^6-1/24*a^2*d/c^2/x^4*(d*x^2+c)^(7/2)-1/16*a^2*
d^2/c^3/x^2*(d*x^2+c)^(7/2)+1/16*a^2*d^3/c^3*(d*x^2+c)^(5/2)+5/48*a^2*d^3/c
^2*(d*x^2+c)^(3/2)-5/16*a^2*d^3/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/
x)+5/16*a^2*d^3/c*(d*x^2+c)^(1/2)-1/2*b^2/c/x^2*(d*x^2+c)^(7/2)+1/2*b^2*d/c
*(d*x^2+c)^(5/2)+5/6*b^2*d*(d*x^2+c)^(3/2)-5/2*b^2*d*c^(3/2)*ln((2*c+2*(d*x
^2+c)^(1/2)*c^(1/2))/x)+5/2*b^2*d*c*(d*x^2+c)^(1/2)-1/2*a*b/c/x^4*(d*x^2+c)
^(7/2)-3/4*a*b*d/c^2/x^2*(d*x^2+c)^(7/2)+3/4*a*b*d^2/c^2*(d*x^2+c)^(5/2)+5/
4*a*b*d^2/c*(d*x^2+c)^(3/2)-15/4*a*b*d^2*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*
c^(1/2))/x)+15/4*a*b*d^2*(d*x^2+c)^(1/2)
```

maxima [A] time = 0.96, size = 353, normalized size = 1.59

$$-\frac{5}{2}\,b^2c^{\frac{3}{2}}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{15}{4}\,ab\sqrt{c}\,d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{5\,a^2d^3\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,\sqrt{c}}+\frac{5}{6}\left(dx^2+c\right)^{\frac{3}{2}}b^2d+\frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2d}{2\,c}+\frac{5}{2}\sqrt{dx^2+c}\,b^2cd+\frac{15}{4}\sqrt{dx^2+c}\,abd^2+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}abd^2}{4\,c^2}+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}abd^2}{4\,c}+\frac{\left(dx^2+c\right)^{\frac{5}{2}}a^2d^3}{16\,c^3}+\frac{5\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3}{48\,c^2}+\frac{5\sqrt{dx^2+c}\,a^2d^3}{16\,c}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}b^2}{2\,cx^2}-\frac{3\left(dx^2+c\right)^{\frac{7}{2}}abd}{4\,c^2x^2}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2d^2}{16\,c^3x^2}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}ab}{2\,cx^4}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2d}{24\,c^2x^4}-\frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{6\,cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] -5/2*b^2*c^(3/2)*d*arcsinh(c/(sqrt(c*d)*abs(x))) - 15/4*a*b*sqrt(c)*d^2*arc
sinh(c/(sqrt(c*d)*abs(x))) - 5/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/sqr
t(c) + 5/6*(d*x^2 + c)^(3/2)*b^2*d + 1/2*(d*x^2 + c)^(5/2)*b^2*d/c + 5/2*sq
rt(d*x^2 + c)*b^2*c*d + 15/4*sqrt(d*x^2 + c)*a*b*d^2 + 3/4*(d*x^2 + c)^(5/2
)*a*b*d^2/c^2 + 5/4*(d*x^2 + c)^(3/2)*a*b*d^2/c + 1/16*(d*x^2 + c)^(5/2)*a^
2*d^3/c^3 + 5/48*(d*x^2 + c)^(3/2)*a^2*d^3/c^2 + 5/16*sqrt(d*x^2 + c)*a^2*d
^3/c - 1/2*(d*x^2 + c)^(7/2)*b^2/(c*x^2) - 3/4*(d*x^2 + c)^(7/2)*a*b*d/(c^2
*x^2) - 1/16*(d*x^2 + c)^(7/2)*a^2*d^2/(c^3*x^2) - 1/2*(d*x^2 + c)^(7/2)*a*
b/(c*x^4) - 1/24*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^4) - 1/6*(d*x^2 + c)^(7/2)*
a^2/(c*x^6)
```

mupad [B] time = 2.72, size = 301, normalized size = 1.36

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{5\,a^2\,c^2\,d^3}{16}+\frac{7\,a\,b\,c^3\,d^2}{4}+\frac{b^2\,c^4\,d}{2}\right)-{\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{5\,a^2\,c\,d^3}{6}+4\,a\,b\,c^2\,d^2+b^2\,c^3\,d\right)+{\left(d\,x^2+c\right)}^{5/2}\,\left(\frac{11\,a^2\,d^3}{16}+\frac{9\,a\,b\,c\,d^2}{4}+\frac{b^2\,c^2\,d}{2}\right)}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}+\left(2\,b\,d\,\left(a\,d-b\,c\right)+4\,b^2\,c\,d\right)\,\sqrt{d\,x^2+c}+\frac{b^2\,d\,{\left(d\,x^2+c\right)}^{3/2}}{3}+\frac{d\,\mathrm{atan}\left(\frac{d\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2+12\,a\,b\,c\,d+8\,b^2\,c^2\right)\,5{}\mathrm{i}}{8\,\sqrt{c}\,\left(\frac{5\,a^2\,d^3}{8}+\frac{15\,a\,b\,c\,d^2}{2}+5\,b^2\,c^2\,d\right)}\right)\,\left(a^2\,d^2+12\,a\,b\,c\,d+8\,b^2\,c^2\right)\,5{}\mathrm{i}}{16\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((b^2*c^4*d)/2 + (5*a^2*c^2*d^3)/16 + (7*a*b*c^3*d^2)/4)
 - (c + d*x^2)^(3/2)*((5*a^2*c*d^3)/6 + b^2*c^3*d + 4*a*b*c^2*d^2) + (c + d
```

```
*x^2)^(5/2)*((11*a^2*d^3)/16 + (b^2*c^2*d)/2 + (9*a*b*c*d^2)/4))/(3*c*(c +
d*x^2)^2 - 3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + (2*b*d*(a*d - b*c) +
4*b^2*c*d)*(c + d*x^2)^(1/2) + (b^2*d*(c + d*x^2)^(3/2))/3 + (d*atan((d*(c
+ d*x^2)^(1/2)*(a^2*d^2 + 8*b^2*c^2 + 12*a*b*c*d)*5i)/(8*c^(1/2)*((5*a^2*d^
3)/8 + 5*b^2*c^2*d + (15*a*b*c*d^2)/2)))*(a^2*d^2 + 8*b^2*c^2 + 12*a*b*c*d)
*5i)/(16*c^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**7,x)
```

```
[Out] Timed out
```

3.618 $\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{c^2\left(48a^2d^2+5bc(7bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}-\frac{cx\sqrt{c+dx^2}\,\left(48a^2d^2+5bc(7bc-16ad)\right)}{128d^4}+\frac{x^3\sqrt{c+dx^2}\,\big(48a^2d^2}{192}$$

Rubi **[A]** time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {464, 459, 321, 217, 206}

$$\frac{c^2\left(48a^2d^2+5bc(7bc-16ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}+\frac{x^3\sqrt{c+dx^2}\,\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)}{192d}-\frac{cx\sqrt{c+dx^2}\,\left(48a^2d^2+5bc(7bc-16ad)\right)}{128d^4}-\frac{bx^5\sqrt{c+dx^2}\,(7bc-16ad)}{48d^2}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -(c*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*x*Sqrt[c + d*x^2])/(128*d^4) + ((
48*a^2 + (5*b*c*(7*b*c - 16*a*d))/d^2)*x^3*Sqrt[c + d*x^2])/(192*d) - (b*(7
*b*c - 16*a*d)*x^5*Sqrt[c + d*x^2])/(48*d^2) + (b^2*x^7*Sqrt[c + d*x^2])/(8
*d) + (c^2*(48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(128*d^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx &= \frac{b^2x^7\sqrt{c+dx^2}}{8d}+\frac{\int \frac{x^4\left(8a^2d-b(7bc-16ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{8d}\\
&= -\frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}-\frac{1}{48}\left(-48a^2-\frac{5bc(7bc-16ad)}{d^2}\right)\int\frac{x^4}{\sqrt{c+dx^2}}\\
&= \frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d}-\frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}-\\
&= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2}+\frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d}-\frac{b(7bc-16ad)x}{48d^2}\\
&= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2}+\frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d}-\frac{b(7bc-16ad)x}{48d^2}\\
&= -\frac{c\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x\sqrt{c+dx^2}}{128d^2}+\frac{\left(48a^2+\frac{5bc(7bc-16ad)}{d^2}\right)x^3\sqrt{c+dx^2}}{192d}-\frac{b(7bc-16ad)x}{48d^2}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 159, normalized size = 0.82

$$\frac{3c^2\left(48a^2d^2-80abcd+35b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+\sqrt{d}\,x\sqrt{c+dx^2}\left(48a^2d^2\left(2dx^2-3c\right)+16abd\left(15c^2-10cdx^2+8d^2x^4\right)+b^2\left(-105c^3+70c^2dx^2-56cd^2x^4+48d^3x^6\right)\right)}{384d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(-3*c + 2*d*x^2) + 16*a*b*d*(15*c^2
- 10*c*d*x^2 + 8*d^2*x^4) + b^2*(-105*c^3 + 70*c^2*d*x^2 - 56*c*d^2*x^4 + 4
8*d^3*x^6)) + 3*c^2*(35*b^2*c^2 - 80*a*b*c*d + 48*a^2*d^2)*Log[d*x + Sqrt[d
]*Sqrt[c + d*x^2]])/(384*d^(9/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 173, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(-144a^2cd^2x+96a^2d^3x^3+240abc^2dx-160abcd^2x^3+128abd^3x^5-105b^2c^3x+70b^2c^2dx^3-56b^2cd^2x^5+48b^2d^3x^7\right)}{384d^4}+\frac{\left(-48a^2c^2d^2+80abc^3d-35b^2c^4\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{128d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(-105*b^2*c^3*x + 240*a*b*c^2*d*x - 144*a^2*c*d^2*x + 70*b
^2*c^2*d*x^3 - 160*a*b*c*d^2*x^3 + 96*a^2*d^3*x^3 - 56*b^2*c*d^2*x^5 + 128*
a*b*d^3*x^5 + 48*b^2*d^3*x^7))/(384*d^4) + ((-35*b^2*c^4 + 80*a*b*c^3*d - 4
8*a^2*c^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(128*d^(9/2))
```

fricas [A] time = 1.17, size = 344, normalized size = 1.77

$$\left[\frac{3\left(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(48\,b^2d^4x^7-8\left(7\,b^2cd^3-16\,abd^4\right)x^5+2\left(35\,b^2c^2d^2-80\,abcd^3+48\,a^2d^4\right)x^3-3\left(35\,b^2c^3d-80\,abc^2d^2+48\,a^2cd^3\right)x\right)\sqrt{dx^2+c}}{768\,d^5},-\frac{3\left(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(48\,b^2d^4x^7-8\left(7\,b^2cd^3-16\,abd^4\right)x^5+2\left(35\,b^2c^2d^2-80\,abcd^3+48\,a^2d^4\right)x^3-3\left(35\,b^2c^3d-80\,abc^2d^2+48\,a^2cd^3\right)x\right)\sqrt{dx^2+c}}{384\,d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 -
16*a*b*d^4)*x^5 + 2*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3
5*b^2*c^3*d - 80*a*b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/38
4*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*
x/sqrt(d*x^2 + c)) - (48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 - 16*a*b*d^4)*x^5 + 2
*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 80*a*
b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

giac [A] time = 0.41, size = 178, normalized size = 0.92

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,b^2x^2}{d}-\frac{7\,b^2cd^5-16\,abd^6}{d^7}\right)x^2+\frac{35\,b^2c^2d^4-80\,abcd^5+48\,a^2d^6}{d^7}\right)x^2-\frac{3\left(35\,b^2c^3d^3-80\,abc^2d^4+48\,a^2cd^5\right)}{d^7}\right)\sqrt{dx^2+c}\,x-\frac{\left(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*x^2/d - (7*b^2*c*d^5 - 16*a*b*d^6)/d^7)*x^2 + (35*b^2*c^
2*d^4 - 80*a*b*c*d^5 + 48*a^2*d^6)/d^7)*x^2 - 3*(35*b^2*c^3*d^3 - 80*a*b*c^
```

```
2*d^4 + 48*a^2*c*d^5)/d^7)*sqrt(d*x^2 + c)*x - 1/128*(35*b^2*c^4 - 80*a*b*c
^3*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

maple [A] time = 0.02, size = 265, normalized size = 1.37

$$\frac{\sqrt{d\,x^2+c}\;b^2x^7}{8d}+\frac{\sqrt{d\,x^2+c}\;ab\,x^5}{3d}-\frac{7\sqrt{d\,x^2+c}\;b^2c\,x^5}{48d^2}+\frac{\sqrt{d\,x^2+c}\;a^2x^3}{4d}-\frac{5\sqrt{d\,x^2+c}\;abc\,x^3}{12d^2}+\frac{35\sqrt{d\,x^2+c}\;b^2c^2x^3}{192d^3}+\frac{3a^2c^2\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)}{8d^{\frac{5}{2}}}-\frac{5ab\,c^3\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)}{8d^{\frac{7}{2}}}+\frac{35b^2c^4\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)}{128d^{\frac{9}{2}}}-\frac{3\sqrt{d\,x^2+c}\;a^2cx}{8d^2}+\frac{5\sqrt{d\,x^2+c}\;ab\,c^2x}{8d^3}-\frac{35\sqrt{d\,x^2+c}\;b^2c^3x}{128d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/8*b^2*x^7*(d*x^2+c)^(1/2)/d-7/48*b^2*c/d^2*x^5*(d*x^2+c)^(1/2)+35/192*b^2
*c^2/d^3*x^3*(d*x^2+c)^(1/2)-35/128*b^2*c^3/d^4*x*(d*x^2+c)^(1/2)+35/128*b^
2*c^4/d^(9/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/3*a*b*x^5/d*(d*x^2+c)^(1/2)-5
/12*a*b*c/d^2*x^3*(d*x^2+c)^(1/2)+5/8*a*b*c^2/d^3*x*(d*x^2+c)^(1/2)-5/8*a*b
*c^3/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/4*a^2*x^3/d*(d*x^2+c)^(1/2)-3/
8*a^2*c/d^2*x*(d*x^2+c)^(1/2)+3/8*a^2*c^2/d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1
/2))
```

maxima [A] time = 0.94, size = 243, normalized size = 1.25

$$\frac{\sqrt{dx^2+c}\,b^2x^7}{8\,d}-\frac{7\,\sqrt{dx^2+c}\,b^2cx^5}{48\,d^2}+\frac{\sqrt{dx^2+c}\,abx^5}{3\,d}+\frac{35\,\sqrt{dx^2+c}\,b^2c^2x^3}{192\,d^3}-\frac{5\,\sqrt{dx^2+c}\,abcx^3}{12\,d^2}+\frac{\sqrt{dx^2+c}\,a^2x^3}{4\,d}-\frac{35\,\sqrt{dx^2+c}\,b^2c^3x}{128\,d^4}+\frac{5\,\sqrt{dx^2+c}\,abc^2x}{8\,d^3}-\frac{3\,\sqrt{dx^2+c}\,a^2cx}{8\,d^2}+\frac{35\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{9}{2}}}-\frac{5\,abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{7}{2}}}+\frac{3\,a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*b^2*x^7/d - 7/48*sqrt(d*x^2 + c)*b^2*c*x^5/d^2 + 1/3*sq
rt(d*x^2 + c)*a*b*x^5/d + 35/192*sqrt(d*x^2 + c)*b^2*c^2*x^3/d^3 - 5/12*sqr
t(d*x^2 + c)*a*b*c*x^3/d^2 + 1/4*sqrt(d*x^2 + c)*a^2*x^3/d - 35/128*sqrt(d*
x^2 + c)*b^2*c^3*x/d^4 + 5/8*sqrt(d*x^2 + c)*a*b*c^2*x/d^3 - 3/8*sqrt(d*x^2
 + c)*a^2*c*x/d^2 + 35/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(9/2) - 5/8*a*b
*c^3*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/d^
(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

sympy [B] time = 28.18, size = 422, normalized size = 2.18

$$-\frac{3a^{2}c^{\frac{3}{2}}x}{8d^{2}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{a^{2}\sqrt{c}\,x^{3}}{8d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{3a^{2}c^{2}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}}+\frac{a^{2}x^{5}}{4\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5abc^{\frac{5}{2}}x}{8d^{3}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{5abc^{\frac{3}{2}}x^{3}}{24d^{2}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{ab\sqrt{c}\,x^{5}}{12d\sqrt{1+\frac{dx^{2}}{c}}}-\frac{5abc^{3}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{7}{2}}}+\frac{abx^{7}}{3\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{35b^{2}c^{\frac{7}{2}}x}{128d^{4}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{35b^{2}c^{\frac{5}{2}}x^{3}}{384d^{3}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{7b^{2}c^{\frac{3}{2}}x^{5}}{192d^{2}\sqrt{1+\frac{dx^{2}}{c}}}-\frac{b^{2}\sqrt{c}\,x^{7}}{48d\sqrt{1+\frac{dx^{2}}{c}}}+\frac{35b^{2}c^{4}\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{128d^{\frac{9}{2}}}+\frac{b^{2}x^{9}}{8\sqrt{c}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**(3/2)*x/(8*d**2*sqrt(1 + d*x**2/c)) - a**2*sqrt(c)*x**3/(8*d*sqr
t(1 + d*x**2/c)) + 3*a**2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + a**2
*x**5/(4*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*a*b*c**(5/2)*x/(8*d**3*sqrt(1 + d*
x**2/c)) + 5*a*b*c**(3/2)*x**3/(24*d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x
**5/(12*d*sqrt(1 + d*x**2/c)) - 5*a*b*c**3*asinh(sqrt(d)*x/sqrt(c))/(8*d**(
7/2)) + a*b*x**7/(3*sqrt(c)*sqrt(1 + d*x**2/c)) - 35*b**2*c**(7/2)*x/(128*d
**4*sqrt(1 + d*x**2/c)) - 35*b**2*c**(5/2)*x**3/(384*d**3*sqrt(1 + d*x**2/c
)) + 7*b**2*c**(3/2)*x**5/(192*d**2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**7
/(48*d*sqrt(1 + d*x**2/c)) + 35*b**2*c**4*asinh(sqrt(d)*x/sqrt(c))/(128*d**
(9/2)) + b**2*x**9/(8*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.619 $\int \frac{x^3\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=112

$$-\frac{b\left(c+dx^2\right)^{5/2}(3bc-2ad)}{5d^4}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}\,(bc-ad)^2}{d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{5/2}(3bc-2ad)}{5d^4}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)(3bc-ad)}{3d^4}-\frac{c\sqrt{c+dx^2}\,(bc-ad)^2}{d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] -((c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4) + ((b*c - a*d)*(3*b*c - a*d)*(c +
d*x^2)^(3/2))/(3*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(5/2))/(5*d^4) + (b^
2*(c + d*x^2)^(7/2))/(7*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(a+bx)^2}{\sqrt{c+dx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^3\sqrt{c+dx}} + \frac{(bc-ad)(3bc-ad)\sqrt{c+dx}}{d^3} - \frac{b(3bc-2ad)(c+dx)^{3/2}}{d^3} + \right.\right. \\
&= -\frac{c(bc-ad)^2\sqrt{c+dx^2}}{d^4} + \frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{3/2}}{3d^4} - \frac{b(3bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^4} +
\end{aligned}
$$

Mathematica **[A]** time = 0.10, size = 99, normalized size = 0.88

$$
\frac{\sqrt{c+dx^2}\left(35a^2d^2\left(dx^2-2c\right)+14abd\left(8c^2-4cdx^2+3d^2x^4\right)-3b^2\left(16c^3-8c^2dx^2+6cd^2x^4-5d^3x^6\right)\right)}{105d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(35*a^2*d^2*(-2*c + d*x^2) + 14*a*b*d*(8*c^2 - 4*c*d*x^2 +
 3*d^2*x^4) - 3*b^2*(16*c^3 - 8*c^2*d*x^2 + 6*c*d^2*x^4 - 5*d^3*x^6)))/(105
*d^4)
```

IntegrateAlgebraic **[A]** time = 0.06, size = 111, normalized size = 0.99

$$
\frac{\sqrt{c+dx^2}\left(-70a^2cd^2+35a^2d^3x^2+112abc^2d-56abcd^2x^2+42abd^3x^4-48b^2c^3+24b^2c^2dx^2-18b^2cd^2x^4+15b^2d^3x^6\right)}{105d^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(-48*b^2*c^3 + 112*a*b*c^2*d - 70*a^2*c*d^2 + 24*b^2*c^2*d
*x^2 - 56*a*b*c*d^2*x^2 + 35*a^2*d^3*x^2 - 18*b^2*c*d^2*x^4 + 42*a*b*d^3*x^
4 + 15*b^2*d^3*x^6))/(105*d^4)
```

fricas **[A]** time = 1.40, size = 103, normalized size = 0.92

$$
\frac{\left(15\,b^2d^3x^6-48\,b^2c^3+112\,abc^2d-70\,a^2cd^2-6\left(3\,b^2cd^2-7\,abd^3\right)x^4+\left(24\,b^2c^2d-56\,abcd^2+35\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{105\,d^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*d^3*x^6 - 48*b^2*c^3 + 112*a*b*c^2*d - 70*a^2*c*d^2 - 6*(3*b^
2*c*d^2 - 7*a*b*d^3)*x^4 + (24*b^2*c^2*d - 56*a*b*c*d^2 + 35*a^2*d^3)*x^2)*
sqrt(d*x^2 + c)/d^4
```

giac [A] time = 0.36, size = 137, normalized size = 1.22

$$-\frac{\left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)\sqrt{dx^2+c}}{d^4} + \frac{15\left(dx^2+c\right)^{\frac{7}{2}}b^2 - 63\left(dx^2+c\right)^{\frac{5}{2}}b^2c + 105\left(dx^2+c\right)^{\frac{3}{2}}b^2c^2 + 42\left(dx^2+c\right)^{\frac{5}{2}}abd - 140\left(dx^2+c\right)^{\frac{3}{2}}abcd + 35\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)/d^4 + 1/105*(15*(d*x^2
 + c)^(7/2)*b^2 - 63*(d*x^2 + c)^(5/2)*b^2*c + 105*(d*x^2 + c)^(3/2)*b^2*c^
2 + 42*(d*x^2 + c)^(5/2)*a*b*d - 140*(d*x^2 + c)^(3/2)*a*b*c*d + 35*(d*x^2
+ c)^(3/2)*a^2*d^2)/d^4
```

maple [A] time = 0.01, size = 108, normalized size = 0.96

$$-\frac{\sqrt{d\,x^2+c}\;\left(-15b^2x^6d^3 - 42ab\,d^3x^4 + 18b^2c\,d^2x^4 - 35a^2d^3x^2 + 56abc\,d^2x^2 - 24b^2c^2d\,x^2 + 70a^2c\,d^2 - 112ab\,c^2d + 48b^2c^3\right)}{105d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/105*(d*x^2+c)^(1/2)*(-15*b^2*d^3*x^6-42*a*b*d^3*x^4+18*b^2*c*d^2*x^4-35*
a^2*d^3*x^2+56*a*b*c*d^2*x^2-24*b^2*c^2*d*x^2+70*a^2*c*d^2-112*a*b*c^2*d+48
*b^2*c^3)/d^4
```

maxima [A] time = 0.91, size = 181, normalized size = 1.62

$$\frac{\sqrt{dx^2+c}\,b^2x^6}{7\,d} - \frac{6\,\sqrt{dx^2+c}\,b^2cx^4}{35\,d^2} + \frac{2\,\sqrt{dx^2+c}\,abx^4}{5\,d} + \frac{8\,\sqrt{dx^2+c}\,b^2c^2x^2}{35\,d^3} - \frac{8\,\sqrt{dx^2+c}\,abcx^2}{15\,d^2} + \frac{\sqrt{dx^2+c}\,a^2x^2}{3\,d} - \frac{16\,\sqrt{dx^2+c}\,b^2c^3}{35\,d^4} + \frac{16\,\sqrt{dx^2+c}\,abc^2}{15\,d^3} - \frac{2\,\sqrt{dx^2+c}\,a^2c}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(d*x^2 + c)*b^2*x^6/d - 6/35*sqrt(d*x^2 + c)*b^2*c*x^4/d^2 + 2/5*sq
rt(d*x^2 + c)*a*b*x^4/d + 8/35*sqrt(d*x^2 + c)*b^2*c^2*x^2/d^3 - 8/15*sqrt(
d*x^2 + c)*a*b*c*x^2/d^2 + 1/3*sqrt(d*x^2 + c)*a^2*x^2/d - 16/35*sqrt(d*x^2
 + c)*b^2*c^3/d^4 + 16/15*sqrt(d*x^2 + c)*a*b*c^2/d^3 - 2/3*sqrt(d*x^2 + c)
*a^2*c/d^2
```

mupad [B] time = 0.66, size = 105, normalized size = 0.94

$$\sqrt{d\,x^2+c}\,\left(\frac{b^2\,x^6}{7\,d} - \frac{70\,a^2\,c\,d^2 - 112\,a\,b\,c^2\,d + 48\,b^2\,c^3}{105\,d^4} + \frac{x^2\,\left(35\,a^2\,d^3 - 56\,a\,b\,c\,d^2 + 24\,b^2\,c^2\,d\right)}{105\,d^4} + \frac{2\,b\,x^4\,\left(7\,a\,d - 3\,b\,c\right)}{35\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((b^2*x^6)/(7*d) - (48*b^2*c^3 + 70*a^2*c*d^2 - 112*a*b*c
^2*d)/(105*d^4) + (x^2*(35*a^2*d^3 + 24*b^2*c^2*d - 56*a*b*c*d^2))/(105*d^4
) + (2*b*x^4*(7*a*d - 3*b*c))/(35*d^2))
```

sympy [A] time = 1.81, size = 240, normalized size = 2.14

$$\begin{cases} -\frac{2a^2c\sqrt{c+dx^2}}{3d^2} + \frac{a^2x^2\sqrt{c+dx^2}}{3d} + \frac{16abc^2\sqrt{c+dx^2}}{15d^3} - \frac{8abcx^2\sqrt{c+dx^2}}{15d^2} + \frac{2abx^4\sqrt{c+dx^2}}{5d} - \frac{16b^2c^3\sqrt{c+dx^2}}{35d^4} + \frac{8b^2c^2x^2\sqrt{c+dx^2}}{35d^3} - \frac{6b^2cx^4\sqrt{c+dx^2}}{35d^2} + \frac{b^2x^6\sqrt{c+dx^2}}{7d} & \text{for}\ d \neq 0 \\ \frac{\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((-2*a**2*c*sqrt(c + d*x**2)/(3*d**2) + a**2*x**2*sqrt(c + d*x**2)
/(3*d) + 16*a*b*c**2*sqrt(c + d*x**2)/(15*d**3) - 8*a*b*c*x**2*sqrt(c + d*x
**2)/(15*d**2) + 2*a*b*x**4*sqrt(c + d*x**2)/(5*d) - 16*b**2*c**3*sqrt(c +
d*x**2)/(35*d**4) + 8*b**2*c**2*x**2*sqrt(c + d*x**2)/(35*d**3) - 6*b**2*c*
x**4*sqrt(c + d*x**2)/(35*d**2) + b**2*x**6*sqrt(c + d*x**2)/(7*d), Ne(d, 0
)), ((a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/sqrt(c), True))
```

3.620 $\int \frac{x^2\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{c\left(8a^2d^2+bc(5bc-12ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}+\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2+bc(5bc-12ad)\right)}{16d^3}-\frac{bx^3\sqrt{c+dx^2}\,(5bc-12ad)}{24d^2}+$$

Rubi [A] time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {464, 459, 321, 217, 206}

$$\frac{x\sqrt{c+dx^2}\,\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)}{16d}-\frac{c\left(8a^2d^2+bc(5bc-12ad)\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}-\frac{bx^3\sqrt{c+dx^2}\,(5bc-12ad)}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*a^2 + (b*c*(5*b*c - 12*a*d))/d^2)*x*Sqrt[c + d*x^2])/(16*d) - (b*(5*b*c
 - 12*a*d)*x^3*Sqrt[c + d*x^2])/(24*d^2) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d)
- (c*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/(16*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx &= \frac{b^2x^5\sqrt{c+dx^2}}{6d}+\frac{\int \frac{x^2\left(6a^2d-b(5bc-12ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{6d}\\
&= -\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}+\frac{1}{8}\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)\int \frac{x^2}{\sqrt{c+dx^2}}\,d\\
&= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d}-\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}-\frac{(c(5b^2c^2}{}\\
&= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d}-\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}-\frac{(c(5b^2c^2}{}\\
&= \frac{\left(8a^2+\frac{bc(5bc-12ad)}{d^2}\right)x\sqrt{c+dx^2}}{16d}-\frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}-\frac{c(5b^2c^2-}{}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 125, normalized size = 0.86

$$\frac{\sqrt{d}\,x\sqrt{c+dx^2}\left(24a^2d^2+12abd\left(2dx^2-3c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)-3c\left(8a^2d^2-12abcd+5b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{48d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(-3*c + 2*d*x^2) + b^2*(1
5*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - 3*c*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)
*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*d^(7/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 132, normalized size = 0.90

$$\frac{\sqrt{c+dx^2}\left(24a^2d^2x-36abcdx+24abd^2x^3+15b^2c^2x-10b^2cdx^3+8b^2d^2x^5\right)}{48d^3}+\frac{\left(8a^2cd^2-12abc^2d+5b^2c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c^2*x - 36*a*b*c*d*x + 24*a^2*d^2*x - 10*b^2*c*d*x
^3 + 24*a*b*d^2*x^3 + 8*b^2*d^2*x^5))/(48*d^3) + ((5*b^2*c^3 - 12*a*b*c^2*d
 + 8*a^2*c*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(16*d^(7/2))
```

fricas [A] time = 0.92, size = 267, normalized size = 1.83

$$\left[\frac{3\left(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2\right)\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(8\,b^2d^3x^5-2\left(5\,b^2cd^2-12\,abd^3\right)x^3+3\left(5\,b^2c^2d-12\,abcd^2+8\,a^2d^3\right)x\right)\sqrt{dx^2+c}}{96\,d^4},\ \frac{3\left(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(8\,b^2d^3x^5-2\left(5\,b^2cd^2-12\,abd^3\right)x^3+3\left(5\,b^2c^2d-12\,abcd^2+8\,a^2d^3\right)x\right)\sqrt{dx^2+c}}{48\,d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b
*d^3)*x^3 + 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/
d^4, 1/48*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b*d^3)*x^3
+ 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

giac [A] time = 0.37, size = 135, normalized size = 0.92

$$\frac{1}{48}\left(2\left(\frac{4\,b^2x^2}{d}-\frac{5\,b^2cd^3-12\,abd^4}{d^5}\right)x^2+\frac{3\left(5\,b^2c^2d^2-12\,abcd^3+8\,a^2d^4\right)}{d^5}\right)\sqrt{dx^2+c}\,x+\frac{\left(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b^2*x^2/d - (5*b^2*c*d^3 - 12*a*b*d^4)/d^5)*x^2 + 3*(5*b^2*c^2*d
^2 - 12*a*b*c*d^3 + 8*a^2*d^4)/d^5)*sqrt(d*x^2 + c)*x + 1/16*(5*b^2*c^3 - 1
2*a*b*c^2*d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

maple [A] time = 0.01, size = 197, normalized size = 1.35

$$\frac{\sqrt{d\,x^2+c}\;b^2x^5}{6d}+\frac{\sqrt{d\,x^2+c}\;ab\,x^3}{2d}-\frac{5\sqrt{d\,x^2+c}\;b^2c\,x^3}{24d^2}-\frac{a^2c\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)}{2d^{\frac{3}{2}}}+\frac{3ab\,c^2\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)}{4d^{\frac{5}{2}}}-\frac{5b^2c^3\ln\left(\sqrt{d}\;x+\sqrt{d\,x^2+c}\right)}{16d^{\frac{7}{2}}}+\frac{\sqrt{d\,x^2+c}\;a^2x}{2d}-\frac{3\sqrt{d\,x^2+c}\;abcx}{4d^2}+\frac{5\sqrt{d\,x^2+c}\;b^2c^2x}{16d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*b^2*x^5*(d*x^2+c)^(1/2)/d-5/24*b^2*c/d^2*x^3*(d*x^2+c)^(1/2)+5/16*b^2*c
^2/d^3*x*(d*x^2+c)^(1/2)-5/16*b^2*c^3/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
+1/2*a*b*x^3/d*(d*x^2+c)^(1/2)-3/4*a*b*c/d^2*x*(d*x^2+c)^(1/2)+3/4*a*b*c^2/
d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/2*a^2*x/d*(d*x^2+c)^(1/2)-1/2*a^2*c
/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 0.92, size = 175, normalized size = 1.20

$$\frac{\sqrt{dx^2+c}\,b^2x^5}{6\,d}-\frac{5\,\sqrt{dx^2+c}\,b^2cx^3}{24\,d^2}+\frac{\sqrt{dx^2+c}\,abx^3}{2\,d}+\frac{5\,\sqrt{dx^2+c}\,b^2c^2x}{16\,d^3}-\frac{3\,\sqrt{dx^2+c}\,abcx}{4\,d^2}+\frac{\sqrt{dx^2+c}\,a^2x}{2\,d}-\frac{5\,b^2c^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{7}{2}}}+\frac{3\,abc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{5}{2}}}-\frac{a^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(d*x^2 + c)*b^2*x^5/d - 5/24*sqrt(d*x^2 + c)*b^2*c*x^3/d^2 + 1/2*sq
rt(d*x^2 + c)*a*b*x^3/d + 5/16*sqrt(d*x^2 + c)*b^2*c^2*x/d^3 - 3/4*sqrt(d*x
^2 + c)*a*b*c*x/d^2 + 1/2*sqrt(d*x^2 + c)*a^2*x/d - 5/16*b^2*c^3*arcsinh(d*
x/sqrt(c*d))/d^(7/2) + 3/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/2*a^2
*c*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

sympy [B] time = 16.35, size = 301, normalized size = 2.06

$$\frac{a^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{a^2c\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}-\frac{3abc^{\frac{3}{2}}x}{4d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{ab\sqrt{c}\,x^3}{4d\sqrt{1+\frac{dx^2}{c}}}+\frac{3abc^2\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{4d^{\frac{5}{2}}}+\frac{abx^5}{2\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{5}{2}}x}{16d^3\sqrt{1+\frac{dx^2}{c}}}+\frac{5b^2c^{\frac{3}{2}}x^3}{48d^2\sqrt{1+\frac{dx^2}{c}}}-\frac{b^2\sqrt{c}\,x^5}{24d\sqrt{1+\frac{dx^2}{c}}}-\frac{5b^2c^3\,\mathrm{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{16d^{\frac{7}{2}}}+\frac{b^2x^7}{6\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - a**2*c*asinh(sqrt(d)*x/sqrt(c))/(
2*d**(3/2)) - 3*a*b*c**(3/2)*x/(4*d**2*sqrt(1 + d*x**2/c)) - a*b*sqrt(c)*x*
```

```
*3/(4*d*sqrt(1 + d*x**2/c)) + 3*a*b*c**2*asinh(sqrt(d)*x/sqrt(c))/(4*d**(5/
2)) + a*b*x**5/(2*sqrt(c)*sqrt(1 + d*x**2/c)) + 5*b**2*c**(5/2)*x/(16*d**3*
sqrt(1 + d*x**2/c)) + 5*b**2*c**(3/2)*x**3/(48*d**2*sqrt(1 + d*x**2/c)) - b
**2*sqrt(c)*x**5/(24*d*sqrt(1 + d*x**2/c)) - 5*b**2*c**3*asinh(sqrt(d)*x/sq
rt(c))/(16*d**(7/2)) + b**2*x**7/(6*sqrt(c)*sqrt(1 + d*x**2/c))
```

3.621 $\int \frac{x\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}\,(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

Rubi **[A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^3}+\frac{\sqrt{c+dx^2}\,(bc-ad)^2}{d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (2*b*(b*c - a*d)*(c + d*x^2)^(3/2))/(
3*d^3) + (b^2*(c + d*x^2)^(5/2))/(5*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx,x,x^2\right)\\
&= \frac{(bc-ad)^2\sqrt{c+dx^2}}{d^3}-\frac{2b(bc-ad)\left(c+dx^2\right)^{3/2}}{3d^3}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.05, size = 66, normalized size = 0.89

$$
\frac{\sqrt{c+dx^2}\left(15a^2d^2+10abd\left(dx^2-2c\right)+b^2\left(8c^2-4cdx^2+3d^2x^4\right)\right)}{15d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(8*c^2 - 4*c*d
*x^2 + 3*d^2*x^4)))/(15*d^3)
```

IntegrateAlgebraic [**A**] time = 0.04, size = 72, normalized size = 0.97

$$
\frac{\sqrt{c+dx^2}\left(15a^2d^2-20abcd+10abd^2x^2+8b^2c^2-4b^2cdx^2+3b^2d^2x^4\right)}{15d^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 4*b^2*c*d*x^2 + 10*
a*b*d^2*x^2 + 3*b^2*d^2*x^4))/(15*d^3)
```

fricas [**A**] time = 1.48, size = 68, normalized size = 0.92

$$
\frac{\left(3\,b^2d^2x^4+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\left(2\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x^2)*sqrt(d*x^2 + c)/d^3
```

giac [A] time = 0.33, size = 84, normalized size = 1.14

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{dx^2+c}}{d^3}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^2-10\left(dx^2+c\right)^{\frac{3}{2}}b^2c+10\left(dx^2+c\right)^{\frac{3}{2}}abd}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)/d^3 + 1/15*(3*(d*x^2 + c)^(
5/2)*b^2 - 10*(d*x^2 + c)^(3/2)*b^2*c + 10*(d*x^2 + c)^(3/2)*a*b*d)/d^3
```

maple [A] time = 0.01, size = 69, normalized size = 0.93

$$\frac{\sqrt{d\,x^2+c}\,\left(3b^2x^4d^2+10ab\,d^2x^2-4b^2cd\,x^2+15a^2d^2-20abcd+8b^2c^2\right)}{15d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(3*b^2*d^2*x^4+10*a*b*d^2*x^2-4*b^2*c*d*x^2+15*a^2*d^2
-20*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 0.89, size = 114, normalized size = 1.54

$$\frac{\sqrt{dx^2+c}\,b^2x^4}{5\,d}-\frac{4\,\sqrt{dx^2+c}\,b^2cx^2}{15\,d^2}+\frac{2\,\sqrt{dx^2+c}\,abx^2}{3\,d}+\frac{8\,\sqrt{dx^2+c}\,b^2c^2}{15\,d^3}-\frac{4\,\sqrt{dx^2+c}\,abc}{3\,d^2}+\frac{\sqrt{dx^2+c}\,a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d*x^2 + c)*b^2*x^4/d - 4/15*sqrt(d*x^2 + c)*b^2*c*x^2/d^2 + 2/3*sq
rt(d*x^2 + c)*a*b*x^2/d + 8/15*sqrt(d*x^2 + c)*b^2*c^2/d^3 - 4/3*sqrt(d*x^2
 + c)*a*b*c/d^2 + sqrt(d*x^2 + c)*a^2/d
```

mupad [B] time = 0.65, size = 68, normalized size = 0.92

$$\sqrt{d\,x^2+c}\,\left(\frac{15\,a^2\,d^2-20\,a\,b\,c\,d+8\,b^2\,c^2}{15\,d^3}+\frac{b^2\,x^4}{5\,d}+\frac{2\,b\,x^2\,\left(5\,a\,d-2\,b\,c\right)}{15\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

```
[Out] (c + d*x^2)^(1/2)*((15*a^2*d^2 + 8*b^2*c^2 - 20*a*b*c*d)/(15*d^3) + (b^2*x^
4)/(5*d) + (2*b*x^2*(5*a*d - 2*b*c))/(15*d^2))
```

sympy [**A**] time = 1.88, size = 158, normalized size = 2.14

$$\begin{cases} \frac{a^2\sqrt{c+dx^2}}{d} - \frac{4abc\sqrt{c+dx^2}}{3d^2} + \frac{2abx^2\sqrt{c+dx^2}}{3d} + \frac{8b^2c^2\sqrt{c+dx^2}}{15d^3} - \frac{4b^2cx^2\sqrt{c+dx^2}}{15d^2} + \frac{b^2x^4\sqrt{c+dx^2}}{5d} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Piecewise((a**2*sqrt(c + d*x**2)/d - 4*a*b*c*sqrt(c + d*x**2)/(3*d**2) + 2*
a*b*x**2*sqrt(c + d*x**2)/(3*d) + 8*b**2*c**2*sqrt(c + d*x**2)/(15*d**3) -
4*b**2*c*x**2*sqrt(c + d*x**2)/(15*d**2) + b**2*x**4*sqrt(c + d*x**2)/(5*d)
, Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/sqrt(c), True))
```

3.622 $\int \frac{\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=107

$$\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}\,(bc-2ad)}{8d^2}+\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}$$

Rubi [**A**] time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 217, 206}

$$\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{3bx\sqrt{c+dx^2}\,(bc-2ad)}{8d^2}+\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*b*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(8*d^2) + (b*x*(a + b*x^2)*Sqrt[c +
d*x^2])/(4*d) + ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(8*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} + \frac{\int \frac{-a(bc-4ad)-3b(bc-2ad)x^2}{\sqrt{c+dx^2}}\,dx}{4d}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} - \frac{(2ad(bc-4ad)-3bc(bc-2ad))\int \frac{1}{\sqrt{c+dx}}}{8d^2}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} - \frac{(2ad(bc-4ad)-3bc(bc-2ad))\,\mathrm{Subst}\Big(}{8d^2}$$

$$= -\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2} + \frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d} + \frac{\left(3b^2c^2-8abcd+8a^2d^2\right)\tanh^{-1}\Big(\frac{\sqrt{d}\,x}{\sqrt{c+dx}}}{8d^{5/2}}$$

Mathematica [A] time = 0.06, size = 91, normalized size = 0.85

$$\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+b\sqrt{d}\,x\sqrt{c+dx^2}\left(8ad-3bc+2bdx^2\right)}{8d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(-3*b*c + 8*a*d + 2*b*d*x^2) + (3*b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(5/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 95, normalized size = 0.89

$$\frac{\left(-8a^2d^2+8abcd-3b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{5/2}}+\frac{\sqrt{c+dx^2}\left(8abdx-3b^2cx+2b^2dx^3\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*b^2*c*x + 8*a*b*d*x + 2*b^2*d*x^3))/(8*d^2) + ((-3*b^2
*c^2 + 8*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(5/
2))
```

fricas [A] time = 1.43, size = 194, normalized size = 1.81

$$\left[\frac{\left(3\,b^2c^2-8\,abcd+8\,a^2d^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(2\,b^2d^2x^3-\left(3\,b^2cd-8\,abd^2\right)x\right)\sqrt{dx^2+c}}{16\,d^3},-\frac{\left(3\,b^2c^2-8\,abcd+8\,a^2d^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(2\,b^2d^2x^3-\left(3\,b^2cd-8\,abd^2\right)x\right)\sqrt{dx^2+c}}{8\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*
x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)*sqr
t(d*x^2 + c))/d^3, -1/8*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)
*sqrt(d*x^2 + c))/d^3]
```

giac [A] time = 0.49, size = 91, normalized size = 0.85

$$\frac{1}{8}\left(\frac{2\,b^2x^2}{d}-\frac{3\,b^2cd-8\,abd^2}{d^3}\right)\sqrt{dx^2+c}\,x-\frac{\left(3\,b^2c^2-8\,abcd+8\,a^2d^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^2*x^2/d - (3*b^2*c*d - 8*a*b*d^2)/d^3)*sqrt(d*x^2 + c)*x - 1/8*(3*
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(
5/2)
```

maple [A] time = 0.01, size = 131, normalized size = 1.22

$$\frac{\sqrt{d\,x^2+c}\,b^2x^3}{4d}+\frac{a^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}-\frac{abc\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{d^{\frac{3}{2}}}+\frac{3b^2c^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8d^{\frac{5}{2}}}+\frac{\sqrt{d\,x^2+c}\,abx}{d}-\frac{3\sqrt{d\,x^2+c}\,b^2cx}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*b^2*x^3/d*(d*x^2+c)^(1/2)-3/8*b^2*c/d^2*x*(d*x^2+c)^(1/2)+3/8*b^2*c^2/d
^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+a*b*x/d*(d*x^2+c)^(1/2)-a*b*c/d^(3/2)*
ln(d^(1/2)*x+(d*x^2+c)^(1/2))+a^2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)
```

maxima [A] time = 0.89, size = 109, normalized size = 1.02

$$\frac{\sqrt{dx^2+c}\,b^2x^3}{4\,d} - \frac{3\,\sqrt{dx^2+c}\,b^2cx}{8\,d^2} + \frac{\sqrt{dx^2+c}\,abx}{d} + \frac{3\,b^2c^2\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}} - \frac{abc\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}} + \frac{a^2\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d*x^2 + c)*b^2*x^3/d - 3/8*sqrt(d*x^2 + c)*b^2*c*x/d^2 + sqrt(d*x^
2 + c)*a*b*x/d + 3/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(5/2) - a*b*c*arcsinh
(d*x/sqrt(c*d))/d^(3/2) + a^2*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2)^2/(c + d*x^2)^(1/2), x)
```

sympy [A] time = 14.27, size = 238, normalized size = 2.22

$$a^2\left(\begin{cases} \frac{\sqrt{-\frac{c}{d}}\,\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d<0 \\ \frac{\sqrt{\frac{c}{d}}\,\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d>0 \\ \frac{\sqrt{-\frac{c}{d}}\,\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\ d>0\wedge c<0 \end{cases}\right) + \frac{ab\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{d} - \frac{abc\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{d^{\frac{3}{2}}} - \frac{3b^2c^{\frac{3}{2}}x}{8d^2\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2\sqrt{c}\,x^3}{8d\sqrt{1+\frac{dx^2}{c}}} + \frac{3b^2c^2\,\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{8d^{\frac{5}{2}}} + \frac{b^2x^5}{4\sqrt{c}\,\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*Piecewise((sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)),
(sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acos
h(x*sqrt(-d/c))/sqrt(-c), (d > 0) & (c < 0))) + a*b*sqrt(c)*x*sqrt(1 + d*x*
*2/c)/d - a*b*c*asinh(sqrt(d)*x/sqrt(c))/d**(3/2) - 3*b**2*c**(3/2)*x/(8*d*
*2*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*x**3/(8*d*sqrt(1 + d*x**2/c)) + 3*b**
2*c**2*asinh(sqrt(d)*x/sqrt(c))/(8*d**(5/2)) + b**2*x**5/(4*sqrt(c)*sqrt(1
+ d*x**2/c))
```

3.623 $\int \frac{\left(a+bx^2\right)^2}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}\,(bc-2ad)}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}$$

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+dx^2}\,(bc-2ad)}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2) + (b^2*(c + d*x^2)^(3/2))/(3*d^2)
 - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{b(bc-2ad)}{d\sqrt{c+dx}}+\frac{a^2}{x\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d}\right)dx, x, x^2\right)\\ &= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{1}{2}a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d}\\ &= -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.84

$$\frac{b\sqrt{c+dx^2}\left(6ad-2bc+bdx^2\right)}{3d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[c + d*x^2]*(-2*b*c + 6*a*d + b*d*x^2))/(3*d^2) - (a^2*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

IntegrateAlgebraic [A] time = 0.07, size = 67, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(6abd-2b^2c+b^2dx^2\right)}{3d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b^2*c + 6*a*b*d + b^2*d*x^2))/(3*d^2) - (a^2*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

fricas [A] time = 1.66, size = 157, normalized size = 2.09

$$\left[\frac{3\,a^{2}\sqrt{c}\,d^{2}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\,\left(b^{2}cdx^{2}-2\,b^{2}c^{2}+6\,abcd\right)\sqrt{dx^{2}+c}}{6\,cd^{2}},\ \frac{3\,a^{2}\sqrt{-c}\,d^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(b^{2}cdx^{2}-2\,b^{2}c^{2}+6\,abcd\right)\sqrt{dx^{2}+c}}{3\,cd^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2)
 + 2*(b^2*c*d*x^2 - 2*b^2*c^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2), 1/3*(3
*a^2*sqrt(-c)*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c*d*x^2 - 2*b^2*c
^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2)]
```

giac [A] time = 0.29, size = 82, normalized size = 1.09

$$\frac{a^{2}\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d^{4}-3\,\sqrt{dx^{2}+c}\,b^{2}cd^{4}+6\,\sqrt{dx^{2}+c}\,abd^{5}}{3\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/3*((d*x^2 + c)^(3/2)*b^2*
d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 + 6*sqrt(d*x^2 + c)*a*b*d^5)/d^6
```

maple [A] time = 0.01, size = 87, normalized size = 1.16

$$\frac{\sqrt{d\,x^{2}+c}\;b^{2}x^{2}}{3d}-\frac{a^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\;\sqrt{c}}{x}\right)}{\sqrt{c}}+\frac{2\sqrt{d\,x^{2}+c}\;ab}{d}-\frac{2\sqrt{d\,x^{2}+c}\;b^{2}c}{3d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*b^2*x^2/d*(d*x^2+c)^(1/2)-2/3*b^2*c/d^2*(d*x^2+c)^(1/2)+2*a*b/d*(d*x^2+
c)^(1/2)-a^2/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.87, size = 75, normalized size = 1.00

$$\frac{\sqrt{dx^2+c}\,b^2x^2}{3\,d}-\frac{a^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{\sqrt{c}}-\frac{2\,\sqrt{dx^2+c}\,b^2c}{3\,d^2}+\frac{2\,\sqrt{dx^2+c}\,ab}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d*x^2 + c)*b^2*x^2/d - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) -
 2/3*sqrt(d*x^2 + c)*b^2*c/d^2 + 2*sqrt(d*x^2 + c)*a*b/d
```

mupad [B] time = 0.72, size = 77, normalized size = 1.03

$$\frac{b^2\left(d\,x^2+c\right)^{3/2}}{3\,d^2}-\frac{a^2\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\sqrt{d\,x^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] (b^2*(c + d*x^2)^(3/2))/(3*d^2) - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^
(1/2) - ((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2)*(c + d*x^2)^(1/2)
```

sympy [A] time = 53.56, size = 76, normalized size = 1.01

$$\frac{a^2\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\,\sqrt{c+dx^2}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b^2\left(c+dx^2\right)^{\frac{3}{2}}}{3d^2}+\frac{b\sqrt{c+dx^2}\,(2ad-bc)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(1/2),x)
```

```
[Out] a**2*atan(1/(sqrt(-1/c)*sqrt(c + d*x**2)))/(c*sqrt(-1/c)) + b**2*(c + d*x**
2)**(3/2)/(3*d**2) + b*sqrt(c + d*x**2)*(2*a*d - b*c)/d**2
```

3.624 $\int \frac{\left(a+bx^2\right)^2}{x^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

Rubi [**A**] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 388, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{cx}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(c*x)) + (b^2*x*Sqrt[c + d*x^2])/(2*d) - (b*(b*c -
4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
```

```
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^2\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{\int \frac{2abc+b^2cx^2}{\sqrt{c+dx^2}}\,dx}{c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{(b(bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{(b(bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{b(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 76, normalized size = 0.93

$$\sqrt{c+dx^2}\left(\frac{b^2x}{2d}-\frac{a^2}{cx}\right)-\frac{b(bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(a^2/(c*x)) + (b^2*x)/(2*d))*Sqrt[c + d*x^2] - (b*(b*c - 4*a*d)*Log[d*x +
 Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(3/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 81, normalized size = 0.99

$$\frac{\sqrt{c+dx^2}\left(b^2cx^2-2a^2d\right)}{2cdx}+\frac{\left(b^2c-4abd\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-2*a^2*d + b^2*c*x^2)*Sqrt[c + d*x^2])/(2*c*d*x) + ((b^2*c - 4*a*b*d)*Log
[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(3/2))
```

fricas [A] time = 0.97, size = 165, normalized size = 2.01

$$\left[-\frac{\left(b^{2} c^{2}-4 \, abcd\right) \sqrt{d} \, x \log \left(-2 \, dx^{2}-2 \, \sqrt{dx^{2}+c} \, \sqrt{d} \, x-c\right)-2 \, \left(b^{2} cdx^{2}-2 \, a^{2} d^{2}\right) \sqrt{dx^{2}+c}}{4 \, cd^{2} x}, \frac{\left(b^{2} c^{2}-4 \, abcd\right) \sqrt{-d} \, x \arctan \left(\frac{\sqrt{-d} \, x}{\sqrt{dx^{2}+c}}\right)+\left(b^{2} cdx^{2}-2 \, a^{2} d^{2}\right) \sqrt{dx^{2}+c}}{2 \, cd^{2} x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2 - 4*a*b*c*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c) - 2*(b^2*c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x), 1/2*(
(b^2*c^2 - 4*a*b*c*d)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*
c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x)]
```

giac [A] time = 0.46, size = 93, normalized size = 1.13

$$\frac{\sqrt{dx^{2}+c} \, b^{2} x}{2 \, d}+\frac{2 \, a^{2} \sqrt{d}}{\left(\sqrt{d} \, x-\sqrt{dx^{2}+c}\right)^{2}-c}+\frac{\left(b^{2} c \sqrt{d}-4 \, abd^{\frac{3}{2}}\right) \log \left(\left(\sqrt{d} \, x-\sqrt{dx^{2}+c}\right)^{2}\right)}{4 \, d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x/d + 2*a^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^
2 - c) + 1/4*(b^2*c*sqrt(d) - 4*a*b*d^(3/2))*log((sqrt(d)*x - sqrt(d*x^2 +
c))^2)/d^2
```

maple [A] time = 0.01, size = 88, normalized size = 1.07

$$\frac{2ab\ln\left(\sqrt{d}\, x+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}-\frac{b^{2}c\ln\left(\sqrt{d}\, x+\sqrt{d\,x^{2}+c}\right)}{2d^{\frac{3}{2}}}+\frac{\sqrt{d\,x^{2}+c}\; b^{2}x}{2d}-\frac{\sqrt{d\,x^{2}+c}\; a^{2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*b^2*x*(d*x^2+c)^(1/2)/d-1/2*b^2*c/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
+2*a*b*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)-a^2*(d*x^2+c)^(1/2)/c/x
```

maxima [A] time = 0.88, size = 73, normalized size = 0.89

$$\frac{\sqrt{dx^{2}+c} \, b^{2} x}{2 \, d}-\frac{b^{2} c \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2 \, d^{\frac{3}{2}}}+\frac{2 \, ab \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}-\frac{\sqrt{dx^{2}+c} \, a^{2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^2*x/d - 1/2*b^2*c*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 2*
a*b*arcsinh(d*x/sqrt(c*d))/sqrt(d) - sqrt(d*x^2 + c)*a^2/(c*x)
```

mupad [B] time = 1.46, size = 125, normalized size = 1.52

$$\begin{cases} \frac{-a^2+2\,a\,b\,x^2+\frac{b^2\,x^4}{3}}{\sqrt{c}\,x} & \text{if } d=0\\ \frac{2\,a\,b\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}+\frac{b^2\,x\,\sqrt{d\,x^2+c}}{2\,d}-\frac{a^2\,\sqrt{d\,x^2+c}}{c\,x}-\frac{b^2\,c\,\ln\left(2\,\sqrt{d}\,x+2\,\sqrt{d\,x^2+c}\right)}{2\,d^{3/2}} & \text{if } d\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(d == 0, (- a^2 + (b^2*x^4)/3 + 2*a*b*x^2)/(c^(1/2)*x), d ~= 0, (2
*a*b*log(d^(1/2)*x + (c + d*x^2)^(1/2)))/d^(1/2) + (b^2*x*(c + d*x^2)^(1/2)
)/(2*d) - (a^2*(c + d*x^2)^(1/2))/(c*x) - (b^2*c*log(2*d^(1/2)*x + 2*(c + d
*x^2)^(1/2)))/(2*d^(3/2)))
```

sympy [A] time = 8.04, size = 155, normalized size = 1.89

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+2ab\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\operatorname{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for } c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\operatorname{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for } c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\operatorname{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for } d>0\wedge c<0\end{cases}\right)+\frac{b^2\sqrt{c}\,x\sqrt{1+\frac{dx^2}{c}}}{2d}-\frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + 2*a*b*Piecewise((sqrt(-c/d)*asin(x*s
qrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asinh(x*sqrt(d/c))/sqrt(
c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))/sqrt(-c), (d > 0) &
 (c < 0))) + b**2*sqrt(c)*x*sqrt(1 + d*x**2/c)/(2*d) - b**2*c*asinh(sqrt(d)
*x/sqrt(c))/(2*d**(3/2))
```

3.625 $\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b^2*Sqrt[c + d*x^2])/d - (a^2*Sqrt[c + d*x^2])/(2*c*x^2) - (a*(4*b*c - a*d
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^3\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-ad)+b^2cx}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{(a(4bc-ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{4c}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}+\frac{(a(4bc-ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2cd}\\
&= \frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 77, normalized size = 0.96

$$
\frac{\frac{\sqrt{c}\,\sqrt{c+dx^2}\left(2b^2cx^2-a^2d\right)}{dx^2}+a(ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[c]*(-(a^2*d) + 2*b^2*c*x^2)*Sqrt[c + d*x^2])/(d*x^2) + a*(-4*b*c + a
*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 79, normalized size = 0.99

$$\frac{\sqrt{c+dx^2}\left(2b^2cx^2-a^2d\right)}{2cdx^2}+\frac{\left(a^2d-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-(a^2*d) + 2*b^2*c*x^2)*Sqrt[c + d*x^2])/(2*c*d*x^2) + ((-4*a*b*c + a^2*d
)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

fricas [A] time = 1.54, size = 175, normalized size = 2.19

$$\left[-\frac{\left(4\,abcd-a^2d^2\right)\sqrt{c}\,x^2\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\left(2\,b^2c^2x^2-a^2cd\right)\sqrt{dx^2+c}}{4\,c^2dx^2},\ \frac{\left(4\,abcd-a^2d^2\right)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(2\,b^2c^2x^2-a^2cd\right)\sqrt{dx^2+c}}{2\,c^2dx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a*b*c*d - a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) - 2*(2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2
), 1/2*((4*a*b*c*d - a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + (2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2)]
```

giac [A] time = 0.49, size = 81, normalized size = 1.01

$$\frac{2\,\sqrt{dx^2+c}\,b^2-\frac{\sqrt{dx^2+c}\,a^2d}{cx^2}+\frac{\left(4\,abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b^2 - sqrt(d*x^2 + c)*a^2*d/(c*x^2) + (4*a*b*c*d - a
^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c))/d
```

maple [A] time = 0.02, size = 100, normalized size = 1.25

$$\frac{a^{2} d \ln \left(\frac{2 c+2 \sqrt{d \,x^{2}+c}\, \sqrt{c}}{x}\right)}{2 c^{\frac{3}{2}}}-\frac{2 a b \ln \left(\frac{2 c+2 \sqrt{d \,x^{2}+c}\, \sqrt{c}}{x}\right)}{\sqrt{c}}+\frac{\sqrt{d \,x^{2}+c}\, b^{2}}{d}-\frac{\sqrt{d \,x^{2}+c}\, a^{2}}{2 c \,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] b^2*(d*x^2+c)^(1/2)/d-1/2*a^2*(d*x^2+c)^(1/2)/c/x^2+1/2*a^2*d/c^(3/2)*ln((2
*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-2*a*b/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^
(1/2))/x)
```

maxima [A] time = 0.91, size = 77, normalized size = 0.96

$$-\frac{2 \, a b \operatorname{arsinh}\left(\frac{c}{\sqrt{c d} {\left| x \right|}}\right)}{\sqrt{c}} + \frac{a^{2} d \operatorname{arsinh}\left(\frac{c}{\sqrt{c d} {\left| x \right|}}\right)}{2 \, c^{\frac{3}{2}}} + \frac{\sqrt{d x^{2} + c} b^{2}}{d} - \frac{\sqrt{d x^{2} + c} a^{2}}{2 \, c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/c^(3/2) + sqrt(d*x^2 + c)*b^2/d - 1/2*sqrt(d*x^2 + c)*a^2/(c*x^
2)
```

mupad [B] time = 0.94, size = 65, normalized size = 0.81

$$\frac{b^2\,\sqrt{d\,x^2+c}}{d}+\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(a\,d-4\,b\,c\right)}{2\,c^{3/2}}-\frac{a^2\,\sqrt{d\,x^2+c}}{2\,c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] (b^2*(c + d*x^2)^(1/2))/d + (a*atanh((c + d*x^2)^(1/2)/c^(1/2))*(a*d - 4*b*
c))/(2*c^(3/2)) - (a^2*(c + d*x^2)^(1/2))/(2*c*x^2)
```

sympy [A] time = 130.45, size = 99, normalized size = 1.24

$$- \frac{a^{2} \sqrt{d} \sqrt{\frac{c}{d x^{2}} + 1}}{2 c x} + \frac{a^{2} d \operatorname{asinh}{\left(\frac{\sqrt{c}}{\sqrt{d} x} \right)}}{2 c^{\frac{3}{2}}} - \frac{2 a b \operatorname{asinh}{\left(\frac{\sqrt{c}}{\sqrt{d} x} \right)}}{\sqrt{c}} + b^{2} \left(\begin{cases} \frac{x^{2}}{2 \sqrt{c}} & \text{for}\: d = 0 \\ \frac{\sqrt{c + d x^{2}}}{d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + a**2*d*asinh(sqrt(c)/(sqrt(d)*
x))/(2*c**(3/2)) - 2*a*b*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) + b**2*Piecewis
e((x**2/(2*sqrt(c)), Eq(d, 0)), (sqrt(c + d*x**2)/d, True))
```

3.626 $\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=84

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}\,(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 451, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a\sqrt{c+dx^2}\,(3bc-ad)}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*c*x^3) - (2*a*(3*b*c - a*d)*Sqrt[c + d*x^2])/(3*c
^2*x) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^4\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}+\frac{\int \frac{2a(3bc-ad)+3b^2cx^2}{x^2\sqrt{c+dx^2}}\,dx}{3c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+b^2\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 72, normalized size = 0.86

$$\frac{b^2\log\left(\sqrt{d}\,\sqrt{c+dx^2}\,+dx\right)}{\sqrt{d}}-\frac{a\sqrt{c+dx^2}\,\left(a\left(c-2dx^2\right)+6bcx^2\right)}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*(a*Sqrt[c + d*x^2]*(6*b*c*x^2 + a*(c - 2*d*x^2)))/(c^2*x^3) + (b^2*Log
[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d]
```

IntegrateAlgebraic [A] time = 0.14, size = 77, normalized size = 0.92

$$\frac{\sqrt{c+dx^2}\,\left(-a^2c+2a^2dx^2-6abcx^2\right)}{3c^2x^3}-\frac{b^2\log\left(\sqrt{c+dx^2}\,-\sqrt{d}\,x\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a^2*c) - 6*a*b*c*x^2 + 2*a^2*d*x^2))/(3*c^2*x^3) - (b^2
*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d]
```

fricas [A] time = 1.54, size = 173, normalized size = 2.06

$$\left[\frac{3 \, b^{2} c^{2} \sqrt{d} x^{3} \log\left(-2 \, d x^{2} - 2 \, \sqrt{d x^{2} + c} \sqrt{d} x - c\right) - 2 \, {\left(a^{2} c d + 2 \, {\left(3 \, a b c d - a^{2} d^{2}\right)} x^{2}\right)} \sqrt{d x^{2} + c}}{6 \, c^{2} d x^{3}}, -\frac{3 \, b^{2} c^{2} \sqrt{-d} x^{3} \arctan\left(\frac{\sqrt{-d} x}{\sqrt{d x^{2} + c}}\right) + {\left(a^{2} c d + 2 \, {\left(3 \, a b c d - a^{2} d^{2}\right)} x^{2}\right)} \sqrt{d x^{2} + c}}{3 \, c^{2} d x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^2*c^2*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 - 2*(a^2*c*d + 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3),
-1/3*(3*b^2*c^2*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d
+ 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3)]
```

giac [B] time = 0.46, size = 156, normalized size = 1.86

$$-\frac{b^{2} \log\left({\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2}\right)}{2 \, \sqrt{d}} + \frac{4 \, {\left(3 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{4} a b \sqrt{d} - 6 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2} a b c \sqrt{d} + 3 \, {\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2} a^{2} d^{\frac{3}{2}} + 3 \, a b c^{2} \sqrt{d} - a^{2} c d^{\frac{3}{2}}\right)}}{3 \, {\left({\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2} - c\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d) + 4/3*(3*(sqrt(d)*x -
 sqrt(d*x^2 + c))^4*a*b*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*s
qrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) + 3*a*b*c^2*sqrt(d)
- a^2*c*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

maple [A] time = 0.01, size = 85, normalized size = 1.01

$$\frac{b^{2} \ln\left(\sqrt{d}\, x + \sqrt{d \,x^{2}+c}\right)}{\sqrt{d}}+\frac{2 \sqrt{d \,x^{2}+c}\, a^{2} d}{3 c^{2} x}-\frac{2 \sqrt{d \,x^{2}+c}\, a b}{c x}-\frac{\sqrt{d \,x^{2}+c}\, a^{2}}{3 c \,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x)
```

```
[Out] b^2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)-2*a*b/c/x*(d*x^2+c)^(1/2)-1/3*a^2
*(d*x^2+c)^(1/2)/c/x^3+2/3*a^2*d/c^2/x*(d*x^2+c)^(1/2)
```

maxima [A] time = 0.88, size = 77, normalized size = 0.92

$$\frac{b^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}-\frac{2\,\sqrt{dx^2+c}\,ab}{cx}+\frac{2\,\sqrt{dx^2+c}\,a^2d}{3\,c^2x}-\frac{\sqrt{dx^2+c}\,a^2}{3\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arcsinh(d*x/sqrt(c*d))/sqrt(d) - 2*sqrt(d*x^2 + c)*a*b/(c*x) + 2/3*sqrt
(d*x^2 + c)*a^2*d/(c^2*x) - 1/3*sqrt(d*x^2 + c)*a^2/(c*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(1/2)), x)
```

sympy [A] time = 5.42, size = 158, normalized size = 1.88

$$-\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}+b^2\left(\begin{cases}\frac{\sqrt{-\frac{c}{d}}\,\mathrm{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d<0\\ \frac{\sqrt{\frac{c}{d}}\,\mathrm{asinh}\left(x\sqrt{\frac{d}{c}}\right)}{\sqrt{c}} & \text{for}\ c>0\wedge d>0\\ \frac{\sqrt{-\frac{c}{d}}\,\mathrm{acosh}\left(x\sqrt{-\frac{d}{c}}\right)}{\sqrt{-c}} & \text{for}\ d>0\wedge c<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 2*a**2*d**(3/2)*sqrt(c/(d*x
**2) + 1)/(3*c**2) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + b**2*Piecewise(
(sqrt(-c/d)*asin(x*sqrt(-d/c))/sqrt(c), (c > 0) & (d < 0)), (sqrt(c/d)*asin
h(x*sqrt(d/c))/sqrt(c), (c > 0) & (d > 0)), (sqrt(-c/d)*acosh(x*sqrt(-d/c))
/sqrt(-c), (d > 0) & (c < 0)))
```

3.627 $\int \frac{\left(a+bx^2\right)^2}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}\,(8bc-3ad)}{8c^2x^2}$$

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 78, 63, 208}

$$-\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a\sqrt{c+dx^2}\,(8bc-3ad)}{8c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(4*c*x^4) - (a*(8*b*c - 3*a*d)*Sqrt[c + d*x^2])/(8*c
^2*x^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/(8*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^5\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-3ad)+2b^2cx}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} + \frac{1}{16}\left(8b^2-\frac{ad(8bc-3ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,d\right.\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} + \frac{\left(8b^2-\frac{ad(8bc-3ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right.}{8d}\\
&= -\frac{a^2\sqrt{c+dx^2}}{4cx^4} - \frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2} - \frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 92, normalized size = 0.87

$$-\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{a\sqrt{c+dx^2}\,\left(2ac-3adx^2+8bcx^2\right)}{8c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/8*(a*Sqrt[c + d*x^2]*(2*a*c + 8*b*c*x^2 - 3*a*d*x^2))/(c^2*x^4) - ((8*b^
2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5/2)
)
```

IntegrateAlgebraic [A] time = 0.18, size = 96, normalized size = 0.91

$$\frac{\left(-3a^2d^2+8abcd-8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{\sqrt{c+dx^2}\left(-2a^2c+3a^2dx^2-8abcx^2\right)}{8c^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c - 8*a*b*c*x^2 + 3*a^2*d*x^2))/(8*c^2*x^4) + ((-8
*b^2*c^2 + 8*a*b*c*d - 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5
/2))
```

fricas [A] time = 0.82, size = 204, normalized size = 1.92

$$\left[\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\sqrt{c}\,x^4\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,\left(2\,a^2c^2+\left(8\,abc^2-3\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{16\,c^3x^4},\ \frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\sqrt{-c}\,x^4\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(2\,a^2c^2+\left(8\,abc^2-3\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{8\,c^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^2*c^2 + (8*a*b*c^2 - 3*a^2*c*d)*x^
2)*sqrt(d*x^2 + c))/(c^3*x^4), 1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqr
t(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2*c^2 + (8*a*b*c^2 - 3*a^
2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^4)]
```

giac [A] time = 0.39, size = 140, normalized size = 1.32

$$\frac{\frac{\left(8\,b^2c^2d-8\,abcd^2+3\,a^2d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}-\frac{8\,\left(dx^2+c\right)^{\frac{3}{2}}abcd^2-8\,\sqrt{dx^2+c}\,abc^2d^2-3\,\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3+5\,\sqrt{dx^2+c}\,a^2cd^3}{c^2d^2x^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((8*b^2*c^2*d - 8*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c
))/(sqrt(-c)*c^2) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^2 + c)*a*b*
c^2*d^2 - 3*(d*x^2 + c)^(3/2)*a^2*d^3 + 5*sqrt(d*x^2 + c)*a^2*c*d^3)/(c^2*d
^2*x^4))/d
```

maple [A] time = 0.01, size = 157, normalized size = 1.48

$$-\frac{3a^{2}d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{8c^{\frac{5}{2}}}+\frac{abd\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{c^{\frac{3}{2}}}-\frac{b^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{\sqrt{c}}+\frac{3\sqrt{d\,x^{2}+c}\,a^{2}d}{8c^{2}x^{2}}-\frac{\sqrt{d\,x^{2}+c}\,ab}{c\,x^{2}}-\frac{\sqrt{d\,x^{2}+c}\,a^{2}}{4c\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*a^2*(d*x^2+c)^(1/2)/c/x^4+3/8*a^2*d/c^2/x^2*(d*x^2+c)^(1/2)-3/8*a^2*d^
2/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-a*b/c/x^2*(d*x^2+c)^(1/2)+a
*b*d/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-b^2/c^(1/2)*ln((2*c+2*(d
*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.89, size = 123, normalized size = 1.16

$$-\frac{b^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{\sqrt{c}}+\frac{abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}-\frac{3\,a^{2}d^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\,c^{\frac{5}{2}}}-\frac{\sqrt{dx^{2}+c}\,ab}{cx^{2}}+\frac{3\,\sqrt{dx^{2}+c}\,a^{2}d}{8\,c^{2}x^{2}}-\frac{\sqrt{dx^{2}+c}\,a^{2}}{4\,cx^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + a*b*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/c^(3/2) - 3/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) - sqrt(d*
x^2 + c)*a*b/(c*x^2) + 3/8*sqrt(d*x^2 + c)*a^2*d/(c^2*x^2) - 1/4*sqrt(d*x^2
 + c)*a^2/(c*x^4)
```

mupad [B] time = 1.06, size = 129, normalized size = 1.22

$$-\frac{\frac{\left(5\,a^{2}\,d^{2}-8\,a\,b\,c\,d\right)\,\sqrt{d\,x^{2}+c}}{8\,c}-\frac{\left(3\,a^{2}\,d^{2}-8\,a\,b\,c\,d\right)\,{\left(d\,x^{2}+c\right)}^{3/2}}{8\,c^{2}}}{{\left(d\,x^{2}+c\right)}^{2}-2\,c\,\left(d\,x^{2}+c\right)+c^{2}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)\,\left(3\,a^{2}\,d^{2}-8\,a\,b\,c\,d+8\,b^{2}\,c^{2}\right)}{8\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] - (((5*a^2*d^2 - 8*a*b*c*d)*(c + d*x^2)^(1/2))/(8*c) - ((3*a^2*d^2 - 8*a*b*
c*d)*(c + d*x^2)^(3/2))/(8*c^2))/((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) -
```

```
(atanh((c + d*x^2)^(1/2)/c^(1/2))*(3*a^2*d^2 + 8*b^2*c^2 - 8*a*b*c*d))/(8*c
^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.628 $\int \frac{\left(a+bx^2\right)^2}{x^6\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{\sqrt{c+dx^2}\,\left(15b^2c^2-4ad(5bc-2ad)\right)}{15c^3x}-\frac{2a\sqrt{c+dx^2}\,(5bc-2ad)}{15c^2x^3}$$

Rubi [**A**] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 453, 264}

$$-\frac{\sqrt{c+dx^2}\,\left(8a^2d^2-20abcd+15b^2c^2\right)}{15c^3x}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a\sqrt{c+dx^2}\,(5bc-2ad)}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*c*x^5) - (2*a*(5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(1
5*c^2*x^3) - ((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Sqrt[c + d*x^2])/(15*c^
3*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^6\sqrt{c+dx^2}}\,dx &= -\frac{a^2\sqrt{c+dx^2}}{5cx^5}+\frac{\int \frac{2a(5bc-2ad)+5b^2cx^2}{x^4\sqrt{c+dx^2}}\,dx}{5c}\\ &= -\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3}-\frac{1}{15}\left(-15b^2+\frac{4ad(5bc-2ad)}{c^2}\right)\int \frac{1}{x^2\sqrt{c+dx^2}}\,dx\\ &= -\frac{a^2\sqrt{c+dx^2}}{5cx^5}-\frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3}-\frac{\left(15b^2-\frac{4ad(5bc-2ad)}{c^2}\right)\sqrt{c+dx^2}}{15cx}\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 0.75

$$-\frac{\sqrt{c+dx^2}\left(a^2\left(3c^2-4cdx^2+8d^2x^4\right)+10abcx^2\left(c-2dx^2\right)+15b^2c^2x^4\right)}{15c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
3*c^2 - 4*c*d*x^2 + 8*d^2*x^4)))/(c^3*x^5)
```

IntegrateAlgebraic [A] time = 0.15, size = 81, normalized size = 0.82

$$\frac{\sqrt{c+dx^2}\left(-3a^2c^2+4a^2cdx^2-8a^2d^2x^4-10abc^2x^2+20abcdx^4-15b^2c^2x^4\right)}{15c^3x^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*a^2*c^2 - 10*a*b*c^2*x^2 + 4*a^2*c*d*x^2 - 15*b^2*c^2*
x^4 + 20*a*b*c*d*x^4 - 8*a^2*d^2*x^4))/(15*c^3*x^5)
```

fricas [A] time = 1.31, size = 73, normalized size = 0.74

$$-\frac{\left(\left(15\,b^2c^2-20\,abcd+8\,a^2d^2\right)x^4+3\,a^2c^2+2\left(5\,abc^2-2\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{15\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*x^4 + 3*a^2*c^2 + 2*(5*a*b*c^2
 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*x^5)
```

giac [**B**] time = 0.46, size = 312, normalized size = 3.15

$$\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c\sqrt{d}+60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6abd^{\frac{3}{2}}+90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^2\sqrt{d}-140\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abcd^{\frac{3}{2}}+80\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^3\sqrt{d}+100\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^2d^{\frac{3}{2}}-40\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2cd^{\frac{5}{2}}+15b^2c^4\sqrt{d}-20abc^3d^{\frac{3}{2}}+8a^2c^2d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*sqrt(d) - 60*(sqrt(d)*x - sqrt
(d*x^2 + c))^6*b^2*c*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*d^(3/
2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 140*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a*b*c*d^(3/2) + 80*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d
^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 100*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a^2*c*d^(5/2) + 15*b^2*c^4*sqrt(d) - 20*a*b*c^3*d^(3/2) + 8*a^2*c^2*d^(5/2
))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

maple [**A**] time = 0.01, size = 78, normalized size = 0.79

$$-\frac{\sqrt{d\,x^2+c}\,\left(8a^2d^2x^4-20abcd\,x^4+15b^2c^2x^4-4a^2cd\,x^2+10ab\,c^2x^2+3a^2c^2\right)}{15c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(8*a^2*d^2*x^4-20*a*b*c*d*x^4+15*b^2*c^2*x^4-4*a^2*c*
d*x^2+10*a*b*c^2*x^2+3*a^2*c^2)/x^5/c^3
```

maxima [**A**] time = 0.91, size = 124, normalized size = 1.25

$$-\frac{\sqrt{dx^2+c}\,b^2}{cx}+\frac{4\,\sqrt{dx^2+c}\,abd}{3\,c^2x}-\frac{8\,\sqrt{dx^2+c}\,a^2d^2}{15\,c^3x}-\frac{2\,\sqrt{dx^2+c}\,ab}{3\,cx^3}+\frac{4\,\sqrt{dx^2+c}\,a^2d}{15\,c^2x^3}-\frac{\sqrt{dx^2+c}\,a^2}{5\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(d*x^2 + c)*b^2/(c*x) + 4/3*sqrt(d*x^2 + c)*a*b*d/(c^2*x) - 8/15*sqrt(
d*x^2 + c)*a^2*d^2/(c^3*x) - 2/3*sqrt(d*x^2 + c)*a*b/(c*x^3) + 4/15*sqrt(d*
x^2 + c)*a^2*d/(c^2*x^3) - 1/5*sqrt(d*x^2 + c)*a^2/(c*x^5)
```

mupad [B] time = 0.73, size = 77, normalized size = 0.78

$$-\frac{\sqrt{d\,x^2+c}\,\left(3\,a^2\,c^2-4\,a^2\,c\,d\,x^2+8\,a^2\,d^2\,x^4+10\,a\,b\,c^2\,x^2-20\,a\,b\,c\,d\,x^4+15\,b^2\,c^2\,x^4\right)}{15\,c^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)^(1/2)),x)
```

```
[Out] -((c + d*x^2)^(1/2)*(3*a^2*c^2 + 8*a^2*d^2*x^4 + 15*b^2*c^2*x^4 + 10*a*b*c^
2*x^2 - 4*a^2*c*d*x^2 - 20*a*b*c*d*x^4))/(15*c^3*x^5)
```

sympy [B] time = 7.48, size = 391, normalized size = 3.95

$$-\frac{3a^2c^4d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2a^2c^3d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{3a^2c^2d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{12a^2cd^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{8a^2d^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{4abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(1/2),x)
```

```
[Out] -3*a**2*c**4*d**(9/2)*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**
5*x**6 + 15*c**3*d**6*x**8) - 2*a**2*c**3*d**(11/2)*x**2*sqrt(c/(d*x**2) +
1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 3*a**2*c**
2*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**
6 + 15*c**3*d**6*x**8) - 12*a**2*c*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)/(15*
c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 8*a**2*d**(17/2)*
x**8*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*
d**6*x**8) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 4*a*b*d**(3/2)
*sqrt(c/(d*x**2) + 1)/(3*c**2) - b**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c
```

3.629 $\int \frac{\left(a+bx^2\right)^2}{x^7\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=151

$$\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{\sqrt{c+dx^2}\,\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^3x^2}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}\,(12}{24c^2x}$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{\sqrt{c+dx^2}\,\left(5a^2d^2-12abcd+8b^2c^2\right)}{16c^3x^2}+\frac{d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}-\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a\sqrt{c+dx^2}\,(12bc-5ad)}{24c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(6*c*x^6) - (a*(12*b*c - 5*a*d)*Sqrt[c + d*x^2])/(24
*c^2*x^4) - ((8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[c + d*x^2])/(16*c^3*
x^2) + (d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/(16*c^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^7\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^4\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(12bc-5ad)+3b^2cx}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)}{6c}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} + \frac{1}{16}\left(8b^2 - \frac{ad(12bc-5ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+d}}\right.$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{\left(8b^2c^2-12abcd+5a^2d^2\right)\sqrt{c+dx^2}}{16c^3x^2} + \left(d\left(-8b^2\right.\right.$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{\left(8b^2c^2-12abcd+5a^2d^2\right)\sqrt{c+dx^2}}{16c^3x^2} + \left(-8b^2+\right.$$

$$= -\frac{a^2\sqrt{c+dx^2}}{6cx^6} - \frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{\left(8b^2c^2-12abcd+5a^2d^2\right)\sqrt{c+dx^2}}{16c^3x^2} + d\left(8b^2-\right.$$

Mathematica [A] time = 0.25, size = 135, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\sqrt{\frac{dx^2}{c}+1}\right)}{\sqrt{\frac{dx^2}{c}+1}} - \frac{c\left(a^2\left(8c^2-10cdx^2+15d^2x^4\right)+12abcx^2\left(2c-3dx^2\right)+24b^2c^2x^4\right)}{x^6}\right)}{48c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-((c*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c - 3*d*x^2) + a^2
*(8*c^2 - 10*c*d*x^2 + 15*d^2*x^4)))/x^6) + (3*d*(8*b^2*c^2 - 12*a*b*c*d +
5*a^2*d^2)*ArcTanh[Sqrt[1 + (d*x^2)/c]])/Sqrt[1 + (d*x^2)/c]))/(48*c^4)
```

IntegrateAlgebraic [A] time = 0.33, size = 135, normalized size = 0.89

$$\frac{\left(5a^2d^3-12abcd^2+8b^2c^2d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}} + \frac{\sqrt{c+dx^2}\left(-8a^2c^2+10a^2cdx^2-15a^2d^2x^4-24abc^2x^2+36abcdx^4-24b^2c^2x^4\right)}{48c^3x^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-8*a^2*c^2 - 24*a*b*c^2*x^2 + 10*a^2*c*d*x^2 - 24*b^2*c^2
*x^4 + 36*a*b*c*d*x^4 - 15*a^2*d^2*x^4))/(48*c^3*x^6) + ((8*b^2*c^2*d - 12*
a*b*c*d^2 + 5*a^2*d^3)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(7/2))
```

fricas [A] time = 1.43, size = 279, normalized size = 1.85

$$\left[\frac{3\left(8\,b^{2}c^{2}d-12\,abcd^{2}+5\,a^{2}d^{3}\right)\sqrt{c}\,x^{6}\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\left(8\,a^{2}c^{3}+3\left(8\,b^{2}c^{3}-12\,abc^{2}d+5\,a^{2}cd^{2}\right)x^{4}+2\left(12\,abc^{3}-5\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{96\,c^{4}x^{6}},-\frac{3\left(8\,b^{2}c^{2}d-12\,abcd^{2}+5\,a^{2}d^{3}\right)\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(8\,a^{2}c^{3}+3\left(8\,b^{2}c^{3}-12\,abc^{2}d+5\,a^{2}cd^{2}\right)x^{4}+2\left(12\,abc^{3}-5\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\,c^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 +
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 - 12*a
*b*c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2
+ c))/(c^4*x^6), -1/48*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(-c)
*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*a^2*c^3 + 3*(8*b^2*c^3 - 12*a*b*
c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c
))/(c^4*x^6)]
```

giac [A] time = 0.34, size = 241, normalized size = 1.60

$$-\frac{\frac{3\left(8\,b^{2}c^{2}d^{2}-12\,abcd^{3}+5\,a^{2}d^{4}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^{3}}+\frac{24\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}c^{2}d^{2}-48\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{3}d^{2}+24\,\sqrt{dx^{2}+c}\,b^{2}c^{4}d^{2}-36\left(dx^{2}+c\right)^{\frac{5}{2}}abcd^{3}+96\left(dx^{2}+c\right)^{\frac{3}{2}}abc^{2}d^{3}-60\,\sqrt{dx^{2}+c}\,abc^{3}d^{3}+15\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d^{4}-40\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cd^{4}+33\,\sqrt{dx^{2}+c}\,a^{2}c^{2}d^{4}}{c^{3}d^{3}x^{6}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 5*a^2*d^4)*arctan(sqrt(d*x^2 + c)/
sqrt(-c))/(sqrt(-c)*c^3) + (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 +
c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 - 36*(d*x^2 + c)^(5/2
)*a*b*c*d^3 + 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 - 60*sqrt(d*x^2 + c)*a*b*c^3
*d^3 + 15*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*d^4 + 33*s
qrt(d*x^2 + c)*a^2*c^2*d^4)/(c^3*d^3*x^6))/d
```

maple [A] time = 0.01, size = 224, normalized size = 1.48

$$\frac{5a^{2}d^{3}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{16c^{\frac{7}{2}}}-\frac{3ab\,d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{4c^{\frac{5}{2}}}+\frac{b^{2}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2c^{\frac{3}{2}}}-\frac{5\sqrt{d\,x^{2}+c}\,a^{2}d^{2}}{16c^{3}x^{2}}+\frac{3\sqrt{d\,x^{2}+c}\,abd}{4c^{2}x^{2}}-\frac{\sqrt{d\,x^{2}+c}\,b^{2}}{2c\,x^{2}}+\frac{5\sqrt{d\,x^{2}+c}\,a^{2}d}{24c^{2}x^{4}}-\frac{\sqrt{d\,x^{2}+c}\,ab}{2c\,x^{4}}-\frac{\sqrt{d\,x^{2}+c}\,a^{2}}{6c\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/6*a^2*(d*x^2+c)^(1/2)/c/x^6+5/24*a^2*d/c^2/x^4*(d*x^2+c)^(1/2)-5/16*a^2*
d^2/c^3/x^2*(d*x^2+c)^(1/2)+5/16*a^2*d^3/c^(7/2)*ln((2*c+2*(d*x^2+c)^(1/2)*
c^(1/2))/x)-1/2*a*b/c/x^4*(d*x^2+c)^(1/2)+3/4*a*b*d/c^2/x^2*(d*x^2+c)^(1/2)
-3/4*a*b*d^2/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/2*b^2/c/x^2*(d
*x^2+c)^(1/2)+1/2*b^2*d/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.92, size = 190, normalized size = 1.26

$$\frac{b^{2}d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{3}{2}}}-\frac{3\,abd^{2}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,c^{\frac{5}{2}}}+\frac{5\,a^{2}d^{3}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{7}{2}}}-\frac{\sqrt{dx^{2}+c}\,b^{2}}{2\,cx^{2}}+\frac{3\,\sqrt{dx^{2}+c}\,abd}{4\,c^{2}x^{2}}-\frac{5\,\sqrt{dx^{2}+c}\,a^{2}d^{2}}{16\,c^{3}x^{2}}-\frac{\sqrt{dx^{2}+c}\,ab}{2\,cx^{4}}+\frac{5\,\sqrt{dx^{2}+c}\,a^{2}d}{24\,c^{2}x^{4}}-\frac{\sqrt{dx^{2}+c}\,a^{2}}{6\,cx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) - 3/4*a*b*d^2*arcsinh(c/(sq
rt(c*d)*abs(x)))/c^(5/2) + 5/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(7/
2) - 1/2*sqrt(d*x^2 + c)*b^2/(c*x^2) + 3/4*sqrt(d*x^2 + c)*a*b*d/(c^2*x^2)
- 5/16*sqrt(d*x^2 + c)*a^2*d^2/(c^3*x^2) - 1/2*sqrt(d*x^2 + c)*a*b/(c*x^4)
+ 5/24*sqrt(d*x^2 + c)*a^2*d/(c^2*x^4) - 1/6*sqrt(d*x^2 + c)*a^2/(c*x^6)
```

mupad [B] time = 1.15, size = 207, normalized size = 1.37

$$\frac{\frac{{\left(d\,x^2+c\right)}^{5/2}\,\left(5\,a^2\,d^3-12\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c^3}-\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(5\,a^2\,d^3-12\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{6\,c^2}+\frac{\sqrt{d\,x^2+c}\,\left(11\,a^2\,d^3-20\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c}}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}+\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(5\,a^2\,d^2-12\,a\,b\,c\,d+8\,b^2\,c^2\right)}{16\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^7*(c + d*x^2)^(1/2)),x)
```

```
[Out] (((c + d*x^2)^(5/2)*(5*a^2*d^3 + 8*b^2*c^2*d - 12*a*b*c*d^2))/(16*c^3) - ((
c + d*x^2)^(3/2)*(5*a^2*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2))/(6*c^2) + ((c +
d*x^2)^(1/2)*(11*a^2*d^3 + 8*b^2*c^2*d - 20*a*b*c*d^2))/(16*c))/(3*c*(c + d
*x^2)^2 - 3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + (d*atanh((c + d*x^2)^(
1/2)/c^(1/2))*(5*a^2*d^2 + 8*b^2*c^2 - 12*a*b*c*d))/(16*c^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.630 $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{9/2}}+\frac{x\sqrt{c+dx^2}\,\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}-\frac{x^3\sqrt{c+dx^2}\,\left(24a^2d^2\right.}{24c}$$

Rubi [A] time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {463, 459, 321, 217, 206}

$$-\frac{x^3\sqrt{c+dx^2}\,\left(24a^2d^2-60abcd+35b^2c^2\right)}{24cd^3}+\frac{x\sqrt{c+dx^2}\,\left(24a^2d^2-60abcd+35b^2c^2\right)}{16d^4}-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{9/2}}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^5\sqrt{c+dx^2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((35*b^2*c^2 - 60*a*b*c*d + 2
4*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*d^4) - ((35*b^2*c^2 - 60*a*b*c*d + 24*a^2
*d^2)*x^3*Sqrt[c + d*x^2])/(24*c*d^3) + (b^2*x^5*Sqrt[c + d*x^2])/(6*d^2) -
 (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(16*d^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} - \frac{\int \frac{x^4\left(-a^2d^2+5(bc-ad)^2-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)\int \frac{x^4}{\sqrt{c+dx^2}}\,dx}{6cd^2}$$

$$= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x^3\sqrt{c+dx^2}}{24cd^3} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} + \frac{\left(35b^2c^2-6\right.}{}$$

$$= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x\sqrt{c+dx^2}}{16d^4} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x}{24cd^3}$$

$$= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x\sqrt{c+dx^2}}{16d^4} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x}{24cd^3}$$

$$= \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x\sqrt{c+dx^2}}{16d^4} - \frac{\left(35b^2c^2-60abcd+24a^2d^2\right)x}{24cd^3}$$

Mathematica [A] time = 0.16, size = 158, normalized size = 0.80

$$\sqrt{c+dx^2}\left(\frac{x\left(8a^2d^2-28abcd+19b^2c^2\right)}{16d^4}+\frac{cx(bc-ad)^2}{d^4\left(c+dx^2\right)}-\frac{bx^3(11bc-12ad)}{24d^3}+\frac{b^2x^5}{6d^2}\right)-\frac{c\left(24a^2d^2-60abcd+35b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{16d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*(((19*b^2*c^2 - 28*a*b*c*d + 8*a^2*d^2)*x)/(16*d^4) - (b*(1
1*b*c - 12*a*d)*x^3)/(24*d^3) + (b^2*x^5)/(6*d^2) + (c*(b*c - a*d)^2*x)/(d^
4*(c + d*x^2))) - (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*Log[d*x + Sqrt[
d]*Sqrt[c + d*x^2]])/(16*d^(9/2))
```

IntegrateAlgebraic [A] time = 0.31, size = 171, normalized size = 0.87

$$\frac{\left(24a^2cd^2-60abc^2d+35b^2c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16d^{9/2}}+\frac{72a^2cd^2x+24a^2d^3x^3-180abc^2dx-60abcd^2x^3+24abd^3x^5+105b^2c^3x+35b^2c^2dx^3-14b^2cd^2x^5+8b^2d^3x^7}{48d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (105*b^2*c^3*x - 180*a*b*c^2*d*x + 72*a^2*c*d^2*x + 35*b^2*c^2*d*x^3 - 60*a
*b*c*d^2*x^3 + 24*a^2*d^3*x^3 - 14*b^2*c*d^2*x^5 + 24*a*b*d^3*x^5 + 8*b^2*d
^3*x^7)/(48*d^4*Sqrt[c + d*x^2]) + ((35*b^2*c^3 - 60*a*b*c^2*d + 24*a^2*c*d
^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(16*d^(9/2))
```

fricas [A] time = 1.44, size = 431, normalized size = 2.19

$$\left[\frac{3\left(35b^2c^4-60abc^3d+24a^2c^2d^2+\left(35b^2c^3d-60abc^2d^2+24a^2cd^3\right)x^2\right)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}\,x-c\right)+2\left(8b^2d^4x^7-2\left(7b^2cd^3-12abd^4\right)x^5+\left(35b^2c^2d^2-60abcd^3+24a^2d^4\right)x^3+3\left(35b^2c^3d-60abc^2d^2+24a^2cd^3\right)x\right)\sqrt{dx^2+c}}{96\left(d^6x^2+cd^5\right)},\ \frac{3\left(35b^2c^4-60abc^3d+24a^2c^2d^2+\left(35b^2c^3d-60abc^2d^2+24a^2cd^3\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(8b^2d^4x^7-2\left(7b^2cd^3-12abd^4\right)x^5+\left(35b^2c^2d^2-60abcd^3+24a^2d^4\right)x^3+3\left(35b^2c^3d-60abc^2d^2+24a^2cd^3\right)x\right)\sqrt{dx^2+c}}{48\left(d^6x^2+cd^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(35*b^2*c^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a
*b*c^2*d^2 + 24*a^2*c*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) + 2*(8*b^2*d^4*x^7 - 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^
2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d
^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^6*x^2 + c*d^5), 1/48*(3*(35*b^2*c
^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^
2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^4*x^7
- 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^
2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2
 + c))/(d^6*x^2 + c*d^5)]
```

giac [A] time = 0.48, size = 175, normalized size = 0.89

$$\frac{\left(\left(2\left(\frac{4\,b^{2}x^{2}}{d}-\frac{7\,b^{2}cd^{5}-12\,abd^{6}}{d^{7}}\right)x^{2}+\frac{35\,b^{2}c^{2}d^{4}-60\,abcd^{5}+24\,a^{2}d^{6}}{d^{7}}\right)x^{2}+\frac{3\left(35\,b^{2}c^{3}d^{3}-60\,abc^{2}d^{4}+24\,a^{2}cd^{5}\right)}{d^{7}}\right)x}{48\,\sqrt{dx^{2}+c}}+\frac{\left(35\,b^{2}c^{3}-60\,abc^{2}d+24\,a^{2}cd^{2}\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^{2}+c}\right|\right)}{16\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*b^2*x^2/d - (7*b^2*c*d^5 - 12*a*b*d^6)/d^7)*x^2 + (35*b^2*c^2*d
^4 - 60*a*b*c*d^5 + 24*a^2*d^6)/d^7)*x^2 + 3*(35*b^2*c^3*d^3 - 60*a*b*c^2*d
^4 + 24*a^2*c*d^5)/d^7)*x/sqrt(d*x^2 + c) + 1/16*(35*b^2*c^3 - 60*a*b*c^2*d
 + 24*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

maple [A] time = 0.02, size = 263, normalized size = 1.34

$$\frac{b^{2}x^{7}}{6\sqrt{d\,x^{2}+c}\,d}+\frac{ab\,x^{5}}{2\sqrt{d\,x^{2}+c}\,d}-\frac{7b^{2}c\,x^{5}}{24\sqrt{d\,x^{2}+c}\,d^{2}}+\frac{a^{2}x^{3}}{2\sqrt{d\,x^{2}+c}\,d}-\frac{5abc\,x^{3}}{4\sqrt{d\,x^{2}+c}\,d^{2}}+\frac{35b^{2}c^{2}x^{3}}{48\sqrt{d\,x^{2}+c}\,d^{3}}+\frac{3a^{2}cx}{2\sqrt{d\,x^{2}+c}\,d^{2}}-\frac{15abc^{2}x}{4\sqrt{d\,x^{2}+c}\,d^{3}}+\frac{35b^{2}c^{3}x}{16\sqrt{d\,x^{2}+c}\,d^{4}}-\frac{3a^{2}c\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{2d^{\frac{5}{2}}}+\frac{15abc^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{4d^{\frac{7}{2}}}-\frac{35b^{2}c^{3}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{16d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/6*b^2*x^7/d/(d*x^2+c)^(1/2)-7/24*b^2*c/d^2*x^5/(d*x^2+c)^(1/2)+35/48*b^2*
c^2/d^3*x^3/(d*x^2+c)^(1/2)+35/16*b^2*c^3/d^4*x/(d*x^2+c)^(1/2)-35/16*b^2*c
^3/d^(9/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/2*a*b*x^5/d/(d*x^2+c)^(1/2)-5/4*
a*b*c/d^2*x^3/(d*x^2+c)^(1/2)-15/4*a*b*c^2/d^3*x/(d*x^2+c)^(1/2)+15/4*a*b*c
^2/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/2*a^2*x^3/d/(d*x^2+c)^(1/2)+3/2*
a^2*c/d^2*x/(d*x^2+c)^(1/2)-3/2*a^2*c/d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 0.96, size = 241, normalized size = 1.22

$$\frac{b^{2}x^{7}}{6\,\sqrt{dx^{2}+c}\,d}-\frac{7\,b^{2}cx^{5}}{24\,\sqrt{dx^{2}+c}\,d^{2}}+\frac{abx^{5}}{2\,\sqrt{dx^{2}+c}\,d}+\frac{35\,b^{2}c^{2}x^{3}}{48\,\sqrt{dx^{2}+c}\,d^{3}}-\frac{5\,abcx^{3}}{4\,\sqrt{dx^{2}+c}\,d^{2}}+\frac{a^{2}x^{3}}{2\,\sqrt{dx^{2}+c}\,d}+\frac{35\,b^{2}c^{3}x}{16\,\sqrt{dx^{2}+c}\,d^{4}}-\frac{15\,abc^{2}x}{4\,\sqrt{dx^{2}+c}\,d^{3}}+\frac{3\,a^{2}cx}{2\,\sqrt{dx^{2}+c}\,d^{2}}-\frac{35\,b^{2}c^{3}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{9}{2}}}+\frac{15\,abc^{2}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{7}{2}}}-\frac{3\,a^{2}c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^7/(sqrt(d*x^2 + c)*d) - 7/24*b^2*c*x^5/(sqrt(d*x^2 + c)*d^2) + 1/
2*a*b*x^5/(sqrt(d*x^2 + c)*d) + 35/48*b^2*c^2*x^3/(sqrt(d*x^2 + c)*d^3) - 5
/4*a*b*c*x^3/(sqrt(d*x^2 + c)*d^2) + 1/2*a^2*x^3/(sqrt(d*x^2 + c)*d) + 35/1
6*b^2*c^3*x/(sqrt(d*x^2 + c)*d^4) - 15/4*a*b*c^2*x/(sqrt(d*x^2 + c)*d^3) +
3/2*a^2*c*x/(sqrt(d*x^2 + c)*d^2) - 35/16*b^2*c^3*arcsinh(d*x/sqrt(c*d))/d^
(9/2) + 15/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(7/2) - 3/2*a^2*c*arcsinh(d*x
/sqrt(c*d))/d^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b x^{2}\right)^{2}}{\left(c + d x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.631 $\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{b\left(c+dx^2\right)^{3/2}(3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}\,(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

Rubi [**A**] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\left(c+dx^2\right)^{3/2}(3bc-2ad)}{3d^4}+\frac{\sqrt{c+dx^2}\,(bc-ad)(3bc-ad)}{d^4}+\frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + ((b*c - a*d)*(3*b*c - a*d)*Sqrt[c
 + d*x^2])/d^4 - (b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^4) + (b^2*(c +
d*x^2)^(5/2))/(5*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-\frac{c(bc-ad)^2}{d^3(c+dx)^{3/2}}+\frac{(bc-ad)(3bc-ad)}{d^3\sqrt{c+dx}}-\frac{b(3bc-2ad)\sqrt{c+dx}}{d^3}+\frac{b^2(c+dx)^{3/2}}{d^3}\right.\right.\\ &= \frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{(bc-ad)(3bc-ad)\sqrt{c+dx^2}}{d^4}-\frac{b(3bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 97, normalized size = 0.90

$$\frac{15a^2d^2\left(2c+dx^2\right)+10abd\left(-8c^2-4cdx^2+d^2x^4\right)+3b^2\left(16c^3+8c^2dx^2-2cd^2x^4+d^3x^6\right)}{15d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (15*a^2*d^2*(2*c + d*x^2) + 10*a*b*d*(-8*c^2 - 4*c*d*x^2 + d^2*x^4) + 3*b^2
*(16*c^3 + 8*c^2*d*x^2 - 2*c*d^2*x^4 + d^3*x^6))/(15*d^4*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.06, size = 111, normalized size = 1.03

$$\frac{30a^2cd^2+15a^2d^3x^2-80abc^2d-40abcd^2x^2+10abd^3x^4+48b^2c^3+24b^2c^2dx^2-6b^2cd^2x^4+3b^2d^3x^6}{15d^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (48*b^2*c^3 - 80*a*b*c^2*d + 30*a^2*c*d^2 + 24*b^2*c^2*d*x^2 - 40*a*b*c*d^2
*x^2 + 15*a^2*d^3*x^2 - 6*b^2*c*d^2*x^4 + 10*a*b*d^3*x^4 + 3*b^2*d^3*x^6)/(
15*d^4*Sqrt[c + d*x^2])
```

fricas [A] time = 0.75, size = 115, normalized size = 1.06

$$\frac{\left(3\,b^2d^3x^6+48\,b^2c^3-80\,abc^2d+30\,a^2cd^2-2\left(3\,b^2cd^2-5\,abd^3\right)x^4+\left(24\,b^2c^2d-40\,abcd^2+15\,a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{15\left(d^5x^2+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*d^3*x^6 + 48*b^2*c^3 - 80*a*b*c^2*d + 30*a^2*c*d^2 - 2*(3*b^2*c
*d^2 - 5*a*b*d^3)*x^4 + (24*b^2*c^2*d - 40*a*b*c*d^2 + 15*a^2*d^3)*x^2)*sqr
t(d*x^2 + c)/(d^5*x^2 + c*d^4)
```

giac [A] time = 0.41, size = 149, normalized size = 1.38

$$\frac{b^2c^3 - 2\,abc^2d + a^2cd^2}{\sqrt{dx^2 + c}\,d^4} + \frac{3\left(dx^2 + c\right)^{\frac{5}{2}}b^2d^{16} - 15\left(dx^2 + c\right)^{\frac{3}{2}}b^2cd^{16} + 45\,\sqrt{dx^2 + c}\,b^2c^2d^{16} + 10\left(dx^2 + c\right)^{\frac{3}{2}}abd^{17} - 60\,\sqrt{dx^2 + c}\,abcd^{17} + 15\,\sqrt{dx^2 + c}\,a^2d^{18}}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(sqrt(d*x^2 + c)*d^4) + 1/15*(3*(d*x^2
+ c)^(5/2)*b^2*d^16 - 15*(d*x^2 + c)^(3/2)*b^2*c*d^16 + 45*sqrt(d*x^2 + c)*
b^2*c^2*d^16 + 10*(d*x^2 + c)^(3/2)*a*b*d^17 - 60*sqrt(d*x^2 + c)*a*b*c*d^1
7 + 15*sqrt(d*x^2 + c)*a^2*d^18)/d^20
```

maple [A] time = 0.01, size = 108, normalized size = 1.00

$$\frac{3b^2x^6d^3 + 10ab\,d^3x^4 - 6b^2c\,d^2x^4 + 15a^2d^3x^2 - 40abc\,d^2x^2 + 24b^2c^2d\,x^2 + 30a^2c\,d^2 - 80ab\,c^2d + 48b^2c^3}{15\sqrt{d\,x^2 + c}\;d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(3*b^2*d^3*x^6+10*a*b*d^3*x^4-6*b^2*c*d^2*x^4+15*a^2*d^3*x^2-40*a*b*c*
d^2*x^2+24*b^2*c^2*d*x^2+30*a^2*c*d^2-80*a*b*c^2*d+48*b^2*c^3)/(d*x^2+c)^(1
/2)/d^4
```

maxima [A] time = 0.92, size = 180, normalized size = 1.67

$$\frac{b^2x^6}{5\,\sqrt{dx^2 + c}\,d} - \frac{2\,b^2cx^4}{5\,\sqrt{dx^2 + c}\,d^2} + \frac{2\,abx^4}{3\,\sqrt{dx^2 + c}\,d} + \frac{8\,b^2c^2x^2}{5\,\sqrt{dx^2 + c}\,d^3} - \frac{8\,abcx^2}{3\,\sqrt{dx^2 + c}\,d^2} + \frac{a^2x^2}{\sqrt{dx^2 + c}\,d} + \frac{16\,b^2c^3}{5\,\sqrt{dx^2 + c}\,d^4} - \frac{16\,abc^2}{3\,\sqrt{dx^2 + c}\,d^3} + \frac{2\,a^2c}{\sqrt{dx^2 + c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^6/(sqrt(d*x^2 + c)*d) - 2/5*b^2*c*x^4/(sqrt(d*x^2 + c)*d^2) + 2/3
*a*b*x^4/(sqrt(d*x^2 + c)*d) + 8/5*b^2*c^2*x^2/(sqrt(d*x^2 + c)*d^3) - 8/3*
a*b*c*x^2/(sqrt(d*x^2 + c)*d^2) + a^2*x^2/(sqrt(d*x^2 + c)*d) + 16/5*b^2*c^
3/(sqrt(d*x^2 + c)*d^4) - 16/3*a*b*c^2/(sqrt(d*x^2 + c)*d^3) + 2*a^2*c/(sqr
t(d*x^2 + c)*d^2)
```

mupad [B] time = 0.88, size = 107, normalized size = 0.99

$$\frac{30\,a^2\,c\,d^2 + 15\,a^2\,d^3\,x^2 - 80\,a\,b\,c^2\,d - 40\,a\,b\,c\,d^2\,x^2 + 10\,a\,b\,d^3\,x^4 + 48\,b^2\,c^3 + 24\,b^2\,c^2\,d\,x^2 - 6\,b^2\,c\,d^2\,x^4 + 3\,b^2\,d^3\,x^6}{15\,d^4\,\sqrt{d\,x^2 + c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] (48*b^2*c^3 + 30*a^2*c*d^2 + 15*a^2*d^3*x^2 + 3*b^2*d^3*x^6 + 24*b^2*c^2*d*
x^2 - 6*b^2*c*d^2*x^4 - 80*a*b*c^2*d + 10*a*b*d^3*x^4 - 40*a*b*c*d^2*x^2)/(
15*d^4*(c + d*x^2)^(1/2))
```

sympy [A] time = 1.99, size = 236, normalized size = 2.19

$$\begin{cases} \frac{2a^2c}{d^2\sqrt{c+dx^2}} + \frac{a^2x^2}{d\sqrt{c+dx^2}} - \frac{16abc^2}{3d^3\sqrt{c+dx^2}} - \frac{8abcx^2}{3d^2\sqrt{c+dx^2}} + \frac{2abx^4}{3d\sqrt{c+dx^2}} + \frac{16b^2c^3}{5d^4\sqrt{c+dx^2}} + \frac{8b^2c^2x^2}{5d^3\sqrt{c+dx^2}} - \frac{2b^2cx^4}{5d^2\sqrt{c+dx^2}} + \frac{b^2x^6}{5d\sqrt{c+dx^2}} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((2*a**2*c/(d**2*sqrt(c + d*x**2)) + a**2*x**2/(d*sqrt(c + d*x**2)
) - 16*a*b*c**2/(3*d**3*sqrt(c + d*x**2)) - 8*a*b*c*x**2/(3*d**2*sqrt(c + d
*x**2)) + 2*a*b*x**4/(3*d*sqrt(c + d*x**2)) + 16*b**2*c**3/(5*d**4*sqrt(c +
 d*x**2)) + 8*b**2*c**2*x**2/(5*d**3*sqrt(c + d*x**2)) - 2*b**2*c*x**4/(5*d
**2*sqrt(c + d*x**2)) + b**2*x**6/(5*d*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2
*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/c**(3/2), True))
```

3.632 $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{\left(8a^2d^2-24abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}-\frac{x\sqrt{c+dx^2}\left(8a^2d^2-24abcd+15b^2c^2\right)}{8cd^3}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+}}{4d^2}$$

Rubi [A] time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {463, 459, 321, 217, 206}

$$-\frac{x\sqrt{c+dx^2}\left(8a^2d^2-24abcd+15b^2c^2\right)}{8cd^3}+\frac{\left(8a^2d^2-24abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2x^3\sqrt{c+dx^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) - ((15*b^2*c^2 - 24*a*b*c*d + 8
*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^3) + (b^2*x^3*Sqrt[c + d*x^2])/(4*d^2)
+ ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(8*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\int \frac{x^2\left(-a^2d^2+3(bc-ad)^2-b^2cdx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd^2}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)\int \frac{x^2}{\sqrt{c+dx^2}}\,dx}{4cd^2}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{\left(15b^2c^2-24abc\right.}{}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{\left(15b^2c^2-24abc\right.}{}$$

$$= \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{\left(15b^2c^2-24abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{\left(15b^2c^2-24abc\right.}{}$$

Mathematica [A] time = 0.12, size = 124, normalized size = 0.82

$$\frac{\left(8a^2d^2-24abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{8d^{7/2}} + \sqrt{c+dx^2}\left(-\frac{x(ad-bc)^2}{d^3\left(c+dx^2\right)} - \frac{bx(7bc-8ad)}{8d^3} + \frac{b^2x^3}{4d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-1/8*(b*(7*b*c - 8*a*d)*x)/d^3 + (b^2*x^3)/(4*d^2) - ((-(b
*c) + a*d)^2*x)/(d^3*(c + d*x^2))) + ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)
*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*d^(7/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 129, normalized size = 0.85

$$\frac{\left(-8a^2d^2+24abcd-15b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{7/2}}+\frac{-8a^2d^2x+24abcdx+8abd^2x^3-15b^2c^2x-5b^2cdx^3+2b^2d^2x^5}{8d^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (-15*b^2*c^2*x + 24*a*b*c*d*x - 8*a^2*d^2*x - 5*b^2*c*d*x^3 + 8*a*b*d^2*x^3
 + 2*b^2*d^2*x^5)/(8*d^3*Sqrt[c + d*x^2]) + ((-15*b^2*c^2 + 24*a*b*c*d - 8*
a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(7/2))
```

fricas [A] time = 1.72, size = 350, normalized size = 2.30

$$\left[\frac{\left(15\,b^2c^3-24\,abc^2d+8\,a^2cd^2+\left(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3\right)x^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,\left(2\,b^2d^3x^5-\left(5\,b^2cd^2-8\,abd^3\right)x^3-\left(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3\right)x\right)\sqrt{dx^2+c}}{16\,\left(d^5x^2+cd^4\right)},-\frac{\left(15\,b^2c^3-24\,abc^2d+8\,a^2cd^2+\left(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3\right)x^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(2\,b^2d^3x^5-\left(5\,b^2cd^2-8\,abd^3\right)x^3-\left(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3\right)x\right)\sqrt{dx^2+c}}{8\,\left(d^5x^2+cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*
d^2 + 8*a^2*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(2*b^2*d^3*x^5 - (5*b^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*
a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/(d^5*x^2 + c*d^4), -1/8*((15*b^2
*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^
3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^3*x^5 - (5*b
^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^3)*x)*sq
rt(d*x^2 + c))/(d^5*x^2 + c*d^4)]
```

giac [A] time = 0.43, size = 131, normalized size = 0.86

$$\frac{\left(\left(\frac{2\,b^2x^2}{d}-\frac{5\,b^2cd^3-8\,abd^4}{d^5}\right)x^2-\frac{15\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4}{d^5}\right)x}{8\,\sqrt{dx^2+c}}-\frac{\left(15\,b^2c^2-24\,abcd+8\,a^2d^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*b^2*x^2/d - (5*b^2*c*d^3 - 8*a*b*d^4)/d^5)*x^2 - (15*b^2*c^2*d^2 -
24*a*b*c*d^3 + 8*a^2*d^4)/d^5)*x/sqrt(d*x^2 + c) - 1/8*(15*b^2*c^2 - 24*a*b
*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

maple [A] time = 0.01, size = 192, normalized size = 1.26

$$\frac{b^{2}x^{5}}{4\sqrt{d\,x^{2}+c}\,d}+\frac{ab\,x^{3}}{\sqrt{d\,x^{2}+c}\,d}-\frac{5b^{2}c\,x^{3}}{8\sqrt{d\,x^{2}+c}\,d^{2}}-\frac{a^{2}x}{\sqrt{d\,x^{2}+c}\,d}+\frac{3abcx}{\sqrt{d\,x^{2}+c}\,d^{2}}-\frac{15b^{2}c^{2}x}{8\sqrt{d\,x^{2}+c}\,d^{3}}+\frac{a^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}-\frac{3abc\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{5}{2}}}+\frac{15b^{2}c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{8d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4*b^2*x^5/d/(d*x^2+c)^(1/2)-5/8*b^2*c/d^2*x^3/(d*x^2+c)^(1/2)-15/8*b^2*c^
2/d^3*x/(d*x^2+c)^(1/2)+15/8*b^2*c^2/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+
a*b*x^3/d/(d*x^2+c)^(1/2)+3*a*b*c/d^2*x/(d*x^2+c)^(1/2)-3*a*b*c/d^(5/2)*ln(
d^(1/2)*x+(d*x^2+c)^(1/2))-a^2*x/d/(d*x^2+c)^(1/2)+a^2/d^(3/2)*ln(d^(1/2)*x
+(d*x^2+c)^(1/2))
```

maxima [A] time = 0.87, size = 170, normalized size = 1.12

$$\frac{b^{2}x^{5}}{4\sqrt{dx^{2}+c}\,d}-\frac{5\,b^{2}cx^{3}}{8\,\sqrt{dx^{2}+c}\,d^{2}}+\frac{abx^{3}}{\sqrt{dx^{2}+c}\,d}-\frac{15\,b^{2}c^{2}x}{8\,\sqrt{dx^{2}+c}\,d^{3}}+\frac{3\,abcx}{\sqrt{dx^{2}+c}\,d^{2}}-\frac{a^{2}x}{\sqrt{dx^{2}+c}\,d}+\frac{15\,b^{2}c^{2}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{7}{2}}}-\frac{3\,abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}+\frac{a^{2}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^5/(sqrt(d*x^2 + c)*d) - 5/8*b^2*c*x^3/(sqrt(d*x^2 + c)*d^2) + a*b
*x^3/(sqrt(d*x^2 + c)*d) - 15/8*b^2*c^2*x/(sqrt(d*x^2 + c)*d^3) + 3*a*b*c*x
/(sqrt(d*x^2 + c)*d^2) - a^2*x/(sqrt(d*x^2 + c)*d) + 15/8*b^2*c^2*arcsinh(d
*x/sqrt(c*d))/d^(7/2) - 3*a*b*c*arcsinh(d*x/sqrt(c*d))/d^(5/2) + a^2*arcsin
h(d*x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}\,{\left(b\,x^{2}+a\right)}^{2}}{{\left(d\,x^{2}+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b x^{2}\right)^{2}}{\left(c + d x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.633 $\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2b\sqrt{c+dx^2}\,(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^3}$$

Rubi [**A**] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2b\sqrt{c+dx^2}\,(bc-ad)}{d^3}-\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -((b*c - a*d)^2/(d^3*Sqrt[c + d*x^2])) - (2*b*(b*c - a*d)*Sqrt[c + d*x^2])/
d^3 + (b^2*(c + d*x^2)^(3/2))/(3*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx,x,x^2\right)\\
&= -\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}}-\frac{2b(bc-ad)\sqrt{c+dx^2}}{d^3}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^3}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.89

$$
\frac{-3a^2d^2+6abd\left(2c+dx^2\right)+b^2\left(-8c^2-4cdx^2+d^2x^4\right)}{3d^3\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (-3*a^2*d^2 + 6*a*b*d*(2*c + d*x^2) + b^2*(-8*c^2 - 4*c*d*x^2 + d^2*x^4))/(
3*d^3*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 71, normalized size = 0.97

$$
\frac{-3a^2d^2+12abcd+6abd^2x^2-8b^2c^2-4b^2cdx^2+b^2d^2x^4}{3d^3\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (-8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 4*b^2*c*d*x^2 + 6*a*b*d^2*x^2 + b^2*
d^2*x^4)/(3*d^3*Sqrt[c + d*x^2])
```

fricas [A] time = 1.56, size = 79, normalized size = 1.08

$$
\frac{\left(b^2d^2x^4-8\,b^2c^2+12\,abcd-3\,a^2d^2-2\left(2\,b^2cd-3\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{3\left(d^4x^2+cd^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^2*x^4 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x^2)*sqrt(d*x^2 + c)/(d^4*x^2 + c*d^3)
```

giac [A] time = 0.45, size = 92, normalized size = 1.26

$$-\frac{b^2c^2 - 2\,abcd + a^2d^2}{\sqrt{dx^2+c}\,d^3} + \frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2d^6 - 6\,\sqrt{dx^2+c}\,b^2cd^6 + 6\,\sqrt{dx^2+c}\,abd^7}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*d^3) + 1/3*((d*x^2 + c)^(
3/2)*b^2*d^6 - 6*sqrt(d*x^2 + c)*b^2*c*d^6 + 6*sqrt(d*x^2 + c)*a*b*d^7)/d^9
```

maple [A] time = 0.01, size = 69, normalized size = 0.95

$$-\frac{-b^2x^4d^2 - 6ab\,d^2x^2 + 4b^2cd\,x^2 + 3a^2d^2 - 12abcd + 8b^2c^2}{3\sqrt{d\,x^2+c}\;d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-b^2*d^2*x^4-6*a*b*d^2*x^2+4*b^2*c*d*x^2+3*a^2*d^2-12*a*b*c*d+8*b^2*c
^2)/(d*x^2+c)^(1/2)/d^3
```

maxima [A] time = 0.84, size = 115, normalized size = 1.58

$$\frac{b^2x^4}{3\,\sqrt{dx^2+c}\,d} - \frac{4\,b^2cx^2}{3\,\sqrt{dx^2+c}\,d^2} + \frac{2\,abx^2}{\sqrt{dx^2+c}\,d} - \frac{8\,b^2c^2}{3\,\sqrt{dx^2+c}\,d^3} + \frac{4\,abc}{\sqrt{dx^2+c}\,d^2} - \frac{a^2}{\sqrt{dx^2+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^4/(sqrt(d*x^2 + c)*d) - 4/3*b^2*c*x^2/(sqrt(d*x^2 + c)*d^2) + 2*a
*b*x^2/(sqrt(d*x^2 + c)*d) - 8/3*b^2*c^2/(sqrt(d*x^2 + c)*d^3) + 4*a*b*c/(s
qrt(d*x^2 + c)*d^2) - a^2/(sqrt(d*x^2 + c)*d)
```

mupad [B] time = 0.67, size = 75, normalized size = 1.03

$$\frac{b^2\,{\left(d\,x^2+c\right)}^2 - 3\,a^2\,d^2 - 3\,b^2\,c^2 - 6\,b^2\,c\,\left(d\,x^2+c\right) + 6\,a\,b\,d\,\left(d\,x^2+c\right) + 6\,a\,b\,c\,d}{3\,d^3\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

```
[Out] (b^2*(c + d*x^2)^2 - 3*a^2*d^2 - 3*b^2*c^2 - 6*b^2*c*(c + d*x^2) + 6*a*b*d*
(c + d*x^2) + 6*a*b*c*d)/(3*d^3*(c + d*x^2)^(1/2))
```

sympy [**A**] time = 1.29, size = 155, normalized size = 2.12

$$\begin{cases} -\frac{a^{2}}{d\sqrt{c+dx^{2}}} + \frac{4abc}{d^{2}\sqrt{c+dx^{2}}} + \frac{2abx^{2}}{d\sqrt{c+dx^{2}}} - \frac{8b^{2}c^{2}}{3d^{3}\sqrt{c+dx^{2}}} - \frac{4b^{2}cx^{2}}{3d^{2}\sqrt{c+dx^{2}}} + \frac{b^{2}x^{4}}{3d\sqrt{c+dx^{2}}} & \text{for}\: d \neq 0 \\ \frac{\frac{a^{2}x^{2}}{2} + \frac{abx^{4}}{2} + \frac{b^{2}x^{6}}{6}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Piecewise((-a**2/(d*sqrt(c + d*x**2)) + 4*a*b*c/(d**2*sqrt(c + d*x**2)) + 2
*a*b*x**2/(d*sqrt(c + d*x**2)) - 8*b**2*c**2/(3*d**3*sqrt(c + d*x**2)) - 4*
b**2*c*x**2/(3*d**2*sqrt(c + d*x**2)) + b**2*x**4/(3*d*sqrt(c + d*x**2)), N
e(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(3/2), True))
```

3.634 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}} \, dx$

Optimal. Leaf size=106

$$-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}+\frac{bx\sqrt{c+dx^2}\,(3bc-2ad)}{2cd^2}-\frac{x\left(a+bx^2\right)(bc-ad)}{cd\sqrt{c+dx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 388, 217, 206}

$$\frac{bx\sqrt{c+dx^2}\,(3bc-2ad)}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(3*b*c - 2*a*d)*x
*Sqrt[c + d*x^2])/(2*c*d^2) - (b*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{\int \frac{abc+b(3bc-2ad)x^2}{\sqrt{c+dx^2}}\,dx}{cd}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{(b(3bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\right.}{2d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{cd\sqrt{c+dx^2}}+\frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2}-\frac{b(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 93, normalized size = 0.88

$$\sqrt{c+dx^2}\left(\frac{x(bc-ad)^2}{cd^2\left(c+dx^2\right)}+\frac{b^2x}{2d^2}\right)-\frac{b(3bc-4ad)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
[Out] Sqrt[c + d*x^2]*((b^2*x)/(2*d^2) + ((b*c - a*d)^2*x)/(c*d^2*(c + d*x^2))) -
 (b*(3*b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(5/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 99, normalized size = 0.93

$$\frac{2a^2d^2x-4abcdx+3b^2c^2x+b^2cdx^3}{2cd^2\sqrt{c+dx^2}}+\frac{\left(3b^2c-4abd\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

```
[Out] (3*b^2*c^2*x - 4*a*b*c*d*x + 2*a^2*d^2*x + b^2*c*d*x^3)/(2*c*d^2*Sqrt[c + d
*x^2]) + ((3*b^2*c - 4*a*b*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(5/
2))
```

fricas [A] time = 1.38, size = 275, normalized size = 2.59

$$\left[-\frac{\left(3\,b^{2}c^{3}-4\,abc^{2}d+\left(3\,b^{2}c^{2}d-4\,abcd^{2}\right)x^{2}\right)\sqrt{d}\,\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-2\,\left(b^{2}cd^{2}x^{3}+\left(3\,b^{2}c^{2}d-4\,abcd^{2}+2\,a^{2}d^{3}\right)x\right)\sqrt{dx^{2}+c}}{4\,\left(cd^{4}x^{2}+c^{2}d^{3}\right)},\ \frac{\left(3\,b^{2}c^{3}-4\,abc^{2}d+\left(3\,b^{2}c^{2}d-4\,abcd^{2}\right)x^{2}\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+\left(b^{2}cd^{2}x^{3}+\left(3\,b^{2}c^{2}d-4\,abcd^{2}+2\,a^{2}d^{3}\right)x\right)\sqrt{dx^{2}+c}}{2\,\left(cd^{4}x^{2}+c^{2}d^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(d)*
log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(b^2*c*d^2*x^3 + (3*b^2
*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)
, 1/2*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c*d^2*x^3 + (3*b^2*c^2*d - 4*a*b
*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)]
```

giac [A] time = 0.46, size = 92, normalized size = 0.87

$$\frac{\left(\frac{b^{2}x^{2}}{d}+\frac{3\,b^{2}c^{2}d-4\,abcd^{2}+2\,a^{2}d^{3}}{cd^{3}}\right)x}{2\,\sqrt{dx^{2}+c}}+\frac{\left(3\,b^{2}c-4\,abd\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^{2}+c}\right|\right)}{2\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2/d + (3*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)/(c*d^3))*x/sqrt(d*
x^2 + c) + 1/2*(3*b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d
^(5/2)
```

maple [A] time = 0.01, size = 123, normalized size = 1.16

$$\frac{b^{2}x^{3}}{2\sqrt{d\,x^{2}+c}\,d}+\frac{a^{2}x}{\sqrt{d\,x^{2}+c}\,c}-\frac{2abx}{\sqrt{d\,x^{2}+c}\,d}+\frac{3b^{2}cx}{2\sqrt{d\,x^{2}+c}\,d^{2}}+\frac{2ab\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}-\frac{3b^{2}c\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{2d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*b^2*x^3/d/(d*x^2+c)^(1/2)+3/2*b^2*c/d^2*x/(d*x^2+c)^(1/2)-3/2*b^2*c/d^(
5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-2*a*b*x/d/(d*x^2+c)^(1/2)+2*a*b/d^(3/2)*
ln(d^(1/2)*x+(d*x^2+c)^(1/2))+a^2*x/c/(d*x^2+c)^(1/2)
```

maxima [A] time = 0.92, size = 108, normalized size = 1.02

$$\frac{b^2x^3}{2\sqrt{dx^2+c}\,d}+\frac{a^2x}{\sqrt{dx^2+c}\,c}+\frac{3\,b^2cx}{2\sqrt{dx^2+c}\,d^2}-\frac{2\,abx}{\sqrt{dx^2+c}\,d}-\frac{3\,b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{5}{2}}}+\frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^3/(sqrt(d*x^2 + c)*d) + a^2*x/(sqrt(d*x^2 + c)*c) + 3/2*b^2*c*x/(
sqrt(d*x^2 + c)*d^2) - 2*a*b*x/(sqrt(d*x^2 + c)*d) - 3/2*b^2*c*arcsinh(d*x/
sqrt(c*d))/d^(5/2) + 2*a*b*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2)^2/(c + d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.635 $\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c - a*d)^2/(c*d^2*Sqrt[c + d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^2 - (a^2*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(bc-ad)^2}{cd(c+dx)^{3/2}}+\frac{b^2}{d\sqrt{c+dx}}+\frac{a^2}{cx\sqrt{c+dx}}\right)dx,x,x^2\right)\\ &= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}\\ &= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{cd}\\ &= \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 62, normalized size = 0.83

$$\frac{a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)+bc\left(-2ad+2bc+bdx^2\right)}{cd^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c*(2*b*c - 2*a*d + b*d*x^2) + a^2*d^2*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ (d*x^2)/c])/(c*d^2*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 78, normalized size = 1.04

$$\frac{a^2d^2-2abcd+2b^2c^2+b^2cdx^2}{cd^2\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2 + b^2*c*d*x^2)/(c*d^2*Sqrt[c + d*x^2]) - (
a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

fricas [A] time = 1.54, size = 232, normalized size = 3.09

$$\left[\frac{\left(a^{2} d^{3} x^{2} + a^{2} c d^{2}\right) \sqrt{c} \log\left(-\frac{d x^{2} - 2 \, \sqrt{d x^{2} + c} \sqrt{c} + 2 \, c}{x^{2}}\right) + 2 \, \left(b^{2} c^{2} d x^{2} + 2 \, b^{2} c^{3} - 2 \, a b c^{2} d + a^{2} c d^{2}\right) \sqrt{d x^{2} + c}}{2 \, \left(c^{2} d^{3} x^{2} + c^{3} d^{2}\right)}, \frac{\left(a^{2} d^{3} x^{2} + a^{2} c d^{2}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2} + c}}\right) + \left(b^{2} c^{2} d x^{2} + 2 \, b^{2} c^{3} - 2 \, a b c^{2} d + a^{2} c d^{2}\right) \sqrt{d x^{2} + c}}{c^{2} d^{3} x^{2} + c^{3} d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(c) + 2*c)/x^2) + 2*(b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2), ((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2)]
```

giac [A] time = 0.33, size = 82, normalized size = 1.09

$$\frac{a^{2} \arctan\left(\frac{\sqrt{d x^{2} + c}}{\sqrt{-c}}\right)}{\sqrt{-c} c} + \frac{\sqrt{d x^{2} + c} b^{2}}{d^{2}} + \frac{b^{2} c^{2} - 2 \, a b c d + a^{2} d^{2}}{\sqrt{d x^{2} + c} c d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) + sqrt(d*x^2 + c)*b^2/d^2
 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c*d^2)
```

maple [A] time = 0.01, size = 102, normalized size = 1.36

$$\frac{b^{2} x^{2}}{\sqrt{d \,x^{2}+c}\, d}-\frac{a^{2} \ln \left(\frac{2 c +2 \sqrt{d \,x^{2}+c}\, \sqrt{c}}{x}\right)}{c^{\frac{3}{2}}}+\frac{a^{2}}{\sqrt{d \,x^{2}+c}\, c}-\frac{2 a b}{\sqrt{d \,x^{2}+c}\, d}+\frac{2 b^{2} c}{\sqrt{d \,x^{2}+c}\, d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x)
```

```
[Out] b^2*x^2/d/(d*x^2+c)^(1/2)+2*b^2*c/d^2/(d*x^2+c)^(1/2)-2*a*b/d/(d*x^2+c)^(1/
2)+a^2/c/(d*x^2+c)^(1/2)-a^2/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.91, size = 90, normalized size = 1.20

$$\frac{b^2x^2}{\sqrt{dx^2+c}\,d}-\frac{a^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}+\frac{a^2}{\sqrt{dx^2+c}\,c}+\frac{2\,b^2c}{\sqrt{dx^2+c}\,d^2}-\frac{2\,ab}{\sqrt{dx^2+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x^2/(sqrt(d*x^2 + c)*d) - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + a
^2/(sqrt(d*x^2 + c)*c) + 2*b^2*c/(sqrt(d*x^2 + c)*d^2) - 2*a*b/(sqrt(d*x^2
+ c)*d)
```

mupad [B] time = 0.86, size = 76, normalized size = 1.01

$$\frac{b^2\,\sqrt{d\,x^2+c}}{d^2}-\frac{a^2\,\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c\,d^2\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x)
```

```
[Out] (b^2*(c + d*x^2)^(1/2))/d^2 - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^(3/2
) + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(c*d^2*(c + d*x^2)^(1/2))
```

sympy [A] time = 36.05, size = 70, normalized size = 0.93

$$\frac{a^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{c\sqrt{-c}}+\frac{b^2\sqrt{c+dx^2}}{d^2}+\frac{(ad-bc)^2}{cd^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(3/2),x)
```

```
[Out] a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(c*sqrt(-c)) + b**2*sqrt(c + d*x**2)/d
**2 + (a*d - b*c)**2/(c*d**2*sqrt(c + d*x**2))
```

3.636 $\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(b^2c^2-2ad(bc-ad)\right)}{c^2d\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

Rubi [**A**] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 385, 217, 206}

$$-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{x\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)}{\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(a^2/(c*x*Sqrt[c + d*x^2])) - ((b^2/d - (2*a*(b*c - a*d))/c^2)*x)/Sqrt[c +
 d*x^2] + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{a^2}{cx\sqrt{c+dx^2}}+\frac{\int \frac{2a(bc-ad)+b^2cx^2}{\left(c+dx^2\right)^{3/2}}\,dx}{c}\\
&= -\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{d}\\
&= -\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{d}\\
&= -\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(\frac{b^2}{d}-\frac{2a(bc-ad)}{c^2}\right)x}{\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.89

$$
\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{3/2}}-\frac{\sqrt{c+dx^2}\left(a^2+\frac{x^2(bc-ad)^2}{d\left(c+dx^2\right)}\right)}{c^2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
[Out] -((Sqrt[c + d*x^2]*(a^2 + ((b*c - a*d)^2*x^2)/(d*(c + d*x^2))))/(c^2*x)) +
(b^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^(3/2)
```

IntegrateAlgebraic [A] time = 0.15, size = 92, normalized size = 1.01

$$
\frac{-a^2cd-2a^2d^2x^2+2abcdx^2-b^2c^2x^2}{c^2dx\sqrt{c+dx^2}}-\frac{b^2\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*c*d) - b^2*c^2*x^2 + 2*a*b*c*d*x^2 - 2*a^2*d^2*x^2)/(c^2*d*x*Sqrt[c
+ d*x^2]) - (b^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^(3/2)
```

fricas [A] time = 1.55, size = 239, normalized size = 2.63

$$\left[\frac{\left(b^{2} c^{2} d x^{3}+b^{2} c^{3} x\right) \sqrt{d} \log \left(-2 d x^{2}-2 \sqrt{d x^{2}+c} \sqrt{d} x-c\right)-2\left(a^{2} c d^{2}+\left(b^{2} c^{2} d-2 a b c d^{2}+2 a^{2} d^{3}\right) x^{2}\right) \sqrt{d x^{2}+c}}{2\left(c^{2} d^{3} x^{3}+c^{3} d^{2} x\right)},-\frac{\left(b^{2} c^{2} d x^{3}+b^{2} c^{3} x\right) \sqrt{-d} \arctan \left(\frac{\sqrt{-d} x}{\sqrt{d x^{2}+c}}\right)+\left(a^{2} c d^{2}+\left(b^{2} c^{2} d-2 a b c d^{2}+2 a^{2} d^{3}\right) x^{2}\right) \sqrt{d x^{2}+c}}{c^{2} d^{3} x^{3}+c^{3} d^{2} x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(d)*x - c) - 2*(a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + 2*a^2*d^3)*x^2)*
sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x), -((b^2*c^2*d*x^3 + b^2*c^3*x)*s
qrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d^2 + (b^2*c^2*d - 2*a*
b*c*d^2 + 2*a^2*d^3)*x^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x)]
```

giac [A] time = 0.47, size = 104, normalized size = 1.14

$$-\frac{b^{2} \log \left(\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2}\right)}{2 d^{\frac{3}{2}}}+\frac{2 a^{2} \sqrt{d}}{\left(\left(\sqrt{d} x-\sqrt{d x^{2}+c}\right)^{2}-c\right) c}-\frac{\left(b^{2} c^{2}-2 a b c d+a^{2} d^{2}\right) x}{\sqrt{d x^{2}+c} c^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2) + 2*a^2*sqrt(d)/(((sq
rt(d)*x - sqrt(d*x^2 + c))^2 - c)*c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(s
qrt(d*x^2 + c)*c^2*d)
```

maple [A] time = 0.01, size = 99, normalized size = 1.09

$$-\frac{2 a^{2} d x}{\sqrt{d \,x^{2}+c}\, c^{2}}+\frac{2 a b x}{\sqrt{d \,x^{2}+c}\, c}-\frac{b^{2} x}{\sqrt{d \,x^{2}+c}\, d}+\frac{b^{2} \ln \left(\sqrt{d}\, x+\sqrt{d \,x^{2}+c}\right)}{d^{\frac{3}{2}}}-\frac{a^{2}}{\sqrt{d \,x^{2}+c}\, c x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2*x/d/(d*x^2+c)^(1/2)+b^2/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+2*a*b*x/
c/(d*x^2+c)^(1/2)-a^2/c/x/(d*x^2+c)^(1/2)-2*a^2*d/c^2*x/(d*x^2+c)^(1/2)
```

maxima [A] time = 0.90, size = 91, normalized size = 1.00

$$\frac{2\,abx}{\sqrt{dx^2+c}\,c}-\frac{b^2x}{\sqrt{dx^2+c}\,d}-\frac{2\,a^2dx}{\sqrt{dx^2+c}\,c^2}+\frac{b^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}-\frac{a^2}{\sqrt{dx^2+c}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*a*b*x/(sqrt(d*x^2 + c)*c) - b^2*x/(sqrt(d*x^2 + c)*d) - 2*a^2*d*x/(sqrt(d
*x^2 + c)*c^2) + b^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) - a^2/(sqrt(d*x^2 + c)*
c*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^2}{x^2\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(3/2)), x)
```

3.637 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{-\frac{3a^2d}{c}+4ab-\frac{2b^2c}{d}}{2c\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 78, 63, 208}

$$\frac{-\frac{3a^2d}{c}+4ab-\frac{2b^2c}{d}}{2c\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] (4*a*b - (2*b^2*c)/d - (3*a^2*d)/c)/(2*c*Sqrt[c + d*x^2]) - a^2/(2*c*x^2*Sq
rt[c + d*x^2]) - (a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^
(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{a^2}{2cx^2\sqrt{c+dx^2}}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-3ad)+b^2cx}{x(c+dx)^{3/2}}\,dx,x,x^2\right)}{2c}\\
&= -\frac{2b^2c^2-4abcd+3a^2d^2}{2c^2d\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}+\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{4c^2}\\
&= -\frac{2b^2c^2-4abcd+3a^2d^2}{2c^2d\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}+\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+}\right.}{2c^2d}\\
&= -\frac{2b^2c^2-4abcd+3a^2d^2}{2c^2d\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.93

$$\frac{a(3ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}-\frac{a^2d\left(c+3dx^2\right)-4abcdx^2+2b^2c^2x^2}{2c^2dx^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/2*(2*b^2*c^2*x^2 - 4*a*b*c*d*x^2 + a^2*d*(c + 3*d*x^2))/(c^2*d*x^2*Sqrt[
c + d*x^2]) + (a*(-4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(5
/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 103, normalized size = 1.00

$$\frac{-a^2cd-3a^2d^2x^2+4abcdx^2-2b^2c^2x^2}{2c^2dx^2\sqrt{c+dx^2}}+\frac{\left(3a^2d-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*c*d) - 2*b^2*c^2*x^2 + 4*a*b*c*d*x^2 - 3*a^2*d^2*x^2)/(2*c^2*d*x^2*S
qrt[c + d*x^2]) + ((-4*a*b*c + 3*a^2*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(
2*c^(5/2))
```

fricas [A] time = 1.19, size = 292, normalized size = 2.83

$$\left[-\frac{\left(\left(4\,abcd^2-3\,a^2d^3\right)x^4+\left(4\,abc^2d-3\,a^2cd^2\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\left(a^2c^2d+\left(2\,b^2c^3-4\,abc^2d+3\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{4\left(c^3d^2x^4+c^4dx^2\right)},\frac{\left(\left(4\,abcd^2-3\,a^2d^3\right)x^4+\left(4\,abc^2d-3\,a^2cd^2\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(a^2c^2d+\left(2\,b^2c^3-4\,abc^2d+3\,a^2cd^2\right)x^2\right)\sqrt{dx^2+c}}{2\left(c^3d^2x^4+c^4dx^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sq
rt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c^2*d +
(2*b^2*c^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4
+ c^4*d*x^2), 1/2*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*
d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (a^2*c^2*d + (2*b^2*c
^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4 + c^4*d*
x^2)]
```

giac [A] time = 0.39, size = 140, normalized size = 1.36

$$\frac{\left(4\,abc-3\,a^{2}d\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}\,c^{2}}-\frac{2\,\left(dx^{2}+c\right)b^{2}c^{2}-2\,b^{2}c^{3}-4\,\left(dx^{2}+c\right)abcd+4\,abc^{2}d+3\,\left(dx^{2}+c\right)a^{2}d^{2}-2\,a^{2}cd^{2}}{2\,\left(\left(dx^{2}+c\right)^{\frac{3}{2}}-\sqrt{dx^{2}+c}\,c\right)c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1
/2*(2*(d*x^2 + c)*b^2*c^2 - 2*b^2*c^3 - 4*(d*x^2 + c)*a*b*c*d + 4*a*b*c^2*d
 + 3*(d*x^2 + c)*a^2*d^2 - 2*a^2*c*d^2)/(((d*x^2 + c)^(3/2) - sqrt(d*x^2 +
c)*c)*c^2*d)
```

maple [A] time = 0.01, size = 135, normalized size = 1.31

$$\frac{3a^{2}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2c^{\frac{5}{2}}}-\frac{2ab\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{c^{\frac{3}{2}}}-\frac{3a^{2}d}{2\sqrt{d\,x^{2}+c}\,c^{2}}+\frac{2ab}{\sqrt{d\,x^{2}+c}\,c}-\frac{b^{2}}{\sqrt{d\,x^{2}+c}\,d}-\frac{a^{2}}{2\sqrt{d\,x^{2}+c}\,c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2/d/(d*x^2+c)^(1/2)-1/2*a^2/c/x^2/(d*x^2+c)^(1/2)-3/2*a^2*d/c^2/(d*x^2+c
)^(1/2)+3/2*a^2*d/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+2*a*b/c/(d*
x^2+c)^(1/2)-2*a*b/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.86, size = 112, normalized size = 1.09

$$-\frac{2\,ab\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{3}{2}}}+\frac{3\,a^{2}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{5}{2}}}+\frac{2\,ab}{\sqrt{dx^{2}+c}\,c}-\frac{b^{2}}{\sqrt{dx^{2}+c}\,d}-\frac{3\,a^{2}d}{2\,\sqrt{dx^{2}+c}\,c^{2}}-\frac{a^{2}}{2\,\sqrt{dx^{2}+c}\,cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + 3/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/c^(5/2) + 2*a*b/(sqrt(d*x^2 + c)*c) - b^2/(sqrt(d*x^2 + c)*d) -
 3/2*a^2*d/(sqrt(d*x^2 + c)*c^2) - 1/2*a^2/(sqrt(d*x^2 + c)*c*x^2)
```

mupad [B] time = 1.08, size = 119, normalized size = 1.16

$$\frac{\frac{a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}}{c}-\frac{\left(d\,x^{2}+c\right)\,\left(3\,a^{2}\,d^{2}-4\,a\,b\,c\,d+2\,b^{2}\,c^{2}\right)}{2\,c^{2}}}{d\,\left(d\,x^{2}+c\right)^{3/2}-c\,d\,\sqrt{d\,x^{2}+c}}+\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{2\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/c - ((c + d*x^2)*(3*a^2*d^2 + 2*b^2*c^2 -
4*a*b*c*d))/(2*c^2))/(d*(c + d*x^2)^(3/2) - c*d*(c + d*x^2)^(1/2)) + (a*ata
nh((c + d*x^2)^(1/2)/c^(1/2))*(3*a*d - 4*b*c))/(2*c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(3/2)), x)
```

3.638 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{a^2}{3cx^3\sqrt{c+dx^2}}+\frac{x\left(3b^2c^2-4ad(3bc-2ad)\right)}{3c^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 453, 191}

$$\frac{x\left(8a^2d^2-12abcd+3b^2c^2\right)}{3c^3\sqrt{c+dx^2}}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(3*c*x^3*Sqrt[c + d*x^2]) - (2*a*(3*b*c - 2*a*d))/(3*c^2*x*Sqrt[c + d*
x^2]) + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{a^2}{3cx^3\sqrt{c+dx^2}}+\frac{\int \frac{2a(3bc-2ad)+3b^2cx^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{3c}\\ &= -\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}-\frac{1}{3}\left(-3b^2+\frac{4ad(3bc-2ad)}{c^2}\right)\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx\\ &= -\frac{a^2}{3cx^3\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}}+\frac{\left(3b^2-\frac{4ad(3bc-2ad)}{c^2}\right)x}{3c\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 0.76

$$\frac{a^2\left(-c^2+4cdx^2+8d^2x^4\right)-6abcx^2\left(c+2dx^2\right)+3b^2c^2x^4}{3c^3x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] (3*b^2*c^2*x^4 - 6*a*b*c*x^2*(c + 2*d*x^2) + a^2*(-c^2 + 4*c*d*x^2 + 8*d^2*
x^4))/(3*c^3*x^3*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.16, size = 81, normalized size = 0.84

$$\frac{-a^2c^2+4a^2cdx^2+8a^2d^2x^4-6abc^2x^2-12abcdx^4+3b^2c^2x^4}{3c^3x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*c^2) - 6*a*b*c^2*x^2 + 4*a^2*c*d*x^2 + 3*b^2*c^2*x^4 - 12*a*b*c*d*x^
4 + 8*a^2*d^2*x^4)/(3*c^3*x^3*Sqrt[c + d*x^2])
```

fricas [A] time = 1.36, size = 85, normalized size = 0.88

$$\frac{\left(\left(3\,b^2c^2-12\,abcd+8\,a^2d^2\right)x^4-a^2c^2-2\left(3\,abc^2-2\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{3\left(c^3dx^5+c^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^4 - a^2*c^2 - 2*(3*a*b*c^2 - 2*
a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d*x^5 + c^4*x^3)
```

giac [B] time = 0.45, size = 199, normalized size = 2.05

$$\frac{\left(b^{2} c^{2}-2\,abcd+a^{2} d^{2}\right) x}{\sqrt{dx^{2}+c}\,c^{3}}+\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4} abc\sqrt{d}\,-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4} a^{2} d^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2} abc^{2}\sqrt{d}\,+12\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2} a^{2} cd^{\frac{3}{2}}+6\,abc^{3}\sqrt{d}\,-5\,a^{2} c^{2} d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3} c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(sqrt(d*x^2 + c)*c^3) + 2/3*(6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 12*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 6*a*b*c^3*sqrt(d) - 5*a^2*c^2*d^(3/
2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^2)
```

maple [A] time = 0.01, size = 77, normalized size = 0.79

$$-\frac{-8a^{2}d^{2}x^{4}+12abcd\,x^{4}-3b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+6ab\,c^{2}x^{2}+a^{2}c^{2}}{3\sqrt{d\,x^{2}+c}\;c^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-8*a^2*d^2*x^4+12*a*b*c*d*x^4-3*b^2*c^2*x^4-4*a^2*c*d*x^2+6*a*b*c^2*x
^2+a^2*c^2)/(d*x^2+c)^(1/2)/x^3/c^3
```

maxima [A] time = 0.88, size = 117, normalized size = 1.21

$$\frac{b^{2}x}{\sqrt{dx^{2}+c}\,c}-\frac{4\,abdx}{\sqrt{dx^{2}+c}\,c^{2}}+\frac{8\,a^{2}d^{2}x}{3\,\sqrt{dx^{2}+c}\,c^{3}}-\frac{2\,ab}{\sqrt{dx^{2}+c}\,cx}+\frac{4\,a^{2}d}{3\,\sqrt{dx^{2}+c}\,c^{2}x}-\frac{a^{2}}{3\,\sqrt{dx^{2}+c}\,cx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x/(sqrt(d*x^2 + c)*c) - 4*a*b*d*x/(sqrt(d*x^2 + c)*c^2) + 8/3*a^2*d^2*x
/(sqrt(d*x^2 + c)*c^3) - 2*a*b/(sqrt(d*x^2 + c)*c*x) + 4/3*a^2*d/(sqrt(d*x^
2 + c)*c^2*x) - 1/3*a^2/(sqrt(d*x^2 + c)*c*x^3)
```

mupad [**B**] time = 0.76, size = 76, normalized size = 0.78

$$-\frac{a^2\,c^2-4\,a^2\,c\,d\,x^2-8\,a^2\,d^2\,x^4+6\,a\,b\,c^2\,x^2+12\,a\,b\,c\,d\,x^4-3\,b^2\,c^2\,x^4}{3\,c^3\,x^3\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x)
```

```
[Out] -(a^2*c^2 - 8*a^2*d^2*x^4 - 3*b^2*c^2*x^4 + 6*a*b*c^2*x^2 - 4*a^2*c*d*x^2 +
 12*a*b*c*d*x^4)/(3*c^3*x^3*(c + d*x^2)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(3/2)), x)
```

3.639 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}-\frac{\left(8b^2c^2-3ad(8bc-5ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{8b^2c^2-3ad(8bc-5ad)}{8c^3\sqrt{c+dx^2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

Rubi [A] time = 0.17, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{a^2}{4cx^4\sqrt{c+dx^2}}+\frac{8b^2-\frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}}-\frac{\left(8b^2c^2-3ad(8bc-5ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (8*b^2 - (3*a*d*(8*b*c - 5*a*d))/c^2)/(8*c*Sqrt[c + d*x^2]) - a^2/(4*c*x^4*
Sqrt[c + d*x^2]) - (a*(8*b*c - 5*a*d))/(8*c^2*x^2*Sqrt[c + d*x^2]) - ((8*b^
2*c^2 - 3*a*d*(8*b*c - 5*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{4cx^4\sqrt{c+dx^2}} + \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}a(8bc-5ad)+2b^2cx}{x^2(c+dx)^{3/2}}\,dx, x, x^2\right)}{4c}$$

$$= -\frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{1}{16}\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\mathrm{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\right.$$

$$= \frac{8b^2 - \frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\mathrm{Subst}\left(\int \right.}{16c}$$

$$= \frac{8b^2 - \frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} + \frac{\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\mathrm{Subst}\left(\int \right.}{8cd}$$

$$= \frac{8b^2 - \frac{3ad(8bc-5ad)}{c^2}}{8c\sqrt{c+dx^2}} - \frac{a^2}{4cx^4\sqrt{c+dx^2}} - \frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}} - \frac{\left(8b^2 - \frac{3ad(8bc-5ad)}{c^2}\right)\tanh^{-1}\left(\right.}{8c^{3/2}}$$

Mathematica [C] time = 0.04, size = 89, normalized size = 0.61

$$\frac{x^4\left(15a^2d^2-24abcd+8b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)+ac\left(-2ac+5adx^2-8bcx^2\right)}{8c^3x^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*c*(-2*a*c - 8*b*c*x^2 + 5*a*d*x^2) + (8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^
2)*x^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])/(8*c^3*x^4*Sqrt[c +
d*x^2])
```

IntegrateAlgebraic [A] time = 0.27, size = 132, normalized size = 0.91

$$\frac{\left(-15a^2d^2+24abcd-8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{-2a^2c^2+5a^2cdx^2+15a^2d^2x^4-8abc^2x^2-24abcdx^4+8b^2c^2x^4}{8c^3x^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2*c^2 - 8*a*b*c^2*x^2 + 5*a^2*c*d*x^2 + 8*b^2*c^2*x^4 - 24*a*b*c*d*x^
4 + 15*a^2*d^2*x^4)/(8*c^3*x^4*Sqrt[c + d*x^2]) + ((-8*b^2*c^2 + 24*a*b*c*d
 - 15*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2))
```

fricas [A] time = 1.00, size = 364, normalized size = 2.51

$$\left[\frac{\left(\left(8\,b^{2}c^{2}d-24\,abcd^{2}+15\,a^{2}d^{3}\right)x^{6}+\left(8\,b^{2}c^{3}-24\,abc^{2}d+15\,a^{2}cd^{2}\right)x^{4}\right)\sqrt{c}\,\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\,\left(2\,a^{2}c^{3}-\left(8\,b^{2}c^{3}-24\,abc^{2}d+15\,a^{2}cd^{2}\right)x^{4}+\left(8\,abc^{3}-5\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{16\,\left(c^{4}dx^{6}+c^{5}x^{4}\right)}, \frac{\left(\left(8\,b^{2}c^{2}d-24\,abcd^{2}+15\,a^{2}d^{3}\right)x^{6}+\left(8\,b^{2}c^{3}-24\,abc^{2}d+15\,a^{2}cd^{2}\right)x^{4}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-\left(2\,a^{2}c^{3}-\left(8\,b^{2}c^{3}-24\,abc^{2}d+15\,a^{2}cd^{2}\right)x^{4}+\left(8\,abc^{3}-5\,a^{2}c^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{8\,\left(c^{4}dx^{6}+c^{5}x^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b
*c^2*d + 15*a^2*c*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) - 2*(2*a^2*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4
 + (8*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4), 1
/8*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*b*c^
2*d + 15*a^2*c*d^2)*x^4)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2
*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4 + (8*a*b*c^3 - 5*a^2*c
^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4)]
```

giac [A] time = 0.35, size = 163, normalized size = 1.12

$$\frac{\left(8\,b^{2}c^{2}-24\,abcd+15\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}\,c^{3}}+\frac{b^{2}c^{2}-2\,abcd+a^{2}d^{2}}{\sqrt{dx^{2}+c}\,c^{3}}-\frac{8\,\left(dx^{2}+c\right)^{\frac{3}{2}}abcd-8\,\sqrt{dx^{2}+c}\,abc^{2}d-7\,\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{2}+9\,\sqrt{dx^{2}+c}\,a^{2}cd^{2}}{8\,c^{3}d^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(sqrt(-c)*c^3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c^3) - 1/
8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 7*(d*x^2 + c
)^(3/2)*a^2*d^2 + 9*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^3*d^2*x^4)
```

maple [A] time = 0.01, size = 211, normalized size = 1.46

$$-\frac{15a^{2}d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{8c^{\frac{7}{2}}}+\frac{3abd\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{c^{\frac{5}{2}}}-\frac{b^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{c^{\frac{3}{2}}}+\frac{15a^{2}d^{2}}{8\sqrt{d\,x^{2}+c}\,c^{3}}-\frac{3abd}{\sqrt{d\,x^{2}+c}\,c^{2}}+\frac{b^{2}}{\sqrt{d\,x^{2}+c}\,c}+\frac{5a^{2}d}{8\sqrt{d\,x^{2}+c}\,c^{2}x^{2}}-\frac{ab}{\sqrt{d\,x^{2}+c}\,c\,x^{2}}-\frac{a^{2}}{4\sqrt{d\,x^{2}+c}\,c\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4*a^2/c/x^4/(d*x^2+c)^(1/2)+5/8*a^2*d/c^2/x^2/(d*x^2+c)^(1/2)+15/8*a^2*d
^2/c^3/(d*x^2+c)^(1/2)-15/8*a^2*d^2/c^(7/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/
```

```
2))/x)-a*b/c/x^2/(d*x^2+c)^(1/2)-3*a*b*d/c^2/(d*x^2+c)^(1/2)+3*a*b*d/c^(5/2
)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+b^2/c/(d*x^2+c)^(1/2)-b^2/c^(3/2)*l
n((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.85, size = 177, normalized size = 1.22

$$-\frac{b^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{3}{2}}}+\frac{3\,abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{5}{2}}}-\frac{15\,a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{8\,c^{\frac{7}{2}}}+\frac{b^2}{\sqrt{dx^2+c}\,c}-\frac{3\,abd}{\sqrt{dx^2+c}\,c^2}+\frac{15\,a^2d^2}{8\,\sqrt{dx^2+c}\,c^3}-\frac{ab}{\sqrt{dx^2+c}\,cx^2}+\frac{5\,a^2d}{8\,\sqrt{dx^2+c}\,c^2x^2}-\frac{a^2}{4\,\sqrt{dx^2+c}\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + 3*a*b*d*arcsinh(c/(sqrt(c*d)*a
bs(x)))/c^(5/2) - 15/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(7/2) + b^2/
(sqrt(d*x^2 + c)*c) - 3*a*b*d/(sqrt(d*x^2 + c)*c^2) + 15/8*a^2*d^2/(sqrt(d*
x^2 + c)*c^3) - a*b/(sqrt(d*x^2 + c)*c*x^2) + 5/8*a^2*d/(sqrt(d*x^2 + c)*c^
2*x^2) - 1/4*a^2/(sqrt(d*x^2 + c)*c*x^4)
```

mupad [B] time = 1.06, size = 179, normalized size = 1.23

$$\frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c}-\frac{\left(d\,x^2+c\right)\,\left(25\,a^2\,d^2-40\,a\,b\,c\,d+16\,b^2\,c^2\right)}{8\,c^2}+\frac{{\left(d\,x^2+c\right)}^2\,\left(15\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^3}}{{\left(d\,x^2+c\right)}^{5/2}-2\,c\,{\left(d\,x^2+c\right)}^{3/2}+c^2\,\sqrt{d\,x^2+c}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(15\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/c - ((c + d*x^2)*(25*a^2*d^2 + 16*b^2*c^2
- 40*a*b*c*d))/(8*c^2) + ((c + d*x^2)^2*(15*a^2*d^2 + 8*b^2*c^2 - 24*a*b*c*
d))/(8*c^3))/((c + d*x^2)^(5/2) - 2*c*(c + d*x^2)^(3/2) + c^2*(c + d*x^2)^(
1/2)) - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(15*a^2*d^2 + 8*b^2*c^2 - 24*a*b*
c*d))/(8*c^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^2\right)^2}{x^5\left(c + dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(3/2)), x)
```

3.640 $\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{2dx\left(15b^2c^2-8ad(5bc-3ad)\right)}{15c^4\sqrt{c+dx^2}}-\frac{15b^2c^2-8ad(5bc-3ad)}{15c^3x\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 271, 191}

$$-\frac{a^2}{5cx^5\sqrt{c+dx^2}}-\frac{2dx\left(15b^2c^2-8ad(5bc-3ad)\right)}{15c^4\sqrt{c+dx^2}}-\frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}}-\frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(5*c*x^5*Sqrt[c + d*x^2]) - (2*a*(5*b*c - 3*a*d))/(15*c^2*x^3*Sqrt[c +
 d*x^2]) - (15*b^2 - (8*a*d*(5*b*c - 3*a*d))/c^2)/(15*c*x*Sqrt[c + d*x^2])
- (2*d*(15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*x)/(15*c^4*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{3/2}}\,dx = -\frac{a^2}{5cx^5\sqrt{c+dx^2}} + \frac{\int \frac{2a(5bc-3ad)+5b^2cx^2}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{5c}$$

$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{1}{15}\left(-15b^2+\frac{8ad(5bc-3ad)}{c^2}\right)\int \frac{1}{x^2\left(c+dx^2\right)^{3/2}}$$

$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}} - \frac{\left(2d\left(15b^2-\frac{8ad(5bc-3ad)}{c^2}\right)\right)\int}{15c}$$

$$= -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2-\frac{8ad(5bc-3ad)}{c^2}}{15cx\sqrt{c+dx^2}} - \frac{2d\left(15b^2-\frac{8ad(5bc-3ad)}{c^2}\right)x}{15c^2\sqrt{c+dx^2}}$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.74

$$\sqrt{c+dx^2}\left(\frac{-33a^2d^2+50abcd-15b^2c^2}{15c^4x} - \frac{a^2}{5c^2x^5} - \frac{dx(bc-ad)^2}{c^4\left(c+dx^2\right)} + \frac{a(9ad-10bc)}{15c^3x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-1/5*a^2/(c^2*x^5) + (a*(-10*b*c + 9*a*d))/(15*c^3*x^3) +
(-15*b^2*c^2 + 50*a*b*c*d - 33*a^2*d^2)/(15*c^4*x) - (d*(b*c - a*d)^2*x)/(c
^4*(c + d*x^2)))
```

IntegrateAlgebraic [A] time = 0.20, size = 120, normalized size = 0.85

$$\frac{-3a^2c^3+6a^2c^2dx^2-24a^2cd^2x^4-48a^2d^3x^6-10abc^3x^2+40abc^2dx^4+80abcd^2x^6-15b^2c^3x^4-30b^2c^2dx^6}{15c^4x^5\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*a^2*c^3 - 10*a*b*c^3*x^2 + 6*a^2*c^2*d*x^2 - 15*b^2*c^3*x^4 + 40*a*b*c^
2*d*x^4 - 24*a^2*c*d^2*x^4 - 30*b^2*c^2*d*x^6 + 80*a*b*c*d^2*x^6 - 48*a^2*d
^3*x^6)/(15*c^4*x^5*Sqrt[c + d*x^2])
```

fricas [A] time = 1.67, size = 121, normalized size = 0.86

$$-\frac{\left(2 \left(15 \, b^{2} c^{2} d - 40 \, a b c d^{2} + 24 \, a^{2} d^{3}\right) x^{6} + 3 \, a^{2} c^{3} + \left(15 \, b^{2} c^{3} - 40 \, a b c^{2} d + 24 \, a^{2} c d^{2}\right) x^{4} + 2 \, \left(5 \, a b c^{3} - 3 \, a^{2} c^{2} d\right) x^{2}\right) \sqrt{d x^{2} + c}}{15 \, \left(c^{4} d x^{7} + c^{5} x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(15*b^2*c^2*d - 40*a*b*c*d^2 + 24*a^2*d^3)*x^6 + 3*a^2*c^3 + (15*b
^2*c^3 - 40*a*b*c^2*d + 24*a^2*c*d^2)*x^4 + 2*(5*a*b*c^3 - 3*a^2*c^2*d)*x^2
)*sqrt(d*x^2 + c)/(c^4*d*x^7 + c^5*x^5)
```

giac [B] time = 0.50, size = 452, normalized size = 3.21

$$-\frac{\left(b^{2} c^{2} d - 2 \, a b c d^{2} + a^{2} d^{3}\right) x}{\sqrt{d x^{2} + c} c^{4}} + \frac{2 \, \left(15 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{8} b^{2} c^{2} \sqrt{d} - 30 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{8} a b c d^{\frac{3}{2}} + 15 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{8} a^{2} d^{\frac{5}{2}} - 60 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{6} b^{2} c^{3} \sqrt{d} + 180 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{6} a b c^{2} d^{\frac{3}{2}} - 90 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{6} a^{2} c d^{\frac{5}{2}} + 90 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} b^{2} c^{4} \sqrt{d} - 320 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} a b c^{3} d^{\frac{3}{2}} + 240 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{4} a^{2} c^{2} d^{\frac{5}{2}} - 60 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} b^{2} c^{5} \sqrt{d} + 220 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} a b c^{4} d^{\frac{3}{2}} - 150 \, \left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)^{2} a^{2} c^{3} d^{\frac{5}{2}} + 15 \, b^{2} c^{6} \sqrt{d} - 50 \, a b c^{5} d^{\frac{3}{2}} + 33 \, a^{2} c^{4} d^{\frac{5}{2}}\right)}{15 \, \left({\left(\sqrt{d} x - \sqrt{d x^{2} + c}\right)}^{2} - c\right)^{5} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x/(sqrt(d*x^2 + c)*c^4) + 2/15*(15*(sq
rt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 30*(sqrt(d)*x - sqrt(d*x^2 +
 c))^8*a*b*c*d^(3/2) + 15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*
(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 180*(sqrt(d)*x - sqrt(d*x
^2 + c))^6*a*b*c^2*d^(3/2) - 90*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c*d^(5/
2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 320*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 240*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^
2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) + 220*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 150*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 50*a*b*c^5*d^(3/2) + 33*a^2
*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^3)
```

maple [A] time = 0.01, size = 117, normalized size = 0.83

$$-\frac{48 a^{2} d^{3} x^{6}-80 a b c \,d^{2} x^{6}+30 b^{2} c^{2} d \,x^{6}+24 a^{2} c \,d^{2} x^{4}-40 a b \,c^{2} d \,x^{4}+15 b^{2} c^{3} x^{4}-6 a^{2} c^{2} d \,x^{2}+10 a b \,c^{3} x^{2}+3 a^{2} c^{3}}{15 \sqrt{d \,x^{2}+c}\, c^{4} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/15*(48*a^2*d^3*x^6-80*a*b*c*d^2*x^6+30*b^2*c^2*d*x^6+24*a^2*c*d^2*x^4-40
*a*b*c^2*d*x^4+15*b^2*c^3*x^4-6*a^2*c^2*d*x^2+10*a*b*c^3*x^2+3*a^2*c^3)/(d*
x^2+c)^(1/2)/x^5/c^4
```

maxima [A] time = 0.83, size = 184, normalized size = 1.30

$$-\frac{2\,b^2 d x}{\sqrt{dx^2+c}\,c^2}+\frac{16\,abd^2 x}{3\,\sqrt{dx^2+c}\,c^3}-\frac{16\,a^2 d^3 x}{5\,\sqrt{dx^2+c}\,c^4}-\frac{b^2}{\sqrt{dx^2+c}\,cx}+\frac{8\,abd}{3\,\sqrt{dx^2+c}\,c^2 x}-\frac{8\,a^2 d^2}{5\,\sqrt{dx^2+c}\,c^3 x}-\frac{2\,ab}{3\,\sqrt{dx^2+c}\,cx^3}+\frac{2\,a^2 d}{5\,\sqrt{dx^2+c}\,c^2 x^3}-\frac{a^2}{5\,\sqrt{dx^2+c}\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^2*d*x/(sqrt(d*x^2 + c)*c^2) + 16/3*a*b*d^2*x/(sqrt(d*x^2 + c)*c^3) - 1
6/5*a^2*d^3*x/(sqrt(d*x^2 + c)*c^4) - b^2/(sqrt(d*x^2 + c)*c*x) + 8/3*a*b*d
/(sqrt(d*x^2 + c)*c^2*x) - 8/5*a^2*d^2/(sqrt(d*x^2 + c)*c^3*x) - 2/3*a*b/(s
qrt(d*x^2 + c)*c*x^3) + 2/5*a^2*d/(sqrt(d*x^2 + c)*c^2*x^3) - 1/5*a^2/(sqrt
(d*x^2 + c)*c*x^5)
```

mupad [B] time = 0.85, size = 116, normalized size = 0.82

$$-\frac{3\,a^2\,c^3-6\,a^2\,c^2\,d\,x^2+24\,a^2\,c\,d^2\,x^4+48\,a^2\,d^3\,x^6+10\,a\,b\,c^3\,x^2-40\,a\,b\,c^2\,d\,x^4-80\,a\,b\,c\,d^2\,x^6+15\,b^2\,c^3\,x^4+30\,b^2\,c^2\,d\,x^6}{15\,c^4\,x^5\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x)
```

```
[Out] -(3*a^2*c^3 + 15*b^2*c^3*x^4 + 48*a^2*d^3*x^6 - 6*a^2*c^2*d*x^2 + 24*a^2*c*
d^2*x^4 + 30*b^2*c^2*d*x^6 + 10*a*b*c^3*x^2 - 40*a*b*c^2*d*x^4 - 80*a*b*c*d
^2*x^6)/(15*c^4*x^5*(c + d*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(3/2)), x)
```

3.641 $\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{a^2}{6cx^6\sqrt{c+dx^2}}+\frac{d\left(24b^2c^2-5ad(12bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}-\frac{d\left(24b^2c^2-5ad(12bc-7ad)\right)}{16c^4\sqrt{c+dx^2}}-\frac{24b^2c^2-5ad(12}{48c^3x^2\sqrt{c -}}$$

Rubi [A] time = 0.22, antiderivative size = 193, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 51, 63, 208}

$$\frac{35a^2d^2-60abcd+24b^2c^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\left(24b^2c^2-5ad(12bc-7ad)\right)}{16c^4x^2}+\frac{d\left(24b^2c^2-5ad(12bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] -a^2/(6*c*x^6*Sqrt[c + d*x^2]) - (a*(12*b*c - 7*a*d))/(24*c^2*x^4*Sqrt[c +
d*x^2]) + (24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)/(24*c^3*x^2*Sqrt[c + d*x^2
]) - ((24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*Sqrt[c + d*x^2])/(16*c^4*x^2) +
 (d*(24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(16*c^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x^4(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{a^2}{6cx^6\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}a(12bc-7ad)+3b^2cx}{x^3(c+dx)^{3/2}}\,dx,x,x^2\right)}{6c}\\
&= -\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}+\frac{1}{48}\left(24b^2-\frac{5ad(12bc-7ad)}{c^2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2(c+dx)}\right.\\
&= -\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}+\frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}}+\frac{\left(24b^2c^2-60abcd+3\right.}{}\\
&= -\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}+\frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{\left(24b^2c^2-60abcd+3\right.}{16c^4x}\\
&= -\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}+\frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{\left(24b^2c^2-60abcd+3\right.}{16c^4x}\\
&= -\frac{a^2}{6cx^6\sqrt{c+dx^2}}-\frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}}+\frac{24b^2c^2-60abcd+35a^2d^2}{24c^3x^2\sqrt{c+dx^2}}-\frac{\left(24b^2c^2-60abcd+3\right.}{16c^4x}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 92, normalized size = 0.48

$$\frac{dx^6\left(-35a^2d^2+60abcd-24b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{dx^2}{c}+1\right)+ac^2\left(-4ac+7adx^2-12bcx^2\right)}{24c^4x^6\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*c^2*(-4*a*c - 12*b*c*x^2 + 7*a*d*x^2) + d*(-24*b^2*c^2 + 60*a*b*c*d - 35
*a^2*d^2)*x^6*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (d*x^2)/c])/(24*c^4*x^6*S
qrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.28, size = 174, normalized size = 0.92

$$\frac{\left(35a^2d^3-60abcd^2+24b^2c^2d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}+\frac{-8a^2c^3+14a^2c^2dx^2-35a^2cd^2x^4-105a^2d^3x^6-24abc^3x^2+60abc^2dx^4+180abcd^2x^6-24b^2c^3x^4-72b^2c^2dx^6}{48c^4x^6\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-8*a^2*c^3 - 24*a*b*c^3*x^2 + 14*a^2*c^2*d*x^2 - 24*b^2*c^3*x^4 + 60*a*b*c
^2*d*x^4 - 35*a^2*c*d^2*x^4 - 72*b^2*c^2*d*x^6 + 180*a*b*c*d^2*x^6 - 105*a^
2*d^3*x^6)/(48*c^4*x^6*Sqrt[c + d*x^2]) + ((24*b^2*c^2*d - 60*a*b*c*d^2 + 3
5*a^2*d^3)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(9/2))
```

fricas [A] time = 1.64, size = 447, normalized size = 2.35

$$\left[\frac{3\left(\left(24\,b^{2}c^{2}d^{2}-60\,abcd^{3}+35\,a^{2}d^{4}\right)x^{8}+\left(24\,b^{2}c^{3}d-60\,abc^{2}d^{2}+35\,a^{2}cd^{3}\right)x^{6}\right)\sqrt{c}\,\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)-2\left(3\left(24\,b^{2}c^{3}d-60\,abc^{2}d^{2}+35\,a^{2}cd^{3}\right)x^{6}+8\,a^{2}c^{4}+\left(24\,b^{2}c^{4}-60\,abc^{3}d+35\,a^{2}c^{2}d^{2}\right)x^{4}+2\left(12\,abc^{4}-7\,a^{2}c^{3}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{96\left(c^{5}dx^{8}+c^{6}x^{6}\right)},\ -\frac{3\left(\left(24\,b^{2}c^{2}d^{2}-60\,abcd^{3}+35\,a^{2}d^{4}\right)x^{8}+\left(24\,b^{2}c^{3}d-60\,abc^{2}d^{2}+35\,a^{2}cd^{3}\right)x^{6}\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(3\left(24\,b^{2}c^{3}d-60\,abc^{2}d^{2}+35\,a^{2}cd^{3}\right)x^{6}+8\,a^{2}c^{4}+\left(24\,b^{2}c^{4}-60\,abc^{3}d+35\,a^{2}c^{2}d^{2}\right)x^{4}+2\left(12\,abc^{4}-7\,a^{2}c^{3}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{48\left(c^{5}dx^{8}+c^{6}x^{6}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d
 - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 +
c)*sqrt(c) + 2*c)/x^2) - 2*(3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3
)*x^6 + 8*a^2*c^4 + (24*b^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(1
2*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6), -1/48
*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d - 60*
a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + (3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + 8*a^2*c^4 + (24*b
^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(12*a*b*c^4 - 7*a^2*c^3*d)*
x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6)]
```

giac [A] time = 0.40, size = 267, normalized size = 1.41

$$-\frac{\left(24\,b^{2}c^{2}d-60\,abcd^{2}+35\,a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{16\,\sqrt{-c}\,c^{4}}-\frac{b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}}{\sqrt{dx^{2}+c}\,c^{4}}-\frac{24\left(dx^{2}+c\right)^{\frac{5}{2}}b^{2}c^{2}d-48\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{3}d+24\,\sqrt{dx^{2}+c}\,b^{2}c^{4}d-84\left(dx^{2}+c\right)^{\frac{5}{2}}abcd^{2}+192\left(dx^{2}+c\right)^{\frac{3}{2}}abc^{2}d^{2}-108\,\sqrt{dx^{2}+c}\,abc^{3}d^{2}+57\left(dx^{2}+c\right)^{\frac{5}{2}}a^{2}d^{3}-136\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}cd^{3}+87\,\sqrt{dx^{2}+c}\,a^{2}c^{2}d^{3}}{48\,c^{4}d^{3}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqr
t(-c))/(sqrt(-c)*c^4) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)/(sqrt(d*x^2 + c
)*c^4) - 1/48*(24*(d*x^2 + c)^(5/2)*b^2*c^2*d - 48*(d*x^2 + c)^(3/2)*b^2*c^
3*d + 24*sqrt(d*x^2 + c)*b^2*c^4*d - 84*(d*x^2 + c)^(5/2)*a*b*c*d^2 + 192*(
d*x^2 + c)^(3/2)*a*b*c^2*d^2 - 108*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 57*(d*x^2
+ c)^(5/2)*a^2*d^3 - 136*(d*x^2 + c)^(3/2)*a^2*c*d^3 + 87*sqrt(d*x^2 + c)*a
^2*c^2*d^3)/(c^4*d^3*x^6)
```

maple [A] time = 0.02, size = 281, normalized size = 1.48

$$\frac{35a^{2}d^{3}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{16c^{\frac{9}{2}}}-\frac{15ab\,d^{2}\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{4c^{\frac{7}{2}}}+\frac{3b^{2}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2c^{\frac{5}{2}}}-\frac{35a^{2}d^{3}}{16\sqrt{d\,x^{2}+c}\,c^{4}}+\frac{15ab\,d^{2}}{4\sqrt{d\,x^{2}+c}\,c^{3}}-\frac{3b^{2}d}{2\sqrt{d\,x^{2}+c}\,c^{2}}-\frac{35a^{2}d^{2}}{48\sqrt{d\,x^{2}+c}\,c^{3}x^{2}}+\frac{5abd}{4\sqrt{d\,x^{2}+c}\,c^{2}x^{2}}-\frac{b^{2}}{2\sqrt{d\,x^{2}+c}\,c\,x^{2}}+\frac{7a^{2}d}{24\sqrt{d\,x^{2}+c}\,c^{2}x^{4}}-\frac{ab}{2\sqrt{d\,x^{2}+c}\,c\,x^{4}}-\frac{a^{2}}{6\sqrt{d\,x^{2}+c}\,c\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/6*a^2/c/x^6/(d*x^2+c)^(1/2)+7/24*a^2*d/c^2/x^4/(d*x^2+c)^(1/2)-35/48*a^2
*d^2/c^3/x^2/(d*x^2+c)^(1/2)-35/16*a^2*d^3/c^4/(d*x^2+c)^(1/2)+35/16*a^2*d^
3/c^(9/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/2*b^2/c/x^2/(d*x^2+c)^(1/
2)-3/2*b^2*d/c^2/(d*x^2+c)^(1/2)+3/2*b^2*d/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2
)*c^(1/2))/x)-1/2*a*b/c/x^4/(d*x^2+c)^(1/2)+5/4*a*b*d/c^2/x^2/(d*x^2+c)^(1/
2)+15/4*a*b*d^2/c^3/(d*x^2+c)^(1/2)-15/4*a*b*d^2/c^(7/2)*ln((2*c+2*(d*x^2+c
)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.95, size = 247, normalized size = 1.30

$$\frac{3\,b^{2}d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{5}{2}}}-\frac{15\,abd^{2}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{4\,c^{\frac{7}{2}}}+\frac{35\,a^{2}d^{3}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{16\,c^{\frac{9}{2}}}-\frac{3\,b^{2}d}{2\,\sqrt{dx^{2}+c}\,c^{2}}+\frac{15\,abd^{2}}{4\,\sqrt{dx^{2}+c}\,c^{3}}-\frac{35\,a^{2}d^{3}}{16\,\sqrt{dx^{2}+c}\,c^{4}}-\frac{b^{2}}{2\,\sqrt{dx^{2}+c}\,cx^{2}}+\frac{5\,abd}{4\,\sqrt{dx^{2}+c}\,c^{2}x^{2}}-\frac{35\,a^{2}d^{2}}{48\,\sqrt{dx^{2}+c}\,c^{3}x^{2}}-\frac{ab}{2\,\sqrt{dx^{2}+c}\,cx^{4}}+\frac{7\,a^{2}d}{24\,\sqrt{dx^{2}+c}\,c^{2}x^{4}}-\frac{a^{2}}{6\,\sqrt{dx^{2}+c}\,cx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) - 15/4*a*b*d^2*arcsinh(c/(s
qrt(c*d)*abs(x)))/c^(7/2) + 35/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(
9/2) - 3/2*b^2*d/(sqrt(d*x^2 + c)*c^2) + 15/4*a*b*d^2/(sqrt(d*x^2 + c)*c^3)
 - 35/16*a^2*d^3/(sqrt(d*x^2 + c)*c^4) - 1/2*b^2/(sqrt(d*x^2 + c)*c*x^2) +
5/4*a*b*d/(sqrt(d*x^2 + c)*c^2*x^2) - 35/48*a^2*d^2/(sqrt(d*x^2 + c)*c^3*x^
2) - 1/2*a*b/(sqrt(d*x^2 + c)*c*x^4) + 7/24*a^2*d/(sqrt(d*x^2 + c)*c^2*x^4)
 - 1/6*a^2/(sqrt(d*x^2 + c)*c*x^6)
```

mupad [B] time = 1.34, size = 246, normalized size = 1.29

$$\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-60\,a\,b\,c\,d+24\,b^2\,c^2\right)}{16\,c^{9/2}}-\frac{\frac{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}{c}-\frac{\left(d\,x^2+c\right)\,\left(77\,a^2\,d^3-132\,a\,b\,c\,d^2+56\,b^2\,c^2\,d\right)}{16\,c^2}+\frac{{\left(d\,x^2+c\right)}^2\,\left(35\,a^2\,d^3-60\,a\,b\,c\,d^2+24\,b^2\,c^2\,d\right)}{6\,c^3}-\frac{{\left(d\,x^2+c\right)}^3\,\left(35\,a^2\,d^3-60\,a\,b\,c\,d^2+24\,b^2\,c^2\,d\right)}{16\,c^4}}{3\,c\,{\left(d\,x^2+c\right)}^{5/2}-{\left(d\,x^2+c\right)}^{7/2}+c^3\,\sqrt{d\,x^2+c}-3\,c^2\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x)
```

```
[Out] (d*atanh((c + d*x^2)^(1/2)/c^(1/2))*(35*a^2*d^2 + 24*b^2*c^2 - 60*a*b*c*d))
/(16*c^(9/2)) - ((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/c - ((c + d*x^2)*(77*a
^2*d^3 + 56*b^2*c^2*d - 132*a*b*c*d^2))/(16*c^2) + ((c + d*x^2)^2*(35*a^2*d
^3 + 24*b^2*c^2*d - 60*a*b*c*d^2))/(6*c^3) - ((c + d*x^2)^3*(35*a^2*d^3 + 2
4*b^2*c^2*d - 60*a*b*c*d^2))/(16*c^4))/(3*c*(c + d*x^2)^(5/2) - (c + d*x^2)
^(7/2) + c^3*(c + d*x^2)^(1/2) - 3*c^2*(c + d*x^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.642 $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=202

$$\frac{\left(8a^2d^2-40abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}-\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2-40abcd+35b^2c^2\right)}{8cd^4}+\frac{x^3\left(8a^2d^2-40abcd+35b^2\right.}{12cd^3\sqrt{c+dx^2}}$$

Rubi [**A**] time = 0.15, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {463, 459, 288, 321, 217, 206}

$$\frac{x^3\left(8a^2d^2-40abcd+35b^2c^2\right)}{12cd^3\sqrt{c+dx^2}}-\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2-40abcd+35b^2c^2\right)}{8cd^4}+\frac{\left(8a^2d^2-40abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}+\frac{x^5(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{b^2x^5}{4d^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d
 + 8*a^2*d^2)*x^3)/(12*c*d^3*Sqrt[c + d*x^2]) + (b^2*x^5)/(4*d^2*Sqrt[c + d
*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(8*c*d^4
) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4 \left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx &= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{x^4\left(-3a^2d^2+5(bc-ad)^2-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2} \\
&= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)\int \frac{x^4}{\left(c+dx^2\right)^{3/2}}\,dx}{12cd^2} \\
&= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8\right.}{4cd} \\
&= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8\right.}{8cd^4} \\
&= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8\right.}{8cd^4} \\
&= \frac{(bc-ad)^2x^5}{3cd^2\left(c+dx^2\right)^{3/2}} + \frac{\left(35b^2c^2-40abcd+8a^2d^2\right)x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{\left(35b^2c^2-40abcd+8\right.}{8cd^4}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 156, normalized size = 0.77

$$\frac{\left(8a^2d^2-40abcd+35b^2c^2\right)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{8d^{9/2}}+\frac{x\left(-8a^2d^2\left(3c+4dx^2\right)+8abd\left(15c^2+20cdx^2+3d^2x^4\right)-\left(b^2\left(105c^3+140c^2dx^2+21cd^2x^4-6d^3x^6\right)\right)\right)}{24d^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(-8*a^2*d^2*(3*c + 4*d*x^2) + 8*a*b*d*(15*c^2 + 20*c*d*x^2 + 3*d^2*x^4)
- b^2*(105*c^3 + 140*c^2*d*x^2 + 21*c*d^2*x^4 - 6*d^3*x^6)))/(24*d^4*(c + d
*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqr
t[c + d*x^2]])/(8*d^(9/2))
```

IntegrateAlgebraic [A] time = 0.35, size = 168, normalized size = 0.83

$$\frac{\left(-8a^2d^2+40abcd-35b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8d^{9/2}}+\frac{-24a^2cd^2x-32a^2d^3x^3+120abc^2dx+160abcd^2x^3+24abd^3x^5-105b^2c^3x-140b^2c^2dx^3-21b^2cd^2x^5+6b^2d^3x^7}{24d^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-105*b^2*c^3*x + 120*a*b*c^2*d*x - 24*a^2*c*d^2*x - 140*b^2*c^2*d*x^3 + 16
0*a*b*c*d^2*x^3 - 32*a^2*d^3*x^3 - 21*b^2*c*d^2*x^5 + 24*a*b*d^3*x^5 + 6*b^
2*d^3*x^7)/(24*d^4*(c + d*x^2)^(3/2)) + ((-35*b^2*c^2 + 40*a*b*c*d - 8*a^2*
d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(9/2))
```

fricas [A] time = 1.36, size = 522, normalized size = 2.58

$$\left[\frac{3\left(35\,b^2c^4-40\,abc^3d+8\,a^2c^2d^2+\left(35\,b^2c^2d^2-40\,abcd^3+8\,a^2d^4\right)x^4+2\left(35\,b^2c^3d-40\,abc^2d^2+8\,a^2cd^3\right)x^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\left(6\,b^2d^4x^7-3\left(7\,b^2cd^3-8\,abd^4\right)x^5-4\left(35\,b^2c^2d^2-40\,abcd^3+8\,a^2d^4\right)x^3-3\left(35\,b^2c^3d-40\,abc^2d^2+8\,a^2cd^3\right)x\right)\sqrt{dx^2+c}}{48\left(d^7x^4+2\,cd^6x^2+c^2d^5\right)},-\frac{3\left(35\,b^2c^4-40\,abc^3d+8\,a^2c^2d^2+\left(35\,b^2c^2d^2-40\,abcd^3+8\,a^2d^4\right)x^4+2\left(35\,b^2c^3d-40\,abc^2d^2+8\,a^2cd^3\right)x^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(6\,b^2d^4x^7-3\left(7\,b^2cd^3-8\,abd^4\right)x^5-4\left(35\,b^2c^2d^2-40\,abcd^3+8\,a^2d^4\right)x^3-3\left(35\,b^2c^3d-40\,abc^2d^2+8\,a^2cd^3\right)x\right)\sqrt{dx^2+c}}{24\left(d^7x^4+2\,cd^6x^2+c^2d^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*b^2*c^4 - 40*a*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*
a*b*c*d^3 + 8*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3
)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d
^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5 - 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3
 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt
(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2*d^5), -1/24*(3*(35*b^2*c^4 - 40*a
*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^4
+ 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5
- 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*
a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2
*d^5)]
```

giac [A] time = 0.51, size = 190, normalized size = 0.94

$$\frac{\left(\left(3\left(\frac{2\,b^2x^2}{d}-\frac{7\,b^2c^2d^5-8\,abcd^6}{cd^7}\right)x^2-\frac{4\left(35\,b^2c^3d^4-40\,abc^2d^5+8\,a^2cd^6\right)}{cd^7}\right)x^2-\frac{3\left(35\,b^2c^4d^3-40\,abc^3d^4+8\,a^2c^2d^5\right)}{cd^7}\right)x}{24\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{\left(35\,b^2c^2-40\,abcd+8\,a^2d^2\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*b^2*x^2/d - (7*b^2*c^2*d^5 - 8*a*b*c*d^6)/(c*d^7))*x^2 - 4*(35*
b^2*c^3*d^4 - 40*a*b*c^2*d^5 + 8*a^2*c*d^6)/(c*d^7))*x^2 - 3*(35*b^2*c^4*d^
3 - 40*a*b*c^3*d^4 + 8*a^2*c^2*d^5)/(c*d^7))*x/(d*x^2 + c)^(3/2) - 1/8*(35*
b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^
(9/2)
```

maple [A] time = 0.02, size = 255, normalized size = 1.26

$$\frac{b^2x^7}{4\left(d\,x^2+c\right)^{\frac{3}{2}}\,d}+\frac{ab\,x^5}{\left(d\,x^2+c\right)^{\frac{3}{2}}\,d}-\frac{7b^2c\,x^5}{8\left(d\,x^2+c\right)^{\frac{3}{2}}\,d^2}-\frac{a^2x^3}{3\left(d\,x^2+c\right)^{\frac{3}{2}}\,d}+\frac{5abc\,x^3}{3\left(d\,x^2+c\right)^{\frac{3}{2}}\,d^2}-\frac{35b^2c^2x^3}{24\left(d\,x^2+c\right)^{\frac{3}{2}}\,d^3}-\frac{a^2x}{\sqrt{d\,x^2+c}\,d^2}+\frac{5abcx}{\sqrt{d\,x^2+c}\,d^3}-\frac{35b^2c^2x}{8\sqrt{d\,x^2+c}\,d^4}+\frac{a^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{d^{\frac{5}{2}}}-\frac{5abc\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{d^{\frac{7}{2}}}+\frac{35b^2c^2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{8d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/4*b^2*x^7/d/(d*x^2+c)^(3/2)-7/8*b^2*c/d^2*x^5/(d*x^2+c)^(3/2)-35/24*b^2*c
^2/d^3*x^3/(d*x^2+c)^(3/2)-35/8*b^2*c^2/d^4*x/(d*x^2+c)^(1/2)+35/8*b^2*c^2/
d^(9/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+a*b*x^5/d/(d*x^2+c)^(3/2)+5/3*a*b*c/d
^2*x^3/(d*x^2+c)^(3/2)+5*a*b*c/d^3*x/(d*x^2+c)^(1/2)-5*a*b*c/d^(7/2)*ln(d^(
1/2)*x+(d*x^2+c)^(1/2))-1/3*a^2*x^3/d/(d*x^2+c)^(3/2)-a^2/d^2*x/(d*x^2+c)^(
1/2)+a^2/d^(5/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [A] time = 0.89, size = 296, normalized size = 1.47

$$\frac{b^{2} x^{7}}{4 \left(d x^{2} + c\right)^{\frac{3}{2}} d} - \frac{7 \, b^{2} c x^{5}}{8 \left(d x^{2} + c\right)^{\frac{3}{2}} d^{2}} + \frac{a b x^{5}}{\left(d x^{2} + c\right)^{\frac{3}{2}} d} - \frac{1}{3} \, a^{2} x \left(\frac{3 \, x^{2}}{\left(d x^{2} + c\right)^{\frac{3}{2}} d} + \frac{2 \, c}{\left(d x^{2} + c\right)^{\frac{3}{2}} d^{2}}\right) - \frac{35 \, b^{2} c^{2} x \left(\frac{3 \, x^{2}}{\left(d x^{2} + c\right)^{\frac{3}{2}} d} + \frac{2 \, c}{\left(d x^{2} + c\right)^{\frac{3}{2}} d^{2}}\right)}{24 \, d^{2}} + \frac{5 \, a b c x \left(\frac{3 \, x^{2}}{\left(d x^{2} + c\right)^{\frac{3}{2}} d} + \frac{2 \, c}{\left(d x^{2} + c\right)^{\frac{3}{2}} d^{2}}\right)}{3 \, d} - \frac{35 \, b^{2} c^{2} x}{24 \, \sqrt{d x^{2} + c} d^{4}} + \frac{5 \, a b c x}{3 \, \sqrt{d x^{2} + c} d^{3}} - \frac{a^{2} x}{3 \, \sqrt{d x^{2} + c} d^{2}} + \frac{35 \, b^{2} c^{2} \operatorname{arsinh}\left(\frac{d x}{\sqrt{c d}}\right)}{8 \, d^{\frac{9}{2}}} - \frac{5 \, a b c \operatorname{arsinh}\left(\frac{d x}{\sqrt{c d}}\right)}{d^{\frac{7}{2}}} + \frac{a^{2} \operatorname{arsinh}\left(\frac{d x}{\sqrt{c d}}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^7/((d*x^2 + c)^(3/2)*d) - 7/8*b^2*c*x^5/((d*x^2 + c)^(3/2)*d^2) +
 a*b*x^5/((d*x^2 + c)^(3/2)*d) - 1/3*a^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2
*c/((d*x^2 + c)^(3/2)*d^2)) - 35/24*b^2*c^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d)
+ 2*c/((d*x^2 + c)^(3/2)*d^2))/d^2 + 5/3*a*b*c*x*(3*x^2/((d*x^2 + c)^(3/2)*
d) + 2*c/((d*x^2 + c)^(3/2)*d^2))/d - 35/24*b^2*c^2*x/(sqrt(d*x^2 + c)*d^4)
 + 5/3*a*b*c*x/(sqrt(d*x^2 + c)*d^3) - 1/3*a^2*x/(sqrt(d*x^2 + c)*d^2) + 35
/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(9/2) - 5*a*b*c*arcsinh(d*x/sqrt(c*d))/
d^(7/2) + a^2*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b x^{2}\right)^{2}}{\left(c + d x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.643 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{b\sqrt{c+dx^2}\,(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4\left(c+dx^2\right)^{3/2}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^4}$$

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{b\sqrt{c+dx^2}\,(3bc-2ad)}{d^4}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{c(bc-ad)^2}{3d^4\left(c+dx^2\right)^{3/2}}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*c - a*d))/(
d^4*Sqrt[c + d*x^2]) - (b*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/d^4 + (b^2*(c +
d*x^2)^(3/2))/(3*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(a+bx)^2}{(c+dx)^{5/2}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\Bigg(\int\Bigg(-\frac{c(bc-ad)^2}{d^3(c+dx)^{5/2}}+\frac{(bc-ad)(3bc-ad)}{d^3(c+dx)^{3/2}}-\frac{b(3bc-2ad)}{d^3\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^3}\Bigg)\,dx,x,x\\
&= \frac{c(bc-ad)^2}{3d^4\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}-\frac{b(3bc-2ad)\sqrt{c+dx^2}}{d^4}+\frac{b^2\left(c+dx^2\right)^{3/2}}{3d^4}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 98, normalized size = 0.89

$$
\frac{-a^2d^2\left(2c+3dx^2\right)+2abd\left(8c^2+12cdx^2+3d^2x^4\right)+b^2\left(-16c^3-24c^2dx^2-6cd^2x^4+d^3x^6\right)}{3d^4\left(c+dx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2*(2*c + 3*d*x^2)) + 2*a*b*d*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4) + b^
2*(-16*c^3 - 24*c^2*d*x^2 - 6*c*d^2*x^4 + d^3*x^6))/(3*d^4*(c + d*x^2)^(3/2
))
```

IntegrateAlgebraic [A] time = 0.06, size = 110, normalized size = 1.00

$$
\frac{-2a^2cd^2-3a^2d^3x^2+16abc^2d+24abcd^2x^2+6abd^3x^4-16b^2c^3-24b^2c^2dx^2-6b^2cd^2x^4+b^2d^3x^6}{3d^4\left(c+dx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-16*b^2*c^3 + 16*a*b*c^2*d - 2*a^2*c*d^2 - 24*b^2*c^2*d*x^2 + 24*a*b*c*d^2
*x^2 - 3*a^2*d^3*x^2 - 6*b^2*c*d^2*x^4 + 6*a*b*d^3*x^4 + b^2*d^3*x^6)/(3*d^
4*(c + d*x^2)^(3/2))
```

fricas [A] time = 1.52, size = 124, normalized size = 1.13

$$
\frac{\left(b^2d^3x^6-16\,b^2c^3+16\,abc^2d-2\,a^2cd^2-6\left(b^2cd^2-abd^3\right)x^4-3\left(8\,b^2c^2d-8\,abcd^2+a^2d^3\right)x^2\right)\sqrt{dx^2+c}}{3\left(d^6x^4+2\,cd^5x^2+c^2d^4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*d^3*x^6 - 16*b^2*c^3 + 16*a*b*c^2*d - 2*a^2*c*d^2 - 6*(b^2*c*d^2 -
 a*b*d^3)*x^4 - 3*(8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(d*x^2 + c
)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

giac [A] time = 0.39, size = 140, normalized size = 1.27

$$-\frac{9\left(dx^2+c\right)b^2c^2-b^2c^3-12\left(dx^2+c\right)abcd+2\,abc^2d+3\left(dx^2+c\right)a^2d^2-a^2cd^2}{3\left(dx^2+c\right)^{\frac{3}{2}}d^4}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2d^8-9\sqrt{dx^2+c}\,b^2cd^8+6\sqrt{dx^2+c}\,abd^9}{3\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x^2 + c)*b^2*c^2 - b^2*c^3 - 12*(d*x^2 + c)*a*b*c*d + 2*a*b*c^2*
d + 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^2 + c)^(3/2)*d^4) + 1/3*((d*x^
2 + c)^(3/2)*b^2*d^8 - 9*sqrt(d*x^2 + c)*b^2*c*d^8 + 6*sqrt(d*x^2 + c)*a*b*
d^9)/d^12
```

maple [A] time = 0.01, size = 108, normalized size = 0.98

$$-\frac{-b^2x^6d^3-6ab\,d^3x^4+6b^2c\,d^2x^4+3a^2d^3x^2-24abc\,d^2x^2+24b^2c^2d\,x^2+2a^2c\,d^2-16ab\,c^2d+16b^2c^3}{3\left(d\,x^2+c\right)^{\frac{3}{2}}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-b^2*d^3*x^6-6*a*b*d^3*x^4+6*b^2*c*d^2*x^4+3*a^2*d^3*x^2-24*a*b*c*d^2
*x^2+24*b^2*c^2*d*x^2+2*a^2*c*d^2-16*a*b*c^2*d+16*b^2*c^3)/(d*x^2+c)^(3/2)/
d^4
```

maxima [A] time = 0.91, size = 181, normalized size = 1.65

$$\frac{b^2x^6}{3\left(dx^2+c\right)^{\frac{3}{2}}d}-\frac{2\,b^2cx^4}{\left(dx^2+c\right)^{\frac{3}{2}}d^2}+\frac{2\,abx^4}{\left(dx^2+c\right)^{\frac{3}{2}}d}-\frac{8\,b^2c^2x^2}{\left(dx^2+c\right)^{\frac{3}{2}}d^3}+\frac{8\,abcx^2}{\left(dx^2+c\right)^{\frac{3}{2}}d^2}-\frac{a^2x^2}{\left(dx^2+c\right)^{\frac{3}{2}}d}-\frac{16\,b^2c^3}{3\left(dx^2+c\right)^{\frac{3}{2}}d^4}+\frac{16\,abc^2}{3\left(dx^2+c\right)^{\frac{3}{2}}d^3}-\frac{2\,a^2c}{3\left(dx^2+c\right)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^6/((d*x^2 + c)^(3/2)*d) - 2*b^2*c*x^4/((d*x^2 + c)^(3/2)*d^2) + 2
*a*b*x^4/((d*x^2 + c)^(3/2)*d) - 8*b^2*c^2*x^2/((d*x^2 + c)^(3/2)*d^3) + 8*
```

```
a*b*c*x^2/((d*x^2 + c)^(3/2)*d^2) - a^2*x^2/((d*x^2 + c)^(3/2)*d) - 16/3*b^
2*c^3/((d*x^2 + c)^(3/2)*d^4) + 16/3*a*b*c^2/((d*x^2 + c)^(3/2)*d^3) - 2/3*
a^2*c/((d*x^2 + c)^(3/2)*d^2)
```

mupad [B] time = 0.78, size = 107, normalized size = 0.97

$$-\frac{2\,a^2\,c\,d^2+3\,a^2\,d^3\,x^2-16\,a\,b\,c^2\,d-24\,a\,b\,c\,d^2\,x^2-6\,a\,b\,d^3\,x^4+16\,b^2\,c^3+24\,b^2\,c^2\,d\,x^2+6\,b^2\,c\,d^2\,x^4-b^2\,d^3\,x^6}{3\,d^4\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] -(16*b^2*c^3 + 2*a^2*c*d^2 + 3*a^2*d^3*x^2 - b^2*d^3*x^6 + 24*b^2*c^2*d*x^2
 + 6*b^2*c*d^2*x^4 - 16*a*b*c^2*d - 6*a*b*d^3*x^4 - 24*a*b*c*d^2*x^2)/(3*d^
4*(c + d*x^2)^(3/2))
```

sympy [A] time = 2.95, size = 454, normalized size = 4.13

$$\begin{cases} - \frac{2a^2cd^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{3a^2d^3x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{16abc^2d}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{24abcd^2x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{6abd^3x^4}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{16b^2c^3}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{24b^2c^2dx^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{6b^2cd^2x^4}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{b^2d^3x^6}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} & \text{for}\: d \neq 0 \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*c*d**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c +
 d*x**2)) - 3*a**2*d**3*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(
c + d*x**2)) + 16*a*b*c**2*d/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(
c + d*x**2)) + 24*a*b*c*d**2*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*
sqrt(c + d*x**2)) + 6*a*b*d**3*x**4/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**
2*sqrt(c + d*x**2)) - 16*b**2*c**3/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2
*sqrt(c + d*x**2)) - 24*b**2*c**2*d*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**
5*x**2*sqrt(c + d*x**2)) - 6*b**2*c*d**2*x**4/(3*c*d**4*sqrt(c + d*x**2) +
3*d**5*x**2*sqrt(c + d*x**2)) + b**2*d**3*x**6/(3*c*d**4*sqrt(c + d*x**2) +
 3*d**5*x**2*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2*x**4/4 + a*b*x**6/3 + b**
2*x**8/8)/c**(5/2), True))
```

3.644 $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{x^3(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

Rubi **[A]** time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {463, 455, 388, 217, 206}

$$\frac{x^3(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2bx(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d)*x)/(d^3*
Sqrt[c + d*x^2]) + (b^2*x*Sqrt[c + d*x^2])/(2*d^3) - (b*(5*b*c - 4*a*d)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx=\frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}-\frac{\int \frac{x^2\left(3bc(bc-2ad)-3b^2cdx^2\right)}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd^2}$$

$$=\frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{\int \frac{-6bcd(bc-ad)+3b^2cd^2x^2}{\sqrt{c+dx^2}}\,dx}{3cd^4}$$

$$=\frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{(b(5bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2d^3}$$

$$=\frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{(b(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,-\right.}{2d^3}$$

$$=\frac{(bc-ad)^2x^3}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}}+\frac{b^2x\sqrt{c+dx^2}}{2d^3}-\frac{b(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}$$

Mathematica [A] time = 0.12, size = 118, normalized size = 0.98

$$\frac{x\left(2a^2d^3x^2-4abcd\left(3c+4dx^2\right)+b^2c\left(15c^2+20cdx^2+3d^2x^4\right)\right)}{6cd^3\left(c+dx^2\right)^{3/2}}+\frac{b(4ad-5bc)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{2d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a^2*d^3*x^2 - 4*a*b*c*d*(3*c + 4*d*x^2) + b^2*c*(15*c^2 + 20*c*d*x^2
+ 3*d^2*x^4)))/(6*c*d^3*(c + d*x^2)^(3/2)) + (b*(-5*b*c + 4*a*d)*Log[d*x +
Sqrt[d]*Sqrt[c + d*x^2]])/(2*d^(7/2))
```

IntegrateAlgebraic [A] time = 0.25, size = 129, normalized size = 1.07

$$\frac{2a^2d^3x^3-12abc^2dx-16abcd^2x^3+15b^2c^3x+20b^2c^2dx^3+3b^2cd^2x^5}{6cd^3\left(c+dx^2\right)^{3/2}}+\frac{\left(5b^2c-4abd\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (15*b^2*c^3*x - 12*a*b*c^2*d*x + 20*b^2*c^2*d*x^3 - 16*a*b*c*d^2*x^3 + 2*a^
2*d^3*x^3 + 3*b^2*c*d^2*x^5)/(6*c*d^3*(c + d*x^2)^(3/2)) + ((5*b^2*c - 4*a*
b*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(7/2))
```

fricas [A] time = 1.23, size = 409, normalized size = 3.38

$$\left[-\frac{3\left(5b^2c^4-4abc^3d+\left(5b^2c^2d^2-4abcd^3\right)x^4+2\left(5b^2c^3d-4abc^2d^2\right)x^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(3b^2cd^3x^5+2\left(10b^2c^2d^2-8abcd^3+a^2d^4\right)x^3+3\left(5b^2c^3d-4abc^2d^2\right)x\right)\sqrt{dx^2+c}}{12\left(cd^6x^4+2c^2d^5x^2+c^3d^4\right)},\ \frac{3\left(5b^2c^4-4abc^3d+\left(5b^2c^2d^2-4abcd^3\right)x^4+2\left(5b^2c^3d-4abc^2d^2\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+\left(3b^2cd^3x^5+2\left(10b^2c^2d^2-8abcd^3+a^2d^4\right)x^3+3\left(5b^2c^3d-4abc^2d^2\right)x\right)\sqrt{dx^2+c}}{6\left(cd^6x^4+2c^2d^5x^2+c^3d^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d^3)*x^4 + 2*
(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) - 2*(3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*b*c*d^3 + a^
2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(c*d^6*x^4
 + 2*c^2*d^5*x^2 + c^3*d^4), 1/6*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d
^2 - 4*a*b*c*d^3)*x^4 + 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*
b*c*d^3 + a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c)
)/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4)]
```

giac [A] time = 0.47, size = 130, normalized size = 1.07

$$\frac{\left(\left(\frac{3\,b^2x^2}{d}+\frac{2\left(10\,b^2c^2d^3-8\,abcd^4+a^2d^5\right)}{cd^5}\right)x^2+\frac{3\left(5\,b^2c^3d^2-4\,abc^2d^3\right)}{cd^5}\right)x}{6\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\left(5\,b^2c-4\,abd\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{2\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*b^2*x^2/d + 2*(10*b^2*c^2*d^3 - 8*a*b*c*d^4 + a^2*d^5)/(c*d^5))*x^2
 + 3*(5*b^2*c^3*d^2 - 4*a*b*c^2*d^3)/(c*d^5))*x/(d*x^2 + c)^(3/2) + 1/2*(5*
b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

maple [A] time = 0.01, size = 185, normalized size = 1.53

$$\frac{b^2x^5}{2\left(d\,x^2+c\right)^{\frac{3}{2}}\,d}-\frac{2ab\,x^3}{3\left(d\,x^2+c\right)^{\frac{3}{2}}\,d}+\frac{5b^2c\,x^3}{6\left(d\,x^2+c\right)^{\frac{3}{2}}\,d^2}-\frac{a^2x}{3\left(d\,x^2+c\right)^{\frac{3}{2}}\,d}+\frac{a^2x}{3\sqrt{d\,x^2+c}\,cd}-\frac{2abx}{\sqrt{d\,x^2+c}\,d^2}+\frac{5b^2cx}{2\sqrt{d\,x^2+c}\,d^3}+\frac{2ab\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{d^{\frac{5}{2}}}-\frac{5b^2c\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{2d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/2*b^2*x^5/d/(d*x^2+c)^(3/2)+5/6*b^2*c/d^2*x^3/(d*x^2+c)^(3/2)+5/2*b^2*c/d
^3*x/(d*x^2+c)^(1/2)-5/2*b^2*c/d^(7/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-2/3*a*
b*x^3/d/(d*x^2+c)^(3/2)-2*a*b/d^2*x/(d*x^2+c)^(1/2)+2*a*b/d^(5/2)*ln(d^(1/2
)*x+(d*x^2+c)^(1/2))-1/3*a^2/d*x/(d*x^2+c)^(3/2)+1/3*a^2/c/d*x/(d*x^2+c)^(1
/2)
```

maxima [B] time = 0.87, size = 211, normalized size = 1.74

$$\frac{b^2x^5}{2\left(dx^2+c\right)^{\frac{3}{2}}d}-\frac{2}{3}\,abx\left(\frac{3\,x^2}{\left(dx^2+c\right)^{\frac{3}{2}}d}+\frac{2\,c}{\left(dx^2+c\right)^{\frac{3}{2}}d^2}\right)+\frac{5\,b^2cx\left(\frac{3\,x^2}{\left(dx^2+c\right)^{\frac{3}{2}}d}+\frac{2\,c}{\left(dx^2+c\right)^{\frac{3}{2}}d^2}\right)}{6\,d}+\frac{5\,b^2cx}{6\,\sqrt{dx^2+c}\,d^3}-\frac{2\,abx}{3\,\sqrt{dx^2+c}\,d^2}-\frac{a^2x}{3\left(dx^2+c\right)^{\frac{3}{2}}d}+\frac{a^2x}{3\,\sqrt{dx^2+c}\,cd}-\frac{5\,b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{7}{2}}}+\frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^5/((d*x^2 + c)^(3/2)*d) - 2/3*a*b*x*(3*x^2/((d*x^2 + c)^(3/2)*d)
+ 2*c/((d*x^2 + c)^(3/2)*d^2)) + 5/6*b^2*c*x*(3*x^2/((d*x^2 + c)^(3/2)*d) +
 2*c/((d*x^2 + c)^(3/2)*d^2))/d + 5/6*b^2*c*x/(sqrt(d*x^2 + c)*d^3) - 2/3*a
*b*x/(sqrt(d*x^2 + c)*d^2) - 1/3*a^2*x/((d*x^2 + c)^(3/2)*d) + 1/3*a^2*x/(s
qrt(d*x^2 + c)*c*d) - 5/2*b^2*c*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 2*a*b*arcs
inh(d*x/sqrt(c*d))/d^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.645 $\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3\left(c+dx^2\right)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{(bc-ad)^2}{3d^3\left(c+dx^2\right)^{3/2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] -(b*c - a*d)^2/(3*d^3*(c + d*x^2)^(3/2)) + (2*b*(b*c - a*d))/(d^3*Sqrt[c +
d*x^2]) + (b^2*Sqrt[c + d*x^2])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}}-\frac{2b(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{b^2}{d^2\sqrt{c+dx}}\right)dx,x,x^2\right)$$
$$=-\frac{(bc-ad)^2}{3d^3\left(c+dx^2\right)^{3/2}}+\frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}}+\frac{b^2\sqrt{c+dx^2}}{d^3}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.93

$$\frac{-a^2d^2-2abd\left(2c+3dx^2\right)+b^2\left(8c^2+12cdx^2+3d^2x^4\right)}{3d^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (-(a^2*d^2) - 2*a*b*d*(2*c + 3*d*x^2) + b^2*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4
))/(3*d^3*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 72, normalized size = 1.00

$$\frac{-a^2d^2-4abcd-6abd^2x^2+8b^2c^2+12b^2cdx^2+3b^2d^2x^4}{3d^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

```
[Out] (8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 12*b^2*c*d*x^2 - 6*a*b*d^2*x^2 + 3*b^2*d
^2*x^4)/(3*d^3*(c + d*x^2)^(3/2))
```

fricas [A] time = 1.12, size = 91, normalized size = 1.26

$$\frac{\left(3\,b^2d^2x^4+8\,b^2c^2-4\,abcd-a^2d^2+6\left(2\,b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}}{3\left(d^5x^4+2\,cd^4x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)/(d^5*x^4 + 2*c*d^4*x^2 + c^2*d^3)
```

giac [A] time = 0.46, size = 79, normalized size = 1.10

$$\frac{\sqrt{dx^2+c}\,b^2}{d^3}+\frac{6\left(dx^2+c\right)b^2c-b^2c^2-6\left(dx^2+c\right)abd+2\,abcd-a^2d^2}{3\left(dx^2+c\right)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + c)*b^2/d^3 + 1/3*(6*(d*x^2 + c)*b^2*c - b^2*c^2 - 6*(d*x^2 + c
)*a*b*d + 2*a*b*c*d - a^2*d^2)/((d*x^2 + c)^(3/2)*d^3)
```

maple [A] time = 0.01, size = 68, normalized size = 0.94

$$-\frac{-3b^2x^4d^2+6ab\,d^2x^2-12b^2cd\,x^2+a^2d^2+4abcd-8b^2c^2}{3\left(d\,x^2+c\right)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-3*b^2*d^2*x^4+6*a*b*d^2*x^2-12*b^2*c*d*x^2+a^2*d^2+4*a*b*c*d-8*b^2*c
^2)/(d*x^2+c)^(3/2)/d^3
```

maxima [A] time = 0.92, size = 114, normalized size = 1.58

$$\frac{b^2x^4}{\left(dx^2+c\right)^{\frac{3}{2}}d}+\frac{4\,b^2cx^2}{\left(dx^2+c\right)^{\frac{3}{2}}d^2}-\frac{2\,abx^2}{\left(dx^2+c\right)^{\frac{3}{2}}d}+\frac{8\,b^2c^2}{3\left(dx^2+c\right)^{\frac{3}{2}}d^3}-\frac{4\,abc}{3\left(dx^2+c\right)^{\frac{3}{2}}d^2}-\frac{a^2}{3\left(dx^2+c\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*x^4/((d*x^2 + c)^(3/2)*d) + 4*b^2*c*x^2/((d*x^2 + c)^(3/2)*d^2) - 2*a*b
*x^2/((d*x^2 + c)^(3/2)*d) + 8/3*b^2*c^2/((d*x^2 + c)^(3/2)*d^3) - 4/3*a*b*
c/((d*x^2 + c)^(3/2)*d^2) - 1/3*a^2/((d*x^2 + c)^(3/2)*d)
```

mupad [B] time = 0.63, size = 76, normalized size = 1.06

$$\frac{3\,b^2\left(d\,x^2+c\right)^2-a^2\,d^2-b^2\,c^2+6\,b^2\,c\,\left(d\,x^2+c\right)-6\,a\,b\,d\,\left(d\,x^2+c\right)+2\,a\,b\,c\,d}{3\,d^3\,\left(d\,x^2+c\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

```
[Out] (3*b^2*(c + d*x^2)^2 - a^2*d^2 - b^2*c^2 + 6*b^2*c*(c + d*x^2) - 6*a*b*d*(c
 + d*x^2) + 2*a*b*c*d)/(3*d^3*(c + d*x^2)^(3/2))
```

sympy [A] time = 1.38, size = 303, normalized size = 4.21

$$\begin{cases} -\frac{a^{2}d^{2}}{3cd^{3}\sqrt{c+dx^{2}}+3d^{4}x^{2}\sqrt{c+dx^{2}}} - \frac{4abcd}{3cd^{3}\sqrt{c+dx^{2}}+3d^{4}x^{2}\sqrt{c+dx^{2}}} - \frac{6abd^{2}x^{2}}{3cd^{3}\sqrt{c+dx^{2}}+3d^{4}x^{2}\sqrt{c+dx^{2}}} + \frac{8b^{2}c^{2}}{3cd^{3}\sqrt{c+dx^{2}}+3d^{4}x^{2}\sqrt{c+dx^{2}}} + \frac{12b^{2}cdx^{2}}{3cd^{3}\sqrt{c+dx^{2}}+3d^{4}x^{2}\sqrt{c+dx^{2}}} + \frac{3b^{2}d^{2}x^{4}}{3cd^{3}\sqrt{c+dx^{2}}+3d^{4}x^{2}\sqrt{c+dx^{2}}} & \text{for } d \neq 0 \\ \frac{\frac{a^{2}x^{2}}{2}+\frac{abx^{4}}{2}+\frac{b^{2}x^{6}}{6}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Piecewise((-a**2*d**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x
**2)) - 4*a*b*c*d/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)
) - 6*a*b*d**2*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**
2)) + 8*b**2*c**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**2)
) + 12*b**2*c*d*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x*
*2)) + 3*b**2*d**2*x**4/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d
*x**2)), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(5/2), Tru
e))
```

3.646 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 217, 206}

$$-\frac{x(bc-ad)(2ad+3bc)}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(3*c*d*(c + d*x^2)^(3/2)) - ((b*c - a*d)*(3*b*
c + 2*a*d)*x)/(3*c^2*d^2*Sqrt[c + d*x^2]) + (b^2*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/d^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx &= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{a(bc+2ad)+3b^2cx^2}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)}{3cd\left(c+dx^2\right)^{3/2}}-\frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 101, normalized size = 0.96

$$\frac{x\left(a^2d^2\left(3c+2dx^2\right)+2abcd^2x^2-b^2c^2\left(3c+4dx^2\right)\right)}{3c^2d^2\left(c+dx^2\right)^{3/2}}+\frac{b^2\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] (x*(2*a*b*c*d^2*x^2 + a^2*d^2*(3*c + 2*d*x^2) - b^2*c^2*(3*c + 4*d*x^2)))/(
3*c^2*d^2*(c + d*x^2)^(3/2)) + (b^2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/d^(
5/2)
```

IntegrateAlgebraic [A] time = 0.19, size = 107, normalized size = 1.02

$$\frac{3a^2cd^2x+2a^2d^3x^3+2abcd^2x^3-3b^2c^3x-4b^2c^2dx^3}{3c^2d^2\left(c+dx^2\right)^{3/2}}-\frac{b^2\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

```
[Out] (-3*b^2*c^3*x + 3*a^2*c*d^2*x - 4*b^2*c^2*d*x^3 + 2*a*b*c*d^2*x^3 + 2*a^2*d
^3*x^3)/(3*c^2*d^2*(c + d*x^2)^(3/2)) - (b^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*
x^2]])/d^(5/2)
```

fricas [A] time = 1.39, size = 321, normalized size = 3.06

$$\left[\frac{3\left(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\left(2\left(2\,b^2c^2d^2-abcd^3-a^2d^4\right)x^3+3\left(b^2c^3d-a^2cd^3\right)x\right)\sqrt{dx^2+c}}{6\left(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3\right)},-\frac{3\left(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(2\left(2\,b^2c^2d^2-abcd^3-a^2d^4\right)x^3+3\left(b^2c^3d-a^2cd^3\right)x\right)\sqrt{dx^2+c}}{3\left(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*
d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*x^4 + 2*c
^3*d^4*x^2 + c^4*d^3), -1/3*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4
)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*(2*b^2*c^2*d^2 - a*b*c*d
^3 - a^2*d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*
x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

giac [A] time = 0.52, size = 105, normalized size = 1.00

$$-\frac{x\left(\frac{2\left(2\,b^2c^2d^2-abcd^3-a^2d^4\right)x^2}{c^2d^3}+\frac{3\left(b^2c^3d-a^2cd^3\right)}{c^2d^3}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^2/(c^2*d^3) + 3*(b^2*c^3*
d - a^2*c*d^3)/(c^2*d^3))/(d*x^2 + c)^(3/2) - b^2*log(abs(-sqrt(d)*x + sqrt
(d*x^2 + c)))/d^(5/2)
```

maple [A] time = 0.01, size = 136, normalized size = 1.30

$$-\frac{b^{2}x^{3}}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}d}+\frac{a^{2}x}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}c}-\frac{2abx}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}d}+\frac{2a^{2}x}{3\sqrt{d\,x^{2}+c}\,c^{2}}+\frac{2abx}{3\sqrt{d\,x^{2}+c}\,cd}-\frac{b^{2}x}{\sqrt{d\,x^{2}+c}\,d^{2}}+\frac{b^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b^2*x^3/d/(d*x^2+c)^(3/2)-b^2/d^2*x/(d*x^2+c)^(1/2)+b^2/d^(5/2)*ln(d^(
1/2)*x+(d*x^2+c)^(1/2))-2/3*a*b/d*x/(d*x^2+c)^(3/2)+2/3*a*b/c/d*x/(d*x^2+c)
^(1/2)+1/3*a^2*x/c/(d*x^2+c)^(3/2)+2/3*a^2/c^2*x/(d*x^2+c)^(1/2)
```

maxima [A] time = 0.95, size = 147, normalized size = 1.40

$$-\frac{1}{3}\,b^{2}x\left(\frac{3\,x^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}}d}+\frac{2\,c}{\left(dx^{2}+c\right)^{\frac{3}{2}}d^{2}}\right)+\frac{2\,a^{2}x}{3\,\sqrt{dx^{2}+c}\,c^{2}}+\frac{a^{2}x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}c}-\frac{b^{2}x}{3\,\sqrt{dx^{2}+c}\,d^{2}}-\frac{2\,abx}{3\left(dx^{2}+c\right)^{\frac{3}{2}}d}+\frac{2\,abx}{3\,\sqrt{dx^{2}+c}\,cd}+\frac{b^{2}\,\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2*c/((d*x^2 + c)^(3/2)*d^2)) + 2/
3*a^2*x/(sqrt(d*x^2 + c)*c^2) + 1/3*a^2*x/((d*x^2 + c)^(3/2)*c) - 1/3*b^2*x
/(sqrt(d*x^2 + c)*d^2) - 2/3*a*b*x/((d*x^2 + c)^(3/2)*d) + 2/3*a*b*x/(sqrt(
d*x^2 + c)*c*d) + b^2*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{2}+a\right)^{2}}{\left(d\,x^{2}+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(5/2),x)
```

```
[Out] int((a + b*x^2)^2/(c + d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{2}}{\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.647 $\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

Rubi [**A**] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b*c - a*d)^2/(3*c*d^2*(c + d*x^2)^(3/2)) + (a^2/c^2 - b^2/d^2)/Sqrt[c + d*
x^2] - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x\left(c+dx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^2}{x(c+dx)^{5/2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{(bc-ad)^2}{cd(c+dx)^{5/2}}+\frac{b^2c^2-a^2d^2}{c^2d(c+dx)^{3/2}}+\frac{a^2}{c^2x\sqrt{c+dx}}\right)dx, x, x^2\right)\\
&= \frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2c^2}\\
&= \frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{c^2d}\\
&= \frac{(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}+\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 67, normalized size = 0.76

$$\frac{a^2d^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)-bc\left(2ad+2bc+3bdx^2\right)}{3cd^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*(2*b*c + 2*a*d + 3*b*d*x^2)) + a^2*d^2*Hypergeometric2F1[-3/2, 1, -1
/2, 1 + (d*x^2)/c])/(3*c*d^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 99, normalized size = 1.12

$$\frac{4a^2cd^2+3a^2d^3x^2-2abc^2d-2b^2c^3-3b^2c^2dx^2}{3c^2d^2\left(c+dx^2\right)^{3/2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-2*b^2*c^3 - 2*a*b*c^2*d + 4*a^2*c*d^2 - 3*b^2*c^2*d*x^2 + 3*a^2*d^3*x^2)/
(3*c^2*d^2*(c + d*x^2)^(3/2)) - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5
/2)
```

fricas [A] time = 1.17, size = 316, normalized size = 3.59

$$\left[\frac{3\left(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2\right)\sqrt{c}\,\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\left(2\,b^2c^4+2\,abc^3d-4\,a^2c^2d^2+3\left(b^2c^3d-a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{6\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)},\ \frac{3\left(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(2\,b^2c^4+2\,abc^3d-4\,a^2c^2d^2+3\left(b^2c^3d-a^2cd^3\right)x^2\right)\sqrt{dx^2+c}}{3\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b^2*c^4 + 2*a*b*c^3*d - 4*a^2
*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4 + 2
*c^4*d^3*x^2 + c^5*d^2), 1/3*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^
2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*b^2*c^4 + 2*a*b*c^3*d - 4
*a^2*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4
 + 2*c^4*d^3*x^2 + c^5*d^2)]
```

giac [A] time = 0.44, size = 102, normalized size = 1.16

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c^2}-\frac{3\left(dx^2+c\right)b^2c^2-b^2c^3+2\,abc^2d-3\left(dx^2+c\right)a^2d^2-a^2cd^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/3*(3*(d*x^2 + c)*b^
2*c^2 - b^2*c^3 + 2*a*b*c^2*d - 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^2
+ c)^(3/2)*c^2*d^2)
```

maple [A] time = 0.01, size = 120, normalized size = 1.36

$$-\frac{b^{2} x^{2}}{\left(d\,x^{2}+c\right)^{\frac{3}{2}} d}+\frac{a^{2}}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}} c}-\frac{2 a b}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}} d}-\frac{2 b^{2} c}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}} d^{2}}-\frac{a^{2} \ln \left(\frac{2 c+2 \sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{c^{\frac{5}{2}}}+\frac{a^{2}}{\sqrt{d\,x^{2}+c}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x)
```

```
[Out] -b^2*x^2/d/(d*x^2+c)^(3/2)-2/3*b^2*c/d^2/(d*x^2+c)^(3/2)-2/3*a*b/d/(d*x^2+c
)^(3/2)+1/3*a^2/c/(d*x^2+c)^(3/2)+a^2/c^2/(d*x^2+c)^(1/2)-a^2/c^(5/2)*ln((2
*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.93, size = 108, normalized size = 1.23

$$-\frac{b^{2} x^{2}}{\left(dx^{2}+c\right)^{\frac{3}{2}} d}-\frac{a^{2} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}+\frac{a^{2}}{\sqrt{dx^{2}+c}\,c^{2}}+\frac{a^{2}}{3\left(dx^{2}+c\right)^{\frac{3}{2}} c}-\frac{2\,b^{2} c}{3\left(dx^{2}+c\right)^{\frac{3}{2}} d^{2}}-\frac{2\,ab}{3\left(dx^{2}+c\right)^{\frac{3}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*x^2/((d*x^2 + c)^(3/2)*d) - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2)
+ a^2/(sqrt(d*x^2 + c)*c^2) + 1/3*a^2/((d*x^2 + c)^(3/2)*c) - 2/3*b^2*c/((d
*x^2 + c)^(3/2)*d^2) - 2/3*a*b/((d*x^2 + c)^(3/2)*d)
```

mupad [B] time = 0.80, size = 90, normalized size = 1.02

$$\frac{\frac{a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}}{3\,c}+\frac{\left(a^{2}\,d^{2}-b^{2}\,c^{2}\right)\left(d\,x^{2}+c\right)}{c^{2}}}{d^{2}\left(d\,x^{2}+c\right)^{3/2}}-\frac{a^{2}\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)}{c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(3*c) + ((a^2*d^2 - b^2*c^2)*(c + d*x^2))/
c^2)/(d^2*(c + d*x^2)^(3/2)) - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^(5/
2)
```

sympy [A] time = 44.36, size = 87, normalized size = 0.99

$$\frac{a^{2} \operatorname{atan}\left(\frac{\sqrt{c+dx^{2}}}{\sqrt{-c}}\right)}{c^{2}\sqrt{-c}}+\frac{\left(ad-bc\right)^{2}}{3cd^{2}\left(c+dx^{2}\right)^{\frac{3}{2}}}+\frac{\left(ad-bc\right)\left(ad+bc\right)}{c^{2}d^{2}\sqrt{c+dx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x/(d*x**2+c)**(5/2),x)
```

```
[Out] a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(c**2*sqrt(-c)) + (a*d - b*c)**2/(3*c*
d**2*(c + d*x**2)**(3/2)) + (a*d - b*c)*(a*d + b*c)/(c**2*d**2*sqrt(c + d*x
**2))
```

3.648 $\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {462, 378, 191}

$$-\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4ax(bc-2ad)}{3c^3\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(a^2/(c*x*(c + d*x^2)^(3/2))) + (x*(2*a*(b*c - 2*a*d) + b^2*c*x^2))/(3*c^2
*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^3*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{5/2}}\,dx &= -\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{2a(bc-2ad)+b^2cx^2}{\left(c+dx^2\right)^{5/2}}\,dx}{c}\\ &= -\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{(4a(bc-2ad))\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{3c^2}\\ &= -\frac{a^2}{cx\left(c+dx^2\right)^{3/2}}+\frac{x\left(2a(bc-2ad)+b^2cx^2\right)}{3c^2\left(c+dx^2\right)^{3/2}}+\frac{4a(bc-2ad)x}{3c^3\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 76, normalized size = 0.84

$$\frac{-a^2\left(3c^2+12cdx^2+8d^2x^4\right)+2abcx^2\left(3c+2dx^2\right)+b^2c^2x^4}{3c^3x\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4 + 2*a*b*c*x^2*(3*c + 2*d*x^2) - a^2*(3*c^2 + 12*c*d*x^2 + 8*d^
2*x^4))/(3*c^3*x*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.17, size = 80, normalized size = 0.89

$$\frac{-3a^2c^2-12a^2cdx^2-8a^2d^2x^4+6abc^2x^2+4abcdx^4+b^2c^2x^4}{3c^3x\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*c^2 + 6*a*b*c^2*x^2 - 12*a^2*c*d*x^2 + b^2*c^2*x^4 + 4*a*b*c*d*x^4
- 8*a^2*d^2*x^4)/(3*c^3*x*(c + d*x^2)^(3/2))
```

fricas [A]   time = 0.99, size = 92, normalized size = 1.02

$$\frac{\left(\left(b^2c^2+4\,abcd-8\,a^2d^2\right)x^4-3\,a^2c^2+6\left(abc^2-2\,a^2cd\right)x^2\right)\sqrt{dx^2+c}}{3\left(c^3d^2x^5+2\,c^4dx^3+c^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*x^4 - 3*a^2*c^2 + 6*(a*b*c^2 - 2*a^2
*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d^2*x^5 + 2*c^4*d*x^3 + c^5*x)
```

giac [A]   time = 0.43, size = 117, normalized size = 1.30

$$\frac{x\left(\frac{\left(b^2c^4d+4\,abc^3d^2-5\,a^2c^2d^3\right)x^2}{c^5d}+\frac{6\left(abc^4d-a^2c^3d^2\right)}{c^5d}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*((b^2*c^4*d + 4*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x^2/(c^5*d) + 6*(a*b*c^4
*d - a^2*c^3*d^2)/(c^5*d))/(d*x^2 + c)^(3/2) + 2*a^2*sqrt(d)/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)*c^2)
```

maple [A]   time = 0.01, size = 78, normalized size = 0.87

$$-\frac{8a^2d^2x^4-4abcd\,x^4-b^2c^2x^4+12a^2cd\,x^2-6ab\,c^2x^2+3a^2c^2}{3\left(d\,x^2+c\right)^{\frac{3}{2}}c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(8*a^2*d^2*x^4-4*a*b*c*d*x^4-b^2*c^2*x^4+12*a^2*c*d*x^2-6*a*b*c^2*x^2+
3*a^2*c^2)/(d*x^2+c)^(3/2)/x/c^3
```

maxima [A]   time = 0.93, size = 132, normalized size = 1.47

$$\frac{4\,abx}{3\sqrt{dx^2+c}\,c^2}+\frac{2\,abx}{3\left(dx^2+c\right)^{\frac{3}{2}}c}-\frac{b^2x}{3\left(dx^2+c\right)^{\frac{3}{2}}d}+\frac{b^2x}{3\sqrt{dx^2+c}\,cd}-\frac{8\,a^2dx}{3\sqrt{dx^2+c}\,c^3}-\frac{4\,a^2dx}{3\left(dx^2+c\right)^{\frac{3}{2}}c^2}-\frac{a^2}{\left(dx^2+c\right)^{\frac{3}{2}}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*a*b*x/(sqrt(d*x^2 + c)*c^2) + 2/3*a*b*x/((d*x^2 + c)^(3/2)*c) - 1/3*b^2
*x/((d*x^2 + c)^(3/2)*d) + 1/3*b^2*x/(sqrt(d*x^2 + c)*c*d) - 8/3*a^2*d*x/(s
qrt(d*x^2 + c)*c^3) - 4/3*a^2*d*x/((d*x^2 + c)^(3/2)*c^2) - a^2/((d*x^2 + c
)^(3/2)*c*x)
```

mupad [B] time = 0.65, size = 77, normalized size = 0.86

$$-\frac{3\,a^2\,c^2+12\,a^2\,c\,d\,x^2+8\,a^2\,d^2\,x^4-6\,a\,b\,c^2\,x^2-4\,a\,b\,c\,d\,x^4-b^2\,c^2\,x^4}{3\,c^3\,x\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] -(3*a^2*c^2 + 8*a^2*d^2*x^4 - b^2*c^2*x^4 - 6*a*b*c^2*x^2 + 12*a^2*c*d*x^2
- 4*a*b*c*d*x^4)/(3*c^3*x*(c + d*x^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2}\right)^{2}}{x^{2} \left(c + dx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(5/2)), x)
```

3.649 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{-\frac{5a^2d}{c}+4ab-\frac{2b^2c}{d}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}$$

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 51, 63, 208}

$$\frac{-\frac{5a^2d}{c}+4ab-\frac{2b^2c}{d}}{6c\left(c+dx^2\right)^{3/2}}-\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}}+\frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}}-\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (4*a*b - (2*b^2*c)/d - (5*a^2*d)/c)/(6*c*(c + d*x^2)^(3/2)) - a^2/(2*c*x^2*
(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d))/(2*c^3*Sqrt[c + d*x^2]) - (a*(4*b*
c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^2}{x^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(4bc-5ad)+b^2cx}{x(c+dx)^{5/2}}\,dx, x, x^2\right)}{2c}$$

$$= -\frac{2b^2c^2-4abcd+5a^2d^2}{6c^2d\left(c+dx^2\right)^{3/2}} - \frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}} + \frac{(a(4bc-5ad))\,\text{Subst}\Big(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, x}{4c^2}$$

$$= -\frac{2b^2c^2-4abcd+5a^2d^2}{6c^2d\left(c+dx^2\right)^{3/2}} - \frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} + \frac{(a(4bc-5ad))\,\text{Subst}\Big(\int}{4c^3}$$

$$= -\frac{2b^2c^2-4abcd+5a^2d^2}{6c^2d\left(c+dx^2\right)^{3/2}} - \frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} + \frac{(a(4bc-5ad))\,\text{Subst}\Bigg(\int}{2c}$$

$$= -\frac{2b^2c^2-4abcd+5a^2d^2}{6c^2d\left(c+dx^2\right)^{3/2}} - \frac{a^2}{2cx^2\left(c+dx^2\right)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} - \frac{a(4bc-5ad)\tanh^{-1}\Bigg(\sqrt{c}}{2c^{7/2}}$$

Mathematica [C] time = 0.05, size = 105, normalized size = 0.80

$$\frac{-c\left(a^2d\left(3c+5dx^2\right)-4abcdx^2+2b^2c^2x^2\right)-3adx^2\left(c+dx^2\right)(5ad-4bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)}{6c^3dx^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(c*(2*b^2*c^2*x^2 - 4*a*b*c*d*x^2 + a^2*d*(3*c + 5*d*x^2))) - 3*a*d*(-4*b
*c + 5*a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])
/(6*c^3*d*x^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.22, size = 130, normalized size = 0.99

$$\frac{-3a^2c^2d-20a^2cd^2x^2-15a^2d^3x^4+16abc^2dx^2+12abcd^2x^4-2b^2c^3x^2}{6c^3dx^2\left(c+dx^2\right)^{3/2}} + \frac{\left(5a^2d-4abc\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*c^2*d - 2*b^2*c^3*x^2 + 16*a*b*c^2*d*x^2 - 20*a^2*c*d^2*x^2 + 12*a*
b*c*d^2*x^4 - 15*a^2*d^3*x^4)/(6*c^3*d*x^2*(c + d*x^2)^(3/2)) + ((-4*a*b*c
+ 5*a^2*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

fricas [A] time = 1.10, size = 426, normalized size = 3.25

$$\left[-\frac{3\left(\left(4\,abcd^{3}-5\,a^{2}d^{4}\right)x^{6}+2\left(4\,abc^{2}d^{2}-5\,a^{2}cd^{3}\right)x^{4}+\left(4\,abc^{3}d-5\,a^{2}c^{2}d^{2}\right)x^{2}\right)\sqrt{c}\log\left(-\frac{dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+2\left(3\,a^{2}c^{3}d-3\left(4\,abc^{2}d^{2}-5\,a^{2}cd^{3}\right)x^{4}+2\left(b^{2}c^{4}-8\,abc^{3}d+10\,a^{2}c^{2}d^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\left(c^{4}d^{3}x^{6}+2\,c^{5}d^{2}x^{4}+c^{6}dx^{2}\right)},\ \frac{3\left(\left(4\,abcd^{3}-5\,a^{2}d^{4}\right)x^{6}+2\left(4\,abc^{2}d^{2}-5\,a^{2}cd^{3}\right)x^{4}+\left(4\,abc^{3}d-5\,a^{2}c^{2}d^{2}\right)x^{2}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-\left(3\,a^{2}c^{3}d-3\left(4\,abc^{2}d^{2}-5\,a^{2}cd^{3}\right)x^{4}+2\left(b^{2}c^{4}-8\,abc^{3}d+10\,a^{2}c^{2}d^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}}{6\left(c^{4}d^{3}x^{6}+2\,c^{5}d^{2}x^{4}+c^{6}dx^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*
x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2
 + c)*sqrt(c) + 2*c)/x^2) + 2*(3*a^2*c^3*d - 3*(4*a*b*c^2*d^2 - 5*a^2*c*d^3
)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c
^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2), 1/6*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*
x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x
^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a^2*c^3*d - 3*(4*a*b*c^2
*d^2 - 5*a^2*c*d^3)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*s
qrt(d*x^2 + c))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2)]
```

giac [A] time = 0.37, size = 128, normalized size = 0.98

$$\frac{\left(4\,abc-5\,a^{2}d\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}\,c^{3}}-\frac{\sqrt{dx^{2}+c}\,a^{2}}{2\,c^{3}x^{2}}-\frac{b^{2}c^{3}-6\left(dx^{2}+c\right)abcd-2\,abc^{2}d+6\left(dx^{2}+c\right)a^{2}d^{2}+a^{2}cd^{2}}{3\left(dx^{2}+c\right)^{\frac{3}{2}}c^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*c - 5*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1
/2*sqrt(d*x^2 + c)*a^2/(c^3*x^2) - 1/3*(b^2*c^3 - 6*(d*x^2 + c)*a*b*c*d - 2
*a*b*c^2*d + 6*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c^3*d)
```

maple [A] time = 0.02, size = 169, normalized size = 1.29

$$-\frac{5a^{2}d}{6\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{2}}+\frac{2ab}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}c}-\frac{b^{2}}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}d}+\frac{5a^{2}d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2c^{\frac{7}{2}}}-\frac{2ab\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{c^{\frac{5}{2}}}-\frac{5a^{2}d}{2\sqrt{d\,x^{2}+c}\,c^{3}}+\frac{2ab}{\sqrt{d\,x^{2}+c}\,c^{2}}-\frac{a^{2}}{2\left(d\,x^{2}+c\right)^{\frac{3}{2}}c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*b^2/d/(d*x^2+c)^(3/2)-1/2*a^2/c/x^2/(d*x^2+c)^(3/2)-5/6*a^2*d/c^2/(d*x
^2+c)^(3/2)-5/2*a^2*d/c^3/(d*x^2+c)^(1/2)+5/2*a^2*d/c^(7/2)*ln((2*c+2*(d*x^
2+c)^(1/2)*c^(1/2))/x)+2/3*a*b/c/(d*x^2+c)^(3/2)+2*a*b/c^2/(d*x^2+c)^(1/2)-
2*a*b/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.96, size = 146, normalized size = 1.11

$$-\frac{2\,ab\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}+\frac{5\,a^2d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,c^{\frac{7}{2}}}+\frac{2\,ab}{\sqrt{dx^2+c}\,c^2}+\frac{2\,ab}{3\left(dx^2+c\right)^{\frac{3}{2}}c}-\frac{b^2}{3\left(dx^2+c\right)^{\frac{3}{2}}d}-\frac{5\,a^2d}{2\sqrt{dx^2+c}\,c^3}-\frac{5\,a^2d}{6\left(dx^2+c\right)^{\frac{3}{2}}c^2}-\frac{a^2}{2\left(dx^2+c\right)^{\frac{3}{2}}cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) + 5/2*a^2*d*arcsinh(c/(sqrt(c*
d)*abs(x)))/c^(7/2) + 2*a*b/(sqrt(d*x^2 + c)*c^2) + 2/3*a*b/((d*x^2 + c)^(3
/2)*c) - 1/3*b^2/((d*x^2 + c)^(3/2)*d) - 5/2*a^2*d/(sqrt(d*x^2 + c)*c^3) -
5/6*a^2*d/((d*x^2 + c)^(3/2)*c^2) - 1/2*a^2/((d*x^2 + c)^(3/2)*c*x^2)
```

mupad [B] time = 0.90, size = 147, normalized size = 1.12

$$\frac{a\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(5\,a\,d-4\,b\,c\right)}{2\,c^{7/2}}-\frac{\frac{\left(d\,x^2+c\right)\,\left(-5\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c}+\frac{d\,{\left(d\,x^2+c\right)}^2\,\left(5\,a^2\,d-4\,a\,b\,c\right)}{2\,c^3}}{d\,{\left(d\,x^2+c\right)}^{5/2}-c\,d\,{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x)
```

```
[Out] (a*atanh((c + d*x^2)^(1/2)/c^(1/2))*(5*a*d - 4*b*c))/(2*c^(7/2)) - (((c + d
*x^2)*(b^2*c^2 - 5*a^2*d^2 + 4*a*b*c*d))/(3*c^2) - (a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)/(3*c) + (d*(c + d*x^2)^2*(5*a^2*d - 4*a*b*c))/(2*c^3))/(d*(c + d*x^
2)^(5/2) - c*d*(c + d*x^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(5/2)), x)
```

3.650 $\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=131

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{2x\left(b^2c^2-8ad(bc-ad)\right)}{3c^4\sqrt{c+dx^2}}+\frac{x\left(b^2c^2-8ad(bc-ad)\right)}{3c^3\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}$$

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 453, 192, 191}

$$-\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{2x\left(b^2c^2-8ad(bc-ad)\right)}{3c^4\sqrt{c+dx^2}}+\frac{x\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)}{3c\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] -a^2/(3*c*x^3*(c + d*x^2)^(3/2)) - (2*a*(b*c - a*d))/(c^2*x*(c + d*x^2)^(3/
2)) + ((b^2 - (8*a*d*(b*c - a*d))/c^2)*x)/(3*c*(c + d*x^2)^(3/2)) + (2*(b^2
*c^2 - 8*a*d*(b*c - a*d))*x)/(3*c^4*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{5/2}}\,dx &= -\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{6a(bc-ad)+3b^2cx^2}{x^2\left(c+dx^2\right)^{5/2}}\,dx}{3c}\\
&= -\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}-\left(-b^2+\frac{8ad(bc-ad)}{c^2}\right)\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx\\
&= -\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c\left(c+dx^2\right)^{3/2}}+\frac{\left(2\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)\right)\int \frac{1}{(c+dx}}{3c}\\
&= -\frac{a^2}{3cx^3\left(c+dx^2\right)^{3/2}}-\frac{2a(bc-ad)}{c^2x\left(c+dx^2\right)^{3/2}}+\frac{\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c\left(c+dx^2\right)^{3/2}}+\frac{2\left(b^2-\frac{8ad(bc-ad)}{c^2}\right)x}{3c^2\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.82

$$
\frac{a^2\left(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6\right)-2abcx^2\left(3c^2+12cdx^2+8d^2x^4\right)+b^2c^2x^4\left(3c+2dx^2\right)}{3c^4x^3\left(c+dx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*x^4*(3*c + 2*d*x^2) - 2*a*b*c*x^2*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4)
 + a^2*(-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6))/(3*c^4*x^3*(c + d*
x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 120, normalized size = 0.92

$$\frac{-a^2c^3+6a^2c^2dx^2+24a^2cd^2x^4+16a^2d^3x^6-6abc^3x^2-24abc^2dx^4-16abcd^2x^6+3b^2c^3x^4+2b^2c^2dx^6}{3c^4x^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(a^2*c^3) - 6*a*b*c^3*x^2 + 6*a^2*c^2*d*x^2 + 3*b^2*c^3*x^4 - 24*a*b*c^2*
d*x^4 + 24*a^2*c*d^2*x^4 + 2*b^2*c^2*d*x^6 - 16*a*b*c*d^2*x^6 + 16*a^2*d^3*
x^6)/(3*c^4*x^3*(c + d*x^2)^(3/2))
```

fricas [A] time = 1.34, size = 130, normalized size = 0.99

$$\frac{\left(2\left(b^2c^2d-8\,abcd^2+8\,a^2d^3\right)x^6-a^2c^3+3\left(b^2c^3-8\,abc^2d+8\,a^2cd^2\right)x^4-6\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{dx^2+c}}{3\left(c^4d^2x^7+2\,c^5dx^5+c^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(b^2*c^2*d - 8*a*b*c*d^2 + 8*a^2*d^3)*x^6 - a^2*c^3 + 3*(b^2*c^3 - 8
*a*b*c^2*d + 8*a^2*c*d^2)*x^4 - 6*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt(d*x^2 + c
)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

giac [B] time = 0.45, size = 258, normalized size = 1.97

$$\frac{x\left(\frac{2\left(b^2c^5d^2-5\,abc^4d^3+4\,a^2c^3d^4\right)x^2}{c^7d}+\frac{3\left(b^2c^6d-4\,abc^5d^2+3\,a^2c^4d^3\right)}{c^7d}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+9\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2cd^{\frac{3}{2}}+3\,abc^3\sqrt{d}-4\,a^2c^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^2*c^5*d^2 - 5*a*b*c^4*d^3 + 4*a^2*c^3*d^4)*x^2/(c^7*d) + 3*(b^2
*c^6*d - 4*a*b*c^5*d^2 + 3*a^2*c^4*d^3)/(c^7*d))/(d*x^2 + c)^(3/2) + 4/3*(3
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a^2*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 9
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 3*a*b*c^3*sqrt(d) - 4*a^2*
c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^3)
```

maple [A] time = 0.01, size = 116, normalized size = 0.89

$$-\frac{-16a^2d^3x^6+16abc\,d^2x^6-2b^2c^2d\,x^6-24a^2c\,d^2x^4+24ab\,c^2d\,x^4-3b^2c^3x^4-6a^2c^2d\,x^2+6ab\,c^3x^2+a^2c^3}{3\left(d\,x^2+c\right)^{\frac{3}{2}}c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-16*a^2*d^3*x^6+16*a*b*c*d^2*x^6-2*b^2*c^2*d*x^6-24*a^2*c*d^2*x^4+24*
a*b*c^2*d*x^4-3*b^2*c^3*x^4-6*a^2*c^2*d*x^2+6*a*b*c^3*x^2+a^2*c^3)/(d*x^2+c
)^(3/2)/x^3/c^4
```

maxima [A] time = 0.93, size = 175, normalized size = 1.34

$$\frac{2\,b^2x}{3\,\sqrt{dx^2+c}\,c^2}+\frac{b^2x}{3\,\left(dx^2+c\right)^{\frac{3}{2}}c}-\frac{16\,abdx}{3\,\sqrt{dx^2+c}\,c^3}-\frac{8\,abdx}{3\,\left(dx^2+c\right)^{\frac{3}{2}}c^2}+\frac{16\,a^2d^2x}{3\,\sqrt{dx^2+c}\,c^4}+\frac{8\,a^2d^2x}{3\,\left(dx^2+c\right)^{\frac{3}{2}}c^3}-\frac{2\,ab}{\left(dx^2+c\right)^{\frac{3}{2}}cx}+\frac{2\,a^2d}{\left(dx^2+c\right)^{\frac{3}{2}}c^2x}-\frac{a^2}{3\,\left(dx^2+c\right)^{\frac{3}{2}}cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*x/(sqrt(d*x^2 + c)*c^2) + 1/3*b^2*x/((d*x^2 + c)^(3/2)*c) - 16/3*a*
b*d*x/(sqrt(d*x^2 + c)*c^3) - 8/3*a*b*d*x/((d*x^2 + c)^(3/2)*c^2) + 16/3*a^
2*d^2*x/(sqrt(d*x^2 + c)*c^4) + 8/3*a^2*d^2*x/((d*x^2 + c)^(3/2)*c^3) - 2*a
*b/((d*x^2 + c)^(3/2)*c*x) + 2*a^2*d/((d*x^2 + c)^(3/2)*c^2*x) - 1/3*a^2/((
d*x^2 + c)^(3/2)*c*x^3)
```

mupad [B] time = 0.73, size = 187, normalized size = 1.43

$$\frac{b^2\,c^4\,x^2-a^2\,c^3\,d-16\,a^2\,d\,{\left(d\,x^2+c\right)}^3+2\,a\,b\,c^4+b^2\,c^3\,x^2\,\left(d\,x^2+c\right)+16\,a\,b\,c\,{\left(d\,x^2+c\right)}^3+6\,a\,b\,c^3\,\left(d\,x^2+c\right)-2\,b^2\,c^2\,x^2\,{\left(d\,x^2+c\right)}^2-24\,a\,b\,c^2\,{\left(d\,x^2+c\right)}^2+24\,a^2\,c\,d\,{\left(d\,x^2+c\right)}^2-6\,a^2\,c^2\,d\,\left(d\,x^2+c\right)}{{\left(d\,x^2+c\right)}^{3/2}\,\left(3\,c^5\,x-3\,c^4\,x\,\left(d\,x^2+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x)
```

```
[Out] (b^2*c^4*x^2 - a^2*c^3*d - 16*a^2*d*(c + d*x^2)^3 + 2*a*b*c^4 + b^2*c^3*x^2
*(c + d*x^2) + 16*a*b*c*(c + d*x^2)^3 + 6*a*b*c^3*(c + d*x^2) - 2*b^2*c^2*x
^2*(c + d*x^2)^2 - 24*a*b*c^2*(c + d*x^2)^2 + 24*a^2*c*d*(c + d*x^2)^2 - 6*
a^2*c^2*d*(c + d*x^2))/((c + d*x^2)^(3/2)*(3*c^5*x - 3*c^4*x*(c + d*x^2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(5/2)), x)
```

3.651 $\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{24c^3\left(c+dx^2\right)^{3/2}}$$

Rubi [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 78, 51, 63, 208}

$$-\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}+\frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}}-\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}-\frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (8*b^2 - (5*a*d*(8*b*c - 7*a*d))/c^2)/(24*c*(c + d*x^2)^(3/2)) - a^2/(4*c*x
^4*(c + d*x^2)^(3/2)) - (a*(8*b*c - 7*a*d))/(8*c^2*x^2*(c + d*x^2)^(3/2)) +
 (8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))/(8*c^4*Sqrt[c + d*x^2]) - ((8*b^2*c^2
- 5*a*d*(8*b*c - 7*a*d))*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2}{x^5\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^2}{x^3(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$= -\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(8bc-7ad)+2b^2cx}{x^2(c+dx)^{5/2}}\,dx,x,x^2\right)}{4c}$$

$$= -\frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{1}{16}\left(8b^2-\frac{5ad(8bc-7ad)}{c^2}\right)\text{Subst}\left(\int \frac{1}{x(c+dx}\right.$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{\left(8b^2-\frac{5ad(8bc-7ad)}{c^2}\right)\text{Subst}\left(\int\right.}{16c}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} + \frac{\left(8b^2-\frac{5}{}\right.}{}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} + \frac{\left(8b^2-\frac{5}{}\right.}{}$$

$$= \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{24c\left(c+dx^2\right)^{3/2}} - \frac{a^2}{4cx^4\left(c+dx^2\right)^{3/2}} - \frac{a(8bc-7ad)}{8c^2x^2\left(c+dx^2\right)^{3/2}} + \frac{8b^2-\frac{5ad(8bc-7ad)}{c^2}}{8c^2\sqrt{c+dx^2}} - \frac{\left(8b^2-\frac{5}{}\right.}{}$$

Mathematica [C] time = 0.04, size = 90, normalized size = 0.49

$$\frac{x^4\left(35a^2d^2-40abcd+8b^2c^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)-3ac\left(2ac-7adx^2+8bcx^2\right)}{24c^3x^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a*c*(2*a*c + 8*b*c*x^2 - 7*a*d*x^2) + (8*b^2*c^2 - 40*a*b*c*d + 35*a^2*
d^2)*x^4*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c])/(24*c^3*x^4*(c +
d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.23, size = 171, normalized size = 0.92

$$\frac{\left(-35a^2d^2+40abcd-8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}+\frac{-6a^2c^3+21a^2c^2dx^2+140a^2cd^2x^4+105a^2d^3x^6-24abc^3x^2-160abc^2dx^4-120abcd^2x^6+32b^2c^3x^4+24b^2c^2dx^6}{24c^4x^4\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-6*a^2*c^3 - 24*a*b*c^3*x^2 + 21*a^2*c^2*d*x^2 + 32*b^2*c^3*x^4 - 160*a*b*
c^2*d*x^4 + 140*a^2*c*d^2*x^4 + 24*b^2*c^2*d*x^6 - 120*a*b*c*d^2*x^6 + 105*
a^2*d^3*x^6)/(24*c^4*x^4*(c + d*x^2)^(3/2)) + ((-8*b^2*c^2 + 40*a*b*c*d - 3
5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

fricas [A] time = 1.33, size = 537, normalized size = 2.90

$$\left[\frac{3\left(\left(8b^2c^2d^2-40abcd^3+35a^2d^4\right)x^8+2\left(8b^2c^3d-40abc^2d^2+35a^2cd^3\right)x^6+\left(8b^2c^4-40abc^3d+35a^2c^2d^2\right)x^4\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(3\left(8b^2c^3d-40abc^2d^2+35a^2cd^3\right)x^6-6a^2c^4+4\left(8b^2c^4-40abc^3d+35a^2c^2d^2\right)x^4-3\left(8abc^4-7a^2c^3d\right)x^2\right)\sqrt{dx^2+c}}{48\left(c^5d^2x^8+2c^6dx^6+c^7x^4\right)}, \frac{3\left(\left(8b^2c^2d^2-40abcd^3+35a^2d^4\right)x^8+2\left(8b^2c^3d-40abc^2d^2+35a^2cd^3\right)x^6+\left(8b^2c^4-40abc^3d+35a^2c^2d^2\right)x^4\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(3\left(8b^2c^3d-40abc^2d^2+35a^2cd^3\right)x^6-6a^2c^4+4\left(8b^2c^4-40abc^3d+35a^2c^2d^2\right)x^4-3\left(8abc^4-7a^2c^3d\right)x^2\right)\sqrt{dx^2+c}}{24\left(c^5d^2x^8+2c^6dx^6+c^7x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*b^2*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d
- 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c
^2*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
2*(3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b
^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x
^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 + 2*c^6*d*x^6 + c^7*x^4), 1/24*(3*((8*b^2
*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d - 40*a*b*c^2*d^2
 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4)*sqr
t(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 +
 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d
^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 +
2*c^6*d*x^6 + c^7*x^4)]
```

giac [A] time = 0.40, size = 210, normalized size = 1.14

$$\frac{\left(8b^2c^2-40abcd+35a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}c^4}+\frac{3\left(dx^2+c\right)b^2c^2+b^2c^3-12\left(dx^2+c\right)abcd-2abc^2d+9\left(dx^2+c\right)a^2d^2+a^2cd^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c^4}-\frac{8\left(dx^2+c\right)^{\frac{3}{2}}abcd-8\sqrt{dx^2+c}\,abc^2d-11\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2+13\sqrt{dx^2+c}\,a^2cd^2}{8c^4d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/
(sqrt(-c)*c^4) + 1/3*(3*(d*x^2 + c)*b^2*c^2 + b^2*c^3 - 12*(d*x^2 + c)*a*b*
```

```
c*d - 2*a*b*c^2*d + 9*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c
^4) - 1/8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 11*(
d*x^2 + c)^(3/2)*a^2*d^2 + 13*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^4*d^2*x^4)
```

maple [A] time = 0.02, size = 265, normalized size = 1.43

$$\frac{35a^2d^2}{24\left(d\,x^2+c\right)^{\frac{3}{2}}c^3}-\frac{5abd}{3\left(d\,x^2+c\right)^{\frac{3}{2}}c^2}+\frac{b^2}{3\left(d\,x^2+c\right)^{\frac{3}{2}}c}-\frac{35a^2d^2\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{8c^{\frac{9}{2}}}+\frac{5abd\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{c^{\frac{7}{2}}}-\frac{b^2\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{c^{\frac{5}{2}}}+\frac{35a^2d^2}{8\sqrt{d\,x^2+c}\,c^4}-\frac{5abd}{\sqrt{d\,x^2+c}\,c^3}+\frac{b^2}{\sqrt{d\,x^2+c}\,c^2}+\frac{7a^2d}{8\left(d\,x^2+c\right)^{\frac{3}{2}}c^2x^2}-\frac{ab}{\left(d\,x^2+c\right)^{\frac{3}{2}}c\,x^2}-\frac{a^2}{4\left(d\,x^2+c\right)^{\frac{3}{2}}c\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x)
```

```
[Out] -a*b/c/x^2/(d*x^2+c)^(3/2)-5/3*a*b*d/c^2/(d*x^2+c)^(3/2)-5*a*b*d/c^3/(d*x^2
+c)^(1/2)+5*a*b*d/c^(7/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/4*a^2/c/x
^4/(d*x^2+c)^(3/2)+7/8*a^2*d/c^2/x^2/(d*x^2+c)^(3/2)+35/24*a^2*d^2/c^3/(d*x
^2+c)^(3/2)+35/8*a^2*d^2/c^4/(d*x^2+c)^(1/2)-35/8*a^2*d^2/c^(9/2)*ln((2*c+2
*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/3*b^2/c/(d*x^2+c)^(3/2)+b^2/c^2/(d*x^2+c)^(1
/2)-b^2/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [A] time = 0.94, size = 231, normalized size = 1.25

$$-\frac{b^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{5}{2}}}+\frac{5\,abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{c^{\frac{7}{2}}}-\frac{35\,a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{8\,c^{\frac{9}{2}}}+\frac{b^2}{\sqrt{dx^2+c}\,c^2}+\frac{b^2}{3\left(dx^2+c\right)^{\frac{3}{2}}c}-\frac{5\,abd}{\sqrt{dx^2+c}\,c^3}-\frac{5\,abd}{3\left(dx^2+c\right)^{\frac{3}{2}}c^2}+\frac{35\,a^2d^2}{8\sqrt{dx^2+c}\,c^4}+\frac{35\,a^2d^2}{24\left(dx^2+c\right)^{\frac{3}{2}}c^3}-\frac{ab}{\left(dx^2+c\right)^{\frac{3}{2}}cx^2}+\frac{7\,a^2d}{8\left(dx^2+c\right)^{\frac{3}{2}}c^2x^2}-\frac{a^2}{4\left(dx^2+c\right)^{\frac{3}{2}}cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) + 5*a*b*d*arcsinh(c/(sqrt(c*d)*a
bs(x)))/c^(7/2) - 35/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(9/2) + b^2/
(sqrt(d*x^2 + c)*c^2) + 1/3*b^2/((d*x^2 + c)^(3/2)*c) - 5*a*b*d/(sqrt(d*x^2
 + c)*c^3) - 5/3*a*b*d/((d*x^2 + c)^(3/2)*c^2) + 35/8*a^2*d^2/(sqrt(d*x^2 +
 c)*c^4) + 35/24*a^2*d^2/((d*x^2 + c)^(3/2)*c^3) - a*b/((d*x^2 + c)^(3/2)*c
*x^2) + 7/8*a^2*d/((d*x^2 + c)^(3/2)*c^2*x^2) - 1/4*a^2/((d*x^2 + c)^(3/2)*
c*x^4)
```

mupad [B] time = 1.11, size = 216, normalized size = 1.17

$$\frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c}+\frac{\left(d\,x^2+c\right)\,\left(7\,a^2\,d^2-8\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c^2}-\frac{5\,{\left(d\,x^2+c\right)}^2\,\left(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{24\,c^3}+\frac{{\left(d\,x^2+c\right)}^3\,\left(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^4}}{{\left(d\,x^2+c\right)}^{7/2}-2\,c\,{\left(d\,x^2+c\right)}^{5/2}+c^2\,{\left(d\,x^2+c\right)}^{3/2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2\right)}{8\,c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(3*c) + ((c + d*x^2)*(7*a^2*d^2 + b^2*c^2
- 8*a*b*c*d))/(3*c^2) - (5*(c + d*x^2)^2*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c
```

```
*d))/(24*c^3) + ((c + d*x^2)^3*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c*d))/(8*c^
4))/((c + d*x^2)^(7/2) - 2*c*(c + d*x^2)^(5/2) + c^2*(c + d*x^2)^(3/2)) - (
atanh((c + d*x^2)^(1/2)/c^(1/2))*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c*d))/(8*
c^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{2}\right)^{2}}{x^{5}\left(c+dx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(5/2)), x)
```

3.652 $\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{8dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^5\sqrt{c+dx^2}}-\frac{4dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^4\left(c+dx^2\right)^{3/2}}-\frac{5b^2c^2-4ad(5bc-4ad)}{5c^3x\left(c+dx^2\right)^{3/2}}-\frac{2a}{15c^2}$$

Rubi [A] time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {462, 453, 271, 192, 191}

$$-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{8dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^5\sqrt{c+dx^2}}-\frac{4dx\left(5b^2c^2-4ad(5bc-4ad)\right)}{15c^4\left(c+dx^2\right)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] -a^2/(5*c*x^5*(c + d*x^2)^(3/2)) - (2*a*(5*b*c - 4*a*d))/(15*c^2*x^3*(c + d
*x^2)^(3/2)) - (5*b^2 - (4*a*d*(5*b*c - 4*a*d))/c^2)/(5*c*x*(c + d*x^2)^(3/
2)) - (4*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^4*(c + d*x^2)^(3/2)
) - (8*d*(5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*x)/(15*c^5*Sqrt[c + d*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{5/2}}\,dx &= -\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{2a(5bc-4ad)+5b^2cx^2}{x^4\left(c+dx^2\right)^{5/2}}\,dx}{5c}\\
&= -\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}-\frac{1}{5}\left(-5b^2+\frac{4ad(5bc-4ad)}{c^2}\right)\int\frac{1}{x^2\left(c+dx^2\right)^{5/}}\\
&= -\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx\left(c+dx^2\right)^{3/2}}-\frac{\left(4d\left(5b^2-\frac{4ad(5bc-4ad)}{c^2}\right)\right)}{5c}\\
&= -\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx\left(c+dx^2\right)^{3/2}}-\frac{4d\left(5b^2-\frac{4ad(5bc-4ad)}{c^2}\right)x}{15c^2\left(c+dx^2\right)^{3/2}}\\
&= -\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}-\frac{2a(5bc-4ad)}{15c^2x^3\left(c+dx^2\right)^{3/2}}-\frac{5b^2-\frac{4ad(5bc-4ad)}{c^2}}{5cx\left(c+dx^2\right)^{3/2}}-\frac{4d\left(5b^2-\frac{4ad(5bc-4ad)}{c^2}\right)x}{15c^2\left(c+dx^2\right)^{3/2}}
\end{aligned}$$

Mathematica [**A**] time = 0.18, size = 142, normalized size = 0.78

$$\frac{-a^2\left(3c^4-8c^3dx^2+48c^2d^2x^4+192cd^3x^6+128d^4x^8\right)+10abcx^2\left(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6\right)-5b^2c^2x^4\left(3c^2+12cdx^2+8d^2x^4\right)}{15c^5x^5\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*b^2*c^2*x^4*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 10*a*b*c*x^2*(-c^3 + 6*c
^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) - a^2*(3*c^4 - 8*c^3*d*x^2 + 48*c^2*d
^2*x^4 + 192*c*d^3*x^6 + 128*d^4*x^8))/(15*c^5*x^5*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.27, size = 161, normalized size = 0.88

$$\frac{-3a^2c^4+8a^2c^3dx^2-48a^2c^2d^2x^4-192a^2cd^3x^6-128a^2d^4x^8-10abc^4x^2+60abc^3dx^4+240abc^2d^2x^6+160abcd^3x^8-15b^2c^4x^4-60b^2c^3dx^6-40b^2c^2d^2x^8}{15c^5x^5\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*c^4 - 10*a*b*c^4*x^2 + 8*a^2*c^3*d*x^2 - 15*b^2*c^4*x^4 + 60*a*b*c^
3*d*x^4 - 48*a^2*c^2*d^2*x^4 - 60*b^2*c^3*d*x^6 + 240*a*b*c^2*d^2*x^6 - 192
*a^2*c*d^3*x^6 - 40*b^2*c^2*d^2*x^8 + 160*a*b*c*d^3*x^8 - 128*a^2*d^4*x^8)/
(15*c^5*x^5*(c + d*x^2)^(3/2))
```

fricas [**A**] time = 1.28, size = 171, normalized size = 0.93

$$-\frac{\left(8\left(5\,b^2c^2d^2-20\,abcd^3+16\,a^2d^4\right)x^8+12\left(5\,b^2c^3d-20\,abc^2d^2+16\,a^2cd^3\right)x^6+3\,a^2c^4+3\left(5\,b^2c^4-20\,abc^3d+16\,a^2c^2d^2\right)x^4+2\left(5\,abc^4-4\,a^2c^3d\right)x^2\right)\sqrt{dx^2+c}}{15\left(c^5d^2x^9+2\,c^6dx^7+c^7x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*b^2*c^2*d^2 - 20*a*b*c*d^3 + 16*a^2*d^4)*x^8 + 12*(5*b^2*c^3*d
- 20*a*b*c^2*d^2 + 16*a^2*c*d^3)*x^6 + 3*a^2*c^4 + 3*(5*b^2*c^4 - 20*a*b*c^
3*d + 16*a^2*c^2*d^2)*x^4 + 2*(5*a*b*c^4 - 4*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c
)/(c^5*d^2*x^9 + 2*c^6*d*x^7 + c^7*x^5)
```

giac [**B**] time = 0.54, size = 509, normalized size = 2.78

$$-\frac{x\left(\frac{\left(5\,b^2c^6d^3-16\,abc^5d^4+11\,a^2c^4d^5\right)x^2}{c^9d}+\frac{6\left(b^2c^7d^2-3\,abc^6d^3+2\,a^2c^5d^4\right)}{c^9d}\right)}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abcd^{\frac{3}{2}}+45\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8a^2d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c^3\sqrt{d}+300\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6abc^2d^{\frac{3}{2}}-240\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6a^2cd^{\frac{5}{2}}+90\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^4\sqrt{d}-500\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc^3d^{\frac{3}{2}}+490\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2c^2d^{\frac{5}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^5\sqrt{d}+340\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^4d^{\frac{3}{2}}-320\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2c^3d^{\frac{5}{2}}+15\,b^2c^6\sqrt{d}-80\,abc^5d^{\frac{3}{2}}+73\,a^2c^4d^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^5c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x*((5*b^2*c^6*d^3 - 16*a*b*c^5*d^4 + 11*a^2*c^4*d^5)*x^2/(c^9*d) + 6*(
b^2*c^7*d^2 - 3*a*b*c^6*d^3 + 2*a^2*c^5*d^4)/(c^9*d))/(d*x^2 + c)^(3/2) + 2
/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 60*(sqrt(d)*x - s
qrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 45*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^
(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 300*(sqrt(d)*x
 - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) - 240*(sqrt(d)*x - sqrt(d*x^2 + c))^6
*a^2*c*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 500*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 490*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt
(d) + 340*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 320*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 80*a*b*c^5*d^(3
/2) + 73*a^2*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^4)
```

maple [A]   time = 0.01, size = 158, normalized size = 0.86

$$-\frac{128a^2d^4x^8-160abc\,d^3x^8+40b^2c^2d^2x^8+192a^2c\,d^3x^6-240ab\,c^2d^2x^6+60b^2c^3d\,x^6+48a^2c^2d^2x^4-60ab\,c^3d\,x^4+15b^2c^4x^4-8a^2c^3d\,x^2+10ab\,c^4x^2+3a^2c^4}{15\left(d\,x^2+c\right)^{\frac{3}{2}}c^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/15*(128*a^2*d^4*x^8-160*a*b*c*d^3*x^8+40*b^2*c^2*d^2*x^8+192*a^2*c*d^3*x
^6-240*a*b*c^2*d^2*x^6+60*b^2*c^3*d*x^6+48*a^2*c^2*d^2*x^4-60*a*b*c^3*d*x^4
+15*b^2*c^4*x^4-8*a^2*c^3*d*x^2+10*a*b*c^4*x^2+3*a^2*c^4)/(d*x^2+c)^(3/2)/x
^5/c^5
```

maxima [A]   time = 0.96, size = 244, normalized size = 1.33

$$-\frac{8\,b^2dx}{3\,\sqrt{dx^2+c}\,c^3}-\frac{4\,b^2dx}{3\left(dx^2+c\right)^{\frac{3}{2}}c^2}+\frac{32\,abd^2x}{3\,\sqrt{dx^2+c}\,c^4}+\frac{16\,abd^2x}{3\left(dx^2+c\right)^{\frac{3}{2}}c^3}-\frac{128\,a^2d^3x}{15\,\sqrt{dx^2+c}\,c^5}-\frac{64\,a^2d^3x}{15\left(dx^2+c\right)^{\frac{3}{2}}c^4}-\frac{b^2}{\left(dx^2+c\right)^{\frac{3}{2}}cx}+\frac{4\,abd}{\left(dx^2+c\right)^{\frac{3}{2}}c^2x}-\frac{16\,a^2d^2}{5\left(dx^2+c\right)^{\frac{3}{2}}c^3x}-\frac{2\,ab}{3\left(dx^2+c\right)^{\frac{3}{2}}cx^3}+\frac{8\,a^2d}{15\left(dx^2+c\right)^{\frac{3}{2}}c^2x^3}-\frac{a^2}{5\left(dx^2+c\right)^{\frac{3}{2}}cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*b^2*d*x/(sqrt(d*x^2 + c)*c^3) - 4/3*b^2*d*x/((d*x^2 + c)^(3/2)*c^2) +
32/3*a*b*d^2*x/(sqrt(d*x^2 + c)*c^4) + 16/3*a*b*d^2*x/((d*x^2 + c)^(3/2)*c^
3) - 128/15*a^2*d^3*x/(sqrt(d*x^2 + c)*c^5) - 64/15*a^2*d^3*x/((d*x^2 + c)^
(3/2)*c^4) - b^2/((d*x^2 + c)^(3/2)*c*x) + 4*a*b*d/((d*x^2 + c)^(3/2)*c^2*x
) - 16/5*a^2*d^2/((d*x^2 + c)^(3/2)*c^3*x) - 2/3*a*b/((d*x^2 + c)^(3/2)*c*x
^3) + 8/15*a^2*d/((d*x^2 + c)^(3/2)*c^2*x^3) - 1/5*a^2/((d*x^2 + c)^(3/2)*c
*x^5)
```

mupad [B]   time = 0.98, size = 298, normalized size = 1.63

$$\frac{2\,a\,\sqrt{d\,x^2+c}\,\left(7\,a\,d-5\,b\,c\right)}{15\,c^4\,x^3}-\frac{\frac{73\,a^2\,c^2\,d^2-80\,a\,b\,c^3\,d+15\,b^2\,c^4}{30\,c^5}-\frac{c\,\left(\frac{d\,\left(73\,a^2\,c^2\,d^2-80\,a\,b\,c^3\,d+15\,b^2\,c^4\right)}{18\,c^6}+\frac{c\,\left(\frac{4\,a\,d^3\,\left(7\,a\,d-5\,b\,c\right)}{45\,c^5}-\frac{a\,d^3\,\left(43\,a\,d-35\,b\,c\right)}{9\,c^5}\right)}{d}+\frac{a\,d^2\,\left(43\,a\,d-35\,b\,c\right)}{15\,c^4}\right)}{d}}{x\,{\left(d\,x^2+c\right)}^{3/2}}-\frac{a^2\,\sqrt{d\,x^2+c}}{5\,c^3\,x^5}-\frac{x^2\,\left(\frac{2\,d\,\left(78\,a^2\,c\,d^2-90\,a\,b\,c^2\,d+20\,b^2\,c^3\right)}{15\,c^6}-\frac{4\,a\,d^2\,\left(7\,a\,d-5\,b\,c\right)}{15\,c^5}\right)+\frac{78\,a^2\,c\,d^2-90\,a\,b\,c^2\,d+20\,b^2\,c^3}{15\,c^5}}{x\,\sqrt{d\,x^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x)
```

```
[Out] (2*a*(c + d*x^2)^(1/2)*(7*a*d - 5*b*c))/(15*c^4*x^3) - ((15*b^2*c^4 + 73*a^
2*c^2*d^2 - 80*a*b*c^3*d)/(30*c^5) - (c*((d*(15*b^2*c^4 + 73*a^2*c^2*d^2 -
80*a*b*c^3*d))/(18*c^6) + (c*((4*a*d^3*(7*a*d - 5*b*c))/(45*c^5) - (a*d^3*(
43*a*d - 35*b*c))/(9*c^5)))/d + (a*d^2*(43*a*d - 35*b*c))/(15*c^4)))/d)/(x*
(c + d*x^2)^(3/2)) - (a^2*(c + d*x^2)^(1/2))/(5*c^3*x^5) - (x^2*((2*d*(20*b
^2*c^3 + 78*a^2*c*d^2 - 90*a*b*c^2*d))/(15*c^6) - (4*a*d^2*(7*a*d - 5*b*c))
/(15*c^5)) + (20*b^2*c^3 + 78*a^2*c*d^2 - 90*a*b*c^2*d)/(15*c^5))/(x*(c + d
*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(5/2)), x)
```

3.653 $\int \frac{x^5}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=72

$$\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 302, 205}

$$\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}-\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[d*x^2])) + x^4/(3*b*Sqrt[d*x^2]) + (a^(3/2)*x*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx &= \frac{x\int \frac{x^4}{a+bx^2}\,dx}{\sqrt{dx^2}} \\
&= \frac{x\int \left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx}{\sqrt{dx^2}} \\
&= -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{\left(a^2x\right)\int \frac{1}{a+bx^2}\,dx}{b^2\sqrt{dx^2}} \\
&= -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{a^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.78

$$
\frac{x\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{b}\,x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b
^(5/2)*Sqrt[d*x^2])
```

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.99

$$
\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{5/2}\sqrt{d}}+\frac{\sqrt{dx^2}\left(bx^2-3a\right)}{3b^2d}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (Sqrt[d*x^2]*(-3*a + b*x^2))/(3*b^2*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d*x^
2])/(Sqrt[a]*Sqrt[d])])/(b^(5/2)*Sqrt[d])
```

fricas [A] time = 1.26, size = 147, normalized size = 2.04

$$\left[\frac{3\,ad\sqrt{-\frac{a}{bd}}\,\log\left(\frac{bx^2+2\,\sqrt{dx^2}\,b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\left(bx^2-3\,a\right)\sqrt{dx^2}}{6\,b^2d},\ \frac{3\,ad\sqrt{\frac{a}{bd}}\,\arctan\left(\frac{\sqrt{dx^2}\,b\sqrt{\frac{a}{bd}}}{a}\right)+\left(bx^2-3\,a\right)\sqrt{dx^2}}{3\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*d*sqrt(-a/(b*d))*log((b*x^2 + 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)
/(b*x^2 + a)) + 2*(b*x^2 - 3*a)*sqrt(d*x^2))/(b^2*d), 1/3*(3*a*d*sqrt(a/(b*
d))*arctan(sqrt(d*x^2)*b*sqrt(a/(b*d))/a) + (b*x^2 - 3*a)*sqrt(d*x^2))/(b^2
*d)]
```

giac [A] time = 0.41, size = 70, normalized size = 0.97

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b^2}+\frac{\sqrt{dx^2}\,b^2d^5x^2-3\,\sqrt{dx^2}\,abd^5}{3\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + 1/3*(sqrt(d*x^2)*
b^2*d^5*x^2 - 3*sqrt(d*x^2)*a*b*d^5)/(b^3*d^6)
```

maple [A] time = 0.03, size = 53, normalized size = 0.74

$$\frac{\left(\sqrt{ab}\;b\,x^3+3a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\sqrt{ab}\;ax\right)x}{3\sqrt{d\,x^2}\;\sqrt{ab}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/3*x*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(1/(a*b)^(1/2)*b*x))
/(d*x^2)^(1/2)/b^2/(a*b)^(1/2)
```

maxima [A] time = 1.83, size = 67, normalized size = 0.93

$$\frac{\frac{3\,a^2d^3\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b^2}+\frac{\left(dx^2\right)^{\frac{3}{2}}bd-3\,\sqrt{dx^2}\,ad^2}{b^2}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*d^3*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + ((d*x^
2)^(3/2)*b*d - 3*sqrt(d*x^2)*a*d^2)/b^2)/d^3
```

mupad [B] time = 0.65, size = 51, normalized size = 0.71

$$\frac{\left(x^2\right)^{3/2}}{3\,b\,\sqrt{d}}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{5/2}\,\sqrt{d}}-\frac{a\,\sqrt{x^2}}{b^2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] (x^2)^(3/2)/(3*b*d^(1/2)) + (a^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(
b^(5/2)*d^(1/2)) - (a*(x^2)^(1/2))/(b^2*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\sqrt{dx^2}\,\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(d*x**2)*(a + b*x**2)), x)
```

3.654 $\int \frac{x^3}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=52

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 321, 205}

$$\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] x^2/(b*Sqrt[d*x^2]) - (Sqrt[a]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt
[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx &= \frac{x\int \frac{x^2}{a+bx^2}\,dx}{\sqrt{dx^2}} \\
&= \frac{x^2}{b\sqrt{dx^2}} - \frac{(ax)\int \frac{1}{a+bx^2}\,dx}{b\sqrt{dx^2}} \\
&= \frac{x^2}{b\sqrt{dx^2}} - \frac{\sqrt{a}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.85

$$\frac{x\left(\sqrt{b}\,x-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*Sqrt[d*x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 60, normalized size = 1.15

$$\frac{\sqrt{dx^2}}{bd}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{dx^2}}{\sqrt{a}\,\sqrt{d}}\right)}{b^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] Sqrt[d*x^2]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*Sqrt[d])
])/(b^(3/2)*Sqrt[d])
```

fricas [A] time = 1.23, size = 126, normalized size = 2.42

$$\left[\frac{d\sqrt{-\frac{a}{bd}}\,\log\left(\frac{bx^2-2\,\sqrt{dx^2}\,b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\,\sqrt{dx^2}}{2\,bd},\,-\frac{d\sqrt{\frac{a}{bd}}\,\arctan\left(\frac{\sqrt{dx^2}\,b\sqrt{\frac{a}{bd}}}{a}\right)-\sqrt{dx^2}}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*sqrt(-a/(b*d))*log((b*x^2 - 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)/(b*
x^2 + a)) + 2*sqrt(d*x^2))/(b*d), -(d*sqrt(a/(b*d))*arctan(sqrt(d*x^2)*b*sq
rt(a/(b*d))/a) - sqrt(d*x^2))/(b*d)]
```

giac [A] time = 0.43, size = 46, normalized size = 0.88

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b}-\frac{\sqrt{dx^2}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*x^2)/b)/d
```

maple [A] time = 0.01, size = 38, normalized size = 0.73

$$\frac{\left(-a\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}\;x\right)x}{\sqrt{d\,x^2}\;\sqrt{ab}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] x*(x*(a*b)^(1/2)-a*arctan(1/(a*b)^(1/2)*b*x))/(d*x^2)^(1/2)/b/(a*b)^(1/2)
```

maxima [A] time = 1.79, size = 49, normalized size = 0.94

$$-\frac{\frac{ad^2\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,b}-\frac{\sqrt{dx^2}\,d}{b}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*d^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*x^2)*d/b
)/d^2
```

mupad [B] time = 0.62, size = 37, normalized size = 0.71

$$\frac{\sqrt{x^2}}{b\,\sqrt{d}}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] (x^2)^(1/2)/(b*d^(1/2)) - (a^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(b^
(3/2)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{dx^2}\,\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(d*x**2)*(a + b*x**2)), x)
```

3.655 $\int \frac{x}{\sqrt{dx^2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=34

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 205}

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{dx^2}\,\left(a+bx^2\right)}\,dx &= \frac{x\int \frac{1}{a+bx^2}\,dx}{\sqrt{dx^2}}\\ &= \frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 42, normalized size = 1.24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{dx^2}}{\sqrt{a}\,\sqrt{d}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*Sqrt[d])]/(Sqrt[a]*Sqrt[b]*Sqrt[d])
```

fricas [A] time = 1.25, size = 94, normalized size = 2.76

$$\left[-\frac{\sqrt{-abd}\,\log\left(\frac{bdx^2-ad-2\,\sqrt{-abd}\,\sqrt{dx^2}}{bx^2+a}\right)}{2\,abd}, \frac{\sqrt{abd}\,\arctan\left(\frac{\sqrt{abd}\,\sqrt{dx^2}}{ad}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b*d)*log((b*d*x^2 - a*d - 2*sqrt(-a*b*d)*sqrt(d*x^2))/(b*x^2
+ a))/(a*b*d), sqrt(a*b*d)*arctan(sqrt(a*b*d)*sqrt(d*x^2)/(a*d))/(a*b*d)]
```

giac [A] time = 0.36, size = 23, normalized size = 0.68

$$\frac{\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

maple [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{x \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{d\,x^2}\,\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/(d*x^2)^(1/2)*x/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 1.97, size = 23, normalized size = 0.68

$$\frac{\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

mupad [B] time = 0.61, size = 23, normalized size = 0.68

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] atan((b^(1/2)*(x^2)^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{dx^2}\,\left(a + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(d*x**2)*(a + b*x**2)), x)
```

3.656 $\int \frac{1}{x\sqrt{dx^2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 325, 205}

$$-\frac{\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}-\frac{1}{a\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -(1/(a*Sqrt[d*x^2])) - (Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqr
t[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{dx^2}\left(a+bx^2\right)}\,dx &= \frac{x\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{\sqrt{dx^2}} \\
&= -\frac{1}{a\sqrt{dx^2}} - \frac{(bx)\int \frac{1}{a+bx^2}\,dx}{a\sqrt{dx^2}} \\
&= -\frac{1}{a\sqrt{dx^2}} - \frac{\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.92

$$
-\frac{dx^2\left(\sqrt{b}\,x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}\left(dx^2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -((d*x^2*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(a^(3/2)*(d*x^2
)^(3/2)))
```

IntegrateAlgebraic [A] time = 0.05, size = 64, normalized size = 1.28

$$
-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{dx^2}}{\sqrt{a}\,\sqrt{d}}\right)}{a^{3/2}\sqrt{d}}-\frac{\sqrt{dx^2}}{adx^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[d*x^2]/(a*d*x^2)) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*S
qrt[d])])/(a^(3/2)*Sqrt[d])
```

fricas [A] time = 0.79, size = 132, normalized size = 2.64

$$
\left[\frac{dx^2\sqrt{-\frac{b}{ad}}\,\log\left(\frac{bx^2-2\,\sqrt{dx^2}\,a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)-2\,\sqrt{dx^2}}{2\,adx^2},\,-\frac{dx^2\sqrt{\frac{b}{ad}}\,\arctan\left(\sqrt{dx^2}\,\sqrt{\frac{b}{ad}}\right)+\sqrt{dx^2}}{adx^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(d*x^2*sqrt(-b/(a*d))*log((b*x^2 - 2*sqrt(d*x^2)*a*sqrt(-b/(a*d)) - a)
/(b*x^2 + a)) - 2*sqrt(d*x^2))/(a*d*x^2), -(d*x^2*sqrt(b/(a*d))*arctan(sqrt
(d*x^2)*sqrt(b/(a*d))) + sqrt(d*x^2))/(a*d*x^2)]
```

giac **[A]** time = 0.28, size = 41, normalized size = 0.82

$$-\frac{b\arctan\left(\frac{\sqrt{dx^2}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,a}-\frac{1}{\sqrt{dx^2}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -b*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*a) - 1/(sqrt(d*x^2)*a)
```

maple **[A]** time = 0.01, size = 36, normalized size = 0.72

$$-\frac{bx\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\sqrt{ab}}{\sqrt{d\,x^2}\,\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] -(b*arctan(1/(a*b)^(1/2)*b*x)*x+(a*b)^(1/2))/(d*x^2)^(1/2)/a/(a*b)^(1/2)
```

maxima **[A]** time = 1.99, size = 35, normalized size = 0.70

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a\sqrt{d}}-\frac{1}{a\sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*sqrt(d)) - 1/(a*sqrt(d)*x)
```

mupad **[B]** time = 0.62, size = 38, normalized size = 0.76

$$-\frac{1}{a\,\sqrt{d}\,\sqrt{x^2}}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] - 1/(a*d^(1/2)*(x^2)^(1/2)) - (b^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))
/(a^(3/2)*d^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{dx^2}\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(d*x**2)*(a + b*x**2)), x)
```

3.657 $\int \frac{1}{x^3\sqrt{dx^2}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=68

$$\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 325, 205}

$$\frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}+\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] b/(a^2*Sqrt[d*x^2]) - 1/(3*a*x^2*Sqrt[d*x^2]) + (b^(3/2)*x*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(a^(5/2)*Sqrt[d*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{dx^2}\left(a+bx^2\right)}\,dx &= \frac{x\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{\sqrt{dx^2}} \\ &= -\frac{1}{3ax^2\sqrt{dx^2}} - \frac{(bx)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a\sqrt{dx^2}} \\ &= \frac{b}{a^2\sqrt{dx^2}} - \frac{1}{3ax^2\sqrt{dx^2}} + \frac{\left(b^2x\right)\int \frac{1}{a+bx^2}\,dx}{a^2\sqrt{dx^2}} \\ &= \frac{b}{a^2\sqrt{dx^2}} - \frac{1}{3ax^2\sqrt{dx^2}} + \frac{b^{3/2}x\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.85

$$\frac{d\left(3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{a}\left(a-3bx^2\right)\right)}{3a^{5/2}\left(dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (d*(-(Sqrt[a]*(a - 3*b*x^2)) + 3*b^(3/2)*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/
(3*a^(5/2)*(d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 75, normalized size = 1.10

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{5/2}\sqrt{d}}+\frac{\sqrt{dx^2}\left(3bx^2-a\right)}{3a^2dx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

```
[Out] (Sqrt[d*x^2]*(-a + 3*b*x^2))/(3*a^2*d*x^4) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[
d*x^2])/(Sqrt[a]*Sqrt[d])])/(a^(5/2)*Sqrt[d])
```

fricas [A] time = 1.37, size = 157, normalized size = 2.31

$$\left[\frac{3\,bdx^{4}\sqrt{-\frac{b}{ad}}\,\log\left(\frac{bx^{2}+2\,\sqrt{dx^{2}}\,a\sqrt{-\frac{b}{ad}}-a}{bx^{2}+a}\right)+2\left(3\,bx^{2}-a\right)\sqrt{dx^{2}}}{6\,a^{2}dx^{4}},\ \frac{3\,bdx^{4}\sqrt{\frac{b}{ad}}\,\arctan\left(\sqrt{dx^{2}}\,\sqrt{\frac{b}{ad}}\right)+\left(3\,bx^{2}-a\right)\sqrt{dx^{2}}}{3\,a^{2}dx^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*d*x^4*sqrt(-b/(a*d))*log((b*x^2 + 2*sqrt(d*x^2)*a*sqrt(-b/(a*d))
- a)/(b*x^2 + a)) + 2*(3*b*x^2 - a)*sqrt(d*x^2))/(a^2*d*x^4), 1/3*(3*b*d*x^
4*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*sqrt(b/(a*d))) + (3*b*x^2 - a)*sqrt(d*x^
2))/(a^2*d*x^4)]
```

giac [A] time = 0.31, size = 60, normalized size = 0.88

$$\frac{b^{2}\arctan\left(\frac{\sqrt{dx^{2}}\,b}{\sqrt{abd}}\right)}{\sqrt{abd}\,a^{2}}+\frac{3\,bdx^{2}-ad}{3\,\sqrt{dx^{2}}\,a^{2}dx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*a^2) + 1/3*(3*b*d*x^2 -
a*d)/(sqrt(d*x^2)*a^2*d*x^2)
```

maple [A] time = 0.01, size = 58, normalized size = 0.85

$$\frac{3b^{2}x^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+3\sqrt{ab}\;b\,x^{2}-\sqrt{ab}\;a}{3\sqrt{d\,x^{2}}\;\sqrt{ab}\;a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x)
```

```
[Out] 1/3/x^2*(3*b^2*arctan(1/(a*b)^(1/2)*b*x)*x^3+3*b*(a*b)^(1/2)*x^2-a*(a*b)^(1
/2))/(d*x^2)^(1/2)/a^2/(a*b)^(1/2)
```

maxima [A] time = 2.01, size = 52, normalized size = 0.76

$$\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}\sqrt{d}}+\frac{3\,b\sqrt{d}\,x^{2}-a\sqrt{d}}{3\,a^{2}dx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sqrt(d)) + 1/3*(3*b*sqrt(d)*x^2 -
a*sqrt(d))/(a^2*d*x^3)
```

mupad [B] time = 0.63, size = 53, normalized size = 0.78

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{a^{5/2}\,\sqrt{d}}-\frac{1}{3\,a\,\sqrt{d}\,{\left(x^2\right)}^{3/2}}+\frac{b\,x^2}{a^2\,\sqrt{d}\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(a^(5/2)*d^(1/2)) - 1/(3*a*d^
(1/2)*(x^2)^(3/2)) + (b*x^2)/(a^2*d^(1/2)*(x^2)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{dx^{2}}\,\left(a + bx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(d*x**2)*(a + b*x**2)), x)
```

3.658 $\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=157

$$\frac{a^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}-\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{x\sqrt{c+dx^2}\,(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

Rubi **[A]** time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {478, 582, 523, 217, 206, 377, 205}

$$-\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}+\frac{a^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}\,(bc-4ad)}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2*d) + (x^3*Sqrt[c + d*x^2])/(4*b) +
 (a^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^3 - ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(8*b^3*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\int \frac{x^2\left(3ac+(-bc+4ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{\int \frac{-ac(bc-4ad)+\left(-b^2c^2-4abcd+8a^2d^2\right)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8b^2d}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{\left(a^2(bc-ad)\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^3}-\frac{\left(b^2c^2+4abcd-8\right.}{8b}\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^3}-\\
&= \frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{a^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}-\frac{\left(b^2c^2+4abcd-8\right.}{8}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 148, normalized size = 0.94

$$
\frac{8a^{3/2}d^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)-\left(-8a^2d^2+4abcd+b^2c^2\right)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)+b\sqrt{d}\,x\sqrt{c+dx^2}\left(-4ad+bc+2bdx^2\right)}{8b^3d^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(b*c - 4*a*d + 2*b*d*x^2) + 8*a^(3/2)*d^(3/2)*
Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] - (b^
2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b^3*d
^(3/2))
```

IntegrateAlgebraic [A] time = 0.40, size = 200, normalized size = 1.27

$$
-\frac{a^{3/2}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{b^3}+\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8b^3d^{3/2}}+\frac{\sqrt{c+dx^2}\left(-4adx+bcx+2bdx^3\right)}{8b^2d}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(b*c*x - 4*a*d*x + 2*b*d*x^3))/(8*b^2*d) - (a^(3/2)*Sqrt[b
*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[
a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/b^3
```

```
 + ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/
(8*b^3*d^(3/2))
```

fricas [A] time = 1.88, size = 857, normalized size = 5.46

$$\left[\frac{4\sqrt{-abc+a^{2}d}\,ad^{2}\log\left(\frac{(b^{2}c^{2}-8abcd+8a^{2}d^{2})x^{4}+a^{2}c^{2}-2(3abc^{2}-4a^{2}cd)x^{2}+4((bc-2ad)x^{3}-acx)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)-(b^{2}c^{2}+4abcd-8a^{2}d^{2})\sqrt{d}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\,\sqrt{d}x-c\right)+2(2b^{2}d^{2}x^{3}+(b^{2}cd-4abd^{2})x)\sqrt{dx^{2}+c}}{16b^{3}d^{2}},\ \frac{2\sqrt{-abc+a^{2}d}\,ad^{2}\log\left(\frac{(b^{2}c^{2}-8abcd+8a^{2}d^{2})x^{4}+a^{2}c^{2}-2(3abc^{2}-4a^{2}cd)x^{2}+4((bc-2ad)x^{3}-acx)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)+(b^{2}c^{2}+4abcd-8a^{2}d^{2})\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)+(2b^{2}d^{2}x^{3}+(b^{2}cd-4abd^{2})x)\sqrt{dx^{2}+c}}{8b^{3}d^{2}},\ \frac{8\sqrt{abc-a^{2}d}\,ad^{2}\arctan\left(\frac{\sqrt{abc-a^{2}d}\,((bc-2ad)x^{2}-ac)\sqrt{dx^{2}+c}}{2((abcd-a^{2}d^{2})x^{3}+(abc^{2}-a^{2}cd)x)}\right)-(b^{2}c^{2}+4abcd-8a^{2}d^{2})\sqrt{d}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\,\sqrt{d}x-c\right)+2(2b^{2}d^{2}x^{3}+(b^{2}cd-4abd^{2})x)\sqrt{dx^{2}+c}}{16b^{3}d^{2}},\ \frac{4\sqrt{abc-a^{2}d}\,ad^{2}\arctan\left(\frac{\sqrt{abc-a^{2}d}\,((bc-2ad)x^{2}-ac)\sqrt{dx^{2}+c}}{2((abcd-a^{2}d^{2})x^{3}+(abc^{2}-a^{2}cd)x)}\right)+(b^{2}c^{2}+4abcd-8a^{2}d^{2})\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)+(2b^{2}d^{2}x^{3}+(b^{2}cd-4abd^{2})x)\sqrt{dx^{2}+c}}{8b^{3}d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (b^
2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqr
t(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/
(b^3*d^2), 1/8*(2*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x
^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^
2)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x
^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*
d^2), 1/16*(8*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x
)*sqrt(d*x^2 + c))/(b^3*d^2), 1/8*(4*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*s
qrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^
2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sq
rt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a
*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.07, size = 1088, normalized size = 6.93

$$\frac{a^{3}d\ln\left(\frac{\frac{2\sqrt{-ab}\,(x-\frac{\sqrt{-ab}}{b})d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{(x-\frac{\sqrt{-ab}}{b})^{2}d+\frac{2\sqrt{-ab}\,(x-\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{3}}-\frac{a^{3}d\ln\left(\frac{-\frac{2\sqrt{-ab}\,(x+\frac{\sqrt{-ab}}{b})d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{(x+\frac{\sqrt{-ab}}{b})^{2}d-\frac{2\sqrt{-ab}\,(x+\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{3}}-\frac{a^{2}c\ln\left(\frac{\frac{2\sqrt{-ab}\,(x-\frac{\sqrt{-ab}}{b})d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{(x-\frac{\sqrt{-ab}}{b})^{2}d+\frac{2\sqrt{-ab}\,(x-\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{a^{2}c\ln\left(\frac{-\frac{2\sqrt{-ab}\,(x+\frac{\sqrt{-ab}}{b})d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{(x+\frac{\sqrt{-ab}}{b})^{2}d-\frac{2\sqrt{-ab}\,(x+\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{a^{2}\sqrt{d}\,\ln\left(\frac{(x+\frac{\sqrt{-ab}}{b})d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{(x+\frac{\sqrt{-ab}}{b})^{2}d-\frac{2\sqrt{-ab}\,(x+\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}\right)}{2b^{3}}+\frac{a^{2}\sqrt{d}\,\ln\left(\frac{(x-\frac{\sqrt{-ab}}{b})d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{(x-\frac{\sqrt{-ab}}{b})^{2}d+\frac{2\sqrt{-ab}\,(x-\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}\right)}{2b^{3}}-\frac{ac\ln\left(\sqrt{d}\,x+\sqrt{dx^{2}+c}\right)}{2b^{2}\sqrt{d}}-\frac{c^{2}\ln\left(\sqrt{d}\,x+\sqrt{dx^{2}+c}\right)}{8bd^{\frac{3}{2}}}-\frac{\sqrt{(x+\frac{\sqrt{-ab}}{b})^{2}d-\frac{2\sqrt{-ab}\,(x+\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}\,a^{2}}{2\sqrt{-ab}\,b^{2}}+\frac{\sqrt{(x-\frac{\sqrt{-ab}}{b})^{2}d+\frac{2\sqrt{-ab}\,(x-\frac{\sqrt{-ab}}{b})d}{b}-\frac{ad-bc}{b}}\,a^{2}}{2\sqrt{-ab}\,b^{2}}-\frac{\sqrt{dx^{2}+c}\,ax}{2b^{2}}-\frac{\sqrt{dx^{2}+c}\,cx}{8bd}+\frac{(dx^{2}+c)^{\frac{3}{2}}x}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b*x*(d*x^2+c)^(3/2)/d-1/8/b*c/d*x*(d*x^2+c)^(1/2)-1/8/b*c^2/d^(3/2)*ln(
d^(1/2)*x+(d*x^2+c)^(1/2))-1/2/b^2*a*x*(d*x^2+c)^(1/2)-1/2/b^2*a*c/d^(1/2)*
ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/2/b^2*a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^
2*d-2*d*(-a*b)^(1/2)/b*(x+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/2/b^3*a^2*d^
(1/2)*ln((-d*(-a*b)^(1/2)/b+d*(x+(-a*b)^(1/2)/b))/d^(1/2)+((x+(-a*b)^(1/2)/
b)^2*d-2*d*(-a*b)^(1/2)/b*(x+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/2/b^3*a^
3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(
x+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*d*(-a*b)
^(1/2)/b*(x+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d+1/2/b
^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2
)/b*(x+(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*d*(
-a*b)^(1/2)/b*(x+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+
1/2/b^2*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x-(-a*
b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/2/b^3*a^2*d^(1/2)*ln((d*(-a*b)^(1/2)/b+d*(
x-(-a*b)^(1/2)/b))/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x-(-
a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))+1/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x-(-a*b)^(1/2)/b)-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-(-a*b)^(1/2)/b)+2*(-(a*d
-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*d*(-a*b)^(1/2)/b*(x-(-a*b)^(1/2)/b
)-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\,x^4}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2), x)
```

3.659 $\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=88

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\sqrt{c+dx^2}}{b^2}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\left(c+dx^2\right)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] -((a*Sqrt[c + d*x^2])/b^2) + (c + d*x^2)^(3/2)/(3*b*d) + (a*Sqrt[b*c - a*d]
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
```

```
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{\left(c+dx^2\right)^{3/2}}{3bd} - \frac{a\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3bd} - \frac{(a(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3bd} - \frac{(a(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{b^2d}\\
&= -\frac{a\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3bd} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 0.97

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}} + \frac{\sqrt{c+dx^2}\,\left(b\left(c+dx^2\right)-3ad\right)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*a*d + b*(c + d*x^2)))/(3*b^2*d) + (a*Sqrt[b*c - a*d]*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.11, size = 96, normalized size = 1.09

$$\frac{\sqrt{c+dx^2}\left(-3ad+bc+bdx^2\right)}{3b^2d}-\frac{a\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(b*c - 3*a*d + b*d*x^2))/(3*b^2*d) - (a*Sqrt[-(b*c) + a*d]
*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/b^(5/2)
```

fricas [A] time = 1.43, size = 295, normalized size = 3.35

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(bdx^2+bc-3\,ad\right)\sqrt{dx^2+c}}{12\,b^2d},\frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+2\left(bdx^2+bc-3\,ad\right)\sqrt{dx^2+c}}{6\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*
sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b*d*
x^2 + b*c - 3*a*d)*sqrt(d*x^2 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b*d*x^2 + b*c - 3*a*d)*sqrt(d*x
^2 + c))/(b^2*d)]
```

giac [A] time = 0.32, size = 96, normalized size = 1.09

$$-\frac{\left(abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2d^2-3\,\sqrt{dx^2+c}\,abd^3}{3\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b*c - a^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^2 + c)*a*b*d^
3)/(b^3*d^3)
```

maple [B] time = 0.01, size = 963, normalized size = 10.94

$$-\frac{a^{2} d \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, b^{3}}-\frac{a^{2} d \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, b^{3}}+\frac{ac \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, b^{2}}+\frac{ac \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, b^{2}}+\frac{\sqrt{-ab}\, a\sqrt{d}\, \ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\, d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b^{3}}-\frac{\sqrt{-ab}\, a\sqrt{d}\, \ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\, d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b^{3}}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\; a}{2b^{2}}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\; a}{2b^{2}}+\frac{\left(d x^{2}+c\right)^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(3/2)/b/d-1/2*a/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln((
(x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2
)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-1/2*a/b^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*a/b^3*d^(1/2)*(-a*b
)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^2/b
^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d+1/2*a/b^2/(-(a*d-b
*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.59, size = 86, normalized size = 0.98

$$\frac{{\left(d\,x^2+c\right)}^{3/2}}{3\,b\,d}-\frac{a\,\sqrt{d\,x^2+c}}{b^2}+\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b*d) - (a*(c + d*x^2)^(1/2))/b^2 + (a*atan((a*b^(1/2)*
(c + d*x^2)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2))/b^
(5/2)
```

sympy [A] time = 7.23, size = 87, normalized size = 0.99

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^2}}{2b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{6b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**2)/(2*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**2)/sqrt((a*d - b*c)/b))/(2*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**2)**(3
/2)/(6*b))/d**2
```

3.660 $\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

Rubi **[A]** time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {478, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}+\frac{x\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^2 + ((b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{x\sqrt{c+dx^2}}{2b}-\frac{\int \frac{ac+(-bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{x\sqrt{c+dx^2}}{2b}+\frac{(bc-2ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2}-\frac{(a(bc-ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^2}\\
&= \frac{x\sqrt{c+dx^2}}{2b}+\frac{(bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}-\frac{(a(bc-ad))\,\mathrm{Subst}\Bigg(\int \frac{1}{a-(-bc+ad)x^2}\,d}{b^2}\\
&= \frac{x\sqrt{c+dx^2}}{2b}-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 108, normalized size = 0.96

$$\frac{\frac{(bc-2ad)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-2\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)+bx\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])] + ((b*c - 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x
^2]])/Sqrt[d])/(2*b^2)
```

IntegrateAlgebraic [A] time = 0.19, size = 167, normalized size = 1.49

$$\frac{(2ad-bc)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^2\sqrt{d}}+\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{b^2}+\frac{x\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b) + (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d
])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[
c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]])/b^2 + ((-(b*c) + 2*a*d)*Log[-(Sqrt[
d]*x) + Sqrt[c + d*x^2]])/(2*b^2*Sqrt[d])
```

fricas [A] time = 1.18, size = 690, normalized size = 6.16

$$\left[\frac{2\sqrt{dx^2+c}\,bdx-(bc-2ad)\sqrt{d}\,\log\left(-2dx^2+2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+\sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)}{4b^2d},\ \frac{2\sqrt{dx^2+c}\,bdx-2(bc-2ad)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)}{4b^2d},\ \frac{2\sqrt{dx^2+c}\,bdx-2\sqrt{abc-a^2d}\,d\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-(bc-2ad)\sqrt{d}\,\log\left(-2dx^2+2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{4b^2d},\ \frac{\sqrt{dx^2+c}\,bdx-\sqrt{abc-a^2d}\,d\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-(bc-2ad)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)}{2b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt
(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a*b*c
*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2
*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(b*c - 2*a*d)*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*sqrt(a*b*c
- a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*
x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (b*c - 2*a*d)
*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d), 1/2*(sqr
t(d*x^2 + c)*b*d*x - sqrt(a*b*c - a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*(
(b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^
2 - a^2*c*d)*x)) - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
))/(b^2*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.01, size = 1010, normalized size = 9.02

$$-\frac{a^{2} d \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{a^{2} d \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{ac \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{ac \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{a\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b^{2}}-\frac{a\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b^{2}}+\frac{c\ln\left(\sqrt{d}\,x+\sqrt{d x^{2}+c}\right)}{2b\sqrt{d}}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;a}{2\sqrt{-ab}\;b}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;a}{2\sqrt{-ab}\;b}+\frac{\sqrt{d x^{2}+c}\;x}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b+1/2/b*c/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/2*a
/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)-1/2*a/b^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/
2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))+1/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x+(-a*b)^(1/2)/b))*d-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x+(-a*b)^(1/2)/b))*c-1/2*a/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*a/b^2*d^(1/2)*l
n(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^2/(-a*b)^(1/
2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d+1/2*a/(-a*b)^(
1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d x^{2} + c}\, x^{2}}{b x^{2} + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2), x)
```

3.661 $\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Rubi [**A**] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)\\ &= \frac{\sqrt{c+dx^2}}{b}+\frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}\\ &= \frac{\sqrt{c+dx^2}}{b}+\frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{bd}\\ &= \frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.00

$$\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.08, size = 74, normalized size = 1.14

$$\frac{\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{b^{3/2}}+\frac{\sqrt{c+dx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

```
[Out] Sqrt[c + d*x^2]/b + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*
Sqrt[c + d*x^2])/(b*c - a*d)])/b^(3/2)
```

fricas [A] time = 1.37, size = 255, normalized size = 3.92

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\sqrt{dx^2+c}}{4\,b},-\frac{\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\,\sqrt{dx^2+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*
x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(d*x^2 +
 c))/b, -1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2
*sqrt(d*x^2 + c))/b]
```

giac [A] time = 0.30, size = 64, normalized size = 0.98

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}+\frac{\sqrt{dx^2+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + sqrt(d*x^2 + c)/b
```

maple [B] time = 0.01, size = 936, normalized size = 14.40

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^2}-\frac{c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b^2}+\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b^2}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{2b}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/2/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)-1/2/b^2*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1
/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
```

```
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))*a*d-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+
1/2/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)+1/2/b^2*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1
/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(
1/2)/b))*a*d-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad **[B]** time = 0.59, size = 53, normalized size = 0.82

$$\frac{\sqrt{d\,x^2+c}}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(1/2)/b - (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2))*
(a*d - b*c)^(1/2))/b^(3/2)
```

sympy [A] time = 4.89, size = 61, normalized size = 0.94

$$\frac{2\left(\frac{d\sqrt{c+dx^2}}{2b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**2)/(2*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**2)/sqrt((a*d
 - b*c)/b))/(2*b**2*sqrt((a*d - b*c)/b)))/d
```

3.662 $\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {402, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sq
rt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx &= \frac{d\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}-\frac{(-bc+ad)\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}\\ &= \frac{d\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b}-\frac{(-bc+ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b}\\ &= \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 1.04

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sq
rt[a]*b) + (Sqrt[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/b
```

IntegrateAlgebraic [A] time = 0.20, size = 138, normalized size = 1.70

$$-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\,b}-\frac{\sqrt{d}\,\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

```
[Out] -((Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^
2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*
d])])/(Sqrt[a]*b)) - (Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/b
```

fricas [A] time = 0.96, size = 596, normalized size = 7.36

$$\left[\frac{2\sqrt{d}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8abcd+8a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3abc^{2}-4a^{2}cd\right)x^{2}-4\left(a^{2}cx-\left(abc-2a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)}{4b}, -\frac{4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8abcd+8a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3abc^{2}-4a^{2}cd\right)x^{2}-4\left(a^{2}cx-\left(abc-2a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)}{4b}, \frac{\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)+\sqrt{d}\log\left(-2dx^{2}-2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)}{2b}, -\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-(b*
c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)
*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b, -1/4*(4*sqrt(-d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*
c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/b, 1/2*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c
^2 - a*c*d)*x)) + sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/
b, -1/2*(2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt((b*c - a*d)/a
)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/
((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.01, size = 948, normalized size = 11.70

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b}+\frac{\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2b}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{2\sqrt{-ab}}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{2\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)+1/2*d^(1/2)/b*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)
/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
```

```
-(a*d-b*c)/b)^(1/2))-1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))*a*d+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+
(-a*b)^(1/2)/b))*c+1/2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*d^(1/2)/b*ln(((x-(-a*b)^(1/2)
/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1
/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b
)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\sqrt{-d}\,\mathrm{asin}\left(x\,\sqrt{-\frac{d}{c}}\right)}{a} & \text{if } ((c+a\,d=0\wedge b=-1)\vee a\,d=b\,c)\wedge d<0 \\ \frac{\sqrt{d}\,\ln\left(2\,\sqrt{d}\,x+2\,\sqrt{d\,x^2+c}\right)}{b}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)\sqrt{b\,c-a\,d}}{\sqrt{a}\,b} & \text{if } c\neq 0\wedge(((c+a\,d\neq 0\vee b\neq -1)\wedge a\,d\neq b\,c)\vee\neg d<0) \\ \int \frac{\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx & \text{if } ((((c+a\,d=0\wedge b=-1)\vee a\,d=b\,c)\wedge d<0)\vee c=0)\wedge(((c+a\,d\neq 0\vee b\neq -1)\wedge a\,d\neq b\,c)\vee\neg d<0) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2),x)
```

```
[Out] piecewise((c + a*d == 0 & b == -1 | a*d == b*c) & d < 0, ((-d)^(1/2)*asin(x
*(-d/c)^(1/2)))/a, c ~= 0 & ((c + a*d ~= 0 | b ~= -1) & a*d ~= b*c | ~d < 0
), (d^(1/2)*log(2*d^(1/2)*x + 2*(c + d*x^2)^(1/2)))/b + (atan((x*(- a*d + b
*c)^(1/2))/(a^(1/2)*(c + d*x^2)^(1/2)))*(- a*d + b*c)^(1/2))/(a^(1/2)*b), (
(c + a*d == 0 & b == -1 | a*d == b*c) & d < 0 | c == 0) & ((c + a*d ~= 0 |
b ~= -1) & a*d ~= b*c | ~d < 0), int((c + d*x^2)^(1/2)/(a + b*x^2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2), x)
```

3.663 $\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Rubi **[A]** time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx,x,x^2\right)\\ &= \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}-\frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2a}\\ &= \frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad}-\frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{ad}\\ &= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.98

$$\frac{\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] (-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/Sqrt[b])/a
```

IntegrateAlgebraic [A] time = 0.08, size = 91, normalized size = 1.14

$$-\frac{\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a\sqrt{b}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(
b*c - a*d)])/(a*Sqrt[b])) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
```

fricas [A] time = 1.53, size = 578, normalized size = 7.22

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4a}, \frac{4\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)}{4a}, \frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{2a}, \frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*
x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(c)*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/4*(4*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) + sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*
b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 +
a^2)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqr
t(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) +
sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/2*(sqrt(-
(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b
*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)))/a]
```

giac [A] time = 0.39, size = 78, normalized size = 0.98

$$-\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a}+\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
a*b*d)*a) + c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [B] time = 0.01, size = 984, normalized size = 12.30

$$\frac{c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{a}+\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2ab}-\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2ab}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{2a}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{2a}+\frac{\sqrt{d\,x^2+c}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x/(b*x^2+a),x)
```

```
[Out] -1/2/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)+1/2/a*d^(1/2)*(-a*b)^(1/2)/b*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(
1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(
1/2)/b))*d+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-1/2
/a*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)-1/2/a*d^(1/2)*(-a*b)^(1/2)/b*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)
/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/
b))*d+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-1/a*c^(1/
2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/a*(d*x^2+c)^(1/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x), x)
```

mupad **[B]** time = 0.73, size = 103, normalized size = 1.29

$$\frac{\operatorname{atanh}\left(\frac{2\,a\,b^2\,c\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b^3\,c^2\,d^3-2\,a^2\,b^2\,c\,d^4}\right)\,\sqrt{b^2\,c-a\,b\,d}}{a\,b}-\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x*(a + b*x^2)),x)
```

```
[Out] (atanh((2*a*b^2*c*d^3*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(2*a*b^3*c^2
*d^3 - 2*a^2*b^2*c*d^4))*(b^2*c - a*b*d)^(1/2))/(a*b) - (c^(1/2)*atanh((c +
 d*x^2)^(1/2)/c^(1/2)))/a
```

sympy [A] time = 9.59, size = 78, normalized size = 0.98

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x/(b*x**2+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*a*b*sqrt((a*d - b*c)/b)))/d
```

3.664 $\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

Rubi [**A**] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {475, 12, 377, 205}

$$-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
```

```
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx &= -\frac{\sqrt{c+dx^2}}{ax}+\frac{\int \frac{-bc+ad}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{\sqrt{c+dx^2}}{ax}+\frac{(-bc+ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{\sqrt{c+dx^2}}{ax}+\frac{(-bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a}\\ &= -\frac{\sqrt{c+dx^2}}{ax}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [**C**] time = 0.02, size = 51, normalized size = 0.73

$$-\frac{\sqrt{c+dx^2}\;{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[c + d*x^2]*Hypergeometric2F1[-1/2, 1, 1/2, ((-(b*c) + a*d)*x^2)/(a*
(c + d*x^2))])/(a*x))
```

IntegrateAlgebraic [**A**] time = 0.20, size = 122, normalized size = 1.74

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*x)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b
*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2
])/(Sqrt[a]*Sqrt[b*c - a*d])])/a^(3/2)
```

fricas [A] time = 1.21, size = 273, normalized size = 3.90

$$\left[\frac{x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{dx^2+c}}{4\,ax},-\frac{x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)+2\sqrt{dx^2+c}}{2\,ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sq
rt(d*x^2 + c))/(a*x), -1/2*(x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c
^2 - a*c*d)*x)) + 2*sqrt(d*x^2 + c))/(a*x)]
```

giac [B] time = 3.65, size = 117, normalized size = 1.67

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a}+\frac{2\,c\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + 2*c*sqrt
(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a)
```

maple [B] time = 0.02, size = 1017, normalized size = 14.53

$$\frac{bc\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{bc\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{a}-\frac{\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2a}-\frac{\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2a}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{2\sqrt{-ab}\,a}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{2\sqrt{-ab}\,a}+\frac{\sqrt{d\,x^2+c}\,dx}{ac}-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x)
```

```
[Out] 1/2*b/a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1
```

```
/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x+(-a*b)^(1/2)/b))*d-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x+(-a*b)^(1/2)/b))*c-1/2*b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a*d^(1/2)*ln(((x-(-a*b
)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-1/a/c/x*(d*x^2+c)^(3/2)+1/a*d/c*x
*(d*x^2+c)^(1/2)+1/a*d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^2\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)), x)
```

3.665 $\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=113

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

Rubi **[A]** time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*a*x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(2*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/a^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2}-\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx\right.}{4a^2}\\
&= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d}-\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{2a^2d}\\
&= -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.17, size = 107, normalized size = 0.95

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)-\frac{a\sqrt{c+dx^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2])/x^2) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.27, size = 122, normalized size = 1.08

$$\frac{\sqrt{b}\,\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{a^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}-\frac{\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(a*x^2) + (Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/a^2 + ((2*b*c - a*d)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*Sqrt[c])
```

fricas [A] time = 1.64, size = 708, normalized size = 6.27

$$\left[\frac{\sqrt{b^2c-abd}\,cx^2\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(bdx^2+2bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-(2bc-ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)-2\sqrt{dx^2+c}\,ac}{4a^2cx^2}, -\frac{2(2bc-ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\sqrt{b^2c-abd}\,cx^2\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(bdx^2+2bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2cx^2}, -\frac{2\sqrt{-b^2c+abd}\,cx^2\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)+(2bc-ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2cx^2}, -\frac{\sqrt{-b^2c+abd}\,cx^2\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)+(2bc-ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{dx^2+c}\,ac}{2a^2cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^
2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b*c - a*d)*
sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*
x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*(2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) - sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*
c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 2*sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*sqrt(-b^2*c + a*b*d)*c*x^2*ar
ctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(c)*x^2*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a
^2*c*x^2), -1/2*(sqrt(-b^2*c + a*b*d)*c*x^2*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a
*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2)]
```

giac [A] time = 0.47, size = 106, normalized size = 0.94

$$\frac{\left(b^{2} c - a b d\right) \arctan\left(\frac{\sqrt{d x^{2} + c}\, b}{\sqrt{-b^{2} c + a b d}}\right)}{\sqrt{-b^{2} c + a b d}\, a^{2}} - \frac{\left(2 \, b c - a d\right) \arctan\left(\frac{\sqrt{d x^{2} + c}}{\sqrt{-c}}\right)}{2 \, a^{2} \sqrt{-c}} - \frac{\sqrt{d x^{2} + c}}{2 \, a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c - a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*a^2) - 1/2*(2*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sq
rt(-c)) - 1/2*sqrt(d*x^2 + c)/(a*x^2)
```

maple [B] time = 0.01, size = 1054, normalized size = 9.33

$$\frac{d \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, a}+\frac{d \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, a}-\frac{bc \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, a^{2}}-\frac{bc \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\, a^{2}}-\frac{d \ln\left(\frac{2c+2\sqrt{d x^{2}+c}\,\sqrt{c}}{x}\right)}{2a\sqrt{c}}+\frac{b\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{d x^{2}+c}\,\sqrt{c}}{x}\right)}{a^{2}}-\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2a^{2}}+\frac{\sqrt{-ab}\,\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2a^{2}}+\frac{\sqrt{d x^{2}+c}\, d}{2ac}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\, b}{2a^{2}}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\, b}{2a^{2}}-\frac{\sqrt{d x^{2}+c}\, b}{a^{2}}-\frac{\left(d x^{2}+c\right)^{\frac{3}{2}}}{2ac\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x)
```

```
[Out] 1/2/a^2*b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)-1/2/a^2*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b
)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b
)^(1/2)/b))*d-1/2/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b)
)*c-1/2/a/c/x^2*(d*x^2+c)^(3/2)-1/2/a*d/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c
^(1/2))/x)+1/2/a*d/c*(d*x^2+c)^(1/2)+1/2/a^2*b*((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/a^2*d^(1/2)*(-a*b)
^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2/a/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d-1/2/a^2*b/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+1/a^2*b*c^(1/2)*ln((2*c+2*(d*x^2+c)^
(1/2)*c^(1/2))/x)-1/a^2*b*(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3), x)
```

mupad [B] time = 0.99, size = 268, normalized size = 2.37

$$\frac{\operatorname{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{2\left(\frac{a\,b^3\,d^5}{2}-\frac{b^4\,c\,d^4}{2}\right)}\right)\sqrt{b^2\,c-a\,b\,d}}{a^2}-\frac{\sqrt{d\,x^2+c}}{2\,a\,x^2}-\frac{\operatorname{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^2+c}}{2\left(\frac{b^4\,c\,d^4}{2}-\frac{3\,a\,b^3\,d^5}{4}+\frac{a^2\,b^2\,d^6}{4\,c}\right)}-\frac{3\,b^3\,d^5\,\sqrt{d\,x^2+c}}{4\,\sqrt{c}\left(\frac{a\,b^2\,d^6}{4\,c}-\frac{3\,b^3\,d^5}{4}+\frac{b^4\,c\,d^4}{2\,a}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^2+c}}{4\,c^{3/2}\left(\frac{b^2\,d^6}{4\,c}-\frac{3\,b^3\,d^5}{4\,a}+\frac{b^4\,c\,d^4}{2\,a^2}\right)}\right)(a\,d-2\,b\,c)}{2\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (atanh((b^3*d^4*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(2*((a*b^3*d^5)/2
- (b^4*c*d^4)/2)))*(b^2*c - a*b*d)^(1/2))/a^2 - (c + d*x^2)^(1/2)/(2*a*x^2)
 - (atanh((b^4*c^(1/2)*d^4*(c + d*x^2)^(1/2))/(2*((b^4*c*d^4)/2 - (3*a*b^3*
d^5)/4 + (a^2*b^2*d^6)/(4*c))) - (3*b^3*d^5*(c + d*x^2)^(1/2))/(4*c^(1/2)*(
(a*b^2*d^6)/(4*c) - (3*b^3*d^5)/4 + (b^4*c*d^4)/(2*a))) + (b^2*d^6*(c + d*x
^2)^(1/2))/(4*c^(3/2)*((b^2*d^6)/(4*c) - (3*b^3*d^5)/(4*a) + (b^4*c*d^4)/(2
*a^2))))*(a*d - 2*b*c))/(2*a^2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)), x)
```

3.666 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=105

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\,(3bc-ad)}{3a^2cx}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {475, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,(3bc-ad)}{3a^2cx}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*a*x^3) + ((3*b*c - a*d)*Sqrt[c + d*x^2])/(3*a^2*c*x) +
(b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a
^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
```

```
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx &= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{\int \frac{-3bc+ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}-\frac{\int -\frac{3bc(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2c}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{(b(bc-ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{(b(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 5.16, size = 93, normalized size = 0.89

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\,\left(3bcx^2-a\left(c+dx^2\right)\right)}{3a^2cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + d*x^2)))/(3*a^2*c*x^3) + (b*Sqrt[b*c -
 a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(5/2)
```

IntegrateAlgebraic [A] time = 0.26, size = 148, normalized size = 1.41

$$\frac{\sqrt{c+dx^2}\left(-ac-adx^2+3bcx^2\right)}{3a^2cx^3}-\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 - a*d*x^2))/(3*a^2*c*x^3) - (b*Sqrt[b*
c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a
]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/a^(5
/2)
```

fricas [A] time = 1.43, size = 325, normalized size = 3.10

$$\left[\frac{3\,bcx^3\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left((3\,bc-ad)x^2-ac\right)\sqrt{dx^2+c}}{12\,a^2cx^3},\ \frac{3\,bcx^3\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)+2\left((3\,bc-ad)x^2-ac\right)\sqrt{dx^2+c}}{6\,a^2cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b*c*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) + 4*((3*b*c - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3), 1/6*(3*b*c*x
^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((3*b*c
 - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3)]
```

giac [B] time = 5.43, size = 215, normalized size = 2.05

$$-\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2}-\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}-ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2) - 2
/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 3*
b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2
)
```

maple [B] time = 0.01, size = 1059, normalized size = 10.09

$$\frac{bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{b^{2}c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{2}}+\frac{b^{2}c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{b\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{a^{2}}+\frac{b\sqrt{d}\,\ln\left(\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)d-\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2a^{2}}+\frac{b\sqrt{d}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)d+\frac{\sqrt{-ab}\,d}{b}}{\sqrt{d}}+\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\right)}{2a^{2}}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^{2}}{2\sqrt{-ab}\,a^{2}}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^{2}}{2\sqrt{-ab}\,a^{2}}-\frac{\sqrt{d\,x^{2}+c}\,bdx}{a^{2}c}+\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}b}{a^{2}cx}-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3ac\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x)
```

```
[Out] -1/2*b^2/a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*b/a^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-
(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(
1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+1/2*b^2/a^2/(-a*b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*b
/a^2*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2*
b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d-1/2*
b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+
1/a^2*b/c/x*(d*x^2+c)^(3/2)-1/a^2*b*d/c*x*(d*x^2+c)^(1/2)-1/a^2*b*d^(1/2)*l
n(d^(1/2)*x+(d*x^2+c)^(1/2))-1/3/a/c/x^3*(d*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^{2}+c}}{\left(bx^{2}+a\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^4\,\left(b\,x^2+a\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)), x)
```

3.667 $\int \frac{x^4\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=210

$$\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{x\sqrt{c+dx^2}\ (8a^2d^2-10}{16b^3d}$$

Rubi [**A**] time = 0.40, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {477, 582, 523, 217, 206, 377, 205}

$$\frac{x\sqrt{c+dx^2}\left(8a^2d^2-10abcd+b^2c^2\right)}{16b^3d}-\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}+\frac{a^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4}+\frac{x^3\sqrt{c+dx^2}\,(7bc-6ad)}{24b^2}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b^2*c^2 - 10*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3*d) + ((7*b*c
 - 6*a*d)*x^3*Sqrt[c + d*x^2])/(24*b^2) + (d*x^5*Sqrt[c + d*x^2])/(6*b) + (
a^(3/2)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b^4 - ((b*c - 2*a*d)*(b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(16*b^4*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx = \frac{dx^5\sqrt{c+dx^2}}{6b} + \frac{\int \frac{x^4\left(c(6bc-5ad)+d(7bc-6ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6b}$$

$$= \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} - \frac{\int \frac{x^2\left(3acd(7bc-6ad)-3d\left(b^2c^2-10abcd+8a^2d^2\right)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{24b^2d}$$

$$= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} + \int \frac{-3acd}{}$$

$$= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} + \frac{\big(a^2(bc}{}$$

$$= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} + \frac{\big(a^2(bc}{}$$

$$= \frac{\left(b^2c^2-10abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} + \frac{a^{3/2}(bc}{}$$

Mathematica [**A**]   time = 0.31, size = 196, normalized size = 0.93

$$\frac{48a^{3/2}d^{3/2}(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+b\sqrt{d}\,x\sqrt{c+dx^2}\left(24a^2d^2-6abd\left(5c+2dx^2\right)+b^2\left(3c^2+14cdx^2+8d^2x^4\right)\right)-3\left(16a^3d^3-24a^2bcd^2+6ab^2c^2d+b^3c^3\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{48b^4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(5*c + 2*d*x^2) + b^2*(3
*c^2 + 14*c*d*x^2 + 8*d^2*x^4)) + 48*a^(3/2)*d^(3/2)*(b*c - a*d)^(3/2)*ArcT
an[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] - 3*(b^3*c^3 + 6*a*b^2*c^
2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(48*
b^4*d^(3/2))
```

IntegrateAlgebraic [**A**]   time = 0.42, size = 234, normalized size = 1.11

$$-\frac{\sqrt{bc-ad}\left(a^{3/2}bc-a^{5/2}d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{b^4}+\frac{\sqrt{c+dx^2}\left(24a^2d^2x-30abcdx-12abd^2x^3+3b^2c^2x+14b^2cdx^3+8b^2d^2x^5\right)}{48b^3d}+\frac{\left(16a^3d^3-24a^2bcd^2+6ab^2c^2d+b^3c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16b^4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b^2*c^2*x - 30*a*b*c*d*x + 24*a^2*d^2*x + 14*b^2*c*d*x^
3 - 12*a*b*d^2*x^3 + 8*b^2*d^2*x^5))/(48*b^3*d) - (Sqrt[b*c - a*d]*(a^(3/2)
*b*c - a^(5/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/
(Sqrt[a]*Sqrt[b*c - a*d])])/b^4 + ((b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^
2 + 16*a^3*d^3)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(16*b^4*d^(3/2))
```

fricas [A] time = 7.62, size = 1119, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*lo
g(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(a*b*c*d^2 - a^2*d^3)*sq
rt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c +
a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(8*b^3*d^3*x^5 + 2
*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*
d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a
^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 12*(
a*b*c*d^2 - a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2
*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
- a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a
*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/96*(48*(a*b*c*d^
2 - a^2*d^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a
*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*
d)*x)) + 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*
log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^3*d^3*x^5 + 2*(7*b
^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*
x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(24*(a*b*c*d^2 - a^2*d^3)*sqrt(a*b*c -
a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(b^3*c^3 + 6*a*
b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*
x^2 + c)) + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2
*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 2081, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*
d*c-1/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)
/b))*d*c-1/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)
^(1/2)/b))*d^2-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-
(-a*b)^(1/2)/b))*c^2+1/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2))/(x+(-a*b)^(1/2)/b))*d^2+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2+1/6/b^2*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/2/b^4*a
^3*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/6/b^
2*a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(3/2)-1/2/b^4*a^3*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b
)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))+1/6/b*x*(d*x^2+c)^(5/2)/d-1/16/b*c^3/d^(3/2)*ln
(d^(1/2)*x+(d*x^2+c)^(1/2))-1/4/b^2*a*x*(d*x^2+c)^(3/2)-1/2/b^3*a^3/(-a*b)^
(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*d+1/2/b^2*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/24/b*c/d*x*(d*x^2+c)^(3/2)
-1/16/b*c^2/d*x*(d*x^2+c)^(1/2)-3/8/b^2*a*c*x*(d*x^2+c)^(1/2)-3/8/b^2*a*c^2
/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/4/b^3*a^2*d*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b^3*a^2*d^
(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/b^3*a
```

```
^3/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)*d-1/2/b^2*a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/4/b^3*a^2*d*((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
)*x+3/4/b^3*a^2*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}x^4}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(d\,x^2+c\right)^{3/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

3.668 $\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=115

$$\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{a\sqrt{c+dx^2}\,(bc-ad)}{b^3}-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}$$

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{c+dx^2}\,(bc-ad)}{b^3}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)*Sqrt[c + d*x^2])/b^3) - (a*(c + d*x^2)^(3/2))/(3*b^2) + (c
 + d*x^2)^(5/2)/(5*b*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)\\
&= \frac{\left(c+dx^2\right)^{5/2}}{5bd}-\frac{a\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{2b}\\
&= -\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}-\frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^2\right)}{2b^2}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3}-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}-\frac{\left(a(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+}}\right.}{2b^3}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3}-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}-\frac{\left(a(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{b^3d}\\
&= -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3}-\frac{a\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5bd}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right.}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 108, normalized size = 0.94

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2-5abd\left(4c+dx^2\right)+3b^2\left(c+dx^2\right)^2\right)}{15b^3d}+\frac{a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*a^2*d^2 + 3*b^2*(c + d*x^2)^2 - 5*a*b*d*(4*c + d*x^2))
)/(15*b^3*d) + (a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[
b*c - a*d]])/b^(7/2)
```

IntegrateAlgebraic [A] time = 0.18, size = 135, normalized size = 1.17

$$\frac{\sqrt{c+dx^2}\left(15a^2d^2-20abcd-5abd^2x^2+3b^2c^2+6b^2cdx^2+3b^2d^2x^4\right)}{15b^3d}+\frac{a(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + 6*b^2*c*d*x^2 - 5*a
*b*d^2*x^2 + 3*b^2*d^2*x^4))/(15*b^3*d) + (a*(-(b*c) + a*d)^(3/2)*ArcTan[(S
qrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/b^(7/2)
```

fricas [A] time = 1.38, size = 397, normalized size = 3.45

$$\left[-\frac{15\left(abcd-a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(3\,b^2d^2x^4+3\,b^2c^2-20\,abcd+15\,a^2d^2+\left(6\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{60\,b^3d},\ \frac{15\left(abcd-a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+2\left(3\,b^2d^2x^4+3\,b^2c^2-20\,abcd+15\,a^2d^2+\left(6\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{30\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2
*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 +
 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) - 4*(3*b^2*d^2*x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*
c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^3*d), 1/30*(15*(a*b*c*d - a^2*d^2
)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(3*b^2*d^2*
x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^2)*sq
rt(d*x^2 + c))/(b^3*d)]
```

giac [A] time = 0.37, size = 151, normalized size = 1.31

$$-\frac{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^3}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^4d^4-5\left(dx^2+c\right)^{\frac{3}{2}}ab^3d^5-15\sqrt{dx^2+c}\,ab^3cd^5+15\sqrt{dx^2+c}\,a^2b^2d^6}{15\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c +
 a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5/2)*b^4*d^4 - 5
*(d*x^2 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^2 + c)*a*b^3*c*d^5 + 15*sqrt(d*x
^2 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

maple [B] time = 0.02, size = 1897, normalized size = 16.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/5*(d*x^2+c)^(5/2)/b/d-1/6*a/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4*a/b^3*(-a*b)^(1/2)*d*((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/
4*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(
1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))*c+1/2*a^2/b^3*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/2*a/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/2*a^2/b^4*d^(3/2)*(-a
*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2*a^3
/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2-a^2/b^3/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c+1/2*a/b^2/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-1/6*a/b^2*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*a/b^3*(-a*b
)^(1/2)*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)*x-3/4*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(
-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2*a^2/b^3*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/2*a/b^2*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/2
*a^2/b^4*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^
(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))+1/2*a^3/b^4/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b)
)*d^2-a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
```

```
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c+1/2*a
/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.65, size = 179, normalized size = 1.56

$$\frac{\left(d\,x^2+c\right)^{5/2}}{5\,b\,d}-\left(d\,x^2+c\right)^{3/2}\,\left(\frac{c}{3\,b\,d}+\frac{a\,d^2-b\,c\,d}{3\,b^2\,d^2}\right)-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{3/2}}{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{7/2}}+\frac{\sqrt{d\,x^2+c}\,\left(a\,d^2-b\,c\,d\right)\,\left(\frac{c}{b\,d}+\frac{a\,d^2-b\,c\,d}{b^2\,d^2}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(5/2)/(5*b*d) - (c + d*x^2)^(3/2)*(c/(3*b*d) + (a*d^2 - b*c*d)/
(3*b^2*d^2)) - (a*atan((a*b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^3
*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))*(a*d - b*c)^(3/2))/b^(7/2) + ((c + d*x^2)^
(1/2)*(a*d^2 - b*c*d)*(c/(b*d) + (a*d^2 - b*c*d)/(b^2*d^2)))/(b*d)
```

sympy [A] time = 46.42, size = 104, normalized size = 0.90

$$-\frac{a\left(c+dx^2\right)^{\frac{3}{2}}}{3b^2}-\frac{a\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^2\right)^{\frac{5}{2}}}{5bd}+\frac{\sqrt{c+dx^2}\left(a^2d-abc\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(3/2)/(3*b**2) - a*(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sq
rt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b)) + (c + d*x**2)**(5/2)/(5*b*d)
 + sqrt(c + d*x**2)*(a**2*d - a*b*c)/b**3
```

3.669 $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=158

$$\frac{\left(8a^2d^2-12abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}-\frac{\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}+\frac{x\sqrt{c+dx^2}\,(5bc-4ad)}{8b^2}+\frac{dx^3\sqrt{c}}{4}$$

Rubi [A] time = 0.25, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {477, 582, 523, 217, 206, 377, 205}

$$\frac{\left(8a^2d^2-12abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}+\frac{x\sqrt{c+dx^2}\,(5bc-4ad)}{8b^2}-\frac{\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*Sqrt[c + d*x^2])/(4*b)
 - (Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^3 + ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/S
qrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx = \frac{dx^3\sqrt{c+dx^2}}{4b} + \frac{\int \frac{x^2\left(c(4bc-3ad)+d(5bc-4ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\int \frac{acd(5bc-4ad)-d\left(3b^2c^2-12abcd+8a^2d^2\right)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8b^2d}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\left(a(bc-ad)^2\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^3} + \left(3b^2c^2-12\right.$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\left(a(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx}}\right.}{b^3}$$

$$= \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^3} + \left(3b^2c^2-\right.$$

Mathematica [A] time = 0.24, size = 139, normalized size = 0.88

$$\frac{\frac{\left(8a^2d^2-12abcd+3b^2c^2\right)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{\sqrt{d}} + bx\sqrt{c+dx^2}\left(-4ad+5bc+2bdx^2\right) - 8\sqrt{a}\,(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(5*b*c - 4*a*d + 2*b*d*x^2) - 8*Sqrt[a]*(b*c - a*d)^(3
/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + ((3*b^2*c^2 - 1
2*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(8*b^3)
```

IntegrateAlgebraic [A] time = 0.34, size = 178, normalized size = 1.13

$$\frac{\sqrt{bc-ad}\,\left(\sqrt{a}\,bc-a^{3/2}d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{b^3} + \frac{\left(-8a^2d^2+12abcd-3b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8b^3\sqrt{d}} + \frac{\sqrt{c+dx^2}\left(-4adx+5bcx+2bdx^3\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(5*b*c*x - 4*a*d*x + 2*b*d*x^3))/(8*b^2) + (Sqrt[b*c - a*d
]*(Sqrt[a]*b*c - a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c
```

```
+ d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/b^3 + ((-3*b^2*c^2 + 12*a*b*c*d - 8*a
^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*b^3*Sqrt[d])
```

fricas [A] time = 2.99, size = 894, normalized size = 5.66

$$\left[\frac{\left(3 b^{2} c^{2}-12 a b c d+8 a^{2} d^{2}\right) \sqrt{d} \log \left(-2 d x^{2}-2 \sqrt{d x^{2}+c} \sqrt{d} x-c\right)-4 \sqrt{-a b c+a^{2} d}\left(b c d-a d^{2}\right) \log \left(\frac{\left(b^{2} c^{2}-8 a b c d+8 a^{2} d^{2}\right) x^{4}+a^{2} c^{2}-2\left(3 a b c^{2}-4 a^{2} c d\right) x^{2}+4\left((b c-2 a d) x^{3}-a c x\right) \sqrt{-a b c+a^{2} d} \sqrt{d x^{2}+c}}{b^{2} x^{4}+2 a b x^{2}+a^{2}}\right)+2\left(2 b^{2} d^{2} x^{3}+\left(5 b^{2} c d-4 a b d^{2}\right) x\right) \sqrt{d x^{2}+c}}{16 b^{3} d},-\frac{\left(3 b^{2} c^{2}-12 a b c d+8 a^{2} d^{2}\right) \sqrt{-d} \arctan \left(\frac{\sqrt{-d} x}{\sqrt{d x^{2}+c}}\right)+2 \sqrt{-a b c+a^{2} d}\left(b c d-a d^{2}\right) \log \left(\frac{\left(b^{2} c^{2}-8 a b c d+8 a^{2} d^{2}\right) x^{4}+a^{2} c^{2}-2\left(3 a b c^{2}-4 a^{2} c d\right) x^{2}+4\left((b c-2 a d) x^{3}-a c x\right) \sqrt{-a b c+a^{2} d} \sqrt{d x^{2}+c}}{b^{2} x^{4}+2 a b x^{2}+a^{2}}\right)-\left(2 b^{2} d^{2} x^{3}+\left(5 b^{2} c d-4 a b d^{2}\right) x\right) \sqrt{d x^{2}+c}}{8 b^{3} d},-\frac{8 \sqrt{a b c-a^{2} d}\left(b c d-a d^{2}\right) \arctan \left(\frac{\sqrt{a b c-a^{2} d}\left((b c-2 a d) x^{2}-a c\right) \sqrt{d x^{2}+c}}{2\left(\left(a b c d-a^{2} d^{2}\right) x^{3}+\left(a b c^{2}-a^{2} c d\right) x\right)}\right)-\left(3 b^{2} c^{2}-12 a b c d+8 a^{2} d^{2}\right) \sqrt{d} \log \left(-2 d x^{2}-2 \sqrt{d x^{2}+c} \sqrt{d} x-c\right)-2\left(2 b^{2} d^{2} x^{3}+\left(5 b^{2} c d-4 a b d^{2}\right) x\right) \sqrt{d x^{2}+c}}{16 b^{3} d},-\frac{4 \sqrt{a b c-a^{2} d}\left(b c d-a d^{2}\right) \arctan \left(\frac{\sqrt{a b c-a^{2} d}\left((b c-2 a d) x^{2}-a c\right) \sqrt{d x^{2}+c}}{2\left(\left(a b c d-a^{2} d^{2}\right) x^{3}+\left(a b c^{2}-a^{2} c d\right) x\right)}\right)+\left(3 b^{2} c^{2}-12 a b c d+8 a^{2} d^{2}\right) \sqrt{-d} \arctan \left(\frac{\sqrt{-d} x}{\sqrt{d x^{2}+c}}\right)-\left(2 b^{2} d^{2} x^{3}+\left(5 b^{2} c d-4 a b d^{2}\right) x\right) \sqrt{d x^{2}+c}}{8 b^{3} d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) - 4*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sq
rt(d*x^2 + c))/(b^3*d), -1/8*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 2*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d
^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/16*(8*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^
2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (3*b^2*c^2 - 12*a*b*c
*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2
*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/8
*(4*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) + (3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-
d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*
x^2 + c))/(b^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.01, size = 1973, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/4/b*x*(d*x^2+c)^(3/2)+3/8/b*c*x*(d*x^2+c)^(1/2)+3/8/b*c^2/d^(1/2)*ln(d^(1
/2)*x+(d*x^2+c)^(1/2))+1/6*a/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*a/b^2*d*((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4*a
/b^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/
2*a^2/(-a*b)^(1/2)/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+1/2*a/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/2*a^2/b^3*d^(
3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^3/(-a*
b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2+a^2/(
-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c-1/
2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2
-1/6*a/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*a/b^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4*a/b^2*d^(1/2)*ln(((x-(-
a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2*a^2/(-a*b)^(1/2)/b^2
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2)*d-1/2*a/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/2*a^2/b^3*d^(3/2)*ln(((x-(-a*b)^(1/
2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2*a^3/(-a*b)^(1/2)/b^3/(-(a*d-b*
c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2-a^2/(-a*b)^(1/2)/b^2/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c+1/2*a/(-a*b)^(1/2)/b/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}x^2}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\left(d\,x^2+c\right)^{3/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

3.670 $\int \frac{x\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{b^2}+\frac{\left(c+dx^2\right)^{3/2}}{3b}$$

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}\,(bc-ad)}{b^2}-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{\left(c+dx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (c + d*x^2)^(3/2)/(3*b) - ((b*c - a*d)^
(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)\\
&= \frac{\left(c+dx^2\right)^{3/2}}{3b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b}\\
&= \frac{(bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3b} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}\\
&= \frac{(bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3b} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{b^2d}\\
&= \frac{(bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{\left(c+dx^2\right)^{3/2}}{3b} - \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.91

$$\frac{\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)}{3b^2} - \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2))/(3*b^2) - ((b*c - a*d)^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.13, size = 93, normalized size = 1.02

$$\frac{\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)}{3b^2} - \frac{(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2))/(3*b^2) - ((-(b*c) + a*d)^(3/2)
*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/b^(5/2)
```

fricas [A] time = 1.62, size = 303, normalized size = 3.33

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(bdx^2+4\,bc-3\,ad\right)\sqrt{dx^2+c}}{12\,b^2},-\frac{3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\left(bdx^2+4\,bc-3\,ad\right)\sqrt{dx^2+c}}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
- a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 4*(b*d*x^2 + 4*b*c - 3*a*d)*sqrt(d*x^2 + c))/b^2, -1/6*(3*(b*c - a*d)*sqr
t(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(b*d*x^2 + 4*b*c
 - 3*a*d)*sqrt(d*x^2 + c))/b^2]
```

giac [A] time = 0.38, size = 112, normalized size = 1.23

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2+3\,\sqrt{dx^2+c}\,b^2c-3\,\sqrt{dx^2+c}\,abd}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2 + 3*sqrt(d*x^2
+ c)*b^2*c - 3*sqrt(d*x^2 + c)*a*b*d)/b^3
```

maple [B] time = 0.01, size = 1856, normalized size = 20.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/6/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(3/2)-1/4/b^2*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
```

```
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln(
((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/2/b^2*((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d+
1/2/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*c+1/2/b^3*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^
(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))*a-1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2
)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a
*b)^(1/2)/b))*a^2*d^2+1/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2
)/b))*a*d*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2+
1/6/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(3/2)+1/4/b^2*(-a*b)^(1/2)*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b^2*d^(1/2)*(-a*b)^(1/2)*ln(
((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/2/b^2*((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d+
1/2/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*c-1/2/b^3*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^
(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))*a-1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*
b)^(1/2)/b))*a^2*d^2+1/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/
b))*a*d*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.62, size = 98, normalized size = 1.08

$$\frac{\left(d\,x^2+c\right)^{3/2}}{3\,b}-\frac{\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}{b^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{3/2}}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b) - ((c + d*x^2)^(1/2)*(a*d - b*c))/b^2 + (atan((b^(1
/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))*(
a*d - b*c)^(3/2))/b^(5/2)
```

sympy [A] time = 33.91, size = 80, normalized size = 0.88

$$\frac{\left(c+dx^2\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx^2}\,\left(-ad+bc\right)}{b^2}+\frac{\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(3/2)/(3*b) + sqrt(c + d*x**2)*(-a*d + b*c)/b**2 + (a*d - b*c
)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

3.671 $\int \frac{\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=113

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^2}+\frac{\sqrt{d}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 523, 217, 206, 377, 205}

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^2}+\frac{\sqrt{d}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b^2) + (Sqrt[d]*(3*b*c - 2*a*d)*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx &= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{\int \frac{c(2bc-ad)+d(3bc-2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(d(3bc-2ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2}+\frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(d(3bc-2ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{(bc-ad)^2\,\mathrm{Subst}\Big(\int \frac{1}{a-(-bc+a}}{b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{2b}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^2}+\frac{\sqrt{d}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 110, normalized size = 0.97

$$
\frac{\sqrt{d}\,(3bc-2ad)\log\left(\sqrt{d}\,\sqrt{c+dx^2}\,+dx\right)+\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}}+bdx\sqrt{c+dx^2}}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2] + (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(3*b*c - 2*a*d)*Log[d*x + Sqrt[
d]*Sqrt[c + d*x^2]])/(2*b^2)
```

IntegrateAlgebraic [A] time = 0.29, size = 173, normalized size = 1.53

$$\frac{\left(2ad^{3/2}-3bc\sqrt{d}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^2}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\,b^2}+\frac{dx\sqrt{c+dx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(2*b) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/S
qrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c +
d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]])/(Sqrt[a]*b^2) + ((-3*b*c*Sqrt[d] + 2*a*
d^(3/2))*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

fricas [A] time = 1.56, size = 721, normalized size = 6.38

$$\left[\frac{2\sqrt{dx^2+c}\,bdx-(3bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-(bc-ad)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2+4\left(a^2cx-\left(abc-2a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)}{4b^2},\ \frac{2\sqrt{dx^2+c}\,bdx-2(3bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(bc-ad)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2+4\left(a^2cx-\left(abc-2a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)}{4b^2},\ \frac{2\sqrt{dx^2+c}\,bdx+2(bc-ad)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)-(3bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{4b^2},\ \frac{\sqrt{dx^2+c}\,bdx-(3bc-2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(bc-ad)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*sqrt(-(b*c - a*d)/a)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(3*b*c
- 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*c - a*d)*sqrt(-(b
*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c
)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^
2 + c)*b*d*x + 2*(b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^
2 - a*c*d)*x)) - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(d)*x - c))/b^2, 1/2*(sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(-d)*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(
1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d -
 a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.01, size = 1875, normalized size = 16.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a),x)
```

```
[Out] -1/6/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(3/2)+1/4/b*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b*d^(1/2)*ln(((x+(-a*b)^(1/2)/b
)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d-1/2/(-a
*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d
)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))*a+1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))*a^2*d^2-1/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x+(-a*b)^(1/2)/b))*a*d*c+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x+(-a*b)^(1/2)/b))*c^2+1/6/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/b*d*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b*d^(1
/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^
2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/2/(-a*b)^
(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*a*d+1/2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/2/b^2*d^(3/2)*ln(((x-(-a*b)^(1
/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a-1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c
)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a^2*d^2+1/(-a*b)^(1/2)/b/(-(a*d-b
*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
```

```
*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d*c-1/2/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2), x)
```

3.672 $\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=96

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 84, 156, 63, 208}

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2])/b - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*
c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(3/
2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx, x, x^2\right)\\
&= \frac{d\sqrt{c+dx^2}}{b} + \frac{\operatorname{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b}\\
&= \frac{d\sqrt{c+dx^2}}{b} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} - \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ab}\\
&= \frac{d\sqrt{c+dx^2}}{b} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+}\right.}{abd}\\
&= \frac{d\sqrt{c+dx^2}}{b} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 102, normalized size = 1.06

$$
\frac{a\sqrt{b}\,d\sqrt{c+dx^2} + (bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right) - b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ab^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (a*Sqrt[b]*d*Sqrt[c + d*x^2] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]] + (b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(a*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 106, normalized size = 1.10

$$\frac{(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{ab^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\sqrt{c+dx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2])/b + ((-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) +
 a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a*b^(3/2)) - (c^(3/2)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/a
```

fricas [A] time = 2.42, size = 682, normalized size = 7.10

$$\left[\frac{2\,bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+4\,\sqrt{dx^2+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,ab},\ \frac{4\,b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+4\,\sqrt{dx^2+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,ab},\ \frac{bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}\,ad+(bc-ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)}{2\,ab},\ \frac{2\,b\sqrt{-c}\,c\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+2\,\sqrt{dx^2+c}\,ad+(bc-ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*s
qrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2
 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x
^2 + a^2)))/(a*b), 1/4*(4*b*sqrt(-c)*c*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 4
*sqrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x
^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b
*x^2 + a^2)))/(a*b), 1/2*(b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
 + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^
2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b), 1/2*(2*b*sqrt(-c)*c*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)
/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b
)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b)]
```

giac [A] time = 0.37, size = 110, normalized size = 1.15

$$\frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{\sqrt{dx^2+c}\,d}{b}-\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)) + sqrt(d*x^2 + c)*d/b - (
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d
))/(sqrt(-b^2*c + a*b*d)*a*b)
```

maple [B] time = 0.01, size = 1919, normalized size = 19.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x/(b*x^2+a),x)
```

```
[Out] -1/6/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(3/2)+1/4/a*(-a*b)^(1/2)/b*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/a/b*d^(1/2)*(-a*b)^(1/2)*ln
(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/b*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/2
/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*c-1/2/b^2*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/
2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)
^(1/2)/b))*d^2-1/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c
+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-1/6/a*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)
-1/4/a*(-a*b)^(1/2)/b*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a/b*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^
(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/b*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/2/a*((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1
/2/b^2*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1
/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^
2-1/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d
```

```
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c+1/2/a/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+1/3/a*(d*x^2+c)^(3/2)-1/a*
c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/a*(d*x^2+c)^(1/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x), x)
```

mupad [B] time = 0.81, size = 711, normalized size = 7.41

$$\frac{d\,\sqrt{d\,x^2+c}}{b}-\frac{\mathrm{atanh}\left(\frac{2\,a^3\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\,a^3\,c^2\,d^6-8\,a^2\,b\,c^3\,d^5+12\,a\,b^2\,c^4\,d^4-6\,b^3\,c^5\,d^3}+\frac{8\,a^2\,c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{8\,a^2\,c^3\,d^5+6\,b^2\,c^5\,d^3-\frac{2\,a^3\,c^2\,d^6}{b}-12\,a\,b\,c^4\,d^4}+\frac{6\,b^2\,c^3\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{8\,a^2\,c^3\,d^5+6\,b^2\,c^5\,d^3-\frac{2\,a^3\,c^2\,d^6}{b}-12\,a\,b\,c^4\,d^4}-\frac{12\,a\,b\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{8\,a^2\,c^3\,d^5+6\,b^2\,c^5\,d^3-\frac{2\,a^3\,c^2\,d^6}{b}-12\,a\,b\,c^4\,d^4}\right)\,\sqrt{c^3}}{a}+\frac{\mathrm{atanh}\left(\frac{6\,c^3\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{6\,b^3\,c^5\,d^3-10\,a^3\,c^2\,d^6-18\,a\,b^2\,c^4\,d^4+20\,a^2\,b\,c^3\,d^5+\frac{2\,a^4\,c\,d^7}{b}}-\frac{6\,a\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{2\,a^4\,c\,d^7-10\,a^3\,b\,c^2\,d^6+20\,a^2\,b^2\,c^3\,d^5-18\,a\,b^3\,c^4\,d^4+6\,b^4\,c^5\,d^3}+\frac{2\,a^2\,c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{2\,a^4\,b\,c\,d^7-10\,a^3\,b^2\,c^2\,d^6+20\,a^2\,b^3\,c^3\,d^5-18\,a\,b^4\,c^4\,d^4+6\,b^5\,c^5\,d^3}\right)\,\sqrt{-b^3\,{\left(a\,d-b\,c\right)}^3}}{a\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x*(a + b*x^2)),x)
```

```
[Out] (d*(c + d*x^2)^(1/2))/b - (atanh((2*a^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/
(2*a^3*c^2*d^6 - 6*b^3*c^5*d^3 + 12*a*b^2*c^4*d^4 - 8*a^2*b*c^3*d^5) + (8*a
^2*c*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^5*d^3 - (2
*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4) + (6*b^2*c^3*d^3*(c + d*x^2)^(1/2)*(c^3)^
(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^5*d^3 - (2*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4)
 - (12*a*b*c^2*d^4*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^
5*d^3 - (2*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4))*(c^3)^(1/2))/a + (atanh((6*c^3
*d^3*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c
^2*d)^(1/2))/(6*b^3*c^5*d^3 - 10*a^3*c^2*d^6 - 18*a*b^2*c^4*d^4 + 20*a^2*b*
c^3*d^5 + (2*a^4*c*d^7)/b) - (6*a*c^2*d^4*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*
b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(2*a^4*c*d^7 + 6*b^4*c^5*
d^3 - 18*a*b^3*c^4*d^4 - 10*a^3*b*c^2*d^6 + 20*a^2*b^2*c^3*d^5) + (2*a^2*c*
d^5*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^
2*d)^(1/2))/(6*b^5*c^5*d^3 - 18*a*b^4*c^4*d^4 + 20*a^2*b^3*c^3*d^5 - 10*a^3
*b^2*c^2*d^6 + 2*a^4*b*c*d^7))*(-b^3*(a*d - b*c)^3)^(1/2))/(a*b^3)
```

sympy [A] time = 29.87, size = 92, normalized size = 0.96

$$\frac{d\sqrt{c+dx^2}}{b}+\frac{c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{(ad-bc)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x/(b*x**2+a),x)
```

```
[Out] d*sqrt(c + d*x**2)/b + c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c)) -
(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt((a*d
 - b*c)/b))
```

3.673 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=102

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {474, 523, 217, 206, 377, 205}

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(a+bx^2\right)}\,dx &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{\int \frac{-c(bc-2ad)+ad^2x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}\\ &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}-\frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{ab}\\ &= -\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}-\frac{(bc-ad)^2\,\text{Subst}\Bigg(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+}}}{ab}\\ &= -\frac{c\sqrt{c+dx^2}}{ax}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.21, size = 105, normalized size = 1.03

$$-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}+\frac{d^{3/2}\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b) + (d^(3/2)*Log[d*x + Sqrt[d]*Sqrt
[c + d*x^2]])/b
```

IntegrateAlgebraic [A] time = 0.27, size = 157, normalized size = 1.54

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{3/2}b}-\frac{c\sqrt{c+dx^2}}{ax}-\frac{d^{3/2}\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] -((c*Sqrt[c + d*x^2])/(a*x)) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/
Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c +
 d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(3/2)*b) - (d^(3/2)*Log[-(Sqrt[d]*x
) + Sqrt[c + d*x^2]])/b
```

fricas [A] time = 1.95, size = 718, normalized size = 7.04

$$\left[\frac{2\,ad^{\frac{3}{2}}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,\sqrt{dx^2+c}\,bc}{4\,abx},\ -\frac{4\,a\sqrt{-d}\,dx\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\sqrt{dx^2+c}\,bc}{4\,abx},\ \frac{ad^{\frac{3}{2}}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-(bc-ad)x\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)-2\,\sqrt{dx^2+c}\,bc}{2\,abx},\ -\frac{2\,a\sqrt{-d}\,dx\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(bc-ad)x\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)+2\,\sqrt{dx^2+c}\,bc}{2\,abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*d^(3/2)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c
- a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^
3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*s
qrt(d*x^2 + c)*b*c)/(a*b*x), -1/4*(4*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) + (b*c - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (
a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) + 4*sqrt(d*x^2 + c)*b*c)/(a*b*x), 1/2*(a*d^(3/2)*x*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*x*sqrt((b*c - a*d)/a)*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b
*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*sqrt(d*x^2 + c)*b*c)/(a*b*x), -
1/2*(2*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*x*sq
rt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt
((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*sqrt(d*x^2 +
 c)*b*c)/(a*b*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 0.02, size = 1956, normalized size = 19.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x)
```

```
[Out] 1/6*b/a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/a*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a*d^(1/2)*ln(((x+(-a*b)^(1/2
)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+1/2*b/a/
(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)*c+1/2/b*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*
d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))-1/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+
(-a*b)^(1/2)/b))*d^2+1/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2
)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a
*b)^(1/2)/b))*d*c-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+
(-a*b)^(1/2)/b))*c^2-1/6*b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/a*d*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a*d^(1/
2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/(-a*b)^(
1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)*d-1/2*b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/2/b*d^(3/2)*ln(((x-(-a*b)^(1/2)
/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2-1/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2
)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
```

```
c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1
/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b
)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-1/a/c/x*(d*x^2+c)^(5/2)+1/a*d/c*x*(d
*x^2+c)^(3/2)+3/2/a*d*x*(d*x^2+c)^(1/2)+3/2/a*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2
+c)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^2+c\right)}^{3/2}}{x^2\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)), x)
```

3.674 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=114

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

Rubi [A] time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$-\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -(c*Sqrt[c + d*x^2])/(2*a*x^2) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^2]/Sqrt[c]])/(2*a^2) - ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^2])/Sqrt[b*c - a*d]])/(a^2*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(a+bx^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{(c(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\right.}{2a^2}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} - \frac{(c(2bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^2d} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{}{a-\frac{bc}{d}}\right.}{a^2}$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2} + \frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}$$

Mathematica [A] time = 0.17, size = 108, normalized size = 0.95

$$\frac{-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}+\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{ac\sqrt{c+dx^2}}{x^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^2])/x^2) + Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]] - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/Sqrt[b])/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.25, size = 128, normalized size = 1.12

$$\frac{\left(2bc^{3/2}-3a\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}-\frac{(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a^2\sqrt{b}}-\frac{c\sqrt{c+dx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*(c*Sqrt[c + d*x^2])/(a*x^2) - ((-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sq
rt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a^2*Sqrt[b]) + ((2*b*c^(3/
2) - 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

fricas [A] time = 3.19, size = 732, normalized size = 6.42

$$\left[-\frac{(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+(2bc-3ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2x^2}, -\frac{2(2bc-3ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2)x^2+4(b^2dx^2+2b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2x^2}, -\frac{2(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2bc-3ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\sqrt{dx^2+c}\,ac}{4a^2x^2}, -\frac{(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2bc-3ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{dx^2+c}\,ac}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - a*d)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
 - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*
c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4*(2*(2*b*c - 3*a*d)*sqrt(-c
)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c - a*d)*x^2*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4
*(2*(b*c - a*d)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d
```

```
)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2
)) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) +
2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/2*((b*c - a*d)*x^2*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b*c - 3*a*d)*sqrt(-
c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + sqrt(d*x^2 + c)*a*c)/(a^2*x^2)]
```

giac [A] time = 0.41, size = 120, normalized size = 1.05

$$\frac{\left(b^{2} c^{2}-2 \, a b c d+a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{d x^{2}+c} \, b}{\sqrt{-b^{2} c+a b d}}\right)}{\sqrt{-b^{2} c+a b d} \, a^{2}}-\frac{\left(2 \, b c^{2}-3 \, a c d\right) \arctan\left(\frac{\sqrt{d x^{2}+c}}{\sqrt{-c}}\right)}{2 \, a^{2} \sqrt{-c}}-\frac{\sqrt{d x^{2}+c} \, c}{2 \, a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/2*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^2
+ c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/2*sqrt(d*x^2 + c)*c/(a*x^2)
```

maple [B] time = 0.01, size = 2003, normalized size = 17.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x)
```

```
[Out] 1/6/a^2*b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(3/2)-1/2/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/3/a^2*b*(d*x^2+c)^(3/2)+1/6/a^2*b*((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/2
/a*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*d+3/2/a*d*(d*x^2+c)^(1/2)+1/2/a^2*b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/2/b/(-(a*d-b*c)/b)^(
1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2+1/a^2*b*c^(3/2)*ln((2*c+2*(d*x^2+c)
^(1/2)*c^(1/2))/x)-1/a^2*b*(d*x^2+c)^(1/2)*c-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2-1/2/a/c/x^2*(d*x^2+c)^(5/2)+1/2/a*d/c*(d*x
^2+c)^(3/2)-3/2/a*d*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/2/a^2*b
*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2)*c+1/2/a/b*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)
/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
```

```
-(a*d-b*c)/b)^(1/2))+1/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))*d*c-1/2/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2+3
/4/a^2*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1
/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))*c-1/2/a/b*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(
1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))+1/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2
)/b))*d*c-1/2/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2
+1/4/a^2*(-a*b)^(1/2)*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/a^2*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a^2*d^(1
/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*
c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3), x)
```

mupad [B] time = 1.16, size = 560, normalized size = 4.91

$$-\frac{c\,\sqrt{d\,x^2+c}}{2\,a\,x^2}-\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{29\,b^2\,c^{3/2}\,d^6\,\sqrt{d\,x^2+c}}{4\,\left(\frac{29\,b^2\,c^2\,d^6}{4}-3\,a\,b\,c\,d^7-\frac{23\,b^3\,c^3\,d^5}{4\,a}+\frac{3\,b^4\,c^4\,d^4}{2\,a^2}\right)}+\frac{23\,b^3\,c^{5/2}\,d^5\,\sqrt{d\,x^2+c}}{4\,\left(\frac{23\,b^3\,c^3\,d^5}{4}-\frac{29\,a\,b^2\,c^2\,d^6}{4}-\frac{3\,b^4\,c^4\,d^4}{2\,a}+3\,a^2\,b\,c\,d^7\right)}+\frac{3\,b^4\,c^{7/2}\,d^4\,\sqrt{d\,x^2+c}}{2\,\left(-3\,a^3\,b\,c\,d^7+\frac{29\,a^2\,b^2\,c^2\,d^6}{4}-\frac{23\,a\,b^3\,c^3\,d^5}{4}+\frac{3\,b^4\,c^4\,d^4}{2}\right)}-\frac{3\,a\,b\,\sqrt{c}\,d^7\,\sqrt{d\,x^2+c}}{\frac{29\,b^2\,c^2\,d^6}{4}-3\,a\,b\,c\,d^7-\frac{23\,b^3\,c^3\,d^5}{4\,a}+\frac{3\,b^4\,c^4\,d^4}{2\,a^2}}\right)\,\left(3\,a\,d-2\,b\,c\right)}{2\,a^2}-\frac{\mathrm{atanh}\left(\frac{3\,b^2\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b\,d^3+3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+b^4\,c^3}}{2\,\left(-2\,a^3\,b\,c\,d^7+\frac{11\,a^2\,b^2\,c^2\,d^6}{2}-5\,a\,b^3\,c^3\,d^5+\frac{3\,b^4\,c^4\,d^4}{2}\right)}+\frac{2\,b\,c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b\,d^3+3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+b^4\,c^3}}{5\,b^3\,c^3\,d^5-\frac{11\,a\,b^2\,c^2\,d^6}{2}-\frac{3\,b^4\,c^4\,d^4}{2\,a}+2\,a^2\,b\,c\,d^7}\right)\,\sqrt{-b\,{\left(a\,d-b\,c\right)}^3}}{a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x)
```

```
[Out] - (c*(c + d*x^2)^(1/2))/(2*a*x^2) - (c^(1/2)*atanh((29*b^2*c^(3/2)*d^6*(c +
 d*x^2)^(1/2))/(4*((29*b^2*c^2*d^6)/4 - 3*a*b*c*d^7 - (23*b^3*c^3*d^5)/(4*a
) + (3*b^4*c^4*d^4)/(2*a^2))) + (23*b^3*c^(5/2)*d^5*(c + d*x^2)^(1/2))/(4*(
```

```
(23*b^3*c^3*d^5)/4 - (29*a*b^2*c^2*d^6)/4 - (3*b^4*c^4*d^4)/(2*a) + 3*a^2*b
*c*d^7)) + (3*b^4*c^(7/2)*d^4*(c + d*x^2)^(1/2))/(2*((3*b^4*c^4*d^4)/2 - (2
3*a*b^3*c^3*d^5)/4 + (29*a^2*b^2*c^2*d^6)/4 - 3*a^3*b*c*d^7)) - (3*a*b*c^(1
/2)*d^7*(c + d*x^2)^(1/2))/((29*b^2*c^2*d^6)/4 - 3*a*b*c*d^7 - (23*b^3*c^3*
d^5)/(4*a) + (3*b^4*c^4*d^4)/(2*a^2)))*(3*a*d - 2*b*c))/(2*a^2) - (atanh((3
*b^2*c^2*d^4*(c + d*x^2)^(1/2)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a
*b^3*c^2*d)^(1/2))/(2*((3*b^4*c^4*d^4)/2 - 5*a*b^3*c^3*d^5 + (11*a^2*b^2*c^
2*d^6)/2 - 2*a^3*b*c*d^7)) + (2*b*c*d^5*(c + d*x^2)^(1/2)*(b^4*c^3 - a^3*b*
d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)^(1/2))/(5*b^3*c^3*d^5 - (11*a*b^2*c^
2*d^6)/2 - (3*b^4*c^4*d^4)/(2*a) + 2*a^2*b*c*d^7))*(-b*(a*d - b*c)^3)^(1/2)
)/(a^2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)), x)
```

3.675 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=102

$$\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}+\frac{\sqrt{c+dx^2}\,(3bc-4ad)}{3a^2x}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {474, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,(3bc-4ad)}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -(c*Sqrt[c + d*x^2])/(3*a*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d*x^2])/(3*a^2*x
) + ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])
])/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
```

```
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)}\,dx &= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{\int \frac{-c(3bc-4ad)-d(2bc-3ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}-\frac{\int -\frac{3c(bc-ad)^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2c}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\
&= -\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.52

$$
-\frac{\left(c+dx^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{3ax^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*((c + d*x^2)^(3/2)*Hypergeometric2F1[-3/2, 1, -1/2, ((-(b*c) + a*d)*x^
2)/(a*(c + d*x^2))])/(a*x^3)
```

IntegrateAlgebraic [A] time = 0.31, size = 144, normalized size = 1.41

$$\frac{\sqrt{c+dx^2}\left(-ac-4adx^2+3bcx^2\right)}{3a^2x^3}-\frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 - 4*a*d*x^2))/(3*a^2*x^3) - ((b*c - a*
d)^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a
]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/a^(5
/2)
```

fricas [A] time = 2.06, size = 331, normalized size = 3.25

$$\left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left((3\,bc-4\,ad)x^2-ac\right)\sqrt{dx^2+c}}{12\,a^2x^3},\ \frac{3\,(bc-ad)x^3\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)+2\left((3\,bc-4\,ad)x^2-ac\right)\sqrt{dx^2+c}}{6\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3), 1/6
*(3*(b*c - a*d)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) + 2*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3)]
```

giac [B] time = 5.35, size = 256, normalized size = 2.51

$$-\frac{\left(b^2c^2\sqrt{d}-2\,abcd^{\frac{3}{2}}+a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2}-\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4bc^2\sqrt{d}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4acd^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc^3\sqrt{d}+6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ac^2d^{\frac{3}{2}}+3\,bc^4\sqrt{d}-4\,ac^3d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x -
 sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d
```

```
 - a^2*d^2)*a^2) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c^2*sqrt(d) - 6
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*b*c^3*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^2*d^(3/2) + 3*b
*c^4*sqrt(d) - 4*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^
2)
```

maple [B] time = 0.01, size = 2089, normalized size = 20.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x)
```

```
[Out] b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c-b/
a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c-1/a
^2*b*d/c*x*(d*x^2+c)^(3/2)+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-1/3/a/c/x^3*(d*x^2+c)^(5/2)+1/6*b^2
/a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(3/2)-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
-(-a*b)^(1/2)/b))*d^2-1/6*b^2/a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/2/(-a*b)^(1/2)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2-1/2*b/a/(-a*b)^(1/2)*((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*d+1/2*b^2/a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/a^2*b/c/x*(d*x^2+c)^(5/2)-3/2/a^2*
b*d*x*(d*x^2+c)^(1/2)-3/2/a^2*b*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-2/3
/a*d/c^2/x*(d*x^2+c)^(5/2)+2/3/a*d^2/c^2*x*(d*x^2+c)^(3/2)+1/a*d^2/c*x*(d*x
^2+c)^(1/2)+1/4*b/a^2*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*b/a^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-
(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2*b/a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/2*b^2/a^2
```

```
/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*c+1/4*b/a^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*b/a^2*d^(1/2)*ln(((x-(-a*b)^(
1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/a*d^(3/2)*ln(d^(1/2)*x+(d*x^
2+c)^(1/2))-1/2/a*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2
)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))-1/2/a*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{x^4\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)), x)
```

3.676 $\int \frac{x^4\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=291

$$\frac{a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^5}+\frac{x^3\sqrt{c+dx^2}\,\left(48a^2d^2-104abcd+59b^2c^2\right)}{192b^3}+\frac{x\sqrt{c+dx^2}\,\left(-64a^3d^3+144a^2bc}{128b^4d}$$

Rubi [A] time = 0.57, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {477, 581, 582, 523, 217, 206, 377, 205}

$$\frac{x^3\sqrt{c+dx^2}\,\left(48a^2d^2-104abcd+59b^2c^2\right)}{192b^3}+\frac{x\sqrt{c+dx^2}\,\left(144a^2bcd^2-64a^3d^3-88ab^2c^2d+5b^3c^3\right)}{128b^4d}-\frac{\left(-240a^2b^2c^2d^2+320a^3bcd^3-128a^4d^4+40ab^3c^3d+5b^4c^4\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{128b^5d^{3/2}}+\frac{a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^5}+\frac{dx^5\sqrt{c+dx^2}\,(11bc-8ad)}{48b^2}+\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^3)*x*Sqrt[c + d*x
^2])/(128*b^4*d) + ((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*d^2)*x^3*Sqrt[c + d*
x^2])/(192*b^3) + (d*(11*b*c - 8*a*d)*x^5*Sqrt[c + d*x^2])/(48*b^2) + (d*x^
5*(c + d*x^2)^(3/2))/(8*b) + (a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b^5 - ((5*b^4*c^4 + 40*a*b^3*c^3*d - 24
0*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(128*b^5*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx &= \frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}+\frac{\int \frac{x^4\sqrt{c+dx^2}\left(c(8bc-5ad)+d(11bc-8ad)x^2\right)}{a+bx^2}\,dx}{8b}\\ &= \frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2}+\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}+\frac{\int \frac{x^4\left(c\left(48b^2c^2-85abcd+40a^2d^2\right)+d\left(59b^2c^2-104abcd\right.\right.}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{48b^2}\\ &= \frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x^3\sqrt{c+dx^2}}{192b^3}+\frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2}+\frac{dx^5\left(c+dx^2\right)^{3}}{8b}\\ &= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d}+\frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x}{192b^3}\\ &= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d}+\frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x}{192b^3}\\ &= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d}+\frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x}{192b^3}\\ &= \frac{\left(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3\right)x\sqrt{c+dx^2}}{128b^4d}+\frac{\left(59b^2c^2-104abcd+48a^2d^2\right)x}{192b^3}\end{aligned}$$

Mathematica [A] time = 0.37, size = 247, normalized size = 0.85

$$\frac{384a^{3/2}(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\frac{bx\sqrt{c+dx^2}\left(-192a^3d^3+48a^2bd^2\left(9c+2dx^2\right)-8ab^2d\left(33c^2+26cdx^2+8d^2x^4\right)+b^3\left(15c^3+118c^2dx^2+136cd^2x^4+48d^3x^6\right)\right)}{d}+\frac{3\left(128a^4d^4-320a^3bcd^3+240a^2b^2c^2d^2-40ab^3c^3d-5b^4c^4\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{d^{3/2}}}{384b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2]*(-192*a^3*d^3 + 48*a^2*b*d^2*(9*c + 2*d*x^2) - 8*a*b^
2*d*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^3*(15*c^3 + 118*c^2*d*x^2 + 136*c
*d^2*x^4 + 48*d^3*x^6)))/d + 384*a^(3/2)*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + (3*(-5*b^4*c^4 - 40*a*b^3*c^3*d + 2
40*a^2*b^2*c^2*d^2 - 320*a^3*b*c*d^3 + 128*a^4*d^4)*Log[d*x + Sqrt[d]*Sqrt[
c + d*x^2]])/d^(3/2))/(384*b^5)
```

IntegrateAlgebraic [A] time = 0.77, size = 320, normalized size = 1.10

$$-\frac{\sqrt{bc-ad}\left(a^{3/2}b^2c^2-2a^{5/2}bcd+a^{7/2}d^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{b^5}+\frac{\sqrt{c+dx^2}\left(-192a^3d^3x+432a^2bcd^2x+96a^2bd^3x^3-264ab^2c^2dx-208ab^2cd^2x^3-64ab^2d^3x^5+15b^3c^3x+118b^3c^2dx^3+136b^3cd^2x^5+48b^3d^3x^7\right)}{384b^4d}+\frac{\left(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}x\right)}{128b^5d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^3*c^3*x - 264*a*b^2*c^2*d*x + 432*a^2*b*c*d^2*x - 19
2*a^3*d^3*x + 118*b^3*c^2*d*x^3 - 208*a*b^2*c*d^2*x^3 + 96*a^2*b*d^3*x^3 +
136*b^3*c*d^2*x^5 - 64*a*b^2*d^3*x^5 + 48*b^3*d^3*x^7))/(384*b^4*d) - (Sqrt
[b*c - a*d]*(a^(3/2)*b^2*c^2 - 2*a^(5/2)*b*c*d + a^(7/2)*d^2)*ArcTan[(a*Sqr
t[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]])/b^5
 + ((5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 1
28*a^4*d^4)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(128*b^5*d^(3/2))
```

fricas **[A]** time = 21.59, size = 1443, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*
d^3 - 128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
- 192*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d)*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*
x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3
*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(
5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(
d*x^2 + c))/(b^5*d^2), 1/384*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c
^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x
^2 + c)) + 96*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8
*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3
 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)
*sqrt(d*x^2 + c))/(b^5*d^2), 1/768*(384*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^
3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^
2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))
 - 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 -
128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(4
8*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 10
4*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 1
44*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(b^5*d^2), 1/384*(192*
(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sq
rt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2
*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a
```

```
^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) + (48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2
*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d -
 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(
b^5*d^2)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [**B**] time = 0.02, size = 3373, normalized size = 11.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -3/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/
b))*d*c^2-3/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b
)^(1/2)/b))*d^2*c+3/2/b^3*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x-(-a*b)^(1/2)/b))*d*c^2+3/2/b^4*a^4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2*c+7/16/b^3*a^2*d*c*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/b^3*a^3/(-a*b
)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*d*c+1/2/b^5*a^5/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x-(-a*b)^(1/2)/b))*d^3-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
```

```
1/2))/(x-(-a*b)^(1/2)/b))*c^3+7/16/b^3*a^2*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/b^3*a^3/(-a*b)^(1/
2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*d*c-1/2/b^5*a^5/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(
-a*b)^(1/2)/b))*d^3+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))/(x+(-a*b)^(1/2)/b))*c^3+1/10/b^2*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+1/2/b^5*a^4*d^(5
/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^
2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/10/b^2*a^2/
(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(5/2)+1/2/b^5*a^4*d^(5/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2
)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))-5/128/b*c^4/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/6
/b^2*a*x*(d*x^2+c)^(5/2)+1/8/b*x*(d*x^2+c)^(7/2)/d-1/6/b^3*a^3/(-a*b)^(1/2)
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(3/2)*d+1/6/b^2*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/4/b^4*a^3*d^2*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4/b^4*a
^3*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/
b^4*a^4/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2+1/2/b^2*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-1/48/b*c/
d*x*(d*x^2+c)^(5/2)-5/192/b*c^2/d*x*(d*x^2+c)^(3/2)-5/128/b*c^3/d*x*(d*x^2+
c)^(1/2)-5/24/b^2*a*c*x*(d*x^2+c)^(3/2)+1/8/b^3*a^2*d*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16/b^3*a^2
*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/6/
b^3*a^3/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(3/2)*d-1/6/b^2*a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/4/b^4*a^3*d
^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*x-5/4/b^4*a^3*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d
^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))*c-1/2/b^4*a^4/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2-1/2/b^2*a^2/(-a*b)^(1/2)
*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2)*c^2-5/16/b^2*a*c^2*x*(d*x^2+c)^(1/2)-5/16/b^2*a*c^3/d^(1/2)*ln(d^(1/
2)*x+(d*x^2+c)^(1/2))+1/8/b^3*a^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16/b^3*a^2*d^(1/2)*ln(((x-(-
a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
```

```
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}x^4}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(d\,x^2+c\right)^{5/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x)
[Out] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^2\right)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a),x)
[Out] Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

3.677 $\int \frac{x^3\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=144

$$\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}-\frac{a\sqrt{c+dx^2}\,(bc-ad)^2}{b^4}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Rubi [A] time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}-\frac{a\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^3}-\frac{a\sqrt{c+dx^2}\,(bc-ad)^2}{b^4}+\frac{a(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] -((a*(b*c - a*d)^2*Sqrt[c + d*x^2])/b^4) - (a*(b*c - a*d)*(c + d*x^2)^(3/2
)/(3*b^3) - (a*(c + d*x^2)^(5/2))/(5*b^2) + (c + d*x^2)^(7/2)/(7*b*d) + (a*
(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(9/
2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{5/2}}{a+bx}\,dx,x,x^2\right)\\
&= \frac{\left(c+dx^2\right)^{7/2}}{7bd}-\frac{a\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx,x,x^2\right)}{2b}\\
&= -\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}-\frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^2\right)}{2b^2}\\
&= -\frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}-\\
&= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4}-\frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}-\\
&= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4}-\frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}-\\
&= -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4}-\frac{a(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^3}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}+
\end{aligned}
$$

Mathematica [A] time = 0.31, size = 136, normalized size = 0.94

$$-\frac{a(bc-ad)\left(\sqrt{b}\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{3b^{9/2}}-\frac{a\left(c+dx^2\right)^{5/2}}{5b^2}+\frac{\left(c+dx^2\right)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] -1/5*(a*(c + d*x^2)^(5/2))/b^2 + (c + d*x^2)^(7/2)/(7*b*d) - (a*(b*c - a*d)
*(Sqrt[b]*Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2) - 3*(b*c - a*d)^(3/2)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]))/(3*b^(9/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 191, normalized size = 1.33

$$\frac{\sqrt{c+dx^2}\left(-105a^3d^3+245a^2bcd^2+35a^2bd^3x^2-161ab^2c^2d-77ab^2cd^2x^2-21ab^2d^3x^4+15b^3c^3+45b^3c^2dx^2+45b^3cd^2x^4+15b^3d^3x^6\right)}{105b^4d}-\frac{a(ad-bc)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^3*c^3 - 161*a*b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*
d^3 + 45*b^3*c^2*d*x^2 - 77*a*b^2*c*d^2*x^2 + 35*a^2*b*d^3*x^2 + 45*b^3*c*d
^2*x^4 - 21*a*b^2*d^3*x^4 + 15*b^3*d^3*x^6))/(105*b^4*d) - (a*(-(b*c) + a*d
)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/b
^(9/2)
```

fricas [A] time = 1.16, size = 527, normalized size = 3.66

$$\left[\frac{105\left(ab^2c^2d-2a^2bcd^2+a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(15b^3d^3x^6+15b^3c^3-161ab^2c^2d+245a^2bcd^2-105a^3d^3+3\left(15b^3cd^2-7ab^2d^3\right)x^4+\left(45b^3c^2d-77ab^2cd^2+35a^2bd^3\right)x^2\right)\sqrt{dx^2+c}}{420b^4d}, \frac{105\left(ab^2c^2d-2a^2bcd^2+a^3d^3\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+2\left(15b^3d^3x^6+15b^3c^3-161ab^2c^2d+245a^2bcd^2-105a^3d^3+3\left(15b^3cd^2-7ab^2d^3\right)x^4+\left(45b^3c^2d-77ab^2cd^2+35a^2bd^3\right)x^2\right)\sqrt{dx^2+c}}{210b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/420*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt((b*c - a*d)/b)*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 161*a*b^2*
c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*d^3)*x^4
+ (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 + c))/(b^4
*d), 1/210*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-(b*c - a*d)/b
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 1
61*a*b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*
```

```
d^3)*x^4 + (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 +
 c))/(b^4*d)]
```

giac [A] time = 0.47, size = 228, normalized size = 1.58

$$-\frac{\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{4}}+\frac{15\left(dx^{2}+c\right)^{\frac{7}{2}}b^{6}d^{6}-21\left(dx^{2}+c\right)^{\frac{5}{2}}ab^{5}d^{7}-35\left(dx^{2}+c\right)^{\frac{3}{2}}ab^{5}cd^{7}-105\,\sqrt{dx^{2}+c}\,ab^{5}c^{2}d^{7}+35\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}b^{4}d^{8}+210\,\sqrt{dx^{2}+c}\,a^{2}b^{4}cd^{8}-105\,\sqrt{dx^{2}+c}\,a^{3}b^{3}d^{9}}{105\,b^{7}d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(sqrt(d*x^2
+ c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/105*(15*(d*x^2
+ c)^(7/2)*b^6*d^6 - 21*(d*x^2 + c)^(5/2)*a*b^5*d^7 - 35*(d*x^2 + c)^(3/2)*
a*b^5*c*d^7 - 105*sqrt(d*x^2 + c)*a*b^5*c^2*d^7 + 35*(d*x^2 + c)^(3/2)*a^2*
b^4*d^8 + 210*sqrt(d*x^2 + c)*a^2*b^4*c*d^8 - 105*sqrt(d*x^2 + c)*a^3*b^3*d
^9)/(b^7*d^7)
```

maple [B] time = 0.01, size = 3127, normalized size = 21.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/6*a/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(3/2)*c-1/2*a^3/b^4*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2-1/2*a/b^2*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-7/16*a/b^3*(-a
*b)^(1/2)*d*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*x+7/16*a/b^3*(-a*b)^(1/2)*d*c*((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/6*a^2/b^3*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)
*d+1/6*a^2/b^3*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(3/2)*d-1/6*a/b^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/2*a^3/b^4*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2-1/2*a/b^2*((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*c^2+1/7*(d*x^2+c)^(7/2)/b/d+15/16*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a
*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+3/2*a^3/b^4/(-(a*d-b*c)
/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-
b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2*c-3/2*a^2/b^3/(-(a*d-b*c)/b)^(
1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
```

```
b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c^2-5/4*a^2/b^4*d^(3/2)*(-a*b)^(1/2)*
ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+3/2*a^3/b^4/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+
2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2*c-3/2*a^2/b^3/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c^2-1/4*a^2/b^4*(-a*b)^(1/2
)*d^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*x+1/8*a/b^3*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/8*a/b^3*(-a*b)^(1/2)*d*((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/
2)*x-15/16*a/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)
/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))*c^2+1/4*a^2/b^4*(-a*b)^(1/2)*d^2*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4*a^2/b^4*d
^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))*c-1/2*a^4/b^5/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^3+1
/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^3+a^2/b^3*(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2)*d*c+1/2*a^3/b^5*d^(5/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(
1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))+a^2/b^3*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*c-1/2*a^3/b^5*d^(5/2)*(-a*b)^(1/2)
*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^4/b^5/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^3+1/2*a/b^2/(-(a*d-b*c)/b
)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*
c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^3-1/10*a/b^2*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/10*a/b^2*((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2
)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.60, size = 251, normalized size = 1.74

$$\frac{\left(d\,x^2+c\right)^{7/2}}{7\,b\,d}-\left(d\,x^2+c\right)^{5/2}\,\left(\frac{c}{5\,b\,d}+\frac{a\,d^2-b\,c\,d}{5\,b^2\,d^2}\right)+\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{5/2}}{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{9/2}}+\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d^2-b\,c\,d\right)\,\left(\frac{c}{b\,d}+\frac{a\,d^2-b\,c\,d}{b^2\,d^2}\right)}{3\,b\,d}-\frac{\sqrt{d\,x^2+c}\,{\left(a\,d^2-b\,c\,d\right)}^2\,\left(\frac{c}{b\,d}+\frac{a\,d^2-b\,c\,d}{b^2\,d^2}\right)}{b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(7/2)/(7*b*d) - (c + d*x^2)^(5/2)*(c/(5*b*d) + (a*d^2 - b*c*d)/
(5*b^2*d^2)) + (a*atan((a*b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(5/2))/(a^4
*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))*(a*d - b*c)^(5/2))/b^(
9/2) + ((c + d*x^2)^(3/2)*(a*d^2 - b*c*d)*(c/(b*d) + (a*d^2 - b*c*d)/(b^2*d
^2)))/(3*b*d) - ((c + d*x^2)^(1/2)*(a*d^2 - b*c*d)^2*(c/(b*d) + (a*d^2 - b*
c*d)/(b^2*d^2)))/(b^2*d^2)
```

sympy [**A**] time = 84.63, size = 144, normalized size = 1.00

$$-\frac{a\left(c+dx^2\right)^{\frac{5}{2}}}{5b^2}+\frac{a\left(ad-bc\right)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^5\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}}{7bd}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(a^2d-abc\right)}{3b^3}+\frac{\sqrt{c+dx^2}\left(-a^3d^2+2a^2bcd-ab^2c^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] -a*(c + d*x**2)**(5/2)/(5*b**2) + a*(a*d - b*c)**3*atan(sqrt(c + d*x**2)/sq
rt((a*d - b*c)/b))/(b**5*sqrt((a*d - b*c)/b)) + (c + d*x**2)**(7/2)/(7*b*d)
 + (c + d*x**2)**(3/2)*(a**2*d - a*b*c)/(3*b**3) + sqrt(c + d*x**2)*(-a**3*
d**2 + 2*a**2*b*c*d - a*b**2*c**2)/b**4
```

3.678 $\int \frac{x^2\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=217

$$\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}-\frac{\sqrt{a}\,(bc-ad}{}$$

Rubi [A] time = 0.39, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {477, 581, 582, 523, 217, 206, 377, 205}

$$\frac{x\sqrt{c+dx^2}\,\left(8a^2d^2-18abcd+11b^2c^2\right)}{16b^3}+\frac{\left(40a^2bcd^2-16a^3d^3-30ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}+\frac{dx^3\sqrt{c+dx^2}\,(3bc-2ad)}{8b^2}-\frac{\sqrt{a}\,(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^4}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^3) + (d*(3*
b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^2) + (d*x^3*(c + d*x^2)^(3/2))/(6*b)
 - (Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/b^4 + ((5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)
*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^4*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx &= \frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}+\frac{\int \frac{x^2\sqrt{c+dx^2}\left(3c(2bc-ad)+3d(3bc-2ad)x^2\right)}{a+bx^2}dx}{6b}\\
&= \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}+\frac{\int \frac{x^2\left(3c\left(8b^2c^2-13abcd+6a^2d^2\right)+3d\left(11b^2c^2-18abcd+8a\right.\right.}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{24b^2}\\
&= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3}+\frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}-\\
&= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3}+\frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}-\\
&= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3}+\frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}-\\
&= \frac{\left(11b^2c^2-18abcd+8a^2d^2\right)x\sqrt{c+dx^2}}{16b^3}+\frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}-
\end{aligned}$$

Mathematica [A] time = 0.16, size = 187, normalized size = 0.86

$$\frac{bx\sqrt{c+dx^2}\left(24a^2d^2-6abd\left(9c+2dx^2\right)+b^2\left(33c^2+26cdx^2+8d^2x^4\right)\right)+\frac{3\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-48\sqrt{a}\,(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{48b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(9*c + 2*d*x^2) + b^2*(33*c^2 +
26*c*d*x^2 + 8*d^2*x^4)) - 48*Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])] + (3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^
2*b*c*d^2 - 16*a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^
4)
```

IntegrateAlgebraic [A] time = 0.47, size = 246, normalized size = 1.13

$$\frac{\sqrt{bc-ad}\left(-2a^{3/2}bcd+a^{5/2}d^2+\sqrt{a}\,b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{b^4}+\frac{\sqrt{c+dx^2}\left(24a^2d^2x-54abcdx-12abd^2x^3+33b^2c^2x+26b^2cdx^3+8b^2d^2x^5\right)}{48b^3}+\frac{\left(16a^3d^3-40a^2bcd^2+30ab^2c^2d-5b^3c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16b^4\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(33*b^2*c^2*x - 54*a*b*c*d*x + 24*a^2*d^2*x + 26*b^2*c*d*x
^3 - 12*a*b*d^2*x^3 + 8*b^2*d^2*x^5))/(48*b^3) + (Sqrt[b*c - a*d]*(Sqrt[a]*
b^2*c^2 - 2*a^(3/2)*b*c*d + a^(5/2)*d^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2
- b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/b^4 + ((-5*b^3*c^3 + 30*
a*b^2*c^2*d - 40*a^2*b*c*d^2 + 16*a^3*d^3)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^
2]])/(16*b^4*Sqrt[d])
```

fricas **[A]** time = 7.14, size = 1161, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c
*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*
(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*
a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d), -1/48*(3*(5*b^3*c^3
 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(-a*b*c + a^
2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*
x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 -
 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt
(d*x^2 + c))/(b^4*d), -1/96*(48*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(a*
b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(
d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^3*c^
3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d)*log(-2*d*x^2 + 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b
^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2
 + c))/(b^4*d), -1/48*(24*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(a*b*c -
a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*b^3*c^3 - 30
*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - (8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*
b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d)]
```

giac **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 3235, normalized size = 14.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 1/6*a/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/8*a/b^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/16/b*c^3/d^(1/2)*ln(d^(1
/2)*x+(d*x^2+c)^(1/2))+5/24/b*c*x*(d*x^2+c)^(3/2)+5/16/b*c^2*x*(d*x^2+c)^(1
/2)+1/10*a/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/2*a^3/b^4*d^(5/2)*ln(((x+(-a*b)^(1/2)/b)*
d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^3/b^4*d^(5/2)*ln(((x-(-a*b)^(1/2)/
b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/2*a^2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c^2-3/2*a^3/(-a*b)^(1/2)/b^3/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2*c+3/2*a^2/(-a*b)^(1/2)
/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c^2+3/2*a^3/(-a
*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2*c-1/1
0*a/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(5/2)+1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))/(x-(-a*b)^(1/2)/b))*c^3-7/16*a/b^2*d*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+a^2/(-a*b)^(1/2)/b^2*((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
)*d*c-1/2*a^4/(-a*b)^(1/2)/b^4/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1
/2)/b))*d^3+1/2*a^4/(-a*b)^(1/2)/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/
```

```
2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(
1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-
a*b)^(1/2)/b))*d^3-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))*c^3-7/16*a/b^2*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-a^2/(-a*b)^(1/2)/b^2*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*c
-1/2*a/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2+1/4*a^2/b^3*d^2*((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4*a^2/b^3*d^(3/2)
*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/2*a^3/(-a*b)
^(1/2)/b^3*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*d^2+1/6/b*x*(d*x^2+c)^(5/2)-1/8*a/b^2*d*((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/6*a^2/(-
a*b)^(1/2)/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(3/2)*d+1/4*a^2/b^3*d^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4*a^2/b^3*d^(3/2)*ln(((x+(
-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2*a^3/(-a*b)^(1/2)/b^
3*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*d^2+1/2*a/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-15/16*a/b^2*d^(1/2)*ln(((x+(-a*b
)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2-15/16*a/b^2*d^(1/2)*ln(((
x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/6*a^2/(-a*b)^(1/
2)/b^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(3/2)*d-1/6*a/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}} x^2}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(d\,x^2+c\right)^{5/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

```
[Out] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx^2\right)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

3.679 $\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\sqrt{c+dx^2}\,(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5b}$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}\,(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{3b^2}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2])/b^3 + ((b*c - a*d)*(c + d*x^2)^(3/2))/(3*b^
2) + (c + d*x^2)^(5/2)/(5*b) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right) \\
&= \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{2b} \\
&= \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{2b^2} \\
&= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{2b^3} \\
&= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} + \frac{(bc-ad)^3\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{b^3d} \\
&= \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2} + \frac{\left(c+dx^2\right)^{5/2}}{5b} - \frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d}}{\sqrt{bc-ad}}\right.}{b^{7/2}}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 113, normalized size = 0.95

$$
\frac{(bc-ad)\left(\sqrt{b}\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}}+\frac{\left(c+dx^2\right)^{5/2}}{5b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (c + d*x^2)^(5/2)/(5*b) + ((b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x^2]*(4*b*c - 3*
a*d + b*d*x^2) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]]))/(3*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 131, normalized size = 1.10

$$\frac{\sqrt{c+dx^2}\,\left(15a^2d^2-35abcd-5abd^2x^2+23b^2c^2+11b^2cdx^2+3b^2d^2x^4\right)}{15b^3}+\frac{(ad-bc)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + 11*b^2*c*d*x^2 - 5
*a*b*d^2*x^2 + 3*b^2*d^2*x^4))/(15*b^3) + ((-(b*c) + a*d)^(5/2)*ArcTan[(Sqr
t[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/b^(7/2)
```

fricas [A] time = 0.93, size = 405, normalized size = 3.40

$$\left[\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(3\,b^2d^2x^4+23\,b^2c^2-35\,abcd+15\,a^2d^2+\left(11\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{60\,b^3},-\frac{15\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\left(3\,b^2d^2x^4+23\,b^2c^2-35\,abcd+15\,a^2d^2+\left(11\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{30\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) + 4*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d
^2 + (11*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3, -1/30*(15*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
 a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)
*x^2)) - 2*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*
c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3]
```

giac [A] time = 0.44, size = 184, normalized size = 1.55

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^3}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^4+5\left(dx^2+c\right)^{\frac{3}{2}}b^4c+15\,\sqrt{dx^2+c}\,b^4c^2-5\left(dx^2+c\right)^{\frac{3}{2}}ab^3d-30\,\sqrt{dx^2+c}\,ab^3cd+15\,\sqrt{dx^2+c}\,a^2b^2d^2}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5
/2)*b^4 + 5*(d*x^2 + c)^(3/2)*b^4*c + 15*sqrt(d*x^2 + c)*b^4*c^2 - 5*(d*x^2
 + c)^(3/2)*a*b^3*d - 30*sqrt(d*x^2 + c)*a*b^3*c*d + 15*sqrt(d*x^2 + c)*a^2
*b^2*d^2)/b^5
```

maple [B] time = 0.01, size = 3078, normalized size = 25.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/8/b^2*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-15/16/b^2*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b
)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/6/b*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c+1/2/b*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*c^2+1/6/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(3/2)*c+1/2/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-1/4/b^3*(-a*b)^(1/2)*d^2*((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x*a-5/4/b
^3*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))*a*c-7/16/b^2*(-a*b)^(1/2)*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4/b^3*(-a*b)^(1/2)*d^2*((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*x*a+5/4/b^3*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d
)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))*a*c-3/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/
2)/b))*a^2*d^2*c+3/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))*a*d*c^2-3/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a^2*
d^2*c+3/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d*c^2+7
/16/b^2*(-a*b)^(1/2)*d*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/6/b^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*a*d+1/2/b^3*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a^2*d^2-1/
2/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^3-1/6/b^2*((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*a
```

```
*d+1/2/b^3*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*a^2*d^2-1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(
1/2)/b))*c^3+1/10/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(5/2)+1/10/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d*c-1/2/b^4*d^(5/2)*(-
a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a^2+1/
8/b^2*(-a*b)^(1/2)*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16/b^2*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(
1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2-1/b^2*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d*c+1/2/b^4*d^
(5/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)*a^2+1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a^3*d^3+1
/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a^3*d^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.61, size = 137, normalized size = 1.15

$$\frac{{\left(d\,x^2+c\right)}^{5/2}}{5\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{5/2}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{7/2}}-\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-b\,c\right)}{3\,b^2}+\frac{\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

```
[Out] (c + d*x^2)^(5/2)/(5*b) - (atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(5/2
))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c)^(5/2))/
b^(7/2) - ((c + d*x^2)^(3/2)*(a*d - b*c))/(3*b^2) + ((c + d*x^2)^(1/2)*(a*d
 - b*c)^2)/b^3
```

sympy [A] time = 50.76, size = 117, normalized size = 0.98

$$\frac{\left(c+dx^2\right)^{\frac{5}{2}}}{5b}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}(-ad+bc)}{3b^2}+\frac{\sqrt{c+dx^2}\left(a^2d^2-2abcd+b^2c^2\right)}{b^3}-\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] (c + d*x**2)**(5/2)/(5*b) + (c + d*x**2)**(3/2)*(-a*d + b*c)/(3*b**2) + sqr
t(c + d*x**2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/b**3 - (a*d - b*c)**3*ata
n(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

3.680 $\int \frac{\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

<u>**Optimal**</u>. Leaf size=156

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3}+\frac{dx\sqrt{c+dx^2}\,(7bc-4ad)}{8b^2}+ dx\,(\ldots$$

<u>**Rubi**</u> [**A**] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {416, 528, 523, 217, 206, 377, 205}

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^3}+\frac{dx\sqrt{c+dx^2}\,(7bc-4ad)}{8b^2}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^2) + (d*x*(c + d*x^2)^(3/2))/(4*
b) + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*b^3) + (Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*ArcTanh[
(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(8*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx &= \frac{dx\left(c+dx^2\right)^{3/2}}{4b}+\frac{\int \frac{\sqrt{c+dx^2}\left(c(4bc-ad)+d(7bc-4ad)x^2\right)}{a+bx^2}\,dx}{4b}\\
&= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}+\frac{\int \frac{c\left(8b^2c^2-9abcd+4a^2d^2\right)+d\left(15b^2c^2-20abcd+8a^2d^2\right)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{8b^2}\\
&= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}+\frac{(bc-ad)^3\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^3}+\frac{\left(d\left(15b^2c^2-2\right.\right.}{}\\
&= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}+\frac{(bc-ad)^3\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^3}\\
&= \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,b^3}+\frac{\sqrt{d}\left(15b^2c^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 140, normalized size = 0.90

$$\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)+bdx\sqrt{c+dx^2}\left(-4ad+9bc+2bdx^2\right)+\frac{8(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}}}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (b*d*x*Sqrt[c + d*x^2]*(9*b*c - 4*a*d + 2*b*d*x^2) + (8*(b*c - a*d)^(5/2)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(15
*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b
^3)
```

IntegrateAlgebraic [A] time = 0.44, size = 199, normalized size = 1.28

$$\frac{\left(-8a^2d^{5/2}+20abcd^{3/2}-15b^2c^2\sqrt{d}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8b^3}-\frac{\sqrt{bc-ad}\left(a^2d^2-2abcd+b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{\sqrt{a}\,b^3}+\frac{\sqrt{c+dx^2}\left(-4ad^2x+9bcdx+2bd^2x^3\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(9*b*c*d*x - 4*a*d^2*x + 2*b*d^2*x^3))/(8*b^2) - (Sqrt[b*c
 - a*d]*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 -
 b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*b^3) + ((-15*b^2
```

```
*c^2*Sqrt[d] + 20*a*b*c*d^(3/2) - 8*a^2*d^(5/2))*Log[-(Sqrt[d]*x) + Sqrt[c
+ d*x^2]])/(8*b^3)
```

fricas [A] time = 3.57, size = 931, normalized size = 5.97

$$\left[\frac{\left(15\,b^{2}c^{2}-20\,abcd+8\,a^{2}d^{2}\right)\sqrt{d}\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+4\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\,\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\,\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+2\,\left(2\,b^{2}d^{2}x^{3}+\left(9\,b^{2}cd-4\,abd^{2}\right)x\right)\sqrt{dx^{2}+c}}{16\,b^{3}},\ -\frac{\left(15\,b^{2}c^{2}-20\,abcd+8\,a^{2}d^{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-2\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\,\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\,\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-\left(2\,b^{2}d^{2}x^{3}+\left(9\,b^{2}cd-4\,abd^{2}\right)x\right)\sqrt{dx^{2}+c}}{8\,b^{3}},\ \frac{8\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2\,\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)+\left(15\,b^{2}c^{2}-20\,abcd+8\,a^{2}d^{2}\right)\sqrt{d}\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+2\,\left(2\,b^{2}d^{2}x^{3}+\left(9\,b^{2}cd-4\,abd^{2}\right)x\right)\sqrt{dx^{2}+c}}{16\,b^{3}},\ -\frac{\left(15\,b^{2}c^{2}-20\,abcd+8\,a^{2}d^{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-4\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2\,\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)-\left(2\,b^{2}d^{2}x^{3}+\left(9\,b^{2}cd-4\,abd^{2}\right)x\right)\sqrt{dx^{2}+c}}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/16*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(
d*x^2 + c)*sqrt(d)*x - c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (9*b^2
*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b*c*d +
 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a
^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) - (2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, 1/16
*(8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c -
2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3
+ (b*c^2 - a*c*d)*x)) + (15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-
2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (9*b^2*c*d
- 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b*c*d + 8*a^
2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 4*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d
*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) -
(2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.01, size = 3101, normalized size = 19.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a),x)
```

```
[Out] 7/16/b*d*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*x-1/6/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*a*d-3/2/(-a*b)^(1/2)/b^2/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a^2*d^2*c+3/2/(-a*b)^(1/2)/b/
(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d*c^2+3/2/(-a*b)^(1/2
)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a^2*d^2*c-3/2/(-
a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a*d*c^2-1
/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)*a*d*c+1/2/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x-(-a*b)^(1/2)/b))*a^3*d^3+1/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d*c-1/2/(-a*b)^(
1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a^3*d^3+1/8/b
*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(3/2)*x+15/16/b*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/
2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))*c^2+1/2/b^3*d^(5/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^
(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))*a^2-1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)
^(1/2)/b))*c^3+1/2/b^3*d^(5/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d
^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))*a^2+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*
b)^(1/2)/b))*c^3+1/8/b*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16/b*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(
-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/10/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2
```

```
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/10/(-a*b)^(1/
2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(5/2)+1/2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-1/6/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/2/(-a*b)^(1/
2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*c^2+1/6/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/4/b^2*d^2*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x*a-5/4/b^2*d^(3/2)
*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a*c+1/6/(-a*b)^(
1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(3/2)*a*d-1/4/b^2*d^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x*a-5/4/b^2*d^(3/2)*ln(((x+(-a*b)^(1/2)/
b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a*c-1/2/(-a*b)^(1/2)/b^2*((x+(-a*b)^(
1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a^2*d^
2+1/2/(-a*b)^(1/2)/b^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a^2*d^2+7/16/b*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/(b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{5/2}}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(a + b*x^2),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2), x)
```

3.681 $\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}+\frac{d\sqrt{c+dx^2}\,(2bc-ad)}{b^2}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}$$

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 84, 154, 156, 63, 208}

$$\frac{d\sqrt{c+dx^2}\,(2bc-ad)}{b^2}+\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{d\left(c+dx^2\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b^2 + (d*(c + d*x^2)^(3/2))/(3*b) - (c^(5
/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*b^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
```

```
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x(a+bx)}\,dx, x, x^2\right)\\
&= \frac{d\left(c+dx^2\right)^{3/2}}{3b} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(bc^2+d(2bc-ad)x\right)}{x(a+bx)}\,dx, x, x^2\right)}{2b}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d\left(c+dx^2\right)^{3/2}}{3b} + \frac{\text{Subst}\left(\int \frac{\frac{b^2c^3}{2}+\frac{1}{2}d\left(3b^2c^2-3abcd+a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{b^2}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d\left(c+dx^2\right)^{3/2}}{3b} + \frac{c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} - \frac{(bc-ad)^3\,\text{Subst}\Big(\int}{2}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d\left(c+dx^2\right)^{3/2}}{3b} + \frac{c^3\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{(bc-ad)^3\,\text{Su}}{}\\
&= \frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2} + \frac{d\left(c+dx^2\right)^{3/2}}{3b} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{(bc-ad)^{5/2}\tanh^{-1}\Big(\frac{\sqrt{b}\,\sqrt{c+}}{\sqrt{bc-a}}}{ab^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 114, normalized size = 0.92

$$
\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}} + \frac{d\sqrt{c+dx^2}\,\left(-3ad+7bc+bdx^2\right)}{3b^2} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (d*Sqrt[c + d*x^2]*(7*b*c - 3*a*d + b*d*x^2))/(3*b^2) - (c^(5/2)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/a + ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/(a*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 129, normalized size = 1.04

$$
-\frac{(ad-bc)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{ab^{5/2}} + \frac{\sqrt{c+dx^2}\,\left(-3ad^2+7bcd+bd^2x^2\right)}{3b^2} - \frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(7*b*c*d - 3*a*d^2 + b*d^2*x^2))/(3*b^2) - ((-(b*c) + a*d)
^(5/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a
*b^(5/2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
```

fricas [A] time = 3.47, size = 837, normalized size = 6.75

$$\left[\frac{6\,b^{2}c^{\frac{5}{2}}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)\sqrt{dx^{2}+c}\,\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(abd^{2}x^{2}+7\,abcd-3\,a^{2}d^{2}\right)\sqrt{dx^{2}+c}}{12\,ab^{2}},\ \frac{12\,b^{2}\sqrt{-c}\,c^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)\sqrt{dx^{2}+c}\,\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(abd^{2}x^{2}+7\,abcd-3\,a^{2}d^{2}\right)\sqrt{dx^{2}+c}}{12\,ab^{2}},\ \frac{3\,b^{2}c^{\frac{5}{2}}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{c}+2\,c}{x^{2}}\right)+3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{dx^{2}+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x^{2}\right)}\right)+2\left(abd^{2}x^{2}+7\,abcd-3\,a^{2}d^{2}\right)\sqrt{dx^{2}+c}}{6\,ab^{2}},\ \frac{6\,b^{2}\sqrt{-c}\,c^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{dx^{2}+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x^{2}\right)}\right)+2\left(abd^{2}x^{2}+7\,abcd-3\,a^{2}d^{2}\right)\sqrt{dx^{2}+c}}{6\,ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^
2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b
^2), 1/12*(12*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2
*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2), 1/6*
(3*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 3*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b
*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*
d^2)*x^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^
2), 1/6*(6*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*b^2)]
```

giac [A] time = 0.38, size = 163, normalized size = 1.31

$$\frac{c^{3}\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,ab^{2}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}d+6\,\sqrt{dx^{2}+c}\,b^{2}cd-3\,\sqrt{dx^{2}+c}\,abd^{2}}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)) - (b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))
/(sqrt(-b^2*c + a*b*d)*a*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d + 6*sqrt(d*x^2
 + c)*b^2*c*d - 3*sqrt(d*x^2 + c)*a*b*d^2)/b^3
```

maple [B] time = 0.02, size = 3148, normalized size = 25.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x/(b*x^2+a),x)
```

```
[Out] 1/3/a*c*(d*x^2+c)^(3/2)-1/a*c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1
/a*(d*x^2+c)^(1/2)*c^2+1/6/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*d-1/6/a*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/2/a*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-1/2/a*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*c^2+1/6/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(3/2)*d-1/6/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-7/16/a*(-a*b)^(1/2)/b*d*c*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+7/16/
a*(-a*b)^(1/2)/b*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/8/a*(-a*b)^(1/2)/b*d*((x+(-a*b)^(1/2)/b)^2*d
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16/a/b*d^(1/
2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c
^2+3/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2*c-1/8
/a*(-a*b)^(1/2)/b*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(3/2)*x-15/16/a/b*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1
/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+3/2*a/b^2/(-(a*d-b*c)/b)^(1/2
)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2*c+1/5/a*(d*x^2+c)^(5/2)-1/10/a*((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1
/10/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(5/2)+1/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)*d*c-1/2*a/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2-1/2*a/b^2*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2+1/b*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*d*c+1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2
*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^3+1/2/a/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
```

```
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^3-1/2*a^2/b^3/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c^2+1/4/b^2*(-a*b)^(1/2)*d^2*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4
/b^2*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2
)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))*c-1/2*a/b^3*d^(5/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(
1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))-1/2*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a
*b)^(1/2)/b))*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
*d*c^2+1/2*a/b^3*d^(5/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)
/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))-1/4/b^2*(-a*b)^(1/2)*d^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4/b^2*d^(3/2)*(-a*b
)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x), x)
```

mupad [B] time = 1.00, size = 2094, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x*(a + b*x^2)),x)
```

```
[Out] (atan((((c^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^
5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 -
```

```
6*a^5*b*c*d^7))/b^3 + (((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c
^2*d^4)/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1
/2))/(a*b^3))*(c^5)^(1/2))/(2*a))*1i)/(2*a) + ((c^5)^(1/2)*((2*(c + d*x^2)^
(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*
a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - (((4*a^4*b^3*c
*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^
2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(2*a))*1i
)/(2*a))/((2*(a^5*c^3*d^8 - 3*b^5*c^8*d^3 + 12*a*b^4*c^7*d^4 - 6*a^4*b*c^4*
d^7 - 19*a^2*b^3*c^6*d^5 + 15*a^3*b^2*c^5*d^6))/b^3 - ((c^5)^(1/2)*((2*(c +
 d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d
^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 + (((4*a
^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 + ((4*a^3*b^5*d^
3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(
2*a)))/(2*a) + ((c^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2
- 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^
2*d^6 - 6*a^5*b*c*d^7))/b^3 - (((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a
^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*
(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(2*a)))/(2*a)))*(c^5)^(1/2)*1i)/a + (d*(
c + d*x^2)^(3/2))/(3*b) + (atan((((-b^5*(a*d - b*c)^5)^(1/2)*((2*(c + d*x^2
)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 2
0*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 + ((-b^5*(a*d
- b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)
/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c)^5)^(1/2)*(c + d
*x^2)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5) + ((-b^5*(a*d - b*c)^5)^(1/
2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^
2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b
^3 - ((-b^5*(a*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12
*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c
)^5)^(1/2)*(c + d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5))*1i)/(2*a*b^5))/((2*(a^5*
c^3*d^8 - 3*b^5*c^8*d^3 + 12*a*b^4*c^7*d^4 - 6*a^4*b*c^4*d^7 - 19*a^2*b^3*c
^6*d^5 + 15*a^3*b^2*c^5*d^6))/b^3 - ((-b^5*(a*d - b*c)^5)^(1/2)*((2*(c + d*
x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4
- 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 + ((-b^5*(a
*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d
^4)/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c)^5)^(1/2)*(c
+ d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5) + ((-b^5*(a*d - b*c)^5)^(1/
2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^
2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b
^3 - ((-b^5*(a*d - b*c)^5)^(1/2)*((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12
*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(-b^5*(a*d - b*c
)^5)^(1/2)*(c + d*x^2)^(1/2))/(a*b^8)))/(2*a*b^5)))/(2*a*b^5)))*(-b^5*(a*d
- b*c)^5)^(1/2)*1i)/(a*b^5) - (d*(c + d*x^2)^(1/2)*(a*d - 2*b*c))/b^2
```

sympy [A] time = 67.14, size = 119, normalized size = 0.96

$$\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx^2}\left(-ad^2+2bcd\right)}{b^2}+\frac{c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}+\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a),x)
```

```
[Out] d*(c + d*x**2)**(3/2)/(3*b) + sqrt(c + d*x**2)*(-a*d**2 + 2*b*c*d)/b**2 + c
**3*atan(sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c)) + (a*d - b*c)**3*atan(sqrt
(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*b**3*sqrt((a*d - b*c)/b))
```

3.682 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}\,(ad+2bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}$$

Rubi **[A]** time = 0.20, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {474, 528, 523, 217, 206, 377, 205}

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2}+\frac{d^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}+\frac{dx\sqrt{c+dx^2}\,(ad+2bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(a*x) -
 ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(a^(3/2)*b^2) + (d^(3/2)*(5*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(2*b^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)}\,dx = -\frac{c\left(c+dx^2\right)^{3/2}}{ax} + \frac{\int \frac{\sqrt{c+dx^2}\left(-c(bc-4ad)+d(2bc+ad)x^2\right)}{a+bx^2}\,dx}{a}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{ax} + \frac{\int \frac{-c\left(2b^2c^2-6abcd+a^2d^2\right)+ad^2(5bc-2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{ax} + \frac{\left(d^2(5bc-2ad)\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2} - \frac{(bc-ad)^3\int \frac{}{(a+bx^2}}{ab^2}$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{ax} + \frac{\left(d^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2b^2} - -$$

$$= \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{ax} - \frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b^2} + \frac{d^{3/2}(5bc-2ad)\,\mathrm{t}}{2b}$$

Mathematica [**A**]   time = 0.09, size = 132, normalized size = 0.91

$$-\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}b^2} + \frac{d^{3/2}(5bc-2ad)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{2b^2} + \sqrt{c+dx^2}\left(\frac{d^2x}{2b} - \frac{c^2}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (-(c^2/(a*x)) + (d^2*x)/(2*b))*Sqrt[c + d*x^2] - ((b*c - a*d)^(5/2)*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*b^2) + (d^(3/2)*(5
*b*c - 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

IntegrateAlgebraic [**A**]   time = 0.40, size = 180, normalized size = 1.24

$$\frac{\sqrt{bc-ad}\left(a^2d^2-2abcd+b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{a^{3/2}b^2} + \frac{\left(2ad^{5/2}-5bcd^{3/2}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^2} + \frac{\sqrt{c+dx^2}\left(ad^2x^2-2bc^2\right)}{2abx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b*c^2 + a*d^2*x^2))/(2*a*b*x) + (Sqrt[b*c - a*d]*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c +
```

```
d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(3/2)*b^2) + ((-5*b*c*d^(3/2) + 2*a
*d^(5/2))*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

fricas [A] time = 2.50, size = 887, normalized size = 6.12

$$\left[-\frac{\left(5\,abcd-2\,a^{2}d^{2}\right)\sqrt{d}\,x\log\left(-2\,dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}+4\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-2\left(abd^{2}x^{2}-2\,b^{2}c^{2}\right)\sqrt{dx^{2}+c}}{4\,ab^{2}x},\,-\frac{2\left(5\,abcd-2\,a^{2}d^{2}\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)-\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}+4\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-2\left(abd^{2}x^{2}-2\,b^{2}c^{2}\right)\sqrt{dx^{2}+c}}{4\,ab^{2}x},\,-\frac{2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)+\left(5\,abcd-2\,a^{2}d^{2}\right)\sqrt{d}\,x\log\left(-2\,dx^{2}+2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)-2\left(abd^{2}x^{2}-2\,b^{2}c^{2}\right)\sqrt{dx^{2}+c}}{4\,ab^{2}x},\,-\frac{\left(5\,abcd-2\,a^{2}d^{2}\right)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)+\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)-\left(abd^{2}x^{2}-2\,b^{2}c^{2}\right)\sqrt{dx^{2}+c}}{2\,ab^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((5*a*b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(d)*x - c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)/a)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*
d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^
2 + c))/(a*b^2*x), -1/4*(2*(5*a*b*c*d - 2*a^2*d^2)*sqrt(-d)*x*arctan(sqrt(-
d)*x/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)
/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(
b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*s
qrt(d*x^2 + c))/(a*b^2*x), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt((
b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*
c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (5*a*b*c*d - 2*a^2
*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(a*b*d^
2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x), -1/2*((5*a*b*c*d - 2*a^2*d^2
)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (a*
b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 3191, normalized size = 22.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x)
```

```
[Out] 3/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^
2*c-3/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))*d^2*c-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)
^(1/2)/b))*d^3+1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*
b)^(1/2)/b))*c^3+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x+(-a*b)^(1/2)/b))*d^3-1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))/(x+(-a*b)^(1/2)/b))*c^3-1/6/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*d+1/6/(-a*b)^(1/2)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*
d+1/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*d*c-1/2/b^2*a*d^(5/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b
)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))+5/4/b*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(
1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))*c-1/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*c-1/2/b^2*a*d^(5/2)*ln(((
x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/10*b/a/(-a*b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(5/2)-1/8/a*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(3/2)*x-15/16/a*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)
/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))*c^2+1/4/b*d^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/a/c/x*(d*x^2+c)^(7/2)+5/4/a*d*
x*(d*x^2+c)^(3/2)+15/8/a*d^(1/2)*c^2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/10*b/a
/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(5/2)-1/8/a*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-15/16/a*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*
d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/4/b*d^2*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4/b*d^(3/2)*ln((
```

```
(x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/2/b*a/(-a*b)^(1/2
)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*d^2-1/2*b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c^2+1/a*d/c*x*(d*x^2+c)^(5/2)+15/8/a
*d*c*x*(d*x^2+c)^(1/2)+1/2/b*a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2+1/2*b/a/(-a*b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*c^2+3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)
/b))*d*c^2-7/16/a*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/6*b/a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-7/16/a*d*c*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*x-1/6*b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2 + c\right)^{\frac{5}{2}}}{\left(bx^2 + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2 + c\right)^{5/2}}{x^2\,\left(b\,x^2 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)), x)
```

3.683 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=144

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}\,(2ad+bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2}$$

Rubi [A] time = 0.24, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 98, 154, 156, 63, 208}

$$-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}+\frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}+\frac{d\sqrt{c+dx^2}\,(2ad+bc)}{2ab}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(2*a*b) - (c*(c + d*x^2)^(3/2))/(2*a*x^2)
 + (c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2) - ((b
*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2*b^
(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{5/2}}{x^2(a+bx)}\,dx, x, x^2\right)$$

$$= -\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}\left(\frac{1}{2}c(2bc-5ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)}\,dx, x, x^2\right)}{2a}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{4}bc^2(2bc-5ad)+\frac{1}{4}d\left(b^2c^2-6abcd+2a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x}{ab}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\left(c^2(2bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2} + (bc$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} - \frac{\left(c^2(2bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^2d}$$

$$= \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2} + \frac{c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{5/2}\,\mathrm{ta}}{a}$$

Mathematica [A] time = 0.14, size = 125, normalized size = 0.87

$$\frac{-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + c^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right) + \frac{a\sqrt{c+dx^2}\left(2ad^2x^2-bc^2\right)}{bx^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(b*c^2) + 2*a*d^2*x^2))/(b*x^2) + c^(3/2)*(2*b*c - 5*
a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]] - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.26, size = 145, normalized size = 1.01

$$\frac{(ad-bc)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a^2b^{3/2}} + \frac{\left(2bc^{5/2}-5ac^{3/2}d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} + \frac{\sqrt{c+dx^2}\left(2ad^2x^2-bc^2\right)}{2abx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c^2) + 2*a*d^2*x^2))/(2*a*b*x^2) + ((-(b*c) + a*d)^(5
/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a^2*
b^(3/2)) + ((2*b*c^(5/2) - 5*a*c^(3/2)*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(2*a^2)
```

fricas [**A**] time = 4.39, size = 891, normalized size = 6.19

$$\left[\frac{\left(b^{2} c^{2} - 2 \, a b c d + a^{2} d^{2}\right)} x^{2} \sqrt{\frac{b c - a d}{b}} \log\left(\frac{b^{2} d^{2} x^{4} + 8 \, b^{2} c^{2} - 8 \, a b c d + a^{2} d^{2} + 2 \, {\left(4 \, b^{2} c d - 3 \, a b d^{2}\right)} x^{2} - 4 \, {\left(b^{2} d x^{2} + 2 \, b^{2} c - a b d\right)} \sqrt{d x^{2} + c} \sqrt{\frac{b c - a d}{b}}}{b^{2} x^{4} + 2 \, a b x^{2} + a^{2}}\right) - {\left(2 \, b^{2} c^{2} - 5 \, a b c d\right)} \sqrt{c} x^{2} \log\left(-\frac{d x^{2} - 2 \, \sqrt{d x^{2} + c} \sqrt{c} + 2 \, c}{x^{2}}\right) + 2 \, {\left(2 \, a^{2} d^{2} x^{2} - a b c^{2}\right)} \sqrt{d x^{2} + c}}{4 \, a^{2} b x^{2}}, -\frac{2 \, {\left(2 \, b^{2} c^{2} - 5 \, a b c d\right)} \sqrt{-c} x^{2} \arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2} + c}}\right) - {\left(b^{2} c^{2} - 2 \, a b c d + a^{2} d^{2}\right)} x^{2} \sqrt{\frac{b c - a d}{b}} \log\left(\frac{b^{2} d^{2} x^{4} + 8 \, b^{2} c^{2} - 8 \, a b c d + a^{2} d^{2} + 2 \, {\left(4 \, b^{2} c d - 3 \, a b d^{2}\right)} x^{2} - 4 \, {\left(b^{2} d x^{2} + 2 \, b^{2} c - a b d\right)} \sqrt{d x^{2} + c} \sqrt{\frac{b c - a d}{b}}}{b^{2} x^{4} + 2 \, a b x^{2} + a^{2}}\right) - 2 \, {\left(2 \, a^{2} d^{2} x^{2} - a b c^{2}\right)} \sqrt{d x^{2} + c}}{4 \, a^{2} b x^{2}}, -\frac{2 \, {\left(b^{2} c^{2} - 2 \, a b c d + a^{2} d^{2}\right)} x^{2} \sqrt{-\frac{b c - a d}{b}} \arctan\left(-\frac{{\left(b d x^{2} + 2 \, b c - a d\right)} \sqrt{d x^{2} + c} \sqrt{-\frac{b c - a d}{b}}}{2 \, {\left(b c^{2} - a c d + {\left(b c d - a d^{2}\right)} x^{2}\right)}}\right) + {\left(2 \, b^{2} c^{2} - 5 \, a b c d\right)} \sqrt{c} x^{2} \log\left(-\frac{d x^{2} - 2 \, \sqrt{d x^{2} + c} \sqrt{c} + 2 \, c}{x^{2}}\right) - 2 \, {\left(2 \, a^{2} d^{2} x^{2} - a b c^{2}\right)} \sqrt{d x^{2} + c}}{4 \, a^{2} b x^{2}}, -\frac{{\left(b^{2} c^{2} - 2 \, a b c d + a^{2} d^{2}\right)} x^{2} \sqrt{-\frac{b c - a d}{b}} \arctan\left(-\frac{{\left(b d x^{2} + 2 \, b c - a d\right)} \sqrt{d x^{2} + c} \sqrt{-\frac{b c - a d}{b}}}{2 \, {\left(b c^{2} - a c d + {\left(b c d - a d^{2}\right)} x^{2}\right)}}\right) + {\left(2 \, b^{2} c^{2} - 5 \, a b c d\right)} \sqrt{-c} x^{2} \arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2} + c}}\right) - {\left(2 \, a^{2} d^{2} x^{2} - a b c^{2}\right)} \sqrt{d x^{2} + c}}{2 \, a^{2} b x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - (2*b^2*c^2 - 5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2
 + c))/(a^2*b*x^2), -1/4*(2*(2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*
d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c -
 a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d
*x^2 + c))/(a^2*b*x^2), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b^2*c^2 - 5*a*b*c*d
)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^
2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2), -1/2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*
sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
 + (2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) -
(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2)]
```

giac [**A**] time = 0.39, size = 158, normalized size = 1.10

$$\frac{\sqrt{d x^{2} + c} d^{2}}{b} - \frac{\sqrt{d x^{2} + c} c^{2}}{2 \, a x^{2}} - \frac{{\left(2 \, b c^{3} - 5 \, a c^{2} d\right)} \arctan\left(\frac{\sqrt{d x^{2} + c}}{\sqrt{-c}}\right)}{2 \, a^{2} \sqrt{-c}} + \frac{{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right)} \arctan\left(\frac{\sqrt{d x^{2} + c} b}{\sqrt{-b^{2} c + a b d}}\right)}{\sqrt{-b^{2} c + a b d} a^{2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + c)*d^2/b - 1/2*sqrt(d*x^2 + c)*c^2/(a*x^2) - 1/2*(2*b*c^3 - 5*
a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) + (b^3*c^3 - 3*a*b
```

```
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b)
```

maple [B] time = 0.02, size = 3247, normalized size = 22.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x)

[Out] 5/4/a/b*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(
1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))*c+7/16/a^2*(-a*b)^(1/2)*d*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/a/b*(-a*b)^(1/2)*d^2*((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*x-5/4/a/b*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*
d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))*c-7/16/a^2*(-a*b)^(1/2)*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4/a/b*(-a*b)^(1/2)*
d^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)*x-1/5/a^2*b*(d*x^2+c)^(5/2)+1/10/a^2*b*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/6/a*((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*d+1/2/b*
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*d^2+1/6/a^2*b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/a*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*c+1/2/a^2*b*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2+1/2/b^2*d^(5/
2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1
/3/a^2*b*c*(d*x^2+c)^(3/2)+1/a^2*b*c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2
))/x)-1/a^2*b*(d*x^2+c)^(1/2)*c^2+1/2/a*d/c*(d*x^2+c)^(5/2)-5/2/a*d*c^(3/2)
*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+5/2/a*d*c*(d*x^2+c)^(1/2)-1/2/a/c/x^
2*(d*x^2+c)^(7/2)+1/6/a^2*b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*c+1/2/a^2*b*((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-1/2/b
^2*d^(5/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))-1/8/a^2*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-15/16/a^2*d^(1/2)*(-a*b)^(1/2)*ln(((x
+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/2*a/b^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
```

```
b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^3-3/2/b/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(
1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2*c+3/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x+(-a*b)^(1/2)/b))*d*c^2-1/2/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(
-a*b)^(1/2)/b))*c^3+1/8/a^2*(-a*b)^(1/2)*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16/a^2*d^(1/2)*(-a*b)
^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2+1/2*a/
b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^3-3/2/b/(-(a*d-b*
c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2*c+3/2/a/(-(a*d-b*c)/b)^(1/2)
*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(
1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c^2-1/2/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x-(-a*b)^(1/2)/b))*c^3+5/6/a*d*(d*x^2+c)^(3/2)+1/10/a^2*b*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/6/a
*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(3/2)*d+1/2/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3), x)
```

mupad [B] time = 1.37, size = 1428, normalized size = 9.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x)
```

```
[Out] (d^2*(c + d*x^2)^(1/2))/b + (atan((a^3*d^9*(c + d*x^2)^(1/2)*(c^3)^(1/2)*5i
)/(5*a^3*c^2*d^9 - (395*b^3*c^5*d^6)/4 + 87*a*b^2*c^4*d^7 - 32*a^2*b*c^3*d^
8 + (185*b^4*c^6*d^5)/(4*a) - (15*b^5*c^7*d^4)/(2*a^2)) + (a^2*c*d^8*(c + d
*x^2)^(1/2)*(c^3)^(1/2)*32i)/(32*a^2*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (185*b
^3*c^6*d^5)/(4*a) - (5*a^3*c^2*d^9)/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c
^4*d^7) + (b^2*c^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2)*395i)/(4*(32*a^2*c^3*d
^8 + (395*b^2*c^5*d^6)/4 - (185*b^3*c^6*d^5)/(4*a) - (5*a^3*c^2*d^9)/b + (1
5*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c^4*d^7)) - (b^3*c^4*d^5*(c + d*x^2)^(1/2)*
(c^3)^(1/2)*185i)/(4*(32*a^3*c^3*d^8 - (185*b^3*c^6*d^5)/4 + (395*a*b^2*c^5
*d^6)/4 - 87*a^2*b*c^4*d^7 + (15*b^4*c^7*d^4)/(2*a) - (5*a^4*c^2*d^9)/b)) +
 (b^4*c^5*d^4*(c + d*x^2)^(1/2)*(c^3)^(1/2)*15i)/(2*(32*a^4*c^3*d^8 + (15*b
^4*c^7*d^4)/2 - (185*a*b^3*c^6*d^5)/4 - 87*a^3*b*c^4*d^7 + (395*a^2*b^2*c^5
*d^6)/4 - (5*a^5*c^2*d^9)/b)) - (a*b*c^2*d^7*(c + d*x^2)^(1/2)*(c^3)^(1/2)*
87i)/(32*a^2*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (185*b^3*c^6*d^5)/(4*a) - (5*a
^3*c^2*d^9)/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c^4*d^7))*(5*a*d - 2*b*c)
*(c^3)^(1/2)*1i)/(2*a^2) - (atan((c^3*d^5*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*
b^3*d^5 + 5*a^4*b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b
^7*c^4*d)^(1/2)*20i)/((185*a*b^3*c^5*d^6)/2 - (85*b^4*c^6*d^5)/2 - 16*a^4*c
^2*d^9 + 56*a^3*b*c^3*d^8 + (2*a^5*c*d^10)/b - (199*a^2*b^2*c^4*d^7)/2 + (1
5*b^5*c^7*d^4)/(2*a)) - (c^2*d^6*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*b^3*d^5 +
 5*a^4*b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b^7*c^4*d)
^(1/2)*10i)/(2*a^4*c*d^10 + (185*b^4*c^5*d^6)/2 - (199*a*b^3*c^4*d^7)/2 - 1
6*a^3*b*c^2*d^9 + 56*a^2*b^2*c^3*d^8 - (85*b^5*c^6*d^5)/(2*a) + (15*b^6*c^7
*d^4)/(2*a^2)) - (c^4*d^4*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*b^3*d^5 + 5*a^4*
b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b^7*c^4*d)^(1/2)*
15i)/(2*(56*a^4*c^3*d^8 + (15*b^4*c^7*d^4)/2 - (85*a*b^3*c^6*d^5)/2 - (199*
a^3*b*c^4*d^7)/2 + (2*a^6*c*d^10)/b^2 + (185*a^2*b^2*c^5*d^6)/2 - (16*a^5*c
^2*d^9)/b)) + (a*c*d^7*(c + d*x^2)^(1/2)*(b^8*c^5 - a^5*b^3*d^5 + 5*a^4*b^4
*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 - 5*a*b^7*c^4*d)^(1/2)*2i)
/((185*b^5*c^5*d^6)/2 - (199*a*b^4*c^4*d^7)/2 + 56*a^2*b^3*c^3*d^8 - 16*a^3
*b^2*c^2*d^9 - (85*b^6*c^6*d^5)/(2*a) + (15*b^7*c^7*d^4)/(2*a^2) + 2*a^4*b*
c*d^10))*(-b^3*(a*d - b*c)^5)^(1/2)*1i)/(a^2*b^3) - (b*c^2*d*(c + d*x^2)^(1
/2))/(2*a*(b*(c + d*x^2) - b*c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^{\frac{5}{2}}}{x^3 \left(a + bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)), x)
```

3.684 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=130

$$\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{c\sqrt{c+dx^2}\,(bc-2ad)}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Rubi [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {474, 580, 523, 217, 206, 377, 205}

$$\frac{c\sqrt{c+dx^2}\,(bc-2ad)}{a^2x}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/(a^2*x) - (c*(c + d*x^2)^(3/2))/(3*a*x^3)
 + ((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(a^(5/2)*b) + (d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)}\,dx &= -\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{\int \frac{\sqrt{c+dx^2}\left(-3c(bc-2ad)+3ad^2x^2\right)}{x^2\left(a+bx^2\right)}\,dx}{3a}\\
&= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{\int \frac{3c\left(b^2c^2-3abcd+3a^2d^2\right)+3a^2d^3x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2}\\
&= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^3\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b}+\frac{(bc-ad)^3\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2b}\\
&= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{d^3\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b}+\frac{(bc-ad)^3\,\mathrm{Sub}}{}\\
&= \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x}-\frac{c\left(c+dx^2\right)^{3/2}}{3ax^3}+\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{c+}}\right.}{b}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 125, normalized size = 0.96

$$
\frac{(bc-ad)^{5/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b}+\frac{c\sqrt{c+dx^2}\left(3bcx^2-a\left(c+7dx^2\right)\right)}{3a^2x^3}+\frac{d^{5/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (c*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + 7*d*x^2)))/(3*a^2*x^3) + ((b*c - a*d
)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*b)
+ (d^(5/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/b
```

IntegrateAlgebraic [A] time = 0.49, size = 172, normalized size = 1.32

$$
\frac{\sqrt{c+dx^2}\left(-ac^2-7acdx^2+3bc^2x^2\right)}{3a^2x^3}-\frac{\sqrt{bc-ad}\left(a^2d^2-2abcd+b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}b}-\frac{d^{5/2}\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c^2) + 3*b*c^2*x^2 - 7*a*c*d*x^2))/(3*a^2*x^3) - (Sqr
t[b*c - a*d]*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*
```

```
x^2 - b*x*Sqrt[c + d*x^2]])/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(5/2)*b) - (d^(5/
2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/b
```

fricas **[A]** time = 2.05, size = 901, normalized size = 6.93

$$\left[\frac{6\,a^{2}d^{\frac{5}{2}}x^{3}\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)+3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{3}\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\left(abc^{2}-\left(3\,b^{2}c^{2}-7\,abcd\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\,a^{2}bx^{3}},\ -\frac{12\,a^{2}\sqrt{-d}\,d^{2}x^{3}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)-3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{3}\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(abc^{2}-\left(3\,b^{2}c^{2}-7\,abcd\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\,a^{2}bx^{3}},\ \frac{3\,a^{2}d^{\frac{5}{2}}x^{3}\log\left(-2\,dx^{2}-2\,\sqrt{dx^{2}+c}\,\sqrt{d}\,x-c\right)+3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{3}\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)-2\left(abc^{2}-\left(3\,b^{2}c^{2}-7\,abcd\right)x^{2}\right)\sqrt{dx^{2}+c}}{6\,a^{2}bx^{3}},\ -\frac{6\,a^{2}\sqrt{-d}\,d^{2}x^{3}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^{2}+c}}\right)-3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x^{3}\sqrt{\frac{bc-ad}{a}}\,\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)+2\left(abc^{2}-\left(3\,b^{2}c^{2}-7\,abcd\right)x^{2}\right)\sqrt{dx^{2}+c}}{6\,a^{2}bx^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^2*d^(5/2)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a
^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*
x^2 + c))/(a^2*b*x^3), -1/12*(12*a^2*sqrt(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqr
t(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*
log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
- a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c
*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), 1/6*(3*a^2*d^(5/2)*x^3*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*
sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sq
rt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*(a*b*c^2 -
 (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), -1/6*(6*a^2*sqr
t(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
+ 2*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3)]
```

giac **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple **[B]** time = 0.02, size = 3346, normalized size = 25.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x)

[Out] 3/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*
c^2-3/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))*d*c^2+1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b
)^(1/2)/b))*c^3+7/16*b/a^2*d*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-15/8/a^2*b*d*c*x*(d*x^2+c)^(1/2)-1/2
*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
^3-1/a^2*b*d/c*x*(d*x^2+c)^(5/2)+7/16*b/a^2*d*c*((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+b/a/(-a*b)^(1/2)*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*d*c-1/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/
2)/b))*d^3-b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*c+1/2/b*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(
1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^3-5/4/a*d^(3/2)*ln(((x+(-a*b)^(1/2)/b
)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/4/a*d^2*((x+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/10*b^2/a^2/(-a*b
)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(5/2)-1/3/a/c/x^3*(d*x^2+c)^(7/2)+5/2/a*d^2*x*(d*x^2+c)^(1/2)+5/2/a
*d^(3/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/10*b^2/a^2/(-a*b)^(1/2)*((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/
4/a*d^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*x-5/4/a*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^
(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))*c-15/8/a^2*b*d^(1/2)*c^2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-4/3/a*d
/c^2/x*(d*x^2+c)^(7/2)+4/3/a*d^2/c^2*x*(d*x^2+c)^(5/2)+5/3/a*d^2/c*x*(d*x^2
+c)^(3/2)+3/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)
/b))*d^2*c+1/8*b/a^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/16*b/a^2*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d
+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
```

```
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2-1/6*b/a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*d+1/6*b^2/
a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(3/2)*c+1/2*b^2/a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2-3/2/(-a*b)^(1/2
)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c
)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2*c+1/8*b/a^2*d*((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/
2)*x+15/16*b/a^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)
+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))*c^2+1/6*b/a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*d-1/6*b^2/a^2/(-a*b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-1
/2*b^2/a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2+1/a^2*b/c/x*(d*x^2+c)^(7/2)-5/4/a^2*b*d*x
*(d*x^2+c)^(3/2)-1/2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2+1/2/b*d^(5/2)*ln(((x+(-a*b)^(1/
2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d^2+1/2/b*d
^(5/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{5/2}}{x^4\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^{\frac{5}{2}}}{x^4 \left(a + bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)), x)
```

3.685 $\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\,(ad+bc)}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}$$

Rubi [A] time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\,(ad+bc)}{b^2d^2}+\frac{\left(c+dx^2\right)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2)) + (c + d*x^2)^(3/2)/(3*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*c
- a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx, x, x^2\right)\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2b^2}\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{b^2d}\\
&= -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 89, normalized size = 0.89

$$\frac{\sqrt{c+dx^2}\left(-3ad-2bc+bdx^2\right)}{3b^2d^2}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b*c - 3*a*d + b*d*x^2))/(3*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*Sqrt[b*c - a*d])
```

IntegrateAlgebraic [A] time = 0.15, size = 99, normalized size = 0.99

$$\frac{\sqrt{c+dx^2}\left(-3ad-2bc+bdx^2\right)}{3b^2d^2}-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*b*c - 3*a*d + b*d*x^2))/(3*b^2*d^2) - (a^2*ArcTan[(Sqr
t[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(b^(5/2)*Sqrt[-(b*c)
 + a*d])
```

fricas [**B**] time = 1.27, size = 390, normalized size = 3.90

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{dx^2+c}}{12\left(b^4cd^2-ab^3d^3\right)},-\frac{3\sqrt{-b^2c+abd}\,a^2d^2\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)+2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{dx^2+c}}{6\left(b^4cd^2-ab^3d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c
*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sq
rt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^3*
c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))
/(b^4*c*d^2 - a*b^3*d^3), -1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(-1/2*
(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 -
 (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

giac [**A**] time = 0.38, size = 105, normalized size = 1.05

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^2+c}\,b^2cd^4-3\,\sqrt{dx^2+c}\,abd^5}{3\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^
2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 - 3*sqrt(
d*x^2 + c)*a*b*d^5)/(b^3*d^6)
```

maple [**B**] time = 0.02, size = 362, normalized size = 3.62

$$-\frac{a^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\;b^3}-\frac{a^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\;b^3}+\frac{\sqrt{d\,x^2+c}\;x^2}{3bd}-\frac{\sqrt{d\,x^2+c}\;a}{b^2d}-\frac{2\sqrt{d\,x^2+c}\;c}{3b\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3/b*x^2/d*(d*x^2+c)^(1/2)-2/3/b*c/d^2*(d*x^2+c)^(1/2)-1/b^2*a/d*(d*x^2+c)
^(1/2)-1/2/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2
/b^3*a^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.69, size = 100, normalized size = 1.00

$$\frac{\left(d\,x^2+c\right)^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}+\frac{a\,d^3-b\,c\,d^2}{b^2\,d^4}\right)\,\sqrt{d\,x^2+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b*d^2) - ((2*c)/(b*d^2) + (a*d^3 - b*c*d^2)/(b^2*d^4))
*(c + d*x^2)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2
)))/(b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.686 $\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=68

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d
]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{c+dx^2}}{bd}-\frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2b}\\ &= \frac{\sqrt{c+dx^2}}{bd}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{bd}\\ &= \frac{\sqrt{c+dx^2}}{bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.00

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d
]])/(b^(3/2)*Sqrt[b*c - a*d])
```

IntegrateAlgebraic [A] time = 0.08, size = 78, normalized size = 1.15

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{b^{3/2}\sqrt{ad-bc}}+\frac{\sqrt{c+dx^2}}{bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/(b*d) + (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^
2])/(b*c - a*d)]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

fricas [B] time = 1.10, size = 306, normalized size = 4.50

$$\left[\frac{\sqrt{b^{2}c-abd}\,ad\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(bdx^{2}+2\,bc-ad\right)\sqrt{b^{2}c-abd}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(b^{2}c-abd\right)\sqrt{dx^{2}+c}}{4\left(b^{3}cd-ab^{2}d^{2}\right)},\ \frac{\sqrt{-b^{2}c+abd}\,ad\arctan\left(-\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{-b^{2}c+abd}\,\sqrt{dx^{2}+c}}{2\left(b^{2}c^{2}-abcd+\left(b^{2}cd-abd^{2}\right)x^{2}\right)}\right)+2\left(b^{2}c-abd\right)\sqrt{dx^{2}+c}}{2\left(b^{3}cd-ab^{2}d^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*
c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c - a*b*d
)*sqrt(d*x^2 + c))/(b^3*c*d - a*b^2*d^2), 1/2*(sqrt(-b^2*c + a*b*d)*a*d*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*
c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 +
c))/(b^3*c*d - a*b^2*d^2)]
```

giac [A] time = 0.34, size = 64, normalized size = 0.94

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b}-\frac{\sqrt{dx^{2}+c}}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - sqrt(d*x^2 + c)/b)/d
```

maple [B] time = 0.01, size = 318, normalized size = 4.68

$$\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{\sqrt{d\,x^{2}+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)/b/d+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)
```

```
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1
/2)/b))+1/2*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.64, size = 57, normalized size = 0.84

$$\frac{\sqrt{d\,x^2+c}}{b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (c + d*x^2)^(1/2)/(b*d) - (a*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(
1/2)))/(b^(3/2)*(a*d - b*c)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.687 $\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*
d]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$
-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,\sqrt{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a*
d]))
```

IntegrateAlgebraic [A] time = 0.05, size = 59, normalized size = 1.20

$$
-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{\sqrt{b}\,\sqrt{ad-bc}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)]/(Sqrt[b]
*Sqrt[-(b*c) + a*d]))
```

fricas [B] time = 1.20, size = 231, normalized size = 4.71

$$
\left[\frac{\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2-4\,\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,\sqrt{b^2c-abd}},-\frac{\sqrt{-b^2c+abd}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)}{2\,\left(b^2c-abd\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*
a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(b^2*c - a*b*d), -1/2*sqrt(-b^2*c + a*b
*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c
)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2))/(b^2*c - a*b*d)]
```

giac [A] time = 0.32, size = 39, normalized size = 0.80

$$\frac{\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

maple [B] time = 0.01, size = 300, normalized size = 6.12

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/b/(-(a*d-b*
c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.65, size = 39, normalized size = 0.80

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^2+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^2)^(1/2))/(a*b*d - b^2*c)^(1/2))/(a*b*d - b^2*c)^(1/2)
```

sympy [A] time = 6.89, size = 36, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b*sqrt((a*d - b*c)/b))
```

3.688 $\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{ad}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.98

$$
\frac{\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^2])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/a
```

IntegrateAlgebraic [A] time = 0.11, size = 90, normalized size = 1.12

$$
\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{a\sqrt{ad-bc}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/
(a*Sqrt[-(b*c) + a*d]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*Sqrt[c])
```

fricas [**A**] time = 1.23, size = 603, normalized size = 7.54

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+4\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{4ac}, -\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)-\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{2ac}, -\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
(b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 2*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2))/(a*c), 1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d
+ a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/
(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*a
rctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d
*x^2 + b*c)) - 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a*c)]
```

giac [**A**] time = 0.30, size = 70, normalized size = 0.88

$$-\frac{b\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a)
+ arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [**B**] time = 0.01, size = 331, normalized size = 4.14

$$\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/2/a/(-(a*d-b*c
)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/a/c^(1/2)*ln((2*c+2*(d*x^2+c)^(
1/2)*c^(1/2))/x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x), x)
```

mupad [**B**] time = 0.88, size = 651, normalized size = 8.14

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a\,\sqrt{c}}-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)\,1\mathrm{i}}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)\,1\mathrm{i}}{a^2\,d-a\,b\,c}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3+\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}}\right)\,\sqrt{b^2\,c-a\,b\,d}\,1\mathrm{i}}{a^2\,d-a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^2)^(1/2)/c^(1/2))/(a*c^(1/2)) - (atan((((b^2*c - a*b*d)^(1
/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 -
((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2)
)/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c) + ((b^2*c
- a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a^2
*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)*(b^2*c - a
*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c)
)/(((b^2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) - ((b^2*c - a*b*d)^(
1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)
*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c))))/(a^2*d
- a*b*c) - ((b^2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) + ((b^2*c -
a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^
```

```
2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c))))
/(a^2*d - a*b*c)))*(b^2*c - a*b*d)^(1/2)*1i)/(a^2*d - a*b*c)
```

sympy [A] time = 14.31, size = 63, normalized size = 0.79

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*sqrt((a*d - b*c)/b)) + atan(
sqrt(c + d*x**2)/sqrt(-c))/(a*sqrt(-c))
```

3.689 $\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*a*c*x^2) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[
c]])/(2*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{4a^2c}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{2a^2cd}$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.38, size = 109, normalized size = 0.95

$$\frac{-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a\sqrt{c+dx^2}}{cx^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((a*Sqrt[c + d*x^2])/(c*x^2)) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/c^(3/2) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/Sqrt[b*c - a*d])/(2*a^2)
```

IntegrateAlgebraic **[A]** time = 0.21, size = 125, normalized size = 1.09

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{a^2\sqrt{ad-bc}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}}-\frac{\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a^2*Sqrt[-(b*c) + a*d]) + ((2*b*c + a*d)*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c^(3/2))
```

fricas **[A]** time = 1.39, size = 734, normalized size = 6.38

$$\left[\frac{bc^2x^2\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+(2\,bc+ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{dx^2+c}\,ac}{4\,a^2c^2x^2}, \frac{bc^2x^2\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-2\,(2\,bc+ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-2\,\sqrt{dx^2+c}\,ac}{4\,a^2c^2x^2}, \frac{2\,bc^2x^2\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+(2\,bc+ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{dx^2+c}\,ac}{4\,a^2c^2x^2}, \frac{bc^2x^2\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)-(2\,bc+ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\sqrt{dx^2+c}\,ac}{2\,a^2c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^
2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*
x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(b
*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2
*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2
*a*b*x^2 + a^2)) - 2*(2*b*c + a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2
+ c)) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(2*b*c^2*x^2*sqrt(-b/(b*c
 - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c -
a*d))/(b*d*x^2 + b*c)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x
^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/2*(b*
c^2*x^2*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
+ c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (2*b*c + a*d)*sqrt(-c)*x^2*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2)]
```

giac [A] time = 0.35, size = 103, normalized size = 0.90

$$\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}\,c}-\frac{\sqrt{dx^2+c}}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^
2) - 1/2*(2*b*c + a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c) -
1/2*sqrt(d*x^2 + c)/(a*c*x^2)
```

maple [B] time = 0.02, size = 385, normalized size = 3.35

$$-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\;a^2}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\;a^2}+\frac{d\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{2a\,c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{a^2\sqrt{c}}-\frac{\sqrt{d\,x^2+c}}{2ac\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2*(d*x^2+
c)^(1/2)/a/c/x^2+1/2/a*d/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/2*
b/a^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/a^2*b/c^(1/2)*l
n((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3), x)
```

mupad [B] time = 1.11, size = 396, normalized size = 3.44

$$\frac{\ln\left(\sqrt{d\,x^2+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{2\,a^3\,d-2\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^2+c}\,{\left(b^4\,c-a\,b^3\,d\right)}^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{2\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^2+c}}{2\,a\,c\,x^2}-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^2+c}\,3{}\mathrm{i}}{2\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{2\,c}+\frac{5\,a\,b^3\,d^5}{4\,c^2}+\frac{a^2\,b^2\,d^6}{4\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{4\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{4\,a}+\frac{b^2\,d^6}{4\,c}+\frac{3\,b^4\,c\,d^4}{2\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^2+c}\,5{}\mathrm{i}}{4\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{2\,a}+\frac{5\,b^3\,d^5}{4\,c}+\frac{a\,b^2\,d^6}{4\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1{}\mathrm{i}}{2\,a^2\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (log((c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(2*a^3*d - 2*a^2*b*c) - (log((c + d*x^2
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(2*(a^3*d - a^2*b*c)) - (c + d*x^2)^(1/2)/(2*a*c*x^2)
- (atan((b^4*d^4*(c + d*x^2)^(1/2)*3i)/(2*(c^3)^(1/2)*((3*b^4*d^4)/(2*c) +
(5*a*b^3*d^5)/(4*c^2) + (a^2*b^2*d^6)/(4*c^3))) + (b^2*d^6*(c + d*x^2)^(1/2
)*1i)/(4*(c^3)^(1/2)*((5*b^3*d^5)/(4*a) + (b^2*d^6)/(4*c) + (3*b^4*c*d^4)/(
2*a^2))) + (b^3*d^5*(c + d*x^2)^(1/2)*5i)/(4*(c^3)^(1/2)*((3*b^4*d^4)/(2*a)
 + (5*b^3*d^5)/(4*c) + (a*b^2*d^6)/(4*c^2))))*(a*d + 2*b*c)*1i)/(2*a^2*(c^3
)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^2\right) \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.690 $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*
Sqrt[c + d*x^2])])/(b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}}{2bd} - \frac{\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2bd}$$

$$= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b^2} - \frac{(bc+2ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^2d}$$

$$= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{(bc+2ad)\,\mathrm{Subst}\Bigl(\int \frac{1}{1-dx^2}\,d}{2b^2d}$$

$$= \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}$$

Mathematica [A] time = 0.18, size = 112, normalized size = 0.98

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{d^{3/2}} + \frac{bx\sqrt{c+dx^2}}{d}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*x*Sqrt[c + d*x^2])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x + Sqrt[d]*Sqrt[c
 + d*x^2]])/d^(3/2))/(2*b^2)
```

IntegrateAlgebraic [A] time = 0.33, size = 170, normalized size = 1.49

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}}+\frac{(2ad+bc)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^2d^{3/2}}+\frac{x\sqrt{c+dx^2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*b*d) - (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c -
a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(S
qrt[a]*Sqrt[b*c - a*d])]])/(b^2*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*Log[-(Sqrt
[d]*x) + Sqrt[c + d*x^2]])/(2*b^2*d^(3/2))
```

fricas [A] time = 1.10, size = 717, normalized size = 6.29

$$\left[\frac{ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,bdx+(bc+2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{4b^2d^2}, \frac{ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2+4\left((b^2c^2-3abcd+2a^2d^2)x^3-(abc^2-a^2cd)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+2\sqrt{dx^2+c}\,bdx+2(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{4b^2d^2}, -\frac{2ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-2\sqrt{dx^2+c}\,bdx-(bc+2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)}{4b^2d^2}, -\frac{ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2(adx^3+acx)}\right)-\sqrt{dx^2+c}\,bdx-(bc+2ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{2b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4
 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + (b*c + 2*a*d)*sqrt(
d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), 1/4*(a*d^2*s
qrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 -
(a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a
*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + 2*(b*c + 2*a*d)*sqrt(-d)*arctan(
sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2), -1/4*(2*a*d^2*sqrt(a/(b*c - a*d))*a
rctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a
*d*x^3 + a*c*x)) - 2*sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt(d)*log(-2*d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), -1/2*(a*d^2*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
- a*d))/(a*d*x^3 + a*c*x)) - sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.01, size = 386, normalized size = 3.39

$$-\frac{a^{2} \ln \left(\frac{\frac{2 \sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right) d}{b}-\frac{2(ad-bc)}{b}+2 \sqrt{-\frac{ad-bc}{b}}\, \sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2} d+\frac{2 \sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right) d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2 \sqrt{-ab}\, \sqrt{-\frac{ad-bc}{b}}\, b^{2}}+\frac{a^{2} \ln \left(\frac{-\frac{2 \sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right) d}{b}-\frac{2(ad-bc)}{b}+2 \sqrt{-\frac{ad-bc}{b}}\, \sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2} d-\frac{2 \sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right) d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2 \sqrt{-ab}\, \sqrt{-\frac{ad-bc}{b}}\, b^{2}}-\frac{a \ln \left(\sqrt{d}\, x+\sqrt{d x^{2}+c}\right)}{b^{2} \sqrt{d}}-\frac{c \ln \left(\sqrt{d}\, x+\sqrt{d x^{2}+c}\right)}{2 b d^{\frac{3}{2}}}+\frac{\sqrt{d x^{2}+c}\, x}{2 b d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b/d-1/2/b*c/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/b
^2*a*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)+1/2/b^2*a^2/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/b^2*a^2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(bx^{2}+a\right)\sqrt{dx^{2}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.691 $\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b
*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx &= \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b} - \frac{a\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 1.04

$$\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b
*c - a*d])) + Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

IntegrateAlgebraic [A] time = 0.20, size = 137, normalized size = 1.67

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}} - \frac{bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{b\sqrt{bc-ad}} - \frac{\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a
]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*S
qrt[b*c - a*d]) - Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

fricas [A] time = 1.26, size = 616, normalized size = 7.51

$$\left[\frac{d \sqrt{-\frac{a}{b c-a d}} \log\left(\frac{\left(b^{2} c^{2}-8 a b c d+8 a^{2} d^{2}\right) x^{4}+a^{2} c^{2}-2 \left(3 a b c^{2}-4 a^{2} c d\right) x^{2}-4 \left(\left(b^{2} c^{2}-3 a b c d+2 a^{2} d^{2}\right) x^{3}-\left(a b c^{2}-a^{2} c d\right) x\right) \sqrt{d x^{2}+c} \sqrt{-\frac{a}{b c-a d}}}{b^{2} x^{4}+2 a b x^{2}+a^{2}}\right)+2 \sqrt{d} \log\left(-2 d x^{2}-2 \sqrt{d x^{2}+c} \sqrt{d} x-c\right)}{4 b d}, \frac{d \sqrt{-\frac{a}{b c-a d}} \log\left(\frac{\left(b^{2} c^{2}-8 a b c d+8 a^{2} d^{2}\right) x^{4}+a^{2} c^{2}-2 \left(3 a b c^{2}-4 a^{2} c d\right) x^{2}-4 \left(\left(b^{2} c^{2}-3 a b c d+2 a^{2} d^{2}\right) x^{3}-\left(a b c^{2}-a^{2} c d\right) x\right) \sqrt{d x^{2}+c} \sqrt{-\frac{a}{b c-a d}}}{b^{2} x^{4}+2 a b x^{2}+a^{2}}\right)-4 \sqrt{-d} \arctan\left(\frac{\sqrt{-d} x}{\sqrt{d x^{2}+c}}\right)}{4 b d}, \frac{d \sqrt{\frac{a}{b c-a d}} \arctan\left(-\frac{\left(\left(b c-2 a d\right) x^{2}-a c\right) \sqrt{d x^{2}+c} \sqrt{\frac{a}{b c-a d}}}{2 \left(a d x^{3}+a c x\right)}\right)+\sqrt{d} \log\left(-2 d x^{2}-2 \sqrt{d x^{2}+c} \sqrt{d} x-c\right)}{2 b d}, \frac{d \sqrt{\frac{a}{b c-a d}} \arctan\left(-\frac{\left(\left(b c-2 a d\right) x^{2}-a c\right) \sqrt{d x^{2}+c} \sqrt{\frac{a}{b c-a d}}}{2 \left(a d x^{3}+a c x\right)}\right)-2 \sqrt{-d} \arctan\left(\frac{\sqrt{-d} x}{\sqrt{d x^{2}+c}}\right)}{2 b d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d
)*x - c))/(b*d), 1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*
b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*
c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)))/(b*d), 1/2*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + sqrt(d)
*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b*d), 1/2*(d*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
- a*d))/(a*d*x^3 + a*c*x)) - 2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))
/(b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.01, size = 337, normalized size = 4.11

$$\frac{a \ln\left(\frac{\frac{2 \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{2(a d-b c)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b}\, \sqrt{-\frac{a d-b c}{b}}\, b}-\frac{a \ln\left(\frac{-\frac{2 \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{2(a d-b c)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{\left(x+\frac{\sqrt{-a b}}{b}\right)^{2} d-\frac{2 \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b}\, \sqrt{-\frac{a d-b c}{b}}\, b}+\frac{\ln\left(\sqrt{d}\, x+\sqrt{d x^{2}+c}\right)}{b \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/b*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)-1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-
b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/2*a/(-a*b)^(1/2)/b/(-(a*d-b*c)/b
)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*
c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.692 $\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {377, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

IntegrateAlgebraic [B] time = 0.00, size = 103, normalized size = 2.10

$$-\frac{\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[
b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]/(Sqrt[a]*Sqr
t[b*c - a*d]))
```

fricas [B] time = 1.02, size = 241, normalized size = 4.92

$$\left[-\frac{\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(abc-a^2d\right)},\ \frac{\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{2\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d
), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
+ c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)
]
```

giac [A] time = 0.33, size = 70, normalized size = 1.43

$$-\frac{\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

maple [B] time = 0.01, size = 306, normalized size = 6.24

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{x\,\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)}{\sqrt{-a\,\left(a\,d-b\,c\right)}} & \text{if } 0<b\,c-a\,d \\ \frac{\ln\left(\frac{\sqrt{a\,\left(d\,x^2+c\right)}+x\,\sqrt{a\,d-b\,c}}{\sqrt{a\,\left(d\,x^2+c\right)}-x\,\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{a\,\left(a\,d-b\,c\right)}} & \text{if } b\,c-a\,d<0 \\ \int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx & \text{if } b\,c-a\,d\notin \mathbb{R}\vee a\,d=b\,c \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^2
)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(1/
2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))/(2
*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/((a +
 b*x^2)*(c + d*x^2)^(1/2)), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{2}\right) \sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.693 $\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}}{acx}-\frac{\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{ac}\\
&= -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a}\\
&= -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{a}\\
&= -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [C] time = 3.94, size = 177, normalized size = 2.39

$$
-\frac{\left(\frac{dx^2}{c}+1\right)\left(\frac{4x^2\left(c+dx^2\right)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{3c^2\left(a+bx^2\right)}+\frac{\left(c+2dx^2\right)\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)}{c\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}}\right)}{x\left(a+bx^2\right)\sqrt{c+dx^2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(((1 + (d*x^2)/c)*(((c + 2*d*x^2)*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))]])/(c*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)]) + (4
*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x^2)
/(c*(a + b*x^2))])/(3*c^2*(a + b*x^2))))/(x*(a + b*x^2)*Sqrt[c + d*x^2]))
```

IntegrateAlgebraic [A] time = 0.21, size = 137, normalized size = 1.85

$$
-\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{3/2}(ad-bc)}-\frac{\sqrt{c+dx^2}}{acx}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/(a*c*x)) - (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sq
rt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d
*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]])/(a^(3/2)*(-(b*c) + a*d))
```

fricas [B] time = 1.31, size = 324, normalized size = 4.38

$$\left[-\frac{\sqrt{-abc + a^2d}\,bcx\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right) + 4\left(abc - a^2d\right)\sqrt{dx^2 + c}}{4\left(a^2bc^2 - a^3cd\right)x}, -\frac{\sqrt{abc - a^2d}\,bcx\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right) + 2\left(abc - a^2d\right)\sqrt{dx^2 + c}}{2\left(a^2bc^2 - a^3cd\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c + a^2*d)*b*c*x*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)
*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*
b*c - a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x), -1/2*(sqrt(a*b*c -
 a^2*d)*b*c*x*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b*c -
 a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x)]
```

giac [A] time = 0.32, size = 111, normalized size = 1.50

$$d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd - a^2d^2}\,ad} + \frac{2}{\left(\left(\sqrt{d}\,x - \sqrt{dx^2 + c}\right)^2 - c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] d^(3/2)*(b*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x - sqrt
(d*x^2 + c))^2 - c)*a*d))
```

maple [B] time = 0.02, size = 334, normalized size = 4.51

$$\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a} - \frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a} - \frac{\sqrt{d\,x^2 + c}}{acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+
1/2*b/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-(d
*x^2+c)^(1/2)/a/c/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.694 $\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=110

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(2ad+3bc)}{3a^2c^2x}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*a*c*x^3) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/(3*a^2*c^2
*x) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*
Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx &= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3ac}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}-\frac{\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3a^2c^2}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a^2}\\
&= -\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 5.12, size = 96, normalized size = 0.87

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\left(-ac+2adx^2+3bcx^2\right)}{3a^2c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 + 2*a*d*x^2))/(3*a^2*c^2*x^3) + (b^2*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*Sqrt[b*c - a
*d])
```

IntegrateAlgebraic [A] time = 0.31, size = 159, normalized size = 1.45

$$\frac{b^2\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{5/2}(ad-bc)}+\frac{\sqrt{c+dx^2}\,\left(-ac+2adx^2+3bcx^2\right)}{3a^2c^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 + 2*a*d*x^2))/(3*a^2*c^2*x^3) + (b^2*S
qrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(
Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]
)/(a^(5/2)*(-(b*c) + a*d))
```

fricas [B] time = 1.45, size = 414, normalized size = 3.76

$$\left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^3\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(a^2bc^2-a^3cd-\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^2\right)\sqrt{dx^2+c}}{12\left(a^3bc^3-a^4c^2d\right)x^3},\ \frac{3\sqrt{abc-a^2d}\,b^2c^2x^3\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)-2\left(a^2bc^2-a^3cd-\left(3\,ab^2c^2-a^2bcd-2\,a^3d^2\right)x^2\right)\sqrt{dx^2+c}}{6\left(a^3bc^3-a^4c^2d\right)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqr
t(d*x^2 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^3), 1/6*(3*sqrt(a*b*c - a^2*d)*b^2
*c^2*x^3*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b*c^2 -
a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(d*x^2 + c))/((a^3
*b*c^3 - a^4*c^2*d)*x^3)]
```

giac [B] time = 5.36, size = 195, normalized size = 1.77

$$-\frac{1}{3}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc-6\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+3\,bc^2+2\,acd\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2
*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 6*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x - sqr
t(d*x^2 + c))^2 - c)^3*a^2*d^2))
```

maple [B] time = 0.02, size = 379, normalized size = 3.45

$$-\frac{b^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{b^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{2\sqrt{d\,x^2+c}\,d}{3a\,c^2x}+\frac{\sqrt{d\,x^2+c}\,b}{a^2cx}-\frac{\sqrt{d\,x^2+c}}{3ac\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))-1/2*b^2/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)
/b))+1/a^2*b/c/x*(d*x^2+c)^(1/2)-1/3*(d*x^2+c)^(1/2)/a/c/x^3+2/3/a*d/c^2/x*
(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^2\right) \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.695 $\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}\,(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {470, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}-\frac{cx}{d\sqrt{c+dx^2}\,(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]]/(b*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{\int \frac{ac+(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{d(bc-ad)}\\
&= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{bd}+\frac{a^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{b(bc-ad)}\\
&= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bd}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{b(bc-ad}\\
&= -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 111, normalized size = 1.02

$$
\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}}+\frac{cx}{d\sqrt{c+dx^2}\,(ad-bc)}+\frac{\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{bd^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x)/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*(b*c - a*d)^(3/2)) + Log[d*x + Sqrt[d]*S
qrt[c + d*x^2]]/(b*d^(3/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 165, normalized size = 1.51

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{b(bc-ad)^{3/2}}+\frac{cx}{d\sqrt{c+dx^2}\,(ad-bc)}-\frac{\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x)/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (a^(3/2)*ArcTan[(Sqrt[a]*Sqrt[d]
)/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c
 + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*(b*c - a*d)^(3/2)) - Log[-(Sqrt[d
]*x) + Sqrt[c + d*x^2]]/(b*d^(3/2))
```

fricas [B] time = 1.74, size = 977, normalized size = 8.96

$$\left[-\frac{4\sqrt{dx^2+c}\,bcdx-2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+\left(ad^3x^2+acd^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(b^2c^2d^2-abcd^3+\left(b^2cd^3-abd^4\right)x^2\right)},\ -\frac{4\sqrt{dx^2+c}\,bcdx+4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(ad^3x^2+acd^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(b^2c^2d^2-abcd^3+\left(b^2cd^3-abd^4\right)x^2\right)},\ -\frac{2\sqrt{dx^2+c}\,bcdx+\left(ad^3x^2+acd^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)-\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{2\left(b^2c^2d^2-abcd^3+\left(b^2cd^3-abd^4\right)x^2\right)},\ -\frac{2\sqrt{dx^2+c}\,bcdx+2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(ad^3x^2+acd^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)}{2\left(b^2c^2d^2-abcd^3+\left(b^2cd^3-abd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*b*c*d*x - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*
sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*d^3*x^2 + a*c*
d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*
x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)
, -1/4*(4*sqrt(d*x^2 + c)*b*c*d*x + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d^2)*sqrt(-
a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*
c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2), -1/2*(2*s
qrt(d*x^2 + c)*b*c*d*x + (a*d^3*x^2 + a*c*d^2)*sqrt(a/(b*c - a*d))*arctan(-
1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3
+ a*c*x)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b
```

```
*d^4)*x^2), -1/2*(2*sqrt(d*x^2 + c)*b*c*d*x + 2*(b*c^2 - a*c*d + (b*c*d - a
*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*
c*d^3 - a*b*d^4)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 720, normalized size = 6.61

$$\frac{a^{2} \ln \left(\frac{\frac{2 \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{2 (a d-b c)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b}\, \left(a d-b c\right) \sqrt{-\frac{a d-b c}{b}}\, b}-\frac{a^{2} \ln \left(\frac{-\frac{2 \sqrt{-a b}\, \left(x+\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{2 (a d-b c)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{\left(x+\frac{\sqrt{-a b}}{b}\right)^{2} d-\frac{2 \sqrt{-a b}\, \left(x+\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b}\, \left(a d-b c\right) \sqrt{-\frac{a d-b c}{b}}\, b}+\frac{a^{2}}{2 \sqrt{-a b}\, \left(a d-b c\right) \sqrt{\left(x+\frac{\sqrt{-a b}}{b}\right)^{2} d-\frac{2 \sqrt{-a b}\, \left(x+\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}\, b}-\frac{a^{2}}{2 \sqrt{-a b}\, \left(a d-b c\right) \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}\, b}+\frac{a^{2} d x}{2 \left(a d-b c\right) \sqrt{\left(x+\frac{\sqrt{-a b}}{b}\right)^{2} d-\frac{2 \sqrt{-a b}\, \left(x+\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}\, b^{2} c}+\frac{a^{2} d x}{2 \left(a d-b c\right) \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right) d}{b}-\frac{a d-b c}{b}}\, b^{2} c}-\frac{a x}{\sqrt{d \,x^{2}+c}\, b^{2} c}-\frac{x}{\sqrt{d \,x^{2}+c}\, b d}+\frac{\ln \left(\sqrt{d}\, x+\sqrt{d \,x^{2}+c}\right)}{b \,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/b*x/d/(d*x^2+c)^(1/2)+1/b/d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/b^2*a*
x/c/(d*x^2+c)^(1/2)+1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/b^2*a^2/(a*d
-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*d*x-1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/b^2
*a^2/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(b x^{2} + a\right)\left(d x^{2} + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.696 $\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{c}{d\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(c/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}}-\frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2(bc-ad)}\\
&= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{d(bc-ad)}\\
&= -\frac{c}{d(bc-ad)\sqrt{c+dx^2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 1.12

$$\frac{c}{d\sqrt{c+dx^2}\,(ad-bc)}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*
Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b]*(-(b*c) + a*d)^2)
```

IntegrateAlgebraic [A] time = 0.12, size = 87, normalized size = 1.13

$$\frac{c}{d\sqrt{c+dx^2}\,(ad-bc)}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (a*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^2])/(b*c - a*d)])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

fricas [B] time = 1.14, size = 428, normalized size = 5.56

$$\left[-\frac{\left(ad^{2}x^{2}+acd\right)\sqrt{b^{2}c-abd}\,\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}-4\left(bdx^{2}+2\,bc-ad\right)\sqrt{b^{2}c-abd}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(b^{2}c^{2}-abcd\right)\sqrt{dx^{2}+c}}{4\left(b^{3}c^{3}d-2\,ab^{2}c^{2}d^{2}+a^{2}bcd^{3}+\left(b^{3}c^{2}d^{2}-2\,ab^{2}cd^{3}+a^{2}bd^{4}\right)x^{2}\right)},\ \frac{\left(ad^{2}x^{2}+acd\right)\sqrt{-b^{2}c+abd}\,\arctan\left(-\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{-b^{2}c+abd}\,\sqrt{dx^{2}+c}}{2\left(b^{2}c^{2}-abcd+\left(b^{2}cd-abd^{2}\right)x^{2}\right)}\right)-2\left(b^{2}c^{2}-abcd\right)\sqrt{dx^{2}+c}}{2\left(b^{3}c^{3}d-2\,ab^{2}c^{2}d^{2}+a^{2}bcd^{3}+\left(b^{3}c^{2}d^{2}-2\,ab^{2}cd^{3}+a^{2}bd^{4}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*d^2*x^2 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
 4*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2), 1/2*((a*d^2*x^2 +
 a*c*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*
c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) -
 2*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*
b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2)]
```

giac [A] time = 0.47, size = 78, normalized size = 1.01

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,(bc-ad)}+\frac{c}{\sqrt{dx^{2}+c}\,(bc-ad)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + c/(sqrt(d*x^2 + c)*(b*c - a*d)))/d
```

maple [B] time = 0.01, size = 653, normalized size = 8.48

$$-\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b}+\frac{a}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}+\frac{a}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}+\frac{\sqrt{-ab}\,adx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^{2}c}-\frac{\sqrt{-ab}\,adx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^{2}c}-\frac{1}{\sqrt{d\,x^{2}+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/b/d/(d*x^2+c)^(1/2)+1/2*a/b/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*a/b^2*(-a*b)^(1/2)/(a*d-
b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)*d*x-1/2*a/b/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*
b)^(1/2)/b))+1/2*a/b/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*a/b^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2)*d*x-1/2*a/b/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b)
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.76, size = 64, normalized size = 0.83

$$\frac{c}{d\,\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}+\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] c/(d*(c + d*x^2)^(1/2)*(a*d - b*c)) + (a*atan((b^(1/2)*(c + d*x^2)^(1/2))/(
a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a + b x^{2}\right) \left(c + d x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.697 $\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x}{\sqrt{c+dx^2}\,(bc-ad)}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {471, 12, 377, 205}

$$\frac{x}{\sqrt{c+dx^2}\,(bc-ad)}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
```

```
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{\int \frac{a}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}\\ &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{a\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}\\ &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}\\ &= \frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.38, size = 111, normalized size = 1.50

$$\frac{x^2(bc-ad)+ac\sqrt{\frac{dx^2}{c}+1}\,\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}\,\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{x\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)*x^2 + a*c*Sqrt[(-(b/a) + d/c)*x^2]*Sqrt[1 + (d*x^2)/c]*ArcTanh
[Sqrt[(-(b/a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/((b*c - a*d)^2*x*Sqrt[c + d
*x^2])
```

IntegrateAlgebraic [A] time = 0.24, size = 126, normalized size = 1.70

$$\frac{x}{\sqrt{c+dx^2}\,(bc-ad)}+\frac{\sqrt{a}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] x/((b*c - a*d)*Sqrt[c + d*x^2]) + (Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*
c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2]
)/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2)
```

fricas [A] time = 1.22, size = 334, normalized size = 4.51

$$\left[-\frac{\left(dx^2+c\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,\sqrt{dx^2+c}\,x}{4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)},\ \frac{\left(dx^2+c\right)\sqrt{\frac{a}{bc-ad}}\,\arctan\left(-\frac{\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)+2\,\sqrt{dx^2+c}\,x}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d
^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d
 + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a
*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)
) + 2*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

giac [A] time = 0.46, size = 103, normalized size = 1.39

$$-\frac{a\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{x}{\sqrt{dx^2+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -a*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + x/(sqrt(d*x^
2 + c)*(b*c - a*d))
```

maple [B] time = 0.02, size = 653, normalized size = 8.82

$$-\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}}+\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}}-\frac{adx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}-\frac{adx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}-\frac{a}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{a}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{x}{\sqrt{d\,x^2+c}\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/b*x/c/(d*x^2+c)^(1/2)-1/2*a/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*a/b/(a*d-b*c
)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)*d*x+1/2*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x+(-a*b)^(1/2)/b))+1/2*a/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*a/b/(a*d-b*c)/c
/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2)*d*x-1/2*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-
(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.698 $\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{\sqrt{c+dx^2}\,(bc-ad)}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$\frac{1}{\sqrt{c+dx^2}\,(bc-ad)}-\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{1}{(bc-ad)\sqrt{c+dx^2}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2(bc-ad)}\\ &= \frac{1}{(bc-ad)\sqrt{c+dx^2}} + \frac{b\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{d(bc-ad)}\\ &= \frac{1}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [**C**] time = 0.02, size = 50, normalized size = 0.69

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{\sqrt{c+dx^2}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^2))/(b*c - a*d)]/((-(b*c) + a
*d)*Sqrt[c + d*x^2]))
```

IntegrateAlgebraic [**A**] time = 0.09, size = 81, normalized size = 1.12

$$\frac{1}{\sqrt{c+dx^2}\,(bc-ad)} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] 1/((b*c - a*d)*Sqrt[c + d*x^2]) + (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*
d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(-(b*c) + a*d)^(3/2)
```

fricas [A] time = 1.12, size = 323, normalized size = 4.49

$$\left[-\frac{\left(dx^2+c\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\,\sqrt{dx^2+c}}{4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)},\ \frac{\left(dx^2+c\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\,\sqrt{dx^2+c}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((d*x^2 + c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b
*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d +
 a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b
^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a
*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b
*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*sqrt(d*
x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

giac [A] time = 0.37, size = 71, normalized size = 0.99

$$\frac{b\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{1}{\sqrt{dx^2+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 1/(sqrt(d*x^2 + c)*(b*c - a*d))
```

maple [B] time = 0.01, size = 618, normalized size = 8.58

$$\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}}-\frac{\sqrt{-ab}\,dx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}+\frac{\sqrt{-ab}\,dx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}-\frac{1}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{1}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)-1/2/b*(-a*b)^(1/2)/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c
)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/(a*d-b*c)/((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1
/2/b*(-a*b)^(1/2)/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*
ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.74, size = 61, normalized size = 0.85

$$-\frac{1}{\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] - 1/((c + d*x^2)^(1/2)*(a*d - b*c)) - (b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1
/2))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

sympy [A] time = 19.96, size = 61, normalized size = 0.85

$$-\frac{1}{\sqrt{c+dx^2}\,(ad-bc)}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -1/(sqrt(c + d*x**2)*(a*d - b*c)) - atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/
b))/(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

3.699 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{bc-ad}\\
&= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}\\
&= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 2.96, size = 236, normalized size = 2.99

$$
-\frac{\frac{15c\left(3c+2dx^2\right)\left(c\left(a+bx^2\right)\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}-a\left(c+dx^2\right)\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)\right)}{\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}}+\frac{4x^4\left(c+dx^2\right)(bc-ad)^2\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}}{15c^3x\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -1/15*((15*c*(3*c + 2*d*x^2)*(c*(a + b*x^2)*Sqrt[(a*(b*c - a*d)*x^2*(c + d*
x^2))/(c^2*(a + b*x^2)^2)] - a*(c + d*x^2)*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c
*(a + b*x^2))]]))/Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)]
 + (4*(b*c - a*d)^2*x^4*(c + d*x^2)*Hypergeometric2F1[2, 2, 7/2, ((b*c - a*
d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2))/(c^3*(-(b*c) + a*d)*x*(a + b*x^2)*Sq
rt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.00, size = 133, normalized size = 1.68

$$
-\frac{dx}{c\sqrt{c+dx^2}\,(bc-ad)}-\frac{b\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - (b*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt
[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x
^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.17, size = 442, normalized size = 5.59

$$\left[-\frac{4\left(abcd - a^2d^2\right)\sqrt{dx^2+c}\,x - \left(bcdx^2 + bc^2\right)\sqrt{-abc + a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\left(ab^2c^4 - 2\,a^2bc^3d + a^3c^2d^2 + \left(ab^2c^3d - 2\,a^2bc^2d^2 + a^3cd^3\right)x^2\right)}, -\frac{2\left(abcd - a^2d^2\right)\sqrt{dx^2+c}\,x - \left(bcdx^2 + bc^2\right)\sqrt{abc - a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{2\left(ab^2c^4 - 2\,a^2bc^3d + a^3c^2d^2 + \left(ab^2c^3d - 2\,a^2bc^2d^2 + a^3cd^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

giac [A] time = 0.31, size = 107, normalized size = 1.35

$$\frac{b\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,(bc-ad)} - \frac{dx}{\left(bc^2-acd\right)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

maple [B] time = 0.01, size = 628, normalized size = 7.95

$$\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}}+\frac{b}{2\sqrt{-ab}\,\,(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{b}{2\sqrt{-ab}\,\,(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{dx}{2\,(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;c}+\frac{dx}{2\,(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/(-a*b)^(1/
2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/(-a
*b)^(1/2)/(a*d-b*c)*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/(-a*b)^(1/2)/(a*d
-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.700 $\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}\,(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 85, 156, 63, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}\,(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*
c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*
(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{2c(bc-ad)}\\
&= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2ac}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d\right.}{2a(bc-ad)}\\
&= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{acd}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{b}{}}\right.}{ad(bc}\\
&= -\frac{d}{c(bc-ad)\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 87, normalized size = 0.81

$$
\frac{(bc-ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)-bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{ac\sqrt{c+dx^2}\,(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^2))/(b*c - a*d)]) + (b*c
 - a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])/(a*c*(b*c - a*d)*Sq
rt[c + d*x^2])
```

IntegrateAlgebraic [**A**] time = 0.20, size = 118, normalized size = 1.10

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a(ad-bc)^{3/2}}-\frac{d}{c\sqrt{c+dx^2}\,(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*Sqrt[c + d*x^2])) - (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c)
 + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]]/(a*c^(3/2))
```

fricas [**B**] time = 2.02, size = 959, normalized size = 8.96

$$\left[-\frac{4\sqrt{dx^2+c}\,acd+\left(bc^2dx^2+bc^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^2\right)},\ -\frac{4\sqrt{dx^2+c}\,acd-4\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(bc^2dx^2+bc^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{4\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^2\right)},\ -\frac{2\sqrt{dx^2+c}\,acd+\left(bc^2dx^2+bc^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)-\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{2\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^2\right)},\ -\frac{2\sqrt{dx^2+c}\,acd+\left(bc^2dx^2+bc^3\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)-2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(b*c^2 -
 a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(
c) + 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/4
*(4*sqrt(d*x^2 + c)*a*c*d - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*
d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b*c^
4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2*sqrt(d*x^2 + c)*a*c
*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c
 - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (b*c^2 - a*
c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
+ 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2
*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
```

```
 b*c)) - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/s
qrt(d*x^2 + c)))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)]
```

giac [A] time = 0.36, size = 110, normalized size = 1.03

$$-\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(abc-a^2d\right)\sqrt{-b^2c+abd}}-\frac{d}{\left(bc^2-acd\right)\sqrt{dx^2+c}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sqrt(-
b^2*c + a*b*d)) - d/((b*c^2 - a*c*d)*sqrt(d*x^2 + c)) + arctan(sqrt(d*x^2 +
 c)/sqrt(-c))/(a*sqrt(-c)*c)
```

maple [B] time = 0.02, size = 681, normalized size = 6.36

$$-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{b}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}+\frac{b}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}+\frac{\sqrt{-ab}\,dx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac}-\frac{\sqrt{-ab}\,dx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac}-\frac{\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{a\,c^{\frac{3}{2}}}+\frac{1}{\sqrt{d\,x^2+c}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/a/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)+1/2/a*(-a*b)^(1/2)/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/a/(a*d
-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/2/a/(a*d-b*c
)*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)-1/2/a*(-a*b)^(1/2)/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/a/(a*d-b*c)*b/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/a/c/(d*x^2+c)^(1/2)-1/a/c^(
3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x), x)
```

mupad [B] time = 1.42, size = 2296, normalized size = 21.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] (atan((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*
a*b^7*c^7*d^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^
6 - 2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(18*a^3*b^6*c^8*d^4
 - 4*a^2*b^7*c^9*d^3 - 32*a^4*b^5*c^7*d^5 + 28*a^5*b^4*c^6*d^6 - 12*a^6*b^3
*c^5*d^7 + 2*a^7*b^2*c^4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2
)*(16*a^2*b^8*c^11*d^2 - 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^
5*b^5*c^8*d^5 + 160*a^6*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^
8))/(4*a*(a*d - b*c)^3)))/(2*a*(a*d - b*c)^3))*1i)/(a*(a*d - b*c)^3) + ((-b
^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7*c^7*d
^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 - 2*a^5*b
^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(4*a^2*b^7*c^9*d^3 - 18*a^3*b^
6*c^8*d^4 + 32*a^4*b^5*c^7*d^5 - 28*a^5*b^4*c^6*d^6 + 12*a^6*b^3*c^5*d^7 -
2*a^7*b^2*c^4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b
^8*c^11*d^2 - 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^5*b^5*c^8*d
^5 + 160*a^6*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^8))/(4*a*(a
*d - b*c)^3)))/(2*a*(a*d - b*c)^3))*1i)/(a*(a*d - b*c)^3))/(2*b^7*c^6*d^3 -
 6*a*b^6*c^5*d^4 + 6*a^2*b^5*c^4*d^5 - 2*a^3*b^4*c^3*d^6 + ((-b^3*(a*d - b*
c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7*c^7*d^3 + 26*a^2*
b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 - 2*a^5*b^3*c^3*d^7))
/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(18*a^3*b^6*c^8*d^4 - 4*a^2*b^7*c^9*d^3 -
32*a^4*b^5*c^7*d^5 + 28*a^5*b^4*c^6*d^6 - 12*a^6*b^3*c^5*d^7 + 2*a^7*b^2*c^
4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^8*c^11*d^2
- 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^5*b^5*c^8*d^5 + 160*a^6
*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^8))/(4*a*(a*d - b*c)^3)
))/(2*a*(a*d - b*c)^3)))/(a*(a*d - b*c)^3) - ((-b^3*(a*d - b*c)^3)^(1/2)*((
(c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7*c^7*d^3 + 26*a^2*b^6*c^6*d^4 -
22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 - 2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a
*d - b*c)^3)^(1/2)*(4*a^2*b^7*c^9*d^3 - 18*a^3*b^6*c^8*d^4 + 32*a^4*b^5*c^7
*d^5 - 28*a^5*b^4*c^6*d^6 + 12*a^6*b^3*c^5*d^7 - 2*a^7*b^2*c^4*d^8 + ((-b^3
*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^8*c^11*d^2 - 88*a^3*b^7*c
^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240*a^5*b^5*c^8*d^5 + 160*a^6*b^4*c^7*d^6 -
 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^5*d^8))/(4*a*(a*d - b*c)^3)))/(2*a*(a*d -
 b*c)^3)))/(a*(a*d - b*c)^3)))*(-b^3*(a*d - b*c)^3)^(1/2)*1i)/(a*(a*d - b*c
)^3) - atanh((6*b^7*c^7*d^3*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c^6*d^3
- 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b^4*c^3*
```

```
d^6 + 12*a^4*b^3*c^2*d^7)) - (24*a*b^6*c^6*d^4*(c + d*x^2)^(1/2))/((c^3)^(1
/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^
5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)) + (38*a^2*b^5*c^5*d^5*(c + d*
x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^
8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)) - (30*a^
3*b^4*c^4*d^6*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5
*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b
^3*c^2*d^7)) + (12*a^4*b^3*c^3*d^7*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(6*b^7*c
^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^5*c^4*d^5 - 30*a^3*b
^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)) - (2*a^5*b^2*c^2*d^8*(c + d*x^2)^(1/2))/(
(c^3)^(1/2)*(6*b^7*c^6*d^3 - 24*a*b^6*c^5*d^4 - 2*a^5*b^2*c*d^8 + 38*a^2*b^
5*c^4*d^5 - 30*a^3*b^4*c^3*d^6 + 12*a^4*b^3*c^2*d^7)))/(a*(c^3)^(1/2)) - d/
((c + d*x^2)^(1/2)*(b*c^2 - a*c*d))
```

sympy [A] time = 21.00, size = 94, normalized size = 0.88

$$\frac{d}{c\sqrt{c+dx^2}\,(ad-bc)}+\frac{b\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d/(c*sqrt(c + d*x**2)*(a*d - b*c)) + b*atan(sqrt(c + d*x**2)/sqrt((a*d - b*
c)/b))/(a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + atan(sqrt(c + d*x**2)/sqrt(-c)
)/(a*c*sqrt(-c))
```

3.701 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(bc-2ad)}{ac^2x(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) - ((b*c - 2*a*d)*Sqrt[c + d*x^2])/(a
*c^2*(b*c - a*d)*x) - (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}+\frac{\int \frac{bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{c(bc-ad)}\\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{\int \frac{b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{ac^2(bc-ad)}\\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{a(bc-ad)}\\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\,\mathrm{Subst}\Bigg(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,}{a(bc-ad)}\\
&= -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}}-\frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 5.21, size = 102, normalized size = 0.82

$$\frac{\frac{d^2x^2}{bc-ad}-\frac{c+dx^2}{a}}{c^2x\sqrt{c+dx^2}}-\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d^2*x^2)/(b*c - a*d) - (c + d*x^2)/a)/(c^2*x*Sqrt[c + d*x^2]) - (b^2*ArcT
an[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(3/
2))
```

IntegrateAlgebraic [A] time = 0.45, size = 164, normalized size = 1.32

$$\frac{b^2\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{3/2}(bc-ad)^{3/2}}+\frac{-acd-2ad^2x^2+bc^2+bcdx^2}{ac^2x\sqrt{c+dx^2}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c^2 - a*c*d + b*c*d*x^2 - 2*a*d^2*x^2)/(a*c^2*(-(b*c) + a*d)*x*Sqrt[c +
d*x^2]) + (b^2*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(
Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])]
)/(a^(3/2)*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.44, size = 560, normalized size = 4.52

$$\left[\frac{\left(b^2c^2dx^3+b^2c^3x\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3\right)x^2\right)\sqrt{dx^2+c}}{4\left(\left(a^2b^2c^4d-2\,a^3bc^3d^2+a^4c^2d^3\right)x^3+\left(a^2b^2c^5-2\,a^3bc^4d+a^4c^3d^2\right)x\right)},-\frac{\left(b^2c^2dx^3+b^2c^3x\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(ab^2c^2d-3\,a^2bcd^2+2\,a^3d^3\right)x^2\right)\sqrt{dx^2+c}}{2\left(\left(a^2b^2c^4d-2\,a^3bc^3d^2+a^4c^2d^3\right)x^3+\left(a^2b^2c^5-2\,a^3bc^4d+a^4c^3d^2\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*
b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c
- 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (a*b^2*c^2*d - 3
*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^4*d - 2*a^3*b*c
^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a^4*c^3*d^2)*x),
 -1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*
c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x
^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (
a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^
4*d - 2*a^3*b*c^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a
^4*c^3*d^2)*x)]
```

giac [A] time = 3.56, size = 152, normalized size = 1.23

$$-\frac{b^{2}\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\sqrt{abcd-a^{2}d^{2}}\left(abc-a^{2}d\right)}+\frac{d^{2}x}{\left(bc^{3}-ac^{2}d\right)\sqrt{dx^{2}+c}}+\frac{2\,\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b^2*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(a*b*c - a^2*d)) + d^2*x/
((b*c^3 - a*c^2*d)*sqrt(d*x^2 + c)) + 2*sqrt(d)/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^2 - c)*a*c)
```

maple [B] time = 0.02, size = 695, normalized size = 5.60

$$-\frac{b^{2}\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{b^{2}\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{b^{2}}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}+\frac{b^{2}}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}-\frac{bdx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac}-\frac{bdx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac}-\frac{2dx}{\sqrt{d\,x^{2}+c}\,a\,c^{2}}-\frac{1}{\sqrt{d\,x^{2}+c}\,acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*b/a/(a*d-b*c)/c/((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2*b
^2/a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2
)/b))+1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*b/a/(a*d-b*c)/c/((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-
1/2*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^
(1/2)/b))-1/a/c/x/(d*x^2+c)^(1/2)-2/a*d/c^2*x/(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)\left(dx^{2}+c\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b x^{2}\right) \left(c + d x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.702 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}\,(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

Rubi [A] time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}-\frac{d(bc-3ad)}{2ac^2\sqrt{c+dx^2}\,(bc-ad)}-\frac{1}{2acx^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d*(b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[c + d*x^2]) - 1/(2*a*c*x^2*Sqr
t[c + d*x^2]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c
^(5/2)) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a^2
*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{\text{Subst}\Bigg(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)-\frac{1}{4}bd(b}{x(a+bx)\sqrt{c+dx}}}{ac^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{b^3\,\text{Subst}\Bigg(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2}{2a^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{b^3\,\text{Subst}\Bigg(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+}}{a^2d(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}} -$$

Mathematica [C] time = 0.04, size = 117, normalized size = 0.75

$$\frac{2b^2c^2x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+(ad-bc)\left(x^2(3ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)+ac\right)}{2a^2c^2x^2\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b^2*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^2))/(b*c - a*d)]
 + (-(b*c) + a*d)*(a*c + (2*b*c + 3*a*d)*x^2*Hypergeometric2F1[-1/2, 1, 1/2
, 1 + (d*x^2)/c]))/(2*a^2*c^2*(b*c - a*d)*x^2*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.31, size = 162, normalized size = 1.04

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a^2(ad-bc)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}}+\frac{-acd-3ad^2x^2+bc^2+bcdx^2}{2ac^2x^2\sqrt{c+dx^2}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*c^2 - a*c*d + b*c*d*x^2 - 3*a*d^2*x^2)/(2*a*c^2*(-(b*c) + a*d)*x^2*Sqrt[
c + d*x^2]) + (b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/
(b*c - a*d)])/(a^2*(-(b*c) + a*d)^(3/2)) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/(2*a^2*c^(5/2))
```

fricas **[B]** time = 2.66, size = 1291, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 +
 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2
*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(
b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - ((2*b^2*c^2*d + a*b*c*d^2 -
3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*log(-(d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c^3 - a^2*c^2*d + (a*
b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x
^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), -1/4*(2*((2*b^2*c^2*d + a*b*c*d^2 - 3*a^
2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(a*b*c^3
 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*
d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/4*(2*(b^2*c^3*d*x^4
+ b^2*c^4*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + ((2*b^2*c^2*d + a*b*c*d
^2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*lo
g(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a*b*c^3 - a^2*c^2*d
+ (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d
^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/2*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*
sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqr
t(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)
*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/
sqrt(d*x^2 + c)) - (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sq
rt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x
^2)]
```

giac [A] time = 0.45, size = 172, normalized size = 1.10

$$\frac{b^{3}\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(a^{2}bc-a^{3}d\right)\sqrt{-b^{2}c+abd}}-\frac{\left(dx^{2}+c\right)bcd-3\left(dx^{2}+c\right)ad^{2}+2\,acd^{2}}{2\left(abc^{3}-a^{2}c^{2}d\right)\left(\left(dx^{2}+c\right)^{\frac{3}{2}}-\sqrt{dx^{2}+c}\,c\right)}-\frac{\left(2\,bc+3\,ad\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,a^{2}\sqrt{-c}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*sqrt(
-b^2*c + a*b*d)) - 1/2*((d*x^2 + c)*b*c*d - 3*(d*x^2 + c)*a*d^2 + 2*a*c*d^2
)/((a*b*c^3 - a^2*c^2*d)*((d*x^2 + c)^(3/2) - sqrt(d*x^2 + c)*c)) - 1/2*(2*
b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

maple [B] time = 0.02, size = 763, normalized size = 4.89

$$\frac{b^{2}\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}+\frac{b^{2}\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{b^{2}}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}}-\frac{b^{2}}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}}-\frac{\sqrt{-ab}\,bdx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}c}+\frac{\sqrt{-ab}\,bdx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}c}+\frac{3d\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{2a\,c^{\frac{5}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{d\,x^{2}+c}\,\sqrt{c}}{x}\right)}{a^{2}c^{\frac{3}{2}}}-\frac{3d}{2\sqrt{d\,x^{2}+c}\,a\,c^{2}}-\frac{b}{\sqrt{d\,x^{2}+c}\,a^{2}c}-\frac{1}{2\sqrt{d\,x^{2}+c}\,ac\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/a^2*b^2/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a^2*b*(-a*b)^(1/2)/(a*d-b*c)/c/((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1
/2/a^2*b^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1
/2/a/c/x^2/(d*x^2+c)^(1/2)-3/2/a*d/c^2/(d*x^2+c)^(1/2)+3/2/a*d/c^(5/2)*ln((
2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/2/a^2*b^2/(a*d-b*c)/((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/a^2*b*(-a
*b)^(1/2)/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/a^2*b^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*
ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/a^2*b/c/(d*x^2+c)^(1/2)+1/a^2*b/c^(3/2)*ln
((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)\left(dx^{2}+c\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3), x)
```

mupad [B] time = 1.98, size = 3025, normalized size = 19.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] (d^2/(b*c^2 - a*c*d) + (d*(c + d*x^2)*(3*a*d - b*c))/(2*a*c^2*(a*d - b*c)))
/(c*(c + d*x^2)^(1/2) - (c + d*x^2)^(3/2)) + (atan((((-b^5*(a*d - b*c)^3)^(
1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*b^9*c^12*d^3 + 16
*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9*d^6 - 544*a^8*b^
5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2 - ((-b^5*(a*d -
b*c)^3)^(1/2)*(416*a^8*b^6*c^12*d^5 - 32*a^6*b^8*c^14*d^3 - 1024*a^9*b^5*c^
11*d^6 + 1056*a^10*b^4*c^10*d^7 - 512*a^11*b^3*c^9*d^8 + 96*a^12*b^2*c^8*d^
9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7*b^8*c^16*d^2 - 2
816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b^5*c^13*d^5 + 512
0*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^2*c^10*d^8))/(4*a
^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3))*1i)/(a^2*(a*d - b*c)^3) + ((-b^5
*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*
b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9
*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2
 - ((-b^5*(a*d - b*c)^3)^(1/2)*(32*a^6*b^8*c^14*d^3 - 416*a^8*b^6*c^12*d^5
+ 1024*a^9*b^5*c^11*d^6 - 1056*a^10*b^4*c^10*d^7 + 512*a^11*b^3*c^9*d^8 - 9
6*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7
*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b
^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^
2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3))*1i)/(a^2*(a*d -
 b*c)^3))/(32*a^2*b^10*c^11*d^3 - 144*a^3*b^9*c^10*d^4 + 96*a^4*b^8*c^9*d^5
 + 256*a^5*b^7*c^8*d^6 - 384*a^6*b^6*c^7*d^7 + 144*a^7*b^5*c^6*d^8 - ((-b^5
*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*
b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9
*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2
 - ((-b^5*(a*d - b*c)^3)^(1/2)*(416*a^8*b^6*c^12*d^5 - 32*a^6*b^8*c^14*d^3
- 1024*a^9*b^5*c^11*d^6 + 1056*a^10*b^4*c^10*d^7 - 512*a^11*b^3*c^9*d^8 + 9
6*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7
*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b
^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^
2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3)))/(a^2*(a*d - b*
c)^3) + ((-b^5*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*
d^2 - 320*a^4*b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 1
```

```
60*a^7*b^6*c^9*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b
^3*c^6*d^9))/2 - ((-b^5*(a*d - b*c)^3)^(1/2)*(32*a^6*b^8*c^14*d^3 - 416*a^8
*b^6*c^12*d^5 + 1024*a^9*b^5*c^11*d^6 - 1056*a^10*b^4*c^10*d^7 + 512*a^11*b
^3*c^9*d^8 - 96*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^
(1/2)*(512*a^7*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4
 - 7680*a^10*b^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7
 + 256*a^13*b^2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3)))/
(a^2*(a*d - b*c)^3)))*(-b^5*(a*d - b*c)^3)^(1/2)*1i)/(a^2*(a*d - b*c)^3) +
(atanh((440*a^4*b^8*c^11*d^5*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c
^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7
+ 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^
10*b^2*c^3*d^11)) - (240*a^3*b^9*c^12*d^4*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(
440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6
*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*
d^10 + 216*a^10*b^2*c^3*d^11)) + (480*a^5*b^7*c^10*d^6*(c + d*x^2)^(1/2))/(
(c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d
^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864
*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d^11)) - (1464*a^6*b^6*c^9*d^7*(c + d*
x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*
a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*
c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d^11)) + (496*a^7*b^5*c^8
*d^8*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^1
0*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 +
936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d^11)) + (936
*a^8*b^4*c^7*d^9*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*c^9*d^5 - 240
*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7 + 496*a^7*b^
5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a^10*b^2*c^3*d
^11)) - (864*a^9*b^3*c^6*d^10*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(440*a^4*b^8*
c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a^6*b^6*c^7*d^7
 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^4*d^10 + 216*a
^10*b^2*c^3*d^11)) + (216*a^10*b^2*c^5*d^11*(c + d*x^2)^(1/2))/((c^5)^(1/2)
*(440*a^4*b^8*c^9*d^5 - 240*a^3*b^9*c^10*d^4 + 480*a^5*b^7*c^8*d^6 - 1464*a
^6*b^6*c^7*d^7 + 496*a^7*b^5*c^6*d^8 + 936*a^8*b^4*c^5*d^9 - 864*a^9*b^3*c^
4*d^10 + 216*a^10*b^2*c^3*d^11)))*(3*a*d + 2*b*c))/(2*a^2*(c^5)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^2\right) \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.703 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\,(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}\,(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\,(3bc-4ad)(2ad+bc)}{3a^2c^3x(bc-ad)}-\frac{\sqrt{c+dx^2}\,(bc-4ad)}{3ac^2x^3(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) - ((b*c - 4*a*d)*Sqrt[c + d*x^2])/
(3*a*c^2*(b*c - a*d)*x^3) + ((3*b*c - 4*a*d)*(b*c + 2*a*d)*Sqrt[c + d*x^2])
/(3*a^2*c^3*(b*c - a*d)*x) + (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} + \frac{\int \frac{bc-4ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{c(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} - \frac{\int \frac{(3bc-4ad)(bc+2ad)+2bd(bc-4ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}}{3ac^2(bc-ad)}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad)\sqrt{c+d}}{3a^2c^3(bc-ad)x}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad)\sqrt{c+d}}{3a^2c^3(bc-ad)x}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad)\sqrt{c+d}}{3a^2c^3(bc-ad)x}$$

$$= -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}} - \frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3} + \frac{(3bc-4ad)(bc+2ad)\sqrt{c+d}}{3a^2c^3(bc-ad)x}$$

Mathematica [A] time = 5.25, size = 124, normalized size = 0.70

$$\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\left(\frac{x^2(5ad+3bc)}{a^2}+\frac{3d^3x^4}{(c+dx^2)(ad-bc)}-\frac{c}{a}\right)}{3c^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(c/a) + ((3*b*c + 5*a*d)*x^2)/a^2 + (3*d^3*x^4)/((-(b*c)
 + a*d)*(c + d*x^2))))/(3*c^3*x^3) + (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

IntegrateAlgebraic [A] time = 0.55, size = 225, normalized size = 1.28

$$\frac{-a^2c^2d+4a^2cd^2x^2+8a^2d^3x^4+abc^3-abc^2dx^2-2abcd^2x^4-3b^2c^3x^2-3b^2c^2dx^4}{3a^2c^3x^3\sqrt{c+dx^2}\,(ad-bc)}-\frac{b^3\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*b*c^3 - a^2*c^2*d - 3*b^2*c^3*x^2 - a*b*c^2*d*x^2 + 4*a^2*c*d^2*x^2 - 3*
b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 + 8*a^2*d^3*x^4)/(3*a^2*c^3*(-(b*c) + a*d)*
x^3*Sqrt[c + d*x^2]) - (b^3*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*S
qrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt
[b*c - a*d])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.75, size = 706, normalized size = 4.01

$$\left[\frac{3\left(b^3c^3dx^5+b^3c^4x^3\right)\sqrt{-abc+a^2d}\,\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2+4\left((bc-2ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2-\left(3ab^3c^3d-a^2b^2c^2d^2-10a^3bcd^3+8a^4d^4\right)x^4-\left(3ab^3c^4-2a^2b^2c^3d-5a^3bc^2d^2+4a^4cd^3\right)x^2\right)\sqrt{dx^2+c}}{12\left(\left(a^3b^2c^5d-2a^4bc^4d^2+a^5c^3d^3\right)x^5+\left(a^3b^2c^6-2a^4bc^5d+a^5c^4d^2\right)x^3\right)},\frac{3\left(b^3c^3dx^5+b^3c^4x^3\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left((abcd-a^2d^2)x^3+(abc^2-a^2cd)x\right)}\right)-2\left(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2-\left(3ab^3c^3d-a^2b^2c^2d^2-10a^3bcd^3+8a^4d^4\right)x^4-\left(3ab^3c^4-2a^2b^2c^3d-5a^3bc^2d^2+4a^4cd^3\right)x^2\right)\sqrt{dx^2+c}}{6\left(\left(a^3b^2c^5d-2a^4bc^4d^2+a^5c^3d^3\right)x^5+\left(a^3b^2c^6-2a^4bc^5d+a^5c^4d^2\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(
(b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) - 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^
3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4
- 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((
a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b
*c^5*d + a^5*c^4*d^2)*x^3), 1/6*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(a*b*c
 - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x
```

```
^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*
b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2
 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a
^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b*c^5*d + a^5*c^4*d^2)*x^3)]
```

giac [A] time = 3.62, size = 275, normalized size = 1.56

$$\frac{b^{3}\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}bc-a^{3}d\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{d^{3}x}{\left(bc^{4}-ac^{3}d\right)\sqrt{dx^{2}+c}}-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}\sqrt{d}-12\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}acd^{\frac{3}{2}}+3\,bc^{3}\sqrt{d}+5\,ac^{2}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] b^3*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a*b*c*d - a^2*d^2)) - d^3*x
/((b*c^4 - a*c^3*d)*sqrt(d*x^2 + c)) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*b*c*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*
c*d^(3/2) + 3*b*c^3*sqrt(d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 +
c))^2 - c)^3*a^2*c^2)
```

maple [B] time = 0.02, size = 762, normalized size = 4.33

$$\frac{b^{3}\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{b^{3}\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}+\frac{b^{3}}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}}-\frac{b^{3}}{2\sqrt{-ab}\,\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}}+\frac{b^{2}dx}{2\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}c}+\frac{b^{2}dx}{2\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a^{2}c}+\frac{8d^{2}x}{3\sqrt{dx^{2}+c}\,ac^{3}}+\frac{2bdx}{\sqrt{dx^{2}+c}\,a^{2}c^{2}}+\frac{4d}{3\sqrt{dx^{2}+c}\,ac^{2}x}+\frac{b}{\sqrt{dx^{2}+c}\,a^{2}cx}-\frac{1}{3\sqrt{dx^{2}+c}\,acx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*b^2/a^2/(a*d-b*c)/c/((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-
1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*
b)^(1/2)/b))-1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*b^2/a^2/(a*d-b*c)
/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*d*x+1/2*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x-(-a*b)^(1/2)/b))+1/a^2*b/c/x/(d*x^2+c)^(1/2)+2/a^2*b*d/c^2*x/(d*x^2
+c)^(1/2)-1/3/a/c/x^3/(d*x^2+c)^(1/2)+4/3/a*d/c^2/x/(d*x^2+c)^(1/2)+8/3/a*d
^2/c^3*x/(d*x^2+c)^(1/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(b\,x^2 + a\right) \left(d\,x^2 + c\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^2\right) \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.704 $\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {470, 527, 12, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}+\frac{x(bc-4ad)}{3d\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(c*x)/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((b*c - 4*a*d)*x)/(3*d*(b*c -
a*d)^2*Sqrt[c + d*x^2]) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt
[c + d*x^2])])/(b*c - a*d)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{cx}{3d(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{\int \frac{ac+(bc-3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3d(bc-ad)}$$

$$= -\frac{cx}{3d(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{\int \frac{3a^2cd}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3cd(bc-ad)^2}$$

$$= -\frac{cx}{3d(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^2\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{cx}{3d(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^2\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x\right.}{(bc-ad)^2}$$

$$= -\frac{cx}{3d(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [A] time = 0.29, size = 160, normalized size = 1.37

$$\frac{x^2\left(a^2d\left(3c+4dx^2\right)-abc\left(3c+5dx^2\right)+b^2c^2x^2\right)-\frac{3a^2\left(c+dx^2\right)^2\sqrt{\frac{x^2(ad-bc)}{ac}}\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{dx^2}{c}+1}}}{3x\left(c+dx^2\right)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^2*(b^2*c^2*x^2 + a^2*d*(3*c + 4*d*x^2) - a*b*c*(3*c + 5*d*x^2)) - (3*a^2
*Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]*(c + d*x^2)^2*ArcTanh[Sqrt[(-(b/a) + d/c)
*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[1 + (d*x^2)/c])/(3*(b*c - a*d)^3*x*(c + d*
x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.44, size = 148, normalized size = 1.26

$$\frac{-3acx-4adx^3+bcx^3}{3\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{a^{3/2}\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*a*c*x + b*c*x^3 - 4*a*d*x^3)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (a^(
3/2)*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sq
rt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a
*d)^(5/2)
```

fricas [B] time = 1.94, size = 524, normalized size = 4.48

$$\left[\frac{3\left(ad^2x^4+2\,acdx^2+ac^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left((bc-4\,ad)x^3-3\,acx\right)\sqrt{dx^2+c}}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)},-\frac{3\left(ad^2x^4+2\,acdx^2+ac^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)-2\left((bc-4\,ad)x^3-3\,acx\right)\sqrt{dx^2+c}}{6\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x
^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((b*c - 4*a*
```

```
d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a
^2*c*d^3)*x^2), -1/6*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(a/(b*c - a*d
))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d)
)/(a*d*x^3 + a*c*x)) - 2*((b*c - 4*a*d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

giac [B] time = 0.45, size = 304, normalized size = 2.60

$$-\frac{a^{2}\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{\left(\frac{\left(b^{3}c^{4}d-6\,ab^{2}c^{3}d^{2}+9\,a^{2}bc^{2}d^{3}-4\,a^{3}cd^{4}\right)x^{2}}{b^{4}c^{5}d-4\,ab^{3}c^{4}d^{2}+6\,a^{2}b^{2}c^{3}d^{3}-4\,a^{3}bc^{2}d^{4}+a^{4}cd^{5}}-\frac{3\left(ab^{2}c^{4}d-2\,a^{2}bc^{3}d^{2}+a^{3}c^{2}d^{3}\right)}{b^{4}c^{5}d-4\,ab^{3}c^{4}d^{2}+6\,a^{2}b^{2}c^{3}d^{3}-4\,a^{3}bc^{2}d^{4}+a^{4}cd^{5}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2
*d^2)) + 1/3*((b^3*c^4*d - 6*a*b^2*c^3*d^2 + 9*a^2*b*c^2*d^3 - 4*a^3*c*d^4)
*x^2/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a
^4*c*d^5) - 3*(a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)/(b^4*c^5*d - 4*
a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2
+ c)^(3/2)
```

maple [B] time = 0.03, size = 1207, normalized size = 10.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/b/d*x/(d*x^2+c)^(3/2)+1/3/b/c/d*x/(d*x^2+c)^(1/2)-1/3/b^2*a*x/c/(d*x^2
+c)^(3/2)-2/3/b^2*a/c^2*x/(d*x^2+c)^(1/2)+1/6/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(3/2)+1/6/b^2*a^2*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/3/b^2*a^2*d/(a*d-b*c)/c^2/((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
-1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/b*a^2/(a*d-b*c)^2/c/((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1
/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
```

```
(1/2)/b))-1/6/b*a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6/b^2*a^2*d/(a*d-b*c)/c/
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(3/2)*x+1/3/b^2*a^2*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^
2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)-1/2/b*a^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2*a^2/(-a*b)^(1/2)/(a*d-b*c)^2/
(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.705 $\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{a}{\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{c}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$-\frac{a}{\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{c}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -c/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - a/((b*c - a*d)^2*Sqrt[c + d*x^2])
+ (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d
)^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{c}{3d(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2(bc-ad)}$$

$$= -\frac{c}{3d(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\operatorname{Subst}\Big(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,}{2(bc-ad)^2}$$

$$= -\frac{c}{3d(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\operatorname{Subst}\Bigg(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,}{d(bc-ad)^2}$$

$$= -\frac{c}{3d(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.75

$$\frac{c(ad-bc)-3ad\left(c+dx^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{3d\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (c*(-(b*c) + a*d) - 3*a*d*(c + d*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c
 + d*x^2))/(b*c - a*d)])/(3*d*(b*c - a*d)^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 109, normalized size = 1.06

$$\frac{-2acd-3ad^2x^2-bc^2}{3d\left(c+dx^2\right)^{3/2}(ad-bc)^2}+\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c^2) - 2*a*c*d - 3*a*d^2*x^2)/(3*d*(-(b*c) + a*d)^2*(c + d*x^2)^(3/2))
 + (a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*
d)])/(-(b*c) + a*d)^(5/2)
```

fricas [B] time = 1.09, size = 535, normalized size = 5.19

$$\left[\frac{3\left(ad^3x^4+2\,acd^2x^2+ac^2d\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(3\,ad^2x^2+bc^2+2\,acd\right)\sqrt{dx^2+c}}{12\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^4+2\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)},-\frac{3\left(ad^3x^4+2\,acd^2x^2+ac^2d\right)\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\left(3\,ad^2x^2+bc^2+2\,acd\right)\sqrt{dx^2+c}}{6\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^4+2\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(b/(b*c - a*d))*log((b^2
*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
+ 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2
+ c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*d^2*x^2 + b
*c^2 + 2*a*c*d)*sqrt(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 +
 (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3
 + a^2*c*d^4)*x^2), -1/6*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(-b/(
b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c
 - a*d))/(b*d*x^2 + b*c)) + 2*(3*a*d^2*x^2 + b*c^2 + 2*a*c*d)*sqrt(d*x^2 +
```

```
c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

giac [**A**] time = 0.32, size = 127, normalized size = 1.23

$$-\frac{\frac{3\,abd\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{bc^2+3\left(dx^2+c\right)ad-acd}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (b*c^2 + 3*(d*x^2 + c)*a*d - a*c
*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2)))/d
```

maple [**B**] time = 0.02, size = 1123, normalized size = 10.90

$$\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)^2\sqrt{-\frac{ad-bc}{b}}}+\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)^2\sqrt{-\frac{ad-bc}{b}}}-\frac{\sqrt{-ab}\,adx}{2\left(ad-bc\right)^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}+\frac{\sqrt{-ab}\,adx}{2\left(ad-bc\right)^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}-\frac{a}{2\left(ad-bc\right)^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{a}{2\left(ad-bc\right)^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{a}{6\left(ad-bc\right)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}b}+\frac{a}{6\left(ad-bc\right)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}b}+\frac{\sqrt{-ab}\,adx}{6\left(ad-bc\right)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}b^2c}-\frac{\sqrt{-ab}\,adx}{6\left(ad-bc\right)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}b^2c}+\frac{\sqrt{-ab}\,adx}{3\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^2c^2}-\frac{\sqrt{-ab}\,adx}{3\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^2c^2}-\frac{1}{3\left(dx^2+c\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3/b/d/(d*x^2+c)^(3/2)+1/6*a/b/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6*a/b^2*(-a*b)^(1/2)*d/(
a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(3/2)*x+1/3*a/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2*a/(a*
d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)-1/2*a/b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2*a/(a*d-b*c)
^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/6*a/b/(a*d-b*c)/(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
3/2)-1/6*a/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/3*a/b^2*(-a*b)^(1/2)*d/
(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*x-1/2*a/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*a/b/(a*d-b*c)^2*(-a*b)^(1
/2)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*d*x+1/2*a/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)
```

```
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)
^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.93, size = 110, normalized size = 1.07

$$\frac{\frac{c}{3\,(a\,d-b\,c)}-\frac{a\,d\,\left(d\,x^2+c\right)}{(a\,d-b\,c)^2}}{d\,\left(d\,x^2+c\right)^{3/2}}-\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{(a\,d-b\,c)^{5/2}}\right)}{(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (c/(3*(a*d - b*c)) - (a*d*(c + d*x^2))/(a*d - b*c)^2)/(d*(c + d*x^2)^(3/2))
 - (a*b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*
d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.706 $\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{x}{3\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}\,b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Rubi **[A]** time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {471, 527, 12, 377, 205}

$$\frac{x(ad+2bc)}{3c\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{x}{3\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}\,b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*b*c + a*d)*x)/(3*c*(b*c - a*d)^2*
Sqrt[c + d*x^2]) - (Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(b*c - a*d)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{a-2bx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3(bc-ad)}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\int \frac{3abc}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{3c(bc-ad)^2}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{(ab)\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x,\right.}{(bc-ad)^2}$$

$$= \frac{x}{3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 3.08, size = 257, normalized size = 2.23

$$\frac{12x^6\left(c+dx^2\right)(bc-ad)^3\,{}_2F_1\left(2,2;\frac{9}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)-\frac{35c\left(a+bx^2\right)\left(5c+2dx^2\right)\left(-3a^2\left(c+dx^2\right)^2\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)-c\left(a+bx^2\right)\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}\left(-3ac-4adx^2+bcx^2\right)\right)}{\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}}}{315c^3x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((-35*c*(a + b*x^2)*(5*c + 2*d*x^2)*(-(c*(a + b*x^2)*Sqrt[(a*(b*c - a*d)*x^
2*(c + d*x^2))/(c^2*(a + b*x^2)^2)]*(-3*a*c + b*c*x^2 - 4*a*d*x^2)) - 3*a^2
*(c + d*x^2)^2*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]]))/Sqrt[(a*(b
*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2)] + 12*(b*c - a*d)^3*x^6*(c +
 d*x^2)*Hypergeometric2F1[2, 2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(3
15*c^3*(b*c - a*d)^2*x*(a + b*x^2)^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.41, size = 156, normalized size = 1.36

$$\frac{ad^2x^3+3bc^2x+2bcdx^3}{3c\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{\sqrt{a}\,b\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*b*c^2*x + 2*b*c*d*x^3 + a*d^2*x^3)/(3*c*(b*c - a*d)^2*(c + d*x^2)^(3/2))
 + (Sqrt[a]*b*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(S
qrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])
/(b*c - a*d)^(5/2)
```

fricas [B] time = 1.79, size = 550, normalized size = 4.78

$$\left[\frac{3\left(bcd^2x^4+2\,bc^2dx^2+bc^3\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(3\,bc^2x+\left(2\,bcd+ad^2\right)x^3\right)\sqrt{dx^2+c}}{12\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^4+2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x^2\right)},\frac{3\left(bcd^2x^4+2\,bc^2dx^2+bc^3\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)+2\left(3\,bc^2x+\left(2\,bcd+ad^2\right)x^3\right)\sqrt{dx^2+c}}{6\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^4+2\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3)*sqrt(-a/(b*c - a*d))*log(((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x
^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt
(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*c^2
```

```
*x + (2*b*c*d + a*d^2)*x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c
^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a
*b*c^3*d^2 + a^2*c^2*d^3)*x^2), 1/6*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3
)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)
*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*(3*b*c^2*x + (2*b*c*d + a*d^2)*
x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 -
 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^
3)*x^2)]
```

giac [**B**] time = 0.45, size = 291, normalized size = 2.53

$$\frac{ab\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{\left(\frac{\left(2\,b^3c^3d^2-3\,ab^2c^2d^3+a^3d^5\right)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}+\frac{3\left(b^3c^4d-2\,ab^2c^3d^2+a^2bc^2d^3\right)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] a*b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2*
d^2)) + 1/3*((2*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + a^3*d^5)*x^2/(b^4*c^5*d - 4
*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5) + 3*(b^3*
c^4*d - 2*a*b^2*c^3*d^2 + a^2*b*c^2*d^3)/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a
^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2 + c)^(3/2)
```

maple [**B**] time = 0.02, size = 1134, normalized size = 9.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3/b*x/c/(d*x^2+c)^(3/2)+2/3/b/c^2*x/(d*x^2+c)^(1/2)-1/6*a/(-a*b)^(1/2)/(a
*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(3/2)-1/6*a/b*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/3*a/b*d/(a*d-b*c)/c^2/((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
+1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*a/(a*d-b*c)^2/c/((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2*a
/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2
```

```
)/b))+1/6*a/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/6*a/b*d/(a*d-b*c)/c/((x-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/3*
a/b*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*a/(a
*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)*d*x+1/2*a/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)
*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(
1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.707 $\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{1}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Rubi [**A**] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{b}{\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{1}{3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] 1/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + b/((b*c - a*d)^2*Sqrt[c + d*x^2]) - (
b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right) \\
&= \frac{1}{3(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2(bc-ad)} \\
&= \frac{1}{3(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{b}{(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\operatorname{Subst}\Big(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2}{2(bc-ad)^2} \\
&= \frac{1}{3(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{b}{(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\operatorname{Subst}\Bigg(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+}}{d(bc-ad)^2} \\
&= \frac{1}{3(bc-ad)\left(c+dx^2\right)^{3/2}}+\frac{b}{(bc-ad)^2\sqrt{c+dx^2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.53

$$
\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{3\left(c+dx^2\right)^{3/2}(bc-ad)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)]/(3*(b*c - a*d
)*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 101, normalized size = 1.03

$$\frac{-ad+4bc+3bdx^2}{3\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (4*b*c - a*d + 3*b*d*x^2)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (b^(3/2)*Ar
cTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(-(b*c) + a
*d)^(5/2)
```

fricas [B] time = 1.15, size = 511, normalized size = 5.21

$$\left[\frac{3\left(bd^2x^4+2\,bcdx^2+bc^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(3\,bdx^2+4\,bc-ad\right)\sqrt{dx^2+c}}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)},\ \frac{3\left(bd^2x^4+2\,bcdx^2+bc^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\left(3\,bdx^2+4\,bc-ad\right)\sqrt{dx^2+c}}{6\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^4+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*
(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)
*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*d*x^2 + 4*b*c -
 a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^
2), 1/6*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) + 2*(3*b*d*x^2 + 4*b*c - a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d -
2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

giac [A] time = 0.35, size = 118, normalized size = 1.20

$$\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\left(dx^2+c\right)b+bc-ad}{3\left(b^2c^2-2\,abcd+a^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(3*(d*x^2 + c)*b + b*c - a*d)/((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2))
```

maple [B] time = 0.01, size = 1086, normalized size = 11.08

$$-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)^{2}\sqrt{-\frac{ad-bc}{b}}}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\left(ad-bc\right)^{2}\sqrt{-\frac{ad-bc}{b}}}+\frac{b}{2\left(ad-bc\right)^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{b}{2\left(ad-bc\right)^{2}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{\sqrt{-ab}\,dx}{2\left(ad-bc\right)^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c}-\frac{\sqrt{-ab}\,dx}{2\left(ad-bc\right)^{2}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c}-\frac{\sqrt{-ab}\,dx}{6\left(ad-bc\right)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}bc}+\frac{\sqrt{-ab}\,dx}{6\left(ad-bc\right)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}bc}-\frac{\sqrt{-ab}\,dx}{3\left(ad-bc\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b\,c^{2}}+\frac{\sqrt{-ab}\,dx}{3\left(ad-bc\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b\,c^{2}}-\frac{1}{6\left(ad-bc\right)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}}-\frac{1}{6\left(ad-bc\right)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(3/2)-1/6/b*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/3/b*(-a*b)^
(1/2)*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/2*b/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)^2*(-a*b
)^(1/2)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)*d*x-1/2*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(
-a*b)^(1/2)/b))-1/6/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6/b*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*
x+1/3/b*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/2*b/(a*d-b*c)^2/((x-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/(a
*d-b*c)^2*(-a*b)^(1/2)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*l
n((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.85, size = 103, normalized size = 1.05

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{1}{3\,\left(a\,d-b\,c\right)}-\frac{b\,\left(d\,x^2+c\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(d\,x^2+c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(
a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (1/(3*(a*d - b*c)) - (b*(c + d*x^2))
/(a*d - b*c)^2)/(c + d*x^2)^(3/2)
```

sympy [A] time = 25.05, size = 85, normalized size = 0.87

$$\frac{b}{\sqrt{c+dx^2}\,(ad-bc)^2}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{1}{3\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] b/(sqrt(c + d*x**2)*(a*d - b*c)**2) + b*atan(sqrt(c + d*x**2)/sqrt((a*d - b
*c)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 1/(3*(c + d*x**2)**(3/2)*(a*
d - b*c))
```

3.708 $\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}-\frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{dx}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Rubi **[A]** time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}-\frac{dx(5bc-2ad)}{3c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{dx}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(5*b*c - 2*a*d)*x)/(3*c^2*(
b*c - a*d)^2*Sqrt[c + d*x^2]) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
rt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}+\frac{\int \frac{3bc-2ad-2bdx^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{3c(bc-ad)}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{\int \frac{3b^2c^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{3c^2(bc-ad)^2}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\,\text{Subst}\Big(\int \frac{1}{a-(-bc+ad)x^2}\,dx,}{(bc-ad)^2}\\
&= -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}}-\frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [C] time = 6.26, size = 1385, normalized size = 11.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(-1575*Sqrt[(a*(b*c - a*d)*x^2*(c
+ d*x^2))/(c^2*(a + b*x^2)^2)] - (2100*d*x^2*Sqrt[(a*(b*c - a*d)*x^2*(c + d
*x^2))/(c^2*(a + b*x^2)^2)])/c - (840*d^2*x^4*Sqrt[(a*(b*c - a*d)*x^2*(c +
d*x^2))/(c^2*(a + b*x^2)^2)])/c^2 + 2100*(((b*c - a*d)*x^2)/(c*(a + b*x^2))
)^(3/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))] + (2800*d*x^2*(((b*c - a*d)*x
^2)/(c*(a + b*x^2)))^(3/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])/c + (1120
*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2)*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))])/c^2 + 1575*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] +
 (2100*d*x^2*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/c + (840*d^2*
x^4*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/c^2 + (1575*(b*c - a*d
)^2*x^4*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)^2
) + (2100*d*(b*c - a*d)^2*x^6*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))
]])/(c^3*(a + b*x^2)^2) + (840*d^2*(b*c - a*d)^2*x^8*ArcSin[Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)^2) - (3150*(b*c - a*d)*x^2*ArcS
in[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (4200*d*(-(b
*c) + a*d)*x^4*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b
*x^2)) + (1680*d^2*(-(b*c) + a*d)*x^6*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a +
 b*x^2))]])/(c^3*(a + b*x^2)) + 96*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2
)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2, 2, 9/2, ((b*c
- a*d)*x^2)/(c*(a + b*x^2))] + (168*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)
))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2, 2, 9/2,
 ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/c + (72*d^2*x^4*(((b*c - a*d)*x^2)/(c*
(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1
[2, 2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/c^2 + 24*(((b*c - a*d)*x^2)
/(c*(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometri
cPFQ[{2, 2, 2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + (48*d*x^2*((
(b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2)
)]*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))
])/c + (24*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((b*c - a*d)
*x^2)/(c*(a + b*x^2))])/c^2))/(315*a*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5
/2)*(c + d*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.44, size = 171, normalized size = 1.40

$$\frac{3acd^2x+2ad^3x^3-6bc^2dx-5bcd^2x^3}{3c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{b^2\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{\sqrt{a}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-6*b*c^2*d*x + 3*a*c*d^2*x - 5*b*c*d^2*x^3 + 2*a*d^3*x^3)/(3*c^2*(b*c - a*
d)^2*(c + d*x^2)^(3/2)) - (b^2*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (
b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*S
qrt[b*c - a*d])]])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

fricas [**B**]   time = 1.61, size = 764, normalized size = 6.26

$$\left[-\frac{3\left(b^{2}c^{2}d^{2}x^{4}+2\,b^{2}c^{3}dx^{2}+b^{2}c^{4}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left((bc-2\,ad)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(\left(5\,ab^{2}c^{2}d^{2}-7\,a^{2}bcd^{3}+2\,a^{3}d^{4}\right)x^{3}+3\left(2\,ab^{2}c^{3}d-3\,a^{2}bc^{2}d^{2}+a^{3}cd^{3}\right)x\right)\sqrt{dx^{2}+c}}{12\left(ab^{3}c^{7}-3\,a^{2}b^{2}c^{6}d+3\,a^{3}bc^{5}d^{2}-a^{4}c^{4}d^{3}+\left(ab^{3}c^{5}d^{2}-3\,a^{2}b^{2}c^{4}d^{3}+3\,a^{3}bc^{3}d^{4}-a^{4}c^{2}d^{5}\right)x^{4}+2\left(ab^{3}c^{6}d-3\,a^{2}b^{2}c^{5}d^{2}+3\,a^{3}bc^{4}d^{3}-a^{4}c^{3}d^{4}\right)x^{2}\right)},\ \frac{3\left(b^{2}c^{2}d^{2}x^{4}+2\,b^{2}c^{3}dx^{2}+b^{2}c^{4}\right)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\sqrt{abc-a^{2}d}\left((bc-2\,ad)x^{2}-ac\right)\sqrt{dx^{2}+c}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)-2\left(\left(5\,ab^{2}c^{2}d^{2}-7\,a^{2}bcd^{3}+2\,a^{3}d^{4}\right)x^{3}+3\left(2\,ab^{2}c^{3}d-3\,a^{2}bc^{2}d^{2}+a^{3}cd^{3}\right)x\right)\sqrt{dx^{2}+c}}{6\left(ab^{3}c^{7}-3\,a^{2}b^{2}c^{6}d+3\,a^{3}bc^{5}d^{2}-a^{4}c^{4}d^{3}+\left(ab^{3}c^{5}d^{2}-3\,a^{2}b^{2}c^{4}d^{3}+3\,a^{3}bc^{3}d^{4}-a^{4}c^{2}d^{5}\right)x^{4}+2\left(ab^{3}c^{6}d-3\,a^{2}b^{2}c^{5}d^{2}+3\,a^{3}bc^{4}d^{3}-a^{4}c^{3}d^{4}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((5*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 +
 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x)*sqrt(d
*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4*d^3 + (
a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*x^4 + 2*
(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x^2), 1/6
*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c
*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((5*a*b^2*c^2*d^2 - 7*a^2*b
*c*d^3 + 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x
)*sqrt(d*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4
*d^3 + (a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*
x^4 + 2*(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x
^2)]
```

giac [**B**]   time = 0.51, size = 321, normalized size = 2.63

$$-\frac{b^{2}\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\frac{\left(5\,b^{3}c^{3}d^{3}-12\,ab^{2}c^{2}d^{4}+9\,a^{2}bcd^{5}-2\,a^{3}d^{6}\right)x^{2}}{b^{4}c^{6}d-4\,ab^{3}c^{5}d^{2}+6\,a^{2}b^{2}c^{4}d^{3}-4\,a^{3}bc^{3}d^{4}+a^{4}c^{2}d^{5}}+\frac{3\left(2\,b^{3}c^{4}d^{2}-5\,ab^{2}c^{3}d^{3}+4\,a^{2}bc^{2}d^{4}-a^{3}cd^{5}\right)}{b^{4}c^{6}d-4\,ab^{3}c^{5}d^{2}+6\,a^{2}b^{2}c^{4}d^{3}-4\,a^{3}bc^{3}d^{4}+a^{4}c^{2}d^{5}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^2
*d^2)) - 1/3*((5*b^3*c^3*d^3 - 12*a*b^2*c^2*d^4 + 9*a^2*b*c*d^5 - 2*a^3*d^6
)*x^2/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 +
a^4*c^2*d^5) + 3*(2*b^3*c^4*d^2 - 5*a*b^2*c^3*d^3 + 4*a^2*b*c^2*d^4 - a^3*c
```

```
*d^5)/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^4 +
a^4*c^2*d^5))*x/(d*x^2 + c)^(3/2)
```

maple [B] time = 0.01, size = 1086, normalized size = 8.90

$$-\frac{b^{2} \ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{-\frac{ad-bc}{b}}}+\frac{b^{2} \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{-\frac{ad-bc}{b}}}-\frac{b^{2}}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}+\frac{b^{2}}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{bdx}{2(ad-bc)^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c}-\frac{bdx}{2(ad-bc)^{2}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c}+\frac{b}{6\sqrt{-ab}\,(ad-bc)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}}-\frac{b}{6\sqrt{-ab}\,(ad-bc)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}}+\frac{dx}{6(ad-bc)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}c}+\frac{dx}{6(ad-bc)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}c}+\frac{dx}{3(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c^{2}}+\frac{dx}{3(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/6/(-a*b)^(1/2)/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/3*d/(a*d-b*c
)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*x-1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*b/(a*d-b*c)^2/c/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*d*x+1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x+(-a*b)^(1/2)/b))-1/6/(-a*b)^(1/2)/(a*d-b*c)*b/((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6*d/(a*d-b*c)/c/((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/
2)*x+1/3*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2
*b/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/(-a*b)^(1/2)*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.709 $\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Rubi [A] time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 85, 152, 156, 63, 208}

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}-\frac{d(2bc-ad)}{c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d))/(c^2*(b*c - a*d)
^2*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*c^(5/2)) + (b^(5/
2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*(b*c - a*d)^(5/2)
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\text{Subst}\Big(\int \frac{-\frac{1}{2}(bc-ad)^2+\frac{1}{2}bd(2bc}{x(a+bx)\sqrt{c+dx}}}{c^2(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2ac^2}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{\text{Subst}\Big(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c}}{ac^2d}$$

$$= -\frac{d}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}} + \frac{b^{5/2}\,\text{ta}}{a}$$

Mathematica [C] time = 0.03, size = 90, normalized size = 0.62

$$\frac{(bc-ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)-bc\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{3ac\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(b*c*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)]) + (b*
c - a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c])/(3*a*c*(b*c - a*d
)*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.33, size = 152, normalized size = 1.05

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a(ad-bc)^{5/2}}+\frac{4acd^2+3ad^3x^2-7bc^2d-6bcd^2x^2}{3c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-7*b*c^2*d + 4*a*c*d^2 - 6*b*c*d^2*x^2 + 3*a*d^3*x^2)/(3*c^2*(b*c - a*d)^2
*(c + d*x^2)^(3/2)) + (b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c +
d*x^2])/(b*c - a*d)])/(a*(-(b*c) + a*d)^(5/2)) - ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]]/(a*c^(5/2))
```

fricas [B] time = 3.20, size = 1711, normalized size = 11.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqr
t(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*(b^2*c^4
 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 +
2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b
*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^
3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*
c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), 1/12*(12*(b^2*c^4 - 2*a*b*c^3*
d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d
- 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + 3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))*log(
(b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*
x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*
x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(7*a*b*c^3*d
 - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b
^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a
^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), -1/
6*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(-b/(b*c - a*d))*arc
tan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x
^2 + b*c)) - 3*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(
c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(7*a*b*c^3*d - 4
*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c
^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c
^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), -1/6*(3
*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 6*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^
```

```
3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*
arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*
b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a
^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2
*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2)]
```

giac [A] time = 0.34, size = 176, normalized size = 1.21

$$-\frac{b^3\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{-b^2c+abd}}-\frac{6\left(dx^2+c\right)bcd+bc^2d-3\left(dx^2+c\right)ad^2-acd^2}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b^2*c^2 - 2*a^2*b*c
*d + a^3*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(6*(d*x^2 + c)*b*c*d + b*c^2*d -
3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^
2 + c)^(3/2)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
```

maple [B] time = 0.02, size = 1186, normalized size = 8.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/6/a/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(3/2)+1/6/a*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/3/a*(-a*
b)^(1/2)*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2/a*b^2/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a*b/(a*d-b
*c)^2*(-a*b)^(1/2)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/6/a/(a*d-b*c)*b/((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/6/a*(-a*b)^(1/2)*d
/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(3/2)*x-1/3/a*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2/a*b^2/(
a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)+1/2/a*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-(-a*b)^(1/2)/b)^2*d
```

```
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/a*b^2/(a*
d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/3/a/c/(d*x^2
+c)^(3/2)+1/a/c^2/(d*x^2+c)^(1/2)-1/a/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(
1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x), x)
```

mupad [B] time = 2.48, size = 4558, normalized size = 31.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (atan((((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 3
2*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4
*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c
^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/
2 + ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4
 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 86
8*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^1
0*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 - ((-b^5*(a*d
- b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20
*d^3 + 800*a^4*b^11*c^19*d^4 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d
^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9
+ 1680*a^10*b^5*c^13*d^10 - 520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12
- 8*a^13*b^2*c^10*d^13))/(4*a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5))*1i)/(a*
(a*d - b*c)^5) + ((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^
16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5
 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240
*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c
^6*d^12))/2 - ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^1
1*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^1
4*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10
 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 + ((
```

```
-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3
*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*
b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6
*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 - 520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*
c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5
))*1i)/(a*(a*d - b*c)^5))/(4*b^12*c^13*d^3 - 26*a*b^11*c^12*d^4 + 72*a^2*b^
10*c^11*d^5 - 110*a^3*b^9*c^10*d^6 + 100*a^4*b^8*c^9*d^7 - 54*a^5*b^7*c^8*d
^8 + 16*a^6*b^6*c^7*d^9 - 2*a^7*b^5*c^6*d^10 + ((-b^5*(a*d - b*c)^5)^(1/2)*
(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^
14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^
7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a
^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/2 + ((-b^5*(a*d - b*c)^5)^(1/2)*(6*
a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*
b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^
12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d
^12 + 2*a^12*b^2*c^8*d^13 - ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(
16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4 - 2280
*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a
^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 - 520*a^1
1*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*a*(a*d
 - b*c)^5)))/(2*a*(a*d - b*c)^5)))/(a*(a*d - b*c)^5) - ((-b^5*(a*d - b*c)^5)
^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*
b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*
c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10
 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^12))/2 - ((-b^5*(a*d - b*c)^5)^(1
/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 5
20*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8
*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 + 110*a^10*b^4*c^10*d^11 - 22*a^11*b^
3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 + ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^
(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4
 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 +
 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 -
520*a^11*b^4*c^12*d^11 + 96*a^12*b^3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*
a*(a*d - b*c)^5)))/(2*a*(a*d - b*c)^5)))/(a*(a*d - b*c)^5)))*(-b^5*(a*d - b
*c)^5)^(1/2)*1i)/(a*(a*d - b*c)^5) - atanh((10*b^12*c^15*d^3*(c + d*x^2)^(1
/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^1
1*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*
b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) + (290*a^2*b^10*c^13*d^5*(c + d*x^2)^(
1/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^
11*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
 660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9
*b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) - (630*a^3*b^9*c^12*d^6*(c + d*x^2)^(
1/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^
11*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
```

```
 660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9
*b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) + (912*a^4*b^8*c^11*d^7*(c + d*x^2)^(
1/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^
11*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
 660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9
*b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) - (922*a^5*b^7*c^10*d^8*(c + d*x^2)^(
1/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^
11*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
 660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9
*b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) + (660*a^6*b^6*c^9*d^9*(c + d*x^2)^(1
/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^1
1*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*
b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) - (330*a^7*b^5*c^8*d^10*(c + d*x^2)^(1
/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^1
1*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*
b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) + (110*a^8*b^4*c^7*d^11*(c + d*x^2)^(1
/2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^1
1*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 +
660*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*
b^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) - (22*a^9*b^3*c^6*d^12*(c + d*x^2)^(1/
2))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^11
*d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 + 6
60*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*b
^3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) + (2*a^10*b^2*c^5*d^13*(c + d*x^2)^(1/2
))/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^11*
d^5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 + 66
0*a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*b^
3*c^4*d^12 + 2*a^10*b^2*c^3*d^13)) - (80*a*b^11*c^14*d^4*(c + d*x^2)^(1/2))
/((c^5)^(1/2)*(10*b^12*c^13*d^3 - 80*a*b^11*c^12*d^4 + 290*a^2*b^10*c^11*d^
5 - 630*a^3*b^9*c^10*d^6 + 912*a^4*b^8*c^9*d^7 - 922*a^5*b^7*c^8*d^8 + 660*
a^6*b^6*c^7*d^9 - 330*a^7*b^5*c^6*d^10 + 110*a^8*b^4*c^5*d^11 - 22*a^9*b^3*
c^4*d^12 + 2*a^10*b^2*c^3*d^13)))/(a*(c^5)^(1/2)) - (d/(3*(b*c^2 - a*c*d))
- (d*(c + d*x^2)*(a*d - 2*b*c))/(b*c^2 - a*c*d)^2)/(c + d*x^2)^(3/2)
```

sympy [A] time = 26.34, size = 133, normalized size = 0.92

$$\frac{d}{3c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d\,(ad-2bc)}{c^2\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{b^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{ac^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d/(3*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + d*(a*d - 2*b*c)/(c**2*sqrt(c + d*
x**2)*(a*d - b*c)**2) - b**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(a*
sqrt((a*d - b*c)/b)*(a*d - b*c)**2) + atan(sqrt(c + d*x**2)/sqrt(-c))/(a*c*
*2*sqrt(-c))
```

3.710 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\,(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.24, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {472, 579, 583, 12, 377, 205}

$$-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\,(bc-4ad)(3bc-2ad)}{3ac^3x(bc-ad)^2}-\frac{d(7bc-4ad)}{3c^2x\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) - (d*(7*b*c - 4*a*d))/(3*c^2*(b*c
- a*d)^2*x*Sqrt[c + d*x^2]) - ((b*c - 4*a*d)*(3*b*c - 2*a*d)*Sqrt[c + d*x^2
])/(3*a*c^3*(b*c - a*d)^2*x) - (b^3*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqr
t[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{3bc-4ad-4bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} + \frac{\int \frac{(bc-4ad)(3bc-2ad)-2bd(7b}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}}{3c^2(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-2ad)}{3ac^3(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-2ad)}{3ac^3(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-2ad)}{3ac^3(bc-ad)^2}$$

$$= -\frac{d}{3c(bc-ad)x\left(c+dx^2\right)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-2ad)}{3ac^3(bc-ad)^2}$$

Mathematica [A] time = 5.27, size = 143, normalized size = 0.80

$$\frac{\sqrt{c+dx^2}\left(\frac{d^2x^2(8bc-5ad)}{\left(c+dx^2\right)(bc-ad)^2}+\frac{cd^2x^2}{\left(c+dx^2\right)^2(bc-ad)}-\frac{3}{a}\right)}{3c^3x}-\frac{b^3\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3/a + (c*d^2*x^2)/((b*c - a*d)*(c + d*x^2)^2) + (d^2*(8*
b*c - 5*a*d)*x^2)/((b*c - a*d)^2*(c + d*x^2))))/(3*c^3*x) - (b^3*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

IntegrateAlgebraic [A] time = 0.61, size = 241, normalized size = 1.35

$$\frac{b^3\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{3/2}(bc-ad)^{5/2}}+\frac{-3a^2c^2d^2-12a^2cd^3x^2-8a^2d^4x^4+6abc^3d+21abc^2d^2x^2+14abcd^3x^4-3b^2c^4-6b^2c^3dx^2-3b^2c^2d^2x^4}{3ac^3x\left(c+dx^2\right)^{3/2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^2*c^4 + 6*a*b*c^3*d - 3*a^2*c^2*d^2 - 6*b^2*c^3*d*x^2 + 21*a*b*c^2*d^
2*x^2 - 12*a^2*c*d^3*x^2 - 3*b^2*c^2*d^2*x^4 + 14*a*b*c*d^3*x^4 - 8*a^2*d^4
*x^4)/(3*a*c^3*(-(b*c) + a*d)^2*x*(c + d*x^2)^(3/2)) + (b^3*ArcTan[(Sqrt[a]
*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*
x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

fricas [**B**] time = 2.17, size = 934, normalized size = 5.25

$$\left[-\frac{3\left(b^{3}c^{3}d^{2}x^{5}+2b^{3}c^{4}dx^{3}+b^{3}c^{5}x\right)\sqrt{-abc+a^{2}d}\log\left(\frac{\left(b^{2}c^{2}-8abcd+8a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3abc^{2}-4a^{2}cd\right)x^{2}+4\left(\left(bc-2ad\right)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\sqrt{dx^{2}+c}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)+4\left(3ab^{3}c^{5}-9a^{2}b^{2}c^{4}d+9a^{3}bc^{3}d^{2}-3a^{4}c^{2}d^{3}+\left(3ab^{3}c^{3}d^{2}-17a^{2}b^{2}c^{2}d^{3}+22a^{3}bcd^{4}-8a^{4}d^{5}\right)x^{4}+3\left(2ab^{3}c^{4}d-9a^{2}b^{2}c^{3}d^{2}+11a^{3}bc^{2}d^{3}-4a^{4}cd^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\left(\left(a^{2}b^{3}c^{6}d^{2}-3a^{3}b^{2}c^{5}d^{3}+3a^{4}bc^{4}d^{4}-a^{5}c^{3}d^{5}\right)x^{5}+2\left(a^{2}b^{3}c^{7}d-3a^{3}b^{2}c^{6}d^{2}+3a^{4}bc^{5}d^{3}-a^{5}c^{4}d^{4}\right)x^{3}+\left(a^{2}b^{3}c^{8}-3a^{3}b^{2}c^{7}d+3a^{4}bc^{6}d^{2}-a^{5}c^{5}d^{3}\right)x\right)}, -\frac{3\left(b^{3}c^{3}d^{2}x^{5}+2b^{3}c^{4}dx^{3}+b^{3}c^{5}x\right)\sqrt{abc-a^{2}d}\arctan\left(\frac{\sqrt{abc-a^{2}d}\left(\left(bc-2ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)+2\left(3ab^{3}c^{5}-9a^{2}b^{2}c^{4}d+9a^{3}bc^{3}d^{2}-3a^{4}c^{2}d^{3}+\left(3ab^{3}c^{3}d^{2}-17a^{2}b^{2}c^{2}d^{3}+22a^{3}bcd^{4}-8a^{4}d^{5}\right)x^{4}+3\left(2ab^{3}c^{4}d-9a^{2}b^{2}c^{3}d^{2}+11a^{3}bc^{2}d^{3}-4a^{4}cd^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}}{6\left(\left(a^{2}b^{3}c^{6}d^{2}-3a^{3}b^{2}c^{5}d^{3}+3a^{4}bc^{4}d^{4}-a^{5}c^{3}d^{5}\right)x^{5}+2\left(a^{2}b^{3}c^{7}d-3a^{3}b^{2}c^{6}d^{2}+3a^{4}bc^{5}d^{3}-a^{5}c^{4}d^{4}\right)x^{3}+\left(a^{2}b^{3}c^{8}-3a^{3}b^{2}c^{7}d+3a^{4}bc^{6}d^{2}-a^{5}c^{5}d^{3}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-a*b*c + a^2
*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x
^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d +
9*a^3*b*c^3*d^2 - 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 2
2*a^3*b*c*d^4 - 8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*
a^3*b*c^2*d^3 - 4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^
3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*a
^3*b^2*c^6*d^2 + 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^3*
b^2*c^7*d + 3*a^4*b*c^6*d^2 - a^5*c^5*d^3)*x), -1/6*(3*(b^3*c^3*d^2*x^5 + 2
*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2
*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a
*b*c^2 - a^2*c*d)*x)) + 2*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d + 9*a^3*b*c^3*d^2
- 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 22*a^3*b*c*d^4 -
8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*a^3*b*c^2*d^3 -
4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^3*b^2*c^5*d^3 +
3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*a^3*b^2*c^6*d^2 +
 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^3*b^2*c^7*d + 3*a^
4*b*c^6*d^2 - a^5*c^5*d^3)*x)]
```

giac [**B**] time = 3.80, size = 366, normalized size = 2.06

$$\frac{b^{3}\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(ab^{2}c^{2}-2a^{2}bcd+a^{3}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{\left(\frac{\left(8b^{3}c^{5}d^{4}-21ab^{2}c^{4}d^{5}+18a^{2}bc^{3}d^{6}-5a^{3}c^{2}d^{7}\right)x^{2}}{b^{4}c^{9}d-4ab^{3}c^{8}d^{2}+6a^{2}b^{2}c^{7}d^{3}-4a^{3}bc^{6}d^{4}+a^{4}c^{5}d^{5}}+\frac{3\left(3b^{3}c^{6}d^{3}-8ab^{2}c^{5}d^{4}+7a^{2}bc^{4}d^{5}-2a^{3}c^{3}d^{6}\right)}{b^{4}c^{9}d-4ab^{3}c^{8}d^{2}+6a^{2}b^{2}c^{7}d^{3}-4a^{3}bc^{6}d^{4}+a^{4}c^{5}d^{5}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^3*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b*c*d -
a^2*d^2)) + 1/3*((8*b^3*c^5*d^4 - 21*a*b^2*c^4*d^5 + 18*a^2*b*c^3*d^6 - 5*a
^3*c^2*d^7)*x^2/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^3*b*
c^6*d^4 + a^4*c^5*d^5) + 3*(3*b^3*c^6*d^3 - 8*a*b^2*c^5*d^4 + 7*a^2*b*c^4*d
^5 - 2*a^3*c^3*d^6)/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^
3*b*c^6*d^4 + a^4*c^5*d^5))*x/(d*x^2 + c)^(3/2) + 2*sqrt(d)/(((sqrt(d)*x -
sqrt(d*x^2 + c))^2 - c)*a*c^2)
```

maple [**B**] time = 0.02, size = 1192, normalized size = 6.70

$$\frac{b^{3}\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{b^{3}\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{b^{3}}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}-\frac{b^{3}}{2\sqrt{-ab}\,(ad-bc)^{2}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,a}+\frac{b^{2}dx}{2(ad-bc)^{2}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac}+\frac{b^{2}dx}{2(ad-bc)^{2}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac}-\frac{b^{2}}{6\sqrt{-ab}\,(ad-bc)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}a}+\frac{b^{2}}{6\sqrt{-ab}\,(ad-bc)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}a}-\frac{bdx}{6(ad-bc)\left(\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}ac}-\frac{bdx}{6(ad-bc)\left(\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}\right)^{\frac{3}{2}}ac}-\frac{bdx}{3(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac^{2}}-\frac{bdx}{3(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ac^{2}}-\frac{4dx}{3\left(dx^{2}+c\right)^{\frac{3}{2}}ac^{2}}-\frac{8dx}{3\sqrt{dx^{2}+c}\,ac^{3}}-\frac{1}{\left(dx^{2}+c\right)^{\frac{3}{2}}acx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/6*b/a*d/(a*d-b*c)/c/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/3*b
/a*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)*x+1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*b^2
/a/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2*b^3/a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/6*b^2/a/(-a*b)^(1/2)/(a*d-b*c)/((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3
/2)-1/6*b/a*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/3*b/a*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2*b^3/a
/(-a*b)^(1/2)/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2*b^2/a/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2*b^3/a
/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b
))-1/a/c/x/(d*x^2+c)^(3/2)-4/3*a*d/c^2*x/(d*x^2+c)^(3/2)-8/3*a*d/c^3*x/(d*x
^2+c)^(1/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{2}+a\right)\left(dx^{2}+c\right)^{\frac{5}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \left(b\,x^2+a\right) \left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a + b x^{2}\right) \left(c + d x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.711 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-$$

Rubi [A] time = 0.32, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{d\left(5a^2d^2-8abcd+b^2c^2\right)}{2ac^3\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}-\frac{d(3bc-5ad)}{6ac^2\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*(3*b*c - 5*a*d))/(6*a*c^2*(b*c - a*d)*(c + d*x^2)^(3/2)) - 1/(2*a*c*x^2
*(c + d*x^2)^(3/2)) - (d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2))/(2*a*c^3*(b*c -
 a*d)^2*Sqrt[c + d*x^2]) + ((2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(2*a^2*c^(7/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
a*d]])/(a^2*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+5ad)+\frac{5bdx}{2}}{x(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{4}(bc-ad)(2bc+5ad)-\frac{3}{4}}{x(a+bx)(c+dx}\right.}{3ac^2(bc-}$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} -$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} +$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} +$$

$$= -\frac{d(3bc-5ad)}{6ac^2(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(c+dx^2\right)^{3/2}} - \frac{d\left(b^2c^2-8abcd+5a^2d^2\right)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} +$$

Mathematica [C] time = 0.05, size = 118, normalized size = 0.56

$$\frac{2b^2c^2x^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+(ad-bc)\left(x^2(5ad+2bc)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)+3ac\right)}{6a^2c^2x^2\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*b^2*c^2*x^2*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)
] + (-(b*c) + a*d)*(3*a*c + (2*b*c + 5*a*d)*x^2*Hypergeometric2F1[-3/2, 1,
-1/2, 1 + (d*x^2)/c]))/(6*a^2*c^2*(b*c - a*d)*x^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.75, size = 237, normalized size = 1.12

$$-\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{a^2(ad-bc)^{5/2}}+\frac{-3a^2c^2d^2-20a^2cd^3x^2-15a^2d^4x^4+6abc^3d+32abc^2d^2x^2+24abcd^3x^4-3b^2c^4-6b^2c^3dx^2-3b^2c^2d^2x^4}{6ac^3x^2\left(c+dx^2\right)^{3/2}(ad-bc)^2}+\frac{(5ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^2*c^4 + 6*a*b*c^3*d - 3*a^2*c^2*d^2 - 6*b^2*c^3*d*x^2 + 32*a*b*c^2*d^
2*x^2 - 20*a^2*c*d^3*x^2 - 3*b^2*c^2*d^2*x^4 + 24*a*b*c*d^3*x^4 - 15*a^2*d^
4*x^4)/(6*a*c^3*(-(b*c) + a*d)^2*x^2*(c + d*x^2)^(3/2)) - (b^(7/2)*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(a^2*(-(b*c) + a*
d)^(5/2)) + ((2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2*c^(7/
2))
```

fricas [B] time = 6.04, size = 2219, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(b/(b*c - a*
d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 3*((2*
b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4
*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^
2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d + 3*a^3*c
^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2*(3*a*b^2
*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2
*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d - 2*a^3*b*
c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^
2), -1/12*(6*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x
^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 +
(2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-c)*a
rctan(sqrt(-c)/sqrt(d*x^2 + c)) - 3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^
3*c^6*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d + 3*a^3*c^3*d^2 + 3*(a*
b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2*(3*a*b^2*c^4*d - 16*a^
2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^6*d^2 - 2*a
^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*
```

```
c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2), 1/12*(6*(b
^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(-b/(b*c - a*d))*arctan
(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2
+ b*c)) + 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^
6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (
2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log
(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b
*c^4*d + 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*
x^4 + 2*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2
 + c))/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*
c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d +
 a^4*c^6*d^2)*x^2), 1/6*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2
)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*s
qrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 3*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 -
8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c
^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*
a^3*c^2*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a*b^2*c^5
- 6*a^2*b*c^4*d + 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^
3*c*d^4)*x^4 + 2*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*s
qrt(d*x^2 + c))/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*
(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*
b*c^7*d + a^4*c^6*d^2)*x^2)]
```

giac [A] time = 0.36, size = 211, normalized size = 1.00

$$\frac{b^{4} \arctan\left(\frac{\sqrt{dx^{2}+c}\, b}{\sqrt{-b^{2}c+abd}}\right)}{\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{9\left(dx^{2}+c\right)bcd^{2}+bc^{2}d^{2}-6\left(dx^{2}+c\right)ad^{3}-acd^{3}}{3\left(b^{2}c^{5}-2\,abc^{4}d+a^{2}c^{3}d^{2}\right)\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{\left(2\,bc+5\,ad\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,a^{2}\sqrt{-c}\,c^{3}}-\frac{\sqrt{dx^{2}+c}}{2\,ac^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] b^4*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^2*c^2 - 2*a^3*b*
c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(9*(d*x^2 + c)*b*c*d^2 + b*c^2*d
^2 - 6*(d*x^2 + c)*a*d^3 - a*c*d^3)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*
(d*x^2 + c)^(3/2)) - 1/2*(2*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(
a^2*sqrt(-c)*c^3) - 1/2*sqrt(d*x^2 + c)/(a*c^3*x^2)
```

maple [B] time = 0.02, size = 1289, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/6/a^2*b^2/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/6/a^2*b*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1
/3/a^2*b*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/2/a^2*b^3/(a*d-b*c)^2/((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
+1/2/a^2*b^2/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/a^2*b^3/(a*d-b*c)^2/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/a/c/x^2/(d*x^2+c)^(
3/2)-5/6/a*d/c^2/(d*x^2+c)^(3/2)-5/2/a*d/c^3/(d*x^2+c)^(1/2)+5/2/a*d/c^(7/2
)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/6/a^2*b^2/(a*d-b*c)/((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6/a^2
*b*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/3/a^2*b*(-a*b)^(1/2)*d/(a*d-b*c)/c^
2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*x+1/2/a^2*b^3/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a^2*b^2/(a*d-b*c)^2*(-a*b)^(1/2
)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)*d*x-1/2/a^2*b^3/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-
a*b)^(1/2)/b))-1/3/a^2*b/c/(d*x^2+c)^(3/2)-1/a^2*b/c^2/(d*x^2+c)^(1/2)+1/a^
2*b/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3), x)
```

mupad [B] time = 3.42, size = 5409, normalized size = 25.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] (atan(((((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 +
 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 +
```

```
 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9
+ 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11
*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) + ((5*a*d + 2*b*c)
*(64*a^6*b^13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19
520*a^9*b^10*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 -
 106624*a^12*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*
d^11 + 16192*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^
12*d^14 - ((c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*
a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 13
8240*a^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^
8 - 115200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^
17*d^11 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(
1/2))))/(4*a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c)*1i)/(4*a^2*(c^7)^(1/2)) + (((c
 + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b
^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b
^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^1
1*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 368
0*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) - ((5*a*d + 2*b*c)*(64*a^6*b^
13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19520*a^9*b^1
0*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 - 106624*a^1
2*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*d^11 + 1619
2*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^12*d^14 + (
(c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^
25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b
^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*
a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3
072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(1/2))))/(4*
a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c)*1i)/(4*a^2*(c^7)^(1/2)))/(32*a^2*b^15*c^1
8*d^3 - 368*a^3*b^14*c^17*d^4 + 1056*a^4*b^13*c^16*d^5 - 5600*a^6*b^11*c^14
*d^7 + 12768*a^7*b^10*c^13*d^8 - 14112*a^8*b^9*c^12*d^9 + 8704*a^9*b^8*c^11
*d^10 - 2880*a^10*b^7*c^10*d^11 + 400*a^11*b^6*c^9*d^12 - (((c + d*x^2)^(1/
2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4
+ 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7
+ 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^
10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^
10*d^13 + 400*a^15*b^3*c^9*d^14) + ((5*a*d + 2*b*c)*(64*a^6*b^13*c^23*d^3 +
 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19520*a^9*b^10*c^20*d^6 -
53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 - 106624*a^12*b^7*c^17*d^
9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*d^11 + 16192*a^15*b^4*c^
14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^12*d^14 - ((c + d*x^2)^(
1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 2560
0*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 -
 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19
*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*
c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(1/2))))/(4*a^2*(c^7)^(1/
```

```
2)))*(5*a*d + 2*b*c))/(4*a^2*(c^7)^(1/2)) + (((c + d*x^2)^(1/2)*(128*a^3*b^
15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^1
2*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^
9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^10 - 34240*a^1
2*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*
a^15*b^3*c^9*d^14) - ((5*a*d + 2*b*c)*(64*a^6*b^13*c^23*d^3 + 64*a^7*b^12*c
^22*d^4 - 3648*a^8*b^11*c^21*d^5 + 19520*a^9*b^10*c^20*d^6 - 53632*a^10*b^9
*c^19*d^7 + 92288*a^11*b^8*c^18*d^8 - 106624*a^12*b^7*c^17*d^9 + 84608*a^13
*b^6*c^16*d^10 - 45760*a^14*b^5*c^15*d^11 + 16192*a^15*b^4*c^14*d^12 - 3392
*a^16*b^3*c^13*d^13 + 320*a^17*b^2*c^12*d^14 + ((c + d*x^2)^(1/2)*(5*a*d +
2*b*c)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^2
4*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 - 182784*a^12*b
^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 + 53760*a
^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d^12 - 25
6*a^18*b^2*c^15*d^13))/(4*a^2*(c^7)^(1/2))))/(4*a^2*(c^7)^(1/2)))*(5*a*d +
2*b*c))/(4*a^2*(c^7)^(1/2))))*(5*a*d + 2*b*c)*1i)/(2*a^2*(c^7)^(1/2)) - ((d
^2*(c + d*x^2)*(5*a*d - 8*b*c))/(3*(b*c^2 - a*c*d)^2) - d^2/(3*(b*c^2 - a*c
*d)) + (d*(c + d*x^2)^2*(5*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(2*a*c^2*(b*c^2
- a*c*d)*(a*d - b*c)))/(c*(c + d*x^2)^(3/2) - (c + d*x^2)^(5/2)) + (atan(((
(-b^7*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704
*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000
*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 4128
0*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 +
 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14
))/2 + ((-b^7*(a*d - b*c)^5)^(1/2)*(32*a^6*b^13*c^23*d^3 + 32*a^7*b^12*c^22
*d^4 - 1824*a^8*b^11*c^21*d^5 + 9760*a^9*b^10*c^20*d^6 - 26816*a^10*b^9*c^1
9*d^7 + 46144*a^11*b^8*c^18*d^8 - 53312*a^12*b^7*c^17*d^9 + 42304*a^13*b^6*
c^16*d^10 - 22880*a^14*b^5*c^15*d^11 + 8096*a^15*b^4*c^14*d^12 - 1696*a^16*
b^3*c^13*d^13 + 160*a^17*b^2*c^12*d^14 - ((-b^7*(a*d - b*c)^5)^(1/2)*(c + d
*x^2)^(1/2)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*b^1
1*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 - 182784*a
^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 + 53
760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d^12
 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(a*d - b*c)^5)))/(2*a^2*(a*d - b*c)^5))*
1i)/(a^2*(a*d - b*c)^5) + ((-b^7*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(
128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4 + 14
40*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7 + 16
864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^10 -
 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^10*d
^13 + 400*a^15*b^3*c^9*d^14))/2 - ((-b^7*(a*d - b*c)^5)^(1/2)*(32*a^6*b^13*
c^23*d^3 + 32*a^7*b^12*c^22*d^4 - 1824*a^8*b^11*c^21*d^5 + 9760*a^9*b^10*c^
20*d^6 - 26816*a^10*b^9*c^19*d^7 + 46144*a^11*b^8*c^18*d^8 - 53312*a^12*b^7
*c^17*d^9 + 42304*a^13*b^6*c^16*d^10 - 22880*a^14*b^5*c^15*d^11 + 8096*a^15
*b^4*c^14*d^12 - 1696*a^16*b^3*c^13*d^13 + 160*a^17*b^2*c^12*d^14 + ((-b^7*
(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^
```

```
12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a
^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 11
5200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^1
1 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(a*d - b*c)^5
)))/(2*a^2*(a*d - b*c)^5))*1i)/(a^2*(a*d - b*c)^5))/(((-b^7*(a*d - b*c)^5)^
(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3 +
1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6 +
2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d^9 +
 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*c^11*
d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14))/2 - ((-b^7*(a*d -
b*c)^5)^(1/2)*(32*a^6*b^13*c^23*d^3 + 32*a^7*b^12*c^22*d^4 - 1824*a^8*b^11*
c^21*d^5 + 9760*a^9*b^10*c^20*d^6 - 26816*a^10*b^9*c^19*d^7 + 46144*a^11*b^
8*c^18*d^8 - 53312*a^12*b^7*c^17*d^9 + 42304*a^13*b^6*c^16*d^10 - 22880*a^1
4*b^5*c^15*d^11 + 8096*a^15*b^4*c^14*d^12 - 1696*a^16*b^3*c^13*d^13 + 160*a
^17*b^2*c^12*d^14 + ((-b^7*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7*
b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^
10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 17
2032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^1
0 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*
d^13))/(4*a^2*(a*d - b*c)^5)))/(2*a^2*(a*d - b*c)^5)))/(a^2*(a*d - b*c)^5)
 - ((-b^7*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 -
704*a^4*b^14*c^20*d^3 + 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6
000*a^7*b^11*c^17*d^6 + 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 4
1280*a^10*b^8*c^14*d^9 + 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^1
1 + 14864*a^13*b^5*c^11*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d
^14))/2 + ((-b^7*(a*d - b*c)^5)^(1/2)*(32*a^6*b^13*c^23*d^3 + 32*a^7*b^12*c
^22*d^4 - 1824*a^8*b^11*c^21*d^5 + 9760*a^9*b^10*c^20*d^6 - 26816*a^10*b^9*
c^19*d^7 + 46144*a^11*b^8*c^18*d^8 - 53312*a^12*b^7*c^17*d^9 + 42304*a^13*b
^6*c^16*d^10 - 22880*a^14*b^5*c^15*d^11 + 8096*a^15*b^4*c^14*d^12 - 1696*a^
16*b^3*c^13*d^13 + 160*a^17*b^2*c^12*d^14 - ((-b^7*(a*d - b*c)^5)^(1/2)*(c
+ d*x^2)^(1/2)*(512*a^7*b^13*c^26*d^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*
b^11*c^24*d^4 - 72960*a^10*b^10*c^23*d^5 + 138240*a^11*b^9*c^22*d^6 - 18278
4*a^12*b^8*c^21*d^7 + 172032*a^13*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 +
 53760*a^15*b^5*c^18*d^10 - 16640*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d
^12 - 256*a^18*b^2*c^15*d^13))/(4*a^2*(a*d - b*c)^5)))/(2*a^2*(a*d - b*c)^5
)))/(a^2*(a*d - b*c)^5) + 32*a^2*b^15*c^18*d^3 - 368*a^3*b^14*c^17*d^4 + 10
56*a^4*b^13*c^16*d^5 - 5600*a^6*b^11*c^14*d^7 + 12768*a^7*b^10*c^13*d^8 - 1
4112*a^8*b^9*c^12*d^9 + 8704*a^9*b^8*c^11*d^10 - 2880*a^10*b^7*c^10*d^11 +
400*a^11*b^6*c^9*d^12))*(-b^7*(a*d - b*c)^5)^(1/2)*1i)/(a^2*(a*d - b*c)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.712 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=245

$$\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}+\frac{\sqrt{c+dx^2}\,(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\,\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3(bc-ad)^2}-\frac{}{c^2x^3}$$

Rubi [A] time = 0.36, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {472, 579, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{3a^2c^4x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\,\left(8a^2d^2-12abcd+b^2c^2\right)}{3ac^3x^3(bc-ad)^2}+\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}-\frac{d(3bc-2ad)}{c^2x^3\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -d/(3*c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) - (d*(3*b*c - 2*a*d))/(c^2*(b*c
- a*d)^2*x^3*Sqrt[c + d*x^2]) - ((b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*Sqrt[c
+ d*x^2])/(3*a*c^3*(b*c - a*d)^2*x^3) + ((b*c - 2*a*d)*(3*b^2*c^2 + 8*a*b*c
*d - 8*a^2*d^2)*Sqrt[c + d*x^2])/(3*a^2*c^4*(b*c - a*d)^2*x) + (b^4*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{3(bc-2ad)-6bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c(bc-ad)}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{\int \frac{3\left(b^2c^2-12abcd+8a^2d^2\right)-1}{x^4\left(a+bx^2\right)\sqrt{c+}}}{3c^2(bc-a}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c}}{3c^2(bc-ad)^2x^3}$$

$$= -\frac{d}{3c(bc-ad)x^3\left(c+dx^2\right)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{\left(\frac{b^2c}{a}-12bd+\frac{8ad^2}{c}\right)\sqrt{c}}{3c^2(bc-ad)^2x^3}$$

Mathematica [A] time = 5.36, size = 160, normalized size = 0.65

$$\frac{b^4\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}} + \frac{\sqrt{c+dx^2}\left(\frac{x^2(8ad+3bc)}{a^2}+\frac{d^3x^4(8ad-11bc)}{\left(c+dx^2\right)(bc-ad)^2}-\frac{cd^3x^4}{\left(c+dx^2\right)^2(bc-ad)}-\frac{c}{a}\right)}{3c^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(c/a) + ((3*b*c + 8*a*d)*x^2)/a^2 - (c*d^3*x^4)/((b*c -
a*d)*(c + d*x^2)^2) + (d^3*(-11*b*c + 8*a*d)*x^4)/((b*c - a*d)^2*(c + d*x^2
))))/(3*c^4*x^3) + (b^4*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(a^(5/2)*(b*c - a*d)^(5/2))
```

IntegrateAlgebraic [A] time = 0.89, size = 319, normalized size = 1.30

$$\frac{-a^3c^3d^2+6a^3c^2d^3x^2+24a^3cd^4x^4+16a^3d^5x^6+2a^2bc^4d-9a^2bc^3d^2x^2-36a^2bc^2d^3x^4-24a^2bcd^4x^6-ab^2c^5+3ab^2c^3d^2x^4+2ab^2c^2d^3x^6+3b^3c^5x^2+6b^3c^4dx^4+3b^3c^3d^2x^6}{3a^2c^4x^3\left(c+dx^2\right)^{3/2}(ad-bc)^2}-\frac{b^4\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{a^{5/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-(a*b^2*c^5) + 2*a^2*b*c^4*d - a^3*c^3*d^2 + 3*b^3*c^5*x^2 - 9*a^2*b*c^3*d
^2*x^2 + 6*a^3*c^2*d^3*x^2 + 6*b^3*c^4*d*x^4 + 3*a*b^2*c^3*d^2*x^4 - 36*a^2
*b*c^2*d^3*x^4 + 24*a^3*c*d^4*x^4 + 3*b^3*c^3*d^2*x^6 + 2*a*b^2*c^2*d^3*x^6
 - 24*a^2*b*c*d^4*x^6 + 16*a^3*d^5*x^6)/(3*a^2*c^4*(-(b*c) + a*d)^2*x^3*(c
+ d*x^2)^(3/2)) - (b^4*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d
]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c
- a*d])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

fricas [B] time = 3.41, size = 1128, normalized size = 4.60

$$\left[-\frac{3\left(b^4c^4d^2x^7+2b^4c^5dx^5+b^4c^6x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(\left(bc-2ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+4\left(a^2b^3c^6-3a^3b^2c^5d+3a^4bc^4d^2-a^5c^3d^3-\left(3ab^4c^4d^2-a^2b^3c^3d^3-26a^3b^2c^2d^4+40a^4bcd^5-16a^5d^6\right)x^6-3\left(2ab^4c^5d-a^2b^3c^4d^2-13a^3b^2c^3d^3+20a^4bc^2d^4-8a^5cd^5\right)x^4-3\left(ab^4c^6-a^2b^3c^5d-3a^3b^2c^4d^2+5a^4bc^3d^3-2a^5c^2d^4\right)x^2\right)\sqrt{dx^2+c}}{12\left(\left(a^3b^3c^7d^2-3a^4b^2c^6d^3+3a^5bc^5d^4-a^6c^4d^5\right)x^7+2\left(a^3b^3c^8d-3a^4b^2c^7d^2+3a^5bc^6d^3-a^6c^5d^4\right)x^5+\left(a^3b^3c^9-3a^4b^2c^8d+3a^5bc^7d^2-a^6c^6d^3\right)x^3\right)},\ \frac{3\left(b^4c^4d^2x^7+2b^4c^5dx^5+b^4c^6x^3\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)-2\left(a^2b^3c^6-3a^3b^2c^5d+3a^4bc^4d^2-a^5c^3d^3-\left(3ab^4c^4d^2-a^2b^3c^3d^3-26a^3b^2c^2d^4+40a^4bcd^5-16a^5d^6\right)x^6-3\left(2ab^4c^5d-a^2b^3c^4d^2-13a^3b^2c^3d^3+20a^4bc^2d^4-8a^5cd^5\right)x^4-3\left(ab^4c^6-a^2b^3c^5d-3a^3b^2c^4d^2+5a^4bc^3d^3-2a^5c^2d^4\right)x^2\right)\sqrt{dx^2+c}}{6\left(\left(a^3b^3c^7d^2-3a^4b^2c^6d^3+3a^5bc^5d^4-a^6c^4d^5\right)x^7+2\left(a^3b^3c^8d-3a^4b^2c^7d^2+3a^5bc^6d^3-a^6c^5d^4\right)x^5+\left(a^3b^3c^9-3a^4b^2c^8d+3a^5bc^7d^2-a^6c^6d^3\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-a*b*c + a
^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d
+ 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a
^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*
b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*
(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^
2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*
b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3*b^3*c^8*d - 3*a^4*b^2*c^7*d^2 + 3*a^5
*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*b^3*c^9 - 3*a^4*b^2*c^8*d + 3*a^5*b*c^
7*d^2 - a^6*c^6*d^3)*x^3), 1/6*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*
c^6*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*
x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x
)) - 2*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*
a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*
a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20
*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^
2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^
3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3
*b^3*c^8*d - 3*a^4*b^2*c^7*d^2 + 3*a^5*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*
b^3*c^9 - 3*a^4*b^2*c^8*d + 3*a^5*b*c^7*d^2 - a^6*c^6*d^3)*x^3)]
```

giac [B] time = 4.33, size = 490, normalized size = 2.00

$$-\frac{b^{4}\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\frac{\left(11\,b^{3}c^{6}d^{5}-30\,ab^{2}c^{5}d^{6}+27\,a^{2}bc^{4}d^{7}-8\,a^{3}c^{3}d^{8}\right)x^{2}}{b^{4}c^{11}d-4\,ab^{3}c^{10}d^{2}+6\,a^{2}b^{2}c^{9}d^{3}-4\,a^{3}bc^{8}d^{4}+a^{4}c^{7}d^{5}}+\frac{3\left(4\,b^{3}c^{7}d^{4}-11\,ab^{2}c^{6}d^{5}+10\,a^{2}bc^{5}d^{6}-3\,a^{3}c^{4}d^{7}\right)}{b^{4}c^{11}d-4\,ab^{3}c^{10}d^{2}+6\,a^{2}b^{2}c^{9}d^{3}-4\,a^{3}bc^{8}d^{4}+a^{4}c^{7}d^{5}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc\sqrt{d}+6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}ad^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}\sqrt{d}-18\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}acd^{\frac{3}{2}}+3\,bc^{3}\sqrt{d}+8\,ac^{2}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -b^4*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b*c*d
 - a^2*d^2)) - 1/3*((11*b^3*c^6*d^5 - 30*a*b^2*c^5*d^6 + 27*a^2*b*c^4*d^7 -
 8*a^3*c^3*d^8)*x^2/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*d^3 - 4*
a^3*b*c^8*d^4 + a^4*c^7*d^5) + 3*(4*b^3*c^7*d^4 - 11*a*b^2*c^6*d^5 + 10*a^2
*b*c^5*d^6 - 3*a^3*c^4*d^7)/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*
d^3 - 4*a^3*b*c^8*d^4 + a^4*c^7*d^5))*x/(d*x^2 + c)^(3/2) - 2/3*(3*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d
^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 18*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d) + 8*a*c^2*d^(3/2))/(((sqrt
(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2*c^3)
```

maple [B] time = 0.02, size = 1285, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/6*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/6*b^2/a^2*d/(a*d-b*c)/c/((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+
1/3*b^2/a^2*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)-1/2*b^3/a^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^
2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/6*b^3/a^2/(-a*b)^(
1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(3/2)+1/6*b^2/a^2*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/3*b^2/a^2*d/(a*d-b
*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*x+1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2*b^3/a^2/(a*d
-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
```

```
d-b*c)/b)^(1/2)*d*x-1/2*b^4/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/a^2*b/c/x/(d*x^2+c)^(3/2)+4/3/a^2*b*d/c
^2*x/(d*x^2+c)^(3/2)+8/3/a^2*b*d/c^3*x/(d*x^2+c)^(1/2)-1/3/a/c/x^3/(d*x^2+c
)^(3/2)+2/a*d/c^2/x/(d*x^2+c)^(3/2)+8/3/a*d^2/c^3*x/(d*x^2+c)^(3/2)+16/3/a*
d^2/c^4*x/(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.713 $\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{a}\,(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

Rubi **[A]** time = 0.17, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {467, 582, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}-\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{x\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/b^2 - (x^3*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) - (Sqrt[a
]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2
*b^3*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
)/(2*b^3*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\left(3c+4dx^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} - \frac{\int \frac{4acd-2d(bc-4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b^2d}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} + \frac{(bc-4ad)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3} - \frac{(a(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b^3}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} + \frac{(bc-4ad)\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2b^3} - \frac{(a(3bc-4ad))\,\mathrm{Subs}}{}$$

$$= \frac{x\sqrt{c+dx^2}}{b^2} - \frac{x^3\sqrt{c+dx^2}}{2b\left(a+bx^2\right)} - \frac{\sqrt{a}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\bigg(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}}{2b^3\sqrt{d}}$$

Mathematica [A] time = 0.18, size = 134, normalized size = 0.89

$$\frac{\frac{bx\left(2a+bx^2\right)\sqrt{c+dx^2}}{a+bx^2} + \frac{(bc-4ad)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{\sqrt{d}} + \frac{\sqrt{a}\,(4ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((b*x*(2*a + b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + (Sqrt[a]*(-3*b*c + 4*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[b*c - a*d] +
((b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^3)
```

IntegrateAlgebraic [A] time = 0.90, size = 169, normalized size = 1.13

$$\frac{\left(3\sqrt{a}\,bc-4a^{3/2}d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2b^3\sqrt{bc-ad}} + \frac{(4ad-bc)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^3\sqrt{d}} + \frac{\left(2ax+bx^3\right)\sqrt{c+dx^2}}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(2*a*x + b*x^3))/(2*b^2*(a + b*x^2)) + ((3*Sqrt[a]*b*c - 4
*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[
a]*Sqrt[b*c - a*d])])/(2*b^3*Sqrt[b*c - a*d]) + ((-(b*c) + 4*a*d)*Log[-(Sqr
t[d]*x) + Sqrt[c + d*x^2]])/(2*b^3*Sqrt[d])
```

fricas [A] time = 1.95, size = 1002, normalized size = 6.68

$$\left[-\frac{2\left(abc-4a^{2}d+\left(b^{2}c-4abd\right)x^{2}\right)\sqrt{d}\log\left(-2dx^{2}+2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+\left(3abcd-4a^{2}d^{2}+\left(3b^{2}cd-4abd^{2}\right)x^{2}\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^{2}c^{2}-8abcd+8a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3abc^{2}-4a^{2}cd\right)x^{2}+4\left(\left(b^{2}c^{2}-3abcd+2a^{2}d^{2}\right)x^{3}-\left(abc^{2}-a^{2}cd\right)x\right)\sqrt{dx^{2}+c}\sqrt{-\frac{a}{bc-ad}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)-4\left(b^{2}dx^{3}+2abdx\right)\sqrt{dx^{2}+c}}{8\left(b^{4}dx^{2}+ab^{3}d\right)}, -\frac{4\left(abc-4a^{2}d+\left(b^{2}c-4abd\right)x^{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)+\left(3abcd-4a^{2}d^{2}+\left(3b^{2}cd-4abd^{2}\right)x^{2}\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^{2}c^{2}-8abcd+8a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3abc^{2}-4a^{2}cd\right)x^{2}+4\left(\left(b^{2}c^{2}-3abcd+2a^{2}d^{2}\right)x^{3}-\left(abc^{2}-a^{2}cd\right)x\right)\sqrt{dx^{2}+c}\sqrt{-\frac{a}{bc-ad}}}{b^{2}x^{4}+2abx^{2}+a^{2}}\right)-4\left(b^{2}dx^{3}+2abdx\right)\sqrt{dx^{2}+c}}{8\left(b^{4}dx^{2}+ab^{3}d\right)}, \frac{\left(3abcd-4a^{2}d^{2}+\left(3b^{2}cd-4abd^{2}\right)x^{2}\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^{3}+acx\right)}\right)-\left(abc-4a^{2}d+\left(b^{2}c-4abd\right)x^{2}\right)\sqrt{d}\log\left(-2dx^{2}+2\sqrt{dx^{2}+c}\sqrt{d}x-c\right)+2\left(b^{2}dx^{3}+2abdx\right)\sqrt{dx^{2}+c}}{4\left(b^{4}dx^{2}+ab^{3}d\right)}, -\frac{2\left(abc-4a^{2}d+\left(b^{2}c-4abd\right)x^{2}\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^{2}+c}}\right)-\left(3abcd-4a^{2}d^{2}+\left(3b^{2}cd-4abd^{2}\right)x^{2}\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^{3}+acx\right)}\right)-2\left(b^{2}dx^{3}+2abdx\right)\sqrt{dx^{2}+c}}{4\left(b^{4}dx^{2}+ab^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2
*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4*a
*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a
^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(
b^4*d*x^2 + a*b^3*d), -1/8*(4*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqr
t(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*
c*d - 4*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*
c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c
- a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^
2 + c))/(b^4*d*x^2 + a*b^3*d), 1/4*((3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4
*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sq
rt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - (a*b*c - 4*a^2*d + (
b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(b^4*d*x^2 + a*b^3*d), -1/4
*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sq
rt(d*x^2 + c)) - (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4*a*b*d^2)*x^2)*sqrt
(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(
a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 +
c))/(b^4*d*x^2 + a*b^3*d)]
```

giac [B] time = 0.60, size = 288, normalized size = 1.92

$$\frac{\sqrt{dx^{2}+c}\,x}{2b^{2}}+\frac{\left(3abc\sqrt{d}-4a^{2}d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{2\sqrt{abcd-a^{2}d^{2}}\,b^{3}}-\frac{\left(bc\sqrt{d}-4ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}\right)}{4b^{3}d}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}d^{\frac{3}{2}}-abc^{2}\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*x/b^2 + 1/2*(3*a*b*c*sqrt(d) - 4*a^2*d^(3/2))*arctan(1/
2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2)
```

```
)/(sqrt(a*b*c*d - a^2*d^2)*b^3) - 1/4*(b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/(b^3*d) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*
c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d)
)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b
*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

maple [B] time = 0.03, size = 2615, normalized size = 17.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*x*(d*x^2+c)^(1/2)/b^2+1/2/b^2*c/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1
/4*a/b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b
*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)-1/4*a^2/b^3*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/
2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^4*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b
)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*
c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+1/4*a/b^2*d/(a*d-b*c)*
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+1/4*a/b^2*d^(1/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b
*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))*c+3/4/b^2*a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/b^3*a*d^(1/2)*ln(((x+
(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/4/b^3*a^2/(-a*b)^(1/2)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c
)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d-3/4/b^2*a/(-a*b)^(1
/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-1/4*a/b^2/(a*d-b
*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*a
^2/b^3*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)
+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))-1/4*a^2/b^4*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
```

```
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x+(-a*b)^(1/2)/b))+1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c
)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+1/4*a/b^2*d/(a*d-b*c)*((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4*a
/b^2*d^(1/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))*c-3/4/b^2*a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/b^3*a*d^(1/2)*ln(((x-(-a*b)^(1/2)/
b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c
)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}\,x^4}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.714 $\int \frac{x^3\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(2bc-3ad)}{2b^2(bc-ad)}+\frac{a\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Rubi [**A**] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{c+dx^2}\,(2bc-3ad)}{2b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(3/2
))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b^2}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+d}\right.}{2b^2d}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{3/2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.86

$$\frac{\frac{(2bc-3ad)\left(\sqrt{b}\sqrt{c+dx^2}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}}+\frac{a(c+dx^2)^{3/2}}{a+bx^2}}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((a*(c + d*x^2)^(3/2))/(a + b*x^2) + ((2*b*c - 3*a*d)*(Sqrt[b]*Sqrt[c + d*x
^2] - Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]))/
b^(3/2))/(2*b*(b*c - a*d))
```

IntegrateAlgebraic [A] time = 0.28, size = 108, normalized size = 0.79

$$\frac{(3ad-2bc)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{5/2}\sqrt{ad-bc}}+\frac{\left(3a+2bx^2\right)\sqrt{c+dx^2}}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((3*a + 2*b*x^2)*Sqrt[c + d*x^2])/(2*b^2*(a + b*x^2)) + ((-2*b*c + 3*a*d)*A
rcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*b^(5/2)
*Sqrt[-(b*c) + a*d])
```

fricas [A] time = 0.97, size = 436, normalized size = 3.21

$$\left[-\frac{\left(2\,abc-3\,a^2d+\left(2\,b^2c-3\,abd\right)x^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2+4\,\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,\left(3\,ab^2c-3\,a^2bd+2\,\left(b^3c-ab^2d\right)x^2\right)\sqrt{dx^2+c}}{8\,\left(ab^4c-a^2b^3d+\left(b^5c-ab^4d\right)x^2\right)},\ -\frac{\left(2\,abc-3\,a^2d+\left(2\,b^2c-3\,abd\right)x^2\right)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\,\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)-2\,\left(3\,ab^2c-3\,a^2bd+2\,\left(b^3c-ab^2d\right)x^2\right)\sqrt{dx^2+c}}{4\,\left(ab^4c-a^2b^3d+\left(b^5c-ab^4d\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2
)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x
^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^2), -1/4*((
2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 -
a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c -
 a*b^2*d)*x^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^
2)]
```

giac [A] time = 0.32, size = 101, normalized size = 0.74

$$\frac{\sqrt{dx^2+c}\,ad}{2\left(\left(dx^2+c\right)b-bc+ad\right)b^2}+\frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b^2}+\frac{\sqrt{dx^2+c}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*a*d/(((d*x^2 + c)*b - b*c + a*d)*b^2) + 1/2*(2*b*c - 3*
a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + sqrt(d*x^2 + c)/b^2
```

maple [B] time = 0.01, size = 2543, normalized size = 18.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)-1/2/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^
(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))+1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b
)^(1/2)/b))*a*d-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b)
)*c+1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4*a/b^2*d/(a*
d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)+1/4*(-a*b)^(1/2)/b^3*a*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b
)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^3*d^2/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4*a/b^2*d/(a*d-b*c)/(-(a*d-b*c)/b
)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*
c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-1/4*(-a*b)^(1/2)/b^2*d/(a*d-b*c)*(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2)*x-1/4*(-a*b)^(1/2)/b^2*d^(1/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*
b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
```

```
)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x+(-a*b)^(1/
2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(3/2)+1/4*a/b^2*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*(-a*b)^(1/2)/b^3*a*d^(3/2)/(a*d
-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^3
*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4*a/b
^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+1/4*(
-a*b)^(1/2)/b^2*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4*(-a*b)^(1/2)/b^2*d^(1/2)/(a*d-b*c)*
ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/2/b^2*((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1
/2/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1
/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))+1/2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d-
1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.90, size = 102, normalized size = 0.75

$$\frac{\sqrt{d\,x^2+c}}{b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-2\,b\,c\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}+\frac{a\,d\,\sqrt{d\,x^2+c}}{2\,\left(b^3\,\left(d\,x^2+c\right)-b^3\,c+a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] (c + d*x^2)^(1/2)/b^2 - (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)
)*(3*a*d - 2*b*c))/(2*b^(5/2)*(a*d - b*c)^(1/2)) + (a*d*(c + d*x^2)^(1/2))/
(2*(b^3*(c + d*x^2) - b^3*c + a*b^2*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.715 $\int \frac{x^2\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=120

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {467, 523, 217, 206, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -(x*Sqrt[c + d*x^2])/(2*b*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*b^2*Sqrt[b*c - a*d]) + (Sqrt
[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{c+2dx^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b}\\
&= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{d\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}+\frac{(bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b^2}\\
&= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{d\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\right.}{2b^2}\\
&= -\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 118, normalized size = 0.98

$$
\frac{-\frac{bx\sqrt{c+dx^2}}{a+bx^2}+\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}+2\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{c+dx^2}\,+dx\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2])/(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a
*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*Sqrt[b*c - a*d]) + 2*Sqrt[d]*Lo
g[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

IntegrateAlgebraic [A] time = 0.60, size = 145, normalized size = 1.21

$$\frac{(2ad-bc)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2\sqrt{a}\,b^2\sqrt{bc-ad}}-\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}-\frac{\sqrt{d}\,\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(x*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + ((-(b*c) + 2*a*d)*ArcTan[(a*Sqrt
[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*S
qrt[a]*b^2*Sqrt[b*c - a*d]) - (Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])
/b^2
```

fricas [B] time = 1.11, size = 1069, normalized size = 8.91

$$\left[-\frac{4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x-4\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{8\left(a^2b^3c-a^3b^2d+\left(ab^4c-a^2b^3d\right)x^2\right)},\ -\frac{4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x+8\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{8\left(a^2b^3c-a^3b^2d+\left(ab^4c-a^2b^3d\right)x^2\right)},\ -\frac{2\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x-\sqrt{abc-a^2d}\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)-2\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{4\left(a^2b^3c-a^3b^2d+\left(ab^4c-a^2b^3d\right)x^2\right)},\ -\frac{2\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x-\sqrt{abc-a^2d}\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+4\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)}{4\left(a^2b^3c-a^3b^2d+\left(ab^4c-a^2b^3d\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - 4*(a^2*b*c - a^3*d + (a*b^
2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c)
 - (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)*x^
2), -1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + 8*(a^2*b*c - a^3*d + (a
*b^2*c - a^2*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b*c
 - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*
c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*
a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)*x^2), -1/4*
(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^2*d)*(a*b*c - 2*a
^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*
```

```
x)) - 2*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b
^3*d)*x^2), -1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^
2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*
d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 4*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-
d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c -
a^2*b^3*d)*x^2)]
```

giac [B] time = 0.49, size = 251, normalized size = 2.09

$$-\frac{\left(bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,b^2}-\frac{\sqrt{d}\,\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2}+\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2) -
 1/2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 + ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2) - b
*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^2)
```

maple [B] time = 0.01, size = 2547, normalized size = 21.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(
1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2))/(x+(-a*b)^(1/2)/b))*a*d+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x+(-a*b)^(1/2)/b))*c+1/4/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/b^2
*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(
```

```
1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^3*(-a*b)^(1/2)*d^2/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a-1/4/b^2*(-a*b)^(1
/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-1/4/
b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*x-1/4/b*d^(1/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*
b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*d^(1/2)*ln(
((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b^2
/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d-1/4/(-a*b)^(1/2)/b
/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+1/4/b/(a*d-b*c)/(x-(
-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(3/2)-1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*a*d^(3/
2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/
b^3*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1
/2)/b))*a+1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x-(-a*b)^(1/2)/b))*c-1/4/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/b*d^(1/2)/(a*d-b*c)*ln(
((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c}\, x^2}{\left(bx^2 + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
[Out] int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{c+dx^{2}}}{\left(a+bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
[Out] Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.716 $\int \frac{x\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Rubi [A] time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*b*(a + b*x^2)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b}\\
&= -\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}+\frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2b}\\
&= -\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{2b^{3/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(b*(a + b*x^2)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/
Sqrt[-(b*c) + a*d]])/(2*b^(3/2)*Sqrt[-(b*c) + a*d])
```

IntegrateAlgebraic [A] time = 0.17, size = 90, normalized size = 1.12

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2b^{3/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/(b*(a + b*x^2)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*b^(3/2)*Sqrt[-(b*c) + a*d])
```

fricas [B] time = 1.11, size = 356, normalized size = 4.45

$$\left[\frac{\left(bdx^2+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(b^2c-abd\right)\sqrt{dx^2+c}}{8\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^2\right)},-\frac{\left(bdx^2+ad\right)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)+2\left(b^2c-abd\right)\sqrt{dx^2+c}}{4\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*
d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b
^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2
), -1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c
- a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d -
 a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d +
 (b^4*c - a*b^3*d)*x^2)]
```

giac [A] time = 0.40, size = 79, normalized size = 0.99

$$\frac{d\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^2+c}\,d}{2\left(\left(dx^2+c\right)b-bc+ad\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*b)
```

maple [B] time = 0.01, size = 1617, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2)+1/4*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)
^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))-1/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x+(-a*b)^(1/2)/b))*c-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4*(-a
*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/
d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))*c-1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/
4/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)-1/4*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^
(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+1/4*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a
*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.74, size = 70, normalized size = 0.88

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}-\frac{d\,\sqrt{d\,x^2+c}}{2\,\left(d\,b^2\,x^2+a\,d\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(3/2)*(a*d - b
*c)^(1/2)) - (d*(c + d*x^2)^(1/2))/(2*(b^2*d*x^2 + a*b*d))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.717 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=82

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {378, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx &= \frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}+\frac{c\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}\\ &= \frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a}\\ &= \frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.29, size = 112, normalized size = 1.37

$$\frac{x\left(\frac{a\left(c+dx^2\right)}{a+bx^2}+\frac{c\sqrt{\frac{dx^2}{c}+1}\,\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{x^2(ad-bc)}{ac}}}\right)}{2a^2\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*((a*(c + d*x^2))/(a + b*x^2) + (c*Sqrt[1 + (d*x^2)/c]*ArcTanh[Sqrt[(-(b/
a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(2*
a^2*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.53, size = 145, normalized size = 1.77

$$\frac{c\sqrt{bc-ad}\,\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{3/2}(ad-bc)}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*
Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x
*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*(-(b*c) + a*d))
```

fricas [B] time = 1.12, size = 369, normalized size = 4.50

$$\left[\frac{4\left(abc-a^{2}d\right)\sqrt{dx^{2}+c}\,x-\left(bcx^{2}+ac\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left((bc-2\,ad)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{8\left(a^{3}bc-a^{4}d+\left(a^{2}b^{2}c-a^{3}bd\right)x^{2}\right)},\ \frac{2\left(abc-a^{2}d\right)\sqrt{dx^{2}+c}\,x+\left(bcx^{2}+ac\right)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\sqrt{abc-a^{2}d}\left((bc-2\,ad)x^{2}-ac\right)\sqrt{dx^{2}+c}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)}{4\left(a^{3}bc-a^{4}d+\left(a^{2}b^{2}c-a^{3}bd\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c + a
^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c - a^4*d + (a^2*b^2*c - a^
3*b*d)*x^2), 1/4*(2*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2 + a*c)*sqr
t(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*s
qrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^3*b*c
 - a^4*d + (a^2*b^2*c - a^3*b*d)*x^2)]
```

giac [B] time = 3.79, size = 218, normalized size = 2.66

$$-\frac{c\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,a}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc\sqrt{d}\,-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad^{\frac{3}{2}}-bc^{2}\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) - ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2) - b
*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a*b)
```

maple [B] time = 0.01, size = 2559, normalized size = 31.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] -1/4/(-a*b)^(1/2)/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a*d^(1/2)/b*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(
1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(
```

```
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x+(-a*b)^(1/2)/b))*d+1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x+(-a*b)^(1/2)/b))*c-1/4/a/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a/b*(-a
*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/b*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*
d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a/b*(-a*b)^(1/2)*d/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+1/4/a*d/(a*d-b*c
)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*x+1/4/a*d^(1/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d
)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))*c-1/4/a/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/a/b*(-a*b)^
(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)-1/4/b*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-
a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/b^2*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+1/4/a*d/(a*d-b*c)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+1/4/a*d^(1/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/
d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/a*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a*d^(1/2)/b*ln(((x-(-a*b)^(1/
2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d-1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1
/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b
)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^2,x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.718 $\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=119

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]])/a^2 + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$

$$= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^2}$$

$$= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c}\right.}{2a^2d}$$

$$= \frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)} - \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.22, size = 112, normalized size = 0.94

$$\frac{\frac{a\sqrt{c+dx^2}}{a+bx^2}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2])/(a + b*x^2) - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]] + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(Sq
rt[b]*Sqrt[b*c - a*d]))/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.53, size = 129, normalized size = 1.08

$$\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2\sqrt{b}\sqrt{ad-bc}}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]/(2*a*(a + b*x^2)) + ((2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b
*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*a^2*Sqrt[b]*Sqrt[-(b*c) + a*d]
) - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2
```

fricas [B] time = 1.51, size = 1054, normalized size = 8.86

$$\left[-\frac{\left(2abc-a^2d+\left(2b^2c-abd\right)x^2\right)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(bdx^2+2bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)-4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}}{8\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^2\right)}, \frac{8\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\left(2abc-a^2d+\left(2b^2c-abd\right)x^2\right)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(bdx^2+2bc-ad\right)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}}{8\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^2\right)}, \frac{\left(2abc-a^2d+\left(2b^2c-abd\right)x^2\right)\sqrt{-b^2c+abd}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)+2\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}}{4\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^2\right)}, \frac{\left(2abc-a^2d+\left(2b^2c-abd\right)x^2\right)\sqrt{-b^2c+abd}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)+4\left(ab^2c-a^2bd+\left(b^3c-ab^2d\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+2\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}}{4\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(c
)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c - a^2*b*
d)*sqrt(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/
8*(8*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) - (2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*
b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*
a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c
```

```
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a*b*c - a^2*d +
(2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a
*b*d^2)*x^2)) + 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(c)*log(-
(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b^2*c - a^2*b*d)*sqrt
(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a
*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d
*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d
 + (b^2*c*d - a*b*d^2)*x^2)) + 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2
)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*
x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2)]
```

giac **[A]** time = 0.36, size = 113, normalized size = 0.95

$$\frac{\sqrt{dx^2+c}\,d}{2\left(\left(dx^2+c\right)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2}+\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*a) - 1/2*(2*b*c - a*d)*a
rctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) +
c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

maple **[B]** time = 0.01, size = 2585, normalized size = 21.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x)
```

```
[Out] -1/2/a^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)+1/2/a^2*d^(1/2)*(-a*b)^(1/2)/b*ln(((x+(-a*b)^(1/2)/b)*d-(-a*
b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2/a/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(
1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-
a*b)^(1/2)/b))*d+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))*c+1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a*d/(a*d-b
*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
```

```
/b)^(1/2)+1/4/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b
)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x+(-a*b)^(1/2)/b))-1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x+(-a*b)^(1/2)/b))*c-1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/(-a*b)
^(1/2)/a*d^(1/2)/(a*d-b*c)*b*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(
1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))*c-1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a
*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)-1/4/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b
)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln
((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x-(-a*b)^(1/2)/b))*c+1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4
/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b
*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))*c-1/2/a^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a^2*d^(1/2)*(-a*b)^(1/2)/b*ln(((x-(-
a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/2/a/b/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x-(-a*b)^(1/2)/b))*c-1/a^2*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+
1/a^2*(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x), x)
```

mupad [B] time = 1.16, size = 996, normalized size = 8.37

$$\frac{d\,\sqrt{d\,x^2+c}}{2\,a\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}-\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a^2}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}-\frac{\left(2\,a\,b^2\,c\,d^3-\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)\,1\mathrm{i}}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}+\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}+\frac{\left(2\,a\,b^2\,c\,d^3+\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)\,1\mathrm{i}}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}}{\frac{b^2\,c^2\,d^3-\frac{a\,b\,c\,d^4}{2}}{a^3}+\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}-\frac{\left(2\,a\,b^2\,c\,d^3-\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}-\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}+\frac{\left(2\,a\,b^2\,c\,d^3+\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,(a\,d-b\,c)}\,(a\,d-2\,b\,c)}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)\,1\mathrm{i}}{2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (d*(c + d*x^2)^(1/2))/(2*a*(b*(c + d*x^2) + a*d - b*c)) - (c^(1/2)*atanh((c
 + d*x^2)^(1/2)/c^(1/2)))/a^2 - (atan((((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*
b^3*c^2*d^2 - 4*a*b^2*c*d^3))/(2*a^2) - ((2*a*b^2*c*d^3 - ((16*a^5*b^2*d^3
- 32*a^4*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))
/(8*a^2*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a
^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c)*1i)/(4*(a^2*b^2*
c - a^3*b*d)) + ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^2 - 4*a*b^2*c
*d^3))/(2*a^2) + ((2*a*b^2*c*d^3 + ((16*a^5*b^2*d^3 - 32*a^4*b^3*c*d^2)*(c
+ d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a^2*b^2*c - a^
3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d)))*(
-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c)*1i)/(4*(a^2*b^2*c - a^3*b*d)))/((b^2*c^
2*d^3 - (a*b*c*d^4)/2)/a^3 + ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^
2 - 4*a*b^2*c*d^3))/(2*a^2) - ((2*a*b^2*c*d^3 - ((16*a^5*b^2*d^3 - 32*a^4*b
^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a
^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c -
 a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d))
 - ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^2 - 4*a*b^2*c*d^3))/(2*a^2
) + ((2*a*b^2*c*d^3 + ((16*a^5*b^2*d^3 - 32*a^4*b^3*c*d^2)*(c + d*x^2)^(1/2
)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a^2*b^2*c - a^3*b*d)))*(-b*
(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c
))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^3*b*d))))*(-b*(a*d - b*c))^(1/2)*
(a*d - 2*b*c)*1i)/(2*(a^2*b^2*c - a^3*b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{x\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x*(a + b*x**2)**2), x)
```

3.719 $\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {469, 583, 12, 377, 205}

$$-\frac{(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-3*Sqrt[c + d*x^2])/(2*a^2*x) + Sqrt[c + d*x^2]/(2*a*x*(a + b*x^2)) - ((3*
b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5
/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
```

```
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)} - \frac{\int \frac{-3c-2dx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x} + \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)} - \frac{\int \frac{c(3bc-2ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2c}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x} + \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)} - \frac{(3bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x} + \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)} - \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a^2}\\
&= -\frac{3\sqrt{c+dx^2}}{2a^2x} + \frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)} - \frac{(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 5.07, size = 101, normalized size = 0.89

$$\frac{(2ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}} + \left(-\frac{bx}{2a^2\left(a+bx^2\right)} - \frac{1}{a^2x}\right)\sqrt{c+dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2))) + ((-3*b*c + 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*Sqrt[
b*c - a*d])
```

IntegrateAlgebraic **[A]**   time = 0.53, size = 133, normalized size = 1.18

$$\frac{\left(-2a-3bx^2\right)\sqrt{c+dx^2}}{2a^2x\left(a+bx^2\right)}-\frac{(3bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{5/2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] ((-2*a - 3*b*x^2)*Sqrt[c + d*x^2])/(2*a^2*x*(a + b*x^2)) - ((3*b*c - 2*a*d)
*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(
Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(-(b*c) + a*d))
```

fricas **[B]**   time = 1.13, size = 458, normalized size = 4.05

$$\left[\frac{\left(\left(3b^2c-2abd\right)x^3+\left(3abc-2a^2d\right)x\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(\left(bc-2ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(2a^2bc-2a^3d+3\left(ab^2c-a^2bd\right)x^2\right)\sqrt{dx^2+c}}{8\left(\left(a^3b^2c-a^4bd\right)x^3+\left(a^4bc-a^5d\right)x\right)},-\frac{\left(\left(3b^2c-2abd\right)x^3+\left(3abc-2a^2d\right)x\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+2\left(2a^2bc-2a^3d+3\left(ab^2c-a^2bd\right)x^2\right)\sqrt{dx^2+c}}{4\left(\left(a^3b^2c-a^4bd\right)x^3+\left(a^4bc-a^5d\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(-a*b*c + a^2*d
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b*c - 2*a^3*d + 3*(a*b^2*c -
 a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d)*x^3 + (a^4*b*c - a^5
*d)*x), -1/4*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(a*b*c
- a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b*c -
2*a^3*d + 3*(a*b^2*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d
)*x^3 + (a^4*b*c - a^5*d)*x)]
```

giac **[B]**   time = 4.07, size = 329, normalized size = 2.91

$$\frac{\left(3bc\sqrt{d}-2ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,a^2}+\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+10\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2acd^{\frac{3}{2}}+3bc^3\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2acd-bc^3\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2)
+ (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 10*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d))/(((sqrt(d)*x
 - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*a^2)
```

maple [B] time = 0.02, size = 2618, normalized size = 23.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] 1/4/a*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))-3/4*b/a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/a^2/c/x*(d*x^2+c)^(3/2)+1/4/a*d^(3/2)
/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/4*b/
a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)-1/4/a^2*d^(1/2)/(a*d-b*c)*b*ln(((x-(-a*b)^(1/2)/b)*d
+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/4/a^2*d/(a*d-b*c)*b*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/a^2*d
^(1/2)/(a*d-b*c)*b*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)*c+3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/
b))*c-3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/
2)/b))*c-1/4/a^2*d/(a*d-b*c)*b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/a^2*d/c*x*(d*x^2+c)^(1/2)+1/4/a^2/
(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-1/4/a^2*(-a*b)^(1/2)*d/(a*
```

```
d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)+3/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))*d+1/4/a^2/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a^2*(-a*b)^(1/
2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)-3/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-
(-a*b)^(1/2)/b))*d-3/4/a^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*
d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))-3/4/a^2*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d
)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))+1/a^2*d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+1/4/a*(-a*b)^(
1/2)*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1
/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))*c-1/4/a*(-a*b)^(1/2)*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^2\,\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{x^2 \left(a + bx^2\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)**2), x)
```

3.720 $\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=159

$$-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 99, 151, 156, 63, 208}

$$-\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)}-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}-\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((b*Sqrt[c + d*x^2])/(a^2*(a + b*x^2))) - Sqrt[c + d*x^2]/(2*a*x^2*(a + b*
x^2)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*Sqrt[c]) -
(Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*a^3*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)}{2a} \\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2(bc-ad)} \\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{(b(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^3} - \frac{(4bc-}{} \\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{(b(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^3d} - \frac{(4}{} \\
&= -\frac{b\sqrt{c+dx^2}}{a^2\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}} - \frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\Bigg(}{2a^3\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 190, normalized size = 1.19

$$
\frac{\sqrt{c}\left(a\left(a+2bx^2\right)\sqrt{c+dx^2}\,(bc-ad)+\sqrt{b}\,x^2\left(a+bx^2\right)(4bc-3ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)-x^2\left(a+bx^2\right)\left(a^2d^2-5abcd+4b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}\,x^2\left(a+bx^2\right)(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*x^2*(a + b*x^2)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]]) + Sqrt[c]*(a*(b*c - a*d)*(a + 2*b*x^2)*Sqrt[c + d*x^2] + Sqrt[b
]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*x^2*(a + b*x^2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]]))/(2*a^3*Sqrt[c]*(-(b*c) + a*d)*x^2*(a + b*x^2))
```

IntegrateAlgebraic [A] time = 0.56, size = 157, normalized size = 0.99

$$
\frac{\left(3a\sqrt{b}\,d-4b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2a^3\sqrt{ad-bc}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}}+\frac{\left(-a-2bx^2\right)\sqrt{c+dx^2}}{2a^2x^2\left(a+bx^2\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

```
[Out] ((-a - 2*b*x^2)*Sqrt[c + d*x^2])/(2*a^2*x^2*(a + b*x^2)) + ((-4*b^(3/2)*c +
 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c -
a*d)])/(2*a^3*Sqrt[-(b*c) + a*d]) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/
Sqrt[c]])/(2*a^3*Sqrt[c])
```

fricas [A] time = 1.64, size = 1043, normalized size = 6.56

$$\left[-\frac{\left(\left(4\,b^{2}c^{2}-3\,abcd\right)x^{4}+\left(4\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(2\,b^{2}c^{2}-3\,abcd+a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}\sqrt{\frac{b}{bc-ad}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+2\left(\left(4\,b^{2}c-abd\right)x^{4}+\left(4\,abc-a^{2}d\right)x^{2}\right)\sqrt{c}\log\left(-\frac{dx^{2}-2\sqrt{dx^{2}+c}\sqrt{c}+2\,c}{x^{2}}\right)+4\left(2\,abcx^{2}+a^{2}c\right)\sqrt{dx^{2}+c}}{8\left(a^{3}bcx^{4}+a^{4}cx^{2}\right)}, -\frac{4\left(\left(4\,b^{2}c-abd\right)x^{4}+\left(4\,abc-a^{2}d\right)x^{2}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(\left(4\,b^{2}c^{2}-3\,abcd\right)x^{4}+\left(4\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(2\,b^{2}c^{2}-3\,abcd+a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}\sqrt{\frac{b}{bc-ad}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\left(2\,abcx^{2}+a^{2}c\right)\sqrt{dx^{2}+c}}{8\left(a^{3}bcx^{4}+a^{4}cx^{2}\right)}, \frac{\left(\left(4\,b^{2}c^{2}-3\,abcd\right)x^{4}+\left(4\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{dx^{2}+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^{2}+bc\right)}\right)-\left(\left(4\,b^{2}c-abd\right)x^{4}+\left(4\,abc-a^{2}d\right)x^{2}\right)\sqrt{c}\log\left(-\frac{dx^{2}-2\sqrt{dx^{2}+c}\sqrt{c}+2\,c}{x^{2}}\right)-2\left(2\,abcx^{2}+a^{2}c\right)\sqrt{dx^{2}+c}}{4\left(a^{3}bcx^{4}+a^{4}cx^{2}\right)}, \frac{\left(\left(4\,b^{2}c^{2}-3\,abcd\right)x^{4}+\left(4\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{dx^{2}+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^{2}+bc\right)}\right)-2\left(\left(4\,b^{2}c-abd\right)x^{4}+\left(4\,abc-a^{2}d\right)x^{2}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)-2\left(2\,abcx^{2}+a^{2}c\right)\sqrt{dx^{2}+c}}{4\left(a^{3}bcx^{4}+a^{4}cx^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 2*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(2*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 +
 c))/(a^3*b*c*x^4 + a^4*c*x^2), -1/8*(4*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c -
 a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c^2 - 3*a*
b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*
x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(
2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*
sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b*c*x^2 + a^2*c)
*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*
x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2
 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ((4
*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 + c))/(a
^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3
*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*((4*b^2*c - a*b*d)*x^
4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(2
*a*b*c*x^2 + a^2*c)*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2)]
```

giac [A] time = 0.36, size = 183, normalized size = 1.15

$$\frac{\left(4\,b^{2}c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{2\,\sqrt{-b^{2}c+abd}\,a^{3}}-\frac{\left(4\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,a^{3}\sqrt{-c}}-\frac{2\,\left(dx^{2}+c\right)^{\frac{3}{2}}bd-2\,\sqrt{dx^{2}+c}\,bcd+\sqrt{dx^{2}+c}\,ad^{2}}{2\,\left(\left(dx^{2}+c\right)^{2}b-2\,\left(dx^{2}+c\right)bc+bc^{2}+\left(dx^{2}+c\right)ad-acd\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*a^3) - 1/2*(4*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))
/(a^3*sqrt(-c)) - 1/2*(2*(d*x^2 + c)^(3/2)*b*d - 2*sqrt(d*x^2 + c)*b*c*d +
sqrt(d*x^2 + c)*a*d^2)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x
^2 + c)*a*d - a*c*d)*a^2)
```

maple [B] time = 0.02, size = 2669, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] -1/4*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/
2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+(-a*b)^(1/2)/b
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(3/2)+1/4*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a
*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-(-a*b)^
(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(3/2)-1/2/a^2/c/x^2*(d*x^2+c)^(3/2)-1/2/a^2*d/c^(1/2)*ln((2*c+2*(d
*x^2+c)^(1/2)*c^(1/2))/x)+1/2/a^2*d/c*(d*x^2+c)^(1/2)+1/a^3*d^(1/2)*(-a*b)^
(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/a^2/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d+2*b/a^3*c^(1/2)*ln((2*c+2*
(d*x^2+c)^(1/2)*c^(1/2))/x)-1/a^3*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/
b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x+(-a*b)^(1/2)/b))*d-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^
2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/
2)/b))*c-1/4*b/a^2*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-b/a^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))*c+b/a^3*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
```

```
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-2*b/a^3*(d*x^2+c)^(1/2)+b/a^3*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b^2
/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c
)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+1/4*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*x+1/4*b^2/a^2/(-a*b)^(1/2)*d^(1/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)
*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/4*b^2/a^2/(-a*b)^(1/2)*d^(1/2)/(a*d
-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4*b/a^2
*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-1/4*b/a^
2*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)-1/4/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3), x)
```

mupad [B] time = 1.69, size = 1193, normalized size = 7.50

$$\frac{\mathrm{atanh}\left(\frac{b^2\,d^6\,\sqrt{d\,x^2+c}}{4\,c^{3/2}\,\left(\frac{b^3\,d^5}{a}-\frac{b^2\,d^6}{4\,c}\right)}-\frac{b^3\,d^5\,\sqrt{d\,x^2+c}}{\sqrt{c}\,\left(b^3\,d^5-\frac{a\,b^2\,d^6}{4\,c}\right)}\right)\,\left(a\,d-4\,b\,c\right)}{2\,a^3\,\sqrt{c}}-\frac{\frac{b\,d\,{\left(d\,x^2+c\right)}^{3/2}}{a^2}+\frac{d\,\sqrt{d\,x^2+c}\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{\left(d\,x^2+c\right)\,\left(a\,d-2\,b\,c\right)+b\,{\left(d\,x^2+c\right)}^2+b\,c^2-a\,c\,d}+\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(\frac{\sqrt{d\,x^2+c}\,\left(5\,a^2\,b^3\,d^4-16\,a\,b^4\,c\,d^3+16\,b^5\,c^2\,d^2\right)}{a^4}-\frac{\left(\frac{2\,a^7\,b^2\,d^4-4\,a^6\,b^3\,c\,d^3}{a^6}-\frac{\left(8\,a^7\,b^2\,d^3-16\,a^6\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,a^4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{4\,\left(a^4\,d-a^3\,b\,c\right)}+\frac{\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(\frac{\sqrt{d\,x^2+c}\,\left(5\,a^2\,b^3\,d^4-16\,a\,b^4\,c\,d^3+16\,b^5\,c^2\,d^2\right)}{a^4}+\frac{\left(\frac{2\,a^7\,b^2\,d^4-4\,a^6\,b^3\,c\,d^3}{a^6}+\frac{\left(8\,a^7\,b^2\,d^3-16\,a^6\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,a^4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{4\,\left(a^4\,d-a^3\,b\,c\right)}}{\frac{\frac{3\,a^2\,b^3\,d^5}{2}-8\,a\,b^4\,c\,d^4+8\,b^5\,c^2\,d^3}{a^6}-\frac{\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(\frac{\sqrt{d\,x^2+c}\,\left(5\,a^2\,b^3\,d^4-16\,a\,b^4\,c\,d^3+16\,b^5\,c^2\,d^2\right)}{a^4}-\frac{\left(\frac{2\,a^7\,b^2\,d^4-4\,a^6\,b^3\,c\,d^3}{a^6}-\frac{\left(8\,a^7\,b^2\,d^3-16\,a^6\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,a^4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\left(3\,a\,d-4\,b\,c\right)}{4\,\left(a^4\,d-a^3\,b\,c\right)}+\frac{\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(\frac{\sqrt{d\,x^2+c}\,\left(5\,a^2\,b^3\,d^4-16\,a\,b^4\,c\,d^3+16\,b^5\,c^2\,d^2\right)}{a^4}+\frac{\left(\frac{2\,a^7\,b^2\,d^4-4\,a^6\,b^3\,c\,d^3}{a^6}+\frac{\left(8\,a^7\,b^2\,d^3-16\,a^6\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,a^4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)}{4\,\left(a^4\,d-a^3\,b\,c\right)}\right)\,\left(3\,a\,d-4\,b\,c\right)}{4\,\left(a^4\,d-a^3\,b\,c\right)}}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(3\,a\,d-4\,b\,c\right)\,1{}\mathrm{i}}{2\,\left(a^4\,d-a^3\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (atan((((-b*(a*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*
c^2*d^2 - 16*a*b^4*c*d^3))/a^4 - (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 -
((8*a^7*b^2*d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2
```

```
)*(3*a*d - 4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d
 - 4*b*c))/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c)*1i)/(4*(a^4*d - a^3*b*c))
 + ((-b*(a*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*
d^2 - 16*a*b^4*c*d^3))/a^4 + (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 + ((8*
a^7*b^2*d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3
*a*d - 4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4
*b*c))/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c)*1i)/(4*(a^4*d - a^3*b*c)))/((
(3*a^2*b^3*d^5)/2 + 8*b^5*c^2*d^3 - 8*a*b^4*c*d^4)/a^6 - ((-b*(a*d - b*c))^
(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*d^2 - 16*a*b^4*c*d^3)
)/a^4 - (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 - ((8*a^7*b^2*d^3 - 16*a^6*
b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*a^4
*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*(a^4*d - a^
3*b*c)))*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c)) + ((-b*(a*d - b*c))^(1/2)*(
((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*d^2 - 16*a*b^4*c*d^3))/a^4 +
 (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 + ((8*a^7*b^2*d^3 - 16*a^6*b^3*c*d
^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*a^4*(a^4*d
 - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c))
)*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c))))*(-b*(a*d - b*c))^(1/2)*(3*a*d -
4*b*c)*1i)/(2*(a^4*d - a^3*b*c)) - ((b*d*(c + d*x^2)^(3/2))/a^2 + (d*(c + d
*x^2)^(1/2)*(a*d - 2*b*c))/(2*a^2))/((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x
^2)^2 + b*c^2 - a*c*d) + (atanh((b^2*d^6*(c + d*x^2)^(1/2))/(4*c^(3/2)*((b^
3*d^5)/a - (b^2*d^6)/(4*c))) - (b^3*d^5*(c + d*x^2)^(1/2))/(c^(1/2)*(b^3*d^
5 - (a*b^2*d^6)/(4*c))))*(a*d - 4*b*c))/(2*a^3*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}}{x^3 \left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)**2), x)
```

3.721 $\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\,(15bc-2ad)}{6a^3cx}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

Rubi **[A]** time = 0.20, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {469, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,(15bc-2ad)}{6a^3cx}+\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}-\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^2])/(6*a^2*x^3) + ((15*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^
3*c*x) + Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + (b*(5*b*c - 4*a*d)*ArcTan[
(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
```

```
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} - \frac{\int \frac{-5c-4dx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{\int \frac{-c(15bc-2ad)-10bcdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2c}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} - \frac{\int -\frac{3bc^2(5bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^3c^2}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{(b(5bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{2a^3}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{(b(5bc-4ad))\,\mathrm{Subst}\Big(\int \frac{1}{a-(-bc}}{2a^3}$$

$$= -\frac{5\sqrt{c+dx^2}}{6a^2x^3} + \frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx} + \frac{\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)} + \frac{b(5bc-4ad)\tan^{-1}\Big(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}}{2a^{7/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 5.14, size = 120, normalized size = 0.82

$$\frac{b(5bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^2}\left(3bx^2\left(\frac{bx^2}{a+bx^2}+4\right)-\frac{2a\left(c+dx^2\right)}{c}\right)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*((-2*a*(c + d*x^2))/c + 3*b*x^2*(4 + (b*x^2)/(a + b*x^2)))
)/(6*a^3*x^3) + (b*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt
[c + d*x^2])])/(2*a^(7/2)*Sqrt[b*c - a*d])
```

IntegrateAlgebraic [A] time = 0.92, size = 170, normalized size = 1.16

$$\frac{\sqrt{bc-ad}\left(5b^2c-4abd\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(ad-bc)}+\frac{\sqrt{c+dx^2}\left(-2a^2c-2a^2dx^2+10abcx^2-2abdx^4+15b^2cx^4\right)}{6a^3cx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c + 10*a*b*c*x^2 - 2*a^2*d*x^2 + 15*b^2*c*x^4 - 2*
a*b*d*x^4))/(6*a^3*c*x^3*(a + b*x^2)) + (Sqrt[b*c - a*d]*(5*b^2*c - 4*a*b*d
)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*
c - a*d])])/(2*a^(7/2)*(-(b*c) + a*d))
```

fricas [B] time = 1.32, size = 602, normalized size = 4.10

$$\left[\frac{3\left(\left(5b^3c^2-4ab^2cd\right)x^5+\left(5ab^2c^2-4a^2bcd\right)x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2+4\left(\left(bc-2ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)-4\left(2a^3bc^2-2a^4cd-\left(15ab^3c^2-17a^2b^2cd+2a^3bd^2\right)x^4-2\left(5a^2b^2c^2-6a^3bcd+a^4d^2\right)x^2\right)\sqrt{dx^2+c}}{24\left(\left(a^4b^2c^2-a^5bcd\right)x^5+\left(a^5bc^2-a^6cd\right)x^3\right)},\frac{3\left(\left(5b^3c^2-4ab^2cd\right)x^5+\left(5ab^2c^2-4a^2bcd\right)x^3\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)-2\left(2a^3bc^2-2a^4cd-\left(15ab^3c^2-17a^2b^2cd+2a^3bd^2\right)x^4-2\left(5a^2b^2c^2-6a^3bcd+a^4d^2\right)x^2\right)\sqrt{dx^2+c}}{12\left(\left(a^4b^2c^2-a^5bcd\right)x^5+\left(a^5bc^2-a^6cd\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c
+ a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b*c^2 - 2
*a^4*c*d - (15*a*b^3*c^2 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2
*c^2 - 6*a^3*b*c*d + a^4*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c
*d)*x^5 + (a^5*b*c^2 - a^6*c*d)*x^3), 1/12*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^
5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
```

```
*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b*c^2 - 2*a^4*c*d - (15*a*b^3*c^2
 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4
*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c*d)*x^5 + (a^5*b*c^2 - a
^6*c*d)*x^3)]
```

giac [B] time = 4.52, size = 361, normalized size = 2.46

$$-\frac{\left(5\,b^{2}c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,a^{3}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abd^{\frac{3}{2}}-b^{2}c^{2}\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)a^{3}}-\frac{2\left(6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}ad^{\frac{3}{2}}-12\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}\sqrt{d}+6\,bc^{3}\sqrt{d}-ac^{2}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c*sqrt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c
))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a*d + b*c^2)*a^3) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt
(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c^2*sqrt(d) + 6*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

maple [B] time = 0.02, size = 2667, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] 5/4*b/a^3*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
-1/3/a^2/c/x^3*(d*x^2+c)^(3/2)+5/4*b^2/a^3/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+5/4*b/a^3*d^(
1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-5/4*b^2/a^3/
(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)-2*b/a^3*d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))+5/4*b^2/a^
3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-5/4*b
/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d+1/
4*b^2/a^3*d^(1/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1
```

```
/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))*c+1/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a^2*(-a*b)^(1/2)*
d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/4*b^2/a
^3*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)*x+1/4*b^2/a^3*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4*b^2/a^3*d^(1/2)/
(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+5/4*b
/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d-5/4
*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
-2*b/a^3*d/c*x*(d*x^2+c)^(1/2)-1/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4
/a^2*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))-1/4*b^2/a^3/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*b/a^2*d^(3/2)/
(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+2*b/a^3
/c/x*(d*x^2+c)^(3/2)-1/4*b^2/a^3/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*b/a
^2*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2))-1/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(
-a*b)^(1/2)/b))*c+1/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}}{x^4\,{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d x^{2}}}{x^{4} \left(a + b x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)**2), x)
```

3.722 $\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=197

$$\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}-\frac{3\sqrt{a}\,(bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^4}+\frac{3x\sqrt{c+dx^2}\,(3bc-4ad)}{8b^3}$$

Rubi [A] time = 0.34, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {467, 581, 582, 523, 217, 206, 377, 205}

$$\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}+\frac{3x\sqrt{c+dx^2}\,(3bc-4ad)}{8b^3}-\frac{3\sqrt{a}\,(bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^4}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*(3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x^3*Sqrt[c + d*x^2])/(
4*b^2) - (x^3*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) - (3*Sqrt[a]*(b*c - 2*a*
d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
2*b^4) + (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(8*b^4*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx=-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{x^2\sqrt{c+dx^2}\left(3c+6dx^2\right)}{a+bx^2}dx}{2b}$$

$$=\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{x^2\left(6c(2bc-3ad)+6d(3bc-4ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}dx}{8b^2}$$

$$=\frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}-\frac{\int \frac{6acd(3bc-4ad)-6d\left(b^2c^2-8abcd+}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{16b^3d}$$

$$=\frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}-\frac{(3a(bc-2ad)(bc-ad))\int}{2b^4}$$

$$=\frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}-\frac{(3a(bc-2ad)(bc-ad))\,\mathrm{Sub}}{}$$

$$=\frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}-\frac{3\sqrt{a}\,(bc-2ad)\sqrt{bc-ad}\ \mathrm{ta}}{2b^4}$$

Mathematica [A] time = 0.17, size = 192, normalized size = 0.97

$$\frac{\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}-\frac{12\sqrt{a}\left(2a^2d^2-3abcd+b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}+\frac{b\sqrt{c+dx^2}\left(-12a^2dx+ab\left(9cx-6dx^3\right)+b^2x^3\left(5c+2dx^2\right)\right)}{a+bx^2}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((b*Sqrt[c + d*x^2]*(-12*a^2*d*x + b^2*x^3*(5*c + 2*d*x^2) + a*b*(9*c*x - 6
*d*x^3)))/(a + b*x^2) - (12*Sqrt[a]*(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[b*c - a*d] + (3*(b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/
(8*b^4)
```

IntegrateAlgebraic [A] time = 0.78, size = 211, normalized size = 1.07

$$\frac{3\sqrt{bc-ad}\left(\sqrt{a}\,bc-2a^{3/2}d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2b^4}-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8b^4\sqrt{d}}+\frac{\sqrt{c+dx^2}\left(-12a^2dx+9abcx-6abdx^3+5b^2cx^3+2b^2dx^5\right)}{8b^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(9*a*b*c*x - 12*a^2*d*x + 5*b^2*c*x^3 - 6*a*b*d*x^3 + 2*b^
2*d*x^5))/(8*b^3*(a + b*x^2)) + (3*Sqrt[b*c - a*d]*(Sqrt[a]*b*c - 2*a^(3/2)
*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(2*b^4) - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Log[-(Sqrt[d]*
x) + Sqrt[c + d*x^2]])/(8*b^4*Sqrt[d])
```

fricas [A] time = 1.84, size = 1249, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*
a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 6
*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*
d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^
2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2 + a*b
^4*d), -1/8*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*
d + 8*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 3*(a*b*
c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^2)*x^3 +
 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2 + a*b^4*d), -
1/16*(12*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a*b*c - a^2
*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c
)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 3*(a*b^2*c^2 - 8*a^2
*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(d)*log
(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^3*d^2*x^5 + (5*b^3*c*
d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b
^5*d*x^2 + a*b^4*d), -1/8*(6*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x
^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) +
3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d
^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^3*d^2*x^5 + (5*
b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 +
c))/(b^5*d*x^2 + a*b^4*d)]
```

giac [B] time = 0.54, size = 394, normalized size = 2.00

$$\frac{1}{8}\sqrt{dx^2+c}\,x\left(\frac{2\,dx^2}{b^2}+\frac{5\,b^7cd^2-8\,ab^6d^3}{b^9d^2}\right)+\frac{3\left(ab^2c^2\sqrt{d}-3\,a^2bcd^{\frac{3}{2}}+2\,a^3d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,b^4}-\frac{3\left(b^2c^2\sqrt{d}-8\,abcd^{\frac{3}{2}}+8\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4d}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^2c^2\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^3d^{\frac{5}{2}}-ab^2c^3\sqrt{d}+a^2bc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*x*(2*d*x^2/b^2 + (5*b^7*c*d^2 - 8*a*b^6*d^3)/(b^9*d^2))
 + 3/2*(a*b^2*c^2*sqrt(d) - 3*a^2*b*c*d^(3/2) + 2*a^3*d^(5/2))*arctan(1/2*(
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(
sqrt(a*b*c*d - a^2*d^2)*b^4) - 3/16*(b^2*c^2*sqrt(d) - 8*a*b*c*d^(3/2) + 8*
a^2*d^(5/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*d) - ((sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2
*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(5/2) - a*b^2*c^3*sq
rt(d) + a^2*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2
)*b^4)
```

maple [B] time = 0.03, size = 4795, normalized size = 24.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -9/8*a^2/b^3*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d
^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))*c+3/4*a^2/b^4*(-a*b)^(1/2)*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4*a/b^2*d/(a
*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(3/2)*x+3/8*a/b^2*d^(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-
a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3/4
/b^4*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*
d^2-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)
/b))*c^2+3/8*a/b^2*d^(1/2)/(a*d-b*c)*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1
/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))+3/4/b^4*a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2
```

```
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x-(-a*b)^(1/2)/b))*d^2+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-
3/8*a^2/b^3*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8*a^2/b^3*d^(3/2)/(a*d-b*c)*ln(((x-(-a*
b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-3/4*a^2/b^4*(-a*b)^(1/2)*d
^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)+1/4*a/b^2*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-3/8*a^2/b^3*d^2/(a*d-b*c)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+1/4/b^2*a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4/b^4*a^2*d^(3/2)*ln(((x+(-a*b)^(1/2
)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/8/b^2*c*x*(d*x^2+c)^(1/2)+3/8/b^2
*c^2/d^(1/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/4/b^2*a/(-a*b)^(1/2)*((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4
/b^4*a^2*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-
3/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/4*a^3/b^5*(-a*b)^(1/2)*d^3/(a*
d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+3/8*a/b^2*d/(a*
d-b*c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)*x+3/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-3/4*a^3/b^5*(-a
*b)^(1/2)*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
+3/8*a/b^2*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/2/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1
/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b
)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c+3/2/b^3*a^2/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c-3/8/b^3*a*d*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8/b^3*a
*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+3/4/b^
```

```
3*a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)*d-3/4/b^2*a/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/4*a/b^2/(a*d-b*
c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(5/2)+3/4*a^3/b^4*d^(5/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/
2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/8/b^3*a*d*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8/b^3*a*d^(1
/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-3/4/b^3*a^2
/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*d+3/4/b^2*a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/4*a/b^2/(a*d-b*c)/(x
-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(5/2)+3/4*a^3/b^4*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)
*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/2*a^2/b^4*(-a*b)^(1/2)*d^2/(a*d-b*c)/
(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-3/4*a/b^3*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-3/2*a
^2/b^4*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b
)^(1/2)/b))*c+3/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x+(-a*b)^(1/2)/b))*c^2+1/4/b^2*x*(d*x^2+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}x^4}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(c + d x^{2}\right)^{\frac{3}{2}}}{\left(a + b x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.723 $\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{\sqrt{c+dx^2}\,(2bc-5ad)}{2b^3}+\frac{\left(c+dx^2\right)^{3/2}(2bc-5ad)}{6b^2(bc-ad)}+\frac{a\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)(bc-a}$$

Rubi **[A]** time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(c+dx^2\right)^{3/2}(2bc-5ad)}{6b^2(bc-ad)}+\frac{\sqrt{c+dx^2}\,(2bc-5ad)}{2b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{a\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^2])/(2*b^3) + ((2*b*c - 5*a*d)*(c + d*x^2)^(3
/2))/(6*b^2*(b*c - a*d)) + (a*(c + d*x^2)^(5/2))/(2*b*(b*c - a*d)*(a + b*x^
2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[b*c - a*d]])/(2*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b^2}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{((2bc-5ad)(}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{((2bc-5ad)(}{}$$

$$= \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\left(c+dx^2\right)^{3/2}}{6b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{5/2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-5ad)\sqrt{}}{}$$

Mathematica [A] time = 0.11, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^2}\left(-15a^2d+ab\left(11c-10dx^2\right)+2b^2x^2\left(4c+dx^2\right)\right)}{6b^3\left(a+bx^2\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(11*c - 10*d*x^2) + 2*b^2*x^2*(4*c + d*x^
2)))/(6*b^3*(a + b*x^2)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.32, size = 150, normalized size = 0.92

$$\frac{\left(-5a^2d^2+7abcd-2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{7/2}\sqrt{ad-bc}} + \frac{\sqrt{c+dx^2}\left(-15a^2d+11abc-10abdx^2+8b^2cx^2+2b^2dx^4\right)}{6b^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(11*a*b*c - 15*a^2*d + 8*b^2*c*x^2 - 10*a*b*d*x^2 + 2*b^2*
d*x^4))/(6*b^3*(a + b*x^2)) + ((-2*b^2*c^2 + 7*a*b*c*d - 5*a^2*d^2)*ArcTan[
(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*b^(7/2)*Sqrt[
-(b*c) + a*d])
```

fricas [A] time = 0.94, size = 413, normalized size = 2.53

$$\left[-\frac{3\left(2\,abc-5\,a^{2}d+\left(2\,b^{2}c-5\,abd\right)x^{2}\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^{2}d^{2}x^{4}+8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}+2\left(4\,b^{2}cd-3\,abd^{2}\right)x^{2}+4\left(b^{2}dx^{2}+2\,b^{2}c-abd\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{b}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\left(2\,b^{2}dx^{4}+11\,abc-15\,a^{2}d+2\left(4\,b^{2}c-5\,abd\right)x^{2}\right)\sqrt{dx^{2}+c}}{24\left(b^{4}x^{2}+ab^{3}\right)},-\frac{3\left(2\,abc-5\,a^{2}d+\left(2\,b^{2}c-5\,abd\right)x^{2}\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^{2}+2\,bc-ad\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^{2}-acd+\left(bcd-ad^{2}\right)x^{2}\right)}\right)-2\left(2\,b^{2}dx^{4}+11\,abc-15\,a^{2}d+2\left(4\,b^{2}c-5\,abd\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\left(b^{4}x^{2}+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(2*a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt((b*c - a*d)/b)
*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*
d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)
/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d +
2*(4*b^2*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/12*(3*(2*
a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2
*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*
d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d + 2*(4*b^2
*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.36, size = 173, normalized size = 1.06

$$\frac{\left(2\,b^{2}c^{2}-7\,abcd+5\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{2\,\sqrt{-b^{2}c+abd}\,b^{3}}+\frac{\sqrt{dx^{2}+c}\,abcd-\sqrt{dx^{2}+c}\,a^{2}d^{2}}{2\left(\left(dx^{2}+c\right)b-bc+ad\right)b^{3}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{4}+3\,\sqrt{dx^{2}+c}\,b^{4}c-6\,\sqrt{dx^{2}+c}\,ab^{3}d}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/2*(sqrt(d*x^2 + c)*a*b*c*d - sqr
t(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^3) + 1/3*((d*x^2 + c)^
(3/2)*b^4 + 3*sqrt(d*x^2 + c)*b^4*c - 6*sqrt(d*x^2 + c)*a*b^3*d)/b^6
```

maple [B] time = 0.02, size = 4673, normalized size = 28.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/4/b^3*(-a*b)^(1/2)*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b^3*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(
1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4*(-a*b)^(1/2)/b^2*d/(a*d-b*
c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(3/2)*x+1/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a*d*c
-3/4*a^3/b^4*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/
b))-1/4*(-a*b)^(1/2)/b^2*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-3/8*(-a*b)^(1/2)/b^2*d^(1/2)/(
a*d-b*c)*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4*
(-a*b)^(1/2)/b^4*a^2*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2
)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))+3/4*a/b^2*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/8*(-a*b)^(1/2)/b^2*d^
(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))+1/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*d*c+3/4*a/b^
2*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*c+3/4*(-a*b)^(1/2)/b^4*a^2*d^(5/2)/(a*d-b*c)*ln(((x+(-a
*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4*a^3/b^4*d^3/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/b^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c
)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-1/2/b^3*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d-1/2/b^2/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-1/2/b^3*((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d+1/6/b^2*
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(3/2)+1/6/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(3/2)-1/2/b^4*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(
-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
```

```
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a-1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x-(-a*b)^(1/2)/b))*a^2*d^2+1/2/b^4*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)
/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a-1/2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x+(-a*b)^(1/2)/b))*a^2*d^2-1/4/b^3*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/b^3*d
^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))*c-1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+1/4*a/b^2*d/(a
*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(3/2)-3/4*a^2/b^3*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4*(-a*b)^(1/2)/b^2/(a*d-b*c
)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(5/2)+1/4*a/b^2*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3/4*a^2/b^3*d^2/(a*d
-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)+3/2*a^2/b^3*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(
-a*b)^(1/2)/b))*c-3/4*a/b^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))*c^2+3/8*(-a*b)^(1/2)/b^2*d/(a*d-b*c)*c*((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8*(-a*b)
^(1/2)/b^3*a*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d
^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))*c-3/8*(-a*b)^(1/2)/b^2*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+9/8*(-a*b)^(1/2)
/b^3*a*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)
+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))*c+3/8*(-a*b)^(1/2)/b^3*a*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/8*(-a*b)^(1/2)/b^3
*a*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*x+3/2*a^2/b^3*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))/(x-(-a*b)^(1/2)/b))*c-3/4*a/b^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln
((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+1/2/b^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
```

```
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+1/2/b^2*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.99, size = 183, normalized size = 1.12

$$\frac{\left(d\,x^2+c\right)^{3/2}}{3\,b^2}-\sqrt{d\,x^2+c}\,\left(\frac{c}{b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}\right)-\frac{\left(\frac{a^2\,d^2}{2}-\frac{a\,b\,c\,d}{2}\right)\,\sqrt{d\,x^2+c}}{b^4\,\left(d\,x^2+c\right)-b^4\,c+a\,b^3\,d}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}\,\left(5\,a\,d-2\,b\,c\right)}{5\,a^2\,d^2-7\,a\,b\,c\,d+2\,b^2\,c^2}\right)\,\sqrt{a\,d-b\,c}\,\left(5\,a\,d-2\,b\,c\right)}{2\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] (c + d*x^2)^(3/2)/(3*b^2) - (c + d*x^2)^(1/2)*(c/b^2 - (2*b^2*c - 2*a*b*d)/
b^4) - (((a^2*d^2)/2 - (a*b*c*d)/2)*(c + d*x^2)^(1/2))/(b^4*(c + d*x^2) - b
^4*c + a*b^3*d) + (atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(1/2)*(5*a*d
 - 2*b*c))/(5*a^2*d^2 + 2*b^2*c^2 - 7*a*b*c*d))*(a*d - b*c)^(1/2)*(5*a*d -
2*b*c))/(2*b^(7/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.724 $\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=149

$$\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^3}+\frac{\sqrt{d}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

Rubi **[A]** time = 0.16, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {467, 528, 523, 217, 206, 377, 205}

$$\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^3}+\frac{\sqrt{d}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{dx\sqrt{c+dx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/b^2 - (x*(c + d*x^2)^(3/2))/(2*b*(a + b*x^2)) + ((b*c
 - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/(2*Sqrt[a]*b^3) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{\sqrt{c+dx^2}\left(c+4dx^2\right)}{a+bx^2}\,dx}{2b}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{\int \frac{2c(bc-2ad)+2d(3bc-4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4b^2}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{(d(3bc-4ad))\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3}+\frac{((bc-4ad)(bc-ad))\int \frac{}{(a}}{2b^3}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{(d(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2b^3}+\frac{((bc-4a}{}\\
&= \frac{dx\sqrt{c+dx^2}}{b^2}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^3}+\frac{\sqrt{d}\,(3bc-4ad}{}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 155, normalized size = 1.04

$$\frac{\frac{\left(4a^2d^2-5abcd+b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}\left(b\left(c-dx^2\right)-2ad\right)}{a+bx^2}+\sqrt{d}\,(3bc-4ad)\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (-((b*x*Sqrt[c + d*x^2]*(-2*a*d + b*(c - d*x^2)))/(a + b*x^2)) + ((b^2*c^2
- 5*a*b*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(Sqrt[a]*Sqrt[b*c - a*d]) + Sqrt[d]*(3*b*c - 4*a*d)*Log[d*x + Sqrt[d]*
Sqrt[c + d*x^2]])/(2*b^3)
```

IntegrateAlgebraic [A] time = 0.66, size = 175, normalized size = 1.17

$$\frac{\left(4ad^{3/2}-3bc\sqrt{d}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^3}-\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2\sqrt{a}\,b^3}+\frac{\sqrt{c+dx^2}\left(2adx-bcx+bdx^3\right)}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c*x) + 2*a*d*x + b*d*x^3))/(2*b^2*(a + b*x^2)) - ((b*
c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c +
 d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*b^3) + ((-3*b*c*Sqrt[d] + 4
*a*d^(3/2))*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^3)
```

<u>fricas</u> [A] time = 1.50, size = 996, normalized size = 6.68

$$\left[-\frac{2\left(3abc - 4a^2d + \left(3b^2c - 4abd\right)x^2\right)\sqrt{d}\log\left(-2dx^2 + 2\sqrt{dx^2 + c}\sqrt{d}x - c\right) + \left(abc - 4a^2d + \left(b^2c - 4abd\right)x^2\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2 - 8abcd + 8a^2d^2\right)x^4 + a^2c^2 - 2\left(3abc^2 - 4a^2cd\right)x^2 + 4\left(a^2cx - \left(abc - 2a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4 + 2abx^2 + a^2}\right) - 4\left(b^2dx^3 - \left(b^2c - 2abd\right)x\right)\sqrt{dx^2 + c}}{8\left(b^4x^2 + ab^3\right)}, -\frac{4\left(3abc - 4a^2d + \left(3b^2c - 4abd\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) + \left(abc - 4a^2d + \left(b^2c - 4abd\right)x^2\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2 - 8abcd + 8a^2d^2\right)x^4 + a^2c^2 - 2\left(3abc^2 - 4a^2cd\right)x^2 + 4\left(a^2cx - \left(abc - 2a^2d\right)x^3\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4 + 2abx^2 + a^2}\right) - 4\left(b^2dx^3 - \left(b^2c - 2abd\right)x\right)\sqrt{dx^2 + c}}{8\left(b^4x^2 + ab^3\right)}, \frac{\left(abc - 4a^2d + \left(b^2c - 4abd\right)x^2\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2ad\right)x^2 - ac\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3 + \left(bc^2-acd\right)x\right)}\right) - \left(3abc - 4a^2d + \left(3b^2c - 4abd\right)x^2\right)\sqrt{d}\log\left(-2dx^2 + 2\sqrt{dx^2 + c}\sqrt{d}x - c\right) + 2\left(b^2dx^3 - \left(b^2c - 2abd\right)x\right)\sqrt{dx^2 + c}}{4\left(b^4x^2 + ab^3\right)}, -\frac{2\left(3abc - 4a^2d + \left(3b^2c - 4abd\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) - \left(abc - 4a^2d + \left(b^2c - 4abd\right)x^2\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2ad\right)x^2 - ac\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3 + \left(bc^2-acd\right)x\right)}\right) - 2\left(b^2dx^3 - \left(b^2c - 2abd\right)x\right)\sqrt{dx^2 + c}}{4\left(b^4x^2 + ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2
 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*
x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*
d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/8*(4*(3
*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-(b*c - a*d)/
a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b
*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 - (b^2*c - 2*a*b*
d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), 1/4*((a*b*c - 4*a^2*d + (b^2*c -
4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt
(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
- (3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x
^2 + c))/(b^4*x^2 + a*b^3), -1/4*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d
)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b*c - 4*a^2*d + (b^
2*c - 4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) - 2*(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b
^3)]
```

<u>giac</u> [B] time = 0.51, size = 336, normalized size = 2.26

$$\frac{\sqrt{dx^2 + c}\,dx}{2\,b^2} - \frac{\left(3\,bc\sqrt{d} - 4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2\right)}{4\,b^3} - \frac{\left(b^2c^2\sqrt{d} - 5\,abcd^{\frac{3}{2}} + 4\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{2\,\sqrt{abcd - a^2d^2}\,b^3} + \frac{\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 b^2c^2\sqrt{d} - 3\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 abcd^{\frac{3}{2}} + 2\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 a^2d^{\frac{5}{2}} - b^2c^3\sqrt{d} + abc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 b - 2\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 bc + 4\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 ad + bc^2\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d*x/b^2 - 1/4*(3*b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt(d
)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^2*c^2*sqrt(d) - 5*a*b*c*d^(3/2) + 4*
```

```
a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) + ((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(
3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*
b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

maple [B] time = 0.02, size = 4685, normalized size = 31.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -3/8/b*d/(a*d-b*c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/8/b^2*a*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+9/8/b^2*a*d^(3
/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-3
/4/b^3*(-a*b)^(1/2)*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a+3/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*c+1/4/(-a*b)^(1/2)/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/
2)/b))*a^2*d^2+3/4/b^3*(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a-3/4/b^2*(-a*b)^(1
/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)*c-3/8/b*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/(-a*b)^(1/2)/b^3/(-
(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+
2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a^2*d^2+3/8/b^2*a*d^2/(a*
d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)*x+9/8/b^2*a*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^
(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))*c+1/8/b^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/8/b^2*d^(1/2)*ln(((x-(-a*b)
^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4/(-a*b)^(1/2)/b*((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1
/4/b^3*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a+
1/4/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+1/4/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)
```

```
*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(5/2)+1/8/b^2*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*x+3/8/b^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1
/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))*c-1/4/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/4/b^3*d^(3/2)*ln(((x
+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*a-1/4/b*d/(a*d-b*c)*((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/
2)*x-3/8/b*d^(1/2)/(a*d-b*c)*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)
/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))+1/2/(-a*b)^(1/2)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a
*b)^(1/2)/b))*a*d*c+3/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*l
n((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-3/2/b^3*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a*c+3/2/b^3*(-a*b)^(1/2)*d^2/
(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2
*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a*c-1/4/b^2*
(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3/4/b^3*a^2*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^
(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(3/2)-3/4/b^3*a^2*d^(5/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)
/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*a*d-1/4/(-a*b)^(1/2)/b/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+1/4/(-a*b)^(1/2)/b^2*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*a*d+1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)
/b))*c^2-1/4/b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-3/8/b*d^(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b
)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/12/(-a*b)^(1/2)/b*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/1
```

```
2/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(3/2)-3/4/b^4*(-a*b)^(1/2)*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a^2-3/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-1/2/(-a*b)^(1/2)/b^2
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c
)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*a*d*c+3/4/b^4*(-a*b)^
(1/2)*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}x^2}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(d\,x^2+c\right)^{3/2}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.725 $\int \frac{x\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=99

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

Rubi **[A]** time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 47, 50, 63, 208}

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}+\frac{3d\sqrt{c+dx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x^2])/(2*b^2) - (c + d*x^2)^(3/2)/(2*b*(a + b*x^2)) - (3*d*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(5
/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^2\right)\\
&= -\frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{(3d(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b^2}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} + \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2b^2}\\
&= \frac{3d\sqrt{c+dx^2}}{2b^2} - \frac{\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)} - \frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.55

$$\frac{d\left(c+dx^2\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{5(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(c + d*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x^2))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^2)
```

IntegrateAlgebraic [A] time = 0.28, size = 106, normalized size = 1.07

$$\frac{3d\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2b^{5/2}}+\frac{\sqrt{c+dx^2}\,\left(3ad-bc+2bdx^2\right)}{2b^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(2*b^2*(a + b*x^2)) + (3*d*S
qrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c
 - a*d)])/(2*b^(5/2))
```

fricas [A] time = 1.31, size = 333, normalized size = 3.36

$$\left[\frac{3\left(bdx^2+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(2\,bdx^2-bc+3\,ad\right)\sqrt{dx^2+c}}{8\left(b^3x^2+ab^2\right)},-\frac{3\left(bdx^2+ad\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\left(2\,bdx^2-bc+3\,ad\right)\sqrt{dx^2+c}}{4\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d*x^2 + a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*
c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2), -1/4*(3
*(b*d*x^2 + a*d)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
- 2*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2)]
```

giac [A] time = 0.43, size = 122, normalized size = 1.23

$$\frac{\sqrt{dx^2+c}\,d}{b^2}+\frac{3\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx^2+c}\,bcd-\sqrt{dx^2+c}\,ad^2}{2\left(\left(dx^2+c\right)b-bc+ad\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + c)*d/b^2 + 3/2*(b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-
b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/2*(sqrt(d*x^2 + c)*b*c*d - s
qrt(d*x^2 + c)*a*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^2)
```

maple [B] time = 0.01, size = 2821, normalized size = 28.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] -3/2*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b)
)*c-3/8*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a
*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/2*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(
1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+
9/8*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2
)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))*c-3/4*(-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln(((x+(-a
*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/4*a^2/b^3*d^3/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+3/4/b*d/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2+1/4*(-a*b)^(1/2)/a/b/(a
*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+3/8*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*((x+(-a
```

```
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
+3/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+
3/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b
))-3/8*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8*(-a*b)^(1/2)/b^2*d^(3/2)/
(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+3/4*(
-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b
*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))-1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-
1/4/b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(3/2)-1/4/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4*a/b^2*d^2/(a*d-b*c)*((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2)+3/4*a/b^2*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/8*(-a*b)^(1/2)
/a/b*d^(1/2)/(a*d-b*c)*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/
2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))+1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-3/4/b*d/(a*d-b*c)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*
c+3/8*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/8*(-a*b)^(1/2)/a/b*d/(a*d-b*
c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.90, size = 117, normalized size = 1.18

$$\frac{\sqrt{d\,x^2+c}\,\left(\frac{a\,d^2}{2}-\frac{b\,c\,d}{2}\right)}{b^3\,\left(d\,x^2+c\right)-b^3\,c+a\,b^2\,d}+\frac{d\,\sqrt{d\,x^2+c}}{b^2}-\frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

```
[Out] ((c + d*x^2)^(1/2)*((a*d^2)/2 - (b*c*d)/2))/(b^3*(c + d*x^2) - b^3*c + a*b^
2*d) + (d*(c + d*x^2)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(c + d*x^2)^(1/2)*(
a*d - b*c)^(1/2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/(2*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.726 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{bc-ad}\,(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}\,(bc-ad)}{2ab\left(a+bx^2\right)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Rubi **[A]** time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {413, 523, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}\,(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{x\sqrt{c+dx^2}\,(bc-ad)}{2ab\left(a+bx^2\right)}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + (Sqrt[b*c - a*d]*(b*c
 + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)
*b^2) + (d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx=\frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{\int \frac{c(bc+ad)+2ad^2x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$=\frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{d^2\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2}+\frac{((bc-ad)(bc+2ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab^2}$$

$$=\frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{d^2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{b^2}+\frac{((bc-ad)(bc+2ad))\,\text{Subst}\left(\int \frac{}{a-(}\right.}{2ab^2}$$

$$=\frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}+\frac{\sqrt{bc-ad}\,(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Mathematica [A] time = 0.16, size = 141, normalized size = 1.08

$$\frac{\frac{\left(-2a^2d^2+abcd+b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}+\frac{bx\sqrt{c+dx^2}\,(bc-ad)}{a\left(a+bx^2\right)}+2d^{3/2}\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*(b*c - a*d)*x*Sqrt[c + d*x^2])/(a*(a + b*x^2)) + ((b^2*c^2 + a*b*c*d -
2*a^2*d^2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*
Sqrt[b*c - a*d]) + 2*d^(3/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^2)
```

IntegrateAlgebraic [A] time = 0.64, size = 155, normalized size = 1.18

$$-\frac{\sqrt{bc-ad}\,(2ad+bc)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{3/2}b^2}-\frac{x\sqrt{c+dx^2}\,(ad-bc)}{2ab\left(a+bx^2\right)}-\frac{d^{3/2}\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((-(b*c) + a*d)*x*Sqrt[c + d*x^2])/(a*b*(a + b*x^2)) - (Sqrt[b*c - a*d
]*(b*c + 2*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(S
qrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*b^2) - (d^(3/2)*Log[-(Sqrt[d]*x) + Sqr
t[c + d*x^2]])/b^2
```

fricas [A] time = 1.68, size = 903, normalized size = 6.89

$$\left[\frac{4\left(b^2c-abd\right)\sqrt{dx^2+c}\,x+4\left(abdx^2+a^2d\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+\left(abc+2a^2d+\left(b^2c+2abd\right)x^2\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(a^2cx-\left(abc-2a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)}{8\left(ab^3x^2+a^2b^2\right)},\ \frac{4\left(b^2c-abd\right)\sqrt{dx^2+c}\,x-8\left(abdx^2+a^2d\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(abc+2a^2d+\left(b^2c+2abd\right)x^2\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(a^2cx-\left(abc-2a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2abx^2+a^2}\right)}{8\left(ab^3x^2+a^2b^2\right)},\ \frac{2\left(b^2c-abd\right)\sqrt{dx^2+c}\,x+\left(abc+2a^2d+\left(b^2c+2abd\right)x^2\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)+2\left(abdx^2+a^2d\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{4\left(ab^3x^2+a^2b^2\right)},\ \frac{2\left(b^2c-abd\right)\sqrt{dx^2+c}\,x-4\left(abdx^2+a^2d\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(abc+2a^2d+\left(b^2c+2abd\right)x^2\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)}{4\left(ab^3x^2+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x + 4*(a*b*d*x^2 + a^2*d)*sqrt(d)*l
og(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c + 2*a^2*d + (b^2*c
+ 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^
2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)
))/(a*b^3*x^2 + a^2*b^2), 1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x - 8*(a*b
*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^
2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x -
(a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a
*b*x^2 + a^2)))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2*c - a*b*d)*sqrt(d*x^2 +
c)*x + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan
(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d)*log(-2*d
*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2
*c - a*b*d)*sqrt(d*x^2 + c)*x - 4*(a*b*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(
```

```
-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*
c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
- a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/(a*b^3*x^2 + a^2*b^2)
]
```

giac [B] time = 0.49, size = 315, normalized size = 2.40

$$-\frac{d^{\frac{3}{2}}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2b^2}-\frac{\left(b^2c^2\sqrt{d}+abcd^{\frac{3}{2}}-2a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}ab^2}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abcd^{\frac{3}{2}}+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{5}{2}}-b^2c^3\sqrt{d}+abc^2d^{\frac{3}{2}}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 - 1/2*(b^2*c^2*sqrt(d
) + a*b*c*d^(3/2) - 2*a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*b^
2) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2
) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b
- 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a*d + b*c^2)*a*b^2)
```

maple [B] time = 0.02, size = 4689, normalized size = 35.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^2,x)
```

```
[Out] 3/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/2/b^2*(-a*b)^(1/2)*d^2/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c+3/4*a/b^3*(-a*b)^(1/
2)*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-3/2/b
^2*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1
/2)/b))*c-3/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/4/b^2*d^(3/2)*ln(((x-(
-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/12/(-a*b)^(1/2)/a*((x-(
```

```
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)
-1/4/b^2*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-
1/12/(-a*b)^(1/2)/a*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/8/a*d/(a
*d-b*c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)*x+1/4/(-a*b)^(1/2)*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x+(-a*b)^(1/2)/b))*d^2-1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x+(-a*b)^(1/2)/b))*d*c+1/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/8/a*d/(a*d
-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*x-1/4/(-a*b)^(1/2)*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(
-a*b)^(1/2)/b))*d^2+1/2/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(
-a*b)^(1/2)/b))*d*c+1/4/(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d-1/4/(-a*b)^(1/2)/a*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1
/4/a/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/4/(-a*b)^(1/2)/b*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+1/4/(-a*
b)^(1/2)/a*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*c-1/4/a/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+1/8/a/b*d*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*x+3/8/a/b*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)*c-1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*
c^2+3/8/a/b*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))*c+1/4/(-a*b)^(1/2)/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b)
)*c^2-3/8/b*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4*a/b^3*(-a*b)^(1/2)*d^3/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
```

```
(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-9/8/b*d^(3/2)/(a*d-b*c)*ln(
((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-3/4/b^2*(-a*b)^(1/
2)*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)+3/4*a/b^2*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d
+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/a*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+3/8/a*d^(1/2)/(
a*d-b*c)*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-9/8/
b*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))*c+3/4/b^2*(-a*b)^(1/2)*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*a/b^2*d^(5/2)/(a*d-b*c)*l
n(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/a*d/(a*d-b*c)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(3/2)*x+3/8/a*d^(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b
*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))+1/8/a/b*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/8/b*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a/b*(-
a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*
c^2+3/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(
1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-
a*b)^(1/2)/b))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^2,x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.727 $\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2ab\left(a+bx^2\right)}$$

Rubi **[A]** time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) - (c^(3/2)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]])/a^2 + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} + \frac{\operatorname{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{((bc-ad)(2bc+ad))\operatorname{Subst}\left(\int \frac{}{(a+bx)}\right.}{4a^2b}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} + \frac{c^2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2d} - \frac{((bc-ad)(2bc+ad))\operatorname{Subst}\left(\int\right.}{2a^2b}$$

$$= \frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2} + \frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}$$

Mathematica [A] time = 0.16, size = 122, normalized size = 0.95

$$\frac{\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}+\frac{a\sqrt{c+dx^2}\,(bc-ad)}{b\left(a+bx^2\right)}-2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^2])/(b*(a + b*x^2)) - 2*c^(3/2)*ArcTanh[Sqrt[c
 + d*x^2]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.33, size = 154, normalized size = 1.19

$$\frac{\left(-a^2d^2-abcd+2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2a^2b^{3/2}\sqrt{ad-bc}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + ((2*b^2*c^2 - a*b*c*d -
 a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])
/(2*a^2*b^(3/2)*Sqrt[-(b*c) + a*d]) - (c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt
[c]])/a^2
```

fricas [A] time = 1.64, size = 883, normalized size = 6.84

$$\left[\frac{\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(b^2cx^2+abc\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+4\left(abc-a^2d\right)\sqrt{dx^2+c}}{8\left(a^2b^2x^2+a^3b\right)},\ \frac{8\left(b^2cx^2+abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(abc-a^2d\right)\sqrt{dx^2+c}}{8\left(a^2b^2x^2+a^3b\right)},\ \frac{\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+2\left(b^2cx^2+abc\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(abc-a^2d\right)\sqrt{dx^2+c}}{4\left(a^2b^2x^2+a^3b\right)},\ \frac{\left(2abc+a^2d+\left(2b^2c+abd\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)+4\left(b^2cx^2+abc\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+2\left(abc-a^2d\right)\sqrt{dx^2+c}}{4\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/b)*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^
2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*s
qrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^
2*b^2*x^2 + a^3*b), 1/8*(8*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqr
t(d*x^2 + c)) + (2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/
b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*
b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*
```

```
d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a
^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt(-(
b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*
c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^2*c*x^2 + a*b*c)*
sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c - a^
2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2
*c + a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*s
qrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2))
+ 4*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a*b*
c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.29, size = 154, normalized size = 1.19

$$\frac{c^{2} \arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{a^{2} \sqrt{-c}} - \frac{\left(2 \, b^{2} c^{2} - abcd - a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{dx^{2}+c} \, b}{\sqrt{-b^{2} c+abd}}\right)}{2 \, \sqrt{-b^{2} c + abd} \, a^{2} b} + \frac{\sqrt{dx^{2} + c} \, bcd - \sqrt{dx^{2} + c} \, ad^{2}}{2 \, \left(\left(dx^{2} + c\right)b - bc + ad\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/2*(2*b^2*c^2 - a*b*
c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*a^2*b) + 1/2*(sqrt(d*x^2 + c)*b*c*d - sqrt(d*x^2 + c)*a*d^2)/(((d*
x^2 + c)*b - b*c + a*d)*a*b)
```

maple [B] time = 0.02, size = 4718, normalized size = 36.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x)
```

```
[Out] -1/4/a^2*(-a*b)^(1/2)/b*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a^2/b*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a
*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-1/a/b/(-(a*d-b*c)/b)^(1/2
)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c-3/4/(-a*b)^(1/2)*a/b*d^(5/2)/(a*d-b*c)
*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4*a*d^3/(a*d-b
*c)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+3/2*d^2/(a*d-b
```

```
*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-3/4/a*d/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2+1/4/(-a*b)^(1/
2)/a/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x
-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(5/2)+3/4/(-a*b)^(1/2)*a/b*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)
^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4*a*d^3/(a*d-b*c)/b^2/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+3/2*d^2/(a*d-b*c)/b/(-(a*d-b*
c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c-3/4/a*d/(a*d-b*c)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-1/a/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)
*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))/(x+(-a*b)^(1/2)/b))*d*c+3/4/a^2/b*d^(1/2)*(-a*b)^(1/2)*ln(((x+(-a*b
)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4/a^2*(-a*b)^(1/2)/b*d*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*x+1/2/a/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*d+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1
/2)/b))*d^2+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^
2+1/2/a/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)*d+1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b
))*d^2+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+1/4/
a*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(3/2)-3/4*d^2/(a*d-b*c)/b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a*d/(a*d-b*c)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3/4*d
```

```
^2/(a*d-b*c)/b*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)+1/3/a^2*(d*x^2+c)^(3/2)-1/6/a^2*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/6/a^2*((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3/
8/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*b*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1
/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/2/a/b^2*
d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2))+1/2/a/b^2*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*
d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))-3/8/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8/(-a*b)^(1/2)*d
^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*
c+3/4/a*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/8/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+9/8/(-a*
b)^(1/2)*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/
2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2))*c+3/4/a*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/8/(-a*b)^(1/2)/a*d^(1/2)/(a*d-b*c)*
b*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^
(1/2)/a*d/(a*d-b*c)*b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+3/8/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*c*((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/
8/(-a*b)^(1/2)/a*d/(a*d-b*c)*b*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2/a^2*((x+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/2/a^2*((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-1/
a^2*c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/a^2*(d*x^2+c)^(1/2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x), x)
```

mupad [B] time = 1.10, size = 488, normalized size = 3.78

$$-\frac{\operatorname{atanh}\left(\frac{d^6\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\left(\frac{c^2\,d^6}{2}+\frac{b\,c^3\,d^5}{a}-\frac{3\,b^2\,c^4\,d^4}{2\,a^2}\right)}+\frac{c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{c^3\,d^5+\frac{a\,c^2\,d^6}{2\,b}-\frac{3\,b\,c^4\,d^4}{2\,a}}-\frac{3\,b\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\left(a\,c^3\,d^5-\frac{3\,b\,c^4\,d^4}{2}+\frac{a^2\,c^2\,d^6}{2\,b}\right)}\right)\sqrt{c^3}}{a^2}-\frac{\operatorname{atanh}\left(\frac{5\,c^2\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{4\left(\frac{a^2\,c\,d^7}{4}+\frac{b^2\,c^3\,d^5}{4}-\frac{3\,b^3\,c^4\,d^4}{2\,a}+a\,b\,c^2\,d^6\right)}+\frac{3\,c^3\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{2\left(a^2\,c^2\,d^6-\frac{3\,b^2\,c^4\,d^4}{2}+\frac{a^3\,c\,d^7}{4\,b}+\frac{a\,b\,c^3\,d^5}{4}\right)}+\frac{c\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{4\left(b^2\,c^2\,d^6+\frac{a\,b\,c\,d^7}{4}+\frac{b^3\,c^3\,d^5}{4\,a}-\frac{3\,b^4\,c^4\,d^4}{2\,a^2}\right)}\right)\left(a\,d+2\,b\,c\right)\sqrt{-b^3\,\left(a\,d-b\,c\right)}}{2\,a^2\,b^3}-\frac{d\,\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}{2\,a\,b\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x)
```

```
[Out] - (atanh((d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(2*((c^2*d^6)/2 + (b*c^3*d^5)/
a - (3*b^2*c^4*d^4)/(2*a^2))) + (c*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(c^3*
d^5 + (a*c^2*d^6)/(2*b) - (3*b*c^4*d^4)/(2*a)) - (3*b*c^2*d^4*(c + d*x^2)^(
1/2)*(c^3)^(1/2))/(2*(a*c^3*d^5 - (3*b*c^4*d^4)/2 + (a^2*c^2*d^6)/(2*b))))*
(c^3)^(1/2))/a^2 - (atanh((5*c^2*d^5*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1
/2))/(4*((a^2*c*d^7)/4 + (b^2*c^3*d^5)/4 - (3*b^3*c^4*d^4)/(2*a) + a*b*c^2*
d^6)) + (3*c^3*d^4*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1/2))/(2*(a^2*c^2*d
^6 - (3*b^2*c^4*d^4)/2 + (a^3*c*d^7)/(4*b) + (a*b*c^3*d^5)/4)) + (c*d^6*(c
+ d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1/2))/(4*(b^2*c^2*d^6 + (a*b*c*d^7)/4 + (
b^3*c^3*d^5)/(4*a) - (3*b^4*c^4*d^4)/(2*a^2))))*(a*d + 2*b*c)*(-b^3*(a*d -
b*c))^(1/2))/(2*a^2*b^3) - (d*(c + d*x^2)^(1/2)*(a*d - b*c))/(2*a*b*(b*(c +
 d*x^2) + a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.728 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{3c\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^2}\,(3bc-ad)}{2a^2bx}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2abx\left(a+bx^2\right)}$$

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {468, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}\,(3bc-ad)}{2a^2bx}-\frac{3c\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2abx\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -((3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*Sqrt[c + d*x^2]
)/(2*a*b*x*(a + b*x^2)) - (3*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)} - \frac{\int \frac{-c(3bc-ad)-2bcdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)} + \frac{\int -\frac{3bc^2(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2bc}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)} - \frac{(3c(bc-ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)} - \frac{(3c(bc-ad))\,\mathrm{Subst}\Bigg(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+}}}{2a^2}$$

$$= -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx\left(a+bx^2\right)} - \frac{3c\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.41

$$-\frac{c\sqrt{c+dx^2}\ {}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{(ad-bc)x^2}{a(dx^2+c)}\right)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -((c*Sqrt[c + d*x^2]*Hypergeometric2F1[-1/2, 2, 1/2, ((-(b*c) + a*d)*x^2)/(
a*(c + d*x^2))])/(a^2*x))
```

IntegrateAlgebraic [A] time = 0.60, size = 155, normalized size = 1.21

$$\frac{3c\sqrt{bc-ad}\ \tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^2}\,\left(-2ac+adx^2-3bcx^2\right)}{2a^2x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(2*a^2*x*(a + b*x^2)) + (3
*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^
2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*
d])])/(2*a^(5/2))
```

fricas [A] time = 0.99, size = 351, normalized size = 2.74

$$\left[\frac{3\left(bcx^3+acx\right)\sqrt{-\frac{bc-ad}{a}}\ \log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(a^2cx-\left(abc-2\,a^2d\right)x^3\right)\sqrt{dx^2+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left((3\,bc-ad)x^2+2\,ac\right)\sqrt{dx^2+c}}{8\left(a^2bx^3+a^3x\right)},-\frac{3\left(bcx^3+acx\right)\sqrt{\frac{bc-ad}{a}}\ \arctan\left(\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^2\right)x^3+\left(bc^2-acd\right)x\right)}\right)+2\left((3\,bc-ad)x^2+2\,ac\right)\sqrt{dx^2+c}}{4\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*x^3 + a*c*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*x^3 + a
^3*x), -1/4*(3*(b*c*x^3 + a*c*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a
*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (
b*c^2 - a*c*d)*x)) + 2*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*
x^3 + a^3*x)]
```

giac [B] time = 3.49, size = 412, normalized size = 3.22

$$\frac{3\left(bc^{2}\sqrt{d}-acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,a^{2}}+\frac{3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b^{2}c^{2}\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}abcd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}a^{2}d^{\frac{5}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c^{3}\sqrt{d}+12\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abc^{2}d^{\frac{3}{2}}-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}cd^{\frac{5}{2}}+3\,b^{2}c^{4}\sqrt{d}-abc^{3}d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}ad+3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}-4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}acd-bc^{3}\right)a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 3/2*(b*c^2*sqrt(d) - a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2)
+ (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(5/2)
 - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 12*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^
(5/2) + 3*b^2*c^4*sqrt(d) - a*b*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 +
 c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*c*d - b*c^3)*a^2*b)
```

maple [B] time = 0.02, size = 4764, normalized size = 37.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -3/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/4*(-a*b)^(1/2)*d^3/(a*d-b*c)/b^
2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c
)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a^2*d/(a*d-b*c)*b
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(3/2)*x-3/8/a^2*d^(1/2)/(a*d-b*c)*b*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1
/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))+3/4/a*(-a*b)^(1/2)*d^2/(a*d-b*c)/b*((x-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/2/a/(-a*
b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d*c+3/4*b/a^2/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-3/8/a^
2*d^(1/2)/(a*d-b*c)*b*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2
)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
```

```
)^(1/2))+3/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-3/4*(-a*b)^(1/2)*d^3/(a*d
-b*c)/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2
*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4/a^2*d/(a
*d-b*c)*b*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(3/2)*x-3/4/a*(-a*b)^(1/2)*d^2/(a*d-b*c)/b*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*b/a^2/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2+3/2/a/(-a*b
)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c-3/8/a^2*d*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x-9/8/a^2*d^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))*c-3/4/a/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+3/4/b/a*d^(3/2)*ln(((x+(-a*b)^(1/2)/b)*
d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4/b*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^(1
/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/a^2/c/x*(d*x^2+c)^(5/2)+3/2/a^2
*d*x*(d*x^2+c)^(1/2)+3/2/a^2*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-1/4*b/
a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(3/2)-3/8/a^2*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8/a^2*d^(1/2)*ln(((x-(-a*b)^(1/2
)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+3/4/a/(-a*b)^(1/2)*((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+3/4/b/
a*d^(3/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-3/4/b*d
^(5/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+
1/4*b/a^2/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a^2/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)*((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-1
/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*d^2+1/a^2*d/c*x*(d*x^2+c)^(3/2)+3/4*b/a^2
/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*c-3/4/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*
```

```
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x+(-a*b)^(1/2)/b))*d^2+3/8/a*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+9/8/a*d^(3/2)/(a*d-b*c)*
ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4/a^2/(a*d-b*
c)*b/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+1/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(
1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/8/a*
d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2)*x+9/8/a*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*
b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))*c-3/4*b/a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/2/a*(-a*b)^
(1/2)*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*
c-3/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*
b)^(1/2)/b))*c^2-3/8/a^2*d/(a*d-b*c)*b*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/a^2*(-a*b)^(1/2)*d/(a*
d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-3/8/a^2*d/(a
*d-b*c)*b*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)*x-3/2/a*(-a*b)^(1/2)*d^2/(a*d-b*c)/b/(-(a*d-b*c)/b)^(1/2)
*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(
1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2 + c\right)^{\frac{3}{2}}}{\left(bx^2 + a\right)^2 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{x^2\,\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)**2), x)
```

3.729 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{\sqrt{c+dx^2}\,(2bc-ad)}{2a^2\left(a+bx^2\right)}-\frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 98, 151, 156, 63, 208}

$$-\frac{\sqrt{c+dx^2}\,(2bc-ad)}{2a^2\left(a+bx^2\right)}+\frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}-\frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*(a + b*x^2)) - (c*Sqrt[c + d*x^2])/
(2*a*x^2*(a + b*x^2)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/(2*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x,\right.}{2a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4a^3} + \frac{((bc}{}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2a^3d}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2\left(a+bx^2\right)} - \frac{c\sqrt{c+dx^2}}{2ax^2\left(a+bx^2\right)} + \frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} - \frac{\sqrt{bc-ad}\,(4bc}{}$$

Mathematica [A] time = 0.32, size = 142, normalized size = 0.84

$$\frac{\frac{a\sqrt{c+dx^2}\left(-ac+adx^2-2bcx^2\right)}{x^2\left(a+bx^2\right)}+\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)-\frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(-(a*c) - 2*b*c*x^2 + a*d*x^2))/(x^2*(a + b*x^2)) + Sqr
t[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]] - (Sqrt[b*c - a*d]*(4
*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/Sqrt[b])/(2
*a^3)
```

IntegrateAlgebraic [A] time = 0.67, size = 179, normalized size = 1.05

$$\frac{\left(4bc^{3/2}-3a\sqrt{c}\,d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}+\frac{\sqrt{c+dx^2}\left(-ac+adx^2-2bcx^2\right)}{2a^2x^2\left(a+bx^2\right)}+\frac{\left(-a^2d^2+5abcd-4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^3\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*c) - 2*b*c*x^2 + a*d*x^2))/(2*a^2*x^2*(a + b*x^2)) +
((-4*b^2*c^2 + 5*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt
[c + d*x^2])/(b*c - a*d)])/(2*a^3*Sqrt[b]*Sqrt[-(b*c) + a*d]) + ((4*b*c^(3/
2) - 3*a*Sqrt[c]*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3)
```

fricas [A] time = 1.56, size = 1034, normalized size = 6.08

$$\left[-\frac{\left(\left(4 b^{2} c - a b d\right) x^{4} + \left(4 a b c - a^{2} d\right) x^{2}\right) \sqrt{\frac{b c - a d}{b}} \log\left(\frac{b^{2} d^{2} x^{4} + 8 b^{2} c^{2} - 8 a b c d + a^{2} d^{2} + 2 \left(4 b^{2} c d - 3 a b d^{2}\right) x^{2} + 4 \left(b^{2} d x^{2} + 2 b^{2} c - a b d\right) \sqrt{d x^{2} + c} \sqrt{\frac{b c - a d}{b}}}{b^{2} x^{4} + 2 a b x^{2} + a^{2}}\right) + 2 \left(\left(4 b^{2} c - 3 a b d\right) x^{4} + \left(4 a b c - 3 a^{2} d\right) x^{2}\right) \sqrt{c} \log\left(-\frac{d x^{2} - 2 \sqrt{d x^{2} + c} \sqrt{c} + 2 c}{x^{2}}\right) + 4 \left(a^{2} c + \left(2 a b c - a^{2} d\right) x^{2}\right) \sqrt{d x^{2} + c}}{8 \left(a^{3} b x^{4} + a^{4} x^{2}\right)}, -\frac{4 \left(\left(4 b^{2} c - 3 a b d\right) x^{4} + \left(4 a b c - 3 a^{2} d\right) x^{2}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2} + c}}\right) + \left(\left(4 b^{2} c - a b d\right) x^{4} + \left(4 a b c - a^{2} d\right) x^{2}\right) \sqrt{\frac{b c - a d}{b}} \log\left(\frac{b^{2} d^{2} x^{4} + 8 b^{2} c^{2} - 8 a b c d + a^{2} d^{2} + 2 \left(4 b^{2} c d - 3 a b d^{2}\right) x^{2} + 4 \left(b^{2} d x^{2} + 2 b^{2} c - a b d\right) \sqrt{d x^{2} + c} \sqrt{\frac{b c - a d}{b}}}{b^{2} x^{4} + 2 a b x^{2} + a^{2}}\right) + 4 \left(a^{2} c + \left(2 a b c - a^{2} d\right) x^{2}\right) \sqrt{d x^{2} + c}}{8 \left(a^{3} b x^{4} + a^{4} x^{2}\right)}, -\frac{\left(\left(4 b^{2} c - a b d\right) x^{4} + \left(4 a b c - a^{2} d\right) x^{2}\right) \sqrt{-\frac{b c - a d}{b}} \arctan\left(-\frac{\left(b d x^{2} + 2 b c - a d\right) \sqrt{d x^{2} + c} \sqrt{-\frac{b c - a d}{b}}}{2 \left(b c^{2} - a c d + \left(b c d - a d^{2}\right) x^{2}\right)}\right) + \left(\left(4 b^{2} c - 3 a b d\right) x^{4} + \left(4 a b c - 3 a^{2} d\right) x^{2}\right) \sqrt{c} \log\left(-\frac{d x^{2} - 2 \sqrt{d x^{2} + c} \sqrt{c} + 2 c}{x^{2}}\right) + 2 \left(a^{2} c + \left(2 a b c - a^{2} d\right) x^{2}\right) \sqrt{d x^{2} + c}}{4 \left(a^{3} b x^{4} + a^{4} x^{2}\right)}, -\frac{\left(\left(4 b^{2} c - a b d\right) x^{4} + \left(4 a b c - a^{2} d\right) x^{2}\right) \sqrt{-\frac{b c - a d}{b}} \arctan\left(-\frac{\left(b d x^{2} + 2 b c - a d\right) \sqrt{d x^{2} + c} \sqrt{-\frac{b c - a d}{b}}}{2 \left(b c^{2} - a c d + \left(b c d - a d^{2}\right) x^{2}\right)}\right) + 2 \left(\left(4 b^{2} c - 3 a b d\right) x^{4} + \left(4 a b c - 3 a^{2} d\right) x^{2}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2} + c}}\right) + 2 \left(a^{2} c + \left(2 a b c - a^{2} d\right) x^{2}\right) \sqrt{d x^{2} + c}}{4 \left(a^{3} b x^{4} + a^{4} x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt((b*c - a*d)/b)*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/
b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c -
3*a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
 4*(a^2*c + (2*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2),
-1/8*(4*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)*arctan
(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2
)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2
+ c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*c + (2*a*b*
c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c - a
*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^
2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c
*d - a*d^2)*x^2)) + ((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqr
t(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c + (2*a*
b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*(((4*b^2*c -
 a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*
x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b
*c*d - a*d^2)*x^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a^2*c + (2*a*b*c - a^2*d)*x
^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2)]
```

giac [A] time = 0.44, size = 216, normalized size = 1.27

$$\frac{\left(4 \, b^{2} c^{2} - 5 \, a b c d + a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{d x^{2} + c} b}{\sqrt{-b^{2} c + a b d}}\right)}{2 \, \sqrt{-b^{2} c + a b d} a^{3}} - \frac{\left(4 \, b c^{2} - 3 \, a c d\right) \arctan\left(\frac{\sqrt{d x^{2} + c}}{\sqrt{-c}}\right)}{2 \, a^{3} \sqrt{-c}} - \frac{2 \, {\left(d x^{2} + c\right)}^{\frac{3}{2}} b c d - 2 \, \sqrt{d x^{2} + c} b c^{2} d - {\left(d x^{2} + c\right)}^{\frac{3}{2}} a d^{2} + 2 \, \sqrt{d x^{2} + c} a c d^{2}}{2 \, {\left({\left(d x^{2} + c\right)}^{2} b - 2 \, {\left(d x^{2} + c\right)} b c + b c^{2} + {\left(d x^{2} + c\right)} a d - a c d\right)} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/2*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/2*(2*(d*x^2 + c)^(3/2)*b*c*d - 2*sq
rt(d*x^2 + c)*b*c^2*d - (d*x^2 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^2 + c)*a*c*d^2
)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)*
a^2)
```

maple [B] time = 0.02, size = 4820, normalized size = 28.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] 1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-3/4*b/a^2*d/(a*
d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)*c-3/2/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b
)^(1/2)/b))*c-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)-3
/4*b/a^2*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*c-3/2/a*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x+(-a*b)^(1/2)/b))*c+b/a^3*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+3/4/(-a*b)^(1/2)*d^(5/2)/(a*d-b*
c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/4/a*d^2/(a*d
-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)-3/2/a^2*d*c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)+1/2/a^2
*d/c*(d*x^2+c)^(3/2)-1/2/a^2/c/x^2*(d*x^2+c)^(5/2)-2*b/a^3*(d*x^2+c)^(1/2)*
c+3/4/a*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/(-a*b)^(1/2)*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b
)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+b/a^3*((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+2*b/a^3*c^(3/2)
*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-3/8*b^2/a^2/(-a*b)^(1/2)*d^(1/2)/(a*
d-b*c)*c^2*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/8*b/
a/(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+9/8*b/a/(-a*b)^(1/2)*d^(3/2)/(a*d-b*c)
```

```
*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+3/4*b/a^2*d/(a
*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-b/a^3/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+1/2/a^3*(-a*b)^(1/2)*d*(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2)*x+3/2/a^3*d^(1/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b
*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))*c-1/b/a^2*d^(3/2)*(-a*b)^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+
(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/b/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(
-a*b)^(1/2)/b))*d^2+2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b
))*d*c-1/4*b/a^2*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4/b*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/a^3*(-a*b)^(1/2)*d*((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/2/a^3*d^(
1/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
*c+1/b/a^2*d^(3/2)*(-a*b)^(1/2)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/
d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))-1/b/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d
^2+2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c-b/a^3/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+
2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-9/8*b/a/(-a*b)^(1/2)*
d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
*c+3/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))*c^2+1/4*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+3/8*b^2/a^2/(-a*b)^(1/2
```

```
)*d^(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))-1/4*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-3/8*b/a/(-a*b)^(1/2)*d^
2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)*x-1/4*b/a^2*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4/b*d^3/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+3/8*b^2/a^2/(-a*b)^(1/2)*d/(a*
d-b*c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)*x-3/8*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/3*b/a^3*(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
3/2)-1/a^2*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*d-2/3*b/a^3*(d*x^2+c)^(3/2)+1/3*b/a^3*((x-(-a*b)^(1/2)/b)^
2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/a^2*((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+
3/2/a^2*d*(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3), x)
```

mupad [B] time = 1.74, size = 441, normalized size = 2.59

$$\frac{\operatorname{atanh}\left(\frac{b^2\,c^2\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{\frac{a^2\,b\,c\,d^7}{4}-\frac{5\,a\,b^2\,c^2\,d^6}{4}+b^3\,c^3\,d^5}-\frac{b\,c\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(\frac{a\,b\,c\,d^7}{4}-\frac{5\,b^2\,c^2\,d^6}{4}+\frac{b^3\,c^3\,d^5}{a}\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-4\,b\,c\right)}{2\,a^3\,b}-\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{3\,b\,\sqrt{c}\,d^7\,\sqrt{d\,x^2+c}}{4\,\left(\frac{3\,b\,c\,d^7}{4}-\frac{7\,b^2\,c^2\,d^6}{4\,a}+\frac{b^3\,c^3\,d^5}{a^2}\right)}-\frac{7\,b^2\,c^{3/2}\,d^6\,\sqrt{d\,x^2+c}}{4\,\left(\frac{3\,a\,b\,c\,d^7}{4}-\frac{7\,b^2\,c^2\,d^6}{4}+\frac{b^3\,c^3\,d^5}{a}\right)}+\frac{b^3\,c^{5/2}\,d^5\,\sqrt{d\,x^2+c}}{\frac{3\,a^2\,b\,c\,d^7}{4}-\frac{7\,a\,b^2\,c^2\,d^6}{4}+b^3\,c^3\,d^5}\right)\,\left(3\,a\,d-4\,b\,c\right)}{2\,a^3}-\frac{\frac{\left(a\,c\,d^2-b\,c^2\,d\right)\,\sqrt{d\,x^2+c}}{a^2}-\frac{d\,{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{\left(d\,x^2+c\right)\,\left(a\,d-2\,b\,c\right)+b\,{\left(d\,x^2+c\right)}^2+b\,c^2-a\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (atanh((b^2*c^2*d^5*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(b^3*c^3*d^5 -
 (5*a*b^2*c^2*d^6)/4 + (a^2*b*c*d^7)/4) - (b*c*d^6*(c + d*x^2)^(1/2)*(b^2*c
 - a*b*d)^(1/2))/(4*((a*b*c*d^7)/4 - (5*b^2*c^2*d^6)/4 + (b^3*c^3*d^5)/a)))
*(-b*(a*d - b*c))^(1/2)*(a*d - 4*b*c))/(2*a^3*b) - (c^(1/2)*atanh((3*b*c^(1
/2)*d^7*(c + d*x^2)^(1/2))/(4*((3*b*c*d^7)/4 - (7*b^2*c^2*d^6)/(4*a) + (b^3
```

```
*c^3*d^5)/a^2)) - (7*b^2*c^(3/2)*d^6*(c + d*x^2)^(1/2))/(4*((3*a*b*c*d^7)/4
 - (7*b^2*c^2*d^6)/4 + (b^3*c^3*d^5)/a)) + (b^3*c^(5/2)*d^5*(c + d*x^2)^(1/
2))/(b^3*c^3*d^5 - (7*a*b^2*c^2*d^6)/4 + (3*a^2*b*c*d^7)/4))*(3*a*d - 4*b*c
))/(2*a^3) - (((a*c*d^2 - b*c^2*d)*(c + d*x^2)^(1/2))/a^2 - (d*(c + d*x^2)^
(3/2)*(a*d - 2*b*c))/(2*a^2))/((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2
+ b*c^2 - a*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.730 $\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{(5bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}\,(15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}\,(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {468, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,(15bc-11ad)}{6a^3x}-\frac{\sqrt{c+dx^2}\,(5bc-3ad)}{6a^2bx^3}+\frac{(5bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}\,(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -((5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b*c - 11*a*d)*Sqrt[
c + d*x^2])/(6*a^3*x) + ((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*x^3*(a + b*x^2
)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(2*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{-c(5bc-3ad)-2d(2bc-ad)x^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ab}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{\int \frac{-bc^2(15bc-11ad)-2bcd(5bc-3ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{6a^2bc}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} - \frac{\int -\frac{3bc^2(5bc-2ad)}{\left(a+bx^2\right)\sqrt{c+}}}{6a^3bc^2}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{((5bc-2ad)(bc}{}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{((5bc-2ad)(bc}{}$$

$$= -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3\left(a+bx^2\right)} + \frac{(5bc-2ad)\sqrt{bc}}{}$$

Mathematica [A] time = 5.16, size = 131, normalized size = 0.79

$$\frac{(5bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\sqrt{c+dx^2}\left(-2a^2\left(c+4dx^2\right)+abx^2\left(10c-11dx^2\right)+15b^2cx^4\right)}{6a^3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(10*c - 11*d*x^2) - 2*a^2*(c + 4*d
*x^2)))/(6*a^3*x^3*(a + b*x^2)) + ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

IntegrateAlgebraic [A] time = 0.63, size = 154, normalized size = 0.93

$$\frac{\sqrt{c+dx^2}\left(-2a^2c-8a^2dx^2+10abcx^2-11abdx^4+15b^2cx^4\right)}{6a^3x^3\left(a+bx^2\right)}-\frac{(5bc-2ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c + 10*a*b*c*x^2 - 8*a^2*d*x^2 + 15*b^2*c*x^4 - 11
*a*b*d*x^4))/(6*a^3*x^3*(a + b*x^2)) - ((5*b*c - 2*a*d)*Sqrt[b*c - a*d]*Arc
Tan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a
*d])])/(2*a^(7/2))
```

fricas [A] time = 1.15, size = 443, normalized size = 2.67

$$\left[-\frac{3\left(\left(5\,b^{2}c-2\,abd\right)x^{5}+\left(5\,abc-2\,a^{2}d\right)x^{3}\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}+4\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\left(\left(15\,b^{2}c-11\,abd\right)x^{4}-2\,a^{2}c+2\left(5\,abc-4\,a^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{24\left(a^{3}bx^{5}+a^{4}x^{3}\right)},\ \frac{3\left(\left(5\,b^{2}c-2\,abd\right)x^{5}+\left(5\,abc-2\,a^{2}d\right)x^{3}\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)+2\left(\left(15\,b^{2}c-11\,abd\right)x^{4}-2\,a^{2}c+2\left(5\,abc-4\,a^{2}d\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\left(a^{3}bx^{5}+a^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c - 11*a*b*d)*
x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^
4*x^3), 1/12*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((15*b^2*c - 11*a
```

```
*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^
5 + a^4*x^3)]
```

giac [B] time = 4.27, size = 442, normalized size = 2.66

$$-\frac{\left(5\,b^{2}c^{2}\sqrt{d}-7\,abcd^{\frac{3}{2}}+2\,a^{2}d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,a^{3}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c^{2}\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abcd^{\frac{3}{2}}+2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}d^{\frac{5}{2}}-b^{2}c^{3}\sqrt{d}+abc^{2}d^{\frac{3}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)a^{3}}-\frac{4\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc^{2}\sqrt{d}-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}acd^{\frac{3}{2}}-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{3}\sqrt{d}+3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ac^{2}d^{\frac{3}{2}}+3\,bc^{4}\sqrt{d}-2\,ac^{3}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*b^2*c^2*sqrt(d) - 7*a*b*c*d^(3/2) + 2*a^2*d^(5/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d)
 - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*b*c^3*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*c^2*d^(3/2) + 3*b*c^4*sqrt(d) - 2*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt
(d*x^2 + c))^2 - c)^3*a^3)
```

maple [B] time = 0.02, size = 4908, normalized size = 29.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] 1/a^2*d^(3/2)*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-5/4/a^2*d^(3/2)*ln(((x-(-a*b)^(
1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-5/4/a^2*d^(3/2)*ln(((x+(-a*b)^(1
/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(3/2)-3/8*b/a^2*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8*b/a^2*d^(3/2)/(a*d-b*c)*ln(((x+
(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1/4*b^2/a^3*d/(a*d-b*c
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(3/2)*x+3/8*b^2/a^3*d^(1/2)/(a*d-b*c)*c^2*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)
^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))-5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln
((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
```

```
)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2-2*b/a^3*d/c*x*(d*x^2+c)^(3/2)+1/4*b/a^3*(-
a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3/8*b/a^2*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-9/8*b/a^2*d^(3
/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+1
/4*b^2/a^3*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+3/8*b^2/a^3*d^(1/2)/(a*d-b*c)*c^2*ln(((x-(-a
*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+5/4*b^2/a^3/(-a*b)^(1/2)/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-3/4*b/a^3*(-a*b)^(1/
2)*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)*c+3/4/b/a*(-a*b)^(1/2)*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-3/2/a^2*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c+3/8*b^2/a^3*d/(a*d-b*c)*
c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*x+5/2*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^
(1/2)/b))*d*c+3/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-3/4/b/a*(-a*b)^(1/
2)*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+3/2/a^
2*(-a*b)^(1/2)*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2
)/b))*c+5/12*b^2/a^3/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4/a*d^(5/2)/(a*d-b*c)*ln(((x+(-a*
b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+3/4/a*d^(5/2)/(a*d-b*c)*ln((
(x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-5/12*b^2/a^3/(-a*b)^(
1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(3/2)-1/3/a^2/c/x^3*(d*x^2+c)^(5/2)+15/8*b/a^3*d^(1/2)*ln(((x+(-a*b)^(
1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+5/4*b/a^2/(-a*b)^(1/2)*((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d
```

```
-5/4*b^2/a^3/(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+5/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x+(-a*b)^(1/2)/b))*d^2+5/8*b/a^3*d*((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4*b^2/a^3/(a*d-b*c
)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(5/2)-3/4/a^2*(-a*b)^(1/2)*d^2/(a*d-b*c)*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b^2
/a^3/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+3/4/a^2*(-a*b)^(1/2)*d^2/(a*d-b*c)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)+5/8*b/a^3*d*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)*x+15/8*b/a^3*d^(1/2)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b
)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))*c-5/4*b/a^2/(-a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d+5/4*b^2/a^3/(-
a*b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*c-5/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-
a*b)^(1/2)/b))*d^2+2*b/a^3/c/x*(d*x^2+c)^(5/2)-3*b/a^3*d*x*(d*x^2+c)^(1/2)-
3*b/a^3*d^(1/2)*c*ln(d^(1/2)*x+(d*x^2+c)^(1/2))-2/3/a^2*d/c^2/x*(d*x^2+c)^(
5/2)+2/3/a^2*d^2/c^2*x*(d*x^2+c)^(3/2)+1/a^2*d^2/c*x*(d*x^2+c)^(1/2)+3/8*b^
2/a^3*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/2*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x+(-a*b)^(1/2)/b))*d*c-3/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^2+3/4*b/a^3*(-a*b)^(1/2)*d/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{3/2}}{x^4\,\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^{2}\right)^{\frac{3}{2}}}{x^{4}\left(a+bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)**2), x)
```

3.731 $\int \frac{x^4 (c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=258

$$\frac{x\sqrt{c+dx^2}\,\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}+\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}-\frac{\sqrt{a}\,(3bc}{}$$

Rubi [A] time = 0.45, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {467, 581, 582, 523, 217, 206, 377, 205}

$$\frac{x\sqrt{c+dx^2}\,\left(32a^2d^2-52abcd+19b^2c^2\right)}{16b^4}+\frac{\left(120a^2bcd^2-64a^3d^3-60ab^2c^2d+5b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}+\frac{dx^3\sqrt{c+dx^2}\,(7bc-8ad)}{8b^3}-\frac{\sqrt{a}\,(3bc-8ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^5}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((19*b^2*c^2 - 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(16*b^4) + (d*(7
*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(8*b^3) + (2*d*x^3*(c + d*x^2)^(3/2))/(3
*b^2) - (x^3*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 8*a*d
)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/
(2*b^5) + ((5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*b^5*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\left(c+dx^2\right)^{3/2}\left(3c+8dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{x^2\sqrt{c+dx^2}\left(6c(3bc-4ad)+6d(7bc-8ad)x^2\right)}{a+bx^2}\,dx}{12b^2}$$

$$= \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2} - \frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} +$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

$$= \frac{\left(19b^2c^2-52abcd+32a^2d^2\right)x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3\left(c+dx^2\right)^{3/2}}{3b^2}$$

Mathematica [A] time = 0.29, size = 219, normalized size = 0.85

$$\frac{bx\sqrt{c+dx^2}\left(72a^2d^2+2bdx^2(13bc-12ad)+\frac{24a(bc-ad)^2}{a+bx^2}-108abcd+33b^2c^2+8b^2d^2x^4\right)+\frac{3\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}+24\sqrt{a}\,(8ad-3bc)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{48b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(33*b^2*c^2 - 108*a*b*c*d + 72*a^2*d^2 + 2*b*d*(13*b*c
 - 12*a*d)*x^2 + 8*b^2*d^2*x^4 + (24*a*(b*c - a*d)^2)/(a + b*x^2)) + 24*Sqr
t[a]*(b*c - a*d)^(3/2)*(-3*b*c + 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]
*Sqrt[c + d*x^2])] + (3*(5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*
a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^5)
```

IntegrateAlgebraic [A] time = 1.19, size = 299, normalized size = 1.16

$$\frac{\sqrt{bc-ad}\left(-11a^{3/2}bcd+8a^{5/2}d^2+3\sqrt{a}\,b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2b^5}+\frac{\left(64a^3d^3-120a^2bcd^2+60ab^2c^2d-5b^3c^3\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{16b^5\sqrt{d}}+\frac{\sqrt{c+dx^2}\left(96a^3d^2x-156a^2bcdx+48a^2bd^2x^3+57ab^2c^2x-82ab^2cdx^3-16ab^2d^2x^5+33b^3c^2x^3+26b^3cdx^5+8b^3d^2x^7\right)}{48b^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(57*a*b^2*c^2*x - 156*a^2*b*c*d*x + 96*a^3*d^2*x + 33*b^3*
c^2*x^3 - 82*a*b^2*c*d*x^3 + 48*a^2*b*d^2*x^3 + 26*b^3*c*d*x^5 - 16*a*b^2*d
^2*x^5 + 8*b^3*d^2*x^7))/(48*b^4*(a + b*x^2)) + (Sqrt[b*c - a*d]*(3*Sqrt[a]
*b^2*c^2 - 11*a^(3/2)*b*c*d + 8*a^(5/2)*d^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*
x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*b^5) + ((-5*b^3*c
^3 + 60*a*b^2*c^2*d - 120*a^2*b*c*d^2 + 64*a^3*d^3)*Log[-(Sqrt[d]*x) + Sqrt
[c + d*x^2]])/(16*b^5*Sqrt[d])
```

fricas [A]   time = 6.21, size = 1697, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 +
(5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 12*(3*a*b^2*c^2*d - 11*
a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)
*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c
 + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(8*b^4*d^3*x^7
+ 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*
a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*
sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(3*(5*a*b^3*c^3 - 60*a^2*b^2*
c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^
2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) - 6*(3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2
*c*d^2 + 8*a^2*b*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*
d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c
*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/96*(24*(
3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2
+ 8*a^2*b*d^3)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
 a^2*c*d)*x)) + 3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^
4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2
)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^4*d^3*x^
7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 4
8*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x
```

```
)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(12*(3*a*b^2*c^2*d - 11*a^2
*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 3*(5*a
*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 6
0*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x
^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2
*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^
5*d)]
```

giac [B] time = 0.60, size = 521, normalized size = 2.02

$$\frac{1}{48}\left(2\left(\frac{4\,d^{2}x^{2}}{b^{2}}+\frac{13\,b^{12}cd^{5}-12\,ab^{11}d^{6}}{b^{14}d^{4}}\right)x^{2}+\frac{3\left(11\,b^{12}c^{2}d^{4}-36\,ab^{11}cd^{5}+24\,a^{2}b^{10}d^{6}\right)}{b^{14}d^{4}}\right)\sqrt{dx^{2}+c}\,x+\frac{\left(3\,ab^{3}c^{3}\sqrt{d}-14\,a^{2}b^{2}c^{2}d^{\frac{3}{2}}+19\,a^{3}bcd^{\frac{5}{2}}-8\,a^{4}d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\,\sqrt{abcd-a^{2}d^{2}}\,b^{5}}-\frac{\left(5\,b^{3}c^{3}\sqrt{d}-60\,ab^{2}c^{2}d^{\frac{3}{2}}+120\,a^{2}bcd^{\frac{5}{2}}-64\,a^{3}d^{\frac{7}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}\right)}{32\,b^{5}d}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{3}c^{3}\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}b^{2}c^{2}d^{\frac{3}{2}}+5\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{3}bcd^{\frac{5}{2}}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{4}d^{\frac{7}{2}}-ab^{3}c^{4}\sqrt{d}+2\,a^{2}b^{2}c^{3}d^{\frac{3}{2}}-a^{3}bc^{2}d^{\frac{5}{2}}}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2/b^2 + (13*b^12*c*d^5 - 12*a*b^11*d^6)/(b^14*d^4))*x^2 +
3*(11*b^12*c^2*d^4 - 36*a*b^11*c*d^5 + 24*a^2*b^10*d^6)/(b^14*d^4))*sqrt(d*
x^2 + c)*x + 1/2*(3*a*b^3*c^3*sqrt(d) - 14*a^2*b^2*c^2*d^(3/2) + 19*a^3*b*c
*d^(5/2) - 8*a^4*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^5) - 1/32*(
5*b^3*c^3*sqrt(d) - 60*a*b^2*c^2*d^(3/2) + 120*a^2*b*c*d^(5/2) - 64*a^3*d^(
7/2))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^5*d) - ((sqrt(d)*x - sqrt(d*x
^2 + c))^2*a*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^
2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(5/2) - 2*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a^4*d^(7/2) - a*b^3*c^4*sqrt(d) + 2*a^2*b^2*c^3*d^(3
/2) - a^3*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*
b^5)
```

maple [B] time = 0.03, size = 7611, normalized size = 29.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{2}+c\right)^{\frac{5}{2}}x^{4}}{\left(bx^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(d\,x^2+c\right)^{5/2}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(c + dx^{2}\right)^{\frac{5}{2}}}{\left(a + bx^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

3.732 $\int \frac{x^3\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=198

$$-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{9/2}}+\frac{\sqrt{c+dx^2}\,(2bc-7ad)(bc-ad)}{2b^4}+\frac{\left(c+dx^2\right)^{3/2}(2bc-7ad)}{6b^3}+\frac{\left(c+dx\right.}{10}$$

Rubi [A] time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(c+dx^2\right)^{5/2}(2bc-7ad)}{10b^2(bc-ad)}+\frac{\left(c+dx^2\right)^{3/2}(2bc-7ad)}{6b^3}+\frac{\sqrt{c+dx^2}\,(2bc-7ad)(bc-ad)}{2b^4}-\frac{(2bc-7ad)(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{9/2}}+\frac{a\left(c+dx^2\right)^{7/2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((2*b*c - 7*a*d)*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^4) + ((2*b*c - 7*a*d)*(c
 + d*x^2)^(3/2))/(6*b^3) + ((2*b*c - 7*a*d)*(c + d*x^2)^(5/2))/(10*b^2*(b*c
 - a*d)) + (a*(c + d*x^2)^(7/2))/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c -
7*a*d)*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]
)/(2*b^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx)^{5/2}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= \frac{a\left(c+dx^2\right)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-7ad)\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{a+bx}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-7ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x\right.}{4b^2}$$

$$= \frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3} + \frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)} + \frac{a\left(c+dx^2\right)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)} + ((2bc-7$$

$$= \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3} + \frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)} +$$

$$= \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3} + \frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)} +$$

$$= \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3} + \frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)} +$$

Mathematica [A] time = 0.43, size = 164, normalized size = 0.83

$$\frac{\left(bc-\frac{7ad}{2}\right)\left(\frac{2(bc-ad)\left(\sqrt{b}\sqrt{c+dx^2}\left(-3ad+4bc+bdx^2\right)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)+\frac{a\left(c+dx^2\right)^{7/2}}{a+bx^2}}{2b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] ((a*(c + d*x^2)^(7/2))/(a + b*x^2) + (b*c - (7*a*d)/2)*((2*(c + d*x^2)^(5/2
))/(5*b) + (2*(b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2
) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])
)/(3*b^(7/2))))/(2*b*(b*c - a*d))
```

IntegrateAlgebraic [A] time = 0.43, size = 205, normalized size = 1.04

$$\frac{\sqrt{ad-bc}\left(7a^2d^2-9abcd+2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{9/2}}+\frac{\sqrt{c+dx^2}\left(105a^3d^2-170a^2bcd+70a^2bd^2x^2+61ab^2c^2-118ab^2cdx^2-14ab^2d^2x^4+46b^3c^2x^2+22b^3cdx^4+6b^3d^2x^6\right)}{30b^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(61*a*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 46*b^3*c^2*x
^2 - 118*a*b^2*c*d*x^2 + 70*a^2*b*d^2*x^2 + 22*b^3*c*d*x^4 - 14*a*b^2*d^2*x
^4 + 6*b^3*d^2*x^6))/(30*b^4*(a + b*x^2)) + (Sqrt[-(b*c) + a*d]*(2*b^2*c^2
- 9*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2]
)/(b*c - a*d)])/(2*b^(9/2))
```

fricas [A] time = 0.88, size = 573, normalized size = 2.89

$$\left[\frac{15\left(2ab^2c^2-9a^2bcd+7a^3d^2+\left(2b^3c^2-9ab^2cd+7a^2bd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(b^2dx^2+2b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2abx^2+a^2}\right)+4\left(6b^3d^2x^6+61ab^2c^2-170a^2bcd+105a^3d^2+2\left(11b^3cd-7ab^2d^2\right)x^4+2\left(23b^3c^2-59ab^2cd+35a^2bd^2\right)x^2\right)\sqrt{dx^2+c}}{120\left(b^5x^2+ab^4\right)},-\frac{15\left(2ab^2c^2-9a^2bcd+7a^3d^2+\left(2b^3c^2-9ab^2cd+7a^2bd^2\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\left(6b^3d^2x^6+61ab^2c^2-170a^2bcd+105a^3d^2+2\left(11b^3cd-7ab^2d^2\right)x^4+2\left(23b^3c^2-59ab^2cd+35a^2bd^2\right)x^2\right)\sqrt{dx^2+c}}{60\left(b^5x^2+ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/120*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d + 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*
d + 7*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c
- a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(6*b^3*d^2*x^6 + 61*a*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3
*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)
*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/60*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d +
 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)
/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b
)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(6*b^3*d^2*x^6 + 61*a*b^2*c^2
- 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^
3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4
)]
```

giac [A] time = 0.40, size = 264, normalized size = 1.33

$$\frac{\left(2b^3c^3-11ab^2c^2d+16a^2bcd^2-7a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^4}+\frac{\sqrt{dx^2+c}\,ab^2c^2d-2\sqrt{dx^2+c}\,a^2bcd^2+\sqrt{dx^2+c}\,a^3d^3}{2\left(\left(dx^2+c\right)b-bc+ad\right)b^4}+\frac{3\left(dx^2+c\right)^{\frac{5}{2}}b^8+5\left(dx^2+c\right)^{\frac{3}{2}}b^8c+15\sqrt{dx^2+c}\,b^8c^2-10\left(dx^2+c\right)^{\frac{3}{2}}ab^7d-60\sqrt{dx^2+c}\,ab^7cd+45\sqrt{dx^2+c}\,a^2b^6d^2}{15b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*c^3 - 11*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(sqrt(d
*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/2*(sqrt(d*
```

```
x^2 + c)*a*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a^2*b*c*d^2 + sqrt(d*x^2 + c)*a^3*
d^3)/(((d*x^2 + c)*b - b*c + a*d)*b^4) + 1/15*(3*(d*x^2 + c)^(5/2)*b^8 + 5*
(d*x^2 + c)^(3/2)*b^8*c + 15*sqrt(d*x^2 + c)*b^8*c^2 - 10*(d*x^2 + c)^(3/2)
*a*b^7*d - 60*sqrt(d*x^2 + c)*a*b^7*c*d + 45*sqrt(d*x^2 + c)*a^2*b^6*d^2)/b
^10
```

maple [B]   time = 0.02, size = 7443, normalized size = 37.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B]   time = 1.17, size = 276, normalized size = 1.39

$$\frac{{\left(d\,x^2+c\right)}^{5/2}}{5\,b^2}-\sqrt{d\,x^2+c}\,\left(\frac{{\left(a\,d-b\,c\right)}^2}{b^4}+\frac{\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(\frac{c}{b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}\right)}{b^2}\right)-{\left(d\,x^2+c\right)}^{3/2}\,\left(\frac{c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{3\,b^4}\right)+\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^3\,d^3}{2}-a^2\,b\,c\,d^2+\frac{a\,b^2\,c^2\,d}{2}\right)}{b^5\,\left(d\,x^2+c\right)-b^5\,c+a\,b^4\,d}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,\left(7\,a\,d-2\,b\,c\right)}{7\,a^3\,d^3-16\,a^2\,b\,c\,d^2+11\,a\,b^2\,c^2\,d-2\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{3/2}\,\left(7\,a\,d-2\,b\,c\right)}{2\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

```
[Out] (c + d*x^2)^(5/2)/(5*b^2) - (c + d*x^2)^(1/2)*((a*d - b*c)^2/b^4 + ((2*b^2*
c - 2*a*b*d)*(c/b^2 - (2*b^2*c - 2*a*b*d)/b^4))/b^2) - (c + d*x^2)^(3/2)*(c
/(3*b^2) - (2*b^2*c - 2*a*b*d)/(3*b^4)) + ((c + d*x^2)^(1/2)*((a^3*d^3)/2 +
 (a*b^2*c^2*d)/2 - a^2*b*c*d^2))/(b^5*(c + d*x^2) - b^5*c + a*b^4*d) - (ata
n((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2)*(7*a*d - 2*b*c))/(7*a^3*d^3
- 2*b^3*c^3 + 11*a*b^2*c^2*d - 16*a^2*b*c*d^2))*(a*d - b*c)^(3/2)*(7*a*d -
2*b*c))/(2*b^(9/2))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.733 $\int \frac{x^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^4}+\frac{dx\sqrt{c+dx^2}\,(11bc}{8b^3}$$

Rubi [A] time = 0.24, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {467, 528, 523, 217, 206, 377, 205}

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8b^4}+\frac{dx\sqrt{c+dx^2}\,(11bc-12ad)}{8b^3}+\frac{(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,b^4}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(8*b^3) + (3*d*x*(c + d*x^2)^(3/2))
/(4*b^2) - (x*(c + d*x^2)^(5/2))/(2*b*(a + b*x^2)) + ((b*c - 6*a*d)*(b*c -
a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a
]*b^4) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(8*b^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = -\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{\left(c+dx^2\right)^{3/2}\left(c+6dx^2\right)}{a+bx^2}\,dx}{2b}$$

$$= \frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2} - \frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{\sqrt{c+dx^2}\left(2c(2bc-3ad)+2d(11bc-12ad)x^2\right)}{a+bx^2}\,dx}{8b^2}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2} - \frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\int \frac{2c\left(4b^2c^2-17abcd+12a^2d^2\right)}{\left(a+bx\right.}}{}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2} - \frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left((bc-6ad)(bc-ad)^2\right)}{2b^4}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2} - \frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left((bc-6ad)(bc-ad)^2\right)}{}$$

$$= \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3} + \frac{3dx\left(c+dx^2\right)^{3/2}}{4b^2} - \frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{(bc-6ad)(bc-ad)^{3/2}}{2\sqrt{a}\,b}$$

Mathematica [A] time = 0.23, size = 173, normalized size = 0.89

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)+bx\sqrt{c+dx^2}\left(-\frac{4(bc-ad)^2}{a+bx^2}+d(9bc-8ad)+2bd^2x^2\right)+\frac{4(bc-6ad)(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(d*(9*b*c - 8*a*d) + 2*b*d^2*x^2 - (4*(b*c - a*d)^2)/(
a + b*x^2)) + (4*(b*c - 6*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 2
4*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b^4)
```

IntegrateAlgebraic [A] time = 1.02, size = 245, normalized size = 1.26

$$\frac{\left(-24a^2d^{5/2}+40abcd^{3/2}-15b^2c^2\sqrt{d}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{8b^4}-\frac{\sqrt{bc-ad}\left(6a^2d^2-7abcd+b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\,b^4}+\frac{\sqrt{c+dx^2}\left(-12a^2d^2x+17abcdx-6abd^2x^3-4b^2c^2x+9b^2cdx^3+2b^2d^2x^5\right)}{8b^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-4*b^2*c^2*x + 17*a*b*c*d*x - 12*a^2*d^2*x + 9*b^2*c*d*x^
3 - 6*a*b*d^2*x^3 + 2*b^2*d^2*x^5))/(8*b^3*(a + b*x^2)) - (Sqrt[b*c - a*d]*
(b^2*c^2 - 7*a*b*c*d + 6*a^2*d^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*S
qrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*b^4) + ((-15*b^2*c^2
*Sqrt[d] + 40*a*b*c*d^(3/2) - 24*a^2*d^(5/2))*Log[-(Sqrt[d]*x) + Sqrt[c + d
*x^2]])/(8*b^4)
```

fricas [A] time = 3.14, size = 1379, normalized size = 7.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c
*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x
- c) + 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*
a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^
2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*
b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/8*((15*a*
b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b
*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b^2*c^2 - 7*a^2
*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt(-(b*c
- a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*s
qrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + 3*(3*b
^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sq
rt(d*x^2 + c))/(b^5*x^2 + a*b^4), 1/16*(4*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*
d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan
(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*
d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*
b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c)
)/(b^5*x^2 + a*b^4), -1/8*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*
b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)) - 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*
c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a
*c*d)*x)) - (2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 -
 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4)]
```

giac [B] time = 0.57, size = 446, normalized size = 2.29

$$\frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,d^2x^2}{b^2}+\frac{9\,b^7cd^3-8\,ab^6d^4}{b^9d^2}\right)x-\frac{\left(15\,b^2c^2\sqrt{d}-40\,abcd^{\frac{3}{2}}+24\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4}-\frac{\left(b^3c^3\sqrt{d}-8\,ab^2c^2d^{\frac{3}{2}}+13\,a^2bcd^{\frac{5}{2}}-6\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,b^4}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d}-4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ab^2c^2d^{\frac{3}{2}}+5\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{5}{2}}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^3d^{\frac{7}{2}}-b^3c^4\sqrt{d}+2\,ab^2c^3d^{\frac{3}{2}}-a^2bc^2d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^2 + c)*(2*d^2*x^2/b^2 + (9*b^7*c*d^3 - 8*a*b^6*d^4)/(b^9*d^2))
*x - 1/16*(15*b^2*c^2*sqrt(d) - 40*a*b*c*d^(3/2) + 24*a^2*d^(5/2))*log((sqr
t(d)*x - sqrt(d*x^2 + c))^2)/b^4 - 1/2*(b^3*c^3*sqrt(d) - 8*a*b^2*c^2*d^(3/
2) + 13*a^2*b*c*d^(5/2) - 6*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2
)*b^4) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*c^4*sqr
t(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2)*b^4)
```

maple [B] time = 0.02, size = 7459, normalized size = 38.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}x^2}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(d\,x^2+c\right)^{5/2}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

```
[Out] int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(c + dx^2\right)^{\frac{5}{2}}}{\left(a + bx^2\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

3.734 $\int \frac{x\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=126

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}+\frac{5d\sqrt{c+dx^2}\,(bc-ad)}{2b^3}-\frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d\left(c+dx^2\right)^{3/2}}{6b^2}$$

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 47, 50, 63, 208}

$$\frac{5d\sqrt{c+dx^2}\,(bc-ad)}{2b^3}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}-\frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d\left(c+dx^2\right)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x^2])/(2*b^3) + (5*d*(c + d*x^2)^(3/2))/(6*b^2)
 - (c + d*x^2)^(5/2)/(2*b*(a + b*x^2)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{(5d)\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^2\right)}{4b}$$

$$= \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{(5d(bc-ad))\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^2\right)}{4b^2}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left(5d(bc-ad)^2\right)\operatorname{Subst}\Bigg(\int \frac{1}{(a+bx)\sqrt{c}}}{4b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} + \frac{\left(5(bc-ad)^2\right)\operatorname{Subst}\Bigg(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}}{2b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3} + \frac{5d\left(c+dx^2\right)^{3/2}}{6b^2} - \frac{\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\Bigg(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}}{2b^{7/2}}$$

Mathematica [C]   time = 0.02, size = 54, normalized size = 0.43

$$\frac{d\left(c+dx^2\right)^{7/2}\ {}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{7(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (d*(c + d*x^2)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*(c + d*x^2))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^2)
```

IntegrateAlgebraic [A]   time = 0.35, size = 182, normalized size = 1.44

$$\frac{\sqrt{c+dx^2}\left(-15a^2d^2+20abcd-10abd^2x^2-3b^2c^2+14b^2cdx^2+2b^2d^2x^4\right)}{6b^3\left(a+bx^2\right)}+\frac{5\left(-a^3d^4+3a^2bcd^3-3ab^2c^2d^2+b^3c^3d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{7/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(-3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 14*b^2*c*d*x^2 - 1
0*a*b*d^2*x^2 + 2*b^2*d^2*x^4))/(6*b^3*(a + b*x^2)) + (5*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sq
rt[c + d*x^2])/(b*c - a*d)])/(2*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

fricas [A]   time = 0.95, size = 453, normalized size = 3.60

$$\left[-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(2\,b^2d^2x^4-3\,b^2c^2+20\,abcd-15\,a^2d^2+2\left(7\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{24\left(b^4x^2+ab^3\right)},-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\left(2\,b^2d^2x^4-3\,b^2c^2+20\,abcd-15\,a^2d^2+2\left(7\,b^2cd-5\,abd^2\right)x^2\right)\sqrt{dx^2+c}}{12\left(b^4x^2+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d^2*x^4 - 3*b^2*c^2 + 20*a*b*
c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2
 + a*b^3), -1/12*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(-(b
*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d^2*x^4 - 3*b^
2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2
 + c))/(b^4*x^2 + a*b^3)]
```

giac [A] time = 0.40, size = 197, normalized size = 1.56

$$\frac{5\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{2\,\sqrt{-b^{2}c+abd}\,b^{3}}-\frac{\sqrt{dx^{2}+c}\,b^{2}c^{2}d-2\,\sqrt{dx^{2}+c}\,abcd^{2}+\sqrt{dx^{2}+c}\,a^{2}d^{3}}{2\left(\left(dx^{2}+c\right)b-bc+ad\right)b^{3}}+\frac{\left(dx^{2}+c\right)^{\frac{3}{2}}b^{4}d+6\,\sqrt{dx^{2}+c}\,b^{4}cd-6\,\sqrt{dx^{2}+c}\,ab^{3}d^{2}}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 5/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/2*(sqrt(d*x^2 + c)*b^2*c^2*d - 2
*sqrt(d*x^2 + c)*a*b*c*d^2 + sqrt(d*x^2 + c)*a^2*d^3)/(((d*x^2 + c)*b - b*c
 + a*d)*b^3) + 1/3*((d*x^2 + c)^(3/2)*b^4*d + 6*sqrt(d*x^2 + c)*b^4*c*d - 6
*sqrt(d*x^2 + c)*a*b^3*d^2)/b^6
```

maple [B] time = 0.01, size = 4363, normalized size = 34.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] 5/2*a/b^2*d^2/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c-5/4*(-a*b)^(1/2)*a^2/b^4*d^(7/2)/(a*d-b*c)*
ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-5/4*a^3/b^4*d^4/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+5/4/b*d/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))*c^3-1/4*(-a*b)^(1/2
)/a/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(7/2)-5/16*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c
)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(3/2)*x-75/32*(-a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d
+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c^2-5/4*a^3/b^4*d^4/(a*d-b*c)/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+5/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c
)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^3+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x
+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
```

```
b*d-(a*d-b*c)/b)^(7/2)+5/16*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+75/32*(-
a*b)^(1/2)/b^2*d^(3/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)
/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2))*c^2+5/2*a/b^2*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*c+5/4*(-a*b)^(1/2)*a^2/b^
4*d^(7/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))-1/4/b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(5/2)-1/4/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)+5/12*a/b^2*d^2/(a*d-b*c)
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(3/2)-5/12/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-5/4*a^2/b^3*d^3/(a*d-b*c)*((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-5/4/b*d/(
a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)*c^2-5/12/b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*c-5/4*a^2/b^3*d^3/(a*d-b*c)*((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2)-5/4/b*d/(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*c^2+5/12*a/b^2*d^2/(a*d-b*c)*((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/16*(-a*b)^(
1/2)/a/b*d/(a*d-b*c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-15/32*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c^2*((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*x+5/16*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*c*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+15/32*(-a*b)^(1/2)/a/b*d/(a*
d-b*c)*c^2*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*x-5/8*(-a*b)^(1/2)*a/b^3*d^3/(a*d-b*c)*((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+35/32*(-a*b
)^(1/2)/b^2*d^2/(a*d-b*c)*c*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-35/32*(-a*b)^(1/2)/b^2*d^2/(a*d-b*c)*c*
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+5/8*(-a*b)^(1/2)*a/b^3*d^3/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+25/8*(-a*b)^(1/2)*a/b^
3*d^(5/2)/(a*d-b*c)*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))*c+1/4*(-a*b)^(1/2)/a/b*d/(a*d-b*c)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(5/2)*x+15/32*(-a*b)^(1/2)/a/b*d^(1/2
)/(a*d-b*c)*c^3*ln(((x-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1
5/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b
))*c-15/4*a/b^2*d^2/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-
```

```
a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^
2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/
2)/b))*c^2-25/8*(-a*b)^(1/2)*a/b^3*d^(5/2)/(a*d-b*c)*ln(((x+(-a*b)^(1/2)/b)
*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))*c+15/4*a^2/b^3*d^3/(a*d-b*c)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c-15/4*a/b^2*d^2/(a*d-b*c)/(-(a*
d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))*c^2-1/4*(-a*b)^(1/2)/a/b*d/
(a*d-b*c)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(5/2)*x-15/32*(-a*b)^(1/2)/a/b*d^(1/2)/(a*d-b*c)*c^3*ln(((x+(-a*b
)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 1.04, size = 172, normalized size = 1.37

$$\frac{d\left(d\,x^2+c\right)^{3/2}}{3\,b^2}-\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^2\,d^3}{2}-a\,b\,c\,d^2+\frac{b^2\,c^2\,d}{2}\right)}{b^4\,\left(d\,x^2+c\right)-b^4\,c+a\,b^3\,d}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{d\,x^2+c}\,{\left(a\,d-b\,c\right)}^{3/2}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{2\,b^{7/2}}+\frac{d\,\sqrt{d\,x^2+c}\,\left(2\,b^2\,c-2\,a\,b\,d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

```
[Out] (d*(c + d*x^2)^(3/2))/(3*b^2) - ((c + d*x^2)^(1/2)*((a^2*d^3)/2 + (b^2*c^2*
d)/2 - a*b*c*d^2))/(b^4*(c + d*x^2) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/2)
*d*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)
)*(a*d - b*c)^(3/2))/(2*b^(7/2)) + (d*(c + d*x^2)^(1/2)*(2*b^2*c - 2*a*b*d)
)/b^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.735 $\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^2\right)^{3/}}{2ab\left(a\right.}$$

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {413, 528, 523, 217, 206, 377, 205}

$$\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3}+\frac{d^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] -(d*(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)
^(3/2))/(2*a*b*(a + b*x^2)) + ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*b^3) + (d^(3/2)*(5*b*
c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*b^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\int \frac{\sqrt{c+dx^2}\left(c(bc+ad)-2d(bc-2ad)x^2\right)}{a+bx^2}\,dx}{2ab}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\int \frac{2c\left(b^2c^2+2abcd-2a^2d^2\right)+2ad^2(5bc-4ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{4ab^2}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(d^2(5bc-4ad)\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2b^3} + \frac{\left(\left(bc-a\right.\right.}{}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{\left(d^2(5bc-4ad)\right)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \right.}{2b^3}$$

$$= -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)} + \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right.}{2a^{3/2}b^3}$$

Mathematica [A] time = 0.21, size = 143, normalized size = 0.82

$$\frac{\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}} + d^{3/2}(5bc-4ad)\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right) + bx\sqrt{c+dx^2}\left(\frac{(bc-ad)^2}{a\left(a+bx^2\right)}+d^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (b*x*Sqrt[c + d*x^2]*(d^2 + (b*c - a*d)^2/(a*(a + b*x^2))) + ((b*c - a*d)^(
3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a
^(3/2) + d^(3/2)*(5*b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(2*b^3
)
```

IntegrateAlgebraic [A] time = 0.80, size = 209, normalized size = 1.20

$$\frac{\sqrt{c+dx^2}\left(2a^2d^2x-2abcdx+abd^2x^3+b^2c^2x\right)}{2ab^2\left(a+bx^2\right)} - \frac{\sqrt{bc-ad}\left(-4a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}b^3} + \frac{\left(4ad^{5/2}-5bcd^{3/2}\right)\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

```
[Out] (Sqrt[c + d*x^2]*(b^2*c^2*x - 2*a*b*c*d*x + 2*a^2*d^2*x + a*b*d^2*x^3))/(2*
a*b^2*(a + b*x^2)) - (Sqrt[b*c - a*d]*(b^2*c^2 + 3*a*b*c*d - 4*a^2*d^2)*Arc
Tan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a
*d])])/(2*a^(3/2)*b^3) + ((-5*b*c*d^(3/2) + 4*a*d^(5/2))*Log[-(Sqrt[d]*x) +
 Sqrt[c + d*x^2]])/(2*b^3)
```

fricas [A] time = 2.37, size = 1228, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d
)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b^2*c^2 + 3*a^2*b*c*
d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d
)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-
(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2
- 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/
8*(4*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-d)*a
rctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 +
(b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 +
 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d +
2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), 1/4*((a*b^2*c^2 + 3
*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (5*a^2*b*c*d - 4*a^
3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^
2 + c)*sqrt(d)*x - c) + 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b
*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/4*(2*(5*a^2*b*c*d - 4*a
^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d
*x^2 + c)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d
- 4*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d
)*x)) - 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*
x^2 + c))/(a*b^4*x^2 + a^2*b^3)]
```

giac [B] time = 0.61, size = 407, normalized size = 2.34

$$\frac{\sqrt{dx^2+c}\,d^2x}{2\,b^2} - \frac{\left(5\,bcd^{\frac{3}{2}} - 4\,ad^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2\right)}{4\,b^3} - \frac{\left(b^3c^3\sqrt{d} + 2\,ab^2c^2d^{\frac{3}{2}} - 7\,a^2bcd^{\frac{5}{2}} + 4\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{2\,\sqrt{abcd - a^2d^2}\,ab^3} - \frac{\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 b^3c^3\sqrt{d} - 4\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 ab^2c^2d^{\frac{3}{2}} + 5\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 a^2bcd^{\frac{5}{2}} - 2\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 a^3d^{\frac{7}{2}} - b^3c^4\sqrt{d} + 2\,ab^2c^3d^{\frac{3}{2}} - a^2bc^2d^{\frac{5}{2}}}{\left(\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}\,x - \sqrt{dx^2+c}\right)^2 ad + bc^2\right)ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*d^2*x/b^2 - 1/4*(5*b*c*d^(3/2) - 4*a*d^(5/2))*log((sqrt
(d)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^3*c^3*sqrt(d) + 2*a*b^2*c^2*d^(3/2
) - 7*a^2*b*c*d^(5/2) + 4*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a*b^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*c^4*sqr
t(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2)*a*b^3)
```

maple [B] time = 0.02, size = 7451, normalized size = 42.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{5/2}}{\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(a + b*x^2)^2,x)
```

```
[Out] int((c + d*x^2)^(5/2)/(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

3.736 $\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=160

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}\,(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Rubi [A] time = 0.22, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 98, 154, 156, 63, 208}

$$\frac{(bc-ad)^{3/2}(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}-\frac{d\sqrt{c+dx^2}\,(bc-3ad)}{2ab^2}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] -(d*(b*c - 3*a*d)*Sqrt[c + d*x^2])/(2*a*b^2) + ((b*c - a*d)*(c + d*x^2)^(3/
2))/(2*a*b*(a + b*x^2)) - (c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2 +
((b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*a^2*b^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{(c+dx)^{5/2}}{x(a+bx)^2}\,dx,x,x^2\right)$$

$$=\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(bc^2-\frac{1}{2}d(bc-3ad)x\right)}{x(a+bx)}\,dx,x,x^2\right)}{2ab}$$

$$=-\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)}+\frac{\text{Subst}\left(\int \frac{\frac{b^2c^3}{2}+\frac{1}{4}d\left(b^2c^2+4abcd-3a^2d^2\right)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x\right.}{ab^2}$$

$$=-\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)}+\frac{c^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2a^2}-\frac{\left((bc-ad\right.}{}$$

$$=-\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)}+\frac{c^3\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{a^2d}-\frac{\left((b\right.}{}$$

$$=-\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2ab\left(a+bx^2\right)}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{(bc-ad)^{3/2}(2bc+}{}$$

Mathematica [A] time = 0.21, size = 158, normalized size = 0.99

$$\frac{\frac{a\sqrt{c+dx^2}\left(3a^2d^2+2abd\left(dx^2-c\right)+b^2c^2\right)}{b^2\left(a+bx^2\right)}+\frac{\sqrt{bc-ad}\left(-3a^2d^2+abcd+2b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-2c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(b^2*c^2 + 3*a^2*d^2 + 2*a*b*d*(-c + d*x^2)))/(b^2*(a +
 b*x^2)) - 2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2
*b^2*c^2 + a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/b^(5/2))/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.35, size = 164, normalized size = 1.02

$$\frac{(ad-bc)^{3/2}(3ad+2bc)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2b^{5/2}}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-2abcd+2abd^2x^2+b^2c^2\right)}{2ab^2\left(a+bx^2\right)}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2 + 2*a*b*d^2*x^2))/(2*a*b^
2*(a + b*x^2)) + ((-(b*c) + a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTan[(Sqrt[b]*Sqrt
[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*a^2*b^(5/2)) - (c^(5/2)*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a^2
```

fricas [A] time = 3.48, size = 1132, normalized size = 7.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d
+ a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)
*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^3
*c^2*x^2 + a*b^2*c^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c
)/x^2) - 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x
^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/8*(8*(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(-c)
*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 +
(2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*
(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3
*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c
*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*
d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)
/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*sq
rt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*b*d^2*
x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 +
a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*
d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)) + 4*(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + 2*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 +
c))/(a^2*b^3*x^2 + a^3*b^2)]
```

giac [A] time = 0.34, size = 206, normalized size = 1.29

$$\frac{c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}+\frac{\sqrt{dx^2+c}\,d^2}{b^2}-\frac{\left(2\,b^3c^3-ab^2c^2d-4\,a^2bcd^2+3\,a^3d^3\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,a^2b^2}+\frac{\sqrt{dx^2+c}\,b^2c^2d-2\,\sqrt{dx^2+c}\,abcd^2+\sqrt{dx^2+c}\,a^2d^3}{2\left(\left(dx^2+c\right)b-bc+ad\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) + sqrt(d*x^2 + c)*d^2/b
^2 - 1/2*(2*b^3*c^3 - a*b^2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(sqrt(
d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b^2) + 1/2*(sq
rt(d*x^2 + c)*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a*b*c*d^2 + sqrt(d*x^2 + c)*a^2
*d^3)/(((d*x^2 + c)*b - b*c + a*d)*a*b^2)
```

maple [B] time = 0.02, size = 7477, normalized size = 46.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x), x)
```

mupad [B] time = 1.31, size = 1321, normalized size = 8.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x)
```

```
[Out] (d^2*(c + d*x^2)^(1/2))/b^2 + (atan((a^2*d^8*(c + d*x^2)^(1/2)*(c^5)^(1/2)*
9i)/(2*((9*a^2*c^3*d^8)/2 + 5*b^2*c^5*d^6 + (10*b^3*c^6*d^5)/a - (15*b^4*c^
7*d^4)/(2*a^2) - 12*a*b*c^4*d^7)) + (c^2*d^6*(c + d*x^2)^(1/2)*(c^5)^(1/2)*
```

```
5i)/(5*c^5*d^6 - (12*a*c^4*d^7)/b + (10*b*c^6*d^5)/a - (15*b^2*c^7*d^4)/(2*
a^2) + (9*a^2*c^3*d^8)/(2*b^2)) + (c^3*d^5*(c + d*x^2)^(1/2)*(c^5)^(1/2)*10
i)/(10*c^6*d^5 + (5*a*c^5*d^6)/b - (15*b*c^7*d^4)/(2*a) - (12*a^2*c^4*d^7)/
b^2 + (9*a^3*c^3*d^8)/(2*b^3)) - (a*c*d^7*(c + d*x^2)^(1/2)*(c^5)^(1/2)*12i
)/(5*b*c^5*d^6 - 12*a*c^4*d^7 + (10*b^2*c^6*d^5)/a + (9*a^2*c^3*d^8)/(2*b)
- (15*b^3*c^7*d^4)/(2*a^2)) - (b*c^4*d^4*(c + d*x^2)^(1/2)*(c^5)^(1/2)*15i)
/(2*(10*a*c^6*d^5 - (15*b*c^7*d^4)/2 + (5*a^2*c^5*d^6)/b - (12*a^3*c^4*d^7)
/b^2 + (9*a^4*c^3*d^8)/(2*b^3))))*(c^5)^(1/2)*1i)/a^2 + ((c + d*x^2)^(1/2)*
(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(2*a*(b^3*(c + d*x^2) - b^3*c + a*b^2*
d)) - (atan((c^4*d^5*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c
*d^2 - 3*a*b^7*c^2*d)^(1/2)*35i)/(4*(9*a^3*b*c^3*d^8 - (25*b^4*c^6*d^5)/4 -
 (85*a*b^3*c^5*d^6)/4 - (81*a^4*c^2*d^9)/4 + (27*a^5*c*d^10)/(4*b) + (49*a^
2*b^2*c^4*d^7)/2 + (15*b^5*c^7*d^4)/(2*a))) - (c^3*d^6*(c + d*x^2)^(1/2)*(b
^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*45i)/(4*((27*
a^4*c*d^10)/4 - (85*b^4*c^5*d^6)/4 + (49*a*b^3*c^4*d^7)/2 - (81*a^3*b*c^2*d
^9)/4 + 9*a^2*b^2*c^3*d^8 - (25*b^5*c^6*d^5)/(4*a) + (15*b^6*c^7*d^4)/(2*a^
2))) + (c^5*d^4*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2
- 3*a*b^7*c^2*d)^(1/2)*15i)/(2*(9*a^4*c^3*d^8 + (15*b^4*c^7*d^4)/2 - (25*a*
b^3*c^6*d^5)/4 + (49*a^3*b*c^4*d^7)/2 + (27*a^6*c*d^10)/(4*b^2) - (85*a^2*b
^2*c^5*d^6)/4 - (81*a^5*c^2*d^9)/(4*b))) + (a^2*c*d^8*(c + d*x^2)^(1/2)*(b^
8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*27i)/(4*((49*a
*b^5*c^4*d^7)/2 - (85*b^6*c^5*d^6)/4 + (27*a^4*b^2*c*d^10)/4 + 9*a^2*b^4*c^
3*d^8 - (81*a^3*b^3*c^2*d^9)/4 - (25*b^7*c^6*d^5)/(4*a) + (15*b^8*c^7*d^4)/
(2*a^2))) - (a*c^2*d^7*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6
*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*27i)/(4*((49*a*b^4*c^4*d^7)/2 - (85*b^5*c^5*d
^6)/4 + 9*a^2*b^3*c^3*d^8 - (81*a^3*b^2*c^2*d^9)/4 - (25*b^6*c^6*d^5)/(4*a)
 + (15*b^7*c^7*d^4)/(2*a^2) + (27*a^4*b*c*d^10)/4)))*(-b^5*(a*d - b*c)^3)^(
1/2)*(3*a*d + 2*b*c)*1i)/(2*a^2*b^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.737 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}\,(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\left(a+bx^2\right)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Rubi **[A]** time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {468, 580, 523, 217, 206, 377, 205}

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}-\frac{c\sqrt{c+dx^2}\,(3bc-ad)}{2a^2bx}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx\left(a+bx^2\right)}+\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c*(3*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*x) + ((b*c - a*d)*(c + d*x^2)^(
3/2))/(2*a*b*x*(a + b*x^2)) - ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b^2
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)} - \frac{\int \frac{\sqrt{c+dx^2}\left(-c(3bc-ad)-2ad^2x^2\right)}{x^2\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)} - \frac{\int \frac{c\left(3b^2c^2-4abcd-a^2d^2\right)-2a^2d^3x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2b}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)} + \frac{d^3\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2} - \frac{\Big((bc-ad)^2(3bc+2ad)}{2a^2b}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{\Big((bc -}{}$$

$$= -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx\left(a+bx^2\right)} - \frac{(bc-ad)^{3/2}(3bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2}$$

Mathematica [A] time = 0.15, size = 150, normalized size = 0.89

$$-\frac{(bc-ad)^{3/2}(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2} + \sqrt{c+dx^2}\left(-\frac{x(bc-ad)^2}{2a^2b\left(a+bx^2\right)} - \frac{c^2}{a^2x}\right) + \frac{d^{5/2}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(c^2/(a^2*x)) - ((b*c - a*d)^2*x)/(2*a^2*b*(a + b*x^2)))
- ((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sq
rt[c + d*x^2])])/(2*a^(5/2)*b^2) + (d^(5/2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^
2]])/b^2
```

IntegrateAlgebraic [A] time = 0.75, size = 203, normalized size = 1.21

$$\frac{\sqrt{c+dx^2}\left(-a^2d^2x^2-2abc^2+2abcdx^2-3b^2c^2x^2\right)}{2a^2bx\left(a+bx^2\right)} + \frac{\sqrt{bc-ad}\left(-2a^2d^2-abcd+3b^2c^2\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}b^2} - \frac{d^{5/2}\log\left(\sqrt{c+dx^2}-\sqrt{d}x\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*b*c^2 - 3*b^2*c^2*x^2 + 2*a*b*c*d*x^2 - a^2*d^2*x^2)
)/(2*a^2*b*x*(a + b*x^2)) + (Sqrt[b*c - a*d]*(3*b^2*c^2 - a*b*c*d - 2*a^2*d
^2)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[
b*c - a*d])])/(2*a^(5/2)*b^2) - (d^(5/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]
])/b^2
```

fricas [**A**] time = 1.85, size = 1184, normalized size = 7.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(d)*x - c) - ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^
2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x -
 (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) - 4*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^
2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/8*(8*(a^2*b*d^2*x^3 + a^3
*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 - a*b^2*c
*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c
 - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b^2*c^2 + (3*b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b
^2*x), -1/4*(((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^
2*b*c*d - 2*a^3*d^2)*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a
*c*d)*x)) - 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x
^2 + c)*sqrt(d)*x - c) + 2*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/4*(4*(a^2*b*d^2*x^
3 + a^3*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 -
a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqr
t((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(
(b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(2*a*b^2*c^2
+ (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3
+ a^3*b^2*x)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 7529, normalized size = 44.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{\left(bx^2+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2+c\right)^{5/2}}{x^2\,\left(b\,x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)**2), x)
```

3.738 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=180

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}\,(bc-ad)(2bc-ad)}{2a^2b\left(a+bx^2\right)}-$$

Rubi [A] time = 0.27, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 98, 149, 156, 63, 208}

$$-\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}+\frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}-\frac{\sqrt{c+dx^2}\,(bc-ad)(2bc-ad)}{2a^2b\left(a+bx^2\right)}-\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -((b*c - a*d)*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*b*(a + b*x^2)) - (c*(c
+ d*x^2)^(3/2))/(2*a*x^2*(a + b*x^2)) + (c^(3/2)*(4*b*c - 5*a*d)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/(2*a^3) - ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(c+dx)^{5/2}}{x^2(a+bx)^2}\,dx, x, x^2\right)$$

$$= -\frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}\left(\frac{1}{2}c(4bc-5ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^2}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} + \frac{\operatorname{Subst}(\int \frac{-\frac{1}{2}bc^2(4bc-5ad)-\frac{1}{2}d(2b^2c^2-2abcd-a}{x(a+bx)\sqrt{c+dx}}}{2a^2b}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} - \frac{\left(c^2(4bc-5ad)\right)\operatorname{Subst}(\int \frac{1}{x\sqrt{c+dx}}\,dx, x}{4a^3}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} - \frac{\left(c^2(4bc-5ad)\right)\operatorname{Subst}(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x,}{2a^3d}$$

$$= -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b\left(a+bx^2\right)} - \frac{c\left(c+dx^2\right)^{3/2}}{2ax^2\left(a+bx^2\right)} + \frac{c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} - \frac{(bc}{}$$

Mathematica [A] time = 0.45, size = 175, normalized size = 0.97

$$-\frac{\frac{a\sqrt{c+dx^2}\left(a^2d^2x^2+abc\left(c-2dx^2\right)+2b^2c^2x^2\right)}{bx^2\left(a+bx^2\right)}+\frac{\sqrt{bc-ad}\left(-a^2d^2-3abcd+4b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}-\left(c^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*((a*Sqrt[c + d*x^2]*(2*b^2*c^2*x^2 + a^2*d^2*x^2 + a*b*c*(c - 2*d*x^2)
))/(b*x^2*(a + b*x^2)) - c^(3/2)*(4*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]] + (Sqrt[b*c - a*d]*(4*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))/a^3
```

IntegrateAlgebraic [A] time = 0.52, size = 231, normalized size = 1.28

$$\frac{\left(4bc^{5/2}-5ac^{3/2}d\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3}+\frac{\sqrt{c+dx^2}\left(-a^2d^2x^2-abc^2+2abcdx^2-2b^2c^2x^2\right)}{2a^2bx^2\left(a+bx^2\right)}+\frac{\left(-a^4d^4-a^3bcd^3+9a^2b^2c^2d^2-11ab^3c^3d+4b^4c^4\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^3b^{3/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(a*b*c^2) - 2*b^2*c^2*x^2 + 2*a*b*c*d*x^2 - a^2*d^2*x^2)
)/(2*a^2*b*x^2*(a + b*x^2)) + ((4*b^4*c^4 - 11*a*b^3*c^3*d + 9*a^2*b^2*c^2*
d^2 - a^3*b*c*d^3 - a^4*d^4)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*
x^2])/(b*c - a*d)])/(2*a^3*b^(3/2)*(-(b*c) + a*d)^(3/2)) + ((4*b*c^(5/2) -
5*a*c^(3/2)*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3)
```

fricas **[A]** time = 3.82, size = 1266, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*
c*d - a^3*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c -
a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)
*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b*c^2 + (2*a*
b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b
*x^2), -1/8*(4*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)
*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^3*c^2 - 3*a*b^2*c*d
 - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt((b*c -
a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c
 - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*
a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2), -1/4*
(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d -
a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (
(4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sqrt(c)*lo
g(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*b*c^2 + (2*a*b^2
*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^
2), -1/4*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2
*b*c*d - a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*
x^2)) + 2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*(a^2*b*c^2 + (2*a*b^2*c^2 -
2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4*b*x^2)]
```

giac [A] time = 0.35, size = 283, normalized size = 1.57

$$-\frac{\left(4\,bc^{3}-5\,ac^{2}d\right)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,a^{3}\sqrt{-c}}+\frac{\left(4\,b^{3}c^{3}-7\,ab^{2}c^{2}d+2\,a^{2}bcd^{2}+a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{2\,\sqrt{-b^{2}c+abd}\,a^{3}b}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}b^{2}c^{2}d-2\,\sqrt{dx^{2}+c}\,b^{2}c^{3}d-2\left(dx^{2}+c\right)^{\frac{3}{2}}abcd^{2}+3\,\sqrt{dx^{2}+c}\,abc^{2}d^{2}+\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}d^{3}-\sqrt{dx^{2}+c}\,a^{2}cd^{3}}{2\left(\left(dx^{2}+c\right)^{2}b-2\left(dx^{2}+c\right)bc+bc^{2}+\left(dx^{2}+c\right)ad-acd\right)a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b*c^3 - 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c))
+ 1/2*(4*b^3*c^3 - 7*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*arctan(sqrt(d*x
^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3*b) - 1/2*(2*(d*x^
2 + c)^(3/2)*b^2*c^2*d - 2*sqrt(d*x^2 + c)*b^2*c^3*d - 2*(d*x^2 + c)^(3/2)*
a*b*c*d^2 + 3*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3 - sqr
t(d*x^2 + c)*a^2*c*d^3)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*
x^2 + c)*a*d - a*c*d)*a^2*b)
```

maple [B] time = 0.02, size = 7590, normalized size = 42.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{2}+c\right)^{\frac{5}{2}}}{\left(bx^{2}+a\right)^{2}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3), x)
```

mupad [B] time = 1.99, size = 1152, normalized size = 6.40

$$\frac{\frac{\sqrt{d\,x^2+c}\,\left(a^2\,c\,d^3-3\,a\,b\,c^2\,d^2+2\,b^2\,c^3\,d\right)}{2\,a^2\,b}-\frac{d\,{\left(d\,x^2+c\right)}^{3/2}\,\left(a^2\,d^2-2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{2\,a^2\,b}}{\left(d\,x^2+c\right)\,\left(a\,d-2\,b\,c\right)+b\,{\left(d\,x^2+c\right)}^2+b\,c^2-a\,c\,d}-\frac{\mathrm{atanh}\left(\frac{5\,d^9\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{4\,\left(\frac{5\,c^2\,d^9}{4}+\frac{4\,b\,c^3\,d^8}{a}-\frac{33\,b^2\,c^4\,d^7}{2\,a^2}+\frac{65\,b^3\,c^5\,d^6}{4\,a^3}-\frac{5\,b^4\,c^6\,d^5}{a^4}\right)}+\frac{4\,c\,d^8\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{4\,c^3\,d^8+\frac{5\,a\,c^2\,d^9}{4\,b}-\frac{33\,b\,c^4\,d^7}{2\,a}+\frac{65\,b^2\,c^5\,d^6}{4\,a^2}-\frac{5\,b^3\,c^6\,d^5}{a^3}}+\frac{65\,b^2\,c^3\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{4\,\left(4\,a^2\,c^3\,d^8+\frac{65\,b^2\,c^5\,d^6}{4}-\frac{5\,b^3\,c^6\,d^5}{a}+\frac{5\,a^3\,c^2\,d^9}{4\,b}-\frac{33\,a\,b\,c^4\,d^7}{2}\right)}-\frac{5\,b^3\,c^4\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{4\,a^3\,c^3\,d^8-5\,b^3\,c^6\,d^5+\frac{65\,a\,b^2\,c^5\,d^6}{4}-\frac{33\,a^2\,b\,c^4\,d^7}{2}+\frac{5\,a^4\,c^2\,d^9}{4\,b}}-\frac{33\,b\,c^2\,d^7\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\,\left(4\,a\,c^3\,d^8-\frac{33\,b\,c^4\,d^7}{2}+\frac{65\,b^2\,c^5\,d^6}{4\,a}+\frac{5\,a^2\,c^2\,d^9}{4\,b}-\frac{5\,b^3\,c^6\,d^5}{a^2}\right)}\right)\,\left(5\,a\,d-4\,b\,c\right)\,\sqrt{c^3}}{2\,a^3}-\frac{\mathrm{atanh}\left(\frac{15\,c^3\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{4\,\left(\frac{7\,a^3\,c^2\,d^9}{4}+\frac{55\,b^3\,c^5\,d^6}{4}-\frac{41\,a\,b^2\,c^4\,d^7}{4}-\frac{a^2\,b\,c^3\,d^8}{2}+\frac{a^4\,c\,d^{10}}{4\,b}-\frac{5\,b^4\,c^6\,d^5}{a}\right)}+\frac{9\,c^2\,d^7\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{4\,\left(\frac{a^3\,c\,d^{10}}{4}-\frac{41\,b^3\,c^4\,d^7}{4}-\frac{a\,b^2\,c^3\,d^8}{2}+\frac{7\,a^2\,b\,c^2\,d^9}{4}+\frac{55\,b^4\,c^5\,d^6}{4\,a}-\frac{5\,b^5\,c^6\,d^5}{a^2}\right)}+\frac{5\,c^4\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{\frac{a^3\,c^3\,d^8}{2}+5\,b^3\,c^6\,d^5-\frac{55\,a\,b^2\,c^5\,d^6}{4}+\frac{41\,a^2\,b\,c^4\,d^7}{4}-\frac{a^5\,c\,d^{10}}{4\,b^2}-\frac{7\,a^4\,c^2\,d^9}{4\,b}}-\frac{c\,d^8\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{4\,\left(\frac{b^3\,c^3\,d^8}{2}-\frac{7\,a\,b^2\,c^2\,d^9}{4}+\frac{41\,b^4\,c^4\,d^7}{4\,a}-\frac{55\,b^5\,c^5\,d^6}{4\,a^2}+\frac{5\,b^6\,c^6\,d^5}{a^3}-\frac{a^2\,b\,c\,d^{10}}{4}\right)}\right)\,\sqrt{-b^3\,{\left(a\,d-b\,c\right)}^3}\,\left(a\,d+4\,b\,c\right)}{2\,a^3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] (((c + d*x^2)^(1/2)*(a^2*c*d^3 + 2*b^2*c^3*d - 3*a*b*c^2*d^2))/(2*a^2*b) -
(d*(c + d*x^2)^(3/2)*(a^2*d^2 + 2*b^2*c^2 - 2*a*b*c*d))/(2*a^2*b))/((c + d*
x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2 + b*c^2 - a*c*d) - (atanh((5*d^9*(c +
d*x^2)^(1/2)*(c^3)^(1/2))/(4*((5*c^2*d^9)/4 + (4*b*c^3*d^8)/a - (33*b^2*c^4
*d^7)/(2*a^2) + (65*b^3*c^5*d^6)/(4*a^3) - (5*b^4*c^6*d^5)/a^4)) + (4*c*d^8
*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*c^3*d^8 + (5*a*c^2*d^9)/(4*b) - (33*b*c^
4*d^7)/(2*a) + (65*b^2*c^5*d^6)/(4*a^2) - (5*b^3*c^6*d^5)/a^3) + (65*b^2*c^
3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*(4*a^2*c^3*d^8 + (65*b^2*c^5*d^6)/4
 - (5*b^3*c^6*d^5)/a + (5*a^3*c^2*d^9)/(4*b) - (33*a*b*c^4*d^7)/2)) - (5*b^
3*c^4*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*a^3*c^3*d^8 - 5*b^3*c^6*d^5 + (
65*a*b^2*c^5*d^6)/4 - (33*a^2*b*c^4*d^7)/2 + (5*a^4*c^2*d^9)/(4*b)) - (33*b
*c^2*d^7*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(2*(4*a*c^3*d^8 - (33*b*c^4*d^7)/2
+ (65*b^2*c^5*d^6)/(4*a) + (5*a^2*c^2*d^9)/(4*b) - (5*b^3*c^6*d^5)/a^2)))*(
5*a*d - 4*b*c)*(c^3)^(1/2))/(2*a^3) - (atanh((15*c^3*d^6*(c + d*x^2)^(1/2)*
(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(4*((7*a^3
*c^2*d^9)/4 + (55*b^3*c^5*d^6)/4 - (41*a*b^2*c^4*d^7)/4 - (a^2*b*c^3*d^8)/2
 + (a^4*c*d^10)/(4*b) - (5*b^4*c^6*d^5)/a)) + (9*c^2*d^7*(c + d*x^2)^(1/2)*
(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(4*((a^3*c
*d^10)/4 - (41*b^3*c^4*d^7)/4 - (a*b^2*c^3*d^8)/2 + (7*a^2*b*c^2*d^9)/4 + (
55*b^4*c^5*d^6)/(4*a) - (5*b^5*c^6*d^5)/a^2)) + (5*c^4*d^5*(c + d*x^2)^(1/2
)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/((a^3*c^
3*d^8)/2 + 5*b^3*c^6*d^5 - (55*a*b^2*c^5*d^6)/4 + (41*a^2*b*c^4*d^7)/4 - (a
^5*c*d^10)/(4*b^2) - (7*a^4*c^2*d^9)/(4*b)) - (c*d^8*(c + d*x^2)^(1/2)*(b^6
*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(4*((b^3*c^3*d
^8)/2 - (7*a*b^2*c^2*d^9)/4 + (41*b^4*c^4*d^7)/(4*a) - (55*b^5*c^5*d^6)/(4*
a^2) + (5*b^6*c^6*d^5)/a^3 - (a^2*b*c*d^10)/4)))*(-b^3*(a*d - b*c)^3)^(1/2)
*(a*d + 4*b*c))/(2*a^3*b^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.739 $\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=176

$$\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}-\frac{c\sqrt{c+dx^2}\,(5bc-3ad)}{6a^2bx^3}+\frac{\sqrt{c+dx^2}\,\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}+\frac{\left(c+dx^2\right)^{3/2}(b}{2abx^3(a+}$$

Rubi [A] time = 0.25, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {468, 580, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,\left(3a^2d^2-20abcd+15b^2c^2\right)}{6a^3bx}-\frac{c\sqrt{c+dx^2}\,(5bc-3ad)}{6a^2bx^3}+\frac{5c(bc-ad)^{3/2}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -(c*(5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(6*a^2*b*x^3) + ((15*b^2*c^2 - 20*a*b*
c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*b*x) + ((b*c - a*d)*(c + d*x^2)^(3
/2))/(2*a*b*x^3*(a + b*x^2)) + (5*c*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{\sqrt{c+dx^2}\left(-c(5bc-3ad)-2bcdx^2\right)}{x^4\left(a+bx^2\right)}\,dx}{2ab}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)} - \frac{\int \frac{c\left(15b^2c^2-20abcd+3a^2d^2\right)+2bcd(5bc-6ad)x^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,d}{6a^2b}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)}$$

$$= -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3} + \frac{\left(15b^2c^2-20abcd+3a^2d^2\right)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{2abx^3\left(a+bx^2\right)}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.31

$$-\frac{c\left(c+dx^2\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{(ad-bc)x^2}{a\left(dx^2+c\right)}\right)}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*(c*(c + d*x^2)^(3/2)*Hypergeometric2F1[-3/2, 2, -1/2, ((-(b*c) + a*d)*
x^2)/(a*(c + d*x^2))])/(a^2*x^3)
```

IntegrateAlgebraic [A] time = 0.93, size = 175, normalized size = 0.99

$$\frac{\sqrt{c+dx^2}\left(-2a^2c^2-14a^2cdx^2+3a^2d^2x^4+10abc^2x^2-20abcdx^4+15b^2c^2x^4\right)}{6a^3x^3\left(a+bx^2\right)} - \frac{5\sqrt{bc-ad}\left(bc^2-acd\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a^2*c^2 + 10*a*b*c^2*x^2 - 14*a^2*c*d*x^2 + 15*b^2*c^2
*x^4 - 20*a*b*c*d*x^4 + 3*a^2*d^2*x^4))/(6*a^3*x^3*(a + b*x^2)) - (5*Sqrt[b
*c - a*d]*(b*c^2 - a*c*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c +
d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2))
```

fricas [**A**] time = 1.17, size = 483, normalized size = 2.74

$$\left[-\frac{15\left(\left(b^{2}c^{2}-abcd\right)x^{5}+\left(abc^{2}-a^{2}cd\right)x^{3}\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}+4\left(a^{2}cx-\left(abc-2\,a^{2}d\right)x^{3}\right)\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\left(\left(15\,b^{2}c^{2}-20\,abcd+3\,a^{2}d^{2}\right)x^{4}-2\,a^{2}c^{2}+2\left(5\,abc^{2}-7\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{24\left(a^{3}bx^{5}+a^{4}x^{3}\right)},\ \frac{15\left(\left(b^{2}c^{2}-abcd\right)x^{5}+\left(abc^{2}-a^{2}cd\right)x^{3}\right)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}\sqrt{\frac{bc-ad}{a}}}{2\left(\left(bcd-ad^{2}\right)x^{3}+\left(bc^{2}-acd\right)x\right)}\right)+2\left(\left(15\,b^{2}c^{2}-20\,abcd+3\,a^{2}d^{2}\right)x^{4}-2\,a^{2}c^{2}+2\left(5\,abc^{2}-7\,a^{2}cd\right)x^{2}\right)\sqrt{dx^{2}+c}}{12\left(a^{3}bx^{5}+a^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a^2*c*d)*x^3)*sqrt(-(b*c -
 a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^
2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sq
rt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c^2 - 20*a*b*
c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^
2 + c))/(a^3*b*x^5 + a^4*x^3), 1/12*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2
 - a^2*c*d)*x^3)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*s
qrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x
)) + 2*((15*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^
2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3)]
```

giac [**B**] time = 4.94, size = 496, normalized size = 2.82

$$-\frac{5\left(b^{2}c^{3}\sqrt{d}-2\,abc^{2}d^{\frac{3}{2}}+a^{2}cd^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{2\sqrt{abcd-a^{2}d^{2}}\,a^{3}}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{3}c^{3}\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{2}c^{2}d^{\frac{3}{2}}+5\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}bcd^{\frac{5}{2}}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{3}d^{\frac{7}{2}}-b^{3}c^{4}\sqrt{d}+2\,ab^{2}c^{3}d^{\frac{3}{2}}-a^{2}bc^{2}d^{\frac{5}{2}}}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)a^{3}b}-\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc^{3}\sqrt{d}-9\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ac^{2}d^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{4}\sqrt{d}+12\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ac^{3}d^{\frac{3}{2}}+6\,bc^{5}\sqrt{d}-7\,ac^{4}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -5/2*(b^2*c^3*sqrt(d) - 2*a*b*c^2*d^(3/2) + a^2*c*d^(5/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt
(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(
7/2) - b^3*c^4*sqrt(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d
)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3*b) - 2/3*(6*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b*c^3*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c^2*d^(3/
2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^4*sqrt(d) + 12*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a*c^3*d^(3/2) + 6*b*c^5*sqrt(d) - 7*a*c^4*d^(3/2))/(((sqrt(
d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

maple [B] time = 0.02, size = 7705, normalized size = 43.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2 + c\right)^{\frac{5}{2}}}{\left(bx^2 + a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^2 + c\right)^{5/2}}{x^4\,\left(b\,x^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^2\right)^{\frac{5}{2}}}{x^4 \left(a + bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a)**2,x)
```

```
[Out] Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)**2), x)
```

3.740 $\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Rubi **[A]** time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - (Sqrt[a]*(3*b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*b^2*(b*c - a*
d)^(3/2)) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{\int \frac{1}{\sqrt{c+dx^2}}\,dx}{b^2} - \frac{(a(3bc-2ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2b^2(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\Bigg(\int}{2b^2(bc}$$

$$= \frac{ax\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

Mathematica [A] time = 0.19, size = 129, normalized size = 0.98

$$\frac{\frac{abx\sqrt{c+dx^2}}{\left(a+bx^2\right)(bc-ad)} + \frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^2}+dx\right)}{\sqrt{d}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2
) + (2*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^2)
```

IntegrateAlgebraic [A] time = 0.95, size = 160, normalized size = 1.21

$$\frac{\left(3\sqrt{a}\,bc-2a^{3/2}d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}-\frac{\log\left(\sqrt{c+dx^2}-\sqrt{d}\,x\right)}{b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) + ((3*Sqrt[a]*b*c - 2*a
^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]
*Sqrt[b*c - a*d])])/(2*b^2*(b*c - a*d)^(3/2)) - Log[-(Sqrt[d]*x) + Sqrt[c +
 d*x^2]]/(b^2*Sqrt[d])
```

fricas [A] time = 1.65, size = 1053, normalized size = 7.98

$$\left[\frac{4\sqrt{dx^2+c}\,abdx+4\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+\left(3abcd-2a^2d^2+\left(3b^2cd-2abd^2\right)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{8\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)},\ \frac{4\sqrt{dx^2+c}\,abdx-8\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(3abcd-2a^2d^2+\left(3b^2cd-2abd^2\right)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2-4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{8\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)},\ \frac{2\sqrt{dx^2+c}\,abdx+\left(3abcd-2a^2d^2+\left(3b^2cd-2abd^2\right)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)+2\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)}{4\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)},\ \frac{2\sqrt{dx^2+c}\,abdx-4\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+\left(3abcd-2a^2d^2+\left(3b^2cd-2abd^2\right)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)}{4\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*d*x + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*s
qrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 2*a^2
*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*
a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^
2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)
*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*c*d - a^2*b^2*d
^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*d*x - 8*(a*b*c
- a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
+ (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d)
)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d
)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a
*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)
*a*b*d*x + (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a/(b*
c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
```

```
 - a*d))/(a*d*x^3 + a*c*x)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(
d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*c*d - a^2*b^2*d^
2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*d*x - 4*(a*b*c -
 a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) +
 (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*
arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(
a*d*x^3 + a*c*x)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

giac [B] time = 0.57, size = 284, normalized size = 2.15

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^{2}d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(b^{3}c-ab^{2}d\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}d^{\frac{3}{2}}-abc^{2}\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)\left(b^{3}c-ab^{2}d\right)}-\frac{\log\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}\right)}{2\,b^{2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2
 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqrt(
a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/2*log((sqrt(d)*x
 - sqrt(d*x^2 + c))^2)/(b^2*sqrt(d))
```

maple [B] time = 0.02, size = 846, normalized size = 6.41

$$\frac{3a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}-\frac{3a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{\sqrt{-ab}\,ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^{3}}-\frac{\sqrt{-ab}\,ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\,b^{3}}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;a}{4\,(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)b^{2}}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;a}{4\,(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)b^{2}}+\frac{\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{b^{2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/b^2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))/d^(1/2)-1/4*a/b^2/(a*d-b*c)/(x-(-a*b)^(
1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)+1/4*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x-(-a*b)^(1/2)/b))-3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x+(-a*b)^(1/2)/b))-1/4*a/b^2/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4
*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
```

```
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))+3/4/b^2*a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^
(1/2)/b))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^2\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.741 $\int \frac{x^3}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}\, dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{a\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{4b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{2bd(bc-ad)}\\
&= \frac{a\sqrt{c+dx^2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.99

$$
\frac{\frac{a\sqrt{b}\sqrt{c+dx^2}}{\left(a+bx^2\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{2b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(2*b^(
3/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 109, normalized size = 1.10

$$\frac{(2bc-ad)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2b^{3/2}(ad-bc)^{3/2}}+\frac{a\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) + ((2*b*c - a*d)*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*b^(3/2)*(-(b*c
) + a*d)^(3/2))
```

fricas [B] time = 0.92, size = 450, normalized size = 4.55

$$\left[\frac{\left(2\,abc-a^2d+\left(2\,b^2c-abd\right)x^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}}{8\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^2\right)},-\frac{\left(2\,abc-a^2d+\left(2\,b^2c-abd\right)x^2\right)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)-2\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}}{4\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2
 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a*b^4*c^2 - 2
*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2), -1
/4*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 -
 a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c
))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^2)]
```

giac [A] time = 0.33, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^2+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^2+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x^2 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^2 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

maple [B] time = 0.02, size = 807, normalized size = 8.15

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^{2}}-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^{2}}-\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,b^{2}}-\frac{\sqrt{-ab}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)b^{2}}+\frac{\sqrt{-ab}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4*(-a*b)^(1
/2)/b^2/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4*a/b^2*d/(a*d-b*c)/(-(a*d-b*c
)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/b^2/(-(a*d-b*c)/b)^(1/2)*ln(
(2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))/(x-(-a*b)^(1/2)/b))+1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/
b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)+1/4*a/b^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1
/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.89, size = 93, normalized size = 0.94

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{2\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^2+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(2*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^2)^(1/2))/(2*b*(a*d - b*c)*(b*(c + d
*x^2) + a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.742 $\int \frac{x^2}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=89

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(x*Sqrt[c + d*x^2])/(2*(b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*
d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx &= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2(bc-ad)}\\
&= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{c\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2(bc-ad)}\\
&= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{c\,\mathrm{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2(bc-ad)}\\
&= -\frac{x\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.44, size = 124, normalized size = 1.39

$$
\frac{\sqrt{c+dx^2}\left(-\frac{x^2(bc-ad)}{a+bx^2}-\frac{c\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^2\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^2}{c}+1}}\right)}{\sqrt{\frac{dx^2}{c}+1}}\right)}{2x(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(((b*c - a*d)*x^2)/(a + b*x^2)) - (c*Sqrt[(-(b/a) + d/c)
*x^2]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[1 + (d*x^
2)/c]))/(2*(b*c - a*d)^2*x)
```

IntegrateAlgebraic [A] time = 0.65, size = 142, normalized size = 1.60

$$-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}-\frac{c\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2\sqrt{a}\,(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) - (c*ArcTan[(Sqrt[a]*Sqr
t[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sq
rt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*(b*c - a*d)^(3/2))
```

fricas [B] time = 1.02, size = 418, normalized size = 4.70

$$\left[-\frac{4\left(abc-a^2d\right)\sqrt{dx^2+c}\,x-\left(bcx^2+ac\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{8\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)},-\frac{2\left(abc-a^2d\right)\sqrt{dx^2+c}\,x-\left(bcx^2+ac\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c +
a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(
d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), -1/4*(2*(a*b*c - a^2*d)
*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

giac [B] time = 3.83, size = 231, normalized size = 2.60

$$\frac{c\sqrt{d}\,\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}\,-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)\left(b^2c-abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + ((sqrt(d)
```

```
*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(
b^2*c - a*b*d))
```

maple [B] time = 0.02, size = 817, normalized size = 9.18

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)b}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4
/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4/(-a*b)^(1/2)/b/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/4/b/(a*d-b*c)/(x-(-a*b)^(1/2
)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)-1/4/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{{\left(b x^{2} + a\right)}^{2} \sqrt{d x^{2} + c}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2+a\right)}^2\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.743 $\int \frac{x}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(2*(b*c - a*d)*(a + b*x^2)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^2\right)}{4(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}-\frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{2(bc-ad)\left(a+bx^2\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.98

$$
\frac{1}{2}\left(\frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]/((-(b*c) + a*d)*(a + b*x^2)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/2
```

IntegrateAlgebraic [A] time = 0.19, size = 97, normalized size = 1.11

$$
-\frac{\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2\sqrt{b}(ad-bc)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2)) - (d*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

fricas [**B**] time = 0.85, size = 404, normalized size = 4.64

$$\left[-\frac{\left(bdx^2+ad\right)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(b^2c-abd\right)\sqrt{dx^2+c}}{8\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)},\ \frac{\left(bdx^2+ad\right)\sqrt{-b^2c+abd}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)-2\left(b^2c-abd\right)\sqrt{dx^2+c}}{4\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a
*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(
b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c
+ a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^
2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(b^2*c - a*b*d)*s
qrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^2)]
```

giac [**A**] time = 0.28, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^2+c}\,d}{2\left(\left(dx^2+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*(b*c - a
*d))
```

maple [**B**] time = 0.01, size = 513, normalized size = 5.90

$$-\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\;b}-\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\;b}+\frac{\sqrt{-ab}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\,(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)ab}-\frac{\sqrt{-ab}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\,(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/b*d/(a*d-b*c)/(
-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4*(-a*b)^(1/2)/a/b/(a
*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)
*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.79, size = 82, normalized size = 0.94

$$\frac{d\,\sqrt{d\,x^2+c}}{2\,\left(a\,d-b\,c\right)\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (d*(c + d*x^2)^(1/2))/(2*(a*d - b*c)*(b*(c + d*x^2) + a*d - b*c)) + (d*atan
((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(1/2)*(a*d - b*c)^(3/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^{2}\right)^{2} \sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.744 $\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}\,dx &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{(bc-2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)} \\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)} \\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 4.09, size = 405, normalized size = 4.05

$$
\frac{x\sqrt{c+dx^2}\left(-30dx^2\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}-45c\sqrt{\frac{ax^2\left(c+dx^2\right)(bc-ad)}{c^2\left(a+bx^2\right)^2}}+16dx^2\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{5/2}{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)+16c\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{5/2}{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)+30dx^2\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)+45c\sin^{-1}\left(\sqrt{\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}}\right)\right)}{30c^2\left(a+bx^2\right)^2\left(\frac{x^2(bc-ad)}{c\left(a+bx^2\right)}\right)^{3/2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*(-45*c*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*
x^2)^2)] - 30*d*x^2*Sqrt[(a*(b*c - a*d)*x^2*(c + d*x^2))/(c^2*(a + b*x^2)^2
)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 30*d*x^2*ArcSin
[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]] + 16*c*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F1[2,
3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 16*d*x^2*(((b*c - a*d)*x^2)/(c
*(a + b*x^2)))^(5/2)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Hypergeometric2F
1[2, 3, 7/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(30*c^2*(((b*c - a*d)*x^2
)/(c*(a + b*x^2)))^(3/2)*(a + b*x^2)^2*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))
])
```

IntegrateAlgebraic [A] time = 0.00, size = 122, normalized size = 1.22

$$
\frac{(2ad-bc)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}-\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b
*c - a*d])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

fricas [B] time = 0.95, size = 459, normalized size = 4.59

$$\left[\frac{4\left(ab^{2}c-a^{2}bd\right)\sqrt{dx^{2}+c}\,x-\left(abc-2\,a^{2}d+\left(b^{2}c-2\,abd\right)x^{2}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left(\left(bc-2\,ad\right)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)}{8\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}+\left(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2}\right)x^{2}\right)},\ \frac{2\left(ab^{2}c-a^{2}bd\right)\sqrt{dx^{2}+c}\,x+\sqrt{abc-a^{2}d}\left(abc-2\,a^{2}d+\left(b^{2}c-2\,abd\right)x^{2}\right)\arctan\left(\frac{\sqrt{abc-a^{2}d}\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)}{4\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}+\left(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2}\right)x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

giac [B] time = 0.55, size = 225, normalized size = 2.25

$$-\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{\left(bc-2\,ad\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(abcd-a^{2}d^{2}\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad-bc^{2}\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)\left(abcd-a^{2}d^{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

maple [B] time = 0.01, size = 823, normalized size = 8.23

$$\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,ab}-\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,ab}-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)a}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{4\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4
/a/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*
(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a/(a*d-b*c)/(x+(-a*b)^(
1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)-1/4/a/b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x+(-a*b)^(1/2)/b))-1/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))
/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.745 $\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=130

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) - ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]]/(a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \frac{}{(a+}\right.}{4a^2(bc-a}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{a^2 d} - \frac{(b(2bc-3ad))\,\text{Subst}}{2a^2}$$

$$= \frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.24, size = 123, normalized size = 0.95

$$\frac{\frac{ab\sqrt{c+dx^2}}{\left(a+bx^2\right)(bc-ad)}+\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) - (2*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.31, size = 144, normalized size = 1.11

$$\frac{\left(3a\sqrt{b}\,d-2b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2a^2(ad-bc)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}-\frac{b\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-2*b^(3/2)*c +
3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a
*d)])/(2*a^2*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*
Sqrt[c])
```

fricas [A] time = 1.86, size = 1037, normalized size = 7.98

$$\left[\frac{4\sqrt{dx^2+c}\,abc+\left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+4\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{8\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^2\right)},\ \frac{4\sqrt{dx^2+c}\,abc+8\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)}{8\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^2\right)},\ \frac{2\sqrt{dx^2+c}\,abc-\left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)}{4\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^2\right)},\ \frac{2\sqrt{dx^2+c}\,abc-\left(2abc^2-3a^2cd+\left(2b^2c^2-3abcd\right)x^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+4\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{4\left(a^3bc^2-a^4cd+\left(a^2b^2c^2-a^3bcd\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*x^2 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(-(
d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a^2*
b^2*c^2 - a^3*b*c*d)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*c + 8*(a*b*c - a^2*d
+ (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*a*b*c
^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*
```

```
d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
 c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c^2 - a^4*c*d
 + (a^2*b^2*c^2 - a^3*b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^
2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*
b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^2 - 3*a^2*c*d + (2*b^2
*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 4*(a*b*c - a^2*
d + (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a^3*b*
c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^2)]
```

giac [**A**] time = 0.31, size = 138, normalized size = 1.06

$$\frac{\sqrt{dx^2+c}\,bd}{2\left(abc-a^2d\right)\left(\left(dx^2+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*d/((a*b*c - a^2*d)*((d*x^2 + c)*b - b*c + a*d)) - 1/2
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*s
qrt(-c))
```

maple [**B**] time = 0.02, size = 838, normalized size = 6.45

$$\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{4\sqrt{-ab}\,\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)a}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b}{4\sqrt{-ab}\,\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)a}+\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-\frac{ad-bc}{b}}\,a^2}-\frac{\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/a/(-a*b)^(
1/2)/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c
)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/2/a^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-
```

```
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
))/(x-(-a*b)^(1/2)/b))-1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)*((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)+1/4/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1
/a^2/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x), x)
```

mupad [B] time = 1.94, size = 3023, normalized size = 23.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^5 - 6*a^5*b^
3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c +
d*x^2)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*
a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2
*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*
c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((
((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(2*(a
^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 +
 2*a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^2)^(1
/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c
*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2
- 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*
(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3
*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/
(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b^3*d
^4)/2 - b^4*c*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^2)^(
```

```
1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(2*(a^4*d^2 + a^2*b
^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2
*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^2)^(1/2)*(3*a*d - 2
*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*
b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)
*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c
)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3
*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2
*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^2)^(1/2)*(13*a^2
*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)) + (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(a^5*d
^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^2)^(1/2)*(3*a*d - 2*b*c)*(-b*(a
*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2
 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 -
 a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d -
 b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)
*1i)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan(
(((((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(2*(a^5*d^2 + a^3
*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^2)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*
c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*a^2*c^(1/2)*(a^4*d^2 +
 a^2*b^2*c^2 - 2*a^3*b*c*d)))/(2*a^2*c^(1/2)) - ((c + d*x^2)^(1/2)*(13*a^2*
b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^
3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*
b^4*c^2*d^3)/(2*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^2)^(1/2)
*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d
^3))/(8*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(2*a^2*c^(1/2))
 + ((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(4
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)))/(((3*a*b^3*d^4)
/2 - b^4*c*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + (((4*a^6*b^2*d^5 -
6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(2*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d)) - ((c + d*x^2)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^
3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))/(2*a^2*c^(1/2)) - ((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d
^2 - 20*a*b^4*c*d^3))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/
2)) + (((4*a^6*b^2*d^5 - 6*a^5*b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(2*(a^5*d^2 +
 a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^2)^(1/2)*(16*a^7*b^2*d^5 - 64*a^6*
b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*a^2*c^(1/2)*(a^4*d
^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(2*a^2*c^(1/2)) + ((c + d*x^2)^(1/2)*(13*
a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(4*(a^4*d^2 + a^2*b^2*c^2 -
2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(a^2*c^(1/2)) - (b*d*(c + d*x^2)^(1/2))/
(2*(a^2*d - a*b*c)*(b*(c + d*x^2) + a*d - b*c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.746 $\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=147

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(3bc-2ad)}{2a^2cx(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)(bc-ad)}$$

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}\,(3bc-2ad)}{2a^2cx(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*x) + (b*Sqrt[c + d*
x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)}-\frac{\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)}-\frac{\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)}-\frac{(b(3bc-4ad))\int \frac{1}{\left(a+bx^2\right)\sqrt{c+d}}}{2a^2(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)}-\frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{}{a-(}\right.}{2a^2(bc-}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x\left(a+bx^2\right)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+d}}\right.}{2a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 5.16, size = 116, normalized size = 0.79

$$\frac{\sqrt{c+dx^2}\left(\frac{b^2x^2}{\left(a+bx^2\right)(ad-bc)}-\frac{2}{c}\right)}{2a^2x}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2/c + (b^2*x^2)/((-(b*c) + a*d)*(a + b*x^2))))/(2*a^2*x)
 - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]
)/(2*a^(5/2)*(b*c - a*d)^(3/2))
```

IntegrateAlgebraic [A] time = 0.95, size = 158, normalized size = 1.07

$$\frac{\left(3b^2c-4abd\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\,\left(-2a^2d+2abc-2abdx^2+3b^2cx^2\right)}{2a^2cx\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^2 - 2*a*b*d*x^2))/(2*a^2*c*
(-(b*c) + a*d)*x*(a + b*x^2)) + ((3*b^2*c - 4*a*b*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(
b*c - a*d)^(3/2))
```

fricas [B] time = 1.11, size = 600, normalized size = 4.08

$$\left[-\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^3+\left(3\,ab^2c^2-4\,a^2bcd\right)x\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left((bc-2\,ad)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2+\left(3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2\right)x^2\right)\sqrt{dx^2+c}}{8\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)x^3+\left(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2\right)x\right)},\ -\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^3+\left(3\,ab^2c^2-4\,a^2bcd\right)x\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\,\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+2\left(2\,a^2b^2c^2-4\,a^3bcd+2\,a^4d^2+\left(3\,ab^3c^2-5\,a^2b^2cd+2\,a^3bd^2\right)x^2\right)\sqrt{dx^2+c}}{4\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)x^3+\left(a^4b^2c^3-2\,a^5bc^2d+a^6cd^2\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt
(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^
2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b^2*c^2 - 4*a
^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqr
t(d*x^2 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2
*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x), -1/4*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3
+ (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c
- a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (
3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*
c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a
^6*c*d^2)*x)]
```

giac [B] time = 3.94, size = 396, normalized size = 2.69

$$\frac{1}{2}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^{2}c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{2\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b^{2}c-4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}abd-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c^{2}+14\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abcd-8\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}a^{2}d^{2}+3\,b^{2}c^{3}-2\,abc^{2}d\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}ad+3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}-4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}acd-bc^{3}\right)\left(a^{2}bcd^{2}-a^{3}d^{3}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt(a
*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c - 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*b*d - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2
+ 14*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d - 8*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*
b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

maple [B] time = 0.01, size = 841, normalized size = 5.72

$$\frac{3b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{3b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{\sqrt{-ab}\,d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}+\frac{\sqrt{-ab}\,d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;b}{4\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)a^{2}}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\;b}{4\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)a^{2}}-\frac{\sqrt{d\,x^{2}+c}}{a^{2}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a^2/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a^2*(-a*b)^(1/2)*d/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-3/4*b/a^2/(-a*b)^(1/
2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/a^2/(a*d-b*c)*b
/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(
1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+3/4*b/a^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^
(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/a^2/c/x*(d*x^2+c)^(1/2)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^2\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.747 $\int \frac{1}{x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}\,(2bc-ad)}{2a^2c\left(a+bx^2\right)(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)}$$

Rubi [A] time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}-\frac{b\sqrt{c+dx^2}\,(2bc-ad)}{2a^2c\left(a+bx^2\right)(bc-ad)}-\frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^2])/(2*a^2*c*(b*c - a*d)*(a + b*x^2)) - Sqrt
[c + d*x^2]/(2*a*c*x^2*(a + b*x^2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]])/(2*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-}{x(a+bx)\sqrt{c+dx}}\right.}{2a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)}\right.}{4a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+}\right.}{2a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)\left(a+bx^2\right)} - \frac{\sqrt{c+dx^2}}{2acx^2\left(a+bx^2\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}} - b$$

Mathematica [A] time = 0.57, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^2}\left(a^2d+ab\left(dx^2-c\right)-2b^2cx^2\right)}{x^2\left(a+bx^2\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*Sqrt[c + d*x^2]*(a^2*d - 2*b^2*c*x^2 + a*b*(-c + d*x^2)))/((b*c - a*d)*
x^2*(a + b*x^2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]
 + (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2))/(2*a^3*c)
```

IntegrateAlgebraic [A] time = 0.74, size = 187, normalized size = 1.01

$$\frac{\left(4b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^3(ad-bc)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}}+\frac{\sqrt{c+dx^2}\left(a^2(-d)+abc-abdx^2+2b^2cx^2\right)}{2a^2cx^2\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(a*b*c - a^2*d + 2*b^2*c*x^2 - a*b*d*x^2))/(2*a^2*c*(-(b*c
) + a*d)*x^2*(a + b*x^2)) + ((4*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*a^3*(-(b*c) + a*d)^(3/
2)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(3/2))
```

fricas [A] time = 2.77, size = 1407, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)
*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2
*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*
d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*
a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) +
 2*c)/x^2) - 4*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d
*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2),
 -1/8*(4*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*
b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - ((4*b^3*c
^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(b/(b*c -
a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^
2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a
^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d*x^2 + c))/((a^3*
b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2), 1/4*(((4*b^3*c^3
 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqrt(-b/(b*c - a
*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d)
)/(b*d*x^2 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*
c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(c) + 2*c)/x^2) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2
)*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*
d)*x^2), 1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2
*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arcta
n(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b
*c*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 -
 a^5*c^2*d)*x^2)]
```

giac [A] time = 0.34, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^2+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^2+c}\,b^2c^2d-\left(dx^2+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^2+c}\,abcd^2-\sqrt{dx^2+c}\,a^2d^3}{2\left(a^2bc^2-a^3cd\right)\left(\left(dx^2+c\right)^2b-2\left(dx^2+c\right)bc+bc^2+\left(dx^2+c\right)ad-acd\right)}-\frac{\left(4\,bc+ad\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/2*(2*(d*x^2 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^2 + c)*b^2*c^2*d - (d*x^2 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^2 + c
)*a*b*c*d^2 - sqrt(d*x^2 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^2 + c)^
2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - 1/2*(4*b*c +
a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

maple [B] time = 0.02, size = 899, normalized size = 4.86

$$-\frac{bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^2}-\frac{bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^2}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^2}{4\sqrt{-ab}\,\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)a^2}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^2}{4\sqrt{-ab}\,\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)a^2}-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{\sqrt{-\frac{ad-bc}{b}}\,a^3}-\frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{\sqrt{-\frac{ad-bc}{b}}\,a^3}+\frac{d\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{2a^2c^{\frac{3}{2}}}+\frac{2b\ln\left(\frac{2c+2\sqrt{d\,x^2+c}\,\sqrt{c}}{x}\right)}{a^3\sqrt{c}}-\frac{\sqrt{d\,x^2+c}}{2a^2c\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4*b^2/a^2/(
-a*b)^(1/2)/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b/a^2*d/(a*d-b*c)/(-(a*d
-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/a^2/c/x^2*(d*x^2+c)^(1/2
)+1/2/a^2*d/c^(3/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/a^3*b/(-(a*d-b*
c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*
c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b/a^2*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x-(-a*b)^(1/2)/b))+2*b/a^3/c^(1/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2
))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3), x)
```

mupad [B] time = 2.85, size = 3837, normalized size = 20.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] (((c + d*x^2)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^2)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2 + b*c^2 - a*c*d) + (atan((((-b
^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 +
32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/
(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^
4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3
*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2
 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b
^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^
2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5
*b*c*d^2)))*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2
)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b
^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c
^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b
^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)
 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^
5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/
(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*
d - 3*a^5*b*c*d^2)))*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2)))/(((5*a^3*b^4*d^6)/4 + 8*b^7*c^3*d^3 - 12*a*b^6*c^2*d^4 + (3*a^
2*b^5*c*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d -
 b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^
4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b
^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
 - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8
*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b
*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7
*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b^2*c^4 +
a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a
```

```
^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*
(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*
b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*
(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*
(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 +
 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a
*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^5*c^5*d^2 -
80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*(a^4*b^2*
c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(
-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3
*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^2)^(1/2)*(a^4*b^3*d^6
 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4
))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((2*a^9*b^2*c*d^6 + 4
*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^
8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^2)^(1/2)*(a*d + 4*b*c)*(32*a^6*b^5*c
^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*
a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c)
)/(4*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^(1/2)) + ((((c + d*x^
2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5
 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (
((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d
^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^2)^(1/2)*(a*d +
 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*
a^9*b^2*c^2*d^5))/(8*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c
^3*d)))*(a*d + 4*b*c))/(4*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^
(1/2)))/((((5*a^3*b^4*d^6)/4 + 8*b^7*c^3*d^3 - 12*a*b^6*c^2*d^4 + (3*a^2*b^5
*c*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((c + d*x^2)^(1/
2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*
a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((2*a^
9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a
^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^2)^(1/2)*(a*d + 4*b*c
)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^
2*c^2*d^5))/(8*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))
)*(a*d + 4*b*c))/(4*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(4*a^3*(c^3)^(1/2)) +
((((c + d*x^2)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a
^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b
*c^3*d)) - (((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a
^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^2)^
(1/2)*(a*d + 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c
^3*d^4 - 16*a^9*b^2*c^2*d^5))/(8*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2
 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(4*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(4*a
^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)/(2*a^3*(c^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.748 $\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{\sqrt{c+dx^2}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{6a^3c^2x(bc-ad)}+\frac{b\sqrt{c+d}}{2ax^3\left(a+bx^2\right)}$$

Rubi [A] time = 0.25, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{6a^3c^2x(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}\,(5bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^2])/(6*a^2*c*(b*c - a*d)*x^3) + ((15*b^2*c^2
 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^2])/(6*a^3*c^2*(b*c - a*d)*x) + (b*S
qrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (b^2*(5*b*c - 6*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)
^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} - \frac{\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)} + \frac{\int \frac{-15b^2c^2+8abcd+4a^2d^2-2bd(5bc-}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}}{6a^2c(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx}}{2a(bc-ad)x^3\left(a\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx}}{2a(bc-ad)x^3\left(a\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx}}{2a(bc-ad)x^3\left(a\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x} + \frac{b\sqrt{c+dx}}{2a(bc-ad)x^3\left(a\right.}$$

Mathematica [A] time = 5.25, size = 136, normalized size = 0.66

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\left(\frac{3b^3x^4}{(a+bx^2)(bc-ad)}+\frac{4x^2(ad+3bc)}{c^2}-\frac{2a}{c}\right)}{6a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*((-2*a)/c + (4*(3*b*c + a*d)*x^2)/c^2 + (3*b^3*x^4)/((b*c
- a*d)*(a + b*x^2))))/(6*a^3*x^3) + (b^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(3/2))
```

IntegrateAlgebraic [A] time = 1.21, size = 216, normalized size = 1.05

$$\frac{(6ab^2d-5b^3c)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^2}\left(-2a^3cd+4a^3d^2x^2+2a^2bc^2+6a^2bcdx^2+4a^2bd^2x^4-10ab^2c^2x^2+8ab^2cdx^4-15b^3c^2x^4\right)}{6a^3c^2x^3(a+bx^2)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(2*a^2*b*c^2 - 2*a^3*c*d - 10*a*b^2*c^2*x^2 + 6*a^2*b*c*d*
x^2 + 4*a^3*d^2*x^2 - 15*b^3*c^2*x^4 + 8*a*b^2*c*d*x^4 + 4*a^2*b*d^2*x^4))/
(6*a^3*c^2*(-(b*c) + a*d)*x^3*(a + b*x^2)) + ((-5*b^3*c + 6*a*b^2*d)*ArcTan
[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d]
)])/(2*a^(7/2)*(b*c - a*d)^(3/2))
```

fricas [A] time = 1.64, size = 758, normalized size = 3.68

$$\left[-\frac{3\left((5b^4c^3-6ab^3c^2d)x^5+(5ab^3c^3-6a^2b^2c^2d)x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4((bc-2ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2abx^2+a^2}\right)+4\left(2a^3b^2c^3-4a^4bc^2d+2a^5cd^2-(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3)x^4-2(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3)x^2\right)\sqrt{dx^2+c}}{24\left((a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2)x^5+(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2)x^3\right)}, \frac{3\left((5b^4c^3-6ab^3c^2d)x^5+(5ab^3c^3-6a^2b^2c^2d)x^3\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2ad)x^2-ac)\sqrt{dx^2+c}}{2((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-2\left(2a^3b^2c^3-4a^4bc^2d+2a^5cd^2-(15ab^4c^3-23a^2b^3c^2d+4a^3b^2cd^2+4a^4bd^3)x^4-2(5a^2b^3c^3-8a^3b^2c^2d+a^4bcd^2+2a^5d^3)x^2\right)\sqrt{dx^2+c}}{12\left((a^4b^3c^4-2a^5b^2c^3d+a^6bc^2d^2)x^5+(a^5b^2c^4-2a^6bc^3d+a^7c^2d^2)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d
)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(
-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^3*b^
2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*
a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4
*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d
 + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^3), 1
/12*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x
```

```
^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) -
2*(2*a^3*b^2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3
*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*
c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^
5*b^2*c^3*d + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d
^2)*x^3)]
```

giac [B] time = 4.75, size = 375, normalized size = 1.82

$$\frac{1}{6}\,d^{\frac{7}{2}}\left(\frac{3\left(5\,b^{3}c-6\,ab^{2}d\right)\arctan\left(-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{3}bcd^{3}-a^{4}d^{4}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{6\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{3}c-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ab^{2}d-b^{3}c^{2}\right)}{\left(a^{3}bcd^{3}-a^{4}d^{4}\right)\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)}-\frac{8\left(3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-6\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc-3\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+3\,bc^{2}+acd\right)}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}d^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4)*s
qrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c - 2*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d^4)*((s
qrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)
^3*a^3*d^3))
```

maple [B] time = 0.02, size = 893, normalized size = 4.33

$$-\frac{5b^{2}\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{3}}+\frac{5b^{2}\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{3}}+\frac{\sqrt{-ab}\,bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{3}}-\frac{\sqrt{-ab}\,bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{3}}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^{2}}{4\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)a^{3}}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,b^{2}}{4\left(ad-bc\right)\left(x-\frac{\sqrt{-ab}}{b}\right)a^{3}}+\frac{2\sqrt{d\,x^{2}+c}\,d}{3a^{2}c^{2}x}+\frac{2\sqrt{d\,x^{2}+c}\,b}{a^{3}cx}-\frac{\sqrt{d\,x^{2}+c}}{3a^{2}c\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 5/4*b^2/a^3/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))-1/4*b^2/a^3/(a*d-b*c)/(x+(-a*b)^(1/2)/b)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b/a^3*(-a*b)^(1/2)*d/
(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2
*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-5/4*b^2/a^3/
(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4*b^2/a^
```

```
3/(a*d-b*c)/(x-(-a*b)^(1/2)/b)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+2*b/a^3/c/x*(d*x^2+c)^(1/2
)-1/3/a^2/c/x^3*(d*x^2+c)^(1/2)+2/3/a^2*d/c^2/x*(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.749 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}-\frac{3\sqrt{a}\,c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

Rubi **[A]** time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {470, 527, 12, 377, 205}

$$\frac{x(ad+2bc)}{2b\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}-\frac{3\sqrt{a}\,c\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((2*b*c + a*d)*x)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (a*x)/(2*b*(b*c - a
*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (3*Sqrt[a]*c*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{ac-2bcx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2b(bc-ad)}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{3abc^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2bc(bc-ad)^2}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3ac)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}}{2(bc-ad)^2}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3ac)\,\text{Subst}\left(\int \frac{}{a-(-b}\right.}{2(bc}$$

$$= \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{3\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{bc-a}}{\sqrt{a}\,\sqrt{c+}}\right.}{2(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.42

$$\frac{cx^5\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{(ad-bc)x^2}{a(dx^2+c)}\right)}{5a^2\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (c*x^5*Hypergeometric2F1[2, 5/2, 7/2, ((-(b*c) + a*d)*x^2)/(a*(c + d*x^2))]
)/(5*a^2*(c + d*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.80, size = 160, normalized size = 1.23

$$\frac{3\sqrt{a}\,c\tan^{-1}\left(\frac{b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}-\frac{bx\sqrt{c+dx^2}}{\sqrt{a}\,\sqrt{bc-ad}}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}+\frac{3acx+adx^3+2bcx^3}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (3*a*c*x + 2*b*c*x^3 + a*d*x^3)/(2*(b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2
]) + (3*Sqrt[a]*c*ArcTan[(Sqrt[a]*Sqrt[d])/Sqrt[b*c - a*d] + (b*Sqrt[d]*x^2
)/(Sqrt[a]*Sqrt[b*c - a*d]) - (b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d
])])/(2*(b*c - a*d)^(5/2))
```

fricas [B] time = 1.57, size = 552, normalized size = 4.25

$$\left[\frac{3\left(bcdx^4+ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+4\left((2\,bc+ad)x^3+3\,acx\right)\sqrt{dx^2+c}}{8\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)},\frac{3\left(bcdx^4+ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left((bc-2\,ad)x^2-ac\right)\sqrt{dx^2+c}\,\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)+2\left((2\,bc+ad)x^3+3\,acx\right)\sqrt{dx^2+c}}{4\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*s
qrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((2*b
*c + a*d)*x^3 + 3*a*c*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*
c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*
d - a^2*b*c*d^2 + a^3*d^3)*x^2), 1/4*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d
)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((2*b*c + a*d)*x^3 + 3*a*c
```

```
*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d -
2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3
*d^3)*x^2)]
```

giac [B] time = 4.33, size = 298, normalized size = 2.29

$$\frac{3\,ac\sqrt{d}\,\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{abcd-a^2d^2}}+\frac{cx}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{dx^2+c}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/2*a*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d -
a^2*d^2)) + c*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - ((sqrt(
d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sq
rt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d +
b*c^2)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2))
```

maple [B] time = 0.03, size = 1498, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/b^2*x/c/(d*x^2+c)^(1/2)-1/4*a/b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*
a/b^2*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*a^2/b^2*d^2/(a*d-b*c)^2/c/((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*
x-3/4*a/b^2*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(
-a*b)^(1/2)/b))-5/4*a/b^2/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-3/4/b*a/(-a*b)^(1/2)/(a*d-b
*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)-5/4/b^2*a/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/(-
(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4*a/b^2/(a*d-b*c)/(x+(
-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)-3/4*a/b^2*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/
```

```
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*a^2/b^2
*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*a/b^2*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)-3/4/b*a/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b
)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.750 $\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^{3/2}} \, dx$

Optimal. Leaf size=134

$$\frac{a}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{5/2}}$$

Rubi [A] time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{ad+2bc}{2b\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(2*b*(b*c - a*d)^2*Sqrt[c + d*x^2]) + a/(2*b*(b*c - a*d)*(a +
 b*x^2)*Sqrt[c + d*x^2]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{4b(bc-ad)}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(2bc+ad)\,\text{Subst}\left(\int\right.}{4(bc-}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(2bc+ad)\,\text{Subst}\left(\int\right.}{2d(b}$$

$$= \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(2bc+ad)\tanh^{-1}\left(\right.}{2\sqrt{b}\,(bc-ad)^5}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.68

$$\frac{\left(a+bx^2\right)(ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+a(bc-ad)}{2b\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (a*(b*c - a*d) + (2*b*c + a*d)*(a + b*x^2)*Hypergeometric2F1[-1/2, 1, 1/2,
(b*(c + d*x^2))/(b*c - a*d)])/(2*b*(b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2
])
```

IntegrateAlgebraic [A] time = 0.31, size = 123, normalized size = 0.92

$$\frac{3ac+adx^2+2bcx^2}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{(-ad-2bc)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2\sqrt{b}\,(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (3*a*c + 2*b*c*x^2 + a*d*x^2)/(2*(b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2])
 + ((-2*b*c - a*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c
 - a*d)])/(2*Sqrt[b]*(-(b*c) + a*d)^(5/2))
```

fricas [B] time = 1.18, size = 732, normalized size = 5.46

$$\left[\frac{\left(\left(2\,b^2cd+abd^2\right)x^4+2\,abc^2+a^2cd+\left(2\,b^2c^2+3\,abcd+a^2d^2\right)x^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(bdx^2+2\,bc-ad\right)\sqrt{b^2c-abd}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(3\,ab^2c^2-3\,a^2bcd+\left(2\,b^3c^2-ab^2cd-a^2bd^2\right)x^2\right)\sqrt{dx^2+c}}{8\left(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^4+\left(b^5c^4-2\,ab^4c^3d+2\,a^3b^2cd^3-a^4bd^4\right)x^2\right)},-\frac{\left(\left(2\,b^2cd+abd^2\right)x^4+2\,abc^2+a^2cd+\left(2\,b^2c^2+3\,abcd+a^2d^2\right)x^2\right)\sqrt{-b^2c+abd}\,\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{-b^2c+abd}\,\sqrt{dx^2+c}}{2\left(b^2c^2-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)-2\left(3\,ab^2c^2-3\,a^2bcd+\left(2\,b^3c^2-ab^2cd-a^2bd^2\right)x^2\right)\sqrt{dx^2+c}}{4\left(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^4+\left(b^5c^4-2\,ab^4c^3d+2\,a^3b^2cd^3-a^4bd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c*d + a*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^2)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a
*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d
)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*
a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(d*x
^2 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (
b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (b^5*c^4
 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2), -1/4*(((2*b^2*c*d + a
*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^2)*
```

```
sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*
d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*
b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqrt(d*x^2
 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^
5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (b^5*c^4 -
 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2)]
```

giac [A] time = 0.39, size = 181, normalized size = 1.35

$$\frac{\frac{\left(2\,bcd+ad^{2}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-b^{2}c+abd}}+\frac{2\left(dx^{2}+c\right)bcd-2\,bc^{2}d+\left(dx^{2}+c\right)ad^{2}+2\,acd^{2}}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(\left(dx^{2}+c\right)^{\frac{3}{2}}b-\sqrt{dx^{2}+c}\,bc+\sqrt{dx^{2}+c}\,ad\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^2 + c)*b*c*d -
2*b*c^2*d + (d*x^2 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*a*d)))/d
```

maple [B] time = 0.02, size = 1456, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/b/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)-1/b^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/b/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4*(-a*b)^(1/2)/
b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*a/b*d/(a*d-b*c)^2/((x-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*(-a*
b)^(1/2)/b^2*a*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4*a/b*d/(a*d-b*c)^2/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+(-a*b)^(1/2)/b^2/(a*d-b*c)/c/((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*
d*x-1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2
```

```
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*a/b*d/(a*d-
b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)+3/4*(-a*b)^(1/2)/b^2*a*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4*a/b*d/(a*
d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2/b/(a*d-b*
c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)+1/2/b/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b)
)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 1.10, size = 142, normalized size = 1.06

$$\frac{\frac{c}{a\,d-b\,c}+\frac{\left(d\,x^2+c\right)\,\left(a\,d+2\,b\,c\right)}{2\,{\left(a\,d-b\,c\right)}^2}}{b\,{\left(d\,x^2+c\right)}^{3/2}+\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)\,\left(a\,d+2\,b\,c\right)}{2\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] (c/(a*d - b*c) + ((c + d*x^2)*(a*d + 2*b*c))/(2*(a*d - b*c)^2))/(b*(c + d*x
^2)^(3/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) + (atan((b^(1/2)*(c + d*x^2)^(1/
2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2))*(a*d + 2*b*c))/(2*b^
(1/2)*(a*d - b*c)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.751 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{5/2}}$$

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {471, 527, 12, 377, 205}

$$-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}-\frac{3dx}{2\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{(2ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d*x)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - x/(2*(b*c - a*d)*(a + b*x^2)*S
qrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx &= -\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{\int \frac{c-2dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2(bc-ad)}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{\int \frac{c(bc+2ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2c(bc-ad)^2}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(bc+2ad)\int \frac{1}{\left(a+bx^2\right)\sqrt{c-}}}{2(bc-ad)^2}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(bc+2ad)\,\mathrm{Subst}\left(\int \frac{}{a-}\right.}{2(bc}\\
&= -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}}-\frac{x}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}+\frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt{bc}}{\sqrt{a}\,\sqrt{}}\right.}{2\sqrt{a}\,(bc-ad)^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 1.18, size = 133, normalized size = 1.08

$$\frac{x^3\left(\frac{8x^2\left(c+dx^2\right)(bc-ad)\,{}_2F_1\left(2,3;\frac{9}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}+7c\left(5c+2dx^2\right)\,{}_2F_1\left(1,2;\frac{7}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)\right)}{105c^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (x^3*(7*c*(5*c + 2*d*x^2)*Hypergeometric2F1[1, 2, 7/2, ((b*c - a*d)*x^2)/(c
*(a + b*x^2))] + (8*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 3, 9/2
, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2)))/(105*c^3*(a + b*x^2)^2*
Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.73, size = 135, normalized size = 1.10

$$\frac{(-2ad-bc)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2\sqrt{a}\,(bc-ad)^{5/2}}+\frac{-2adx-bcx-3bdx^3}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*x) - 2*a*d*x - 3*b*d*x^3)/(2*(b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^
2]) + ((-(b*c) - 2*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*
x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*(b*c - a*d)^(5/2))
```

fricas [B] time = 1.73, size = 744, normalized size = 6.05

$$\left[-\frac{\left(\left(b^2cd+2\,abd^2\right)x^4+abc^2+2\,a^2cd+\left(b^2c^2+3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(3\left(ab^2cd-a^2bd^2\right)x^3+\left(ab^2c^2+a^2bcd-2\,a^3d^2\right)x\right)\sqrt{dx^2+c}}{8\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(ab^4c^3d-3\,a^2b^3c^2d^2+3\,a^3b^2cd^3-a^4bd^4\right)x^4+\left(ab^4c^4-2\,a^2b^3c^3d+2\,a^4bcd^3-a^5d^4\right)x^2\right)},\frac{\left(\left(b^2cd+2\,abd^2\right)x^4+abc^2+2\,a^2cd+\left(b^2c^2+3\,abcd+2\,a^2d^2\right)x^2\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)-2\left(3\left(ab^2cd-a^2bd^2\right)x^3+\left(ab^2c^2+a^2bcd-2\,a^3d^2\right)x\right)\sqrt{dx^2+c}}{4\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(ab^4c^3d-3\,a^2b^3c^2d^2+3\,a^3b^2cd^3-a^4bd^4\right)x^4+\left(ab^4c^4-2\,a^2b^3c^3d+2\,a^4bcd^3-a^5d^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((b^2*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*
c*d + 2*a^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3
 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2)*
x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*
c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d^4)*x^4
```

```
 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2), 1/4*(((b^2
*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*c*d + 2*a^2*
d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*
x)) - 2*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2
)*x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^
5*c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d^4)*x
^4 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2)]
```

giac [B] time = 4.45, size = 299, normalized size = 2.43

$$-\frac{dx}{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{dx^{2}+c}}-\frac{\left(bc\sqrt{d}+2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad^{\frac{3}{2}}-bc^{2}\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -d*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - 1/2*(b*c*sqrt(d) +
 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^
2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^
4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*d + b*c^2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

maple [B] time = 0.02, size = 1453, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4/b/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/4/(-a*b)^(1/
2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/b/(a*
d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4/b*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/
b*a*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/b*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*c)/
b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-
b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d
```

```
-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3
/4/b/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*d*x+1/4/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*
ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/4/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4
/b*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/b*a*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b
*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)
/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.752 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{3d}{2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{1}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$-\frac{3d}{2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{1}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-3*d)/(2*(b*c - a*d)^2*Sqrt[c + d*x^2]) - 1/(2*(b*c - a*d)*(a + b*x^2)*Sqr
t[c + d*x^2]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a
*d]])/(2*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^2\right)$$

$$= -\frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3d)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx,x,x^2\right)}{4(bc-ad)}$$

$$= -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3bd)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{}}\right.}{4(bc-ad}$$

$$= -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{2(bc-a}$$

$$= -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+d}}{\sqrt{bc-a}}\right.}{2(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.46

$$-\frac{d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{\sqrt{c+dx^2}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x^2))/(-(b*c) + a*d))])/((
-(b*c) + a*d)^2*Sqrt[c + d*x^2]))
```

IntegrateAlgebraic [A] time = 0.29, size = 113, normalized size = 1.00

$$\frac{-2ad-bc-3bdx^2}{2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{3\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(b*c) - 2*a*d - 3*b*d*x^2)/(2*(b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2])
+ (3*Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a
*d)])/(2*(-(b*c) + a*d)^(5/2))
```

fricas [B] time = 1.00, size = 537, normalized size = 4.75

$$\left[\frac{3\left(bd^2x^4+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2+4\left(2\,b^2c^2-3\,abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\,\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)-4\left(3\,bdx^2+bc+2\,ad\right)\sqrt{dx^2+c}}{8\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)},-\frac{3\left(bd^2x^4+acd+\left(bcd+ad^2\right)x^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\,\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\left(3\,bdx^2+bc+2\,ad\right)\sqrt{dx^2+c}}{4\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^4+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((
b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x
^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*b*d*x^2 +
b*c + 2*a*d)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3
*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*
d^2 + a^3*d^3)*x^2), -1/4*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt
(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b
/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*b*d*x^2 + b*c + 2*a*d)*sqrt(d*x^2 + c
))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^
2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)]
```

giac [A] time = 0.39, size = 153, normalized size = 1.35

$$-\frac{3\,bd\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}-\frac{3\left(dx^2+c\right)bd-2\,bcd+2\,ad^2}{2\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^2+c\right)^{\frac{3}{2}}b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -3/2*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/2*(3*(d*x^2 + c)*b*d - 2*b*c*d + 2*
a*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 +
 c)*b*c + sqrt(d*x^2 + c)*a*d))
```

maple [**B**]   time = 0.01, size = 989, normalized size = 8.75

$$\frac{3d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{4(ad-bc)^2\sqrt{-\frac{ad-bc}{b}}}+\frac{3d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{4(ad-bc)^2\sqrt{-\frac{ad-bc}{b}}}+\frac{3\sqrt{-ab}\,d^2x}{4(ad-bc)^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}-\frac{3\sqrt{-ab}\,d^2x}{4(ad-bc)^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,bc}-\frac{3d}{4(ad-bc)^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{3d}{4(ad-bc)^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}-\frac{\sqrt{-ab}\,dx}{2(ad-bc)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,abc}+\frac{\sqrt{-ab}\,dx}{2(ad-bc)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,abc}-\frac{\sqrt{-ab}}{4(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ab}+\frac{\sqrt{-ab}}{4(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-\frac{2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*d/(a*d-b*c)^2/
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)+3/4*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*d/(a*d-b*c)^2/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/2*(-a*b)^(1/2)/a/b/(a*d-b*c
)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)*d*x+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*d
/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)-3/4*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*d/(a*d-
b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/2*(-a*b)^(1/2
)/a/b/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)*d*x
```

maxima [**F(-2)**]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.99, size = 130, normalized size = 1.15

$$-\frac{\frac{d}{a\,d-b\,c}+\frac{3\,b\,d\,\left(d\,x^2+c\right)}{2\,{\left(a\,d-b\,c\right)}^2}}{b\,{\left(d\,x^2+c\right)}^{3/2}+\sqrt{d\,x^2+c}\,\left(a\,d-b\,c\right)}-\frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{2\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] - (d/(a*d - b*c) + (3*b*d*(c + d*x^2))/(2*(a*d - b*c)^2))/(b*(c + d*x^2)^(3
/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*x^
2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(2*(a*d - b*c
)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

3.753 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.10, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 205}

$$\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{dx(2ad+bc)}{2ac\sqrt{c+dx^2}\,(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + (b*x)/(2*a*(b*c
 - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-bc+2ad-2bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\int \frac{bc(bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx}{2ac(bc-ad)^2}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(b(bc-4ad))\int \frac{\quad}{(a+}}{2a(bc-a}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{(b(bc-4ad))\,\mathrm{Subs}}{2}$$

$$= \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{b(bc-4ad)\tan^{-1}\left(\right.}{2a^{3/2}(bc-a}$$

Mathematica [A] time = 5.25, size = 120, normalized size = 0.85

$$\frac{1}{2}\left(\frac{b(bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}} + \frac{x\sqrt{c+dx^2}\left(\frac{b^2}{a^2+abx^2}+\frac{2d^2}{c^2+cdx^2}\right)}{(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((x*Sqrt[c + d*x^2]*(b^2/(a^2 + a*b*x^2) + (2*d^2)/(c^2 + c*d*x^2)))/(b*c -
 a*d)^2 + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(a^(3/2)*(b*c - a*d)^(5/2)))/2
```

IntegrateAlgebraic [**A**] time = 0.76, size = 163, normalized size = 1.15

$$\frac{\left(4abd-b^2c\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{5/2}}+\frac{2a^2d^2x+2abd^2x^3+b^2c^2x+b^2cdx^3}{2ac\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*x + 2*a^2*d^2*x + b^2*c*d*x^3 + 2*a*b*d^2*x^3)/(2*a*c*(-(b*c) + a*
d)^2*(a + b*x^2)*Sqrt[c + d*x^2]) + ((-(b^2*c) + 4*a*b*d)*ArcTan[(a*Sqrt[d]
 + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3
/2)*(b*c - a*d)^(5/2))
```

fricas [**B**] time = 1.75, size = 854, normalized size = 6.01

$$\left[\frac{\left(ab^2c^3-4\,a^2bc^2d+\left(b^3c^2d-4\,ab^2cd^2\right)x^4+\left(b^3c^3-3\,ab^2c^2d-4\,a^2bcd^2\right)x^2\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2+4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\,\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\left(\left(ab^3c^2d+a^2b^2cd^2-2\,a^3bd^3\right)x^3+\left(ab^3c^3-a^2b^2c^2d+2\,a^3bcd^2-2\,a^4d^3\right)x\right)\sqrt{dx^2+c}}{8\left(a^3b^3c^5-3\,a^4b^2c^4d+3\,a^5bc^3d^2-a^6c^2d^3+\left(a^2b^4c^4d-3\,a^3b^3c^3d^2+3\,a^4b^2c^2d^3-a^5bcd^4\right)x^4+\left(a^2b^4c^5-2\,a^3b^3c^4d+2\,a^5bc^2d^3-a^6cd^4\right)x^2\right)},\ \frac{\left(ab^2c^3-4\,a^2bc^2d+\left(b^3c^2d-4\,ab^2cd^2\right)x^4+\left(b^3c^3-3\,ab^2c^2d-4\,a^2bcd^2\right)x^2\right)\sqrt{abc-a^2d}\,\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+2\left(\left(ab^3c^2d+a^2b^2cd^2-2\,a^3bd^3\right)x^3+\left(ab^3c^3-a^2b^2c^2d+2\,a^3bcd^2-2\,a^4d^3\right)x\right)\sqrt{dx^2+c}}{4\left(a^3b^3c^5-3\,a^4b^2c^4d+3\,a^5bc^3d^2-a^6c^2d^3+\left(a^2b^4c^4d-3\,a^3b^3c^3d^2+3\,a^4b^2c^2d^3-a^5bcd^4\right)x^4+\left(a^2b^4c^5-2\,a^3b^3c^4d+2\,a^5bc^2d^3-a^6cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c
^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) + 4*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 +
 (a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^2 + c)
)/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (a^2*b^4
*c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 + (a^2*b^
4*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2), 1/4*((a*b^2*c^
3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 3*a*b^2*c^
2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d
)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b
*c^2 - a^2*c*d)*x)) + 2*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^3 +
(a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^2 + c))
/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (a^2*b^4*
```

```
c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 + (a^2*b^4
*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2)]
```

giac [B] time = 4.53, size = 318, normalized size = 2.24

$$\frac{d^{2}x}{\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)\sqrt{dx^{2}+c}}-\frac{\left(b^{2}c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c\sqrt{d}-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}abd^{\frac{3}{2}}-b^{2}c^{2}\sqrt{d}}{\left(\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}\,x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)\left(ab^{2}c^{2}-2\,a^{2}bcd+a^{3}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*x/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)) - 1/2*(b^2*c*sq
rt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt
(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2))
```

maple [B] time = 0.01, size = 1461, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4/a*(-a*b)^(1/2)*d/(a*d-b*c)^2/
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)+3/4*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a/(a*d-b*c)/c/((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1
/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/4/a/(-a*b
)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/
4/a/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/a*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2)+3/4*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
```

```
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/a*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)+1/4/a/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a
*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^2+a\right)}^2\,{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.754 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}\,(bc-ad)}$$

Rubi [A] time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b}{2a\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{d(2ad+bc)}{2ac\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a*d
)*(a + b*x^2)*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(3
/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c
- a*d]])/(2*a^2*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^2\right)}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad}{x(a}\right.}{ac(}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{2a^2c}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{a^2c}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}$$

Mathematica [C] time = 0.13, size = 123, normalized size = 0.72

$$\frac{\frac{b(2bc-5ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{a(ad-bc)} + \left(\frac{2b}{a}-\frac{2d}{c}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right) + \frac{b}{a+bx^2}}{2a\sqrt{c+dx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b/(a + b*x^2) + (b*(2*b*c - 5*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c +
 d*x^2))/(b*c - a*d)])/(a*(-(b*c) + a*d)) + ((2*b)/a - (2*d)/c)*Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c])/(2*a*(b*c - a*d)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.71, size = 181, normalized size = 1.06

$$\frac{\left(2b^{5/2}c-5ab^{3/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2(ad-bc)^{5/2}} + \frac{2a^2d^2+2abd^2x^2+b^2c^2+b^2cdx^2}{2ac\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2 + 2*a^2*d^2 + b^2*c*d*x^2 + 2*a*b*d^2*x^2)/(2*a*c*(-(b*c) + a*d)^2
*(a + b*x^2)*Sqrt[c + d*x^2]) + ((2*b^(5/2)*c - 5*a*b^(3/2)*d)*ArcTan[(Sqrt
[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*a^2*(-(b*c) + a*d)
^(5/2)) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(3/2))
```

fricas [B] time = 5.88, size = 1992, normalized size = 11.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(b/(b*c - a*d))*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d
*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^
2*c^2*d + 2*a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d
 + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 +
(a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), 1/8*(8*(a
*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d
^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-c)*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d
 - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c
*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^
2 + a^2)) + 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^2)
*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4
*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d
- a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), -1/4*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (
2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c
^2*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d
*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2*(a*b^2*c^3 - 2*a^2*b*c^
2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 -
a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^
2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*
a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d
^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5
```

```
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2), -1/4*((2*a*b^2*c^4 -
5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*
c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*
b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 4*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a
^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^
2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^4 + (a^2*b^3*c^5
- a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2)]
```

giac [A] time = 0.35, size = 225, normalized size = 1.32

$$-\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{-b^2c+abd}}+\frac{\left(dx^2+c\right)b^2cd+2\left(dx^2+c\right)abd^2-2\,abcd^2+2\,a^2d^3}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left(\left(dx^2+c\right)^{\frac{3}{2}}b-\sqrt{dx^2+c}\,bc+\sqrt{dx^2+c}\,ad\right)}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/2*((d*x^2 +
 c)*b^2*c*d + 2*(d*x^2 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + s
qrt(d*x^2 + c)*a*d)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c)
```

maple [B] time = 0.02, size = 1672, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/2/a^2/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/a^2*(-a*b)^(1/2)/(a*d-b*c)/c/((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/a^
2/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b
*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/(-a*
b)^(1/2)/a/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4/a*d/(a*d-b*c)^2*b/((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
-3/4/(-a*b)^(1/2)*b*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a*d/(a*d-b*c)^2*b/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
```

```
(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/2/(-a*b)^(1/2)/a/(a*d-b*c)*
b/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(1/2)*d*x+1/2/a^2/(a*d-b*c)*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/2/a^2*(-a*b)^(1/2)/(a*d-b*c)/c/((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)*d*x-1/2/a^2/(a*d-b*c)*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)
/b))-1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^
2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4/a*d/(a*d-b
*c)^2*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)+3/4/(-a*b)^(1/2)*b*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-3/4/a*d/(a*d-b*c
)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/2/(-a*b)^(1/2)/
a/(a*d-b*c)*b/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)*d*x+1/a^2/c/(d*x^2+c)^(1/2)-1/a^2/c^(3/2)*ln((2*c+2*(
d*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x), x)
```

mupad **[B]** time = 3.41, size = 5227, normalized size = 30.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] atanh((240*a^3*b^11*c^11*d^4*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c
*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^
6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 -
 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12))
- (2080*a^4*b^10*c^10*d^5*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^
13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 +
 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10
```

```
160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (
7760*a^5*b^9*c^9*d^6*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 -
240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 1638
4*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a
^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (16384
*a^6*b^8*c^8*d^7*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*
a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^
6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b
^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (21584*a^7
*b^7*c^7*d^8*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*
b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^
8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c
^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (18400*a^8*b^6
*c^6*d^9*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11
*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^
7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d
^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (10160*a^9*b^5*c^5
*d^10*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^
10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d
^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10
 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (3520*a^10*b^4*c^4*d^
11*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10*
d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7
- 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 +
3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) + (704*a^11*b^3*c^3*d^12*(
c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10*d^4
+ 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7 - 21
584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520
*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)) - (64*a^12*b^2*c^2*d^13*(c + d
*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*c*d^13 - 240*a^3*b^11*c^10*d^4 + 208
0*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a
^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10
*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^12)))/(a^2*(c^3)^(1/2)) - (d^2/(b*c^2 -
a*c*d) + (d*(c + d*x^2)*(b^2*c + 2*a*b*d))/(2*a*(b*c^2 - a*c*d)*(a*d - b*c)
))/(b*(c + d*x^2)^(3/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) - (atan((((-b^3*(a
*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*((c + d*x^2)^(1/2)*(128*a^3*b^13*c^13*d^
2 - 1344*a^4*b^12*c^12*d^3 + 6160*a^5*b^11*c^11*d^4 - 16160*a^6*b^10*c^10*d
^5 + 26800*a^7*b^9*c^9*d^6 - 29312*a^8*b^8*c^8*d^7 + 21424*a^9*b^7*c^7*d^8
- 10400*a^10*b^6*c^6*d^9 + 3280*a^11*b^5*c^5*d^10 - 640*a^12*b^4*c^4*d^11 +
 64*a^13*b^3*c^3*d^12) + ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*(64*a^
6*b^12*c^14*d^3 - 896*a^7*b^11*c^13*d^4 + 4992*a^8*b^10*c^12*d^5 - 15360*a^
9*b^9*c^11*d^6 + 29568*a^10*b^8*c^10*d^7 - 37632*a^11*b^7*c^9*d^8 + 32256*a
^12*b^6*c^8*d^9 - 18432*a^13*b^5*c^7*d^10 + 6720*a^14*b^4*c^6*d^11 - 1408*a
^15*b^3*c^5*d^12 + 128*a^16*b^2*c^4*d^13 - ((-b^3*(a*d - b*c)^5)^(1/2)*(c +
 d*x^2)^(1/2)*(5*a*d - 2*b*c)*(512*a^7*b^13*c^16*d^2 - 5376*a^8*b^12*c^15*d
```

```
^3 + 25600*a^9*b^11*c^14*d^4 - 72960*a^10*b^10*c^13*d^5 + 138240*a^11*b^9*c
^12*d^6 - 182784*a^12*b^8*c^11*d^7 + 172032*a^13*b^7*c^10*d^8 - 115200*a^14
*b^6*c^9*d^9 + 53760*a^15*b^5*c^8*d^10 - 16640*a^16*b^4*c^7*d^11 + 3072*a^1
7*b^3*c^6*d^12 - 256*a^18*b^2*c^5*d^13))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*
b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4))))/(4*
(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*
c^2*d^3 - 5*a^6*b*c*d^4)))*1i)/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d
- 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4)) + ((-b^3*(a*d -
 b*c)^5)^(1/2)*(5*a*d - 2*b*c)*((c + d*x^2)^(1/2)*(128*a^3*b^13*c^13*d^2 -
1344*a^4*b^12*c^12*d^3 + 6160*a^5*b^11*c^11*d^4 - 16160*a^6*b^10*c^10*d^5 +
 26800*a^7*b^9*c^9*d^6 - 29312*a^8*b^8*c^8*d^7 + 21424*a^9*b^7*c^7*d^8 - 10
400*a^10*b^6*c^6*d^9 + 3280*a^11*b^5*c^5*d^10 - 640*a^12*b^4*c^4*d^11 + 64*
a^13*b^3*c^3*d^12) - ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*(64*a^6*b^
12*c^14*d^3 - 896*a^7*b^11*c^13*d^4 + 4992*a^8*b^10*c^12*d^5 - 15360*a^9*b^
9*c^11*d^6 + 29568*a^10*b^8*c^10*d^7 - 37632*a^11*b^7*c^9*d^8 + 32256*a^12*
b^6*c^8*d^9 - 18432*a^13*b^5*c^7*d^10 + 6720*a^14*b^4*c^6*d^11 - 1408*a^15*
b^3*c^5*d^12 + 128*a^16*b^2*c^4*d^13 + ((-b^3*(a*d - b*c)^5)^(1/2)*(c + d*x
^2)^(1/2)*(5*a*d - 2*b*c)*(512*a^7*b^13*c^16*d^2 - 5376*a^8*b^12*c^15*d^3 +
 25600*a^9*b^11*c^14*d^4 - 72960*a^10*b^10*c^13*d^5 + 138240*a^11*b^9*c^12*
d^6 - 182784*a^12*b^8*c^11*d^7 + 172032*a^13*b^7*c^10*d^8 - 115200*a^14*b^6
*c^9*d^9 + 53760*a^15*b^5*c^8*d^10 - 16640*a^16*b^4*c^7*d^11 + 3072*a^17*b^
3*c^6*d^12 - 256*a^18*b^2*c^5*d^13))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*
c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4))))/(4*(a^7
*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*
d^3 - 5*a^6*b*c*d^4)))*1i)/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10
*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4)))/(((-b^3*(a*d - b*c
)^5)^(1/2)*(5*a*d - 2*b*c)*((c + d*x^2)^(1/2)*(128*a^3*b^13*c^13*d^2 - 1344
*a^4*b^12*c^12*d^3 + 6160*a^5*b^11*c^11*d^4 - 16160*a^6*b^10*c^10*d^5 + 268
00*a^7*b^9*c^9*d^6 - 29312*a^8*b^8*c^8*d^7 + 21424*a^9*b^7*c^7*d^8 - 10400*
a^10*b^6*c^6*d^9 + 3280*a^11*b^5*c^5*d^10 - 640*a^12*b^4*c^4*d^11 + 64*a^13
*b^3*c^3*d^12) - ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*(64*a^6*b^12*c
^14*d^3 - 896*a^7*b^11*c^13*d^4 + 4992*a^8*b^10*c^12*d^5 - 15360*a^9*b^9*c^
11*d^6 + 29568*a^10*b^8*c^10*d^7 - 37632*a^11*b^7*c^9*d^8 + 32256*a^12*b^6*
c^8*d^9 - 18432*a^13*b^5*c^7*d^10 + 6720*a^14*b^4*c^6*d^11 - 1408*a^15*b^3*
c^5*d^12 + 128*a^16*b^2*c^4*d^13 + ((-b^3*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^
(1/2)*(5*a*d - 2*b*c)*(512*a^7*b^13*c^16*d^2 - 5376*a^8*b^12*c^15*d^3 + 256
00*a^9*b^11*c^14*d^4 - 72960*a^10*b^10*c^13*d^5 + 138240*a^11*b^9*c^12*d^6
- 182784*a^12*b^8*c^11*d^7 + 172032*a^13*b^7*c^10*d^8 - 115200*a^14*b^6*c^9
*d^9 + 53760*a^15*b^5*c^8*d^10 - 16640*a^16*b^4*c^7*d^11 + 3072*a^17*b^3*c^
6*d^12 - 256*a^18*b^2*c^5*d^13))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*
d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4))))/(4*(a^7*d^5
 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3
- 5*a^6*b*c*d^4))))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^
3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4)) - ((-b^3*(a*d - b*c)^5)^(1
/2)*(5*a*d - 2*b*c)*((c + d*x^2)^(1/2)*(128*a^3*b^13*c^13*d^2 - 1344*a^4*b^
```

```
12*c^12*d^3 + 6160*a^5*b^11*c^11*d^4 - 16160*a^6*b^10*c^10*d^5 + 26800*a^7*
b^9*c^9*d^6 - 29312*a^8*b^8*c^8*d^7 + 21424*a^9*b^7*c^7*d^8 - 10400*a^10*b^
6*c^6*d^9 + 3280*a^11*b^5*c^5*d^10 - 640*a^12*b^4*c^4*d^11 + 64*a^13*b^3*c^
3*d^12) + ((-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*(64*a^6*b^12*c^14*d^3
 - 896*a^7*b^11*c^13*d^4 + 4992*a^8*b^10*c^12*d^5 - 15360*a^9*b^9*c^11*d^6
+ 29568*a^10*b^8*c^10*d^7 - 37632*a^11*b^7*c^9*d^8 + 32256*a^12*b^6*c^8*d^9
 - 18432*a^13*b^5*c^7*d^10 + 6720*a^14*b^4*c^6*d^11 - 1408*a^15*b^3*c^5*d^1
2 + 128*a^16*b^2*c^4*d^13 - ((-b^3*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(
5*a*d - 2*b*c)*(512*a^7*b^13*c^16*d^2 - 5376*a^8*b^12*c^15*d^3 + 25600*a^9*
b^11*c^14*d^4 - 72960*a^10*b^10*c^13*d^5 + 138240*a^11*b^9*c^12*d^6 - 18278
4*a^12*b^8*c^11*d^7 + 172032*a^13*b^7*c^10*d^8 - 115200*a^14*b^6*c^9*d^9 +
53760*a^15*b^5*c^8*d^10 - 16640*a^16*b^4*c^7*d^11 + 3072*a^17*b^3*c^6*d^12
- 256*a^18*b^2*c^5*d^13))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*
a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4))))/(4*(a^7*d^5 - a^2*
b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6
*b*c*d^4))))/(4*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d
^2 + 10*a^5*b^2*c^2*d^3 - 5*a^6*b*c*d^4)) + 32*a^2*b^12*c^11*d^3 - 208*a^3*
b^11*c^10*d^4 + 416*a^4*b^10*c^9*d^5 + 80*a^5*b^9*c^8*d^6 - 1600*a^6*b^8*c^
7*d^7 + 2768*a^7*b^7*c^6*d^8 - 2272*a^8*b^6*c^5*d^9 + 944*a^9*b^5*c^4*d^10
- 160*a^10*b^4*c^3*d^11))*(-b^3*(a*d - b*c)^5)^(1/2)*(5*a*d - 2*b*c)*1i)/(2
*(a^7*d^5 - a^2*b^5*c^5 + 5*a^3*b^4*c^4*d - 10*a^4*b^3*c^3*d^2 + 10*a^5*b^2
*c^2*d^3 - 5*a^6*b*c*d^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.755 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{}{2acx}$$

Rubi [A] time = 0.27, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {472, 579, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}\left(4a^2d^2-4abcd+3b^2c^2\right)}{2a^2c^2x(bc-ad)^2}-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}+\frac{d(2ad+bc)}{2acx\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x*Sqrt[c + d*x^2]) + b/(2*a*(b*c - a
*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) - ((3*b^2*c^2 - 4*a*b*c*d + 4*a^2*d^2)*S
qrt[c + d*x^2])/(2*a^2*c^2*(b*c - a*d)^2*x) - (3*b^2*(b*c - 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2)
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-3bc+2ad-4bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-3b^2c^2+4abcd-4}{x^2(a+b}}{2ac(b}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd\right.}{2a^2c^2(}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd\right.}{2a^2c^2(}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd\right.}{2a^2c^2(}$$

$$= \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(3b^2c^2-4abcd\right.}{2a^2c^2(}$$

Mathematica [A] time = 5.37, size = 145, normalized size = 0.71

$$\sqrt{c+dx^2}\left(-\frac{\frac{b^3x}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{1}{c^2x}}{a^2}-\frac{d^3x}{c^2\left(c+dx^2\right)(bc-ad)^2}\right)-\frac{3b^2(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-((d^3*x)/(c^2*(b*c - a*d)^2*(c + d*x^2))) - (1/(c^2*x) +
(b^3*x)/(2*(b*c - a*d)^2*(a + b*x^2)))/a^2) - (3*b^2*(b*c - 2*a*d)*ArcTan[(
Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(5/2)
)
```

IntegrateAlgebraic [A] time = 1.19, size = 244, normalized size = 1.19

$$\frac{3\left(b^3c-2ab^2d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{5/2}}+\frac{-2a^3cd^2-4a^3d^3x^2+4a^2bc^2d+2a^2bcd^2x^2-4a^2bd^3x^4-2ab^2c^3+2ab^2c^2dx^2+4ab^2cd^2x^4-3b^3c^3x^2-3b^3c^2dx^4}{2a^2c^2x\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2 - 3*b^3*c^3*x^2 + 2*a*b^2*c^2*d
*x^2 + 2*a^2*b*c*d^2*x^2 - 4*a^3*d^3*x^2 - 3*b^3*c^2*d*x^4 + 4*a*b^2*c*d^2*
x^4 - 4*a^2*b*d^3*x^4)/(2*a^2*c^2*(-(b*c) + a*d)^2*x*(a + b*x^2)*Sqrt[c + d
*x^2]) + (3*(b^3*c - 2*a*b^2*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqr
t[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(b*c - a*d)^(5/2))
```

fricas [B] time = 1.83, size = 1018, normalized size = 4.97

$$\left[\frac{3\left(\left(b^{4}c^{3}d-2\,ab^{3}c^{2}d^{2}\right)x^{5}+\left(b^{4}c^{4}-ab^{3}c^{3}d-2\,a^{2}b^{2}c^{2}d^{2}\right)x^{3}+\left(ab^{3}c^{4}-2\,a^{2}b^{2}c^{3}d\right)x\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{4}+a^{2}c^{2}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{2}-4\left(\left(bc-2\,ad\right)x^{3}-acx\right)\sqrt{-abc+a^{2}d}\,\sqrt{dx^{2}+c}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\left(2\,a^{2}b^{3}c^{4}-6\,a^{3}b^{2}c^{3}d+6\,a^{4}bc^{2}d^{2}-2\,a^{5}cd^{3}+\left(3\,ab^{4}c^{3}d-7\,a^{2}b^{3}c^{2}d^{2}+8\,a^{3}b^{2}cd^{3}-4\,a^{4}bd^{4}\right)x^{4}+\left(3\,ab^{4}c^{4}-5\,a^{2}b^{3}c^{3}d+6\,a^{4}bcd^{3}-4\,a^{5}d^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}}{8\left(\left(a^{3}b^{4}c^{5}d-3\,a^{4}b^{3}c^{4}d^{2}+3\,a^{5}b^{2}c^{3}d^{3}-a^{6}bc^{2}d^{4}\right)x^{5}+\left(a^{3}b^{4}c^{6}-2\,a^{4}b^{3}c^{5}d+2\,a^{6}bc^{3}d^{3}-a^{7}c^{2}d^{4}\right)x^{3}+\left(a^{4}b^{3}c^{6}-3\,a^{5}b^{2}c^{5}d+3\,a^{6}bc^{4}d^{2}-a^{7}c^{3}d^{3}\right)x\right)},\ -\frac{3\left(\left(b^{4}c^{3}d-2\,ab^{3}c^{2}d^{2}\right)x^{5}+\left(b^{4}c^{4}-ab^{3}c^{3}d-2\,a^{2}b^{2}c^{2}d^{2}\right)x^{3}+\left(ab^{3}c^{4}-2\,a^{2}b^{2}c^{3}d\right)x\right)\sqrt{abc-a^{2}d}\,\arctan\left(\frac{\sqrt{abc-a^{2}d}\left(\left(bc-2\,ad\right)x^{2}-ac\right)\sqrt{dx^{2}+c}}{2\left(\left(abcd-a^{2}d^{2}\right)x^{3}+\left(abc^{2}-a^{2}cd\right)x\right)}\right)+2\left(2\,a^{2}b^{3}c^{4}-6\,a^{3}b^{2}c^{3}d+6\,a^{4}bc^{2}d^{2}-2\,a^{5}cd^{3}+\left(3\,ab^{4}c^{3}d-7\,a^{2}b^{3}c^{2}d^{2}+8\,a^{3}b^{2}cd^{3}-4\,a^{4}bd^{4}\right)x^{4}+\left(3\,ab^{4}c^{4}-5\,a^{2}b^{3}c^{3}d+6\,a^{4}bcd^{3}-4\,a^{5}d^{4}\right)x^{2}\right)\sqrt{dx^{2}+c}}{4\left(\left(a^{3}b^{4}c^{5}d-3\,a^{4}b^{3}c^{4}d^{2}+3\,a^{5}b^{2}c^{3}d^{3}-a^{6}bc^{2}d^{4}\right)x^{5}+\left(a^{3}b^{4}c^{6}-2\,a^{4}b^{3}c^{5}d+2\,a^{6}bc^{3}d^{3}-a^{7}c^{2}d^{4}\right)x^{3}+\left(a^{4}b^{3}c^{6}-3\,a^{5}b^{2}c^{5}d+3\,a^{6}bc^{4}d^{2}-a^{7}c^{3}d^{3}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^4*c^4 - a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d)*x)*sqrt(-a*b*c + a^2*d)*l
og(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*
c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 +
c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a
^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2
*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3
- 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*
a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a
^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 + (a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*
c^4*d^2 - a^7*c^3*d^3)*x), -1/4*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^
4*c^4 - a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d
)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2
- a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) +
 2*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*
b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b
^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3 - 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))
/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x
^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 +
(a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*c^4*d^2 - a^7*c^3*d^3)*x)]
```

giac [B] time = 4.77, size = 554, normalized size = 2.70

$$-\frac{d^{3}x}{\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\sqrt{dx^{2}+c}}+\frac{3\left(b^{3}c\sqrt{d}-2\,ab^{2}d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{3}c^{2}\sqrt{d}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ab^{2}cd^{\frac{3}{2}}+2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}bd^{\frac{5}{2}}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{3}c^{3}\sqrt{d}+18\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{2}c^{2}d^{\frac{3}{2}}-20\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}bcd^{\frac{5}{2}}+8\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{3}d^{\frac{7}{2}}+3\,b^{3}c^{4}\sqrt{d}-4\,ab^{2}c^{3}d^{\frac{3}{2}}+2\,a^{2}bc^{2}d^{\frac{5}{2}}}{\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad+3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}-4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}acd-bc^{3}\right)\left(a^{2}b^{2}c^{3}-2\,a^{3}bc^{2}d+a^{4}cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -d^3*x/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(d*x^2 + c)) + 3/2*(b^3*c
*sqrt(d) - 2*a*b^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^
2)*sqrt(a*b*c*d - a^2*d^2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^3*c^2*sq
rt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c*d^(3/2) + 2*(sqrt(d)*x -
sqrt(d*x^2 + c))^4*a^2*b*d^(5/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^
3*sqrt(d) + 18*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) - 20*(sqrt
(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c*d^(5/2) + 8*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a^3*d^(7/2) + 3*b^3*c^4*sqrt(d) - 4*a*b^2*c^3*d^(3/2) + 2*a^2*b*c^2*d^(
5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))
^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*(a^2*b^2*c
^3 - 2*a^3*b*c^2*d + a^4*c*d^2))
```

maple [B] time = 0.02, size = 1524, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/4/a^2/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/a^2*(-a*b)^(1/2)*d/(a*d-b*
c)^2*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)-3/4/a*b*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/a^2*(-a*b)^(1/2)*d/(a*d-
b*c)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a^2/(a*d-b
*c)*b/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*d*x-3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b/a^2/(a*d-b
*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*d*x+3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2
)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2))/(x+(-a*b)^(1/2)/b))+1/4/a^2/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)/((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3
/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)^2*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4/a*b*d^2/(a*d-b*c)^2/c/((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
-3/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(
-a*b)^(1/2)/b))+3/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+
```

```
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^2/(-a*b)
^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/a^2/
c/x/(d*x^2+c)^(1/2)-2/a^2*d/c^2*x/(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.756 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{b(2b}{2a^2c\left(a+bx^2\right)}$$

Rubi [A] time = 0.34, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{2a^2c^2\sqrt{c+dx^2}\,(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(2*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^2]) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2
]) - 1/(2*a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^3*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{1}{2acx^2\left(a+bx^2\right)\sqrt{c+dx^2}} -$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} -$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} -$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} -$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}} -$$

Mathematica [C] time = 0.13, size = 189, normalized size = 0.78

$$\frac{b^2c^2x^2\left(a+bx^2\right)(4bc-7ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)-(ad-bc)\left(x^2\left(a+bx^2\right)\left(3a^2d^2+abcd-4b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^2}{c}+1\right)+ac\left(a^2d+ab\left(dx^2-c\right)-2b^2cx^2\right)\right)}{2a^3c^2x^2\left(a+bx^2\right)\sqrt{c+dx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 7*a*d)*x^2*(a + b*x^2)*Hypergeometric2F1[-1/2, 1, 1/2, (b
*(c + d*x^2))/(b*c - a*d)] - (-(b*c) + a*d)*(a*c*(a^2*d - 2*b^2*c*x^2 + a*b
*(-c + d*x^2)) + (-4*b^2*c^2 + a*b*c*d + 3*a^2*d^2)*x^2*(a + b*x^2)*Hyperge
ometric2F1[-1/2, 1, 1/2, 1 + (d*x^2)/c]))/(2*a^3*c^2*(b*c - a*d)^2*x^2*(a +
 b*x^2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A]   time = 0.99, size = 272, normalized size = 1.13

$$\frac{\left(7ab^{5/2}d-4b^{7/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2a^3(ad-bc)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}}+\frac{-a^3cd^2-3a^3d^3x^2+2a^2bc^2d+a^2bcd^2x^2-3a^2bd^3x^4-ab^2c^3+ab^2c^2dx^2+2ab^2cd^2x^4-2b^3c^3x^2-2b^3c^2dx^4}{2a^2c^2x^2\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(a*b^2*c^3) + 2*a^2*b*c^2*d - a^3*c*d^2 - 2*b^3*c^3*x^2 + a*b^2*c^2*d*x^2
 + a^2*b*c*d^2*x^2 - 3*a^3*d^3*x^2 - 2*b^3*c^2*d*x^4 + 2*a*b^2*c*d^2*x^4 -
3*a^2*b*d^3*x^4)/(2*a^2*c^2*(-(b*c) + a*d)^2*x^2*(a + b*x^2)*Sqrt[c + d*x^2
]) + ((-4*b^(7/2)*c + 7*a*b^(5/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqr
t[c + d*x^2])/(b*c - a*d)])/(2*a^3*(-(b*c) + a*d)^(5/2)) + ((4*b*c + 3*a*d)
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(5/2))
```

fricas [B]   time = 11.55, size = 2554, normalized size = 10.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b*c -
 a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d -
3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x
^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(
(4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^
4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4
*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(c)*
log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b^2*c^4 - 2*a^
3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^
3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*s
qrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^6 +
(a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4 + (a^4*b^2*
c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), -1/8*(4*((4*b^4*c^3*d - 5*a*b^3*c^
2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a
^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^
3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x
^2 + c)) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*
d - 7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b
*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*
d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*
```

```
c^2*d^2 + 3*a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2
 + 3*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^
3*d^3)*x^4 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), 1/4*(((4*b^4
*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*
d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan
(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2
+ b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*
x^6 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^
4)*x^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^
2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3
*a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*
d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4
+ (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), 1/4*(((4*b^4*c^4*d - 7*
a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^4 +
(4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x
^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - 2
*((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*
b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 +
(4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-
c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^4 + (2*a*b^
3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/
((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^6 + (a^3*b^3*c^6 - a
^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^4 + (a^4*b^2*c^6 - 2*a^5*b*c^
5*d + a^6*c^4*d^2)*x^2)]
```

giac [A] time = 0.36, size = 367, normalized size = 1.52

$$\frac{\left(4\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^2+c\right)^2b^3c^2d-2\left(dx^2+c\right)b^3c^3d-2\left(dx^2+c\right)^2ab^2cd^2+3\left(dx^2+c\right)ab^2c^2d^2+3\left(dx^2+c\right)^2a^2bd^3-7\left(dx^2+c\right)a^2bcd^3+2\,a^2bc^2d^3+3\left(dx^2+c\right)a^3d^4-2\,a^3cd^4}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(\left(dx^2+c\right)^{\frac{5}{2}}b-2\left(dx^2+c\right)^{\frac{3}{2}}bc+\sqrt{dx^2+c}\,bc^2+\left(dx^2+c\right)^{\frac{3}{2}}ad-\sqrt{dx^2+c}\,acd\right)}-\frac{(4\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/2*(2*(d*x^2
+ c)^2*b^3*c^2*d - 2*(d*x^2 + c)*b^3*c^3*d - 2*(d*x^2 + c)^2*a*b^2*c*d^2 +
3*(d*x^2 + c)*a*b^2*c^2*d^2 + 3*(d*x^2 + c)^2*a^2*b*d^3 - 7*(d*x^2 + c)*a^2
*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^2 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^2 + c)^(5/2)*b - 2*(d*x^2 + c)^(3
/2)*b*c + sqrt(d*x^2 + c)*b*c^2 + (d*x^2 + c)^(3/2)*a*d - sqrt(d*x^2 + c)*a
```

```
*c*d)) - 1/2*(4*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)
*c^2)
```

maple [B] time = 0.02, size = 1778, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] -b^2/a^3/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)-b/a^3*(-a*b)^(1/2)/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+b^2/a^3/(
a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4*b^2/a^2/(
-a*b)^(1/2)/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^2*d/(a*d-b*c)^2/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)+3/4*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*b^2/a^2*d/(a*d-b*c
)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2*b^2/a^2/(-a*b)
^(1/2)/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/2/a^2/c/x^2/(d*x^2+c)^(1/2)-3/2/a^2*d/c^2/(d
*x^2+c)^(1/2)+3/2/a^2*d/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-b^2/a
^3/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)+b/a^3*(-a*b)^(1/2)/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+b^2/a^3/(a*d-b*
c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/4*b^2/a^2/(-a*b)^(
1/2)/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^2*d/(a*d-b*c)^2/((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4
*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*b^2/a^2*d/(a*d-b*c)^2/(-(
a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/2*b^2/a^2/(-a*b)^(1/2)/(
a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*d*x-2*b/a^3/c/(d*x^2+c)^(1/2)+2*b/a^3/c^(3/2)*ln((2*c+2*(d
*x^2+c)^(1/2)*c^(1/2))/x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3), x)
```

mupad [B] time = 4.90, size = 4286, normalized size = 17.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] (atan((((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*
a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 3814
4*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 +
 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10
- 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*d^13
+ 144*a^18*b^3*c^6*d^14) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128
*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9
792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 -
18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^5*c^11*d^
11 + 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*b^2*c^8*d^
14 - ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*c)*(512*a^1
2*b^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*a^14*b^11*c^19*d^4 - 7296
0*a^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 - 182784*a^17*b^8*c^16*d^7
+ 172032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*d^9 + 53760*a^20*b^5*c^13
*d^10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c^11*d^12 - 256*a^23*b^2*c
^10*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2
 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4
*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)))*1i)
/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*
b^2*c^2*d^3 - 5*a^7*b*c*d^4)) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)
*((c + d*x^2)^(1/2)*(512*a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824
*a^8*b^13*c^16*d^4 - 38144*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 3
1808*a^11*b^10*c^13*d^7 + 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 -
 3536*a^14*b^7*c^10*d^10 - 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12
- 1056*a^17*b^4*c^7*d^13 + 144*a^18*b^3*c^6*d^14) - ((-b^5*(a*d - b*c)^5)^(
1/2)*(7*a*d - 4*b*c)*(128*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 48
00*a^12*b^11*c^17*d^5 - 9792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 +
```

```
2688*a^15*b^8*c^14*d^8 - 18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10
 - 18048*a^18*b^5*c^11*d^11 + 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d
^13 + 192*a^21*b^2*c^8*d^14 + ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)
*(7*a*d - 4*b*c)*(512*a^12*b^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*
a^14*b^11*c^19*d^4 - 72960*a^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 -
182784*a^17*b^8*c^16*d^7 + 172032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*
d^9 + 53760*a^20*b^5*c^13*d^10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c
^11*d^12 - 256*a^23*b^2*c^10*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c
^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*
d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d
^3 - 5*a^7*b*c*d^4)))*1i)/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*
a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)))/(((-b^5*(a*d - b*c)
^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*a^6*b^15*c^18*d^2 - 4608*
a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38144*a^9*b^12*c^15*d^5 + 476
80*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 + 4624*a^12*b^9*c^12*d^8 +
 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10 - 2560*a^15*b^6*c^9*d^11
+ 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*d^13 + 144*a^18*b^3*c^6*d^14)
- ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128*a^10*b^13*c^19*d^3 - 121
6*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9792*a^13*b^10*c^16*d^6 +
9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 - 18816*a^16*b^7*c^13*d^9 +
 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^5*c^11*d^11 + 7744*a^19*b^4*c^10*d
^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*b^2*c^8*d^14 + ((-b^5*(a*d - b*c)^5
)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*c)*(512*a^12*b^13*c^21*d^2 - 5376*a^
13*b^12*c^20*d^3 + 25600*a^14*b^11*c^19*d^4 - 72960*a^15*b^10*c^18*d^5 + 13
8240*a^16*b^9*c^17*d^6 - 182784*a^17*b^8*c^16*d^7 + 172032*a^18*b^7*c^15*d^
8 - 115200*a^19*b^6*c^14*d^9 + 53760*a^20*b^5*c^13*d^10 - 16640*a^21*b^4*c^
12*d^11 + 3072*a^22*b^3*c^11*d^12 - 256*a^23*b^2*c^10*d^13))/(4*(a^8*d^5 -
a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5
*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c
^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 +
 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)
) - ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512*a^6
*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38144*a
^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^7 + 46
24*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d^10 - 2
560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*d^13 + 1
44*a^18*b^3*c^6*d^14) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*(128*a^
10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*d^5 - 9792
*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^14*d^8 - 188
16*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^5*c^11*d^11
+ 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*b^2*c^8*d^14
- ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*c)*(512*a^12*b
^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*a^14*b^11*c^19*d^4 - 72960*a
^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 - 182784*a^17*b^8*c^16*d^7 + 1
72032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*d^9 + 53760*a^20*b^5*c^13*d^
```

```
10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c^11*d^12 - 256*a^23*b^2*c^10
*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 +
10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^
4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a
^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^
2*d^3 - 5*a^7*b*c*d^4)) + 256*a^4*b^15*c^16*d^3 - 2048*a^5*b^14*c^15*d^4 +
7216*a^6*b^13*c^14*d^5 - 14672*a^7*b^12*c^13*d^6 + 18424*a^8*b^11*c^12*d^7
- 12992*a^9*b^10*c^11*d^8 + 1288*a^10*b^9*c^10*d^9 + 7024*a^11*b^8*c^9*d^10
 - 6968*a^12*b^7*c^8*d^11 + 2976*a^13*b^6*c^7*d^12 - 504*a^14*b^5*c^6*d^13)
)*(-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*1i)/(2*(a^8*d^5 - a^3*b^5*c^5
+ 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4
)) - (d^3/(b*c^2 - a*c*d) + (d*(c + d*x^2)^2*(2*b^3*c^2 + 3*a^2*b*d^2 - 2*a
*b^2*c*d))/(2*a^2*(b*c^2 - a*c*d)^2) + (d*(c + d*x^2)*(a*d - 2*b*c)*(3*a^2*
d^2 + b^2*c^2 - a*b*c*d))/(2*a^2*(b*c^2 - a*c*d)^2))/(b*(c + d*x^2)^(5/2) +
 (c + d*x^2)^(1/2)*(b*c^2 - a*c*d) + (c + d*x^2)^(3/2)*(a*d - 2*b*c)) - (at
an((a^3*b^11*c^21*d^3*(c + d*x^2)^(1/2)*140i - a^14*c^10*d^14*(c + d*x^2)^(
1/2)*27i - a^4*b^10*c^20*d^4*(c + d*x^2)^(1/2)*1015i + a^5*b^9*c^19*d^5*(c
+ d*x^2)^(1/2)*2996i - a^6*b^8*c^18*d^6*(c + d*x^2)^(1/2)*4375i + a^7*b^7*c
^17*d^7*(c + d*x^2)^(1/2)*2561i + a^8*b^6*c^16*d^8*(c + d*x^2)^(1/2)*1316i
- a^9*b^5*c^15*d^9*(c + d*x^2)^(1/2)*3073i + a^10*b^4*c^14*d^10*(c + d*x^2)
^(1/2)*1694i + a^11*b^3*c^13*d^11*(c + d*x^2)^(1/2)*35i - a^12*b^2*c^12*d^1
2*(c + d*x^2)^(1/2)*441i + a^13*b*c^11*d^13*(c + d*x^2)^(1/2)*189i)/(c^5*(c
^5)^(1/2)*(c^5*(c^5*(2561*a^7*b^7*d^7 - 4375*a^6*b^8*c*d^6 + 140*a^3*b^11*c
^4*d^3 - 1015*a^4*b^10*c^3*d^4 + 2996*a^5*b^9*c^2*d^5) - 441*a^12*b^2*d^12
+ 35*a^11*b^3*c*d^11 + 1316*a^8*b^6*c^4*d^8 - 3073*a^9*b^5*c^3*d^9 + 1694*a
^10*b^4*c^2*d^10) - 27*a^14*c^3*d^14 + 189*a^13*b*c^4*d^13)))*(3*a*d + 4*b*
c)*1i)/(2*a^3*(c^5)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^2\right)^2 \left(c + dx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.757 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d\right.}{6a^3c^3x(bc-ad)^2}$$

Rubi [A] time = 0.40, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {472, 579, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\left(-8a^2bcd^2+16a^3d^3-14ab^2c^2d+15b^3c^3\right)}{6a^3c^3x(bc-ad)^2}-\frac{\sqrt{c+dx^2}\left(8a^2d^2-4abcd+5b^2c^2\right)}{6a^2c^2x^3(bc-ad)^2}+\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}+\frac{d(2ad+bc)}{2acx^3\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(2*a*c*(b*c - a*d)^2*x^3*Sqrt[c + d*x^2]) + b/(2*a*(b*c -
 a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - ((5*b^2*c^2 - 4*a*b*c*d + 8*a^2*d^
2)*Sqrt[c + d*x^2])/(6*a^2*c^2*(b*c - a*d)^2*x^3) + ((15*b^3*c^3 - 14*a*b^2
*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^3*c^3*(b*c - a*d
)^2*x) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx &= \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-5bc+2ad-6bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{2a(bc-ad)} \\
&= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\int \frac{-5b^2c^2+4ab}{x^4}}{2} \\
&= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4a\right.}{6a^2} \\
&= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4a\right.}{6a^2} \\
&= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4a\right.}{6a^2} \\
&= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4a\right.}{6a^2} \\
&= \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\sqrt{c+dx^2}} - \frac{\left(5b^2c^2-4a\right.}{6a^2}
\end{aligned}$$

Mathematica [A] time = 5.49, size = 167, normalized size = 0.60

$$\frac{b^3(5bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}} + \sqrt{c+dx^2}\left(\frac{\frac{b^4x}{2\left(a+bx^2\right)(bc-ad)^2}+\frac{2b}{c^2x}}{a^3} - \frac{c-5dx^2}{3a^2c^3x^3} + \frac{d^4x}{c^3\left(c+dx^2\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-1/3*(c - 5*d*x^2)/(a^2*c^3*x^3) + (d^4*x)/(c^3*(b*c - a*d
)^2*(c + d*x^2)) + ((2*b)/(c^2*x) + (b^4*x)/(2*(b*c - a*d)^2*(a + b*x^2)))/
a^3) + (b^3*(5*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*
x^2])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

IntegrateAlgebraic [**A**] time = 1.93, size = 324, normalized size = 1.17

$$\frac{\left(8ab^3d-5b^4c\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{5/2}}+\frac{-2a^4c^2d^2+8a^4cd^3x^2+16a^4d^4x^4+4a^3bc^3d-6a^3bc^2d^2x^2+16a^3bd^4x^6-2a^2b^2c^4-12a^2b^2c^3dx^2-18a^2b^2c^2d^2x^4-8a^2b^2cd^3x^6+10ab^3c^4x^2-4ab^3c^3dx^4-14ab^3c^2d^2x^6+15b^4c^4x^4+15b^4c^3dx^6}{6a^3c^3x^3\left(a+bx^2\right)\sqrt{c+dx^2}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-2*a^2*b^2*c^4 + 4*a^3*b*c^3*d - 2*a^4*c^2*d^2 + 10*a*b^3*c^4*x^2 - 12*a^2
*b^2*c^3*d*x^2 - 6*a^3*b*c^2*d^2*x^2 + 8*a^4*c*d^3*x^2 + 15*b^4*c^4*x^4 - 4
*a*b^3*c^3*d*x^4 - 18*a^2*b^2*c^2*d^2*x^4 + 16*a^4*d^4*x^4 + 15*b^4*c^3*d*x
^6 - 14*a*b^3*c^2*d^2*x^6 - 8*a^2*b^2*c*d^3*x^6 + 16*a^3*b*d^4*x^6)/(6*a^3*
c^3*(-(b*c) + a*d)^2*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) + ((-5*b^4*c + 8*a*b^
3*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt
[b*c - a*d])])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

fricas [**B**] time = 3.02, size = 1252, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((5*b^5*c^4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d
- 8*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sq
rt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b^3*c^5 - 6*a^4*b^2*
c^4*d + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^4*c^3*
d^2 + 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15*a*b^5*
c^5 - 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + 16*a^5*b
*c*d^4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*
c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^4*b^4*c^
6*d - 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7 + (a^4*b^4
*c^7 - 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (a^5*b^3*c^7
- 3*a^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3), 1/12*(3*((5*b^5*c^
4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d - 8*a^2*b^3*c^3*d^2
)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d -
a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^3*c^5 - 6*a^4*b^2*c^4*d
 + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^4*c^3*d^2 +
 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15*a*b^5*c^5 -
 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 + 16*a^5*b*c*d^
4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d
^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^4*b^4*c^6*d -
 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7 + (a^4*b^4*c^7
```

```
- 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (a^5*b^3*c^7 - 3*a
^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3)]
```

giac [A] time = 5.81, size = 486, normalized size = 1.75

$$\frac{d^{4}x}{\left(b^{2}c^{5}-2\,abc^{4}d+a^{2}c^{3}d^{2}\right)\sqrt{dx^{2}+c}}-\frac{\left(5\,b^{4}c\sqrt{d}-8\,ab^{3}d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{4}c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{3}d^{\frac{3}{2}}-b^{4}c^{2}\sqrt{d}}{\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}\right)\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)}-\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}\sqrt{d}-12\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}acd^{\frac{3}{2}}+6\,bc^{3}\sqrt{d}+5\,ac^{2}d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}-c\right)^{3}a^{3}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] d^4*x/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(d*x^2 + c)) - 1/2*(5*b^4*
c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
- b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d
^2)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^4*c*sqrt(
d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^3*d^(3/2) - b^4*c^2*sqrt(d))/((a
^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d
 + b*c^2)) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c
^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 6*b*c^3*sqrt(
d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3*c^2)
```

maple [B] time = 0.02, size = 1608, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x)
```

```
[Out] 5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^3/(a*d-b*c)/c/((x+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-
5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*
b)^(1/2)/b))-1/4*b^2/a^3/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*b^2/a^3*(-a
*b)^(1/2)*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4*b^2
/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))-5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^3/(a*d-b*c)/c/
```

```
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*d*x+5/4*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x-(-a*b)^(1/2)/b))-1/4*b^2/a^3/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*b^
2/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*b^2/a^2*d^2/(a*d-b*c)^2/c/((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*
x-3/4*b^2/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
-(-a*b)^(1/2)/b))+2*b/a^3/c/x/(d*x^2+c)^(1/2)+4*b/a^3*d/c^2*x/(d*x^2+c)^(1/
2)-1/3/a^2/c/x^3/(d*x^2+c)^(1/2)+4/3/a^2*d/c^2/x/(d*x^2+c)^(1/2)+8/3/a^2*d^
2/c^3*x/(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
[Out] Integral(1/(x**4*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.758 $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}\,(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{b}}{\sqrt{a}\,\sqrt{}}\right.}{2(bc-ad)^{7/2}}$$

Rubi [A] time = 0.21, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {470, 527, 12, 377, 205}

$$\frac{x(11ad+4bc)}{6\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{x(3ad+2bc)}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{a}\,(2ad+3bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((2*b*c + 3*a*d)*x)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (a*x)/(2*b*(b*c
 - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b*c + 11*a*d)*x)/(6*(b*c - a*d
)^3*Sqrt[c + d*x^2]) - (Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(2*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{ac-2(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2b(bc-ad)}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{5abc^2-2bc(2bc+3a}{\left(a+bx^2\right)\left(c+dx^2\right)^{3}}}{6bc(bc-ad}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)}{6(bc-ad)^3\sqrt{c+}}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)}{6(bc-ad)^3\sqrt{c+}}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)}{6(bc-ad)^3\sqrt{c+}}$$

$$= \frac{(2bc+3ad)x}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{ax}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(4bc+11ad)}{6(bc-ad)^3\sqrt{c+}}$$

Mathematica [**C**] time = 1.17, size = 133, normalized size = 0.76

$$\frac{x^5\left(\frac{8x^2\left(c+dx^2\right)(bc-ad)\,{}_2F_1\left(2,3;\frac{11}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)}{a+bx^2}+9c\left(7c+2dx^2\right)\,{}_2F_1\left(1,2;\frac{9}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)\right)}{315c^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^5*(9*c*(7*c + 2*d*x^2)*Hypergeometric2F1[1, 2, 9/2, ((b*c - a*d)*x^2)/(c
*(a + b*x^2))] + (8*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 3, 11/
2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2)))/(315*c^3*(a + b*x^2)^2
*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 1.43, size = 189, normalized size = 1.09

$$\frac{\left(2a^{3/2}d+3\sqrt{a}\,bc\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{-6a^2cdx-8a^2d^2x^3-9abc^2x-16abcdx^3-11abd^2x^5-6b^2c^2x^3-4b^2cdx^5}{6\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(-9*a*b*c^2*x - 6*a^2*c*d*x - 6*b^2*c^2*x^3 - 16*a*b*c*d*x^3 - 8*a^2*d
^2*x^3 - 4*b^2*c*d*x^5 - 11*a*b*d^2*x^5)/((b*c - a*d)^3*(a + b*x^2)*(c + d*
x^2)^(3/2)) + ((3*Sqrt[a]*b*c + 2*a^(3/2)*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*
x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*(b*c - a*d)^(7/2)
)
```

fricas [B] time = 2.59, size = 1008, normalized size = 5.79

$$\left[-\frac{3\left(\left(3b^2cd^2+2abd^3\right)x^6+3abc^3+2a^2c^2d+\left(6b^2c^2d+7abcd^2+2a^2d^3\right)x^4+\left(3b^2c^3+8abc^2d+4a^2cd^2\right)x^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8abcd+8a^2d^2\right)x^4+a^2c^2-2\left(3abc^2-4a^2cd\right)x^2+4\left(\left(b^2c^2-3abcd+2a^2d^2\right)x^3-\left(abc^2-a^2cd\right)x\right)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-4\left(\left(4b^2cd+11abd^2\right)x^5+2\left(3b^2c^2+8abcd+4a^2d^2\right)x^3+3\left(3abc^2+2a^2cd\right)x\right)\sqrt{dx^2+c}}{24\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^6+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x^2\right)},\ \frac{3\left(\left(3b^2cd^2+2abd^3\right)x^6+3abc^3+2a^2c^2d+\left(6b^2c^2d+7abcd^2+2a^2d^3\right)x^4+\left(3b^2c^3+8abc^2d+4a^2cd^2\right)x^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{\left(\left(bc-2ad\right)x^2-ac\right)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\left(adx^3+acx\right)}\right)+2\left(\left(4b^2cd+11abd^2\right)x^5+2\left(3b^2c^2+8abcd+4a^2d^2\right)x^3+3\left(3abc^2+2a^2cd\right)x\right)\sqrt{dx^2+c}}{12\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^6+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2
*c^2*d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*
d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
 a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^4 + 2*a*b*x^2 + a^2)) - 4*((4*b^2*c*d + 11*a*b*d^2)*x^5 + 2*(3*b^2*c^2
+ 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x)*sqrt(d*x^2 + c)
)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d
^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*
a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 -
 a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/1
2*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^2*c^2*
d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*c*d^2)*
x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((4*b^2*c*d + 11*a*b*d^2)*x^
5 + 2*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x
)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2
*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 +
(2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)
*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4
*c*d^4)*x^2)]
```

giac [B] time = 4.49, size = 594, normalized size = 3.41

$$\frac{\left(\frac{2\left(b^4c^5d^2-ab^3c^4d^3-3\,a^2b^2c^3d^4+5\,a^3bc^2d^5-2\,a^4cd^6\right)x^2}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}+\frac{3\left(b^4c^6d-2\,ab^3c^5d^2+2\,a^3bc^3d^4-a^4c^2d^5\right)}{b^6c^7d-6\,ab^5c^6d^2+15\,a^2b^4c^5d^3-20\,a^3b^3c^4d^4+15\,a^4b^2c^3d^5-6\,a^5bc^2d^6+a^6cd^7}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\left(3\,abc\sqrt{d}+2\,a^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^4*c^5*d^2 - a*b^3*c^4*d^3 - 3*a^2*b^2*c^3*d^4 + 5*a^3*b*c^2*d^5 -
 2*a^4*c*d^6)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^
3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 3*(b^4*
c^6*d - 2*a*b^3*c^5*d^2 + 2*a^3*b*c^3*d^4 - a^4*c^2*d^5)/(b^6*c^7*d - 6*a*b
^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 -
 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) + 1/2*(3*a*b*c*sqrt(d) +
 2*a^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt
(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*a*d + b*c^2))
```

maple [B] time = 0.03, size = 2463, normalized size = 14.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+
1/3/b^2*x/c/(d*x^2+c)^(3/2)+2/3/b^2/c^2*x/(d*x^2+c)^(1/2)-5/12*a/b^2*(-a*b)
^(1/2)*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4*a/b*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+5/12*a/
b^2*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/4*a/b*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/
2)-1/4*a/b^2/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/b*a/(-a*b)^(1/2)/(a*d-b
*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(3/2)-3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*
b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
```

```
x+(-a*b)^(1/2)/b))-1/4*a/b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/b*a/(-a
*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(3/2)+3/4*a/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+5/4*a/b*(-a*b)^(1/2)*d/(a*d-b*c)^3/(-(a*d
-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-7/12*a/b^2*d/(a*d-b*c)/c/((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2
)*x-7/6*a/b^2*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/12*a^2/b^2*d^2/(a*d-b*c)^2/c/((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*
x+5/6*a^2/b^2*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4*a^2/b*d^2/(a*d-b*c)^3/c/((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
*x-5/4*a/b*(-a*b)^(1/2)*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(
-a*b)^(1/2)/b))+3/4/b*a/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-7/12/b^2*a*d/(a*d-b*c)/c/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3
/2)*x-7/6/b^2*a*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/4/b*a/(a*d-b*c)^2/c/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+5/1
2*a^2/b^2*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/6*a^2/b^2*d^2/(a*d-b*c)^2/c^2/((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
-5/4*a^2/b*d^2/(a*d-b*c)^3/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(a + bx^{2}\right)^{2} \left(c + dx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.759 $\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{a}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{3ad+2bc}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{b}\,(3ad+2bc)\tanh^{-1}\Bigl(}{2(bc-ad)^{7/2}}$$

Rubi [A] time = 0.16, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{3ad+2bc}{2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{3ad+2bc}{6b\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{b}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*b*c + 3*a*d)/(6*b*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + a/(2*b*(b*c - a*d)*
(a + b*x^2)*(c + d*x^2)^(3/2)) + (2*b*c + 3*a*d)/(2*(b*c - a*d)^3*Sqrt[c +
d*x^2]) - (Sqrt[b]*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{5/2}}\,dx, x, x^2\right)$$

$$= \frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+3ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx, x, x^2\right.}{4b(bc-ad)}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{(2bc+3ad)\,\mathrm{Su}}{}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{2bc+3ad}{2(bc-ad)^3\sqrt{c}}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{2bc+3ad}{2(bc-ad)^3\sqrt{c}}$$

$$= \frac{2bc+3ad}{6b(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{a}{2b(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{2bc+3ad}{2(bc-ad)^3\sqrt{c}}$$

Mathematica [C] time = 0.03, size = 93, normalized size = 0.55

$$\frac{\left(a+bx^2\right)(3ad+2bc)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)+3a(bc-ad)}{6b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a*(b*c - a*d) + (2*b*c + 3*a*d)*(a + b*x^2)*Hypergeometric2F1[-3/2, 1, -
1/2, (b*(c + d*x^2))/(b*c - a*d)])/(6*b*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^
2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.48, size = 185, normalized size = 1.09

$$\frac{\left(-3a\sqrt{b}\,d-2b^{3/2}c\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2(bc-ad)^3\sqrt{ad-bc}}-\frac{-4a^2cd-6a^2d^2x^2-11abc^2-16abcdx^2-9abd^2x^4-8b^2c^2x^2-6b^2cdx^4}{6\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/6*(-11*a*b*c^2 - 4*a^2*c*d - 8*b^2*c^2*x^2 - 16*a*b*c*d*x^2 - 6*a^2*d^2*
x^2 - 6*b^2*c*d*x^4 - 9*a*b*d^2*x^4)/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)
^(3/2)) + ((-2*b^(3/2)*c - 3*a*Sqrt[b]*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d
]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*(b*c - a*d)^3*Sqrt[-(b*c) + a*d])
```

fricas [**B**] time = 1.54, size = 993, normalized size = 5.84

$$\left[-\frac{3\left(\left(2b^2cd^2+3abd^3\right)x^6+2abc^3+3a^2c^2d+\left(4b^2c^2d+8abcd^2+3a^2d^3\right)x^4+\left(2b^2c^3+7abc^2d+6a^2cd^2\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2+4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)-4\left(3\left(2b^2cd+3abd^2\right)x^4+11abc^2+4a^2cd+2\left(4b^2c^2+8abcd+3a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{24\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^6+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x^2\right)}, \frac{3\left(\left(2b^2cd^2+3abd^3\right)x^6+2abc^3+3a^2c^2d+\left(4b^2c^2d+8abcd^2+3a^2d^3\right)x^4+\left(2b^2c^3+7abc^2d+6a^2cd^2\right)x^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\left(3\left(2b^2cd+3abd^2\right)x^4+11abc^2+4a^2cd+2\left(4b^2c^2+8abcd+3a^2d^2\right)x^2\right)\sqrt{dx^2+c}}{12\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^6+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2
*c^2*d + 8*a*b*c*d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*
d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^
2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
+ (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
2*a*b*x^2 + a^2)) - 4*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2 + 4*a^2*c
*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5
 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3
*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^
2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d
 - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/12*(3*((2*b^2
*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2*c^2*d + 8*a*b*c*
d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*d^2)*x^2)*sqrt(-b
/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b
*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2
 + 4*a^2*c*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/
(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2
 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*
b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a
*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

giac [**A**] time = 0.36, size = 260, normalized size = 1.53

$$\frac{\frac{3\sqrt{dx^2+c}\,abd^2}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\left(\left(dx^2+c\right)b-bc+ad\right)}+\frac{3\left(2b^2cd+3abd^2\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}+\frac{2\left(3\left(dx^2+c\right)bcd+bc^2d+3\left(dx^2+c\right)ad^2-acd^2\right)}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\left(dx^2+c\right)^{\frac{3}{2}}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(3*sqrt(d*x^2 + c)*a*b*d^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*((d*x^2 + c)*b - b*c + a*d)) + 3*(2*b^2*c*d + 3*a*b*d^2)*arctan(sq
rt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*(3*(d*x^2 + c)*b*c*d + b*c^2*d +
3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3)*(d*x^2 + c)^(3/2)))/d
```

maple **[B]** time = 0.02, size = 2400, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/12*a/b*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4*a*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2))/(x+(-a*b)^(1/2)/b))-5/4*a*d/(a*d-b*c)^3/((x-(-a*b)^(1/2)/b)^2*d+2*(-
a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-5/4*a*d/(a*d-b*c)^3/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)+1/2/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+5/4*(-a*b)^(1/2)/b*a*d^2/(a
*d-b*c)^3/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)*x+5/12*(-a*b)^(1/2)/b^2*a*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/6*(
-a*b)^(1/2)/b^2*a*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4*(-a*b)^(1/2)/b*a*d^2/(a*d
-b*c)^3/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)*x-5/12*(-a*b)^(1/2)/b^2*a*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-5/6*(-a
*b)^(1/2)/b^2*a*d^2/(a*d-b*c)^2/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4*a*d/(a*d-b*c)^3/(-(a*d-b*c)
/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-
b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/6/b/(a*d-b*c)/((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/2/(a*d-b*c
)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/6/b/(a*d-b*c)/((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/
2)-1/2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
```

```
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/2
/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/b^2*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/
((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+1/2/b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2*(-a*b)^(1/2)/b^2*d/(
a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(3/2)*x+(-a*b)^(1/2)/b^2*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/2/b/(a*d-b*c)
^2*(-a*b)^(1/2)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x-(-a*b)^(1/2)/
b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(3/2)-1/4*(-a*b)^(1/2)/b^2/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/12*a/b*d
/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 1.43, size = 193, normalized size = 1.14

$$-\frac{\frac{\left(d\,x^2+c\right)\,\left(3\,a\,d+2\,b\,c\right)}{3\,{\left(a\,d-b\,c\right)}^2}-\frac{c}{3\,\left(a\,d-b\,c\right)}+\frac{b\,{\left(d\,x^2+c\right)}^2\,\left(3\,a\,d+2\,b\,c\right)}{2\,{\left(a\,d-b\,c\right)}^3}}{b\,{\left(d\,x^2+c\right)}^{5/2}+{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-b\,c\right)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^{7/2}}\right)\,\left(3\,a\,d+2\,b\,c\right)}{2\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] - (((c + d*x^2)*(3*a*d + 2*b*c))/(3*(a*d - b*c)^2) - c/(3*(a*d - b*c)) + (b
*(c + d*x^2)^2*(3*a*d + 2*b*c))/(2*(a*d - b*c)^3))/(b*(c + d*x^2)^(5/2) + (
c + d*x^2)^(3/2)*(a*d - b*c)) - (b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2))*(3*a*
d + 2*b*c))/(2*(a*d - b*c)^(7/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.760 $\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5dx}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx}}\right.}{2\sqrt{a}\,(bc-ad)^{7/2}}$$

Rubi [A] time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {471, 527, 12, 377, 205}

$$-\frac{dx(2ad+13bc)}{6c\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5dx}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{b(4ad+bc)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d*x)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - x/(2*(b*c - a*d)*(a + b*x^2)
*(c + d*x^2)^(3/2)) - (d*(13*b*c + 2*a*d)*x)/(6*c*(b*c - a*d)^3*Sqrt[c + d*
x^2]) + (b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = -\frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{c-4dx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{c(3bc+2ad)-10bc}{\left(a+bx^2\right)\left(c+dx^2\right)^{3}}}{6c(bc-ad)}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{d(13bc+2ad}{6c(bc-ad)^3\sqrt{c}}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{d(13bc+2ad}{6c(bc-ad)^3\sqrt{c}}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{d(13bc+2ad}{6c(bc-ad)^3\sqrt{c}}$$

$$= -\frac{5dx}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{x}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{d(13bc+2ad}{6c(bc-ad)^3\sqrt{c}}$$

Mathematica [C] time = 2.43, size = 211, normalized size = 1.29

$$\frac{x^3\left(16x^2\left(c+dx^2\right)^2(bc-ad)\,{}_3F_2\left(2,2,3;1,\frac{11}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)+48x^2\left(2c^2+3cdx^2+d^2x^4\right)(bc-ad)\,{}_2F_1\left(2,3;\frac{11}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)+9c\left(a+bx^2\right)\left(35c^2+28cdx^2+8d^2x^4\right)\,{}_2F_1\left(1,2;\frac{9}{2};\frac{(bc-ad)x^2}{c\left(bx^2+a\right)}\right)\right)}{945c^4\left(a+bx^2\right)^3\left(c+dx^2\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x^3*(9*c*(a + b*x^2)*(35*c^2 + 28*c*d*x^2 + 8*d^2*x^4)*Hypergeometric2F1[1
, 2, 9/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))] + 48*(b*c - a*d)*x^2*(2*c^2 +
3*c*d*x^2 + d^2*x^4)*Hypergeometric2F1[2, 3, 11/2, ((b*c - a*d)*x^2)/(c*(a
+ b*x^2))] + 16*(b*c - a*d)*x^2*(c + d*x^2)^2*HypergeometricPFQ[{2, 2, 3},
{1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(945*c^4*(a + b*x^2)^3*(c +
 d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 1.27, size = 198, normalized size = 1.21

$$\frac{-2a^2d^3x^3-12abc^2dx-10abcd^2x^3-2abd^3x^5-3b^2c^3x-18b^2c^2dx^3-13b^2cd^2x^5}{6c\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^3}+\frac{\left(b^2(-c)-4abd\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2\sqrt{a}\,(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^2*c^3*x - 12*a*b*c^2*d*x - 18*b^2*c^2*d*x^3 - 10*a*b*c*d^2*x^3 - 2*a^
2*d^3*x^3 - 13*b^2*c*d^2*x^5 - 2*a*b*d^3*x^5)/(6*c*(b*c - a*d)^3*(a + b*x^2
)*(c + d*x^2)^(3/2)) + ((-(b^2*c) - 4*a*b*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*
x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*(b*c - a*
d)^(7/2))
```

fricas [B] time = 2.93, size = 1292, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*d^3)*x^6 + (
2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 6*a*b^2*c^3
*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - 4*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^5 + 2*(9*
a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*x^3 + 3*(a*b^3*c
^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(a^2*b^4*c^7 -
```

```
4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^4 + (a*b^
5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4 - 4*a^4*b^2*c^2*d^5 + a^5
*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^2 + 8*a^3*b^3*c^4*d^3 - 2*
a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^4 + (a*b^5*c^7 - 2*a^2*b^4
*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 - 7*a^5*b*c^3*d^4 + 2*a^6*c^
2*d^5)*x^2), 1/12*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*
d^3)*x^6 + (2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 +
 6*a*b^2*c^3*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(
a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^
2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3
- 2*a^3*b*d^4)*x^5 + 2*(9*a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 -
 a^4*d^4)*x^3 + 3*(a*b^3*c^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d
*x^2 + c))/(a^2*b^4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4
*d^3 + a^6*c^3*d^4 + (a*b^5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4
 - 4*a^4*b^2*c^2*d^5 + a^5*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^
2 + 8*a^3*b^3*c^4*d^3 - 2*a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^
4 + (a*b^5*c^7 - 2*a^2*b^4*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 -
7*a^5*b*c^3*d^4 + 2*a^6*c^2*d^5)*x^2)]
```

giac [**B**] time = 4.52, size = 595, normalized size = 3.65

$$-\frac{\left(\frac{\left(5\,b^{4}c^{4}d^{3}-14\,ab^{3}c^{3}d^{4}+12\,a^{2}b^{2}c^{2}d^{5}-2\,a^{3}bcd^{6}-a^{4}d^{7}\right)x^{2}}{b^{6}c^{7}d-6\,ab^{5}c^{6}d^{2}+15\,a^{2}b^{4}c^{5}d^{3}-20\,a^{3}b^{3}c^{4}d^{4}+15\,a^{4}b^{2}c^{3}d^{5}-6\,a^{5}bc^{2}d^{6}+a^{6}cd^{7}}+\frac{6\left(b^{4}c^{5}d^{2}-3\,ab^{3}c^{4}d^{3}+3\,a^{2}b^{2}c^{3}d^{4}-a^{3}bc^{2}d^{5}\right)}{b^{6}c^{7}d-6\,ab^{5}c^{6}d^{2}+15\,a^{2}b^{4}c^{5}d^{3}-20\,a^{3}b^{3}c^{4}d^{4}+15\,a^{4}b^{2}c^{3}d^{5}-6\,a^{5}bc^{2}d^{6}+a^{6}cd^{7}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{\left(b^{2}c\sqrt{d}+4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{2}c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}abd^{\frac{3}{2}}-b^{2}c^{2}\sqrt{d}}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ad+bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((5*b^4*c^4*d^3 - 14*a*b^3*c^3*d^4 + 12*a^2*b^2*c^2*d^5 - 2*a^3*b*c*d^
6 - a^4*d^7)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3
*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 6*(b^4*c
^5*d^2 - 3*a*b^3*c^4*d^3 + 3*a^2*b^2*c^3*d^4 - a^3*b*c^2*d^5)/(b^6*c^7*d -
6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*
d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) - 1/2*(b^2*c*sqrt(d
) + 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*
a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*s
qrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - sqrt(d*x^
2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a*d + b*c^2))
```

maple [**B**] time = 0.02, size = 2369, normalized size = 14.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)

[Out] -1/4/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b*c)/b)^
(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c
)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+5/4*a*d^2/(a*d-b*c)^3/c/((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/12/
b*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(3/2)*x+5/6/b*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4*a*d^2/(a*d-b*c)^3
/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(1/2)*x+5/12/b*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/6/b*d/(a*d-b*c)/c^2/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4/(
-a*b)^(1/2)*b/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/
4*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(
-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)
-5/4*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-
5/4*(-a*b)^(1/2)*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1
/2)/b))-1/4/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/4/(-a*b)^(1/2)*b/(a*d-b*c)^2/(-(a*d-b
*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+5/4*(-a*b)^(1/2)*d/(a*d-b*c)^3/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+5/12/b*(-a*b)^(1/2)*d/
(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(3/2)-5/12/b*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/6/b*a*d^2/(a*d-
b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*x-5/12/b*a*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-5/12/b*a*d^2/(a*d-b*c
)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(3/2)*x-5/6/b*a*d^2/(a*d-b*c)^2/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/12/(-a*b)^(1/2)/(a*d-b*
c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
```

```
b)^(3/2)-1/12/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.761 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {444, 51, 63, 208}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}-\frac{5bd}{2\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{1}{2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{5d}{6\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-5*d)/(6*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x^2)*(
c + d*x^2)^(3/2)) - (5*b*d)/(2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (5*b^(3/2)*
d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*(b*c - a*d)^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$=-\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{(5d)\,\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx,x,x^2\right)}{4(bc-ad)}$$

$$=-\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{(5bd)\,\text{Subst}\left(\int\right.}{4(b}$$

$$=-\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{5bd}{2(bc-ad)^3\sqrt{c+}}$$

$$=-\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{5bd}{2(bc-ad)^3\sqrt{c+}}$$

$$=-\frac{5d}{6(bc-ad)^2\left(c+dx^2\right)^{3/2}}-\frac{1}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}-\frac{5bd}{2(bc-ad)^3\sqrt{c+}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.39

$$-\frac{d\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{b\left(dx^2+c\right)}{ad-bc}\right)}{3\left(c+dx^2\right)^{3/2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(d*Hypergeometric2F1[-3/2, 2, -1/2, -((b*(c + d*x^2))/(-(b*c) + a*d))]
)/((-(b*c) + a*d)^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.36, size = 151, normalized size = 1.08

$$\frac{2a^2d^2-14abcd-10abd^2x^2-3b^2c^2-20b^2cdx^2-15b^2d^2x^4}{6\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^3}-\frac{5b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2(ad-bc)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2 - 20*b^2*c*d*x^2 - 10*a*b*d^2*x^2 - 15
*b^2*d^2*x^4)/(6*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (5*b^(3/2)*
d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*(-(b
*c) + a*d)^(7/2))
```

fricas [B] time = 1.14, size = 895, normalized size = 6.39

$$\left[-\frac{15\left(b^2d^3x^6+abc^2d+\left(2b^2cd^2+abd^3\right)x^4+\left(b^2c^2d+2abcd^2\right)x^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2\left(4b^2cd-3abd^2\right)x^2-4\left(2b^2c^2-3abcd+a^2d^2+\left(b^2cd-abd^2\right)x^2\right)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2abx^2+a^2}\right)+4\left(15b^2d^2x^4+3b^2c^2+14abcd-2a^2d^2+10\left(2b^2cd+abd^2\right)x^2\right)\sqrt{dx^2+c}}{24\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^6+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x^2\right)},-\frac{15\left(b^2d^3x^6+abc^2d+\left(2b^2cd^2+abd^3\right)x^4+\left(b^2c^2d+2abcd^2\right)x^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{\left(bdx^2+2bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\left(bdx^2+bc\right)}\right)+2\left(15b^2d^2x^4+3b^2c^2+14abcd-2a^2d^2+10\left(2b^2cd+abd^2\right)x^2\right)\sqrt{dx^2+c}}{12\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5\right)x^6+\left(2b^4c^4d-5ab^3c^3d^2+3a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^4+\left(b^4c^5-ab^3c^4d-3a^2b^2c^3d^2+5a^3bc^2d^3-2a^4cd^4\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^
2*d + 2*a*b*c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*
c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^2*d^2*x^4 + 3*b^2*c^2 + 14*a*b*c*
d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3*c^5 -
 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c
^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2
+ 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d -
 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), -1/12*(15*(b^2*d^
3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^2*d + 2*a*b*c*d^2)
*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(15*b^2*d^2*x^4 + 3*b^2*c^2 +
14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*
b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 -
3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3
```

```
*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^
3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)]
```

giac [A] time = 0.56, size = 226, normalized size = 1.61

$$-\frac{5\,b^{2}d\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{\sqrt{dx^{2}+c}\,b^{2}d}{2\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(\left(dx^{2}+c\right)b-bc+ad\right)}-\frac{6\left(dx^{2}+c\right)bd+bcd-ad^{2}}{3\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left(dx^{2}+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -5/2*b^2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/2*sqrt(d*x^2
+ c)*b^2*d/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x^2 + c
)*b - b*c + a*d)) - 1/3*(6*(d*x^2 + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^(3/2))
```

maple [B] time = 0.02, size = 1639, normalized size = 11.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/12*d/(a*d-b*c)^2
/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(3/2)+5/12*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/6*(-a*b)^(1/2)/b*d^2
/(a*d-b*c)^2/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*x+5/4*b*d/(a*d-b*c)^3/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-5/4*(-a*b)^(1/2)*d^2/(a*
d-b*c)^3/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)*x-5/4*b*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1
/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(
-a*b)^(1/2)/b))-1/3*(-a*b)^(1/2)/a/b*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-2/3*(-a*b)^(1/2)
/a/b*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x+(-a*b)^(1/2)
/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)
/b)^(3/2)-5/12*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/12*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3
/2)*x-5/6*(-a*b)^(1/2)/b*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
```

```
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4*b*d/(a*d-b*c)^3/((
x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2)+5/4*(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4*b*d/(a*d-b*c)^3/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(
a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/3*(-a*b)^(1/2)/a/b*d/(a*d-b*
c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(3/2)*x+2/3*(-a*b)^(1/2)/a/b*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 1.24, size = 171, normalized size = 1.22

$$\frac{\frac{5\,b^2\,d\,\left(d\,x^2+c\right)^2}{2\,\left(a\,d-b\,c\right)^3}-\frac{d}{3\,\left(a\,d-b\,c\right)}+\frac{5\,b\,d\,\left(d\,x^2+c\right)}{3\,\left(a\,d-b\,c\right)^2}}{b\,\left(d\,x^2+c\right)^{5/2}+\left(d\,x^2+c\right)^{3/2}\,\left(a\,d-b\,c\right)}+\frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{\left(a\,d-b\,c\right)^{7/2}}\right)}{2\,\left(a\,d-b\,c\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((5*b^2*d*(c + d*x^2)^2)/(2*(a*d - b*c)^3) - d/(3*(a*d - b*c)) + (5*b*d*(c
+ d*x^2))/(3*(a*d - b*c)^2))/(b*(c + d*x^2)^(5/2) + (c + d*x^2)^(3/2)*(a*d
- b*c)) + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^3*d^3 - b^3*c^3 +
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(2*(a*d - b*c)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

3.762 $\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=201

$$\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{dx(2}{6ac\left(c+d\right.}$$

Rubi [A] time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {414, 527, 12, 377, 205}

$$\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{6ac^2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{dx(2ad+3bc)}{6ac\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d)*x)/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 16*a*b*c*d - 4
*a^2*d^2)*x)/(6*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (b^2*(b*c - 6*a*d)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d
)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-bc+2ad-4bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-3b^2c^2+12abcd-}{(a+bx^2}}{6ac(}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16ab\right.}{6ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16ab\right.}{6ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16ab\right.}{6ac^2(bc-ad}$$

$$= \frac{d(3bc+2ad)x}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(3b^2c^2+16ab\right.}{6ac^2(bc-ad}$$

Mathematica [A] time = 5.51, size = 170, normalized size = 0.85

$$\frac{1}{6}\left(\frac{3b^2(bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{7/2}}+x\sqrt{c+dx^2}\left(-\frac{3b^3}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{4d^2(4bc-ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{2d^2}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (x*Sqrt[c + d*x^2]*((-3*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2)/(c*
(b*c - a*d)^2*(c + d*x^2)^2) + (4*d^2*(4*b*c - a*d))/(c^2*(b*c - a*d)^3*(c
+ d*x^2))) + (3*b^2*(b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(a^(3/2)*(b*c - a*d)^(7/2)))/6
```

IntegrateAlgebraic [A] time = 1.47, size = 251, normalized size = 1.25

$$\frac{\left(6ab^2d-b^3c\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{7/2}}+\frac{6a^3cd^3x+4a^3d^4x^3-18a^2bc^2d^2x-10a^2bcd^3x^3+4a^2bd^4x^5-18ab^2c^2d^2x^3-16ab^2cd^3x^5-3b^3c^4x-6b^3c^3dx^3-3b^3c^2d^2x^5}{6ac^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^3*c^4*x - 18*a^2*b*c^2*d^2*x + 6*a^3*c*d^3*x - 6*b^3*c^3*d*x^3 - 18*a
*b^2*c^2*d^2*x^3 - 10*a^2*b*c*d^3*x^3 + 4*a^3*d^4*x^3 - 3*b^3*c^2*d^2*x^5 -
 16*a*b^2*c*d^3*x^5 + 4*a^2*b*d^4*x^5)/(6*a*c^2*(-(b*c) + a*d)^3*(a + b*x^2
)*(c + d*x^2)^(3/2)) + ((-(b^3*c) + 6*a*b^2*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d
]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*(b*c -
a*d)^(7/2))
```

fricas [B] time = 3.22, size = 1434, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^
6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4
*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*
((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4
+ 2*a*b*x^2 + a^2)) - 4*((3*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2
*c*d^4 + 4*a^4*b*d^5)*x^5 + 2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^
2*c^2*d^3 - 7*a^4*b*c*d^4 + 2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3*c^4*d +
```

```
 6*a^3*b^2*c^3*d^2 - 8*a^4*b*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2 + c))/(a^
3*b^4*c^8 - 4*a^4*b^3*c^7*d + 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3 + a^7*c^4
*d^4 + (a^2*b^5*c^6*d^2 - 4*a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4 - 4*a^5*b^2
*c^3*d^5 + a^6*b*c^2*d^6)*x^6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^6*d^2 + 8*a^
4*b^3*c^5*d^3 - 2*a^5*b^2*c^4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*d^6)*x^4 + (a
^2*b^5*c^8 - 2*a^3*b^4*c^7*d - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*c^5*d^3 - 7*a^
6*b*c^4*d^4 + 2*a^7*c^3*d^5)*x^2), 1/12*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (
b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^
2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*
sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*
a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2*c*d^4 + 4*a^4*b*d^5)*x^5 +
2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 - 7*a^4*b*c*d^4 +
2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - 8*a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2 + c))/(a^3*b^4*c^8 - 4*a^4*b^3*c^7*d
+ 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3 + a^7*c^4*d^4 + (a^2*b^5*c^6*d^2 - 4*
a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4 - 4*a^5*b^2*c^3*d^5 + a^6*b*c^2*d^6)*x^
6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^6*d^2 + 8*a^4*b^3*c^5*d^3 - 2*a^5*b^2*c^
4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*d^6)*x^4 + (a^2*b^5*c^8 - 2*a^3*b^4*c^7*d
 - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*c^5*d^3 - 7*a^6*b*c^4*d^4 + 2*a^7*c^3*d^5)
*x^2)]
```

giac [B] time = 4.91, size = 619, normalized size = 3.08

$$\frac{\left(\frac{2\left(4\,b^4c^4d^4-13\,ab^3c^3d^5+15\,a^2b^2c^2d^6-7\,a^3bcd^7+a^4d^8\right)x^2}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7}+\frac{3\left(3\,b^4c^5d^3-10\,ab^3c^4d^4+12\,a^2b^2c^3d^5-6\,a^3bc^2d^6+a^4cd^7\right)}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\left(b^3c\sqrt{d}-6\,ab^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^2d^{\frac{3}{2}}-b^3c^2\sqrt{d}}{\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(4*b^4*c^4*d^4 - 13*a*b^3*c^3*d^5 + 15*a^2*b^2*c^2*d^6 - 7*a^3*b*c*d
^7 + a^4*d^8)*x^2/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^
3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7) + 3*(3*
b^4*c^5*d^3 - 10*a*b^3*c^4*d^4 + 12*a^2*b^2*c^3*d^5 - 6*a^3*b*c^2*d^6 + a^4
*c*d^7)/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3*c^5*
d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7))*x/(d*x^2 + c)^(3
/2) + 1/2*(b^3*c*sqrt(d) - 6*a*b^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^3*c^3 - 3*a^2
*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*
x - sqrt(d*x^2 + c))^2*b^3*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*
b^2*d^(3/2) - b^3*c^2*sqrt(d))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^
2 - a^4*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2))
```

maple [B] time = 0.02, size = 2405, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -5/4*b*d^2/(a*d-b*c)^3/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4/a*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/2/a*d/(a*d-b
*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*x+5/4/a*b*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-5/4*b*d^2/(a*d-b*c
)^3/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2)*x-1/4/a*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-1/2/a*d/(a*d-b*c)/c^2/((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4
/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/
b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^
(1/2)/b))-1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a
*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x-(-a*b)^(1/2)/b))-5/4/a*b*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a/(a*d-b
*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/a/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4/(
-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/12/(-a*b)^(1/2)/a/(a*d-b*c)*b/((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1
/12/(-a*b)^(1/2)/a/(a*d-b*c)*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/12/a*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2
)+5/6*d^2/(a*d-b*c)^2/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/12*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-5/4/a*b*(-
a*b)^(1/2)*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b
))-1/4/a*b/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+5/4/a*b*(-a*b)^(1/2)*d/(a*d-b*c)^3/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/4/a*b/(a*d-b*c)^2/c/((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*
d*x+5/12/a*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2
```

```
)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/12*d^2/(a*d-b*c)^2/c/((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+
5/6*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/4/(-a*b)^(1/2)/a*b^2/(a*d-b*c)^2/((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.763 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}+\frac{d\left(-2a^2d^2+6abcd+b^2c^2\right)}{2ac^2\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(b}$$

Rubi [A] time = 0.33, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{d\left(-2a^2d^2+6abcd+b^2c^2\right)}{2ac^2\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{b^{5/2}(2bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{b}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6ac\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + b/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 6*a*b*c*d - 2*a^2*d^2)
)/(2*a*c^2*(b*c - a*d)^3*Sqrt[c + d*x^2]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
]/(a^2*c^(5/2)) + (b^(5/2)*(2*b*c - 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[b*c - a*d]])/(2*a^2*(b*c - a*d)^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{5/2}}\,dx,x,x^2\right)$$

$$= \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{5bdx}{2}}{x(a+bx)(c+dx)^{5/2}}\,dx,x,x^2\right)}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}\left(\int -\frac{3}{2}\right.}{}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6a\right.}{2ac^2(bc-a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6a\right.}{2ac^2(bc-a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6a\right.}{2ac^2(bc-a}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2+6a\right.}{2ac^2(bc-a}$$

Mathematica [C] time = 0.12, size = 114, normalized size = 0.51

$$\frac{-\frac{b(2bc-7ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)}{(bc-ad)^2}+\frac{3ab}{\left(a+bx^2\right)(bc-ad)}+\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)}{c}}{6a^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] ((3*a*b)/((b*c - a*d)*(a + b*x^2)) - (b*(2*b*c - 7*a*d)*Hypergeometric2F1[-
3/2, 1, -1/2, (b*(c + d*x^2))/(b*c - a*d)])/(b*c - a*d)^2 + (2*Hypergeometr
ic2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c])/c)/(6*a^2*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.65, size = 276, normalized size = 1.23

$$\frac{\left(2b^{7/2}c-7ab^{5/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}\sqrt{ad-bc}}{bc-ad}\right)}{2a^2(bc-ad)^3\sqrt{ad-bc}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}}+\frac{8a^3cd^3+6a^3d^4x^2-20a^2bc^2d^2-10a^2bcd^3x^2+6a^2bd^4x^4-20ab^2c^2d^2x^2-18ab^2cd^3x^4-3b^3c^4-6b^3c^3dx^2-3b^3c^2d^2x^4}{6ac^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (-3*b^3*c^4 - 20*a^2*b*c^2*d^2 + 8*a^3*c*d^3 - 6*b^3*c^3*d*x^2 - 20*a*b^2*c
^2*d^2*x^2 - 10*a^2*b*c*d^3*x^2 + 6*a^3*d^4*x^2 - 3*b^3*c^2*d^2*x^4 - 18*a*
b^2*c*d^3*x^4 + 6*a^2*b*d^4*x^4)/(6*a*c^2*(-(b*c) + a*d)^3*(a + b*x^2)*(c +
 d*x^2)^(3/2)) + ((2*b^(7/2)*c - 7*a*b^(5/2)*d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c)
 + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*a^2*(b*c - a*d)^3*Sqrt[-(b*c) + a
*d]) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a^2*c^(5/2))
```

fricas [B] time = 15.72, size = 3403, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3)
*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*c^
6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d
^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 +
4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 12*(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(c)*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(3*a*b^3*c^5 + 20*a^3*b*c^3*d^2 -
8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*c*d^4)*x^4 +
 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a^4*c*d^4)*x^2
)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c
^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c
^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^
5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c
^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2), 1/24*(24*(a*b^3*c^5 - 3*a^2
*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3
 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^
2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2
*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)
/sqrt(d*x^2 + c)) + 3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a
```

```
*b^3*c^3*d^3)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^
4 + (2*b^4*c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d
))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*
b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*
b^3*c^5 + 20*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c
^2*d^3 - 2*a^3*b*c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3
*b*c^2*d^3 - 3*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^
7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3
+ 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6
*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8
- a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2)
, -1/12*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^
3)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*
c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(
1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
 b*c)) - 6*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (
b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (
b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*
x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b
^3*c^5 + 20*a^3*b*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^
2*d^3 - 2*a^3*b*c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 3*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7
*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 +
 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*
d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 -
 a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2),
 -1/12*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3
)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*c
^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1
/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 +
b*c)) - 12*(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (
b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (
b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*
x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 2*(3*a*b^3*c^5 + 20*a^3*b*
c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*c*
d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a^4*
c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d
^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4
 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^
5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3
*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2)]
```

giac [A] time = 0.42, size = 298, normalized size = 1.32

$$\frac{\sqrt{dx^2+c}\,b^3d}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\left(\left(dx^2+c\right)b-bc+ad\right)}-\frac{\left(2\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\sqrt{-b^2c+abd}}+\frac{9\left(dx^2+c\right)bcd^2+bc^2d^2-3\left(dx^2+c\right)ad^3-acd^3}{3\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)\left(dx^2+c\right)^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b^3*d/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a
^4*d^3)*((d*x^2 + c)*b - b*c + a*d)) - 1/2*(2*b^4*c - 7*a*b^3*d)*arctan(sqr
t(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^
4*b*c*d^2 - a^5*d^3)*sqrt(-b^2*c + a*b*d)) + 1/3*(9*(d*x^2 + c)*b*c*d^2 + b
*c^2*d^2 - 3*(d*x^2 + c)*a*d^3 - a*c*d^3)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2
*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^(3/2)) + arctan(sqrt(d*x^2 + c)/sqrt(
-c))/(a^2*sqrt(-c)*c^2)
```

maple [B] time = 0.02, size = 2837, normalized size = 12.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/4/(-a*b)^(1/2)/a/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4/a*d/(a*d-b*c)^3
*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-1/4/(-a*b)^(1/2)/
a/(a*d-b*c)*b/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4/a*d/(a*d-b*c)^3*b^2/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b
*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+1/6/a^2/(a*d-b*c)*b/((x-(-a*b)^(1/2
)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/2/a^2*b
^2/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2)+1/6/a^2/(a*d-b*c)*b/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/2/a^2*b^2/(a*d-b*c)^2/((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
)+1/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+2/3/(-a*b)^(1/2)/a*d/(a*d-b*c)
*b/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2)*x+1/2/a^2*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x-(-a*b)^(1/2)/b)^2*d
+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-1/3/(-a*b)^(1
/2)/a*d/(a*d-b*c)*b/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2
```

```
)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-2/3/(-a*b)^(1/2)/a*d/(a*d-b*c)*b/c^2/((x-(-a*
b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-
1/2/a^2*b/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/2/a^2*b^2/(a*d-b*c)^2/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2
*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+5/12/a*d/(a*d-b*c)^2*b/((x
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/
2)-5/4/a*d/(a*d-b*c)^3*b^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/2/a^2*b^2/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/6/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-(-
a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*
x-1/3/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/12/(-a*b)^(1/2)*b*d^2/(a*d
-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(3/2)*x-5/6/(-a*b)^(1/2)*b*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4/(-a*b)
^(1/2)*b^2*d^2/(a*d-b*c)^3/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4/(-a*b)^(1/2)*b^2*d^2/(a*d-b*c)^3/c
/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2)*x+5/12/a*d/(a*d-b*c)^2*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/4/a*d/(a*d-b*c)^3*b^2/((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+5/12/(-
a*b)^(1/2)*b*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/6/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/c/(
(x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
3/2)*x+1/3/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/6/(-a*b)^(1/2)*b*d^2/
(a*d-b*c)^2/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b
*d-(a*d-b*c)/b)^(1/2)*x-1/a^2/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)
+1/3/a^2/c/(d*x^2+c)^(3/2)+1/a^2/c^2/(d*x^2+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x), x)
```

mupad [B] time = 5.14, size = 8467, normalized size = 37.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((d^2*(c + d*x^2)*(3*a*d - 8*b*c))/(3*(b*c^2 - a*c*d)^2) - d^2/(3*(b*c^2 -
a*c*d)) + (d*(c + d*x^2)^2*(b^3*c^2 - 2*a^2*b*d^2 + 6*a*b^2*c*d))/(2*a*c*(b
*c^2 - a*c*d)*(a*d - b*c)^2))/(b*(c + d*x^2)^(5/2) + (c + d*x^2)^(3/2)*(a*d
 - b*c)) - atanh((560*a^3*b^16*c^19*d^4*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(56
0*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 14
9184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d^
9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^8
*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^1
4*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17*
b^2*c^3*d^18)) - (7280*a^4*b^15*c^18*d^5*(c + d*x^2)^(1/2))/((c^5)^(1/2)*(5
60*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 - 1
49184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12*d
^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*b^
8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*a^
14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^17
*b^2*c^3*d^18)) + (42560*a^5*b^14*c^17*d^6*(c + d*x^2)^(1/2))/((c^5)^(1/2)*
(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6 -
 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c^12
*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^11*
b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 35840*
a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64*a^
17*b^2*c^3*d^18)) - (149184*a^6*b^13*c^16*d^7*(c + d*x^2)^(1/2))/((c^5)^(1/
2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^
6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c
^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a^
11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 358
40*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 + 64
*a^17*b^2*c^3*d^18)) + (351904*a^7*b^12*c^15*d^8*(c + d*x^2)^(1/2))/((c^5)^
(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15
*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^1
1*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008
*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 -
35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 +
 64*a^17*b^2*c^3*d^18)) - (593440*a^8*b^11*c^14*d^9*(c + d*x^2)^(1/2))/((c^
5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c
^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*
b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505
008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14
 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^1
7 + 64*a^17*b^2*c^3*d^18)) + (741120*a^9*b^10*c^13*d^10*(c + d*x^2)^(1/2))/
```

```
((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^
14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*
a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 +
 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*
d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4
*d^17 + 64*a^17*b^2*c^3*d^18)) - (699840*a^10*b^9*c^12*d^11*(c + d*x^2)^(1/
2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^
5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593
440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^
11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*
c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3
*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (505008*a^11*b^8*c^11*d^12*(c + d*x^2)
^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 4256
0*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 -
 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^1
0*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*
b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16
*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) - (278768*a^12*b^7*c^10*d^13*(c + d*
x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 +
42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d
^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9
*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a
^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*
a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (116480*a^13*b^6*c^9*d^14*(c +
 d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5
 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^1
3*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*
b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 11648
0*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 10
24*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) - (35840*a^14*b^5*c^8*d^15*(c
 + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d
^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c
^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^1
0*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116
480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 -
1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (7680*a^15*b^4*c^7*d^16*(
c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*
d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*
c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^
10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 11
6480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 -
 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) - (1024*a^16*b^3*c^6*d^17*
(c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16
*d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12
*c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a
```

```
^10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 1
16480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16
- 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)) + (64*a^17*b^2*c^5*d^18*(
c + d*x^2)^(1/2))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*
d^5 + 42560*a^5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*
c^13*d^8 - 593440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^
10*b^9*c^10*d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 11
6480*a^13*b^6*c^7*d^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 -
 1024*a^16*b^3*c^4*d^17 + 64*a^17*b^2*c^3*d^18)))/(a^2*(c^5)^(1/2)) + (atan
((((-b^5*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(128*a^3*b^18*c^21*d^2 - 1
984*a^4*b^17*c^20*d^3 + 13840*a^5*b^16*c^19*d^4 - 57680*a^6*b^15*c^18*d^5 +
 161280*a^7*b^14*c^17*d^6 - 322560*a^8*b^13*c^16*d^7 + 480928*a^9*b^12*c^15
*d^8 - 550560*a^10*b^11*c^14*d^9 + 494400*a^11*b^10*c^13*d^10 - 352640*a^12
*b^9*c^12*d^11 + 199696*a^13*b^8*c^11*d^12 - 88144*a^14*b^7*c^10*d^13 + 291
20*a^15*b^6*c^9*d^14 - 6720*a^16*b^5*c^8*d^15 + 960*a^17*b^4*c^7*d^16 - 64*
a^18*b^3*c^6*d^17) - ((-b^5*(a*d - b*c)^7)^(1/2)*(7*a*d - 2*b*c)*(1536*a^7*
b^16*c^22*d^4 - 64*a^6*b^17*c^23*d^3 - 13952*a^8*b^15*c^21*d^5 + 71040*a^9*
b^14*c^20*d^6 - 235968*a^10*b^13*c^19*d^7 + 551936*a^11*b^12*c^18*d^8 - 948
992*a^12*b^11*c^17*d^9 + 1229184*a^13*b^10*c^16*d^10 - 1214400*a^14*b^9*c^1
5*d^11 + 918016*a^15*b^8*c^14*d^12 - 528000*a^16*b^7*c^13*d^13 + 227456*a^1
7*b^6*c^12*d^14 - 71232*a^18*b^5*c^11*d^15 + 15360*a^19*b^4*c^10*d^16 - 204
8*a^20*b^3*c^9*d^17 + 128*a^21*b^2*c^8*d^18 + ((-b^5*(a*d - b*c)^7)^(1/2)*(
c + d*x^2)^(1/2)*(7*a*d - 2*b*c)*(512*a^7*b^18*c^26*d^2 - 7936*a^8*b^17*c^2
5*d^3 + 57600*a^9*b^16*c^24*d^4 - 259840*a^10*b^15*c^23*d^5 + 815360*a^11*b
^14*c^22*d^6 - 1886976*a^12*b^13*c^21*d^7 + 3331328*a^13*b^12*c^20*d^8 - 45
76000*a^14*b^11*c^19*d^9 + 4942080*a^15*b^10*c^18*d^10 - 4209920*a^16*b^9*c
^17*d^11 + 2818816*a^17*b^8*c^16*d^12 - 1467648*a^18*b^7*c^15*d^13 + 582400
*a^19*b^6*c^14*d^14 - 170240*a^20*b^5*c^13*d^15 + 34560*a^21*b^4*c^12*d^16
- 4352*a^22*b^3*c^11*d^17 + 256*a^23*b^2*c^10*d^18))/(4*(a^9*d^7 - a^2*b^7*
c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^
3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6))))/(4*(a^9*d^7 - a^2*b^7*c^
7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^3*
c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)))*(7*a*d - 2*b*c)*1i)/(4*(a^9
*d^7 - a^2*b^7*c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*
d^3 - 35*a^6*b^3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)) + ((-b^5*(a
*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(128*a^3*b^18*c^21*d^2 - 1984*a^4*b^1
7*c^20*d^3 + 13840*a^5*b^16*c^19*d^4 - 57680*a^6*b^15*c^18*d^5 + 161280*a^7
*b^14*c^17*d^6 - 322560*a^8*b^13*c^16*d^7 + 480928*a^9*b^12*c^15*d^8 - 5505
60*a^10*b^11*c^14*d^9 + 494400*a^11*b^10*c^13*d^10 - 352640*a^12*b^9*c^12*d
^11 + 199696*a^13*b^8*c^11*d^12 - 88144*a^14*b^7*c^10*d^13 + 29120*a^15*b^6
*c^9*d^14 - 6720*a^16*b^5*c^8*d^15 + 960*a^17*b^4*c^7*d^16 - 64*a^18*b^3*c^
6*d^17) - ((-b^5*(a*d - b*c)^7)^(1/2)*(7*a*d - 2*b*c)*(64*a^6*b^17*c^23*d^3
 - 1536*a^7*b^16*c^22*d^4 + 13952*a^8*b^15*c^21*d^5 - 71040*a^9*b^14*c^20*d
^6 + 235968*a^10*b^13*c^19*d^7 - 551936*a^11*b^12*c^18*d^8 + 948992*a^12*b^
11*c^17*d^9 - 1229184*a^13*b^10*c^16*d^10 + 1214400*a^14*b^9*c^15*d^11 - 91
```

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8016*a^15*b^8*c^14*d^12 + 528000*a^16*b^7*c^13*d^13 - 227456*a^17*b^6*c^12*
d^14 + 71232*a^18*b^5*c^11*d^15 - 15360*a^19*b^4*c^10*d^16 + 2048*a^20*b^3*
c^9*d^17 - 128*a^21*b^2*c^8*d^18 + ((-b^5*(a*d - b*c)^7)^(1/2)*(c + d*x^2)^
(1/2)*(7*a*d - 2*b*c)*(512*a^7*b^18*c^26*d^2 - 7936*a^8*b^17*c^25*d^3 + 576
00*a^9*b^16*c^24*d^4 - 259840*a^10*b^15*c^23*d^5 + 815360*a^11*b^14*c^22*d^
6 - 1886976*a^12*b^13*c^21*d^7 + 3331328*a^13*b^12*c^20*d^8 - 4576000*a^14*
b^11*c^19*d^9 + 4942080*a^15*b^10*c^18*d^10 - 4209920*a^16*b^9*c^17*d^11 +
2818816*a^17*b^8*c^16*d^12 - 1467648*a^18*b^7*c^15*d^13 + 582400*a^19*b^6*c
^14*d^14 - 170240*a^20*b^5*c^13*d^15 + 34560*a^21*b^4*c^12*d^16 - 4352*a^22
*b^3*c^11*d^17 + 256*a^23*b^2*c^10*d^18))/(4*(a^9*d^7 - a^2*b^7*c^7 + 7*a^3
*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^3*c^3*d^4 +
 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6))))/(4*(a^9*d^7 - a^2*b^7*c^7 + 7*a^3*b
^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^3*c^3*d^4 + 2
1*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)))*(7*a*d - 2*b*c)*1i)/(4*(a^9*d^7 - a^2*
b^7*c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^
6*b^3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)))/(208*a^3*b^16*c^17*d^
4 - 32*a^2*b^17*c^18*d^3 + 304*a^4*b^15*c^16*d^5 - 7040*a^5*b^14*c^15*d^6 +
 31200*a^6*b^13*c^14*d^7 - 75936*a^7*b^12*c^13*d^8 + 118944*a^8*b^11*c^12*d
^9 - 126528*a^9*b^10*c^11*d^10 + 92640*a^10*b^9*c^10*d^11 - 46000*a^11*b^8*
c^9*d^12 + 14768*a^12*b^7*c^8*d^13 - 2752*a^13*b^6*c^7*d^14 + 224*a^14*b^5*
c^6*d^15 + ((-b^5*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(128*a^3*b^18*c^2
1*d^2 - 1984*a^4*b^17*c^20*d^3 + 13840*a^5*b^16*c^19*d^4 - 57680*a^6*b^15*c
^18*d^5 + 161280*a^7*b^14*c^17*d^6 - 322560*a^8*b^13*c^16*d^7 + 480928*a^9*
b^12*c^15*d^8 - 550560*a^10*b^11*c^14*d^9 + 494400*a^11*b^10*c^13*d^10 - 35
2640*a^12*b^9*c^12*d^11 + 199696*a^13*b^8*c^11*d^12 - 88144*a^14*b^7*c^10*d
^13 + 29120*a^15*b^6*c^9*d^14 - 6720*a^16*b^5*c^8*d^15 + 960*a^17*b^4*c^7*d
^16 - 64*a^18*b^3*c^6*d^17) - ((-b^5*(a*d - b*c)^7)^(1/2)*(7*a*d - 2*b*c)*(
1536*a^7*b^16*c^22*d^4 - 64*a^6*b^17*c^23*d^3 - 13952*a^8*b^15*c^21*d^5 + 7
1040*a^9*b^14*c^20*d^6 - 235968*a^10*b^13*c^19*d^7 + 551936*a^11*b^12*c^18*
d^8 - 948992*a^12*b^11*c^17*d^9 + 1229184*a^13*b^10*c^16*d^10 - 1214400*a^1
4*b^9*c^15*d^11 + 918016*a^15*b^8*c^14*d^12 - 528000*a^16*b^7*c^13*d^13 + 2
27456*a^17*b^6*c^12*d^14 - 71232*a^18*b^5*c^11*d^15 + 15360*a^19*b^4*c^10*d
^16 - 2048*a^20*b^3*c^9*d^17 + 128*a^21*b^2*c^8*d^18 + ((-b^5*(a*d - b*c)^7
)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 2*b*c)*(512*a^7*b^18*c^26*d^2 - 7936*a^8
*b^17*c^25*d^3 + 57600*a^9*b^16*c^24*d^4 - 259840*a^10*b^15*c^23*d^5 + 8153
60*a^11*b^14*c^22*d^6 - 1886976*a^12*b^13*c^21*d^7 + 3331328*a^13*b^12*c^20
*d^8 - 4576000*a^14*b^11*c^19*d^9 + 4942080*a^15*b^10*c^18*d^10 - 4209920*a
^16*b^9*c^17*d^11 + 2818816*a^17*b^8*c^16*d^12 - 1467648*a^18*b^7*c^15*d^13
 + 582400*a^19*b^6*c^14*d^14 - 170240*a^20*b^5*c^13*d^15 + 34560*a^21*b^4*c
^12*d^16 - 4352*a^22*b^3*c^11*d^17 + 256*a^23*b^2*c^10*d^18))/(4*(a^9*d^7 -
 a^2*b^7*c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 -
35*a^6*b^3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6))))/(4*(a^9*d^7 - a
^2*b^7*c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35
*a^6*b^3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)))*(7*a*d - 2*b*c))/(
4*(a^9*d^7 - a^2*b^7*c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^
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```
4*c^4*d^3 - 35*a^6*b^3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)) - ((-
b^5*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(128*a^3*b^18*c^21*d^2 - 1984*a
^4*b^17*c^20*d^3 + 13840*a^5*b^16*c^19*d^4 - 57680*a^6*b^15*c^18*d^5 + 1612
80*a^7*b^14*c^17*d^6 - 322560*a^8*b^13*c^16*d^7 + 480928*a^9*b^12*c^15*d^8
- 550560*a^10*b^11*c^14*d^9 + 494400*a^11*b^10*c^13*d^10 - 352640*a^12*b^9*
c^12*d^11 + 199696*a^13*b^8*c^11*d^12 - 88144*a^14*b^7*c^10*d^13 + 29120*a^
15*b^6*c^9*d^14 - 6720*a^16*b^5*c^8*d^15 + 960*a^17*b^4*c^7*d^16 - 64*a^18*
b^3*c^6*d^17) - ((-b^5*(a*d - b*c)^7)^(1/2)*(7*a*d - 2*b*c)*(64*a^6*b^17*c^
23*d^3 - 1536*a^7*b^16*c^22*d^4 + 13952*a^8*b^15*c^21*d^5 - 71040*a^9*b^14*
c^20*d^6 + 235968*a^10*b^13*c^19*d^7 - 551936*a^11*b^12*c^18*d^8 + 948992*a
^12*b^11*c^17*d^9 - 1229184*a^13*b^10*c^16*d^10 + 1214400*a^14*b^9*c^15*d^1
1 - 918016*a^15*b^8*c^14*d^12 + 528000*a^16*b^7*c^13*d^13 - 227456*a^17*b^6
*c^12*d^14 + 71232*a^18*b^5*c^11*d^15 - 15360*a^19*b^4*c^10*d^16 + 2048*a^2
0*b^3*c^9*d^17 - 128*a^21*b^2*c^8*d^18 + ((-b^5*(a*d - b*c)^7)^(1/2)*(c + d
*x^2)^(1/2)*(7*a*d - 2*b*c)*(512*a^7*b^18*c^26*d^2 - 7936*a^8*b^17*c^25*d^3
 + 57600*a^9*b^16*c^24*d^4 - 259840*a^10*b^15*c^23*d^5 + 815360*a^11*b^14*c
^22*d^6 - 1886976*a^12*b^13*c^21*d^7 + 3331328*a^13*b^12*c^20*d^8 - 4576000
*a^14*b^11*c^19*d^9 + 4942080*a^15*b^10*c^18*d^10 - 4209920*a^16*b^9*c^17*d
^11 + 2818816*a^17*b^8*c^16*d^12 - 1467648*a^18*b^7*c^15*d^13 + 582400*a^19
*b^6*c^14*d^14 - 170240*a^20*b^5*c^13*d^15 + 34560*a^21*b^4*c^12*d^16 - 435
2*a^22*b^3*c^11*d^17 + 256*a^23*b^2*c^10*d^18))/(4*(a^9*d^7 - a^2*b^7*c^7 +
 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^3*c^3
*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6))))/(4*(a^9*d^7 - a^2*b^7*c^7 + 7
*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^3*c^3*d
^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6)))*(7*a*d - 2*b*c))/(4*(a^9*d^7 - a
^2*b^7*c^7 + 7*a^3*b^6*c^6*d - 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35
*a^6*b^3*c^3*d^4 + 21*a^7*b^2*c^2*d^5 - 7*a^8*b*c*d^6))))*(-b^5*(a*d - b*c)
^7)^(1/2)*(7*a*d - 2*b*c)*1i)/(2*(a^9*d^7 - a^2*b^7*c^7 + 7*a^3*b^6*c^6*d -
 21*a^4*b^5*c^5*d^2 + 35*a^5*b^4*c^4*d^3 - 35*a^6*b^3*c^3*d^4 + 21*a^7*b^2*
c^2*d^5 - 7*a^8*b*c*d^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.764 $\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{6ac^2x\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{\sqrt{c+dx^2}\,\left(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+\right.}{6a^2c^3x(bc-ad)^3}$$

Rubi [A] time = 0.44, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {472, 579, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^2}\,\left(40a^2bcd^2-16a^3d^3-18ab^2c^2d+9b^3c^3\right)}{6a^2c^3x(bc-ad)^3}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{6ac^2x\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x*(c + d*x^2)^(3/2)) + b/(2*a*(b*c
 - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(3*b^2*c^2 + 20*a*b*c*d - 8*a
^2*d^2))/(6*a*c^2*(b*c - a*d)^3*x*Sqrt[c + d*x^2]) - ((9*b^3*c^3 - 18*a*b^2
*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c - a*
d)^3*x) - (b^3*(3*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-3bc+2ad-6bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-9b^2c^2+}{}}{}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\,(3b^2c^2}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\,(3b^2c^2}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\,(3b^2c^2}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\,(3b^2c^2}{6ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\,(3b^2c^2}{6ac^2(bc}$$

Mathematica [A] time = 5.59, size = 188, normalized size = 0.67

$$\sqrt{c+dx^2}\left(\frac{b^4x}{2a^2\left(a+bx^2\right)(ad-bc)^3} - \frac{1}{a^2c^3x} + \frac{d^3x(5ad-11bc)}{3c^3\left(c+dx^2\right)(bc-ad)^3} - \frac{d^3x}{3c^2\left(c+dx^2\right)^2(bc-ad)^2}\right) - \frac{b^3(3bc-8ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*(-(1/(a^2*c^3*x)) + (b^4*x)/(2*a^2*(-(b*c) + a*d)^3*(a + b*
x^2)) - (d^3*x)/(3*c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-11*b*c + 5*a*d
)*x)/(3*c^3*(b*c - a*d)^3*(c + d*x^2))) - (b^3*(3*b*c - 8*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

IntegrateAlgebraic [A] time = 1.57, size = 368, normalized size = 1.32

$$\frac{\left(3b^4c-8ab^3d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}\,x^2}{\sqrt{a}\,\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{7/2}}+\frac{-6a^4c^2d^3-24a^4cd^4x^2-16a^4d^5x^4+18a^3bc^3d^2+54a^3bc^2d^3x^2+16a^3bcd^4x^4-16a^3bd^5x^6-18a^2b^2c^4d-18a^2b^2c^3d^2x^2+42a^2b^2c^2d^3x^4+40a^2b^2cd^4x^6+6ab^3c^5-6ab^3c^4dx^2-30ab^3c^3d^2x^4-18ab^3c^2d^3x^6+9b^4c^5x^2+18b^4c^4dx^4+9b^4c^3d^2x^6}{6a^2c^3x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (6*a*b^3*c^5 - 18*a^2*b^2*c^4*d + 18*a^3*b*c^3*d^2 - 6*a^4*c^2*d^3 + 9*b^4*
c^5*x^2 - 6*a*b^3*c^4*d*x^2 - 18*a^2*b^2*c^3*d^2*x^2 + 54*a^3*b*c^2*d^3*x^2
 - 24*a^4*c*d^4*x^2 + 18*b^4*c^4*d*x^4 - 30*a*b^3*c^3*d^2*x^4 + 42*a^2*b^2*
c^2*d^3*x^4 + 16*a^3*b*c*d^4*x^4 - 16*a^4*d^5*x^4 + 9*b^4*c^3*d^2*x^6 - 18*
a*b^3*c^2*d^3*x^6 + 40*a^2*b^2*c*d^4*x^6 - 16*a^3*b*d^5*x^6)/(6*a^2*c^3*(-(
b*c) + a*d)^3*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((3*b^4*c - 8*a*b^3*d)*Arc
Tan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a
*d])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

fricas [B] time = 3.96, size = 1662, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*
c^4*d^2 - 8*a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*
c^4*d^2)*x^3 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(-a*b*c + a^2*d)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d
)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d + 36*a^
4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2 - 27*a^
2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^6)*x^6 +
 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^
2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*b^4*c^5*d
- 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*a^6*c*d^5)*
x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c^5
*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8*d - 7*a^4*b^
4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c^4*d^5 + a^8*c
^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7*d^2 + 8*a^6*b^
2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4*c^9 - 4*a^5*b^3
*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4)*x), -1/12*(3*((
3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4*c^4*d^2 - 8*
a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2)*x^3
 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a
*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2
)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d + 36*
```

```
a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2 - 27*
a^2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^6)*x^6
 + 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a^4*b^2*
c^2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*b^4*c^5*
d - 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*a^6*c*d^5
)*x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6*a^5*b^3*c
^5*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8*d - 7*a^4*
b^4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c^4*d^5 + a^8
*c^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7*d^2 + 8*a^6*
b^2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4*c^9 - 4*a^5*b
^3*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4)*x)]
```

giac [B] time = 5.34, size = 938, normalized size = 3.36

$$-\frac{\left(\frac{\left(11\,b^{4}c^{6}d^{5}-38\,ab^{3}c^{5}d^{6}+48\,a^{2}b^{2}c^{4}d^{7}-26\,a^{3}bc^{3}d^{8}+5\,a^{4}c^{2}d^{9}\right)x^{2}}{b^{6}c^{11}d-6\,ab^{5}c^{10}d^{2}+15\,a^{2}b^{4}c^{9}d^{3}-20\,a^{3}b^{3}c^{8}d^{4}+15\,a^{4}b^{2}c^{7}d^{5}-6\,a^{5}bc^{6}d^{6}+a^{6}c^{5}d^{7}}+\frac{6\left(2\,b^{4}c^{7}d^{4}-7\,ab^{3}c^{6}d^{5}+9\,a^{2}b^{2}c^{5}d^{6}-5\,a^{3}bc^{4}d^{7}+a^{4}c^{3}d^{8}\right)}{b^{6}c^{11}d-6\,ab^{5}c^{10}d^{2}+15\,a^{2}b^{4}c^{9}d^{3}-20\,a^{3}b^{3}c^{8}d^{4}+15\,a^{4}b^{2}c^{7}d^{5}-6\,a^{5}bc^{6}d^{6}+a^{6}c^{5}d^{7}}\right)x}{3\left(dx^{2}+c\right)^{\frac{3}{2}}}+\frac{\left(3\,b^{4}c\sqrt{d}-8\,ab^{3}d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{2\left(a^{2}b^{3}c^{3}-3\,a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-a^{5}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}b^{4}c^{3}\sqrt{d}-8\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ab^{3}c^{2}d^{\frac{3}{2}}+6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{2}b^{2}cd^{\frac{5}{2}}-2\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}a^{3}bd^{\frac{7}{2}}-6\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}b^{4}c^{4}\sqrt{d}+22\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}ab^{3}c^{3}d^{\frac{3}{2}}-36\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{2}b^{2}c^{2}d^{\frac{5}{2}}+28\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{3}bcd^{\frac{7}{2}}-8\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}a^{4}d^{\frac{9}{2}}+3\,b^{4}c^{5}\sqrt{d}-6\,ab^{3}c^{4}d^{\frac{3}{2}}+6\,a^{2}b^{2}c^{3}d^{\frac{5}{2}}-2\,a^{3}bc^{2}d^{\frac{7}{2}}}{\left(a^{2}b^{3}c^{5}-3\,a^{3}b^{2}c^{4}d+3\,a^{4}bc^{3}d^{2}-a^{5}c^{2}d^{3}\right)\left(\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{6}b-3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}bc+4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{4}ad+3\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}bc^{2}-4\left(\sqrt{d}x-\sqrt{dx^{2}+c}\right)^{2}acd-bc^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((11*b^4*c^6*d^5 - 38*a*b^3*c^5*d^6 + 48*a^2*b^2*c^4*d^7 - 26*a^3*b*c^
3*d^8 + 5*a^4*c^2*d^9)*x^2/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*
d^3 - 20*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d
^7) + 6*(2*b^4*c^7*d^4 - 7*a*b^3*c^6*d^5 + 9*a^2*b^2*c^5*d^6 - 5*a^3*b*c^4*
d^7 + a^4*c^3*d^8)/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*d^3 - 20
*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d^7))*x/(
d*x^2 + c)^(3/2) + 1/2*(3*b^4*c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqr
t(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2
*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b*c*d - a^2*d^
2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^4*c^3*sqrt(d) - 8*(sqrt(d)*x - s
qrt(d*x^2 + c))^4*a*b^3*c^2*d^(3/2) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*b^2*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b*d^(7/2) - 6*(sqrt(
d)*x - sqrt(d*x^2 + c))^2*b^4*c^4*sqrt(d) + 22*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a*b^3*c^3*d^(3/2) - 36*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^2*d^(5
/2) + 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(7/2) - 8*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a^4*d^(9/2) + 3*b^4*c^5*sqrt(d) - 6*a*b^3*c^4*d^(3/2) + 6
*a^2*b^2*c^3*d^(5/2) - 2*a^3*b*c^2*d^(7/2))/((a^2*b^3*c^5 - 3*a^3*b^2*c^4*d
 + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(s
qrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d +
 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*c*d - b*c^3))
```

maple [B] time = 0.02, size = 2513, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -5/12/a^2*(-a*b)^(1/2)*d/(a*d-b*c)^2*b/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4/a^2*(-a*b)^(1/2)*d/(a*d-b*
c)^3*b^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)+5/12/a^2*(-a*b)^(1/2)*d/(a*d-b*c)^2*b/((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/4/a^2*(-a*b)^(
1/2)*d/(a*d-b*c)^3*b^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2)-3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*
c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a
*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+3/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b
*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/a^2/c/x/(d*x^2+
c)^(3/2)-1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+3/4*b^3/a^2/(-a*b)^(1/2)/
(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)+1/4/a^2/(a*d-b*c)*b/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b
)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4/a^2/(a*d
-b*c)*b/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-3
/4*b^3/a^2/(-a*b)^(1/2)/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-4/3/a^2*d/c^2*x/(d*x^2+c)^(3/2)-8
/3/a^2*d/c^3*x/(d*x^2+c)^(1/2)-5/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)^3*b^2/(-(a*
d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))+5/4/a*b^2*d^2/(a*d-b*c)^3/c/
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x+3/4*b^2/a^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x-5/12/a*b*d^2/(a*d-b*c)^2/c/((x+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2
)*x-5/6/a*b*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+5/4/a*b^2*d^2/(a*d-b*c)^3/c/((x+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
+5/4/a^2*(-a*b)^(1/2)*d/(a*d-b*c)^3*b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^
(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
+(-a*b)^(1/2)/b))+1/12*b/a^2*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)
^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/6*b/a^2*d/(a*d-b*c)/c^
2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*x-5/12/a*b*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-5/6/a*b*d^2/(a*d-b*c)^2/c^2/((
x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
```

```
/2)*x+1/12/a^2*d/(a*d-b*c)*b/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/6/a^2*d/(a*d-b*c)*b/c^2/((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+3/
4*b^2/a^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.765 $\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=304

$$-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}}-\frac{d\left(5a^2d^2-6abcd+6b^2c^2\right)}{6a^2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{d(2bc-ad)\,(5}{2a^2c^3\sqrt{c}}$$

Rubi [A] time = 0.48, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{2a^2c^3\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{d\left(5a^2d^2-6abcd+6b^2c^2\right)}{6a^2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}-\frac{b^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}+\frac{(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}}-\frac{b(2bc-ad)}{2a^2c\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*(6*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2))/(6*a^2*c^2*(b*c - a*d)^2*(c + d*x^
2)^(3/2)) - (b*(2*b*c - a*d))/(2*a^2*c*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^
(3/2)) - 1/(2*a*c*x^2*(a + b*x^2)*(c + d*x^2)^(3/2)) - (d*(2*b*c - a*d)*(b^
2*c^2 - a*b*c*d + 5*a^2*d^2))/(2*a^2*c^3*(b*c - a*d)^3*Sqrt[c + d*x^2]) + (
(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3*c^(7/2)) - (b^(7/2
)*(4*b*c - 9*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(2*a^
3*(b*c - a*d)^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}}\, dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{5/2}}\, dx, x, x^2\right)$$

$$= -\frac{1}{2acx^2 \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+5ad)+\frac{7bdx}{2}}{x(a+bx)^2(c+dx)^{5/2}}\, dx, x, x^2\right)}{2ac}$$

$$= -\frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{1}{2acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\text{Subst}}{}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{}{2acx^2}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{}{2acx^2}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{}{2acx^2}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{}{2acx^2}$$

$$= -\frac{d\left(6b^2c^2-6abcd+5a^2d^2\right)}{6a^2c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{}{2acx^2}$$

Mathematica [C] time = 0.14, size = 190, normalized size = 0.62

$$\frac{b^2c^2x^2\left(a+bx^2\right)(4bc-9ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\left(dx^2+c\right)}{bc-ad}\right)-(bc-ad)\left(x^2\left(a+bx^2\right)\left(-5a^2d^2+abcd+4b^2c^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{dx^2}{c}+1\right)-3ac\left(a^2d+ab\left(dx^2-c\right)-2b^2cx^2\right)\right)}{6a^3c^2x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 9*a*d)*x^2*(a + b*x^2)*Hypergeometric2F1[-3/2, 1, -1/2, (
b*(c + d*x^2))/(b*c - a*d)] - (b*c - a*d)*(-3*a*c*(a^2*d - 2*b^2*c*x^2 + a*
b*(-c + d*x^2)) + (4*b^2*c^2 + a*b*c*d - 5*a^2*d^2)*x^2*(a + b*x^2)*Hyperge
ometric2F1[-3/2, 1, -1/2, 1 + (d*x^2)/c]))/(6*a^3*c^2*(b*c - a*d)^2*x^2*(a
+ b*x^2)*(c + d*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 1.33, size = 397, normalized size = 1.31

$$\frac{\left(4b^{9/2}c-9ab^{7/2}d\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}\,\sqrt{ad-bc}}{bc-ad}\right)}{2a^3(ad-bc)^{7/2}}+\frac{(5ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}}+\frac{-3a^4c^2d^3-20a^4cd^4x^2-15a^4d^5x^4+9a^3bc^3d^2+41a^3bc^2d^3x^2+13a^3bcd^4x^4-15a^3bd^5x^6-9a^2b^2c^4d-9a^2b^2c^3d^2x^2+35a^2b^2c^2d^3x^4+33a^2b^2cd^4x^6+3ab^3c^5-3ab^3c^4dx^2-15ab^3c^3d^2x^4-9ab^3c^2d^3x^6+6b^4c^5x^2+12b^4c^4dx^4+6b^4c^3d^2x^6}{6a^2c^3x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a*b^3*c^5 - 9*a^2*b^2*c^4*d + 9*a^3*b*c^3*d^2 - 3*a^4*c^2*d^3 + 6*b^4*c^
5*x^2 - 3*a*b^3*c^4*d*x^2 - 9*a^2*b^2*c^3*d^2*x^2 + 41*a^3*b*c^2*d^3*x^2 -
20*a^4*c*d^4*x^2 + 12*b^4*c^4*d*x^4 - 15*a*b^3*c^3*d^2*x^4 + 35*a^2*b^2*c^2
*d^3*x^4 + 13*a^3*b*c*d^4*x^4 - 15*a^4*d^5*x^4 + 6*b^4*c^3*d^2*x^6 - 9*a*b^
3*c^2*d^3*x^6 + 33*a^2*b^2*c*d^4*x^6 - 15*a^3*b*d^5*x^6)/(6*a^2*c^3*(-(b*c)
 + a*d)^3*x^2*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((4*b^(9/2)*c - 9*a*b^(7/2)*
d)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x^2])/(b*c - a*d)])/(2*a^3
*(-(b*c) + a*d)^(7/2)) + ((4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/(2*a^3*c^(7/2))
```

fricas [B] time = 29.74, size = 4115, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*c
^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c^5
*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(b*c - a*d))*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^
2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*((4*b^5*c^4*d^
2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x
^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*
d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*
c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b
^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2
*d^4)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*
(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*(2*a
*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*x^6
+ (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*c^2*
```

```
d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^4*d^
2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^7*
d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a^3*b
^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*
d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d
^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2
- a^7*c^6*d^3)*x^2), -1/24*(12*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^
3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c
^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)
*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 +
17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a
^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - 3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*
c^6*d - 14*a*b^4*c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*
d - 18*a^2*b^3*c^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c
*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
+ 4*(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*
(2*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*
x^6 + (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*
c^2*d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^
4*d^2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*
c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a
^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*
c^4*d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c
^6*d^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*
d^2 - a^7*c^6*d^3)*x^2), 1/12*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (
8*b^5*c^6*d - 14*a*b^4*c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^
4*c^6*d - 18*a^2*b^3*c^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sq
rt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 3*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*
a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a
*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^
5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*
d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(c)*log(-
(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a^2*b^3*c^6 - 9*a^3*b
^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3*(2*a*b^4*c^4*d^2 - 3*a^2*b^3
*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)*x^6 + (12*a*b^4*c^5*d - 15*a
^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b*c^2*d^4 - 15*a^5*c*d^5)*x^4
+ (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c^4*d^2 + 41*a^4*b*c^3*d^3 - 2
0*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3
+ 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7
*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^6 + (a^3*b^4*c^9
```

```
- a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^4
+ (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^2), 1/1
2*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*c^5*d
^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c^5*d^2
)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/
2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b
*c)) - 6*((4*b^5*c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2
*c*d^5 - 5*a^4*b*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^
3*d^3 + 19*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*
b^4*c^5*d - 17*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*
a^5*c*d^5)*x^4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^
4*b*c^3*d^3 - 5*a^5*c^2*d^4)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 - 2*(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3*d^3 + 3
*(2*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4*b*c*d^5)
*x^6 + (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3 + 13*a^4*b
*c^2*d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d - 9*a^3*b^2*c
^4*d^2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4
*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^8 + (2*
a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7
*c^4*d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*
c^6*d^3 - 2*a^7*c^5*d^4)*x^4 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7
*d^2 - a^7*c^6*d^3)*x^2)]
```

giac [A]   time = 0.42, size = 505, normalized size = 1.66

$$\frac{\left(4\,b^{5}c-9\,ab^{4}d\right)\arctan\left(\frac{\sqrt{dx^{2}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{2\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{2\left(dx^{2}+c\right)^{\frac{3}{2}}b^{4}c^{3}d-2\sqrt{dx^{2}+c}\,b^{4}c^{4}d-3\left(dx^{2}+c\right)^{\frac{3}{2}}ab^{3}c^{2}d^{2}+4\sqrt{dx^{2}+c}\,ab^{3}c^{3}d^{2}+3\left(dx^{2}+c\right)^{\frac{3}{2}}a^{2}b^{2}cd^{3}-6\sqrt{dx^{2}+c}\,a^{2}b^{2}c^{2}d^{3}-\left(dx^{2}+c\right)^{\frac{3}{2}}a^{3}bd^{4}+4\sqrt{dx^{2}+c}\,a^{3}bcd^{4}-\sqrt{dx^{2}+c}\,a^{4}d^{5}}{2\left(a^{2}b^{3}c^{6}-3\,a^{3}b^{2}c^{5}d+3\,a^{4}bc^{4}d^{2}-a^{5}c^{3}d^{3}\right)\left(\left(dx^{2}+c\right)^{2}b-2\left(dx^{2}+c\right)bc+bc^{2}+\left(dx^{2}+c\right)ad-acd\right)}-\frac{12\left(dx^{2}+c\right)bcd^{3}+bc^{2}d^{3}-6\left(dx^{2}+c\right)ad^{4}-acd^{4}}{3\left(b^{3}c^{6}-3\,ab^{2}c^{5}d+3\,a^{2}bc^{4}d^{2}-a^{3}c^{3}d^{3}\right)\left(dx^{2}+c\right)^{\frac{3}{2}}}-\frac{(4\,bc+5\,ad)\arctan\left(\frac{\sqrt{dx^{2}+c}}{\sqrt{-c}}\right)}{2\,a^{3}\sqrt{-c}\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*b^5*c - 9*a*b^4*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(-b^2*c + a*b*
d)) - 1/2*(2*(d*x^2 + c)^(3/2)*b^4*c^3*d - 2*sqrt(d*x^2 + c)*b^4*c^4*d - 3*
(d*x^2 + c)^(3/2)*a*b^3*c^2*d^2 + 4*sqrt(d*x^2 + c)*a*b^3*c^3*d^2 + 3*(d*x^
2 + c)^(3/2)*a^2*b^2*c*d^3 - 6*sqrt(d*x^2 + c)*a^2*b^2*c^2*d^3 - (d*x^2 + c
)^(3/2)*a^3*b*d^4 + 4*sqrt(d*x^2 + c)*a^3*b*c*d^4 - sqrt(d*x^2 + c)*a^4*d^5
)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*((d*x^2
+ c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - 1/3*(12*
(d*x^2 + c)*b*c*d^3 + b*c^2*d^3 - 6*(d*x^2 + c)*a*d^4 - a*c*d^4)/((b^3*c^6
- 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^(3/2)) - 1/2*(
4*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c^3)
```

maple [B]   time = 0.02, size = 2980, normalized size = 9.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] -5/4*b^3/a^2*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)
/b))-5/4*b^3/a^2*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)
^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1
/2)/b))+1/4*b^2/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x-(-a*b)^(1/2)/b)/((x-(-a*b)^(1
/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-1/4*b^2
/a^2/(-a*b)^(1/2)/(a*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-
a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-2*b/a^3/c^2/(d*x^2+c)^
(1/2)+2*b/a^3/c^(5/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/2/a^2/c/x^2/(
d*x^2+c)^(3/2)-5/6/a^2*d/c^2/(d*x^2+c)^(3/2)-5/2/a^2*d/c^3/(d*x^2+c)^(1/2)+
5/2/a^2*d/c^(7/2)*ln((2*c+2*(d*x^2+c)^(1/2)*c^(1/2))/x)-1/3*b^2/a^3/(a*d-b*
c)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
b)^(3/2)+b^3/a^3/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/3*b^2/a^3/(a*d-b*c)/((x-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+b^3/a^3/(a*d
-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)-2/3*b/a^3/c/(d*x^2+c)^(3/2)-b^2/a^3/(a*d-b*c)^2*(-a*b)^(1/2)/
c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(1/2)*d*x-5/12*b^2/a^2*d/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/
2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4*b^3/a^2*d/(a*d-b*c)^3/((x-
(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2
)-b^3/a^3/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))-
5/12*b^2/a^2*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4*b^3/a^2*d/(a*d-b*c)^3/((x+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-b^3/a^3/(a
*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-5/12*b^2/a/(-
a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-5/6*b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2
/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)*x+5/4*b^3/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x+1/3*b^2/a^2/(
-a*b)^(1/2)*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^
(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+2/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c^2/
((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)*x-1/3*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
```

```
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-2/3*b/a^3*(-a*b)^(1/2)
*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(1/2)*x+b^2/a^3/(a*d-b*c)^2*(-a*b)^(1/2)/c/((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+5/12*
b^2/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/6*b^2/a/(-a*b)^(1/2)*d^2/(a*
d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a/(-a*b)^(1/2)*d^2/(a*d-b*c)^3/c/((x+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/3*b
^2/a^2/(-a*b)^(1/2)*d/(a*d-b*c)/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x-2/3*b^2/a^2/(-a*b)^(1/2)*d/(a*d-b
*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*x+1/3*b/a^3*(-a*b)^(1/2)*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+2/3*b/a^3*(-a*
b)^(1/2)*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(
1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{5}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3), x)
```

mupad [B] time = 6.58, size = 5800, normalized size = 19.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] ((5*d^3*(c + d*x^2)*(a*d - 2*b*c))/(3*(b*c^2 - a*c*d)^2) - d^3/(3*(b*c^2 -
a*c*d)) + (d*(c + d*x^2)^2*(15*a^4*d^4 + 6*b^4*c^4 + 64*a^2*b^2*c^2*d^2 - 1
2*a*b^3*c^3*d - 58*a^3*b*c*d^3))/(6*a^2*(b*c^2 - a*c*d)^3) + (d*(c + d*x^2)
^3*(a*d - 2*b*c)*(b^3*c^2 + 5*a^2*b*d^2 - a*b^2*c*d))/(2*a^2*(b*c^2 - a*c*d
)^3))/(b*(c + d*x^2)^(7/2) + (c + d*x^2)^(3/2)*(b*c^2 - a*c*d) + (c + d*x^2
)^(5/2)*(a*d - 2*b*c)) - (atan((a^19*c^15*d^19*(c + d*x^2)^(1/2)*125i + a^3
*b^16*c^31*d^3*(c + d*x^2)^(1/2)*420i - a^4*b^15*c^30*d^4*(c + d*x^2)^(1/2)
*4515i + a^5*b^14*c^29*d^5*(c + d*x^2)^(1/2)*20916i - a^6*b^13*c^28*d^6*(c
+ d*x^2)^(1/2)*52836i + a^7*b^12*c^27*d^7*(c + d*x^2)^(1/2)*71070i - a^8*b^
11*c^26*d^8*(c + d*x^2)^(1/2)*19530i - a^9*b^10*c^25*d^9*(c + d*x^2)^(1/2)*
107740i + a^10*b^9*c^24*d^10*(c + d*x^2)^(1/2)*212608i - a^11*b^8*c^23*d^11
```

```
*(c + d*x^2)^(1/2)*184563i + a^12*b^7*c^22*d^12*(c + d*x^2)^(1/2)*40965i +
a^13*b^6*c^21*d^13*(c + d*x^2)^(1/2)*91560i - a^14*b^5*c^20*d^14*(c + d*x^2
)^(1/2)*126720i + a^15*b^4*c^19*d^15*(c + d*x^2)^(1/2)*87276i - a^16*b^3*c^
18*d^16*(c + d*x^2)^(1/2)*37776i + a^17*b^2*c^17*d^17*(c + d*x^2)^(1/2)*104
40i - a^18*b*c^16*d^18*(c + d*x^2)^(1/2)*1700i)/(c^7*(c^7)^(1/2)*(c^7*(c^7*
(212608*a^10*b^9*d^10 - 107740*a^9*b^10*c*d^9 + 420*a^3*b^16*c^7*d^3 - 4515
*a^4*b^15*c^6*d^4 + 20916*a^5*b^14*c^5*d^5 - 52836*a^6*b^13*c^4*d^6 + 71070
*a^7*b^12*c^3*d^7 - 19530*a^8*b^11*c^2*d^8) + 10440*a^17*b^2*d^17 - 37776*a
^16*b^3*c*d^16 - 184563*a^11*b^8*c^6*d^11 + 40965*a^12*b^7*c^5*d^12 + 91560
*a^13*b^6*c^4*d^13 - 126720*a^14*b^5*c^3*d^14 + 87276*a^15*b^4*c^2*d^15) +
125*a^19*c^5*d^19 - 1700*a^18*b*c^6*d^18)))*(5*a*d + 4*b*c)*1i)/(2*a^3*(c^7
)^(1/2)) + (atan((((-b^7*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(512*a^6*b
^20*c^26*d^2 - 6656*a^7*b^19*c^25*d^3 + 38560*a^8*b^18*c^24*d^4 - 129920*a^
9*b^17*c^23*d^5 + 275920*a^10*b^16*c^22*d^6 - 363440*a^11*b^15*c^21*d^7 + 2
35312*a^12*b^14*c^20*d^8 + 85360*a^13*b^13*c^19*d^9 - 316400*a^14*b^12*c^18
*d^10 + 205840*a^15*b^11*c^17*d^11 + 152384*a^16*b^10*c^16*d^12 - 430816*a^
17*b^9*c^15*d^13 + 444080*a^18*b^8*c^14*d^14 - 281680*a^19*b^7*c^13*d^15 +
118640*a^20*b^6*c^12*d^16 - 32656*a^21*b^5*c^11*d^17 + 5360*a^22*b^4*c^10*d
^18 - 400*a^23*b^3*c^9*d^19) + ((-b^7*(a*d - b*c)^7)^(1/2)*(9*a*d - 4*b*c)*
(128*a^10*b^18*c^28*d^3 - 1792*a^11*b^17*c^27*d^4 + 10624*a^12*b^16*c^26*d^
5 - 33280*a^13*b^15*c^25*d^6 + 47936*a^14*b^14*c^24*d^7 + 40448*a^15*b^13*c
^23*d^8 - 368896*a^16*b^12*c^22*d^9 + 948992*a^17*b^11*c^21*d^10 - 1531200*
a^18*b^10*c^20*d^11 + 1754368*a^19*b^9*c^19*d^12 - 1485440*a^20*b^8*c^18*d^
13 + 939008*a^21*b^7*c^17*d^14 - 439616*a^22*b^6*c^16*d^15 + 148480*a^23*b^
5*c^15*d^16 - 34304*a^24*b^4*c^14*d^17 + 4864*a^25*b^3*c^13*d^18 - 320*a^26
*b^2*c^12*d^19 - ((-b^7*(a*d - b*c)^7)^(1/2)*(c + d*x^2)^(1/2)*(9*a*d - 4*b
*c)*(512*a^12*b^18*c^31*d^2 - 7936*a^13*b^17*c^30*d^3 + 57600*a^14*b^16*c^2
9*d^4 - 259840*a^15*b^15*c^28*d^5 + 815360*a^16*b^14*c^27*d^6 - 1886976*a^1
7*b^13*c^26*d^7 + 3331328*a^18*b^12*c^25*d^8 - 4576000*a^19*b^11*c^24*d^9 +
 4942080*a^20*b^10*c^23*d^10 - 4209920*a^21*b^9*c^22*d^11 + 2818816*a^22*b^
8*c^21*d^12 - 1467648*a^23*b^7*c^20*d^13 + 582400*a^24*b^6*c^19*d^14 - 1702
40*a^25*b^5*c^18*d^15 + 34560*a^26*b^4*c^17*d^16 - 4352*a^27*b^3*c^16*d^17
+ 256*a^28*b^2*c^15*d^18))/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 2
1*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^
2*d^5 - 7*a^9*b*c*d^6))))/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 21
*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^2
*d^5 - 7*a^9*b*c*d^6)))*(9*a*d - 4*b*c)*1i)/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*
a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^
4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)) + ((-b^7*(a*d - b*c)^7)^(1/2)*((c
+ d*x^2)^(1/2)*(512*a^6*b^20*c^26*d^2 - 6656*a^7*b^19*c^25*d^3 + 38560*a^8*
b^18*c^24*d^4 - 129920*a^9*b^17*c^23*d^5 + 275920*a^10*b^16*c^22*d^6 - 3634
40*a^11*b^15*c^21*d^7 + 235312*a^12*b^14*c^20*d^8 + 85360*a^13*b^13*c^19*d^
9 - 316400*a^14*b^12*c^18*d^10 + 205840*a^15*b^11*c^17*d^11 + 152384*a^16*b
^10*c^16*d^12 - 430816*a^17*b^9*c^15*d^13 + 444080*a^18*b^8*c^14*d^14 - 281
680*a^19*b^7*c^13*d^15 + 118640*a^20*b^6*c^12*d^16 - 32656*a^21*b^5*c^11*d^
```

```
17 + 5360*a^22*b^4*c^10*d^18 - 400*a^23*b^3*c^9*d^19) - ((-b^7*(a*d - b*c)^
7)^(1/2)*(9*a*d - 4*b*c)*(128*a^10*b^18*c^28*d^3 - 1792*a^11*b^17*c^27*d^4
+ 10624*a^12*b^16*c^26*d^5 - 33280*a^13*b^15*c^25*d^6 + 47936*a^14*b^14*c^2
4*d^7 + 40448*a^15*b^13*c^23*d^8 - 368896*a^16*b^12*c^22*d^9 + 948992*a^17*
b^11*c^21*d^10 - 1531200*a^18*b^10*c^20*d^11 + 1754368*a^19*b^9*c^19*d^12 -
 1485440*a^20*b^8*c^18*d^13 + 939008*a^21*b^7*c^17*d^14 - 439616*a^22*b^6*c
^16*d^15 + 148480*a^23*b^5*c^15*d^16 - 34304*a^24*b^4*c^14*d^17 + 4864*a^25
*b^3*c^13*d^18 - 320*a^26*b^2*c^12*d^19 + ((-b^7*(a*d - b*c)^7)^(1/2)*(c +
d*x^2)^(1/2)*(9*a*d - 4*b*c)*(512*a^12*b^18*c^31*d^2 - 7936*a^13*b^17*c^30*
d^3 + 57600*a^14*b^16*c^29*d^4 - 259840*a^15*b^15*c^28*d^5 + 815360*a^16*b^
14*c^27*d^6 - 1886976*a^17*b^13*c^26*d^7 + 3331328*a^18*b^12*c^25*d^8 - 457
6000*a^19*b^11*c^24*d^9 + 4942080*a^20*b^10*c^23*d^10 - 4209920*a^21*b^9*c^
22*d^11 + 2818816*a^22*b^8*c^21*d^12 - 1467648*a^23*b^7*c^20*d^13 + 582400*
a^24*b^6*c^19*d^14 - 170240*a^25*b^5*c^18*d^15 + 34560*a^26*b^4*c^17*d^16 -
 4352*a^27*b^3*c^16*d^17 + 256*a^28*b^2*c^15*d^18))/(4*(a^10*d^7 - a^3*b^7*
c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^
3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6))))/(4*(a^10*d^7 - a^3*b^7*c
^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3
*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)))*(9*a*d - 4*b*c)*1i)/(4*(a^
10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^
4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)))/(256*a^4
*b^20*c^23*d^3 - 2944*a^5*b^19*c^22*d^4 + 16048*a^6*b^18*c^21*d^5 - 55160*a
^7*b^17*c^20*d^6 + 130000*a^8*b^16*c^19*d^7 - 206112*a^9*b^15*c^18*d^8 + 18
2808*a^10*b^14*c^17*d^9 + 23664*a^11*b^13*c^16*d^10 - 332160*a^12*b^12*c^15
*d^11 + 519200*a^13*b^11*c^14*d^12 - 460544*a^14*b^10*c^13*d^13 + 260936*a^
15*b^9*c^12*d^14 - 93712*a^16*b^8*c^11*d^15 + 19520*a^17*b^7*c^10*d^16 - 18
00*a^18*b^6*c^9*d^17 - ((-b^7*(a*d - b*c)^7)^(1/2)*((c + d*x^2)^(1/2)*(512*
a^6*b^20*c^26*d^2 - 6656*a^7*b^19*c^25*d^3 + 38560*a^8*b^18*c^24*d^4 - 1299
20*a^9*b^17*c^23*d^5 + 275920*a^10*b^16*c^22*d^6 - 363440*a^11*b^15*c^21*d^
7 + 235312*a^12*b^14*c^20*d^8 + 85360*a^13*b^13*c^19*d^9 - 316400*a^14*b^12
*c^18*d^10 + 205840*a^15*b^11*c^17*d^11 + 152384*a^16*b^10*c^16*d^12 - 4308
16*a^17*b^9*c^15*d^13 + 444080*a^18*b^8*c^14*d^14 - 281680*a^19*b^7*c^13*d^
15 + 118640*a^20*b^6*c^12*d^16 - 32656*a^21*b^5*c^11*d^17 + 5360*a^22*b^4*c
^10*d^18 - 400*a^23*b^3*c^9*d^19) + ((-b^7*(a*d - b*c)^7)^(1/2)*(9*a*d - 4*
b*c)*(128*a^10*b^18*c^28*d^3 - 1792*a^11*b^17*c^27*d^4 + 10624*a^12*b^16*c^
26*d^5 - 33280*a^13*b^15*c^25*d^6 + 47936*a^14*b^14*c^24*d^7 + 40448*a^15*b
^13*c^23*d^8 - 368896*a^16*b^12*c^22*d^9 + 948992*a^17*b^11*c^21*d^10 - 153
1200*a^18*b^10*c^20*d^11 + 1754368*a^19*b^9*c^19*d^12 - 1485440*a^20*b^8*c^
18*d^13 + 939008*a^21*b^7*c^17*d^14 - 439616*a^22*b^6*c^16*d^15 + 148480*a^
23*b^5*c^15*d^16 - 34304*a^24*b^4*c^14*d^17 + 4864*a^25*b^3*c^13*d^18 - 320
*a^26*b^2*c^12*d^19 - ((-b^7*(a*d - b*c)^7)^(1/2)*(c + d*x^2)^(1/2)*(9*a*d
- 4*b*c)*(512*a^12*b^18*c^31*d^2 - 7936*a^13*b^17*c^30*d^3 + 57600*a^14*b^1
6*c^29*d^4 - 259840*a^15*b^15*c^28*d^5 + 815360*a^16*b^14*c^27*d^6 - 188697
6*a^17*b^13*c^26*d^7 + 3331328*a^18*b^12*c^25*d^8 - 4576000*a^19*b^11*c^24*
d^9 + 4942080*a^20*b^10*c^23*d^10 - 4209920*a^21*b^9*c^22*d^11 + 2818816*a^
```

```
22*b^8*c^21*d^12 - 1467648*a^23*b^7*c^20*d^13 + 582400*a^24*b^6*c^19*d^14 -
 170240*a^25*b^5*c^18*d^15 + 34560*a^26*b^4*c^17*d^16 - 4352*a^27*b^3*c^16*
d^17 + 256*a^28*b^2*c^15*d^18))/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*
d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b
^2*c^2*d^5 - 7*a^9*b*c*d^6))))/(4*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d
 - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^
2*c^2*d^5 - 7*a^9*b*c*d^6)))*(9*a*d - 4*b*c))/(4*(a^10*d^7 - a^3*b^7*c^7 +
7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*
d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)) + ((-b^7*(a*d - b*c)^7)^(1/2)*((
c + d*x^2)^(1/2)*(512*a^6*b^20*c^26*d^2 - 6656*a^7*b^19*c^25*d^3 + 38560*a^
8*b^18*c^24*d^4 - 129920*a^9*b^17*c^23*d^5 + 275920*a^10*b^16*c^22*d^6 - 36
3440*a^11*b^15*c^21*d^7 + 235312*a^12*b^14*c^20*d^8 + 85360*a^13*b^13*c^19*
d^9 - 316400*a^14*b^12*c^18*d^10 + 205840*a^15*b^11*c^17*d^11 + 152384*a^16
*b^10*c^16*d^12 - 430816*a^17*b^9*c^15*d^13 + 444080*a^18*b^8*c^14*d^14 - 2
81680*a^19*b^7*c^13*d^15 + 118640*a^20*b^6*c^12*d^16 - 32656*a^21*b^5*c^11*
d^17 + 5360*a^22*b^4*c^10*d^18 - 400*a^23*b^3*c^9*d^19) - ((-b^7*(a*d - b*c
)^7)^(1/2)*(9*a*d - 4*b*c)*(128*a^10*b^18*c^28*d^3 - 1792*a^11*b^17*c^27*d^
4 + 10624*a^12*b^16*c^26*d^5 - 33280*a^13*b^15*c^25*d^6 + 47936*a^14*b^14*c
^24*d^7 + 40448*a^15*b^13*c^23*d^8 - 368896*a^16*b^12*c^22*d^9 + 948992*a^1
7*b^11*c^21*d^10 - 1531200*a^18*b^10*c^20*d^11 + 1754368*a^19*b^9*c^19*d^12
 - 1485440*a^20*b^8*c^18*d^13 + 939008*a^21*b^7*c^17*d^14 - 439616*a^22*b^6
*c^16*d^15 + 148480*a^23*b^5*c^15*d^16 - 34304*a^24*b^4*c^14*d^17 + 4864*a^
25*b^3*c^13*d^18 - 320*a^26*b^2*c^12*d^19 + ((-b^7*(a*d - b*c)^7)^(1/2)*(c
+ d*x^2)^(1/2)*(9*a*d - 4*b*c)*(512*a^12*b^18*c^31*d^2 - 7936*a^13*b^17*c^3
0*d^3 + 57600*a^14*b^16*c^29*d^4 - 259840*a^15*b^15*c^28*d^5 + 815360*a^16*
b^14*c^27*d^6 - 1886976*a^17*b^13*c^26*d^7 + 3331328*a^18*b^12*c^25*d^8 - 4
576000*a^19*b^11*c^24*d^9 + 4942080*a^20*b^10*c^23*d^10 - 4209920*a^21*b^9*
c^22*d^11 + 2818816*a^22*b^8*c^21*d^12 - 1467648*a^23*b^7*c^20*d^13 + 58240
0*a^24*b^6*c^19*d^14 - 170240*a^25*b^5*c^18*d^15 + 34560*a^26*b^4*c^17*d^16
 - 4352*a^27*b^3*c^16*d^17 + 256*a^28*b^2*c^15*d^18))/(4*(a^10*d^7 - a^3*b^
7*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*
b^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6))))/(4*(a^10*d^7 - a^3*b^7
*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b
^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6)))*(9*a*d - 4*b*c))/(4*(a^1
0*d^7 - a^3*b^7*c^7 + 7*a^4*b^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4
*d^3 - 35*a^7*b^3*c^3*d^4 + 21*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6))))*(-b^7*(a
*d - b*c)^7)^(1/2)*(9*a*d - 4*b*c)*1i)/(2*(a^10*d^7 - a^3*b^7*c^7 + 7*a^4*b
^6*c^6*d - 21*a^5*b^5*c^5*d^2 + 35*a^6*b^4*c^4*d^3 - 35*a^7*b^3*c^3*d^4 + 2
1*a^8*b^2*c^2*d^5 - 7*a^9*b*c*d^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.766 $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=362

$$\frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}+\frac{d\left(-4a^2d^2+8abcd+b^2c^2\right)}{2ac^2x^3\sqrt{c+dx^2}\,(bc-ad)^3}-\frac{\sqrt{c+dx^2}\,(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3}{6a^2c^3x^3(bc-ad)^3}$$

Rubi [A] time = 0.60, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {472, 579, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^2}\left(-12a^2b^2c^2d^2+64a^3bcd^3-32a^4d^4-20ab^3c^3d+15b^4c^4\right)}{6a^3c^4x(bc-ad)^3}-\frac{\sqrt{c+dx^2}\left(32a^2bcd^2-16a^3d^3-6ab^2c^2d+5b^3c^3\right)}{6a^2c^3x^3(bc-ad)^3}+\frac{d\left(-4a^2d^2+8abcd+b^2c^2\right)}{2ac^2x^3\sqrt{c+dx^2}\,(bc-ad)^3}+\frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{d(2ad+3bc)}{6acx^3\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (d*(3*b*c + 2*a*d))/(6*a*c*(b*c - a*d)^2*x^3*(c + d*x^2)^(3/2)) + b/(2*a*(b
*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (d*(b^2*c^2 + 8*a*b*c*d - 4*
a^2*d^2))/(2*a*c^2*(b*c - a*d)^3*x^3*Sqrt[c + d*x^2]) - ((5*b^3*c^3 - 6*a*b
^2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(6*a^2*c^3*(b*c -
a*d)^3*x^3) + ((15*b^4*c^4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b
*c*d^3 - 32*a^4*d^4)*Sqrt[c + d*x^2])/(6*a^3*c^4*(b*c - a*d)^3*x) + (5*b^4*
(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
7/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}}\,dx = \frac{b}{2a(bc-ad)x^3 \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-5bc+2ad-8bdx^2}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{2a(bc-ad)}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} - \int \frac{-3\left(5b^2c\right.}{}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2\right.}{2ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2\right.}{2ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2\right.}{2ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2\right.}{2ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2\right.}{2ac^2(bc}$$

$$= \frac{d(3bc+2ad)}{6ac(bc-ad)^2x^3\left(c+dx^2\right)^{3/2}} + \frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}} + \frac{d\left(b^2c^2\right.}{2ac^2(bc}$$

Mathematica [A] time = 5.87, size = 210, normalized size = 0.58

$$\frac{5b^4(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}} + \frac{\sqrt{c+dx^2}\left(-\frac{3b^5x^4}{a^3\left(a+bx^2\right)(ad-bc)^3} + \frac{4x^2(4ad+3bc)}{a^3c^4} - \frac{2}{a^2c^3} + \frac{4d^4x^4(7bc-4ad)}{c^4\left(c+dx^2\right)(bc-ad)^3} + \frac{2d^4x^4}{c^3\left(c+dx^2\right)^2(bc-ad)^2}\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2/(a^2*c^3) + (4*(3*b*c + 4*a*d)*x^2)/(a^3*c^4) - (3*b^5
*x^4)/(a^3*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^4*x^4)/(c^3*(b*c - a*d)^2*(
c + d*x^2)^2) + (4*d^4*(7*b*c - 4*a*d)*x^4)/(c^4*(b*c - a*d)^3*(c + d*x^2))
))/(6*x^3) + (5*b^4*(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(2*a^(7/2)*(b*c - a*d)^(7/2))
```

IntegrateAlgebraic [A] time = 2.08, size = 480, normalized size = 1.33

$$\frac{-2a^5c^3d^3+12a^5c^2d^4x^2+48a^5cd^5x^4+32a^5d^6x^6+6a^4bc^4d^2-26a^4bc^3d^3x^2-84a^4bc^2d^4x^4-16a^4bcd^5x^6+32a^4bd^6x^8-6a^3b^2c^5d+6a^3b^2c^4d^2x^2-6a^3b^2c^3d^3x^4-84a^3b^2c^2d^4x^6-64a^3b^2cd^5x^8+2a^2b^3c^6+18a^2b^3c^5dx^2+42a^2b^3c^4d^2x^4+38a^2b^3c^3d^3x^6+12a^2b^3c^2d^4x^8-10ab^4c^6x^2+30ab^4c^4d^2x^6+20ab^4c^3d^3x^8-15b^5c^6x^4-30b^5c^5dx^6-15b^5c^4d^2x^8}{6a^3c^4x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(ad-bc)^3}-\frac{5\left(b^5c-2ab^4d\right)\tan^{-1}\left(\frac{a\sqrt{d}-bx\sqrt{c+dx^2}+b\sqrt{d}x^2}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

```
[Out] (2*a^2*b^3*c^6 - 6*a^3*b^2*c^5*d + 6*a^4*b*c^4*d^2 - 2*a^5*c^3*d^3 - 10*a*b
^4*c^6*x^2 + 18*a^2*b^3*c^5*d*x^2 + 6*a^3*b^2*c^4*d^2*x^2 - 26*a^4*b*c^3*d^
3*x^2 + 12*a^5*c^2*d^4*x^2 - 15*b^5*c^6*x^4 + 42*a^2*b^3*c^4*d^2*x^4 - 6*a^
3*b^2*c^3*d^3*x^4 - 84*a^4*b*c^2*d^4*x^4 + 48*a^5*c*d^5*x^4 - 30*b^5*c^5*d*
x^6 + 30*a*b^4*c^4*d^2*x^6 + 38*a^2*b^3*c^3*d^3*x^6 - 84*a^3*b^2*c^2*d^4*x^
6 - 16*a^4*b*c*d^5*x^6 + 32*a^5*d^6*x^6 - 15*b^5*c^4*d^2*x^8 + 20*a*b^4*c^3
*d^3*x^8 + 12*a^2*b^3*c^2*d^4*x^8 - 64*a^3*b^2*c*d^5*x^8 + 32*a^4*b*d^6*x^8
)/(6*a^3*c^4*(-(b*c) + a*d)^3*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (5*(b^5*
c - 2*a*b^4*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sq
rt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2)*(b*c - a*d)^(7/2))
```

fricas [B] time = 5.63, size = 1890, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5*c^
5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*b^5
*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a
*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2
 + a^2)) + 4*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2*c^5*d^2 - 8*a^6*
b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5*c^4*d^3 + 8*a^3*
b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a^6*b*d^7)*x^8 - 2
*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + 61*a^4*b^3*c^3*
d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^6 - 3*(5*a*b^6*c^
7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*d^3 + 26*a^5*b^2*
c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*b^5*c^7 - 14*a^3*
b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a
^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^5*b^4*c^7*d^3 + 6
*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9 + (2*a^4*b^5*c^9*
```

```
d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*d^4 - 2*a^8*b*c^5
*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d - 2*a^6*b^3*c^8*d
^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)*x^5 + (a^5*b^4*c^
10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d^3 + a^9*c^6*d^4)*x
^3), 1/12*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5
*c^5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*
b^5*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c -
 a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2
*c^5*d^2 - 8*a^6*b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5
*c^4*d^3 + 8*a^3*b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a
^6*b*d^7)*x^8 - 2*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3
+ 61*a^4*b^3*c^3*d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^
6 - 3*(5*a*b^6*c^7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*
d^3 + 26*a^5*b^2*c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*
b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^
6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^
5*b^4*c^7*d^3 + 6*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9
+ (2*a^4*b^5*c^9*d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*
d^4 - 2*a^8*b*c^5*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d
- 2*a^6*b^3*c^8*d^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)*
x^5 + (a^5*b^4*c^10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d^3
 + a^9*c^6*d^4)*x^3)]
```

giac [B] time = 6.41, size = 789, normalized size = 2.18

$$\frac{\left(\frac{2\left(7b^4c^7d^6-25ab^3c^6d^7+33a^2b^2c^5d^8-19a^3bc^4d^9+4a^4c^3d^{10}\right)x^2}{b^6c^{13}d-6ab^5c^{12}d^2+15a^2b^4c^{11}d^3-20a^3b^3c^{10}d^4+15a^4b^2c^9d^5-6a^5bc^8d^6+a^6c^7d^7}+\frac{3\left(5b^4c^8d^5-18ab^3c^7d^6+24a^2b^2c^6d^7-14a^3bc^5d^8+3a^4c^4d^9\right)}{b^6c^{13}d-6ab^5c^{12}d^2+15a^2b^4c^{11}d^3-20a^3b^3c^{10}d^4+15a^4b^2c^9d^5-6a^5bc^8d^6+a^6c^7d^7}\right)x}{3\left(dx^2+c\right)^{\frac{3}{2}}}-\frac{5\left(b^5c\sqrt{d}-2ab^4d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3\right)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^5c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^4d^{\frac{3}{2}}-b^5c^2\sqrt{d}}{\left(a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3\right)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)}-\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}-9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2acd^{\frac{3}{2}}+3bc^3\sqrt{d}+4ac^2d^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*(7*b^4*c^7*d^6 - 25*a*b^3*c^6*d^7 + 33*a^2*b^2*c^5*d^8 - 19*a^3*b*c^
4*d^9 + 4*a^4*c^3*d^10)*x^2/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c^1
1*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6*c^
7*d^7) + 3*(5*b^4*c^8*d^5 - 18*a*b^3*c^7*d^6 + 24*a^2*b^2*c^6*d^7 - 14*a^3*
b*c^5*d^8 + 3*a^4*c^4*d^9)/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c^11
*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6*c^7
*d^7))*x/(d*x^2 + c)^(3/2) - 5/2*(b^5*c*sqrt(d) - 2*a*b^4*d^(3/2))*arctan(1
/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2
))/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b*c*d
- a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^5*c*sqrt(d) - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*b^4*d^(3/2) - b^5*c^2*sqrt(d))/((a^3*b^3*c^3 - 3*a^
4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b -
 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
```

```
a*d + b*c^2)) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b
*c^2*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt
(d) + 4*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3*c^3)
```

maple [B] time = 0.02, size = 2623, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x)
```

```
[Out] 5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/
2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x+(-a*b)^(
1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x+(-
a*b)^(1/2)/b))+8/3*b/a^3*d/c^2*x/(d*x^2+c)^(3/2)+16/3*b/a^3*d/c^3*x/(d*x^2+
c)^(1/2)-5/12*b^2/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4*b^3/a^3*(-a*b)^(
1/2)*d/(a*d-b*c)^3/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)+5/12*b^2/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^2/((x-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-5/4*
b^3/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^3/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)
^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))-1/3/a^2/c/x^3/(d*x^2
+c)^(3/2)-5/4*b^3/a^3/(a*d-b*c)^2/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*
(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+5/12*b^2/a^2*d^2/(a*d-b*c)^2/
c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b
)^(3/2)*x+5/6*b^2/a^2*d^2/(a*d-b*c)^2/c^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a^2*d^2/(a*d-b*c)
^3/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)*x+5/6*b^2/a^2*d^2/(a*d-b*c)^2/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a^2*d^2/(a*d-b
*c)^3/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2)*x-5/4*b^3/a^3*(-a*b)^(1/2)*d/(a*d-b*c)^3/(-(a*d-b*c)/b)^(1/2)
*ln((-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*
c)/b)^(1/2))/(x+(-a*b)^(1/2)/b))+1/12*b^2/a^3*d/(a*d-b*c)/c/((x+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+1/6*b^2
/a^3*d/(a*d-b*c)/c^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)
/b)/b*d-(a*d-b*c)/b)^(1/2)*x-5/4*b^3/a^3/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*d*x+1/12*b^2/a
^3*d/(a*d-b*c)/c/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/
b*d-(a*d-b*c)/b)^(3/2)*x+1/6*b^2/a^3*d/(a*d-b*c)/c^2/((x-(-a*b)^(1/2)/b)^2*
```

```
d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)*x-1/4*b^2/a^3/(a
*d-b*c)/(x+(-a*b)^(1/2)/b)/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/12*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x+(-
a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)-
5/4*b^4/a^3/(-a*b)^(1/2)/(a*d-b*c)^2/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*b^2/a^3/(a*d-b*c)/(x-(-a*b)^
(1/2)/b)/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(3/2)-5/12*b^3/a^3/(-a*b)^(1/2)/(a*d-b*c)/((x-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)+5/4*b^4/a^3/(-a*b)
^(1/2)/(a*d-b*c)^2/((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2)+2*b/a^3/c/x/(d*x^2+c)^(3/2)+2/a^2*d/c^2/x/(d*x^2+c
)^(3/2)+8/3/a^2*d^2/c^3*x/(d*x^2+c)^(3/2)+16/3/a^2*d^2/c^4*x/(d*x^2+c)^(1/2
)+5/12*b^2/a^2*d^2/(a*d-b*c)^2/c/((x+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(3/2)*x+5/4*b^3/a^3*(-a*b)^(1/2)*d/(a*d-b*
c)^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(b\,x^2+a\right)^2\left(d\,x^2+c\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.767 $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}}{}$$

Rubi [A] time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+2abcd+5b^2c^2\right)}{16b^2d^3}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{24b^2d^2}+\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b^2
*d^3) - ((5*b*c + 3*a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b^2*d^2) +
(x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)*(5*b^2*c^2 +
 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])])/(16*b^(5/2)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{6bd}$$

$$= -\frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{6bd} + \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\mathrm{S}}{16b}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+\right.}{}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+\right.}{}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+\right.}{}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2\left(a+\right.}{}$$

Mathematica [A] time = 0.45, size = 187, normalized size = 0.89

$$\frac{-3\left(a^2d^2+2abcd+5b^2c^2\right)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(3a^2d^2-2abd\left(dx^2-2c\right)+b^2\left(-15c^2+10cdx^2-8d^2x^4\right)\right)}{48b^3d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(3*a^2*d^2 - 2*a*b*d*(-2*c + d*x^2
) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - 3*(b*c - a*d)^(3/2)*(5*b^2*c
^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d
]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*b^3*d^(7/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [F] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2], x]
```

fricas [A] time = 1.96, size = 442, normalized size = 2.11

$$\left[-\frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}\right)-4\left(8b^3d^3x^4+15b^3c^2d-4ab^2cd^2-3a^2bd^3-2\left(5b^3cd^2-ab^2d^3\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{192b^3d^4}, \frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)+2\left(8b^3d^3x^4+15b^3c^2d-4ab^2cd^2-3a^2bd^3-2\left(5b^3cd^2-ab^2d^3\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{96b^3d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2
 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4
*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d
^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4), 1/96*(3*(5
*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*
b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^
4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c^2*d -
 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c))/(b^3*d^4)]
```

giac [A] time = 0.48, size = 226, normalized size = 1.08

$$\frac{\left(\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{b^3d}-\frac{5b^7cd^3+7ab^6d^4}{b^9d^5}\right)+\frac{3\left(5b^8c^2d^2+2ab^7cd^3+a^2b^6d^4\right)}{b^9d^5}\right)+\frac{3\left(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3\right)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^3}\right)b}{48\left|b\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b^3*d) - (5*b^7*c*d^3 + 7*a*b^6*d^4)/(b^9*d^5)) + 3*(5*b^8*
c^2*d^2 + 2*a*b^7*c*d^3 + a^2*b^6*d^4)/(b^9*d^5)) + 3*(5*b^3*c^3 - 3*a*b^2*
c^2*d - a^2*b*c*d^2 - a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b/abs(b)
```

maple [B] time = 0.08, size = 532, normalized size = 2.55

$$\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\left(16\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,b^2d^2x^4+3a^3d^3\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3a^2bcd^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+9ab^2c^2d\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-15b^3c^3\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+4\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,abd^2x^2-20\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,b^2cdx^2-6\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,a^2d^2-8\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,abcd+30\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,b^2c^2\right)}{96\sqrt{x^4bd+x^2ad+bcx^2+ac}\,\sqrt{bd}\,b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*x^4*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*d^2
*(b*d)^(1/2)-20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)
+3*d^3*ln(1/2*(2*x^2*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*x^2*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+9*ln(1/2*(2*x^2*b*d+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c
^2*b^2*d-15*b^3*ln(1/2*(2*x^2*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^
2*d^2*(b*d)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)
+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/d^3/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)/b^2/(b*d)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 54.78, size = 993, normalized size = 4.75

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,b^{5/2}\,d^{7/2}}-\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(\frac{a^3\,b^3\,d^3}{8}+\frac{a^2\,b^4\,c\,d^2}{8}+\frac{3\,a\,b^5\,c^2\,d}{8}-\frac{5\,b^6\,c^3}{8}\right)}{d^9\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^5\,\left(\frac{19\,a^3\,b\,d^3}{4}+\frac{275\,a^2\,b^2\,c\,d^2}{4}+\frac{313\,a\,b^3\,c^2\,d}{4}+\frac{33\,b^4\,c^3}{4}\right)}{d^7\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^5}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^7\,\left(\frac{19\,a^3\,d^3}{4}+\frac{275\,a^2\,b\,c\,d^2}{4}+\frac{313\,a\,b^2\,c^2\,d}{4}+\frac{33\,b^3\,c^3}{4}\right)}{d^6\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^7}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(\frac{17\,a^3\,b^2\,d^3}{24}+\frac{91\,a^2\,b^3\,c\,d^2}{8}+\frac{17\,a\,b^4\,c^2\,d}{8}-\frac{85\,b^5\,c^3}{24}\right)}{d^8\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^{11}\,\left(\frac{a^3\,d^3}{8}+\frac{a^2\,b\,c\,d^2}{8}+\frac{3\,a\,b^2\,c^2\,d}{8}-\frac{5\,b^3\,c^3}{8}\right)}{b^2\,d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^{11}}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^9\,\left(\frac{17\,a^3\,d^3}{24}+\frac{91\,a^2\,b\,c\,d^2}{8}+\frac{17\,a\,b^2\,c^2\,d}{8}-\frac{85\,b^3\,c^3}{24}\right)}{b\,d^5\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^9}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^8\,\left(16\,d\,a^2+48\,b\,c\,a\right)}{d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^8}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4\,\left(16\,d\,a^2\,b^2+48\,c\,a\,b^3\right)}{d^6\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6\,\left(32\,a^2\,b\,d^2+\frac{352\,a\,b^2\,c\,d}{3}+64\,b^3\,c^2\right)}{d^6\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}}{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^{12}}{{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^{12}}+\frac{b^6}{d^6}-\frac{6\,b^5\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d^5\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}+\frac{15\,b^4\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4}{d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}-\frac{20\,b^3\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6}{d^3\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}+\frac{15\,b^2\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^8}{d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^8}-\frac{6\,b\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^{10}}{d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] (atanh((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2)
 - c^(1/2))))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8*b^(5/2)*d^(7
/2)) - ((((a + b*x^2)^(1/2) - a^(1/2))*((a^3*b^3*d^3)/8 - (5*b^6*c^3)/8 + (
a^2*b^4*c*d^2)/8 + (3*a*b^5*c^2*d)/8))/(d^9*((c + d*x^2)^(1/2) - c^(1/2)))
 - (((a + b*x^2)^(1/2) - a^(1/2))^5*((33*b^4*c^3)/4 + (19*a^3*b*d^3)/4 + (27
5*a^2*b^2*c*d^2)/4 + (313*a*b^3*c^2*d)/4))/(d^7*((c + d*x^2)^(1/2) - c^(1/2
))^5) - (((a + b*x^2)^(1/2) - a^(1/2))^7*((19*a^3*d^3)/4 + (33*b^3*c^3)/4 +
 (313*a*b^2*c^2*d)/4 + (275*a^2*b*c*d^2)/4))/(d^6*((c + d*x^2)^(1/2) - c^(1
/2))^7) - (((a + b*x^2)^(1/2) - a^(1/2))^3*((17*a^3*b^2*d^3)/24 - (85*b^5*c
^3)/24 + (91*a^2*b^3*c*d^2)/8 + (17*a*b^4*c^2*d)/8))/(d^8*((c + d*x^2)^(1/2
) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^11*((a^3*d^3)/8 - (5*b^3*c
^3)/8 + (3*a*b^2*c^2*d)/8 + (a^2*b*c*d^2)/8))/(b^2*d^4*((c + d*x^2)^(1/2) -
```

```
 c^(1/2))^11) - (((a + b*x^2)^(1/2) - a^(1/2))^9*((17*a^3*d^3)/24 - (85*b^3
*c^3)/24 + (17*a*b^2*c^2*d)/8 + (91*a^2*b*c*d^2)/8))/(b*d^5*((c + d*x^2)^(1
/2) - c^(1/2))^9) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^8*(16*a^
2*d + 48*a*b*c))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^8) + (a^(1/2)*c^(1/2)*(
(a + b*x^2)^(1/2) - a^(1/2))^4*(16*a^2*b^2*d + 48*a*b^3*c))/(d^6*((c + d*x^
2)^(1/2) - c^(1/2))^4) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^6*(
64*b^3*c^2 + 32*a^2*b*d^2 + (352*a*b^2*c*d)/3))/(d^6*((c + d*x^2)^(1/2) - c
^(1/2))^6))/(((a + b*x^2)^(1/2) - a^(1/2))^12/((c + d*x^2)^(1/2) - c^(1/2))
^12 + b^6/d^6 - (6*b^5*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^5*((c + d*x^2)^(
1/2) - c^(1/2))^2) + (15*b^4*((a + b*x^2)^(1/2) - a^(1/2))^4)/(d^4*((c + d*
x^2)^(1/2) - c^(1/2))^4) - (20*b^3*((a + b*x^2)^(1/2) - a^(1/2))^6)/(d^3*((
c + d*x^2)^(1/2) - c^(1/2))^6) + (15*b^2*((a + b*x^2)^(1/2) - a^(1/2))^8)/(
d^2*((c + d*x^2)^(1/2) - c^(1/2))^8) - (6*b*((a + b*x^2)^(1/2) - a^(1/2))^1
0)/(d*((c + d*x^2)^(1/2) - c^(1/2))^10))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.768 $\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+3bc)}{8bd^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd}$$

Rubi [A] time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+3bc)}{8bd^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b*d^2) + ((a + b*x^2)^(
3/2)*Sqrt[c + d*x^2])/(4*b*d) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(3/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{8bd}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \right.}{16bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \right.}{8b^2d^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{((bc-ad)(3bc+ad))\,\text{Subst}\left(\int \right.}{8b^2d^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4bd} + \frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{b}}\right.}{8b^{3/2}d^{5/2}}$$

Mathematica [A] time = 0.31, size = 138, normalized size = 1.01

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(ad-3bc+2bdx^2\right)+(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{8b^2d^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(-3*b*c + a*d + 2*b*d*x^2) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^2*d^(5/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 1.09, size = 227, normalized size = 1.66

$$\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8b^{3/2}d^{5/2}} - \frac{\sqrt{c+dx^2}\left(-\frac{a^2bd^2\left(c+dx^2\right)}{a+bx^2}-a^2d^3+\frac{3b^3c^2\left(c+dx^2\right)}{a+bx^2}-\frac{2ab^2cd\left(c+dx^2\right)}{a+bx^2}+6abcd^2-5b^2c^2d\right)}{8bd^2\sqrt{a+bx^2}\left(\frac{b\left(c+dx^2\right)}{a+bx^2}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] -1/8*(Sqrt[c + d*x^2]*(-5*b^2*c^2*d + 6*a*b*c*d^2 - a^2*d^3 + (3*b^3*c^2*(c
 + d*x^2))/(a + b*x^2) - (2*a*b^2*c*d*(c + d*x^2))/(a + b*x^2) - (a^2*b*d^2
*(c + d*x^2))/(a + b*x^2)))/(b*d^2*Sqrt[a + b*x^2]*(-d + (b*(c + d*x^2))/(a
 + b*x^2))^2) + ((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(8*b^(3/2)*d^(5/2))
```

fricas [A] time = 1.42, size = 334, normalized size = 2.44

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)-4\left(2\,b^2d^2x^2-3\,b^2cd+abd^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,b^2d^3},-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)-2\left(2\,b^2d^2x^2-3\,b^2cd+abd^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2
*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c
 + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(2*b^2*d^2*x^2 - 3*b
^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3), -1/16*((3*b^2
*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/(b^2*d^3)]
```

giac [A] time = 0.56, size = 159, normalized size = 1.16

$$\frac{\left(\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}\left(\frac{2\left(bx^2+a\right)}{b^2d}-\frac{3\,b^3cd+ab^2d^2}{b^4d^3}\right)-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b^2*d) - (3*b^3*c*d + a*b^2*d^2)/(b^4*d^3)) - (3*b^2*c^2 - 2*a*b*c*d - a^2*
d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d)))/(sqrt(b*d)*b*d^2))*b/abs(b)
```

maple [B] time = 0.02, size = 339, normalized size = 2.47

$$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(a^2d^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2abcd\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-3b^2c^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-4\sqrt{bd}\,\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,bd\,x^2-2\sqrt{bd}\,\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,ad+6\sqrt{bd}\,\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,bc\right)}{16\sqrt{x^4bd+x^2ad+bc\,x^2+ac}\,\sqrt{bd}\,b\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*x^2*b*d+d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2+2*ln(1/2*(2*b*d*x^2+a*d+b*c+2
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*c*b*d-3*b^
2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2))/(b*d)^(1/2))*c^2-2*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d+
6*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)/d^2/b/(b*d)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 25.18, size = 639, normalized size = 4.66

$$\frac{\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)\left(\frac{a^2 b^2 d^2}{4}+\frac{a b^3 c d}{2}-\frac{3 b^4 c^2}{4}\right)}{d^6\left(\sqrt{d x^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^3\left(\frac{7 a^2 b d^2}{4}+\frac{23 a b^2 c d}{2}+\frac{11 b^3 c^2}{4}\right)}{d^5\left(\sqrt{d x^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^5\left(\frac{7 a^2 d^2}{4}+\frac{23 a b c d}{2}+\frac{11 b^2 c^2}{4}\right)}{d^4\left(\sqrt{d x^2+c}-\sqrt{c}\right)^5}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^7\left(\frac{a^2 d^2}{4}+\frac{a b c d}{2}-\frac{3 b^2 c^2}{4}\right)}{b d^3\left(\sqrt{d x^2+c}-\sqrt{c}\right)^7}-\frac{4 a^{3/2}\sqrt{c}\left(\sqrt{b x^2+a}-\sqrt{a}\right)^6}{d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^6}-\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{b x^2+a}-\sqrt{a}\right)^4\left(16 c b^2+8 a d b\right)}{d^4\left(\sqrt{d x^2+c}-\sqrt{c}\right)^4}-\frac{4 a^{3/2} b^2\sqrt{c}\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2}{d^4\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^8}{\left(\sqrt{d x^2+c}-\sqrt{c}\right)^8}+\frac{b^4}{d^4}-\frac{4 b^3\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2}{d^3\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}+\frac{6 b^2\left(\sqrt{b x^2+a}-\sqrt{a}\right)^4}{d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^4}-\frac{4 b\left(\sqrt{b x^2+a}-\sqrt{a}\right)^6}{d\left(\sqrt{d x^2+c}-\sqrt{c}\right)^6}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{d}\left(\sqrt{b x^2+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d x^2+c}-\sqrt{c}\right)}\right)\left(a d-b c\right)\left(a d+3 b c\right)}{4 b^{3/2} d^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*((a^2*b^2*d^2)/4 - (3*b^4*c^2)/4 + (a*b^3*c
*d)/2))/(d^6*((c + d*x^2)^(1/2) - c^(1/2))) + (((a + b*x^2)^(1/2) - a^(1/2)
)^3*((11*b^3*c^2)/4 + (7*a^2*b*d^2)/4 + (23*a*b^2*c*d)/2))/(d^5*((c + d*x^2
)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^5*((7*a^2*d^2)/4 + (
11*b^2*c^2)/4 + (23*a*b*c*d)/2))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^5) + ((
(a + b*x^2)^(1/2) - a^(1/2))^7*((a^2*d^2)/4 - (3*b^2*c^2)/4 + (a*b*c*d)/2))
/(b*d^3*((c + d*x^2)^(1/2) - c^(1/2))^7) - (4*a^(3/2)*c^(1/2)*((a + b*x^2)^
(1/2) - a^(1/2))^6)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^6) - (a^(1/2)*c^(1/2
)*((a + b*x^2)^(1/2) - a^(1/2))^4*(16*b^2*c + 8*a*b*d))/(d^4*((c + d*x^2)^(
1/2) - c^(1/2))^4) - (4*a^(3/2)*b^2*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2
)/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^8/(
(c + d*x^2)^(1/2) - c^(1/2))^8 + b^4/d^4 - (4*b^3*((a + b*x^2)^(1/2) - a^(1
/2))^2)/(d^3*((c + d*x^2)^(1/2) - c^(1/2))^2) + (6*b^2*((a + b*x^2)^(1/2) -
 a^(1/2))^4)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^4) - (4*b*((a + b*x^2)^(1/2
) - a^(1/2))^6)/(d*((c + d*x^2)^(1/2) - c^(1/2))^6)) - (atanh((d^(1/2)*((a
```

```
+ b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d -
b*c)*(a*d + 3*b*c))/(4*b^(3/2)*d^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.769 $\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 63, 217, 206}

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{4d}\\
&= \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{2bd}\\
&= \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2bd}\\
&= \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2\sqrt{b}\,d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 116, normalized size = 1.35

$$
\frac{b\sqrt{d}\,\sqrt{a+bx^2}\,\left(c+dx^2\right)-(bc-ad)^{3/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{2bd^{3/2}\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2) - (b*c - a*d)^(3/2)*Sqrt[(b*(c + d*x
^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(2*b*
d^(3/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.50, size = 117, normalized size = 1.36

$$\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2\sqrt{b}\,d^{3/2}}+\frac{\sqrt{c+dx^2}\,(ad-bc)}{2d\sqrt{a+bx^2}\left(d-\frac{b\left(c+dx^2\right)}{a+bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((-(b*c) + a*d)*Sqrt[c + d*x^2])/(2*d*Sqrt[a + b*x^2]*(d - (b*(c + d*x^2))/
(a + b*x^2))) + ((-(b*c) + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*
Sqrt[a + b*x^2])])/(2*Sqrt[b]*d^(3/2))
```

fricas [A] time = 1.62, size = 259, normalized size = 3.01

$$\left[\frac{4\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)}{8\,bd^2},\ \frac{2\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd+(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)}{4\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*
(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b*d^2)
, 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arcta
n(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b
^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b*d^2)]
```

giac [A] time = 0.55, size = 106, normalized size = 1.23

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}}{bd}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b*((b*c - a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*
sqrt(b*x^2 + a)/(b*d))/abs(b)
```

maple [B] time = 0.01, size = 198, normalized size = 2.30

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(ad\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+x^{2}ad+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-bc\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+x^{2}ad+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{x^{4}bd+x^{2}ad+bc\,x^{2}+ac}\,\sqrt{bd}\right)}{4\sqrt{x^{4}bd+x^{2}ad+bc\,x^{2}+ac}\,\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(a*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*d-b*ln(1/2*(2*b*d*x^2
+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)/d/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 5.00, size = 280, normalized size = 3.26

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\,\left(a\,d+b\,c\right)}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(c\,b^2+a\,d\,b\right)}{d^3\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{4\,\sqrt{a}\,b\,\sqrt{c}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}+\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(a\,d-b\,c\right)}{\sqrt{b}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))^3*(a*d + b*c))/(d^2*((c + d*x^2)^(1/2) - c^
(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))*(b^2*c + a*b*d))/(d^3*((c + d*x^
```

```
2)^(1/2) - c^(1/2))) - (4*a^(1/2)*b*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2
)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^4/(
(c + d*x^2)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^2)^(1/2) - a^(1/2
))^2)/(d*((c + d*x^2)^(1/2) - c^(1/2))^2)) + (atanh((d^(1/2)*((a + b*x^2)^(
1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(
1/2)*d^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.770 $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

Rubi [**A**] time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sq
rt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2
])])/Sqrt[d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
```

```
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^2\right)+\frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= a\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)+\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^2}\right)\\
&= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)\\
&= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 129, normalized size = 1.40

$$
\frac{\sqrt{bc-ad}\,\sqrt{\frac{b(c+dx^2)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,\sqrt{c+dx^2}}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTanh[(S
qrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c]
```

IntegrateAlgebraic [A] time = 0.64, size = 92, normalized size = 1.00

$$
\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/Sq
rt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2
])])/Sqrt[d]
```

fricas [B] time = 1.88, size = 777, normalized size = 8.45

$$\left[\frac{1}{4}\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2bd^2x^2+bcd+ad^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right)+\frac{1}{4}\sqrt{\frac{a}{c}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(2ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right),-\frac{1}{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^4+abc+\left(b^2c+abd\right)x^2\right)}\right)+\frac{1}{4}\sqrt{\frac{a}{c}}\log\left(\frac{\left(b^2c^2+6abcd+a^2d^2\right)x^4+8a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(2ac^2+\left(bc^2+acd\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right),\frac{1}{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{a}{c}}}{2\left(abdx^4+a^2c+\left(abc+a^2d\right)x^2\right)}\right)+\frac{1}{4}\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2bd^2x^2+bcd+ad^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right),\frac{1}{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{a}{c}}}{2\left(abdx^4+a^2c+\left(abc+a^2d\right)x^2\right)}\right)-\frac{1}{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^4+abc+\left(b^2c+abd\right)x^2\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*sqrt(b/d)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^
4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^
2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), -1/2*sqrt(-b/d)*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2
*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 +
 (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), 1/2*
sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 1/4*sqrt(b/d)*
log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x
^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b
/d)), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 1/2*
sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.03, size = 177, normalized size = 1.92

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(-\sqrt{bd}\,a\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+\sqrt{ac}\,b\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)\right)}{2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(b*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)-a*ln((x^2
*a*d+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*
(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 19.83, size = 4638, normalized size = 50.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] (2*atanh((20*a*b^7*(b*d)^(1/2))/(34*a^(1/2)*b^8*c^(1/2) - (33*a^(3/2)*b^7*d
)/c^(1/2) - (54*b^8*c*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c
^(1/2)) + (25*b^9*c^(3/2))/(2*a^(1/2)*d) + (4*a^(5/2)*b^6*d^2)/c^(3/2) - (1
8*b^10*c^(5/2))/(a^(3/2)*d^2) + (a^(7/2)*b^5*d^3)/(2*c^(5/2)) + (20*a*b^7*d
*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) + (10*a^2*b^6
*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (23
*b^9*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d*((c + d*x^2)^(1/2) - c^(1/2)))
 - (3*a^3*b^5*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) -
c^(1/2))) + (4*b^10*c^3*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^2*((c + d*x^2
)^(1/2) - c^(1/2)))) - (54*b^8*(b*d)^(1/2))/((25*b^9*c^(1/2))/(2*a^(1/2)) +
 (34*a^(1/2)*b^8*d)/c^(1/2) - (54*b^8*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c
+ d*x^2)^(1/2) - c^(1/2)) - (33*a^(3/2)*b^7*d^2)/c^(3/2) - (18*b^10*c^(3/2)
)/(a^(3/2)*d) + (4*a^(5/2)*b^6*d^3)/c^(5/2) + (a^(7/2)*b^5*d^4)/(2*c^(7/2))
 + (23*b^9*c*((a + b*x^2)^(1/2) - a^(1/2)))/(a*((c + d*x^2)^(1/2) - c^(1/2)
)) + (10*a^2*b^6*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2)
 - c^(1/2))) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d*((c + d*x^
2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^4*((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*(
(c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*d^2*((a + b*x^2)^(1/2) - a^(1/2))
)/(c*((c + d*x^2)^(1/2) - c^(1/2)))) + (23*b^9*c*(b*d)^(1/2))/((25*a^(1/2)*
b^9*c^(1/2)*d)/2 - (18*b^10*c^(3/2))/a^(1/2) + (34*a^(3/2)*b^8*d^2)/c^(1/2)
```

```
 - (33*a^(5/2)*b^7*d^3)/c^(3/2) + (4*a^(7/2)*b^6*d^4)/c^(5/2) + (a^(9/2)*b^
5*d^5)/(2*c^(7/2)) - (54*a*b^8*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x
^2)^(1/2) - c^(1/2)) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a*((c +
d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c +
 d*x^2)^(1/2) - c^(1/2)) + (20*a^2*b^7*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(
c*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^3*b^6*d^4*((a + b*x^2)^(1/2) - a^(
1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^4*b^5*d^5*((a + b*x^2)^(1
/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2)))) + (10*a^2*b^6*(b*d)^(1
/2))/((4*a^(5/2)*b^6*d)/c^(1/2) - 33*a^(3/2)*b^7*c^(1/2) + (34*a^(1/2)*b^8*
c^(3/2))/d + (25*b^9*c^(5/2))/(2*a^(1/2)*d^2) + (a^(7/2)*b^5*d^2)/(2*c^(3/2
)) - (18*b^10*c^(7/2))/(a^(3/2)*d^3) + (10*a^2*b^6*d*((a + b*x^2)^(1/2) - a
^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (54*b^8*c^2*((a + b*x^2)^(1/2) - a
^(1/2)))/(d*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*c*((a + b*x^2)^(1/2)
 - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (3*a^3*b^5*d^2*((a + b*x^2)^(1
/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c^3*((a + b*x^2
)^(1/2) - a^(1/2)))/(a*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^4*((a
 + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^3*((c + d*x^2)^(1/2) - c^(1/2)))) - (3*a
^3*b^5*(b*d)^(1/2))/(4*a^(5/2)*b^6*c^(1/2) + (a^(7/2)*b^5*d)/(2*c^(1/2)) -
(33*a^(3/2)*b^7*c^(3/2))/d + (34*a^(1/2)*b^8*c^(5/2))/d^2 + (25*b^9*c^(7/2)
)/(2*a^(1/2)*d^3) - (18*b^10*c^(9/2))/(a^(3/2)*d^4) + (10*a^2*b^6*c*((a + b
*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (3*a^3*b^5*d*((a +
b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (54*b^8*c^3*((a +
b*x^2)^(1/2) - a^(1/2)))/(d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c^4*
((a + b*x^2)^(1/2) - a^(1/2)))/(a*d^3*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b
^10*c^5*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^4*((c + d*x^2)^(1/2) - c^(1/2
))) + (20*a*b^7*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) -
c^(1/2)))) + (4*b^10*c^2*(b*d)^(1/2))/((25*a^(3/2)*b^9*c^(1/2)*d^2)/2 - 18*
a^(1/2)*b^10*c^(3/2)*d + (34*a^(5/2)*b^8*d^3)/c^(1/2) - (33*a^(7/2)*b^7*d^4
)/c^(3/2) + (4*a^(9/2)*b^6*d^5)/c^(5/2) + (a^(11/2)*b^5*d^6)/(2*c^(7/2)) +
(4*b^10*c^2*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))
- (54*a^2*b^8*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/
2)) + (20*a^3*b^7*d^4*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2)
- c^(1/2))) + (10*a^4*b^6*d^5*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x
^2)^(1/2) - c^(1/2))) - (3*a^5*b^5*d^6*((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*
((c + d*x^2)^(1/2) - c^(1/2))) + (23*a*b^9*c*d^2*((a + b*x^2)^(1/2) - a^(1/
2)))/((c + d*x^2)^(1/2) - c^(1/2))) + (34*a^(1/2)*b^7*(b*d)^(1/2)*((a + b*x
^2)^(1/2) - a^(1/2)))/(c^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))*((34*a^(1/2)*b
^8)/c^(1/2) - (54*b^8*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c
^(1/2)) - (33*a^(3/2)*b^7*d)/c^(3/2) + (25*b^9*c^(1/2))/(2*a^(1/2)*d) - (18
*b^10*c^(3/2))/(a^(3/2)*d^2) + (4*a^(5/2)*b^6*d^2)/c^(5/2) + (a^(7/2)*b^5*d
^3)/(2*c^(7/2)) + (10*a^2*b^6*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c +
 d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(
c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/
2)))/(a^2*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*d*((a + b*x^2)^(1/
2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c*((a + b*x^2)^(
```

```
1/2) - a^(1/2)))/(a*d*((c + d*x^2)^(1/2) - c^(1/2))))) - (33*a^(3/2)*b^6*(b
*d)^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(c^(3/2)*((c + d*x^2)^(1/2) - c^(1
/2))*((4*a^(5/2)*b^6*d)/c^(5/2) - (33*a^(3/2)*b^7)/c^(3/2) + (34*a^(1/2)*b^
8)/(c^(1/2)*d) + (25*b^9*c^(1/2))/(2*a^(1/2)*d^2) - (18*b^10*c^(3/2))/(a^(3
/2)*d^3) + (a^(7/2)*b^5*d^2)/(2*c^(7/2)) - (54*b^8*((a + b*x^2)^(1/2) - a^(
1/2)))/(d*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*((a + b*x^2)^(1/2) - a
^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^2*((a + b*x^2)^(1
/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^2*((a + b*x
^2)^(1/2) - a^(1/2)))/(a^2*d^3*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^2*b^6
*d*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) + (23
*b^9*c*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d^2*((c + d*x^2)^(1/2) - c^(1/2)))
)) + (25*b^8*c^(1/2)*(b*d)^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(2*a^(1/2)*
((c + d*x^2)^(1/2) - c^(1/2))*((25*b^9*c^(1/2))/(2*a^(1/2)) + (34*a^(1/2)*b
^8*d)/c^(1/2) - (54*b^8*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2)
 - c^(1/2)) - (33*a^(3/2)*b^7*d^2)/c^(3/2) - (18*b^10*c^(3/2))/(a^(3/2)*d)
 + (4*a^(5/2)*b^6*d^3)/c^(5/2) + (a^(7/2)*b^5*d^4)/(2*c^(7/2)) + (23*b^9*c*(
(a + b*x^2)^(1/2) - a^(1/2)))/(a*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^2*b
^6*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) +
 (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d*((c + d*x^2)^(1/2) - c^(
1/2))) - (3*a^3*b^5*d^4*((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1
/2) - c^(1/2))) + (20*a*b^7*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x
^2)^(1/2) - c^(1/2))))) + (4*a^(5/2)*b^5*(b*d)^(1/2)*((a + b*x^2)^(1/2) - a
^(1/2)))/(c^(5/2)*((c + d*x^2)^(1/2) - c^(1/2))*((4*a^(5/2)*b^6)/c^(5/2) +
(a^(7/2)*b^5*d)/(2*c^(7/2)) + (34*a^(1/2)*b^8)/(c^(1/2)*d^2) + (25*b^9*c^(1
/2))/(2*a^(1/2)*d^3) - (33*a^(3/2)*b^7)/(c^(3/2)*d) - (18*b^10*c^(3/2))/(a^
(3/2)*d^4) - (54*b^8*((a + b*x^2)^(1/2) - a^(1/2)))/(d^2*((c + d*x^2)^(1/2)
 - c^(1/2))) + (10*a^2*b^6*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)
^(1/2) - c^(1/2))) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^4*((
c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*((a + b*x^2)^(1/2) - a^(1/2)))/(c*
d*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d*((a + b*x^2)^(1/2) - a^(1/2
)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c*((a + b*x^2)^(1/2) - a^
(1/2)))/(a*d^3*((c + d*x^2)^(1/2) - c^(1/2))))) - (18*b^9*c^(3/2)*(b*d)^(1/
2)*((a + b*x^2)^(1/2) - a^(1/2)))/(a^(3/2)*((c + d*x^2)^(1/2) - c^(1/2))*((
25*b^9*c^(1/2)*d)/(2*a^(1/2)) - (18*b^10*c^(3/2))/a^(3/2) + (34*a^(1/2)*b^8
*d^2)/c^(1/2) - (33*a^(3/2)*b^7*d^3)/c^(3/2) + (4*a^(5/2)*b^6*d^4)/c^(5/2)
 + (a^(7/2)*b^5*d^5)/(2*c^(7/2)) - (54*b^8*d^2*((a + b*x^2)^(1/2) - a^(1/2))
)/((c + d*x^2)^(1/2) - c^(1/2)) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2))
)/(a^2*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^2*b^6*d^4*((a + b*x^2)^(1/2)
 - a^(1/2)))/(c^2*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^5*((a + b*x^
2)^(1/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (23*b^9*c*d*((a
 + b*x^2)^(1/2) - a^(1/2)))/(a*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*d^
3*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2))))) + (a^(
7/2)*b^4*(b*d)^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(2*c^(7/2)*((c + d*x^2)
^(1/2) - c^(1/2))*((a^(7/2)*b^5)/(2*c^(7/2)) + (34*a^(1/2)*b^8)/(c^(1/2)*d^
3) + (25*b^9*c^(1/2))/(2*a^(1/2)*d^4) - (33*a^(3/2)*b^7)/(c^(3/2)*d^2) + (4
```

```
*a^(5/2)*b^6)/(c^(5/2)*d) - (18*b^10*c^(3/2))/(a^(3/2)*d^5) - (54*b^8*((a +
 b*x^2)^(1/2) - a^(1/2)))/(d^3*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*
((a + b*x^2)^(1/2) - a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (10*a^
2*b^6*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*d*((c + d*x^2)^(1/2) - c^(1/2)))
+ (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^5*((c + d*x^2)^(1/2) -
c^(1/2))) + (20*a*b^7*((a + b*x^2)^(1/2) - a^(1/2)))/(c*d^2*((c + d*x^2)^(1
/2) - c^(1/2))) + (23*b^9*c*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d^4*((c + d*x
^2)^(1/2) - c^(1/2))))))*(b*d)^(1/2))/d - (a^(1/2)*log(((a + b*x^2)^(1/2) -
 a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2))) - a^(1/2)*log(((c^(1/2)*(a + b*x^2
)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b*x^2)^(
1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c + d*x^2)^(1/2) - c^(1/
2))))/(2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x*sqrt(c + d*x**2)), x)
```

3.771 $\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

Rubi [A] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 94, 93, 208}

$$-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - ((b*c - a*d)*ArcTanh[(Sqrt[c
]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*c^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4c}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2c}\\
&= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}\,c^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.00

$$
\frac{1}{2}\left(-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,c^{3/2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2)))/2
```

IntegrateAlgebraic [A] time = 0.91, size = 117, normalized size = 1.31

$$
\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2\sqrt{a}\,c^{3/2}} - \frac{\sqrt{c+dx^2}\,(bc-ad)}{2c\sqrt{a+bx^2}\left(c-\frac{a\left(c+dx^2\right)}{a+bx^2}\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*((b*c - a*d)*Sqrt[c + d*x^2])/(c*Sqrt[a + b*x^2]*(c - (a*(c + d*x^2))/
(a + b*x^2))) + ((-(b*c) + a*d)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*
Sqrt[a + b*x^2])])/(2*Sqrt[a]*c^(3/2))
```

fricas [A] time = 1.64, size = 280, normalized size = 3.15

$$\left[-\frac{\sqrt{ac}\,(bc-ad)x^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2+4\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)+4\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{8\,ac^2x^2},\ \frac{\sqrt{-ac}\,(bc-ad)x^2\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\left(abcdx^4+a^2c^2+\left(abc^2+a^2cd\right)x^2\right)}\right)-2\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{4\,ac^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*a*c)/(a*c^2*x^2), 1/4*(sqrt(-a*c)*(b*c - a*d)*x^2*arctan(1/2*((b*c + a*d
)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^
2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c)/
(a*c^2*x^2)]
```

giac [B] time = 2.10, size = 434, normalized size = 4.88

$$-\frac{b\left(\frac{\left(\sqrt{bd}\,b^2c-\sqrt{bd}\,abd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\,\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc}+\frac{2\left(\sqrt{bd}\,b^4c^2-2\,\sqrt{bd}\,ab^3cd+\sqrt{bd}\,a^2b^2d^2-\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^2c-\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2abd\right)}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^4\right)c}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*((sqrt(b*d)*b^2*c - sqrt(b*d)*a*b*d)*arctan(-1/2*(b^2*c + a*b*d - (s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(
-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c) + 2*(sqrt(b*d)*b^4*c^2 - 2*sqrt(b*d)*a*b
^3*c*d + sqrt(b*d)*a^2*b^2*d^2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - sqrt(b*d)*(sqrt(b*x^2 + a)*sq
rt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d)/((b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*
x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^4)*c))/abs(b)
```

maple [B] time = 0.03, size = 207, normalized size = 2.33

$$\frac{\sqrt{b\,x^{2}+a}\;\sqrt{d\,x^{2}+c}\;\left(ad\,x^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\;\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-bc\,x^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\;\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-2\sqrt{ac}\;\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\right)}{4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\;\sqrt{ac}\;c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1
/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a*d-ln((a*d*x^2+b*c*x^2+2
*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*b*c-2*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)/x^2/(a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 6.41, size = 477, normalized size = 5.36

$$\frac{\frac{\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)\left(\frac{c\,b^{2}}{8}+\frac{a\,d\,b}{8}\right)}{\sqrt{a}\;c^{3/2}\,d\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\right)}-\frac{b^{2}}{8\,c\,d}+\frac{\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)^{2}\left(\frac{a^{2}\,d^{2}}{8}-\frac{3\,a\,b\,c\,d}{8}+\frac{b^{2}\,c^{2}}{8}\right)}{a\,c^{2}\,d\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\right)^{2}}}{\frac{\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)^{3}}{\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\right)^{3}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)^{2}(a\,d+b\,c)}{\sqrt{a}\;\sqrt{c}\;d\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\right)^{2}}}-\frac{d\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)}{8\,c\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\right)}-\frac{\ln\left(\frac{\sqrt{b\,x^{2}+a}-\sqrt{a}}{\sqrt{d\,x^{2}+c}-\sqrt{c}}\right)\left(\sqrt{a}\;b\,c^{3/2}-a^{3/2}\,\sqrt{c}\;d\right)}{4\,a\,c^{2}}+\frac{\ln\left(\frac{\left(\sqrt{c}\;\sqrt{b\,x^{2}+a}-\sqrt{a}\;\sqrt{d\,x^{2}+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\;d\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\right)}{\sqrt{d\,x^{2}+c}-\sqrt{c}}\right)}{\sqrt{d\,x^{2}+c}-\sqrt{c}}\right)\left(\sqrt{a}\;b\,c^{3/2}-a^{3/2}\,\sqrt{c}\;d\right)}{4\,a\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*((b^2*c)/8 + (a*b*d)/8))/(a^(1/2)*c^(3/2)*d
*((c + d*x^2)^(1/2) - c^(1/2))) - b^2/(8*c*d) + (((a + b*x^2)^(1/2) - a^(1/
2))^2*((a^2*d^2)/8 + (b^2*c^2)/8 - (3*a*b*c*d)/8))/(a*c^2*d*((c + d*x^2)^(1
/2) - c^(1/2))^2))/((((a + b*x^2)^(1/2) - a^(1/2))^3/((c + d*x^2)^(1/2) - c^
(1/2))^3 + (b*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c^(1/2
))) - (((a + b*x^2)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c
+ d*x^2)^(1/2) - c^(1/2))^2)) - (d*((a + b*x^2)^(1/2) - a^(1/2)))/(8*c*((c
+ d*x^2)^(1/2) - c^(1/2))) - (log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2
```

```
)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(4*a*c^2) + (l
og(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a
^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c
 + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(4*a*c
^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**3*sqrt(c + d*x**2)), x)
```

3.772 $\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=143

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3ad+bc)}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 96, 94, 93, 208}

$$\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3ad+bc)}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*c + 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*a*c^2*x^2) - ((a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(4*a*c*x^4) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^3\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}-\frac{\left(\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{4ac}\\
&= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}-\frac{((bc-ad)(bc+3ad))\,\text{Subst}\left(\int\right.}{16ac^2}\\
&= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}-\frac{((bc-ad)(bc+3ad))\,\text{Subst}\left(\int\right.}{8ac^2}\\
&= \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4acx^4}+\frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a}}\right.}{8a^{3/2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 125, normalized size = 0.87

$$
\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-bcx^2\right)}{8ac^2x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2))/(8*a*c^2*x
^4) + ((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/
(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

IntegrateAlgebraic [F] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 1.86, size = 358, normalized size = 2.50

$$\left[-\frac{\left(b^{2}c^{2}+2\,abcd-3\,a^{2}d^{2}\right)\sqrt{ac}\,x^{4}\log\left(\frac{\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)x^{4}+8\,a^{2}c^{2}+8\,\left(abc^{2}+a^{2}cd\right)x^{2}-4\,\left(\left(bc+ad\right)x^{2}+2\,ac\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{ac}}{x^{4}}\right)+4\,\left(2\,a^{2}c^{2}+\left(abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{32\,a^{2}c^{3}x^{4}},-\frac{\left(b^{2}c^{2}+2\,abcd-3\,a^{2}d^{2}\right)\sqrt{-ac}\,x^{4}\arctan\left(\frac{\left(\left(bc+ad\right)x^{2}+2\,ac\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{-ac}}{2\,\left(abcdx^{4}+a^{2}c^{2}+\left(abc^{2}+a^{2}cd\right)x^{2}\right)}\right)+2\,\left(2\,a^{2}c^{2}+\left(abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{16\,a^{2}c^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a
*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*(2*a^2
*c^2 + (a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3
*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*(
(b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*
d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*(2*a^2*c^2 + (a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3*x^4)]
```

giac [B] time = 2.27, size = 1107, normalized size = 7.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*((sqrt(b*d)*b^3*c^2 + 2*sqrt(b*d)*a*b^2*c*d - 3*sqrt(b*d)*a^2*b*d^2)*
arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a*b*c^2) - 2*(
```

```
sqrt(b*d)*b^9*c^5 - 7*sqrt(b*d)*a*b^8*c^4*d + 18*sqrt(b*d)*a^2*b^7*c^3*d^2
- 22*sqrt(b*d)*a^3*b^6*c^2*d^3 + 13*sqrt(b*d)*a^4*b^5*c*d^4 - 3*sqrt(b*d)*a
^5*b^4*d^5 - 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^2*b^7*c^4 + 16*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^6*c^3*d - 14*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^5*c^2*d^2
 - 8*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^2*a^3*b^4*c*d^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^3*d^4 + 3*sqrt(b*d)*(sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^5*c^3 - 7*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^
4*c^2*d - 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^4*a^2*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^2*d^3 - sqrt(b*d)*(sqrt(b*x^2
 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^3*c^2 - 2*sqrt
(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6
*a*b^2*c*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^6*a^2*b*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c -
 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*
b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4
)^2*a*c^2))/abs(b)
```

maple [B] time = 0.03, size = 355, normalized size = 2.48

$$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-2abcd\,x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-b^{2}c^{2}x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-6\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,ad\,x^{2}+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,bc\,x^{2}+4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,ac\right)}{16\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,a\,c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2*(3*ln((a*d*x^2+b*c*x^2+2*a*c+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*d^2-2*ln((a*d
*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*
x^4*a*b*c*d-ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2))/x^2)*x^4*b^2*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^
2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+4*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)/x^4/(a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 21.52, size = 955, normalized size = 6.68

$$\frac{\ln\left(\frac{\sqrt{b x^2+a}-\sqrt{a}}{\sqrt{d x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b^2\,c^{5/2}-3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^2\,c^3}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{b\,d}{8\,a\,c}-\frac{3\,d\,(a\,d+b\,c)}{32\,a\,c^2}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}-\frac{\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^5\left(\frac{a^2 d^2}{8}-\frac{11\,a\,b\,c\,d}{32}+\frac{5\,b^2 c^2}{32}\right)}{a\,c^3\left(\sqrt{d x^2+c}-\sqrt{c}\right)^5}-\frac{b^4}{64\,\sqrt{a}\,c^{3/2}\,d^2}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2\left(\frac{11\,a^2 b^2 d^2}{64}+\frac{a\,b^3 c\,d}{16}-\frac{5\,b^4 c^2}{64}\right)}{a^{3/2}\,c^{5/2}\,d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^3\left(\frac{a^3 b\,d^3}{32}-\frac{9\,a^2 b^2 c\,d^2}{16}+\frac{3\,a\,b^3 c^2 d}{16}+\frac{b^4 c^3}{32}\right)}{a^2\,c^3\,d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)\left(\frac{b^4 c}{16}-\frac{a\,b^3 d}{16}\right)}{a\,c^2\,d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^4\left(-\frac{7\,a^4 d^4}{64}+\frac{a^3 b\,c\,d^3}{4}+\frac{21\,a^2 b^2 c^2 d^2}{64}-\frac{a\,b^3 c^3 d}{4}+\frac{b^4 c^4}{64}\right)}{a^{5/2}\,c^{7/2}\,d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^4}}{\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^6}{\left(\sqrt{d x^2+c}-\sqrt{c}\right)^6}+\frac{b^2\left(\sqrt{b x^2+a}-\sqrt{a}\right)^2}{d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^2}-\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^3\left(2\,c\,b^2+2\,a\,d\,b\right)}{\sqrt{a}\,\sqrt{c}\,d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^3}-\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^5\left(2\,a\,d+2\,b\,c\right)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d x^2+c}-\sqrt{c}\right)^5}+\frac{\left(\sqrt{b x^2+a}-\sqrt{a}\right)^4\left(a^2 d^2+4\,a\,b\,c\,d+b^2 c^2\right)}{a\,c\,d^2\left(\sqrt{d x^2+c}-\sqrt{c}\right)^4}}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b x^2+a}-\sqrt{a}\,\sqrt{d x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b x^2+a}-\sqrt{a}\right)}{\sqrt{d x^2+c}-\sqrt{c}}\right)}{\sqrt{d x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b^2\,c^{5/2}-3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^2\,c^3}+\frac{d^2\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{64\,\sqrt{a}\,c^{3/2}\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] (log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*
b^2*c^(5/2) - 3*a^(5/2)*c^(1/2)*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^2*c^3)
- (((a + b*x^2)^(1/2) - a^(1/2))*((b*d)/(8*a*c) - (3*d*(a*d + b*c))/(32*a*c
^2)))/((c + d*x^2)^(1/2) - c^(1/2)) - ((((a + b*x^2)^(1/2) - a^(1/2))^5*((a
^2*d^2)/8 + (5*b^2*c^2)/32 - (11*a*b*c*d)/32))/(a*c^3*((c + d*x^2)^(1/2) -
c^(1/2))^5) - b^4/(64*a^(1/2)*c^(3/2)*d^2) + (((a + b*x^2)^(1/2) - a^(1/2))
^2*((11*a^2*b^2*d^2)/64 - (5*b^4*c^2)/64 + (a*b^3*c*d)/16))/(a^(3/2)*c^(5/2
)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) + (((a + b*x^2)^(1/2) - a^(1/2))^3*(
(b^4*c^3)/32 + (a^3*b*d^3)/32 - (9*a^2*b^2*c*d^2)/16 + (3*a*b^3*c^2*d)/16))
/(a^2*c^3*d^2*((c + d*x^2)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1
/2))*((b^4*c)/16 - (a*b^3*d)/16))/(a*c^2*d^2*((c + d*x^2)^(1/2) - c^(1/2)))
 + (((a + b*x^2)^(1/2) - a^(1/2))^4*((b^4*c^4)/64 - (7*a^4*d^4)/64 + (21*a^
2*b^2*c^2*d^2)/64 - (a*b^3*c^3*d)/4 + (a^3*b*c*d^3)/4))/(a^(5/2)*c^(7/2)*d^
2*((c + d*x^2)^(1/2) - c^(1/2))^4))/(((a + b*x^2)^(1/2) - a^(1/2))^6/((c +
d*x^2)^(1/2) - c^(1/2))^6 + (b^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^2*((c
+ d*x^2)^(1/2) - c^(1/2))^2) - (((a + b*x^2)^(1/2) - a^(1/2))^3*(2*b^2*c +
2*a*b*d))/(a^(1/2)*c^(1/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^3) - (((a + b*
x^2)^(1/2) - a^(1/2))^5*(2*a*d + 2*b*c))/(a^(1/2)*c^(1/2)*d*((c + d*x^2)^(1
/2) - c^(1/2))^5) + (((a + b*x^2)^(1/2) - a^(1/2))^4*(a^2*d^2 + b^2*c^2 + 4
*a*b*c*d))/(a*c*d^2*((c + d*x^2)^(1/2) - c^(1/2))^4)) - (log(((c^(1/2)*(a +
 b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b*
x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c + d*x^2)^(1/2) -
 c^(1/2)))*(a^(1/2)*b^2*c^(5/2) - 3*a^(5/2)*c^(1/2)*d^2 + 2*a^(3/2)*b*c^(3/
2)*d))/(16*a^2*c^3) + (d^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(64*a^(1/2)*c^(
3/2)*((c + d*x^2)^(1/2) - c^(1/2))^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b x^{2}}}{x^{5} \sqrt{c + d x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(x**5*sqrt(c + d*x**2)), x)
```

3.773 $\int \frac{x^5\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=276

$$-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)}{128b^2d^4}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,\left(3a^2d^2+10abcd+35b^2c^2\right)}{192b^2d^3}+\frac{(b}{}$$

Rubi **[A]** time = 0.33, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,\left(3a^2d^2+10abcd+35b^2c^2\right)}{192b^2d^3}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)}{128b^2d^4}+\frac{(bc-ad)^2\left(3a^2d^2+10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{128b^{5/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\,(3ad+7bc)}{48b^2d^2}+\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])/(128*b^2*d^4) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(192*b^2*d^3) - ((7*b*c + 3*a*d)*(a + b*x^2)^(5/2)
*Sqrt[c + d*x^2])/(48*b^2*d^2) + (x^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(8
*b*d) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(5/2)*d^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{8bd}$$

$$= -\frac{(7bc+3ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48b^2d^2} + \frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{8bd} + \big(35b^2c^2+10abcd+3a$$

$$= \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192b^2d^3} - \frac{(7bc+3ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48b^2d^2} +$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\big(}{192b^2d^3}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\big(}{192b^2d^3}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\big(}{192b^2d^3}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)\big(}{192b^2d^3}$$

Mathematica [A] time = 0.60, size = 231, normalized size = 0.84

$$\frac{3(bc-ad)^{5/2}\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(9a^3d^3+3a^2bd^2\left(5c-2dx^2\right)+ab^2d\left(-145c^2+92cdx^2-72d^2x^4\right)+b^3\left(105c^3-70c^2dx^2+56cd^2x^4-48d^3x^6\right)\right)}{384b^3d^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^2) + a*b^2*d*(-145*c^2 + 92*c*d*x^2 - 72*d^2*x^4) + b^3*(105*c^3 - 70*c
^2*d*x^2 + 56*c*d^2*x^4 - 48*d^3*x^6))) + 3*(b*c - a*d)^(5/2)*(35*b^2*c^2 +
 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(384*b^3*d^(9/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [F] time = 2.84, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(a + bx^2\right)^{3/2}}{\sqrt{c + dx^2}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2], x]
```

fricas [A] time = 1.03, size = 574, normalized size = 2.08

$$\left[\frac{3\left(35\,b^4c^4 - 60\,ab^3c^3d + 18\,a^2b^2c^2d^2 + 4\,a^3bcd^3 + 3\,a^4d^4\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,bdx^2 + bc + ad\right)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}\,\sqrt{bd}\right) + 4\left(48\,b^4d^4x^6 - 105\,b^4c^3d + 145\,ab^3c^2d^2 - 15\,a^2b^2cd^3 - 9\,a^3bd^4 - 8\left(7\,b^4cd^3 - 9\,ab^3d^4\right)x^4 + 2\left(35\,b^4c^2d^2 - 46\,ab^3cd^3 + 3\,a^2b^2d^4\right)x^2\right)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}}{1536\,b^3d^5}, -\frac{3\left(35\,b^4c^4 - 60\,ab^3c^3d + 18\,a^2b^2c^2d^2 + 4\,a^3bcd^3 + 3\,a^4d^4\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right) - 2\left(48\,b^4d^4x^6 - 105\,b^4c^3d + 145\,ab^3c^2d^2 - 15\,a^2b^2cd^3 - 9\,a^3bd^4 - 8\left(7\,b^4cd^3 - 9\,ab^3d^4\right)x^4 + 2\left(35\,b^4c^2d^2 - 46\,ab^3cd^3 + 3\,a^2b^2d^4\right)x^2\right)\sqrt{bx^2 + a}\,\sqrt{dx^2 + c}}{768\,b^3d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)*sqrt(b*d)) + 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*b^3*c^2
*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^4 +
 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c))/(b^3*d^5), -1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c
*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*
b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4
)*x^4 + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c))/(b^3*d^5)]
```

giac [A] time = 0.65, size = 305, normalized size = 1.11

$$\frac{\left(\sqrt{b^2c + \left(bx^2 + a\right)bd - abd}\,\sqrt{bx^2 + a}\left(2\left(bx^2 + a\right)\left(4\left(bx^2 + a\right)\left(\frac{6\left(bx^2+a\right)}{b^3d} - \frac{7\,b^7cd^5+9\,ab^6d^6}{b^9d^7}\right) + \frac{35\,b^8c^2d^4+10\,ab^7cd^5+3\,a^2b^6d^6}{b^9d^7}\right) - \frac{3\left(35\,b^9c^3d^3-25\,ab^8c^2d^4-7\,a^2b^7cd^5-3\,a^3b^6d^6\right)}{b^9d^7}\right) - \frac{3\left(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4\right)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^4}\right)b}{384\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b^3*d) - (7*b^7*c*d^5 + 9*a*b^6*d^6)/(b^9*d
^7)) + (35*b^8*c^2*d^4 + 10*a*b^7*c*d^5 + 3*a^2*b^6*d^6)/(b^9*d^7)) - 3*(35
*b^9*c^3*d^3 - 25*a*b^8*c^2*d^4 - 7*a^2*b^7*c*d^5 - 3*a^3*b^6*d^6)/(b^9*d^7
```

```
)) - 3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 +
3*a^4*d^4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*b/abs(b)
```

maple [B] time = 0.04, size = 770, normalized size = 2.79

$$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(96\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,b^3d^3x^6+144\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,a b^2d^3x^4-112\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,b^3c\,d^2x^4+9a^4d^4\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+12a^3bc\,d^3\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+54a^2b^2c^2d^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-180a\,b^3c^3d\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+105b^4c^4\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+12\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,a^2b\,d^3x^2-184\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,a b^2c\,d^2x^2+140\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,b^3c^2d\,x^2-18\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,a^3d^3-30\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,a^2bc\,d^2+290\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,a b^2c^2d-210\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,b^3c^3\right)}{768\sqrt{x^4bd+ad x^2+bc x^2+ac}\,\sqrt{bd}\,b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(96*x^6*b^3*d^3*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(b*d)^(1/2)+144*x^4*a*b^2*d^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(b*d)^(1/2)-112*x^4*b^3*c*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^2*b*d^3*(b*d)^(1/2)-1
84*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*c*b^2*d^2*(b*d)^(1/2)+140*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c^2*b^3*d*(b*d)^(1/2)+9*d^4*ln(1/2*(2*b
*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/
2))*a^4+12*a^3*c*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b*d^3+54*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2*c^2*b^2*d^2-180
*a*c^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2))/(b*d)^(1/2))*b^3*d+105*b^4*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*c^4-18*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*a^3*d^3*(b*d)^(1/2)-30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*a^2*c*b*d^2*(b*d)^(1/2)+290*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c^2*b^
2*d*(b*d)^(1/2)-210*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^3*b^3*(b*d)^(1/2)
)/b^2/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(a + bx^2\right)^{\frac{3}{2}}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.774 $\int \frac{x^3\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)(ad+5bc)}{16bd^3}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(ad+5}{24bd^2}$$

Rubi [A] time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 63, 217, 206}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(ad+5bc)}{24bd^2}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)(ad+5bc)}{16bd^3}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*b*d^3) - ((
5*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*b*d^2) + ((a + b*x^2)^(
5/2)*Sqrt[c + d*x^2])/(6*b*d) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(16*b^(3/2)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6bd} - \frac{(5bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{12bd}$$

$$= -\frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6bd} + \frac{((bc-ad)(5bc+ad))\,\text{Subs}}{16bd^2}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}}{6b}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}}{6b}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}}{6b}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}}{6b}$$

Mathematica [A] time = 0.44, size = 173, normalized size = 0.93

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(3a^2d^2+2abd\left(7dx^2-11c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)-3(bc-ad)^{5/2}(ad+5bc)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{48b^2d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - 3*(b*c - a*d)^(5/2)*(5*b*c +
a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sq
rt[b*c - a*d]])/(48*b^2*d^(7/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 2.08, size = 242, normalized size = 1.29

$$\frac{\sqrt{c+dx^2}\,(bc-ad)^2\left(\frac{15b^3c\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}+\frac{3ab^2d\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}-\frac{40b^2cd\left(c+dx^2\right)}{a+bx^2}-\frac{8abd^2\left(c+dx^2\right)}{a+bx^2}-3ad^3+33bcd^2\right)}{48bd^3\sqrt{a+bx^2}\left(\frac{b\left(c+dx^2\right)}{a+bx^2}-d\right)^3}-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x^2]*(33*b*c*d^2 - 3*a*d^3 - (40*b^2*c*d*(c + d*x
^2))/(a + b*x^2) - (8*a*b*d^2*(c + d*x^2))/(a + b*x^2) + (15*b^3*c*(c + d*x
^2)^2)/(a + b*x^2)^2 + (3*a*b^2*d*(c + d*x^2)^2)/(a + b*x^2)^2))/(48*b*d^3*
Sqrt[a + b*x^2]*(-d + (b*(c + d*x^2))/(a + b*x^2))^3) - ((b*c - a*d)^2*(5*b
*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(16
*b^(3/2)*d^(7/2))
```

fricas [A] time = 1.53, size = 440, normalized size = 2.35

$$\left[\frac{3\left(5 b^{3} c^{3}-9 a b^{2} c^{2} d+3 a^{2} b c d^{2}+a^{3} d^{3}\right) \sqrt{b d} \log \left(8 b^{2} d^{2} x^{4}+b^{2} c^{2}+6 a b c d+a^{2} d^{2}+8\left(b^{2} c d+a b d^{2}\right) x^{2}-4\left(2 b d x^{2}+b c+a d\right) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c} \sqrt{b d}\right)+4\left(8 b^{3} d^{3} x^{4}+15 b^{3} c^{2} d-22 a b^{2} c d^{2}+3 a^{2} b d^{3}-2\left(5 b^{3} c d^{2}-7 a b^{2} d^{3}\right) x^{2}\right) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{192 b^{2} d^{4}}, \frac{3\left(5 b^{3} c^{3}-9 a b^{2} c^{2} d+3 a^{2} b c d^{2}+a^{3} d^{3}\right) \sqrt{-b d} \arctan \left(\frac{\left(2 b d x^{2}+b c+a d\right) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c} \sqrt{-b d}}{2\left(b^{2} d^{2} x^{4}+a b c d+\left(b^{2} c d+a b d^{2}\right) x^{2}\right)}\right)+2\left(8 b^{3} d^{3} x^{4}+15 b^{3} c^{2} d-22 a b^{2} c d^{2}+3 a^{2} b d^{3}-2\left(5 b^{3} c d^{2}-7 a b^{2} d^{3}\right) x^{2}\right) \sqrt{b x^{2}+a} \sqrt{d x^{2}+c}}{96 b^{2} d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) +
4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c
*d^2 - 7*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4), 1/96*(
3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b*d)*arctan(1
/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*
d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c
^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4)]
```

giac [A] time = 0.50, size = 225, normalized size = 1.20

$$\frac{\left(\sqrt{b^{2} c+\left(b x^{2}+a\right) b d-a b d} \sqrt{b x^{2}+a}\left(2\left(b x^{2}+a\right)\left(\frac{4\left(b x^{2}+a\right)}{b^{2} d}-\frac{5 b^{3} c d^{3}+a b^{2} d^{4}}{b^{4} d^{5}}\right)+\frac{3\left(5 b^{4} c^{2} d^{2}-4 a b^{3} c d^{3}-a^{2} b^{2} d^{4}\right)}{b^{4} d^{5}}\right)+\frac{3\left(5 b^{3} c^{3}-9 a b^{2} c^{2} d+3 a^{2} b c d^{2}+a^{3} d^{3}\right) \log \left(\left|-\sqrt{b x^{2}+a} \sqrt{b d}+\sqrt{b^{2} c+\left(b x^{2}+a\right) b d-a b d}\right|\right)}{\sqrt{b d} b d^{3}}\right) b}{48|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b^2*d) - (5*b^3*c*d^3 + a*b^2*d^4)/(b^4*d^5)) + 3*(5*b^4*c^
2*d^2 - 4*a*b^3*c*d^3 - a^2*b^2*d^4)/(b^4*d^5)) + 3*(5*b^3*c^3 - 9*a*b^2*c^
2*d + 3*a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*b/abs(b)
```

maple [B] time = 0.02, size = 532, normalized size = 2.84

$$-\frac{\sqrt{b x^{2}+a} \sqrt{d x^{2}+c}\left(-16 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} b^{2} d^{2} x^{4}+3 a^{3} d^{3} \ln \left(\frac{2 b d x^{2}+a d+b c+2 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d}}{2 \sqrt{b d}}\right)+9 a^{2} b c d^{2} \ln \left(\frac{2 b d x^{2}+a d+b c+2 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d}}{2 \sqrt{b d}}\right)-27 a b^{2} c^{2} d \ln \left(\frac{2 b d x^{2}+a d+b c+2 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d}}{2 \sqrt{b d}}\right)+15 b^{3} c^{3} \ln \left(\frac{2 b d x^{2}+a d+b c+2 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d}}{2 \sqrt{b d}}\right)-28 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} a b d^{2} x^{2}+20 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} b^{2} c d x^{2}-6 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} a^{2} d^{2}+44 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} a b c d-30 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} b^{2} c^{2}\right)}{96 \sqrt{x^{4} b d+a d x^{2}+b c x^{2}+a c} \sqrt{b d} b d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(b*d)^(1/2)*b^2*d^2*x^4-28*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1
/2)*a*b*d^2*x^2+20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*d*
x^2+3*a^3*d^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2))/(b*d)^(1/2))+9*a^2*b*c*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-27*a*b^2*c^2*d*ln
(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/
(b*d)^(1/2))+15*b^3*c^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2)*a^2*d^2+44*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*
b*c*d-30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^2)/b/d^3/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

maxima **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b x^{2}\right)^{\frac{3}{2}}}{\sqrt{c + d x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.775 $\int \frac{x\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=125

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8\sqrt{b}\,d^{5/2}}-\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)}{8d^2}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)}{8d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8\sqrt{b}\,d^{5/2}}+\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + ((a + b*x^2)^(3/
2)*Sqrt[c + d*x^2])/(4*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^
2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*Sqrt[b]*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{8d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}(\int \frac{}{\sqrt{a+bx}}}{16d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}(\int \frac{1}{\sqrt{c-\frac{ad}{b}}}}{8bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}(\int \frac{1}{1-\frac{dx^2}{b}}}{8bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8d^2} + \frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{3(bc-ad)^2\tanh^{-1}(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}}{8\sqrt{b}\,d^{5/2}}$$

Mathematica [A]   time = 0.38, size = 131, normalized size = 1.05

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(5ad-3bc+2bdx^2\right)+\frac{3(bc-ad)^{5/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{b}}{8d^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(-3*b*c + 5*a*d + 2*b*d*x^2) + (3*(b*c
 - a*d)^(5/2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b
*x^2])/Sqrt[b*c - a*d]])/b)/(8*d^(5/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A]   time = 1.21, size = 150, normalized size = 1.20

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8\sqrt{b}\,d^{5/2}}+\frac{(ad-bc)^2\left(\frac{5d\sqrt{c+dx^2}}{\sqrt{a+bx^2}}-\frac{3b\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\right)}{8d^2\left(d-\frac{b\left(c+dx^2\right)}{a+bx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((-(b*c) + a*d)^2*((5*d*Sqrt[c + d*x^2])/Sqrt[a + b*x^2] - (3*b*(c + d*x^2)
^(3/2))/(a + b*x^2)^(3/2)))/(8*d^2*(d - (b*(c + d*x^2))/(a + b*x^2))^2) + (
3*(b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])
])/(8*Sqrt[b]*d^(5/2))
```

fricas [A]   time = 0.98, size = 334, normalized size = 2.67

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\left(2\,b^2d^2x^2-3\,b^2cd+5\,abd^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,bd^3},-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)-2\left(2\,b^2d^2x^2-3\,b^2cd+5\,abd^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c
+ a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3*b^
2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3), -1/16*(3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d
```

```
+ a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b*d^3)]
```

giac [A] time = 0.52, size = 149, normalized size = 1.19

$$\frac{\left(\sqrt{b^{2}c + \left(bx^{2} + a\right)bd - abd}\,\sqrt{bx^{2} + a}\left(\frac{2\left(bx^{2}+a\right)}{bd} - \frac{3\left(bcd-ad^{2}\right)}{bd^{3}}\right) - \frac{3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(\left|-\sqrt{bx^{2}+a}\,\sqrt{bd}+\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,d^{2}}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b*d) - 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(a
bs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sq
rt(b*d)*d^2))*b/abs(b)
```

maple [B] time = 0.02, size = 337, normalized size = 2.70

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\,\sqrt{bd}}\right)-6abcd\ln\left(\frac{2bd\,x^{2}+ad+bc+2\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\,\sqrt{bd}}\right)+3b^{2}c^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\,\sqrt{bd}}\right)+4\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,bd\,x^{2}+10\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,ad-6\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,bc\right)}{16\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*b*d*x^2+3*a^2*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-6*a*b*c*d*ln(1/2*(2*b*d*x^2+a
*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*b^
2*c^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2))/(b*d)^(1/2))+10*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d
-6*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)/d^2/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(b\,x^2+a\right)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.776 $\int \frac{\left(a+bx^2\right)^{3/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=133

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d}$$

Rubi [**A**] time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {446, 102, 157, 63, 217, 206, 93, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2d^{3/2}}+\frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - (a^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (Sqrt[b]*(b*c - 3*a*d)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} + \frac{\text{Subst}\left(\int \frac{a^2d-\frac{1}{2}b(bc-3ad)x}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{2d}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right) - \frac{(b(bc-3ad))\,\text{Subst}\,(}{}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} + a^2\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right) - \frac{(bc-3ad)\,\text{Subst}\Big(\int \frac{}{\sqrt{c-}}}{2}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2d}$$

$$= \frac{b\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2d} - \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}} - \frac{\sqrt{b}\,(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

Mathematica [A] time = 0.77, size = 195, normalized size = 1.47

$$\frac{\sqrt{d}\left(b\sqrt{a+bx^2}\,\left(c+dx^2\right) - \frac{2a^{3/2}d\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{c}}\right) - \frac{\left(3a^2d^2-4abcd+b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}}{2d^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*Arc
Sinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]) + Sqrt[d]
*(b*Sqrt[a + b*x^2]*(c + d*x^2) - (2*a^(3/2)*d*Sqrt[c + d*x^2]*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c]))/(2*d^(3/2)*Sqrt
[c + d*x^2])
```

IntegrateAlgebraic [A] time = 1.43, size = 169, normalized size = 1.27

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}+\frac{\left(3a\sqrt{b}\,d-b^{3/2}c\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+dx^2}\,(bc-ad)}{2d\sqrt{a+bx^2}\left(d-\frac{b\left(c+dx^2\right)}{a+bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*(b*c - a*d)*Sqrt[c + d*x^2])/(d*Sqrt[a + b*x^2]*(d - (b*(c + d*x^2)
)/(a + b*x^2))) - (a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[
a + b*x^2])])/Sqrt[c] + ((-(b^(3/2)*c) + 3*a*Sqrt[b]*d)*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(2*d^(3/2))
```

fricas [A] time = 3.09, size = 918, normalized size = 6.90

$$\left[\frac{2ad\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6abcd+a^2d^2)x^4+8a^2c^2+8(abc^2+a^2cd)x^2-4(2ac^2+(bc^2+acd)x^2)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right)-(bc-3ad)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8(b^2cd+abd^2)x^2+4(2bd^2x^2+bcd+ad^2)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right)+4\sqrt{bx^2+a}\sqrt{dx^2+c}\,b}{8d},\ \frac{ad\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6abcd+a^2d^2)x^4+8a^2c^2+8(abc^2+a^2cd)x^2-4(2ac^2+(bc^2+acd)x^2)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right)+(bc-3ad)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2(b^2dx^4+abc+(b^2c+abd)x^2)}\right)+2\sqrt{bx^2+a}\sqrt{dx^2+c}\,b}{4d},\ \frac{4ad\sqrt{-\frac{a}{c}}\arctan\left(\frac{((bc+ad)x^2+2ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{a}{c}}}{2(abdx^4+a^2c+(abc+a^2d)x^2)}\right)-(bc-3ad)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8(b^2cd+abd^2)x^2+4(2bd^2x^2+bcd+ad^2)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right)+4\sqrt{bx^2+a}\sqrt{dx^2+c}\,b}{8d},\ \frac{2ad\sqrt{-\frac{a}{c}}\arctan\left(\frac{((bc+ad)x^2+2ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{a}{c}}}{2(abdx^4+a^2c+(abc+a^2d)x^2)}\right)+(bc-3ad)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2(b^2dx^4+abc+(b^2c+abd)x^2)}\right)+2\sqrt{bx^2+a}\sqrt{dx^2+c}\,b}{4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
+ 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/4*(a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 +
(b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (b*c
 - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 2*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/8*(4*a*d*sqrt(-a/c)*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a
^2*c + (a*b*c + a^2*d)*x^2)) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2*d^2*x^4 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2
+ b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*b)/d, 1/4*(2*a*d*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) + (b*c - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b
^2*c + a*b*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.02, size = 287, normalized size = 2.16

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(-2\sqrt{bd}\,a^{2}d\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+3\sqrt{ac}\,abd\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-\sqrt{ac}\,b^{2}c\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{bd}\,\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,b\right)}{4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*a*b*d-ln(
1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(
b*d)^(1/2))*(a*c)^(1/2)*b^2*c-2*(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*
c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a^2*d+2*(b*d)^(1/2)*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x*sqrt(c + d*x**2)), x)
```

3.777 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=136

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2}$$

Rubi [**A**] time = 0.14, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*x^2) - (Sqrt[a]*(3*b*c - a*d)*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) + (b
^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt[d
]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)-b^2cx}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + \frac{1}{2}b^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^2\right) + \frac{(a(3bc-ad))\,\text{Subst}}{}$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + b\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^2}\right) + \frac{(a(3bc-ad))\,\text{Subs}}{}$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}} + b\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a}}{\sqrt{c}}\right.$$

$$= -\frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}\,(3bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

Mathematica [A] time = 1.01, size = 172, normalized size = 1.26

$$\frac{\sqrt{a}\,(ad-3bc)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2c^{3/2}} - \frac{a\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2cx^2} + \frac{(bc-ad)^{3/2}\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2) + ((b*c - a*d)^(3/2)*((b*(
c + d*x^2))/(b*c - a*d))^(3/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c -
 a*d]])/(Sqrt[d]*(c + d*x^2)^(3/2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2))
```

IntegrateAlgebraic [A] time = 1.66, size = 167, normalized size = 1.23

$$\frac{\left(a^{3/2}d-3\sqrt{a}\,bc\right)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{2c^{3/2}} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{\sqrt{d}} + \frac{a\sqrt{c+dx^2}\,(ad-bc)}{2c\sqrt{a+bx^2}\,\left(c-\frac{a\left(c+dx^2\right)}{a+bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*(-(b*c) + a*d)*Sqrt[c + d*x^2])/(2*c*Sqrt[a + b*x^2]*(c - (a*(c + d*x^2)
)/(a + b*x^2))) + ((-3*Sqrt[a]*b*c + a^(3/2)*d)*ArcTanh[(Sqrt[a]*Sqrt[c + d
*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)) + (b^(3/2)*ArcTanh[(Sqrt[b]*
Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/Sqrt[d]
```

fricas [B] time = 3.08, size = 958, normalized size = 7.04

$$\left[\frac{2\,bcx^{2}\sqrt{\frac{b}{d}}\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)x^{2}+4\left(2\,bd^{2}x^{2}+bcd+ad^{2}\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{\frac{b}{d}}\right)-\left(3\,bc-ad\right)x^{2}\sqrt{\frac{a}{c}}\log\left(\frac{\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)x^{4}+8\,a^{2}c^{2}+8\left(abc^{2}+a^{2}cd\right)x^{2}+4\left(2\,ac^{2}+\left(bc^{2}+acd\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{\frac{a}{c}}}{x^{4}}\right)-4\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\,a}{8\,cx^{2}},\ -\frac{4\,bcx^{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2\,bdx^{2}+bc+ad\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{-\frac{b}{d}}}{2\left(b^{2}dx^{4}+abc+\left(b^{2}c+abd\right)x^{2}\right)}\right)+\left(3\,bc-ad\right)x^{2}\sqrt{\frac{a}{c}}\log\left(\frac{\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)x^{4}+8\,a^{2}c^{2}+8\left(abc^{2}+a^{2}cd\right)x^{2}+4\left(2\,ac^{2}+\left(bc^{2}+acd\right)x^{2}\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{\frac{a}{c}}}{x^{4}}\right)+4\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\,a}{8\,cx^{2}},\ \frac{bcx^{2}\sqrt{\frac{b}{d}}\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)x^{2}+4\left(2\,bd^{2}x^{2}+bcd+ad^{2}\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{\frac{b}{d}}\right)+\left(3\,bc-ad\right)x^{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{\left(\left(bc+ad\right)x^{2}+2\,ac\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{-\frac{a}{c}}}{2\left(abdx^{4}+a^{2}c+\left(abc+a^{2}d\right)x^{2}\right)}\right)-2\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\,a}{4\,cx^{2}},\ -\frac{2\,bcx^{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2\,bdx^{2}+bc+ad\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{-\frac{b}{d}}}{2\left(b^{2}dx^{4}+abc+\left(b^{2}c+abd\right)x^{2}\right)}\right)-\left(3\,bc-ad\right)x^{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{\left(\left(bc+ad\right)x^{2}+2\,ac\right)\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\sqrt{-\frac{a}{c}}}{2\left(abdx^{4}+a^{2}c+\left(abc+a^{2}d\right)x^{2}\right)}\right)+2\sqrt{bx^{2}+a}\sqrt{dx^{2}+c}\,a}{4\,cx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (3*b*c - a*d)*x^2*sqrt(a/c)*log(((b^2*c^2
+ 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*(2*a
*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4)
 - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/8*(4*b*c*x^2*sqrt(-b/d)
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b
/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + (3*b*c - a*d)*x^2*sqrt(a/c
)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c
*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*sqrt(a/c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), 1/4*(b*c*x
^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d
 + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)*sqrt(b/d)) + (3*b*c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/4*
(2*b*c*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (3*b*
c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 2
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2)]
```

giac [B] time = 0.75, size = 498, normalized size = 3.66

$$-\frac{\left(\frac{\sqrt{bd}\,b\log\left(\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{d}+\frac{\left(3\,\sqrt{bd}\,ab^2c-\sqrt{bd}\,a^2bd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\,\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc}+\frac{2\left(\sqrt{bd}\,ab^4c^2-2\,\sqrt{bd}\,a^2b^3cd+\sqrt{bd}\,a^3b^2d^2-\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2ab^2c-\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2a^2bd\right)}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^4\right)c}\right)b}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(b*d)*b*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^2)/d + (3*sqrt(b*d)*a*b^2*c - sqrt(b*d)*a^2*b*d)*arctan(-1/2
*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c) + 2*(sqrt(b*d)*a*b^4
*c^2 - 2*sqrt(b*d)*a^2*b^3*c*d + sqrt(b*d)*a^3*b^2*d^2 - sqrt(b*d)*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^2*c - sqr
t(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2*a^2*b*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*c))*b/abs(b)
```

maple [**B**] time = 0.02, size = 298, normalized size = 2.19

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(\sqrt{bd}\,a^{2}d\,x^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-3\sqrt{bd}\,abc\,x^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+2\sqrt{ac}\,b^{2}c\,x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,a\right)}{4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,c\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^2*c*(a*c)^(1/
2)+ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2))/x^2)*x^2*a^2*d*(b*d)^(1/2)-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a*b*c*(b*d)^(1/2)-2*a*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*(a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/x^2/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x^3\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x^3*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**3*sqrt(c + d*x**2)), x)
```

3.778 $\int \frac{\left(a+bx^2\right)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=131

$$-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}\,c^{5/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 94, 93, 208}

$$-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)}{8c^2x^2}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}\,c^{5/2}}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - ((a + b*x^2)
^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[
a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*Sqrt[a]*c^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^3\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}+\frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{8c}\\
&= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}+\frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c}}\right.}{16c^2}\\
&= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}+\frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx\right.}{8c^2}\\
&= -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}\,c^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 110, normalized size = 0.84

$$
\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-5bcx^2\right)}{8c^2x^4}-\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}\,c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 5*b*c*x^2 + 3*a*d*x^2))/(8*c^2*x
^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
d*x^2])])/(8*Sqrt[a]*c^(5/2))
```

IntegrateAlgebraic [F] time = 2.57, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 2.13, size = 360, normalized size = 2.75

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{ac}\,x^4\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left(\left(bc+ad\right)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)-4\left(2\,a^2c^2+\left(5\,abc^2-3\,a^2cd\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{32\,ac^3x^4},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-ac}\,x^4\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\left(abcdx^4+a^2c^2+\left(abc^2+a^2cd\right)x^2\right)}\right)-2\left(2\,a^2c^2+\left(5\,abc^2-3\,a^2cd\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{16\,ac^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a*
b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*
d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a^2*
c^2 + (5*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*
x^4), 1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*
x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 + (5*a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*x^4)]
```

giac [B] time = 2.29, size = 1101, normalized size = 8.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*b*(3*(sqrt(b*d)*b^3*c^2 - 2*sqrt(b*d)*a*b^2*c*d + sqrt(b*d)*a^2*b*d^2)
*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*
x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c^2) + 2*(5
*sqrt(b*d)*b^9*c^5 - 23*sqrt(b*d)*a*b^8*c^4*d + 42*sqrt(b*d)*a^2*b^7*c^3*d^
2 - 38*sqrt(b*d)*a^3*b^6*c^2*d^3 + 17*sqrt(b*d)*a^4*b^5*c*d^4 - 3*sqrt(b*d)
*a^5*b^4*d^5 - 15*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^2*b^7*c^4 + 28*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^6*c^3*d - 2*sqrt(b*d)*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^5*c^2*d
```

```
^2 - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d))^2*a^3*b^4*c*d^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(
b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^3*d^4 + 15*sqrt(b*d)*(sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^5*c^3 + sqrt(b*
d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*
b^4*c^2*d + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^4*a^2*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^2*d^3 - 5*sqrt(b*d)*(sqrt(b
*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^3*c^2 - 6*
sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^6*a*b^2*c*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^6*a^2*b*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 -
2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2
*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^4)^2*c^2))/abs(b)
```

maple [B] time = 0.02, size = 352, normalized size = 2.69

$$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-6abcd\,x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+3b^{2}c^{2}x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-6\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,ad\,x^{2}+10\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,bc\,x^{2}+4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,ac\right)}{16\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(3*a^2*d^2*x^4*ln((a*d*x^2+b*c*x^
2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-6*a*b*c*d*x
^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2))/x^2)+3*b^2*c^2*x^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^
(1/2)*a*d*x^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*c*x^2+4*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*c)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/x^4/(a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{3/2}}{x^5\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(3/2)/(x^5*(c + d*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(x**5*sqrt(c + d*x**2)), x)
```

3.779 $\int \frac{x^5\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=340

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)}{256b^2d^5}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(bc-ad)\bigl(3a^2d^2+14abcd+63b}{384b^2d^4}$$

Rubi [A] time = 0.42, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}\left(3a^2d^2+14abcd+63b^2c^2\right)}{480b^2d^3}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)}{384b^2d^4}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)}{256b^2d^5}-\frac{(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{256b^{5/2}d^{11/2}}-\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}\,(ad+3bc)}{80b^2d^2}+\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])/(256*b^2*d^5) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^
2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(384*b^2*d^4) + ((63*b^2*c^2 + 14*a*b
*c*d + 3*a^2*d^2)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(480*b^2*d^3) - (3*(3*
b*c + a*d)*(a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(80*b^2*d^2) + (x^2*(a + b*x^
2)^(7/2)*Sqrt[c + d*x^2])/(10*b*d) - ((b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*
d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]
)/(256*b^(5/2)*d^(11/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5 \left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^2\right)}{10bd}$$

$$= -\frac{3(3bc+ad)\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{80b^2d^2} + \frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{10bd} + \left(63b^2c^2 + 14abcd + 3a\right.$$

$$= \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{480b^2d^3} - \frac{3(3bc+ad)\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{80b^2d^2} +$$

$$= -\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{384b^2d^4} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right.}{480b}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+\right.}{384}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+\right.}{384}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+\right.}{384}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} - \frac{(bc-ad)\left(63b^2c^2+14abcd+\right.}{384}$$

Mathematica [**A**] time = 1.20, size = 271, normalized size = 0.80

$$\frac{\sqrt{c+dx^2}\left(\frac{5(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{16d^3\left(a+bx^2\right)^3}{15(bc-ad)^3}-\frac{4d^2\left(a+bx^2\right)^2}{3(bc-ad)^2}+\frac{2d\left(a+bx^2\right)}{bc-ad}-\frac{2\sqrt{d}\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}}\right)}{4bd^5}-\frac{24\left(a+bx^2\right)^4(ad+3bc)}{bd}+64x^2\left(a+bx^2\right)^4\right)}{640bd\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[c + d*x^2]*((-24*(3*b*c + a*d)*(a + b*x^2)^4)/(b*d) + 64*x^2*(a + b*x
^2)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((2*d*(a + b
*x^2))/(b*c - a*d) - (4*d^2*(a + b*x^2)^2)/(3*(b*c - a*d)^2) + (16*d^3*(a +
 b*x^2)^3)/(15*(b*c - a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(
b*c - a*d)]))))/(4*b*d^5)))/(640*b*d*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [F] time = 3.34, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2], x]
```

fricas [A] time = 1.05, size = 734, normalized size = 2.16

$$\left[-\frac{15 \left(63 b^5 c^5 - 175 a b^4 c^4 d + 150 a^2 b^3 c^3 d^2 - 30 a^3 b^2 c^2 d^3 - 5 a^4 b c d^4 - 3 a^5 d^5\right) \sqrt{bd} \log\left(8 b^2 d^2 x^4 + b^2 c^2 + 6 abcd + a^2 d^2 + 8 \left(b^2 cd + abd^2\right) x^2 + 4 \left(2 bdx^2 + bc + ad\right) \sqrt{bx^2+a}\, \sqrt{dx^2+c}\, \sqrt{bd}\right) - 4 \left(384 b^5 d^5 x^8 + 945 b^5 c^4 d - 2310 a b^4 c^3 d^2 + 1564 a^2 b^3 c^2 d^3 - 90 a^3 b^2 c d^4 - 45 a^4 b d^5 - 144 \left(3 b^5 c d^4 - 7 a b^4 d^5\right) x^6 + 8 \left(63 b^5 c^2 d^3 - 148 a b^4 c d^4 + 93 a^2 b^3 d^5\right) x^4 - 2 \left(315 b^5 c^3 d^2 - 749 a b^4 c^2 d^3 + 481 a^2 b^3 c d^4 - 15 a^3 b^2 d^5\right) x^2\right) \sqrt{bx^2+a}\, \sqrt{dx^2+c}}{15360\, b^3 d^6}, \frac{15 \left(63 b^5 c^5 - 175 a b^4 c^4 d + 150 a^2 b^3 c^3 d^2 - 30 a^3 b^2 c^2 d^3 - 5 a^4 b c d^4 - 3 a^5 d^5\right) \sqrt{-bd} \arctan\left(\frac{\left(2 bdx^2 + bc + ad\right) \sqrt{bx^2+a}\, \sqrt{dx^2+c}\, \sqrt{-bd}}{2 \left(b^2 d^2 x^4 + abcd + \left(b^2 cd + abd^2\right) x^2\right)}\right) + 2 \left(384 b^5 d^5 x^8 + 945 b^5 c^4 d - 2310 a b^4 c^3 d^2 + 1564 a^2 b^3 c^2 d^3 - 90 a^3 b^2 c d^4 - 45 a^4 b d^5 - 144 \left(3 b^5 c d^4 - 7 a b^4 d^5\right) x^6 + 8 \left(63 b^5 c^2 d^3 - 148 a b^4 c d^4 + 93 a^2 b^3 d^5\right) x^4 - 2 \left(315 b^5 c^3 d^2 - 749 a b^4 c^2 d^3 + 481 a^2 b^3 c d^4 - 15 a^3 b^2 d^5\right) x^2\right) \sqrt{bx^2+a}\, \sqrt{dx^2+c}}{7680\, b^3 d^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/15360*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*
b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c
+ a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(384*b^5*d^5*x^8 + 94
5*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4
- 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*d^3 -
148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*
d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c))/(b^3*d^6), 1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*
d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(-b*d)*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^
2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(384*b^5*d^5*x^8 + 945*b^5*
c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a
^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*d^3 - 148*a*
b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 +
481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(
b^3*d^6)]
```

giac [A] time = 0.68, size = 398, normalized size = 1.17

$$\frac{\left(\sqrt{b^2 c + \left(bx^2 + a\right) bd - abd}\, \sqrt{bx^2 + a} \left(2 \left(bx^2 + a\right) \left(4 \left(bx^2 + a\right) \left(6 \left(bx^2 + a\right) \left(\frac{8 \left(bx^2+a\right)}{b^3 d} - \frac{9 b^7 c d^7 + 11 a b^6 d^8}{b^9 d^9}\right) + \frac{63 b^8 c^2 d^6 + 14 a b^7 c d^7 + 3 a^2 b^6 d^8}{b^9 d^9}\right) - \frac{5 \left(63 b^9 c^3 d^5 - 49 a b^8 c^2 d^6 - 11 a^2 b^7 c d^7 - 3 a^3 b^6 d^8\right)}{b^9 d^9}\right) + \frac{15 \left(63 b^{10} c^4 d^4 - 112 a b^9 c^3 d^5 + 38 a^2 b^8 c^2 d^6 + 8 a^3 b^7 c d^7 + 3 a^4 b^6 d^8\right)}{b^9 d^9}\right) + \frac{15 \left(63 b^5 c^5 - 175 a b^4 c^4 d + 150 a^2 b^3 c^3 d^2 - 30 a^3 b^2 c^2 d^3 - 5 a^4 b c d^4 - 3 a^5 d^5\right) \log\left(\left|-\sqrt{bx^2+a}\, \sqrt{bd} + \sqrt{b^2 c + \left(bx^2+a\right) bd - abd}\right|\right)}{\sqrt{bd}\, b^2 d^5}\right) b}{3840\, |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a
)*(4*(b*x^2 + a)*(6*(b*x^2 + a)*(8*(b*x^2 + a)/(b^3*d) - (9*b^7*c*d^7 + 11*
a*b^6*d^8)/(b^9*d^9)) + (63*b^8*c^2*d^6 + 14*a*b^7*c*d^7 + 3*a^2*b^6*d^8)/(
b^9*d^9)) - 5*(63*b^9*c^3*d^5 - 49*a*b^8*c^2*d^6 - 11*a^2*b^7*c*d^7 - 3*a^3
*b^6*d^8)/(b^9*d^9)) + 15*(63*b^10*c^4*d^4 - 112*a*b^9*c^3*d^5 + 38*a^2*b^8
*c^2*d^6 + 8*a^3*b^7*c*d^7 + 3*a^4*b^6*d^8)/(b^9*d^9)) + 15*(63*b^5*c^5 - 1
75*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 -
 3*a^5*d^5)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b
*d - a*b*d)))/(sqrt(b*d)*b^2*d^5))*b/abs(b)
```

maple [B] time = 0.04, size = 1054, normalized size = 3.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/7680*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(768*x^8*b^4*d^4*(b*d)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2016*x^6*a*b^3*d^4*(b*d)^(1/2)*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)-864*x^6*b^4*c*d^3*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+1488*x^4*a^2*b^2*d^4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)-2368*x^4*a*b^3*c*d^3*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+
1008*x^4*b^4*c^2*d^2*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+60*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^3*b*d^4*(b*d)^(1/2)-1924*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a^2*c*b^2*d^3*(b*d)^(1/2)+2996*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*x^2*a*c^2*b^3*d^2*(b*d)^(1/2)-1260*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^2*c^3*b^4*d*(b*d)^(1/2)+45*d^5*ln(1/2*(2*b*d*x^2+a*d+b*c
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^5+75*ln(
1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(
b*d)^(1/2))*a^4*c*b*d^4+450*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^3*c^2*b^2*d^3-2250*ln(1/2*(2*b
*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/
2))*a^2*c^3*b^3*d^2+2625*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*c^4*a*b^4*d-945*b^5*ln(1/2*(2*b*d*x
^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*
c^5-90*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^4*d^4*(b*d)^(1/2)-180*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^3*c*b*d^3*(b*d)^(1/2)+3128*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*a^2*c^2*b^2*d^2*(b*d)^(1/2)-4620*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*a*c^3*b^3*d*(b*d)^(1/2)+1890*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*c^4*b^4*(b*d)^(1/2))/b^2/d^5/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1
/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(a + b x^{2}\right)^{\frac{5}{2}}}{\sqrt{c + d x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.780 $\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=237

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2(ad+7bc)}{128bd^4}+\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc}{192bd^3}$$

Rubi [A] time = 0.23, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}-\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(ad+7bc)}{48bd^2}+\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)(ad+7bc)}{192bd^3}-\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2(ad+7bc)}{128bd^4}+\frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(128*b*d^4
) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(192*b*
d^3) - ((7*b*c + a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(48*b*d^2) + ((a +
 b*x^2)^(7/2)*Sqrt[c + d*x^2])/(8*b*d) + (5*(b*c - a*d)^3*(7*b*c + a*d)*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(128*b^(3/2)*d^(
9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd} - \frac{(7bc+ad)\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{16bd}$$

$$= -\frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + \frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd} + \frac{(5(bc-ad)(7bc+ad))\operatorname{Sub}}{96bd}$$

$$= \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + \frac{\left(a+bx\right.}{}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3}$$

Mathematica [A] time = 0.59, size = 214, normalized size = 0.90

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(15a^3d^3+a^2bd^2\left(118dx^2-191c\right)+ab^2d\left(265c^2-172cdx^2+136d^2x^4\right)+b^3\left(-105c^3+70c^2dx^2-56cd^2x^4+48d^3x^6\right)\right)+15(ad+7bc)(bc-ad)^{7/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{384b^2d^{9/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^2) + a*b^2*d*(265*c^2 - 172*c*d*x^2 + 136*d^2*x^4) + b^3*(-105*c^3 +
70*c^2*d*x^2 - 56*c*d^2*x^4 + 48*d^3*x^6)) + 15*(b*c - a*d)^(7/2)*(7*b*c +
a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sq
rt[b*c - a*d]])/(384*b^2*d^(9/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [F] time = 3.10, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2], x]
```

fricas [A] time = 1.35, size = 574, normalized size = 2.42

$$\left[-\frac{15\left(7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}\right)-4\left(48b^4d^4x^6-105b^4c^3d+265ab^3c^2d^2-191a^2b^2cd^3+15a^3bd^4-8\left(7b^4cd^3-17ab^3d^4\right)x^4+2\left(35b^4c^2d^2-86ab^3cd^3+59a^2b^2d^4\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{1536b^2d^5},\ -\frac{15\left(7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)-2\left(48b^4d^4x^6-105b^4c^3d+265ab^3c^2d^2-191a^2b^2cd^3+15a^3bd^4-8\left(7b^4cd^3-17ab^3d^4\right)x^4+2\left(35b^4c^2d^2-86ab^3cd^3+59a^2b^2d^4\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{768b^2d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/1536*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c)*sqrt(b*d)) - 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a*b^3*c^2*
d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^4
 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/(b^2*d^5), -1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2
+ b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a
*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3
*d^4)*x^4 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5)]
```

giac [A] time = 0.57, size = 304, normalized size = 1.28

$$\frac{\left(\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(4\left(bx^2+a\right)\left(\frac{6\left(bx^2+a\right)}{b^2d}-\frac{7b^3cd^5+ab^2d^6}{b^4d^7}\right)+\frac{5\left(7b^4c^2d^4-6ab^3cd^5-a^2b^2d^6\right)}{b^4d^7}\right)-\frac{15\left(7b^5c^3d^3-13ab^4c^2d^4+5a^2b^3cd^5+a^3b^2d^6\right)}{b^4d^7}\right)-\frac{15\left(7b^4c^4-20ab^3c^3d+18a^2b^2c^2d^2-4a^3bcd^3-a^4d^4\right)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,bd^4}\right)b}{384\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b^2*d) - (7*b^3*c*d^5 + a*b^2*d^6)/(b^4*d^7
)) + 5*(7*b^4*c^2*d^4 - 6*a*b^3*c*d^5 - a^2*b^2*d^6)/(b^4*d^7)) - 15*(7*b^5
*c^3*d^3 - 13*a*b^4*c^2*d^4 + 5*a^2*b^3*c*d^5 + a^3*b^2*d^6)/(b^4*d^7)) - 1
```

```
5*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^
4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*b*d^4))*b/abs(b)
```

maple [B] time = 0.02, size = 770, normalized size = 3.25

$$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(-96\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b^3 d^3 x^6-272\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a b^2 d^3 x^4+112\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b^3 c d^2 x^4+15 a^4 d^4 \ln\left(\frac{2 b d x^2+a d+b c+2\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2\sqrt{b d}}\right)+60 a^3 b c d^3 \ln\left(\frac{2 b d x^2+a d+b c+2\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2\sqrt{b d}}\right)-270 a^2 b^2 c^2 d^2 \ln\left(\frac{2 b d x^2+a d+b c+2\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2\sqrt{b d}}\right)+300 a b^3 c^3 d \ln\left(\frac{2 b d x^2+a d+b c+2\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2\sqrt{b d}}\right)-105 b^4 c^4 \ln\left(\frac{2 b d x^2+a d+b c+2\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}}{2\sqrt{b d}}\right)-236\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a^2 b d^3 x^2+344\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a b^2 c d^2 x^2-140\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b^3 c^2 d x^2-30\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a^3 d^3+382\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a^2 b c d^2-530\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,a b^2 c^2 d+210\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b^3 c^3\right)}{768\sqrt{x^4 b d+a d x^2+b c x^2+a c}\,\sqrt{b d}\,b d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/768*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-96*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2)*b^3*d^3*x^6-272*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)*a*b^2*d^3*x^4+112*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^3
*c*d^2*x^4-236*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*b*d^3*x^
2+344*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b^2*c*d^2*x^2-140*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^3*c^2*d*x^2+15*a^4*d^4*ln(
1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(
b*d)^(1/2))+60*a^3*b*c*d^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-270*a^2*b^2*c^2*d^2*ln(1/2*(2*b*d
*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2)
)+300*a*b^3*c^3*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-105*b^4*c^4*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-30*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^3*d^3+382*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*a^2*b*c*d^2-530*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2)*a*b^2*c^2*d+210*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b
^3*c^3)/b/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(a + bx^2\right)^{\frac{5}{2}}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.781 $\int \frac{x\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=164

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}\,d^{7/2}}+\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(bc-ad)}{24d^2}+\frac{\left(a+bx^2\right)^{5/}}{6}$$

Rubi [A] time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 63, 217, 206}

$$\frac{5\sqrt{a+bx^2}\sqrt{c+dx^2}\,(bc-ad)^2}{16d^3}-\frac{5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,(bc-ad)}{24d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}\,d^{7/2}}+\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(16*d^3) - (5*(b*c - a*d)
*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(24*d^2) + ((a + b*x^2)^(5/2)*Sqrt[c +
d*x^2])/(6*d) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]
*Sqrt[c + d*x^2])])/(16*Sqrt[b]*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{12d}$$

$$= -\frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt{a}}{\sqrt{c}}\right.}{16d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+d}}{6d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+d}}{6d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+d}}{6d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+d}}{6d}$$

Mathematica [A] time = 0.56, size = 164, normalized size = 1.00

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(33a^2d^2+2abd\left(13dx^2-20c\right)+b^2\left(15c^2-10cdx^2+8d^2x^4\right)\right)-\frac{15(bc-ad)^{7/2}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{b}}{48d^{7/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x^
2) + b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)) - (15*(b*c - a*d)^(7/2)*Sqrt[(b
*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d
]])/b)/(48*d^(7/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 1.86, size = 180, normalized size = 1.10

$$\frac{(ad-bc)^3\left(\frac{15b^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}+\frac{33d^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}-\frac{40bd\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\right)}{48d^3\left(d-\frac{b\left(c+dx^2\right)}{a+bx^2}\right)^3}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16\sqrt{b}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((-(b*c) + a*d)^3*((33*d^2*Sqrt[c + d*x^2])/Sqrt[a + b*x^2] - (40*b*d*(c +
d*x^2)^(3/2))/(a + b*x^2)^(3/2) + (15*b^2*(c + d*x^2)^(5/2))/(a + b*x^2)^(5
/2)))/(48*d^3*(d - (b*(c + d*x^2))/(a + b*x^2))^3) - (5*(b*c - a*d)^3*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(16*Sqrt[b]*d^(7/2
))
```

fricas [A] time = 1.49, size = 440, normalized size = 2.68

$$\left[-\frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}\right)-4\left(8\,b^3d^3x^4+15\,b^3c^2d-40\,ab^2cd^2+33\,a^2bd^3-2\left(5\,b^3cd^2-13\,ab^2d^3\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{192\,bd^4},\ \frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)+2\left(8\,b^3d^3x^4+15\,b^3c^2d-40\,ab^2cd^2+33\,a^2bd^3-2\left(5\,b^3cd^2-13\,ab^2d^3\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{96\,bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) -
4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*
c*d^2 - 13*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4), 1/96*(
15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/
```

```
2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d
^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3*c^
2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4)]
```

giac [A] time = 0.50, size = 210, normalized size = 1.28

$$\frac{\left(\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(\frac{4\left(bx^2+a\right)}{bd}-\frac{5\left(bcd^3-ad^4\right)}{bd^5}\right)+\frac{15\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)}{bd^5}\right)+\frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,d^3}\right)b}{48\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)*
(4*(b*x^2 + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d)))/(sqrt(b*d)*d^3))*b/abs(b)
```

maple [B] time = 0.02, size = 529, normalized size = 3.23

$$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(16\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2d^2x^4+15a^3d^3\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-45a^2bc\,d^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+45a\,b^2c^2d\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-15b^3c^3\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+52\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,ab\,d^2x^2-20\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2cd\,x^2+66\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,a^2d^2-80\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,abcd+30\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2c^2\right)}{96\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2)*b^2*d^2*x^4+52*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)*a*b*d^2*x^2-20*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*d*x^
2+15*a^3*d^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2))/(b*d)^(1/2))-45*a^2*b*c*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+45*a*b^2*c^2*d*ln
(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/
(b*d)^(1/2))-15*b^3*c^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2)*a^2*d^2-80*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a
*b*c*d+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^2)/d^3/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(b\,x^2+a\right)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^2\right)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.782 $\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3bc-7ad)}{8d^2}$$

Rubi [**A**] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {446, 102, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3bc-7ad)}{8d^2}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -(b*(3*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*d^2) + (b*(a + b*x^
2)^(3/2)*Sqrt[c + d*x^2])/(4*d) - (a^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2]
)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d +
15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(
8*d^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(2a^2d-\frac{1}{2}b(3bc-7ad)x\right)}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{4d}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{\text{Subst}\left(\int \frac{2a^3d^2+\frac{1}{4}b\left(3b^2c^2-10a}{x\sqrt{a+bx}\sqrt{c-}}\right.}{4d^2}$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{1}{2}a^3\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{}}\right.$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} + a^3\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x\right.$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} +$$

$$= -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} +$$

Mathematica [A] time = 0.67, size = 213, normalized size = 1.14

$$\frac{1}{8}\left(-\frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} + \frac{\left(-15a^3d^3+25a^2bcd^2-13ab^2c^2d+3b^3c^3\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{d^{5/2}\sqrt{c+dx^2}\sqrt{bc-ad}} + \frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\left(9ad-3bc+2bdx^2\right)}{d^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] ((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 9*a*d + 2*b*d*x^2))/d^2 + ((3
*b^3*c^3 - 13*a*b^2*c^2*d + 25*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[(b*(c + d*x^2
))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(d^(5/2
)*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]) - (8*a^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b
*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c])/8
```

IntegrateAlgebraic [A] time = 2.27, size = 276, normalized size = 1.48

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}+\frac{\left(15a^2\sqrt{b}d^2-10ab^{3/2}cd+3b^{5/2}c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8d^{5/2}}-\frac{b\sqrt{c+dx^2}\left(\frac{7a^2bd^2(c+dx^2)}{a+bx^2}-9a^2d^3+\frac{3b^3c^2(c+dx^2)}{a+bx^2}-\frac{10ab^2cd(c+dx^2)}{a+bx^2}+14abcd^2-5b^2c^2d\right)}{8d^2\sqrt{a+bx^2}\left(d-\frac{b(c+dx^2)}{a+bx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/8*(b*Sqrt[c + d*x^2]*(-5*b^2*c^2*d + 14*a*b*c*d^2 - 9*a^2*d^3 + (3*b^3*c
^2*(c + d*x^2))/(a + b*x^2) - (10*a*b^2*c*d*(c + d*x^2))/(a + b*x^2) + (7*a
^2*b*d^2*(c + d*x^2))/(a + b*x^2)))/(d^2*Sqrt[a + b*x^2]*(d - (b*(c + d*x^2
))/(a + b*x^2))^2) - (a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sq
rt[a + b*x^2])])/Sqrt[c] + ((3*b^(5/2)*c^2 - 10*a*b^(3/2)*c*d + 15*a^2*Sqrt
[b]*d^2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(8*d
^(5/2))
```

fricas [A] time = 10.42, size = 1075, normalized size = 5.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(8*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2
*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^
2*d^2)*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2
*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b/d)) + 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c))/d^2, 1/16*(4*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^
2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (3*b^2*c^
2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*
b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/d^2, 1/32*(16*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c
```

```
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c +
a^2*d)*x^2)) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(b/d)*log(8*b^2*d^
2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*
d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(2*
b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/d^2, 1/16*(
8*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - (3*b
^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c
 + a*b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/d^2]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [**B**] time = 0.02, size = 446, normalized size = 2.39

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(-8\sqrt{bd}\,a^{3}d^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+15\sqrt{ac}\,a^{2}b\,d^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-10\sqrt{ac}\,a\,b^{2}cd\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3\sqrt{ac}\,b^{3}c^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,b^{2}d\,x^{2}+18\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,abd-6\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,b^{2}c\right)}{16\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,\sqrt{ac}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(b*d)^(1/2)*(a*c)^(1/2)*x^2*b^2*d+15*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*a^2*b*d^2-
10*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1
/2))/(b*d)^(1/2))*(a*c)^(1/2)*a*b^2*c*d+3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*b^3*c^
2-8*(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2))/x^2)*a^3*d^2+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2)*(a*c)^(1/2)*a*b*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*
(a*c)^(1/2)*b^2*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d^2/(b*d)^(1/2)/(a*c
)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{x\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x*sqrt(c + d*x**2)), x)
```

3.783 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=187

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\,\sqrt{}}{}$$

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {446, 98, 154, 157, 63, 217, 206, 93, 208}

$$-\frac{a^{3/2}(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}}-\frac{b^{3/2}(bc-5ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad+bc)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*c*d) - (a*(a + b*x^2)^(3
/2)*Sqrt[c + d*x^2])/(2*c*x^2) - (a^(3/2)*(5*b*c - a*d)*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*c^(3/2)) - (b^(3/2)*(b*c - 5*
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*d^(3/
2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
```

```
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-ad)-b(bc+ad)x\right)}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2c}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\text{Subst}\Bigg(\int \frac{-\frac{1}{2}a^2d(5bc-ad)+\frac{1}{2}b^2c(bc-5}{x\sqrt{a+bx}\sqrt{c+dx}}}{2cd}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{\left(b^2(bc-5ad)\right)\text{Subst}\Big(\int \frac{1}{\sqrt{a+bx}}}{4d}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{(b(bc-5ad))\text{Subst}\Bigg(\int \frac{1}{\sqrt{c-\frac{ad}{b}+}}}{2d}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\tanh^{-1}\Bigg(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+dx}}}{2c^{3/2}}$$

$$= \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\tanh^{-1}\Bigg(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+dx}}}{2c^{3/2}}$$

Mathematica [A] time = 1.06, size = 196, normalized size = 1.05

$$\frac{1}{2}\left(\frac{a^{3/2}(ad-5bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{c^{3/2}}+\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(b^2cx^2-a^2d\right)}{cdx^2}-\frac{(bc-5ad)(bc-ad)^{3/2}\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{d^{3/2}\left(c+dx^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[a + b*x^2]*(-(a^2*d) + b^2*c*x^2)*Sqrt[c + d*x^2])/(c*d*x^2) - ((b*c
 - 5*a*d)*(b*c - a*d)^(3/2)*((b*(c + d*x^2))/(b*c - a*d))^(3/2)*ArcSinh[(Sq
```

```
rt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(d^(3/2)*(c + d*x^2)^(3/2)) + (a^(
3/2)*(-5*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])])/c^(3/2))/2
```

IntegrateAlgebraic [A] time = 2.41, size = 269, normalized size = 1.44

$$\frac{\left(a^{5/2}d-5a^{3/2}bc\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}}-\frac{\sqrt{c+dx^2}\,(bc-ad)\left(-\frac{a^2bd\left(c+dx^2\right)}{a+bx^2}+a^2d^2-\frac{ab^2c\left(c+dx^2\right)}{a+bx^2}+b^2c^2\right)}{2cd\sqrt{a+bx^2}\left(c-\frac{a\left(c+dx^2\right)}{a+bx^2}\right)\left(d-\frac{b\left(c+dx^2\right)}{a+bx^2}\right)}+\frac{\left(5ab^{3/2}d-b^{5/2}c\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*((b*c - a*d)*Sqrt[c + d*x^2]*(b^2*c^2 + a^2*d^2 - (a*b^2*c*(c + d*x^2)
)/(a + b*x^2) - (a^2*b*d*(c + d*x^2))/(a + b*x^2)))/(c*d*Sqrt[a + b*x^2]*(c
 - (a*(c + d*x^2))/(a + b*x^2))*(d - (b*(c + d*x^2))/(a + b*x^2))) + ((-5*a
^(3/2)*b*c + a^(5/2)*d)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a +
 b*x^2])])/(2*c^(3/2)) + ((-(b^(5/2)*c) + 5*a*b^(3/2)*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(2*d^(3/2))
```

fricas [A] time = 8.11, size = 1097, normalized size = 5.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*
d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (5*a*b*c*d - a^2*d^2)*x^2
*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^
2 + a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(a/c))/x^4) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c))/(c*d*x^2), 1/8*(2*(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan(1/2*(
2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^
4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (5*a*b*c*d - a^2*d^2)*x^2*sqrt(a/c)*log
(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x
^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt
(a/c))/x^4) + 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x
^2), 1/8*(2*(5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c +
(a*b*c + a^2*d)*x^2)) - (b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x
^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2
*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(b^2*c
```

```
*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2), 1/4*((5*a*b*c*d -
 a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) +
(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d
)*x^2)) + 2*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

giac [B] time = 0.87, size = 558, normalized size = 2.98

$$\frac{b\left(\frac{2\sqrt{b^2c+(bx^2+a)bd-abd}\,\sqrt{bx^2+a}\,b}{d}+\frac{\left(\sqrt{bd}\,b^2c-5\sqrt{bd}\,abd\right)\log\left(\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d^2}-\frac{2\left(5\sqrt{bd}\,a^2b^2c-\sqrt{bd}\,a^3bd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc}-\frac{4\left(\sqrt{bd}\,a^2b^4c^2-2\sqrt{bd}\,a^3b^3cd+\sqrt{bd}\,a^4b^2d^2-\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2b^2c-\sqrt{bd}\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^3bd\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4\right)c}\right)}{4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*b/d + (sqrt(
b*d)*b^2*c - 5*sqrt(b*d)*a*b*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^2)/d^2 - 2*(5*sqrt(b*d)*a^2*b^2*c - sqrt(b*d)*
a^3*b*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c)
- 4*(sqrt(b*d)*a^2*b^4*c^2 - 2*sqrt(b*d)*a^3*b^3*c*d + sqrt(b*d)*a^4*b^2*d^
2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2*a^2*b^2*c - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d))^2*a^3*b*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 -
2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2
*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^4)*c))/abs(b)
```

maple [B] time = 0.02, size = 423, normalized size = 2.26

$$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(\sqrt{bd}\,a^3d^2x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)-5\sqrt{bd}\,a^2bcd\,x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+5\sqrt{ac}\,a\,b^2cd\,x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-\sqrt{ac}\,b^3c^2x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2c\,x^2-2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,a^2d\right)}{4\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,\sqrt{ac}\,cd\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c*(5*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b^2*c*d*(a*c)
^(1/2)-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2))/(b*d)^(1/2))*x^2*b^3*c^2*(a*c)^(1/2)+ln((a*d*x^2+b*c*x^2+2*a*c+2*(
a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^3*d^2*(b*d)^(1/2
)-5*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2))/x^2)*x^2*a^2*b*c*d*(b*d)^(1/2)+2*x^2*b^2*c*(a*c)^(1/2)*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-2*a^2*d*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*
```

```
x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^2/(b*d)^(
1/2)/(a*c)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^2+a\right)^{5/2}}{x^3\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x^3*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**3/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**3*sqrt(c + d*x**2)), x)
```

3.784 $\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}\,(7bc-3ad)}{8c^2x^2}$$

Rubi [A] time = 0.20, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {446, 98, 149, 157, 63, 217, 206, 93, 208}

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}\,(7bc-3ad)}{8c^2x^2}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a*(7*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*c^2*x^2) - (a*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(4*c*x^4) - (Sqrt[a]*(15*b^2*c^2 - 10*a*b*c*d
 + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])
/(8*c^(5/2)) + (b^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c +
 d*x^2])])/Sqrt[d]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^3\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$= -\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(7bc-3ad)-2b^2cx\right)}{x^2\sqrt{c+dx}}\,dx,x,x^2\right)}{4c}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{4}a\left(15b^2c^2-10abcd+3\right.}{x\sqrt{a+bx}\sqrt{c}}\right.}{4c^2}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4} + \frac{1}{2}b^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c}}\right.$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4} + b^2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}\left(15b^2c^2-10abcd+3a^2\right.}{8c^{5/2}}$$

$$= -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}\left(15b^2c^2-10abcd+3a^2\right.}{8c^{5/2}}$$

Mathematica [A] time = 1.25, size = 206, normalized size = 1.07

$$-\frac{\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}+\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-2ac+3adx^2-9bcx^2\right)}{8c^2x^4}+\frac{(bc-ad)^{5/2}\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{5/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 9*b*c*x^2 + 3*a*d*x^2))/(8*c^2
*x^4) + ((b*c - a*d)^(5/2)*((b*(c + d*x^2))/(b*c - a*d))^(5/2)*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[d]*(c + d*x^2)^(5/2)) - (Sqrt
[a]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])
/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*c^(5/2))
```

IntegrateAlgebraic [F] time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 6.71, size = 1123, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(8*b^2*c^2*x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^
2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)
*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/32*(16*b^2*c^2*x^4*sqrt(-b/d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/
(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*c*d + 3*a
^2*d^2)*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
+ 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), 1/16*(4*b^2*c^2*x^4*sqrt(b/d)
*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(
b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^
4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/16*(8*b^2*c^2*x^4*sqrt(-b/d)*
arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/
d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*c*d +
3*a^2*d^2)*x^4*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 +
```

```
 a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) +
 2*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^
2*x^4)]
```

giac [B] time = 1.18, size = 1175, normalized size = 6.12

$$-\frac{\left(\frac{4\sqrt{bd}b^2\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d}+\frac{\left(15\sqrt{bd}ab^3c^2-10\sqrt{bd}a^2b^2cd+3\sqrt{bd}a^3bd^2\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,bc^2}+\frac{2\left(9\sqrt{bd}ab^9c^5-39\sqrt{bd}a^2b^8c^4d+66\sqrt{bd}a^3b^7c^3d^2-54\sqrt{bd}a^4b^6c^2d^3+21\sqrt{bd}a^5b^5cd^4-3\sqrt{bd}a^6b^4d^5-27\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ab^7c^4+40\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^2b^6c^3d+10\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^3b^5c^2d^2-32\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^4b^4cd^3+9\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2a^5b^3d^4+27\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4ab^5c^3+9\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4a^2b^4c^2d+21\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4a^3b^3cd^2-9\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4a^4b^2d^3-9\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^6ab^3c^2-10\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^6a^2b^2cd+3\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^6a^3bd^2\right)}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^4\right)^2c^2}\right)b}{8|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(b*d)*b^2*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2
+ a)*b*d - a*b*d))^2)/d + (15*sqrt(b*d)*a*b^3*c^2 - 10*sqrt(b*d)*a^2*b^2*c*
d + 3*sqrt(b*d)*a^3*b*d^2)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sq
rt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sq
rt(-a*b*c*d)*b*c^2) + 2*(9*sqrt(b*d)*a*b^9*c^5 - 39*sqrt(b*d)*a^2*b^8*c^4*d
 + 66*sqrt(b*d)*a^3*b^7*c^3*d^2 - 54*sqrt(b*d)*a^4*b^6*c^2*d^3 + 21*sqrt(b*
d)*a^5*b^5*c*d^4 - 3*sqrt(b*d)*a^6*b^4*d^5 - 27*sqrt(b*d)*(sqrt(b*x^2 + a)*
sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^7*c^4 + 40*sqrt(b*
d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^
2*b^6*c^3*d + 10*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d))^2*a^3*b^5*c^2*d^2 - 32*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^4*c*d^3 + 9*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^5*
b^3*d^4 + 27*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^4*a*b^5*c^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^4*c^2*d + 21*sqrt(b*d)*(sqrt(b*x
^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^3*c*d^2
- 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^4*a^4*b^2*d^3 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
+ (b*x^2 + a)*b*d - a*b*d))^6*a*b^3*c^2 - 10*sqrt(b*d)*(sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2*b^2*c*d + 3*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^3*
b*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*s
qrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)^2*c^2))*b/abs(b)
```

maple [B] time = 0.02, size = 464, normalized size = 2.42

$$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(-3\sqrt{bd}\,a^3d^2x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+10\sqrt{bd}\,a^2bcd\,x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)-15\sqrt{bd}\,a\,b^2c^2x^4\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+8\sqrt{ac}\,b^3c^2x^4\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+6\sqrt{bd}\,\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,a^2d\,x^2-18\sqrt{bd}\,\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,abc\,x^2-4\sqrt{bd}\,\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,a^2c\right)}{16\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,\sqrt{ac}\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2*(8*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^4*b^3*c^2*(a*c
)^(1/2)-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2))/x^2)*x^4*a^3*d^2*(b*d)^(1/2)+10*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*
c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*b*c*d*(b*d)^(1/2
)-15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2))/x^2)*x^4*a*b^2*c^2*(b*d)^(1/2)+6*x^2*a^2*d*(b*d)^(1/2)*(a*c)^(1/2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-18*x^2*a*b*c*(b*d)^(1/2)*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-4*a^2*c*(b*d)^(1/2)*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^4/(b*d)^(1
/2)/(a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{x^5\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(x^5*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{2}\right)^{\frac{5}{2}}}{x^{5}\sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/x**5/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(x**5*sqrt(c + d*x**2)), x)
```

3.785 $\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{36}\sqrt{2-3x^2}\,\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{3x^2-1}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.231, Rules used = {446, 80, 50, 53, 619, 216}

$$-\frac{1}{36}\sqrt{2-3x^2}\,\left(3x^2-1\right)^{3/2}-\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{3x^2-1}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-7*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/72 - (Sqrt[2 - 3*x^2]*(-1 + 3*x^2)^(3
/2))/36 - (7*ArcSin[3 - 6*x^2])/144
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx,x,x^2\right)\\
&= -\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2}+\frac{7}{24}\operatorname{Subst}\left(\int \frac{\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx,x,x^2\right)\\
&= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2}+\frac{7}{48}\operatorname{Subst}\Bigg(\int \frac{1}{\sqrt{2-3x}\,\sqrt{-1+3x}}\\
&= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2}+\frac{7}{48}\operatorname{Subst}\Bigg(\int \frac{1}{\sqrt{-2+9x-9x^2}}\,d\\
&= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2}-\frac{7}{432}\operatorname{Subst}\Bigg(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,9\big(\\
&= -\frac{7}{72}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{36}\sqrt{2-3x^2}\,\left(-1+3x^2\right)^{3/2}-\frac{7}{144}\sin^{-1}\left(3-6x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.68

$$\frac{1}{72}\left(-7\sin^{-1}\left(\sqrt{2-3x^2}\right)-\sqrt{-9x^4+9x^2-2}\,\left(6x^2+5\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-((5 + 6*x^2)*Sqrt[-2 + 9*x^2 - 9*x^4]) - 7*ArcSin[Sqrt[2 - 3*x^2]])/72
```

IntegrateAlgebraic [A] time = 0.53, size = 96, normalized size = 1.48

$$-\frac{\sqrt{2-3x^2}\left(\frac{7\left(2-3x^2\right)}{3x^2-1}+9\right)}{72\sqrt{3x^2-1}\left(\frac{2-3x^2}{3x^2-1}+1\right)^2}-\frac{7}{72}\tan^{-1}\left(\frac{\sqrt{2-3x^2}}{\sqrt{3x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -1/72*(Sqrt[2 - 3*x^2]*(9 + (7*(2 - 3*x^2))/(-1 + 3*x^2)))/(Sqrt[-1 + 3*x^2
]*(1 + (2 - 3*x^2)/(-1 + 3*x^2))^2) - (7*ArcTan[Sqrt[2 - 3*x^2]/Sqrt[-1 + 3
*x^2]])/72
```

fricas [A] time = 0.71, size = 72, normalized size = 1.11

$$-\frac{1}{72}\left(6\,x^2+5\right)\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}-\frac{7}{144}\,\arctan\left(\frac{3\,\sqrt{3\,x^2-1}\left(2\,x^2-1\right)\sqrt{-3\,x^2+2}}{2\left(9\,x^4-9\,x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 7/144*arctan(3/2*sqrt(
3*x^2 - 1)*(2*x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

giac [A] time = 0.38, size = 40, normalized size = 0.62

$$-\frac{1}{72}\left(6\,x^2+5\right)\sqrt{3\,x^2-1}\,\sqrt{-3\,x^2+2}+\frac{7}{72}\,\arcsin\left(\sqrt{3\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 7/72*arcsin(sqrt(3*x^2
 - 1))
```

maple [A] time = 0.03, size = 81, normalized size = 1.25

$$\frac{\sqrt{3x^2-1}\;\sqrt{-3x^2+2}\;\left(-12\sqrt{-9x^4+9x^2-2}\;x^2+7\arcsin\left(6x^2-3\right)-10\sqrt{-9x^4+9x^2-2}\right)}{144\sqrt{-9x^4+9x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/144*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(-12*x^2*(-9*x^4+9*x^2-2)^(1/2)+7*ar
csin(6*x^2-3)-10*(-9*x^4+9*x^2-2)^(1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

maxima [A] time = 2.01, size = 46, normalized size = 0.71

$$-\frac{1}{12}\sqrt{-9\,x^4+9\,x^2-2}\,x^2-\frac{5}{72}\sqrt{-9\,x^4+9\,x^2-2}+\frac{7}{144}\arcsin\left(6\,x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(-9*x^4 + 9*x^2 - 2)*x^2 - 5/72*sqrt(-9*x^4 + 9*x^2 - 2) + 7/144*
arcsin(6*x^2 - 3)
```

mupad [B] time = 12.04, size = 414, normalized size = 6.37

$$-\frac{7\,\mathrm{atan}\left(\frac{\sqrt{3\,x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3\,x^2}}\right)}{36}+\frac{\frac{7\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}+\frac{143\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^3}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^3}-\frac{143\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^5}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^5}-\frac{7\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^7}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^7}+\frac{\sqrt{2}\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^2 4\mathrm{i}}{9\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^2}-\frac{\sqrt{2}\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^4 40\mathrm{i}}{9\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^4}+\frac{\sqrt{2}\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^6 4\mathrm{i}}{9\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^6}}{\frac{4\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^2}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^2}+\frac{6\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^4}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^4}+\frac{4\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^6}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^6}+\frac{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^8}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^8}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

```
[Out] ((7*((3*x^2 - 1)^(1/2) - 1i))/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))) + (143*((3
*x^2 - 1)^(1/2) - 1i)^3)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^3) - (143*((3*x^
2 - 1)^(1/2) - 1i)^5)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^5) - (7*((3*x^2 - 1
)^(1/2) - 1i)^7)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^7) + (2^(1/2)*((3*x^2 -
1)^(1/2) - 1i)^2*4i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^2) - (2^(1/2)*((3*x^2
 - 1)^(1/2) - 1i)^4*40i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^4) + (2^(1/2)*((3
*x^2 - 1)^(1/2) - 1i)^6*4i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^6))/((4*((3*x^
2 - 1)^(1/2) - 1i)^2)/(2^(1/2) - (2 - 3*x^2)^(1/2))^2 + (6*((3*x^2 - 1)^(1/
2) - 1i)^4)/(2^(1/2) - (2 - 3*x^2)^(1/2))^4 + (4*((3*x^2 - 1)^(1/2) - 1i)^6
)/(2^(1/2) - (2 - 3*x^2)^(1/2))^6 + ((3*x^2 - 1)^(1/2) - 1i)^8/(2^(1/2) - (
2 - 3*x^2)^(1/2))^8 + 1) - (7*atan(((3*x^2 - 1)^(1/2) - 1i)/(2^(1/2) - (2 -
 3*x^2)^(1/2))))/36
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

3.786 $\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{3x^2-1}\;-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 50, 53, 619, 216}

$$-\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{3x^2-1}\;-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2])/6 - ArcSin[3 - 6*x^2]/12
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
 1, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{-1+3x}}{\sqrt{2-3x}}\,dx,x,x^2\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{2-3x}\,\sqrt{-1+3x}}\,dx,x,x^2\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-2+9x-9x^2}}\,dx,x,x^2\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{36}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,9\left(1-2x^2\right)\right)\\ &= -\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{-1+3x^2}-\frac{1}{12}\sin^{-1}\left(3-6x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.95

$$\frac{1}{6}\left(-\sin^{-1}\left(\sqrt{2-3x^2}\right)-\sqrt{-9x^4+9x^2-2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] (-Sqrt[-2 + 9*x^2 - 9*x^4] - ArcSin[Sqrt[2 - 3*x^2]])/6
```

IntegrateAlgebraic [A] time = 0.16, size = 59, normalized size = 1.51

$$\frac{1}{3}\tan^{-1}\left(\frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}-1}\right)-\frac{1}{6}\sqrt{2-3x^2}\,\sqrt{3x^2-1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

```
[Out] -1/6*(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2]) + ArcTan[Sqrt[-1 + 3*x^2]/(-1 + Sqr
t[2 - 3*x^2])]/3
```

fricas [B] time = 1.25, size = 65, normalized size = 1.67

$$-\frac{1}{6}\sqrt{3x^2-1}\sqrt{-3x^2+2}-\frac{1}{12}\arctan\left(\frac{3\sqrt{3x^2-1}\left(2x^2-1\right)\sqrt{-3x^2+2}}{2\left(9x^4-9x^2+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 1/12*arctan(3/2*sqrt(3*x^2 - 1)*(2*
x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

giac [A] time = 0.45, size = 33, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{1}{6}\arcsin\left(\sqrt{3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 1/6*arcsin(sqrt(3*x^2 - 1))
```

maple [A] time = 0.01, size = 60, normalized size = 1.54

$$\frac{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}\,\left(\arcsin\left(6x^2-3\right)-2\sqrt{-9x^4+9x^2-2}\right)}{12\sqrt{-9x^4+9x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/12*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(arcsin(6*x^2-3)-2*(-9*x^4+9*x^2-2)^(
1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

maxima [A] time = 2.00, size = 27, normalized size = 0.69

$$-\frac{1}{6}\sqrt{-9x^4+9x^2-2}+\frac{1}{12}\arcsin\left(6x^2-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(-9*x^4 + 9*x^2 - 2) + 1/12*arcsin(6*x^2 - 3)
```

mupad [B] time = 2.74, size = 206, normalized size = 5.28

$$-\frac{\mathrm{atan}\left(\frac{\sqrt{3\,x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3\,x^2}}\right)}{3}-\frac{-\frac{\sqrt{3\,x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3\,x^2}}+\frac{{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^3}+\frac{\sqrt{2}\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^2\,4\mathrm{i}}{3\,{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^2}}{\frac{2\,{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^2}+\frac{{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

```
[Out] - atan(((3*x^2 - 1)^(1/2) - 1i)/(2^(1/2) - (2 - 3*x^2)^(1/2)))/3 - (((3*x^2
 - 1)^(1/2) - 1i)^3/(2^(1/2) - (2 - 3*x^2)^(1/2))^3 - ((3*x^2 - 1)^(1/2) -
1i)/(2^(1/2) - (2 - 3*x^2)^(1/2)) + (2^(1/2)*((3*x^2 - 1)^(1/2) - 1i)^2*4i)
/(3*(2^(1/2) - (2 - 3*x^2)^(1/2))^2))/((2*((3*x^2 - 1)^(1/2) - 1i)^2)/(2^(1
/2) - (2 - 3*x^2)^(1/2))^2 + ((3*x^2 - 1)^(1/2) - 1i)^4/(2^(1/2) - (2 - 3*x
^2)^(1/2))^4 + 1)
```

sympy [A] time = 6.11, size = 66, normalized size = 1.69

$$\frac{\begin{cases} -\frac{\sqrt{2-3x^2}\,\sqrt{3x^2-1}}{2}+\frac{\operatorname{asin}\left(\sqrt{3x^2-1}\right)}{2} & \text{for}\: \left(x \geq \frac{\sqrt{3}}{3} \wedge x < \frac{\sqrt{6}}{3}\right) \vee \left(x \leq -\frac{\sqrt{3}}{3} \wedge x > -\frac{\sqrt{6}}{3}\right) \end{cases}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2 - 3*x**2)*sqrt(3*x**2 - 1)/2 + asin(sqrt(3*x**2 - 1))/2,
 ((x >= sqrt(3)/3) & (x < sqrt(6)/3)) | ((x <= -sqrt(3)/3) & (x > -sqrt(6)/
3))))/3
```

3.787 $\int \frac{x^5}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+bc)}{8b^2d^2}+\frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd}$$

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 90, 80, 63, 217, 206}

$$-\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+bc)}{8b^2d^2}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}+\frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(8*b^2*d^2) + (x^2*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(4*b*d) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
```

```
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= \frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{4bd}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{Su}}{16}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{Su}}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{Su}}{8b}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8b^2d^2} + \frac{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4bd} - \frac{\left(4abcd-3(bc+ad)^2\right)\text{tan}}{8b^{5/2}d^{5/2}}$$

Mathematica [A] time = 0.27, size = 154, normalized size = 1.09

$$\frac{\sqrt{bc-ad}\,\left(3a^2d^2+2abcd+3b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\,\sqrt{a+bx^2}\,\left(c+dx^2\right)\left(-3ad-3bc+2bdx^2\right)}{8b^3d^{5/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(-3*b*c - 3*a*d + 2*b*d*x^2) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^3*d^(5/2)*Sq
rt[c + d*x^2])
```

IntegrateAlgebraic [F] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 1.54, size = 336, normalized size = 2.38

$$\left[\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{4}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+8\,\left(b^{2}cd+abd^{2}\right)x^{2}+4\,\left(2\,bdx^{2}+bc+ad\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{bd}\right)+4\,\left(2\,b^{2}d^{2}x^{2}-3\,b^{2}cd-3\,abd^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{32\,b^{3}d^{3}}, -\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^{2}+bc+ad\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{-bd}}{2\,\left(b^{2}d^{2}x^{4}+abcd+\left(b^{2}cd+abd^{2}\right)x^{2}\right)}\right)-2\,\left(2\,b^{2}d^{2}x^{2}-3\,b^{2}cd-3\,abd^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{16\,b^{3}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3*
b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3), -1/16*((3*
b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^
2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3)]
```

giac [A] time = 0.55, size = 160, normalized size = 1.13

$$\frac{\left(\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\,\sqrt{bx^{2}+a}\left(\frac{2\left(bx^{2}+a\right)}{b^{3}d}-\frac{3\,b^{6}cd+5\,ab^{5}d^{2}}{b^{8}d^{3}}\right)-\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\log\left(\left|-\sqrt{bx^{2}+a}\,\sqrt{bd}+\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,b^{2}d^{2}}\right)b}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/(
b^3*d) - (3*b^6*c*d + 5*a*b^5*d^2)/(b^8*d^3)) - (3*b^2*c^2 + 2*a*b*c*d + 3*
a^2*d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^2*d^2))*b/abs(b)
```

maple [B] time = 0.04, size = 340, normalized size = 2.41

$$\frac{\left(3a^{2}d^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\,\sqrt{bd}}\right)+2abcd\ln\left(\frac{2bd\,x^{2}+ad+bc+2\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\,\sqrt{bd}}\right)+3b^{2}c^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\,\sqrt{bd}}\right)+4\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,bd\,x^{2}-6\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,ad-6\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,bc\right)\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{16\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,b^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/16*(4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*d*x^2+3*a^2*d^2*l
n(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))
/(b*d)^(1/2))+2*a*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^
```

```
2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*b^2*c^2*ln(1/2*(2*b*d*x^2+a*d+b*c+
2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-6*(b*d)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d-6*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/d^2/b^2/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 22.54, size = 550, normalized size = 3.90

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{4\,b^{5/2}\,d^{5/2}}-\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(\frac{3\,a^2\,b\,d^2}{4}+\frac{a\,b^2\,c\,d}{2}+\frac{3\,b^3\,c^2}{4}\right)}{d^6\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(\frac{11\,a^2\,d^2}{4}+\frac{25\,a\,b\,c\,d}{2}+\frac{11\,b^2\,c^2}{4}\right)}{d^5\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^7\,\left(\frac{3\,a^2\,d^2}{4}+\frac{a\,b\,c\,d}{2}+\frac{3\,b^2\,c^2}{4}\right)}{b^2\,d^3\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^7}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^5\,\left(\frac{11\,a^2\,d^2}{4}+\frac{25\,a\,b\,c\,d}{2}+\frac{11\,b^2\,c^2}{4}\right)}{b\,d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^5}+\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4\,\left(16\,a\,d+16\,b\,c\right)}{d^4\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}}{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^8}{{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^8}+\frac{b^4}{d^4}-\frac{4\,b^3\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d^3\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}+\frac{6\,b^2\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4}{d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}-\frac{4\,b\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6}{d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (atanh((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2)
- c^(1/2))))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(4*b^(5/2)*d^(5/2)) - (((
(a + b*x^2)^(1/2) - a^(1/2))*((3*b^3*c^2)/4 + (3*a^2*b*d^2)/4 + (a*b^2*c*d)
/2))/(d^6*((c + d*x^2)^(1/2) - c^(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2))^3
*((11*a^2*d^2)/4 + (11*b^2*c^2)/4 + (25*a*b*c*d)/2))/(d^5*((c + d*x^2)^(1/2
) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^7*((3*a^2*d^2)/4 + (3*b^2*
c^2)/4 + (a*b*c*d)/2))/(b^2*d^3*((c + d*x^2)^(1/2) - c^(1/2))^7) - (((a + b
*x^2)^(1/2) - a^(1/2))^5*((11*a^2*d^2)/4 + (11*b^2*c^2)/4 + (25*a*b*c*d)/2)
)/(b*d^4*((c + d*x^2)^(1/2) - c^(1/2))^5) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(
1/2) - a^(1/2))^4*(16*a*d + 16*b*c))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^4))
/(((a + b*x^2)^(1/2) - a^(1/2))^8/((c + d*x^2)^(1/2) - c^(1/2))^8 + b^4/d^4
 - (4*b^3*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^3*((c + d*x^2)^(1/2) - c^(1/2
))^2) + (6*b^2*((a + b*x^2)^(1/2) - a^(1/2))^4)/(d^2*((c + d*x^2)^(1/2) - c
^(1/2))^4) - (4*b*((a + b*x^2)^(1/2) - a^(1/2))^6)/(d*((c + d*x^2)^(1/2) -
c^(1/2))^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{a + b x^{2}}\,\sqrt{c + d x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.788 $\int \frac{x^3}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*d^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^2\right)}{4bd}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^2}\right)}{2b^2d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2b^2d}\\
&= \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 123, normalized size = 1.40

$$\frac{b\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)-\sqrt{bc-ad}\,(ad+bc)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{2b^2d^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*
d]])/(2*b^2*d^(3/2)*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 1.07, size = 122, normalized size = 1.39

$$\frac{(-ad-bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{2b^{3/2}d^{3/2}}-\frac{\sqrt{c+dx^2}\,(ad-bc)}{2bd\sqrt{a+bx^2}\left(\frac{b\left(c+dx^2\right)}{a+bx^2}-d\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*((-(b*c) + a*d)*Sqrt[c + d*x^2])/(b*d*Sqrt[a + b*x^2]*(-d + (b*(c + d*
x^2))/(a + b*x^2))) + ((-(b*c) - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sq
rt[d]*Sqrt[a + b*x^2])])/(2*b^(3/2)*d^(3/2))
```

fricas [A] time = 1.20, size = 256, normalized size = 2.91

$$\left[\frac{4\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd+(bc+ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)}{8\,b^2d^2},\ \frac{2\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,bd+(bc+ad)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)}{4\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*
(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b^2*d^
2), 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/
(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b^2*d^2)]
```

giac [A] time = 0.40, size = 104, normalized size = 1.18

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^2+a}\,\sqrt{bd}+\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}}{bd}}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((b*c + a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sq
rt(b*x^2 + a)/(b*d))/abs(b)
```

maple [B] time = 0.02, size = 200, normalized size = 2.27

$$-\frac{\left(ad\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+bc\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{4\sqrt{bd}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(a*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
b*d)^(1/2))/(b*d)^(1/2))+b*c*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d/(b*d)^(1/2)/b/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 5.15, size = 279, normalized size = 3.17

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)(a\,d+b\,c)}{d^3\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3(a\,d+b\,c)}{b\,d^2\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}-\frac{4\,\sqrt{a}\,\sqrt{c}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)(a\,d+b\,c)}{b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*(a*d + b*c))/(d^3*((c + d*x^2)^(1/2) - c^(1
/2))) + (((a + b*x^2)^(1/2) - a^(1/2))^3*(a*d + b*c))/(b*d^2*((c + d*x^2)^(
1/2) - c^(1/2))^3) - (4*a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d
^2*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^4/((c +
 d*x^2)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^2)^(1/2) - a^(1/2))^2
)/(d*((c + d*x^2)^(1/2) - c^(1/2))^2)) - (atanh((d^(1/2)*((a + b*x^2)^(1/2)
```

```
 - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d + b*c))/(b^(3/2)
*d^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.789 $\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}
$$

Mathematica [**A**] time = 0.07, size = 82, normalized size = 1.82

$$
\frac{\sqrt{c+dx^2}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[
d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

IntegrateAlgebraic [**A**] time = 0.53, size = 45, normalized size = 1.00

$$
\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{d}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[
d])
```

fricas [B] time = 1.24, size = 194, normalized size = 4.31

$$\left[\frac{\sqrt{bd} \log\left(8 \, b^{2} d^{2} x^{4} + b^{2} c^{2} + 6 \, abcd + a^{2} d^{2} + 8 \, \left(b^{2} cd + abd^{2}\right) x^{2} + 4 \, \left(2 \, bdx^{2} + bc + ad\right) \sqrt{bx^{2} + a} \sqrt{dx^{2} + c} \sqrt{bd}\right)}{4 \, bd}, -\frac{\sqrt{-bd} \arctan\left(\frac{\left(2 \, bdx^{2} + bc + ad\right) \sqrt{bx^{2} + a} \sqrt{dx^{2} + c} \sqrt{-bd}}{2 \, \left(b^{2} d^{2} x^{4} + abcd + \left(b^{2} cd + abd^{2}\right) x^{2}\right)}\right)}{2 \, bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^2))/(b*d)]
```

giac [A] time = 0.42, size = 54, normalized size = 1.20

$$-\frac{b \log\left({\left| -\sqrt{bx^{2} + a} \sqrt{bd} + \sqrt{b^{2} c + \left(bx^{2} + a\right) bd - abd} \right|}\right)}{\sqrt{bd} {\left| b \right|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -b*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d)))/(sqrt(b*d)*abs(b))
```

maple [B] time = 0.02, size = 103, normalized size = 2.29

$$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)}{2\sqrt{bd}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2))/(b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 1.22, size = 49, normalized size = 1.09

$$-\frac{2\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -(2*atan((b*((c + d*x^2)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^2)^(1/2)
 - a^(1/2)))))/(-b*d)^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^2}\,\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.790 $\int \frac{1}{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^2\right)\\ &= \mathrm{Subst}\left(\int \frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{c}}\end{aligned}$$

Mathematica [**A**] time = 0.01, size = 46, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

IntegrateAlgebraic [**A**] time = 0.77, size = 46, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c+dx^2}}{\sqrt{c}\,\sqrt{a+bx^2}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[a]*Sqr
t[c]))
```

fricas [**B**] time = 1.22, size = 204, normalized size = 4.43

$$\left[\frac{\sqrt{ac}\,\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2-4\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)}{4\,ac},\ \frac{\sqrt{-ac}\,\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\left(abcdx^4+a^2c^2+\left(abc^2+a^2cd\right)x^2\right)}\right)}{2\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x
^2 + c)*sqrt(a*c))/x^4)/(a*c), 1/2*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^2 +
 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^2))/(a*c)]
```

giac [**B**] time = 0.43, size = 89, normalized size = 1.93

$$-\frac{\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\,\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a
bs(b))
```

maple [**B**] time = 0.02, size = 103, normalized size = 2.24

$$-\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)}{2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2))/x^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 3.45, size = 136, normalized size = 2.96

$$-\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{2\,\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2))) - log(((
c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a^(1/2
)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c + d*
x^2)^(1/2) - c^(1/2))))/(2*a^(1/2)*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.791 $\int \frac{1}{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2acx^2}$$

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 96, 93, 208}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}-\frac{(bc+ad)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^2\right)}{4ac}\\ &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}-\frac{(bc+ad)\operatorname{Subst}\left(\int\frac{1}{-a+cx^2}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{2ac}\\ &= -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 1.00

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^2) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

IntegrateAlgebraic [A] time = 1.14, size = 120, normalized size = 1.32

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2a^{3/2}c^{3/2}}-\frac{\sqrt{c+dx^2}\,(ad-bc)}{2ac\sqrt{a+bx^2}\left(\frac{a\left(c+dx^2\right)}{a+bx^2}-c\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*((-(b*c) + a*d)*Sqrt[c + d*x^2])/(a*c*Sqrt[a + b*x^2]*(-c + (a*(c + d*
x^2))/(a + b*x^2))) + ((b*c + a*d)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[
c]*Sqrt[a + b*x^2])])/(2*a^(3/2)*c^(3/2))
```

fricas [A] time = 1.38, size = 278, normalized size = 3.05

$$\left[\frac{\sqrt{ac}\,(bc+ad)x^2\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^2+4\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{ac}}{x^4}\right)-4\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{8\,a^2c^2x^2}, -\frac{\sqrt{-ac}\,(bc+ad)x^2\arctan\left(\frac{\left((bc+ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-ac}}{2\left(abcdx^4+a^2c^2+\left(abc^2+a^2cd\right)x^2\right)}\right)+2\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,ac}{4\,a^2c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c)*(b*c + a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
)*a*c)/(a^2*c^2*x^2), -1/4*(sqrt(-a*c)*(b*c + a*d)*x^2*arctan(1/2*((b*c + a
*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 +
a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*c
)/(a^2*c^2*x^2)]
```

giac [B] time = 0.48, size = 413, normalized size = 4.54

$$\frac{\sqrt{bd}\,b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\,\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,ab^3cd}-\frac{2\left(b^3c^2-2\,ab^2cd+a^2bd^2-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2bc-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2ad\right)}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^2c-2\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2abd+\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^4\right)ab^2cd}\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

maple [B] time = 0.02, size = 209, normalized size = 2.30

$$\frac{\left(ad\,x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)+bc\,x^2\ln\left(\frac{ad\,x^2+bc\,x^2+2ac+2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}}{x^2}\right)-2\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\right)\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{4\sqrt{ac}\,\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,ac\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/a/c*(a*d*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2))/x^2)+b*c*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/x^2/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 6.70, size = 481, normalized size = 5.29

$$\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{8}+\frac{a\,d\,b}{8}\right)}{a^{3/2}\,c^{3/2}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{b^2}{8\,a\,c\,d}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{8}-\frac{3\,a\,b\,c\,d}{8}+\frac{b^2\,c^2}{8}\right)}{a^2\,c^2\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\,(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{4\,a^2\,c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{4\,a^2\,c^2}-\frac{d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{8\,a\,c\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((((a + b*x^2)^(1/2) - a^(1/2))*((b^2*c)/8 + (a*b*d)/8))/(a^(3/2)*c^(3/2)*d
*((c + d*x^2)^(1/2) - c^(1/2))) - b^2/(8*a*c*d) + (((a + b*x^2)^(1/2) - a^(
1/2))^2*((a^2*d^2)/8 + (b^2*c^2)/8 - (3*a*b*c*d)/8))/(a^2*c^2*d*((c + d*x^2
)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^3/((c + d*x^2)^(1/2)
- c^(1/2))^3 + (b*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c^
(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*
((c + d*x^2)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^2)^(1/2) - a^(1/2))/((c
+ d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(4*a^2*
c^2) - (log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(
1/2) - (a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/
2))))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d
))/(4*a^2*c^2) - (d*((a + b*x^2)^(1/2) - a^(1/2)))/(8*a*c*((c + d*x^2)^(1/2
) - c^(1/2)))
```

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.792 $\int \frac{1}{x^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=149

$$\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+bc)}{8a^2c^2x^2}-\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4}$$

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 103, 151, 12, 93, 208}

$$-\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}+\frac{3\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad+bc)}{8a^2c^2x^2}-\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(4*a*c*x^4) + (3*(b*c + a*d)*Sqrt[a + b*
x^2]*Sqrt[c + d*x^2])/(8*a^2*c^2*x^2) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2
)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*
c^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right) \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+ad)+bdx}{x^2\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{4ac} \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\text{Subst}\left(\int \frac{3b^2c^2+2abcd+3a^2d^2}{4x\sqrt{a+bx}\,\sqrt{c+dx}}\right.}{4a^2c^2} \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)}{} \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8a^2c^2x^2} + \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)}{} \\
&= -\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{4acx^4} + \frac{3(bc+ad)\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{8a^2c^2x^2} - \frac{\left(3b^2c^2+2abcd+3a^2d^2\right)}{8a^{5/2}c^{5}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 126, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,\left(-2ac+3adx^2+3bcx^2\right)}{8a^2c^2x^4} - \frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c + 3*b*c*x^2 + 3*a*d*x^2))/(8*a^2*c
^2*x^4) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(5/2))
```

IntegrateAlgebraic [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 2.04, size = 360, normalized size = 2.42

$$\left[\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\sqrt{ac}\,x^{4}\log\left(\frac{\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)x^{4}+8\,a^{2}c^{2}+8\left(abc^{2}+a^{2}cd\right)x^{2}-4\left((bc+ad)x^{2}+2\,ac\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{ac}}{x^{4}}\right)-4\left(2\,a^{2}c^{2}-3\left(abc^{2}+a^{2}cd\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{32\,a^{3}c^{3}x^{4}},\ \frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\sqrt{-ac}\,x^{4}\arctan\left(\frac{\left((bc+ad)x^{2}+2\,ac\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}\,\sqrt{-ac}}{2\left(abcdx^{4}+a^{2}c^{2}+\left(abc^{2}+a^{2}cd\right)x^{2}\right)}\right)-2\left(2\,a^{2}c^{2}-3\left(abc^{2}+a^{2}cd\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{16\,a^{3}c^{3}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a^
2*c^2 - 3*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^
3*x^4), 1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*
c*d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 - 3*(a*b*c^2 +
 a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^3*x^4)]
```

giac [B] time = 1.97, size = 1015, normalized size = 6.81

$$-\frac{\sqrt{bd}\,b^{6}d^{2}\left(\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(-\frac{b^{2}c+abd-\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}}{2\,\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,a^{2}b^{5}c^{2}d^{2}}-\frac{2\left(3\,b^{8}c^{5}-9\,ab^{7}c^{4}d+6\,a^{2}b^{6}c^{3}d^{2}+6\,a^{3}b^{5}c^{2}d^{3}-9\,a^{4}b^{4}cd^{4}+3\,a^{5}b^{3}d^{5}-9\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}b^{6}c^{4}-4\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}ab^{5}c^{3}d+26\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}a^{2}b^{4}c^{2}d^{2}-4\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}a^{3}b^{3}cd^{3}-9\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}a^{4}b^{2}d^{4}+9\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}b^{4}c^{3}+15\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}ab^{3}c^{2}d+15\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}a^{2}b^{2}cd^{2}+9\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}a^{3}bd^{3}-3\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{6}b^{2}c^{2}-2\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{6}abcd-3\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{6}a^{2}d^{2}\right)}{\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}-2\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}b^{2}c-2\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}abd+\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}\right)^{2}a^{2}b^{4}c^{2}d^{2}}\right)}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(b*d)*b^6*d^2*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(-1/2*(b^
2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a^2*b^5*c^2*d^2) - 2*(3*b^8*c
^5 - 9*a*b^7*c^4*d + 6*a^2*b^6*c^3*d^2 + 6*a^3*b^5*c^2*d^3 - 9*a^4*b^4*c*d^
4 + 3*a^5*b^3*d^5 - 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^2*b^6*c^4 - 4*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^2*a*b^5*c^3*d + 26*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(
b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c^2*d^2 - 4*(sqrt(b*x^2 + a)*sq
rt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^3*c*d^3 - 9*(sqrt(
b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^2*d^4
 + 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*
b^4*c^3 + 15*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^4*a*b^3*c^2*d + 15*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^4*a^2*b^2*c*d^2 + 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b*d^3 - 3*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c^2 - 2*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b*c*d - 3*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2*d^2)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
```

```
c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b
^2*c + (b*x^2 + a)*b*d - a*b*d))^4)^2*a^2*b^4*c^2*d^2))/abs(b)
```

maple [B] time = 0.03, size = 355, normalized size = 2.38

$$-\frac{\left(3a^{2}d^{2}x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+2abcd\,x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)+3b^{2}c^{2}x^{4}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2ac+2\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}}{x^{2}}\right)-6\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,ad\,x^{2}-6\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,bc\,x^{2}+4\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,ac\right)\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}}{16\sqrt{ac}\,\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,a^{2}c^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/16/a^2/c^2*(3*a^2*d^2*x^4*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)+2*a*b*c*d*x^4*ln((a*d*x^2+b*c*x^2+2*a*c
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)+3*b^2*c^2*x^4*ln((
a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^
2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*d*x^2-6*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*c*x^2+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(a*c)^(1/2)*a*c)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/x^4/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 21.58, size = 962, normalized size = 6.46

$$\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(3\,\sqrt{a}\,b^2\,c^{5/2}+3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^3\,c^3}-\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(3\,\sqrt{a}\,b^2\,c^{5/2}+3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^3\,c^3}-\frac{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2\,\left(\frac{11\,a^2\,b^2\,d^2}{64}+\frac{5\,a\,b^3\,c\,d}{16}+\frac{11\,b^4\,c^2}{64}\right)}{a^{5/2}\,c^{5/2}\,d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}-\frac{b^4}{64\,a^{3/2}\,c^{3/2}\,d^2}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(\frac{a^3\,b\,d^3}{32}-\frac{9\,a^2\,b^2\,c\,d^2}{16}-\frac{9\,a\,b^3\,c^2\,d}{16}+\frac{b^4\,c^3}{32}\right)}{a^3\,c^3\,d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{c\,b^4}{16}+\frac{a\,d\,b^3}{16}\right)}{a^2\,c^2\,d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^5\,\left(\frac{a^3\,d^3}{8}-\frac{7\,a^2\,b\,c\,d^2}{32}-\frac{7\,a\,b^2\,c^2\,d}{32}+\frac{b^3\,c^3}{8}\right)}{a^3\,c^3\,d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^5}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4\,\left(-\frac{7\,a^4\,d^4}{64}+\frac{a^3\,b\,c\,d^3}{8}+\frac{45\,a^2\,b^2\,c^2\,d^2}{64}+\frac{a\,b^3\,c^3\,d}{8}-\frac{7\,b^4\,c^4}{64}\right)}{a^{7/2}\,c^{7/2}\,d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}}{\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^6}{{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^6}+\frac{b^2\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^3\,\left(2\,c\,b^2+2\,a\,d\,b\right)}{\sqrt{a}\,\sqrt{c}\,d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^3}-\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^5\,\left(2\,a\,d+2\,b\,c\right)}{\sqrt{a}\,\sqrt{c}\,d\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^5}+\frac{{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^4\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{a\,c\,d^2\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^4}}+\frac{d^2\,{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}^2}{64\,a^{3/2}\,c^{3/2}\,{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}^2}+\frac{3\,d\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(a\,d+b\,c\right)}{32\,a^2\,c^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) -
(a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/(
(c + d*x^2)^(1/2) - c^(1/2)))*(3*a^(1/2)*b^2*c^(5/2) + 3*a^(5/2)*c^(1/2)*d^
2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^3*c^3) - (log(((a + b*x^2)^(1/2) - a^(1/2
))/((c + d*x^2)^(1/2) - c^(1/2)))*(3*a^(1/2)*b^2*c^(5/2) + 3*a^(5/2)*c^(1/2
```

```
)*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^3*c^3) - ((((a + b*x^2)^(1/2) - a^(1/
2))^2*((11*b^4*c^2)/64 + (11*a^2*b^2*d^2)/64 + (5*a*b^3*c*d)/16))/(a^(5/2)*
c^(5/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) - b^4/(64*a^(3/2)*c^(3/2)*d^2)
 + (((a + b*x^2)^(1/2) - a^(1/2))^3*((b^4*c^3)/32 + (a^3*b*d^3)/32 - (9*a^2
*b^2*c*d^2)/16 - (9*a*b^3*c^2*d)/16))/(a^3*c^3*d^2*((c + d*x^2)^(1/2) - c^(
1/2))^3) - (((a + b*x^2)^(1/2) - a^(1/2))*((b^4*c)/16 + (a*b^3*d)/16))/(a^2
*c^2*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (((a + b*x^2)^(1/2) - a^(1/2))^5*
((a^3*d^3)/8 + (b^3*c^3)/8 - (7*a*b^2*c^2*d)/32 - (7*a^2*b*c*d^2)/32))/(a^3
*c^3*d*((c + d*x^2)^(1/2) - c^(1/2))^5) + (((a + b*x^2)^(1/2) - a^(1/2))^4*
((45*a^2*b^2*c^2*d^2)/64 - (7*b^4*c^4)/64 - (7*a^4*d^4)/64 + (a*b^3*c^3*d)/
8 + (a^3*b*c*d^3)/8))/(a^(7/2)*c^(7/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^4)
)/(((a + b*x^2)^(1/2) - a^(1/2))^6/((c + d*x^2)^(1/2) - c^(1/2))^6 + (b^2*(
(a + b*x^2)^(1/2) - a^(1/2))^2)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) - (((
a + b*x^2)^(1/2) - a^(1/2))^3*(2*b^2*c + 2*a*b*d))/(a^(1/2)*c^(1/2)*d^2*((c
 + d*x^2)^(1/2) - c^(1/2))^3) - (((a + b*x^2)^(1/2) - a^(1/2))^5*(2*a*d + 2
*b*c))/(a^(1/2)*c^(1/2)*d*((c + d*x^2)^(1/2) - c^(1/2))^5) + (((a + b*x^2)^
(1/2) - a^(1/2))^4*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/(a*c*d^2*((c + d*x^2)^(
1/2) - c^(1/2))^4)) + (d^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(64*a^(3/2)*c^(
3/2)*((c + d*x^2)^(1/2) - c^(1/2))^2) + (3*d*((a + b*x^2)^(1/2) - a^(1/2))*
(a*d + b*c))/(32*a^2*c^2*((c + d*x^2)^(1/2) - c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.793 $\int \frac{x^5}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=129

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}\,(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d}$$

Rubi [A] time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 89, 80, 63, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}\,(bc-ad)}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2])) + (Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(2*b^2*d) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(5/2)*d^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, x^2\right)}{b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,d}{4b^2d}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx}{2b^3d}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,}{2b^3d}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}$$

Mathematica [**A**] time = 0.35, size = 185, normalized size = 1.43

$$\frac{\sqrt{a+bx^2}\,\sqrt{bc-ad}\,\left(-3a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - b\sqrt{d}\,\left(c+dx^2\right)\left(-3a^2d+ab\left(c-dx^2\right)+b^2cx^2\right)}{2b^3d^{3/2}\sqrt{a+bx^2}\,\sqrt{c+dx^2}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^2)*(-3*a^2*d + b^2*c*x^2 + a*b*(c - d*x^2))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*
x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(2*b
^3*d^(3/2)*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 2.33, size = 169, normalized size = 1.31

$$\frac{\sqrt{c+dx^2}\left(-\frac{2a^2bd\left(c+dx^2\right)}{a+bx^2}+3a^2d^2-2abcd+b^2c^2\right)}{2b^2d\sqrt{a+bx^2}\,(bc-ad)\left(\frac{b\left(c+dx^2\right)}{a+bx^2}-d\right)}+\frac{(-3ad-bc)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{2b^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2 - (2*a^2*b*d*(c + d*x^2))
/(a + b*x^2)))/(2*b^2*d*(b*c - a*d)*Sqrt[a + b*x^2]*(-d + (b*(c + d*x^2))/(
a + b*x^2))) + ((-(b*c) - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]
*Sqrt[a + b*x^2])])/(2*b^(5/2)*d^(3/2))
```

fricas [B] time = 1.45, size = 498, normalized size = 3.86

$$\left[\frac{\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2-4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)+4\left(ab^2cd-3\,a^2bd^2+\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{8\left(ab^4cd^2-a^2b^3d^3+\left(b^5cd^2-ab^4d^3\right)x^2\right)},\ \frac{\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2+\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)+2\left(ab^2cd-3\,a^2bd^2+\left(b^3cd-ab^2d^2\right)x^2\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{4\left(ab^4cd^2-a^2b^3d^3+\left(b^5cd^2-ab^4d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2
*b*d^2)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b*d)) + 4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*
x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d
^2 - a*b^4*d^3)*x^2), 1/4*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2
+ 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x^2)) + 2*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2
)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c
*d^2 - a*b^4*d^3)*x^2)]
```

giac [A] time = 0.54, size = 192, normalized size = 1.49

$$-\frac{2\,\sqrt{bd}\,a^2}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)b|b|}+\frac{\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\,\sqrt{bx^2+a}\,|b|}{2\,b^4d}+\frac{\left(\sqrt{bd}\,bc+3\,\sqrt{bd}\,ad\right)\log\left(\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{4\,b^2d^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*d)*a^2/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
+ (b*x^2 + a)*b*d - a*b*d))^2)*b*abs(b)) + 1/2*sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d)*sqrt(b*x^2 + a)*abs(b)/(b^4*d) + 1/4*(sqrt(b*d)*b*c + 3*sqrt(b*d)
*a*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)/(b^2*d^2*abs(b))
```

maple [B] time = 0.05, size = 553, normalized size = 4.29

$$-\frac{\left(3a^{2}b\,d^{2}x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2a\,b^{2}cd\,x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-b^{3}c^{2}x^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3a^{3}d^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2a^{2}bcd\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-a\,b^{2}c^{2}\ln\left(\frac{2bd\,x^{2}+ad+bc+2\sqrt{x^{4}bd+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}\,abd\,x^{2}+2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}\,b^{2}c\,x^{2}-6\sqrt{bd}\,\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,a^{2}d+2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}\,abc\right)\sqrt{d\,x^{2}+c}}{4\sqrt{b\,x^{2}+a}\,\sqrt{bd}\,\left(ad-bc\right)\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b^2*c*d-ln(1/2*
(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)
^(1/2))*x^2*b^3*c^2-2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)*x^2*a*b*d+2*(
(b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)*x^2*b^2*c+3*ln(1/2*(2*b*d*x^2+a*d+b*
c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^3*d^2-2
*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
))/(b*d)^(1/2))*a^2*b*c*d-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a*b^2*c^2-6*(b*d)^(1/2)*((b*x^2+a)
*(d*x^2+c))^(1/2)*a^2*d+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)*a*b*c)*(d
*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)/d/(b*d)^(1/2)/(a*d-b*c)/((b*x^2+a)*(d*x^2
+c))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.794 $\int \frac{x^3}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}\,(bc-ad)}$$

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 78, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[d]*Sqrt
[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(3/2)*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx,x,x^2\right)}{2b}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^2}\right)}{b^2}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\right)}{b^2}\\
&= \frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}
\end{aligned}
$$

Mathematica [A] time = 0.52, size = 118, normalized size = 1.42

$$
\frac{\frac{ab\left(c+dx^2\right)}{\sqrt{a+bx^2}\,(bc-ad)}+\frac{\sqrt{bc-ad}\,\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}}{b^2\sqrt{c+dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((a*b*(c + d*x^2))/((b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[b*c - a*d]*Sqrt[(b
*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d
]])/Sqrt[d])/(b^2*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 1.75, size = 83, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^2}}{\sqrt{d}\,\sqrt{a+bx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*Sqrt
[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])]/(b^(3/2)*Sqrt[d])
```

fricas [B] time = 1.44, size = 367, normalized size = 4.42

$$\left[\frac{4\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,abd+\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}\right)}{4\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)},\ \frac{2\,\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,abd-\left(abc-a^2d+\left(b^2c-abd\right)x^2\right)\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)}{2\left(ab^3cd-a^2b^2d^2+\left(b^4cd-ab^3d^2\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d + (a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*
(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*
x^2 + c)*sqrt(b*d)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2),
 1/2*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d - (a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2))
)/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

giac [B] time = 0.48, size = 135, normalized size = 1.63

$$\frac{\frac{4\,\sqrt{bd}\,ab}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)|b|}-\frac{\sqrt{bd}\,\log\left(\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{d|b|}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(b*d)*a*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2)*abs(b)) - sqrt(b*d)*log((sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(d*abs(b)))/b
```

maple [B] time = 0.03, size = 320, normalized size = 3.86

$$\frac{\left(abd\,x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-b^2c\,x^2\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+a^2d\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-abc\ln\left(\frac{2bd\,x^2+ad+bc+2\sqrt{x^4bd+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-2\sqrt{bd}\,\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,a\right)\sqrt{d\,x^2+c}}{2\sqrt{b\,x^2+a}\,\sqrt{bd}\,\left(ad-bc\right)\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2))/(b*d)^(1/2))*x^2*a*b*d-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^2*c+ln(1/2*(2*b*d*x^2+a*
d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2*d
-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
))/(b*d)^(1/2))*a*b*c-2*(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)*a)/b*(d*x^2
+c)^(1/2)/(b*x^2+a)^(1/2)/(b*d)^(1/2)/(a*d-b*c)/((b*x^2+a)*(d*x^2+c))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.795 $\int \frac{x}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {444, 37}

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.97

$$\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Sqrt[c + d*x^2]/((-(b*c) + a*d)*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.10, size = 34, normalized size = 1.00

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

fricas [A] time = 1.18, size = 48, normalized size = 1.41

$$-\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{abc-a^2d+\left(b^2c-abd\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

giac [B] time = 0.49, size = 70, normalized size = 2.06

$$-\frac{2\sqrt{bd}\,b}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\,\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^2)*abs(b))
```

maple [**A**] time = 0.00, size = 30, normalized size = 0.88

$$\frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,\left(a d-b c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*d-b*c)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 1.21, size = 45, normalized size = 1.32

$$\frac{d\,x^2+c}{\left(a\,d\,\sqrt{d\,x^2+c}-b\,c\,\sqrt{d\,x^2+c}\right)\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] (c + d*x^2)/((a*d*(c + d*x^2)^(1/2) - b*c*(c + d*x^2)^(1/2))*(a + b*x^2)^(1
/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^{2}\right)^{\frac{3}{2}} \sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.796 $\int \frac{x^5}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^2}\,(3bc-2ad)}{3b^2\sqrt{a+bx^2}\,(bc-ad)^2}$$

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 89, 78, 63, 217, 206}

$$-\frac{a^2\sqrt{c+dx^2}}{3b^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}\,(3bc-2ad)}{3b^2\sqrt{a+bx^2}\,(bc-ad)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*a*(3*b*c
- 2*a*d)*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(b^(5/2)*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{3b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,\right.}{2b^2}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x\right.}{b^3}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+}}{\sqrt{c+}}\right.}{b^3}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

Mathematica [**A**] time = 0.48, size = 214, normalized size = 1.56

$$\frac{\sqrt{c+dx^2}\left(\frac{\left(a+bx^2\right)\left(3b^2c^2-a^2d^2\right)}{d(bc-ad)^2}+\frac{a^2}{ad-bc}-\frac{3\left(a+bx^2\right)\left(\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}-\sqrt{d}\sqrt{a+bx^2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{d\sqrt{bc-ad}\sqrt{\frac{b\left(c+dx^2\right)}{bc-ad}}}\right)}{3b^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^2))/
(d*(b*c - a*d)^2) - (3*(a + b*x^2)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b
*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt
[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])))/(3*b
^2*(a + b*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 2.74, size = 114, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{b^{5/2}\sqrt{d}}-\frac{a\sqrt{c+dx^2}\left(\frac{ab(c+dx^2)}{a+bx^2}+3ad-6bc\right)}{3b^2\sqrt{a+bx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/3*(a*Sqrt[c + d*x^2]*(-6*b*c + 3*a*d + (a*b*(c + d*x^2))/(a + b*x^2)))/(
b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqr
t[d]*Sqrt[a + b*x^2])]/(b^(5/2)*Sqrt[d])
```

fricas [B] time = 1.71, size = 706, normalized size = 5.15

$$\left[\frac{3\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(b^4c^2-2ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^2\right)\sqrt{bd}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{bd}\right)+4\left(5a^2b^2cd-3a^3bd^2+2\left(3ab^3cd-2a^2b^2d^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{12\left(a^2b^5c^2d-2a^3b^4cd^2+a^4b^3d^3+\left(b^7c^2d-2ab^6cd^2+a^2b^5d^3\right)x^4+2\left(ab^6c^2d-2a^2b^5cd^2+a^3b^4d^3\right)x^2\right)}, -\frac{3\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(b^4c^2-2ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^2\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)-2\left(5a^2b^2cd-3a^3bd^2+2\left(3ab^3cd-2a^2b^2d^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{6\left(a^2b^5c^2d-2a^3b^4cd^2+a^4b^3d^3+\left(b^7c^2d-2ab^6cd^2+a^2b^5d^3\right)x^4+2\left(ab^6c^2d-2a^2b^5cd^2+a^3b^4d^3\right)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^
2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) +
4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3
+ (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d - 2*a^2*b^
5*c*d^2 + a^3*b^4*d^3)*x^2), -1/6*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 +
 (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x^2)) - 2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x
^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4
*b^3*d^3 + (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d -
 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^2)]
```

giac [B] time = 0.86, size = 333, normalized size = 2.43

$$-\frac{\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)}{2\sqrt{bd}\,b|b|}+\frac{4\left(3\sqrt{bd}\,ab^4c^2-5\sqrt{bd}\,a^2b^3cd+2\sqrt{bd}\,a^3b^2d^2-6\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2ab^2c+3\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2a^2bd+3\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^4a\right)}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)^3b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)
)^2)/(sqrt(b*d)*b*abs(b)) + 4/3*(3*sqrt(b*d)*a*b^4*c^2 - 5*sqrt(b*d)*a^2*b^
3*c*d + 2*sqrt(b*d)*a^3*b^2*d^2 - 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^2*c + 3*sqrt(b*d)*(sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b*d + 3*sqrt(
b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*
a)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*
b*d - a*b*d))^2)^3*b*abs(b))
```

maple [B] time = 0.05, size = 651, normalized size = 4.75

$$\frac{\left(-8\sqrt{bd}\,a^2bd^2x^4+12\sqrt{bd}\,ab^2cdx^4+3\sqrt{\left(bx^2+a\right)\left(dx^2+c\right)}\,a^2bd^2x^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-6\sqrt{\left(bx^2+a\right)\left(dx^2+c\right)}\,ab^2cdx^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3\sqrt{\left(bx^2+a\right)\left(dx^2+c\right)}\,b^3c^2x^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-6\sqrt{bd}\,a^3d^2x^2+2\sqrt{bd}\,a^2bcdx^2+12\sqrt{bd}\,ab^2c^2x^2+3\sqrt{\left(bx^2+a\right)\left(dx^2+c\right)}\,a^3d^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-6\sqrt{\left(bx^2+a\right)\left(dx^2+c\right)}\,a^2bcd\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)+3\sqrt{\left(bx^2+a\right)\left(dx^2+c\right)}\,ab^2c^2\ln\left(\frac{2bdx^2+ad+bc+2\sqrt{x^4bd+adx^2+bcx^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)-6\sqrt{bd}\,a^3cd+10\sqrt{bd}\,a^2bc^2\right)\sqrt{dx^2+c}}{6\sqrt{bx^2+a}\,\sqrt{bd}\,\left(ad-bc\right)^2\left(x^4bd+adx^2+bcx^2+ac\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*(-8*(b*d)^(1/2)*x^4*a^2*b*d^2+12*x^4*a*b^2*c*d*(b*d)^(1/2)+3*((b*x^2+a)
*(d*x^2+c))^(1/2)*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a^2*b*d^2-6*((b*x^2+a)*(d*x^2+c))^(1/2
)*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2))/(b*d)^(1/2))*x^2*a*b^2*c*d+3*((b*x^2+a)*(d*x^2+c))^(1/2)*ln(1/2*(2*b*d*
x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))
*x^2*b^3*c^2-6*x^2*a^3*d^2*(b*d)^(1/2)+2*(b*d)^(1/2)*x^2*a^2*b*c*d+12*x^2*a
*b^2*c^2*(b*d)^(1/2)+3*((b*x^2+a)*(d*x^2+c))^(1/2)*ln(1/2*(2*b*d*x^2+a*d+b*
c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^3*d^2-6
*((b*x^2+a)*(d*x^2+c))^(1/2)*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a^2*b*c*d+3*((b*x^2+a)*(d*x^2+c
))^(1/2)*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2))/(b*d)^(1/2))*a*b^2*c^2-6*a^3*c*d*(b*d)^(1/2)+10*(b*d)^(1/2)*a^2*
b*c^2)*(d*x^2+c)^(1/2)/b^2/(b*x^2+a)^(1/2)/(b*d)^(1/2)/(a*d-b*c)^2/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^2+a\right)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.797 $\int \frac{x^3}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{a\sqrt{c+dx^2}}{3b\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{c+dx^2}\,(3bc-ad)}{3b\sqrt{a+bx^2}\,(bc-ad)^2}$$

Rubi [**A**] time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 78, 37}

$$\frac{a\sqrt{c+dx^2}}{3b\left(a+bx^2\right)^{3/2}(bc-ad)}-\frac{\sqrt{c+dx^2}\,(3bc-ad)}{3b\sqrt{a+bx^2}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((3*b*c - a*d)*Sq
rt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= \frac{a\sqrt{c+dx^2}}{3b(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{(3bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{6b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^2}}{3b(bc-ad)\left(a+bx^2\right)^{3/2}}-\frac{(3bc-ad)\sqrt{c+dx^2}}{3b(bc-ad)^2\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.61

$$\frac{\sqrt{c+dx^2}\left(-2ac+adx^2-3bcx^2\right)}{3\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^
2)^(3/2))
```

IntegrateAlgebraic [A] time = 2.28, size = 64, normalized size = 0.72

$$\frac{\frac{a\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}-\frac{3c\sqrt{c+dx^2}}{\sqrt{a+bx^2}}}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-3*c*Sqrt[c + d*x^2])/Sqrt[a + b*x^2] + (a*(c + d*x^2)^(3/2))/(a + b*x^2)
^(3/2))/(3*(b*c - a*d)^2)
```

fricas [A] time = 1.55, size = 128, normalized size = 1.44

$$-\frac{\left((3\,bc-ad)x^2+2\,ac\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c
^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*
(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

giac [B] time = 0.63, size = 214, normalized size = 2.40

$$-\frac{2\left(3\sqrt{bd}\,b^{5}c^{2}-4\sqrt{bd}\,ab^{4}cd+\sqrt{bd}\,a^{2}b^{3}d^{2}-6\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}b^{3}c+3\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}b\right)}{3\left(b^{2}c-abd-\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}\right)^{3}b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(b*d)*b^5*c^2 - 4*sqrt(b*d)*a*b^4*c*d + sqrt(b*d)*a^2*b^3*d^2 -
 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^2*b^3*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^4*b)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*b*abs(b))
```

maple [A] time = 0.01, size = 63, normalized size = 0.71

$$-\frac{\sqrt{d\,x^{2}+c}\,\left(-ad\,x^{2}+3bc\,x^{2}+2ac\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/3*(d*x^2+c)^(1/2)*(-a*d*x^2+3*b*c*x^2+2*a*c)/(b*x^2+a)^(3/2)/(a^2*d^2-2*
a*b*c*d+b^2*c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.41, size = 139, normalized size = 1.56

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{2\,a\,c^2}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{x^2\,\left(3\,b\,c^2+a\,d\,c\right)}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{x^4\,\left(a\,d^2-3\,b\,c\,d\right)}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{d\,x^2+c}+\frac{a^2\,\sqrt{d\,x^2+c}}{b^2}+\frac{2\,a\,x^2\,\sqrt{d\,x^2+c}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*((2*a*c^2)/(3*b^2*(a*d - b*c)^2) + (x^2*(3*b*c^2 + a*c*
d))/(3*b^2*(a*d - b*c)^2) - (x^4*(a*d^2 - 3*b*c*d))/(3*b^2*(a*d - b*c)^2)))
/(x^4*(c + d*x^2)^(1/2) + (a^2*(c + d*x^2)^(1/2))/b^2 + (2*a*x^2*(c + d*x^2
)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.798 $\int \frac{x}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=74

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}\,(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 45, 37}

$$\frac{2d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}\,(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{3\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*d*Sqrt[c + d*x^2])/
(3*(b*c - a*d)^2*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}}-\frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{3(bc-ad)}\\
&= -\frac{\sqrt{c+dx^2}}{3(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{2d\sqrt{c+dx^2}}{3(bc-ad)^2\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [**A**] time = 0.02, size = 52, normalized size = 0.70

$$
\frac{\sqrt{c+dx^2}\left(3ad-bc+2bdx^2\right)}{3\left(a+bx^2\right)^{3/2}(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^2)
^(3/2))
```

IntegrateAlgebraic [**A**] time = 1.81, size = 65, normalized size = 0.88

$$
\frac{\frac{3d\sqrt{c+dx^2}}{\sqrt{a+bx^2}}-\frac{b\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}}{3(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((3*d*Sqrt[c + d*x^2])/Sqrt[a + b*x^2] - (b*(c + d*x^2)^(3/2))/(a + b*x^2)^
(3/2))/(3*(b*c - a*d)^2)
```

fricas [**B**] time = 1.43, size = 126, normalized size = 1.70

$$
\frac{\left(2\,bdx^2-bc+3\,ad\right)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c^2
- 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*
b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

giac [**B**] time = 0.45, size = 129, normalized size = 1.74

$$\frac{4\left(b^{2}c-abd-3\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}\right)\sqrt{bd}\,b^{2}d}{3\left(b^{2}c-abd-\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}\right)^{3}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(b^2*c - a*b*d - 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*abs(b))
```

maple [**A**] time = 0.01, size = 60, normalized size = 0.81

$$\frac{\sqrt{d\,x^{2}+c}\,\left(2bd\,x^{2}+3ad-bc\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(2*b*d*x^2+3*a*d-b*c)/(b*x^2+a)^(3/2)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.34, size = 137, normalized size = 1.85

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^2\,\left(3\,a\,d^2+b\,c\,d\right)}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{b\,c^2-3\,a\,c\,d}{3\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,d^2\,x^4}{3\,b\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{d\,x^2+c}+\frac{a^2\,\sqrt{d\,x^2+c}}{b^2}+\frac{2\,a\,x^2\,\sqrt{d\,x^2+c}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((x^2*(3*a*d^2 + b*c*d))/(3*b^2*(a*d - b*c)^2) - (b*c^2
- 3*a*c*d)/(3*b^2*(a*d - b*c)^2) + (2*d^2*x^4)/(3*b*(a*d - b*c)^2)))/(x^4*(
c + d*x^2)^(1/2) + (a^2*(c + d*x^2)^(1/2))/b^2 + (2*a*x^2*(c + d*x^2)^(1/2)
)/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.799 $\int \frac{x^5}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}\,(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}\,(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Rubi [A] time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 89, 78, 37}

$$-\frac{\sqrt{c+dx^2}\left(3a^2d^2-10abcd+15b^2c^2\right)}{15b^2\sqrt{a+bx^2}\,(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{5b^2\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}\,(5bc-3ad)}{15b^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(5*b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) + (2*a*(5*b*c
 - 3*a*d)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - ((15*b
^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(15*b^2*(b*c - a*d)^3*Sqr
t[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(5bc-ad)+\frac{5}{2}b(bc-ad)x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{5b^2(bc-ad)}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\left(15b^2c^2-10abcd+3\right.}{}$$

$$= -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{\left(15b^2c^2-10abcd+3\right.}{15b^2(bc-ad)^3}$$

Mathematica [A]   time = 0.06, size = 91, normalized size = 0.59

$$-\frac{\sqrt{c+dx^2}\left(a^2\left(8c^2-4cdx^2+3d^2x^4\right)+10abcx^2\left(2c-dx^2\right)+15b^2c^2x^4\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(2*c - d*x^2) + a^2*(
8*c^2 - 4*c*d*x^2 + 3*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)^(5/2))
```

IntegrateAlgebraic [F] time = 3.71, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]), x]
```

fricas [A] time = 1.58, size = 262, normalized size = 1.70

$$-\frac{\left(\left(15\,b^{2}c^{2}-10\,abcd+3\,a^{2}d^{2}\right)x^{4}+8\,a^{2}c^{2}+4\,\left(5\,abc^{2}-a^{2}cd\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{15\,\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{6}+3\,\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3}\right)x^{4}+3\,\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 8*a^2*c^2 + 4*(5*a*b*c^2
 - a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2
 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^
4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*x^2)
```

giac [B] time = 0.87, size = 597, normalized size = 3.88

$$-\frac{2\left(15\sqrt{bd}\,b^{8}c^{4}-40\sqrt{bd}\,ab^{7}c^{3}d+38\sqrt{bd}\,a^{2}b^{6}c^{2}d^{2}-16\sqrt{bd}\,a^{3}b^{5}cd^{3}+3\sqrt{bd}\,a^{4}b^{4}d^{4}-60\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}b^{6}c^{3}+80\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}ab^{5}c^{2}d-20\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}a^{2}b^{4}cd^{2}+90\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}b^{4}c^{2}-40\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}ab^{3}cd+30\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}a^{2}b^{2}d^{2}-60\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{6}b^{2}c+15\sqrt{bd}\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{8}\right)}{15\left(b^{2}c-abd-\left(\sqrt{bx^{2}+a}\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}\right)^{5}b{\left|b\right|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(b*d)*b^8*c^4 - 40*sqrt(b*d)*a*b^7*c^3*d + 38*sqrt(b*d)*a^2*b
^6*c^2*d^2 - 16*sqrt(b*d)*a^3*b^5*c*d^3 + 3*sqrt(b*d)*a^4*b^4*d^4 - 60*sqrt
(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2
*b^6*c^3 + 80*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^2*a*b^5*c^2*d - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 90*sqrt(b*d)*(sqrt
(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c^2 -
40*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^4*a*b^3*c*d + 30*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 60*sqrt(b*d)*(sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c + 15*sqrt(b*d)*(sqr
```

```
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8)/((b^2*c
- a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)^5*b*abs(b))
```

maple **[A]** time = 0.01, size = 119, normalized size = 0.77

$$\frac{\sqrt{d\,x^2+c}\,\left(3a^2d^2x^4-10abcd\,x^4+15b^2c^2x^4-4a^2cd\,x^2+20ab\,c^2x^2+8a^2c^2\right)}{15\left(b\,x^2+a\right)^{\frac{5}{2}}\left(a^3d^3-3a^2c\,d^2b+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(3*a^2*d^2*x^4-10*a*b*c*d*x^4+15*b^2*c^2*x^4-4*a^2*c*d
*x^2+20*a*b*c^2*x^2+8*a^2*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)
```

maxima **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad **[B]** time = 1.60, size = 220, normalized size = 1.43

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2\,c^3}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^4\,\left(-a^2\,c\,d^2+10\,a\,b\,c^2\,d+15\,b^2\,c^3\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^6\,\left(3\,a^2\,d^3-10\,a\,b\,c\,d^2+15\,b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,a\,c^2\,x^2\,\left(a\,d+5\,b\,c\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((8*a^2*c^3)/(15*b^3*(a*d - b*c)^3) + (x^4*(15*b^2*c^3 -
 a^2*c*d^2 + 10*a*b*c^2*d))/(15*b^3*(a*d - b*c)^3) + (x^6*(3*a^2*d^3 + 15*b
^2*c^2*d - 10*a*b*c*d^2))/(15*b^3*(a*d - b*c)^3) + (4*a*c^2*x^2*(a*d + 5*b*
c))/(15*b^3*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c + d*x^2)^(1/2
))/b^3 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^(1/2))/b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.800 $\int \frac{x^3}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=138

$$\frac{a\sqrt{c+dx^2}}{5b\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2d\sqrt{c+dx^2}\,(5bc-ad)}{15b\sqrt{a+bx^2}\,(bc-ad)^3}-\frac{\sqrt{c+dx^2}\,(5bc-ad)}{15b\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Rubi [A] time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 78, 45, 37}

$$\frac{a\sqrt{c+dx^2}}{5b\left(a+bx^2\right)^{5/2}(bc-ad)}+\frac{2d\sqrt{c+dx^2}\,(5bc-ad)}{15b\sqrt{a+bx^2}\,(bc-ad)^3}-\frac{\sqrt{c+dx^2}\,(5bc-ad)}{15b\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((5*b*c - a*d)*Sq
rt[c + d*x^2])/(15*b*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (2*d*(5*b*c - a*d)*
Sqrt[c + d*x^2])/(15*b*(b*c - a*d)^3*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx, x, x^2\right)\\
&= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}}+\frac{(5bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^2\right)}{10b(bc-ad)}\\
&= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}}-\frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\left(a+bx^2\right)^{3/2}}-\frac{(d(5bc-ad))\,\text{Subst}\left(\int \frac{}{(a+}\right.}{15b(bc-a}\\
&= \frac{a\sqrt{c+dx^2}}{5b(bc-ad)\left(a+bx^2\right)^{5/2}}-\frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2\left(a+bx^2\right)^{3/2}}+\frac{2d(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^3\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 91, normalized size = 0.66

$$\frac{\sqrt{c+dx^2}\left(-5a^2d\left(dx^2-2c\right)-2ab\left(c^2-13cdx^2+d^2x^4\right)-5b^2cx^2\left(c-2dx^2\right)\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-5*b^2*c*x^2*(c - 2*d*x^2) - 5*a^2*d*(-2*c + d*x^2) - 2*a
*b*(c^2 - 13*c*d*x^2 + d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 2.56, size = 119, normalized size = 0.86

$$\frac{\frac{3ab\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}-\frac{5bc\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}-\frac{5ad\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}+\frac{15cd\sqrt{c+dx^2}}{\sqrt{a+bx^2}}}{15(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((15*c*d*Sqrt[c + d*x^2])/Sqrt[a + b*x^2] - (5*b*c*(c + d*x^2)^(3/2))/(a +
b*x^2)^(3/2) - (5*a*d*(c + d*x^2)^(3/2))/(a + b*x^2)^(3/2) + (3*a*b*(c + d*
x^2)^(5/2))/(a + b*x^2)^(5/2))/(15*(b*c - a*d)^3)
```

fricas [B] time = 2.09, size = 269, normalized size = 1.95

$$\frac{\left(2\left(5b^2cd-abd^2\right)x^4-2abc^2+10a^2cd-\left(5b^2c^2-26abcd+5a^2d^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}}{15\left(a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3+\left(b^6c^3-3ab^5c^2d+3a^2b^4cd^2-a^3b^3d^3\right)x^6+3\left(ab^5c^3-3a^2b^4c^2d+3a^3b^3cd^2-a^4b^2d^3\right)x^4+3\left(a^2b^4c^3-3a^3b^3c^2d+3a^4b^2cd^2-a^5bd^3\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*b^2*c*d - a*b*d^2)*x^4 - 2*a*b*c^2 + 10*a^2*c*d - (5*b^2*c^2 - 2
6*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 -
3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^
2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3
*c*d^2 - a^4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*
d^2 - a^5*b*d^3)*x^2)
```

giac [B] time = 0.91, size = 472, normalized size = 3.42

$$\frac{4\left(5\sqrt{bd}b^8c^3d-11\sqrt{bd}ab^7c^2d^2+7\sqrt{bd}a^2b^6cd^3-\sqrt{bd}a^3b^5d^4-25\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2b^6c^2d+30\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2ab^5cd^2-5\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2a^2b^4d^3+35\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^4b^4cd+5\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^4ab^3d^2-15\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^6b^2d\right)}{15\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2\right)^5b|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*sqrt(b*d)*b^8*c^3*d - 11*sqrt(b*d)*a*b^7*c^2*d^2 + 7*sqrt(b*d)*a^2*
b^6*c*d^3 - sqrt(b*d)*a^3*b^5*d^4 - 25*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^6*c^2*d + 30*sqrt(b*d)*(sqrt(
b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c*d^2
 - 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^2*a^2*b^4*d^3 + 35*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c*d + 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^3*d^2 - 15*sqrt(b*d)*(
```

```
sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*d)
/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d))^2)^5*b*abs(b))
```

maple [**A**] time = 0.01, size = 125, normalized size = 0.91

$$-\frac{\sqrt{d\,x^2+c}\,\left(-2ab\,d^2x^4+10b^2cd\,x^4-5a^2d^2x^2+26abcd\,x^2-5b^2c^2x^2+10a^2cd-2ab\,c^2\right)}{15\left(b\,x^2+a\right)^{\frac{5}{2}}\left(a^3d^3-3a^2c\,d^2b+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(-2*a*b*d^2*x^4+10*b^2*c*d*x^4-5*a^2*d^2*x^2+26*a*b*c
*d*x^2-5*b^2*c^2*x^2+10*a^2*c*d-2*a*b*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 1.54, size = 227, normalized size = 1.64

$$-\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^2\,\left(5\,a^2\,c\,d^2+24\,a\,b\,c^2\,d-5\,b^2\,c^3\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^4\,\left(-5\,a^2\,d^3+24\,a\,b\,c\,d^2+5\,b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^2\,x^6\,\left(a\,d-5\,b\,c\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,a\,c^2\,\left(5\,a\,d-b\,c\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -((a + b*x^2)^(1/2)*((x^2*(5*a^2*c*d^2 - 5*b^2*c^3 + 24*a*b*c^2*d))/(15*b^3
*(a*d - b*c)^3) + (x^4*(5*b^2*c^2*d - 5*a^2*d^3 + 24*a*b*c*d^2))/(15*b^3*(a
*d - b*c)^3) - (2*d^2*x^6*(a*d - 5*b*c))/(15*b^2*(a*d - b*c)^3) + (2*a*c^2*
(5*a*d - b*c))/(15*b^3*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c +
d*x^2)^(1/2))/b^3 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^
(1/2))/b^2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^2\right)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.801 $\int \frac{x}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=113

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}\,(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{5/2}(bc-ad)}$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 45, 37}

$$-\frac{8d^2\sqrt{c+dx^2}}{15\sqrt{a+bx^2}\,(bc-ad)^3}+\frac{4d\sqrt{c+dx^2}}{15\left(a+bx^2\right)^{3/2}(bc-ad)^2}-\frac{\sqrt{c+dx^2}}{5\left(a+bx^2\right)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -Sqrt[c + d*x^2]/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*d*Sqrt[c + d*x^2])/
(15*(b*c - a*d)^2*(a + b*x^2)^(3/2)) - (8*d^2*Sqrt[c + d*x^2])/(15*(b*c - a
*d)^3*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx,x,x^2\right)$$

$$=-\frac{\sqrt{c+dx^2}}{5(bc-ad)\left(a+bx^2\right)^{5/2}}-\frac{(2d)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{5(bc-ad)}$$

$$=-\frac{\sqrt{c+dx^2}}{5(bc-ad)\left(a+bx^2\right)^{5/2}}+\frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2\left(a+bx^2\right)^{3/2}}+\frac{\left(4d^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c}}\right.}{15(bc-ad)^2}$$

$$=-\frac{\sqrt{c+dx^2}}{5(bc-ad)\left(a+bx^2\right)^{5/2}}+\frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2\left(a+bx^2\right)^{3/2}}-\frac{8d^2\sqrt{c+dx^2}}{15(bc-ad)^3\sqrt{a+bx^2}}$$

Mathematica [A] time = 0.03, size = 83, normalized size = 0.73

$$-\frac{\sqrt{c+dx^2}\left(15a^2d^2-10abd\left(c-2dx^2\right)+b^2\left(3c^2-4cdx^2+8d^2x^4\right)\right)}{15\left(a+bx^2\right)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/15*(Sqrt[c + d*x^2]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x^2) + b^2*(3*c^2 -
4*c*d*x^2 + 8*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 2.15, size = 95, normalized size = 0.84

$$\frac{-\frac{3b^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}-\frac{15d^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}+\frac{10bd\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}}{15(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-15*d^2*Sqrt[c + d*x^2])/Sqrt[a + b*x^2] + (10*b*d*(c + d*x^2)^(3/2))/(a
+ b*x^2)^(3/2) - (3*b^2*(c + d*x^2)^(5/2))/(a + b*x^2)^(5/2))/(15*(b*c - a*
d)^3)
```

fricas [B] time = 2.09, size = 259, normalized size = 2.29

$$-\frac{\left(8\,b^{2}d^{2}x^{4}+3\,b^{2}c^{2}-10\,abcd+15\,a^{2}d^{2}-4\left(b^{2}cd-5\,abd^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{15\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}+\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}\right)x^{6}+3\left(ab^{5}c^{3}-3\,a^{2}b^{4}c^{2}d+3\,a^{3}b^{3}cd^{2}-a^{4}b^{2}d^{3}\right)x^{4}+3\left(a^{2}b^{4}c^{3}-3\,a^{3}b^{3}c^{2}d+3\,a^{4}b^{2}cd^{2}-a^{5}bd^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^2*d^2*x^4 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2
*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
 a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*
b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*x^2)
```

giac [B] time = 0.48, size = 243, normalized size = 2.15

$$-\frac{16\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}-5\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}b^{2}c+5\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}abd+10\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{4}\right)\sqrt{bd}\,b^{3}d^{2}}{15\left(b^{2}c-abd-\left(\sqrt{bx^{2}+a}\,\sqrt{bd}-\sqrt{b^{2}c+\left(bx^{2}+a\right)bd-abd}\right)^{2}\right)^{5}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -16/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*x^2 + a)*sqrt(b*d)
- sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((
b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^2)^5*abs(b))
```

maple [A] time = 0.01, size = 113, normalized size = 1.00

$$\frac{\sqrt{d\,x^{2}+c}\;\left(8b^{2}d^{2}x^{4}+20ab\,d^{2}x^{2}-4b^{2}cd\,x^{2}+15a^{2}d^{2}-10abcd+3b^{2}c^{2}\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(a^{3}d^{3}-3a^{2}c\,d^{2}b+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4+20*a*b*d^2*x^2-4*b^2*c*d*x^2+15*a^2*d^2
-10*a*b*c*d+3*b^2*c^2)/(b*x^2+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d
-b^3*c^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.47, size = 216, normalized size = 1.91

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{15\,a^2\,c\,d^2-10\,a\,b\,c^2\,d+3\,b^2\,c^3}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^3\,x^6}{15\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,\left(15\,a^2\,d^3+10\,a\,b\,c\,d^2-b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,d^2\,x^4\,\left(5\,a\,d+b\,c\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((3*b^2*c^3 + 15*a^2*c*d^2 - 10*a*b*c^2*d)/(15*b^3*(a*d
- b*c)^3) + (8*d^3*x^6)/(15*b*(a*d - b*c)^3) + (x^2*(15*a^2*d^3 - b^2*c^2*d
 + 10*a*b*c*d^2))/(15*b^3*(a*d - b*c)^3) + (4*d^2*x^4*(5*a*d + b*c))/(15*b^
2*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c + d*x^2)^(1/2))/b^3 + (
3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^(1/2))/b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^{2}\right)^{\frac{7}{2}} \sqrt{c + dx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.802 $\int \frac{x^5}{\left(a+bx^2\right)^{9/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=217

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}\,(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a}{35b^2}$$

Rubi [A] time = 0.27, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 89, 78, 45, 37}

$$\frac{2d\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\sqrt{a+bx^2}\,(bc-ad)^4}-\frac{\sqrt{c+dx^2}\left(3a^2d^2-14abcd+35b^2c^2\right)}{105b^2\left(a+bx^2\right)^{3/2}(bc-ad)^3}-\frac{a^2\sqrt{c+dx^2}}{7b^2\left(a+bx^2\right)^{7/2}(bc-ad)}+\frac{2a\sqrt{c+dx^2}\,(7bc-4ad)}{35b^2\left(a+bx^2\right)^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -(a^2*Sqrt[c + d*x^2])/(7*b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) + (2*a*(7*b*c
- 4*a*d)*Sqrt[c + d*x^2])/(35*b^2*(b*c - a*d)^2*(a + b*x^2)^(5/2)) - ((35*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(105*b^2*(b*c - a*d)^3*(a
 + b*x^2)^(3/2)) + (2*d*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^
2])/(105*b^2*(b*c - a*d)^4*Sqrt[a + b*x^2])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(a+bx^2\right)^{9/2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx,x,x^2\right)\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}}+\frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(7bc-ad)+\frac{7}{2}b(bc-ad)x}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx,x,x^2\right)}{7b^2(bc-ad)}\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}}+\frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}}+\frac{\left(35b^2c^2-14abcd+3a\right.}{}\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}}+\frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}}-\frac{\left(35b^2c^2-14abcd+3a\right.}{105b^2(bc-ad)^3\left(a\right.}\\
&= -\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)\left(a+bx^2\right)^{7/2}}+\frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2\left(a+bx^2\right)^{5/2}}-\frac{\left(35b^2c^2-14abcd+3a\right.}{105b^2(bc-ad)^3\left(a\right.}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 151, normalized size = 0.70

$$
\frac{\sqrt{c+dx^2}\left(7a^3d\left(8c^2-4cdx^2+3d^2x^4\right)+a^2b\left(-8c^3+200c^2dx^2-101cd^2x^4+6d^3x^6\right)-7ab^2cx^2\left(4c^2-37cdx^2+4d^2x^4\right)-35b^3c^2x^4\left(c-2dx^2\right)\right)}{105\left(a+bx^2\right)^{7/2}(bc-ad)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*(-35*b^3*c^2*x^4*(c - 2*d*x^2) + 7*a^3*d*(8*c^2 - 4*c*d*x^
2 + 3*d^2*x^4) - 7*a*b^2*c*x^2*(4*c^2 - 37*c*d*x^2 + 4*d^2*x^4) + a^2*b*(-8
*c^3 + 200*c^2*d*x^2 - 101*c*d^2*x^4 + 6*d^3*x^6)))/(105*(b*c - a*d)^4*(a +
 b*x^2)^(7/2))
```

IntegrateAlgebraic [F] time = 3.84, size = 0, normalized size = 0.00

$$
\int \frac{x^5}{\left(a+bx^2\right)^{9/2}\sqrt{c+dx^2}}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]), x]
```

fricas [B] time = 3.46, size = 451, normalized size = 2.08

$$\frac{\left(2\left(35\,b^{3}c^{2}d-14\,ab^{2}cd^{2}+3\,a^{2}bd^{3}\right)x^{6}-8\,a^{2}bc^{3}+56\,a^{3}c^{2}d-\left(35\,b^{3}c^{3}-259\,ab^{2}c^{2}d+101\,a^{2}bcd^{2}-21\,a^{3}d^{3}\right)x^{4}-4\left(7\,ab^{2}c^{3}-50\,a^{2}bc^{2}d+7\,a^{3}cd^{2}\right)x^{2}\right)\sqrt{bx^{2}+a}\,\sqrt{dx^{2}+c}}{105\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)x^{8}+4\left(ab^{7}c^{4}-4\,a^{2}b^{6}c^{3}d+6\,a^{3}b^{5}c^{2}d^{2}-4\,a^{4}b^{4}cd^{3}+a^{5}b^{3}d^{4}\right)x^{6}+6\left(a^{2}b^{6}c^{4}-4\,a^{3}b^{5}c^{3}d+6\,a^{4}b^{4}c^{2}d^{2}-4\,a^{5}b^{3}cd^{3}+a^{6}b^{2}d^{4}\right)x^{4}+4\left(a^{3}b^{5}c^{4}-4\,a^{4}b^{4}c^{3}d+6\,a^{5}b^{3}c^{2}d^{2}-4\,a^{6}b^{2}cd^{3}+a^{7}bd^{4}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(35*b^3*c^2*d - 14*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^6 - 8*a^2*b*c^3 +
56*a^3*c^2*d - (35*b^3*c^3 - 259*a*b^2*c^2*d + 101*a^2*b*c*d^2 - 21*a^3*d^3
)*x^4 - 4*(7*a*b^2*c^3 - 50*a^2*b*c^2*d + 7*a^3*c*d^2)*x^2)*sqrt(b*x^2 + a)
*sqrt(d*x^2 + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7
*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b
^5*c*d^3 + a^4*b^4*d^4)*x^8 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^
2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^6 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3
*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^4 + 4*(a^3*b^5*c^
4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x^2)
```

giac [B] time = 1.24, size = 1036, normalized size = 4.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 4/105*(35*sqrt(b*d)*b^10*c^5*d - 119*sqrt(b*d)*a*b^9*c^4*d^2 + 150*sqrt(b*d
)*a^2*b^8*c^3*d^3 - 86*sqrt(b*d)*a^3*b^7*c^2*d^4 + 23*sqrt(b*d)*a^4*b^6*c*d
^5 - 3*sqrt(b*d)*a^5*b^5*d^6 - 245*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^8*c^4*d + 588*sqrt(b*d)*(sqrt(b*x
^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^7*c^3*d^2
 - 462*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^2*a^2*b^6*c^2*d^3 + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^5*c*d^4 - 21*sqrt(b*d)*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^4*d^5 +
 630*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^4*b^6*c^3*d - 714*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
 + (b*x^2 + a)*b*d - a*b*d))^4*a*b^5*c^2*d^2 + 42*sqrt(b*d)*(sqrt(b*x^2 + a
)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^4*c*d^3 + 42*s
qrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)
)^4*a^3*b^3*d^4 - 770*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^6*b^4*c^2*d + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b^3*c*d^2 - 210*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a^2*
b^2*d^3 + 455*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^8*b^2*c*d + 105*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
```

```
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8*a*b*d^2 - 105*sqrt(b*d)*(sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^10*d)/((b^2*c - a*b*
d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^
7*abs(b))
```

maple [A] time = 0.01, size = 213, normalized size = 0.98

$$\frac{\sqrt{d\,x^{2}+c}\,\left(6a^{2}b\,d^{3}x^{6}-28a\,b^{2}c\,d^{2}x^{6}+70b^{3}c^{2}d\,x^{6}+21a^{3}d^{3}x^{4}-101a^{2}bc\,d^{2}x^{4}+259a\,b^{2}c^{2}d\,x^{4}-35b^{3}c^{3}x^{4}-28a^{3}c\,d^{2}x^{2}+200a^{2}b\,c^{2}d\,x^{2}-28a\,b^{2}c^{3}x^{2}+56a^{3}c^{2}d-8a^{2}b\,c^{3}\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}c^{2}d^{2}b^{2}-4a\,c^{3}d\,b^{3}+c^{4}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(6*a^2*b*d^3*x^6-28*a*b^2*c*d^2*x^6+70*b^3*c^2*d*x^6+
21*a^3*d^3*x^4-101*a^2*b*c*d^2*x^4+259*a*b^2*c^2*d*x^4-35*b^3*c^3*x^4-28*a^
3*c*d^2*x^2+200*a^2*b*c^2*d*x^2-28*a*b^2*c^3*x^2+56*a^3*c^2*d-8*a^2*b*c^3)/
(b*x^2+a)^(7/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*
c^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.86, size = 336, normalized size = 1.55

$$\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(21\,a^3\,d^4-95\,a^2\,b\,c\,d^3+231\,a\,b^2\,c^2\,d^2+35\,b^3\,c^3\,d\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}-\frac{x^4\,\left(7\,a^3\,c\,d^3-99\,a^2\,b\,c^2\,d^2-231\,a\,b^2\,c^3\,d+35\,b^3\,c^4\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{8\,a^2\,c^3\,\left(7\,a\,d-b\,c\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,d^2\,x^8\,\left(3\,a^2\,d^2-14\,a\,b\,c\,d+35\,b^2\,c^2\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,a\,c^2\,x^2\,\left(7\,a^2\,d^2+48\,a\,b\,c\,d-7\,b^2\,c^2\right)}{105\,b^4\,{\left(a\,d-b\,c\right)}^4}\right)}{x^8\,\sqrt{d\,x^2+c}+\frac{a^4\,\sqrt{d\,x^2+c}}{b^4}+\frac{4\,a\,x^6\,\sqrt{d\,x^2+c}}{b}+\frac{6\,a^2\,x^4\,\sqrt{d\,x^2+c}}{b^2}+\frac{4\,a^3\,x^2\,\sqrt{d\,x^2+c}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^2)^(9/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*((x^6*(21*a^3*d^4 + 35*b^3*c^3*d + 231*a*b^2*c^2*d^2 - 9
5*a^2*b*c*d^3))/(105*b^4*(a*d - b*c)^4) - (x^4*(35*b^3*c^4 + 7*a^3*c*d^3 -
99*a^2*b*c^2*d^2 - 231*a*b^2*c^3*d))/(105*b^4*(a*d - b*c)^4) + (8*a^2*c^3*(
7*a*d - b*c))/(105*b^4*(a*d - b*c)^4) + (2*d^2*x^8*(3*a^2*d^2 + 35*b^2*c^2
- 14*a*b*c*d))/(105*b^3*(a*d - b*c)^4) + (4*a*c^2*x^2*(7*a^2*d^2 - 7*b^2*c^
```

```
2 + 48*a*b*c*d))/(105*b^4*(a*d - b*c)^4)))/(x^8*(c + d*x^2)^(1/2) + (a^4*(c
 + d*x^2)^(1/2))/b^4 + (4*a*x^6*(c + d*x^2)^(1/2))/b + (6*a^2*x^4*(c + d*x^
2)^(1/2))/b^2 + (4*a^3*x^2*(c + d*x^2)^(1/2))/b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^2\right)^{\frac{9}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**2+a)**(9/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**(9/2)*sqrt(c + d*x**2)), x)
```

3.803 $\int \frac{x}{\sqrt{a-bx^2}\,\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Rubi [**A**] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {444, 63, 217, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt
[d]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx, x, x^2\right)\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx^2}\right)}{b}\\ &= -\frac{\operatorname{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}\end{aligned}$$

Mathematica [B] time = 0.09, size = 108, normalized size = 2.30

$$\frac{\sqrt{-b}\sqrt{-ad-bc}\sqrt{\frac{b\left(c+dx^2\right)}{ad+bc}}\,\sin^{-1}\left(\frac{\sqrt{-b}\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{-ad-bc}}\right)}{b^{3/2}\sqrt{d}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[-b]*Sqrt[-(b*c) - a*d]*Sqrt[(b*(c + d*x^2))/(b*c + a*d)]*ArcSin[(Sqrt
[-b]*Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[-(b*c) - a*d])])/(b^(3/2)*Sqrt[
d]*Sqrt[c + d*x^2])
```

IntegrateAlgebraic [A] time = 0.56, size = 46, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a - b*x^2])]/(Sqrt[b]*Sqrt[d
])
```

fricas [B] time = 1.39, size = 201, normalized size = 4.28

$$\left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^4 + b^2c^2 - 6\,abcd + a^2d^2 + 8\left(b^2cd - abd^2\right)x^2 - 4\left(2\,bdx^2 + bc - ad\right)\sqrt{-bx^2 + a}\,\sqrt{dx^2 + c}\,\sqrt{-bd}\right)}{4\,bd},\ -\frac{\sqrt{bd}\,\arctan\left(\frac{\left(2\,bdx^2+bc-ad\right)\sqrt{-bx^2+a}\,\sqrt{dx^2+c}\,\sqrt{bd}}{2\left(b^2d^2x^4-abcd+\left(b^2cd-abd^2\right)x^2\right)}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 8*(b^2
*c*d - a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(d*x^2
 + c)*sqrt(-b*d))/(b*d), -1/2*sqrt(b*d)*arctan(1/2*(2*b*d*x^2 + b*c - a*d)*
sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)/(b^2*d^2*x^4 - a*b*c*d + (b^2*c*
d - a*b*d^2)*x^2))/(b*d)]
```

giac [A] time = 0.43, size = 57, normalized size = 1.21

$$\frac{b\log\left(\left|-\sqrt{-bx^2 + a}\,\sqrt{-bd} + \sqrt{b^2c + \left(bx^2 - a\right)bd + abd}\right|\right)}{\sqrt{-bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-sqrt(-b*x^2 + a)*sqrt(-b*d) + sqrt(b^2*c + (b*x^2 - a)*b*d + a*b
*d)))/(sqrt(-b*d)*abs(b))
```

maple [B] time = 0.05, size = 108, normalized size = 2.30

$$\frac{\sqrt{-b\,x^2 + a}\,\sqrt{d\,x^2 + c}\,\arctan\left(\frac{\sqrt{bd}\,\left(2bd\,x^2-ad+bc\right)}{2\sqrt{-x^4bd+ad\,x^2-bc\,x^2+ac}\,bd}\right)}{2\sqrt{bd}\,\sqrt{-x^4bd + ad\,x^2 - bc\,x^2 + ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(b*d)^(1/2)*(2*b*d*x^2-a*d+b*c)/b/d/(-b*d*x^4+a*d*x^2-b*c*x^
2+a*c)^(1/2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/(-b*d*x^4+a*d*x^
2-b*c*x^2+a*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.27, size = 48, normalized size = 1.02

$$-\frac{2\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{a-b\,x^2}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] -(2*atan((d*((a - b*x^2)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x^2)^(1/2)
- c^(1/2)))))/(b*d)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a - bx^2}\,\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

3.804 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 63, 217, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{a-bx^2}\,\sqrt{c-dx^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx}\,\sqrt{c-dx}}\,dx,x,x^2\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a-bx^2}\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\right)}{b}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx^2}}{\sqrt{b}\,\sqrt{c-dx^2}}\right)}{\sqrt{b}\,\sqrt{d}}
\end{aligned}
$$

Mathematica [B] time = 0.09, size = 109, normalized size = 2.27

$$
\frac{\sqrt{-b}\,\sqrt{ad-bc}\,\sqrt{\frac{b\left(c-dx^2\right)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{-b}\,\sqrt{d}\,\sqrt{a-bx^2}}{\sqrt{b}\,\sqrt{ad-bc}}\right)}{b^{3/2}\sqrt{d}\,\sqrt{c-dx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[-b]*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c - d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[-b]*Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[-(b*c) + a*d])])/(b^(3/2)*Sqrt
[d]*Sqrt[c - d*x^2])
```

IntegrateAlgebraic [A] time = 0.57, size = 48, normalized size = 1.00

$$
-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c-dx^2}}{\sqrt{d}\,\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{d}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sqrt[c - d*x^2])/(Sqrt[d]*Sqrt[a - b*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

fricas [B] time = 1.46, size = 203, normalized size = 4.23

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 - 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,bdx^2 - bc - ad\right)\sqrt{-bx^2 + a}\,\sqrt{-dx^2 + c}\,\sqrt{bd}\right)}{4\,bd}, -\frac{\sqrt{-bd}\,\arctan\left(\frac{\left(2\,bdx^2 - bc - ad\right)\sqrt{-bx^2 + a}\,\sqrt{-dx^2 + c}\,\sqrt{-bd}}{2\left(b^2d^2x^4 + abcd - \left(b^2cd + abd^2\right)x^2\right)}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 - b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(-d*x^2
+ c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 - b*c - a*d)*s
qrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d - (b^2*c
*d + a*b*d^2)*x^2))/(b*d)]
```

giac [A] time = 0.38, size = 57, normalized size = 1.19

$$\frac{b\log\left(\left|-\sqrt{-bx^2 + a}\,\sqrt{bd} + \sqrt{b^2c - \left(bx^2 - a\right)bd - abd}\right|\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-sqrt(-b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c - (b*x^2 - a)*b*d - a*b*
d)))/(sqrt(b*d)*abs(b))
```

maple [B] time = 0.05, size = 111, normalized size = 2.31

$$\frac{\sqrt{-b\,x^2 + a}\,\sqrt{-d\,x^2 + c}\,\ln\left(\frac{2bd\,x^2 - ad - bc + 2\sqrt{x^4bd - ad\,x^2 - bc\,x^2 + ac}\,\sqrt{bd}}{2\sqrt{bd}}\right)}{2\sqrt{bd}\,\sqrt{x^4bd - ad\,x^2 - bc\,x^2 + ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] 1/2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4-a*d*x^2-b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)-a*d
-b*c)/(b*d)^(1/2))*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(b*d)^(1/2)/(b*d*x^4-a
*d*x^2-b*c*x^2+a*c)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 1.25, size = 51, normalized size = 1.06

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{c-d\,x^2}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a-b\,x^2}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

```
[Out] (2*atan((b*((c - d*x^2)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a - b*x^2)^(1/2)
- a^(1/2)))))/(-b*d)^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a - bx^2}\,\sqrt{c - dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

3.805 $\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=109

$$\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{3}{2}\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*Sqrt[3]*ArcTan[(1 + (2
- 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - (9*Log[3 + x^2])/(4*2^(2/3)) + (27*
Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{2}{\sqrt[3]{1-x}}-(1-x)^{2/3}+\frac{9}{\sqrt[3]{1-x}\,(3+x)}\right)dx, x, x^2\right)\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{9}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}-\frac{9\log\left(3+x^2\right)}{4\;2^{2/3}}+\frac{27}{4}\,\mathrm{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,\right.\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}-\frac{9\log\left(3+x^2\right)}{4\;2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\;2^{2/3}}-\frac{27\,\mathrm{Subst}}{}\\
&= \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{9\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\;2^{2/3}}-\frac{9\log\left(3+x^2\right)}{4\;2^{2/3}}+\frac{27\log\left(2^{2}\right.}{4}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 106, normalized size = 0.97

$$-\frac{3}{40}\left(4\left(1-x^2\right)^{2/3}x^2-24\left(1-x^2\right)^{2/3}+15\sqrt[3]{2}\log\left(x^2+3\right)-45\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-30\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(-24*(1 - x^2)^(2/3) + 4*x^2*(1 - x^2)^(2/3) - 30*2^(1/3)*Sqrt[3]*ArcTa
n[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 15*2^(1/3)*Log[3 + x^2] - 45*2^(1/3)*L
og[2^(2/3) - (1 - x^2)^(1/3)]))/40
```

IntegrateAlgebraic [A] time = 0.16, size = 140, normalized size = 1.28

$$-\frac{3}{10}\left(1-x^2\right)^{2/3}\left(x^2-6\right)+\frac{9\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{2\,2^{2/3}}-\frac{9\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{4\,2^{2/3}}+\frac{9\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(1 - x^2)^(2/3)*(-6 + x^2))/10 + (9*Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)
*(1 - x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) + (9*Log[-2 + 2^(1/3)*(1 - x^2)^(1/
3)])/(2*2^(2/3)) - (9*Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)
^(2/3)])/(4*2^(2/3))
```

fricas [A] time = 0.92, size = 102, normalized size = 0.94

$$-\frac{3}{10}\left(x^2-6\right)\left(-x^2+1\right)^{\frac{2}{3}}+\frac{9}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -3/10*(x^2 - 6)*(-x^2 + 1)^(2/3) + 9/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*s
qrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3))) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)
*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 +
1)^(1/3))
```

giac [A] time = 0.42, size = 108, normalized size = 0.99

$$\frac{9}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}}-\frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{3}{2}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] 9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1
)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) +
 3/2*(-x^2 + 1)^(2/3)
```

maple [C] time = 7.18, size = 655, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 3/10*(x^2-6)*(x^2-1)/(-x^2+1)^(1/3)-9/4*ln(-(8*x^2*RootOf(_Z^3-2)^3*RootOf(
RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-16*RootOf(RootOf(_Z^3-2)^2+4*
_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-21*RootOf(_Z^3-2)^2*(-x^2
+1)^(1/3)+5*RootOf(_Z^3-2)*x^2-10*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-
2)+16*_Z^2)*x^2-21*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)+42*RootOf(RootOf(_Z^3-2
)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(x^2+3))*RootOf(_Z^3-2)-9*ln(-(8*x^2*Root
Of(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-16*RootOf
(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-21*Ro
otOf(_Z^3-2)^2*(-x^2+1)^(1/3)+5*RootOf(_Z^3-2)*x^2-10*RootOf(RootOf(_Z^3-2)
^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-21*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)+42*
RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(x^2+3))*RootOf(RootO
f(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+9*RootOf(RootOf(_Z^3-2)^2+4*_Z*Roo
tOf(_Z^3-2)+16*_Z^2)*ln((24*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*
_Z*RootOf(_Z^3-2)+16*_Z^2)+32*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+1
6*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-3*RootOf(
_Z^3-2)*x^2-4*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+42*(
-x^2+1)^(2/3)+63*RootOf(_Z^3-2)+84*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3
-2)+16*_Z^2))/(x^2+3))
```

maxima [A] time = 1.94, size = 108, normalized size = 0.99

$$\frac{9}{8} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}} - \frac{9}{16} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{9}{8} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{3}{2}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1
)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
+ 3/2*(-x^2 + 1)^(2/3)
```

mupad [B] time = 0.89, size = 128, normalized size = 1.17

$$\frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}}{4}\right)}{4}+\frac{3\,{\left(1-x^2\right)}^{2/3}}{2}+\frac{3\,{\left(1-x^2\right)}^{5/3}}{10}+\frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}-\frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (9*2^(1/3)*log((729*(1 - x^2)^(1/3))/4 - (729*2^(2/3))/4))/4 + (3*(1 - x^2)
^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*2^(1/3)*log((729*(1 - x^2)^(1/3))/4
 - (729*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 - (9*2^(1/3)*lo
g((729*(1 - x^2)^(1/3))/4 - (729*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1
i + 1))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.806 $\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)} \, dx$

Optimal. Leaf size=94

$$-\frac{3}{4} \left(1-x^2\right)^{2/3}+\frac{3 \log \left(x^2+3\right)}{4\ 2^{2/3}}-\frac{9 \log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3 \sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{2-2 x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Rubi [**A**] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 55, 617, 204, 31}

$$-\frac{3}{4} \left(1-x^2\right)^{2/3}+\frac{3 \log \left(x^2+3\right)}{4\ 2^{2/3}}-\frac{9 \log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{3 \sqrt{3} \tan ^{-1}\left(\frac{\sqrt[3]{2-2 x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(2*2^(2/3)) + (3*Log[3 + x^2])/(4*2^(2/3)) - (9*Log[2^(2/3) - (1 - x^2)^(
1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,\left(3+x^2\right)}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} + \frac{3\log\left(3+x^2\right)}{4\,2^{2/3}} - \frac{9}{4}\,\mathrm{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{9\,\mathrm{S}}{}$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} + \frac{3\log\left(3+x^2\right)}{4\,2^{2/3}} - \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}} + \frac{9\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.}{2\,2^{2/3}}$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{3\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} + \frac{3\log\left(3+x^2\right)}{4\,2^{2/3}} - \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 0.96

$$-\frac{3}{8}\left(2\left(1-x^2\right)^{2/3} - \sqrt[3]{2}\,\log\left(x^2+3\right) + 3\sqrt[3]{2}\,\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right) + 2\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/S
qrt[3]] - 2^(1/3)*Log[3 + x^2] + 3*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)]))
/8
```

IntegrateAlgebraic [A] time = 0.13, size = 135, normalized size = 1.44

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{3\log\left(\sqrt[3]{2}\,\sqrt[3]{1-x^2}-2\right)}{2\,2^{2/3}}+\frac{3\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\,\sqrt[3]{1-x^2}+4\right)}{4\,2^{2/3}}-\frac{3\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2}\,\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (3*Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(
1/3))/Sqrt[3]])/(2*2^(2/3)) - (3*Log[-2 + 2^(1/3)*(1 - x^2)^(1/3)])/(2*2^(2
/3)) + (3*Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)])/(4*
2^(2/3))
```

fricas [A] time = 1.32, size = 122, normalized size = 1.30

$$-\frac{3}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\,(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)-\frac{3}{16}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{3}{8}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -3/4*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*(-1)^(1/3)*(-
x^2 + 1)^(1/3) - 4^(1/3))) - 3/16*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(-1)^(2/3)
*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 3/8*4^(2/3)*(-
1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

giac [A] time = 0.51, size = 97, normalized size = 1.03

$$-\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] -3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) - 3/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

maple [C] time = 7.98, size = 754, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 3/4*(x^2-1)/(-x^2+1)^(1/3)+3/4*RootOf(_Z^3+2)*ln(-(96*RootOf(RootOf(_Z^3+2)
^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)^2*RootOf(_Z^3+2)^2*x^2+8*RootOf(RootOf(_Z^3
+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*RootOf(_Z^3+2)^3*x^2+168*RootOf(_Z^3+2)*
(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)-60*Root
Of(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*x^2+42*RootOf(_Z^3+2)^2*(-
x^2+1)^(1/3)-5*RootOf(_Z^3+2)*x^2-42*(-x^2+1)^(2/3)+252*RootOf(RootOf(_Z^3+
2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+21*RootOf(_Z^3+2))/(x^2+3))-3/4*ln((64*Ro
otOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-8
*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*RootOf(_Z^3+2)^3*x^2-
168*RootOf(_Z^3+2)*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+
2)+16*_Z^2)+8*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*x^2-42*R
ootOf(_Z^3+2)^2*(-x^2+1)^(1/3)-RootOf(_Z^3+2)*x^2+42*(-x^2+1)^(2/3)-168*Roo
tOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+21*RootOf(_Z^3+2))/(x^2+3
))*RootOf(_Z^3+2)-3*ln((64*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_
Z^2)^2*RootOf(_Z^3+2)^2*x^2-8*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+1
6*_Z^2)*RootOf(_Z^3+2)^3*x^2-168*RootOf(_Z^3+2)*(-x^2+1)^(1/3)*RootOf(RootO
f(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+8*RootOf(RootOf(_Z^3+2)^2+4*_Z*Roo
tOf(_Z^3+2)+16*_Z^2)*x^2-42*RootOf(_Z^3+2)^2*(-x^2+1)^(1/3)-RootOf(_Z^3+2)*
x^2+42*(-x^2+1)^(2/3)-168*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z
^2)+21*RootOf(_Z^3+2))/(x^2+3))*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)
+16*_Z^2)
```

maxima [A] time = 2.00, size = 97, normalized size = 1.03

$$-\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)-\frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^{2}+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] -3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) - 3/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3)
```

mupad [B] time = 0.86, size = 117, normalized size = 1.24

$$-\frac{3\,2^{1/3}\,\ln\left(\frac{81\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{81\,2^{2/3}}{4}\right)}{4}-\frac{3\,{\left(1-x^2\right)}^{2/3}}{4}-\frac{3\,2^{1/3}\,\ln\left(\frac{81\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{81\,2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}+\frac{3\,2^{1/3}\,\ln\left(\frac{81\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{81\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/4 - (81*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)
*(3^(1/2)*1i + 1))/8 - (3*(1 - x^2)^(2/3))/4 - (3*2^(1/3)*log((81*(1 - x^2)
^(1/3))/4 - (81*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 - (3*2^
(1/3)*log((81*(1 - x^2)^(1/3))/4 - (81*2^(2/3))/4))/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.807 $\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)} \, dx$

Optimal. Leaf size=79

$$-\frac{\log \left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3 \log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2-2 x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log \left(x^2+3\right)}{4\ 2^{2/3}}+\frac{3 \log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}+\frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt[3]{2-2 x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3)) - Log[3 + x^2
]/(4*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{\log\left(3+x^2\right)}{4\,2^{2/3}} + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{3\,\text{Subst}\left(\int \frac{1}{2^{2/3}-x}\,d\right.}{4\,2^{2/3}}$$

$$= -\frac{\log\left(3+x^2\right)}{4\,2^{2/3}} + \frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\sqrt[3]{2-2x^2}\right)}{2\,2^{2/3}}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\,2^{2/3}} - \frac{\log\left(3+x^2\right)}{4\,2^{2/3}} + \frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\,2^{2/3}}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.85

$$\frac{-\log\left(x^2+3\right)+3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{4\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - Log[3 + x^2] + 3*Log[2
^(2/3) - (1 - x^2)^(1/3)])/(4*2^(2/3))
```

IntegrateAlgebraic [A] time = 0.10, size = 120, normalized size = 1.52

$$\frac{\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{2\,2^{2/3}}-\frac{\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{4\,2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/(2*2^(2/3))
 + Log[-2 + 2^(1/3)*(1 - x^2)^(1/3)]/(2*2^(2/3)) - Log[4 + 2*2^(1/3)*(1 - x
^2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)]/(4*2^(2/3))
```

fricas [A] time = 1.40, size = 86, normalized size = 1.09

$$\frac{1}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3
))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3
)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

giac [A] time = 0.47, size = 86, normalized size = 1.09

$$\frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] 1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/
3)) + 1/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3))
```

maple [C] time = 6.32, size = 460, normalized size = 5.82

$$\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)\ln\left(\frac{64x^2\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)^2\operatorname{RootOf}\left(\_Z^3-2\right)^2+24x^2\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)\operatorname{RootOf}\left(\_Z^3-2\right)^3+40x^2\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)+15x^2\operatorname{RootOf}\left(\_Z^3-2\right)+168\left(-x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)\operatorname{RootOf}\left(\_Z^3-2\right)-168\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)-63\operatorname{RootOf}\left(\_Z^3-2\right)+42\left(-x^2+1\right)^{\frac{2}{3}}}{x^2+3}\right)+\frac{\operatorname{RootOf}\left(\_Z^3-2\right)\ln\left(-\frac{48x^2\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)^2\operatorname{RootOf}\left(\_Z^3-2\right)^2+8x^2\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)\operatorname{RootOf}\left(\_Z^3-2\right)^3-6x^2\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)-x^2\operatorname{RootOf}\left(\_Z^3-2\right)-84\left(-x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)\operatorname{RootOf}\left(\_Z^3-2\right)+126\operatorname{RootOf}\left(16\_Z^2+4\_Z\operatorname{RootOf}\left(\_Z^3-2\right)+\operatorname{RootOf}\left(\_Z^3-2\right)^2\right)+21\operatorname{RootOf}\left(\_Z^3-2\right)-21\left(-x^2+1\right)^{\frac{2}{3}}}{x^2+3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 1/4*RootOf(_Z^3-2)*ln(-(48*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_
Z^2)^2*RootOf(_Z^3-2)^2*x^2+8*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+
4*_Z*RootOf(_Z^3-2)+16*_Z^2)-84*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z
*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)-6*RootOf(RootOf(_Z^3-2)^2+4*_Z*Root
Of(_Z^3-2)+16*_Z^2)*x^2-RootOf(_Z^3-2)*x^2-21*(-x^2+1)^(2/3)+126*RootOf(Roo
tOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+21*RootOf(_Z^3-2))/(x^2+3))+Root
Of(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*ln((64*RootOf(RootOf(_Z^3-
2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+24*x^2*RootOf(_Z^3
-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+168*(-x^2+1)^(1/
3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)+40*R
ootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+15*RootOf(_Z^3-2)*x
^2+42*(-x^2+1)^(2/3)-168*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^
2)-63*RootOf(_Z^3-2))/(x^2+3))
```

maxima [A] time = 2.02, size = 86, normalized size = 1.09

$$\frac{1}{8} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2 \left(-x^{2} + 1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{16} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(-x^{2} + 1\right)^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{2}{3}}\right) + \frac{1}{8} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] 1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/
3)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

mupad [B] time = 1.05, size = 106, normalized size = 1.34

$$\frac{2^{1/3} \ln\left(\frac{9 \left(1-x^2\right)^{1/3}}{4} - \frac{9\, 2^{2/3}}{4}\right)}{4} + \frac{2^{1/3} \ln\left(\frac{9 \left(1-x^2\right)^{1/3}}{4} - \frac{9\, 2^{2/3} \left(-1+\sqrt{3}\, 1\mathrm{i}\right)^2}{16}\right) \left(-1 + \sqrt{3}\, 1\mathrm{i}\right)}{8} - \frac{2^{1/3} \ln\left(\frac{9 \left(1-x^2\right)^{1/3}}{4} - \frac{9\, 2^{2/3} \left(1+\sqrt{3}\, 1\mathrm{i}\right)^2}{16}\right) \left(1 + \sqrt{3}\, 1\mathrm{i}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (2^(1/3)*log((9*(1 - x^2)^(1/3))/4 - (9*2^(2/3))/4))/4 + (2^(1/3)*log((9*(1
 - x^2)^(1/3))/4 - (9*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 -
 (2^(1/3)*log((9*(1 - x^2)^(1/3))/4 - (9*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)*(3
^(1/2)*1i + 1))/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{- (x - 1) (x + 1)} \left(x^{2} + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(x/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.808 $\int \frac{1}{x\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=136

$$\frac{\log\left(x^2+3\right)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}$$

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 55, 618, 204, 31, 617}

$$\frac{\log\left(x^2+3\right)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(1 +
2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - Log[x]/6 + Log[3 + x^2]/(12*2^(2/
3)) + Log[1 - (1 - x^2)^(1/3)]/4 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(4*2^(2/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,\left(3+x^2\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(3+x)}\,dx, x, x^2\right)$$

$$= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^2\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{\log(x)}{6} + \frac{\log\left(3+x^2\right)}{12\;2^{2/3}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^2}\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x}\right.$$

$$= -\frac{\log(x)}{6} + \frac{\log\left(3+x^2\right)}{12\;2^{2/3}} + \frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\;2^{2/3}} - \frac{1}{2}\,\text{Subst}\left(\int\right.$$

$$= -\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\;2^{2/3}\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\log(x)}{6} + \frac{\log\left(3+x^2\right)}{12\;2^{2/3}} + \frac{1}{4}\log\left(1-\sqrt[3]{1-x}\right.$$

Mathematica [A] time = 0.04, size = 127, normalized size = 0.93

$$\frac{1}{24}\left(\sqrt[3]{2}\log\left(x^2+3\right)+6\log\left(1-\sqrt[3]{1-x^2}\right)-3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 4*Sqrt[3]*Arc
Tan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 4*Log[x] + 2^(1/3)*Log[3 + x^2] + 6*
Log[1 - (1 - x^2)^(1/3)] - 3*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)])/24
```

IntegrateAlgebraic [A] time = 0.19, size = 201, normalized size = 1.48

$$\frac{1}{6}\log\left(\sqrt[3]{1-x^2}-1\right)-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{6\,2^{2/3}}-\frac{1}{12}\log\left(\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1\right)+\frac{\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{12\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\,2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[1/Sqrt[3] + (2*(1 - x^2)^(1/3))/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[1/Sqrt
[3] + (2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]) + Log[-1 + (1
- x^2)^(1/3)]/6 - Log[-2 + 2^(1/3)*(1 - x^2)^(1/3)]/(6*2^(2/3)) - Log[1 + (
1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/12 + Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) +
2^(2/3)*(1 - x^2)^(2/3)]/(12*2^(2/3))
```

fricas [A] time = 1.37, size = 177, normalized size = 1.30

$$-\frac{1}{12}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}\,(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{48}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{12}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/12*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)^(1/3)*(
-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) - 1/48*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(
-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 1/24*
4^(2/3)*(-1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) + 1/6*sqrt(3
)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/12*log((-x^2 + 1)^
(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/6*log((-x^2 + 1)^(1/3) - 1)
```

giac [A] time = 0.41, size = 149, normalized size = 1.10

$$-\frac{1}{24}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{48}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{1}{12}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{6}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] -1/24*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 1/48*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) - 1/24*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/12*log((-x^2 + 1)^(2/3) + (-x^2 +
1)^(1/3) + 1) + 1/6*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}+3\right)\left(-x^{2}+1\right)^{\frac{1}{3}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x), x)
```

mupad [B] time = 0.95, size = 256, normalized size = 1.88

$$\frac{\ln\left(\frac{405}{8}-\frac{405\,{\left(1-x^2\right)}^{1/3}}{8}\right)}{6}+\ln\left({\left(-\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)}^3\,\left(393660\,{\left(-\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)}^2-\frac{37179\,{\left(1-x^2\right)}^{1/3}}{4}\right)-\frac{243\,{\left(1-x^2\right)}^{1/3}}{32}\right)\,\left(-\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)-\ln\left(-{\left(\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)}^3\,\left(393660\,{\left(\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)}^2-\frac{37179\,{\left(1-x^2\right)}^{1/3}}{4}\right)-\frac{243\,{\left(1-x^2\right)}^{1/3}}{32}\right)\,\left(\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)-\frac{2^{1/3}\,\ln\left(\frac{405\,{\left(1-x^2\right)}^{1/3}}{128}-\frac{405\,2^{2/3}}{128}\right)}{12}+\frac{{\left(-1\right)}^{1/3}\,2^{1/3}\,\ln\left(\frac{405\,{\left(1-x^2\right)}^{1/3}}{128}-\frac{405\,{\left(-1\right)}^{2/3}\,2^{2/3}}{128}\right)}{12}-\frac{{\left(-1\right)}^{1/3}\,2^{1/3}\,\ln\left(-\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^3\,\left(\frac{37179\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{10935\,{\left(-1\right)}^{2/3}\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)}{6912}-\frac{243\,{\left(1-x^2\right)}^{1/3}}{32}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] log(405/8 - (405*(1 - x^2)^(1/3))/8)/6 + log(((3^(1/2)*1i)/12 - 1/12)^3*(39
3660*((3^(1/2)*1i)/12 - 1/12)^2 - (37179*(1 - x^2)^(1/3))/4) - (243*(1 - x^
2)^(1/3))/32)*((3^(1/2)*1i)/12 - 1/12) - log(- ((3^(1/2)*1i)/12 + 1/12)^3*(
393660*((3^(1/2)*1i)/12 + 1/12)^2 - (37179*(1 - x^2)^(1/3))/4) - (243*(1 -
x^2)^(1/3))/32)*((3^(1/2)*1i)/12 + 1/12) - (2^(1/3)*log((405*(1 - x^2)^(1/3
))/128 - (405*2^(2/3))/128))/12 + ((-1)^(1/3)*2^(1/3)*log((405*(1 - x^2)^(1
/3))/128 - (405*(-1)^(2/3)*2^(2/3))/128))/12 - ((-1)^(1/3)*2^(1/3)*log(- ((
3^(1/2)*1i + 1)^3*((37179*(1 - x^2)^(1/3))/4 - (10935*(-1)^(2/3)*2^(2/3)*(3
```

```
^(1/2)*1i + 1)^2)/16))/6912 - (243*(1 - x^2)^(1/3))/32)*(3^(1/2)*1i + 1))/2
4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.809 $\int \frac{1}{x^3\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

Optimal. Leaf size=97

$$-\frac{\left(1-x^2\right)^{2/3}}{6x^2}-\frac{\log\left(x^2+3\right)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 103, 12, 55, 617, 204, 31}

$$-\frac{\left(1-x^2\right)^{2/3}}{6x^2}-\frac{\log\left(x^2+3\right)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(1 - x^2)^(2/3)/(6*x^2) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(6*2^(2/
3)*Sqrt[3]) - Log[3 + x^2]/(36*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/3)]/(1
2*2^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(3+x)}\,dx, x, x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2} - \frac{1}{6}\,\text{Subst}\left(\int -\frac{1}{3\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2} + \frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2} - \frac{\log\left(3+x^2\right)}{36\ 2^{2/3}} + \frac{1}{12}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{\text{Sul}}{}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2} - \frac{\log\left(3+x^2\right)}{36\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\sqrt[3]{2\ -}\right.}{6\ 2^{2/3}}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2} + \frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}} - \frac{\log\left(3+x^2\right)}{36\ 2^{2/3}} + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 93, normalized size = 0.96

$$
\frac{1}{72}\left(-\frac{12\left(1-x^2\right)^{2/3}}{x^2} - \sqrt[3]{2}\log\left(x^2+3\right) + 3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right) + 2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ((-12*(1 - x^2)^(2/3))/x^2 + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]] - 2^(1/3)*Log[3 + x^2] + 3*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3
)])/72
```

IntegrateAlgebraic [A] time = 0.17, size = 138, normalized size = 1.42

$$
-\frac{\left(1-x^2\right)^{2/3}}{6x^2} + \frac{\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{18\ 2^{2/3}} - \frac{\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{36\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/6*(1 - x^2)^(2/3)/x^2 + ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(1/3))/Sqr
t[3]]/(6*2^(2/3)*Sqrt[3]) + Log[-2 + 2^(1/3)*(1 - x^2)^(1/3)]/(18*2^(2/3))
- Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)]/(36*2^(2/3))
```

fricas [A] time = 1.13, size = 115, normalized size = 1.19

$$\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3} x^2 \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(4^{\frac{1}{3}} \sqrt{3} + 2\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - 4^{\frac{2}{3}} x^2 \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + 2 \cdot 4^{\frac{2}{3}} x^2 \log\left(-4^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{1}{3}}\right) - 24\left(-x^2+1\right)^{\frac{2}{3}}}{144\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/144*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*sqrt(3
)*(-x^2 + 1)^(1/3))) - 4^(2/3)*x^2*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) +
 (-x^2 + 1)^(2/3)) + 2*4^(2/3)*x^2*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 24*(-
x^2 + 1)^(2/3))/x^2
```

giac [A] time = 0.37, size = 100, normalized size = 1.03

$$\frac{1}{72} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3}\left(4^{\frac{1}{3}} + 2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{144} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{72} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{1}{3}} - \left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{\left(-x^2+1\right)^{\frac{2}{3}}}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] 1/72*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/144*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/72*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/6*(-x^2 + 1)^(2/3)
/x^2
```

maple [C] time = 6.05, size = 754, normalized size = 7.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] 1/6*(x^2-1)/x^2/(-x^2+1)^(1/3)-1/36*ln((64*RootOf(RootOf(_Z^3-2)^2+4*_Z*Roo
tOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*x^2*RootOf(_Z^3-2)^3*RootOf(R
ootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-168*(-x^2+1)^(1/3)*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)-42*RootOf(_Z^3-2)^
2*(-x^2+1)^(1/3)-8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2
+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+4*_Z*Root
Of(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))*RootOf(_Z^3-2)-1/9*ln((64*R
ootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-
8*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)
```

```
-168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*Ro
otOf(_Z^3-2)-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-8*RootOf(RootOf(_Z^3-2)^2+4
*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*Ro
otOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+
3))*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+1/36*RootOf(_Z^3-2
)*ln((-96*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^
3-2)^2*x^2-8*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-
2)+16*_Z^2)-168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+
16*_Z^2)*RootOf(_Z^3-2)-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-60*RootOf(RootOf
(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-5*RootOf(_Z^3-2)*x^2+42*(-x^2+1
)^(2/3)+252*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+21*RootOf(
_Z^3-2))/(x^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3), x)
```

mupad [B] time = 0.89, size = 120, normalized size = 1.24

$$\frac{2^{1/3}\ln\left(\frac{\left(1-x^2\right)^{1/3}}{36}-\frac{2^{2/3}}{36}\right)}{36}-\frac{\left(1-x^2\right)^{2/3}}{6\,x^2}+\frac{2^{1/3}\ln\left(\frac{\left(1-x^2\right)^{1/3}}{36}-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{144}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{72}-\frac{2^{1/3}\ln\left(\frac{\left(1-x^2\right)^{1/3}}{36}-\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{144}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] (2^(1/3)*log((1 - x^2)^(1/3)/36 - 2^(2/3)/36))/36 - (1 - x^2)^(2/3)/(6*x^2)
 + (2^(1/3)*log((1 - x^2)^(1/3)/36 - (2^(2/3)*(3^(1/2)*1i - 1)^2)/144)*(3^(
1/2)*1i - 1))/72 - (2^(1/3)*log((1 - x^2)^(1/3)/36 - (2^(2/3)*(3^(1/2)*1i +
 1)^2)/144)*(3^(1/2)*1i + 1))/72
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.810 $\int \frac{1}{x^5 \sqrt[3]{1-x^2} \left(3+x^2\right)} \, dx$

Optimal. Leaf size=172

$$-\frac{\left(1-x^2\right)^{2/3}}{18x^2}+\frac{\log\left(x^2+3\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{9\sqrt{3}}$$

Rubi [**A**] time = 0.13, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 151, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{\left(1-x^2\right)^{2/3}}{12x^4}+\frac{\log\left(x^2+3\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -(1 - x^2)^(2/3)/(12*x^4) - (1 - x^2)^(2/3)/(18*x^2) - ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]]/(18*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/S
qrt[3]]/(9*Sqrt[3]) - Log[x]/27 + Log[3 + x^2]/(108*2^(2/3)) + Log[1 - (1 -
 x^2)^(1/3)]/18 - Log[2^(2/3) - (1 - x^2)^(1/3)]/(36*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^3(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{1}{12}\,\text{Subst}\left(\int \frac{-2-\frac{4x}{3}}{\sqrt[3]{1-x}\,x^2(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\left(1-x^2\right)^{2/3}}{18x^2}+\frac{1}{36}\,\text{Subst}\left(\int \frac{4+\frac{2x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)+\frac{1}{27}\,\text{Subst}\Bigg(\int\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{\log(x)}{27}+\frac{\log\left(3+x^2\right)}{108\ 2^{2/3}}-\frac{1}{36}\,\text{Subst}\Bigg(\int \frac{1}{2\sqrt[3]{2}\ +2^{2/3}x+}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{\log(x)}{27}+\frac{\log\left(3+x^2\right)}{108\ 2^{2/3}}+\frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\Bigg(}{}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4}-\frac{\left(1-x^2\right)^{2/3}}{18x^2}-\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\log(x)}{27}+\frac{\text{log}}{1}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 178, normalized size = 1.03

$$
-\frac{8x^4\log(x)+12\left(1-x^2\right)^{2/3}x^2+18\left(1-x^2\right)^{2/3}-\sqrt[3]{2}\,x^4\log\left(x^2+3\right)-12x^4\log\left(1-\sqrt[3]{1-x^2}\right)+3\sqrt[3]{2}\,x^4\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt[3]{2}\,\sqrt{3}\,x^4\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)-8\sqrt{3}\,x^4\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{216x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -1/216*(18*(1 - x^2)^(2/3) + 12*x^2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*x^4
*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 8*Sqrt[3]*x^4*ArcTan[(1 + 2*(1 -
 x^2)^(1/3))/Sqrt[3]] + 8*x^4*Log[x] - 2^(1/3)*x^4*Log[3 + x^2] - 12*x^4*Lo
g[1 - (1 - x^2)^(1/3)] + 3*2^(1/3)*x^4*Log[2^(2/3) - (1 - x^2)^(1/3)])/x^4
```

IntegrateAlgebraic [A] time = 0.33, size = 226, normalized size = 1.31

$$
\frac{1}{27}\log\left(\sqrt[3]{1-x^2}-1\right)-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{54\ 2^{2/3}}-\frac{1}{54}\log\left(\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1\right)+\frac{\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{108\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{9\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}}+\frac{\left(1-x^2\right)^{2/3}\left(-2x^2-3\right)}{36x^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ((-3 - 2*x^2)*(1 - x^2)^(2/3))/(36*x^4) + ArcTan[1/Sqrt[3] + (2*(1 - x^2)^(
1/3))/Sqrt[3]]/(9*Sqrt[3]) - ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(1/3))/S
qrt[3]]/(18*2^(2/3)*Sqrt[3]) + Log[-1 + (1 - x^2)^(1/3)]/27 - Log[-2 + 2^(1
/3)*(1 - x^2)^(1/3)]/(54*2^(2/3)) - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/
3)]/54 + Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)]/(108*
2^(2/3))
```

fricas [A] time = 1.17, size = 217, normalized size = 1.26

$$-\frac{4 \cdot 4^{\frac{1}{6}} \sqrt{3} \left(-1\right)^{\frac{1}{3}} x^{4} \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(2 \sqrt{3} \left(-1\right)^{\frac{1}{3}} \left(-x^{2} + 1\right)^{\frac{1}{3}} - 4^{\frac{1}{3}} \sqrt{3}\right)\right) + 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x^{4} \log\left(4^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} \left(-x^{2} + 1\right)^{\frac{1}{3}} - 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{2}{3}}\right) - 2 \cdot 4^{\frac{2}{3}} \left(-1\right)^{\frac{1}{3}} x^{4} \log\left(-4^{\frac{1}{3}} \left(-1\right)^{\frac{2}{3}} + \left(-x^{2} + 1\right)^{\frac{1}{3}}\right) - 16 \sqrt{3} x^{4} \arctan\left(\frac{2}{3} \sqrt{3} \left(-x^{2} + 1\right)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) + 8 x^{4} \log\left(\left(-x^{2} + 1\right)^{\frac{2}{3}} + \left(-x^{2} + 1\right)^{\frac{1}{3}} + 1\right) - 16 x^{4} \log\left(\left(-x^{2} + 1\right)^{\frac{1}{3}} - 1\right) + 12 \left(2 x^{2} + 3\right) \left(-x^{2} + 1\right)^{\frac{2}{3}}}{432 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/432*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)
^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 4^(2/3)*(-1)^(1/3)*x^4*log(4^
(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3))
- 2*4^(2/3)*(-1)^(1/3)*x^4*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 16
*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 8*x^4*log
((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 16*x^4*log((-x^2 + 1)^(1/3) - 1
) + 12*(2*x^2 + 3)*(-x^2 + 1)^(2/3))/x^4
```

giac [A] time = 0.45, size = 177, normalized size = 1.03

$$-\frac{1}{216} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2 \left(-x^{2} + 1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{432} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(-x^{2} + 1\right)^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{2}{3}}\right) - \frac{1}{216} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{1}{3}} - \left(-x^{2} + 1\right)^{\frac{1}{3}}\right) + \frac{1}{27} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(-x^{2} + 1\right)^{\frac{1}{3}} + 1\right)\right) + \frac{2 \left(-x^{2} + 1\right)^{\frac{5}{3}} - 5 \left(-x^{2} + 1\right)^{\frac{2}{3}}}{36 x^{4}} - \frac{1}{54} \log\left(\left(-x^{2} + 1\right)^{\frac{2}{3}} + \left(-x^{2} + 1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{27} \log\left(-\left(-x^{2} + 1\right)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] -1/216*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 1/432*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 1/216*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/27*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/36*(2*(-x^2 + 1)^(5/3) - 5*(-x
^2 + 1)^(2/3))/x^4 - 1/54*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/
27*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [F] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right) x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5), x)
```

mupad [B] time = 0.98, size = 397, normalized size = 2.31

$$\frac{\ln\left(\frac{11}{486}-\frac{11\,{\left(1-x^2\right)}^{1/3}}{486}\right)}{27}-\frac{2^{1/3}\,\ln\left(-\frac{2^{2/3}\,\left(\frac{2^{1/3}\,\left(\frac{135\,2^{2/3}}{4}-\frac{1755\,{\left(1-x^2\right)}^{1/3}}{4}\right)}{108}+\frac{7}{2}\right)}{11664}-\frac{{\left(1-x^2\right)}^{1/3}}{2916}\right)}{108}+\ln\left(\left({\left(-\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)}^2\,\left(\left(-\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)\,\left(393660\,{\left(-\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)}^2-\frac{1755\,{\left(1-x^2\right)}^{1/3}}{4}\right)-\frac{7}{2}\right)-\frac{{\left(1-x^2\right)}^{1/3}}{2916}\right)\,\left(-\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)\right)-\ln\left(-\left({\left(\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)}^2\,\left(\left(\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)\,\left(393660\,{\left(\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)}^2-\frac{1755\,{\left(1-x^2\right)}^{1/3}}{4}\right)+\frac{7}{2}\right)-\frac{{\left(1-x^2\right)}^{1/3}}{2916}\right)\,\left(\frac{1}{54}+\frac{\sqrt{3}\,1{}\mathrm{i}}{54}\right)\right)-\frac{\frac{5\,{\left(1-x^2\right)}^{2/3}}{36}-\frac{{\left(1-x^2\right)}^{5/3}}{18}}{{\left(x^2-1\right)}^2+2\,x^2-1}+\frac{{\left(-1\right)}^{1/3}\,2^{1/3}\,\ln\left(\frac{{\left(-1\right)}^{2/3}\,2^{2/3}\,\left(\frac{{\left(-1\right)}^{1/3}\,2^{1/3}\,\left(\frac{135\,{\left(-1\right)}^{2/3}\,2^{2/3}}{4}-\frac{1755\,{\left(1-x^2\right)}^{1/3}}{4}\right)}{108}-\frac{7}{2}\right)}{11664}-\frac{{\left(1-x^2\right)}^{1/3}}{2916}\right)}{108}-\frac{{\left(-1\right)}^{1/3}\,2^{1/3}\,\ln\left(-\frac{{\left(1-x^2\right)}^{1/3}}{2916}+\frac{{\left(-1\right)}^{2/3}\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{{\left(-1\right)}^{1/3}\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(\frac{1755\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{135\,{\left(-1\right)}^{2/3}\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)}{216}-\frac{7}{2}\right)}{46656}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] log(11/486 - (11*(1 - x^2)^(1/3))/486)/27 - (2^(1/3)*log(- (2^(2/3)*((2^(1/
3)*((135*2^(2/3))/4 - (1755*(1 - x^2)^(1/3))/4))/108 + 7/2))/11664 - (1 - x
^2)^(1/3)/2916))/108 + log(((3^(1/2)*1i)/54 - 1/54)^2*(((3^(1/2)*1i)/54 - 1
/54)*(393660*((3^(1/2)*1i)/54 - 1/54)^2 - (1755*(1 - x^2)^(1/3))/4) - 7/2)
 - (1 - x^2)^(1/3)/2916)*((3^(1/2)*1i)/54 - 1/54) - log(- ((3^(1/2)*1i)/54 +
 1/54)^2*(((3^(1/2)*1i)/54 + 1/54)*(393660*((3^(1/2)*1i)/54 + 1/54)^2 - (17
55*(1 - x^2)^(1/3))/4) + 7/2) - (1 - x^2)^(1/3)/2916)*((3^(1/2)*1i)/54 + 1/
54) - ((5*(1 - x^2)^(2/3))/36 - (1 - x^2)^(5/3)/18)/((x^2 - 1)^2 + 2*x^2 -
1) + ((-1)^(1/3)*2^(1/3)*log(((-1)^(2/3)*2^(2/3)*(((-1)^(1/3)*2^(1/3)*((135
*(-1)^(2/3)*2^(2/3))/4 - (1755*(1 - x^2)^(1/3))/4))/108 - 7/2))/11664 - (1
- x^2)^(1/3)/2916))/108 - ((-1)^(1/3)*2^(1/3)*log(((-1)^(2/3)*2^(2/3)*(3^(1
/2)*1i + 1)^2*(((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*((1755*(1 - x^2)^(1/3))
/4 - (135*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2)/16))/216 - 7/2))/46656 - (
1 - x^2)^(1/3)/2916)*(3^(1/2)*1i + 1))/216
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\sqrt[3]{-(x-1)(x+1)}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3),x)

[Out] Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.811 $\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Rubi [**A**] time = 0.01, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {393}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 393

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b/a), 2]}, Simp[(q*ArcTan[Sqrt[3]/(q*x)])/(2*2^(2/3)*Sqrt[3]*a^
(1/3)*d), x] + (Simp[(q*ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2
)^(1/3))])/(2*2^(2/3)*a^(1/3)*d), x] - Simp[(q*ArcTanh[q*x])/(6*2^(2/3)*a^(
1/3)*d), x] + Simp[(q*ArcTan[(Sqrt[3]*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))
)/(a^(1/3)*q*x)])/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d), x])] /; FreeQ[{a, b, c, d}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

Mathematica [C] time = 0.04, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((1 - x^2)^(1/3)*(3 + x^2)), x]
```

fricas [B] time = 4.82, size = 1943, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/20736*432^(5/6)*sqrt(3)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 +
27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x)
+ 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 -
 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/20736*432
^(5/6)*sqrt(3)*log(2592*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) + 144*432
^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) + 216*2^(1/3)
*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4 + 24*x^3 - 18*x^2 - 9*x
)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3
)*log(10368*(6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(
3)*(x^5 - x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^
2))*(-x^2 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)
^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/20736*432^(5/6)*sqrt(3)*log(2592*(
6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 - x^3
) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1
```

```
)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6
+ 9*x^4 + 27*x^2 + 27)) - 1/1296*432^(5/6)*arctan(1/36*(432^(5/6)*(x^5 - 18
*x^3 + 9*x)*(-x^2 + 1)^(1/3) + sqrt(3)*2^(1/3)*(432^(5/6)*(x^4 + 9*x^2)*(-x
^2 + 1)^(2/3) - 288*sqrt(3)*(2*x^4 - 3*x^2)*(-x^2 + 1)^(1/3) + 6*432^(1/6)*
(x^6 + 141*x^4 - 153*x^2 + 27)) - 648*432^(1/6)*(3*x^3 - x)*(-x^2 + 1)^(2/3
) - 72*sqrt(3)*(7*x^5 + 6*x^3 - 9*x))/(x^6 - 225*x^4 + 243*x^2 - 27)) - 1/2
592*432^(5/6)*arctan(-1/18*(sqrt(2)*(18*sqrt(3)*2^(2/3)*(29*x^11 + 879*x^9
- 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) - 2*(-x^2 + 1)^(2/3)*(432^(5/6)
*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) - 216*sqrt(3)*2^(1/3)*(31*x^9 - 297*
x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)*(sqrt(3)*(x^11 + 1167*x^9 - 1
3158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) - 8*sqrt(3)*(13*x^10 - 6*x^8 - 140
4*x^6 + 1350*x^4 - 81*x^2)) - 3*432^(1/6)*(x^12 + 7620*x^10 - 92115*x^8 + 1
69776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sqrt((6*2^(2/3)*(x^6 + 225*x^4 -
 189*x^2 + 27) + 144*432^(1/6)*sqrt(3)*(x^5 - x^3) + (432^(5/6)*sqrt(3)*(7*
x^3 - 3*x) + 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2 + 1)^(2/3) - 72*(x^5 + 18*x^4
 + 24*x^3 - 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^6 + 2808*x^4 - 243*x^2) - 3*43
2^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^3 + 27*x))*(-x^2 + 1)^(2/3) -
18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 42660*x^6 + 58239*x^4 - 14094*x^2
 + 729) + 144*sqrt(3)*(11*x^11 - 807*x^9 + 4518*x^7 - 5238*x^5 + 3807*x^3 -
 243*x) - (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 - 1215*x^9 + 11754*x^7 - 21006*
x^5 + 5589*x^3 - 243*x) - 432*sqrt(3)*2^(1/3)*(13*x^10 - 120*x^8 + 1242*x^6
 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 110727*x^8 - 301860*x^6 + 18783
9*x^4 - 21870*x^2 + 729)) - 1/2592*432^(5/6)*arctan(1/18*(sqrt(2)*(18*sqrt(
3)*2^(2/3)*(29*x^11 + 879*x^9 - 12078*x^7 + 10638*x^5 - 3807*x^3 + 243*x) +
 2*(-x^2 + 1)^(2/3)*(432^(5/6)*(x^10 + 153*x^8 - 1701*x^6 + 459*x^4) + 216*
sqrt(3)*2^(1/3)*(31*x^9 - 297*x^7 - 27*x^5 - 27*x^3)) - 36*(-x^2 + 1)^(1/3)
*(sqrt(3)*(x^11 + 1167*x^9 - 13158*x^7 + 17550*x^5 - 4779*x^3 + 243*x) + 8*
sqrt(3)*(13*x^10 - 6*x^8 - 1404*x^6 + 1350*x^4 - 81*x^2)) + 3*432^(1/6)*(x^
12 + 7620*x^10 - 92115*x^8 + 169776*x^6 - 109269*x^4 + 16524*x^2 - 729))*sq
rt((6*2^(2/3)*(x^6 + 225*x^4 - 189*x^2 + 27) - 144*432^(1/6)*sqrt(3)*(x^5 -
 x^3) - (432^(5/6)*sqrt(3)*(7*x^3 - 3*x) - 216*2^(1/3)*(x^4 + 3*x^2))*(-x^2
 + 1)^(2/3) + 72*(x^5 - 18*x^4 + 24*x^3 + 18*x^2 - 9*x)*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 216*(sqrt(3)*2^(2/3)*(x^10 + 144*x^8 - 918*x^
6 + 2808*x^4 - 243*x^2) + 3*432^(1/6)*(31*x^9 - 568*x^7 + 1710*x^5 - 432*x^
3 + 27*x))*(-x^2 + 1)^(2/3) - 18*sqrt(3)*(x^12 - 366*x^10 + 14535*x^8 - 426
60*x^6 + 58239*x^4 - 14094*x^2 + 729) - 144*sqrt(3)*(11*x^11 - 807*x^9 + 45
18*x^7 - 5238*x^5 + 3807*x^3 - 243*x) + (-x^2 + 1)^(1/3)*(432^(5/6)*(x^11 -
 1215*x^9 + 11754*x^7 - 21006*x^5 + 5589*x^3 - 243*x) + 432*sqrt(3)*2^(1/3)
*(13*x^10 - 120*x^8 + 1242*x^6 - 1728*x^4 + 81*x^2)))/(x^12 - 8334*x^10 + 1
10727*x^8 - 301860*x^6 + 187839*x^4 - 21870*x^2 + 729))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

maple [C] time = 42.31, size = 938, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] -1/432*ln((RootOf(_Z^6+108)^4*x^6+72*RootOf(_Z^6+108)^4*x^5+36*RootOf(_Z^6+
108)^2*(-x^2+1)^(1/3)*x^5+225*RootOf(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^2
*(-x^2+1)^(1/3)*x^4-72*RootOf(_Z^6+108)^4*x^3+648*(-x^2+1)^(2/3)*x^4+864*Ro
otOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3-189*RootOf(_Z^6+108)^4*x^2+4536*(-x^2+1
)^(2/3)*x^3-648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/3)*x
^2-324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x+27*RootOf(_Z^6+108)^4-1944*x*(-x
^2+1)^(2/3))/(x^2+3)^3)*RootOf(_Z^6+108)^4+1/72*ln((RootOf(_Z^6+108)^4*x^6+
72*RootOf(_Z^6+108)^4*x^5+36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5+225*Root
Of(_Z^6+108)^4*x^4+648*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^4-72*RootOf(_Z^6
+108)^4*x^3+648*(-x^2+1)^(2/3)*x^4+864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^
3-189*RootOf(_Z^6+108)^4*x^2+4536*(-x^2+1)^(2/3)*x^3-648*RootOf(_Z^6+108)^2
*(-x^2+1)^(1/3)*x^2+1944*(-x^2+1)^(2/3)*x^2-324*RootOf(_Z^6+108)^2*(-x^2+1)
^(1/3)*x+27*RootOf(_Z^6+108)^4-1944*x*(-x^2+1)^(2/3))/(x^2+3)^3)*RootOf(_Z^
6+108)+1/36*RootOf(_Z^6+108)*ln((1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/3
)*x^3+3888*(-x^2+1)^(2/3)*x^2-486*RootOf(_Z^6+108)-3888*x*(-x^2+1)^(2/3)+18
9*RootOf(_Z^6+108)^4*x^2+3402*RootOf(_Z^6+108)*x^2+72*RootOf(_Z^6+108)^4*x^
3+1296*RootOf(_Z^6+108)*x^3-1296*RootOf(_Z^6+108)*x^5-4050*RootOf(_Z^6+108)
*x^4-RootOf(_Z^6+108)^4*x^6-72*RootOf(_Z^6+108)^4*x^5-225*RootOf(_Z^6+108)^
4*x^4-18*RootOf(_Z^6+108)*x^6-144*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^3+108
*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^2+54*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5
*x-36*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^5-648*RootOf(_Z^6+108)^2*(-x^2+1)
^(1/3)*x^4-864*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x^3+648*RootOf(_Z^6+108)^2
*(-x^2+1)^(1/3)*x^2+324*RootOf(_Z^6+108)^2*(-x^2+1)^(1/3)*x-6*(-x^2+1)^(1/3
)*RootOf(_Z^6+108)^5*x^5-108*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^4-27*RootO
f(_Z^6+108)^4)/(x^2+3)^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)\left(-x^2+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^2\right)^{1/3}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

```
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,\left(x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.812 $\int \frac{x^7}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{9\left(1-x^2\right)^{2/3}\left(14x^2+69\right)}{40\left(x^2+3\right)}-\frac{99\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{99\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}-\frac{3\left(1-x^2\right)^{2/3}x^4}{10\left(x^2+3\right)}$$

Rubi [A] time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 100, 146, 55, 617, 204, 31}

$$-\frac{3\left(1-x^2\right)^{2/3}x^4}{10\left(x^2+3\right)}+\frac{9\left(1-x^2\right)^{2/3}\left(14x^2+69\right)}{40\left(x^2+3\right)}-\frac{99\log\left(x^2+3\right)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{99\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*x^4*(1 - x^2)^(2/3))/(10*(3 + x^2)) + (9*(1 - x^2)^(2/3)*(69 + 14*x^2))
/(40*(3 + x^2)) + (99*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2
^(2/3)) - (99*Log[3 + x^2])/(16*2^(2/3)) + (297*Log[2^(2/3) - (1 - x^2)^(1/
3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)\\
&= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} - \frac{3}{10}\,\text{Subst}\left(\int \frac{x(-6+7x)}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)\\
&= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} + \frac{99}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)\\
&= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} - \frac{99\log\left(3+x^2\right)}{16\ 2^{2/3}} + \frac{297}{16}\,\text{Subst}\Bigg(\int \frac{}{2\sqrt[3]{2}}\\
&= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} - \frac{99\log\left(3+x^2\right)}{16\ 2^{2/3}} + \frac{297\log\Big(2^{2/3}-\sqrt[3]{1-}}{16\ 2^{2/3}}\\
&= -\frac{3x^4\left(1-x^2\right)^{2/3}}{10\left(3+x^2\right)} + \frac{9\left(1-x^2\right)^{2/3}\left(69+14x^2\right)}{40\left(3+x^2\right)} + \frac{99\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} - \frac{99\log\Big(3+}{16\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 120, normalized size = 0.90

$$
\frac{3}{80}\left(\frac{6\left(1-x^2\right)^{2/3}\left(14x^2+69\right)}{x^2+3} + \frac{165\left(-\log\left(x^2+3\right)+3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)}{2^{2/3}} - \frac{8\left(1-x^2\right)^{2/3}x^4}{x^2+3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((-8*x^4*(1 - x^2)^(2/3))/(3 + x^2) + (6*(1 - x^2)^(2/3)*(69 + 14*x^2))/
(3 + x^2) + (165*(2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - Log[3
 + x^2] + 3*Log[2^(2/3) - (1 - x^2)^(1/3)]))/2^(2/3)))/80
```

IntegrateAlgebraic [A] time = 0.21, size = 154, normalized size = 1.16

$$
\frac{99\log\left(\sqrt[3]{2}\,\sqrt[3]{1-x^2}-2\right)}{8\ 2^{2/3}} - \frac{99\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\,\sqrt[3]{1-x^2}+4\right)}{16\ 2^{2/3}} + \frac{99\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2}\,\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{8\ 2^{2/3}} - \frac{3\left(1-x^2\right)^{2/3}\left(4x^4-42x^2-207\right)}{40\left(x^2+3\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3)*(-207 - 42*x^2 + 4*x^4))/(40*(3 + x^2)) + (99*Sqrt[3]*A
rcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (99*Log
[-2 + 2^(1/3)*(1 - x^2)^(1/3)])/(8*2^(2/3)) - (99*Log[4 + 2*2^(1/3)*(1 - x^
2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)])/(16*2^(2/3))
```

fricas [A] time = 1.03, size = 133, normalized size = 1.00

$$\frac{3\left(660\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^{2}+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)-165\cdot 4^{\frac{2}{3}}\left(x^{2}+3\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)+330\cdot 4^{\frac{2}{3}}\left(x^{2}+3\right)\log\left(-4^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)-8\left(4x^{4}-42x^{2}-207\right)\left(-x^{2}+1\right)^{\frac{2}{3}}\right)}{320\left(x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 3/320*(660*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) +
2*(-x^2 + 1)^(1/3))) - 165*4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 +
1)^(1/3) + (-x^2 + 1)^(2/3)) + 330*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 +
 1)^(1/3)) - 8*(4*x^4 - 42*x^2 - 207)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

giac [A] time = 0.48, size = 126, normalized size = 0.95

$$\frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{3}{10}\left(-x^{2}+1\right)^{\frac{5}{3}}-\frac{99}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)+\frac{99}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^{2}+1\right)^{\frac{1}{3}}\right)+\frac{15}{4}\left(-x^{2}+1\right)^{\frac{2}{3}}+\frac{27\left(-x^{2}+1\right)^{\frac{2}{3}}}{8\left(x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/
3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

maple [C] time = 5.98, size = 770, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] 3/40*(4*x^6-46*x^4-165*x^2+207)/(x^2+3)/(-x^2+1)^(1/3)+99/16*RootOf(_Z^3-2)
*ln((-96*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3
-2)^2*x^2-8*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2
)+16*_Z^2)-168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+1
6*_Z^2)*RootOf(_Z^3-2)-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-60*RootOf(RootOf(
_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-5*RootOf(_Z^3-2)*x^2+42*(-x^2+1)
```

```
^(2/3)+252*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+21*RootOf(_
Z^3-2))/(x^2+3))-99/16*ln((64*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+1
6*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)
^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+
4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1
/3)-8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+RootOf(_Z^3-
2)*x^2+42*(-x^2+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16
*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))*RootOf(_Z^3-2)-99/4*ln((64*RootOf(RootOf
(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*x^2*RootOf
(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-168*(-x^2+1
)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)
-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_
Z^3-2)+16*_Z^2)*x^2+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*RootOf(RootOf(
_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))*RootOf(R
ootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)
```

maxima [A] time = 1.98, size = 126, normalized size = 0.95

$$\frac{99}{32} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2 \left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{10} \left(-x^2+1\right)^{\frac{5}{3}} - \frac{99}{64} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(-x^2+1\right)^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{99}{32} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{15}{4} \left(-x^2+1\right)^{\frac{2}{3}} + \frac{27 \left(-x^2+1\right)^{\frac{2}{3}}}{8 \left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1
/3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

mupad [B] time = 0.92, size = 148, normalized size = 1.11

$$\frac{99\, 2^{1/3} \ln\left(\frac{88209 \left(1-x^2\right)^{1/3}}{64} - \frac{88209\, 2^{2/3}}{64}\right)}{16} + \frac{27 \left(1-x^2\right)^{2/3}}{8 \left(x^2+3\right)} + \frac{15 \left(1-x^2\right)^{2/3}}{4} + \frac{3 \left(1-x^2\right)^{5/3}}{10} + \frac{99\, 2^{1/3} \ln\left(\frac{88209 \left(1-x^2\right)^{1/3}}{64} - \frac{88209\, 2^{2/3} \left(-1+\sqrt{3}\, 1\mathrm{i}\right)^2}{256}\right) \left(-1+\sqrt{3}\, 1\mathrm{i}\right)}{32} - \frac{99\, 2^{1/3} \ln\left(\frac{88209 \left(1-x^2\right)^{1/3}}{64} - \frac{88209\, 2^{2/3} \left(1+\sqrt{3}\, 1\mathrm{i}\right)^2}{256}\right) \left(1+\sqrt{3}\, 1\mathrm{i}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (99*2^(1/3)*log((88209*(1 - x^2)^(1/3))/64 - (88209*2^(2/3))/64))/16 + (27*
(1 - x^2)^(2/3))/(8*(x^2 + 3)) + (15*(1 - x^2)^(2/3))/4 + (3*(1 - x^2)^(5/3
))/10 + (99*2^(1/3)*log((88209*(1 - x^2)^(1/3))/64 - (88209*2^(2/3)*(3^(1/2
)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (99*2^(1/3)*log((88209*(1 - x^2)^(
1/3))/64 - (88209*2^(2/3)*(3^(1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.813 $\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\,2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\,2^{2/3}}$$

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 89, 80, 55, 617, 204, 31}

$$-\frac{3}{4}\left(1-x^2\right)^{2/3}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{21\log\left(x^2+3\right)}{16\,2^{2/3}}-\frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}-\frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3))/4 - (9*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (21*Sqrt[3]*Ar
cTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (21*Log[3 + x^2])/(16*
2^(2/3)) - (63*Log[2^(2/3) - (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)$$

$$= -\frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{-9+4x}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{21}{8}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}} - \frac{63}{16}\,\mathrm{Subst}\Bigg(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}} - \frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}} + 63\,\mathrm{Subs}$$

$$= -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{9\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{21\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} + \frac{21\log\left(3+x^2\right)}{16\ 2^{2/3}} - 63\log\Big(2$$

Mathematica [A] time = 0.17, size = 110, normalized size = 0.95

$$\frac{3}{32}\left(-8\left(1-x^2\right)^{2/3} - \frac{12\left(1-x^2\right)^{2/3}}{x^2+3} + 7\sqrt[3]{2}\log\left(x^2+3\right) - 21\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right) - 14\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(-8*(1 - x^2)^(2/3) - (12*(1 - x^2)^(2/3))/(3 + x^2) - 14*2^(1/3)*Sqrt[3
]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 7*2^(1/3)*Log[3 + x^2] - 21*2^(
1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)]))/32
```

IntegrateAlgebraic [A] time = 0.16, size = 149, normalized size = 1.28

$$-\frac{3\left(1-x^2\right)^{2/3}\left(2x^2+9\right)}{8\left(x^2+3\right)} - \frac{21\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{8\ 2^{2/3}} + \frac{21\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{16\ 2^{2/3}} - \frac{21\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{8\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3)*(9 + 2*x^2))/(8*(3 + x^2)) - (21*Sqrt[3]*ArcTan[1/Sqrt[
3] + (2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) - (21*Log[-2 + 2^(1/3)
*(1 - x^2)^(1/3)])/(8*2^(2/3)) + (21*Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^
(2/3)*(1 - x^2)^(2/3)])/(16*2^(2/3))
```

fricas [A] time = 1.15, size = 153, normalized size = 1.32

$$-\frac{3\left(28\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(x^{2}+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\left(-1\right)^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)+7\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{2}+3\right)\log\left(4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)-14\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^{2}+3\right)\log\left(-4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)+8\left(2x^{2}+9\right)\left(-x^{2}+1\right)^{\frac{2}{3}}\right)}{64\left(x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -3/64*(28*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(
2*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3))) + 7*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*
log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(
2/3)) - 14*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1
)^(1/3)) + 8*(2*x^2 + 9)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

giac [A] time = 0.42, size = 115, normalized size = 0.99

$$-\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right)+\frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)-\frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^{2}+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^{2}+1\right)^{\frac{2}{3}}-\frac{9\left(-x^{2}+1\right)^{\frac{2}{3}}}{8\left(x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] -21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 21/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2
/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

maple [C] time = 6.77, size = 490, normalized size = 4.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] 3/8*(2*x^4+7*x^2-9)/(x^2+3)/(-x^2+1)^(1/3)+21/16*RootOf(_Z^3+2)*ln(-(48*Roo
tOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)^2*RootOf(_Z^3+2)^2*x^2+8*
RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*RootOf(_Z^3+2)^3*x^2-8
4*RootOf(_Z^3+2)*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)
+16*_Z^2)+6*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)*x^2+RootOf
(_Z^3+2)*x^2-21*(-x^2+1)^(2/3)-126*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3
```

```
+2)+16*_Z^2)-21*RootOf(_Z^3+2))/(x^2+3))+21/4*RootOf(RootOf(_Z^3+2)^2+4*_Z*
RootOf(_Z^3+2)+16*_Z^2)*ln((64*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+
16*_Z^2)^2*RootOf(_Z^3+2)^2*x^2+24*RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3
+2)+16*_Z^2)*RootOf(_Z^3+2)^3*x^2+168*RootOf(_Z^3+2)*(-x^2+1)^(1/3)*RootOf(
RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)-40*RootOf(RootOf(_Z^3+2)^2+4*
_Z*RootOf(_Z^3+2)+16*_Z^2)*x^2-15*RootOf(_Z^3+2)*x^2+42*(-x^2+1)^(2/3)+168*
RootOf(RootOf(_Z^3+2)^2+4*_Z*RootOf(_Z^3+2)+16*_Z^2)+63*RootOf(_Z^3+2))/(x^
2+3))
```

maxima [A] time = 1.96, size = 115, normalized size = 0.99

$$-\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}-\frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 21/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(
2/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

mupad [B] time = 0.89, size = 137, normalized size = 1.18

$$-\frac{21\,2^{1/3}\ln\left(\frac{3969\left(1-x^2\right)^{1/3}}{64}-\frac{3969\,2^{2/3}}{64}\right)}{16}-\frac{9\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\left(1-x^2\right)^{2/3}}{4}-\frac{21\,2^{1/3}\ln\left(\frac{3969\left(1-x^2\right)^{1/3}}{64}-\frac{3969\,2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32}+\frac{21\,2^{1/3}\ln\left(\frac{3969\left(1-x^2\right)^{1/3}}{64}-\frac{3969\,2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (21*2^(1/3)*log((3969*(1 - x^2)^(1/3))/64 - (3969*2^(2/3)*(3^(1/2)*1i + 1)^
2)/256)*(3^(1/2)*1i + 1))/32 - (9*(1 - x^2)^(2/3))/(8*(x^2 + 3)) - (3*(1 -
x^2)^(2/3))/4 - (21*2^(1/3)*log((3969*(1 - x^2)^(1/3))/64 - (3969*2^(2/3)*(
3^(1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (21*2^(1/3)*log((3969*(1 - x
^2)^(1/3))/64 - (3969*2^(2/3))/64))/16
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.814 $\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=101

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\log\left(x^2+3\right)}{16\,2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\,2^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 55, 617, 204, 31}

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3\log\left(x^2+3\right)}{16\,2^{2/3}}+\frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3
))/Sqrt[3]])/(8*2^(2/3)) - (3*Log[3 + x^2])/(16*2^(2/3)) + (9*Log[2^(2/3) -
 (1 - x^2)^(1/3)])/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(3+x)^2}\,dx, x, x^2\right)$$

$$= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx, x, x^2\right)$$

$$= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{3\log\left(3+x^2\right)}{16\,2^{2/3}} + \frac{9}{16}\,\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx, x, \sqrt[3]{1-x^2}\right) - \frac{9\,\mathrm{S}}{\ }$$

$$= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} - \frac{3\log\left(3+x^2\right)}{16\,2^{2/3}} + \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}} - \frac{9\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.}{8\,2^{2/3}}$$

$$= \frac{3\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{3\sqrt{3}\,\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\,2^{2/3}} - \frac{3\log\left(3+x^2\right)}{16\,2^{2/3}} + \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}$$

Mathematica [A]   time = 0.11, size = 97, normalized size = 0.96

$$\frac{3}{32}\left(\frac{4\left(1-x^2\right)^{2/3}}{x^2+3}-\sqrt[3]{2}\log\left(x^2+3\right)+3\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*((4*(1 - x^2)^(2/3))/(3 + x^2) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^
2)^(1/3))/Sqrt[3]] - 2^(1/3)*Log[3 + x^2] + 3*2^(1/3)*Log[2^(2/3) - (1 - x^
2)^(1/3)]))/32
```

IntegrateAlgebraic [A]   time = 0.16, size = 142, normalized size = 1.41

$$\frac{3\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{8\,2^{2/3}}-\frac{3\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{16\,2^{2/3}}+\frac{3\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{8\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (3*(1 - x^2)^(2/3))/(8*(3 + x^2)) + (3*Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(1/3)*
(1 - x^2)^(1/3))/Sqrt[3]])/(8*2^(2/3)) + (3*Log[-2 + 2^(1/3)*(1 - x^2)^(1/3
)])/(8*2^(2/3)) - (3*Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^
(2/3)])/(16*2^(2/3))
```

fricas [A]   time = 1.04, size = 121, normalized size = 1.20

$$\frac{3\left(4\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+2\cdot 4^{\frac{2}{3}}\left(x^2+3\right)\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+8\left(-x^2+1\right)^{\frac{2}{3}}\right)}{64\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 3/64*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(
-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3
) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)
) + 8*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

giac [A]   time = 0.41, size = 104, normalized size = 1.03

$$\frac{3}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{3}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{3}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) + 3/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2/3)/
(x^2 + 3)
```

maple [C] time = 5.05, size = 758, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] -3/8*(x^2-1)/(x^2+3)/(-x^2+1)^(1/3)+3/16*RootOf(_Z^3-2)*ln((-96*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*x^2*Root
Of(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-168*(-x^2
+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-
2)-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-60*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootO
f(_Z^3-2)+16*_Z^2)*x^2-5*RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+252*RootOf(Ro
otOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+21*RootOf(_Z^3-2))/(x^2+3))-3/1
6*ln((64*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3
-2)^2*x^2-8*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2
)+16*_Z^2)-168*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+1
6*_Z^2)*RootOf(_Z^3-2)-42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-8*RootOf(RootOf(_
Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+RootOf(_Z^3-2)*x^2+42*(-x^2+1)^(2
/3)+168*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3
-2))/(x^2+3))*RootOf(_Z^3-2)-3/4*ln((64*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf
(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-8*x^2*RootOf(_Z^3-2)^3*RootOf(Root
Of(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-168*(-x^2+1)^(1/3)*RootOf(RootOf(
_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*RootOf(_Z^3-2)-42*RootOf(_Z^3-2)^2*(
-x^2+1)^(1/3)-8*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2+Ro
otOf(_Z^3-2)*x^2+42*(-x^2+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(
_Z^3-2)+16*_Z^2)-21*RootOf(_Z^3-2))/(x^2+3))*RootOf(RootOf(_Z^3-2)^2+4*_Z*R
ootOf(_Z^3-2)+16*_Z^2)
```

maxima [A] time = 1.95, size = 104, normalized size = 1.03

$$\frac{3}{32} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2 \left(-x^{2} + 1\right)^{\frac{1}{3}}\right)\right) - \frac{3}{64} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(-x^{2} + 1\right)^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{2}{3}}\right) + \frac{3}{32} \cdot 4^{\frac{2}{3}} \log\left(-4^{\frac{1}{3}} + \left(-x^{2} + 1\right)^{\frac{1}{3}}\right) + \frac{3 \left(-x^{2} + 1\right)^{\frac{2}{3}}}{8 \left(x^{2} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) + 3/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2/3)
/(x^2 + 3)
```

mupad [B] time = 0.86, size = 126, normalized size = 1.25

$$\frac{3\,2^{1/3}\,\ln\left(\frac{81\,\left(1-x^2\right)^{1/3}}{64}-\frac{81\,2^{2/3}}{64}\right)}{16}+\frac{3\,\left(1-x^2\right)^{2/3}}{8\,\left(x^2+3\right)}+\frac{3\,2^{1/3}\,\ln\left(\frac{81\,\left(1-x^2\right)^{1/3}}{64}-\frac{81\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32}-\frac{3\,2^{1/3}\,\ln\left(\frac{81\,\left(1-x^2\right)^{1/3}}{64}-\frac{81\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/64 - (81*2^(2/3))/64))/16 + (3*(1 - x^2
)^(2/3))/(8*(x^2 + 3)) + (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/64 - (81*2^(2/
3)*(3^(1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (3*2^(1/3)*log((81*(1 -
x^2)^(1/3))/64 - (81*2^(2/3)*(3^(1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)(x+1)}\,\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.815 $\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{\log\left(x^2+3\right)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

Rubi **[A]** time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {444, 51, 55, 617, 204, 31}

$$-\frac{\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{\log\left(x^2+3\right)}{48\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -(1 - x^2)^(2/3)/(8*(3 + x^2)) + ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8
*2^(2/3)*Sqrt[3]) - Log[3 + x^2]/(48*2^(2/3)) + Log[2^(2/3) - (1 - x^2)^(1/
3)]/(16*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{1}{24}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{\log\left(3+x^2\right)}{48\,2^{2/3}}+\frac{1}{16}\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+2^{2/3}x+x^2}\,dx,x,\sqrt[3]{1-x^2}\right)-\text{Su}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}-\frac{\log\left(3+x^2\right)}{48\,2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\sqrt[3]{2 -}\right.}{8\,2^{2/3}}\\
&= -\frac{\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\,2^{2/3}\sqrt{3}}-\frac{\log\left(3+x^2\right)}{48\,2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\,2^{2/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.34

$$-\frac{3}{64}\left(1-x^2\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{1}{4}\left(1-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-3*(1 - x^2)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, (1 - x^2)/4])/64
```

IntegrateAlgebraic [A] time = 0.13, size = 142, normalized size = 1.41

$$-\frac{\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{24\ 2^{2/3}}-\frac{\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{48\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] -1/8*(1 - x^2)^(2/3)/(3 + x^2) + ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(1/3
))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]) + Log[-2 + 2^(1/3)*(1 - x^2)^(1/3)]/(24*2^(
2/3)) - Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)]/(48*2^
(2/3))
```

fricas [A] time = 0.96, size = 125, normalized size = 1.24

$$\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^2+3\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}\left(x^2+3\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+2\cdot 4^{\frac{2}{3}}\left(x^2+3\right)\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-24\left(-x^2+1\right)^{\frac{2}{3}}}{192\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*
sqrt(3)*(-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 +
 1)^(1/3)) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

giac [A] time = 0.39, size = 104, normalized size = 1.03

$$\frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/96*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(2/3)
/(x^2 + 3)
```

maple [C] time = 4.95, size = 657, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] 1/8*(x^2-1)/(x^2+3)/(-x^2+1)^(1/3)-1/48*ln(-(8*x^2*RootOf(_Z^3-2)^3*RootOf(
RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-16*RootOf(RootOf(_Z^3-2)^2+4*
_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-21*RootOf(_Z^3-2)^2*(-x^2
+1)^(1/3)+5*RootOf(_Z^3-2)*x^2-10*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-
2)+16*_Z^2)*x^2-21*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)+42*RootOf(RootOf(_Z^3-2
)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(x^2+3))*RootOf(_Z^3-2)-1/12*ln(-(8*x^2*R
ootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)-16*Roo
tOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2-21
*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)+5*RootOf(_Z^3-2)*x^2-10*RootOf(RootOf(_Z^3
-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^2-21*(-x^2+1)^(2/3)-21*RootOf(_Z^3-2)+
42*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2))/(x^2+3))*RootOf(Ro
otOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+1/12*RootOf(RootOf(_Z^3-2)^2+4*
_Z*RootOf(_Z^3-2)+16*_Z^2)*ln((24*x^2*RootOf(_Z^3-2)^3*RootOf(RootOf(_Z^3-2
)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)+32*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^
3-2)+16*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+42*RootOf(_Z^3-2)^2*(-x^2+1)^(1/3)-3*R
ootOf(_Z^3-2)*x^2-4*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootOf(_Z^3-2)+16*_Z^2)*x^
2+42*(-x^2+1)^(2/3)+63*RootOf(_Z^3-2)+84*RootOf(RootOf(_Z^3-2)^2+4*_Z*RootO
f(_Z^3-2)+16*_Z^2))/(x^2+3))
```

maxima [A] time = 2.03, size = 104, normalized size = 1.03

$$\frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/96*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(2/3
)/(x^2 + 3)
```

mupad [B] time = 0.86, size = 126, normalized size = 1.25

$$\frac{2^{1/3}\ln\left(\frac{\left(1-x^2\right)^{1/3}}{64}-\frac{2^{2/3}}{64}\right)}{48}-\frac{\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}+\frac{2^{1/3}\ln\left(\frac{\left(1-x^2\right)^{1/3}}{64}-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{96}-\frac{2^{1/3}\ln\left(\frac{\left(1-x^2\right)^{1/3}}{64}-\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (2^(1/3)*log((1 - x^2)^(1/3)/64 - 2^(2/3)/64))/48 - (1 - x^2)^(2/3)/(8*(x^2
 + 3)) + (2^(1/3)*log((1 - x^2)^(1/3)/64 - (2^(2/3)*(3^(1/2)*1i - 1)^2)/256
)*(3^(1/2)*1i - 1))/96 - (2^(1/3)*log((1 - x^2)^(1/3)/64 - (2^(2/3)*(3^(1/2
)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/96
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(x/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.816 $\int \frac{1}{x\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

Optimal. Leaf size=158

$$\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)}+\frac{5\log\left(x^2+3\right)}{144\,2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\,2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Rubi **[A]** time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 55, 618, 204, 31, 617}

$$\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)}+\frac{5\log\left(x^2+3\right)}{144\,2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\,2^{2/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{24\,2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) - (5*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]
)/(24*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(6*Sqrt[3]
) - Log[x]/18 + (5*Log[3 + x^2])/(144*2^(2/3)) + Log[1 - (1 - x^2)^(1/3)]/1
2 - (5*Log[2^(2/3) - (1 - x^2)^(1/3)])/(48*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(3+x)^2}\,dx,x,x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}+\frac{1}{24}\,\text{Subst}\left(\int \frac{4-\frac{x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,x,x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}+\frac{1}{18}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^2\right)-\frac{5}{72}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(3+x)}\,dx,x\right.\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}-\frac{\log(x)}{18}+\frac{5\log\left(3+x^2\right)}{144\;2^{2/3}}-\frac{1}{12}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^2}\right)+\frac{1}{12}\,\text{S}\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}-\frac{\log(x)}{18}+\frac{5\log\left(3+x^2\right)}{144\;2^{2/3}}+\frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{5\log\left(2^{2/3}-\sqrt[3]{1-\right.}{48\;2^{2/3}}\\
&= \frac{\left(1-x^2\right)^{2/3}}{24\left(3+x^2\right)}-\frac{5\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{24\;2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\log(x)}{18}+\frac{5\log\left(3+x^2\right)}{144\;2^{2/3}}+\end{aligned}$$

Mathematica [A] time = 0.24, size = 148, normalized size = 0.94

$$\frac{1}{288}\left(\frac{12\left(1-x^2\right)^{2/3}}{x^2+3}+5\sqrt[3]{2}\log\left(x^2+3\right)+24\log\left(1-\sqrt[3]{1-x^2}\right)-15\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-10\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)+16\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-16\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((12*(1 - x^2)^(2/3))/(3 + x^2) - 10*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2
)^(1/3))/Sqrt[3]] + 16*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 16
*Log[x] + 5*2^(1/3)*Log[3 + x^2] + 24*Log[1 - (1 - x^2)^(1/3)] - 15*2^(1/3)
*Log[2^(2/3) - (1 - x^2)^(1/3)])/288
```

IntegrateAlgebraic [A] time = 0.26, size = 223, normalized size = 1.41

$$\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)}+\frac{1}{18}\log\left(\sqrt[3]{1-x^2}-1\right)-\frac{5\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{72\;2^{2/3}}-\frac{1}{36}\log\left(\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1\right)+\frac{5\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{144\;2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{24\;2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2)^(2/3)/(24*(3 + x^2)) + ArcTan[1/Sqrt[3] + (2*(1 - x^2)^(1/3))/Sqr
t[3]]/(6*Sqrt[3]) - (5*ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]
])/(24*2^(2/3)*Sqrt[3]) + Log[-1 + (1 - x^2)^(1/3)]/18 - (5*Log[-2 + 2^(1/3
)*(1 - x^2)^(1/3)])/(72*2^(2/3)) - Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3
)]/36 + (5*Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^(2/3)])/(1
44*2^(2/3))
```

fricas [A] time = 1.29, size = 227, normalized size = 1.44

$$-\frac{20 \cdot 4^{\frac{1}{6}} \sqrt{3} \, (-1)^{\frac{1}{3}} \left(x^{2}+3\right) \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}} \left(2 \sqrt{3} \, (-1)^{\frac{1}{3}} \left(-x^{2}+1\right)^{\frac{1}{3}} - 4^{\frac{1}{3}} \sqrt{3}\right)\right) + 5 \cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \left(x^{2}+3\right) \log\left(4^{\frac{1}{3}} (-1)^{\frac{2}{3}} \left(-x^{2}+1\right)^{\frac{1}{3}} - 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} + \left(-x^{2}+1\right)^{\frac{2}{3}}\right) - 10 \cdot 4^{\frac{2}{3}} (-1)^{\frac{1}{3}} \left(x^{2}+3\right) \log\left(-4^{\frac{1}{3}} (-1)^{\frac{2}{3}} + \left(-x^{2}+1\right)^{\frac{1}{3}}\right) - 32 \sqrt{3} \left(x^{2}+3\right) \arctan\left(\frac{2}{3} \sqrt{3} \left(-x^{2}+1\right)^{\frac{1}{3}} + \frac{1}{3} \sqrt{3}\right) + 16 \left(x^{2}+3\right) \log\left(\left(-x^{2}+1\right)^{\frac{2}{3}} + \left(-x^{2}+1\right)^{\frac{1}{3}} + 1\right) - 32 \left(x^{2}+3\right) \log\left(\left(-x^{2}+1\right)^{\frac{1}{3}} - 1\right) - 24 \left(-x^{2}+1\right)^{\frac{2}{3}}}{576 \left(x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/576*(20*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(2*sqrt(
3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 5*4^(2/3)*(-1)^(1/3)*(
x^2 + 3)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x
^2 + 1)^(2/3)) - 10*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) +
(-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^2 + 3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3
) + 1/3*sqrt(3)) + 16*(x^2 + 3)*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1
) - 32*(x^2 + 3)*log((-x^2 + 1)^(1/3) - 1) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

giac [A] time = 0.55, size = 167, normalized size = 1.06

$$-\frac{5}{288} \cdot 4^{\frac{2}{3}} \sqrt{3} \arctan\left(\frac{1}{12} \cdot 4^{\frac{2}{3}} \sqrt{3} \left(4^{\frac{1}{3}} + 2 \left(-x^{2}+1\right)^{\frac{1}{3}}\right)\right) + \frac{5}{576} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{2}{3}} + 4^{\frac{1}{3}} \left(-x^{2}+1\right)^{\frac{1}{3}} + \left(-x^{2}+1\right)^{\frac{2}{3}}\right) - \frac{5}{288} \cdot 4^{\frac{2}{3}} \log\left(4^{\frac{1}{3}} - \left(-x^{2}+1\right)^{\frac{1}{3}}\right) + \frac{1}{18} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \left(-x^{2}+1\right)^{\frac{1}{3}} + 1\right)\right) + \frac{\left(-x^{2}+1\right)^{\frac{2}{3}}}{24 \left(x^{2}+3\right)} - \frac{1}{36} \log\left(\left(-x^{2}+1\right)^{\frac{2}{3}} + \left(-x^{2}+1\right)^{\frac{1}{3}} + 1\right) + \frac{1}{18} \log\left(-\left(-x^{2}+1\right)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] -5/288*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 5/576*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 5/288*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/18*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/24*(-x^2 + 1)^(2/3)/(x^2 + 3)
- 1/36*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/18*log(-(-x^2 + 1)^
(1/3) + 1)
```

maple [F] time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x), x)
```

mupad [B] time = 0.93, size = 375, normalized size = 2.37

$$\frac{\ln\left(\frac{127}{512}-\frac{127\left(1-x^2\right)^{1/3}}{512}\right)}{18}-\frac{5\,2^{1/3}\ln\left(-\frac{25\,2^{2/3}\left(\frac{5\,2^{1/3}\left(\frac{30375\,2^{2/3}}{64}-\frac{68283\left(1-x^2\right)^{1/3}}{64}\right)}{144}-\frac{1647}{128}\right)}{20736}-\frac{25\left(1-x^2\right)^{1/3}}{384}\right)}{144}+\ln\left(\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2\left(\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)\left(393660\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2-\frac{68283\left(1-x^2\right)^{1/3}}{64}\right)+\frac{1647}{128}\right)-\frac{25\left(1-x^2\right)^{1/3}}{384}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)-\ln\left(-\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2\left(\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)\left(393660\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2-\frac{68283\left(1-x^2\right)^{1/3}}{64}\right)-\frac{1647}{128}\right)-\frac{25\left(1-x^2\right)^{1/3}}{384}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)+\frac{\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)}+\frac{5\,(-1)^{1/3}\,2^{1/3}\ln\left(\frac{25\,(-1)^{2/3}\,2^{2/3}\left(\frac{5\,(-1)^{1/3}\,2^{1/3}\left(\frac{30375\,(-1)^{2/3}\,2^{2/3}}{64}-\frac{68283\left(1-x^2\right)^{1/3}}{64}\right)}{144}+\frac{1647}{128}\right)}{20736}-\frac{25\left(1-x^2\right)^{1/3}}{384}\right)}{144}-\frac{5\,(-1)^{1/3}\,2^{1/3}\ln\left(-\frac{25\left(1-x^2\right)^{1/3}}{384}+\frac{25\,(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2\left(\frac{5\,(-1)^{1/3}\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(\frac{68283\left(1-x^2\right)^{1/3}}{64}-\frac{30375\,(-1)^{2/3}\,2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)}{288}+\frac{1647}{128}\right)}{82944}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] log(127/512 - (127*(1 - x^2)^(1/3))/512)/18 - (5*2^(1/3)*log(- (25*2^(2/3)*
((5*2^(1/3)*((30375*2^(2/3))/64 - (68283*(1 - x^2)^(1/3))/64))/144 - 1647/1
28))/20736 - (25*(1 - x^2)^(1/3))/384))/144 + log(((3^(1/2)*1i)/36 - 1/36)^
2*(((3^(1/2)*1i)/36 - 1/36)*(393660*((3^(1/2)*1i)/36 - 1/36)^2 - (68283*(1
- x^2)^(1/3))/64) + 1647/128) - (25*(1 - x^2)^(1/3))/384)*((3^(1/2)*1i)/36
- 1/36) - log(- ((3^(1/2)*1i)/36 + 1/36)^2*(((3^(1/2)*1i)/36 + 1/36)*(39366
0*((3^(1/2)*1i)/36 + 1/36)^2 - (68283*(1 - x^2)^(1/3))/64) - 1647/128) - (2
5*(1 - x^2)^(1/3))/384)*((3^(1/2)*1i)/36 + 1/36) + (1 - x^2)^(2/3)/(24*(x^2
 + 3)) + (5*(-1)^(1/3)*2^(1/3)*log((25*(-1)^(2/3)*2^(2/3)*((5*(-1)^(1/3)*2^
(1/3)*((30375*(-1)^(2/3)*2^(2/3))/64 - (68283*(1 - x^2)^(1/3))/64))/144 + 1
647/128))/20736 - (25*(1 - x^2)^(1/3))/384))/144 - (5*(-1)^(1/3)*2^(1/3)*lo
g((25*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*((5*(-1)^(1/3)*2^(1/3)*(3^(1/2)
*1i + 1)*((68283*(1 - x^2)^(1/3))/64 - (30375*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1
i + 1)^2)/256))/288 + 1647/128))/82944 - (25*(1 - x^2)^(1/3))/384)*(3^(1/2)
*1i + 1))/288
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\left(x^2+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.817 $\int \frac{1}{x^3 \sqrt[3]{1-x^2} \left(3+x^2\right)^2} \, dx$

Optimal. Leaf size=183

$$-\frac{\left(1-x^2\right)^{2/3}}{6 x^2 \left(x^2+3\right)}-\frac{5 \left(1-x^2\right)^{2/3}}{72 \left(x^2+3\right)}-\frac{\log \left(x^2+3\right)}{48\ 2^{2/3}}-\frac{1}{36} \log \left(1-\sqrt[3]{1-x^2}\right)+\frac{\log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3} \sqrt{3}}-$$

Rubi [A] time = 0.13, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 151, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^2\right)^{2/3}}{6 x^2 \left(x^2+3\right)}-\frac{5 \left(1-x^2\right)^{2/3}}{72 \left(x^2+3\right)}-\frac{\log \left(x^2+3\right)}{48\ 2^{2/3}}-\frac{1}{36} \log \left(1-\sqrt[3]{1-x^2}\right)+\frac{\log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{8\ 2^{2/3} \sqrt{3}}-\frac{\tan^{-1}\left(\frac{2 \sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18 \sqrt{3}}+\frac{\log (x)}{54}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (-5*(1 - x^2)^(2/3))/(72*(3 + x^2)) - (1 - x^2)^(2/3)/(6*x^2*(3 + x^2)) + A
rcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]) - ArcTan[(1 + 2*
(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + Log[x]/54 - Log[3 + x^2]/(48*2^(2/
3)) - Log[1 - (1 - x^2)^(1/3)]/36 + Log[2^(2/3) - (1 - x^2)^(1/3)]/(16*2^(2
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[3]{1-x^2}\,\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)}-\frac{1}{6}\,\text{Subst}\left(\int \frac{1-\frac{4x}{3}}{\sqrt[3]{1-x}\,x(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)}-\frac{1}{72}\,\text{Subst}\left(\int \frac{4-\frac{5x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,x,x^2\right)\\
&= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)}-\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^2\right)+\frac{1}{24}\,\text{Subst}\Bigg(\int\\
&= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)}+\frac{\log(x)}{54}-\frac{\log\left(3+x^2\right)}{48\ 2^{2/3}}+\frac{1}{36}\,\text{Subst}\Bigg(\int \frac{1}{1-x}\,dx,x,\\
&= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)}+\frac{\log(x)}{54}-\frac{\log\left(3+x^2\right)}{48\ 2^{2/3}}-\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)+\\
&= -\frac{5\left(1-x^2\right)^{2/3}}{72\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{6x^2\left(3+x^2\right)}+\frac{\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\log(x)}{54}-
\end{aligned}$$

Mathematica [A] time = 0.18, size = 171, normalized size = 0.93

$$\frac{1}{864}\left(-\frac{144\left(1-x^2\right)^{2/3}}{x^2\left(x^2+3\right)}-\frac{60\left(1-x^2\right)^{2/3}}{x^2+3}-9\sqrt[3]{2}\log\left(x^2+3\right)-24\log\left(1-\sqrt[3]{1-x^2}\right)+27\sqrt[3]{2}\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)+18\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)-16\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)+16\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((-60*(1 - x^2)^(2/3))/(3 + x^2) - (144*(1 - x^2)^(2/3))/(x^2*(3 + x^2)) +
18*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 16*Sqrt[3]*Arc
Tan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] + 16*Log[x] - 9*2^(1/3)*Log[3 + x^2] -
```

```
 24*Log[1 - (1 - x^2)^(1/3)] + 27*2^(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)])/8
64
```

IntegrateAlgebraic [A]   time = 0.34, size = 233, normalized size = 1.27

$$\frac{(1-x^2)^{2/3}\left(-5x^2-12\right)}{72x^2\left(x^2+3\right)}-\frac{1}{54}\log\left(\sqrt[3]{1-x^2}-1\right)+\frac{\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{24\,2^{2/3}}+\frac{1}{108}\log\left((1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1\right)-\frac{\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{48\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{18\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{8\,2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((-12 - 5*x^2)*(1 - x^2)^(2/3))/(72*x^2*(3 + x^2)) - ArcTan[1/Sqrt[3] + (2*
(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) + ArcTan[1/Sqrt[3] + (2^(1/3)*(1 - x
^2)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]) - Log[-1 + (1 - x^2)^(1/3)]/54 + Lo
g[-2 + 2^(1/3)*(1 - x^2)^(1/3)]/(24*2^(2/3)) + Log[1 + (1 - x^2)^(1/3) + (1
 - x^2)^(2/3)]/108 - Log[4 + 2*2^(1/3)*(1 - x^2)^(1/3) + 2^(2/3)*(1 - x^2)^
(2/3)]/(48*2^(2/3))
```

fricas [A]   time = 1.17, size = 238, normalized size = 1.30

$$\frac{36\cdot 4^{\frac{1}{6}}\sqrt{3}\left(x^4+3x^2\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-9\cdot 4^{\frac{2}{3}}\left(x^4+3x^2\right)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+18\cdot 4^{\frac{2}{3}}\left(x^4+3x^2\right)\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-32\sqrt{3}\left(x^4+3x^2\right)\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+16\left(x^4+3x^2\right)\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)-32\left(x^4+3x^2\right)\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)-24\left(5x^2+12\right)\left(-x^2+1\right)^{\frac{2}{3}}}{1728\left(x^4+3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1728*(36*4^(1/6)*sqrt(3)*(x^4 + 3*x^2)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3
) + 2*sqrt(3)*(-x^2 + 1)^(1/3))) - 9*4^(2/3)*(x^4 + 3*x^2)*log(4^(2/3) + 4^
(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 18*4^(2/3)*(x^4 + 3*x^2)*log(-
4^(1/3) + (-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^4 + 3*x^2)*arctan(2/3*sqrt(3)*(
-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 16*(x^4 + 3*x^2)*log((-x^2 + 1)^(2/3) + (-
x^2 + 1)^(1/3) + 1) - 32*(x^4 + 3*x^2)*log((-x^2 + 1)^(1/3) - 1) - 24*(5*x^
2 + 12)*(-x^2 + 1)^(2/3))/(x^4 + 3*x^2)
```

giac [A]   time = 0.42, size = 190, normalized size = 1.04

$$\frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)+\frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)-\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{5\left(-x^2+1\right)^{\frac{5}{3}}-17\left(-x^2+1\right)^{\frac{2}{3}}}{72\left(\left(x^2-1\right)^2+5x^2-1\right)}+\frac{1}{108}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{54}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/96*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/54*sqrt(3)*arctan(
1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/72*(5*(-x^2 + 1)^(5/3) - 17*(-x^2
```

```
 + 1)^(2/3))/((x^2 - 1)^2 + 5*x^2 - 1) + 1/108*log((-x^2 + 1)^(2/3) + (-x^2
 + 1)^(1/3) + 1) - 1/54*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [F] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}+3\right)^{2} \left(-x^{2}+1\right)^{\frac{1}{3}} x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3), x)
```

mupad [B] time = 0.99, size = 409, normalized size = 2.23

$$\frac{2^{1/3}\ln\left(\frac{2^{2/3}\left(\frac{2^{1/3}\left(\frac{10935\,2^{2/3}}{64}-\frac{9099\,(1-x^2)^{1/3}}{64}\right)}{48}-\frac{665}{128}\right)}{2304}+\frac{(1-x^2)^{1/3}}{576}\right)}{48}-\frac{\ln\left(\frac{985\,(1-x^2)^{1/3}}{373248}-\frac{985}{373248}\right)}{54}+\ln\left(\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)^2\left(\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)\left(393660\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)^2-\frac{9099\,(1-x^2)^{1/3}}{64}\right)-\frac{665}{128}\right)+\frac{(1-x^2)^{1/3}}{576}\right)\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)-\ln\left(\frac{(1-x^2)^{1/3}}{576}-\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)^2\left(\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)\left(393660\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)^2-\frac{9099\,(1-x^2)^{1/3}}{64}\right)+\frac{665}{128}\right)\right)\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)-\frac{\frac{17\,(1-x^2)^{2/3}}{72}-\frac{5\,(1-x^2)^{5/3}}{72}}{(x^2-1)^2+5\,x^2-1}+\frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{576}+\frac{2^{2/3}\,(-1+\sqrt{3}\,1\mathrm{i})^2\left(\frac{2^{1/3}\,(-1+\sqrt{3}\,1\mathrm{i})\left(\frac{10935\,2^{2/3}\,(-1+\sqrt{3}\,1\mathrm{i})^2}{256}-\frac{9099\,(1-x^2)^{1/3}}{64}\right)}{96}-\frac{665}{128}\right)}{9216}\right)(-1+\sqrt{3}\,1\mathrm{i})}{96}-\frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{576}-\frac{2^{2/3}\,(1+\sqrt{3}\,1\mathrm{i})^2\left(\frac{2^{1/3}\,(1+\sqrt{3}\,1\mathrm{i})\left(\frac{10935\,2^{2/3}\,(1+\sqrt{3}\,1\mathrm{i})^2}{256}-\frac{9099\,(1-x^2)^{1/3}}{64}\right)}{96}+\frac{665}{128}\right)}{9216}\right)(1+\sqrt{3}\,1\mathrm{i})}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] (2^(1/3)*log((2^(2/3)*((2^(1/3)*((10935*2^(2/3))/64 - (9099*(1 - x^2)^(1/3)
)/64))/48 - 665/128))/2304 + (1 - x^2)^(1/3)/576))/48 - log((985*(1 - x^2)^
(1/3))/373248 - 985/373248)/54 + log(((3^(1/2)*1i)/108 + 1/108)^2*(((3^(1/2
)*1i)/108 + 1/108)*(393660*((3^(1/2)*1i)/108 + 1/108)^2 - (9099*(1 - x^2)^(
1/3))/64) - 665/128) + (1 - x^2)^(1/3)/576)*((3^(1/2)*1i)/108 + 1/108) - lo
g((1 - x^2)^(1/3)/576 - ((3^(1/2)*1i)/108 - 1/108)^2*(((3^(1/2)*1i)/108 - 1
/108)*(393660*((3^(1/2)*1i)/108 - 1/108)^2 - (9099*(1 - x^2)^(1/3))/64) + 6
65/128))*((3^(1/2)*1i)/108 - 1/108) - ((17*(1 - x^2)^(2/3))/72 - (5*(1 - x^
2)^(5/3))/72)/((x^2 - 1)^2 + 5*x^2 - 1) + (2^(1/3)*log((1 - x^2)^(1/3)/576
+ (2^(2/3)*(3^(1/2)*1i - 1)^2*((2^(1/3)*(3^(1/2)*1i - 1)*((10935*2^(2/3)*(3
^(1/2)*1i - 1)^2)/256 - (9099*(1 - x^2)^(1/3))/64))/96 - 665/128))/9216)*(3
^(1/2)*1i - 1))/96 - (2^(1/3)*log((1 - x^2)^(1/3)/576 - (2^(2/3)*(3^(1/2)*1
```

```
i + 1)^2*((2^(1/3)*(3^(1/2)*1i + 1)*((10935*2^(2/3)*(3^(1/2)*1i + 1)^2)/256
 - (9099*(1 - x^2)^(1/3))/64))/96 + 665/128))/9216)*(3^(1/2)*1i + 1))/96
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[3]{-(x-1)(x+1)} \left(x^2+3\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.818 $\int \frac{1}{x^5 \sqrt[3]{1-x^2} \left(3+x^2\right)^2} \, dx$

Optimal. Leaf size=208

$$-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(x^2+3\right)}+\frac{\left(1-x^2\right)^{2/3}}{216\left(x^2+3\right)}+\frac{13\log\left(x^2+3\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}-\frac{13\tan^{-1}\left(\frac{\sqrt[3]{2-2x}}{\sqrt{3}}\right.}{216\ 2^{2/3}\sqrt{3}}$$

Rubi [A] time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 151, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(x^2+3\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(x^2+3\right)}+\frac{\left(1-x^2\right)^{2/3}}{216\left(x^2+3\right)}+\frac{13\log\left(x^2+3\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}-\frac{13\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{\log(x)}{54}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] (1 - x^2)^(2/3)/(216*(3 + x^2)) - (1 - x^2)^(2/3)/(12*x^4*(3 + x^2)) - (1 -
 x^2)^(2/3)/(36*x^2*(3 + x^2)) - (13*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]
])/(216*2^(2/3)*Sqrt[3]) + ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt
[3]) - Log[x]/54 + (13*Log[3 + x^2])/(1296*2^(2/3)) + Log[1 - (1 - x^2)^(1/
3)]/36 - (13*Log[2^(2/3) - (1 - x^2)^(1/3)])/(432*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^3(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{1}{12}\,\text{Subst}\left(\int \frac{-1-\frac{7x}{3}}{\sqrt[3]{1-x}\,x^2(3+x)^2}\,dx,x,x^2\right)\\
&= -\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)}+\frac{1}{36}\,\text{Subst}\left(\int \frac{6+\frac{4x}{3}}{\sqrt[3]{1-x}\,x(3+x)^2}\,dx,x,x^2\right)\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)}+\frac{1}{432}\,\text{Subst}\left(\int \frac{24-\frac{2x}{3}}{\sqrt[3]{1-x}\,x(3+x)}\,dx,\right.\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)}+\frac{1}{54}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx,x,x^2\right)-\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)}-\frac{\log(x)}{54}+\frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}}-\frac{1}{36}\,\text{Sub}\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)}-\frac{\log(x)}{54}+\frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}}+\frac{1}{36}\log\\
&= \frac{\left(1-x^2\right)^{2/3}}{216\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{12x^4\left(3+x^2\right)}-\frac{\left(1-x^2\right)^{2/3}}{36x^2\left(3+x^2\right)}-\frac{13\tan^{-1}\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2}{}\right.}{18\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 194, normalized size = 0.93

$$\frac{-\frac{72\left(1-x^2\right)^{2/3}}{x^2\left(x^2+3\right)}+\frac{12\left(1-x^2\right)^{2/3}}{x^2+3}+13\sqrt[3]{2}\,\log\left(x^2+3\right)+72\log\left(1-\sqrt[3]{1-x^2}\right)-39\sqrt[3]{2}\,\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)-26\sqrt[3]{2}\,\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)+48\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{216\left(1-x^2\right)^{2/3}}{x^4\left(x^2+3\right)}-48\log(x)}{2592}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((12*(1 - x^2)^(2/3))/(3 + x^2) - (216*(1 - x^2)^(2/3))/(x^4*(3 + x^2)) - (
72*(1 - x^2)^(2/3))/(x^2*(3 + x^2)) - 26*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]] + 48*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]]
- 48*Log[x] + 13*2^(1/3)*Log[3 + x^2] + 72*Log[1 - (1 - x^2)^(1/3)] - 39*2^
(1/3)*Log[2^(2/3) - (1 - x^2)^(1/3)])/2592
```

IntegrateAlgebraic [A] time = 0.37, size = 236, normalized size = 1.13

$$\frac{1}{54}\log\left(\sqrt[3]{1-x^2}-1\right)-\frac{13\log\left(\sqrt[3]{2}\sqrt[3]{1-x^2}-2\right)}{648\ 2^{2/3}}-\frac{1}{108}\log\left(\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1\right)+\frac{13\log\left(2^{2/3}\left(1-x^2\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{1-x^2}+4\right)}{1296\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{18\sqrt{3}}-\frac{13\tan^{-1}\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}}+\frac{\left(1-x^2\right)^{2/3}\left(x^4-6x^2-18\right)}{216x^4\left(x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

```
[Out] ((1 - x^2)^(2/3)*(-18 - 6*x^2 + x^4))/(216*x^4*(3 + x^2)) + ArcTan[1/Sqrt[3
] + (2*(1 - x^2)^(1/3))/Sqrt[3]]/(18*Sqrt[3]) - (13*ArcTan[1/Sqrt[3] + (2^(
1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/(216*2^(2/3)*Sqrt[3]) + Log[-1 + (1 - x^2)^
(1/3)]/54 - (13*Log[-2 + 2^(1/3)*(1 - x^2)^(1/3)])/(648*2^(2/3)) - Log[1 +
(1 - x^2)^(1/3) + (1 - x^2)^(2/3)]/108 + (13*Log[4 + 2*2^(1/3)*(1 - x^2)^(1
/3) + 2^(2/3)*(1 - x^2)^(2/3)])/(1296*2^(2/3))
```

fricas [A] time = 0.70, size = 265, normalized size = 1.27

$$-\frac{52\cdot 4^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(x^6+3x^4\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}\left(-1\right)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+13\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^6+3x^4\right)\log\left(4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-26\cdot 4^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\left(x^6+3x^4\right)\log\left(-4^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)-96\sqrt{3}\left(x^6+3x^4\right)\arctan\left(\frac{2}{3}\sqrt{3}\left(-x^2+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+48\left(x^6+3x^4\right)\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)-96\left(x^6+3x^4\right)\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)-24\left(x^4-6x^2-18\right)\left(-x^2+1\right)^{\frac{2}{3}}}{5184\left(x^6+3x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/5184*(52*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^6 + 3*x^4)*arctan(1/6*4^(1/6)*(2*
sqrt(3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 13*4^(2/3)*(-1)^(
1/3)*(x^6 + 3*x^4)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(
1/3) + (-x^2 + 1)^(2/3)) - 26*4^(2/3)*(-1)^(1/3)*(x^6 + 3*x^4)*log(-4^(1/3)
*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 96*sqrt(3)*(x^6 + 3*x^4)*arctan(2/3*sqrt(
3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 48*(x^6 + 3*x^4)*log((-x^2 + 1)^(2/3)
+ (-x^2 + 1)^(1/3) + 1) - 96*(x^6 + 3*x^4)*log((-x^2 + 1)^(1/3) - 1) - 24*(
x^4 - 6*x^2 - 18)*(-x^2 + 1)^(2/3))/(x^6 + 3*x^4)
```

giac [A] time = 0.45, size = 181, normalized size = 0.87

$$-\frac{13}{2592}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)+\frac{13}{5184}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)-\frac{13}{2592}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{216\left(x^2+3\right)}-\frac{\left(-x^2+1\right)^{\frac{2}{3}}}{36x^4}-\frac{1}{108}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)+\frac{1}{54}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

```
[Out] -13/2592*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1
)^(1/3))) + 13/5184*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2
+ 1)^(2/3)) - 13/2592*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/54*sqrt(3
)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/216*(-x^2 + 1)^(2/3)/(x^
2 + 3) - 1/36*(-x^2 + 1)^(2/3)/x^4 - 1/108*log((-x^2 + 1)^(2/3) + (-x^2 + 1
)^(1/3) + 1) + 1/54*log(-(-x^2 + 1)^(1/3) + 1)
```

maple [F]   time = 1.69, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

```
[Out] int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{2}+3\right)^{2} \left(-x^{2}+1\right)^{\frac{1}{3}} x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5), x)
```

mupad [B]   time = 1.01, size = 416, normalized size = 2.00

$$\frac{\ln\left(\frac{9109}{10077696}-\frac{9109\,{\left(1-x^2\right)}^{1/3}}{10077696}\right)}{54}-\frac{13\,2^{1/3}\,\ln\left(-\frac{169\,2^{2/3}\,\left(\frac{13\,2^{1/3}\,\left(\frac{2535\,2^{2/3}}{64}-\frac{7419\,{\left(1-x^2\right)}^{1/3}}{64}\right)}{1296}-\frac{469}{3456}\right)}{1679616}-\frac{845\,{\left(1-x^2\right)}^{1/3}}{5038848}\right)}{1296}+\frac{\frac{{\left(1-x^2\right)}^{5/3}}{54}-\frac{23\,{\left(1-x^2\right)}^{2/3}}{216}+\frac{{\left(1-x^2\right)}^{8/3}}{216}}{6\,{\left(x^2-1\right)}^2+{\left(x^2-1\right)}^3+9\,x^2-5}+\ln\left({\left(-\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)}^2\,\left(\left(-\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)\,\left(393660\,{\left(-\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)}^2-\frac{7419\,{\left(1-x^2\right)}^{1/3}}{64}\right)+\frac{469}{3456}\right)-\frac{845\,{\left(1-x^2\right)}^{1/3}}{5038848}\right)\,\left(-\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)-\ln\left(-{\left(\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)}^2\,\left(\left(\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)\,\left(393660\,{\left(\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)}^2-\frac{7419\,{\left(1-x^2\right)}^{1/3}}{64}\right)-\frac{469}{3456}\right)-\frac{845\,{\left(1-x^2\right)}^{1/3}}{5038848}\right)\,\left(\frac{1}{108}+\frac{\sqrt{3}\,1{}\mathrm{i}}{108}\right)+\frac{13\,{\left(-1\right)}^{1/3}\,2^{1/3}\,\ln\left(\frac{169\,{\left(-1\right)}^{2/3}\,2^{2/3}\,\left(\frac{13\,{\left(-1\right)}^{1/3}\,2^{1/3}\,\left(\frac{2535\,{\left(-1\right)}^{2/3}\,2^{2/3}}{64}-\frac{7419\,{\left(1-x^2\right)}^{1/3}}{64}\right)}{1296}+\frac{469}{3456}\right)}{1679616}-\frac{845\,{\left(1-x^2\right)}^{1/3}}{5038848}\right)}{1296}-\frac{13\,{\left(-1\right)}^{1/3}\,2^{1/3}\,\ln\left(-\frac{845\,{\left(1-x^2\right)}^{1/3}}{5038848}+\frac{169\,{\left(-1\right)}^{2/3}\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(\frac{13\,{\left(-1\right)}^{1/3}\,2^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(\frac{7419\,{\left(1-x^2\right)}^{1/3}}{64}-\frac{2535\,{\left(-1\right)}^{2/3}\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{256}\right)}{2592}+\frac{469}{3456}\right)}{6718464}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2592}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

```
[Out] log(9109/10077696 - (9109*(1 - x^2)^(1/3))/10077696)/54 - (13*2^(1/3)*log(-
 (169*2^(2/3)*((13*2^(1/3)*((2535*2^(2/3))/64 - (7419*(1 - x^2)^(1/3))/64))
/1296 - 469/3456))/1679616 - (845*(1 - x^2)^(1/3))/5038848))/1296 + ((1 - x
^2)^(5/3)/54 - (23*(1 - x^2)^(2/3))/216 + (1 - x^2)^(8/3)/216)/(6*(x^2 - 1)
^2 + (x^2 - 1)^3 + 9*x^2 - 5) + log(((3^(1/2)*1i)/108 - 1/108)^2*(((3^(1/2)
*1i)/108 - 1/108)*(393660*((3^(1/2)*1i)/108 - 1/108)^2 - (7419*(1 - x^2)^(1
/3))/64) + 469/3456) - (845*(1 - x^2)^(1/3))/5038848)*((3^(1/2)*1i)/108 - 1
/108) - log(- ((3^(1/2)*1i)/108 + 1/108)^2*(((3^(1/2)*1i)/108 + 1/108)*(393
```

```
660*((3^(1/2)*1i)/108 + 1/108)^2 - (7419*(1 - x^2)^(1/3))/64) - 469/3456) -
 (845*(1 - x^2)^(1/3))/5038848)*((3^(1/2)*1i)/108 + 1/108) + (13*(-1)^(1/3)
*2^(1/3)*log((169*(-1)^(2/3)*2^(2/3)*((13*(-1)^(1/3)*2^(1/3)*((2535*(-1)^(2
/3)*2^(2/3))/64 - (7419*(1 - x^2)^(1/3))/64))/1296 + 469/3456))/1679616 - (
845*(1 - x^2)^(1/3))/5038848))/1296 - (13*(-1)^(1/3)*2^(1/3)*log((169*(-1)^
(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*((13*(-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(
(7419*(1 - x^2)^(1/3))/64 - (2535*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2)/25
6))/2592 + 469/3456))/6718464 - (845*(1 - x^2)^(1/3))/5038848)*(3^(1/2)*1i
+ 1))/2592
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\,\left(x^{2}+3\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.819 $\int \frac{x^7}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=136

$$\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2}}\right.$$

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {440, 261, 266, 43, 439}

$$\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}+\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(1
1/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*
x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)
*(2 - 3*x^2)^(1/4))])/81
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,\left(4-3x^2\right)}\,dx = \int \left(-\frac{16x}{27\sqrt[4]{2-3x^2}} - \frac{4x^3}{9\sqrt[4]{2-3x^2}} - \frac{x^5}{3\sqrt[4]{2-3x^2}} + \frac{64x}{27\sqrt[4]{2-3x^2}\,\left(4-3x^2\right)}\right)dx$$

$$= -\left(\frac{1}{3}\int \frac{x^5}{\sqrt[4]{2-3x^2}}\,dx\right) - \frac{4}{9}\int \frac{x^3}{\sqrt[4]{2-3x^2}}\,dx - \frac{16}{27}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx + \frac{64}{27}\int \frac{}{\sqrt[4]{2-3x}}$$

$$= \frac{32}{243}\left(2-3x^2\right)^{3/4} + \frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{32}{81}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3}}{2^{3/4}\sqrt[4]{2-3x}}\right.$$

$$= \frac{32}{243}\left(2-3x^2\right)^{3/4} + \frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{32}{81}\sqrt[4]{2}\,\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3}}{2^{3/4}\sqrt[4]{2-3x}}\right.$$

$$= \frac{56}{243}\left(2-3x^2\right)^{3/4} - \frac{16}{567}\left(2-3x^2\right)^{7/4} + \frac{2}{891}\left(2-3x^2\right)^{11/4} + \frac{32}{81}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2}}{2^{3/4}\sqrt[4]{2-}}\right.$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.34

$$\frac{2\left(2-3x^2\right)^{3/4}\left(-2464\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)+189x^4+540x^2+1712\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(1712 + 540*x^2 + 189*x^4 - 2464*Hypergeometric2F1[3/4
, 1, 7/4, -1 + (3*x^2)/2]))/18711
```

IntegrateAlgebraic [**A**] time = 0.18, size = 124, normalized size = 0.91

$$-\frac{32}{81}\sqrt[4]{2}\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)+\frac{2\left(2-3x^2\right)^{3/4}\left(189x^4+540x^2+1712\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(1712 + 540*x^2 + 189*x^4))/18711 - (32*2^(1/4)*ArcTan
[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)])/81 + (32*2^(1/4)
*ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/81
```

fricas [**B**] time = 1.28, size = 253, normalized size = 1.86

$$\frac{2}{18711}\left(189\,x^4+540\,x^2+1712\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3\,x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{32}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3\,x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{8}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3\,x^2+2}\right)-\frac{8}{81}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3\,x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 540*x^2 + 1712)*(-3*x^2 + 2)^(3/4) + 32/81*8^(1/4)*sqrt(
2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*s
qrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 1)
+ 32/81*8^(1/4)*sqrt(2)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8^(3/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + 1) + 8/81*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*
x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 8/81*8^(1/4)*sqrt(2)*l
og(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)
)
```

giac [**A**] time = 0.45, size = 160, normalized size = 1.18

$$\frac{2}{891}\left(3\,x^2-2\right)^2\left(-3\,x^2+2\right)^{\frac{3}{4}}-\frac{16}{567}\left(-3\,x^2+2\right)^{\frac{7}{4}}-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{56}{243}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 2)^2*(-3*x^2 + 2)^(3/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*
2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1/4
)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log
(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*l
og(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x
^2 + 2)^(3/4)
```

maple [C] time = 2.85, size = 217, normalized size = 1.60

$$\frac{16\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\ln\left(\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\mathrm{RootOf}\left(\_Z^4+8\right)^2-6x^2+2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4+8\right)^2-4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)}{3x^2-4}\right)}{81}-\frac{16\,\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+8\right)^3-6x^2-2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4+8\right)^2+4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+8\right)}{3x^2-4}\right)}{81}-\frac{2\left(189x^4+540x^2+1712\right)\left(3x^2-2\right)}{18711\left(-3x^2+2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] -2/18711*(189*x^4+540*x^2+1712)*(3*x^2-2)/(-3*x^2+2)^(1/4)-16/81*RootOf(_Z^
4+8)*ln((RootOf(_Z^4+8)^3*(-3*x^2+2)^(3/4)-2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1
/2)+4*RootOf(_Z^4+8)*(-3*x^2+2)^(1/4)-6*x^2)/(3*x^2-4))+16/81*RootOf(_Z^2+R
ootOf(_Z^4+8)^2)*ln((RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(-3*x^2
+2)^(3/4)+2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2+RootOf(_Z^4+8)^
2)*(-3*x^2+2)^(1/4)-6*x^2)/(3*x^2-4))
```

maxima [A] time = 1.96, size = 151, normalized size = 1.11

$$\frac{2}{891}\left(-3x^2+2\right)^{\frac{11}{4}}-\frac{16}{567}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{32}{81}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{16}{81}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{56}{243}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] 2/891*(-3*x^2 + 2)^(11/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*2^(1/4)*arcta
n(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1/4)*arctan(-1/2
*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log(2^(3/4)*(-3*
x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*log(-2^(3/4)*(
-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2 + 2)^(3/4)
```

mupad [B] time = 0.91, size = 82, normalized size = 0.60

$$\frac{56\left(2-3x^2\right)^{3/4}}{243}-\frac{16\left(2-3x^2\right)^{7/4}}{567}+\frac{2\left(2-3x^2\right)^{11/4}}{891}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{32}{81}+\frac{32}{81}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{32}{81}-\frac{32}{81}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (56*(2 - 3*x^2)^(3/4))/243 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 +
1i/2))*(32/81 + 32i/81) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/
2))*(32/81 - 32i/81) - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(11/4))/
891
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.820 $\int \frac{x^5}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {440, 261, 266, 43, 439}

$$-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)*ArcTan[(S
qrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27 + (8*2^(1/4)*Arc
Tanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt[4]{2-3x^2}\,\left(4-3x^2\right)}\,dx &= \int \left(-\frac{4x}{9\sqrt[4]{2-3x^2}}-\frac{x^3}{3\sqrt[4]{2-3x^2}}+\frac{16x}{9\sqrt[4]{2-3x^2}\,\left(4-3x^2\right)}\right)dx\\
&= -\left(\frac{1}{3}\int \frac{x^3}{\sqrt[4]{2-3x^2}}\,dx\right)-\frac{4}{9}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx+\frac{16}{9}\int \frac{x}{\sqrt[4]{2-3x^2}\,\left(4-3x^2\right)}\,dx\\
&= \frac{8}{81}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\,\tanh^{-1}\Bigg(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\\
&= \frac{8}{81}\left(2-3x^2\right)^{3/4}+\frac{8}{27}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\,\tanh^{-1}\Bigg(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\\
&= \frac{4}{27}\left(2-3x^2\right)^{3/4}-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{8}{27}\sqrt[4]{2}\,\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\,\tanh
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.35

$$\frac{2}{567}\left(2-3x^2\right)^{3/4}\left(9\left(x^2+4\right)-56\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(9*(4 + x^2) - 56*Hypergeometric2F1[3/4, 1, 7/4, -1 +
(3*x^2)/2]))/567
```

IntegrateAlgebraic [A] time = 0.15, size = 117, normalized size = 0.97

$$\frac{2}{63}\left(2-3x^2\right)^{3/4}\left(x^2+4\right)-\frac{8}{27}\sqrt[4]{2}\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4)*(4 + x^2))/63 - (8*2^(1/4)*ArcTan[(-2^(-1/4) + Sqrt[2
- 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)])/27 + (8*2^(1/4)*ArcTanh[(2*2^(1/4)*(2
 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/27
```

fricas [B] time = 1.17, size = 246, normalized size = 2.03

$$\frac{2}{63}\left(x^2+4\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{8}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)-\frac{2}{27}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/63*(x^2 + 4)*(-3*x^2 + 2)^(3/4) + 8/27*8^(1/4)*sqrt(2)*arctan(1/4*8^(1/4)
*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^
2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 1) + 8/27*8^(1/4)*sqrt(2
)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1
6*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) +
 1) + 2/27*8^(1/4)*sqrt(2)*log(4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sq
rt(2) + 16*sqrt(-3*x^2 + 2)) - 2/27*8^(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*
(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2))
```

giac [A] time = 0.52, size = 140, normalized size = 1.16

$$-\frac{2}{189}\left(-3x^2+2\right)^{\frac{7}{4}}-\frac{2}{27}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{4}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) - 2/27*8^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 2/27*8^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
+ 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

maple [C] time = 2.51, size = 211, normalized size = 1.74

$$-\frac{4\,\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)\ln\left(-\frac{\left(-3x^{2}+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}+6x^{2}-2\sqrt{-3x^{2}+2}\;\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}-4\left(-3x^{2}+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)}{3x^{2}-4}\right)}{27}-\frac{4\,\mathrm{RootOf}\left(\_Z^{4}+8\right)\ln\left(\frac{\left(-3x^{2}+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{4}+8\right)^{3}-6x^{2}-2\sqrt{-3x^{2}+2}\;\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}+4\left(-3x^{2}+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+8\right)}{3x^{2}-4}\right)}{27}-\frac{2\left(x^{2}+4\right)\left(3x^{2}-2\right)}{63\left(-3x^{2}+2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] -2/63*(x^2+4)*(3*x^2-2)/(-3*x^2+2)^(1/4)-4/27*RootOf(_Z^4+8)*ln((RootOf(_Z^
4+8)^3*(-3*x^2+2)^(3/4)-2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)+4*RootOf(_Z^4+8
)*(-3*x^2+2)^(1/4)-6*x^2)/(3*x^2-4))-4/27*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(
-(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(-3*x^2+2)^(3/4)-2*RootOf(
_Z^4+8)^2*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(-3*x^2+2)^(1/4)
+6*x^2)/(3*x^2-4))
```

maxima [A] time = 2.07, size = 140, normalized size = 1.16

$$-\frac{2}{189}\left(-3\,x^{2}+2\right)^{\frac{7}{4}}-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{27}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^{2}+2}\right)-\frac{4}{27}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^{2}+2}\right)+\frac{4}{27}\left(-3\,x^{2}+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/189*(-3*x^2 + 2)^(7/4) - 8/27*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
+ 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

mupad [B] time = 0.17, size = 71, normalized size = 0.59

$$\frac{4\left(2-3\,x^{2}\right)^{3/4}}{27}-\frac{2\left(2-3\,x^{2}\right)^{7/4}}{189}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3\,x^{2}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{8}{27}+\frac{8}{27}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3\,x^{2}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{8}{27}-\frac{8}{27}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (4*(2 - 3*x^2)^(3/4))/27 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i
/2))*(8/27 + 8i/27) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*
(8/27 - 8i/27) - (2*(2 - 3*x^2)^(7/4))/189
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{5}}{3x^{2}\sqrt[4]{2-3x^{2}}-4\sqrt[4]{2-3x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.821 $\int \frac{x^3}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=106

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {440, 261, 439}

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2
^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^
2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx &= \int \left(-\frac{x}{3\sqrt[4]{2-3x^2}}+\frac{4x}{3\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\right)dx\\ &= -\left(\frac{1}{3}\int \frac{x}{\sqrt[4]{2-3x^2}}\,dx\right)+\frac{4}{3}\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx\\ &= \frac{2}{27}\left(2-3x^2\right)^{3/4}+\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)\end{aligned}$$

Mathematica [**C**] time = 0.01, size = 36, normalized size = 0.34

$$-\frac{2}{27}\left(2-3x^2\right)^{3/4}\left(2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-2*(2 - 3*x^2)^(3/4)*(-1 + 2*Hypergeometric2F1[3/4, 1, 7/4, -1 + (3*x^2)/2
]))/27
```

IntegrateAlgebraic [**A**] time = 0.13, size = 112, normalized size = 1.06

$$\frac{2}{27}\left(2-3x^2\right)^{3/4}-\frac{2}{9}\sqrt[4]{2}\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 - (2*2^(1/4)*ArcTan[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2
^(3/4))/(2 - 3*x^2)^(1/4)])/9 + (2*2^(1/4)*ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(
1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/9
```

fricas [**B**] time = 1.14, size = 241, normalized size = 2.27

$$\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{2}{9}\cdot 8^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{18}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)-\frac{1}{18}\cdot 8^{\frac{1}{4}}\sqrt{2}\log\left(-4\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+16\sqrt{2}+16\sqrt{-3x^2+2}\right)+\frac{2}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/9*8^(1/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4) - 1) + 2/9*8^(1/4)*sqrt(2)*arctan(1/8*8^(1/4)*sqrt(2)*sqrt(-4*8
^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/2
*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) + 1/18*8^(1/4)*sqrt(2)*log(4*8^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*sqrt(-3*x^2 + 2)) - 1/18*8^
(1/4)*sqrt(2)*log(-4*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 16*sqrt(2) + 16*s
qrt(-3*x^2 + 2)) + 2/27*(-3*x^2 + 2)^(3/4)
```

giac [A] time = 0.43, size = 129, normalized size = 1.22

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*lo
g(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)*lo
g(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2
+ 2)^(3/4)
```

maple [C] time = 1.72, size = 206, normalized size = 1.94

$$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\ln\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\mathrm{RootOf}\left(\_Z^4+8\right)^2+6x^2-2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4+8\right)^2-4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)}{3x^2-4}\right)}{9}-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+8\right)^3-6x^2-2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4+8\right)^2+4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+8\right)}{3x^2-4}\right)}{9}-\frac{2\left(3x^2-2\right)}{27\left(-3x^2+2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] -2/27*(3*x^2-2)/(-3*x^2+2)^(1/4)-1/9*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-(Roo
tOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_Z^4+8)^2*(-3*x^2+2)^(3/4)-2*RootOf(_Z^4+
8)^2*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2+RootOf(_Z^4+8)^2)*(-3*x^2+2)^(1/4)+6*x^
2)/(3*x^2-4))-1/9*RootOf(_Z^4+8)*ln((RootOf(_Z^4+8)^3*(-3*x^2+2)^(3/4)-2*Ro
otOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)+4*RootOf(_Z^4+8)*(-3*x^2+2)^(1/4)-6*x^2)/(3
*x^2-4))
```

maxima [A] time = 2.04, size = 129, normalized size = 1.22

$$-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\left(-3x^2+2\right)^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*lo
g(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)*lo
g(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*x^2
+ 2)^(3/4)
```

mupad [B] time = 0.15, size = 60, normalized size = 0.57

$$\frac{2\left(2-3x^2\right)^{3/4}}{27}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{2}{9}+\frac{2}{9}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{2}{9}-\frac{2}{9}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (2*(2 - 3*x^2)^(3/4))/27 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i
/2))*(2/9 + 2i/9) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(2
/9 - 2i/9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^3}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.822 $\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=91

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Rubi [A] time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {439}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3/4))
 + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3
/4))
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.37

$$-\frac{1}{9}\left(2-3x^2\right)^{3/4}\ {}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{1}{2}\left(3x^2-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/9*((2 - 3*x^2)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, (-2 + 3*x^2)/2])
```

IntegrateAlgebraic **[A]** time = 0.12, size = 97, normalized size = 1.07

$$\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)}{3\,2^{3/4}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)}{3\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/3*ArcTan[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)]/2^(3/4
) + ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])]/(3
*2^(3/4))
```

fricas **[B]** time = 1.40, size = 189, normalized size = 2.08

$$\frac{1}{3}\cdot 2^{\frac{1}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{3}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/3*2^(1/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 1/3*2^(1/4)*arctan(1/2*2^
(1/4)*sqrt(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
- 2^(1/4)*(-3*x^2 + 2)^(1/4) + 1) + 1/12*2^(1/4)*log(4*2^(3/4)*(-3*x^2 + 2)
^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*log(-4*2^(3/4)*(-3*
x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2))
```

giac **[A]** time = 0.37, size = 118, normalized size = 1.30

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

maple [C] time = 3.96, size = 189, normalized size = 2.08

$$-\frac{\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)\ln\left(-\frac{\left(-3x^{2}+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}+6x^{2}-2\sqrt{-3x^{2}+2}\ \mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}-4\left(-3x^{2}+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)}{3x^{2}-4}\right)}{12}+\frac{\mathrm{RootOf}\left(\_Z^{4}+8\right)\ln\left(-\frac{\left(-3x^{2}+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{4}+8\right)^{3}+6x^{2}+2\sqrt{-3x^{2}+2}\ \mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}+4\left(-3x^{2}+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+8\right)}{3x^{2}-4}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] 1/12*RootOf(_Z^4+8)*ln(-(RootOf(_Z^4+8)^3*(-3*x^2+2)^(3/4)+2*RootOf(_Z^4+8)
^2*(-3*x^2+2)^(1/2)+4*RootOf(_Z^4+8)*(-3*x^2+2)^(1/4)+6*x^2)/(3*x^2-4))-1/1
2*RootOf(_Z^2+RootOf(_Z^4+8)^2)*ln(-(RootOf(_Z^2+RootOf(_Z^4+8)^2)*RootOf(_
Z^4+8)^2*(-3*x^2+2)^(3/4)-2*RootOf(_Z^4+8)^2*(-3*x^2+2)^(1/2)-4*RootOf(_Z^2
+RootOf(_Z^4+8)^2)*(-3*x^2+2)^(1/4)+6*x^2)/(3*x^2-4))
```

maxima [A] time = 1.90, size = 118, normalized size = 1.30

$$-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^(
1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

mupad [B] time = 0.15, size = 49, normalized size = 0.54

$$2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^{2}\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^{2}\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/6 - 1i/6) - 2^(1/
4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/2))*(1/6 + 1i/6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{3x^{2}\sqrt[4]{2-3x^{2}}-4\sqrt[4]{2-3x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.823 $\int \frac{1}{x\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=145

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}$$

Rubi [**A**] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {440, 266, 63, 298, 203, 206, 439}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcTanh[(2 - 3*x^2)^(1/4
)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 -
3*x^2)^(1/4))]/(4*2^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx &= \int \left(\frac{1}{4x\sqrt[4]{2-3x^2}}-\frac{3x}{4\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\right)dx \\
&= \frac{1}{4}\int \frac{1}{x\sqrt[4]{2-3x^2}}\,dx-\frac{3}{4}\int \frac{x}{\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\,dx \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt[4]{2-3x}\,x}\,dx,x,x^2\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{x^2}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{2-3}\right. \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right.}{4\,2^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 73, normalized size = 0.50

$$\frac{1}{24}\left(3\,2^{3/4}\left(\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)-\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)\right)-2\left(2-3x^2\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (3*2^(3/4)*(ArcTan[(1 - (3*x^2)/2)^(1/4)] - ArcTanh[(1 - (3*x^2)/2)^(1/4)])
 - 2*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, -1 + (3*x^2)/2])/24
```

IntegrateAlgebraic [A] time = 0.19, size = 151, normalized size = 1.04

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) - ArcTan[(-2^(-1/4) + Sqrt[2
- 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)]/(4*2^(3/4)) - ArcTanh[(2 - 3*x^2)^(1/4
)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]
*Sqrt[2 - 3*x^2])]/(4*2^(3/4))
```

fricas [B] time = 1.47, size = 270, normalized size = 1.86

$$-\frac{1}{4}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-3x^{2}+2}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)-\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{1}{4}}+\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}-2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^{2}+2}}-2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+1\right)+\frac{1}{16}\cdot 2^{\frac{1}{4}}\log\left(4\cdot 2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^{2}+2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\log\left(-4\cdot 2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^{2}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/4*2^(3/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-3*x^2 + 2)) - 1/2*2^(3
/4)*(-3*x^2 + 2)^(1/4)) - 1/16*2^(3/4)*log(2^(1/4) + (-3*x^2 + 2)^(1/4)) +
1/16*2^(3/4)*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) + 1/4*2^(1/4)*arctan(2^(1/4
)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4)*(
-3*x^2 + 2)^(1/4) - 1) + 1/4*2^(1/4)*arctan(1/2*2^(1/4)*sqrt(-4*2^(3/4)*(-3
*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1
/4) + 1) + 1/16*2^(1/4)*log(4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sq
rt(-3*x^2 + 2)) - 1/16*2^(1/4)*log(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2
) + 4*sqrt(-3*x^2 + 2))
```

giac [A] time = 0.42, size = 216, normalized size = 1.49

$$-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\log\left(4^{\frac{1}{8}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{-3x^{2}+2}+4^{\frac{1}{4}}\right)-\frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\log\left(-4^{\frac{1}{8}}\sqrt{2}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{-3x^{2}+2}+4^{\frac{1}{4}}\right)+\frac{1}{8}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)+\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(-\left(-3x^{2}+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)-\frac{1}{16}\cdot 4^{\frac{3}{8}}\log\left(\left(-3x^{2}+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/16*4^(3/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*x
^2 + 2)^(1/4))) - 1/16*4^(3/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/8)
*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) + 1/32*4^(3/8)*sqrt(2)*log(4^(1/8)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/32*4^(3/8)*sqrt(2)*l
og(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/8*
4^(1/8)*sqrt(2)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) + 1/16*4^(1/8)*sqrt(
2)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8)) - 1/16*4^(3/8)*log((-3*x^2 + 2)^(1/4)
 + 4^(1/8))
```

maple [F] time = 11.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^{2}+2\right)^{\frac{1}{4}}\left(-3x^{2}+4\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x), x)
```

mupad [B] time = 0.97, size = 91, normalized size = 0.63

$$\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{8}+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/8 - 2^(1/4)*atan(2^(1/4)*(2 -
 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)
^(1/4)*(1/2 + 1i/2))*(1/8 + 1i/8) + (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4
)*1i)/2)*1i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^3\sqrt[4]{2-3x^2}-4x\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(2 - 3*x**2)**(1/4) - 4*x*(2 - 3*x**2)**(1/4)), x)
```

3.824 $\int \frac{1}{x^3 \sqrt[4]{2-3x^2}\left(4-3x^2\right)} \, dx$

Optimal. Leaf size=163

$$-\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

Rubi **[A]** time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {440, 266, 51, 63, 298, 203, 206, 439}

$$-\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(3/4)/(16*x^2) + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(
1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])
/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(1/4)) + (3*Ar
cTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(3/4)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 439

```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
-Simp[ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(
1/4))]/(Sqrt[2]*Rt[a, 4]*d), x] - Simp[(1*ArcTanh[(Rt[a, 4]^2 + Sqrt[a + b*
x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))])/(Sqrt[2]*Rt[a, 4]*d), x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

Rule 440

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx &= \int \left(\frac{1}{4x^3\sqrt[4]{2-3x^2}}+\frac{3}{16x\sqrt[4]{2-3x^2}}-\frac{9x}{16\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\right)dx \\
&= \frac{3}{16}\int \frac{1}{x\sqrt[4]{2-3x^2}}\,dx+\frac{1}{4}\int \frac{1}{x^3\sqrt[4]{2-3x^2}}\,dx-\frac{9}{16}\int \frac{x}{\sqrt[4]{2-3x^2}\left(-4+3x^2\right)}\,dx \\
&= \frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3}{32}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{2-3x}\,x}\,dx,x,x\right. \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3}{64}\,\mathrm{Subst}\left(\int \frac{}{\sqrt[4]{}}\right. \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{1}{16}\,\mathrm{Subst}\left(\int \frac{}{\frac{2}{3}}\right. \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\sqrt[4]{2}}+ \\
&= -\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+
\end{aligned}$$

Mathematica [C] time = 0.05, size = 102, normalized size = 0.63

$$-\frac{4\left(2-3x^2\right)^{3/4}x^2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};\frac{3x^2}{2}-1\right)+4\left(2-3x^2\right)^{3/4}-9\,2^{3/4}x^2\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+9\,2^{3/4}x^2\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)}{64x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/64*(4*(2 - 3*x^2)^(3/4) - 9*2^(3/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)] +
9*2^(3/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + 4*x^2*(2 - 3*x^2)^(3/4)*Hype
rgeometric2F1[3/4, 1, 7/4, -1 + (3*x^2)/2])/x^2
```

IntegrateAlgebraic [A]   time = 0.32, size = 169, normalized size = 1.04

$$-\frac{\left(2-3x^2\right)^{3/4}}{16x^2}+\frac{9\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}-\frac{9\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/16*(2 - 3*x^2)^(3/4)/x^2 + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(
1/4)) - (3*ArcTan[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)])
/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(1/4)) + (3*Ar
cTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/(16*2^(
3/4))
```

fricas [B]   time = 1.27, size = 307, normalized size = 1.88

$$-\frac{36\cdot 2^{\frac{3}{4}}x^2\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)+9\cdot 2^{\frac{3}{4}}x^2\log\left(2^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right)-9\cdot 2^{\frac{3}{4}}x^2\log\left(-2^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right)-24\cdot 2^{\frac{1}{4}}x^2\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)-24\cdot 2^{\frac{1}{4}}x^2\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-6\cdot 2^{\frac{1}{4}}x^2\log\left(4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)+6\cdot 2^{\frac{1}{4}}x^2\log\left(-4\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}\right)+8\left(-3x^2+2\right)^{\frac{3}{4}}}{128x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/128*(36*2^(3/4)*x^2*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-3*x^2 + 2))
 - 1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) + 9*2^(3/4)*x^2*log(2^(1/4) + (-3*x^2 + 2
)^(1/4)) - 9*2^(3/4)*x^2*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) - 24*2^(1/4)*x^
2*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 +
2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) - 24*2^(1/4)*x^2*arctan(1/2*2^(1/4)*s
qrt(-4*2^(3/4)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 2^(1/
4)*(-3*x^2 + 2)^(1/4) + 1) - 6*2^(1/4)*x^2*log(4*2^(3/4)*(-3*x^2 + 2)^(1/4)
 + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 6*2^(1/4)*x^2*log(-4*2^(3/4)*(-3*x^2 +
 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) + 8*(-3*x^2 + 2)^(3/4))/x^2
```

giac [A]   time = 0.59, size = 192, normalized size = 1.18

$$\frac{9}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{9}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right)+\frac{9}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{3}{64}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{3}{64}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 9/64*2^(3/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 9/128*2^(3/4)*log(2^(
1/4) + (-3*x^2 + 2)^(1/4)) + 9/128*2^(3/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/4)
) - 3/32*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/3
```

```
2*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 3/64*2^(1
/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 3/64*2^(
1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/16*(
-3*x^2 + 2)^(3/4)/x^2
```

maple [F] time = 9.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}\left(-3x^2+4\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3), x)
```

mupad [B] time = 1.01, size = 109, normalized size = 0.67

$$\frac{9\,2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3\,x^2\right)^{1/4}}{2}\right)}{64}-\frac{\left(2-3\,x^2\right)^{3/4}}{16\,x^2}+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3\,x^2\right)^{1/4}\,1\mathrm{i}}{2}\right)\,9\mathrm{i}}{64}-\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,\left(2-3\,x^2\right)^{1/4}\,1\mathrm{i}}{2}\right)\,3\mathrm{i}}{32}-\frac{{\left(-1\right)}^{3/4}\,2^{3/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{3/4}\,2^{3/4}\,\left(2-3\,x^2\right)^{1/4}\,1\mathrm{i}}{2}\right)\,3\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] (9*2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/64 - (2 - 3*x^2)^(3/4)/(16*
x^2) + (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*9i)/64 - ((-1)^(1/4)
*2^(3/4)*atan(((-1)^(1/4)*2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*3i)/32 - ((-1)^(
3/4)*2^(3/4)*atan(((-1)^(3/4)*2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*3i)/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^5\sqrt[4]{2-3x^2}-4x^3\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(2 - 3*x**2)**(1/4) - 4*x**3*(2 - 3*x**2)**(1/4)), x)
```

3.825 $\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {397}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 397

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, -Simp[(b*ArcTan[(b + q^2*Sqrt[a + b*x^2])/(q^3*x*(a +
 b*x^2)^(1/4))])/(2*a*d*q), x] - Simp[(b*ArcTanh[(b - q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))])/(2*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{2-3x^2}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Mathematica [C] time = 0.03, size = 135, normalized size = 1.12

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\sqrt[4]{2-3x^2}\left(3x^2-4\right)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x^2)^(1/4)*
(-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*A
ppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2
, (3*x^2)/2, (3*x^2)/4])))
```

IntegrateAlgebraic [A] time = 0.00, size = 137, normalized size = 1.14

$$\frac{\tan^{-1}\left(\frac{\frac{\sqrt{3}x^2}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}}}{x\sqrt[4]{2-3x^2}}\right)}{4\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}{3\sqrt{2}x^2+4\sqrt{2-3x^2}}\right)}{4\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[((Sqrt[3]*x^2)/(2*2^(1/4)) - (2^(1/4)*Sqrt[2 - 3*x^2])/Sqrt[3])/(x*(
2 - 3*x^2)^(1/4))]/(4*2^(3/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 -
3*x^2)^(1/4))/(3*Sqrt[2]*x^2 + 4*Sqrt[2 - 3*x^2])]/(4*2^(3/4)*Sqrt[3])
```

fricas [B] time = 6.60, size = 553, normalized size = 4.61

$$\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3+54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x+12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}-72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}-72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)-48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)-\frac{1}{72}\cdot 18^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{6\cdot 18^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x^3-54x^4+24\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}x-12\sqrt{2}\left(3x^2-4\right)\sqrt{-3x^2+2}+72x^2+\left(18^{\frac{3}{4}}\sqrt{2}\left(3x^3+4x\right)\sqrt{-3x^2+2}+72\left(-3x^2+2\right)^{\frac{1}{4}}x^2-6\cdot 18^{\frac{1}{4}}\sqrt{2}\left(3x^3-4x\right)+48\sqrt{2}\left(-3x^2+2\right)^{\frac{3}{4}}\right)\sqrt{-\frac{3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}}{3x^2-4}}}{6\left(9x^4+24x^2-16\right)}\right)+\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2+2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)-\frac{1}{288}\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{36\left(3\sqrt{2}x^2-2\cdot 18^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+4\sqrt{-3x^2+2}\right)}{3x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/72*18^(3/4)*sqrt(2)*arctan(-1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 + 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x + 12*sqrt(2)*(3*x^2 -
 4)*sqrt(-3*x^2 + 2) - 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) - 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) - 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) -
 1/72*18^(3/4)*sqrt(2)*arctan(1/6*(6*18^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x^
3 - 54*x^4 + 24*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(3/4)*x - 12*sqrt(2)*(3*x^2 -
```

```
 4)*sqrt(-3*x^2 + 2) + 72*x^2 + (18^(3/4)*sqrt(2)*(3*x^3 + 4*x)*sqrt(-3*x^2
 + 2) + 72*(-3*x^2 + 2)^(1/4)*x^2 - 6*18^(1/4)*sqrt(2)*(3*x^3 - 4*x) + 48*s
qrt(2)*(-3*x^2 + 2)^(3/4))*sqrt(-(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)))/(9*x^4 + 24*x^2 - 16)) +
 1/288*18^(3/4)*sqrt(2)*log(-36*(3*sqrt(2)*x^2 + 2*18^(1/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x + 4*sqrt(-3*x^2 + 2))/(3*x^2 - 4)) - 1/288*18^(3/4)*sqrt(2)*l
og(-36*(3*sqrt(2)*x^2 - 2*18^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 4*sqrt(-3
*x^2 + 2))/(3*x^2 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

maple [C] time = 2.06, size = 186, normalized size = 1.55

$$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(\frac{3x\,\mathrm{RootOf}\left(\_Z^4+72\right)^2+\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)\mathrm{RootOf}\left(\_Z^4+72\right)^2+18\sqrt{-3x^2+2}\,x+6\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+72\right)^2\right)}{3x^2-4}\right)}{24}+\frac{\mathrm{RootOf}\left(\_Z^4+72\right)\ln\left(\frac{-3x\,\mathrm{RootOf}\left(\_Z^4+72\right)^2-\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)^3+18\sqrt{-3x^2+2}\,x+6\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+72\right)}{3x^2-4}\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] 1/24*RootOf(_Z^4+72)*ln((6*(-3*x^2+2)^(3/4)*RootOf(_Z^4+72)-(-3*x^2+2)^(1/4
)*RootOf(_Z^4+72)^3+18*(-3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
+1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln((6*(-3*x^2+2)^(3/4)*RootOf(_Z^2+Roo
tOf(_Z^4+72)^2)+(-3*x^2+2)^(1/4)*RootOf(_Z^4+72)^2*RootOf(_Z^2+RootOf(_Z^4+
72)^2)+18*(-3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\left(2-3\,x^2\right)^{1/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

```
[Out] -int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.826 $\int \frac{x^7}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 63, 298, 203, 206}

$$\frac{2}{891}\left(3x^2-1\right)^{11/4}+\frac{8}{567}\left(3x^2-1\right)^{7/4}+\frac{14}{243}\left(3x^2-1\right)^{3/4}+\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(3/4))/243 + (8*(-1 + 3*x^2)^(7/4))/567 + (2*(-1 + 3*x^2)^
(11/4))/891 + (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(
1/4)])/81
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\Bigg(\int \Bigg(\frac{7}{27\sqrt[4]{-1+3x}}+\frac{8}{27(-2+3x)\sqrt[4]{-1+3x}}+\frac{4}{27}(-1+3x)^{3/4}+\frac{1}{27}(-1\\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{4}{27}\text{Subst}\Bigg(\int \\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{16}{81}\text{Subst}\Bigg(\int \\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}-\frac{8}{81}\text{Subst}\Bigg(\int \\
&= \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4}+\frac{8}{81}\tan^{-1}\Big(\sqrt[4]{-1
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.73

$$
\frac{2\left(924\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-924\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)+\left(3x^2-1\right)^{3/4}\left(189x^4+270x^2+428\right)\right)}{18711}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(3/4)*(428 + 270*x^2 + 189*x^4) + 924*ArcTan[(-1 + 3*x^2)^
(1/4)] - 924*ArcTanh[(-1 + 3*x^2)^(1/4)]))/18711
```

IntegrateAlgebraic [A] time = 0.04, size = 60, normalized size = 0.77

$$\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{2\left(3x^2-1\right)^{3/4}\left(189x^4+270x^2+428\right)}{18711}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4)*(428 + 270*x^2 + 189*x^4))/18711 + (8*ArcTan[(-1 + 3*
x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(1/4)])/81
```

fricas [A] time = 0.90, size = 64, normalized size = 0.82

$$\frac{2}{18711}\left(189\,x^4+270\,x^2+428\right)\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/18711*(189*x^4 + 270*x^2 + 428)*(3*x^2 - 1)^(3/4) + 8/81*arctan((3*x^2 -
1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) -
1)
```

giac [A] time = 0.31, size = 75, normalized size = 0.96

$$\frac{2}{891}\left(3\,x^2-1\right)^{\frac{11}{4}}+\frac{8}{567}\left(3\,x^2-1\right)^{\frac{7}{4}}+\frac{14}{243}\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{8}{81}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\,\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3/
4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 1.08, size = 148, normalized size = 1.90

$$\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\sqrt{3x^2-1}}{3x^2-2}\right)}{81}+\frac{4\ln\left(\frac{-3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}-2\sqrt{3x^2-1}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{81}+\frac{2\left(189x^4+270x^2+428\right)\left(3x^2-1\right)^{\frac{3}{4}}}{18711}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] 2/18711*(189*x^4+270*x^2+428)*(3*x^2-1)^(3/4)+4/81*RootOf(_Z^2+1)*ln(-(2*Ro
otOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(
1/2)+3*x^2)/(3*x^2-2))+4/81*ln((2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)-3*x^2+2
*(3*x^2-1)^(1/4))/(3*x^2-2))
```

maxima [A] time = 1.86, size = 74, normalized size = 0.95

$$\frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}} + \frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}} + \frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}} + \frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3/
4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log((3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.13, size = 62, normalized size = 0.79

$$\frac{8\,\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{81} + \frac{14\left(3x^2-1\right)^{3/4}}{243} + \frac{8\left(3x^2-1\right)^{7/4}}{567} + \frac{2\left(3x^2-1\right)^{11/4}}{891} + \frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,8\mathrm{i}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (8*atan((3*x^2 - 1)^(1/4)))/81 + (atan((3*x^2 - 1)^(1/4)*1i)*8i)/81 + (14*(
3*x^2 - 1)^(3/4))/243 + (8*(3*x^2 - 1)^(7/4))/567 + (2*(3*x^2 - 1)^(11/4))/
891
```

sympy [A] time = 24.08, size = 88, normalized size = 1.13

$$\frac{2\left(3x^2-1\right)^{\frac{11}{4}}}{891} + \frac{8\left(3x^2-1\right)^{\frac{7}{4}}}{567} + \frac{14\left(3x^2-1\right)^{\frac{3}{4}}}{243} + \frac{4\log\left(\sqrt[4]{3x^2-1}-1\right)}{81} - \frac{4\log\left(\sqrt[4]{3x^2-1}+1\right)}{81} + \frac{8\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(11/4)/891 + 8*(3*x**2 - 1)**(7/4)/567 + 14*(3*x**2 - 1)**(
3/4)/243 + 4*log((3*x**2 - 1)**(1/4) - 1)/81 - 4*log((3*x**2 - 1)**(1/4) +
1)/81 + 8*atan((3*x**2 - 1)**(1/4))/81
```

3.827 $\int \frac{x^5}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 63, 298, 203, 206}

$$\frac{2}{189}\left(3x^2-1\right)^{7/4}+\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*(-1 + 3*x^2)^(7/4))/189 + (4*ArcTan[(-1 + 3*
x^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{3\sqrt[4]{-1+3x}}+\frac{4}{9(-2+3x)\sqrt[4]{-1+3x}}+\frac{1}{9}(-1+3x)^{3/4}\right)dx, x, x^2\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{2}{9}\operatorname{Subst}\Bigg(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x,\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{8}{27}\operatorname{Subst}\Bigg(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}-\frac{4}{27}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4}+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\Bigg(
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.81

$$\frac{2}{189}\left(3\left(3x^2-1\right)^{3/4}\left(x^2+2\right)+14\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-14\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(3*(2 + x^2)*(-1 + 3*x^2)^(3/4) + 14*ArcTan[(-1 + 3*x^2)^(1/4)] - 14*Arc
Tanh[(-1 + 3*x^2)^(1/4)]))/189
```

IntegrateAlgebraic [A] time = 0.04, size = 53, normalized size = 0.84

$$\frac{2}{63}\left(3x^2-1\right)^{3/4}\left(x^2+2\right)+\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(2 + x^2)*(-1 + 3*x^2)^(3/4))/63 + (4*ArcTan[(-1 + 3*x^2)^(1/4)])/27 - (
4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

fricas [A] time = 0.91, size = 57, normalized size = 0.90

$$\frac{2}{63}\left(3x^2-1\right)^{\frac{3}{4}}\left(x^2+2\right)+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/63*(3*x^2 - 1)^(3/4)*(x^2 + 2) + 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/27*lo
g((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.50, size = 64, normalized size = 1.02

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4) -
 1))
```

maple [C] time = 0.96, size = 141, normalized size = 2.24

$$\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\sqrt{3x^2-1}}{3x^2-2}\right)}{27}+\frac{2\ln\left(\frac{-3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}-2\sqrt{3x^2-1}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{27}+\frac{2\left(x^2+2\right)\left(3x^2-1\right)^{\frac{3}{4}}}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] 2/63*(x^2+2)*(3*x^2-1)^(3/4)+2/27*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(3*x
^2-1)^(3/4)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)+3*x^2)/(3*x^
2-2))+2/27*ln((-3*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)+2*(3*x^2-1)^(1/4)
)/(3*x^2-2))
```

maxima [A] time = 1.93, size = 63, normalized size = 1.00

$$\frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.85, size = 51, normalized size = 0.81

$$\frac{4\,\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{27}+\frac{2\left(3x^2-1\right)^{3/4}}{27}+\frac{2\left(3x^2-1\right)^{7/4}}{189}+\frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,4\mathrm{i}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (4*atan((3*x^2 - 1)^(1/4)))/27 + (atan((3*x^2 - 1)^(1/4)*1i)*4i)/27 + (2*(3
*x^2 - 1)^(3/4))/27 + (2*(3*x^2 - 1)^(7/4))/189
```

sympy [A] time = 18.98, size = 75, normalized size = 1.19

$$\frac{2\left(3x^2-1\right)^{\frac{7}{4}}}{189}+\frac{2\left(3x^2-1\right)^{\frac{3}{4}}}{27}+\frac{2\log\left(\sqrt[4]{3x^2-1}-1\right)}{27}-\frac{2\log\left(\sqrt[4]{3x^2-1}+1\right)}{27}+\frac{4\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(7/4)/189 + 2*(3*x**2 - 1)**(3/4)/27 + 2*log((3*x**2 - 1)**
(1/4) - 1)/27 - 2*log((3*x**2 - 1)**(1/4) + 1)/27 + 4*atan((3*x**2 - 1)**(1
/4))/27
```

3.828 $\int \frac{x^3}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 63, 298, 203, 206}

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(
-1 + 3*x^2)^(1/4)])/9
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(-2+3x)\sqrt[4]{-1+3x}}\,dx,x,x^2\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx,x,x^2\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{4}{9}\,\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx,x,\sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}-\frac{2}{9}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{-1+3x^2}\right)+\frac{2}{9}\,\text{Subst}\left(\int \frac{1}{1+x^2}\right.\\
&= \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.92

$$\frac{2}{27}\left(\left(3x^2-1\right)^{3/4}+3\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-3\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(3/4) + 3*ArcTan[(-1 + 3*x^2)^(1/4)] - 3*ArcTanh[(-1 + 3*x
^2)^(1/4)]))/27
```

IntegrateAlgebraic [A] time = 0.03, size = 48, normalized size = 1.00

$$\frac{2}{27}\left(3x^2-1\right)^{3/4}+\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(3/4))/27 + (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(
-1 + 3*x^2)^(1/4)])/9
```

fricas [A] time = 0.95, size = 52, normalized size = 1.08

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.48, size = 53, normalized size = 1.10

$$\frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 1.01, size = 136, normalized size = 2.83

$$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)+2\sqrt{3x^2-1}}{3x^2-2}\right)}{9}-\frac{\ln\left(-\frac{3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}+2\sqrt{3x^2-1}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{9}+\frac{2\left(3x^2-1\right)^{\frac{3}{4}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] 2/27*(3*x^2-1)^(3/4)-1/9*ln(-(2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/2)+3*x^2+2*(
3*x^2-1)^(1/4))/(3*x^2-2))-1/9*RootOf(_Z^2+1)*ln((2*RootOf(_Z^2+1)*(3*x^2-1
)^(3/4)+2*(3*x^2-1)^(1/2)-2*RootOf(_Z^2+1)*(3*x^2-1)^(1/4)-3*x^2)/(3*x^2-2)
)
```

maxima [A] time = 1.99, size = 52, normalized size = 1.08

$$\frac{2}{27}\left(3\,x^{2}-1\right)^{\frac{3}{4}}+\frac{2}{9}\,\arctan\left(\left(3\,x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\,\log\left(\left(3\,x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\,\log\left(\left(3\,x^{2}-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.86, size = 36, normalized size = 0.75

$$\frac{2\,\mathrm{atan}\left(\left(3\,x^{2}-1\right)^{1/4}\right)}{9}-\frac{2\,\mathrm{atanh}\left(\left(3\,x^{2}-1\right)^{1/4}\right)}{9}+\frac{2\,\left(3\,x^{2}-1\right)^{3/4}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (2*atan((3*x^2 - 1)^(1/4)))/9 - (2*atanh((3*x^2 - 1)^(1/4)))/9 + (2*(3*x^2
 - 1)^(3/4))/27
```

sympy [A] time = 14.21, size = 58, normalized size = 1.21

$$\frac{2\left(3x^{2}-1\right)^{\frac{3}{4}}}{27}+\frac{\log\left(\sqrt[4]{3x^{2}-1}-1\right)}{9}-\frac{\log\left(\sqrt[4]{3x^{2}-1}+1\right)}{9}+\frac{2\,\mathrm{atan}\left(\sqrt[4]{3x^{2}-1}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] 2*(3*x**2 - 1)**(3/4)/27 + log((3*x**2 - 1)**(1/4) - 1)/9 - log((3*x**2 - 1
)**(1/4) + 1)/9 + 2*atan((3*x**2 - 1)**(1/4))/9
```

3.829 $\int \frac{x}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=33

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 63, 298, 203, 206}

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{-1+3x}\right.\\
&= \frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.00

$$\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

fricas [A] time = 0.90, size = 41, normalized size = 1.24

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((3
*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.36, size = 42, normalized size = 1.27

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(ab
s((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 0.93, size = 125, normalized size = 3.79

$$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\sqrt{3x^2-1}}{3x^2-2}\right)}{6}+\frac{\ln\left(\frac{-3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}-2\sqrt{3x^2-1}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] 1/6*ln((-3*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)+2*(3*x^2-1)^(1/4))/(3*x^
2-2))+1/6*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*RootOf(_Z^
2+1)*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)+3*x^2)/(3*x^2-2))
```

maple

maxima [A] time = 2.02, size = 41, normalized size = 1.24

$$\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] 1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((3
*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.16, size = 25, normalized size = 0.76

$$\frac{\operatorname{atan}\left(\left(3\,x^2-1\right)^{1/4}\right)}{3}-\frac{\operatorname{atanh}\left(\left(3\,x^2-1\right)^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] atan((3*x^2 - 1)^(1/4))/3 - atanh((3*x^2 - 1)^(1/4))/3
```

sympy [A] time = 8.99, size = 42, normalized size = 1.27

$$\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6}+\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 + atan((3*x
**2 - 1)**(1/4))/3
```

3.830 $\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=173

$$-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\right.}{2\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {446, 86, 63, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1 +
 3*x^2)^(1/4)]/2 - Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(
4*Sqrt[2]) + Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqrt
[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{(-2+3x)\sqrt[4]{-1+3x}}\,dx, x\right.\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right)+\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x}\right.\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx\right.\\
&= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{\log\left(1-\sqrt{2}\,\sqrt[4]{-1+3x^2}+\sqrt{-}\right.}{4\sqrt{2}}\\
&= \frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt[4]{-1+3x}\right.}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.36

$$-\frac{1}{3}\left(3x^2-1\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-3x^2\right)+\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 - ((-1 + 3*x^2
)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - 3*x^2])/3
```

IntegrateAlgebraic [A] time = 0.13, size = 122, normalized size = 0.71

$$\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{3x^2-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3x^2-1}}{\sqrt{3x^2-1}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(-1 + 3*x^2)^(1/4)]/2 - ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + 3*x^2]/Sqrt
[2])/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 + ArcT
anh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt[-1 + 3*x^2])]/(2*Sqrt[2])
```

fricas [A] time = 0.77, size = 215, normalized size = 1.24

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqr
t(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(2)*(3*x^2 -
1)^(1/4) + 1) + 1/8*sqrt(2)*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2
- 1) + 4) - 1/8*sqrt(2)*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1
) + 4) + 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1
/4*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.36, size = 155, normalized size = 0.90

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)+\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 1/8*sqrt(2)*log(
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/8*sqrt(2)*log(-sqrt(2)
*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/2*arctan((3*x^2 - 1)^(1/4)) -
 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 5.78, size = 301, normalized size = 1.74

$$\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(-\frac{3x^{2}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}+2\left(3x^{2}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}-2\sqrt{3x^{2}-1}\;\mathrm{RootOf}\left(\_Z^{4}+1\right)+2\left(3x^{2}-1\right)^{\frac{3}{4}}}{x^{2}}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}\ln\left(-\frac{-3x^{2}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+2\sqrt{3x^{2}-1}\;\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+2\left(3x^{2}-1\right)^{\frac{3}{4}}-2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(\frac{-3x^{2}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)+2\sqrt{3x^{2}-1}\;\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}+2\left(3x^{2}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)-2\left(3x^{2}-1\right)^{\frac{3}{4}}}{x^{2}}\right)}{4}+\frac{\ln\left(-\frac{-3x^{2}+2\left(3x^{2}-1\right)^{\frac{3}{4}}-2\sqrt{3x^{2}-1}+2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] 1/4*RootOf(_Z^4+1)*ln((2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)^3+2*RootOf(_Z^4+1)^
2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x^2-2*(3*x^2-1)^(3/4)+2*RootOf(_Z^4+1))/
x^2)+1/4*RootOf(_Z^4+1)^3*ln(-(3*RootOf(_Z^4+1)^3*x^2-2*RootOf(_Z^4+1)^3+2*
RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)+2*(3*x^2-
1)^(3/4))/x^2)+1/4*ln(-(-3*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)+2*(3*x^2
-1)^(1/4))/(3*x^2-2))+1/4*RootOf(_Z^4+1)^2*ln(-(2*(3*x^2-1)^(1/2)*RootOf(_Z
^4+1)^2-3*RootOf(_Z^4+1)^2*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/4))/(3*x^2-
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^{2}-1\right)^{\frac{1}{4}}\left(3\,x^{2}-2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x), x)
```

mupad [B] time = 0.19, size = 77, normalized size = 0.45

$$\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{2}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] atan((3*x^2 - 1)^(1/4))/2 + (atan((3*x^2 - 1)^(1/4)*1i)*1i)/2 - 2^(1/2)*ata
n(2^(1/2)*(3*x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 - 1i/4) - 2^(1/2)*atan(2^(1/
2)*(3*x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 + 1i/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.831 $\int \frac{1}{x^3\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{\left(3x^2-1\right)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {446, 103, 156, 63, 297, 1162, 617, 204, 1165, 628, 298, 203, 206}

$$-\frac{\left(3x^2-1\right)^{3/4}}{4x^2}-\frac{9\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{9\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{9\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(3/4)/(4*x^2) + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (9*ArcTan[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (9*ArcTan[1 + Sqrt[2]*(-1 +
3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 - (9*Log[1 -
 Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) + (9*Log[1 +
Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{-\frac{9}{2}+\frac{9x}{4}}{x(-2+3x)\sqrt[4]{-1+3x}}\,dx, x, x^2\right)\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} - \frac{9}{16}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{-1+3x}}\,dx, x, x^2\right) + \frac{9}{8}\operatorname{Subst}\left(\int \frac{}{(-2+3}\right.\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} - \frac{3}{4}\operatorname{Subst}\left(\int \frac{x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) + \frac{3}{2}\operatorname{Subst}\left(\int \frac{x^2}{-1+}\right.\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{8}\operatorname{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{3}{8}\operatorname{Subst}\left(\int \frac{1+x}{\frac{1}{3}+}\right.\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{9}{16}\operatorname{Subs}\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{9\log\left(1\right.}{}\\
&= -\frac{\left(-1+3x^2\right)^{3/4}}{4x^2} + \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{9\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}} - \frac{9\,\mathrm{t}}{}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.40

$$
\frac{1}{4}\left(-3\left(3x^2-1\right)^{3/4}{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};1-3x^2\right) - \frac{\left(3x^2-1\right)^{3/4}}{x^2} + 3\tan^{-1}\left(\sqrt[4]{3x^2-1}\right) - 3\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (-((-1 + 3*x^2)^(3/4)/x^2) + 3*ArcTan[(-1 + 3*x^2)^(1/4)] - 3*ArcTanh[(-1 +
 3*x^2)^(1/4)] - 3*(-1 + 3*x^2)^(3/4)*Hypergeometric2F1[3/4, 1, 7/4, 1 - 3*
x^2])/4
```

IntegrateAlgebraic [A] time = 0.22, size = 140, normalized size = 0.73

$$-\frac{\left(3x^2-1\right)^{3/4}}{4x^2}+\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{9\tan^{-1}\left(\frac{\frac{\sqrt{3x^2-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{3x^2-1}}\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{9\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3x^2-1}}{\sqrt{3x^2-1}+1}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/4*(-1 + 3*x^2)^(3/4)/x^2 + (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 - (9*ArcTan[
(-(1/Sqrt[2]) + Sqrt[-1 + 3*x^2]/Sqrt[2])/(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2])
- (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 + (9*ArcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1/4
))/(1 + Sqrt[-1 + 3*x^2])])/(8*Sqrt[2])
```

fricas [A] time = 1.03, size = 252, normalized size = 1.32

$$\frac{36\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+36\sqrt{2}x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+9\sqrt{2}x^2\log\left(4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-9\sqrt{2}x^2\log\left(-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+24x^2\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-12x^2\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+12x^2\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)-8\left(3x^2-1\right)^{\frac{3}{4}}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/32*(36*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3
*x^2 - 1) + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 36*sqrt(2)*x^2*arctan(1/2
*sqrt(2)*sqrt(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(
2)*(3*x^2 - 1)^(1/4) + 1) + 9*sqrt(2)*x^2*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) +
 4*sqrt(3*x^2 - 1) + 4) - 9*sqrt(2)*x^2*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) +
4*sqrt(3*x^2 - 1) + 4) + 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3*x
^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(3/4
))/x^2
```

giac [A] time = 0.50, size = 169, normalized size = 0.88

$$-\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{9}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)+\frac{9}{32}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{9}{32}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{\left(3x^2-1\right)^{\frac{3}{4}}}{4x^2}+\frac{3}{4}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{3}{8}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] -9/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 9/16*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 9/32*sqrt(2)*l
og(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 9/32*sqrt(2)*log(-sqr
t(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(3/4)/x^2 +
 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

maple [C]   time = 5.09, size = 315, normalized size = 1.65

$$-\frac{9 \mathrm{RootOf}\left(\_Z^4+1\right)^3 \ln\left(-\frac{-3x^2 \mathrm{RootOf}\left(\_Z^4+1\right)^3+2 \mathrm{RootOf}\left(\_Z^4+1\right)^3+2\left(3x^2-1\right)^{\frac{1}{4}} \mathrm{RootOf}\left(\_Z^4+1\right)^2+2\sqrt{3x^2-1}\ \mathrm{RootOf}\left(\_Z^4+1\right)+2\left(3x^2-1\right)^{\frac{3}{4}}}{x^2}\right)}{16}+\frac{3 \mathrm{RootOf}\left(\_Z^4+1\right)^2 \ln\left(-\frac{-3x^2 \mathrm{RootOf}\left(\_Z^4+1\right)^2+2\sqrt{3x^2-1}\ \mathrm{RootOf}\left(\_Z^4+1\right)^2+2\left(3x^2-1\right)^{\frac{3}{4}}-2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{8}+\frac{9 \mathrm{RootOf}\left(\_Z^4+1\right) \ln\left(\frac{-3x^2 \mathrm{RootOf}\left(\_Z^4+1\right)+2\sqrt{3x^2-1}\ \mathrm{RootOf}\left(\_Z^4+1\right)^3+2\left(3x^2-1\right)^{\frac{1}{4}} \mathrm{RootOf}\left(\_Z^4+1\right)^2+2 \mathrm{RootOf}\left(\_Z^4+1\right)-2\left(3x^2-1\right)^{\frac{3}{4}}}{x^2}\right)}{16}+\frac{3\ln\left(-\frac{-3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}-2\sqrt{3x^2-1}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{8}-\frac{\left(3x^2-1\right)^{\frac{3}{4}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] -1/4*(3*x^2-1)^(3/4)/x^2+9/16*RootOf(_Z^4+1)*ln((2*(3*x^2-1)^(1/2)*RootOf(_
Z^4+1)^3+2*RootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x^2-2*(3*x^2-1
)^(3/4)+2*RootOf(_Z^4+1))/x^2)-9/16*RootOf(_Z^4+1)^3*ln(-(-3*RootOf(_Z^4+1)
^3*x^2+2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)+2*RootOf(_Z^4+1)^
2*(3*x^2-1)^(1/4)+2*RootOf(_Z^4+1)^3)/x^2)+3/8*RootOf(_Z^4+1)^2*ln(-(2*(3*x
^2-1)^(1/2)*RootOf(_Z^4+1)^2-3*RootOf(_Z^4+1)^2*x^2+2*(3*x^2-1)^(3/4)-2*(3*
x^2-1)^(1/4))/(3*x^2-2))+3/8*ln(-(-3*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2
)+2*(3*x^2-1)^(1/4))/(3*x^2-2))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3), x)
```

mupad [B]   time = 0.23, size = 82, normalized size = 0.43

$$\frac{3\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{4}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{4}-\frac{{\left(3\,x^2-1\right)}^{3/4}}{4\,x^2}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{8}-\frac{{\left(-1\right)}^{3/4}\,\mathrm{atan}\left({\left(-1\right)}^{3/4}\,{\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] (3*atan((3*x^2 - 1)^(1/4)))/4 + (atan((3*x^2 - 1)^(1/4)*1i)*3i)/4 - (3*x^2
- 1)^(3/4)/(4*x^2) - ((-1)^(1/4)*atan((-1)^(1/4)*(3*x^2 - 1)^(1/4)*1i)*9i)/
8 - ((-1)^(3/4)*atan((-1)^(3/4)*(3*x^2 - 1)^(1/4)*1i)*9i)/8
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(3x^{2}-2\right)\sqrt[4]{3x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.832 $\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {398}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 398

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-(b^2/a), 4]}, Simp[(b*ArcTan[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])
/(2*Sqrt[2]*a*d*q), x] + Simp[(b*ArcTanh[(q*x)/(Sqrt[2]*(a + b*x^2)^(1/4))]
)/(2*Sqrt[2]*a*d*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &
& NegQ[b^2/a]
```

Rubi steps

$$\int \frac{1}{\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [**C**] time = 0.02, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(-1 + 3*x^
2)^(1/4)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[
3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*
x^2)/2])))
```

IntegrateAlgebraic [**A**] time = 0.00, size = 63, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{3x^2-1}}{x}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x]/(2*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

fricas [**B**] time = 6.60, size = 104, normalized size = 1.70

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{24}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-1\right)^{\frac{1}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

maple [C] time = 1.29, size = 137, normalized size = 2.25

$$-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{3\sqrt{3x^2-1}\,x+3x+\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)+\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)}{3x^2-2}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln\left(\frac{3\sqrt{3x^2-1}\,x-3x+\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)-\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)}{3x^2-2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] -1/12*RootOf(_Z^2+6)*ln((RootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x
-RootOf(_Z^2+6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))-1/12*RootOf(_Z^2-6)*ln((Roo
tOf(_Z^2-6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1)^(1
/4)+3*x)/(3*x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(3\,x^2-1\right)^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

```
[Out] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.833 $\int \frac{x^2}{\left(2+3x^2\right)^{3/4}\left(4+3x^2\right)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{3x^2+2}}{2\sqrt{3}\,x\,\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{3x^2+2}+2\,2^{3/4}}{2\sqrt{3}\,x\,\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{3x^2+2}}{2\sqrt{3}\,x\,\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{3x^2+2}+2\,2^{3/4}}{2\sqrt{3}\,x\,\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1
/4))]/(3*2^(1/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2])
/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2+3x^2\right)^{3/4}\left(4+3x^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+3x^2}}{2\sqrt{3}\,x\,\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+3x^2}}{2\sqrt{3}\,x\,\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}$$

Mathematica [**C**]   time = 0.04, size = 37, normalized size = 0.29

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/2, (-3*x^2)/4])/(12*2^(3/4))
```

IntegrateAlgebraic [**A**]   time = 2.18, size = 136, normalized size = 1.05

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{2}\sqrt{3x^2+2}}{\sqrt{3}}-\frac{\sqrt{3}x^2}{2\sqrt[4]{2}}}{x\sqrt[4]{3x^2+2}}\right)}{6\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3}x\sqrt[4]{3x^2+2}}{3x^2+2\sqrt{2}\sqrt{3x^2+2}}\right)}{6\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[(-1/2*(Sqrt[3]*x^2)/2^(1/4) + (2^(1/4)*Sqrt[2 + 3*x^2])/Sqrt[3]
)/(x*(2 + 3*x^2)^(1/4))]/(2^(1/4)*Sqrt[3]) - ArcTanh[(2*2^(1/4)*Sqrt[3]*x*(
2 + 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[2]*Sqrt[2 + 3*x^2])]/(6*2^(1/4)*Sqrt[3])
```

fricas [**B**]   time = 1.08, size = 282, normalized size = 2.19

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{-\frac{72^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}x^2-24\sqrt{3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}+36x}{36x}\right)-\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{3x^2+2}\right)}{x^2}\right)+\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}x^2-24\sqrt{3x^2+2}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(3*x^2 + 2))/x^2)
- 12*72^(1/4)*sqrt(2)*(3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt(2)
*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(3*x^2 + 2
)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(3*x^2 + 2))/x^2) - 12*72^(1/4)*sqrt(2)
*(3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/4)*sqr
t(2)*(3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(3*x^2 + 2))/x^2) + 1/86
4*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(3*x^2 + 2)^(1/4)*x - 18*sqrt(
2)*x^2 - 24*sqrt(3*x^2 + 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+4\right)\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

maple [C] time = 7.44, size = 186, normalized size = 1.44

$$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\ln\left(\frac{3x\,\mathrm{RootOf}\left(\_Z^4+18\right)^2+\left(3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\mathrm{RootOf}\left(\_Z^4+18\right)^2+9\sqrt{3x^2+2}\,x-6\left(3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)}{3x^2+4}\right)}{18}-\frac{\mathrm{RootOf}\left(\_Z^4+18\right)\ln\left(-\frac{3x\,\mathrm{RootOf}\left(\_Z^4+18\right)^2+\left(3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^3-9\sqrt{3x^2+2}\,x+6\left(3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)}{3x^2+4}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x)
```

```
[Out] -1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln((RootOf(_Z^4+18)^2*RootOf(_Z^2+Root
Of(_Z^4+18)^2)*(3*x^2+2)^(3/4)+3*RootOf(_Z^4+18)^2*x+9*(3*x^2+2)^(1/2)*x-6*
RootOf(_Z^2+RootOf(_Z^4+18)^2)*(3*x^2+2)^(1/4))/(3*x^2+4))-1/18*RootOf(_Z^4
+18)*ln(-((3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3+3*RootOf(_Z^4+18)^2*x-9*(3*x^2+
2)^(1/2)*x+6*(3*x^2+2)^(1/4)*RootOf(_Z^4+18))/(3*x^2+4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+4\right)\left(3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(3\,x^2+2\right)}^{3/4}\,\left(3\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 + 2)^(3/4)*(3*x^2 + 4)),x)
```

```
[Out] int(x^2/((3*x^2 + 2)^(3/4)*(3*x^2 + 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3x^2+2\right)^{\frac{3}{4}}\left(3x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+2)**(3/4)/(3*x**2+4),x)
```

```
[Out] Integral(x**2/((3*x**2 + 2)**(3/4)*(3*x**2 + 4)), x)
```

3.834 $\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

Mathematica [C] time = 0.04, size = 37, normalized size = 0.31

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(3/4))
```

IntegrateAlgebraic [A] time = 2.24, size = 136, normalized size = 1.13

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}}-\frac{\sqrt{3}x^2}{2\sqrt[4]{2}}}{x\sqrt[4]{2-3x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}{3x^2+2\sqrt{2}\sqrt{2-3x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[(-1/2*(Sqrt[3]*x^2)/2^(1/4) + (2^(1/4)*Sqrt[2 - 3*x^2])/Sqrt[3]
)/(x*(2 - 3*x^2)^(1/4))]/(2^(1/4)*Sqrt[3]) - ArcTanh[(2*2^(1/4)*Sqrt[3]*x*(
2 - 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[2]*Sqrt[2 - 3*x^2])]/(6*2^(1/4)*Sqrt[3])
```

fricas [B] time = 1.08, size = 282, normalized size = 2.35

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{-\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}x^2-24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+36x}{36x}\right)-\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{-3x^2+2}\right)}{x^2}\right)+\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}x^2-24\sqrt{-3x^2+2}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) - 12*72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt
(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2) - 12*72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) + 1/864*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x -
 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

maple [C] time = 7.25, size = 186, normalized size = 1.55

$$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\ln\left(\frac{-3x\,\mathrm{RootOf}\left(\_Z^4+18\right)^2+\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\mathrm{RootOf}\left(\_Z^4+18\right)^2+9\sqrt{-3x^2+2}\,x+6\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)}{3x^2-4}\right)}{18}-\frac{\mathrm{RootOf}\left(\_Z^4+18\right)\ln\left(-\frac{-3x\,\mathrm{RootOf}\left(\_Z^4+18\right)^2+\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^3-9\sqrt{-3x^2+2}\,x-6\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)}{3x^2-4}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] -1/18*RootOf(_Z^4+18)*ln(-((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3-3*RootOf(_Z^4
+18)^2*x-9*(-3*x^2+2)^(1/2)*x-6*RootOf(_Z^4+18)*(-3*x^2+2)^(1/4))/(3*x^2-4)
)-1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln((RootOf(_Z^4+18)^2*RootOf(_Z^2+Roo
tOf(_Z^4+18)^2)*(-3*x^2+2)^(3/4)-3*RootOf(_Z^4+18)^2*x+9*(-3*x^2+2)^(1/2)*x
+6*RootOf(_Z^2+RootOf(_Z^4+18)^2)*(-3*x^2+2)^(1/4))/(3*x^2-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{{\left(2-3\,x^2\right)}^{3/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] -int(x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.835 $\int \frac{x^2}{\left(2+bx^2\right)^{3/4}\left(4+bx^2\right)}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{bx^2+2}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{bx^2+2}+2\,2^{3/4}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{bx^2+2}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt{bx^2+2}+2\,2^{3/4}}{2\sqrt{b}\,x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] -(ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(
1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2])
/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2+bx^2\right)^{3/4}\left(4+bx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{2\,2^{3/4}+2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}+\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\,\sqrt{2+bx^2}}{2\sqrt{b}\,x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Mathematica [C] time = 0.05, size = 39, normalized size = 0.31

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, -1/2*(b*x^2), -1/4*(b*x^2)])/(12*2^(3/4))
```

IntegrateAlgebraic [A] time = 2.29, size = 136, normalized size = 1.10

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{2}\sqrt{bx^2+2}}{\sqrt{b}}-\frac{\sqrt{b}x^2}{2\sqrt[4]{2}}}{x\sqrt[4]{bx^2+2}}\right)}{2\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{b}x\sqrt[4]{bx^2+2}}{bx^2+2\sqrt{2}\sqrt{bx^2+2}}\right)}{2\sqrt[4]{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(-1/2*(Sqrt[b]*x^2)/2^(1/4) + (2^(1/4)*Sqrt[2 + b*x^2])/Sqrt[b]
)/(x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2)) - ArcTanh[(2*2^(1/4)*Sqrt[b]*x*(
2 + b*x^2)^(1/4))/(b*x^2 + 2*Sqrt[2]*Sqrt[2 + b*x^2])]/(2*2^(1/4)*b^(3/2))
```

fricas [B] time = 1.10, size = 393, normalized size = 3.17

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{x^2}}\frac{1}{b^6}^{\frac{3}{4}}x-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(bx^2+2\right)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}-x}{x}\right)+\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^4\sqrt{\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}(bx^2+2)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{x^2}}\frac{1}{b^6}^{\frac{3}{4}}x-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(bx^2+2\right)^{\frac{1}{4}}b^4\frac{1}{b^6}^{\frac{3}{4}}+x}{x}\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(bx^2+2\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{2x^2}\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\frac{1}{b^6}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^4\sqrt{\frac{1}{b^6}}x^2-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(bx^2+2\right)^{\frac{1}{4}}b^2\frac{1}{b^6}^{\frac{1}{4}}x+2\sqrt{bx^2+2}}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*
b^4*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2
)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2)*(b^(-6))^(3/4)*x - 8
*sqrt(2)*(1/8)^(3/4)*(b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) - x)/x) + sqrt(2)
*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*b^4*sqr
t((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)
*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x^2)*(b^(-6))^(3/4)*x - 8*sqrt(2
)*(1/8)^(3/4)*(b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) + x)/x) - 1/4*sqrt(2)*(1
/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2
)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(b*x^2 + 2))/x
^2) + 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^
```

```
(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x +
2*sqrt(b*x^2 + 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^2+4)(bx^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^2+2)^{\frac{3}{4}}\,(b\,x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

```
[Out] int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^2+4)(bx^2+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(b\,x^2+2)^{3/4}\,(b\,x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2 + 2)^(3/4)*(b*x^2 + 4)),x)
```

```
[Out] int(x^2/((b*x^2 + 2)^(3/4)*(b*x^2 + 4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+2\right)^{\frac{3}{4}}\left(bx^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+2)**(3/4)/(b*x**2+4),x)
```

```
[Out] Integral(x**2/((b*x**2 + 2)**(3/4)*(b*x**2 + 4)), x)
```

3.836 $\int \frac{x^2}{\left(2-bx^2\right)^{3/4}\left(4-bx^2\right)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2^(1/4)*b^(3/2)) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]
*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2-bx^2\right)^{3/4}\left(4-bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}\,x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}\,b^{3/2}}$$

Mathematica [C] time = 0.05, size = 39, normalized size = 0.33

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)}{12\ 2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/2, (b*x^2)/4])/(12*2^(3/4))
```

IntegrateAlgebraic [A] time = 2.31, size = 140, normalized size = 1.18

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{2}\sqrt{2-bx^2}}{\sqrt{b}}-\frac{\sqrt{b}x^2}{2\sqrt[4]{2}}}{x\sqrt[4]{2-bx^2}}\right)}{2\sqrt[4]{2}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}{bx^2+2\sqrt{2}\sqrt{2-bx^2}}\right)}{2\sqrt[4]{2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(-1/2*(Sqrt[b]*x^2)/2^(1/4) + (2^(1/4)*Sqrt[2 - b*x^2])/Sqrt[b]
)/(x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2)) - ArcTanh[(2*2^(1/4)*Sqrt[b]*x*(
2 - b*x^2)^(1/4))/(b*x^2 + 2*Sqrt[2]*Sqrt[2 - b*x^2])]/(2*2^(1/4)*b^(3/2))
```

fricas [B] time = 0.95, size = 403, normalized size = 3.39

$$\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}{\frac{1}{b^{6}}}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^{4}\sqrt{\frac{\sqrt{\frac{1}{2}}b^{4}\sqrt{\frac{1}{b^{6}}}x^{2}+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^{2}+2\right)^{\frac{1}{4}}b^{2}{\frac{1}{b^{6}}}^{\frac{1}{4}}x+2\sqrt{-bx^{2}+2}}{x^{2}}}{\frac{1}{b^{6}}}^{\frac{3}{4}}x-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(-bx^{2}+2\right)^{\frac{1}{4}}b^{4}{\frac{1}{b^{6}}}^{\frac{3}{4}}-x}{x}\right)+\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}{\frac{1}{b^{6}}}^{\frac{1}{4}}\arctan\left(\frac{8\sqrt{2}\sqrt{\frac{1}{2}}\left(\frac{1}{8}\right)^{\frac{3}{4}}b^{4}{\frac{1}{b^{6}}}^{\frac{3}{4}}x\sqrt{\frac{\sqrt{\frac{1}{2}}b^{4}\sqrt{\frac{1}{b^{6}}}x^{2}-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^{2}+2\right)^{\frac{1}{4}}b^{2}{\frac{1}{b^{6}}}^{\frac{1}{4}}x+2\sqrt{-bx^{2}+2}}{x^{2}}}-8\sqrt{2}\left(\frac{1}{8}\right)^{\frac{3}{4}}\left(-bx^{2}+2\right)^{\frac{1}{4}}b^{4}{\frac{1}{b^{6}}}^{\frac{3}{4}}+x}{x}\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}{\frac{1}{b^{6}}}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^{4}\sqrt{\frac{1}{b^{6}}}x^{2}+2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^{2}+2\right)^{\frac{1}{4}}b^{2}{\frac{1}{b^{6}}}^{\frac{1}{4}}x+2\sqrt{-bx^{2}+2}}{2x^{2}}\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}{\frac{1}{b^{6}}}^{\frac{1}{4}}\log\left(\frac{\sqrt{\frac{1}{2}}b^{4}\sqrt{\frac{1}{b^{6}}}x^{2}-2\sqrt{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-bx^{2}+2\right)^{\frac{1}{4}}b^{2}{\frac{1}{b^{6}}}^{\frac{1}{4}}x+2\sqrt{-bx^{2}+2}}{2x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*
b^4*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 +
2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)*(b^(-6))^(3/4)*x -
 8*sqrt(2)*(1/8)^(3/4)*(-b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) - x)/x) + sqrt
(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*arctan((8*sqrt(2)*sqrt(1/2)*(1/8)^(3/4)*b^4*
(b^(-6))^(3/4)*x*sqrt((sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/
4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2) - 8*s
qrt(2)*(1/8)^(3/4)*(-b*x^2 + 2)^(1/4)*b^4*(b^(-6))^(3/4) + x)/x) - 1/4*sqrt
(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4*sqrt(b^(-6))*x^2 + 2*
sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(1/4)*x + 2*sqrt(-b*x^2
 + 2))/x^2) + 1/4*sqrt(2)*(1/8)^(1/4)*(b^(-6))^(1/4)*log(1/2*(sqrt(1/2)*b^4
```

```
*sqrt(b^(-6))*x^2 - 2*sqrt(2)*(1/8)^(1/4)*(-b*x^2 + 2)^(1/4)*b^2*(b^(-6))^(
1/4)*x + 2*sqrt(-b*x^2 + 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+2\right)^{\frac{3}{4}}\left(-b\,x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

```
[Out] int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(bx^2-4\right)\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{{\left(2-b\,x^2\right)}^{3/4}\,\left(b\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - b*x^2)^(3/4)*(b*x^2 - 4)),x)
```

```
[Out] -int(x^2/((2 - b*x^2)^(3/4)*(b*x^2 - 4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{bx^2\left(-bx^2+2\right)^{\frac{3}{4}}-4\left(-bx^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+2)**(3/4)/(-b*x**2+4),x)
[Out] -Integral(x**2/(b*x**2*(-b*x**2 + 2)**(3/4) - 4*(-b*x**2 + 2)**(3/4)), x)
```

3.837 $\int \frac{x^2}{\left(a+3x^2\right)^{3/4}\left(2a+3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\,\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\,\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4
))]/(3*Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))/(
Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a+3x^2\right)^{3/4}\left(2a+3x^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Mathematica [C] time = 0.06, size = 65, normalized size = 0.54

$$\frac{x^3\left(\frac{a+3x^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{6a\left(a+3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] (x^3*((a + 3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/a, (-3*x^2)/
(2*a)])/(6*a*(a + 3*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.26, size = 136, normalized size = 1.13

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{a+3x^2}}{\sqrt{3}}-\frac{\sqrt{3}x^2}{2\sqrt[4]{a}}}{x\sqrt[4]{a+3x^2}}\right)}{6\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{2\sqrt{3}\sqrt[4]{a}\,x\sqrt[4]{a+3x^2}}{2\sqrt{a}\sqrt{a+3x^2}+3x^2}\right)}{6\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[(-1/2*(Sqrt[3]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a + 3*x^2])/Sqrt[3]
)/(x*(a + 3*x^2)^(1/4))]/(Sqrt[3]*a^(1/4)) - ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(
a + 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])]/(6*Sqrt[3]*a^(1/4))
```

fricas [A] time = 0.92, size = 171, normalized size = 1.42

$$-\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}ax\left(-\frac{1}{a}\right)^{\frac{3}{4}}\sqrt{\frac{3x^2\sqrt{-\frac{1}{a}}+2\sqrt{3x^2+a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(3x^2+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(3x^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a*x*(-1/a)
^(3/4)*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(3*x^2 + a))/x^2) - (1/36)^(3/4)*(3*x
^2 + a)^(1/4)*a*(-1/a)^(3/4))/x) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/
36)^(1/4)*x*(-1/a)^(1/4) + (3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/a)^
(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (3*x^2 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

maple **[F]** time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3x^2+a\right)^{\frac{3}{4}}\left(3x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

```
[Out] int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2+2\,a\right)\left(3\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(3\,x^2+2\,a\right)\,{\left(3\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*a + 3*x^2)*(a + 3*x^2)^(3/4)),x)
```

```
[Out] int(x^2/((2*a + 3*x^2)*(a + 3*x^2)^(3/4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+3x^2\right)^{\frac{3}{4}}\left(2a+3x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2+a)**(3/4)/(3*x**2+2*a),x)
[Out] Integral(x**2/((a + 3*x**2)**(3/4)*(2*a + 3*x**2)), x)
```

3.838 $\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\,\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}\,x\,\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}\,x\,\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\,\sqrt[4]{a}}$$

Mathematica [C] time = 0.06, size = 65, normalized size = 0.54

$$\frac{x^3 \left(\frac{a-3x^2}{a}\right)^{3/4} F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{6a\left(a-3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] (x^3*((a - 3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/a, (3*x^2)/(2
*a)])/(6*a*(a - 3*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.23, size = 136, normalized size = 1.13

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{a-3x^2}}{\sqrt{3}}-\frac{\sqrt{3}x^2}{2\sqrt[4]{a}}}{x\sqrt[4]{a-3x^2}}\right)}{6\sqrt{3}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}{2\sqrt{a}\sqrt{a-3x^2}+3x^2}\right)}{6\sqrt{3}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[(-1/2*(Sqrt[3]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a - 3*x^2])/Sqrt[3]
)/(x*(a - 3*x^2)^(1/4))]/(Sqrt[3]*a^(1/4)) - ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(
a - 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])]/(6*Sqrt[3]*a^(1/4))
```

fricas [A] time = 0.82, size = 171, normalized size = 1.42

$$-\frac{2}{3}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}ax\left(-\frac{1}{a}\right)^{\frac{3}{4}}\sqrt{\frac{3x^2\sqrt{-\frac{1}{a}}+2\sqrt{-3x^2+a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(-3x^2+a\right)^{\frac{1}{4}}a\left(-\frac{1}{a}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(-3x^2+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(-3x^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(1/36)^(1/4)*(-1/a)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a*x*(-1/a)
^(3/4)*sqrt((3*x^2*sqrt(-1/a) + 2*sqrt(-3*x^2 + a))/x^2) - (1/36)^(3/4)*(-3
*x^2 + a)^(1/4)*a*(-1/a)^(3/4))/x) - 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(
1/36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/
a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-1/a)^(1/4) - (-3*x^2 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(3x^2-2a\right)\left(-3x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="giac")
[Out] integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3x^2+a\right)^{\frac{3}{4}} \left(-3x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
[Out] int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(3\,x^2-2\,a\right)\left(-3\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="maxima")
[Out] -integrate(x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(2\,a-3\,x^2\right)\,{\left(a-3\,x^2\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*a - 3*x^2)*(a - 3*x^2)^(3/4)),x)
[Out] int(x^2/((2*a - 3*x^2)*(a - 3*x^2)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{-2a\left(a-3x^2\right)^{\frac{3}{4}}+3x^2\left(a-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+a)**(3/4)/(-3*x**2+2*a),x)
[Out] -Integral(x**2/(-2*a*(a - 3*x**2)**(3/4) + 3*x**2*(a - 3*x**2)**(3/4)), x)
```

3.839 $\int \frac{x^2}{\left(a+bx^2\right)^{3/4}\left(2a+bx^2\right)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {441}

$$\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] -(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/
4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))/(
Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^2\right)^{3/4}\left(2a+bx^2\right)}\,dx=-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.58

$$\frac{x^3\left(\frac{a+bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{6a\left(a+bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] (x^3*((a + b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -1/2*(b
*x^2)/a])/(6*a*(a + b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.38, size = 136, normalized size = 1.18

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{a+bx^2}}{\sqrt{b}}-\frac{\sqrt{b}x^2}{2\sqrt[4]{a}}}{x\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}{2\sqrt{a}\sqrt{a+bx^2}+bx^2}\right)}{2\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(-1/2*(Sqrt[b]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a + b*x^2])/Sqrt[b]
)/(x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2)) - ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(
a + b*x^2)^(1/4))/(b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2])]/(2*a^(1/4)*b^(3/2))
```

fricas [B] time = 0.91, size = 207, normalized size = 1.80

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{bx^2+a}}{x^2}}\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(bx^2+a\right)^{\frac{1}{4}}ab^4\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*s
qrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(b*x^2 + a))/x^2)*(-1/(a*b^6))^(3/4)
- (1/4)^(3/4)*(b*x^2 + a)^(1/4)*a*b^4*(-1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1
/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (b*x^2 +
 a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*
(-1/(a*b^6))^(1/4) - (b*x^2 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+2\,a\right)\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="giac")
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{3}{4}}\left(b\,x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
[Out] int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2+2\,a\right)\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{3/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x)
[Out] int(x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^2\right)^{\frac{3}{4}}\left(2a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2+a)**(3/4)/(b*x**2+2*a),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**(3/4)*(2*a + b*x**2)), x)
```

3.840 $\int \frac{x^2}{\left(a-bx^2\right)^{3/4}\left(2a-bx^2\right)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(Sqr
t[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(a-bx^2\right)^{3/4}\left(2a-bx^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}\,x\,\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [C] time = 0.07, size = 68, normalized size = 0.57

$$\frac{x^3 \left(\frac{a-bx^2}{a}\right)^{3/4} F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{6a\left(a-bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] (x^3*((a - b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/a, (b*x^2)/(2
*a)])/(6*a*(a - b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.36, size = 140, normalized size = 1.18

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{a}\sqrt{a-bx^2}}{\sqrt{b}}-\frac{\sqrt{b}x^2}{2\sqrt[4]{a}}}{x\sqrt[4]{a-bx^2}}\right)}{2\sqrt[4]{a}b^{3/2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}{2\sqrt{a}\sqrt{a-bx^2}+bx^2}\right)}{2\sqrt[4]{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(-1/2*(Sqrt[b]*x^2)/a^(1/4) + (a^(1/4)*Sqrt[a - b*x^2])/Sqrt[b]
)/(x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2)) - ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(
a - b*x^2)^(1/4))/(b*x^2 + 2*Sqrt[a]*Sqrt[a - b*x^2])]/(2*a^(1/4)*b^(3/2))
```

fricas [B] time = 0.96, size = 211, normalized size = 1.77

$$-2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{-\frac{1}{ab^6}}+2\sqrt{-bx^2+a}}{x^2}}\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(-bx^2+a\right)^{\frac{1}{4}}ab^4\left(-\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(-bx^2+a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(-bx^2+a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*s
qrt((b^4*x^2*sqrt(-1/(a*b^6)) + 2*sqrt(-b*x^2 + a))/x^2)*(-1/(a*b^6))^(3/4)
 - (1/4)^(3/4)*(-b*x^2 + a)^(1/4)*a*b^4*(-1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^
(1/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (-b*x^
2 + a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2
*x*(-1/(a*b^6))^(1/4) - (-b*x^2 + a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(bx^2-2\,a\right)\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{3}{4}}\left(-b\,x^2+2a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

```
[Out] int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(bx^2-2\,a\right)\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a-b\,x^2\right)}^{3/4}\,\left(2\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x)
```

```
[Out] int(x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{-2a\left(a-bx^{2}\right)^{\frac{3}{4}}+bx^{2}\left(a-bx^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2+a)**(3/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(x**2/(-2*a*(a - b*x**2)**(3/4) + b*x**2*(a - b*x**2)**(3/4)), x)
```

3.841 $\int \frac{x^7}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=188

$$\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\Big($$

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {443, 261, 266, 43, 444, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{16}{81}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{16}{81}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/
4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81 + (16*2^(3/4)*ArcTa
n[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2
- 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2
 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int \bigg(-\frac{16x}{27\left(2-3x^2\right)^{3/4}}-\frac{4x^3}{9\left(2-3x^2\right)^{3/4}}-\frac{x^5}{3\left(2-3x^2\right)^{3/4}}+\frac{64x}{27\left(2-3x^2\right)^{3/4}\left(4-3x^2\right.}\\
&= -\left(\frac{1}{3}\int \frac{x^5}{\left(2-3x^2\right)^{3/4}}\,dx\right)-\frac{4}{9}\int \frac{x^3}{\left(2-3x^2\right)^{3/4}}\,dx-\frac{16}{27}\int \frac{x}{\left(2-3x^2\right)^{3/4}}\,dx+\frac{64}{27}\int\\
&= \frac{32}{81}\sqrt[4]{2-3x^2}-\frac{1}{6}\operatorname{Subst}\left(\int \frac{x^2}{(2-3x)^{3/4}}\,dx,x,x^2\right)-\frac{2}{9}\operatorname{Subst}\bigg(\int \frac{x}{(2-3x)^{3/4}}\,dx,x,\\
&= \frac{32}{81}\sqrt[4]{2-3x^2}-\frac{1}{6}\operatorname{Subst}\left(\int \left(\frac{4}{9(2-3x)^{3/4}}-\frac{4}{9}\sqrt[4]{2-3x}+\frac{1}{9}(2-3x)^{5/4}\right)dx,x,x^2\right)\\
&= \frac{56}{81}\sqrt[4]{2-3x^2}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{1}{81}\left(32\sqrt{2}\right)\operatorname{Subst}\bigg(\int \frac{\sqrt{2}}{2+}\\
&= \frac{56}{81}\sqrt[4]{2-3x^2}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{1}{81}\left(16\sqrt{2}\right)\operatorname{Subst}\bigg(\int \frac{}{\sqrt{2}}\\
&= \frac{56}{81}\sqrt[4]{2-3x^2}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{2}{729}\left(2-3x^2\right)^{9/4}+\frac{8}{81}2^{3/4}\log\bigg(\sqrt{2}-2^{3/4}\sqrt[4]{2-}\\
&= \frac{56}{81}\sqrt[4]{2-3x^2}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{81}2^{3/4}\tan^{-1}\bigg(1+\sqrt[4]{4-6x}
\end{aligned}$$

Mathematica [A]   time = 0.11, size = 179, normalized size = 0.95

$$\frac{2\left(156\sqrt[4]{2-3x^2}\,x^2+1136\sqrt[4]{2-3x^2}+180\,2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-180\,2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+360\,2^{3/4}\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-360\,2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+45\sqrt[4]{2-3x^2}\,x^4\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(1136*(2 - 3*x^2)^(1/4) + 156*x^2*(2 - 3*x^2)^(1/4) + 45*x^4*(2 - 3*x^2)
^(1/4) + 360*2^(3/4)*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 360*2^(3/4)*ArcTan[1 +
 (4 - 6*x^2)^(1/4)] + 180*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) +
 Sqrt[2 - 3*x^2]] - 180*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + S
qrt[2 - 3*x^2]]))/3645
```

IntegrateAlgebraic [**A**] time = 0.17, size = 124, normalized size = 0.66

$$-\frac{16}{81}2^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)-\frac{16}{81}2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)+\frac{2\sqrt[4]{2-3x^2}\left(45x^4+156x^2+1136\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4)*(1136 + 156*x^2 + 45*x^4))/3645 - (16*2^(3/4)*ArcTan[(
-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)])/81 - (16*2^(3/4)*A
rcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/81
```

fricas [**A**] time = 0.83, size = 198, normalized size = 1.05

$$\frac{32}{81}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{32}{81}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}}-2^{\frac{1}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{3645}\left(45\,x^4+156\,x^2+1136\right)\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 32/81*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sq
rt(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 32/81*2^(3/4)*arctan(2^
(1/4)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1
/4)*(-3*x^2 + 2)^(1/4) + 1) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4)
 + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/3645*(45*x^4 + 156*x^2 + 1136)*(-3*x^2 +
 2)^(1/4)
```

giac [**A**] time = 0.35, size = 160, normalized size = 0.85

$$\frac{2}{729}\left(3\,x^2-2\right)^2\left(-3\,x^2+2\right)^{\frac{1}{4}}-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{16}{405}\left(-3\,x^2+2\right)^{\frac{5}{4}}+\frac{56}{81}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 2)^2*(-3*x^2 + 2)^(1/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(
2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/
4) - 2*(-3*x^2 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4)
 + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2
+ 2)^(1/4)
```

maple [C] time = 2.97, size = 541, normalized size = 2.88

$$-\frac{2\left(45x^4+156x^2+1136\right)\left(3x^2-2\right)}{3645\left(-3x^2+2\right)^{\frac{3}{4}}}-\frac{\left(\frac{16\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{27x^6-18\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)-36x^4-6\sqrt{-27x^6+54x^4-36x^2+8}\,x^2\,\mathrm{RootOf}\left(\_Z^4+2\right)^2+24\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)+2\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\mathrm{RootOf}\left(\_Z^4+2\right)^2+12x^2+4\sqrt{-27x^6+54x^4-36x^2+8}\,\mathrm{RootOf}\left(\_Z^4+2\right)^2-8\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)}{\left(3x^2-4\right)\left(3x^2-2\right)^2}\right)}{81}+\frac{16\,\mathrm{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{-27x^6+18\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^4+2\right)+36x^4-6\sqrt{-27x^6+54x^4-36x^2+8}\,x^2\,\mathrm{RootOf}\left(\_Z^4+2\right)^2-24\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^4+2\right)+2\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+2\right)^3-12x^2+4\sqrt{-27x^6+54x^4-36x^2+8}\,\mathrm{RootOf}\left(\_Z^4+2\right)^2+8\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+2\right)}{\left(3x^2-4\right)\left(3x^2-2\right)^2}\right)}{81}\right)\left(-\left(3x^2-2\right)^3\right)^{\frac{1}{4}}}{\left(-3x^2+2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] -2/3645*(45*x^4+156*x^2+1136)*(3*x^2-2)/(-3*x^2+2)^(3/4)-(16/81*RootOf(_Z^4
+2)*ln((2*RootOf(_Z^4+2)^3*(-27*x^6+54*x^4-36*x^2+8)^(3/4)-6*RootOf(_Z^4+2)
^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*x^2+18*RootOf(_Z^4+2)*(-27*x^6+54*x^4-36
*x^2+8)^(1/4)*x^4-27*x^6+4*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)
-24*RootOf(_Z^4+2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*x^2+36*x^4+8*RootOf(_Z^4
+2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)-12*x^2)/(3*x^2-4)/(3*x^2-2)^2)+16/81*Ro
otOf(_Z^2+RootOf(_Z^4+2)^2)*ln(-(2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^
4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(3/4)-18*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(-2
7*x^6+54*x^4-36*x^2+8)^(1/4)*x^4-6*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+
8)^(1/2)*x^2+27*x^6+24*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(-27*x^6+54*x^4-36*x^2
+8)^(1/4)*x^2+4*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)-36*x^4-8*R
ootOf(_Z^2+RootOf(_Z^4+2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)+12*x^2)/(3*x^2
-4)/(3*x^2-2)^2))/(-3*x^2+2)^(3/4)*(-(3*x^2-2)^3)^(1/4)
```

maxima [A] time = 2.01, size = 151, normalized size = 0.80

$$\frac{2}{729}\left(-3x^2+2\right)^{\frac{9}{4}}-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{16}{81}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{8}{81}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{16}{405}\left(-3x^2+2\right)^{\frac{5}{4}}+\frac{56}{81}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] 2/729*(-3*x^2 + 2)^(9/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-
3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2
 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2 + 2)^(1/4)
```

mupad [B] time = 0.88, size = 82, normalized size = 0.44

$$\frac{56\left(2-3x^2\right)^{1/4}}{81}-\frac{16\left(2-3x^2\right)^{5/4}}{405}+\frac{2\left(2-3x^2\right)^{9/4}}{729}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{16}{81}-\frac{16}{81}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{16}{81}+\frac{16}{81}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^7/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (56*(2 - 3*x^2)^(1/4))/81 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1
i/2))*(16/81 - 16i/81) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2
))*(16/81 + 16i/81) - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/4))/72
9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**7/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.842 $\int \frac{x^5}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=173

$$-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\bigg(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2}$$

Rubi [A] time = 0.18, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {443, 261, 266, 43, 444, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{4}{27}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{4}{27}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*ArcTan[1 +
 (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])
/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]
)/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]
])/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int \left(-\frac{4x}{9\left(2-3x^2\right)^{3/4}}-\frac{x^3}{3\left(2-3x^2\right)^{3/4}}+\frac{16x}{9\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\right)dx\\
&= -\left(\frac{1}{3}\int \frac{x^3}{\left(2-3x^2\right)^{3/4}}\,dx\right)-\frac{4}{9}\int \frac{x}{\left(2-3x^2\right)^{3/4}}\,dx+\frac{16}{9}\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx\\
&= \frac{8}{27}\sqrt[4]{2-3x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{x}{(2-3x)^{3/4}}\,dx,x,x^2\right)+\frac{8}{9}\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3\right.\\
&= \frac{8}{27}\sqrt[4]{2-3x^2}-\frac{1}{6}\text{Subst}\left(\int \left(\frac{2}{3(2-3x)^{3/4}}-\frac{1}{3}\sqrt[4]{2-3x}\right)dx,x,x^2\right)-\frac{32}{27}\text{Subst}\left(\int\right.\\
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}\left(2-3x^2\right)^{5/4}-\frac{1}{27}\left(8\sqrt{2}\right)\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx,x,\sqrt[4]{2-3x^2}\right)\\
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}\left(2-3x^2\right)^{5/4}-\frac{1}{27}\left(4\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx,x,\sqrt[4]{2}\right.\\
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}\left(2-3x^2\right)^{5/4}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)-\\
&= \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}\left(2-3x^2\right)^{5/4}-\frac{4}{27}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{4}{27}2^{3/4}\tan^{-1}\left(1\right.
\end{aligned}$$

Mathematica [A] time = 0.07, size = 163, normalized size = 0.94

$$\frac{2}{135}\left(3\sqrt[4]{2-3x^2}\,x^2+28\sqrt[4]{2-3x^2}+5\,2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-5\,2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+10\,2^{3/4}\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-10\,2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(28*(2 - 3*x^2)^(1/4) + 3*x^2*(2 - 3*x^2)^(1/4) + 10*2^(3/4)*ArcTan[1 -
(4 - 6*x^2)^(1/4)] - 10*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)] + 5*2^(3/4)*L
og[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] - 5*2^(3/4)*Log[S
qrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]))/135
```

IntegrateAlgebraic [A] time = 0.16, size = 119, normalized size = 0.69

$$\frac{2}{135}\sqrt[4]{2-3x^2}\left(3x^2+28\right)-\frac{4}{27}2^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)-\frac{4}{27}2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4)*(28 + 3*x^2))/135 - (4*2^(3/4)*ArcTan[(-2^(-1/4) + Sqr
t[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)])/27 - (4*2^(3/4)*ArcTanh[(2*2^(1/4
)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/27
```

fricas [A] time = 1.05, size = 193, normalized size = 1.12

$$\frac{8}{27}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{8}{27}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{135}\left(3x^2+28\right)\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 8/27*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqr
t(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 8/27*2^(3/4)*arctan(2^(1
/4)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4
)*(-3*x^2 + 2)^(1/4) + 1) - 2/27*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) +
 sqrt(2) + sqrt(-3*x^2 + 2)) + 2/135*(3*x^2 + 28)*(-3*x^2 + 2)^(1/4)
```

giac [A] time = 0.42, size = 140, normalized size = 0.81

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{2}{135}\left(-3x^2+2\right)^{\frac{5}{4}}+\frac{4}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/4)
```

```
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3
*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

maple [C] time = 2.16, size = 536, normalized size = 3.10

$$-\frac{2\left(3x^2+28\right)\left(3x^2-2\right)}{135\left(-3x^2+2\right)^{\frac{3}{4}}}-\frac{\left(\frac{4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{27x^6-18\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)-36x^4-6\sqrt{-27x^6+54x^4-36x^2+8}\,x^2\,\mathrm{RootOf}\left(\_Z^4+2\right)^2+24\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)+2\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\mathrm{RootOf}\left(\_Z^4+2\right)^2+12x^2+4\sqrt{-27x^6+54x^4-36x^2+8}\,\mathrm{RootOf}\left(\_Z^4+2\right)^2-8\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)}{\left(3x^2-4\right)\left(3x^2-2\right)^2}\right)}{27}+\frac{4\,\mathrm{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{-27x^6+18\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^4+2\right)+36x^4-6\sqrt{-27x^6+54x^4-36x^2+8}\,x^2\,\mathrm{RootOf}\left(\_Z^4+2\right)^2-24\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^4+2\right)+2\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+2\right)^3-12x^2+4\sqrt{-27x^6+54x^4-36x^2+8}\,\mathrm{RootOf}\left(\_Z^4+2\right)^2+8\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+2\right)}{\left(3x^2-4\right)\left(3x^2-2\right)^2}\right)}{27}\right)\left(-\left(3x^2-2\right)^3\right)^{\frac{1}{4}}}{\left(-3x^2+2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] -2/135*(3*x^2+28)*(3*x^2-2)/(-3*x^2+2)^(3/4)-(4/27*RootOf(_Z^4+2)*ln((2*Roo
tOf(_Z^4+2)^3*(-27*x^6+54*x^4-36*x^2+8)^(3/4)-6*RootOf(_Z^4+2)^2*(-27*x^6+5
4*x^4-36*x^2+8)^(1/2)*x^2+18*RootOf(_Z^4+2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)
*x^4-27*x^6+4*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)-24*RootOf(_Z
^4+2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*x^2+36*x^4+8*RootOf(_Z^4+2)*(-27*x^6+
54*x^4-36*x^2+8)^(1/4)-12*x^2)/(3*x^2-4)/(3*x^2-2)^2)+4/27*RootOf(_Z^2+Root
Of(_Z^4+2)^2)*ln(-(2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(-27*x^
6+54*x^4-36*x^2+8)^(3/4)-18*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(-27*x^6+54*x^4-3
6*x^2+8)^(1/4)*x^4-6*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*x^2+2
7*x^6+24*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*x^2+
4*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)-36*x^4-8*RootOf(_Z^2+Roo
tOf(_Z^4+2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)+12*x^2)/(3*x^2-4)/(3*x^2-2)^
2))/(-3*x^2+2)^(3/4)*(-(3*x^2-2)^3)^(1/4)
```

maxima [A] time = 2.08, size = 140, normalized size = 0.81

$$-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{2}{135}\left(-3\,x^2+2\right)^{\frac{5}{4}}+\frac{4}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135*(-3
*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

mupad [B] time = 0.22, size = 71, normalized size = 0.41

$$\frac{4\left(2-3\,x^2\right)^{1/4}}{9}-\frac{2\left(2-3\,x^2\right)^{5/4}}{135}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3\,x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{4}{27}-\frac{4}{27}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3\,x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{4}{27}+\frac{4}{27}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^5/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (4*(2 - 3*x^2)^(1/4))/9 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/
2))*(4/27 - 4i/27) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(
4/27 + 4i/27) - (2*(2 - 3*x^2)^(5/4))/135
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^5}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**5/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.843 $\int \frac{x^3}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=158

$$\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-\right.$$

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {443, 261, 444, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/9 + (2^(3
/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x
^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int \left(-\frac{x}{3\left(2-3x^2\right)^{3/4}}+\frac{4x}{3\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\right)dx \\
&= -\left(\frac{1}{3}\int \frac{x}{\left(2-3x^2\right)^{3/4}}\,dx\right)+\frac{4}{3}\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx \\
&= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}\,dx,x,x^2\right) \\
&= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{8}{9}\operatorname{Subst}\left(\int \frac{1}{2+x^4}\,dx,x,\sqrt[4]{2-3x^2}\right) \\
&= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}\left(2\sqrt{2}\right)\operatorname{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx,x,\sqrt[4]{2-3x^2}\right)-\frac{1}{9}\left(2\sqrt{2}\right)\operatorname{Subst}\left(\right. \\
&= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\operatorname{Subst}\left(\int \frac{2^{3/4}+2x}{-\sqrt{2}-2^{3/4}x-x^2}\,dx,x,\sqrt[4]{2-3x^2}\right)}{9\sqrt[4]{2}}+\frac{\operatorname{Subst}\left(\int \frac{2^{3/4}-2x}{-\sqrt{2}+2^{3/4}x-x^2}\,d\right.}{9\sqrt[4]{2}} \\
&= \frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}\right.}{9\sqrt[4]{2}} \\
&= \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}2^{3/4}\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)+\frac{1}{9}2^{3/4}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 146, normalized size = 0.92

$$\frac{1}{18}\left(4\sqrt[4]{2-3x^2}+2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+2\,2^{3/4}\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-2\,2^{3/4}\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (4*(2 - 3*x^2)^(1/4) + 2*2^(3/4)*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*2^(3/4)*
ArcTan[1 + (4 - 6*x^2)^(1/4)] + 2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(
1/4) + Sqrt[2 - 3*x^2]] - 2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) +
 Sqrt[2 - 3*x^2]])/18
```

IntegrateAlgebraic [A] time = 0.13, size = 112, normalized size = 0.71

$$\frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}2^{3/4}\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)-\frac{1}{9}2^{3/4}\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3
/4))/(2 - 3*x^2)^(1/4)])/9 - (2^(3/4)*ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))
/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/9
```

fricas [A] time = 0.97, size = 186, normalized size = 1.18

$$\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{2}{9}\cdot 2^{\frac{3}{4}}\arctan\left(2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}-2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 2/9*2^(3/4)*arctan(2^(1/4)*sqrt(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt
(-3*x^2 + 2)) - 2^(1/4)*(-3*x^2 + 2)^(1/4) - 1) + 2/9*2^(3/4)*arctan(2^(1/4
)*sqrt(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2^(1/4)*
(-3*x^2 + 2)^(1/4) + 1) - 1/18*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqr
t(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + s
qrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2 + 2)^(1/4)
```

giac [A] time = 0.46, size = 129, normalized size = 0.82

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^(
3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2
 + 2)^(1/4)
```

maple [C] time = 2.39, size = 528, normalized size = 3.34

$$-\frac{2\left(3x^2-2\right)}{9\left(-3x^2+2\right)^{\frac{3}{4}}}-\frac{\left(-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(\frac{-27x^6-18\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^4\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)+36x^4+6\sqrt{-27x^6+54x^4-36x^2+8}\,x^2\operatorname{RootOf}\left(\_Z^4+2\right)^2+24\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^2\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)+2\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)\operatorname{RootOf}\left(\_Z^4+2\right)^2-12x^2-4\sqrt{-27x^6+54x^4-36x^2+8}\,\operatorname{RootOf}\left(\_Z^4+2\right)^2-8\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)}{\left(3x^2-4\right)\left(3x^2-2\right)^2}\right)}{9}+\frac{\operatorname{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{-27x^6+18\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^4\operatorname{RootOf}\left(\_Z^4+2\right)+36x^4-6\sqrt{-27x^6+54x^4-36x^2+8}\,x^2\operatorname{RootOf}\left(\_Z^4+2\right)^2-24\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}x^2\operatorname{RootOf}\left(\_Z^4+2\right)+2\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^4+2\right)^3-12x^2+4\sqrt{-27x^6+54x^4-36x^2+8}\,\operatorname{RootOf}\left(\_Z^4+2\right)^2+8\left(-27x^6+54x^4-36x^2+8\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+2\right)}{\left(3x^2-4\right)\left(3x^2-2\right)^2}\right)}{9}\right)\left(-\left(3x^2-2\right)^3\right)^{\frac{1}{4}}}{\left(-3x^2+2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] -2/9*(3*x^2-2)/(-3*x^2+2)^(3/4)-(1/9*RootOf(_Z^4+2)*ln((2*RootOf(_Z^4+2)^3*
(-27*x^6+54*x^4-36*x^2+8)^(3/4)-6*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8
)^(1/2)*x^2+18*RootOf(_Z^4+2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*x^4-27*x^6+4*
RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)-24*RootOf(_Z^4+2)*(-27*x^6
+54*x^4-36*x^2+8)^(1/4)*x^2+36*x^4+8*RootOf(_Z^4+2)*(-27*x^6+54*x^4-36*x^2+
8)^(1/4)-12*x^2)/(3*x^2-4)/(3*x^2-2)^2)-1/9*RootOf(_Z^2+RootOf(_Z^4+2)^2)*l
n((2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+
8)^(3/4)-18*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*x
^4+6*RootOf(_Z^4+2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*x^2-27*x^6+24*RootOf(
_Z^2+RootOf(_Z^4+2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*x^2-4*RootOf(_Z^4+2)
^2*(-27*x^6+54*x^4-36*x^2+8)^(1/2)+36*x^4-8*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(
-27*x^6+54*x^4-36*x^2+8)^(1/4)-12*x^2)/(3*x^2-4)/(3*x^2-2)^2))/(-3*x^2+2)^(
3/4)*(-(3*x^2-2)^3)^(1/4)
```

maxima [A] time = 2.01, size = 129, normalized size = 0.82

$$-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^(
3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)*l
og(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/4)*
log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3*x^2
 + 2)^(1/4)
```

mupad [B] time = 0.17, size = 60, normalized size = 0.38

$$\frac{2\left(2-3x^2\right)^{1/4}}{9}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{9}-\frac{1}{9}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{9}+\frac{1}{9}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (2*(2 - 3*x^2)^(1/4))/9 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/
2))*(1/9 - 1i/9) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/
9 + 1i/9)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{3}}{3x^{2}\left(2-3x^{2}\right)^{\frac{3}{4}}-4\left(2-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x**3/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.844 $\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=143

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}(}{}$$

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {444, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{12\sqrt[4]{2}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{6\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -ArcTan[1 + (4 - 6*x^2)^(1/4)]/(6*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)
^(1/4)]/(6*2^(1/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*
x^2]]/(12*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x
^2]]/(12*2^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}(4-3x)}\,dx, x, x^2\right) \\
&= -\left(\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)\right) \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)}{3\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{2+x^4}\,dx, x, \sqrt[4]{2-3x^2}\right)}{3\sqrt{2}} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{6\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+2^{3/4}x+x^2}\,dx, x, \sqrt[4]{2-3x^2}\right)}{6\sqrt{2}} \\
&= \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}} \\
&= -\frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{6\sqrt[4]{2}} + \frac{\tan^{-1}\left(1-\sqrt[4]{2}\,\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}} + \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}\right.}{12\sqrt[4]{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 117, normalized size = 0.82

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+2\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-2\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{12\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (2*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*ArcTan[1 + (4 - 6*x^2)^(1/4)] + Log[Sq
rt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] - Log[Sqrt[2] + 2^(3/4
)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/(12*2^(1/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 97, normalized size = 0.68

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)}{6\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\,\sqrt[4]{2-3x^2}}{\sqrt{2}\,\sqrt{2-3x^2}+2}\right)}{6\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)]/2^(1/4
) - ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])]/(6
*2^(1/4))
```

fricas [B] time = 1.05, size = 230, normalized size = 1.61

$$\frac{1}{24}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{24}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{16}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{1}{96}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+\frac{1}{96}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/24*8^(3/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) - 1) + 1/24*8^(3/4)*sqrt(2)*arctan(1/16*8^(1/4)*sqrt(2)*sqrt(-
16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) -
 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 1/96*8^(3/4)*sqrt(2)*log(16*
8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 1/
96*8^(3/4)*sqrt(2)*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2)
+ 64*sqrt(-3*x^2 + 2))
```

giac [A] time = 0.38, size = 118, normalized size = 0.83

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

maple [C] time = 1.80, size = 189, normalized size = 1.32

$$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(\frac{2\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\mathrm{RootOf}\left(\_Z^4+2\right)^2-3x^2-2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4+2\right)^2-2\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)}{3x^2-4}\right)}{12}-\frac{\mathrm{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{2\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+2\right)^3-3x^2+2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4+2\right)^2+2\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+2\right)}{3x^2-4}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] 1/12*RootOf(_Z^2+RootOf(_Z^4+2)^2)*ln((2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*Root
Of(_Z^4+2)^2*(-3*x^2+2)^(1/4)-2*RootOf(_Z^2+RootOf(_Z^4+2)^2)*(-3*x^2+2)^(3
/4)-2*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1/2)-3*x^2)/(3*x^2-4))-1/12*RootOf(_Z^4+
2)*ln((2*RootOf(_Z^4+2)^3*(-3*x^2+2)^(1/4)+2*RootOf(_Z^4+2)^2*(-3*x^2+2)^(1
/2)+2*RootOf(_Z^4+2)*(-3*x^2+2)^(3/4)-3*x^2)/(3*x^2-4))
```

maxima [A]   time = 2.02, size = 118, normalized size = 0.83

$$-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(3/4
)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

mupad [B]   time = 0.96, size = 49, normalized size = 0.34

$$2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/12 + 1i/12) - 2^(
3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/2))*(1/12 - 1i/12)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(x/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.845 $\int \frac{1}{x\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=197

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}(\sqrt[4]{4-6x}}{8\sqrt[4]{2}}$$

Rubi **[A]** time = 0.19, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {443, 266, 63, 212, 206, 203, 444, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) - ArcTan[1 + (4 - 6*x^2)^(1/
4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]/(8*2^(1/4)) - ArcTa
nh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^
2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2
)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx &= \int\left(\frac{1}{4x\left(2-3x^2\right)^{3/4}}-\frac{3x}{4\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\right)dx \\
&= \frac{1}{4}\int\frac{1}{x\left(2-3x^2\right)^{3/4}}\,dx-\frac{3}{4}\int\frac{x}{\left(2-3x^2\right)^{3/4}\left(-4+3x^2\right)}\,dx \\
&= \frac{1}{8}\text{Subst}\left(\int\frac{1}{(2-3x)^{3/4}x}\,dx,x,x^2\right)-\frac{3}{8}\text{Subst}\left(\int\frac{1}{(2-3x)^{3/4}(-4+3x)}\,dx,x,x^2\right. \\
&= -\left(\frac{1}{6}\text{Subst}\left(\int\frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right)\right)+\frac{1}{2}\text{Subst}\left(\int\frac{1}{-2-x^4}\,dx,x,\sqrt[4]{2-3x^2}\right. \\
&= -\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{2-3x^2}\right)}{4\sqrt{2}}-\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2}+x^2}\,dx,x,\sqrt[4]{2-3x^2}\right)}{4\sqrt{2}}+ \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2}-2^{3/4}x+x^2}\,dx,x,\sqrt[4]{2-3x^2}\right)}{8\sqrt{2}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{16\sqrt[4]{2}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{8\sqrt[4]{2}}+\frac{\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}-
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 156, normalized size = 0.79

$$
\frac{-4\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)-4\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+\sqrt{2}\left(\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+2\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)-2\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)\right)}{16\,2^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*ArcTan[(1 - (3*x^2)/2)^(1/4)] - 4*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + Sqrt
[2]*(2*ArcTan[1 - (4 - 6*x^2)^(1/4)] - 2*ArcTan[1 + (4 - 6*x^2)^(1/4)] + Lo
g[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] - Log[Sqrt[2] + 2^
(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]))/(16*2^(3/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 151, normalized size = 0.77

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)}{8\sqrt[4]{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)}{8\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/4*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/2^(3/4) - ArcTan[(-2^(-1/4) + Sqrt[2
 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)]/(8*2^(1/4)) - ArcTanh[(2 - 3*x^2)^(1/
4)/2^(1/4)]/(4*2^(3/4)) - ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2
]*Sqrt[2 - 3*x^2])]/(8*2^(1/4))
```

fricas [B] time = 1.01, size = 315, normalized size = 1.60

$$\frac{1}{32}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+\frac{1}{32}\cdot 8^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{16}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\frac{1}{128}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+\frac{1}{128}\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{\sqrt{2}+\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(8^{\frac{3}{4}}+4\left(-3x^2+2\right)^{\frac{1}{4}}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(-8^{\frac{3}{4}}+4\left(-3x^2+2\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/32*8^(3/4)*sqrt(2)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^
2 + 2)^(1/4) - 1) + 1/32*8^(3/4)*sqrt(2)*arctan(1/16*8^(1/4)*sqrt(2)*sqrt(-
16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) -
 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 1/128*8^(3/4)*sqrt(2)*log(16
*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 1
/128*8^(3/4)*sqrt(2)*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2
) + 64*sqrt(-3*x^2 + 2)) + 1/16*8^(3/4)*arctan(1/2*8^(1/4)*sqrt(sqrt(2) + s
qrt(-3*x^2 + 2)) - 1/2*8^(1/4)*(-3*x^2 + 2)^(1/4)) - 1/64*8^(3/4)*log(8^(3/
4) + 4*(-3*x^2 + 2)^(1/4)) + 1/64*8^(3/4)*log(-8^(3/4) + 4*(-3*x^2 + 2)^(1/
4))
```

giac [A] time = 0.50, size = 210, normalized size = 1.07

$$-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right)+\frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(-4^{\frac{1}{8}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right)-\frac{1}{8}\cdot 4^{\frac{1}{8}}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{1}{16}\cdot 4^{\frac{1}{8}}\log\left(\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)+\frac{1}{16}\cdot 4^{\frac{1}{8}}\log\left(-\left(-3x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -1/16*4^(1/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*x
^2 + 2)^(1/4))) - 1/16*4^(1/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/8)
```

```
*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) - 1/32*4^(1/8)*sqrt(2)*log(4^(1/8)*sqrt(2
)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/32*4^(1/8)*sqrt(2)*l
og(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/8*
4^(1/8)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) - 1/16*4^(1/8)*log((-3*x^2 +
 2)^(1/4) + 4^(1/8)) + 1/16*4^(1/8)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8))
```

maple [C]   time = 15.08, size = 560, normalized size = 2.84

$$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2\ln\left(\frac{3x^2\,\mathrm{RootOf}\left(\_Z^4-2\right)-4\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4-2\right)^3+4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-2\right)^2-4\,\mathrm{RootOf}\left(\_Z^4-2\right)+4\left(-3x^2+2\right)^{\frac{3}{4}}}{3x^2-4}\right)}{32}-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2\ln\left(\frac{3x^2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)+3x^2\,\mathrm{RootOf}\left(\_Z^4-2\right)+2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2-2\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4-2\right)^3-4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)+4\left(-3x^2+2\right)^{\frac{3}{4}}}{3x^2-4}\right)}{16}+\frac{\mathrm{RootOf}\left(\_Z^4-2\right)^3\ln\left(\frac{3x^2\,\mathrm{RootOf}\left(\_Z^4-2\right)-4\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4-2\right)^3+4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-2\right)^2-4\,\mathrm{RootOf}\left(\_Z^4-2\right)+4\left(-3x^2+2\right)^{\frac{3}{4}}}{3x^2-4}\right)}{32}+\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(\frac{3x^2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2-4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2+4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-2\right)^2+4\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)-4\left(-3x^2+2\right)^{\frac{3}{4}}}{x^2}\right)}{16}+\frac{\mathrm{RootOf}\left(\_Z^4-2\right)\ln\left(-\frac{3x^2\,\mathrm{RootOf}\left(\_Z^4-2\right)^3-4\,\mathrm{RootOf}\left(\_Z^4-2\right)^3+4\left(-3x^2+2\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4-2\right)^2-4\sqrt{-3x^2+2}\,\mathrm{RootOf}\left(\_Z^4-2\right)+4\left(-3x^2+2\right)^{\frac{3}{4}}}{x^2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] 1/16*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((3*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootO
f(_Z^4-2)^2)*x^2+4*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(-3*x^2+2)^(1/2)-4*RootOf(
_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)-4*(-3*x^2+2)^(3/4)+4*RootOf(_Z^4-2)
^2*(-3*x^2+2)^(1/4))/x^2)+1/16*RootOf(_Z^4-2)*ln(-(3*RootOf(_Z^4-2)^3*x^2+4
*(-3*x^2+2)^(3/4)-4*(-3*x^2+2)^(1/2)*RootOf(_Z^4-2)+4*RootOf(_Z^4-2)^2*(-3*
x^2+2)^(1/4)-4*RootOf(_Z^4-2)^3)/x^2)+1/32*ln((-4*RootOf(_Z^4-2)^3*(-3*x^2+
2)^(1/2)+4*(-3*x^2+2)^(3/4)+4*RootOf(_Z^4-2)^2*(-3*x^2+2)^(1/4)+3*RootOf(_Z
^4-2)*x^2-4*RootOf(_Z^4-2))/(3*x^2-4))*RootOf(_Z^4-2)^3+1/32*ln((-4*RootOf(
_Z^4-2)^3*(-3*x^2+2)^(1/2)+4*(-3*x^2+2)^(3/4)+4*RootOf(_Z^4-2)^2*(-3*x^2+2)
^(1/4)+3*RootOf(_Z^4-2)*x^2-4*RootOf(_Z^4-2))/(3*x^2-4))*RootOf(_Z^4-2)^2*R
ootOf(_Z^2+RootOf(_Z^4-2)^2)-1/16*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-
2)^2)*ln((2*(-3*x^2+2)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2
-2*RootOf(_Z^4-2)^3*(-3*x^2+2)^(1/2)-4*(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(
_Z^4-2)^2)*RootOf(_Z^4-2)+3*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+4*(-3*x^2+2)^
(3/4)+3*RootOf(_Z^4-2)*x^2)/(3*x^2-4))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x), x)
```

mupad [B]   time = 0.24, size = 91, normalized size = 0.46

$$-\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{8}+\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,1{}\mathrm{i}}{8}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{16}-\frac{1}{16}{}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{16}+\frac{1}{16}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (2^(1/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*1i)/8 - (2^(1/4)*atan((2^(3
/4)*(2 - 3*x^2)^(1/4))/2))/8 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2
- 1i/2))*(1/16 + 1i/16) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/
2))*(1/16 - 1i/16)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^{3}\left(2-3x^{2}\right)^{\frac{3}{4}}-4x\left(2-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**3*(2 - 3*x**2)**(3/4) - 4*x*(2 - 3*x**2)**(3/4)), x)
```

3.846 $\int \frac{1}{x^3\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3}}{\sqrt[4]{2}}\right.}{32\ 2^{3/4}}$$

Rubi [A] time = 0.24, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = $\{443, 266, 51, 63, 212, 206, 203, 444, 211, 1165, 628, 1162, 617, 204\}$

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-\frac{3\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -(2 - 3*x^2)^(1/4)/(16*x^2) - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^
(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4)) + (3*ArcTan[1 - 2^(
1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/
4)])/(32*2^(3/4)) + (3*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3
*x^2]])/(64*2^(1/4)) - (3*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
- 3*x^2]])/(64*2^(1/4))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 443

```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
```

```
tegerQ[m/2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \left(2-3x^2\right)^{3/4} \left(4-3x^2\right)}\,dx &= \int \left(\frac{1}{4x^3 \left(2-3x^2\right)^{3/4}}+\frac{3}{16x \left(2-3x^2\right)^{3/4}}-\frac{9x}{16 \left(2-3x^2\right)^{3/4} \left(-4+3x^2\right)}\right)dx \\
&= \frac{3}{16}\int \frac{1}{x \left(2-3x^2\right)^{3/4}}\,dx+\frac{1}{4}\int \frac{1}{x^3 \left(2-3x^2\right)^{3/4}}\,dx-\frac{9}{16}\int \frac{x}{\left(2-3x^2\right)^{3/4} (-4+3x} \\
&= \frac{3}{32}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}x}\,dx,x,x^2\right)+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}x^2}\,dx,x,x^2\right)-\frac{9}{32} \\
&= -\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right)+\frac{9}{64}\,\text{Subst}\left(\int \frac{1}{(2-3x)^{3/4}x} \right. \\
&= -\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{3}{16}\,\text{Subst}\left(\int \frac{1}{\frac{2}{3}-\frac{x^4}{3}}\,dx,x,\sqrt[4]{2-3x^2}\right)-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\right.}{16\sqrt{2}} \\
&= -\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\,2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{16\,2^{3/4}}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-2^{3/4}x+x^2}\right.}{32\sqrt{2}} \\
&= -\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}+\frac{3\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2\right.}}{64} \\
&= -\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\,2^{3/4}}-\frac{3\tan^{-1}\left(1+\sqrt[4]{4-6x^2}\right)}{32\sqrt[4]{2}}+\frac{3\tan^{-1}\left(1-\sqrt[4]{2}\right.}{32\sqrt[4]{2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 210, normalized size = 0.98

$$-\frac{8\sqrt[4]{2-3x^2}-3\,2^{3/4}x^2\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+3\,2^{3/4}x^2\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)+30\sqrt[4]{2}\,x^2\tan^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)-6\,2^{3/4}x^2\tan^{-1}\left(1-\sqrt[4]{4-6x^2}\right)+6\,2^{3/4}x^2\tan^{-1}\left(\sqrt[4]{4-6x^2}+1\right)+30\sqrt[4]{2}\,x^2\tanh^{-1}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)}{128x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/128*(8*(2 - 3*x^2)^(1/4) + 30*2^(1/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)]
- 6*2^(3/4)*x^2*ArcTan[1 - (4 - 6*x^2)^(1/4)] + 6*2^(3/4)*x^2*ArcTan[1 + (4
 - 6*x^2)^(1/4)] + 30*2^(1/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] - 3*2^(3/4
)*x^2*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]] + 3*2^(3/4
)*x^2*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/x^2
```

IntegrateAlgebraic [A] time = 0.31, size = 169, normalized size = 0.79

$$-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\tan^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\, 2^{3/4}}-\frac{3\tan^{-1}\left(\frac{\frac{\sqrt{2-3x^2}}{2^{3/4}}-\frac{1}{\sqrt[4]{2}}}{\sqrt[4]{2-3x^2}}\right)}{32\sqrt[4]{2}}-\frac{15\tanh^{-1}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\, 2^{3/4}}-\frac{3\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt[4]{2-3x^2}}{\sqrt{2}\sqrt{2-3x^2}+2}\right)}{32\sqrt[4]{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/16*(2 - 3*x^2)^(1/4)/x^2 - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^
(3/4)) - (3*ArcTan[(-2^(-1/4) + Sqrt[2 - 3*x^2]/2^(3/4))/(2 - 3*x^2)^(1/4)]
)/(32*2^(1/4)) - (15*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(3/4)) - (3*
ArcTanh[(2*2^(1/4)*(2 - 3*x^2)^(1/4))/(2 + Sqrt[2]*Sqrt[2 - 3*x^2])])/(32*2
^(1/4))
```

fricas [B] time = 0.96, size = 352, normalized size = 1.64

$$\frac{12\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+4\sqrt{2}+4\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-1\right)+12\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\arctan\left(\frac{1}{16}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-3\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\log\left(16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+3\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\log\left(-16\cdot 8^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+64\sqrt{2}+64\sqrt{-3x^2+2}\right)+60\cdot 8^{\frac{3}{4}}x^2\arctan\left(\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{\sqrt{2}+\sqrt{-3x^2+2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-15\cdot 8^{\frac{3}{4}}x^2\log\left(8^{\frac{3}{4}}+4\left(-3x^2+2\right)^{\frac{1}{4}}\right)+15\cdot 8^{\frac{3}{4}}x^2\log\left(-8^{\frac{3}{4}}+4\left(-3x^2+2\right)^{\frac{1}{4}}\right)-32\left(-3x^2+2\right)^{\frac{1}{4}}}{512x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/512*(12*8^(3/4)*sqrt(2)*x^2*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(8^(3/4)*sqrt(
2)*(-3*x^2 + 2)^(1/4) + 4*sqrt(2) + 4*sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*sqrt(
2)*(-3*x^2 + 2)^(1/4) - 1) + 12*8^(3/4)*sqrt(2)*x^2*arctan(1/16*8^(1/4)*sqr
t(2)*sqrt(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*
x^2 + 2)) - 1/2*8^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 1) - 3*8^(3/4)*sqrt(2)
*x^2*log(16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) + 64*sqrt(2) + 64*sqrt(-3*x^
2 + 2)) + 3*8^(3/4)*sqrt(2)*x^2*log(-16*8^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)
+ 64*sqrt(2) + 64*sqrt(-3*x^2 + 2)) + 60*8^(3/4)*x^2*arctan(1/2*8^(1/4)*sqr
t(sqrt(2) + sqrt(-3*x^2 + 2)) - 1/2*8^(1/4)*(-3*x^2 + 2)^(1/4)) - 15*8^(3/4
)*x^2*log(8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 15*8^(3/4)*x^2*log(-8^(3/4) + 4
*(-3*x^2 + 2)^(1/4)) - 32*(-3*x^2 + 2)^(1/4))/x^2
```

giac [A] time = 0.58, size = 192, normalized size = 0.89

$$-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)+\frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right)-\frac{15}{64}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{15}{128}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right)+\frac{15}{128}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}-\left(-3x^2+2\right)^{\frac{1}{4}}\right)-\frac{\left(-3x^2+2\right)^{\frac{1}{4}}}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] -3/64*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/64*2
^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 3/128*2^(3/4
```

```
)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 3/128*2^(3
/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 15/64*2
^(1/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 15/128*2^(1/4)*log(2^(1/4)
+ (-3*x^2 + 2)^(1/4)) + 15/128*2^(1/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/4)) -
1/16*(-3*x^2 + 2)^(1/4)/x^2
```

maple [C] time = 23.05, size = 1540, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] 1/16*(3*x^2-2)/x^2/(-3*x^2+2)^(3/4)-(15/128*RootOf(_Z^2+RootOf(_Z^4-2)^2)*l
n((6*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_
Z^4-2)^2*x^2-27*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^6-18*(-27*x^6+54*x^4-36*x^2
+8)^(1/4)*RootOf(_Z^4-2)^2*x^4-4*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^
2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2+72*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^4+2
4*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^2-60*RootOf(_Z^2+RootO
f(_Z^4-2)^2)*x^2+4*(-27*x^6+54*x^4-36*x^2+8)^(3/4)-8*RootOf(_Z^4-2)^2*(-27*
x^6+54*x^4-36*x^2+8)^(1/4)+16*RootOf(_Z^2+RootOf(_Z^4-2)^2))/(3*x^2-2)^2/x^
2)+15/128*RootOf(_Z^4-2)*ln((-6*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^4
-2)^3*x^2+18*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^4-27*RootOf
(_Z^4-2)*x^6+4*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^4-2)^3-24*(-27*x^6
+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^2+72*RootOf(_Z^4-2)*x^4+4*(-27*x
^6+54*x^4-36*x^2+8)^(3/4)+8*RootOf(_Z^4-2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/4
)-60*RootOf(_Z^4-2)*x^2+16*RootOf(_Z^4-2))/(3*x^2-2)^2/x^2)+3/128*ln(-(-27*
x^6*RootOf(_Z^4-2)^3+36*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^
4+72*RootOf(_Z^4-2)^3*x^4-24*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^4-2)
*x^2-48*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^2-60*RootOf(_Z^4
-2)^3*x^2+8*(-27*x^6+54*x^4-36*x^2+8)^(3/4)+16*(-27*x^6+54*x^4-36*x^2+8)^(1
/2)*RootOf(_Z^4-2)+16*RootOf(_Z^4-2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/4)+16*R
ootOf(_Z^4-2)^3)/(3*x^2-4)/(3*x^2-2)^2)*RootOf(_Z^4-2)^3+3/128*ln(-(-27*x^6
*RootOf(_Z^4-2)^3+36*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^4+7
2*RootOf(_Z^4-2)^3*x^4-24*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^4-2)*x^
2-48*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)^2*x^2-60*RootOf(_Z^4-2)
^3*x^2+8*(-27*x^6+54*x^4-36*x^2+8)^(3/4)+16*(-27*x^6+54*x^4-36*x^2+8)^(1/2)
*RootOf(_Z^4-2)+16*RootOf(_Z^4-2)^2*(-27*x^6+54*x^4-36*x^2+8)^(1/4)+16*Root
Of(_Z^4-2)^3)/(3*x^2-4)/(3*x^2-2)^2)*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z
^4-2)^2)-3/64*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln((27*x^6*Roo
tOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)^2-27*x^6*RootOf(_Z^4-2)^3-36*Root
Of(_Z^2+RootOf(_Z^4-2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4-2)*x^
4-36*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^4+36*RootOf(_Z^4-2)^3
*x^4+12*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*x^2+4
8*RootOf(_Z^2+RootOf(_Z^4-2)^2)*(-27*x^6+54*x^4-36*x^2+8)^(1/4)*RootOf(_Z^4
```

```
-2)*x^2+12*RootOf(_Z^4-2)^2*RootOf(_Z^2+RootOf(_Z^4-2)^2)*x^2+12*(-27*x^6+5
4*x^4-36*x^2+8)^(1/2)*RootOf(_Z^4-2)*x^2-12*RootOf(_Z^4-2)^3*x^2-8*(-27*x^6
+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^2+RootOf(_Z^4-2)^2)-16*(-27*x^6+54*x^4-36
*x^2+8)^(1/4)*RootOf(_Z^2+RootOf(_Z^4-2)^2)*RootOf(_Z^4-2)-8*(-27*x^6+54*x^
4-36*x^2+8)^(3/4)-8*(-27*x^6+54*x^4-36*x^2+8)^(1/2)*RootOf(_Z^4-2))/(3*x^2-
4)/(3*x^2-2)^2))/(-3*x^2+2)^(3/4)*(-(3*x^2-2)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(3\,x^{2}-4\right)\left(-3\,x^{2}+2\right)^{\frac{3}{4}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3), x)
```

mupad [B] time = 1.01, size = 107, normalized size = 0.50

$$-\frac{\left(2-3\,x^2\right)^{1/4}}{16\,x^2}-\frac{15\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{64}+\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,15{}\mathrm{i}}{64}+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{3}{64}-\frac{3}{64}{}\mathrm{i}\right)+\frac{{\left(-1\right)}^{1/4}\,2^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)\,3{}\mathrm{i}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] (2^(1/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*15i)/64 - (15*2^(1/4)*atan(
(2^(3/4)*(2 - 3*x^2)^(1/4))/2))/64 - (2 - 3*x^2)^(1/4)/(16*x^2) - 2^(3/4)*a
tan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(3/64 + 3i/64) + ((-1)^(1/4)*2^
(1/4)*atan(((-1)^(1/4)*2^(3/4)*(2 - 3*x^2)^(1/4))/2)*3i)/32
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^{5}\left(2-3x^{2}\right)^{\frac{3}{4}}-4x^{3}\left(2-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**5*(2 - 3*x**2)**(3/4) - 4*x**3*(2 - 3*x**2)**(3/4)), x)
```

3.847 $\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\,\sqrt{2-3x^2}}{\sqrt[4]{2}\,\sqrt{3}\,x\,\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{2-3x^2}+2}{\sqrt[4]{2}\,\sqrt{3}\,x\,\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {441}

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\,\sqrt{2-3x^2}}{\sqrt[4]{2}\,\sqrt{3}\,x\,\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{2-3x^2}+2}{\sqrt[4]{2}\,\sqrt{3}\,x\,\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

Rule 441

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a
 + b*x^2)^(1/4))])/(a*d*Rt[b^2/a, 4]^3), x] + Simp[(b*ArcTanh[(b - Rt[b^2/a
, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))])/(a*d*Rt[b^2/
a, 4]^3), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

Rubi steps

$$\int \frac{x^2}{\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx=\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\,\sqrt{2-3x^2}}{\sqrt[4]{2}\,\sqrt{3}\,x\,\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\,\sqrt{2-3x^2}}{\sqrt[4]{2}\,\sqrt{3}\,x\,\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\,\sqrt{3}}$$

Mathematica [C] time = 0.03, size = 37, normalized size = 0.31

$$\frac{x^3F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\,2^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] (x^3*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(3/4))
```

IntegrateAlgebraic [A] time = 0.00, size = 136, normalized size = 1.13

$$-\frac{\tan^{-1}\left(\frac{\frac{\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}}-\frac{\sqrt{3}x^2}{2\sqrt[4]{2}}}{x\sqrt[4]{2-3x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}{3x^2+2\sqrt{2}\sqrt{2-3x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

```
[Out] -1/6*ArcTan[(-1/2*(Sqrt[3]*x^2)/2^(1/4) + (2^(1/4)*Sqrt[2 - 3*x^2])/Sqrt[3]
)/(x*(2 - 3*x^2)^(1/4))]/(2^(1/4)*Sqrt[3]) - ArcTanh[(2*2^(1/4)*Sqrt[3]*x*(
2 - 3*x^2)^(1/4))/(3*x^2 + 2*Sqrt[2]*Sqrt[2 - 3*x^2])]/(6*2^(1/4)*Sqrt[3])
```

fricas [B] time = 1.14, size = 282, normalized size = 2.35

$$\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}-36x}{36x}\right)+\frac{1}{216}\cdot 72^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{72^{\frac{1}{4}}\sqrt{6}\sqrt{2}x\sqrt{-\frac{72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}x^2-24\sqrt{-3x^2+2}}{x^2}}-12\cdot 72^{\frac{1}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}+36x}{36x}\right)-\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x+18\sqrt{2}x^2+24\sqrt{-3x^2+2}\right)}{x^2}\right)+\frac{1}{864}\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(-\frac{96\left(72^{\frac{3}{4}}\sqrt{2}\left(-3x^2+2\right)^{\frac{1}{4}}x-18\sqrt{2}x^2-24\sqrt{-3x^2+2}\right)}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/216*72^(3/4)*sqrt(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt((72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) - 12*72^(1/4)*sqrt(2)*(-3*x^2 + 2)^(1/4) - 36*x)/x) + 1/216*72^(3/4)*sqrt
(2)*arctan(1/36*(72^(1/4)*sqrt(6)*sqrt(2)*x*sqrt(-(72^(3/4)*sqrt(2)*(-3*x^2
 + 2)^(1/4)*x - 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2) - 12*72^(1/4)*sq
rt(2)*(-3*x^2 + 2)^(1/4) + 36*x)/x) - 1/864*72^(3/4)*sqrt(2)*log(96*(72^(3/
4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x + 18*sqrt(2)*x^2 + 24*sqrt(-3*x^2 + 2))/x^2
) + 1/864*72^(3/4)*sqrt(2)*log(-96*(72^(3/4)*sqrt(2)*(-3*x^2 + 2)^(1/4)*x -
 18*sqrt(2)*x^2 - 24*sqrt(-3*x^2 + 2))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

maple [C] time = 0.00, size = 186, normalized size = 1.55

$$-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+18\right)^2\right)\ln\left(\frac{-3x\operatorname{RootOf}\left(\_Z^4+18\right)^2+\left(-3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+18\right)^2\right)\operatorname{RootOf}\left(\_Z^4+18\right)^2+9\sqrt{-3x^2+2}\,x+6\left(-3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+18\right)^2\right)}{3x^2-4}\right)}{18}-\frac{\operatorname{RootOf}\left(\_Z^4+18\right)\ln\left(-\frac{-3x\operatorname{RootOf}\left(\_Z^4+18\right)^2+\left(-3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^4+18\right)^3-9\sqrt{-3x^2+2}\,x-6\left(-3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+18\right)}{3x^2-4}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

```
[Out] -1/18*RootOf(_Z^4+18)*ln(-((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3-3*RootOf(_Z^4
+18)^2*x-9*(-3*x^2+2)^(1/2)*x-6*RootOf(_Z^4+18)*(-3*x^2+2)^(1/4))/(3*x^2-4)
)-1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln((RootOf(_Z^4+18)^2*RootOf(_Z^2+Roo
tOf(_Z^4+18)^2)*(-3*x^2+2)^(3/4)-3*RootOf(_Z^4+18)^2*x+9*(-3*x^2+2)^(1/2)*x
+6*RootOf(_Z^2+RootOf(_Z^4+18)^2)*(-3*x^2+2)^(1/4))/(3*x^2-4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{{\left(2-3\,x^2\right)}^{3/4}\,\left(3\,x^2-4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

```
[Out] -int(x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{3x^2 \left(2-3x^2\right)^{\frac{3}{4}} - 4 \left(2-3x^2\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
[Out] -Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.848 $\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Rubi **[A]** time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] time = 0.04, size = 52, normalized size = 0.85

$$-\frac{x^3\left(1-3x^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{6\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(
-1 + 3*x^2)^(3/4)
```

IntegrateAlgebraic [A] time = 2.05, size = 61, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

fricas [B] time = 0.87, size = 104, normalized size = 1.70

$$-\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}\,x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6
)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

maple [C] time = 1.28, size = 137, normalized size = 2.25

$$-\frac{\mathrm{RootOf}\left(\_Z^{2}-6\right) \ln \left(\frac{3 \sqrt{3 x^{2}-1}\, x+3 x+\left(3 x^{2}-1\right)^{\frac{3}{4}} \mathrm{RootOf}\left(\_Z^{2}-6\right)+\left(3 x^{2}-1\right)^{\frac{1}{4}} \mathrm{RootOf}\left(\_Z^{2}-6\right)}{3 x^{2}-2}\right)}{18}+\frac{\mathrm{RootOf}\left(\_Z^{2}+6\right) \ln \left(\frac{3 \sqrt{3 x^{2}-1}\, x-3 x+\left(3 x^{2}-1\right)^{\frac{3}{4}} \mathrm{RootOf}\left(\_Z^{2}+6\right)-\left(3 x^{2}-1\right)^{\frac{1}{4}} \mathrm{RootOf}\left(\_Z^{2}+6\right)}{3 x^{2}-2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 1/18*RootOf(_Z^2+6)*ln((RootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x-
RootOf(_Z^2+6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))-1/18*RootOf(_Z^2-6)*ln((Root
Of(_Z^2-6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1)^(1/
4)+3*x)/(3*x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(3 \, x^{2} - 1\right)^{\frac{3}{4}} \left(3 \, x^{2} - 2\right)}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(3\,x^2-1\right)}^{3/4}\,\left(3\,x^2-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(3 x^{2} - 2\right) \left(3 x^{2} - 1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.849 $\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2-3x^2\right)\left(-1-3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] time = 0.10, size = 52, normalized size = 0.85

$$-\frac{x^3\left(3x^2+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)}{6\left(-3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 + 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -3*x^2, (-3*x^2)/2])
/(-1 - 3*x^2)^(3/4)
```

IntegrateAlgebraic [A] time = 2.11, size = 79, normalized size = 1.30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x\left(-3x^2-1\right)^{3/4}}{3x^2+1}\right)}{3\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x\left(-3x^2-1\right)^{3/4}}{3x^2+1}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*ArcTan[(Sqrt[3/2]*x*(-1 - 3*x^2)^(3/4))/(1 + 3*x^2)]/Sqrt[6] + ArcTanh
[(Sqrt[3/2]*x*(-1 - 3*x^2)^(3/4))/(1 + 3*x^2)]/(3*Sqrt[6])
```

fricas [C] time = 0.93, size = 115, normalized size = 1.89

$$-\frac{1}{36}\sqrt{6}\log\left(\frac{\sqrt{6}\,x+2\left(-3\,x^2-1\right)^{\frac{1}{4}}}{2\,x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{\sqrt{6}\,x-2\left(-3\,x^2-1\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{36}i\sqrt{6}\log\left(\frac{i\sqrt{6}\,x+2\left(-3\,x^2-1\right)^{\frac{1}{4}}}{2\,x}\right)+\frac{1}{36}i\sqrt{6}\log\left(\frac{-i\sqrt{6}\,x+2\left(-3\,x^2-1\right)^{\frac{1}{4}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*log(1/2*(sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*sqrt(6)*
log(-1/2*(sqrt(6)*x - 2*(-3*x^2 - 1)^(1/4))/x) - 1/36*I*sqrt(6)*log(1/2*(I*
sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*I*sqrt(6)*log(1/2*(-I*sqrt(6)*x
 + 2*(-3*x^2 - 1)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(3\,x^2+2\right)\left(-3\,x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

maple [C] time = 1.32, size = 138, normalized size = 2.26

$$\frac{\mathrm{RootOf}\left(\_Z^{2}-6\right)\ln\left(-\frac{-3\sqrt{-3x^{2}-1}\,x+3x+\left(-3x^{2}-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-6\right)-\left(-3x^{2}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-6\right)}{3x^{2}+2}\right)}{18}+\frac{\mathrm{RootOf}\left(\_Z^{2}+6\right)\ln\left(\frac{3\sqrt{-3x^{2}-1}\,x+3x+\left(-3x^{2}-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}+6\right)+\left(-3x^{2}-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+6\right)}{3x^{2}+2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x)
```

```
[Out] 1/18*RootOf(_Z^2+6)*ln(((-3*x^2-1)^(3/4)*RootOf(_Z^2+6)+3*(-3*x^2-1)^(1/2)*
x+RootOf(_Z^2+6)*(-3*x^2-1)^(1/4)+3*x)/(3*x^2+2))+1/18*RootOf(_Z^2-6)*ln(-(
(-3*x^2-1)^(3/4)*RootOf(_Z^2-6)-3*(-3*x^2-1)^(1/2)*x-RootOf(_Z^2-6)*(-3*x^2
-1)^(1/4)+3*x)/(3*x^2+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{\left(3\,x^{2}+2\right)\left(-3\,x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^{2}}{{\left(-3\,x^{2}-1\right)}^{3/4}\,\left(3\,x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((- 3*x^2 - 1)^(3/4)*(3*x^2 + 2)),x)
```

```
[Out] -int(x^2/((- 3*x^2 - 1)^(3/4)*(3*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{3x^{2}\left(-3x^{2}-1\right)^{\frac{3}{4}}+2\left(-3x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2)/(-3*x**2-1)**(3/4),x)
[Out] -Integral(x**2/(3*x**2*(-3*x**2 - 1)**(3/4) + 2*(-3*x**2 - 1)**(3/4)), x)
```

3.850 $\int \frac{x^2}{\left(-2+bx^2\right)\left(-1+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+bx^2\right)\left(-1+bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

Mathematica [C] time = 0.05, size = 54, normalized size = 0.75

$$-\frac{x^3\left(1-bx^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)}{6\left(bx^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - b*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, b*x^2, (b*x^2)/2])/(
-1 + b*x^2)^(3/4)
```

IntegrateAlgebraic [A] time = 2.18, size = 72, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

fricas [B] time = 0.95, size = 275, normalized size = 3.82

$$\left[-\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}\,x}\right)-\sqrt{2}\,\sqrt{b}\,\log\left(-\frac{b^2x^4-2\,\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\,\sqrt{bx^2-1}\,bx^2+4\,bx^2-4\,\sqrt{2}\left(bx^2-1\right)^{\frac{3}{4}}\sqrt{b}\,x-4}{b^2x^4-4\,bx^2+4}\right)}{4\,b^2},\ \frac{2\,\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{b^2x^4-2\,\sqrt{2}\left(bx^2-1\right)^{\frac{1}{4}}\sqrt{-b}\,bx^3-4\,\sqrt{bx^2-1}\,bx^2+4\,bx^2+4\,\sqrt{2}\left(bx^2-1\right)^{\frac{3}{4}}\sqrt{-b}\,x-4}{b^2x^4-4\,bx^2+4}\right)}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - sq
rt(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*s
qrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)
/(b^2*x^4 - 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x
^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 2*sqrt(2)*
(b*x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 + 4*sq
rt(2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2-1\right)^{\frac{3}{4}}\left(bx^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{2}-2\right) \left(b\,x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
[Out] int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{{\left(bx^{2}-1\right)}^{\frac{3}{4}} {\left(bx^{2}-2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(b\,x^2-1\right)}^{3/4}\,\left(b\,x^2-2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)),x)
[Out] int(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(bx^{2} - 2\right) \left(bx^{2} - 1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2-2)/(b*x**2-1)**(3/4),x)
[Out] Integral(x**2/((b*x**2 - 2)*(b*x**2 - 1)**(3/4)), x)
```

3.851 $\int \frac{x^2}{\left(-2-bx^2\right)\left(-1-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTan
h[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2-bx^2\right)\left(-1-bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}\,b^{3/2}}$$

Mathematica [C] time = 0.06, size = 55, normalized size = 0.74

$$-\frac{x^3\left(bx^2+1\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)}{6\left(-bx^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 + b*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -(b*x^2), -1/2*(b*x^
2)])/(-1 - b*x^2)^(3/4)
```

IntegrateAlgebraic [A] time = 2.20, size = 92, normalized size = 1.24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x\left(-bx^2-1\right)^{3/4}}{\sqrt{2}\left(bx^2+1\right)}\right)}{\sqrt{2}\,b^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x\left(-bx^2-1\right)^{3/4}}{\sqrt{2}\left(bx^2+1\right)}\right)}{\sqrt{2}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*x*(-1 - b*x^2)^(3/4))/(Sqrt[2]*(1 + b*x^2))]/(Sqrt[2]*b^(
3/2))) + ArcTanh[(Sqrt[b]*x*(-1 - b*x^2)^(3/4))/(Sqrt[2]*(1 + b*x^2))]/(Sqr
t[2]*b^(3/2))
```

fricas [B] time = 0.81, size = 274, normalized size = 3.70

$$\left[-\frac{2\,\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(-bx^2-1\right)^{\frac{1}{4}}}{\sqrt{b}\,x}\right)-\sqrt{2}\,\sqrt{b}\,\log\left(-\frac{b^2x^4+4\,\sqrt{-bx^2-1}\,bx^2-4\,bx^2-2\,\sqrt{2}\left(\left(-bx^2-1\right)^{\frac{1}{4}}bx^3+2\left(-bx^2-1\right)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^2x^4+4\,bx^2+4}\right)}{4\,b^2},\ \frac{2\,\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\left(-bx^2-1\right)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{b^2x^4-4\,\sqrt{-bx^2-1}\,bx^2-4\,bx^2-2\,\sqrt{2}\left(\left(-bx^2-1\right)^{\frac{1}{4}}bx^3-2\left(-bx^2-1\right)^{\frac{3}{4}}x\right)\sqrt{-b}-4}{b^2x^4+4\,bx^2+4}\right)}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - s
qrt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(
2)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^
4 + 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)
^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1
)*b*x^2 - 4*b*x^2 - 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3
/4)*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(bx^2+2\right)\left(-bx^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="giac")
[Out] integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(-b\,x^{2}-2\right)\left(-b\,x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
[Out] int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{\left(bx^{2}+2\right)\left(-bx^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="maxima")
[Out] -integrate(x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^{2}}{{\left(-b\,x^{2}-1\right)}^{3/4}\,\left(b\,x^{2}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((- b*x^2 - 1)^(3/4)*(b*x^2 + 2)),x)
[Out] -int(x^2/((- b*x^2 - 1)^(3/4)*(b*x^2 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{bx^{2}\left(-bx^{2}-1\right)^{\frac{3}{4}}+2\left(-bx^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2-2)/(-b*x**2-1)**(3/4),x)
[Out] -Integral(x**2/(b*x**2*(-b*x**2 - 1)**(3/4) + 2*(-b*x**2 - 1)**(3/4)), x)
```

3.852 $\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a+3x^2\right)\left(-a+3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

Mathematica [C] time = 0.11, size = 66, normalized size = 0.78

$$-\frac{x^3\left(1-\frac{3x^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{6a\left(3x^2-a\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - (3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/a, (3*x^
2)/(2*a)])/(a*(-a + 3*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.21, size = 87, normalized size = 1.02

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,\sqrt[4]{a}\,\sqrt[4]{3x^2-a}}{x}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[2/3]*a^(1/4)*(-a + 3*x^2)^(1/4))/x]/(3*Sqrt[6]*a^(1/4))
```

fricas [B] time = 0.84, size = 145, normalized size = 1.71

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x\sqrt{\frac{\frac{3x^2}{\sqrt{a}}+2\sqrt{3x^2-a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(3x^2-a\right)^{\frac{1}{4}}a^{\frac{1}{4}}\right)}{x}\right)}{3a^{\frac{1}{4}}}-\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+\left(3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-\left(3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a^(1/4)*x*sqrt((3*x^2/sq
rt(a) + 2*sqrt(3*x^2 - a))/x^2) - (1/36)^(3/4)*(3*x^2 - a)^(1/4)*a^(1/4))/x
)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (3*x^2 - a)^(1
/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (3*x^2
- a)^(1/4))/x)/a^(1/4)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2-a\right)^{\frac{3}{4}}\left(3\,x^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

maple **[F]** time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3x^2-2a\right)\left(3x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2-a\right)^{\frac{3}{4}}\left(3\,x^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(2\,a-3\,x^2\right)\,{\left(3\,x^2-a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2*a - 3*x^2)*(3*x^2 - a)^(3/4)),x)
```

```
[Out] -int(x^2/((2*a - 3*x^2)*(3*x^2 - a)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-2a+3x^2\right)\left(-a+3x^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2*a)/(3*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + 3*x**2)*(-a + 3*x**2)**(3/4)), x)
```

3.853 $\int \frac{x^2}{\left(-2a-3x^2\right)\left(-a-3x^2\right)^{3/4}} \, dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a-3x^2\right)\left(-a-3x^2\right)^{3/4}} \, dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.79

$$-\frac{x^3 \left(\frac{a+3x^2}{a}\right)^{3/4} F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{6a\left(-a-3x^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*((a + 3*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (-3*x^2)/a, (-3*
x^2)/(2*a)])/(a*(-a - 3*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.21, size = 87, normalized size = 1.02

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\,\sqrt[4]{a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\,\sqrt[4]{a}\,\sqrt[4]{-a-3x^2}}{x}\right)}{3\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - Ar
cTanh[(Sqrt[2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x]/(3*Sqrt[6]*a^(1/4))
```

fricas [B] time = 0.85, size = 145, normalized size = 1.71

$$\frac{2\left(\frac{1}{36}\right)^{\frac{1}{4}}\arctan\left(\frac{12\left(\sqrt{\frac{1}{2}}\left(\frac{1}{36}\right)^{\frac{3}{4}}a^{\frac{1}{4}}x\sqrt{\frac{\frac{3x^2}{\sqrt{a}}+2\sqrt{-3x^2-a}}{x^2}}-\left(\frac{1}{36}\right)^{\frac{3}{4}}\left(-3x^2-a\right)^{\frac{1}{4}}a^{\frac{1}{4}}\right)}{x}\right)}{3a^{\frac{1}{4}}}-\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+\left(-3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}+\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-\left(-3x^2-a\right)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3*(1/36)^(1/4)*arctan(12*(sqrt(1/2)*(1/36)^(3/4)*a^(1/4)*x*sqrt((3*x^2/sq
rt(a) + 2*sqrt(-3*x^2 - a))/x^2) - (1/36)^(3/4)*(-3*x^2 - a)^(1/4)*a^(1/4))
/x)/a^(1/4) - 1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (-3*x^2 - a)
^(1/4))/x)/a^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (-3*
x^2 - a)^(1/4))/x)/a^(1/4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(3\,x^2+2\,a\right)\left(-3\,x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-3x^2-2a\right)\left(-3x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

```
[Out] int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(3\,x^2+2\,a\right)\left(-3\,x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(3\,x^2+2\,a\right)\,\left(-3\,x^2-a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((2*a + 3*x^2)*(- a - 3*x^2)^(3/4)),x)
```

```
[Out] -int(x^2/((2*a + 3*x^2)*(- a - 3*x^2)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^{2}}{2a\left(-a-3x^{2}\right)^{\frac{3}{4}}+3x^{2}\left(-a-3x^{2}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3*x**2-2*a)/(-3*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - 3*x**2)**(3/4) + 3*x**2*(-a - 3*x**2)**(3/4)), x)
```

3.854 $\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx$

Optimal. Leaf size=96

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a+bx^2\right)\left(-a+bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [C] time = 0.07, size = 68, normalized size = 0.71

$$-\frac{x^3\left(1-\frac{bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{6a\left(bx^2-a\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, (b*x^2)/a, (b*x^
2)/(2*a)])/(a*(-a + b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.32, size = 98, normalized size = 1.02

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{bx^2-a}}{\sqrt{b}\,x}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

fricas [B] time = 0.73, size = 207, normalized size = 2.16

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{bx^2-a}}{x^2}}\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(bx^2-a\right)^{\frac{1}{4}}ab^4\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*sqr
t((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(b*x^2 - a))/x^2)*(1/(a*b^6))^(3/4) - (1
/4)^(3/4)*(b*x^2 - a)^(1/4)*a*b^4*(1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1/4)*(
1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (b*x^2 - a)^(1/
4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*(1/(a*b^
6))^(1/4) - (b*x^2 - a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2-a\right)^{\frac{3}{4}}\left(bx^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="giac")
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^2-2a\right)\left(b\,x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
[Out] int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^2-a\right)^{\frac{3}{4}}\left(bx^2-2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(b\,x^2-a\right)^{3/4}\,\left(2\,a-b\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((b*x^2 - a)^(3/4)*(2*a - b*x^2)),x)
[Out] -int(x^2/((b*x^2 - a)^(3/4)*(2*a - b*x^2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-2a+bx^2\right)\left(-a+bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**2-2*a)/(b*x**2-a)**(3/4),x)
```

```
[Out] Integral(x**2/((-2*a + b*x**2)*(-a + b*x**2)**(3/4)), x)
```

3.855 $\int \frac{x^2}{\left(-2a-bx^2\right)\left(-a-bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=98

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Rubi [**A**] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2a-bx^2\right)\left(-a-bx^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\,\sqrt[4]{a}\,b^{3/2}}$$

Mathematica [C] time = 0.06, size = 70, normalized size = 0.71

$$-\frac{x^3\left(\frac{a+bx^2}{a}\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{6a\left(-a-bx^2\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*((a + b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -1
/2*(b*x^2)/a])/(a*(-a - b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 2.32, size = 100, normalized size = 1.02

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*b
^(3/2)) - ArcTanh[(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)]/(Sqrt[2
]*a^(1/4)*b^(3/2))
```

fricas [B] time = 0.96, size = 211, normalized size = 2.15

$$2\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\arctan\left(\frac{4\left(\sqrt{\frac{1}{2}}\left(\frac{1}{4}\right)^{\frac{3}{4}}ab^4x\sqrt{\frac{b^4x^2\sqrt{\frac{1}{ab^6}}+2\sqrt{-bx^2-a}}{x^2}}\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}-\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(-bx^2-a\right)^{\frac{1}{4}}ab^4\left(\frac{1}{ab^6}\right)^{\frac{3}{4}}\right)}{x}\right)-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(-bx^2-a\right)^{\frac{1}{4}}}{x}\right)+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(-bx^2-a\right)^{\frac{1}{4}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*arctan(4*(sqrt(1/2)*(1/4)^(3/4)*a*b^4*x*sqr
t((b^4*x^2*sqrt(1/(a*b^6)) + 2*sqrt(-b*x^2 - a))/x^2)*(1/(a*b^6))^(3/4) - (
1/4)^(3/4)*(-b*x^2 - a)^(1/4)*a*b^4*(1/(a*b^6))^(3/4))/x) - 1/2*(1/4)^(1/4)
*(1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (-b*x^2 - a)^
(1/4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^(1/4)*b^2*x*(1/(a
*b^6))^(1/4) - (-b*x^2 - a)^(1/4))/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(bx^2+2\,a\right)\left(-bx^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="giac")
[Out] integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-b\,x^2-2a\right)\left(-b\,x^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
[Out] int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(bx^2+2\,a\right)\left(-bx^2-a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="maxima")
[Out] -integrate(x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(-b\,x^2-a\right)^{3/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((- a - b*x^2)^(3/4)*(2*a + b*x^2)),x)
[Out] -int(x^2/((- a - b*x^2)^(3/4)*(2*a + b*x^2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{2a\left(-a-bx^2\right)^{\frac{3}{4}}+bx^2\left(-a-bx^2\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-b*x**2-2*a)/(-b*x**2-a)**(3/4),x)
```

```
[Out] -Integral(x**2/(2*a*(-a - b*x**2)**(3/4) + b*x**2*(-a - b*x**2)**(3/4)), x)
```

3.856 $\int \frac{x^7}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 63, 212, 206, 203}

$$\frac{2}{729}\left(3x^2-1\right)^{9/4}+\frac{8}{405}\left(3x^2-1\right)^{5/4}+\frac{14}{81}\sqrt[4]{3x^2-1}-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (14*(-1 + 3*x^2)^(1/4))/81 + (8*(-1 + 3*x^2)^(5/4))/405 + (2*(-1 + 3*x^2)^(
9/4))/729 - (8*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(1/
4)])/81
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{7}{27(-1+3x)^{3/4}} + \frac{8}{27(-2+3x)(-1+3x)^{3/4}} + \frac{4}{27}\sqrt[4]{-1+3x} + \frac{1}{27}\right.\right.\\
&= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} + \frac{4}{27}\,\text{Subst}\left(\int \frac{}{(-2+}\right.\\
&= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} + \frac{16}{81}\,\text{Subst}\left(\int \frac{1}{-1+}\right.\\
&= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} - \frac{8}{81}\,\text{Subst}\left(\int \frac{1}{1-x}\right.\\
&= \frac{14}{81}\sqrt[4]{-1+3x^2} + \frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4} - \frac{8}{81}\tan^{-1}\left(\sqrt[4]{-1+3}\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.74

$$\frac{-360\tan^{-1}\left(\sqrt[4]{3x^2-1}\right) - 360\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right) + 2\sqrt[4]{3x^2-1}\left(45x^4+78x^2+284\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(284 + 78*x^2 + 45*x^4) - 360*ArcTan[(-1 + 3*x^2)^(1/
4)] - 360*ArcTanh[(-1 + 3*x^2)^(1/4)])/3645
```

IntegrateAlgebraic [A] time = 0.05, size = 60, normalized size = 0.77

$$-\frac{8}{81}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{81}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{2\sqrt[4]{3x^2-1}\left(45x^4+78x^2+284\right)}{3645}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(284 + 78*x^2 + 45*x^4))/3645 - (8*ArcTan[(-1 + 3*x^2
)^(1/4)])/81 - (8*ArcTanh[(-1 + 3*x^2)^(1/4)])/81
```

fricas [A] time = 0.80, size = 64, normalized size = 0.82

$$\frac{2}{3645}\left(45\,x^4+78\,x^2+284\right)\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/3645*(45*x^4 + 78*x^2 + 284)*(3*x^2 - 1)^(1/4) - 8/81*arctan((3*x^2 - 1)^
(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.28, size = 75, normalized size = 0.96

$$\frac{2}{729}\left(3\,x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3\,x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4)
 - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*
log(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 0.86, size = 424, normalized size = 5.44

$$\frac{2\left(45x^4+78x^2+284\right)\left(3x^2-1\right)^{\frac{1}{4}}}{3645}+\frac{\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-27x^6-18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^2+1\right)+18x^4+12\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+1\right)+6\sqrt{27x^6-27x^4+9x^2-1}\,x^2-3x^2+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\sqrt{27x^6-27x^4+9x^2-1}}{\left(3x^2-2\right)\left(3x^2-1\right)^2}\right)}{81}-\frac{4\ln\left(-\frac{27x^6+18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4-18x^4+6\sqrt{27x^6-27x^4+9x^2-1}\,x^2-12\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^2+3x^2+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{3}{4}}-2\sqrt{27x^6-27x^4+9x^2-1}+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}}{\left(3x^2-2\right)\left(3x^2-1\right)^2}\right)}{81}\right)\left(\left(3x^2-1\right)^3\right)^{\frac{1}{4}}}{\left(3x^2-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 2/3645*(45*x^4+78*x^2+284)*(3*x^2-1)^(1/4)+(-4/81*ln(-(27*x^6+18*(27*x^6-27
*x^4+9*x^2-1)^(1/4)*x^4+6*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^2-18*x^4+2*(27*x^
6-27*x^4+9*x^2-1)^(3/4)-12*(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^2-2*(27*x^6-27*x
^4+9*x^2-1)^(1/2)+3*x^2+2*(27*x^6-27*x^4+9*x^2-1)^(1/4))/(3*x^2-2)/(3*x^2-1
)^2)-4/81*RootOf(_Z^2+1)*ln((-18*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(1/
4)*x^4-27*x^6+2*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(3/4)+6*(27*x^6-27*x
^4+9*x^2-1)^(1/2)*x^2+12*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^2+1
8*x^4-2*(27*x^6-27*x^4+9*x^2-1)^(1/2)-2*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2
-1)^(1/4)-3*x^2)/(3*x^2-2)/(3*x^2-1)^2))/(3*x^2-1)^(3/4)*((3*x^2-1)^3)^(1/4
)
```

maxima [A] time = 2.02, size = 74, normalized size = 0.95

$$\frac{2}{729}\left(3x^2-1\right)^{\frac{9}{4}}+\frac{8}{405}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{14}{81}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4)
 - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*
log((3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.11, size = 62, normalized size = 0.79

$$\frac{14\left(3x^2-1\right)^{1/4}}{81}-\frac{8\,\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{81}+\frac{8\left(3x^2-1\right)^{5/4}}{405}+\frac{2\left(3x^2-1\right)^{9/4}}{729}+\frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,8\mathrm{i}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*8i)/81 - (8*atan((3*x^2 - 1)^(1/4)))/81 + (14*(
3*x^2 - 1)^(1/4))/81 + (8*(3*x^2 - 1)^(5/4))/405 + (2*(3*x^2 - 1)^(9/4))/72
9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^7}{\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**7/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.857 $\int \frac{x^5}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 63, 212, 206, 203}

$$\frac{2}{135}\left(3x^2-1\right)^{5/4}+\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 + (2*(-1 + 3*x^2)^(5/4))/135 - (4*ArcTan[(-1 + 3*x
^2)^(1/4)])/27 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\Bigg(\int \left(\frac{1}{3(-1+3x)^{3/4}}+\frac{4}{9(-2+3x)(-1+3x)^{3/4}}+\frac{1}{9}\sqrt[4]{-1+3x}\right)dx, x, x\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}+\frac{2}{9}\operatorname{Subst}\Bigg(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x,\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}+\frac{8}{27}\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}-\frac{4}{27}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)-\\
&= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{2}{135}\left(-1+3x^2\right)^{5/4}-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\tanh^{-1}\Big(\sqrt[4]{-
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.84

$$\frac{1}{135}\left(2\sqrt[4]{3x^2-1}\left(3x^2+14\right)-20\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-20\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(14 + 3*x^2) - 20*ArcTan[(-1 + 3*x^2)^(1/4)] - 20*Arc
Tanh[(-1 + 3*x^2)^(1/4)])/135
```

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.87

$$\frac{2}{135}\sqrt[4]{3x^2-1}\left(3x^2+14\right)-\frac{4}{27}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{4}{27}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4)*(14 + 3*x^2))/135 - (4*ArcTan[(-1 + 3*x^2)^(1/4)])/27
 - (4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

fricas [A] time = 0.94, size = 59, normalized size = 0.94

$$\frac{2}{135}\left(3x^2+14\right)\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/135*(3*x^2 + 14)*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/2
7*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.44, size = 64, normalized size = 1.02

$$\frac{2}{135}\left(3x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(
1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4) -
1))
```

maple [C] time = 0.83, size = 420, normalized size = 6.67

$$\frac{2\left(3x^2+14\right)\left(3x^2-1\right)^{\frac{1}{4}}}{135}+\frac{\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{27x^6-18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^2+1\right)-18x^4+12\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+1\right)-6\sqrt{27x^6-27x^4+9x^2-1}\,x^2+3x^2+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)+2\sqrt{27x^6-27x^4+9x^2-1}}{\left(3x^2-2\right)\left(3x^2-1\right)^2}\right)}{27}-\frac{2\ln\left(-\frac{27x^6+18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4-18x^4+6\sqrt{27x^6-27x^4+9x^2-1}\,x^2-12\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^2+3x^2+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{3}{4}}-2\sqrt{27x^6-27x^4+9x^2-1}+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}}{\left(3x^2-2\right)\left(3x^2-1\right)^2}\right)}{27}\right)\left(\left(3x^2-1\right)^3\right)^{\frac{1}{4}}}{\left(3x^2-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 2/135*(3*x^2+14)*(3*x^2-1)^(1/4)+(-2/27*ln(-(27*x^6+18*(27*x^6-27*x^4+9*x^2
-1)^(1/4)*x^4-18*x^4+6*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^2-12*(27*x^6-27*x^4+
9*x^2-1)^(1/4)*x^2+3*x^2+2*(27*x^6-27*x^4+9*x^2-1)^(3/4)-2*(27*x^6-27*x^4+9
*x^2-1)^(1/2)+2*(27*x^6-27*x^4+9*x^2-1)^(1/4))/(3*x^2-2)/(3*x^2-1)^2)+2/27*
RootOf(_Z^2+1)*ln(-(-18*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^4+27
*x^6+2*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(3/4)-6*(27*x^6-27*x^4+9*x^2-
1)^(1/2)*x^2+12*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^2-18*x^4+2*(
27*x^6-27*x^4+9*x^2-1)^(1/2)-2*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(1/4)
+3*x^2)/(3*x^2-2)/(3*x^2-1)^2))/(3*x^2-1)^(3/4)*((3*x^2-1)^3)^(1/4)
```

maxima [A] time = 1.95, size = 63, normalized size = 1.00

$$\frac{2}{135}\left(3\,x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}}-\frac{4}{27}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(
1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.12, size = 51, normalized size = 0.81

$$\frac{2\,{\left(3\,x^2-1\right)}^{1/4}}{9}-\frac{4\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{27}+\frac{2\,{\left(3\,x^2-1\right)}^{5/4}}{135}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,4{}\mathrm{i}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*4i)/27 - (4*atan((3*x^2 - 1)^(1/4)))/27 + (2*(3
*x^2 - 1)^(1/4))/9 + (2*(3*x^2 - 1)^(5/4))/135
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(3x^{2}-2\right)\left(3x^{2}-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**5/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.858 $\int \frac{x^3}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=48

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 63, 212, 206, 203}

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(-
1 + 3*x^2)^(1/4)])/9
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\ &= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)\\ &= \frac{2}{9}\sqrt[4]{-1+3x^2}+\frac{4}{9}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+3x^2}\right)\\ &= \frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\text{Subst}\Bigg(\int \frac{1}{1+x^2}\,d\\ &= \frac{2}{9}\sqrt[4]{-1+3x^2}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.92

$$\frac{2}{9}\left(\sqrt[4]{3x^2-1}-\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*((-1 + 3*x^2)^(1/4) - ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^
(1/4)]))/9
```

IntegrateAlgebraic [A] time = 0.04, size = 48, normalized size = 1.00

$$\frac{2}{9}\sqrt[4]{3x^2-1}-\frac{2}{9}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{2}{9}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (2*(-1 + 3*x^2)^(1/4))/9 - (2*ArcTan[(-1 + 3*x^2)^(1/4)])/9 - (2*ArcTanh[(-
1 + 3*x^2)^(1/4)])/9
```

fricas [A] time = 0.89, size = 52, normalized size = 1.08

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.41, size = 53, normalized size = 1.10

$$\frac{2}{9}\left(3x^2-1\right)^{\frac{1}{4}}-\frac{2}{9}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 0.76, size = 412, normalized size = 8.58

$$\frac{2\left(3x^2-1\right)^{\frac{1}{4}}}{9}+\frac{\left(\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{27x^6-18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4\,\mathrm{RootOf}\left(\_Z^2+1\right)-18x^4+12\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+1\right)-6\sqrt{27x^6-27x^4+9x^2-1}\,x^2+3x^2+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)-2\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+1\right)+2\sqrt{27x^6-27x^4+9x^2-1}}{\left(3x^2-2\right)\left(3x^2-1\right)^2}\right)}{9}+\frac{\ln\left(\frac{-27x^6+18\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^4+18x^4-6\sqrt{27x^6-27x^4+9x^2-1}\,x^2-12\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}x^2-3x^2+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{3}{4}}+2\sqrt{27x^6-27x^4+9x^2-1}+2\left(27x^6-27x^4+9x^2-1\right)^{\frac{1}{4}}}{\left(3x^2-2\right)\left(3x^2-1\right)^2}\right)}{9}\right)\left(\left(3x^2-1\right)^3\right)^{\frac{1}{4}}}{\left(3x^2-1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 2/9*(3*x^2-1)^(1/4)+(1/9*RootOf(_Z^2+1)*ln(-(-18*RootOf(_Z^2+1)*(27*x^6-27*
x^4+9*x^2-1)^(1/4)*x^4+27*x^6+2*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^2-1)^(3/4
)-6*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^2+12*RootOf(_Z^2+1)*(27*x^6-27*x^4+9*x^
2-1)^(1/4)*x^2-18*x^4+2*(27*x^6-27*x^4+9*x^2-1)^(1/2)-2*RootOf(_Z^2+1)*(27*
x^6-27*x^4+9*x^2-1)^(1/4)+3*x^2)/(3*x^2-2)/(3*x^2-1)^2)+1/9*ln((-27*x^6+18*
(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^4-6*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^2+18*x^
4+2*(27*x^6-27*x^4+9*x^2-1)^(3/4)-12*(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^2+2*(2
7*x^6-27*x^4+9*x^2-1)^(1/2)-3*x^2+2*(27*x^6-27*x^4+9*x^2-1)^(1/4))/(3*x^2-2
)/(3*x^2-1)^2))/(3*x^2-1)^(3/4)*((3*x^2-1)^3)^(1/4)
```

maxima [A] time = 1.81, size = 52, normalized size = 1.08

$$\frac{2}{9}\left(3\,x^{2}-1\right)^{\frac{1}{4}}-\frac{2}{9}\,\arctan\left(\left(3\,x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\,\log\left(\left(3\,x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\,\log\left(\left(3\,x^{2}-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1)
^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.12, size = 36, normalized size = 0.75

$$\frac{2\left(3\,x^{2}-1\right)^{1/4}}{9}-\frac{2\,\mathrm{atanh}\left(\left(3\,x^{2}-1\right)^{1/4}\right)}{9}-\frac{2\,\mathrm{atan}\left(\left(3\,x^{2}-1\right)^{1/4}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (2*(3*x^2 - 1)^(1/4))/9 - (2*atanh((3*x^2 - 1)^(1/4)))/9 - (2*atan((3*x^2 -
 1)^(1/4)))/9
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(3x^{2}-2\right)\left(3x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(x**3/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.859 $\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 63, 212, 206, 203}

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\,dx,x,x^2\right)\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt[4]{-1+3x^2}\right)\\ &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[4]{-1+3x^2}\right)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt[4]{-1+\right.\right.\\ &= -\frac{1}{3}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.00

$$-\frac{1}{3}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{3}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/3*ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

fricas [A] time = 0.74, size = 41, normalized size = 1.24

$$-\frac{1}{3}\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.42, size = 42, normalized size = 1.27

$$-\frac{1}{3}\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^{2}-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 0.53, size = 125, normalized size = 3.79

$$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{3x^{2}+2\left(3x^{2}-1\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^{2}+1\right)-2\left(3x^{2}-1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^{2}+1\right)-2\sqrt{3x^{2}-1}}{3x^{2}-2}\right)}{6}+\frac{\ln\left(\frac{-3x^{2}+2\left(3x^{2}-1\right)^{\frac{3}{4}}-2\sqrt{3x^{2}-1}+2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 1/6*ln((-3*x^2+2*(3*x^2-1)^(3/4)-2*(3*x^2-1)^(1/2)+2*(3*x^2-1)^(1/4))/(3*x^
2-2))-1/6*RootOf(_Z^2+1)*ln(-(2*RootOf(_Z^2+1)*(3*x^2-1)^(3/4)-2*RootOf(_Z^
2+1)*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)+3*x^2)/(3*x^2-2))
```

maxima [A] time = 1.85, size = 41, normalized size = 1.24

$$-\frac{1}{3}\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

mupad [B] time = 0.87, size = 25, normalized size = 0.76

$$-\frac{\operatorname{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{3}-\frac{\operatorname{atanh}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] - atan((3*x^2 - 1)^(1/4))/3 - atanh((3*x^2 - 1)^(1/4))/3
```

sympy [A] time = 9.73, size = 42, normalized size = 1.27

$$\frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6}-\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 - atan((3*x
**2 - 1)**(1/4))/3
```

3.860 $\int \frac{1}{x\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\,\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\,\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{}\right.}{2\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {446, 86, 63, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$\frac{\log\left(\sqrt{3x^2-1}-\sqrt{2}\,\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{\log\left(\sqrt{3x^2-1}+\sqrt{2}\,\sqrt[4]{3x^2-1}+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{3x^2-1}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\,\sqrt[4]{3x^2-1}+1\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*S
qrt[2]) - ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1
+ 3*x^2)^(1/4)]/2 + Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/
(4*Sqrt[2]) - Log[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]/(4*Sqr
t[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$

$$= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(-1+3x)^{3/4}}\,dx, x, x^2\right)\right) + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{(-2+3x)(-1+3x)^{3/4}}\right.$$

$$= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) + \operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt[4]{-1+}\right.$$

$$= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right)\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1+x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+}\right.$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x}\right.$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{2}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{\log\left(1-\sqrt{2}\,\sqrt[4]{-1+3x^2}\right. +}{4\sqrt{2}}$$

$$= -\frac{1}{2}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\,\sqrt[4]{-1+}\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 150, normalized size = 0.87

$$\frac{1}{8}\left(-4\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-4\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)+\sqrt{2}\left(\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)-\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)+2\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)-2\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] (-4*ArcTan[(-1 + 3*x^2)^(1/4)] - 4*ArcTanh[(-1 + 3*x^2)^(1/4)] + Sqrt[2]*(2
*ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)] - 2*ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)
^(1/4)] + Log[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]] - Log[1 +
Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]]))/8
```

IntegrateAlgebraic [A] time = 0.13, size = 122, normalized size = 0.71

$$-\frac{1}{2}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{\tan^{-1}\left(\frac{\frac{\sqrt{3x^2-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3x^2-1}}{\sqrt{3x^2-1}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/2*ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTan[(-(1/Sqrt[2]) + Sqrt[-1 + 3*x^2]/S
qrt[2])/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - ArcTanh[(-1 + 3*x^2)^(1/4)]/2 - A
rcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt[-1 + 3*x^2])]/(2*Sqrt[2])
```

fricas [A] time = 0.82, size = 215, normalized size = 1.24

$$\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+1\right)-\frac{1}{8}\sqrt{2}\log\left(4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+\frac{1}{8}\sqrt{2}\log\left(-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1)
 + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqr
t(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(2)*(3*x^2 -
1)^(1/4) + 1) - 1/8*sqrt(2)*log(4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2
- 1) + 4) + 1/8*sqrt(2)*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1
) + 4) - 1/2*arctan((3*x^2 - 1)^(1/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1
/4*log((3*x^2 - 1)^(1/4) - 1)
```

giac [A] time = 0.41, size = 155, normalized size = 0.90

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 1/8*sqrt(2)*log(
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt(2)
*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/2*arctan((3*x^2 - 1)^(1/4)) -
 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 3.50, size = 299, normalized size = 1.73

$$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3 \ln\left(\frac{-3x^2\,\mathrm{RootOf}\left(\_Z^4+1\right)+2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3+2\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2+2\,\mathrm{RootOf}\left(\_Z^4+1\right)-2\left(3x^2-1\right)^{\frac{3}{4}}}{x^2}\right)}{4}+\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^2 \ln\left(\frac{-3x^2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2+2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^2-2\left(3x^2-1\right)^{\frac{3}{4}}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{4}-\frac{\mathrm{RootOf}\left(\_Z^4+1\right) \ln\left(\frac{3x^2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-2\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^4+1\right)^2-2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)-2\left(3x^2-1\right)^{\frac{3}{4}}}{x^2}\right)}{4}-\frac{\ln\left(-\frac{3x^2+2\left(3x^2-1\right)^{\frac{3}{4}}+2\sqrt{3x^2-1}+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 1/4*RootOf(_Z^4+1)^3*ln((2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)^3+2*RootOf(_Z^4+1
)^2*(3*x^2-1)^(1/4)-3*RootOf(_Z^4+1)*x^2-2*(3*x^2-1)^(3/4)+2*RootOf(_Z^4+1)
)/x^2)-1/4*RootOf(_Z^4+1)*ln((3*RootOf(_Z^4+1)^3*x^2-2*RootOf(_Z^4+1)^3-2*R
ootOf(_Z^4+1)^2*(3*x^2-1)^(1/4)-2*(3*x^2-1)^(1/2)*RootOf(_Z^4+1)-2*(3*x^2-1
)^(3/4))/x^2)-1/4*ln(-(3*x^2+2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/2)+2*(3*x^2-1
)^(1/4))/(3*x^2-2))+1/4*RootOf(_Z^4+1)^2*ln((2*(3*x^2-1)^(1/2)*RootOf(_Z^4+
1)^2-3*RootOf(_Z^4+1)^2*x^2-2*(3*x^2-1)^(3/4)+2*(3*x^2-1)^(1/4))/(3*x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(3\,x^2-1\right)}^{\frac{3}{4}}\left(3\,x^2-2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x), x)
```

mupad [B] time = 0.91, size = 77, normalized size = 0.45

$$-\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{2}+\frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}-\frac{1}{4}{}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,{\left(3\,x^2-1\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{1}{4}+\frac{1}{4}{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*1i)/2 - atan((3*x^2 - 1)^(1/4))/2 - 2^(1/2)*ata
n(2^(1/2)*(3*x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 + 1i/4) - 2^(1/2)*atan(2^(1/
2)*(3*x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 - 1i/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.861 $\int \frac{1}{x^3\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Rubi [A] time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {446, 103, 156, 63, 211, 1165, 628, 1162, 617, 204, 212, 206, 203}

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}+\frac{15\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{15\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{16\sqrt{2}}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+\frac{15\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{8\sqrt{2}}-\frac{15\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -(-1 + 3*x^2)^(1/4)/(4*x^2) - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 + (15*ArcTan
[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2]) - (15*ArcTan[1 + Sqrt[2]*(-1
+ 3*x^2)^(1/4)])/(8*Sqrt[2]) - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 + (15*Log[
1 - Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2]) - (15*Log[
1 + Sqrt[2]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/(16*Sqrt[2])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^3\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^2(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{-\frac{15}{2}+\frac{27x}{4}}{x(-2+3x)(-1+3x)^{3/4}}\,dx, x, x^2\right)$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{15}{16}\,\mathrm{Subst}\left(\int \frac{1}{x(-1+3x)^{3/4}}\,dx, x, x^2\right) + \frac{9}{8}\,\mathrm{Subst}\Big(\int \frac{}{(-2+}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) + \frac{3}{2}\,\mathrm{Subst}\Big(\int \frac{1}{-1+}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{5}{8}\,\mathrm{Subst}\left(\int \frac{1-x^2}{\frac{1}{3}+\frac{x^4}{3}}\,dx, x, \sqrt[4]{-1+3x^2}\right) - \frac{5}{8}\,\mathrm{Subst}\Big(\int \frac{1+x}{\frac{1}{3}+\frac{x}{3}}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{15}{16}\,\mathrm{Subs}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) - \frac{3}{4}\tanh^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{15\log(1}{}$$

$$= -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\tan^{-1}\left(\sqrt[4]{-1+3x^2}\right) + \frac{15\tan^{-1}\left(1-\sqrt{2}\,\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}} - \frac{15}{}$$

Mathematica [A] time = 0.05, size = 181, normalized size = 0.95

$$\frac{1}{32}\left(-\frac{8\sqrt[4]{3x^2-1}}{x^2}+15\sqrt{2}\log\left(\sqrt{3x^2-1}-\sqrt{2}\sqrt[4]{3x^2-1}+1\right)-15\sqrt{2}\log\left(\sqrt{3x^2-1}+\sqrt{2}\sqrt[4]{3x^2-1}+1\right)-24\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)+30\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)-30\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)-24\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ((-8*(-1 + 3*x^2)^(1/4))/x^2 - 24*ArcTan[(-1 + 3*x^2)^(1/4)] + 30*Sqrt[2]*A
rcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)] - 30*Sqrt[2]*ArcTan[1 + Sqrt[2]*(-1 +
 3*x^2)^(1/4)] - 24*ArcTanh[(-1 + 3*x^2)^(1/4)] + 15*Sqrt[2]*Log[1 - Sqrt[2
]*(-1 + 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]] - 15*Sqrt[2]*Log[1 + Sqrt[2]*(-1 +
 3*x^2)^(1/4) + Sqrt[-1 + 3*x^2]])/32
```

IntegrateAlgebraic [A] time = 0.24, size = 140, normalized size = 0.73

$$-\frac{\sqrt[4]{3x^2-1}}{4x^2}-\frac{3}{4}\tan^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{15\tan^{-1}\left(\frac{\frac{\sqrt{3x^2-1}}{\sqrt{2}}-\frac{1}{\sqrt{2}}}{\sqrt[4]{3x^2-1}}\right)}{8\sqrt{2}}-\frac{3}{4}\tanh^{-1}\left(\sqrt[4]{3x^2-1}\right)-\frac{15\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{3x^2-1}}{\sqrt{3x^2-1}+1}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/4*(-1 + 3*x^2)^(1/4)/x^2 - (3*ArcTan[(-1 + 3*x^2)^(1/4)])/4 - (15*ArcTan
[(-(1/Sqrt[2]) + Sqrt[-1 + 3*x^2]/Sqrt[2])/(-1 + 3*x^2)^(1/4)])/(8*Sqrt[2])
 - (3*ArcTanh[(-1 + 3*x^2)^(1/4)])/4 - (15*ArcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1
/4))/(1 + Sqrt[-1 + 3*x^2])])/(8*Sqrt[2])
```

fricas [A] time = 0.84, size = 252, normalized size = 1.32

$$\frac{60\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}-1\right)+60\sqrt{2}x^2\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4}-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+1\right)-15\sqrt{2}x^2\log\left(4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)+15\sqrt{2}x^2\log\left(-4\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+4\sqrt{3x^2-1}+4\right)-24x^2\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-12x^2\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+12x^2\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)-8\left(3x^2-1\right)^{\frac{1}{4}}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/32*(60*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3
*x^2 - 1) + 1) - sqrt(2)*(3*x^2 - 1)^(1/4) - 1) + 60*sqrt(2)*x^2*arctan(1/2
*sqrt(2)*sqrt(-4*sqrt(2)*(3*x^2 - 1)^(1/4) + 4*sqrt(3*x^2 - 1) + 4) - sqrt(
2)*(3*x^2 - 1)^(1/4) + 1) - 15*sqrt(2)*x^2*log(4*sqrt(2)*(3*x^2 - 1)^(1/4)
+ 4*sqrt(3*x^2 - 1) + 4) + 15*sqrt(2)*x^2*log(-4*sqrt(2)*(3*x^2 - 1)^(1/4)
+ 4*sqrt(3*x^2 - 1) + 4) - 24*x^2*arctan((3*x^2 - 1)^(1/4)) - 12*x^2*log((3
*x^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(1
/4))/x^2
```

giac [A] time = 0.50, size = 169, normalized size = 0.88

$$-\frac{15}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{15}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)-\frac{15}{32}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)+\frac{15}{32}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{\left(3x^2-1\right)^{\frac{1}{4}}}{4x^2}-\frac{3}{4}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{3}{8}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{3}{8}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] -15/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 15/16*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 15/32*sqrt(2
)*log(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 15/32*sqrt(2)*log(
-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(1/4)/x
^2 - 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*l
og(abs((3*x^2 - 1)^(1/4) - 1))
```

maple [C] time = 5.96, size = 916, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] -1/4*(3*x^2-1)^(1/4)/x^2+(15/16*RootOf(_Z^4+1)^3*ln((27*x^6*RootOf(_Z^4+1)^
3-36*RootOf(_Z^4+1)^3*x^4+18*RootOf(_Z^4+1)^2*(27*x^6-27*x^4+9*x^2-1)^(1/4)
*x^4+15*RootOf(_Z^4+1)^3*x^2-12*RootOf(_Z^4+1)^2*(27*x^6-27*x^4+9*x^2-1)^(1
/4)*x^2+6*RootOf(_Z^4+1)*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^2-2*RootOf(_Z^4+1)
^3+2*RootOf(_Z^4+1)^2*(27*x^6-27*x^4+9*x^2-1)^(1/4)-2*(27*x^6-27*x^4+9*x^2-
1)^(1/2)*RootOf(_Z^4+1)+2*(27*x^6-27*x^4+9*x^2-1)^(3/4))/x^2/(3*x^2-1)^2)+1
5/16*RootOf(_Z^4+1)*ln((6*RootOf(_Z^4+1)^3*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^
2-18*RootOf(_Z^4+1)^2*(27*x^6-27*x^4+9*x^2-1)^(1/4)*x^4+27*x^6*RootOf(_Z^4+
1)-2*(27*x^6-27*x^4+9*x^2-1)^(1/2)*RootOf(_Z^4+1)^3+12*RootOf(_Z^4+1)^2*(27
*x^6-27*x^4+9*x^2-1)^(1/4)*x^2-36*RootOf(_Z^4+1)*x^4-2*RootOf(_Z^4+1)^2*(27
*x^6-27*x^4+9*x^2-1)^(1/4)+15*RootOf(_Z^4+1)*x^2+2*(27*x^6-27*x^4+9*x^2-1)^
(3/4)-2*RootOf(_Z^4+1))/x^2/(3*x^2-1)^2)+3/8*ln((-27*x^6+18*(27*x^6-27*x^4+
9*x^2-1)^(1/4)*x^4+18*x^4-6*(27*x^6-27*x^4+9*x^2-1)^(1/2)*x^2-12*(27*x^6-27
*x^4+9*x^2-1)^(1/4)*x^2-3*x^2+2*(27*x^6-27*x^4+9*x^2-1)^(3/4)+2*(27*x^6-27*
x^4+9*x^2-1)^(1/2)+2*(27*x^6-27*x^4+9*x^2-1)^(1/4))/(3*x^2-2)/(3*x^2-1)^2)+
3/8*RootOf(_Z^4+1)^2*ln((-27*RootOf(_Z^4+1)^2*x^6+6*RootOf(_Z^4+1)^2*(27*x^
6-27*x^4+9*x^2-1)^(1/2)*x^2+18*RootOf(_Z^4+1)^2*x^4-18*(27*x^6-27*x^4+9*x^2
-1)^(1/4)*x^4-2*RootOf(_Z^4+1)^2*(27*x^6-27*x^4+9*x^2-1)^(1/2)-3*RootOf(_Z^
4+1)^2*x^2+2*(27*x^6-27*x^4+9*x^2-1)^(3/4)+12*(27*x^6-27*x^4+9*x^2-1)^(1/4)
*x^2-2*(27*x^6-27*x^4+9*x^2-1)^(1/4))/(3*x^2-1)^2/(3*x^2-2)))/(3*x^2-1)^(3/
4)*((3*x^2-1)^3)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3), x)
```

mupad [B] time = 0.21, size = 81, normalized size = 0.42

$$-\frac{3\,\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\right)}{4}+\frac{\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{4}-\frac{\left(3\,x^2-1\right)^{1/4}}{4\,x^2}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\left(3\,x^2-1\right)^{1/4}\right)\,15{}\mathrm{i}}{8}-\frac{{\left(-1\right)}^{3/4}\,\mathrm{atan}\left({\left(-1\right)}^{3/4}\,\left(3\,x^2-1\right)^{1/4}\right)\,15{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] (atan((3*x^2 - 1)^(1/4)*1i)*3i)/4 - (3*atan((3*x^2 - 1)^(1/4)))/4 - (3*x^2
- 1)^(1/4)/(4*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*(3*x^2 - 1)^(1/4))*15i)/8
- ((-1)^(3/4)*atan((-1)^(3/4)*(3*x^2 - 1)^(1/4))*15i)/8
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(3x^2-2\right) \left(3x^2-1\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

```
[Out] Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.862 $\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {442}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

Rule 442

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> -Simp[(b*ArcTan[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))])/(Sqrt[2
]*a*d*Rt[-(b^2/a), 4]^3), x] + Simp[(b*ArcTanh[(Rt[-(b^2/a), 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))])/(Sqrt[2]*a*d*Rt[-(b^2/a), 4]^3), x] /; FreeQ[{a, b, c
, d}, x] && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

Rubi steps

$$\int \frac{x^2}{\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.85

$$-\frac{x^3\left(1-3x^2\right)^{3/4}F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)}{6\left(3x^2-1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] -1/6*(x^3*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^2, (3*x^2)/2])/(
-1 + 3*x^2)^(3/4)
```

IntegrateAlgebraic [A] time = 0.00, size = 61, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\,x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

```
[Out] ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x
)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

fricas [B] time = 0.65, size = 104, normalized size = 1.70

$$-\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}\,x^2-4\sqrt{6}\left(3x^2-1\right)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6
)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3x^2-1\right)^{\frac{3}{4}}\left(3x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

maple [C] time = 0.00, size = 137, normalized size = 2.25

$$-\frac{\mathrm{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{3\sqrt{3x^2-1}\,x+3x+\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)+\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-6\right)}{3x^2-2}\right)}{18}+\frac{\mathrm{RootOf}\left(\_Z^2+6\right)\ln\left(\frac{3\sqrt{3x^2-1}\,x-3x+\left(3x^2-1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)-\left(3x^2-1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+6\right)}{3x^2-2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

```
[Out] 1/18*RootOf(_Z^2+6)*ln((RootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x-
RootOf(_Z^2+6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))-1/18*RootOf(_Z^2-6)*ln((Root
Of(_Z^2-6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1)^(1/
4)+3*x)/(3*x^2-2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{\left(3\,x^2-1\right)}^{3/4}\,\left(3\,x^2-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

```
[Out] int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(3x^{2}-2\right)\left(3x^{2}-1\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
[Out] Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.863 $\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=173

$$\frac{3ae^{5/2}(8bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}\,(8bc-7ad)}{16b^2}+d(ex$$

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 321, 329, 331, 298, 205, 208}

$$\frac{3ae^{5/2}(8bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3ae^{5/2}(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}+\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}\,(8bc-7ad)}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((8*b*c - 7*a*d)*e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(16*b^2) + (d*(e*x)^(7/2)
*(a + b*x^2)^(1/4))/(4*b*e) + (3*a*(8*b*c - 7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*
Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^(11/4)) - (3*a*(8*b*c - 7*a*
d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(32*b^
(11/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(-4bc+\frac{7ad}{2}\right)\int \frac{(ex)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{4b}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(3a(8bc-7ad)e^2\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{32b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3}}\right.}{16b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{(3a(8bc-7ad)e)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,d\right.}{16b^2}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} - \frac{\left(3a(8bc-7ad)e^3\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x}\right.}{32b^{5/2}}$$

$$= \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2} + \frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} + \frac{3a(8bc-7ad)e^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right.}{32b^{11/4}}$$

Mathematica [A] time = 0.17, size = 131, normalized size = 0.76

$$\frac{(ex)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}\left(-7ad+8bc+4bdx^2\right)-3a(7ad-8bc)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+3a(7ad-8bc)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{32b^{11/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((e*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4)*(8*b*c - 7*a*d + 4*b*d*x^
2) - 3*a*(-8*b*c + 7*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3*a
*(-8*b*c + 7*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(32*b^(11/
4)*x^(5/2))
```

IntegrateAlgebraic [A] time = 2.20, size = 206, normalized size = 1.19

$$-\frac{3e^{5/2}\left(7a^2d-8abc\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{32b^{11/4}}+\frac{3e^{5/2}\left(7a^2d-8abc\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{32b^{11/4}}+\frac{\sqrt[4]{a+bx^2}\left(-7ade^2(ex)^{3/2}+8bce^2(ex)^{3/2}+4bd(ex)^{7/2}\right)}{16b^2e}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] ((a + b*x^2)^(1/4)*(8*b*c*e^2*(e*x)^(3/2) - 7*a*d*e^2*(e*x)^(3/2) + 4*b*d*(
e*x)^(7/2)))/(16*b^2*e) - (3*(-8*a*b*c + 7*a^2*d)*e^(5/2)*ArcTan[(b^(1/4)*e
^(3/2)*Sqrt[e*x]*(a + b*x^2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(32*b^(11/4)) + (
3*(-8*a*b*c + 7*a^2*d)*e^(5/2)*ArcTanh[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^
2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(32*b^(11/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{5/2}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

```
[Out] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 41.58, size = 94, normalized size = 0.54

$$\frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(3/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(15/4))
```

3.864 $\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{e}\,(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

Rubi [A] time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 329, 331, 298, 205, 208}

$$-\frac{\sqrt{e}\,(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{\sqrt{e}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(7/4)) + ((4*b*c -
3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4
*b^(7/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx = \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} - \frac{\left(-2bc+\frac{3ad}{2}\right)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{2b}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{2be}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{2be}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} + \frac{((4bc-3ad)e)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{4b^{3/2}} - \frac{((4bc-3ad)e)\,\text{Sub}}{}$$

$$= \frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be} - \frac{(4bc-3ad)\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} + \frac{(4bc-3ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{e}}{\sqrt{e}\,\sqrt[4]{a+b}}\right.}{4b^{7/4}}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.82

$$\frac{\sqrt{ex}\left(2b^{3/4}dx^{3/2}\sqrt[4]{a+bx^2}+(3ad-4bc)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (Sqrt[e*x]*(2*b^(3/4)*d*x^(3/2)*(a + b*x^2)^(1/4) + (-4*b*c + 3*a*d)*ArcTan
[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + (4*b*c - 3*a*d)*ArcTanh[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)]))/(4*b^(7/4)*Sqrt[x])
```

IntegrateAlgebraic [A] time = 1.20, size = 168, normalized size = 1.24

$$-\frac{\sqrt{e}(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{b}e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{4b^{7/4}}+\frac{\sqrt{e}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{4b^{7/4}}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

```
[Out] (d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) - ((4*b*c - 3*a*d)*Sqrt[e]*ArcTan
[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(4*b^(
7/4)) + ((4*b*c - 3*a*d)*Sqrt[e]*ArcTanh[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*
x^2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(4*b^(7/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{2}+c\right)\sqrt{ex}}{\left(bx^{2}+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

```
[Out] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

sympy [C] time = 5.83, size = 92, normalized size = 0.68

$$\frac{c\,(ex)^{\frac{3}{2}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e\Gamma\left(\frac{7}{4}\right)}+\frac{d\,(ex)^{\frac{7}{2}}\,\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{3}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

```
[Out] c*(e*x)**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/
a)/(2*a**(3/4)*e*gamma(7/4)) + d*(e*x)**(7/2)*gamma(7/4)*hyper((3/4, 7/4),
(11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*e**3*gamma(11/4))
```

3.865 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=113

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {451, 329, 331, 298, 205, 208}

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2)) + (d*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(3/4)*e^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{d\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{e^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{e^3}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{e^3}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}+\frac{d\,\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{b}\,e}-\frac{d\,\text{Subst}\left(\int \frac{1}{e+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right.}{\sqrt{b}\,e}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}$$

Mathematica [A] time = 0.03, size = 100, normalized size = 0.88

$$\frac{x\left(-2b^{3/4}c\sqrt[4]{a+bx^2}-ad\sqrt{x}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+ad\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{ab^{3/4}(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (x*(-2*b^(3/4)*c*(a + b*x^2)^(1/4) - a*d*Sqrt[x]*ArcTan[(b^(1/4)*Sqrt[x])/(
a + b*x^2)^(1/4)] + a*d*Sqrt[x]*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)
]))/(a*b^(3/4)*(e*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.95, size = 145, normalized size = 1.28

$$-\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{b^{3/4}e^{3/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{b^{3/4}e^{3/2}}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) - (d*ArcTan[(b^(1/4)*e^(3/2)*Sqrt[
e*x]*(a + b*x^2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(b^(3/4)*e^(3/2)) + (d*ArcTan
h[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(b^(3
/4)*e^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2 + c}{(ex)^{\frac{3}{2}} \left(b\,x^2 + a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)), x)
```

sympy [C] time = 10.42, size = 85, normalized size = 0.75

$$\frac{\sqrt[4]{b}\,c\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(3/2)*gamma(3/4)) +
d*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(3/4)*e**(3/2)*gamma(7/4))
```

3.866 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt[4]{a+bx^2}\,(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$\frac{2\sqrt[4]{a+bx^2}\,(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)*(a + b*x^
2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}} - \frac{(4bc-5ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx}{5ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}} + \frac{2(4bc-5ad)\sqrt[4]{a+bx^2}}{5a^2e^3\sqrt{ex}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.66

$$-\frac{2x\sqrt[4]{a+bx^2}\,\left(a\left(c+5dx^2\right)-4bcx^2\right)}{5a^2(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*x*(a + b*x^2)^(1/4)*(-4*b*c*x^2 + a*(c + 5*d*x^2)))/(5*a^2*(e*x)^(7/2))
```

IntegrateAlgebraic [A] time = 1.66, size = 55, normalized size = 0.82

$$-\frac{2\sqrt[4]{a+bx^2}\,\left(ace^2+5ade^2x^2-4bce^2x^2\right)}{5a^2e^3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4)*(a*c*e^2 - 4*b*c*e^2*x^2 + 5*a*d*e^2*x^2))/(5*a^2*e^3
*(e*x)^(5/2))
```

fricas [A] time = 1.16, size = 43, normalized size = 0.64

$$\frac{2\left((4\,bc-5\,ad)x^2-ac\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{5\,a^2e^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c - 5*a*d)*x^2 - a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*e^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.58

$$-\frac{2 \left(b\, x^2 + a\right)^{\frac{1}{4}} \left(5ad\, x^2 - 4bc\, x^2 + ac\right) x}{5\, (ex)^{\frac{7}{2}}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/5*(b*x^2+a)^(1/4)*x*(5*a*d*x^2-4*b*c*x^2+a*c)/a^2/(e*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

mupad [B] time = 1.17, size = 49, normalized size = 0.73

$$-\frac{\left(\frac{2\,c}{5\,a\,e^3} + \frac{x^2\,(10\,a\,d - 8\,b\,c)}{5\,a^2\,e^3}\right) \left(b\,x^2 + a\right)^{1/4}}{x^2\,\sqrt{e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -(((2*c)/(5*a*e^3) + (x^2*(10*a*d - 8*b*c))/(5*a^2*e^3))*(a + b*x^2)^(1/4))
/(x^2*(e*x)^(1/2))
```

sympy [B] time = 59.81, size = 121, normalized size = 1.81

$$-\frac{\sqrt[4]{b}\,c\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{8ae^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt[4]{b}\,d\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{b^{\frac{5}{4}}c\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^2e^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] -b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*e**(7/2)*x**2*gamma(3/
4)) + b**(1/4)*d*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(7/2)*gamma(3/
4)) + b**(5/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*e**(7/2)*gamma
(3/4))
```

3.867 $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{8\left(a+bx^2\right)^{5/4}(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}\,(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{8\left(a+bx^2\right)^{5/4}(8bc-9ad)}{45a^3e^3(ex)^{5/2}}+\frac{2\sqrt[4]{a+bx^2}\,(8bc-9ad)}{9a^2e^3(ex)^{5/2}}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) + (2*(8*b*c - 9*a*d)*(a + b*x^
2)^(1/4))/(9*a^2*e^3*(e*x)^(5/2)) - (8*(8*b*c - 9*a*d)*(a + b*x^2)^(5/4))/(
45*a^3*e^3*(e*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} - \frac{(8bc-9ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx}{9ae^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} + \frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}} + \frac{(4(8bc-9ad))\int \frac{\sqrt[4]{a+bx^2}}{(ex)^{7/2}}\,dx}{9a^2e^2}$$

$$= -\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}} + \frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}} - \frac{8(8bc-9ad)\left(a+bx^2\right)^{5/4}}{45a^3e^3(ex)^{5/2}}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.69

$$-\frac{2\sqrt{ex}\,\sqrt[4]{a+bx^2}\,\left(a^2\left(5c+9dx^2\right)-4abx^2\left(2c+9dx^2\right)+32b^2cx^4\right)}{45a^3e^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(a + b*x^2)^(1/4)*(32*b^2*c*x^4 - 4*a*b*x^2*(2*c + 9*d*x^2) +
 a^2*(5*c + 9*d*x^2)))/(45*a^3*e^6*x^5)
```

IntegrateAlgebraic [A] time = 1.34, size = 84, normalized size = 0.81

$$\frac{2\sqrt[4]{a+bx^2}\,\left(-5a^2ce^4-9a^2de^4x^2+8abce^4x^2+36abde^4x^4-32b^2ce^4x^4\right)}{45a^3e^5(ex)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (2*(a + b*x^2)^(1/4)*(-5*a^2*c*e^4 + 8*a*b*c*e^4*x^2 - 9*a^2*d*e^4*x^2 - 32
*b^2*c*e^4*x^4 + 36*a*b*d*e^4*x^4))/(45*a^3*e^5*(e*x)^(9/2))
```

fricas [A] time = 1.55, size = 66, normalized size = 0.63

$$-\frac{2\left(4\left(8\,b^2c-9\,abd\right)x^4+5\,a^2c-\left(8\,abc-9\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{45\,a^3e^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/45*(4*(8*b^2*c - 9*a*b*d)*x^4 + 5*a^2*c - (8*a*b*c - 9*a^2*d)*x^2)*(b*x^
2 + a)^(1/4)*sqrt(e*x)/(a^3*e^6*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.60

$$-\frac{2 \left(b\, x^2 + a\right)^{\frac{1}{4}} \left(-36abd\, x^4 + 32b^2c\, x^4 + 9a^2d\, x^2 - 8abc\, x^2 + 5c\, a^2\right) x}{45\, (ex)^{\frac{11}{2}}\, a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/45*(b*x^2+a)^(1/4)*x*(-36*a*b*d*x^4+32*b^2*c*x^4+9*a^2*d*x^2-8*a*b*c*x^2
+5*a^2*c)/a^3/(e*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

mupad [B] time = 1.21, size = 75, normalized size = 0.72

$$-\frac{\left(b\, x^2 + a\right)^{1/4} \left(\frac{2\, c}{9\, a\, e^5} + \frac{x^2 \left(18\, a^2\, d - 16\, a\, b\, c\right)}{45\, a^3\, e^5} + \frac{x^4 \left(64\, b^2\, c - 72\, a\, b\, d\right)}{45\, a^3\, e^5}\right)}{x^4\, \sqrt{e\, x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(9*a*e^5) + (x^2*(18*a^2*d - 16*a*b*c))/(45*a^3*
e^5) + (x^4*(64*b^2*c - 72*a*b*d))/(45*a^3*e^5)))/(x^4*(e*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

3.868 $\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal. Leaf size=141

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}\,(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

Rubi [A] time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{64\left(a+bx^2\right)^{9/4}(12bc-13ad)}{585a^4e^3(ex)^{9/2}}-\frac{16\left(a+bx^2\right)^{5/4}(12bc-13ad)}{65a^3e^3(ex)^{9/2}}+\frac{2\sqrt[4]{a+bx^2}\,(12bc-13ad)}{13a^2e^3(ex)^{9/2}}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) + (2*(12*b*c - 13*a*d)*(a +
b*x^2)^(1/4))/(13*a^2*e^3*(e*x)^(9/2)) - (16*(12*b*c - 13*a*d)*(a + b*x^2)^
(5/4))/(65*a^3*e^3*(e*x)^(9/2)) + (64*(12*b*c - 13*a*d)*(a + b*x^2)^(9/4))/
(585*a^4*e^3*(e*x)^(9/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx &= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}-\frac{(12bc-13ad)\int \frac{1}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx}{13ae^2}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}+\frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}}+\frac{(8(12bc-13ad))\int \frac{\sqrt[4]{a+bx^2}}{(ex)^{11/2}}\,dx}{13a^2e^2}\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}+\frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}}-\frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}}-(32(12\\
&= -\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}+\frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}}-\frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}}+64(12b
\end{aligned}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 0.67

$$-\frac{2\sqrt{ex}\,\sqrt[4]{a+bx^2}\,\left(5a^3\left(9c+13dx^2\right)-4a^2bx^2\left(15c+26dx^2\right)+32ab^2x^4\left(3c+13dx^2\right)-384b^3cx^6\right)}{585a^4e^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(a + b*x^2)^(1/4)*(-384*b^3*c*x^6 + 32*a*b^2*x^4*(3*c + 13*d*
x^2) + 5*a^3*(9*c + 13*d*x^2) - 4*a^2*b*x^2*(15*c + 26*d*x^2)))/(585*a^4*e^
8*x^7)
```

IntegrateAlgebraic [A] time = 2.23, size = 114, normalized size = 0.81

$$-\frac{2\sqrt[4]{a+bx^2}\,\left(45a^3ce^6+65a^3de^6x^2-60a^2bce^6x^2-104a^2bde^6x^4+96ab^2ce^6x^4+416ab^2de^6x^6-384b^3ce^6x^6\right)}{585a^4e^7(ex)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4)*(45*a^3*c*e^6 - 60*a^2*b*c*e^6*x^2 + 65*a^3*d*e^6*x^2
 + 96*a*b^2*c*e^6*x^4 - 104*a^2*b*d*e^6*x^4 - 384*b^3*c*e^6*x^6 + 416*a*b^2
*d*e^6*x^6))/(585*a^4*e^7*(e*x)^(13/2))
```

fricas [A] time = 0.70, size = 90, normalized size = 0.64

$$\frac{2\left(32\left(12\,b^3c-13\,ab^2d\right)x^6-8\left(12\,ab^2c-13\,a^2bd\right)x^4-45\,a^3c+5\left(12\,a^2bc-13\,a^3d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{585\,a^4e^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/585*(32*(12*b^3*c - 13*a*b^2*d)*x^6 - 8*(12*a*b^2*c - 13*a^2*b*d)*x^4 - 4
5*a^3*c + 5*(12*a^2*b*c - 13*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^4*e
^8*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

maple [A] time = 0.01, size = 86, normalized size = 0.61

$$-\frac{2\left(b\,x^2+a\right)^{\frac{1}{4}}\left(416a\,b^2d\,x^6-384b^3c\,x^6-104a^2bd\,x^4+96a\,b^2c\,x^4+65a^3d\,x^2-60a^2bc\,x^2+45c\,a^3\right)x}{585\,(ex)^{\frac{15}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x)
```

```
[Out] -2/585*(b*x^2+a)^(1/4)*x*(416*a*b^2*d*x^6-384*b^3*c*x^6-104*a^2*b*d*x^4+96*
a*b^2*c*x^4+65*a^3*d*x^2-60*a^2*b*c*x^2+45*a^3*c)/a^4/(e*x)^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

mupad [**B**] time = 1.26, size = 100, normalized size = 0.71

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2\,c}{13\,a\,e^7}+\frac{x^2\,\left(130\,a^3\,d-120\,a^2\,b\,c\right)}{585\,a^4\,e^7}-\frac{x^6\,\left(768\,b^3\,c-832\,a\,b^2\,d\right)}{585\,a^4\,e^7}-\frac{16\,b\,x^4\,\left(13\,a\,d-12\,b\,c\right)}{585\,a^3\,e^7}\right)}{x^6\,\sqrt{e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(13*a*e^7) + (x^2*(130*a^3*d - 120*a^2*b*c))/(58
5*a^4*e^7) - (x^6*(768*b^3*c - 832*a*b^2*d))/(585*a^4*e^7) - (16*b*x^4*(13*
a*d - 12*b*c))/(585*a^3*e^7)))/(x^6*(e*x)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(15/2)/(b*x**2+a)**(3/4),x)
```

```
[Out] Timed out
```

3.869 $\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=171

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}\,(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 288, 329, 240, 212, 208, 205}

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}+\frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}-\frac{e\sqrt{ex}\,(4bc-5ad)}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] -((4*b*c - 5*a*d)*e*Sqrt[e*x])/(2*b^2*(a + b*x^2)^(1/4)) + (d*(e*x)^(5/2))/
(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e
*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4)) + ((4*b*c - 5*a*d)*e^(3/2)*A
rcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(9/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx &= \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} - \frac{\left(-2bc+\frac{5ad}{2}\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{2b} \\
&= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{\left((4bc-5ad)e^2\right)\int \frac{1}{\sqrt{ex}\,\sqrt[4]{a+bx^2}}\,dx}{4b^2} \\
&= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{((4bc-5ad)e)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{2b^2} \\
&= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{((4bc-5ad)e)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{2b^2} \\
&= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{\left((4bc-5ad)e^2\right)\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{4b^2} + \\
&= -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{(4bc-5ad)e^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} + \frac{(4bc-5ad)e}{}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 77, normalized size = 0.45

$$\frac{x(ex)^{3/2}\left(\sqrt[4]{\frac{bx^2}{a}+1}\,(4bc-5ad)\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)+5ad\right)}{10ab\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] (x*(e*x)^(3/2)*(5*a*d + (4*b*c - 5*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometri
c2F1[5/4, 5/4, 9/4, -((b*x^2)/a)]))/(10*a*b*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 3.91, size = 213, normalized size = 1.25

$$\frac{e^{3/2}(4bc-5ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{4b^{9/4}} + \frac{e^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{4b^{9/4}} + \frac{\left(a+bx^2\right)^{3/4}\left(5ade^3\sqrt{ex}-4bce^3\sqrt{ex}+bde(ex)^{5/2}\right)}{2b^2\left(ae^2+be^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

```
[Out] ((a + b*x^2)^(3/4)*(-4*b*c*e^3*Sqrt[e*x] + 5*a*d*e^3*Sqrt[e*x] + b*d*e*(e*x
)^(5/2)))/(2*b^2*(a*e^2 + b*e^2*x^2)) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTan[(b^
(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^2)^(3/4))/(a*e^2 + b*e^2*x^2)])/(4*b^(9/4)
) + ((4*b*c - 5*a*d)*e^(3/2)*ArcTanh[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^2)
^(3/4))/(a*e^2 + b*e^2*x^2)])/(4*b^(9/4))
```

fricas [**B**]   time = 1.10, size = 916, normalized size = 5.36

$$\frac{4\left(bdex^2-(4bc-5ad)e\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}+4\left(b^3x^2+ab^2\right)\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(4b^8c-5ab^7d\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,e\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{3}{4}}+\left(b^8x^2+ab^7\right)\sqrt{\frac{\left(16b^2c^2-40abcd+25a^2d^2\right)\sqrt{bx^2+a}\,e^3x+\left(b^5x^2+ab^4\right)\sqrt{\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}}}{bx^2+a}}\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{3}{4}}}{\left(256b^5c^4-1280ab^4c^3d+2400a^2b^3c^2d^2-2000a^3b^2cd^3+625a^4bd^4\right)e^6x^2+\left(256ab^4c^4-1280a^2b^3c^3d+2400a^3b^2c^2d^2-2000a^4bcd^3+625a^5d^4\right)e^6}\right)+\left(b^3x^2+ab^2\right)\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}(4bc-5ad)\sqrt{ex}\,e+\left(b^3x^2+ab^2\right)\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{1}{4}}}{bx^2+a}\right)-\left(b^3x^2+ab^2\right)\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}(4bc-5ad)\sqrt{ex}\,e-\left(b^3x^2+ab^2\right)\left(\frac{\left(256b^4c^4-1280ab^3c^3d+2400a^2b^2c^2d^2-2000a^3bcd^3+625a^4d^4\right)e^6}{b^9}\right)^{\frac{1}{4}}}{bx^2+a}\right)}{8\left(b^3x^2+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*d*e*x^2 - (4*b*c - 5*a*d)*e)*(b*x^2 + a)^(3/4)*sqrt(e*x) + 4*(b^3
*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 200
0*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*arctan(((4*b^8*c - 5*a*b^7*d)*(
b*x^2 + a)^(3/4)*sqrt(e*x)*e*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^
2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(3/4) + (b^8*x^2 + a*b
^7)*sqrt(((16*b^2*c^2 - 40*a*b*c*d + 25*a^2*d^2)*sqrt(b*x^2 + a)*e^3*x + (b
^5*x^2 + a*b^4)*sqrt((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2
 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9))/(b*x^2 + a))*((256*b^4*c^4 - 1
280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^
6/b^9)^(3/4))/((256*b^5*c^4 - 1280*a*b^4*c^3*d + 2400*a^2*b^3*c^2*d^2 - 200
0*a^3*b^2*c*d^3 + 625*a^4*b*d^4)*e^6*x^2 + (256*a*b^4*c^4 - 1280*a^2*b^3*c^
3*d + 2400*a^3*b^2*c^2*d^2 - 2000*a^4*b*c*d^3 + 625*a^5*d^4)*e^6)) + (b^3*x
^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*
a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c -
5*a*d)*sqrt(e*x)*e + (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2
400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4))/(b*x^
2 + a)) - (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2
*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x^2 + a)
^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e - (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280
*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b
^9)^(1/4))/(b*x^2 + a)))/(b^3*x^2 + a*b^2)
```

giac [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{2}+c\right)(ex)^{\frac{3}{2}}}{\left(bx^{2}+a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
```

```
[Out] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

sympy [C] time = 32.83, size = 94, normalized size = 0.55

$$\frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}+\frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((5/4
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(13/4))
```

3.870 $\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=122

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}\,(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 329, 240, 212, 208, 205}

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{2\sqrt{ex}\,(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (d*ArcTan[(b^(1/4)*Sq
rt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e]) + (d*ArcTanh[(b^(1
/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(b^(5/4)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}} + \frac{d\int \frac{1}{\sqrt{ex}\,\sqrt[4]{a+bx^2}}\,dx}{b}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{be}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}} + \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{be}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b} + \frac{d\,\mathrm{Subst}\left(\int \frac{1}{e+\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right.}{b}$$

$$= \frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}} + \frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}} + \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}$$

Mathematica [**C**] time = 0.06, size = 68, normalized size = 0.56

$$\frac{2\left(dx^3\sqrt[4]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};-\frac{bx^2}{a}\right)+5cx\right)}{5a\sqrt{ex}\,\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(5*c*x + d*x^3*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, -(
(b*x^2)/a)]))/(5*a*Sqrt[e*x]*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [**A**] time = 2.36, size = 168, normalized size = 1.38

$$\frac{d\tan^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{b^{5/4}\sqrt{e}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{b}\,e^{3/2}\sqrt{ex}\left(a+bx^2\right)^{3/4}}{ae^2+be^2x^2}\right)}{b^{5/4}\sqrt{e}}-\frac{2e\sqrt{ex}\left(a+bx^2\right)^{3/4}(ad-bc)}{ab\left(ae^2+be^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*(-(b*c) + a*d)*e*Sqrt[e*x]*(a + b*x^2)^(3/4))/(a*b*(a*e^2 + b*e^2*x^2))
 + (d*ArcTan[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^2)^(3/4))/(a*e^2 + b*e^2*x
^2)])/(b^(5/4)*Sqrt[e]) + (d*ArcTanh[(b^(1/4)*e^(3/2)*Sqrt[e*x]*(a + b*x^2)
^(3/4))/(a*e^2 + b*e^2*x^2)])/(b^(5/4)*Sqrt[e])
```

fricas [**B**] time = 1.06, size = 384, normalized size = 3.15

$$\frac{4\left(bx^2+a\right)^{\frac{3}{4}}(bc-ad)\sqrt{ex}-4\left(ab^2ex^2+a^2be\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,b^4de\left(\frac{d^4}{b^5e^2}\right)^{\frac{3}{4}}-\left(b^5ex^2+ab^4e\right)\sqrt{\frac{\sqrt{bx^2+a}\,d^2ex+\left(b^3e^2x^2+ab^2e^2\right)\sqrt{\frac{d^4}{b^5e^2}}}{bx^2+a}}\left(\frac{d^4}{b^5e^2}\right)^{\frac{3}{4}}}{bd^4x^2+ad^4}\right)+\left(ab^2ex^2+a^2be\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,d+\left(b^2ex^2+abe\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)-\left(ab^2ex^2+a^2be\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,d-\left(b^2ex^2+abe\right)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)}{2\left(ab^2ex^2+a^2be\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(b*x^2 + a)^(3/4)*(b*c - a*d)*sqrt(e*x) - 4*(a*b^2*e*x^2 + a^2*b*e)*
(d^4/(b^5*e^2))^(1/4)*arctan(-((b*x^2 + a)^(3/4)*sqrt(e*x)*b^4*d*e*(d^4/(b^
5*e^2))^(3/4) - (b^5*e*x^2 + a*b^4*e)*sqrt((sqrt(b*x^2 + a)*d^2*e*x + (b^3*
e^2*x^2 + a*b^2*e^2)*sqrt(d^4/(b^5*e^2)))/(b*x^2 + a))*(d^4/(b^5*e^2))^(3/4
))/(b*d^4*x^2 + a*d^4)) + (a*b^2*e*x^2 + a^2*b*e)*(d^4/(b^5*e^2))^(1/4)*log
(((b*x^2 + a)^(3/4)*sqrt(e*x)*d + (b^2*e*x^2 + a*b*e)*(d^4/(b^5*e^2))^(1/4)
)/(b*x^2 + a)) - (a*b^2*e*x^2 + a^2*b*e)*(d^4/(b^5*e^2))^(1/4)*log(((b*x^2
```

```
+ a)^(3/4)*sqrt(e*x)*d - (b^2*e*x^2 + a*b*e)*(d^4/(b^5*e^2))^(1/4))/(b*x^2
+ a)))/(a*b^2*e*x^2 + a^2*b*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{5}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{d\,x^2 + c}{\sqrt{ex}\,\left(b\,x^2 + a\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{5}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^2 + c}{\sqrt{e\,x}\,\left(b\,x^2 + a\right)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/4)), x)
```

sympy [C] time = 16.06, size = 83, normalized size = 0.68

$$\frac{c\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*gamma(1/4)/(2*a*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)) + d*
x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(5/4)*sqrt(e)*gamma(9/4))
```

3.871 $\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2\sqrt{ex}\,(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Rubi [**A**] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2\sqrt{ex}\,(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)*Sqrt[e*x]
)/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{(4bc-3ad)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx}{3ae^2}$$
$$= -\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{2(4bc-3ad)\sqrt{ex}}{3a^2e^3\sqrt[4]{a+bx^2}}$$

Mathematica [**A**] time = 0.02, size = 45, normalized size = 0.67

$$\frac{x\left(-2ac+6adx^2-8bcx^2\right)}{3a^2(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (x*(-2*a*c - 8*b*c*x^2 + 6*a*d*x^2))/(3*a^2*(e*x)^(5/2)*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [**A**] time = 1.83, size = 72, normalized size = 1.07

$$\frac{2\left(a+bx^2\right)^{3/4}\left(-ace^2+3ade^2x^2-4bce^2x^2\right)}{3a^2e(ex)^{3/2}\left(ae^2+be^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(a + b*x^2)^(3/4)*(-(a*c*e^2) - 4*b*c*e^2*x^2 + 3*a*d*e^2*x^2))/(3*a^2*e
*(e*x)^(3/2)*(a*e^2 + b*e^2*x^2))
```

fricas [**A**] time = 0.82, size = 57, normalized size = 0.85

$$-\frac{2\left((4\,bc-3\,ad)x^2+ac\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{3\left(a^2be^3x^4+a^3e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/3*((4*b*c - 3*a*d)*x^2 + a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*b*e^3*x^4
 + a^3*e^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.58

$$-\frac{2\left(-3ad\,x^2 + 4bc\,x^2 + ac\right)x}{3\left(b\,x^2 + a\right)^{\frac{1}{4}} (ex)^{\frac{5}{2}}\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/3*x*(-3*a*d*x^2+4*b*c*x^2+a*c)/(b*x^2+a)^(1/4)/a^2/(e*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{5}{4}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

mupad [B] time = 1.20, size = 70, normalized size = 1.04

$$-\frac{\left(b\,x^2 + a\right)^{3/4} \left(\frac{2\,c}{3\,a\,b\,e^2} - \frac{x^2\,(6\,a\,d - 8\,b\,c)}{3\,a^2\,b\,e^2}\right)}{x^3\,\sqrt{e\,x} + \frac{a\,x\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(3*a*b*e^2) - (x^2*(6*a*d - 8*b*c))/(3*a^2*b*e^2
)))/(x^3*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

sympy [A] time = 64.29, size = 117, normalized size = 1.75

$$c\left(\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}\,e^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^2e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}\right)+\frac{d\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\,e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] c*(gamma(-3/4)/(8*a*b**(1/4)*e**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/
4)) + b**(3/4)*gamma(-3/4)/(2*a**2*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5
/4))) + d*gamma(1/4)/(2*a*b**(1/4)*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5
/4))
```

3.872 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Rubi [**A**] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{8\left(a+bx^2\right)^{3/4}(8bc-7ad)}{21a^3e^3(ex)^{3/2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - (2*(8*b*c - 7*a*d))/(7*a^2*e
^3*(e*x)^(3/2)*(a + b*x^2)^(1/4)) + (8*(8*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(
21*a^3*e^3*(e*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx &= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{(8bc-7ad)\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx}{7ae^2}\\
&= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{(4(8bc-7ad))\int \frac{1}{(ex)^{5/2}\sqrt[4]{a+bx^2}}\,dx}{7a^2e^2}\\
&= -\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}+\frac{8(8bc-7ad)\left(a+bx^2\right)^{3/4}}{21a^3e^3(ex)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.68

$$
-\frac{2\sqrt{ex}\left(a^2\left(3c+7dx^2\right)+ab\left(28dx^4-8cx^2\right)-32b^2cx^4\right)}{21a^3e^5x^4\sqrt[4]{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*Sqrt[e*x]*(-32*b^2*c*x^4 + a^2*(3*c + 7*d*x^2) + a*b*(-8*c*x^2 + 28*d*x
^4)))/(21*a^3*e^5*x^4*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 2.53, size = 100, normalized size = 0.96

$$
-\frac{2\left(a+bx^2\right)^{3/4}\left(3a^2ce^4+7a^2de^4x^2-8abce^4x^2+28abde^4x^4-32b^2ce^4x^4\right)}{21a^3e^3(ex)^{7/2}\left(ae^2+be^2x^2\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4)*(3*a^2*c*e^4 - 8*a*b*c*e^4*x^2 + 7*a^2*d*e^4*x^2 - 32
*b^2*c*e^4*x^4 + 28*a*b*d*e^4*x^4))/(21*a^3*e^3*(e*x)^(7/2)*(a*e^2 + b*e^2*
x^2))
```

fricas [A] time = 1.20, size = 80, normalized size = 0.77

$$\frac{2\left(4\left(8\,b^{2}c-7\,abd\right)x^{4}-3\,a^{2}c+\left(8\,abc-7\,a^{2}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{ex}}{21\left(a^{3}be^{5}x^{6}+a^{4}e^{5}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(4*(8*b^2*c - 7*a*b*d)*x^4 - 3*a^2*c + (8*a*b*c - 7*a^2*d)*x^2)*(b*x^2
 + a)^(3/4)*sqrt(e*x)/(a^3*b*e^5*x^6 + a^4*e^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{5}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.60

$$-\frac{2\left(28abd\,x^{4}-32b^{2}c\,x^{4}+7a^{2}d\,x^{2}-8abc\,x^{2}+3c\,a^{2}\right)x}{21\left(b\,x^{2}+a\right)^{\frac{1}{4}}(ex)^{\frac{9}{2}}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/21*x*(28*a*b*d*x^4-32*b^2*c*x^4+7*a^2*d*x^2-8*a*b*c*x^2+3*a^2*c)/(b*x^2+
a)^(1/4)/a^3/(e*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{5}{4}}(ex)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

mupad [B] time = 1.23, size = 101, normalized size = 0.97

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2\,c}{7\,a\,b\,e^4}+\frac{x^2\,\left(14\,a^2\,d-16\,a\,b\,c\right)}{21\,a^3\,b\,e^4}-\frac{x^4\,\left(64\,b^2\,c-56\,a\,b\,d\right)}{21\,a^3\,b\,e^4}\right)}{x^5\,\sqrt{e\,x}+\frac{a\,x^3\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(7*a*b*e^4) + (x^2*(14*a^2*d - 16*a*b*c))/(21*a^
3*b*e^4) - (x^4*(64*b^2*c - 56*a*b*d))/(21*a^3*b*e^4)))/(x^5*(e*x)^(1/2) +
(a*x^3*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

3.873 $\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.06, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{64\left(a+bx^2\right)^{7/4}(12bc-11ad)}{231a^4e^3(ex)^{7/2}}+\frac{16\left(a+bx^2\right)^{3/4}(12bc-11ad)}{33a^3e^3(ex)^{7/2}}-\frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - (2*(12*b*c - 11*a*d))/(11*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (16*(12*b*c - 11*a*d)*(a + b*x^2)^
(3/4))/(33*a^3*e^3*(e*x)^(7/2)) - (64*(12*b*c - 11*a*d)*(a + b*x^2)^(7/4))/
(231*a^4*e^3*(e*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{(12bc-11ad)\int \frac{1}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx}{11ae^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} - \frac{(8(12bc-11ad))\int \frac{1}{(ex)^{9/2}\sqrt[4]{a+bx^2}}}{11a^2e^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)\left(a+bx^2\right)^{3/4}}{33a^3e^3(ex)^{7/2}} +$$

$$= -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)\left(a+bx^2\right)^{3/4}}{33a^3e^3(ex)^{7/2}} -$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.48

$$\frac{2x\left(-21a^3c-x^2\left(-3a^2+8abx^2+32b^2x^4\right)(12bc-11ad)\right)}{231a^4(ex)^{13/2}\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*x*(-21*a^3*c - (12*b*c - 11*a*d)*x^2*(-3*a^2 + 8*a*b*x^2 + 32*b^2*x^4)))
/(231*a^4*(e*x)^(13/2)*(a + b*x^2)^(1/4))
```

IntegrateAlgebraic [A] time = 3.97, size = 130, normalized size = 0.92

$$\frac{2\left(a+bx^2\right)^{3/4}\left(-21a^3ce^6-33a^3de^6x^2+36a^2bce^6x^2+88a^2bde^6x^4-96ab^2ce^6x^4+352ab^2de^6x^6-384b^3ce^6x^6\right)}{231a^4e^5(ex)^{11/2}\left(ae^2+be^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

```
[Out] (2*(a + b*x^2)^(3/4)*(-21*a^3*c*e^6 + 36*a^2*b*c*e^6*x^2 - 33*a^3*d*e^6*x^2
 - 96*a*b^2*c*e^6*x^4 + 88*a^2*b*d*e^6*x^4 - 384*b^3*c*e^6*x^6 + 352*a*b^2*
d*e^6*x^6))/(231*a^4*e^5*(e*x)^(11/2)*(a*e^2 + b*e^2*x^2))
```

fricas [A] time = 1.09, size = 105, normalized size = 0.74

$$-\frac{2\left(32\left(12\,b^{3}c-11\,ab^{2}d\right)x^{6}+8\left(12\,ab^{2}c-11\,a^{2}bd\right)x^{4}+21\,a^{3}c-3\left(12\,a^{2}bc-11\,a^{3}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{ex}}{231\left(a^{4}be^{7}x^{8}+a^{5}e^{7}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

```
[Out] -2/231*(32*(12*b^3*c - 11*a*b^2*d)*x^6 + 8*(12*a*b^2*c - 11*a^2*b*d)*x^4 +
21*a^3*c - 3*(12*a^2*b*c - 11*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^4*
b*e^7*x^8 + a^5*e^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{5}{4}}\left(ex\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 86, normalized size = 0.61

$$-\frac{2\left(-352 a\,b^{2} d\,x^{6}+384 b^{3} c\,x^{6}-88 a^{2} b d\,x^{4}+96 a\,b^{2} c\,x^{4}+33 a^{3} d\,x^{2}-36 a^{2} b c\,x^{2}+21 c\,a^{3}\right) x}{231\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(ex\right)^{\frac{13}{2}} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

```
[Out] -2/231*x*(-352*a*b^2*d*x^6+384*b^3*c*x^6-88*a^2*b*d*x^4+96*a*b^2*c*x^4+33*a
^3*d*x^2-36*a^2*b*c*x^2+21*a^3*c)/(b*x^2+a)^(1/4)/a^4/(e*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{5}{4}}\left(ex\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

mupad [B] time = 1.27, size = 125, normalized size = 0.89

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2\,c}{11\,a\,b\,e^6}-\frac{16\,x^4\,\left(11\,a\,d-12\,b\,c\right)}{231\,a^3\,e^6}+\frac{x^2\,\left(66\,a^3\,d-72\,a^2\,b\,c\right)}{231\,a^4\,b\,e^6}+\frac{x^6\,\left(768\,b^3\,c-704\,a\,b^2\,d\right)}{231\,a^4\,b\,e^6}\right)}{x^7\,\sqrt{e\,x}+\frac{a\,x^5\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(11*a*b*e^6) - (16*x^4*(11*a*d - 12*b*c))/(231*a
^3*e^6) + (x^2*(66*a^3*d - 72*a^2*b*c))/(231*a^4*b*e^6) + (x^6*(768*b^3*c -
 704*a*b^2*d))/(231*a^4*b*e^6)))/(x^7*(e*x)^(1/2) + (a*x^5*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

```
[Out] Timed out
```

3.874 $\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=184

$$-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}\,(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}}{3abe\big(a}$$

Rubi [A] time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {457, 321, 329, 331, 298, 205, 208}

$$-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}-\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}\,(4bc-7ad)}{6ab^2}+\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c - 7*a*d)*
e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(6*a*b^2) - ((4*b*c - 7*a*d)*e^(5/2)*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(11/4)) + ((4*b*c
- 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/
(4*b^(11/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{\left(2\left(-2bc+\frac{7ad}{2}\right)\right)\int\frac{(ex)^{5/2}}{\left(a+bx^2\right)^{3/4}}\,dx}{3ab}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{\left((4bc-7ad)e^2\right)\int\frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{4b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e)\,\text{Subst}\Bigg(\int\frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,d}{2b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{((4bc-7ad)e)\,\text{Subst}\Bigg(\int\frac{x^2}{1-\frac{bx^4}{e^2}}\,dx,x}{2b^2}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} + \frac{\left((4bc-7ad)e^3\right)\text{Subst}\Big(\int\frac{1}{e-\sqrt{b}\,x^2}\,d}{4b^{5/2}}$$

$$= \frac{2(bc-ad)(ex)^{7/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2} - \frac{(4bc-7ad)e^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}$$

Mathematica [C] time = 0.13, size = 77, normalized size = 0.42

$$\frac{x(ex)^{5/2}\left(\left(\frac{bx^2}{a}+1\right)^{3/4}(4bc-7ad)\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};-\frac{bx^2}{a}\right)+7ad\right)}{14ab\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (x*(e*x)^(5/2)*(7*a*d + (4*b*c - 7*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometri
c2F1[7/4, 7/4, 11/4, -((b*x^2)/a)]))/(14*a*b*(a + b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 23.70, size = 220, normalized size = 1.20

$$\frac{\sqrt{e}\,\sqrt[4]{a+bx^2}\left(-\frac{e^{5/2}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{4b^{11/4}}+\frac{e^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{4b^{11/4}}+\frac{7ade^{5/2}(ex)^{3/2}-4bce^{5/2}(ex)^{3/2}+3bd\sqrt{e}\,(ex)^{7/2}}{6b^2\left(ae^2+be^2x^2\right)^{3/4}}\right)}{\sqrt[4]{ae^2+be^2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (Sqrt[e]*(a + b*x^2)^(1/4)*((-4*b*c*e^(5/2)*(e*x)^(3/2) + 7*a*d*e^(5/2)*(e*
x)^(3/2) + 3*b*d*Sqrt[e]*(e*x)^(7/2))/(6*b^2*(a*e^2 + b*e^2*x^2)^(3/4)) - (
(4*b*c - 7*a*d)*e^(5/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(a*e^2 + b*e^2*x^2)^(1/4
)])/(4*b^(11/4)) + ((4*b*c - 7*a*d)*e^(5/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(a*
e^2 + b*e^2*x^2)^(1/4)])/(4*b^(11/4))))/(a*e^2 + b*e^2*x^2)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{5}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{5}{2}}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{5/2}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

```
[Out] int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

sympy [C] time = 166.39, size = 94, normalized size = 0.51

$$\frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(7/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hyper(
(7/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(15/4))
```

3.875 $\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=125

$$-\frac{d\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 329, 331, 298, 205, 208}

$$-\frac{d\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - (d*Sqrt[e]*ArcTan
[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4) + (d*Sqrt[e]*Arc
Tanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/b^(7/4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{d\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{3/4}}\,dx}{b}$$

$$= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{\left(a+\frac{bx^4}{e^2}\right)^{3/4}}\,dx, x, \sqrt{ex}\right)}{be}$$

$$= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{be}$$

$$= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^{3/2}} - \frac{(de)\,\text{Subst}\left(\int \frac{1}{e+\sqrt{b}\,x^2}\,dx, x,\right.}{b^{3/2}}$$

$$= \frac{2(bc-ad)(ex)^{3/2}}{3abe\left(a+bx^2\right)^{3/4}} - \frac{d\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}} + \frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}$$

Mathematica [**C**] time = 0.06, size = 69, normalized size = 0.55

$$\frac{2\sqrt{ex}\left(3dx^3\left(\frac{bx^2}{a}+1\right)^{3/4}{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};-\frac{bx^2}{a}\right)+7cx\right)}{21a\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (2*Sqrt[e*x]*(7*c*x + 3*d*x^3*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/4,
7/4, 11/4, -((b*x^2)/a)]))/(21*a*(a + b*x^2)^(3/4))
```

IntegrateAlgebraic [**A**] time = 22.95, size = 173, normalized size = 1.38

$$\frac{\sqrt{e}\,\sqrt[4]{a+bx^2}\left(-\frac{d\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{b^{7/4}}+\frac{d\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{b^{7/4}}-\frac{2\sqrt{e}\,(ex)^{3/2}(ad-bc)}{3ab\left(ae^2+be^2x^2\right)^{3/4}}\right)}{\sqrt[4]{ae^2+be^2x^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

```
[Out] (Sqrt[e]*(a + b*x^2)^(1/4)*((-2*(-(b*c) + a*d)*Sqrt[e]*(e*x)^(3/2))/(3*a*b*
(a*e^2 + b*e^2*x^2)^(3/4)) - (d*Sqrt[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(a*e^2 +
 b*e^2*x^2)^(1/4)])/b^(7/4) + (d*Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(a*e^2
 + b*e^2*x^2)^(1/4)])/b^(7/4)))/(a*e^2 + b*e^2*x^2)^(1/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

```
[Out] int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)\sqrt{ex}}{\left(bx^2+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

```
[Out] int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

sympy [C] time = 21.77, size = 87, normalized size = 0.70

$$\frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}{2a^{\frac{7}{4}}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*sqrt(e)*x**(3/2)*gamma(3/4)/(2*a**(7/4)*(1 + b*x**2/a)**(3/4)*gamma(7/4))
 + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(7/4)*gamma(11/4))
```

3.876 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=65

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3\left(a+bx^2\right)^{3/4}}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)^(3/2))/(3
*a^2*e^3*(a + b*x^2)^(3/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}} - \frac{(4bc-ad)\int \frac{\sqrt{ex}}{\left(a+bx^2\right)^{7/4}}\,dx}{ae^2}$$
$$= -\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-ad)(ex)^{3/2}}{3a^2e^3\left(a+bx^2\right)^{3/4}}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.68

$$\frac{2x\left(-3ac+adx^2-4bcx^2\right)}{3a^2(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
[Out] (2*x*(-3*a*c - 4*b*c*x^2 + a*d*x^2))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 24.38, size = 71, normalized size = 1.09

$$\frac{2\sqrt[4]{a+bx^2}\left(-3ace^2+ade^2x^2-4bce^2x^2\right)}{3a^2e\sqrt{ex}\left(ae^2+be^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
[Out] (2*(a + b*x^2)^(1/4)*(-3*a*c*e^2 - 4*b*c*e^2*x^2 + a*d*e^2*x^2))/(3*a^2*e*S
qrt[e*x]*(a*e^2 + b*e^2*x^2))
```

fricas [A] time = 1.74, size = 56, normalized size = 0.86

$$-\frac{2\left((4\,bc-ad)x^2+3\,ac\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{3\left(a^2be^2x^3+a^3e^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
[Out] -2/3*((4*b*c - a*d)*x^2 + 3*a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*b*e^2*x^3
 + a^3*e^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

maple [A] time = 0.01, size = 40, normalized size = 0.62

$$-\frac{2\left(-ad\,x^2 + 4bc\,x^2 + 3ac\right)x}{3\left(b\,x^2 + a\right)^{\frac{3}{4}} (ex)^{\frac{3}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/3*x*(-a*d*x^2+4*b*c*x^2+3*a*c)/(b*x^2+a)^(3/4)/a^2/(e*x)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{7}{4}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

mupad [B] time = 1.18, size = 69, normalized size = 1.06

$$-\frac{\left(b\,x^2 + a\right)^{1/4} \left(\frac{2\,c}{a\,b\,e} - \frac{x^2\,(2\,a\,d - 8\,b\,c)}{3\,a^2\,b\,e}\right)}{x^2\,\sqrt{e\,x} + \frac{a\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(a*b*e) - (x^2*(2*a*d - 8*b*c))/(3*a^2*b*e)))/(x
^2*(e*x)^(1/2) + (a*(e*x)^(1/2))/b)
```

sympy [A] time = 79.59, size = 119, normalized size = 1.83

$$c\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{8ab^{\frac{3}{4}}e^{\frac{3}{2}}x^{2}\left(\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt[4]{b}\,\Gamma\left(-\frac{1}{4}\right)}{2a^{2}e^{\frac{3}{2}}\left(\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right)+\frac{d\Gamma\left(\frac{3}{4}\right)}{2ab^{\frac{3}{4}}e^{\frac{3}{2}}\left(\frac{a}{bx^{2}}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] c*(3*gamma(-1/4)/(8*a*b**(3/4)*e**(3/2)*x**2*(a/(b*x**2) + 1)**(3/4)*gamma(
7/4)) + b**(1/4)*gamma(-1/4)/(2*a**2*e**(3/2)*(a/(b*x**2) + 1)**(3/4)*gamma
(7/4))) + d*gamma(3/4)/(2*a*b**(3/4)*e**(3/2)*(a/(b*x**2) + 1)**(3/4)*gamma
(7/4))
```

3.877 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=104

$$\frac{8\sqrt[4]{a+bx^2}\,(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\,\left(a+bx^2\right)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Rubi [**A**] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{8\sqrt[4]{a+bx^2}\,(8bc-5ad)}{15a^3e^3\sqrt{ex}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\,\left(a+bx^2\right)^{3/4}}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - (2*(8*b*c - 5*a*d))/(15*a^2*
e^3*Sqrt[e*x]*(a + b*x^2)^(3/4)) + (8*(8*b*c - 5*a*d)*(a + b*x^2)^(1/4))/(1
5*a^3*e^3*Sqrt[e*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx &= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{(8bc-5ad)\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx}{5ae^2}\\
&= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}-\frac{(4(8bc-5ad))\int \frac{1}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,d}{15a^2e^2}\\
&= -\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\left(a+bx^2\right)^{3/4}}+\frac{8(8bc-5ad)\sqrt[4]{a+bx^2}}{15a^3e^3\sqrt{ex}}
\end{aligned}
$$

Mathematica [**A**] time = 0.03, size = 66, normalized size = 0.63

$$
\frac{x\left(-6a^2\left(c+5dx^2\right)+8abx^2\left(6c-5dx^2\right)+64b^2cx^4\right)}{15a^3(ex)^{7/2}\left(a+bx^2\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (x*(64*b^2*c*x^4 + 8*a*b*x^2*(6*c - 5*d*x^2) - 6*a^2*(c + 5*d*x^2)))/(15*a^
3*(e*x)^(7/2)*(a + b*x^2)^(3/4))
```

IntegrateAlgebraic [**A**] time = 27.33, size = 100, normalized size = 0.96

$$
-\frac{2\sqrt[4]{a+bx^2}\left(3a^2ce^4+15a^2de^4x^2-24abce^4x^2+20abde^4x^4-32b^2ce^4x^4\right)}{15a^3e^3(ex)^{5/2}\left(ae^2+be^2x^2\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(1/4)*(3*a^2*c*e^4 - 24*a*b*c*e^4*x^2 + 15*a^2*d*e^4*x^2 -
32*b^2*c*e^4*x^4 + 20*a*b*d*e^4*x^4))/(15*a^3*e^3*(e*x)^(5/2)*(a*e^2 + b*e^
2*x^2))
```

fricas [A] time = 2.42, size = 81, normalized size = 0.78

$$\frac{2\left(4\left(8\,b^{2}c-5\,abd\right)x^{4}-3\,a^{2}c+3\left(8\,abc-5\,a^{2}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{ex}}{15\left(a^{3}be^{4}x^{5}+a^{4}e^{4}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/15*(4*(8*b^2*c - 5*a*b*d)*x^4 - 3*a^2*c + 3*(8*a*b*c - 5*a^2*d)*x^2)*(b*x
^2 + a)^(1/4)*sqrt(e*x)/(a^3*b*e^4*x^5 + a^4*e^4*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{7}{4}}\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.60

$$-\frac{2\left(20abd\,x^{4}-32b^{2}c\,x^{4}+15a^{2}d\,x^{2}-24abc\,x^{2}+3c\,a^{2}\right)x}{15\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(ex\right)^{\frac{7}{2}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/15*x*(20*a*b*d*x^4-32*b^2*c*x^4+15*a^2*d*x^2-24*a*b*c*x^2+3*a^2*c)/(b*x^
2+a)^(3/4)/a^3/(e*x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{7}{4}}\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

mupad [B] time = 1.21, size = 101, normalized size = 0.97

$$-\frac{\left(b\,x^2+a\right)^{1/4}\,\left(\frac{2\,c}{5\,a\,b\,e^3}+\frac{x^2\,\left(30\,a^2\,d-48\,a\,b\,c\right)}{15\,a^3\,b\,e^3}-\frac{x^4\,\left(64\,b^2\,c-40\,a\,b\,d\right)}{15\,a^3\,b\,e^3}\right)}{x^4\,\sqrt{e\,x}+\frac{a\,x^2\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(5*a*b*e^3) + (x^2*(30*a^2*d - 48*a*b*c))/(15*a^
3*b*e^3) - (x^4*(64*b^2*c - 40*a*b*d))/(15*a^3*b*e^3)))/(x^4*(e*x)^(1/2) +
(a*x^2*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

3.878 $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal. Leaf size=141

$$-\frac{64\left(a+bx^2\right)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}\,(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

Rubi [A] time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{64\left(a+bx^2\right)^{5/4}(4bc-3ad)}{45a^4e^3(ex)^{5/2}}+\frac{16\sqrt[4]{a+bx^2}\,(4bc-3ad)}{9a^3e^3(ex)^{5/2}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - (2*(4*b*c - 3*a*d))/(9*a^2*e
^3*(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (16*(4*b*c - 3*a*d)*(a + b*x^2)^(1/4))/
(9*a^3*e^3*(e*x)^(5/2)) - (64*(4*b*c - 3*a*d)*(a + b*x^2)^(5/4))/(45*a^4*e^
3*(e*x)^(5/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{(4bc-3ad)\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx}{3ae^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} - \frac{(8(4bc-3ad))\int \frac{1}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}}{9a^2e^2}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} + \frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}} + \frac{(32}{}$$

$$= -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}} - \frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}} + \frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}} - \frac{64(}{}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 0.62

$$\frac{4x^3\left(3a^2-24abx^2-32b^2x^4\right)\left(6bc-\frac{9ad}{2}\right)}{135a^4(ex)^{11/2}\left(a+bx^2\right)^{3/4}} - \frac{2cx}{9a(ex)^{11/2}\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (-2*c*x)/(9*a*(e*x)^(11/2)*(a + b*x^2)^(3/4)) + (4*(6*b*c - (9*a*d)/2)*x^3*
(3*a^2 - 24*a*b*x^2 - 32*b^2*x^4))/(135*a^4*(e*x)^(11/2)*(a + b*x^2)^(3/4))
```

IntegrateAlgebraic [A] time = 39.69, size = 130, normalized size = 0.92

$$\frac{2\sqrt[4]{a+bx^2}\left(-5a^3ce^6-9a^3de^6x^2+12a^2bce^6x^2+72a^2bde^6x^4-96ab^2ce^6x^4+96ab^2de^6x^6-128b^3ce^6x^6\right)}{45a^4e^5(ex)^{9/2}\left(ae^2+be^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

```
[Out] (2*(a + b*x^2)^(1/4)*(-5*a^3*c*e^6 + 12*a^2*b*c*e^6*x^2 - 9*a^3*d*e^6*x^2 -
 96*a*b^2*c*e^6*x^4 + 72*a^2*b*d*e^6*x^4 - 128*b^3*c*e^6*x^6 + 96*a*b^2*d*e
^6*x^6))/(45*a^4*e^5*(e*x)^(9/2)*(a*e^2 + b*e^2*x^2))
```

fricas [A] time = 1.16, size = 105, normalized size = 0.74

$$-\frac{2\left(32\left(4\,b^{3}c-3\,ab^{2}d\right)x^{6}+24\left(4\,ab^{2}c-3\,a^{2}bd\right)x^{4}+5\,a^{3}c-3\left(4\,a^{2}bc-3\,a^{3}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{1}{4}}\sqrt{ex}}{45\left(a^{4}be^{6}x^{7}+a^{5}e^{6}x^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

```
[Out] -2/45*(32*(4*b^3*c - 3*a*b^2*d)*x^6 + 24*(4*a*b^2*c - 3*a^2*b*d)*x^4 + 5*a^
3*c - 3*(4*a^2*b*c - 3*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^4*b*e^6*x
^7 + a^5*e^6*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

maple [A] time = 0.01, size = 86, normalized size = 0.61

$$-\frac{2\left(-96a\,b^{2}d\,x^{6}+128b^{3}c\,x^{6}-72a^{2}bd\,x^{4}+96a\,b^{2}c\,x^{4}+9a^{3}d\,x^{2}-12a^{2}bc\,x^{2}+5c\,a^{3}\right)x}{45\left(b\,x^{2}+a\right)^{\frac{3}{4}}(ex)^{\frac{11}{2}}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x)
```

```
[Out] -2/45*x*(-96*a*b^2*d*x^6+128*b^3*c*x^6-72*a^2*b*d*x^4+96*a*b^2*c*x^4+9*a^3*
d*x^2-12*a^2*b*c*x^2+5*a^3*c)/(b*x^2+a)^(3/4)/a^4/(e*x)^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

mupad [B] time = 1.26, size = 125, normalized size = 0.89

$$-\frac{{\left(b\,x^2+a\right)}^{1/4}\,\left(\frac{2\,c}{9\,a\,b\,e^5}-\frac{16\,x^4\,\left(3\,a\,d-4\,b\,c\right)}{15\,a^3\,e^5}+\frac{x^2\,\left(18\,a^3\,d-24\,a^2\,b\,c\right)}{45\,a^4\,b\,e^5}+\frac{x^6\,\left(256\,b^3\,c-192\,a\,b^2\,d\right)}{45\,a^4\,b\,e^5}\right)}{x^6\,\sqrt{e\,x}+\frac{a\,x^4\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x)
```

```
[Out] -((a + b*x^2)^(1/4)*((2*c)/(9*a*b*e^5) - (16*x^4*(3*a*d - 4*b*c))/(15*a^3*e
^5) + (x^2*(18*a^3*d - 24*a^2*b*c))/(45*a^4*b*e^5) + (x^6*(256*b^3*c - 192*
a*b^2*d))/(45*a^4*b*e^5)))/(x^6*(e*x)^(1/2) + (a*x^4*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(7/4),x)
```

```
[Out] Timed out
```

3.879 $\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=221

$$\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e^3\sqrt{ex}\,(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}$$

Rubi [A] time = 0.13, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {457, 285, 288, 329, 240, 212, 208, 205}

$$-\frac{e^3\sqrt{ex}\,(4bc-9ad)}{2b^3\sqrt[4]{a+bx^2}}+\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}-\frac{e(ex)^{5/2}(4bc-9ad)}{10ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c - 9*a*d)*
e^3*Sqrt[e*x])/(2*b^3*(a + b*x^2)^(1/4)) - ((4*b*c - 9*a*d)*e*(e*x)^(5/2))/
(10*a*b^2*(a + b*x^2)^(1/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTan[(b^(1/4)*Sqr
t[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/
2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(4*b^(13/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 285

```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(2*c*(
c*x)^(m - 1))/(b*(2*m - 3)*(a + b*x^2)^(1/4)), x] - Dist[(2*a*c^2*(m - 1))/
(b*(2*m - 3)), Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(-2bc+\frac{9ad}{2}\right)\right)\int \frac{(ex)^{7/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^2\right)\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{4b^2}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^4\right)\int \frac{}{\sqrt{ex}\,\sqrt[4]{}}}{4b^3}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^3\right)\text{Subst}\Bigg(}{2b}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^3\right)\text{Subst}\Bigg(}{2b}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{\left((4bc-9ad)e^4\right)\text{Subst}\big(}{4}$$

$$= \frac{2(bc-ad)(ex)^{9/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{(4bc-9ad)e^{7/2}\tan^{-1}\Big(}{4b^{13/4}}$$

Mathematica [C] time = 0.12, size = 91, normalized size = 0.41

$$\frac{e^3x^4\sqrt{ex}\left(9a^2d+\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}\,(4bc-9ad)\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};-\frac{bx^2}{a}\right)\right)}{18a^2b\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^3*x^4*Sqrt[e*x]*(9*a^2*d + (4*b*c - 9*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(
1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, -((b*x^2)/a)]))/(18*a^2*b*(a + b*x^2
)^(5/4))
```

IntegrateAlgebraic [A] time = 30.80, size = 260, normalized size = 1.18

$$\frac{e^{3/2}\left(a+bx^2\right)^{3/4}\left(\frac{45a^2de^{11/2}\sqrt{ex}-20abce^{11/2}\sqrt{ex}+54abde^{7/2}(ex)^{5/2}-24b^2ce^{7/2}(ex)^{5/2}+5b^2de^{3/2}(ex)^{9/2}}{10b^3\left(ae^2+be^2x^2\right)^{5/4}}+\frac{e^{7/2}(4bc-9ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{4b^{13/4}}+\frac{e^{7/2}(4bc-9ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{4b^{13/4}}\right)}{\left(ae^2+be^2x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^(3/2)*(a + b*x^2)^(3/4)*((-20*a*b*c*e^(11/2)*Sqrt[e*x] + 45*a^2*d*e^(11/
2)*Sqrt[e*x] - 24*b^2*c*e^(7/2)*(e*x)^(5/2) + 54*a*b*d*e^(7/2)*(e*x)^(5/2)
+ 5*b^2*d*e^(3/2)*(e*x)^(9/2))/(10*b^3*(a*e^2 + b*e^2*x^2)^(5/4)) + ((4*b*c
 - 9*a*d)*e^(7/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(a*e^2 + b*e^2*x^2)^(1/4)])/(4
*b^(13/4)) + ((4*b*c - 9*a*d)*e^(7/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(a*e^2 +
b*e^2*x^2)^(1/4)])/(4*b^(13/4))))/(a*e^2 + b*e^2*x^2)^(3/4)
```

fricas [B] time = 1.75, size = 997, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/40*(4*(5*b^2*d*e^3*x^4 - 6*(4*b^2*c - 9*a*b*d)*e^3*x^2 - 5*(4*a*b*c - 9*a
^2*d)*e^3)*(b*x^2 + a)^(3/4)*sqrt(e*x) + 20*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^
3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*
d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*arctan(((4*b^11*c - 9*a*b^10*d)*(b*x^2
 + a)^(3/4)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664
*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(3/4)*sqrt(e*x)*e^3 + (b^11*x^2 + a
*b^10)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*
b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(3/4)*sqrt(((16*b^2*c^2 - 72*a*b*c*d + 8
1*a^2*d^2)*sqrt(b*x^2 + a)*e^7*x + (b^7*x^2 + a*b^6)*sqrt((256*b^4*c^4 - 23
04*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e
^14/b^13))/(b*x^2 + a)))/((256*b^5*c^4 - 2304*a*b^4*c^3*d + 7776*a^2*b^3*c^
2*d^2 - 11664*a^3*b^2*c*d^3 + 6561*a^4*b*d^4)*e^14*x^2 + (256*a*b^4*c^4 - 2
304*a^2*b^3*c^3*d + 7776*a^3*b^2*c^2*d^2 - 11664*a^4*b*c*d^3 + 6561*a^5*d^4
)*e^14)) + 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c
^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^
(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 + (b^4*x^2 + a*
b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*
c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4))/(b*x^2 + a)) - 5*(b^5*x^4 + 2*a*b^4
```

```
*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 1
1664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-((b*x^2 + a)^(3/4)*(
4*b*c - 9*a*d)*sqrt(e*x)*e^3 - (b^4*x^2 + a*b^3)*((256*b^4*c^4 - 2304*a*b^3
*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13
)^(1/4))/(b*x^2 + a)))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{7}{2}}\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{7}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(e\,x)}^{7/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

```
[Out] int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

3.880 $\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=149

$$\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {452, 288, 329, 240, 212, 208, 205}

$$\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*d*e*Sqrt[e*x])
/(b^2*(a + b*x^2)^(1/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(
a + b*x^2)^(1/4))])/b^(9/4) + (d*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[
e]*(a + b*x^2)^(1/4))])/b^(9/4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx &= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{d\int \frac{(ex)^{3/2}}{\left(a+bx^2\right)^{5/4}}\,dx}{b} \\
&= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{\left(de^2\right)\int \frac{1}{\sqrt{ex}\sqrt[4]{a+bx^2}}\,dx}{b^2} \\
&= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(2de)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{a+\frac{bx^4}{e^2}}}\,dx, x, \sqrt{ex}\right)}{b^2} \\
&= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{(2de)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^4}{e^2}}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^2} \\
&= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{\left(de^2\right)\text{Subst}\left(\int \frac{1}{e-\sqrt{b}\,x^2}\,dx, x, \frac{\sqrt{ex}}{\sqrt[4]{a+bx^2}}\right)}{b^2} + \frac{\left(de^2\right)\text{Subs}}{} \\
&= \frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}} - \frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}} + \frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{9/4}} + \frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{ex}}{\sqrt{e}\,\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 77, normalized size = 0.52

$$\frac{2x(ex)^{3/2}\left(5dx^2\left(a+bx^2\right)\sqrt[4]{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};-\frac{bx^2}{a}\right)+9ac\right)}{45a^2\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (2*x*(e*x)^(3/2)*(9*a*c + 5*d*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[9/4, 9/4, 13/4, -((b*x^2)/a)]))/(45*a^2*(a + b*x^2)^(5/4))
```

IntegrateAlgebraic [A] time = 28.61, size = 206, normalized size = 1.38

$$\frac{e^{3/2}\left(a+bx^2\right)^{3/4}\left(-\frac{2\left(5a^2de^{7/2}\sqrt{ex}+6abde^{3/2}(ex)^{5/2}-b^2ce^{3/2}(ex)^{5/2}\right)}{5ab^2\left(ae^2+be^2x^2\right)^{5/4}}+\frac{de^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{ae^2+be^2x^2}}\right)}{b^{9/4}}\right)}{\left(ae^2+be^2x^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

```
[Out] (e^(3/2)*(a + b*x^2)^(3/4)*((-2*(5*a^2*d*e^(7/2)*Sqrt[e*x] - b^2*c*e^(3/2)*
(e*x)^(5/2) + 6*a*b*d*e^(3/2)*(e*x)^(5/2)))/(5*a*b^2*(a*e^2 + b*e^2*x^2)^(5
/4)) + (d*e^(3/2)*ArcTan[(b^(1/4)*Sqrt[e*x])/(a*e^2 + b*e^2*x^2)^(1/4)])/b^
(9/4) + (d*e^(3/2)*ArcTanh[(b^(1/4)*Sqrt[e*x])/(a*e^2 + b*e^2*x^2)^(1/4)])/
b^(9/4)))/(a*e^2 + b*e^2*x^2)^(3/4)
```

fricas [B] time = 1.53, size = 448, normalized size = 3.01

$$-\frac{4\left(5a^2de-\left(b^2c-6abd\right)ex^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}+20\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,b^7de\left(\frac{d^4e^6}{b^9}\right)^{\frac{3}{4}}-\left(b^8x^2+ab^7\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{3}{4}}\sqrt{\frac{\sqrt{bx^2+a}\,d^2e^3x+\left(b^5x^2+ab^4\right)\sqrt{\frac{d^4e^6}{b^9}}}{bx^2+a}}}{bd^4e^6x^2+ad^4e^6}\right)-5\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,de+\left(b^3x^2+ab^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}}{bx^2+a}\right)+5\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}\,de-\left(b^3x^2+ab^2\right)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}}{bx^2+a}\right)}{10\left(ab^4x^4+2a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/10*(4*(5*a^2*d*e - (b^2*c - 6*a*b*d)*e*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)
+ 20*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*arctan(-((b*
x^2 + a)^(3/4)*sqrt(e*x)*b^7*d*e*(d^4*e^6/b^9)^(3/4) - (b^8*x^2 + a*b^7)*(d
^4*e^6/b^9)^(3/4)*sqrt((sqrt(b*x^2 + a)*d^2*e^3*x + (b^5*x^2 + a*b^4)*sqrt(
d^4*e^6/b^9))/(b*x^2 + a)))/(b*d^4*e^6*x^2 + a*d^4*e^6)) - 5*(a*b^4*x^4 + 2
*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x
)*d*e + (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))/(b*x^2 + a)) + 5*(a*b^4*x^4
+ 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(
e*x)*d*e - (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))/(b*x^2 + a)))/(a*b^4*x^4
+ 2*a^2*b^3*x^2 + a^3*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{2}+c\right)(ex)^{\frac{3}{2}}}{\left(bx^{2}+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,\left(d\,x^2+c\right)}{\left(b\,x^2+a\right)^{9/4}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

```
[Out] int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

3.881 $\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt{ex}\,(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}\,(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Rubi [**A**] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 264}

$$\frac{2\sqrt{ex}\,(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}\,(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c + a*d)*Sq
rt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}\,dx = \frac{2(bc-ad)\sqrt{ex}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{\left(2\left(2bc+\frac{ad}{2}\right)\right)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx}{5ab}$$
$$= \frac{2(bc-ad)\sqrt{ex}}{5abe\left(a+bx^2\right)^{5/4}} + \frac{2(4bc+ad)\sqrt{ex}}{5a^2be\sqrt[4]{a+bx^2}}$$

Mathematica [A]   time = 0.05, size = 44, normalized size = 0.56

$$\frac{2x\left(5ac+adx^2+4bcx^2\right)}{5a^2\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(5*a*c + 4*b*c*x^2 + a*d*x^2))/(5*a^2*Sqrt[e*x]*(a + b*x^2)^(5/4))
```

IntegrateAlgebraic [A]   time = 64.35, size = 89, normalized size = 1.13

$$\frac{2c\sqrt{ex}\left(5ae^2+4be^2x^2\right)}{5a^2e^3\left(a+bx^2\right)^{5/4}} + \frac{2de(ex)^{5/2}\left(a+bx^2\right)^{3/4}}{5a\left(ae^2+be^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*d*e*(e*x)^(5/2)*(a + b*x^2)^(3/4))/(5*a*(a*e^2 + b*e^2*x^2)^2) + (2*c*Sq
rt[e*x]*(5*a*e^2 + 4*b*e^2*x^2))/(5*a^2*e^3*(a + b*x^2)^(5/4))
```

fricas [A]   time = 1.33, size = 62, normalized size = 0.78

$$\frac{2\left((4\,bc+ad)x^2+5\,ac\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{5\left(a^2b^2ex^4+2\,a^3bex^2+a^4e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/5*((4*b*c + a*d)*x^2 + 5*a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*b^2*e*x^4
+ 2*a^3*b*e*x^2 + a^4*e)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

maple [A] time = 0.01, size = 39, normalized size = 0.49

$$\frac{2\left(ad\,x^2 + 4bc\,x^2 + 5ac\right)x}{5\left(b\,x^2 + a\right)^{\frac{5}{4}}\sqrt{ex}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] 2/5*x*(a*d*x^2+4*b*c*x^2+5*a*c)/(b*x^2+a)^(5/4)/a^2/(e*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{(bx^2 + a)^{\frac{9}{4}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

mupad [B] time = 1.21, size = 79, normalized size = 1.00

$$\frac{\left(b\,x^2 + a\right)^{3/4}\left(\frac{x^3\,(2\,a\,d+8\,b\,c)}{5\,a^2\,b^2} + \frac{2\,c\,x}{a\,b^2}\right)}{x^4\,\sqrt{e\,x} + \frac{a^2\,\sqrt{e\,x}}{b^2} + \frac{2\,a\,x^2\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((x^3*(2*a*d + 8*b*c))/(5*a^2*b^2) + (2*c*x)/(a*b^2)))/(
x^4*(e*x)^(1/2) + (a^2*(e*x)^(1/2))/b^2 + (2*a*x^2*(e*x)^(1/2))/b)
```

sympy [B] time = 165.87, size = 230, normalized size = 2.91

$$c\left(\frac{5a\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}\sqrt{e}\,x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}+\frac{4bx^2\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}\sqrt{e}\,x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}\right)+\frac{d\Gamma\left(\frac{5}{4}\right)}{\frac{2a^2\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}{x^2}+2ab^{\frac{5}{4}}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] c*(5*a*gamma(1/4)/(8*a**3*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/
4) + 8*a**2*b**(5/4)*sqrt(e)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)) + 4*b
*x**2*gamma(1/4)/(8*a**3*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4
) + 8*a**2*b**(5/4)*sqrt(e)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4))) + d*g
amma(5/4)/(2*a**2*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)/x**2
+ 2*a*b**(5/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4))
```

3.882 $\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=104

$$-\frac{8\sqrt{ex}\,(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}\,(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{8\sqrt{ex}\,(8bc-3ad)}{15a^3e^3\sqrt[4]{a+bx^2}}-\frac{2\sqrt{ex}\,(8bc-3ad)}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (2*(8*b*c - 3*a*d)*Sqrt[e*x]
)/(15*a^2*e^3*(a + b*x^2)^(5/4)) - (8*(8*b*c - 3*a*d)*Sqrt[e*x])/(15*a^3*e^
3*(a + b*x^2)^(1/4))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx &= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{(8bc-3ad)\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}\,dx}{3ae^2}\\ &= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{(4(8bc-3ad))\int \frac{1}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}\,dx}{15a^2e^2}\\ &= -\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\left(a+bx^2\right)^{5/4}}-\frac{8(8bc-3ad)\sqrt{ex}}{15a^3e^3\sqrt[4]{a+bx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.62

$$\frac{x\left(-10a^2\left(c-3dx^2\right)+ab\left(24dx^4-80cx^2\right)-64b^2cx^4\right)}{15a^3(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
[Out] (x*(-64*b^2*c*x^4 - 10*a^2*(c - 3*d*x^2) + a*b*(-80*c*x^2 + 24*d*x^4)))/(15
*a^3*(e*x)^(5/2)*(a + b*x^2)^(5/4))
```

IntegrateAlgebraic [A] time = 27.47, size = 100, normalized size = 0.96

$$\frac{2\left(a+bx^2\right)^{3/4}\left(-5a^2ce^4+15a^2de^4x^2-40abce^4x^2+12abde^4x^4-32b^2ce^4x^4\right)}{15a^3e(ex)^{3/2}\left(ae^2+be^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
[Out] (2*(a + b*x^2)^(3/4)*(-5*a^2*c*e^4 - 40*a*b*c*e^4*x^2 + 15*a^2*d*e^4*x^2 -
32*b^2*c*e^4*x^4 + 12*a*b*d*e^4*x^4))/(15*a^3*e*(e*x)^(3/2)*(a*e^2 + b*e^2*
x^2)^2)
```

fricas [A] time = 1.15, size = 95, normalized size = 0.91

$$-\frac{2\left(4\left(8\,b^2c-3\,abd\right)x^4+5\,a^2c+5\left(8\,abc-3\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{15\left(a^3b^2e^3x^6+2\,a^4be^3x^4+a^5e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/15*(4*(8*b^2*c - 3*a*b*d)*x^4 + 5*a^2*c + 5*(8*a*b*c - 3*a^2*d)*x^2)*(b*
x^2 + a)^(3/4)*sqrt(e*x)/(a^3*b^2*e^3*x^6 + 2*a^4*b*e^3*x^4 + a^5*e^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.60

$$-\frac{2\left(-12abd\,x^4+32b^2c\,x^4-15a^2d\,x^2+40abc\,x^2+5c\,a^2\right)x}{15\left(b\,x^2+a\right)^{\frac{5}{4}}(ex)^{\frac{5}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/15*x*(-12*a*b*d*x^4+32*b^2*c*x^4-15*a^2*d*x^2+40*a*b*c*x^2+5*a^2*c)/(b*x
^2+a)^(5/4)/a^3/(e*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

mupad [B] time = 1.31, size = 115, normalized size = 1.11

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2\,c}{3\,a\,b^2\,e^2}-\frac{x^2\,\left(30\,a^2\,d-80\,a\,b\,c\right)}{15\,a^3\,b^2\,e^2}+\frac{x^4\,\left(64\,b^2\,c-24\,a\,b\,d\right)}{15\,a^3\,b^2\,e^2}\right)}{x^5\,\sqrt{e\,x}+\frac{2\,a\,x^3\,\sqrt{e\,x}}{b}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(3*a*b^2*e^2) - (x^2*(30*a^2*d - 80*a*b*c))/(15*
a^3*b^2*e^2) + (x^4*(64*b^2*c - 24*a*b*d))/(15*a^3*b^2*e^2)))/(x^5*(e*x)^(1
/2) + (2*a*x^3*(e*x)^(1/2))/b + (a^2*x*(e*x)^(1/2))/b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

3.883 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=141

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Rubi [A] time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$\frac{64\left(a+bx^2\right)^{3/4}(12bc-7ad)}{105a^4e^3(ex)^{3/2}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (2*(12*b*c - 7*a*d))/(35*a^2
*e^3*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - (16*(12*b*c - 7*a*d))/(35*a^3*e^3*(e*
x)^(3/2)*(a + b*x^2)^(1/4)) + (64*(12*b*c - 7*a*d)*(a + b*x^2)^(3/4))/(105*
a^4*e^3*(e*x)^(3/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{(12bc-7ad)\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx}{7ae^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{(8(12bc-7ad))\int \frac{1}{(ex)^{5/2}\left(a+bx^2\right.}}{35a^2e^2}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}} - \frac{(3}{}$$

$$= -\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}} - \frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}} + \frac{64}{}$$

Mathematica [A] time = 0.05, size = 94, normalized size = 0.67

$$\frac{\sqrt{ex}\left(-10a^3\left(3c+7dx^2\right)+40a^2bx^2\left(3c-14dx^2\right)+64ab^2x^4\left(15c-7dx^2\right)+768b^3cx^6\right)}{105a^4e^5x^4\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (Sqrt[e*x]*(768*b^3*c*x^6 + 40*a^2*b*x^2*(3*c - 14*d*x^2) + 64*a*b^2*x^4*(1
5*c - 7*d*x^2) - 10*a^3*(3*c + 7*d*x^2)))/(105*a^4*e^5*x^4*(a + b*x^2)^(5/4
))
```

IntegrateAlgebraic [A] time = 38.39, size = 130, normalized size = 0.92

$$-\frac{2\left(a+bx^2\right)^{3/4}\left(15a^3ce^6+35a^3de^6x^2-60a^2bce^6x^2+280a^2bde^6x^4-480ab^2ce^6x^4+224ab^2de^6x^6-384b^3ce^6x^6\right)}{105a^4e^3(ex)^{7/2}\left(ae^2+be^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*(a + b*x^2)^(3/4)*(15*a^3*c*e^6 - 60*a^2*b*c*e^6*x^2 + 35*a^3*d*e^6*x^2
 - 480*a*b^2*c*e^6*x^4 + 280*a^2*b*d*e^6*x^4 - 384*b^3*c*e^6*x^6 + 224*a*b^
2*d*e^6*x^6))/(105*a^4*e^3*(e*x)^(7/2)*(a*e^2 + b*e^2*x^2)^2)
```

fricas [A] time = 1.26, size = 119, normalized size = 0.84

$$\frac{2\left(32\left(12\,b^{3}c-7\,ab^{2}d\right)x^{6}+40\left(12\,ab^{2}c-7\,a^{2}bd\right)x^{4}-15\,a^{3}c+5\left(12\,a^{2}bc-7\,a^{3}d\right)x^{2}\right)\left(bx^{2}+a\right)^{\frac{3}{4}}\sqrt{ex}}{105\left(a^{4}b^{2}e^{5}x^{8}+2\,a^{5}be^{5}x^{6}+a^{6}e^{5}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/105*(32*(12*b^3*c - 7*a*b^2*d)*x^6 + 40*(12*a*b^2*c - 7*a^2*b*d)*x^4 - 15
*a^3*c + 5*(12*a^2*b*c - 7*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^4*b^2
*e^5*x^8 + 2*a^5*b*e^5*x^6 + a^6*e^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{9}{4}}\left(ex\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

maple [A] time = 0.01, size = 86, normalized size = 0.61

$$-\frac{2\left(224a\,b^{2}d\,x^{6}-384b^{3}c\,x^{6}+280a^{2}bd\,x^{4}-480a\,b^{2}c\,x^{4}+35a^{3}d\,x^{2}-60a^{2}bc\,x^{2}+15c\,a^{3}\right)x}{105\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(ex\right)^{\frac{9}{2}}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/105*x*(224*a*b^2*d*x^6-384*b^3*c*x^6+280*a^2*b*d*x^4-480*a*b^2*c*x^4+35*
a^3*d*x^2-60*a^2*b*c*x^2+15*a^3*c)/(b*x^2+a)^(5/4)/a^4/(e*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^{2}+c}{\left(bx^{2}+a\right)^{\frac{9}{4}}\left(ex\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

mupad [**B**] time = 1.34, size = 144, normalized size = 1.02

$$-\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{2\,c}{7\,a\,b^2\,e^4}+\frac{16\,x^4\,\left(7\,a\,d-12\,b\,c\right)}{21\,a^3\,b\,e^4}+\frac{x^2\,\left(70\,a^3\,d-120\,a^2\,b\,c\right)}{105\,a^4\,b^2\,e^4}-\frac{x^6\,\left(768\,b^3\,c-448\,a\,b^2\,d\right)}{105\,a^4\,b^2\,e^4}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x^3\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^5\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] -((a + b*x^2)^(3/4)*((2*c)/(7*a*b^2*e^4) + (16*x^4*(7*a*d - 12*b*c))/(21*a^
3*b*e^4) + (x^2*(70*a^3*d - 120*a^2*b*c))/(105*a^4*b^2*e^4) - (x^6*(768*b^3
*c - 448*a*b^2*d))/(105*a^4*b^2*e^4)))/(x^7*(e*x)^(1/2) + (a^2*x^3*(e*x)^(1
/2))/b^2 + (2*a*x^5*(e*x)^(1/2))/b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

3.884 $\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx$

Optimal. Leaf size=178

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5}}$$

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{256\left(a+bx^2\right)^{7/4}(16bc-11ad)}{385a^5e^3(ex)^{7/2}}+\frac{64\left(a+bx^2\right)^{3/4}(16bc-11ad)}{55a^4e^3(ex)^{7/2}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - (2*(16*b*c - 11*a*d))/(55*
a^2*e^3*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - (24*(16*b*c - 11*a*d))/(55*a^3*e^3
*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (64*(16*b*c - 11*a*d)*(a + b*x^2)^(3/4))/
(55*a^4*e^3*(e*x)^(7/2)) - (256*(16*b*c - 11*a*d)*(a + b*x^2)^(7/4))/(385*a
^5*e^3*(e*x)^(7/2))
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{(16bc-11ad)\int \frac{1}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx}{11ae^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{(12(16bc-11ad))\int \frac{}{(ex)^{9/2}(}}{55a^2e^2}$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} -$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} +$$

$$= -\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}} - \frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}} - \frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} +$$

Mathematica [A]   time = 0.06, size = 79, normalized size = 0.44

$$\frac{2x\left(ax^2\left(-5a^3+20a^2bx^2+160ab^2x^4+128b^3x^6\right)(11ad-16bc)-35a^5c\right)}{385a^6(ex)^{13/2}\left(a+bx^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*x*(-35*a^5*c + a*(-16*b*c + 11*a*d)*x^2*(-5*a^3 + 20*a^2*b*x^2 + 160*a*b
^2*x^4 + 128*b^3*x^6)))/(385*a^6*(e*x)^(13/2)*(a + b*x^2)^(5/4))
```

IntegrateAlgebraic [A]   time = 45.91, size = 160, normalized size = 0.90

$$\frac{2\left(a+bx^2\right)^{3/4}\left(-35a^4ce^8-55a^4de^8x^2+80a^3bce^8x^2+220a^3bde^8x^4-320a^2b^2ce^8x^4+1760a^2b^2de^8x^6-2560ab^3ce^8x^6+1408ab^3de^8x^8-2048b^4ce^8x^8\right)}{385a^5e^5(ex)^{11/2}\left(ae^2+be^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

```
[Out] (2*(a + b*x^2)^(3/4)*(-35*a^4*c*e^8 + 80*a^3*b*c*e^8*x^2 - 55*a^4*d*e^8*x^2
 - 320*a^2*b^2*c*e^8*x^4 + 220*a^3*b*d*e^8*x^4 - 2560*a*b^3*c*e^8*x^6 + 176
0*a^2*b^2*d*e^8*x^6 - 2048*b^4*c*e^8*x^8 + 1408*a*b^3*d*e^8*x^8))/(385*a^5*
e^5*(e*x)^(11/2)*(a*e^2 + b*e^2*x^2)^2)
```

fricas [A] time = 1.08, size = 143, normalized size = 0.80

$$-\frac{2\left(128\left(16\,b^4c-11\,ab^3d\right)x^8+160\left(16\,ab^3c-11\,a^2b^2d\right)x^6+35\,a^4c+20\left(16\,a^2b^2c-11\,a^3bd\right)x^4-5\left(16\,a^3bc-11\,a^4d\right)x^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{385\left(a^5b^2e^7x^{10}+2\,a^6be^7x^8+a^7e^7x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/385*(128*(16*b^4*c - 11*a*b^3*d)*x^8 + 160*(16*a*b^3*c - 11*a^2*b^2*d)*x
^6 + 35*a^4*c + 20*(16*a^2*b^2*c - 11*a^3*b*d)*x^4 - 5*(16*a^3*b*c - 11*a^4
*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^5*b^2*e^7*x^10 + 2*a^6*b*e^7*x^8 +
a^7*e^7*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}\left(ex\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

maple [A] time = 0.01, size = 110, normalized size = 0.62

$$-\frac{2\left(-1408a\,b^3d\,x^8+2048b^4c\,x^8-1760a^2b^2d\,x^6+2560a\,b^3c\,x^6-220a^3bd\,x^4+320a^2b^2c\,x^4+55a^4d\,x^2-80a^3bc\,x^2+35c\,a^4\right)x}{385\left(b\,x^2+a\right)^{\frac{5}{4}}\left(ex\right)^{\frac{13}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x)
```

```
[Out] -2/385*x*(-1408*a*b^3*d*x^8+2048*b^4*c*x^8-1760*a^2*b^2*d*x^6+2560*a*b^3*c*
x^6-220*a^3*b*d*x^4+320*a^2*b^2*c*x^4+55*a^4*d*x^2-80*a^3*b*c*x^2+35*a^4*c)
/(b*x^2+a)^(5/4)/a^5/(e*x)^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\left(bx^2 + a\right)^{\frac{9}{4}} (ex)^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

mupad [B] time = 1.41, size = 156, normalized size = 0.88

$$\frac{\left(b\,x^2+a\right)^{3/4}\,\left(\frac{64\,x^6\,\left(11\,a\,d-16\,b\,c\right)}{77\,a^4\,e^6}-\frac{2\,c}{11\,a\,b^2\,e^6}+\frac{8\,x^4\,\left(11\,a\,d-16\,b\,c\right)}{77\,a^3\,b\,e^6}-\frac{x^2\,\left(110\,a^4\,d-160\,a^3\,b\,c\right)}{385\,a^5\,b^2\,e^6}+\frac{256\,b\,x^8\,\left(11\,a\,d-16\,b\,c\right)}{385\,a^5\,e^6}\right)}{x^9\,\sqrt{e\,x}+\frac{a^2\,x^5\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^7\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x)
```

```
[Out] ((a + b*x^2)^(3/4)*((64*x^6*(11*a*d - 16*b*c))/(77*a^4*e^6) - (2*c)/(11*a*b
^2*e^6) + (8*x^4*(11*a*d - 16*b*c))/(77*a^3*b*e^6) - (x^2*(110*a^4*d - 160*
a^3*b*c))/(385*a^5*b^2*e^6) + (256*b*x^8*(11*a*d - 16*b*c))/(385*a^5*e^6)))
/(x^9*(e*x)^(1/2) + (a^2*x^5*(e*x)^(1/2))/b^2 + (2*a*x^7*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(9/4),x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)



GradeAntiderivative[result_,optimal_] :=
```

```
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0, expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
   gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
   Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
   Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
   Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
   [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
   isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```